



Final Examination 2023

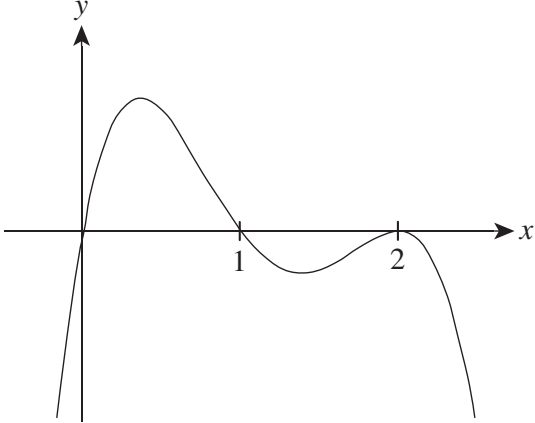
NSW Year 11 Mathematics Extension 1

Solutions and Marking Guidelines

SECTION I

Answer and explanation	Syllabus content, outcomes and targeted performance bands
<p>Question 1 D</p> <p>D is not equal to $\cos 2\theta$ and is therefore the correct response.</p> <p>Substituting the basic trigonometric identity $(\sin^2 \theta + \cos^2 \theta = 1)$ into the double angle formula for cosine does not result in $1 - 2\cos^2 \theta$; it results in $2\cos^2 \theta - 1$.</p> <p>A is incorrect. This can be obtained by applying the compound angle formula.</p> <p>B and C are incorrect. These can be obtained by substituting in values from the basic trigonometric identity $\sin^2 \theta + \cos^2 \theta = 1$.</p>	<p>ME–T2 Further Trigonometric Identities ME11–3</p> <p style="text-align: right;">Band E2</p>
<p>Question 2 B</p> $N = A e^{kt} + 1000$ $\frac{dN}{dt} = k \times A e^{kt}$ $= k (A e^{kt} + 1000 - 1000)$ $= k (N - 1000)$	<p>ME–C1 Rates of Change ME11–4</p> <p style="text-align: right;">Band E2</p>
<p>Question 3 C</p> <p>C is correct. In the equation, the basic curve is shifted left by two and stretched vertically by a factor of two to obtain the correct curve. This is shown in option C.</p> <p>A is incorrect. This graph represents the equation $y = 0.5 \cos^{-1}(x - 2)$.</p> <p>B is incorrect. This graph represents the equation $y = 2 \cos^{-1}(x - 2)$.</p> <p>D is incorrect. This graph represents the equation $y = 0.5 \cos^{-1}(x + 2)$.</p>	<p>ME–T1 Inverse Trigonometric Functions ME11–1</p> <p style="text-align: right;">Band E2</p>

Answer and explanation	Syllabus content, outcomes and targeted performance bands
<p>Question 4 D</p> $\alpha + \beta + \gamma = -\frac{b}{a}$ $= -\frac{-6}{3}$ $= 2$ $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = \frac{3}{3} = 1$ $\alpha\beta\gamma = -\frac{d}{a} = -\frac{-7}{3} = \frac{7}{3}$ $3\alpha + 3\beta + 3\gamma + 4 = 3(\alpha + \beta + \gamma) + 4$ $= 3(2) + 4$ $= 10$	<p>ME–F2 Polynomials ME11–2</p> <p style="text-align: right;">Band E2</p>
<p>Question 5 A</p> <p>A is correct. Squaring the key points shows that option A is the correct graph. When the original graph is -2 (the minimum point), the squared graph will be $(-2)^2 = 4$.</p> <p>B, C and D are incorrect. These do not correctly represent the graph of $(f(x))^2$.</p>	<p>ME–F1 Further Work with Functions ME11–1</p> <p style="text-align: right;">Band E3</p>
<p>Question 6 C</p> <p>Placing the two instances of the letter S in the first and last position gives:</p> <p>S _ _ _ _ _ S</p> <p>Therefore, the number of ways to arrange the remaining letters is $\frac{6!}{2!}$, accounting for the duplicate letter T.</p>	<p>ME–A1 Working with Combinatorics ME11–5</p> <p style="text-align: right;">Band E2</p>

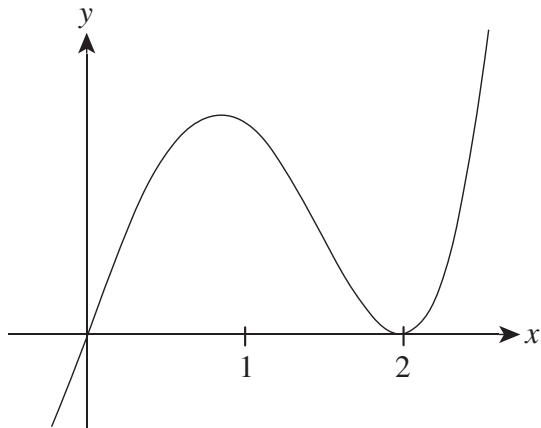
Answer and explanation	Syllabus content, outcomes and targeted performance bands
<p>Question 7 D</p> <p>Method 1:</p> <p>Evaluating $P(1.9)$, $P(2)$ and $P(2.1)$ for each equation gives:</p> <p>A: $P(1.9) > 0$, $P(2) = 0$, $P(2.1) > 0$.</p> <p>B: $P(1.9) > 0$, $P(2) = 0$, $P(2.1) > 0$.</p> <p>C: $P(1.9) < 0$, $P(2) = 0$, $P(2.1) < 0$.</p> <p>D: $P(1.9) < 0$, $P(2) = 0$, $P(2.1) < 0$.</p> <p>Either options C or D can represent the graph because the curve is below the x-axis at $x = 2$. However, C describes an equation of degree 2 and is therefore incorrect.</p> <p>Method 2:</p> <p>D is correct. The degree of this polynomial is 4, the leading coefficient is negative and there are three roots (with a double root at $x = 2$). Thus, sketching $P(x) = x(1 - x)(x - 2)^2$ gives:</p>  <p>The section of the graph around $x = 2$ matches the sketch given in the question. Therefore, D is correct.</p> <p>(continues on next page)</p>	<p>ME-F1 Further Work with Functions ME11-2</p> <p>Band E2</p>

Answer and explanation

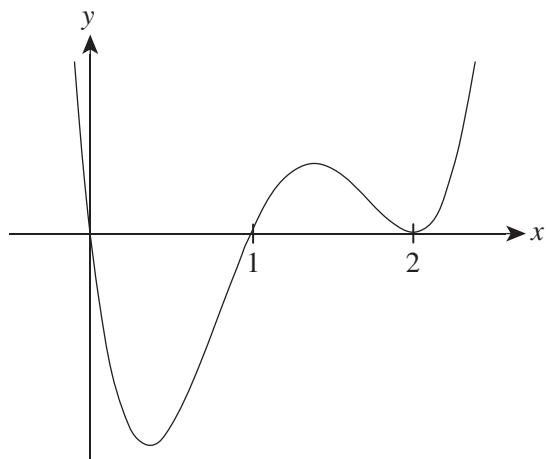
Syllabus content, outcomes and targeted performance bands

(continued)

A is incorrect. Sketching $P(x) = x(x+1)(x-2)^2$ shows that the area around $x=2$ in this graph is above the x -axis.



B is incorrect. Sketching $P(x) = x(x-1)(x-2)^2$ shows that the area around $x=2$ in this graph is above the x -axis.



C is incorrect. This equation has a degree of 2.

Question 8 **B**

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$\frac{dr}{dt} = 0.5$$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$$= 2\pi r \times 0.5$$

$$= \pi r$$


Given that $r = 10$ cm, $\frac{dA}{dt} = 10\pi$ cm²/s.

ME-C1 Rates of Change
ME11-4

Band E3

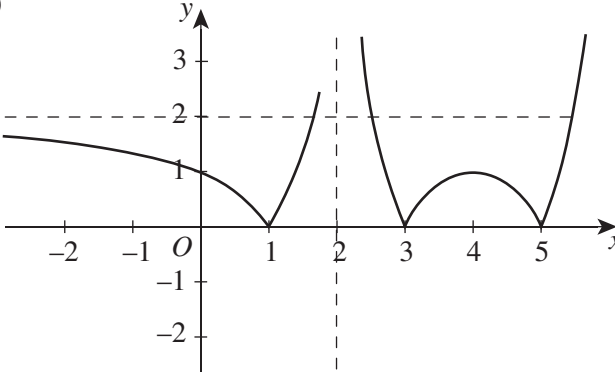
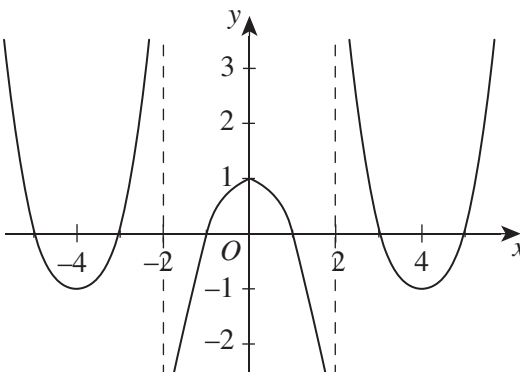
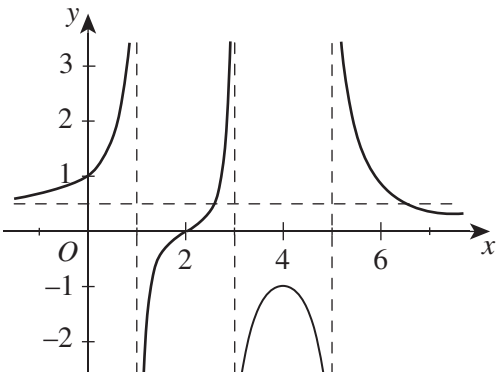
Answer and explanation	Syllabus content, outcomes and targeted performance bands
<p>Question 9 D</p> <p>Let the roots be α and 2α.</p> $3\alpha = -\frac{b}{a}$ $= \frac{18}{2}$ $= 9$ $\alpha = 3$ $2\alpha^2 = \frac{c}{a}$ $= \frac{c}{2}$ $2(3)^2 = \frac{c}{2}$ $c = 36$	<p>ME–F2 Polynomials ME11–2</p> <p style="text-align: right;">Band E3</p>
<p>Question 10 A</p> <p>A is correct. The inverse function is the reflection of the graph of the original function across the line $y = x$. This can be seen in graph A.</p> <p>B, C and D are incorrect. These graphs do not accurately show the inverse of $f(x)$.</p>	<p>ME–F1 Further Work with Functions ME11–2</p> <p style="text-align: right;">Band E3</p>

SECTION II

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>Question 11</p> <p>(a) $\frac{3}{2x-5} > 6$ $3(2x-5) > 6(2x-5)^2$ $3(2x-5) - 6(2x-5)^2 > 0$ $3(2x-5)(1-2(2x-5)) > 0$ $3(2x-5)(1-4x+10) > 0$ $3(2x-5)(11-4x) > 0$ <p>The LHS describes a concave down parabola with zeros at 2.75 and 2.5. The curve is greater than 0 between the zeros.</p> $\therefore \frac{5}{2} < x < \frac{11}{4} \text{ (or } 2.5 < x < 2.75)$  </p>	<p>ME-F1 Further Work with Functions ME11-2 Band E2</p> <ul style="list-style-type: none"> Provides the correct solution3 <hr/> <ul style="list-style-type: none"> Provides the correct critical points but incorrect region(s).2 <hr/> <ul style="list-style-type: none"> Shows some understanding of the problem1
<p>(b) $x = \sqrt{2t+4}$ $x^2 = 2t+4$ $x^2 - 4 = 2t$ $t = \frac{x^2 - 4}{2}$ $y = 2t + 1$ $y = 2\left(\frac{x^2 - 4}{2}\right) + 1$ $= x^2 - 4 + 1$ $= x^2 - 3$ <p>The domain for t is $-2 \leq t \leq 6$. Finding the domain for x: When $t = -2$, $x = 0$. When $t = 6$, $x = 4$. $\therefore y = x^2 - 3, 0 \leq x \leq 4$</p> </p>	<p>ME-F1 Further Work with Functions ME11-2 Band E3</p> <ul style="list-style-type: none"> Provides the correct solution3 <hr/> <ul style="list-style-type: none"> Provides the correct solution but gives the incorrect domain.2 <hr/> <ul style="list-style-type: none"> Shows some understanding of the problem1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(c) (i) Method 1: Arranging 8 people in a line with 10 positions gives: ${}^{10}P_8 = 1\,814\,400$</p> <p>Method 2: Selecting 8 people from 10 people and arranging them in a line gives: ${}^{10}C_8 \times 8! = 1\,814\,400$</p>	<p>ME–A1 Working with Combinatorics ME11–5 Band E2</p> <ul style="list-style-type: none"> Provides the correct solution1
<p>(ii) The four unrestricted friends can be arranged to act as ‘barriers’ for Ahmad and the group of three friends.</p> <p>These ‘barriers’ form 5 positions to place Ahmad and the group of three friends.</p> <p>Ahmad can be placed in any of the 5 positions, and the group of three can then be placed in any of the remaining 4 positions.</p> <p>Finally, the group of three can be permuted amongst themselves in 3! ways.</p> <p>Thus, the number of ways to arrange all the friends is: $4! \times 5 \times 4 \times 3! = 2800$</p>	<p>ME–A1 Working with Combinatorics ME11–5 Band E3</p> <ul style="list-style-type: none"> Provides the correct solution2 <hr/> <ul style="list-style-type: none"> Shows some understanding of the problem1
<p>(d) $\sin \frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$</p> <p>$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \frac{3\pi}{4}, 0 \leq x \leq \pi$</p>	<p>ME–T1 Inverse Trigonometric Functions ME11–5 Band E2</p> <ul style="list-style-type: none"> Provides the correct solution2 <hr/> <ul style="list-style-type: none"> Shows some understanding of the problem1
<p>(e) (i) $\cos 105^\circ = \cos(60^\circ + 45^\circ)$ $= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$ $= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$ $= \frac{1 - \sqrt{3}}{2\sqrt{2}}$ $= \frac{\sqrt{2} - \sqrt{6}}{4}$</p>	<p>ME–T2 Further Trigonometric Identities ME11–3 Band E3</p> <ul style="list-style-type: none"> Provides the correct solution2 <hr/> <ul style="list-style-type: none"> Makes some progress using $\cos(\alpha + \beta)$. OR Shows some understanding of the problem1

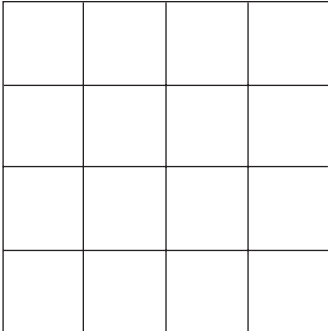
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(ii) $\cos 105^\circ = \cos(2 \times 52.5^\circ)$</p> $\frac{\sqrt{2} - \sqrt{6}}{4} = 2\cos^2 52.5^\circ - 1$ $2\cos^2 52.5^\circ = \frac{4 + \sqrt{2} - \sqrt{6}}{4}$ $\cos^2 52.5^\circ = \frac{4 + \sqrt{2} - \sqrt{6}}{8}$ $\cos 52.5^\circ = \frac{\sqrt{4 + \sqrt{2} - \sqrt{6}}}{2\sqrt{2}}, \text{ as } \cos 52.5^\circ > 0$ <p><i>Note: Consequential on answer to Question 11(e)(i).</i></p>	<p>ME–T2 Further Trigonometric Identities ME11–3 Band E3</p> <ul style="list-style-type: none"> Provides the correct solution 2 <hr/> <ul style="list-style-type: none"> Makes some progress using the double angle formula. OR Shows some understanding of the problem 1
Question 12	
<p>(a) $P(x) = (x + 1)(x - 2)Q(x) + 4x + k$</p> <p>Evaluating $P(-1)$:</p> $-2 = 4x + k$ $k = 2$ $P(2) = 4(2) + 2$ $= 10$	<p>ME–F2 Polynomials ME11–2 Band E3</p> <ul style="list-style-type: none"> Provides the correct solution 2 <hr/> <ul style="list-style-type: none"> Makes some progress using the remainder theorem to find k. OR Shows some understanding of the problem 1
<p>(b) $\sin\left(\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{\pi}{3}\right)$</p> $= \left(\sin\theta \cos\frac{\pi}{6} + \cos\theta \sin\frac{\pi}{6}\right) + \left(\cos\theta \cos\frac{\pi}{3} - \sin\theta \sin\frac{\pi}{3}\right)$ $= \frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta + \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta$ $= \cos\theta$	<p>ME–T2 Further Trigonometric Identities ME11–3 Band E3</p> <ul style="list-style-type: none"> Provides the correct solution 3 <hr/> <ul style="list-style-type: none"> Uses compound angle formulae for BOTH terms 2 <hr/> <ul style="list-style-type: none"> Shows some understanding of the problem 1
<p>(c) Using the binomial theorem:</p> $(1 - 2x)^5 = 1 - 5(2x) + 10(4x^2) - 10(8x^3) + \dots$ $= 1 - 10x + 40x^2 - 80x^3 + \dots$ <p>The term in x^3 is given by:</p> $1(-80x^3) + x(40x^2) + 2x^2(-10x)$ <p>Thus, coefficient of x^3 is:</p> $-80 + 40 - 20 = -60$	<p>ME–A1 Working with Combinatorics ME11–5 Band E3</p> <ul style="list-style-type: none"> Provides the correct solution 3 <hr/> <ul style="list-style-type: none"> Expands $(1 - 2x)^5$ correctly 2 <hr/> <ul style="list-style-type: none"> Shows some understanding of the problem 1

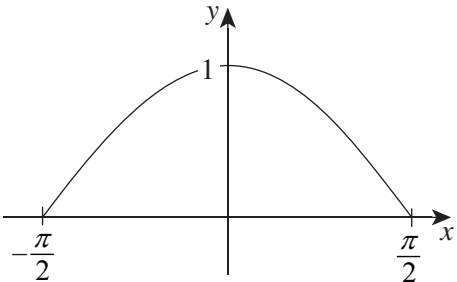
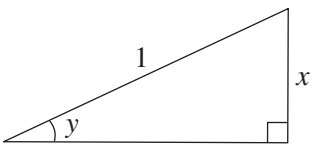
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(d) (i)</p> 	<p>ME–F1 Further Work with Functions ME11–1 Band E2</p> <ul style="list-style-type: none"> • Sketches the correct graph. 2 <hr/> <ul style="list-style-type: none"> • Provides some correct features of the graph. OR • Equivalent merit 1
<p>(ii)</p> 	<p>ME–F1 Further Work with Functions ME11–1 Band E2</p> <ul style="list-style-type: none"> • Sketches the correct graph. 2 <hr/> <ul style="list-style-type: none"> • Provides some correct features of the graph. OR • Equivalent merit 1
<p>(iii)</p> 	<p>ME–F1 Further Work with Functions ME11–1 Band E4</p> <ul style="list-style-type: none"> • Sketches the correct graph. 3 <hr/> <ul style="list-style-type: none"> • Finds the reciprocal of one branch. OR • Finds all asymptotes. OR • Makes other substantial progress. . . 2 <hr/> <ul style="list-style-type: none"> • Provides some correct features of the graph. OR • Equivalent merit 1

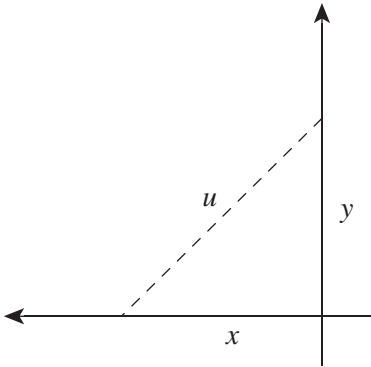
Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 13	
<p>(a) There are three pairs of repeated letters (A, M and T) and five non-repeated letters (C, E, H, I and S).</p> <p>Case 1: All five letters are different:</p> $\binom{8}{5} \times 5! = 6720$ <p>Case 2: One pair of repeated letters:</p> $\binom{3}{1} \times \binom{7}{3} \times \frac{5!}{2!} = 6300$ <p>Case 3: Two pairs of repeated letters:</p> $\binom{3}{2} \times \binom{6}{1} \times \frac{5!}{2!2!} = 540$ <p>Summing all three cases gives:</p> $6720 + 6300 + 540 = 13\,560$	<p>ME–A1 Working with Combinatorics ME11–5 Band E4</p> <ul style="list-style-type: none"> • Provides the correct solution 4 <hr/> <ul style="list-style-type: none"> • Makes substantial progress towards the correct solution. 3 <hr/> <ul style="list-style-type: none"> • Recognises the problem involves cases. AND • Makes some progress towards the correct solution. 2 <hr/> <ul style="list-style-type: none"> • Shows some understanding of the problem 1
<p>(b) (i) $P(t) = Ae^{kt}$ $P'(t) = k \times Ae^{kt}$ $= k \times P(t)$</p>	<p>ME–C1 Rates of Change ME11–4 Band E2</p> <ul style="list-style-type: none"> • Provides the correct solution 1
<p>(ii) $\frac{A}{2} = Ae^{50k}$ $0.5 = e^{50k}$ $\ln 0.5 = 50k$ $k = \frac{\ln 0.5}{50}$ $\approx -1.39\%$</p> <p>The rate of change is approximately -1.39% per year.</p>	<p>ME–C1 Rates of Change ME11–4 Band E3</p> <ul style="list-style-type: none"> • Provides the correct solution 2 <hr/> <ul style="list-style-type: none"> • Shows some understanding of the problem 1
<p>(iii) $P(80) = 3000e^{80k}$ $k = \frac{\ln 0.5}{50}$ $P(80) = 3000e^{80\left(\frac{\ln 0.5}{50}\right)}$ ≈ 989</p> <p>Thus, there will be approximately 989 animals in the population after 80 years.</p>	<p>ME–C1 Rates of Change ME11–4 Band E3</p> <ul style="list-style-type: none"> • Provides the correct solution 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(iv) Population:</p> $P(t) = 3000e^{\left(\frac{\ln 0.5}{50}\right)t}$, where $\left(\frac{\ln 0.5}{50}\right)$ is negative. <p>As $t \rightarrow \infty$, $e^{\left(\frac{\ln 0.5}{50}\right)t} \rightarrow 0$, $3000e^{\left(\frac{\ln 0.5}{50}\right)t}$ will also approach 0. Thus, the population will decrease until the animal becomes extinct.</p> <p>Rate of change:</p> $\frac{d}{dt}P(t) = k \times P(t)$ $= \left(\frac{\ln 0.5}{50}\right)e^{\left(\frac{\ln 0.5}{50}\right)t}$ <p>$e^{\left(\frac{\ln 0.5}{50}\right)t}$ will approach 0. This will not be changed by $\left(\frac{\ln 0.5}{50}\right)$. Thus, $\frac{d}{dt}P(t)$ will approach 0 and the rate of change will also approach 0.</p> <p>(If the animal becomes extinct, then the rate of change will be 0 by definition.)</p>	<p>ME–C1 Rates of Change ME11–4 Band E4</p> <ul style="list-style-type: none"> • Explains what will happen to the population AND rate of change with reference to limits (or equivalent) 2 <hr/> <ul style="list-style-type: none"> • Shows some understanding of the problem 1
<p>(c) (i) $P(t) = 500 + Ce^{kt}$ $P'(t) = k \times Ce^{kt}$ $= k(Ce^{kt} + 500 - 500)$ $= k(P(t) - 500)$</p>	<p>ME–C1 Rates of Change ME11–4 Band E2</p> <ul style="list-style-type: none"> • Provides the correct solution 1
<p>(ii) In the new equation, $P(t) = 500 + Ce^{kt}$, Ce^{kt} will approach 0 because k is negative. Thus, $P(t)$ will approach 500 as $t \rightarrow \infty$ because Ce^{kt} will approach 0.</p> <p>$P'(t) = k \times Ce^{kt}$, $P' \rightarrow 0$ as $t \rightarrow \infty$, since k is negative; thus, the rate of change will be effectively 0.</p> <p>Therefore, the population will become static at around 500 animals.</p>	<p>ME–C1 Rates of Change ME11–4 Band E3</p> <ul style="list-style-type: none"> • Explains what will happen to population AND rate of change with reference to limits (or equivalent) 2 <hr/> <ul style="list-style-type: none"> • Shows some understanding of the problem 1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(iii) $3000 = 500 + Ce^{k(0)}$ $= 500 + C$ $C = 2500$ $P(t) = 500 + 2500e^{kt}$ The population halves in 50 years: $1500 = 500 + 2500e^{k(50)}$ $\frac{1000}{2500} = e^{50k}$ $\ln\left(\frac{2}{5}\right) = 50k$ $k = \frac{\ln\left(\frac{2}{5}\right)}{50}$ The population after 80 years: $P(80) = 500 + 2500e^{k(80)}$ ≈ 1077 Thus, the population is expected to be around 1077 after 80 years.</p>	<p>ME–C1 Rates of Change ME11–4 Band E4</p> <ul style="list-style-type: none"> • Provides the correct solution2 <hr/> <ul style="list-style-type: none"> • Shows some understanding of the problem1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
Question 14	
<p>(a) Since 33 points are to be drawn inside a square with sides of 4 cm, the square must have an area of $4 \times 4 = 16 \text{ cm}^2$.</p> <p>Dividing the square into 16 equally sized squares of 1 cm^2 gives:</p> <div style="text-align: center;">  </div> <p>If three or more points are assigned within one of the 16 squares, these points will be guaranteed to form a triangle with an area less than 1 cm^2.</p> <p>Assigning two points to each square gives 32 points. The 33rd point, regardless of where it is placed, will lie in a square which already has two points. Thus, there will be three points within an area of 1 cm^2.</p> <p>Thus, it is guaranteed to be possible to find three points that form a triangle with an area less than 1 cm^2.</p>	<p>ME–A1 Working with Combinatorics ME11–5 Band E4</p> <ul style="list-style-type: none"> • Provides the correct solution3 <hr/> <ul style="list-style-type: none"> • Makes substantial progress towards the correct solution.2 <hr/> <ul style="list-style-type: none"> • Makes some progress towards recognising this as a pigeonhole principle question.1
<p>(b) Possible integer roots will be factors of -2. Thus, check $P(0)$, $P(1)$, $P(-1)$, $P(2)$ and $P(-2)$. Of these, it is found that 2 and -1 are roots of the polynomial.</p> <p>Checking the multiplicity of 2:</p> $P'(x) = 4x^3 + 3x^2 - 6x - 5$ $P'(2) \neq 0$ <p>Thus, 2 is a root with multiplicity 1.</p> <p>Checking the multiplicity of -1:</p> $P'(-1) = 0$ $P''(x) = 12x^2 + 6x - 6$ $P''(-1) = 0$ $P'''(x) = 24x + 6$ $P'''(-1) \neq 0$ <p>Therefore, -1 is a root with multiplicity 3 of $P(x)$.</p>	<p>ME–F2 Polynomials ME11–2 Band E4</p> <ul style="list-style-type: none"> • Provides the correct solution3 <hr/> <ul style="list-style-type: none"> • Differentiates at least once to show the link between differentiation and root multiplicity2 <hr/> <ul style="list-style-type: none"> • Shows some understanding of the problem1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(c) (i) $y = \sin^{-1} x$ $\cos y = \cos(\sin^{-1} x)$</p> <p>Consider $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$.</p> <p>Sketching the cosine curve in the region $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ gives:</p>  <p>$\sin^{-1} x$ will take a value between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, and $\cos \theta$ for these values will be between 0 and 1; thus, it can be concluded that $0 \leq \cos(\sin^{-1} x) \leq 1$.</p>	<p>ME–T1 Inverse Trigonometric Functions ME11–3 Band E4</p> <ul style="list-style-type: none"> Provides the correct solution2 <hr/> <ul style="list-style-type: none"> Shows some understanding of the problem1
<p>(ii) As $y = \sin^{-1} x$, $x = \sin y$.</p>  <p>Using Pythagoras' theorem, the third side is equal to $\sqrt{1-x^2}$.</p> <p>Hence, $\cos y = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$</p>	<p>ME–T1 Inverse Trigonometric Functions ME11–3 Band E4</p> <ul style="list-style-type: none"> Provides the correct solution3 <hr/> <ul style="list-style-type: none"> Makes substantial progress towards finding the length of the third side2 <hr/> <ul style="list-style-type: none"> Shows some understanding of the problem1
<p>(d) (i) $\frac{d}{dt} = \frac{d}{dx} \times \frac{dx}{dt}$</p> <p>$\therefore \frac{d}{dt}(x^2) = 2x \times \frac{dx}{dt}$</p>	<p>ME–C1 Rates of Change ME11–4 Band E4</p> <ul style="list-style-type: none"> Provides the correct solution1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(ii) Similar to $\frac{d}{dt}(x^2) = 2x \times \frac{dx}{dt}$, $\frac{d}{dt}(u^2) = 2u \times \frac{du}{dt}$ and $\frac{d}{dt}(y^2) = 2y \times \frac{dy}{dt}$.</p> <p>Representing the scenario described in the question gives:</p>  <p>Using Pythagoras' theorem, the distance between the boats is:</p> $u^2 = x^2 + y^2 \quad (1)$ <p>Differentiating (1) in terms of time:</p> $\frac{d}{dt}(u^2) = \frac{d}{dt}(x^2) + \frac{d}{dt}(y^2)$ $2u \times \frac{du}{dt} = 2x \times \frac{dx}{dt} + 2y \times \frac{dy}{dt}$ $u \frac{du}{dt} = x \frac{dx}{dt} + y \frac{dy}{dt}$ <p>Given that $\frac{dx}{dt} = 3$ and $\frac{dy}{dt} = 5$:</p> $u \frac{du}{dt} = 3x + 5y \quad (2)$ <p>(continues on next page)</p>	<p>ME–C1 Rates of Change ME11–4 Band E4</p> <ul style="list-style-type: none"> • Provides the correct solution3 <hr/> <ul style="list-style-type: none"> • Makes substantial progress using related rates2 <hr/> <ul style="list-style-type: none"> • Shows some understanding of the problem, such as finding the distance travelled by each boat OR $u^2 = x^2 + y^2$1

Sample answer	Syllabus content, outcomes, targeted performance bands and marking guide
<p>(continued)</p> <p>Finding x, y and u:</p> <p>At 2:30 pm, the first boat has travelled for 2.5 hours at 3 km/h. Thus, it has travelled 7.5 km.</p> <p>At 2:30 pm, the second boat has travelled for 1.5 hours at 5 km/h. Thus, it has also travelled 7.5 km.</p> <p>Since $u^2 = x^2 + y^2$, $x = 7.5$ and $y = 7.5$, using Pythagoras' theorem to find u gives:</p> $u = \frac{\sqrt{225}}{\sqrt{2}}$ $= \frac{15}{\sqrt{2}}$ <p>Substituting 7.5, 7.5 and $\frac{15}{\sqrt{2}}$ into (2):</p> $\frac{15}{\sqrt{2}} \frac{du}{dt} = 3(7.5) + 5(7.5)$ $\frac{du}{dt} \approx 5.64 \text{ km/h}$ <p>Therefore, the rate of change is 5.64 km/h.</p>	