Neap

Final Examination 2023

NSW Year 11 Mathematics Extension 1

General Instructions	
	• Reading time – 10 minutes • Working time – 2 hours • Write using black pen • Calculators approved by NESA may be used • A reference sheet is provided at the back of this paper • For questions in Section II, show all relevant mathematical reasoning and/or calculations

Total Marks: 70	
	Section I – 10 marks (pages 2–6) • Attempt Questions 1–10 • Allow about 15 minutes for this section **Section II – 60 marks (pages 7–10)** • Attempt Questions 11–14 • Allow about 1 hour and 45 minutes for this section

SECTION I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Which of the following is NOT equal to $\cos 2\theta$?

A. $\cos^2\theta - \sin^2\theta$

B. $1 - 2\sin^2\theta$

C. $2\cos^2\theta - 1$

D. $1 - 2\cos^2\theta$

2 The population of koalas, P, in a certain region at time t is given by $N = Ae^{kt} + 1000$, where A and k are positive constants.
Which of the following differential equations is correct?

A. $\dfrac{dN}{dt} = -k(N - 1000)$

B. $\dfrac{dN}{dt} = k(N - 1000)$

C. $\dfrac{dN}{dt} = -k(N + 1000)$

D. $\dfrac{dN}{dt} = k(N + 1000)$

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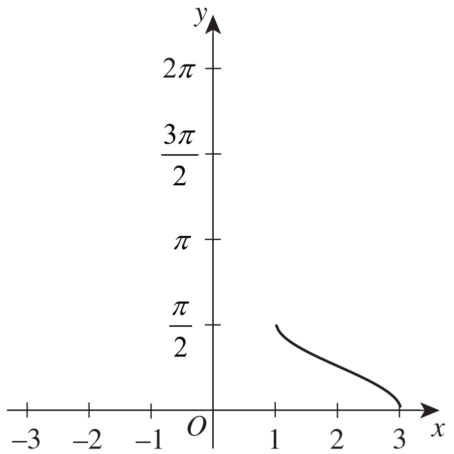
3 Consider the equation $y = 2\cos^{-1}(x+2)$.

Which of the following graphs best represents this equation?

A.



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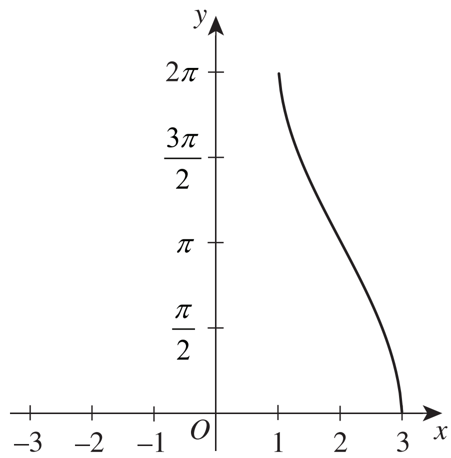

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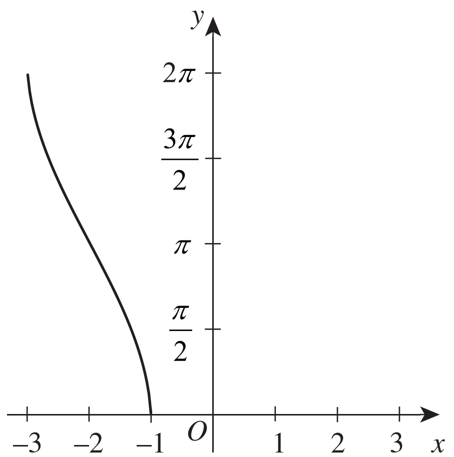
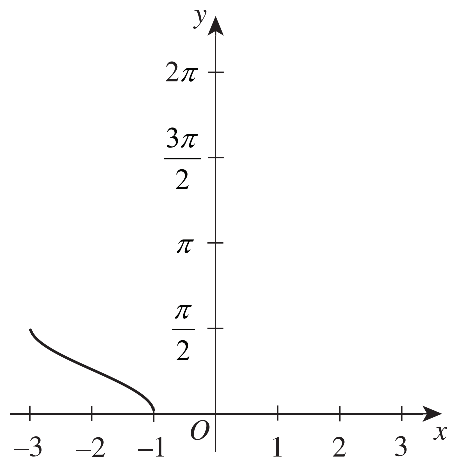


D.



4 The polynomial $P(x) = 3x^3 - 6x^2 + 3x - 7$ has roots α, β and γ.

What is the value of $3\alpha + 3\beta + 3\gamma + 4$?

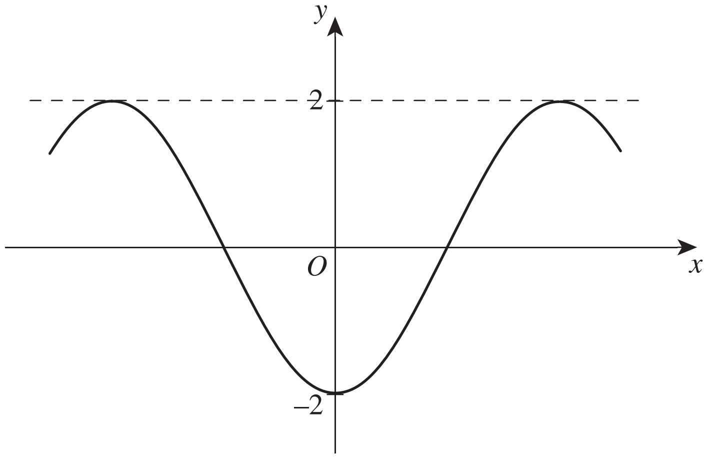
A. –6

B. 2

C. 6

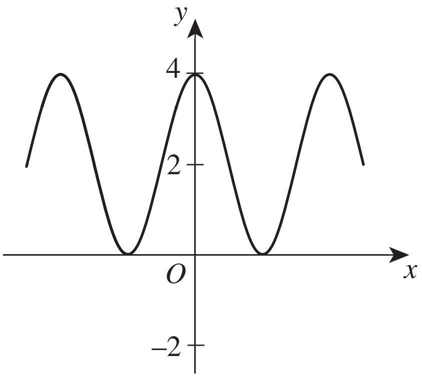
D. 10

5 The graph of the function $f(x)$ is shown.



Which of the following best represents the graph of $(f(x))^2$?

A.



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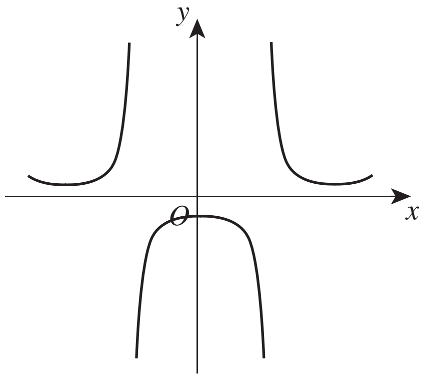
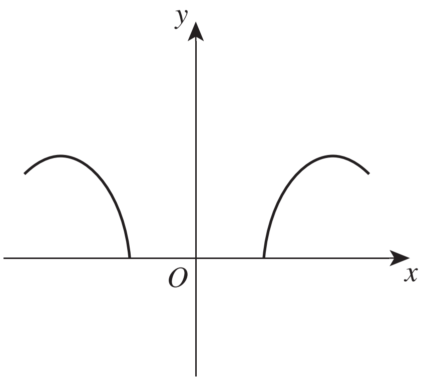
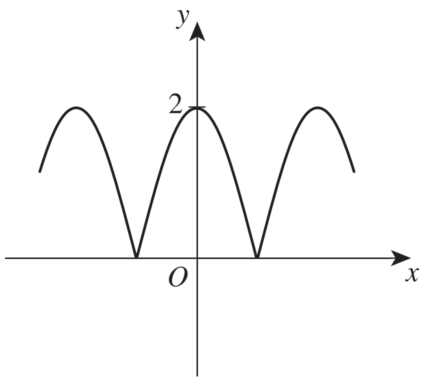


C.



D.



6 How many arrangements that begin and end with the letter S can be made from the letters of STATIONS?

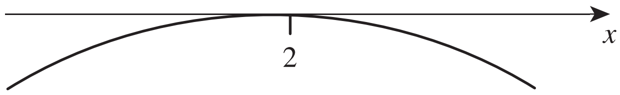
A. $\dfrac{6!}{(2!)^2}$

B. $\dfrac{8!}{(2!)^2}$

C. $\dfrac{6!}{2!}$

D. $\dfrac{8!}{2!}$

7 Let $P(x)$ be a polynomial with a degree of 4. The part of the graph near $x = 2$ is shown.



Which of the following could be the equation of the polynomial $P(x)$?

A. $P(x) = x(x + 1)(x - 2)^2$

B. $P(x) = x(x - 1)(x - 2)^2$

C. $P(x) = -(x - 2)^2$

D. $P(x) = x(1 - x)(x - 2)^2$

8 A tap is leaking droplets of water, which form a circular pool below the tap. The pool is expanding at a rate of 0.5 cm per second.
When the radius of the pool is 10 cm, what is the rate at which the area of the pool is increasing?

A. 0.25π cm^2/s

B. 10π cm^2/s

C. 20π cm^2/s

D. 100π cm^2/s

9 Consider the quadratic equation $2x^2 - 18x + c = 0$, where c is a constant such that the quadratic equation has one root that is twice the other root.
Which of the following is the value of c?

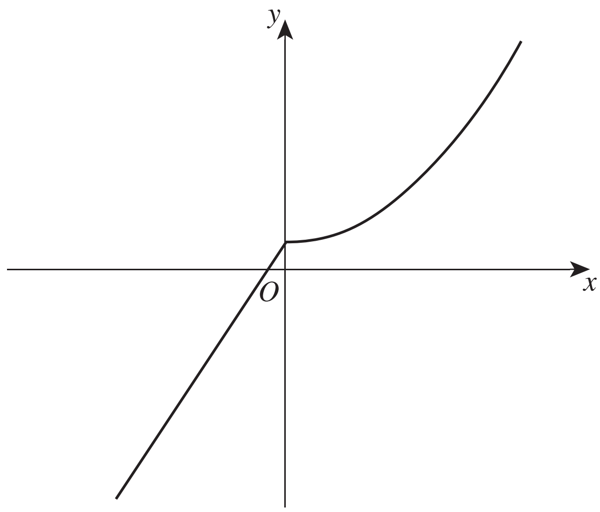
A. 3

B. 9

C. 18

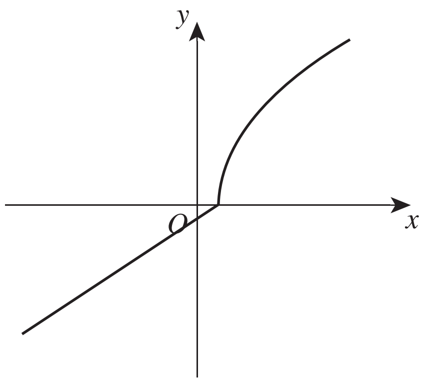
D. 36

10 The graph of the function $y = f(x)$ is shown below.

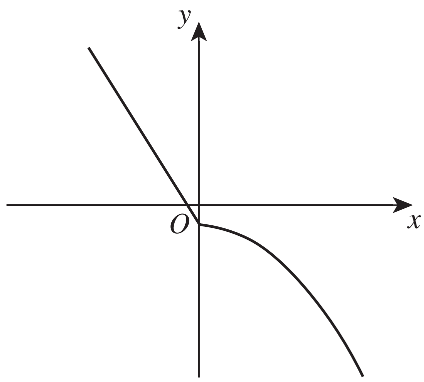


Which of the following graphs most accurately shows the inverse of $f(x)$?

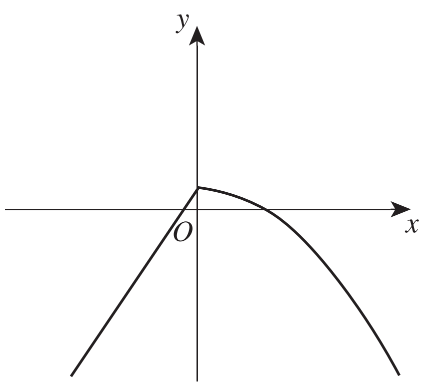
A.



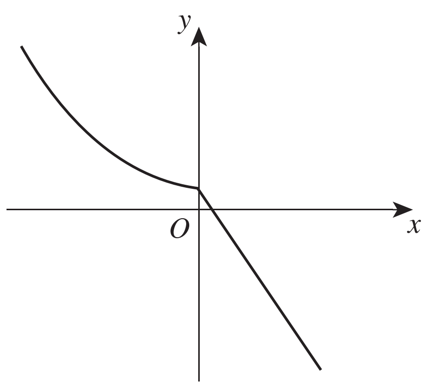
B.



C.



D.



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SECTION II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Solve $\dfrac{3}{2x-5} > 6$. **3**

(b) Consider the equations. **3**

$$x = \sqrt{2t+4}, \quad y = 2t+1, \quad -2 \le t \le 6$$

Convert the equations from parametric form to Cartesian form.

(c) Ahmad goes to the cinema with seven friends to see a movie.

(i) There are ten empty seats in a particular row of the cinema. **1**

Find the number of ways Ahmad and his seven friends can be seated in this row if there are no restrictions.

(ii) After watching the movie, Ahmad and his friends go to a sushi bar for dinner. They are seated in a row with eight seats. **2**

Find the number of ways Ahmad and his seven friends can be seated if three of the friends must be seated together, but none of those three are able to sit beside Ahmad.

(d) Evaluate $\cos^{-1}\left(\sin\dfrac{5\pi}{4}\right)$. **2**

(e) (i) Prove that $\cos 105° = \dfrac{\sqrt{2}-\sqrt{6}}{4}$. **2**

(ii) Hence, or otherwise, find the exact value of $\cos 52.5°$. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the polynomial $P(x) = (x+1)(x-2)Q(x) + 4x + k$ for some polynomial $Q(x)$ and some constant k. **2**

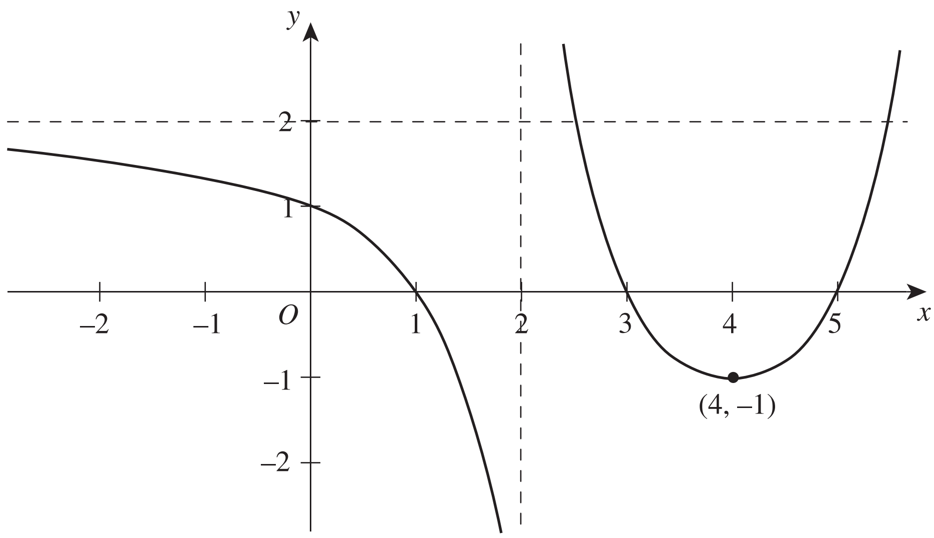
When $P(x)$ is divided by $(x + 1)$, the remainder is -2.

Find the remainder when $P(x)$ is divided by $(x - 2)$.

(b) Prove that $\cos\theta = \sin\left(\theta + \frac{\pi}{6}\right) + \cos\left(\theta + \frac{\pi}{3}\right)$. **3**

(c) Find the coefficient of x^3 in the expansion of $\left(1 + x + 2x^2\right)(1 - 2x)^5$. **3**

(d) The graph of $y = f(x)$ is shown.



Sketch the graph of each of the following.

(i) $y = |f(x)|$ **2**

(ii) $y = f(|x|)$ **2**

(iii) $y = \dfrac{1}{f(x)}$ **3**

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Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) Ibrahim is making a five-letter password and decides to use the letters from the word MATHEMATICS. **4**

Find the number of passwords it is possible for him to choose.

(b) The rate of change of the population, $P(t)$, of a native Australian animal in a certain region is given by $\frac{d}{dt}P(t) = k \times P(t)$, where k is a negative constant and t years is $t \geq 0$.

(i) Show that $P(t) = Ae^{kt}$ satisfies the equation $\frac{d}{dt}P(t)$. **1**

(ii) As a result of deforestation, the population is predicted to halve in 50 years. **2**

Find the rate of change as a percentage. Give your answer correct to two decimal places.

(iii) Estimate the population after 80 years if the initial population was 3000 animals. **1**

(iv) Explain what will happen to the population and the rate of change as time passes. **2**

(c) Due to the creation of a protected reserve, it was found that the rate of change of the population of the native Australian animal can be described more accurately by the equation $\frac{d}{dt}P(t) = k\left(P(t) - 500\right)$, where k is a negative constant.

(i) Verify that for any constant, C, the expression $P(t) = 500 + Ce^{kt}$ satisfies the new equation. **1**

(ii) Explain what will happen to the population and the rate of change as time passes with the new equation. **2**

(iii) If the initial population was 3000 animals and the population halved in 50 years, estimate the population after 80 years using the new equation. **2**

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Thirty-three points are drawn inside a square with sides of 4 cm. **3**

Show that it is possible to find three points that form a triangle with an area less than 1 cm^2.

(b) Consider the polynomial $P(x) = x^4 + x^3 - 3x^2 - 5x - 2$. This polynomial has a multiple root. **3**

Find this root and its multiplicity.

(c) Let $y = \sin^{-1} x$.

(i) Explain why $0 \le \cos y \le 1$. **2**

(ii) Prove that $\cos y = \sqrt{1 - x^2}$. **3**

(d) A buoy is a floating object that sits fixed on the ocean's surface. It resists waves, tides and currents, and is used to warn ships and boats of danger.

A boat passes a buoy at 12:00 pm, heading west at 3 km per hour, while another boat passes the same buoy at 1:00 pm heading due north at 5 km per hour.

Let time t be measured in hours after 12.00 pm. Let x be the number of kilometres that the first boat is west of the buoy at time t. Let y be the number of kilometres that the second boat is north of the buoy at time t. Let u be the distance in kilometres between the boats at time t.

(i) Use the chain rule to evaluate $\frac{d}{dt}\left(x^2\right)$. **1**

(ii) By differentiating u^2 and y^2 with respect to t, or otherwise, evaluate the rate of change of the distance between the boats at 2:30 pm. **3**

End of paper

MATHEMATICS ADVANCED
MATHEMATICS EXTENSION 1
MATHEMATICS EXTENSION 2
REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a+b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x-h)^2 + (y-k)^2 = r^2$$

Financial Mathematics

$$A = P(1+r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, \ r \neq 1$$

$$S = \frac{a}{1-r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \dfrac{\text{opp}}{\text{hyp}}, \quad \cos A = \dfrac{\text{adj}}{\text{hyp}}, \quad \tan A = \dfrac{\text{opp}}{\text{adj}}$

$A = \dfrac{1}{2}ab\sin C$

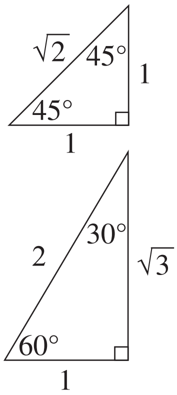
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$

$A = \dfrac{1}{2}r^2\theta$



Trigonometric identities

$\sec A = \dfrac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \dfrac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \dfrac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1 + t^2}$

$\cos A = \dfrac{1 - t^2}{1 + t^2}$

$\tan A = \dfrac{2t}{1 - t^2}$

$\cos A\cos B = \dfrac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A\sin B = \dfrac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A\cos B = \dfrac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A\sin B = \dfrac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \dfrac{1}{2}(1 - \cos 2nx)$

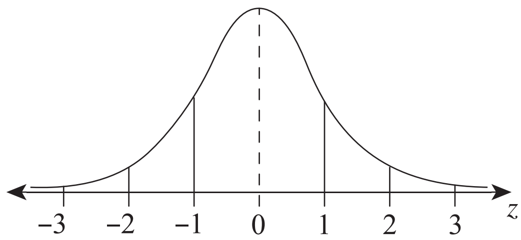
$\cos^2 nx = \dfrac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \dfrac{x - \mu}{\sigma}$

An outlier is a score less than $Q_1 - 1.5 \times IQR$ or more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \le r) = \displaystyle\int_a^r f(x)\,dx$

$P(a < X < b) = \displaystyle\int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r\, p^r(1 - p)^{n-r}$

$X \sim \text{Bin}(n, p)$

$\Rightarrow P(X = x)$

$= \dbinom{n}{x} p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\operatorname{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$
where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$
$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \ldots + f\left(x_{n-1}\right)\right]\right\}$$
where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \ldots + \binom{n}{r}x^{n-r}a^{r} + \ldots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + x\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$[r(\cos\theta + i\sin\theta)]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$

Final Examination 2023

NSW Year 11 Mathematics Extension 1

DIRECTIONS:

Write your name in the space provided.

Write your student number in the boxes provided below. Then, in the columns of digits below each box, fill in the oval which has the same number as you have written in the box. Fill in **one** oval only in each column.

Read each question and its suggested answers. Select the alternative A, B, C, or D that best answers the question. Fill in the response oval completely, using blue or black pen. Mark only **one** oval per question.

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and draw an arrow as follows.

correct ↓

A ⊗ B ⊗ C ○ D ○

STUDENT NAME: ______________________________

STUDENT NUMBER:

1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9
0	0	0	0	0	0	0	0	0

SECTION I
MULTIPLE-CHOICE ANSWER SHEET

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

STUDENTS SHOULD NOW CONTINUE WITH SECTION II

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NSW Year 11 Mathematics Extension 1

Section II Writing Booklet

Question Number

Student Name/Number: ______________________________

Instructions

Use a separate writing booklet for each question in Section II.

Write the number of this booklet and the total number of booklets that you have used for this question (e.g. 1 of 3) ⇒ [ ] of [ ]

this booklet — number of booklets for this question

Write using black pen.

You may ask for an extra writing booklet if you need more space.

If you have not attempted the question(s), you must still hand in a writing booklet, with 'NOT ATTEMPTED' written clearly on the front cover.

You may NOT take any writing booklets, used or unused, from the examination room.

Tick this box if you have continued this answer in another writing booklet. ☐