Neap

Final Examination 2022

NSW Year 11 Mathematics Advanced

Solutions and Marking Guidelines

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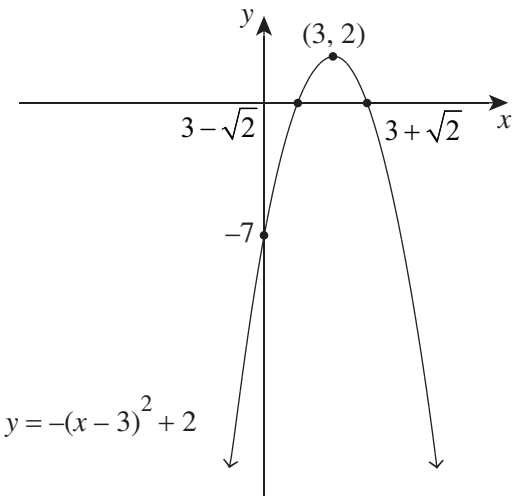
SECTION I

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 1 **B** **B** is correct. This graph represents a one-to-many relationship. **A** is incorrect. This graph represents a one-to-one relationship. **C** and **D** are incorrect. These graphs both represent many-to-one relationships.	MA–F1 Working with Functions MA11–2 Bands 2–3
Question 2 **B** $(x-2)(x+3)=0$ $x^2+x-6=0$	MA–F1 Working with Functions MA11–2 Bands 2–3
Question 3 **C** **C** is correct. This Venn diagram represents $A \cup \overline{B}$. **A** is incorrect. This Venn diagram represents A only; that is, $A \cap \overline{B}$. **B** is incorrect. This Venn diagram represents not B; that is, $\overline{B}$. **D** is incorrect. This Venn diagram represents $\overline{A} \cap \overline{B}$.	MA–S1 Probability and Discrete Probability Distributions MA11–7 Bands 3–4
Question 4 **C** **C** is correct. $\frac{\sin A}{15}=\frac{\sin 53°}{25}$ $\sin A=\frac{15\sin 53°}{25}$ $\angle A=28°37'55''$ $\approx 29°$ $\angle C=180°-(53°+29°)$ $\approx 98°$ $\frac{AB}{\sin 98°}=\frac{25}{\sin 53°}$ $AB=\frac{25\sin 98°}{\sin 53°}$ $=30.9987\ldots$ ≈ 31 cm perimeter $\approx 25+15+31$ ≈ 71 cm **A** is incorrect. The longest side is opposite the largest angle. Therefore, AB is the longest side as C is the largest angle. **B** is incorrect. The triangle is an obtuse-angled triangle. **D** is incorrect. area $\approx \frac{1}{2}\times 31\times 15\times \sin 53°$ ≈ 185.7 cm^2	MA–T1 Trigonometry and Measure of Angles MA11–3 Bands 4–5

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 5 D Substituting (1, 15) into the equation gives: $15 = ka \quad (1)$ Substituting (2, 90) into the equation gives: $90 = ka^2 \quad (2)$ Substituting (1) into (2) gives: $90 = 15a$ $a = 6$ Substituting $a = 6$ into (1) gives: $6k = 15$ $k = \frac{5}{2}$ $= 2.5$	MA–F1 Working with Functions MA11–1, 11–2 Bands 3–4
Question 6 A If A and B are mutually exclusive, then $P(A) + P(B) \leq 1$. As $P(B) + P(\overline{B}) = 1$, $P(B) = 1 - P(\overline{B})$. $P(A) + 1 - P(\overline{B}) \leq 1$ $P(A) - P(\overline{B}) \leq 0$ $P(A) \leq P(\overline{B})$	MA–S1 Probability and Discrete Probability Distributions MA11–7, 11–9 Bands 4–5
Question 7 C **C** is correct. For odd functions, the points need to have point symmetry about the origin. Option **C** has point symmetry about the origin and satisfies $f(x) = f(-x)$. Therefore, it represents an odd function. **A** and **D** are incorrect. These options do not have point symmetry about the origin. **B** is incorrect. This option is symmetrical about the y-axis and satisfies $f(x) = f(-x)$. Therefore, it represents an even function.	MA–F1 Working with Functions MA11–2, 11–9 Bands 4–5
Question 8 C As this is an upper half semicircle with its centre at the origin and a radius of 4 units, the equation of the semicircle is $y = \sqrt{16 - x^2}$.	MA–F1 Working with Functions MA11–1, 11–2 Bands 2–3

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 9 **C** **C** is not a true statement and is therefore the required response. $\Delta = (-8)^2 - 4 \times 2 \times 6$ $= 16$ Δ is a perfect square, so there are two real solutions; displacement is zero at these times. **A** is a true statement and is therefore not the required response. As the equation of displacement produces a concave up parabola, displacement increases after 2 seconds. At times, displacement is not only zero but also negative (between the two real roots). **B** is a true statement and is therefore not the required response. The particle's displacement is modelled using a concave up quadratic function, so it has a minimum turning point. Therefore, the particle changes direction once. The axis of symmetry when $t = -\frac{-8}{2 \times 2}$ is 2. **D** is a true statement and is therefore not the required response. $s = 2t^2 - 8t + 6$ $v = 4t - 8$ $a = 4$	MA–C1 Introduction to Differentiation MA11–5 Bands 4–5
Question 10 **D** $2x + 3y + 4 = 0$ $3y = -2x - 4$ $y = -\frac{2}{3}x - \frac{4}{3}$ $m = \tan\theta$ $\tan\theta = -\frac{2}{3}$ $\theta \approx -33.69$ $\approx 146°$	MA–F1 Working with Functions MA11–2 Bands 3–4

SECTION II

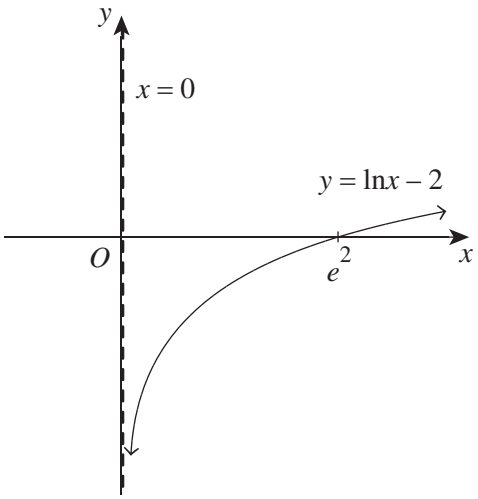
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 11	
$\frac{x}{2}+4=2-3x$ $x+8=4-6x$ $7x=-4$ $x=-\frac{4}{7}$	MA–F1 Working with Functions MA11–2 Bands 2–3 • Provides the correct solution . . . . . 2 • Shows progress towards eliminating the denominator OR equivalent merit. 1
Question 12	
$y=-(x-3)^2+2$ is in vertex form with the following features. • concave down parabola • vertex at (3, 2) Finding the x-intercepts using $y=0$: $-(x-3)^2+2=0$ $(x-3)^2=2$ $x-3=\pm\sqrt{2}$ $x=3\pm\sqrt{2}$ $\approx 4.41, 1.59$ Finding the y-intercept using $x=0$: $y=-(0-3)^2+2$ $=-7$  	MA–F1 Working with Functions MA11–2 Bands 3–4 • Determines the vertex. AND • Determines the x-intercepts. AND • Sketches the correct graph that shows the: – vertex – x- and y-intercepts . . . . . 3 • Any TWO of the above points. . . . . 2 • Any ONE of the above points 1

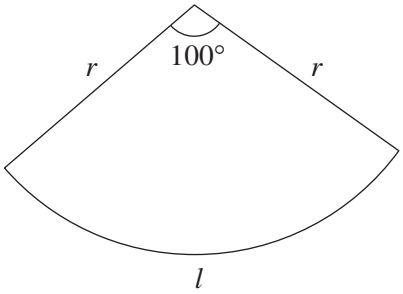
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 13	
When $x = 1$: $6(1) - 8y + 15 = 0$ $8y = 21$ $y = \frac{21}{8}$ Therefore, the point has the coordinates $\left(1, \frac{21}{8}\right)$. $6x - 8y + 15 = 0$ $8y = 6x + 15$ $y = \frac{3}{4}x + \frac{15}{8}$ $m_1 = \frac{3}{4}$ $m_2 = \frac{-1}{\frac{3}{4}}$ $= -\frac{4}{3}$ Finding the equation of the line using $m = -\frac{4}{3}$ and the point $\left(1, \frac{21}{8}\right)$: $y - \frac{21}{8} = -\frac{4}{3}(x - 1)$ Multiplying both sides by the lowest common multiple of 24: $24y - 63 = -32(x - 1)$ $24y - 63 = -32x + 32$ $32x + 24y - 95 = 0$	MA–F1 Working with Functions MA11–2 Bands 3–4 • Provides the correct solution . . . . . 3 --- • Finds the coordinates of the point of intersection. AND • Calculates the gradient of the line . . 2 --- • Finds the coordinates of the point of intersection. OR • Calculates the gradient of the line . . 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 14	
$y = x^2 - 2x \quad (1)$ $y = 6x - x^2 \quad (2)$ Subtracting (2) from (1) gives: $0 = 2x^2 - 8x$ $2x(x-4) = 0$ $x = 0,\ 4$ When $x = 0$: $y = 0^2 - 2(0)$ $= 0$ When $x = 4$: $y = 4^2 - 2(4)$ $= 8$	MA–F1 Working with Functions MA11–2 Bands 3–4 • Provides the correct solution . . . . . 3 • Provides the x-coordinates. AND • Provides ONE of the y-coordinates . . . . . 2 • Provides the x-coordinates. 1
Question 15	
(a) $\begin{array}{c\|c} S\checkmark & A \\ \hline T\checkmark & C \end{array}$ $2\cos\theta = -\sqrt{3}$ $\cos\theta = -\frac{\sqrt{3}}{2}$ related angle $= \frac{\pi}{6}$ $\theta = \pi - \frac{\pi}{6}, -\pi + \frac{\pi}{6}$ $= \frac{5\pi}{6}, -\frac{5\pi}{6}$	MA–T2 Trigonometric Functions and Identities MA11–4 Bands 4–5 • Provides the correct solution in radians . . . . . 2 • Calculates the correct related acute angle OR one of the correct solutions in radians 1
(b) $y = \tan x$ is undefined for: $x = 90°,\ 270°,\ 360° + 90°,\ 720° - 90°$ $= 90°,\ 270°,\ 450°,\ 630°$	MA–T2 Trigonometric Functions and Identities MA11–1, 11–4 Bands 4–5 • Provides the correct solution in degrees. . . . . 2 • Provides any TWO values of x in degrees 1

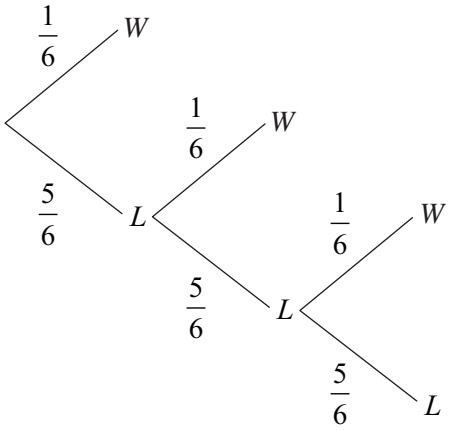
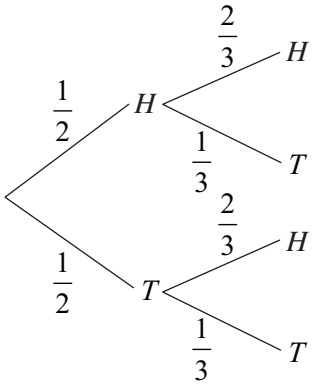
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 16	
$\frac{d}{dx}\left(\left(x^2-5\right)\left(9x-2x^3\right)\right)=\frac{d}{dx}\left(9x^3-2x^5-45x+10x^3\right)$ $=27x^2-10x^4-45+30x^2$ $=-10x^4+57x^2-45$	MA–C1 Introduction to Differentiation MA11–5 Bands 3–4 • Provides the correct solution . . . . . 2 • Attempts to use the product rule. OR • Expands the binomial product 1
Question 17	
$f(x+h)=(x+h)^2+5(x+h)$ $=x^2+2xh+h^2+5x+5h$ $f'(x)=\lim_{h\to 0}\frac{x^2+2xh+h^2+5x+5h-\left(x^2+5x\right)}{h}$ $=\lim_{h\to 0}\frac{2xh+h^2+5h}{h}$ $=\lim_{h\to 0}\frac{h(2x+h+5)}{h}$ $=\lim_{h\to 0}2x+h+5$ $=2x+5$	MA–C1 Introduction to Differentiation MA11–5 Bands 4–5 • Provides the correct solution . . . . . 2 • Substitutes into the definition OR equivalent merit. 1
Question 18	
$\frac{dy}{dx}=5\left(2x^2-3\right)^4\times 4x$ $=20x\left(2x^2-3\right)^4$	MA–C1 Introduction to Differentiation MA11–5 Bands 3–4 • Provides the correct solution . . . . . 2 • Substitutes into the definition OR equivalent merit. 1

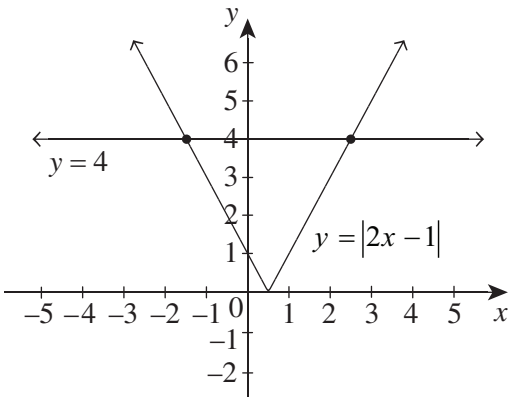
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 19	
$\text{LHS} = \dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta}$ $= \dfrac{\sin^2\theta + (1+\cos\theta)^2}{\sin\theta(1+\cos\theta)}$ $= \dfrac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{\sin\theta(1+\cos\theta)}$ $= \dfrac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}$ $= \dfrac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$ $= \dfrac{2}{\sin\theta}$ $= 2\text{cosec}\,\theta$ $= \text{RHS}$ $\therefore \dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = 2\text{cosec}\,\theta$	MA–T2 Trigonometric Functions and Identities MA11–4 Bands 4–5 • Provides the correct solution . . . . . 3 • Shows substantial progress by simplifying the numerator . . . . . 2 • Writes the expression under a common denominator 1
Question 20	
$u = 5x + 2$ $\dfrac{du}{dx} = 5$ $v = 3x + 1$ $\dfrac{dv}{dx} = 3$ $\dfrac{dy}{dx} = \dfrac{5(3x+1) - 3(5x+2)}{(3x+1)^2}$ $= \dfrac{15x + 5 - 15x - 6}{(3x+1)^2}$ $= \dfrac{-1}{(3x+1)^2}$ When $x = 1$: $\dfrac{dy}{dx} = \dfrac{-1}{(3\times 1+1)^2}$ $= -\dfrac{1}{16}$	MA–C1 Introduction to Differentiation MA11–5 Bands 4–5 • Provides the correct solution . . . . . 3 • Finds the derivative . . . . . 2 • Attempts to use the quotient rule OR equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 21	
$y = \ln x - 2$ is the graph of $y = \ln x$ translated two units down. Finding the x-intercept using $y = 0$: $\ln x - 2 = 0$ $\ln x = 2$ $x = e^2$  	MA–E1 Logarithms and Exponentials MA11–6 Bands 3–4 • Sketches the correct graph that shows the: – x-intercept – vertical asymptote. . . . . 2 • Determines the x-intercept. 1

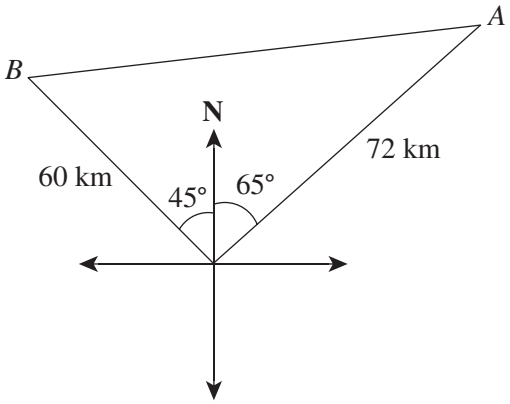
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 22	
 $l + 2r = 120$ $\frac{100°}{360°} \times 2\pi r + 2r = 120$ $\frac{5\pi r}{9} + 2r = 120$ $r\left(\frac{5\pi}{9} + 2\right) = 120$ $r = \frac{120}{\frac{5\pi}{9} + 2}$ $= \frac{1080}{5\pi + 18}$ cm $A = \frac{5}{18} \times \pi \times \left(\frac{1080}{5\pi + 18}\right)^2$ $= 895.8387\ldots$ cm^2 ≈ 896 cm^2	MA–T1 Trigonometry and Measure of Angles MA11–3 Bands 4–5 • Provides the correct solution . . . . . 4 • Shows progress towards calculating the area . . . . . 3 • Calculates the radius . . . . . 2 • Derives an equation for the perimeter in terms of r. 1
Question 23	
(a) 90 minutes = 1.5 hours When $t = 1.5$: $P = 1000e^{-0.15 \times 1.5} + 300e^{0.04 \times 1.5}$ $= 1117$ microbes	MA–E1 Logarithms and Exponentials MA11–6 Bands 2–3 • Provides the correct solution . . . . . 2 • Substitutes $t = 1.5$ into the expression 1
(b) $\frac{dP}{dt} = -150e^{-0.15t} + 12e^{0.04t}$ When $t = 12$: $\frac{dP}{dt} = -150e^{-0.15 \times 12} + 12e^{0.04 \times 12}$ $= -5.4$ microbes per hour	MA–E1 Logarithms and Exponentials MA11–6 Bands 3–4 • Provides the correct solution. AND • Includes a negative sign OR indicates that the rate of change is a decrease . . . . . 2 • Finds the derivative 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 24	
(a) $f(x)-g(x)=4x^2-9-(2x^2-x-3)$ $=4x^2-9-2x^2+x+3$ $=2x^2+x-6$	MA–F1 Working with Functions MA11–2 Bands 2–3 • Provides the correct solution 1
(b) $f(x)g(x)=(4x^2-9)(2x^2-x-3)$ $=8x^4-4x^3-12x^2-18x^2+9x+27$ $=8x^4-4x^3-30x^2+9x+27$	MA–F1 Working with Functions MA11–2 Bands 2–3 • Provides the correct solution . . . . . 2 • Shows substantial progress towards expanding the product 1
(c) $\frac{f(x)}{g(x)}=\frac{4x^2-9}{2x^2-x-3}$ $=\frac{(2x+3)(2x-3)}{(2x-3)(x+1)}$ $=\frac{2x+3}{x+1}$	MA–F1 Working with Functions MA11–2 Bands 3–4 • Provides the correct solution . . . . . 2 • Factorises the numerator AND denominator 1
(d) $f(h(x))=4(3x+1)^2-9$ $=4(9x^2+6x+1)-9$ $=36x^2+24x-5$ $=[2(3x+1)+3][2(3x+1)-3]$ $=(6x+5)(6x-1)$	MA–F1 Working with Functions MA11–2 Bands 4–5 • Provides the correct solution . . . . . 2 • Shows progress towards expanding the brackets 1
Question 25	
$\frac{\log_3 x}{\log_x 3}=4$ $\log_3 x=4\log_x 3$ $=\frac{4\log_3 3}{\log_3 x}$ $=\frac{4}{\log_3 x}$ Let $m=\log_3 x$: $m=\frac{4}{m}$ $m^2=4$ $m=\pm 2$ $\log_3 x=2, \log_3 x=-2$ $x=9, \frac{1}{9}$	MA–E1 Logarithms and Exponentials MA11–6 Bands 4–6 • Provides the correct solution . . . . . 3 • Finds the solution(s) for $\log_3 x$ . . . . . 2 • Uses the logarithm laws for change of base OR equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 26	
(a)  $\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{25}{216}$	MA–S1 Probability and Discrete Probability Distributions MA11–7 Bands 4–5 • Provides the correct solution . . . . . 2 • Draws a tree diagram OR equivalent merit 1
(b) $P(\text{no more than 3 rolls}) = P(W) + P(LW) + P(LLW)$ $= \frac{1}{6} + \frac{5}{6}\times\frac{1}{6} + \frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}$ $= \frac{91}{216}$ *Note: Consequential on answer to **Question 26(a)**.*	MA–S1 Probability and Discrete Probability Distributions MA11–7 Bands 3–4 • Provides the correct solution 1
Question 27	
(a) 	MA–S1 Probability and Discrete Probability Distributions MA11–7 Bands 2–3 • Provides the correct solution . . . . . 2 • Draws a one-stage tree diagram OR equivalent merit. 1
(b) $P(\text{second tails given first tails}) = \frac{P(TT)}{P(T \text{ first})}$ $= \frac{\frac{1}{2}\times\frac{1}{3}}{\frac{1}{2}}$ $= \frac{1}{3}$ *Note: Consequential on answer to **Question 27(a)**.*	MA–S1 Probability and Discrete Probability Distributions MA11–7 Bands 4–5 • Provides the correct solution . . . . . 2 • Demonstrates an understanding of conditional probability OR equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 28	
 The solutions are $x = 2.5$ and $x = -1.5$. $A = \frac{1}{2}bh$ $= \frac{1}{2} \times 4 \times 4$ $= 8 \text{ units}^2$	MA–F1 Working with Functions MA11–1, 11–2 Bands 3–4 • Provides the correct solution . . . . . 3 • Provides the solutions for x . . . . . 2 • Graphs $y = \|2x - 1\|$ OR equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 29	
$mx + 5 = x^2 - 5x + 14$ $x^2 + (-5 - m)x + 9 = 0$ $\Delta = (-5 - m)^2 - 4 \times 1 \times 9$ $= 25 + 10m + m^2 - 36$ $= m^2 + 10m - 11$ $= (m + 11)(m - 1)$ For the tangent, $\Delta = 0$. $(m + 11)(m - 1) = 0$ $m = -11, 1$ When $m = -11$: $x^2 + (-5 + 11)x + 0 = 0$ $x^2 + 6x + 9 = 0$ $(x + 3)^2 = 0$ $x = -3$ $y = (-3)^2 - 5(-3) + 14$ $= 38$ When $m = 1$: $x^2 + (-5 - 1)x + 9 = 0$ $x^2 - 6x + 9 = 0$ $(x - 3)^2 = 0$ $x = 3$ $y = (3)^2 - 5(3) + 14$ $= 8$ Therefore: If $m = -11$, the point is $(-3, 38)$. If $m = 1$, the point is $(3, 8)$.	MA–F1 Working with Functions MA11–2 Bands 4–6 • Provides the correct solution . . . . . 5 • Provides ONE of the points . . . . . 4 • Finds the correct solutions for m . . . 3 • Provides an expression for the discriminant . . . . . 2 • Shows progress towards equating the equations 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 30	
After three hours: ship $A = 24 \times 3$ $= 72$ km ship $B = 20 \times 3$ $= 60$ km  $AB^2 = 60^2 + 72^2 - 2 \times 60 \times 72 \times \cos 110°$ $AB^2 = 11\,739.05$ $AB = 108.3469\ldots$ km ≈ 108 km	MA–T1 Trigonometry and Measure of Angles MA11–3 Bands 4–5 • Provides the correct solution . . . . . 3 • Shows progress towards finding the length of AB. . . . . 2 • Draws a diagram with relevant information OR equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 31	

$$P(1)=\frac{3}{8}$$

$$P(2)=\frac{5}{8}\times\frac{3}{7}$$
$$=\frac{15}{56}$$

$$P(3)=\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}$$
$$=\frac{5}{28}$$

$$P(4)=\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{3}{5}$$
$$=\frac{3}{28}$$

$$P(5)=\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\times\frac{3}{4}$$
$$=\frac{3}{56}$$

$$P(6)=\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}\times\frac{3}{3}$$
$$=\frac{1}{56}$$

X	1	2	3	4	5	6
$P(X)$	$\frac{3}{8}$	$\frac{15}{56}$	$\frac{5}{28}$	$\frac{3}{28}$	$\frac{3}{56}$	$\frac{1}{56}$

$$E(X)=1\times\frac{3}{8}+2\times\frac{15}{56}+3\times\frac{5}{28}+4\times\frac{3}{28}+5\times\frac{3}{56}+6\times\frac{1}{56}$$
$$=2.25$$

$$\text{Var}(X)=1^2\times\frac{3}{8}+2^2\times\frac{15}{56}+3^2\times\frac{5}{28}+4^2\times\frac{3}{28}+5^2\times\frac{3}{56}$$
$$+6^2\times\frac{1}{56}-2.25^2$$
$$=\frac{27}{16}$$
$$=1.6875$$

MA–S1 Probability and Discrete Probability Distributions
MA11–7 Bands 5–6

- Provides the correct solution 5
- Shows progress towards calculating Var(X) 4
- Shows progress towards calculating $E(X)$ 3
- Completes the table 2
- Calculates ONE of $P(2)$, $P(4)$, $P(5)$ or $P(6)$. 1