CSSA

2024

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MARKING GUIDELINES

Mathematics Advanced

The Marking Guidelines show the criteria to be applied to responses along with the marks to be awarded in line with the quality of responses. These guidelines are suggested and not prescriptive. This is not intended to be an exhaustive list but rather an indication of the considerations that students could include in their responses.

Section I

10 marks
Questions 1–10 (1 mark each)

Question	Answer	Outcomes Assessed	Targeted Performance Bands
1	C	MA12-1	2–3
2	D	MA11-1	3
3	A	MA11-4	3
4	B	MA12-8	3–4
5	A	MA12-2	3–4
6	C	MA12-8	4
7	D	MA11-6	4–5
8	B	MA11-7	5
9	B	MA12-3	5–6
10	C	MA12-7	6

Section II

90 marks
Question 11 (2 marks)
Outcomes Assessed: MA11-1
Targeted Performance Bands: 2–3

Criteria	Marks
• Provides correct solution	2
• Incorrect solution but with one correct step, or equivalent merit	1

Sample Answer:

$$5 - x = \frac{2x}{3}$$
$$3(5 - x) = 2x$$
$$15 - 3x = 2x$$
$$15 = 5x$$
$$\therefore x = 3$$

Question 12 (2 marks)
Outcomes Assessed: MA11-5
Targeted Performance Bands: 3

Criteria	Marks
• Provides correct solution	2
• Finds correct expression for $f(x+h)-f(x)$, or equivalent merit	1

Sample Answer:

$$f(x) = 3x^2 + 5x$$

$$f(x+h) = 3(x+h)^2 + 5(x+h)$$
$$= 3x^2 + 6hx + 3h^2 + 5x + 5h$$

$$f(x+h) - f(x) = 3x^2 + 6hx + 3h^2 + 5x + 5h - (3x^2 + 5x)$$
$$= 6xh + 3h^2 + 5h$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} \frac{6xh + 3h^2 + 5h}{h}$$
$$= \lim_{h\to 0} 6x + 3h + 5$$

Apply the limit by letting $h = 0$:

$$\therefore \frac{dy}{dx} = 6x + 5$$

Question 13 (2 marks)
Outcomes Assessed: MA11-1
Targeted Performance Bands: 3–4

Criteria	Marks
• Provides correct solution	2

• Some progress toward solution
  ○ Finds solution based on inverse variation equation with incorrect constant of variation
  ○ Finds solution based on direct linear variation, or equivalent merit
	1

Sample Answer:

Let x be number of people and y be cost per person in dollars ($).

Inverse variation equation is of the form

$$y = \frac{k}{x}$$

When $x = 50, y = 70$, hence

$k = xy$

$= 50 \times 70$

$= 3500$

When $x = 125$:

$$y = \frac{3500}{125}$$

$= 28$

∴ Cost is $28 per person if 125 people attend.

Question 14 (3 marks)
Outcomes Assessed: MA11-2, MA12-1
Targeted Performance Bands: 4

Criteria	Marks
• Provides correct solution	3
• Determines vertical asymptote and domain, or equivalent merit	2
• Finds correct expression for $h(x)$, or equivalent merit	1

Sample Answer:

$$h(x) = \frac{1}{x-2} + 3$$

Graph of $y = h(x)$ is a hyperbola with vertical asymptote at $x = 2$ and horizontal asymptote at $y = 3$.

$\therefore$ Domain is $(-\infty, 2) \cup (2, \infty)$ and range is $(-\infty, 3) \cup (3, \infty)$

Question 15 (3 marks)
Question 15 (a) (1 mark)
Outcomes Assessed: MA12-6
Targeted Performance Bands: 3–4

Criteria	Mark
• Provides correct solution	1

Sample Answer:

$$\frac{dy}{dx} = \sec^2(x^2) \times \frac{d}{dx}(x^2)$$
$$= 2x\sec^2(x^2)$$

Question 15 (b) (2 marks)
Outcomes Assessed: MA11-4, MA12-7
Targeted Performance Bands: 4

Criteria	Marks
• Provides correct solution	2
• Expresses integrand with appropriate Pythagorean identity, or equivalent merit	1

Sample Answer:

$$\int x\tan^2(x^2)\,dx = \int x(\sec^2(x^2) - 1)\,dx$$
$$= \frac{1}{2}\int 2x(\sec^2(x^2) - 1)\,dx$$
$$= \frac{1}{2}\int (2x\sec^2(x^2) - 2x)\,dx$$
$$= \frac{1}{2}(\tan(x^2) - x^2) + C \text{ [Using result from (a)]}$$

Question 16 (3 marks)
Outcomes Assessed: MA12-10
Targeted Performance Bands: 4

Criteria	Marks
• Provides correct solution	3
• Compares one summary statistic and shape of distribution, or two summary statistics without comparing shape of distribution, or equivalent merit	2
• Compares either one summary statistic or shape of distribution, or equivalent merit	1

Sample Answer:

Sample answer should include a reference to the shape of the distribution, for example:

- Shapes of distributions for Amy and Beth are symmetrical and positively skewed respectively.

Sample answer should include at least two or more points from those shown below:

- Median score for Amy is 40, which is 10 higher than the median for Beth
- Range of scores for Beth is 95, which is 30 higher than the range of scores for Amy
- Interquartile range (IQR) for Beth is 30, which is 10 higher than the IQR for Amy
- Half of Beth's scores are below the first quartile for Amy
- One quarter of Amy's and Beth's scores are above 50

Other points may be accepted if they adequately interpret and compare any summary statistic that may be found from the box-plots.

Points that are unacceptable include references to the mean, mode, standard deviation etc as they cannot be calculated or interpreted from a box-plot.

Question 17 (3 marks)
Outcomes Assessed: MA12-8
Targeted Performance Bands: 4

Criteria	Marks
• Provides correct solution	3
• Correctly uses the empirical rule to find percentage of scores that correspond to weights above 640 grams, or equivalent merit	2
• Calculates correct z-score for 640 grams, or equivalent merit	1

Sample Answer:

$\mu = 650, \sigma = 5$

$$z = \frac{640 - 650}{5}$$

$$= -2$$

Using empirical rule:

$P(z > -2) = 47.5\% + 50\%$

$= 97.5\%$

$97.5\% \times 1\,000\,000 = 975\,000$

∴Expected number of boxes that are fit for sale is 975 000

Question 18 (4 marks)
Question 18 (a) (2 marks)
Outcomes Assessed: MA12-2
Targeted Performance Bands: 3–4

Criteria	Marks
• Provides correct solution	2
• Finds correct monthly repayment from table, or equivalent merit	1

Sample Answer:

From the table, monthly repayment is \$683

Total repayments is given by $683 \times 12 \times 5 = 40980$

Interest paid is given by $40980 - 30000 = 10980$

$\therefore$ Interest paid on the loan is \$10980

Question 18 (b) (2 marks)
Outcomes Assessed: MA12-2
Targeted Performance Bands: 4

Criteria	Marks
• Provides correct solution	2
• Correct substitution into simple interest formula and attempts to solve for r, or equivalent merit	1

Sample Answer:

$I = P \times r \times n$, where $I = 10980, P = 30000, n = 5$

Solve for r the equation

$10980 = 30000 \times r \times 5$

$r = 10980 \div 30000 \div 5$

$= 0.0732$

$= 7.32\%$

$\therefore$ Percentage flat rate of interest is 7.32% per annum

Question 19 (5 marks)
Outcomes Assessed: MA12-8
Targeted Performance Bands: 4

Criteria	Marks
• Provides correct solution	5
• Correctly finds perimeter of triangle with incorrect rounding, or equivalent merit	4
• Uses sine rule to find either side AC or BC, or equivalent merit	3
• Correctly finds all angles inside the triangle, or equivalent merit	2
• Correctly finds $\angle BAC$, or equivalent merit	1

Sample Answer:

$\angle BAC = \angle BCA = 30°$ (alternate angles)

$\angle CBA = 180° - 84° - 30° = 66°$ ($\angle$ sum of Δ)

$$\frac{AC}{\sin(66°)} = \frac{18}{\sin(30°)}$$

$$AC = \frac{18}{\sin(30°)} \times \sin(66°)$$

$$= 32.88763648$$

$$\frac{BC}{\sin(84°)} = \frac{18}{\sin(30°)}$$

$$BC = \frac{18}{\sin(30°)} \times \sin(84°)$$

$$= 35.80278823$$

Perimeter of triangle $= 18 + 32.88763648 + 35.80278823$

$= 86.6904 \ldots$

$= 86.7$ (3 sig. fig.)

$\therefore$ Total length of race is 86.7 km, correct to three significant figures

Question 20 (3 marks)
Outcomes Assessed: MA12-3, MA12-6
Targeted Performance Bands: 4–5

Criteria	Marks
• Provides correct solution	3
• Finds the equation of the tangent instead of the normal, or finds the gradient of the normal but not the equation of the normal, or equivalent merit	2
• Correctly applies quotient rule, or equivalent merit	1

Sample Answer:

$$y = \frac{\ln(x-1)}{x-1}$$

Using quotient rule:

$$y' = \frac{\frac{1}{x-1} \times (x-1) - \ln(x-1)}{(x-1)^2}$$

$$= \frac{1 - \ln(x-1)}{(x-1)^2}$$

At the point where $x = 2$, gradient of tangent is given by

$$\frac{1 - \ln(2-1)}{(2-1)^2} = 1$$

Hence, the gradient of the normal is -1.

Using the point-gradient formula to find the equation of the normal:

$y - 0 = -1(x - 2)$

$\therefore y = 2 - x$

Question 21 (4 marks)
Outcomes Assessed: MA11-7, MA12-3
Targeted Performance Bands: 4–5

Criteria	Marks
• Provides correct solution	4
• Correctly finds the area of the lake and the rectangle, or equivalent merit	3
• Correctly finds the area of the lake using the Trapezoidal rule, or equivalent merit	2
• Recognises that the probability of success is found by dividing the area of the lake by the area of the rectangle, or equivalent merit	1

Sample Answer:

Rectangle is 450 m wide and 700 m long.

Centre measurement of the lake is given by $450 - 70 - 80 = 300$

Using the Trapezoidal rule, the area of the lake is approximately

$$\frac{350}{2}(80 + 2 \times 300 + 100) = 136\,500 \text{ m}^2$$

Area of the rectangle is given by $700 \times 450 = 315\,000 \text{ m}^2$

$$P(\text{success}) = \frac{\text{Area of lake}}{\text{Area of rectangle}}$$

$$= \frac{136500}{315000}$$

$$= \frac{13}{30}$$

∴ Probability that monitoring system will drop onto the lake is $\frac{13}{30}$ or around 43.3%

Question 22 (5 marks)
Question 22 (a) (2 marks)
Outcomes Assessed: MA12-4
Targeted Performance Bands: 4

Criteria	Marks
• Provides correct solution	2
• Uses correct formula to get side length of 24th square but does not multiply by 4 to obtain perimeter, or equivalent merit	1

Sample Answer:

Using side lengths of consecutive squares, the arithmetic sequence is 5, 8, 11, …

First term, a, is 5 and common difference, d, is 3.

Using formula for nth term of arithmetic sequence $T_n = a + (n-1)d$ for $n = 24$:

$T_n = 5 + (24 - 1)3$

$= 74$

Side length of 24th square is 74 cm and its perimeter is 4 times the side length

∴ Perimeter of 24th square is 296 cm

Question 22 (b) (1 mark)
Outcomes Assessed: MA12-4
Targeted Performance Bands: 4

Criteria	Mark
• Provides correct solution	1

Sample Answer:

Perimeter of first square is 20 cm, perimeter of 24th square is 296 cm

Total perimeter of all 24 squares is given by arithmetic sum formula $S_n = \frac{n}{2}(a + l)$

$S_{24} = \frac{24}{2}(20 + 296)$

$= 3792$

∴ Perimeter sum of first 24 squares is 3792 cm

Question 22 (c) (2 marks)
Outcomes Assessed: MA12-4, MA12-10
Targeted Performance Bands: 4–5

Criteria	Marks
• Provides correct solution	2
• Attempts to solve for n using the appropriate sum formula without regard to the perimeter of each square, or correctly forms an arithmetic series based on the perimeter of each successive square & attempts to solve for n, or equivalent merit	1

Sample Answer:

12 000 cm represents total perimeter robot can draw, so divide by 4 so series is in terms of side length, not perimeter.

Total sum of side lengths is given by $12\,000 \div 4 = 3000$

Solve for n the sum formula $S_n = \frac{n}{2}(2a + (n-1)d)$ where $S_n = 3000, a = 5, d = 3$

$$3000 = \frac{n}{2}(2 \times 5 + (n-1)3)$$

$$6000 = n(10 + 3n - 3)$$

$$6000 = 3n^2 + 7n$$

$$3n^2 + 7n - 6000 = 0$$

Using quadratic formula, $n = -45.9$ or $n = 43.6$ but $n > 0$ so negative answer in invalid

43.6 means robot draws 43 complete squares plus part of the 44th square.

∴ Robot can draw 43 complete squares with one pen

Question 23 (4 marks)
Outcomes Assessed: MA12-8, MA12-9
Targeted Performance Bands: 4–5

Criteria	Marks
• Provides correct solution	4
• Correctly finds value for $\ln(y)$ in the 7th year	3
• Correctly finds equation of least-squares regression line relating $\ln(y)$ and x, or equivalent merit	2
• Attempts to find equation of least-squares regression line based on transformed data with either correct gradient or correct y-intercept, or equivalent merit	1

Sample Answer:

Based on the transformed data and using REG mode on calculator, equation of the least-squares regression is

$\ln(y) = 1.124x + 2.85$

The value of $\ln(y)$ in the 7th year is given by

$\ln(y) = 1.124 \times 7 + 2.85$

$= 10.718$

Hence, the value of y is given by

$y = e^{10.718}$

$= 45161.48948$

∴ Steve predicts that the YouTube channel will achieve 45 161 hours of view time in the 7th year, correct to the nearest hour.

Question 24 (9 marks)
Question 24 (a) (3 marks)
Outcomes Assessed: MA12-7
Targeted Performance Bands: 4–5

Criteria	Marks
• Provides correct solution	3
• Finds an expression for $f(x)$ by integration and incorrectly evaluates the constant of integration, or equivalent merit	2
• Correctly expands expression for $f'(x)$, or equivalent merit	1

Sample Answer:

$f'(x) = 3(x^2 + 2x - 3)$

$= 3x^2 + 6x - 9$

$f(x) = \int 3x^2 + 6x - 9\,dx$

$= x^3 + 3x^2 - 9x + C$

Apply boundary condition $f(2) = -8$ to find C:

$-8 = (2)^3 + 3(2)^2 - 9(2) + C$

$-8 = 2 + C$, hence $C = -10$

$\therefore f(x) = x^3 + 3x^2 - 9x - 10$

Question 24 (b) (2 marks)
Outcomes Assessed: MA11-5, MA12-3
Targeted Performance Bands: 4

Criteria	Marks
• Provides correct solution	2
• Finds either the values of $f(x)$ at interval endpoints, or at the stationary points at $x = -3$ and $x = 1$, or equivalent merit	1

Sample Answer:

At interval endpoints: $f(-4) = 10, f(4) = 66$

At stationary points: $f(-3) = 17, f(1) = -15$

$\therefore$Minimum value is -15, maximum value is 66

Question 24 (c) (2 marks)
Outcomes Assessed: MA11-5, MA12-3, MA12-6
Targeted Performance Bands: 4

Criteria	Marks
• Provides correct solution	2
• Finds $f''(x)$ and finds value of x where $f''(x) = 0$, or equivalent merit	1

Sample Answer:

$f'(x) = 3x^2 + 6x - 9$

$f''(x) = 6x + 6$

$f''(-1) = 0$ hence a point of inflection may exist at $x = -1$

Testing for sign change of $f''(x)$ around $x = -1$:

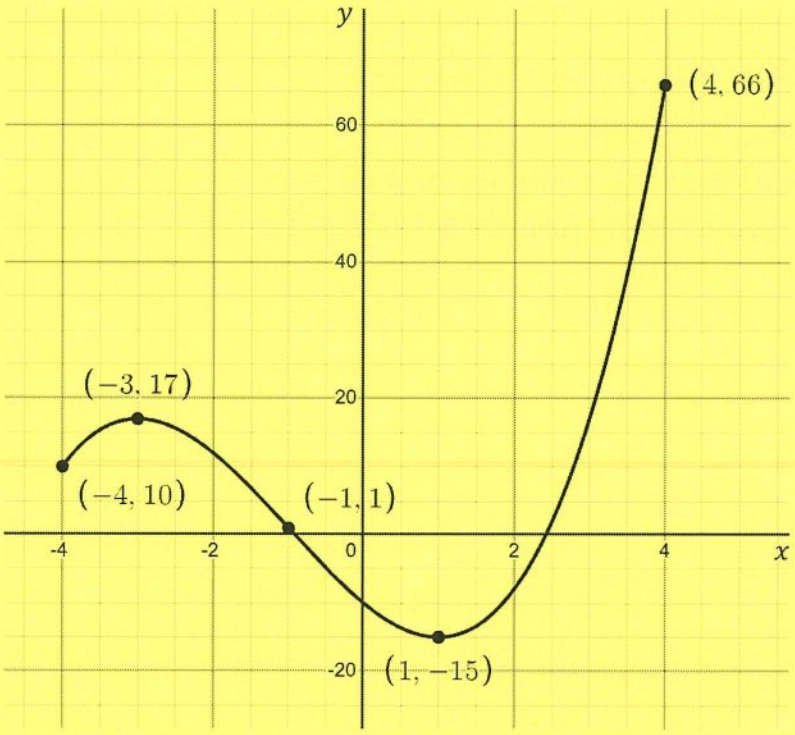
$f''(-2) = -6, f''(0) = 6$ hence point of inflection exists at $x = -1$

At $x = -1, f(-1) = 1$ $\therefore$ coordinates of point of inflection is $(-1, 1)$

Question 24 (d) (2 marks)
Outcomes Assessed: MA12-3
Targeted Performance Bands: 4

Criteria	Marks
• Provides correct solution	2
• Draws graph with correct shape without showing locations of stationary points and point of inflection, or equivalent merit	1

Sample Answer:



Question 25 (4 marks)
Question 25 (a) (2 marks)
Outcomes Assessed: MA11-7
Targeted Performance Bands: 4–5

Criteria	Marks
• Provides correct solution	2
• Finds two correct equations that involves the sum of the probabilities and the expected value but does not solve them simultaneously to find a and b, or equivalent merit	1

Sample Answer:

$0.3 + a + b + 0.1 = 1$ hence $a + b = 0.6$ **[1]**

$0.6 + 3a + 4b + 0.5 = 3.1$ hence $3a + 4b = 2$ **[2]**

Solving **[1]** and **[2]** simultaneously using an appropriate method gives $a = 0.4, b = 0.2$

Question 25 (b) (2 marks)
Outcomes Assessed: MA11-7
Targeted Performance Bands: 4–5

Criteria	Marks
• Provides correct solution	2
• Attempts to use the formula to find the variance, OR • finds the variance using an alternative formula, OR • finds the variance by using STAT mode and squaring the standard deviation, or equivalent merit	1

Sample Answer:

$$\text{Var}(X) = 0.3(2 - 3.1)^2 + 0.4(3 - 3.1)^2 + (0.2(4 - 3.1)^2 + 0.1(5 - 3.1)^2$$
$$= 0.89$$

Question 26 (6 marks)
Question 26 (a) (2 marks)
Outcomes Assessed: MA11-6
Targeted Performance Bands: 4–5

Criteria	Marks
• Provides correct solution	2
• Correctly substitutes values for C and t into equation with some manipulation and attempts to use logarithms to solve for k, or equivalent merit	1

Sample Answer:

$$35 = 100\left(1 - 2^{-50k}\right)$$

$$0.35 = 1 - 2^{-50k}$$

$$2^{-50k} = 0.65$$

$$-50k \ln(2) = \ln(0.65)$$

$$k = \frac{\ln(0.65)}{-50 \ln(2)} = 0.0124297 \ldots$$

$\therefore k = 0.01243$ (4 sig. fig.)

Question 26 (b) (2 marks)
Outcomes Assessed: MA11-6
Targeted Performance Bands: 4–5

Criteria	Marks
• Provides correct solution	2
• Attempts to solve for t when $C = 90$ with $k = 0.01243$ and showing some algebraic manipulation, or equivalent merit	1

Sample Answer:

$$90 = 100(1 - 2^{-0.01243t})$$

$$0.1 = 2^{-0.01243t}$$

$$-0.01243t \ln(2) = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.01243 \ln(2)} = 267.2508 \ldots$$

$\therefore$ Battery will be on charge for 267.25 minutes (2 dec. pl.) OR 4 hours 27 minutes.

Question 26 (c) (2 marks)
Outcomes Assessed: MA12-3, MA12-6
Targeted Performance Bands: 5

Criteria	Marks
• Provides correct solution	2
• Provides a brief description of the behaviour of the function by either considering the rate of change over time OR the behaviour of C, or equivalent merit	1

Sample Answer:

Initially, $C = 0$.

$2^{-kt} \to 0$ as $t \to \infty$ for $k > 0$

Hence, $C \to 100$ as $t \to \infty$

Describing rate of change by considering the first and second derivatives:

$C' = 100k \ln(2) \times 2^{-kt}$

$C'' = -100k^2(\ln 2)^2 \times 2^{-kt}$

Given that $k > 0$, $C' > 0$ and $C'' < 0$ for all $t > 0$.

Hence, the battery charge capacity increases from 0 at a decreasing rate over time, but can never (theoretically) reach 100% charge.

Question 27 (4 marks)
Question 27 (a) (2 marks)
Outcomes Assessed: MA12-8, MA12-10
Targeted Performance Bands: 5

Criteria	Marks
• Provides correct solution	2
• Correctly sets up integral equation with some progress toward solving equation, or equivalent merit	1

Sample Answer:

$$A\int_1^5 (3x+1)^{-\frac{1}{2}}\,dx = 1$$

$$\frac{2A}{3} \times \left[\sqrt{(3x+1)}\right]_1^5 = 1$$

$$\frac{2A}{3}(4-2) = 1$$

$$\frac{4A}{3} = 1 \ \therefore A = \frac{3}{4}.$$

Question 27 (b) (2 marks)
Outcomes Assessed: MA12-8
Targeted Performance Bands: 5–6

Criteria	Marks
• Provides correct solution	2
• Correctly finds cumulative distribution function $F(x)$ and sets up equation with some progress, or equivalent merit	1

Sample Answer:

$F(x) = \frac{1}{2}\sqrt{3x+1} - 1, 1 \leq x \leq 5$

Solve for c the equation

$F(c) = 3\big(1 - F(c)\big)$

$F(c) = 3 - 3F(c)$

$4F(c) = 3$

$F(c) = \frac{3}{4}$

$\frac{1}{2}\sqrt{3c+1} - 1 = \frac{3}{4}$

$\sqrt{3c+1} = \frac{7}{2}$

$3c + 1 = \frac{49}{4}$

$3c = \frac{45}{4}$

$\therefore c = \frac{15}{4}$

Question 28 (4 marks)
Question 28 (a) (1 mark)
Outcomes Assessed: MA12-1
Targeted Performance Bands: 4

Criteria	Marks
• Provides correct solution	1

Sample Answer:

Total surface area comprises the 4 sides and the base, and is equal to k cm^2.

$$k = 2lx + lx + lx + 2x^2 + 2x^2$$

$$= 4lx + 4x^2$$

$$4lx = k - 4x^2$$

$$\therefore l = \frac{k - 4x^2}{4x}$$

Question 28 (b) (3 marks)
Outcomes Assessed: MA12-3
Targeted Performance Bands: 5

Criteria	Marks
• Provides correct solution	3
• Correctly finds and differentiates an expression for the volume in terms of x and finds the value of x that maximises volume, or equivalent merit	2
• Finds an expression for the volume in terms of x, or equivalent merit	1

Sample Answer:

Let V represent the volume of the prism.

$V = 2x^2 l$

$= 2x^2\left(\frac{1200 - 4x^2}{4x}\right)$

$= 600x - 2x^3$

$V' = 600 - 6x^2$ and $V'' = -12x$

$V' = 0$ when $x = \pm 10$, but $x > 0$ since it represents a side length.

$V'' < 0$ when $x = 10$ hence volume is maximised when $x = 10$.

$$V = 2 \times (10)^2 \times \left(\frac{1200 - 4 \times (10)^2}{4 \times (10)}\right) = 4000$$

$\therefore$ Maximum volume of the box is 4000 cm^3.

Question 29 (6 marks)
Question 29 (a) (1 mark)
Outcomes Assessed: MA12-1
Targeted Performance Bands: 3–4

Criteria	Marks
• Provides correct solution	1

Sample Answer:

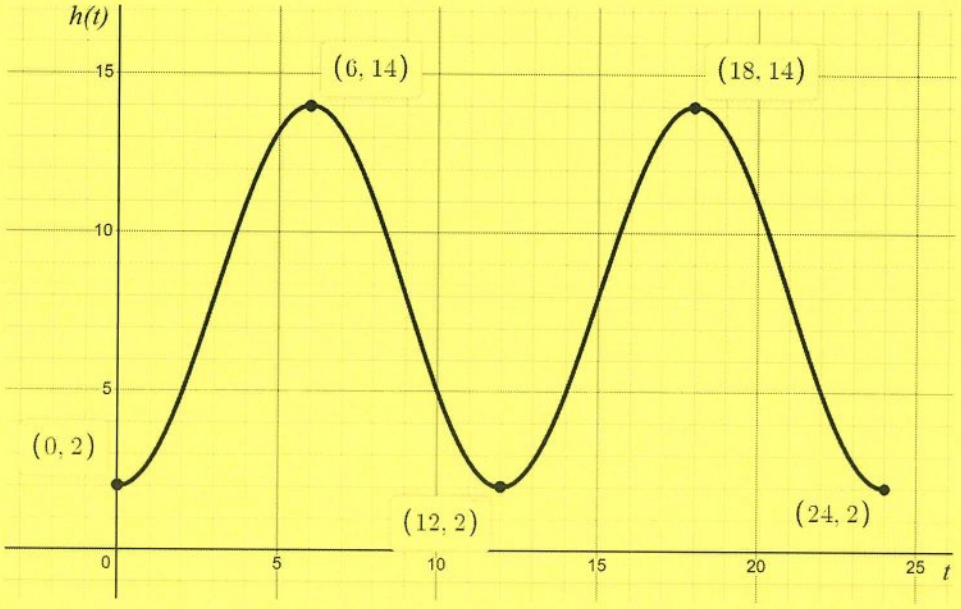
Minimum height is $(8 - 6) \times 1000 = 2000$ km

Maximum height is $(8 + 6) \times 1000 = 14\,000$ km

Question 29 (b) (2 marks)
Outcomes Assessed: MA12-5
Targeted Performance Bands: 4–5

Criteria	Marks
• Provides correct solution	2
• Sketches graph with correct shape and period but with errors such as incorrect heights and phase, or equivalent merit	1

Sample Answer:



Question 29 (c) (3 marks)
Outcomes Assessed: MA12-5, MA12-10
Targeted Performance Bands: 5–6

Criteria	Marks
• Provides correct solution	3
• Correctly finds the times in the day when satellite is exactly 11 000 km above the ground, or equivalent merit	2
• Understands that the answer are the intervals where $h(t) \geq 11$ and attempts to solve the equation $h(t) = 11$ to find the times where the satellite is exactly 11 000 km above the ground, or equivalent merit	1

Sample Answer:

Solve for t the equation $h(t) = 11$

$$6\sin\left(\frac{\pi}{6}(t-15)\right) + 8 = 11$$

$$6\sin\left(\frac{\pi}{6}(t-15)\right) = 3$$

$$\sin\left(\frac{\pi}{6}(t-15)\right) = 0.5$$

$$\frac{\pi}{6}(t-15) = -\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$$

$$t - 15 = -11, -7, 1, 5$$

$$t = 4, 8, 16, 20$$

From the graph in (b), satellite is above 11 000 km in the intervals $4 < t < 8$ and $16 < t < 20$

∴ Satellite is out of range for 8 hours each day.

Question 30 (5 marks)
Question 30 (a) (2 marks)
Outcomes Assessed: MA11-2
Targeted Performance Bands: 4–5

Criteria	Marks
• Provides correct solution	2
• Finds an equation in the form $y = mx + c$ with correct gradient and incorrect y-intercept, or equivalent merit	1

Sample Answer:

At $t = 0$: $v_Q = v_P = 4$

At $t = 4$: $v_Q = v_P = 3(4)^2 - 6(4) + 4$

$= 28$

Equation is in the form $y = mx + c$

$$m = \frac{28 - 4}{4 - 0} = 6, c = 4$$

$\therefore v_Q = 6t + 4$

Question 30 (b) (3 marks)
Outcomes Assessed: MA12-7
Targeted Performance Bands: 5–6

Criteria	Marks
• Provides correct solution	3
• Finds an integral that represents the area between P and Q and attempts to find a time t $(t > 0)$ when relative displacement is zero, or equivalent merit	2
• Understands that relative displacement between both cars is represented by the signed area between P and Q, or equivalent merit.	1

Sample Answer:

Signed area between curves P and Q represents the relative displacement between both cars.

Cars are at exactly the same displacement from starting point when displacement between cars is zero.

Solve for a:

$$\int_0^a 3t^2 - 6t + 4 - (6t + 4)\, dt = 0$$

$$\int_0^a 3t^2 - 12t\, dt = 0$$

$$[t^3 - 6t^2]_0^a = 0$$

$$a^3 - 6a^2 = 0$$

$$a^2(a - 6) = 0, a > 0$$

Hence cars are at the same distance away from the start at $t = 6$.

The integrand $3t^2 - 12t$ represents velocity of P relative to Q.

For $t > 4$, $v_p > v_Q$ Hence car P will pass car Q after 6 seconds from start of race.

Question 31 (4 marks)
Outcomes Assessed: MA11-5, MA11-6, MA12-4, MA12-7
Targeted Performance Bands: 5–6

Criteria	Marks
• Provides correct solution	4
• Finds an expression for $f(x)$ by integrating S_∞, or equivalent merit	3
• Recognises that the expression for $f'(x)$ forms a geometric series with limiting sum with first term -1 and common ratio x, or equivalent merit	2
• Finds an expression for $f'(x)$, or equivalent merit	1

Sample Answer:

$f'(x) = -1 + x - x^2 + x^3 - \cdots$

This is s geometric series with first term -1 and common ratio $-x$

Since $0 < x < 1$, the geometric series will have a limiting sum.

Hence,

$$f'(x) = S_\infty = \frac{-1}{1+x}$$

$$f(x) = -\int \frac{1}{1+x}dx$$

$$= -\ln(1+x) + C$$

$$f(a) - f(b) = -\ln(1+a) - (-\ln(1+b))$$

$$= \ln\left(\frac{1}{1+a}\right) - \ln\left(\frac{1}{1+b}\right)$$

$$= \ln\left(\frac{1}{1+a} \div \frac{1}{1+b}\right)$$

$$= \ln\left(\frac{1}{1+a} \times \frac{1+b}{1}\right)$$

$$= \ln\left(\frac{1+b}{1+a}\right)$$

Question 32 (5 marks)
Question 32 (a) (3 marks)
Outcomes Assessed: MA11-1, MA11-2, MA12-1
Targeted Performance Bands: 6

Criteria	Marks
• Provides correct solution	3
• Recognises that discriminant > 0 for 2 distinct real solutions and attempts to use quadratic formula to find values of d, or equivalent merit	2
• Finds an equation involving x and d by solving equation of ℓ and C simultaneously, or equivalent merit	1

Sample Answer:

Solve simultaneously the equations $y = x + d$ and $(x-1)^2 + (y-3)^2 = 9$

By substitution:

$(x-1)^2 + (x+d-3)^2 = 9$

$(x-1)^2 + \big(x+(d-3)\big)^2 = 9$

$x^2 - 2x + 1 + x^2 + 2x(d-3) + (d+3)^2 = 9$

$x^2 - 2x + 1 + x^2 + 2dx - 6x + d^2 - 6d + 9 = 9$

$2x^2 + 2dx - 8x + d^2 - 6d + 1 = 0$

$2x^2 + (2d-8)x + (d^2 - 6d + 1) = 0$

This is a non-monic quadratic equation with 2 distinct real solutions if discriminant $D > 0$.

$D = (2d-8)^2 - 4(2)(d^2 - 6d + 1)$

$= 4d^2 - 32d + 64 - 8d^2 + 48d - 8$

$= -4d^2 + 16d + 56$

Use quadratic formula to find values of d when $D = 0$:

$$d = \frac{-16 \pm \sqrt{(16)^2 + 4(4)(56)}}{2(-4)} = \frac{-16 \pm \sqrt{1152}}{-8}$$

$$= \frac{-16 \pm 24\sqrt{2}}{-8} = 2 \pm 3\sqrt{2}$$

Graph of D is a concave down parabola with roots $2 \pm 3\sqrt{2}$ hence $D > 0$ in the interval

$2 - 3\sqrt{2} < d < 2 + 3\sqrt{2}$

$\therefore \ell$ intersects C at exactly two points if $2 - 3\sqrt{2} < d < 2 + 3\sqrt{2}$.

Question 32 (b) (2 marks)
Outcomes Assessed: MA11-1, MA11-2, MA12-1
Targeted Performance Bands: 6

Criteria	Marks
• Provides correct solution	2
• Finds the coordinates of A and B in terms of d, or equivalent merit	1

Sample Answer:

Solving the equation from part (a) to find x coordinates of A and B:

$2x^2 + (2d - 8)x + d^2 - 6d + 1 = 0$

$$x_1 = \frac{8 - 2d - \sqrt{D}}{4} = 2 - \frac{d}{2} - \frac{\sqrt{D}}{4}$$

$$x_2 = \frac{8 - 2d + \sqrt{D}}{4} = 2 - \frac{d}{2} + \frac{\sqrt{D}}{4}$$

Using the equation $y = x + d$ to find corresponding y-coordinates:

$$y_1 = 2 + \frac{d}{2} - \frac{\sqrt{D}}{4}, y_2 = 2 + \frac{d}{2} + \frac{\sqrt{D}}{4}$$

Using the distance formula:

$$\sqrt{\left(\frac{\sqrt{D}}{2}\right)^2 + \left(\frac{\sqrt{D}}{2}\right)^2} = 2\sqrt{3}$$

$$\left(\frac{\sqrt{D}}{2}\right)^2 + \left(\frac{\sqrt{D}}{2}\right)^2 = 12$$

$D = 24$

Solving $-4d^2 + 16d + 56 = 24$ by first dividing all terms by (-4):

$d^2 - 4d - 8 = 0$

$d = 2 \pm 2\sqrt{3}$

∴Possible values of d are $2 - 2\sqrt{3}$ or $2 + 2\sqrt{3}$.