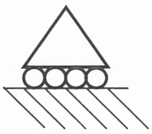
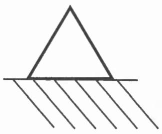
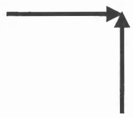
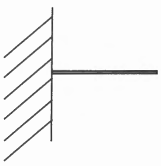
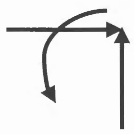
Chapter 1 - Truss Analysis

A truss is a triangular framework made by joining lengths of timber or steel bars (commonly known as members) by either welding or riveting.

There are 3 different types of supports for beams and trusses. These are roller, pin/hinge and fixed supports, all of which have different force configurations.

Support	Symbol	Free body diagram	Characteristics	Reason behind its use
Roller			Always acts 90° to the surface of the roller support. Allows translation and rotational movement.	Allows side movement for expansion and contraction of bridge decks due to temperature changes and deformation from applied loads.
Pin			Can support horizontal and vertical forces Allows rotation but no translation movement.	Prevents lateral movement whilst allowing slight rotation.
Fixed			Can support moments and vertical and horizonal forces.	Securely anchors a structure in position.

Determining Reactions at the Supports

Before we can find the forces in the members of a truss, we must first determine the reaction forces of the supports on both sides of the truss. The following sequence of steps should be followed.

Steps:

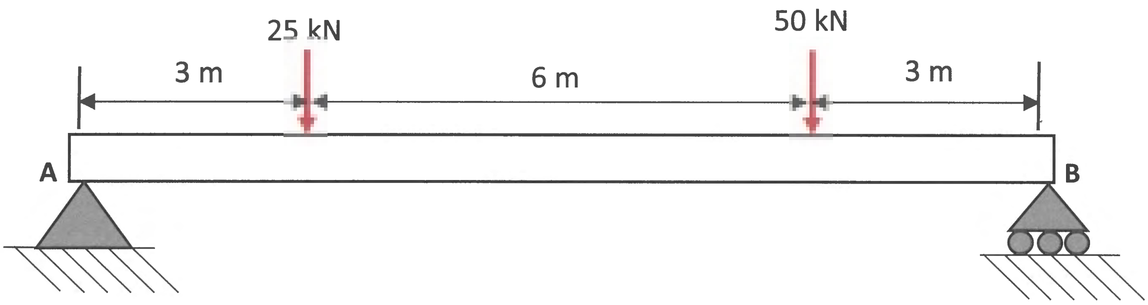
1. A <u>free body diagram</u> (FBD) should be drawn to document all the forces and dimensions present.
2. Apply <u>sum of moments</u> about one of the reaction supports. Assume the anticlockwise direction to be positive.
3. Apply <u>sum of forces</u> in both the vertical and horizontal directions to find the remaining reaction forces. Assume the upwards vertical direction to be positive and the horizontal force directed towards the right to be positive.

4. Find the resultant force acting at the pin joint. To do this, draw a vector triangle with the resultant making a head-head connection with one of the components and a tail-tail connection with the other component.
5. Apply Pythagoras theorem to the vector triangle to find the resultant force.
6. Apply inverse tan to find the direction of the resultant vector.

Note: The words horizontal component and x-component are used interchangeably (they mean the same thing). Likewise, with the vertical component and y-component.

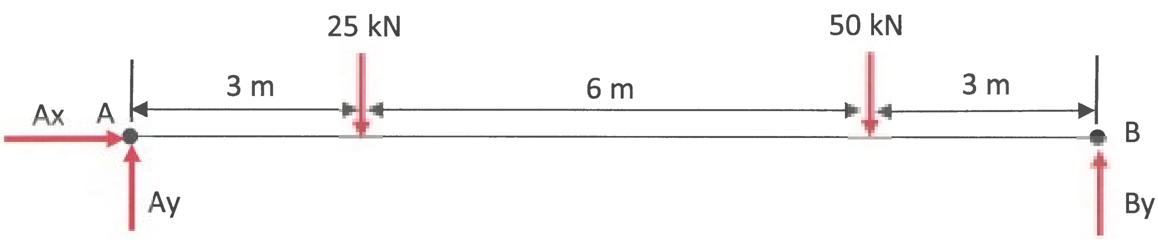
Worked Example 1:

Find the reactions at the supports for the beam below:



Solution:

Step 1 – Draw FBD of the above beam.



Step 2 – Applying sum of moments about A will eliminate A_y and A_x as both forces go through A.

$$+\curvearrowleft \sum M_A = 0$$
$$(-25 \times 3) - (50 \times 9) + B_y(12) = 0$$
$$B_y = \frac{525}{12}$$
$$\therefore B_y = 43.75\ kN \uparrow$$

Step 3 – Apply sum of forces in the vertical and horizontal directions to determine the remaining reaction forces.

$$+\uparrow \Sigma F_y = 0$$
$$A_y + B_y - 25 - 50 = 0$$
$$A_y + 43.75 - 25 - 50 = 0$$
$$\therefore A_y = 31.25\ kN \uparrow$$

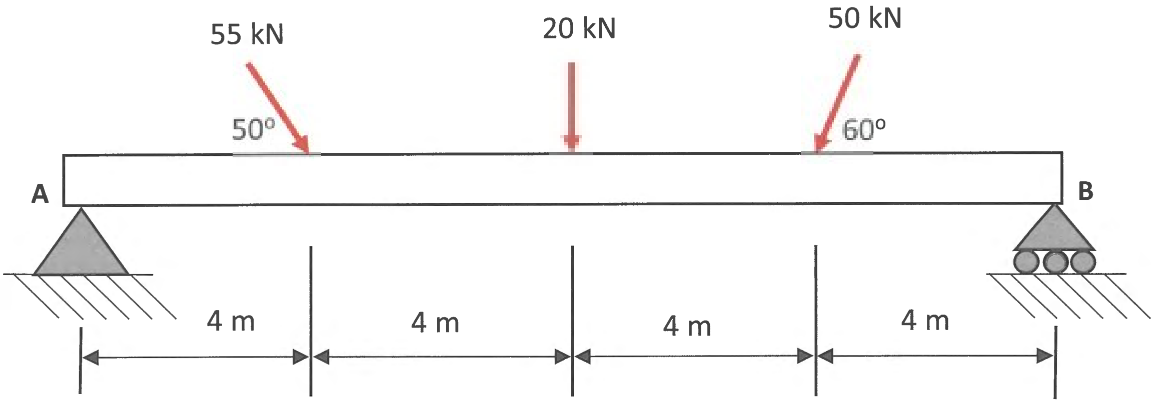
$$\overset{\rightarrow}{+}\sum F_H = 0$$
$$A_x = 0$$

Therefore, the reaction at the pin joint is 31.25 kN and the reaction at the roller joint is 43.75 kN.

Note: This was a simple problem as there were no other external horizontal forces. As such, finding the resultant was not required.

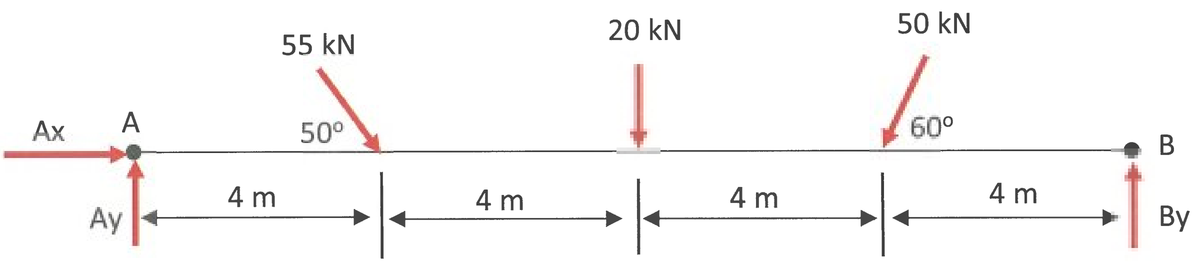
Worked Example 2:

A beam can be seen resting on 2 supports with 3 loads acting on it. Determine the reactions at both A and B.



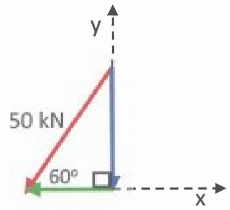
Solution:

Step 1 – Draw the FBD of the beam above.



Step 2 – Apply moments about joint A. This will eradicate the forces acting on joint A along with the horizontal components of the 55 kN and 50 kN force as they go through A.

Note: When breaking forces into its components, we consider the tail as the starting position and the head as the end destination. For example, with the 55kN force, we move down from the tail and then move right to the head. The upwards vector is assumed to be positive as well as the vector pointing to the right.

Force Vector Triangle	X – component	Y – component
	$cos50 = \frac{F_x}{55}$ $F_x = 55cos50$ $F_x = 35.35$ kN	$sin50 = \frac{F_y}{55}$ $F_y = 55sin50$ $F_y = -42.13$ kN
	$cos60 = \frac{F_x}{50}$ $F_x = 50cos60$ $F_x = -25$ kN	$sin60 = \frac{F_y}{50}$ $F_y = 50sin60$ $F_y = -43.30$ kN

$$+\curvearrowleft \Sigma M_A = 0$$
$$(-55sin50 \times 4) - (20 \times 8) - (50sin60 \times 12) + B_y(16) = 0$$
$$B_y = \frac{848.1}{16}$$
$$\therefore B_y = 53\ kN \uparrow$$

Therefore, the reaction at the roller support is 53 kN.

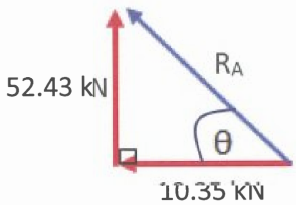
Step 3 – Apply sum of forces in the vertical (y) and horizontal (x) direction to find the remaining forces.

$$+\uparrow \Sigma F_y = 0$$
$$A_y + B_y - 42.13 - 43.30 - 20 = 0$$
$$A_y + 53 - 42.13 - 43.30 - 20 = 0$$
$$\therefore A_y = 52.43\ kN \uparrow$$

$$\overrightarrow{+} \Sigma F_x = 0$$
$$A_x + 35.35 - 25 = 0$$
$$A_x = -10.35\ \text{kN}$$
$$\therefore\ A_x = 10.35\ kN\ \leftarrow$$

As above, our first A_x result was a negative number. This means that we have assumed its sense incorrectly. Therefore, we need to flip its sense so instead of acting towards the right, it is in fact acting to the left with a value of 10.35 kN.

Step 4 – Form a vector triangle to find the resultant force (using Pythagoras) acting on the pin joint then use trigonometry to find its direction.



$$R_A = \sqrt{52.43^2 + 10.35^2}$$

$$R_A = 53.44\ kN$$

$$\tan\theta = \frac{52.43}{10.35}$$

$$\theta = \tan^{-1}\left(\frac{52.43}{10.35}\right)$$

$$\theta = 79^{\circ}$$

Note: It is imperative that the reaction forces are determined before analysing truss members.

Analysing Truss Members

There are 2 methods that one can use when calculating forces in members. These are 'method of joints' and 'method of sections'. In most cases, method of sections is the faster method.

Method of Joints:

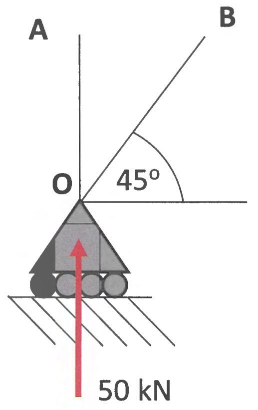
When applying this method, we must select a joint that has no more than 2 unknown forces. Assumptions can be made on whether the member is in tension or compression. This can then be altered once the calculations have been completed. For example, if the force in a member is calculated to be a negative value, then this simply means that the force is opposite in action to that of the assumed sense. We usually begin with the reaction joints and work our way around the truss. In the below detail, a step by step procedure has been provided to simplify the process followed with an example.

Steps:

1. Draw a free body diagram (FBD) of all the forces acting on the joint that you are analysing.
2. Break up each force into its horizontal and vertical components where necessary.
3. Apply $\Sigma F_x = 0$ and/or $\Sigma F_y = 0$ by determining how many unknown forces there are in each direction. If there are 2 unknown forces in the x – direction then we cannot use the ΣF_x equation. However, if there is 1 unknown force in the y-direction then we can apply ΣF_y equation to solve for the unknown. Once this is solved, we can then apply ΣF_x to find the remaining force.
4. Determine whether the member is in tension or compression using the following rule. If a member force points away from the joint it is attached to then we say that the force is in tension. If that force points towards the joint then we say it is in compression.
5. Repeat the above two steps to find the force in every remaining member.

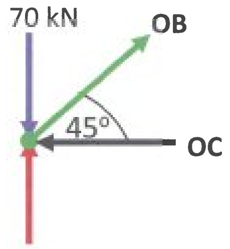
Worked Example 3:

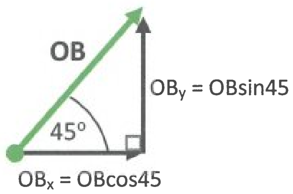
The reaction force on the roller joint is 50 kN acting vertically upwards and the force in member OA is 70kN in compression. Determine the force in members OB and OC by analysing joint O and state their nature.



Solution:

Step 1 – Draw FBD by having the sense of OA facing joint O since it is under compression. Assume the sense for the rest of forces acting on Joint O.





Step 2 – Break up OB into its x and y components by using trigonometry as shown above.

$$OB_x = OBcos45$$
$$OB_y = OBsin45$$

Step 3 – We have 2 unknowns in the horizontal direction and 1 in the vertical direction. Therefore, you must sum the forces in the vertical direction first then the horizontal direction.

$$+\uparrow \Sigma F_y = 0$$
$$50 - 70 + OBsin45 = 0$$
$$\text{OB} = \frac{20}{sin45}$$
$$\therefore \boldsymbol{OB = 28.28\ kN} \nearrow (T)$$

$$\overrightarrow{+} \sum F_x = 0$$
$$-\text{OC} + \text{OBcos45} = 0$$
$$\text{OC} = 28.28\text{cos}45$$
$$\therefore\ \boldsymbol{OC = 20\ kN} \leftarrow \text{(C)}$$

Step 4 – OB is in tension as the force is directed away from the joint. OC is in compression as the force is acting towards the joint.

Method of Sections

- This method involves analytically cutting the truss into sections and using the static equilibrium equations to solve for each section.

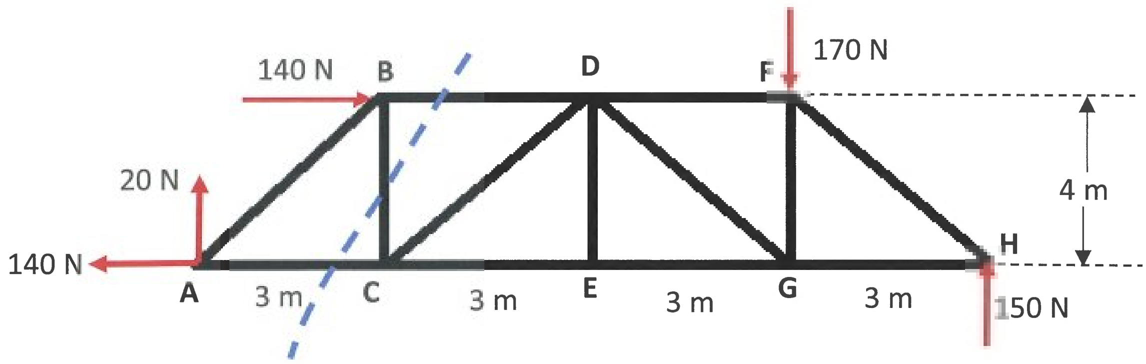
- Cutting through the members will expose the force inside the members.
- When applying method of joints, we utilise the static equilibrium equations at a point so we only use the 2 sum of forces equations and not the moment equation.
- Contrastingly, a section has a fixed size which allows us to use the moment equations to solve the unknown forces. Hence we can solve up to 3 unknown forces at one time. Due to this, one should choose sections that involve cutting through no more than 3 members at one time.
- If a member force points away from the joint it is attached to then we say that the force is in tension. If that force points towards the joint then we say it is in compression.

Steps:

1. Make a cut through the members in question
2. Members that have been cut now become forces
3. Choose either the left or right side of the cut to analyse. Usually we take the side with less forces. Either side is correct
4. Draw a labelled free body diagram with all the necessary information. Assume the force's sense.
5. Apply the appropriate static equilibrium equations to solve for the unknown forces. If the value is negative, then the correct sense is opposite to the assumed sense.
6. Determine nature of force by stating either tension or compression.

Worked Example 4:

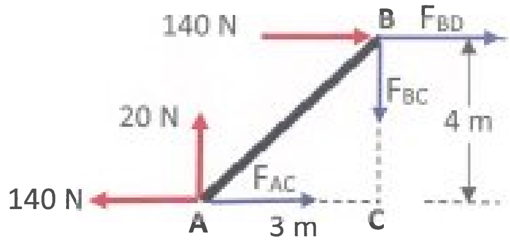
Determine the force in members BD, BC and AC and state their nature (tension or compression). The reaction forces have been predetermined.



Solution:

Step 1 – Cut through members BD, BC and AC (as shown by the blue line above)

Step 2 – Analyse the left hand side (LHS) by drawing the free body diagram (FBD). Remember that the cut members become forces.



Step 3: Apply appropriate static equilibrium equations to determine the unknown forces

When analysing the above FBD, you need to see which equation applies. In this question, you have 2 unknown forces - F_{AC} and F_{BD} - acting in the horizontal direction. Hence the ΣF_x equation CANNOT be used due to having 2 unknowns.

However, we have 1 unknown force - F_{BC} - acting in the vertical direction and hence the ΣF_y equation CAN be used due to having 1 unknown.

We have assumed the upwards direction to be positive.

$$+\uparrow \Sigma F_y = 0$$
$$-F_{BC} + 20 = 0$$
$$\boldsymbol{F_{BC} = 20\ N\ \downarrow}$$

We have guessed the sense of F_{BC} correctly due to the positive value obtained. Since F_{BC} is directed away from the joint it is attached to (joint B), then we say that F_{BC} is in **tension**.

Now, remember we cannot apply the ΣF_x equation just yet due to the 2 unknowns. However, we can apply moments about a certain point to find one of the unknowns.

To find the force in member AC (F_{AC}), we need to apply the ΣM equation about point B. By taking moments about this point, we eliminate F_{BC}, F_{BD} and the 140 N force due to the lines of these forces passing through that point. This will leave us with one unknown (F_{AC}).

We have assumed the anticlockwise moment to be positive.

Note: Moment = Force × Perpendicular distance to the point (in this case, B).

$$+\circlearrowleft \Sigma M_B = 0$$
$$(F_{AC} \times 4) - (140 \times 4) - (20 \times 3) = 0$$
$$4F_{AC} = 620\ N$$
$$\boldsymbol{F_{AC} = 155 \rightarrow}$$

We have guessed the sense of F_{AC} correctly due to the positive value obtained. Since F_{AC} is directed away from the joint it is attached to (joint A) then we say that F_{AC} is in **tension.**

As we now have 1 of the horizontal forces, we can apply the ΣF_x equation. This leaves us with 1 unknown.

We have assumed the force directed to the right to be positive.

$$\overset{+}{\rightarrow} \sum F_x = 0$$
$$-140 + F_{AC} + F_{BD} + 140 = 0$$
$$-140 + 155 + F_{BD} + 140 = 0$$
$$F_{BD} = -155\ N$$

We have guessed the sense of F_{BD} incorrectly since the value obtained was negative. Hence instead of F_{BD} acting towards the right, it is actually acting towards the left.

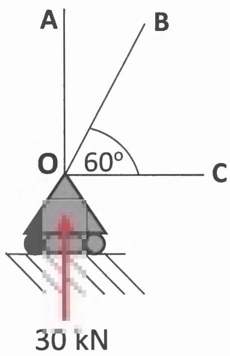
Hence $\boldsymbol{F_{BD} = 155\ N} \leftarrow$

Since F_{BD} is directed towards the joint it is attached to (joint B) then we say that F_{BD} is in **compression**.

Questions

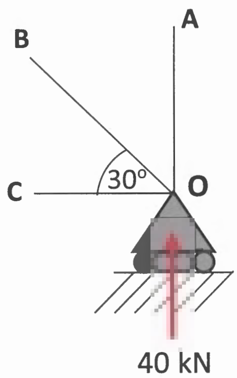
Section I

1. The reaction force on the joint is 30 kN acting vertically upwards and the force in member OC is 50kN in compression. Determine the force in member OA and state its nature.



(A) 80 kN (C)
(B) 117 kN (C)
(C) 80 kN (T)
(D) 117 kN (T)

2. The reaction force on the joint is 40 kN acting vertically upwards and the force in member OC is 20kN in tension. Determine the force in member OB and state its nature.



(A) 23 kN (C)
(B) 23 kN (T)
(C) 30 kN (C)
(D) 30 kN (T)

3. How many support reaction(s) does a roller and a pin joint have?
(A) Roller – 1, Pin – 2
(B) Roller – 2, Pin – 1
(C) Roller – 1, Pin – 1
(D) Roller – 2, Pin – 2

4. The direction of the reaction at a roller support is always:
   (A) Vertical
   (B) Horizontal
   (C) Diagonal
   (D) Perpendicular to the surface the joint sits on

5. The purpose of having pin supports for a bridge is:
   (A) To prevent lateral movement whilst allowing slight rotation
   (B) To allow side movement for expansion and contraction of bridge deck
   (C) To securely anchor a structure in position
   (D) None of the above

6. The purpose of having roller joints for a bridge is:
   (A) To allow side movement for expansion and contraction of bridge deck
   (B) To securely anchor a structure in position
   (C) To prevent lateral movement whilst allowing slight rotation
   (D) None of the above

7. Determine the correct order for the steps listed below in determining the reaction forces at the supports.

   1. Apply sum of forces in both the vertical and horizontal directions to find the remaining reaction forces.
   2. Draw a vector triangle to find the resultant acting on a pin joint.
   3. Apply inverse tan to find the direction of the resultant vector.
   4. A free body diagram (FBD) is drawn to document all forces and dimensions present.
   5. Apply moments about one of the reaction supports.
   6. Apply Pythagoras theorem to find the resultant force.

   (A) 1,3,4,6,5,2
   (B) 4,5,1,2,6,3
   (C) 4,1,5,6,3,2
   (D) 5,3,1,4,2,6

8. The members in a truss bridge experience what type of forces?
   (A) Bending and shear
   (B) Shear and compression
   (C) Shear and tension
   (D) Compression and tension

9. Determine the correct order for effectively utilising the method of joints.

1. Break up forces into horizontal and vertical components where necessary.
2. Draw a free body diagram (FBD) of all the forces acting on the joint
3. Apply $\Sigma F_x = 0$ and/or $\Sigma F_y = 0$
4. Determine whether the member is in tension or compression
5. Determine how many unknown forces you have in each direction.

(A) 1,3,2,5,4
(B) 1,2,3,5,4
(C) 2,1,5,3,4
(D) 2,1,3,5,4

10. Determine the correct order for effectively utilising the method of sections.

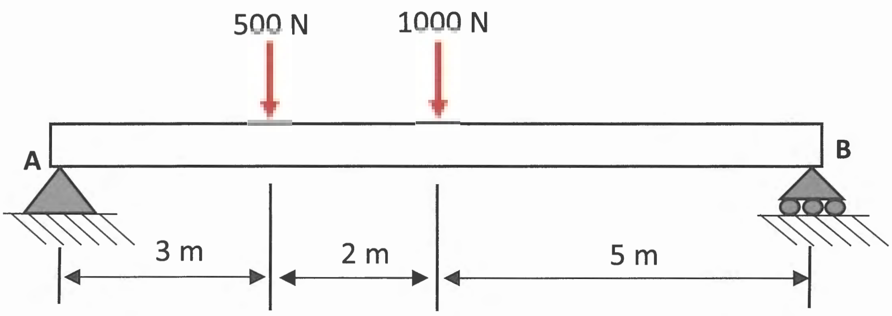
1. Draw a labelled free body diagram with all the necessary information.
2. Choose either the left or right side of the cut to analyse.
3. Members that have been cut now become forces.
4. Make a cut through the members in question.
5. Determine nature of force.
6. Apply appropriate static equilibrium equations to solve for the unknown forces.

(A) 4,3,2,1,6,5
(B) 1,3,2,4,6,5
(C) 1,4,2,3,6,5
(D) 4,1,2,3,5,6

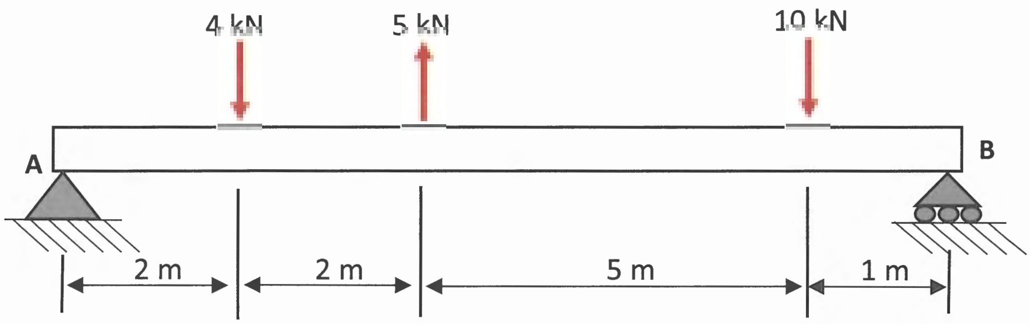
Section II

Reaction Forces

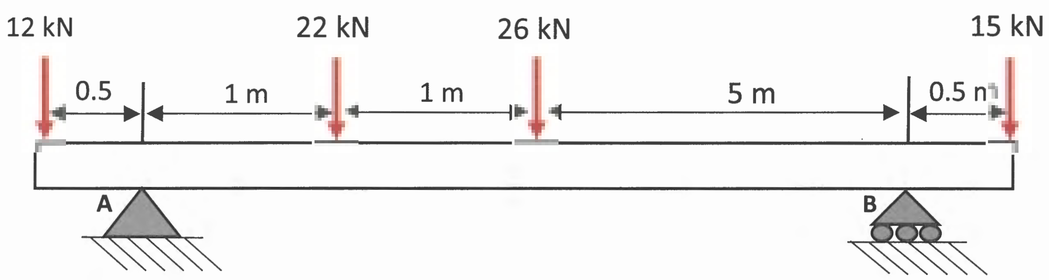
11. Determine the reaction forces on supports A and B.



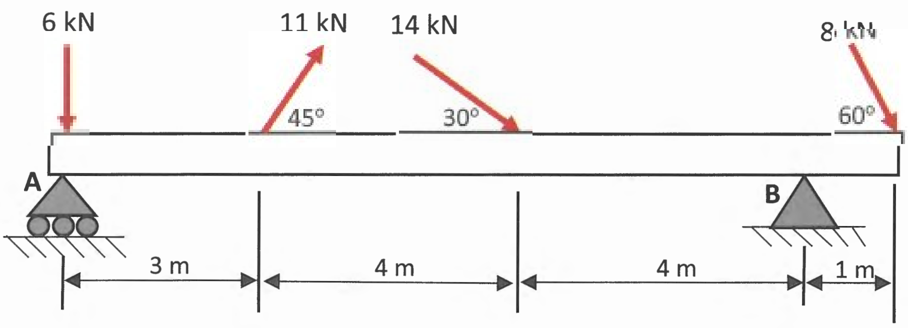
12. Determine the reactions at the supports of the simply supported beam shown below.



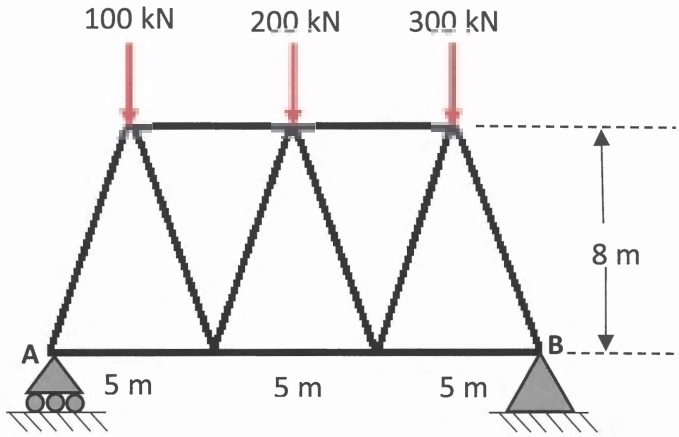
13. Determine the reactions at supports A and B



14. Determine the reactions at the supports of the simply supported beam shown below.

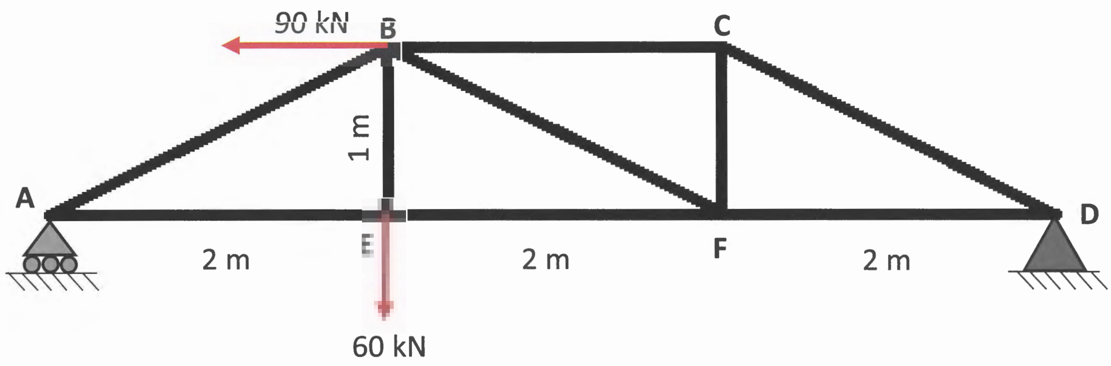


15. The truss is subjected to 3 external forces as shown below. Determine the reaction force at the roller joint.

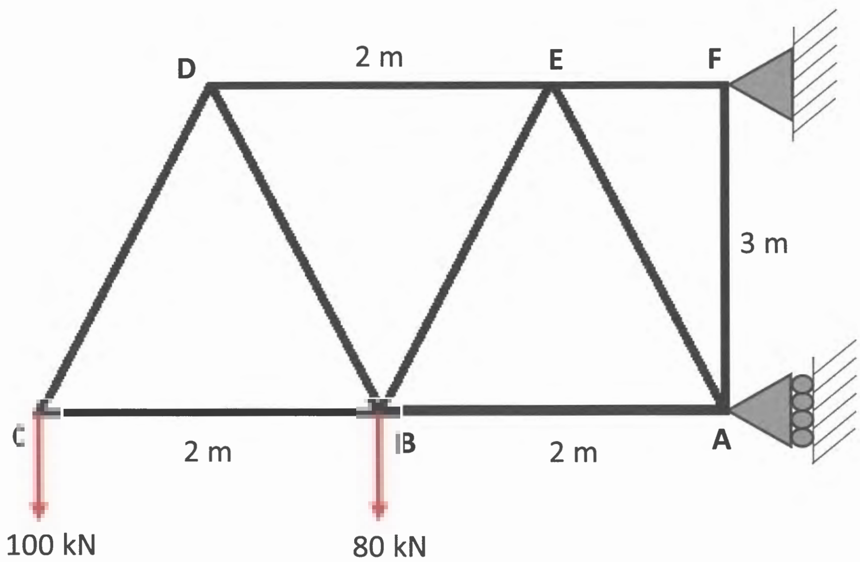


Method of Joints

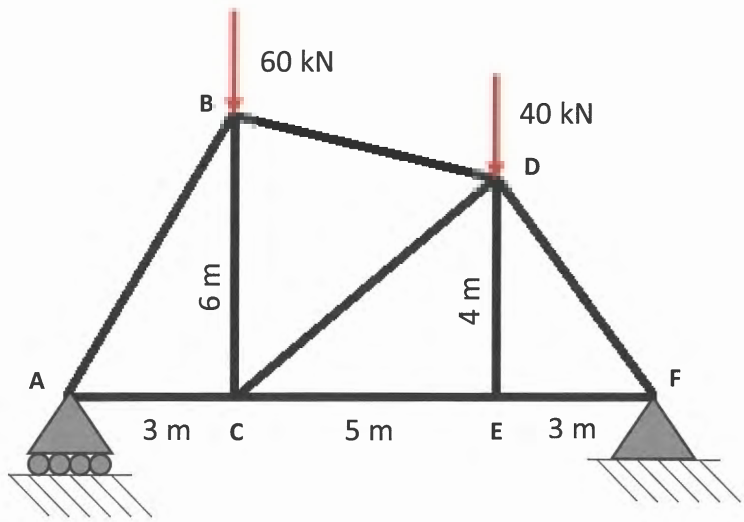
16. Find the force acting in all members of the truss shown below.



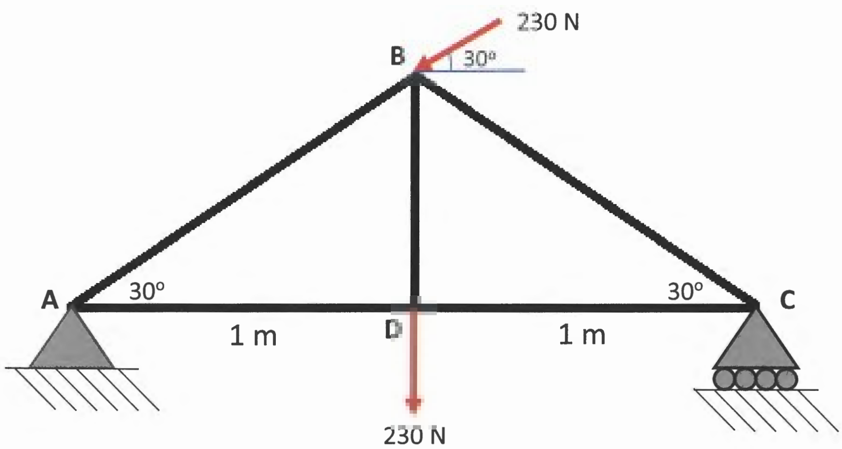
17. Find the force in each of the members by analysing the truss below.



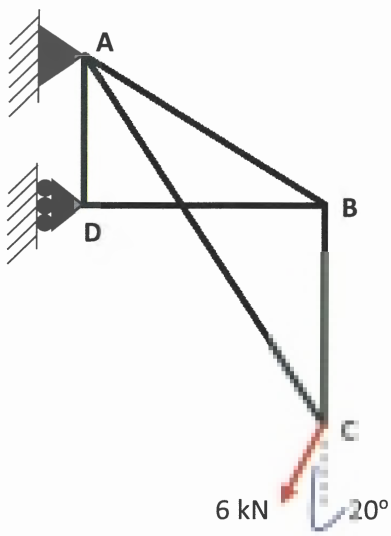
18. Determine the forces present in each member of the truss shown below.



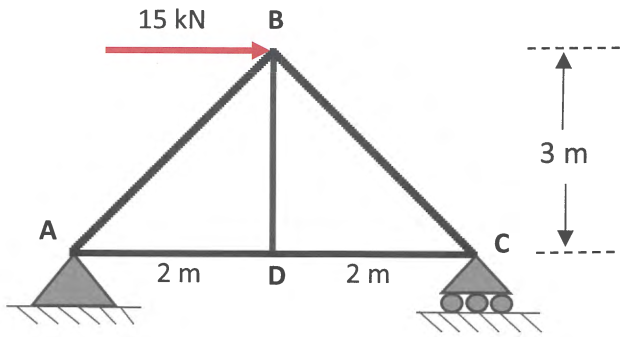
19. Determine the forces in each member of the roof truss shown below.



20. In the loaded truss shown below, AD has a length of 3m, DB has a length of 5m and BC has a length of 4m. Determine the force in each member of the loaded truss and state their nature.

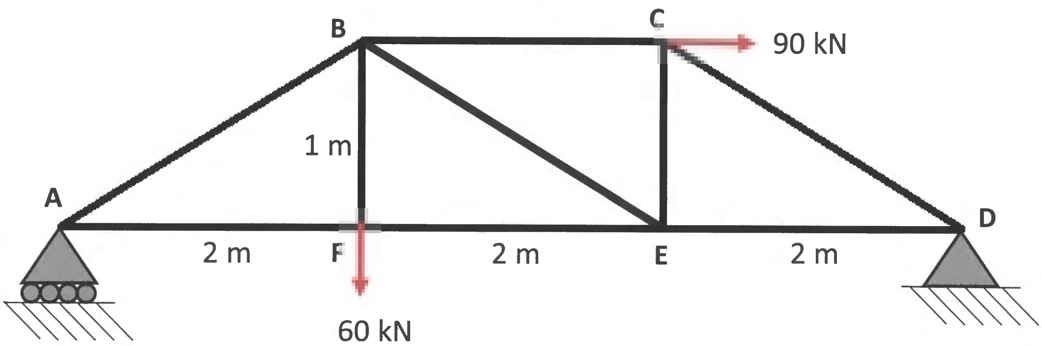


21. For the truss shown, determine the force in member BD.

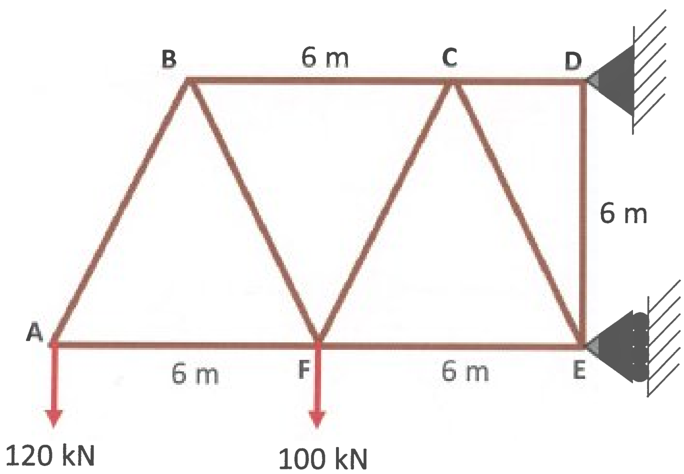


Method of Sections

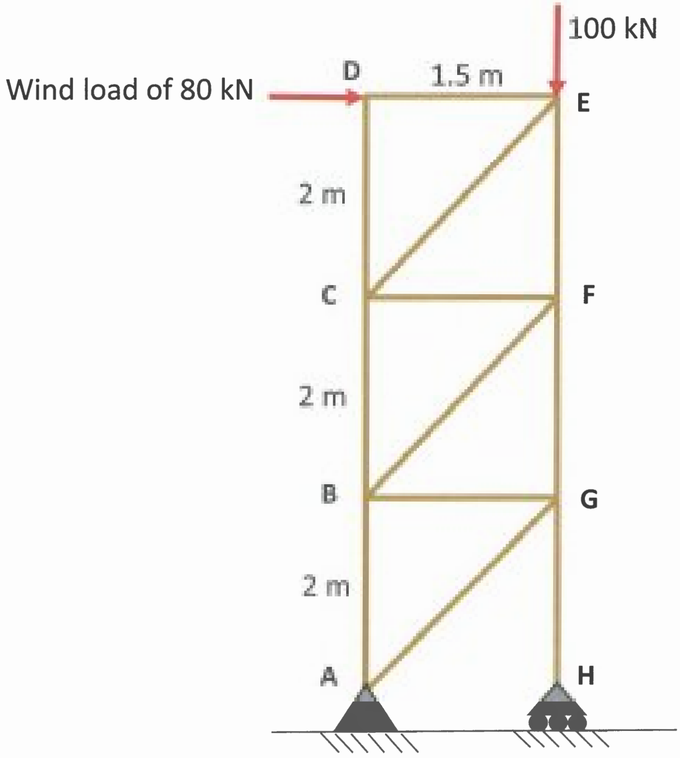
22. Using the method of sections, determine the force in members BC, BE and FE. State the nature of their forces.



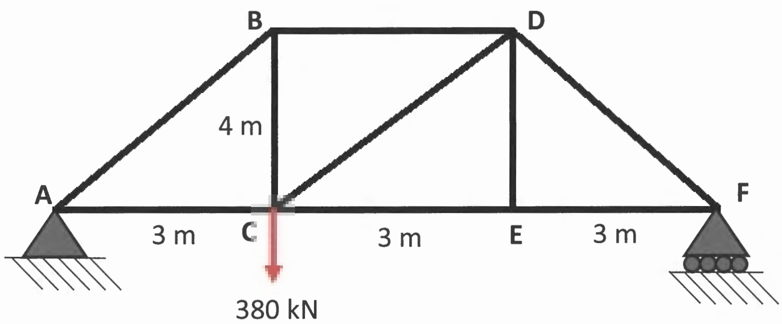
23. The truss is supported by a roller joint at point E and pinned to the wall at point D. Calculate the force in members BC, FC, and FE and state their nature (tension or compression).



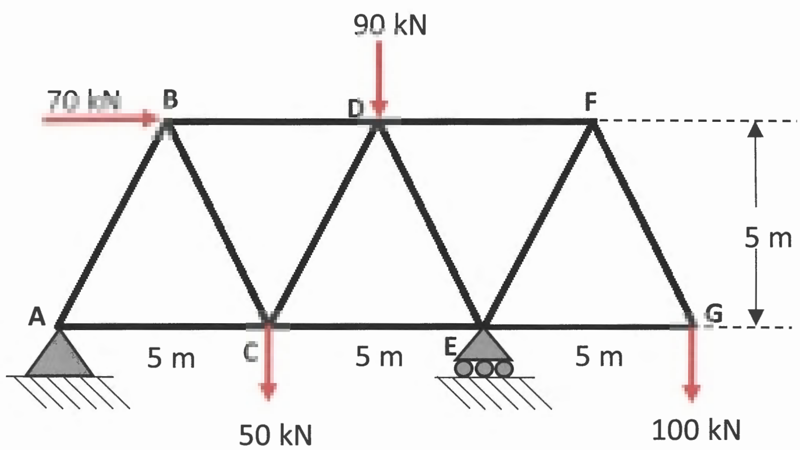
24. A pin jointed truss is loaded as shown below. Determine the magnitude of the vertical reactions at A and H and the horizontal reaction at A. Calculate the force in member BF and state its nature.



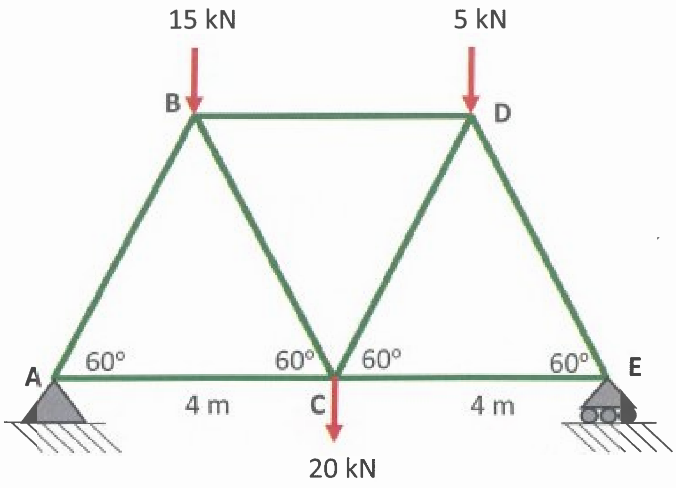
25. Using the method of sections for the roof truss shown below, determine the force in members BD, CD and CE.



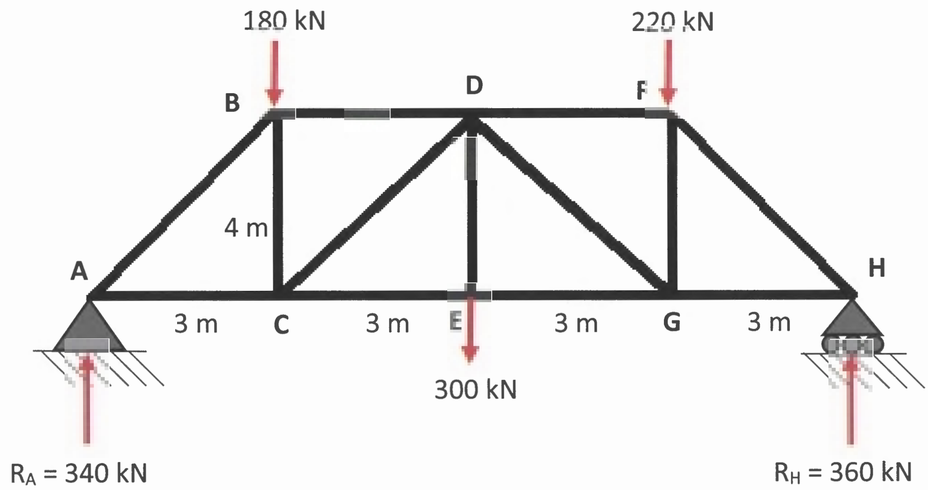
26. A loaded warren truss is supported by a pin joint at A and a roller joint at E. Determine the force in members BC, DF and CE.



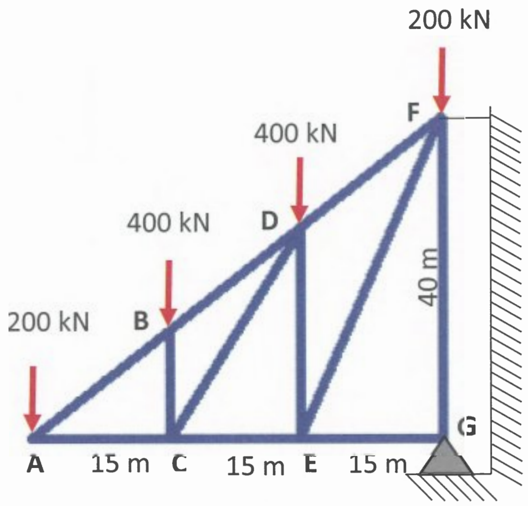
27. Using the method of sections, determine the force in members BD, CD and CE of the Warren truss as shown below. State the nature of these forces.



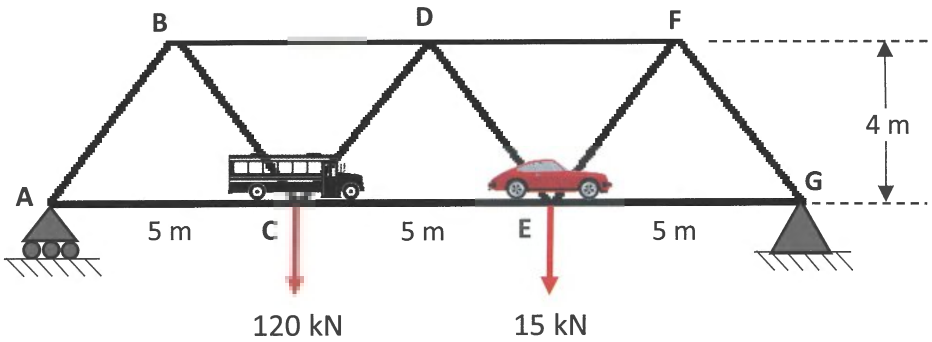
28. In the Howe truss shown below, determine the force in members DF, DG and EG and state their nature. The reaction forces have been provided.



29. By the cantilever truss shown below, determine the force in members DF, EF and EG State their nature.



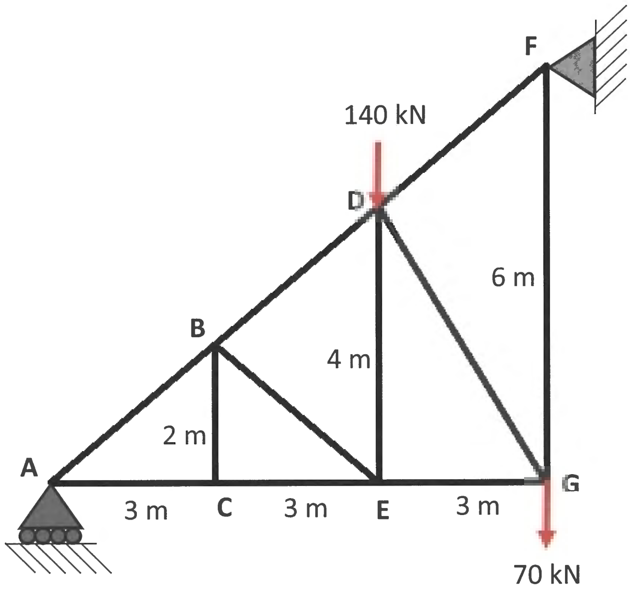
30. The below truss is subjected to a 12-tonne bus and a 1.5-tonne car acting on it. Determine the force in member CD by first calculating the reactions at the supports.



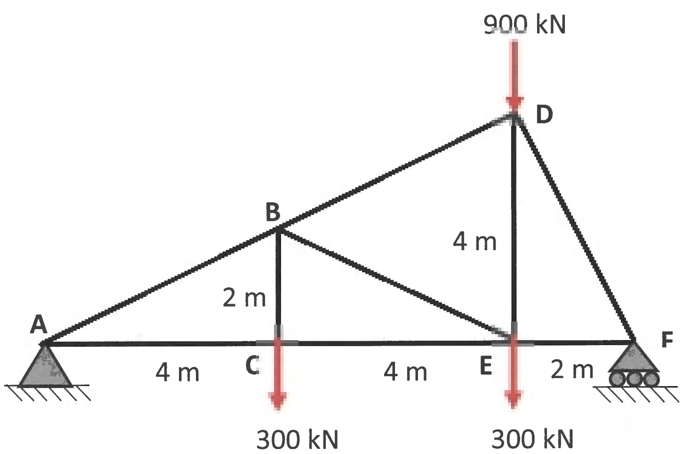
31. In the truss below, determine the following

a) The reaction forces acting on supports A and F

b) The force in members DF, DG and EG

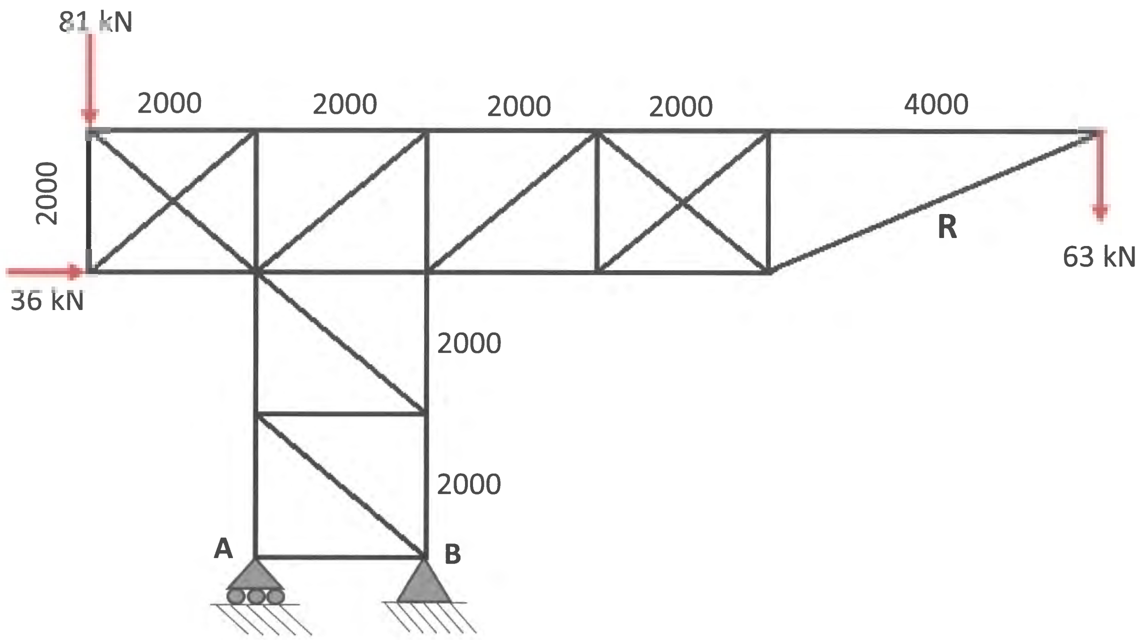


32. Determine the force in members BD, BE, and CE by first calculating the reaction forces at the supports.



Miscellaneous

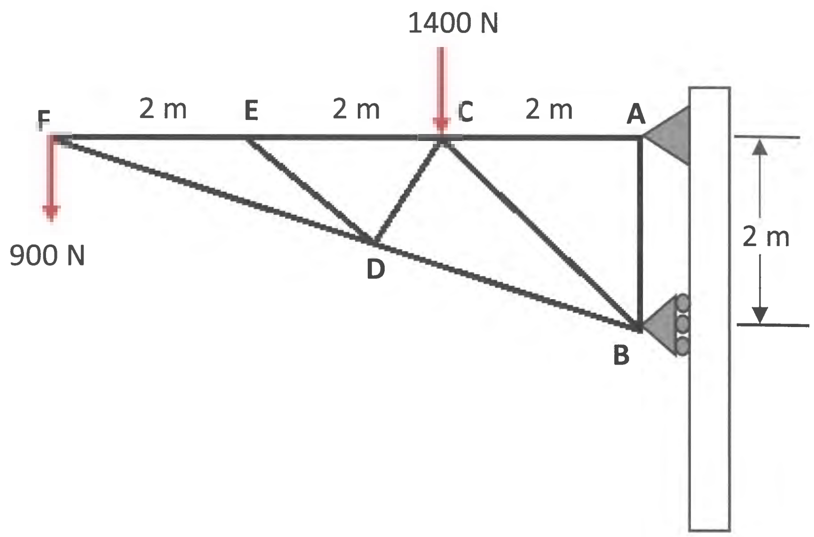
33. A hammerhead crane consisting of a pin jointed truss is used to lift a load of 63 kN.



a) Calculate the reactions at supports A and B

b) Determine the magnitude and nature of the force in member R.

34. A cantilever truss supporting overhead railway cables is shown below.



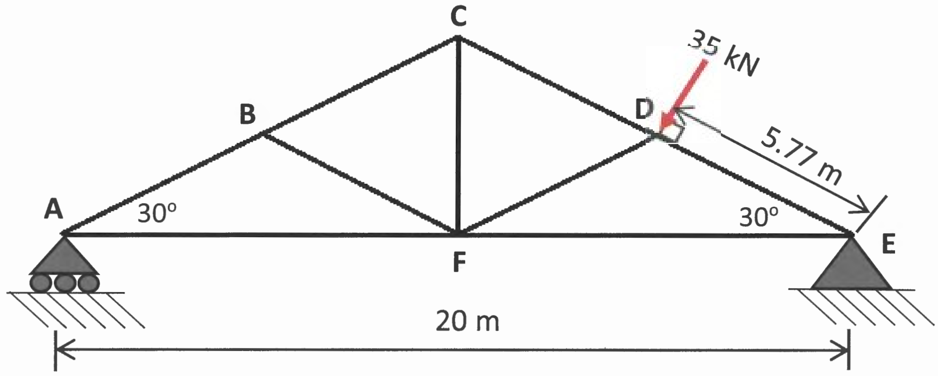
A weight of 1400 N is acting at joint C and the weight of the cable is acting at joint F with a value of 900 N.

a) Calculate the magnitude of the reaction at support B

b) Calculate the magnitude and direction of the reaction at the pin joint A.

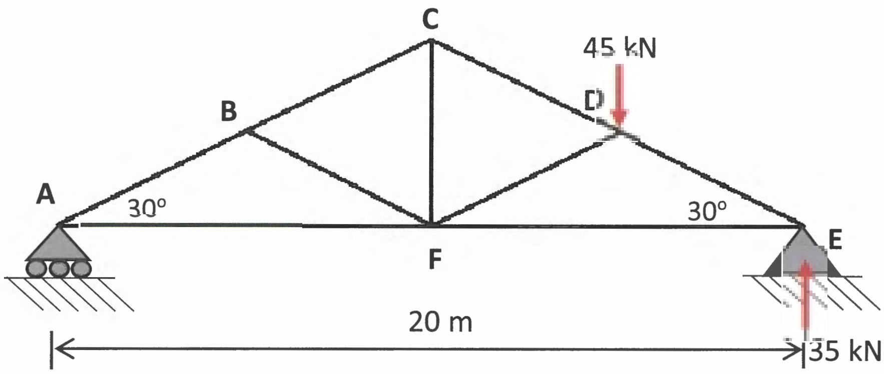
c) Calculate the magnitude of the force in member BC. State its nature.

35. A roof truss supported on each end is shown below.



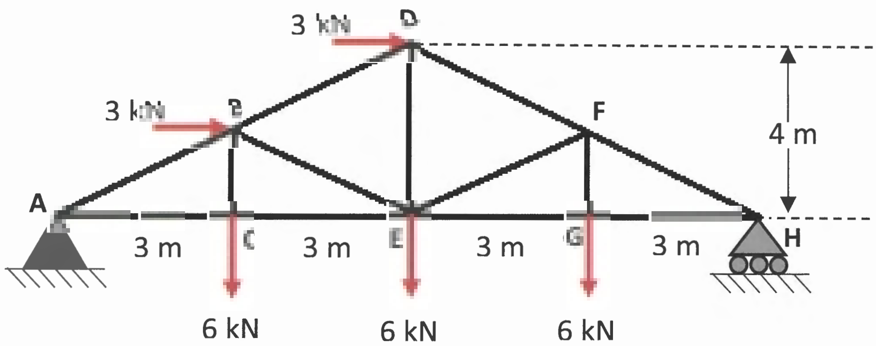
a) A force of 35 kN is applied at joint D. Calculate the magnitude and direction of the reaction at pin joint E.

b) The below truss now experiences a 45 kN vertical force acting on joint D and a reaction at pin joint E of 35 kN.



Determine the magnitude and nature of the forces in members CD and BF.

36. The following truss diagram has a pin support at A, a roller support at H and five external loads as shown below.



a) Calculate the support reactions at A and H

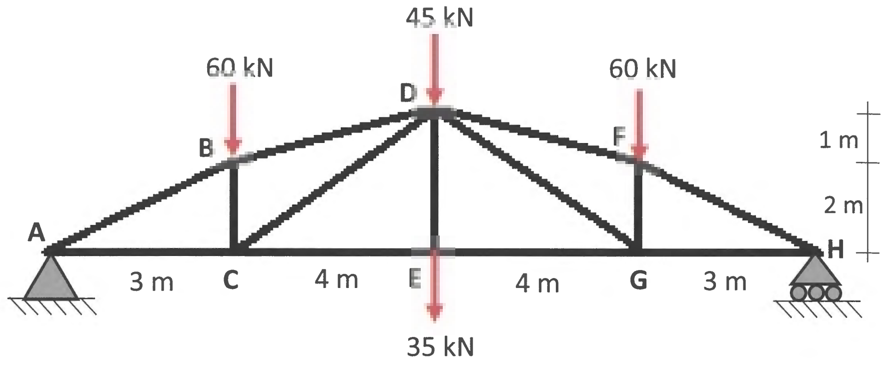
b) Calculate the magnitude of the force in members FH, GH and FG using the method of joints. State their nature.

c) Calculate the magnitude of the force in members BD, BE and CE using the method of sections. State their nature.

37. For the truss shown below, determine the following:

a) The reaction forces at the 2 supports.

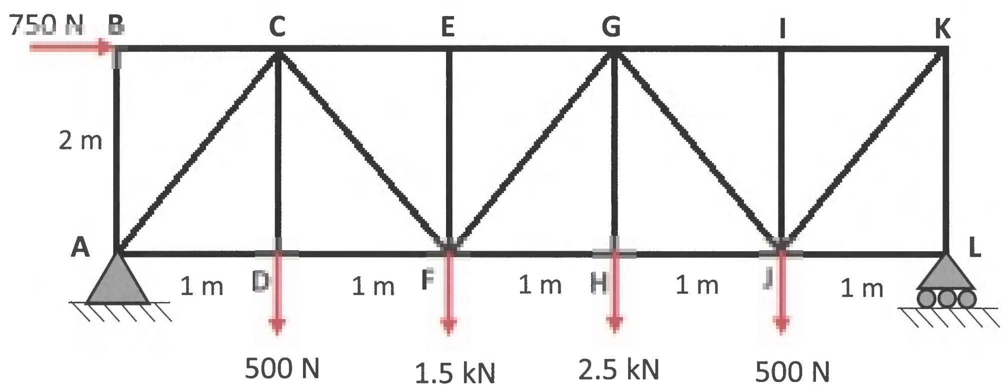
b) Force in members BD, CD and CE. State their nature.



38. The pin jointed truss below has 4 external loads acting at joint B, D, F and J.

a) Determine which members are redundant.

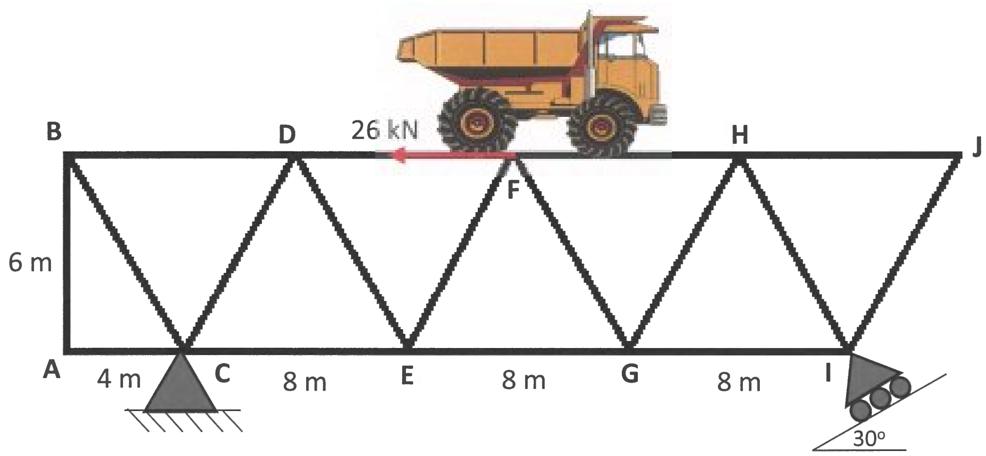
b) Determine the force in member FG.



39. A dump truck with a mass of 30 tonnes is used to carry 20 tonnes of soil across the pin jointed truss. It's centre of gravity in this instance is located above joint F.

a) Determine the reaction force that is created at both supports due to this dump truck.

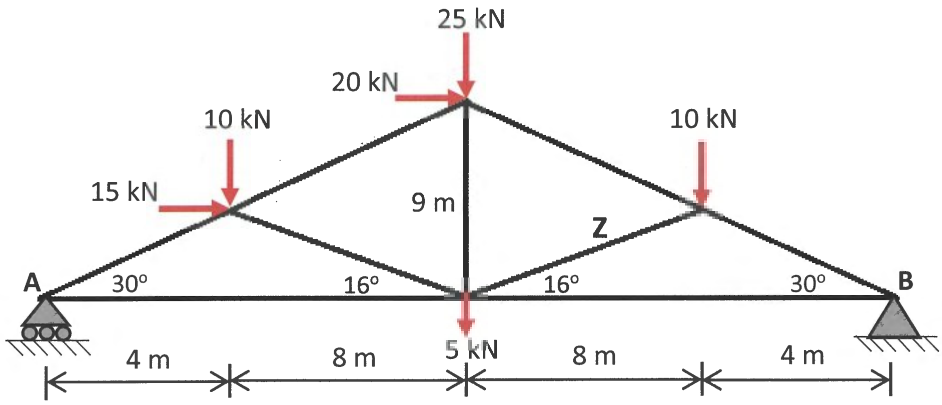
b) Determine the magnitude of the force in member DF and state its nature.



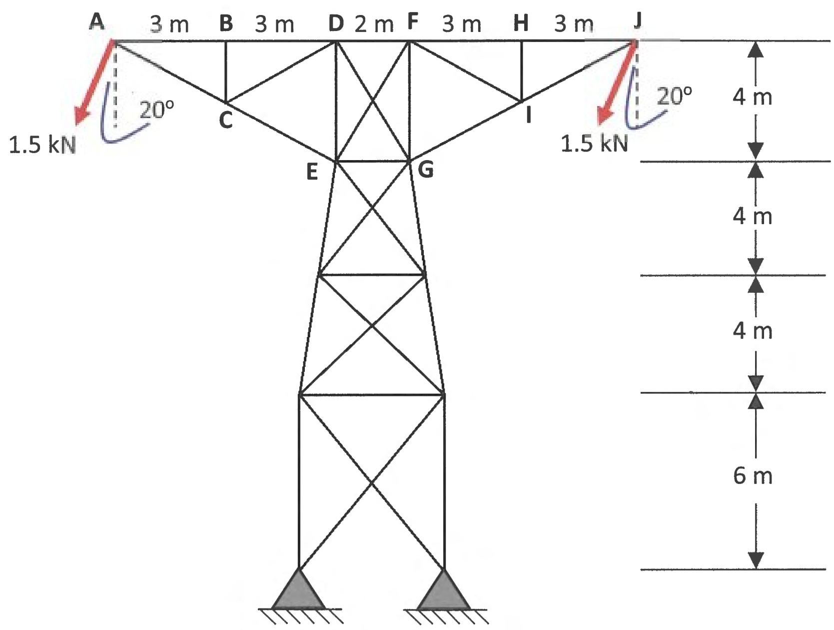
40. The roof truss below is subjected to 6 external loads due to wind and various weights.

a) Calculate the reaction at pin joint B.

b) Calculate the magnitude of the force in member Z. State its nature.



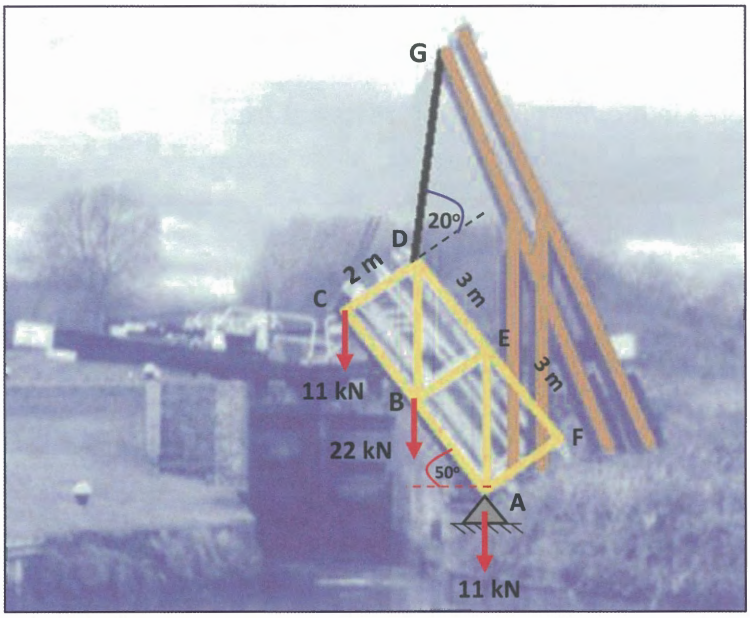
41. A tower for a transmission line is shown with 2 external loads of 1.5 kN acting at an angle of 20° to the vertical axis. Determine the force in members IJ, HI and FI.



42. A drawbridge is being raised by a cable DG. The 3 forces shown are due to the weight of the roadway.

a) Determine the force in cable DG.

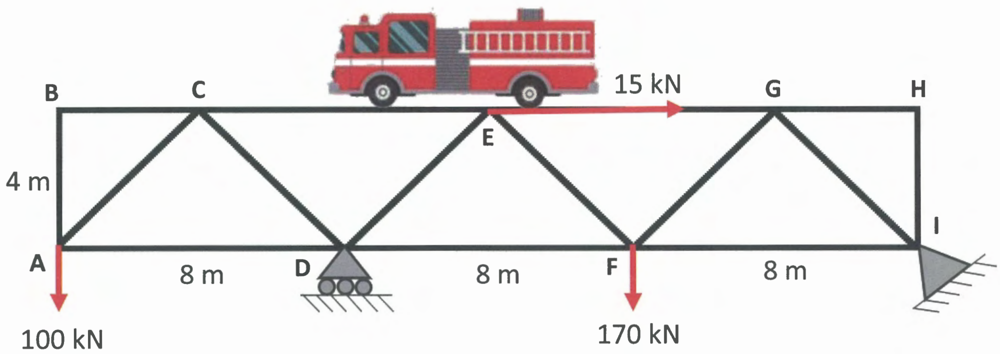
b) Determine the force in members DE, DC, DB, CB and EF.



43. The driver of the fire truck decides to cross the truss bridge shown below to get to his destination. Mass of the vehicle is 2 tonnes with its centre of gravity acting on joint E.

a) Calculate the magnitude and direction of the reactions at D and I.

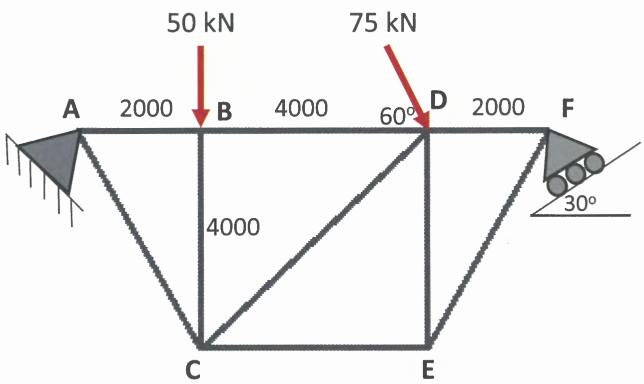
b) Calculate the magnitude of the force in member CE. State its nature.



44. A deck truss bridge is shown below.

a) Determine the reaction forces at joint A and F.

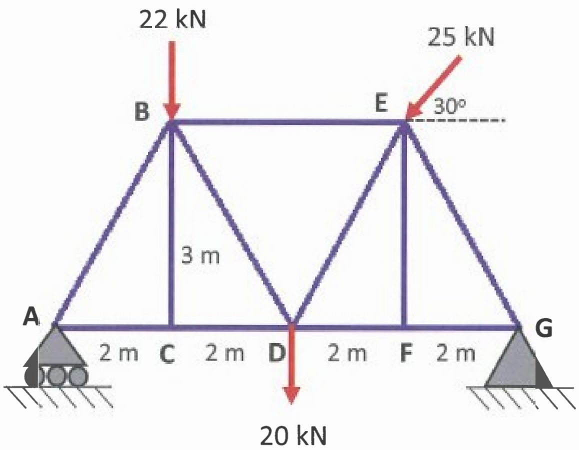
b) Determine the forces in member CD and CE. State their nature.



45. The below truss has 3 external loads acting on it as shown.

a) Determine the reaction force on the pin joint.

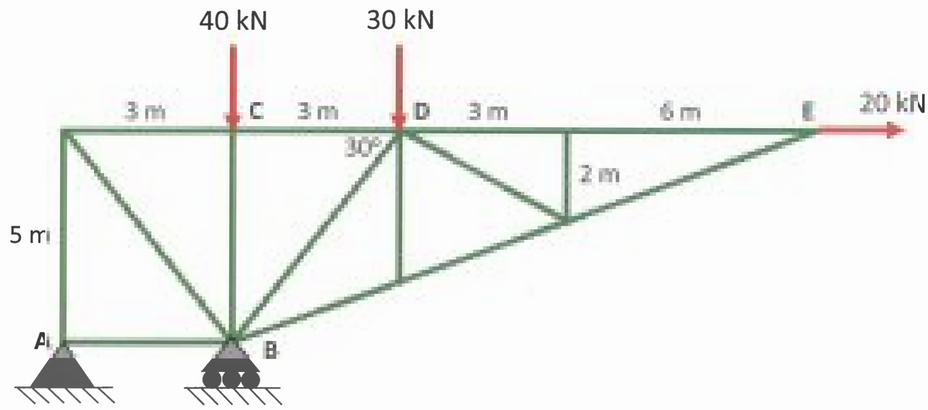
b) Calculate the force in member BE and state its nature.



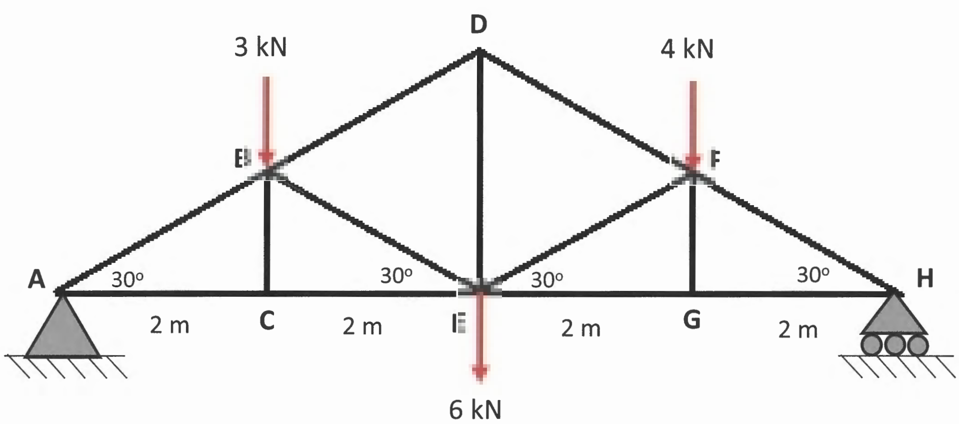
46. A cantilever truss bridge has 3 external loads acting at C, D and E.

a) Determine the force in the roller and pin reaction.

b) Determine the magnitude and nature of the force in member BD.



47. The below Howe truss has 3 external forces acting on joints B, E and F. Determine the forces in AB, AC, BC, BD and BE. State their nature.



Chapter 2 - Stress and Strain

Normal Stress

Normal stress (or simply, stress) is a measure of the internal reaction that occurs in a material in response to a load that is applied externally. It is used to define the loading in terms of the force applied to a cross-sectional area of the object. When looking at stress from a loading point of view, it is these forces that deform a body. This stress can either be a tensile stress, whereby the material is elongated, or a compressive stress, where the material is compressed or shortened. These stresses are shown in the figure below. The forces that are perpendicular (90^o) to the surface of the object indicate that the object is under normal stress.

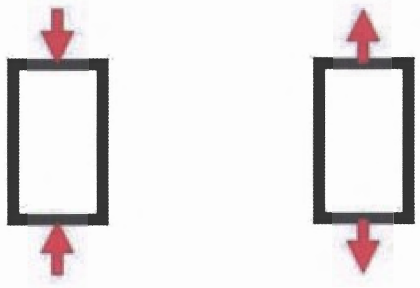


Figure 2.1: *Left - Compressive stress, Right - Tensile stress*

The equation to calculate stress is:

$$\sigma = \frac{F}{A}$$

Where:

σ = Stress
F = Applied force
A = Cross-sectional area

The units for implementing the stress equation are crucial. There are 2 situations which give different units for stress. If the force is in N and the area is in m^2 then our stress is in Pascals (Pa). If the force is in N and the area is in mm^2 then our stress is in Megapascals (MPa).

Worked Example 1:
Calculate the stress in a member when the axial load is 40 kN and cross-sectional area is 200 mm^2.

Solution:

Convert kN to N by multiplying it by 1000 then apply the stress formula.

$$\sigma = \frac{F}{A} = \frac{40 \times 1000}{200} = 200\ MPa$$

Shear Stress

Shear stress results from the application of opposing forces that causes layers to slide past one another in opposite directions. One common example is the action of chopping vegetables or scissors cutting paper. The forces that are parallel to the surface of the object indicate that the object is under shear stress. This can be seen in the figure below.

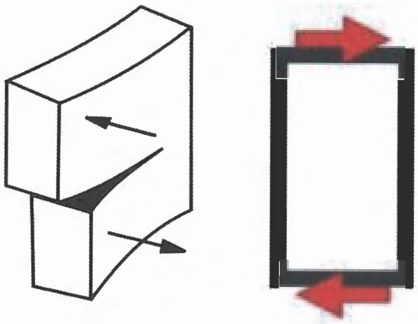


Figure 2.2: *Object under shear stress*

The equation to calculate the shear stress (τ) is the same as the normal stress; however, the area will depend on the number of surfaces that are in shear. For example, the left image below has 1 surface in shear (single shear) so the calculation of the shear stress will be the standard force divided by area equation. The image on the right, however, has 2 surfaces that are in shear (double shear) so the shear stress is now the force divided by two times the area.

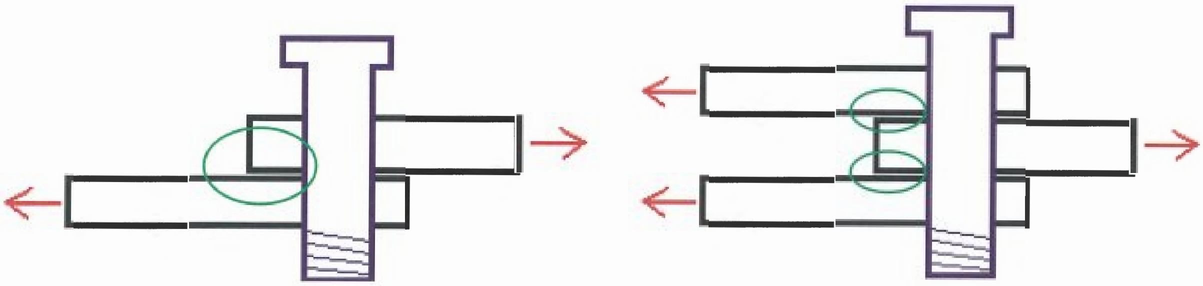
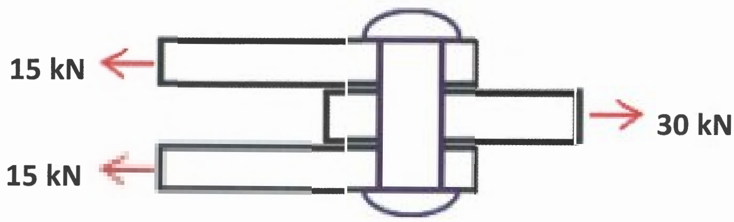


Figure 2.3: *Bolt undergoing single shear (left) and double shear (right)*

Worked Example 2:

A single rivet holds sheets of steel together and is loaded as shown. Determine the minimum diameter of the rivet if the maximum allowable shear strength of the rivet material is 50 MPa.



Solution:

This is a double shear problem whereby the rivet will shear in two places (contacts two surfaces). Hence when applying the shear stress equation, the area is doubled.
Note: Using **A = πr^2** will provide the same answer.

$$\tau = \frac{F}{2A}$$

$$50 = \frac{30000}{2 \times (\frac{\pi}{4} \times d^2)}$$

$$25\pi d^2 = 30000$$

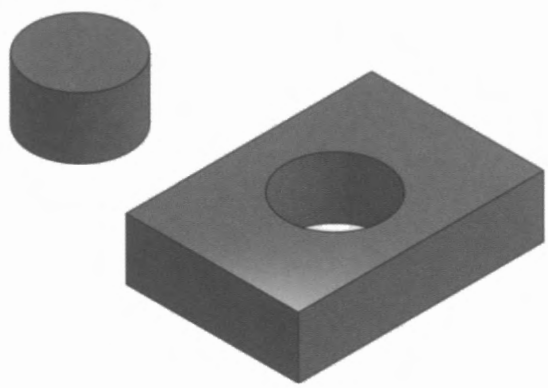
$$d = \sqrt{\frac{30000}{25\pi}} = 19.5\ mm$$

Punching shear stress

If the shearing process involves punching a hole out then the shearing area will become the perimeter of the hole multiplied by the thickness of the material.

$$\textbf{Punching Shear Stress } (\tau) = \frac{\textbf{Force (F)}}{\textbf{Perimeter (P) x Thickness (t)}}$$

Worked Example 3:
A force of 200kN is required to punch a hole with a diameter of 40mm through a 25mm thick plate. Determine the punching shear stress. **Note**: Perimeter of circle: **P = πd**



Solution:

$$\text{Punching Shear Stress} = \frac{200 \times 1000\ \text{N}}{(\pi \times 40) \times 25\ mm^2} = \mathbf{63.7\ MPa}$$

Engineering and True Stress

Engineering stress is found by dividing the applied load by the **ORIGINAL** cross-sectional area of a material. This is the same as the normal stress.

True stress is found by dividing the applied load by the cross-sectional area at that point in time (**ACTUAL** cross-sectional area).

Strain

Strain is the amount of deformation resulting from an applied force divided by the initial length of a material. Strain has no units and can be given as a ratio or percentage. Mathematically, it is represented as:

$$\textit{Strain}\ (\varepsilon) = \frac{\textit{Change in length}\ (\Delta L)}{\textit{Original length}\ (L_o)}\ \textit{or}\ \frac{e}{L}\ \text{(per the HSC formula sheet)}$$

The below image shows (a) undergoing an extension of ΔL when a force is applied and (b) being compressed a distance of ΔL when a force is applied in the opposite direction. The ΔL value is assumed to be small in this case. A large enough value would cause the cross sections of both (a) and (b) to change where (a) would get thinner decreasing its cross-sectional area and (b) would bulge out increasing its cross-sectional area.

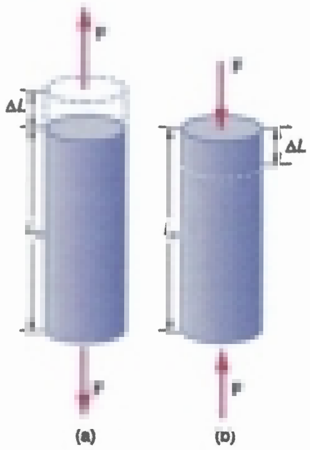


Figure 2.4: *Specimen undergoing deformation*

By combining the normal stress and strain equations, we get a relationship known as the FLEA formula. This will significantly reduce the amount of calculations required.

$$E = \frac{FL}{eA}\ \text{or}\ e = \frac{FL}{EA}$$

Where:

e = Extension
F = Force
L = Length
E = Modulus of elasticity
A = Cross sectional area

Factor of safety (FoS)

A factor of safety (FoS) is applied to design calculations to increase the safety of people and reduce the risk of failure of a product. The FoS is vital when it comes to safety equipment and fall protection. If a structure fails, there is a risk of injury and death as well as a company's financial loss.

Increasing the FoS increases the cost of the product so companies need to determine whether it is viable to spend a certain amount of money to reach the desired FoS. Companies will ultimately need to strike a balance between cost reduction and safety to efficiently run their business.

Put simply, the FoS is how much stronger a system is than required. A FoS of 1 indicates that a structure or component will fail once it reaches design load. A FoS of less than 1 is not practical. A FoS of 2 for example will mean that the component will fail at twice the design load.

Buildings commonly use a factor of safety of 2.0 for each structural member. This value is relatively low due to the loads being well understood as well as most structures being redundant. Pressure vessels use 3.5 to 4.0 and automobiles use 3.0.

In the aviation industry, the FoS is kept as low as possible (about 1.5) due to reducing the mass of the aircraft and reducing cost of manufacturing. Applying a larger safety factor would result in the structures being made stronger and stiffer. This means more material would go into making the parts hence increasing mass and cost.

The equations below are used to calculate the maximum allowable/working stress when a FoS is applied. Ductile materials use the **yield strength** value to determine the safety factor whereas brittle materials use the **ultimate strength** value.

For ductile materials – $\textbf{Max Allowable/Working Stress } \sigma_a = \dfrac{\textbf{Yield Stress } (\sigma_y)}{\textbf{FoS}}$

For brittle materials - $\textbf{Max Allowable/Working Stress } \sigma_a = \dfrac{\textbf{Ultimate strength } (\sigma_{UTS})}{\textbf{FoS}}$

Worked Example 4:

Four cables each with a diameter of 16 mm are used to support an elevator. There is a sign inside the elevator that shows the safe working load.

a) Determine the factor of safety if the cables have a yield stress of 130 MPa
b) Calculate the elongation of the steel cable in mm if it has an unrestrained length of 16m and a Young's modulus of 200 GPa.

Solution:

a) The four cables are required to support a load of 9000 N. Hence each cable will need to withstand a load of 2250 N (9000/4).

Calculating the working stress will give us the following:

$$\sigma = \frac{F}{A}$$

$$\sigma = \frac{2250}{\frac{\pi}{4} \times 16^2} = 11.2\ MPa$$

We now apply the working stress equation where we make the FoS the subject:

$$\text{FoS} = \frac{\sigma_{yield}}{\sigma_{allowable}}$$

$$FoS = \frac{130}{11.2} = 11.6$$

b) Using the FLEA formula, we can quickly determine the elongation. To get the elongation in mm, we need to convert m to mm by multiplying by 1000 and converting GPa to MPa by multiplying by 1000.

$$e = \frac{FL}{EA}$$

$$e = \frac{2250 \ \times 16000}{200000 \times \frac{\pi}{4} \times 16^2} = \mathbf{0.9\ mm}$$

Load/Extension and Stress/Strain diagrams

We can obtain various information about mechanical properties of a material by performing a tensile test. This test will produce a load-extension graph which can then be converted to a stress-strain graph. There are 2 regions; elastic region (green) whereby the material will return to its original shape if the load is removed and the plastic region (pink) where permanent deformation will occur even if the load is removed.

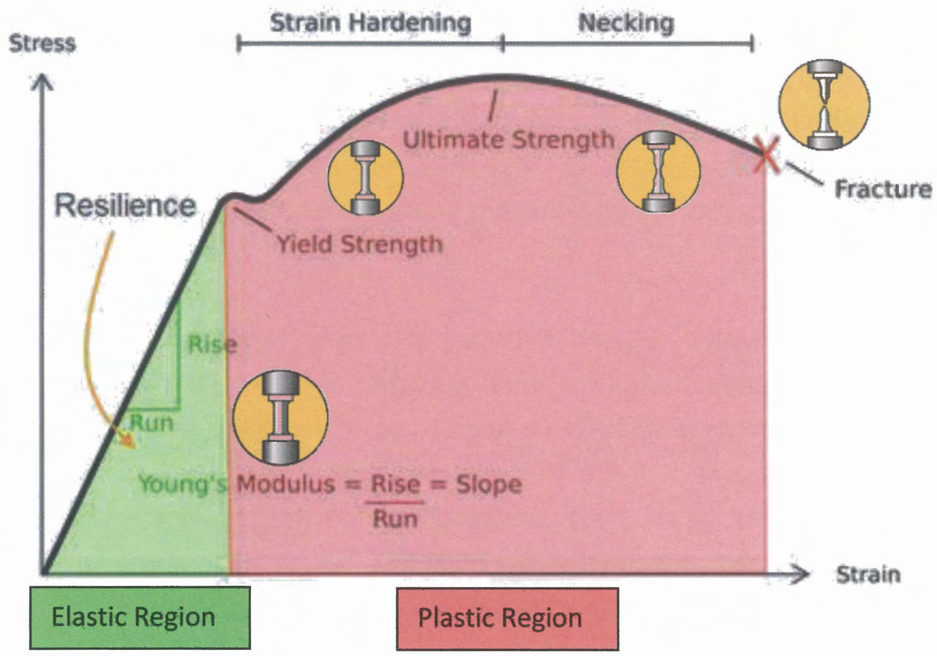


Figure 2.5: *Stress strain graph*

- **Young's Modulus** – Represents the stiffness of the material. This is determined by finding the gradient of the elastic region (straight line) of a stress-strain graph.
- **Toughness** – Represents how much energy/impact a material can absorb. This is determined by finding the area below the entire curve (not required in this course).
- **Yield strength/stress** – Represents the stress at which a material begins to deform plastically. This is found at the point just past the elastic limit point.
- **Proof stress** – For graphs that show a progressive yield point and not a definite yield point then the proof stress terminology is used. This is determined by drawing a line parallel to the elastic region (straight line) of the graph from an offset of 0.2% of strain.
- **Ultimate Tensile Strength** or **Tensile Strength** – Represents the maximum tensile stress a material can take before it fractures. This can be determined by looking at the highest point on a stress-strain curve.
- **Resilience** – Ability of a material to absorb energy when it is deformed elastically and to release that energy upon unloading. This is determined by finding the area underneath the elastic region (represented in green) of the graph.
- **Ductility** – Represents how much plastic deformation a material will undergo before fracturing. This is the preferable way of deformation as it gives an indication that the material is on its way to failure. Ductility can be seen by how much strain a material undergoes before it fractures.
- **Brittleness** – If the material does not undergo plastic deformation (red region in the graph above) then the material is said to be brittle. This is not ideal as the material will fracture without any warning.
- **Proportional limit stress** – Represents the highest stress at which stress is directly proportional to strain (i.e. highest stress at which the stress-strain curve is a straight line).
- **Elastic limit** – Represents the point on the stress-strain curve up to which the deformation is elastic in nature. This point is after the proportional limit point.

Shear Force Diagrams (SFD) and Bending Moment Diagrams (BMD)

Shear force is the force in the beam acting perpendicular to its longitudinal axis (x-axis). For design purposes, a beam's ability to resist shear forces is more critical than its ability to resist an axial force (force in the beam acting parallel to the x-axis). A **shear force diagram** represents the sum of the shear force reactions that act at given points along the beam in response to externally applied loads.

Bending moment is the reaction induced in a structural member caused by an external force acting perpendicular to the member, resulting in a bend in the beam. A **bending moment diagram** represents the sum of the moments acting at a given point along the beam.

Both shear force and bending moment are essential for the analysis and design of structural members. When choosing an appropriate material, it is necessary to check the member for shear and bending moments so as to withstand these internal forces and bending moments. These are vital to avoid failure of structural members.

Bending Stress

When a member is being loaded, bending stress (or flexure stress) will occur. When a beam experiences load and it sags in a "U-shape" then the top fibres of the beam will undergo a normal compressive stress and the bottom fibres of the beam will undergo a normal tensile stress. The stress at the neutral axis (NA) is zero and this is the point where compressive stress transitions into a tensile stress or vice versa. This can be seen in the diagram below where max stress is at the top and bottom of the beam.

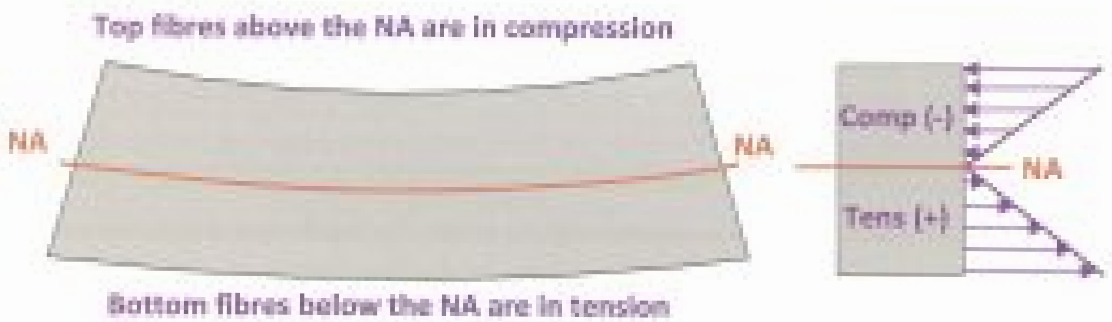


Figure 2.6: *Beam undergoing bending whereby top is in compression and bottom in tension. NA in the middle.*

The bending stress value will vary with the distance from the neutral axis and is calculated via the following equation:

$$\sigma = \frac{My}{I}$$

Where:

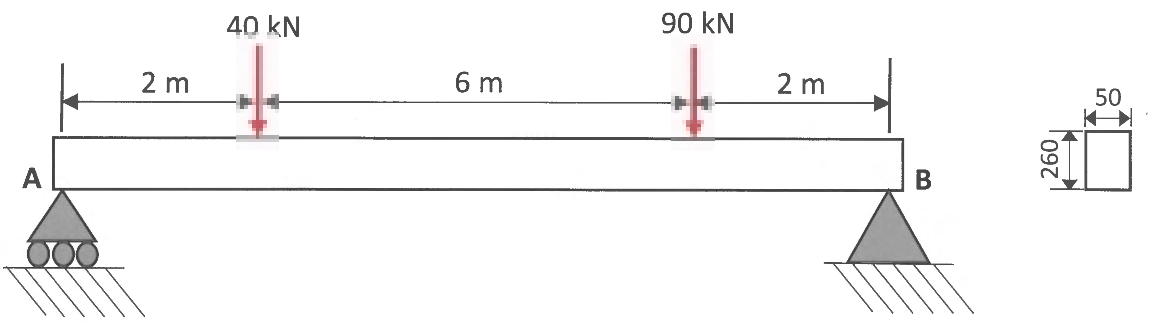
σ = bending stress either tensile or compressive - (MPa)
M = bending moment (found using the bending moment diagram) - (Nmm)
y = vertical distance from the NA. In a rectangular cross section, NA passes through the middle of the section hence the y-value to top/bottom surface is half of the vertical dimension – (mm)
I = Second moment of area which is a property of a structural shape (value is usually given) – (mm^4)

Note: The second moment of area, I, is a measure of the efficiency of a cross sectional shape to resist bending caused by loading. I-beams are widely used in structural engineering due to its high second moment of area value.

Drawing a Shear Force Diagram for a loaded beam

There are usually 2 methods in calculating the shear forces and the bending moments along a beam. In this book the "follow the forces" and "area under shear force diagram" method will be explored, and it will be all you need to answer questions of this sort. This will be demonstrated below.

Worked Example 5:



a) Draw the shear force diagram (SFD) for the loaded beam shown above.
b) Draw the bending moment diagram (BMD) for the loaded beam.
c) Determine the maximum bending stress in the beam if the second moment of area, I, is 210×10^6 mm^4.

Solution:

a) **Step 1** - Calculate reaction forces at supports A and B by applying sum of moments about one of the supports then applying sum of forces in the vertical direction to find the force at the other support.

$$+\curvearrowleft \sum M_B = 0$$
$$-A(10) + (40 \times 8) + (90 \times 2) = 0$$
$$10A = 500$$
$$\therefore A = 50\ kN \uparrow$$

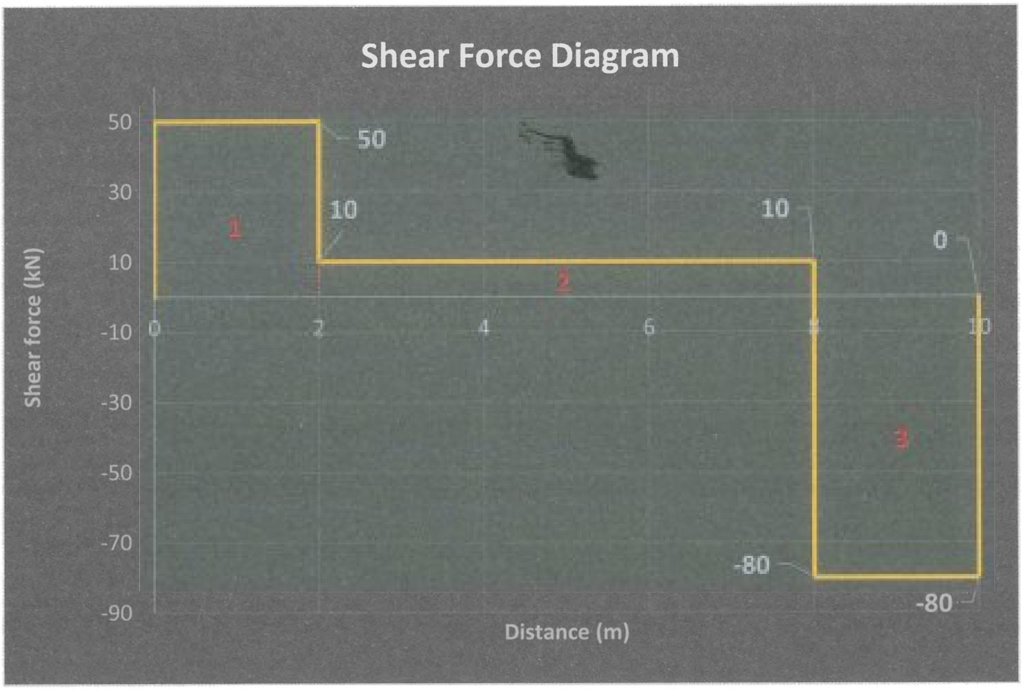
$$+\uparrow \Sigma F_v = 0$$
$$A - 40 - 90 + B = 0$$
$$B = 40 + 90 - 50$$
$$\therefore B = 80\ kN\ \uparrow$$

Step 2 – Start the SFD at the first value of the force acting on the beam which in this case is 50 kN due to the reaction at support A.

Step 3 – Move across the beam until you see an external force acting on the beam. When you get to this force, add its value to the SFD. In this case, after 2 metres we encounter a negative 40 kN force (due to it acting downwards). Hence, we minus 40 kN from the previous 50kN which gives us 50 – 40 kN = 10kN.

Step 3 (repeated) – Continue moving across for another 6 metres where you will encounter a negative 90 kN force. Now we minus 90 kN from the previous value of 10kN which gives us 10 – 90 = - 80 kN. Move across for another 2 metres where you will encounter the reaction of 80 kN acting upwards on support B. Thus, we now have 80 kN added to the previous value of -80 kN giving us a value of 0 kN.

Note: Your SFD should always equal to 0 at both ends of the beam.



Drawing a Bending Moment Diagram for a loaded beam

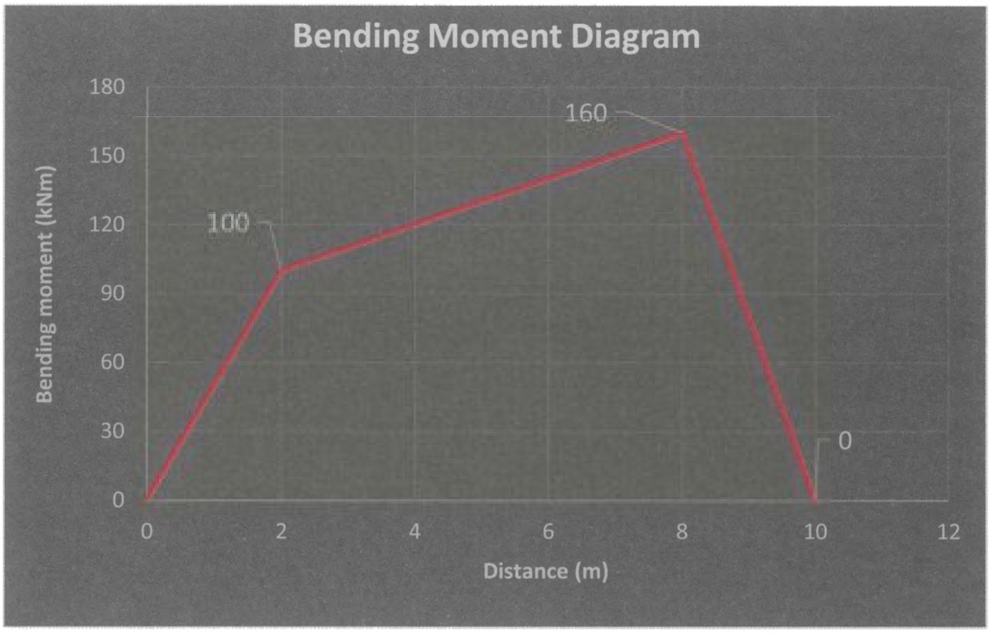
The ***bending moment*** at any point along the beam is equal to the area under the shear force diagram up to that point. To find the bending moment, we break up the SFD into regions and then sum up the area in each region. The example above has 3 labelled regions. If the graph is above the x-axis, we add the moments. If the graph is below the x-axis, then we minus the moments.

b) Step 1 - Area of the first region is $50 \times 2 = 100\ kNm$

Step 2 - Area of the second region is $10 \times 6 = 60\ kNm$. So now our total moment is $100 + 60 = 160\ kNm$. We add the second moment since it is above the x-axis.

Step 3 - Area of the third region is $80 \times 2 = -160\ kNm$. This is a negative value due to the region being below the x-axis. Hence the total moment is now $160 - 160 = 0\ kNm$.

Note: For a simply-supported beam, the bending moment at the ends will always be equal to zero.



c) Step 1 – Before applying the equation, we need to figure out the maximum bending moment from the BMD above. This value upon inspection is 160 kNm.

Step 2 – Convert 160 kNm to Nmm. To do this we simply multiply the value by 1000 to get kN to N and then multiply by 1000 again to convert m to mm. Hence our moment value is now 160 x 10^6 Nm.

Step 3 – Our y-value will be half of 260 mm since the neutral axis is acting across the centre of the rectangular cross section. Hence our y-value is now 130 mm.

Note: Always convert kN to N. Depending on the units of the I value, it is preferable to convert m to mm since the I is in mm^4. In the case you do want to convert mm^4 to m^4, then you would divide the value by $(1000)^4$ so 10^{12}

$$\sigma = \frac{My}{I}$$

$$\sigma = \frac{(160 \times 1000 \times 1000) \times 130}{210 \times 10^6}$$

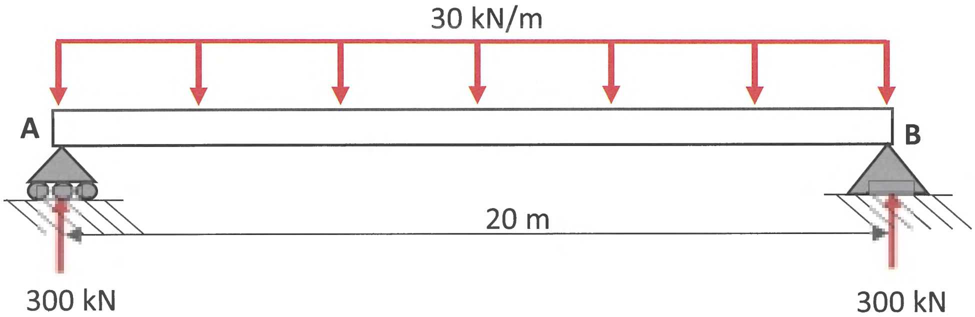
$$\sigma = 99\ MPa$$

Uniformly Distributed Loads (UDL)

A uniformly distributed load (UDL) is a load of the same magnitude that is spread across a region of an element such as a beam or slab. For this course, you are required to understand the effect of a UDL on a beam without using calculations. However, calculations will be used to explain the UDL concept.

Worked Example 6:

Determine the shape of the shear force and bending moment diagrams for the beam that experiences a uniformly distributed load as shown below.

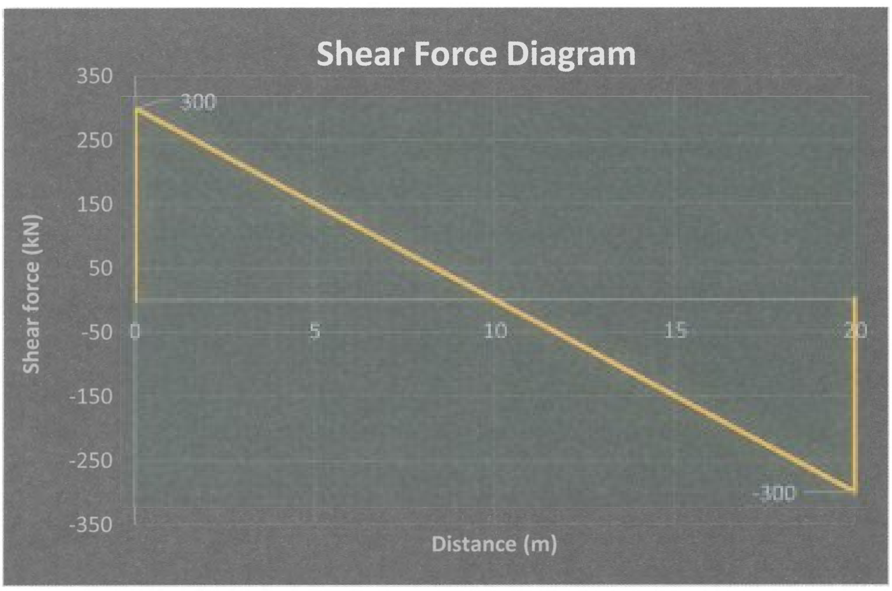


Shear Force Diagram

1. Start the SFD at the first value of the force acting on the beam which in this case is 300 kN due to the reaction at support A.

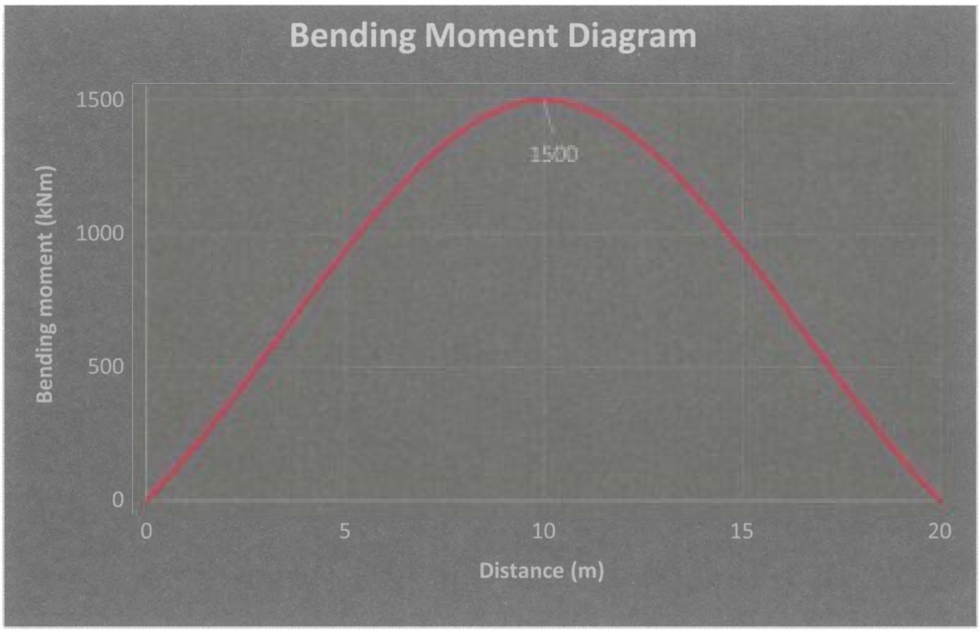
2. The value of the shear force drops 30 kN for every metre along the beam. So after 1 metre, the SF value is at 300 – 30 = 270 kN and at 2 metres it drops a further 30 kN with the SF value now at 240 kN. At the halfway point along the beam (10m), the SF value would drop by 300 kN where now the total SF value is 300 – 300 = 0 kN. At 20m, the total shear value is now at 300 – 600 = -300 kN. Thus, it can be seen that the SF value is dropping in a linear fashion.

3. The reaction at support B is 300 kN so the SF value rises up vertically from -300 to 0.



Bending Moment Diagram

1. The bending moment is the area below the straight line. Finding the area below a straight line becomes a parabola mathematically.
2. If the SF value is above the x-axis then the bending moment will increase. If the SF value is below the x-axis then the bending moment will decrease.



Questions

Section I

1. Toughness is best defined as the
   (A) energy absorbed by a material without yielding.
   (B) area underneath the stress-strain graph up to the yielding point
   (C) energy absorbed by a material without fracturing
   (D) ability of a material to resist deformation.

2. A material's toughness is equal to the area under which part of the stress-strain curve.
   (A) Elastic
   (B) Plastic
   (C) Both
   (D) None

3. Hooke's law holds true up until the
   (A) yield point
   (B) proportional limit
   (C) breaking point
   (D) elastic limit

4. An applied load on a wire causes its radius to double. Determine the effect this will have on the Young's modulus.
   (A) Doubled
   (B) Halved
   (C) Quadrupled
   (D) No effect

5. The tensile strength of a material is obtained by dividing the maximum load during the test by the
   (A) minimum area after fracture
   (B) area at the time of fracture
   (C) original cross-sectional area
   (D) average of (b) and (c)

6. If a part is heated and its movement is restricted, what stress will it experience as a result
   (A) no stress
   (B) tensile stress
   (C) shear stress
   (D) compressive stress

7. What can we deduce about a material if a test piece returns to its original shape after the load has been removed
   (A) It is plastic
   (B) It is elastic
   (C) It does not obey Hooke's Law
   (D) It has high stiffness

8. A material that does not give any indication of deformation when a stress is applied is known as a
   (A) composite
   (B) brittle material
   (C) polymer
   (D) ductile material

9. Factor of safety for a ductile material is the ratio of
   (A) Ultimate stress to working stress
   (B) Ultimate stress to yield stress
   (C) Yield stress to working stress
   (D) Breaking stress to working stress

10. For metals which have a progressive yield point, the proof stress is determined by drawing a line parallel to the elastic region at a strain of
   (A) 0.2 %
   (B) 0.5 %
   (C) 1 %
   (D) 2 %

11. What changes occur to a mild steel specimen after it has undergone tensile testing?

	Cross sectional area	Gauge length (just before fracture)
(A)	Decreases	Increases
(B)	Increases	Decreases
(C)	Increases	Increases
(D)	Decreases	Decreases

12. True stress is defined as the:
   (A) Force divided by the original cross-sectional area
   (B) Young's modulus divided by the strain
   (C) Force divided by the cross-sectional area at that point in time
   (D) Force multiplied by the cross-sectional area

13. In a brittle material, which two stresses are used to calculate the factor of safety (FOS)?

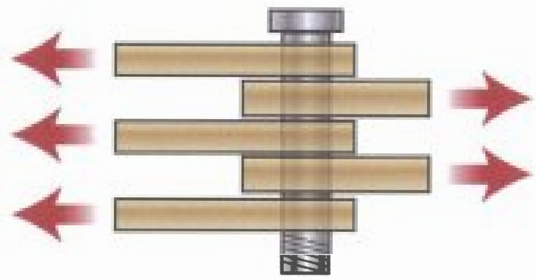
(A) Working and yield

(B) Working and UTS

(C) Bending and compressive

(D) Yield and UTS

14. The diagram below shows a steel bolt undergoing a particular shearing stress.



What type of shear stress is the bolt experiencing?

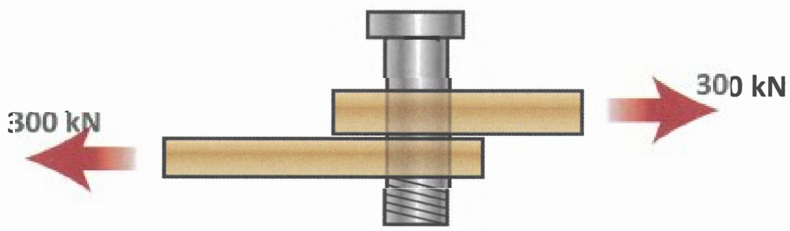
(A) Single

(B) Double

(C) Triple

(D) Quadruple

15. A 300 kN force is applied to the plates as shown below.



The shear stress in the ϕ25 mm diameter bolt is closest to

(A) 12 MPa

(B) 153 MPa

(C) 306 MPa

(D) 611 MPa

16. A Uniformly Distributed Load (UDL):

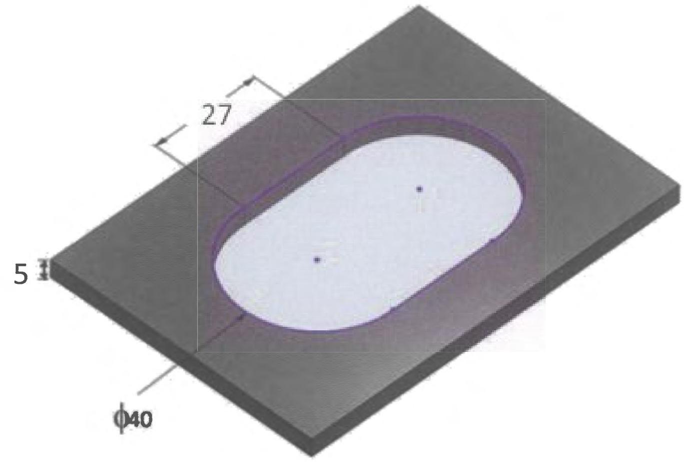
(A) has the same magnitude along the beam.

(B) has the same SFD and BMD shape when concentrated loads are placed along the length of the beam

(C) changes uniformly in magnitude along the beam

(D) has no effect on calculations on a simple beam

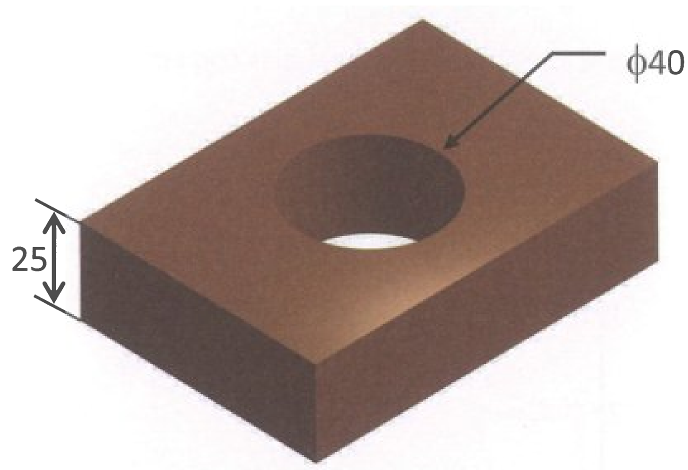
17. A 5 mm thick metal plate has a hole punched through it as shown



Determine the force required to punch the hole if the ultimate shear stress of the plate is 750 MPa.

(A) 674 kN
(B) 94 kN
(C) 471 kN
(D) 4712 kN

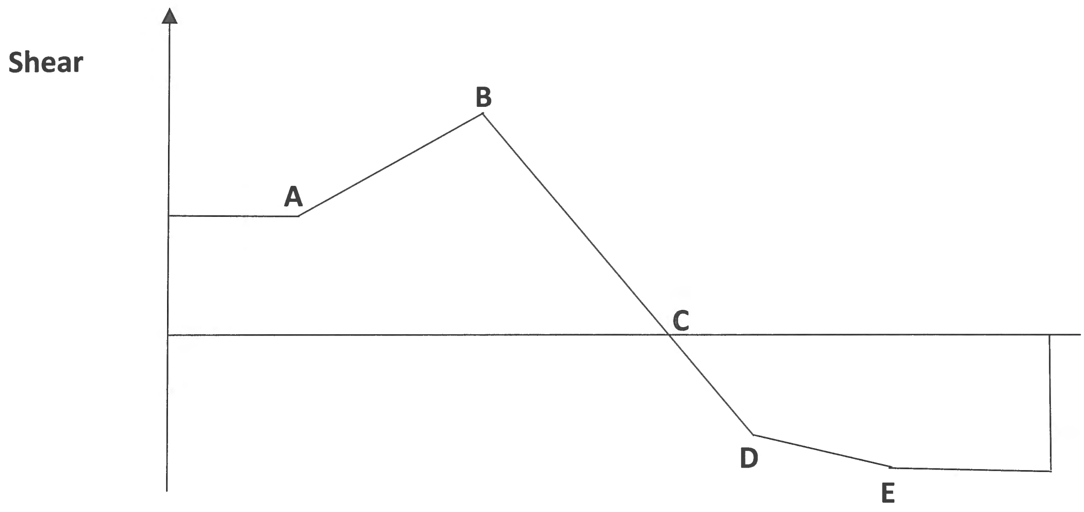
18. The ultimate shear stress of the metal plate shown below is 250 MPa.



Calculate the force that is needed to punch the hole.

(A) 785 kN
(B) 314 kN
(C) 250 kN
(D) 7854 kN

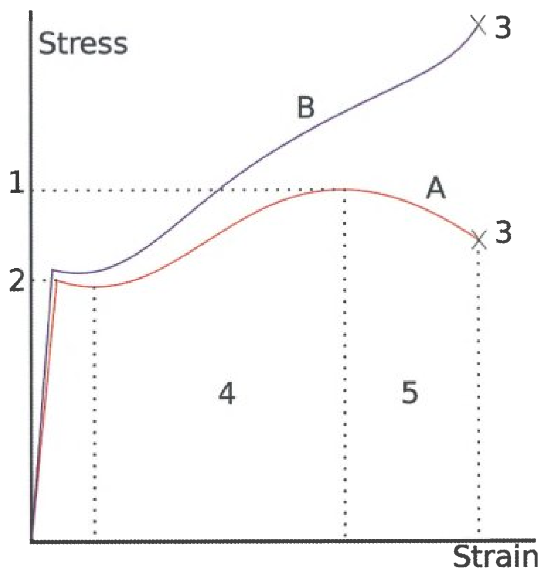
19. A shear force diagram for a loaded beam is drawn below.



Which point of the beam will the maximum bending moment occur?

(A) A
(B) B
(C) C
(D) D
(E) E

20. Based on the diagram below, determine which label is correct.



(A) A – True stress-strain curve & 5 – Toughness
(B) A – Engineering stress-strain curve & 5 – Toughness
(C) B – True stress-strain curve & 5 – Necking occurs
(D) B – Engineering stress-strain curve & 5 – Necking occurs

Section II

21. A rubber strip of length 80 mm is stretched until it is 110 mm long. What is the tensile strain?

22. A marble cylindrical column with a 450 mm diameter is used to support a load of 30 kN. Determine the compressive stress in the column. Give your answer in kPa.

23. A 2.5 m copper wire with a diameter of 0.35 mm is suspended from the ceiling. The wire extends 0.8 mm when a 600g mass is suspended at the bottom of the wire. Determine the following:
    - **a)** Strain of the wire
    - **b)** Stress in the wire
    - **c)** Young's modulus of the Copper wire

24. The heels on a pair of women's shoes have been designed to have a radius of 5 mm at the bottom. If 20% of the weight of a woman whose mass is 62kg is supported by each heel, determine the stress on each heel.

25. A 5m long copper wire with a Young's Modulus of 120 GPa will stretch 5 mm when a load of 2000N is hung at the end of the wire. Find the diameter of this copper wire.

26. An abseiler uses a kernmantle rope that has been specifically designed for climbing.

The rope is 50m long with a diameter of 11mm and stretches 3m when supporting a 750 N load. Determine the modulus of elasticity for this type of rope. Give your answer in MPa.

27. Human bone has a Young's Modulus of 20 GPa. Under compression, it can withstand a stress of about 210 MPa before fracturing. Determine how much compression the bone can withstand before breaking assuming a human adult thigh bone has a length of 0.48m.

28. A 3-tonne statue sits on top of a concrete square pillar which has side lengths of 1m and a height of 5m. Determine how much the pillar has been compressed? Assume Young's modulus of concrete to be 35 GPa.

29. A 2m copper wire with a diameter of 3mm and a Young's modulus of 116 GPa is used to support a 4kg mass. Determine the following:

a) The stress in the wire

b) The elongation of the wire.

30. A patient's leg was put into traction, stretching the femur from a length of 0.479 m to 0.48 m. The femur has a diameter of 2.34cm and the Young's modulus of bone is said to be 21 GPa. Determine the force required to stretch the femur?

31. Gondolas at ski resorts are carried by suspension cables as shown in the figure below.

Calculate the amount of extension in the ϕ 50mm steel cable if it can withstand a maximum tension of 4MN. Assume that the cable has an unsupported span length of 4.5 km and a modulus of elasticity of 205 GPa.

32. A metal wire is 3 m long and has a diameter of 3.5 mm. It stretches 0.25 mm when a force of 12N is applied. Determine the:

a) Stress in the wire (MPa)

b) Strain in the wire

33. A 4 m long steel bar with a diameter of 20 mm undergoes an extension of 0.3 mm when a force of 35 kN is applied. Calculate:

a) Stress in the rod (MPa)

b) Strain in the rod

34. A force of 50 kN has extended a 450 mm bar to a length of 455 mm. Determine the following if the bar has a diameter of 15mm:

a) Stress
b) Strain
c) Modulus of elasticity (assuming it is still within the elastic region)

35. A 250 mm long bar with a diameter of 50 mm has been stressed to a value of 320 MPa. Considering the Young's modulus of this bar is 205 GPa, determine the following:

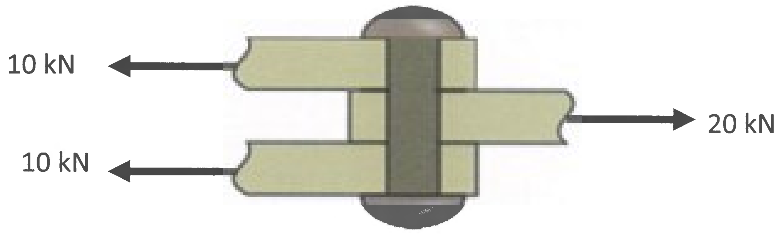
a) Strain in the bar
b) Force required (kN)

36. A 200 mm metal rod increases in length by 0.08 mm when subjected to a 20 kN axial force. Determine the diameter of this rod if it has a stiffness value of 190 GPa.

37. A 2 m circular metal column is used to support a load of 650 tonnes. It has been designed in a way that it does not compress more than 0.15 mm. Calculate the diameter of this 200 GPa metal column.

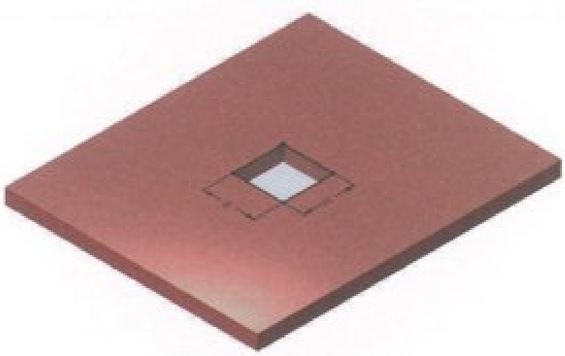
38. Calculate the force required to punch a 40mm diameter hole in 4 mm thick sheet metal given that the ultimate shear stress is 80 MPa

39. A single rivet holds three sheets of metal together and is loaded as shown. Find the minimum rivet diameter if the maximum shear stress allowed is 150 MPa and a factor of safety of 1.5 is applied.



40. A material must resist a load of 500 N when impacted by a 10 x 10 square punch. The maximum allowable shear stress in the material is 5 MPa.

Calculate the minimum thickness of the material required to prevent the punch from piercing through.



41. Explain the difference between elastic deformation and plastic deformation.

42. The photograph below shows a 'ductile' labelled manhole used to cover an opening that leads to a sewer.

Ductile iron has been chosen over the more brittle cast iron material as it can resist the same traffic loads with less self-weight.

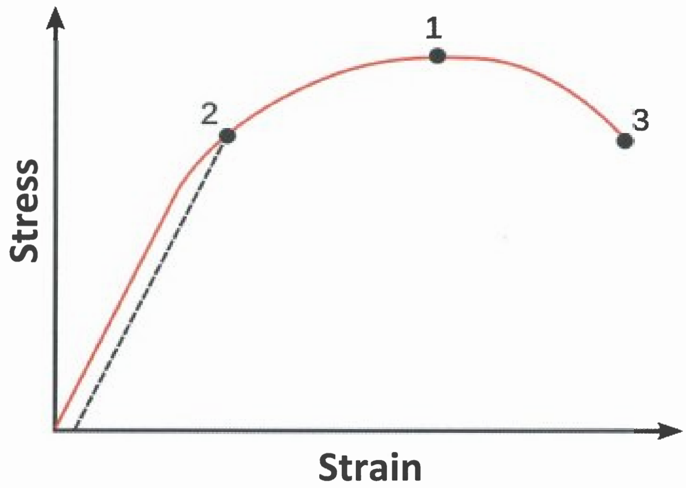
a) Describe the following terms and illustrate your answer by sketching the appropriate stress-strain diagram.

i) Ductile

ii) Brittle

b) Determine the stress the 600 x 450 manhole would have to endure if it can withstand a load of 50 kN.

43. A stress strain diagram is shown below. Label points 1,2 and 3 and state their significance.

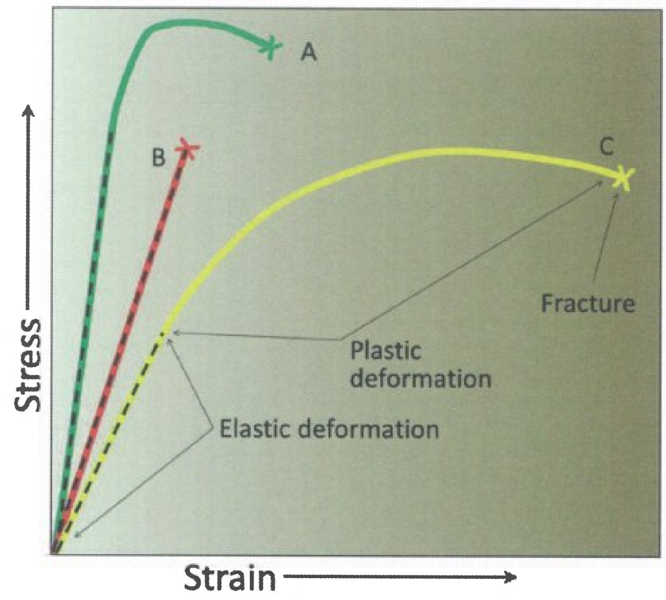


1 –

2 –

3 –

44. A stress strain diagram is shown below depicting 3 possible materials.



Determine which material, A, B or C:

a) Is most likely glass: _____

b) Is most likely a low carbon steel: ________

c) Is most likely a high carbon steel: ________

d) Is the stiffest: ______

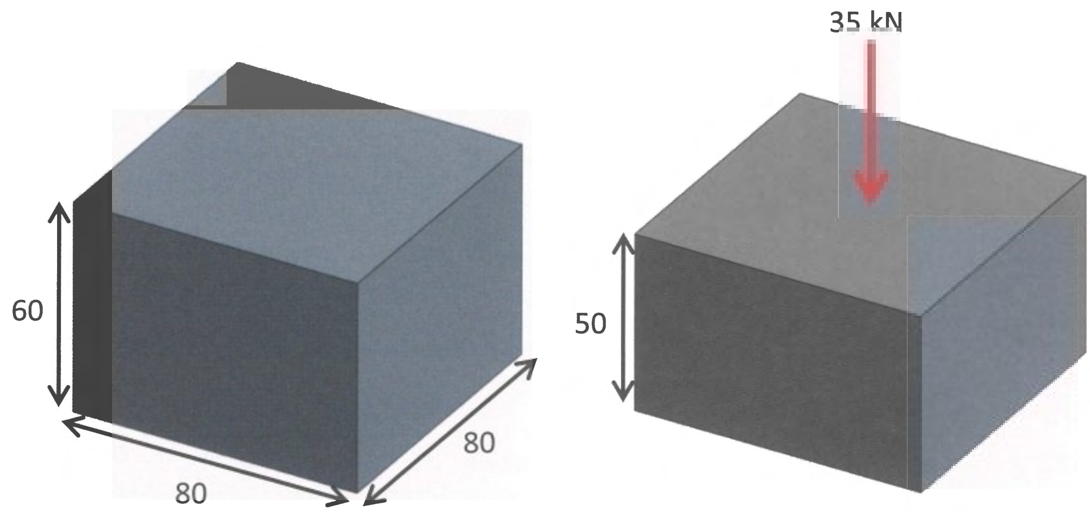
e) Is the toughest: _______

f) Is the most ductile: ________

g) Exhibits the highest strength: ________

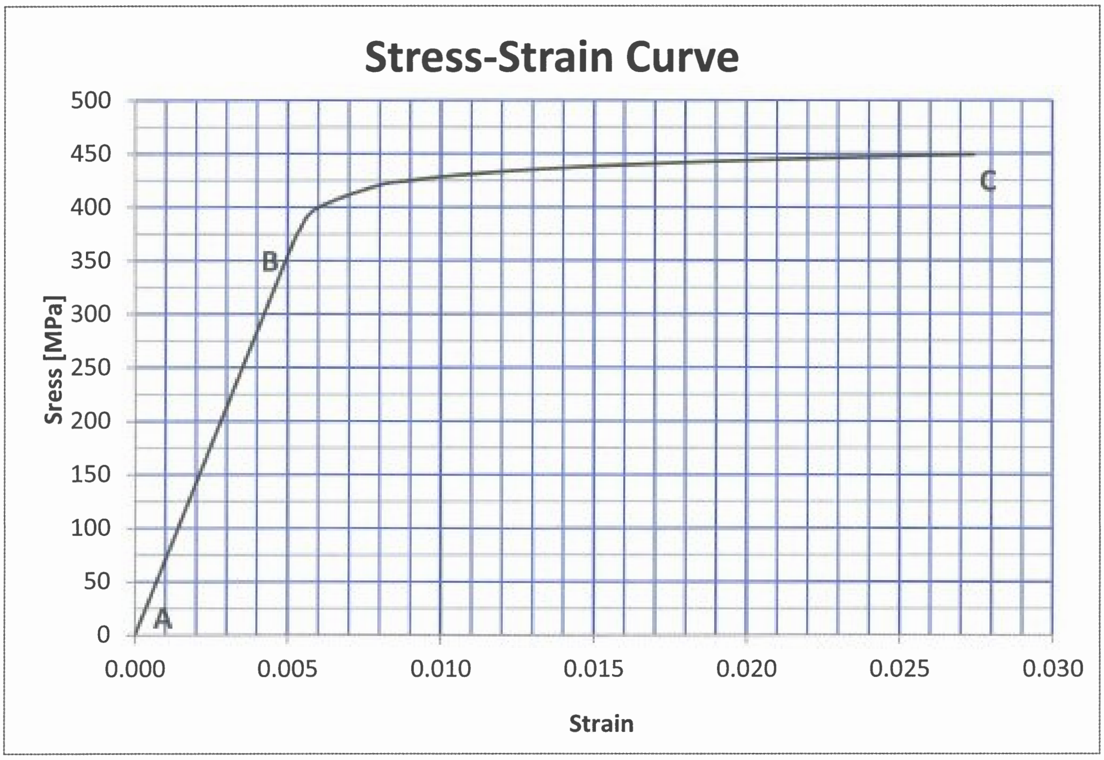
h) Exhibits brittleness: ______

45. A 35 kN force compresses an 80 x 80 square base concrete prism reducing its height from 60 mm to 50 mm. Assume the rest of the dimensions stay constant.



Calculate the concrete's modulus of elasticity.

46. The below figure shows the stress-strain graph obtained from a tensile test performed on an aluminium alloy.



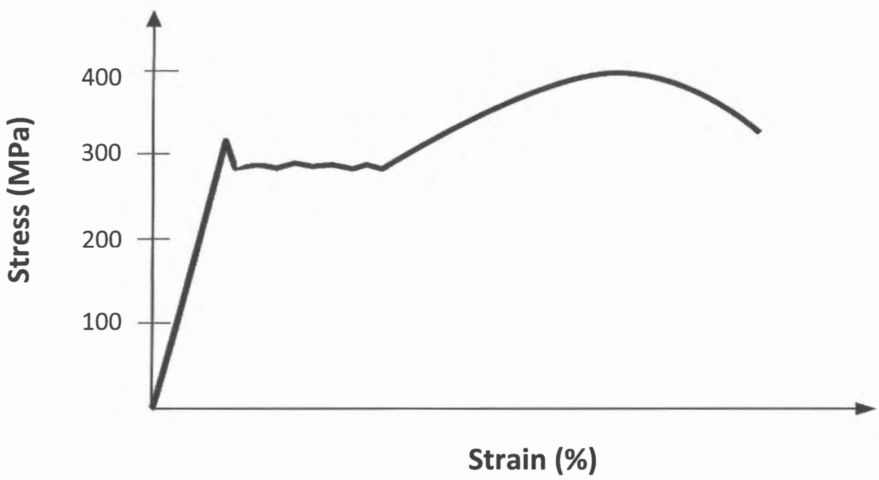
a) Using the above figure, determine the Young's Modulus and the yield strength of the material.

b) Describe the behaviour of the material in the following regions:

i. AB

ii. BC

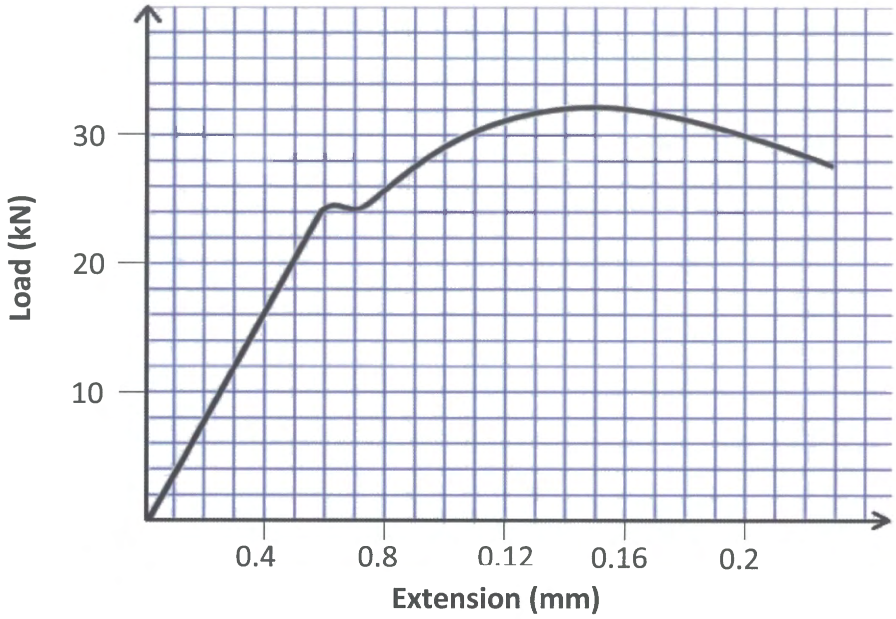
47. A bolt is subjected to a tensile test which resulted in the stress strain graph shown below.



a) Using the graph above, determine the minimum diameter of the bolt that will support a tensile load of 85 kN if a factor of safety of 1.6 is assumed.

b) Describe how you would obtain 5 different mechanical properties by analysing a typical stress strain diagram like the one above.

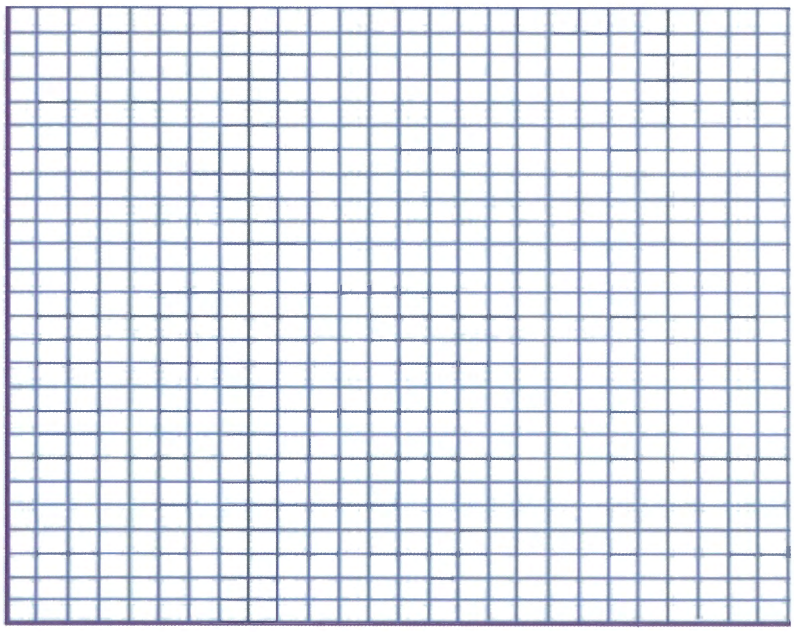
48. The steel used to manufacture the cables on a bridge was tested using a tensometer and the resulting load extension graph was produced.



The initial gauge length of the steel test piece was measured to be 65 mm and its diameter to be 4.5 mm. Using the results from the load extension graph above, determine the Young's modulus of steel in GPa.

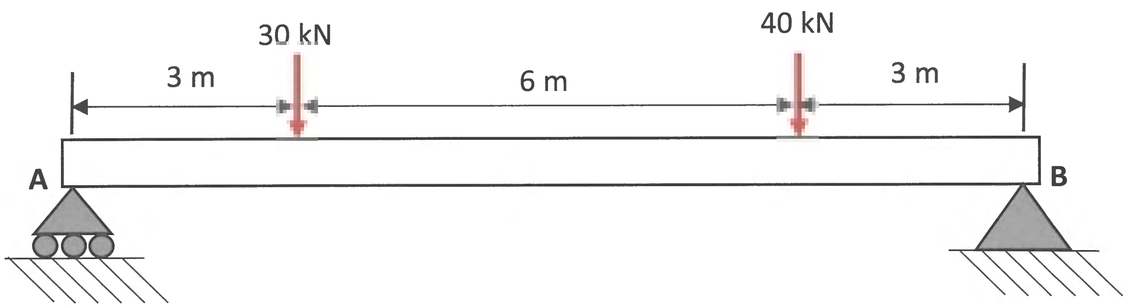
49. A tensile test performed on a test piece 40 mm in gauge length and a cross sectional area of 120 mm^2 yielded the following results.

Extension (mm)	0	0.40	0.80	1.20	1.40	2.2	3.2	3.6	4.0	5.0
Load (kN)	0	30	60	90	93	100	115	120	115	110 (failure)



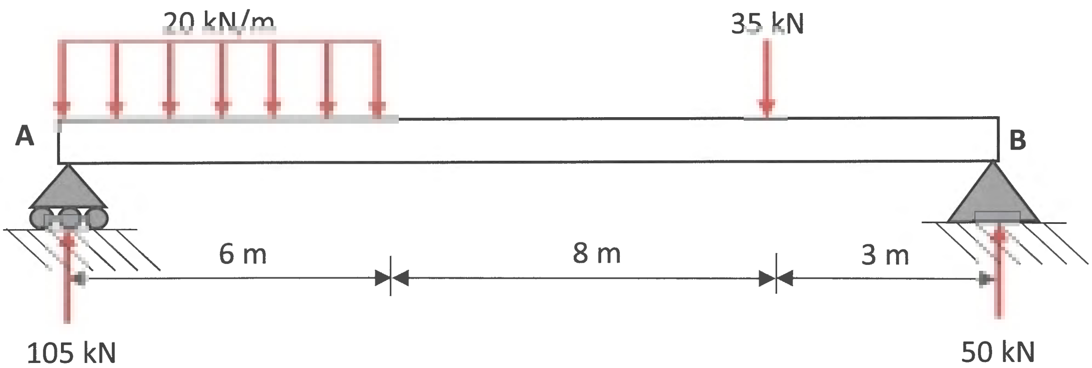
a) Plot the load extension diagram on the grid provided above
b) Calculate the modulus of elasticity (Young's Modulus) for this test piece in GPa.
c) Calculate the material's Ultimate Tensile Strength (UTS) in GPa
d) Calculate the material's fracture strength in MPa.
e) Calculate the stress at the proportional limit in MPa.

50. A loaded beam is shown below.

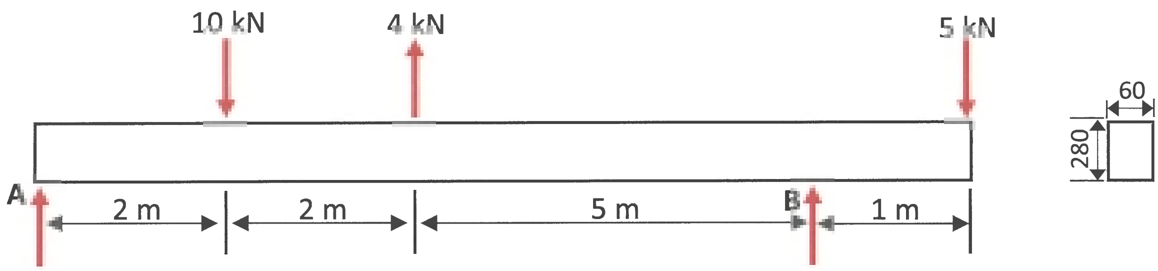


a) Determine the magnitude of the reaction at A and B.
b) Draw the shear force diagram and the bending moment diagram.

51. Draw the shear force diagram for the partially uniformly distributed loaded beam.

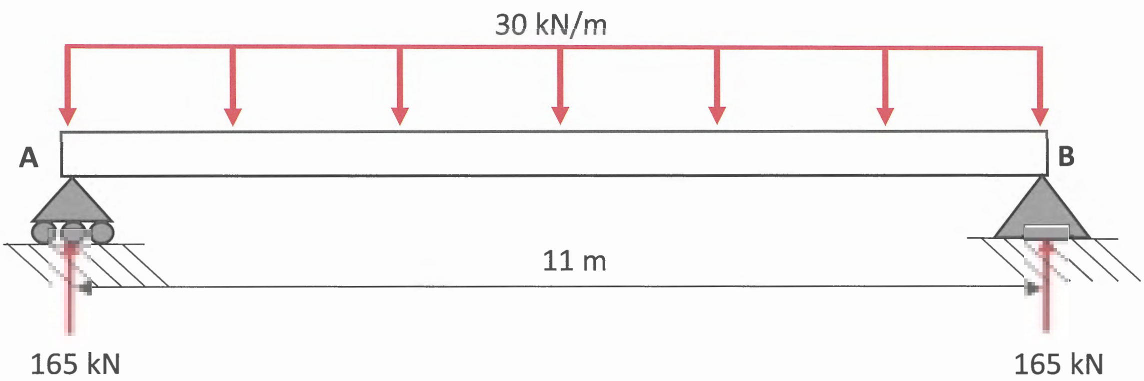


52. A beam that is 60 mm wide, 280 mm deep and 10 m long is loaded as shown with supports at A and B.

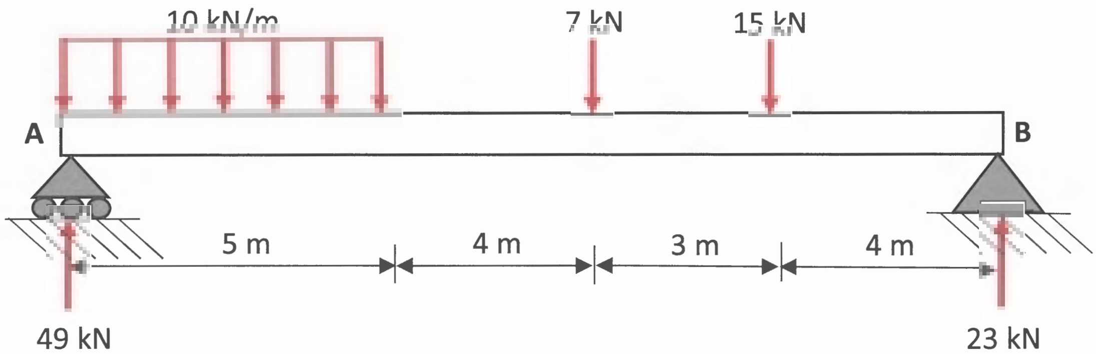


a) Draw the shear force diagram AND the bending moment diagram for the beam. Include labels in your diagram.
b) Determine the maximum bending stress in the beam if the second moment of area, I, is 120 x 10^6 mm^4. The mass of the beam is neglected.

53. Draw the shear force and bending moment for the beam that has a uniformly distributed load.



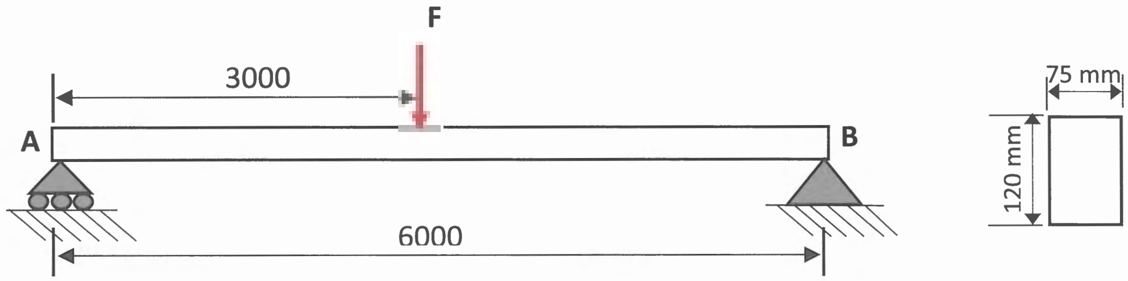
54. Draw the shear force diagram for the loaded beam shown below.



55. A beam with a square cross-section of 200 mm x 200 mm is shown. The beam has a maximum bending moment of 76 kNm and a second moment of area of 200×10^6 mm^4.

Determine the maximum value of the bending stress (MPa) in the beam.

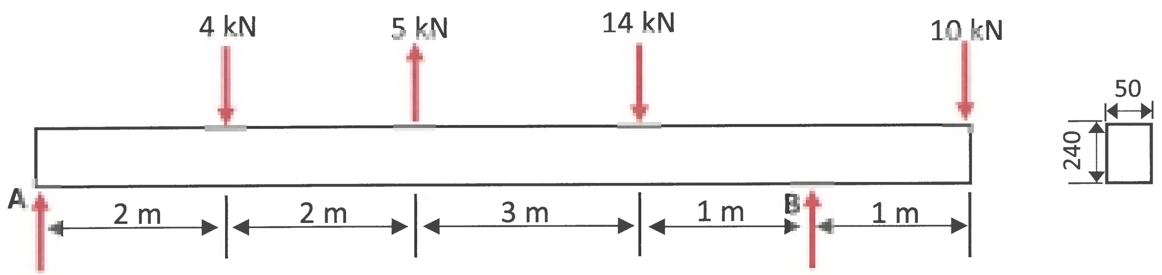
56. A force is applied in the middle of a 6-metre beam with its cross section shown on the right.



The second moment of area, I, for this beam is 5.26×10^6 mm^4.

Calculate the maximum value of the force (F) if the maximum bending stress of the beam is not to exceed 65 MPa.

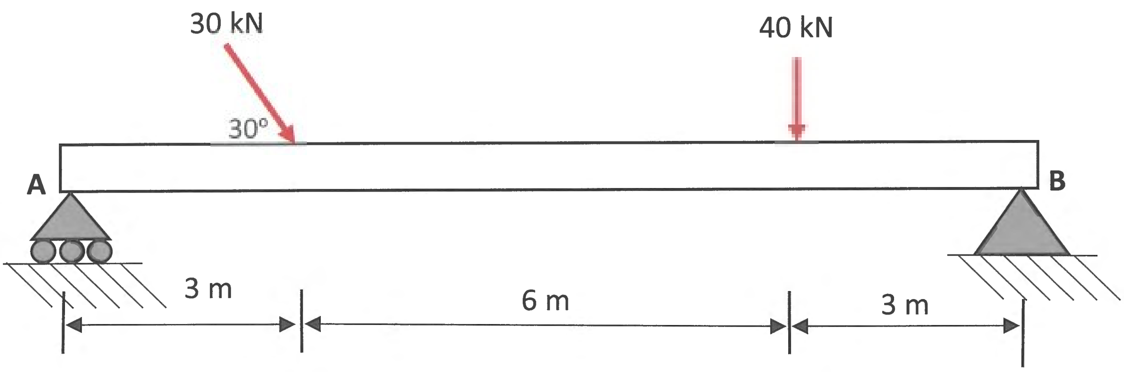
57. A loaded beam with supports at A and B is 50 mm wide, 240 mm deep and 9 m long.



a) Draw the shear force diagram AND the bending moment diagram for the beam. Include labels in your diagram.

b) Determine the maximum bending stress in the beam if the second moment of area, I, is 150 x 10^{-6} m^4. The mass of the beam is neglected.

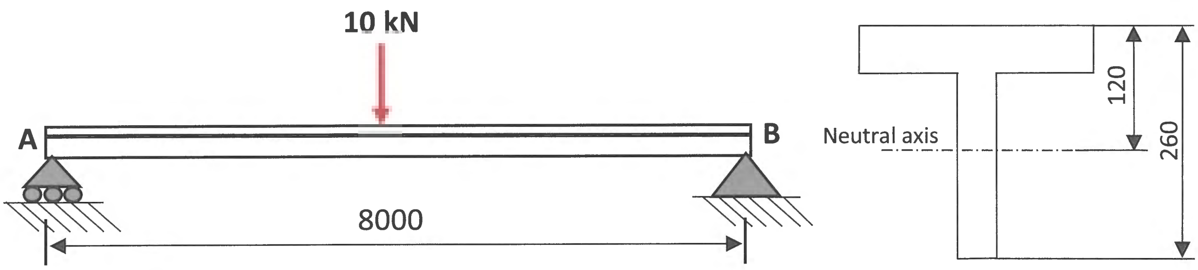
58. A 12 m long beam is loaded as shown below.



a) Determine the magnitude of the reaction at A.

b) Draw the shear force diagram and the bending moment diagram.

59. An 8-metre beam has a T cross section as shown below.

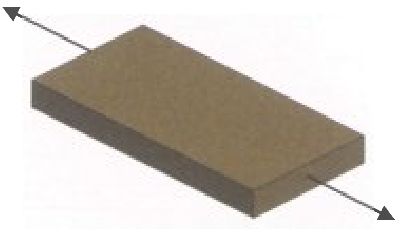


A 10kN force acts in the middle of the beam which has a second moment of area (I_{xx}) of 110 x 10^6 mm^4.

Calculate the maximum tensile stress in the beam.

60. A tensile test is performed on an ingot of brass which is 30 mm wide and 7 mm thick. At fracture these dimensions are reduced to 5.5 mm thick by 25 mm wide.

Calculate both the engineering stress and true stress at fracture, if the ingot failed when loaded with a force of 15.7 kN.

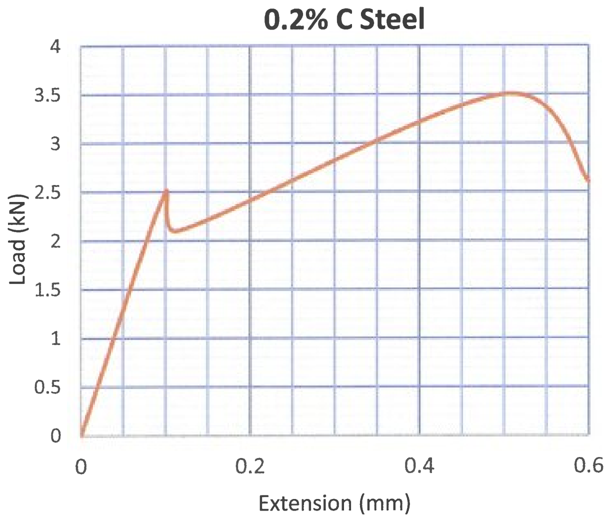


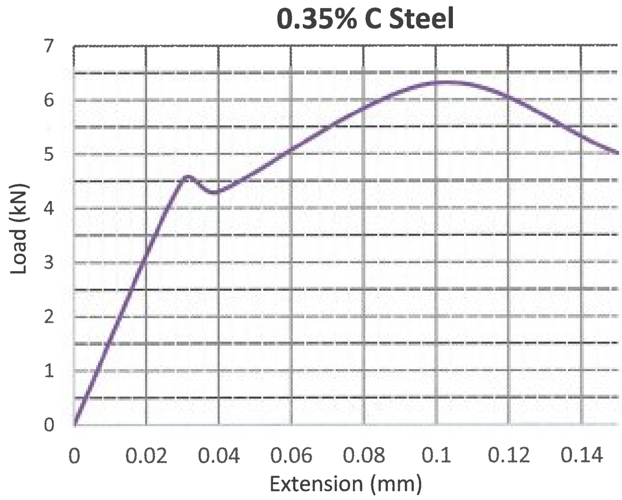
61. The tower of the Rainha Santa Isabel cable-stayed bridge in Portugal can support the bridge deck via several steel cables. A structural analysis of the bridge reports that one of the cables experiences a load of 368 kN.

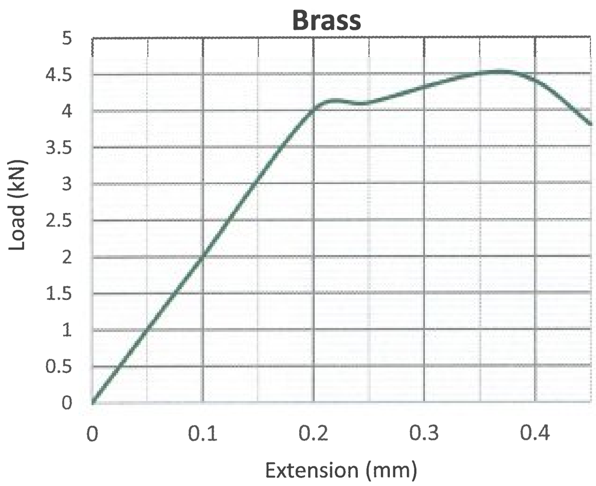
Select the most appropriate material for this application from the 3 options below by considering the desired mechanical properties. Once selected, determine the minimum diameter required to safely withstand the load if a factor of safety of 2.5 is used.

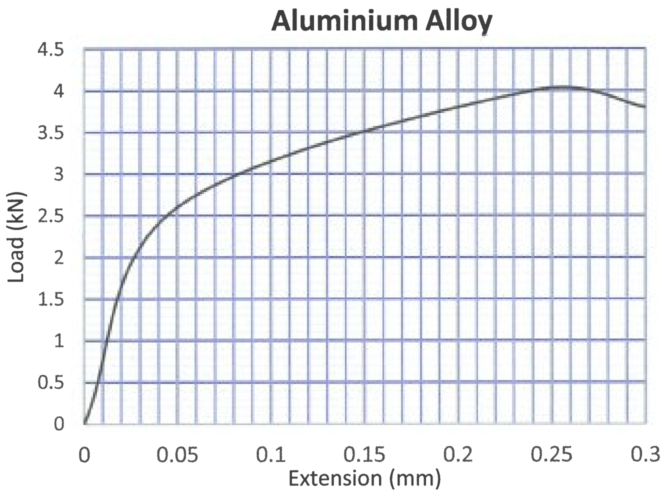
Material	Yield Strength (MPa)	Ultimate Tensile Strength (MPa)	Mechanical Properties
Low carbon steel (Steel alloy 1008)	285	340	Low strength and high ductility
Medium carbon steel (Steel alloy 4140)	470	745	Good strength and excellent toughness
Cast iron	Fractures before yielding	520	High hardness and brittle

62. Complete the following table by analysing the graphs below.









	0.2% C Steel	0.35% C Steel	Brass	Aluminium Alloy
Original Diameter	6 mm	6 mm	6 mm	6 mm
Gauge Length	30 mm	30 mm	30 mm	30 mm
Young's Modulus				
Ultimate Tensile Strength				

Chapter 3 - Simple Machines

A simple machine is a machine that makes work easier to perform by achieving one or more of the following functions:

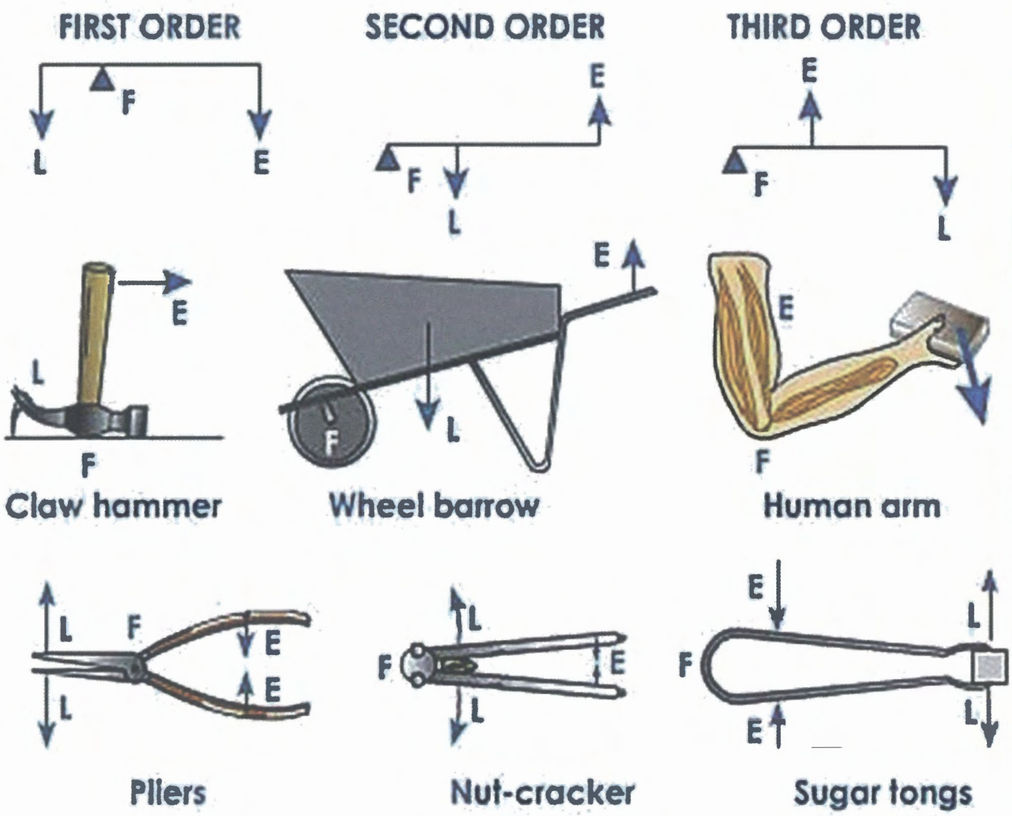
- Changing the direction of a force
- Increasing the magnitude of a force
- Increasing the distance or speed of a force
- Transferring a force from one place to another

The 6 main simple machines that will be explored in this book are:

- Levers
- Gears
- Pulleys
- Inclined planes
- Screws and
- Wheel and axle

Levers

A lever consists of a rigid bar that pivots on a fixed hinge or fulcrum. An input force known as the effort is applied to the lever that produces an output force to move the load. There are 3 classes of levers as shown below:



If the below components of a lever are positioned in the middle, then we can easily identify the class of lever using the anagram FLE.

F (Fulcrum)
→ 1st class

L (Load/resistance)
→ 2nd class

E (Effort)
→ 3rd class

Figure 3.1 – *Three different classes of levers*

Mechanical Advantage

The mechanical advantage (MA) gives us an indication of how beneficial a simple machine is in carrying a task. This value can be obtained by:

$$MA = \frac{Load}{Effort} = \frac{L}{E}$$

The less effort required to move a load, the greater the MA. The MA represents the amount in which the machine amplifies the effort force. MA is not a constant value as it is affected by friction and other losses in the system.

Velocity Ratio

The velocity ratio (VR) is represented as the ratio of the distance that the effort moves compared to the distance that the load moves.

$$VR = \frac{distance\ that\ the\ effort\ moves}{distance\ that\ the\ load\ moves} = \frac{d_E}{d_L}$$

The higher the VR, the greater the distance that the operator must move. This, however, coincides with a lower effort that would be needed in completing the operation. VR, unlike MA, is not affected by any losses in the system.

Efficiency

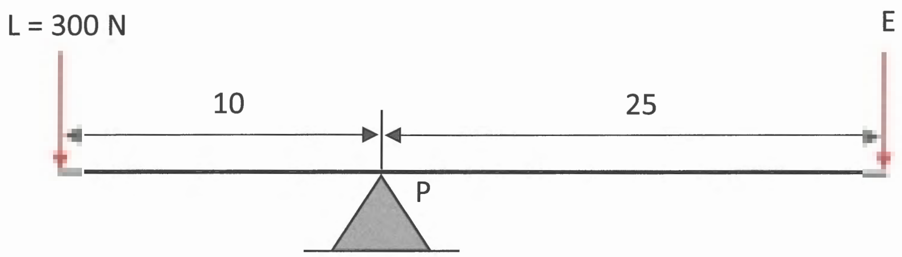
In the case of an ideal machine with no losses (100% efficient) the MA is equal to the VR. Realistically, machines are never 100% efficient due to losses that occur, namely friction. This means that the output force (load) will always be less than the input force (effort). Efficiency can be obtained by applying the following equation:

$$\eta = \frac{MA}{VR} \times 100$$

Worked Example 1:

For the 1st class lever below, calculate the following:

- **a)** MA
- **b)** VR
- **c)** Efficiency
- **d)** If the efficiency of the lever was 90%, calculate the MA required and hence the effort needed.



Solution:

a) Calculate the effort force. To do this, we apply sum of moments about the pivot/fulcrum.

$$+\curvearrowleft \sum M_P = 0$$
$$(300 \ x \ 10) - E(25) = 0$$
$$E = \frac{3000}{25}$$
$$\therefore E = 120 \ N \uparrow$$

Substitute the effort and load force into the MA to find the answer.

$$MA = \frac{L}{E}$$
$$MA = \frac{300}{120}$$
$$\therefore \boldsymbol{MA = 2.5}$$

b) VR is calculated by dividing the distance that the effort moves by the distance that the load moves. The effort and load force both move in a circular motion about the fulcrum and hence the distance they travel is the circumference of a circle.

$$VR = \frac{d_E}{d_L}$$
$$VR = \frac{2\pi R}{2\pi r}$$
$$VR = \frac{R}{r}$$
$$\therefore \boldsymbol{VR = \frac{25}{10} = 2.5}$$

c) Efficiency is simply MA divided by the VR.

$$\eta = \frac{MA}{VR}$$
$$\eta = \frac{2.5}{2.5} \times 100$$
$$\therefore \boldsymbol{\eta = 100\%}$$

d) We know that VR is not affected by any losses, hence VR stays constant. Use the efficiency formula to find the MA. Note: Convert percentage to a decimal.

$$\eta = \frac{MA}{VR}$$
$$MA = \eta \times VR$$
$$\therefore MA = 0.9 \times 2.5 = 2.25$$

Now that we have MA, we can find the effort force using the MA equation.

$$MA = \frac{L}{E}$$
$$E = \frac{L}{MA}$$
$$\therefore \boldsymbol{E = \frac{300}{2.25} = 133.3 \ N}$$

Gears

Gears are designed to interlock or mesh with other gears. This design prevents slippage, allowing engineering mechanisms to change speed, torque and rotational direction.
In a gear system, you typically have a ***driver*** gear which is the first gear in the system and is the one that has an energy source attached to it. It moves the system. The driver gear transports its energy to the ***driven*** gear which is moved in the system. When these 2 gears mesh together, they will rotate in opposite directions. An **idler** (also known as intermediate) gear is a gear that is inserted between the driver and driven gear. One of the reasons for its use is to keep the driver and driven gear rotating in the same direction. A key note to remember is that the idler gear(s), regardless of their size have NO effect on the gear ratio. The ratio depends only on the first (driver) gear and the last (driven) gear.

Reducing gears:
In this configuration, the driver gear (24 teeth) is smaller than the driven gear (48 teeth), meaning that with every 2 rotations of the smaller gear, the bigger gear would complete 1 rotation. This reducing gear acts as a force multiplier and is very useful when climbing up a large hill. However, it will have the effect of reducing speed.

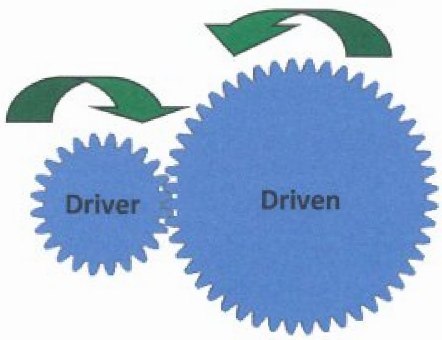


Figure 3.2 – *Reducing gears*

Multiplying gears:
In this configuration, the driver gear (48 teeth) is bigger than the driven gear (24 teeth), meaning that with every rotation of the larger gear, the smaller gear would complete 2 rotations. This multiplying gear acts as a speed multiplier and is very useful when wanting to increase speed whilst riding your bicycle on a flat horizontal surface.

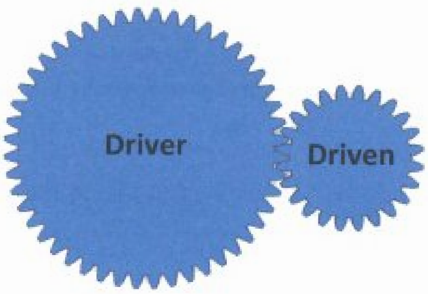


Figure 3.3 – *Multiplying gears*

Velocity Ratio/Gear Ratio

The velocity ratio or gear ratio is determined using the following equation:

$$VR = \frac{radius, diameter, circumference\ or\ no.of\ teeth\ on\ driven\ gear}{radius, diameter, circumference\ or\ no.of\ teeth\ on\ driver\ gear}$$

Speed Ratio (SR)

To determine how many revolutions per minute a gear experiences, we simply multiply the input speed by the speed ratio. The speed ratio is the reciprocal of VR:

$$SR = \frac{radius, diameter, circumference\ or\ no.of\ teeth\ on\ driver\ gear}{radius, diameter, circumference\ or\ no.of\ teeth\ on\ driven\ gear}$$

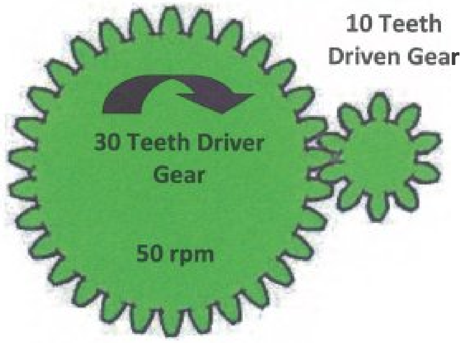
$$RPM = input\ speed\ \times\ SR$$

Worked Example 2:

Determine the following for the gear train shown below:

a) Velocity ratio

b) Speed of the driven gear in rpm and its rotational direction



Solution:

a) $VR = \frac{no.of\ teeth\ on\ driven\ gear}{no.of\ teeth\ on\ driver\ gear}$

$VR = \frac{10}{30} = \frac{1}{3} = \mathbf{1:3}$

b) $SR = \frac{no.of\ teeth\ on\ driver\ gear}{no.of\ teeth\ on\ driven\ gear} = \frac{30}{10} = 3:1$

Therefore the driven gear will rotate at 3 times the speed of the driver gear.

$$RPM_{Driven} = Input\ speed\ \times\ SR = 50\ \times\ 3 = \mathbf{150\ rpm}$$

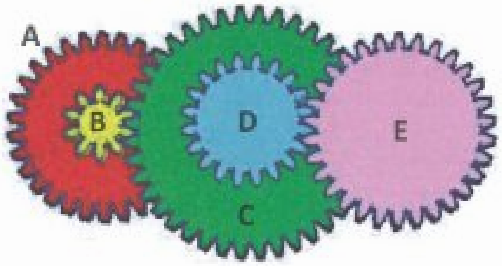
The driven gear will rotate in an <u>**anticlockwise direction**</u> since the driver gear is rotating in a clockwise direction.

Compound gear trains

A compound gear consists of 2 differently sized gears that rotate around the same shaft. 2 or more of these compound gears make up a compound gear train.

Worked Example 3:

Gear A, B, C, D and E have 30, 10, 40, 20 and 30 teeth respectively.



Determine how many revs/min occurs at E and in what direction does Gear E rotate if Gear A rotates in an anti-clockwise direction at 240 revs/min?

Solution:

Gears A and B share the same shaft so they will rotate at the same speed of 240 rpm. Gear B meshes with gear C. Gear C is larger than B so it will rotate at a slower rpm.

$$SR = \frac{no.of\ teeth\ on\ \mathbf{driver\ gear}}{no.of\ teeth\ on\ \mathbf{driven\ gear}}$$

$$SR_{BC} = \frac{10}{40} = 0.25$$

$$RPM = input\ speed\ (gear\ B) \times SR$$

$$RPM_C = 240 \times 0.25 = 60\ rpm$$

Since gears C and D share the same shaft then the RPM for both C and D will be the same value of 60 rpm. Now gear D meshes with gear E.

$$SR_{DE} = \frac{20}{30} = \frac{2}{3}$$

$$RPM_E = 60 \times \frac{2}{3} = \mathbf{40\ rpm}$$

Gears A and B rotate anti-clockwise (same shaft). B meshes with C so C will rotate in the opposite i.e. clockwise. Gears C and D share the same shaft so D will also rotate clockwise. D and E now mesh together so E will rotate in the opposite direction which is anti-clockwise.

Pulleys

A **pulley** consists of one or more wheels with a grooved rim carrying a rope. This rope is pulled making it easier to lift things. You'll find that increasing the number of wheels will increase your ability to lift heavier objects.

One wheel

In a one-wheel pulley configuration, if an object of mass 50 kg (weight of 500 N) is to be lifted then a force of 500 N is required to lift the object. Thus, the mechanical advantage for this configuration is 1. If we want to raise the mass 2m into the air, then we would need to pull the end of the rope a total distance of 2m at the other end. Note: 100% efficiency is assumed.

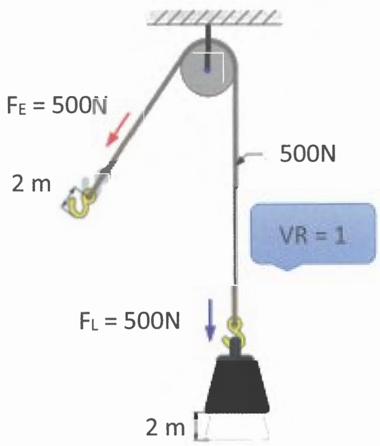


Figure 3.4 – *A pulley system with a VR of 1*

Two wheels

Adding an extra wheel will reduce the effort required to lift the load by a **half**. The 50 kg mass (weight of 500 N) is now supported by two portions of the same rope instead of one (ignoring the end of the rope you are pulling with) as in the one-wheel pulley configuration. Due to this, we can apply a force of only 250 N to lift the object. Thus, the mechanical advantage for this configuration is 2. The greater the mechanical advantage, the less effort required. However, this force would have to be applied over a greater distance. So, if we want to raise the mass 2m into the air then we need to pull the end of the rope a total of 4m. Note: 100% efficiency is assumed.

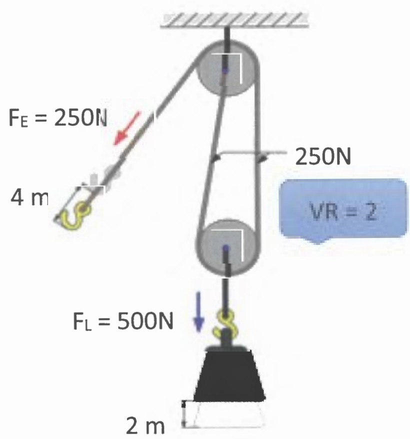


Figure 3.5 - *A pulley system with a VR of 2*

Four wheels

A four-wheel pulley configuration will reduce the effort required to lift the load by a **quarter** as the 50 kg mass (weight of 500 N) is now supported by four portions of the same rope (ignoring the end of the rope you are pulling with). Due to this, we can apply a force of only 125 N to lift the object. Thus, the mechanical advantage for this configuration is 4. If we want to raise the mass 2m upwards, then we need to pull the end of the rope four times as much so a total of 8m would be required. Note: 100% efficiency is assumed.

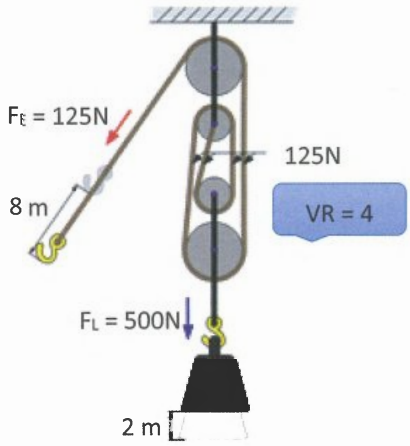


Figure 3.6 – *Pulley system with a VR of 4*

Therefore, pulleys are used to aid through mechanical advantage and to change the direction of a force. In all the above pulley arrangements, the work required to perform the task is the same. It is, however, easier to complete the task using a four-wheel pulley configuration.

Velocity Ratio

The velocity ratio (VR) for a pulley system is:

$$VR = \textit{number of rope sections supporting the load}$$

When looking at the one-wheel configuration, we can see that there is only 1 rope supporting the load, hence the VR is 1. Similarly, the two-wheel configuration has 2 sections of the rope supporting the load so the VR is 2 and the four-wheel configuration has 4 sections of the rope supporting the load so the VR is 4.

Pulley Belts

Pulley belts transfer power by transmitting rotary motion from a motor to a shaft in machines.

$$VR = \frac{\textit{radius, diameter or circumference on driven gear}}{\textit{radius, diameter or circumference on driver gear}}$$

To achieve a faster output speed, the driver pulley must be larger than the driven pulley. This is the same concept detailed previously in the multiplying gears section.

To determine the pulley's **direction of rotation**, the below rule is followed:

- If the belts are crossed, then the direction of rotation are opposite to each other
- If the belts are NOT crossed, then the direction will be the same for both pulleys.

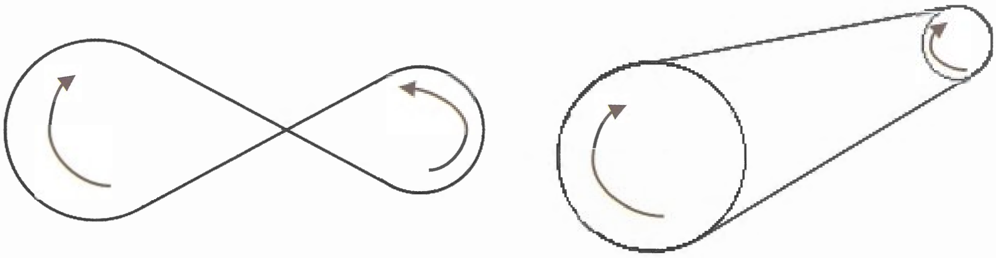


Figure 3.7: *Crossed Pulley belt (left) and Uncrossed Pulley belt (right)*

Inclined Planes

An inclined plane is a surface that is at an angle to the horizontal. This is considered a simple machine as it is much easier (requires less effort) to push a load along the inclined surface than lifting an object vertically. The work done is the same regardless of how the box is lifted to its final position.

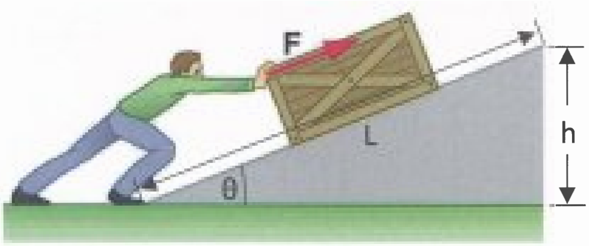


Figure 3.8 – *Man pushing a box up an incline*

Velocity Ratio

When the effort moves along the plane over a distance of L, the load moves through a vertical distance of h (direction in which the load's weight acts). When applying trigonometry, we get h = L$\sin\theta$.

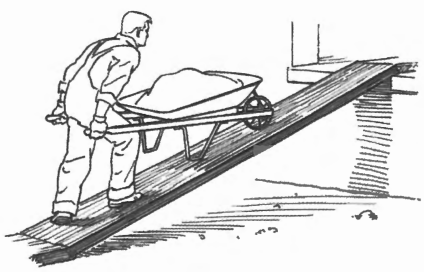
$$VR = \frac{\text{distance that the effort moves}}{\text{distance that the load moves}} = \frac{L}{h} = \frac{L}{L\sin\theta} = \frac{1}{\sin\theta}$$

Mechanical Advantage (Ideal case assuming no friction)

$$MA = \frac{Load}{Effort} = \frac{mg}{mg\sin\theta} = \frac{1}{\sin\theta}$$

Worked Example 4:

A man applies an effort force of 200N to push a wheelbarrow of mass 10 kg with a load of 30 kg up a 20° incline. The vertical height of the ramp is 2m.



Calculate the efficiency at which this operation was performed with.

Solution:

The wheelbarrow and its contents make up a total of 40 kg which amounts to 400N. This represents the load. We then substitute our load and effort values to find the MA.

$$MA = \frac{Load}{Effort} = \frac{400}{200} = 2$$

We now need the VR to help us calculate the efficiency. Substitute the angle that the plane is inclined at to find the VR.

$$VR = \frac{1}{sin\theta} = \frac{1}{sin20} = 2.92$$

Now, substitute the above two values to find the efficiency.

$$\eta = \frac{MA}{VR} \times 100 = \frac{2}{2.92} \times 100 = \mathbf{68.4\%}$$

Screws

A screw is a type of inclined plane with a circular or helical ramp with threads cut into the outside of the ramp. The most common use of a screw is to hold objects together.

Velocity Ratio

$$VR = \frac{\boldsymbol{distance\ that\ the\ effort\ moves}}{\boldsymbol{distance\ that\ the\ load\ moves}} = \frac{\boldsymbol{Circumference\ (2\pi r\ or\ \pi d)}}{\boldsymbol{pitch\ (p)}}$$

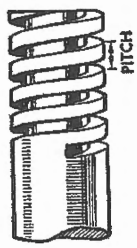


Figure 3.9 – *Screw*

where the pitch represents the distance between the threads.

Screw Jacks

A screw jack is typically used to raise and lower the horizontal stabiliser of an aircraft and to lift heavy weights such as vehicles. A screw jack used in a machine press is shown in Fig 3.10 below.

The velocity ratio of this simple machine is typically very high. This is due to the pitch of the screw being very small compared to the length of the rod that is used to operate the screw. A high velocity ratio will hence mean a high mechanical advantage that is only affected by the frictional forces involved.


Figure 3.10 – *Screw Jack*

Worked Example 5:

Determine the VR for an M18 x 2 bolt.

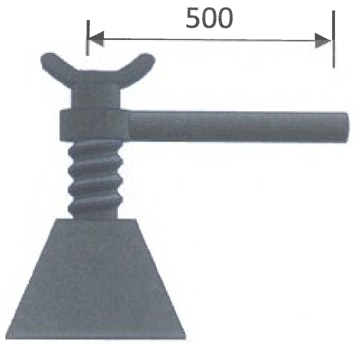
Solution:

The 18 value represents the bolt's diameter and the 2 value represents the bolt's pitch. Substituting this into the equation will yield the following calculation:

$$VR = \frac{\pi d}{pitch} = \frac{\pi \times 18}{2} = \mathbf{28.27}$$

Worked Example 6:

A screw jack used to lift a car has an efficiency of 71% and a VR of 314.16.



Determine the following:

a) The pitch
b) The effort required to lift a load of mass 405 kg

Solution:

a) $VR = \frac{2\pi r}{pitch}$

$$314.16 = \frac{2 \times \pi \times 500}{pitch}$$

$$pitch = \frac{2 \times \pi \times 500}{314.16} = \mathbf{10\ mm}$$

b)

$$\eta = \frac{MA}{VR}$$

$$0.71 = \frac{MA}{314.16}$$

$$MA = 0.71 \times 314.16 = 223.0536$$

$$MA = \frac{Load}{Effort} = \frac{L}{E}$$

$$223.0536 = \frac{4050}{E}$$

$$E = \frac{4050}{223.0536} = \mathbf{18.2\ N}$$

Wheel and Axle

In a wheel and axle mechanism, there is an effort force applied to the circumference of the wheel and a load is lifted by a rope that is wounded around the axle which has a significantly smaller diameter than the wheel.

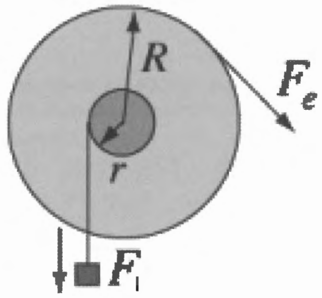


Figure 3.11 – *Wheel and axle*

Velocity Ratio

$$VR = \frac{distance\ that\ the\ effort\ moves}{distance\ that\ the\ load\ moves} = \frac{2\pi R}{2\pi r} = \frac{R}{r}\ or\ \frac{\pi D}{\pi d} = \frac{D}{d}$$

Worked Example 7:

The efficiency of a wheel and axle system is 85% in which the radius of the wheel is 750mm and the radius of the axle is 250mm. Determine the effort required to lift a load of 30 kg.

Solution:

We can utilise the 2 radii to find the velocity ratio.

$$VR = \frac{R}{r} = \frac{750}{250} = 3$$

We then use the efficiency formula to find the mechanical advantage.

$$\eta = \frac{MA}{VR}$$

$$0.85 = \frac{MA}{3}$$

$$MA = 3 \times 0.85 = 2.55$$

Now that we have the mechanical advantage, we can find the effort force required.

$$MA = \frac{L}{E}$$

$$2.55 = \frac{300}{E}$$

$$E = \frac{300}{2.55} = \mathbf{117.6\ N}$$

Hydraulics

- Hydraulic systems have a wide range of applications. They are used in car's braking systems, construction machines, airplanes, amusement park rides, elevators and the list goes on.
- Hydraulics operate according to Pascal's Principle which states that a pressure applied to a fluid in a closed container is transmitted equally to every point of the fluid and the walls of the container. Thus, the pressure underneath one piston must equal to the pressure underneath the other piston. This leads to the formulas stated below.

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = \frac{F_1}{A_1} \times A_2$$

- Based on the equation above and the diagram below, if we apply a force onto piston 1, at the narrow end of the tube on the left, then there must be a large force acting upward on the larger piston (10 times bigger) on the right to keep the pressure equal. Hence, hydraulics are known as force magnifiers.

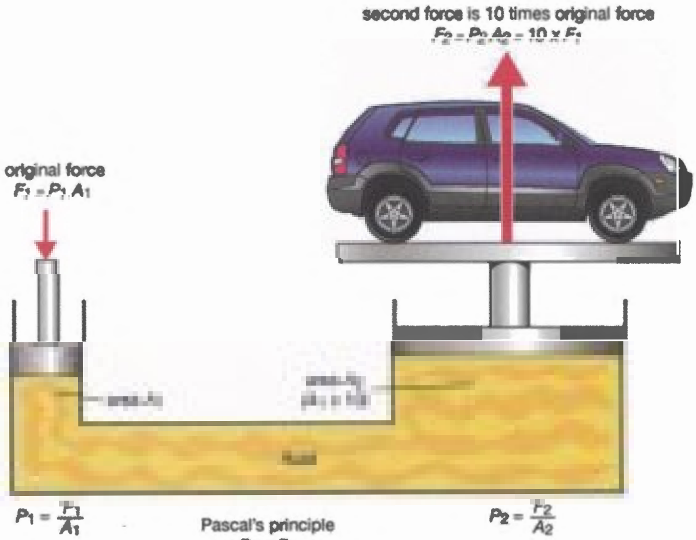


Figure 3.12 – *Hydraulic system showing the magnification of force*

Velocity Ratio

$$VR = \frac{Distance\ that\ the\ input\ piston\ moves}{Distance\ that\ the\ output\ piston\ moves} = \frac{(Diameter\ of\ output\ piston)^2}{(Diameter\ of\ input\ piston)^2}$$

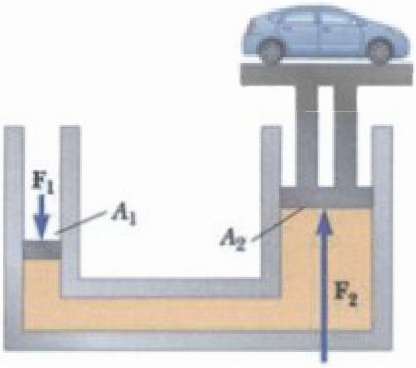
Or

$$VR = \frac{Distance\ that\ the\ input\ piston\ moves}{Distance\ that\ the\ output\ piston\ moves} = \frac{Area\ of\ output\ piston}{Area\ of\ input\ piston}$$

$$\therefore VR = \frac{\phi_{output}^{\ 2}}{\phi_{input}^{\ 2}}\ Or\ \frac{A_{output}}{A_{input}}$$

Worked Example 8:

A hydraulic car jack is used to lift a vehicle as shown below. The area of piston 1 in which the mechanic applies a force of 150 N is 0.08 m^2. The area of piston 2 which is holding the car has an area of 0.6 m^2.



a) Calculate the force that is exerted onto the car by piston 2.

b) Calculate the distance that piston 1 moves if piston 2 moves 100 mm.

Solution:

a) We apply Pascal's principle such that pressure underneath piston 1 is equal to pressure underneath piston 2.

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\frac{150}{0.08} = \frac{F_2}{0.6}$$

$$\therefore F_2 = \frac{0.6 \times 150}{0.08} = \mathbf{1125\ N}$$

b) Applying the following equation will help in solving the problem. Note: convert 100 mm to m since the area is in m^2.

$$\frac{\textit{Distance that the input piston moves}}{\textit{Distance that the output piston moves}} = \frac{\textit{Area of output piston}}{\textit{Area of input piston}}$$

$$\frac{d_1}{d_2} = \frac{A_2}{A_1}$$

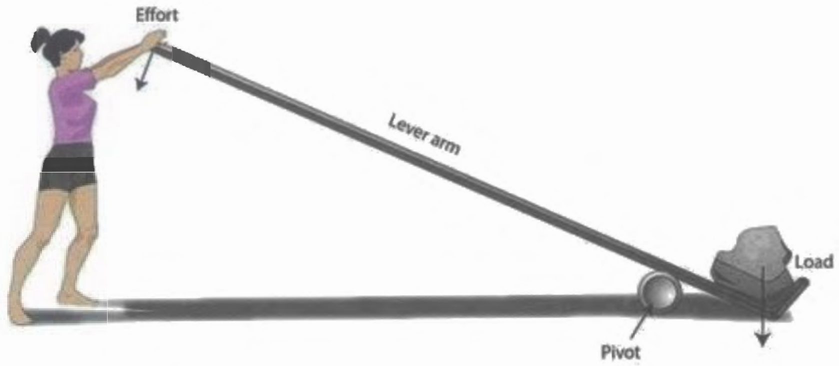
$$\frac{d_1}{0.1} = \frac{0.6}{0.08}$$

$$d_1 = \frac{0.1 \times 0.6}{0.08} = 0.75\ m = \mathbf{750\ mm}$$

Questions

Section I

1. Sarah applies a force to the lever to lift the rock. However, she finds great difficulty in doing so. Determine which option(s) would allow her to lift the rock with greater ease.



(A) Increase the length of the lever
(B) Move the pivot closer to the girl
(C) Apply a greater force on the lever
(D) Push the lever upwards

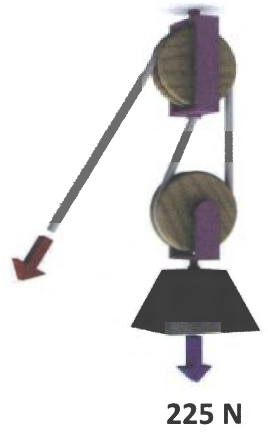
2. The wheelbarrow can be classified as what type of lever?
(A) 1^{st} class
(B) 2^{nd} class
(C) 3^{rd} class
(D) 4^{th} class

3. Which of the following is not a simple machine?
(A) Spring
(B) Pulley
(C) Screw
(D) Ramp

4. Calculate the mechanical advantage of a screw jack which has a pitch of 30 mm and a handle radius of 50 cm. Assume 100% efficiency.
(A) 105
(B) 10
(C) 1.7
(D) 0.6

5. Which of the following is true if the mechanical advantage of a simple machine is 7?
(A) The output work is 7 times greater than the input work
(B) The input force is 7 times greater than the output force
(C) The output force is 7 times greater than the input force
(D) The efficiency is 700%

6. Determine the effort required to lift a 22.5 kg object if the efficiency of the pulley system is 90%.



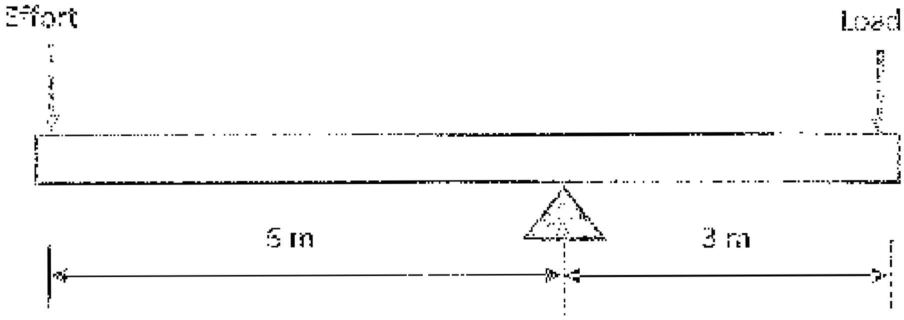
(A) 405 N
(B) 125 N
(C) 101 N
(D) 182 N

7. A 550 N effort is required to lift a 176 kg load. Determine the efficiency of this system.

(A) 8 %
(B) 78 %
(C) 64 %
(D) 80 %

8. What is the velocity ratio of a pulley system?
(A) Always equal to the mechanical advantage
(B) Equal to the amount of pulleys present
(C) Equal to the number of rope sections supporting the load
(D) Equal to the system's efficiency

9. Find the velocity ratio of the lever shown below.

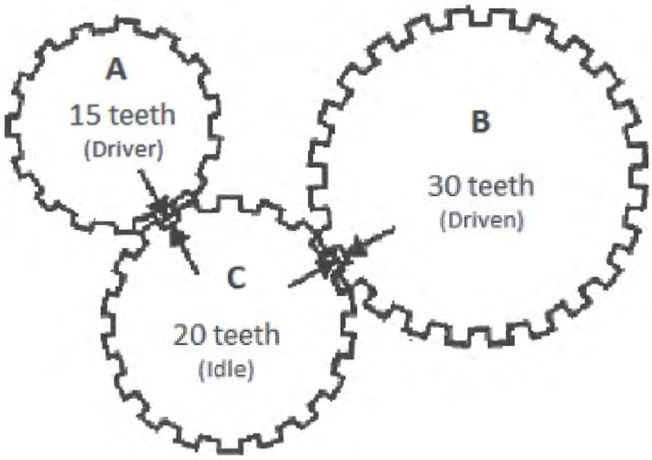


(A) 0.5
(B) 2
(C) 9
(D) 3

10. Which of the following statements is correct?
(A) High MA → less effort is required – High VR → force is applied for a longer time
(B) High MA → more effort is required – High VR → force is applied for a longer time
(C) Low MA → more effort is required – Low VR → force is applied for a longer time
(D) Low MA → less effort is required – Low VR → force is applied for a shorter time

Section II

11. The diagram below shows a gear train.

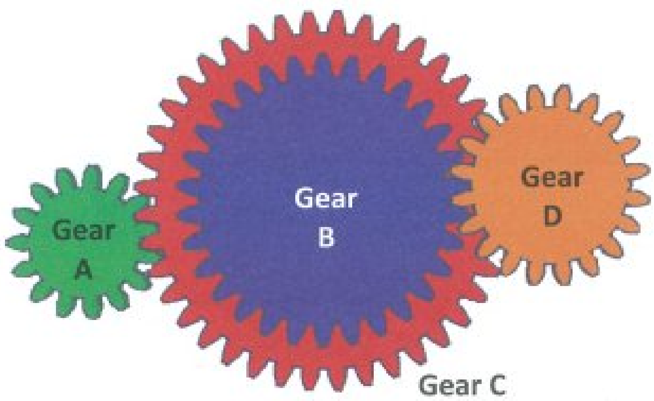


a) Calculate the mechanical advantage of the gear train if it is 90% efficient

b) If Gear A rotates at 200 rpm anticlockwise, what will be the speed and direction of rotation for gear B.

12. A hydraulic system has a mechanical advantage of 2.5 and is 90% efficient. Calculate the diameter of the input cylinder if the diameter of the output cylinder is 30 mm.

13. Gear A has 15 teeth, Gear B has 30 teeth, Gear C has 40 teeth and Gear D has 20 teeth. Determine how many revs/min occurs at D and in what direction does Gear D rotate if Gear A rotates in a clockwise direction at 128 revs/min? **Note**: Gear B and C share the same shaft.

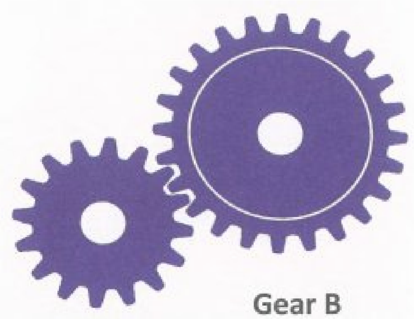


14. A 250 kg mass lies on a plane that is inclined at 15° to the horizontal.

a) Calculate the effort (parallel to the plane) required to move this mass if the coefficient of friction is 0.2.

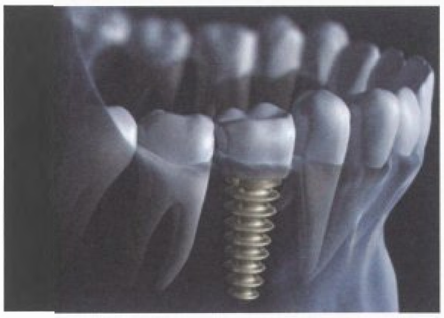
b) Hence calculate the efficiency.

15. In the gear system shown below, the driving Gear B has 24 teeth and the driven Gear A has 16 teeth. Calculate the mechanical advantage of the system if it has an efficiency of 80%.



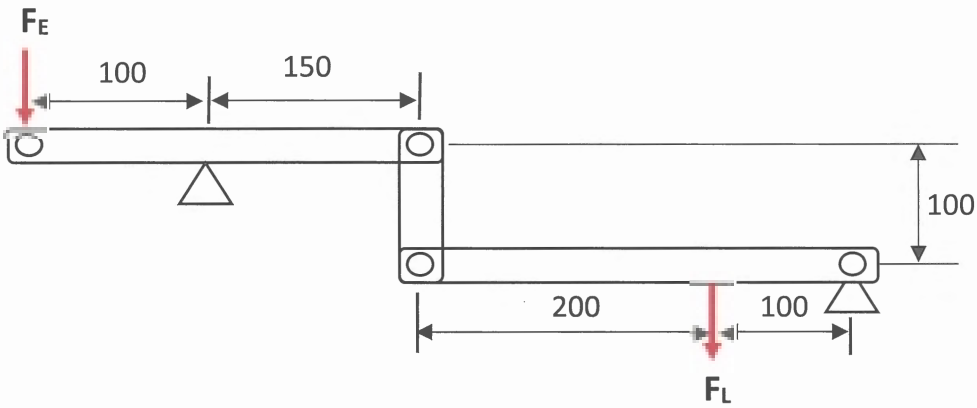
Gear A

16. The dental implant shown below has a pitch of 4 mm. The radius of the lever used to drive the screw is 45mm.



Calculate the mechanical advantage of the above system if the operation is 85% efficient.

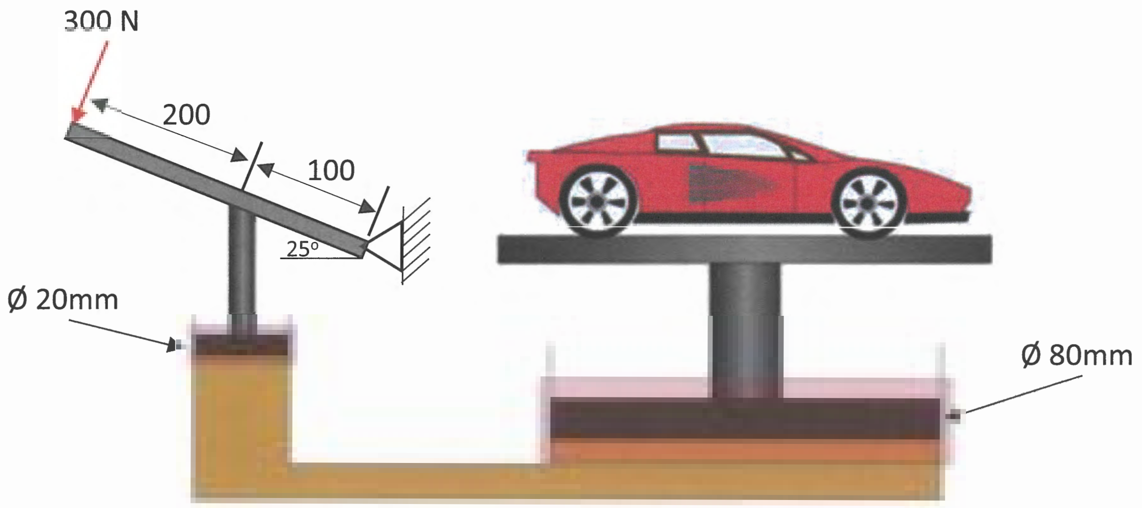
17. A combination of two levers connected as shown below was utilised in operating a valve.



If the load is 40 N and the frictional effort is 30% of the actual effort, calculate the:

a) Velocity ratio
b) Mechanical Advantage
c) Actual effort

18. A jack is used to lift a car.

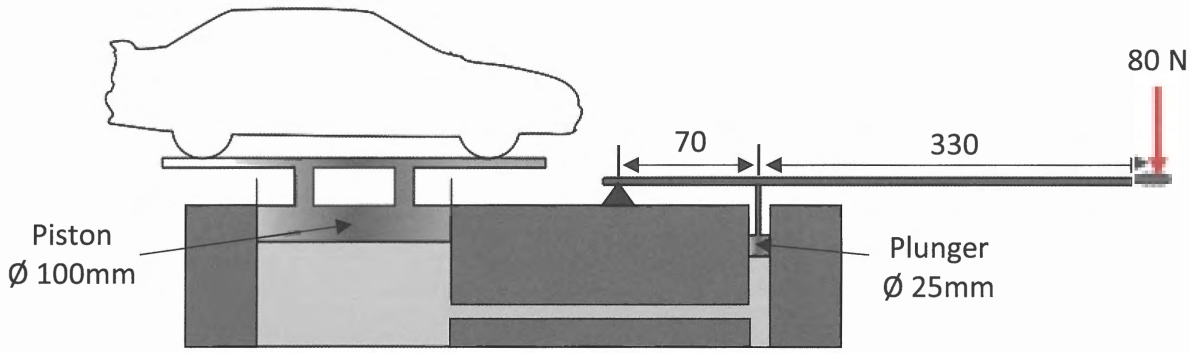


Determine whether a car of mass 1500 kg can be raised when an effort of 300 N is applied to the jack.

19. A wheel and axle with a handle has a velocity ratio of 15 and an efficiency of 45%. What mass can be raised if an effort of 100N is applied?

20. A rope pulley block system with an efficiency of 75% is used to lift a mass of 250 kg. Calculate the effort required if the velocity ratio is 8.

21. A hydraulic jack is shown below.



a) Calculate the force that the plunger applies onto the fluid when an effort of 80 N is used.

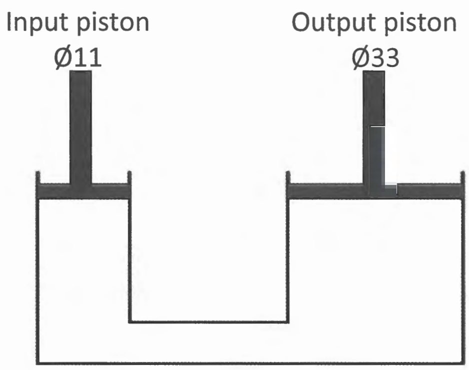
b) Determine the pressure in the system

c) Calculate the load that the hydraulic jack can lift if the piston has a diameter of 100mm.

22. A hydraulic system has a mechanical advantage of 8 and an efficiency of 80%. The diameter of the output cylinder is 45 mm.

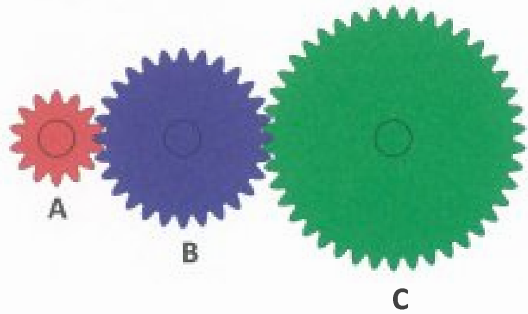
Calculate the distance that the input piston moves so that the output piston moves 20 mm.

23. A hydraulic system is shown below.



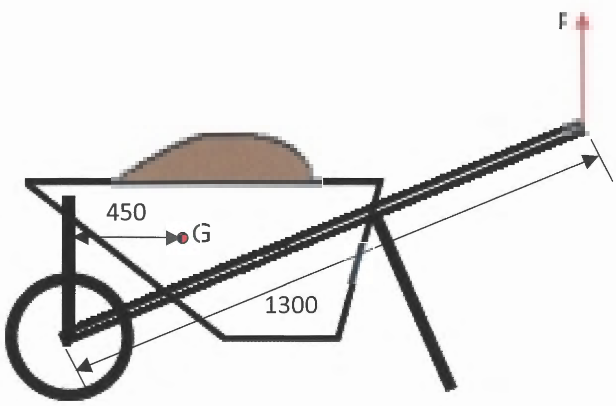
Calculate how far the input piston will move if the output piston moves a distance of 6 mm.

24. In the gear train below, Gear A (driver), B and C (driven) have 14, 28 and 42 teeth respectively.



Determine the gear ratio.

25. The combined mass of the wheelbarrow and its load is 25 kg with the centre of mass located at G.

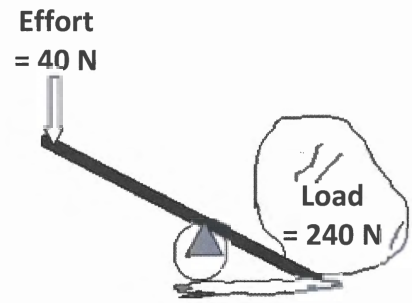


a) Calculate the vertical force required by the gardener to lift the wheelbarrow off the ground if the handle is inclined at an angle of 30°.

b) Hence, find the mechanical advantage of the wheelbarrow.

26.

a) Find the mechanical advantage of the lever pictured below.

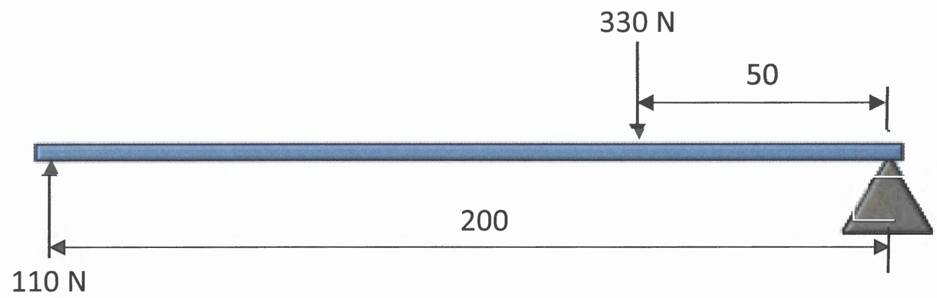


b) At what distance should the effort be applied in order to lift the large rock that is positioned 3 m from the fulcrum and inclined at 30°.

c) Calculate the velocity ratio of the system.

d) If the efficiency of the lever is 80%, calculate the mechanical advantage and the effort required to lift the rock.

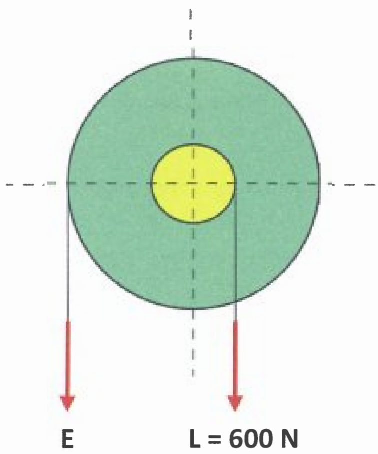
27.



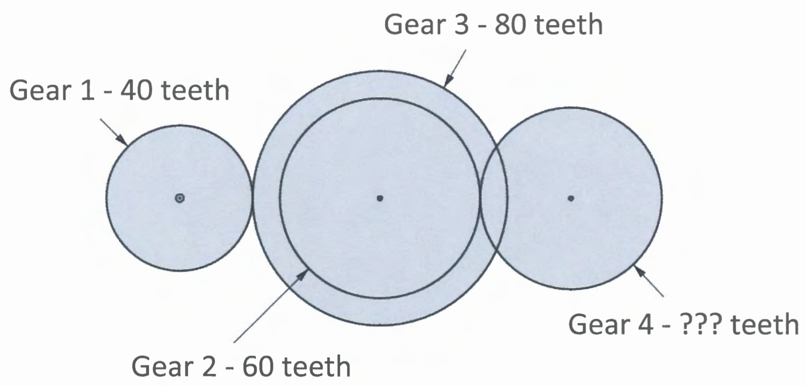
a) Calculate the velocity ratio for the 2nd class lever shown above.

b) Determine the efficiency for this lever.

28. n the wheel and axle shown below, the diameter of the wheel is 500 mm and 80 mm for the axle. Calculate the mechanical advantage for this simple machine.

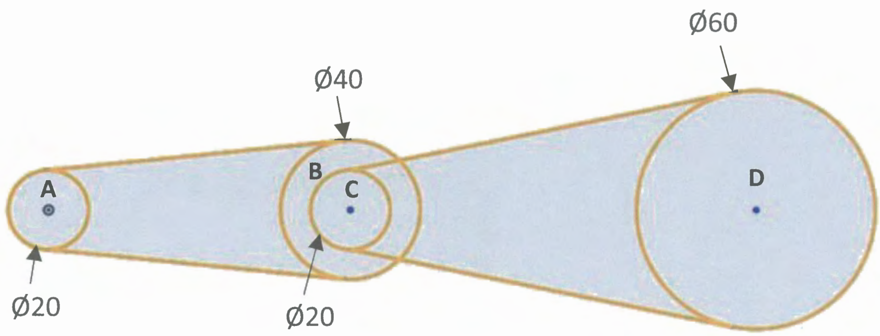


29. In the gear system shown below, Gear 1 is the driver gear, Gear 4 is the driven gear and Gears 2 and 3 lie on the same shaft. **Note**: Gear 2 meshes with Gear 4.

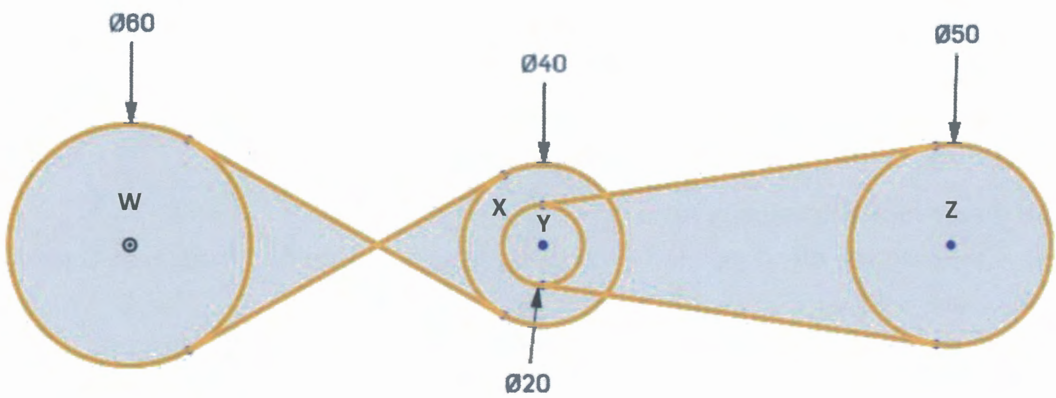


Calculate the number of teeth on the driven gear if the mechanical advantage of the system is 1.35 and it operates with an efficiency of 75%.

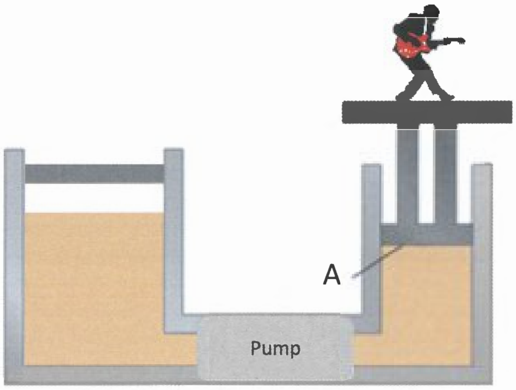
30. Determine the speed and direction of rotation of Pulley D if Pulley A rotates in a clockwise direction at 200 rpm.



31. Determine the speed and direction of rotation of Pulley Z if Pulley W rotates in an anti-clockwise direction at 175 rpm.



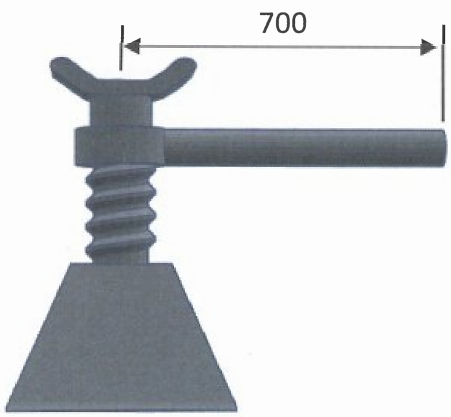
32. A guitarist performs on a moving platform that is hydraulically operated as shown below.



A pressure of 90 kPa, created via a pump, is required to lift the platform upwards.

Calculate the force applied on the pistons if the area underneath the platform (A) is 1.85×10^4 mm^2.

33. A screw jack with a pitch of 6 mm is used to raise a car of mass 1.2 tonnes. The length of the handle is 700 mm long.



a) What is the mechanical advantage of the screw jack? (Exclude any frictional forces).

b) Calculate the force required to hold up the car.

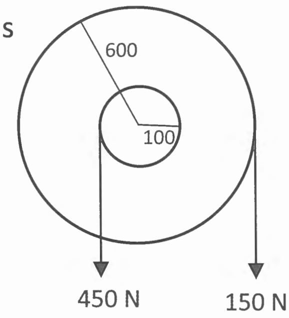
34. A screw jack has a pitch of 5 mm and is operated by a lever which has an effective radius of 450 mm.

a) Determine the velocity ratio

b) Calculate the effort needed in order to raise a section of a truck that weighs 650 kg if the screw jack's efficiency is 35%?

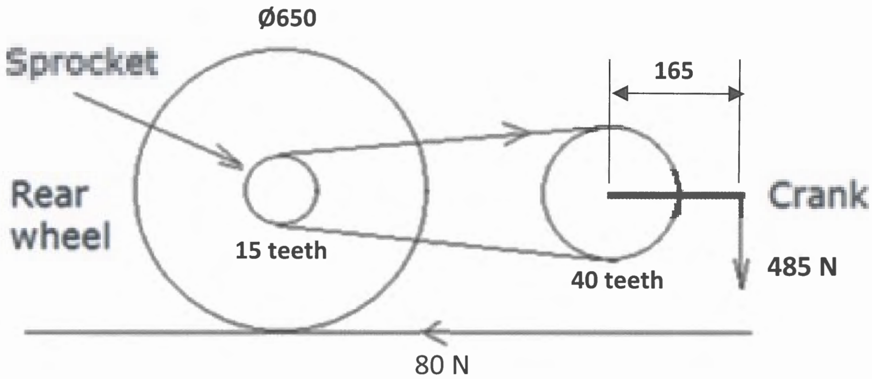
c) Determine the mechanical advantage if a 400 kg load is lifted with the same effort?

35. A machine consists of a wheel of radius 600 mm and an axle of radius 100 mm. An effort of 150 N is used to lift a load of 450 N. Calculate the following:



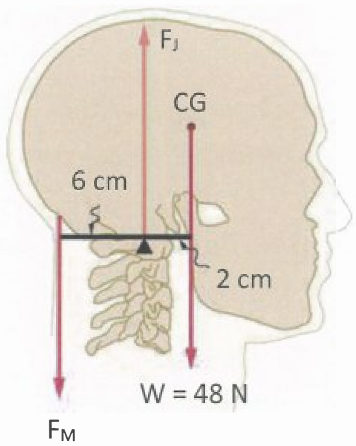
a) Mechanical advantage
b) Velocity ratio
c) Efficiency

36. A bicycle drive mechanism is shown below. A force of 485 N acting vertically downwards on the crank is just enough for the rear wheel to rotate against a resistance of 80 N.



Calculate the efficiency of this mechanism.

37. In the diagram below, the muscles at the back of the neck need to apply a force to keep the head erect since its centre of gravity is not located on the pivot point.



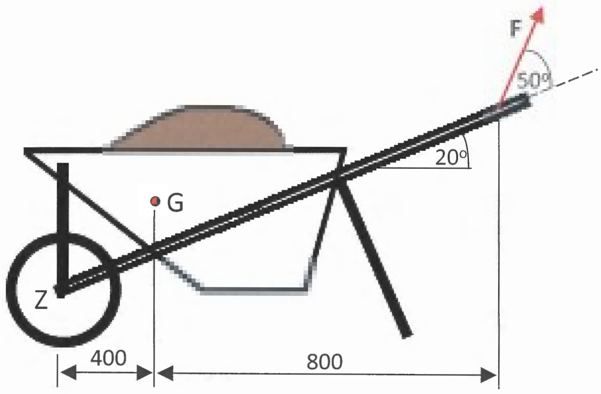
Calculate the following:

a) The force exerted by these muscles (F_M).
b) The force exerted by the pivot on the head (F_J)?

38. A 2.2 tonne Tesla Model S runs on an electric motor and travels 4 km up a 3° slope.

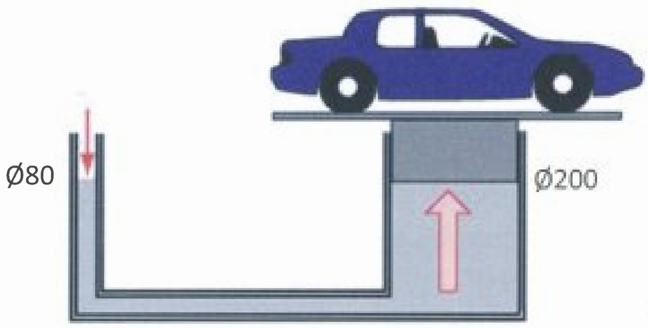
Calculate the net force necessary to drive the motor up the incline if the car is 85% efficient.

39. The combined mass of the wheelbarrow and its load is 70 kg with the centre of mass located at G.



Calculate the force that the man must exert on the handle that will allow the net moment about Point Z to be zero.

40. A 1.2 tonne car is to be lifted by a 200 mm diameter hydraulic piston.

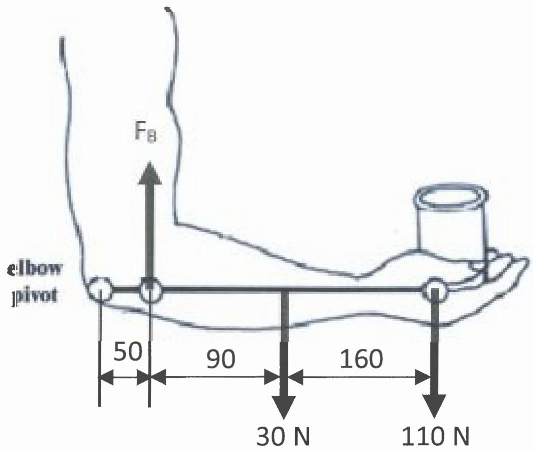


a) What force needs to be applied to an 80 mm diameter piston that will enable the lifting of the car.

b) Determine the pressure in the hydraulic system in kPa

c) Calculate how far the piston needs to move in order to lift the car by 0.8 m.

41. A man is holding a weight of 110 N. The 30 N force is the weight of the forearm acting at its centre of mass.



Calculate the force in the bicep.

Chapter 4 - Friction

- ***Friction*** is the force that acts opposite to the body's direction of motion. It slows the body's motion or makes a machine less efficient in its operation.

- ***Static friction*** is the force that keeps an object at rest or is just at the point of moving. Once the object starts moving, it will experience kinetic friction. For the purpose of this course, only static friction will be explored.

- The friction force is mainly determined by two parameters - the normal force and the coefficient of friction. The ***coefficient of friction (μ_s)*** is a value that depends upon the nature of surfaces that are in contact. It is the ratio of the frictional resistance force to the normal force. This value is usually obtained through experimentation. The following table shows the average coefficient of static friction for a range of surfaces.

Surfaces in contact (Dry)	Coefficient of static friction (μ_s)
Car tyre on asphalt	0.72
Car tyre on grass	0.35
Glass on glass	0.9 – 1
Glass on metal	0.5 – 0.7
Horseshoe on concrete	0.58
Leather on wood	0.3 - 0.4
Paper on cast iron	0.2
Rubber on rubber	1.16
Wood on wood	0.25 – 0.5
Teflon on steel	0.04
Skin on metals	0.8 – 1
Wood on concrete	0.62

- The ***normal force (N)*** is the force that acts perpendicular (90^0) to the surface on which the object is resting.

- Thus, linking the above two parameters, we get the following equation:

$$F_f = \mu_s N$$

where:

F_f = frictional force
μ_s = coefficient of static friction
N = normal force

- If we have a surface material with a high coefficient of friction then our frictional force will also be high as the two parameters are directly proportional. Friction can be both beneficial and hindering. Without friction, cars would have immense difficulty in coming to a stop via brakes. Friction also provides traction which allows cars to turn a corner without sliding onto oncoming traffic. However, friction can be a hindrance during the operation of a machine by reducing its efficiency in conducting the required work.

- A typical free body diagram (FBD) for a box on level ground and a box on an incline is shown below.

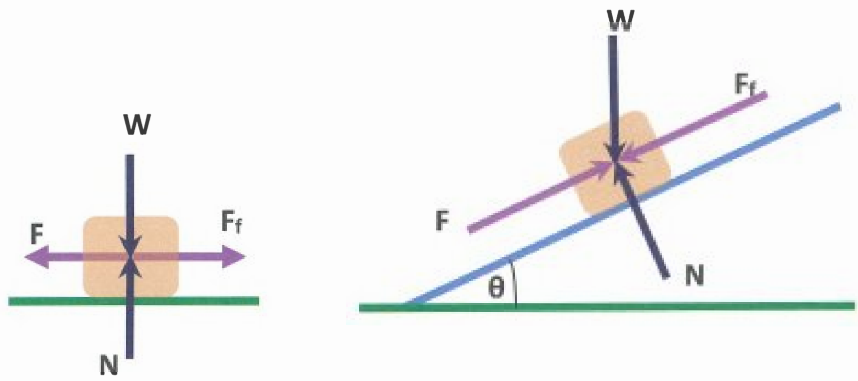


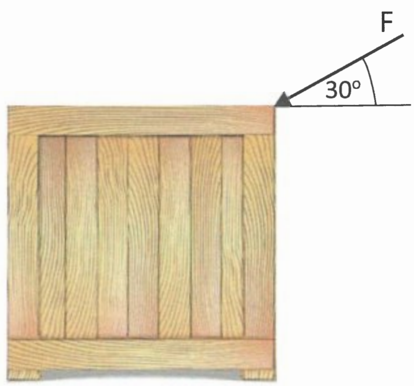
Figure 4.1 – *FBD of a box on level ground (left) and on an incline (right)*

The box on level ground is pulled to the left hence the friction force will act in the opposite direction i.e. to the right. The weight force is always directed downwards, and the normal force is acting 90° to the surface i.e. upwards.

The box on an incline is being pushed up the incline so the friction force acts in the opposite direction i.e. down the incline. The weight force is always directed downwards, and the normal force is acting 90° to the surface i.e. perpendicular to both the force applied and the friction force.

Worked Example 1:

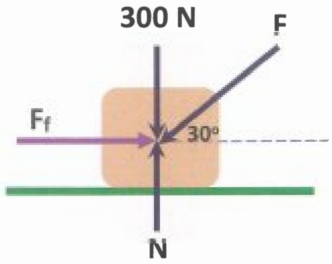
A 30 kg wooden crate rests on a surface with a coefficient of friction of 0.2.



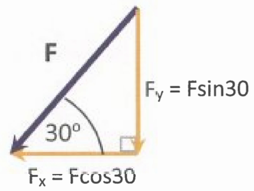
Determine the magnitude of force F that will allow the crate to start moving.

Solution:

1. Draw the FBD as shown below



2. Break up force F into its x and y components.



3. Sum the forces in the vertical direction and equate to 0 to find the normal force in terms of F.

$$+\uparrow \Sigma F_V = 0$$
$$-300 + N - Fsin30 = 0$$
$$N = Fsin30 + 300$$

4. Sum the forces in the horizontal direction and equate to 0. Substitute the frictional force equation (μN) into F_f where μ is equal to 0.2. Substitute the normal force equation into N to get an equation in terms of F. Factorise F out and solve.

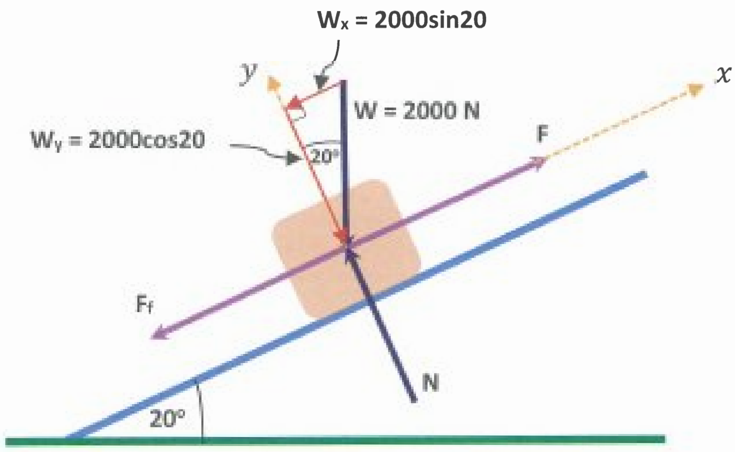
$$\overrightarrow{+}\Sigma F_H = 0$$
$$F_f - Fcos30 = 0$$
$$(0.2N) - Fcos30 = 0$$
$$(0.2 \times (Fsin30 + 300) - Fcos30 = 0$$
$$0.2Fsin30 + 60 - Fcos30 = 0$$
$$F(0.2sin30 - cos30) = -60$$
$$F = \frac{-60}{0.2sin30 - cos30} = \frac{-60}{-0.766} = 78.3\ N$$

Worked Example 2:

A winch mechanism is used to pull a 200 kg crate up a 20° inclined ramp. Calculate the required force in the cable for the crate to start moving if the coefficient of friction between the ramp and the crate is 0.4

Solution:

1. Draw the FBD as shown below. The winch is pulling the box up hence the force in the cable, F, is acting up the ramp and so the friction force will be acting in the opposite direction to the intended motion. Weight force always acts directly downwards, and the normal force is 90° to the ramp's surface.
2. Assign x and y axes. The x-axis is drawn along the ramp and the y-axis is 90° to the x-axis in line with the normal force. This is denoted in orange.
3. Break the weight force into its x and y components. Ensure that the x-component is parallel to the x-axis and the y-component is in line with the y-axis. These components are denoted in red.



4. Sum the forces in the y-direction and equate to 0 to find the normal force.

$$+\nwarrow \Sigma F_y = 0$$
$$-2000\cos20 + N = 0$$
$$N = 2000\cos20$$
$$\therefore N = 1879.4\ N$$

5. Sum the force in the x-direction and equate to 0 to find the force in the cable. Make F the subject and substitute the formula for the frictional force (μN) into F_f to find the final answer.

$$+\nearrow \Sigma F_x = 0$$
$$-2000\sin20 - F_f + F = 0$$
$$F = F_f + 2000\sin20 \qquad \text{Now } F_f = \mu N$$
$$\therefore F = (0.4 \times 1879.4) + 2000\sin20$$
$$\therefore F = 1435.8\ N$$

Angle of Repose

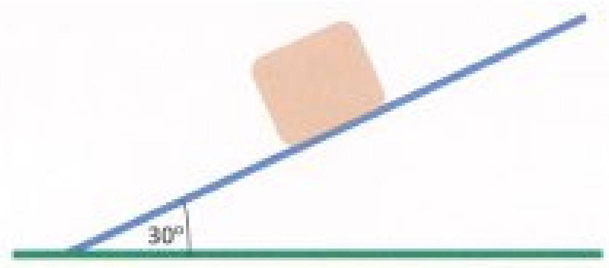
- The ***angle of repose*** is the angle of the inclined plane at which the object begins to slide down without any external force acting on it. The object would not move if it was initially sitting on flat ground. However, as the angle of the incline increases, the weight component acting down the plane (Wsinθ) will also increase up to the point where this component is just above the value of the frictional force causing the object to slide. We can determine whether an object will slide by using the below formula (ignoring the derivations to get to this formula):

$$\mu = tan\emptyset$$
$$\therefore \emptyset = tan^{-1}\mu$$

- If the angle found using this equation is **LOWER** than the angle at which the plane/ramp is inclined, then the object will slide down.

Worked Example 3:

Determine whether the crate will slide down the plane or remain stationary if it is inclined at 30° and the coefficient of friction between the crate and the plane is 0.6.



Solution:

$$\mu = \tan\theta$$
$$\theta = \tan^{-1}(\mu)$$
$$\theta = \tan^{-1}(0.6)$$
$$\boldsymbol{\theta = 30.96^{\circ}}$$

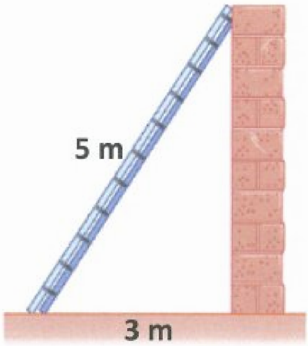
An angle of 30.96° would be required for the crate to move down the ramp. However, the ramp is inclined at 30° (lower angle). This means that the box will remain stationary. If the plane was inclined at an angle of 31° then the crate would start sliding.

Friction involving ladders

Ladders are a good example of objects that require friction to be safe and effective. In most problems you will find that the wall is assumed to be a smooth surface, i.e. no friction. This will simplify the calculation required. However, in practice, the ladder will experience friction with the wall. If the surface is smooth, then the reaction force will be perpendicular to the surface. If the surface is rough, then you will have a normal force (perpendicular to the surface) and a friction force.

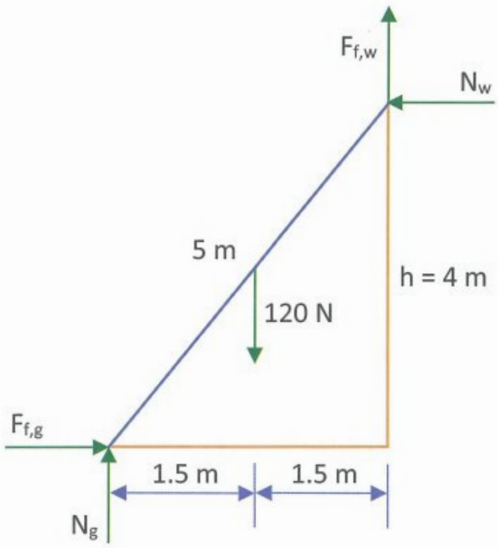
Worked Example 4:

A 12 kg ladder of length 5m leans against a rough vertical wall and a rough horizontal ground. The coefficient of friction between the wall and the ladder is 0.3. Determine the reaction forces at both the wall and the ground.



Solution:

1. Draw the FBD of the forces acting on the ladder as shown below. The ladder tends to slide down and to the left. The friction force will act in the opposing direction i.e. up at the wall and right at the ground. The normal forces will be acting 90° to the wall and ground surface as both surfaces are rough.



2. Find the height of the wall using Pythagoras.

$$5^2 = h^2 + 3^2$$
$$h = \sqrt{25 - 9}$$
$$\therefore h = 4\,m$$

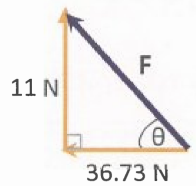
3. Apply sum of moments about the foot of the ladder to find the normal force at the wall. Substitute the friction force formula (µN) into $F_{f,w}$ to get it in terms of N from which its value can determined

$$+\curvearrowleft \Sigma M_{foot} = 0$$
$$F_{f,w}(3) + N_w(4) - 120(1.5) = 0$$
$$(0.3N_w)(3) + 4N_w - 180 = 0 \qquad \text{Now } F_{f,w} = \mu N_w$$
$$4.9N_w = 180$$
$$\therefore N_w = 36.73\,N \leftarrow$$

4. Find the friction force acting on the wall.

$$F_{f,w} = \mu N_w$$
$$F_{f,w} = 0.3 \times 36.73$$
$$\therefore F_{f,w} = 11 \text{ N} \uparrow$$

5. Draw the vector triangle connecting the vectors head to tail with the resultant having a tail-tail and head-head configuration. Use Pythagoras to find the resultant force acting at the wall and trig to find the direction.



$$R = \sqrt{11^2 + 36.73^2}$$
$$\boldsymbol{R \approx 38.3\ N}$$

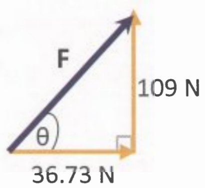
$$\tan\theta = \frac{11}{36.73}$$
$$\theta = \tan^{-1}\left(\frac{11}{36.73}\right)$$
$$\boldsymbol{\theta \approx 16.7^\circ}$$

6. Find the 2 forces acting on the ground by applying our equilibrium equations ($\Sigma F_H = 0$ & $\Sigma F_V = 0$)

$$\overset{\rightarrow}{+} \Sigma F_H = 0$$
$$F_{f,g} - N_w = 0$$
$$F_{f,g} = 36.73\ N \rightarrow$$

$$+\uparrow \Sigma F_v = 0$$
$$-120 + N_g + F_{f,w} = 0$$
$$\therefore N_g = 120 - 11 = 109 \text{ N} \uparrow$$

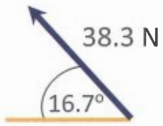
7. Draw the vector triangle similar to step 5.



$$R = \sqrt{109^2 + 36.73^2}$$
$$\boldsymbol{R \approx 115\ N}$$

$$\tan\theta = \frac{109}{36.73}$$
$$\theta = \tan^{-1}\left(\frac{109}{36.73}\right)$$
$$\boldsymbol{\theta \approx 71.4^{\circ}}$$

∴ **Reaction force at the wall**



∴ **Reaction force at the ground**

Flight mechanics

An aircraft at level flight with a constant velocity has 4 forces that act through the aircraft's centre of gravity. The FBD for this situation is shown below (albeit simplified).

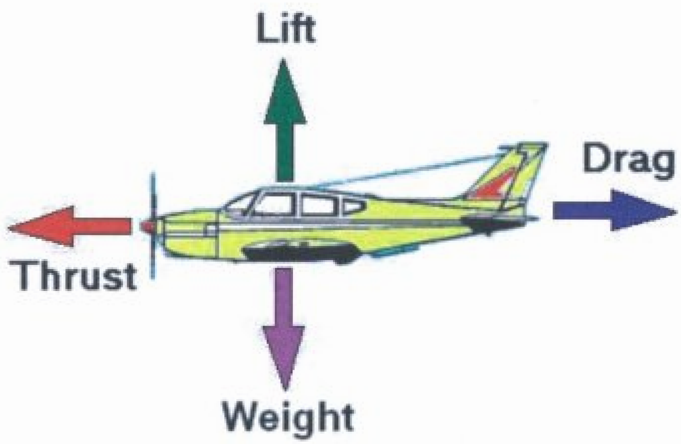


Figure 4.2 – *Forces acting on a plane in level flight*

These forces are defined as such:

- **Lift** is the aerodynamic force produced via the airplane's motion through the air. It holds the airplane in the air, maintaining level flight due to the lift force directly opposing the weight force. Lift is mostly generated by the wings.
- **Thrust** is the force produced by the engines of the aircraft via a propulsion system. The thrust force directly opposes the drag force to maintain a constant velocity.
- **Drag** is the aerodynamic force that opposes an airplane's motion through the air. It is a type of friction force known as fluid friction.
- **Weight** is defined as the force of gravity on the object and is always directed downwards.

The simplified FBD (shown below) for an aircraft undergoing a steady climb at a constant velocity differs slightly to the aircraft travelling at level flight.

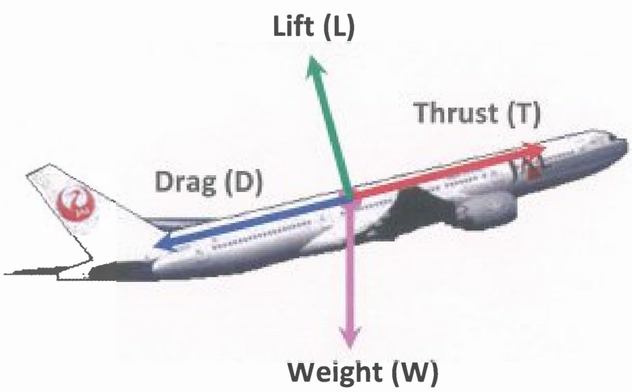


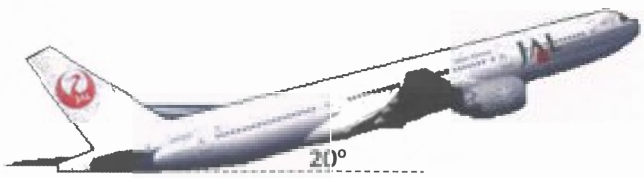
Figure 4.3 – *Forces acting on plane that is climbing*

- Thrust force is always in the direction of flight
- Drag force always opposes the direction of thrust
- Weight force is always directed downwards
- Lift force is always 90° to the thrust/drag direction.

Note: In the case of a glider, there is NO thrust force.

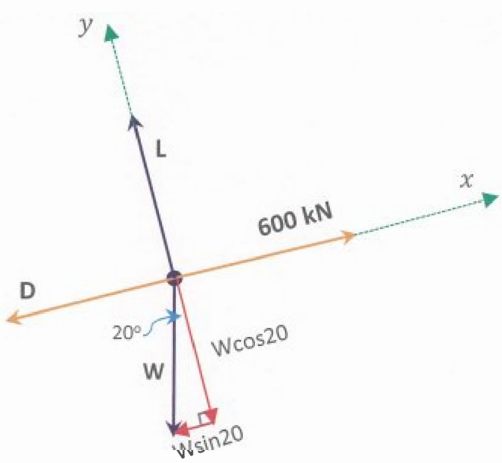
Worked Example 5:

An aircraft with mass 150 tonnes is climbing at an angle of 20°. A thrust of 600 kN is required for the aircraft to maintain a constant velocity.



Calculate the lift to drag ratio.

Solution:



$$+\nwarrow \Sigma F_y = 0$$
$$L - W\cos 20 = 0$$
$$L = 1500\cos 20$$
$$L = 1409.5\ kN$$

$$+\nearrow \sum F_x = 0$$
$$600 - D - W\sin 20 = 0$$
$$\mathrm{D} = 600 - 1500\sin 20$$
$$D = 87\ kN$$

$$\therefore \frac{L}{D} = \frac{1409.5}{87}$$

$$\therefore \mathbf{L:D = 16.2 : 1}$$

Note: The climbing angle is always positioned between the weight force vector and the y component of the weight force vector.

Note: We multiply the value by 10 to convert tonnes to kN.

A higher lift to drag ratio is one of the major design goals in the aviation industry. An aircraft's lift is heavily influenced by its weight so delivering the required lift with lower drag would lead directly to an improvement in climb performance and better fuel economy in the aircraft.

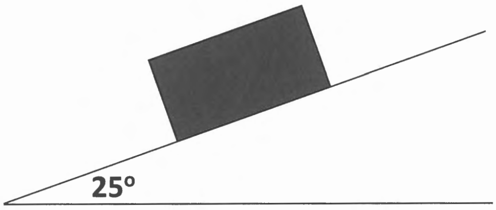
Questions

Section I

1. A 30 kg wooden crate is resting on a flat surface. Calculate the minimum horizontal force, F, needed for the crate to start sliding if the coefficient of friction between the crate and surface is 0.4.

(A) 12 N
(B) 120 N
(C) 24 N
(D) 240 N

2. A box is positioned on a plane that is inclined at 25°. The coefficient of friction is equal to 0.3.



Determine which of the options below is true?
(A) The box will slide down the plane at an increasing rate
(B) The box will slide down the plane at a decreasing rate
(C) The box will remain stationary
(D) The box will slide down the plane at a constant rate

3. The direction of the friction force is always
(A) perpendicular to the plane
(B) parallel to the plane
(C) opposite to the direction of motion
(D) both B and C

4. When calculating the frictional force, the normal force is
(A) always horizontal
(B) always vertical
(C) always smaller than the frictional force
(D) always perpendicular to the surface

5. A 4WD is parked on a slanted surface where the coefficient of friction between the tyres and the surface is 0.5.

Determine which of the angles is the largest possible value of θ before the car starts to slide back down the slope.

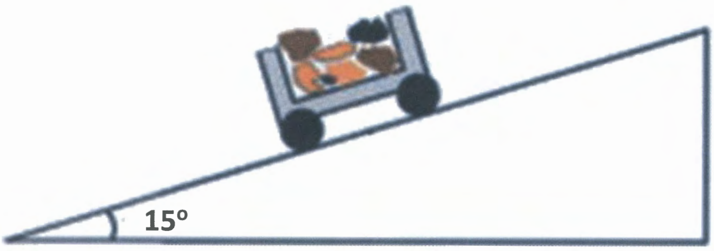
(A) 30°

(B) 26°

(C) 22°

(D) 15°

6. The cart begins to slide when the plane is inclined at an angle of 15°.



What is the coefficient of friction between the cart and the plane?

(A) 0.27

(B) 0.26

(C) 0.28

(D) 0.97

7. Lift, weight, thrust, and drag are the four forces that act on an aircraft. Which statement is correct?

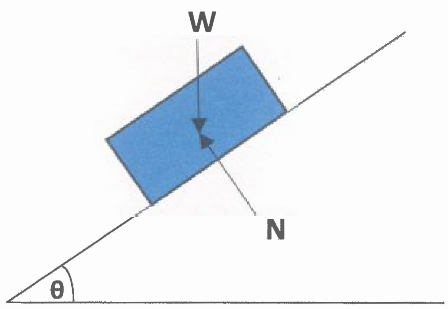
(A) The thrust must equal to drag to maintain constant velocity.

(B) On a conventional aircraft, the propeller or jet engine is used to overcome gravity.

(C) The four forces must be unequal to maintain straight and level flight.

(D) Only the weight of the aircraft and its fuel must be against gravity

8. The diagram below shows an object on a slope.



What is the component of weight, W, that is parallel to the slope's surface?

(A) Wsinθ

(B) Wcosθ

(C) 1

(D) 0

9. The lift force for a climbing aircraft is always:

(A) Directed vertically upwards

(B) In the opposite direction to the weight force

(C) Opposes the drag force

(D) Perpendicular to the thrust/drag force

10. How many forces act on a descending glider?

(A) 1

(B) 2

(C) 3

(D) 4

11. Which of the following will decrease the frictional force between two surfaces?

(A) Lubricate the two surfaces

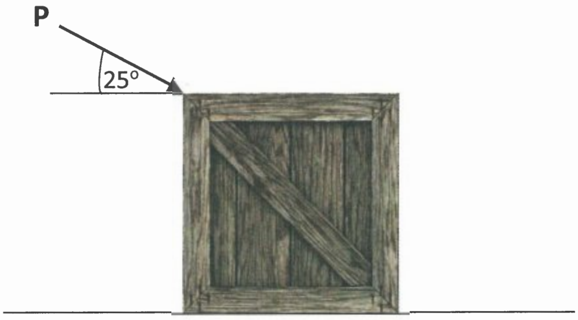
(B) Roughen the two surfaces

(C) Increase the coefficient of friction of a surface

(D) Increase the normal reaction of the two surfaces

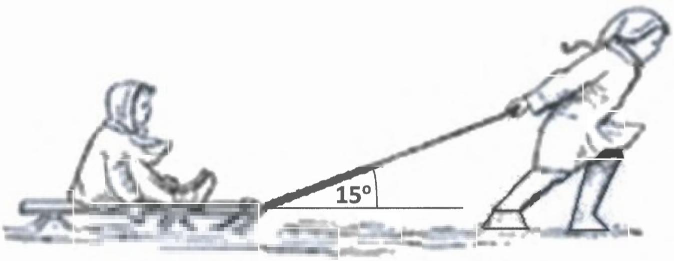
Section II

12. A wooden crate of mass 25 kg rests on a surface with a coefficient of friction of 0.25.



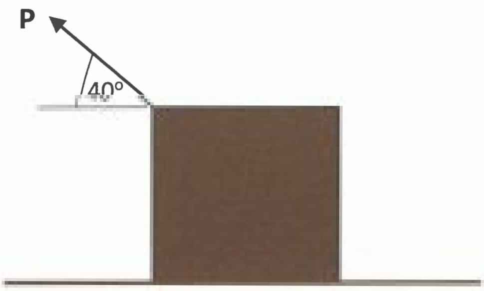
Determine the magnitude of the force, P, that will have the crate on the point of sliding.

13. A child and a sled are being pulled along a flat snowy surface using a rope. The combined mass of the child and sled is 26kg and the coefficient of static friction between the sled and the snow surface is 0.18.



Determine the tension in the rope just before the sled starts to move. Include a free body diagram in your answer.

14. A 17 kg box rests on a wooden surface which has a coefficient of 0.6.



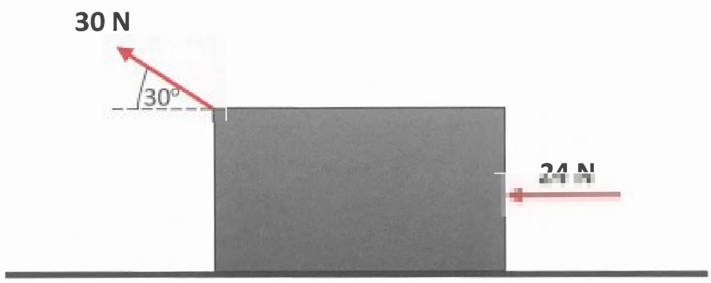
Determine the force, P, required to just get the box moving. (MCQ?)

15. An 18 kg sled is pulled across the ground at a constant speed with 10 N of force by a rope making a 30° angle above the horizontal.

Calculation the coefficient of friction between the sled and the ground?

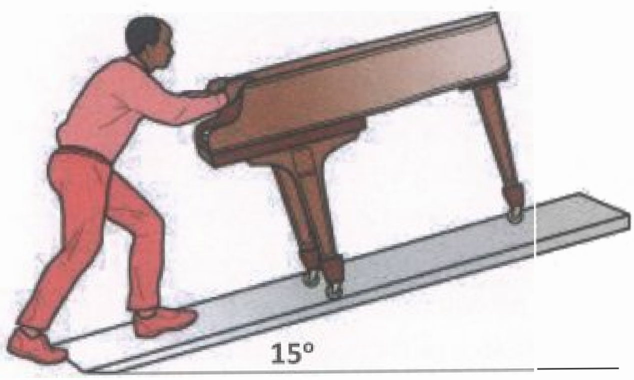
16. A trash can of mass 10 kg is standing on flat ground. If the coefficient of friction is 0.45, determine whether the trash can will move if it is pushed by a horizontal force of 50N.

17. The diagram shows a box of weight, *W*, resting on a horizontal surface. It is being pulled with a force of 30 N at an angle of 30° above the horizontal and pushed with a force of 24 N.



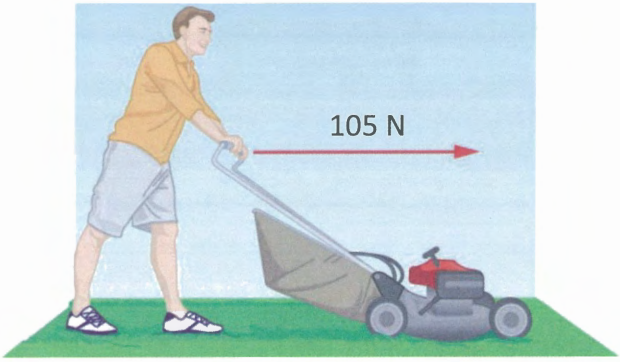
a) Draw a free body diagram showing all the forces acting on the box.
b) Show that the frictional force acting on the box is close to 50N.
c) Calculate the normal reaction force if the coefficient of friction between the box and surface is 0.25.
d) Determine the mass of the box.

18. A ramp leading to a furniture removal truck is inclined at 15°. Calculate the force Mark must apply (parallel to the ramp) to prevent the 400 kg piano from rolling down the ramp if the coefficient of friction been the wheels and the surface is 0.25. Include a free body diagram in your answer.

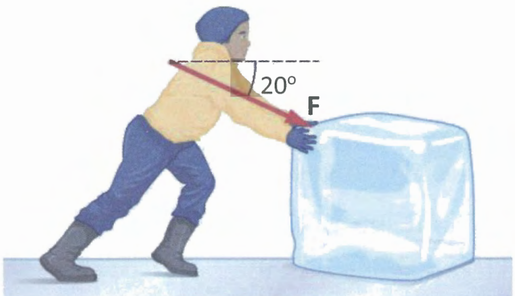


19. A 20 kg block rests on a 20° inclined plane. A man applies a force parallel to the slope to just get the block moving up the inclined plane. Calculate the magnitude of this force if the coefficient of friction is 0.4.

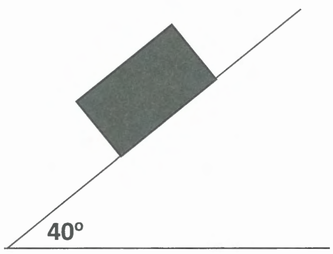
20. Ayrat requires a horizontal force of 105 N to just get the lawn mower moving. Determine the mass of the lawnmower if the coefficient of friction between the grass and wheels is 0.3.



21. Bailey pushes a 40 kg block of ice across a frozen lake as shown below. Calculate the force F he must apply in order to just get the block moving. The coefficient of friction in this instance is 0.08.



22. A 10 kg block is placed on a ramp that is inclined at 40°. The block is at rest in this position.

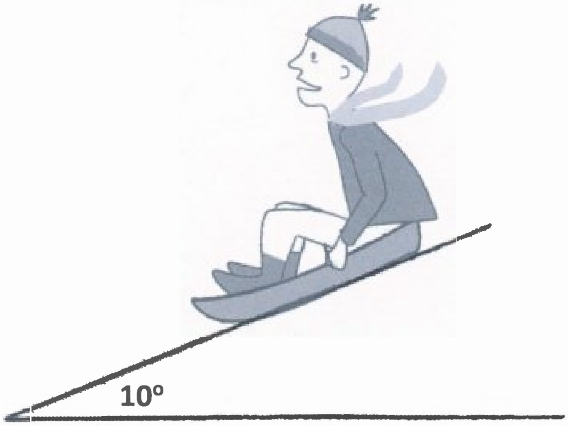


a) Draw a free body diagram showing all the forces acting on the block.
b) Determine the value of the normal reaction force acting on the block
c) Calculate the frictional force acting on the block
d) Determine the coefficient of friction between the block and the ramp

23. A horizontal force of 2500 N is required to just get the 450 kg steel crate to move across a flat steel beam. What is the coefficient of friction for the steel on steel combination?

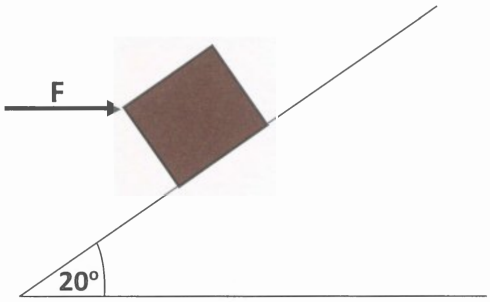
24. A 15 kg crate initially resting on horizontal ground requires a 55 N horizontal force to set it in motion. Find the coefficient of friction between the crate and the ground.

25. The combined mass of the man and sled is 100 kg. The man requires a force of 100 N to just get the sled moving downhill on a decline of 10°. Determine the sled material and the conditions required for the sled to start moving downhill by referring to the table below.



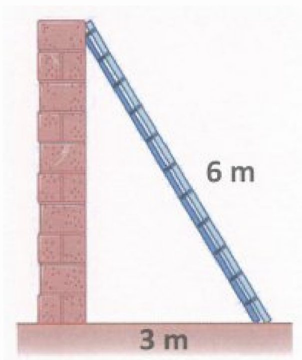
Material in contact	Coefficient of friction
Waxed wood on Wet snow	0.14
Waxed wood on Dry snow	0.4
Rubber on Dry snow	0.25
Rubber on Wet snow	0.5

26. A horizontal force of F is applied to prevent the 30kg block from sliding down a 20° decline. The coefficient of friction between the block and the slope is 0.25.



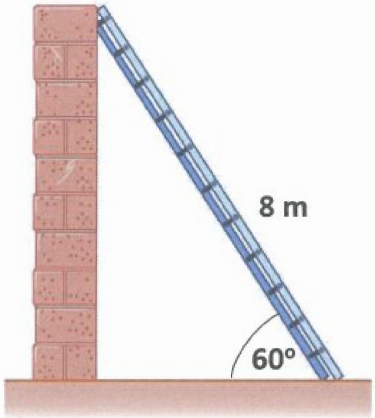
Determine the minimum force, F, required for the block to start moving up the slope.

27. A 20 kg ladder of length 6m is leaning against a smooth vertical wall and a rough horizontal ground. The bottom of the ladder is 3m from the wall.



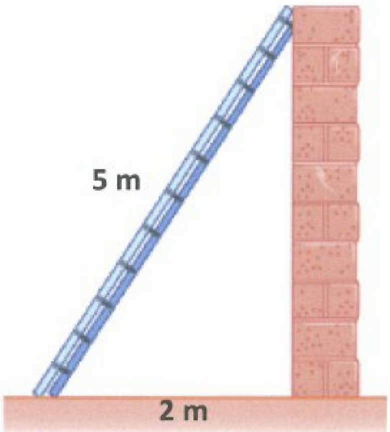
a) Determine the frictional force between the ladder and the ground.
b) Find the normal reaction at the ground.
c) What is the coefficient of friction between the ladder and ground?

28. A 25 kg ladder of length 8m leans against a smooth vertical wall and a rough horizontal ground.

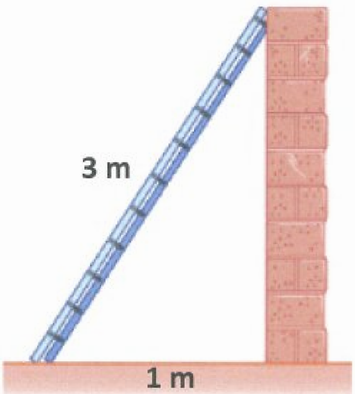


Determine how far a 100 kg man can climb up the ladder before it begins to slip if the coefficient of friction between the ladder and the ground is 0.45.

29. A 15 kg ladder of length 5m leans against a smooth vertical wall and a rough horizontal ground. Determine the reaction forces at both the wall and the ground.



30. A 10 kg ladder of length 3m leans against a rough vertical wall and a rough horizontal ground. The coefficient of friction between the wall and the ladder is 0.25. Determine the coefficient of friction between the ladder and the ground.

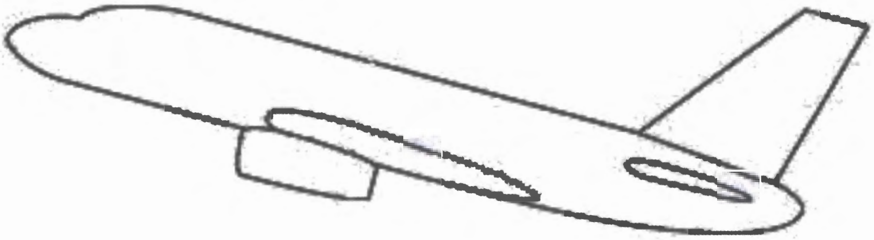


31. A Boeing 767 aircraft of mass 80 tonnes is travelling with a constant velocity at an incline of 15°.

a) Calculate the value of lift produced.

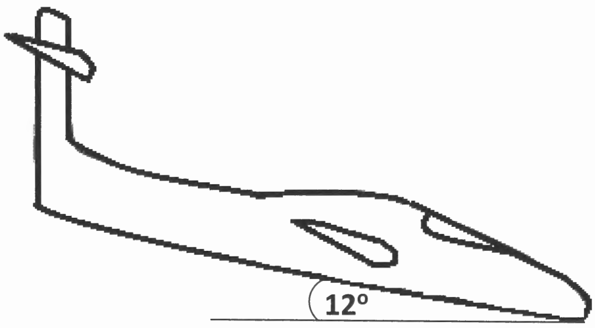
b) Determine the thrust required to produce a lift to drag ratio of 16:1.

32. A 12 tonne aircraft requires a thrust of 75 kN to maintain constant velocity when ascending at an incline of 18°.



Calculate the lift to drag ratio (L:D).

33. A glider descends at 12° whilst maintaining a constant velocity. The pilot has a mass of 75 kg. and the glider experiences a lift force of 5566 N.



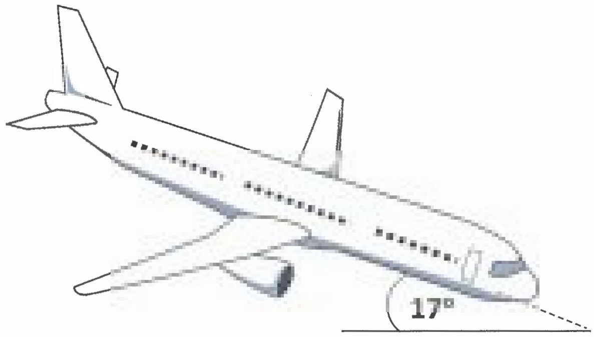
a) Calculate the mass of the glider.

b) Calculate the lift to drag ratio.

34. A 300 tonne Boeing 777 is powered by 2 of the world's most powerful GE90 engines. Each engine produces a thrust value of 580 kN. The aircraft maintains a constant velocity whilst climbing at 20°.

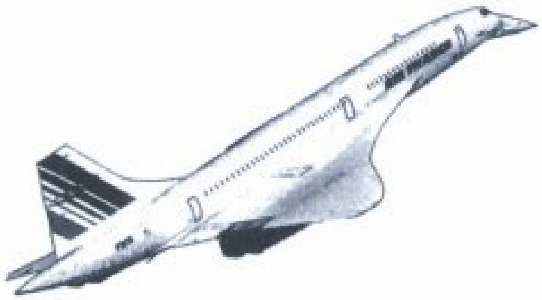
Calculate the induced drag for the Boeing 777.

35. An aircraft of mass 200 tonnes undertakes a descent of 17° to the horizontal whilst maintaining constant velocity.



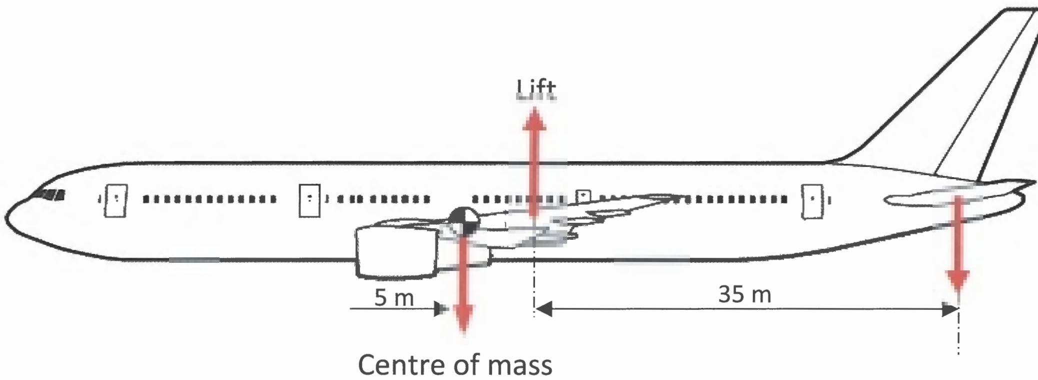
Calculate the drag experienced by the aircraft if it has a lift to drag ratio of 6:1.

36. The Concorde has a mass of 102 tonnes and experiences a drag of 40 kN.



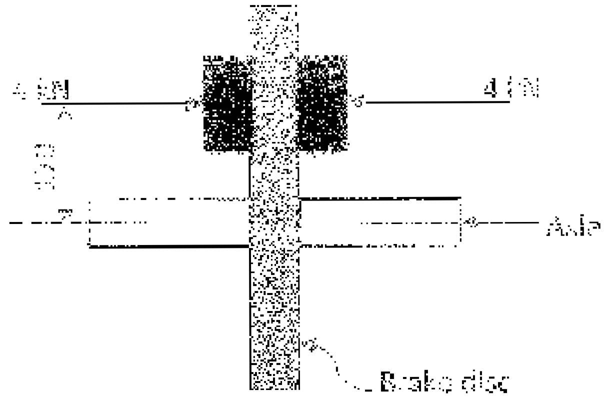
Calculate the amount of thrust required to keep this aircraft climbing at 15° whilst maintaining a constant velocity.

37. The below image shows a plane of mass 500 tonnes cruising in level flight. A weight force, lift force and the force at the horizontal stabiliser/elevator are shown.



a) Calculate the magnitude of the lift force required to keep the plane in level flight.
b) Calculate the force at the horizontal stabiliser/elevator.

28 A braking force of 4 kN is applied to each brake pad as shown below.



Determine the moment needed to overcome the braking force so the car at rest can start moving. Assume a coefficient of friction of 0.35.

Chapter 5 - Work, Energy and Power

Work

- **Work** is defined as the energy that is transferred by a force. Therefore, for work to be done, the force applied must be able to move an object a certain distance. For example, work is done when an applied force causes a box to move.
- If the force has a component in the same direction as the object's displacement, then we say that **positive work** has been done. A common example is the work done by acceleration forces.
- If the force has a component in the direction opposite to the object's displacement (opposing the object's motion) then we say **negative work** has been done. A common example is the work done by the friction force.
- When calculating the work done, it is imperative that the component of the applied force is acting in the same direction as the distance travelled. The units for work is Joules. The work equation is:

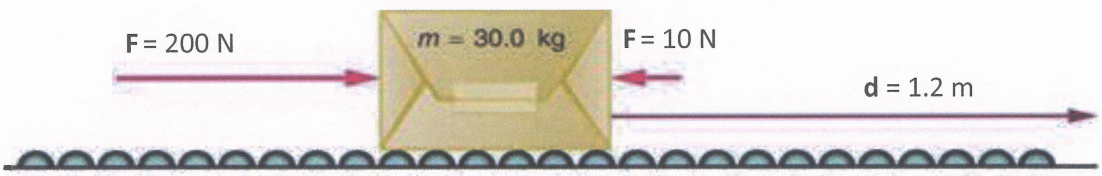
$$W = Fs$$

Where:

W = work (J)
F = applied force (N)
s = distance (m)

Worked Example 1:

A 200 N force is applied on a 30 kg package over a distance of 1.2 m with an average friction force of 10 N. Calculate the work done on the package.



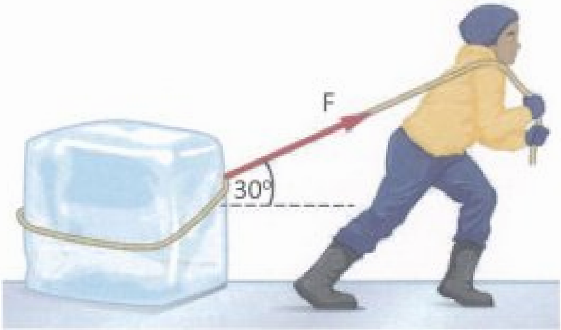
Solution:

The force in the work equation is the total force that causes the box to move in its intended direction. That is, net force equals applied force minus the friction force.

$$W = Fs$$
$$W = (200 - 10) \times 1.2$$
$$\therefore W = 228\ J$$

Worked Example 2:

Avneel applies a force of 200 N at an angle of 30° above the horizontal to drag a 30 kg block of ice.

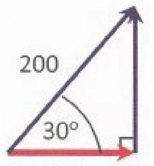


Calculate the work he must exert to move the block 10m across the ice. Assume no friction.

Solution:

When calculating the work done, the force we need is the force that is parallel to the surface. Therefore, we break up the 200N force into its horizontal component.

$$W = Fs$$
$$W = (200\cos 30) \times 10$$
$$\therefore W = 1732\ J$$



Energy

- **Energy** is defined as the ability to do work. Two types of energy that are of interest in this course are potential and kinetic energy.
- **Potential energy** is the energy held by an object due to its position relative to other objects. It is more commonly known as stored energy. Examples of potential energy include a raised weight and a stretched spring in a pinball machine.
- The formula to calculate potential energy is:

$$PE = mgh$$

Where:

PE = potential energy (J)
m = mass of object (kg)
g = acceleration due to gravity (ms^{-2})
h = height of the object (m)

- **Kinetic energy** is the energy that an object possesses due to its motion. Examples of kinetic energy include an aircraft during flight and a car travelling down a road.
- The formula to calculate kinetic energy is:

$$KE = \frac{1}{2}mv^2$$

Where:

KE = kinetic energy (J)
m = mass of object (kg)
v = velocity of object (ms^{-1})

Kinetic energy – Potential energy relationship

These two types of mechanical energy are closely related. Potential energy is stored energy – as an object begins to move, that potential energy gets transformed into kinetic energy as per the conservation of energy law. An example of this is when a book is resting on a table. Relative to the ground, this book has maximum potential energy and zero kinetic energy. When the book gets knocked to the ground, it picks up kinetic energy due to its motion and loses potential energy (max kinetic energy is achieved just before it hits the ground and potential energy is zero hence potential has been converted to kinetic).

Another example are the roller coaster carts on a roller coaster. When the cart reaches the peak, it has maximum potential energy and zero kinetic energy. As it descends to the bottom of the coaster (at ground level), it now has maximum kinetic energy (as it picks up speed with the help of gravity pushing it down) and zero potential energy. Hence potential energy has been converted to kinetic energy.

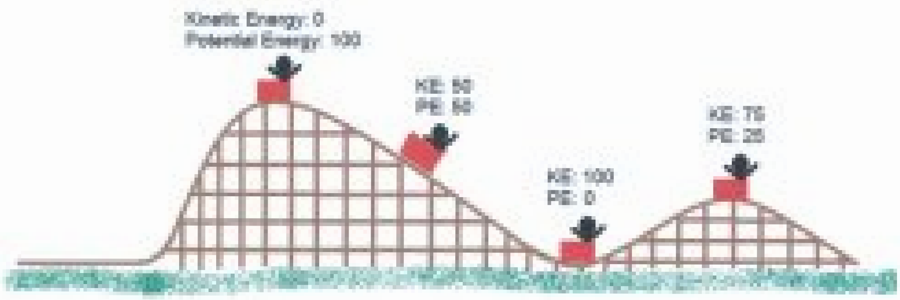


Figure 5.1: *KE and PE relationship applied to a roller coaster*

Taking the above into account, we can find the work possessed by a system using the below formula:

$$W = \Delta KE + \Delta PE$$

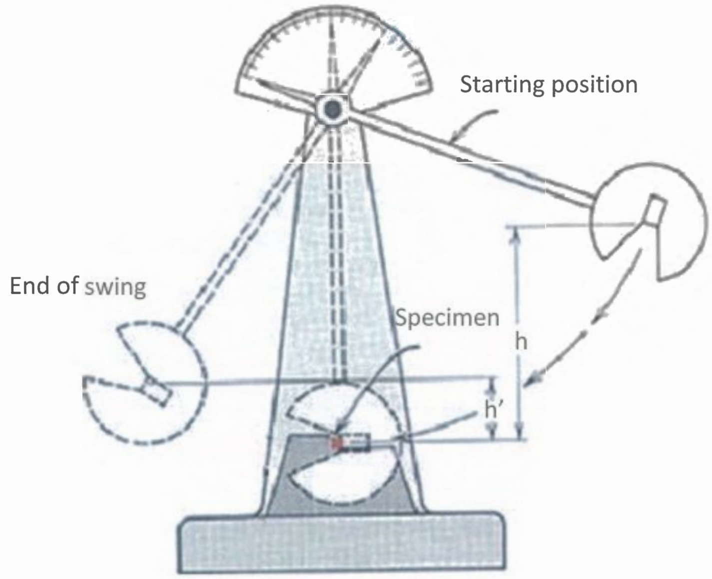
$$\therefore W = (KE_f - KE_i) + (PE_f - PE_i)$$

A positive work value will suggest that the system has gained mechanical energy. A negative work value will suggest that the system has lost mechanical energy. A zero-work value will occur when the only force doing work is the gravitational force. This is usually implied when a rider coasts down a hill and the friction is assumed to be zero.

It is recommended that when working on these problems that you write the full work equation and work your way by analysing what terms in the equation are zero.

Worked Example 3:

In a Charpy Impact Test, a hammer of mass 9kg is released from a vertical height of 750 mm. If the energy lost in fracturing the steel specimen is 65 J, to what vertical height will the pendulum rise to?



Solution:

The velocity is zero at 2 points - the start of the swing and at the end of the swing where it momentarily stops before coming back down. Hence, the change in KE is zero. Work value is negative as energy is being LOST.

$$Work = \Delta PE + \Delta KE$$
$$W = (mgh_f - mgh_i) + 0$$
$$-65 = (9 \times 10 \times h_f) - (9 \times 10 \times 0.75)$$

$$-65 = 90h_f - 67.5$$
$$90h_f = 2.5$$
$$h_f = \frac{2.5}{90} = 0.0278\ m$$
$$\therefore \mathbf{h_f = 27.8\ mm}$$

Worked Example 4:

A cyclist coasts down a 40m tall hill starting from rest. Determine his speed at the bottom of the hill assuming there is no friction. Give your answer in km/h.

Solution:

W = 0 as there is no pedalling or any energy input ("coasting") and no friction. Velocity is initially 0 since it starts at rest. The final height is 0 since it is at the bottom of the hill (our reference point). **Note**: To convert m/s to km/h, we multiply its value by 3.6.

$$W = \Delta PE + \Delta KE = (\mathrm{mg}h_f - \mathrm{mg}h_i) + \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right)$$
$$0 = 0 - (m \times 10 \times 40) + \left(\frac{1}{2} \times m \times v_f^2\right) - 0$$
$$400m = \frac{mv_f^2}{2}$$
$$800m = mv_f^2$$
$$v_f^2 = 800$$
$$v_f = \sqrt{800} = 28.28\ \mathrm{ms}^{-1}$$
$$\therefore v_f = 28.28 \times 3.6 = \mathbf{101.8\ km/h}$$

Power

Power is the rate at which work is done and is expressed in Watts (W) which is equivalent to Joules per second (Js^{-1}). The formulas for power are as follows:

$$\boldsymbol{P = \frac{W}{t} = \frac{\Delta KE + \Delta PE}{t} = \frac{Fs}{t} = Fv}$$

Worked Example 5:

A man is trying to lose weight by working out on a rowing machine. Each time he moves the rowing bar towards himself, it moves a distance of 1.1m in 1.4 seconds. The digital readout indicates that he is producing an average power of 94 W. Determine what force he is exerting on the rowing bar.

Solution:

We are given the distance (s), the time (t) and the power (P). Hence, we can use the below the equation to solve for F by making F the subject.

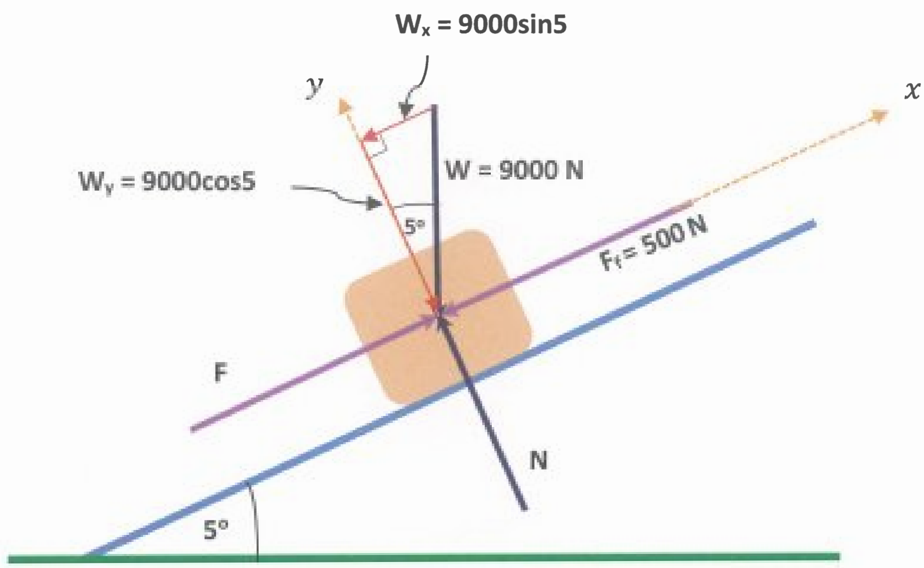
$$P = \frac{Fs}{t}$$
$$94 = \frac{F \times 1.1}{1.4}$$
$$F = \frac{94 \times 1.4}{1.1} = \mathbf{119.6\ N}$$

Worked Example 6:

Calculate the power output needed for a 900 kg car to climb a 5-degree slope at a constant 90 km/h while encountering wind resistance and friction totalling 500 N.

Solution:

We first need to find the force that the engine must provide the wheels for them to overcome the total resistance forces and the weight component that is acting against the car's motion. To do this, we need to sum the forces in the x-direction i.e. along the plane. A FBD is drawn depicting the forces at play.



$$+\nearrow \Sigma F_x = 0$$
$$F - 9000\sin 5 - 500 = 0$$
$$F = 500 + 9000 \sin 5$$
$$\therefore \mathbf{F = 1284.4\ N}$$

We can now determine the power required as we have the force and the velocity at which this car is travelling. Note: 90 km/h must be converted to m/s by dividing it by 3.6.

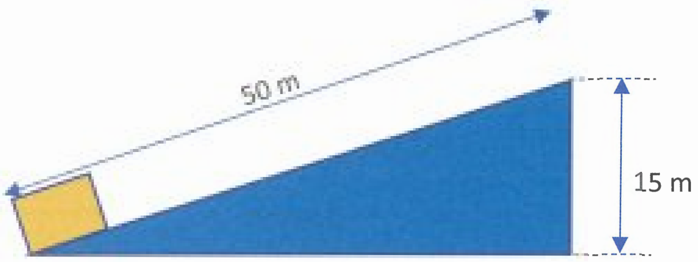
$$P = Fv$$
$$P = 1284.4 \times \left(\frac{90}{3.6}\right)$$
$$\therefore P = 32{,}110\ W = \mathbf{32.1\ kW}$$

Questions

Section I

1. A shopping trolley is pushed for 20m at a constant speed on level ground with a 35.0 N frictional force opposing its motion. The shopper pushes in a direction 25° below the horizontal. What is the total work done?
   (A) 700 N
   (B) 700 J
   (C) 38.6 N
   (D) 0

2. A constant force of 10 kN, parallel to the inclined surface, moves a box of weight 30kN through a distance of 50 m up a slope. The box achieves a final position of 15m above the ground surface.



   Calculate how much of the work done is dissipated as heat.
   (A) 50 kJ
   (B) 100 kJ
   (C) 150 kJ
   (D) 200 kJ

Use the information below to answer Q3 and 4.

A force of 180 N is applied to a model steam locomotive which has a mass of 300 kg. Initially, the train is at rest and can travel 50m in 13.5 secs under constant acceleration on a frictionless surface.

3. What is the work done by the locomotive's steam engine?
   (A) 9 kJ
   (B) 90 kJ
   (C) 15 kJ
   (D) 150 kJ

4. What is the power produced by the locomotive's steam engine?
   (A) 0.667 W
   (B) 667 W
   (C) 1111 W
   (D) 1.111 W

5. A car is accelerating up an inclined road.

Which of the following options is correct.

	Potential Energy	Kinetic energy
(A)	Increases	Decreases
(B)	Increases	Increases
(C)	Stays the same	Decreases
(D)	Stays the same	Increases

6. Abdul lifts 10 bags of Concrete Mix onto the tray of a ute that is 1.4 m high. He takes a total time of 70 secs and each bag has a mass of 20 kg.
Determine how much power he produces.
(A) 20 W
(B) 80 W
(C) 2800 W
(D) 40 W

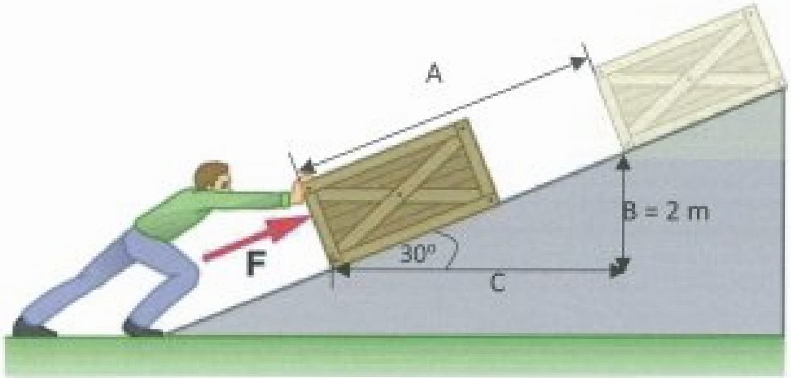
7. A lamp uses an input power of 5 W. After 5 seconds, the lamp has lost 4 J of energy.
Determine the lamp's efficiency.
(A) 10 %
(B) 90 %
(C) 16 %
(D) 84 %

8. A bicycle and rider with a combined mass of 90 kg have a kinetic energy of 2.5 kJ. What is the speed at which the rider is traveling at?
(A) 2.1 km/h
(B) 112.5 km/h
(C) 15.4 km/h
(D) 26.8 km/h

9. A wind turbine receives an input of 250 kW of wind power. This input generates 90 kW of useful power. What is the efficiency of this turbine?
(A) 64 %
(B) 278 %
(C) 36 %
(D) 56 %

Use the information below to answer Q10 and 11.

A man applies a force onto the crate to push it to the top of the slope.



The work done is directly proportional to the magnitude of the force and a distance.

10. Which is the relevant distance required to calculate the work done?

(A) A
(B) B
(C) C
(D) B + C

11. The man pushes the 15 kg crate up the 30° slope through a height of 2 m. The friction force developed as a result of the crate moving is 120 N.

What is the work exerted by the man for the crate to move slowly at a constant speed?

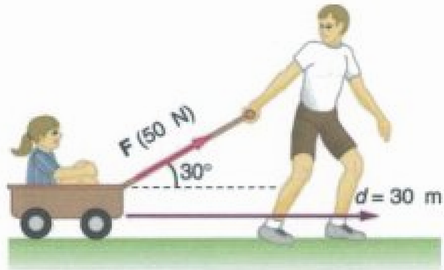
(A) 180 J
(B) 300 J
(C) 420 J
(D) 660 J

12. The power output of a motor depends on the input power and its efficiency. Determine which option is correct

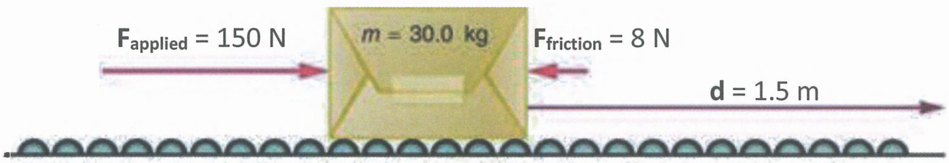
	Power Input	Power Output	Efficiency
a)	5 kW	4 kW	1.25
b)	3 kW	6 kW	0.5
c)	8 kW	5 kW	0.625
d)	10 kW	8 kW	0.75

Section II

13. Calculate the amount of work done by the boy pulling his sister 30 m in a wagon across level ground. Assume that no friction is acting on the wagon.



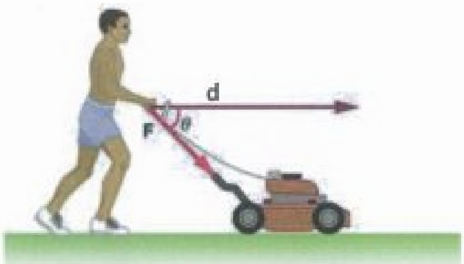
14.



a) Determine the kinetic energy for a 30 kg package which is moving at 0.5 m/s on a roller belt conveyor system.

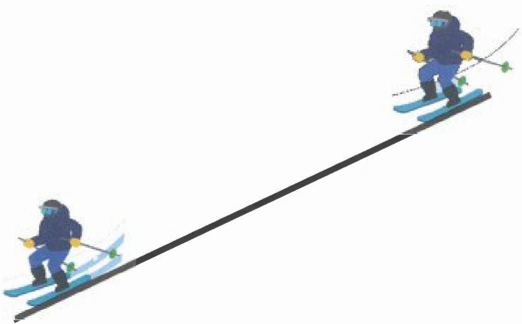
b) A constant force of 150 N is applied on the 30 kg package over a distance of 1.5 m with an average friction force of 8 N. Calculate the work done on the package.

15. A lawn mower is pushed with a resistance of 15 N. The man applies a constant force of 80 N at an angle of 30° below the horizontal to move the lawn mower 25 m on level ground.

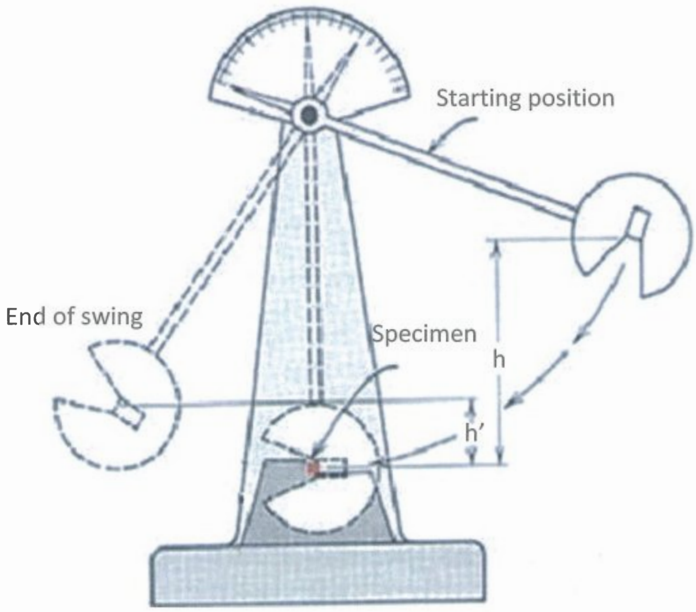


Determine how much work is exerted on the lawn mower.

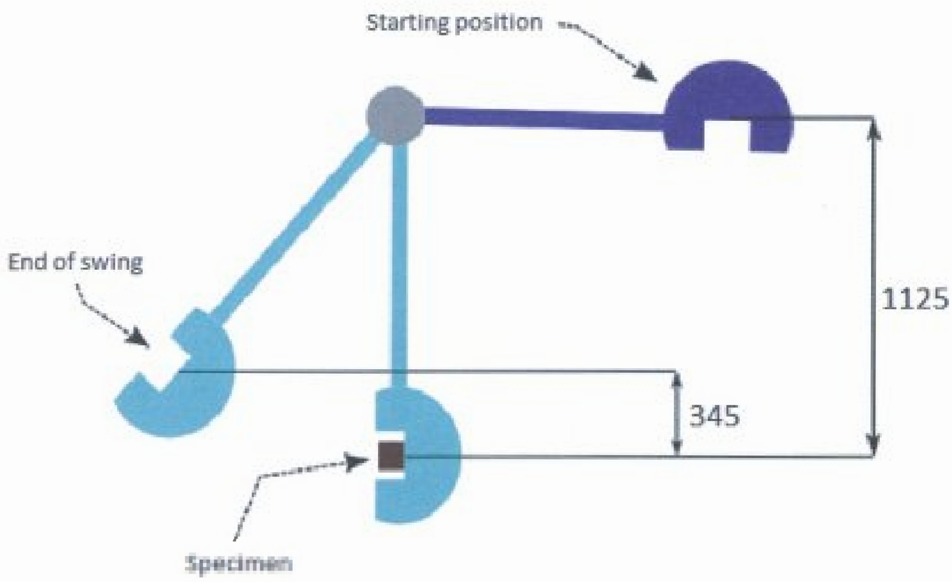
16. A skier weighing 80 kg skis from rest down a 2.2 km strip of unpacked snow with a friction of 30 N. Determine the altitude lost by the skier if he ends up down the hill with a speed of 30m/s.



17. In a Charpy Impact Test, a hammer of mass 11kg is released from a vertical height of 750 mm. If the energy lost in fracturing the steel specimen is 52.75 J, to what vertical height will the pendulum rise to?



18. An impact test was conducted to determine the toughness of the steel specimen. A hammer of mass 8 kg was released from a height of 1125 mm. The hammer strikes and fractures the steel specimen at the bottom of the arc and then continues to swing through to a height of 345 mm.



Determine the amount of energy lost in fracturing the steel specimen. Assume no friction loss in the machine.

19. A 2100 kg car accelerates on a flat horizontal road from rest to a speed of 120km/h in 5.9 seconds. Calculate the power generated by the car in kW. Ignore all resistances.

20. The mass of a locomotive and its contents are 300 tonnes. It possesses a total kinetic energy of 25 MJ.

Calculate the velocity of the locomotive in m/s.

21. A force of 4500 N is required for a 1750 kg car to accelerate from a speed of 20m/s to a speed of 45 m/s. Calculate the distance covered due to this acceleration.

22. A motor vehicle travels on level ground at a constant velocity of 110 km/h. The car experiences an air resistance of 8 kN and a friction force of 12 kN.

Calculate the power (in kW) needed to maintain this constant velocity.

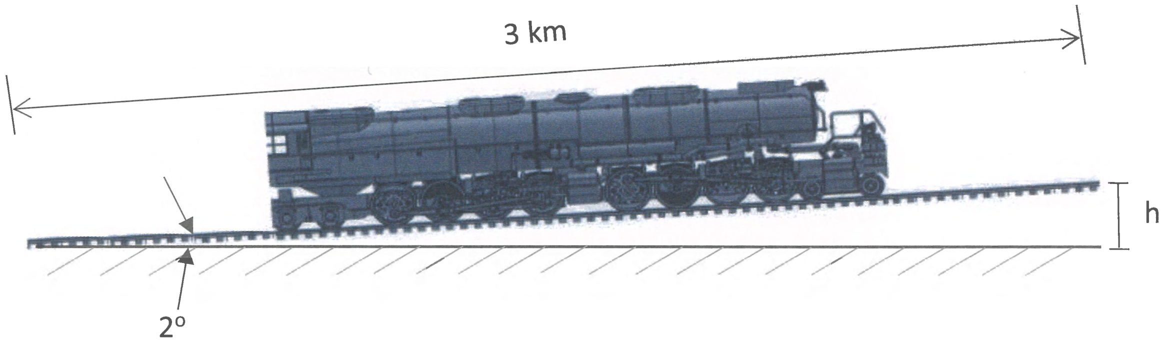
23. A 72 kg cyclist riding on an 8 kg bicycle is travelling at 35 km/h at the top of a 120m hill of slope 30°. The cyclist coasts down to the bottom of the hill where his speed is now 65 km/h. Calculate the resistance to motion whilst the cyclist travels down the 120m hill.

24. How high can a 1100kg car coast up a hill if the work done by friction is negligible and its initial speed is 100 km/h?

25. A vehicle travels at a constant velocity for 2 kilometres on level road. It takes 2 minutes to complete its journey. Calculate the resistance to motion if the vehicle requires a power of 360 kW to maintain this velocity.

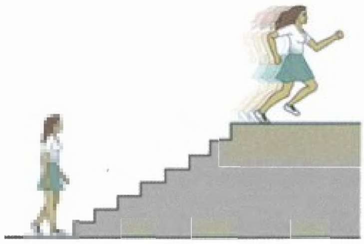
26. A cyclist coasts down a 30m tall hill starting from rest. How fast will he be riding at the bottom of the hill, assuming there is no friction?

27. A 110-tonne locomotive climbs up an incline of 2° for 3 km. Its initial velocity before the climb is 100 km/h. The tractive effort exerted by the engine is 58 kN and the tractive resistance is 92 N/tonne. Calculate the final speed of the locomotive after the climb.



28. A 1500 kg car is driven 2000 m up a 15-degree incline at 20 m/s. How much power is required if both friction and wind resistance apply a 750 N force on the car?

29. A woman with a mass of 55 kg runs up a 3.5 m high flight of stairs in 4 secs. Calculate her power output if she starts from rest and has a final speed of 1.5 m/s.



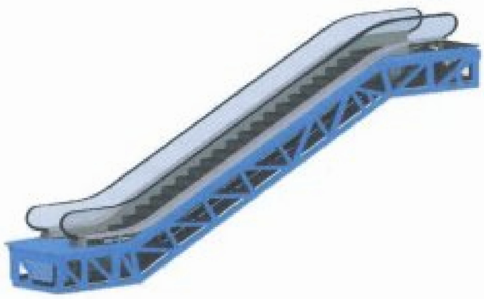
30. A box with a mass of 30 kg is pushed 50 m up a frictionless incline that makes an angle of 30° with the horizontal. How much work did it take for the box to move slowly at a constant speed?

31. A 30 kg box is pushed 50 m up a 20° incline in which the coefficient of friction is equal to 0.20. How much work did it take for the box to move slowly at a constant speed?

32. 80 bags each with a mass of 30 kg, are raised 5 m by a conveyor belt onto an aircraft. A total of 12 minutes was required to complete the task.

Calculate the power required for such a task.

33. A 24 m escalator inclined at 30° is used to transport people from an underground parking lot to the shopping centre's ground floor. At one point in time, 30 shoppers, each with an average mass of 65 kg, are on the escalator.



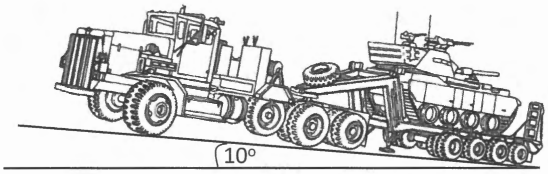
Calculate how much power is required to deliver the shoppers to the ground floor in 20 seconds.

34. A 26 kg box slides down a 30° incline from a starting height of 13m. What will be the speed of the box at the bottom of the incline if the coefficient of friction is 0.15?

35. A motor with an efficiency of 70% is used to drive a conveyor system with a mechanical efficiency of 80%.

Calculate how much power is delivered to the motor for the conveyor to produce 3 kW of mechanical lift.

36. Calculate the velocity at which the 30-tonne tractor truck can pull the 50 tonne tank up a 10° incline with a total resistance of 1.2 kN. The truck's motor produces a power output of 80 kW.

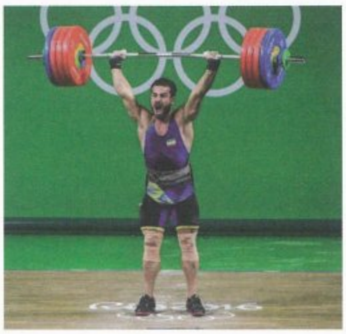


37. A truck with a total pulling power of 500 kW is used to transport logs to a paper mill. The truck and its trailer have a combined mass of 25 tonnes and each log has an average mass of 1200 kg.

To reach its destination on time, the truck will be required to climb a 150m long road that is inclined at 12° in 25 seconds at a constant velocity. Assume no rolling resistance.

Calculate the amount of logs this truck can carry.

38. An Iranian weightlifter exerted 4400 J in the 'clean and jerk' weightlifting category to bring home the 2016 Olympic gold medal.

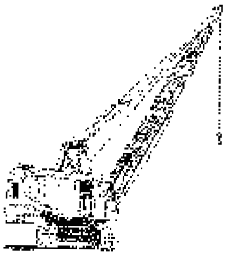


Find the mass that he successfully lifted 2 metres above the ground.

39. A 3-tonne truck travelling at 80 km/h has its brakes applied as it approaches a red traffic light.

a) What is the work done by the brakes to bring the truck to a complete stop?
b) How much power is developed if it takes 12 secs to come to a stop?
c) Calculate the distance taken for the bus to stop (braking distance) if the brakes apply a resistance force of 20 kN.

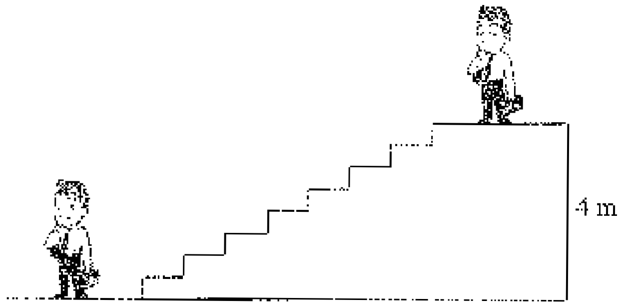
40. A 300 kg mass is lifted vertically off the ground by a motor driven crane.



After 15 seconds and an upwards velocity of 1.5 ms^{-1}, the mass is now 16 metres off the ground.

Calculate the average power required by the motor for this lifting operation to occur if the motor has an efficiency of 70%.

41. A 75 kg man carries his 3 kg suitcase up a 4 m staircase in 10 secs.



Calculate the power required for this task.

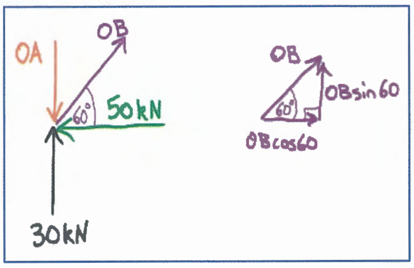
42. The world's steepest passenger railway is used to transport tourists in the Blue Mountains. An electric winch is employed to haul the 16.1 tonne carriage up the 52° rails at a constant velocity of 14km/h.

Calculate the power required when the efficiency of the winch system is 90%. Assume the coefficient of friction between the rails and the wheels to be 0.25.

43. A 30-tonne truck climbs a 20° slope at a constant velocity of 10m/s. The coefficient of friction between the truck and the road is 0.4. Calculate the power required if the efficiency of the power unit is 75%.

Chapter 1 Solutions

1. (B)



1 unknown force in the horizontal direction as opposed to 2 in the vertical. So sum the forces in the horizontal direction first and equate to 0.

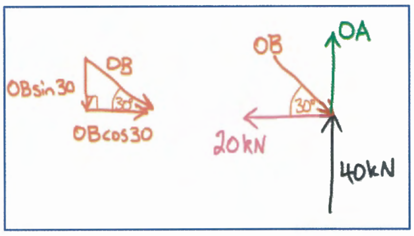
$$\overset{\rightarrow}{+} \Sigma F_H = 0$$
$$OB\cos60 - 50 = 0$$
$$OB = \frac{50}{\cos60} = 100\ kN$$

Now sum the forces in the vertical direction and equate to 0 to find OA.

$$+\uparrow \Sigma F_v = 0$$
$$30 + OB\sin60 - OA = 0$$
$$OA = 30 + 100\sin60 = 117\ kN \downarrow (C)$$

Since OA is directed towards the joint then OA is in compression.

2. (A)

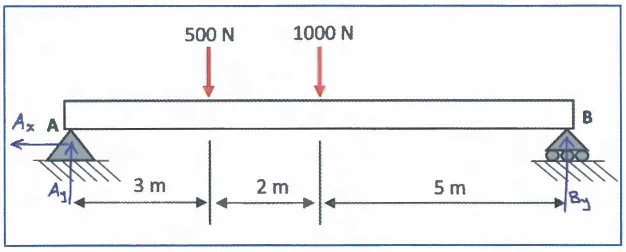


$$\overset{\rightarrow}{+} \Sigma F_H = 0$$
$$-20 + OB\cos30 = 0$$
$$OB = \frac{20}{\cos30} = 23\ kN \searrow (\mathrm{C})$$

Since OB is directed towards the joint then OB is in compression.

3. (A)
4. (D)
5. (A)
6. (A)
7. (B)
8. (D)
9. (C)
10. (A)

11.



1. Apply sum of moments about A to find the force at the roller joint.

$$+\circlearrowleft \Sigma M_A = 0$$
$$-(500 \times 3) - (1000 \times 5) + B_y(10) = 0$$
$$B_y = \frac{6500}{10} N$$
$$\therefore B_y = 650\ N \uparrow$$

2. Apply sum of forces to find the remaining reaction force.

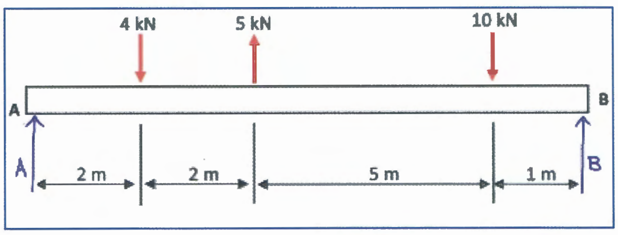
$$+\uparrow \Sigma F_v = 0$$
$$A_y - 500 - 1000 + By = 0$$
$$A_y - 500 - 1000 + 650 = 0$$
$$A_y = 850\ N\ \uparrow$$

$$\overset{\rightarrow}{+} \Sigma F_H = 0$$
$$-A_x = 0$$
$$A_x = 0$$

Note: A_x = 0 as there are no other horizontal forces acting on the beam.

Therefore, the reaction forces at the roller joint and pin joint are 650N and 850N respectively.

12.



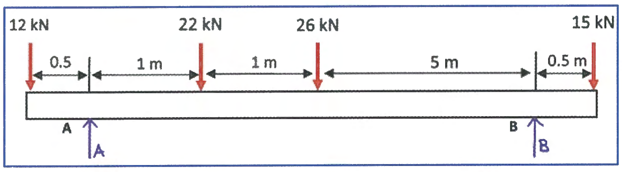
Note: There are no other external horizontal forces so the horizontal force at the pin joint would equal to 0 hence not shown in the above FBD.

Sum the moments about B to find force at A then sum the forces in the vertical direction to find force at B.

$$+\circlearrowleft \Sigma M_B = 0$$
$$-(A \times 10) + (4 \times 8) - (5 \times 6) + (10 \times 1) = 0$$
$$10A = 12$$
$$\therefore A = 1.2\ kN \uparrow$$

$$+\uparrow \Sigma F_v = 0$$
$$A - 4 + 5 - 10 + B = 0$$
$$B = 4 - 5 + 10 - 1.2$$
$$\therefore B = 7.8\ kN \uparrow$$

13.



Note: There are no other external horizontal forces so the horizontal force at the pin joint would equal to 0 hence not shown in the above FBD.

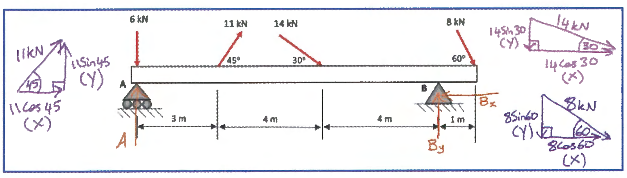
Sum the moments about B to find force at A then sum the forces in the vertical direction to find force at B.

$$+\circlearrowleft \Sigma M_B = 0$$
$$(12 \times 7.5) - (A \times 7) + (22 \times 6)$$
$$+ (26\ \times 5) - (15\ \times 0.5) = 0$$

$$7A = 344.5$$
$$\therefore A = 49.2\ kN \uparrow$$

$$+\uparrow \Sigma F_v = 0$$
$$-12 + A - 22 - 26 + B - 15 = 0$$
$$B = 12 - 49..2 + 22 + 26 + 15$$
$$\therefore B = 25.8\ kN \uparrow$$

14.



Applying sum of moments about B to find force at the roller joint will eliminate the forces at the pin joint.

$$+\circlearrowleft \Sigma M_B = 0$$
$$-(A \times 11) + (6 \times 11) - (11\sin 45 \times 8)$$
$$+ (14 sin30\ \times\ 4) - (8 sin60\ \times\ 1) = 0$$

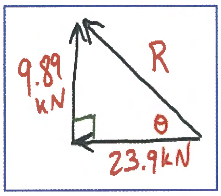
$$11A = 24.8$$
$$\therefore A = 2.26\ kN \uparrow$$

Apply sum of forces in the vertical and horizontal directions to find the pin joint components.

$$+\uparrow \Sigma F_v = 0$$
$$A - 6 + 11\sin 45 - 14\sin 30 - 8\sin 60 + B_y = 0$$
$$B_y = 6 - 11\sin 45 + 14\sin 30 + 8\sin 60 - 2.26$$
$$B_y = 9.89\ kN \uparrow$$

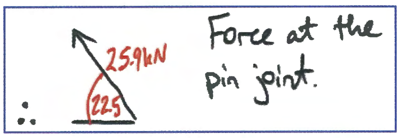
$$\overrightarrow{+} \Sigma F_H = 0$$
$$(11\cos 45) + (14\cos 30) + (8\cos 60) - B_x = 0$$
$$B_x = 23.9\ kN \leftarrow$$

Draw a vector triangle to find the resultant force acting at the pin joint by applying Pythagoras and use trigonometry to find the direction.

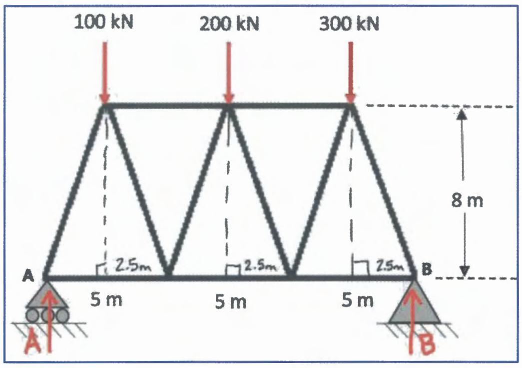


$$R = \sqrt{9.89^2 + 23.9^2}$$
$$\therefore R = 25.9\ kN$$

$$\tan\theta = \frac{9.89}{23.9}$$
$$\therefore \theta = \tan^{-1}\left(\frac{9.89}{23.9}\right)$$
$$\therefore \theta = 22.5^0$$



15.



Note: Horizontal component of the pin joint has been excluded due to no external horizontal forces acting on the truss.

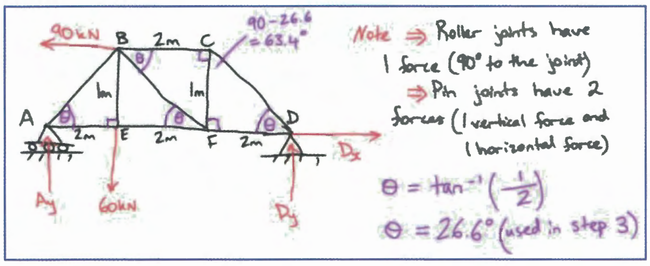
Sum the moments about B to find force at the roller joint keeping in mind that moment is equal to the force multiplied by the perpendicular distance to B.

$$+\circlearrowleft \Sigma M_B = 0$$
$$-15\text{A} + (100 \times 12.5) + (200 \times 7.5) + (300 \times 2.5) = 0$$
$$A = \frac{3500}{15}$$
$$\therefore A = 233.3\ kN \uparrow$$

Sum the forces in the vertical direction to find force at the pin joint.

$$+\uparrow \Sigma F_v = 0$$
$$A - 100 - 200 - 300 + B = 0$$
$$B = 100 + 200 + 300 - 233.3$$
$$B = 366.67\ kN \uparrow$$

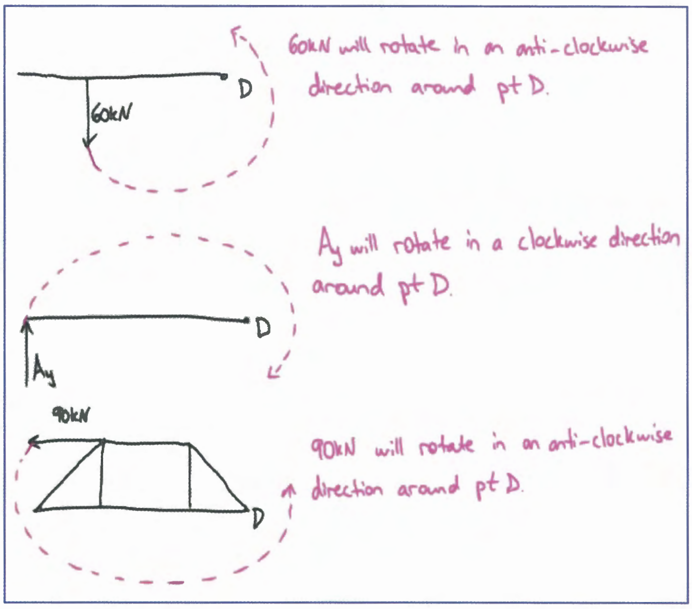
16.



1. Determine support reactions by first summing the moments about either A or D. Anticlockwise direction has been assumed to be positive.

$$+\circlearrowleft \Sigma M_D = 0$$
$$-A_y(6) + (90 \times 1) + (60 \times 4) = 0$$
$$6A_y = 330$$
$$A_y = \frac{330}{6} = 55\ kN \uparrow$$

The following diagram shows the direction of the moment produced by these forces about point D.

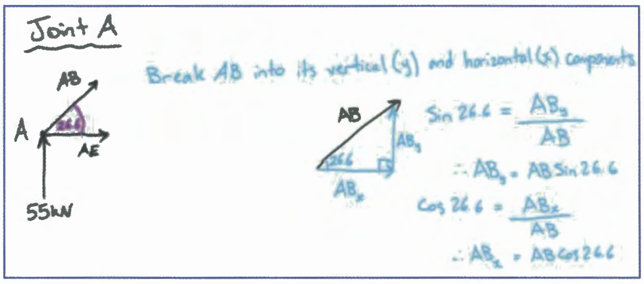


2. Now sum the forces forces in both the vertical and horizontal directions to find the remaining reaction forces.

$$+\uparrow \Sigma F_v = 0$$
$$A_y - 60 + D_y = 0$$
$$55 - 60 + D_y = 0$$
$$D_y = 5\ kN \uparrow$$

$$\overrightarrow{+}\Sigma F_H = 0$$
$$-90 + D_x = 0$$
$$D_x = 90\ kN \rightarrow$$

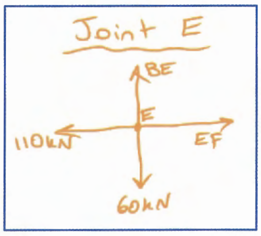
3. We now need to analyse each joint to find the force in each member. To do this, we draw a FBD of each joint and use the sum of forces equations.



$$+\uparrow \Sigma F_v = 0$$
$$55 + AB_y = 0$$
$$55 + AB\sin 26.6 = 0$$
$$AB = \frac{-55}{\sin 26.6} = -123\ kN$$
$$\therefore AB = 123\ kN \swarrow (C)$$

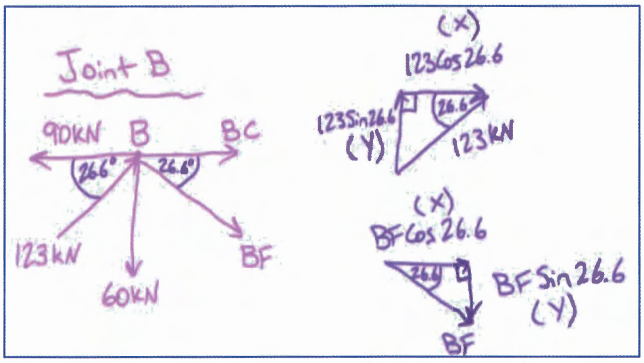
$$\overset{+}{\rightarrow} \Sigma F_H = 0$$
$$AE - AB_x = 0$$
$$AE = 123\cos 26.6$$
$$\therefore AE = 110\ kN \rightarrow (T)$$

Note: Sense of AB has been flipped due to the negative value so horizontal component is now directed to the left instead of to the right.



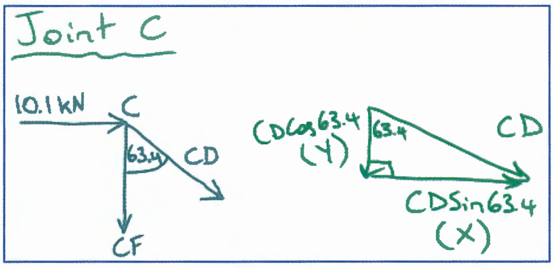
$$+\uparrow \Sigma F_v = 0$$
$$BE - 60 = 0$$
$$BE = 60\ kN \uparrow (T)$$

$$\overset{+}{\rightarrow} \Sigma F_H = 0$$
$$-110 + EF = 0$$
$$EF = 110\ kN \rightarrow (T)$$



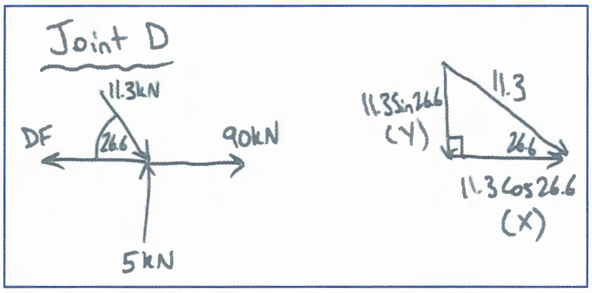
$$+\uparrow \Sigma F_v = 0$$
$$123\sin 26.6 - 60 - BF\sin 26.6 = 0$$
$$BF = \frac{-4.9}{\sin 26.6} = -11kN$$
$$\therefore BF = 11\ kN \nwarrow (c)$$

$$\overset{+}{\rightarrow} \Sigma F_H = 0$$
$$-90 + 123\cos 26.6 - BF\cos 26.6 + BC = 0$$
$$BC = -10.1kN$$
$$\therefore BC = 10.1\ kN \leftarrow (C)$$



$$\overset{+}{\rightarrow} \Sigma F_H = 0$$
$$10.1 + CD\sin 63.4 = 0$$
$$CD = \frac{-10.1}{\sin 63.4} = -11.3\ kN$$
$$\therefore CD = 11.3\ kN \nwarrow (C)$$

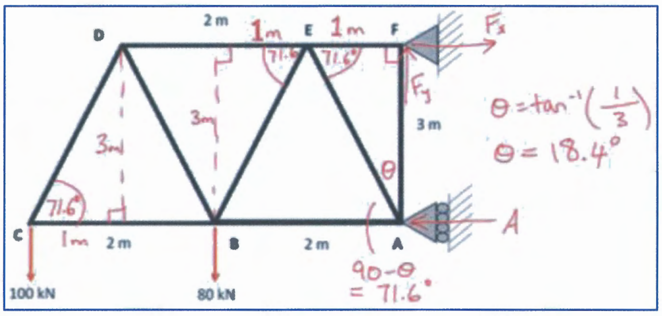
$$+\uparrow \Sigma F_v = 0$$
$$-CF + CD\cos 63.4 = 0$$
$$CF = 11.3\cos 63.4$$
$$\therefore CF = 5\ kN \downarrow (T)$$



$$\overset{+}{\rightarrow} \Sigma F_H = 0$$
$$90 - DF + (11.3\cos 26.6) = 0$$
$$DF = 100\ kN \leftarrow (T)$$

$$\therefore AB = 123\ kN(C)$$
$$\therefore AE = 110\ kN(T)$$
$$\therefore BE = 60\ kN(T)$$
$$\therefore EF = 110\ \ kN(T)$$
$$\therefore BF = 11\ kN(C)$$
$$\therefore BC = 10.1\ kN(C)$$
$$\therefore CD = 11.3\ kN(C)$$
$$\therefore CF = 5\ kN(T)$$
$$\therefore DF = 100\ kN(T)$$

17.



Find force at roller joint by applying moments about F.

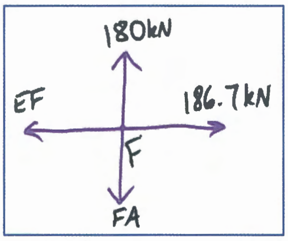
$$+\circlearrowleft \sum M_F = 0$$
$$(100 \times 4) + (80 \times 2) - (A \times 3) = 0$$
$$A = \frac{560}{3} = 186.7\ kN \leftarrow$$

Apply sum of forces in the vertical and horizontal directions to find the remaining reaction forces.

$$+\uparrow \Sigma F_v = 0$$
$$F_y - 100 - 80 = 0$$
$$F_y = 180\ N \uparrow$$

$$\overset{\rightarrow}{+} \sum F_H = 0$$
$$F_x - A = 0$$
$$\therefore F_x = 186.7\ kN \rightarrow$$

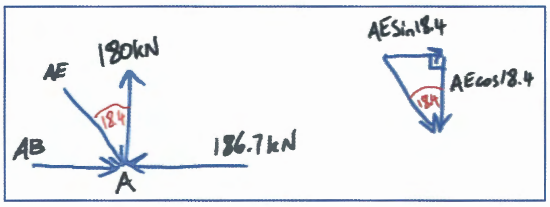
Analyse joint F. Draw FBD and apply the sum of forces equations.



$$+\uparrow \Sigma F_v = 0$$
$$180 - FA = 0$$
$$\therefore FA = 180\ kN \downarrow$$

$$\overset{\rightarrow}{+} \sum F_H = 0$$
$$-EF + 186.7 = 0$$
$$\therefore EF = 186.7\ kN \leftarrow$$

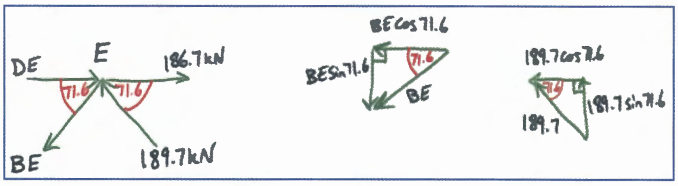
Analyse joint A. Apply sum of forces in the vertical direction first then in the horizontal direction.



$$+\uparrow \Sigma F_v = 0$$
$$180 - AE\cos 18.4 = 0$$
$$AE = \frac{180}{\cos 18.4} = 189.7\ kN \searrow$$

$$\overset{\rightarrow}{+} \sum F_H = 0$$
$$AB + AE\sin 18.4 - 186.7 = 0$$
$$AB = 186.7 - 189.7\sin 18.4$$
$$AB = 126.8\ kN \rightarrow$$

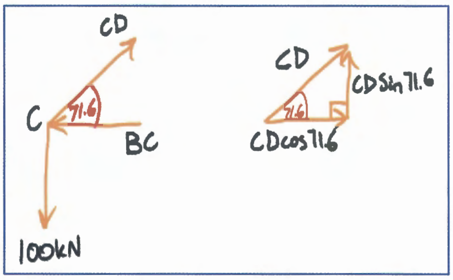
Analyse joint E. Apply sum of forces in the vertical direction first then in the horizontal direction.



$$+\uparrow \Sigma F_v = 0$$
$$(189.7\sin 71.6) - BE\sin 71..6 = 0$$
$$\therefore BE = 189.7kN \swarrow$$

$$\overset{\rightarrow}{+} \sum F_H = 0$$
$$DE + 186.7 - 189.7\cos 71.6 - BE\cos 71.6 = 0$$
$$DE = 189.7\cos 71.6 + 189.7\cos 71.6 - 186.7 = -66.9kN$$
$$\therefore DE = 66.9kN \leftarrow$$

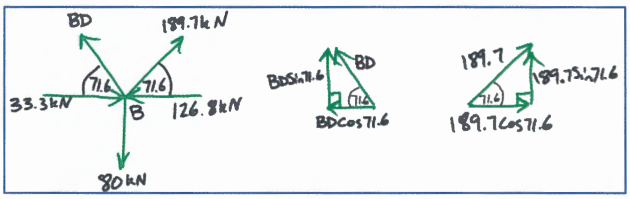
Analyse joint C. Apply sum of forces in the vertical direction first then in the horizontal direction.



$$+\uparrow \Sigma F_v = 0$$
$$-100 + CD\sin 71.6 = 0$$
$$CD = \frac{100}{\sin 71.6} = 105.4\ kN \nearrow$$

$$\overset{\rightarrow}{+} \sum F_H = 0$$
$$-BC + CD\cos 71{,}6 = 0$$
$$BC = 105.4\cos 71.6 = 33.3 \text{ kN} \leftarrow$$

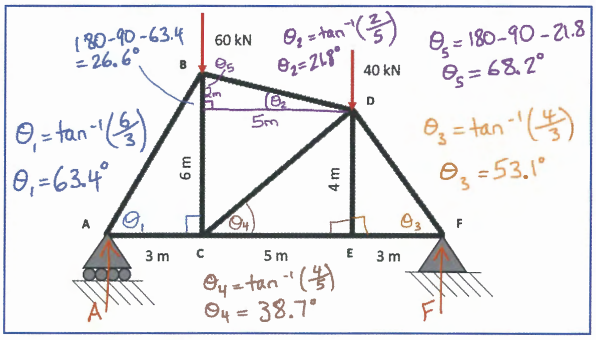
Analyse joint B. Apply sum of forces in the vertical direction to find BD.



$$+\uparrow \Sigma F_v = 0$$
$$BD\sin 71.6 + 189.7\sin 71.6 - 80 = 0$$
$$BD = \frac{-100}{\sin 71..6} = -105.4kN$$
$$\therefore BD = 105.4kN \searrow$$

Ensure that the nature of forces are applied to all other joints. For example, when analysing joint A, AE was found to be in compression. So when analysing joint E, AE must also be acting in compression.

18.



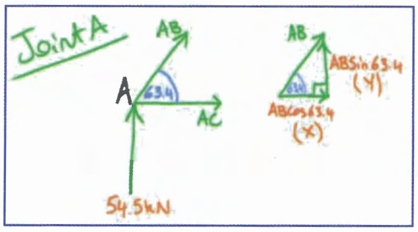
Note: There are no other external horizontal forces so the horizontal force at the pin joint would equal to 0.

First, find the reaction forces so apply sum of moments about F to find the force at the roller joint. Apply sum of forces in the vertical direction to find the remaining reaction force at the pin joint.

$$+\circlearrowleft \Sigma M_F = 0$$
$$-(A \times 11) + (60 \times 8) + (40 \times 3) = 0$$
$$A = \frac{600}{11} = 54.5\,kN\ \uparrow$$

$$+\uparrow \Sigma F_v = 0$$
$$A - 60 - 40 + F = 0$$
$$F = 60 + 40 - 54.5$$
$$\therefore F = 45.5kN \uparrow$$

We now need to analyse each joint. First, start with joint A and draw the FBD.

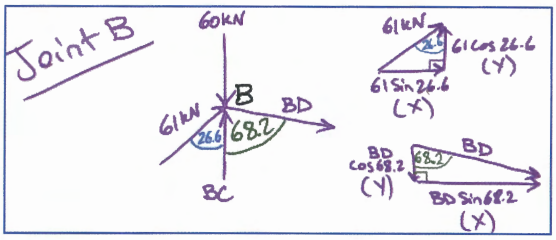


Apply sum of forces in the vertical direction first as there is only one unknown then apply sum of forces in the horizontal direction.

$$+\uparrow \Sigma F_v = 0$$
$$54.5 + AB\sin 63.4 = 0$$
$$AB = \frac{-54.5}{\sin 63.4} = -61\,kN$$
$$\therefore AB = 61\,kN \swarrow$$

$$\overrightarrow{+}\Sigma F_H = 0$$
$$AC - AB\cos 63.4 = 0$$
$$AC = 61\cos 63.4 = 27.3\text{ kN} \rightarrow$$

Note: AB direction has changed so the horizontal component is now directed to the left instead of to the right. AC is in tension and AB is in compression which must stay the same for other joints.

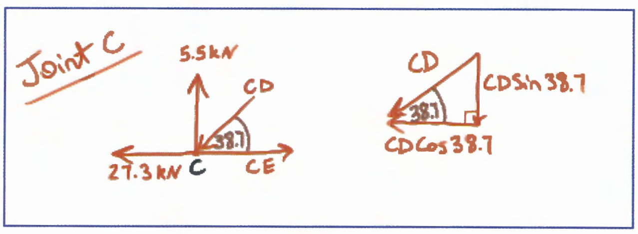


Apply sum of forces in the horizontal direction first as there is only one unknown then apply sum of forces in the vertical direction.

$$\overrightarrow{+}\Sigma F_H = 0$$
$$61\sin 26.6 + BD\sin 68.2 = 0$$
$$BD = \frac{-27.31}{\sin 68.2} = -29.4\,kN$$
$$\therefore BD = 29.4\,kN \nwarrow$$

$$+\uparrow \Sigma F_v = 0$$
$$61\cos 26.6 + BC + BD\cos 68.2 - 60 = 0$$
$$BC = -61\cos 26.6 - 29.4\cos 68.2 + 60$$
$$BC = -5.5\,kN$$
$$\therefore BC = 5.5\,kN \downarrow$$

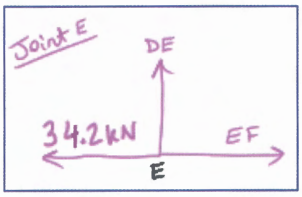
Note: BD direction has changed so the vertical component is now directed upwards. instead of downwards. BC is in tension and BD is in compression which must stay the same for other joints.



Apply sum of forces in the vertical direction first as there is only one unknown then apply sum of forces in the horizontal direction.

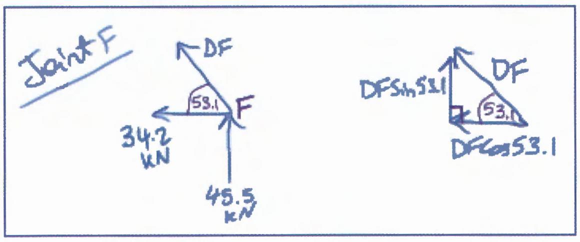
$$+\uparrow \Sigma F_v = 0$$
$$5.5 - CD\sin 38.7 = 0$$
$$CD = \frac{5.5}{\sin 38.7}$$
$$\therefore CD = 8.8\ kN \swarrow$$

$$\overset{+}{\rightarrow} \Sigma F_H = 0$$
$$-27.3 - CD\cos 38.7 + CE = 0$$
$$CE = 27.3 + 8.8\cos 38.7$$
$$\therefore CE = 34.2\ kN \rightarrow$$



$$+\uparrow \Sigma F_v = 0$$
$$DE = 0\ kN$$

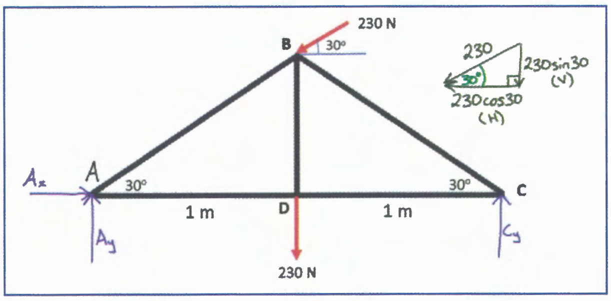
$$\overset{+}{\rightarrow} \Sigma F_H = 0$$
$$-34.2 + EF = 0$$
$$\therefore EF = 34.2\ kN \rightarrow$$



Sum the forces in the vertical direction to find DF.

$$+\uparrow \Sigma F_v = 0$$
$$DF\sin 53.1 + 45.5 = 0$$
$$DF = \frac{-45.5}{\sin 53.1} = -56.9\ kN$$
$$\therefore DF = 56.9\ kN \searrow$$

19.



1. Find reaction forces first by summing moments about either A or C. We will do it about A. Summing moments about A will eliminate the 230N as it goes through point A leaving us with a simple calculation. Alternatively, you could break 230N into its horizontal and vertical components to find moments about A. Doing this will cancel each of the components out.

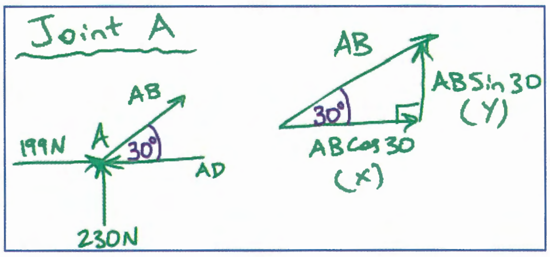
$$+\circlearrowleft \Sigma M_A = 0$$
$$(C_y \times 2) - (230 \times 1) = 0$$
$$C_y = \frac{230}{2}$$
$$\therefore C_y = 115\ N \uparrow$$

2. Sum the forces in both vertical and horizontal directions to find remaining reaction forces.

$$+\uparrow \Sigma F_v = 0$$
$$A_y + C_y - 230 - 230\sin 30 = 0$$
$$A_y = 230 + 230\sin 30 - 115 = 0 + 230$$
$$A_y = 230\ N \uparrow$$

$$\overset{+}{\rightarrow} \Sigma F_H = 0$$
$$A_x - 230\cos 30 = 0$$
$$A_x = 199\ N \rightarrow$$

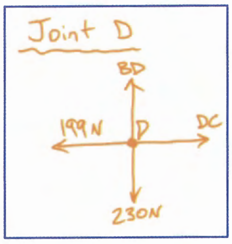
3. Now analyse every joint by drawing its FBD (that is every force attached to the joint) then applying appropriate sum of forces equation according to the number of unknowns we have.



$$+\uparrow \Sigma F_v = 0$$
$$230 + ABSin30 = 0$$
$$AB = \frac{-230}{\sin 30} = 460N$$
$$\therefore AB = 460\ N \swarrow (C)$$

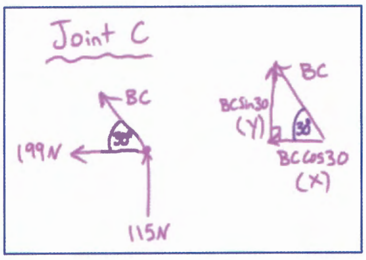
$$\overset{+}{\rightarrow} \Sigma F_H = 0$$
$$199 - AB\cos 30 - AD = 0$$
$$AD = 199 - 460\cos 30 = -199N$$
$$\therefore AD = 199\ N \rightarrow (T)$$

Note: AB's sense has changed so the horizontal component is now directed to the left instead of to the right. AD is in tension and must stay the same for other joints.



$$+\uparrow \Sigma F_v = 0$$
$$BD - 230 = 0$$
$$\therefore BD = 230\ N \uparrow (T)$$

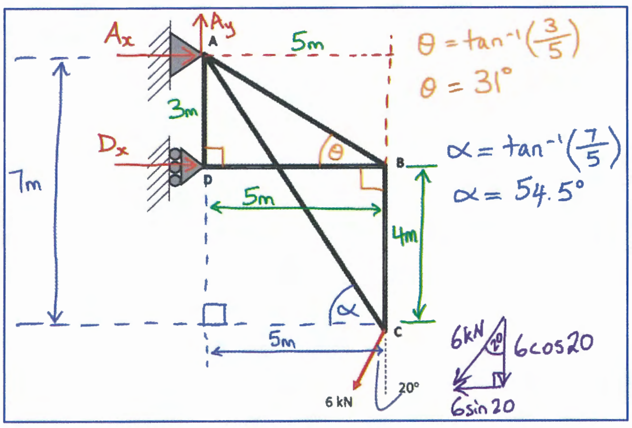
$$\overrightarrow{+} \Sigma F_H = 0$$
$$-199 + DC = 0$$
$$DC = 199\ N \rightarrow (T)$$



$$+\uparrow \Sigma F_v = 0$$
$$115 + BC\sin 30 = 0$$
$$BC = \frac{-115}{\sin 30} = -230N$$
$$\therefore BC = 230\ N \searrow (C)$$

$$\therefore AB = 460\ N\ (C)$$
$$\therefore AD = 199\ N\ (T)$$
$$\therefore BC = 230\ N\ (C)$$
$$\therefore DC = 199\ N\ (T)$$

20.



Sum the moments about A to find the force at the roller joint. Keep in mind that moment equals force multiplied by the perpendicular distance to joint A. Force at C needs to be broken down into its horizontal (x) and vertical (y) components. The perpendicular distance to A for the x component is 4 + 3 m and the y component has a perpendicular distance of 5 m.

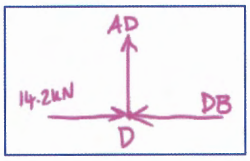
$$+\circlearrowleft \Sigma M_A = 0$$
$$(D_x \times 3) - (6 \times \cos 20 \times 5) - (6 \times \sin 20 \times 7) = 0$$
$$D_x = \frac{42.56}{3} = 14.2\ kN\ \rightarrow$$

Apply sum of forces in the x and y directions to find the remaining reaction forces.

$$\overrightarrow{+} \Sigma F_x = 0$$
$$A_x + D_x - 6\sin 20 = 0$$
$$A_x = 6\sin 20 - 14.2$$
$$A_x = -12.1kN$$
$$\therefore A_x = 12.1\ kN \leftarrow$$

$$+\uparrow \Sigma F_y = 0$$
$$A_y - 6\cos 20 = 0$$
$$A_y = 5.6\ kN\ \uparrow$$

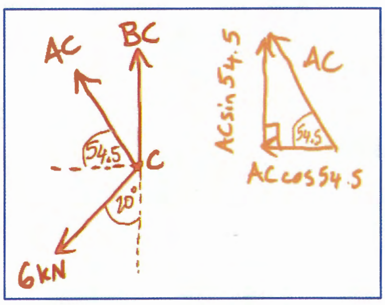
Analyse joint D and draw FBD.



$$\overrightarrow{+} \Sigma F_H = 0$$
$$14.2 - DB = 0$$
$$DB = 14.2kN \leftarrow$$

$$+\uparrow \Sigma F_v = 0$$
$$AD = 0$$

Analyse joint C and draw FBD. Break up AC into its vertical and horizontal components.



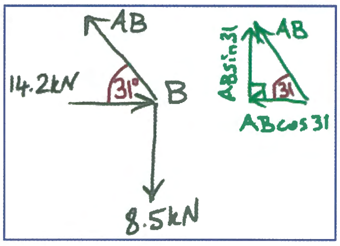
$$\overrightarrow{+}\sum F_H = 0$$
$$-AC\cos 54.5 - 6\sin 20 = 0$$
$$AC = \frac{-6\sin 20}{\cos 54.5} = -3.5\ kN$$
$$\therefore AC = 3.5\ kN \searrow$$

Note: AC direction has changed so the vertical component is now directed in the opposite direction. Hence it is now negative.

$$+\uparrow \Sigma F_v = 0$$
$$-AC\sin 54.5 + BC - 6\cos 20 = 0$$
$$BC = 6\cos 20 + 3.5\sin 54.5$$
$$\therefore BC = 8.5\ kN \uparrow$$

BC is in tension when analysing joint C. Hence it must also be in tension when analysing joint B. DB is in compression when analysing joint D. Hence it must also be in compression when analysing joint B.

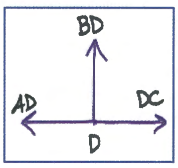
Analyse joint B and draw FBD and sum the forces in the vertical direction.



$$+\uparrow \sum F_v = 0$$
$$AB\sin 31 - 8.5 = 0$$
$$AB = \frac{8.5}{\sin 31}$$
$$\therefore AB = 16.5kN \nwarrow$$

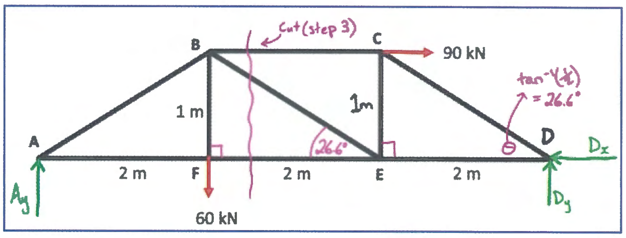
21.

Analyse joint D. Draw FBD and sum the forces in the vertical direction.



$$+\uparrow \Sigma F_v = 0$$
$$BD = 0$$

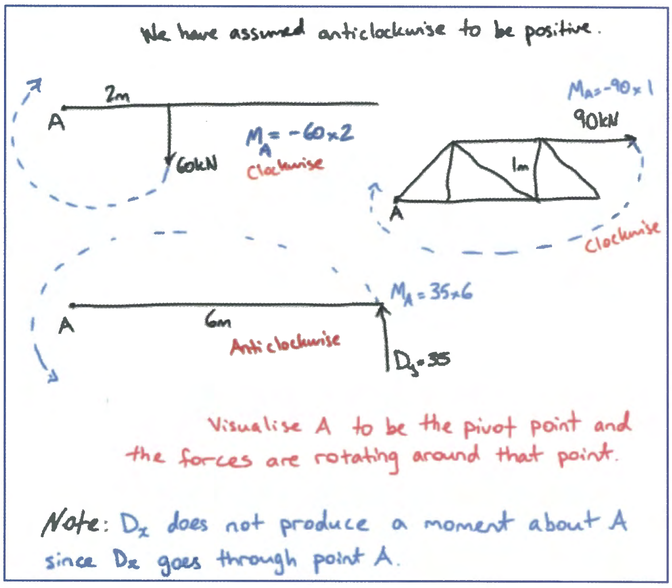
22.



1. Determine the reaction forces by applying sum of moments about either A or D.

$$+\circlearrowleft \sum M_A = 0$$
$$(-60 \times 2) - (90 \times 1) + D_y(6) = 0$$
$$6D_y = 210$$
$$D_y = \frac{210}{6} = 35\text{kN} \uparrow$$

The sketch below will explain how we get the moment's direction



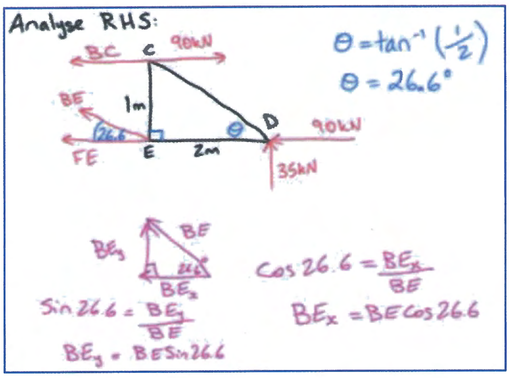
2. Find the remaining reaction forces by applying sum of forces in both the vertical (y) and horizontal (x) direction.

$$+\uparrow \Sigma F_v = 0$$
$$A_y - 60 + D_y = 0$$
$$A_y - 60 + 35 = 0$$
$$A_y = 25\ \text{kN} \uparrow$$

$$\overrightarrow{+}\Sigma F_H = 0$$
$$90 - D_x = 0$$
$$D_x = 90\ \text{kN} \leftarrow$$

Now that we have found the reaction forces, we can go on and apply the method of sections to find the 3 forces in question.

3. Make a cut through the members in question (BC, BE and FE) as seen in the top sketch. These members when cut become forces. Choose either the left side of the cut or the right side and draw the FBD.



4. Summing moments about E will eliminate BE, FE and 90kN force as they all go through point E.

$$+\circlearrowleft \sum M_E = 0$$
$$BC(1) - (90 \times 1) + (35 \times 2) = 0$$
$$BC = 20\ kN \leftarrow$$

5. To find BE and FE, we sum the forces in the vertical direction as we have 1 unknown force in that direction as opposed to 2 in the horizontal direction. Once you do that, we then sum the forces in the horizontal direction.

$$+\uparrow \Sigma F_v = 0$$
$$35 + BESin26.6 = 0$$
$$BE = \frac{-35}{\sin 26.6} = -78\ kN$$
$$\therefore BE = 78\ kN \searrow$$

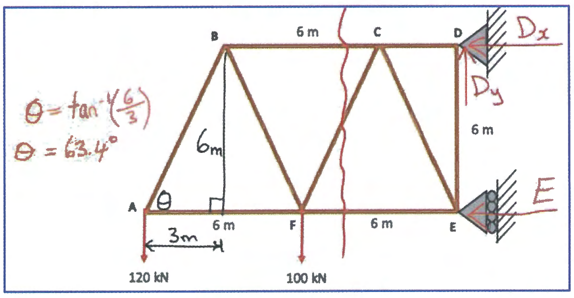
$$\overrightarrow{+} \Sigma F_H = 0$$
$$-BC - FE - BE\cos 26.6 + 90 - 90 = 0$$
$$-20 - FE - -78\cos 26.6 = 0$$
$$FE = -20 + 78\cos 26.6$$
$$\therefore FE = 50\ kN \leftarrow$$

Note: You could either substitute the negative value of BE into the equation or you can adjust its sense and use the new horizontal component which would be positive. Both methods give the same results.

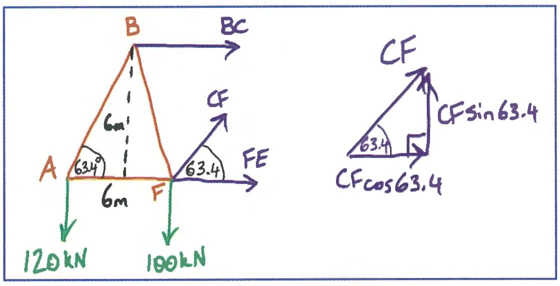
6. The negative value of BE means we assumed its sense incorrectly. Hence, we flip its sense so now BE is directed towards the joint. Therefore, BE is in COMPRESSION, FE is directed away from the joint so it is in TENSION and BC is in TENSION.

$$\therefore BC = 20\ kN(T)$$
$$\therefore BE = 78\ kN(C)$$
$$\therefore FE = 50\ kN(T)$$

23.



Make a cut through members BC, CF and FE to find the force in these members. We will consider the left side of the cut and so the reaction forces are not needed. Alternatively, you can analyse the right side which will require the reaction forces to be found first. The cut members become forces.



Sum moments about F. This will remove the 100 kN force, CF and FE leaving BC as the only unknown force.

$$+\circlearrowleft \Sigma M_F = 0$$
$$(120 \times 6) - (BC \times 6) = 0$$
$$BC = \frac{720}{6}$$
$$\therefore BC = 120\ kN \rightarrow (T)$$

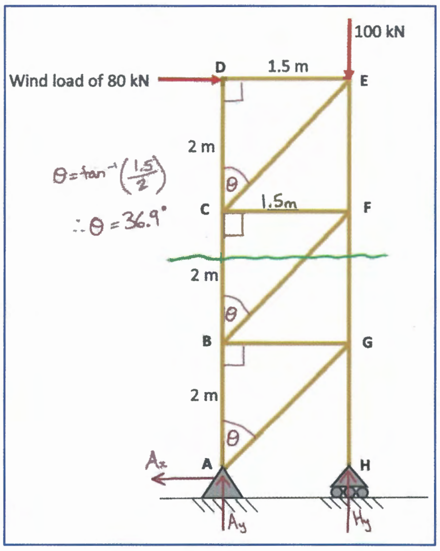
Sum the forces in the vertical direction. This is done due to 1 unknown force in the vertical direction as opposed to 2 in the horizontal direction.

$$+\uparrow \sum F_v = 0$$
$$-120 - 100 + CF\sin 63.4 = 0$$
$$CF = \frac{220}{\sin 63.4}$$
$$\therefore CF = 246\ kN \nearrow$$

Apply sum of forces in the horizontal direction to find the remaining unknown force.

$$\overset{\rightarrow}{+}\Sigma F_H = 0$$
$$BC + CF\cos 63.4 + FE = 0$$
$$120 + (246 \times \cos 63.4) + FE = 0$$
$$FE = -230.2kN$$
$$\therefore FE = 230.2\ kN \leftarrow$$

24.



1. Apply sum of moments about either A or H to find one of the reaction forces.

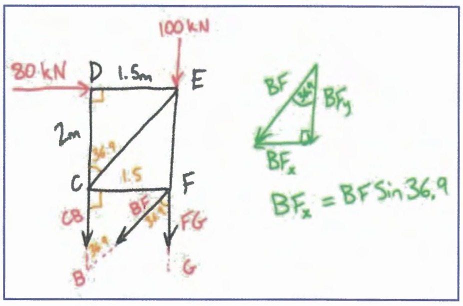
$$+\circlearrowleft \Sigma M_A = 0$$
$$H_y(1.5) - (100 \times 1.5) - (80 \times 6) = 0$$
$$H_y = \frac{630}{1.5}$$
$$\therefore H_y = 420\ kN \uparrow$$

2. Apply sum of forces in both the vertical and horizontal directions to find remaining reaction forces.

$$+\uparrow \Sigma F_v = 0$$
$$A_y + H_y - 100 = 0$$
$$A_y = 100 - 420$$
$$\therefore A_y = -320kN$$
$$\therefore A_y = 320\ kN \downarrow$$

$$\overset{\rightarrow}{+}\Sigma F_H = 0$$
$$80 - A_x = 0$$
$$A_x = 80\ kN \leftarrow$$

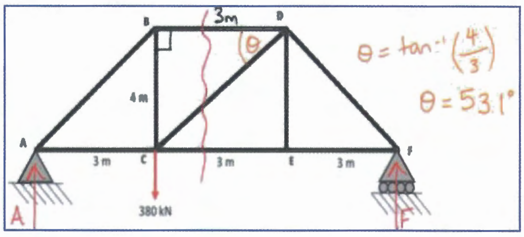
3. Make a cut through member BF and analyse the side above the cut. Draw FBD.



4. Apply sum of forces in the horizontal direction (x)as we only have 1 unknown force (BF) in that direction.

$$\overset{\rightarrow}{+}\Sigma F_H = 0$$
$$80 - BFSin36.9 = 0$$
$$BF = \frac{80}{\sin 36.9}$$
$$\therefore BF = 133\ kN \swarrow (T)$$

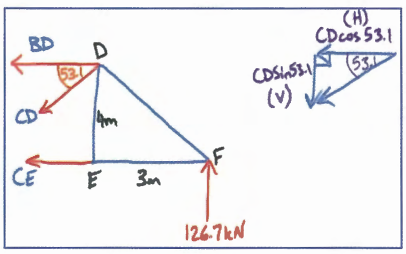
25.



Sum moments about A to find the force at the roller joint. No external horizontal forces so horizontal force acting on pin joint is 0 (not shown above).

$$+\circlearrowleft \Sigma M_A = 0$$
$$(-380 \times 3) + (F \times 9) = 0$$
$$F = \frac{114}{9}$$
$$\therefore F = 126.7\ kN \uparrow$$

Cut through members BD, CD and CE to find the force in these members. Consider the right side of the cut so the force at the pin joint is not required. Alternatively, you can analyse the left side which will require the pin reaction force to be found first. The cut members become forces.



Apply sum of moments about D. This will remove BD and CD leaving CE as the only unknown force.

$$+\circlearrowleft \sum M_D = 0$$
$$(-CE \times 4) + (126.7 \times 3) = 0$$
$$CE = \frac{380.1}{4}$$
$$\therefore CE = 95\ kN \leftarrow$$

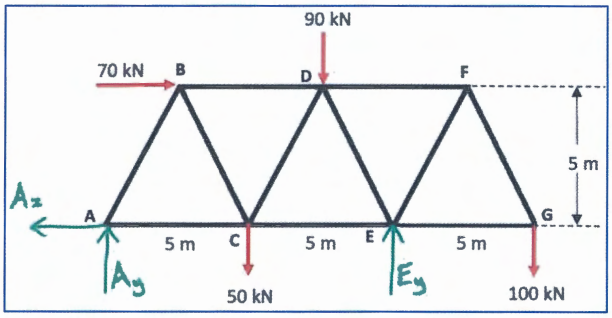
Sum the forces in the vertical direction. This is done due to having only one unknown force in the vertical direction as opposed to 2 in the horizontal direction.

$$+\uparrow \sum F_v = 0$$
$$-CD\sin 53.1 + 126.7 = 0$$
$$CD = \frac{126.7}{\sin 53.1}$$
$$\therefore CD = 158.4\ kN \swarrow$$

Apply sum of forces in the horizontal direction to find the remaining unknown force.

$$\overset{\rightarrow}{+} \sum F_H = 0$$
$$-BD - CE - CD\cos 53 - 1 = 0$$
$$BD = -95 - 158.4\cos 53.1 = -190.1 kN$$
$$\therefore BD = 190.1\ kN \rightarrow$$

26.



1. Apply sum of moments about either A or E to find one of the reaction forces.

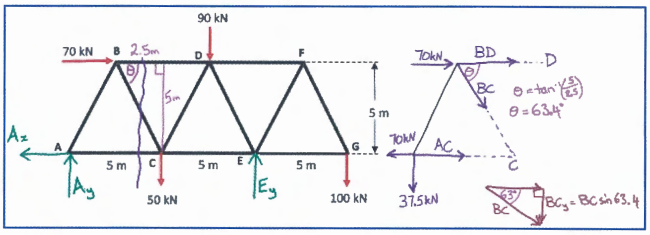
$$+\circlearrowleft \sum M_A = 0$$
$$(-70 \times 5) - (90 \times 7.5) - (50 \times 5)$$
$$-(100 \times 15) + (E_y \times 10) = 0$$

$$E_y = \frac{2775}{10} = 277.5\ kN \uparrow$$

2. Sum the forces in both the vertical and horizontal directions to find remaining reaction forces.

$$+\uparrow \Sigma F_v = 0$$
$$A_y - 50 + E_y - 100 - 90 = 0$$
$$A_y = 90 + 100 + 50 - 277.5$$
$$A_y = -37.5\ kN$$
$$\therefore A_y = 37.5\ kN \downarrow$$

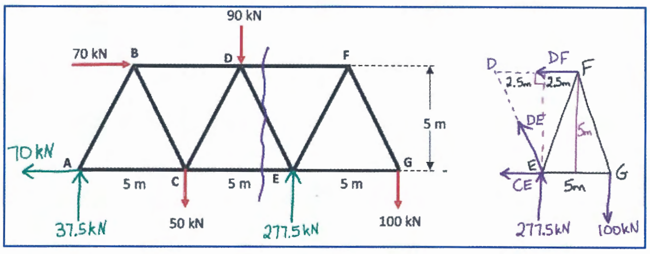
$$\overset{\rightarrow}{+} \sum F_H = 0$$
$$70 - A_x = 0$$
$$A_x = 70\ kN \leftarrow$$



3. Make a cut through member BC (shown above) to determine the force in BC. Analyse left of the cut and draw FBD (shown above).
4. When determining how many unknowns we have, we can see that we have one unknown in the vertical direction hence sum of forces in the vertical direction should be employed.

$$+\uparrow \sum F_v = 0$$
$$-37.5 - BCSin63.4 = 0$$
$$BC = \frac{-37.5}{\sin 63 \cdot 4} = -41.9 kN$$
$$\therefore BC = 41.9\ kN \nwarrow$$

5. Cut through member DF and CE to find the force in these members. Analyse right side of cut and draw FBD.



6. Apply sum of moments about point E which will eliminate the 2 unknowns DE and CE as well as the 277.5 kN force leaving us with DF.

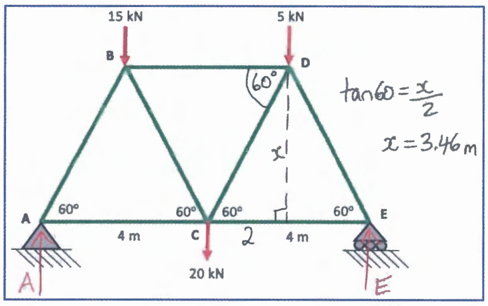
$$+\circlearrowleft \sum M_E = 0$$
$$(DF \times 5) - (100 \times 5) = 0$$
$$DF = \frac{500}{5} = 100\ kN\ \leftarrow$$

7. Sum the moments about point D to eliminate DE and DF leaving us with CE as the only unknown. Alternatively, you could find DE first by summing the vertical forces and then the horizontal forces to find CE. Albeit a longer process.

$$+\circlearrowleft \Sigma M_D = 0$$
$$(-CE \times 5) + (277 \cdot 5 \times 2.5) - (100 \times 7.5) = 0$$
$$CE = -\frac{56.25}{5} = -11.25$$
$$\therefore CE = 11.25\ kN \rightarrow$$

$$\therefore BC = 41.9\ kN(C)$$
$$\therefore DF = 100\ kN(T)$$
$$\therefore CE = 11.25\ kN(C)$$

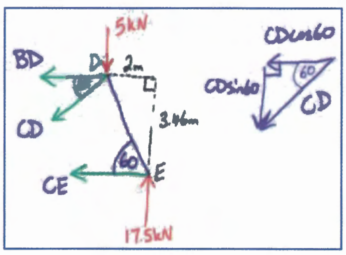
27.



Find reaction forces by summing the moments about A to find the force at the roller joint. No external horizontal forces so the horizontal force acting on the pin joint is 0 hence not shown above.

$$+\circlearrowleft \sum M_A = 0$$
$$(-15 \times 2) - (5 \times 6) - (20 \times 4) + (E \times 8) = 0$$
$$E = \frac{140}{8}$$
$$\therefore E = 17.5\ kN$$

Make a cut through members BD, CD and CE to find the force in these members. We will consider the right side of the cut and so the force at the pin joint is not required. Alternatively, you can analyse the left side which will require the pin reaction force to be found first. The cut members become forces.



Apply sum of moments about D. This will remove BD and CD leaving CE as the only unknown force.

$$+\circlearrowleft \Sigma M_D = 0$$
$$(17.5 \times 2) - (CE \times 3.46) = 0$$
$$CE = \frac{35}{3.46}$$
$$\therefore CE = 10.1\ kN$$

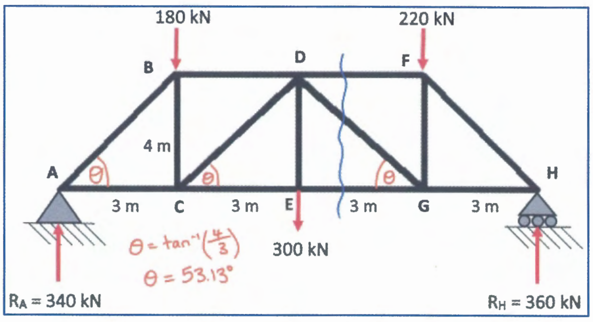
Sum the forces in the vertical direction. This is done due to having only one unknown force in the vertical direction as opposed to 2 in the horizontal direction.

$$+\uparrow \Sigma F_v = 0$$
$$-CD\sin 60 + 17.5 - 5 = 0$$
$$CD = \frac{12.5}{\sin 60}$$
$$\therefore CD = 14.4\ kN \swarrow$$

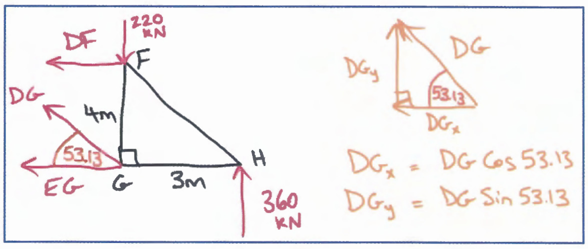
Apply sum of forces in the horizontal direction to find the remaining unknown force.

$$\overrightarrow{+} \sum F_H = 0$$
$$-BD - CD\cos 60 - CE = 0$$
$$BD = -14.4\cos 60 - 10.1 = -17.3kN$$
$$\therefore BD = 17.3\ kN \rightarrow$$

28.



1. Make a cut through members DF, DG, EG and analyse RHS by drawing the FBD.



2. Apply sum of moments about G to find DF then apply sum of forces in the vertical direction to find DG and horizontal direction to find EG.

$$+\circlearrowleft \Sigma M_G = 0$$
$$DF(4) + (360 \times 3) = 0$$
$$DF = -\frac{1080}{4} = -270 \text{ kN}$$
$$\therefore DF = 270\ kN \rightarrow \text{(C)}$$

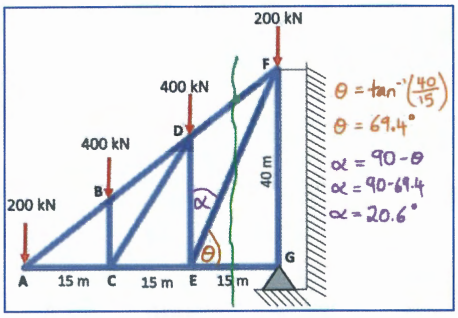
$$+\uparrow \sum F_v = 0$$
$$DGsin53.13 + 360 - 220 = 0$$
$$DG = \frac{-140}{\sin 53.13} = -175\ kN$$
$$\therefore DG = 175 \text{ kN} \searrow \text{(C)}$$

$$\overrightarrow{+} \Sigma F_H = 0$$
$$270 + DGcos53.13 - EG = 0$$
$$270 + 175cos53.13 - EG = 0$$
$$\therefore EG = 375\ kN\ \leftarrow \text{(T)}$$

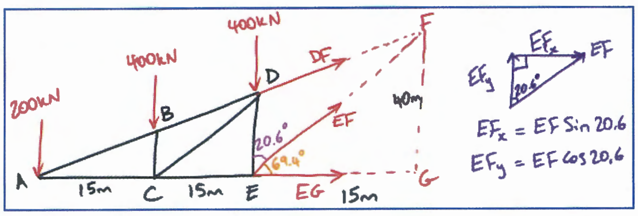
Note: Due to DG being negative, its sense was flipped hence its horizontal component is now positive.

$$\therefore DF = 270\ \ kN(C)$$
$$\therefore DG = 175\ kN(C)$$
$$\therefore EG = 375\ kN(T)$$

29.



1. Cut through members DF, EF and EG. Consider LHS for analysis. Calculate theta and alpha as these will be needed to carry out the calculations.

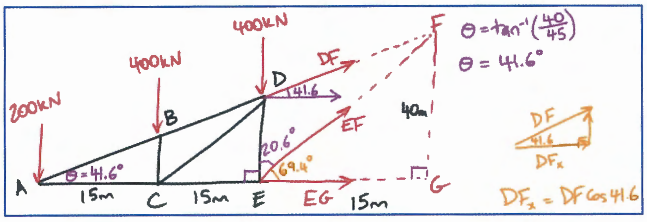


2. Summing the moments about A will eradicate EG, DF, 200kN force and the horizontal component of EF as they all go through point A. This leaves us with 1 unknown, EF.

$$+\circlearrowleft \sum M_A = 0$$
$$(-400 \times 15) - (400 \times 30) + (EF\cos 20.6 \times 30) = 0$$
$$30EF\cos 20.6 = 18000$$
$$EF = 641\ kN \nearrow \text{(T)}$$

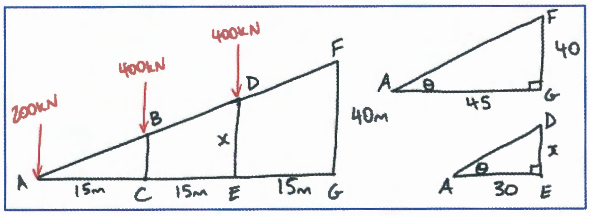
3. Apply sum of moments about F. This will eliminate DF and EF leaving EG as the only unknown force.

$$+\circlearrowleft \sum M_F = 0$$
$$(EG \times 40) + (400 \times 15) + (400 \times 30) + (200 \times 45) = 0$$
$$40EG = -27000$$
$$\therefore EG = -675kN$$
$$\therefore EG = 675\ kN \leftarrow \text{(C)}$$



Note: Angle between DF and horizontal component of DF is equal to the theta angles as these angles are corresponding angles, hence they are equal.

4. Sum the moments about E. This eliminates EG, EF and the vertical component of DF leaving DF as the only unknown force. But first we need to find the length of member DE by using similar triangles.

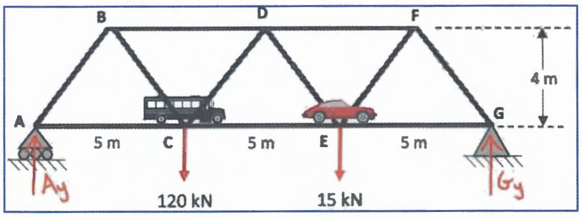


$$\frac{x}{40} = \frac{30}{45}$$
$$45x = 1200$$
$$x = 26.67\ m$$
$$\therefore DE = 26.67\ m$$

$$+\circlearrowleft \sum M_E = 0$$
$$(-DF\cos 41.6 \times 26.67) + (400 \times 15) + (200 \times 30) = 0$$
$$DF = \frac{12000}{26.67\cos 41.6}$$
$$\therefore DF = 602\ kN \nearrow \text{(T)}$$

$$\therefore DF = 602\ kN(T)$$
$$\therefore EG = 675\ kN(C)$$
$$\therefore EF = 641\ kN(T)$$

30.



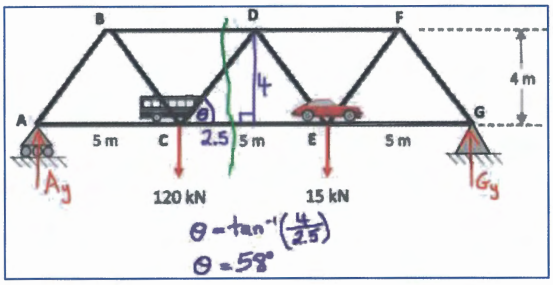
Note: Pin joint will only have a vertical reaction force due to no external horizontal forces.

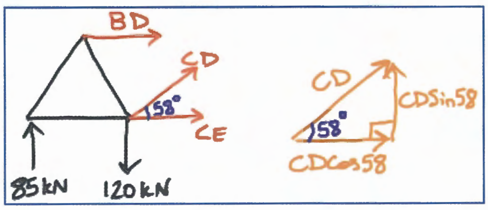
Apply moments about A to find G_y and then sum the forces in the vertical direction to find A_y.

$$+\circlearrowleft \sum M_A = 0$$
$$-(120 \times 5) - (15 \times 10) + G_y(15) = 0$$
$$G_y = \frac{750}{15} = 50\ kN\ \uparrow$$

$$+\uparrow \sum F_v = 0$$
$$A_y - 120 - 15 + G, = 0$$
$$A_y = 120 + 15 - 50 = 85\text{ kN}\ \uparrow$$

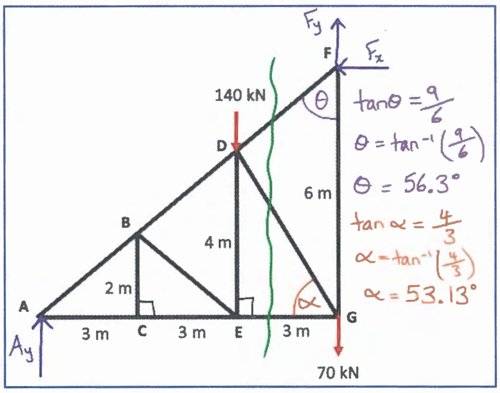
Make a cut through member CD and analyse left side of cut. Draw FBD. Apply sum of forces in the vertical direction since BD and CE have no vertical component. Thus, we only have the vertical component of CD as the only unknown.





$$+\uparrow \Sigma F_v = 0$$
$$85 - 120 + CD\sin 58 = 0$$
$$CD = \frac{35}{\sin 58} = 41\ kN \nearrow$$

31.



1. Apply sum of moments about F to find A_y. F_x, F_y and the 70 kN force are eliminated as they go through point F.

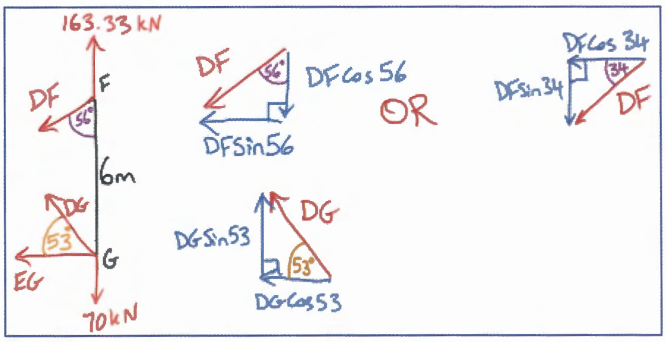
$$+\circlearrowleft \sum M_F = 0$$
$$-A_y(9) + (140 \times 3) = 0$$
$$A_y = \frac{420}{9}$$
$$\therefore A_y = 46..67\ kN \uparrow$$

2. F_x = 0 due to no other external horizontal force. Sum the forces in the vertical direction to find F_y.

$$+\uparrow \Sigma F_v = 0$$
$$F_y + A_y - 140 - 70 = 0$$
$$F_y + 46.67 - 140 - 70 = 0$$
$$F_y = 163.33\ kN \uparrow$$

Therefore, roller reaction is 46.67 kN and the pin reaction is 163.33 kN.

3. Make a cut through member DF, DG and EG (as shown above). Analyse the right side of the cut and draw FBD.



4. Apply sum of moments about G. This eliminates EG, DG, 70 kN and 163.33 kN force and the vertical component of DF as they all go through point G.

$$+\circlearrowleft \sum M_G = 0$$
$$(DF\sin 56 \times 6) = 0$$
$$DF = 0$$

5. Sum the forces in the vertical direction and then the horizontal direction.

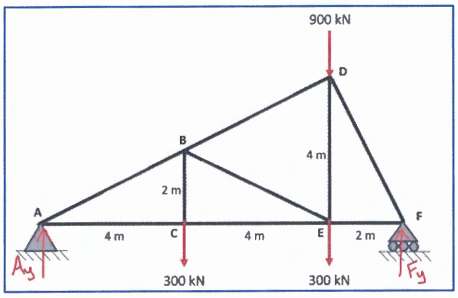
$$+\uparrow \Sigma F_v = 0$$
$$-70 + DG\sin 53 + 163.33 = 0$$
$$DG = \frac{-93.33}{\sin 53} = -117kN$$
$$\therefore DG = 117\ kN \searrow$$

$$\overset{+}{\rightarrow} \sum F_H = 0$$
$$-EG + DG\cos 53 = 0$$
$$EG = 117 \times \cos 53$$
$$\therefore EG = 70\ kN \leftarrow$$

Note: Horizontal component of DG is now directed to the right. The negative value tells us that we assumed the sense incorrectly hence we flip the vector.

$$\therefore DF = 0$$
$$\therefore DG = 117\ kN \searrow$$
$$\therefore EG = 70\ kN \leftarrow$$

32.

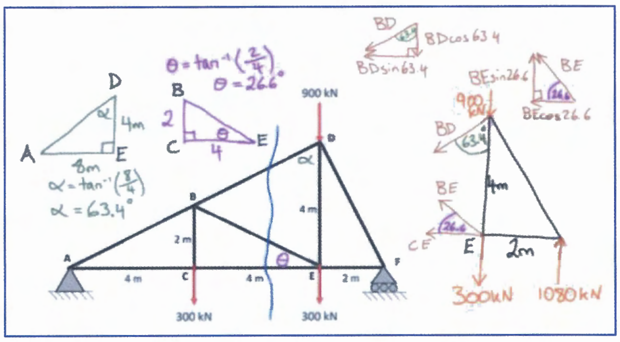


Since there are no external horizontal forces, only a vertical reaction force acts at pin joint A. Find the reaction forces, apply moments about A to find the force at roller joint. Then apply the sum of forces in the vertical direction to find the force at the pin joint.

$$+\circlearrowleft \sum M_A = 0$$
$$-(300 \times 4) - (300 \times 8) - (900 \times 8) + F_y(10) = 0$$
$$F_y = \frac{10800}{10}$$
$$\therefore F_y = 1080\ kN \uparrow$$

$$+\uparrow \Sigma F_y = 0$$
$$A_y - 300 - 300 - 900 + F_y = 0$$
$$A_y = 300 + 300 + 900 - 1080$$
$$\therefore A_y = 420\ kN \uparrow$$

Make a cut through members BD, BE and CE and analyse the right side of the cut. Draw FBD shown below and apply sum of moments about E. This will remove BE, CE, 300 kN force, 900 kN force and the vertical component of BD as they all go through point E.



$$+\circlearrowleft \Sigma M_E = 0$$
$$(BD\sin 63.4 \times 4) + (1080 \times 2) = 0$$
$$BD = \frac{-2160}{4\sin 63.4} = -604\ kN$$
$$\therefore BD = 604\ kN \nearrow$$

Apply sum of forces in vertical direction as there is now only 1 unknown force in that direction.

Note: We assumed direction of BD incorrectly, so the vertical component is now going up instead of down and the horizontal component is directed towards the right instead of to the left. This is reflected in the calculations below.

$$+\uparrow \Sigma F_v = 0$$
$$BD\cos 63.4 + BE\sin 26.6 - 300 + 1080 - 900 = 0$$
$$BE = \frac{300 - 1080 - (604 \times \cos 63.4) + 900}{\sin 26.6} = -336kN$$
$$\therefore BE = 336kN \searrow$$

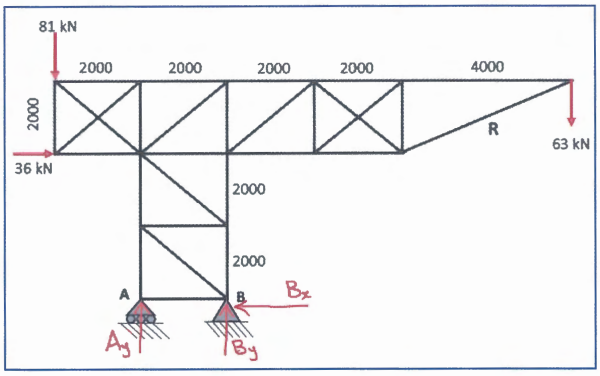
Apply sum of forces in horizontal direction as there is now only 1 unknown force in that direction.

Note: We assumed direction of BE incorrectly, so the horizontal component is now going to the right instead of to the left. This is reflected in the calculation below.

$$\overset{+}{\rightarrow} \sum F_H = 0$$
$$BD\sin 63.4 + BE\cos 26.6 - CE = 0$$
$$CE = 604\sin 63.4 + 2346\cos 26.6$$

$$\therefore CE = 2638\ kN \leftarrow$$

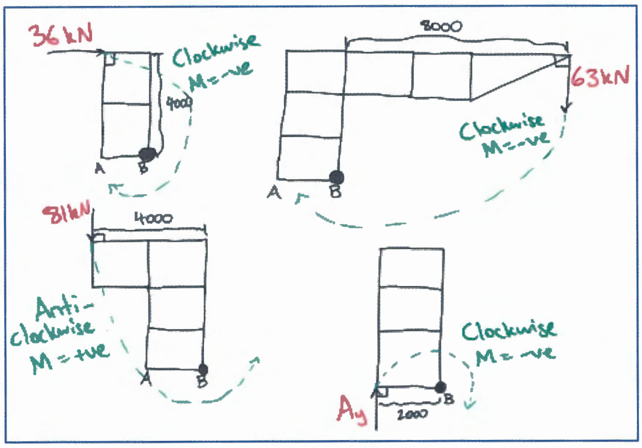
33. a)



1. Apply sum of moments about either A or B to find one of the reaction forces. Measurements with no units are always in mm. They are converted to metres below.

$$+\circlearrowleft \Sigma M_B = 0$$
$$(81 \times 4) - (36 \times 4) - (63 \times 8) - A_y(2) = 0$$
$$A_y = \frac{-324}{2} = -162\ kN$$
$$\therefore A_y = 162\ kN \downarrow$$

Note: Moment = force × perpendicular distance to B. The sketch below provides further clarification on the moments produced about B.

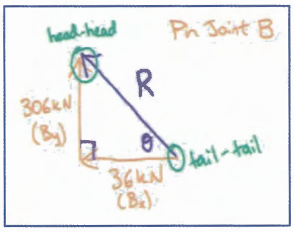


2. Apply sum of forces in both vertical and horizontal directions to find the remaining reaction forces.

$$+\uparrow \Sigma F_v = 0$$
$$-81 - 63 - 162 + B_y = 0$$
$$B_y = 306\ kN \uparrow$$

$$\overset{\rightarrow}{+} \Sigma F_H = 0$$
$$36 - B_x = 0$$
$$B_x = 36\ kN \leftarrow$$

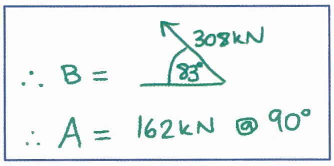
3. To find the reaction force at the pin joint B, we need to find the resultant force. To do this, we draw a vector triangle and apply Pythagoras. We then use trigonometry to find the direction (i.e. the angle).



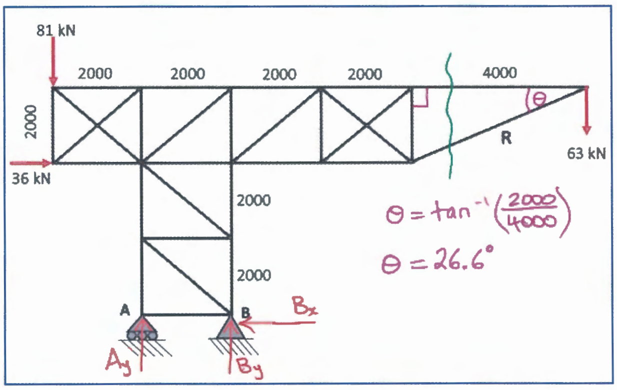
Note: When drawing vectors, the resultant needs to have a head-head and a tail-tail connection. The other 2 vectors need to be connected head-tail.

$$R = \sqrt{306^2 + 36^2}$$
$$R = 308\ kN$$

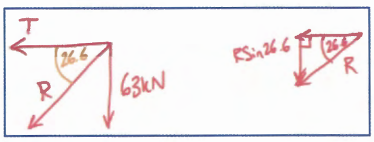
$$\theta = tan^{-1}\left(\frac{306}{36}\right)$$
$$\theta = 83^o$$



b)



1. Make a cut through member R and analyse the right side of the cut. Draw FBD. This then becomes a method of joints Q.

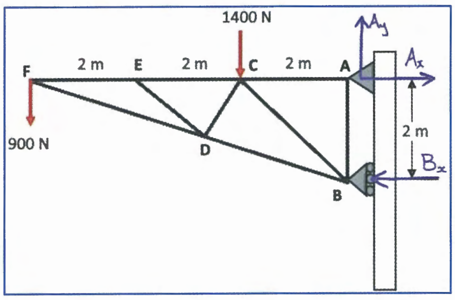


2. Now decide which sum of forces equation we need to use. Here we have two unknown forces in the x-direction and only one unknown force in the y-direction. Hence we can sum the forces in the y-direction (vertical direction).

$$+\uparrow \Sigma F_v = 0$$
$$-R\sin 26.6 - 63 = 0$$
$$R = \frac{-63}{\sin 26.6} = -141\ kN$$
$$\therefore R = 141\ kN \nearrow (C)$$

Note: Since R provided us with a negative value we flip its sense so its now directed towards the joint which means R will experience compression.

34. a)



Apply sum of moments about A to find the reaction force at support B.

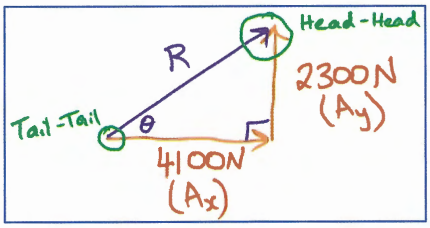
$$+\curvearrowleft \Sigma M_A = 0$$
$$-B_x(2) + (1400 \times 2) + (900 \times 6) = 0$$
$$B_x = \frac{8200}{2} = 4100$$
$$\therefore B_x = 4100\ N \leftarrow$$

b) Apply sum of forces in vertical and horizontal directions to find the reaction forces on A.

$$+\uparrow \Sigma F_v = 0$$
$$A_y - 900 - 1400 = 0$$
$$A_y = 2300\ N \uparrow$$

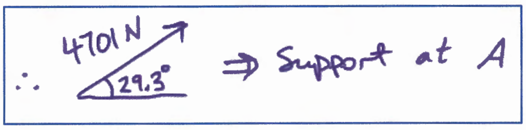
$$\overrightarrow{+} \Sigma F_H = 0$$
$$-4100 + A_x = 0$$
$$A_x = 4100\ N\ \rightarrow$$

Draw up the vector triangle to find the resultant force acting on the pin support A by applying Pythagoras and trigonometry to find its direction.

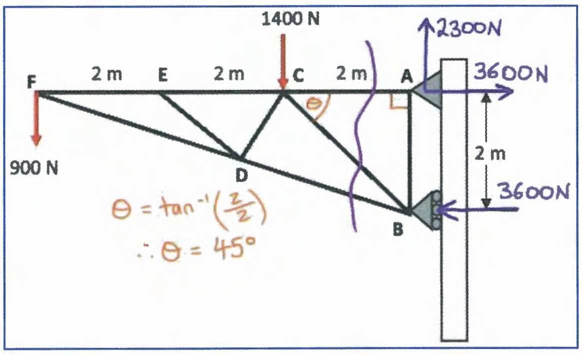


$$R = \sqrt{4100^2 + 2300^2}$$
$$R = 4701\ N$$

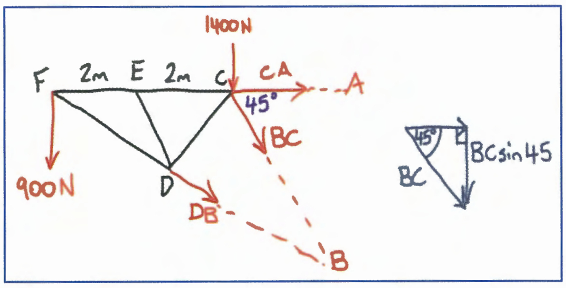
$$\theta = \tan^{-1}\left(\frac{2300}{4100}\right)$$
$$\theta = 29.3^\circ$$



c)



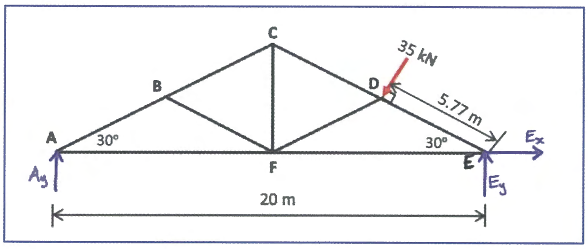
Make a cut through BC and analyse the left side of the cut. Draw the FBD.



Apply sum of moments about F. This will eliminate DB, CA, 900N force and the horizontal component of BC as they all go through point F.

$$+\curvearrowleft \Sigma M_F = 0$$
$$(-1400 \times 4) - (BC\sin 45 \times 4) = 0$$
$$BC = \frac{-5600}{4\sin 45} = -1980\ N$$
$$\therefore BC = 1980\ N \nwarrow (C)$$

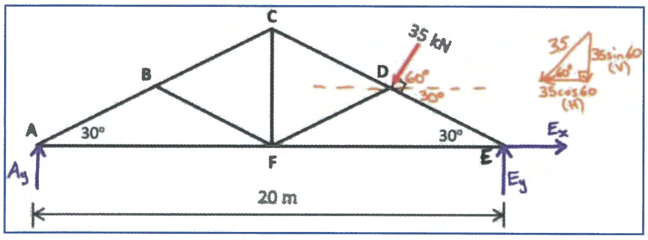
35. a)



1. Apply sum of moments about A or E. In this case, it is much easier to apply moments about E as the perpendicular distance from the 35kN force to E is given to us. Doing this will remove the pin joint forces from the calculation as they go through point E. This will leave us with the force at the roller joint.

$$+\circlearrowleft \sum M_E = 0$$
$$(35 \times 5.77) - Ay(20) = 0$$
$$Ay = \frac{201.95}{20}$$
$$\therefore A_y = 10.1\ kN \uparrow$$

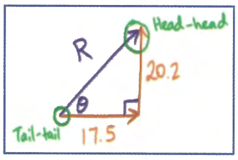
2. Now to find the components of the pin joint forces, we need to sum the forces in both the vertical and horizontal directions. To do this, we need to break the 35 kN force into its x and y components. Create a horizontal line that goes through point D. The angle between this line and DE is 30 due to alternate angles and angle in interest is 60 due to complementary angles. From this we can now easily determine its components.



$$+\uparrow \Sigma F_v = 0$$
$$A_y + E_y - 35\sin 60 = 0$$
$$10.1 + E_y - 35\sin 60 = 0$$
$$\therefore E_y = 20.2\ kN \uparrow$$

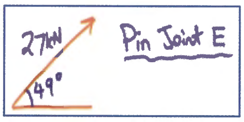
$$\overset{\rightarrow}{+} \sum F_H = 0$$
$$E_x - 35\cos 60 = 0$$
$$E_x = 17.5\ kN \rightarrow$$

3. We now need to draw a vector triangle to find the resultant force acting on the pin joint using Pythagoras and to determine its direction using trigonometry.

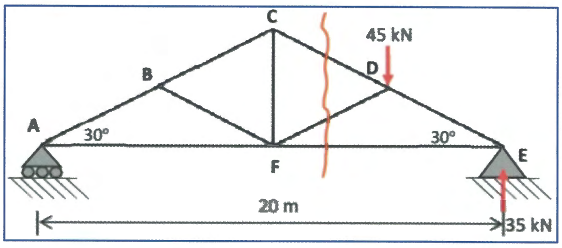


$$R = \sqrt{17.5^2 + 20.2^2}$$
$$R = 27\ kN$$

$$\theta = \tan^{-1}\left(\frac{20.2}{17.5}\right)$$
$$\theta = 49°$$

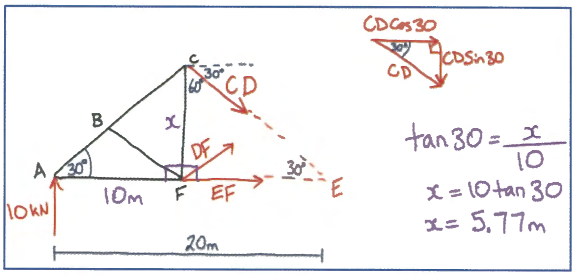


b)



1. Make a cut through member CD. We will analyse the left side of the cut by drawing the FBD. Before doing this, we need to find the force acting on the roller joint.

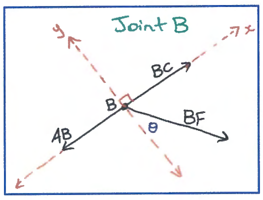
$$+\uparrow \Sigma F_v = 0$$
$$A_y - 45 + 35 = 0$$
$$A_y = 10\ kN \uparrow$$



2. Sum moments about F. This eliminates DF, EF and vertical component of CD as they all go through point F leaving CD as the only unknown force.

$$+\curvearrowleft \Sigma M_F = 0$$
$$(-CD\cos 30 \times 5.77) - (10 \times 10) = 0$$
$$CD = \frac{-100}{5.77\cos 30} = -20\ kN$$
$$\therefore CD = 20\ kN \nwarrow (C)$$

3. To find BF, use method of joints by analysing joint B and drawing the FBD. You will see that BF=0 from observation. This will be clarified below.

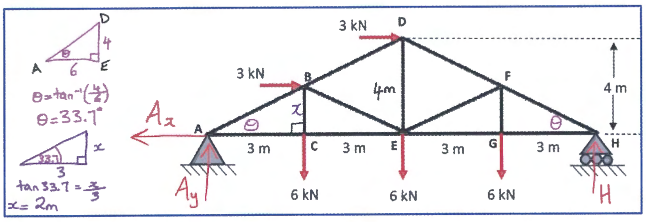


4. Assign a new x-y axis, keeping in mind that the 2 axes intersect at right angles. BC and AB only have x-components and NO y-components. Sum the forces in the y-direction to get the following:

$$+\nwarrow \Sigma F_y = 0$$
$$-BF\cos\theta = 0$$
$$BF = 0$$

Therefore, BF is a redundant member (zero force member) which helps in adding stiffness to the truss.

36. a)



Apply sum of moments about A to find the reaction force at the roller joint. To do this, find the perpendicular distance of the 3kN force by using trig as shown above or you can apply similar triangles to determine BC (faster method).

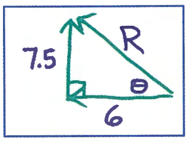
$$+\curvearrowleft \Sigma M_A = 0$$
$$(-3 \times 2) - (6 \times 3) - (6 \times 6) - (6 \times 9)$$
$$-(3 \times 4) + (H \times 12) = 0$$
$$H = \frac{126}{12} = 10.5\ kN\ \uparrow$$

Sum the forces in the vertical direction to find Ay and sum the forces in the horizontal direction to find A_x.

$$+\uparrow \Sigma F_v = 0$$
$$A_y - 6 - 6 - 6 + H = 0$$
$$A_y = 6 + 6 + 6 - 10.5$$
$$\therefore A_y = 7.5kN \uparrow$$

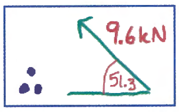
$$\overrightarrow{+} \Sigma F_H = 0$$
$$-A_x + 3 + 3 = 0$$
$$\therefore A_x = 6kN \leftarrow$$

Draw a vector triangle and apply Pythagoras to find the resultant force acting at the pin joint and use trig to find its direction.

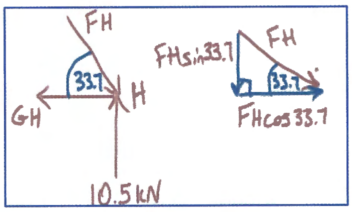


$$R = \sqrt{6^2 + 7.5^2}$$
$$R = 9.6kN$$

$$\theta = \tan^{-1}\left(\frac{7.5}{6}\right)$$
$$\therefore \theta = 51.3^0$$



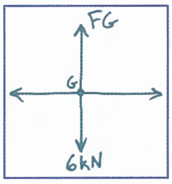
b) Analyse joint H and draw the FBD. Apply the sum of forces equations to find the appropriate member forces. To determine which force equation you need to use, you need to determine the amount of unknown forces in a specific direction in which we want 1 unknown force.



$$+\uparrow \Sigma F_v = 0$$
$$10.5 - FH\sin 33.7 = 0$$
$$FH = \frac{10.5}{\sin 33.7} = 18.9\ kN\ \searrow$$

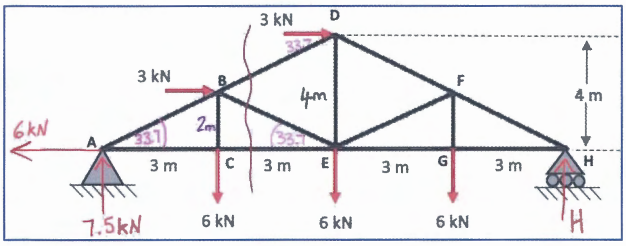
$$\overrightarrow{+} \Sigma F_H = 0$$
$$FH\cos 33.7 - GH = 0$$
$$GH = 18.9\cos 33.7 = 15.7\ \text{kN}\ \leftarrow$$

Analyse joint G and draw FBD.

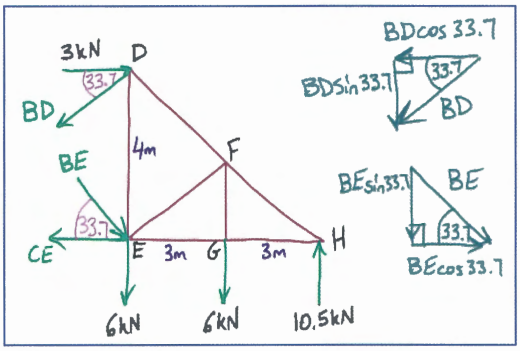


$$+\uparrow \sum F_v = 0$$
$$FG - 6 = 0$$
$$\therefore FG = 6\ kN \uparrow$$

c)



Make a cut through members BD, BE and CE and analyse the right side of the cut. Draw the FBD with the cut members becoming forces.



Summing moments about E will eliminate BE, CE and the 6 kN force leaving BD as the only unknown force.

$$+\circlearrowleft \sum M_E = 0$$
$$(-3 \times 4) + (BD\cos 33.7 \times 4) - (6 \times 3) + (10.5 \times 6) = 0$$
$$BD = \frac{-33}{4\cos 33.7} = -9.9kN$$
$$\therefore BD = 9.9\ kN \nearrow$$

Apply sum of forces in the vertical direction as there is only 1 unknown force in that direction.

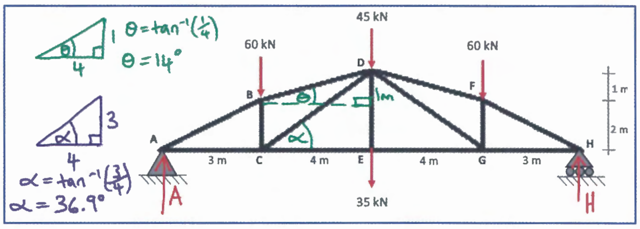
Note: BD direction has changed so the vertical component is now directed up instead of down. Horizontal direction is now directed to the right instead of to the left.

$$+\uparrow \sum F_v = 0$$
$$BD\sin 33.7 - BE\sin 33.7 - 6 - 6 + 10.5 = 0$$
$$BE = \frac{(9.9\sin 33.7) - 6 - 6 + 10.5}{\sin 33.7} = 7.2\ kN \searrow$$

Apply sum of forces in the horizontal direction as there is only 1 unknown force in that direction keeping in mind the new BD horizontal direction.

$$\overset{\rightarrow}{+} \sum F_H = 0$$
$$3 + BD\cos 33.7 + BE\cos 33.7 - CE = 0$$
$$CE = 3 + (9.9\cos 33.7) + (7.2\cos 33.7) = 17.2\ kN \leftarrow$$

37. a)



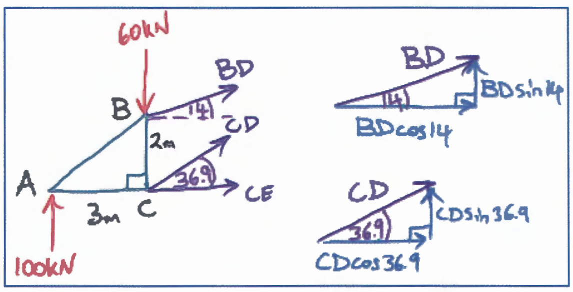
Determine the reaction forces by first applying sum of moments about A to find the force at the roller joint. There are no external horizontal forces so the horizontal force acting on the pin joint is 0 hence not shown above.

$$+\circlearrowleft \sum M_A = 0$$
$$(-60 \times 3) - (45 \times 7) - (35 \times 7)$$
$$-(60 \times 11) + (H \times 14) = 0$$
$$H = \frac{1400}{14} = 100\ kN\ \uparrow$$

Apply sum of forces in the vertical direction to find the force at the pin joint.

$$+\uparrow \sum F_v = 0$$
$$-60 + A - 45 - 35 - 60 + H = 0$$
$$A = 45 + 35 + 60 + 60 - 100$$
$$\therefore A = 100\ kN \uparrow$$

b)



Make a cut through members BD, CD and CE. These cut members become forces. Analyse LHS and draw the FBD (see above). Find the necessary angles (see top image) to break them down into its components.

Apply sum of moments about C. This will eliminate CD, CE, 60kN force and the vertical component of BD leaving BD as the only unknown.

$$+\circlearrowleft \sum M_C = 0$$
$$(-100 \times 3) - (BD\cos14 \times 2) = 0$$
$$BD = \frac{-300}{2\cos14} = -154.6\ kN$$
$$\therefore BD = 154.6\ kN \swarrow (C)$$

We now have 1 unknown in the vertical direction (vertical component of CD) as opposed to 2 unknowns in the horizontal direction. So apply sum of forces in the vertical direction first then apply it in the horizontal direction to find CD and CE respectively.

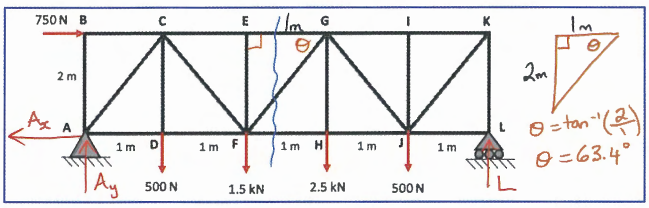
Note: BD direction has changed so the vertical component is now directed down instead of up. Horizontal direction is now directed to the left instead of to the right.

$$+\uparrow \Sigma F_v = 0$$
$$-BD\sin14 + CD\sin36.9 + 100 - 60 = 0$$
$$CD = \frac{(154.6\sin14) - 100 + 60}{\sin36.9} = -4.3kN$$
$$\therefore CD = 4.3\ kN \swarrow (C)$$

Note: CD direction has changed so the horizontal direction is now directed to the left instead of to the right.

$$\overrightarrow{+} \sum F_H = 0$$
$$-BD\cos14 - CD\cos36.9 + CE = 0$$
$$CE = (154.6\cos14) + (4.3\cos36.9) = 153.5\ kN \rightarrow (T)$$

38. a)



By observation, redundant members are AB, EF, IJ and JL. Alternatively, you can analyse joints B, E and I. Draw their FBD and sum the forces in the vertical direction and you'll find that the abovementioned members experience 0 N.

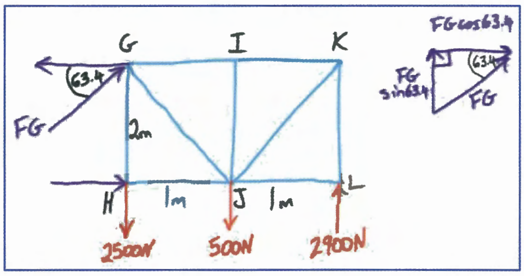
Similarly, analyse joint L and draw its FBD and sum the forces in the horizontal direction.

b) Before we can find the force in FG, we need to find the reaction forces. Apply moments about A first to find the force at the roller joint.

Note: Convert kN to N to keep the units consistent.

$$+\circlearrowleft \sum M_A = 0$$
$$(-750 \times 2) - (500 \times 1) - (1500 \times 2)$$
$$-(2500 \times 3) - (500 \times 4) + (L \times 5) = 0$$
$$L = \frac{14500}{5} = 2900\ N \uparrow$$

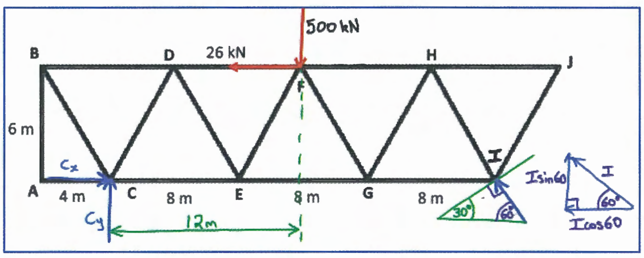
Cut through member FG. Cut members become forces. Analyse RHS and draw FBD. No need to find the forces at the pin joint since RHS will be analysed.



Due to only 1 unknown force in the vertical direction, we can sum the forces in that direction to find FG. FG has been broken up into its components above.

$$+\uparrow \Sigma F_v = 0$$
$$FG\sin63.4 - 2500 - 500 + 2900 = 0$$
$$FG = \frac{100}{\sin63.4} = 112\ N \nearrow$$

39. a)



Total mass is equal to mass of the contents (soil) + mass of the dump truck which equals 50 tonnes. This equates to 50,000kg (1 tonne = 1000kg) which we multiply by 10 to convert it to a force giving us 500,000 N which is 500 kN (1000N = 1 kN).

Force at the roller joint must act 90° to the joint. This force then needs to be broken down into its horizontal and vertical components.

Apply sum of moments about C to find force at the roller joint. This will remove C_x, C_y and horizontal component of I as they all go through joint C.

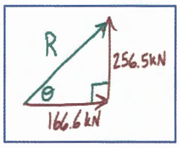
$$+\circlearrowleft \sum M_c = 0$$
$$(26 \times 6) - (500 \times 12) + (I\sin 60 \times 24) = 0$$
$$I = \frac{5844}{24\sin 60}$$
$$\therefore I = 281.2kN \nwarrow$$

Apply sum of forces in both horizontal and vertical directions to find the forces at the pin joint.

$$+\uparrow \Sigma F_v = 0$$
$$C_y - 500 + I\sin 60 = 0$$
$$C_y = 500 - (281.2\sin 60)$$
$$\therefore C_y = 256.5kN \uparrow$$

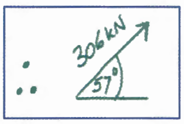
$$\overrightarrow{+} \Sigma F_H = 0$$
$$C_x - 26 - I\cos 60 = 0$$
$$C_x = 26 + (281.2\cos 60)$$
$$C_x = 166.6kN \rightarrow$$

Draw vector triangle of pin joint components to find resultant force using Pythagoras and its direction using trigonometry.

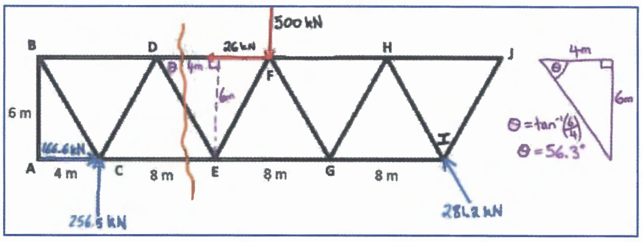


$$R = \sqrt{166.6^2 + 256.5^2}$$
$$R \approx 306kN$$

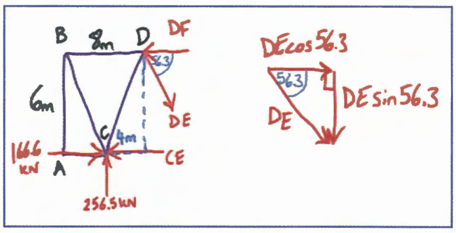
$$\theta = \tan^{-1}\left(\frac{256.5}{166.6}\right)$$
$$\theta = 57°$$



b)



Make a cut through member DF and analyse the left side of the cut. Draw FBD.



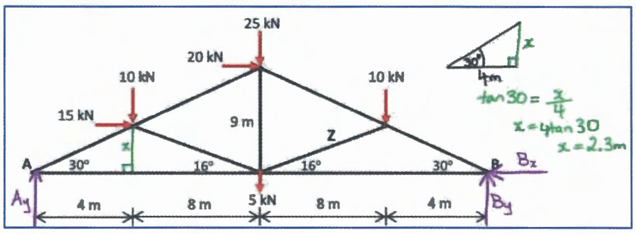
To find DF, we first find DE by summing forces in the vertical direction and equating it to 0.

$$+\uparrow \Sigma F_v = 0$$
$$256.5 - DE\sin 56.3 = 0$$
$$DE = \frac{256.5}{\sin 56.3}$$
$$\therefore DE = 308.3kN \searrow$$

Apply sum of moments about C. This will eliminate CE, 256.5kN and 166.6kN force leaving DF as the only unknown force. Vertical and horizontal component of DE will create moment about C.

$$+\circlearrowleft \sum M_c = 0$$
$$(DF \times 6) - (DE\cos 56.3 \times 6) - (DE\sin 56.3 \times 4) = 0$$
$$DF = \frac{(308.3\cos 56.3 \times 6) + (308.3\sin 56.3 \times 4)}{6}$$
$$\therefore DF = 342kN \leftarrow (C)$$

40. a)



Apply sum of moments about B to find the force at the roller joint. This will eradicate B_y and B_x.

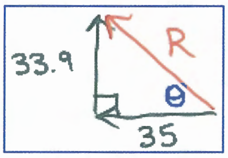
$$+\curvearrowleft \sum M_B = 0$$
$$-(A_y \times 24) + (10 \times 20) - (15 \times 2.3) - (20 \times 9) + (25 \times 12) + (10 \times 4) + (5 \times 12) = 0$$
$$A_y = \frac{385.5}{24}$$
$$\therefore Ay = 16.1kN \uparrow$$

Apply sum of forces in the vertical and horizontal directions to find By and Bx.

$$+\uparrow \Sigma F_v = 0$$
$$A_y - 10 - 25 - 5 - 10 + B_y = 0$$
$$16.1 - 10 - 25 - 5 - 10 + B_y = 0$$
$$B_y = 33.9kN \uparrow$$

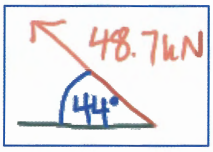
$$\overrightarrow{+} \sum F_H = 0$$
$$15 + 20 - B_x = 0$$
$$B_x = 35kN \leftarrow$$

Draw a vector triangle to find the resultant force acting at the pin joint by using Pythagoras and find the direction using trigonometry.

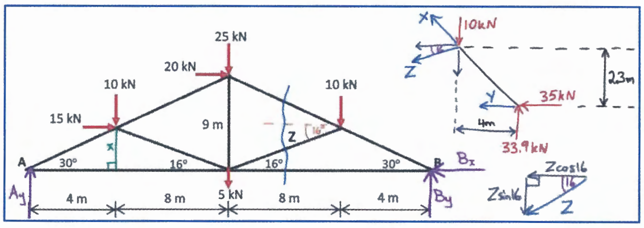


$$R = \sqrt{35^2 + 33.9^2}$$
$$R = 48.7kN$$

$$\theta = \tan^{-1}\left(\frac{33.9}{35}\right)$$
$$\therefore \theta = 44^0$$



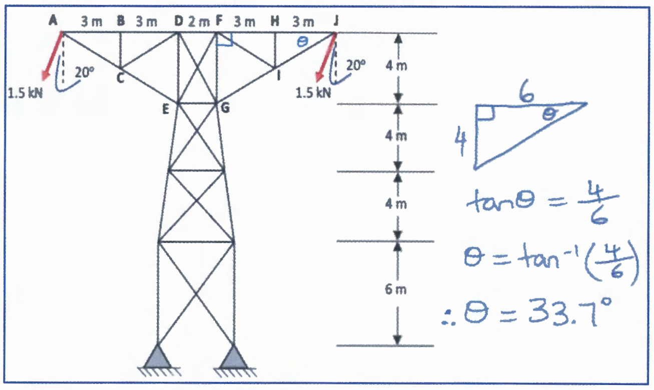
b) Make a cut through member Z and analyse right side of cut. Apply sum of moments about B. This will eliminate X, Y, 35 kN and 33.9 kN force as they all go through point B. This leaves Z as the only unknown force which needs to be broken into its x and y components as they create moments about B.



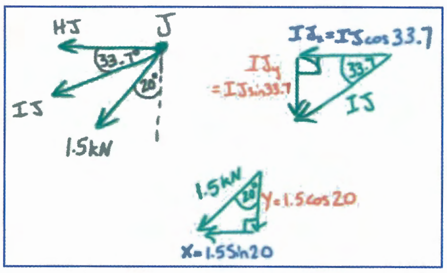
$$+\curvearrowleft \Sigma M_B = 0$$
$$(Z\cos 16 \times 2.3) + (Z\sin 16 \times 4) + (10 \times 4) = 0$$
$$Z[(2.3\cos 16) + (4\sin 16)] = -40$$
$$Z = \frac{-40}{(2.3\cos 16) + (4\sin 16)} = -12kN$$
$$\therefore Z = 12kN \nearrow (C)$$

Since Z is acting towards the joint then we say it is in compression.

41.

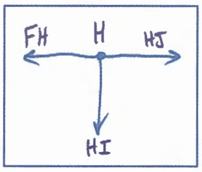


Analyse Joint J to find the force in member IJ. Draw the FBD and apply sum of forces in the vertical direction (y direction) as there is only 1 unknown in that direction.

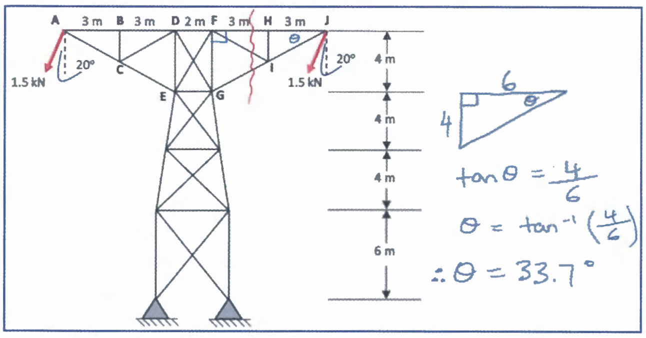


$$+\uparrow \Sigma F_y = 0$$
$$-IJ\sin 33.7 - (1.5\cos 20) = 0$$
$$IJ = \frac{-1.4}{\sin 33.7} = -1.8kN$$
$$\therefore IJ = 2.5kN \nearrow$$

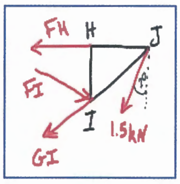
Analyse Joint H and apply sum of forces in vertical direction to find force in member HI.



$$+\uparrow \Sigma F_y = 0$$
$$HI = 0$$

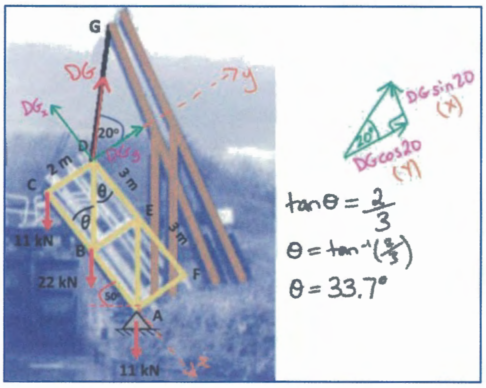


Make a cut through member FI. Analyse RHS and draw the FBD. The reaction forces are not required.

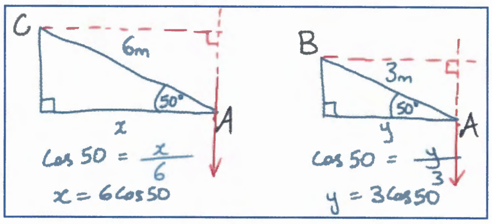


Apply moments about J. This will eliminate FH, 1.5kN force and GI leaving FI as the only force. Thus, FI multiplied by its perpendicular distance equals 0. For this to happen FI must equal to 0, hence it is redundant.

42. a)



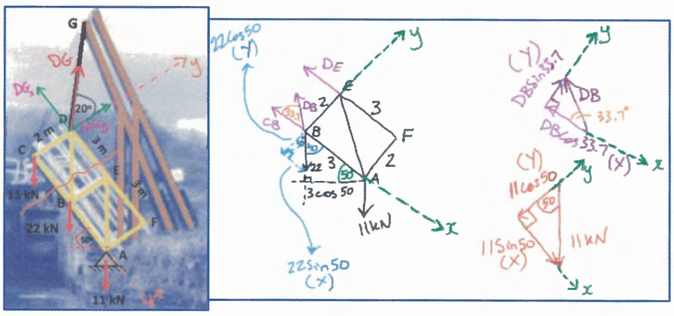
1. To find force in the cable DG, apply sum of moments about A. An x and y axis has been assigned as shown above. To find moment produced by DG, we need to break it up into its x and y components as shown above and then multiply it by its perpendicular distance.
2. To find the moment produced by the forces at B and C, we can draw a horizontal line from A and a vertical line down from B and C creating a right-angled triangle from which we can find the horizontal distance using trig. This can be seen in the sketch below. **Note**: Moment is equal to the force times the perpendicular distance so now we multiply the force by this horizontal distance.



$$+\circlearrowleft \sum M_A = 0$$
$$-(DG\cos 20 \times 6) + (DG\sin 20 \times 2)$$
$$+ (11 \times 6\cos 50) + (22 \times 3 \times \cos 50) = 0$$

$$(6DG\cos 20) - (2DG\sin 20) = 84.85$$
$$DG\big((6\cos 20) - (2\sin 20)\big) = 84.85$$
$$DG = \frac{84 \cdot 85}{(6\cos 20) - (2\sin 20)}$$
$$\therefore DG = 17\ kN \uparrow$$

b) Make a cut through members DE, DB and CB to find the force in those members. Apply Sum of moments about B to eliminate the forces acting at B. DB and the 11kN force has been broken up into its components.



$$+\circlearrowleft \sum M_B = 0$$
$$DE(2) - (11 \times 3\cos 50) = 0$$
$$DE = \frac{33\cos 50}{2}$$
$$\therefore DE = 10.6\ kN \nwarrow$$

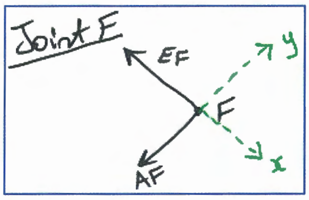
Apply sum of moments about A. This will remove the 11kN force, CB and the X-component of DB as they all go through point A leaving DB as the only unknown.

$$+\circlearrowleft \sum M_A = 0.$$
$$DE(2) - (DB\sin 33.7 \times 3) + (22 \times 3 \times \cos 50) = 0$$
$$(10.6 \times 2) - (DB\sin 33.7 \times 3) + (22 \times 3 \times \cos 50) = 0$$
$$DB = \frac{63.62}{3Sin33.7} = 38.2\ kN\ \nwarrow$$

Apply sum of forces in the x-direction to find CB. This will require both the 22kN force and the 11kN force to be broken down into its x-component.

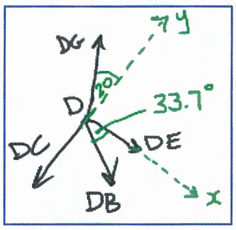
$$+\searrow \sum F_x = 0$$
$$-CB - DE - DB\cos 33.7 + (22\sin 50) + (11\sin 50) = 0$$
$$CB = -10.6 - (38.2\cos 33.7) + (22\sin 50) + (11\sin 50)$$
$$CB = -17.1\ \text{kN} \quad \therefore CB = 17.1\ kN \searrow$$

Analyse joint F to determine force in EF. Draw FBD and apply sum of forces in the x-direction to find EF.



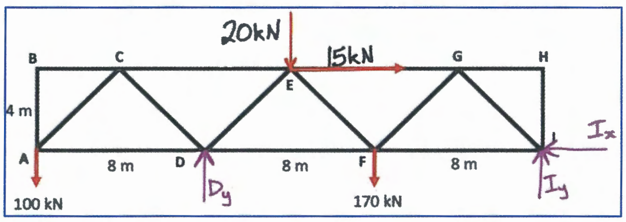
$$+\searrow \Sigma F_x = 0$$
$$-EF = 0$$
$$\therefore EF = 0$$

Analyse joint D to find force in member DC and draw FBD. Sum the forces in the y-direction to find DC.



$$+\nearrow \sum F_y = 0$$
$$DG\cos 20 - DC - DBSin33.7 = 0$$
$$DC = (17\cos 20) - (38.2\sin 33.7) = -5.2kN$$
$$\therefore DC = 5.2kN \nearrow$$

43. a)



Note: Although the pin joint is at an angle, its components will always be directly vertical and horizontal. Also, when converting 2 tonnes to kilograms, we multiply by 1000 so we have 2000kg. Multiply this value by 10 to get force so 20000N. Divide by 1000 to convert to kN gives us 20kN.

Sum moments about I to find force at roller joint D.

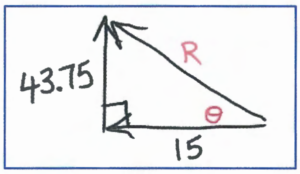
$$+\circlearrowleft \Sigma M_I = 0$$
$$(100 \times 24) - D_y(16) + (20 \times 12) - (15 \times 4) + (170 \times 8) =$$
$$D_y = \frac{3940}{16} = 246.25\ kN\ \uparrow$$

Apply sum of forces in the vertical and horizontal directions to find the pin joint component forces.

$$+\uparrow \Sigma F_v = 0$$
$$-100 + 246.25 - 20 - 170 + I_y = 0$$
$$I_y = 43.75\ kN \uparrow$$

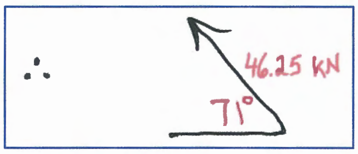
$$\overrightarrow{+} \sum F_H = 0$$
$$15 - I_x = 0$$
$$I_x = 15kN \leftarrow$$

Draw a vector triangle to find the resultant force at the pin joint along with its direction. Ensure that the resultant vector has a tail-tail and head- head connection with the other 2 vectors.

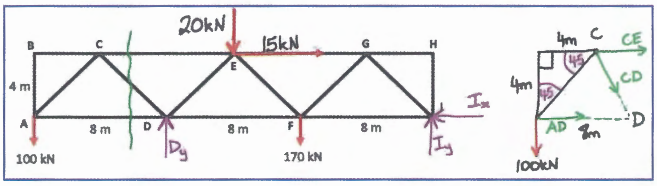


$$R = \sqrt{15^2 + 43.75^2} = 46.25\ kN$$

$$\theta = \tan^{-1}\left(\frac{43.75}{15}\right) = 71^\circ$$



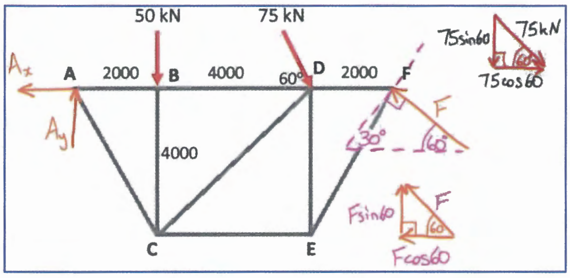
b) Cut through member CE and analyse the left side of the cut due to fewer forces on that side. Draw FBD. Apply moments about D eliminating AD and CD as they go through point D leaving CE as the only unknown.



$$+\circlearrowleft \Sigma M_D = 0$$
$$(-CE \times 4) + (100 \times 8) = 0$$
$$CE = \frac{800}{4} = 200\ kN\ \rightarrow (T)$$

Note: CE is in tension due to CE being directed away from the joint.

44. a)



Force at the roller joint must act 90° to the joint. This force then needs to be broken down into its horizontal and vertical components.

An angled pin joint, no matter what the angle, will always have a vertical and a horizontal component as shown in the FBD above.

To find the reaction forces apply sum of moments about A to find the force at the roller joint. Keep in mind that the horizontal component of F and D goes through joint A along with the pin joint forces and so they will not create a moment about A.

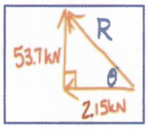
$$+\circlearrowleft \Sigma M_A = 0$$
$$(-50 \times 2) - (75 \times \sin 60 \times 6) + (F\sin 60 \times 8) = 0$$
$$F = \frac{489.7}{8 \times \sin 60} = 70.7\ kN\ \nwarrow$$

Apply sum of forces in the vertical and horizontal directions to determine the forces at the pin joint.

$$+\uparrow \Sigma F_v = 0$$
$$A_y - 50 - (75\sin 60) + F\sin 60 = 0$$
$$A_y = 50 + (75\sin 60) - (70.7\sin 60) = 53.7\ kN\ \uparrow$$

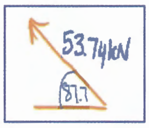
$$\overrightarrow{+} \Sigma F_H = 0$$
$$-A_x + (75\cos 60) - (F\cos 60) = 0$$
$$A_x = (75\cos 60) - (70.7\cos 60) = 2.15\ kN\ \leftarrow$$

Draw vector triangle and use Pythagoras to find resultant force acting at the pin joint and use trig to find direction.

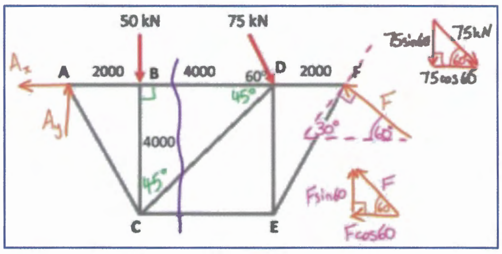


$$R = \sqrt{2.15^2 + 53.7^2}$$
$$R = 53.74kN$$

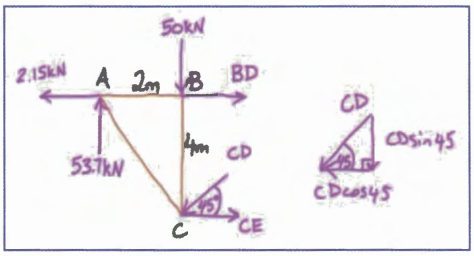
$$\tan\theta = \frac{53.7}{2.15}$$
$$\theta = \tan^{-1}\left(\frac{53.7}{2.15}\right)$$
$$\theta = 87.7°$$



b)



Make a cut through member CD and CE. Cut members become forces. Analyse the left side of the cut (easier) and draw the FBD.



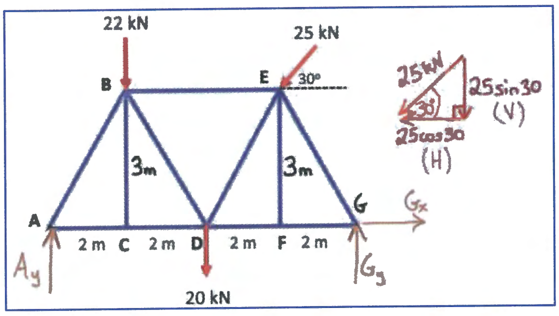
Apply sum of forces in the vertical direction to find CD.

$$+\uparrow \Sigma F_v = 0$$
$$53.7 - 50 - CD\sin 45 = 0$$
$$CD = \frac{3.7}{\sin 45} = 5.5\ kN\ \swarrow$$

Apply sum of moments about B. This removes the 2.15 kN force, 50 kN force, BD and vertical component of CD leaving CE as the unknown force.

$$+\circlearrowleft \sum M_B = 0$$
$$(-53.7 \times 2) - (CD\cos 45 \times 4) + (CE \times 4) = 0$$
$$CE = \frac{107.4 + (5.2\cos 45 \times 4)}{4}$$
$$\therefore CE = 30.5\ kN \rightarrow$$

45. a)



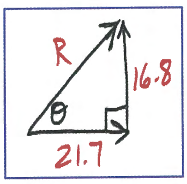
1. Sum moments about A to find vertical component of pin reaction. G_x produces no moment about A as it goes through point A. Break 25kN force into its vertical and horizontal components.

$$+\circlearrowleft \sum M_A = 0$$
$$(-22 \times 2) - (20 \times 4) + G_y(8)$$
$$-(25\sin 30 \times 6) + (25\cos 30 \times 3) = 0$$

$$G_y = \frac{134.048}{8}$$
$$\therefore G_y = 16.8\ kN \uparrow$$

2. Sum forces in the horizontal direction to find G_x.

$$\overset{+}{\rightarrow} \sum F_H = 0$$
$$-25\cos 30 + G_x = 0$$
$$\therefore G_x = 21.7\ kN \rightarrow$$



3. Draw up the vector triangle to find the resultant force acting on the pin joint by using Pythagoras and find the direction using trigonometry.

Note: Resultant force makes a tail-tail connection and a head-head connection with the other 2 vectors.

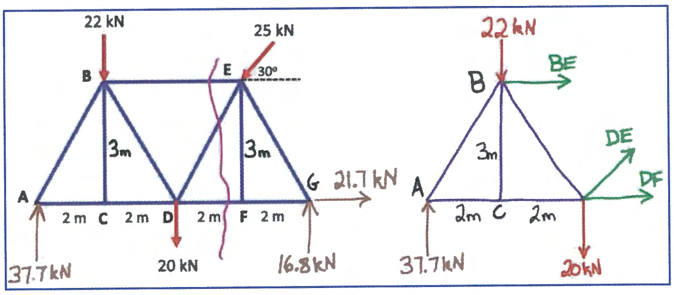
$$R = \sqrt{21.7^2 + 16.8^2}$$
$$R = 27.4\ kN$$

$$\tan\theta = \frac{16.8}{21.7}$$
$$\theta = \tan^{-1}\left(\frac{16.8}{21.7}\right)$$
$$\theta = 38°$$

b) Before we determine the force in member BE we need to calculate the roller force as this will be used in the following calculations.

$$+\uparrow \Sigma F_v = 0$$
$$A_y - 22 - 20 - 25\sin 30 + 16.8 = 0$$
$$A_y = 37.7\ kN \uparrow$$

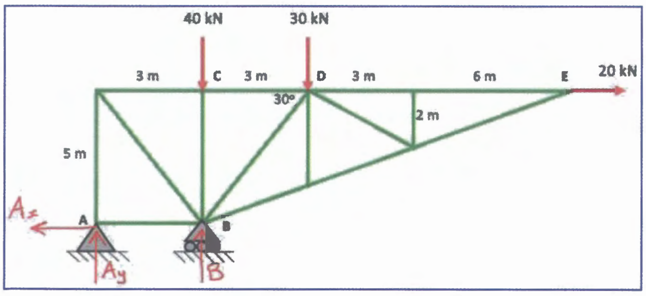
Make a cut through member BE and analyse the left side of the cut. Draw FBD as shown above. Apply sum of moments about D as this will remove DE, DF and 20 kN force as they all go through point D. This will leave BE as the only unknown force.



$$+\circlearrowleft \sum M_D = 0$$
$$-(BE \times 3) + (22 \times 2) - (37.7 \times 4) = 0$$
$$BE = -\frac{106.8}{3} = -35.6\ kN$$
$$\therefore BE = 35.6\ kN \leftarrow (C)$$

Note: If the force is pointed towards the joint then it is in compression.

46. a)



Apply moments about A to find force at the roller joint.

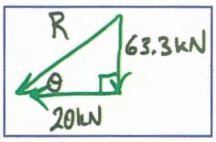
$$+\curvearrowleft \sum M_A = 0$$
$$(B \times 3) - (40 \times 3) - (30 \times 6) - (20 \times 5) = 0$$
$$B = \frac{400}{3} = 133.3\ kN\ \uparrow$$

Apply sum of forces in both the vertical and horizontal directions to find the forces at the pin joint.

$$+\uparrow \Sigma F_v = 0$$
$$A_y + 133.3 - 40 - 30 = 0$$
$$A_y = -63.3\ kN$$
$$\therefore A_y = 63.3\ kN \downarrow$$

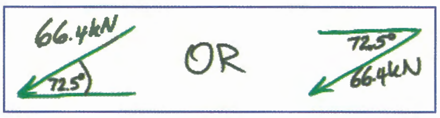
$$\overrightarrow{+} \sum F_H = 0$$
$$-A_x + 20 = 0$$
$$A_x = 20\ kN \leftarrow$$

Draw a vector triangle of the pin joint components to find the resultant force using Pythagoras and its direction using trigonometry.

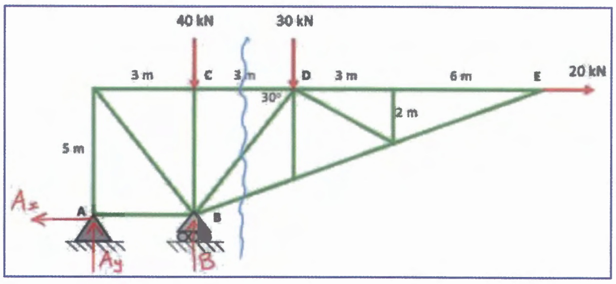


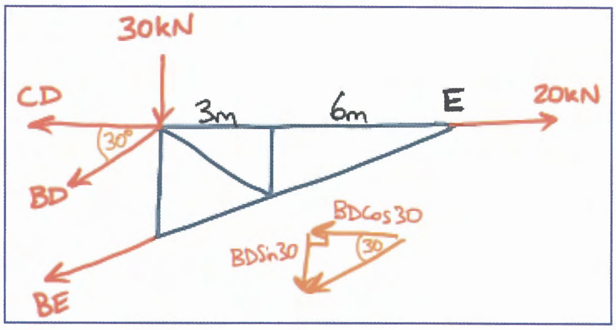
$$R = \sqrt{20^2 + 63.3^2} = 66.4\ kN$$

$$\theta = \tan^{-1}\left(\frac{63.3}{20}\right) = 72.5°$$



b) Make a cut through member BD and analyse the right side of the cut. Draw FBD.



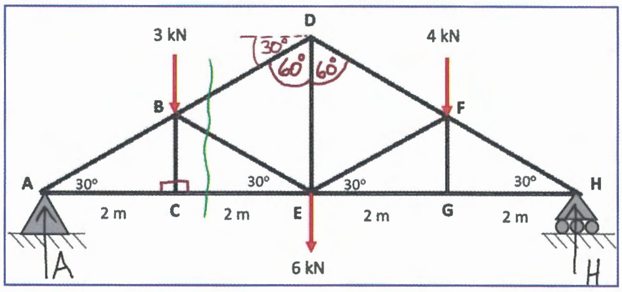


Sum moments about E. This will remove the 20 kN force, CD, BE and the horizontal component of BD.

$$+\curvearrowleft \sum M_E = 0$$
$$(30 \times 9) + (BD\sin 30 \times 9) = 0$$
$$BD = \frac{-270}{9\sin 30} = -60kN$$
$$\therefore BD = 60\ kN \nearrow (C)$$

Since BD is directed towards the joint, we say that it is in compression.

47.

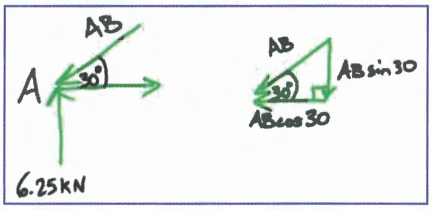


Find reaction forces first by applying sum of moments about H. No horizontal forces acting on the pin joint as there are no external horizontal forces.

$$+\curvearrowleft \sum M_H = 0$$
$$(4 \times 2) + (6 \times 4) + (3 \times 6) - (A \times 8) = 0$$
$$A = \frac{50}{8} = 6.25\ kN\ \uparrow$$

$$+\uparrow \Sigma F_v = 0$$
$$A - 3 - 6 - 4 + H = 0$$
$$H = 4 + 6 + 3 - 6.25$$
$$\therefore H = 6.75\ kN \uparrow$$

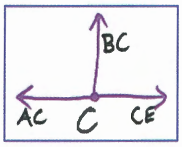
Analyse joint A and apply sum of forces in the vertical and horizontal direction to find force in AB and AC respectively.



$$+\uparrow \Sigma F_v = 0$$
$$6.25 - AB\sin30 = 0$$
$$AB = \frac{6.25}{\sin30} = 12.5\ kN\ \swarrow \text{(C)}$$

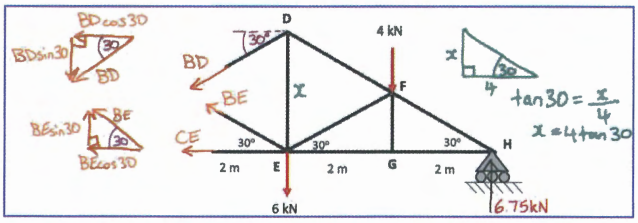
$$\overrightarrow{+}\Sigma F_H = 0$$
$$AC - AB\cos30 = 0$$
$$AC = AB\cos30 = 12.5\cos30$$
$$\therefore AC = 10.83\ kN \rightarrow \text{(T)}$$

Analyse joint C to find BC and CE.



$$+\uparrow \Sigma F_v = 0$$
$$BC = 0\ \text{(Redundant)}$$

Use method of sections by making a cut through members BD, BE and CE. Analyse right side of cut and draw FBD.



Sum the moments about E. This will eliminate BE, CE, vertical component of BD and the 6 kN force as they all go through point E. BD and BE are broken down into its components shown above.

$$+\circlearrowleft \sum M_E = 0$$
$$BD\cos30 \times (4\tan30) - (4 \times 2) + (6.75 \times 4) = 0$$
$$BD = \frac{-(6.75 \times 4) + 8}{4\tan30\cos30} = -9.5\ kN$$

$$\therefore BD = 9.5\ kN \nearrow \text{(C)}$$

Note: BD direction has changed so the y-component is now directed in the opposite direction. Hence the y-component is now positive.

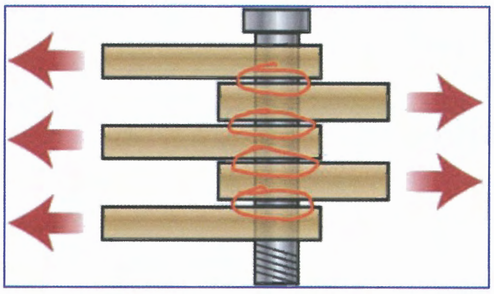
Apply sum of forces in the y-direction to find the remaining force in member BE.

$$+\uparrow \sum F_y = 0$$
$$BD\sin30 + BE\sin30 - 6 - 4 + 6.75 = 0$$
$$BE = \frac{6 + 4 - 6.75 - (9.5\sin30)}{\sin30} = -3\ kN$$

$$\therefore BE = 3\ kN \searrow \text{(C)}$$

Chapter 2 Solutions

1. (C)
2. (C)
3. (B)
4. (D)
5. (C)
6. (D)
7. (B)
8. (B)
9. (C)
10. (A)
11. (A)
12. (C)
13. (B)
14. (D)



4 pairs of shear surfaces (surfaces sliding against one another) shown as circles in the image above. Hence bolt will experience quadruple the shear stress.

15. (D)

$$\tau = \frac{F}{A}$$

$$\tau = \frac{300 \times 10^3}{\frac{\pi}{4} \times 25^2}$$

$$\tau = 611 \text{ MPa}$$

16. (C)

17. (A)

$$\tau = 750MPa$$
$$t = 5\text{mm}$$
$$d = 40\text{mm}$$
$$P = \pi d \text{ (2 semi circles)} + 2L\ (side\ lengths)$$
$$P = (\pi \times 40) + (2 \times 27)$$

$$\tau_{punching} = \frac{F}{P \times t}$$
$$F = \tau \times P \times t$$
$$F = 750 \times \big((\pi \times 40) + (2 \times 27)\big) \times 5$$
$$F = 673{,}738.9\ N$$
$$\therefore F = 674\ kN$$

18. (A)

$$\tau_{punching} = \frac{F}{P \times t}$$

Solve for F and divide by 1000 to convert N to kN.

$$250 = \frac{F}{(\pi \times 40) \times 25}$$
$$F = 250 \times (\pi \times 40) \times 25$$
$$\therefore F = 785{,}398N = 785\ kN$$

19. (C) – Max BM occurs when the SF is 0. Alternatively, since A and B are above the x-axis, the BM will increase until C. From there, it will drop down (below the x-axis) to 0.

20. (C) – True stress is the force divided by the cross-sectional area at that point in time. When the specimen undergoes tensile testing, its diameter reduces hence decreasing its area. According to the stress formula, decreasing the area will increase the stress hence the shape of curve B. Necking occurs after the UTS

21.

$$\varepsilon = \frac{\Delta L}{L_0}$$
$$\varepsilon = \frac{110 - 80}{80}$$
$$\therefore \varepsilon = 0.375 = 37.5\ \%$$

22. We convert our force to N by multiplying the value by 1000. Remember: N/mm^2 gives MPa. To convert to MPa to kPa, we multiply by 1000.

$$\sigma = \frac{F}{A}$$
$$\sigma = \frac{30 \times 1000}{\frac{\pi}{4} \times (450)^2}$$
$$\sigma = 0.188628\ MPa$$
$$\therefore \sigma = 188.6\ kPa$$

23. a)

$$\varepsilon = \frac{e}{L_0} = \frac{0.8\text{mm}}{(2.5 \times 1000)\text{mm}}$$
$$\therefore \varepsilon = 0.00032$$

b)

$$\sigma = \frac{F}{A}$$
$$\sigma = \frac{0.6 \times 10N}{\frac{\pi}{4} \times (0.35)^2\text{mm}^2}$$
$$\sigma = 62.4\ MPa$$

c)

$$E = \frac{\sigma}{\varepsilon}$$
$$E = \frac{62.4 \text{ MPa}}{0.00032}$$
$$E = 194883.603\ MPa$$
$$\therefore E \approx 195\ GPa$$

24.

$$Weight = 62 \times 10 = 620N$$
$$F = \frac{20}{100} \times 620$$
$$\therefore F = 124\ N$$

$$\sigma = \frac{F}{A}$$
$$\sigma = \frac{124N}{\pi \times 5^2\text{mm}^2}$$
$$\sigma = 1.6\ MPa$$

25.

$$\varepsilon = \frac{e}{L_0}$$
$$\varepsilon = \frac{5\text{mm}}{5 \times 1000\text{mm}}$$
$$\therefore \varepsilon = 0.001$$

Convert 120 GPa to MPa by multiplying by 10^3.

$$E = \frac{\sigma}{\varepsilon}$$
$$\sigma = E\varepsilon = 120 \times 10^3 \times 0.001$$
$$\therefore \ \sigma = 120MPa$$

$$\sigma = \frac{F}{A}$$
$$120 = \frac{2000}{A}$$
$$\therefore \ A = \frac{2000}{120} = 16.67\text{mm}^2$$

$$A = \frac{\pi}{4}d^2$$
$$4A = \pi d^2$$
$$d^2 = \frac{4A}{\pi}$$
$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 16.67}{\pi}}$$
$$\therefore \ d = 4.6\ mm$$

26.

$$\sigma = \frac{F}{A} = \frac{750N}{\frac{\pi}{4} \times 11^2\text{mm}^2}$$
$$\therefore \sigma = 7.9MPa$$

$$\varepsilon = \frac{e}{L_0}$$
$$\varepsilon = \frac{3}{50} = 0.06$$

$$E = \frac{\sigma}{\varepsilon}$$
$$E = \frac{7.9}{0.06} = 132\ MPa$$

OR

Convert e to mm, L to mm and use the FLeA formula

$$E = \frac{FL}{eA}$$
$$E = \frac{750 \times 50 \times 1000}{3 \times 1000 \times \frac{\pi}{4} \times 11^2}$$
$$E \approx 132\ MPa$$

27.

$$E = \frac{\sigma}{\varepsilon}$$
$$\varepsilon = \frac{\sigma}{E}$$
$$\varepsilon = \frac{210}{20 \times 10^3} = 0.0105$$

$$\varepsilon = \frac{e}{L_0}$$
$$e = \varepsilon \times L_0$$
$$e = 0.0105 \times 0.48 = 0.00504m$$
$$\therefore e = 5.04\ \text{mm}$$

28. Convert 3 tonnes to Newtons

$$F = 3 \times 1000 \times 10 = 30000N$$
$$A = 1 \times 1 = 1m^2$$
$$\sigma = \frac{F}{A} = \frac{30000}{1} = 30000\ Pa$$

Note: N and m² will give answer in Pa.

$$E = \frac{\sigma}{\varepsilon}$$
$$\varepsilon = \frac{\sigma}{E} = \frac{30000}{35 \times 10^9}$$
$$\therefore \varepsilon = 8.571 \times 10^{-7}$$

$$\varepsilon = \frac{e}{L_0}$$
$$8.571 \times 10^{-7} = \frac{e}{5000}$$
$$e = 0.004\ \text{mm}$$

29. a)

$$\sigma = \frac{F}{A} = \frac{4 \times 10\ N}{\frac{\pi}{4} \times 3^2\ \text{mm}^2}$$
$$\sigma = 5.66\ MPa$$

b)

$$E = \frac{\sigma}{\varepsilon}$$
$$\varepsilon = \frac{\sigma}{E} = \frac{5.66}{116 \times 10^3}$$
$$\varepsilon = 0.000048$$

$$\varepsilon = \frac{e}{L_0}$$
$$e = \varepsilon \times L_0$$
$$e = 0.000048 \times 2 \times 1000$$
$$e = 0.098 \text{ mm}$$

30.

$$L_0 = 0.479m = 479\text{mm}$$
$$e = 0.48 - 0.479 = 0.001m = 1\text{mm}$$
$$d = 2.34\text{cm} = 23.4\text{mm}$$
$$E = 21GPa = 21000MPa$$

$$\varepsilon = \frac{e}{L_0}$$
$$\varepsilon \simeq \frac{1}{479} \simeq 0.002087$$

$$E = \frac{\sigma}{\varepsilon}$$
$$\sigma = E\varepsilon$$
$$\sigma = 21000 \times 0.002087$$
$$\sigma = 43.8 \; MPa$$

$$\sigma = \frac{F}{A}$$
$$F = \sigma \times A$$
$$F = 43.8 \times \left(\frac{\pi}{4} \times 23.4^2\right)$$
$$F = 18854N = 18.9 \; kN$$

31.

$$d = 50\text{mm}$$
$$F = 4MN = 4 \times 10^6 N$$
$$L_0 = 4.5\text{km} = 4500m$$
$$E = 205GPa = 205 \times 10^3 MPa$$
$$e = ?$$

$$\sigma = \frac{F}{A}$$
$$\sigma = \frac{4 \times 10^6}{\frac{\pi}{4} \times (50^2)}$$
$$\sigma = 2037.18 \; MPa$$

$$E = \frac{\sigma}{\varepsilon}$$
$$\varepsilon = \frac{\sigma}{E}$$
$$\varepsilon = \frac{2037.18}{205 \times 10^3}$$
$$\varepsilon = 0.0099$$

$$\varepsilon = \frac{e}{L_0}$$
$$e = \varepsilon \times L_0$$
$$e = 0.0099 \times 4500m$$
$$e = 44.72 \; m$$

This is quite a significant number but it is only 0.99% of the unsupported length which may be vital when taking the effects of temperature into account.

32. a)

$$\sigma = \frac{F}{A}$$
$$\sigma = \frac{12}{\frac{\pi}{4} \times 3.5^2}$$
$$\sigma = 1.247 \; MPa$$

b)

$$\varepsilon = \frac{e}{L_0}$$
$$\varepsilon = \frac{0.25\text{mm}}{3000\text{mm}}$$
$$\varepsilon = 0.000083 = 0.0083 \;\%$$

33. a)

$$\sigma = \frac{F}{A}$$
$$\sigma = \frac{35 \times 10^3}{\frac{\pi}{4} \times (20^2)}$$
$$\sigma = 111.4 \; MPa$$

b)

$$\varepsilon = \frac{e}{L_0}$$
$$\varepsilon = \frac{0.3}{4 \times 1000}$$
$$\varepsilon = 0.000075 = 0.0075 \;\%$$

34. a)

$$\sigma = \frac{F}{A}$$
$$\sigma = \frac{50 \times 10^3}{\frac{\pi}{4} \times 15^2}$$
$$\sigma = 282.9 \; MPa$$

b)

$$\varepsilon = \frac{e}{L_0}$$
$$\varepsilon = \frac{455 - 450}{450}$$
$$\varepsilon = 0.011$$

c)

$$E = \frac{\sigma}{\varepsilon}$$
$$E = \frac{282.9}{0.011} = 25464.79MPa$$
$$\therefore E \approx 25\ GPa$$

35. a)

$$E = \frac{\sigma}{\varepsilon}$$
$$\varepsilon = \frac{\sigma}{E}$$
$$\varepsilon = \frac{320}{205 \times 10^3}$$
$$\varepsilon = 0.00156$$

b)

$$\sigma = \frac{F}{A}$$
$$F = \sigma \times A$$
$$F = 320 \times \left(\frac{\pi}{4} \times 50^2\right)$$
$$F = 628318.5N$$
$$\therefore F \approx 628\ kN$$

36.

$$L_0 = 200\text{mm}$$
$$e = 0.08\text{mm}$$
$$F = 20kN = 20 \times 10^3 N$$
$$E = 190GPa = 190 \times 10^3 MPa$$

$$\varepsilon = \frac{e}{L_0}$$
$$\varepsilon = \frac{0.08}{200}$$
$$\varepsilon = 0.0004$$

$$E = \frac{\sigma}{\varepsilon}$$
$$\sigma = E \times \varepsilon$$
$$\sigma = 190 \times 10^3 \times 0.0004$$
$$\sigma = 76\ MPa$$

$$\sigma = \frac{F}{A}$$
$$A = \frac{F}{\sigma}$$
$$A = \frac{20 \times 10^3 N}{76MPa}$$
$$A = 263.16\ \text{mm}^2$$

$$A = \frac{\pi}{4} d^2$$
$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 263.16}{\pi}}$$
$$d = 18.3\ \text{mm}$$

37.

$$L_0 = 2m = 2 \times 10^3\text{mm}$$
$$F = 650 \times 1000\text{kg} \times 10 = 6{,}500{,}000N$$
$$e = 0.15\text{mm}$$
$$E = 200GPa = 200 \times 10^3\ MPa$$

$$e = \frac{FL}{EA}$$
$$\therefore A = \frac{FL}{eE} = \frac{6.5 \times 10^6 \times 2 \times 10^3}{0.15 \times 200 \times 10^3}$$

$$A = 433333.3\ \text{mm}^2$$

$$A = \frac{\pi}{4} d^2$$
$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 433333.3}{\pi}}$$
$$d = 742.79\ \text{mm}$$

38.

$$\tau = 80MPa$$
$$t = 4\text{mm}$$
$$d = 40\text{mm}$$
$$P = \pi d$$
$$P = \pi \times 40$$

$$\tau_{punching} = \frac{F}{P \times t}$$
$$F = \tau \times P \times t$$
$$F = 80 \times \pi \times 40 \times 4$$
$$\therefore F = 40212.4N = 40\ kN$$

39.

$$\sigma_{allowable} = \frac{\sigma_{uss}}{F_0 S}$$
$$\sigma_{allowable} = \frac{150}{1.5}$$
$$\sigma_{allowable} = 100\ MPa$$

$$\sigma = \frac{F}{2A}$$
$$A = \frac{F}{2\sigma}$$
$$A = \frac{20 \times 10^3}{2 \times 100}$$
$$A = 100 \text{ mm}^2$$

$$A = \frac{\pi}{4} d^2$$
$$d = \sqrt{\frac{4A}{\pi}}$$
$$d \approx 11.28 \text{ mm}$$

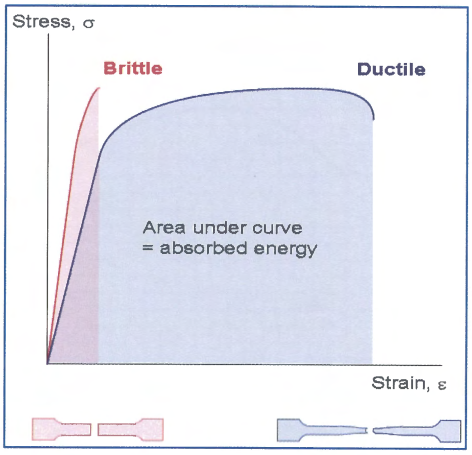
40.
$$\tau_{punching} = \frac{F}{P \times t}$$

$$5 = \frac{500}{(10 \times 4) \times t}$$
$$t = \frac{500}{5 \times (10 \times 4)}$$
$$t = 2.5 \text{ mm}$$

41. Elastic deformation – Original dimensions of a deformed body are recovered when the load is removed. Plastic deformation – If elastic limit is exceeded, body will permanently deform once load is removed.

42. a)i) Brittle is when a material fractures without any significant plastic deformation. They absorb very little energy (low toughness). Fracture occurs without any indication of failure.

ii) Ductile is when a material undergoes plastic deformation and can withstand large amounts of impact (high toughness). Ductile materials are desirable as they provide us with an indication that failure is imminent.



b)
$$\sigma = \frac{F}{A}$$
$$\sigma = \frac{50000}{600 \times 450} = 0.18519 \, MPa$$

$$\therefore \sigma = 185 \text{ kPa}$$

43. 1 - Ultimate Tensile Strength (UTS) gives us the maximum stress a material can handle without fracturing. Crucial for brittle members. 2 - Proof stress provides us with the value of stress that can be applied before a material starts to "yield" or change size/shape. Crucial for ductile members (brittle members don't yield). 3 – Breaking stress represents the stress at which the material will fracture.

44. a) B - Glass undergoes brittle fracture, hence will not experience plastic deformation.

b) C - Decreasing carbon content of steel increases its ductility.

c) A - Increasing carbon content of steel decreases its ductility.

d) A - The Young's modulus represents the stiffness of the material. This can be obtained by the gradient (slope) of the elastic region. Since A has the steepest slope, it will be the stiffest.

e) C - Toughness is obtained by area underneath the curve. Hence C has the largest toughness.

f) C - Has the longest tail of all the graphs hence most ductile.

g) A -The highest point of all the curves is located on curve A and this represents maximum strength.

h) B - It does not exhibit any plastic deformation hence brittle.

45.
$$\sigma = \frac{F}{A}$$
$$\therefore \sigma = \frac{35000}{80 \times 80} = 5.47 \, MPa$$

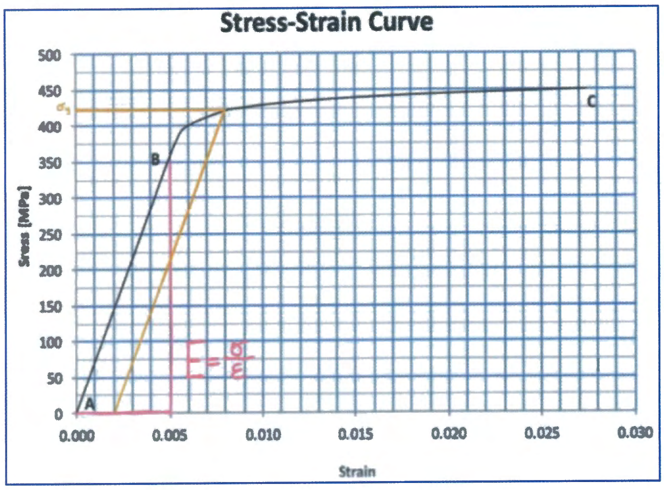
$$\varepsilon = \frac{\Delta L}{L_0}$$
$$\therefore \varepsilon = \frac{10}{60} \simeq 0.167$$

$$E = \frac{\sigma}{\varepsilon}$$
$$E = \frac{5.47}{0.167} = 32.8\ MPa$$

Or

$$E = \frac{FL}{eA} = \frac{35000 \times 60}{10 \times (80 \times 80)} = 32.8\ MPa$$

46. a)



Young's modulus is determined by finding the gradient of the elastic region (straight line). Point B is at the end of the straight portion and will be the point of choice.

$$E = \frac{\sigma}{\varepsilon}$$
$$E \approx \frac{350}{0.005} \approx 70000 MPa$$
$$\therefore E = 70\ GPa$$

This stress strain graph has a progressive yield point. To find the yield stress, we draw a line from 0.2% strain (0.002) that is parallel to the elastic portion of the graph. The point where the line intersects the graph is projected to the stress axis which gives us the yield strength known as the proof stress. See graph above.

$$\sigma_{yield} = 425\ MPa$$

b) i) AB is the elastic region. This is the portion of the graph where the material will extend then return to its original length when the force is removed.

ii) BC is the plastic region. This is the portion of the graph where the material will permanently deform when force is removed.

47. a) The yield point is the point after the elastic limit at which strain is developed without much increase of stress. This point is the yield strength which in this graph is approx. 300MPa. Since we are given the factor of safety, we need to use the graph to obtain the yield strength so we can find the allowable stress.

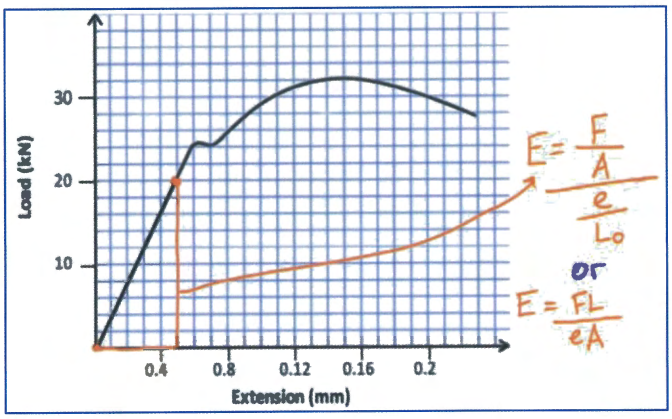
$$\sigma_{allowable} = \frac{\sigma_{yield}}{FS}$$
$$\sigma_{allowable} = \frac{300}{1.6} = 187.5\ MPa$$

$$\sigma = \frac{F}{A}$$
$$187.5 = \frac{85000}{A}$$
$$A = \frac{85000}{187.5} = 453.33\ mm^2$$

$$A = \frac{\pi}{4} d^2$$
$$4A = \pi d^2$$
$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 453.33}{\pi}} = 24\ \text{mm}$$

b) **Toughness** – Area under the curve. **Stiffness** – Gradient of the elastic region. **Ultimate Tensile Strength** – Maximum stress on a stress-strain curve. **Resilience** – Area under the elastic region. **Yield strength** – Clear increase in strain without the corresponding increase in stress (usually the point just above the elastic limit).

48.



To find Young's modulus (E), analyse the elastic region of the graph by calculating the gradient of the elastic region (straight line). Pick any point within the straight line. The point (0.5, 20) has been chosen. These must be converted to stress and strain respectively or can be used directly in the FLEA formula. **Note**: Divide by 1000 to convert MPa to GPa.

$$\sigma = \frac{F}{A}$$
$$\sigma \simeq \frac{20 \times 1000}{\frac{\pi}{4} \times (4.5)^2} \simeq 1257.5\ MPa$$

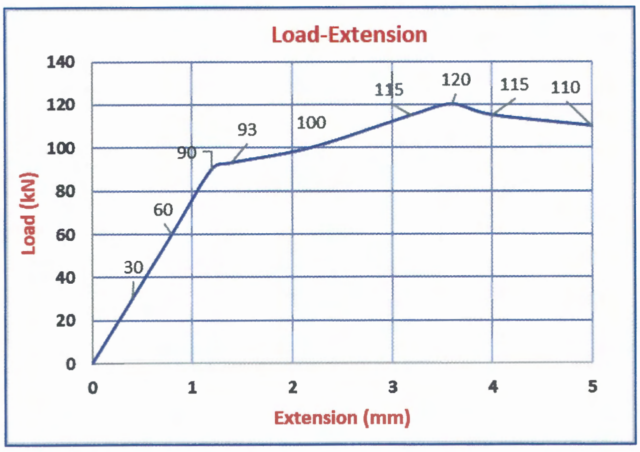
$$\varepsilon = \frac{e}{L_0}$$
$$\varepsilon \simeq \frac{0.5}{65} \simeq 0.00769$$

$$E = \frac{\sigma}{\varepsilon}$$
$$E \simeq \frac{1257.5}{0.00769} \simeq 163477.67 MPa$$
$$\therefore E \approx 163\ GPa$$

or

$$E = \frac{FL}{eA}$$
$$E = \frac{20 \times 1000 \times 65}{0.5 \times \left(\frac{\pi}{4} \times 4.5^2\right)} = 163477.67 MPa$$
$$\therefore E = 163\ GPa$$

49. a)



b) Find the gradient of the straight-line section (elastic region). Pick any 2 points within this straight line and apply rise over run principle. We have chosen the points (0,0) and (0.4,30). However, we must convert the load in question to stress and the extension to strain first before finding Young's modulus.

$$\sigma = \frac{F}{A}$$
$$\sigma = \frac{30000}{120} = 250 MPa$$
$$\varepsilon = \frac{e}{L_0}$$
$$\varepsilon = \frac{0.4}{40} = 0.01$$

$$E = \frac{\sigma}{\varepsilon}$$
$$E = \frac{250}{0.01} = 25000\ MPa$$
$$\therefore E = \frac{25000}{1000} = 25\ GPa$$

c)

$$UTS = \frac{F_{\max}}{A}$$
$$UTS = \frac{120000}{120} = 1000\ MPa$$

$$\therefore UTS = 1\ GPa$$

d)

$$\sigma_{fracture} = \frac{F_{fracture}}{A}$$

$$\sigma_{fracture} = \frac{110000}{120} = 917\ MPa$$

e)

$$\sigma_{proportional} = \frac{F_{proportional}}{A}$$

$$\sigma_{proportional} = \frac{90000}{120} = 750\ MPa$$

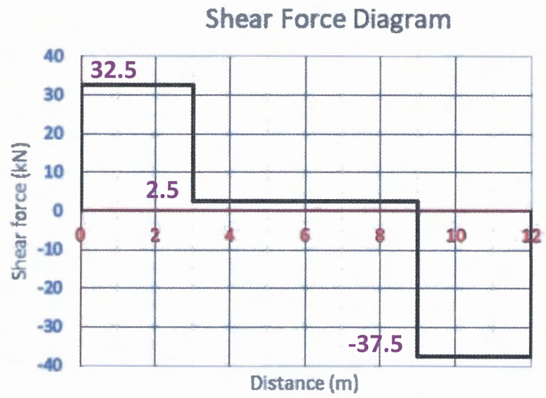
50. a) To find the force at the roller joint, apply sum of moments about the pin joint (B).

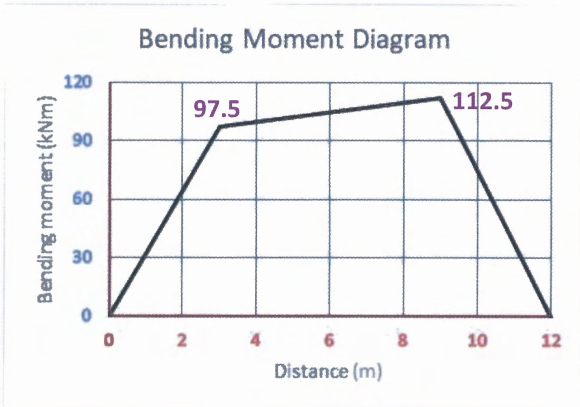
$$+\circlearrowleft \Sigma M_B = 0$$
$$-A(12) + (30 \times 9) + (40 \times 3) = 0$$
$$12A = 390$$
$$\therefore A = 32.5\ kN \uparrow$$

Apply sum of forces in the vertical direction to find the remaining reaction force.

$$+\uparrow \sum F_v = 0$$
$$32.5 - 30 - 40 + B = 0$$
$$B = 37.5\ kN \uparrow$$

b) To draw the shear force diagram, we simply follow the forces. Initially, we have a force of 32.5 kN acting at A so we draw a vertical line to 32.5 kN. We now draw a horizontal line until we encounter another force which is at 3m. The 30 kN force is directed downwards so now the total shear force at that point is 32.5 – 30 which gives us a total of 2.5 kN. After another 6m, the total shear force is again reduced this time by 40 kN (directed downwards) and so our total is now -37.5 kN. After another 3 m, the total shear force increases by 37.5 kN which now gives us a total shear force of 0 at the end of the beam. If the shear force does not equal to 0 at the end of the beam, then a calculation error has occurred.





To draw the bending moment diagram, we simply find the area underneath the graph, so we break up the shear force diagram into rectangular regions of which we have 3 in this case. Finding the area of these rectangles will give us the bending moment values.

Note: If these rectangular regions are above the x-axis then we add to the previous bending moment value. If it is below the x-axis, then we subtract from the previous bending moment value.

1 → 32.5 x 3 = 97.52

2 → 2.5 x 6 = 15

Total bending moment is now 97.52 + 15 = 112.5

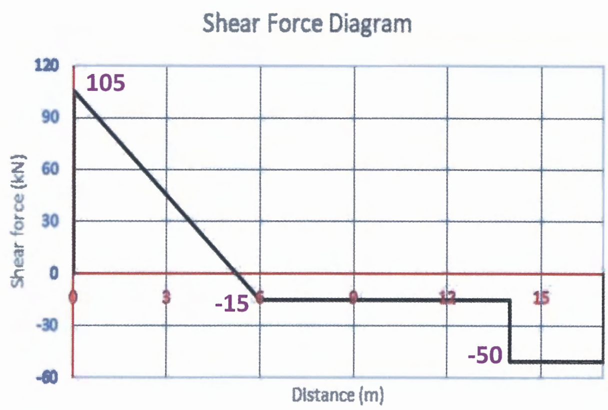
3 → 37.5 x 3 = 112.5 (below the x-axis)

Total bending moment is now 112.5 – 112.5 = 0

Just like the shear force, if the bending moment is not 0 at the end of the beam then a calculation error has occurred.

51. There will initially be a 105 kN force rising up due to joint A. The UDL means that there will be a force of 20 kN acting over 6 metres of the beam so after 6 m, a total of 120 kN (20*6) would be acting down hence the total force will now be 105 – 120 = -15 kN. After another 8 metres, there is a force of 35 kN acting down so the total force is now -15-35 = -50 kN. After another 3m, the shear force increases by 50 kN to 0.

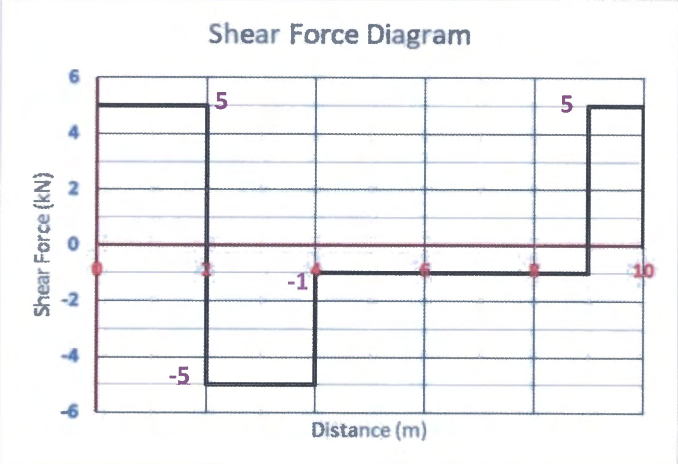
It is important that you understand the shape of the UDL whereby the shear force will be in the shape of a linear graph.

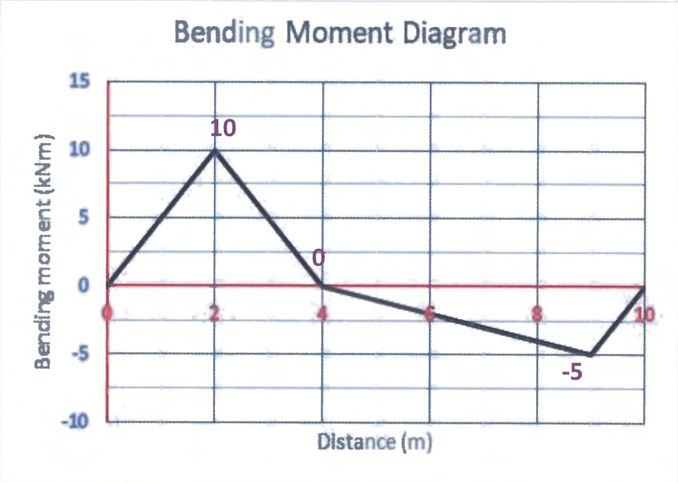


52. a)

$$+\circlearrowleft \Sigma M_B = 0$$
$$-A(9) + (10 \times 7) - (4 \times 5) - (5 \times 1) = 0$$
$$9A = 45$$
$$A = 5\ kN\ \uparrow$$

$$+\uparrow \sum F_v = 0$$
$$A - 10 + 4 + B - 5 = 0$$
$$B = 5 - 4 + 10 - 5 = 6\ kN \uparrow$$



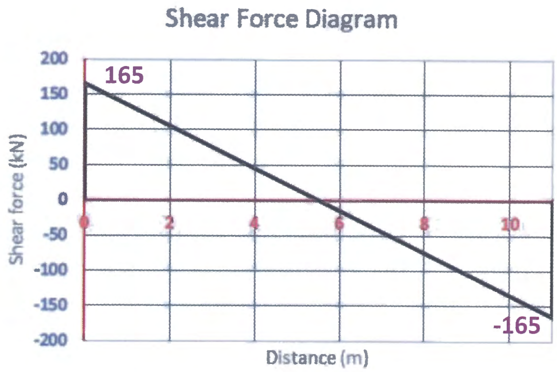


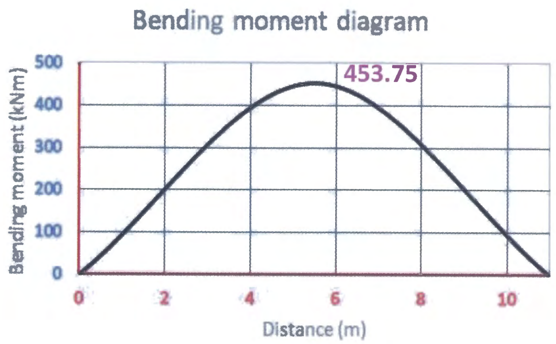
b)

$$\sigma = \frac{My}{I}$$
$$\sigma = \frac{10 \times 1000 \times 1000 \times 140}{120 \times 10^6}$$

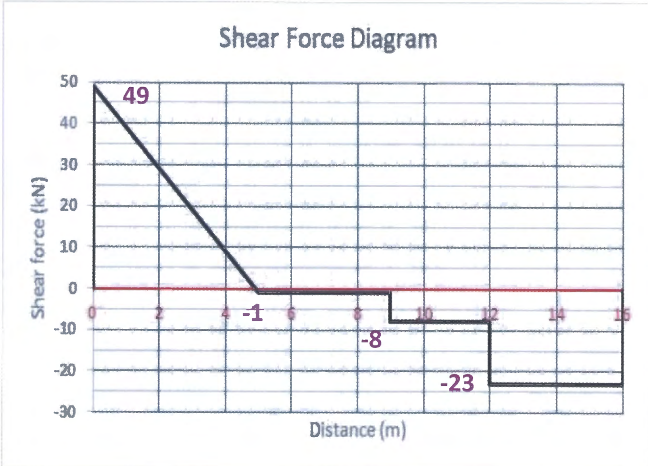
$$\therefore \sigma = 11.67\ MPa$$

53.





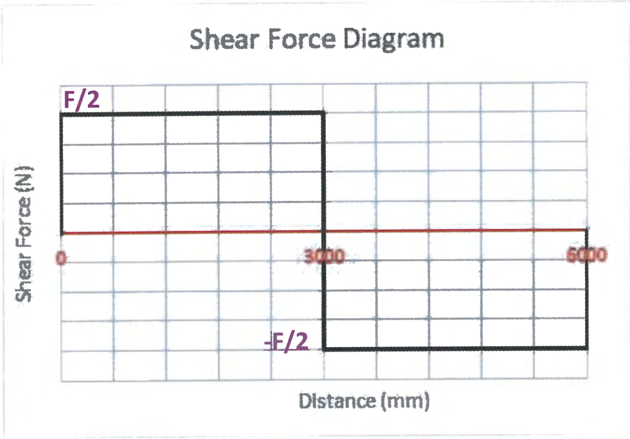
54.

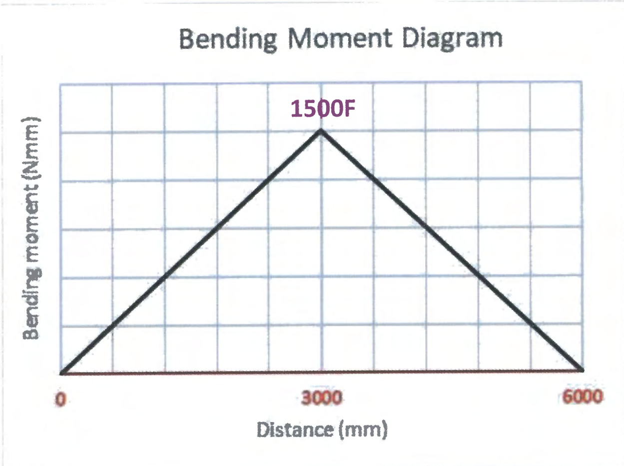


55.

$$\sigma = \frac{My}{I}$$
$$\sigma = \frac{76 \times 1000 \times 1000 \times 100}{200 \times 10^6}$$
$$\sigma = 38\ MPa$$

56. Force is acting on the middle of the beam hence it is symmetrical. Therefore, the forces at the joints will be the same and half of the applied force.





Calculating the above moment by finding the area under the graph (area of the rectangle = F/2 x 3000 = 1500F). This gives us the max moment.

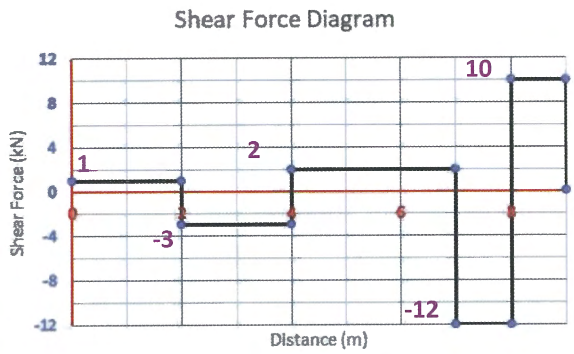
$$\sigma = \frac{My}{I}$$
$$65 = \frac{1500F \times 60}{5.26 \times 10^6}$$
$$3.419 \times 10^8 = 90000F$$

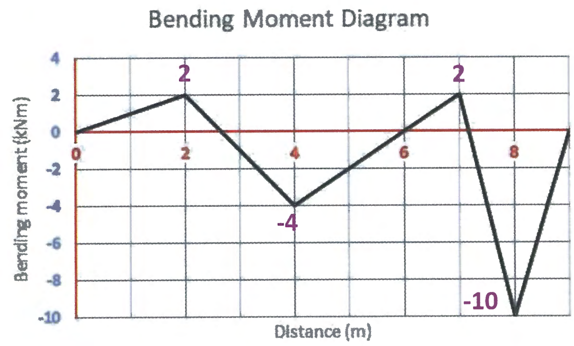
$$\therefore F = \frac{3.419 \times 10^8}{90000} = 3799\ N$$

57. a)

$$+\circlearrowleft \sum M_B = 0$$
$$-A(8) + (4 \times 6) - (5 \times 4) + (14 \times 1) - (10 \times 1) = 0$$
$$8A = 8$$
$$A = 1\ kN \uparrow$$

$$+\uparrow \Sigma F_v = 0$$
$$A - 4 + 5 - 14 + B - 10 = 0$$
$$B = 10 + 14 - 5 + 4 - 1 = 22\ kN \uparrow$$





b)

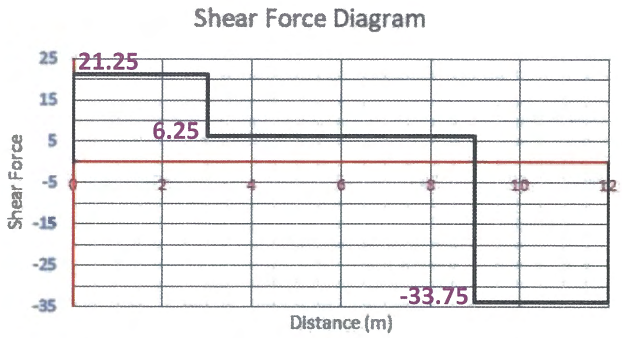
$$\sigma = \frac{My}{I}$$
$$\sigma = \frac{10 \times 1000 \times 0.120}{150 \times 10^{-6}}$$
$$\sigma = 8000000Pa$$
$$\therefore \sigma = 8\ MPa$$

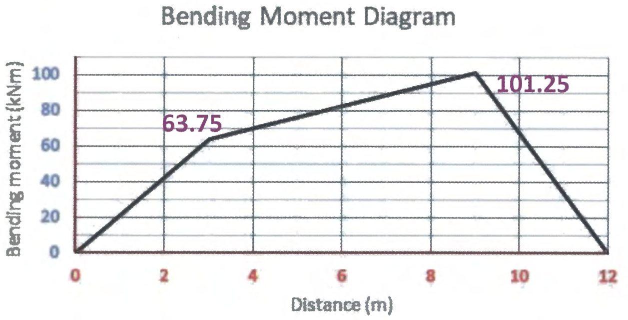
58. a)

$$+\circlearrowleft \sum M_B = 0$$
$$-A(12) + (30 \times \sin 30 \times 9) + (40 \times 3) = 0$$
$$12A = 255$$
$$\therefore A = 21.25\ kN \uparrow$$

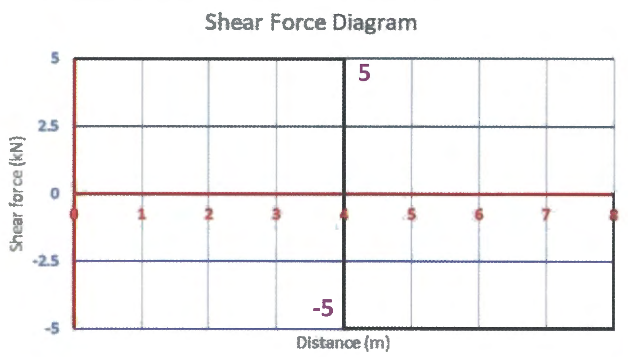
b) We are not concerned with the horizontal component of the pin joint since this is not a shear force. However, we need to find the vertical component of the pin joint as this is a shear force and will be included in the diagram.

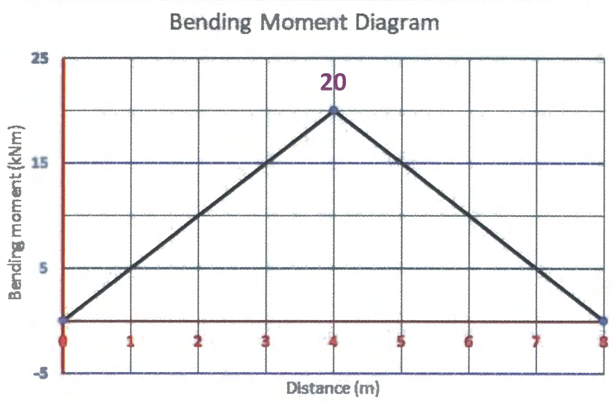
$$+\uparrow \Sigma F_v = 0$$
$$A - (30 \times \sin 30) - 40 + B = 0$$
$$B = (30 \times \sin 30) + 40 - 21.25 = 33.75\ kN \uparrow$$





59.





$$\sigma = \frac{My}{I}$$

$$\sigma = \frac{20 \times 1000 \times 1000 \times 140}{110 \times 10^6}$$

$$\sigma = 25.5\ MPa$$

60. Engineering stress is the force divided by the original cross-sectional area. True stress is the force divided by the cross-sectional area at that point in time.

$$\sigma_{engineering} = \frac{F}{A}$$

$$\sigma_{engineering} = \frac{15700}{30 \times 7} = 74.8\ MPa$$

$$\sigma_{true} = \frac{F}{A}$$

$$\sigma_{true} = \frac{15700}{5.5 \times 25} = 114.2\ MPa$$

61. Medium carbon steel is chosen due to its mixture of superior strength and excellent toughness (ductile). The yield stress value is the critical stress value for ductile materials and so it will be used in the factor of safety equation.

$$\sigma_{allowable} = \frac{\sigma_{yield}}{FS}$$

$$\delta_{allowable} = \frac{470}{2.5} = 188\ MPa$$

$$\sigma = \frac{F}{A}$$

$$188 = \frac{368000}{A}$$

$$A = \frac{368000}{188} = 1957.4\ \text{mm}^2$$

$$A = \frac{\pi}{4} d^2$$

$$d = \sqrt{\frac{4A}{\pi}}$$

$$d = \sqrt{\frac{4 \times 1957.4}{\pi}} = 50\ \text{mm}$$

62.

$$E = \frac{FL}{eA}$$

$$E_{0.2\%\ \mathrm{C}} = \frac{1250 \times 30}{0.05 \times \frac{\pi}{4} \times 6^2} = 26526\ MPa$$

$$\therefore E_{0.2\%\ \mathrm{C}} = 27\ GPa$$

$$E_{0.35\%\,\mathrm{C}} = \frac{3500 \times 30}{0.0225 \times \frac{\pi}{4} \times 6^2} = 165050\ MPa$$

$$\therefore E_{0.35\%\,\mathrm{C}} = 165\ GPa$$

$$E_{Brass} = \frac{1000 \times 30}{0.05 \times \frac{\pi}{4} \times 6^2} = 21221\ MPa$$

$$\therefore E_{Brass} = 21\ GPa$$

$$E_{Al} = \frac{750 \times 30}{0.01 \times \frac{\pi}{4} \times 6^2} = 79577\ MPa$$

$$\therefore E_{Al} = 80\ GPa$$

$$UTS = \frac{F_{\max}}{A}$$

$$UTS_{0.2\%\,\mathrm{C}} = \frac{3500}{\frac{\pi}{4} \times 6^2} = 124\ MPa$$

$$UTS_{0.35\%\,\mathrm{C}} = \frac{6300}{\frac{\pi}{4} \times 6^2} = 223\ MPa$$

$$UTS_{Brass} = \frac{4500}{\frac{\pi}{4} \times 6^2} = 159\ MPa$$

$$UTS_{Al} = \frac{4100}{\frac{\pi}{4} \times 6^2} = 145\ MPa$$

Chapter 3 – Solutions

1. (A) & (C)
2. (B) – Load is in between the fulcrum and the effort force
3. (A)
4. (A) - $VR = \frac{2\pi r}{pitch} = \frac{2 \times \pi \times 500\ mm}{30\ mm} = 105$. Since its 100% efficient, MA = VR = 105.
5. (C)
6. (B) -

$$VR = 2$$
$$\eta = \frac{MA}{VR}$$
$$0.9 = \frac{MA}{2}$$
$$\therefore MA = 1.8$$

$$MA = \frac{L}{E}$$
$$1.8 = \frac{225}{E}$$
$$E = \frac{225}{1.8} = 125\ N$$

7. (D) -

$$MA = \frac{L}{E}$$
$$MA = \frac{1760}{550} = 3.2$$

$$VR = 4$$
$$\eta = \frac{MA}{VR}$$
$$\eta = \frac{3.2}{4} = 0.8$$
$$\therefore \eta = 0.8 \times 100$$
$$\therefore \eta = 80\ \%$$

8. (C)
9. (B) - $VR = \frac{d_E}{d_L}$ $\therefore VR = \frac{6}{3} = 2$
10. (A)
11. a)

$$VR = \frac{Driven}{Driver} \qquad \eta = \frac{MA}{VR}$$
$$VR = \frac{30}{15} = 2 \qquad 0.9 = \frac{MA}{2}$$
$$\therefore MA = 1.8$$

b) Gear A and C are meshed together. Since gear C is larger than gear A, gear C will rotate less per minute.

Apply the speed ratio formula to find the RPM which is equal to:

$$RPM = \text{input speed x } \frac{driver}{driven}$$

$$RPM_C = 200 \times \frac{15}{20} = 150\ rpm$$

Now gear C is meshed with gear B. Since gear B is larger than gear C, gear B will rotate less.

$$RPM_B = 150 \times \frac{20}{30} = 100\ rpm$$

Gear A rotates CCW so C will rotate in the opposite direction CW. This will cause B to rotate CCW. Alternatively, the idler gear allows the driver and driven gear to rotate in the same direction hence CCW.

12.

$$\eta = \frac{MA}{VR}$$
$$0.9 = \frac{2.5}{VR}$$
$$VR = \frac{2.5}{0.9} = 2.78$$

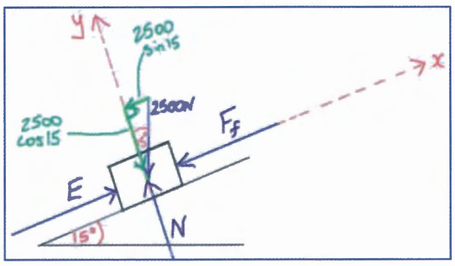
$$VR_{hydraulics} = \frac{\phi^2_{output}}{\phi^2_{input}}$$
$$2.78 = \frac{30^2}{\phi^2_{input}}$$
$$\phi^2_{input} = \frac{30^2}{2.78} = 324$$
$$\therefore \phi_{input} = \sqrt{324} = 18 \text{ mm}$$

13.

$$SR = \frac{driver}{driven}$$
$$SR_{AC} = \frac{15}{40} = 0.375$$
$$\therefore RPM_c = 128 \times 0.375 = 48\, rpm$$
$$\therefore RPM_B = 48rpm$$
$$SR_{BD} = \frac{30}{20} = 1.5$$
$$\therefore RPM_D = 48 \times 1.5 = 72\, rpm$$

Gear A meshes with gear C. Gear C is larger than A so it will rotate at a slower rpm. Gear B and C share the same shaft so B will rotate at the same speed as B which now meshes with D. Gear D is smaller than B so it will rotate at a faster speed to keep up with the bigger gear B. Gear A rotates clockwise so gear C will rotate anticlockwise. B will rotate in the same direction as C since B and C lie on the same shaft. Now D will rotate opposite to B hence **clockwise.**

14. a)



$$+\nwarrow \Sigma F_y = 0$$
$$-2500\cos 15 + N = 0$$
$$\therefore N = 2414.81\, N$$

$$F_f = \mu N$$
$$F_f = 0.2 \times 2414.81 = 482.96\, N$$

$$+\nearrow \sum F_{x=0}$$
$$E - F_f - 2500\sin 15 = 0$$
$$E = 482.96 + 2500\sin 15 = 1130\, N$$

b)

$$VR_{ramp} = \frac{1}{\sin\theta}$$
$$VR_{ramp} = \frac{1}{\sin 15} = 3.86$$

$$MA = \frac{L}{E}$$
$$MA = \frac{2500}{1130} = 2.21$$

$$\eta = \frac{MA}{VR} \times 100$$
$$\eta = \frac{2.21}{3.86} \times 100 = 57.3\,\%$$

15.

$$VR = \frac{driven}{driver}$$
$$VR = \frac{16}{24} = 0.67$$

$$\eta = \frac{MA}{VR}$$
$$0.8 = \frac{MA}{0.67}$$
$$\therefore MA = 0.53$$

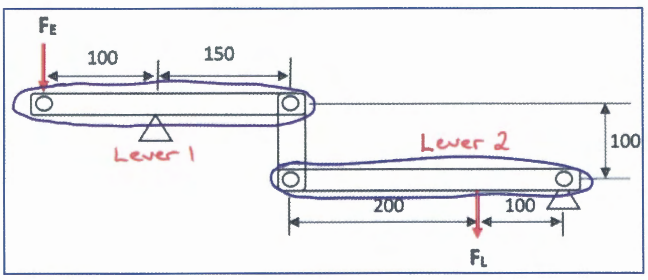
This gear system is a speed multiplier since MA is less than 1.

16.

$$VR_{screw} = \frac{2\pi r}{pitch}$$
$$VR_{screw} = \frac{2\times\pi\times 45}{4} = 70.69$$

$$\eta = \frac{MA}{VR}$$
$$0.85 = \frac{MA}{70.69}$$
$$\therefore MA = 70.69 \times 0.85 = 60.1$$

17. a)



We treat this problem as 2 separate levers, find their velocity ratio and multiply them together to calculate the velocity ratio of the compound machine.

$$VR = \frac{d_E}{d_L}$$

$$VR_1 = \frac{100}{150} = \frac{2}{3}$$

$$VR_2 = \frac{300}{100} = 3$$

$$VR_{Total} = VR_1 \times VR_2$$
$$\therefore VR_{Total} = \frac{2}{3} \times 3 = 2$$

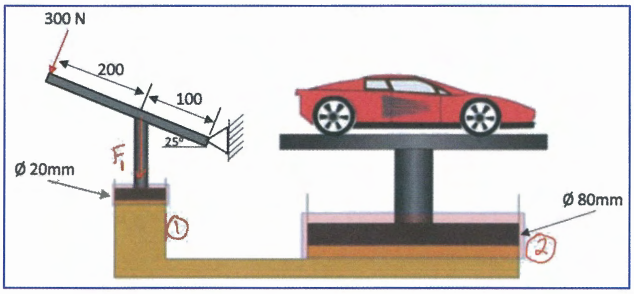
b) Since frictional effort accounts for 30% of actual effort then efficiency of the machine is 100-30 = 70%.

$$\eta = \frac{MA}{VR}$$
$$0.7 = \frac{MA}{2}$$
$$\therefore MA = 0.7 \times 2 = 1.4$$

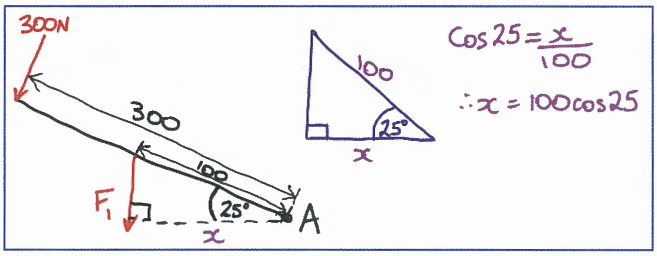
c)

$$MA = \frac{L}{E}$$
$$1.4 = \frac{40}{E}$$
$$E = \frac{40}{1.4} = 28.6N$$

18.



Calculate the force that is exerted by the input piston. To do this, sum the moments about the pin joint. **Note**: Moment = force × perpendicular distance.



$$+\circlearrowleft \sum M_A = 0$$
$$(300 \times 300) + F_1(100\cos 25) = 0$$
$$F_1 = \frac{-90000}{100\cos 25} = -993N$$
$$\therefore F_1 = 993N$$

We now apply Pascal's principle whereby the pressure applied to an enclosed fluid is the same at every point in the fluid. Hence pressure underneath piston 1 is equal to the pressure underneath piston 2.

$$P_1 = P_2$$
$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\frac{993}{\left(\frac{\pi}{4} \times 20^2\right)} = \frac{F_2}{\left(\frac{\pi}{4} \times 80^2\right)}$$

$$F_2 = \frac{993 \times 80^2 \times \frac{\pi}{4}}{20^2 \times \frac{\pi}{4}} = 15888.6N$$

Mass of the car is 1500kg which equates to 15000 N (less than the force produced by piston 2). Therefore, an effort of 300N is enough to lift the car.

19.

$$\eta = \frac{MA}{VR}$$
$$0.45 = \frac{MA}{15}$$
$$\therefore MA = 15 \times 0.45 = 6.75$$

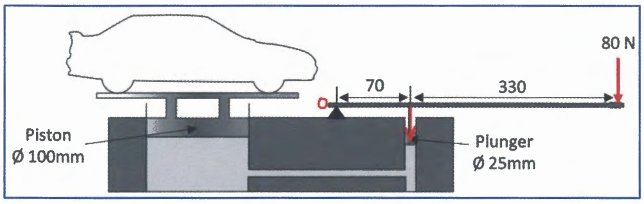
$$MA = \frac{L}{E}$$
$$6.75 = \frac{L}{100}$$
$$\therefore L = 100 \times 6.75 = 675N$$

20.

$$\eta = \frac{MA}{VR}$$
$$0.75 = \frac{MA}{8}$$
$$MA = 8 \times 0.75 = 6$$

$$MA = \frac{L}{E}$$
$$6 = \frac{2500}{E}$$
$$E = \frac{2500}{6} = 416.7N$$

21. a)



Analyse the lever and apply sum of moments about the pivot O to find the magnitude of the force in the plunger.

$$+\circlearrowleft \Sigma M_0 = 0$$
$$(-80 \times 400) - (F_p \times 70) = 0$$
$$F_p = -\frac{32000}{70} = -457.1N$$
$$\therefore F_p = 457.1N$$

b) According to Pascal's principle, the pressure underneath piston 1 will be the same pressure applied to the fluid at all points.

$$P_1 = \frac{F}{A}$$
$$P_1 = \frac{457.1}{\left(\frac{\pi}{4} \times 25^2\right)} = 0.93\ MPa$$

c)

$$P_1 = P_2 = 0.93\ MPa$$
$$P_2 = \frac{F_2}{A_2}$$
$$F_2 = PA_2$$
$$\therefore F_2 = 0.93 \times \left(\frac{\pi}{4} \times 100^2\right) = 7304.2\ N$$

Note: Pressure is in MPa so area needs to be in mm to get our force in Newtons.

22.

$$\eta = \frac{MA}{VR}$$
$$\therefore VR = \frac{MA}{\eta} = \frac{8}{0.8} = 10$$

$$VR = \frac{d_i}{d_0}$$

$$d_i = d_0 \times VR$$
$$\therefore d_i = 20 \times 10 = 200\text{mm}$$

23.

$$\frac{d_{in}}{d_{out}} = \frac{\phi_{out}^2}{\phi_{in}^2}$$
$$\frac{d_{in}}{6} = \frac{33^2}{11^2}$$
$$d_{in} = 54\text{mm}$$

24.

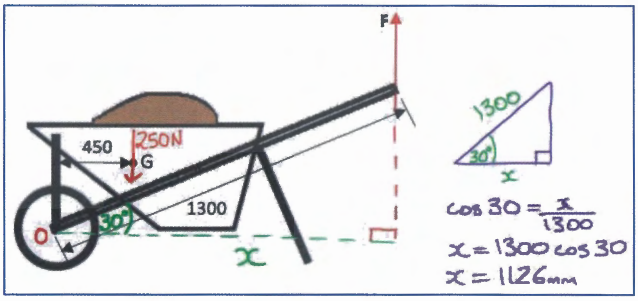
$$GR = \frac{driven}{driver} = \frac{42}{14} = 3$$
$$\therefore GR = 3:1$$

OR

$$GR = \frac{GearB}{GearA} \times \frac{Gearc}{GearB}$$

$$GR = \frac{28}{14} \times \frac{42}{28}$$
$$\therefore GR = 3:1$$

25. a)



Apply sum of moments about O to find F. Note: Find the horizontal distance (x) by applying trig as above. This will serve as the perpendicular distance to F. Convert distances to metres.

$$+\circlearrowleft \sum M_0 = 0$$
$$(-250 \times 0.450) + F(1.126) = 0$$
$$F = \frac{112.5}{1.126} = 100\ N$$

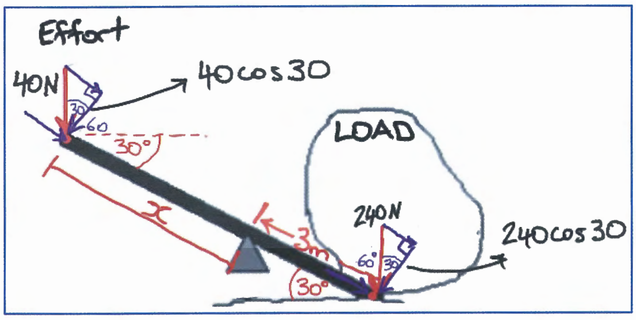
b)

$$MA = \frac{L}{E}$$
$$MA = \frac{250}{100} = 2.5$$

26. a)

$$MA = \frac{L}{E}$$
$$MA = \frac{240}{40} = 6$$

b)



$$+\circlearrowleft \Sigma M_0 = 0$$
$$(40\cos30 \times x) - (240\cos30 \times 3) = 0$$
$$x = \frac{240\cos30 \times 3}{40\cos30} = 18m$$

c)

$$VR = \frac{d_E}{d_L}$$
$$VR = \frac{18}{3} = 6$$

d) Only MA is affected by friction and any other losses. VR remains constant.

$$\eta = \frac{MA}{VR}$$
$$0.8 = \frac{MA}{6}$$
$$MA = 4.8$$

$$MA = \frac{L}{E}$$
$$4.8 = \frac{240}{E}$$
$$E = \frac{240}{4.8} = 50N$$

27. a) Since this is a 2nd order lever, the load is placed between the effort and the fulcrum. Therefore, 330N represents the load and 110N represents the effort.

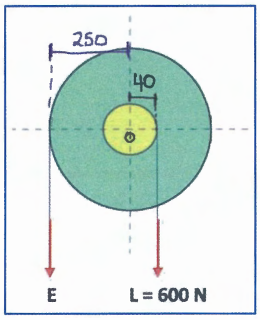
$$VR = \frac{d_E}{d_L}$$
$$VR = \frac{200}{50} = 4$$

b)

$$MA = \frac{L}{E}$$
$$MA = \frac{330}{110} = 3$$

$$\eta = \frac{MA}{VR} \times 100$$
$$\therefore \eta = \frac{3}{4} \times 100 = 75\%$$

28.



Apply sum of moments about the centre of the axle to find the effort force.

$$+\circlearrowleft \Sigma M_0 = 0$$
$$E(0.250) - (600 \times 0.040) = 0$$
$$E = 96N$$

Now that we have the effort force, we use the MA equation to find the mechanical advantage.

$$MA = \frac{L}{E}$$
$$MA = \frac{600}{96} = 6.25$$

29.

$$\eta = \frac{MA}{VR}$$
$$0.75 = \frac{1.35}{VR}$$
$$\therefore VR = \frac{1.35}{0.75} = 1.8$$

To calculate velocity ratio for compound gear train, we multiply velocity ratio of the meshed gears where VR = no. of teeth on driven gear divided by no. of teeth on driver gear. By doing this, we can find the number of teeth on gear 4.

$$VR_T = VR_{1-3} \times VR_2 - 4$$

$$1.8 = \frac{80}{40} \times \frac{driven}{60}$$

$$1.8 = \frac{2driven}{60}$$

$$driven = \frac{1.8 \times 60}{2} = 54\ teeth$$

30.

$$RPM_B = input \times \frac{driver}{driven}$$

$$RPM_B = 200 \times \frac{20}{40} = 100\ rpm$$

Pulley B and C share the same shaft hence they will rotate at the same speed.

$$RPM_D = 100 \times \frac{20}{60} = 33.3\ rpm$$

Since the pulley belts are not crossed, they will continue to rotate in the original direction. Hence clockwise.

31.

$$RPM_x = Input \times \frac{driver}{driven}$$

$$RPM_x = 175 \times \frac{60}{40} = 262.5\ rpm$$

Pulley X and Y share the same shaft hence rotate at the same speed.

$$RPM_z = 262.5 \times \frac{20}{50} = 105\ rpm$$

If pulley W rotates anticlockwise then X will rotate clockwise due to the pulley belt crossing. Y shares the same shaft as X so will rotate clockwise as well. Z will now rotate in a clockwise direction since the pulley belt between y and Z is not crossed.

32. Convert 90 kPa to MPa by dividing it by 1000 as we are dealing with mm. Apply pressure formula to find the force required.

$$P = \frac{F}{A}$$

$$0.09 = \frac{F}{1.85 \times 10^4}$$

$$F = 0.09 \times 1.85 \times 10^4 = 1665\ N$$

33. a)

$$VR_{screw} = \frac{2\pi r}{pitch}$$

$$VR_{screw} = \frac{2 \times \pi \times 700}{6} = 733.04$$

Since there are no frictional forces, we can assume 100% efficiency hence VR = MA

$$\therefore MA = VR = 733.04$$

b) Convert 1.2 tonnes to N by multiplying it by 10000 (x 1000 → kgs, x 10 → N.

$$MA = \frac{L}{E}$$

$$733.04 = \frac{12000}{E}$$

$$E = \frac{12000}{733.04} = 16.4\ N$$

34. a)

$$VR_{screw} = \frac{2\pi r}{pitch}$$

$$VR_{screw} = \frac{2 \times \pi \times 450}{5} = 565.5$$

b)

$$\eta = \frac{MA}{VR}$$

$$0.35 = \frac{MA}{565.5}$$

$$MA = 0.35 \times 565.5 = 197.9$$

$$MA = \frac{L}{E}$$

$$197.9 = \frac{6500}{E}$$

$$E = \frac{6500}{197.9} = 32.84\ N$$

c)

$$MA = \frac{L}{E}$$

$$MA = \frac{4000}{32.84} = 121.8$$

35.a)

$$MA = \frac{L}{E}$$

$$MA = \frac{450}{150} = 3$$

b) $VR = \frac{d_E}{d_L} = \frac{R}{r}$

$$VR = \frac{600}{100} = 6$$

c)

$$\eta = \frac{MA}{VR} \times 100$$
$$\eta = \frac{3}{6} \times 100 = 50\ \%$$

36. Calculate MA then VR. Substitute these values into the efficiency formula to find efficiency.

$$MA = \frac{L}{E}$$
$$MA = \frac{80}{485} = 0.165$$

The load is also known as the resistance hence load equals 80N.

$$VR = \frac{d_E}{d_L} = \frac{d}{D \times N_{revs}}$$

$$VR = \frac{2 \times \pi \times 165}{\pi \times 650 \times \frac{40}{15}} = 0.190$$

Effort and load move in a circular motion hence distance it covers will be the circumference of that circle. Due to the bike's gearing, for every rotation of the crank, the rear wheel (attached to the load) will rotate $\frac{40}{15}$ times. **Note**: The smaller gear will always rotate more than the bigger gear.

$$\eta = \frac{MA}{VR} \times 100$$
$$\eta = \frac{0.165}{0.190} \times 100 = 87\%$$

37.a)

$$+\circlearrowleft \sum M_{pivot} = 0$$
$$(-48 \times 2) + (F_m \times 6) = 0$$
$$6F_m = 96$$
$$F_m = \frac{96}{6} = 16\ N \downarrow$$

b)

$$+\uparrow \sum F_v = 0$$
$$F_J - 48 - F_m = 0$$
$$F_J = 48 + 16 = 64\ N$$

38.

$$VR_{ramp} = \frac{1}{\sin\theta}$$
$$\therefore VR = \frac{1}{\sin 3} = 19.11$$

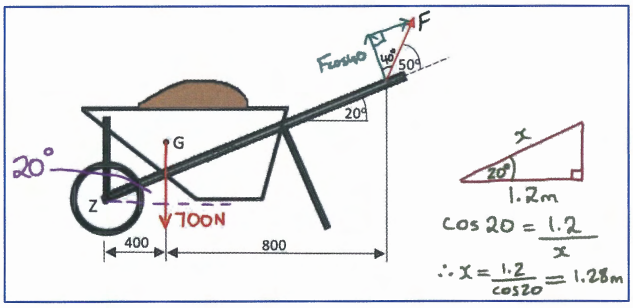
$$\eta = \frac{MA}{VR}$$
$$MA = \eta \times VR$$

$$\therefore MA = 0.85 \times 19.11 = 16.24$$

$$MA = \frac{L}{E}$$

$$\therefore E = \frac{L}{MA} = \frac{22000}{16.24} = 1355\ N$$

39.



Convert F into the component that is perpendicular to the handle arm. We then need to find the length of the handle arm using simple trig shown above. We can now apply the sum of moments equation about Z. Keeping the distances in mm will yield the same result. **Note**: Moment = Force x perpendicular distance.

$$+\circlearrowleft \Sigma M_z = 0$$
$$-700(0.4) + F\cos 40(1.28) = 0$$
$$F = \frac{280}{1.28\cos 40} = 286\ N$$

40.a)

$$P_1 = P_2$$
$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$
$$\frac{F_1}{\frac{\pi}{4} \times (80^2)} = \frac{12000}{\frac{\pi}{4} \times (200^2)}$$
$$F_1 = \frac{12000 \times \frac{\pi}{4} \times 80^2}{\frac{\pi}{4} \times 200^2} = 1920\ N$$

b)

$$P = \frac{F}{A}$$
$$P = \frac{12000}{\frac{\pi}{4} \times 200^2} = 0.382\ MPa$$
$$\therefore P = 382\ kPa$$

Divide by 1000 to convert MPa to kPa

c)

$$\frac{d_{input}}{d_{output}} = \frac{\phi^2_{output}}{\phi^2_{input}}$$
$$\frac{d}{800} = \frac{200^2}{80^2}$$
$$d = \frac{200^2 \times 800}{80^2} = 5000 \text{ mm}$$

$$\therefore d = 5\ m$$

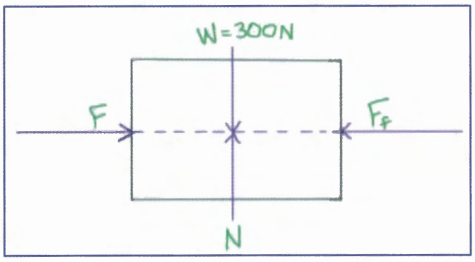
41.

$$+\sum M_{elbow} = 0$$
$$-110(300) - 30(140) + F_B(50) = 0$$

$$F_B = \frac{110(300) + 30(140)}{50} = 744\ N$$

Chapter 4 Solutions

1. (B)



$$+\uparrow \sum F_v = 0$$
$$-300 + N = 0$$
$$\therefore N = 300N$$

$$F_f = \mu N$$
$$F_f = 0.4 \times 300$$
$$\therefore F_f = 120N$$

$$\overset{\rightarrow}{+} \sum F_H = 0$$
$$F - F_f = 0$$
$$\therefore F = 120N$$

2. (A) -

$$\mu = \tan\theta$$
$$\theta = \tan^{-1}(\mu)$$
$$\theta = \tan^{-1}(0.3)$$
$$\theta = 16.7°$$

An angle of 16.7 would be required for the block to move down the ramp. The ramp is inclined at 25 (steeper angle) and hence will cause the box to slide. An acceleration due to gravity of 10m/s^2 pushes the box down the ramp at an increasing rate.

3. (D) - Friction Force is always parallel to the plane and opposite to the motion's direction whether it's on flat ground or on an incline.

4. (D) - Normal force is always perpendicular (at 90^O) to the surface.

5. (B) -

$$\mu = \tan\theta$$
$$\theta = \tan^{-1}(\mu)$$
$$\theta = \tan^{-1}(0.5)$$
$$\theta = 26.6°$$

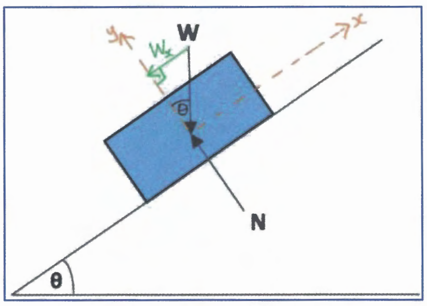
Therefore, the largest possible angle before the car starts to slide down is 26.

6. (A) -

$$\mu = \tan\theta = \tan 15$$
$$\mu = 0.27$$

7. (A)

8. (A) -



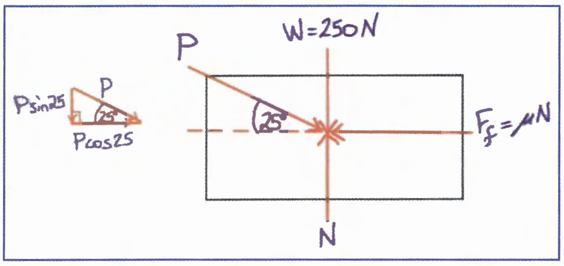
$$\sin\theta = \frac{W_x}{W}$$
$$W_x = W\sin\theta$$

9. (D)

10. (C) – Weight, drag and lift are the forces – no thrust.

11. (A) - Lubricating the 2 surfaces will lower the coefficient of friction hence reducing the frictional force.

12.



Sum the forces in the vertical direction and equate to 0 to find the normal force in terms of P.

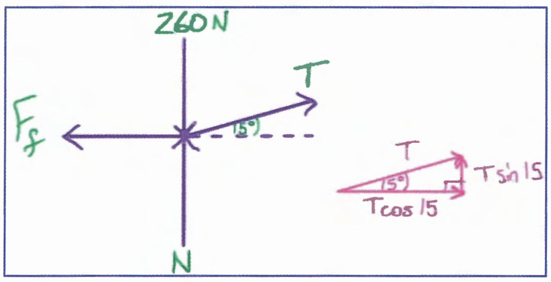
$$+\uparrow \Sigma F_v = 0$$
$$-250 + N - P\sin 25 = 0$$
$$N = P\sin 25 + 250$$

Sum the forces in the horizontal direction and equate to 0. Sub the frictional force equation into F_f and then substitute the normal equation into N. Factorise P out and solve. Treat this as a simultaneous equation problem.

$$\overrightarrow{+}\ \sum F_H = 0$$
$$P\cos 25 - F_f = 0$$
$$P\cos 25 - (0.25N) = 0$$
$$P\cos 25 - 0.25(P\sin 25 + 250) = 0$$
$$P\cos 25 - 0.25P\sin 25 - 62.5 = 0$$
$$P(\cos 25 - 0.25\sin 25) = 62.5$$

$$P = \frac{62.5}{\cos 25 - 0.25\sin 25} = 78\ N$$

13.



Sum the forces in the vertical direction to find the normal force in terms of T.

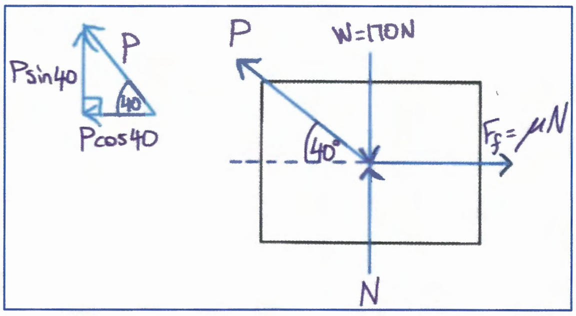
$$+\uparrow \Sigma F_v = 0$$
$$-260 + N + T\sin 15 = 0$$
$$N = 260 - T\sin 15$$

Sum the forces in the horizontal direction. Sub the frictional force equation into F_f and then substitute the normal equation into N. Factorise T out and solve.

$$\overrightarrow{+}\ \sum F_H = 0$$
$$T\cos 15 - F_f = 0$$
$$T\cos 15 - (0.18N) = 0$$
$$T\cos 15 - 0.18(260 - T\sin 15) = 0$$
$$T\cos 15 - 46.8 + 0.18T\sin 15 = 0$$
$$T(\cos 15 + 0.18\sin 15) = 46.8$$

$$T = \frac{46.8}{\cos 15 + 0.18\sin 15} = 46.2\ N$$

14.



Sum the forces in the vertical direction to find the normal force in terms of P.

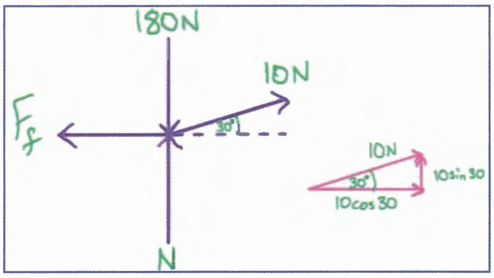
$$+\uparrow \sum F_v = 0$$
$$-170 + N + P\sin 40 = 0$$
$$N = 170 - P\sin 40$$

Sum the forces in the horizontal direction. Sub the frictional force equation into F_f and then substitute the normal equation into N. Factorise P out and solve.

$$\overrightarrow{+}\ \sum F_H = 0$$
$$-P\cos 40 + F_f = 0$$
$$-P\cos 40 + (0.6N) = 0$$
$$-P\cos 40 + 0.6(170 - P\sin 40) = 0$$
$$-P\cos 40 + 102 - 0.6P\sin 40 = 0$$
$$P(0.6\sin 40 + \cos 40) = 102$$

$$P = \frac{102}{0.6\sin 40 + \cos 40} = 88.6\ N$$

15.



Sum the forces in the vertical direction to find the normal force.

$$+\uparrow \Sigma F_v = 0$$
$$-180 + N + 10\sin 30 = 0$$
$$N = 180 - 10\sin 30 = 175\ \text{N}$$

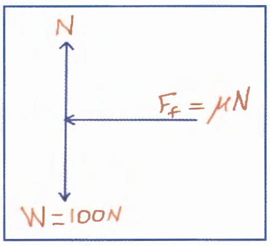
Sum the forces in the horizontal direction to find the frictional force.

$$\overrightarrow{+}\ \Sigma F_H = 0$$
$$-F_f + 10\cos 30 = 0$$
$$F_f = 10\cos 30 = 8.66\ \text{N}$$

Use the frictional force equation to find the coefficient of friction.

$$F_f = \mu \text{N}$$
$$8.66 = \mu(175)$$
$$\mu = \frac{8.66}{175} = 0.05$$
$$\therefore \mu = 0.05$$

16. A FBD of the normal force, weight force and the friction force should be drawn. If the applied force is greater than the frictional force then the trash can will move.

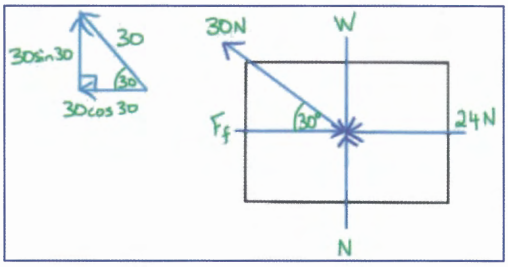


$$+\uparrow \Sigma F_v = 0$$
$$N - 100 = 0$$
$$N = 100\ N$$

$$F_f = \mu \text{N}$$
$$F_f = 0.45 \times 100$$
$$\therefore F_f = 45\ N$$

The trash can will move since the applied force of 50 N is greater than the frictional force.

17. a)



b)

$$\overrightarrow{+}\ \Sigma F_H = 0$$
$$F_f - 24 - 30\cos 30 = 0$$
$$F_f = 24 + 30\cos 30$$
$$F_f = 50\ N$$

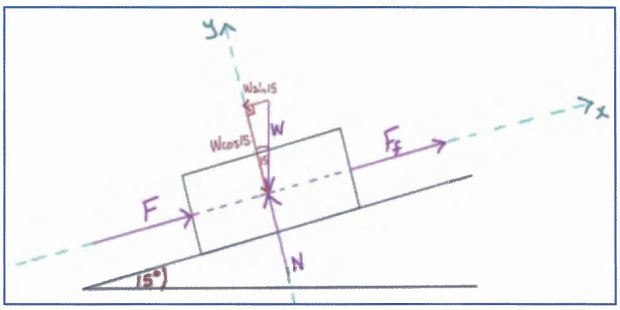
c)

$$F_f = \mu \text{N}$$
$$50 = 0.25N$$
$$N = \frac{50}{0.25}$$
$$\therefore N = 200\ N$$

d)

$$+\uparrow \Sigma F_v = 0$$
$$30\sin 30 - W + N = 0$$
$$W = 30\sin 30 + 200$$
$$\therefore W = 215\ N$$
$$Mass = W \div 10$$
$$\therefore Mass = 21.5\ \text{kg}$$

18.



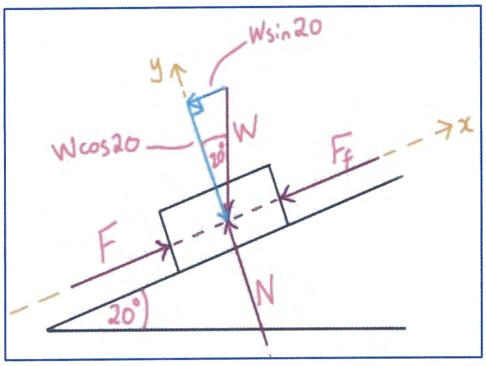
The piano has the tendency to slip hence the frictional force will be directed opposite to its motion (up the ramp). We need to calculate the force required to stop it from slipping.

$$+\nwarrow \sum F_y = 0$$
$$-W\cos 15 + N = 0$$
$$N = 4000 \times \cos 15 = 3863.7 \text{ N}$$

Remember to sub μN into F_f where μ = 0.25.

$$+\nearrow \sum F_x = 0$$
$$F_f + F - W\sin 15 = 0$$
$$F = W\sin 15 - (0.25N)$$
$$F = 4000\sin 15 - (0.25 \times 3863.7)$$
$$\therefore F \approx 69\ N$$

19.

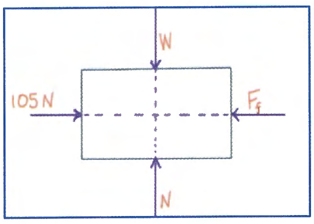


$$+\nwarrow \Sigma F_y = 0$$
$$-W\cos 20 + N = 0$$
$$N = W\cos 20 = 200\cos 20 = 188\ N$$

$$F_f = \mu N$$
$$F_f = 0.4 \times 188 = 75.2\ N$$

$$+\nearrow \sum F_x = 0$$
$$F - F_f - W\sin 20 = 0$$
$$F = F_f + W\sin 20$$
$$\therefore F = 75.2 + 200\sin 20 = 143.6\ N$$

20.



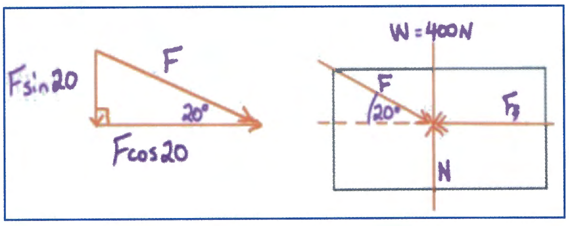
$$\overrightarrow{+} \Sigma F_x = 0$$
$$105 - F_f = 0$$
$$F_f = 105\ N$$

$$F_f = \mu N$$
$$105 = 0.3N$$
$$\therefore N = \frac{105}{0.3} = 350\ N$$

$$+\uparrow \sum F_y = 0$$
$$-W + N = 0$$
$$W = N$$
$$\therefore W = 350\ N$$

$$\therefore mass = \frac{350}{10} = 35\ kg$$

21.



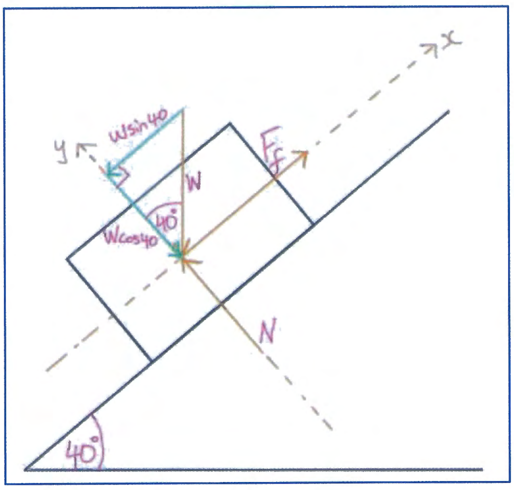
The 20-degree angle is positioned between the x-axis and the force vector due to alternate angles. The force vector is broken up into its components shown above. Apply the sum of forces equation in both the vertical and horizontal directions to solve for F using simultaneous equations (by substitution).

$$+\uparrow \Sigma F_v = 0$$
$$-F\sin 20 - 400 + N = 0$$
$$N = F\sin 20 + 400$$

$$\overrightarrow{+} \Sigma F_H = 0$$
$$F\cos 20 - F_F = 0$$
$$F\cos 20 - (0.08N) = 0$$
$$F\cos 20 - 0.08(F\sin 20 + 400) = 0$$
$$F\cos 20 - 0.08F\sin 20 - 32 = 0$$
$$F(\cos 20 - 0.08\sin 20) = 32$$

$$F = \frac{32}{\cos 20 - 0.08\sin 20} = 35\ N$$

22. a)



b)

$$+\nwarrow \Sigma F_y = 0$$
$$-W\cos40 + N = 0$$
$$N = 100 \times \cos40$$
$$\therefore N = 76.6\ N$$

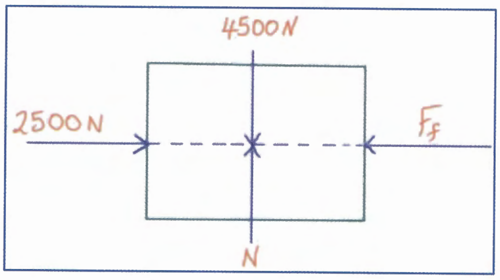
c)

$$+\nearrow \sum F_x = 0$$
$$F_f - W\sin40 = 0$$
$$F_f = 100 \times \sin40$$
$$F_f = 64.3\ N$$

d)

$$F_f = \mu\text{N}$$
$$64.3 = \mu(76.6)$$
$$\mu = \frac{64.3}{76.6}$$
$$\mu = 0.84$$
$$OR$$
$$\mu = \tan\theta = \tan 40$$
$$\mu = 0.84$$

23.

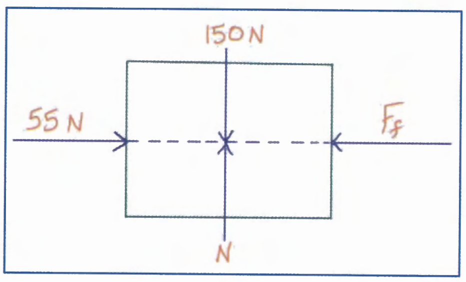


$$+\uparrow \Sigma F_v = 0$$
$$-4500 + N = 0$$
$$N = 4500\ N$$

$$\overset{\rightarrow}{+} \sum F_H = 0$$
$$2500 - F_f = 0$$
$$F_f = 2500\ N$$

$$F_f = \mu\text{N}$$
$$\mu = \frac{F_f}{N} = \frac{2500}{4500}$$
$$\therefore \mu = 0.56$$

24.

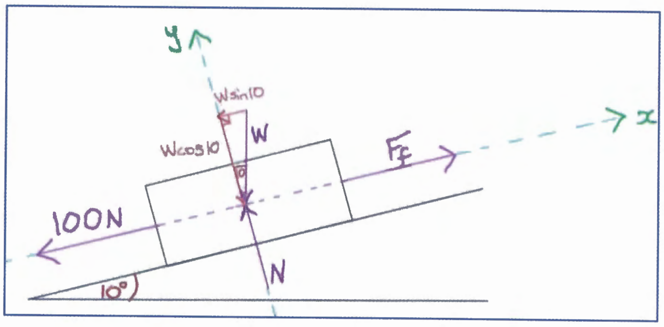


$$+\uparrow \Sigma F_v = 0$$
$$-150 + N = 0$$
$$N = 150\ N$$

$$\overset{\rightarrow}{+} \sum F_H = 0$$
$$55 - F_f = 0$$
$$F_f = 55\ N$$

$$F_f = \mu\text{N}$$
$$\mu = \frac{F_f}{N} = \frac{55}{150}$$
$$\therefore \mu = 0.37$$

25.



$$+\nwarrow \Sigma F_y = 0$$
$$-W\cos10 + N = 0$$
$$N = W\cos10 = 1000 \times \cos10$$

$$\therefore N = 984.8\ N$$

$$+\nearrow \Sigma F_x = 0$$
$$-W\sin10 + F_f - 50 = 0$$
$$F_f = 100 + W\sin10 = 100 + (1000 \times \sin10)$$

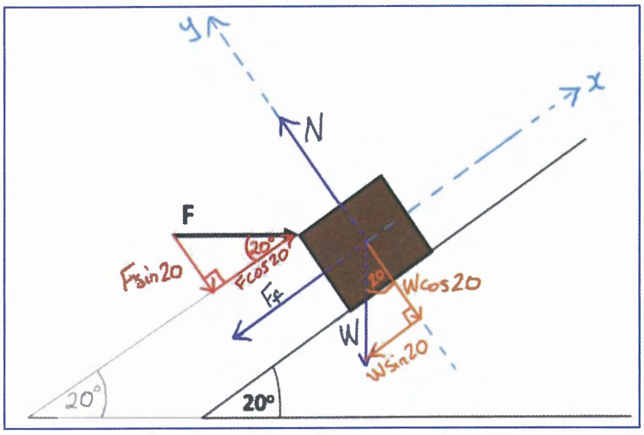
$$\therefore F_f = 273.6\ N$$

$$F_f = \mu\text{N}$$
$$\mu = \frac{F_f}{N}$$

$$\mu = \frac{273.6}{984.8} = 0.28$$

Sled will be waxed wood on wet snow or rubber on dry snow since they both have lower coefficient of friction than what is needed. Hence, it will have a smaller frictional force and so sled will slide.

26.



$$+\nwarrow \Sigma F_y = 0$$
$$N - W\cos 20 - F\sin 20 = 0$$
$$N = F\sin 20 + 300\cos 20$$

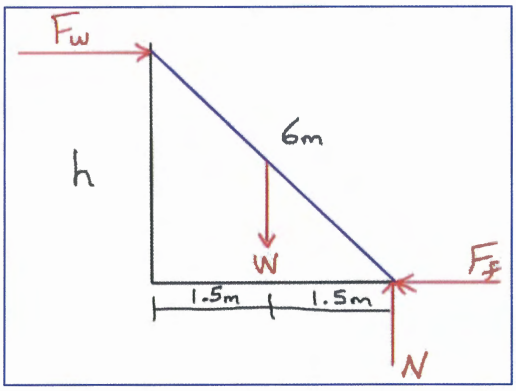
$$+\nearrow \Sigma F_x = 0$$
$$F\cos 20 - F_f - W\sin 20 = 0$$
$$F_f = F\cos 20 - 300\sin 20$$
$$0.25N = F\cos 20 - 300\sin 20$$

Substitute the equation for N into the above equation and solve for F by making it the subject.

$$0.25 \times (F\sin 20 + 300\cos 20) = F\cos 20 - 300\sin 20$$
$$0.25F\sin 20 + 70.48 = F\cos 20 - 102.61$$
$$F(\cos 20 - 0.25\sin 20) = 70.48 + 102.61$$
$$F = \frac{173.08}{\cos 20 - 0.25\sin 20}$$

$$\therefore F \approx 203\ N$$

27. a)



Use Pythagoras to determine the height of the wall.

$$6^2 = h^2 + 3^2$$
$$36 = h^2 + 9$$
$$h = \sqrt{36 - 9} = 5.2\ \mathrm{m}$$

To find the frictional force, we need to first find the reaction force at the wall (smooth means only one force). To do this, apply sum of moments about the foot of the ladder.

$$+\curvearrowleft \Sigma M_{foot} = 0$$
$$-F_w(5.2) + W(1.5) = 0$$
$$F_\omega = \frac{200 \times 1.5}{5.2} = 57.7\ N$$

$$\overset{\rightarrow}{+} \Sigma F_H = 0$$
$$F_w - F_f = 0$$
$$F_f = 57.7\ N$$

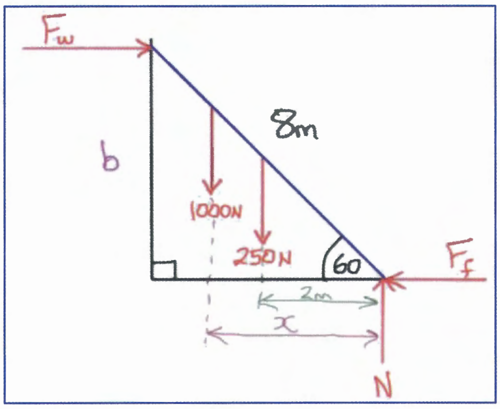
b)

$$+\uparrow \Sigma F_v = 0$$
$$-W + N = 0$$
$$N = 200\ N$$

c)

$$F_f = \mu N$$
$$\mu = \frac{F_f}{N} = \frac{57.7}{200}$$
$$\mu = 0.29$$

28.



Our first step is to find the frictional force and then the force at the wall.

$$+\uparrow \Sigma F_v = 0$$
$$-1000 - 250 + N = 0$$
$$N = 1250\ N$$

$$F_f = \mu N = 0.45 \times 1250$$
$$\therefore F_f = 562.5\ \mathrm{N}$$

$$\overset{\rightarrow}{+} \Sigma F_H = 0$$
$$F_w - F_f = 0$$
$$F_w = 562.5\ \mathrm{N}$$

We now need to find the perpendicular distances of F_w and the weight forces. Use trig to do this.

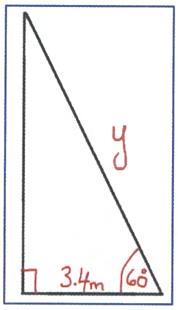
$$\cos 60 = \frac{a}{8}$$
$$a = 8 \times \cos 60 = 4 \text{ m}$$

$$\sin 60 = \frac{b}{8}$$
$$b = 8 \times \sin 60 = 6.93 \text{ m}$$

We now apply sum of moments about the foot of the ladder to find the horizontal distance from the man to the base of the ladder.

$$+\circlearrowleft \Sigma M_{foot} = 0$$
$$-F_w(b) + 1000x + (250 \times 2) = 0$$
$$1000x = (562.5 \times 6.93) - 500$$
$$x = 3.40\ m$$

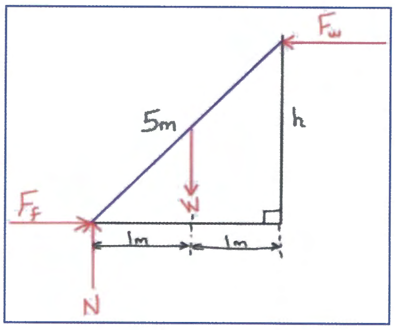
Use trig to find how far up the ladder the man is.



$$\cos 60 = \frac{3.4}{y}$$
$$y = \frac{3.4}{\cos 60}$$
$$y = 6.8\ \ m$$

Therefore, he can climb 6.8M up the ladder before the ladder starts to slip.

29.



Find the height of the wall using Pythagoras.

$$5^2 = h^2 + 2^2$$
$$h = \sqrt{25 - 4}$$
$$\therefore h = 4.58\ m$$

Apply sum of moments about the foot of the ladder to find the reaction force at the wall.

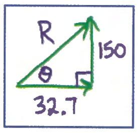
$$+\circlearrowleft \Sigma M_{foot} = 0$$
$$F_w(4.58) - W(1) = 0$$
$$F_w = \frac{150}{4.58}$$
$$\therefore F_w = 32.7\ N$$

Find the reaction forces at the ground.

$$+\uparrow \Sigma F_v = 0$$
$$-W + N = 0$$
$$N = 150\ N$$

$$\overrightarrow{+} \Sigma F_H = 0$$
$$F_f - F_w = 0$$
$$F_f = 32.7\ N$$

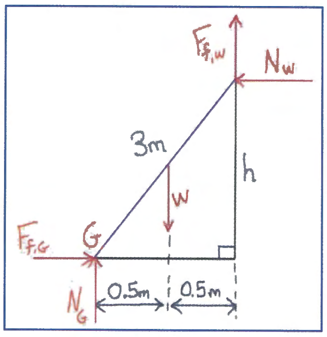
Draw a vector triangle to find the resultant force at the ground and its direction.



$$R = \sqrt{32.7^2 + 150^2}$$
$$R \approx 154\ N$$

$$\tan\theta = \frac{150}{32.7}$$
$$\theta = \tan^{-1}\left(\frac{150}{32.7}\right)$$
$$\theta \approx 78°$$

30.



In the above diagram, N_G is the normal force at the ground and $F_{f,G}$ is the friction force between ground and ladder. $F_{f,G}$ is directed to the right since the ladder has tendency to slip to the left. N_w is the normal force at the wall and $F_{f,w}$ is the friction force between wall and ladder. It is directed upwards as it has tendency to slip downwards. W represents the 100 N weight force.

Use Pythagoras to find the height of the wall.

$$3^2 = h^2 + 1^2$$
$$h = \sqrt{9-1} = 2.8\,m$$

Apply sum of moments about G.

$$+\Sigma M_G = 0$$
$$-100(0.5) + N_w(2.8) + F_{f,w}(1) = 0$$

Using the friction force formula gives us the following relationship. From this, we can then substitute it into the above equation to find N_w.

$$F_f = \mu N$$
$$F_{f,w} = 0.25N_w$$

$$-100(0.5) + N_w(2.8) + 0.25N_w(1) = 0$$
$$3.05N_w = 50$$
$$N_w = \frac{50}{3.05} = 16.4\ \ N$$

We can now find $F_{f,w}$ by substituting N_w into the below equation.

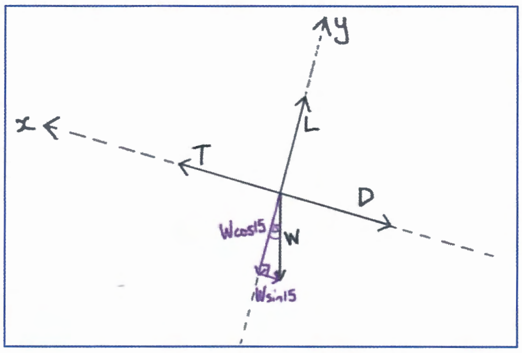
$$F_{f,w} = 0.25 \times N_w$$
$$\therefore F_{f,w} = 0.25 \times 16.4 = 4.1\ N$$

$$\overrightarrow{+}\Sigma F_H = 0$$
$$F_{f,g} - Nw = 0$$
$$\therefore F_{f,g} = 16.4\ N$$

$$+\uparrow \Sigma F_v = 0$$
$$N_G - W + F_{f,w} = 0$$
$$N_G = 100 - 4.1 = 95.9\ \mathrm{N}$$

$$F_{f,w} = \mu N_G$$
$$\mu = \frac{F_{f,w}}{N_G} = \frac{16.4}{95.9} = 0.17$$

31. a)



To convert 80 tonnes to kN, multiply by 10.

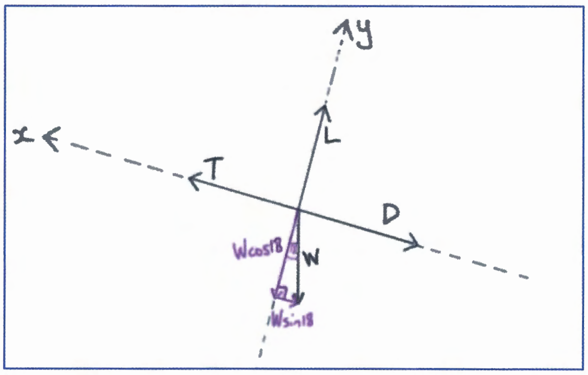
$$+\nearrow \Sigma F_y = 0$$
$$L - W\cos 15 = 0$$
$$L = 800 \times \cos 15$$
$$L \approx 773\ kN$$

b) Use the lift to drag ratio to find drag which you'll use to find the thrust.

$$\frac{L}{D} = \frac{16}{1}$$
$$L = 16D$$
$$D = \frac{L}{16} = \frac{223}{16}$$
$$\therefore D \approx 48\ kN$$

$$+\nwarrow \Sigma F_x = 0$$
$$T - D - W\sin 15 = 0$$
$$T = 48 + 800\sin 15$$
$$T = 255\ kN$$

32.



To convert 12 tonnes to kN, multiply by 10.

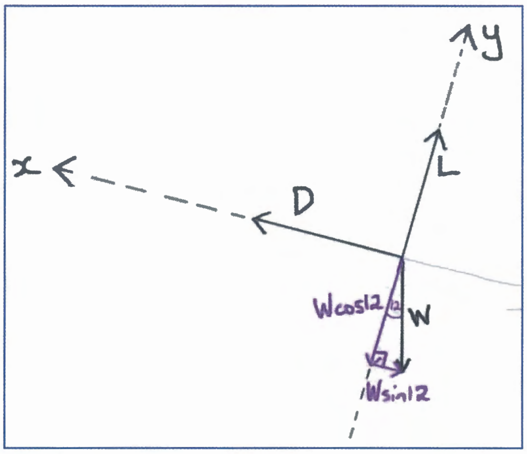
$$+\nearrow \Sigma F_y = 0$$
$$L - W\cos 18 = 0$$
$$L = 120\cos 18 = 114\ \mathrm{kN}$$

$$+\nwarrow \sum F_x = 0$$
$$T - D - W\sin15 = 0$$
$$D = 75 - 120\sin15 = 44 \text{ kN}$$

$$\frac{L}{D} = \frac{114}{44} = \frac{2.6}{1}$$

$$\therefore L:D = 2.6:1$$

33. a)



$$+\nearrow \sum F_y = 0$$
$$L - W\cos12 = 0$$
$$W = \frac{L}{\cos12} = \frac{5566}{\cos12}$$
$$\therefore W \approx 5690\ N$$
$$\therefore m_{total} = 569 \text{ kg}$$
$$m_{glider} = 569 - 75$$

$$\therefore m_{glider} = 494 \text{ kg}$$

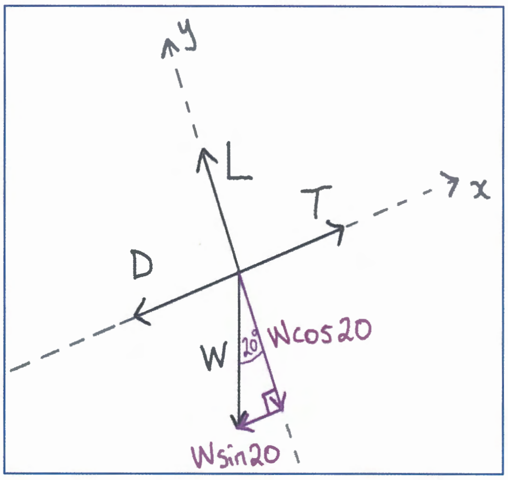
b) A glider has no thrust.

$$+\nwarrow \sum F_x = 0$$
$$-D + WSin12 = 0$$
$$D = 5690\sin12$$
$$\therefore D = 1183N$$

$$\frac{L}{D} = \frac{5566}{1183}$$

$$\therefore L:D = 4.7:1$$

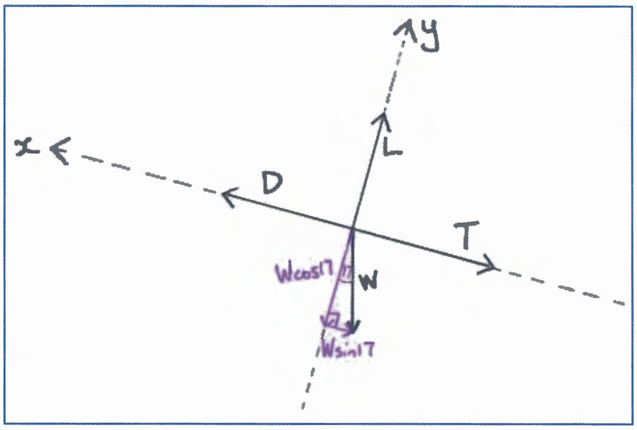
34.



$$+\nearrow \sum F_x = 0$$
$$T - D - W\sin20 = 0$$
$$D = T - W\sin20$$
$$D = (2 \times 580) - (3000\sin20)$$
$$\therefore D = 134\ kN$$

Note: Due to the 2 engines, we multiply the thrust value by 2 to find the total thrust.

35.

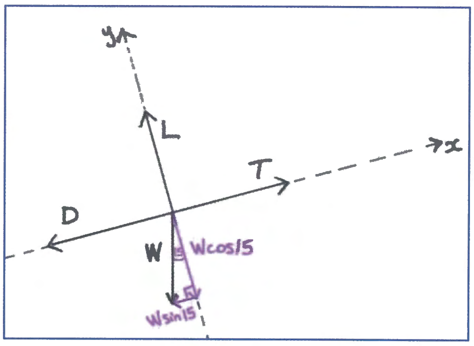


$$+\nearrow \sum F_y = 0$$
$$L - W\cos17 = 0$$
$$L = 2000\cos17$$
$$L = 1912.6\ kN$$

$$\frac{L}{D} = \frac{6}{1}$$
$$\frac{1912.6}{D} = \frac{6}{1}$$
$$D = \frac{19126}{6}$$

$$\therefore D = 318.8\ kN$$

36.



$$+\nwarrow \Sigma F_y = 0$$
$$L - W\cos15 = 0$$
$$L = 1020\cos15 = 985 \text{ kN}$$

$$+\nearrow \sum F_x = 0$$
$$T - D - W\sin15 = 0$$
$$T = 40 + 1020\sin15 = 304 \text{ kN}$$

37.

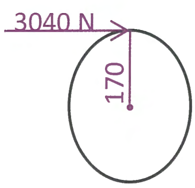
$$+\circlearrowleft \sum M_{tail} = 0$$
$$(5000 \times 40) - L(35) = 0$$
$$L = \frac{200000}{35} = 5714\ kN$$

$$+\uparrow \Sigma F_v = 0$$
$$-5000 + 5714 - F_{stabiliser} = 0$$
$$\therefore F_{stabiliser} = 714\ kN$$

38. Find friction force produced by the braking pads impacting the brake disc. Multiply friction force by 2 due to having 2 normal forces applied to the brake pads.

$$F_f = \mu N$$
$$F_f = 0.38 \times 4000 \times 2 = 3040\ N$$

This friction force acts at a tangent to the rotary motion of the wheel slowing it down.



Multiply frictional force by the distance to the axle's centre to find its moment.

$$M = Fd$$
$$M = 3040 \times 0.170 = 516.8\ Nm$$

Chapter 5 Solutions

1. (D) - Work is equal to the sum of the change in potential energy and the change in kinetic energy. Since the trolley is pushed on level ground (no change in height hence 0 PE) and at a constant speed (no change in speed hence 0 KE), the total work done will be 0.

2. (A) - Work done by the constant force of 10 kN to move the body a distance of 50 m up the slope is:

$$W = Fs$$
$$W = 10 \times 50 = 500\ kJ$$

Work done due to potential energy (mgh) only since there is no change in speed hence no change in KE:

$$W = \Delta PE$$
$$W = \text{mg}h_f - \text{mg}h_i$$
$$W = (30 \times 15) - 0$$
$$W = 450\ kJ$$

The work done by the constant force is converted to PE. The energy loss due to heat will hence be 500-450 = 50 kJ.

3. (A) -
$$W = Fs$$
$$W = 180 \times 50 = 9000\ J$$
$$\therefore W = 9\ kJ$$

4. (B) -
$$P = \frac{W}{t} = \frac{9000}{13.5}$$
$$\therefore P = 667\ W$$

5. (B) - Inclined road suggests an increase in height hence PE increases. Accelerating suggests an increase in speed hence KE increases.

6. (D) - Barbara lifts a total of 200kg which accounts to a weight of 2000N.

$$mass = 20 \times 10 = 200 \text{ kg}$$
$$F_{weight} = 200 \times 10 = 2000\ N$$

She produces a power of:

$$P = \frac{Fs}{t}$$
$$P = \frac{2000 \times 1.4}{70} = 40\ W$$

7. (B) - 4J of energy represents the work done. We can use the power equation to determine how much power was lost in 5 secs. From this, we can determine the power output after 5 secs. We then use the efficiency formula to solve the Q.

$$P_{lost} = \frac{W}{t} = \frac{4}{5} = 0.8\ W$$
$$\therefore P_{output} = 8 - 0.8 = 7.2\ W$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{7.2}{8} \times 100$$
$$\therefore \eta = 90\ \%$$

8. (D) → To convert m/s to km/h, we multiply by 3.6 and converting kJ to J, we multiply by 1000.

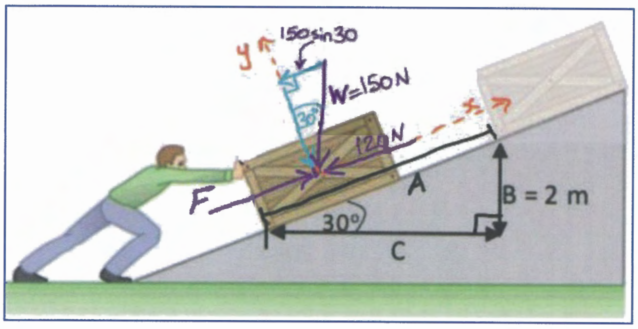
$$KE = \frac{1}{2} mv^2$$
$$2.5 \times 10^3 = \frac{1}{2} \times 90 \times v^2$$
$$v^2 = \frac{2500}{45} = 55.6$$
$$v = \sqrt{55.6} = 7.45\ \mathrm{ms^{-1}}$$

$$\therefore v = 7.45 \times 3.6 = 26.83\ \mathrm{km/h}$$

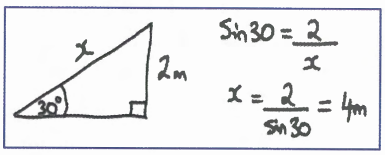
9. (C) -
$$\eta = \frac{P_{out}}{P_{in}} \times 100$$
$$\eta = \frac{90}{250} \times 100 = 36\ \%$$

10. (A) - The distance required is the one that is in line with the force's line of action.

11. (B)



First find the distance that this box covers if it gains a height of 2m.



Find the force required to just get this box moving.

$$+\nearrow \sum F_x = 0$$
$$F - 150\sin 30 - 120 = 0$$
$$F = 150\sin 30 + 120 = 195\ N$$

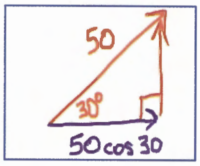
We now find the work done.

$$W = Fd$$
$$W = 195 \times 4 = 780\ \mathrm{J}$$

12. (C) - obeys the efficiency formula. Although B is correct mathematically, logically and practically, it is impossible to have an efficiency of greater than 1 or 100%.

$$\eta = \frac{P_{out}}{p_{in}}$$
$$\eta = \frac{5}{8} = 0.625$$

13. When calculating the work done, the force we need is the force that is parallel to the surface. Hence we break the 50N force into its horizontal component.



$$W = Fs$$
$$W = 50\cos 30 \times 30$$
$$\therefore W = 1299\ \mathrm{J}$$

14. a)
$$KE = \frac{1}{2} mv^2$$
$$KE = \frac{1}{2} \times 30 \times 0.5^2$$
$$\therefore KE = 3.75\ J$$

b) The force in the work equation is the total force that causes the box to move in its intended direction. That is, net force equals applied force minus friction force.

$$\mathrm{W = Fs}$$
$$\mathrm{W} = (150 - 8) \times 1.5$$
$$\therefore \mathrm{W} = 213\ \mathrm{J}$$

15. When calculating the work done, the force we need is the force that is parallel to the surface.

Hence we break the 80N force into its horizontal component.

The work done is calculated by multiplying the total force (force applied parallel to the surface minus the resistance force) by the amount of movement of the object.

$$Work = Fd$$
$$W = [(80 \times \cos 30) - (15)] \times 25$$
$$W = 1357\,J$$

16. At the bottom of the hill, the height is 0 due to it being our reference point. At rest means that the skier is not moving hence initial velocity is 0. The work done by friction is a negative as it opposes the motion of the skier over the length of the hill.

$$W = Fd$$
$$W = \Delta PE + \Delta KE = (\mathrm{mg}h_f - \mathrm{mg}h_i) + \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right)$$
$$\therefore Fd = (\mathrm{mg}h_f - \mathrm{mg}h_i) + \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right)$$
$$-30 \times 2200 = 0 - (80 \times 10 \times h_i) + \left(\frac{1}{2} \times 80 \times 30^2 - 0\right)$$

$$-66000 = -800h_i + 36000$$
$$800h_i = 102000$$
$$h_i = \frac{102000}{800} = 127.5\,m$$

17. The velocity is 0 at the start of the swing and at the end of the swing where it momentarily stops before coming back down. Hence the change in KE is 0. Work value is a negative since the energy is LOST.

$$Work = \Delta PE + \Delta KE$$
$$W = (\mathrm{mg}h_f - \mathrm{mg}h_i) + 0$$
$$-52.75 = (11 \times 10 \times h_f) - (11 \times 10 \times 0.75)$$

$$-52.75 = 110h_f - 82.5$$
$$110h_f = 29.75$$
$$h_f = \frac{29.75}{110} = 0.270\,m$$
$$\therefore h_f = 270\text{ mm}$$

18. Change in KE is 0 due to velocity being 0 at the start and end of the swing.

$$W = \Delta PE + \Delta KE$$
$$W = (\mathrm{mg}h_f - \mathrm{mg}h_i) + 0$$
$$W = (8 \times 10 \times 0.345) - (8 \times 10 \times 1.125)$$

$$W = -62.4\,J$$

Therefore, 62.4 J is lost.

19. Flat horizontal road means no change in height hence no change in potential energy. However, there is a change in KE as the car accelerates from 0 to 120km/h. 120km/h is converted to m/s by dividing it by 3.6 and W is converted to kW by dividing it by 1000.

$$P = \frac{W}{t} = \frac{\Delta PE + \Delta KE}{t}$$

$$P = \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{t}$$

$$P = \frac{\frac{1}{2} \times 2100 \times \left(\frac{120}{3.6}\right)^2 - 0}{5.9} = 197740\,W$$

$$\therefore P = 197.74\,kW$$

20. Convert KE to Joules and mass to kilograms to get velocity in m/s.

$$KE = \frac{1}{2}mv^2$$
$$KE = 25\,MJ = 25 \times 10^6\,J$$
$$m = 300\,tonnes = 300 \times 10^3\text{ kg}$$

$$25 \times 10^6 = \frac{1}{2} \times (300 \times 10^3) \times v^2$$
$$v^2 = \frac{25 \times 10^6}{150 \times 10^3} = 166.6\text{ J}$$
$$\therefore V = \sqrt{166.67} = 12.9\text{ ms}^{-1}$$

21.

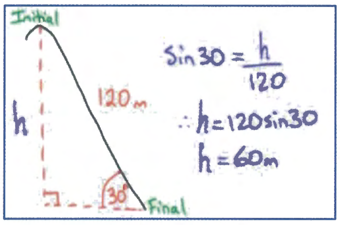
$$W = Fs$$
$$W = \Delta PE + \Delta KE$$
$$\therefore Fs = \Delta PE + \Delta KE$$

$$4500s = \left(\frac{1}{2} \times 1750 \times 45^2\right) - \left(\frac{1}{2} \times 1750 \times 20^2\right)$$
$$4500s = 1421875$$
$$\therefore s = \frac{1421875}{4500} = 316\ m$$

22. The power of the engine does work against the total force resisting motion i.e. air resistance and the frictional force. Velocity must be converted to m/s by dividing it by 3.6 and converting kN to N by multiplying it by 1000.

$$P = Fv$$
$$P = (8000 + 12000) \times \left(\frac{110}{3.6}\right)$$
$$P = 611{,}111W$$
$$\therefore P = 611\ kW$$

23. The work done by the resistance to motion is a negative as it opposes the motion.



$$W = \Delta PE + \Delta KE$$
$$W = Fs$$
$$-Fs = \Delta PE + \Delta KE$$

Note: Final height is 0 hence $mgh_f = 0$

$$-F(120) = -(80 \times 10 \times 60) + \left(\frac{1}{2} \times 80 \times \left(\frac{65}{3.6}\right)^2\right) - \left(\frac{1}{2} \times 80 \times \left(\frac{35}{3.6}\right)^2\right)$$

$$-120F = -38740.74$$

$$F \simeq \frac{-38740.74}{-120} = 322.8\ N$$

24. Since work done by friction is negligible and there is no input (not being driven by the engine) due to the car coasting then work equals 0. As the car coasts up the hill, its final velocity will be 0. The car initially starts from the bottom of the hill, hence initial height is 0.

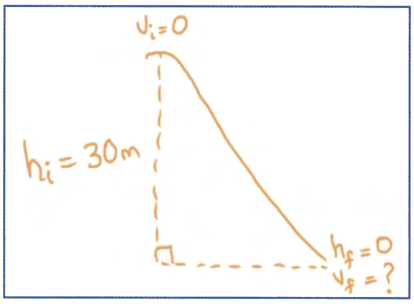
$$W = \Delta PE + \Delta KE = (mgh_f - mgh_i) + \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right)$$
$$0 = (1100 \times 10 \times h_f) - \left(\frac{1}{2} \times 1100 \times \left(\frac{100}{3.6}\right)^2\right)$$
$$0 = 11000h_f - 424382.72$$
$$h_f = \frac{424382.72}{11000} = 38.58\ m$$

25.

$$s = 2\text{km} = 2000\ m$$
$$P = 360kW = 360{,}000\ W$$
$$t = 2\ \text{mins} = 120\ \text{secs}$$
$$F = ?$$

$$P = \frac{Fs}{t}$$
$$360{,}000 = \frac{F \times 2000}{120}$$
$$F = \frac{360000 \times 120}{2000} = 21600\ N$$

26.



W = 0 since there is no pedalling or any energy input and no friction. Initially velocity is 0 since it starts at rest. The final height is 0 since it is at the bottom of the hill (our reference point).

$$W = \Delta PE + \Delta KE = (mgh_f - mgh_i) + \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right)$$
$$0 = 0 - (m \times 10 \times 30) + \frac{1}{2}mv_f^2 - 0$$
$$300m = \frac{mv_f^2}{2}$$
$$600m = mv_f^2$$
$$v_f^2 = 600$$
$$v_f = \sqrt{600} = 24.5\ \text{ms}^{-1}$$

27.

$$110\ tonne = 110{,}000\ \text{kg}$$
$$\sin 2 = \frac{h}{3000}$$
$$h = 3000 \times \sin 2 = 104.7\ m$$

$$F_{resistance} = 92 \times 110 = 10120\ \text{N}$$

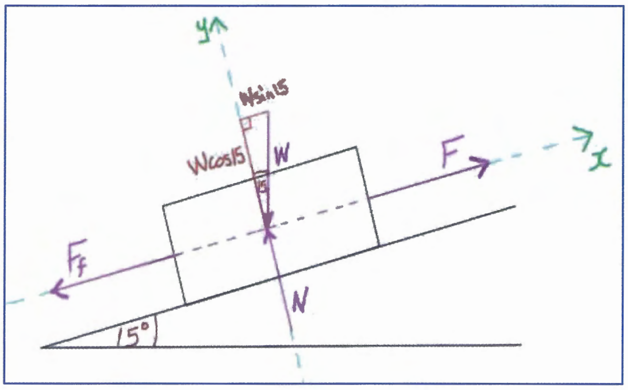
$$W = \Delta PE + \Delta KE$$
$$Fs = (\mathrm{mg}h_f - \mathrm{mg}h_i) + \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right)$$
$$(58000 - 10120) \times 3000 = (110000 \times 10 \times 104.7) - 0$$
$$+\left(\frac{1}{2} \times 110000 \times V_f^2\right) - \left(\frac{1}{2} \times 110000 \times \left(\frac{100}{3.6}\right)^2\right)$$

$$143{,}640{,}000 = 115{,}170{,}000 + 55000v_f^2 - 42438271.6$$
$$55000V_f^2 = 70908271.6$$

$$V_f = \sqrt{\frac{70908271.6}{55000}} = 35.9 \text{ ms}^{-1}$$

28. Find the force that the engine needs to provide the wheels for them to overcome the total resistances and the weight component that is acting against the car's motion.

To do this, draw FBD and sum the forces in the x-direction i.e. along the plane.



$$+\nearrow \sum F_x = 0$$
$$F - W\sin 15 - F_f = 0$$
$$F = W\sin 15 + F_f$$
$$\therefore F = 15000\sin 15 + 750 = 4632.3\ N$$

Since we have force and velocity, we can now determine the power required.

$$P = Fv$$
$$\therefore P = 4632.3 \times 20 = 92.6\ kW$$

29. Initial PE is 0 due to ground (our reference point) being the initial starting position. KE is 0 since she starts from rest.

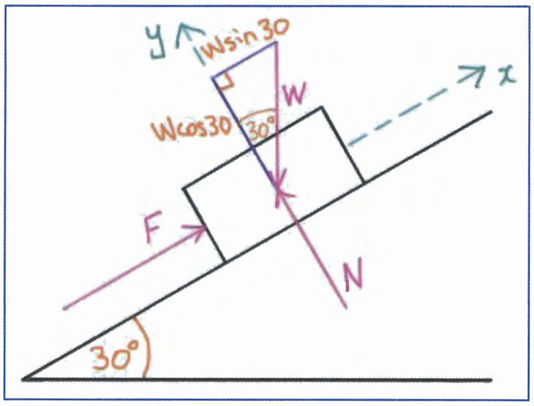
$$Power = \frac{Work}{time} = \frac{\Delta PE + \Delta KE}{time}$$

$$\Delta PE = \mathrm{mg}h_f - \mathrm{mg}h_i = (55 \times 10 \times 3.5) - 0$$
$$\Delta PE = 1925\ J$$

$$\Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2} \times 55 \times 1.5^2 - 0$$
$$\Delta KE = 61.875\ J$$

$$\therefore P = \frac{1925 + 61.875}{4} = 496.7\ W$$

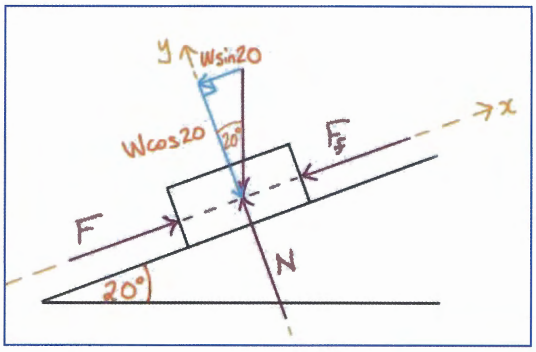
30. There is no friction force since it is a frictionless incline. Draw FBD and calculate the force that is required to push this box at a constant speed.



$$+\nearrow \sum F_x = 0$$
$$F - W\sin 30 = 0$$
$$F = W\sin 30$$
$$\therefore F = 300\sin 30 = 150\ N$$

$$W = Fd$$
$$\therefore W = 150 \times 50 = 7500\ J$$

31.



To find the friction force, we need to first find the normal force by applying the sum of forces in the y-direction equation.

$$+\nwarrow \Sigma F_y = 0$$
$$-W\cos 20 + N = 0$$
$$N = W\cos 20$$
$$N = 300\cos 20 = 281.9\ N$$

$$F_f = \mu N$$
$$F_f = 0.2 \times 281.9 = 56.4\ N$$

$$+\nearrow \sum F_x = 0$$
$$F - W\sin 20 - F_f = 0$$
$$F = W\sin 20 + F_f$$
$$\therefore F = 300\sin 20 + 56.4 = 159\ N$$

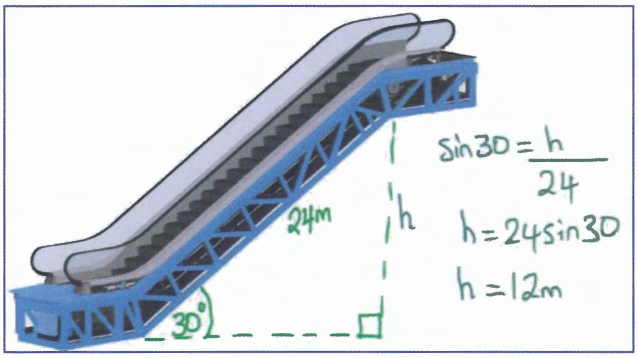
$$W = Fd$$
$$W = 159 \times 50 = 7950\ J$$

32. $m_{Total} = 80 \times 30 = 2400$ kg

No change in KE but there is a change in PE. Initial PE is 0. Multiply by 60 to convert minutes to secs.

$$P = \frac{W}{t} = \frac{\Delta PE}{t} = \frac{\text{mgh}}{t}$$
$$P = \frac{2400 \times 10 \times 5}{12 \times 60} = 167\ W$$

33.

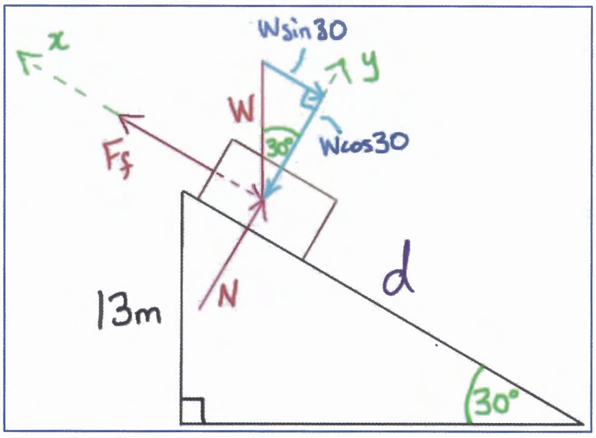


There is no change in kinetic energy but there is a change in potential energy. We need to use trig to find the vertical height which we then substitute into the PE equation. This will be the final PE. Initial PE is 0

$$m_{Total} = 65 \times 30 = 1950 \text{ kg}$$

$$P = \frac{W}{t} = \frac{\Delta PE}{t} = \frac{mgh}{t}$$
$$P = \frac{1950 \times 10 \times 12}{20} = 11700\ W$$

34.



The frictional force provides negative work on the box. So we need to find this force first. Apply sum of forces in the y-direction to find the normal force then apply the frictional force equation.

$$+\nearrow \Sigma F_y = 0$$
$$N - w\cos 30 = 0$$
$$N = 260 \times \cos 30 = 225.1\ N$$

$$F_f = \mu N$$
$$F_f = 0.15 \times 225.1 = 33.77\ N$$

Find the distance the box travels down the incline then apply the work formula to find the velocity at the bottom of the slope.

$$\sin 30 = \frac{13}{d}$$

$$d = \frac{13}{\sin 30} = 26m$$

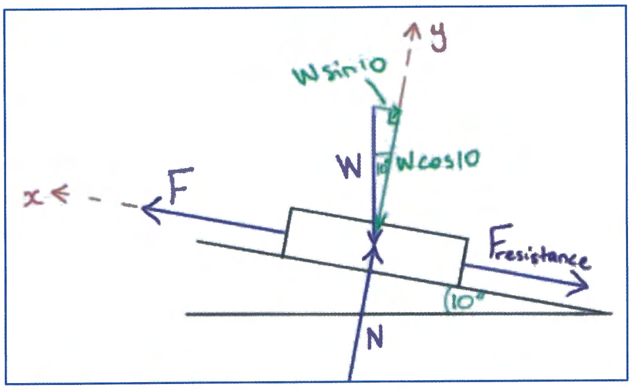
$$W = \Delta PE + \Delta KE$$
$$-F_f d = (\text{mg}h_f - \text{mg}h_i) + \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right)$$
$$-33.77 \times 26 = 0 - 26(10)(13) + \frac{1}{2}(26)v_f^2 - 0$$
$$-878 = -3380 + 13v_f^2$$
$$v_f^2 = \frac{3380 - 878}{13} = 192.46$$

$$\therefore V_f = \sqrt{192.46} = 13.9\ \text{ms}^{-1}$$

35. We multiply the efficiencies when dealing with connected machines.

$$\eta_{total} = 0.7 \times 0.8 = 0.56$$
$$\eta = \frac{P_{out}}{P_{in}}$$
$$0.56 = \frac{3}{P_{in}}$$
$$P_{in} = \frac{3}{0.56} = 5.357\ kW$$

36. Total mass is 80 tonnes which equals 80000 kg. So the weight force is 800,000 N.



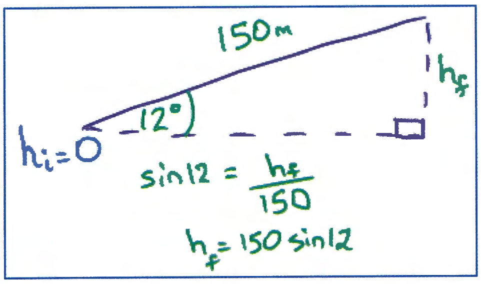
We need to find the force required to get the 2 vehicles moving up the incline.

$$+\nwarrow \sum F_x = 0$$
$$F - 1200 - (800000 \times \sin 10) = 0$$
$$F = 1200 + (800000 \times \sin 10) \simeq 140118.5\ N$$

$$P = Fv$$
$$80 \times 10^3 = 140118.5v$$
$$v = \frac{80000}{140118.5} = 0.57\ \mathrm{ms}^{-1}$$

$$v = 0.57 \times 3.6 = 2.1\ \mathrm{km}/h$$

37.



There is no change in KE due to the truck moving at a constant velocity. Initial PE is 0 since base of the climb is our reference point. Apply trig to find the final height as shown above.

$$P = \frac{W}{t} = \frac{\Delta PE}{t} = \frac{PE_f - PE_i}{t}$$
$$\therefore P = \frac{\mathrm{mg}h_f}{t}$$

$$500 \times 10^3 = \frac{m \times 10 \times (150 \times \sin 12)}{25}$$
$$m = \frac{500000 \times 25}{1500 \times \sin 12} = 40081.12\ \mathrm{kg}$$

$$\therefore m = 40.08\ tonnes$$

Convert the above mass to tonnes by dividing by 1000. We then subtract the mass of truck and trailer from the value of above. This will give us the total amount that the truck can carry.

$$m_{contents} = 40.08 - 25 = 15.08\ tonnes$$

We now divide the above value by the mass of each log to find how many logs the truck can carry.

$$\mathrm{No.\,of\,logs} = \frac{15.08}{1.2} = 12.6$$
$$\therefore \mathrm{logs} = 12$$

∴ Truck can carry a total of 12 full sized logs.

38. Initial height (ground) is 0 and there is no KE before lift and end of lift.

$$W = \Delta PE + \Delta KE$$
$$W = \mathrm{mgh}$$
$$4400 = m \times 10 \times 2$$
$$\therefore m = 220\ \mathrm{kg}$$

39. a)

$$W = \Delta PE + \Delta KE$$
$$W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$W = 0 - \left(\frac{1}{2} \times 3000 \times \left(\frac{80}{3.6}\right)^2\right) = -740740.74\ J$$
$$\therefore W = -740.74\ kJ$$

Therefore approximately 741 kJ is expended by the brakes to bring the truck to a halt.

b)

$$P = \frac{W}{t} = \frac{740740.74}{12} = 61728.4W = 61.73\ kW$$

c)

$$W = Fs$$
$$-740740.74 = -20000s$$
$$s = \frac{-740740.74}{-20000} = 37.04\,m$$

40.

$$P = \frac{W}{t} = \frac{\Delta PE + \Delta KE}{t}$$
$$\Delta PE = (300 \times 10 \times 16) - 0 = 48000\text{ J}$$
$$\Delta KE = \frac{1}{2} \times 300 \times 1.5^2 - 0 = 337.5\text{ J}$$

$$\therefore P = \frac{48000 + 337.5}{15} = 3222.5\,W$$

This is the power required for the mass to be lifted. Now take motor's efficiency into account.

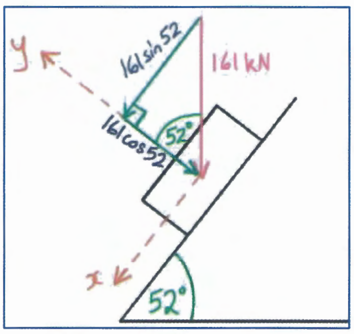
$$\eta = \frac{P_{out}}{P_{in}}$$
$$0.7 = \frac{3222.5}{p_{in}}$$
$$\therefore P_{in} = \frac{3222.5}{0.7} = 4604W = 4.6\,kW$$

41.

$$P = \frac{W}{t} = \frac{\Delta PE}{t} = \frac{\text{mgh}}{t}$$

$$P = \frac{78 \times 10 \times 4}{10} = 312\,W$$

42.



$$+\nwarrow \Sigma F_y = 0$$
$$-161\cos52 + N = 0$$
$$\therefore N = 161\cos52 = 99.1\,kN$$

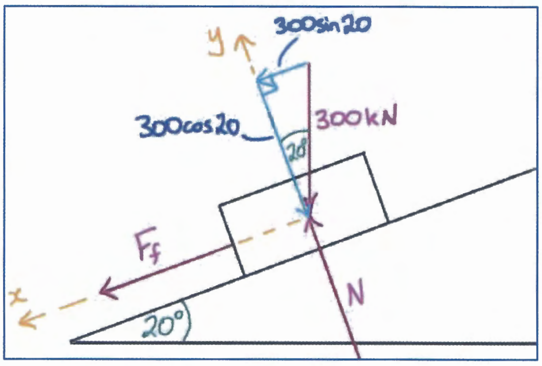
$$F_f = \mu\text{N}$$
$$F_f = 0.25 \times 99.1 = 24.8\,kN$$

$$+\swarrow \sum F_x = 161\sin52 + 24.8 = 151.7\text{ kN}$$

$$P = Fv$$
$$\therefore P = 151.7 \times \left(\frac{14}{3.6}\right) = 589.8\,kW$$

$$\eta = \frac{P_{out}}{P_{in}}$$
$$P_{in} = \frac{P_{out}}{\eta} = \frac{589.8}{0.9} = 655\,kW$$

43.



$$+\nwarrow \Sigma F_y = 0$$
$$-300 \times \cos20 + N = 0$$
$$\therefore N = 300 \times \cos20 = 281.9\,kN$$

$$F_f = \mu\text{N}$$
$$F_f = 0.4 \times 281.9 = 112.8\,kN$$

Summing the forces in the x-direction will give us the total resistance force.

$$\swarrow + \sum F_x = 300\sin20 + 112.8 = 215.4\text{ kN}$$

We can now calculate the power required to keep the truck travelling at a constant velocity of 10 m/s.

$$P = Fv$$
$$\therefore P = 215.4 \times 10 = 2154\,kW$$

Taking in the efficiency, we can calculate the power required to set a power output of 2154 kW.

$$\eta = \frac{P_{out}}{P_{in}}$$

$$P_{in} = \frac{P_{out}}{\eta} = \frac{2154}{0.75} = 2872\,kW$$