Neap

Final Examination 2021

NSW Year 11 Mathematics Extension 1

Solutions and marking guidelines

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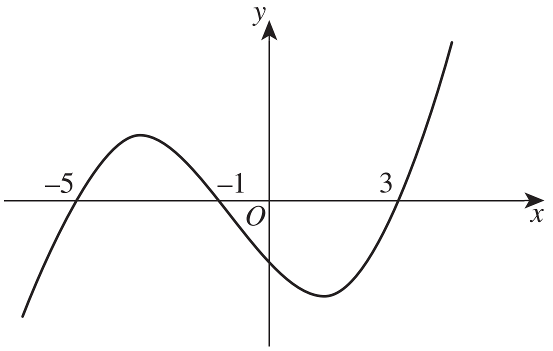
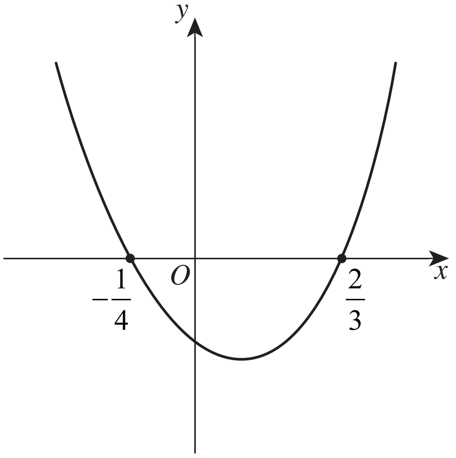
SECTION I

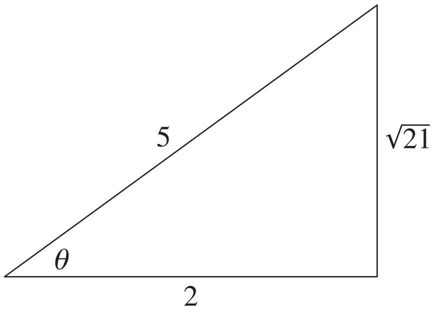
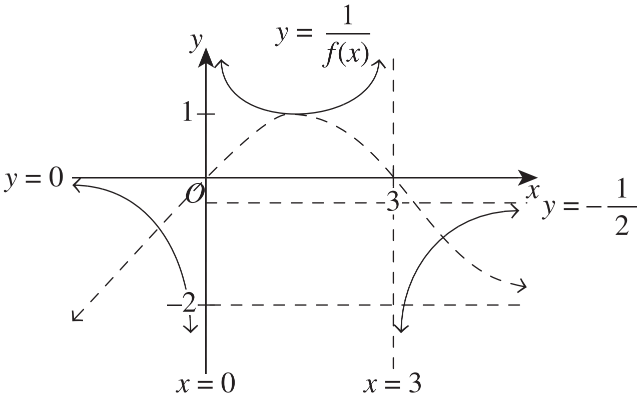
Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 1 A $t \to \infty,\ P = 100 + 300e^{-0.2t} \to 100^+$	ME–C1 Rates of Change ME11–4 Band E2
Question 2 B 1, 2, 3, 4, 5 $4 \times 3 \times 3$ since last digit can only be a 1, 2 or 5. Therefore, there are 36 options.	ME–A1 Working with Combinatorics ME11–5 Band E2
Question 3 C $2\sin\left(x+\frac{\pi}{3}\right) = 2\sin x\cos\frac{\pi}{3} + 2\cos x\sin\frac{\pi}{3}$ $= 2\sin x \times \frac{1}{2} + 2\cos x \times \frac{\sqrt{3}}{2}$ $= \sin x + \sqrt{3}\cos x$	ME–T2 Further Trigonometric Identities ME11–3 Band E2
Question 4 D $y = 3x^3 + 4$ Interchanging x and y, $x = 3y^3 + 4$ $3y^3 = x - 4$ $y = \sqrt[3]{\frac{x-4}{3}}$	ME–F1 Further Work with Functions ME11–2 Band E3
Question 5 B $y^2 = f(x) \Rightarrow y = \pm\sqrt{f(x)}$ y-intercept should be $\pm\sqrt{3} \Rightarrow$ **A** is incorrect. For $2 < x < 3$, y is undefined $\Rightarrow$ **C** is incorrect. x-intercept should be 3 and vertical asymptote at $x = 2$ $\Rightarrow$ **D** is incorrect.	ME–F1 Further Work with Functions ME11–2 Band E2
Question 6 D $\frac{\text{cosec}^2\theta}{1+\tan^2\theta} = \frac{\text{cosec}^2\theta}{\sec^2\theta}$ $= \frac{\cos^2\theta}{\sin^2\theta}$ $= \cot^2\theta$ $= \left(\frac{1-t^2}{2t}\right)^2$ $= \frac{\left(1-t^2\right)^2}{4t^2}$	ME–T2 Further Trigonometric Identities ME11–3 Band E3

Answer and explanation	**Syllabus content, outcomes and targeted performance bands**
Question 7 C $x=\dfrac{t}{1+t} \qquad y=\dfrac{t}{1-t}$ $x\left(1+x\right)=t \qquad y\left(1-y\right)=t$ $t\left(1-x\right)=x \qquad t\left(1+y\right)=y$ $t=\dfrac{x}{1-x} \qquad t=\dfrac{y}{1+y}$ $\dfrac{x}{1-x}=\dfrac{y}{1+y}$ $x\left(1+y\right)=y\left(x-1\right)$ $x+xy=y-xy$ $x+2xy-y=0$	ME–F1 Further Work with Functions ME11–3 Band E3
Question 8 A $f(x)=\cos^{-1}x+\sin^{-1}x+\tan^{-1}x$ Domain: [–1,1] $\cos^{-1}x+\sin^{-1}x=\dfrac{\pi}{2}$ $f(x)=\dfrac{\pi}{2}+\tan^{-1}x$ $x=-1$ $f(-1)=\dfrac{\pi}{2}+\tan^{-1}(-1)=\dfrac{\pi}{4}$ $x=1$ $f(1)=\dfrac{\pi}{2}+\tan^{-1}(1)=\dfrac{3\pi}{4}$ $\therefore \dfrac{\pi}{4}\le y\le\dfrac{3\pi}{4}$	ME–T1 Inverse Trigonometric Functions ME11–3 Band E3
Question 9 D 1, 3, 5, 7, …, 97, 99 (3, 99), (5, 97), (7, 95), …, (49, 53) $\Rightarrow$ 24 pairs + number 1 + middle number (51) + extra = 24 + 1 + 1 + 1 = 27	ME–A1 Working with Combinatorics ME11–5 Bands E3–E4
Question 10 D $\left(1+x\right)^{10}=\binom{10}{0}+\binom{10}{1}x+\binom{10}{2}x^2+...+\binom{10}{10}x^{10}$ Let $x = 3$. $4^{10}=\binom{10}{0}+3\binom{10}{1}+3^2\binom{10}{2}+3^3\binom{10}{3}+...+3^{10}\binom{10}{10}$ $4^{10}-1=3\binom{10}{1}+3^2\binom{10}{2}+3^3\binom{10}{3}+...+3^{10}\binom{10}{10}$	ME–A1 Working with Combinatorics ME11–5 Band E4

SECTION II

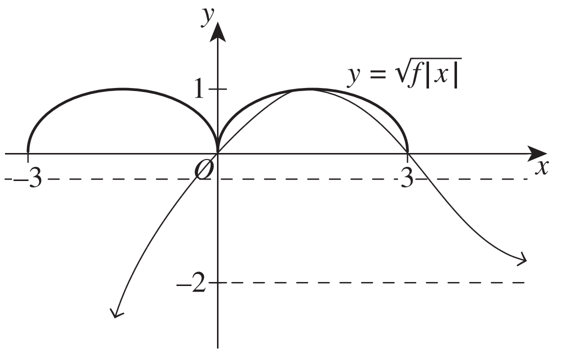
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 11	
(a) (i) Let $P(x)=x^3+3x^2-13x-15.$ $P(-1)=(-1)^3+3(-1)^2-13(-1)-15$ $=-1+3+13-15$ $=0$ $\therefore x+1$ is a factor of $P(x)$. $P(x)=x^3+3x^2-13x-15$ $=(x+1)(x^2+2x-15)$ (By inspection) $=(x+1)(x+5)(x-3)$	ME–F2 Polynomials ME11–2 Band E2 • Gives the correct solution .... 2 • Attempts to factorise the polynomial expression using the root $x=-1,-5$ or 3 OR equivalent merit 1
(ii)  $x^3+3x^2-13x-15>0$ $-5<x<-1, x>3 \left(\text{or } (-5,-1)\cup(3,\infty)\right)$	ME–F1 Further Work with Functions ME11–2 Bands E2–E3 • Gives the correct solution .... 2 • Attempts to apply result from part (a)(i) OR equivalent merit 1
(b) $\left(\frac{2x-5}{3x-2}\le 2\right)(3x-2)^2, x\neq\frac{2}{3}$ $(2x-5)(3x-2)\le 2(3x-2)^2$ $2(3x-2)^2-(2x-5)(3x-2)\ge 0$ $(3x-2)\left[2(3x-2)-(2x-5)\right]\ge 0$ $(3x-2)(4x+1)\ge 0$  $x\le-\frac{1}{4}, x>\frac{2}{3}$ since $x\neq\frac{2}{3}$	ME–F1 Further Work with Functions ME11–4 Bands E2–E3 • Gives the correct solution .... 3 • Gives the solution but concludes $x\le-\frac{1}{4}, x\ge\frac{2}{3}$ ...... 2 • Shows some understanding of the problem 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) $^{8}C_k = 2\times {}^{7}C_k$ $\dfrac{8!}{(8-k)!k!} = 2\times\dfrac{7!}{(7-k)!k!}$ $\dfrac{8\times 7!}{(8-k)(7-k)!k!} = 2\times\dfrac{7!}{(7-k)!k!}$ $4 = 8-k$ $k = 4$	ME–A1 Working with Combinatorics ME11–5 Bands E2–E3 • Gives the correct solution .... 2 • Shows some understanding of the problem 1
(d) $\left(2+\dfrac{x}{4}\right)^6 = \dbinom{6}{0}2^6 + \dbinom{6}{1}2^5\left(\dfrac{x}{4}\right) +$ $\dbinom{6}{2}2^4\left(\dfrac{x}{4}\right)^2 + \dbinom{6}{3}2^3\left(\dfrac{x}{4}\right)^3 +$ $\dbinom{6}{4}2^2\left(\dfrac{x}{4}\right)^4 + \dbinom{6}{5}2^1\left(\dfrac{x}{4}\right)^5 + \dbinom{6}{6}\left(\dfrac{x}{4}\right)^6$ The coefficient of $x^4 = \dbinom{6}{4}2^2\left(\dfrac{1}{4}\right)^4$. $\dbinom{6}{4}2^2\left(\dfrac{1}{4}\right)^4 = 15\times 2^2\times\dfrac{1}{2^8}$ $= \dfrac{15}{64}$	ME–A1 Working with Combinatorics ME11–5 Band E3 • Gives the correct solution .... 2 • Uses the binomial theorem OR equivalent merit 1
(e) Prove $\tan\theta\tan\dfrac{\theta}{2} = \sec\theta - 1$. Let $t = \tan\dfrac{\theta}{2}, \tan\theta = \dfrac{2t}{1-t^2}, \sec\theta = \dfrac{1+t^2}{1-t^2}$. $\text{LHS} = \tan\theta\tan\dfrac{\theta}{2}$ $= \dfrac{2t}{1-t^2}\times t$ $= \dfrac{2t^2}{1-t^2}$ $\text{RHS} = \sec\theta - 1$ $= \dfrac{1+t^2}{1-t^2} - 1$ $= \dfrac{1+t^2-(1-t^2)}{1-t^2}$ $= \dfrac{2t^2}{1-t^2}$ $\therefore \text{LHS} = \text{RHS}$	ME–T2 Further Trigonometric Identities ME11–3 Bands E2–E3 • Gives the correct solution .... 2 • Correctly substitutes to form an expression in terms of *t* OR equivalent merit.................. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(f) $\sin\left(2\cos^{-1}\left(-\frac{2}{5}\right)\right)$ Let $\theta=\cos^{-1}\left(-\frac{2}{5}\right)$. $\cos\theta=-\frac{2}{5}$ (second quadrant)  $\sin\theta=-\frac{\sqrt{21}}{5}$ $\sin 2\theta=2\sin\theta\cos\theta$ $\sin\left(2\cos^{-1}\left(-\frac{2}{5}\right)\right)=2\times\frac{\sqrt{21}}{5}\times\left(-\frac{2}{5}\right)$ $=-\frac{4\sqrt{21}}{25}$	ME–T1 Inverse Trigonometric Functions ME11–3 Band E3 • Gives the correct solution . . . . 2 • Shows some understanding of the problem 1
Question 12	
(a) (i) 	ME–F1 Further Work with Functions ME11–2 Band E2 • Gives the correct sketch. . . . . 2 • Gives some correct features of the graph OR equivalent merit 1

<table>
<tr><th>Sample answer</th><th>Syllabus content, outcomes, targeted performance bands and marking guide</th></tr>
<tr><td>

(ii)



</td><td>

ME–F1 Further Work with Functions
ME11–2 Band E3

- Gives the correct sketch........... 2

- Graphs a square-root graph based on the given graph, or $y = f|x|$ based on the given graph.

OR

- Gives some correct features of the graph OR equivalent merit.............. 1

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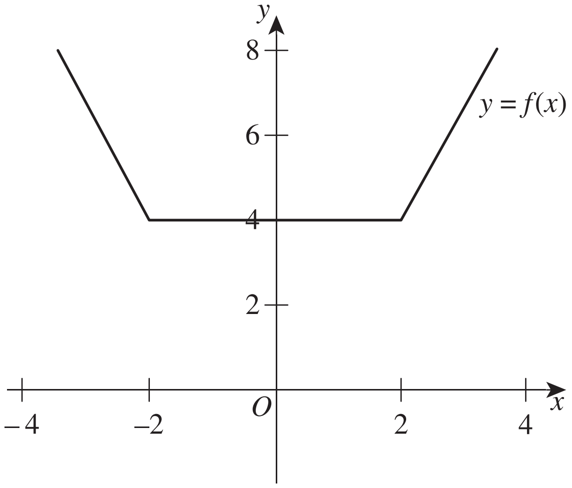
<tr><td>

(b) (i) The graph of $f(x) = |x+2| + |x-2|$:



$$|x+2| = \begin{cases} x+2, x \geq -2 \\ -x-2, x < -2 \end{cases}$$

$$|x-2| = \begin{cases} x-2, x \geq 2 \\ -x+2, x < 2 \end{cases}$$

$$f(x) = \begin{cases} 2x, x \geq 2 \\ 4, -2 \leq x \leq 2 \\ -2x, x < -2 \end{cases}$$

</td><td>

ME–F1 Further Work with Functions
ME11–2 Bands E2–E3

- Gives the correct sketch........... 2

- Gives some correct features of the graph OR equivalent merit.............. 1

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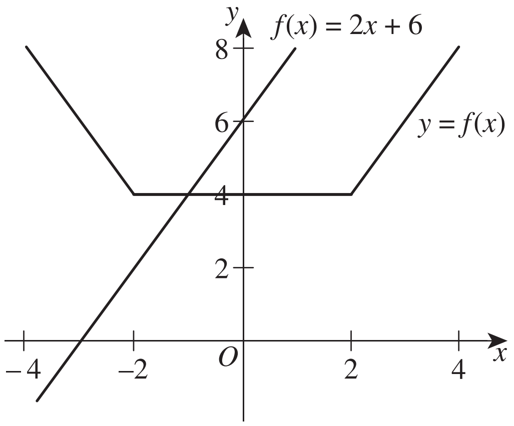
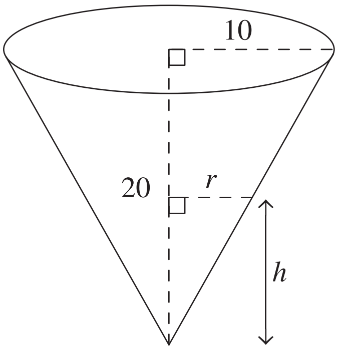
</table>

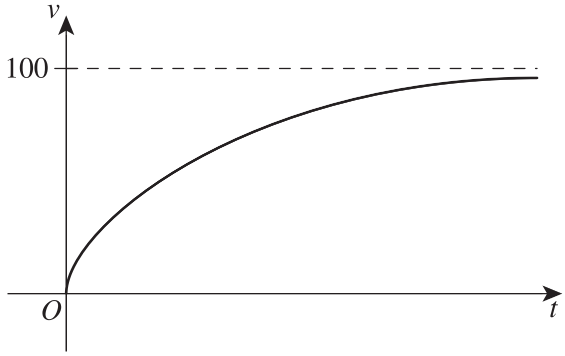
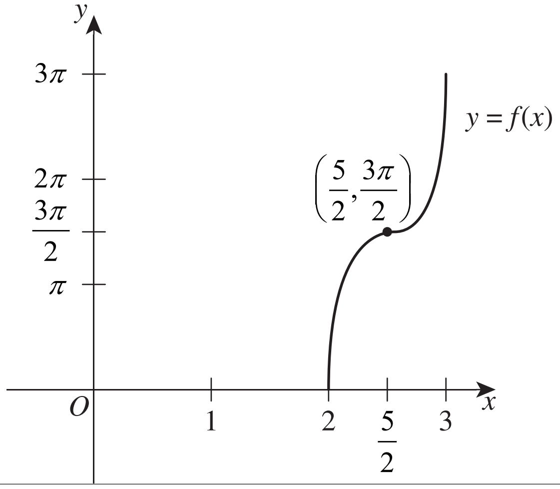
Sample answer	**Syllabus content, outcomes, targeted performance bands and marking guide**
(ii) 	ME–F1 Further Work with Functions ME11–2 Band E2 • Gives the correct sketch. 1
(iii) For $f(x) = h(x)$, $2x + 6 = 4$ $x = -1$ $f(x) \ge h(x) \Rightarrow x \le -1$	ME–F1 Further Work with Functions ME11–3 Bands E2–E3 • Gives the correct solution 1
(c)  Given: $\frac{dV}{dt} = 1$ cm^3/s $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$ $= 1 \times \frac{4}{\pi h^2}$ $h = 2,\ \frac{dh}{dt} = \frac{4}{\pi 2^2}$ $= \frac{1}{\pi}$ $V = \frac{\pi}{3} r^2 h$ Similar triangles $\Rightarrow \frac{r}{h} = \frac{10}{20}$ $r = \frac{h}{2}$ $V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2 h$ $= \frac{\pi h^3}{12}$ $\frac{dV}{dh} = \frac{\pi h^2}{4}$ $\therefore$ The sand level is rising at $\frac{1}{\pi}$ cm/s when it is 2 cm deep.	ME–C1 Rates of Change ME11–4 Band E3 • Gives the correct solution . . . . 3 • Makes some correct use of the chain rule to attempt to find the rate of change. 2 • Establishes that $V = \frac{\pi h^3}{12}$ OR equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(d) $\tan^{-1}x=\tan^{-1}\left(\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{3}\right)$ $x=\tan\left(\tan^{-1}\left(\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{3}\right)\right)$ $=\dfrac{\tan\left(\tan^{-1}\left(\frac{1}{2}\right)\right)-\tan\left(\tan^{-1}\left(\frac{1}{3}\right)\right)}{1+\tan\left(\tan^{-1}\left(\frac{1}{2}\right)\right)\times\tan\left(\tan^{-1}\left(\frac{1}{3}\right)\right)}$ $=\dfrac{\frac{1}{2}-\frac{1}{3}}{1+\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}$ $=\dfrac{\frac{1}{6}}{\frac{7}{6}}$ $=\dfrac{1}{7}$	ME–F1 Further Work with Functions ME–T2 Further Trigonometric Identities ME11–2, 11–3 Band E3 • Gives the correct solution .... 2 • Shows some understanding of the problem 1
(e) Total arrangement = $\dfrac{8!}{2!}$ Number of arrangements when all the vowels are together: $\dfrac{3!6!}{2!}$ $\therefore$ Number of arrangements when all the vowels are not together: $\dfrac{8!}{2!}-\dfrac{3!6!}{2!}=18\,000$	ME–A1 Working with Combinatorics ME11–5 Band E3 • Gives the correct solution .... 2 • Finds the number of arrangements where all the vowels are together OR equivalent merit.................. 1

Sample answer	**Syllabus content, outcomes, targeted performance bands and marking guide**
Question 13	
(a) Let $P(x)=x^4-3x^3-6x^2+ax+b$ $P'(x)=4x^3-9x^2-12x+a$ $P''(x)=12x^2-18x-12=0$ (triple root) $2x^2-3x-2=0$ $(2x+1)(x-2)=0$ $x=-\frac{1}{2},\ x=2$ ∴ $x = 2$ is the triple root since $P(x)$ is MONIC. $P'(2)=0 \Rightarrow 4(2)^3-9(2)^2-12(2)+a=0$ $-28+a=0$ $a=28$ $P(2) \Rightarrow 2^4-3(2)^3-6(2)^2+2a+b=0$ $-32+2a+b=0$ Substitute $a=28 \Rightarrow -32+2(28)+b=0$ $b=-24$	ME–F2 Polynomials ME11–2 Bands E2–E3 • Gives the correct solution .... 3 • Obtains a correct value for *a* or *b* OR equivalent merit .... 2 • Obtains ONE correct equation OR equivalent merit 1
(b) $4x^3+7x^2+kx+24=0$ Let $\alpha, \frac{1}{\alpha}$ and β be the roots. $\alpha\left(\frac{1}{\alpha}\right)\beta=-6$ $\beta=-6$ $\alpha+\frac{1}{\alpha}+\beta=-\frac{7}{4}$ Substitute $\beta=-6 \Rightarrow \alpha+\frac{1}{\alpha}-6=-\frac{7}{4}$ $\alpha+\frac{1}{\alpha}-\frac{17}{4}=0$ $4\alpha^2-17\alpha+4=0$ $(4\alpha-1)(\alpha-4)=0$ $\therefore \alpha=\frac{1}{4}$ or $\alpha=4$ Hence, the three roots are $\frac{1}{4}$, 4 and -6. $\left(\frac{1}{4}\right)(4)+\left(\frac{1}{4}\right)(-6)+4(-6)=\frac{k}{4}$ $-24\frac{1}{2}=\frac{k}{4}$ $k=-98$	ME–F2 Polynomials ME11–2 Bands E2–E3 • Gives the correct solution .... 3 • Obtains THREE correct roots. OR • Obtains ONE correct root AND *k* value. OR • Equivalent merit .... 2 • Obtains ONE correct root. OR • Obtains TWO correct equations. OR • Equivalent merit 1

<table>
<tr><th>Sample answer</th><th>Syllabus content, outcomes, targeted performance bands and marking guide</th></tr>
<tr><td>(c) (i) The inverse of $f(x)$ is NOT a function since $f(x)$ is NOT a one-to-one function.</td><td>ME–F1 Further Work with Functions
ME11–2 Bands E2–E3
• Gives the correct solution .... 1</td></tr>
<tr><td>(ii) $$\begin{aligned} f(x) &= x^2 - 6x + 10 \\ &= x^2 - 6x + 9 + 1 \\ &= (x-3)^2 + 1 \end{aligned}$$ The largest domain of $f(x)$ containing $x = 0$ for which $f^{-1}(x)$ exists is $x \le 3$ or $(-\infty, 3]$.</td><td>ME–F1 Further Work with Functions
ME11–2 Bands E2–E3
• Gives the correct solution .... 1</td></tr>
<tr><td>(iii) Let $y = (x-3)^2 + 1$.
Domain $f(x)$: $(-\infty, 3] \Rightarrow$ Range $f^{-1}(x) : (-\infty, 3]$
Range $f(x)$: $[1, \infty) \Rightarrow$ Domain $f^{-1}(x) : [1, \infty)$
Interchanging *x* and *y*: $$\begin{aligned} x &= (y-3)^2 + 1 \\ (y-3)^2 &= x - 1 \\ y &= 3 \pm \sqrt{x-1} \end{aligned}$$ $\therefore f^{-1}(x) = 3 - \sqrt{x-1}$ since range $f^{-1}(x) : (-\infty, 3]$</td><td>ME–F1 Further Work with Functions
ME11–2 Bands E2–E3
• Gives the correct solution 2
• Shows some understanding of the problem .... 1</td></tr>
<tr><td>(iv) For $f(x) = f^{-1}(x)$
$\Rightarrow f(x) = f^{-1}(x) = x$
So $x^2 - 6x + 10 = x$ $$\begin{aligned} x^2 - 7x + 10 &= 0 \\ (x-2)(x-5) &= 0 \\ x &= 2, 5 \end{aligned}$$ $\Rightarrow x = 2$ since $x \le 3$
$\therefore$ The point of intersection is $(2, 2)$.</td><td>ME–F1 Further Work with Functions
ME11–2 Bands E2–E3
• Gives the correct solution .... 1</td></tr>
<tr><td>(d) 13 cards from 52
Combinations:
<table>
<tr><th>Aces</th><th>Kings</th><th>Others</th></tr>
<tr><td>3</td><td>3</td><td>7</td></tr>
<tr><td>4</td><td>3</td><td>6</td></tr>
<tr><td>3</td><td>4</td><td>6</td></tr>
<tr><td>4</td><td>4</td><td>5</td></tr>
</table>
$$\begin{aligned} \text{Number of ways} &= \binom{4}{3}\binom{4}{3}\binom{44}{7} + 2 \times \binom{4}{3}\binom{4}{4}\binom{44}{6} + \\ &\quad \binom{4}{4}\binom{4}{4}\binom{44}{5} \\ &= 670\,687\,512 \end{aligned}$$</td><td>ME–A1 Working with Combinatorics
ME11–5 Band E3
• Gives the correct solution 2
• Obtains at least TWO correct combinations OR equivalent merit .... 1</td></tr>
</table>

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(e) Prove $\sin 40° + \cos 70° = \cos 10°$. $\text{LHS} = \sin 40° + \cos 70°$ $= \sin 40° + \sin 20°$ $= 2\sin\left(\frac{40° + 20°}{2}\right)\cos\left(\frac{40° - 20°}{2}\right)$ $= 2\sin 30° \cos 10°$ $= 2 \times \frac{1}{2} \times \cos 10°$ $= \cos 10°$ $= \text{RHS}$	ME–T2 Further Trigonometric Identities ME11–3 Band E3 • Gives the correct solution .... 2 • Uses the sum to produce identities 1
Question 14	
(a) (i) $v = 100 - 100e^{-kt}$ $\frac{dv}{dt} = -100e^{-kt} \times -k$ (chain rule) $= -k(100 - 100e^{-kt} - 100)$ $= -k(v - 100)$ $\therefore\ v = 100 - 100e^{-kt}$ is a possible equation to this differential equation.	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Gives the correct proof............ 1
(ii) $v = 100 - 100e^{-kt}$ $t = 10$ s, $v = 40$ m/s $\Rightarrow 40 = 100 - 100e^{-10k}$ $100e^{-10k} = 60$ $e^{-10k} = 0.6$ $k = \frac{\ln(0.6)}{-10}$ $= 0.0511$ (correct to two decimal places)	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Gives the correct solution 1
(iii) $v = 100 - 100e^{-0.0511t}$ $t = 25$ s $\Rightarrow v = 100 - 100e^{-0.0511(25)}$ $= 72.13$ m/s (correct to four decimal places)	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Gives the correct solution 1
(iv) $t \to \infty,\ 100e^{-0.0511t} \to 0$ $v = 100 - 100e^{-0.0511t} \to 100^{-}$ $\therefore v = 100$ m/s	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Gives the correct solution 1

Sample answer	**Syllabus content, outcomes, targeted performance bands and marking guide**
(v) 	ME–C1 Rates of Change ME11–4 Bands E2–E3 • Gives the correct sketch........... 1
(b) $f(x) = 3\cos^{-1}(5-2x)$ Domain: $-1 \le 5-2x \le 1$ $-6 \le -2x \le -4$ $3 \ge x \ge 2$ $\therefore 2 \le x \le 3$ Range: $0 \le \frac{y}{3} \le \pi$ $\therefore 0 \le y \le 3\pi$ When $x = 2$, $f(2) = 3\cos^{-1}(1) = 0$ 	ME–T1 Inverse Trigonometric Functions ME11–3 Bands E2–E3 • Gives the correct solution .... 3 • Obtains correct domain and range OR equivalent merit.............. 2 • Obtains correct domain and range 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) $(1+x)^{15} = (1+x)^{12}(1+x)^3$ $\text{LHS} = (1+x)^{15}$ $= \binom{15}{0} + \binom{15}{1}x + \binom{15}{2}x^2 +$ $\binom{15}{3}x^3 + \binom{15}{4}x^4 + \ldots + \binom{15}{15}x^{15}$ $\binom{15}{4}$ is the coefficient of x^4. $\text{RHS} = (1+x)^{12}(1+x)^3$ $(1+x)^{12} = \binom{12}{0} + \binom{12}{1}x + \binom{12}{2}x^2 +$ $\binom{12}{3}x^3 + \binom{12}{4}x^4 + \ldots + \binom{12}{15}x^{15}$ $(1+x)^3 = \binom{3}{0} + \binom{3}{1}x + \binom{3}{2}x^2 + \binom{3}{3}x^3$ Coefficient of $x^4 = \binom{3}{0}\binom{12}{4} + \binom{3}{1}\binom{12}{3} +$ $\binom{3}{2}\binom{12}{2} + \binom{3}{3}\binom{12}{1}$ $\therefore \binom{15}{4} = \binom{12}{4} + \binom{3}{1}\binom{12}{3} + \binom{3}{2}\binom{12}{2} + \binom{12}{1}$	ME–A1 Working with Combinatorics ME11–5 Bands E3–E4 • Gives the correct solution .... 3 • Makes substantial progress towards finding the coefficient of x^4 in the expansion of $(1+x)^{12}(1+x)^3$ OR equivalent merit.................. 2 • Gives correct explanation that $\binom{15}{4}$ is the coefficient of x^4 1
(d) (i) Prove $\cos 3\theta = 4\cos^3\theta - 3\cos\theta.$ $\cos 3\theta = \cos(2\theta + \theta)$ $= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$ $= (2\cos^2\theta - 1)\cos\theta - (2\sin\theta\cos\theta)\sin\theta$ $= 2\cos^3\theta - \cos\theta - 2\cos\theta\sin^2\theta$ $= 2\cos^3\theta - \cos\theta - 2\cos\theta(1-\cos^2\theta)$ $= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$ $= 4\cos^3\theta - 3\cos\theta$	ME–T1 Inverse Trigonometric Functions ME11–3 Band E3 • Gives the correct solution .... 2 • Makes substantial progress in expanding $\cos 3\theta$ 1

Sample answer	**Syllabus content, outcomes, targeted performance bands and marking guide**
(ii) $\sin 2\theta = \cos(90^\circ - 2\theta)$ Let $\theta = 18^\circ$. $\Rightarrow \sin\left(2\times 18^\circ\right) = \cos\left(90^\circ - \left(2\times 18^\circ\right)\right)$ $= \cos\left(3\times 18^\circ\right)$ $2\sin 18^\circ \cos 18^\circ = 4\cos^3 18^\circ - 3\cos 18^\circ$ (from part (d)(i)) $2\sin 18^\circ = 4\cos^2 18^\circ - 3$ $2\sin 18^\circ = 4\left(1 - \sin^2 18^\circ\right) - 3$ $4\sin^2 18^\circ + 2\sin 18^\circ - 1 = 0$ $\sin 18^\circ = \dfrac{-2 \pm \sqrt{2^2 - 4(4)(-1)}}{2(4)}$ $= \dfrac{-2 \pm \sqrt{20}}{8}$ $= \dfrac{-2 \pm 2\sqrt{5}}{8}$ $= \dfrac{-1 \pm \sqrt{5}}{4}$ $\therefore \sin 18^\circ = \dfrac{\sqrt{5} - 1}{4}$ since $\sin 18^\circ > 0$ as it is in the first quadrant.	ME–T1 Inverse Trigonometric Functions ME11–3 Band E4 - Gives the correct solution .... 2 - Attempts to apply answer from part (d)(i) OR equivalent merit 1