Neap

Trial Examination 2022

HSC Year 12 Mathematics Extension 2

Solutions and Marking Guidelines

SECTION I

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 1 **B** **B** is correct. Multiplication by $-i$ will cause a rotation of 90° clockwise, and multiplication by 3 will cause an enlargement factor of 3. **A** is incorrect. This option would cause a rotation of 90° anticlockwise. **C** and **D** are incorrect. The correct solution in Euler form is $\omega = -3e^{\frac{\pi}{2}i}$.	MEX–N1 Introduction to Complex Numbers MEX12–1, 12–4 Band E2
Question 2 **C** $x = 5\sin 3t + 12\cos 3t$ $= 13\left(\frac{5}{13}\sin 3t + \frac{12}{13}\cos 3t\right)$ $= 13\left(\sin\theta \sin 3t + \cos\theta \cos 3t\right)$, where $\sin\theta = \frac{5}{13}$ and $\cos\theta = \frac{12}{13}$ $= 13\cos\left(3t + \theta\right)$ $\therefore \dot{x} = -39\sin\left(3t + \theta\right)$ $\therefore \ddot{x} = -117\cos\left(3t + \theta\right)$ $0 = -117\cos\left(3t + \theta\right)$ $\left(3t + \theta\right) = \frac{\pi}{2}$ $\therefore \dot{x} = -39$ Hence, the particle's greatest speed (and its speed as it passes through the centre of its motion) is 39 m/s.	MEX–M1 Applications of Calculus to Mechanics MEX12–6, 12–7 Band E3

Answer and explanation	**Syllabus content, outcomes and targeted performance bands**
Question 3 **A** $\dfrac{12x-3}{(x-2)(x^2-3x+2)} = \dfrac{12x-3}{(x-2)(x-1)(x-2)}$ $= \dfrac{12x-3}{(x-2)^2(x-1)}$ $= \dfrac{A}{x-1} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$ $\therefore 12x-3 = A(x-2)^2 + B(x-1)(x-2) + C(x-1)$ When $x = 2$, $C = 21$. When $x = 1$, $A = 9$. When $x = 0$: $-3 = 4(9) + 2B - 21$ $B = -9$ $\therefore \dfrac{12x-3}{(x-2)(x^2-3x+2)} = \dfrac{9}{x-1} - \dfrac{9}{x-2} + \dfrac{21}{(x-2)^2}$	MEX–C1 Further Integration MEX12–5 Band E3
Question 4 **C** **C** is correct. $\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = -2 + 6 - 4$ $= 0$ $x: 1 = -1 + 2\lambda$ $\lambda = 1$ $y: 3 = 0 + 3\lambda$ $\lambda = 1$ $z: -2 = 2 - 4\lambda$ $\lambda = 1$ As the dot product of the direction vectors is zero, the two vectors are perpendicular. As the value of λ is consistent for x, y and z, the line passes through point (1, 3, –2). **A** and **B** are incorrect. The direction vectors do not produce a dot product of zero. **D** is incorrect. Solving for x gives $1 = -1 - \lambda$, $\lambda = -2$. However, substituting this value into the y-coordinate gives $y = -2$, which is inconsistent with the y value of the given point.	MEX–V1 Further Work with Vectors MEX12–3 Bands E2–E3

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 5 D Rewriting the locus gives: $\arg\left(\frac{z-2}{z+2i}\right)=\arg(z-2)-\arg(z+2i)$ $=\frac{\pi}{4}$ Therefore, the angle formed by ray $z-2$ has to be bigger than that of $z+2i$.	MEX–N2 Using Complex Numbers MEX12–4 Band E3
Question 6 C $a=e^{-3t}$ $v=\int a\,dt$ $=-\frac{1}{3}e^{-3t}+C$ $t=0,\ v=2,\ C=2\frac{1}{3}$ $\therefore v=-\frac{1}{3}e^{-3t}+2\frac{1}{3}$ $\lim_{x\to\infty} v=2\frac{1}{3}$ $=2.33\text{ m/s}$	MEX–M1 Applications of Calculus to Mechanics MEX12–6 Bands E2–E3
Question 7 C **C** is correct. $x:1+2\lambda=2-\mu$ $y:-2+\lambda=3+\mu$ $\therefore \lambda=2,\ \mu=-3$ $z:4(2)\neq 1-2(-3)$ As the solutions are inconsistent, the lines will not intersect. $\begin{pmatrix}2\\1\\4\end{pmatrix}\bullet\begin{pmatrix}-1\\1\\-2\end{pmatrix}=-9$ $\cos\theta=\frac{-9}{\sqrt{2^2+1^2+4^2}\sqrt{(-1)^2+1^2+(-2)^2}}$ $\theta=2.50\text{ rad}$ **A** and **B** are incorrect. These options state that the lines will intersect, which is incorrect. **D** is incorrect. Two skew lines will still have an angle between them.	MEX–V1 Further Work with Vectors MEX12–3 Bands E2–E3

Answer and explanation	Syllabus content, outcomes and targeted performance bands
Question 8 **C** **C** is correct. In a negation statement, the phrases 'there exist' and 'for all' must be interchanged, 'and' and 'or' must be interchanged, '=' must change to '≠' and '>' must change to '≤'. **A**, **B** and **D** are incorrect. These options do not show an accurate negation of the statement.	MEX–P1 The Nature of Proof MEX12–2, 12–8 Band E2
Question 9 **C** **C** is correct. This option is the contrapositive of the original statement. As the contrapositive of a statement '*A* implies *B*' is 'not *B* implies not *A*', this option is the contrapositive of the original statement and, hence, is logically equivalent. **A** is incorrect. This option is the converse of the original statement. **B** is incorrect. This option is the inverse of the original statement. **D** is incorrect. The correct statement is 'I will have the flu only if you have the flu'.	MEX–P1 The Nature of Proof MEX12–2, 12–8 Band E2
Question 10 **A** **A** is correct. The particle's path is a spiral as the amplitudes of the sine and cosine functions are increasing with t. When $t=\frac{\pi}{2}$, $x=\frac{\pi}{2}$ and $y=0$. When $t=\pi$, $x=0$ and $y=-\pi$. Hence, the direction of movement is clockwise. **B** is incorrect. This option gives the incorrect direction of travel. **C** and **D** are incorrect. The path is not a helix.	MEX–V1 Further Work with Vectors MEX12–1, 12–3 Bands E3–E4

SECTION II

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 11	
(a) LHS – RHS: $\cos x - 1 + x$ Let $f(x) = \cos x - 1 + x$ $f'(x) = -\sin x + 1$ $-1 \le \sin x \le 1$ $-1 \le -\sin x \le 1$ $0 \le -\sin x + 1 \le 2$ $\therefore f'(x) \ge 0$ Hence, $f(x)$ is either increasing or stationary. Finding the minimum point at $x = 0$ gives: $f(0) = \cos(0) - 1 + 0$ $= 0$ $\therefore f(x) > 0$ when $x > 0$ Since $\cos x - 1 - x > 0$, $\cos x > 1 - x$.	MEX–P1 The Nature of Proof MEX12–1, 12–2, 12–8 Band E3 • Provides the correct solution . . . . . 3 • Proves that $f'(x) \ge 0$. AND • Provides the correct reasoning. . . . 2 • Proves that $f'(x) \ge 0$ OR equivalent merit. 1
(b) $\left(1 - \sqrt{3}i\right)^n = 2^n cis\left(-\frac{n\pi}{3}\right)$ $\because \text{Re}(z) = 0$ $\cos\left(-\frac{n\pi}{3}\right) = 0$ $\cos\left(\frac{n\pi}{3}\right) = 0$ $\frac{n\pi}{3} = \frac{\pi}{2} \pm k\pi,\ k \in \mathbb{Z}$ $n\pi = \frac{3\pi \pm 6k\pi}{2}$ $n = \frac{3 \pm 6k}{2},\ k \in \mathbb{Z}$	MEX–N2 Using Complex Numbers MEX12–1, 12–4 Band E3 • Provides the correct solution . . . . . 3 • Applies De Moivre's theorem. AND • Provides the correct general solution for cosine . . . . 2 • Applies De Moivre's theorem. OR • Provides the correct general solution for cosine. OR • Equivalent merit 1

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Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) Let $\overrightarrow{OQ}=a+bi$. $\overrightarrow{QP}=\overrightarrow{OP}-\overrightarrow{OQ}$ $=(-1+5i)-(a+bi)$ $\because \overrightarrow{QO}=i\overrightarrow{QP}$: $-(a+bi)=i\left[(-1+5i)-(a+bi)\right]$ $=(b-5)+(-1-a)i$ $-a=b-5$ $-b=-1-a$ $a=2,\ b=3$ $\therefore \overrightarrow{OQ}=2+3i$	MEX–N1 Introduction to Complex Numbers MEX12–1, 12–4 Band E3 • Provides the correct solution . . . . . 2 • Uses $\overrightarrow{QO}=i\overrightarrow{QP}$ AND equates the real and imaginary parts. OR • Equivalent merit 1
(d) $\int \frac{4x-1}{x^2+2x+6}dx=\int \frac{4x+4-5}{x^2+2x+6}dx$ $=2\int \frac{2x+2}{x^2+2x+6}dx$ $-5\int \frac{1}{x^2+2x+6}dx$ $=2\ln\left\|x^2+2x+6\right\|$ $-5\int \frac{1}{(x+1)^2+5}dx$ $=2\ln\left\|x^2+2x+6\right\|$ $-\sqrt{5}\tan^{-1}\left(\frac{x+1}{\sqrt{5}}\right)+C$	MEX–C1 Further Integration MEX12–5 Band E3 • Provides the correct solution . . . . . 3 • Separates the expression into two fractions. AND • Derives $2\ln\left\|x^2+2x+6\right\|$ . . . . 2 • Separates the expression into two fractions. OR • Derives $2\ln\left\|x^2+2x+6\right\|$. OR • Equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(e) $F = ma,\ m = 1,\ \therefore F = a$ $a_{\text{resistance}} = -kv,\ a = 40,\ v = 10\ \ \therefore k = -4$ $\therefore \dfrac{dv}{dt} = g - 4v,\ \dfrac{dt}{dv} = \dfrac{1}{g-4v}$ $\int dt = \int \dfrac{1}{g-4v}\,dv$ $t = \dfrac{-1}{4}\ln\lvert g-4v\rvert + C$ $t = 0,\ v = 0$ $\therefore 0 = \dfrac{-1}{4}\ln\lvert 10\rvert + C,\ C = \dfrac{1}{4}\ln 10$ $t = \dfrac{-1}{4}\ln\lvert 10-4v\rvert + \dfrac{1}{4}\ln 10$ $= \dfrac{1}{4}\ln\left\lvert\dfrac{10}{10-4v}\right\rvert$ $\dfrac{10}{10-4v} = e^{4t}$ $10 - 4v = \dfrac{10}{e^{4t}}$ $v = \dfrac{5}{2} - \dfrac{5}{2e^{4t}}$	MEX–M1 Applications of Calculus to Mechanics MEX12–6, 12–7 Band E3 • Provides the correct solution . . . . . 4 • Provides the correct integral. AND • Provides the correct integration. AND • Provides the correct value of *t*. AND • Provides the correct expression of *v* in terms of *t*. . . . . 3 • Provides the correct integral. AND • Provides the correct integration. OR • Provides the correct value of *t*. OR • Provides the correct expression of *v* in terms of *t*. . . . . 2 • Provides the correct integral. OR • Provides the correct integration. OR • Provides the correct value of *t*. OR • Provides the correct expression of *v* in terms of *t*. OR • Equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 12	
(a) Let $P(z)=z^3+6z-4\sqrt{2}i$. $P'(z)=3z^2+6$ $0=3z^2+6$ $z=\pm\sqrt{2}i$ $P(\sqrt{2}i)=(\sqrt{2}i)^3+6(\sqrt{2}i)-4\sqrt{2}i$ $=-2\sqrt{2}i+6\sqrt{2}i-4\sqrt{2}i$ $=0$ $P(-\sqrt{2}i)=(-\sqrt{2}i)^3+6(-\sqrt{2}i)-4\sqrt{2}i$ $=2\sqrt{2}i-6\sqrt{2}i-4\sqrt{2}i$ $\neq 0$ $\therefore \sqrt{2}i$ is a root $\therefore \alpha=\sqrt{2}i$ $(z-\alpha)^2=z^2-2(\sqrt{2}i)z+(\sqrt{2}i)^2$ $=z^2-2\sqrt{2}zi-2$ $z^3+6z-4\sqrt{2}i=(z^2-2\sqrt{2}zi-2)(z+2\sqrt{2}i)$ $\therefore \beta=-2\sqrt{2}i$	MEX–N2 Using Complex Numbers MEX12–1, 12–4, 12–7 Band E3 • Provides the correct solution . . . . . 3 • Equates the differential to zero. AND • Provides the correct value for α. AND • Provides the correct value for β . . 2 • Equates the differential to zero. OR • Provides the correct value for α. OR • Provides the correct value for β. OR • Equivalent merit 1
(b) (i) If the square of the sum of x and y is odd, then x is odd and y is even.	MEX–P1 The Nature of Proof MEX12–2 Band E2 • Provides the correct solution 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) Proving the original statement 'if *x* is odd and *y* is even, then the square of the sum of *x* and *y* is also odd': Let $x = 2M + 1$ and $y = 2N$, where $M, N \in \mathbb{Z}$. $(x+y)^2 = ((2M+1)+2N)^2$ $= (2M+1)^2 + 4(2M+1)(2N) + (2N)^2$ $= 4M^2 + 4M + 1 + 4((2M+1)(2N) + N^2)$ $= 2(2M^2 + 2M + 2((2M+1)(2N) + N^2)) + 1$ $\therefore (x+y)^2$ is odd Proving the converse statement 'if the square of the sum of *x* and *y* is odd, then *x* is odd and *y* is even': The contrapositive statement is 'if *x* and *y* are both even or both odd, then the square of the sum of *x* and *y* is even'. Let $x = 2M$ and $y = 2N$, where $M, N \in \mathbb{Z}$. $(x+y)^2 = (2M+2N)^2$ $= 4M^2 + 4MN + 4N^2$ $= 2(2M^2 + 2MN + 2N^2)$ $\therefore (x+y)^2$ is even Let $x = 2M + 1$ and $y = 2N + 1$, $M, N \in \mathbb{Z}$. $(x+y)^2 = ((2M+1)+(2N+1))^2$ $= (2M+1)^2 + 2(2M+1)(2N+1) + (2N+1)^2$ $= (4M^2 + 4M + 1) + 2(4MN + 2M + 2N + 1) + (4N^2 + 4N + 1)$ $= 4M^2 + 8M + 8MN + 4N^2 + 8N + 4$ $= 2(2M^2 + 4M + 4MN + 2N^2 + 4N + 2)$ $\therefore (x+y)^2$ is even Therefore, the statement is true by contrapositive.	MEX–P1 The Nature of Proof MEX12–2, 12–8 Band E3 • Provides the correct solution . . . . . 3 • Proves the original statement. AND • Proves that if *x* and *y* are both even, then $(x+y)^2$ must be even. AND • Proves that if *x* and *y* are both odd, then $(x+y)^2$ must be even . . . . . 2 • Proves the original statement OR equivalent merit. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) Let $u=\sin^{-1}x,\ \therefore du=\dfrac{1}{\sqrt{1-x^2}}dx$ $\dfrac{dv}{dx}=x,\ \therefore v=\dfrac{1}{2}x^2$ $\int x\sin^{-1}x\,dx=\dfrac{1}{2}x^2\sin^{-1}x-\int\dfrac{1}{2}x^2\times\dfrac{1}{\sqrt{1-x^2}}dx$ $=\dfrac{1}{2}x^2\sin^{-1}x-\dfrac{1}{2}\int\dfrac{x^2}{\sqrt{1-x^2}}dx$ Let $x=\sin\theta,\ \therefore dx=\cos\theta d\theta$ $\int\dfrac{x^2}{\sqrt{1-x^2}}dx=\int\dfrac{\sin^2\theta}{\sqrt{1-\sin^2\theta}}\cos\theta d\theta$ $=\int\sin^2\theta d\theta$ $=\dfrac{1}{2}\int(1-\cos 2\theta)d\theta$ $=\dfrac{1}{2}\theta-\dfrac{1}{4}\sin 2\theta+C_1$ $=\dfrac{1}{2}\sin^{-1}x-\dfrac{1}{2}x\sqrt{1-x^2}+C_2$ $\int x\sin^{-1}x\,dx=\dfrac{1}{2}x^2\sin^{-1}x-\dfrac{1}{2}\left(\dfrac{1}{2}\sin^{-1}x-\dfrac{1}{2}x\sqrt{1-x^2}+C_2\right)$ $=\dfrac{1}{2}x^2\sin^{-1}x-\dfrac{1}{4}\sin^{-1}x+\dfrac{1}{4}x\sqrt{1-x^2}+C_3$	MEX–C1 Further Integration MEX12–5 Band E3 • Provides the correct solution . . . . . 4 • Changes θ back into x and arrives at $\dfrac{1}{2}\sin^{-1}x-\dfrac{1}{2}x\sqrt{1-x^2}+C_2$ . . . 3 • Uses by parts AND substitution. AND • Integrates and arrives at $\dfrac{1}{2}\theta-\dfrac{1}{4}\sin 2\theta+C$ . . . . 2 • Uses by parts OR substitution. OR • Equivalent merit 1
(d) (i) $\overrightarrow{OA}\bullet\overrightarrow{OB}=\begin{pmatrix}3\\3\\3\end{pmatrix}\bullet\begin{pmatrix}6\\0\\0\end{pmatrix}$ $=18$ $18=\sqrt{27}\sqrt{36}\cos\angle AOB$ $\therefore\cos\angle AOB=\dfrac{1}{\sqrt{3}}$ $\sin\angle AOB=\dfrac{\sqrt{6}}{3}$	MEX–V1 Further Work with Vectors MEX12–3 Band E3 • Provides the correct solution . . . . . 2 • Uses the dot product to find the cos angle OR equivalent merit. . . . 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) $\text{area} = \frac{1}{2}\lvert OA\rvert\lvert OB\rvert \sin\angle AOB$ $= \frac{1}{2}\times\sqrt{27}\times 6\times\frac{\sqrt{6}}{3}$ $= \frac{1}{2}\lvert OB\rvert h$ $\therefore \frac{1}{2}\times\sqrt{27}\times 6\times\frac{\sqrt{6}}{3} = \frac{1}{2}\times\sqrt{36}\times h$ $h = 3\sqrt{2}$ *Note: Consequential on answer to* ***Question 12(d)(i)****.*	MEX–V1 Further Work with Vectors MEX12–1, 12–3, 12–7, 12–8 Band E3 • Provides the correct solution . . . . . 2 • Uses the area of the triangle to find h OR equivalent merit. 1
Question 13	
(a) (i) $L: (3+t)\underset{\sim}{i} + (4-2t)\underset{\sim}{j} + (t-1)\underset{\sim}{k}$ $P: x = 3+t,\ y = 4-2t,\ z = t-1$ $\overrightarrow{PQ} = \begin{pmatrix}2\\1\\3\end{pmatrix} - \begin{pmatrix}3+t\\4-2t\\t-1\end{pmatrix}$ $= \begin{pmatrix}-1-t\\-3+2t\\4-t\end{pmatrix}$ $\because \overrightarrow{PQ}\bullet\begin{pmatrix}1\\-2\\1\end{pmatrix} = 0$ $(-1-t) - 2(-3+2t) + (4-t) = 0$ $9-6t = 0$ $t = \frac{3}{2}$ $\lvert\overrightarrow{PQ}\rvert = \left\lvert\begin{pmatrix}-\frac{5}{2}\\0\\\frac{5}{2}\end{pmatrix}\right\rvert$ $= \frac{5\sqrt{2}}{2}$	MEX–V1 Further Work with Vectors MEX12–3 Band E3 • Provides the correct solution . . . . . 3 • Finds the vectors $\overrightarrow{PQ}$. AND • Uses the dot product of $\overrightarrow{PQ}$ and the direction vector of line L to find t . . . . 2 • Finds the vectors $\overrightarrow{PQ}$. OR • Uses the dot product of $\overrightarrow{PQ}$ and the direction vector of line L to find t. OR • Equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) $\lvert PQ\rvert : \lvert PR\rvert = 1:3$ If $\overrightarrow{PR}$ is in the same direction as $\overrightarrow{PQ}$, $\lvert PQ\rvert : \lvert QR\rvert = 1:2$. $\therefore \overrightarrow{QR} = 2\overrightarrow{PQ}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 2\begin{pmatrix} -\frac{5}{2} \\ 0 \\ \frac{5}{2} \end{pmatrix}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 8 \end{pmatrix}$ If $\overrightarrow{PR}$ is in the opposite direction to $\overrightarrow{PQ}$, $\overrightarrow{PR} = -3\overrightarrow{PQ}$. P: $x = 3 + \frac{3}{2} = \frac{9}{2}$, $y = 4 - 2\left(\frac{3}{2}\right) = 1$, $z = \frac{3}{2} - 1 = \frac{1}{2}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} \frac{9}{2} \\ 1 \\ \frac{1}{2} \end{pmatrix} = -3\begin{pmatrix} -\frac{5}{2} \\ 0 \\ \frac{5}{2} \end{pmatrix}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 1 \\ -7 \end{pmatrix}$ $\therefore R: (-3, 1, 8)$ or $(12, 1, -7)$ *Note: Consequential on answer to* ***Question 13(a)(i).***	MEX–V1 Further Work with Vectors MEX12–3 Band E3 • Provides the correct solution . . . . . 2 • Finds ONE possible coordinate of *R* OR equivalent merit. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(b) Let $t = \tan\frac{x}{2}$ $\therefore \frac{dt}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$ $\frac{dx}{dt} = 2\cos^2\frac{x}{2}$ $= \frac{2}{t^2+1}$ $\int \frac{dx}{1+2\sin x - \cos x} = \int \frac{\frac{2}{t^2+1}dt}{1+2\left(\frac{2t}{1+t^2}\right)-\left(\frac{1-t^2}{1+t^2}\right)}$ $= \int \frac{\frac{2}{t^2+1}dt}{\frac{1+t^2+4t-1+t^2}{1+t^2}}$ $= \int \frac{1}{t^2+2t}dt$ $= \int \frac{1}{t(t+2)}dt$ Let $\frac{1}{t(t+2)} = \frac{A}{t} + \frac{B}{t+2}$ $1 = A(t+2) + B(t)$ $\therefore A = \frac{1}{2},\ B = -\frac{1}{2}$ $\int \frac{dx}{1+2\sin x - \cos x} = \frac{1}{2}\int \frac{1}{t} - \frac{1}{t+2}dt$ $= \frac{1}{2}\ln\left\|\frac{t}{t+2}\right\| + C_1$ $= \frac{1}{2}\ln\left\|\frac{\tan\frac{x}{2}}{\tan\frac{x}{2}+2}\right\| + C_2$	MEX–C1 Further Integration MEX12–5 Band E3 • Provides the correct solution . . . . . 4 • Substitutes t and arrives at $\int \frac{\frac{2}{t^2+1}dt}{1+2\left(\frac{2t}{1+t^2}\right)-\left(\frac{1-t^2}{1+t^2}\right)}$. AND • Performs the correct simplification AND factorisation, arriving at $\int \frac{1}{t(t+2)}dt$. AND • Uses partial fractions to integrate into $\frac{1}{2}\ln\left\|\frac{t}{t+2}\right\| + C_1$ . . . . 3 • Substitutes t and arrives at $\int \frac{\frac{2}{t^2+1}dt}{1+2\left(\frac{2t}{1+t^2}\right)-\left(\frac{1-t^2}{1+t^2}\right)}$. AND • Performs the correct simplification AND factorisation, arriving at $\int \frac{1}{t(t+2)}dt$. OR • Uses partial fractions to integrate into $\frac{1}{2}\ln\left\|\frac{t}{t+2}\right\| + C_1$ . . . . 2 • Substitutes t and arrives at $\int \frac{\frac{2}{t^2+1}dt}{1+2\left(\frac{2t}{1+t^2}\right)-\left(\frac{1-t^2}{1+t^2}\right)}$. OR • Equivalent merit 1

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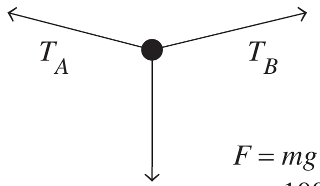
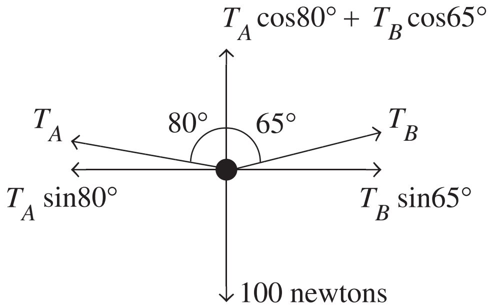
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) (i)  $F = mg$ $= 100$ newtons	MEX–M1 Applications of Calculus to Mechanics MEX12–6 Band E2 • Draws a correct diagram with all forces labelled. 1
(ii)  Vertical forces: $T_A\cos 80° + T_B\cos 65° = 100$ Horizontal forces: $T_A\sin 80° = T_B\sin 65°$ $T_A = \dfrac{T_B\sin 65°}{\sin 80°}$ $\therefore \left(\dfrac{T_B\sin 65°}{\sin 80°}\right)\cos 80° + T_B\cos 65° = 100$ $T_B = \dfrac{100}{\left(\dfrac{\sin 65°}{\sin 80°}\right)\cos 80° + \cos 65°}$ $= 171.70$ newtons $T_A = 158.01$ newtons	MEX–M1 Applications of Calculus to Mechanics MEX12–6, 12–7 Bands E2–E3 • Provides the correct solution . . . . . 2 • Provides the correct equation for the vertical forces and the correct equation for the horizontal forces. OR • Equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(d) $\ddot{x} = 9x^2$ $= \frac{d}{dx}\left(\frac{1}{2}\dot{x}^2\right)$ $\frac{1}{2}\dot{x}^2 = \int 9x^2 dx$ $\dot{x}^2 = 6x^3 + C$ $\dot{x} = -\sqrt{6},\ x = 1,\ C = 0$ $\therefore \dot{x} = -\sqrt{6x^3}$ $\because \frac{dx}{dt} = -\sqrt{6x^3},\ \frac{dt}{dx} = -\frac{1}{\sqrt{6x^3}}$ $t = -\frac{1}{\sqrt{6}}\int \frac{1}{\sqrt{x^3}} dx$ $= \frac{2}{\sqrt{6}} x^{-\frac{1}{2}} + C$ $t = 0,\ x = 1,\ C = -\frac{2}{\sqrt{6}}$ $\therefore t = \frac{2}{\sqrt{6x}} - \frac{2}{\sqrt{6}}$ $t + \frac{2}{\sqrt{6}} = \frac{2}{\sqrt{6x}}$ $\frac{2\sqrt{6}}{2 + t\sqrt{6}} = \sqrt{6x}$ $x = \frac{4}{\left(2 + t\sqrt{6}\right)^2}$	MEX–M1 Applications of Calculus to Mechanics MEX12–6, 12–7 Band E3 • Provides the correct solution . . . . . 3 ——— • Provides the correct integration and substitution of limits to derive $\dot{x} = -\sqrt{6x^3}$. AND • Provides the correct integration and substitution of limits to derive $t = \frac{2}{\sqrt{6x}} - \frac{2}{\sqrt{6}}$ . . . . 2 ——— • Provides the correct integration and substitution of limits to derive $\dot{x} = -\sqrt{6x^3}$ OR $t = \frac{2}{\sqrt{6x}} - \frac{2}{\sqrt{6}}$. OR • Equivalent merit 1

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Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 14	
(a) (i) Let $u=\ln^n x$, $\dfrac{du}{dx}=\dfrac{n\ln^{n-1}x}{x}$. $\dfrac{dv}{dx}=x^m$, $v=\dfrac{x^{m+1}}{m+1}$ $\displaystyle\int x^m \ln^n x\,dx=\frac{x^{m+1}}{m+1}\ln^n x-\int\frac{x^{m+1}}{m+1}\frac{n\ln^{n-1}x}{x}dx$ $\displaystyle=\frac{x^{m+1}}{m+1}\ln^n x-\frac{n}{m+1}\int x^m \ln^{n-1} x\,dx$ $\displaystyle=\frac{x^{m+1}}{m+1}\ln^n x-\frac{n}{m+1}I_{n-1}$	MEX–C1 Further Integration MEX12–5 Bands E3–E4 • Provides the correct solution . . . . . 2 • Provides the correct integration by parts. OR • Provides the correct algebra for the proof. OR • Equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) $\int_1^2 x^3\ln^4 x\,dx=\left[\frac{x^4}{4}\ln^4 x\right]_1^2-I_3$ $I_3=\left[\frac{x^4}{4}\ln^3 x\right]_1^2-\frac{3}{4}I_2$ $I_2=\left[\frac{x^4}{4}\ln^2 x\right]_1^2-\frac{1}{2}I_1$ $I_1=\left[\frac{x^4}{4}\ln x\right]_1^2-\frac{1}{4}I_0$ $I_0=\int_0^1 x^3dx$ $=\frac{1}{4}\left[x^4\right]_1^2$ $=\frac{15}{4}$ $I_1=\left(\frac{2^4}{4}\ln 2-\frac{1^4}{4}\ln 1\right)-\left(\frac{1}{4}\left(\frac{15}{4}\right)\right)$ $=4\ln 2-\frac{15}{16}$ $I_2=\left(\frac{2^4}{4}\ln^2 2-\frac{1^4}{4}\ln^2 1\right)-\left(\frac{1}{2}\left(4\ln 2-\frac{15}{16}\right)\right)$ $=4\ln^2 2-2\ln 2+\frac{15}{32}$ $I_3=\left(\frac{2^4}{4}\ln^3 2-\frac{1^4}{4}\ln^3 1\right)-\left(\frac{3}{4}\left(4\ln^2 2-2\ln 2+\frac{15}{32}\right)\right)$ $=4\ln^3 2-3\ln^2 2+\frac{3}{2}\ln 2-\frac{45}{128}$ $I_4=\left(\frac{2^4}{4}\ln^4 2-\frac{1^4}{4}\ln^4 1\right)-\left(4\ln^3 2-3\ln^2 2+\frac{3}{2}\ln 2-\frac{45}{128}\right)$ $=4\ln^4 2-4\ln^3 2+3\ln^2 2-\frac{3}{2}\ln 2+\frac{45}{128}$	MEX–C1 Further Integration MEX12–5 Bands E3–E4 • Provides the correct solution . . . . . 3 • Provides TWO correct applications of the recursive formula. . . . . 2 • Calculates $I_0=\frac{15}{4}$ OR equivalent merit 1

<table>
<tr><th>Sample answer</th><th>Syllabus content, outcomes, targeted performance bands and marking guide</th></tr>
<tr><td>(b) $\omega^4 = (\omega - 2)^4$
$e^{i2k\pi}\omega^4 = (\omega - 2)^4,\ k = 1,\ 2,\ 3$
$e^{i\frac{k\pi}{2}}\omega = \omega - 2$
$\omega\left(1 - e^{i\frac{k\pi}{2}}\right) = 2$
$\omega = \dfrac{2}{1 - e^{i\frac{k\pi}{2}}}$
$\omega_1 = \dfrac{2}{1 - i}$
$= 1 + i$
$\omega_2 = \dfrac{2}{1 + 1}$
$= 1$
$\omega_3 = \dfrac{2}{1 + i}$
$= 1 - i$

Note: Alternatively, responses can expand the polynomial and solve for the roots.</td><td>MEX–N2 Using Complex Numbers
MEX12–1, 12–4, 12–7 Bands E3–E4
• Provides the correct solution 3
<hr>• Multiplies by $e^{i2k\pi}$.
AND
• Uses the correct method to find ω. 2
<hr>• Multiplies by $e^{i2k\pi}$.
OR
• Uses the correct method to find ω.
OR
• Equivalent merit . . . . 1</td></tr>
<tr><td>(c) (i) $a = \underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}$
$v = \int \underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}\,dt$
$= t\underset{\sim}{i} - t\underset{\sim}{j} - 2t\underset{\sim}{k} + C$
$t = 0,\ v = 3\underset{\sim}{i} + 4\underset{\sim}{j} + 10\underset{\sim}{k}$
$\therefore C = 3\underset{\sim}{i} + 4\underset{\sim}{j} + 10\underset{\sim}{k}$
$v = (t + 3)\underset{\sim}{i} + (4 - t)\underset{\sim}{j} + (10 - 2t)\underset{\sim}{k}$</td><td>MEX–M1 Applications of Calculus to Mechanics
MEX–V1 Further Work with Vectors
MEX12–1, 12–3, 12–6, 12–7
Bands E3–E4
• Provides the correct solution 2
<hr>• Integrates a to obtain v OR equivalent merit. . . . . 1</td></tr>
</table>

	Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii)	$\underset{\sim}{v} = (t+3)\underset{\sim}{i} + (4-t)\underset{\sim}{j} + (10-2t)\underset{\sim}{k}$ (from part (i)) $\underset{\sim}{x} = \int (t+3)\underset{\sim}{i} + (4-t)\underset{\sim}{j} + (10-2t)\underset{\sim}{k}\,dt$ $= \frac{1}{2}(t+3)^2\underset{\sim}{i} - \frac{1}{2}(4-t)^2\underset{\sim}{j} - \frac{1}{4}(10-2t)^2\underset{\sim}{k} + C$ $t = 0,\ \underset{\sim}{x} = 1.2\underset{\sim}{k}$ $\frac{6}{5}\underset{\sim}{k} = \frac{9}{2}\underset{\sim}{i} - 8\underset{\sim}{j} - 25\underset{\sim}{k} + C$ $C = -\frac{9}{2}\underset{\sim}{i} + 8\underset{\sim}{j} + \frac{131}{5}\underset{\sim}{k}$ $\therefore \underset{\sim}{x} = \left(\frac{1}{2}(t+3)^2 - \frac{9}{2}\right)\underset{\sim}{i}$ $- \left(\frac{1}{2}(4-t)^2 - 8\right)\underset{\sim}{j}$ $- \left(\frac{1}{4}(10-2t)^2 - \frac{131}{5}\right)\underset{\sim}{k}$ $\frac{1}{4}(10-2t)^2 - \frac{131}{5} = 0$ $10 - 2t = \pm 2\sqrt{\frac{131}{5}}$ $= 10.237$ $t = -0.12$ (rej), $t = 10.12$ seconds *Note: Consequential on answer to* ***Question 14(c)(i)***.	MEX–M1 Applications of Calculus to Mechanics MEX–V1 Further Work with Vectors MEX12–1, 12–3, 12–6, 12–7 Band E4 • Provides the correct solution, with the negative value rejected . . 3 • Integrates to obtain *x*. AND • Substitutes the initial conditions to obtain the correct expression for *x*. AND • Equates the *k* component to 0 . . . . 2 • Integrates to obtain *x* OR equivalent merit. 1
(iii)	$10 - 2t = 0$ (from part (ii)) $t = 5$ $-\frac{1}{4}(10 - 2(0))^2 + \frac{131}{5} = \frac{131}{5}$ $= 26.2$ metres *Note: Consequential on answer to* ***Question 14(c)(ii)***.	MEX–M1 Applications of Calculus to Mechanics MEX–V1 Further Work with Vectors MEX12–1, 12–3, 12–6, 12–7 Bands E3–E4 • Provides the correct solution 1
(iv)	$\lvert x_{\text{horizontal}} \rvert = \left\lvert \left(\frac{1}{2}(10.12+3)^2 - \frac{9}{2}\right)\underset{\sim}{i} - \left(\frac{1}{2}(4-10.12)^2 - 8\right)\underset{\sim}{j} \right\rvert$ (from part (ii)) $= 82.27$ metres *Note: Consequential on answer to* ***Question 14(c)(ii)***.	MEX–M1 Applications of Calculus to Mechanics MEX–V1 Further Work with Vectors MEX12–1, 12–3, 12–6, 12–7 Bands E3–E4 • Applies Pythagoras to the *i* and *j* components of the displacement function to obtain the correct solution 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 15	
(a) (i) $\left(\sqrt{a}-\sqrt{b}\right)^2 \geq 0$ $a-2\sqrt{ab}+b \geq 0$ $a+b \geq 2\sqrt{ab}$	MEX–P1 The Nature of Proof MEX12–2, 12–8 Band E2 • Provides the correct proof 1
(ii) $a+b \geq 2\sqrt{ab}$ $a^4b^2+b^4c^2 \geq 2\sqrt{a^4b^6c^2}$ $a^4b^2+b^4c^2 \geq 2a^2b^3c$ Without loss of generality: $b^4c^2+c^4a^2 \geq 2ab^2c^3$ $c^4a^2+a^4b^2 \geq 2a^3bc^2$ Adding all three: $a^4b^2+b^4c^2+b^4c^2+c^4a^2+c^4a^2+a^4b^2$ $\geq 2a^2b^3c+2ab^2c^3+2a^3bc^2$ $2\left(a^4b^2+b^4c^2+c^4a^2\right)$ $\geq 2\left(a^2b^3c+ab^2c^3+a^3bc^2\right)$ $\therefore a^4b^2+b^4c^2+c^4a^2$ $\geq ab^2c^3+a^2b^3c+a^3bc^2$	MEX–P1 The Nature of Proof MEX12–2, 12–7, 12–8 Bands E3–E4 • Provides the correct proof for $a^4b^2+b^4c^2 \geq 2a^2b^3c$. AND • Adds all three inequalities. AND • Provides the correct algebra to finish the proof . . . . 2 — • Provides the correct proof for $a^4b^2+b^4c^2 \geq 2a^2b^3c$ OR equivalent merit. 1
(b) (i) $(1+i\tan\theta)^n+(1-i\tan\theta)^n$ $=\left(\dfrac{\cos\theta}{\cos\theta}+\dfrac{i\sin\theta}{\cos\theta}\right)^n+\left(\dfrac{\cos\theta}{\cos\theta}-\dfrac{i\sin\theta}{\cos\theta}\right)^n$ $=\left(\dfrac{\cos\theta+i\sin\theta}{\cos\theta}\right)^n+\left(\dfrac{\cos\theta-i\sin\theta}{\cos\theta}\right)^n$ $=\dfrac{\text{cis}n\theta+\text{cis}(-n\theta)}{\cos^n\theta}$ $=\dfrac{2\cos n\theta}{\cos^n\theta}$	MEX–N2 Using Complex Numbers MEX12–1, 12–4, 12–7, 12–8 Bands E3–E4 • Provides the correct solution . . . . . 2 — • Uses trigonometric identity. OR • Uses de Moivre's theorem to derive the required result. OR • Equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) Let $z = i\tan\theta$. $(1+i\tan\theta)^2+(1-i\tan\theta)^2=\dfrac{2\cos 2\theta}{\cos^2\theta}$ $\therefore 2\cos 2\theta = 0$ $2\theta=\dfrac{\pi}{2}\pm k\pi,\ k\in\mathbb{Z}$ $\theta=\dfrac{\pi\pm 2k\pi}{4}$ $\theta_1=\dfrac{\pi}{4},\ \theta_2=\dfrac{3\pi}{4}$ $\therefore z_1=i\tan\dfrac{\pi}{4}$ $z_2=i\tan\dfrac{3\pi}{4}$ $=i\tan\left(\dfrac{-\pi}{4}\right)$ $=-i\tan\dfrac{\pi}{4}$	MEX–N2 Using Complex Numbers MEX12–1, 12–4, 12–7, 12–8 Bands E3–E4 • Provides the correct solution . . . . . 2 • Uses $\dfrac{2\cos 2\theta}{\cos^2\theta}$ to derive $2\cos 2\theta = 0$. OR • Solves the cosine function to derive the required solutions. OR • Equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) $P(2): T_1 = a,\ T_2 = a \times 2a^2b$ $\therefore T_1 + T_2 = a + 2a^3b = a\left(1+2a^2b\right)$ $\sum_{n=1}^{2} T_n = \frac{a - 2^2 a^{2(2)+1} b^2}{1-2a^2b}$ $= \frac{a\left(1-2^2a^4b^2\right)}{1-2a^2b}$ $= \frac{a\left(1-2a^2b\right)\left(1+2a^2b\right)}{1-2a^2b}$ $= a\left(1+2a^2b\right)$ $P(2)$ is true. If $P(k)$ is true, $\sum_{n=1}^{k} T_k = \frac{a - 2^k a^{2k+1} b^k}{1-2a^2b}$. $P(k + 1)$: $\sum_{n=1}^{k+1} T_n = \left(\sum_{n=1}^{k} T_n\right) + T_{k+1}$ $= \frac{a - 2^2 a^{2k+1} b^2}{1-2a^2b} + \left(T_k \times 2a^2b\right)$ This is a geometric progression series with $T_1 = a$ and $r = 2a^2b,\ T_k = a \times \left(2a^2b\right)^{k-1}$. (continues on next page)	MEX–P2 Further Proof by Mathematical Induction MEX12–1, 12–2, 12–7, 12–8 Band E4 • Provides the correct solution . . . . . 4 • Provides the correct proof for $P(2)$. AND • Expands the summation notation AND uses $P(K)$ in $P(k + 1)$. AND • Finds the closed form for T_k using geometric progression or equivalent. AND • Provides the correct algebraic manipulation of powers to prove $P(k + 1)$ . . . . 3 • Provides the correct proof for $P(2)$. AND • Expands the summation notation AND uses $P(K)$ in $P(k + 1)$. AND • Finds the closed form for T_k using geometric progression or equivalent. OR • Provides the correct algebraic manipulation of powers to prove $P(k + 1)$ . . . . 2 • Provides the correct proof for $P(2)$ OR equivalent merit. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) (continued) $\sum_{n=1}^{k+1} T_n$ $= \frac{a - 2^k a^{2k+1} b^k}{1 - 2a^2 b} + \left(\left(a \times \left(2a^2 b\right)^{k-1}\right) \times 2a^2 b\right)$ $= \frac{a - 2^k a^{2k+1} b^k}{1 - 2a^2 b} + \left(a \times \left(2a^2 b\right)^k\right)$ $= \frac{a - 2^k a^{2k+1} b^k}{1 - 2a^2 b} + 2^k a^{2k+1} b^k$ $= \frac{a - 2^k a^{2k+1} b^k}{1 - 2a^2 b} + \frac{2^k a^{2k+1} b^k \left(1 - 2a^2 b\right)}{1 - 2a^2 b}$ $= \frac{a - 2^k a^{2k+1} b^k + 2^k a^{2k+1} b^k \left(1 - 2a^2 b\right)}{1 - 2a^2 b}$ $= \frac{a - 2^k a^{2k+1} b^k + 2^k a^{2k+1} b^k - 2^{k+1} a^{2k+3} b^{k+1}}{1 - 2a^2 b}$ $= \frac{a - 2^{k+1} a^{2k+3} b^{k+1}}{1 - 2a^2 b}$ $= \frac{a - 2^{k+1} a^{2(k+1)+1} b^{k+1}}{1 - 2a^2 b}$ As $P(2)$ is true, and $P(k)$ implies $P(k+1)$, $P(n)$ is true for all $n > 1$.	
(d) $\overrightarrow{BA} = \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}$ $\left\|\overrightarrow{BA}\right\| = 3$, $\left\|\overrightarrow{BC}\right\| = 5$ Therefore, the angle bisector of ABC will have a direction vector of: $\overrightarrow{BA} + \frac{3}{5}\overrightarrow{BC} = \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} + \frac{3}{5}\begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}$ $= \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$ $\therefore L: \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$	MEX–V1 Further Work with Vectors MEX12–3, 12–7 Bands E3–E4 • Provides the correct solution . . . . . 2 • Uses the correct method to find the direction vector of the line by looking at the diagonal of a kite OR equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(e) Let t be the time after 12:00 pm when one submarine is directly above the other. $A: \begin{pmatrix} -120 \\ 210 \\ -265 \end{pmatrix} + t\begin{pmatrix} 30 \\ 45 \\ -10 \end{pmatrix}$ $B: \begin{pmatrix} 100 \\ 458 \\ -151 \end{pmatrix} + (t-0.5)\begin{pmatrix} -15 \\ -\frac{9}{2} \\ 13 \end{pmatrix}$ For one submarine to be directly above the other, the x and y coordinates must be equal. For x: $-120+30t = 100+(t-0.5)(-15)$ $-120+30t = 100+7.5-15t$ $t = \frac{227.5}{45}$ $= \frac{455}{90}$ $= 5.0\dot{5}$ Checking with y: $210+45t = 458+(t-0.5)\left(-\frac{9}{2}\right)$ $45t = 248-\frac{9}{2}t+\frac{9}{4}$ $\frac{99}{2}t = \frac{1001}{4}$ $t = \frac{1001}{198}$ $= 5.0\dot{5}$ Therefore, one submarine will be above the other at 17:03 (05:03 pm).	MEX–V1 Further Work with Vectors MEX12–3 Band E3 • Equates the x and y coordinates. AND • Arrives at a correct equation for the relationship between the t values for each submarine. AND • Provides the correct time . . . . 2 ——— • Equates the x and y coordinates OR equivalent merit. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 16	
(a) Let $I = \int \sec^3 x\, dx$ $= \int \sec x \sec^2 x\, dx$ Let $u = \sec x,\ \frac{du}{dx} = \sec x \tan x,\ \frac{dv}{dx} = \sec^2 x,\ v = \tan x$ $I = \sec x \tan x - \int \tan^2 x \sec x dx$ $= \sec x \tan x - \int \left(\sec^2 x - 1\right) \sec x dx$ $= \sec x \tan x - \int \sec^3 x dx + \int \sec x dx$ $\therefore 2I = \sec x \tan x + \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\, dx$ $= \sec x \tan x + \ln\lvert \tan x + \sec x \rvert + C$ $I = \frac{1}{2}\left(\sec x \tan x + \ln\lvert \tan x + \sec x \rvert\right) + C$	MEX–C1 Further Integration MEX12–5 Bands E3–E4 • Provides the correct solution . . . . . 3 ——— • Provides the correct integration by parts. AND • Uses trigonometric identity to find the correct expression for $2I$. AND • Provides the correct integration of $\sec x$ . . . . 2 ——— • Provides the correct integration by parts OR equivalent merit. 1
(b) (i) $x = \lambda + 1$ $y = \lambda$ $z = 2\lambda + 3$ $\lvert v - c \rvert = \sqrt{29}$ $\left\lvert \begin{pmatrix} \lambda + 1 \\ \lambda \\ 2\lambda + 3 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \right\rvert = \sqrt{29}$ $\left\lvert \begin{pmatrix} \lambda - 1 \\ \lambda + 1 \\ 2\lambda + 3 \end{pmatrix} \right\rvert = \sqrt{29}$ $(\lambda - 1)^2 + (\lambda + 1)^2 + (2\lambda + 3)^2 = 29$ $6\lambda^2 + 12\lambda - 18 = 0$ $\lambda^2 + 2\lambda - 3 = 0$ $\lambda = -3,\ \lambda = 1$ $\therefore P: (-2, -3, -3),\ Q: (2, 1, 5)$	MEX–V1 Further Work with Vectors MEX12–1, 12–3 Band E3 • Provides the correct solution . . . . . 2 ——— • Forms the correct quadratic equation OR equivalent merit 1

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Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(ii) Since the line is parallel to L and tangent to the sphere at R, CR will be perpendicular to L and bisect PQ. Let M be the mid-point of PQ and, hence, the intersection point of PQ and CR. Therefore: $P:(-2,-3,-3),\ Q:(2,1,5),\ C:(2,-1,0)$ $M:(0,-1,1)$ $\overrightarrow{CM}=\begin{pmatrix}-2\\0\\1\end{pmatrix}$ As $\overrightarrow{CR}$ is an extension of $\overrightarrow{CM}$ with a length of $\sqrt{29}$: $\overrightarrow{CR}=\sqrt{\frac{29}{5}}\begin{pmatrix}-2\\0\\1\end{pmatrix}$ $\overrightarrow{OR}=\sqrt{\frac{29}{5}}\begin{pmatrix}-2\\0\\1\end{pmatrix}+\begin{pmatrix}2\\-1\\0\end{pmatrix}$ $R:\left(2-2\sqrt{\frac{29}{5}},\ -1,\ \sqrt{\frac{29}{5}}\right)$ *Note: Consequential on answer to* ***Question 16(b)(i)***.	MEX–V1 Further Work with Vectors MEX12–1, 12–3, 12–7 Band E4 • Provides the correct solution . . . . . 2 • Finds the direction vector of $\overrightarrow{CR}$ OR equivalent merit. 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(c) $z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ $= e^{i\frac{\pi}{3}}$ $z_2 = i$ $= e^{i\frac{\pi}{2}}$ $\frac{z_1+z_2}{z_1-z_2} = \frac{e^{i\frac{\pi}{3}}+e^{i\frac{\pi}{2}}}{e^{i\frac{\pi}{3}}-e^{i\frac{\pi}{2}}}$ To obtain: $i\cot\frac{\pi}{12} = \frac{i\cos\frac{\pi}{12}}{\sin\frac{\pi}{12}}$ $= \frac{\cos\left(-\frac{\pi}{12}\right)}{-i\sin\left(-\frac{\pi}{12}\right)}$ We want: $\frac{z+\overline{z}}{z-\overline{z}} = \frac{e^{-i\frac{\pi}{12}}+e^{i\frac{\pi}{12}}}{e^{-i\frac{\pi}{12}}-e^{-i\frac{\pi}{12}}}$ $\therefore \frac{\pi}{3}+\theta = -\frac{\pi}{12},\ \frac{\pi}{2}+\theta = -\frac{\pi}{12},\ \theta = -\frac{5\pi}{12}$ $\frac{e^{i\frac{\pi}{3}}+e^{i\frac{\pi}{2}}}{e^{i\frac{\pi}{3}}-e^{i\frac{\pi}{2}}} \times \frac{e^{-i\frac{5\pi}{12}}}{e^{-i\frac{5\pi}{12}}} = \frac{e^{-i\frac{\pi}{12}}+e^{i\frac{\pi}{12}}}{e^{-i\frac{\pi}{12}}-e^{-i\frac{\pi}{12}}}$ $= \frac{2\cos\left(-\frac{\pi}{12}\right)}{-2i\sin\left(-\frac{\pi}{12}\right)}$ $= \frac{\cos\left(\frac{\pi}{12}\right)}{i\sin\left(\frac{\pi}{12}\right)}$ $= i\cot\frac{\pi}{12}$	MEX–N2 Using Complex Numbers MEX12–1, 12–4, 12–7, 12–8 Band E4 • Provides the correct solution, using algebra to cancel out the negative in the argument using the fact cosine is even and sine is odd . . . . 4 • Expresses the complex numbers in cis OR Euler form. AND • Multiplies by $\frac{e^{-i\frac{5\pi}{12}}}{e^{-i\frac{5\pi}{12}}}$. AND • Uses $\frac{2\,\mathrm{Re}(z)}{2i\,\mathrm{Im}(z)}$ . . . . 3 • Expresses the complex numbers in cis OR Euler form. AND • Multiplies by $\frac{e^{-i\frac{5\pi}{12}}}{e^{-i\frac{5\pi}{12}}}$. OR • Uses $\frac{2\,\mathrm{Re}(z)}{2i\,\mathrm{Im}(z)}$ . . . . 2 • Expresses the complex numbers in cis OR Euler form. OR • Equivalent merit 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
(d) Let $u = \frac{1}{x}$, $x = \frac{1}{u}$ $\therefore \frac{dx}{du} = \frac{-1}{u^2}$ When $x = 0$, $u = \infty$ and when $x = 1$, $u = 1$. $\int_0^1 \frac{x^3+1}{x^5+1}dx = \int_\infty^1 \frac{\left(\frac{1}{u}\right)^3+1}{\left(\frac{1}{u}\right)^5+1}\left(\frac{-1}{u^2}\right)du$ $= \int_1^\infty \frac{\frac{1}{u^3}+1}{\frac{1}{u^3}+u^2}du$ $= \int_1^\infty \frac{\frac{1+u^3}{u^3}}{\frac{1+u^5}{u^3}}du$ $= \int_1^\infty \frac{1+u^3}{1+u^5}du$ $\int_0^1 \frac{x^3+1}{x^5+1}dx = \int_1^\infty \frac{1+u^3}{1+u^5}du$ $\left[f(x)\right]_0^1 = \left[f(u)\right]_1^\infty$ Therefore, the area under the graph for $f(x)$ between 0 to 1 is half that from 0 to ∞.	MEX–C1 Further Integration MEX–P1 The Nature of Proof MEX12–2, 12–5, 12–7, 12–8 Band E4 • Provides the correct solution 4
	• Substitutes to derive $\int_\infty^1 \frac{\left(\frac{1}{u}\right)^3+1}{\left(\frac{1}{u}\right)^5+1}\left(\frac{-1}{u^2}\right)du$. AND • Changes the limits. AND • Uses algebra to derive $\int_1^\infty \frac{1+u^3}{1+u^5}du$ 3
	• Substitutes to derive $\int_\infty^1 \frac{\left(\frac{1}{u}\right)^3+1}{\left(\frac{1}{u}\right)^5+1}\left(\frac{-1}{u^2}\right)du$. AND • Changes the limits. OR • Uses algebra to derive $\int_1^\infty \frac{1+u^3}{1+u^5}du$ 2
	• Substitutes to derive $\int_\infty^1 \frac{\left(\frac{1}{u}\right)^3+1}{\left(\frac{1}{u}\right)^5+1}\left(\frac{-1}{u^2}\right)du$ OR equivalent merit. 1