Neap

Final Examination 2021

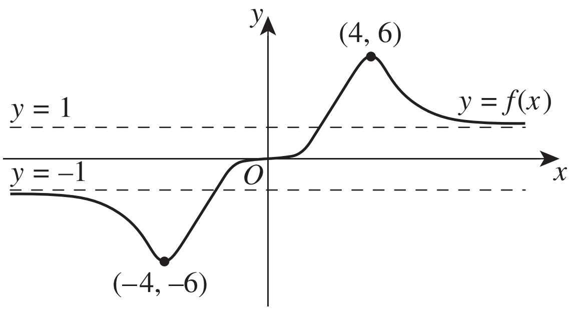
NSW Year 11 Mathematics Advanced

Solutions and marking guidelines

SECTION I

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 1 **A** Using the quadratic formula: $x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-4)}}{2 \times 2}$ $= \frac{-3 \pm \sqrt{41}}{4}$	MA-F1 Working with Functions MA11–1 Band 2
Question 2 **C** **C** is correct. This is a many-to-one relation as two students have the same teacher and no student has more than one teacher. **A**, **B** and **D** are incorrect. The relations specified in these options do not reflect the information shown in the diagram.	MA-F1 Working with Functions MA11–2 Bands 2–3
Question 3 **D** Domain: $2 - x \geq 0$ $2 \geq x$ $x \leq 2$ $\therefore (-\infty, 2]$ Range: $f(x) \geq 0$ $\therefore [0, \infty)$	MA-F1 Working with Functions MA11–1 Bands 3–4
Question 4 **B** $\frac{d}{dx}\left(\left(5x^2+2\right)^4\right) = 4\left(5x^2+2\right)^3 \times 10x$ $= 40x\left(5x^2+2\right)^3$	MA-C1 Introduction to Differentiation MA11–5 Bands 3–4
Question 5 **D** **D** is correct. The tangent is vertical at point *D*. **A**, **B** and **C** are incorrect. A function is said to be differentiable at point $x = a$ if it is smooth and continuous at $x = a$. *Note: A function is differentiable at point $x = a$ if, and only if, a non-vertical tangent exists at $x = a$.*	MA-C1 Introduction to Differentiation MA11–5 Bands 3–5
Question 6 **B** If *P*, *Q* and *R* are collinear, then $m_{PQ} = m_{PR} = m_{QR}$. $\frac{2-4}{1-3} = \frac{b-4}{a-3}$ $1 = \frac{b-4}{a-3}$ $a - 3 = b - 4$ $a - b = -1$	MA-F1 Working with Functions MA11–1 Bands 3–5

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 7 **B** $P(\text{different colours}) = P(BR) + P(RB)$ $= \frac{26}{52} \times \frac{26}{51} + \frac{26}{52} \times \frac{26}{51}$ $= \frac{26}{51}$	MA-S1 Probability and Discrete Probability Distributions MA11–7 Bands 3–4
Question 8 **D** $50°24' = 50°24' \times \frac{\pi}{180°}$ $= 0.87964594\ldots$ radians ≈ 0.880 radians (to 3 significant figures)	MA-T1 Trigonometry and Measure of Angles MA11–3 Bands 2–3
Question 9 **A** sector angle $= 2\pi - \frac{\pi}{3}$ $= \frac{5\pi}{3}$ $A_{\text{sector}} = \frac{1}{2} \times 12^2 \times \frac{5\pi}{3}$ $= 120\pi \text{ cm}^2$ $A_{\text{triangle}} = \frac{1}{2} \times 12^2 \times \sin\left(\frac{\pi}{3}\right)$ $= 36\sqrt{3} \text{ cm}^2$ $\therefore$ total area $= 120\pi + 36\sqrt{3} \text{ cm}^2$	MA-T1 Trigonometry and Measure of Angles MA11–3 Bands 4–5

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 10 **C** Odd functions have point symmetry about the origin. Examine the complete graph of $y = f(x)$.  **C** is correct. As the gradient of the tangent at $x = -3$ appears to be positive, **C** is possible. Through the process of elimination, **C** is the correct answer. **A** is incorrect. As $y = f(x)$ has two horizontal asymptotes, **A** is not possible. **B** is incorrect. **B** is not possible, as $f = (-4) = -6$. **D** is incorrect. **D** is not possible, as $f(x)$ is decreasing for $x > 4$ but increasing for $-4 < x < 0$. *Note: Options **A**, **B** and **D** would be correct if $y = f(x)$ had been an even function, which would have been the case if the graph had been symmetrical about the y-axis.*	MA-F1 Working with Functions MA-C1 Introduction to Differentiation MA11–2, MA11–5 Bands 4–6

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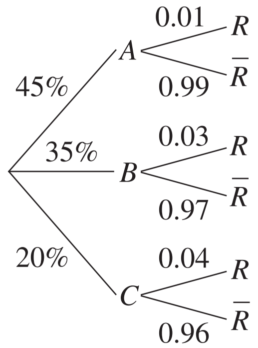
SECTION II

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 11	
$\sqrt{x} = 6\sqrt{2} + \sqrt{128} - 3\sqrt{32}$ $= 6\sqrt{2} + 8\sqrt{2} - 12\sqrt{2}$ $= 2\sqrt{2}$ $= \sqrt{8}$ $x = 8$	MA-F1 Working with Functions MA11–1 Bands 2–4 • Gives the correct solution . . . . 2 • Makes progress by writing all surds as like surds OR equivalent merit. 1
Question 12	
$3x - 2y + 4 = 0$ $2y = 3x + 4$ $y = \frac{3}{2}x + 2$ $\therefore m = \frac{3}{2}$ Equation of line using $m = \frac{3}{2}$ and point (4, –5): $y - (-5) = \frac{3}{2}(x - 4)$ $2(y + 5) = 3(x - 4)$ $2y + 10 = 3x - 12$ $3x - 2y - 22 = 0$	MA-F1 Working with Functions MA11–1 Bands 2–3 • Gives the correct solution . . . . 2 • Calculates the gradient OR equivalent merit. 1
Question 13	
(a) $\lvert 3x + 4 \rvert = 7$ $3x + 4 = 7$ $\quad$ $3x + 4 = -7$ $3x = 3$ $\quad$ $3x = -11$ $x = 1$ $\quad$ $x = -\frac{11}{3}$ $\therefore x = 1, -\frac{11}{3}$	MA-F1 Working with Functions MA11–1 Bands 3–4 • Gives the correct solution . . . . 2 • Attempts to deal with the absolute value OR equivalent merit. 1
(b) When taking the absolute value, you cannot have a negative result; that is, $\lvert 3x + 4 \rvert \geq 0$ for all values of *x*. $\lvert 3x + 4 \rvert = -7$ has no solutions, so it is not true that it is the same as $\lvert 3x + 4 \rvert = 7$.	MA-F1 Working with Functions MA11–9 Bands 3–5 • Gives the correct solution 1

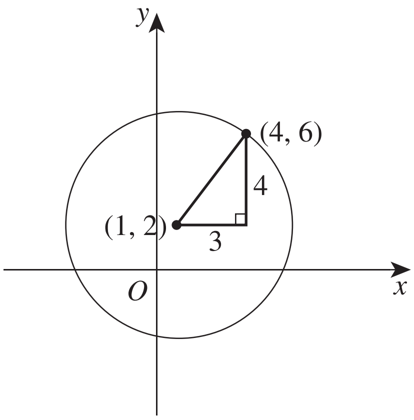
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 14	
The function has x-intercepts of -1 and 3. So the equation is in the form $y = a(x+1)(x-3)$. Substituting the y-intercept into this equation gives: $-6 = a(0+1)(0-3)$ $-6 = -3a$ $a = 2$ $\therefore y = 2(x+1)(x-3)$	MA-F1 Working with Functions MA11–1 Bands 2–4 • Gives the correct solution . . . . 2 • Provides a quadratic equation with the zeros 1
Question 15	
(a) $2\cos\theta = -\sqrt{2}$ $\cos\theta = -\frac{\sqrt{2}}{2}$ related angle $\theta = \frac{\pi}{4}$ As cos < 0, angles are in the second and third quadrants. $\theta = \pi - \frac{\pi}{4},\ \pi + \frac{\pi}{4}$ $= \frac{3\pi}{4},\ \frac{5\pi}{4}$	MA-T1 Trigonometry and Measure of Angles MA11–3 Bands 3–4 • Gives the correct solution . . . . 2 • Finds the related angle in radians OR equivalent merit. 1
(b) As $-180° \le x \le -180°$, then $-360° \le 2x \le -360°$. $\sin^2 2x = \frac{1}{4}$ $\sin 2x = \pm\frac{1}{2}$ related angle $2x = 30°$ Angles are in all four quadrants: $2x = 30°,\ 180° - 30°,\ 180° + 30°,\ 360° - 30°,$ $-30°,\ -180° + 30°,\ -180° - 30°,\ -360° + 30°$ $= 30°,\ 150°,\ 210°,\ 330°,$ $-30°,\ -150°,\ -210°,\ -330°$ $x = \pm 15°,\ \pm 75°,\ \pm 105°,\ \pm 165°$	MA-T1 Trigonometry and Measure of Angles MA11–3 Bands 3–4 • Gives the correct solution . . . . 3 • Finds the angles for $2x$ in the domain $-360° \le 2x \le -360°$. . . . . 2 • Finds the related angle $2x$ in degrees OR equivalent merit. 1

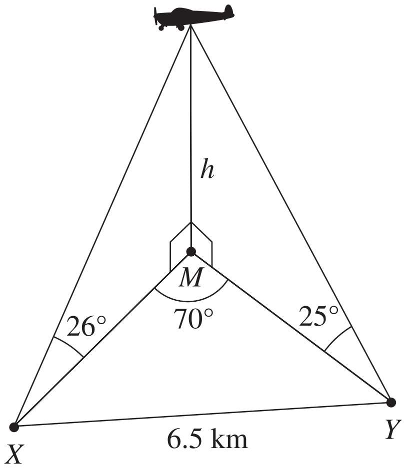
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 16	
$\frac{5x}{5x+3}=\frac{u}{v}$ $u=5x \qquad v=5x+3$ $\frac{du}{dx}=5 \qquad \frac{dv}{dx}=5$ $\frac{d}{dx}\left(\frac{5x}{5x+3}\right)=\frac{5(5x+3)-5(5x)}{(5x+3)^2}$ $=\frac{\cancel{25x}+15-\cancel{25x}}{(5x+3)^2}$ $=\frac{15}{(5x+3)^2}$	MA-C1 Introduction to Differentiation MA11–5 Bands 2–3 • Gives the correct solution . . . . 2 • Attempts to use the quotient rule OR equivalent merit 1
Question 17	
$f'(x)=\lim_{h\to 0}\frac{(x+h)^2-3(x+h)-(x^2-3x)}{h}$ $=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2-\cancel{3x}-3h-\cancel{x^2}+\cancel{3x}}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2-3h}{h}$ $=\lim_{h\to 0}\frac{\cancel{h}(2x+h-3)}{\cancel{h}}$ $=\lim_{h\to 0}2x+h-3$ $=2x+0-3$ $=2x-3$	MA-C1 Introduction to Differentiation MA11–5 Bands 4–5 • Gives the correct solution . . . . 2 • Correctly substitutes into the definition OR equivalent merit. 1
Question 18	
$\angle QRP=180°-(39°+65°)$ $=76°$ The longest side, PQ, is opposite the largest angle, $\angle QRP$. $\frac{PQ}{\sin 76°}=\frac{94}{\sin 39°}$ $PQ=\frac{94\sin 76°}{\sin 39°}$ $=144.93...$ cm ≈ 145 cm (to the nearest cm)	MA-T1 Trigonometry and Measure of Angles MA11–5 Bands 4–5 • Gives the correct solution . . . . 2 • Correctly substitutes into the sine rule OR equivalent merit. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 19	
$\frac{dy}{dx} = 3e^x - 3ex^2$ At $x = -1$: $m_{\text{tangent}} = 3e^{-1} - 3e(-1)^2$ $= \frac{3}{e} - 3e$ $m_{\text{normal}} = \frac{-1}{\frac{3}{e} - 3e}$ $= \frac{-e}{3 - 3e^2}$ or $\frac{e}{3e^2 - 3}$	MA-C1 Introduction to Differentiation MA-E1 Logarithms and Exponentials MA11–5 Bands 3–5 • Gives the correct solution . . . . 3 • Calculates the gradient of the tangent OR equivalent merit . . . . . . 2 • Calculates the derivative 1
Question 20	
$\text{LHS} = \frac{\tan A}{1 + \sec A} - \frac{\tan A}{1 - \sec A}$ $= \frac{\tan A(1 - \sec A) - \tan A(1 + \sec A)}{(1 + \sec A)(1 - \sec A)}$ $= \frac{\cancel{\tan A} - \tan A \sec A - \cancel{\tan A} - \tan A \sec A}{1 - \sec^2 A}$ $= \frac{-2 \tan A \sec A}{1 - (1 + \tan^2 A)}$ $= \frac{\cancel{-}2 \cancel{\tan A} \sec A}{\cancel{-} \tan^{\cancel{2}} A}$ $= \frac{2 \sec A}{\tan A}$ $= \frac{\frac{2}{\cos A}}{\frac{\sin A}{\cos A}}$ $= \frac{2}{\sin A}$ $= 2\text{cosec}A$ $= \text{RHS}$ $\therefore \frac{\tan A}{1 + \sec A} - \frac{\tan A}{1 - \sec A} = 2\text{cosec}A$	MA-T2 Trigonometric Functions and Identities MA11–1, 11–4 Bands 4–6 • Gives the correct solution . . . . 3 • Makes substantial progress by cancelling common factors. . . . . 2 • Writes the expression under a common denominator 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 21	
Equate RHS of both equations: $mx - 4 = \frac{x^2}{2}$ $2mx - 8 = x^2$ $x^2 - 2mx + 8 = 0$ $\Delta = (-2m)^2 - 4 \times 1 \times 8$ $= 4m^2 - 32$ If the line is a tangent, then $\Delta = 0$. $4m^2 - 32 = 0$ $4m^2 = 32$ $m^2 = 8$ $m = \pm\sqrt{8}$ $= \pm 2\sqrt{2}$	MA-F1 Working with Functions MA-C1 Introduction to Differentiation MA11–1, MA11–5 Bands 4–5 • Gives the correct solution . . . . 3 • Calculates the determinant. . . . . 2 • Constructs a correct quadratic equation OR equivalent merit. 1
Question 22	
$y = f(x)$ has been reflected on the x-axis, and translated two units right and one unit up.	MA-E1 Logarithms and Exponentials MA11–6 Bands 3–5 • Gives the correct solution . . . . 2 • Correctly references reflection OR translation 1
Question 23	
(a) 	MA-S1 Probability and Discrete Probability Distributions MA11–7 Bands 3–4 • Draws the correct probability tree diagram . . . . 2 • Draws a one-stage probability tree diagram OR equivalent merit. 1
(b) $P(\text{rotten}) = P(AR) + P(BR) + P(CR)$ $= 45\% \times 0.01 + 35\% \times 0.03 + 20\% \times 0.04$ $= 0.023$	MA-S1 Probability and Discrete Probability Distributions MA11–7 Bands 3–4 • Gives the correct solution . . . . 2 • Calculates some of the probabilities, demonstrating an understanding of the addition rule OR equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) $P\left(B \mid \overline{\text{rotten}}\right) = \dfrac{P\left(B \cap \overline{R}\right)}{P\left(\overline{\text{rotten}}\right)}$ $= \dfrac{35\% \times 0.97}{1 - 0.023}$ $= \dfrac{679}{1954}$	MA-S1 Probability and Discrete Probability Distributions MA11–7 Bands 4–5 • Gives the correct solution . . . . 2 • Demonstrates an understanding of conditional probability OR equivalent merit 1
Question 24	
Let θ be the acute angle that interval AB makes with the x-axis. Then $\angle ABC = \theta + 90°$, as BC is perpendicular to the x-axis. $m_{AB} = \dfrac{2-0}{2+4}$ $= \dfrac{1}{3}$ $\tan\theta = \dfrac{1}{3}$ $\theta = \tan^{-1}\left(\dfrac{1}{3}\right)$ $\approx 18°$ (to the nearest degree) $\angle ABC \approx 18° + 90°$ $\approx 108°$	MA-F1 Working with Functions MA-C1 Introduction to Differentiation MA11–1 Bands 3–4 • Gives the correct solution . . . . 2 • Calculates the gradient of AB OR equivalent merit 1
Question 25	
(a) $\log_7 63 = \log_7(7 \times 9)$ $= \log_7 7 + \log_7 9$ $= 1 + 2\log_7 3$ $= 1 + 2a$	MA-E1 Logarithms and Exponentials MA11–6 Bands 3–4 • Gives the correct solution. . . . . 2 • Uses logarithm laws OR equivalent merit. 1
(b) $\log_7 \dfrac{16}{27} = \log_7 16 - \log_7 27$ $= \log_7 2^4 - \log_7 3^3$ $= \log_7 \left(2^3\right)^{\frac{4}{3}} - 3\log_7 3$ $= \dfrac{4}{3}\log_7 8 - 3\log_7 3$ $= \dfrac{4}{3}b - 3a$	MA-E1 Logarithms and Exponentials MA11–6 Bands 4–5 • Gives the correct solution. . . . . 3 • Obtains an expression for $\log_7 16$, in terms of $\log_7 8$. . . . . 2 • Uses logarithm laws OR equivalent merit. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 26	
 Using Pythagoras' theorem: $r = \sqrt{3^2 + 4^2}$ $= 5$ Equation of circle with centre (1, 2) and radius 5 units: $(x-1)^2 + (y-2)^2 = 5^2$ $x^2 - 2x + 1 + y^2 - 4y + 4 = 25$ $x^2 - 2x + y^2 - 4y - 20 = 0$	MA-F1 Working with Functions MA11–1 Bands 3–4 • Gives the correct solution. . . . . 2 • Calculates the radius. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 27	
Let h be the plane's altitude above M.  $\tan 26° = \dfrac{h}{XM}$ $XM = \dfrac{h}{\tan 26°}$ $XM = h\cot 26°$ Similarly, $YM = h\cot 25°$. Using the cosine rule in triangle XMY: $(h\cot 25°)^2 + (h\cot 26°)^2 - 2\times h\cot 25° \times h\cot 26° \times \cos 70° = 6.5^2$ $h^2\cot^2 25° + h^2\cot^2 26° - 2h^2\cot 25°\cot 26°\cos 70° = 6.5^2$ $h^2\left(\cot^2 25° + \cot^2 26° - 2\cot 25°\cot 26°\cos 70°\right) = 6.5^2$ $h^2 = \dfrac{6.5^2}{\cot^2 25° + \cot^2 26° - 2\cot 25°\cot 26°\cos 70°}$ $h = \dfrac{6.5}{\sqrt{\cot^2 25° + \cot^2 26° - 2\cot 25°\cot 26°\cos 70°}}$ $= 2.70014\ldots$ km ≈ 2.7 km ≈ 2700 m	MA-T1 Trigonometry and Measure of Angles MA11–1, MA11–3 Bands 4–6 • Gives the correct solution. . . . . 4 • Obtains a correct expression for h^2 OR equivalent merit. . . . . 3 • Uses the cosine rule correctly to write a valid equation in triangle XMY OR equivalent merit. . . . . 2 • Provides an expression for XM or YM OR equivalent merit. 1

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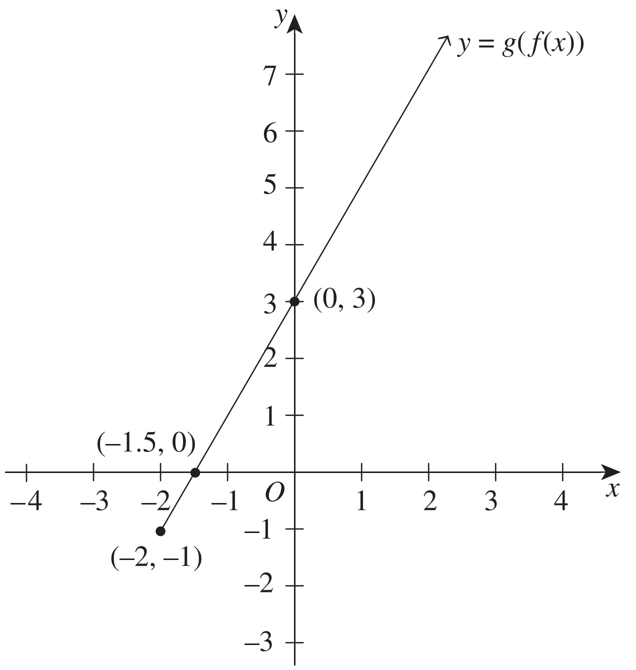
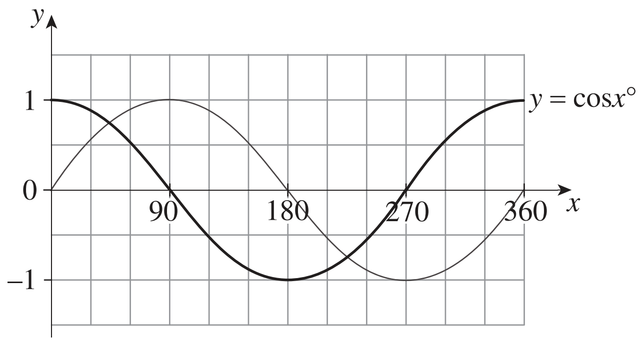
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 28	
(a) When $t = 10$, $M = 62$: $62 = 75e^{-10k}$ $\frac{62}{75} = e^{-10k}$ $-10k = \ln\left(\frac{62}{75}\right)$ $k = -\frac{1}{10}\ln\left(\frac{62}{75}\right)$	MA-E1 Logarithms and Exponentials MA11–6 Bands 3–4 • Gives the correct solution. . . . . 2 • Substitutes the given values into the equation OR equivalent merit. 1
(b) When $t = 5$: $M = 75e^{-\left(-\frac{1}{10}\ln\left(\frac{62}{75}\right)\right)\times 5}$ $= 68.190\ldots$ g reduction $= 75$ g $- 68.190\ldots$ g $= 6.809\ldots$ g ≈ 6.8 g (to 1 decimal place)	MA-E1 Logarithms and Exponentials MA11–6 Band 3 • Gives the correct solution. . . . . 2 • Calculates mass when $t = 5$.. 1
Question 29	
(a) $g\left(f(x)\right) = \left(\sqrt{2x+4}\right)^2 - 1$ $= 2x + 4 - 1$ $= 2x + 3$	MA-F1 Working with Functions MA11–1 Bands 3–4 • Gives the correct solution. . . . . 2 • Attempts to work with the correct composite function OR equivalent merit. 1
(b) $2x + 4 \geq 0$ $2x \geq -4$ $x \geq -2$ domain: $\left[-2, \infty\right)$	MA-F1 Working with Functions MA11–1 Bands 4–5 • Gives the correct solution. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) 	MA-F1 Working with Functions MA11–1 Bands 3–4 • Draws the correct graph. . . . . 2 • Draws the correct graph without domain restrictions or intercepts. 1
Question 30	
$\frac{1}{2}\times 8^2\theta = 80$ $32\theta = 80$ $\theta = \frac{5}{2}$ $l = 8\times\frac{5}{2}$ $= 20$ cm perimeter $= 8+8+20$ $= 36$ cm	MA-T1 Trigonometry and Measure of Angles MA11–3 Bands 3–4 • Gives the correct solution. . . . . 3 • Calculates the arc length OR equivalent merit. . . . . 2 • Calculates the sector angle OR equivalent merit. 1
Question 31	
(a) 	MA-T2 Trigonometric Functions and Identities MA11–4 Bands 3–4 • Gives the correct solution. . . . . 2 • Makes progress with drawing the correct graph OR correct points plotted. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) $x = 45, 225$	MA-F1 Working with Functions MA-T2 Trigonometric Functions and Identities MA11–1, MA11–4 Band 4 • Gives the correct solution. 1
Question 32	

For discrete probability distribution:

$$k + k^2 + 2k^2 + k = 1$$
$$3k^2 + 2k = 1$$
$$3k^2 + 2k - 1 = 0$$
$$(3k - 1)(k + 1) = 0$$
$$k = \frac{1}{3}, -1$$

$$0 \le P(X = x) \le 1$$

$$\therefore k = \frac{1}{3} \text{ only}$$

X	a	$2a$	$3a$	$4a$
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{3}$

$$E(X) = a \times \frac{1}{3} + 2a \times \frac{1}{9} + 3a \times \frac{2}{9} + 4a \times \frac{1}{3}$$
$$= \frac{23a}{9}$$

$$\frac{23a}{9} = \frac{23}{18}$$
$$a = \frac{1}{2}$$

X	0.5	1	1.5	2
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{3}$

$$\text{Var}(X) = \left(0.5 - \frac{23}{18}\right)^2 \times \frac{1}{3} + \left(1 - \frac{23}{18}\right)^2 \times \frac{1}{9} + \left(1.5 - \frac{23}{18}\right)^2 \times \frac{2}{9}$$
$$+ \left(2 - \frac{23}{18}\right)^2 \times \frac{1}{3}$$
$$= \frac{32}{81}$$

MA-S1 Probability and Discrete Probability Distributions
MA-F1 Working with Functions
MA11–7, MA11–9 Bands 4–6

- Gives the correct solution. 5
- Substitutes values to calculate variance. 4
- Solves for a. 3
- Calculates the probabilities using $k = \frac{1}{3}$. 2
- Constructs the correct quadratic equation in terms of k. 1