Neap

Final Examination 2022

NSW Year 11 Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant reasoning and/or calculations.

Total Marks: 80

SECTION I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

SECTION II – 70 marks (pages 7–26)

- Attempt Questions 11–31
- Allow about 1 hour and 45 minutes for this section

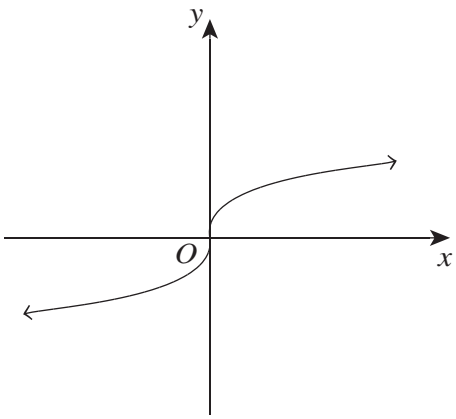
SECTION I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1 Which of the following graphs represents a one-to-many relationship?

A.



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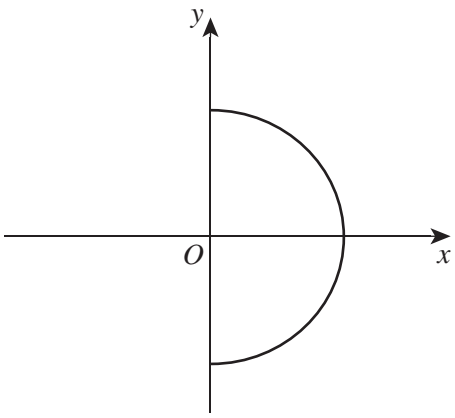

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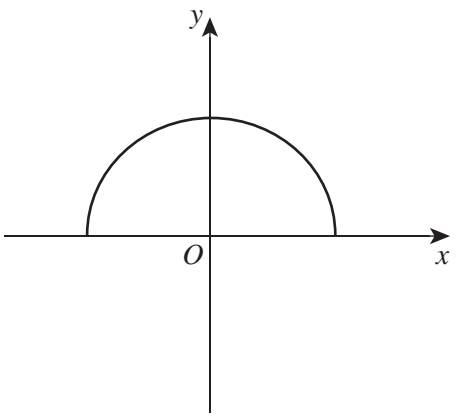

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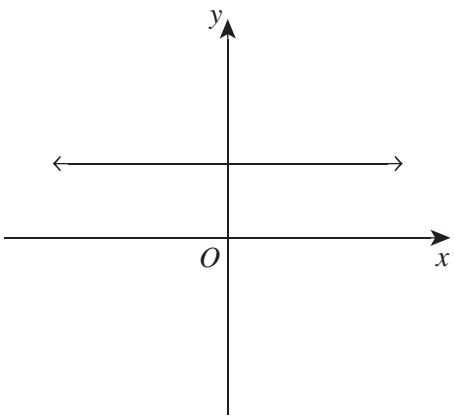


2 Which of the following equations has the solutions $x = 2$ and $x = -3$?

A. $x^2 - x - 6 = 0$

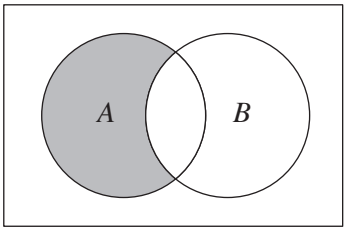
B. $x^2 + x - 6 = 0$

C. $x^2 + 5x + 6 = 0$

D. $x^2 - 5x + 6 = 0$

3 Which of the following Venn diagrams represents $A \cup \overline{B}$?

A.



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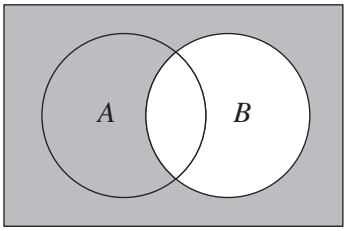

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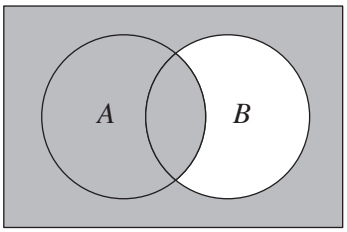

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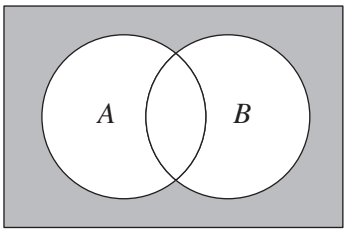
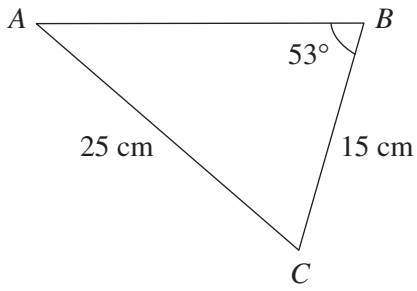


4 The triangle ABC is shown.



NOT TO SCALE

Which of the following statements is correct?

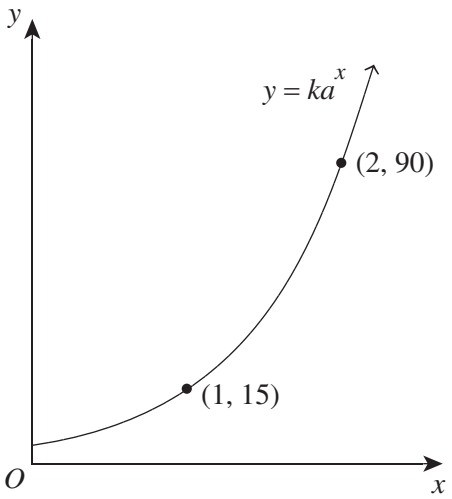
A. AC is the longest side of the triangle.

B. The triangle is right-angled.

C. The perimeter of the triangle is approximately 71 cm.

D. The area of the triangle is approximately 187.5 cm^2.

5 The graph shows $y = ka^x$.



What are the values of a and k?

A. $a = 15, k = 6$

B. $a = 6, k = 15$

C. $a = 2.5, k = 6$

D. $a = 6, k = 2.5$

6 A and B are mutually exclusive events.
Which of the following statements is true?

A. $P(A) \leq P(\overline{B})$

B. $P(A) \geq P(\overline{B})$

C. $P(A) < P(\overline{B})$

D. $P(A) > P(\overline{B})$

7 Which of the following ordered pairs represents an odd function?

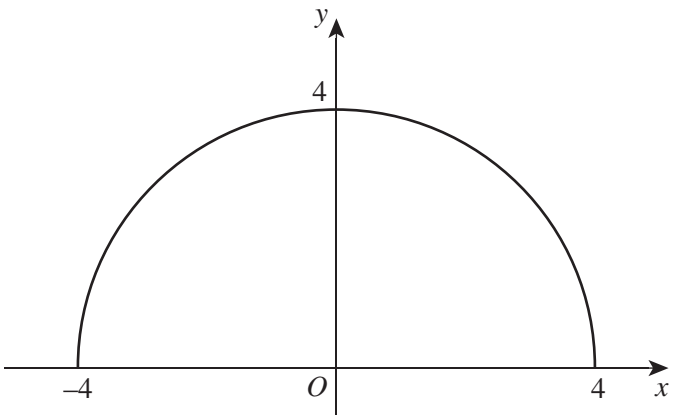
A. {(2, 3), (3, 2), (5, 7), (7, 5)}

B. {(–2, 3), (2, 3), (–5, 7), (7, 5)}

C. {(2, 3), (–2, –3), (5, 7), (–5, –7)}

D. {(2, 3), (3, 5), (5, 7), (7, 2)}

8 A semicircle is shown.



What is the equation of the semicircle?

A. $y = \sqrt{4 - x^2}$

B. $y = -\sqrt{4 - x^2}$

C. $y = \sqrt{16 - x^2}$

D. $y = -\sqrt{16 - x^2}$

9 The displacement of a particle, in centimetres, is given by $s = 2t^2 - 8t + 6$, where t is time in seconds. Which of the following statements is NOT true?

A. The particle's displacement is increasing after 2 seconds.

B. The particle does not change direction.

C. The particle's displacement is always positive.

D. The particle's acceleration is constant.

10 The equation $2x + 3y + 4 = 0$ makes an angle with the positive x-axis. Which of the following is closest to the size of the angle?

A. 34°

B. 56°

C. 124°

D. 146°

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NSW Year 11 Mathematics Advanced

Section II Answer Booklet 1

SECTION II

70 marks
Attempt Questions 11–31
Allow about 1 hour and 45 minutes for this section

Booklet 1 — Attempt Questions 11–21 (29 marks)
Booklet 2 — Attempt Questions 22–31 (41 marks)

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on page 14 of Booklet 1. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (2 marks)

Solve $\frac{x}{2} + 4 = 2 - 3x$. **2**

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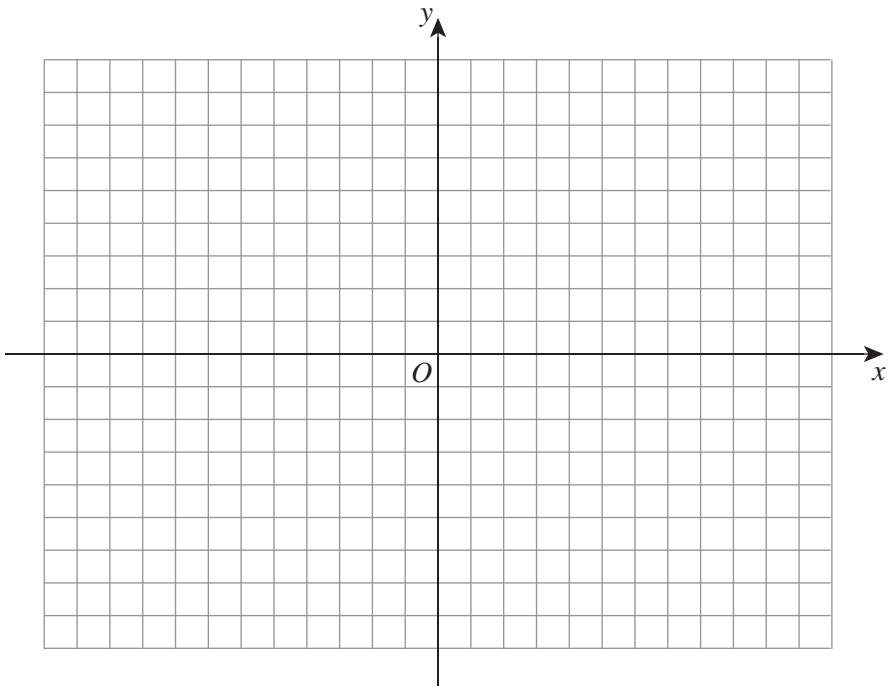
Question 12 (3 marks)

Sketch the graph of $y = -(x-3)^2 + 2$, labelling the vertex and any x- and y-intercepts. **3**



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Question 13 (3 marks)

Find the equation of the line that is perpendicular to $6x - 8y + 15 = 0$ at a point that has an x-coordinate of 1. **3**

Question 14 (3 marks)

Solve the equations $y = x^2 - 2x$ and $y = 6x - x^2$ simultaneously. **3**

Question 15 (4 marks)

(a) Solve $2\cos\theta = -\sqrt{3}$ for $[-\pi, \pi]$. **2**

(b) For what values of x is $y = \tan x$ undefined for $0° \leq x \leq 720°$? **2**

Question 16 (2 marks)

Differentiate $(x^2 - 5)(9x - 2x^3)$ with respect to x. **2**

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Question 17 (2 marks)

Differentiate $f(x) = x^2 + 5x$ using $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. **2**

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Question 18 (2 marks)

Find $\frac{dy}{dx}$ if $y = (2x^2 - 3)^5$. **2**

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Question 19 (3 marks)

Show that $\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\text{cosec}\,\theta.$ **3**

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Question 20 (3 marks)

Find the exact gradient of the tangent to the curve $y = \dfrac{5x+2}{3x+1}$ when $x = 1$. **3**

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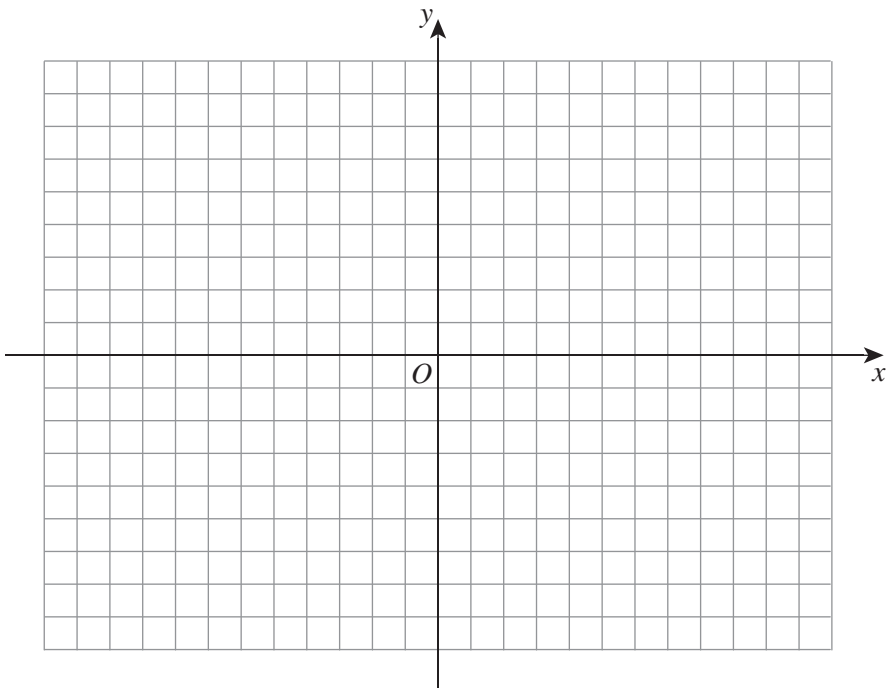
Question 21 (2 marks)

Sketch the graph of $y = \ln x - 2$, labelling the x-intercept and vertical asymptote. **2**



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NSW Year 11 Mathematics Advanced

Section II Answer Booklet 2

Booklet 2 — Attempt Questions 22–31 (41 marks)

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 25–26 of Booklet 2. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 22 (4 marks)

A piece of wire that is 120 cm long is bent into a sector of a circle. The arc of the sector subtends an angle of 100° at the centre. **4**

Find the area of this sector, correct to the nearest square centimetre.

Question 23 (4 marks)

The population of a type of microbe in a petri dish can be modelled using the formula

$$P = 1000e^{-0.15t} + 300e^{0.04t},$$

where t is the time in hours.

(a) What is the population after 90 minutes? Give your answer correct to the nearest whole number. **2**

(b) What is the rate of change when $t = 12$? Give your answer correct to one decimal place. **2**

Question 24 (7 marks)

Consider the functions $f(x) = 4x^2 - 9$, $g(x) = 2x^2 - x - 3$ and $h(x) = 3x + 1$.

(a) Find a simplified expression for $y = f(x) - g(x)$. **1**

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(b) Find a simplified expression for $y = f(x)g(x)$. **2**

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(c) Find a simplified expression for $y = \dfrac{f(x)}{g(x)}$. **2**

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(d) Find a simplified expression for $y = f(h(x))$. **2**

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Question 25 (3 marks)

Solve $\dfrac{\log_3 x}{\log_x 3} = 4$ for x. **3**

Question 26 (3 marks)

Bindu is playing a game where she rolls a fair, six-sided die. If the die lands on the number 2, Bindu wins and the game ends. Bindu plays the game until she wins.

(a) By using a probability tree diagram or otherwise, calculate the probability that Bindu ends the game after the third roll. **2**

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(b) Calculate the probability that Bindu rolls the die no more than three times in a single game. **1**

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Question 27 (4 marks)

Sheena tosses a fair coin then a biased coin. The probability of obtaining tails when tossing the biased coin is $\frac{1}{3}$.

(a) Draw a probability tree diagram representing this information. **2**

(b) Find the probability of Sheena obtaining tails on the second toss, given that the first toss landed on tails. **2**

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Question 28 (3 marks)

By drawing the graphs of $y = |2x - 1|$ and $y = 4$ on the same number plane, or otherwise, calculate the area of the triangle formed by the two graphs. **3**

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Question 29 (5 marks)

The line $y = mx + 5$ is a tangent to the curve $y = x^2 - 5x + 14$. **5**

By finding the possible values of m, determine the coordinates of the points where the line touches the curve.

Question 30 (3 marks)

Two ships, A and B, depart a port at the same time. Ship A travels at 24 km/h on a bearing of 065° and ship B travels at 20 km/h on a bearing of 315°. **3**

Calculate the distance between ships A and B after three hours. Give your answer correct to the nearest kilometre.

Question 31 (5 marks)

Vicki has eight A4-sized notebooks on her bookshelf with identical covers. Three of the notebooks are blank. She does not know if a notebook is blank unless she opens the notebook. **5**

Vicki wants a blank notebook, so she randomly pulls out one notebook at a time and does not replace them on the bookshelf until she finds a blank notebook. The random variable X is the number of notebooks that she needs to pull out before she finds a blank notebook.

By completing the probability distribution for X, calculate $\text{Var}(X)$.

X	1	2	3	4	5	6
$P(X)$			$\frac{5}{28}$			

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Section II extra writing space

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MATHEMATICS ADVANCED
MATHEMATICS EXTENSION 1
MATHEMATICS EXTENSION 2
REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1},\ r \neq 1$$

$$S = \frac{a}{1-r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

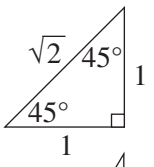
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}},\ \cos A = \frac{\text{adj}}{\text{hyp}},\ \tan A = \frac{\text{opp}}{\text{adj}}$$

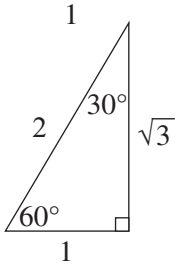
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A},\ \cos A \neq 0$$

$$\text{cosec} A = \frac{1}{\sin A},\ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A},\ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

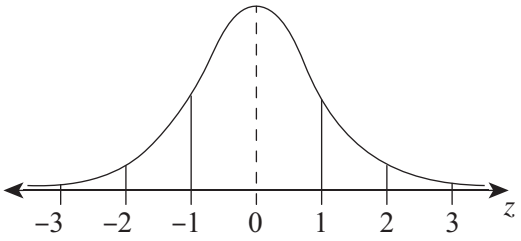
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 - 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X-\mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)dx$$

$$P(a < X < b) = \int_a^b f(x)dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1-p)^{n-x},\ x = 0,\ 1,\ \ldots,\ n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c \quad \text{where } n \neq -1$$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 - \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \ldots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \ldots + \binom{n}{r}x^{n-r}a^{r} + \ldots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + x\underset{\sim}{j}\right| = \sqrt{x^{2} + y^{2}}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r\left(\cos\theta + i\sin\theta\right)$$
$$= re^{i\theta}$$

$$\left[r\left(\cos\theta + i\sin\theta\right)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^{2}(x - c)$$

Final Examination 2022

NSW Year 11 Mathematics Advanced

DIRECTIONS:

Write your name in the space provided.
Write your student number in the boxes provided below. Then, in the columns of digits below each box, fill in the oval which has the same number as you have written in the box. Fill in **one** oval only in each column.
Read each question and its suggested answers. Select the alternative A, B, C, or D that best answers the question. Fill in the response oval completely, using blue or black pen. Mark only **one** oval per question.

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and draw an arrow as follows.

correct ↓

A ⊗ B ⊗ C ○ D ○

STUDENT NAME: _____

STUDENT NUMBER:

1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9
0	0	0	0	0	0	0	0	0

SECTION I

MULTIPLE-CHOICE ANSWER SHEET

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

STUDENTS SHOULD NOW CONTINUE WITH SECTION II