



CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NSW

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Centre Number

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Student Number

DO NOT REMOVE PAPER FROM EXAM ROOM

2020
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics Extension 1

Afternoon Session
Friday, 28 August 2020

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A Reference Sheet is provided at the back of this paper
- In Questions 11–14, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks – 70

Section I Pages 2 – 6

10 marks

- Attempt Questions 1 – 10
- Allow 15 minutes for this section

Section II Pages 7 – 13

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Disclaimer

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6200-1

Section I

10 marks

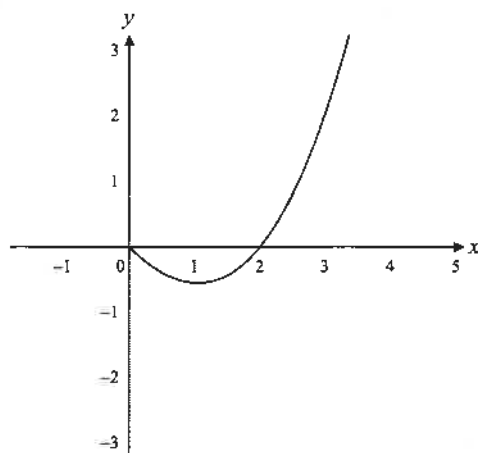
Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

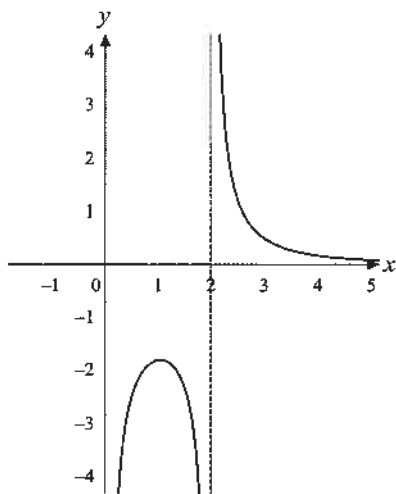
- 1 In a group of 110 students, what is the minimum number of students that have a birthday in at least one of the months of the year?
- (A) 9
- (B) 10
- (C) 11
- (D) 12
-
- 2 A polynomial $P(x)$ has a triple root at $x = -2$.
- Which of the following statements is true?
- (A) $(x+2)^2$ is a factor of $P'(x)$.
- (B) $(x-2)^2$ is a factor of $P'(x)$.
- (C) $(x+2)^3$ is a factor of $P'(x)$.
- (D) $(x-2)^3$ is a factor of $P'(x)$.

3 The diagram shows the graph of the function $y = f(x)$.

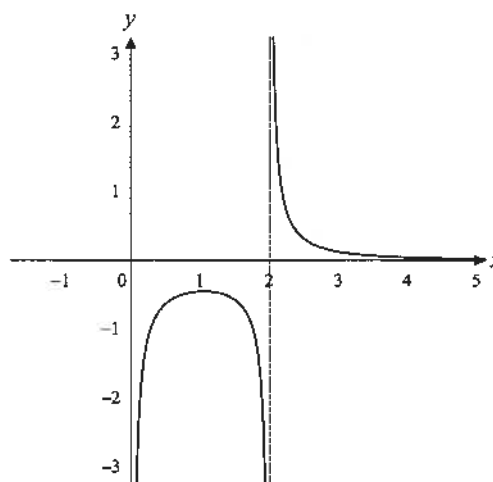


If the local minimum is $\left(1, -\frac{1}{2}\right)$, which of the following is the graph of $y = \frac{1}{f(x)}$?

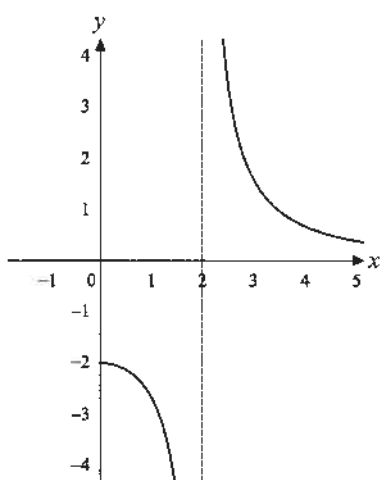
(A)



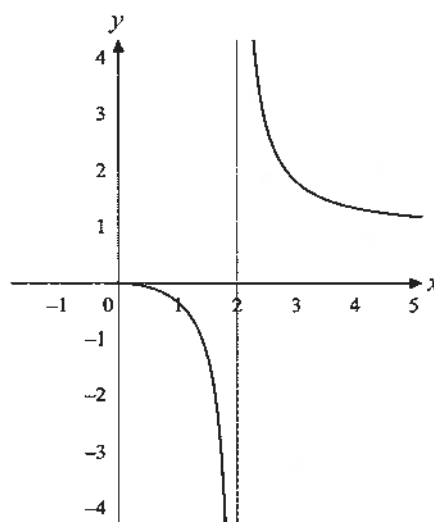
(B)



(C)



(D)



4 Using the substitution $t = \tan \frac{\theta}{2}$, simplify the expression $\frac{\cos \theta - 1}{2 \sin \theta}$.

(A) t

(B) $-t$

(C) $\frac{t}{2}$

(D) $-\frac{t}{2}$

5 In how many ways can the letters of the word TROOPS be arranged if the two Os are to be separated?

(A) 60

(B) 240

(C) 300

(D) 600

6 What is the variance of the following Bernoulli distribution?

X	0	1
$P(X = x)$	0.2	0.8

(A) 0.16

(B) 0.20

(C) 0.40

(D) 0.64

7 Which expression is equal to $\int(\sin^2 x + x^2) dx$?

(A) $\frac{1}{2}x - \frac{1}{4}\sin 2x + 2x + c$

(B) $x - \frac{1}{2}\sin 2x + \frac{x^3}{3} + c$

(C) $\frac{1}{2}x - \frac{1}{4}\sin 2x + \frac{x^3}{3} + c$

(D) $x - \frac{1}{2}\sin 2x + 2x + c$

8 Given the vectors $\underline{a} = 2\underline{i} - 5\underline{j}$ and $\underline{b} = 3\underline{i} + 4\underline{j}$, what is $\text{proj}_{\underline{a}}\underline{b}$?

(A) $\frac{14}{29}(2\underline{i} - 5\underline{j})$

(B) $\frac{14}{\sqrt{29}}(2\underline{i} - 5\underline{j})$

(C) $\frac{-14}{29}(2\underline{i} - 5\underline{j})$

(D) $\frac{-14}{\sqrt{29}}(2\underline{i} - 5\underline{j})$

9 The graph of the function $y = \cos^{-1} x + 1$ is transformed by being dilated vertically by a scale factor of 2 and then translated up by 3.

What is the equation of the transformed graph?

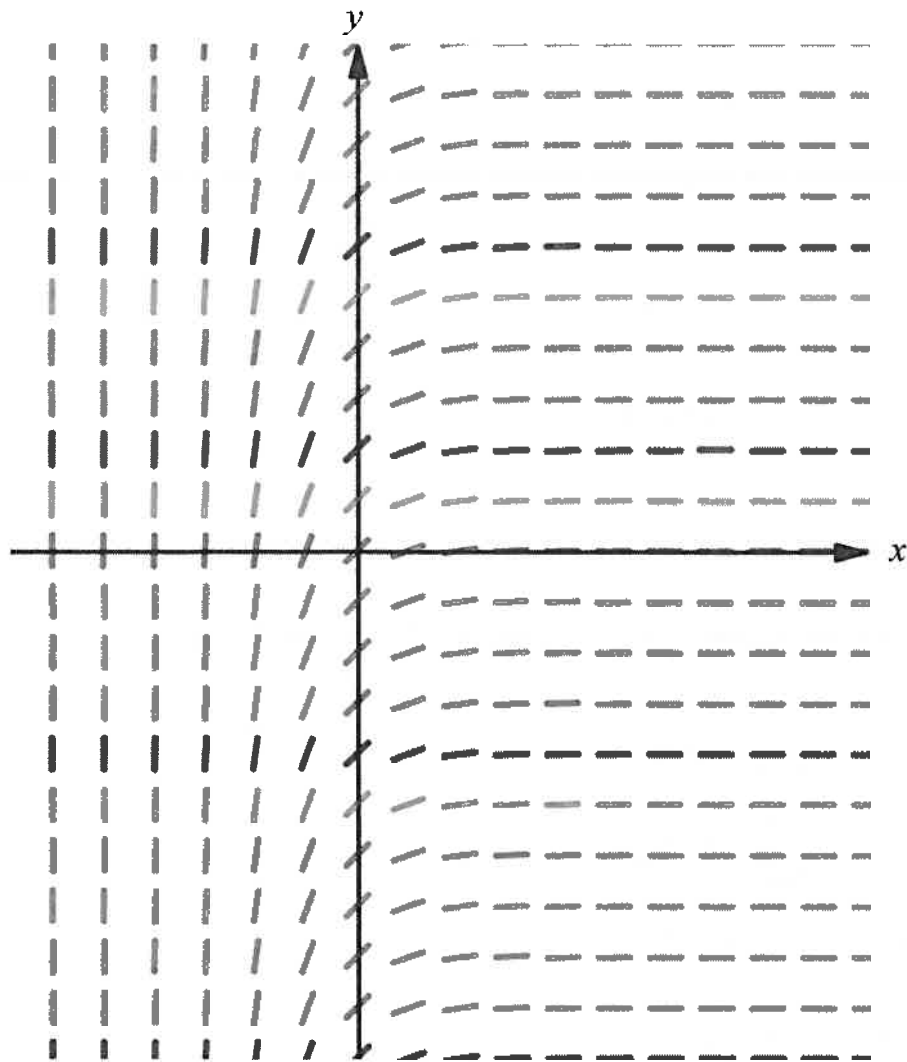
(A) $y = 2\cos^{-1} x + 4$

(B) $y = 2\cos^{-1} x + 5$

(C) $y = 2\cos^{-1} x + 6$

(D) $y = 2\cos^{-1} x + 8$

- 10 Which of the following could be the differential equation represented by the slope field below?



- (A) $\frac{dy}{dx} = \ln x$
- (B) $\frac{dy}{dx} = -\ln x$
- (C) $\frac{dy}{dx} = e^x$
- (D) $\frac{dy}{dx} = e^{-x}$

End of Section I

Section II

60 marks

Attempt Questions 11 - 14

Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

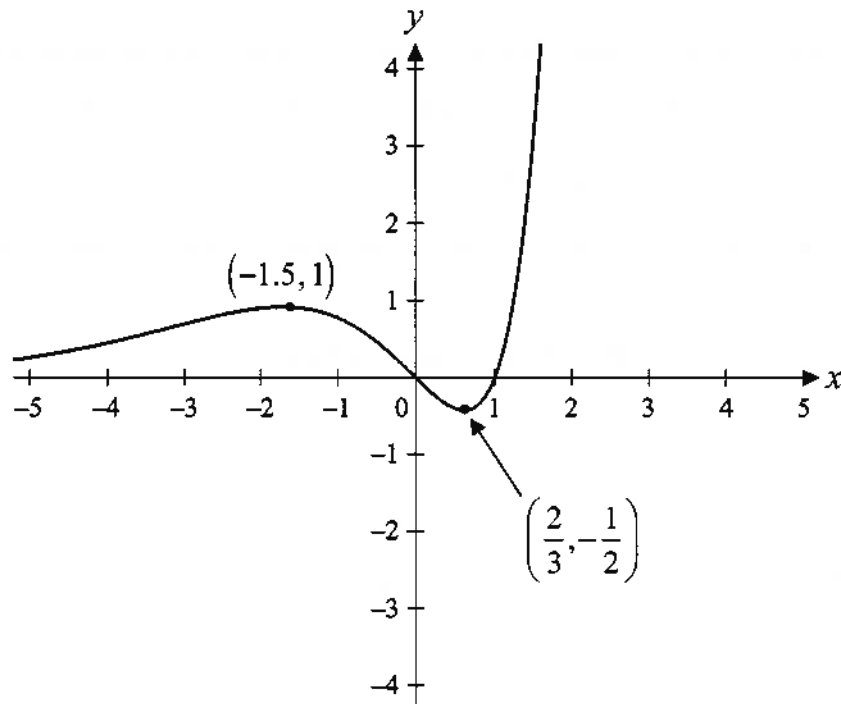
Question 11 (15 marks) Use a SEPARATE writing booklet.

- (a) Solve $\frac{2x}{x+3} \leq 1$. 3
- (b) The polynomial $P(x) = 2x^3 + kx^2 - 1$ is divided by $x + 2$ and the remainder is 7. 2
Find the value of k .
- (c) A luxury bed and breakfast has 6 standard and 5 executive rooms. On any day the probability that a standard room is unoccupied is 6.4% and the probability that an executive room is unoccupied is 13.1%. 3
On any random day, find the probability that 2 standard rooms and 2 executive rooms are unoccupied.
- (d) A committee of five is to be chosen from four men and six women. 1
(i) How many committees are possible if there are no restrictions? 1
(ii) How many committees are possible if the majority of members are to be women? 2
- (e) Find the exact magnitude and direction (to the nearest degree) of vector \overline{AB} where 2
 $\overline{OA} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\overline{OB} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$.
- (f) Find the value of x if the vectors $\underline{a} = 2\underline{i} + 3\underline{j}$ and $\underline{b} = -3\underline{i} + x\underline{j}$ are perpendicular. 2

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) The diagram shows the graph of $y = f(x)$.



Draw a separate half-page diagram for each of the following, showing all asymptotes and intercepts.

(i) $y^2 = f(x)$

2

(ii) $y = x f(x)$

2

(b) Solve $\sin x + \sqrt{3} \cos x = 1$ for $x \in [0, 2\pi]$.

3

(c) Find $\int_0^{\frac{4}{3}} \frac{dx}{16 + 9x^2}$.

3

Question 12 continues on page 9

Question 12 (continued)

(d) Use the substitution $u = x^3 - 3$ to find the exact volume of the solid formed when the curve $y = x(x^3 - 3)^3$ is rotated about the x -axis between $x = 0$ and $x = 1$. **3**

(e) The relationship between the volume, V , measured in cm^3 , of a balloon and the internal air pressure, P , measured in grams/cm^2 , is given by $PV = 45\,000$. **2**

The volume of a balloon is increasing at a rate of $100 \text{ cm}^3\text{s}^{-1}$. Find the rate of change of the air pressure inside the balloon when the volume is $4\,000 \text{ cm}^3$.

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

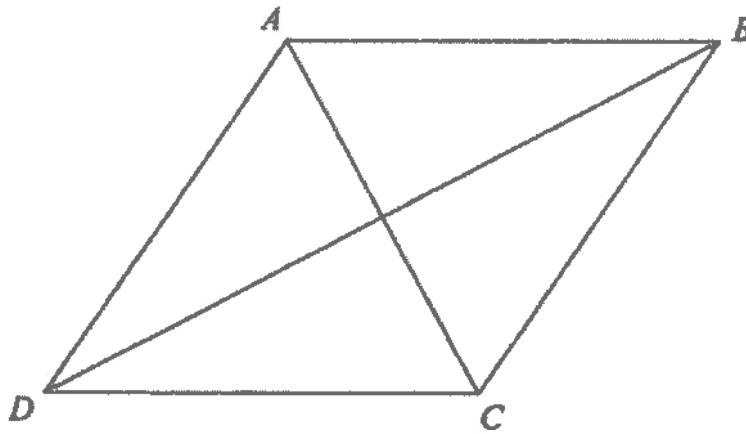
- (a) Find the Cartesian equation for the curve with parametric equations $x = 1 + 2 \cos 2t$ and $y = 2 + 2 \sin 2t$. 2
- (b) Jerry stands at the top of a building 30 m tall and throws a ball with a velocity of 15 m s^{-1} at an angle of 45° to the horizontal. The ball eventually reaches the ground.
- (i) Derive the vector displacement of the ball in the form $\underline{s} = x\underline{i} + y\underline{j}$. You may assume that $g = 10 \text{ m s}^{-2}$. 3
- (ii) Find the time taken for the ball to reach the ground. 1
- (c) (i) The functions f and g are defined by $f(x) = \sqrt{2x-1}$ and $g(x) = x+2$. Find the inverse of the composite function $f \circ g$. 2
- (ii) Sketch the graph of $y = f^{-1}(g(x))$. 2
- (d) Prove using mathematical induction that $7^n - 3^n$ is divisible by 4 for $n \geq 1$. 3

Question 13 continues on page 11

Question 13 (continued)

- (e) In the diagram $ABCD$ is a parallelogram. If $AB = BC$, prove that the vectors \overline{AC} and \overline{BD} are perpendicular.

2



End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) (i) Prove that $\frac{d}{dx}(x \cos^{-1} x) = \cos^{-1} x - \frac{x}{\sqrt{1-x^2}}$. **1**

(ii) Hence use the substitution $x = \sin \theta$ to show that **3**

$$\int_0^{\frac{1}{2}} \cos^{-1} x \, dx = \frac{\pi}{6} + \left(1 - \frac{\sqrt{3}}{2}\right).$$

(b) Solve the equation $\sec x \frac{dy}{dx} = \frac{e^{\sin x}}{y}$, given $x=0$ when $y=0$. **3**

(c) The population, P , of kangaroos in a reservation is modelled by the logistic equation $\frac{dP}{dt} = 2P \left(5 - \frac{P}{10000}\right)$ where t is measured in days. It is estimated that initially the population of the kangaroos is 15% of the carrying capacity of the reservation.

(i) What is the initial population of the kangaroos? **2**

(ii) What is the population of the kangaroos when the rate of increase is a maximum? **1**

Question 14 continues on page 13

Question 14 (continued)

- (d) A standard quality check found that out of 200 light globes, 6 had a defect.
- (i) If the sample proportion is approximately normally distributed, show that the mean and sample standard deviation are 0.03 and 0.01206 respectively. 2
 - (ii) Find the z -score if 4 light globes in this sample are defective. 1
 - (iii) Use the section of the table showing the values of $P(Z < z)$ to find the probability that the number of defective light globes in this sample was between 4 and 5. 2

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7258	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7	0.7580	0.7612	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7996	0.8023	0.8051	0.8079	0.8106	0.8133

End of Paper

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EXAMINERS

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Mathematics Extension 1 Replacement Questions for Trial HSC Examination**Replacement Question 1** on page 2

12 people are running for election as school captain. There are 109 people voting.

What is the least number of votes needed for someone to win the election?

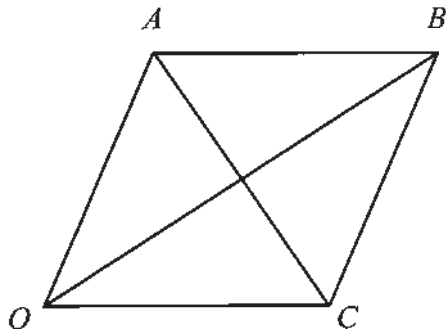
- (A) 9
- (B) 10
- (C) 11
- (D) 12

Replacement Question 13(c)(i) on Page 10

- (c) (i) The functions f and g are defined by $f(x) = \sqrt{2x-1}$ and $g(x) = x+2$. Find the inverse of the composite function $f(g(x))$. 2
- (ii) Sketch the graph of $y = f^{-1}(g(x))$. 2

Replacement Question 13(e) on Page 11

- (e) In the diagram $OABC$ is a parallelogram. Given $OA = OC$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{b}$. 2
- Prove that the vectors, \overrightarrow{OB} and \overrightarrow{AC} are perpendicular.



Replacement Question 14(a)(ii) on Page 12

(a) (i) Prove that $\frac{d}{dx}(x \cos^{-1} x) = \cos^{-1} x - \frac{x}{\sqrt{1-x^2}}$.

1

(ii) Hence use the substitution $x = \sin \theta$, or otherwise, to show that

3

$$\int_0^{\frac{1}{2}} \cos^{-1} x \, dx = \frac{\pi}{6} + \left(1 - \frac{\sqrt{3}}{2}\right).$$

