



Trial Examination 2022

HSC Year 12 Mathematics Extension 1

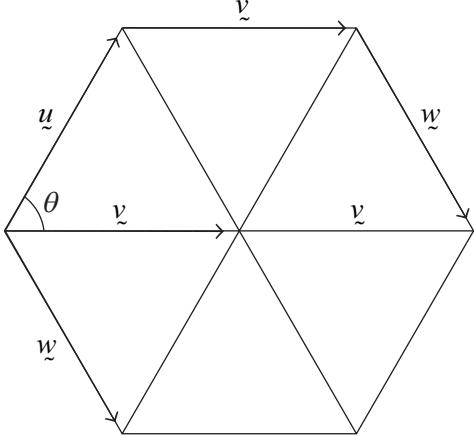
Solutions and Marking Guidelines

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SECTION I

Answer and explanation	Syllabus content, outcomes and targeted performance bands
<p>Question 1 C</p> $\tan\left(\theta + \frac{\pi}{3}\right) = \frac{\tan\theta + \tan\frac{\pi}{3}}{1 - \tan\theta \tan\frac{\pi}{3}}$ $= \frac{\frac{1}{3} + \sqrt{3}}{1 - \frac{\sqrt{3}}{3}} \times \frac{3}{3}$ $= \frac{1 + 3\sqrt{3}}{3 - \sqrt{3}}$ <p>Alternatively:</p> $\tan\theta = \frac{1}{3}$ $\theta = \tan^{-1}\left(\frac{1}{3}\right)$ $= 0.32175\dots$ <p>Going through each option to check gives:</p> <p>Option A:</p> $\tan\left(\theta + \frac{\pi}{3}\right) = \frac{\sqrt{3} + 3}{3\sqrt{3} - 1}$ $\theta + \frac{\pi}{3} = \tan^{-1}\left(\frac{\sqrt{3} + 3}{3\sqrt{3} - 1}\right)$ $\theta = -0.20184\dots$ <p>Option B:</p> $\tan\left(\theta + \frac{\pi}{3}\right) = \frac{\sqrt{3} - 3}{3\sqrt{3} + 1}$ $\theta + \frac{\pi}{3} = \tan^{-1}\left(\frac{\sqrt{3} - 3}{3\sqrt{3} + 1}\right)$ $\theta = -1.24904\dots$ <p>Option C:</p> $\tan\left(\theta + \frac{\pi}{3}\right) = \frac{1 + 3\sqrt{3}}{3 - \sqrt{3}}$ $\theta + \frac{\pi}{3} = \tan^{-1}\left(\frac{1 + 3\sqrt{3}}{3 - \sqrt{3}}\right)$ $\theta = 0.32175\dots$ <p>(continues on next page)</p>	ME-T2 Further Trigonometric Identities ME11-1 Band E2

Answer and explanation	Syllabus content, outcomes and targeted performance bands
<p>(continued)</p> <p>Option D:</p> $\tan\left(\theta + \frac{\pi}{3}\right) = \frac{1-3\sqrt{3}}{3+\sqrt{3}}$ $\theta + \frac{\pi}{3} = \tan^{-1}\left(\frac{1-3\sqrt{3}}{3+\sqrt{3}}\right)$ $\theta = -1.77265\dots$ <p>Therefore, C is correct.</p>	
<p>Question 2 A</p> <p>Let $u = (\ln x)^2$.</p> $\therefore du = 2(\ln x) \times \frac{1}{x} dx$ <p><i>Note: Although the limits are the same for each option, the limits should always be changed when using the substitution method.</i></p> <p>When $x = e$, $u = (\ln e)^2$ $= 1$</p> <p>When $x = e^2$, $u = (\ln e^2)^2$ $= 4$</p> <p>Rewriting the integral gives:</p> $\int_e^{e^2} \frac{(\ln x)^3}{x} dx = \frac{1}{2} \int_e^{e^2} (\ln x)^2 \frac{2(\ln x)}{x} dx$ $\therefore \frac{1}{2} \int_1^4 u du$	<p>ME–C2 Further Calculus Skills ME12–1 Bands E2–E3</p>
<p>Question 3 C</p> $\int \frac{4}{\sqrt{9-x^2}} dx = 4 \int \frac{1}{\sqrt{(3)^2 - x^2}} dx$ $= 4 \sin^{-1}\left(\frac{x}{3}\right) + c$	<p>ME–C2 Further Calculus Skills ME12–1 Bands E2–E3</p>

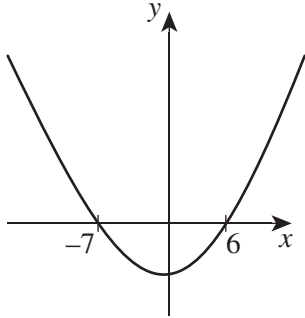
Answer and explanation	Syllabus content, outcomes and targeted performance bands
<p>Question 4 B</p>  <p>As shown on the diagram through vector addition, $\underline{u} + \underline{v} + \underline{w} = 2\underline{v}$.</p> $\therefore \underline{u} \cdot (\underline{u} + \underline{v} + \underline{w}) = \underline{u} \cdot 2\underline{v}$ $\cos \theta = \frac{\underline{u} \cdot 2\underline{v}}{ \underline{u} 2\underline{v} }$ $\therefore \underline{u} \cdot 2\underline{v} = \underline{u} 2\underline{v} \cos \theta$ <p>Since all the triangles are equilateral,</p> $\theta = \frac{\pi}{3}, \underline{u} = 4 \text{ and } 2\underline{v} = 8.$ $\therefore \underline{u} \cdot 2\underline{v} = 4 \times 8 \times \cos \frac{\pi}{3}$ $= 16$	<p>ME–V1 Introduction to Vectors ME12–2 Bands E2–E3</p>
<p>Question 5 A</p> <p>A is correct. This option is reached through a process of elimination. When $x = 1$ and $y = 1$, the gradient is 0. Therefore, we can eliminate B and C. When $x = -1$ and $y = 1$, the gradient is negative. Therefore, we can eliminate D, leaving A as the only viable option.</p>	<p>ME–C3 Applications of Calculus ME12–4 Bands E2–E3</p>

Answer and explanation	Syllabus content, outcomes and targeted performance bands
<p>Question 6 D</p> <p>For points of intersection, let $f(x) = g(x)$.</p> $x^3 - 2x^2 + 3 = x^3 + 3x^2 - 2$ $5x^2 = 5$ $x^2 = 1$ $\therefore x = -1, 1$ $\text{area} = \int_{-1}^1 (x^3 - 2x^2 + 3) - (x^3 + 3x^2 - 2) dx$ $= \int_{-1}^1 x^3 - 2x^2 + 3 - x^3 - 3x^2 + 2 dx$ $= \int_{-1}^1 -5x^2 + 5 dx$	<p>ME-C3 Applications of Calculus ME12-1 Bands E2-E3</p>
<p>Question 7 D</p> $2 \cos x - 3 \sin x = R \cos(x + \theta)$ $= R \cos x \cos \theta - R \sin x \sin \theta$ <p>Equating both sides: $2 \cos x = R \cos x \cos \theta$ and $3 \sin x = R \sin x \sin \theta$</p> <p>Therefore:</p> $R \cos \theta = 2 \quad (1)$ $R \sin \theta = 3 \quad (2)$ $\frac{(2)}{(1)}$ $\therefore \tan \theta = \frac{3}{2}$	<p>ME-T3 Trigonometric Equations ME12-3 Bands E2-E3</p>
<p>Question 8 C</p> <p>The ‘worst-case scenario’ is if five students receive an A grade, five students receive a B grade, five students receive a C grade, five students receive a D grade and five students receive an E grade. The next student must receive a grade of A, B, C, D, or E, so that they will be the sixth student to receive that grade.</p> $\therefore 5 \times 5 + 1 = 26 \text{ students}$	<p>ME-A1 Working with Combinatorics ME11-5 Bands E2-E3</p>

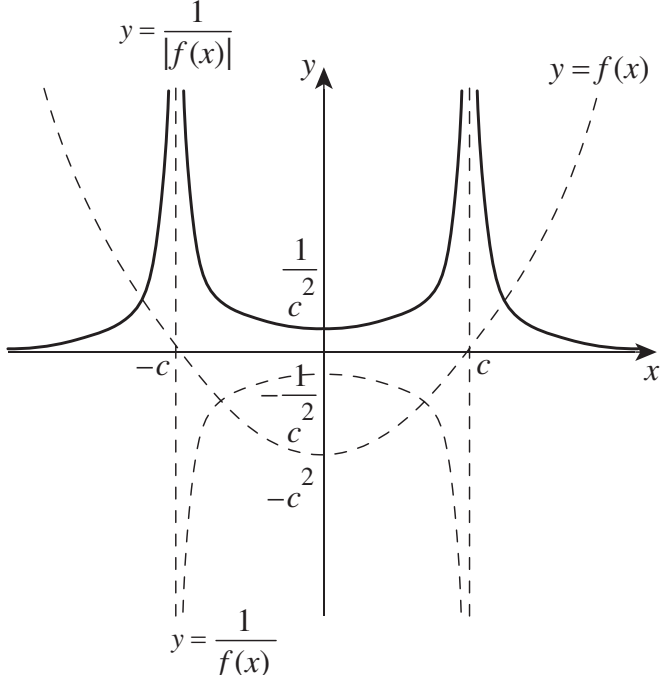
Answer and explanation	Syllabus content, outcomes and targeted performance bands
<p>Question 9 B</p> <p>For \underline{p} and \underline{q} to be parallel: $\underline{p} = k\underline{q}$, where k is a constant.</p> $\begin{pmatrix} t-8 \\ 6 \end{pmatrix} = k \begin{pmatrix} 3 \\ 2t \end{pmatrix}$ $t-8 = 3k \quad (1)$ $6 = 2kt \quad (2)$ <p>From (2):</p> $k = \frac{3}{t} \quad (3)$ <p>Substituting (3) into (1):</p> $t-8 = 3\left(\frac{3}{t}\right)$ $t^2 - 8t = 9$ $t^2 - 8t - 9 = 0$ $(t-9)(t+1) = 0$ $t = -1, 9$	<p>ME-V1 Introduction to Vectors ME12-2 Bands E2-E3</p>
<p>Question 10 C</p> <p>5 glasses: $\binom{9}{5}$</p> <p>4 glasses, 1 no glasses: $\binom{9}{4} \times \binom{3}{1}$</p> <p>3 glasses, 2 no glasses: $\binom{9}{3} \times \binom{3}{2}$</p> <p>total = $\binom{9}{5} + \binom{9}{4} \times \binom{3}{1} + \binom{9}{3} \times \binom{3}{2}$ = 756</p>	<p>ME-A1 Working with Combinatorics ME11-5 Bands E2-E3</p>

SECTION II

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>Question 11</p> <p>(a) $P(x) = 8x^4 - 38x^3 + 9x^2 + ax + b$ $P(3) = 0$ and $P'(3) = 0$</p> $P(3) = 8(3)^4 - 38(3)^3 + 9(3)^2 + 3a + b$ $0 = 648 - 1026 + 81 + 3a + b$ $3a + b = 297 \quad (1)$ $P'(x) = 32x^3 - 114x^2 + 18x + a$ $P'(3) = 32(3)^3 - 114(3)^2 + 18(3) + a$ $0 = 864 - 1026 + 54 + a$ $a = 108$ <p>Substitute $a = 108$ into (1):</p> $3(108) + b = 297$ $b = -27$	<p>ME–F2 Polynomials ME11–1 Bands E2–E3</p> <ul style="list-style-type: none"> Provides the correct solution3 <hr/> <ul style="list-style-type: none"> Substitutes $x = 3$ into $P(x)$. <p>AND</p> <ul style="list-style-type: none"> Substitutes $x = 3$ into $P'(x)$.2 <hr/> <ul style="list-style-type: none"> Substitutes $x = 3$ into $P(x)$. <p>OR</p> <ul style="list-style-type: none"> Substitutes $x = 3$ into $P'(x)$.1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(b) $\binom{5n+3}{5n+1} = \frac{(5n+3)!}{(5n+3-(5n+1))!(5n+1)!}$</p> $= \frac{(5n+3)!}{(2)!(5n+1)!}$ $= \frac{(5n+3)(5n+2)(5n+1)!}{2(5n+1)!}$ $= \frac{(5n+3)(5n+2)}{2}$ $\frac{(5n+3)(5n+2)}{2} \geq 528$ $25n^2 + 25n + 6 \geq 1056$ $n^2 + n - 42 \geq 0$ $(n-6)(n+7) \geq 0$ <p>Sketching the graph of $(n-6)(n+7) \geq 0$ gives:</p>  <p>Reading from the graph, the sections where $y \geq 0$ occur when $x \leq -7$ and $x \geq 6$.</p> <p>Therefore, $n \leq -7, n \geq 6$.</p> <p>As n is a positive integer, $n \geq 6$.</p>	<p>ME–A1 Working with Combinatorics ME11–5 Bands E2–E3</p> <ul style="list-style-type: none"> Provides the correct solution3 <hr/> <ul style="list-style-type: none"> Uses the formula for $\binom{n}{k}$ AND simplifies to the correct quadratic2 <hr/> <ul style="list-style-type: none"> Uses the formula for $\binom{n}{k}$ AND makes some progress toward simplifying the correct quadratic1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(c) (i) Volume of a cylinder:</p> $V = \pi r^2 h$ $= \pi \times 0.4^2 \times h$ $= 0.16\pi h$ $\frac{dV}{dh} = 0.16\pi$ <p>Using the chain rule:</p> $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $k\sqrt{h} = 0.16\pi \times \frac{dh}{dt}$ $0.16\pi \times \frac{1}{\sqrt{h}} \times \frac{dh}{dt} = k$ $0.16\pi \int \frac{1}{\sqrt{h}} dh = \int k dt$ $0.16\pi \int h^{-\frac{1}{2}} dh = kt + c$ $0.16\pi \left[\frac{h^{\frac{1}{2}}}{\frac{1}{2}} \right] = kt + c$ $0.32\pi\sqrt{h} = kt + c$ <p>When $t = 0, h = 1$.</p> $0.32\pi\sqrt{1} = c$ $c = 0.32\pi$ $0.32\pi\sqrt{h} = kt + 0.32\pi$ <p>When $t = 20, h = 0.36$.</p> $0.32\pi\sqrt{0.36} = 20k + 0.32\pi$ $0.192\pi = 20k + 0.32\pi$ $20k = -0.128\pi$ $k = -\frac{4}{625}\pi$	<p>ME–C1 Rates of Change ME–C3 Applications of Calculus ME11–4, 12–4 Bands E2–E4</p> <ul style="list-style-type: none"> Provides the correct solution 3 <hr/> <ul style="list-style-type: none"> Uses the chain rule, integrates and finds the values of the constant 2 <hr/> <ul style="list-style-type: none"> Differentiates the volume formula and attempts to use the chain rule 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(ii) $0.32\pi\sqrt{h} = -\frac{4}{625}\pi t + 0.32\pi$</p> <p>When $h = 0$:</p> $0.32\pi\sqrt{0} = -\frac{4}{625}\pi t + 0.32\pi$ $-\frac{4}{625}\pi t + 0.32\pi = 0$ $\frac{4}{625}\pi t = 0.32\pi$ $t = 50 \text{ minutes}$ <p><i>Note: Consequential on answer to Question 11(c)(i).</i></p>	<p>ME–C1 Rates of Change ME11–4 Bands E2–E3</p> <ul style="list-style-type: none"> Provides the correct solution 1
<p>(d) $T_4 = \binom{9}{3}(2x)^6(-p)^3$</p> $= \binom{9}{3} \times 64x^6 \times -p^3$ $= -5376p^3x^6$ $-5376p^3 = -672\,000$ $p^3 = 125$ $p = 5$	<p>ME–A1 Working with Combinatorics ME11–5 Bands E2–E3</p> <ul style="list-style-type: none"> Provides the correct solution 2 <hr/> <ul style="list-style-type: none"> Provides the correct solution of T_4 1
<p>(e)</p> 	<p>ME–F1 Further Work with Functions ME11–1, 11–7 Bands E2–E3</p> <ul style="list-style-type: none"> Provides the correct solution 3 <hr/> <ul style="list-style-type: none"> Draws the graph of $\frac{1}{ f(x) }$ without turning points OR without asymptotes. <p>OR</p> <ul style="list-style-type: none"> Draws the graph of $\frac{1}{f(x)}$ with turning points AND asymptotes 2 <hr/> <ul style="list-style-type: none"> Draws the graph of $\frac{1}{f(x)}$ without turning points OR without asymptotes 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>Question 12</p> <p>(a) Step 1: Proving the statement is true for $n = 1$ gives: $\text{LHS} = 1 \times 2^{-(1-1)}$ $= 1$ $\text{RHS} = \frac{2^{1+1} - 1 - 2}{2^{1-1}}$ $= \frac{4 - 1 - 2}{1}$ $= 1$ $\text{LHS} = \text{RHS}$ Therefore, the statement is true for $n = 1$. Step 2: Assuming the statement is true for $n = k$ gives: $1 + 2 \times 2^{-1} + 3 \times 2^{-2} + 4 \times 2^{-3}$ $+ \dots + k \times 2^{-(k-1)} = \frac{2^{k+1} - k - 2}{2^{k-1}}$ Step 3: Proving that the statement is true for $n = k + 1$ gives: $1 + 2 \times 2^{-1} + 3 \times 2^{-2} + 4 \times 2^{-3} + \dots + k \times 2^{-(k-1)}$ $+ (k + 1) \times 2^{-k} = \frac{2^{k+2} - (k + 1) - 2}{2^k}$ $= \frac{2^{k+2} - k - 3}{2^k}$ $\text{LHS} = 1 + 2 \times 2^{-1} + 3 \times 2^{-2} + 4 \times 2^{-3} + \dots$ $+ k \times 2^{-(k-1)} + (k + 1) \times 2^{-k}$ $= \frac{2^{k+1} - k - 2}{2^{k-1}} + (k + 1) \times 2^{-k} \text{ (by assumption)}$ $= \frac{2^{k+1} - k - 2}{2^{k-1}} + \frac{k + 1}{2^k}$ $= \frac{2^{k+2} - 2k - 4}{2^k} + \frac{k + 1}{2^k}$ $= \frac{2^{k+2} - 2k - 4 + k + 1}{2^k}$ $= \frac{2^{k+2} - k - 3}{2^k}$ $= \text{RHS}$ If $n = k$ is true, then $n = k + 1$ is true. Therefore, by mathematical induction, the statement is true for $n \geq 1$. </p>	<p>ME–P1 Proof by Mathematical Induction ME12–1 Bands E2–E3</p> <ul style="list-style-type: none"> Provides the correct proof for all steps. 3 <hr/> <ul style="list-style-type: none"> Provides the correct proof for step 1. <p>AND</p> <ul style="list-style-type: none"> Makes some progress using the assumption for step 3. 2 <hr/> <ul style="list-style-type: none"> Provides the correct proof for step 1 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(b) (i) $\frac{d}{dx}(-\cot x) = \frac{d}{dx}\left(-\frac{\cos x}{\sin x}\right)$</p> <p>Let $u = -\cos x$ and $v = \sin x$.</p> $\frac{du}{dx} = \sin x$ $\frac{dv}{dx} = \cos x$ <p>Using the quotient rule gives:</p> $\frac{d}{dx}\left(-\frac{\cos x}{\sin x}\right) = \frac{(\sin x)(\sin x) - (-\cos x)(\cos x)}{(\sin x)^2}$ $= \frac{\sin^2 x + \cos^2 x}{\sin^2 x}$ $= \frac{1}{\sin^2 x}$	<p>ME–C2 Further Calculus Skills ME12–1 Bands E2–E3</p> <ul style="list-style-type: none"> Provides the correct solution 2 <hr/> <ul style="list-style-type: none"> Attempts to the use quotient rule OR equivalent merit 1
<p>(ii) Let $x = 4\sin\theta$ and $dx = 4\cos\theta d\theta$.</p> <p>When $x = 2$, $4\sin\theta = 2$.</p> $\sin\theta = \frac{1}{2}$ $\theta = \frac{\pi}{6}$ <p>When $x = 2\sqrt{3}$, $4\sin\theta = 2\sqrt{3}$.</p> $\sin\theta = \frac{\sqrt{3}}{2}$ $\theta = \frac{\pi}{3}$ $\int_2^{2\sqrt{3}} \frac{1}{x^2\sqrt{16-x^2}} dx$ $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4\cos\theta}{(4\sin\theta)^2\sqrt{16-(4\sin\theta)^2}} d\theta$ $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4\cos\theta}{16\sin^2\theta\sqrt{16-16\sin^2\theta}} d\theta$ <p>(continues on next page)</p>	<p>ME–C2 Further Calculus Skills ME12–1 Bands E2–E3</p> <ul style="list-style-type: none"> Provides the correct solution 4 <hr/> <ul style="list-style-type: none"> Finds the complete integrand in terms of θ. 3 <hr/> <ul style="list-style-type: none"> Finds $\frac{dx}{d\theta}$. <p>AND</p> <ul style="list-style-type: none"> Changes the limits 2 <hr/> <ul style="list-style-type: none"> Finds $\frac{dx}{d\theta}$. <p>OR</p> <ul style="list-style-type: none"> Changes the limits 1

Sample answer

Syllabus content, outcomes, targeted performance bands and marking guide

(ii) (continued)

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4 \cos \theta}{16 \sin^2 \theta \times 4 \sqrt{1 - \sin^2 \theta}} d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4 \cos \theta}{16 \sin^2 \theta \times 4 \sqrt{\cos^2 \theta}} d\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{4 \cos \theta}{16 \sin^2 \theta \times 4 \cos \theta} d\theta$$

$$= \frac{1}{16} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sin^2 \theta} d\theta$$

$$= \frac{1}{16} [-\cot \theta]_{\frac{\pi}{6}}^{\frac{\pi}{3}} \text{ (using part (i))}$$

$$= -\frac{1}{16} \left[\frac{1}{\tan \theta} \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= -\frac{1}{16} \left(\frac{1}{\tan \frac{\pi}{3}} - \frac{1}{\tan \frac{\pi}{6}} \right)$$

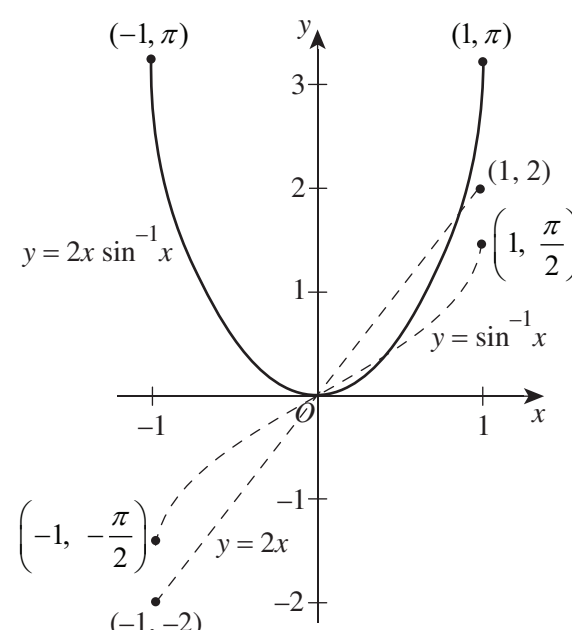
$$= -\frac{1}{16} \left(\frac{1}{\sqrt{3}} - \sqrt{3} \right)$$

$$= -\frac{1}{16} \left(\frac{\sqrt{3}}{3} - \frac{3\sqrt{3}}{3} \right)$$

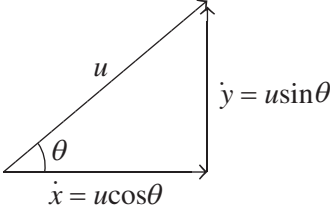
$$= \frac{2\sqrt{3}}{48}$$

$$= \frac{\sqrt{3}}{24}$$

Note: Consequential on answer to **Question 12(b)(i)**.

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(c) (i) $f(x) = 2x \sin^{-1} x$ $f(-x) = 2(-x) \sin^{-1}(-x)$ $= -2x \sin^{-1}(-x)$</p> <p>Since the function is odd, $\sin^{-1}(-x) = -\sin^{-1} x$.</p> <p>$f(-x) = 2x \sin^{-1} x$ $= f(x)$</p> <p>Therefore, $f(x)$ is an even function.</p>	<p>ME-F1 Further Work with Functions ME-T1 Inverse Trigonometric Functions ME11-1, 11-3 Bands E2-E3</p> <ul style="list-style-type: none"> Provides the correct solution 1
<p>(ii)</p> 	<p>ME-F1 Further Work with Functions ME-T1 Inverse Trigonometric Functions ME11-1, 11-3 Bands E2-E3</p> <ul style="list-style-type: none"> Provides the correct solution 2 <hr/> <p>OR</p> <ul style="list-style-type: none"> Sketches the graph. Provides the coordinates of the endpoints 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(d) (i) $E(\hat{p}) = E\left(\frac{X}{n}\right)$</p> $= \frac{E(X)}{n}$ $= \frac{np}{n}$ $= p$ $= 0.03$ $\sigma(\hat{p}) = \sigma\left(\frac{X}{n}\right)$ $= \frac{\sigma(X)}{n}$ $= \frac{\sqrt{npq}}{n}$ $= \frac{\sqrt{250 \times 0.03 \times 0.97}}{250}$ $= 0.0108$	<p>ME–S1 The Binomial Distribution ME12–5 Bands E2–E3</p> <ul style="list-style-type: none"> Finds the mean AND standard deviation 1
<p>(ii) For the z-score:</p> $z = \frac{0.02 - 0.03}{0.0108}$ $= -0.9269$ ≈ -0.93 <p>Using the table:</p> $P(Z < -0.93) = 1 - P(Z < 0.93)$ $= 1 - 0.8238$ $= 0.1762$ <p>Therefore, the probability that 2% of the noodle packets in the sample weigh less than 82 grams is 0.1762.</p> <p><i>Note: Consequential on answer to Question 12(d)(i).</i></p>	<p>ME–S1 The Binomial Distribution ME12–5 Bands E2–E3</p> <ul style="list-style-type: none"> Provides the correct solution 2 <hr/> <ul style="list-style-type: none"> Uses the z-score table 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>Question 13</p> <p>(a) (i)</p>  <p> $\dot{x} = u \cos \theta$ $\ddot{x} = 0, \dot{x} = C_1$ When $t = 0, \dot{x} = u \cos \theta.$ $C_1 = u \cos \theta$ $\dot{x} = u \cos \theta$ $x = ut \cos \theta + C_2$ When $t = 0, x = 0.$ $C_2 = 0$ $x = ut \cos \theta$ $\ddot{y} = -10$ $\dot{y} = -10t + C_3$ When $t = 0, \dot{y} = u \sin \theta.$ $C_3 = u \sin \theta$ $\dot{y} = -10t + u \sin \theta$ $y = -5t^2 + ut \sin \theta + C_4$ When $t = 0, y = 147.$ $C_4 = 147$ $y = -5t^2 + ut \sin \theta + 147$ </p>	<p>ME-V1 Introduction to Vectors ME12-2 Bands E2-E3</p> <ul style="list-style-type: none"> Provides the correct solution3 <hr/> <ul style="list-style-type: none"> Finds the equations of motion in either the horizontal OR vertical direction AND makes some progress in the other direction2 <hr/> <ul style="list-style-type: none"> Makes some progress in deriving the horizontal OR vertical equations of motion1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(ii) When $t = T, y = 0$.</p> $-5T^2 + uT \sin \theta + 147 = 0$ $uT \sin \theta = 5T^2 - 147 \quad (1)$ <p>When $t = T, v = 4u$.</p> $v = \sqrt{\dot{x}^2 + \dot{y}^2}$ $4u = \sqrt{(u \cos \theta)^2 + (-10T + u \sin \theta)^2}$ $16u^2 = u^2 \cos^2 \theta + 100T^2 - 20uT \sin \theta + u^2 \sin^2 \theta$ $= u^2 (\cos^2 \theta + \sin^2 \theta) + 100T^2 - 20uT \sin \theta$ $= u^2 + 100T^2 - 20uT \sin \theta$ <p>(as $\cos^2 \theta + \sin^2 \theta = 1$)</p> $15u^2 = 100T^2 - 20uT \sin \theta \quad (2)$ <p>Substituting (1) into (2):</p> $15u^2 = 100T^2 - 20(5T^2 - 147)$ $= 100T^2 - 100T^2 + 2940$ $= 2940$ $u^2 = 196$ $u = 14 \text{ (as } u > 0)$ <p><i>Note: Consequential on answer to Question 13(a)(i).</i></p>	<p>ME–V1 Introduction to Vectors ME12–2 Bands E2–E4</p> <ul style="list-style-type: none"> Provides the correct solution3 <hr/> <ul style="list-style-type: none"> Obtains the equation $uT \sin \theta = 5T^2 - 147$. <p>AND</p> <ul style="list-style-type: none"> Obtains the equation $15u^2 = 100T^2 - 20uT \sin \theta$2 <hr/> <ul style="list-style-type: none"> Obtains the equation $uT \sin \theta = 5T^2 - 147$. <p>OR</p> <ul style="list-style-type: none"> Obtains the equation $15u^2 = 100T^2 - 20uT \sin \theta$1
<p>(iii) From (2):</p> $15u^2 = 100T^2 - 20uT \sin \theta$ <p>Substituting $u = 14$:</p> $15(14)^2 = 100T^2 - 20(14)T \sin \theta$ $2940 = 100T^2 - 280T \sin \theta$ $280T \sin \theta = 100T^2 - 2940$ $\sin \theta = \frac{100T^2 - 2940}{280T}$ $= \frac{5T^2 - 147}{14T}$ <p><i>Note: Consequential on answer to Question 13(a)(ii).</i></p>	<p>ME–V1 Introduction to Vectors ME12–2 Bands E2–E3</p> <ul style="list-style-type: none"> Provides the correct solution2 <hr/> <ul style="list-style-type: none"> Uses the initial velocity from part (a)(ii) AND makes some progress toward the solution1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(iv) Minimum value of T is when $\sin\theta = 0$ $\left(\text{as } 0 < \theta < \frac{\pi}{2} \right)$.</p> $\frac{5T^2 - 147}{14T} = 0$ $5T^2 - 147 = 0$ $5T^2 = 147$ $T^2 = \frac{147}{5}$ $T = \pm \frac{\sqrt{147}}{\sqrt{5}}$ $= \pm \frac{\sqrt{735}}{5}$ <p>$T > 0$, therefore, the minimum value is $\frac{\sqrt{735}}{5}$.</p> <p>Maximum value of T is when $\sin\theta = 1$.</p> $\frac{5T^2 - 147}{14T} = 1$ $5T^2 - 147 = 14T$ $5T^2 - 14T - 147 = 0$ $T = \frac{14 \pm \sqrt{(-14)^2 - 4(5)(-147)}}{2(5)}$ $= -\frac{21}{5}, 7$ <p>$T > 0$, therefore, the maximum value is 7.</p> $\therefore \frac{\sqrt{735}}{5} < T < 7$	<p>ME-T3 Trigonometric Equations ME12-3 Bands E2-E4</p> <ul style="list-style-type: none"> Provides the correct solution3 <hr/> <ul style="list-style-type: none"> Attempts to solve for $\sin\theta = 0$ AND $\sin\theta = 1$2 <hr/> <ul style="list-style-type: none"> Solves for $\sin\theta = 0$. <p>OR</p> <ul style="list-style-type: none"> Solves for $\sin\theta = 1$1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(b) (i) $y = x \int f(x) dx$</p> <p>Let $u = x$ and $v = \int f(x) dx$.</p> $\frac{du}{dx} = 1$ $\frac{dv}{dx} = f(x)$ <p>Using the product rule gives:</p> $\frac{dy}{dx} = xf(x) + \int f(x) dx$ <p>Multiplying by x gives:</p> $x \frac{dy}{dx} = x^2 f(x) + x \int f(x) dx$ $x \frac{dy}{dx} = x^2 f(x) + y$ $x \frac{dy}{dx} - y = x^2 f(x)$	<p>ME–C3 Applications of Calculus ME12–4 Bands E3–E4</p> <ul style="list-style-type: none"> Provides the correct solution 2 <hr/> <ul style="list-style-type: none"> Attempts to use the product rule OR equivalent merit 1
<p>(ii) $x \frac{dy}{dx} - y = x^5$</p> $x^2 f(x) = x^5 \text{ (using part (b)(i))}$ $f(x) = x^3$ $y = x \int f(x) dx$ $= x \int x^3 dx$ $= x \times \frac{x^4}{4} + c$ $= \frac{x^5}{4} + c$ <p>When $x = 2$ and $y = 5$:</p> $5 = \frac{2^5}{4} + c$ $c = -3$ $\therefore y = \frac{x^5}{4} - 3$ <p><i>Note: Consequential on answer to Question 13(b)(i).</i></p>	<p>ME–C3 Applications of Calculus ME12–4 Bands E3–E4</p> <ul style="list-style-type: none"> Provides the correct solution 2 <hr/> <ul style="list-style-type: none"> Finds $f(x)$ using part (b)(i) OR equivalent merit. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>Question 14</p> <p>(a) To find the y-intercept, let $x = 0$.</p> $y = \sqrt{m - 3(0)}$ $= \sqrt{m}$ <p>Rearranging to make x the subject gives:</p> $y = \sqrt{m - 3x}$ $y^2 = m - 3x$ $3x = m - y^2$ $x = \frac{1}{3}(m - y^2)$ <p>volume = $\pi \int_0^{\sqrt{m}} \left(\frac{1}{3}(m - y^2) \right)^2 dy$</p> $= \frac{\pi}{9} \int_0^{\sqrt{m}} (m - y^2)^2 dy$ $= \frac{\pi}{9} \int_0^{\sqrt{m}} m^2 - 2my^2 + y^4 dy$ $= \frac{\pi}{9} \left[m^2 y - \frac{2my^3}{3} + \frac{y^5}{5} \right]_0^{\sqrt{m}}$ $= \frac{\pi}{9} \left(m^2 \times \sqrt{m} - \frac{2m \times (\sqrt{m})^3}{3} + \frac{(\sqrt{m})^5}{5} - (0 - 0 + 0) \right)$ $= \frac{\pi}{9} \left(m^2 \sqrt{m} - \frac{2}{3} m^2 \sqrt{m} + \frac{1}{5} m^2 \sqrt{m} \right)$ $= \frac{\pi}{9} \times \frac{8}{15} m^2 \sqrt{m}$ $= \frac{8\pi}{135} m^2 \sqrt{m}$ $\frac{8\pi}{135} m^2 \sqrt{m} = \frac{5000\pi}{27}$ $m^2 \sqrt{m} = 3125$ $m^{\frac{5}{2}} = 3125$ $m = (3125)^{\frac{2}{5}}$ $= 25$	<p>ME–C3 Applications of Calculus ME12–4 Bands E2–E4</p> <ul style="list-style-type: none"> Provides the correct solution4 <hr/> <ul style="list-style-type: none"> Finds the integrand for the volume of solid of revolution3 <hr/> <ul style="list-style-type: none"> Finds the y-intercept. <p>AND</p> <ul style="list-style-type: none"> Rearranges the equation to make x the subject2 <hr/> <ul style="list-style-type: none"> Finds the y-intercept. <p>OR</p> <ul style="list-style-type: none"> Rearranges the equation to make x the subject1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(b) (i) For an inverse function to exist, the function needs to be monotonically increasing or decreasing. Solving $f'(x) = 0$ to find the minimum turning point gives:</p> $f(x) = x - 4x^{\frac{1}{2}} - 1$ $f'(x) = 1 - 2x^{-\frac{1}{2}}$ $= 1 - \frac{2}{\sqrt{x}}$ $1 - \frac{2}{\sqrt{x}} = 0$ $\sqrt{x} - 2 = 0$ $\sqrt{x} = 2$ $x = 4$ $\therefore k = 4$	<p>ME-F1 Further Work with Functions ME11-1 Bands E3-E4</p> <ul style="list-style-type: none"> Provides the correct solution 2 <hr/> <ul style="list-style-type: none"> Attempts to find the minimum turning point OR equivalent merit. 1
<p>(ii) $f(x) = x - 4\sqrt{x} - 1, x \geq 4$ For $f^{-1}(x)$: $x = y - 4\sqrt{y} - 1, y \geq 4$ Completing the square (in terms of \sqrt{y}) gives:</p> $x = (y - 4\sqrt{y} + 4) - 4 - 1$ $= (\sqrt{y} - 2)^2 - 5$ $(\sqrt{y} - 2)^2 = x + 5$ $\sqrt{y} - 2 = \pm\sqrt{x + 5}$ $\sqrt{y} = 2 \pm \sqrt{x + 5}$ $y = (2 \pm \sqrt{x + 5})^2$ <p>Since $y \geq 4$: $\therefore f^{-1}(x) = (2 + \sqrt{x + 5})^2$ <i>Note: Consequential on answer to Question 14(b)(i).</i></p>	<p>ME-F1 Further Work with Functions ME11-1 Bands E3-E4</p> <ul style="list-style-type: none"> Provides the correct solution 2 <hr/> <ul style="list-style-type: none"> Attempts to complete the square OR equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(iii) For the minimum point on $f(x)$:</p> $f(4) = 4 - 4\sqrt{4} - 1$ $= -5$ <p>For $f(x)$:</p> <p>$D: x \geq 4$ and $R: y \geq -5$</p> <p>For $f^{-1}(x)$:</p> <p>$D: x \geq -5$ and $R: y \geq 4$</p> <p><i>Note: Consequential on answer to Question 14(b)(i).</i></p>	<p>ME–F1 Further Work with Functions ME11–1 Bands E2–E3</p> <ul style="list-style-type: none"> Provides the correct solution2 <hr/> <ul style="list-style-type: none"> Finds coordinates of the minimum turning point. <p>OR</p> <ul style="list-style-type: none"> Finds the domain and range of $f(x)$.1
<p>(c) (i) $(x + \tan \theta)(x - \cot \theta) = 0$</p> $x^2 - (\cot \theta)x + (\tan \theta)x - (\tan \theta)(\cot \theta) = 0$ $x^2 - (\cot \theta - \tan \theta)x - (\tan \theta)\left(\frac{1}{\tan \theta}\right) = 0$ $x^2 - \left(\frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta}\right)x - 1 = 0$ $x^2 - \left(\frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta}\right)x - 1 = 0$ $x^2 - 2\left(\frac{\cos^2 \theta - \sin^2 \theta}{2 \sin \theta \cos \theta}\right)x - 1 = 0$ <p>Using the double angle formulae gives:</p> $x^2 - 2\left(\frac{\cos 2\theta}{\sin 2\theta}\right)x - 1 = 0$ $x^2 - 2(\cot 2\theta)x - 1 = 0$	<p>ME–F2 Polynomials ME–T2 Further Trigonometric Identities ME11–1, 11–2 Bands E2–E3</p> <ul style="list-style-type: none"> Provides the correct solution2 <hr/> <ul style="list-style-type: none"> Expands $(x + \tan \theta)(x - \cot \theta)$ AND makes some progress toward the solution1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(ii) $x = -\tan\theta$ and $x = \cot\theta$ are roots of $x^2 - 2(\cot 2\theta)x - 1 = 0$</p> <p>Substitute $\theta = \frac{\pi}{8}$.</p> <p>So, $x = -\tan\left(\frac{\pi}{8}\right)$ and $x = \cot\left(\frac{\pi}{8}\right)$ are roots of:</p> $x^2 - 2\left(\cot\left(\frac{\pi}{4}\right)\right)x - 1 = 0$ $x^2 - 2x - 1 = 0 \quad \left(\text{as } \cot\left(\frac{\pi}{4}\right) = 1\right)$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2}$ $= \frac{2 \pm \sqrt{8}}{2}$ $= \frac{2 \pm 2\sqrt{2}}{2}$ $= 1 \pm \sqrt{2}$ <p>$\therefore -\tan\left(\frac{\pi}{8}\right) = 1 - \sqrt{2}$ (as $-\tan\left(\frac{\pi}{8}\right) < 0$)</p> <p>and $\cot\left(\frac{\pi}{8}\right) = 1 + \sqrt{2}$</p> <p>$\therefore \tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1$</p>	<p>ME-F2 Polynomials ME-T2 Further Trigonometric Identities ME11-1, 11-2, 11-7 Bands E3-E4</p> <ul style="list-style-type: none"> Provides the correct solution 2 <hr/> <ul style="list-style-type: none"> Substitutes $\theta = \frac{\pi}{8}$ into the equation and makes some progress 1
<p>(iii) Substitute $\theta = \frac{\pi}{16}$.</p> <p>So, $x = -\tan\left(\frac{\pi}{16}\right)$ and $x = \cot\left(\frac{\pi}{16}\right)$ are roots of:</p> $x^2 - 2\left(\cot\left(\frac{\pi}{8}\right)\right)x - 1 = 0$ <p>Sum of roots:</p> $-\tan\left(\frac{\pi}{16}\right) + \cot\left(\frac{\pi}{16}\right) = \frac{2\left(\cot\frac{\pi}{8}\right)}{1}$ $\cot\left(\frac{\pi}{16}\right) - \tan\left(\frac{\pi}{16}\right) = 2(1 + \sqrt{2})$ <p>(from part (c)(ii))</p> <p><i>Note: Consequential on answer to Question 14(c)(ii).</i></p>	<p>ME-F2 Polynomials ME-T2 Further Trigonometric Identities ME11-1, 11-2, 11-7 Bands E3-E4</p> <ul style="list-style-type: none"> Provides the correct solution using part (c)(ii) 1