



**CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW
2020 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION
MATHEMATICS EXTENSION 1 - MARKING GUIDELINES**

Section I

10 Marks

Multiple-choice Answer Key

Question	Answer
1	B
2	A
3	A
4	D
5	B
6	A
7	C
8	C
9	B
10	D

Question 1 (1 mark)

Outcomes Assessed: ME11-5

Targeted Performance Bands: E2

Solution	Mark
Using the pigeonhole principle. $\frac{110}{12} = 9\frac{1}{6}$ <p>\therefore if there are 110 students, the minimum number of students that have a birthday in at least one month is 10. Hence (B).</p>	1

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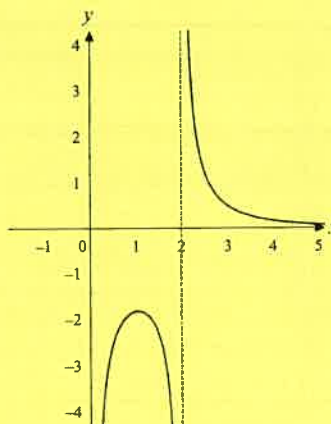
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Question 2 (1 mark)**Outcomes Assessed:** ME11-2**Targeted Performance Bands:** E2

Solution	Mark
<p>If a polynomial $P(x)$ has a triple root at $x = -2$ it can be written as:</p> $P(x) = (x+2)^3 Q(x)$ $P'(x) = 3(x+2)^2 Q(x) + (x+2)^3 Q'(x)$ $= (x+2)^2 [3Q(x) + (x+2)Q'(x)]$ <p>$\therefore (x+2)^2$ is a factor of $P'(x)$</p> <p>Hence (A).</p>	1

Question 3 (1 mark)**Outcomes Assessed:** ME11-1**Targeted Performance Bands:** E2-E3

Solution	Mark
<p>at $x = 0^+$, $\frac{1}{f(x)} \rightarrow -\infty$</p> <p>at $x = 2^-$, $\frac{1}{f(x)} \rightarrow -\infty$</p> <p>at $x = 2^+$, $\frac{1}{f(x)} \rightarrow \infty$</p> <p>at $x = 1$, $\frac{1}{f(x)} = -2$</p> <p>Hence (A)</p>	1

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Question 4 (1 mark)**Outcomes Assessed:** ME11-3**Targeted Performance Bands:** E2-E3

Solution	Mark
$\text{let } t = \tan \frac{\theta}{2}$ $\therefore \cos \theta = \frac{1-t^2}{1+t^2} \text{ and } \sin \theta = \frac{2t}{1+t^2}$ $\frac{\cos \theta - 1}{2 \sin \theta} = \frac{\frac{1-t^2}{1+t^2} - 1}{2 \times \frac{2t}{1+t^2}}$ $= \frac{1-t^2 - (1+t^2)}{4t}$ $= \frac{-2t^2}{4t}$ $= \frac{-t}{2}$ <p>Hence (D)</p>	1

Question 5 (1 mark)**Outcomes Assessed:** ME11-5**Targeted Performance Bands:** E2-E3

Solution	Mark
$n(\text{two Os separated}) = n(\text{unrestricted}) - n(\text{two Os together})$ $= \frac{6!}{2!} - 5!$ $= 240 \text{ ways}$ <p>Hence (B)</p>	1

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Question 6 (1 mark)**Outcomes Assessed: ME12-5****Targeted Performance Bands: E3**

Solution	Mark
The variance of a Bernoulli distribution is given by: $\sigma^2 = p(1-p)$ $= 0.8(1-0.8)$ $= 0.16$ Hence (A)	1

Question 7 (1 mark)**Outcomes Assessed: ME12-4****Targeted Performance Bands: E3**

Solution	Mark
$\cos^2 x - \sin^2 x = \cos 2x$ $\int \sin^2 x \, dx = \frac{1}{2} \int (1 - \cos 2x) \, dx$ $= \frac{1}{2} \left[x - \frac{1}{2} \sin 2x \right] + c$ $= \frac{1}{2} x - \frac{1}{4} \sin 2x + c$ $\therefore \int (\sin^2 x + x^2) \, dx = \frac{1}{2} x - \frac{1}{4} \sin 2x + \frac{x^3}{3} + c$ Hence (C)	1

Question 8 (1 mark)**Outcomes Assessed: ME12-2****Targeted Performance Bands: E3**

Solution	Mark
$\underline{a} = 2\underline{i} - 5\underline{j}, \underline{b} = 3\underline{i} + 4\underline{j}$ $\text{proj}_{\underline{a}} \underline{b} = \frac{\underline{a} \cdot \underline{b}}{ \underline{a} ^2} \underline{a}$ $= \frac{2 \times 3 + (-5 \times 4)}{(\sqrt{2^2 + 5^2})^2} [2\underline{i} - 5\underline{j}]$ $= \frac{-14}{29} [2\underline{i} - 5\underline{j}]$ Hence (C)	1

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Question 9 (1 mark)**Outcomes Assessed:** ME11-3**Targeted Performance Bands:** E3-E4

Solution	Mark
<p>The graph of $y = \cos^{-1} x + 1$ is dilated vertically by a scale factor of 2</p> $y = 2(\cos^{-1} x + 1)$ $= 2\cos^{-1} x + 2$ <p>The graph of $y = 2\cos^{-1} x + 2$ is then shifted up 3 units</p> $y = 2\cos^{-1} x + 2 + 3$ $= 2\cos^{-1} x + 5$ <p>Hence (B)</p>	1

Question 10 (1 mark)**Outcomes Assessed:** ME12-4**Targeted Performance Bands:** E3-E4

Solution	Mark
<p>The slope field follows the following pattern:</p> <p>as $x \rightarrow -\infty$, $\frac{dy}{dx} \rightarrow \infty$</p> <p>for $x \in (-\infty, \infty)$, $\frac{dy}{dx} > 0$ and decreasing</p> <p>as $x \rightarrow \infty$, $\frac{dy}{dx} \rightarrow 0$</p> $\therefore \frac{dy}{dx} = e^{-x}$ <p>Hence (D)</p>	1

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Section II
60 marks

Question 11 (15 marks)

11 (a) (3 marks)

Outcomes assessed: ME11-2

Targeted Performance Bands: E2

Criteria	Marks
• Correct solution	3
• Multiplies both sides of the inequality by $(x+3)^2$ or equivalent merit	2
• Recognises that $x = -3$ cannot be part of solution	1

Sample Answer:

$$\frac{2x}{x+3} \leq 1 \quad \text{Note } x \neq -3$$

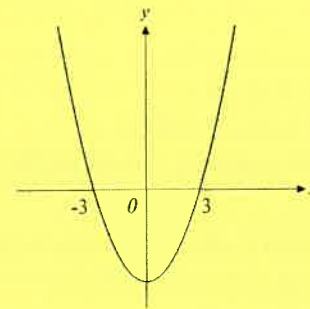
$$2x(x+3) \leq (x+3)^2$$

$$2x(x+3) - (x+3)^2 \leq 0$$

$$(x+3)(2x - (x+3)) \leq 0$$

$$(x+3)(x-3) \leq 0$$

From graph, solution is $-3 < x \leq 3$.



11 (b) (2 marks)

Outcomes Assessed: ME11-2

Targeted Performance Bands: E2

Criteria	Marks
• Correct solution	2
• Attempts to use the remainder theorem	1

Sample Answer:

$$P(x) = 2x^3 + kx^2 - 1$$

$$\text{Given } P(-2) = 7$$

$$7 = 2(-2)^3 + k(-2)^2 - 1$$

$$7 = -17 + 4k$$

$$\therefore k = 6$$

11 (c) (3 marks)

Outcomes Assessed: ME12-5

Targeted Performance Bands: E2-E3

Criteria	Marks
• Correct solution	3
• Two correct binomial probabilities	2
• Attempted use of binomial probability	1

Sample Answer:

Standard Room: $X \sim B(6, 0.064)$

$$\begin{aligned} P(X = 2) &= {}^6C_2 (0.064)^2 (0.936)^4 \\ &= 0.0471 \quad (4 \text{ dp}) \end{aligned}$$

Executive Room: $X \sim B(5, 0.131)$

$$\begin{aligned} P(X = 2) &= {}^5C_2 (0.131)^2 (0.869)^3 \\ &= 0.1126 \quad (4 \text{ dp}) \end{aligned}$$

$$\begin{aligned} P(2 \text{ Standard and 2 Executive unoccupied}) &= 0.0471 \times 0.1126 \\ &= 0.0053 \quad (4 \text{ dp}) \end{aligned}$$

11 (d) (i) (1 mark)

Outcomes Assessed: ME11-5

Targeted Performance Bands: E2-E3

Criteria	Mark
• Correct solution	1

Sample Answer:

$${}^{10}C_5 = 252$$

\therefore there are 252 possible committees that can be formed with no restrictions.

11 (d)(ii) (2 marks)

Outcomes Assessed: ME11-5

Targeted Performance Bands: E2-E3

Criteria	Marks
• Correct solution	2
• Makes some progress towards solution	1

Sample Answer:

Majority of women can be formed with 5 women, 4 women or 3 women.

$${}^6C_4 \times {}^4C_1 = 60$$

$${}^6C_3 \times {}^4C_2 = 120$$

$${}^6C_5 \times {}^4C_0 = 6$$

∴ there are 186 possible committees that can be formed if the majority of members are women.

11 (e) (2 marks)

Outcomes Assessed: ME12-2

Targeted Performance Bands: E2-E3

Criteria	Marks
• Correct solution	2
• Correct magnitude or direction	1

Sample Answer:

$$|AB| = \sqrt{(3-2)^2 + (-1-1)^2}$$
$$= \sqrt{5}$$

$$\tan \theta = \frac{-1-1}{3-1}$$

$$\theta = \tan^{-1}(-2)$$

$$= 297^\circ \text{ (nearest degree)}$$

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11 (f) (2 marks)

Outcomes Assessed: ME12-2

Targeted Performance Bands: E2-E3

Criteria	Marks
• Correct solution	2
• Attempts to use the dot product to calculate the value of x	1

Sample Answer:

If \underline{a} and \underline{b} are perpendicular then $\underline{a} \cdot \underline{b} = 0$

$$2 \times (-3) + (3) \times x = 0$$

$$\therefore x = 2$$

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Question 12 (15 marks)

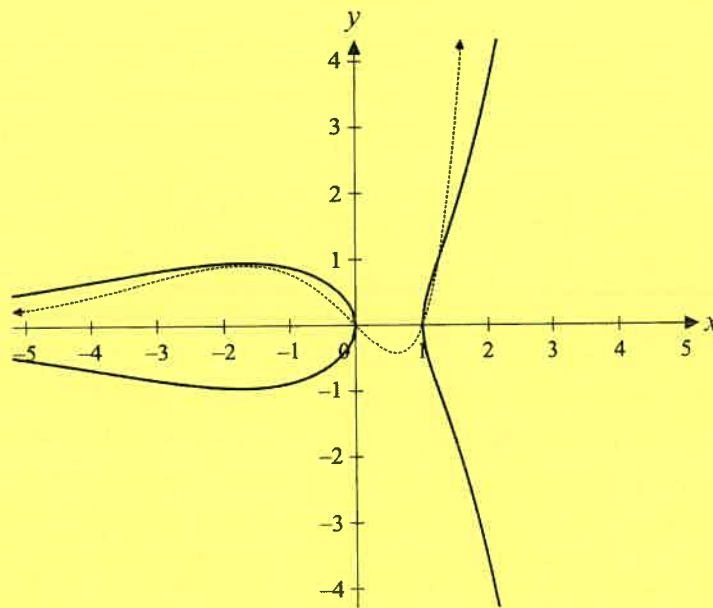
12 (a)(i) (2 marks)

Outcomes assessed: ME11-2

Targeted Performance Bands: E2-E3

Criteria	Marks
• Correct solution	2
• Shows some features of the graph of $y^2 = f(x)$ or the correct graph of $y = \sqrt{f(x)}$	1

Sample Answer:



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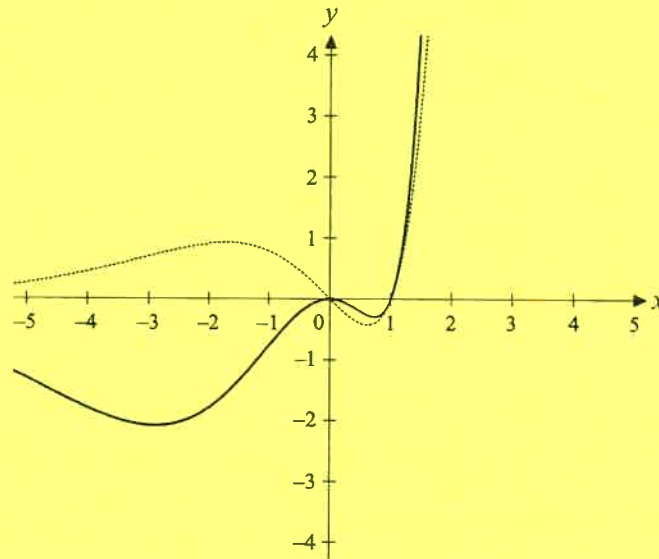
12 (a)(ii) (2 marks)

Outcomes assessed: ME11-2

Targeted Performance Bands: E2-E3

Criteria	Marks
• Correct solution	2
• Shows some features of the graph of $y = x f(x)$	1

Sample Answer:



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12 (b) (3 marks)

Outcomes Assessed: ME12-3

Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	3
• Finds correct values of x outside of the specified domain	2
• Forms a correct equation using the auxiliary method	1

Sample Answer:

$$\sin x + \sqrt{3} \cos x = 2 \sin \left(x + \frac{\pi}{3} \right)$$

$\therefore \sin x + \sqrt{3} \cos x = 1$ becomes

$$2 \sin \left(x + \frac{\pi}{3} \right) = 1$$

$$\sin \left(x + \frac{\pi}{3} \right) = \frac{1}{2}$$

$$\therefore x = \frac{\pi}{2}, \frac{11\pi}{6} \text{ for } x \in [0, 2\pi]$$

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12 (c) (3 marks)

Outcomes assessed: ME12-4

Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	3
• Correct integration	2
• Attempts to form an integral involving $\tan^{-1} x$	1

Sample Answer:

$$\begin{aligned} & \int_0^{\frac{4}{3}} \frac{dx}{16+9x^2} \\ &= \frac{1}{3} \int_0^{\frac{4}{3}} \frac{3dx}{16+(3x)^2} \\ &= \frac{1}{4} \times \frac{1}{3} \left[\tan^{-1} \frac{3x}{4} \right]_0^{\frac{4}{3}} \\ &= \frac{1}{12} \left[\tan^{-1} \left(\frac{3}{4} \times \frac{4}{3} \right) - \tan^{-1}(0) \right] \\ &= \frac{1}{12} \tan^{-1} 1 \\ &= \frac{\pi}{48} \end{aligned}$$

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12 (d) (3 marks)

Outcomes Assessed: ME12-4

Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	3
• Correct substitution and change of variable or equivalent merit	2
• Forms correct integral	1

Sample Answer:

$$V = \pi \int_0^1 y^2 dx$$

$$= \pi \int_0^1 x^2 (x^3 - 3)^6 dx$$

$$= \pi \int_0^1 x^2 (x^3 - 3)^6 dx$$

$$= \pi \int_{-3}^{-2} u^6 \cdot \frac{1}{3} du$$

$$= \frac{\pi}{3} \left[\frac{u^7}{7} \right]_{-3}^{-2}$$

$$= \frac{\pi}{3} \left[\frac{2059}{7} \right]$$

$$= \frac{2059\pi}{21} \text{ units}^3$$

$$u = x^3 - 3$$

$$\frac{du}{dx} = 3x^2$$

$$\frac{1}{3} du = x^2 dx$$

$$x = 1, u = -2$$

$$x = 0, u = -3$$

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12 (e) (2 marks)

Outcomes Assessed: ME11-4

Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	2
• Obtains expression for $\frac{dP}{dt}$	1

Sample Answer:

$$\frac{dV}{dt} = 100$$

$$\frac{dP}{dt} = \frac{dV}{dt} \times \frac{dP}{dV}$$

$$PV = 45000 \rightarrow P = \frac{45000}{V}$$

$$\frac{dP}{dV} = \frac{-45000}{V^2}$$

$$\therefore \frac{dP}{dt} = 100 \times \frac{-45000}{V^2}$$

$$\begin{aligned} \text{When } V = 4000, \frac{dP}{dt} &= 100 \times \frac{-45000}{4000^2} \\ &= -0.28 \end{aligned}$$

\therefore the air pressure is decreasing at a rate of $0.28 \text{ g/cm}^2/\text{s}$.

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Question 13 (15 marks)

13 (a) (2 marks)

Outcomes assessed: ME11-2

Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	2
• Attempts to eliminate parameter by squaring both equations	1

Sample Answer:

$$x = 1 + 2\cos 2t \text{ and } y = 2 + 2\sin 2t$$

$$x - 1 = 2\cos 2t \quad (1)$$

$$y - 2 = 2\sin 2t \quad (2)$$

$(1)^2 + (2)^2$ gives:

$$\begin{aligned}(x-1)^2 + (y-2)^2 &= 4\cos^2 2t + 4\sin^2 2t \\ &= 4(\cos^2 2t + \sin^2 2t) \\ &= 4\end{aligned}$$

\therefore the Cartesian equation is $(x-1)^2 + (y-2)^2 = 4$

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13 (b)(i) (3 marks)

Outcomes Assessed: ME12-2

Targeted Performance Bands: E4

Criteria	Marks
• Correct solution	3
• Correct integration to displacement	2
• Correct components of velocity	1

Sample Answer:

Find the components of displacement

$$\ddot{y} = -g$$

$$\dot{y} = -gt + c_1$$

$$\text{when } t = 0, \dot{y} = \frac{15\sqrt{2}}{2} \therefore c_1 = \frac{15\sqrt{2}}{2}$$

$$\therefore \dot{y} = -gt + \frac{15\sqrt{2}}{2}$$

$$y = -\frac{gt^2}{2} + \frac{15\sqrt{2}}{2}t + c_2$$

$$\text{when } t = 0, y = 30 \therefore c_2 = 30$$

$$\therefore y = -\frac{gt^2}{2} + \frac{15\sqrt{2}}{2}t + 30$$

$$\therefore y = -5t^2 + \frac{15\sqrt{2}}{2}t + 30 \quad \text{using } g = 10$$

$$\therefore \underline{s} = \left(\frac{15\sqrt{2}}{2}t \right) \underline{i} + \left(-5t^2 + \frac{15\sqrt{2}}{2}t + 30 \right) \underline{j}$$

$$\ddot{x} = 0$$

$$\dot{x} = c_3$$

$$\text{when } t = 0, \dot{x} = \frac{15\sqrt{2}}{2} \therefore c_3 = \frac{15\sqrt{2}}{2}$$

$$\therefore \dot{x} = \frac{15\sqrt{2}}{2}$$

$$x = \frac{15\sqrt{2}}{2}t + c_4$$

$$\text{when } t = 0, x = 0 \therefore c_4 = 0$$

$$\therefore x = \frac{15\sqrt{2}}{2}t$$

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13 (b)(ii) (1 mark)

Outcomes Assessed: ME12-2

Targeted Performance Bands: E3

Criteria	Mark
• Correct solution	1

Sample Answer:

For the ball to reach the ground the component of j must equal zero.

$$\begin{aligned} -5t^2 + \frac{15\sqrt{2}}{2}t + 30 &= 0 \\ t &= \frac{-\frac{15\sqrt{2}}{2} \pm \sqrt{\left(-\frac{15\sqrt{2}}{2}\right)^2 - 4(-5)(30)}}{2(-5)} \\ &= \frac{-\frac{15\sqrt{2}}{2} \pm \sqrt{712.5}}{-10} \\ &= 3.7 \text{ seconds (1 dp)} \quad \text{taking } t > 0 \end{aligned}$$

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13 (c)(i) (2 marks)

Outcomes Assessed: ME11-1

Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	2
• Correct equation of f^{-1} without specifying domain	1

Sample Answer:

$$\begin{aligned} f \circ g &= \sqrt{2(x+2)-1} \\ &= \sqrt{2x+3} \end{aligned}$$

$(f \circ g)^{-1}$ is given by rearranging:

$$x = \sqrt{2y+3}$$

$$\therefore y = \frac{x^2-3}{2}, \text{ for } x \geq 0$$

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13 (c)(ii) (2 marks)

Outcomes Assessed: ME11-1

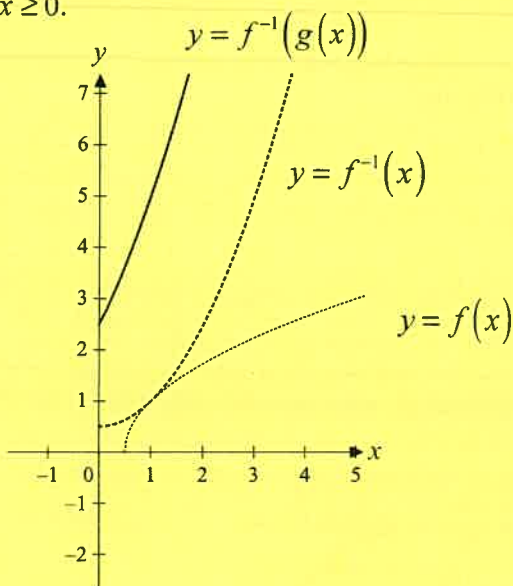
Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	2
• Correct graph without correct domain	1

Sample Answer:

The domain of $f^{-1}(x)$ is $x \geq 0$

\therefore the domain of $f^{-1}(g(x))$ is also $x \geq 0$.



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13 (d) (3 marks)

Outcomes Assessed: ME12-1

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct solution	3
• Correctly sets up the statement that needs to be proved	2
• Shows that $P(n)$ is true for $n = 1$	1

Sample Answer:

Let $P(n)$ be the proposition that $7^n - 3^n$ is divisible by 4 for $n \geq 1$.

When $n = 1$, $LHS = 7^1 - 3^1 = 4$

$$= 4 \times 1$$

$\therefore P(n)$ is true when $n = 1$.

Assume that $P(n)$ is true when $n = k$.

i.e. assume that $7^k - 3^k = 4M$, where $M \in \mathbb{Z}^+$

Required to prove that $P(n)$ is true when $n = k + 1$.

i.e. required to prove that $7^{k+1} - 3^{k+1} = 4P$, where $P \in \mathbb{Z}^+$

$$LHS = 7^{k+1} - 3^{k+1}$$

$$= 7 \cdot 7^k - 3 \cdot 3^k$$

$$= 7(4M + 3^k) - 3 \cdot 3^k \quad \text{by assumption}$$

$$= 7 \times 4M + 7 \cdot 3^k - 3 \cdot 3^k$$

$$= 7 \times 4M + 4 \cdot 3^k$$

$$= 4(7M + 3^k)$$

$$= 4P, \quad \text{where } P \in \mathbb{Z}^+$$

\therefore if $P(k)$ is true then $P(k+1)$ is true.

\therefore by the process of mathematical induction $P(n)$ is true for $n \geq 1$.

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13 (e) (2 marks)

Outcomes Assessed: ME12-2

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct solution	2
• Progress towards solution using the dot product	1

Sample Answer:

let $\overrightarrow{AB} = \underline{a}$ and $\overrightarrow{BC} = \underline{b}$

since $ABCD$ is a parallelogram

$\overrightarrow{CD} = -\underline{a}$ and $\overrightarrow{DA} = -\underline{b}$

$\therefore \overrightarrow{AC} = \underline{a} + \underline{b}$ and $\overrightarrow{DB} = \underline{a} - \underline{b}$

to prove \overrightarrow{AC} and \overrightarrow{DB} are perpendicular we show $\overrightarrow{AC} \cdot \overrightarrow{DB} = 0$

$$\begin{aligned}(\underline{a} + \underline{b}) \cdot (\underline{a} - \underline{b}) &= |\underline{a}|^2 - |\underline{b}|^2 \\ &= 0 \quad \text{since } AB = BC\end{aligned}$$

$\therefore \overrightarrow{AC}$ and \overrightarrow{DB} are perpendicular

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Question 14 (15 marks)

14 (a)(i) (1 mark)

Outcomes assessed: ME12-4

Targeted Performance Bands: E3

Criteria	Mark
• Correct solution	1

Sample Answer:

using the product rule:

$$\begin{aligned}\frac{d}{dx}(x \cos^{-1} x) &= x \times \frac{-1}{\sqrt{1-x^2}} + 1 \times \cos^{-1} x \\ &= \cos^{-1} x - \frac{x}{\sqrt{1-x^2}}\end{aligned}$$

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14 (a)(ii) (3 marks)

Outcomes assessed: ME12-4

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct solution	3
• Correctly evaluates one integral	2
• Integration of all terms between $x = 0$ and $x = \frac{1}{2}$	1

Sample Answer:

integrating both sides of $\frac{d}{dx}(x \cos^{-1} x) = \cos^{-1} x - \frac{x}{\sqrt{1-x^2}}$

$$\int_0^{\frac{1}{2}} \frac{d}{dx}(x \cos^{-1} x) dx = \int_0^{\frac{1}{2}} \cos^{-1} x dx - \int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}} dx$$

$$\left[x \cos^{-1} x \right]_0^{\frac{1}{2}} = \int_0^{\frac{1}{2}} \cos^{-1} x dx - \int_0^{\frac{\pi}{6}} \frac{\sin \theta}{\sqrt{1-\sin^2 \theta}} \cos \theta d\theta \quad \text{using } x = \sin \theta$$

$$\left[\frac{1}{2} \cos^{-1} \frac{1}{2} - 0 \cos^{-1} 0 \right] = \int_0^{\frac{1}{2}} \cos^{-1} x dx - \int_0^{\frac{\pi}{6}} \frac{\sin \theta}{|\cos \theta|} \cos \theta d\theta$$

$$\frac{\pi}{6} = \int_0^{\frac{1}{2}} \cos^{-1} x dx - \int_0^{\frac{\pi}{6}} \sin \theta d\theta \quad \sqrt{\cos^2 \theta} > 0 \text{ since } 0 \leq \theta \leq \frac{\pi}{2}$$

$$\frac{\pi}{6} = \int_0^{\frac{1}{2}} \cos^{-1} x dx - \left(-\frac{\sqrt{3}}{2} + 1 \right)$$

$$\therefore \int_0^{\frac{1}{2}} \cos^{-1} x dx = \frac{\pi}{6} + \left(1 - \frac{\sqrt{3}}{2} \right)$$

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14 (b) (3 marks)

Outcomes assessed: ME12-4

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct solution	3
• Correct integration	2
• Correctly separates the differential equation	1

Sample Answer:

$$\sec x \frac{dy}{dx} = \frac{e^{\sin x}}{y}$$

$$y dy = \cos x e^{\sin x} dx$$

integrating both sides:

$$\int y dy = \int \cos x e^{\sin x} dx$$

$$\frac{y^2}{2} = e^{\sin x} + c$$

$$\text{since } \frac{d}{dx}(e^{\sin x}) = \cos x e^{\sin x}$$

given $x = 0, y = 0$

$$\frac{0^2}{2} = e^{\sin 0} + c$$

$$c = -1$$

$$\therefore y^2 = 2e^{\sin x} - 2$$

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14 (c)(i) (2 marks)

Outcomes assessed: ME12-4

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct solution	2
• Attempts to let $\frac{dP}{dt} = 0$	1

Sample Answer:

The carrying capacity is found by letting $\frac{dP}{dt} = 0$

$$\text{i.e. } P = 0 \text{ or } 5 - \frac{P}{10000} = 0$$

$\therefore P = 50000$ is the carrying capacity.

The initial population of kangaroos is 15% of the carrying capacity.

$$\begin{aligned}\therefore P_0 &= 0.15 \times 50000 \\ &= 7500\end{aligned}$$

14 (c)(ii) (1 mark)

Outcomes assessed: ME11-4

Targeted Performance Bands: E3-E4

Criteria	Mark
• Correct solution	1

Sample Answer:

The rate of increase is a maximum when the population is half the carrying capacity.

$\therefore \frac{dP}{dt}$ is a maximum when $P = 25000$ kangaroos.

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14 (d)(i) (2 marks)

Outcomes assessed: ME12-5

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct solution	2
• Finds the correct value of μ or σ .	1

Sample Answer:

$$\hat{p} = \frac{6}{200}$$
$$= 0.03$$

$$\mu_{\text{proportion}} = \hat{p}$$
$$= 0.03$$

$$\sigma_{\text{proportion}} = \frac{\sigma}{n}$$
$$= \frac{\sqrt{np(1-p)}}{n}$$
$$= \frac{\sqrt{200 \times 0.03 \times 0.97}}{200}$$
$$= 0.01206 \text{ (5 decimal places)}$$

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14 (d)(ii) (1 mark)

Outcomes assessed: ME12-5

Targeted Performance Bands: E3

Criteria	Mark
• Correct solution	1

Sample Answer:

$$\begin{aligned}\text{For 4 defective globes } \hat{p} &= \frac{4}{200} \\ &= 0.02\end{aligned}$$

$$\begin{aligned}z &= \frac{0.02 - 0.03}{0.01206} \\ &= -0.829187\dots \\ &= -0.83 \text{ (2 decimal places)}\end{aligned}$$

14 (d)(iii) (2 marks)

Outcomes assessed: ME12-5

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct solution	2
• Attempts use of the table of $P(Z < z)$ values	1

Sample Answer:

To find $P(4 < X < 5)$ we need to calculate the z -score of 5 defective globes

$$x = \frac{5}{200} = 0.025$$

$$z = \frac{0.025 - 0.03}{0.01206}$$

$$= -0.41459\dots$$

$$= -0.41 \text{ (2 decimal places)}$$

$$\begin{aligned}\therefore P(4 < X < 5) &= P(-0.83 < z < -0.41) \\ &= P(z < -0.41) - P(z < -0.83) \\ &= (1 - P(z < 0.41)) - (1 - P(z < 0.83)) \\ &= (1 - 0.6591) - (1 - 0.7967) \\ &= 0.1376\end{aligned}$$

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