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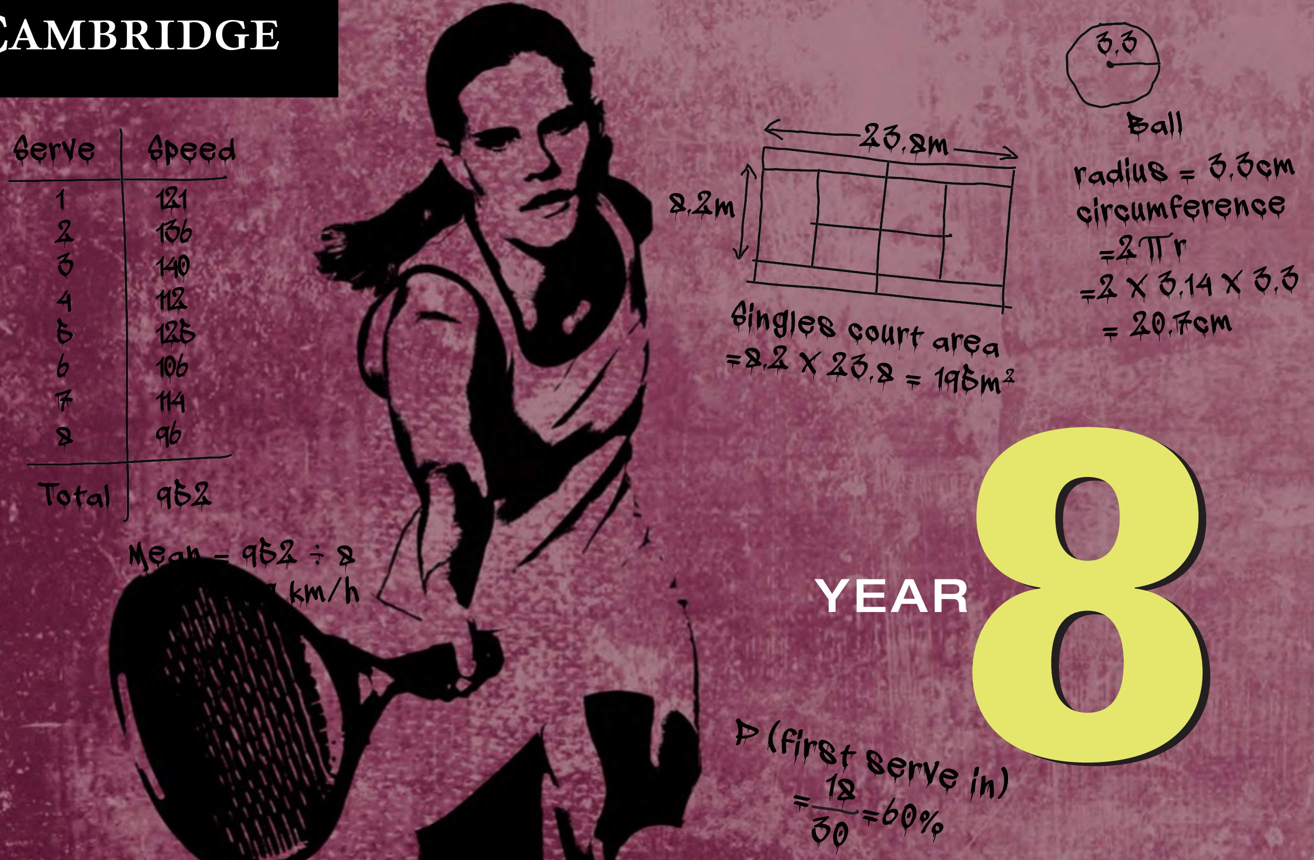

YEAR 8

# CambridgeMATHS

## NSW SYLLABUS FOR THE AUSTRALIAN CURRICULUM

STAGE 3/4

## GOLD

Interactive Textbook included

STUART PALMER | DAVID GREENWOOD
BRYN HUMBERSTONE | JENNY GOODMAN
KAREN McDAID | JENNIFER VAUGHAN | MARGARET POWELL

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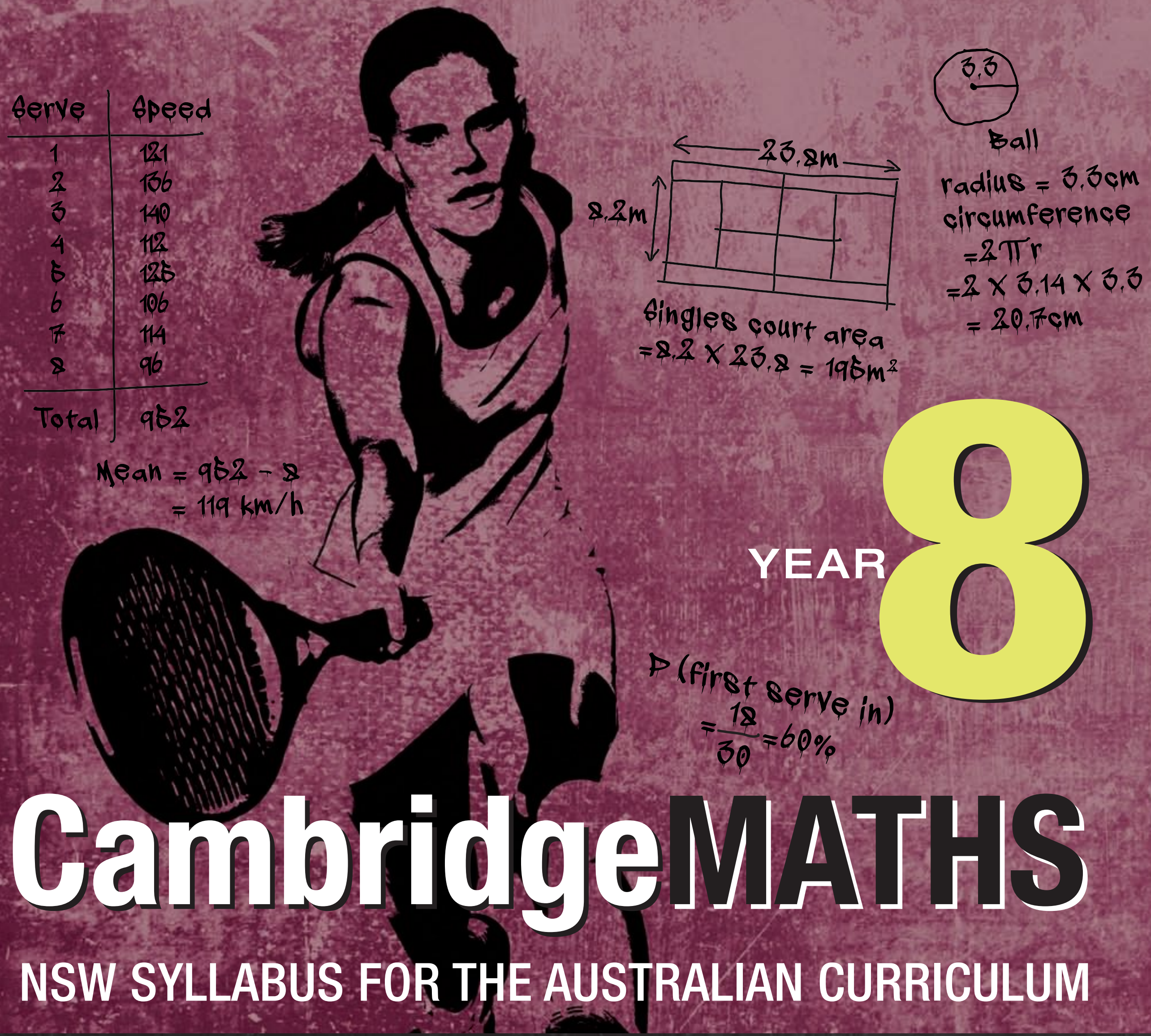

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Additional resources online

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# Contents

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**Note:** Students who require additional revision of 'Computation with integers' may find **Appendix 1** useful. This can be accessed online at **www.cambridge.edu.au/GO**.

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# About the authors

**Stuart Palmer** was born and educated in NSW. He is a high school mathematics teacher with more than 25 years' experience teaching students from all walks of life in a variety of schools. Stuart has taught all the current NSW Mathematics courses in Stages 4, 5 and 6 numerous times. He has been a head of department in two schools and is now an educational consultant who conducts professional development workshops for teachers all over NSW and beyond. He also works with pre-service teachers at the University of Sydney and the University of Western Sydney.

**David Greenwood** is the head of Mathematics at Trinity Grammar School in Melbourne and has 19 years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 20 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.

**Bryn Humberstone** graduated from University of Melbourne with an Honours degree in Pure Mathematics, and is currently teaching both junior and senior mathematics in Victoria. Bryn is particularly passionate about writing engaging mathematical investigations and effective assessment tasks for students with a variety of backgrounds and ability levels.

**Jenny Goodman** has worked for 20 years in comprehensive state and selective high schools in NSW and has a keen interest in teaching students of differing ability levels. She was awarded the Jones Medal for Education at the University of Sydney and the Bourke Prize for Mathematics. She has written for Cambridge NSW and was involved in the *Spectrum* and *Spectrum Gold* series.

**Karen McDaid** is an academic and lecturer in Mathematics Education in the School of Education at the University of Western Sydney. She has taught mathematics to both primary and high school students and is currently teaching undergraduate students on their way to becoming primary school teachers.

**Jennifer Vaughan** has taught secondary mathematics for over 30 years in NSW, Western Australia, Queensland and New Zealand and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has taught special needs students and has had extensive experience in developing resources that make mathematical concepts more accessible, hence facilitating student confidence, achievement and an enjoyment of maths.

## Consultant

**Margaret Powell** has 23 years of experience in teaching special needs students in Sydney and London. She has been head teacher of the support unit at a NSW comprehensive high school for 12 years. She is one of the authors of *Spectrum Maths Gold Year 7* and *Year 8*. Margaret is passionate about ensuring that students with learning difficulties achieve in their academic careers by providing learning materials that are engaging and accessible.

# Introduction and guide to this book

Thank you for choosing *CambridgeMATHS Gold*. This book is one component of an integrated package of resources designed for the NSW Syllabus for the Australian Curriculum. *CambridgeMATHS Gold* follows on from the standard CambridgeMATHS series published in 2013–14, and the two series have been structured so that they can be used in the same classroom. Mapping documents showing the relationship between the series can be found on *Cambridge GO*.

Whereas the standard *CambridgeMATHS* books for Years 7 and 8 begin at Stage 4, the *Gold* books for Years 7 and 8 are intended for students who need to consolidate Stage 3 learning prior to progressing to Stage 4. The aim is to develop Understanding and Fluency in core mathematical skills. Clear explanations of concepts, worked examples embedded in each exercise and carefully graded questions contribute to this goal. Problem-solving, Reasoning and Communicating are also developed through carefully constructed activities, exercises and enrichment.

An important component of *CambridgeMATHS Gold* is a set of worksheets called Drilling for Gold. These are engaging, innovative, skill-and-drill style tasks that provide the kind of practice and consolidation of the skills required for each topic without adding hundreds of pages to the textbook.

Literacy issues can be a barrier for learning mathematics, especially in the transition from primary to secondary school. As such, the relationship between literacy and maths is a major focus of *CambridgeMATHS Gold*. Key words and concepts are defined using student-friendly language; real-world contexts and applications of mathematics help students connect these concepts to everyday life; and a host of literacy activities can be downloaded from the website. In the interactive version of this book, definitions are enhanced by audio pronunciation, visual definitions and examples. More information about the interactive version can be found on page xi.

A suite of accompanying resources, including Drilling for Gold worksheets and Literacy activities, can be downloaded from *Cambridge GO*.

Chapter introductions provide real-world context for students.

**What you will learn** gives an overview of the chapter contents.

A summary of the chapter connects the topic to the NSW Syllabus. Detailed mapping documents to the NSW Syllabus can be found in the teaching program on *Cambridge GO*.

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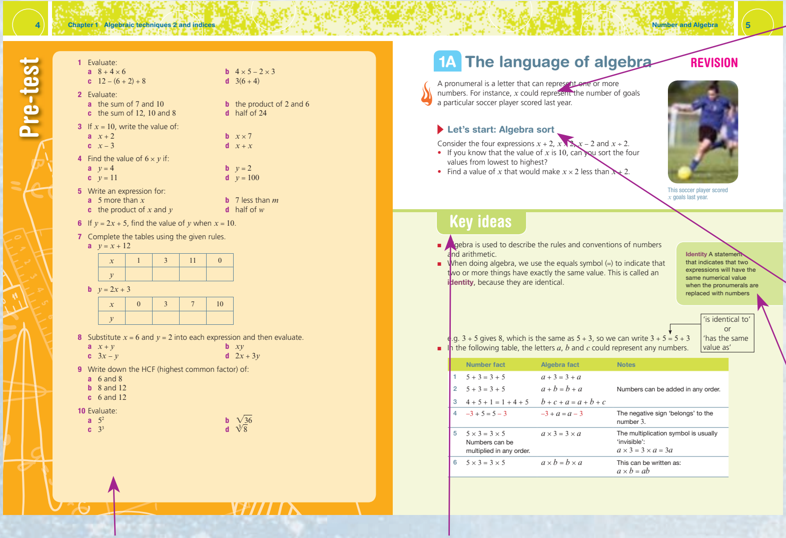


Topic introductions relate the topic to mathematics in the wider world.

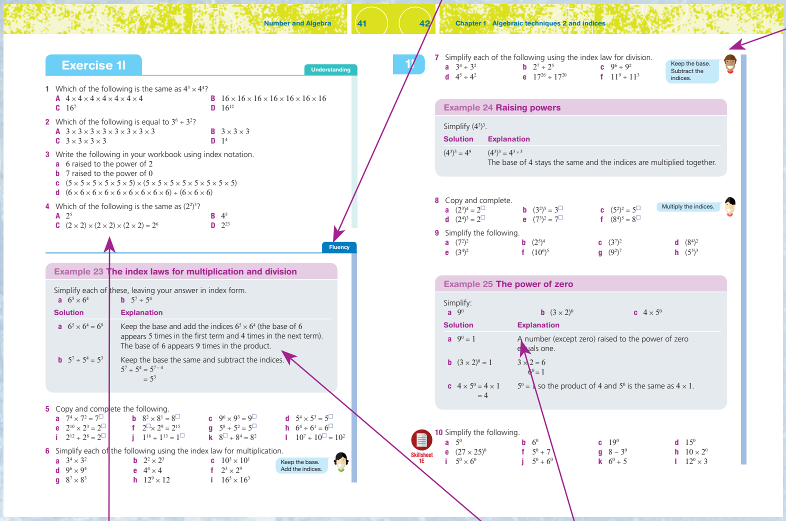
**Let's start** activities provide an engaging way to begin thinking about the topic.

Every important term in the **Key ideas** contains a simple-language definition.

A **Pre-test** for each chapter establishes prior knowledge.

**Key ideas** summarises the knowledge and skills for the topic.

Exercises are structured according to the four Working Mathematically strands: Understanding, Fluency, Problem-solving and Reasoning, with Communicating present in each of the other three. **Enrichment** questions at the end of the exercise challenge students to reach further.



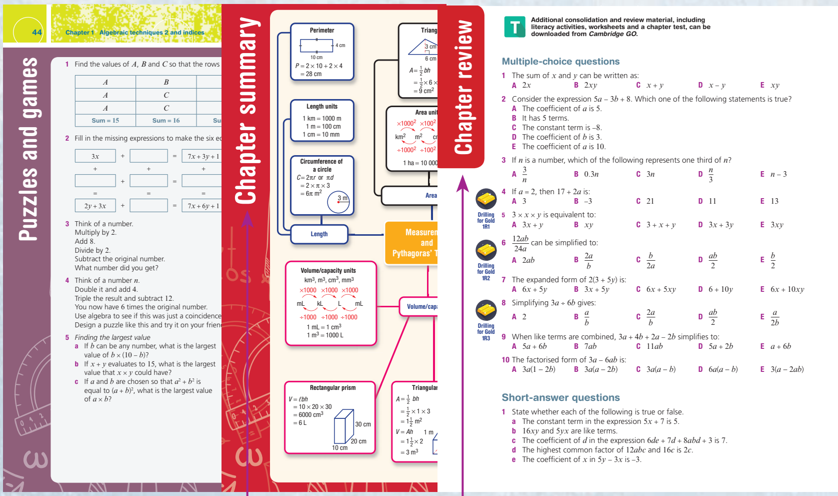
Hint boxes give hints and advice for tackling questions.

Within each Working Mathematically strand, questions are further carefully graded from easier to challenging.

**Examples** with worked solutions and explanations are embedded in the exercises immediately before the relevant question/s.

**Puzzles and games** allow students to have fun with the mathematics contained in the chapter.

**Chapter summaries** give mind maps of key concepts and the interconnections between them.



44 Chapter 1 Algebraic techniques 2 and indices

Puzzles and games

1 Find the values of $A$, $B$ and $C$ so that the rows

| | | |
|---|---|---|
| $A$ | $B$ | |
| $A$ | $C$ | |
| $A$ | $C$ | |
| Sum = 15 | Sum = 16 | Su |

2 Fill in the missing expressions to make the six ec

| | | | | |
|---|---|---|---|---|
| $3x$ | + | | = | $7x + 3y + 1$ |
| + | | + | | + |
| | + | | = | |
| = | | = | | = |
| $2y + 3x$ | + | | = | $7x + 6y + 1$ |

3 Think of a number.
Multiply by 2.
Add 8.
Divide by 2.
Subtract the original number.
What number did you get?

4 Think of a number $n$.
Double it and add 4.
Triple the result and subtract 12.
You now have 6 times the original number.
Use algebra to see if this was just a coincidence
Design a puzzle like this and try it on your frien

5 *Finding the largest value*
- a If $b$ can be any number, what is the largest value of $b \times (10 - b)$?
- b If $x + y$ evaluates to 15, what is the largest value that $x \times y$ could have?
- c If $a$ and $b$ are chosen so that $a^2 + b^2$ is equal to $(a + b)^2$, what is the largest value of $a \times b$?

Chapter summary

Chapter review

**T** **Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.**

Multiple-choice questions

1 The sum of $x$ and $y$ can be written as:
A $2x$ B $2xy$ C $x + y$ D $x - y$ E $xy$

2 Consider the expression $5a - 3b + 8$. Which one of the following statements is true?
A The coefficient of $a$ is 5.
B It has 5 terms.
C The constant term is –8.
D The coefficient of $b$ is 3.
E The coefficient of $a$ is 10.

3 If $n$ is a number, which of the following represents one third of $n$?
A $\frac{3}{n}$ B $0.3n$ C $3n$ D $\frac{n}{3}$ E $n - 3$

4 If $a = 2$, then $17 + 2a$ is:
A 3 B –3 C 21 D 11 E 13

Drilling for Gold 1R1

5 $3 \times x \times y$ is equivalent to:
A $3x + y$ B $xy$ C $3 + x + y$ D $3x + 3y$ E $3xy$

6 $\frac{12ab}{24a}$ can be simplified to:
A $2ab$ B $\frac{2a}{b}$ C $\frac{b}{2a}$ D $\frac{ab}{2}$ E $\frac{b}{2}$

Drilling for Gold 1R2

7 The expanded form of $2(3 + 5y)$ is:
A $6x + 5y$ B $3x + 5y$ C $6x + 5xy$ D $6 + 10y$ E $6x + 10xy$

8 Simplifying $3a \div 6b$ gives:
A 2 B $\frac{a}{b}$ C $\frac{2a}{b}$ D $\frac{ab}{2}$ E $\frac{a}{2b}$

Drilling for Gold 1R3

9 When like terms are combined, $3a + 4b + 2a - 2b$ simplifies to:
A $5a + 6b$ B $7ab$ C $11ab$ D $5a + 2b$ E $a + 6b$

10 The factorised form of $3a - 6ab$ is:
A $3a(1 - 2b)$ B $3a(a - 2b)$ C $3a(a - b)$ D $6a(a - b)$ E $3(a - 2ab)$

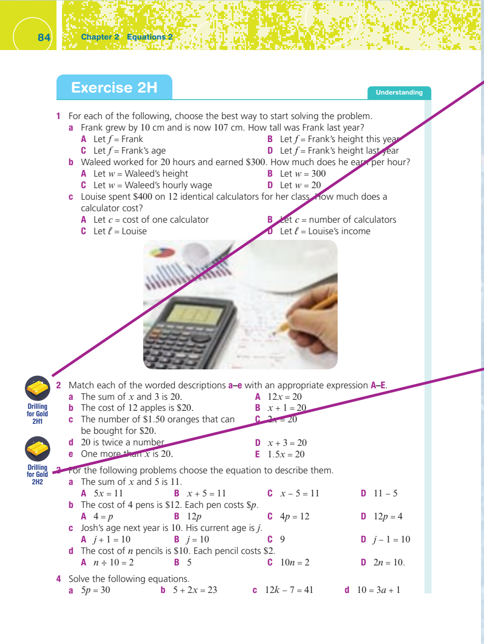
Short-answer questions

1 State whether each of the following is true or false.
- a The constant term in the expression $5x + 7$ is 5.
- b $16xy$ and $5yx$ are like terms.
- c The coefficient of $d$ in the expression $6de + 7d + 8abd + 3$ is 7.
- d The highest common factor of $12abc$ and $16c$ is $2c$.
- e The coefficient of $x$ in $5y - 3x$ is –3.

**Chapter reviews** test comprehension with multiple-choice, short-answer and extended-response questions.

## Drilling for Gold

Drilling for Gold is a collection of engaging and motivating learning resources that provide opportunities for students to repeatedly practise routine mathematical skills. Their purpose is to improve automaticity, fluency and understanding through 'hands-on' resources, games, competitions, puzzles, investigations and sets of closed questions. These activities are designed to be used as if they were part of the textbook; each one is referenced in the pages of the textbook via a 'gold' icon and unique reference number. The Drilling for Gold resources can be downloaded via the *Cambridge GO* website.



84 Chapter 2 Equations 2

Exercise 2H

Understanding

1 For each of the following, choose the best way to start solving the problem.
- a Frank grew by 10 cm and is now 107 cm. How tall was Frank last year?
  - A Let $f$ = Frank
  - B Let $f$ = Frank's height this year
  - C Let $f$ = Frank's age
  - D Let $f$ = Frank's height last year
- b Waleed worked for 20 hours and earned \$300. How much does he earn per hour?
  - A Let $w$ = Waleed's height
  - B Let $w = 300$
  - C Let $w$ = Waleed's hourly wage
  - D Let $w = 20$
- c Louise spent \$400 on 12 identical calculators for her class. How much does a calculator cost?
  - A Let $c$ = cost of one calculator
  - B Let $c$ = number of calculators
  - C Let $\ell$ = Louise
  - D Let $\ell$ = Louise's income

Drilling for Gold 2H1

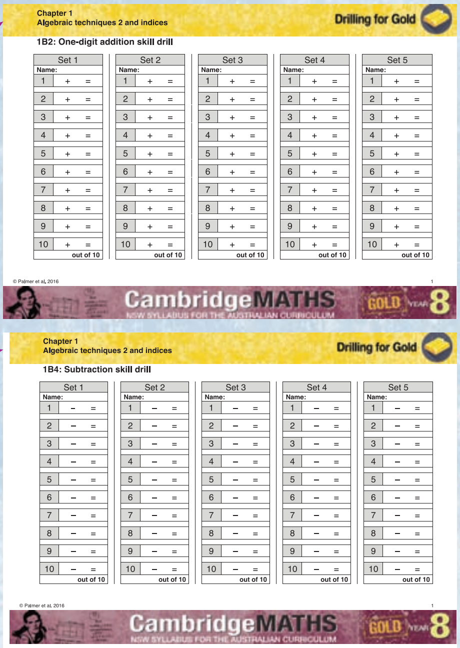
2 Match each of the worded descriptions **a–e** with an appropriate expression **A–E**.

| | |
|---|---|
| a The sum of $x$ and 3 is 20. | A $12x = 20$ |
| b The cost of 12 apples is \$20. | B $x + 1 = 20$ |
| c The number of \$1.50 oranges that can be bought for \$20. | C $2x = 20$ |
| d 20 is twice a number. | D $x + 3 = 20$ |
| e One more than $x$ is 20. | E $1.5x = 20$ |

Drilling for Gold 2H2

3 For the following problems choose the equation to describe them.
- a The sum of $x$ and 5 is 11.
  A $5x = 11$ B $x + 5 = 11$ C $x - 5 = 11$ D $11 - 5$
- b The cost of 4 pens is \$12. Each pen costs \$$p$.
  A $4 = p$ B $12p$ C $4p = 12$ D $12p = 4$
- c Josh's age next year is 10. His current age is $j$.
  A $j + 1 = 10$ B $j = 10$ C 9 D $j - 1 = 10$
- d The cost of $n$ pencils is \$10. Each pencil costs \$2.
  A $n \div 10 = 2$ B 5 C $10n = 2$ D $2n = 10$.

4 Solve the following equations.
a $5p = 30$ b $5 + 2x = 23$ c $12k - 7 = 41$ d $10 = 3a + 1$



Chapter 1
Algebraic techniques 2 and indices

Drilling for Gold

1B2: One-digit addition skill drill

| Set 1 | Set 2 | Set 3 | Set 4 | Set 5 |
|---|---|---|---|---|
| Name: | Name: | Name: | Name: | Name: |
| 1 + = | 1 + = | 1 + = | 1 + = | 1 + = |
| 2 + = | 2 + = | 2 + = | 2 + = | 2 + = |
| 3 + = | 3 + = | 3 + = | 3 + = | 3 + = |
| 4 + = | 4 + = | 4 + = | 4 + = | 4 + = |
| 5 + = | 5 + = | 5 + = | 5 + = | 5 + = |
| 6 + = | 6 + = | 6 + = | 6 + = | 6 + = |
| 7 + = | 7 + = | 7 + = | 7 + = | 7 + = |
| 8 + = | 8 + = | 8 + = | 8 + = | 8 + = |
| 9 + = | 9 + = | 9 + = | 9 + = | 9 + = |
| 10 + = | 10 + = | 10 + = | 10 + = | 10 + = |
| out of 10 | out of 10 | out of 10 | out of 10 | out of 10 |

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Chapter 1
Algebraic techniques 2 and indices

Drilling for Gold

1B4: Subtraction skill drill

| Set 1 | Set 2 | Set 3 | Set 4 | Set 5 |
|---|---|---|---|---|
| Name: | Name: | Name: | Name: | Name: |
| 1 – = | 1 – = | 1 – = | 1 – = | 1 – = |
| 2 – = | 2 – = | 2 – = | 2 – = | 2 – = |
| 3 – = | 3 – = | 3 – = | 3 – = | 3 – = |
| 4 – = | 4 – = | 4 – = | 4 – = | 4 – = |
| 5 – = | 5 – = | 5 – = | 5 – = | 5 – = |
| 6 – = | 6 – = | 6 – = | 6 – = | 6 – = |
| 7 – = | 7 – = | 7 – = | 7 – = | 7 – = |
| 8 – = | 8 – = | 8 – = | 8 – = | 8 – = |
| 9 – = | 9 – = | 9 – = | 9 – = | 9 – = |
| 10 – = | 10 – = | 10 – = | 10 – = | 10 – = |
| out of 10 | out of 10 | out of 10 | out of 10 | out of 10 |

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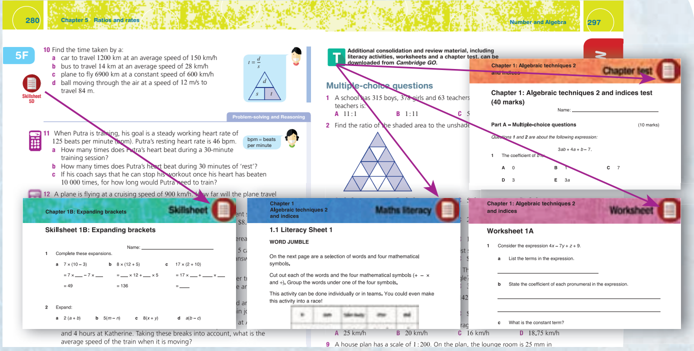
## Other resources



In addition to Drilling for Gold, a host of other resources for each chapter can be downloaded from *Cambridge GO*:

- **Skillsheets** provide practise of the key skills learned across the entirety of the chapter, and are linked to the later sections via their own icon and reference number.
- **Maths literacy** worksheets familiarise students with mathematical English via cloze activities, games, group activities, crosswords and much more.
- A **chapter test** provides exam-style assessment, with multiple-choice, short-answer and extended-response questions.
- **Worksheets** cover multiple topics within a chapter and can be done in class or completed as homework.

## About your Interactive Textbook

An interactive digital textbook is included with your print textbook and is an integral part of the *CambridgeMATHS Gold* learning package. As well as being an attractive, easy-to-navigate digital version of the textbook, it contains many features that enhance learning in ways not possible with a print book:

- **Roll-over definitions** give short, simple-language definitions of key terms at the start of a topic
- **Clickable 'Enhanced' definitions** containing diagrams, illustrations, examples and audio pronunciation provide instant assistance and revision within exercises and worked examples
- **Roll-over hints** for selected questions are provided within exercises by rolling your mouse over the cartoon faces
- **Matching *HOTmaths* lessons** can be accessed by clicking the flame at the start of each topic
- **Additional teacher resources** can be accessed by clicking the 'T' icon in the chapter review
- **Drilling for Gold** and **Skillsheets** can be downloaded by clicking on the respective icons in the margins
- **Fill-the-gap** and **drag-and-drop** activities at the end of each chapter provide a fun way of learning key concepts and consolidating knowledge
- **Answers** for Exercises, Pre-tests, Puzzles and Games and Chapter reviews can be conveniently accessed by clicking the 'Show Answers' button at the bottom of the page
- **Font size** can be increased or decreased as required
- **Annotations** can be added to words, phrases, questions or whole paragraphs to allow critical engagement with the textbook.

A more detailed guide to using the Interactive Textbook can be found on *Cambridge GO*.

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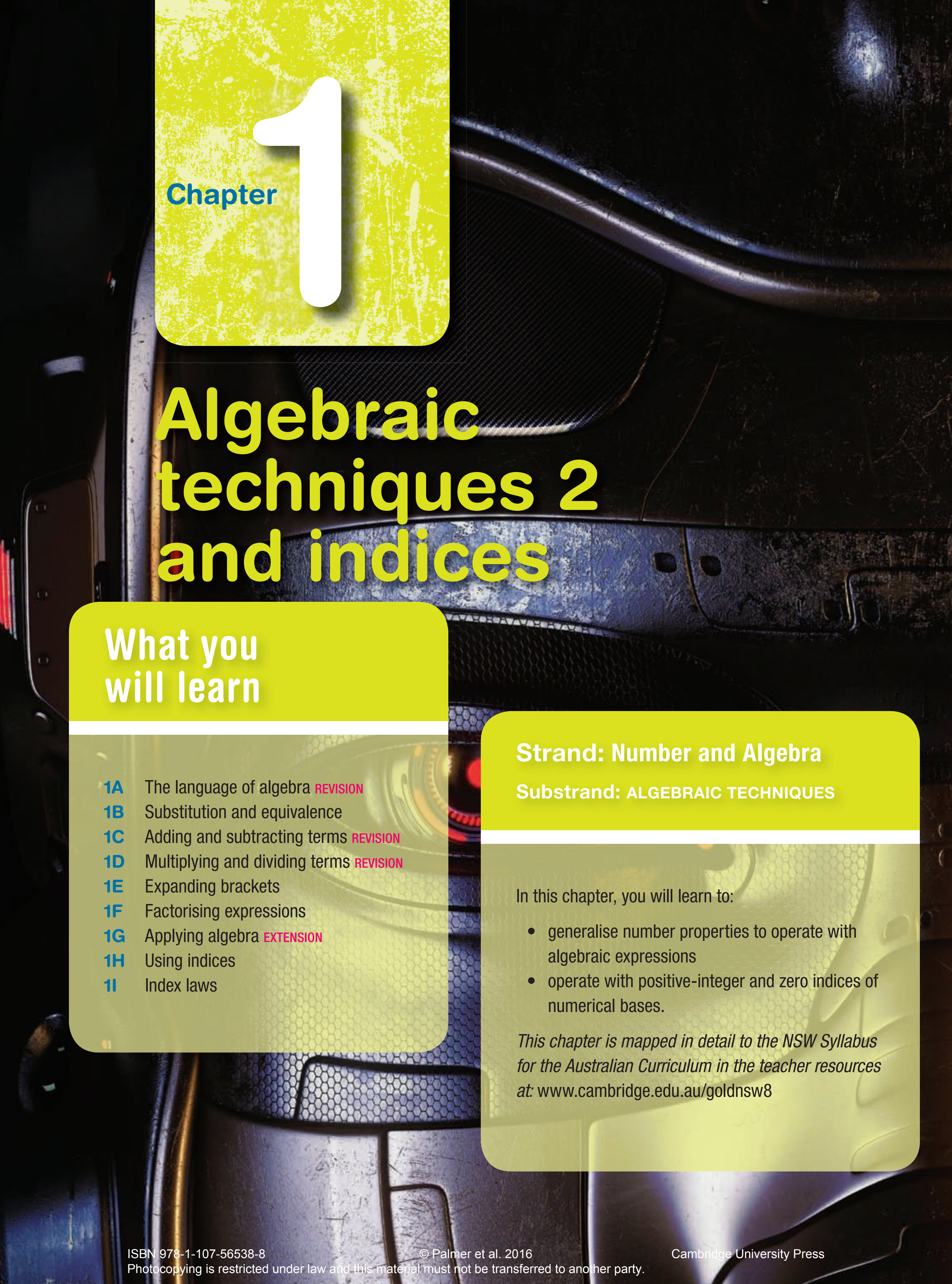

Chapter

# 1

# Algebraic techniques 2 and indices

## What you will learn

**Strand: Number and Algebra**

**Substrand: ALGEBRAIC TECHNIQUES**

In this chapter, you will learn to:

- generalise number properties to operate with algebraic expressions
- operate with positive-integer and zero indices of numerical bases.

*This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at:* www.cambridge.edu.au/goldnsw8

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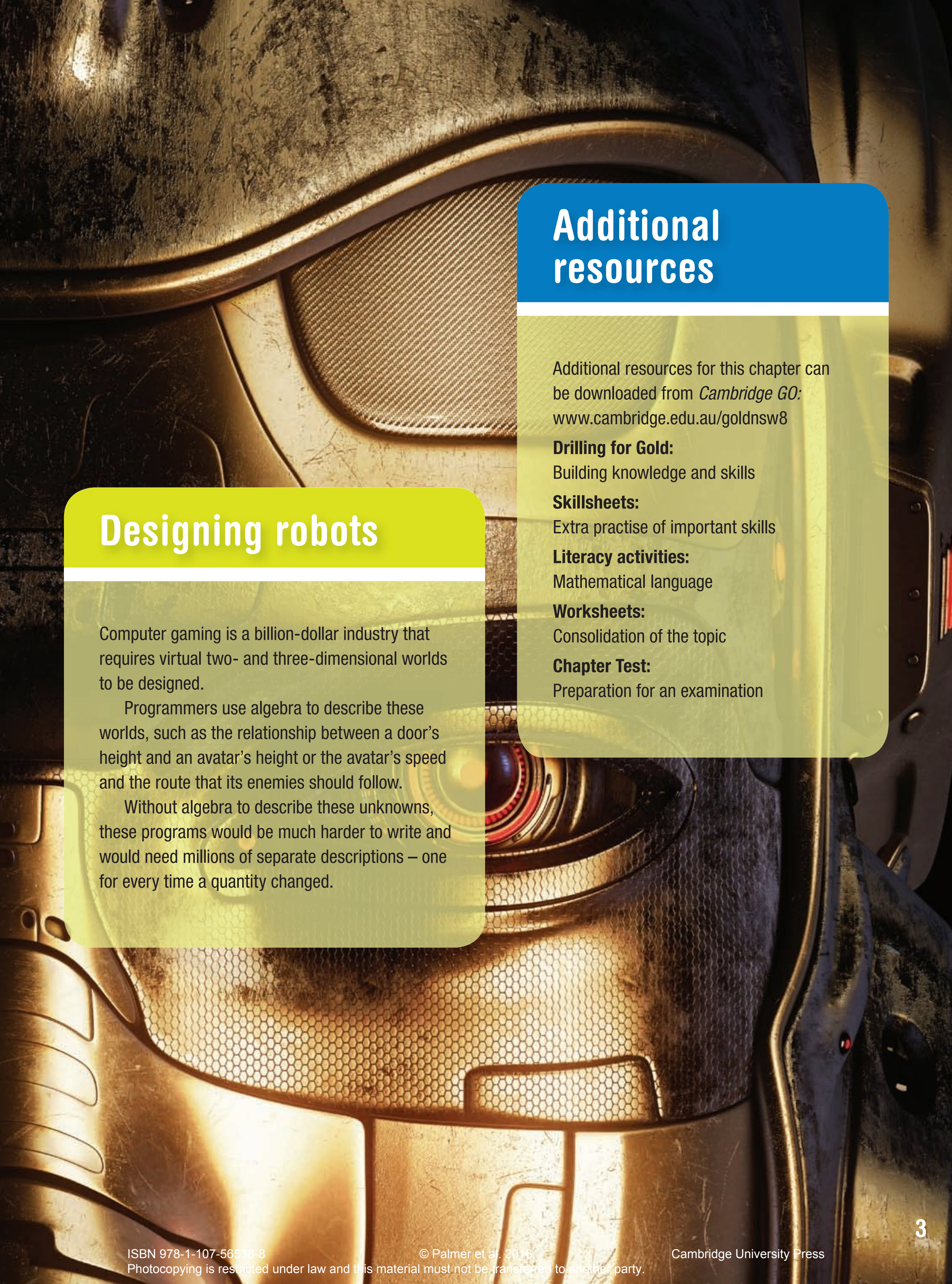

## Designing robots

Computer gaming is a billion-dollar industry that requires virtual two- and three-dimensional worlds to be designed.

Programmers use algebra to describe these worlds, such as the relationship between a door's height and an avatar's height or the avatar's speed and the route that its enemies should follow.

Without algebra to describe these unknowns, these programs would be much harder to write and would need millions of separate descriptions – one for every time a quantity changed.

## Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO:* www.cambridge.edu.au/goldnsw8

**Drilling for Gold:**
Building knowledge and skills

**Skillsheets:**
Extra practise of important skills

**Literacy activities:**
Mathematical language

**Worksheets:**
Consolidation of the topic

**Chapter Test:**
Preparation for an examination

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## Pre-test

**1** Evaluate:

**a** $8 + 4 \times 6$

**b** $4 \times 5 - 2 \times 3$

**c** $12 - (6 + 2) + 8$

**d** $3(6 + 4)$

**2** Evaluate:

**a** the sum of 7 and 10

**b** the product of 2 and 6

**c** the sum of 12, 10 and 8

**d** half of 24

**3** If $x = 10$, write the value of:

**a** $x + 2$

**b** $x \times 7$

**c** $x - 3$

**d** $x + x$

**4** Find the value of $6 \times y$ if:

**a** $y = 4$

**b** $y = 2$

**c** $y = 11$

**d** $y = 100$

**5** Write an expression for:

**a** 5 more than $x$

**b** 7 less than $m$

**c** the product of $x$ and $y$

**d** half of $w$

**6** If $y = 2x + 5$, find the value of $y$ when $x = 10$.

**7** Complete the tables using the given rules.

**a** $y = x + 12$

| $x$ | 1 | 3 | 11 | 0 |
|---|---|---|---|---|
| $y$ | | | | |

**b** $y = 2x + 3$

| $x$ | 0 | 3 | 7 | 10 |
|---|---|---|---|---|
| $y$ | | | | |

**8** Substitute $x = 6$ and $y = 2$ into each expression and then evaluate.

**a** $x + y$

**b** $xy$

**c** $3x - y$

**d** $2x + 3y$

**9** Write down the HCF (highest common factor) of:

**a** 6 and 8

**b** 8 and 12

**c** 6 and 12

**10** Evaluate:

**a** $5^2$

**b** $\sqrt{36}$

**c** $3^3$

**d** $\sqrt[3]{8}$

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# 1A The language of algebra

**REVISION**

A pronumeral is a letter that can represent one or more numbers. For instance, $x$ could represent the number of goals a particular soccer player scored last year.

This soccer player scored $x$ goals last year.

## Let's start: Algebra sort

Consider the four expressions $x + 2$, $x \times 2$, $x - 2$ and $x \div 2$.

- If you know that the value of $x$ is $10$, can you sort the four values from lowest to highest?
- Find a value of $x$ that would make $x \times 2$ less than $x + 2$.

## Key ideas

- Algebra is used to describe the rules and conventions of numbers and arithmetic.
- When doing algebra, we use the equals symbol (=) to indicate that two or more things have exactly the same value. This is called an **identity**, because they are identical.

**Identity** A statement that indicates that two expressions will have the same numerical value when the pronumerals are replaced with numbers

e.g. $3 + 5$ gives $8$, which is the same as $5 + 3$, so we can write $3 + 5 = 5 + 3$

'is identical to' or 'has the same value as'

- In the following table, the letters $a$, $b$ and $c$ could represent any numbers.

| | Number fact | Algebra fact | Notes |
|---|---|---|---|
| 1 | $5 + 3 = 3 + 5$ | $a + 3 = 3 + a$ | |
| 2 | $5 + 3 = 3 + 5$ | $a + b = b + a$ | Numbers can be added in any order. |
| 3 | $4 + 5 + 1 = 1 + 4 + 5$ | $b + c + a = a + b + c$ | |
| 4 | $-3 + 5 = 5 - 3$ | $-3 + a = a - 3$ | The negative sign 'belongs' to the number 3. |
| 5 | $5 \times 3 = 3 \times 5$<br>Numbers can be multiplied in any order. | $a \times 3 = 3 \times a$ | The multiplication symbol is usually 'invisible':<br>$a \times 3 = 3 \times a = 3a$ |
| 6 | $5 \times 3 = 3 \times 5$ | $a \times b = b \times a$ | This can be written as:<br>$a \times b = ab$ |

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| | Number fact | Algebra fact | Notes |
|---|---|---|---|
| 7 | $5 + 5 + 5 = 3 \times 5$<br>That is '3 lots of 5' | $a + a + a = 3 \times a$ | This can be written as:<br>$a + a + a = 3a$ |
| 8 | $2 \div 8 = \frac{2}{8}$ | $a \div 8 = \frac{a}{8}$ | Division and fractions are related to each other. The first number is the numerator. |
| 9 | $2 \div 8 = \frac{2}{8}$ | $a \div b = \frac{a}{b}$ | |

| Terminology | Example | Definition |
|---|---|---|
| pronumeral | $a$ | A letter of an alphabet or a symbol used to represent one or more numerical values |
| variable | $a$ | A pronumeral that represents more than one numerical value |
| expression or algebraic expression | $3a + 5$ | A statement containing numbers and pronumerals connected by mathematical operations, but containing no equals sign |
| term | The expression $3a + 5$ contains two terms. | One of the components of an expression |
| like terms | $3a$ and $5a$ are like terms.<br>$3a$ and $5b$ are not like terms.<br>$3a$ and $5a^2$ are not like terms. | Two or more terms that contain the same pronumerals and indices |
| constant<br>or<br>constant term | In the expression $3a + 5$, the number $5$ is called the constant or the constant term. | The part of an expression or equation without any pronumerals |
| coefficient | In the expression $3a + b + 5$ the:<br>• coefficient of $a$ is $3$<br>• coefficient of $b$ is $1$. | A numeral placed before a pronumeral to indicate that the pronumeral is multiplied by that factor |
| equivalent expressions | $3a + 5$ and $5 + 3a$ | Expressions that will have the same numerical value as each other when the pronumerals are replaced with any number |
| simplify | $3a + 5a$ simplifies to $8a$.<br>$3a + 5$ can't be simplified. | To find the simplest possible equivalent expression. |
| identity | $3a + 5 = 5 + 3a$<br>or<br>$3a + 5 \equiv 5 + 3a$ | A statement that indicates that two expressions will have the same numerical value when the pronumerals are replaced with numbers. The symbol $\equiv$ is sometimes used in identities. |
| substitute | If $a = 3$, $a + 5$ becomes $3 + 5$. | To replace pronumerals with numerical values |
| substitution | If $a = 3$, the value of $a + 5$ is $8$. | A process in which pronumerals are replaced with numbers |
| evaluate | Evaluate $a + 5$ when $a = 3$. | To calculate the numerical value of an expression in which all the pronumerals have been given a value |

Drilling for Gold 1A1

## Exercise 1A

**Understanding**

**1** Are the following identities true for all values of $x$? (Yes/No)

**a** $x + 2 = 2 + x$
**b** $x \times 2 = 2 \times x$
**c** $x - 2 = 2 - x$
**d** $x \div 2 = 2 \div x$
**e** $x \times 1 = x$
**f** $x \times 0 = 0$

### Example 1 Using the language of algebra

**a** List the individual terms in the expression $4a + b - 12c + 5$.

**b** In the expression $4a + b - 12c + 5$ state the coefficients of $a$, $b$ and $c$.

**c** What is the constant term in $4a + b - 12c + 5$?

| Solution | Explanation |
|---|---|
| **a** There are four terms: $4a$, $b$, $12c$ and $5$. | Each part of an expression is a term. Terms are added (or subtracted) to make an expression. |
| **b** The coefficient of $a$ is $4$.<br>The coefficient of $b$ is $1$.<br>The coefficient of $c$ is $-12$. | The coefficient is the number in front of a pronumeral. For $b$ the coefficient is $1$ because $b$ is the same as $1 \times b$. For $c$ the coefficient is $-12$ because this term is being subtracted. |
| **c** $5$ | A constant term is the part of an expression without any pronumerals. |

**2** The expression $3a + 2b + 5c$ has three terms.

**a** List the terms.
**b** State the coefficient of:
  **i** $a$  **ii** $b$  **iii** $c$
**c** Write another expression with three terms.

**3** **a** List the terms in $4x + 6y + 2z$.
**b** Which of the pronumerals ($x$, $y$ or $z$) has a coefficient of 6?

**4** The expression $5a + 7b + c - 3ab + 6$ has five terms.

**a** State the constant term.
**b** State the coefficient of:
  **i** $a$  **ii** $b$  **iii** $c$
**c** Write another expression that has five terms.

A constant term has no pronumeral.

**Fluency**

**5** State the number of terms in each expression.

**a** $7a + 2b + c$
**b** $19y - 52x + 32$
**c** $a + 2b$
**d** $7u - 3v + 2a + 123c$
**e** $10f + 2be$
**f** $9 - 2b + 4c + d + e$

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**6** For each of the following expressions, state the coefficient of $b$.

**a** $3a + 2b + c$ **b** $3a + b + 2c$ **c** $4a + 9b + 2c + d$

**d** $3a - 2b + f$ **e** $5a - 6b + c$ **f** $7a - b + c$

Coefficients are negative if the term is subtracted.

## Example 2 Creating expressions from a description

Write an expression for each of the following.

**a** The sum of 3 and $k$ **b** The product of $m$ and 7

**c** 5 is added to half of $k$ **d** The sum of $a$ and $b$ is doubled

| Solution | Explanation |
|---|---|
| **a** $3 + k$ | The word 'sum' means +. |
| **b** $m \times 7$ or $7m$ | The word 'product' means ×. |
| **c** $\frac{1}{2}k + 5$ or $\frac{k}{2} + 5$ | Half of $k$ can be written $\frac{1}{2} \times k$ (because 'of' means ×), or $\frac{k}{2}$ because $k$ is being divided by two. |
| **d** $(a + b) \times 2$ or $2(a + b)$ | The values of $a$ and $b$ are added, then the result is multiplied by 2. Brackets are required to multiply the whole result by two. |

Drilling for Gold 1A2

**7** Match each of the following worded statements with the correct mathematical expression.

| | |
|---|---|
| **a** The sum of $x$ and 7 | **A** $3 - x$ |
| **b** 3 less than $x$ | **B** $\frac{x}{3}$ |
| **c** Half of $x$ | **C** $x - 3$ |
| **d** $x$ is tripled | **D** $3x$ |
| **e** $x$ is subtracted from 3 | **E** $\frac{x}{2}$ |
| **f** $x$ is divided by 3 | **F** $x + 7$ |

**8** Write an expression for each of the following.

**a** 7 more than $y$ **b** 3 less than $x$

**c** The sum of $a$ and $b$ **d** The product of 4 and $p$

**e** Half of $q$ is subtracted from 4 **f** One third of $r$ is added to 10

**g** The sum of $b$ and $c$ multiplied by 2 **h** The sum of $b$ and twice the value of $c$

**9** Describe each of the following expressions in words.

**a** $3 + x$ **b** $a + b$ **c** $2 \times k$ **d** $\frac{m}{2}$

**10** Describe each of the following expressions in words.

**a** $4 \times b \times c$ **b** $2a + b$ **c** $(4 - b) \times 2$ **d** $4 - 2b$

Problem-solving and Reasoning

**11** Write an expression for the:

- **a** total cost of buying 10 litres of petrol at $\$x$ per litre.
- **b** time spent shopping if you spend $A$ minutes in the supermarket and $B$ minutes in the department store.
- **c** difference, in age, between Oliver, who is 22 years old, and his younger cousin, Ben, who is $k$ years old.
- **d** volume of water left in a 50-litre vat after $x$ litres are removed.

If petrol is $2 per litre, then the cost of 10 litres is $20.

**12** Marcela buys 7 plants from the local nursery.

- **a** If the cost is $10 for each plant, what is the total cost?
- **b** If the cost is $\$x$ for each plant, write an expression for the total cost in dollars.
- **c** If the cost of each plant is decreased by $3 during a sale, write an expression for the new:
  - **i** cost per plant in dollars
  - **ii** total cost in dollars of the 7 plants.

**13** Francine earns $\$p$ per week for her job. She works for 48 weeks each year. Write an expression for the amount she earns in:

- **a** a fortnight
- **b** one year (of 48 weeks)
- **c** one year if her wage is increased by $20 per week after she has already worked 30 weeks in the year.

## Enrichment: DVD dilemma

**14** Tom would like to purchase some DVDs of two television shows.

- **a** Write an expression for the total cost of:
  - **i** 4 seasons of Numbers
  - **ii** 7 seasons of Proof by Induction
  - **iii** 5 seasons of both shows
  - **iv** all 7 seasons of both shows, if the total price is halved in a sale.
- **b** If $a$ is 20 and $b$ is 30, how many DVDs could he buy for $200?

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# 1B Substitution and equivalence

Replacing pronumerals with numbers is called *substitution*. We can *evaluate* (find the value of) an expression once we substitute the numbers.

If two expressions always evaluate to the same number, they are called *equivalent*. For instance, $4 + x$ and $x + 4$ are equivalent.

## Let's start: AFL algebra

In the sport of AFL a goal scores 6 points and a behind scores 1 point. The team score is given by $6x + y$, where $x$ is the number of goals and $y$ is the number of behinds.

- State the score if $x = 3$ and $y = 4$.
- If the score is 29, what are the values of $x$ and $y$? Try to list all the possibilities.
- If $y = 9$ and the score is a 2-digit number, what are the possible values of $x$?

## Key ideas

**Evaluate** Find (calculate) the numerical value of

**Substitute** Replace pronumerals with numerical values

**Equivalent** Having the same values

- To **evaluate** an expression or to **substitute** values means to replace each pronumeral in an expression with a number to obtain a final value.
  e.g. If $a = 3$, then we can evaluate the expression $7a + 13$:
  $$\begin{aligned} 7a + 13 &= 7 \times 3 + 13 \\ &= 21 + 13 \\ &= 34 \end{aligned}$$
- Two expressions are **equivalent** if they have equal values regardless of the number that is substituted for each pronumeral.

## Exercise 1B

**Understanding**

Brackets first then division and multiplication, then addition and subtraction.

**1** State the value of:
**a** $5 + 3 \times 2$ **b** $5 \times 3 + 2$
**c** $17 - 2 \times 4$ **d** $20 \div 5 + 3$

**2** If $x = 6$, determine the value of each expression.
**a** $x + 5$ **b** $x \times 2$ **c** $x - 3$ **d** $x \div 2$

**3** Find the value of $x + 11$ if:
**a** $x = 5$ **b** $x = 10$ **c** $x = 100$ **d** $x = 59$

**4** Fill in the blanks.
Two expressions that are always equal are called __________ __________.

**Fluency**

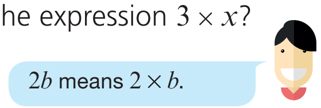
### Example 3 Substituting into an expression

Substitute $x = 3$ to evaluate $5x$.

| **Solution** | **Explanation** |
|---|---|
| $5x = 5 \times 3$<br>$= 15$ | Remember that $5x$ means $5 \times x$. |

Drilling for Gold 1B1

**5** **a** What number is obtained when $x = 5$ is substituted into the expression $3 \times x$?
**b** What is the result of evaluating $20 - b$ if $b$ is equal to $12$?
**c** What is the value of $2b$ if $b$ is equal to $10$?



**6** **a** State the value of $4 + 2x$ if $x = 5$.
**b** State the value of $40 - 2x$ if $x = 5$.
**c** Are $4 + 2x$ and $40 - 2x$ equivalent expressions?

**7** Substitute the following values of $x$ into the expression $7x + 2$.
**a** $4$ **b** $5$ **c** $2$ **d** $8$

**8** If $y = 4$, find the value of:
**a** $y + 3$ **b** $9 - y$ **c** $3y - 2$ **d** $5y + 3$

### Example 4 Substituting two numbers

Substitute $x = 3$ and $y = 6$ to evaluate $3x + 2y$.

| **Solution** | **Explanation** |
|---|---|
| $3x + 2y = 3 \times 3 + 2 \times 6$<br>$= 9 + 12$<br>$= 21$ | Replace all pronumerals with their values. Evaluate in the correct order (multiplication before addition). |

**9** If $a = 4$ and $b = 7$, evaluate:
**a** $3a + 2$ **b** $2b - 1$ **c** $a + b$ **d** $6 + ab$
**e** $3a + b$ **f** $2a + 3b$ **g** $b - a$ **h** $3b - a$

**10** Evaluate the expression $2x - 3y$ when:
**a** $x = 10$ and $y = 4$ **b** $x = 4$ and $y = 2$

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### Example 5 Deciding if expressions are equivalent

**a** Are $x - 3$ and $3 - x$ equivalent expressions?
**b** Are $a + b$ and $b + a$ equivalent expressions?

| Solution | Explanation |
|---|---|
| **a** No | The two expressions are equal if $x = 3$ (both equal zero).<br>But if $x = 7$ then $x - 3 = 4$ and $3 - x = -4$.<br>Because they are not equal for every single value of $x$, they are not equivalent. |
| **b** Yes | Regardless of the values of $a$ and $b$ substituted, the two expressions are equal. This is because it does not matter the order in which numbers are added. |

**11** For the following state whether they are equivalent (E) or not (N).

**a** $x + y$ and $y + x$
**b** $3 \times x$ and $x \times 3$
**c** $4a + b$ and $4b + a$
**d** $4 + 2x$ and $2 + 4x$
**e** $\frac{1}{2} \times a$ and $\frac{a}{2}$
**f** $3 + 6y$ and $3(2y + 1)$

Try different values to see if the expressions are always equal.

**Problem-solving and Reasoning**

**12** Which four of the following expressions represent a number added to itself?
$a + 2,\ a + a,\ \sqrt{a},\ a + a + a,\ 2 \times a,\ a - a,\ 2 + a,\ 2a,\ a^2$

**13 a** A number is substituted for $k$ in the expression $7k$ and the result is 56. What is the value of $k$?

**b** The pronumeral $m$ is chosen so that $4m$ is a two-digit number and $4 + m$ is a single-digit number. List all possible values of $m$.

**14** The expressions $ab$ and $a + b$ are not equivalent.

**a** Explain why they are not equivalent.

**b** If $a = 0$ and $b = 0$, the two expressions are equal. Give an example of another pair of values that make them equal.

Find values for $a$ and $b$ where $ab$ and $a + b$ are not equal.

**c** Explain why $a + 2$ and $a - 2$ are not equivalent.

**d** Will $a + 2$ and $a - 2$ ever evaluate to the same number? Why/why not?

## Enrichment: Missing values

**15** Copy and complete the following tables.

**a**

| $x$ | 3 | | | | 4 | 2 |
|---|---|---|---|---|---|---|
| $y$ | 8 | 7 | | –3 | | |
| $x + y$ | | 12 | 5 | | | |
| $x - 2y$ | | | –4 | | 8 | |
| $xy$ | | | | 0 | | 12 |

**b**

| $a$ | 10 | 0 | 2 | | | |
|---|---|---|---|---|---|---|
| $a + 2$ | | | | | | |
| $2a$ | | | | 24 | 10 | |
| $a^2$ | | | | | | |
| $2 - a$ | | | | | | |
| $\frac{a}{2}$ | | | | | | 0.5 |

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# 1C Adding and subtracting terms

**REVISION**

Two terms with the same pronumerals are called *like terms*. They can be collected and combined. For example, $2a + 6a$ can be simplified to $8a$ because $2a$ and $6a$ are like terms.

The order of the pronumerals does not matter, so $3ab$ and $5ba$ are like terms because they both include $a$ and $b$.

| | Like terms? |
|---|---|
| $3a$ and $4b$ | × |
| $3a$ and $4a$ | ✓ |
| $3a$ and $4a^2$ | × |
| $3ab$ and $4ba$ | ✓ |

## Let's start: Like terms

The terms $2abc$ and $5cab$ are like terms, and $2abc + 5cab = 7abc$.

- How many ways can you fill in the boxes?

  $\square + \square = 7abc$
- Can you explain why $abc$ and $cab$ are equivalent?

## Key ideas

**Like terms**
Terms with the same pronumerals and same powers

- **Like terms** contain exactly the same pronumerals with the same powers; the pronumerals do not need to be in the same order, e.g. $4ab$ and $7ba$ are like terms.
- Like terms can be combined when they are added or subtracted to simplify an expression, e.g. $3xy + 5xy = 8xy$.

– sign stays with following term

$$\begin{aligned}&3x + 7y \boxed{-2x} + 3y + x \boxed{-4y}\\ &= 3x - 2x + x + 7y + 3y - 4y\\ &= 2x + 6y\end{aligned}$$

- A subtraction symbol stays in front of a term even when it is moved.

## Exercise 1C

**Understanding**

**1** For each term below, state all the pronumerals that it contains.

**a** $7a$ **b** $4ac$ **c** $2xy$ **d** $3wz$

**2** Fill in the blanks.

**a** Two terms with exactly the same pronumerals are called __________ __________.

**b** If two expressions are always equal when evaluated, they are called __________ expressions.

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**3** **a** If $x = 3$, evaluate $5x + 2x$.
**b** If $x = 3$, evaluate $7x$.
**c** $5x + 2x$ is equivalent to $7x$. True or false?

**4** **a** If $x = 3$ and $y = 4$, evaluate $5x + 2y$.
**b** If $x = 3$ and $y = 4$, evaluate $7xy$.
**c** $5x + 2y$ is equivalent to $7xy$. True or false?

**5** **a** List the pronumerals that occur in $3abc$.
**b** List the pronumerals that occur in $7bca$.
**c** Are $3abc$ and $7bca$ like terms?

Like terms have the same pronumerals, possibly in a different order.

**Fluency**

## Example 6 Identifying like terms

Classify the following pairs as like terms (L) or not like terms (N).
**a** $3x$ and $12x$
**b** $5y$ and $7z$

| **Solution** | **Explanation** |
|---|---|
| **a** L | Both $3x$ and $12x$ have the same pronumeral ($x$) so they are like terms. |
| **b** N | $5y$ and $7z$ have different pronumerals so they are not like terms. |

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1C1

**6** Classify the following pairs as like terms (L) or not like terms (N).
**a** $5x$ and $2x$ **b** $5x$ and $2y$ **c** $3k$ and $4k$ **d** $5x$ and $5x^2$

## Example 7 Identifying like terms with multiple pronumerals

Classify the following pairs as like terms (L) or not like terms (N).
**a** $2ab$ and $3ba$ **b** $4x$ and $2xy$

| **Solution** | **Explanation** |
|---|---|
| **a** L | They have the same pronumerals (order does not matter). |
| **b** N | $4x$ has the pronumeral $x$.<br>$2xy$ has the pronumerals $x$ and $y$.<br>Since the terms have different pronumerals they are not like terms. |

**7** Classify the following pairs as like terms (L) or not like terms (N).
**a** $4pq$ and $3pq$ **b** $2ab$ and $5bc$ **c** $7rs$ and $12sr$ **d** $5ab$ and $6a$
**e** $7abc$ and $2cba$ **f** $8x$ and $8xy$ **g** $12ab$ and $14ba$ **h** $8xyz$ and $9yzx$

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## Example 8 Simplifying by combining like terms

Simplify the following by combining like terms.

**a** $7t + 2t - 3t$ **b** $4x + 3y + 2x + 7y$ **c** $8b + 7ac - 5b + 2ca$

| **Solution** | **Explanation** |
|---|---|
| **a** $7t + 2t - 3t$<br>$= 6t$ | These are like terms, so they can be combined:<br>$7 + 2 - 3 = 6$. |
| **b** $4x + 3y + 2x + 7y$<br>$= 4x + 2x + 3y + 7y$ | Move the like terms next to each other. |
| $= 6x + 10y$ | Combine the like terms. |
| **c** $8b + 7ac - 5b + 2ca$<br>$= 8b - 5b + 7ac + 2ca$ | Move like terms together.<br>The subtraction sign stays in front of $5b$ when it is moved. |
| $= 3b + 9ac$ | $8 - 5 = 3$ and $7 + 2 = 9$ |

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**8** Simplify the following by combining like terms.

**a** $3x + 2x$ **b** $7a + 12a$ **c** $15x - 6x$ **d** $9y - 2y$

**e** $4xy + 3xy$ **f** $16uv - 3uv$ **g** $10ab + 4ba$ **h** $3pq + 12pq$

**i** $x + x$ **j** $6x + x$ **k** $x - x$ **l** $6x - x$

**m** $x + x - x$ **n** $2x - x - x$ **o** $x - 2x$ **p** $6x - 5x$

**9** Simplify the following by combining like terms.

**a** $7f + 2f + 8 + 4$ **b** $10x + 3x + 5y + 3y$

**c** $2a + 5a + 13b - 2b$ **d** $10a + 5b + 3a + 4b$

**e** $10 + 5x + 2 + 7x$ **f** $10a + 3 + 4b - 2a - b$

**g** $10x + 31y - y + 4x$ **h** $11a + 4 - 2a + 12a$

**i** $2b + 4c + 3b + 5c$ **j** $3a - b + 4b - a$

Pair up the like terms
Note: $ab = ba$

**10** For each expression choose an equivalent expression from the options listed.

**a** $7x + 2x$ **A** $10y + 3x$

**b** $12y + 3x - 2y$ **B** $9xy$

**c** $3x + 3y$ **C** $9x$

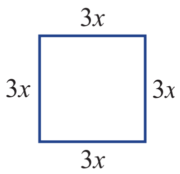
**d** $8y - 2x + 6y - x$ **D** $3y + 3x$

**e** $4xy + 5yx$ **E** $14y - 3x$

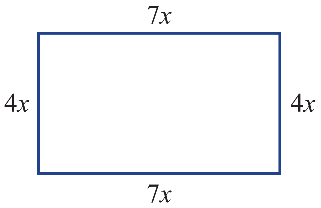
Problem-solving and Reasoning

**11** Write expressions (in simplest form) for the perimeters of the following shapes.

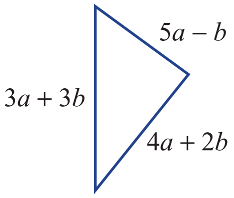
**a**



**b**



**c**



Perimeter = total distance around shape

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**12** Towels cost $\$c$ each at a shop.

- **a** John buys 3 towels, Mary buys 6 towels and Naomi buys 4 towels. Write a fully simplified expression for the total amount spent on towels.
- **b** On another occasion, Chris buys $n$ towels, David buys twice as many as Chris and Edward buys 3 times as many as David. Write a simplified expression for the total amount they spent on towels.

**13** **a** Make a substitution to prove that $4a + 3b$ is not equivalent to $7ab$.

- **b** Is $4a + 3b$ ever equal to $7ab$? Try to find some values of $a$ and $b$ to make $4a + 3b = 7ab$ a true equation.
- **c** Is $4a + 3a$ ever not equal to $7a$? Explain your answer.

## Enrichment: Filling in the blanks

**14** The expression $4a + 7b + 6a$ is equivalent to $10a + 7b$.

- **a** Give another way to fill in the blanks to make this statement true:
  $\square a + \square b + \square a = 10a + 7b$
- **b** Assuming the blanks above must be filled by positive integers, how many ways could they be filled to make a true statement?

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# 1D Multiplying and dividing terms

**REVISION**

Recall that $4ab$ is shorthand for $4 \times a \times b$. Observing this helps us to see how we can multiply terms.

$$\begin{aligned} 4ab \times 3c &= 4 \times a \times b \times 3 \times c \\ &= 4 \times 3 \times a \times b \times c \\ &= 12abc \end{aligned}$$

Division is written as a fraction, so $\frac{12ab}{9ad}$ means $(12ab) \div (9ad)$. To simplify a division, we look for common factors.

$$\frac{{}^{4}\cancel{12} \times \cancel{a} \times b}{{}^{3}\cancel{9} \times \cancel{a} \times d} = \frac{4b}{3d}$$

$a \div a = 1$ for any value of $a$ except $0$

so $\frac{a}{a}$ cancels to $1$.

## Let's start: Multiple ways

Multiplying $4a \times 6b$ gives you $24ab$.

- In how many ways can positive integers fill the blanks in $\square a \times \square b = 24ab$?
- Can you explain why there are more ways to fill in the blanks for $\square a \times \square b = 24ab$ than for $\square a \times \square b = 25ab$?

## Key ideas

- $12abc$ means $12 \times a \times b \times c$.
- When multiplying, the order is not important: $2 \times a \times 4 \times b = 2 \times 4 \times a \times b$.
- $x \times x$ can be written as $x^2$.
- When dividing, cancel any common factors.

  For example: $\frac{{}^{3}\cancel{15}x\cancel{y}}{{}^{4}\cancel{20}\cancel{y}z} = \frac{3x}{4z}$

## Exercise 1D

**Understanding**

**1** Are the following true or false?

- **a** $3 \times a$ can be written as $3a$.
- **b** $k \times 5$ can be written as $5k$.
- **c** $2x$ is short for $2 + x$.
- **d** $4ab$ could also be written as $4a \div b$.
- **e** $q \times q$ can be written as $q^2$.

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**2** Which is the correct way to write $3 \times a \times b \times b$?

**A** $3ab$ **B** $3ab^2$ **C** $ab^3$ **D** $3a^2b$

**3** Which expression is equivalent to $xy^2$?

**a** $x \times y \times y$ **b** $x \times y \times 2$ **c** $x \times x \times y \times y$ **d** $(xy)^2$

**4** Write these without multiplication signs.

**a** $3 \times x \times y$ **b** $5 \times a \times b \times c$ **c** $12 \times a \times b \times b$ **d** $4 \times a \times c \times c \times c$

**Fluency**

## Example 9 Multiplying terms

Simplify $7a \times 2bc \times 3d$.

| **Solution** | **Explanation** |
|---|---|
| $7a \times 2bc \times 3d = 7 \times a \times 2 \times b \times c \times 3 \times d$<br>$= 7 \times 2 \times 3 \times a \times b \times c \times d$<br>$= 42abcd$ | Write the expression with multiplication signs and bring the numbers to the front.<br>Simplify: $7 \times 2 \times 3 = 42$ and $a \times b \times c \times d = abcd$ |

**5** Simplify the following.

**a** $7d \times 9$ **b** $5a \times 2$ **c** $3 \times 12x$

**d** $4k \times 6$ **e** $3 \times 2q$ **f** $3x \times 10y$

**g** $4a \times 2b \times cd$ **h** $3a \times 10bc \times 2d$ **i** $4a \times 6de \times 2b$

## Example 10 Multiplying terms with squares

Simplify $3xy \times 5xz$.

| **Solution** | **Explanation** |
|---|---|
| $3xy \times 5xz = 3 \times x \times y \times 5 \times x \times z$<br>$= 3 \times 5 \times x \times x \times y \times z$<br>$= 15x^2yz$ | Write the expression with multiplication signs and bring the numbers to the front.<br>Simplify, remembering that $x \times x = x^2$. |

**6** Simplify the following.

**a** $x \times x$ **b** $a \times a$ **c** $3d \times d$

**d** $5d \times 2d \times e$ **e** $7x \times 2y \times x$ **f** $5xy \times 2x$

**g** $4xy \times 2xz$ **h** $4abc \times 2abd$ **i** $12xy \times 4x$

**j** $9ab \times 2a$ **k** $3xy \times 2x \times 4y$ **l** $2ab \times 4a \times 3b$

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**7** Write each expression without a division sign.

**a** $k \div 4$  **b** $x \div 5$  **c** $2q \div 5$  **d** $3k \div 10$

**e** $5 \div a$  **f** $a \div b$  **g** $x \div y$  **h** $12 \div g$

$\frac{k}{4}$ is the same as $k \div 4$.

## Example 11 Simplifying fractions

Simplify $\frac{10}{15}$.

**Solution**

$$\frac{10}{15} = \frac{{}^{2}\cancel{10}}{{}^{3}\cancel{15}}$$
$$= \frac{2}{3}$$

**Explanation**

Divide the numerator and Denominator by highest common factor (HCF) to expess the fraction in its simplest form.

**8** Simplify these fractions.

**a** $\frac{12}{20}$  **b** $\frac{5}{15}$  **c** $\frac{12}{8}$  **d** $\frac{15}{25}$

## Example 12 Dividing terms

Simplify $\frac{10ab}{15bc}$.

**Solution**

$$\frac{10ab}{15bc} = \frac{{}^{2}\cancel{10} \times a \times \cancel{b}}{{}^{3}\cancel{15} \times \cancel{b} \times c}$$
$$= \frac{2a}{3c}$$

**Explanation**

Write the numerator and denominator in full, with multiplication signs. Cancel any common factors and remove the multiplication signs.

Skillsheet 1A

**9** Simplify the following divisions by cancelling any common factors.

**a** $\frac{5a}{10a}$  **b** $\frac{7x}{14y}$  **c** $\frac{10xy}{12y}$  **d** $\frac{ab}{4b}$

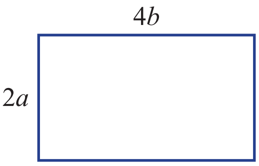
**e** $\frac{7xyz}{21yz}$  **f** $\frac{2}{12x}$  **g** $\frac{4xy}{7x}$  **h** $\frac{3abc}{6b}$

Cancel numbers and pronumerals where possible

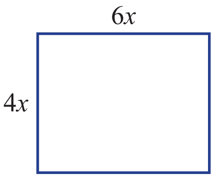
**Problem-solving and Reasoning**

**10** Write a simplified expression for the area of the following shapes. Recall that the area of a rectangle is given by length multiplied by breadth.

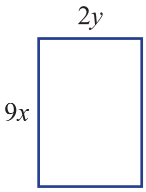
**a**



**b**



**c**

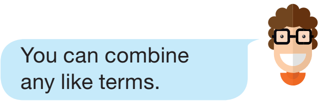


**11** Simplify the following completely.

**a** $2a \times 3b + 5ab$

**b** $6q \times 2r + 4q \times 3r$

**c** $10x \times 2y - 3y \times 6x$



**12** Fill in the missing terms to make the following identities true.

**a** $3x \times \square \times z = 6xyz$

**b** $4a \times \square = 12ab$

**c** $\dfrac{\square}{4r} = 7s$

**d** $\dfrac{\square}{2ab} = 4b$

**13** Joanne claims that the following three expressions are equivalent: $\dfrac{2a}{5}, \ \dfrac{2}{5} \times a, \ \dfrac{2}{5a}$.

**a** Is she correct? Try different values of $a$.

**b** Which two expressions are equivalent?

**c** There are two values of $a$ that make all three expressions are equal. State one of them.

## Enrichment: Missing coefficients

**14 a** Simplify $2a \times 3b + 5b \times 2a$ to a single term.

**b** State another way to fill in the blanks to make the simplification correct:
$\square a \times \square b + \square b \times \square a = 16ab$

**c** Give an example of an even longer expression that is equivalent to $16ab$.

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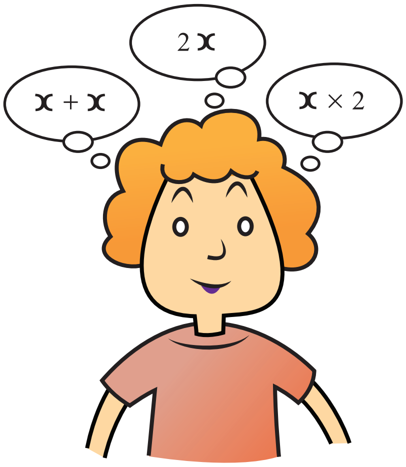
## 1E Expanding brackets

Expressions can look different and still be equivalent, like $x + x$, $2x$ and $x \times 2$.

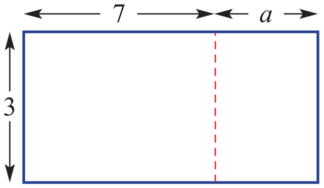
Expressions involving brackets can also be written in various ways.



### Let's start: Equivalent areas

What is the total area of this shape?

Try to write two expressions: one with brackets and the other without brackets.



## Key ideas

**Expand** Remove grouping symbols (brackets)

**Distributive law** Adding numbers and then multiplying the total gives the same answer as multiplying each number first and then adding the product

- The expression for '3 lots of (7+*a*)' is $3 \times (7 + a) = 3(7 + a)$, so

$$\begin{aligned} 3(7 + a) &= 7 + a + 7 + a + 7 + a \\ &= 21 + 3a \end{aligned}$$

This means that $3(7 + a)$ and $21 + 3a$ are equivalent.

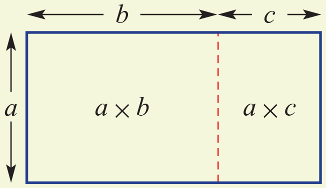
- **Expanding** brackets involves writing an equivalent expression without brackets:

$$\begin{aligned} 2(a + b) &= a + b + a + b \\ &= 2a + 2b \end{aligned}$$

or

$$\begin{aligned} 2(a + b) &= 2 \times a + 2 \times b \\ &= 2a + 2b \end{aligned}$$

- To expand an expressions, you can use the **distributive law**, which states that:
  - $a(b + c) = ab + ac$
  - $a(b - c) = ab - ac$
- The distributive law can be demonstrated by considering rectangle areas:



Area = $a(b + c)$ or Area = $ab + ac$

$\therefore \ a(b + c) = ab + ac$

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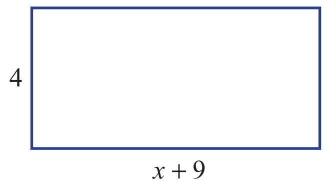
## Exercise 1E

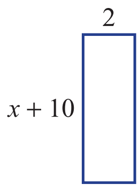
**Understanding**

**1** Write an expression involving brackets the area of each of the following rectangles?

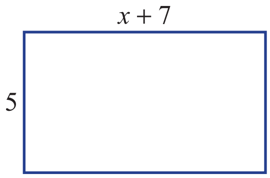
**a**



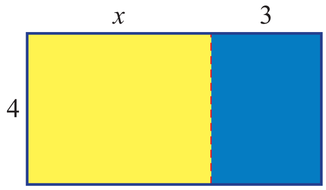
**b**



**c**



**2** Consider the rectangle below. Which expressions give the area of the rectangle? There is more than one correct answer.



**A** $4 \times x + 3$ **B** $4 \times (x + 3)$ **C** $4 \times x + 4 \times 3$

**D** $4x + 12$ **E** $(x + 3) \times 4$ **F** $4(x + 3)$

### Example 13 Expanding brackets by simplifying repeated terms

Write the expression $3(2m + 5)$ in full without brackets and simplify the result.

| **Solution** | **Explanation** |
|---|---|
| $3(2m + 5) = 2m + 5 + 2m + 5 + 2m + 5$<br>$= 6m + 15$ | $3(2m + 5)$ means 3 'lots of' $2m + 5$.<br>Simplify by collecting the like terms. |

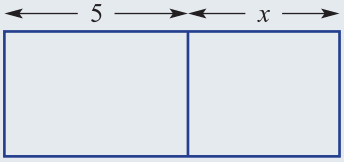
**3** The expression $3(a + 2)$ can be written as $(a + 2) + (a + 2) + (a + 2)$.
- **a** Simplify this expression by collecting like terms.
- **b** Write $2(x + y)$ out in full without brackets and simplify the result.
- **c** Write $4(p + 1)$ out in full without brackets and simplify the result.
- **d** Write $3(4a + 2b)$ out in full without brackets and simplify the result.

**4** Copy and complete.
- **a** $3(2 + 5) = 3 \times \square + \square \times \square$
- **b** $3(x + 2) = 3 \times \square + \square \times \square$
- **c** $a(b + c) = \square \times \square + \square \times \square$

**Fluency**

### Example 14 Expanding brackets using rectangle areas

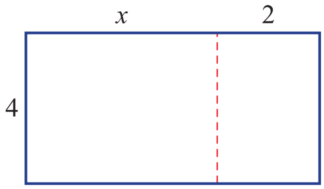
Write two equivalent expressions for the total area of the rectangle shown: one with brackets and the other without brackets.



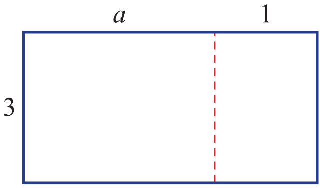
| **Solution** | **Explanation** |
|---|---|
| Using brackets: $2(5 + x)$ | The dimensions are 2 and $(5 + x)$ |
| Without brackets: $10 + 2x$ | $2 \times 5 = 10$ and $2 \times x = 2x$ |

**5** For each of the following rectangles, write two equivalent expressions for the total area.

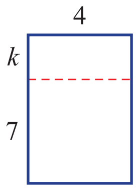
**a**



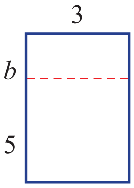
**b**



**c**



**d**



One of the expressions should have brackets.

### Example 15 Expanding using the distributive law

Expand the following expressions.

**a** $5(x + 3)$ **b** $3(a - 4)$ **c** $2(3p - 7q)$

| **Solution** | **Explanation** |
|---|---|
| **a** $5(x + 3) = 5x + 5 \times 3$ | Using the distributive law<br>$5(x + 3) = 5x + 5 \times 3$ |
| $= 5x + 15$ | Simplify the result. |

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| **Solution** | **Explanation** |
|---|---|
| **b** $3(a-4) = 3a - 3 \times 4$ | Using the distributive law<br>$3(a-4) = 3a - 3 \times 4$ |
| $= 3a - 12$ | Simplify the result. |
| **c** $2(3p - 7q) = 2 \times 3p - 2 \times 7q$ | Using the distributive law<br>$2(3p - 7q) = 2 \times 3p - 2 \times 7q$ |
| $= 6p - 14q$ | Simplify the result, remembering $2 \times 3p = 6p$ and $2 \times 7q = 14q$. |

**6** Use the distributive law to expand the following.
**a** $6(y + 8)$ **b** $7(\ell + 4)$ **c** $9(a + 7)$ **d** $2(t + 6)$

**7** Use the distributive law to expand the following.
**a** $2(m - 10)$ **b** $8(y - 3)$ **c** $3(e - 7)$ **d** $7(e - 3)$

**8** Use the distributive law to expand the following.
**a** $10(6g - 7)$ **b** $5(3e - 8)$ **c** $5(7w + 10)$ **d** $5(2u + 5)$
**e** $7(8x - 2)$ **f** $3(9v - 4)$ **g** $7(2q - 4)$ **h** $4(5c - v)$
**i** $4(2 + 5x)$ **j** $3(7 + 2y)$ **k** $8(9 - 3x)$ **l** $11(2 - 4k)$

Skillsheet 1B

**9** Fill in the missing number in the following expansions.
**a** $4(x + 5) = 4x + \square$ **b** $3(x + 2) = 3x + \square$
**c** $5(3a + 2) = 15a + \square$ **d** $7(4x - 2) = 28x - \square$

**Problem-solving and Reasoning**

**10** The perimeter of a rectangle is given by the expression $2(\ell + b)$ where $\ell$ is the length and $b$ is the breadth. What is an equivalent expression for this?

**11** Expand the brackets in the following and then simplify the result.
**a** $3(x + 2) + 4x$ **b** $4(a + 3) - 2a$ **c** $5(3b - 2) + 10$
**d** $6(2c + 4) - 2c$ **e** $6 + 2(x + 2)$ **f** $9 + 4(x - 1)$

You can combine like terms.

**12** Write an expression for each of the following and then expand it.
**a** A number $x$ has 3 added to it and the result is multiplied by 5.
**b** A number $b$ has 6 added to it and the result is doubled.
**c** A number $z$ has 4 subtracted from it and the result is multiplied by 3.
**d** A number $y$ is subtracted from 10 and the result is multiplied by 7.

**13** When expanded, $4(2a + 6b)$ gives $8a + 24b$. Find two other expressions that expand to $8a + 24b$.

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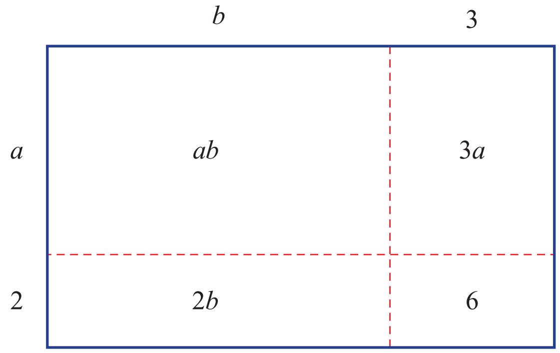
## Enrichment: Bigger expansions

**14** The diagram below helps to demonstrate that $(a + 2)(b + 3) = ab + 2b + 3a + 6$.

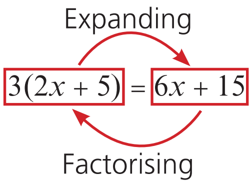


Use a diagram like the one above to expand the following expressions.

**a** $(a + 4)(b + 2)$
**b** $(x + 3)(y + 5)$
**c** $(2a + 5)(3c + 2)$
**d** $(4a + 1)(5b + 3)$

# 1F Factorising expressions

Factorising is the opposite procedure to expanding. The expression $3(2x + 5)$ expands to $6x + 15$, so the factorised form of $6x + 15$ is $3(2x + 5)$.



## Let's start: Expanding gaps

- Try to fill in the gaps to make the following equivalence true: ☐(☐ + ☐) = $12x + 24$.
- In how many ways can this be done? Try to find as many ways as possible.
- If the aim is to make the term outside the brackets as large as possible, what is the best possible solution to the puzzle?

## Key ideas

- The **highest common factor (HCF)** of two terms is the largest factor that divides into each term.
  e.g. HCF of $15x$ and $21y$ is 3.
  HCF of $10a$ and $20c$ is 10.
  HCF of $12x$ and $18xy$ is $6x$.
- To **factorise** an expression, first take the HCF of the terms outside the brackets and divide each term by it, leaving the result in brackets.

e.g. $10x + 15y$

HCF = 5

$$\underset{\text{HCF}}{5}(\underset{10x \div 5}{2x} + \underset{15y \div 5}{3y})$$

**Highest common factor (HCF)** The largest term that is a factor of all the given terms

**Factorise** To write an expression as a product

## Exercise 1F

**Understanding**

1 Fill in the blanks to make these equations true.

| | | | |
|---|---|---|---|
| **a** $5 \times$ ☐ $= 20$ | **b** ☐ $\times 4 = 12$ | **c** $10 \times$ ☐ $= 20$ | **d** ☐ $\times 4 = 24$ |

2 The factors of 14 are 1, 2, 7 and 14. The factors of 26 are 1, 2, 13 and 26. What is the highest factor that these two numbers have in common?

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### Example 16 Finding the highest common factor (HCF)

Find the highest common factor (HCF) of 20 and 35.

| Solution | Explanation |
|---|---|
| 5 | 5 is the largest number that divides into 20 and 35. |

**3** Find the highest common factor of the following pairs of numbers.
**a** 12 and 18 **b** 15 and 25 **c** 40 and 60 **d** 24 and 10

**4** Fill in the blanks.
**a** $5x \times \square = 15x$ **b** $7 \times \square\, a = 28a$ **c** $3 \times \square = 6b$ **d** $2 \times \square = 14x$

**5** Fill in the blanks to make these expansions correct.
**a** $3(4x + 1) = \square x + 3$ **b** $5(7 - 2x) = \square - 10x$
**c** $6(2 + 5y) = \square + \square y$ **d** $7(2a - 3b) = \square - \square$
**e** $3(2a + \square) = 6a + 21$ **f** $4(\square - 2y) = 12 - 8y$
**g** $7(\square + \square) = 14 + 7q$ **h** $\square(2x + 3y) = 8x + 12y$

$a(b + c) = ab + ac$

Fluency

### Example 17 Finding the highest common factor (HCF)

Find the highest common factor (HCF) of:
**a** $12k$ and 20 **b** $18x$ and $24xy$

| Solution | Explanation |
|---|---|
| **a** 4 | There are no pronumerals in common, so choose the highest common factor of 12 and 20. |
| **b** $6x$ | 6 is the largest number that divides into 18 and 24, and $x$ is in both terms. |

**6** Find the highest common factor (HCF) of the following pairs of terms.
**a** 15 and $10x$ **b** $20a$ and 12 **c** 27 and $9b$
**d** $7y$ and $14x$ **e** $3a$ and $6b$ **f** $12x$ and $18y$

The HCF can include pronumerals.

**7** Find the HCF of the following pairs of terms.
**a** $12x$ and $18xy$ **b** $8a$ and $16ab$ **c** $9bc$ and $12b$
**d** $36xy$ and $24y$ **e** $10q$ and $12qr$ **f** $8p$ and $20pq$

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## Example 18 Factorising expressions

Factorise the following expressions.

**a** $6x + 15$ **b** $12a - 18ab$

| **Solution** | **Explanation** |
|---|---|
| **a** $6x + 15 = 3(2x + 5)$ | HCF of $6x$ and 15 is 3. $6x \div 3 = 2x$ and $15 \div 3 = 5$ |
| **b** $12a - 18ab = 6a(2 - 3b)$ | HCF of $12a$ and $18ab$ is $6a$. $12a \div 6a = 2$ and $18ab \div 6a = 3b$ |

**8** Factorise the following by first finding the highest common factor.

**a** $3x + 6$ **b** $8v + 40$ **c** $15x + 35$ **d** $10z + 25$
**e** $40 + 4w$ **f** $5j - 20$ **g** $9b - 15$ **h** $12 - 16f$
**i** $5d - 30$ **j** $10x + 5$ **k** $6k - 12$ **l** $18p + 20$

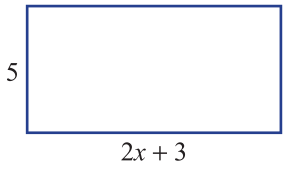
**Skillsheet 1C**

**9** Factorise the following.

**a** $10cn + 12n$ **b** $24y + 8ry$ **c** $14jn + 10n$ **d** $24g + 20gj$
**e** $10h + 4z$ **f** $30u - 20n$ **g** $21p - 6c$ **h** $12a + 15b$

**Problem-solving and Reasoning**

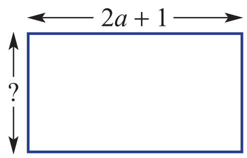
**10** The rectangle shown has an area of $10x + 15$. Draw a rectangle that would have an area $12x + 16$.



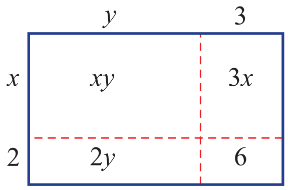
**11** The area of the rectangle shown is $10a + 5$. One side's measurement is unknown.

**a** What is the value of the unknown measurement?

**b** Write a simplified expression for the perimeter of the rectangle.



**12** Consider the diagram shown to the right. What is the factorised form of $xy + 3x + 2y + 6$?



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## Enrichment: The factorising photographer

**13** A group of students lines up for a photo. They are in $6$ rows with $x$ students in each row. Another $18$ students join the photo.

- **a** Write an expression for the total number of students in the photo.
- **b** Factorise the expression.
- **c** How many students would be in each of the $6$ rows now? Write an expression.
- **d** If the photographer wanted just $3$ rows, how many students would be in each row? Write an expression.
- **e** If the photographer wanted just $2$ rows, how many students would be in each row? Write an expression.

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# 1G Applying algebra

**EXTENSION**

The skills of algebra can be applied to many situations involving unknown or varying quantities.

Algebra can be applied to both the engineering of a carnival ride and the price of tickets.

## Let's start: Carnival conundrum

Alwin, Bryson and Calvin have each been offered special deals for the local carnival.

- Alwin can pay \$50 to go on all the rides all day.
- Bryson can pay \$20 to enter the carnival and then pay \$2 per ride.
- Calvin can enter the carnival at no cost and then pay \$5 per ride.

- Which of them do you think has the best deal?
- In the end, they each went on 12 rides. Who paid the most? Who paid the least?

## Key ideas

- Different situations can be **modelled** with algebraic expressions.
- To apply a rule, the pronumerals should first be clearly defined.
  e.g. total cost is $2 \times n + 3 \times d$

  $n$ = number of minutes

  $d$ = distance in km

**Modelling**
Representing a real-life situation using an algebraic expression

## Exercise 1G

**Understanding**

**1** The cost of a newspaper is \$2 and the cost of an ice-cream is \$3. Find the cost of:
  **a** 5 newspapers
  **b** 4 ice-creams
  **c** 10 newspapers and 2 ice-creams.

**2** An episode of Joshua's favourite television program lasts 30 minutes.
  **a** How long would it take him (in minutes) to watch:
    **i** 2 episodes? **ii** 5 episodes? **iii** 10 episodes?
  **b** Which of the following expressions gives the total time to watch $n$ episodes?
    **A** $n + 30$ **B** $30n$ **C** $n \div 30$ **D** $30 - n$

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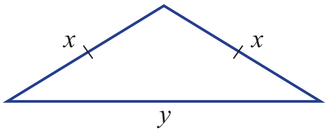
**3** Evaluate the expression $3d + 5$ when:
**a** $d = 10$ **b** $d = 12$ **c** $d = 0$

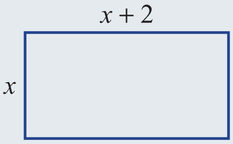
**4** Consider the isosceles triangle shown.
**a** Write an expression for the perimeter of the triangle.
**b** Find the perimeter when $x = 3$ and $y = 2$.



**Fluency**

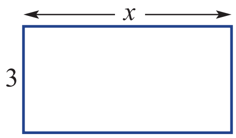
## Example 19 Writing expressions from descriptions

Write an expression for the following situations.
**a** The total cost of $k$ bottles if each bottle cost \$4
**b** The perimeter of a rectangle if its breadth is 2 cm more than its length and its length is $x$ cm
**c** The total cost of hiring a plumber for $n$ hours if he charges \$40 call-out fee and \$70 per hour



| Solution | Explanation |
|---|---|
| **a** $4 \times k = 4k$<br>∴ The cost is $\$(4k)$. | Each bottle costs \$4 so the total cost is \$4 multiplied by the number of bottles purchased. |
| **b** $x + x + 2 + x + x + 2 = 4x + 4$<br>∴ The perimeter is $(4x + 4)$ cm. | Length $= x$ so breadth $= x + 2$.<br>The perimeter is length + breadth + length + breadth. |
| **c** $40 + 70n$<br>∴ The cost is $\$(40 + 7n)$. | \$70 per hour means that the cost to hire the plumber would be $70 \times n$. Additionally \$40 is added for the call-out fee, which is charged regardless of how long the plumber stays. |

**5** **a** Write an expression for the total perimeter of this rectangle.
**b** If $x = 9$, what is the perimeter?
**c** Write an expression for the area.



**6** Pens cost \$3 each.
**a** How much would 10 pens cost?
**b** Write an expression for the total cost of $n$ pens.
**c** If $n = 12$, find the total cost.

Rectangle area = length × breadth

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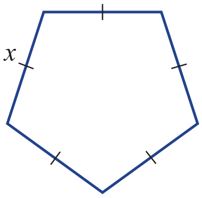
**7** An electrician charges a call-out fee of \$30 and \$90 per hour.

- **a** How much does a 2-hour visit cost?
- **b** Which of the following represents the total cost (in dollars) for $x$ hours?
  - **A** $x(30 + 90)$
  - **B** $30x + 90$
  - **C** $30 + 90x$
  - **D** $120x$

**Skillsheet 1D**



**8** **a** Give an expression for the perimeter of this regular pentagon.

- **b** If each side length were doubled, what would the perimeter be?
- **c** If each side length were increased by 3, write a new expression for the perimeter.

**Problem-solving and Reasoning**

**9** An indoor soccer pitch costs \$40 per hour to hire plus a \$30 booking fee.

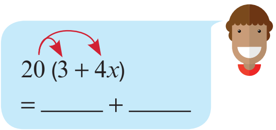
- **a** Write an expression for the cost of hiring the pitch for $x$ hours.
- **b** What is the cost of hiring the pitch for an 8-hour tournament?

**10** A plumber says that the cost in dollars to hire her for $x$ hours is $50 + 60x$.

- **a** What is her call-out fee?
- **b** How much does she charge per hour?
- **c** How much does a 3-hour visit cost?

**11** A repairman says the cost in dollars to hire his services for $x$ hours is $20(3 + 4x)$.

- **a** How much would it cost to hire him for 1 hour?
- **b** Expand the expression he has given you.
- **c** What is:
  - **i** his call-out fee?
  - **ii** the amount he charges per hour?



**12** Tamir notes that whenever he hires an electrician, they charge a call-out fee $\$F$ and an hourly rate of $\$H$ per hour.

Your expressions should involve $F$ and/or $H$.

**a** Write an expression for the cost of hiring an electrician for one hour.
**b** Write an expression for the cost of hiring an electrician for two hours.
**c** Write an expression for the cost of hiring an electrician for $30$ minutes.

## Enrichment: Ticket sales

**13** Three deals are available at a fair.

Deal $1$: Pay $\$10$, rides cost $\$4$ each.
Deal $2$: Pay $\$20$, rides cost $\$1$ each.
Deal $3$: Pay $\$30$, all rides are free.

**a** Write an expression for the total cost of $n$ rides using deal $1$. (The total cost includes the entry fee of $\$10$.)
**b** Write an expression for the total cost of $n$ rides using deal $2$.
**c** Write an expression for the total cost of $n$ rides using deal $3$.
**d** Which of the three deals is best for someone going on just two rides?
**e** Which of the three deals is best for someone going on $20$ rides?
**f** Fill in the gaps.
  **i** Deal $1$ is best for people wanting up to ______ rides.
  **ii** Deal $2$ is best for people wanting between ______ and ______ rides.
  **iii** Deal $3$ is best for people wanting more than ______ rides.

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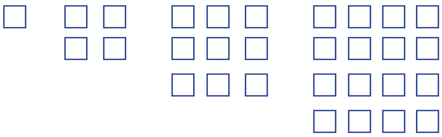
# 1H Using indices

The expression $4^2$ means $4 \times 4$.
The expression $4^3$ means $4 \times 4 \times 4$.
Indices are used to describe repeated multiplication and to record very large numbers, for example $2^{24} = 16\ 777\ 216$.

## Let's start: Square numbers



Can you explain why we call the numbers 1, 4, 9 and 16 square numbers?
Draw the next two square numbers in your book.
Use centicubes to build the first three cube numbers. Write down the next cube number.

## Key ideas

- In the statement $4^3 = 8^2$, the 3 and 2 are called **indices**. This is the plural of 'index'.
- **Index** notation

  index or power

  $3^4$

  base

  $3^4 = 3 \times 3 \times 3 \times 3$

  index form expanded form

  - $5 \times 5$ and $5^2$ are read as '5 to the power of 2' or '5 squared'.
  - $5 \times 5 \times 5$ and $5^3$ are read as '5 to the power of 3' or '5 cubed'.
  - $5 \times 5 \times 5 \times 5$ and $5^4$ are read as '5 to the power of 4'.
- The opposite of squaring is finding the **square root** of a number. The symbol $\sqrt{\ }$ means square root.

  $3^2 = 9$ so $\sqrt{9} = 3$

  - The square root of a number is always positive.
- The opposite of cubing is taking the **cube root** of a number. The symbol for cube root is $\sqrt[3]{\ }$.

  $2^3 = 8$, so $\sqrt[3]{8} = 2$

| | |
|---|---|
| **Index** | The number of times the base number appears in the product |
| **Base** | The number that is being raised to a power |
| **Square** | To multiply a number by itself |
| **Square root** | The opposite operation of squaring |
| **Cube root** | The opposite operation of cubing |

## Exercise 1H

**Understanding**

**1** Write each of the following in index form.

**a** $2 \times 2$
**b** $4 \times 4$
**c** $5 \times 5$
**d** $5 \times 5 \times 5$
**e** $6 \times 6 \times 6 \times 6$
**f** $7 \times 7 \times 7$

**2** Match each expression **a–f** to an expression in symbols **I–VI**.

**a** The square of 10
**b** The cube of 1
**c** The square of 12
**d** The square root of 1
**e** The cube root of 1
**f** The square root of 16

**I** $\sqrt{16}$
**II** $\sqrt[3]{1}$
**III** $\sqrt{1}$
**IV** $10^2$
**V** $1^3$
**VI** $12^2$

The cube of 2 is
$2^3 = 2 \times 2 \times 2 = 8$

**3** Copy and complete.

$1^2 = 1 \times 1 = 1$
$2^2 = 2 \times 2 = 4$
$3^2 =$
$4^2 =$
$5^2 =$
$6^2 =$
$7^2 =$
$8^2 =$
$9^2 =$
$10^2 =$

**4** Copy and complete.

$1^3 = 1 \times 1 \times 1 = 1$
$2^3 = 2 \times 2 \times 2 = 8$
$3^3 =$
$4^3 =$
$5^3 =$
$6^3 =$

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## Example 20 Using index notation

Write each product in index notation.

**a** $8 \times 8 \times 8$ **b** $7 \times 7 \times 7 \times 7 \times 7 \times 7$

| **Solution** | **Explanation** |
|---|---|
| **a** $8 \times 8 \times 8 = 8^3$ | The number 8 appears 3 times. We write 8 to the power of 3. |
| **b** $7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^6$ | The 7 appears 6 times. We write 7 to the power of 6. |

**5** Write each of the following products in index notation.

**a** $7 \times 7 \times 7$ **b** $10 \times 10 \times 10 \times 10$ **c** $8 \times 8$
**d** $4 \times 4 \times 4$ **e** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ **f** $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$
**g** $12 \times 12$ **h** $5 \times 5 \times 5 \times 5 \times 5 \times 5$ **i** $6$

**Fluency**

## Example 21 Expanded notation and evaluating index notation

**a** Write $5^4$ in expanded form. **b** Find the value of $5^4$.

| **Solution** | **Explanation** |
|---|---|
| **a** $5^4 = 5 \times 5 \times 5 \times 5$ | The power of 4 tells us that the number 5 repeats in multiplication 4 times.<br>$5^4 = 5 \times 5 \times 5 \times 5$ |
| **b** $5^4 = 625$ | $5^4 = 5 \times 5 \times 5 \times 5$<br>$= 25 \times 5 \times 5$<br>$= 125 \times 5$<br>$= 625$ |

**6** Write each expression in expanded form.

**a** $8^5$ **b** $3^4$ **c** $9^2$
**d** $4^4$ **e** $2^8$ **f** $11^2$

$5 \times 5 \times 5$ is the expanded form of $5^3$.

**7** Find the value of each expression.

**a** $2^3$ **b** $2^4$ **c** $3^3$
**d** $10^4$ **e** $5^3$ **f** $1^4$

### Example 22 Finding squares, cubes, square roots and cube roots

Evaluate the following.

**a** $6^2$ **b** $\sqrt{81}$ **c** $3^3$ **d** $\sqrt[3]{64}$

| **Solution** | **Explanation** |
|---|---|
| **a** $6^2 = 6 \times 6$<br>$= 36$ | Find the product of 6 with itself. |
| **b** $\sqrt{81} = 9$ | $9^2 = 9 \times 9 = 81$ so $\sqrt{81} = 9$ |
| **c** $3^3 = 3 \times 3 \times 3$<br>$= 27$ | The number 3 appears 3 times |
| **d** $\sqrt[3]{64} = 4$ | $4^3 = 4 \times 4 \times 4 = 64$ so $\sqrt[3]{64} = 4$ |

**8** Evaluate these squares and square roots.

**a** $4^2$ **b** $10^2$ **c** $13^2$ **d** $15^2$

**e** $100^2$ **f** $20^2$ **g** $\sqrt{25}$ **h** $\sqrt{49}$

**i** $\sqrt{121}$ **j** $\sqrt{100}$ **k** $\sqrt{144}$ **l** $\sqrt{16 \times 16}$

$3^2 = 9$ and $\sqrt{9} = 3$.

**9** Evaluate these cubes and cube roots.

**a** $2^3$ **b** $4^3$ **c** $7^3$ **d** $5^3$

**e** $6^3$ **f** $10^3$ **g** $\sqrt[3]{27}$ **h** $\sqrt[3]{8}$

**i** $\sqrt[3]{125}$ **j** $\sqrt[3]{512}$ **k** $\sqrt[3]{729}$ **l** $\sqrt[3]{1000000}$

Hint: Use a calculator for parts **i**, **j**, **k** and **l**.

**Problem-solving and Reasoning**

**10** Decide which of the following is larger.

**a** $2^3$ or $3^2$ **b** $2^4$ or $3^2$ **c** $2^5$ or $5^2$

**11** Copy and complete.

**a** If $13^2 = 169$, then $\sqrt{169} = \square$

**b** If $15^2 = 225$, then $\sqrt{225} = \square$

**c** If $\sqrt{625} = 25$, then $25^2 = \square$

**d** If $9^3 = 729$, then $\sqrt[3]{729} = \square$

**e** If $\sqrt[3]{1331} = 11$, then $11^3 = \square$

**12** Given $5 \times 5 \times 5 \times 4 \times 4$ is written as $5^3 \times 4^2$ (the different bases of 5 and 4 are kept separate), write each of the following in index form.

**a** $6 \times 6 \times 7 \times 7 \times 7 \times 7$ **b** $5 \times 5 \times 5 \times 5 \times 2 \times 2$

**c** $3 \times 3 \times 8 \times 8$ **d** $11 \times 9 \times 9 \times 9 \times 9$

**e** $12 \times 12 \times 4 \times 4 \times 4$ **f** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

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## Enrichment: Algebraic bases

**13** Write each of the following in index form. Remember, different bases cannot be collected.

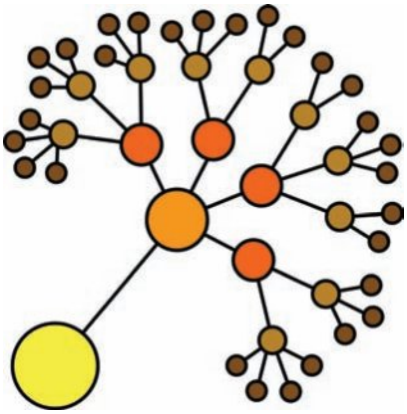
$$\underbrace{a\times a}\times\underbrace{b\times b\times b}$$
$$\underbrace{a^2\times b^3}$$
$$a^2b^3$$

**a** $m \times m \times m$
**b** $a \times a \times a \times a \times a$
**c** $n \times n \times n \times n \times n \times n \times n$
**d** $p \times p \times p \times p \times p \times p \times p \times p \times p \times p$
**e** $p \times p \times p \times q \times q$
**f** $a \times a \times a \times a \times b \times b$
**g** $a \times a \times b \times b \times b \times b$
**h** $x \times x \times x \times x \times y$

# 1I Index laws

In this section we will look at the rules that can be used when working with numbers written in index notation.

We call these rules the index laws.



Index notation is used in science, economics and computer applications.

## Let's start: Investigating the first two rules

Write out $3^7$ in expanded notation.
Now write out $3^4$ in expanded notation.
What do you get when $3^7$ is multiplied by $3^4$?
How many times does the base of 3 appear in this product?
What do you get when $3^7$ is divided by $3^4$? How many times does the base of 3 appear in this quotient?

## Key ideas

**Index notation** Method of writing numbers that are multiplied by themselves

The index law for multiplication:

- Use when multiplying numbers written in **index notation**. If the base is the same, keep the base and add the indices together.
  - e.g. $2^3 \times 2^2 = (2 \times 2 \times 2 \times 2 \times 2)$
    $= 2^5$ (here the base of 2 appears 5 times $(3 + 2)$)

The index law for division:

- Use when dividing numbers written in index notation. If the base is the same, keep the base and subtract the indices.
  - e.g. $2^6 \div 2^2 = (2 \times 2 \times 2 \times 2 \times 2 \times 2) \div (2 \times 2)$
    $= \dfrac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2}$
    $= 2^4$ (here the base of 2 appears 4 times $(6 - 2)$)

The index law for power of a power:

- Use when a number written in index notation is raised to another power. The base remains the same and the two indices are multiplied together.
  - e.g. $(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$
    $= 2^{3+3+3+3}$
    $= 2^{12}$ (here the base of 2 appears in total 12 times $(3 \times 4)$)

The zero index:

- Any non-zero number raised to the power of zero gives an answer of one.
  - e.g. $2^0 = 1$
    e.g. $2^3 \div 2^3 = 2^{3-3} = 2^0$ (but $2^3 \div 2^3 = 1$ so this must mean that $2^0 = 1$)

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## Exercise 1I

**Understanding**

**1** Which of the following is the same as $4^3 \times 4^4$?
- **A** $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
- **B** $16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16$
- **C** $16^7$
- **D** $16^{12}$

**2** Which of the following is equal to $3^6 \div 3^2$?
- **A** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
- **B** $3 \times 3 \times 3$
- **C** $3 \times 3 \times 3 \times 3$
- **D** $1^4$

**3** Write the following in your workbook using index notation.
- **a** 6 raised to the power of 2
- **b** 7 raised to the power of 0
- **c** $(5 \times 5 \times 5 \times 5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5)$
- **d** $(6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6) \div (6 \times 6 \times 6)$

**4** Which of the following is the same as $(2^2)^3$?
- **A** $2^5$
- **B** $4^5$
- **C** $(2 \times 2) \times (2 \times 2) \times (2 \times 2) = 2^6$
- **D** $2^{23}$

**Fluency**

### Example 23 The index laws for multiplication and division

Simplify each of these, leaving your answer in index form.

**a** $6^5 \times 6^4$ **b** $5^7 \div 5^4$

| **Solution** | **Explanation** |
|---|---|
| **a** $6^5 \times 6^4 = 6^9$ | Keep the base and add the indices $6^5 \times 6^4$ (the base of 6 appears 5 times in the first term and 4 times in the next term).<br>The base of 6 appears 9 times in the product. |
| **b** $5^7 \div 5^4 = 5^3$ | Keep the base the same and subtract the indices.<br>$5^7 \div 5^4 = 5^{7-4}$<br>$= 5^3$ |

**5** Copy and complete the following.

**a** $7^4 \times 7^2 = 7^{\square}$ **b** $8^2 \times 8^1 = 8^{\square}$ **c** $9^6 \times 9^3 = 9^{\square}$ **d** $5^4 \times 5^3 = 5^{\square}$

**e** $2^{10} \times 2^3 = 2^{\square}$ **f** $2^{\square} \times 2^9 = 2^{15}$ **g** $5^8 \div 5^2 = 5^{\square}$ **h** $6^4 \div 6^1 = 6^{\square}$

**i** $2^{12} \div 2^8 = 2^{\square}$ **j** $1^{16} \div 1^{13} = 1^{\square}$ **k** $8^{\square} \div 8^4 = 8^2$ **l** $10^7 \div 10^{\square} = 10^2$

**6** Simplify each of the following using the index law for multiplication.

**a** $3^4 \times 3^2$ **b** $2^2 \times 2^3$ **c** $10^3 \times 10^1$

**d** $9^6 \times 9^4$ **e** $4^4 \times 4$ **f** $2^3 \times 2^9$

**g** $8^7 \times 8^3$ **h** $12^9 \times 12$ **i** $16^5 \times 16^3$

Keep the base.
Add the indices.

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**7** Simplify each of the following using the index law for division.

**a** $3^4 \div 3^2$ **b** $2^7 \div 2^5$ **c** $9^6 \div 9^2$
**d** $4^5 \div 4^2$ **e** $17^{26} \div 17^{20}$ **f** $11^9 \div 11^3$

Keep the base. Subtract the indices.

## Example 24 Raising powers

Simplify $(4^3)^3$.

| Solution | Explanation |
|---|---|
| $(4^3)^3 = 4^9$ | $(4^3)^3 = 4^{3 \times 3}$<br>The base of 4 stays the same and the indices are multiplied together. |

**8** Copy and complete.

**a** $(2^3)^4 = 2^{\square}$ **b** $(3^2)^5 = 3^{\square}$ **c** $(5^2)^2 = 5^{\square}$
**d** $(2^4)^3 = 2^{\square}$ **e** $(7^3)^2 = 7^{\square}$ **f** $(8^4)^5 = 8^{\square}$

Multiply the indices.

**9** Simplify the following.

**a** $(7^2)^2$ **b** $(2^5)^4$ **c** $(3^7)^2$ **d** $(8^4)^2$
**e** $(3^4)^2$ **f** $(10^6)^5$ **g** $(9^2)^7$ **h** $(5^5)^3$

## Example 25 The power of zero

Simplify:

**a** $9^0$ **b** $(3 \times 2)^0$ **c** $4 \times 5^0$

| Solution | Explanation |
|---|---|
| **a** $9^0 = 1$ | A number (except zero) raised to the power of zero equals one. |
| **b** $(3 \times 2)^0 = 1$ | $3 \times 2 = 6$<br>$6^0 = 1$ |
| **c** $4 \times 5^0 = 4 \times 1$<br>$= 4$ | $5^0 = 1$ so the product of 4 and $5^0$ is the same as $4 \times 1$. |

Skillsheet 1E

**10** Simplify the following.

**a** $5^0$ **b** $6^0$ **c** $19^0$ **d** $15^0$
**e** $(27 \times 25)^0$ **f** $5^0 + 7$ **g** $8 - 3^0$ **h** $10 \times 2^0$
**i** $5^0 \times 6^0$ **j** $5^0 + 6^0$ **k** $6^0 + 5$ **l** $12^0 \times 3$

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### Problem-solving and Reasoning

**11** Complete the following.

**a** Given $4 = 2^2$, write the product $2^7 \times 4$ as $2^{\square}$.

**b** Write $5^4 \times 25$ as $5^{\square}$.

**c** Write down the numerical value of $6^{14} \div 6^{12}$.

**d** What do you notice about $(3^4)^2$ and $(3^2)^4$?

**e** Write down the numerical value of $4^2 \times 3^2$. Is it the same as $7^2$ or $12^2$?

**12** Simplify the following.

**a** $2^7 \times 2^4 \div 2^3$

**b** $(2^3)^3 \times 2^4$

**c** $10^7 \div 10^2 \div 10^2$

**d** $7^9 \times 7^3 \times 7^2$

**e** $6^4 \times 6^5 \div 6^8$

**f** $3^7 \times 3 \times 3$

Combine the index laws where required.

## Enrichment: Algebraic bases

**13** Use the four index laws to complete these index law questions involving pronumeral bases.

**a** $a^7 \times a^4$

**b** $m^4 \times m^3$

**c** $a^5 \times a^4$

**d** $x^5 \times x^8$

**e** $n^7 \times n^4$

**f** $m^6 \times m^7 \times m$

**g** $n^9 \div n^3$

**h** $a^{10} \div a^7$

**i** $m^6 \div m^4$

**j** $a^7 \times a^2 \times a^3$

**k** $w^{12} \div w^3$

**l** $p^8 \times p^2 \div p^6$

Remember, the base stays the same.

$m^{20} \times m^4$

$= m^{20+4}$

$= m^{24}$

**14** Simplify these using the given hint.

**a** $5m^4 \times m^3$

**b** $6m^2 \times 4m^6$

**c** $8m^6 \times 2m^4$

**d** $3a^2 \times 4a^7$

**e** $7x^3 \times 3x^4$

**f** $5x^9 \times 4x^3$

$5x^7 \times 3x^2$

$= 5 \times 3 \times x^7 \times x^2$

$= 15 \times x^{7+2}$

$= 15x^9$

# Puzzles and games

**1** Find the values of $A$, $B$ and $C$ so that the rows and columns add up correctly.

| | | | |
|---|---|---|---|
| $A$ | $B$ | $C$ | Sum = 14 |
| $A$ | $C$ | $B$ | Sum = 14 |
| $A$ | $C$ | $B$ | Sum = 14 |
| Sum = 15 | Sum = 16 | Sum = 11 | |

**2** Fill in the missing expressions to make the six equivalences true.

| | | | | |
|---|---|---|---|---|
| $3x$ | + | | = | $7x + 3y + 1$ |
| + | | + | | + |
| | + | | = | |
| = | | = | | = |
| $2y + 3x$ | + | | = | $7x + 6y + 1$ |

**3** Think of a number.
Multiply by 2.
Add 8.
Divide by 2.
Subtract the original number.
What number did you get?

**4** Think of a number $n$.
Double it and add 4.
Triple the result and subtract 12.
You now have 6 times the original number.
Use algebra to see if this was just a coincidence.
Design a puzzle like this and try it on your friends.

**5** *Finding the largest value*

**a** If $b$ can be any number, what is the largest value of $b \times (10 - b)$?

**b** If $x + y$ evaluates to 15, what is the largest value that $x \times y$ could have?

**c** If $a$ and $b$ are chosen so that $a^2 + b^2$ is equal to $(a + b)^2$, what is the largest value of $a \times b$?

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# Chapter summary

## Algebraic techniques 2 and indices

**Pronumeral:** a letter that stands for a numerical value

### Language

| + | − | × | ÷ |
|---|---|---|---|
| sum<br>more than<br>added<br>increased | difference<br>less than<br>minus<br>decreased | product<br>times<br>double (2×)<br>twice (2×)<br>triple (3×) | quotient<br>divide<br>one third<br>one half<br>quarter |

$a \times b = ab$

$a \div b = \frac{a}{b}$

an expression

terms

$5x - 7xy - 8$

5 is the coefficient of $x$

−7 is the coefficient of $xy$

constant term is −8

### Like terms

Have the same pronumerals

$6x$ and $5x$ ✓

$3x$ and $7y$ ✗

can be in different order

$6ab$ and $12ba$

### Adding and subtracting like terms

$\boxed{6a}\ \boxed{+12b}\ \boxed{+3a}\ \boxed{-7b}$

$= 6a + 3a + 12b - 7b$

$= 9a + 5b$

### Substitution

'evaluate' 'substitute'

Replace pronumerals with numbers and calculate answer

If $a = 3$ then

$5a + 7 = 5 \times 3 + 7$

$= 15 + 7$

$= 22$

### Expanding

$3(2x + 5y) = 6x + 15y$

$2a(5 - 7b) = 10a - 14ab$

### Equivalent expressions

Always evaluate to the same number

e.g. $2x$ and $x + x$

### Factorising

$12x + 6$ (HCF = 6)

$= 6(2x + 1)$

### Squares and square roots

$4^2 = 16$, so $\sqrt{16} = 4$

### Cubes and cube roots

$4^3 = 64$, so $\sqrt[3]{64} = 4$

### Multiplying and dividing terms

$3a \times 2b = 3 \times a \times 2 \times b$

$= 6 \times a \times b$

$= 6ab$

$\frac{\overset{2}{\cancel{12}}a}{\underset{3}{\cancel{18}}b} = \frac{2a}{3b}$

### Indices

$3^0 = 1$

$3^1 = 3$

$3^2 = 3 \times 3$

$3^3 = 3 \times 3 \times 3$

$3^5 \times 3^3 = 3^8$

$3^5 \div 3^3 = 3^2$

$(3^5)^3 = 3^{15}$

**Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.**

## Multiple-choice questions

**1** The sum of $x$ and $y$ can be written as:
**A** $2x$ **B** $2xy$ **C** $x + y$ **D** $x - y$ **E** $xy$

**2** Consider the expression $5a - 3b + 8$. Which one of the following statements is true?
**A** The coefficient of $a$ is 5.
**B** It has 5 terms.
**C** The constant term is $-8$.
**D** The coefficient of $b$ is 3.
**E** The coefficient of $a$ is 10.

**3** If $n$ is a number, which of the following represents one third of $n$?
**A** $\frac{3}{n}$ **B** $0.3n$ **C** $3n$ **D** $\frac{n}{3}$ **E** $n - 3$

**4** If $a = 2$, then $17 + 2a$ is:
**A** 3 **B** $-3$ **C** 21 **D** 11 **E** 13

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**5** $3 \times x \times y$ is equivalent to:
**A** $3x + y$ **B** $xy$ **C** $3 + x + y$ **D** $3x + 3y$ **E** $3xy$

**6** $\frac{12ab}{24a}$ can be simplified to:
**A** $2ab$ **B** $\frac{2a}{b}$ **C** $\frac{b}{2a}$ **D** $\frac{ab}{2}$ **E** $\frac{b}{2}$

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**7** The expanded form of $2(3 + 5y)$ is:
**A** $6x + 5y$ **B** $3x + 5y$ **C** $6x + 5xy$ **D** $6 + 10y$ **E** $6x + 10xy$

**8** Simplifying $3a \div 6b$ gives:
**A** 2 **B** $\frac{a}{b}$ **C** $\frac{2a}{b}$ **D** $\frac{ab}{2}$ **E** $\frac{a}{2b}$

Drilling for Gold 1R3

**9** When like terms are combined, $3a + 4b + 2a - 2b$ simplifies to:
**A** $5a + 6b$ **B** $7ab$ **C** $11ab$ **D** $5a + 2b$ **E** $a + 6b$

**10** The factorised form of $3a - 6ab$ is:
**A** $3a(1 - 2b)$ **B** $3a(a - 2b)$ **C** $3a(a - b)$ **D** $6a(a - b)$ **E** $3(a - 2ab)$

## Short-answer questions

**1** State whether each of the following is true or false.
**a** The constant term in the expression $5x + 7$ is 5.
**b** $16xy$ and $5yx$ are like terms.
**c** The coefficient of $d$ in the expression $6de + 7d + 8abd + 3$ is 7.
**d** The highest common factor of $12abc$ and $16c$ is $2c$.
**e** The coefficient of $x$ in $5y - 3x$ is $-3$.

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**2** For the expression $6xy + 2x - 4y + 3$, state the:

**a** coefficient of $x$ **b** constant term

**c** number of terms **d** coefficient of $xy$.

**3** Substitute the following values of $a$ to evaluate the expression $12 - 2a$.

**a** $1$ **b** $2$ **c** $4$ **d** $6$

**4** Substitute $A = 2$ and $B = 5$ into the following expressions.

**a** $10A$ **b** $A + B$ **c** $B - A$ **d** $3A + 2B$

**5** Substitute $x = 2$ and $y = 3$ into each of the following.

**a** $2y + 3$ **b** $3x + y$ **c** $xy + y$ **d** $4x - 2y$

**6** Simplify each of these expressions by collecting like terms.

**a** $7m + 9m$

**b** $3a + 5b - a$

**c** $3y - x + y + 1$

**d** $5x + 3y + 2x + 4y$

**e** $7x - 4xy + 5xy + 2x$

**f** $7m - 2n + 3m - 4n$

**7** Simplify:

**a** $9a \times 4b$ **b** $30 \times x \times y$ **c** $2x \times 5y \times 3z$

**8** Simplify:

**a** $\frac{10x}{5}$ **b** $\frac{12ab}{4b}$ **c** $\frac{4xz}{20xy}$

**9** Expand and simplify when necessary.

**a** $3(x - 4)$ **b** $2(5 + x)$ **c** $3(2y + 4)$ **d** $10(2x + 7)$

**e** $3(x - 5)$ **f** $11(z - 2)$ **g** $4(3a - 11)$ **h** $2(6b - 3)$

**10** Find the HCF of:

**a** $12x$ and $16$

**b** $14ab$ and $21a$.

**11** Factorise fully.

**a** $2x + 6$ **b** $24 - 16g$ **c** $12x + 3xy$ **d** $7a + 14ab$

**12** If apricots cost $\$a$ each and pears cost $\$p$ each, write an expression for the cost of:

**a** 5 apricots

**b** 3 pears

**c** 5 apricots and 3 pears.

**13** Greg runs 10 km each day.

**a** How far (in km) does he run in one week (7 days)?

**b** Write an expression for how far he runs in $n$ days.

**14** Evaluate the following.

**a** $2^2$ **b** $2^3$ **c** $\sqrt{4}$ **d** $\sqrt[3]{8}$ **e** $2^0$

**15** Simplify (in index form):

**a** $2 \times 2 \times 2$ **b** $2^7 \times 2^5$ **c** $2^7 \div 2^5$ **d** $(2^7)^5$

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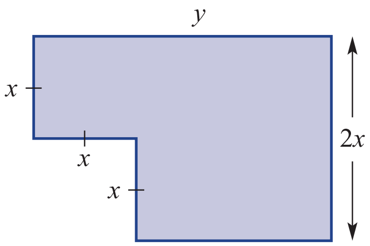
## Extended-response questions

**1** Two bus companies have different pricing structures.

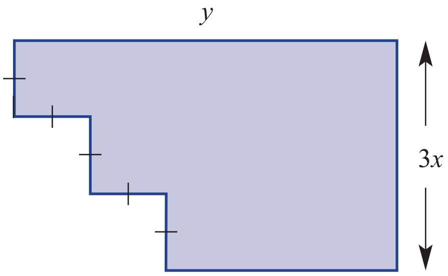
| Company A | Company B |
|---|---|
| $\$120$ call-out fee, plus $\$80$ per hour | $\$80$ call-out fee, plus $\$100$ per hour |

- **a** Write an expression for the total cost of travelling for $n$ hours with company A.
- **b** Write an expression for the total cost of travelling for $n$ hours with company B.
- **c** What is the cost of travelling for 3 hours with each company?
- **d** For how long would you need to hire a bus to make company A the cheaper option?
- **e** If a school hired one bus from each company for $n$ hours, what would the total cost be?

**2** Consider the floor plan shown.



- **a** Write an expression for the floor's area in terms of $x$ and $y$.
- **b** Using that expression, find the floor's area if $x = 3$ metres and $y = 7$ metres.
- **c** Write an expression for the floor's perimeter in terms of $x$ and $y$.
- **d** Using that expression, find the floor's perimeter if $x = 3$ metres and $y = 7$ metres.
- **e** Another floor plan is shown below. Write an expression for the floor's area and an expression for its perimeter.



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**3** Congratulations! You have won a very special prize. You have two options.

Option 1: Take \$1 million now.

Option 2: Take 1 cent at the end of this year, 2 cents at the end of next year, 4 cents at the end of the year after, 8 cents one year later and keep doubling.

- **a** If you choose Option 2, how much will you receive in the tenth year?
- **b** If you choose Option 2, how much in total will you have by the end of the tenth year?
- **c** How long will it take for Option 2 to overtake Option 1?

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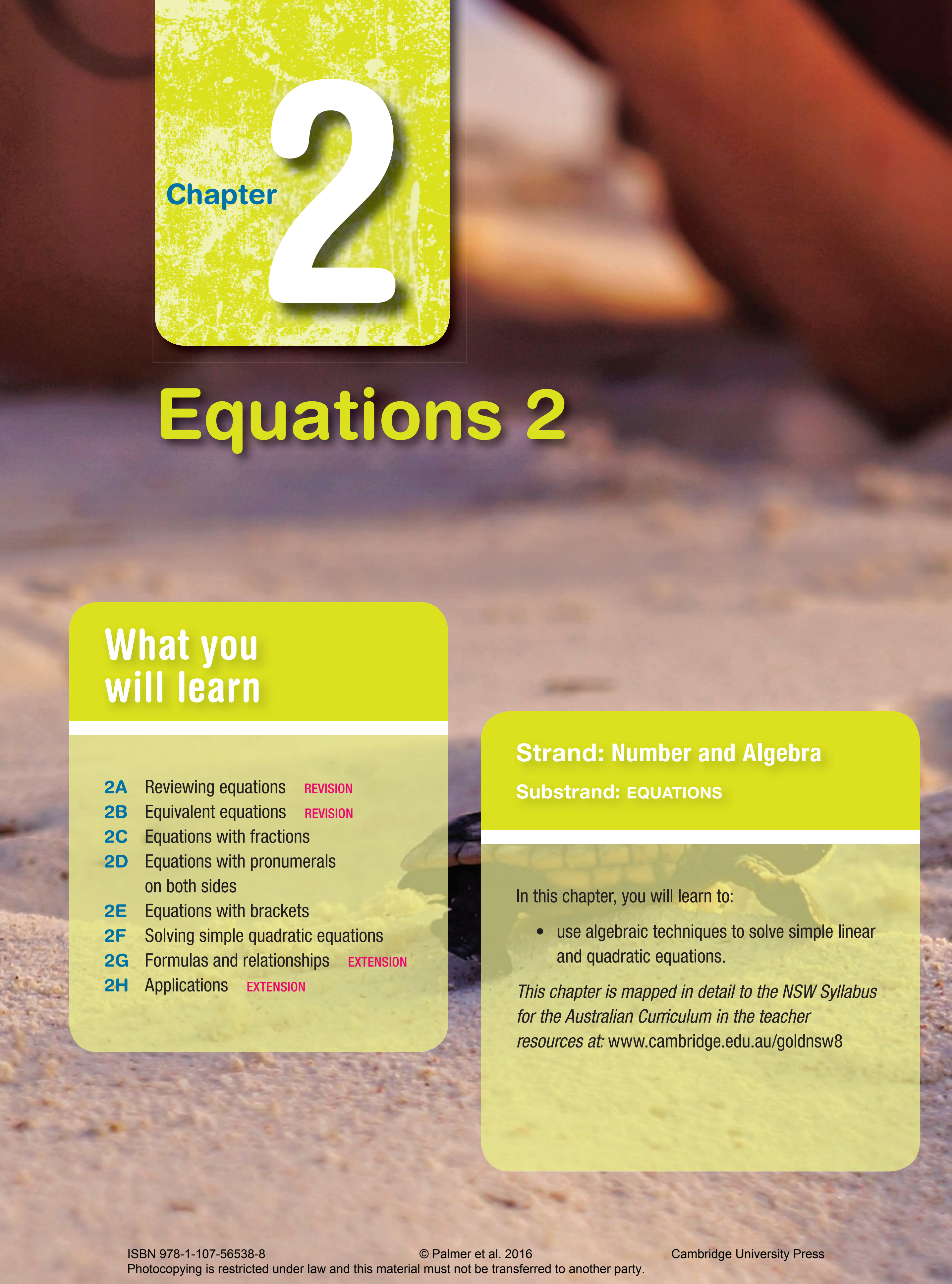


## Chapter 2

# Equations 2

## What you will learn

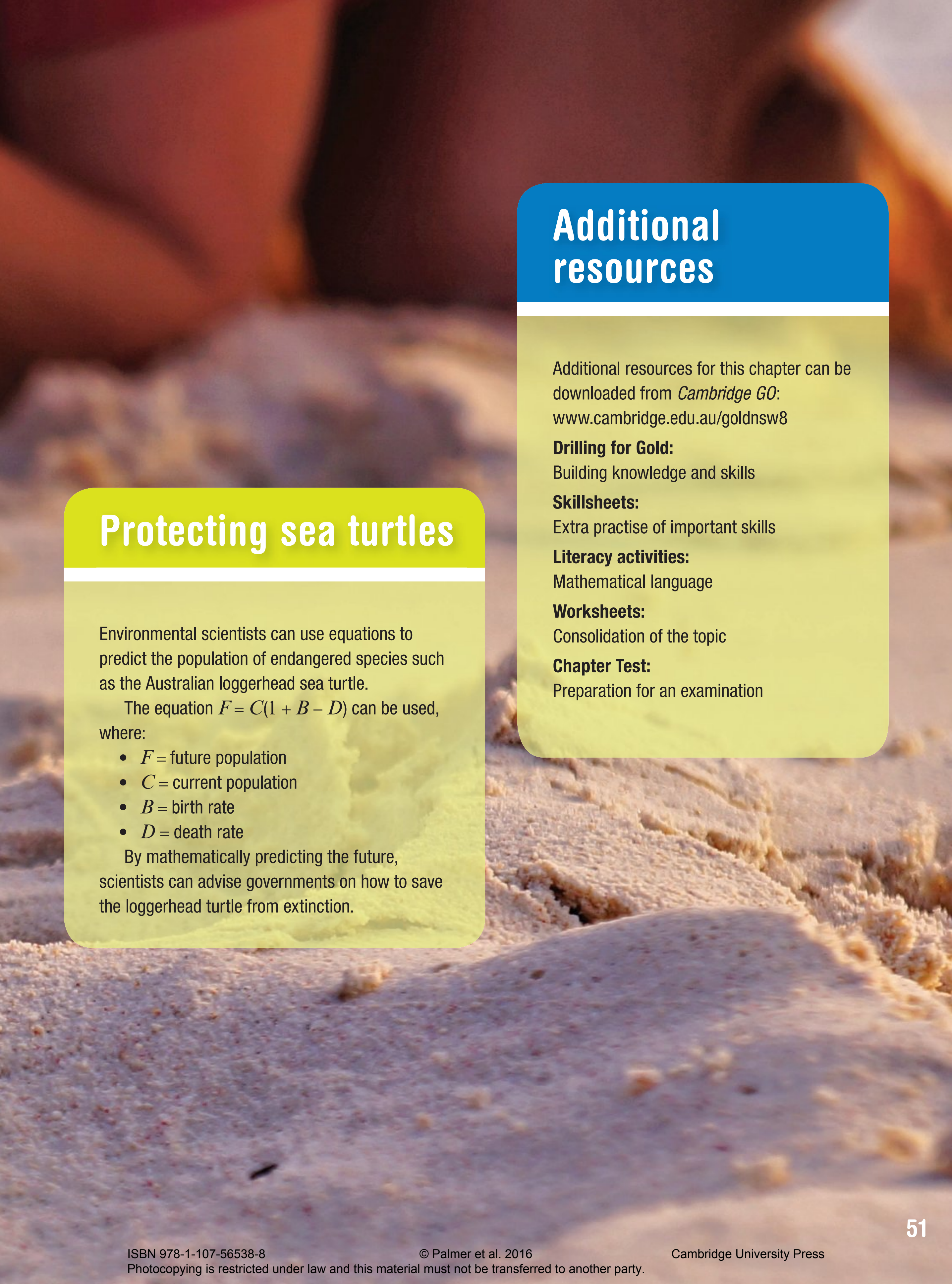
### Strand: Number and Algebra

### Substrand: EQUATIONS

In this chapter, you will learn to:

- use algebraic techniques to solve simple linear and quadratic equations.

*This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at:* www.cambridge.edu.au/goldnsw8



## Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw8

**Drilling for Gold:**
Building knowledge and skills

**Skillsheets:**
Extra practise of important skills

**Literacy activities:**
Mathematical language

**Worksheets:**
Consolidation of the topic

**Chapter Test:**
Preparation for an examination

## Protecting sea turtles

Environmental scientists can use equations to predict the population of endangered species such as the Australian loggerhead sea turtle.

The equation $F = C(1 + B - D)$ can be used, where:

- $F$ = future population
- $C$ = current population
- $B$ = birth rate
- $D$ = death rate

By mathematically predicting the future, scientists can advise governments on how to save the loggerhead turtle from extinction.

# Pre-test

**1** Fill in the missing number in these equations.

**a** $5 + 7 = \square$ **b** $3 \times 9 = \square$

**c** $12 \div 4 = \square$ **d** $5 \times 2 = \square$

**2** Find the value of $\triangle$ to make these equations true.

**a** $4 + \triangle = 12$ **b** $6 \times \triangle = 12$

**c** $\triangle + 14 = 19$ **d** $\triangle - 4 = 11$

**3** If $x = 6$, find the value of:

**a** $x + 2$ **b** $x \times 7$

**c** $x - 2$ **d** $8 - x$

**4** Simplify these algebraic expressions.

**a** $9m + 2m$ **b** $4a - 3a$ **c** $7n + 3n - n$

**d** $8a + 2a - 10$ **e** $4x + 2 + 7x$ **f** $5b + 4 + 3b$

**5** Expand these algebraic expressions using the distributive law.

**a** $3(m + 4)$ **b** $2(a + 6)$

**c** $3(x + 7)$ **d** $4(k - 6)$

**6** I think of a number, double it, and then add three to get 27. What is the number?

**7** If $x = 5$, are the following equations true or false?

**a** $x + 2 = 7$ **b** $3x = 35$

**c** $x - 1 = 6$ **d** $2x = 10$

**8** Solve each of the following equations by inspection or using guess and check.

**a** $x + 8 = 12$ **b** $4x = 32$

**c** $m - 6 = -2$ **d** $3m = 18$

**9** State the opposite operation of each of the following.

**a** $\times 5$ **b** $+2$

**c** $\div 3$ **d** $-3$

**10** The sum of $k$ and 3 is written as $k + 3$. Write expressions for:

**a** the sum of $p$ and 10 **b** the product of 4 and $x$

**c** double $z$ **d** 6 less than $q$.

**11** True or false?

**a** $x = 3$ is a solution of $3x = 0$.

**b** $x = 3$ is a solution of $3 - x = 0$.

**c** $x = 3$ is a solution of $x^2 = 9$.

**d** $x = -3$ is a solution of $x^2 = 9$.

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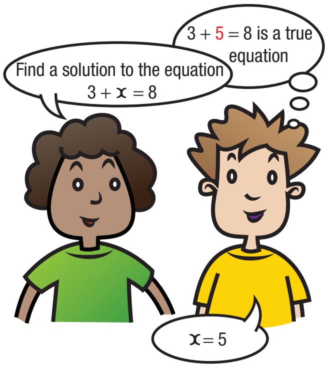
# 2A Reviewing equations

**REVISION**

Equations are mathematical statements saying that two things are equal. For example, $2 + 2 = 4$ is an equation.

If there is a pronumeral involved, then a solution is a value that makes the equation true.



## Let's start: What's missing?

Rory has erased a number in each of the equations below.

- If the equations were originally true, find the missing values:
  $10 + \square = 57$ $\square - 31 = 40$ $2 \times \square + 5 = 19$
- In one equation he erased two numbers to get $\square \times 2 = \square$.
  Is it possible to find the missing values? Why or why not?

## Key ideas

- An **equation** is a mathematical statement that two expressions are equal, such as $3 \times 5 = 15$ (which is true) or $2 + 2 = 100$ (which is false).
- The parts of an equation are: $3 + 7 = 2 \times 5$

  left-hand side LHS
  equals sign
  right-hand side RHS

- A **solution** to an equation is a value that makes an equation true. The process of finding a solution is called **solving**.
- Simple equations can be solved by inspection but a methodical approach is needed for complex equations.

Drilling for Gold 2A1

**Equation** A mathematical statement that two expressions (numeric or algebraic) have the same value

**Solution** The value/s that give a true statement when substituted for the unknown in an equation

**Solving** Finding the value(s) of the unknown(s) so that the equation is a true statement

## Exercise 2A

**Understanding**

**1** State the value of:

**a** $7 + 12$ **b** $5 \times 3$
**c** $2 \times 8$ **d** $10 - 4$
**e** $2 \times 5 + 1$ **f** $3 + 5 \times 2$
**g** $(3 + 2) \times 7$ **h** $5 \div (4 + 1)$

Remember: Brackets first, then Division and Multiplication, then Addition and Subtraction.

**2A**

### Example 1 Classifying equations as true or false

For each of the following equations, state whether they are true or false.

**a** $3 + 8 = 15 - 4$ **b** $7 \times 3 = 20 + 5$

| **Solution** | **Explanation** |
|---|---|
| **a** True | Left-hand side (LHS) is $3 + 8$, which is 11.<br>Right-hand side (RHS) is $15 - 4$, which is also 11.<br>Since LHS equals RHS, the equation is true. |
| **b** False | $\text{LHS} = 7 \times 3 = 21$<br>$\text{RHS} = 20 + 5 = 25$<br>Since LHS and RHS are different, the equation is false. |

**2** Classify these equations as true or false.

**a** $5 \times 3 = 15$ **b** $7 + 2 = 12$ **c** $5 + 3 = 16 \div 2$
**d** $8 - 6 = 6$ **e** $4 \times 3 = 12 \times 1$ **f** $2 = 8 - 3 - 3$

**3** Find the value of $A + 5$ if:

**a** $A = 3$ **b** $A = 7$ **c** $A = 10$ **d** $A = 40$

**4** If the value of $x$ is 3, what is the value of the following?

**a** $10 + x$ **b** $3x$ **c** $5 - x$ **d** $6 \div x$

$3x$ means $3 \times x$.

**5** State the value of the missing number to make the following equations true.

**a** $5 + \square = 12$ **b** $10 \times \square = 90$
**c** $\square - 3 = 12$ **d** $3 + 5 = \square$

**Fluency**

### Example 2 Classifying equations as true or false by substitution

If $x = 10$, is the equation $x + 20 = 3 \times x$ true or false?

| **Solution** | **Explanation** |
|---|---|
| True | $\text{LHS} = x + 20 = 10 + 20 = 30$<br>$\text{RHS} = 3 \times x = 3 \times 10 = 30$<br>LHS equals RHS, so the equation is true. |

**6** If $x = 2$, state whether the following equations are true or false.

**a** $x + 4 = 6$ **b** $10x = 5$ **c** $8 = 10 - x$
**d** $7x = 8 + 3x$ **e** $10 - x = 4x$ **f** $3x = 5 - x$

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**7** If $a = 3$, state whether the following equations are true or false.
**a** $7 + a = 10$ **b** $2a + 4 = 12$ **c** $8 - a = 5$
**d** $4a - 3 = 9$ **e** $7a + 2 = 8a$ **f** $a = 6 - a$

**8** For each equation below, choose the correct solution from the table on the right.
**a** $x + 12 = 20$ **b** $10x + 5 = 35$
**c** $12 = x + 5$ **d** $10 + x = 3x + 2$
**e** $3 + 2x = 5$ **f** $6 - x = 1$

| | |
|---|---|
| $x = 5$ | $x = 1$ |
| $x = 7$ | $x = 3$ |
| $x = 8$ | $x = 4$ |

## Example 3 Stating a solution to an equation

State a solution to each of the following equations.
**a** $4 + x = 25$ **b** $5y = 45$

| **Solution** | **Explanation** |
|---|---|
| **a** $x = 21$ | We need to find a value of $x$ that makes the equation true. If $4 + 21 = 25$ is a true equation, $x = 21$ is a solution. |
| **b** $y = 9$ | If $y = 9$ then $5y = 5 \times 9 = 45$, so the equation is true. |

**Drilling for Gold 2A2**

**9** State a solution to each of the following equations.
**a** $5 + x = 12$ **b** $3 = x - 10$ **c** $4u = 28$
**d** $17 = p - 2$ **e** $10x = 20$ **f** $77 = 7k$

**Problem-solving and Reasoning**

## Example 4 Writing equations from a description

Write equations for the following.
**a** The number $k$ is doubled, then three is added and the result is 52.
**b** Akira works $n$ hours, earning \$12 per hour. The total she earned was \$156.

| **Solution** | **Explanation** |
|---|---|
| **a** $2k + 3 = 52$ | The number $k$ is doubled, giving $k \times 2$. This is the same as $2k$. If 3 is added, the left-hand side is $2k + 3$, which must be equal to 52 according to the description. |
| **b** $12n = 156$ | If Akira works $n$ hours at \$12 per hour, the total amount earned is $12 \times n$, or $12n$. |

**10** 'A number $x$ is tripled and the result is 12.' Which of the following equations describes this?
**A** $x + 3 = 12$ **B** $12x = 3$ **C** $3x = 12$ **D** $12 - x = 3$

**11** Write equations to describe the following scenarios. You do not need to solve the equations.

- **a** The number $k$ is increased by 4 and the result is 20.
- **b** A number $x$ is doubled and then 7 is added. The result is 10.
- **c** The sum of $x$ and half of $x$ is 12.
- **d** Fel's height is $h$ cm and her brother Pat is 30 cm taller. Pat's height is 147 cm.
- **e** Coffee costs \$$c$ per cup and tea costs \$3. Four cups of coffee and two cups of tea cost a total of \$22.
- **f** Chairs cost \$$c$ each. To purchase 8 chairs and a \$2000 table costs a total of \$3600.

**12** Find the value of the number for the following problems.

- **a** A number is tripled to obtain the result 21.
- **b** Half of a number is 21.
- **c** Six less than a number is 7.
- **d** A number is doubled and the result is 52.

**13** Berkeley buys $x$ kg of oranges at \$3.20 per kg. He spends a total of \$9.60.

- **a** Write an equation involving $x$ to describe this situation.
- **b** State a solution to this equation.

## Enrichment: More than one unknown

**14 a** There are six equations in the square below. Find the values of $a$, $b$, $c$, $d$ and $e$ to make all six equations true.

$$\begin{array}{ccccc}
\boxed{a} & + & \boxed{12} & = & \boxed{22} \\
\times & & \div & & - \\
\boxed{2} & \times & \boxed{b} & = & \boxed{c} \\
= & & = & & = \\
\boxed{d} & \div & \boxed{e} & = & \boxed{10}
\end{array}$$

**b** If the four numbers above (2, 10, 12, 22) are doubled, what would the values of $a$, $b$, $c$, $d$ and $e$ become?

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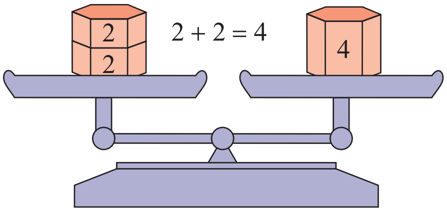
# 2B Equivalent equations

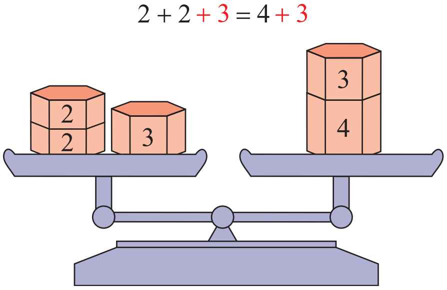
**REVISION**

Sometimes it is helpful to think of an equation as two weights balancing on scales.

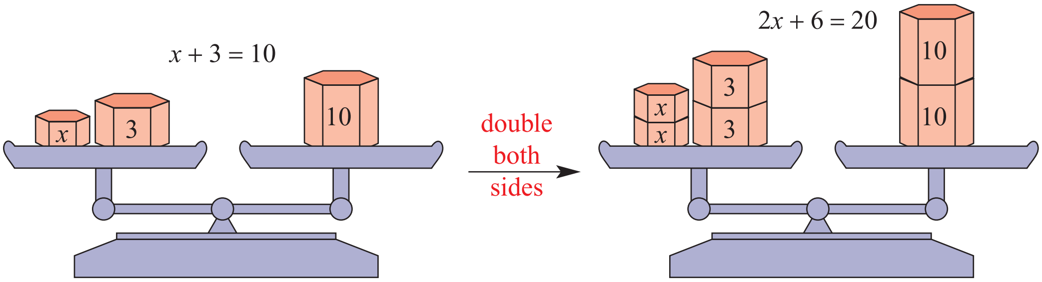


An equation is like this old-fashioned pan balance. When both sides are of equal value, the pans are balanced.

If the same weight is added to both sides, the scales still balance.



We can also subtract a value from both sides, or multiply/divide both sides by the same value, and the scales will still balance.



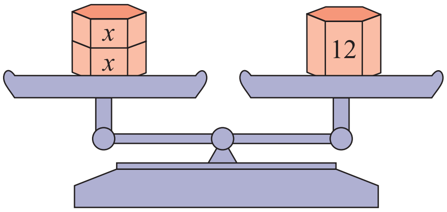
Equations are called *equivalent* if you can get from one to the other by performing the same operations on the LHS and RHS.

The operations can be written next to arrows, like this:

$$\begin{tikzcd} x + 3 = 10 \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\ 2x + 6 = 20 \end{tikzcd}$$

## Let's start: Equivalent equations

- Write down 5 equations that are equivalent to $2x = 12$.
- For one equation that you wrote down, show it as a pair of scales like this diagram.



- Show one of them with arrows like this diagram.

$$\begin{tikzcd} 2x = 12 \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ \_\_\_\_ = \_\_\_\_ \end{tikzcd}$$

- What is the simplest equation that is equivalent to $2x = 12$?

## Key ideas

**Equivalent** Having the same values

- Two equations are **equivalent** if you can get from one to the other by repeatedly:
  - adding a number to both sides
  - subtracting a number from both sides
  - multiplying both sides by a number other than zero
  - dividing both sides by a number other than zero
  - swapping the left-hand side and right-hand sides of the equation.
- To solve an equation using the balance method, you should repeatedly find an equivalent equation that is simpler. For example:

$$\begin{tikzcd} 5x + 2 = 32 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\ 5x = 30 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ x = 6 \end{tikzcd}$$

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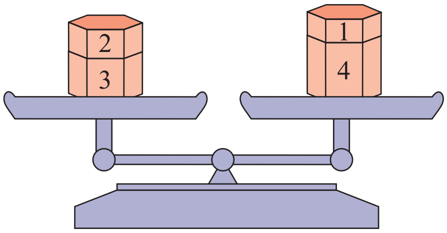
## Exercise 2B

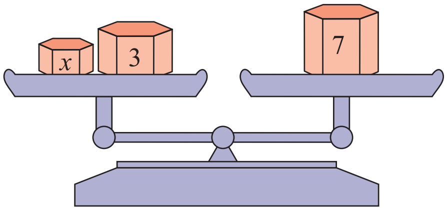
**Understanding**

**1** Write an equation for each of the balancing scales below.

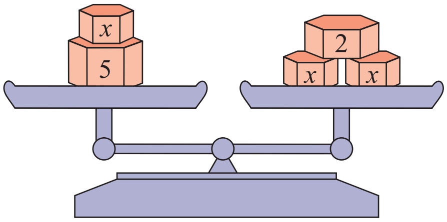
**a**



**b**



**c**



An example could be $7 + 2 = 6 + 1$ or $3x + 1 = x + 4$

**2** Write the equivalent equations to $2x = 12$ by filling in the blanks.

**a** $2x = 12$

$\times 3$ $\times 3$

$\_\_\_\_ = \_\_\_\_$

**b** $2x = 12$

$+1$ $+1$

$\_\_\_\_ = \_\_\_\_$

**c** $2x = 12$

$\div 2$ $\div 2$

$\_\_\_\_ = \_\_\_\_$

**3** For each equation fill in the blank to get an equivalent equation.

**a** $5x = 10$

$+2$ $+2$

$5x + 2 = \_\_\_\_$

**b** $10 - 2x = 20$

$+5$ $+5$

$15 - 2x = \_\_\_\_$

**c** $3q + 4 = 16$

$-4$ $-4$

$3q = \_\_\_\_$

**4** Consider the equation $4x = 32$.

**a** Copy and complete the following working.

$4x = 32$

$\div 4$ $\div 4$

$x = \_\_\_\_$

**b** What is the solution to the equation $4x = 32$?

A solution is a value of $x$ that makes the equation true.

**5** To solve the equation $10x + 6 = 45$, which of the following operations would you first apply to both sides?

**A** Divide by 6
**B** Subtract 6
**C** Divide by 10
**D** Subtract 45

**Fluency**

## Example 5 Finding equivalent equations

Show the result of applying the given operation to both sides of these equations.

**a** $8y = 40\ [\div 8]$ **b** $10 + 2x = 36\ [-10]$ **c** $5a - 3 = 12\ [+3]$

**Solution** | **Explanation**

**a**

$$\begin{tikzcd} 8y = 40 \arrow[d, bend right=60, "\div 8"'] \arrow[d, bend left=60, "\div 8"] \\ y = 5 \end{tikzcd}$$

Write the equation and then divide both sides by 8.
$40 \div 8$ is 5 and $8y \div 8$ is $y$.

**b**

$$\begin{tikzcd} 10 + 2x = 36 \arrow[d, bend right=60, "-10"'] \arrow[d, bend left=60, "-10"] \\ 2x = 26 \end{tikzcd}$$

Write the equation and then subtract 10 from both sides.
$36 - 10$ is 26
$10 + 2x - 10$ is $2x$

**c**

$$\begin{tikzcd} 5a - 3 = 12 \arrow[d, bend right=60, "+3"'] \arrow[d, bend left=60, "+3"] \\ 5a = 15 \end{tikzcd}$$

Write the equation and then add 3 to both sides.
$12 + 3$ is 15
$5a - 3 + 3$ is $5a$

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**6** For each equation, show the result of applying the given operation to both sides.

**a** $10 + 2x = 30\ [-10]$ **b** $4 + q = 12\ [-2]$ **c** $13 = 12 - q\ \ [+5]$
**d** $4x = 8\ [\times 3]$ **e** $7p = 2p + 4\ [+6]$ **f** $3q + 1 = 2q + 1\ [-1]$

**7** Copy and complete the following to solve the given equations using the balancing method.

**a**

$$\begin{tikzcd} 10x = 30 \arrow[d, bend right=60, "\div 10"'] \arrow[d, bend left=60, "\div 10"] \\ x = \_\_\_ \end{tikzcd}$$

**b**

$$\begin{tikzcd} q + 5 = 12 \arrow[d, bend right=60, "-5"'] \arrow[d, bend left=60, "-5"] \\ \_\_\_ = \_\_\_ \end{tikzcd}$$

**c**

$$\begin{tikzcd} k - 3 = 8 \arrow[d, bend right=60, "+3"'] \arrow[d, bend left=60, "+3"] \\ \_\_\_ = \_\_\_ \end{tikzcd}$$

**d**

$$\begin{tikzcd} 4x + 2 = 22 \arrow[d, bend right=60, "-2"'] \arrow[d, bend left=60, "-2"] \\ 4x = \_\_\_ \arrow[d, bend right=60, "\div 4"'] \arrow[d, bend left=60, "\div 4"] \\ \_\_\_ = \_\_\_ \end{tikzcd}$$

**e**

$$\begin{tikzcd} 7p + 2 = 30 \arrow[d, bend right=60, "-2"'] \arrow[d, bend left=60, "-2"] \\ \_\_\_ = \_\_\_ \arrow[d, bend right=60, "\square"'] \arrow[d, bend left=60, "\square"] \\ \_\_\_ = \_\_\_ \end{tikzcd}$$

**f**

$$\begin{tikzcd} 26 = 10x - 4 \arrow[d, bend right=60, "+4"'] \arrow[d, bend left=60, "+4"] \\ \_\_\_ = \_\_\_ \arrow[d, bend right=60, "\square"'] \arrow[d, bend left=60, "\square"] \\ \_\_\_ = \_\_\_ \end{tikzcd}$$

## Example 6 Solving equations using the balancing method

Solve the following equations.

**a** $x - 4 = 16$ **b** $2u + 7 = 17$ **c** $10 = 3k - 11$

**Solution** | **Explanation**

**a**

$$\begin{tikzcd} x - 4 = 16 \arrow[d, bend right=60, "+4"'] \arrow[d, bend left=60, "+4"] \\ x = 20 \end{tikzcd}$$

So the solution is $x = 20$.

By adding 4 to both sides of the equation, we get an equivalent equation.
Check solution:
LHS $= 20 - 4 = 16$ RHS $= 16$

**Solution**

**b**

$$\begin{tikzcd} 2u + 7 = 17 \arrow[d, bend right, "-7"'] \arrow[d, bend left, "-7"] \\ 2u = 10 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ u = 5 \end{tikzcd}$$

So the solution is $u = 5$.

**Explanation**

To get rid of the +7, we subtract 7 from both sides.

Finally we divide by 2 to reverse the $2u$.

Remember that $2u$ means $2 \times u$.

Check solution:

LHS = $2 \times 5 + 7 = 17$ RHS = $17$

**Solution**

**c**

$$\begin{tikzcd} 10 = 3k - 11 \arrow[d, bend right, "+11"'] \arrow[d, bend left, "+11"] \\ 21 = 3k \arrow[d, bend right, "\div 3"'] \arrow[d, bend left, "\div 3"] \\ 7 = k \end{tikzcd}$$

So the solution is $k = 7$.

**Explanation**

First add 11 to 'undo' the $-11$

Then divide by 3 since $3k$ means $k \times 3$.

Write $7 = k$ as $k = 7$.

Check solution:

LHS = $10$ RHS = $3 \times 7 - 11 = 10$

**8** Solve the following equations.

**a** $a + 5 = 8$ **b** $t \times 2 = 14$ **c** $q - 2 = 7$ **d** $k + 2 = 11$

**e** $x + 9 = 19$ **f** $3h = 30$ **g** $9l = 36$ **h** $g \div 3 = 3$

**9** Solve the following equations.

**a** $9h + 5 = 32$ **b** $9u - 6 = 30$ **c** $5s - 2 = 13$ **d** $3w - 6 = 18$

**e** $8 + 5x = 28$ **f** $6 + 10w = 56$ **g** $8a - 8 = 8$ **h** $4y - 8 = 40$

**10** Solve the following equations.

**a** $10 = 5x$ **b** $12 = k + 7$ **c** $30 = x - 12$ **d** $5 = x \div 4$

**e** $32 = 4k + 4$ **f** $50 = 2x - 10$ **g** $12 = 3y - 6$ **h** $14 = x \div 2 + 4$

**Problem-solving and Reasoning**

**11** The solutions to the following equations are negative numbers. Solve the equations to find them.

**a** $x + 10 = 4$ **b** $7a = -21$ **c** $3x + 4 = -26$ **d** $2k + 20 = 10$

**e** $7 = 2k + 15$ **f** $1 = 7p + 8$ **g** $-2 = p \div 8$ **h** $-3 = 2x + 7$

**12** For each of the following, write an equation and solve.

**a** The sum of $p$ and 8 is 15.

**b** The product of $q$ and 3 is 12.

**c** 4 is subtracted from double the value of $k$ and the result is 18.

**d** When $r$ is tripled and 4 is added the result is 34.

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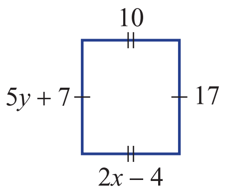
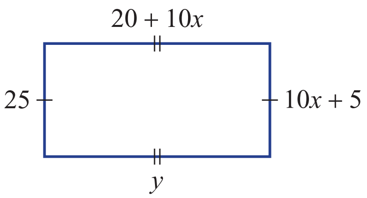
**13** The following shapes are rectangles. By solving equations, find the value of the pronumerals.

**a**

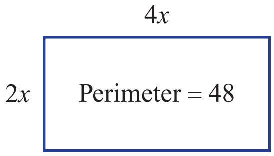


**b**



Find the value of $x$ first.

**c**



**14** Solve the following equations. More than two steps are involved.

**a** $14 \times (4x + 2) = 140$ **b** $8 = (10x - 4) \div 2$ **c** $3 + (2x + 1) \times 4 = 47$

---

## Enrichment: From solutions to equations

**15** A student has taken the equation $x = 5$ and performed some operations to both sides:

$$\begin{tikzcd} x = 5 \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\ 4x = 20 \arrow[d, bend right, "+3"'] \arrow[d, bend left, "+3"] \\ 4x + 3 = 23 \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\ (4x + 3) \times 2 = 46 \end{tikzcd}$$

**a** Solve $(4x + 3) \times 2 = 46$.

**b** Describe how the steps you used in your solution compare with the steps the student used.

**c** Give an example of another equation that has $x = 5$ as its solution.

# 2C Equations with fractions

A fraction such as $\frac{x}{3}$ represents $x \div 3$. This means that to solve an equation with $\frac{x}{3}$ on one side, we should first multiply both sides by 3. For example:

$$\begin{tikzcd} \frac{x}{3} = 10 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ x = 30 \end{tikzcd} \qquad \begin{tikzcd} 20 = \frac{x}{5} \arrow[d, bend right, "\times 5"'] \arrow[d, bend left, "\times 5"] \\ 100 = x \end{tikzcd}$$

$$\therefore\ x = 100$$

### Let's start: Practising with fractions

- If $x = 10$, find out what each of these expressions would equal:

$$\frac{2x + 1}{2} \qquad 2\left(\frac{x}{2} + 1\right) \qquad \frac{2}{x + 1} \qquad \frac{2 + 2x}{2} \qquad 2\left(x + \frac{1}{2}\right)$$

- Which of the above expressions are equal if $x = 0$?

## Key ideas

- $\frac{a}{b}$ means $a \div b$.
- To solve an equation with a fraction on one side, multiply both sides by the denominator.

$$\begin{tikzcd} \frac{q}{4} = 12 \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\ q = 48 \end{tikzcd}$$

## Exercise 2C

**Understanding**

**1** Which of the following expressions represents '$x$ divided by 5'?

**A** $x + 5$ **B** $\frac{x}{5}$ **C** $\frac{5}{x}$ **D** $5x$

**2** If $x = 20$, state whether the following equations are true or false.

**a** $\frac{x}{4} = 5$ **b** $\frac{x}{2} = 40$ **c** $\frac{x}{5} = 5$ **d** $\frac{x}{10} = 2$

**3** **a** If $x = 4$, find the value of $\frac{x}{2} + 6$.

**b** If $x = 4$, find the value of $\frac{x + 6}{2}$.

**c** Are $\frac{x}{2} + 6$ and $\frac{x + 6}{2}$ equivalent expressions?

Expressions are equivalent if they are always equal.

**4** Fill in the missing steps to solve these equations.

**a**
$$\begin{tikzcd} \frac{x}{3} = 10 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ x = \_\_ \end{tikzcd}$$

**b**
$$\begin{tikzcd} \frac{m}{5} = 2 \arrow[d, bend right, "\times 5"'] \arrow[d, bend left, "\times 5"] \\ m = \_\_ \end{tikzcd}$$

**c**
$$\begin{tikzcd} 11 = \frac{q}{2} \arrow[d, bend right, "\square"'] \arrow[d, bend left, "\square"] \\ \_\_ = q \end{tikzcd}$$

**d**
$$\begin{tikzcd} \frac{p}{10} = 7 \arrow[d, bend right, "\square"'] \arrow[d, bend left, "\square"] \\ p = \_\_ \end{tikzcd}$$

**Fluency**

## Example 7 Solving equations with fractions

Solve the following equations.

**a** $\frac{k}{10} = 4$

**b** $\frac{4x}{3} = 8$

| **Solution** | **Explanation** |
|---|---|
| **a** $\begin{tikzcd} \frac{k}{10} = 4 \arrow[d, bend right, "\times 10"'] \arrow[d, bend left, "\times 10"] \\ k = 40 \end{tikzcd}$ | Multiplying both sides by 10 removes the denominator of 10. |
| **b** $\begin{tikzcd} \frac{4x}{3} = 8 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ 4x = 24 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ x = 6 \end{tikzcd}$ | Multiplying both sides by 3 removes the denominator of 3.<br>Both sides are divided by 4 to solve the equation. |

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**5** Solve the following equations.

**a** $\frac{b}{5} = 4$ **b** $\frac{g}{10} = 2$ **c** $\frac{a}{5} = 3$ **d** $\frac{k}{6} = 3$

**e** $5 = \frac{x}{7}$ **f** $10 = \frac{x}{10}$ **g** $2 = \frac{t}{4}$ **h** $4 = \frac{t}{2}$

**Drilling for Gold 2C1**

**6** Solve the following equations.

Multiply both sides by a chosen number.

**a** $\frac{2\ell}{5} = 8$ **b** $\frac{7w}{10} = -7$ **c** $\frac{3s}{2} = -9$

**d** $\frac{5v}{4} = 15$ **e** $\frac{3m}{7} = 6$ **f** $\frac{3n}{7} = 6$

**g** $\frac{-6j}{5} = 6$ **h** $\frac{-6f}{5} = -24$

## Example 8 Solving more complex equations with fractions

Solve the equation: $\frac{4y + 15}{9} = 3$

| Solution | Explanation |
|---|---|
| $\frac{4y+15}{9} = 3$<br>$\times 9 \quad \times 9$<br>$4y + 15 = 27$<br>$-15 \quad -15$<br>$4y = 12$<br>$\div 4 \quad \div 4$<br>$y = 3$ | Multiplying both sides by 9 removes the denominator of 9.<br>The equation $4y + 15 = 27$ is solved in the usual fashion (subtract 15, divide by 4). |

**7** Solve the following equations.

First multiply.

**a** $\frac{t-8}{2} = 10$ **b** $\frac{h+10}{3} = 4$ **c** $\frac{a+12}{5} = 5$

**d** $\frac{c-7}{2} = 5$ **e** $\frac{s-2}{8} = 1$ **f** $\frac{5j+6}{8} = 2$

**g** $7 = \frac{x-9}{5}$ **h** $8 = \frac{2x+4}{6}$ **i** $0 = \frac{2x-4}{5}$

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## Example 9 Solving more equations with fractions

Solve the equation: $4 + \frac{5x}{2} = 29$

| **Solution** | **Explanation** |
|---|---|
| $$\begin{tikzcd} 4 + \frac{5x}{2} = 29 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\ \frac{5x}{2} = 25 \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\ 5x = 50 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ x = 10 \end{tikzcd}$$ | We must subtract 4 first because we do not have a fraction by itself on the left-hand side. Once there is a fraction by itself, multiply by the denominator (2). |

**8** Solve the following equations.

**a** $\frac{v}{10} + 3 = 5$

**b** $2 + \frac{x}{4} = 7$

**c** $\frac{y}{2} - 6 = 1$

**d** $\frac{2x}{5} + 6 = 10$

**e** $\frac{6p}{7} - 4 = 2$

**f** $9 + \frac{3k}{2} = 18$

**9** Match each of these equations with the correct first step to solve it.

**a** $\frac{x}{4} = 7$

**b** $\frac{x - 4}{2} = 5$

**c** $\frac{x}{2} - 4 = 7$

**d** $\frac{x}{4} + 4 = 3$

**A** Multiply both sides by 2.

**B** Add 4 to both sides.

**C** Multiply both sides by 4.

**D** Subtract 4 from both sides.

**Skillsheet 2A**

**10** Solve the following equations.

**a** $\frac{g - 3}{5} = 1$

**b** $\frac{2x}{7} = 4$

**c** $\frac{k}{3} + 1 = 6$

**d** $\frac{x}{4} = 9$

**e** $3 = \frac{q}{2} - 2$

**f** $15 = \frac{3 + x}{2}$

**g** $2 = \frac{5p}{15}$

**h** $\frac{2x + 7}{3} = 3$

**i** $9 = \frac{2r}{4} - 1$

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Problem-solving and Reasoning

Drilling for Gold 2C2

**11** For the following puzzles, write an equation and solve it to find the unknown number.

- **a** A number $x$ is divided by 5 and the result is 7.
- **b** Half of $y$ is 12.
- **c** A number $p$ is doubled and then divided by 7. The result is 4.
- **d** Four is added to $x$. This is halved to get a result of 10.
- **e** $x$ is halved and then 4 is added to get a result of 10.
- **f** A number $k$ is doubled and then 6 is added. This result is halved to obtain 14.

**12** The average of two numbers can be found by adding them and then dividing the result by 2.

- **a** Find the average of 9 and 5.
- **b** If the average of $x$ and 5 is 12, what is $x$? Solve the equation $\frac{x+5}{2} = 12$ to find out.
- **c** The average of 7 and $p$ is 5. Find $p$ by writing and solving an equation.
- **d** The average of a number and double that number is 18. What is that number?
- **e** The average of $4x$ and 6 is 19. What is the average of $6x$ and 4? (Hint: Find $x$ first.)

**13** A restaurant bill is to be paid. Blake puts in \$40, which is one third of the amount in his wallet.

- **a** Write an equation to describe this situation, if $b$ represents the amount in Blake's wallet before he pays.
- **b** Solve the equation to find out how much money Blake has in his wallet.

## Enrichment: Unknown denominators

**14** In these equations, the unknown is the denominator. Solve them by inspection.

**a** $\frac{12}{x} = 2$ **b** $\frac{15}{x} = 5$ **c** $\frac{20}{x} = 4$

**d** $4 + \frac{20}{x} = 14$ **e** $\frac{16}{x} + 1 = 3$ **f** $\frac{12}{x} = 1$

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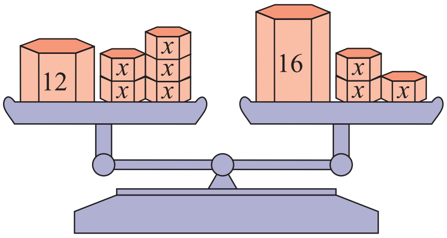
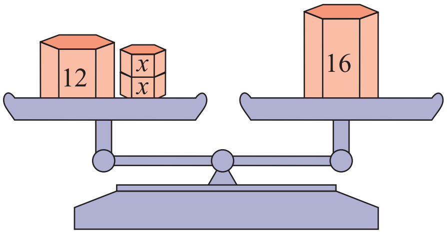
# 2D Equations with pronumerals on both sides

All the equations we have considered so far have involved a pronumeral either on the left-hand side, e.g. $2x + 3 = 11$, or on the right side, e.g. $15 = 10 - 2x$. How can you solve an equation with pronumerals on both sides, e.g. $12 + 5x = 16 + 3x$? The idea is to look for an equivalent equation with pronumerals on just one side.

The equation $12 + 5x = 16 + 3x$ can be thought of as balancing scales.



Then $3x$ can be removed from both sides of this equation to get:



The equation $12 + 2x = 16$ is straightforward to solve.

## Let's start: Moving pronumerals

You are given the equation $11 + 5x = 7 + 3x$.

- Can you find an equivalent equation with $x$ just on the left-hand side?
- Can you find an equivalent equation with $x$ just on the right-hand side?
- Try to find an equivalent equation with $9x$ on the left-hand side.
- Do all of these equations have the same solution? Try to find it.

## Key ideas

- If both sides of an equation have a pronumeral added or subtracted, the new equation will be **equivalent** to the original equation.
- If pronumerals are on both sides of an equation, add or subtract something to both sides so that the pronumeral appears on only one side. For example:

$$\begin{tikzcd} 10 + 5a = 13 + 2a \arrow[d, bend right, "-2a"'] \arrow[d, bend left, "-2a"] \\ 10 + 3a = 13 \end{tikzcd} \qquad \begin{tikzcd} 4b + 12 = 89 - 3b \arrow[d, bend right, "+3b"'] \arrow[d, bend left, "+3b"] \\ 7b + 12 = 89 \end{tikzcd}$$

- Sometimes it is wise to swap the left-hand side and right-hand side.

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## Exercise 2D

Understanding

**1** If $x = 3$, are the following equations true or false?

**a** $5 + 2x = 4x - 1$
**b** $7x = 6x + 5$
**c** $2 + 8x = 12x$
**d** $9x - 7 = 3x + 11$

**2** Fill in the blanks for these equivalent equations.

**a**
$$\begin{tikzcd} 5x + 3 = 2x + 8 \arrow[d, bend right, "-2x"'] \arrow[d, bend left, "-2x"] \\ \_\_\_ = 8 \end{tikzcd}$$

**b**
$$\begin{tikzcd} 9q + 5 = 12q + 21 \arrow[d, bend right, "-9q"'] \arrow[d, bend left, "-9q"] \\ \_\_\_ = 3q + 21 \end{tikzcd}$$

**c**
$$\begin{tikzcd} 2p + 9 = 5 - 2p \arrow[d, bend right, "+2p"'] \arrow[d, bend left, "+2q"] \\ \_\_\_ = \_\_\_ \end{tikzcd}$$

**d**
$$\begin{tikzcd} 15k + 12 = 13 - 7k \arrow[d, bend right, "+7k"'] \arrow[d, bend left, "+7k"] \\ \_\_\_ = \_\_\_ \end{tikzcd}$$

**3** To solve the equation $12x + 2 = 8x + 16$, which one of the following first steps will ensure that $x$ is only on one side of the equation?

**A** Subtract 2
**B** Subtract $8x$
**C** Add $12x$
**D** Subtract 16
**E** Add $20x$

Fluency

### Example 10 Solving equations with pronumerals on both sides

Solve the following equations.

**a** $7t + 4 = 5t + 10$
**b** $6x + 4 = 22 - 3x$
**c** $2u = 7u - 20$

| Solution | Explanation |
|---|---|
| **a** $\begin{tikzcd} 7t + 4 = 5t + 10 \arrow[d, bend right, "-5t"'] \arrow[d, bend left, "-5t"] \\ 2t + 4 = 10 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\ 2t = 6 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ t = 3 \end{tikzcd}$ | Pronumerals are on both sides of the equation, so subtract $5t$ from both sides.<br>Once $5t$ is subtracted, the usual procedure is applied for solving equations.<br>Check the solution:<br>LHS $= 7 \times 3 + 4$ $= 25$ RHS $= 5 \times 3 + 10$ $= 25$ |
| **b** $\begin{tikzcd} 6x + 4 = 22 - 3x \arrow[d, bend right, "+3x"'] \arrow[d, bend left, "+3x"] \\ 9x + 4 = 22 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\ 9x = 18 \arrow[d, bend right, "\div 9"'] \arrow[d, bend left, "\div 9"] \\ x = 2 \end{tikzcd}$ | Pronumerals are on both sides. To remove $3x$, we add $3x$ to both sides of the equation.<br>Once pronumerals are just on the LHS, the usual procedure is applied for solving equations.<br>Check the solution:<br>LHS $= 6 \times 2 + 4$ $= 16$ RHS $= 22 - 3 \times 2$ $= 16$ |

| **Solution** | **Explanation** |
|---|---|
| **c** $\begin{tikzcd} 2u \arrow[d, bend right, "-2u"'] & = & 7u - 20 \arrow[d, bend left, "-2u"] \\ 0 \arrow[d, bend right, "+20"'] & = & 5u - 20 \arrow[d, bend left, "+20"] \\ 20 \arrow[d, bend right, "\div 5"'] & = & 5u \arrow[d, bend left, "\div 5"] \\ 4 & = & u \end{tikzcd}$<br>$\therefore\ u = 4$ | Choose to remove $2u$ by subtracting it.<br>Note that $2u - 2u$ is equal to $0$, so the LHS of the new equation is $0$.<br>Check the solution:<br>LHS $= 2 \times 4$ $= 8$ RHS $= 7 \times 4 - 20$ $= 8$ |

**4** Solve the following equations systematically and check your solutions.

**a** $10f + 3 = 23 + 6f$
**b** $10y + 5 = 26 + 3y$
**c** $7s + 7 = 19 + 3s$
**d** $9j + 4 = 4j + 14$
**e** $2t + 8 = 8t + 20$
**f** $4 + 3n = 10n + 39$
**g** $4 + 8y = 10y + 14$
**h** $5 + 3t = 6t + 17$
**i** $7 + 5q = 19 + 9q$

**5** Solve the following equations systematically, checking your solutions using substitution.

**a** $9 + 4t = 7t + 15$
**b** $2c - 2 = 4c - 6$
**c** $6t - 3 = 7t - 8$
**d** $7z - 1 = 8z - 4$
**e** $8t - 24 = 2t - 6$
**f** $2q - 5 = 3q - 3$
**g** $5x + 8 = 6x - 1$
**h** $8w - 15 = 6w + 3$
**i** $6j + 4 = 5j - 1$

**6** Solve the following equations systematically. Your solutions should be checked using substitution.

**a** $1 - 4a = 7 - 6a$
**b** $6 - 7g = 2 - 5g$
**c** $12 - 8n = 8 - 10n$
**d** $2 + 8u = 37 + 3u$
**e** $21 - 3h = 6 - 6h$
**f** $37 - 4j = 7 - 10j$
**g** $13 - 7c = 8c - 2$
**h** $10 + 4n = 4 - 2n$
**i** $10a + 32 = 2a$
**j** $10v + 14 = 8v$
**k** $18 + 8c = 2c$
**l** $2t + 7 = 22 - 3t$
**m** $6n - 47 = 9 - 8n$
**n** $3n = 15 + 8n$
**o** $38 - 10\ell = 10 + 4\ell$

**7** Solve the following equations systematically. Your answers should be given as fractions.

**a** $3x + 5 = x + 6$
**b** $5k - 2 = 2k$
**c** $3 + m = 6 + 3m$
**d** $9j + 4 = 5j + 14$
**e** $3 - j = 4 + j$
**f** $2z + 3 = 4z - 8$

**Problem-solving and Reasoning**

**8** Write an equation and solve it systematically to find the unknown number in these problems.

**a** Doubling $x$ and adding 3 is the same as tripling $x$ and adding 1.
**b** If $z$ is increased by 9, this is the same as doubling the value of $z$.
**c** The product of 7 and $y$ is the same as the sum of $y$ and 12.
**d** When a number is increased by 10, this has the same effect as tripling the number and subtracting 6.

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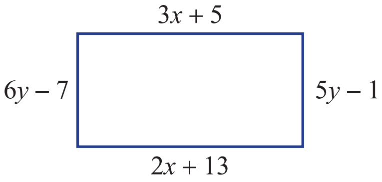
**9** At a newsagency, Preeta bought 4 pens and a \$1.50 newspaper, while her husband Levy bought 2 pens and a \$4.90 magazine. To their surprise the cost was the same.

- **a** Write an equation to describe this, using $p$ for the cost of a single pen.
- **b** Solve the equation to find the cost of pens.
- **c** If Fred has a \$20 note, what is the maximum number of pens that he can purchase?

**10** To solve the equation $12 + 3x = 5x + 2$ you can first subtract $3x$ or subtract $5x$.

- **a** Solve the equation above by first subtracting $3x$.
- **b** Solve the equation above by first subtracting $5x$.
- **c** What is the difference between the two methods?

**11** Prove that the rectangular shape, to the right, must be a square. (Hint: First find the values of $x$ and $y$.)



**12** **a** Try to solve the equation $4x + 3 = 10 + 4x$.

- **b** This tells you that the equation you are trying to solve has no solutions (because $10 = 3$ is never true). Prove that $2x + 3 = 7 + 2x$ has no solutions.
- **c** Give an example of another equation that has no solutions.

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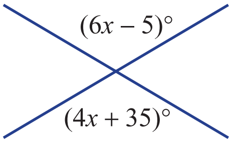
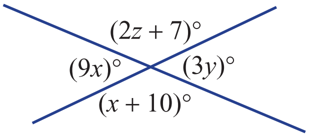
## Enrichment: Geometric equations

**13** Find the values of the pronumerals in the following geometric diagrams.

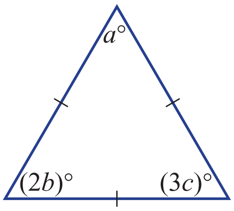
**a**



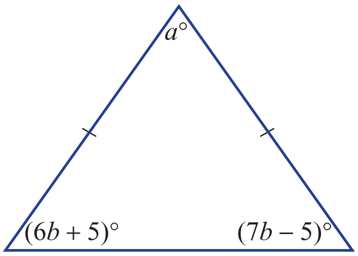
**b**



**c**



**d**



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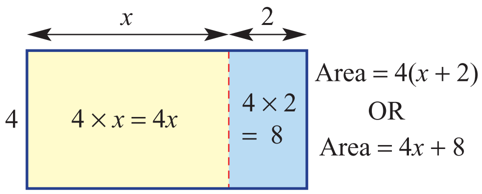
# 2E Equations with brackets

In Chapter 1 it was noted that expressions with brackets could be expanded by considering rectangle areas.

So $4(x + 2) = 4x + 8$



Demonstration that $4x + 8$ and $4(x + 2)$ are equivalent

## Let's start: Tank tops and shorts

Harrison buys two sporting outfits at a shop where shorts cost $5 more than T-shirts.

- If each pair of shorts is $10, how much does the outfit cost?
- If the two outfits cost $60 in total, can you give the cost of each item?
- Try to find an expression for the total cost of the outfits.

## Key ideas

- To expand brackets, use the **distributive law**, which states that:
  - $a(b + c) = ab + ac$. e.g. $3(x + 4) = 3x + 12$.
  - $a(b - c) = ab - ac$. e.g. $4(b - 2) = 4b - 8$.
- **Like terms** are terms that contain exactly the same pronumeral and can be collected to simplify expressions. For example, $5x + 10 + 7x$ can be simplified to $12x + 10$.
- Equations involving brackets can be solved by first expanding brackets and collecting like terms.

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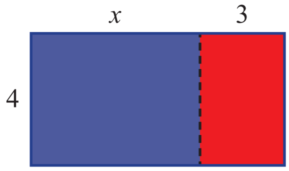
## Exercise 2E

**Understanding**

**1** Which of the following expressions give the area of the rectangle? (There is more than one correct answer.)



**A** $4 \times x + 3$ **B** $4 + x + 3$ **C** $4 \times (x + 3)$
**D** $(x + 3) \times 4$ **E** $4(x + 3)$ **F** $4x + 12$

### Example 11 Expanding brackets

Expand the brackets for:
**a** $2(5x + 3)$ **b** $3(k - 4)$

| **Solution** | **Explanation** |
|---|---|
| **a** $2(5x + 3) = 10x + 6$ | $2 \times 5x = 10x$ and $2 \times 3 = 6$ |
| **b** $3(k - 4) = 3k - 12$<br>Note the minus sign remains | $3 \times k = 3k$ and $3 \times 4 = 12$ |

**2** Fill in the missing numbers.
**a** $4(y + 3) = 4y + \square$ **b** $7(2p - 5) = \square p - 35$
**c** $2(4x + 5) = \square x + \square$ **d** $10(5 + 3q) = \square + \square q$

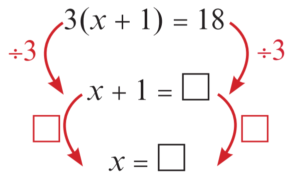
**3** Match each expression (**a**–**d**) with its expanded form (**A**–**D**).

| | |
|---|---|
| **a** $2(x + 4)$ | **A** $4x + 8$ |
| **b** $4(x + 2)$ | **B** $2x + 4$ |
| **c** $2(2x + 1)$ | **C** $2x + 8$ |
| **d** $2(x + 2)$ | **D** $4x + 2$ |

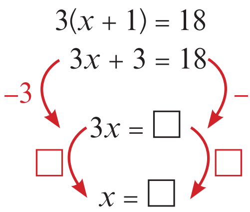
**4** If $x = 5$, state whether the following equations are true or false.
**a** $3(x + 1) = 18$ **b** $4(x - 2) = 16$ **c** $2(2x + 1) = 22$ **d** $5(x - 1) = 20$

**5** Copy and complete.

**a**



**b**



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Fluency

## Example 12 Solving equations with brackets

Solve the following equations by first expanding any brackets.

**a** $3(p + 4) = 18$

**b** $4(2x - 5) + 3x = 57$

| **Solution** | **Explanation** |
|---|---|
| **a** $\begin{array}{rrcll} & 3(p+4) &=& 18 & \\ -12 & 3p+12 &=& 18 & -12 \\ & 3p &=& 6 & \\ \div 3 & & & & \div 3 \\ & p &=& 2 & \end{array}$ | Use the distributive law to expand the brackets.<br>Check:<br>LHS $= 3(p + 4)$ RHS $= 18$<br>$= 3 \times (2 + 4)$<br>$= 18$ |
| **b** $\begin{array}{rrcll} & 4(2x-5)+3x &=& 57 & \\ & 8x-20+3x &=& 57 & \\ +20 & 11x-20 &=& 57 & +20 \\ & 11x &=& 77 & \\ \div 11 & & & & \div 11 \\ & x &=& 7 & \end{array}$ | Use the distributive law to expand the brackets.<br>Combine the like terms: $8x + 3x = 11x$.<br>Check:<br>LHS $= 4(2x - 5) + 3x$ RHS $= 57$<br>$= 4 \times 9 + 3 \times 7$<br>$= 57$ |

**6** Solve the following equations by first expanding the brackets, as in the examples above.

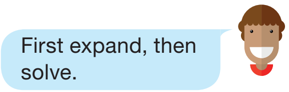
**a** $4(x + 1) = 24$
**b** $3(k + 5) = 18$
**c** $2(r - 7) = 20$
**d** $2(4u + 2) = 52$
**e** $3(3j - 4) = 15$
**f** $5(2p - 4) = 40$
**g** $15 = 5(2m - 5)$
**h** $2(5n + 5) = 60$
**i** $26 = 2(3a + 4)$

**7** Repeat Question **6**, but do a division as the first step, as in Question **5b**.

Skillsheet 2B

**8** Solve the following equations by expanding then combining like terms.

**a** $2(x + 3) + x = 30$
**b** $3(x - 1) + 2x = 47$
**c** $5(r - 2) + r = 50$
**d** $4(3y + 2) + 2y = 50$
**e** $5(2\ell - 5) + 3\ell = 1$
**f** $4(5 + 3w) + 5 = 49$
**g** $49 = 5(3c + 5) - 3c$
**h** $28 = 4(3d + 3) - 4d$
**i** $58 = 4(2w - 5) + 5w$
**j** $23 = 4(2p - 3) + 3$
**k** $44 = 5(3k + 2) + 2k$
**l** $49 = 3(2c - 5) + 4$



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### Problem-solving and Reasoning

**9** A number is increased by 5 and then the result is doubled.

**a** If the number is $n$, write an expression for the final result.

**b** If the final result equals 40, which of the following equations describes this?

**A** $n + 5 \times 2 = 40$ **B** $2(n + 5) = 40$

**C** $2n + 5 = 40$ **D** $40(n + 2) = 5$

**c** What was the original number?

**10** Desmond notes that in 4 years' time his age when doubled will give the number 50. Desmond's current age is $d$.

**a** Write an expression for Desmond's age in 4 years' time.

**b** Write an expression for double his age in 4 years' time.

**c** Write an equation to describe the situation described above.

**d** Solve the equation to find his current age.

**11** Amos buys 3 shirts and 2 pairs of trousers for a total of \$225. Each pair of trousers costs \$20 more than a shirt.

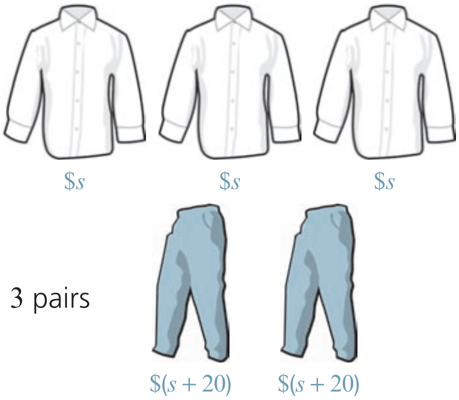
**a** Explain why the total cost is $3s + 2(s + 20)$ if $\$s$ is the cost of one shirt.

**b** Solve the equation $3s + 2(s + 20) = 225$

**c** How much does one shirt cost?

**d** How much does one pair of trousers cost?

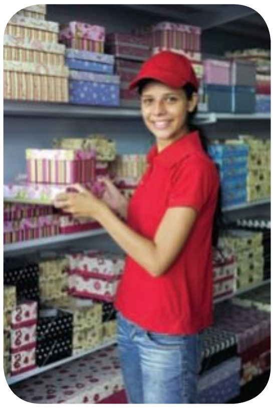
**e** What would the total cost be for 5 shirts and 3 pairs of trousers?



**12** Rahda's usual hourly wage is $\$w$. She works for 5 hours at this wage and then 3 more hours at an increased wage of $\$(w + 4)$.

**a** Write an expression for the total amount Rahda earns for the 8 hours.

**b** Rahda earns \$104 for the 8 hours. Write and solve an equation to find her usual hourly wage.



## Enrichment: Into the negative zone

**13** The following equations involve negative numbers. Solve them.

**a** $2(x + 1) = -10$ **b** $3(p - 2) = -18$

**c** $10(q + 9) = -100$ **d** $-2(r + 1) = -10$

**e** $-5(r + 6) = -40$ **f** $2(x + 5) = -12$

**g** $3(k + 1) + k = -37$ **h** $-10(s - 5) = 50$

$-10(s - 5) = -10s + 50$

$(-10 \times -5)$

# 2F Solving simple quadratic equations

Equation 1 has exactly one solution. In the left-hand side, the unknown is multiplied by 2. Equations like this, which are linear, usually have one solution.

Equation 2 is different. In the left-hand side, the unknown is multiplied by itself. Equations like this, which are called quadratic, can have two solutions, one solution or no solutions.

| **Equation 1:** | **Equation 2:** |
|---|---|
| $2x = 16$ | $x^2 = 16$ |
| **Solution:** | **Solution:** |
| $x = 8$ | $x = ?$ |

## Let's start: Squaring numbers

- What is $4^2$?
- What is $(-4)^2$?
- What number(s) go in the box?

  $\square^2 = 16$

## Key ideas

- Simple quadratic equations:
  - $x^2 = 9$ has two solutions, because 9 is a positive number.

$$\begin{aligned} &x^2 = 9 \\ x = \sqrt{9},&\quad x = -\sqrt{9} \\ x = 3,&\quad x = -3 \\ &x = \pm 3 \end{aligned}$$

Note: $3^2 = 9$ and $(-3)^2 = 9$

  - $x^2 = 0$ has one solution ($x = 0$), because $0^2 = 0$.
  - $x^2 = -9$ has no solutions, because the square of any number is 0 or positive.

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## Exercise 2F

**Understanding**

**1** **a** Calculate the following.

**i** $3^2$ and $(-3)^2$ **ii** $6^2$ and $(-6)^2$ **iii** $1^2$ and $(-1)^2$ **iv** $10^2$ and $(-10)^2$

**b** What do you notice about the answers to each pair?

**2** **a** Use a calculator to multiply these numbers by themselves. Recall that a negative × negative = positive.

**i** $-3$ **ii** $7$ **iii** $13$ **iv** $-8$

**b** Did you obtain any negative numbers in part **a**?

**3** Write in the missing numbers.

**a** $(-3)^2 =$ ______ and $3^2 = 9$ so if $x^2 = 9$ then $x =$ ______ or $x =$ ______

**b** $(5)^2 =$ ______ and $(-5)^2 = 25$ so if $x^2 = 25$ then $x =$ ______ or $x =$ ______

**c** $(11)^2 = 121$ and $(-11)^2 =$ ______ so if $x^2 = 121$ then $x =$ ______ or $x =$ ______

### Example 13 Solving $x^2 = c$ if $c > 0$

Solve the following equations. Round to 2 decimal places in part **b** by using a calculator to assist.

**a** $x^2 = 81$ **b** $x^2 = 23$

| **Solution** | **Explanation** |
|---|---|
| **a** $x = 9$ or $x = -9$ | Since 81 is a positive number, the equation has two solutions. Both 9 and –9 square to give 81. |
| **b** $x = \sqrt{23} = 4.80$<br>(to 2 decimal places)<br>or $x = -\sqrt{23} = -4.80$<br>(to 2 decimal places) | The number 23 is not a perfect square so $\sqrt{23}$ can be rounded if required. |

**Fluency**

**4** Solve the following equations.

**a** $x^2 = 4$ **b** $x^2 = 49$ **c** $x^2 = 100$

**d** $x^2 = 64$ **e** $x^2 = 1$ **f** $x^2 = 144$

**g** $x^2 = 36$ **h** $x^2 = 121$ **i** $x^2 = 169$

**j** $x^2 = 256$ **k** $x^2 = 900$ **l** $x^2 = 10000$

Hint: Use a calculator for parts **h–l**.

**5** Solve the following and round to 2 decimal places.

**a** $x^2 = 6$ **b** $x^2 = 12$ **c** $x^2 = 37$ **d** $x^2 = 41$

**e** $x^2 = 104$ **f** $x^2 = 317$ **g** $x^2 = 390$ **h** $x^2 = 694$

### Example 14 Stating the number of solutions

State the number of solutions for $x$ in these equations.

**a** $x^2 = -3$ **b** $x^2 = 0$ **c** $x^2 = 7$

| **Solution** | **Explanation** |
|---|---|
| **a** 0 solutions | In $x^2 = c$, if $c < 0$ there are no solutions because any number squared is positive or zero. |
| **b** 1 solution | $x = 0$ is the only solution to $x^2 = 0$ |
| **c** 2 solutions | Both $\sqrt{7}$ and $-\sqrt{7}$ square to give 7. |

**6** State the number of solutions for these equations.

**a** $x^2 = 10$ **b** $x^2 = 4$ **c** $x^2 = 3917$ **d** $x^2 = -4$

**e** $x^2 = -94$ **f** $x^2 = 0$ **g** $a^2 = 0$ **h** $y^2 = 1$

**Problem-solving and Reasoning**

**7** The area of a square is 25 m². Find its perimeter.

**8** A square mirror has an area of 1 m². Find its perimeter.

**9** By first dividing both sides by the coefficient of $x^2$, solve these simple quadratic equations.

**a** $2x^2 = 8$ **b** $3x^2 = 3$

**c** $5x^2 = 45$ **d** $-3x^2 = -12$

**e** $-2x^2 = -50$ **f** $7x^2 = 0$

**g** $-6x^2 = -216$ **h** $-10x^2 = -1000$

**10** Explain why:

**a** $x^2 = 0$ has only one solution

**b** $x^2 = c$ has no solutions if $c$ is a negative number.

**11** The exact value solutions to $x^2 = 5$, for example, are written as $x = \sqrt{5}$ or $-\sqrt{5}$. Alternatively, we can write $x = \pm\sqrt{5}$.

Write the exact value solutions to these equations.

**a** $x^2 = 11$ **b** $x^2 = 17$ **c** $x^2 = 33$ **d** $x^2 = 156$

## Enrichment: Solving more complex quadratic equations

**12** Solve these quadratic equations.

**a** $2x^2 + 1 = 9$ **b** $5x^2 - 2 = 3$ **c** $3x^2 - 4 = 23$

**d** $-x^2 + 1 = 0$ **e** $-2x^2 + 8 = 0$ **f** $7x^2 - 6 = 169$

**g** $4 - x^2 = 0$ **h** $27 - 3x^2 = 0$ **i** $38 - 2x^2 = -34$

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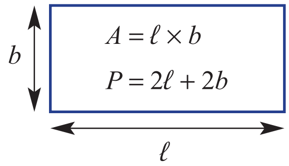
# 2G Formulas and relationships EXTENSION

Formulas occur in many areas of maths and science.
Formulas are a special type of equation that relate to two or more variables.
The formula $E = mc^2$ relates energy and mass.

## Let's start: Rectangular dimensions

You know that the area and perimeter of a rectangle are given by $A = \ell \times b$ and $P = 2\ell + 2b$.

- If $\ell = 10$ and $b = 7$, find the perimeter and the area.
- If $\ell = 2$ and $b = 8$, find the perimeter and the area.
- Notice that sometimes the area is bigger than the perimeter and sometimes the area is less than the perimeter. If $\ell = 10$, is it possible to make the area and the perimeter equal?
- If $\ell = 2$ can you make the area and the perimeter equal? Discuss.



Two formulas used in measurement

## Key ideas

- The **subject** of an equation is a pronumeral that occurs by itself on the left-hand side, e.g. $V$ is the subject of $V = 3x + 2y$.
- A **formula** or *rule* is an equation containing two or more pronumerals, one of which is the subject of the equation.
- To use a formula, substitute all the known values and then solve the equation to find the value of the unknown.

**Subject** The pronumeral on the left-hand side of the equals sign in an equation

**Formula** A general rule for finding the value of one quantity given the values of others

## Exercise 2G

**Understanding**

**1** Fill in the blanks.

**a** A ________ or rule is an equation relating two or more pronumerals.
**b** A pronumeral by itself on the left-hand side of an equation is called the ________.
**c** The formula A = $\ell \times b$ is used to find the ________ of a rectangle.

**2** **a** Substitute $x = 4$ into the expression $x + 7$.
**b** Substitute $a = 2$ into the expression $3a$.
**c** Substitute $p = 5$ into the expression $2p - 3$.
**d** Substitute $r = -4$ into the expression $7r$.

**3** If you substitute $\ell = 5$ and $b = 3$ into the formula $A = \ell \times b$, which of the following equations would you get?
**A** $A = 5 + 3$ **B** $A = 53$ **C** $A = 5 \times 3$ **D** $A = 5 - 3$

**4** If you substitute $P = 10$ and $x = 2$ into the formula $P = 3m + x$, which of the following equations would you get?
**A** $10 = 6 + x$ **B** $10 = 3m + 2$ **C** $2 = 3m + 10$ **D** $P = 30 + 2$

**5** If you substitute $k = 10$ and $L = 12$ into the formula $L = 4k + Q$, which of the following equations would you get?
**A** $12 = 40 + Q$ **B** $L = 40 + 12$ **C** $12 = 410 + Q$ **D** $10 = 48 + Q$

**Fluency**

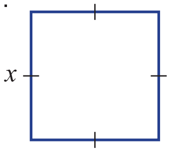
### Example 15 Applying a formula

Apply the formula for a rectangle's perimeter $P = 2\ell + 2b$ to find:
**a** $P$ when $\ell = 4$ and $b = 7$ **b** $\ell$ when $P = 40$ and $b = 3$.

| Solution | Explanation |
|---|---|
| **a** $P = 2\ell + 2b$ | Write the formula. |
| $P = 2 \times 4 + 2 \times 7$ | Substitute in the values for $\ell$ and $b$. |
| $P = 22$ | Simplify the result. |
| **b** $P = 2\ell + 2b$ | Write the formula. |
| $40 = 2\ell + 2 \times 3$ | Substitute in the values for $P$ and $b$ obtain an equation. |
| $\begin{tikzcd} 40 = 2\ell + 6 \arrow[d, bend right, "-6"'] \arrow[d, bend left, "-6"] \\ 34 = 2\ell \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div"] \\ 17 = \ell \end{tikzcd}$<br>$\therefore \ell = 17$ | Solve the equation to obtain the value of $\ell$. |

**6** Consider the rule $A = 4p + 7$.
**a** Find $A$ if $p = 3$. **b** Find $A$ if $p = 11$.
**c** Find $A$ if $p = 0$. **d** Find $A$ if $p = 100$.

**7** The perimeter of a square is given by $P = 4x$, where $x$ is the side length.



**a** Find the value of $P$ if $x$ is:
**i** 10 **ii** 3 **iii** 7.5
**b** Solve the equation $44 = 4x$.
**c** If $P = 44$, what is the side length of the square?

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**8** Look at the rule $U = 8a + 4$.

- **a** Find the value of $a$ if $U = 20$. Set up and solve an equation.
- **b** Find $a$ if $U = 44$. Set up and solve an equation.
- **c** Find $a$ if $U = 92$. Set up and solve an equation.

**9** Look at the relationship $y = 2x + 4$.

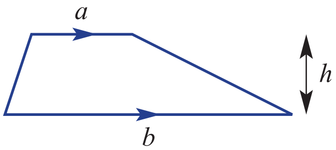
- **a** Find $y$ if $x = 3$.
- **b** By solving an appropriate equation, find the value of $x$ that makes $y = 16$.
- **c** Find the value of $x$ if $y = 0$.

Your answer for part **c** will be a negative number.

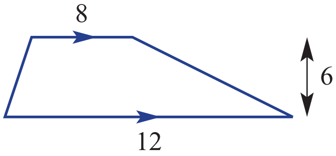
**10** Use the formula $P = mv$ to find the value of $m$ when $P = 22$ and $v = 4$.

**Problem-solving and Reasoning**

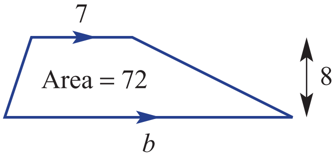
**11** The formula for the area of a trapezium is $A = \frac{1}{2}h(a + b)$.



- **a** Find the area of the trapezium shown below.

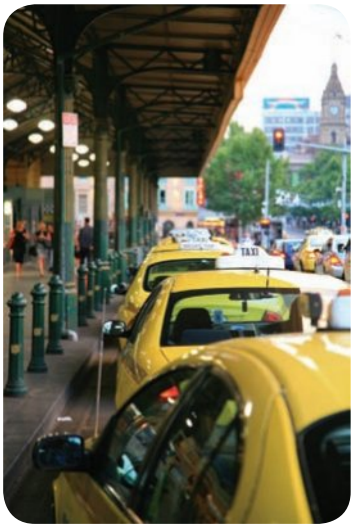


- **b** Find the value of $h$ if $A = 20$, $a = 3$ and $b = 7$.
- **c** Find the missing value in the trapezium shown below.



**12** The cost $\$C$ to hire a taxi for a trip of length $d$ km is $C = 3 + 2d$.

- **a** Find the cost of a 10 km trip (i.e. for $d = 10$).
- **b** A trip has a total cost of $\$161$.
  - **i** Set up an equation by substituting $C = 161$.
  - **ii** Solve the equation algebraically.
  - **iii** How far did the taxi travel? (Give your answer in km.)



**13** Look at the rule $G = 120 - 4p$.

- **a** If $p$ is between 7 and 11, what is the largest value of $G$?
- **b** Is it possible to make $G$ equal to zero? What would $p$ equal?

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2G

## Enrichment: Mobile phone plans

**14** Two companies have mobile phone plans where the cost of a call depends on how much time ($t$ minutes) you talk for.

Company $A$'s cost in dollars: $A = 0.3 + 0.5t$
Company $B$'s cost in dollars: $B = 0.6t$

Use trial and error to solve this equation.

**a** Find the cost of a 10-minute call with each company.
**b** If company $A$ charged $6.30 for a call, how long did it take?
**c** If company $B$ charged $6.30 for a call, how long did it take?
**d** How long would a call have to be if the cost for company $A$ and company $B$ is the same?

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# 2H Applications

**EXTENSION**

An equation can be used when two values are known to be equal.

The challenge in applying equations is recognising when two things equal each other.

Problems involving two people's ages can be expressed as an equation.

## Let's start: Sibling sum

John and his elder sister are 4 years apart in their ages.

- If the sum of their ages is 32, describe how you could work out how old they are.
- Could you write an equation to describe the situation above, if $x$ is used for John's age?
- How would the equation change if the *product* of their ages is 32?

## Key ideas

- An equation can be used to describe any situation in which two values are equal.
- To solve a problem follow these steps.

**1** Define pronumerals to stand for unknown numbers.

**1** Let $x$ = John's age.

**2** Write an equation to describe the problem.

**2** $x + x + 4 = 32$

**3** Solve the equation by inspection or systematically.

**3**

$$\begin{tikzcd} 2x + 4 = 32 \arrow[d, bend right, "-4"'] \arrow[d, bend left, "-4"] \\ 2x = 28 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ x = 14 \end{tikzcd}$$

**4** Make sure you answer the original question, including the correct units (e.g. dollars, years, cm).

**4** John is 14 years old and his sister is 18.

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## Exercise 2H

Understanding

**1** For each of the following, choose the best way to start solving the problem.

**a** Frank grew by 10 cm and is now 107 cm. How tall was Frank last year?

**A** Let $f$ = Frank **B** Let $f$ = Frank's height this year
**C** Let $f$ = Frank's age **D** Let $f$ = Frank's height last year

**b** Waleed worked for 20 hours and earned \$300. How much does he earn per hour?

**A** Let $w$ = Waleed's height **B** Let $w = 300$
**C** Let $w$ = Waleed's hourly wage **D** Let $w = 20$

**c** Louise spent \$400 on 12 identical calculators for her class. How much does a calculator cost?

**A** Let $c$ = cost of one calculator **B** Let $c$ = number of calculators
**C** Let $\ell$ = Louise **D** Let $\ell$ = Louise's income

Drilling for Gold 2H1

**2** Match each of the worded descriptions **a–e** with an appropriate expression **A–E**.

**a** The sum of $x$ and 3 is 20. **A** $12x = 20$
**b** The cost of 12 apples is \$20. **B** $x + 1 = 20$
**c** The number of \$1.50 oranges that can be bought for \$20. **C** $2x = 20$
**d** 20 is twice a number. **D** $x + 3 = 20$
**e** One more than $x$ is 20. **E** $1.5x = 20$

Drilling for Gold 2H2

**3** For the following problems choose the equation to describe them.

**a** The sum of $x$ and 5 is 11.

**A** $5x = 11$ **B** $x + 5 = 11$ **C** $x - 5 = 11$ **D** $11 - 5$

**b** The cost of 4 pens is \$12. Each pen costs \$$p$.

**A** $4 = p$ **B** $12p$ **C** $4p = 12$ **D** $12p = 4$

**c** Josh's age next year is 10. His current age is $j$.

**A** $j + 1 = 10$ **B** $j = 10$ **C** 9 **D** $j - 1 = 10$

**d** The cost of $n$ pencils is \$10. Each pencil costs \$2.

**A** $n \div 10 = 2$ **B** 5 **C** $10n = 2$ **D** $2n = 10$.

**4** Solve the following equations.

**a** $5p = 30$ **b** $5 + 2x = 23$ **c** $12k - 7 = 41$ **d** $10 = 3a + 1$

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Fluency

**5** The combined age of twin girls is 26. Let $a$ = the age of one girl.
  **a** Solve the equation $a + a = 26$.
  **b** How old is each girl?

## Example 16 Solving a problem using equations

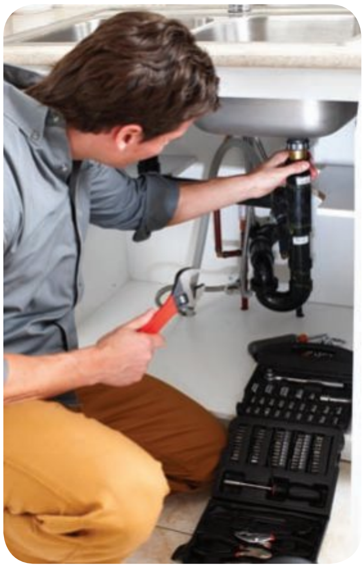
The weight of 6 identical books is 1.2 kg. What is the weight of one book?

| Solution | Explanation |
|---|---|
| Let $b$ = weight of one book.<br>$6b = 1.2$ | **1** Define a pronumeral to stand for the unknown number.<br>**2** Write an equation to describe the situation. |
| $\div 6 \left( \begin{array}{c} 6b = 1.2 \\ \downarrow \\ b = 0.2 \end{array} \right) \div 6$ | **3** Solve the equation. |
| The books weigh 0.2 kg each, or 200 g each. | **4** Answer the original question. It is not enough to give a final answer as 0.2; this is not the weight of a book, it is just a number. |

**6** Jerry buys 4 cups of coffee for $14.
  **a** Choose a pronumeral to stand for the cost of one cup of coffee.
  **b** Write an equation to describe the problem.
  **c** Solve the equation.
  **d** What is the cost of one cup of coffee?

Remember to include the $ sign in answer.

**7** A plumber charges a $70 call-out fee and $80 per hour.
The total cost of a particular visit was $310.
  **a** Define a pronumeral to stand for the length of the visit in hours.
  **b** Write an equation to describe the problem.
  **c** Solve the equation.
  **d** What is the length of the plumber's visit?



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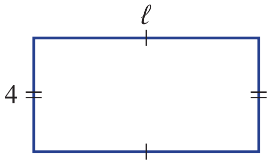
**8** When 6 chairs are bought, a 'bulk buy' discount reduces the final price by $200. The total becomes $1300.

**a** Define a pronumeral for the cost of one chair.
**b** Write an equation to describe the problem.
**c** Solve the equation.
**d** What is the cost of one chair?

**9** The perimeter of this rectangle is 72 cm.

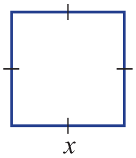
**a** Write an equation to describe the problem, using $\ell$ for the length.
**b** Solve the equation.
**c** What is the length of the rectangle?



**Problem-solving and Reasoning**

**10** A square has a perimeter of 24 cm.

**a** Solve an equation to find its side length.
**b** What is the area of the square?



Perimeter = 24 cm

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## Example 17 Solving problems with two related unknowns

Jane and Luke have a combined age of 60. Given that Jane is twice as old as Luke, find the ages of Luke and Jane.

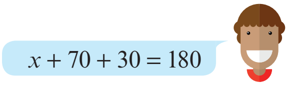
| **Solution** | **Explanation** |
|---|---|
| Let $\ell$ = Luke's age. | **1** Define a pronumeral for the unknown. Once Luke's age is found, we can double it to find Jane's age. |
| $\ell + 2\ell = 60$ | **2** Write an equation to describe the situation. Note that Jane's age is $2\ell$ because she is twice as old as Luke. |
| $$\begin{tikzcd} 3\ell = 60 \arrow[d, bend right, "\div 3"'] \arrow[d, bend left, "\div 3"] \\ \ell = 20 \end{tikzcd}$$ | **3** Solve the equation by first combining like terms. |
| Luke is 20 years old and Jane is 40 years old. | **4** Answer the original question. Include units. |

**11** Alison and Flynn's combined age is 40. Flynn is 4 years older than Alison.
  - **a** Write an equation and solve it to find Alison's age.
  - **b** How old is Flynn?

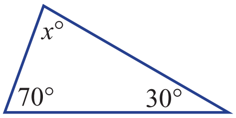
**12** The length of a rectangular pool is 5 metres longer than the breadth. The perimeter of the pool is 58 metres.
  - **a** Draw a diagram of this situation.
  - **b** Use an equation to find the pool's breadth.
  - **c** What is the area of the pool?

## Enrichment: Equational geometry

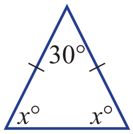
**13** The sum of angles in a triangle is 180° and the sum of angles in a quadrilateral is 360°. Find the value of $x$ in the shapes below by first solving an equation.


$x + 70 + 30 = 180$

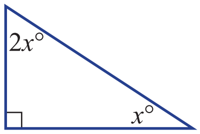
**a**



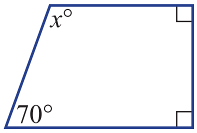
**b**



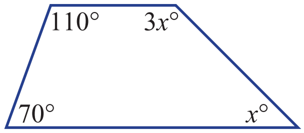
**c**



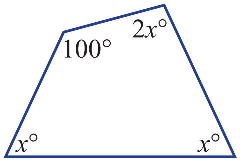
**d**



**e**



**f**



# Puzzles and games

**1** Find the value of $\square$, $\triangle$ and $\bigcirc$ using the following clues.

- $\square \times \triangle = 24$
- $\bigcirc + \bigcirc + \bigcirc + \bigcirc = 36$
- $\square - \triangle = \bigcirc + 1$
- $\square + \triangle + \triangle + \triangle = \bigcirc + \bigcirc$

**2** Find the unknown value in the following puzzles.

**a** A number is halved, then halved again, then halved again. The result is 11.

**b** A number is tripled, then it is added to itself. The result is 24.

**c** A number is increased by 2, then doubled, then increased by 3 and then tripled. The result is 99.

**d** The price of a shirt is increased by 10% for GST and then decreased by 10% on a sale. The new price is \$44. What was the original price?

**e** The average of a number and double that number is 50.

**3** Consider the following 'proof' that $0 = 1$.

$$\begin{tikzcd} 2x + 5 = 3x + 5 \arrow[d, bend right, "-5"'] \arrow[d, bend left, "-5"] \\ 2x = 3x \arrow[d, bend right, "\div x"'] \arrow[d, bend left, "\div x"] \\ 2 = 3 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\ 0 = 1 \end{tikzcd}$$

**a** Which step caused the problem in this proof? (Hint: Consider the actual solution to the equation.)

**b** Prove that $0 = 1$ is equivalent to the equation $22 = 50$ by adding, subtracting, multiplying and dividing both sides.

**4** Consider the expressions below.

$$4x + 2 \qquad 2(x + 4) \qquad 2x + 4 \qquad 4(x + 2) \qquad 4\left(x + \frac{1}{2}\right)$$

**a** If $x = 0$, which pairs are equal?

**b** Use two of the expressions above to form an equation that is always true.

**c** Use two of the expressions to form an equation that is never true.

**5** A certain pair of scales only registers weights between 100 kg and 150 kg, but it allows more than one person to get on at a time.

**a** If three people weigh themselves in pairs and the first pair weighs 117 kg, the second pair weighs 120 kg and the third pair weighs 127 kg, what are their individual weights?

**b** If another three people weigh themselves in pairs and get weights of 108 kg, 118 kg and 130 kg, what are their individual weights?

**c** A group of four children who all weigh less than 50 kg, weigh themselves in groups of three, getting the weights 122 kg, 128 kg, 125 kg and 135 kg. How much do they each weigh?

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# Chapter summary

**Equations with pronumerals on both sides**

$-2x$ $\quad 6x+2=2x+10 \quad$ $-2x$

$-2$ $\quad 4x+2=10 \quad$ $-2$

$\div 4$ $\quad 4x=8 \quad$ $\div 4$

$x=2$

**Solving by inspection**

- Choose value to make equation true e.g. $x+5=12$ solution: $x=7$

**Solving systematically**

- Same operation to both sides

$\div 3$ $\quad 3x=12 \quad$ $\div 3$

$x=4$

$-6$ $\quad 4k+6=42 \quad$ $-6$

$\div 4$ $\quad 4k=36 \quad$ $\div 4$

$k=9$

**Equations**

- A statement that two values are equal
- $2+2=4$

LHS Equal sign RHS

**Checking solutions**

Put solutions in to see if equation is true.
Is $x=3$ a solution to $4x+2=14$?
LHS $=4\times 3+2$ RHS $=14$
$=14$
$\therefore$ true

**Equations with fractions**

- Multiply by denominator

$\times 4$ $\quad \frac{x}{4}=10 \quad$ $\times 4$

$x=40$

$\times 7$ $\quad \frac{k+3}{7}=9 \quad$ $\times 7$

$-3$ $\quad k+3=63 \quad$ $-3$

$k=60$

**Formulas**

- Equations with 2 or more variables, one on the LHS by itself e.g. $F=ma$ $S=2x+3$
- Substitute known values to get unknown

**Equations with brackets**

Expand using distributive law

$2(x+4)=14$

$-8$ $\quad 2x+8=14 \quad$ $-8$

$\div 2$ $\quad 2x=6 \quad$ $\div 2$

$x=3$

Combine like terms after expanding.

$4(x+3)+2x=15$

$4x+12+2x=15$

$6x+12=15$

**Applications**

Whenever two things are equal

| | |
|---|---|
| 1. Define pronumeral | Let $c$ = car cost |
| 2. Write equation | $2c=60000$ |
| 3. Solve equation | $c=30000$ |
| 4. Answer question | A car costs \$30000. |

| Simple | quadratic | equations |
|---|---|---|
| $x^2=25$ | $x^2=0$ | $x^2=-9$ |
| $x^2=\pm 5$ | $x=0$ | no solution |

**Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.**

## Multiple-choice questions

**1** If $x = 3$, then the value of $2x + 5$ is:

**A** 28 **B** 11 **C** 7 **D** 25 **E** 1

**2** If $a = 10$, which one of the following equations is true?

**A** $a + 5 = 10$ **B** $10 - a = 20$ **C** $a + a = 20$
**D** $3 = a - 5$ **E** $10 = a + 10$

**3** Which one of the following equations does not have the solution $x = 9$?

**A** $4x = 36$ **B** $x + 7 = 16$ **C** $\frac{x}{3} = 3$
**D** $x + 9 = 0$ **E** $14 - x = 5$

**4** The solution to the equation $6 = 2x$ is:

**A** $x = 12$ **B** $x = 3$ **C** $x = 6$ **D** $x = 4$ **E** $x = 8$

**5** The solution to the equation $3a + 8 = 29$ is:

**A** $a = 21$ **B** $a = 12\frac{1}{3}$ **C** $a = 7$ **D** $a = 18$ **E** $a = 3$

**6** 'Three less than half a number is 4' can be expressed as an equation by:

**A** $\frac{x}{2} - 3 = 4$ **B** $\frac{(x - 3)}{2} = 4$ **C** $2x - 3 = 4$
**D** $\frac{x}{2} + 3 = 4$ **E** $\frac{x}{2} - 3 + 4$

**7** Which equations has two solutions?

**A** $2x = 9$ **B** $x^2 = -9$ **C** $x^2 = 0$ **D** $x^2 = 9$ **E** $x^2 + 9 = 0$

**8** The solution to the equation $3(m + 4) + m = 24$ is:

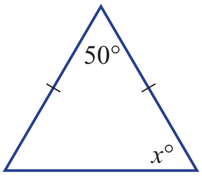
**A** $m = 7$ **B** $m = 8$ **C** $m = 4$ **D** $m = 1$ **E** $m = 3$

**9** Using the formula $F = 3k + b$, if $b = 7$ and $F = 34$, then $k$ equals:

**A** 27 **B** 3 **C** 9 **D** 14 **E** 13

**10** An equation that could be used to find $x$ in this isosceles triangle is:

**A** $50 + x = 180$ **B** $50 + 2x = 180$
**C** $2x = 180$ **D** $x = x$
**E** $50 = x$



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## Short-answer questions

**1** Are the following equations true or false?

- **a** If $x = 3$, then $3x = 6$.
- **b** If $a = 21$, then $a - 14 = \frac{a}{3}$.
- **c** $5 \times 4 = 10 + 10$

**2** State the solutions to these equations.

**a** $4m = 16$ **b** $m + 5 = 11$ **c** $20 = 4q$ **d** $z - 10 = 40$

**3** Write an equation to represent each of the following statements. You do not need to solve the equations.

- **a** Double $m$ plus 3 equals 27.
- **b** The sum of $n$ and four is tripled; the answer is 18.
- **c** The sum of two consecutive numbers, the first being $x$, is 7.

**4** Solve the following equations.

**a** $3x + 2 = 14$ **b** $4u + 5 = 21$ **c** $3d - 5 = 13$

**d** $2b - 1 = 13$ **e** $6f - 2 = 16$ **f** $12k + 3 = 27$

**5** Copy and complete the following equivalent equations.

**a**

$$\begin{tikzcd} 3x + 2 = 14 \arrow[d, bend right, "-2"'] \arrow[d, bend left, "-2"] \\ \_\_ = \_\_ \end{tikzcd}$$

**b**

$$\begin{tikzcd} 2b - 1 = 13 \arrow[d, bend right, "+1"'] \arrow[d, bend left, "+1"] \\ \_\_ = \_\_ \end{tikzcd}$$

**c**

$$\begin{tikzcd} 4x = 20 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ \_\_ = \_\_ \end{tikzcd}$$

**6** For each equation below, state the first operation you would apply to both sides.

**a** $15 + 2x = 45$ **b** $\frac{x}{2} - 5 = 6$ **c** $\frac{3a + 1}{2} = 11$

**7** Solve the following equations.

**a** $7a + 3 = 38$ **b** $4b - 10 = 14$ **c** $2n + 9 = 41$

**d** $12 = 4c + 4$ **e** $12 = 3 + x$ **f** $10 = 8x - 6$

**8** Solve the following equations.

**a** $\frac{m}{3} = 2$ **b** $\frac{5x}{2} = 20$ **c** $5 = \frac{k}{6}$

**d** $\frac{2y}{3} = 12$ **e** $\frac{k + 3}{11} = 5$ **f** $10 = \frac{x - 2}{3}$

**9** Solve the following equations.

**a** $2x + 4 = x + 6$ **b** $x + 4 = 2x + 6$ **c** $2x - 4 = x - 6$ **d** $x - 4 = 2x - 6$

**10** Solve the following equations.

**a** $2(x + 5) = 16$ **b** $3(x + 1) = 9$ **c** $5(p + 2) + p = 46$

**d** $18 = 2(2x - 1)$ **e** $3(2x + 1) + 4 = 67$ **f** $5(k - 2) + 2k = 74$

**11** Look at the formula $F = ma$, relating force, mass and acceleration.

- **a** Find $F$, if $m = 10$ and $a = 3$.
- **b** Find $m$, if $F = 20$ and $a = 5$.
- **c** If $F = 100$ and $a = 100$, what is the value of $m$?

**12** **a** If $P = 2(I + b)$, find $I$ when $P = 48$ and $b = 3$.

**b** If $M = \dfrac{f}{f - d}$, find $M$ when $f = 12$ and $d = 8$.

**c** If $F = \dfrac{5c}{2} + 20$, find $c$ when $F = 30$.

**13** Hugo buys 4 mangoes and a $20 gift voucher from the supermarket, giving a total cost of $26.

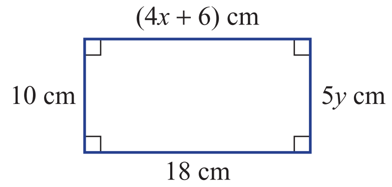
**a** Let $m$ = the cost of a mango. Which of the following equations describes this situation?

**A** $m = 20$
**B** $20m + 4 = 26$
**C** $4m = 20$
**D** $4m + 20 = 26$
**E** $4m + 26 = 20$

**b** Solve the equation chosen in part **a**.

**c** What is the cost of a mango?

**14** **a** Find the value of $x$ and $y$ for this rectangle.



**b** The sum of three consecutive numbers is 39. First write an equation and then find the value of the smallest number.

**c** The difference between a number and three times that number is 17. What is the number?

## Extended-response questions

**1** At a theme park, customers pay $10 entry fee and then $5 for each ride.

**a** Write an expression for the total cost to go on $n$ rides.

**b** Inga spent a total of $55 one afternoon at the theme park.

**i** Write an equation to describe how much she spent.

**ii** Solve the equation.

**iii** How many rides did Inga go on?

A parent and three children visit the park together. The parent does not go on any rides so a formula for the total cost is:

$$T = 3(\underbrace{5n + 10}_{\text{each child's entry}}) + 10$$

parent's entry

**c** If the children go on 4 rides together ($n = 4$), what is the total cost?

**d** If the total cost was $145, how many rides did the children go on?

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**2** To upload an advertisement to the www.searches.com.au website costs \$20 and then 12 cents whenever someone clicks on it.

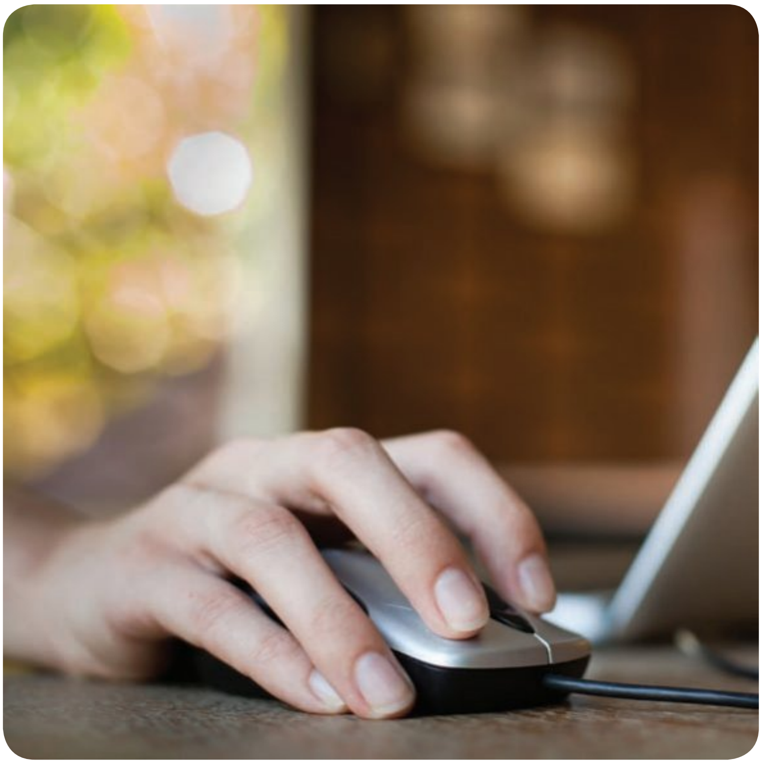
**a** Write a formula relating the total cost (\$$S$) and the number of clicks ($n$) on the advertisement.

**b** If the total cost is \$23.60, write and solve an equation to find out how many times the advertisement has been clicked on.

**c** To upload to the www.yousearch.com.au website costs \$15 initially and then 20 cents for every click. Write a formula for the total cost \$$Y$ when the advertisement has been clicked $n$ times.

**d** If a person has at most \$20 to spend, what is the maximum number of clicks they can afford on their advertisement at www.yousearch.com.au?

**e** Use trial and error to find the minimum number of clicks for which the total cost of posting an advertisement to searches.com.au is less than the cost of posting to www.yousearch.com.au.



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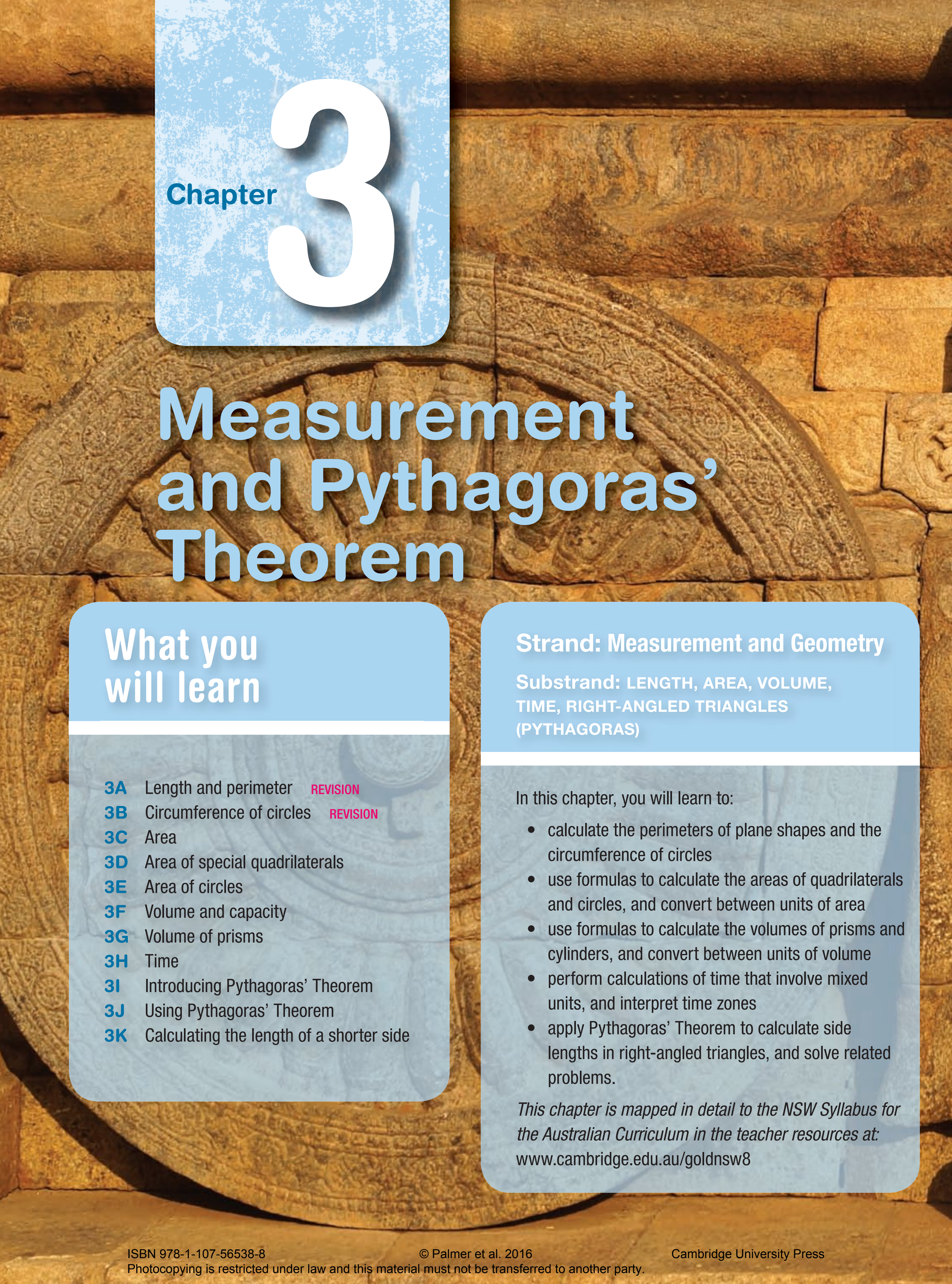


Chapter 3

# Measurement and Pythagoras' Theorem

## What you will learn

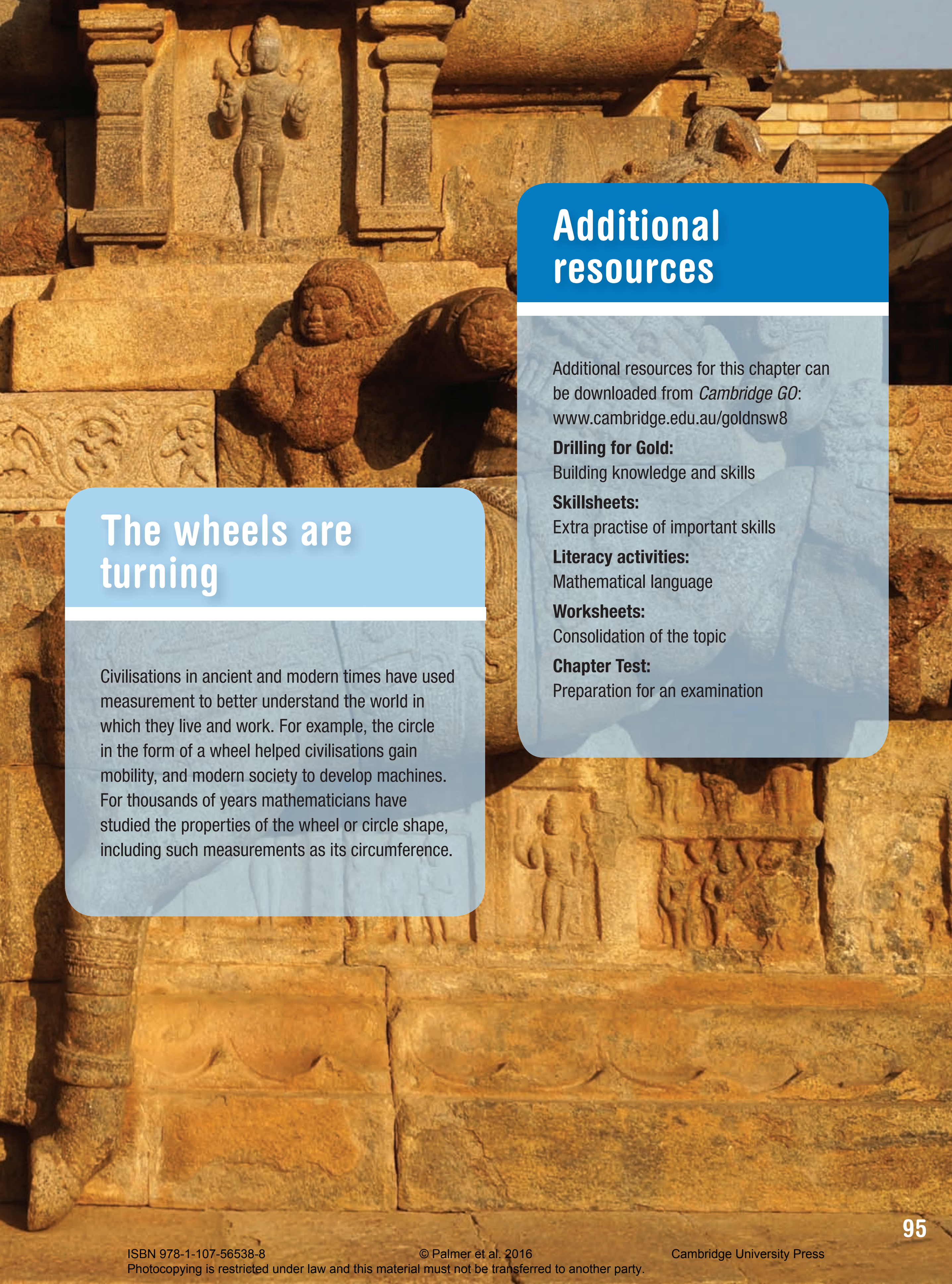
## Strand: Measurement and Geometry

**Substrand:** LENGTH, AREA, VOLUME, TIME, RIGHT-ANGLED TRIANGLES (PYTHAGORAS)

In this chapter, you will learn to:

- calculate the perimeters of plane shapes and the circumference of circles
- use formulas to calculate the areas of quadrilaterals and circles, and convert between units of area
- use formulas to calculate the volumes of prisms and cylinders, and convert between units of volume
- perform calculations of time that involve mixed units, and interpret time zones
- apply Pythagoras' Theorem to calculate side lengths in right-angled triangles, and solve related problems.

*This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at:* www.cambridge.edu.au/goldnsw8



## The wheels are turning

Civilisations in ancient and modern times have used measurement to better understand the world in which they live and work. For example, the circle in the form of a wheel helped civilisations gain mobility, and modern society to develop machines. For thousands of years mathematicians have studied the properties of the wheel or circle shape, including such measurements as its circumference.

## Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw8

**Drilling for Gold:**
Building knowledge and skills

**Skillsheets:**
Extra practise of important skills

**Literacy activities:**
Mathematical language

**Worksheets:**
Consolidation of the topic

**Chapter Test:**
Preparation for an examination

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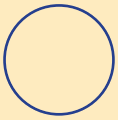
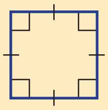
# Pre-test

**1** Name these shapes.

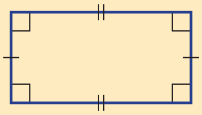
**a**



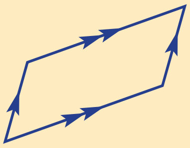
**b**



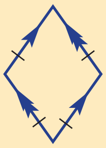
**c**



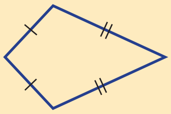
**d**



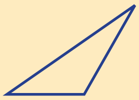
**e**



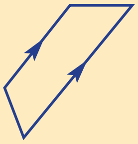
**f**



**g**

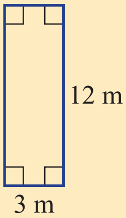


**h**

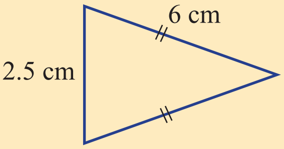


**2** Find the perimeter (distance around the outside) of these shapes.

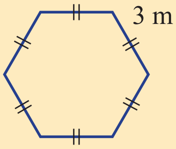
**a**



**b**



**c**



**3** Evaluate the following.

**a** $\frac{1}{2} \times 5 \times 4$

**b** $\frac{6}{2}(2 + 7)$

**c** $5^2$

**d** $11^2$

**4** Convert these measurements to the units shown in the brackets.

**a** 3 m (cm)

**b** 20 cm (mm)

**c** 1.8 km (m)

**d** 0.25 m (cm)

**e** 35 mm (cm)

**f** 4200 m (km)

**g** 500 cm (m)

**h** 100 mm (m)

**i** 2 minutes (seconds)

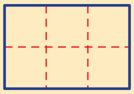
**j** 3 L (mL)

**k** 4000 mL (L)

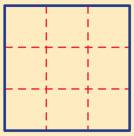
**l** 3000 g (kg)

**5** Count squares to find the area of these shapes.

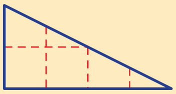
**a**



**b**



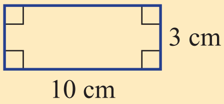
**c**



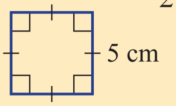
**6** Find the area of these rectangles and triangles.

Remember: Area (rectangle) = $\ell \times b$ and Area (triangle) = $\frac{1}{2}bh$

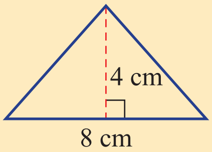
**a**



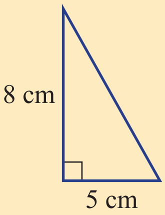
**b**



**c**



**d**



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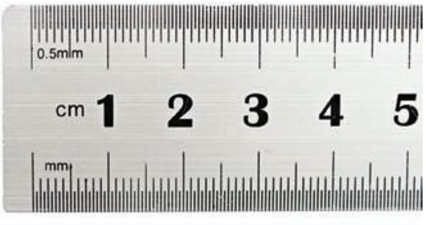
# 3A Length and perimeter

**REVISION**

Developed in France in the 1790s, the metric system for measurement includes length units such as millimetre, centimetre, metre and kilometre.

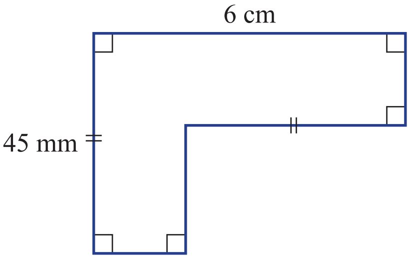
We use such units to describe, for example, the distance between two towns, the perimeter of a block of land, the depth of the ocean or the length of a racetrack.



## Let’s start: Provide the perimeter

In this diagram some of the lengths are given.
Three students were asked to find the perimeter.

- Will says that you cannot work out some lengths and so the perimeter cannot be found.
- Sally says that there is enough information and the answer is $9 + 12 = 21$ cm.
- Greta says that there is enough information but the answer is $90 + 12 = 102$ cm.



Who is correct?
Discuss how each person arrived at their answer.

## Key ideas

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- The common metric units of length include:
  - **kilometre** (km) 1 km = 1000 m
  - **metre** (m) 1 m = 100 cm
  - **centimetre** (cm) 1 cm = 10 mm
  - **millimetre** (mm)

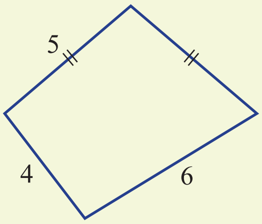
$$\begin{tikzcd}
\text{km} \arrow[r, bend left, "\times 1000"] & \text{m} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 100"] & \text{cm} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10"] & \text{mm} \arrow[l, bend left, "\div 10"]
\end{tikzcd}$$

**Perimeter** The total distance (length) around the outside of a figure

Drilling for Gold 3A2

- **Perimeter** is the distance around a closed shape.
  - All units must be of the same type when calculating the perimeter.
  - Sides with the same markings are of equal length.

e.g.

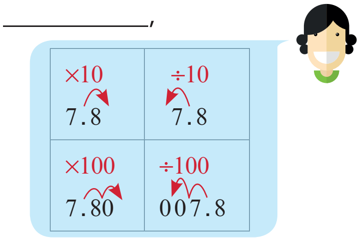


$$\begin{aligned} P &= 2 \times 5 + 4 + 6 \\ &= 20 \end{aligned}$$

## Exercise 3A

**Understanding**

**1** Write the missing words.
- **a** The commonly used measurement system used today is called the _______ system.
- **b** The common metric units for length include millimetres, ___________, ___________ and ___________.

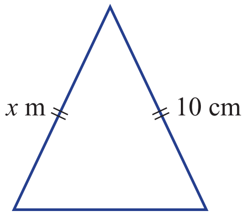


**2** Evaluate the following.
- **a** $2 \times 100$ **b** $5.2 \times 1000$ **c** $7.8 \times 10$
- **d** $840 \div 100$ **e** $9610 \div 10$ **f** $41\,200 \div 1000$

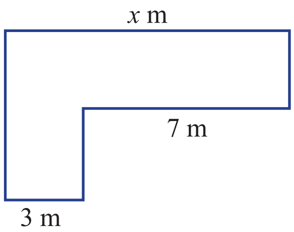
**3** Write the missing number in each of these sentences.
- **a** There are ___ mm in 1 cm.
- **b** There are ___ cm in 1 m.
- **c** There are ___ m in 1 km.
- **d** There are ___ cm in 1 km.
- **e** There are ___ mm in 1 m.
- **f** There are ___ mm in 1 km.

**4** Find the value of $x$ in these diagrams.

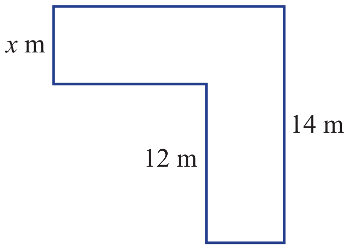
**a**



**b**



**c**



**Fluency**

### Example 1 Converting length measurements

Convert these lengths to the units shown in the brackets.
- **a** 5.2 cm (mm) **b** 2400 m (km)

| **Solution** | **Explanation** |
|---|---|
| **a** 5.2 cm = $5.2 \times 10$<br>= 52 mm | 1 cm = 10 mm so multiply by 10.<br>$\text{cm} \xrightarrow{\times 10} \text{mm}$ (From diagram in Key ideas) |
| **b** 2400 m = $2400 \div 1000$<br>= 2.4 km | 1 km = 1000 m so divide 1000. |

Use the diagram in the Key ideas.

**5** Convert these measurements to the units shown in the brackets.
- **a** 3 cm (mm) **b** 6.1 m (cm) **c** 8.93 km (m) **d** 3 m (cm)
- **e** 0.0021 km (m) **f** 320 mm (cm) **g** 9620 m (km) **h** 38 000 cm (m)
- **i** 48 mm (cm) **j** 0.2 cm (mm) **k** 4.2 cm (m) **l** 0.4 m (cm)
- **m** 3700 m (km) **n** 600 m (km) **o** 0.71 km (m) **p** 0.02 m (cm)

Drilling for Gold 3A3

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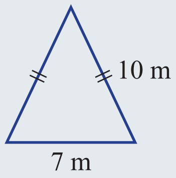
## Example 2 Finding perimeters

Find the perimeter of this shape.



**Solution**

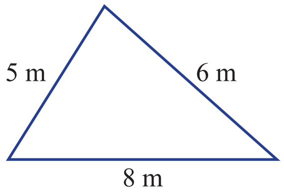
$P = 2 \times 10 + 7$
$= 27$ m

**Explanation**

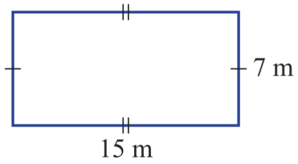
There are two equal 10 m lengths and one 7 m length to add up.

**6** Find the perimeters of these shapes.

**a**

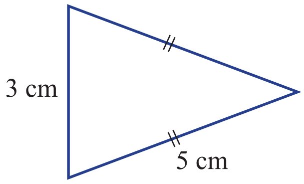


**b**

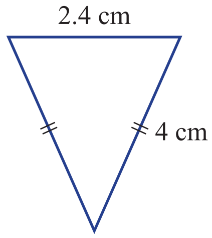


Sides with the same markings have the same length.

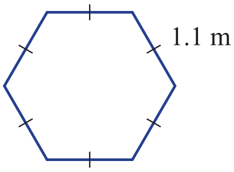
**c**



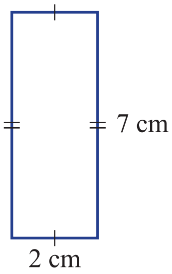
**d**



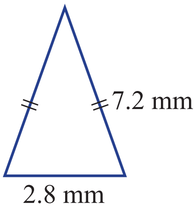
**e**



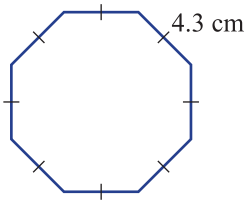
**f**



**g**



**h**



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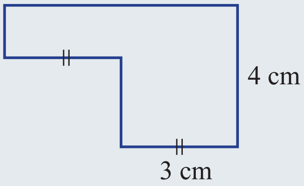
## Example 3 Finding perimeters of rectangular shapes

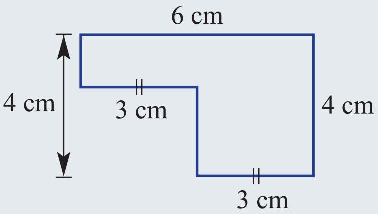
Find the perimeter of this shape.



**Solution**

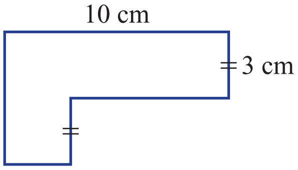
$P = 2 \times (3 + 3) + 2 \times 4$
$\quad = 12 + 8$
$\quad = 20$ cm

**Explanation**

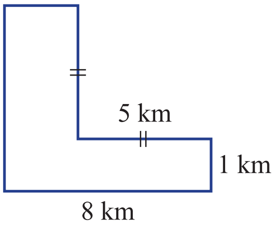


**7** Find the perimeters of these shapes.

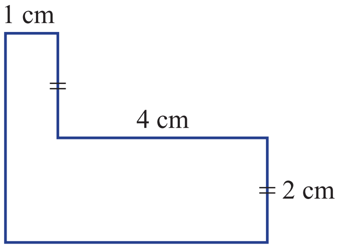
**a**



**b**



**c**



**Problem-solving and Reasoning**

**8** Convert these measurement to the units shown in the brackets.

**a** 0.0043 m (mm) **b** 0.0204 km (cm) **c** 23 098 mm (m)
**d** 342 000 cm (km) **e** 194 300 mm (m) **f** 10 000 mm (km)

Use the diagram in the Key ideas.

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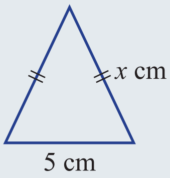
## Example 4 Finding an unknown length

Find the unknown value $x$ in this triangle if the perimeter is 19 cm.

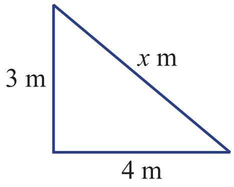


$P = 19$ cm

| **Solution** | **Explanation** |
|---|---|
| $2x + 5 = 19$ | $2x + 5$ makes up the perimeter, which is 19. |
| $2x = 14$ | Solve the equation to find the value of $x$. |
| $x = 7$ | |

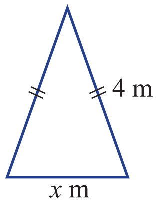
**9** Find the value of $x$ in these shapes with the given perimeters ($P$).

**a**



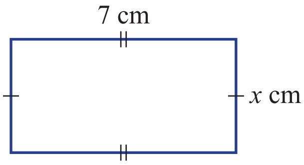
$P = 12$ m

**b**



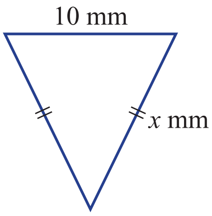
$P = 10$ m

**c**



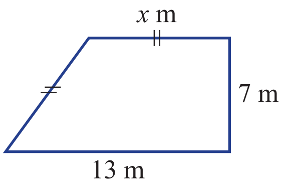
$P = 22$ cm

**d**



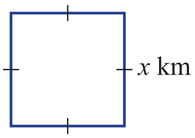
$P = 46$ mm

**e**



$P = 39$ m

**f**



$P = 26$ km

Use the given perimeter to find the value of $x$.

**10** Jennifer needs to fence her block of land to keep her dog in. The block is a rectangle with length 50 m and breadth 42 m. Fencing costs \$13 per metre. What will be the total cost of fencing?

**11** Gillian can jog 100 metres in 24 seconds. How long will it take her to jog 2 km? Give your answer in minutes.

**12** A rectangular picture of length 65 cm and breadth 35 cm is surrounded by a frame of width 5 cm. What is the perimeter of the framed picture?

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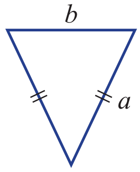
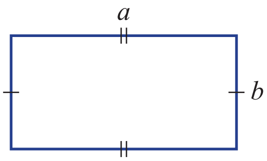
**13** Write down rules using the given letters for the perimeters of these shapes, e.g. $P = a + 2b$.

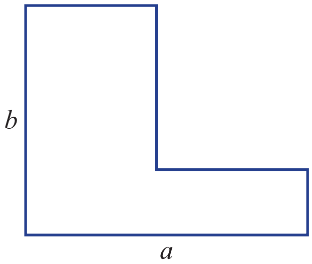
**a**



**b**



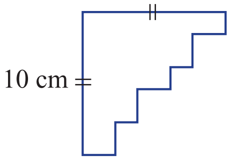
**c**



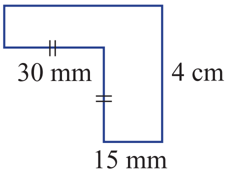
## Enrichment: Perimeter challenge

**14** Find the perimeters of these shapes. Give your answers in cm.

**a**

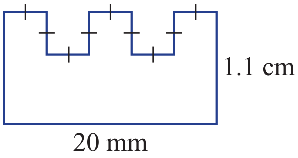


**b**

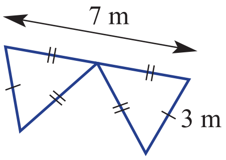


Check to make sure the units are all the same.

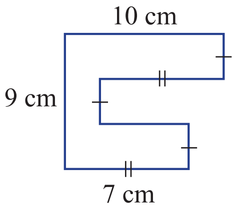
**c**



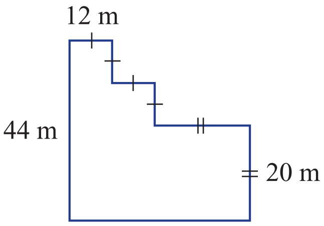
**d**



**e**



**f**



# 3B Circumference of circles

**REVISION**

The distance around the outside of a circle, known as the circumference, is connected to the diameter through a special number called *pi*.

The symbol for pi is π, and as a decimal π = 3.141 59 … There is no exact fraction for pi, which is why we often use calculators when working with this number.

An easy way to find circumference is to multiply the diameter by π.

## Let's start: Discovering pi

Drilling for Gold 3B1

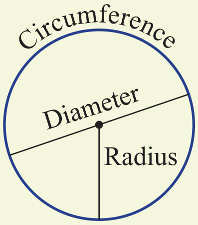
Drilling for Gold 3B2

Steps:

1 Find a circular object like a dinner plate or a wheel.
2 Use a tape measure or string to measure the circumference (in mm).
3 Measure the diameter (in mm).
4 Use a calculator to divide the circumference by the diameter. (It should be about 3.14.)
5 Repeat Steps 1 to 4 with a larger or smaller object.

## Key ideas

- Features of a circle
  - **Diameter** ($d$) is the distance across the centre of a circle.
  - **Radius** ($r$) is the distance from the centre to the circle. Note $d = 2r$.
- **Circumference** ($C$) is the distance around a circle.



**Formula:** $C = 2\pi r$ **or** $C = \pi d$

- **Pi** ($\pi$) $\approx 3.14159$ (correct to 5 decimal places)
  - Pi is an irrational number, because it cannot be converted to a fraction.
  - Common approximations include 3.14 and $\frac{22}{7}$.
  - A more precise estimate for pi can be found on most calculators or on the internet. Find the π button on your calculator.

**Radius** The distance from the centre of a circle to its outside edge

**Circumference** The curved boundary of a circle

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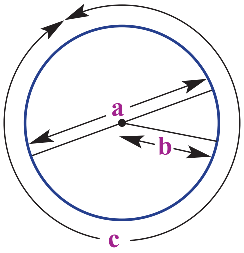
## Exercise 3B

**Understanding**

**1** Name the features of the circle as shown.



**2** **a** Find the diameter of a circle if its radius is:
  **i** 5 m **ii** 11 cm **iii** 2.3 mm

**b** Find the radius of a circle if its diameter is:
  **i** 12 cm **ii** 31 mm **iii** 0.42 m

**3** Write down the value of π correct to:
**a** 1 decimal place **b** 2 decimal places
**c** 3 decimal places.

**4** Evaluate the following using a calculator and round to 2 decimal places.
**a** $\pi \times 5$ **b** $\pi \times 13$ **c** $2 \times \pi \times 3$ **d** $2 \times \pi \times 37$

**Fluency**

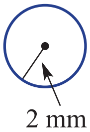
### Example 5 Finding the circumference using the radius

Find the circumference of this circle, correct to 2 decimal places. Use a calculator for the value of pi.

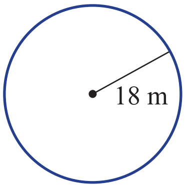
| Solution | Explanation |
|---|---|
| $C = 2\pi r$ | Since $r$ is given, you can use $C = 2\pi r$. |
| $= 2 \times \pi \times 3.5$ | |
| $= 7\pi$ | $7\pi$ is an exact value for the circumference. |
| $= 21.99$ m (to 2 decimal places) | |

**5** Find the circumference of each circle correct to 2 decimal places. Use a calculator for the value of pi.

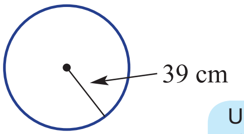
**a**



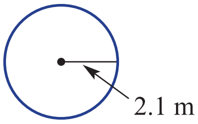
**b**



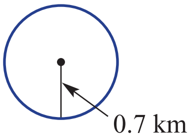
**c**



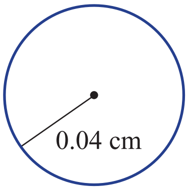
**d**



**e**



**f**



Use the rule $C = 2\pi r$ and substitute the value of $r$.

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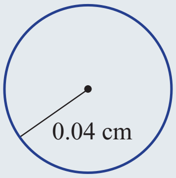
## Example 6 Finding the circumference using the diameter

Find the circumference of this circle, correct to 2 decimal places.



**Solution**

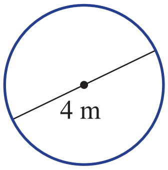
$$\begin{aligned} C &= \pi d \\ &= \pi \times 4 \\ &= 4\pi \\ &= 12.57 \text{ cm (to 2 decimal places)} \end{aligned}$$

**Explanation**

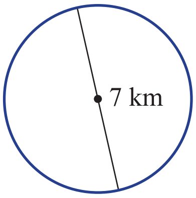
Substitute $d = 4$ into the rule $C = \pi d$ or use $C = 2\pi r$ with $r = 2$.

**6** Find the circumferences of these circles, correct to 2 decimal places.

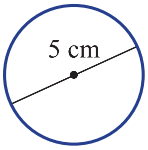
**a**



**b**

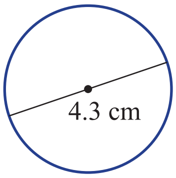


**c**

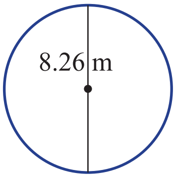


Use the rule $C = \pi d$ and substitute the value of $d$.

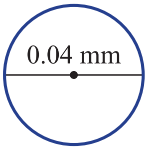
**d**



**e**



**f**



**Problem-solving and Reasoning**

**7** The diameter of a metal drum is 80 cm. Find its circumference, correct to the nearest centimetre.

**8** A water tank has a diameter of 3.5 m. Find its circumference, correct to 1 decimal place.

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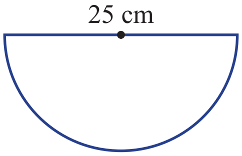
**9** A wheel of radius 28 cm rolls one full turn. Find how far it rolls, correct to the nearest centimetre.

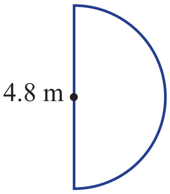
**10** An athlete trains on a circular track of radius 40 m and jogs 10 laps each day, 5 days a week. How far does he jog each week? Round the answer to the nearest whole number of metres.

**11** These shapes are semicircles. Find the perimeter of these shapes including the straight edge and round the answer to 2 decimal places.

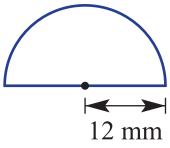
**a**



**b**



**c**



The perimeter is half of the circumference of a full circle plus the diameter.

**12** Here are some student's approximate circle measurements. Which students have incorrect measurements?

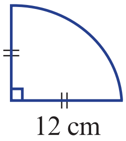
| | $r$ | $C$ |
|---|---|---|
| **Mick** | 4 cm | 25.1 cm |
| **Svenya** | 3.5 m | 44 m |
| **Andre** | 1.1 m | 13.8 m |

**13** Explain why the rule $C = 2\pi r$ is equivalent (i.e. the same as) $C = \pi d$.

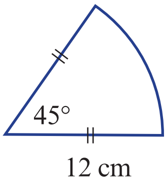
## Enrichment: Curved perimeters

**14** Find the perimeters of these shapes. Leave your answers in terms of π, e.g. $(3\pi + 25)$ cm

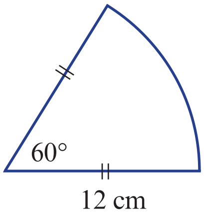
**a**



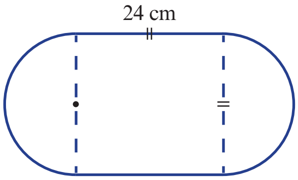
**b**



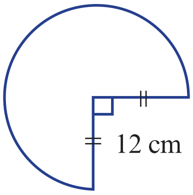
**c**



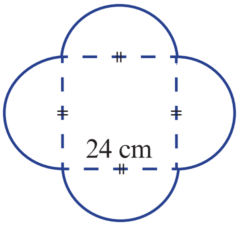
**d**



**e**



**f**



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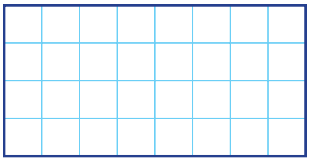
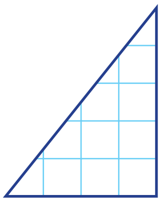
# 3C Area

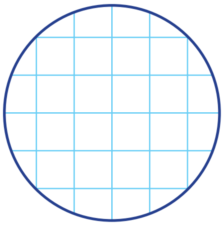
For international competition, a basketball court is 28 metres by 15 metres, so the area is 420 square metres. Area is useful when calculating the cost of building and landscaping.

## Let's start: Estimating area

By counting squares, or by using an estimate, you can find the area of a shape.
For the following shapes, find or estimate their area. Explain your method for each one.



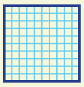




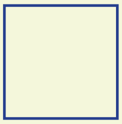
## Key ideas

- The common metric units for area include:
  - square millimetres (mm²)
  - square centimetres (cm²)
  - square metres (m²)
  - square kilometres (km²)
  - hectares (ha).

$1\text{ cm}^2$
$= 10 \times 10$
$= 100\text{ mm}^2$



$1\text{ m}^2$
$= 100 \times 100$
$= 10\ 000\text{ cm}^2$

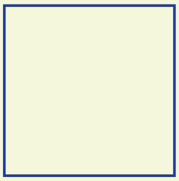


- Flowcharts for converting units

$$\begin{tikzcd}
\text{km}^2 \arrow[r, bend left, "\times 1000^2"] & \text{m}^2 \arrow[l, bend left, "\div 1000^2"] \arrow[r, bend left, "\times 100^2"] & \text{cm}^2 \arrow[l, bend left, "\div 100^2"] \arrow[r, bend left, "\times 10^2"] & \text{mm}^2 \arrow[l, bend left, "\div 10^2"]
\end{tikzcd}$$

$$\begin{tikzcd}
\text{ha} \arrow[r, bend left, "\times 10\,000"] & \text{m}^2 \arrow[l, bend left, "\div 10\,000"]
\end{tikzcd}$$

$1\text{ km}^2$
$= 1000 \times 1000$
$= 1\ 000\ 000\text{ m}^2$



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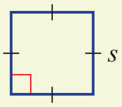
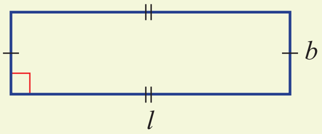
- Formulas for area of squares, rectangles and triangles
  - Square

    $A = s^2$



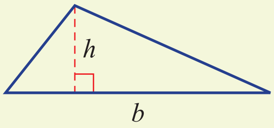
  - Rectangle

    $A = \ell b$



  - Triangle

    $A = \frac{1}{2}bh$



**Perpendicular** At right angles to another line or surface

The dashed line that gives the height is **perpendicular** (at right angles) to the base.

## Exercise 3C

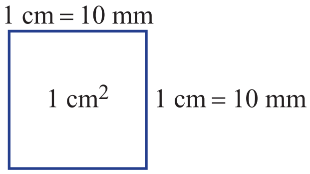
**Understanding**

**1** Which would be the most appropriate unit for measuring the area of this page?

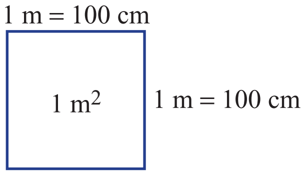
**A** mm$^2$ **B** cm$^2$ **C** m$^2$ **D** ha **E** km$^2$

**2** By considering the given diagrams answer the questions.

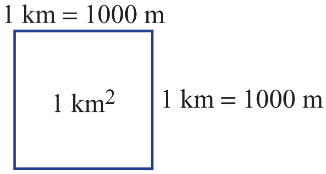
**a** **i** How many mm$^2$ in 1 cm$^2$?
  **ii** How many mm$^2$ in 4 cm$^2$?



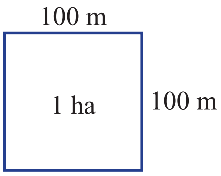
**b** **i** How many cm$^2$ in 1 m$^2$?
  **ii** How many cm$^2$ in 7 m$^2$?



**c** **i** How many m$^2$ in 1 km$^2$?
  **ii** How many m$^2$ in 5 km$^2$?



**d** **i** How many m$^2$ in 1 ha?
  **ii** How many m$^2$ in 3 ha?



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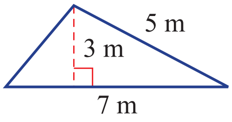
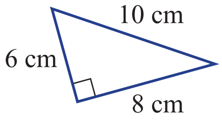
**3** Which measurements would be used for the *base* and the *height* of these triangles?

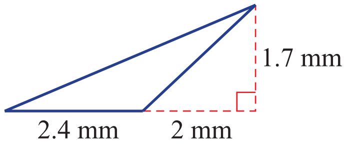
**a**



**b**



**c**



Recall that the base and height are perpendicular (at 90°).

**Fluency**

## Example 7 Converting units of area

Convert these area measurements to the units shown in the brackets.

**a** 0.248 m² (cm²) **b** 3100 mm² (cm²)

| **Solution** | **Explanation** |
|---|---|
| **a** $0.248\ \text{m}^2 = 0.248 \times 10\,000$<br>$= 2480\ \text{cm}^2$ | From the diagram in Key ideas: $\begin{tikzcd} \text{m}^2 \arrow[r, bend left, "\times 100^2"] & \text{cm}^2 \end{tikzcd}$ |
| **b** $3100\ \text{mm}^2 = 3100 \div 100$<br>$= 31\ \text{cm}^2$ | From the diagram in Key ideas: $\begin{tikzcd} \text{cm}^2 & \text{mm}^2 \arrow[l, bend left, "\div 10^2"] \end{tikzcd}$ |

**Drilling for Gold 3C1**

**4** Convert these area measurements to the units shown in the brackets.

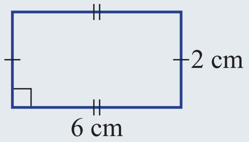
**a** 2 cm² (mm²) **b** 7 m² (cm²)
**c** 0.5 km² (m²) **d** 3 ha (m²)
**e** 0.34 cm² (mm²) **f** 700 cm² (m²)
**g** 3090 mm² (cm²) **h** 0.004 km² (m²)
**i** 2000 cm² (m²) **j** 450 000 m² (km²)
**k** 4000 m² (ha) **l** 3210 mm² (cm²)
**m** 320 000 m² (ha) **n** 0.0051 m² (cm²)
**o** 0.043 cm² (mm²) **p** 4802 cm² (m²)
**q** 19 040 m² (ha) **r** 2933 m² (ha)
**s** 0.0049 ha (m²) **t** 7.7 ha (m²)

Use the diagram in the Key ideas.

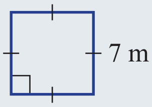
## Example 8 Finding areas of rectangles and squares

Find the areas of these shapes.

**a**



**b**



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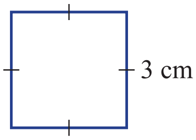
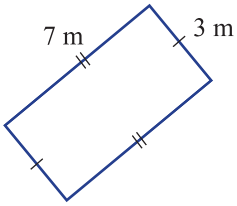
| **Solution** | **Explanation** |
|---|---|
| **a** $A = \ell b$<br>$= 6 \times 2$<br>$= 12\text{ cm}^2$ | Write the formula for the area of a rectangle and substitute $\ell = 6$ and $b = 2$. |
| **b** $A = s^2$<br>$= 7^2$<br>$= 49\text{ m}^2$ | For a square, multiply the length of a side by itself to get the area. |

**5** Find the areas of these squares and rectangles.

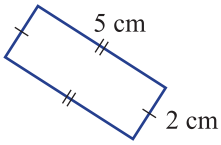
**a**



**b**

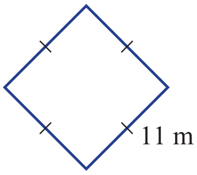


**c**

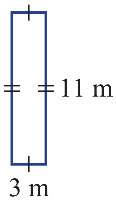


Use $A = \ell \times b$ or $A = s^2$.

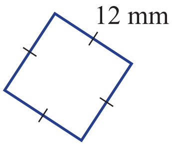
**d**



**e**



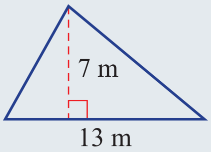
**f**



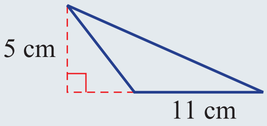
## Example 9 Finding the area of triangles

Find the areas of these triangles.

**a**



**b**



| **Solution** | **Explanation** |
|---|---|
| **a** $A = \frac{1}{2}bh$<br>$= \frac{1}{2} \times 13 \times 7$<br>$= 45.5\text{ m}^2$ | Remember that the height is measured using a line that is perpendicular to the base. |
| **b** $A = \frac{1}{2}bh$<br>$= \frac{1}{2} \times 11 \times 5$<br>$= 27.5\text{ cm}^2$ | The base is 11 cm and the height is 5 cm so use $b = 11$ and $h = 5$. |

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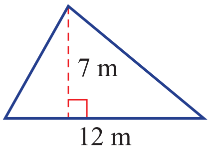
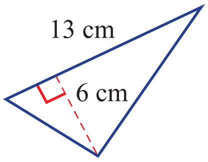
**6** Find the areas of these triangles.

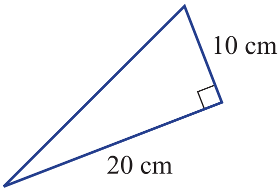
**a**



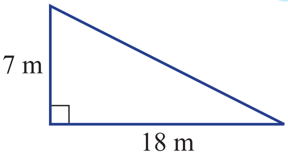
**b**



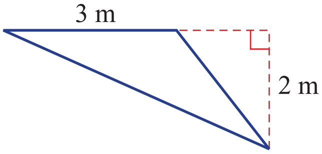
**c**



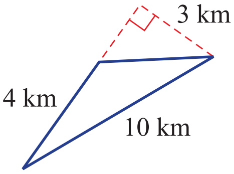
**d**



**e**



**f**



Use $A = \frac{1}{2}bh$ and choose the base and height so they are perpendicular (at 90°).

**Problem-solving and Reasoning**

**7** A rectangular park has length 100 m and area 5000 m$^2$. What is its breadth?

**8** A triangle has area 20 cm$^2$ and base 4 cm. Find its height.

**9** Find the side length of a square if its area is:

**a** 36 m$^2$ **b** 2.25 cm$^2$

**10 a** Find the area of a square if its perimeter is 20 m.

**b** Find the area of a square if its perimeter is 18 cm.

**c** Find the perimeter of a square if its area is 49 cm$^2$.

**d** Find the perimeter of a square if its area is 169 m$^2$.

First find the side length of the square.

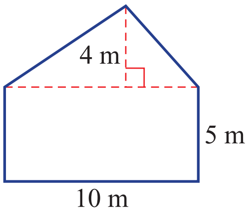
**11** Paint costs $12 per litre and can only be purchased in a full number of litres. One litre of paint covers an area of 10 m$^2$. A rectangular wall is 6.5 m long and 3 m high and needs two coats of paint. What will be the cost of paint for the wall?

12 The cost of building a house is $1305 per square metre of floor space.
A basketball court is 28 m by 15 m.
How much will it cost to build a house where the floor space is the same size as a basketball court?

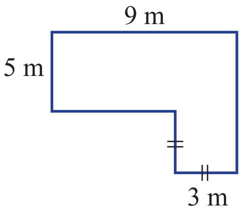
## Enrichment: Composite shapes

13 Find the areas of these composite shapes by using addition or subtraction.

**a**

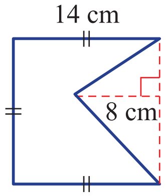


**b**

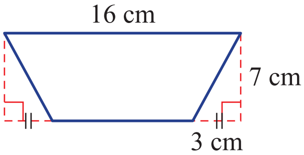


Divide into two or more shapes then add or subtract.

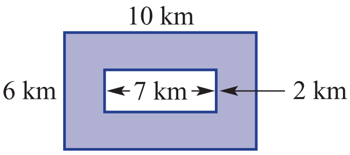
**c**



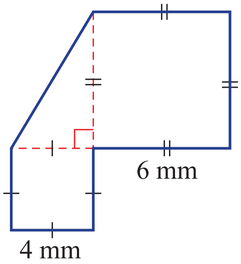
**d**



**e**



**f**



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# 3D Area of special quadrilaterals

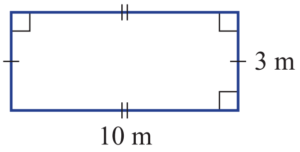
In this section, we will develop and use formulas for the area of a:

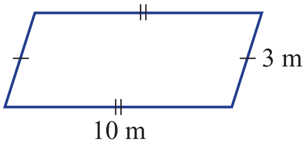
- parallelogram
- rhombus
- kite
- trapezium.

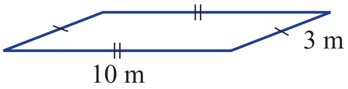
The area of each quadrilateral needs to be calculated to estimate the required number of pavers.

## Let's start: How is a parallelogram like a rectangle?

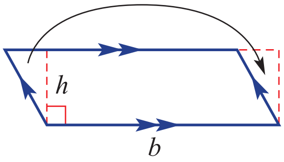
Do rectangles and parallelograms with the same side lengths have the same area? Use these diagrams to help in your discussion.







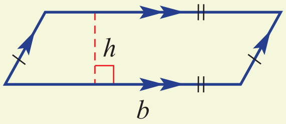
- How does this diagram help you work out the area of a parallelogram?
- Can you write the rule for the area of a parallelogram?



## Key ideas

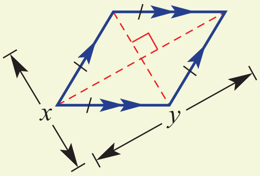
- Area of a **parallelogram**

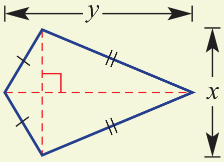
  $A = bh$



- Area of a **rhombus** and **kite**

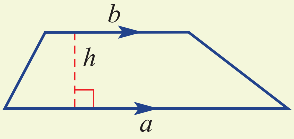
  $A = \frac{1}{2}xy$





- Area of a **trapezium**

  $A = \frac{1}{2}h(a+b)$



**Parallelogram** A quadrilateral with both pairs of opposite sides parallel

**Rhombus** A quadrilateral with both pairs of opposite sides parallel and all sides equal

**Kite** A quadrilateral with two pairs of adjacent sides equal

**Trapezium** A quadrilateral with at least one pair of parallel sides

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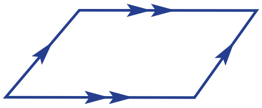
## Exercise 3D

**Understanding**

**1** Match each formula with a shape.

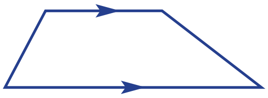
**a** $A = \ell b$ **b** $A = \frac{1}{2}xy$ **c** $A = bh$ **d** $A = \frac{1}{2}h(a + b)$

**A**

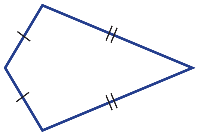


**B**

**C**



**D**



**2** Find the value of $A$ using these formulas and given values. Substitute the given values into the formulas.

**a** $A = bh \quad (b = 2, h = 3)$

**b** $A = \frac{1}{2}xy \quad (x = 5, y = 12)$

**c** $A = \frac{1}{2}(a + b)h \quad (a = 2, b = 7, h = 3)$

**d** $A = \frac{1}{2}h(a + b) \quad (a = 7, b = 4, h = 6)$

**3** Complete these sentences.

**a** The angle between two perpendicular lines is ___________ degrees.

**b** In a parallelogram, you find the area using a base and the __________________.

**c** The two diagonals in a kite or a rhombus are __________________.

**d** To find the area of a trapezium, multiply $\frac{1}{2}$ by the __________________ height, then multiply by the sum of the two __________________ sides.

**e** The two special quadrilaterals that have the same area formula using diagonal lengths $x$ and $y$ are the __________________ and the __________________.

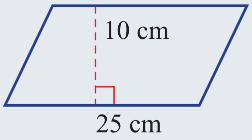
Choose from: height, 90, parallel, kite, rhombus, perpendicular.

**Fluency**

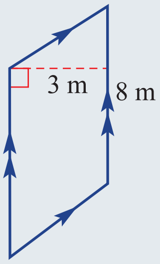
### Example 10 Finding areas of parallelograms

Find the areas of these parallelograms.

**a**



**b**



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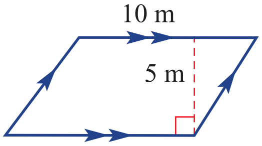
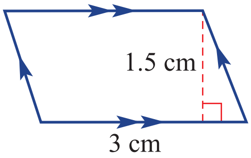
| **Solution** | **Explanation** |
|---|---|
| **a** $A = bh$<br>$= 25 \times 10$<br>$= 250\text{ cm}^2$ | Use $A = bh$ with $b = 25$ and $h = 10$ |
| **b** $A = bh$<br>$= 8 \times 3$<br>$= 24\text{ m}^2$ | The height is measured at right angles to the base. |

**4** Find the areas of these parallelograms.

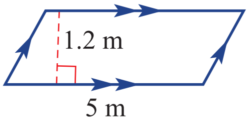
**a**



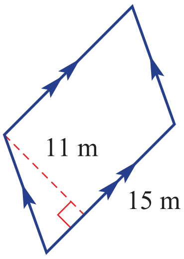
**b**



**c**

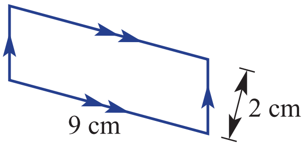


**d**

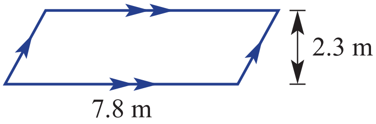


Use $A = bh$ and choose your base and perpendicular height.

**e**



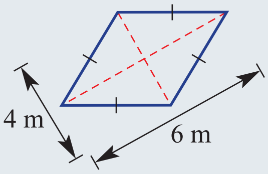
**f**



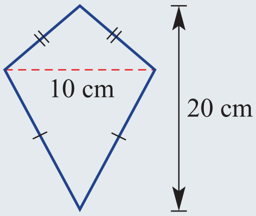
## Example 11 Finding areas of rhombuses and kites

Find the areas of this rhombus and kite.

**a**



**b**



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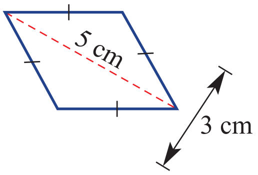
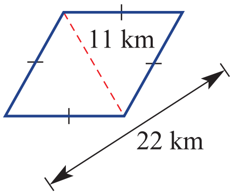
| Solution | Explanation |
|---|---|
| **a** $A = \frac{1}{2}xy$<br>$= \frac{1}{2} \times 6 \times 4$<br>$= 12\text{ m}^2$ | Use $A = \frac{1}{2}xy$ when the diagonals are given with $x = 6$ and $y = 4$ (or vice versa) |
| **b** $A = \frac{1}{2}xy$<br>$= \frac{1}{2} \times 10 \times 20$<br>$= 100\text{ cm}^2$ | Use the formula $A = \frac{1}{2}xy$ since both diagonals are given. This formula can also be used for a rhombus. |

**5** Find the areas of these rhombuses and kites.

**a**

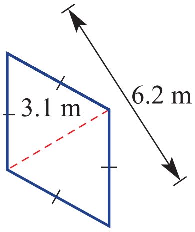


**b**

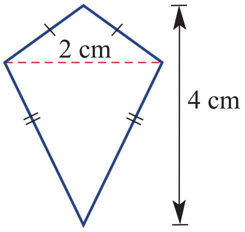


Recall that $A = \frac{1}{2}xy$ for both rhombuses and kites with $x$ and $y$ as the diagonals.

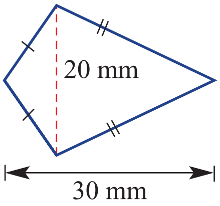
**c**



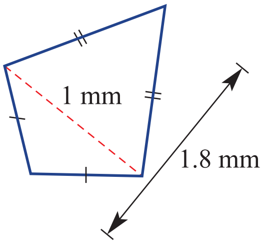
**d**



**e**

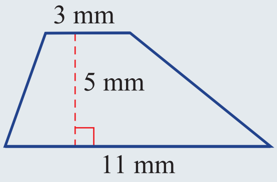


**f**



## Example 12 Finding areas of trapezia

Find the area of this trapezium.



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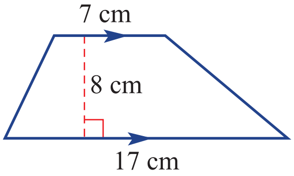
| **Solution** | **Explanation** |
|---|---|
| $A = \frac{1}{2}h(a + b)$<br>$= \frac{1}{2} \times 5 \times (11 + 3)$<br>$= \frac{1}{2} \times 5 \times 14$<br>$= 35\text{ mm}^2$ | The two parallel sides are 11 mm and 3 mm in length.<br>The perpendicular height is 5 mm. |

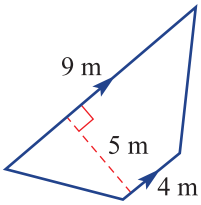
**6** Find the areas of these trapezia.

In $A = \frac{1}{2}h(a + b)$, where $a$ and $b$ are the lengths of the parallel sides.

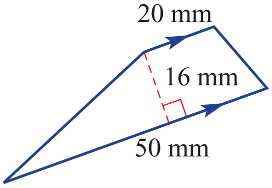
**a**



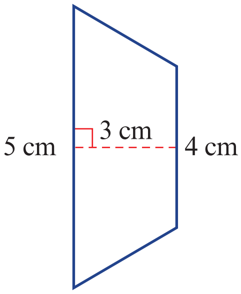
**b**



**c**



**d**

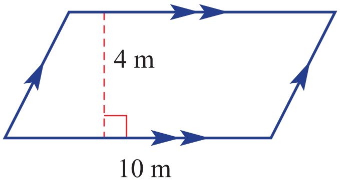


Skillsheet 3A

Drilling for Gold 3D1

**Problem-solving and Reasoning**

**7** A special type of paint costs \$3 per square metre and is to be used to paint a wall in the shape of a parallelogram with base 10 m and height 4 m. How much does it cost to paint the wall?



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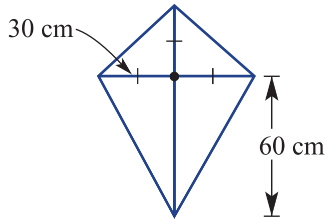
**8** A kite is made from 4 centre rods all connected near the middle of the kite as shown. What area of plastic, in square metres, is needed to cover the kite?

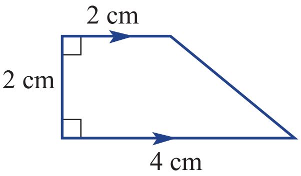


**9** A parallelogram has an area of 26 m$^2$ and its base length is 13 m. What is its perpendicular height?

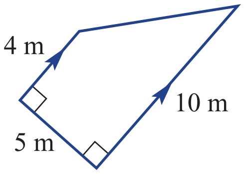
**10** A landscape gardener charges $20 per square metre of lawn. A lawn area is in the shape of a rhombus and its diagonals are 8 m and 14.5 m. What would be the cost of laying this lawn?

**11** These trapezia have one side at right angles to the two parallel sides. Find the area of each.

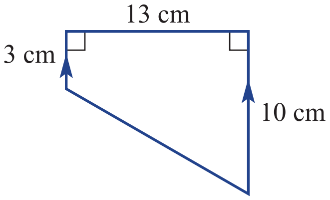
**a**



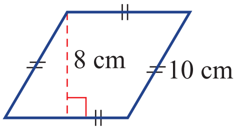
**b**



**c**



**12** Would you use the formula $A = \frac{1}{2}xy$ to find the area of this rhombus? Explain.



## Enrichment: Deriving formulas

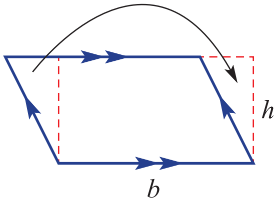
**13** Copy and complete these proofs to give the formula for the area of a parallelogram, a rhombus and a trapezium.

**a** Parallelogram

$A$ = base × perpendicular height

$= \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$



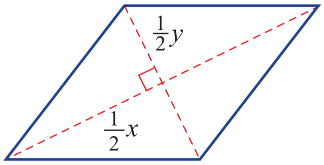
**b** Rhombus

$A$ = 4 triangle areas

$= 4 \times \frac{1}{2} \times \text{base} \times \text{height}$

$= 4 \times \frac{1}{2} \times \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$



**c** Trapezium

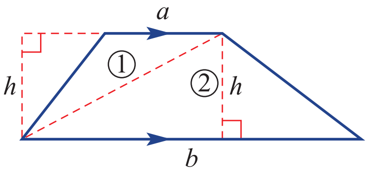
$A$ = Area (triangle 1) + Area (triangle 2)

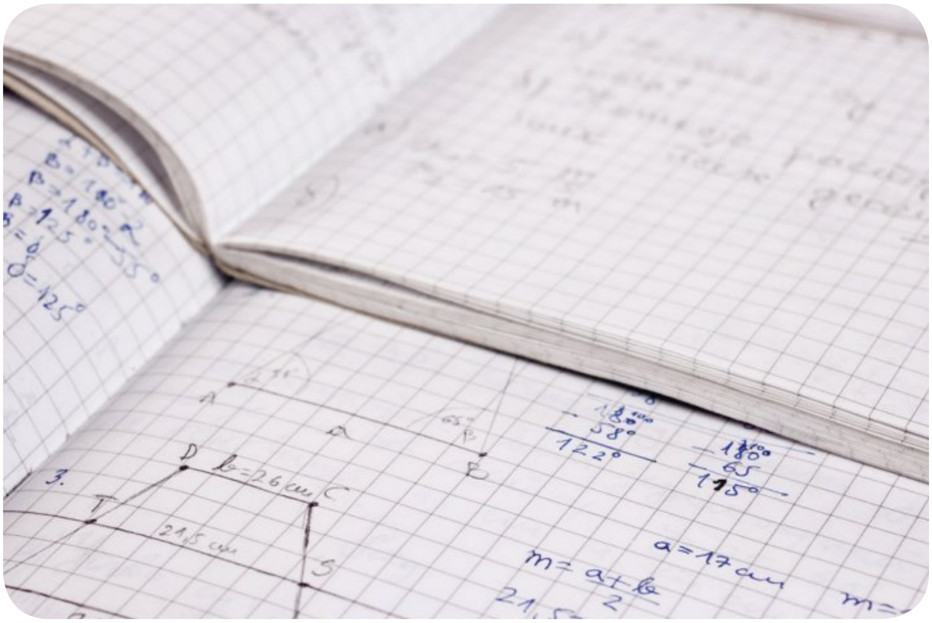
$= \frac{1}{2} \times \text{base}_1 \times \text{height}_1 + \frac{1}{2} \times \text{base}_2 \times \text{height}_2$

$= \frac{1}{2} \times \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} + \frac{1}{2} \times \underline{\hspace{2cm}} \times \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}} + \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$





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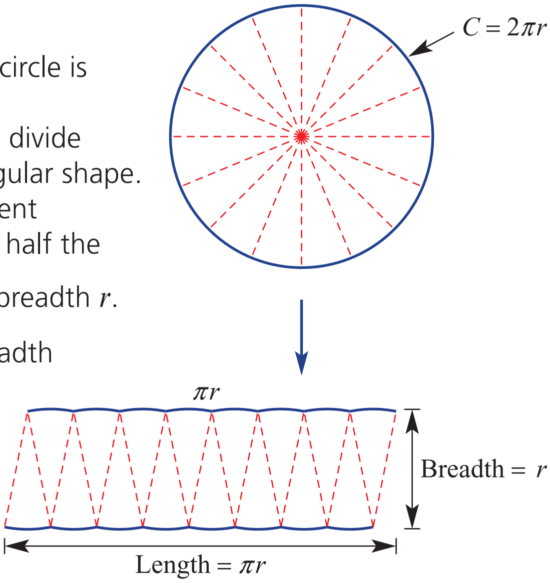
# 3E Area of circles

Like the circumference of a circle, the area of a circle is linked to the number pi ($\pi$).

One way to consider the area of a circle is to divide it into sectors, then arrange them into a rectangular shape. If very thin sectors are used, then the arrangement will be close to a rectangle with a length that is half the circumference of the circle, or $\frac{1}{2} \times 2\pi r = \pi r$ and breadth $r$.

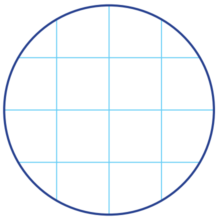
This leads to the area formula: 
$$\begin{aligned} A &= \text{length} \times \text{breadth} \\ &= \pi r \times r \\ &= \pi r^2 \end{aligned}$$



## Let's start: Counting squares

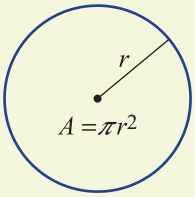
To find an estimate for the area of a circle you can count the number of squares.

- Count squares to estimate the area of this circle in cm$^2$.
- Ask your teacher to give you an accurate measure of its area. How close was your estimate?



## Key ideas

- The area of a circle is given by the formula $A = \pi r^2$.
  - Substitute the radius into the formula to find the area.
    e.g. If $r = 2$ then
    $$\begin{aligned} A &= \pi \times 2^2 \\ &= 4\pi \\ &\approx 12.57 \text{ (to 2 d.p.)} \end{aligned}$$



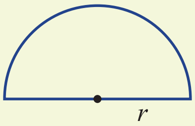
Note:
$4\pi$ is the exact value
12.57 is an approximation

**Semicircle** Half a circle

**Quadrant** A sector that is one quarter of a circle

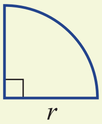
- A half circle is called a **semicircle**.

$$A = \frac{1}{2}\pi r^2$$



- A quarter circle is called a **quadrant**.

$$A = \frac{1}{4}\pi r^2$$



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## Exercise 3E

**Understanding**

**1** Write the formula for the:

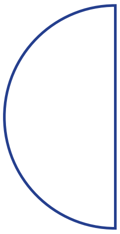
**a** circumference of a circle **b** area of a circle.

**2** Use a calculator to evaluate these to 2 decimal places.

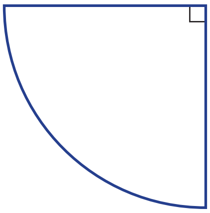
**a** $\pi \times 5^2$ **b** $\pi \times 13^2$ **c** $\pi \times 3.1^2$ **d** $\pi \times 9.8^2$

**3** What fraction of a full circle is shown here?

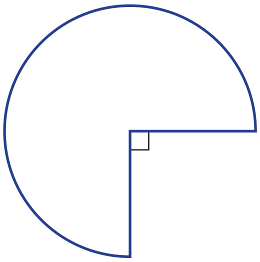
**a**



**b**

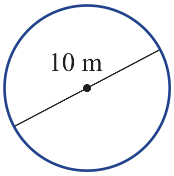


**c**

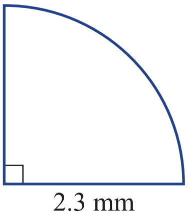


**4** What is the length of the radius in these shapes?

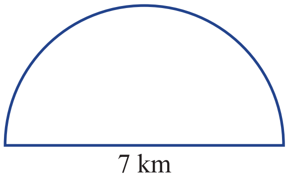
**a**



**b**



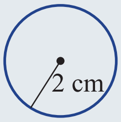
**c**



**Fluency**

### Example 13 Finding circle areas using a radius

Find the area of this circle, correct to 2 decimal places.



| Solution | Explanation |
|---|---|
| $A = \pi r^2$<br>$= \pi \times 2^2$<br>$= 12.57$ cm$^2$ (to 2 decimal places) | Use the $\pi$ button on the calculator and enter $\pi \times 2^2$ or $\pi \times 4$.<br>Note: The exact value is $4\pi$ cm$^2$ |

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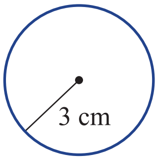
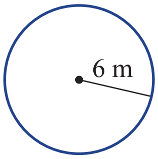
**5** Find the areas of these circles correct to 2 decimal places.

Substitute the radius for $r$ in $A = \pi r^2$.

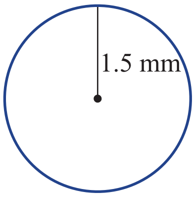
**a**



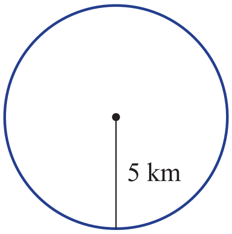
**b**



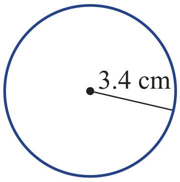
**c**



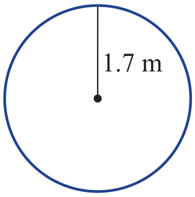
**d**



**e**

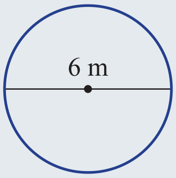


**f**



## Example 14 Finding circle areas using a diameter

Find the area of this circle, correct to 2 decimal places.



| **Solution** | **Explanation** |
|---|---|
| $r = 6 \div 2 = 3$ | First work out the radius as half of the diameter. |
| $A = \pi r^2$<br>$= \pi \times 3^2$<br>$= 28.27$ m$^2$ (to 2 decimal places) | Substitute $r = 3$ into the rule, then round to 2 decimal places.<br>Note: The exact value is $9\pi$ cm$^2$. |

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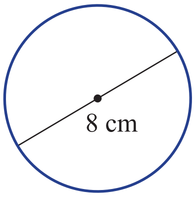
Skillsheet 3B

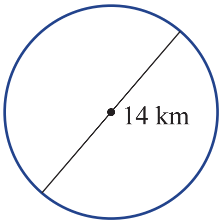
The radius is half the diameter.

**6** Find the areas of these circles, correct to 2 decimal places.

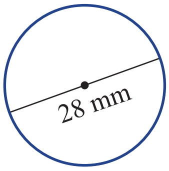
**a**



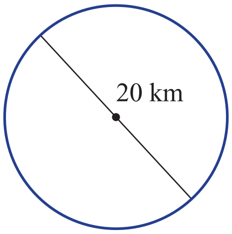
**b**



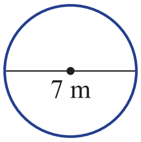
**c**



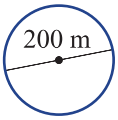
**d**



**e**

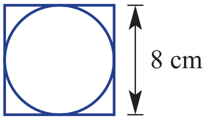


**f**

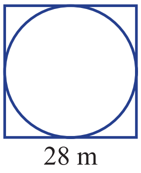


**7** Find the exact areas of the circles inside these shapes.

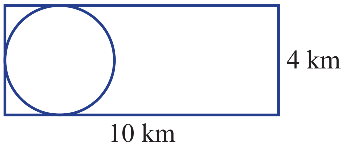
**a**



**b**



**c**



**Problem-solving and Reasoning**

**8** A pizza tray has a diameter of 30 cm. Calculate the exact area.

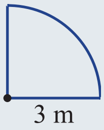
**9** A tree trunk is cut to show a circular cross-section of radius 60 cm. Is the area of the cross-section more than 1 $\text{m}^2$ and, if so, by how much? Round your answer to the nearest square centimetre.

**10** A circular oil slick has a diameter of 1 km. The newspaper reported an area of more than 1 $\text{km}^2$. Is the newspaper correct?

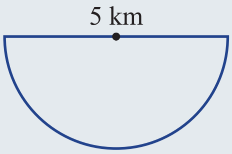
## Example 15 Finding areas of quadrants and semicircles

Find the areas of this quadrant and semicircle, correct to 2 decimal places.

**a**



**b**

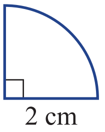


| Solution | Explanation |
|---|---|
| **a** $A = \frac{1}{4} \times \pi r^2$<br>$= \frac{1}{4} \times \pi \times 3^2$<br>$= 7.07$ m² (to 2 decimal places) | The area of a quadrant is $\frac{1}{4}$ the area of a circle with the same radius. |
| **b** $r = \frac{5}{2} = 2.5$ | The radius is half the diameter. |
| $A = \frac{1}{2} \times \pi r^2$<br>$= \frac{1}{2} \times \pi \times 2.5^2$<br>$= 9.82$ km² (to 2 decimal places) | The area of a semicircle is $\frac{1}{2}$ the area of a circle with the same radius. |

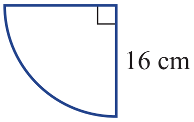
**11** Find the areas of these quadrants and semicircles, correct to 2 decimal places.

The radius is half the diameter.

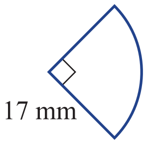
**a**



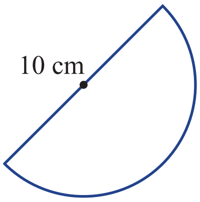
**b**



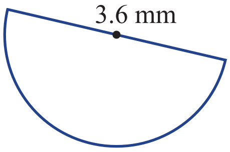
**c**



**d**



**e**



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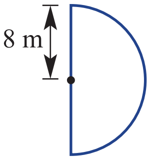


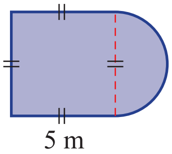
**12** Two circular plates have radii 12 cm and 13 cm. Find the difference in their areas, correct to 2 decimal places.

**13** A square of side length 10 cm has a hole in the middle. The diameter of the hole is 5 cm. What is the area remaining? Round the answer to the nearest whole number.

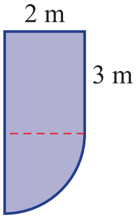
## Enrichment: Composite problems

**14** Find the areas of the shaded regions of these composite shapes using addition or subtraction. Round the answers to 2 decimal places.

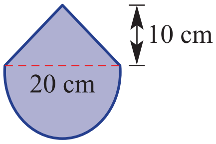
**a**



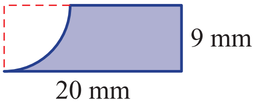
**b**



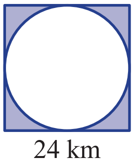
**c**



**d**



**e**



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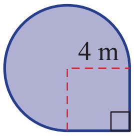
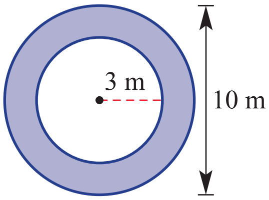
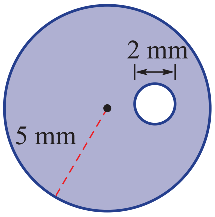


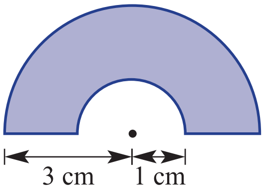
**g**



**h**



**i**



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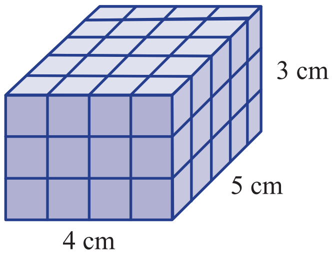
# 3F Volume and capacity

The photo shows a cube with edges 10 cm long. Volume is the amount of space inside the cube. Capacity is the amount of liquid it can hold. This cube has a volume of 1000 cubic centimetres and capacity of 1000 millilitres (1 litre).

## Let's start: Counting cubes quickly

This rectangular prism is made up of small blocks (1 $cm^3$ cubes). The prism is 4 cm wide, 5 cm long and 3 cm high.

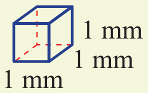
- How many 1 $cm^3$ cubes are there in one horizontal layer? Explain how you worked this out.
- How many 1 $cm^3$ cubes are there in total?
- What is the quickest way to find the total number of cubes (i.e. the volume)?

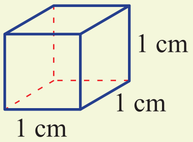


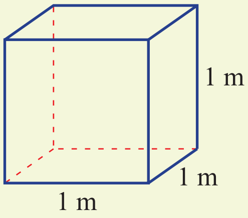
## Key ideas

**Volume** The amount of three-dimensional space in an object

- **Volume** is measured in cubic units.
- The common metric units for volume include:
  - cubic millimetres ($mm^3$)
  - cubic centimetres ($cm^3$)
  - cubic metres ($m^3$)







- Conversions for volume

$$\begin{tikzcd}
km^3 \arrow[r, bend left, "\times 1000^3"] & m^3 \arrow[l, bend left, "\div 1000^3"] \arrow[r, bend left, "\times 100^3"] & cm^3 \arrow[l, bend left, "\div 100^3"] \arrow[r, bend left, "\times 10^3"] & mm^3 \arrow[l, bend left, "\div 10^3"]
\end{tikzcd}$$

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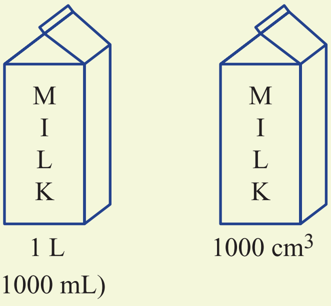
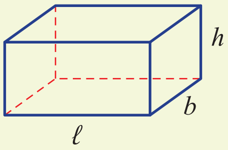
- **Capacity** is the volume of fluid or gas that a container can hold. Common metric units are:
  - millilitre (mL)
  - litre (L)
  - kilolitre (kL)
  - megalitre (ML)

$$\begin{tikzcd}
\text{ML} \arrow[r, bend left, "\times 1000"] & \text{kL} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{L} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{mL} \arrow[l, bend left, "\div 1000"]
\end{tikzcd}$$

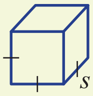
- Some common conversions are:
  - 1 mL = 1 $cm^3$
  - 1 L = 1000 mL
  - 1 kL = 1000 L = 1 $m^3$



- One litre of water has mass 1 kilogram.
- Volume of a rectangular prism
  - Volume = length × breadth × height
    $V = \ell bh$



- Volume of a cube
  $V = s^3$



**Capacity** The amount of liquid or gas a container can hold

## Exercise 3F

**Understanding**

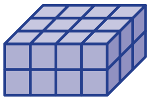
**1** State if the following are units for length, area, volume or capacity.

**a** cm **b** mL **c** $mm^3$ **d** ha
**e** $cm^2$ **f** $mm^2$ **g** km **h** $cm^3$
**i** $m^3$ **j** $m^2$ **k** ML **l** mm
**m** m **n** kL **o** L **p** $km^2$

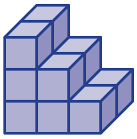
Drilling for Gold 3F2

**2** How many cubic units are shown in these stacks?

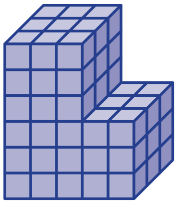
**a**



**b**



**c**



**3** Complete these sentences.

**a** 1 $cm^3$ contains ______ mL.

**b** 1 L contains ______ mL.

**c** A cube with edges 10 cm long has a volume of ______ $cm^3$ and contains ______ mL, which is ______ L.

**d** A cube with edges 1 m long has a volume of ______ $m^3$ and contains ______ L, which is ______ kL.

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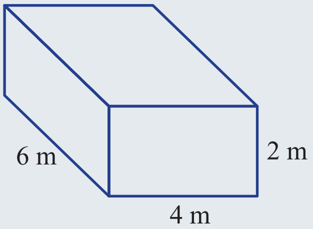
Fluency

## Example 16 Calculating the volume of a rectangular prism

Find the volume of this rectangular prism.

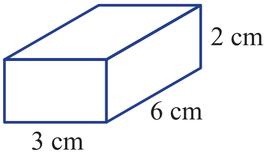


| Solution | Explanation |
|---|---|
| $V = \ell bh$<br>$= 6 \times 4 \times 2$<br>$= 48\ \text{m}^3$ | First write the rule and then substitute for the length, breadth and height. Any order will do since $6 \times 4 \times 2 = 4 \times 6 \times 2 = 2 \times 4 \times 6$ etc. |

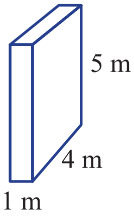
**4** Find the volumes of these rectangular prisms.

Prism: $V = \ell bh$
Cube: $V = s^3$

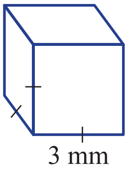
**a**



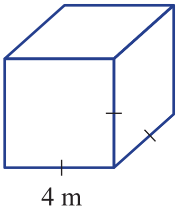
**b**



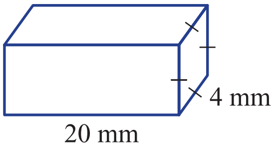
**c**



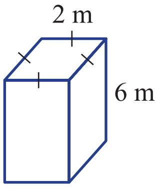
**d**



**e**



**f**



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### Example 17 Converting units for capacity

Convert to the units shown in brackets.

**a** 500 mL (L) **b** 3 kL (L)

| **Solution** | **Explanation** |
|---|---|
| **a** $500 \text{ mL} = (500 \div 1000) \text{ L}$<br>$= 0.5 \text{ L}$ | From Key ideas:<br>$\begin{tikzcd} \text{L} & \text{mL} \arrow[l, bend left, "\div 1000"] \end{tikzcd}$ |
| **b** $3 \text{ kL} = (3 \times 1000) \text{ L}$<br>$= 3000 \text{ L}$ | From Key ideas:<br>$\begin{tikzcd} \text{kL} \arrow[r, bend left, "\times 1000"] & \text{L} \end{tikzcd}$ |

**5** Convert to the units shown in brackets.

**a** 400 mL (L)
**b** 700 mL (L)
**c** 2000 L (kL)
**d** 36 000 L (kL)
**e** 4000 kL (ML)
**f** 500 kL (ML)
**g** 2 L (mL)
**h** 0.1 L (mL)
**i** 6 ML (kL)
**j** 3 ML (kL)
**k** 24 kL (L)
**l** 38 kL (L)
**m** 2000 L (kL)
**n** 3500 mL (L)
**o** 70 000 mL (L)
**p** 2500 kL (ML)
**q** 0.257 L (mL)
**r** 9320 mL (L)
**s** 3.847 ML (kL)
**t** 47 000 L (kL)
**u** 5800 kL (ML)

See the flowchart in the Key ideas.

### Example 18 Finding capacity

Find the capacity, in litres, for a container that is a rectangular prism 20 cm long, 10 cm wide and 15 cm high.

| **Solution** | **Explanation** |
|---|---|
| $V = \ell b h$<br>$= 20 \times 10 \times 15$<br>$= 3000 \text{ cm}^3$<br>$= 3000 \div 1000$<br>$= 3 \text{ L}$ | First calculate the volume of the container in $\text{cm}^3$.<br>Then convert to litres using $1 \text{ L} = 1000 \text{ cm}^3$. |

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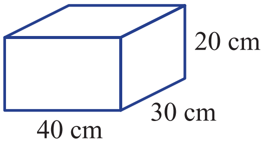
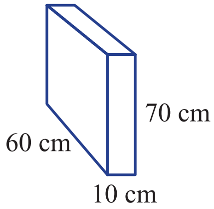
**6** Find the capacities of these containers, converting your answer to litres.

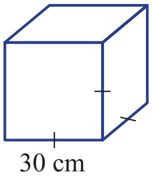
**a**



**b**

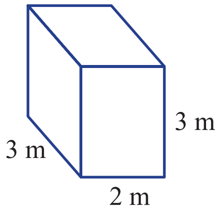


**c**

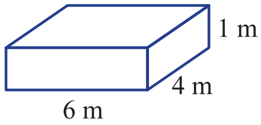


First find the volume in cm$^3$ using $V = \ell bh$, then divide by 1000 to convert to litres.

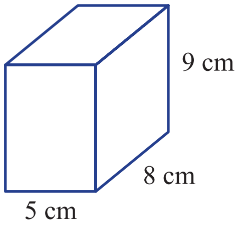
**d**



**e**



**f**



**Problem-solving and Reasoning**

**7** Choose the capacity (**A**–**F**) that best matches the container (**a**–**f**).

| | |
|---|---|
| **a** teaspoon | **A** 18 L |
| **b** cup | **B** 250 mL |
| **c** bottle | **C** 10 kL |
| **d** kitchen sink | **D** 20 mL |
| **e** water tank | **E** 45 ML |
| **f** water in a lake | **F** 0.8 L |

**8** A dose of 12 mL of medicine is to be taken twice each day from a 0.36 L bottle. How many days will it take to finish the medicine?

Remember, there are 1000 mL in 1 L.

**9** An oil tanker has a volume of 60 000 m$^3$.

Use 1 m$^3$ = 1000 L.

**a** What is the ship's capacity in:
  - **i** litres?
  - **ii** kilolitres?
  - **iii** megalitres?

**b** If the ship leaks oil at a rate of 300 000 litres per day, how long will it take for all the oil to leak out?

**10** Every litre of water weighs one kilogram. What is the mass of water in a full container that is a cube with side length 2 m?

**11** Water is being poured into a fish tank at a rate of 2 L every 10 seconds. The tank is 1.2 m long by 1 m wide by 80 cm high. How long will it take to fill the tank? Give the answer in minutes.

**12** How many cubic containers (with side lengths that are a whole number of centimetres) have a capacity of less than 1 litre?

## Enrichment: Water issues

**13** A swimming pool in the shape of a rectangular prism has length 50 m, breadth 25 m and depth 2 m. Find the swimming pool's:

**a** volume, in $m^3$ **b** capacity, in L.

**14** A dripping tap leaks about 10 mL every minute.

**a** If there are 50 drips per minute, find the volume of one drip.

**b** Find the approximate volume of water, in litres, that has leaked from a tap after:

- **i** 100 minutes
- **ii** 1 hour
- **iii** 1 day
- **iv** 1 year.

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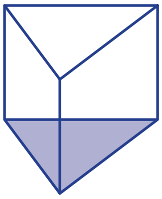
# 3G Volume of prisms

In the previous section we used $V = \ell bh$ to calculate the volume of a rectangular prism.

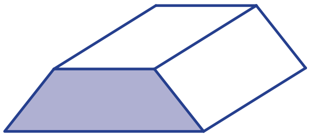
In this section we will look at other prisms.

Prisms are named using the shape of their cross-section, which has a constant shape and size along the entire length of the prism.

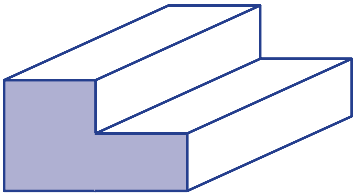
Here are some examples of prisms, with the cross-section shaded.



Cross-section is a triangle
Triangular prism



Cross-section is a trapezium
Trapezoidal prism



Cross-section is a hexagon
Hexagonal prism

## Let's start: Drawing prisms

Try to draw prisms that have the following shapes as their cross-sections.

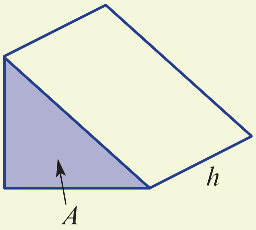
- Triangle
- Trapezium
- Pentagon
- Parallelogram

## Key ideas

**Prism** A solid with flat faces and whose cross-section is the same along the entire edge

- A **prism** is a solid with flat faces and a constant (uniform) cross-section.
- The height ($h$) is always measured perpendicular to the cross-section.
- Volume of a prism = area of cross-section × perpendicular height.

$$V = Ah$$



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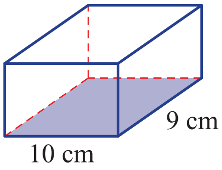
## Exercise 3G

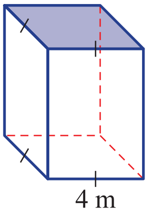
**Understanding**

**1** What is the shape of the cross-section in each prism (shaded)?

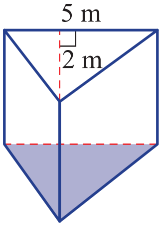
**a**



**b**



**c**



**2** What is the area of each shaded cross-section in Question **1**?

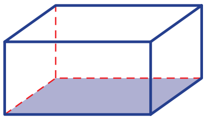
**3** For each solid:

**i** state whether or not it is a prism

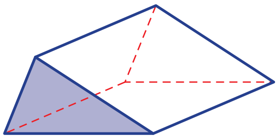
**ii** if it is a prism, state the shape of its cross-section.

A prism has a constant cross-section.

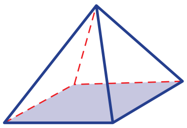
**a**



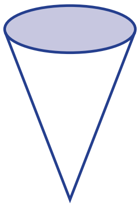
**b**



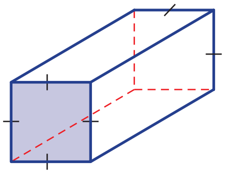
**c**



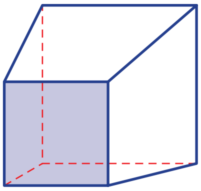
**d**



**e**



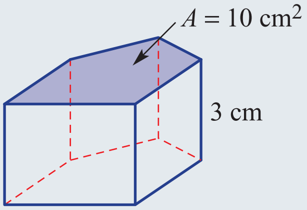
**f**



**Fluency**

### Example 19 Finding the volume of prisms given the cross-section

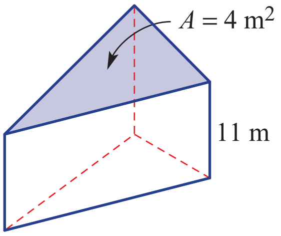
Find the volume of this prism using $V = Ah$.



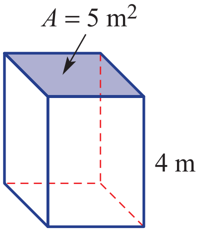
| **Solution** | **Explanation** |
|---|---|
| $V = Ah$ | Write the rule and substitute the given values of $A$ and $h$, where $A$ is the area of the cross-section. |
| $= 10 \times 3$ | |
| $= 30\text{ cm}^3$ | Note: Volume is recorded using cubic units. |

**4** Find the volumes of these solids using $V = Ah$.

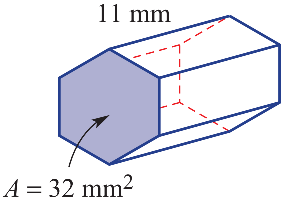
**a**



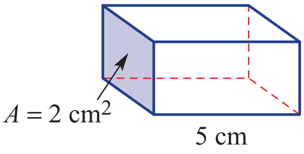
**b**



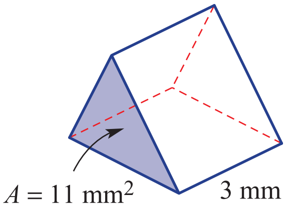
**c**



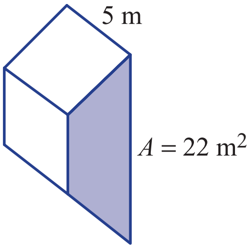
**d**



**e**

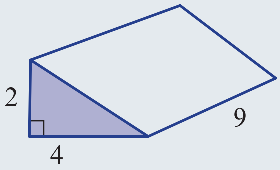


**f**



## Example 20 Finding the volumes of prisms

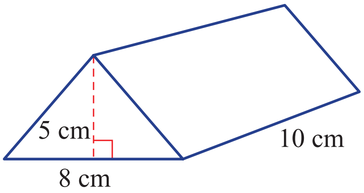
Find the volume of this prism.



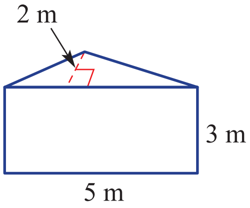
| Solution | Explanation |
|---|---|
| $A = \frac{1}{2}bh$<br>$= \frac{1}{2} \times 4 \times 2$<br>$= 4\text{ m}^2$ | The cross-section is a triangle, so use $A = \frac{1}{2}bh$ with base 4 m and height 2 m. |
| $V = Ah$<br>$= 4 \times 9$<br>$= 36\text{ m}^3$ | We need to multiply $A$ by $h$.<br>The perpendicular height is 9, so $h = 9$. |

**5** Find the volumes of these prisms.

**a**



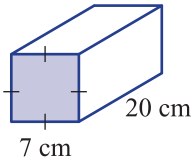
**b**



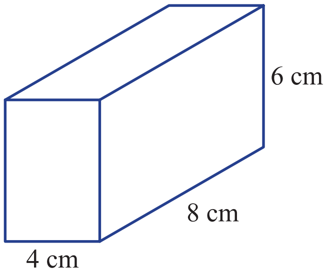
First find the area of the cross-section, then multiply by $h$.

Skillsheet 3C

**c**



**d**



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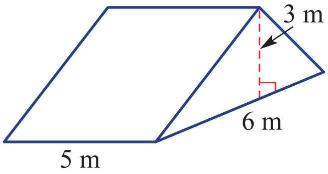
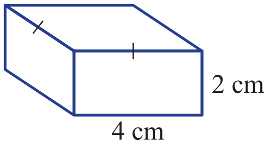
**e**



**f**



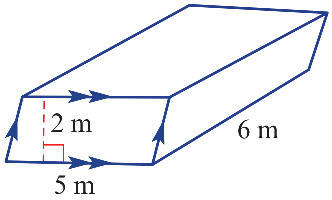
**6** A rectangular drain pipe has a cross-sectional area of 4 $m^2$ and is 10 m long. Find its volume.

## Problem-solving and Reasoning

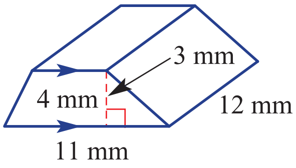
**7** These solids have cross-sections that are parallelograms, trapezia, rhombuses or kites. Find their volumes.

First find the area of the cross-section using:
$A = bh$
$A = \frac{1}{2}h(a + b)$
$A = \frac{1}{2}xy$

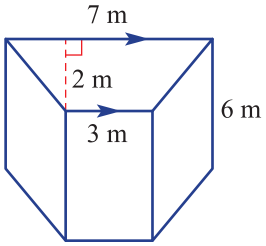
**a**



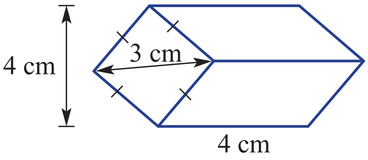
**b**



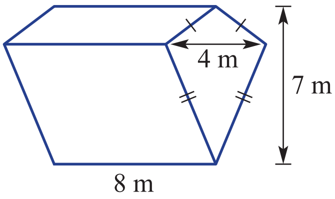
**c**



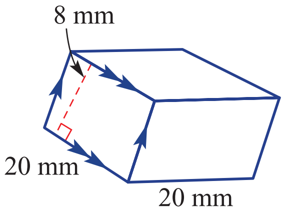
**d**



**e**



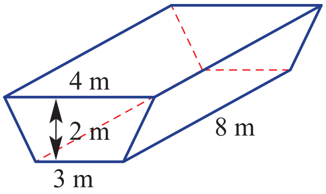
**f**



**8** A swimming pool is a prism with a cross-section that is a trapezium. The pool is being filled at a rate of 1000 litres per hour.

**a** Find the capacity of the pool in litres.

**b** How long will it take to fill the pool?



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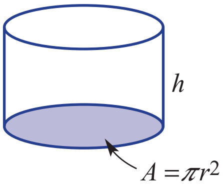
3G

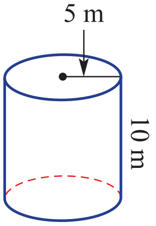
## Enrichment: Volume of a cylinder

**9** Although a cylinder is not a prism, the volume of a cylinder can be calculated using $V = Ah$ where $A = \pi r^2$ so $V = \pi r^2 h$.

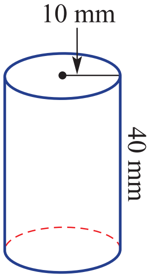


Find the volumes of these cylinders. Round the answers to 2 decimal places.

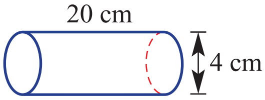
**a**



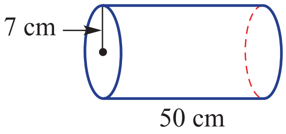
**b**



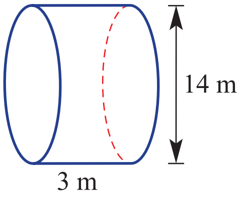
**c**



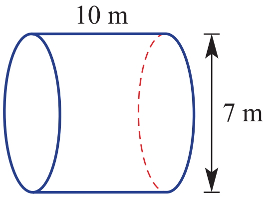
**d**



**e**



**f**



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# 3H Time

This topic was covered in detail in Year 7. This section will consolidate and extend the material covered in Year 7.

## Let's start: Knowledge of time

Do you know the answers to these questions about time and the calendar?

- How many seconds in a minute?
- How many hours in two days?
- How many months in a year?
- When is the next leap year?
- Which months have 31 days?
- Is it earlier or later in Perth?

## Key ideas

- Units of time include:
  - 1 **minute** (min) = 60 **seconds** (s)
  - 1 **hour** (h) = 60 **minutes** (min)
  - 1 **day** = 24 **hours** (h)
  - 1 **week** = 7 **days**
  - 1 **year** = 12 **months**

$$\begin{tikzcd}
\text{day} \arrow[r, bend left, "\times 24"] & \text{hour} \arrow[l, bend left, "\div 24"] \arrow[r, bend left, "\times 60"] & \text{minute} \arrow[l, bend left, "\div 60"] \arrow[r, bend left, "\times 60"] & \text{second} \arrow[l, bend left, "\div 60"]
\end{tikzcd}$$

- a.m. means before midday and p.m. means after midday.
- **24-hour time** shows the number of hours and minutes after midnight.
  - 0330 is 3:30 a.m.
  - 1530 is 3:30 p.m.
  - 1121 is 11:21 a.m.
  - 2247 is 10:47 p.m.
- Earth is divided into **time zones**.
  - Twenty-four 15° lines of longitude divide Earth into its time zones. (See map on pages 140–141 for details.)
  - Time is based on the time in Greenwich, United Kingdom, and this is called Coordinated Universal Time (UTC) or Greenwich Mean Time (GMT).
  - Places east of Greenwich are ahead in time.
  - Places west of Greenwich are behind in time.
- Australia has three **time zones**:
  - Eastern Standard Time (EST), which is UTC plus 10 hours.
  - Central Standard Time (CST), which is UTC plus 9.5 hours.
  - Western Standard Time (WST), which is UTC plus 8 hours.
- Some states have daylight saving over summer when clocks are moved forward by 1 hour.

**Time zone** Any geographic region of the world in which all places have the same time

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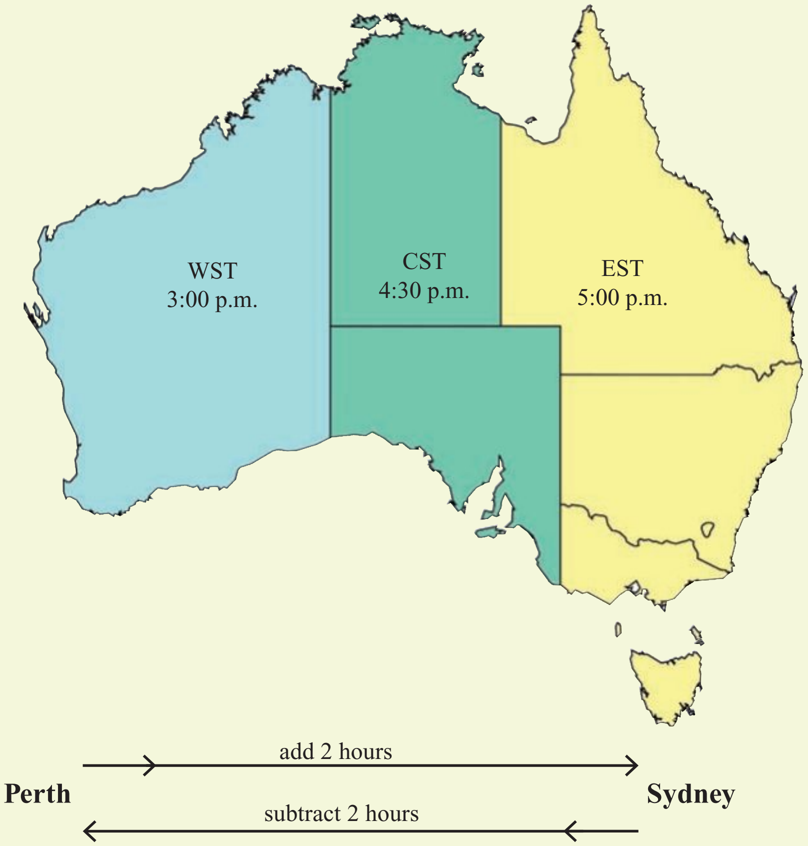

## Exercise 3H

**Understanding**

**1** Write the missing numbers.

**a** 1 minute = ____ seconds
**b** ____ days = 1 week
**c** ____ hours = 1 day
**d** 2 hours = ____ minutes
**e** 240 seconds = ____ minutes
**f** March has ____ days

Drilling for Gold 3H1

**2** Find the number of:

**a** seconds in 2 minutes
**b** minutes in 180 seconds
**c** hours in 120 minutes
**d** minutes in 4 hours
**e** hours in 3 days
**f** days in 48 hours
**g** weeks in 35 days
**h** days in 40 weeks.

**3** What is the time difference between these times? Use your calculator to check your answers.

**a** 12:00 noon and 6:30 p.m.
**b** 12:00 midnight and 10:45 a.m.
**c** 12:00 midnight and 4:20 p.m.
**d** 11:00 a.m. and 3:30 p.m.

Use the 'degrees, minutes, seconds' button.

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**Fluency**

### Example 21 Converting units of time

Convert these times to the units shown in brackets.

**a** 3 days (minutes) **b** 30 months (years)

| **Solution** | **Explanation** |
|---|---|
| **a** 3 days = 3 × 24 × 60 min<br>= 4320 min | 1 day = 24 hours<br>1 hour = 60 minutes |
| **b** 30 months = 30 ÷ 12 years<br>$= 2\frac{1}{2}$ years | There are 12 months in 1 year. |

**4** Convert these times to the units shown in brackets.

**a** 2 min (s) **b** 48 h (days) **c** 21 days (weeks)
**d** 3 h (min) **e** 10.5 min (s) **f** 240 s (min)
**g** 90 min (h) **h** 6 days (h) **i** 72 h (days)
**j** 1 week (h) **k** 1 day (min) **l** $3\frac{1}{2}$h (min)

Use the flowchart in the Key ideas.

Drilling for Gold 3H2

**5** Write the times for these descriptions.

**a** 4 hours after 2:30 p.m. **b** 10 hours before 7:00 p.m.
**c** $3\frac{1}{2}$ hours before 10:00 p.m. **d** $7\frac{1}{2}$ hours after 9:00 a.m.
**e** $6\frac{1}{4}$ hours after 11:15 a.m. **f** $1\frac{3}{4}$ hours before 1:25 p.m.

### Example 22 Using 24-hour time

Write these times using the system given in brackets.

**a** 4:30 p.m. (24-hour time) **b** 1945 (a.m./p.m.)

| **Solution** | **Explanation** |
|---|---|
| **a** 4:30 p.m. = 1200 + 0430<br>= 1630 hours | Since the time is p.m., add 12 hours to 0430 hours. |
| **b** 1945 hours = 7:45 p.m. | Since the time is after 1200 hours, subtract 12 hours. |

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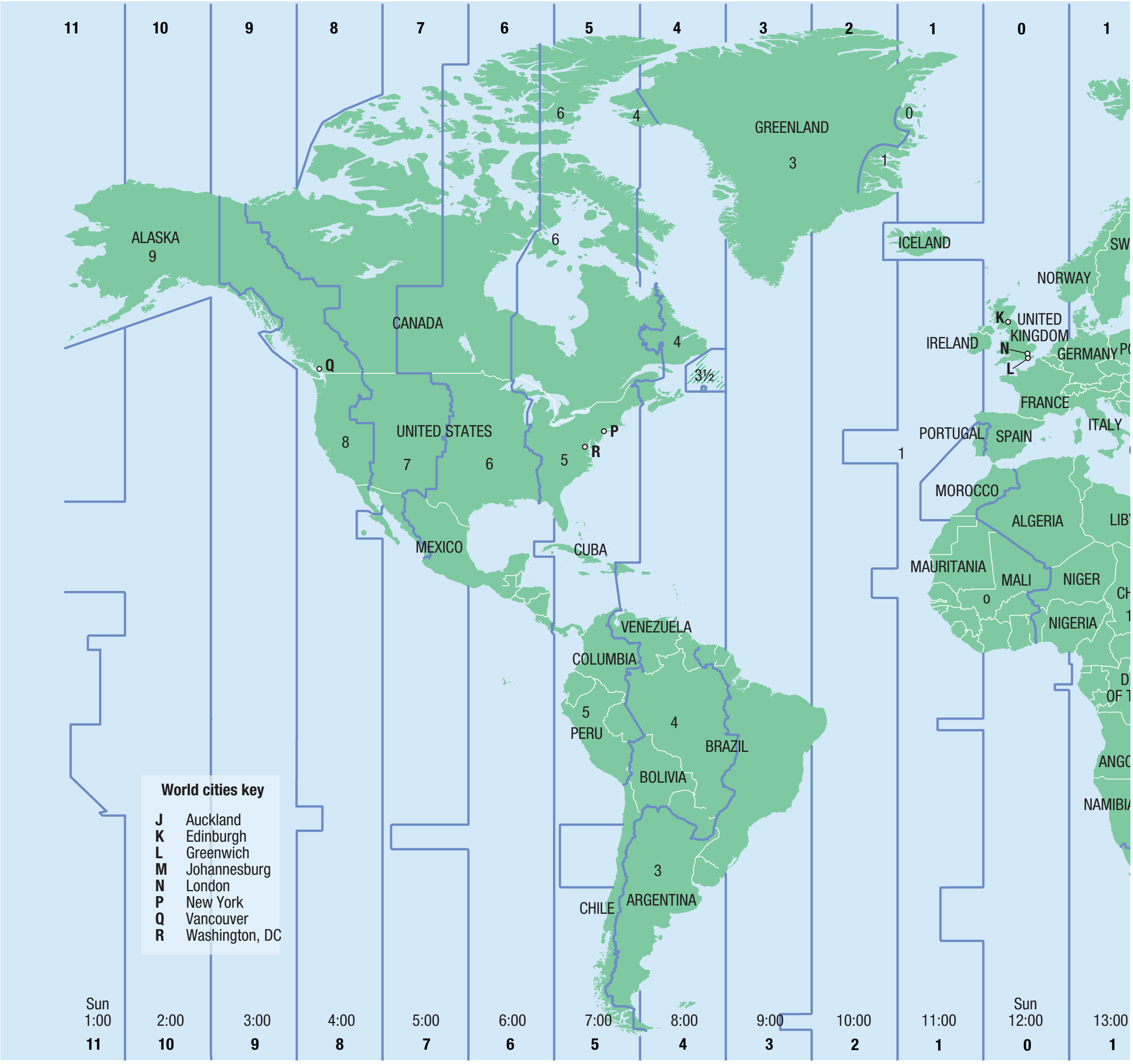
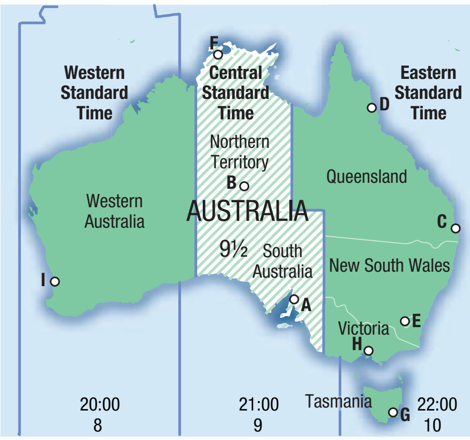


**Australian cities key**

| | |
|---|---|
| A | Adelaide |
| B | Alice Springs |
| C | Brisbane |
| D | Cairns |
| E | Canberra, ACT |
| F | Darwin |
| G | Hobart |
| H | Melbourne |
| I | Perth |



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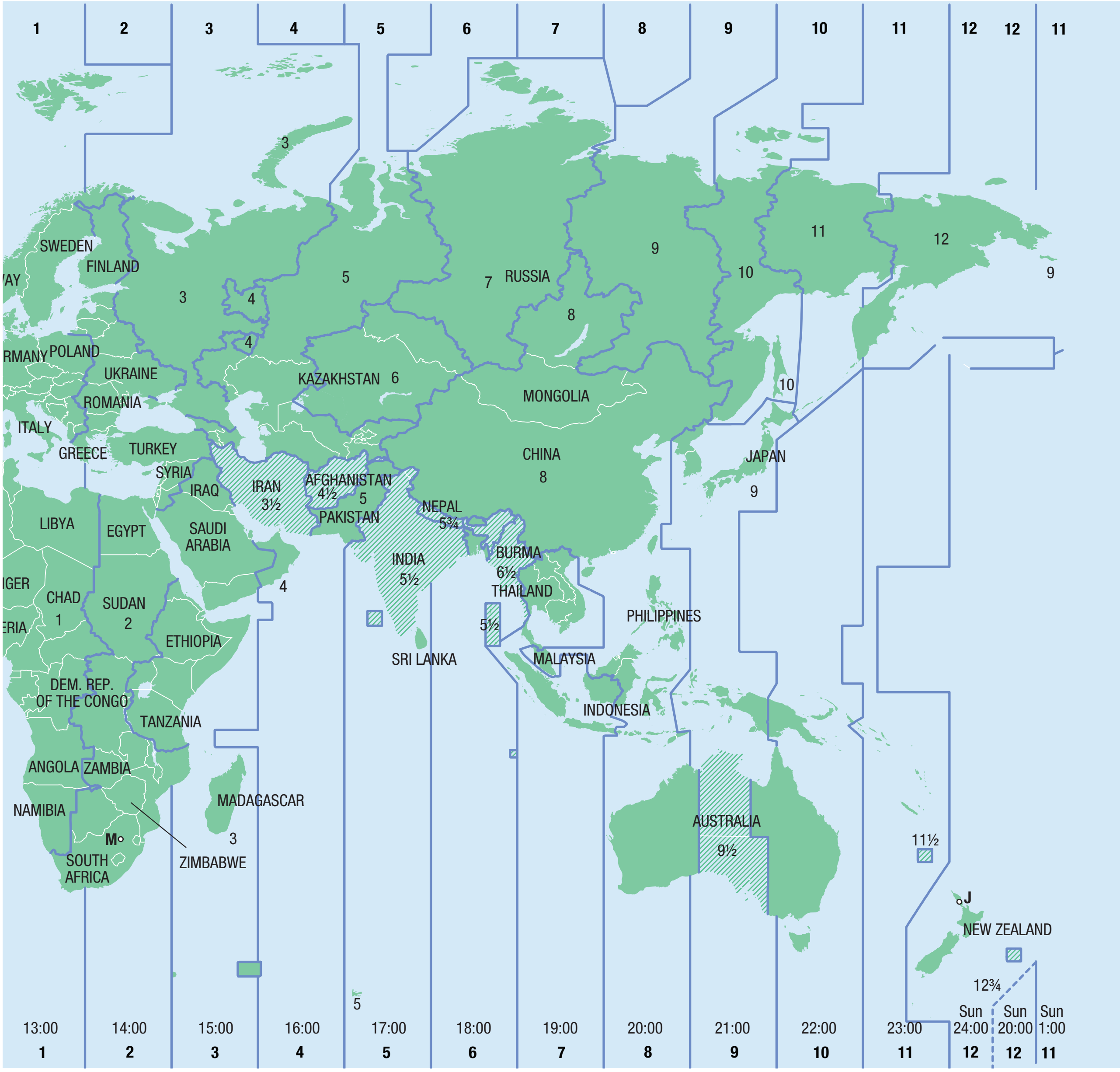
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ROMANIA
KAZAKHSTAN 6
MONGOLIA
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ITALY
GREECE
TURKEY
SYRIA
IRAQ
IRAN
3½
AFGHANISTAN
4½
5
PAKISTAN
NEPAL
5¾
CHINA
8
JAPAN
9
LIBYA
EGYPT
SAUDI
ARABIA
INDIA
5½
BURMA
6½
THAILAND
IGER
CHAD
1
SUDAN
2
ERIA
4
ETHIOPIA
5½
SRI LANKA
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PHILIPPINES
DEM. REP.
OF THE CONGO
TANZANIA
INDONESIA
ANGOLA
ZAMBIA
NAMIBIA
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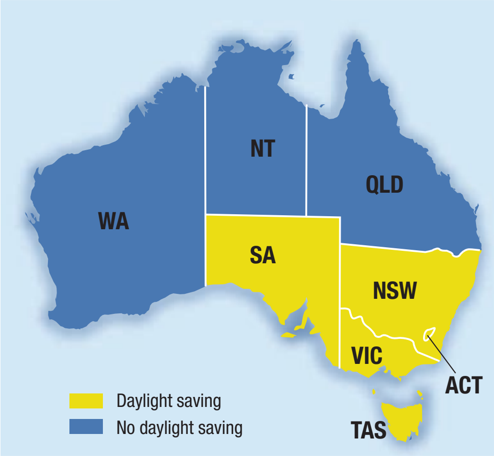
NT
QLD
WA
SA
NSW
VIC
ACT
TAS
🟨 Daylight saving
🟦 No daylight saving

**6** Write these times using the system shown in brackets.

**a** 1:30 p.m. (24-hour) **b** 8:15 p.m. (24-hour)
**c** 10:23 a.m. (24-hour) **d** 11:59 p.m. (24-hour)
**e** 0630 hours (a.m./p.m.) **f** 1300 hours (a.m./p.m.)
**g** 1429 hours (a.m./p.m.) **h** 1938 hours (a.m./p.m.)
**i** 2351 hours (a.m./p.m.) **j** 0426 hours (a.m./p.m.)
**k** 6:47 p.m. (24-hour) **l** 4:32 a.m. (24-hour)

6:00 a.m. is 0600 hours. 12:00 noon is 1200 hours. 6:00 p.m. is 1800 hours.

**7** Round these times to the nearest hour.

**a** 1:32 p.m. **b** 5:28 a.m. **c** 1219 hours **d** 1749 hours

## Example 23 Using time zones near Australia

Use the world time zone map (on pages 140–141) to answer the following.

**a** When it is 2:00 p.m. EST (Eastern Standard Time), find the time in these places.
  **i** Adelaide **ii** Perth **iii** Queensland **iv** Phillipines
**b** When it is 9:35 a.m. in Western Australia, Australia, find the time in these places.
  **i** Alice Springs **ii** Tasmania **iii** Brisbane **iv** China

| **Solution** | **Explanation** |
|---|---|
| **a i** 1:30 p.m. | Adelaide is in the Central Standard Time zone, which is $\frac{1}{2}$ hour behind Eastern Standard Time. |
| **ii** 12:00 noon | Perth is in the WST zone, 2 hours behind EST. |
| **iii** 2:00 p.m. | Queensland is in the Eastern Standard Time zone. |
| **iv** 12:00 noon | Phillipines is in the same zone as Western Australia. |
| **b i** 11:05 a.m. | Alice Springs uses Central Standard Time, which is $1\frac{1}{2}$ hours ahead of Western Standard Time. |
| **ii** 11:35 a.m. | Tasmania uses Eastern Standard Time, which is 2 hours ahead Western Standard Time. |
| **iii** 11:35 a.m. | Brisbane is in the EST zone 2 hours ahead of WST. |
| **iv** 9:35 a.m. | China is in the same zone as Western Australia. |

**8** Use the time zone map on pages 140–141 to find the time in the following places, when it is 10:00 a.m. EST.

**a** Melbourne **b** Darwin
**c** Adelaide **d** Perth
**e** Sydney **f** Tasmania
**g** China **h** Papua New Guinea

CST is $\frac{1}{2}$ an hour behind EST, WST is 2 hours behind EST.

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Drilling for Gold 3H3

**9** Use the time zone map on pages 140–141 to find the time in these places, when it is 3:30 p.m. in Perth.

**a** Melbourne **b** Phillipines **c** Sydney
**d** China **e** Hobart **f** Queensland
**g** Alice Springs **h** New Zealand **i** Japan

**Problem-solving and Reasoning**

**10** Match each time unit **A**–**F** with the most appropriate description **a**–**f**.

| | |
|---|---|
| **a** single heartbeat | **A** 1 hour |
| **b** 40 hours of work | **B** 1 minute |
| **c** duration of a university lecture | **C** 1 day |
| **d** bank term deposit | **D** 1 week |
| **e** 200-m run | **E** 1 year |
| **f** flight from Australia to the UK | **F** 1 second |

**11** What is the time difference between these time periods? Use your calculator to check your answers.

**a** 10:30 a.m. and 1:20 p.m.
**b** 9:10 a.m. and 3:30 p.m.
**c** 2:37 p.m. and 5:21 p.m.
**d** 10:42 p.m. and 7:32 a.m. (the following day)
**e** 1451 and 2310 hours
**f** 1940 and 0629 hours (the following day)

**12** Three essays are marked by a teacher. The first takes 4 minutes and 32 seconds to mark, the second takes 7 minutes and 19 seconds, and the third takes 5 minutes and 37 seconds. What is the total time taken to complete marking the essays?

**13** Adrian arrives at school at 8:09 a.m. and leaves at 3:37 p.m. How many hours and minutes is Adrian at school?

**14** On a flight to Europe, Janelle spends 8 hours and 36 minutes on a flight from Melbourne to Kuala Lumpur, Malaysia, 2 hours and 20 minutes at the airport at Kuala Lumpur, and then 12 hours and 19 minutes on a flight to Geneva, Switzerland. What is Janelle's total travel time?

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**15** A phone plan charges 11 cents per 30 seconds. The 11 cents are added to the bill at the beginning of every 30-second block of time.

- **a** What is the cost of a 70-second call?
- **b** What is the cost of a call that lasts 6 minutes and 20 seconds?

**16** A doctor earns $180 000 working 40 weeks per year, 5 days per week, 10 hours per day. What does the doctor earn in each of these time periods (while working)?

- **a** per day
- **b** per hour
- **c** per minute
- **d** per second (in cents)

## Enrichment: World time zones

**17** Use the time zone map to find the times in the following places if it is 3:30 p.m. in Victoria.

- **a** United Kingdom
- **b** Libya
- **c** Sweden
- **d** Perth
- **e** Japan
- **f** Central Greenland
- **g** Alice Springs
- **h** New Zealand

**18** Use the time zone map to find the times in the following places if it is 10:00 a.m. UTC in England.

- **a** Spain
- **b** Turkey
- **c** Tasmania
- **d** Darwin
- **e** Argentina
- **f** Peru
- **g** Alaska
- **h** Portugal

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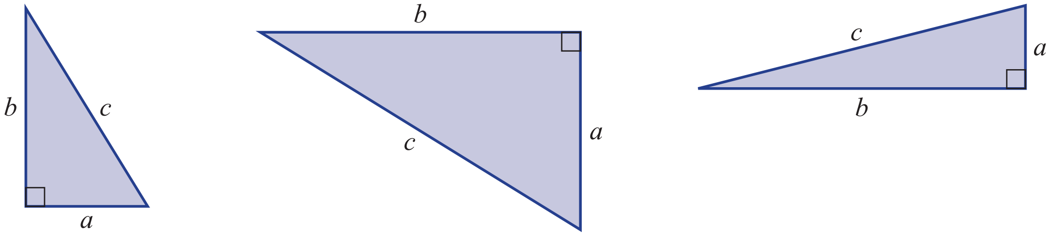
# 3I Introducing Pythagoras' Theorem

Pythagoras was a philosopher who lived in the sixth century BC. He studied astronomy, mathematics and music and found Pythagorean triads, which are sets of three whole numbers that make up the sides of right-angled triangles.

A thousand years before Pythagoras' time, the ancient Babylonians and the Egyptians also found a relationship between the sides of a right-angled triangle. However, Pythagoras' explanation was easier to understand.

## Let's start: Discovering Pythagoras' Theorem

Use a ruler to measure the sides of these right-angled triangles to the nearest mm. Then complete the table.

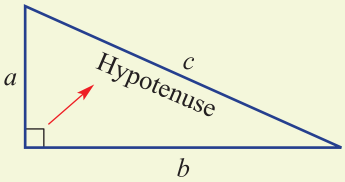


| $a$ | $b$ | $c$ | $a^2$ | $b^2$ | $c^2$ |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |

- Can you see any relationship between the numbers in the columns for $a^2$ and $b^2$ and the number in the column for $c^2$?
- Can you write down this relationship as an equation?

## Key ideas

- The **hypotenuse**
  - It is the longest side of a right-angled triangle.
  - It is opposite the right angle.



**Hypotenuse** The longest side of a right triangle

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- Pythagoras' Theorem
  - The square of the hypotenuse is the sum of the squares of the other two shorter sides.
  - $a^2 + b^2 = c^2$ or $c^2 = a^2 + b^2$
- A Pythagorean triad is a set of three integers that satisfy Pythagoras' Theorem.

## Exercise 3I

**Understanding**

**1** Calculate these squares and sums of squares.

**a** $3^2$ **b** $5^2$ **c** $12^2$ **d** $1.5^2$

**e** $2^2 + 4^2$ **f** $3^2 + 7^2$ **g** $6^2 + 11^2$ **h** $12^2 + 15^2$

**2** Decide if these equations are true or false.

**a** $2^2 + 3^2 = 4^2$ **b** $6^2 + 8^2 = 10^2$ **c** $7^2 + 24^2 = 25^2$

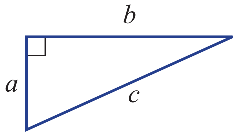
**d** $5^2 - 3^2 = 4^2$ **e** $6^2 - 3^2 = 2^2$ **f** $10^2 - 5^2 = 5^2$

**3** Write the missing words in this sentence.

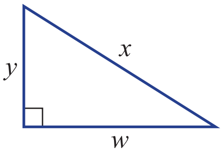
The ____________ is the longest side of a right-angled ____________.

**4** Which letters mark the length of the hypotenuse in these triangles?

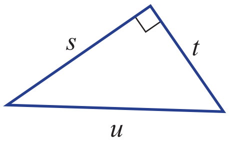
**a**



**b**



**c**



**Fluency**

### Example 24 Checking Pythagorean triads

Decide if the following are Pythagorean triads

**a** 6, 8, 10 **b** 4, 5, 9

| **Solution** | **Explanation** |
|---|---|
| **a** $a^2 + b^2 = 62 + 82$<br>$= 36 + 64$<br>$= 100\ (= 10^2)$ | Let $a = 6$, $b = 8$ and $c = 10$ and check that $a^2 + b^2 = c^2$. |
| **b** $a^2 + b^2 = 4^2 + 5^2$<br>$= 16 + 25$<br>$= 41$<br>$\neq 9^2$ | $a^2 + b^2 = 41$ and $c^2 = 81$ so the set of numbers are not a Pythagorean triad. |

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**5** Decide if the following are Pythagorean triads.

**a** 3, 4, 6 **b** 4, 2, 5 **c** 3, 4, 5

**d** 9, 12, 15 **e** 5, 12, 13 **f** 2, 5, 6

**g** 9, 40, 41 **h** 10, 12, 20 **i** 4, 9, 12

**6** Complete this table and answer the questions.

| $a$ | $b$ | $c$ | $a^2$ | $b^2$ | $a^2 + b^2$ | $c^2$ |
|---|---|---|---|---|---|---|
| 3 | 4 | 5 | | | | |
| 6 | 8 | 10 | | | | |
| 8 | 15 | 17 | | | | |

**a** Which two columns give equal results?

**b** What would be the value of $c^2$ if:

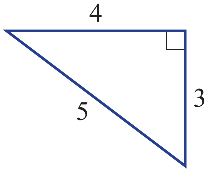
**i** $a^2 = 4$ and $b^2 = 9$? **ii** $a^2 = 7$ and $b^2 = 13$?

**c** What would be the value of $a^2 + b^2$ if:

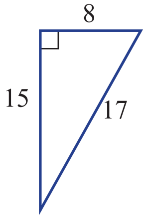
**i** $c^2 = 25$? **ii** $c^2 = 110$?

**7** Check that $a^2 + b^2 = c^2$ for all these right-angled triangles.

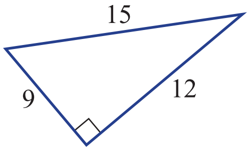
**a**



**b**



**c**



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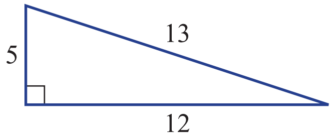
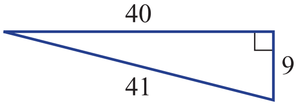
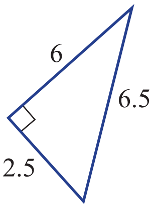


**e**



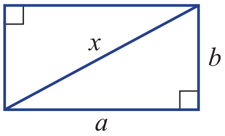
**f**



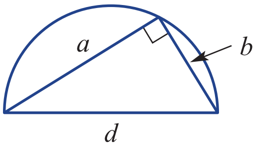
### Problem-solving and Reasoning

**8** Write down Pythagoras' Theorem using the letters given these diagrams.

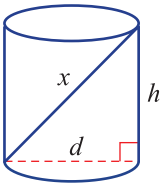
**a**



**a**



**b**



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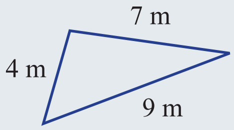
**9** A cable connects the top of a 30 m mast to a point on the ground. The cable is 40 m long and connects to a point 20 m from the base of the mast.

**a** Using $c = 40$, decide if $a^2 + b^2 = c^2$.

**b** Do you think the triangle formed by the mast and the cable is right angled? Give a reason.

## Example 25 Deciding if a triangle has a right angle

Decide if this triangle has a right angle.

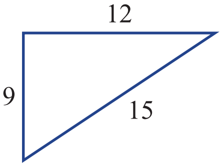


| **Solution** | **Explanation** |
|---|---|
| $a^2 + b^2 = 4^2 + 7^2$<br>$= 16 + 49$<br>$= 65$<br>$\neq 9^2$ | Check to see if $a^2 + b^2 = c^2$. In this case $a^2 + b^2 = 65$ and $c^2 = 81$ so the triangle is not right angled. |

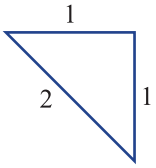
## Enrichment: Pythagorean triads

**10** If $a^2 + b^2 = c^2$, we know that the triangle must have a right angle. Which of these triangles must have a right angle?

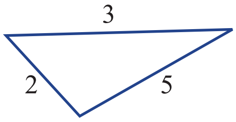
**a**



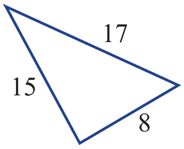
**b**



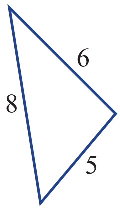
**c**



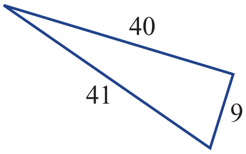
**d**



**e**



**f**



**11** (3, 4, 5) and (5, 12, 13) are Pythagorean triads since $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$. Find 10 more Pythagorean triads using whole numbers less than 100.

Extension: Find the total number of Pythagorean triads with whole numbers of less than 100.

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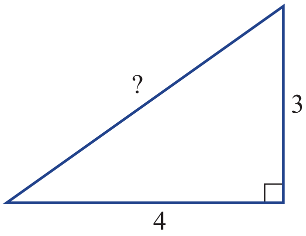
# 3J Using Pythagoras' Theorem

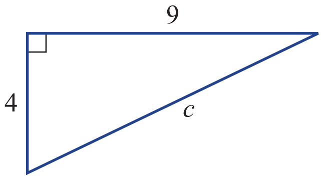
In this section we will use the lengths of the two short sides to calculate the length of the hypotenuse.



## Let's start: Correct layout

Two students who are trying to find the value of $c$ in this triangle by using Pythagoras' Theorem write their solutions on a board. There are only very minor differences between each solution and the answer is rounded to 2 decimal places.

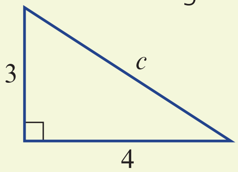
- Which student has all the steps correct?
- Give reasons why the other solution is not laid out correctly.



| Student 1 | Student 2 |
|---|---|
| $c^2 = a^2 + b^2$<br>$= 4^2 + 9^2$<br>$= 97$<br>$= \sqrt{97}$<br>$= 9.85$ | $c^2 = a^2 + b^2$<br>$= 4^2 + 9^2$<br>$= 97$<br>$\therefore c = \sqrt{97}$<br>$= 9.85$ |

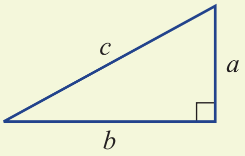
## Key ideas

- To find the length of the hypotenuse:



$$\begin{aligned} c^2 &= 3^2 + 4^2 \\ c^2 &= 9 + 16 \\ c^2 &= 25 \\ \therefore\ c &= \sqrt{25} \\ &= 5 \end{aligned}$$

- Note that the final step may not always result in a whole number. For example, $\sqrt{3}$ and $\sqrt{24}$ are not whole numbers.
- **Surds** are numbers that have a $\sqrt{\phantom{x}}$ sign when written in simplest form.
  - They are not a whole number and cannot be written as a fraction.
  - Written as a decimal, the decimal places would continue forever with no repeated pattern (just like the number pi), so surds are irrational numbers.
  - Examples of surds are $\sqrt{2}$ and $\sqrt{5}$.



**Surd** An irrational number, which cannot be simplified to remove a square root

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## Exercise 3J

**Understanding**

**1** Decide if these numbers written with a $\sqrt{\ }$ sign simplify to a whole number. Answer yes or no.

**a** $\sqrt{9}$ **b** $\sqrt{11}$ **c** $\sqrt{20}$ **d** $\sqrt{121}$

**2** Round these surds, correct to 2 decimal places using a calculator.

**a** $\sqrt{10}$ **b** $\sqrt{26}$ **c** $\sqrt{65}$ **d** $\sqrt{230}$

**3** Copy and complete each working out.

**a**
$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 5^2 + 12^2 \\ &= \underline{\qquad} \\ \therefore c &= \sqrt{\underline{\qquad}} \\ &= \underline{\qquad} \end{aligned}$$

**b**
$$\begin{aligned} c^2 &= \underline{\qquad\qquad} \\ &= 9^2 + 40^2 \\ &= \underline{\qquad} \\ \therefore c &= \sqrt{\underline{\qquad}} \\ &= \underline{\qquad} \end{aligned}$$

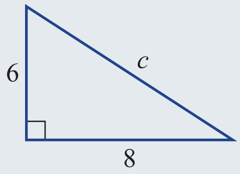
**c**
$$\begin{aligned} \underline{\quad} &= \underline{\qquad\qquad} \\ &= 9^2 + 12^2 \\ &= \underline{\qquad} \\ \therefore c &= \sqrt{\underline{\qquad}} \\ &= \underline{\qquad} \end{aligned}$$

**Fluency**

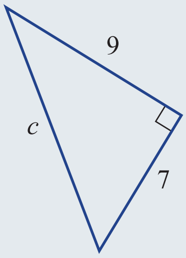
### Example 26 Finding the length of the hypotenuse

Find the length of the hypotenuse for these right-angled triangles. Round the answer for part **b** to 2 decimal places.

**a**



**b**



| **Solution** | **Explanation** |
|---|---|
| **a** $c^2 = a^2 + b^2$<br>$= 6^2 + 8^2$<br>$= 100$<br>$\therefore c = \sqrt{100}$<br>$= 10$ | Write the equation for Pythagoras' Theorem and substitute the values for the shorter sides.<br>Find $c$ by taking the square root. |
| **b** $c^2 = a^2 + b^2$<br>$= 7^2 + 9^2$<br>$= 130$ | First calculate the value of $7^2 + 9^2$. |
| $\therefore c = \sqrt{130}$<br>$= 11.40$ (to 2 decimal places) | $\sqrt{130}$ is a surd (the exact answer), so round the answer as required. |

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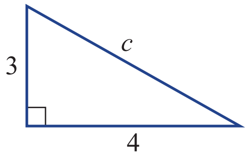
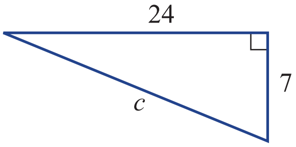
**4** Find the length of the hypotenuse ($c$) of each right-angled triangle.

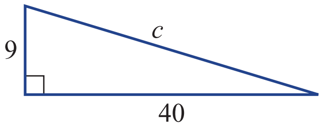
**a**



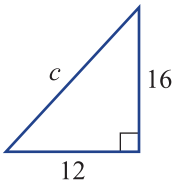
**b**



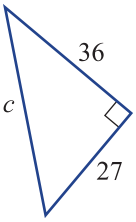
**c**



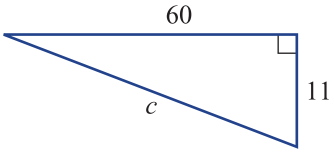
**d**



**e**

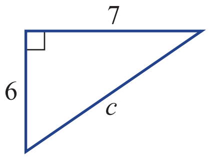


**f**

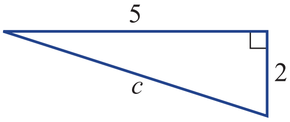


**5** Find the length of the hypotenuse ($c$) of each right-angled triangle, correct to 2 decimal places.

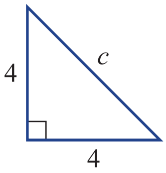
**a**



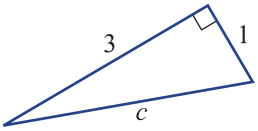
**b**



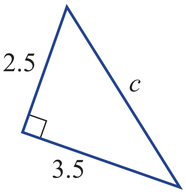
**c**



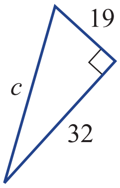
**d**



**e**



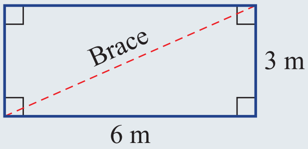
**f**



**Problem-solving and Reasoning**

## Example 27 Applying Pythagoras' Theorem

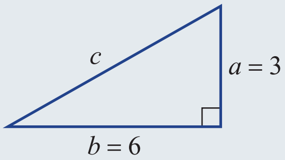
A rectangular wall is to be strengthened by a diagonal brace. The wall is 6 m wide and 3 m high. Find the length of brace required, correct to the nearest cm.



**Solution**

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 3^2 + 6^2 \\ &= 45 \\ \therefore\ c &= \sqrt{45} \\ &= 6.71 \text{ m or } 671 \text{ cm} \quad \text{(nearest cm)} \end{aligned}$$

**Explanation**



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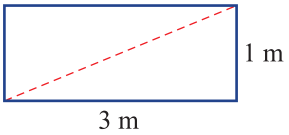
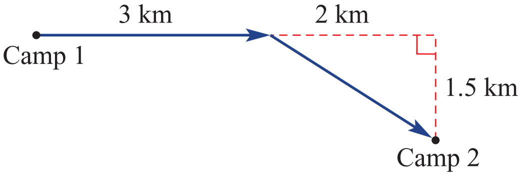
**6** A rectangular board is to be cut along one of its diagonals. The board is 1 m wide and 3 m high. What will be the length of the cut, correct to the nearest cm?

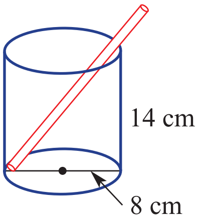


**7** The size of a television screen is determined by its diagonal length. Find the size of a television screen that is 1.2 m wide and 70 cm high. Round the answer to the nearest cm.

**8** Here is a diagram showing the path of a bushwalker from camp 1 to camp 2. Find the total distance calculated to 1 decimal place.



**9** A 20 cm straw sits in a cylindrical glass as shown. What length of straw sticks above the top of the glass? Round the answer to 2 decimal places.



**10** Explain the error in each set of working.

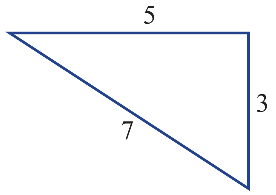
**a** $c^2 = 2^2 + 3^2$
$\therefore c = 2 + 3$
$= 5$

**b** $c^2 = 3^2 + 4^2$
$= 7^2$
$= 49$
$\therefore c = 7$

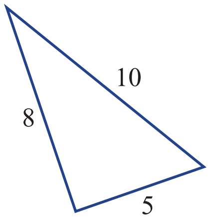
**c** $c^2 = 2^2 + 5^2$
$= 4 + 25$
$= 29$
$= \sqrt{29}$

**11** Prove that these are not right-angled triangles.

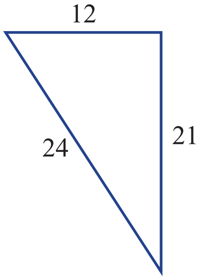
**a**



**b**



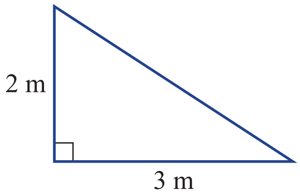
**c**



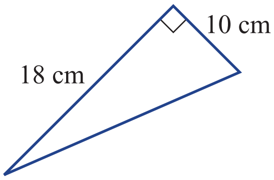
## Enrichment: Perimeter and Pythagoras

**12** Find the perimeters of these shapes, correct to 2 decimal places.

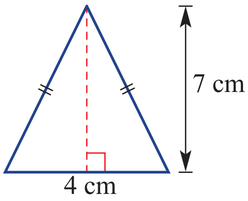
**a**



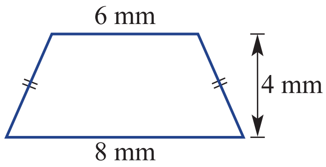
**b**



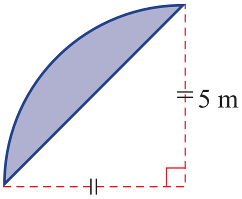
**c**



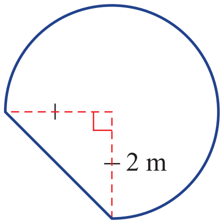
**d**



**e**

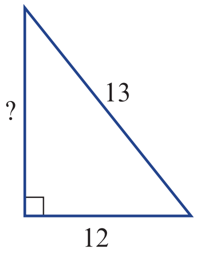


**f**



# 3K Calculating the length of a shorter side

In this section we will see that any two sides can be used to calculate the length of a third side.

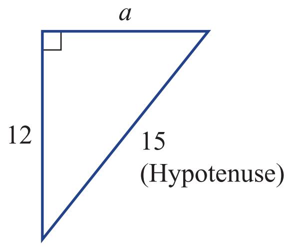


## Let's start: What's the setting out?

The triangle shown has a hypotenuse length of 15 and one shorter side length of 12. Here is the setting out to find the length of the unknown side $a$.

Can you fill in the missing gaps and explain what is happening at each step?

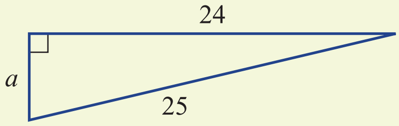
$$
\begin{aligned}
a^2 + b^2 &= c^2\\
a^2 + \underline{\quad}^2 &= \underline{\quad}^2\\
\square \curvearrowleft\ a^2 + \underline{\quad} &= \underline{\quad}\ \curvearrowright \square\\
a^2 &= \underline{\quad}\\
\therefore a &= \sqrt{\underline{\quad}}\\
&= \underline{\quad}
\end{aligned}
$$



## Key ideas

- Pythagoras' Theorem can be used to find the length of the shorter sides of a right-angled triangle if the hypotenuse and another side are known.
- Use subtraction to make the unknown the subject of the equation.

$$
\begin{aligned}
a^2 + b^2 &= c^2\\
a^2 + 24^2 &= 25^2\\
-576 \curvearrowleft\ a^2 + 576 &= 625\ \curvearrowright -576\\
a^2 &= 49\\
\therefore a &= \sqrt{49}\\
&= 7
\end{aligned}
$$



## Exercise 3K

Understanding

**1** Find the value of $a$ in these equations. (Assume $a$ is a positive number.)

**a** $a^2 = 16$  **b** $a^2 = 49$  **c** $a^2 + 16 = 25$

**d** $a^2 + 9 = 25$  **e** $a^2 + 36 = 100$  **f** $a^2 + 441 = 841$

**g** $10 + a^2 = 19$  **h** $6 + a^2 = 31$  **i** $25 + a^2 = 650$

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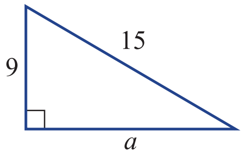
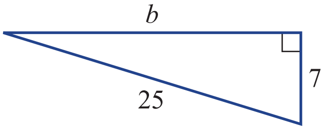
**2** Copy and complete the missing steps.

**a**



$$\begin{aligned} a^2 + b^2 &= c^2\\ a^2 + 9^2 &= \underline{\qquad}\\ a^2 + \underline{\qquad} &= 225\\ a^2 &= \underline{\qquad}\\ \therefore a &= \sqrt{\underline{\qquad}}\\ &= \underline{\qquad} \end{aligned}$$

**b**

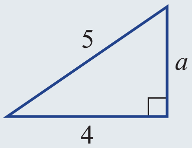


$$\begin{aligned} a^2 + b^2 &= c^2\\ 7^2 + b^2 &= \underline{\qquad}\\ \underline{\qquad} + b^2 &= \underline{\qquad}\\ b^2 &= 576\\ \therefore b &= \sqrt{\underline{\qquad}}\\ &= \underline{\qquad} \end{aligned}$$

**Fluency**

## Example 28 Finding the length of a shorter side

Find the value of $a$ in this right-angled triangle.



**Solution**

$$\begin{aligned} a^2 + b^2 &= c^2\\ a^2 + 4^2 &= 5^2\\ a^2 + 16 &= 25\\ -16 \quad a^2 &= 9 \quad -16\\ \therefore a &= \sqrt{9}\\ &= 3 \end{aligned}$$

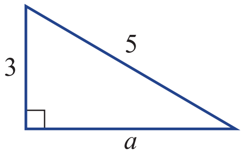
**Explanation**

Write the equation for Pythagoras' Theorem and substitute the known values.

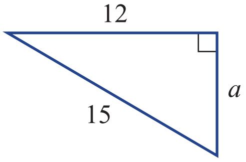
Subtract 16 from both sides.

**3** Find the lengths of the unknown sides in these right-angled triangles.

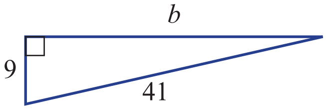
**a**



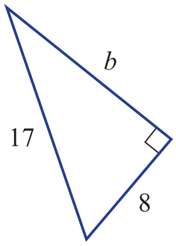
**b**



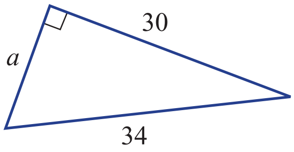
**c**



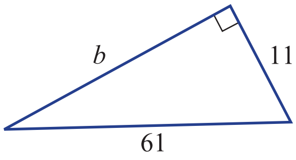
**d**



**e**

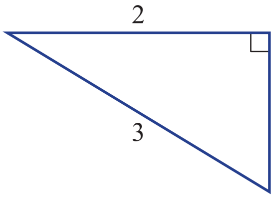


**f**

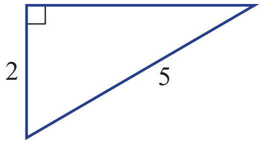


**4** Find the lengths of the unknown sides in these right-angled triangles, giving the answer correct to 2 decimal places.

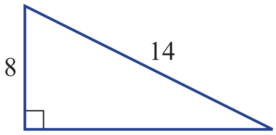
**a**



**b**

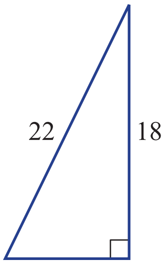


**c**

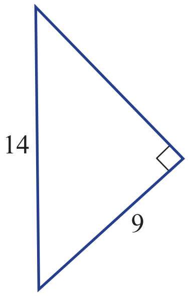


Drilling for Gold 3K1

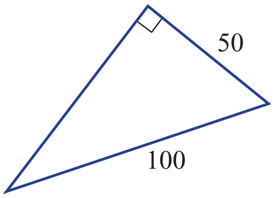
**d**



**e**



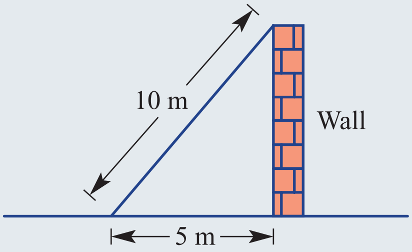
**f**



**Problem-solving and Reasoning**

## Example 29 Applying Pythagoras to find a shorter side

A 10 m steel brace holds up a concrete wall. The bottom of the brace is 5 m from the base of the wall. Find the height of the concrete wall, correct to 2 decimal places.



| Solution | Explanation |
|---|---|
| Let $a$ metres be the height of the wall. | Choose a pronumeral for the unknown height. |
| $a^2 + b^2 = c^2$ | |
| $a^2 + 5^2 = 10^2$ | Substitute into Pythagoras' Theorem. |
| $-25$ $a^2 + 25 = 100$ $-25$<br>$a^2 = 75$ | Subtract 25 from both sides. |
| $\therefore a = \sqrt{75}$ | $\sqrt{75}$ is the exact answer. |
| $= 8.66$ (to 2 decimal places) | Round as required. |
| The height of the wall is 8.66 metres. | Answer a worded problem using a full sentence. |

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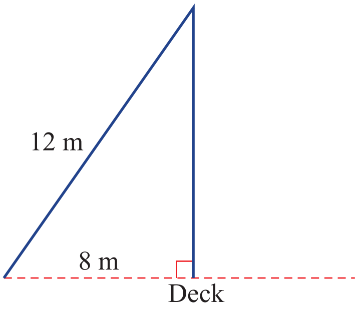
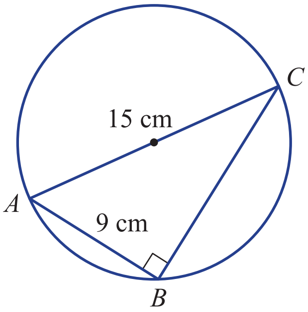
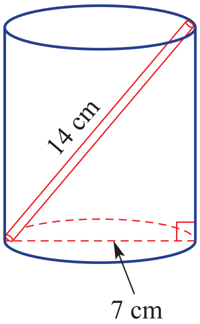
**5** A yacht's mast is supported by a 12 m cable attached to its top. On the deck of the yacht, the cable is 8 m from the base of the mast. How tall is the mast? Round the answer to 2 decimal places.



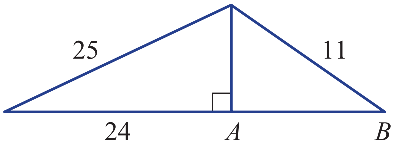
**6** A circle's diameter $AC$ is 15 cm and the chord $AB$ is 9 cm. Angle $ABC$ is 90°. Find the length of the chord $BC$.



**7** A 14 cm drinking straw just fits into a can as shown. The diameter of the can is 7 cm. Find the height of the can, correct to 2 decimal places.



**8** Find the length $AB$ is this diagram. Round to 2 decimal places.



**9** Describe what is wrong with the second line of working in each step.

**a** $a^2 + 10 = 24$
$a^2 = 34$

**b** $a^2 = 25$
$= 5$

**c** $a^2 + 25 = 36$
$a + 5 = 6$

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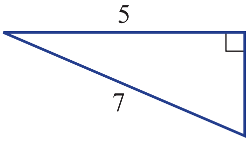
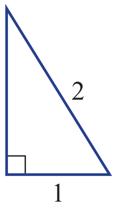
**10** The number $\sqrt{11}$ is an example of a surd that is written as an exact value. Find the surd that describes the exact lengths of the unknown sides of these triangles.

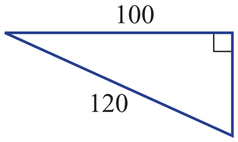
**a**



**b**

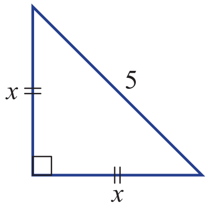


**c**



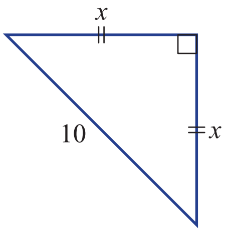
**11** Show how Pythagoras' Theorem can be used to find the unknown lengths in these isosceles triangles. Complete the solution for part **a** and then try the others. Round to 2 decimal places.

**a**

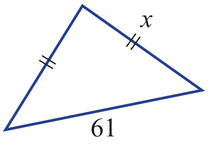


$$a^2 + b^2 = c^2$$
$$x^2 + x^2 = 5^2$$
$$2x^2 = 25$$
$$x^2 = \_\_\_$$
$$\therefore\ x = \sqrt{\_\_\_}$$

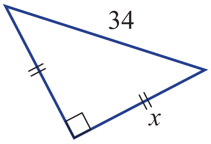
**b**



**c**



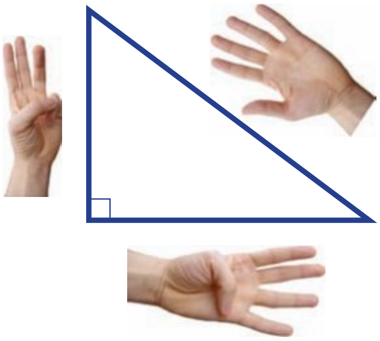
**d**



## Enrichment: Pythagorean families

**12** (3, 4, 5) is called a Pythagorean triad because the numbers 3, 4 and 5 satisfy Pythagoras' Theorem ($3^2 + 4^2 = 5^2$).

- **a** Explain why (6, 8, 10) is also a Pythagorean triad.
- **b** Explain why (6, 8, 10) is considered to be in the same family as (3, 4, 5).
- **c** List three other Pythagorean triads in the same family as (3, 4, 5) and (6, 8, 10).
- **d** Find another triad not in the same family as (3, 4, 5), but which has all three numbers less than 20.
- **e** List five triads that are each the smallest triad of five different families.



3, 4, 5 is the best known of an infinite number of Pythagorean triads.

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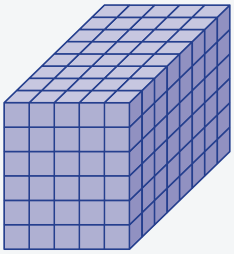
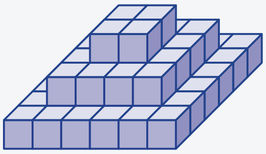
# Puzzles and games

**1** How many cubes are in each solid stack?

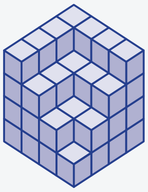
**a**



**b**

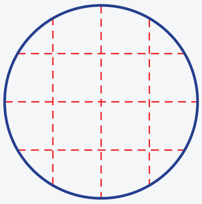


**c**

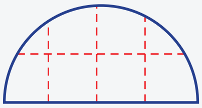


**2** Estimate the areas of these shapes by counting squares.

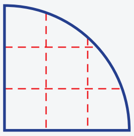
**a**



**b**



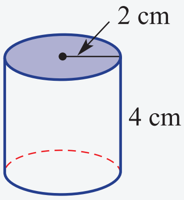
**c**



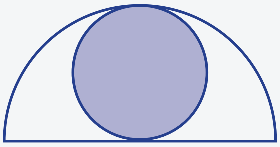
**3** A cube has capacity 1 L. What are its dimensions in cm?

**4** A fish tank is 60 cm long, 30 cm wide, 40 cm high and contains 70 L of water. A rock with a volume of 3000 cm$^3$ is placed into the tank. Will the tank overflow?

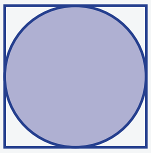
**5** Find the total surface area of this cylinder.



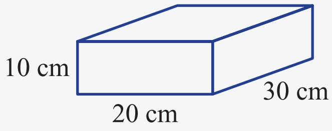
**6** What proportion (fraction or percentage) of the semicircle does the full circle occupy?



**7** A circle just fits inside a square. What percentage of the square is occupied by the circle?



**8** 1.8 L of water is poured into this container. What will be the depth of the water?



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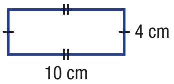
# Chapter summary

## Measurement and Pythagoras' Theorem

### Length

#### Perimeter



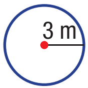
$P = 2 \times 10 + 2 \times 4$
$= 28$ cm

#### Length units

1 km = 1000 m
1 m = 100 cm
1 cm = 10 mm

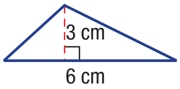
#### Circumference of a circle

$C = 2\pi r$ or $\pi d$
$= 2 \times \pi \times 3$
$= 6\pi$ m$^2$



### Area

#### Triangle



$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 6 \times 3$
$= 9$ cm$^2$

#### Quadrilaterals

- Square $A = s^2$
- Rectangle $A = \ell b$
- Parallelogram $A = bh$
- Rhombus $A = \frac{1}{2}xy$
- Kite $A = \frac{1}{2}xy$
- Trapezium $A = \frac{1}{2}h(a + b)$

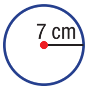
#### Area units

$\times 1000^2$ $\times 100^2$ $\times 10^2$
km$^2$ m$^2$ cm$^2$ mm$^2$
$\div 1000^2$ $\div 100^2$ $\div 10^2$

1 ha = 10 000 m$^2$

#### Circle area

$A = \pi r^2$
$= \pi \times 7^2$
$= 49\pi$ cm$^2$



### Time

1 min = 60 s
1 h = 60 min
1day = 24h
1week = 7days
1 year = 12 months

0311 is 03:11 a.m.
2049 is 08:49 p.m.

### Volume/capacity

#### Volume/capacity units

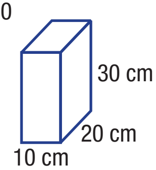
km$^3$, m$^3$, cm$^3$, mm$^3$

$\times 1000$ $\times 1000$ $\times 1000$
mL kL L mL
$\div 1000$ $\div 1000$ $\div 1000$

1 mL = 1 cm$^3$
1 m$^3$ = 1000 L

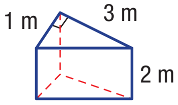
#### Rectangular prism

$V = \ell bh$
$= 10 \times 20 \times 30$
$= 6000$ cm$^3$
$= 6$ L



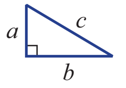
#### Triangular prism

$A = \frac{1}{2}\ bh$
$= \frac{1}{2} \times 1 \times 3$
$= 1\frac{1}{2}$ m$^2$
$V = Ah$
$= 1\frac{1}{2} \times 2$
$= 3$ m$^3$



### Pythagoras' Theorem

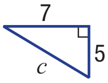
#### Theorem



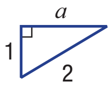
$a^2 + b^2 = c^2$

#### Finding $c$

$c^2 = 5^2 + 7^2$
$= 74$
$\therefore c = \sqrt{74}$



#### A shorter side

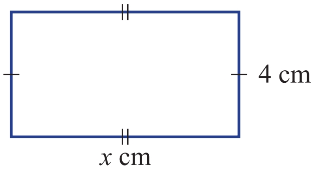


$a^2 + 1^2 = 2^2$
$a^2 + 1 = 4$
$a^2 + 1 - 1 = 4 - 1$
$a^2 = 3$
$a = \sqrt{3}$

**T** **Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.**

## Multiple-choice questions

**1** The perimeter of this rectangle is 20 cm.



The unknown value $x$ is:

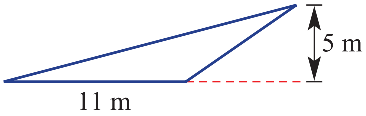
**A** 4 **B** 16 **C** 5 **D** 10 **E** 6

**2** A wheel has a diameter of 2 m. Its circumference and area (in that order) are given by:

**A** $\pi, \pi^2$ **B** $2\pi, \pi$ **C** $4\pi, 4\pi$ **D** 2, 1 **E** 4, 4

**3** The area of this triangle is:

**A** 27.5 $m^2$ **B** 55 m **C** 55 $m^2$
**D** 110 m **E** 16 $m^2$



**4** Using $\pi = 3.14$, the area of a circular oil slick with radius 100 m is:

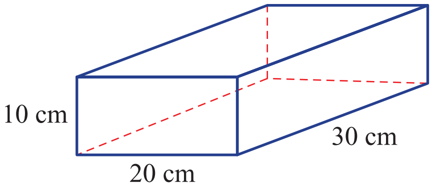
**A** 7850 $m^2$ **B** 314 $m^2$ **C** 31 400 $m^2$ **D** 78.5 $m^2$ **E** 628 $m^2$

**5** 2.5 L is the same as:

**A** 250 mL **B** 2500 mL **C** 1 ML **D** 0.025 kL **E** 25 000 mL

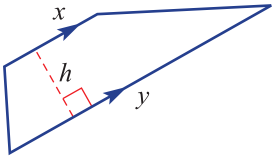
**6** The volume of this rectangular prism is:

**A** 60 L **B** 60 cm **C** 6 $m^3$ **D** 600 $cm^3$ **E** 6000 $cm^3$



**7** The rule for the area of the trapezium shown is:

**A** $\frac{1}{2}xh$ **B** $\frac{1}{2}(x+y)$ **C** $\frac{1}{2}xy$ **D** $\pi xy^2$ **E** $\frac{1}{2}h(x+y)$



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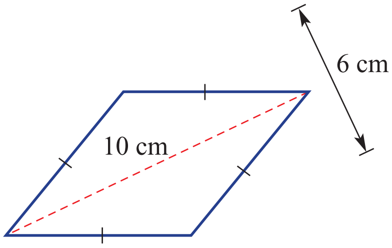
**8** The volume of a rectangular prism is 48 cm$^3$. If its breadth is 4 cm and height 3 cm, its length would be:
**A** 3 cm **B** 4 cm **C** 2 cm **D** 12 cm **E** 96 cm

**9** The diagonals of a rhombus measure 10 cm and 6 cm. Its area is:
**A** 120 cm$^2$ **B** 16 cm$^2$ **C** 15 cm$^2$
**D** 30 cm$^2$ **E** 60 cm$^2$

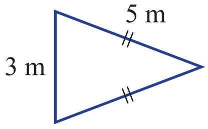


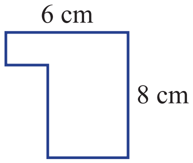
**10** A square has area 49 m$^2$. Its side length is:
**A** 5 m **B** 8 m **C** 49 m
**D** 7 m **E** 4 m

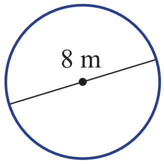
## Short-answer questions

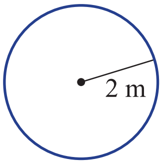
**1** Convert these measurements to the units given in the brackets.
**a** 2 m (mm) **b** 50 000 cm (m) **c** 320 m (km) **d** 0.04 km (m)
**e** 3 cm$^2$ (mm$^2$) **f** 4000 cm$^2$ (m$^2$) **g** 0.01 km$^2$ (m$^2$) **h** 350 mm$^2$ (cm$^2$)
**i** 4000 mL (L) **j** 3 cm$^3$ (mm$^3$) **k** 400 cm$^3$ (L) **l** 4300 kL (ML)

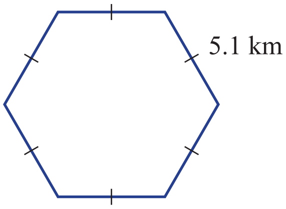
**2** Find the perimeters/circumferences of these shapes. Round the answers to 2 decimal places where necessary.

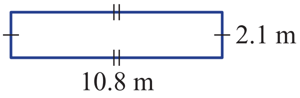
**a**


**b**


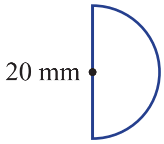
**c**


**d**


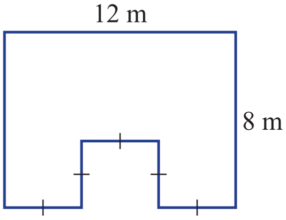
**e**


**f**


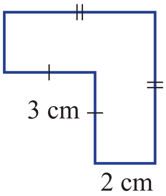
**g**



**h**

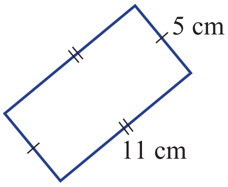


**i**

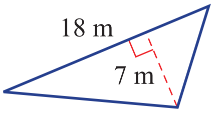


**3** Find the areas of these shapes. Round the answers to 2 decimal places where necessary.

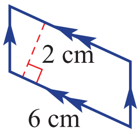
**a**



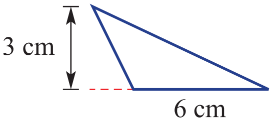
**b**



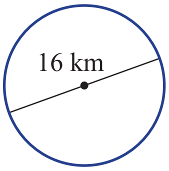
**c**



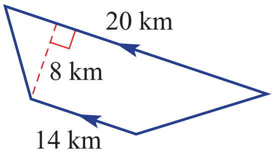
**d**



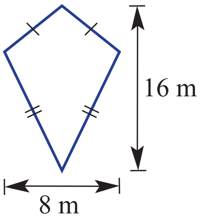
**e**



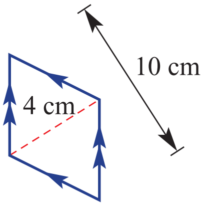
**f**



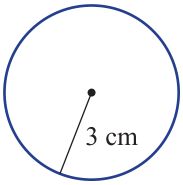
**g**



**h**

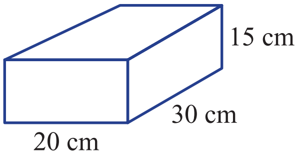


**i**

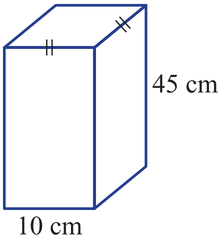


**4** Find the volumes of these rectangular prisms in litres. Recall 1 L = 1000 $\text{cm}^3$.

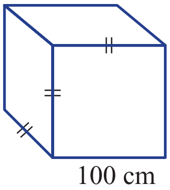
**a**



**b**



**c**



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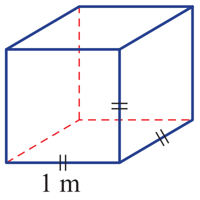
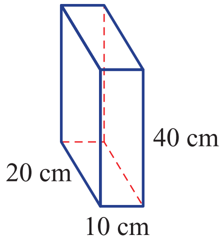
**5** Find the volume of each prism.

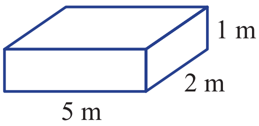
**a**



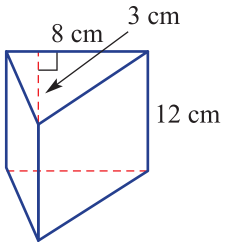
**b**



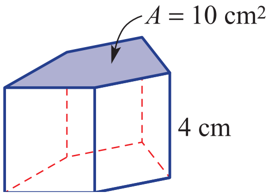
**c**



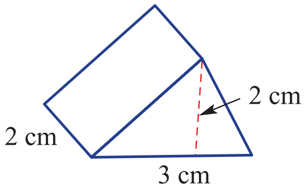
**d**



**e**

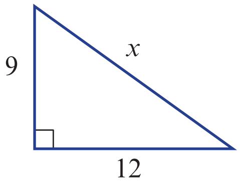


**f**

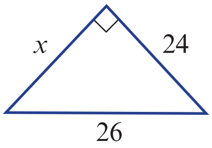


**6** Use Pythagoras' Theorem to find the values of $x$.

**a**



**b**



**7** **a** What is the time difference between 4:20 a.m. and 2:37 p.m.?
**b** Write 2145 hours in a.m./p.m. time.
**c** Write 11:31 p.m. in 24-hour time.

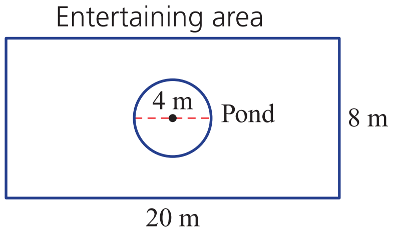
**8** When it is 4:30 p.m. in Western Australia, state the time in each of these places.

**a** New South Wales
**b** Adelaide
**c** Darwin
**d** China
**e** Perth
**f** Phillipines
**g** New Zealand
**h** Tasmania
**i** Queensland

## Extended-response questions

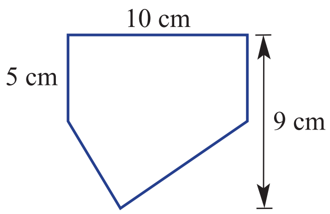
**1** A rectangular entertaining area is to be tiled with square tiles 10 cm by 10 cm. The entertaining area is 20 m by 8 m.
A circular pond of diameter 4 m is to be built in the centre.

- **a** Find the total area of the entertaining area in $m^2$.
- **b** Find the perimeter of the entertaining area.
- **c** Find the area of the pond, correct to 2 decimal places.
- **d** Find the area to be tiled (not including the pond area), correct to 2 decimal places.
- **e** Find the area of one tile in:
  - **i** $cm^2$
  - **ii** $m^2$
- **f** Find the minimum number of tiles required for the job.
- **g** Why would a tiler need more tiles than the minimum number?

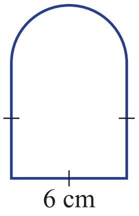


**2** Find the areas of these composite shapes.

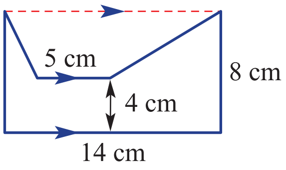
**a**

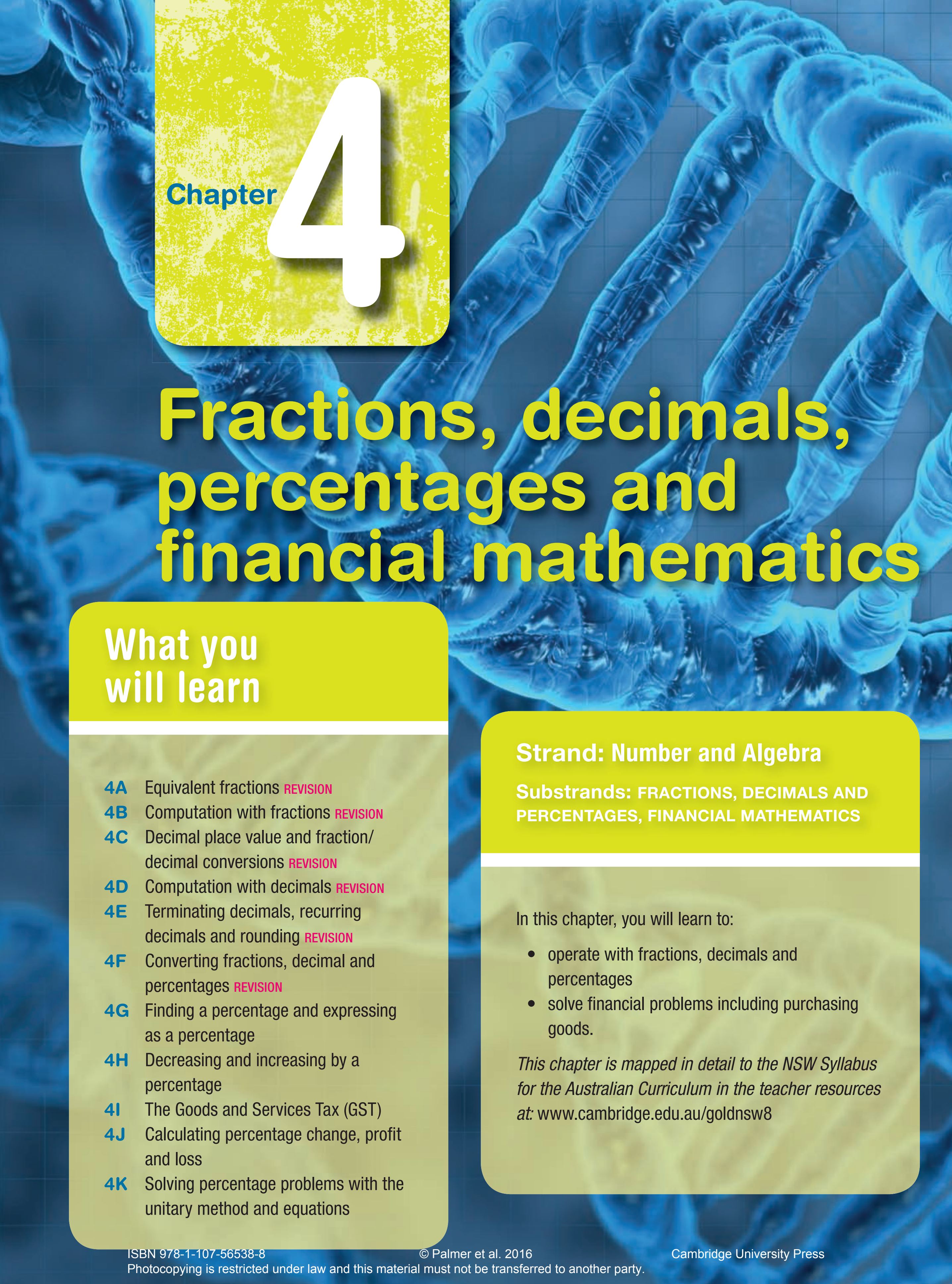


**b**



**c**





# Chapter 4

# Fractions, decimals, percentages and financial mathematics

## What you will learn

### Strand: Number and Algebra

**Substrands:** FRACTIONS, DECIMALS AND PERCENTAGES, FINANCIAL MATHEMATICS

In this chapter, you will learn to:

- operate with fractions, decimals and percentages
- solve financial problems including purchasing goods.

*This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at:* www.cambridge.edu.au/goldnsw8

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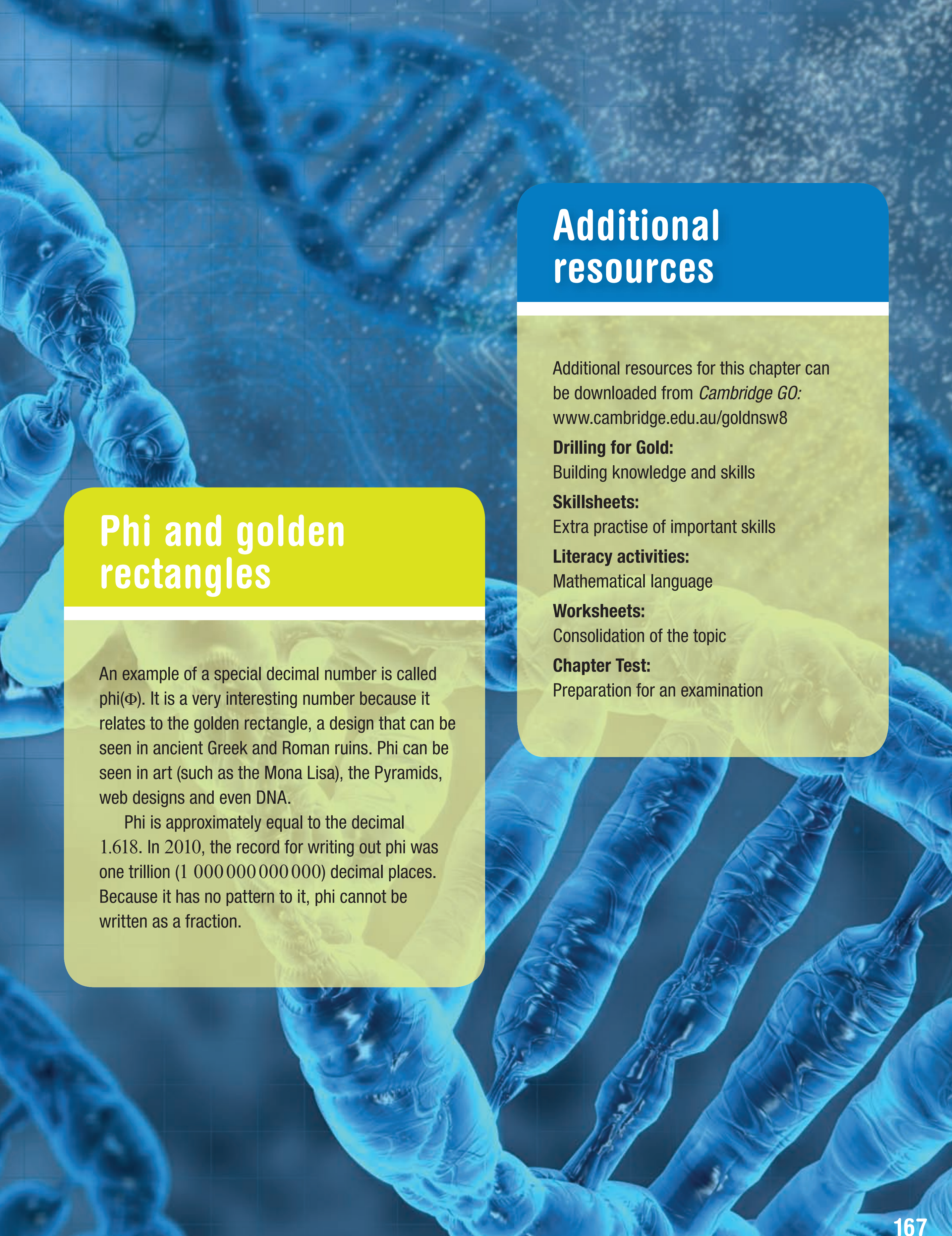


## Phi and golden rectangles

An example of a special decimal number is called phi($\Phi$). It is a very interesting number because it relates to the golden rectangle, a design that can be seen in ancient Greek and Roman ruins. Phi can be seen in art (such as the Mona Lisa), the Pyramids, web designs and even DNA.

Phi is approximately equal to the decimal $1.618$. In $2010$, the record for writing out phi was one trillion ($1\ 000\,000\,000\,000$) decimal places. Because it has no pattern to it, phi cannot be written as a fraction.

## Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO:* www.cambridge.edu.au/goldnsw8

**Drilling for Gold:**
Building knowledge and skills

**Skillsheets:**
Extra practise of important skills

**Literacy activities:**
Mathematical language

**Worksheets:**
Consolidation of the topic

**Chapter Test:**
Preparation for an examination

# Pre-test

**1** Match the following words to the types of fractions: whole number, improper fraction, proper fraction, mixed numeral

**a** $1\frac{2}{5}$ **b** $\frac{3}{7}$ **c** $5$ **d** $\frac{7}{4}$

**2** How many quarters are in:

**a** 1 whole? **b** 2 wholes? **c** 5 wholes?

**3** Complete the following.

**a** $1\frac{1}{2} = \frac{\square}{2}$ **b** $2\frac{1}{4} = \frac{\square}{4}$ **c** $1\frac{2}{3} = \frac{5}{\square}$ **d** $1\frac{3}{5} = \frac{\square}{5}$

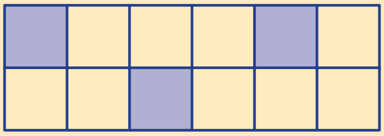
**4** Fill in the blanks.

**a** $\frac{3}{4} = \frac{75}{\square}$ **b** $\frac{3}{6} = \frac{\square}{2}$ **c** $\frac{2}{3} = \frac{\square}{6}$ **d** $\frac{20}{100} = \frac{\square}{5}$

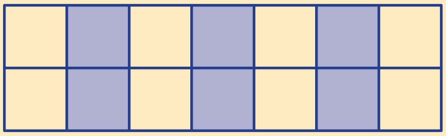
**e** $\frac{3}{10} = \frac{\square}{100}$ **f** $\frac{3}{5} = \frac{\square}{100}$ **g** $\frac{7}{20} = \frac{35}{\square}$ **h** $\frac{1}{25} = \frac{\square}{100}$

**5** What fraction is shaded?

**a**



**b**



**6** Match the fractions on the left-hand side to their decimal form on the right.

| | |
|---|---|
| **a** $\frac{1}{2}$ | **A** 3.75 |
| **b** $\frac{1}{100}$ | **B** 0.25 |
| **c** $\frac{3}{20}$ | **C** 0.01 |
| **d** $3\frac{3}{4}$ | **D** 0.5 |
| **e** $\frac{1}{4}$ | **E** 0.15 |

**7** Find:

**a** $\frac{1}{2} + \frac{1}{4}$ **b** $0.5 + \frac{1}{2}$ **c** $3 - 1\frac{1}{3}$

**d** $0.3 + 0.2 + 0.1$ **e** $2.4 \div 2$ **f** $0.5 \times 6$

**8** Write as: **i** simple fractions **ii** decimals.

**a** 10% **b** 25% **c** 50% **d** 75%

**9** Find 10% of:

**a** \$50 **b** \$66 **c** 8 km **d** 6900 m

**10** Find:

**a** 25% of 40 **b** 75% of 24 **c** 90% of \$1

**11** Copy and complete the following table.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Fraction** | $\frac{3}{4}$ | | | $\frac{2}{5}$ | | | | 2 |
| **Decimal** | | 0.2 | | | 0.99 | | 1.6 | |
| **Percentage** | | | 15% | | | 100% | | |

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# 4A Equivalent fractions

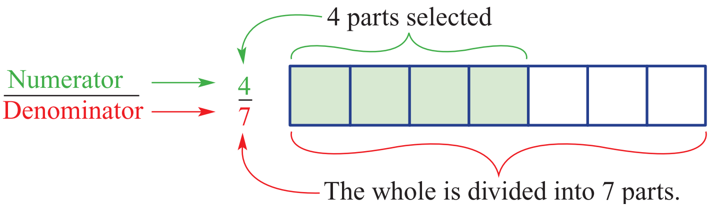
REVISION

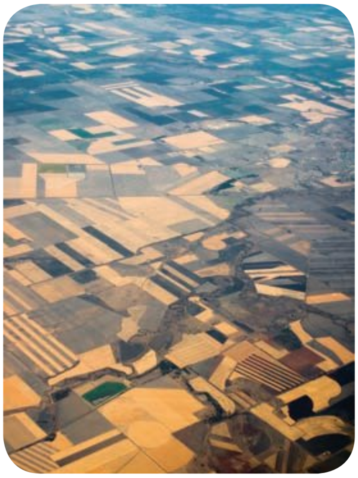
Fractions are made when whole numbers are divided into equal parts.

This diagram shows the parts of a fraction.

4 → n**u**merator: parts taken from the whole
7 → **d**enominator: number of equal parts the whole is broken into

Think ‘**u**’ for ‘**u**p the top’ and ‘**d**’ for ‘**d**own the bottom’.

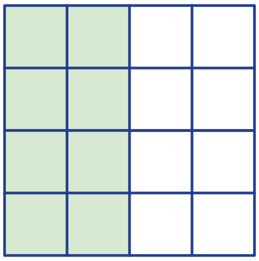




Aerial view of farmland. Each paddock is a fraction of the farmer’s land.

There are 7 equal parts in the whole and 4 of them are shaded.

Equivalent fractions are fractions that represent equal portions of a whole amount and so are equal in value. The skill of generating equivalent fractions is needed whenever you add or subtract fractions with different denominators.



$$\underbrace{\frac{8}{16} = \frac{2}{4} = \frac{1}{2}}$$

These equivalent fractions all mean one half.

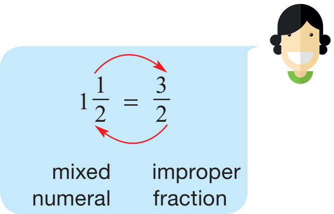
Fractions are very important whenever we measure or compare. Chefs use them when baking. Builders use them when mixing concrete. Musicians use fractions when composing music.

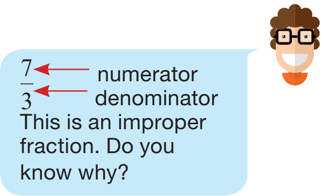
## Let’s start: Know your terminology

It is important to know and understand key terms associated with the study of fractions.

As a class give a definition or example of each of the following key terms.

- Numerator
- Equivalent fraction
- Improper fraction
- Multiples
- Lowest common multiple
- Lowest common denominator
- Denominator
- Proper fraction
- Mixed numeral
- Factors
- Highest common factor
- Vinculum





This is an improper fraction. Do you know why?

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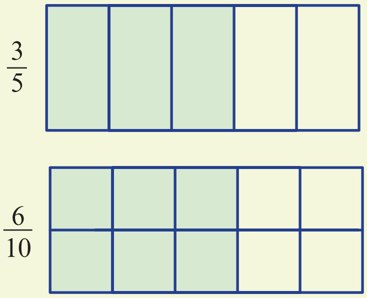
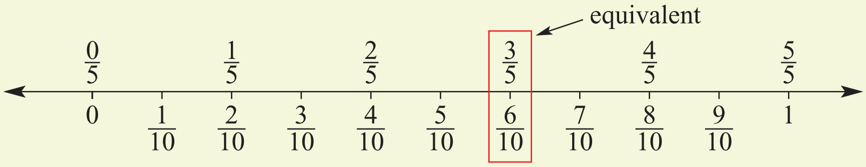
## Key ideas

**Equivalent** Equal in value

**Simplify** Find the simplest possible expression for

- **Equivalent** fractions are equal in value. They mark the same place on a number line or cover the same space in a shape.
  For example, $\frac{3}{5}$ and $\frac{6}{10}$ are equivalent fractions.





- Equivalent fractions are made by multiplying or dividing the numerator **and** denominator by the same number.
  e.g.

$$\underset{\times 2}{\overset{\times 2}{\frac{3}{4} = \frac{6}{8}}} \qquad \underset{\times 5}{\overset{\times 5}{\frac{2}{7} = \frac{10}{35}}} \qquad \underset{\div 3}{\overset{\div 3}{\frac{6}{21} = \frac{2}{7}}}$$

- A fraction can be **broken down** (**simplified**) if the top (numerator) **and** bottom (denominator) have a common factor, other than one.
  e.g.

$$\underset{\div 6}{\overset{\div 6}{\frac{12}{18} = \frac{2}{3}}} \quad \text{OR} \quad \frac{12}{18} = \frac{2 \times 6}{3 \times 6} = \frac{2}{3}$$

The HCF of 12 and 18 is 6.

$\frac{6}{6}$ 'cancels' to 1 because $6 \div 6 = 1$

- Two fractions are equivalent if they have the same simplest form.

e.g. $\frac{6}{8} = \frac{3}{4}$ and $\frac{30}{40} = \frac{3}{4}$ $\quad \therefore \frac{6}{8}$ is equivalent to $\frac{30}{40}$

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## Exercise 4A

**Understanding**

**1** Fill in the missing numbers to complete the following strings of equivalent fractions.

**a**

$$\begin{tikzcd} \frac{3}{5} \arrow[rr, bend left, "\times 2"] \arrow[rrrr, bend left=50, "\times 3"] \arrow[rr, bend right, "\times 2"'] \arrow[rrrr, bend right=50, "\times 3"'] & = & \frac{\square}{\square} & = & \frac{\triangle}{\triangle} \end{tikzcd}$$

**b**

$$\begin{tikzcd} \frac{4}{7} \arrow[rr, bend left, "\times 2"] \arrow[rr, bend right, "\times 2"'] & = & \frac{8}{\square} \arrow[rr, bend left, "\times 2"] \arrow[rr, bend right, "\times 2"'] & = & \frac{\square}{28} \arrow[rr, bend left, "\times 2"] \arrow[rr, bend right, "\times 2"'] & = & \frac{\square}{\square} \end{tikzcd}$$

Remember always
× or ÷ the top and
the bottom by the
same number!
$\frac{a}{a} = 1$

**c** $\frac{50}{100} = \frac{25}{\square} = \frac{10}{\square} = \frac{\square}{10} = \frac{1}{\square}$

**d** $\frac{1}{3} = \frac{2}{\square} = \frac{3}{\square} = \frac{4}{\square}$

**2** Which fraction is equivalent to $\frac{2}{3}$: $\frac{10}{15}$ or $\frac{3}{4}$?

**3** Which three of the following fractions can be broken down (simplified)?

**a** $\frac{3}{7}$ **b** $\frac{10}{12}$ **c** $\frac{8}{6}$ **d** $\frac{5}{9}$ **e** $\frac{3}{9}$

**4** Are the following statements true or false?

**a** $\frac{1}{2}$ and $\frac{1}{4}$ are equivalent fractions.

**b** $\frac{3}{6}$ and $\frac{1}{2}$ are equivalent fractions.

**c** The fraction $\frac{8}{9}$ is written in its simplest form.

**d** $\frac{14}{21}$ can be simplified to $\frac{2}{3}$.

**e** $\frac{11}{99}$ and $\frac{1}{9}$ and $\frac{2}{18}$ are all equivalent fractions.

**f** $\frac{4}{5}$ can be simplified to $\frac{2}{5}$.

## Example 1 Generation equivalent fractions

Rewrite the following fractions with a denominator of 20.

**a** $\frac{3}{5}$ **b** $\frac{1}{2}$ **c** $\frac{36}{120}$

| Solution | Explanation |
|---|---|
| **a** $\frac{3}{5} = \frac{12}{20}$ | $\underset{\times 4}{\overset{\times 4}{\frac{3}{5} = \frac{\square}{20}}}$ Multiply the fraction by $\frac{4}{4}$, which is 1. |
| **b** $\frac{1}{2} = \frac{10}{20}$ | $\underset{\times 10}{\overset{\times 10}{\frac{1}{2} = \frac{\square}{20}}}$ Multiply numerator and denominator by 10. |
| **c** $\frac{36}{120} = \frac{6}{20}$ | $\underset{\div 6}{\overset{\div 6}{\frac{36}{120} = \frac{\square}{20}}}$ Divide top and bottom by 6. |

**5** Rewrite the following fractions with a denominator of 24.

**a** $\frac{1}{3} = \frac{\square}{24}$ **b** $\frac{2}{8} = \frac{\square}{24}$ **c** $\frac{1}{2} = \frac{\square}{24}$ **d** $\frac{5}{12} = \frac{\square}{24}$

**e** $\frac{5}{6} = \frac{\square}{24}$ **f** $\frac{5}{1} = \frac{\square}{24}$ **g** $\frac{3}{4} = \frac{\square}{24}$ **h** $\frac{7}{8} = \frac{\square}{24}$

Multiply or divide top and bottom by the same number.

**6** Rewrite the following fractions with a denominator of 30.

**a** $\frac{1}{5} = \frac{\square}{30}$ **b** $\frac{2}{6} = \frac{\square}{30}$ **c** $\frac{5}{10} = \frac{\square}{30}$ **d** $\frac{3}{1} = \frac{\square}{30}$

**e** $\frac{2}{3} = \frac{\square}{30}$ **f** $\frac{22}{60} = \frac{\square}{30}$ **g** $\frac{5}{2} = \frac{\square}{30}$ **h** $\frac{150}{300} = \frac{\square}{30}$

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**7** Find the missing value to make the equation true.

**a** $\frac{1}{5} = \frac{\square}{10}$ **b** $\frac{1}{5} = \frac{\square}{100}$ **c** $\frac{2}{5} = \frac{4}{\square}$ **d** $\frac{3}{4} = \frac{\square}{40}$

**e** $\frac{2}{3} = \frac{12}{\square}$ **f** $\frac{3}{2} = \frac{6}{\square}$ **g** $\frac{15}{10} = \frac{\square}{2}$ **h** $\frac{90}{100} = \frac{\square}{10}$

**i** $\frac{2}{5} = \frac{\square}{15}$ **j** $\frac{7}{9} = \frac{14}{\square}$ **k** $\frac{7}{14} = \frac{1}{\square}$ **l** $\frac{21}{30} = \frac{\square}{10}$

**m** $\frac{4}{3} = \frac{\square}{21}$ **n** $\frac{8}{5} = \frac{80}{\square}$ **o** $\frac{3}{12} = \frac{\square}{60}$ **p** $\frac{7}{11} = \frac{28}{\square}$

## Example 2 Converting to simplest form

Write the following fractions in simplest form.

**a** $\frac{8}{20}$ **b** $\frac{25}{15}$

| **Solution** | **Explanation** |
|---|---|
| **a** $$\begin{tikzcd} 8 \arrow[rr, bend left, "\div 4"] & & 2 \\ 20 \arrow[r, phantom, "="] \arrow[rr, bend right, "\div 4"'] & & 5 \end{tikzcd}$$ | The HCF of 8 and 20 is 4.<br>Divide the numerator and denominator by 4. |
| **b** $$\begin{tikzcd} 25 \arrow[rr, bend left, "\div 5"] & & 5 \\ 15 \arrow[r, phantom, "="] \arrow[rr, bend right, "\div 5"'] & & 3 \end{tikzcd}$$ | The HCF of 25 and 15 is 5.<br>Divide the numerator and denominator by 5. |

**8** Write the following fractions in simplest form. Use a calculator to check your answer.

**a** $\frac{2}{4}$ **b** $\frac{3}{6}$ **c** $\frac{8}{10}$ **d** $\frac{14}{20}$

**e** $\frac{3}{9}$ **f** $\frac{4}{8}$ **g** $\frac{10}{12}$ **h** $\frac{15}{18}$

**i** $\frac{11}{44}$ **j** $\frac{12}{20}$ **k** $\frac{16}{18}$ **l** $\frac{25}{35}$

**m** $\frac{15}{9}$ **n** $\frac{22}{20}$ **o** $\frac{120}{100}$ **p** $\frac{64}{48}$

First find the HCF of the numerator or denominator.

**9** A calculator gives the simplified fraction after you press equals. Use a calculator to simplify these fractions.

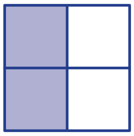
**a** $\frac{36}{40}$ **b** $\frac{16}{12}$ **c** $\frac{14}{56}$

**d** $\frac{28}{52}$ **e** $\frac{32}{48}$ **f** $\frac{156}{312}$

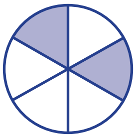
**Problem-solving and reasoning**

**10** Each diagram below shows a fraction of the whole. Write each fraction shaded in more than one way.

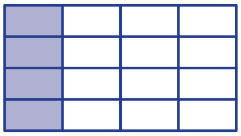
**a**



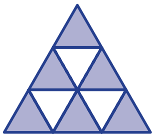
**b**



**c**



**d**



**11** **a** Thomas ate $\frac{1}{4}$ of a 250-gram block of chocolate. Mary ate $\frac{3}{6}$ of her 250-gram block. Who ate the most chocolate?

**b** A pizza is cut into eight equal pieces. Sian ate 2 slices of pizza, Callum had 4 slices. What fraction of the pizza is left?

**12** Write down four fractions that simplify to $\frac{1}{5}$.

**13** Which fraction is the odd one out?

$\frac{75}{100}$ $\frac{15}{20}$ $\frac{18}{28}$ $\frac{3}{4}$ $\frac{12}{16}$

## Enrichment: Bigger or smaller?

**14** If you multiply the numerator by 2 and denominator by 2, you get equivalent fractions.

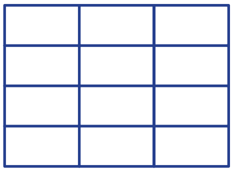
E.g. $\frac{1}{2} \underset{\times 2}{\overset{\times 2}{=}} \frac{2}{4}$

What happens If you *add* 2 to the numerator and denominator?
Does the fraction get bigger or smaller?
(Hint: Try a variety of fractions, including some improper fractions.)

# 4B Computation with fractions REVISION

This section reviews the different techniques involved in adding, subtracting, multiplying and dividing fractions.
Proper fractions, improper fractions and mixed numerals will be considered for each of the four mathematical operations.

## Let's start: Shading fractions



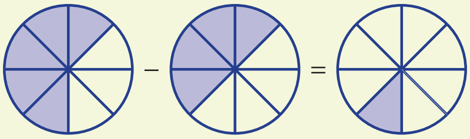
In pairs, draw and shade this grid to evaluate:

- $\frac{1}{2} + \frac{1}{3}$
- $\frac{7}{12} - \frac{1}{3}$
- $\frac{1}{2}$ of $\frac{1}{2}$

What rules do you know about adding, subtracting, multiplying and dividing fractions?

## Key ideas

- Adding and subtracting fractions
  - To add or subtract fractions, convert to the same denominator. When the denominator is the same, just add or subtract the numerator.

e.g. 

$$\frac{5}{8} - \frac{4}{8} = \frac{1}{8}$$

  - The **lowest common multiple** of the denominators is used if the denominators are different. This is called the **lowest common denominator** (LCD).

$$\text{e.g. } \frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$$
$$= \frac{5}{6}$$

**Lowest common multiple (LCM)** The smallest number that two or more numbers divide into evenly

**Lowest common denominator** The lowest common multiple of the denominators of two or more fractions

- Multiplying fractions
  - Convert to improper fractions.
  - Multiply the numerators.
  - Multiply the denominators.
  - Simplify your answer.

$$\text{e.g. } 1\frac{1}{2} \times \frac{4}{7} = \frac{3}{2} \times \frac{4}{7}$$
$$= \frac{12}{14}$$
$$= \frac{6}{7}$$

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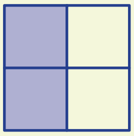
**Reciprocal** The result of swapping the numerator and denominator

- Dividing fractions
  - To divide by a fraction, turn that fraction sign upside down, then multiply.

    e.g. $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1}$

    $= 2$
  - To turn a fraction $\frac{a}{b}$ upside down $\frac{b}{a}$, is called taking its **reciprocal**.



There are 2 quarters in one half.

## Exercise 4B

Understanding

1 **a** Which two operations require the denominators to be the same?
  **b** Which two operations do not require the denominators to be the same?

2 State the lowest common denominator for the following pairs of fractions.

**a** $\frac{1}{5} + \frac{3}{4}$ **b** $\frac{2}{9} + \frac{5}{3}$ **c** $\frac{11}{25} + \frac{7}{10}$ **d** $\frac{5}{12} + \frac{13}{8}$

3 Copy and complete.

**a** $\frac{2}{3} + \frac{1}{4}$
$= \frac{8}{12} + \frac{\square}{12}$
$= \frac{11}{\square}$

**b** $\frac{7}{8} - \frac{9}{16}$
$= \frac{\square}{16} - \frac{9}{16}$
$= \frac{\square}{16}$

**c** $1\frac{4}{7} \times \frac{3}{5}$
$= \frac{\square}{7} \times \frac{3}{5}$
$= \frac{\square}{35}$

**d** $\frac{5}{7} \div \frac{2}{3}$
$= \frac{5}{7} \square \frac{3}{2}$
$= \frac{15}{\square} = \square\frac{\square}{14}$

4 State the reciprocals of the following fractions.

**a** $\frac{5}{8}$ **b** $\frac{3}{2}$ **c** $3\frac{1}{4}$ **d** $1\frac{1}{11}$

If the fraction is a mixed numeral, convert it to improper.

Fluency

### Example 3 Adding and subtracting fractions

Evaluate:

**a** $\frac{3}{5} + \frac{4}{5}$ **b** $\frac{5}{3} - \frac{3}{4}$

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**Solution**

**Explanation**

**a** $\frac{3}{5}+\frac{4}{5}=\frac{7}{5}$

$=1\frac{2}{5}$

The denominators are the same, simply add the numerators. Three *fifths* plus four *fifths* equals seven *fifths*.
The final answer can be written as a mixed numeral.

**b** $\frac{5}{3}-\frac{3}{4}=\frac{20}{12}-\frac{9}{12}$

$=\frac{11}{12}$

LCM of 3 and 4 is 12.
Write equivalent fractions with a LCD of 12.
The denominators are now the same, so subtract the numerators.

**5** Evaluate, then check your answers with a calculator.

**a** $\frac{1}{3}+\frac{1}{3}$ **b** $\frac{1}{3}+\frac{1}{6}$ **c** $\frac{7}{12}-\frac{1}{2}$ **d** $\frac{11}{10}-\frac{7}{10}$

**e** $\frac{1}{5}+\frac{2}{5}$ **f** $\frac{7}{9}-\frac{2}{9}$ **g** $\frac{5}{8}+\frac{7}{8}$ **h** $\frac{24}{7}-\frac{11}{7}$

**i** $\frac{3}{4}+\frac{2}{5}$ **j** $\frac{3}{10}+\frac{4}{5}$ **k** $\frac{5}{7}-\frac{2}{3}$ **l** $\frac{11}{18}-\frac{1}{6}$

Look at denominators first! Same or different?

## Example 4 Adding and subtracting mixed numerals

Evaluate:

**a** $3\frac{5}{8}+2\frac{3}{4}$ **b** $2\frac{1}{2}-1\frac{5}{6}$

**Solution**

**Explanation**

**a** $3\frac{5}{8}+2\frac{3}{4}=\frac{29}{8}+\frac{11}{4}$

$=\frac{29}{8}+\frac{22}{8}$

$=\frac{51}{8}$

$=6\frac{3}{8}$

Convert mixed numerals to improper fractions.
The LCM of 8 and 4 is 8.
Write equivalent fractions with LCD.
Add numerators together, denominator remains the same. Convert the answer back to a mixed numeral.

**b** $2\frac{1}{2}-1\frac{5}{6}=\frac{5}{2}-\frac{11}{6}$

$=\frac{15}{6}-\frac{11}{6}$

$=\frac{4}{6}$

$=\frac{2}{3}$

Convert mixed numerals to improper fractions.
The LCD of 2 and 6 is 6.
Write equivalent fractions with LCD.
Subtract numerators and simplify the answer.

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**6** Evaluate, then check your answers with a calculator.

**a** $3\frac{1}{7}+1\frac{3}{7}$ **b** $7\frac{2}{5}+2\frac{1}{5}$ **c** $3\frac{5}{8}-1\frac{2}{8}$

**d** $8\frac{5}{11}-7\frac{3}{11}$ **e** $5\frac{1}{3}+4\frac{1}{6}$ **f** $17\frac{5}{7}+4\frac{1}{2}$

**g** $6\frac{1}{2}-2\frac{3}{4}$ **h** $4\frac{2}{5}-2\frac{5}{6}$

You can add the wholes first if you like.

$$
\begin{aligned}
3\frac{5}{8}+2\frac{3}{4} &= 3+2+\frac{5}{8}+\frac{3}{4}\\
&= 5+\frac{5}{8}+\frac{6}{8}\\
&= 5+\frac{11}{8} = 6\frac{3}{8}
\end{aligned}
$$

## Example 5 Multiplying fractions

Evaluate:

**a** $\frac{2}{5}\times\frac{3}{7}$ **b** $\frac{8}{5}\times 1\frac{3}{4}$

| **Solution** | **Explanation** |
|---|---|
| **a** $\frac{2}{5}\times\frac{3}{7}=\frac{6}{35}$ | Multiply the numerators.<br>Multiply the denominators.<br>Check that the answer is in its simplest form. |
| **b** $\frac{8}{5}\times 1\frac{3}{4}=\frac{\cancel{8}^{2}}{5}\times\frac{7}{\cancel{4}^{1}}$<br>$=\frac{14}{5}$<br>$=2\frac{4}{5}$ | Convert mixed numerals to improper fraction.<br>Cancel any numerator with any denominator:<br>8 and 4 can both divided by their HCF (4).<br>Multiply the numerators: $2\times 7=14$<br>Multiply the denominators: $5\times 1=5$<br>Convert to a mixed numeral and check it is simplified. |

**7** Evaluate, then check your answers with a calculator.

**a** $\frac{3}{5}\times\frac{1}{4}$ **b** $\frac{2}{9}\times\frac{5}{7}$ **c** $\frac{7}{5}\times\frac{6}{5}$ **d** $\frac{5}{3}\times\frac{8}{9}$

**e** $\frac{4}{9}\times\frac{3}{8}$ **f** $\frac{12}{10}\times\frac{5}{16}$ **g** $\frac{12}{9}\times\frac{2}{5}$ **h** $\frac{24}{8}\times\frac{5}{3}$

Look to cancel first if possible.

**8** Evaluate, then check your answers with a calculator.

**a** $2\frac{3}{4}\times 1\frac{1}{3}$ **b** $3\frac{2}{7}\times\frac{1}{3}$ **c** $4\frac{1}{6}\times 3\frac{3}{5}$ **d** $10\frac{1}{2}\times 3\frac{1}{3}$

## Example 6 Dividing fractions

Evaluate:

**a** $\frac{2}{5} \div \frac{3}{7}$

**b** $2\frac{1}{4} \div 1\frac{1}{3}$

| Solution | Explanation |
|---|---|
| **a** $\frac{2}{5} \div \frac{3}{7} = \frac{2}{5} \times \frac{7}{3}$<br>$= \frac{14}{15}$ | Change ÷ sign to a × sign and invert the divisor.<br>Multiply by the reciprocal.<br>Multiply numerators.<br>Multiply denominators. |
| **b** $2\frac{1}{4} \div 1\frac{1}{3} = \frac{9}{4} \div \frac{4}{3}$<br>$= \frac{9}{4} \times \frac{3}{4}$<br>$= \frac{27}{16}$<br>$= 1\frac{11}{16}$ | Convert mixed numerals to improper fractions.<br>Change ÷ sign to × sign and invert the divisor.<br>The reciprocal of $\frac{4}{3}$ is $\frac{3}{4}$.<br>Multiply.<br>Simplify. |

**9** Evaluate, then check your answers with a calculator.

**a** $\frac{2}{9} \div \frac{3}{5}$ **b** $\frac{1}{3} \div \frac{2}{5}$ **c** $\frac{8}{7} \div \frac{11}{2}$ **d** $\frac{11}{3} \div \frac{5}{2}$

**e** $\frac{3}{4} \div \frac{6}{7}$ **f** $\frac{10}{15} \div \frac{1}{3}$ **g** $\frac{6}{5} \div \frac{9}{10}$ **h** $\frac{22}{35} \div \frac{11}{63}$

Multiply by the reciprocal.
$\frac{2}{9} \div \frac{3}{5} = \frac{2}{9} \times \frac{5}{3}$

**10** Evaluate, then check your answers with a calculator.

**a** $1\frac{4}{7} \div 1\frac{2}{3}$ **b** $3\frac{1}{5} \div 8\frac{1}{3}$ **c** $3\frac{1}{5} \div 2\frac{2}{7}$ **d** $6\frac{2}{4} \div 2\frac{1}{6}$

Convert to improper fractions first.

Skillsheet 4A

### Problem-solving and Reasoning

**11** There are 30 students in my class. How many students are in each group?

**a** $\frac{1}{3}$ of the class had brown hair.

**b** $\frac{1}{2}$ of the class came to school by bus.

**c** $\frac{5}{6}$ of the class spoke English at home.

**d** $\frac{1}{10}$ of the class liked Maths.

**12** Max and Tanya are painting two adjacent walls of equal area.

Max has painted $\frac{3}{7}$ of his wall and Tanya has painted $\frac{2}{5}$ of her wall.

**a** What fraction of the two walls have Max and Tanya painted in total?

**b** What fraction of the two walls remains to be painted?

## Enrichment: Multiple fractions

**13** Use a calculator to evaluate:

**a** $\frac{2}{3} \times \frac{1}{4} \div 1\frac{1}{2}$

**b** $1\frac{2}{3} + 4\frac{4}{5} - \frac{3}{8}$

**c** $1\frac{1}{4} \div \frac{2}{3} - \frac{5}{7}$

**d** $\left(1\frac{1}{2} + 2\frac{2}{3}\right) \times \frac{4}{5}$

**14** What is the lowest common denominator for:
$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$?

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# 4C Decimal place value and fraction/decimal conversions

REVISION

Decimals are fractions in which the denominator is 10 or 100 or 1000 or any power of 10. The decimal point is used to separate the whole number from the fraction.

$$3\frac{17}{100} = 3.17$$

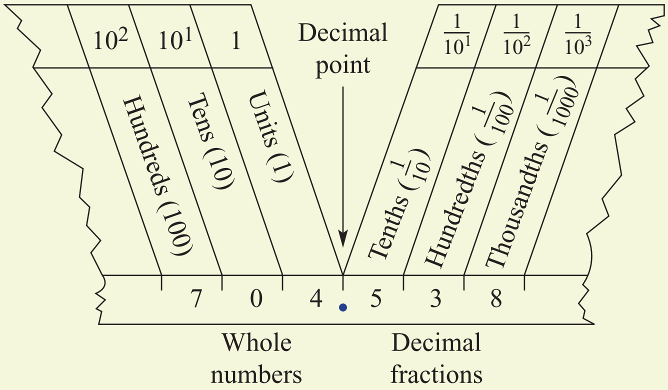
Why do we use base 10?

## Let's start: Decimals around us

List five 'real-life' examples of the use of decimals. Give a specific example for each one giving a decimal number.

## Key ideas

- The **place value** table extended for decimals:



$$704.538 = 704\frac{538}{1000} = 7 \times 100 + 4 \times 1 + \frac{5}{10} + \frac{3}{100} + \frac{8}{1000}$$

- Place value

In 704.538, the place value of the 3 is 3 hundredths or $\frac{3}{100}$ or 0.03.

**Place value** The value of a digit in a number as determined by its position

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- **Comparing and ordering decimals**
  To compare two numbers:
  - line up the decimal points and every digit
  - compare the digits from left to right.

  e.g. Compare 362.581 and 362.549.

  362.581
  362.549
  ↑
  the 1st digit that is different: $8 > 4$
  So $362.581 > 362.549$

- **Converting decimals to fractions (non-calculator)**
  - Count the number of digits to the right of the decimal point.
  - This is the number of zeroes that you must place in the denominator.
  - Simplify the fraction if required.

  e.g. $0.64 = \frac{64}{100} = \frac{16}{25}$

  Note: 0.64 means '64 hundredths'

Drilling for Gold 4C1

- **Converting fractions to decimals (non-calculator)**
  Begin by looking at the denominator.
  - If the denominator is a power of 10, simply change the fraction directly to a decimal from your knowledge of its place value.

  e.g. $\frac{3}{10} = 0.3, \frac{3}{100} = 0.03, \frac{13}{100} = 0.13, \frac{13}{1000} = 0.013$

  - If the denominator is not a power of 10, convert to tenths, hundredths or thousandths and then convert to a decimal.

  e.g. $\frac{3}{20} \overset{\times 5}{\underset{\times 5}{=}} \frac{15}{100} = 0.15$

  - If the above two methods are not suitable, divide the bottom (denominator) into the top (numerator).

  e.g. $\frac{1}{8} = \begin{array}{r} 0.\,1\,2\,5 \\ 8\overline{)1.{}^{1}0{}^{2}0{}^{4}00} \end{array}$

- Calculators can do fraction/decimal conversions.
  Try this on *your* calculator.
  - Convert $3\frac{2}{5}$ to 3.4.
  - Convert 3.25 to $\frac{13}{4}$ to $3\frac{1}{4}$.

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## Exercise 4C

Understanding

**1** Which of the following is the mixed numeral equivalent of 8.17?

**A** $8\frac{1}{7}$ **B** $8\frac{17}{10}$ **C** $8\frac{1}{17}$ **D** $8\frac{17}{1000}$ **E** $8\frac{17}{100}$

**2** Which of the following is the mixed numerals equivalent of 5.75?

**A** $5\frac{75}{10}$ **B** $5\frac{25}{50}$ **C** $5\frac{3}{4}$ **D** $5\frac{15}{25}$ **E** $5\frac{75}{1000}$

### Example 7 Comparing decimals

Which is larger?
57.89342 or 57.89631

| Solution | Explanation |
|---|---|
| 57.89631 is larger. | Write underneath each other, with decimal points lined up.<br>$57.89\textcircled{3}42$<br>$57.89\textcircled{6}31$<br>↑<br>1st digit different from left to right $6 > 3$<br>$\frac{3}{1000} < \frac{6}{1000}$ |

**3** Write down the larger decimal in each pair.

**a** 36.485 37.123
**b** 21.953 21.864
**c** 0.0372 0.0375
**d** 4.21753 4.21809
**e** 65.4112 64.8774
**f** 9.5281352 9.5281347

Compare digits from left to right.

**4** Are these statements true or false?

**a** $\frac{3}{5} = 0.6$
**b** $\frac{11}{20} = \frac{55}{100} = 5.5$
**c** $\frac{1}{100} = 0.01$
**d** $3.6 < 0.36$
**e** $0.504 > 0.54$
**f** $0.65 < 0.645$

### Example 8 Converting decimals to fractions

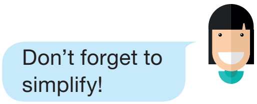
Convert the following decimals to fractions in their simplest form.

**a** 0.007 **b** 5.12

| **Solution** | **Explanation** |
|---|---|
| **a** $0.007 = \frac{7}{1000}$ | Three decimal places, therefore three zeroes in denominator.<br>0.007 means '7 thousandths'. |
| **b** $5.12 = 5\frac{12}{100}$ | Two decimal places, therefore two zeroes in denominator. |
| $= 5\frac{3}{25}$ | 0.12 means '12 hundredths'. |

**5** Convert the following decimals to fractions in their simplest form. Use a calculator to check

**a** 0.3 **b** 0.03 **c** 0.003 **d** 1.3
**e** 0.13 **f** 0.103 **g** 0.013 **h** 0.2
**i** 0.02 **j** 0.25 **k** 0.75 **l** 0.8



### Example 9 Converting fractions to decimals

Convert the following fractions to decimals.

**a** $\frac{239}{100}$ **b** $\frac{9}{25}$

| **Solution** | **Explanation** |
|---|---|
| **a** $\frac{239}{100} = 2\frac{39}{100} = 2.39$ | Convert improper fraction to a mixed numeral.<br>Denominator is a power of 10. |
| **b** $\frac{9}{25} = \frac{36}{100} = 0.36$ | $\frac{9}{25} = \frac{9}{25} \xrightarrow[\times 4]{\times 4} \frac{36}{100}$ |

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**6** Convert the following fractions to decimals.

**a** $\frac{17}{100}$ **b** $\frac{301}{1000}$ **c** $\frac{45}{100}$ **d** $\frac{6}{10}$

**e** $\frac{67}{100}$ **f** $\frac{674}{1000}$ **g** $\frac{15}{100}$ **h** $\frac{79}{100}$

**i** $\frac{7}{10}$ **j** $\frac{17}{10}$ **k** $\frac{118}{100}$ **l** $\frac{41}{1000}$

**7** Convert the following fractions to decimals. Check with a calculator.

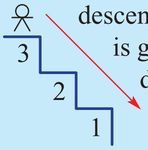
**a** $\frac{3}{5}$ **b** $\frac{1}{2}$ **c** $\frac{3}{2}$ **d** $\frac{7}{5}$

**e** $\frac{11}{50}$ **f** $\frac{1}{4}$ **g** $\frac{3}{4}$ **h** $\frac{32}{50}$

Convert to a denominator of 10, 100 or 1000.

**8** Convert the following mixed numerals to decimals and then place them in descending order.

$2\frac{2}{5}$, $2\frac{1}{4}$, $2\frac{9}{50}$, $2\frac{3}{10}$



descending is going down

from biggest to smallest

**Problem-solving and Reasoning**

**9** The distances from Nam's locker to his six different classrooms are listed below:

- Locker to room B5 (0.186 km)
- Locker to room A1 (0.119 km)
- Locker to room P9 (0.254 km)
- Locker to gym (0.316 km)
- Locker to room C07 (0.198 km)
- Locker to BW Theatre (0.257 km)

List Nam's six classrooms in order of distance of his locker from the closest classroom to the one furthest away.

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**10** Sophie scored:

Maths 60 out of 80
English 38 out of 50
Science 54 out of 75

Use fractions and decimals to rank her results from highest to lowest.

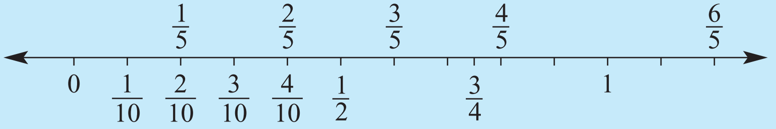
**11** What is the best whole number to place in the box?

$\frac{\square}{60}$ is approximately equal to 0.235.

**12** Write down a *decimal* that lies between the pairs of fractions.

- **a** $\frac{1}{2}$ and $\frac{9}{10}$
- **b** $\frac{3}{4}$ and 1
- **c** $\frac{1}{4}$ and $\frac{3}{4}$
- **d** $\frac{1}{5}$ and $\frac{7}{10}$

A number line may help



## Enrichment: Magic squares

**13** Complete the following magic squares using a mixture of fractions and decimals

**a**

| | | |
|---|---|---|
| 2.6 | | $1\frac{4}{5}$ |
| | $\frac{6}{2}$ | |
| 4.2 | | |

**b**

| | | | |
|---|---|---|---|
| 0.8 | 1.8 | | 3.2 |
| 3.0 | | 2.0 | 0.6 |
| 2.8 | | | |
| 0.2 | | | 2.6 |

Each row, column and diagonal add to the same number in each magic square. It's the MAGIC number!

# 4D Computation with decimals REVISION

This section reviews the different techniques involved in adding, subtracting, multiplying and dividing decimals.

Reminder:

$3.6 \div 2 = 1.8$

dividend divisor quotient

Electronic measuring instruments usually use decimals.

## Let's start: Match the phrases

There are seven different sentence beginnings and seven different sentence endings below. Your task is to match each sentence beginning with its correct ending. When you have done this, write the seven correct sentences in your book.

| Sentence beginnings | Sentence endings |
|---|---|
| When adding or subtracting decimals | the decimal point appears to move two places to the right. |
| When multiplying decimals | the decimal point in the quotient goes directly above the decimal point in the dividend. |
| When multiplying decimals by 100 | make sure you line up the decimal points. |
| When dividing decimals by decimals | the number of decimal places in the question must equal the number of decimal places in the answer. |
| When multiplying decimals | the decimal point appears to move two places to the left. |
| When dividing by 100 | start by ignoring the decimal points. |
| When dividing decimals by a whole number | we start by changing the question so that the divisor is a whole number. |

## Key ideas

- Before any computation, look at the numbers and the operator and make an estimate.
- Adding and subtracting decimals
  - Ensure the decimal points are lined up directly under one another.

$37.56 + 5.231$

$$\begin{array}{r} 37.560 \\ \underline{+\ 05.231} \end{array} \ ✓ \qquad \begin{array}{r} 37.56 \\ \underline{5.231} \end{array} \ ×$$

  - Ensure digits are correctly aligned in similar place value columns.
  - Fill every empty space with a zero.
  - Add or subtract as usual.
  - Ensure that the answer matches your estimate.

- **Multiplying and dividing decimals by powers of 10**
  - When multiplying, the decimal point appears to move to the *right* the same number of places as there are zeroes in the multiplier.
    e.g. $13.753 \times 100 = 1375.3$

    $13.7\overset{\curvearrowright\curvearrowright}{53}$

    Multiply by 10 twice.
  - When dividing, the decimal point appears to move to the *left* the same number of places as there are zeroes in the divisor.
    e.g. $586.92 \div 10 = 58.692$

    $58\overset{\curvearrowleft}{6}.92$

    Divide by 10 once.

- **Multiplying by a decimal**
  - Initially ignore the decimal points and multiply the numbers.
  - Place the decimal point into the answer using the rule:
    'The number of decimal places in the answer must equal the total number of decimal places in the question.'

e.g. $5.73 \times 8.6 \longrightarrow \begin{array}{r} 573 \\ \times \quad 86 \\ \hline 49278 \end{array} \longrightarrow 5.\underline{73} \times 8.\underline{6} = 49.\underline{278}$

(3 decimal places in question, 3 decimal places in answer).

**Divisor** The number you are dividing by

**Dividend** The number being divided

- **Dividing decimals by a whole number**
  The decimal point in the quotient goes directly above the decimal point in the **dividend**.

e.g. $56.34 \div 3$

$$\textbf{Divisor} \longrightarrow \begin{array}{r} 18.78 \\ 3\overline{)56.34} \end{array} \begin{array}{l} \longleftarrow \text{Quotient (answer)} \\ \longleftarrow \textbf{Dividend} \end{array}$$

- **Dividing a number by a decimal**
  - When dividing a decimal without a calculator, we change the *divisor* into a whole number.
  - Multiply both the divisor *and* the dividend by a power of 10.
  - In this example, we multiply by 100 or $10^2$.

$$\text{dividend} \longrightarrow 27.\overset{\overset{100}{\curvearrowright\curvearrowright}}{35}4 \div 0.\overset{\overset{100}{\curvearrowright\curvearrowright}}{02} \longleftarrow \text{divisor}$$

$$2735.4 \div 2$$

Use a calculator to calculate the following.

$27.354 \div 0.02$
$273.54 \div 0.2$
$2735.4 \div 2$
$27354 \div 20$

What do you notice?

- After any computation, ensure that the answer seems realistic and matches your original estimate.

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## Exercise 4D

**Understanding**

**1** Which of the following is the correct set-up for the following addition problem?
5.386 + 53.86 + 538.6

**A** $\begin{array}{r} 5.386 \\ 53.86 \\ +\ 538.6 \\ \hline \end{array}$ **B** $\begin{array}{r} 5.386 \\ 53.860 \\ +\ 538.600 \\ \hline \end{array}$ **C** $\begin{array}{lr} & 5.386 \\ & 53.86 \\ + & 538.6 \\ \hline \end{array}$ **D** $\begin{array}{r} 538 + 53 + 5 \\ +\ \ 0.386 + 0.86 + 0.6 \\ \hline \end{array}$

**2** The correct value of 2.731 ÷ 1000 is:
**A** 2731 **B** 27.31 **C** 2.731 **D** 0.02731 **E** 0.002731

**3** If 56 × 37 = 2072, the correct value of 5.6 × 3.7 is:
**A** 207.2 **B** 2072 **C** 20.72 **D** 2.072 **E** 0.2072

**4** Which of the following divisions is equivalent to 62.5314 ÷ 0.03?
**A** 625.314 ÷ 3 **B** 6253.14 ÷ 3 **C** 0.625314 ÷ 3 **D** 625314 ÷ 3

**Fluency**

### Example 10 Adding and subtracting decimals

Calculate:
**a** 23.07 + 9.8 **b** 9.7 – 2.86

| **Solution** | **Explanation** |
|---|---|
| **a** $\begin{array}{r} 23.07 \\ +\ \ 9.80 \\ \hline 32.87 \end{array}$ | Line up the decimal points. Fill in empty places with zeros.<br>Carry out addition as usual |
| **b** $\begin{array}{r} {}^{8}\cancel{9}.\ {}^{16}\cancel{7}\,{}^{1}\cancel{0} \\ -\ 2.\ 8\ 6 \\ \hline 6.\ 8\ 4 \end{array}$ | Align decimal points directly under one another and fill empty places with zeroes.<br>Carry out subtraction following the same procedure as for subtraction of whole numbers. |

**5** Evaluate, then check your answers with a calculator.
**a** 5.6 + 1.2 **b** 8.4 + 2.1 **c** 18.6 + 3.3
**d** 4.9 + 5.3 **e** 8.1 + 8.2 **f** 9.3 + 3.9
**g** 23.57 + 39.14 **h** 64.28 + 213.71 **i** 5.623 + 18.34
**j** 92.3 + 1.872 **k** 56.3 + 4.41 **l** 0.61 + 6.1

Align digits in similar place value columns.

**6** Evaluate, both without and with a calculator.
**a** 5.6 – 1.2 **b** 8.4 – 2.1 **c** 18.6 – 3.3 **d** 7.9 – 3.8
**e** 15.6 – 9.5 **f** 10.4 – 6.4 **g** 38.52 – 24.11 **h** 76.74 – 53.62
**i** 123.8 – 39.21 **j** 14.57 – 9.8 **k** 96.3 – 4.2 **l** 85.631 – 5.22

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## Example 11 Multiplying and dividing by powers of 10

Calculate:

**a** $27.58 \times 10\,000$ **b** $9.753 \div 100$

| **Solution** | **Explanation** |
|---|---|
| **a** $27.58 \times 10\,000 = 275\,800$ | Multiplying by 10 000 (4 zeroes), therefore the decimal point appears to move four places to the right. Additional zeroes are inserted as necessary.<br>27.5800. |
| **a** $9.753 \div 100 = 0.09753$ | Dividing by 100 (2 zeroes), therefore the decimal point appears to move two places to the left. Additional zeroes are inserted as necessary.<br>009.753 |

**7** Evaluate:

**a** $9.61 \times 10$
**b** $9.61 \times 100$
**c** $15.463 \times 1000$
**d** $19.4 \div 10$
**e** $19.4 \div 100$
**f** $27.4 \div 10$
**g** $27.4 \div 1000$
**h** $1.6 \times 1000$
**i** $36.5173 \times 100$
**j** $0.08155 \times 1000$
**k** $7.5 \div 10$
**l** $3.812 \div 100$
**m** $634.8 \times 10\,000$
**n** $1.0615 \div 1000$
**o** $0.003 \times 10\,000$
**p** $0.452 \div 1000$

Move right for ×.
Move right for ÷.

## Example 12 Multiplying decimals

Find the product of 25.7 and 0.3

| **Solution** | **Explanation** |
|---|---|
| $\begin{array}{r} {}^{1}2{}^{2}57 \\ \times \quad 3 \\ \hline 771 \end{array}$<br>$25.7 \times 0.3 = 7.71$ | Perform multiplication ignoring the decimal point.<br>$(257 \times 3 = 771)$<br>There are 2 decimal places in the question, so 2 decimal places in the answer. |

Decimal numbers are frequently encountered when dealing with money.

**8** Evaluate these products, then check your answers with a calculator.

**a** $0.8 \times 7$ **b** $0.8 \times 0.7$ **c** $15 \times 0.1$
**d** $0.4 \times 0.3$ **e** $15.4 \times 2$ **f** $1.2 \times 0.3$
**g** $0.8 \times 0.4$ **h** $0.8 \times 0.04$ **i** $15 \times 0.2$
**j** $24.5 \times 0.2$ **k** $0.9 \times 9$ **l** $1.2 \times 1.2$

First ignore the decimal point.

## Example 13 Dividing decimals

Calculate:

**a** $35.756 \div 4$ **b** $64.137 \div 0.03$

| **Solution** | **Explanation** |
|---|---|
| **a** 8.939 <br> $\begin{array}{r} 8.\,9\,3\,9 \\ 4\overline{)35.^{3}7^{1}5^{3}6} \end{array}$ | The divisor (4) is a whole number. <br> Carry out division, remembering that the decimal point in the answer is placed directly above the decimal point in the dividend. |
| **b** $64.137 \div 0.03$ <br> $= 6413.7 \div 3$ <br> $= 2137.9$ <br> $\begin{array}{r} 2\,1\,3\,7.\,9 \\ 3\overline{)6\,4^{1}1^{2}3.^{2}7} \end{array}$ | The divisor (0.03) is *not* a whole number. <br> Instead of dividing by 0.03, multiply both numbers by 100 so that the divisor is a whole number (3). <br> (Move both decimal points two places to the right.) <br> Carry out the division: $6413.7 \div 3$. |

**9** **a** $24.54 \div 2$ **b** $17.64 \div 3$ **c** $0.0485 \div 5$ **d** $347.55 \div 7$
**e** $133.44 \div 2$ **f** $4912.6 \div 4$ **g** $2.58124 \div 8$ **h** $17.31 \div 5$

**10** Complete these divisions by filling in the missing numbers.

**a** $15.6 \div 0.3 = 156 \div 3 =$ ☐
**b** $12.4 \div 0.02 = 1240 \div 2 =$ ☐
**c** $15.06 \div 0.2 =$ ☐ $\div 2 =$ ☐
**d** $45.9 \div 0.03 = 4590 \div$ ☐ $=$ ☐
**e** $0.484 \div 0.4 =$ ☐ $\div 4 =$ ☐

**Problem-solving and Reasoning**

**11** The heights of five children are 1.34 m, 1.92 m, 0.7 m, 1.5 m, and 1.66 m. If the children laid down in a row, how long would the row be?

**12**

| **Canteen prices** | | |
|---|---|---|
| pie \$2.80 | chips \$1.70 | juice \$3.40 |
| cola \$3.20 | chocolate \$2.20 | sandwich \$2.60 |
| sauce \$0.60 | apple \$0.50 | milk \$1.85 |

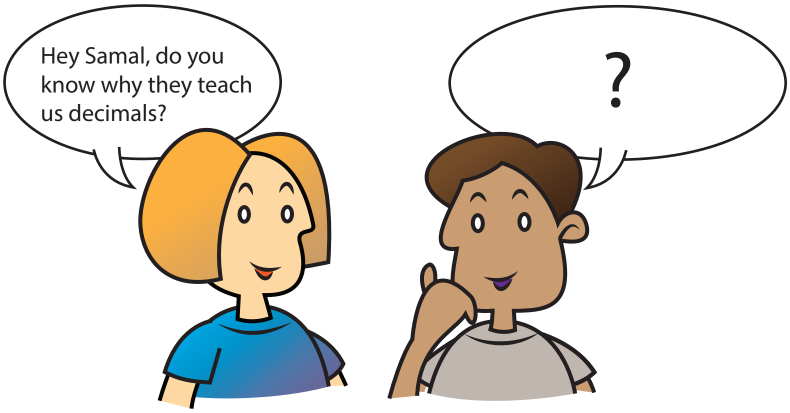
Drilling for Gold 4D1

**a** Find the cost of each person's lunch.

| Vaughn | Charlotte | Reece |
|---|---|---|
| 1 pie | 1 sandwich | 1 pie |
| 1 sauce | 1 chocolate | 2 colas |
| 1 apple | 1 juice | 1 sandwich |
| 2 milks | | 1 chips |

**b** How much change from $20 should each person receive?

## Enrichment: Secret code



Answer each of the 12 questions below to unlock the code and find out how Samal answers Sally's question.

| | | | | | | |
|---|---|---|---|---|---|---|
| 20.7 | 12.2 | 4.4 | | 4.4 | 0.3 | |

| | | | | | | |
|---|---|---|---|---|---|---|
| 12.2 | 4.75 | 14.4 | 12.2 | | 3.2 | 160 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 24.2 | 0.3 | 12.2 | 4.75 | | 1.32 | 160 | 12.2 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 12.2 | 4.75 | 160 | | 17.97 | 0.3 | 20.7 | 0.72 | 12.2 | ! |

| | | | |
|---|---|---|---|
| I $3.2 + 17.5$ | O $1.5 \times 0.2$ | H $47.5 \div 10$ | E $96 \div 0.6$ |
| A $1.2 \times 12$ | T $15.8 - 3.6$ | N $0.9 \times 0.8$ | B $5.9 + 18.3$ |
| W $9.6 \div 3$ | P $18.57 - 0.6$ | S $9 - 4.6$ | G $1.2 + 0.12$ |

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# 4E Terminating decimals, recurring decimals and rounding

**REVISION**

Not all fractions convert to the same type of decimal. For example:

$\frac{1}{2} = 1 \div 2 = 0.5$ (only has 1 decimal place)

$\frac{1}{3} = 1 \div 3 = 0.33333\ldots$ (keeps going and going)

$\frac{1}{11} = 1 \div 11 = 0.090909\ldots$ (the pattern repeats)

| |
|---|
| $\pi = 3.141592653589\ldots$ |
| $\sqrt{2} = 1.414213562\ldots$ |

Some decimals never end and don't have a pattern that repeats.

Decimals that stop (or terminate) are known as terminating decimals, whereas decimals that have a pattern that repeats forever are known as repeating or recurring decimals.

## Let's start: Decimal patterns

Use a calculator to perform these divisions. Can you see a pattern?

- $\frac{1}{9} = 1 \div 9 = 0.1111\ldots$
- $\frac{2}{9}$
- $\frac{3}{9}$
- $\frac{4}{9}$

Without your calculator, write down $\frac{5}{9}$ and $\frac{6}{9}$ as decimals. What do we call these types of decimals?

## Key ideas

- A **terminating decimal** has a fixed number of decimal places (i.e. it terminates).

  e.g. $\frac{5}{8} = 5 \div 8 = 0.625$ $\quad 8\overline{)5.{}^{5}0{}^{2}0{}^{4}0}$ with quotient $0.\ 6\ 2\ 5$ ⟵ Terminating decimal with 3 decimal places.

- A **recurring decimal** (or **repeating decimal**) keeps going and the decimal places repeat.

  e.g. $\frac{1}{3} = 1 \div 3 = 0.333\ldots$ $\quad 3\overline{)1.{}^{1}0{}^{1}0{}^{1}0{}^{1}0}$ with quotient $0.\ 3\ 3\ 3\ldots$ ⟵ Recurring decimal

- A convention is to use 1 or 2 dots above the digits to show the start and finish of a repeating cycle of digits.

  e.g. $0.55555\ldots = 0.\dot{5}$ and $0.3412412412\ldots = 0.3\dot{4}1\dot{2}$

**Terminating decimal**
A decimal that contains a finite number of digits

**Recurring decimal**
A decimal in which a single digit, or a group of digits, repeats

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Another convention is to use a horizontal bar above the digits to show the repeating cycle of digits.
e.g. $0.55555\ldots = 0.\overline{5}$ and $0.3412412412\ldots = 0.3\overline{412}$

- **Rounding decimals**
  Decimals with many decimal places can be approximated with fewer decimal places by rounding.
- **Rounding** involves approximating a decimal number using fewer digits.

**Rounding** To make an approximation of a number with fewer digits

- Rounding to 2 decimal places:
  In the following decimals, more than 2 decimals are given.
  A blue line has been drawn after 2 decimal places.
  The 'critical digit' is circled.
  If the critical digit is 0, 1, 2, 3 or 4, then round *down*.


Ignore all digits to the right of the critical digit.

For example: $185.26\,|\textcircled{3}$ = 185.26 (to 2 d.p.)
$185.26\,|\textcircled{0}\,0\,5$ = 185.26 (to 2 d.p.)
$185.26\,|\textcircled{4}\,4\,9\,9$ = 185.26 (to 2 d.p.)

If the critical digit is 5, 6, 7, 8 or 9, then round *up*.
For example, $185.26\,|\textcircled{5}$ = 185.27 (to 2 d.p.)
$185.26\,|\textcircled{6}\,0\,5$ = 185.27 (to 2 d.p.)
$185.26\,|\textcircled{9}\,4\,9\,9$ = 185.27 (to 2 d.p.)

## Exercise 4E

**Understanding**

**1** State whether the following are terminating decimals (T) or recurring decimals (R).

**a** $5.47$ **b** $3.1541\dot{5}\ldots$ **c** $8.\dot{6}$ **d** $7.1834$
**e** $0.333$ **f** $0.\dot{5}3\dot{4}$ **g** $0.5615$ **h** $0.32727\ldots$

**2** Express the following recurring decimals using the convention of dots or a bar to indicate the start and finish of the repeating cycle.

**a** $0.33333\ldots$ **b** $6.21212121\ldots$
**c** $8.5764444\ldots$ **d** $2.135635635\ldots$
**e** $11.2857328573\ldots$ **f** $0.003523523\ldots$


Write $0.7555\ldots$ as $0.7\dot{5}$

**3** Write down the 'critical' digit (the digit immediately after the rounding digit) for each of the following.

**a** $3.5724$ (rounding to 3 decimal places)
**b** $15.89154$ (rounding to 1 decimal place)
**c** $0.004571$ (rounding to 4 decimal places)
**d** $5432.726$ (rounding to 2 decimal places)

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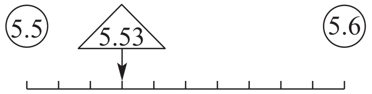
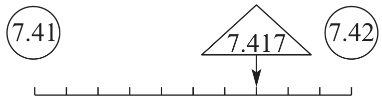
**4** For each line given, which circled decimal is the decimal in the triangle closest to?

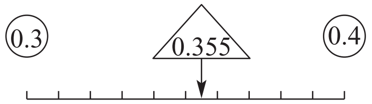
**a**



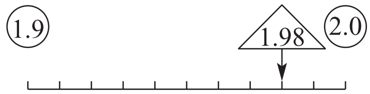
**b**



**c**



**d**



**Fluency**

### Example 14 Writing terminating decimals

Convert the following fractions to decimals.

**a** $\frac{1}{4}$ **b** $\frac{7}{8}$

| **Solution** | **Explanation** | |
|---|---|---|
| **a** $\frac{1}{4} = 0.25$ | $\begin{array}{r} 0.\,2\,5 \\ 4\overline{)1.{}^{1}0{}^{2}0} \end{array}$ | Write 1 as 1.00.<br>Divide the bottom (denominator) into the top (numerator). |
| **b** $\frac{7}{8} = 0.875$ | $\begin{array}{r} 0.\,8\,7\,5 \\ 8\overline{)7.{}^{7}0{}^{6}0{}^{4}0} \end{array}$ | Write 7 as 7.000.<br>Divide the bottom (denominator) into the top (numerator). |

**5** Convert the following fractions to decimals.

**a** $\frac{3}{5}$ **b** $\frac{3}{4}$ **c** $\frac{1}{8}$ **d** $\frac{11}{20}$

**e** $\frac{1}{2}$ **f** $\frac{4}{5}$ **g** $\frac{1}{25}$ **h** $\frac{9}{50}$

These are all terminating decimals.

### Example 15 Writing recurring decimals

Express the following fractions as recurring decimals.

**a** $\frac{2}{3}$ **b** $3\frac{5}{7}$

| **Solution** | **Explanation** | |
|---|---|---|
| **a** $\frac{2}{3} = 0.\dot{6}$ | $\begin{array}{r} 0.\,6\,\,6... \\ 3\overline{)2.{}^{2}0{}^{2}0{}^{2}0} \end{array}$ | This pattern continues, it is a repeating decimal. |
| **b** $3\frac{5}{7} = 3.\dot{7}1428\dot{5}$ or $3.\overline{714285}$ | $\begin{array}{r} 0.\,7\,1\,4\,2\,8\,5\,7... \\ 7\overline{)5.{}^{5}0{}^{1}0{}^{3}0{}^{2}0{}^{6}0{}^{4}0{}^{5}0{}^{1}0} \end{array}$ | This pattern continues. |

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**6** Express the following fractions as recurring decimals. Check with a calculator.

**a** $\frac{1}{3}$ **b** $\frac{5}{9}$ **c** $\frac{5}{6}$ **d** $\frac{7}{9}$

**e** $\frac{3}{7}$ **f** $\frac{1}{6}$ **g** $\frac{4}{3}$ **h** $1\frac{6}{7}$

Remember to use the repeating notation. $0.444... = 0.\dot{4}$

## Example 16 Rounding decimals

**a** Round 14.568 to 1 decimal place.
**b** Round 0.671 to 2 decimal places.

| Solution | Explanation |
|---|---|
| **a** 14.6 | 14.5 \|⑥ 8 1 decimal place, look at next digit (5).<br>Critical digit is 6. Round up 14.568 ≈ 14.6 |
| **b** 0.67 | 0.67 \|① 2 decimal places, look at the next digit (1).<br>Critical digit is 1. Round down 0.671 ≈ 0.67. |

**7** Round each of the following decimals to 1 decimal place.

**a** 0.57 **b** 0.83 **c** 1.49
**d** 8.16 **e** 9.47 **f** 8.33
**g** 1.487 **h** 3.444 **i** 0.333

The first decimal place is also called the tenths column.

**8** Write each of the following decimals, correct to 2 decimal places (or the nearest hundredth).

**a** 0.783 **b** 0.666 **c** 1.478
**d** 0.893 **e** 15.488 **f** 9.035
**g** 9.4163 **h** 8.7499 **i** 1.7891

**9** **a** Choose the correct answer to each of the following.
- **i** Is 7.9 closer to 7 or 8?
- **ii** Is 7.99 closer to 7.9 or 8.0?
- **iii** Is 4.95 closer to 4.9 or 5.0?

**b** Round the following to 1 decimal place.
**i** 4.96 **ii** 8.941 **iii** 5.999

Skillsheet 4B

**Problem-solving and Reasoning**

**10** Copy and complete this table for recurring decimals.

| Fraction | Decimal | One dec pl | Two dec pl |
|---|---|---|---|
| $\frac{1}{3}$ | 0.333... | 0.3 | 0.33 |
| $\frac{2}{3}$ | | | |
| $\frac{5}{9}$ | | | |
| $\frac{1}{6}$ | | | |

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**11** A race was timed using hundredths of seconds (i.e. 2 decimal places). Simone ran 100 m in 12.83 seconds, while Greer ran it in 12.77 seconds.

- **a** Who came first, and by how much?
- **b** Round each time to 1 decimal place. Can you still decide who came first?
- **c** What other times would round to 12.8 seconds?

**12** Write down three different decimals that, when written to 2 decimal places, become 3.45.

## Enrichment: Buying petrol (and water)

**13** Petrol prices go up or down almost every day. At any given moment, the price of petrol can vary from place to place.

A website contains the following information about petrol prices every day in Sydney:

Lowest price today: 107.9 cents per litre, in Blakehurst
Highest price today: 132.9 cents per litre, in Bondi

Lucy is going to buy petrol. She is going to pay with cash, so the final price will be rounded to the nearest five cents.
Give your answers to parts **a**, **b** and **c** in the form $17.25.
Note: When using your calculator, 125.9 cents can be entered as 1.259 dollars.

- **a** How much will it cost for 40 litres at the lowest price?
- **b** How much *extra* will it cost for 40 litres at the highest price?
- **c** **i** What is the difference per litre between the lowest price and highest price?
  - **ii** Lucy buys 40 litres of petrol every week. The difference between the lowest and highest prices is usually 25 cents. How much could she save in a year by 'shopping around'?
- **d** **i** If Lucy only has $20, how many litres (to 1 decimal place) can she buy at the highest price?
  - **ii** How many *extra* litres could she buy at the lowest price?
- **e** The petrol station in Bondi sells water in bottles.
  The 600 mL bottle is on special for $1.
  Which is cheaper, per litre, the petrol or the water?

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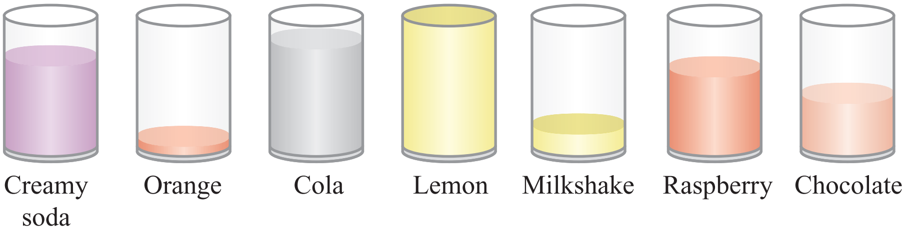
# 4F Converting fractions, decimals and percentages

REVISION

A *percentage* is a particular fraction in which the denominator is always 100.
*Percent* is Latin for 'out of 100'.

$$7\% = 7 \text{ percent} = 7 \text{ out of } 100 = \frac{7}{100} = 0.07$$

## Let's start: Estimating percentages



- List the drinks in order from the most to the least amount left in the glass.
- Estimate the percentage of drink remaining in each of the glasses shown above.
- Discuss your estimations with a partner.

## Key ideas

- A percent sign (%) means 'out of 100'.

  $23\% = \frac{23}{100}$
- Percentages can be converted to fractions and decimals.

  $35\% = 0.35 = \frac{35}{100} = \frac{7}{20}$
- Fractions and decimals can be converted to percentages.

  $\frac{3}{8} \times 100\% = 37.5\%$

  $0.375 \times 100\% = 37.5\%$

**Drilling for Gold 4F1**

- Percentage–decimal–fraction facts

  Note: It is useful to memorise these

  $100\% = 1$

  $50\% = 0.5 = \frac{1}{2}$

  $33\frac{1}{3}\% = 0.\dot{3} = \frac{1}{3} \xrightarrow{\times 2} 66\frac{2}{3}\% = 0.\dot{6} = \frac{2}{3}$

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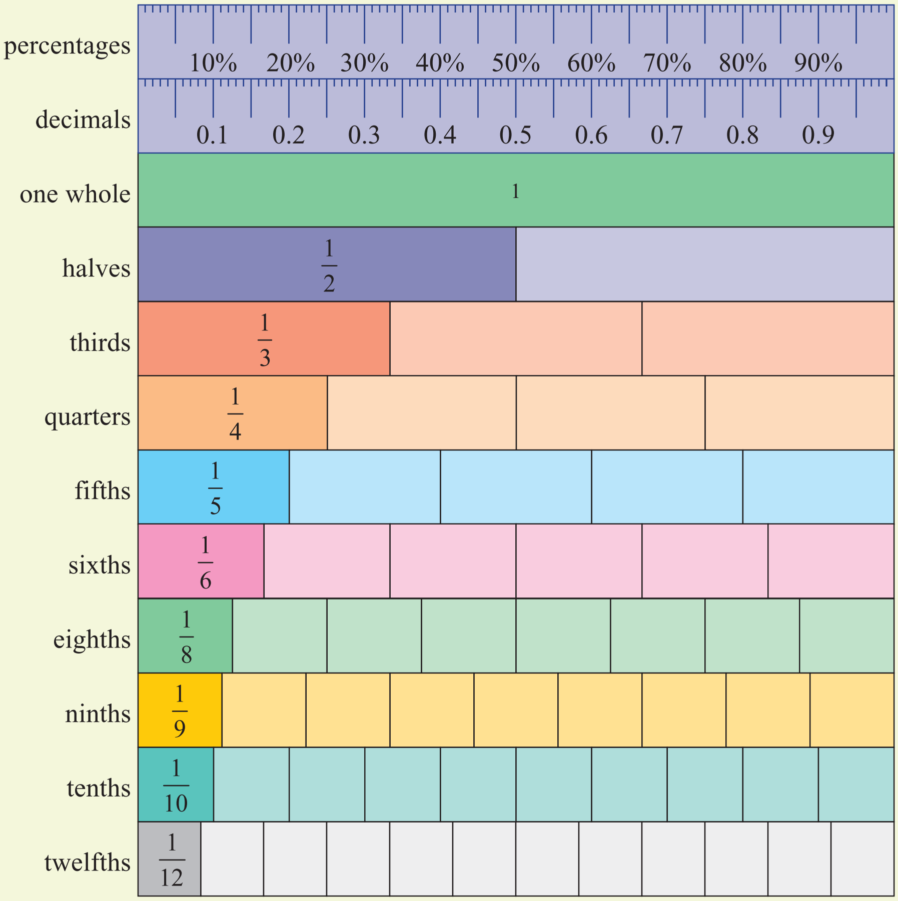
$$25\% = 0.25 = \frac{1}{4} \xrightarrow{\times 3} 75\% = 0.75 = \frac{3}{4}$$

$$\begin{tikzcd} & 20\% = 0.2 = \frac{2}{10} = \frac{1}{5} \\ 10\% = 0.1 = \frac{1}{10} \arrow[ur, "\times 2"] \arrow[dr, "\times 3"'] & \\ & 30\% = 0.3 = \frac{3}{10} \end{tikzcd}$$

$$1\% = 0.01 = \frac{1}{100} \xrightarrow{\times 5} 5\% = 0.05 = \frac{5}{100} = \frac{1}{20}$$

- It is also important to understand the relationships and connections between fractions, decimals and percentages. The ‘fraction wall’ diagram below shows these very clearly.



**Drilling for Gold 4F2**

## Exercise 4F

**Understanding**

Remember if you see a % sign, it means out of 100.

**1** The fraction equivalent of 27% is:

**A** $\frac{2}{7}$ **B** $\frac{27}{100}$ **C** 2700 **D** $\frac{13.5}{10}$

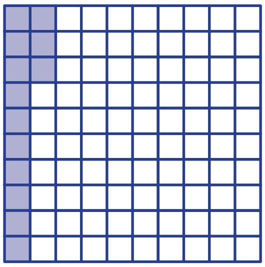
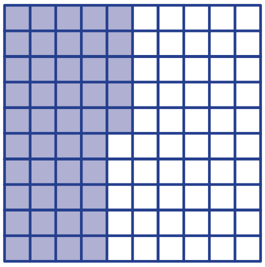
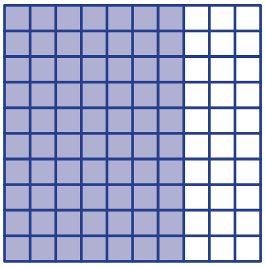
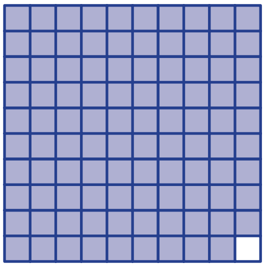
**2** The decimal equivalent of 37% is:

**A** 0.037 **B** 0.37 **C** 3.7 **D** 37.00

**3** The percentage equivalent of $\frac{47}{100}$ is:

**A** 0.47% **B** 4.7% **C** 47% **D** 470%

**4**

| | | Fraction shaded | Decimal in words | Decimal in figures | Percent in words | Percent in figures |
|---|---|---|---|---|---|---|
| **a** |  | $\frac{13}{100}$ | thirteen hundredths | | | |
| **b** |  | | | 0.45 | | |
| **c** |  | | | | seventy percent | |
| **d** |  | | | | | 99% |

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**Fluency**

## Example 17 Converting percentages to fractions without a calculator

Write 60% as a:

**a** simple fraction **b** decimal.

**Solution** | **Explanation**

**a** $60\% = \frac{60}{100} \underset{\div 10}{\overset{\div 10}{=}} \frac{6}{10} \underset{\div 2}{\overset{\div 2}{=}} \frac{3}{5}$

60% means '60 out of 100'
Reduce to simplest form.

**b** $60\% = 60 \div 100$
$= 0.6$

To change a % to a decimal, divide by 100.
Alternatively 10% = 0.1, so 60% = 0.6

**5** Write these percentages as simple fractions.

**a** 39% **b** 11% **c** 17% **d** 99%
**e** 20% **f** 70% **g** 75% **h** 55%

**6** Write these percentages as decimals.

**a** 39% **b** 11% **c** 17% **d** 99%
**e** 20% **f** 70% **g** 75% **h** 55%
**i** 7% **j** 1% **k** 10% **l** 47%

Write each number out of 100, then simplify if possible.

Divide by 100.

## Example 18 Converting fractions to percentages without a calculator

Write the following as percentages.

**a** $\frac{7}{20}$ **b** 0.81

**Solution** | **Explanation**

**a** $\frac{7}{20} \underset{\times 5}{\overset{\times 5}{=}} \frac{35}{100} = 35\%$

20 goes into 100 five times.
Multiply denominator by 5.
∴ Multiply numerator by 5.

**b** $0.81 \times 100\% = 81\%$

Multiply 0.81 by 100.
0.81 = 81%

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**4F**

**7** Write these fractions as percentages, without using a calculator.

**a** $\frac{77}{100}$ **b** $\frac{49}{100}$ **c** $\frac{3}{4}$ **d** $\frac{4}{5}$
**e** $\frac{7}{25}$ **f** $\frac{9}{20}$ **g** $\frac{11}{20}$ **h** $\frac{19}{50}$
**i** $\frac{47}{50}$ **j** $\frac{7}{10}$ **k** $\frac{12}{10}$ **l** $\frac{3}{2}$

Use a memorised factor to convert to $\frac{\square}{100}$.

**8** Write these decimals as percentages.

**a** 0.16 **b** 0.79 **c** 0.83 **d** 0.97
**e** 0.03 **f** 0.33 **g** 0.91 **h** 0.09
**i** 0.125 **j** 0.375 **k** 1.25 **l** 1.06

$0.73 = 73\%$

**9** Use a calculator to complete the table.

| | Percentage | Decimal | Fraction | Simple fraction |
|---|---|---|---|---|
| **a** | 85% | | | |
| **b** | | 0.35 | | |
| **c** | | | $\frac{80}{100}$ | |
| **d** | | | $\frac{125}{100}$ | |
| **e** | 37.5% | | | |
| **f** | | | | $\frac{1}{6}$ |

**Problem-solving and Reasoning**

**10 a** If $\frac{1}{10} = 10\%$, what does $\frac{7}{10}$ equal as a percentage?

**b** If $\frac{1}{5} = 20\%$, what does $\frac{3}{5}$ equal as a percentage?

**c** If $\frac{1}{8} = 12.5\%$, what does $\frac{7}{8}$ equal as a percentage?

**d** If $\frac{1}{2} = 50\%$, what does $1\frac{1}{2}$ equal as a percentage?

Think: What can I multiply each fraction by?

**11** Consider the 'halving' pattern $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

**a** Write down the first ten numbers in the pattern.
**b** Convert the first ten numbers from fractions to decimals.
**c** Find the sum of the first ten numbers, using a calculator or spreadsheet.

**12** The Sharks team has won 13 out of 17 games for the season to date. The team still has three games to play. What is the smallest and the largest percentage of games the Sharks could win for the season?

## Enrichment: Money and percentages

**13** Copy and complete this table. Can you see a connection?

| Cents per 100 cents | Cents in the dollar | Percentage of $1 |
|---|---|---|
| 5c | $0.05 | |
| 10c | | |
| | $0.09 | |
| | | 17% |
| | | 25% |
| | $0.70 | |
| | | 90% |
| 75c | | |
| 100c | | |
| | $2 | |

One dollar equals 100 *cents*.
One *century* is 100 years.

# 4G Finding a percentage and expressing as a percentage

Percentages are useful when comparing two quantities.
For example, Huen's report card could be written as 'marks out of' or in percentages:

| | |
|---|---|
| French test<br>14 out of 20 | French test<br>70% |
| German test<br>54 out of 75 | German test<br>72% |

In this section we look at expressing a number as a percentage of another number as well as finding a percentage of an amount.

## Let's start: What percentage has passed?

Estimate the following, using a percentage.

- What percentage of this day has passed?
- What percentage of the current month has passed?
- What percentage of the calendar year has passed?
- What percentage of your school year has passed?
- What percentage of your school education has passed?
- When you turned 5, what percentage of your life was 1 year?
- When you turn 40, what percentage of your life was 1 year?

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## Key ideas

- **To express one quantity as a percentage of another**
  1. Write the quantities as a fraction.
     e.g. 14 out of 20 is $\frac{14}{20}$.
  2. Convert the fraction to a percentage.
     - Calculator method:
       $$\frac{14}{20} \times 100 = 70$$
       $$\text{so } \frac{14}{20} = 70\%$$
     - Non-calculator method:
       ×5
       $$\frac{14}{20} = \frac{70}{100} = 70\%$$
       ×5

Drilling for Gold 4G1

- **To find a certain percentage of a quantity**
  1. Express the required percentage as a fraction. (You can use decimals.)
  2. Change the 'of' to a multiplication sign.
  3. Express the number as a fraction.
  4. Complete the computation.

     e.g. find 20% of 80.
     - Calculator method:
       $$\begin{aligned} 20\% \text{ of } 80 &= \frac{20}{100} \times 80 \\ &= 16 \end{aligned}$$
     - Non-calculator method:
       $$20\% \text{ of } 80 = \frac{20}{100} \times \frac{80}{1} = \frac{{}^{4}\cancel{20}}{{}_{1}\cancel{5}\,\cancel{100}} \times \frac{{}^{4}\cancel{80}}{1} = 16$$
       or 10% of 80 = 8
       ∴ 20% of 80 = 16

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## Exercise 4G

Understanding

**1** The correct working line to express 42 as a percentage of 65 is:

**A** $\frac{42}{100} \times 65\%$ **B** $\frac{65}{42} \times 100\%$ **C** $\frac{100}{42} \times 65\%$ **D** $\frac{42}{65} \times 100\%$

**2** The correct working line to find 42% of 65 is:

**A** $\frac{42}{100} \times 65$ **B** $\frac{65}{42} \times 100$ **C** $\frac{100}{42} \times 65$ **D** $\frac{42}{65} \times 100$

The number with the percent sign is written with the 100 in the denominator.

**3** What percentage is a mark of:

**a** 20 out of 40?
**b** 0 out of 10?
**c** 50 out of 50?

**4** Copy and complete the following sentences.

**a** Finding 1% of a quantity is the same as dividing the quantity by ______.
**b** Finding 10% of a quantity is the same as dividing the quantity by ______.
**c** Finding 20% of a quantity is the same as dividing the quantity by ______.
**d** Finding 50% of a quantity is the same as dividing the quantity by ______.
**e** Finding 25% of a quantity is the same as dividing the quantity by ______.

Fluency

### Example 19 Expressing one quality as a percentage of another

Express the following as a percentage.
34 out of 40

| **Solution** | **Explanation** |
|---|---|
| Non-calculator method<br>$\frac{34}{40} \times \frac{100}{1}\% = \frac{17}{{}_{1}\cancel{20}} \times \frac{\cancel{100}^{5}}{1}\%$<br>$= \frac{17}{1} \times \frac{5}{1}\%$<br>$= 85\%$ | Write as a fraction, with the first quantity as the numerator and second quantity as the denominator. Multiply by 100%.<br>Cancel and simplify. |
| Calculator method<br>$34 \div 40 \times 100 = 85\%$ | Divide the first quantity by the second quantity, then multiply by 100. |

**5** Express each of the following as a percentage. Check with your calculator.

Multiply by 100%.

**a** 20 out of 25 **b** 13 out of 20 **c** 39 out of 50
**d** 17 out of 25 **e** 12 out of 20 **f** 49 out of 50
**g** 7 out of 10 **h** 12 out of 30 **i** 15 out of 20
**j** 32 out of 40 **k** 54 out of 90 **l** 18 out of 24

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## Example 20 Converting units before expressing as a percentage

Express:
60c as a percentage of $5

**Solution**

$$\begin{aligned}\frac{60}{500}\times\frac{100}{1}\% &= \frac{60}{5}\% \\ &= 12\%\end{aligned}$$

**Explanation**

Units need to be the same.
Convert $5 to 500 cents.
Write quantities as a fraction and multiply by 100%.
Cancel and simplify.
Note: $60 \div 500 \times 100 = 12$, by calculator

**6** Express (using a calculator if necessary):

- **a** 40c as a percentage of $8
- **b** 50c as a percentage of $2
- **c** 3 mm as a percentage of 6 cm
- **d** 400 m as a percentage of 1.6 km
- **e** 200 g as a percentage of 5 kg
- **f** 200 m as a percentage of 8 km.

Remember:
1 km = 1000 m
1 cm = 10 mm
1 kg = 1000 g
$1 = 100 cents

**7** Express each quantity as a percentage of the total.

- **a** 28 laps of a 50 lap race completed
- **b** Saved $450 towards a $600 guitar
- **c** 172 fans in a train carriage of 200 people
- **d** Level 7 completed of a 28 level video game
- **e** 36 students absent out of 90 total
- **f** 21 km mark of a 42 km marathon

## Example 21 Finding a certain percentage of a quantity, without a calculator

Find:

**a** 25% of 128

**b** 70% of 400

**Solution**

**a** $$\begin{aligned}25\% \text{ of } 128 &= \frac{25}{100}\times\frac{128}{1} \\ &= \frac{1}{4}\times\frac{128}{1} = 32\end{aligned}$$

**Explanation**

Write the percentage as a fraction over 100.
'of' means multiply.
Cancel and simplify.

Alternatively,

$$\begin{aligned}25\% \text{ of } 128 &= 128 \div 4 \\ &= 32\end{aligned}$$

25% is one quarter,
so divide by 4.

**b** 70% of 400:
10% is 40
∴ 70% is 280

First, find 10%.
Then, multiply by 7 to find 70%.

Skillsheet 4C

**8** Find (without using a calculator):

**a** 50% of 36 **b** 10% of 80 **c** 30% of 500
**d** 9% of 200 **e** 20% of 40 **f** 25% of 48
**g** 75% of 80 **h** 25% of 88 **i** 50% of 25
**j** 5% of 60 **k** 5% of 6000 **l** 1% of 720
**m** 2% of 150 **n** 99% of 200 **o** 75% of 960

50% of 36
36 ÷ 2

**9** Calculate (to 2 decimal places if necessary):

**a** 12.3% of \$196 **b** 6.7% of \$35 000
**c** 12.5% of \$6.75 **d** 0.2% of \$1000 000

**Problem-solving and Reasoning**

**10** Find (without using a calculator):

**a** 10% of \$750
**b** 5% of 2 km
**c** 30% of 150 kg
**d** 20% of 90 minutes
**e** 10% of 5 litres
**f** 25% of one hour
**g** 50% of \$6.50
**h** 2% of \$8
**i** 7% of $\frac{1}{2}$ kg

You may like to change the units in the question to make it easier to work with 3% of 1 km = 3% of 1000 metres.

Remember to put the units in your answer
10% of \$50
$= \frac{10}{100} \times \$50$
= \$5

**11** Copy and complete the table of sporting choices.

| Sport | Number of students | Fraction of total | Percentage |
|---|---|---|---|
| Tennis | 40 | | |
| Golf | 30 | | |
| AFL | 70 | | |
| NRL | 50 | | |
| Swimming | 10 | | |
| Total | 200 | 1 | 100% |

**12** Most banks require a 10% deposit before lending you any money. Ashlee and Matt have 7% of the \$450 000 their home costs.

**a** How much do Ashlee and Matt have as their deposit?
**b** How much do the banks need them to have?

**c** How much more do they need to save?
**d** If they get a government grant of $14 000, will they have the 10% needed?

## Enrichment: Calculator percentages

**13** Calculators make working with percentages easier. Use a calculator to answer these questions.
- **a** Find 8% of $8.40.
- **b** Find 13% of 2 km.
- **c** Find $7\frac{1}{4}\%$ of $500.
- **d** Find 24% of 1 hour.
- **e** Find 31.5% of $45 960.
- **f** 4% of a class of 25 students are away with the flu. How many students are at school?
- **g** 49.5% of babies born at the local hospital are girls. Of the 200 born in the month, how many were boys?
- **h** Sean pays 42% of his $86 400 income in tax. How much is left after he pays his tax?

# 4H Decreasing and increasing by a percentage

When the price of an item is decreased, figures such as 20% are used to describe the discount.

The original price was 100%. The new price will be 80% of the original price.

## Let's start: What does it mean?

In pairs, answer the following.

- What does it mean to buy a pair of shoes 'on sale'?
- What does it mean if the sale is '20% off'?
- What does it mean to 'pay the marked price'?
- What does it mean to you to buy an item on sale? Is '$10 off' better than '10% off'? Discuss.

## Key ideas

- **Decreasing by a percentage**

  The original price is 100%.
  If the decrease is 5%, the new value is 95% of the original value.
  Words used to describe this are: *discount*, *sale*, *mark-down*, *decrease*, *loss*, *deflation* and *depreciation*.
  There are two ways to decrease by 5%:

| | |
|---|---|
| *Method 1:*<br>Calculate 5%.<br>Subtract that amount from the value. | *Method 2:*<br>Subtract 5% from 100% to give 95%.<br>Calculate 95% of the value. |

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- Increasing by a percentage
  The original price is $100\%$.
  If the increase is $5\%$, the new value is $105\%$ of the original value.
  Words used to describe this are: **mark-up**, **increase**, **profit**, **inflation**, **surcharge** and **appreciation**.

| *Method 1:*<br>Calculate $5\%$.<br>Add that amount to the value. | *Method 2:*<br>Add $5\%$ to $100\%$ to give $105\%$.<br>Calculate $105\%$ of the value. |
|---|---|

- In the retail industry:
  - **Cost price** is the amount for the retailer or shopkeeper the goods.
  - **Selling price**, **marked-up price** or **retail price** is the advertised amount for which the customer can purchase the goods.
  - **Discounted price** or **marked-down price** is an amount that the retailer is willing to accept.
  - **Profit (or loss)** is the difference between the cost price and the price for which the item is sold.

## Exercise 4H

Understanding

**1** Decide if each of these shows an increase or a decrease.
**a** Mark's $\$1650$ return airfare to Los Angeles was reduced by $10\%$.
**b** Sonya made $15\%$ profit when she sold her house.
**c** The shop discounted all of its computers by $10\%$.
**d** Thomas received a pay rise of $5\%$ on his wage of $\$570$ per week.
**e** A tax of $15\%$ is added to the cost of everything in the United Kingdom.

**2** Add or subtract these percentages.
**a** $100\% + 20\%$ **b** $100\% + 15\%$
**c** $100\% - 10\%$ **d** $100\% - 15\%$

**3** Calculate the new price when an item marked at:
**a** $\$15$ is discounted by $\$3$
**b** $\$25.99$ is marked up by $\$8$
**c** $\$17$ is reduced by $\$2.50$
**d** $\$180$ is increased by $\$45$.

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**Fluency**

## Example 22 Finding new values: increasing

Find the new value when:
$160 is increased by 40%

| **Solution** | **Explanation** |
|---|---|
| Method 1:<br>40% of \$160 = \$64<br>New price = \$160 + \$64<br>= \$224 | Calculate 40% of \$160, with or without a calculator.<br>New price = original price + increase |
| Method 2:<br>100% + 40% = 140% = 1.4<br>New price = 140% of \$160<br>= \$224 | Add 40% to 100%.<br>By calculator, 160 × 1.4 = 224 |

**4** Find the new value when:

**a** \$400 is increased by 10%
**b** \$240 is increased by 10%
**c** \$250 is increased by 10%
**d** \$700 is increased by 20%
**e** \$500 is increased by 1%
**f** \$800 is increased by 25%
**g** \$84 is increased by 25%
**h** \$90 is increased by 50%.

Add the increase on to the original amount.

## Example 23 Finding new values: decreasing

Find the new value when:
$63 is decreased by 20%

| **Solution** | **Explanation** |
|---|---|
| Method 1:<br>20% of \$63 = \$12.60<br>New price = \$63 – \$12.60<br>= \$50.40 | Calculate 20% of \$63, with or without a calculator.<br>New price = original price – decrease |
| Method 2:<br>100% – 20% = 80% = 0.8<br>New price = 80% of \$63 = \$50.40 | Subtract 20% from 100%.<br>By calculator, 63 × 0.8 = 50.4 |

**5** Find the new value when:

**a** \$400 is decreased by 10%
**b** \$240 is decreased by 10%
**c** \$250 is decreased by 10%
**d** \$90 is decreased by 20%
**e** \$200 is decreased by 15%
**f** \$840 is decreased by 25%
**g** \$1000 is decreased by 50%
**h** \$60 is decreased by 15%.

**6** **a** Find 8% of \$2500. **b** Increase \$2500 by 8%. **c** Decrease \$2500 by 8%.

## Example 24 Calculating discounts

Find the cost of a $860 television that has been discounted by 25%.

| **Solution** | **Explanation** |
|---|---|
| Discount = 25% of $860<br>$= \frac{25}{100} \times \frac{860}{1} = \$215$ | Calculate 25% discount.<br>Cancel and simplify. |
| Selling price = $860 – $215<br>= $645 | Selling price = cost price – discount |

**7** Find the cost of the following.

- **a** A $600 television that has been discounted by 20%
- **b** A $150 shirt that has been reduced by 15%
- **c** A $52 jumper that has depreciated by 25%

**8** Calculate the selling prices of the following items if they are to be reduced by 25%.

- **a** $16 thongs
- **b** $32 sunhat
- **c** $50 sunglasses
- **d** $85 bathers
- **e** $130 boogie board
- **f** $6.60 surfboard wax

## Example 25 Calculating mark-ups

Find the cost of a $250 microwave oven that has been marked up by 12%.

| **Solution** | **Explanation** |
|---|---|
| Method 1:<br>Mark-up = 12% of $250 = $30 | Calculate 12% of $250, with or without a calculator. |
| Selling price = $250 + $30<br>= $280 | Selling price = cost price + mark-up |
| Method 2:<br>100% + 12% = 112% = 1.12<br>New price = 112% of $250<br>= $280 | Add 12% to 100%.<br>By calculator: 250 × 1.12 = 280 |

**9** Find the cost of the following.

- **a** A $80 framed poster that has been marked up by 10%
- **b** A $14 meal that has been increased by 10%
- **c** A $420 stereo that has been marked down by 50%

**Problem-solving and Reasoning**

**10** Solve these problems.

- **a** Anne's annual salary was $\$86\,000$. Her new salary is $5\%$ more. What is Anne's new salary?
- **b** The state government increases the cost of a $\$9.60$ train trip by $5\%$. What is the new fare?
- **c** A car worth $\$47\,000$ dropped in value by $20\%$ during the year. What is the car now worth?
- **d** A $10\%$ surcharge needs to be added to the cost of the meal. What does a $\$74$ meal cost, including the surcharge?
- **e** Tax of $40\%$ reduces Saul's wage of $\$1600$. What amount does Saul receive?
- **f** Sally makes a $24\%$ profit on her house. She paid $\$500\,000$. For how much did she sell it?

**11** Two shops advertise the same bike. Both have a recommended retail price of $\$1800$. Shop one offers a $10\%$ discount. Shop two offers $\$200$ off all bikes.

- **a** How much discount does shop one offer on this bike?
- **b** How much do you pay for the bike at each shop?
- **c** What shop would you recommend and why?
- **d** If the same deal applies to a:
  - **i** $\$2000$ bike, would you still buy it from the same shop?
  - **ii** $\$2200$ bike, would you still buy it from the same shop?

Find the price of each bike at shop one and two before answering part **d**. Are you surprised by your answers?

**12** If a price is increased by $10\%$, then that price is decreased by $10\%$, it does *not* go back to the original price.
Will the new price be higher or lower than the original price?
Give an example to explain your answer.

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## Enrichment: Depreciation

**13** The word *depreciation* is used when the value of an item, such as a car, boat or a set of golf clubs, decreases each year.

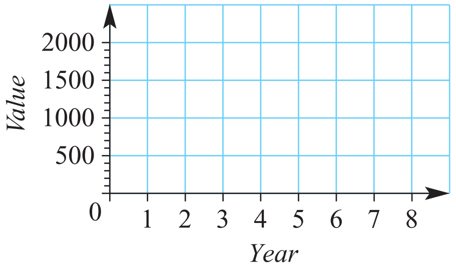
**a** Rick's set of golf clubs, worth $2000, depreciates at a rate of $250 a year.

**i** Copy and complete the table showing how the value of the clubs changes over time.

| End of year | Value |
|---|---|
| 0 | 2000 |
| 1 | 1750 |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |

Drilling for Gold 4H1

**ii** Draw up a set of axes (like those shown below) and graph the values shown in the table.



**iii** What shape is your graph?

**iv** After how many years is the value of the clubs zero?

**b** Rick's wife has a set of golf clubs, also valued at $2000, that depreciate at $12\frac{1}{2}\%$ each year.

**i** Complete a similar table showing how the value of her clubs changes.

| End of year | Value |
|---|---|
| 0 | 2000 |
| 1 | 1750 |
| 2 | $1531.25 |
| 3 | |
| 4 | |

Use a calculator to help you find the values in this table!

**ii** Will her clubs ever be worthless?

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# 4I The Goods and Services Tax (GST)

The Goods and Services Tax or GST is added to the cost of most goods and services in Australia.

The advertised price of the goods in shops, restaurants and other businesses must include the GST. At present in Australia, the GST is set at $10\%$.

Not all goods and services are taxed under the GST. Items that are exempt from the goods and services tax include most basic foods, some education courses and some medical and healthcare products and services.

## Let's start: GST

Look at the prices before and after GST was included.

$$\begin{tikzcd}\boxed{\begin{array}{c}\text{Before}\\ \text{GST}\\ \$180\end{array}} \arrow[r, "\times \square"] & \boxed{\begin{array}{c}\text{After}\\ \text{GST}\\ \$198\end{array}}\end{tikzcd}$$

- How much GST was paid?
- What percentage is the GST?
- What number could be placed in the box?
- If the GST was $12\%$, what number would go in the box?

## Key ideas

**Unitary method**
Calculating the value of one unit of an item and then using this to calculate the value of a number of items

- The GST is $10\%$ of the sale price. It is paid by the consumer and passed on to the government by the supplier.
- The final advertised price, inclusive of the GST represents $110\%$ of the value of the product: the cost $(100\%)$ plus the GST of $10\%$ gives $110\%$.
- The **unitary method** can be used to find the GST included in the price of an item or the pre-tax price.
- The unitary method involves finding the value of 'one unit', usually $1\%$, then using this information to answer the question.
- Alternatively, this chart could be used:

$$\begin{tikzcd}\boxed{\begin{array}{c}\text{Before}\\ \text{GST}\\ (100\%)\end{array}} \arrow[r, shift left, "\times 1.1"] & \boxed{\begin{array}{c}\text{After}\\ \text{GST}\\ (110\%)\end{array}} \arrow[l, shift left, "\div 1.1"]\end{tikzcd}$$

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## Exercise 4I

**Understanding**

**1** Without using a calculator, evaluate the following.

**a** 10% of \$50 **b** 10% of \$160
**c** 10% of \$250 **d** 10% of \$700
**e** 10% of \$15 **f** 10% of \$88
**g** 10% of \$5 **h** 10% of \$2.50

**2** Complete this table, without using a calculator.

| | Price (no GST) | 10% GST | Price (inc. GST) |
|---|---|---|---|
| **a** | \$100 | | |
| **b** | \$50 | | |
| **c** | \$150 | | |
| **d** | \$5 | | |
| **e** | \$1 | | |
| **f** | \$120 | | |

**3** If the GST is \$8, what was the original price?

**4** The final advertised price of a pair of shoes is \$99. The GST included in this price is:
**A** \$9.90 **B** \$90 **C** \$9 **D** \$89.10

**Fluency**

### Example 26 Calculating the GST

Calculate the GST payable on a:
**a** table that a manufacturer values at \$289
**b** bill from a landscape gardener of \$2190.

| Solution | Explanation |
|---|---|
| **a** $\frac{10}{100} \times 289 = 28.9$ | GST is 10% of the value. |
| The GST is \$28.90. | Find 10% of \$289. ($289 \div 10$) |
| **b** $\frac{10}{100} \times 2190 = 219$ | Find 10% of \$2190. ($2190 \div 10$) |
| The GST is \$219. | |

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**5** Calculate the GST payable on goods priced at:

**a** \$680 **b** \$4000 **c** \$550
**d** \$28 **e** \$357 **f** \$5.67

### Example 27 Using the unitary method to find the full amount

A bike has GST of \$38 added to its price. What is the price of the bike:
**a** before the GST? **b** after the GST?

| **Solution** | **Explanation** |
|---|---|
| **a** $10\% = 38$<br>$1\% = 3.8$<br>$100\% = 380$<br>Before the GST was added, the cost of the bike was \$380. | Divide by 10 to find the value of $1\%$.<br>Multiply by 100 to find $100\%$. |
| **b** After the GST, the price is \$418. | Add the GST onto the pre-GST price<br>$\$380 + \$38 = \$418$ |

**6** Calculate the final price, including the GST, on items priced at:

**a** \$700 **b** \$3000 **c** \$450
**d** \$34 **e** \$56 700 **f** \$4.90

### Example 28 Using the unitary method to find the pre-GST price

The final price at a café including the $10\%$ GST was \$137.50, what was the pre-GST price of the meal?

| **Solution** | **Explanation** |
|---|---|
| $110\% = 137.50$<br>$1\% = 1.25$<br>$100\% = 125$ | The GST adds $10\%$ to the cost of the meal<br>$100\% + 10\% = 110\%$<br>$\therefore$ final price is $110\%$. |
| The pre-GST cost is \$125.<br>Alternative method<br>$\$137.50 \div 1.1 = \$125$ | Divide by 110 to find the value of $1\%$.<br>Multiply by 100 to find $100\%$.<br>Dividing by 1.1 gives the original price excluding GST. |

**7** The final price to the consumer includes the $10\%$ GST. Calculate the pre-GST price if the final price was:

**a** \$220 **b** \$66 **c** \$8800
**d** \$121 **e** \$110 **f** \$0.99

**8** Calculate the pre-GST price if the final cost to the consumer was:

**a** \$352 **b** \$1064.80 **c** \$506
**d** \$52.25 **e** \$10791 **f** \$6.16

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**9** Copy and complete the table.

| Pre-GST price | $10\%$ GST | Final cost including the $10\%$ GST |
|---|---|---|
| $599 | | |
| | $68 | |
| | $70 | |
| | | $660 |
| $789 | | |
| | $89.20 | |
| | | $709.50 |
| | | $95.98 |

**Problem-solving and Reasoning**

**10** Here are three real-life receipts (with the names of shops changed). GST rate is $10\%$. Answer the following based on each one.

### Superbarn

**a** How much was spent at Superbarn?

**b** How many kilograms of tomatoes were bought?

**c** Which item included the GST, and how do you tell by looking at the receipt?

**d** What was the cost of the item if the GST is not included?

TAX INVOICE

SUPERBARN

SUPERBARN GYMEA

| Description | Total $ |
|---|---|
| O/E PASO TACO KITS 290GM | 6.09 |
| TOMATOES LARGE KILO<br>0.270kg @$4.99/kg | 1.35 |
| WATERMELON SEEDLESS WHOLE KILO<br>1.675kg @$2.99/kg | 5.01 |
| LETTUCE ICEBERG EACH | 2.49 |
| *PAS M/MALLOWS 250GM | 1.89 |
| SubTotal | $16.83 |
| Rounding | $0.02 |
| **TOTAL** (Inc GST)<br>5 Items | $16.85 |
| Cash Tendered | $20.00 |
| Change Due | $3.15 |
| GST Amount | $0.17 |

* Signifies items(s) with GST

Thank you for shopping at Superbarn

### Gymea Fruit Market

**a** What was the cost of bananas per kilogram?

**b** On what date was the purchase made?

**c** What does ROUND mean?

**d** What was the total paid for the items?

**e** How much tax was included in the bill?

**f** What percentage of the bill was the tax?

GYMEA FRUIT MARKET

HAVE A NICE DAY

DATE 05/07/2011 TUES TIME 11:21

| | |
|---|---|
| 0.090 KG @ $14.99/kg<br>BANANA SUGAR | $1.35 |
| SL MUSHROOM | $2.49 |
| PISTACCHIO 11 | $6.00 |
| ROUND | $0.01 |
| **TOTAL** | $9.85 |
| CASH | $9.85 |
| TAX 1 | $0.55 |

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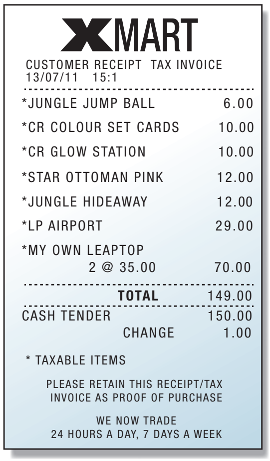
4I

**Xmart**

**a** How many toys were purchased?
**b** What was the cost of the most expensive item?
**c** Which of the toys attracted GST?
**d** How much GST was paid in total?
**e** What percentage of the total bill was the GST?



XMART

CUSTOMER RECEIPT TAX INVOICE
13/07/11 15:1

| | |
|---|---|
| *JUNGLE JUMP BALL | 6.00 |
| *CR COLOUR SET CARDS | 10.00 |
| *CR GLOW STATION | 10.00 |
| *STAR OTTOMAN PINK | 12.00 |
| *JUNGLE HIDEAWAY | 12.00 |
| *LP AIRPORT | 29.00 |
| *MY OWN LEAPTOP<br>2 @ 35.00 | 70.00 |
| TOTAL | 149.00 |
| CASH TENDER | 150.00 |
| CHANGE | 1.00 |

* TAXABLE ITEMS

PLEASE RETAIN THIS RECEIPT/TAX INVOICE AS PROOF OF PURCHASE

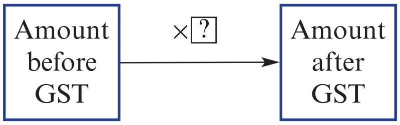
WE NOW TRADE
24 HOURS A DAY, 7 DAYS A WEEK

**11** A plumber's quote for installing a dishwasher is $\$264$ including the GST.
**a** Use the unitary method to calculate the GST included in this price.
**b** Calculate the pre-GST price for installing the dishwasher.
**c** Divide the $\$264$ by $11$. What do you notice? Explain why this works.
**d** Divide the $\$264$ by $1.1$. Why does this give the same answer as part **b**?

**12** Use the technique outlined in Question **11** to find the GST already paid on goods and services costing:
**a** $\$616$ **b** $\$1067$ **c** $\$8679$ **d** $\$108.57$

**13** The cost of a lounge suite is $\$990$ and includes $\$90$ in GST. Find $10\%$ of $\$990$ and explain why it is more than the GST included in the price.

**14** In Singapore the GST is $7\%$. What number goes in the box?



## Enrichment: GST and the manufacturing process

The final consumer of the product pays GST. Consider the following.

A fabric merchant sells fabric at $66 (including the 10% GST) to a dressmaker. The merchant makes $60 on the sale and passes the $6 GST onto the Australian Tax Office (ATO).

A dressmaker uses the fabric and sells the product onto a retail store for $143 (this includes the $13 GST). As the dressmaker has already paid $6 in GST when they bought the fabric they have a GST tax credit of $6 and they pass on the $7 to the ATO.

The retailer sells the dress for $220, including $20 GST. As the retailer has already paid $13 in GST to the dressmaker, they are required to forward the extra $7 to the ATO.

The consumer who buys the dress bears the $20 GST included in the price.

**15** Copy and complete the following.

*Raw material* $110 (includes the 10% GST)

GST on sale = ________

GST credit = $0

Net GST to pay = ________

*Production stage* $440 (includes the 10% GST)

GST on sale = ________

GST credit = ________

Net GST to pay = ________

*Distribution stage* $572 (includes the 10% GST)

GST on sale = ________

GST credit = ________

Net GST to pay = ________

*Retail stage* $943.80 (includes the 10% GST)

GST on sale = ________

GST credit = ________

Net GST to pay = ________

GST paid by the final consumer = ________

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# 4J Calculating percentage change, profit and loss

People who sell things like to make a *profit*. This is when you sell it for more than you paid for it.

Unfortunately, people often do the opposite and make a *loss*.

The percentage change depends on what the item is originally worth. For example:

| | |
|---|---|
| Car bought for \$1000<br>Car sold for \$200 | Car bought for \$16 000<br>Car sold for \$15 200 |
|  |  |
| Loss \$800, percentage loss 80% | Loss \$800, percentage loss 5% |

## Let's start: Hang on!

Discuss how you could check if the correct price for the jeans had been paid.

## Key ideas

- **Profit** = selling price – cost price
- **Loss** = cost price – selling price
- Calculating a percentage change involves the technique of expressing one quantity as a percentage of another.

$$\textbf{Percentage change} = \frac{\text{change}}{\text{original value}} \times 100\%$$

$$\textbf{Percentage profit} = \frac{\text{profit}}{\text{original value}} \times 100\%$$

$$\textbf{Percentage loss} = \frac{\text{loss}}{\text{original value}} \times 100\%$$

**Profit** The amount of money made by selling for more than the cost

**Loss** The amount of money lost by selling for less than the cost

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## Exercise 4J

**Understanding**

**1** Decide whether each of the following represents a profit or a loss.

**a**

bought = $250 000
sold = $280 000

**b**

bought = $795
sold = $210

**c**

bought = $1200
sold = $500

**d**

bought = $2000
sold = $4500

**e**

bought = $1.40
sold = $3.20

**2** Calculate the profit made in each of the following situations.

- **a** Cost price = $14, Sale price = $21
- **b** Cost price = $75, Sale price = $103
- **c** Cost price = $25.50, Sale price = $28.95
- **d** Cost price = $499, Sale price = $935

**3** Calculate the loss made in each of the following situations.

- **a** Cost price = $22, Sale price = $9
- **b** Cost price = $92, Sale price = $47
- **c** Cost price = $71.10, Sale price = $45.20
- **d** Cost price = $1121, Sale price = $874

**4** Which of the following is the correct formula for working out percentage change?

- **A** $\%\text{ change} = \dfrac{\text{change}}{\text{original value}}$
- **B** $\%\text{ change} = \dfrac{\text{original value}}{\text{change}} \times 100\%$
- **C** $\%\text{ change} = \text{change} \times 100\%$
- **D** $\%\text{ change} = \dfrac{\text{change}}{\text{original value}} \times 100\%$

**Fluency**

### Example 29 Calculating percentage change: profit

Calculate the percentage change when:
$25 becomes $32

**Solution**

Profit = $7

$$\%\text{ Profit} = \frac{7}{25} \times \frac{100}{1}\%$$
$$= 28\%$$

**Explanation**

This is percentage *profit* because it was sold for more than the original $25.
Profit = $32 – $25

$$\text{Percentage profit} = \frac{\text{profit}}{\text{original value}} \times 100\%$$

**5** Find the percentage change as a percentage profit when:

**a** \$20 becomes \$36 **b** \$10 becomes \$13
**c** \$40 becomes \$50 **d** \$25 becomes \$30
**e** \$12 becomes \$20 **f** \$8 becomes \$11
**g** \$10 becomes \$15 **h** \$6 becomes \$12.

$$\% \text{ change} = \frac{\text{change}}{\text{original}} \times \frac{100}{1}$$

## Example 30 Calculating percentage change: loss

Calculate the percentage change when:
\$60 becomes \$48

**Solution**

Loss = \$12

$$\% \text{ Loss} = \frac{12}{60} \times \frac{100}{1}\%$$
$$= 20\%$$

**Explanation**

This is percentage *loss* because it was sold for less than the original \$60.
Loss = \$60 – \$48

$$\text{Percentage loss} = \frac{\text{loss}}{\text{original value}} \times 100\%$$

**6** Find the percentage change as a percentage loss when:

**a** \$40 becomes \$30 **b** \$25 becomes \$21
**c** \$6 becomes \$3 **d** \$8 becomes \$2
**e** \$12 becomes \$8 **f** \$10 becomes \$9
**g** \$25 becomes \$20 **h** \$20 becomes \$18.

$$\% \text{ loss} = \frac{\text{loss}}{\text{original}} \times \frac{100}{1}$$

## Example 31 Solving worded problems

Ross buys a ticket to a concert for \$125, but is later unable to go. He sells it to his friend for \$75. Calculate the percentage loss Ross made.

**Solution**

Loss = \$125 – \$75 = \$50

$$\% \text{ Loss} = \frac{50}{125} \times \frac{100}{1}\%$$
$$= 40\%$$

Ross made a 40% loss on the concert ticket.

**Explanation**

Loss = cost price – selling price

$$\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$

**7** Copy and complete the tables below.

**a**

| Cost price \$ | Selling price \$ | Profit \$ | % profit |
|---|---|---|---|
| 4 | 5 | | |
| 10 | 12 | | |
| 24 | 30 | | |
| 100 | 127 | | |

$$\% \text{ profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

**b**

| Cost price \$ | Selling price \$ | Loss \$ | % loss |
|---|---|---|---|
| 10 | 7 | | |
| 16 | 12 | | |
| 50 | 47 | | |
| 100 | 93 | | |

$$\% \text{ loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$

**8** Find the percentage change (increase or decrease) when:

**a** 15°C becomes 18°C

**b** 18°C becomes 15°C

**c** 4°C becomes 24°C

**d** 12°C becomes 30°C.

Skillsheet 4D

**9** Find the percentage change in population when a:

**a** town of 4000 becomes a town of 5000

**b** city of 750 000 becomes a city of 900 000

**c** country of 5 000 000 becomes a country of 12 000 000.

## Problem-solving and Reasoning

**10** Gari buys a ticket to a concert for \$90, but is unable to go. He sells it to his friend for \$72. Calculate the percentage loss Gari made.

**11** Xavier purchased materials for \$48 and made a dog kennel. He later sold the dog kennel for \$84.

**a** Calculate the profit Xavier made.

**b** Calculate the percentage profit Xavier made.

**12** Gemma purchased a \$400 foal, which she later sold for \$720.

**a** Calculate the profit Gemma made.

**b** Calculate the percentage profit Gemma made.

**13** Lee-Sen purchased a \$5000 car, which she later sold for \$2800.

**a** Calculate the loss Lee-Sen made.

**b** Calculate the percentage loss Lee-Sen made.

**c** What should Lee-Sen sell the car for to make a 10% profit?

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4J

## Enrichment: Growth rate for Australia

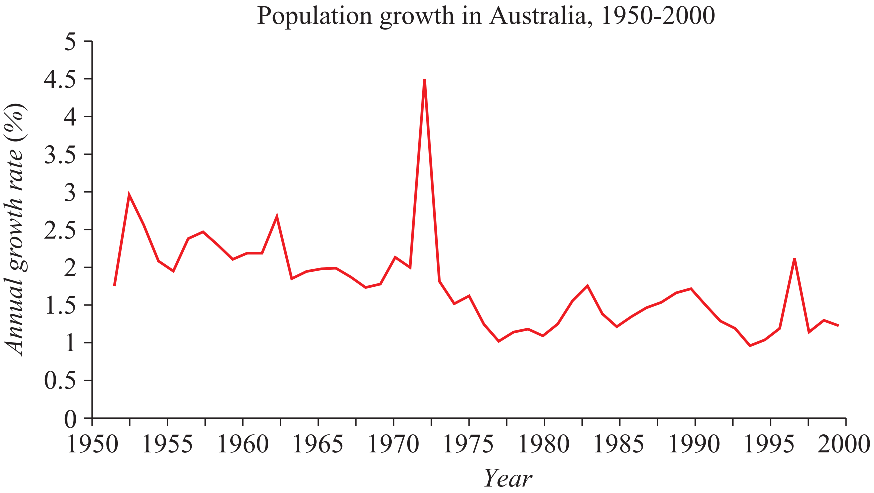
**14** The Australian Bureau of Statistics tracks the population growth of the country and of each individual state and territory.

**a** Copy and complete the table below.

| | March 2011 | Change in the past 12 months | % change |
|---|---|---|---|
| NSW | 7 287 600 | 82 100 | |
| VIC | 5 605 600 | 81 600 | |
| QLD | 4 561 700 | 73 200 | |
| SA | 1 654 200 | 13 900 | |
| WA | 2 331 500 | 51 000 | |
| TAS | 510 200 | 3200 | |
| NT | 229 200 | 900 | |
| ACT | 363 800 | 6400 | |
| AUSTRALIA | 22 546 300 | 312 400 | |

Use a calculator to help you with this question.

**b** Research the current growth rate of Australia and one other country of your choice.



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# 4K Solving percentage problems with the unitary method and equations

In the GST section, we did calculations like this:

$10\% = 38$

$1\% = 3.8$

$100\% = 380$

Once we know what 1% is worth, we can find any percentage amount. This is called the *unitary method*.

## Let's start: Using the unitary method

- Four tickets to a concert cost \$100. What does one ticket cost? How much will three tickets cost?
- Six small pizzas cost \$54. What does one pizza cost? How much would ten pizzas cost?
- If eight pairs of socks cost \$64, how much would 11 pairs of socks cost?
- Five passionfruit cost \$2.00. How much will nine passionfruit cost?

## Key ideas

- The **unitary method** involves finding the value of 'one unit' and then using this information to find other values.
- When dealing with percentage, finding 'one unit' means finding one percent (1%).
- Once the value of 1% of an amount Is known, it can be multiplied to find the value of any desired percentage, often 100%.

**Unitary method** Calculating the value of one unit of an item and then using this to calculate the value of a number of items

## Exercise 4K

Understanding

**1** **a** What do you divide by to go from 8% to 1%?
   **b** What do you divide by to go from 25% to 1%?
   **c** What do you multiply by to go from 1% to 100%?
   **d** What do you multiply by to go from 1% to 50%?

**2** If 1% of an amount is \$3, what is:
   **a** 2% of the amount? **b** 10% of the amount? **c** 100% of the amount?

**3** If 1% of an amount is \$8, what is:
   **a** 10% of the amount? **b** 100% of the amount?

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**Fluency**

**4** Copy and complete.

4% = $16
then 1% = ☐
and 100% = ☐

## Example 32 Using the unitary method to find the full amount

If 8% of an amount of money is $48, what is the full amount of money?

| **Solution** | **Explanation** |
|---|---|
| ÷8 8% of amount is $48 ÷8<br>×100 1% of amount is $6 ×100<br>100% of amount is $600<br>Full amount of money is $600. | Remember to find 1% first.<br>Divide by 8 to find the value of 1% (48 ÷ 8 = 6).<br>Multiply by 100 to find the value of 100% (6 × 100 = 600). |
| Alternative solution:<br>8% of ‘amount’ is $48<br>$8\%$ of $A = \$48$<br>÷0.08 $0.08 \times A = 48$ ÷0.08<br>$A = 48 \div 0.08$<br>$A = 600$ | Form an equation.<br>Divide both sides by 0.08. |

**5** Calculate the full amount of money for each of the following.

- **a** 3% of an amount of money is $27.
- **b** 5% of an amount of money is $40.
- **c** 12% of an amount of money is $132.
- **d** 60% of an amount of money is $300.
- **e** 8% of an amount of money is $44.
- **f** 6% of an amount of money is $15.

First find the value of 1%.

## Example 33 Using the unitary method to find a new percentage

If 11% of the food bill was $77, how much is 25% of the food bill?

| **Solution** | **Explanation** |
|---|---|
| 11% of food bill is $77<br>∴ 1% of food bill is $7<br>∴ 25% of food bill is $175 | Find 1% first.<br>Divide by 11 to find the value of 1% (77 ÷ 11 = 7).<br>Multiply by 25 to find the value of 25% (7 × 25 = 175). |

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| **Solution** | **Explanation** |
|---|---|
| Alternative solution: | |
| $11\%$ of $B = 77$ | |
| $\div 0.11$ $0.11 \times B = 77$ $\div 0.11$ | Form an equation. |
| $B = 77 \div 0.11$ | Solve the equation. |
| $B = 700$ | |
| $0.25 \times B = 175$ | We need $25\%$ of $B$. |

**6** If $4\%$ of the total bill is $\$12$, how much is $30\%$ of the bill?

First find the value of $1\%$ of the bill.

**7** Calculate:

- **a** $20\%$ of the bill, if $6\%$ of the total bill is $\$36$
- **b** $80\%$ of the bill, if $15\%$ of the total bill is $\$45$
- **c** $3\%$ of the bill, if $40\%$ of the total bill is $\$200$
- **d** $7\%$ of the bill, if $25\%$ of the total bill is $\$75$.

Problem-solving and reasoning

## Example 34 Using the unitary method to find the original price

A pair of shoes has been discounted by $20\%$. If the sale price was $\$160$, what was the original price of the shoes?

| **Solution** | **Explanation** |
|---|---|
| Only paying $80\%$ of original price: | $20\%$ discount, so paying $(100 - 20)\%$. |
| $\therefore$ $80\%$ of original price is $\$160$ | We pay $80\%$ after the $20\%$ discount. |
| $\therefore$ $1\%$ of original price is $\$2$ | Divide by 80 to find the value of $1\%$ $(160 \div 80 = 2)$. |
| $\therefore$ $100\%$ of original price is $\$200$<br>The original price of the shoes was $\$200$. | Multiply by 100 to find the value of $100\%$ $(2 \times 100 = 200)$. |
| Alternative solution: | |
| $80\%$ of $P = \$160$ | |
| $\div 0.8$ $0.8 \times P = 160$ $\div 0.8$ | Form an equation. |
| $P = 160 \div 0.8$ | |
| $P = 200$ | Solve the equation. |

**8** A necklace in a jewellery store has been discounted by $20\%$. If the sale price is $\$240$, what was the original price of the necklace?

$100\% - 20\% = 80\%$

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**9** Find the original price of the following items.

**a** A pair of jeans discounted by 40% has a sale price of $30 (you pay 60%).

**b** A hockey stick discounted by 30% has a sale price of $105 (you pay 70%).

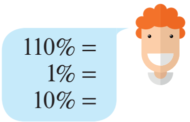
**c** A second-hand computer discounted by 85% has a sale price of $90 (you pay 15%).

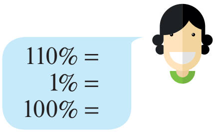
**d** A second-hand textbook discounted by 80% has a sale price of $6.

**e** A standard rose bush discounted by 15% has a sale price of $8.50.

**f** A motorbike discounted by 25% has a sale price of $1500.

**10** Jen paid for dinner with a credit card. She was charged a $1.50 surcharge, which was 2% of the cost. How much did the meal cost without the surcharge?





**11** Elsie received a 6% discount and saved $18. How much did she pay?

**12** If 22% of an amount is $8540, which of the following would give the value of 1% of the amount?

**A** $8540 × 100
**B** $8540 ÷ 100
**C** $8540 × 22
**D** $8540 ÷ 22

## Enrichment: Lots of ups and downs

**13** A shirt is currently selling for a price of $120.

**a** Increase the price by 10%, then increase that price by 10%. Is this the same as increasing the price by 20%?

**b** Increase the price of $120 by 10%, then decrease that price by 10%. Did the price go back to $120?

**c** The price of $120 is increased by 10%. By what percentage must you decrease the price for it to go back to $120?

**d** If the price of $120 goes up by 10% at the end of every year, how many years does it take for the price to double?

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# Puzzles and games

**1** Write down four decimals that when rounded to 2 decimal places give 2.67.

**2** Jill has 5 coins in her pocket. A \$2 coin, \$1 coin, 50 c piece, 20 c piece, and one 10 cent coin.
If Jill chooses just two coins from her pocket without looking at them, or noticing their size or shape, how many different amounts could she arrive at?

**3** Write one half in ten different ways.

**4** Complete these magic squares. All rows, columns and the two diagonals sum to the same total.

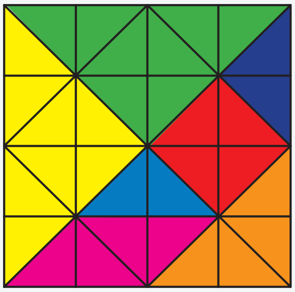
**a**

| | | |
|---|---|---|
| $\frac{4}{3}$ | | |
| 1 | $1\frac{2}{3}$ | |
| $2\frac{2}{3}$ | | |

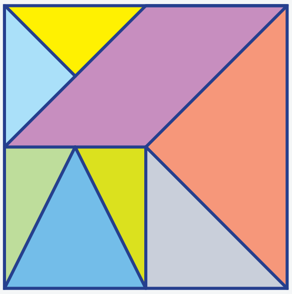
**b**

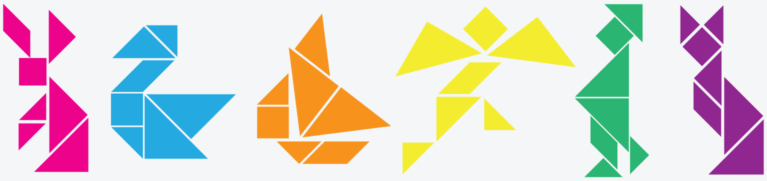
| | | |
|---|---|---|
| $\frac{5}{3}$ | | |
| | $\frac{11}{6}$ | $2\frac{1}{6}$ |
| | | 2 |

**5** A tangram consists of seven geometric shapes (tans) as shown. The tangram puzzle is precisely constructed using vertices, midpoints and straight edges.



- **a** Write each of the separate tan pieces as a percentage, a fraction and a decimal amount of the entire puzzle.
- **b** Check your seven tans add up to a total of 100%.
- **c** Starting with a square, make a new version of a 'modern' tangram puzzle. You must have at least six pieces in your puzzle.
  An example of a modern puzzle is shown.
- **d** Write each of the separate pieces of your new puzzle as a percentage, a fraction and a decimal amount of the whole puzzle.
- **e** Separate pieces of tangrams can be arranged to make more than 300 creative shapes and designs, some of which are shown. You may like to research tangrams and attempt to make some of the images.





# Chapter summary

## Fractions

### What is a fraction?

A fraction is part of a whole.

$\frac{1}{4}$

$\frac{1}{4}$ 1 → numerator (parts from the whole); 4 → denominator (parts in the whole)

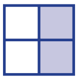
### Simplifying fractions

Simplify fractions by dividing the numerator and denominator by their HCF.

$$\underset{\div 4}{\overset{\div 4}{\frac{8}{20} = \frac{2}{5}}}$$

### Equivalent fractions

- represent the same amount, they are the same decimal.



$\frac{2}{4} = \frac{1}{2} = 0.5$

- create equivalent fractions by multiplying or dividing the top (numerator) and bottom (denominator) by the same number.

$$\underset{\times 3}{\overset{\times 3}{\frac{2}{5} \qquad \frac{6}{15}}}$$

### Mixed and improper

$$\text{Mixed } 1\frac{1}{3} \xrightarrow{\frac{1 \times 3 + 1}{3}} \text{Improper } \frac{4}{3}$$

$$\frac{4}{3} \xrightarrow{4 \div 3 = 1 \text{ rem } 1} 1\frac{1}{3}$$

### Adding or subtracting fractions

To add or subtract fractions you should use the lowest common denominator.

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$$
$$= \frac{5}{6}$$

### Multiplying fractions

- use proper or improper fractions $\frac{3}{2} \times \frac{1}{4}$
- cancel any numerator with any denominator
- multiply the numerators
- multiply the denominators
- simplify $\frac{3}{8}$

### Reciprocal

To take the reciprocal of a fraction is to swap the numerator and denominator.

### Dividing fractions

- keep the first
- change the ÷ to ×
- invert the following fraction (its reciprocal) and multiply.

# Chapter summary

## Decimals

**Fractions to decimals**

(1) Change denominator to 10, 100 or 1000.

or (2) Divide the denominator into the numerator.

**Types of decimals**

Decimals can terminate (stop).

0·5, 0·75, 0·125

Decimals can repeat/ recurr.

$0\cdot3333.... = 0\cdot\dot{3}$

$0\cdot616161... = 0\cdot\dot{6}\dot{1}$

**Multiplying by tens**

The decimal moves the same number of places as the number of zeroes.

$6\cdot413 \times 100 = 641\cdot3$

**Adding and subtracting**

When adding and subtracting with decimals make sure the decimal points line up.

**Dividing by tens**

$71\cdot3 \div 100 = 0\cdot713$

**Dividing decimals**

Multiply both numbers by 10, 100, 1000 etc. This is dividing by a whole number.

$$\frac{12\cdot66}{0\cdot3} \xrightarrow{\times 10} \frac{126\cdot6}{3} = 42\cdot2$$

**Multiplying decimals**

The number of decimal places in the question is the same as in the answer

$0\cdot04 \times 0\cdot3 = 0\cdot012$

**Rounding decimals**

If the digit after the place you want is 0, 1, 2, 3 or 4 round down; if the digit 5, 6, 7, 8 or 9 round up.

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# Chapter summary

**Drilling for Gold 4R1**

## Percentages

### Percentages and decimals

$0.45 = 45\%$

$20\% = 0.2$

### Unitary method
**1 unit is 1%**

If / find 80%

$$\begin{tikzcd} 6\% \text{ is } \$420 \arrow[d, bend right, "\div 6"'] \arrow[d, bend left, "\div 6"] \\ 1\% \text{ is } \$70 \arrow[d, bend right, "\times 80"'] \arrow[d, bend left, "\times 80"] \\ 80\% \text{ is } \$5600 \end{tikzcd}$$

### Percentages and fractions

$$\frac{3}{5} = \frac{3}{5} \times 100\%$$
$$= 60\%$$

$$35\% = \frac{35}{100} = \frac{7}{20}$$

### Common conversions

$\frac{1}{2} = 0.5 = 50\%$

$\frac{1}{10} = 0.1 = 10\%$

$\frac{1}{100} = 0.01 = 1\%$

$\frac{1}{4} = 0.25 = 25\%$

$\frac{3}{4} = 0.75 = 75\%$

$\frac{1}{3} = 0.\dot{3} = 33\frac{1}{3}\%$

$\frac{2}{3} = 0.\dot{6} = 66\frac{2}{3}\%$

### 'A' as a percentage of 'B'

$$\frac{A}{B} \times 100\%$$

25 g as a percentage of 1 kg

$$\frac{25}{1000} \times 100\% = 2.5\%$$

Same units

### Percentage profit

$$= \frac{\text{Profit}}{\text{Cost price}} \times 100\%$$

### Percentage loss

$$= \frac{\text{Loss}}{\text{Cost price}} \times 100\%$$

### % of quantity

$$7\% \text{ of } 20 = \frac{7}{100} \times \frac{20}{1}$$
$$= 1.4$$

### Percentage increase

- mark-up
- profit

Selling price = Cost price + Mark-up (Profit)

$12\frac{1}{2}\%$ mark-up on $300

- Increase $= \frac{12\frac{1}{2}}{100} \times 300 = \$37.50$
- New price $= 300 + 37.50$
  $= \$337.50$

alternatively,

$100\% + 12.5\% = 112.5\%$
$= 1.125$

$\$300 \times 1.125 = \$337.50$

### Percentage Decrease

- Discount
- Reduce
- Depreciate

New price = Orginal price – Decrease

$1700 TV discounted by 10%

Discount $= 1700 \times \frac{10}{100} = \$170$

Sale price $= 1700 - 170$
$= \$1530$

alternatively,

$100\% - 10\% = 90\%$
$= 0.9$

$\$1700 \times 0.9 = \$1530$

### Goods and Services Tax (GST)

Current GST rate in Australia is 10%

$$\begin{tikzcd} \text{Ⓐ Price (excluding GST)} \arrow[r, bend left, "\times 1.1"] & \text{Ⓑ Price (including GST)} \arrow[l, bend left, "\div 1.1"] \end{tikzcd}$$

GST = 10% of A or B – A or B ÷ 11

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**Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.**

## Multiple-choice questions

**1** 0.36 expressed as a fraction is:

**A** $\frac{36}{10}$ **B** $\frac{36}{100}$ **C** $\frac{3}{6}$ **D** $\frac{9}{20}$

**2** $\frac{1}{8}+\frac{5}{8}$ is equal to:

**A** $\frac{6}{64}$ **B** $\frac{6}{16}$ **C** $\frac{15}{8}$ **D** $\frac{6}{8}$

**3** When 21.63 is multiplied by 13.006, the number of decimal places in the answer is:

**A** 2 **B** 3 **C** 4 **D** 5

**4** $2\frac{1}{3}$ is the same as:

**A** 7 **B** $\frac{3}{7}$ **C** $\frac{7}{3}$ **D** 2.3

**5** The reciprocal of $\frac{3}{4}$ is:

**A** $\frac{4}{3}$ **B** $\frac{1}{4}$ **C** $\frac{1}{3}$ **D** $1\frac{1}{2}$

**6** Which decimal has the largest value?

**A** 6.0061 **B** 6.06 **C** 6.016 **D** 6.0006

**7** $9.46 \times 1000$ is:

**A** 94 600 000 **B** 9460 **C** 94 605 **D** 0.000 094 6

**8** 75% of 84 is the same as:

**A** $\frac{84}{4} \times 3$ **B** $\frac{84}{3} \times 4$ **C** $84 \times 100 \div 75$ **D** $\frac{(0.75 \times 84)}{100}$

**9** If 1% equals 8, then 5% equals:

**A** 800 **B** 80 **C** 40 **D** 4

**10** \$790 increased by 10% gives:

**A** \$79 **B** \$880 **C** \$771 **D** \$869

## Short-answer questions

**1** Copy and complete.

**a** $\frac{7}{20}=\frac{\square}{60}$ **b** $\frac{25}{40}=\frac{5}{\square}$ **c** $\frac{4}{7}=\frac{\square}{21}$

**2** Simplify:

**a** $\frac{25}{45}$ **b** $\frac{36}{12}$ **c** $\frac{16}{12}$

**3** Evaluate:

**a** $\frac{1}{4}+\frac{1}{4}$ **b** $\frac{5}{6}-\frac{4}{6}$ **c** $\frac{7}{8}-\frac{4}{8}$ **d** $\frac{7}{10}+\frac{1}{10}$

**e** $\frac{7}{8}-\frac{3}{4}$ **f** $\frac{1}{4}+\frac{1}{2}$ **g** $\frac{5}{12}+\frac{1}{4}$ **h** $\frac{3}{5}+\frac{7}{10}$

**4** Find:

**a** $3-1\frac{1}{4}$ **b** $1\frac{1}{2}+2\frac{1}{2}$ **c** $10-3\frac{1}{2}$ **d** $3\frac{4}{5}+1\frac{2}{5}$

**5** Find:

**a** $\frac{2}{3}$ of 6 **b** $\frac{1}{5}$ of 10 **c** $\frac{2}{3}\times 12$ **d** $\frac{3}{5}\times 20$

**6** Find:

**a** $\frac{1}{2}\times\frac{1}{3}$ **b** $\frac{2}{5}\times\frac{1}{4}$ **c** $\frac{7}{8}\times\frac{2}{5}$ **d** $1\frac{1}{2}\times\frac{2}{9}$

**7** Calculate these divisions.

**a** $6\div\frac{1}{2}$ **b** $\frac{2}{3}\div\frac{1}{3}$ **c** $\frac{4}{5}\div\frac{1}{2}$ **d** $1\frac{1}{2}\div\frac{3}{4}$

**8** Convert these fractions to decimals.

**a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{5}$ **d** $\frac{117}{1000}$

**9** Write these decimals as simple fractions.

**a** 0.6 **b** 0.12 **c** 0.04 **d** 0.95

**10** Evaluate:

**a** $12.6+7.4$ **b** $8.59+5.6$ **c** $9.4-1.2$

**d** $10-5.4$ **e** $9.6+10.1+3.21$ **f** $12.4-6.22$

**11** Evaluate:

**a** $3\times 2$ **b** $0.3\times 0.2$ **c** $1.2\times 4$ **d** $0.12\times 0.4$

**e** $1.5\times 0.4$ **f** $7.164\times 100$ **g** $9.6\times 10$ **h** $0.06\times 7$

**12** Find:

**a** $12\div 0.3$ **b** $18.6\div 3$ **c** $14.22\div 0.2$

**13** Round these decimals to 3 decimal places:

**a** 0.666666 … **b** 3.579 64 **c** 0.005 496 31

**14** Copy and complete this table of conversions.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0.1 | | | | | 0.75 | | |
| | $\frac{1}{100}$ | | | $\frac{1}{4}$ | | $\frac{1}{3}$ | $\frac{1}{8}$ |
| | | 5% | 50% | | | | |

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**15** Find:
  **a** 10% of \$50  **b** 25% of \$64  **c** 5% of 700 g

**16** Express each of the following as a percentage.
  **a** \$35 out of \$40  **b** 6 out of 24
  **c** \$1.50 out of \$1  **d** 16 cm out of 4 m

**17** **a** Increase \$560 by 10%.

  **b** Decrease \$4000 by 15%.

**18** If 6% of an amount is \$18, what is the amount?

**19** Toni bought a \$194 dress on sale for 20% off. What did Toni pay for the dress?

**20** Sally earned \$84 000 last year. This year she got 5% more. What did Sally earn this year?

## Extended-response question

**1** What is my number?
  **a** 5% of my number is 60. What is my number?
  **b** When I multiply my number by 2.5, the result is 60. What is my number?
  **c** $\frac{1}{3}$ of $\frac{1}{2}$ of my number is 60. What is my number?
  **d** When my number is decreased by 60%, the result is 50. What is my number?
  **e** My number was increased by 50% to give 60. What is my number?

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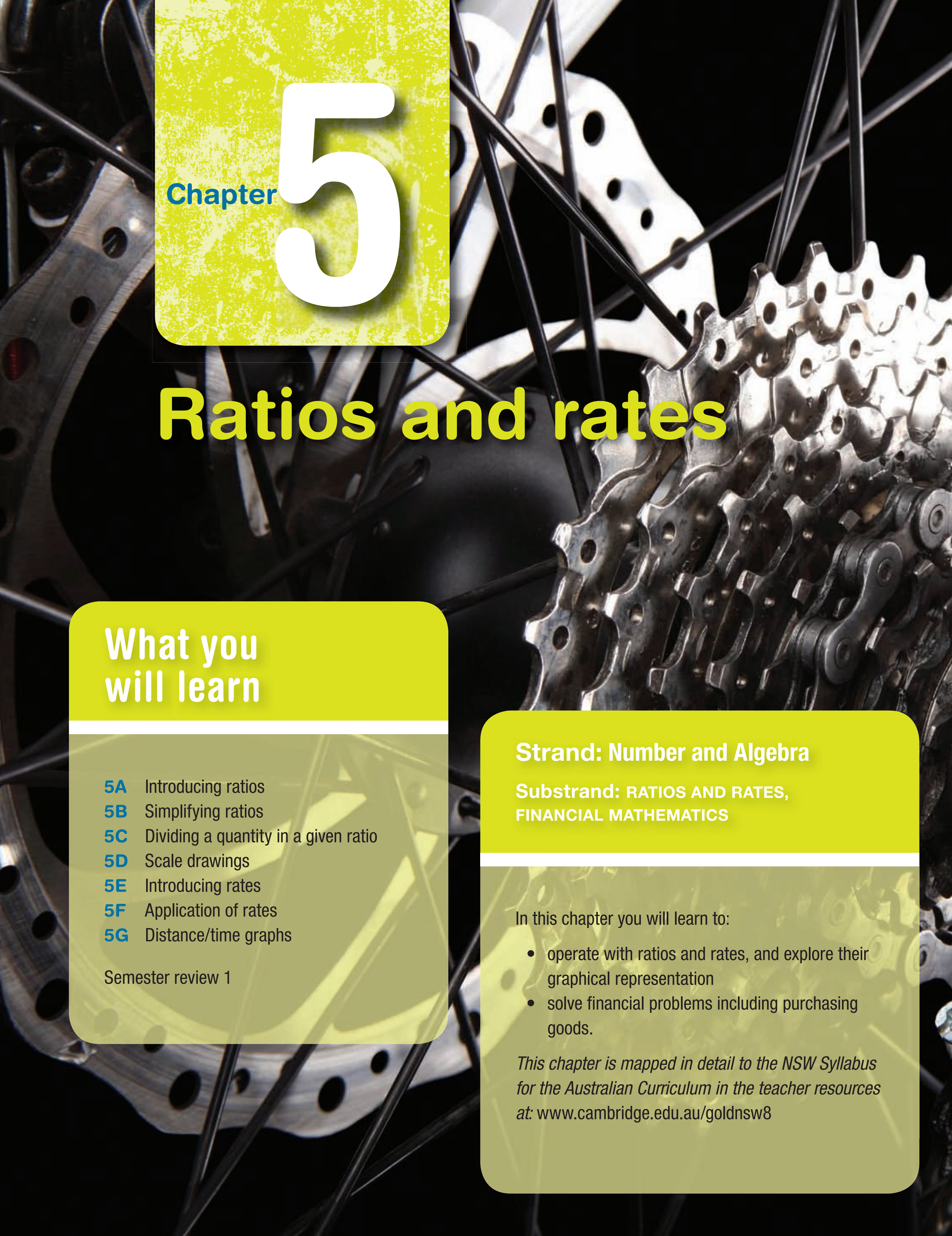


## Chapter 5

# Ratios and rates

## What you will learn

## Strand: Number and Algebra

**Substrand:** RATIOS AND RATES, FINANCIAL MATHEMATICS

In this chapter you will learn to:

- operate with ratios and rates, and explore their graphical representation
- solve financial problems including purchasing goods.

*This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at:* www.cambridge.edu.au/goldnsw8

## Bicycle gear ratios

Bicycle gears make riding much easier. The ‘gear ratio’ compares the number of times the pedals go around to the number of times the bicycle wheels revolve.

Riding up a hill using a low gear ratio of $2:1$ means that the pedals go around twice each time the wheels go around once. When riding fast down a hill, a high gear ratio is used so the rider does not have to pedal as fast as the wheels are turning. For example, a gear ratio of $1:3$ means that for one turn of the pedals, the wheels will rotate three times.

## Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw8

**Drilling for Gold:**
Building knowledge and skills

**Skillsheets:**
Extra practise of important skills

**Literacy activities:**
Mathematical language

**Worksheets:**
Consolidation of the topic

**Chapter Test:**
Preparation for an examination

# Pre-test

**1** Copy and complete.

**a** $$\underset{\div 2}{\overset{\div 2}{\frac{2}{4} = \frac{1}{\square}}}$$

**b** $$\underset{\div 5}{\overset{\div 5}{\frac{15}{20} = \frac{\square}{4}}}$$

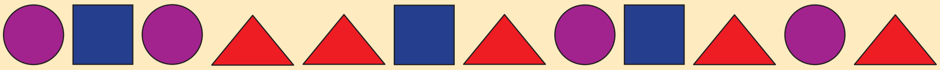
**c** $$\underset{\div ?}{\overset{\div ?}{\frac{12}{15} = \frac{\square}{5}}}$$

**2** Copy and complete.

**a** $$2:5 \xrightarrow{\times 2} 4:\square$$

**b** $$20:28 \xrightarrow{\div 4} \square:7$$

**c** $$3:2 \xrightarrow{\times ?} 12:\square$$

**3** 

Copy and complete.

**a** The ratio of squares to circles is __ : __

**b** The ratio of circles to triangles is __ : __

**c** The ratio of triangles to other shapes is __ : __

**4** Convert:

**a** 5 m = ____ cm

**b** 6 km = ____ m

**c** 500 cm = ____ m

**d** 80 mm = ____ cm

**e** 120 cm = ____ m

**f** 15 000 m = ____ km

10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

**5** Write these ratios in the same units and simplify.

**a** 3 cm : 15 mm

**b** 45 cm : 1m

**c** 45 minutes : 1 hour

**d** 10 minutes : $\frac{1}{2}$ hour

**e** 2 km : 500 m

**f** 40 m : $\frac{1}{2}$ km

**6** If 4 mangoes cost $6, how much would:

**a** one mango cost?

**b** 12 mangoes cost?

$$\text{4 mangoes cost \$6} \xrightarrow{\div 4} \text{1 mango costs \$?}$$

**7** The cost of 1 kg of bananas is $4.99. Find the cost of:

**a** 2 kg **b** 5 kg **c** 10 kg **d** $\frac{1}{2}$ kg

**8** Kevin walks 3 km in one hour. How far did he walk in 30 minutes?

**9** Tao earns $240 for working 8 hours. How much did Tao earn each hour?

**10** A car travels at an average speed of 60 km per hour.

**a** How far does it travel in the following times?

**i** 2 hours **ii** 5 hours **iii** $\frac{1}{2}$ hour

**b** How long would it take to travel the following distances? (Answer in fractions of hours.)

**i** 180 km **ii** 90 km **iii** 20 km

**c** If the car's speed was 70 km per hour, how many minutes would it take to travel 7 km?

$$\text{70 km in 60 minutes} \xrightarrow{\div 10} \text{7 km in ? minutes}$$

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# 5A Introducing ratios

So far you have used fractions, decimals and percentages to describe situations like the seating plan in this diagram.

In this chapter you will learn how to use ratio to compare two or more quantities.

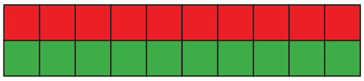
|  |  |  |  |  |
|---|---|---|---|---|
| G | G | G | B | B |
| G | G | G | B | B |
| G | G | G | B | B |
| G | G | G | B | B |

The seating plan for my class
(G is 1 girl and B is 1 boy)

The simplest ratio is 1 to 1.

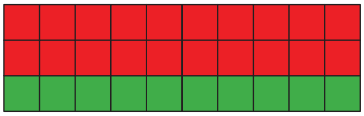
In this diagram the ratio of **red squares** to **green squares** is **1 : 1**.

This is pronounced '1 to 1'.



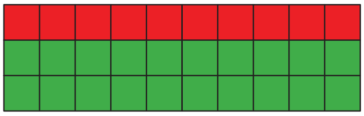
**Half of the squares are red** and **half are green**.

In this diagram the ratio of **red squares** to **green squares** is **2 : 1**.



**Two thirds are red** and **one third is green**.

In this diagram the ratio of **red squares** to **green squares** is **1 : 2**.



**One third is red** and **two thirds are green**.

## Let's start

Consider the seating plan at the top of the page.

- What fraction of the class are girls?
- What fraction of the class are boys?
- What is the ratio of girls to boys?
- What is the ratio of boys to girls?

## Key ideas

- A **ratio** shows the relationship between two or more amounts.
  - The amounts are separated by a colon (:).
  - The amounts are measured using the same units.
    e.g. 1 mm : 100 mm is written as 1 : 100.
  - The ratio 1 : 100 is read as 'the ratio of 1 to 100'.
  - The order in which the numbers are written is important.
    e.g. teachers : students = 1 : 20 means '1 teacher for every 20 students'.

**Ratio** A method used to compare two or more quantities measured in the same units

- It is possible to have three or more numbers in a ratio.
  e.g. flour : water : milk = $2:3:1$
- All the amounts in a ratio can be multiplied (or divided) by the same number to give an **equivalent ratio**. For example:

$$\begin{tikzcd}
1:3 \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] & 1:3 \arrow[d, bend right, "\times 10"'] \arrow[d, bend left, "\times 10"] & 8:12 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] & 8:12 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\
2:6 & 10:30 & 4:6 & 2:3
\end{tikzcd}$$

## Exercise 5A

**Understanding**

### Example 1 Understanding the order of numbers in a ratio

In the diagram below, what is the ratio of:

**a** red squares to blue squares?
**b** blue squares to red squares?

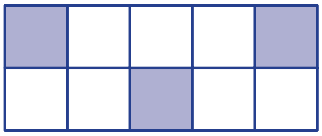
| **Solution** | **Explanation** |
|---|---|
| **a** $3:1$ | Red goes first and blue goes second. |
| **b** $1:3$ | Blue goes first and red goes second. |

**1** In the diagram in Example $1$, what is the ratio of:
  **a** blue squares to green squares?
  **b** blue squares to all squares?
  **c** non-blue squares to blue squares?

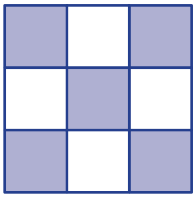
**Drilling for Gold 5A1**

**2** Write down the ratio of shaded parts to unshaded parts for each grid.

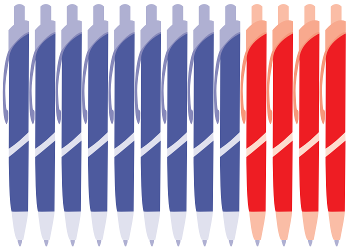
**a**



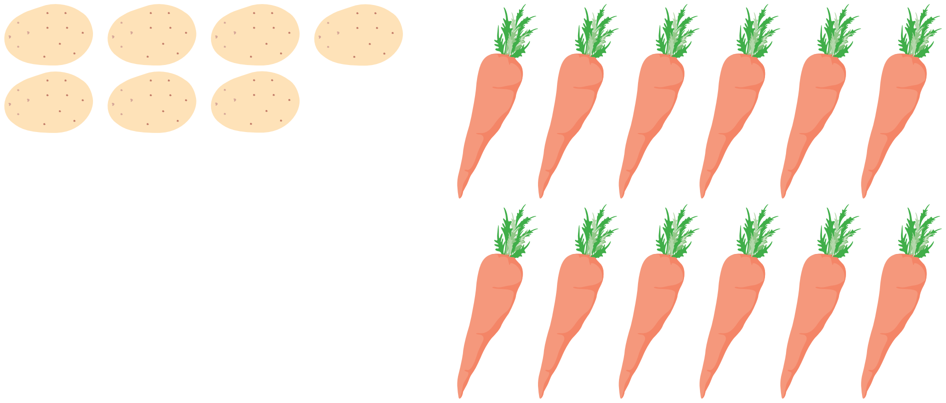
**b**



**3** **a** What is the ratio of blue pens to red pens?



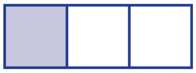
**b** What is the ratio of potatoes to carrots?



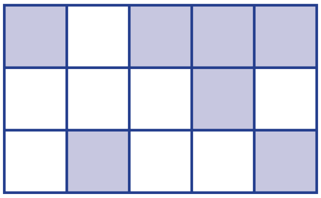
**c** What is the ratio of cordial to water?

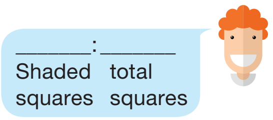
**4** Write down the ratio of shaded parts to total parts for each grid.

**a**



**b**





**Fluency**

## Example 2 Writing ratios

A sample of mixed nuts contains 5 cashews and 12 peanuts.
Write down the:

- **a** ratio of cashews to peanuts
- **b** total number of nuts
- **c** ratio of cashews to the total number of nuts.

| Solution | Explanation |
|---|---|
| **a** 5 : 12 | cashews : peanuts |
| **b** 17 | 5 + 12 = 17 |
| **c** 5 : 17 | 5 cashews, total nuts 17 |

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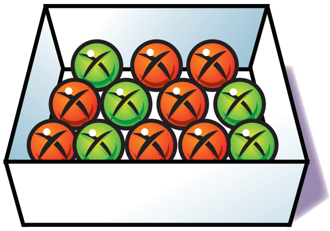
5 A box contains 5 green and 7 red marbles.

**a** Write the ratio of green marbles to red marbles.

**b** What is the total number of marbles?

**c** Write the ratio of green marbles to the total number of marbles.

Remember the order of numbers is important in a ratio.



6 Over the past fortnight, it has rained on eight days and it has snowed on three days.

| Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
|---|---|---|---|---|---|---|
| fine | fine | rain | rain | snow | snow | snow |
| rain | rain | rain | rain | rain | rain | fine |

Write down the ratio of:

**a** rainy days to snowy days

**b** snowy days to total days

**c** fine days to rainy and snowy days

**d** rainy days to non-rainy days.

7 In a box of 40 ice blocks there were 13 lime, 9 lemonade, 11 raspberry and 7 orange. Write down the ratio of:

**a** lime ice blocks to orange ice blocks

**b** raspberry ice blocks to lemonade ice blocks

**c** the four different flavours of ice blocks; lime : lemonade : raspberry : orange

**d** lime and orange ice blocks to raspberry and lemonade ice blocks.

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**8** Copy and complete to make the ratios equivalent (using multiplication).

**a**
$$\begin{tikzcd} 2:3 \arrow[d, bend right=60, "\times 4"'] \arrow[d, bend left=60, "\times 4"] \\ 8:\square \end{tikzcd}$$

**b**
$$\begin{tikzcd} 2:3 \arrow[d, bend right=60, "\times 2"'] \arrow[d, bend left=60, "\times 2"] \\ 4:\square \end{tikzcd}$$

**c**
$$\begin{tikzcd} 2:3 \arrow[d, bend right=60, "?"'] \arrow[d, bend left=60, "?"] \\ 12:\square \end{tikzcd}$$

**d**
$$\begin{tikzcd} 2:3 \arrow[d, bend right=60, "?"'] \arrow[d, bend left=60, "?"] \\ \square:12 \end{tikzcd}$$

**e**
$$\begin{tikzcd} 4:1 \arrow[d, bend right=60, "?"'] \arrow[d, bend left=60, "?"] \\ 8:\square \end{tikzcd}$$

**f**
$$\begin{tikzcd} 4:1 \arrow[d, bend right=60, "?"'] \arrow[d, bend left=60, "?"] \\ \square:8 \end{tikzcd}$$

See the example in Key ideas.

**9** Copy and complete to make the ratios equivalent (using division).

**a**
$$\begin{tikzcd} 8:12 \arrow[d, bend right=60, "\div 2"'] \arrow[d, bend left=60, "\div 2"] \\ 4:\square \end{tikzcd}$$

**b**
$$\begin{tikzcd} 8:12 \arrow[d, bend right=60, "\div 4"'] \arrow[d, bend left=60, "\div 4"] \\ 2:\square \end{tikzcd}$$

**c**
$$\begin{tikzcd} 6:12 \arrow[d, bend right=60, "?"'] \arrow[d, bend left=60, "?"] \\ 3:\square \end{tikzcd}$$

**d**
$$\begin{tikzcd} 6:12 \arrow[d, bend right=60, "?"'] \arrow[d, bend left=60, "?"] \\ 2:\square \end{tikzcd}$$

**e**
$$\begin{tikzcd} 6:12 \arrow[d, bend right=60, "?"'] \arrow[d, bend left=60, "?"] \\ \square:2 \end{tikzcd}$$

**f**
$$\begin{tikzcd} 9:12 \arrow[d, bend right=60, "?"'] \arrow[d, bend left=60, "?"] \\ \square:4 \end{tikzcd}$$

## Example 3 Producing equivalent ratios

Complete each pair of equivalent ratios.

**a** $4:9 = 16:\square$ **b** $30:15 = \square:5$ **c** $2:4:7 = \square:12:\square$

**Solution** **Explanation**

**a**
$$\begin{tikzcd} 4:9 \arrow[d, bend right=60, "\times 4"'] \arrow[d, bend left=60, "\times 4"] \\ 16:36 \end{tikzcd}$$

$4 \times 4 = 16$
$9 \times 4 = 36$

**b**
$$\begin{tikzcd} 30:15 \arrow[d, bend right=60, "\div 3"'] \arrow[d, bend left=60, "\div 3"] \\ 10:5 \end{tikzcd}$$

$15 \div 3 = 5$
$30 \div 3 = 10$

**c**
$$\begin{tikzcd} 2:4:7 \arrow[d, bend right=60, "\times 3"'] \arrow[d, "\times 3"] \arrow[d, bend left=60, "\times 3"] \\ 6:12:21 \end{tikzcd}$$

$4 \times 3 = 12$ so multiply each number by 3.

**10** Copy and complete each pair of equivalent ratios.

**a** $1:3 = 4:\square$

**b** $1:7 = 2:\square$

**c** $2:5 = \square:10$

**d** $3:7 = \square:21$

**e** $5:10 = 1:\square$

**f** $12:16 = 3:\square$

**g** $12:18 = \square:3$

**h** $20:50 = \square:25$

**i** $2:3:5 = 4:\square:\square$

**j** $4:12:16 = \square:6:\square$

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**Problem-solving and Reasoning**

**11** Write three equivalent ratios for each of the following ratios.
**a** $1:2$ **b** $2:5$ **c** $8:6$ **d** $9:3$

Multiply or divide both parts of the ratio by the same number.

**12** Sort the following ratios into three pairs of equivalent ratios.
$2:5$, $6:12$, $7:4$, $1:2$, $4:10$, $70:40$

**13** Write the ratio of vowels to consonants for each of the following words.
**a** Queensland **b** Canberra **c** Wagga Wagga **d** Australia

The vowels are a, e, i, o, u.

**14** There are four groups of students with 12 students in each group. From the ratios given below, work out the number of boys and the number of girls in each group.

**a** Group A
boys : girls = $2:1$

**b** Group B
girls : boys = $2:1$

**c** Group C
girls : boys = $1:3$

**d** Group D
boys : girls = $1:5$

For group A, find how many lots of 2 boys and 1 girl are needed to make a group of 12 children.

**15** Consider the diagram below.

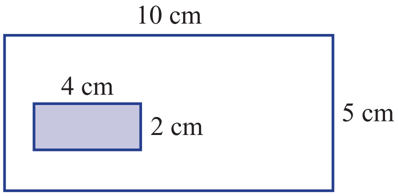
**a** At the moment, the ratio of red:blue = ___ $:1$
**b** How many blue squares need to be added so that the ratio is $1:4$?

## Enrichment: Area ratios

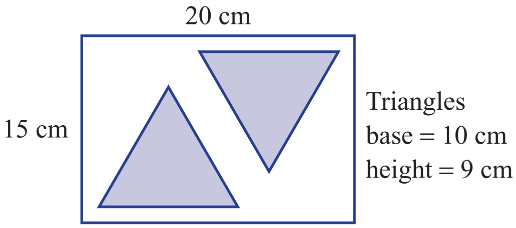
**16** Using the dimensions provided, find the ratio of the shaded area to the unshaded area for each of the following diagrams.

Area of a triangle = $\frac{1}{2} \times b \times h$

**a**



**b**



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**17** Use your ruler to measure the length and breadth of three rectangular objects that are on your desk. Measure in mm and round each answer to the nearest 5 mm. For example: calculator, pencil case, exercise book, textbook and desk.

**a** For each object, find these ratios and simplify.
  **i** Length : breadth
  **ii** Length : area

**b** What do you notice about your answers to part **ii** when each ratio is simplified?

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# 5B Simplifying ratios

Using the diagram below, there are several equivalent ways to write the ratio of girls to boys.

| | | | | | |
|---|---|---|---|---|---|
| Row 1 | G | G | G | B | B |
| Row 2 | G | G | G | B | B |
| Row 3 | G | G | G | B | B |
| Row 4 | G | G | G | B | B |

The seating plan for my class
(G is 1 girl and B is 1 boy)

| | Ratio of girls to boys |
|---|---|
| Using the whole class | $12:8$ |
| Using three rows | $9:6$ |
| Using two rows | $6:4$ |
| Using one row | $3:2$ |
| Using 1 boy | $1.5:1$ |
| Using 1 girl | $1:\frac{2}{3}$ |

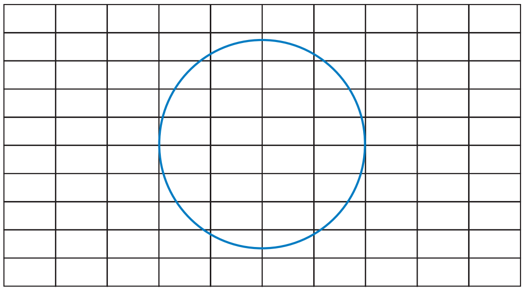
In these equivalent ratios, the *simplest form* is $3:2$ because:

- both numbers are whole numbers, and
- the highest common factor (HCF) is 1.

In this class, there are 3 girls for every 2 boys.

## Let's start: The national flag of Rationia guessing competition

The mathemagical nation of Rationia has a new national flag.



Without doing any calculations, use a simplified ratio such as $2:3$ to *estimate* the following ratios.

- Area inside blue circle : area outside blue circle
- Area inside blue circle : total area of flag

Compare your estimates with others in your class.
Maybe there will be a prize for the best estimator.

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## Key ideas

- The quantities in ratios must be expressed using the same unit.
  e.g. 20 minutes : 1 hour
  $= 20:60$
- **Simplifying ratios**
  A ratio is simplified by dividing all numbers in the ratio by their highest common factor (HCF).
  e.g. the ratio $15:25$ can be simplified to $3:5$.

$$\begin{tikzcd} 15:25 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ 3:5 \end{tikzcd}$$

- Ratios in simplest form use whole numbers only.
- In the following list of ratios, $2:3$ is expressed in simplest form:

$$2:3 = 1:1\frac{1}{2} = 5:7.5 = 4:6 = 20:30$$

## Exercise 5B

Understanding

**1** Copy and complete writing these fractions in simplest form.

**a** $\overset{\div 5}{\underset{\div 5}{\frac{5}{10} = \frac{\square}{\square}}}$

**b** $\overset{\div 4}{\underset{\div 4}{\frac{12}{20} = \frac{\square}{\square}}}$

**c** $\overset{?}{\underset{?}{\frac{6}{18} = \frac{\square}{\square}}}$

**d** $\overset{?}{\underset{?}{\frac{15}{35} = \frac{\square}{\square}}}$

**e** $\overset{?}{\underset{?}{\frac{80}{50} = \frac{\square}{\square}}}$

Drilling for Gold 5B1

**2** Copy and complete writing these ratios in simplest form.

**a** $$\begin{tikzcd} 5:10 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ = \square:\square \end{tikzcd}$$

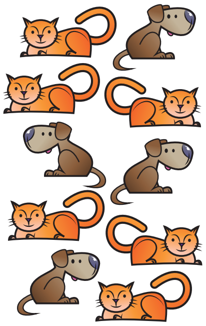
**b** $$\begin{tikzcd} 12:20 \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ = \square:\square \end{tikzcd}$$

**c** $$\begin{tikzcd} 6:18 \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ = \square:\square \end{tikzcd}$$

**d** $$\begin{tikzcd} 15:35 \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ = \square:\square \end{tikzcd}$$

**e** $$\begin{tikzcd} 80:50 \arrow[d, bend right, "?"'] \arrow[d, bend left, "?"] \\ = \square:\square \end{tikzcd}$$

**3** Write the ratio of cats to dogs in simplest form.



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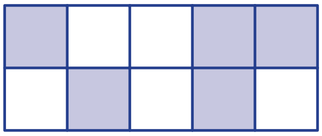
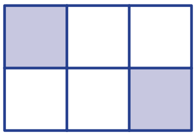
**4** Write down the simplified ratio of shaded parts to unshaded parts for each of the following.

**a**



**b**



**Fluency**

## Example 4 Simplifying ratios

Simplify the following ratios.

**a** $7:21$ **b** $80:60$

HCF means highest common factor.

| **Solution** | **Explanation** |
|---|---|
| **a** $$\begin{tikzcd} 7:21 \arrow[d, bend right, "\div 7"'] \arrow[d, bend left, "\div 7"] \\ 1:3 \end{tikzcd}$$ | HCF of 7 and 21 is 7.<br>Divide both quantities by 7. |
| **b** $$\begin{tikzcd} 80:60 \arrow[d, bend right, "\div 20"'] \arrow[d, bend left, "\div 20"] \\ 4:30 \end{tikzcd}$$ | HCF of 80 and 60 is 20.<br>Divide both quantities by 20. |

Alternative solution:

$$\begin{tikzcd} 80:60 \arrow[d, bend right, "\div 10"'] \arrow[d, bend left, "\div 10"] \\ 8:6 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\ 4:3 \end{tikzcd}$$

Divide by 10, then divide by 2.

**5** Simplify the following ratios.

**a** $2:8$ **b** $10:50$ **c** $4:24$ **d** $6:18$
**e** $8:10$ **f** $25:40$ **g** $21:28$ **h** $24:80$
**i** $18:14$ **j** $26:13$ **k** $45:35$ **l** $81:27$
**m** $51:17$ **n** $20:180$ **o** $300:550$ **p** $150:75$
**q** $1200:100$ **r** $70:420$ **s** $200:125$ **t** $90:75$

Divide both numbers by the highest common factor.

**6** Simplify the following ratios.

**a** $2:4:6$ **b** $12:21:33$
**c** $42:60:12$ **d** $85:35:15$
**e** $12:24:36$ **f** $100:300:250$
**g** $270:420:60$ **h** $24:48:84$

Divide all three numbers by the HCF.

## Example 5 Simplifying ratios that have different units

First change the quantities to the same unit, by changing the larger unit to the smaller unit. Then express each pair of quantities as a ratio in simplest form.

**a** 4 mm to 2 cm

**b** 25 minutes to 2 hours

| **Solution** | **Explanation** |
|---|---|
| **a** 4 mm to 2 cm = 4 mm to 20 mm<br>$= 4:20$<br>$= 1:5$ | 2 cm = 20 mm<br>Once in same unit, write as a ratio.<br>Simplify ratio by dividing by HCF of 4. |
| **b** 25 minutes to 2 hours<br>= 25 minutes to 120 minutes<br>$= 25:120$<br>$= 5:24$ | 2 hours = 120 minutes<br>Once in same unit, write as a ratio.<br>Simplify ratio by dividing by HCF of 5. |

**7** First change the quantities to the same unit, and then express each pair of quantities as a ratio in simplest form.

- **a** 12 mm to 3 cm
- **b** 7 cm to 5 mm
- **c** 120 m to 1 km
- **d** 60 mm to 2.1 m
- **e** 3 kg to 450 g
- **f** 200 g to 2.5 kg
- **g** 2 tonnes to 440 kg
- **h** 1.25 L to 250 mL
- **i** 400 mL to 1 L
- **j** 20 minutes to 2 hours
- **k** 3 hours to 15 minutes
- **l** 3 days to 8 hours
- **m** 180 minutes to 2 days
- **n** 8 months to 3 years
- **o** 4 days to 4 weeks
- **p** 8 weeks to 12 days
- **q** 50 cents to $4
- **r** $7.50 to 25 cents

1 tonne = 1000 kg
1 L = 1000 mL
10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

**Skillsheet 5A**

**8** Simple ratios do *not* contain decimals or fractions. Simplify the following ratios using the hints provided.

**a**
$$\begin{tikzcd} \frac{1}{2}:1 \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\ ?:? \end{tikzcd}$$

**b**
$$\begin{tikzcd} 2:\frac{1}{4} \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\ ?:? \end{tikzcd}$$

**c**
$$\begin{tikzcd} \frac{1}{2}:\frac{1}{4} \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\ ?:? \end{tikzcd}$$

**d**
$$\begin{tikzcd} 0.7:1 \arrow[d, bend right, "\times 10"'] \arrow[d, bend left, "\times 10"] \\ ?:? \end{tikzcd}$$

**e**
$$\begin{tikzcd} 1:0.1 \arrow[d, bend right, "\times 10"'] \arrow[d, bend left, "\times 10"] \\ ?:? \end{tikzcd}$$

**f**
$$\begin{tikzcd} 1.1:1.3 \arrow[d, bend right, "\times 10"'] \arrow[d, bend left, "\times 10"] \\ ?:? \end{tikzcd}$$

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**g**

$$1.5:1$$

convert to fractions

$$1\frac{1}{2}:1$$

make a common denominator

$$\times 2 \quad \frac{?}{2}:\frac{2}{2} \quad \times 2$$

**h**

$$1\frac{1}{2}:1\frac{1}{4}$$

convert to improper

$$\frac{?}{2}:\frac{?}{4}$$

common denominator

$$\times 4 \quad \frac{?}{4}:\frac{?}{4} \quad \times 4$$

$$?:?$$

**i**

$$1:2.25$$

convert to fractions

$$1:2\frac{1}{4}$$

$$\times 4 \quad \frac{4}{4}:\frac{?}{4} \quad \times 4$$

$$4:?$$

**Problem-solving and Reasoning**

**9** To express the ratio $4:16$ in simplest form, you would:

**A** multiply both quantities by 2

**B** subtract 4 from both quantities

**C** divide both quantities by 2

**D** divide both quantities by 4.

**10** Decide which of the following ratios is not written in simplest form.

**A** $1:5$ **B** $3:9$ **C** $2:5$ **D** $11:17$

Find the ratio that has a common factor.

**11** Decide which of the following ratios is written in simplest form.

**A** $2:28$ **B** $15:75$ **C** $14:45$ **D** $13:39$

Find the ratio that does not have a common factor.

**12** When Lisa makes fruit salad for her family, she uses 5 bananas, 5 apples, 2 passionfruit, 4 oranges, 3 pears, 1 lemon (for juice) and 20 strawberries.

**a** Write the ratio of the fruits in Lisa's fruit salad (in the same order as given in the question).

**b** Lisa wanted to make four times the amount of fruit salad to take to a party. Write an equivalent ratio that shows how many of each fruit Lisa would need.

**c** Write these ratios in simplest form.

- **i** Bananas to strawberries
- **ii** Strawberries to other fruits

The ratio in part **a** will have 7 numbers.

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**13** Andrew incorrectly simplified 12 cm to 3 mm as a ratio of 4 : 1. What was Andrew's mistake and what is the correct simplified ratio?

**14** **a** Write two quantities of time, in different units, which have a ratio of 2 : 5.

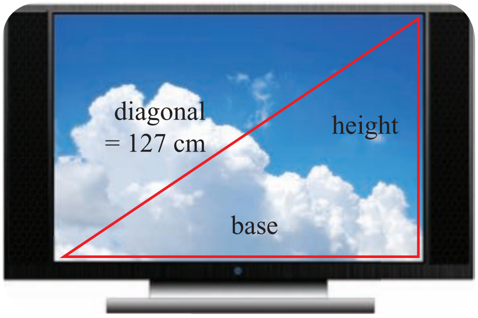
**b** Write two quantities of distance, in different units, which have a ratio of 4 : 3.

First write 12 cm : 3 mm with the same units.

2 hours : 5 hours
2 hours : ☐ minutes

## Enrichment: Aspect ratios

**15** Aspect ratio is the relationship between the base and height of the image as displayed on a screen. The aspect ratio of a rectangle is the ratio of the base to the height.



Size of TV = diagonal base of image
Aspect ratio = base : height

Investigate aspect ratios and create a poster or PowerPoint slide show that demonstrates:

**a** how to calculate the aspect ratio for a rectangular image
**b** examples of aspect ratios.

Research these examples of aspect ratios.

**1** Use the internet or a newspaper to find an advertisement for the various enlargements available from a local photo print shop. State the aspect ratio for each of these enlargements.

**2** What is the difference between the size of a television (e.g. 127 cm) and the aspect ratio of the television?

**3** Find out the aspect ratio of:

- analogue televisions
- high-definition digital televisions
- widescreen movies shown on television
- old cinema movies
- modern cinema movies
- computer screens.

**4** Calculate the aspect ratio for different-sized newspaper pages.

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# 5C Dividing a quantity in a given ratio

When two people share an amount of money they usually split it evenly. This is a 1 : 1 ratio or a 50–50 split. Each person gets half the money.

Sometimes one person deserves a larger share than the other. The diagram to the right shows \$20 being divided in the ratio 3 : 2. Gia's share is \$12 and Ben's is \$8.

| Gia's share | | | Ben's share | |
|---|---|---|---|---|
| \$1 | \$1 | \$1 | \$1 | \$1 |
| \$1 | \$1 | \$1 | \$1 | \$1 |
| \$1 | \$1 | \$1 | \$1 | \$1 |
| \$1 | \$1 | \$1 | \$1 | \$1 |

Gia and Ben divided \$20 in the ratio 3 : 2

## Let's start: Is there a shortcut?

Take 15 counters or blocks and use them to represent \$1 each.

Working with a partner:

- Share the \$15 evenly. How much did each person get?
- Start again. This time share \$15 in the ratio 2 : 1, like this: '2 for me, 1 for you; 2 for me, 1 for you …'. How much did each person get?
- Start again. This time share \$15 in the ratio 3 : 2, like this: '3 for me, 2 for you; 3 for me, for you …'. How much did each person get?
- Is there a shortcut? Without using the counters, can you work out how to share \$15 in the ratio 4 : 1?
- Does your shortcut work correctly for the example at the top of this page?
- Have a class discussion about different shortcuts.
- Use your favourite shortcut (and a calculator) to divide \$80 in the ratio 3 : 2. Write down the steps you used.

## Key ideas

- When Gia and Ben divide $20 in the ratio 3 : 2, for every $3 given to Gia $2 is given to Ben.
- There are various ways to do the calculation, such as:
  - **Using the unitary method to find each part**
    3 : 2 implies 3 parts and 2 parts, which makes 5 parts.
    $20 divided by 5 gives $4 for each part.
    Gia's share is $3 \times \$4 = \$12$
    Ben's share is $2 \times \$4 = \$8$
  - **Using fractions of the amount**
    3 : 2 implies 3 parts and 2 parts, which makes 5 parts.
    Gia gets 3 of the 5 parts (i.e. three fifths of $20).
    Ben gets 2 of the 5 parts (i.e. two fifths of $20).
    Gia's share is: $\frac{3}{5} \times \$20 = \$20 \div 5 \times 3 = \$12$
    Ben's share is: $\frac{2}{5} \times \$20 = \$20 \div 5 \times 2 = \$8$

## Exercise 5C

**Understanding**

**1** Find the total number of parts in the following ratios.
**a** 3 : 7  **b** 1 : 5  **c** 11 : 3  **d** 2 : 3 : 4

Add the numbers in the ratio to find the total parts.

**2** Marta and Joshua earned $30 between them. They want to share it in the ratio Marta : Joshua = 3 : 2. Copy and complete these steps.

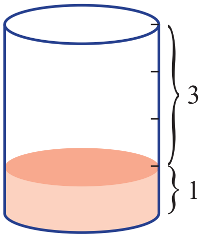
**a** In the ratio 3 : 2, the total parts = ___ + ___ = ___
**b** ___ parts = $30, so 1 part = ___
**c** Marta gets 3 parts, so Marta gets 3 × $ ___ = $ ___
**d** Joshua gets 2 parts, so Joshua gets 2 × $ ___ = $ ___

Drilling for Gold 5C1

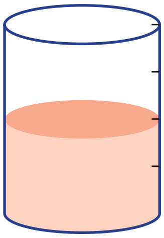
**3** The diagram shows four glasses that contain different amounts of cordial. Water is then added to fill each glass right to the top. For each drink shown, what is the ratio of cordial to water?

Cordial : water
$\square$ : 3

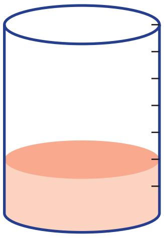
**a**



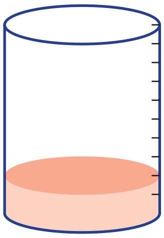
**b**



**c**



**d**



Write ratios in simplest form.

**4** What is the total number of parts for each ratio in Question **3**?

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**5C**

**Fluency**

### Example 6 Dividing a quantity in a particular ratio

Drilling for Gold 5C2

Divide 54 m in a ratio of $4:5$.

| **Solution** | **Explanation** |
|---|---|
| **Unitary method**<br>Total number of parts = 9 | Total number of parts = $4 + 5 = 9$<br>Value of 1 part = $54 \text{ m} \div 9 = 6 \text{ m}$ |

$$\begin{tikzcd}
9 \text{ parts} = 54 \text{ m} \arrow[d, "\div 9"', "\div 9"] \\
1 \text{ part} = 6 \text{ m} \arrow[d, "\times 4"', "\times 4"] \\
4 \text{ parts} = 24 \text{ m}
\end{tikzcd}
\qquad
\begin{tikzcd}
1 \text{ part} = 6 \text{ m} \arrow[d, "\times 5"', "\times 5"] \\
5 \text{ parts} = 30 \text{ m}
\end{tikzcd}$$

The amounts are 24 m and 30 m.

Check numbers add to total:
$24 + 30 = 54$
Write the answers with units.

Alternative solution:
$4 + 5 = 9$

$\frac{4}{9} \times 54 = 24, \qquad \frac{5}{9} \times 54 = 30$

The amounts are 24 m and 30 m.

The amounts are:
- 4 out of 9 parts
- 5 out of 9 parts

**5** Divide (using a calculator where necessary):
- **a** \$60 in the ratio of $2:3$
- **b** \$110 in the ratio of $7:4$
- **c** \$1000 in the ratio of $3:17$
- **d** 48 kg in the ratio of $1:5$
- **e** 14 kg in the ratio of $4:3$
- **f** 360 kg in the ratio of $5:7$
- **g** 72 m in the ratio of $1:2$
- **h** 40 m in the ratio of $3:5$
- **i** 155 m in the ratio of $4:1$.

**6** Share \$400 in the ratio:
- **a** $1:3$
- **b** $2:3$
- **c** $3:5$
- **d** $9:11$

Start by finding the:
- total parts
- value of one part.

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## Example 7 Dividing a quantity in a ratio with three numbers

Divide \$300 in the ratio of 2 : 1 : 3.

| **Solution** | **Explanation** |
|---|---|
| **Unitary method**<br>Total number of parts = 6 | Total number of parts = 2 + 1 + 3 = 6<br>Value of 1 part = \$300 ÷ 6 = \$50 |

$$\begin{tikzcd} 6 \text{ parts} = \$300 \arrow[d, bend right, "\div 6"'] \arrow[d, bend left, "\div 6"] \\ 1 \text{ part} = \$50 \arrow[d, bend right, "\times 2"'] \arrow[d, bend left, "\times 2"] \\ 2 \text{ parts} = \$100 \end{tikzcd} \qquad \begin{tikzcd} 1 \text{ part} = \$50 \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ 3 \text{ parts} = \$150 \end{tikzcd}$$

The three amounts are \$100, \$50 and \$150.

Check numbers add to total:
\$100 + \$50 + \$150 = \$300

Alternatively,
$2 + 1 + 3 = 6$

$\frac{2}{6} \times 300 = 100,\quad \frac{1}{6} \times 300 = 50,\quad \frac{3}{6} \times 300 = 150$

The amounts are \$100, \$50, \$150

The amounts are:
- 2 out of 6 parts
- 1 out of 6 parts
- 3 out of 6 parts

**7** Divide:
**a** \$200 in the ratio of 1 : 2 : 2
**b** \$400 in the ratio of 1 : 3 : 4
**c** 12 kg in the ratio of 1 : 2 : 3
**d** 88 kg in the ratio of 2 : 1 : 5
**e** 320 kg in the ratio of 12 : 13 : 15
**f** \$50 000 in the ratio of 1 : 2 : 3 : 4.

What is the total number of parts?

**8** Share 600 lollies in the ratio:
**a** 1 : 9
**b** 2 : 1 : 3
**c** 2 : 5 : 5
**d** 12 : 7 : 8 : 3

Write units in the answers.

**Skillsheet 5B**

**Problem-solving and Reasoning**

**9** Evergreen Fertiliser is made up of the three vital nutrients nitrogen, potassium and phosphorus in a ratio of 4 : 5 : 3. How many grams of each nutrient is in a 1.5 kg bag?

First change 1.5 kg to g.

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**10** The angles of a triangle are in the ratio of $2:3:4$.
Using a calculator, find the size of each angle.

The angles in a traingle add to $180^\circ$.

## Example 8 Finding a total quantity from a given ratio

The ratio of boys to girls at Birdsville College is $2:3$. If there are 246 boys at the school, how many students attend Birdsville College?

**Solution**

**Unitary method**

$$\begin{tikzcd}
2 \text{ parts} = 246 \arrow[d, bend right, "\div 2"'] \arrow[d, bend left, "\div 2"] \\
1 \text{ parts} = 123 \arrow[d, bend right, "\times 5"'] \arrow[d, bend left, "\times 5"] \\
5 \text{ parts} = 615
\end{tikzcd}$$

615 students attend Birdsville College.

**Explanation**

Ratio of boys : girls is $2:3$
Boys have '2 parts' $= 246$
Value of 1 part $= 246 \div 2 = 123$
Total parts $= 2 + 3 = 5$ parts
5 parts $= 5 \times 123 = 615$

**Solution**

**Equivalent ratios method**

$$\begin{tikzcd}
\text{boys}:\text{girls} \\
= 2:3 \arrow[d, bend right, "\times 123"'] \arrow[d, bend left, "\times 123"] \\
= 246:369
\end{tikzcd}$$

615 students attend Birdsville College.

**Explanation**

Use equivalent ratios.
$246 \div 2 = 123$ so $3 \times 123 = 369$ girls
Total number of students
$= 246$ boys $+ 369$ girls $= 615$

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**11** Copy and complete the table.

| | a | b | c |
|---|---|---|---|
| **Ratio of boys to girl is:** | $2:3$ | $4:3$ | $5:6$ |
| If there are $20$ boys, how many girls are there? | | | |

**12** In Year $8$, the ratio of boys to girls is $5:7$. If there are $140$ girls in Year $8$, what is the total number of students in Year $8$?

Think: How many parts of the total equals $140$ girls?

**13** A textbook has three chapters and the ratio of pages in these chapters is $3:2:5$. If there are $24$ pages in the smallest chapter, how many pages are in the textbook?

The smallest chapter $= 2$ parts of the total.

## Enrichment: Changing ratios

**14** The ratio of the cost of a shirt to the cost of a jacket is $2:5$. If the jacket cost \$240 more than the shirt, find the cost of the shirt and the cost of the jacket.

Try out some amounts for one parts of the ratio.

**15** In a class of $24$ students, the ratio of girls to boys is $1:2$.

**a** On one day, the ratios of girls to boys was $3:7$. How many boys and how many girls were absent?

**b** If $4$ more girls and $4$ more boys joined the original class, what would be the new ratio of girls : boys?

How many boys and girls are in the original class?

# 5D Scale drawings

A scale drawing is used when the:

- actual object is too large to be drawn on the page or
- actual object is very small and needs to be enlarged so we can see details clearly.

Scale drawings are used for:

- house plans
- maps
- drawings of tiny animals.

This house plan has a scale of $1:200$. Every millimetre in this plan is $200$ mm in the house.

This picture of a dragonfly is $5$ times larger than a real dragonfly. The real dragonfly is $\frac{1}{5}$ of the size of this picture.

## Let's start: Enlarging photographs

Photo A

Photo B

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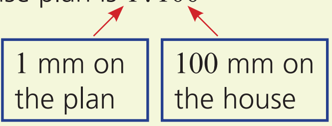
- Measure the bases of Photos A and B. What is the simplified ratio of the bases?
- Measure the heights of Photos A and B. What is the simplified ratio of the heights?
- Rachel said: 'If you double the side lengths you, will double the area.' Use calculations to explain why Rachel is incorrect.
- What happens to the area when the side lengths are tripled?

## Key ideas

- A scale drawing has exactly the same shape as the original object, but it is a different size.
- The **scale** on a drawing is written as a ratio of the drawing length : actual (real) length.
  e.g. if the scale on a house plan is 1 : 100

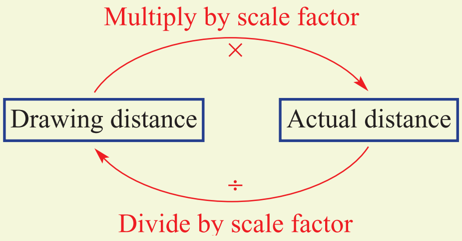


- Scales should begin with 1. Then the second number in the ratio is called the **scale factor**.

e.g.

$$\begin{tikzcd}[row sep=small] 5:25000 \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ 1:5000 \end{tikzcd}$$

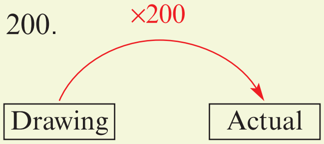
The scale factor is 5000.



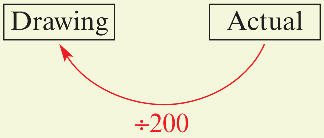
**Scale** A ratio that compares a drawing or a model to the real object

**Scale factor** The number you multiply each length by to enlarge or reduce a shape

- To change a drawing distance to an actual (real) distance, multiply by the scale factor.
  e.g. A room is 2 cm wide on a house plan with scale 1 : 200.
  The real room will be 2 cm × 200 = 400 cm wide
  = 400 ÷ 100
  = 4 m wide



- To convert an actual (real) distance to a drawing distance, divide by the scale factor.
  e.g. A real house is 12 m wide and the house plan has scale 1 : 200.
  The house plan will be 12 m ÷ 200 = 0.06 m wide
  = 0.06 × 100
  = 6 cm wide



- It is important to remember how to correctly convert units of length when working with scale drawings.

$$\begin{tikzcd} \text{km} \arrow[r, bend left, "\times 1000"] & \text{m} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 100"] & \text{cm} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10"] & \text{mm} \arrow[l, bend left, "\div 10"] \end{tikzcd}$$

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## Exercise 5D

Understanding

**1** **a** Convert $10000$ cm to:
  - **i** mm **ii** m **iii** km

**b** Convert $560$ m to:
  - **i** km **ii** cm **iii** mm

### Example 9 Comparing scale distance to real distance

A doll's house has a toy lounge chair $2$ cm wide. The real lounge chair is $1$ m wide.

**a** Write both toy width and real chair width in cm.
**b** How much bigger is the real chair than the toy chair?
**c** What is the scale for toy chair : real chair?
**d** What is the scale factor?

| **Solution** | **Explanation** |
|---|---|
| **a** 2 cm, 100 cm | $1 \text{ m} \times 100 = 100 \text{ cm}$ |
| **b** 50 times bigger | $2 \text{ cm} \times 50 = 100 \text{ cm}$ |
| **c** $1:50$ | A scale always starts with $1$.<br>The second number shows how many times larger the real object is. |
| **d** 50 | A scale is a ratio of two numbers but a scale factor is one number. |

**2** Here are pictures of a real classic racing car and a model racing car.

model car length = 4 cm

real car length = 4 m

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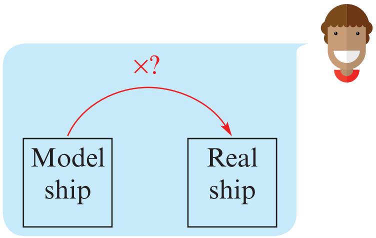
**a** Write the model car length in cm and real car length in cm.
**b** How many times longer than the toy car is the real car?
**c** What is the scale for the model car : real car?
**d** What is the scale factor?

**3** A model ship is 60 cm long and the real ship is 300 m long.
- **a** Write model length in cm and real ship length in cm.
- **b** How many times longer is the real ship compared to the model?
- **c** What is the scale for the model ship : real ship?



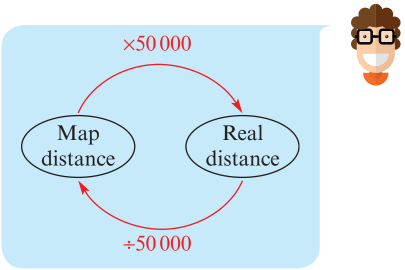
## Example 10 Calculating the real distance

**Drilling for Gold**
**5D1**

A map has a scale ratio of 1 : 20 000.
- **a** What is the scale factor?
- **b** What actual real distance in cm would 1 cm on the map represent?
- **c** What actual real distance in mm would 5 mm on the map represent?

| **Solution** | **Explanation** |
|---|---|
| **a** 20 000 | The scale factor is the second number in the scale ratio |
| **b** 20 000 cm | 1 cm × 20 000 = 20 000 cm |
| **c** 100 000 mm | 5 mm × 20 000 = 100 000 mm |

**4** A map has a scale ratio of 1 : 50 000.
- **a** What is the scale factor?
- **b** What actual real distance in cm would 1 cm on the map represent?
- **c** What actual real distance in m would 1 m on the map represent?
- **d** What actual real distance in mm would 5 mm on the map represent?
- **e** What actual real distance in cm would 5 cm on the map represent?



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**Fluency**

### Example 11 Converting scale distance to actual distance

A map has a scale of $1:20\,000$.
Find the actual distance in m for each scaled distance (map distance).

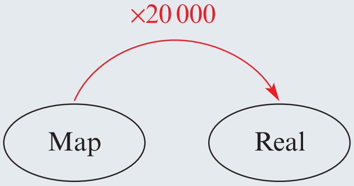
**a** 2 cm  **b** 5 mm

**Solution**

**a** Actual distance $= 2$ cm $\times 20\,000$
$= 40\,000$ cm
$= 400$ m

**Explanation**

Scale factor $= 20\,000$



$\div 100$ to convert cm to m.

**Solution**

**b** Actual distance $= 5$ mm $\times 20\,000$
$= 100\,000$ mm
$= 100$ m

**Explanation**

Multiply scaled distance by scale factor.
5 mm times scale factor gives the answer in mm.
$\div 1000$ to convert mm to m.

**5** Find the actual distance for each of the following scaled distances. Give your answer in the unit that is in brackets after each question.

**a** Scale $1:2$  **i** 310 cm (cm)  **ii** 2.5 mm (mm)
**b** Scale $1:10\,000$  **i** 2 cm (m)  **ii** 4 mm (m)
**c** Scale $1:20\,000$  **i** 80 cm (km)  **ii** 1.25 m (km)
**d** Scale $1:400$  **i** 16 mm (m)  **ii** 72 cm (m)
**e** Scale $1:0.01$  **i** 3 cm (mm)  **ii** 0.815 m (mm)

- Multiply by the scale factor.
- Keep the units the same as the question.
- Then convert to the required units.

### Example 12 Converting actual distance to scaled distance

A model boat has a scale of $1:500$.
Find the scaled length in mm for each of these actual lengths.

**a** 50 m (mm)  **b** 4550 mm (mm)

**Solution**

**a** Scaled length $= 50$ m $\div 500$
$= 0.01$ m
$= 10$ cm
$= 100$ mm

**Explanation**

Divide by the scale factor, 500.
50 m $\div$ 500 gives the answer in m.
$\times$ 100 to convert m to cm.
$\times$ 10 to convert cm to mm.

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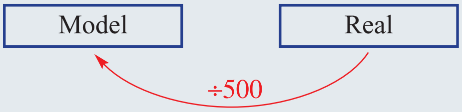
| Solution | Explanation |
|---|---|
| **b** Scaled length = 4550 mm ÷ 500 | Divide actual distance by scale factor. |
| = 45.5 mm ÷ 5 | Shortcut: ÷100, then ÷5 (or vice versa) |
| = 9.1 mm | The answer is in mm. |



**6** Find the scaled length for each of these actual lengths. Give your final answer in the unit that is in brackets after each question.

**a** Scale 1 : 200 **i** 200 m (m) **ii** 4 km (m)
**b** Scale 1 : 500 **i** 10 000 m (m) **ii** 1 km (m)
**c** Scale 1 : 10 000 **i** 1350 m (cm) **ii** 736.5 m (cm)
**d** Scale 1 : 250 000 **i** 5000 m (mm) **ii** 1250 m (mm)
**e** Scale 1 :0.05 **i** 7.5 cm (m) **ii** 8.2 mm (m)

- Divide by the scale factor.
- Keep the units the same as the question.
- Then convert to the required units.

**7** Change the two measurements provided in each scale into the same unit and then write the scale as a ratio of two numbers in simplest form.

**a** 2 cm : 200 m **b** 5 mm : 500 cm **c** 12 mm : 360 cm
**d** 4 mm : 600 m **e** 4 cm : 5 m **f** 1 cm : 2 km
**g** 28 mm : 2800 m **h** 3 cm : 0.6 mm **i** 1.1 m : 0.11 mm

- Convert larger unit to smaller unit.
- Divide by HCF.

## Example 13 Determining the scale and scale factor

State the scale factor in the following situations.

**a** 4 mm on a scale drawing represents an actual distance of 50 cm.
**b** An actual length of 0.1 mm is represented by 3 cm on a scaled drawing.

| Solution | Explanation |
|---|---|
| **a** Scale = 4 mm : 50 cm | Write the ratio drawing length : actual length. |
| = 4 mm : 500 mm | Convert to ‘like’ units. |
| Scale = 4 : 500 | Write the scale without units. |
| = 1 : 125 | Divide both numbers by 4. (HCF = 4) |
| Scale factor = 125 | Ratio is now in the form 1 : scale factor.<br>The actual size is 125 times larger than the scaled drawing. |

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| **Solution** | **Explanation** |
|---|---|
| **b** Scale $= 3\text{ cm} : 0.1\text{ mm}$ | Write the ratio drawing length : actual length. |
| $= 30\text{ mm} : 0.1\text{ mm}$ | Convert to 'like' units. |
| Scale $= 30 : 0.1$ | Write the scale without units. |
| $= 300 : 1$ | Multiply both numbers by 10 so both numbers are whole. |
| $= 1 : \frac{1}{300}$ | Divide both numbers by 300. |
| Scale factor $= \frac{1}{300}$ | Ratio is now in the form 1 : scale factor.<br>The actual size is 300 times smaller than the scaled drawing. |

**Skillsheet 5C**

**8** Find the scale and the scale factor for each of the following.

- **a** 2 mm on a scale drawing represents an actual distance of 50 cm.
- **b** 4 cm on a scale drawing represents an actual distance of 2 km.
- **c** 1.2 cm on a scale drawing represents an actual distance of 0.6 km.
- **d** 5 cm on a scale drawing represents an actual distance of 900 m.
- **e** An actual length of 7 mm is represented by 4.9 cm on a scaled drawing.
- **f** An actual length of 0.2 mm is represented by 12 cm on a scaled drawing.

- Same units
- Scale ratios start with 1
- Write scale factors as whole numbers or fractions

**Problem-solving and Reasoning**

**9** A model city has a scale ratio of 1 : 1000.

- **a** Find the actual height in m of a skyscraper that has a scaled height of 8 cm.
- **b** Find the scaled length in cm of a train platform that is 45 m long in real life.

Scaled height means model height.

**10** Blackbottle and Toowoola are 17 cm apart on a map with a scale of 1 : 50 000. How many km apart are the towns in real life?

**11** The house plan opposite has a scale of 1 : 150. For each room listed below do these two steps.

- Use a ruler to measure the internal length and width in mm.
- Use the scale to calculate the real dimensions in m to 1 decimal place.

**a** Bedroom 1 **b** Family room **c** Patio

Length in mm × scale factor = real length in mm

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Scale 1 : 150

12 For each question below do these two steps.
- Use a ruler to measure the straight line map distance in cm to the nearest millimetre.
- Use the scale to calculate the real distance and give each answer to the nearest 100 km.

**a** Hobart to Cairns
**b** Perth to Sydney
**c** Darwin to Adelaide
**d** Brisbane to Melbourne
**e** Australia's furthest point west to furthest point east.

Scale 1 : 50 000 000

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## Enrichment: Design a bedroom

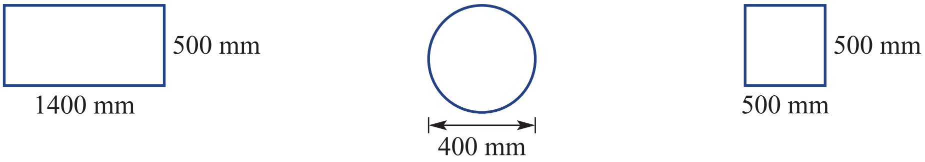
**13** For this activity you will design and draw the floor plan of a bedroom. Builders use millimetres for units so keep all units in millimetres for this activity.

This bedroom has length = 3000 mm and width = 4000 mm.

The furniture in the bedroom is illustrated here. These pictures are *not* shown to scale. The real dimensions are given for the 'top view'. The 'top view' is how it is seen looking down from above.

How many times larger is the bedroom length than your page length? Choose a whole number for the scale factor. *scale* = 1 : *scale factor*

Top views with real dimensions



Top views with real dimensions

You are to design a scaled drawing of a bedroom including this furniture.

- **a** Find a scale that will allow the drawing of this bedroom to fit on one page.
- **b** Use your scale to change all the real dimensions to scaled lengths and widths in mm.
- **c** Draw a scaled rectangle for the bedroom.
- **d** Choose where each piece of furniture will be placed in the bedroom and draw the scaled top view of each.
- **e** Choose where a window and a door will be placed in the bedroom and draw the scaled top view of each.
- **f** Label dimensions with the real measurements in mm.
- **g** Write the scale next to your bedroom plan.

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# 5E Introducing rates

A *ratio* shows the relationship between the same type of quantities with the same units, but a *rate* shows the relationship between two different types of quantities with different units.

The following are all examples of rates.

- Cost of petrol was $1.45 per litre.
- Rump steak was on special for $18/kg.
- Dad drove to school at an average speed of 52 km/h.
- After the match, your heart rate was 140 beats/minute.

A *ratio* compares two amounts of the same type and units, so a ratio does not include units.

For example, the ratio of girls to boys in a group was 4 : 5.

A *rate* compares different types of quantities so both units must be shown.

For example, the average rate of growth of a teenage boy is 6 cm/year.

A knowledge of heart rates is important for understanding fitness.

## Let's start: State the rate

For each of the following statements, write down a corresponding rate.

- We travelled 400 km in 5 hours. What was our average speed in km/h?
- Gary was paid $98 for a 4-hour shift at work. What is his rate of pay in $/h?
- Felicity spent $600 in two days. What was her spending rate in $/day?
- Max grew 9 cm in three months. What is Max's growth rate in cm/month?
- Paul cycled a total distance of 350 km for the week. At what rate did Paul cycle in km/day?

What was the rate (in questions/minute) at which you answered these questions?

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## Key ideas

**Rate** The number of units of one quantity for each single unit of another quantity

- **Rates** compare quantities measured in different units.
- The two different units are separated by a slash '/', which is the mathematical symbol for 'per'. e.g. 20 km/h = 20 km per hour = 20 km for each hour.
- In a simplified rate, the second quantity is usually 1.
  e.g. earned $45 in 3 hours = $45 in 3 hours ← Non-simplified rate
  ↓÷3 ÷3
  = $15 in 1 hour
  = $15/h ← Simplified rate
- An exception to this is petrol consumption.
  e.g. 13 litres per 100 km = 13 L/100 km
- The *average rate* is calculated by dividing the total change in the first quantity by the total change in the second quantity.
  e.g. reading a 400-page book in 4 days
  Average reading rate = 400 pages in 4 days
  ↓÷4 ÷4
  = 100 pages in 1 day
  Average reading rate = 100 pages/day.

## Exercise 5E

**Understanding**

Drilling for Gold 5E1

**1** Which of the following are examples of rates?
**A** $5.50 **B** 180 mL/min **C** $60/h **D** $\frac{5}{23}$
**E** 4.2 runs/over **F** 0.6 g/L **G** 200 cm$^2$ **H** 84 c/L

Remember that rates have two different units.

**2** Match each rate in the first column with its most likely rate in the second column.

| | | |
|---|---|---|
| **a** | Employee's wage | 90 people/day |
| **b** | Speed of a car | $2100/m$^2$ |
| **c** | Cost of building a new home | 68 km/h |
| **d** | Population growth | 64 beats/min |
| **e** | Resting heart rate | $15/h |

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**3** Select from this list the most typical units for each of the following rates.
$/L mg/tablet $/kg kJ/serve runs/over words/minute goals/shots (on goal) L/minute
- **a** Price of sausages
- **b** Petrol costs
- **c** Typing speed
- **d** Goal conversion rate
- **e** Energy nutrition information
- **f** Water usage in the shower
- **g** Pain relief medication
- **h** Cricket team's run rate

**Fluency**

## Example 14 Writing simplified rates

Express each of the following as a simplified rate.
- **a** 12 students for two teachers
- **b** $28 for 4 kilograms

Drilling for Gold 5E2

| **Solution** | **Explanation** |
|---|---|
| **a** 12 students/2 teachers<br>= 6 students/teacher | Divide both quantities by the second amount.<br>12 ÷ 2 = 6 students for 1 teacher.<br>Include both units seperated by / |
| **b** $28/4 kg<br>= $7/kg | 28 ÷ 4 = $7 for 1 kg<br>When writing a cost rate, the $ sign is written before the number. |

**4** Write each of the following as a simplified rate.
- **a** 12 days in 4 years
- **b** 15 goals in 3 games
- **c** $180 in 6 hours
- **d** $17.50 for 5 kilograms
- **e** $126 000 to purchase 9 acres
- **f** 36 000 cans in 8 hours
- **g** 12 000 revolutions in 10 minutes
- **h** 80 mm rainfall in 5 days
- **i** 60 minutes to run 15 kilometres
- **j** 15 kilometres run in 60 minutes

Divide both amounts by the second number. The answer includes both units separated by (/).

## Example 15 Finding average rates

Find the average rate for each situation.
- **a** 15 000 revolutions in 5 minutes
- **b** 30 minutes to run 6 km

| **Solution** | **Explanation** |
|---|---|
| **a** Average rate = 15 000 rev/<br>5 min<br>= 3000 rev/min | Divide both quantities by the second amount<br>15 000 ÷ 5 = 3000 revolutions for 1 minute on average |

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| **Solution** | **Explanation** |
|---|---|
| **b** Average rate = 30 min/6 km<br>= 5 min/km | 30 ÷ 6 = 5 minutes for 1 km on average<br>Include both units seperated by a / |

**5** Find the average rate for each situation.

**a** Relma drove 6000 kilometres in 20 days.
**b** Holly saved $420 over three years.
**c** A cricket team scored 78 runs in 12 overs.
**d** Saskia grew 120 centimetres in 16 years.
**e** Russell gained 6 kilograms in 4 years.
**f** The temperature dropped 5°C in 2 hours.

In your answer, write the units in the same order as the question.

## Example 16 Finding average rates

Tom was 120 cm tall when he turned 10 years old. He was 185 cm tall when he turned 20 years old. Find Tom's average rate of growth per year between 10 and 20 years of age.

| **Solution** | **Explanation** |
|---|---|
| Average rate = 65 cm/10 years<br>= 6.5 cm/year | Growth = 185 − 120 = 65 cm<br>Divide both numbers by 10. |

**6** **a** Liam was 150 cm tall at 10 years old and 188 cm tall when 20 years old. Find Liam's average rate of growth per year between 10 and 20 years of age.
**b** Brittany was 140 cm tall at 10 years old and 164 cm at 18 years old. Find Brittany's average rate of growth per year between 10 and 18 years of age.

**Problem-solving and Reasoning**

**7** A dripping tap filled a 9 litre bucket in 3 hours.

**a** What was the dripping rate of the tap in litres/hour?
**b** How long would it take the tap to fill a 21 litre bucket?

$$\begin{tikzcd} \text{? litres in 1 hour} \arrow[d, bend right, "\times ?"'] \arrow[d, bend left, "\times ?"] \\ \text{21 litres in ? hours} \end{tikzcd}$$

**8** Martine grew at an average rate of 6 cm/year for the first 18 years of her life. If Martine was 50 cm long when she was born, how tall was Martine when she turned 18?

$$\begin{tikzcd} \text{6 cm in 1 year} \arrow[d, bend right, "\times ?"'] \arrow[d, bend left, "\times ?"] \\ \text{? cm in 18 years} \end{tikzcd}$$

**9** If 30 salad rolls were bought to feed 20 people at a picnic, and the total cost was $120, find the following rates.

**a** Salad rolls/person **b** Cost/person **c** Cost/roll

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**10** Harvey finished a 10-kilometre race in 37 minutes and 30 seconds. Jacques finished a 16-kilometre race in 53 minutes and 20 seconds. Calculate the running rate of each runner in min/km. Which runner had a faster running pace?

First write the distance in metres and the time in minutes.

**11** The Tungamah Football Club had 12 000 members. After five successful years they now have 18 000 members.

**a** What has been the average rate of membership growth per year for the past 5 years?

**b** If this membership growth rate continues, how many more years will it take for the club to have 32 400 members?

**12** A car uses 24 L of petrol to travel 216 km. Express these quantities as a simplified rate in:

**a** km/L **b** L/100 km

Start with 216 km uses 24 litres.

## Enrichment: Target 155

**13** Due to repeated droughts, the government has urged all people to save water. The goal was set for each person to use no more than 155 litres of water per day.

**a** How many people live in your household?

**b** According to the government, how many litres of water can your household use per day?

Use the following rates of water flow for the questions below.

| | |
|---|---|
| Shower rate (10 L/min) | Washing machine (100 L/load) |
| Hose (24 L/min) | Toilet (4.5 L/flush, 3 L/half flush) |
| Running tap (16 L/min) | Drinking water (3 L/day) |
| Dishwasher (20 L/wash) | Water for food preparation (15 L/day) |

Draw a table.

**c** Estimate the average daily rate of water usage for your household.

**d** Ask your parents for a recent water bill and find out what your family household water usage rate was for the past three months.

**e** What is the rate at which your family is charged for its water usage?

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# 5F Application of rates

A rate that we come across almost every day is speed. Speed is the rate of distance travelled per unit of time.

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

## Let's start: Fastest to slowest

Rank these speeds from fastest to slowest.

- **A** The average speed of the fastest runner in the 100 m sprint at the Olympic Games
- **B** The top speed of a hippopotamus running on dry land
- **C** The average speed of the fastest swimmer in the 100 m race at the Olympic Games
- **D** The cruising speed of a passenger jet
- **E** The speed at which a person standing on the equator rotates around the centre of Earth
- **F** The speed at which Earth travels around the Sun
- **G** Cathy Freeman's average speed when she won the gold medal in the 400 m race at the Sydney Olympics
- **H** The speed at which a saltwater crocodile can swim

Use the internet to check your ranking.

## Key ideas

**Speed** A measure of how fast an object is moving

- When a rate is provided, a change in one quantity implies that an equivalent change must occur in the other quantity.
e.g. Patrick earns $20/hour. How much will he earn in 6 hours?

$$\begin{tikzcd} \$20 \text{ for } 1 \text{ hour} \arrow[d, bend right, "\times 6"'] \arrow[d, bend left, "\times 6"] \\ \$120 \text{ for } 6 \text{ hour} \end{tikzcd}$$

e.g. Patrick earns $20/hour. How long will it take him to earn $60?

$$\begin{tikzcd} \$20 \text{ for } 1 \text{ hour} \arrow[d, bend right, "\times 3"'] \arrow[d, bend left, "\times 3"] \\ \$60 \text{ for } 3 \text{ hour} \end{tikzcd}$$

- Carefully consider the units involved in each question and answer.
- **Speed** is a measure of how fast an object is travelling.
- If the speed of an object does not change over time, the object is travelling at a **constant speed**. 'Cruise control' helps a car travel at a constant speed.

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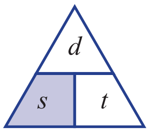
- When speed is not constant, due to acceleration or deceleration, we are often interested to know the **average speed** of the object.
- Average speed is calculated by the formula:

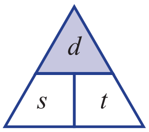
$$\textbf{Average speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

- The above formula can be rearranged to make $d$ or $t$ the subject. Three formulas involving speed ($s$), distance ($d$) and time ($t$) are found below. The diagrams beneath each formula may help you to memorise them.

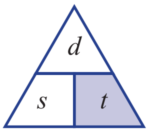
$$s = \frac{d}{t}$$



$$d = s \times t$$



$$t = \frac{d}{s}$$



- Care must be taken with units for speed, and on occasions units will need to be converted. Common units of speed are m/s and km/h.

## Exercise 5F

**Understanding**

**1** Fill in the gaps.

**a** 60 km in 1 hour
×3 ×3
180 km in ____

**b** $25 in 1 hour
×5 ____
$125 in ____

**c** 7 questions in 3 minutes
____ ____
70 questions in____

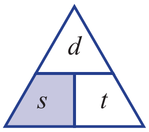
**d** 120 litres in 1 minute
____ ____
____ in 6 minutes

**2** Fill in the gaps.

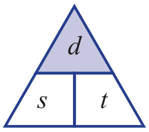
**a** $36 for 3 hours
÷3 ÷3
____ for 1 hour
×5 ____
____ for 5 hours

**b** 150 rotations in 5 minutes
___ ___
_______ in 1 minute
___ ___
_______ in 7 minutes

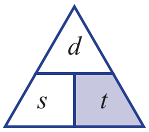
**3** Copy and complete by writing in the missing words.

**a** 

speed = $\frac{?}{\text{time}}$

**b** 

distance = $\frac{?}{\phantom{x}}$ × time

**c** 

time $\frac{?}{?}$

Use the triangles to help memorise each rule.

**4** Which of the following is not a unit of speed?

**A** m/s
**B** km/h
**C** cm/h
**D** L/kg
**E** m/min

Units of speed have a length unit and a time unit.

**Fluency**

**Drilling for Gold 5F1**

## Example 17 Solving rate problems

**a** Rachael can type 74 words/minute. How many words can she type in 15 minutes?

**b** Leanne sells on average 30 donuts every 15 minutes. How long is it likely to take her to sell 400 donuts?

| **Solution** | **Explanation** |
|---|---|
| **a** 74 words in 1 minute<br>×15 ×15<br>1110 words in 15 minutes<br>Rachael can type 1110 words in 15 minutes. | Calculate 74 × 15 = 1110 |
| **b** 30 donuts in 15 minutes<br>÷15 ÷15<br>2 donuts in 1 minute<br>×200 ×200<br>400 donunts in 200 minutes<br>Leanne is likely to take 3 hours and 20 minutes to sell 400 donuts. | Selling rate = 30 donuts/15 minutes<br>Divide both quantities by 15.<br>Multiply both quantities by 200 so that the first quantity will be 400.<br>Convert answer to hours and minutes. |

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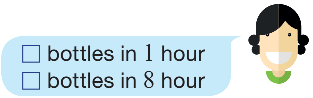
**5** **a** Lewis can type 80 words/minute. How many words can he type in 20 minutes?
**b** Robbie, on average, sells 4 loaves of bread every 10 minutes. How long it will it take him to sell 20 loaves of bread?

**6** A factory produces 40 bottles/minute.
**a** How many bottles can the factory produce in 60 minutes?
**b** How many bottles can the factory produce in an 8-hour day of operation?



## Example 18 Same product in jars of various sizes

**1** Monaco instant coffee is sold in three different jars. Use a calculator to find the 'per 100 g' price of each.

- $5.59 for 50 grams
- $17.50 for 200 grams
- $22.99 for 450 grams

**2** Which is the best value for money?

| **Solution** | **Explanation** |
|---|---|
| ×2 $5.59 for 50 grams is equivalent to ×2<br>$11.18 for 100 grams<br>∴ The cost is $11.18/100 g | We need to multiply 50 g by 2 to get 100 g.<br>Multiply both numbers by 2. |
| ÷2 $17.50 for 200 grams is equivalent to ÷2<br>$8.75 for 100 grams<br>∴ The cost is $8.75/100 g | We need to divide 200 g by 2 to get 100 g.<br>Divide both numbers by 2. |
| ÷4.5 $22.99 for 450 grams is equivalent to ÷4.5<br>$5.108… for 100 grams<br>∴ The cost is $5.11/100 g (nearest cent)<br>This is the best value for money, assuming it can be consumed before the use-by date. | 450 divided by 100 is 4.5, so divide both numbers by 4.5. |

**Drilling for Gold 5F1**

**7** Use the information given in Example 18 for the following questions.
**a** The makers of Monaco coffee have introduced a new size of 750 grams for $28.99. Calculate the 'per 100 g' price, correct to the nearest cent.
**b** A shop offers a 'buy one get one free' promotion for the 200-gram jar. Does this make it cheaper (per 100 g) than the 450-gram jar?
**c** A caterer estimates that she needs at least 1 kilogram of coffee for a function.
 **i** Using only the jars in the example, what is the cheapest way to do this?
 **ii** What is the 'per 100 g' price?

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## Example 19 Finding average speed

Find the average speed in km/h of a:

**a** cyclist who travels 140 km in 5 hours

**b** runner who travels 3 km in 15 minutes.

| **Solution** | **Explanation** |
|---|---|
| **a** $s = \frac{d}{t}$<br>$= \frac{140\text{ km}}{5\text{ h}}$<br>$= 28$ km/h | The unknown value is speed.<br>Write the formula for speed.<br>Distance travelled = 140 km<br>Time taken = 5 h. Calculate 140 ÷ 5<br>Speed unit is km/h. |
| Alternative unitary method<br>$\begin{tikzcd} \text{140 km in 5 hours} \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ \text{28 km in 1 hours} \end{tikzcd}$<br>Average speed = 28 km/h | Write down the rate provided in the question.<br>Divide both quantities by 5 so that the second quantity will be 1 |
| **a** $s = \frac{d}{t}$<br>$= \frac{3\text{ km}}{15\text{ min}}$<br>$= \frac{1}{5}$ km/min<br>$= 12$ km/h | Distance travelled = 3 km<br>$\begin{tikzcd} \frac{1}{5}\text{ km in 1 minute} \arrow[d, bend right, "\times 60"'] \arrow[d, bend left, "\times 60"] \\ \text{12 km in 60 minutes} \end{tikzcd}$ |
| Alternative unitary method<br>$\begin{tikzcd} \text{3 km in 15 minutes} \arrow[d, bend right, "\times 4"'] \arrow[d, bend left, "\times 4"] \\ \text{12 km in 60 minutes} \end{tikzcd}$<br>Average speed = 12 km/h | Write down the rate provided in the question.<br>15 × 4 = 60 minutes = 1 hour<br>Multiply both quantities by 4 so that the second quantity will be 60. |

**8** Find the average speed of a:

**a** sprinter running 200 m in 20 seconds (in m/s)

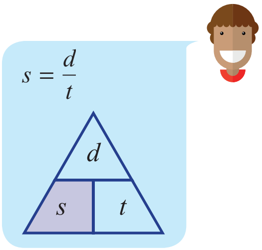
**b** skateboarder travelling 840 m in 120 seconds (in m/s)

**c** car travelling 180 km in 3 hours (in km/h)

**d** truck travelling 400 km in 8 hours (in km/h)

**e** train travelling 60 km in 30 minutes (in km/min and km/h)

**f** tram travelling 15 km in 20 minutes (in km/min and km/h).



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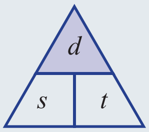
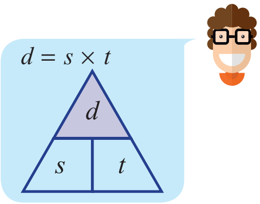
## Example 20 Finding the distance travelled

**1** Find the distance travelled by a truck travelling for 15 hours at an average speed of 95 km/h.

| **Solution** | **Explanation** |
|---|---|
| $d = s \times t$<br>$= 95\text{ km/h} \times 15\text{ h}$<br>$= 1425\text{ km}$ | The unknown value is distance.<br>Write the formula for distance.<br>Distance unit is km.<br> |
| Alternative unitary method: | |
| $\begin{tikzcd} \text{95 km in 1 hour} \arrow[d, bend right, "\times 15"'] \arrow[d, bend left, "\times 15"] \\ \text{1425 km in 15 hours} \end{tikzcd}$ | Write the rate provided in the question.<br>Multiply both quantities by 15 so that the second quantity will be 50. |
| Truck travels 1425 km in 15 hours. | |

**9** Find the distance travelled by:

**a** a cyclist travelling at 12 m/s for 90 seconds
**b** an ant travelling at 2.5 cm/s for 3 minutes
**c** a bushwalker who has walked for 8 hours at an average speed of 4.5 km/h
**d** a tractor ploughing fields for 2.5 hours at an average speed of 20 km/h



## Example 21 Finding the time taken

**1** Find the time taken for a hiker walking at 4 km/h to travel 15 km.

| **Solution** | **Explanation** |
|---|---|
| $t = \frac{d}{s}$<br>$= \frac{15\text{ km}}{4\text{ km/h}}$<br>$= 3.75\text{ h}$<br>$= 3\text{ h }45\text{ min}$ | The unknown value is time.<br>Write the formula with $t$ as the subject.<br>The time unit is h.<br>Leave answer as a decimal or convert to hours and minutes.<br>$0.75\text{ h} = 0.75 \times 60 = 45\text{ min}$ |
| Alternative unitary method: | |
| $\begin{tikzcd} \text{4 km in 1 hour} \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ \text{1 km in } \frac{1}{4} \text{ hour} \arrow[d, bend right, "\times 15"'] \arrow[d, bend left, "\times 15"] \\ \text{15 km in } \frac{15}{4} \text{ hours} \end{tikzcd}$ | Express the rate as provided in the question.<br>Divide both quantities by 4.<br>Multiply both quantities by 15 so that the first quantity will be 15. |
| It takes 3 h 45 min to travel 15 km. | |

Skillsheet 5D

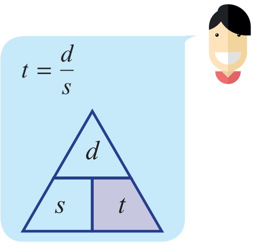
**10** Find the time taken by a:

**a** car to travel 1200 km at an average speed of 150 km/h

**b** bus to travel 14 km at an average speed of 28 km/h

**c** plane to fly 6900 km at a constant speed of 600 km/h

**d** ball moving through the air at a speed of 12 m/s to travel 84 m.



**Problem-solving and Reasoning**

**11** When Putra is training, his goal is a steady working heart rate of 125 beats per minute (bpm). Putra's resting heart rate is 46 bpm.

bpm = beats per minute

**a** How many times does Putra's heart beat during a 30-minute training session?

**b** How many times does Putra's heart beat during 30 minutes of 'rest'?

**c** If his coach says that he can stop his workout once his heart has beaten 10 000 times, for how long would Putra need to train?

**12** A plane is flying at a cruising speed of 900 km/h. How far will the plane travel from 11:15 a.m. to 1:30 p.m. on the same day?

**13** The Mighty Oats breakfast cereal is sold in boxes of three different sizes: small (400 g) for \$5.00, medium (600 g) for \$7.20 and large (750 g) for \$8.25.

**a** Find the value of each box in \$/100 g.

**b** What is the cheapest way to buy a minimum of 4 kg of the cereal?

**14** A 700-gram can of dog food is usually \$2.19. Today you can buy 5 cans for \$8.89. By how much does this reduce the 'per 100 g' price? Give your answer correct to the nearest cent.

**15** You can board the Ghan train in Adelaide and 2979 km later, after travelling via Alice Springs, you arrive in Darwin. For these questions round the answers to 1 decimal place.

**a** If you board the Ghan in Adelaide on Sunday at 2:20 p.m. and arrive in Darwin on Tuesday at 5:30 p.m., what is the average speed of the train journey?

**b** There are two major rest breaks. The train stops for $4\frac{1}{4}$ hours at Alice Springs and 4 hours at Katherine. Taking these breaks into account, what is the average speed of the train when it is moving?

## Enrichment: Speed research

**16** Carry out research to find answers to the following questions.

### Light and sound

- **a** What is the speed of sound in m/s?
- **b** What is the speed of light in m/s?
- **c** How long would it take sound to travel 100 m?
- **d** How long would it take light to travel 100 km?
- **e** How many times quicker is the speed of light than the speed of sound?
- **f** What is a Mach number?

### Spacecraft

- **g** What is the escape velocity needed by a spacecraft to 'break free' of Earth's gravitational pull? Give this answer in km/h and also km/s.
- **h** What is the orbital speed of planet Earth around the Sun? Give your answer in km/h and km/s.
- **i** What is the average speed of a space shuttle on a journey from Earth to the International Space Station?

### Knots

Wind speed and boat speed are often given in terms of knots (kt).

- **j** What does a knot stand for?
- **k** What is the link between nautical miles and a system of locating positions on Earth?
- **l** How do you convert a speed in knots to a speed in km/h?

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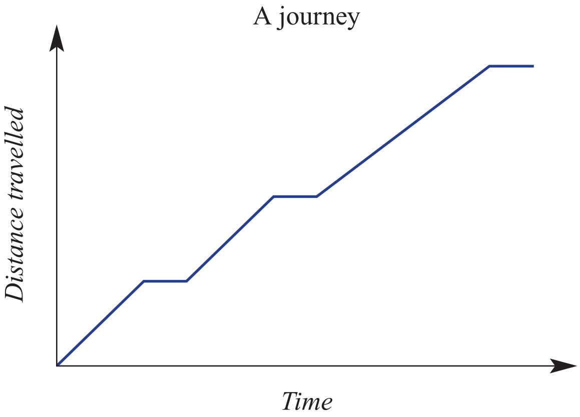
# 5G Distance/time graphs

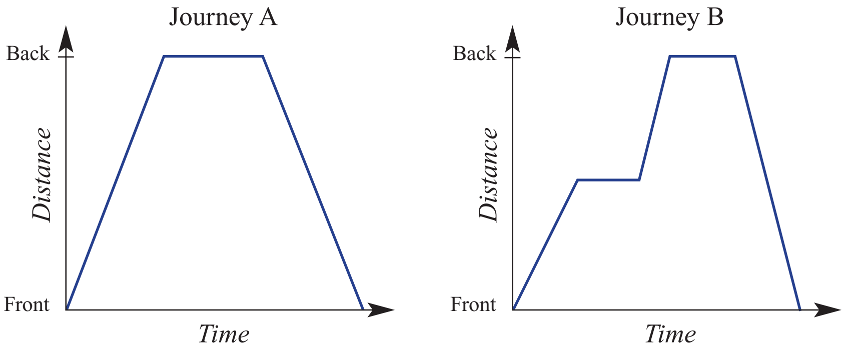
The distance/time graph to the right shows a journey. Distance/time graphs usually show the distance on the vertical axis and the time on the horizontal axis.

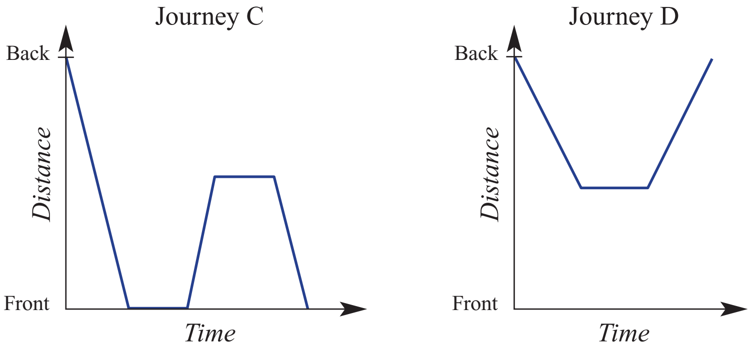
- When an object moves at a constant speed, the graph will be a straight line segment.
- The steepness of a line segment shows the speed of that part of the journey.
- A flat line segment shows that there is no movement. Several different line segments joined together can make up a journey.



## Let's start: Matching graphs and journeys

Work in pairs to match each graph with the student who walked that journey.





- Ella walked from the back of the room, stopped for a short time and then turned around and walked to the back of the room again.
- Jasmine walked from the front of the room to the back of the room where she stopped for a short time. She then turned and walked to the front of the room.

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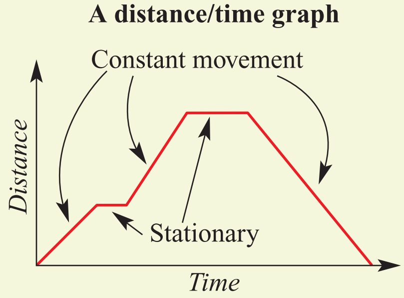
- Lucas walked from the back to the front of the room where he stopped for a short time. He then turned and walked halfway to the back, briefly stopped and then turned again and walked to the front.
- Riley walked from the front of the room, stopped briefly partway and then completed his walk to the back of the room where he stopped for a short time. He then turned and walked to the front.

## Key ideas

- In a distance/time graph, time is on the horizontal axis and distance is on the vertical axis.
- Distance/time graphs may consist of line segments.



- Each segment shows whether the object is moving or at rest.
- The steepness of a line segment shows the speed.
- Steeper lines show greater speed than less steep lines.
- Horizontal lines show that the person or vehicle is stationary.
- Speed = $\dfrac{\text{distance}}{\text{time}}$

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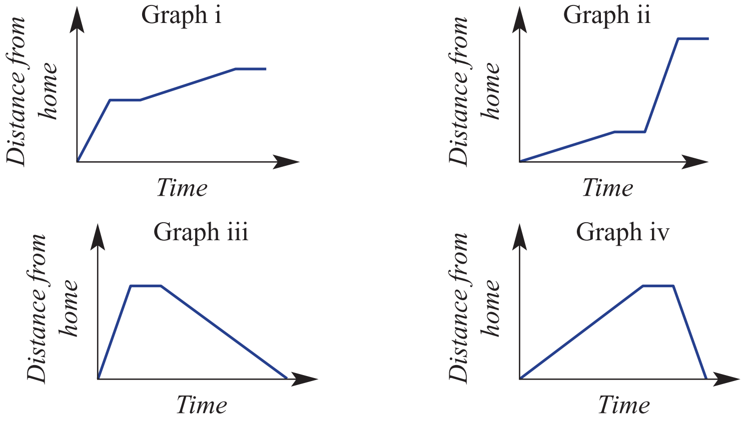
## Exercise 5G

Understanding

1 Each of these distance/time graphs show a person's journey from home. Match each graph with the correct description of that journey.



Journey A: A man walks slowly away from home, stops for a short time, then continues walking away from home at a fast pace and finally stops again.

Journey B: A girl walks slowly away from home and then stops for a short time. She then turns around and walks at a fast pace back to her home.

Journey C: A boy walks quickly away from home, briefly stops, then continues slowly walking away from home and finally stops again.

Journey D: A woman walks quickly away from home and then stops briefly. She then turns around and slowly walks back home.

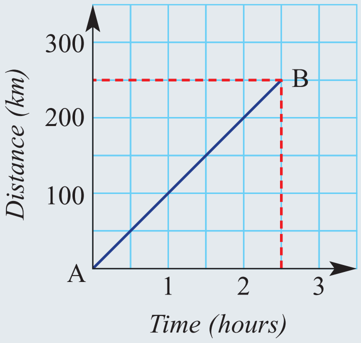
### Example 22 Reading information from a graph

This graph shows the journey of a car from one town (A) to another (B).

**a** How far did the car travel?

**b** How long did it take the car to complete the journey?

**c** What was the average speed of the car?



| Solution | Explanation |
|---|---|
| **a** 250 km | Draw an line from point B to the vertical axis, i.e. 250 km. |
| **b** 2.5 hours | Draw an imaginary line from point B to the horizontal axis; i.e. 2.5 hours. |

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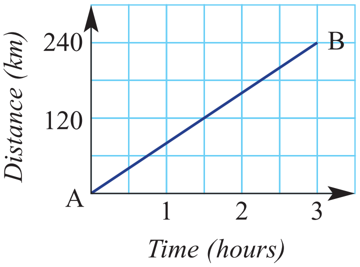
| **Solution** | **Explanation** |
|---|---|
| **c** $s = \frac{d}{t}$<br>$= \frac{250\text{ km}}{2.5\text{ h}}$<br>$= 100$ km/h | Speed = $\frac{\text{distance}}{\text{time}}$<br>Distance = 250 km<br>Time = 2.5 hours |
| Alternatively,<br>÷2.5 250 km in 2.5 hours ÷2.5<br>100 km in 1 hour | Write the rate.<br>Divide both quantities by 2.5 so that the second quantity will be 1. |

**2** This graph shows a train journey from one town (A) to another town (B).

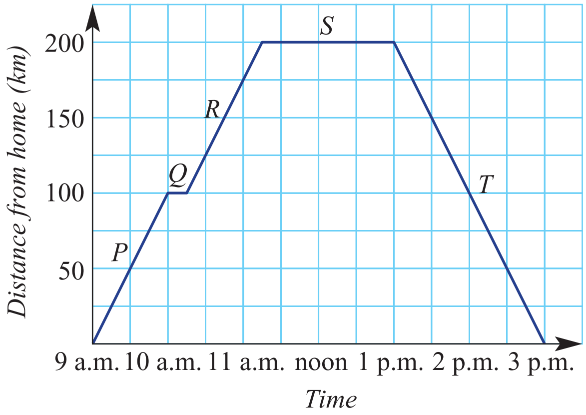
**a** How far did the train travel?

**b** How long did it take to complete this journey?

**c** What was the average speed of the train?



**3** The Wilson family drove from their home to a relative's place for lunch and then back home again. This distance/time graph shows their journey.



For each description below, choose the line segment of the graph that matches it.

**a** A $\frac{1}{4}$ hour rest break is taken from 10 a.m. to 10:15 a.m.

**b** The Wilson's drove 100 km in the first hour.

**c** The car is stopped for $1\frac{3}{4}$ hours.

**d** The car travels from 100 km to 200 km away from home in this section of the journey.

**e** At the end of this segment, the Wilsons start their drive back to their home.

**f** The Wilsons travel from 1 p.m. to 3 p.m. without stopping.

**g** At the end of this segment, the Wilsons have arrived back home again.

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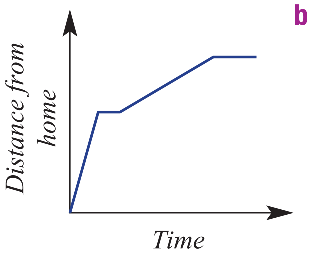
Fluency

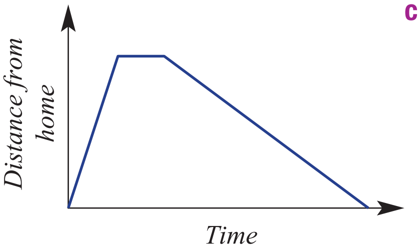
**4** Each of these distance/time graphs show a person's journey. Using sentences explain the meaning of each straight-line segment describing whether the person:

- travelled slowly or quickly or was stopped
- travelled away from home or towards home.

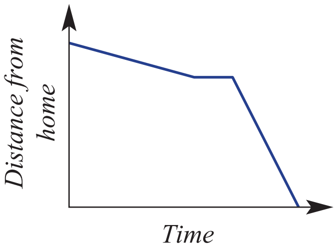
**a**



**b**



**c**



**5** This distance/time graph shows Levi's walk from his home to school one morning.

**a** How far did Levi walk in the first minute?

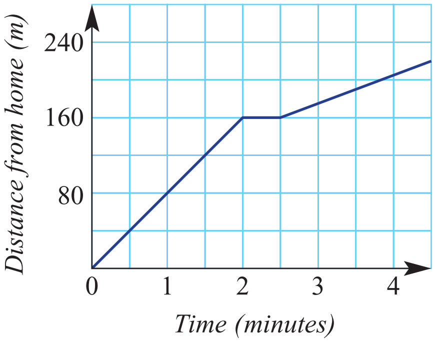
**b** How long did it take Levi to walk 120 m from home?

**c** How far had Levi walked when he stopped to talk to a friend?

**d** For how long did Levi stop?

**e** How far had Levi walked after 3 minutes?

**f** How far was the school from Levi's home?



## Example 23 Calculations from a distance/time graph

This distance/time graph shows Noah's cycle journey from home one morning.

**a** Find Noah's speed in the first hour of the journey.

**b** How many minutes was Noah's first rest break?

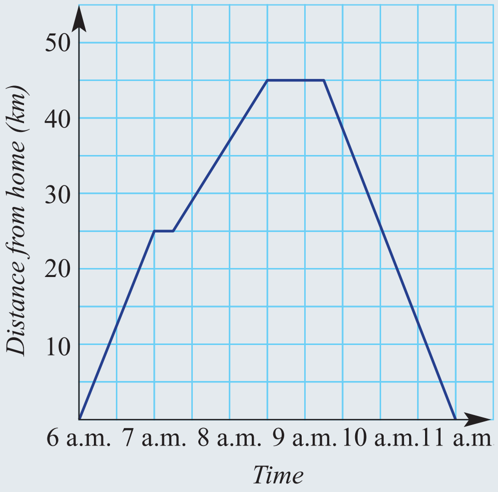
**c** Find Noah's speed between 7:15 a.m. and 8:30 a.m.

**d** At what time did Noah start and finish his second rest break?

**e** At approximately what time had Noah ridden a total of 65 km?

**f** Find Noah's speed on the return journey to 1 decimal place.

**g** What was Noah's average speed for the whole journey?



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| | **Solution** | **Explanation** |
|---|---|---|
| **a** | 25 km/h | The first hour is 6 a.m. to 7 a.m. Noah cycles 25 km in one hour. |
| **b** | 15 minutes | The first horizontal line segment is at 7 a.m. to 7:15 a.m. |
| **c** | $s = \frac{d}{t}$<br>$= \frac{20 \text{ km}}{1.25 \text{ h}}$<br>$= 16$ km/h | Distance travelled = 45 km – 25 km.<br>Time taken = 7:15 a.m. to 8:30 a.m. = 1.25 hours |
| | Alternatively,<br>$\begin{tikzcd} \text{20 km in 1.25 hours} \arrow[d, bend right, "\div 1.25"'] \arrow[d, bend left, "\div 1.25"] \\ \text{16 km in 1 hour} \end{tikzcd}$<br>$= 16$ km/h | Divide both quantities by 1.25 so that the second quantity will be 1. |
| **d** | 8:30 a.m. to 9:15 a.m. | A horizontal line segment shows that Noah is stopped. |
| **e** | 10 a.m. | 45 km plus 20 km of the return trip or 25 km from home. |
| **f** | $s = \frac{d}{t}$<br>$= \frac{45 \text{ km}}{1.75 \text{ h}}$<br>$= 25.7$ km/h (to 1 d.p.) | Distance travelled = 45 km.<br>Time taken = 9:15 a.m. to 11 a.m. = 1.75 hours |
| | Alternatively,<br>$\begin{tikzcd} \text{45 km in 1.75 hours} \arrow[d, bend right, "\div 1.75"'] \arrow[d, bend left, "\div 1.75"] \\ \text{25.7 km in 1 hour} \end{tikzcd}$<br>$= 25.7$ km/h (to 1 d.p.) | Divide both quantities by 1.75 so that the second quantity is 1. |
| **g** | $s = \frac{d}{t}$<br>$= \frac{90 \text{ km}}{5 \text{ h}}$<br>$= 18$ km/h | Average speed $= \frac{\text{distance travelled}}{\text{time taken}}$<br>Distance travelled $= 2 \times 45 = 90$ km.<br>Time taken = 6 a.m. to 11 a.m. = 5 hours. |
| | Alternatively,<br>$\begin{tikzcd} \text{90 km in 5 hours} \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ \text{18 km in 1 hour} \end{tikzcd}$ | Divide both quantities by 5 so that the second quantity is 1. |

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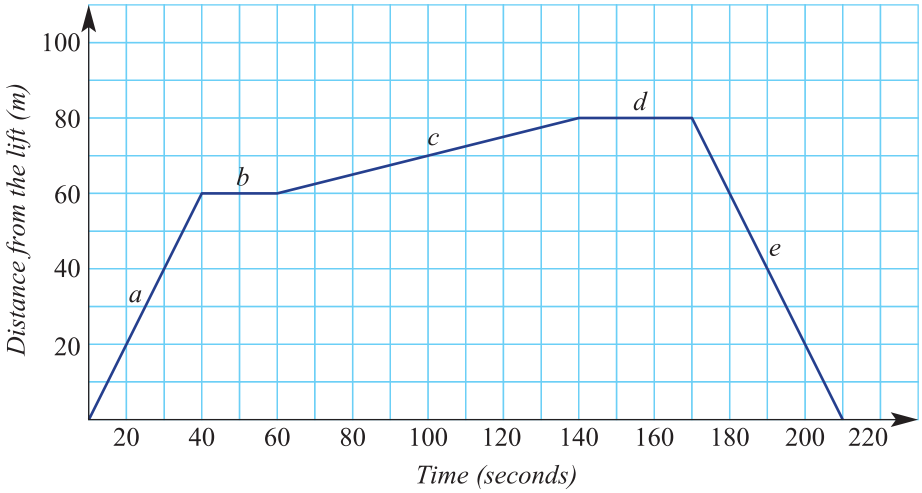
**6** This distance/time graph shows Ava's short walk in a shopping centre from the lift and back to the lift.



**a** Which line segments show that Ava was stopped?
**b** For how long was Ava not walking?
**c** What distance had Ava travelled by 20 seconds?
**d** How long did Ava take to walk 70 m?
**e** When did Ava turn around to start to walk back towards the lift?
**f** At what times was Ava 60 m from the lift?
**g** What was the total distance that Ava walked from the lift and back again?
**h** For which section of Ava's walk does the line segment have the flattest slope? What does this tell you about Ava's speed for this section?
**i** For which section of Ava's walk is the line segment steepest? What does this tell you about Ava's speed for this section?

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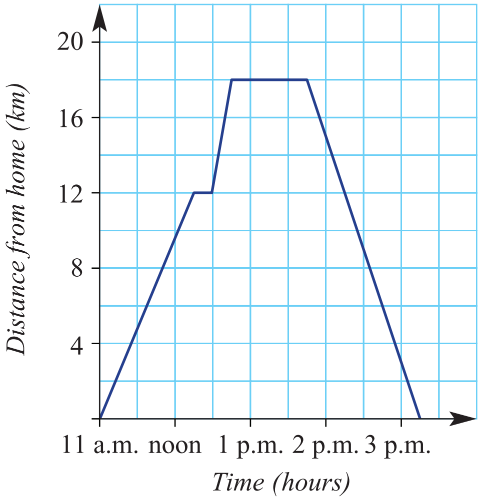
Problem-solving and Reasoning

**7** Enzo cycles from home to a friend's place. After catching up with his friend he then cycles back home. On the way to his friend's place, Enzo stopped briefly at a shop. His journey is shown on this distance/time graph.



**a** At what time did Enzo arrive at the shop?
**b** Find Enzo's speed (to 1 decimal place) between his home and the shop.
**c** Find Enzo's speed when cycling between the shop and his friend's place.
**d** What time did Enzo arrive at and leave his friend's place?
**e** What was the total time that Enzo was stopped on this journey?
**f** At what times was Enzo 12 km from home?
**g** At what time had Enzo ridden a total of 30 km?
**h** At what speed did Enzo travel when returning home from his friend's place?
**i** Calculate Enzo's average speed (to 1 decimal place) over the whole journey.

**8** A train travels 5 km in 8 minutes, stops at a station for 2 minutes, travels 12 km in 10 minutes, stops at another station for 2 minutes and then completes the journey by travelling 10 km in 15 minutes.
**a** Calculate the speed of the train in km/h (to 1 decimal place) for each section of the journey.
**b** Explain why the average speed over the whole journey is not the average of these three speeds.
**c** Calculate the average speed of the train in km/h (to 1 decimal place) over the whole journey.
**d** On graph paper, draw an accurate distance/time graph for the journey.

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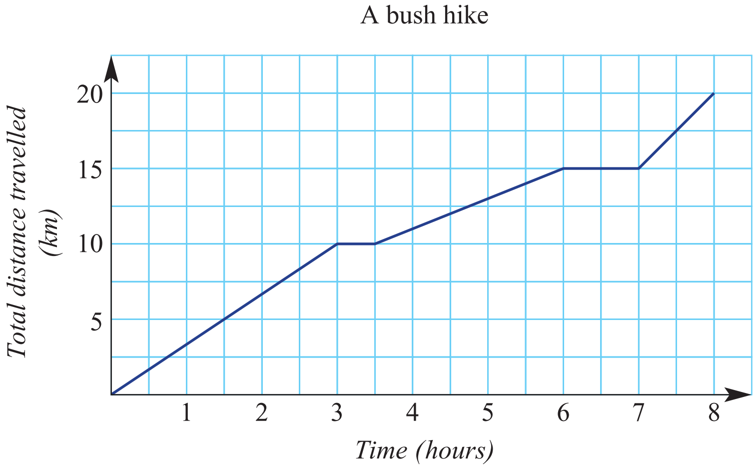
**9** A 20 km bush hike is shown by this graph of 'total distance travelled' versus time.

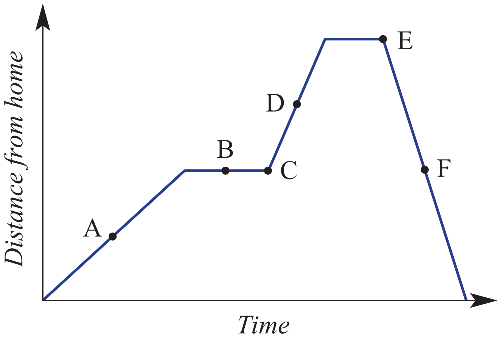


**a** What were the fastest and slowest speeds (in km/h) of the hikers? Suggest a feature of the hiking route that could have made these speeds so different.

**b** Write a story of the journey shown by this graph. In your story, use sentences to describe the features of the hike shown by each straight line segment of the graph, including the time taken and distance travelled. Also include a sentence comparing the average speeds for the different sections of the hike.

**c** Suppose the hikers turn back after their $\frac{1}{2}$ hour rest at 10 km and return to where they started from. Redraw the graph above showing the same sections with the same average speeds but with 'distance from start' versus time.

**10** This distance/time graph shows Deanna's bike journey from home one morning. State whether the following statements are true or false and give a reason for your answer.



**a** Deanna was the same distance from home at B and F.

**b** Deanna was not cycling at B.

**c** Deanna was cycling faster at A than she was at D.

**d** Deanna was facing the same direction at C and E.

**e** Deanna was cycling faster at F than she was at A.

**f** Deanna was further from home at F than at D.

**g** Deanna had ridden further at F than at D.

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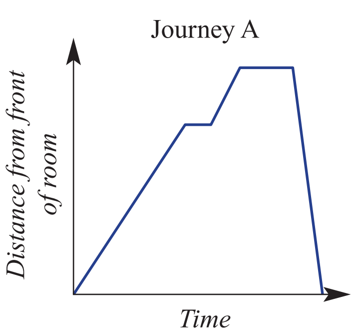
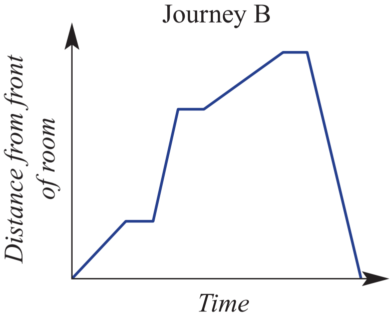
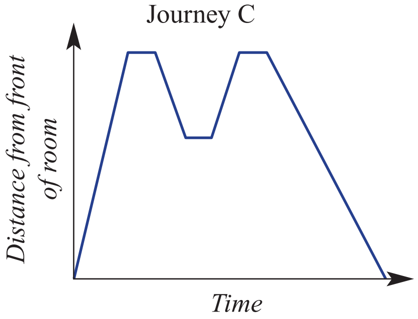
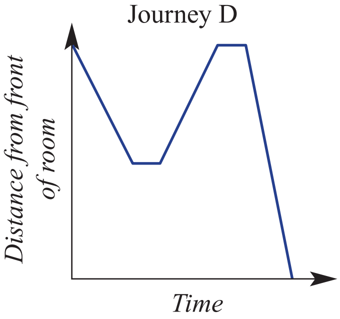
**11** Match each distance/time graph with the student below who correctly walked the journey shown by that graph. Explain the reasons for your choice.









Adam: Walks from the front, stops, a fast walk, stops, slow walk to the back, stops, and walks to the front.

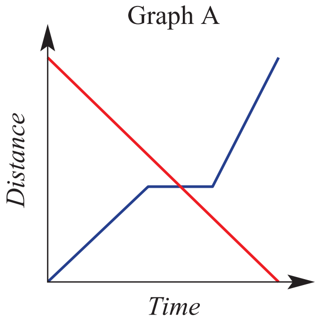
Max: Walks from the back, stops, walks to the front, stops, walks to the back.

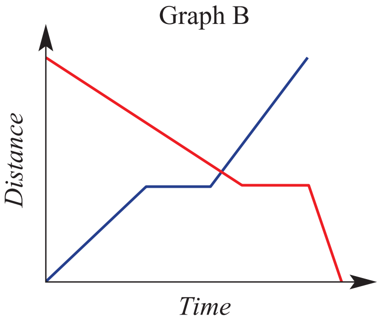
Ruby: Walks from the back, stops halfway, turns, walks to the back, stops, walks to the front.

Conner: Fast walk from front to back, stops, walks towards the front, stops, walks to the back, stops, slower walk to the front.

Isla: Walks from the front, stops, walks to the back, stops, walks at a fast pace to the front.

**12** Jayden and Cooper cycled towards each other along the same track. Jayden was resting when Cooper caught up to him and stopped for a chat and then they both continued their ride. Which of these distance/time graphs show their journeys? Explain why.





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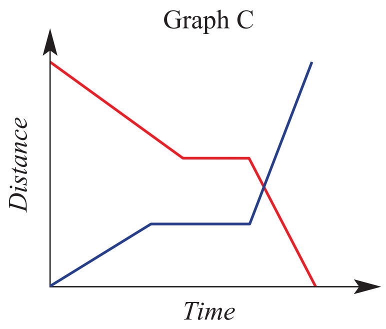
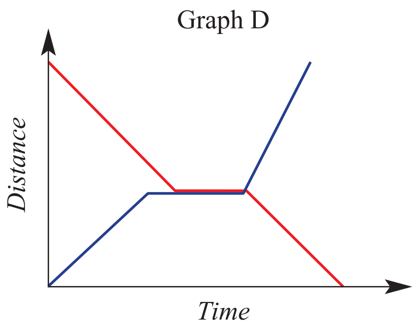


Graph C



Graph D

## Enrichment: More than one journey

**13** Northbrook is 160 km north of Gurang. Archie leaves Gurang to drive north and at the same time Heidi leaves Northbrook to drive south. After 40 minutes, Archie is halfway to Northbrook when his car collides with a tree. Five minutes later Heidi sees Archie's car and stops. Heidi immediately calls an ambulance, which comes from 15 km away and arrives in 10 minutes.

Using graph paper, draw a distance/time graph to show the journey of the two cars and the ambulance.

# Puzzles and games

**1** Write these ratios in simplest form to solve the riddles below.

**A** 4 : 8
**C** 4 : 16
**E** 6 : 10
**F** 4 : 12
**H** 8 : 12
**I** 20 : 16
**K** 10 : 4
**L** 12 : 3
**O** 9 : 6
**P** 15 : 5
**R** 25 : 15
**S** 20 : 10
**T** 35 : 25
**V** 2 : 12

**a** What do termites eat for dessert?

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| 7 : 5 | 3 : 2 | 3 : 2 | 7 : 5 | 2 : 3 | 3 : 1 | 5 : 4 | 1 : 4 | 5 : 2 | 2 : 1 |

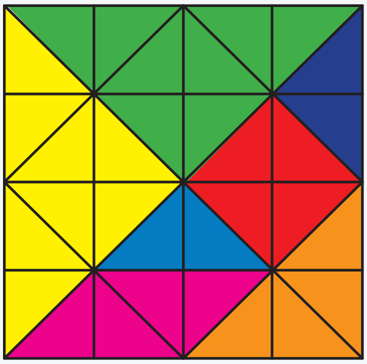
**b** Where do geologists go to have a good time?

| | |
|---|---|
| | |
| 7 : 5 | 3 : 2 |

| | | | |
|---|---|---|---|
| | | | |
| 5 : 3 | 3 : 2 | 1 : 4 | 5 : 2 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| 1 : 3 | 3 : 5 | 2 : 1 | 7 : 5 | 5 : 4 | 1 : 6 | 1 : 2 | 4 : 1 | 2 : 1 |

**2** The Ancient Chinese puzzle known as a tangram consists of 7 geometric shapes (tans) as shown.



**a** Write the ratio of the areas of the seven shapes in this tangram. Write the ratio in simplest form in ascending order.

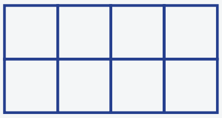
**b** The pieces (tans) of a tangram can be arranged to make many creative shapes and designs. Use the diagrams below to find the ratios of the areas of the:

**i** yacht's sails to the boat hull
**ii** cat's head to the rest of the body.

**3** Hannah is 14 years old and her brother is 9 years old. Find their ages when the ratio of Hannah's age to Blake's age is:

**a** 3 : 2
**b** 5 : 4
**c** 11 : 10

**4** This diagram is made up of 8 equal-sized squares.



How many squares need to be shaded if the ratio of shaded squares to unshaded squares is:

**a** 1 : 3?
**b** 2 : 3?
**c** 1 : 2?

Give each answer as a mixed numeral if necessary.

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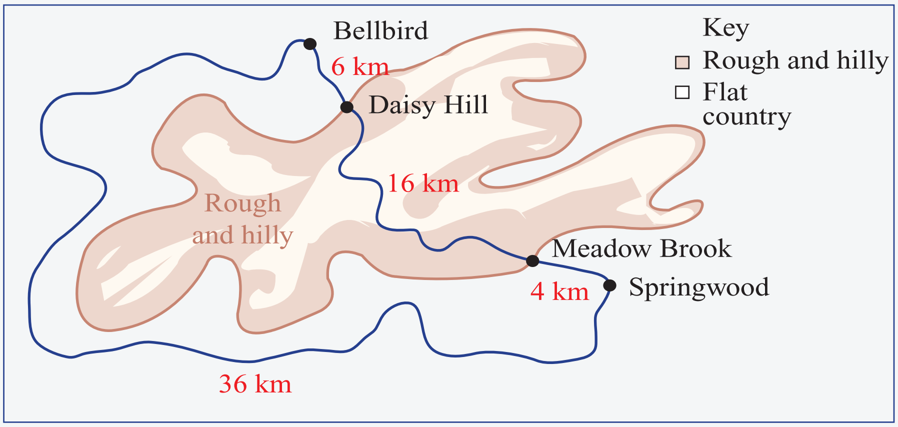
**5** Bottle A has 1 L of cordial drink with a cordial to water ratio of 3 : 7. Bottle B has 1 L of cordial drink with a cordial to water ratio of 1 : 4. The drink from both bottles is combined to form a 2 L drink. What is the new cordial to water ratio?

**6** A group of cyclists decide to have a race from Springwood to Bellbird. The towns and distances are shown on the sketch map below. Over flat country a cyclist averages 20 km/h but through hilly country the average is 12 km/h. Which route would be fastest and by how much?



**7** Brothers Marco and Matthew start riding from home into town, which is 30 km away. Marco rode at 10 km/h and Matthew took 20 minutes longer to complete the trip. Assuming that they both rode at a constant speed, how fast was Matthew riding?

**8** Solve the questions below to find the answer to the riddle:
*Why did the monkey put a steak under himself?*

5 m 4 m 8 m 2 m 1 : 8000 1 : 4000 4 m 70 cm 4 m

1 : 3000 70 cm 1 m 1 : 8000 90 cm 70 cm 1 : 3000

70 cm 4 m 1 : 80 2 m 1:400 2 m

90 cm 1 : 500 10 cm 25 cm 25 cm 4 m 1 : 500

If the scale is 1 : 100, find the real length in metres shown by:
**a** 2 cm **b** 5 cm **c** 8 cm **d** 6 cm **e** 4 cm
If the scale is 1 : 10, find the real length shown by:
**f** 3 cm **g** 9 cm **h** 7 cm **i** 1 cm **j** 15 cm
If the scale is 1 : 5, find the real length shown by:
**k** 3 cm **l** 5 cm **m** 10 cm **n** 30 cm **o** 20 cm
Write each scale in the simplest ratio form.
**p** 1 cm to 2 m **q** 1 cm to 10 m **r** 1 cm to 5 m **s** 1 m to 4 km
**t** 1 m to 3 km **u** 1 m to 8 km **v** 1 mm to 3 cm **w** 1 mm to 8 cm
**x** 1 mm to 15 cm **y** 1 mm to 6 cm **z** 1 mm to 2 cm

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# Chapter summary

## Ratios

Ratios are writen in
- simplest form
- with whole numbers

**Simplest form**

Eliminate decimals

HCF 4

0.8 : 1.2 (×10)
8 : 12 (÷4)
2 : 3

Same units

25 minutes: $1\frac{1}{4}$ hours
25 : 75 (÷25)
1 : 3

**Equivalent ratios**

2 : 3 (×4)
8 : 12 (÷2)
4 : 6

*a* : *b*
*a* is to *b*
*a* to *b*

Comparison of two quantities of the
- same type and
- same unit

Dividing a quantity in a given ratio

Divide \$180 in the ratio 4 : 5

Unitary method

Divide \$180 in the ratio 4 : 5

Total number of parts → 4 + 5 = 9

One part →
9 parts = \$180 (÷9)
1 part = \$20 (×4)
4 parts = \$80

1 part = \$20 (×5)
5 parts = \$100

Sentence answer → \$180 divided in the ratio 4 : 5 is \$80 and \$100

Finding a quantity from a ratio

Cows to horses in ratio 5 : 2
There are 8 horses, how many cows?

**Unitary method**
cows : horses
5 : 2
2 parts = 8 horses
1 part = 4 horses
5 parts = 20 cows

**Equivalent ratio method**
C : H = 5 : 2
C : H = 5 : 2 (×4)
= 20 : 8
There are 20 cows.

**Scale ratios**

Drawing length : Actual length
= 1 : scale factor

Actual size from model

Model car 17 cm long
Scale 1 : 25
Scale factor = 25
Actual car = 25 × 17 cm
= 425 cm ← same units
= 4.25 m

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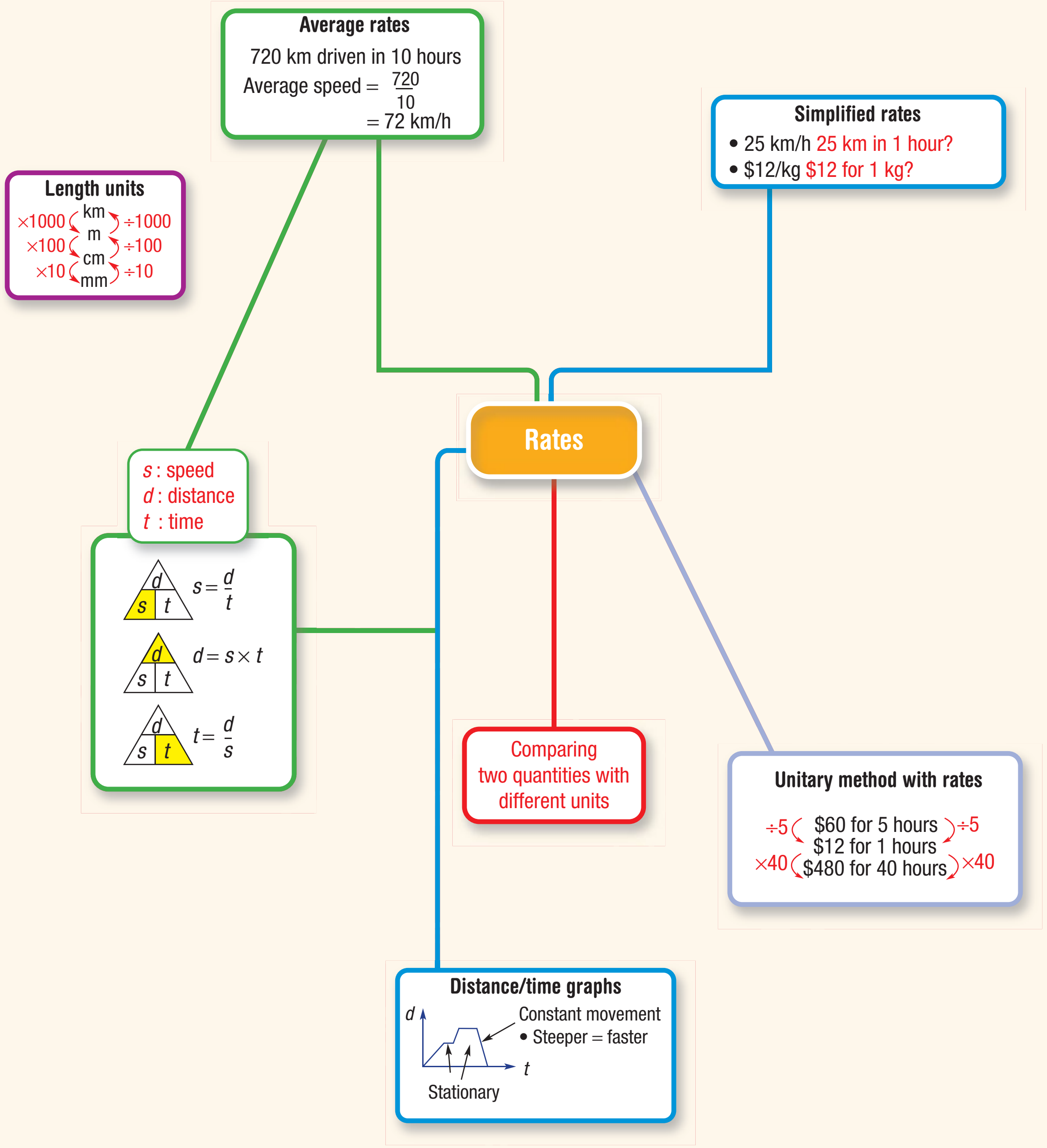
# Chapter summary



## Rates

**Average rates**

720 km driven in 10 hours

$$\text{Average speed} = \frac{720}{10}$$
$$= 72 \text{ km/h}$$

**Simplified rates**

- 25 km/h 25 km in 1 hour?
- \$12/kg \$12 for 1 kg?

**Length units**

$$\begin{tikzcd}
\text{km} \arrow[d, bend left, "\div 1000"] \\
\text{m} \arrow[u, bend left, "\times 1000"] \arrow[d, bend left, "\div 100"] \\
\text{cm} \arrow[u, bend left, "\times 100"] \arrow[d, bend left, "\div 10"] \\
\text{mm} \arrow[u, bend left, "\times 10"]
\end{tikzcd}$$

$s$ : speed
$d$ : distance
$t$ : time

$s = \frac{d}{t}$

$d = s \times t$

$t = \frac{d}{s}$

Comparing two quantities with different units

**Unitary method with rates**

$$\begin{tikzcd}
\text{\$60 for 5 hours} \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\
\text{\$12 for 1 hours} \arrow[d, bend right, "\times 40"'] \arrow[d, bend left, "\times 40"] \\
\text{\$480 for 40 hours}
\end{tikzcd}$$

**Distance/time graphs**

Constant movement
- Steeper = faster

Stationary

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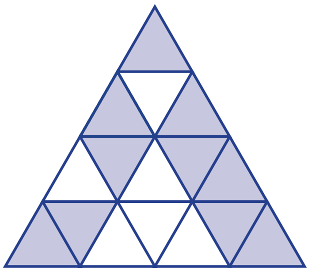
**Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.**

## Multiple-choice questions

**1** A school has 315 boys, 378 girls and 63 teachers. The ratio of students to teachers is:
**A** 11 : 1 **B** 1 : 11 **C** 5 : 6 **D** 6 : 5

**2** Find the ratio of the shaded area to the unshaded area in this triangle.



**A** 3 : 5 **B** 8 : 5 **C** 5 : 3 **D** 5 : 8

**3** The ratio 500 mm to 20 cm is the same as:
**A** 50 : 2 **B** 2500 : 1 **C** 2 : 5 **D** 5 : 2

**4** The ratio 1 hour : 30 minutes simplifies to:
**A** 2 : 1 **B** 1 : 2 **C** 1 : 30 **D** 4 : 3

**5** \$750 is divided in the ratio 1 : 3 : 2. The smallest share is:
**A** \$250 **B** \$125 **C** \$375 **D** \$750

**6** The ratio of the areas of two triangles is 5 : 2. The area of the larger triangle is 60 $cm^2$. What is the area of the smaller triangle?
**A** 12 $cm^2$ **B** 24 $cm^2$ **C** 30 $cm^2$ **D** 17 $cm^2$

**7** Callum fills his car with 28 litres of petrol at 142.7 cents per litre. His change from \$50 cash is:
**A** \$10 **B** \$39.95 **C** \$10.05 **D** \$40

**8** Madison cycled 20 km in 1.25 hours. Her average speed was:
**A** 25 km/h **B** 20 km/h **C** 16 km/h **D** 18.75 km/h

**9** A house plan has a scale of 1 : 200. On the plan, the lounge room is 25 mm in length. The real length of the lounge room would be:
**A** 50 m **B** 5 m **C** 50 cm **D** 8 m

**10** On a map, Sydney and Melbourne are 143.2 mm apart. If the cities are 716 km apart, what scale has been used?
**A** 1 : 5 **B** 1 : 5000 **C** 1 : 50 000 **D** 1 : 5 000 000

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## Short-answer questions

**1** In Lao's pencil case there are 6 coloured pencils, 2 black pens, 1 red pen and 3 lead pencils. Find the ratio of:
- **a** lead pencils to coloured pencils
- **b** black pens to red pens
- **c** all pens to all pencils.

**2** True or false?
- **a** $1:4 = 3:6$
- **b** The ratio $2:3$ is the same as $3:2$.
- **c** The ratio $3:5$ is written in simplest form.
- **d** 40 cm:1 m is written as $40:1$ in simplest form.

**3** Copy and complete.
- **a** $4:50 = 2:\square$
- **b** $3:7 = \square:21$
- **c** $\square:12 = 8:3$
- **d** $1:\square:5 = 5:15:25$

**4** Simplify the following ratios.
- **a** $10:40$ **b** $36:24$ **c** $75:100$ **d** $8:64$ **e** $27:9$
- **f** $5:25$ **g** $6:4$ **h** $52:26$ **i** $6:9$ **j** $8:4:20$

**5** Simplify the following ratios by first changing to the same units.
- **a** 2 cm : 8 mm **b** 5 mm : 1.5 cm **c** 3 L : 7500 mL **d** 30 min : 1 h
- **e** 400 kg : 2 tonnes **f** 6 h : 1 day **g** 120 m : 1 km **h** 45 min : $2\frac{1}{2}$ h

**6** Divide:
- **a** \$80 in the ratio $7:9$ **b** 200 kg in the ratio $4:1$
- **c** 40 m in the ratio $6:2$ **d** \$1445 in the ratio $4:7:6$.
- **e** \$100 in the ratio $3:1:1$

**7** Orange juice, pineapple juice and guava juice are mixed in the ratio $4:3:2$. If 250 mL of guava juice is used, how many litres of drink does this make?

**8** A map has a scale of $1:20\,000$. Find the real distance for each of these scaled distances.
- **a** 3 cm (answer in m) **b** 12 cm (answer in km)

**9** For each of these situations, find the scale ratio and also state the scale factor.
- **a** 5 mm on a scale drawing represents a real length of 1 m.
- **b** 4 cm on a map represents an actual length of 10 km.

**10** Two towns are 5 km apart. How many millimetres apart are they on a map that has a scale of $1:100\,000$?

**11** Express each rate in simplest form.
- **a** 10 km in 2 hours (? km/h)
- **b** \$650 for 13 hours (\$?/h)
- **c** 2800 km in 20 days (? km/day)

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**12** Copy and complete.

**a**

$$\begin{tikzcd} \text{7 km uses 1 L of fuel} \arrow[d, bend right, "\times ?"'] \arrow[d, bend left, "\times ?"] \\ \text{280 km uses? L of fuel} \end{tikzcd}$$

**b**

$$\begin{tikzcd} \text{60 words typed in 1 minute} \arrow[d, bend right, "\times ?"'] \arrow[d, bend left, "\times ?"] \\ \text{? words typed in 10 minutes} \end{tikzcd}$$

**13 a** A truck uses 12 litres of petrol to travel 84 km. How far will it travel on:

- **i** 1 L of petrol?
- **ii** 42 L of petrol?

**b** Samira earns $67.20 for a 12-hour shift. How much will she earn for:

- **i** 1 hour?
- **ii** 7 hours?

**14 a** Sandra drives to her mother's house. It takes 2 hours. Calculate Sandra's average speed km/h if her mother lives 150 km away.

**b** How long does it take Ari to drive 180 km along the freeway to work if he manages to average 100 km/h for the trip? Give your answer in hours.

**c** How far does Siri ride his bike if he rides at 12 km/h for 45 minutes?

## Extended-response question

From Canberra to Melbourne, it is 660 km. Two families, the Harrisons and the Nguygens, both leave Canberra at 8 a.m. to drive to Melbourne.

*The Harrison family's trip*

- The Harrison's 17-year-old son drives for the first 2 hours at an average speed of 80 km/h.
- Then they stop for a rest of 1.5 hours.
- Mr Harrison drives the rest of the way to Melbourne with no more stops.

**a** How far did the Harrison's son drive?

**b** How far did Mr Harrison drive?

**c** At what time did the Harrison family finish their morning rest break?

**d** If the Harrisons arrive in Melbourne at 4:30 p.m., for how long did Mr Harrison drive?

**e** What was Mr Harrison's average speed?

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*The Nguygen family's trip*

- The Nguygen family drove to Melbourne with one 30-minute break.
- It took them $8\frac{1}{4}$ hours in total.

**f** At what time did the Nguygen family arrive in Melbourne?

**g** Calculate the average speed that the Nguygen family drove at.

*Comparing the cost of each trip*

**h** Using the information below, calculate the cost of each car's fuel for the trip.
Petrol costs 152.7 cents/L.
The Harrison family's car uses 8 L/100 km.
The Nguygen family's car uses 11 L/100 km.

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## Chapter 1: Algebraic techniques 2 and indices

### Multiple-choice questions

**1** If $x = 3$, then $7x + 2$ equals:

**A** $21$ **B** $75$ **C** $73$ **D** $23$

**2** $4x + 5 + 3x$ simplifies to:

**A** $7x + 5$ **B** $12x$ **C** $12 + x^2$ **D** $2x + 12$

**3** $2(6x + 5)$ expands to:

**A** $12x + 5$ **B** $12x + 10$ **C** $6x + 10$ **D** $60x$

**4** $12m + 18$ factorises to:

**A** $2(6m - 9)$ **B** $-6(2m - 3)$ **C** $6(3 - 2m)$ **D** $6(2m + 3)$

**5** $2^3 \times 2^5$ is equal to:

**A** $2^{15}$ **B** $4^{15}$ **C** $2^8$ **D** $4^8$ **E** $2^{35}$

### Short-answer questions

**1** Write an expression for:

**a** the sum of $p$ and $q$

**b** the product of $p$ and 3

**c** half the square of $m$

**d** the sum of $x$ and $y$, divided by 2.

**2** Find the value of $7k - 2$ if:

**a** $k = 3$ **b** $k = 10$ **c** $k = 5$ **d** $k = 100$

**3** If $a = 6$, $b = 4$ and $c = 1$, evaluate:

**a** $a + b + c$

**b** $ab - c$

**c** $a(b - c)$

**d** $3a + 2b$

**e** $abc$

**f** $a - 2b + 3c$

**4** Simplify each algebraic expression.

**a** $4 \times 6k$

**b** $a + a + a$

**c** $a \times a \times a$

**d** $7p \div 14$

**e** $3ab + 2 + 4ab$

**f** $7x + 9 - 6x - 10$

**g** $18xy \div 9x$

**h** $m + n - 3m + n$

**5** Simplify:

**a** $\dfrac{5xy}{5}$ **b** $\dfrac{30x}{21y}$ **c** $\dfrac{2w}{10}$ **d** $\dfrac{17abc}{5bc}$

**6** Expand, and simplify where necessary.

**a** $2(x + 5)$ **b** $6(2m - 3)$ **c** $10 + 2(m - 3)$

**7** Factorise:

**a** $18a - 12$ **b** $6mn + 12m$ **c** $8x + 12$

**8** **a** $\sqrt{49} =$

**b** $12^2 =$

**c** $5 \times 5 \times 5 = 125$, so $\sqrt[3]{125} =$

**d** $6^3 = 216$, so $\sqrt[3]{126} =$

**e** $5^7 \times 5^2 = 5^{\square}$

**f** $5^7 \div 5^2 = 5^{\square}$

**g** $(5^7)^2 = 5^{\square}$

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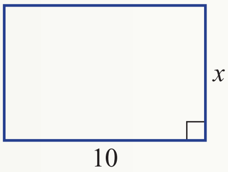
**9** Write an expression for the rectangle's:

**a** perimeter
**b** area



**10** If pens cost \$2 each and notepads cost \$3 each, write expressions for the:

**a** cost of $x$ pens.
**b** cost of $y$ notepads
**c** total cost of $x$ pens and $y$ notepads.

### Extended-response question

A repairman charges a \$60 call-out fee plus \$80 per hour.

**a** Find the cost of a 2-hour visit.
**b** Write an expression for the cost of an $n$-hour visit.
**c** Another repairman charges no call-out fee but \$100 per hour.
  **i** Write an expression for this repairman's total cost.
  **ii** For how many hours were they hired if the total cost was the same for both repairman?

## Chapter 2: Equations 2

### Multiple-choice questions

**1** If $x = 5$, which one of these equations is true?
**A** $x + 3 = 2$ **B** $7x = 75$ **C** $7 - x = 2$ **D** $2x = 20$

**2** The sum of a number and three is doubled. The result is 12. This can be written as:
**A** $x + 3 \times 2 = 12$ **B** $2(x + 3) = 12$ **C** $2x + 3 = 12$ **D** $x + 3 = 24$

**3** The solution to the equation $2m - 4 = 48$ is:
**A** $m = 8$ **B** $m = 22$ **C** $m = 20$ **D** $m = 26$

**4** The solution to the equation $4k + 3 = 39$ is:
**A** $k = 36$ **B** $k = 9$ **C** $k = 10$ **D** $k = 4$

**5** The solution to $\frac{x}{4} - 3 = 4$ is:
**A** $x = 20$ **B** $x = 28$ **C** $x = 4$ **D** $x = 24$

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## Short-answer questions

**1** Solve these equations.

**a** $3w = 27$
**b** $12 = m + 5$
**c** $2x - 1 = 9$
**d** $4a + 2 = 10$
**e** $2w + 6 = 32$
**f** $4 = 6x - 2$

**2** Solve these equations.

**a** $\frac{x}{3} = 10$
**b** $\frac{2q}{5} = 4$
**c** $3 = \frac{p}{5}$
**d** $\frac{x+2}{4} = 3$
**e** $\frac{r-3}{12} = 1$
**f** $2 = \frac{3a-4}{10}$

**3** Solve the following equations.

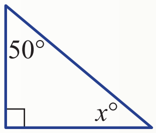
**a** $2(x + 3) = 16$
**b** $4(2k + 1) = 84$
**c** $3(r + 2) + r = 6$
**d** $10(z - 4) = 80 - 2z$

**4** The formula $S = 6g + b$ relates an AFL score ($S$) to the number of goals ($g$) and behinds ($b$).

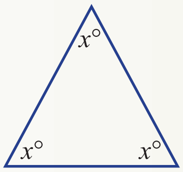
**a** Find $S$ if $g = 3$ and $b = 2$.
**b** Find $b$ if $S = 62$ and $g = 10$.
**c** Find $g$ if $S = 50$ and $b = 8$.

**5** Using the fact that angles of a triangle add to 180°, find $x$ in the following triangles.

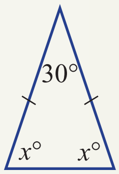
**a**



**b**



**c**



## Extended-response question

EM Publishing has fixed costs of \$1500 and production costs of \$5 per book.

**a** Write an expression for the cost of producing $n$ books.
**b** The total costs for one year were \$2000. Use an equation to find how many books were produced.
**c** Write an expression for the money made by selling $n$ books, if they sell each book for \$20.
**d** If the total revenue is \$1000, find the number of books sold.
**e** Given that the profit is given by the formula, $P = 15n - 1500$, find the:
  **i** profit if 200 books are sold.
  **ii** profit if 1000 books are sold.
  **iii** number of books sold if the profit is \$0.
**f** If $n = 50$, the profit is \$–750. Explain what this means for EM Publishing.

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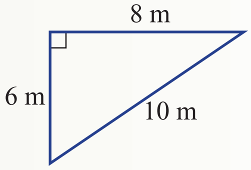
## Chapter 3: Measurement and Pythagoras' Theorem

### Multiple-choice questions

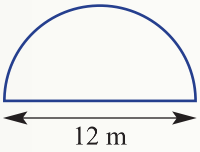
**1** A cube has a volume of 8 cubic metres. The side length of the cube is:
**A** 8 m **B** 4 m **C** 2 m **D** 16 m

**2** The area of this triangle is:



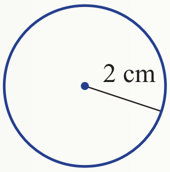
**A** 48 $m^2$
**B** 24 $m^2$
**C** 30 $m^2$
**D** 40 $m^2$

**3** The perimeter of this semicircle is closest to:



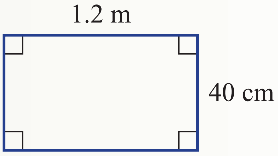
**A** 38 cm
**B** 30 cm
**C** 19 cm
**D** 31 cm

**4** The exact area of this circle is given by:



**A** $8\pi$ $cm^2$
**B** $4\pi$ $cm^2$
**C** $2\pi$ $cm^2$
**D** $\pi$ $cm^2$

**5** The area of this rectangle is:



**A** 48 $m^2$
**B** 48 000 $cm^2$
**C** 480 $cm^2$
**D** 0.48 $m^2$

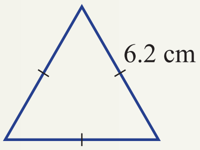
### Short-answer questions

**1** Complete these conversions.

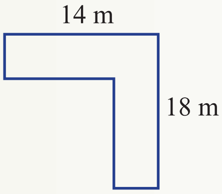
**a** 5 m = _______ cm
**b** 1.8 m = _______ cm
**c** 9 $m^2$ = _______ $cm^2$
**d** 1800 mm = _______ cm
**e** 4 L = _______ mL
**f** 0.01 $km^2$ = _______ $m^2$

**2** Find the perimeter of these shapes.

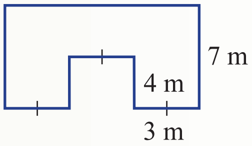
**a**



**b**

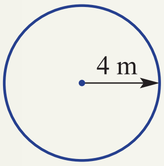


**c**

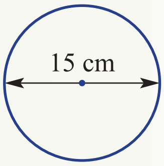


**3** Find correct to 2 decimal places:
**i** the circumference **ii** the area

**a**



**b**



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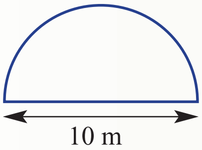
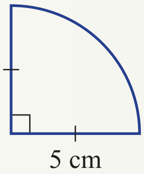
**4** Find correct to two decimal places:

**i** the perimeter **ii** the area

**a**

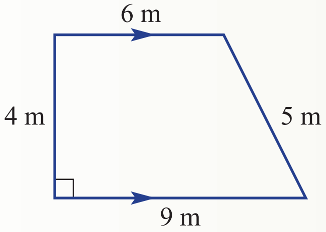


**b**

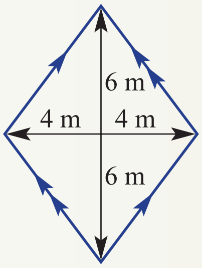


**5** Find the area of these shapes.

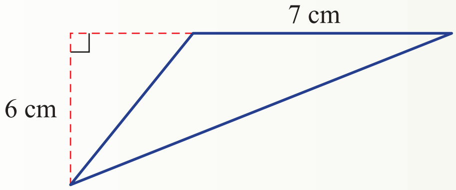
**a**



**b**

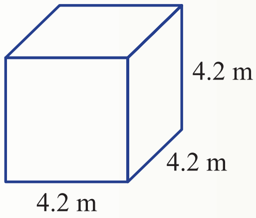


**c**

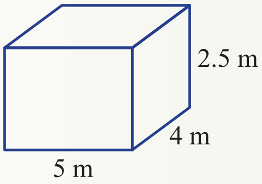


**6** Find the volume of these solids.

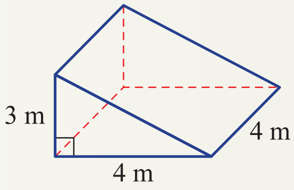
**a**



**b**



**c**

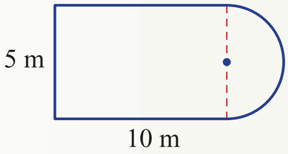


**7** Write these times using 24-hour time.

**a** 3:30 pm **b** 7:35 am

## Extended-response question

A paved area is in the shape of a rectangle with a semicircular end as shown.



**a** What is the radius of the semicircle?

**b** What is the area of the semicircle correct to two decimal places?

**c** What is the total area of the paved area correct to two decimal places?

**d** A special brick border is to go around the perimeter of the area. Find this length correct to the nearest metre.

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## Chapter 4: Fractions, decimals, percentages and financial mathematics

### Multiple-choice questions

**1** $\frac{150}{350}$ simplifies to:

**A** $\frac{6}{14}$ **B** $\frac{3}{70}$ **C** $\frac{15}{35}$ **D** $\frac{3}{7}$

**2** Sienna spends $\frac{3}{7}$ of $\$280$ her income on clothes and saves the rest. She saves:

**A** $\$470$ **B** $\$120$ **C** $\$160$ **D** $\$2613$

**3** $0.008 \times 0.07$ is equal to:

**A** $0.056$ **B** $0.0056$ **C** $0.00056$ **D** $56$

**4** $0.24$ expressed as a fraction is:

**A** $\frac{1}{24}$ **B** $\frac{6}{25}$ **C** $\frac{12}{5}$ **D** $\frac{24}{10}$

**5** If $5\%$ of $x$ is $8$, then $10\%$ of $x$ equals:

**A** $4$ **B** $16$ **C** $64$ **D** $80$

### Short-answer questions

**1** Copy and complete these equivalent fractions.

**a** $\frac{3}{5} = \frac{\square}{30}$ **b** $\frac{\square}{11} = \frac{5}{55}$ **c** $1\frac{4}{6} = \frac{\square}{3}$

**2** Evaluate each of the following.

**a** $\frac{3}{4} - \frac{1}{2}$ **b** $\frac{4}{5} + \frac{3}{5}$ **c** $1\frac{1}{2} + 1\frac{3}{4}$

**d** $\frac{4}{7} - \frac{2}{3}$ **e** $\frac{4}{9} \times \frac{3}{4}$ **f** $1\frac{1}{2} \times \frac{3}{5}$

**3** Write the reciprocal of:

**a** $\frac{2}{5}$ **b** $8$ **c** $4\frac{1}{5}$

**4** Evaluate:

**a** $2\frac{1}{2} \times 1\frac{4}{5}$ **b** $1\frac{1}{2} \div 2$ **c** $3 - 2\frac{1}{3}$

**5** Calculate each of the following.

**a** $3.84 + 3.09$ **b** $10.85 - 3.27$ **c** $12.09 \div 3$

**d** $6.59 - 0.08$ **e** $96.37 \times 40$ **f** $15.84 \div 0.02$

**6** Evaluate:

**a** $5.3 \times 100$ **b** $9.6 \times 1000$ **c** $61.4 \div 100$

**7** Copy and complete this table of decimals, fractions and percentages.

| **Fraction** | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{5}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Decimal** | | | | | | | | 0.99 | 0.005 |
| **Percentage** | | | | | | 80% | 95% | | |

**8** Find:

**a** 10% of 56 **b** 12% of 98 **c** 15% of 570 m

**d** 99% of \$2 **e** 25% of \$840 **f** 50% of 8500 g

**9 a** Increase \$560 by 25%.

**b** Decrease \$980 by 10%.

**10** A \$348 dress sold for \$261.

**a** What was the saving?

**b** What was the percentage discount?

### Extended-response question

A laptop decreases in value by 20% every year.

**a** Find the value of a \$2000 laptop at the end of:

**i** 1 year **ii** 2 years **iii** 3 years.

**b** After how many years is the laptop worth less than \$800?

**c** Will the laptop ever have a value of zero dollars? Explain.

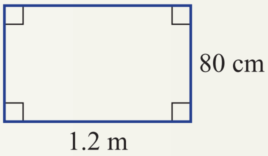
## Chapter 5: Ratios and rates

### Multiple-choice questions

**1** The ratio of the length to the breadth of this rectangle is:

**A** 12 : 80 **B** 3 : 20

**C** 3 : 2 **D** 20 : 3



**2** Which of the following ratios is not written in simplest form?

**A** 2 : 3 **B** 5 : 10 **C** 11 : 3 **D** 3 : 7

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**3** \$18 is divided in the ratio 2 : 3. The larger part is:

**A** \$3.60 **B** \$7.20 **C** \$10.80 **D** \$12

**4** Calvin spent \$3 on his mobile phone plan for every \$4 he spent on his internet connection. Calvin spent \$420 on his phone last year. How much did he spend for his internet connection the same year?

**A** \$140 **B** \$315 **C** \$560 **D** \$240

**5** A boat sailed 30 kilometres in 90 minutes. What was the average speed of the boat?

**A** 15 km/h **B** 45 km/h **C** 3 km/h **D** 20 km/h

### Short-answer questions

**1** Simplify these ratios.

**a** 24 to 36
**b** 15 : 30 : 45
**c** 0.6 m to 70 cm
**d** 15 cents to \$2
**e** 2 kg to 400 g
**f** 30 seconds to $1\frac{2}{3}$ minutes

**2** **a** Divide 960 cm in the ratio of 3 : 2.
**b** Divide \$4000 in the ratio of 3 : 5.
**c** Divide \$8 in the ratio of 2 : 5 : 3.

**3** A business has a ratio of profit to costs of 5 : 8. If the costs were \$12 400, how much profit was made?

**4** A map has a scale 1 : 10 000. Find the real distance in cm and also in m between two towns that are 6 cm apart on the map.

**5** Write each of the following as a simplified rate.
**a** 84 mm rainfall in 7 days
**b** 18 goals in 6 games
**c** \$15 for 750 g of meat

**6** A shop sells $1\frac{1}{2}$ kg bags of apples for \$3.40. Find the cost of a one kilogram at this rate.

**7** A family travels the 1070 km road from Rockhampton to Cairns, in 12.5 hours. Calculate their average speed.

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### Extended-response question

A small car uses 30 litres of petrol to travel 495 km.

**a** What is the average distance travelled per litre?

**b** At this rate, what is the maximum distance a small car can travel on 45 litres of petrol?

**c** Find the number of litres used to travel 100 km, correct to 1 decimal place.

**d** Petrol costs 117.9 cents/litre. Find the cost of petrol for the 495-km trip.

**e** A larger car uses 42 litres of petrol to travel 378 km. The smaller car holds 36 litres of petrol while the larger car holds 68 litres. How much further can the larger car travel on a full tank of petrol?

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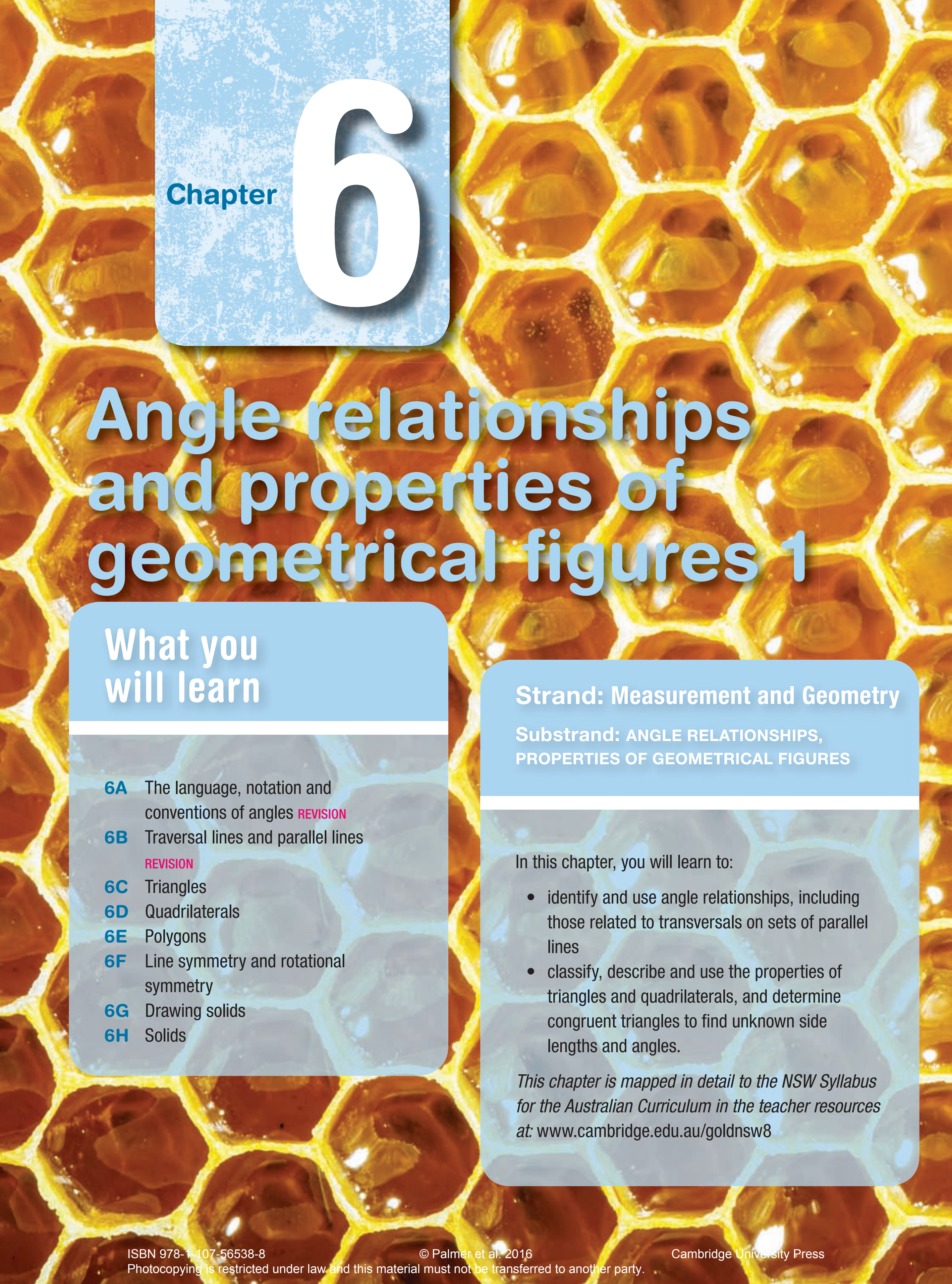


# Chapter 6

# Angle relationships and properties of geometrical figures 1

## What you will learn

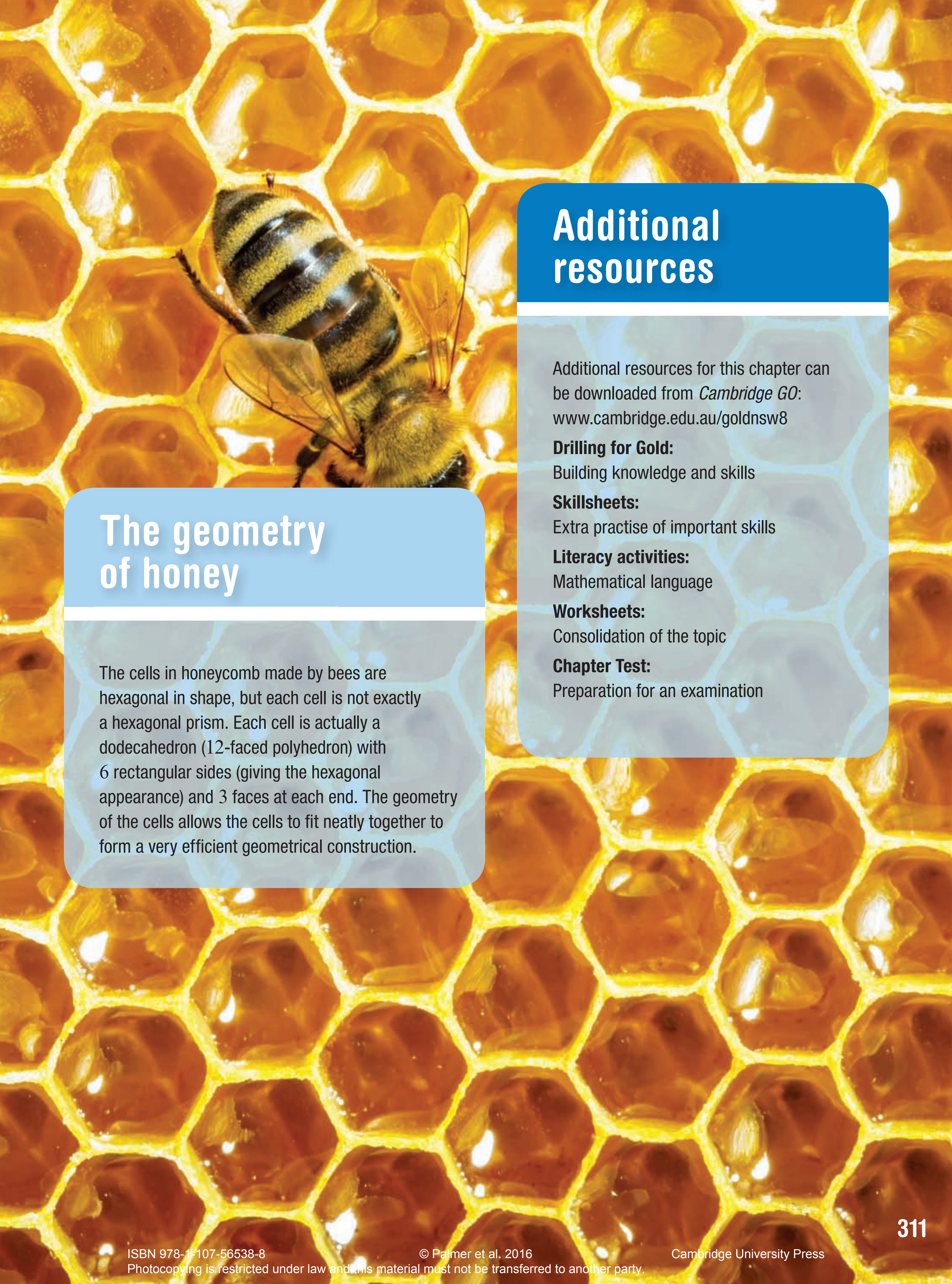
## Strand: Measurement and Geometry

### Substrand: ANGLE RELATIONSHIPS, PROPERTIES OF GEOMETRICAL FIGURES

In this chapter, you will learn to:

- identify and use angle relationships, including those related to transversals on sets of parallel lines
- classify, describe and use the properties of triangles and quadrilaterals, and determine congruent triangles to find unknown side lengths and angles.

*This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at:* www.cambridge.edu.au/goldnsw8



## Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw8

**Drilling for Gold:**
Building knowledge and skills

**Skillsheets:**
Extra practise of important skills

**Literacy activities:**
Mathematical language

**Worksheets:**
Consolidation of the topic

**Chapter Test:**
Preparation for an examination

## The geometry of honey

The cells in honeycomb made by bees are hexagonal in shape, but each cell is not exactly a hexagonal prism. Each cell is actually a dodecahedron ($12$-faced polyhedron) with $6$ rectangular sides (giving the hexagonal appearance) and $3$ faces at each end. The geometry of the cells allows the cells to fit neatly together to form a very efficient geometrical construction.

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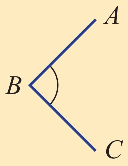
# Pre-test

**1** Name these objects. Choose from: line $AB$, segment $AB$, point $A$ or angle $ABC$.

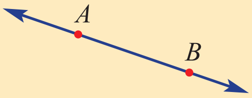
**a**

**b**

**c**

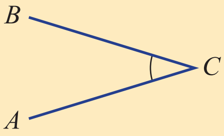


**d**

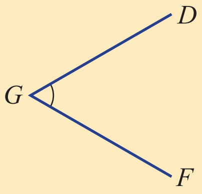


**2** Name these angles using the given letters. For example $\angle EFG$.

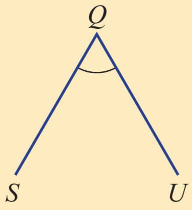
**a**



**b**



**c**



**3** Name these angles as acute, right, obtuse, straight, reflex or revolution.

**a** 360° **b** 90° **c** 37°

**d** 149° **e** 180° **f** 301°

**4** Name the triangle that fits the description. Choose from *scalene*, *isosceles*, *equilateral*, *acute*, *right* or *obtuse*. Draw an example of each triangle to help.

**a** One obtuse angle **b** Two equal length sides

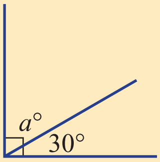
**c** All angles acute **d** Three different side lengths

**e** Three equal 60° angles **f** One right angle

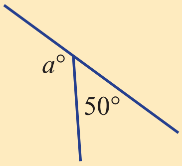
**5** Name the 6 special quadrilaterals. Draw one example of each.

**6** Find the value of $a$ in these diagrams.

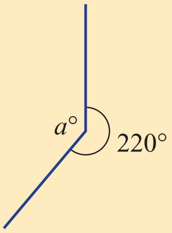
**a**



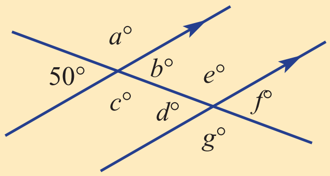
**b**



**c**



**7** This diagram includes a pair of parallel lines and a third line (transversal).



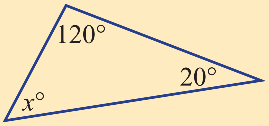
**a** What is the value of $a$?

**b** Which pronumerals ($b$, $c$, $d$, $e$, $f$ or $g$) are equal to $a$?

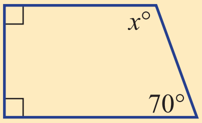
**c** Which pronumerals ($b$, $c$, $d$, $e$, $f$ or $g$) are equal to 50?

**8** Find the value of $x$ in these shapes, using the given angle sum.

**a** Angle sum = 180°



**b** Angle sum = 360°



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# 6A The language, notation and conventions of angles REVISION

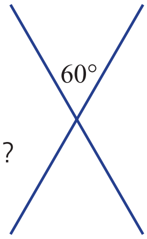
From three simple objects – point, line and plane – we can develop all the elements of Geometry, just as the Greek mathematician Euclid did over 2000 years ago.

We can start by looking at the angles formed when lines meet at a point.

## Let's start: How many angles?

When two lines cross, different angles are formed, like in this example.

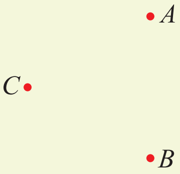
- Is there another 60° angle? Where?
- What is the size of one of the obtuse angles? How did you work this out?
- Are there any straight angles in the diagram?
- Are there any reflex angles in the diagram?
- What is the sum of the four angles?



## Key ideas

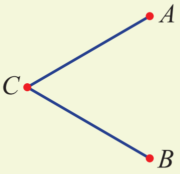
Drilling for Gold 6A2

- A **point** represents a position.
  - It is shown using a dot and labelled with an upper case letter.
  - The diagram shows points $A$, $B$ and $C$.



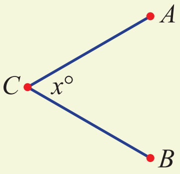
Drilling for Gold 6A3

- This diagram shows **intervals** $AC$ and $CB$. These are sometimes called *line segments*.
  - $AC$ and $CB$ form two **angles**. One is *acute* and one is *reflex*.
  - $C$ is called the **vertex**. The plural is **vertices**.



Drilling for Gold 6A4

- This diagram shows acute angle $ACB$. It can be written as: $\angle C$ or $\angle ACB$ or $\angle BCA$ or $A\hat{C}B$ or $B\hat{C}A$
  - $CA$ and $CB$ are sometimes called **arms**.
  - The pronumeral $x$ represents the number of degrees in the angle.



| | |
|---|---|
| **Point** | A position in space, marked with a dot and named with a capital letter |
| **Interval** | A section of a line with two end points |
| **Vertex** | A point from which two lines, or 'arms' extend in different directions |
| **Angle** | A measure of the space between two lines, usually measured in degrees |
| **Arm** | One of two line intervals joined at a vertex to form an angle |

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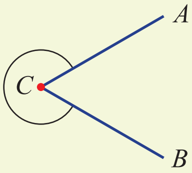
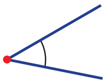
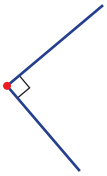
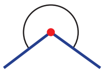
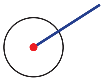
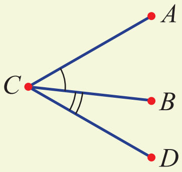
- This diagram shows reflex $\angle ACB$.



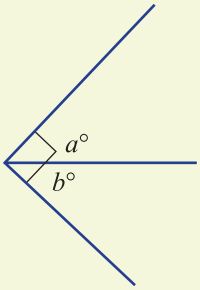
| Type of angle | Size of angle | Diagram |
|---|---|---|
| acute | greater than 0° but less than 90° |  |
| right | exactly 90° |  |
| obtuse | greater than 90° but less than 180° |  |
| straight | exactly 180° |  |
| reflex | greater than 180° but less than 360° |  |
| revolution | exactly 360° |  |

- This diagram shows two angles sharing a vertex and an arm. They are called **adjacent** angles.



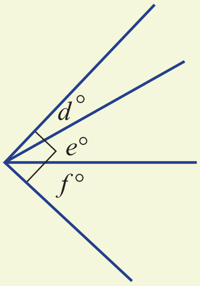
- This diagram shows two angles in a right angle. They are *adjacent complementary angles*. $a°$ is the **complement** of $b°$.

$$a + b = 90$$



- It is possible to have three or more angles in a right angle. They are not complementary.

$$d + e + f = 90$$



**Adjacent** Two angles that are next to each other: they share a common arm and vertex

**Complementary angles** Two angles with a sum of 90°. Each angle is the complement of the other.

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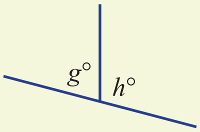
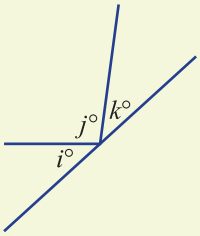
- This diagram shows two angles on a straight line. They are *adjacent supplementary* angles. $g^\circ$ is the **supplement** of $h^\circ$.

$$g + h = 180$$



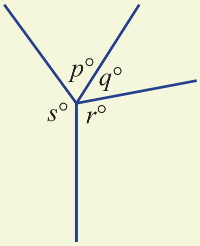
- It is possible to have three or more angles on a straight line. They are not supplementary.

$$i + j + k = 180$$



- This diagram shows *angles at a point* and *angles in a revolution*.

$$p + q + r + s = 360$$

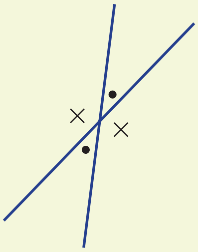


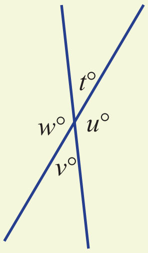
**Supplementary angles**
Two angles with a sum of 180°. Each angle is the supplement of the other.

**Vertically opposite**
A pair of angles (always equal) that are opposite each other across a common vertex

**Perpendicular**
At right angles (90°) to each other

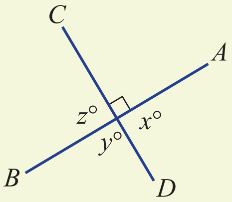
- When two straight lines meet they form two pairs of **vertically opposite angles**. Vertically opposite angles are equal.





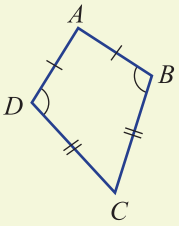
$$t = v$$
$$u = w$$

- If one of the four angles in vertically opposite angles is a right angle, then all four angles are right angles.



$$x = 90$$
$$y = 90$$
$$z = 90$$

  - $AB$ and $CD$ are **perpendicular lines**. This is written as $AB \perp CD$.
- The markings in this diagram indicates that:
  - $AB = AD$
  - $BC = CD$
  - $\angle ABC = \angle ADC$



## Exercise 6A

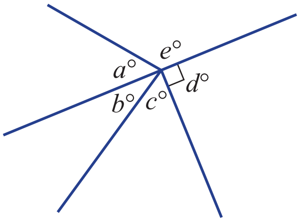
**Understanding**

**1** Write the missing word.
- **a** Angles that add to $90^\circ$ are called __________ angles.
- **b** Angles that add to $180^\circ$ are called __________ angles.
- **c** If two lines meet at right angles ($90^\circ$), then they are said to be __________.
- **d** Vertically opposite angles are __________.

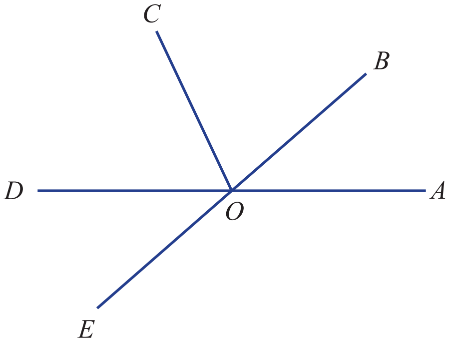
**2** What type of angle are the following?
- **a** $27^\circ$
- **b** $317^\circ$
- **c** $180^\circ$
- **d** $90^\circ$
- **e** $360^\circ$
- **f** $139^\circ$

Choose from:
acute, right, obtuse,
straight, reflex or
revolution.

**3** Complete these sentences for this diagram.
- **a** $b$ and $c$ are ______________ angles.
- **b** $a$ and $e$ are ______________ angles.
- **c** $a$, $b$, $c$, $d$ and $e$ form a ______________.

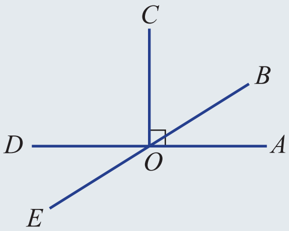


**4** Estimate the size of these angles, then measure them with a protractor.
- **a** $\angle AOB$
- **b** $\angle AOC$
- **c** Reflex $\angle AOE$



### Example 1 Naming angles

Name the angle that is:
- **a** vertically opposite to $\angle DOE$
- **b** complementary to $\angle COB$
- **c** supplementary to $\angle EOA$.



| **Solution** | **Explanation** |
|---|---|
| **a** $\angle AOB$ | $AD$ and $BE$ are straight lines that intersect at $O$. |
| **b** $\angle BOA$ | $\angle COB$ and $\angle BOA$ are adjacent angles inside a right angle. |
| **c** $\angle DOE$ (or $\angle AOB$) | Pairs of angles on a straight line are supplementary (add to $180^\circ$). |

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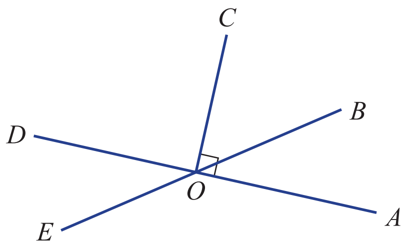
**5** Name an angle that is:
- **a** vertically opposite to $\angle DOE$
- **b** complementary to $\angle COB$
- **c** supplementary to $\angle EOA$
- **d** adjacent to $\angle BOA$
- **e** acute
- **f** obtuse
- **g** straight
- **h** 90 degrees
- **i** equal to $\angle COA$.

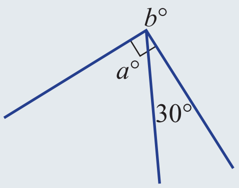


**Fluency**

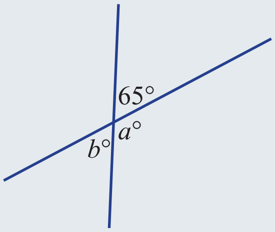
### Example 2 Finding the value of pronumerals in geometrical figures

Determine the value of the pronumeral in these diagrams, giving reasons.

**a**



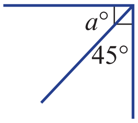
**b**



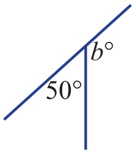
| Solution | Explanation |
|---|---|
| **a** $a + 30 = 90$ (angles in a right angle)<br>$a = 60$<br>$b + 90 = 360$ (angles in a revolution)<br>$b = 270$ | $a°$ and $30°$ are adjacent complementary angles.<br>$b°$ and $90°$ make a revolution. |
| **b** $a + 65 = 180$ (angles on a straight line)<br>$a = 115$<br>$b = 65$ (vertically opposite angles) | $a°$ and $65°$ are adjacent supplementary angles.<br>$b°$ and $65°$ are vertically opposite angles. |

**6** State the values of the pronumerals in these diagrams.

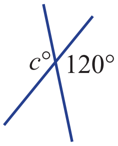
**a**



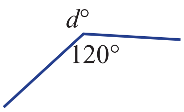
**b**



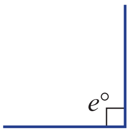
**c**



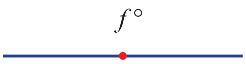
**d**



**e**



**f**



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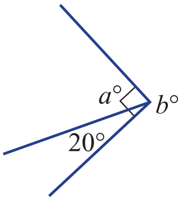
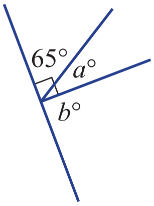
Skillsheet 6A

**7** Determine the values of the pronumerals, giving reasons.

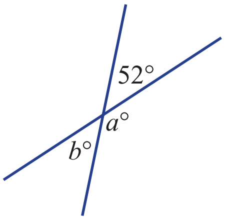
**a**



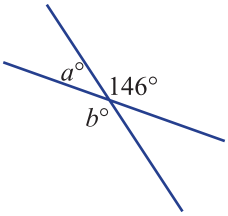
**b**



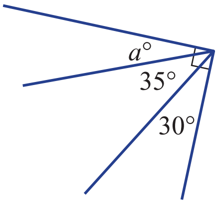
**c**



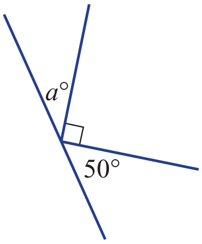
**d**



**e**

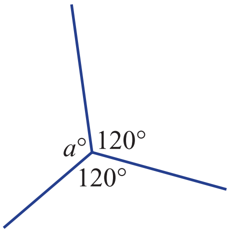


**f**

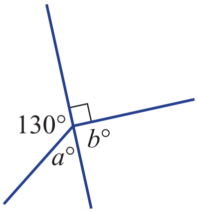


Choose from the following reasons.
- Angles in a right angle
- Angles on a straight line
- Angles in a revolution
- Vertically opposite angle

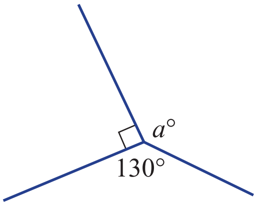
**g**



**h**

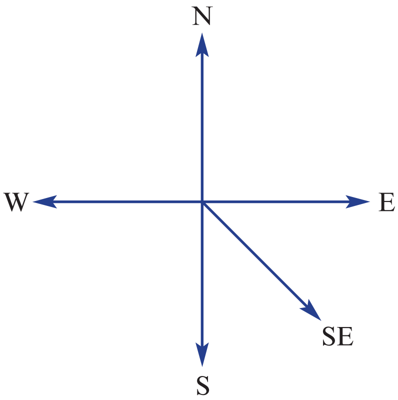


**i**



**8** **a** What is the obtuse angle between north and south-east?

**b** What is the reflex angle between north and south-east?



**Problem-solving and Reasoning**

**9** A round birthday cake is cut into sectors for nine friends (including Jack) at Jack's birthday party. After the cake is cut there is no cake remaining. What will be the angle at the centre of the cake for Jack's piece if:

**a** everyone receives an equal share?

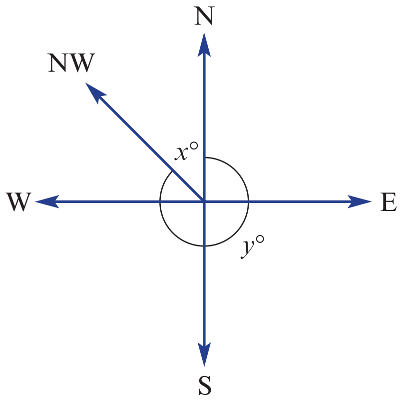
**b** Jack receives twice as much as everyone else? (In parts **b**, **c** and **d** assume his friends have equal shares of the rest.)

**c** Jack receives four times as much as everyone else?

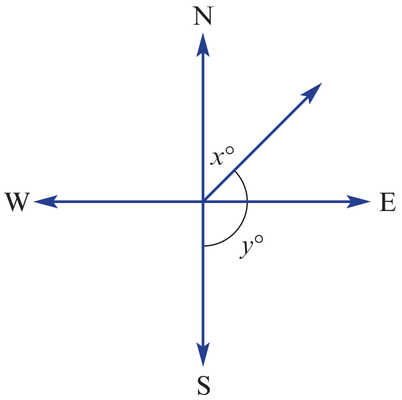
**d** Jack receives ten times as much as everyone else?

**10** Find the values of $x$ and $y$.

**a**

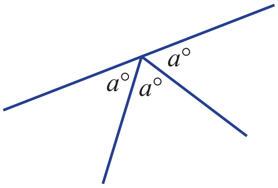


**b**

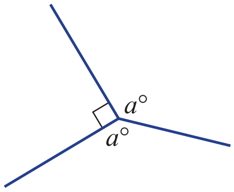


**11** Find the values of the pronumerals in these diagrams.

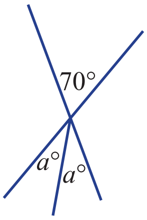
**a**



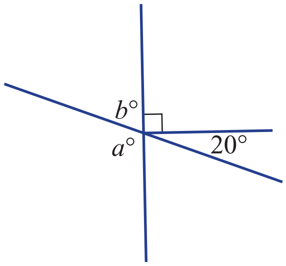
**b**



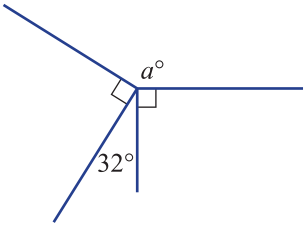
**c**



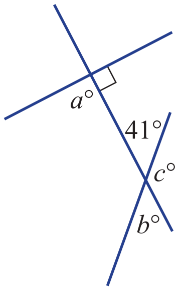
**d**



**e**

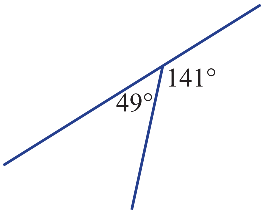


**f**

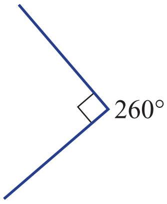


**12** Explain, with reasons, what is wrong with these diagrams.

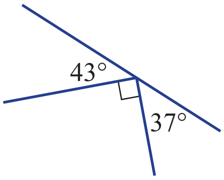
**a**



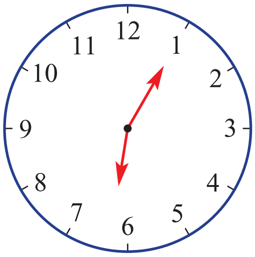
**b**



**c**



## Enrichment: Clock geometry



**13** Here is a clock face with an hour hand (short arrow) and minute hand (long arrow). The time shown is 6:05.

- **a** How many degrees does the hour hand turn in:
  - **i** 6 hours?
  - **ii** 12 hours?
  - **iii** 1 hour?
  - **iv** 3 hours?
- **b** How many degrees does the minute hand turn in:
  - **i** 1 hour?
  - **ii** 30 minutes?
  - **iii** 5 minutes?
  - **iv** 20 minutes?

**14** What is the smallest angle between the hour hand and minute hand on a clock at these times?

- **a** 3:00
- **b** 6:00
- **c** 1:00
- **d** 4:00

# 6B Transversal lines and parallel lines

REVISION

Parallel lines do not intersect or meet.

If two lines are parallel and are cut by a third line called a transversal, special pairs of angles are created.

Parallel lines never intersect.

## Let's start: How many *different* angles?

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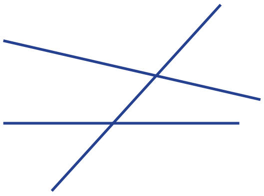
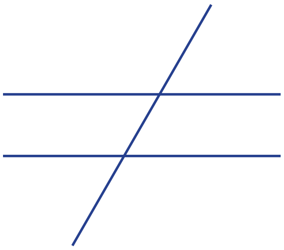
The diagram below shows a pair of lines crossed by a transversal.

- Carefully measure all eight angles with a protractor.
- How many different angles did you find?



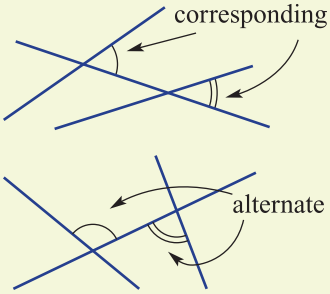
The diagram below shows a pair of *parallel* lines crossed by a transversal.

- Carefully measure all eight angles with a protractor.
- How many different angles did you find?



## Key ideas

- A **transversal** is a line cutting two or more other lines.
- When a transversal crosses two or more lines, pairs of angles can be:
  - **corresponding** (in corresponding positions)
  - **alternate** (on opposite sides of the transversal and inside the other two lines)



**Transversal** A line that cuts two or more other lines

**Corresponding** Angles in the same relative position between a transversal and an intersecting line

**Alternate** Angles on opposite sides of the transversal, inside the other two intersecting lines

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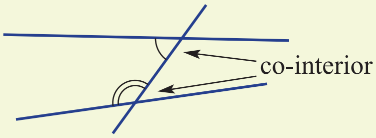
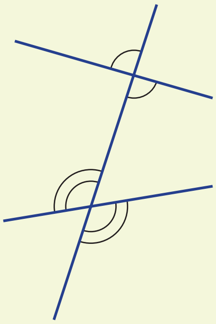
**Co-interior** Angles on the same side of the transversal and inside the other two intersecting lines

**Parallel** Two lines that are the same distance apart at every point and never meet

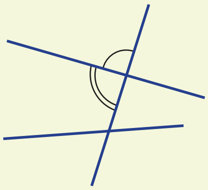
- **co-interior** (on the same side of the transversal and inside the other two lines)



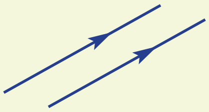
- vertically opposite



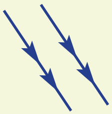
- angles on a straight line.



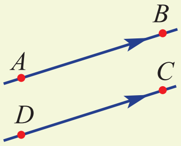
- Lines are **parallel** if they do not intersect.
  - Parallel lines are marked with the same number of arrows.



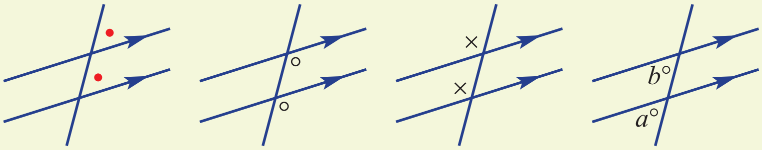
or



  - In the diagram below, it is acceptable to write $AB \parallel DC$ or $BA \parallel CD$ but *not* $AB \parallel CD$.

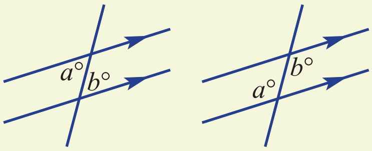


- If two parallel lines are cut by a transversal the:
  - corresponding angles are equal
    Note: There are four pairs of corresponding angles.



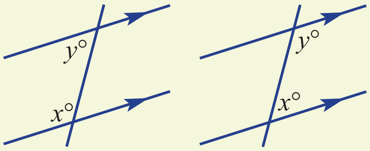
$a = b$

  - alternate angles are equal
    Note: There are two pairs of alternate angles.



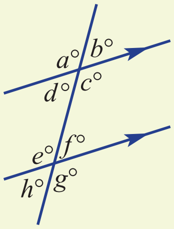
$a = b$

– co-interior angles are supplementary (sum to 180°).
Note: There are two pairs of co-interior angles.



$x + y = 180$

- The eight angles can be grouped in the following way:
  In this diagram:
  $a = c = e = g$
  $b = d = f = h$



## Exercise 6B

**Understanding**

**1** Two parallel lines are cut by a transversal. Write the missing word.
  **a** Corresponding angles are ________________.
  **b** Co-interior angles are ________________.
  **c** Alternate angles are ________________.

Choose from: **equal** or **supplementary**.

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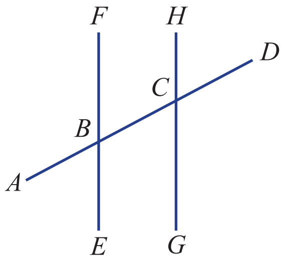
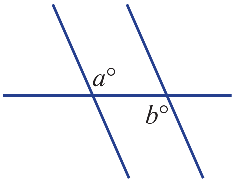
**2** Name the angle that is:
  **a** corresponding to $\angle ABF$
  **b** corresponding to $\angle BCG$
  **c** alternate to $\angle FBC$
  **d** alternate to $\angle CBE$
  **e** co-interior to $\angle HCB$
  **f** co-interior to $\angle EBC$
  **g** vertically opposite to $\angle ABE$
  **h** vertically opposite to $\angle HCB$.



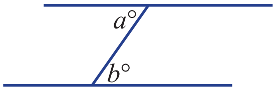
Name angles like this $\angle ABC$ or $\angle DEF$.

**3** State whether the following marked angles are corresponding, alternate or co-interior.

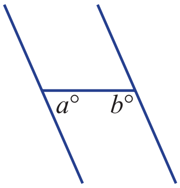
**a**



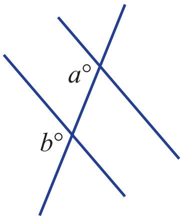
**b**



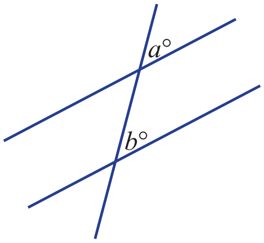
**c**



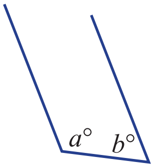
**d**



**e**



**f**



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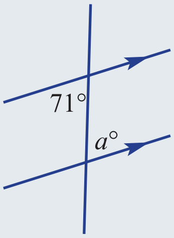
Fluency

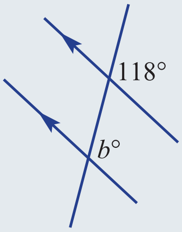
## Example 3 Working with parallel lines

Find the values of the pronumerals in these diagrams. Give a reason for each answer.

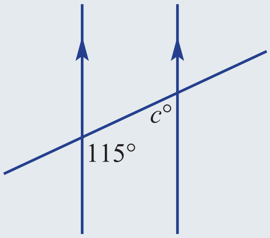
**a**



**b**



**c**

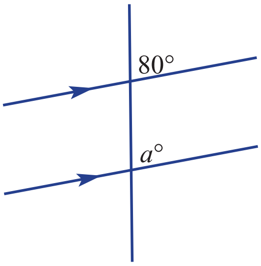


| Solution | Explanation |
|---|---|
| **a** $a = 71$ (alternate angles on parallel lines) | Alternate angles on parallel lines are equal. |
| **b** $b = 118$ (corresponding angles on parallel lines) | Corresponding angles on parallel lines are equal. |
| **c** $c + 115 = 180$ (co-interior angles on parallel lines)<br>$c = 180 - 115$<br>$c = 65$ | Co-interior angles on parallel lines are supplementary. |

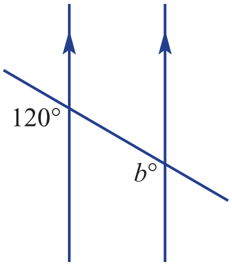
**4** Find the value of the pronumerals in these diagrams.
Give a reason for each answer from this list:

- corresponding angles on parallel lines
- alternate angles on parallel lines
- co-interior angles on parallel lines.

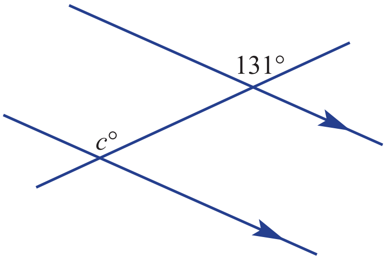
**a**



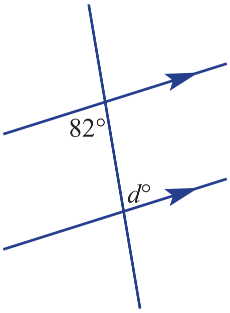
**b**



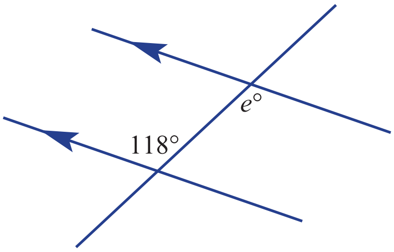
**c**



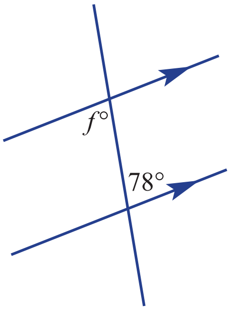
**d**



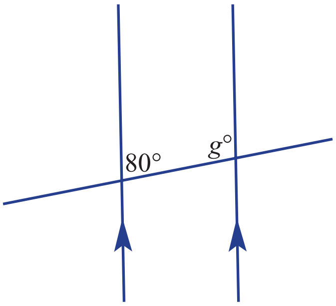
**e**



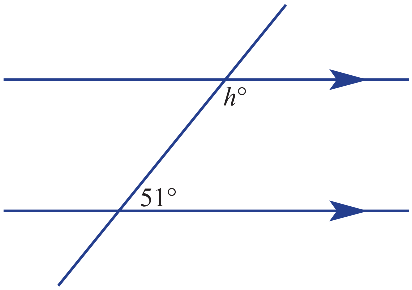
**f**



**g**

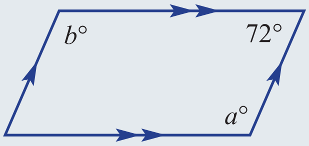


**h**



## Example 4 Using parallel lines in shapes

Find the values of the pronumerals in this diagram, stating reasons.



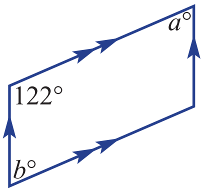
| **Solution** | **Explanation** |
|---|---|
| $a + 72 = 180$ (co-interior angles on parallel lines)<br>$a = 108$ | The pairs of angles are co-interior, which are supplementary because the lines are parallel. |
| $b + 72 = 180$ (co-interior angles on parallel lines)<br>$b = 108$ | |

**6B**

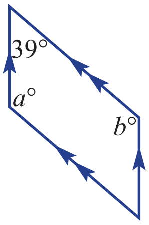
**Skillsheet 6B**

**5** Find the values of the pronumerals in these diagrams.

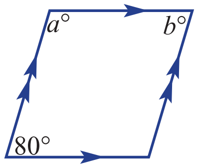
**a**



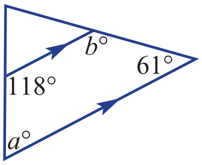
**b**



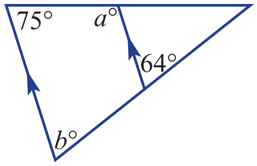
**c**



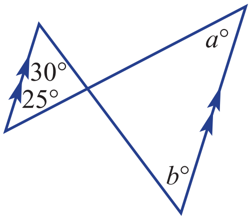
**d**



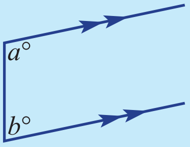
**e**



**f**



Co-interior angles add to 180°.

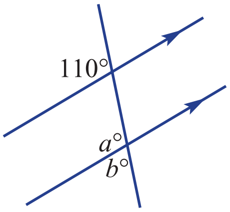


$a + b = 180$

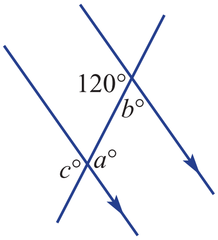
**Problem-solving and Reasoning**

**6** Find the values of the pronumerals in these diagrams, stating reasons.

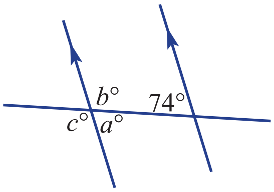
**a**



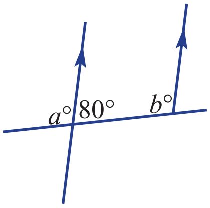
**b**



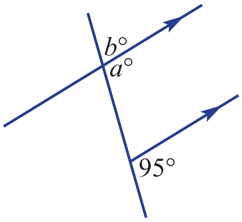
**c**



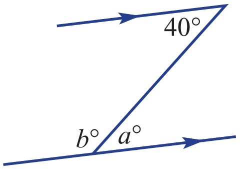
**d**



**e**

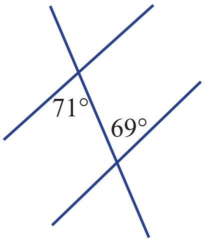


**f**

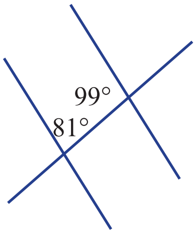


**7** Decide if the following diagrams include a pair of parallel lines. Give a reason for each answer.

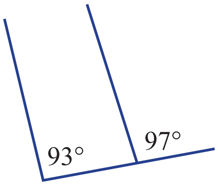
**a**



**b**



**c**



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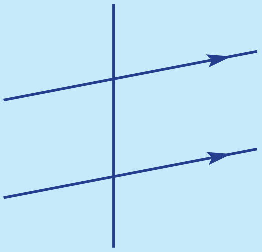
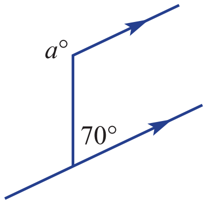
**8** Find the values of $a$ in these diagrams.
(Hint: It may be necessary to extend lines and fill in some of the unknown angles.)

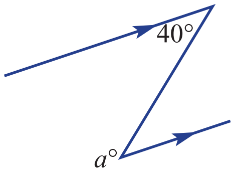
Extend lines so it is easier to see this type of diagram.



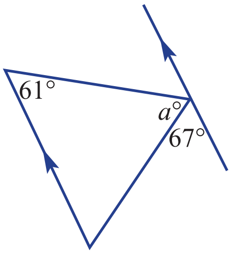
**a**



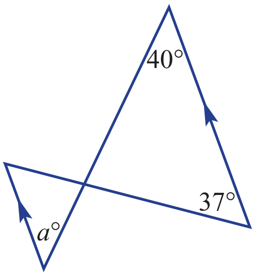
**b**



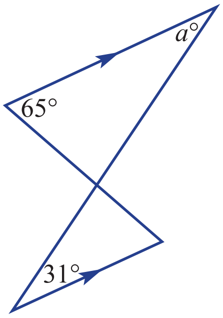
**c**



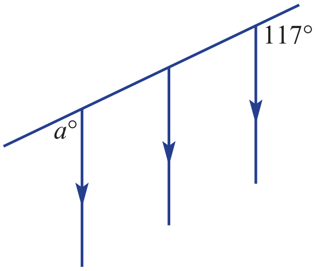
**d**



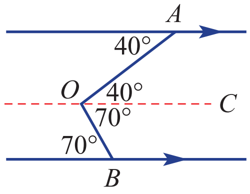
**e**



**f**

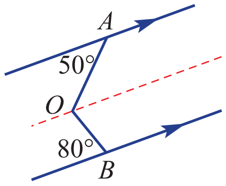


**9** Sometimes parallel lines can be added to a diagram to help find an unknown angle. For example, $\angle AOB$ can be found in this diagram by first drawing the dashed line and finding $\angle AOC$ (40°) and $\angle COB$ (70°). So $\angle AOB = 40° + 70° = 110°$.

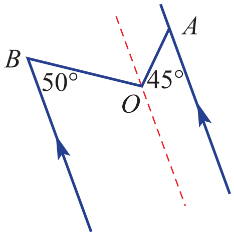


Apply a similar technique to find $\angle AOB$ in these diagrams.

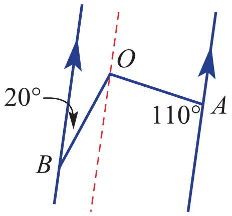
**a**



**b**



**c**



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6B

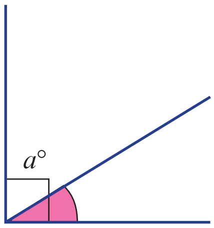
## Enrichment: How big are you?

**10** My measure is $a°$. How big are you?
The following diagrams all contain an angle that measures $a°$.
They also contain a shaded angle.
Choose, from the table below, a name for each shaded angle.

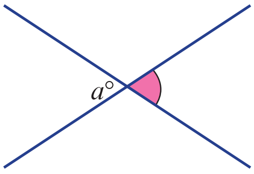
| | | | | | | |
|---|---|---|---|---|---|---|
| $a°$ | $(90 - a)°$ | $(180 - a)°$ | $(360 - a)°$ | $(90 + a)°$ | $(180 - 2a)°$ | $(2a)°$ |

Hint: You could make up a number for $a$ in every part.

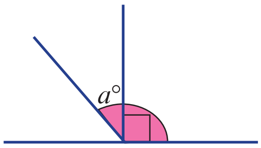
**a**



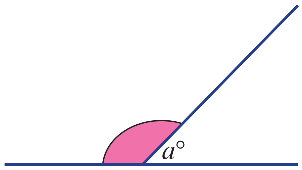
**b**



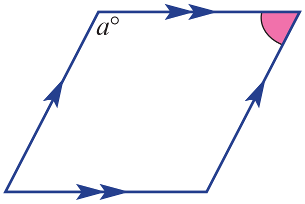
**c**



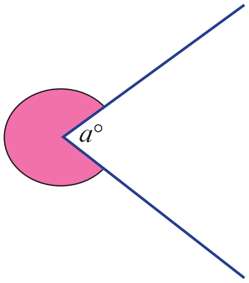
**d**



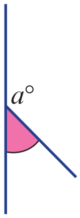
**e**



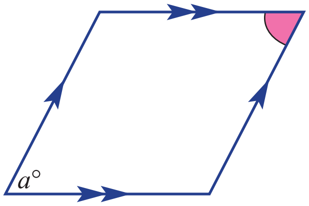
**f**



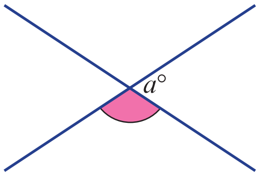
**g**



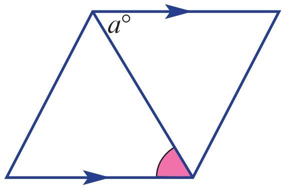
**h**



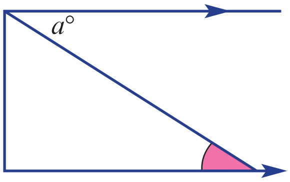
**i**



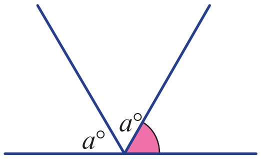
**j**



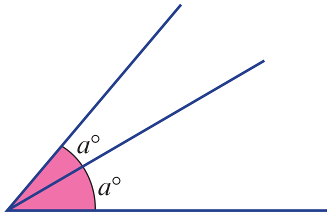
**k**



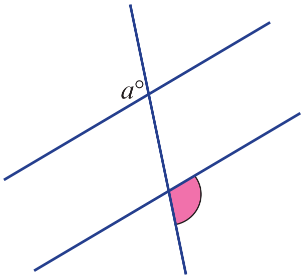
**l**



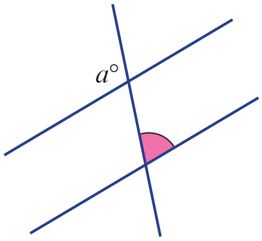
**m**



**n**



**o**



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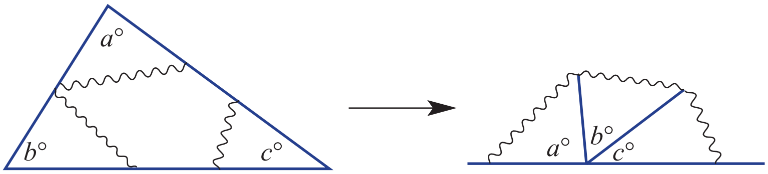
# 6C Triangles

A triangle is a polygon with three sides. The triangle is a very rigid shape and this leads to its use in the construction of houses and bridges. It is one of the most commonly used shapes in design and construction.

Triangular shapes are often used in architecture.

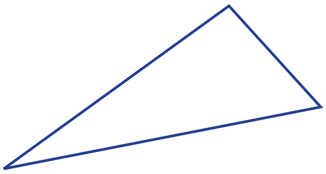
## Let's start: Investigating the angle sum of a triangle

Use a ruler to draw any triangle. Cut out the triangle and tear off the three corners. Then place the three corners together.



What do you notice and what does this tell you about the three angles in the triangle? Compare your results with those of others. Does this work for other triangles?

You can complete this task using a pencil and ruler or using dynamic geometry software.



- Draw a large triangle and measure each interior angle and all three sides.
- Add all three angles to find the angle sum of your triangle.
- Compare your angle sum with the results of others. What do you notice?
- Add the two shorter sides together and compare that sum to the longest side.
- Is the longest side opposite the largest angle?

If dynamic geometry is used, drag one of the vertices to alter the interior angles. Now check to see if your conclusions remain the same.

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## Key ideas

**Scalene triangle**
A triangle where all sides are different lengths and all angles are different

**Isosceles triangle**
A triangle where two sides have equal lengths and two angles are equal. The equal angles are the ones joining each same-length side to the third side.

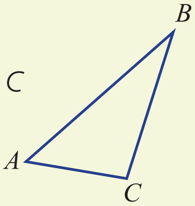
**Equilateral triangle**
A triangle where all angles and all sides are equal (all angles are 60°)

**Acute triangle**
A triangle where one of the interior angles is an acute angle (less than 90°)

**Right triangle**
A triangle where one of the interior angles is a right angle (90° exactly)

**Obtuse triangle**
A triangle where one of the interior angles is obtuse (more than 90° but less than 180°)

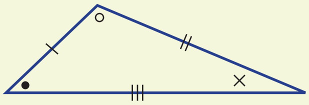
- A *triangle* has:
  - 3 sides: *AB*, *BC* and *AC*
  - 3 vertices (the plural of vertex): *A*, *B* and *C*
  - 3 interior angles.



This triangle could be called $\Delta ABC$ or $\Delta ACB$.

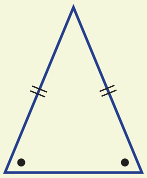
- Triangles classified by side lengths
  - Sides with the same number of dashes are of equal length.

**Scalene**



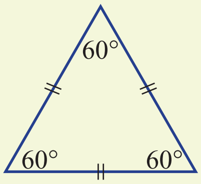
3 unequal sides
3 unequal angles

**Isosceles**



2 equal sides
2 equal angles
The equal sides are opposite the equal angles.

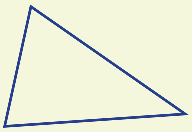
**Equilateral**



3 equal sides
3 equal $60^\circ$ angles

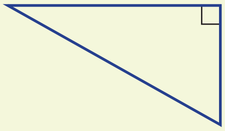
- Triangles classified by interior angles

**Acute**



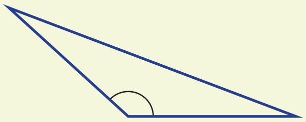
All angles acute

**Right**



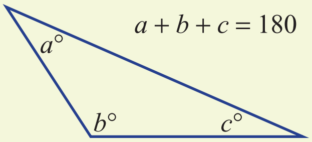
One right angle

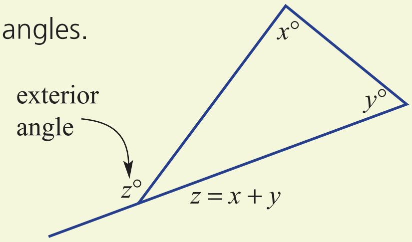
**Obtuse**



One obtuse angle

- The sum of the interior angles of a triangle is $180^\circ$.
- An exterior angle of a triangle is formed by extending one of the sides.
- The *exterior angle theorem*:
  The exterior angle of a triangle is equal to the sum of the two opposite interior angles.





## Exercise 6C

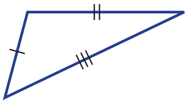
**Understanding**

**1** Give the common name of a triangle with these properties.

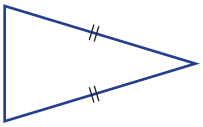
| | |
|---|---|
| **a** One right angle | **b** Two equal side lengths |
| **c** All angles acute | **d** All angles 60° |
| **e** One obtuse angle | **f** Three equal side lengths |
| **g** Two equal angles | **h** Three different side lengths |

**2** State whether these triangles are scalene, isosceles or equilateral.

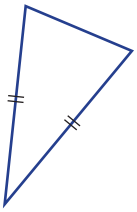
**a**



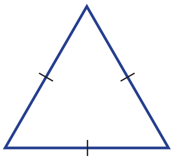
**b**



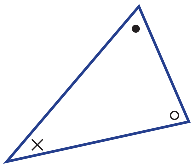
**c**



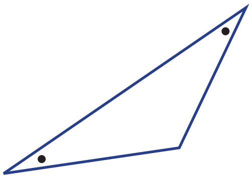
**d**



**e**



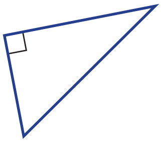
**f**



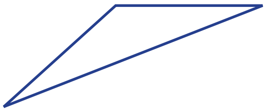
**Drilling for Gold 6C1**

**3** State whether these triangles are acute, right or obtuse.

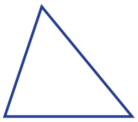
**a**



**b**



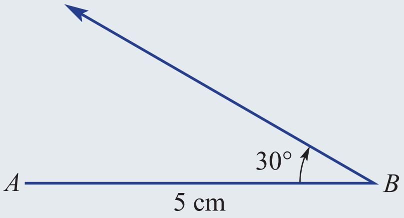
**c**



### Example 5 Drawing triangles

Draw a triangle $ABC$ with $AB = 5$ cm, $\angle ABC = 30°$ and $\angle BAC = 45°$.

**Solution**



**Explanation**

First, measure and draw line segment $AB$. Then use a protractor to form the angle 30° at point $B$.

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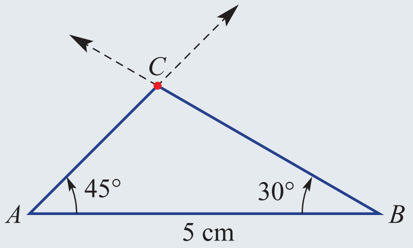
**Solution**



**Explanation**

Next, use a protractor to form the angle 45° at point $A$.
Mark point $C$ and join with $A$ and $B$.

Drilling for Gold 6C2

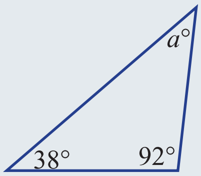
**4** Use a protractor and ruler to draw:

- **a** triangle $ABC$ with $AB = 5$ cm, $\angle ABC = 40°$ and $\angle BAC = 30°$
- **b** triangle $DEF$ with $DE = 6$ cm, $\angle DEF = 50°$ and $\angle EDF = 25°$
- **c** triangle $GHI$ with $GH = 4$ cm, $\angle GHI = 70°$ and $\angle HGI = 50°$.

**Fluency**

## Example 6 Using the angle sum of triangle

Find the value of $a$ in this triangle.



**Solution**

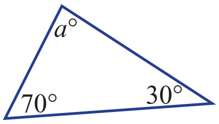
$$\begin{aligned} a + 38 + 92 &= 180 \\ a + 130 &= 180 \\ a &= 50 \end{aligned}$$

**Explanation**

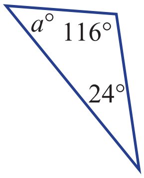
The angle sum of the three interior angles of a triangle is 180°. Also $38 + 92 = 130$ and $180 - 130 = 50$.

**5** Use the angle sum of a triangle to help find the unknown angle in these triangles.

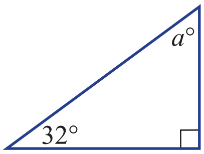
**a**



**b**

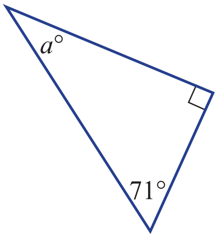


**c**

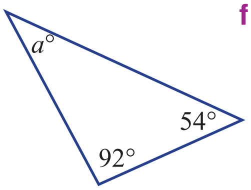


For each one start with an equation like $a + 36 + 48 = 180$. Then find the value of $a$.

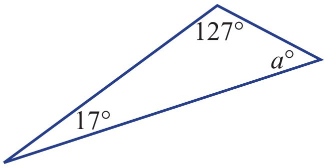
**d**



**e**



**f**



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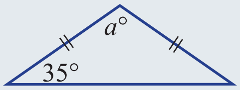
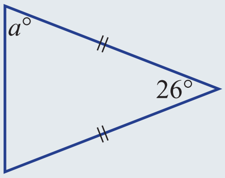
## Example 7 Working with isosceles triangles

Find the value of $a$ in these isosceles triangles.

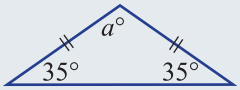
**a**

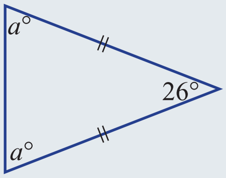


**b**



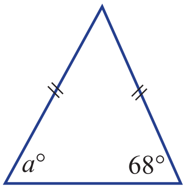
| **Solution** | **Explanation** |
|---|---|
| **a** $a + 35 + 35 = 180$<br>$a + 70 = 180$<br>$a = 110$ | The two base angles are equal. |
| **b** $2a + 26 = 180$<br>$2a = 154$<br>$a = 77$ | The two base angles in an isosceles triangle are equal. |



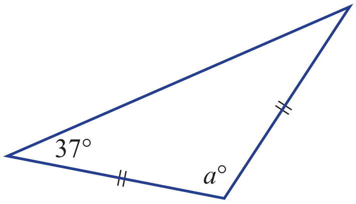


**6** These triangles are isosceles. Find the value of $a$.

**a**

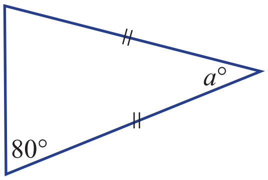


**b**

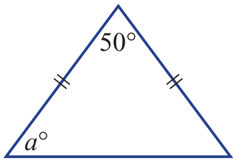


The two base angles in an isosceles triangle are equal.

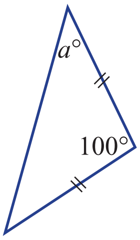
**c**



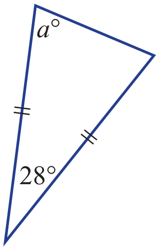
**d**



**e**



**f**



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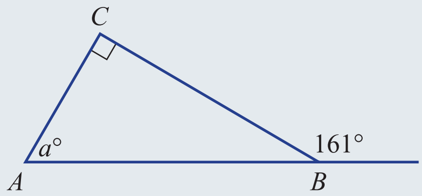
## Example 8 Using the exterior angle theorem

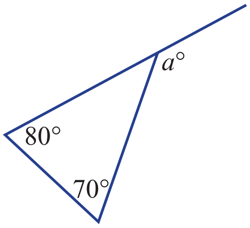
Find the value of $a$.



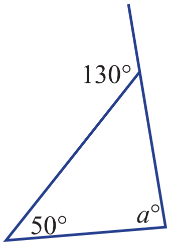
| **Solution** | **Explanation** |
|---|---|
| $a + 90 = 161$<br>$a = 161 - 90$<br>$a = 71$ | Use the exterior angle theorem for a triangle.<br>The exterior angle (161°) is equal to the sum of the two opposite interior angles. |

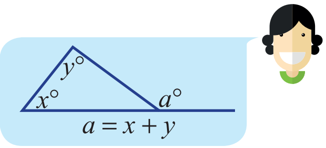
**7** Find the value of $a$.

**a**

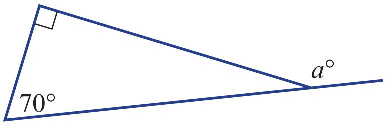


**b**

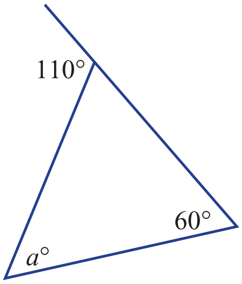




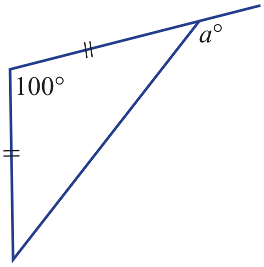
**c**



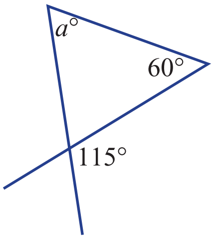
**d**



**e**



**f**



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## Problem-solving and Reasoning

**8** Decide if it is possible to draw a triangle with the given description. Draw a diagram to support your answer.

- **a** A scalene right-angled triangle
- **b** An equilateral obtuse-angled triangle
- **c** An isosceles right-angled triangle
- **d** An isosceles acute-angled triangle
- **e** An equilateral acute-angled triangle
- **f** An isosceles obtuse-angled triangle
- **g** A scalene acute-angled triangle

**9** Try to draw an example of a triangle that fits the triangle type in both the row and column. Are there any cells in the table for which it is impossible to draw a triangle?

Each triangle must suit both the column (scalene, isosceles or equilateral) *and* the row (acute, right or obtuse).

| Triangles | scalene | isosceles | equilateral |
|---|---|---|---|
| acute | | | |
| right | | | |
| obtuse | | | |

**10** Use a protractor and ruler to draw a:

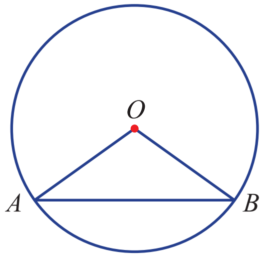
- **a** triangle $ABC$ with $AB = 5$ cm, $\angle ABC = 35^\circ$ and $BC = 4$ cm
- **b** triangle $DEF$ with $AB = 4$ cm, $\angle CAB = 90^\circ$ and $CA = 3$ cm
- **c** triangle $GHI$ with $HI = 4$ cm, $\angle GHI = 55^\circ$ and $GH = 3.5$ cm.

**11** Try drawing triangles with the following characteristics. Then explain why they are impossible.

- **a** Three sides: 4 cm, 3 cm and 9 cm
- **b** Three angles: $70^\circ$, $80^\circ$ and $60^\circ$
- **c** Two obtuse angles

**12** A triangle is constructed using a circle and two radius lengths.

- **a** What type of triangle is $\Delta AOB$ and why?
- **b** Name two angles that are equal.
- **c** Find $\angle ABO$ if $\angle BAO$ is $30^\circ$.
- **d** Find $\angle AOB$ if $\angle OAB$ is $36^\circ$.
- **e** Find $\angle ABO$ if $\angle AOB$ is $100^\circ$.



What can you say about the lengths $OA$ and $OB$?

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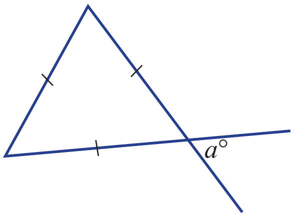
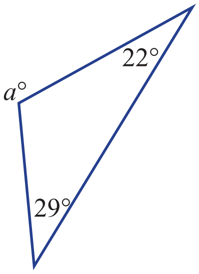
**13** Find the value of $a$ in these diagrams.

What is the size of all the angles in an equilateral triangle?

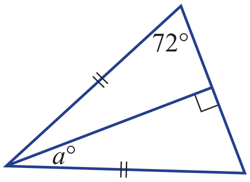
**a**



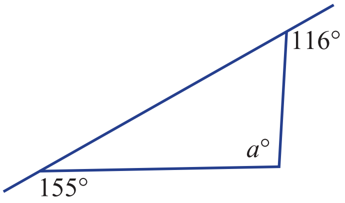
**b**



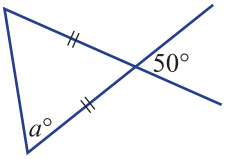
**c**



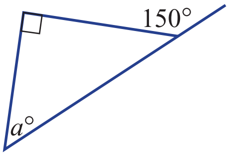
**d**



**e**

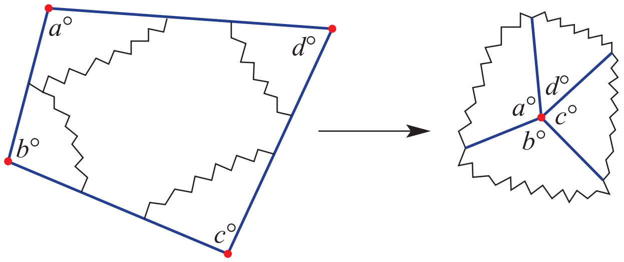


**f**



## Enrichment: What about quadrilaterals?

**14** Use a ruler to draw any convex quadrilateral with four different sides longer than 10 cm. Label the four angles $a$, $b$, $c$, $d$ as in the diagram. Cut it out and tear off the corners. Arrange them to meet at a point.



**a** What does the arrangement tell you about the angles inside a quadrilateral?

**b** Compare your results with those of others in the class.

**c** Now draw a non-convex quadrilateral and tear off the corners. Arrange them to meet at a point. What do you see?

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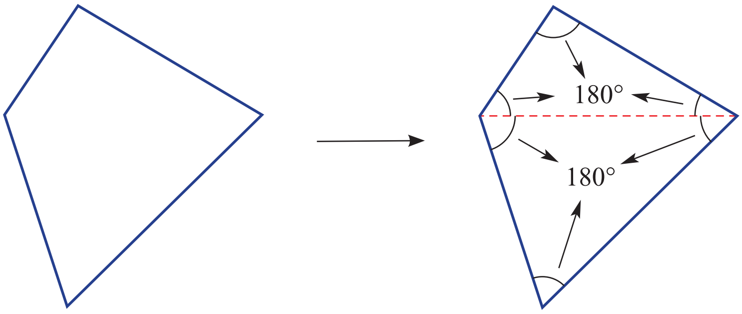
# 6D Quadrilaterals

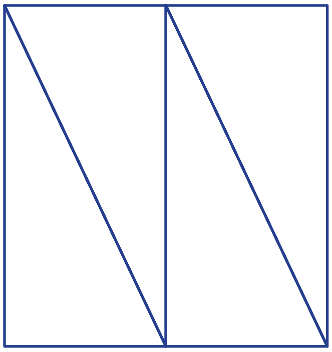
Quadrilaterals are four-sided polygons. All quadrilaterals can be divided into two triangles. Since the six angles inside the two triangles make up the four angles of the quadrilateral, the angle sum is $2 \times 180^\circ = 360^\circ$.



## Let’s start: Making special quadrilaterals from triangles

There are six special quadrilaterals described in the Key ideas section on the next page.

Start with a square piece of paper, then fold it in half and cut it into four identical triangles like this:



- Use four triangles to make a trapezium.
- Use three triangles to make a trapezium.
- Use two triangles to make a kite.
- Use two triangles to make a parallelogram.
- Use two triangles to make a rectangle.
- Use four triangles to make a rectangle.
- Use four triangles to make a rhombus.

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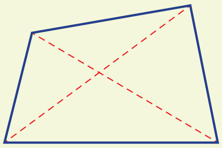
## Key ideas

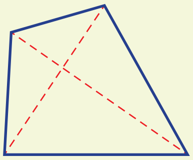
**Quadrilateral**
A two-dimensional shape with four joined edges of any lengths

- Every quadrilateral has two diagonals.
  - In some quadrilaterals the diagonals bisect each other (i.e. cut each other in half)



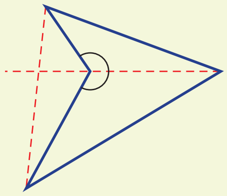
- **Quadrilaterals** can be convex or non-convex.
  - Convex quadrilaterals have all vertices pointing outwards.
  - Non-convex quadrilaterals have one vertex pointing inwards.
  - Both diagonals of convex quadrilaterals lie inside the figure.

Convex



All interior angles less than 180°

Non-convex



One reflex interior angle

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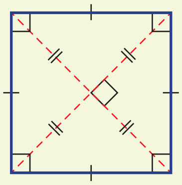
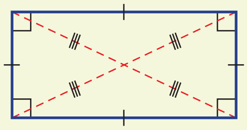
- Special quadrilaterals

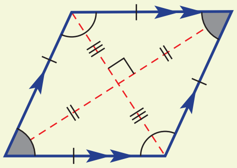
Square



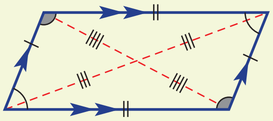
Rectangle



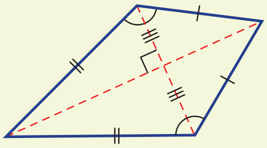
Rhombus



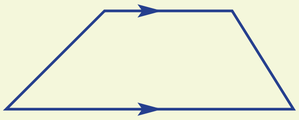
Parallelogram



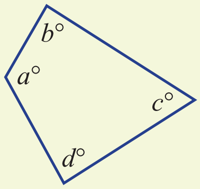
Kite

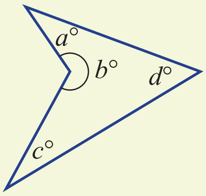


Trapezium



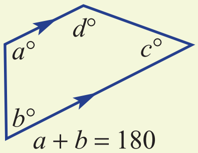
- The angle sum of any quadrilateral is 360°.





$$a + b + c + d = 360$$

- Quadrilaterals with parallel sides contain two pairs of co-interior angles.



$$c + d = 180$$

$$a + b = 180$$

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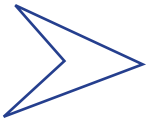
# Exercise 6D

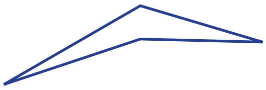
**Understanding**

**1** Decide if these quadrilaterals are convex or non-convex.

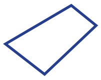
**a**



**b**

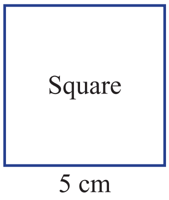


**c**

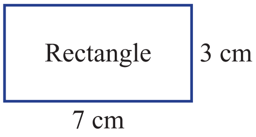


**2** Use a ruler and protractor to make a neat and accurate drawing of these special quadrilaterals.

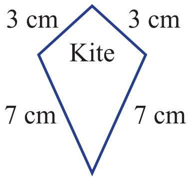
**a**



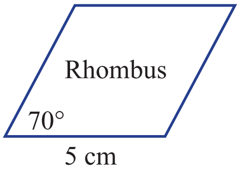
**b**



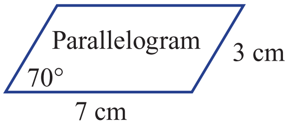
**c**



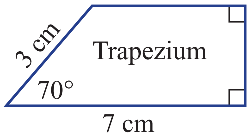
**d**



**e**



**f**



Use your shapes to write YES in the cells in the table, for the statements that are definitely true.

| Property | Trapezium | Kite | Parallelogram | Rectangle | Rhombus | Square |
|---|---|---|---|---|---|---|
| The opposite sides are parallel. | | | | | | |
| All sides are equal. | | | | | | |
| The adjacent sides are perpendicular. | | | | | | |
| The opposite sides are equal. | | | | | | |
| The diagonals are equal. | | | | | | |

| Property | Trapezium | Kite | Parallelogram | Rectangle | Rhombus | Square |
|---|---|---|---|---|---|---|
| The diagonals bisect each other. | | | | | | |
| The diagonals bisect each other at right angles. | | | | | | |
| The diagonals bisect the angles of the quadrilateral. | | | | | | |

**3** Write the missing number or word.

**a** The angle sum of a quadrilateral is ________________.

**b** The side lengths of a rhombus are ________________ in length.

**c** A kite has ________________ pairs of equal sides.

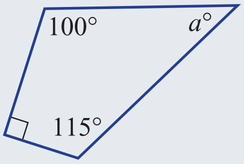
**d** The diagonals of squares, rhombuses and kites intersect at ________________.

**Fluency**

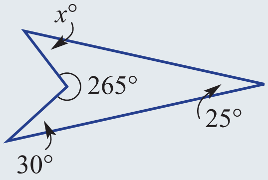
## Example 9 Using the angle sum of a quadrilateral

Find the value of the pronumerals in these quadrilaterals.

**a**



**b**



| Solution | Explanation |
|---|---|
| **a** $a + 100 + 90 + 115 = 360$<br>$a + 305 = 360$<br>$a = 360 - 305$<br>$a = 55$ | The sum of angles in a quadrilateral is 360°.<br>Simplify then solve for $a$. |
| **b** $x + 265 + 30 + 25 = 360$<br>$x + 320 = 360$<br>$x = 360 - 320$<br>$x = 40$ | Use the angle sum of a quadrilateral. |

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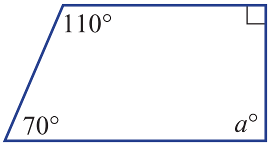
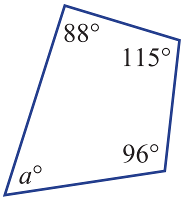
**4** Use the quadrilateral angle sum to find the value of $a$ in these quadrilaterals.

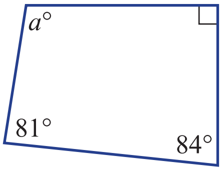
**a**



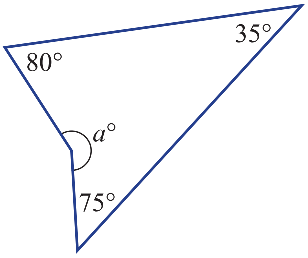
**b**



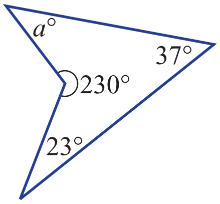
**c**



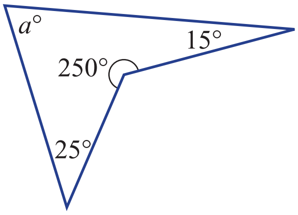
**d**



**e**

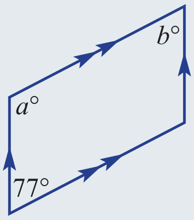


**f**



## Example 10 Working with parallelograms

Find the value of $a$ and $b$ in this parallelogram.



| **Solution** | **Explanation** |
|---|---|
| $a + 77 = 180$<br>$a = 103$ | Two angles inside parallel lines are co-interior and therefore sum to 180°. |
| $b = 180 - 103 = 77$ | Opposite angles in a parallelogram are equal. |

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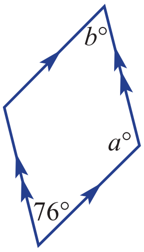
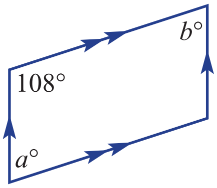
**5** Find the value of the pronumerals in these quadrilaterals.

**a**

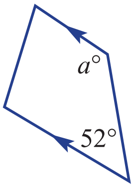


**b**

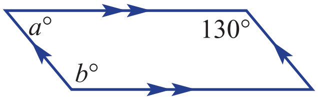


Opposite angles in a parallelogram are equal. Other pairs of angles are co-interior (add to 180°).

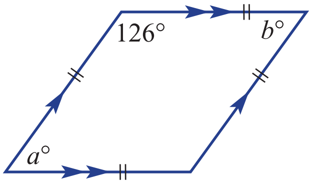
**c**



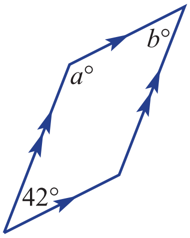
**d**



**e**



**f**



**Problem-solving and Reasoning**

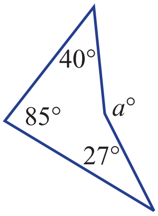
**6** Name all the special quadrilaterals that definitely have:

**a** all sides of equal length
**b** two pairs of parallel sides
**c** opposite of equal sides
**d** diagonals meeting at right angles
**e** diagonals of equal length
**f** four right angles
**g** two pairs of equal opposite angles
**h** diagonals that bisect each other.

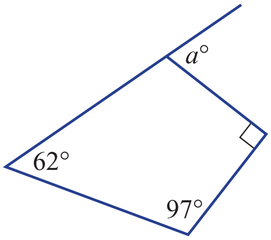
- Trapezium
- Parallelogram
- Rhombus
- Rectangle
- Kite
- Square

**7** Use your knowledge of geometry from the previous sections to find the values of $a$.

**a**

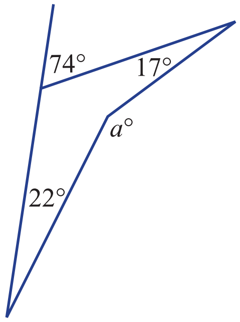


**b**

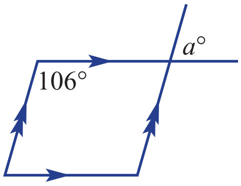


If may be helpful to fill in some of the unknown angles

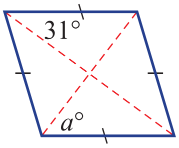
**c**



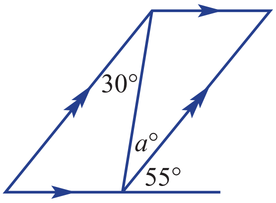
**d**



**e**



**f**



**8** Consider the properties of special quadrilaterals. Decide if the following are always true.

**a** A square is a type of rectangle.
**b** A rectangle is a type of square.
**c** A square is a type of rhombus.
**d** A rectangle is a type of parallelogram.
**e** A parallelogram is a type of square.
**f** A rhombus is a type of parallelogram.

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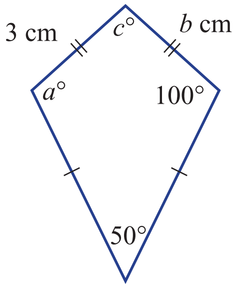
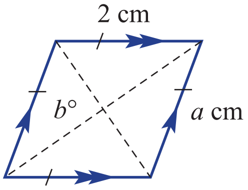
**9** Consider the properties of the given quadrilaterals. Give the values of the pronumerals.

**a** A kite
**b** A rhombus
**c** A parallelogram

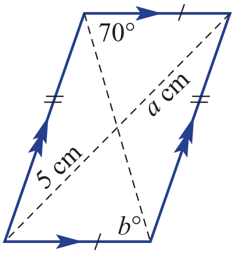
**a**



**b**

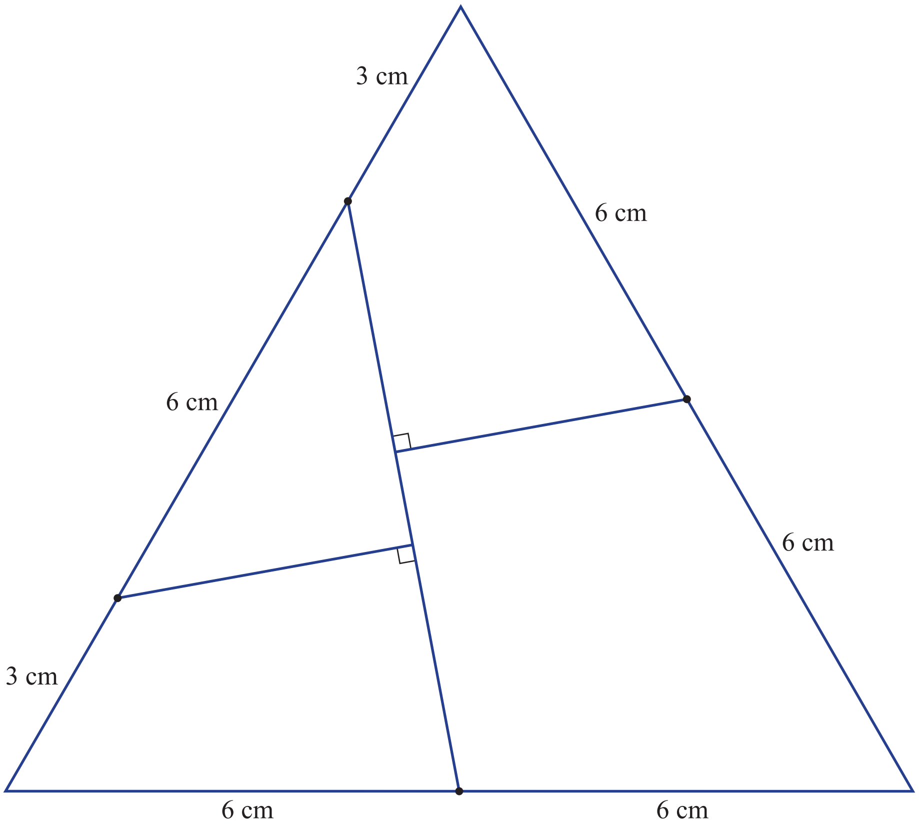


**c**



## Enrichment: A triangle makes a square!

**10** Photocopy or print the triangle on a piece of paper and cut it into the four pieces shown. Can you form a square with all four pieces?



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# 6E Polygons

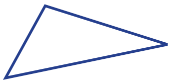
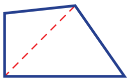
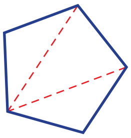
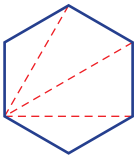
The word 'polygon' comes from the Greek words *poly*, meaning 'many', and *gonia*, meaning 'angles'. The number of interior angles equals the number of sides.

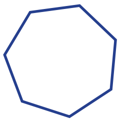
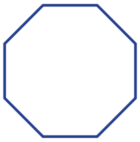
The Pentagon is a famous government office building in Washington, USA.

## Let's start: Developing the rule

Drilling for Gold 6E1

The following procedure uses the fact that the angle sum of a triangle is 180°. Complete the table and try to write the general rule in the final row.

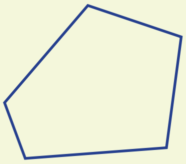
| Shape | Number of sides | Number of triangles | Angle sum |
|---|---|---|---|
| Triangle  | 3 | 1 | $1 \times 180^\circ = 180^\circ$ |
| Quadrilateral  | 4 | 2 | $\_\_\_\_ \times 180^\circ = \_\_\_\_$ |
| Pentagon  | 5 | | |
| Hexagon  | 6 | | |

| Shape | Number of sides | Number of triangles | Angle sum |
|---|---|---|---|
| Heptagon  | 7 | | |
| Octagon  | 8 | | |
| $n$-sided polygon | $n$ | | ( ____ ) × 180° |

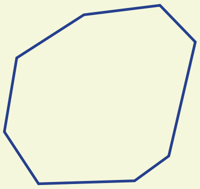
## Key ideas

**Polygon** A plane figure bounded by line segments

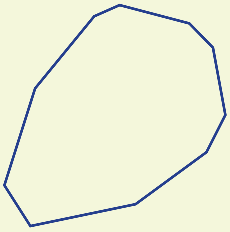
- **Polygons** are shapes with straight sides.



Pentagon



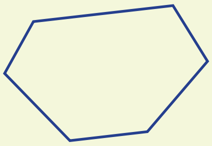
Octagon



Decagon

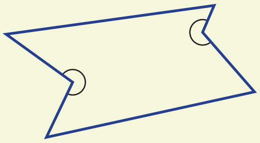
- **Polygons** can be convex or non-convex.
  - Convex polygons have all vertices pointing outwards.
  - Non-convex polygons have at least one vertex pointing inwards.

Convex



This means all interior angles are less than 180°.

Non-convex



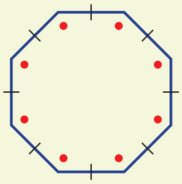
This means there is one or more reflex interior angle.

- Polygons are named according to their number of sides.

| Number of sides | Name |
|---|---|
| 3 | Triangle |
| 4 | Quadrilateral |
| 5 | Pentagon |
| 6 | Hexagon |
| 7 | Heptagon |
| 8 | Octagon |
| 9 | Nonagon |
| 10 | Decagon |
| 11 | Undecagon |
| 12 | Dodecagon |

**Regular polygon** A polygon with all sides equal and all angles equal

- The angle sum $S$ of a polygon with $n$ sides is given by the rule: $S = (n - 2) \times 180^\circ$.
- A **regular polygon** has sides of equal length and equal interior angles.



A regular octagon

## Exercise 6E

**Understanding**

**1** Name the polygons with the following number of sides.
**a** 7 **b** 3 **c** 8 **d** 9
**e** 12 **f** 10 **g** 4 **h** 11

**2** State the number of sides on these polygons.
**a** Hexagon **b** Quadrilateral **c** Decagon
**d** Heptagon **e** Pentagon **f** Dodecagon

**3** Evaluate $(n - 2) \times 180^\circ$ if:
**a** $n = 6$ **b** $n = 10$ **c** $n = 22$

**4** What is the common name given to these polygons?
**a** Regular quadrilateral **b** Regular triangle

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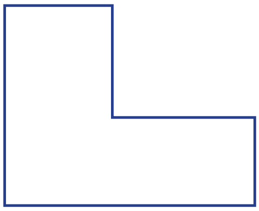
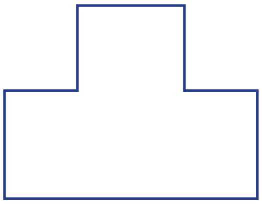
**5** Name these polygons.

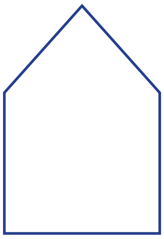
**a**



**b**



**c**



**Fluency**

## Example 11 Finding the angle sum

Find the angle sum of a heptagon.

| **Solution** | **Explanation** |
|---|---|
| $S = (n - 2) \times 180^\circ$<br>$= (7 - 2) \times 180^\circ$<br>$= 5 \times 180^\circ$<br>$= 900^\circ$ | A heptagon has 7 sides so $n = 7$.<br>Simplify $(7 - 2)$ before multiplying by $180^\circ$. |

**6** Find the angle sum of these polygons.

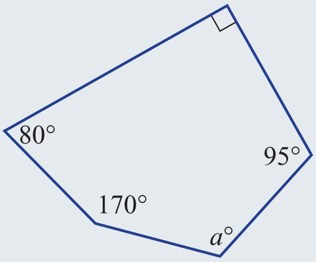
**a** Pentagon ($n = 5$)
**b** Octagon ($n = 8$)
**c** Decagon ($n = 10$)
**d** Hexagon
**e** Nonagon
**f** Heptagon

Use $S = (n - 2) \times 180$

## Example 12 Finding angles in polygons

Find the value of $a$ in this pentagon by using the given angle sum.

Angle sum = 540°

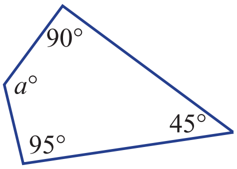


| **Solution** | **Explanation** |
|---|---|
| $a + 170 + 80 + 90 + 95 = 540$<br>$a + 435 = 540$<br>$a = 540 - 435$<br>$a = 105$ | Sum all the angles and set this equal to the angle sum of 540°. Then simplify and solve for $a$. |

**7** Find the value of $a$ in these polygons, by using the given angle sum.

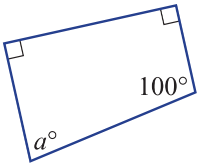
Write an equation using the given angle sum, then find the value of $a$.

**a**



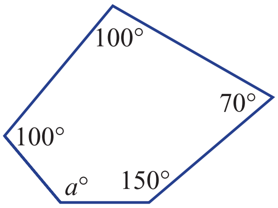
Angle sum = 360°

**b**



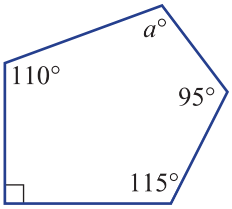
Angle sum = 360°

**c**



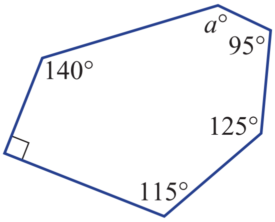
Angle sum = 540°

**d**



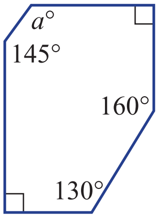
Angle sum = 540°

**e**



Angle sum = 720°

**f**

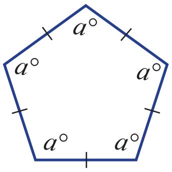


Angle sum = 720°

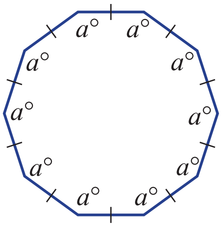
**8** Regular polygons have equal interior angles. Find the size of an interior angle for these regular polygons with the given angle sum.

First find the angle sum then divide by the number of sides.

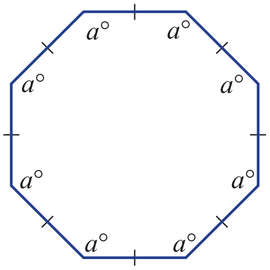
**a** Pentagon (540°)



**b** Decagon (1440°)



**c** Octagon (1080°)



## Example 13 Finding interior angles of regular polygons

Find the size of an interior angle in a regular octagon by firstly finding the angle sum.

| Solution | Explanation |
|---|---|
| $S = (n - 2) \times 180°$<br>$= (8 - 2) \times 180°$<br>$= 6 \times 180° = 1080°$ | First calculate the angle sum of a octagon using $n = 8$ and $S = (n - 2) \times 180°$. |
| Angle size $= 1080 \div 8$<br>$= 135°$ | All 8 angles are equal in size so divide the angle sum by 8. |

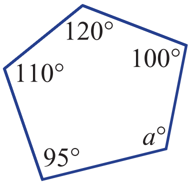
**9** Find the size of an interior angle of these regular polygons by firstly finding the angle sum. Round the answer to 1 decimal place where necessary. Use a calculator to help you.

**a** Regular pentagon **b** Regular heptagon **c** Regular hexagon
**d** Regular decagon **e** Regular octagon **f** Regular undecagon

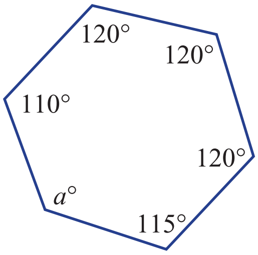
**Problem-solving and Reasoning**

**10** Find the value of $a$ in these shapes by firstly finding the angle sum.

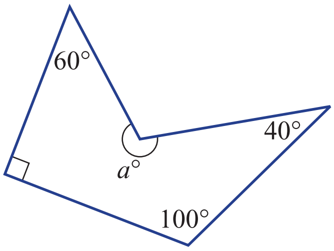
**a**



**b**



**c**



**11** Find the number of sides of a polygon with the given angle sums.

**a** 1260° **b** 2340° **c** 3420° **d** 29 700°

The angle sum rule is $S = (n - 2) \times 180°$

**12** Find the number of sides of a regular polygon if each interior angle is:

**a** 120° **b** 162° **c** 147.272 727…°

**13 a** A 50-cent piece has 12 equal sides and 12 equal angles. Calculate the size of each angle.

**b** A regular icosagon has 20 equal sides and 20 equal angles. Calculate the size of each angle.

**14 a** Draw a shape with four equal sides in which the angles are not equal.

**b** Repeat part **a** for a shape with 5 equal sides.

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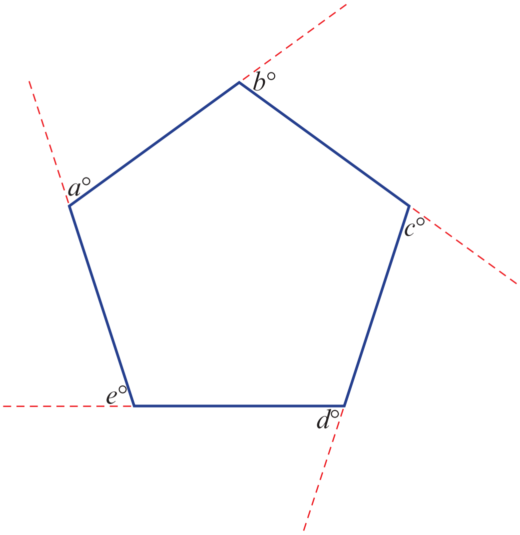
## Enrichment: Exterior angles

**15** On a blank sheet of A4 paper, use a ruler to draw a large convex pentagon with 5 unequal sides. Make each side more than 5 cm long, such as the one below.



Extend each side to make 5 exterior angles.
Measure the angles and add then together.
Did you get 360°? You should!
Try it again, this time with a convex hexagon. What is the sum? 360°?

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# 6F Line symmetry and rotational symmetry

The most familiar form of symmetry, called line symmetry, is connected to the idea of reflection. A flower, for example, might have one of more lines of symmetry.

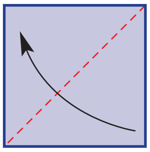
The flower might also have rotational symmetry, which will also be studied in this section.

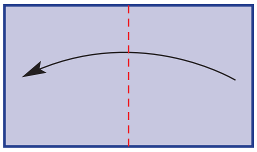
Is this flower perfectly symmetrical?

## Let's start: Working with symmetry

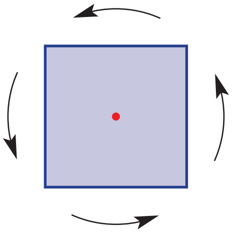
Cut out a paper square (with side lengths of about $10$ cm) and a rectangle (with length of about $15$ cm and breadth of about $10$ cm).

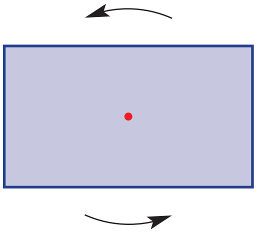
- How many ways can you fold each shape in half so that the two halves match exactly? The number of creases formed will be the number of lines of symmetry.





- Now locate the centre of each shape and place a sharp pencil on this point. Rotate the shape $360°$. How many times does the shape match its original position exactly? This number describes the rotational symmetry.





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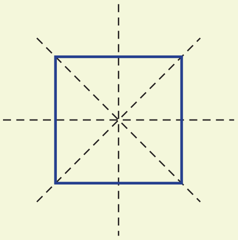
## Key ideas

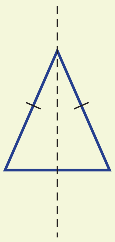
- An axis or **line of symmetry** divides a shape into two equal parts. It acts as a mirror line, with each half of the shape being a reflection of the other.

Square



4 lines of symmetry

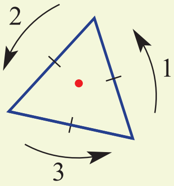
Isosceles triangle



1 line of symmetry

- The **order of rotational symmetry** is the number of times a shape matches its original position exactly during rotation of 360°.

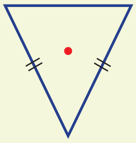
Equilateral triangle



rotational symmetry of order 3

  - We say that there is no **rotational symmetry** if the order is equal to 1.

Isosceles triangle



no rotational symmetry

**Line of symmetry** The line (axis) along which a figure could be folded to produce identical halves

**Order of rotational symmetry** The number of times a figure matches its original position during rotation of 360°

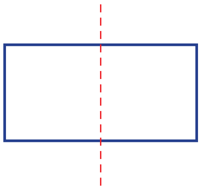
**Rotational symmetry** When a figure rotated less than 360° matches its original position

## Exercise 6F

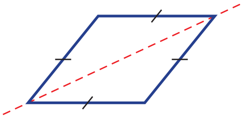
**Understanding**

**1** For each shape, decide if the dashed line is a line of symmetry.

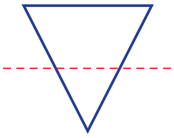
**a**



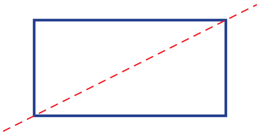
**b**



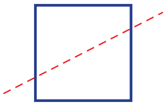
**c**



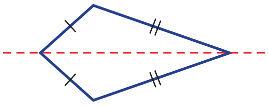
**d**



**e**

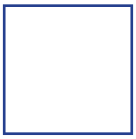


**f**



**2** In how many ways could you fold each of these shapes in half so that the two halves match exactly?

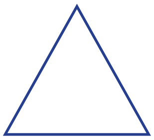
**a** Square



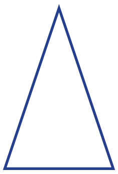
**b** Rectangle

Try cutting out similar shapes and folding them.

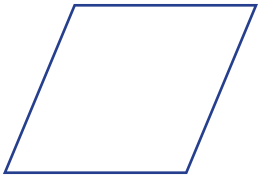
**c** Equilateral triangle



**d** Isosceles triangle

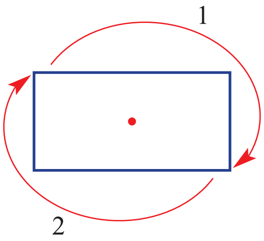


**e** Rhombus



**f** Parallelogram

**3** Look again at the shapes in Question **2**, and imagine rotating them $360^\circ$ about their centre. How many times would you get an exact match of the original position?



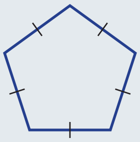
Fluency

## Example 14 Finding the symmetry of shapes

Find the number of lines of symmetry and the order of rotational symmetry for each of these shapes.

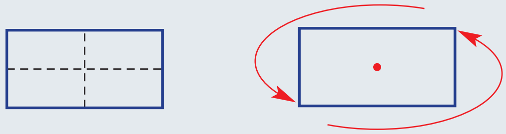
**a** Rectangle

**b** Regular pentagon

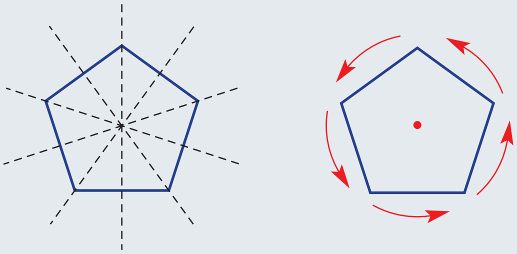


**Solution** **Explanation**

**a** 2 lines of symmetry and rotational symmetry of order 2



**b** 5 lines of symmetry and rotational symmetry of order 5

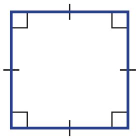


**Drilling for Gold 6F1**

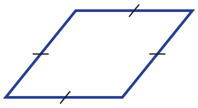
**4** State the number of lines of symmetry and the order of rotational symmetry for each shape.

The order of rotation is the number of exact 'matches' during a turn of 360°.

**a**

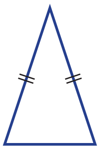


**b**

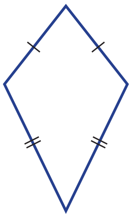


**c**

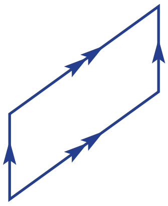
**d**



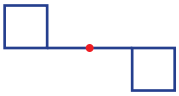
**e**



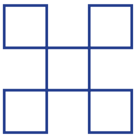
**f**



**g**



**h**



**i**

**5** List the quadrilaterals that have these properties.

**a** Number of lines of symmetry:

**i** 1 **ii** 2 **iii** 4

**b** Rotational symmetry of order:

**i** 1 **ii** 2 **iii** 4

Choose from: *square, rectangle, parallelogram, kite, trapezium, rhombus.*

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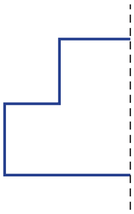
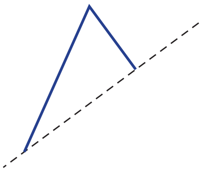
**6** Copy each shape and draw the other half for the given axis of symmetry.

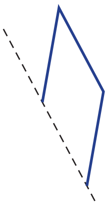
**a**



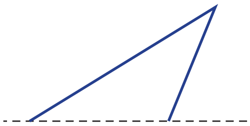
**b**



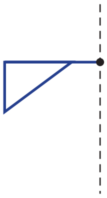
**c**



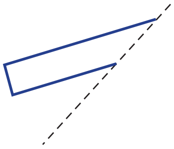
**d**



**e**



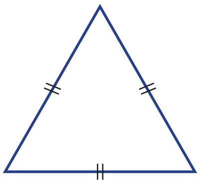
**f**



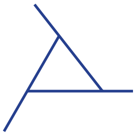
**Problem-solving and Reasoning**

**7** State the number of lines of symmetry and order of rotational symmetry for each of the following.

**a**

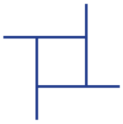


**b**

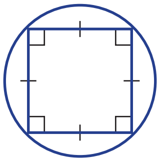


**c**

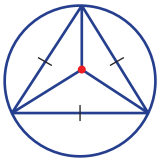
**d**



**e**



**f**



**8** Name a type of triangle that has:

**a** 3 lines of symmetry and order of rotational symmetry 3

**b** 1 line of symmetry and no rotational symmetry

**c** no line or rotational symmetry.

**9** Consider these capital letters.

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

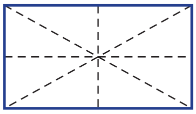
List the letters that have:

**a** 1 line of symmetry
**b** 2 lines of symmetry
**c** rotational symmetry of order 2.

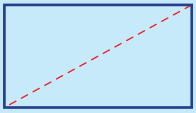
Think carefully about **K** and **Q**.

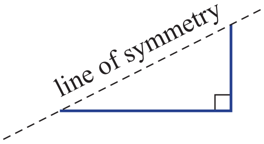
**10** Many people think a rectangle has four lines of symmetry, including the diagonals.

**a** Complete the other half of this diagram to show that this is not true.
**b** Using the same method as that used in part **a**, show that the diagonals of a parallelogram are not lines of symmetry.



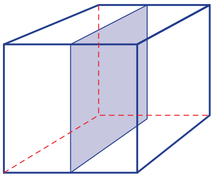
The answer for part **a** is *not*:





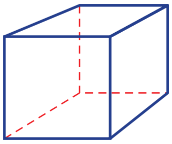
## Enrichment: Symmetry in 3D

**11** Some solid objects also have symmetry. Rather than line symmetry, they have plane symmetry. This cube shows one plane of symmetry, but there are more that could be drawn.

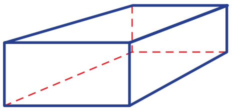


State the number of planes of symmetry for each of these solids.

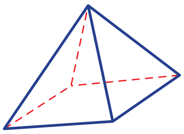
**a** Cube



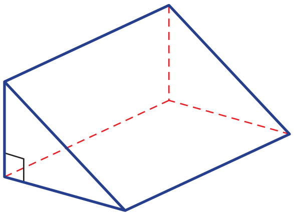
**b** Rectangular prism



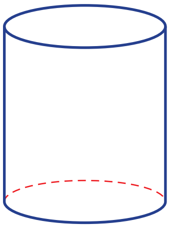
**c** Right square pyramid



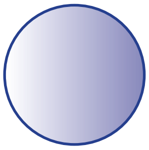
**d** Right triangular prism



**e** Cylinder



**f** Sphere



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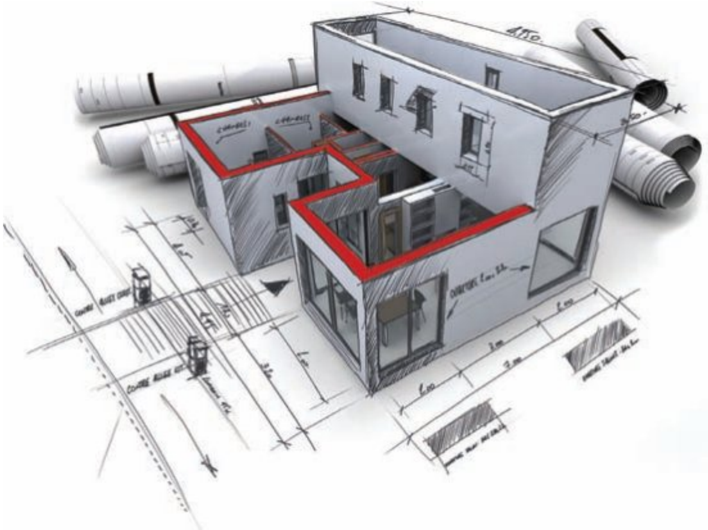
# 6G Drawing solids

Three-dimensional solids can be represented as a drawing on a two-dimensional surface (such as paper or a computer screen).

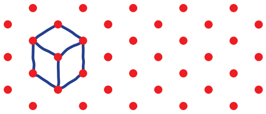


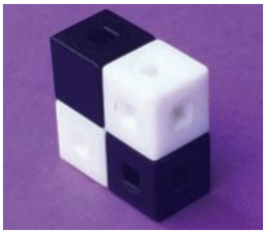
Architects create 3D models of building plans by hand or using a computer.

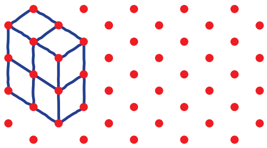
## Let’s start: Drawing solids on isometric dot paper

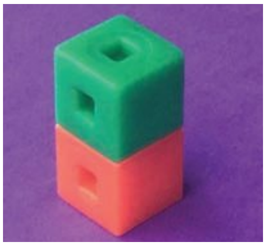
Isometric dot paper makes it easy to draw solids. Notice that in the drawings below, there are some vertical lines but no horizontal lines.

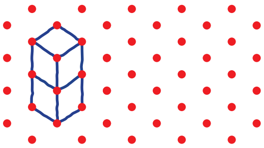
- The photos below have been drawn. Try drawing them with a pencil. It is a good idea to start at either the highest point or the lowest point.

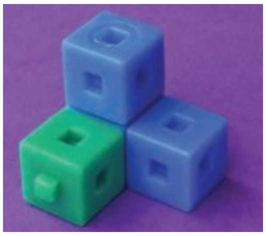


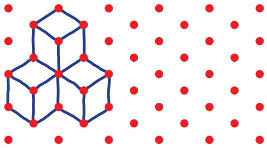






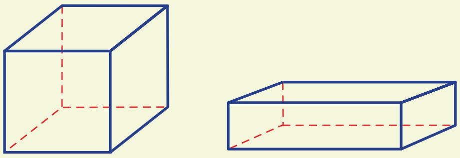




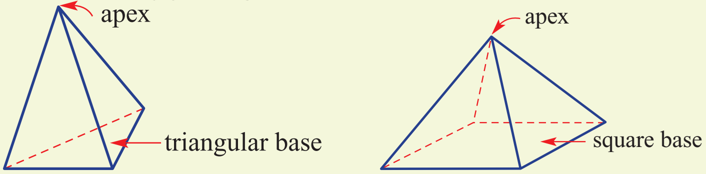


## Key ideas

- **Cubes** and rectangular **prisms** can be drawn by keeping:
  - parallel edges pointing in the same direction
  - parallel edges the same length.

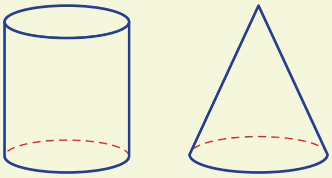


- Draw **pyramids** by joining the apex with the vertices on the base.



triangular pyramid (tetrahedron) square pyramid

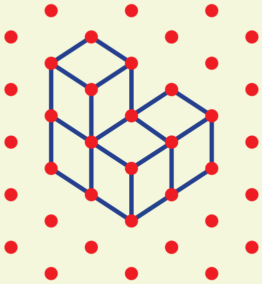
- In the above drawings, note the way in which broken lines are used for the sides that are at the back of the object.
- Draw **cylinders** and **cones** by using an oval shape for the circular faces.



cylinder cone

- Isometric dot paper can help you to draw solids accurately. Drawings made on isometric dot paper clearly show the cubes that make up the solid.

Isometric dot paper



- When using isometric dot paper:
  - no horizontal lines are drawn
  - only draw the visible edges
  - it is best to start from the highest point or the lowest point.

**Cube** A solid with six square faces where all edges are equal length and all angles are right angles

**Prism** A solid where each cross-section in a particular direction is exactly the same and all faces are polygons

**Pyramid** A solid in which the base is a polygon and the other faces are formed by triangles with a common vertex

**Cylinder** A solid with two circles forming its opposing end faces

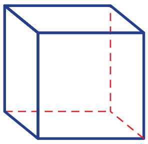
**Cone** A solid in which the base is a circle whose sides narrow smoothly to form a vertex at the opposite end

## Exercise 6G

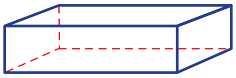
**Understanding**

**1** Name these solids.

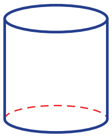
**a**



**b**

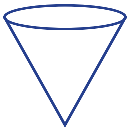


**c**

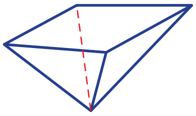


Choose from: *cone*, *cylinder*, *triangular pyramid*, *square pyramid*, *cube*, *rectangular prism*.

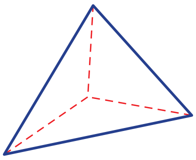
**d**



**e**

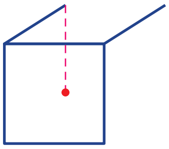


**f**

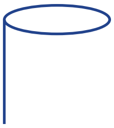


**2** Copy these diagrams and add lines to complete the solid. Use dashed lines for invisible edges.

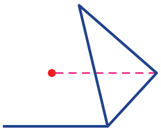
**a** Cube



**b** Cylinder

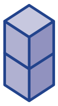


**c** Square pyramid

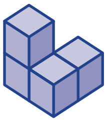


**3** Cubes are stacked to form these solids. How many cubes are there in each solid?

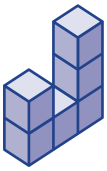
**a**



**b**



**c**

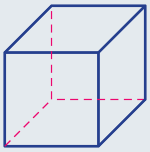


**Fluency**

### Example 15 Drawing solids

Draw these solids.

**a** Cube **b** Cone **c** Square pyramid

| **Solution** | **Explanation** |
|---|---|
| **a**  | Draw the square front.<br>Draw edges parallel to the top and the right edge.<br>Connect, using dashed lines for hidden edges. |

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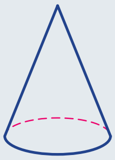
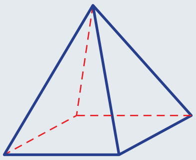
**Solution** | **Explanation**

**b**



Draw an oval shape for the base.
Draw the apex point above the centre of the oval.
Join the apex to the sides of the base and erase part of the oval to create a dashed (hidden) edge.

**c**



Draw a rhombus for the base.
Pick a central point above the base for the axis, then connect.
Use dashed lines for hidden edges.

**4** On plain paper, draw an example of these common solids.

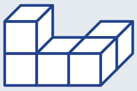
- **a** Cube
- **b** Cylinder
- **c** Triangular pyramid
- **d** Cone
- **e** Rectangular prism
- **f** Square pyramid

## Example 16 Using isometric dot paper

Draw these solids on isometric dot paper.

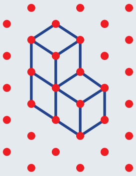
**a**

**b**

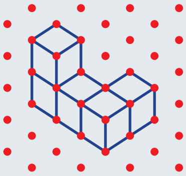


**Solution**

**a**



**b**



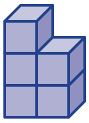
**Explanation**

Rotate the solids slightly. Then start at the top or bottom and draw every line you can see. There are vertical lines but no horizontal lines.

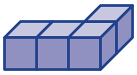
Drilling for Gold 6G1

**5** Draw these solids on isometric dot paper.

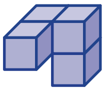
**a**



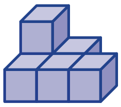
**b**



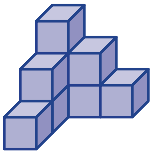
**c**



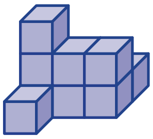
**d**



**e**



**f**



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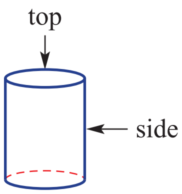
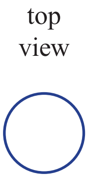
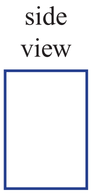
**Problem-solving and Reasoning**

**6** Here is a cylinder with its top view (circle) and side view (rectangle).

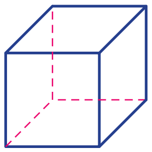




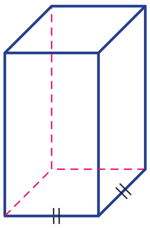


Draw the shapes that are the top view and side view of these solids.

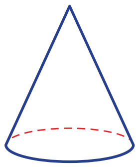
**a** Cube



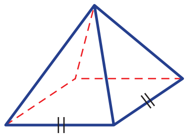
**b** Square-based prism



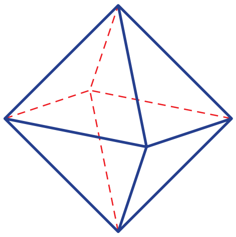
**c** Cone



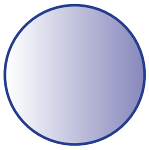
**d** Square pyramid



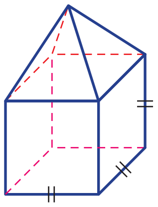
**e** Octahedron



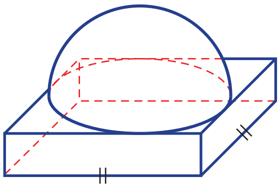
**f** Sphere



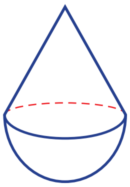
**g** Square pyramid on cube



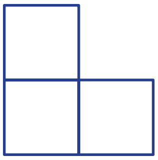
**h** Hemisphere (half sphere) on square prism

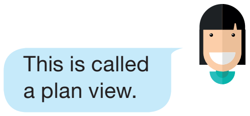


**i** Cone on hemisphere



**7** Here is the top (or bird's eye) view of a stack of 5 cubes. How many different stacks of 5 cubes could this represent?
Hint: Check using centicubes.





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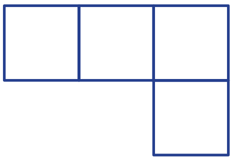
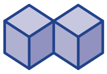
**8** Here is the top view of a stack of 7 cubes.
How many different stacks of 7 cubes could this represent?
Hint: Check using centicubes.

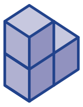


**9** Match each solid (**a** to **d**) with an identical solid chosen from **A**, **B**, **C** and **D**.
Hint: Check using centicubes.

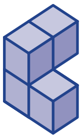
**a**



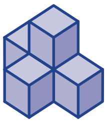
**b**



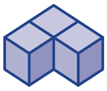
**c**



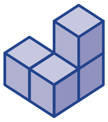
**d**



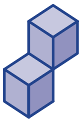
**A**



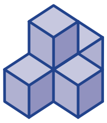
**B**



**C**

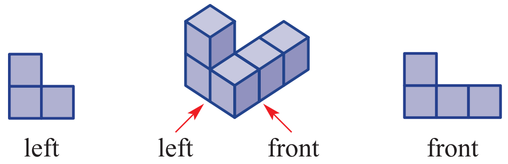


**D**



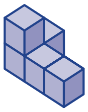
## Enrichment: Three view points

**10** These diagrams show the front and left views of a solid.

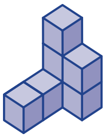


**a** Draw the front, left and top views of these solids.

**i**

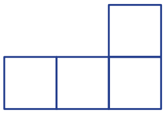


**ii**

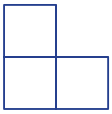


**b** Use blocks to make the solids that have these views, then draw them on isometric dot paper.

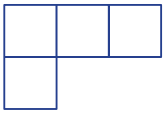
**i** Front



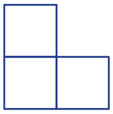
Left



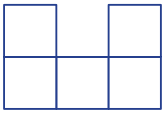
Top



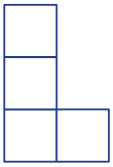
**ii** Front



Left



Top

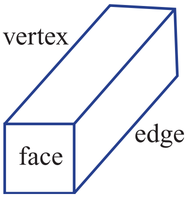


# 6H Solids

A solid is a an object that occupies three-dimensional space. The outside surfaces could be flat or curved. A solid with all flat surfaces is called a polyhedron, plural *polyhedra* or *polyhedrons*. The word 'polyhedron' comes from the Greek words *poly*, meaning 'many', and *hedron*, meaning 'faces'.

The top of this Canary Wharf building in London (left) is a large, complex polyhedron. Polyhedra also occur in nature, particularly in rock or mineral crystals such as quartz (right).

## Let's start: Amazing names!



Polyhedra have faces (shapes), vertices (points) and edges (line segments).

Use the internet to fill in the table.

| | | Number of faces | Number of vertices | Number of edges |
|---|---|---|---|---|
| 1 | Dodecahedron | | | |
| 2 | Icosahedron | | | |
| 3 | Octagonal prism | | | |
| 4 | Octagonal pyramid | | | |
| 5 | Rhombicosidodecahedron | | | |
| 6 | Truncated cube | | | |
| 7 | Rhombic triacontahedron | | | |
| 8 | Icosidodecahedron | | | |

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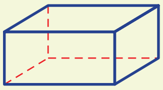
## Key ideas

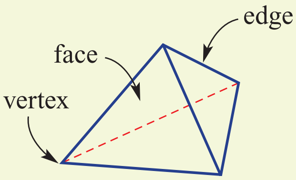
- A **polyhedron** is a solid figure bounded by plane polygonal faces.
  - Two adjacent faces intersect at an edge.
  - Each edge joins two vertices.

Rectangular prism



- 6 faces
- 8 vertices
- 12 edges

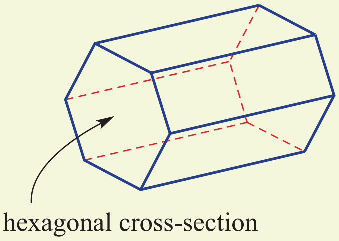
Tetrahedron (or triangular pyramid)



- 4 faces
- 4 vertices
- 6 edges.

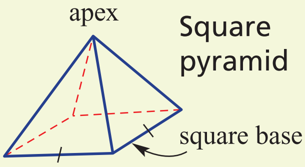
- **Prisms** are polyhedra with two identical (congruent) ends. The congruent ends define the **cross-section** of the prism and also its name.

Hexagonal prism



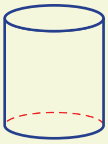
- **Pyramids** are polyhedra with a polygonal base. All other faces meet at the same vertex point called the apex. They are named by the shape of the base.

Square pyramid

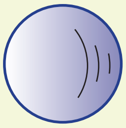


- Some solids have **curved** surfaces. Common examples include:

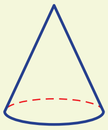
Cylinder



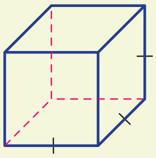
Sphere



Cone



- A cube has six identical square faces.



**Polyhedron** A three-dimensional figure made by joining polygons at their edges

**Prism** A solid where each cross-section in a particular direction is exactly the same and all faces are polygons

**Cross-section** A surface exposed by making a straight cut through a shape at a right angle to the edge

**Pyramid** A solid in which the base is a polygon and the other faces are formed by triangles with a common vertex

Natural hexagonal prisms of rock at the Giant's Causeway in Northern Ireland formed when lava cooled quickly and cracked in semi-regular patterns.

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## Exercise 6H

**Understanding**

**1** Write the missing word or number in these sentences.

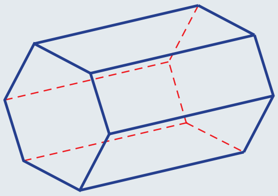
- **a** A rectangular prism has _____________ faces.
- **b** The flat face at the base of a cylinder is a _____________.
- **c** A regular solid with six square faces is called a _____________.
- **d** A polyhedron has faces, _____________ and edges.
- **e** A hexagonal pyramid has _______ faces.
- **f** A prism has two _____________ ends.
- **g** A pentagonal prism has _____________ faces.
- **h** The base of a pyramid has 8 sides. The pyramid is called an _____________ pyramid.

**2** Name three solids that have curved surfaces.

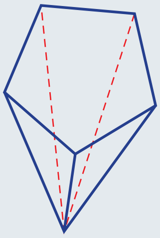
### Example 17 Counting faces, vertices and edges

State the number of faces, vertices and edges for these solids.

**a** Hexagonal prism



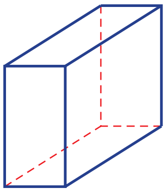
**b** Pentagonal pyramid



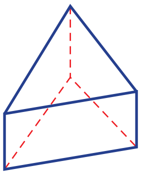
| **Solution** | **Explanation** |
|---|---|
| **a** 8 faces | Faces are the flat surfaces. |
| 12 vertices | Vertices are the corners. |
| 18 edges | Edges are the lines on the diagram. |
| **b** 6 faces | There is one pentagonal face and five triangular faces. |
| 6 vertices | Five vertices are on the base plus one apex. |
| 10 edges | Five edges are on the base and five meet the apex. |

**3** Count the number of faces, vertices and edges (in that order) on these polyhedra.

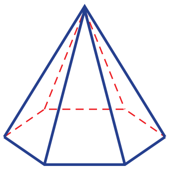
**a**



**b**



**c**



- Faces are flat.
- Vertices are points.
- Edges are lines.

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**4** Which of these solids are polyhedra (i.e. have only flat surfaces)?

| | | |
|---|---|---|
| **A** Cube | **B** Pyramid | **C** Cone |
| **D** Sphere | **E** Cylinder | **F** Rectangular prism |
| **G** Tetrahedron | **H** Hexahedron | |

**Fluency**

**5** Copy and complete the table.

| | Shape of cross-section | Name of prism |
|---|---|---|
| **a** | Triangle | |
| **b** | Rectangle | |
| **c** | Trapezium | |
| **d** | Pentagon | |
| **e** | Hexagon | |
| **f** | Octagon | |

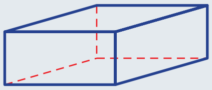
**6** Copy and complete the table.

| | Name of pyramid | Faces | Vertices | Edges |
|---|---|---|---|---|
| **a** | Triangular pyramid | | | |
| **b** | Square pyramid | | | |
| **c** | Pentagonal pyramid | | | |
| **d** | Hexagonal pyramid | | | |
| **e** | Octagonal pyramid | | | |

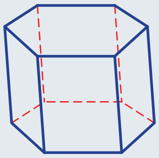
## Example 18 Naming prisms

Name these solids as types of prisms.

**a**



**b**



**Solution**

**Explanation**

| Solution | Explanation |
|---|---|
| **a** Rectangular prism | The cross-section is a rectangle. |
| **b** Hexagonal prism | The cross-section is a hexagon. |

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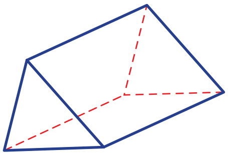
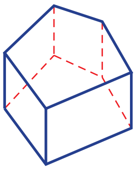
**7** Name these prisms.

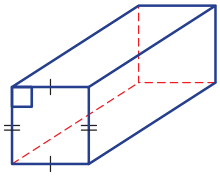
**a**



**b**



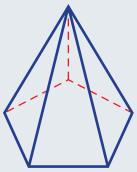
**c**



Name using the shape of the cross-section.

## Example 19 Naming pyramids

Name this solid as a type of pyramid.

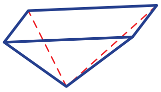


| **Solution** | **Explanation** |
|---|---|
| Pentagonal pyramid | The base is a pentagon. |

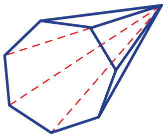
Skillsheet 6C

**8** Name these pyramids.

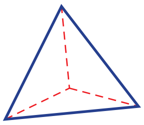
**a**



**b**



**c**

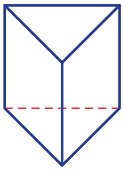


Name using the shape of the base.

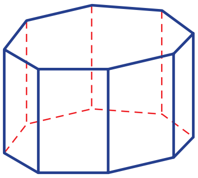
**Problem-solving and Reasoning**

**9** Name each of these solids.

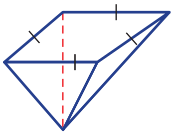
**a**



**b**



**c**



As an example: a pentagonal pyramid.

**10** Decide if the following statements are true or false. Make drawings to help.

- **a** All pyramids have some triangular faces.
- **b** All solids with curved surfaces are cylinders.
- **c** A cube and a rectangular prism have the same number of edges.
- **d** A cylinder is a prism.
- **e** There are no solids with $0$ vertices.
- **f** There are no polyhedra with $3$ surfaces.
- **g** All pyramids will have an odd number of faces.

**11** Explain the difference between a hexagonal prism and a hexagonal pyramid. Name the shapes that are the faces.

**12** Investigate if this statement is true or false.
For all pyramids, the number of faces is equal to the number of vertices.

## Enrichment: Euler's rule

**13 a** Copy and complete this table.

| Solid | Number of faces ($F$) | Number of vertices ($V$) | Number of edges ($E$) | $F + V$ |
|---|---|---|---|---|
| Cube | | | | |
| Square pyramid | | | | |

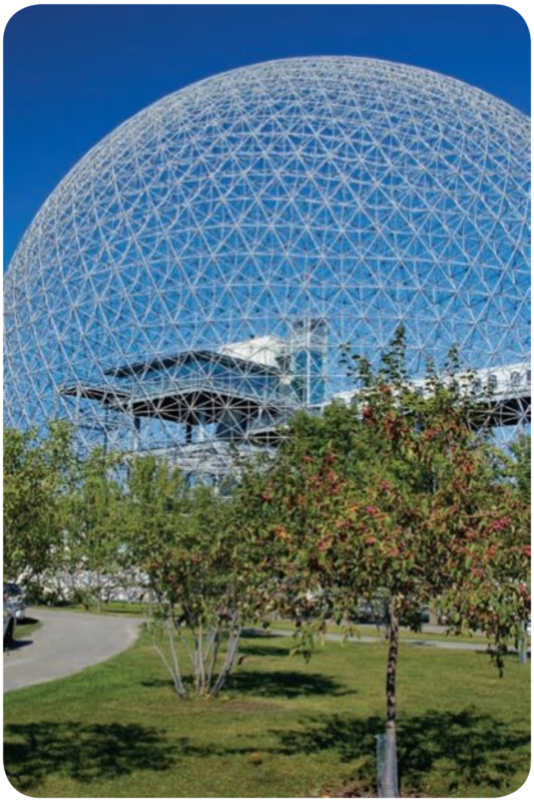
**b** Compare the number of edges ($E$) with the value $F + V$ for each polyhedron. What do you notice?

**c** Does the relationship work for the polyhedra in the Let's start section?

**14 a** A polyhedron has 16 faces and 12 vertices. How many edges does it have?

**b** A polyhedron has 18 edges and 9 vertices. How many faces does it have?

**c** A polyhedron has 34 faces and 60 edges. How many vertices does it have?



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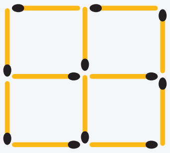
# Puzzles and games



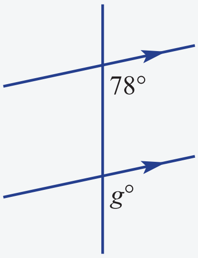
**1** This shape includes 12 matchsticks. (To solve these puzzles all matches remaining must connect to other matches at both ends.)

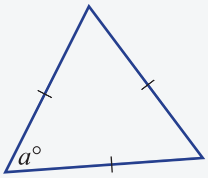
- **a** Remove 2 matchsticks to form 2 squares.
- **b** Move 3 matchsticks to form 3 squares.

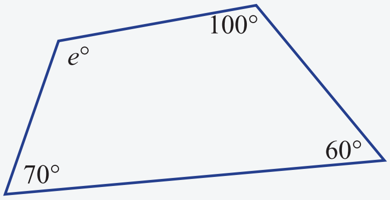
**2**
- **a** Use 9 matchsticks to form 5 equilateral triangles.
- **a** Use 6 matchsticks to form 4 equilateral triangles.

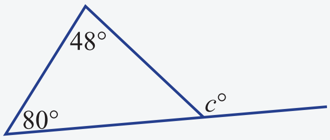
**3** Who am I?

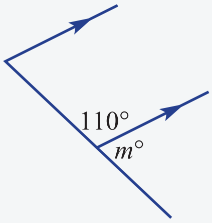
I was a female mathematician famous for her work and publications on geometry. Use your answers to the following to unlock the puzzle code.

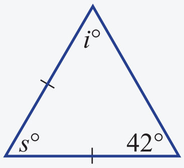


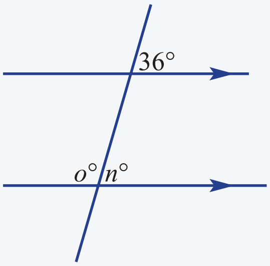


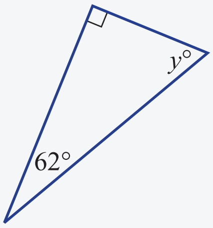


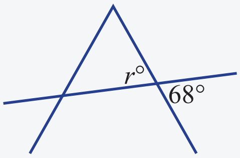


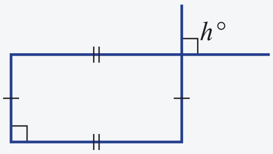


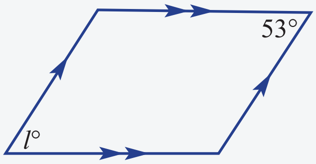


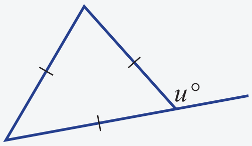












78 68 60 128 130 128 90 42 96 90 144 53 70 28 144 120 36 78

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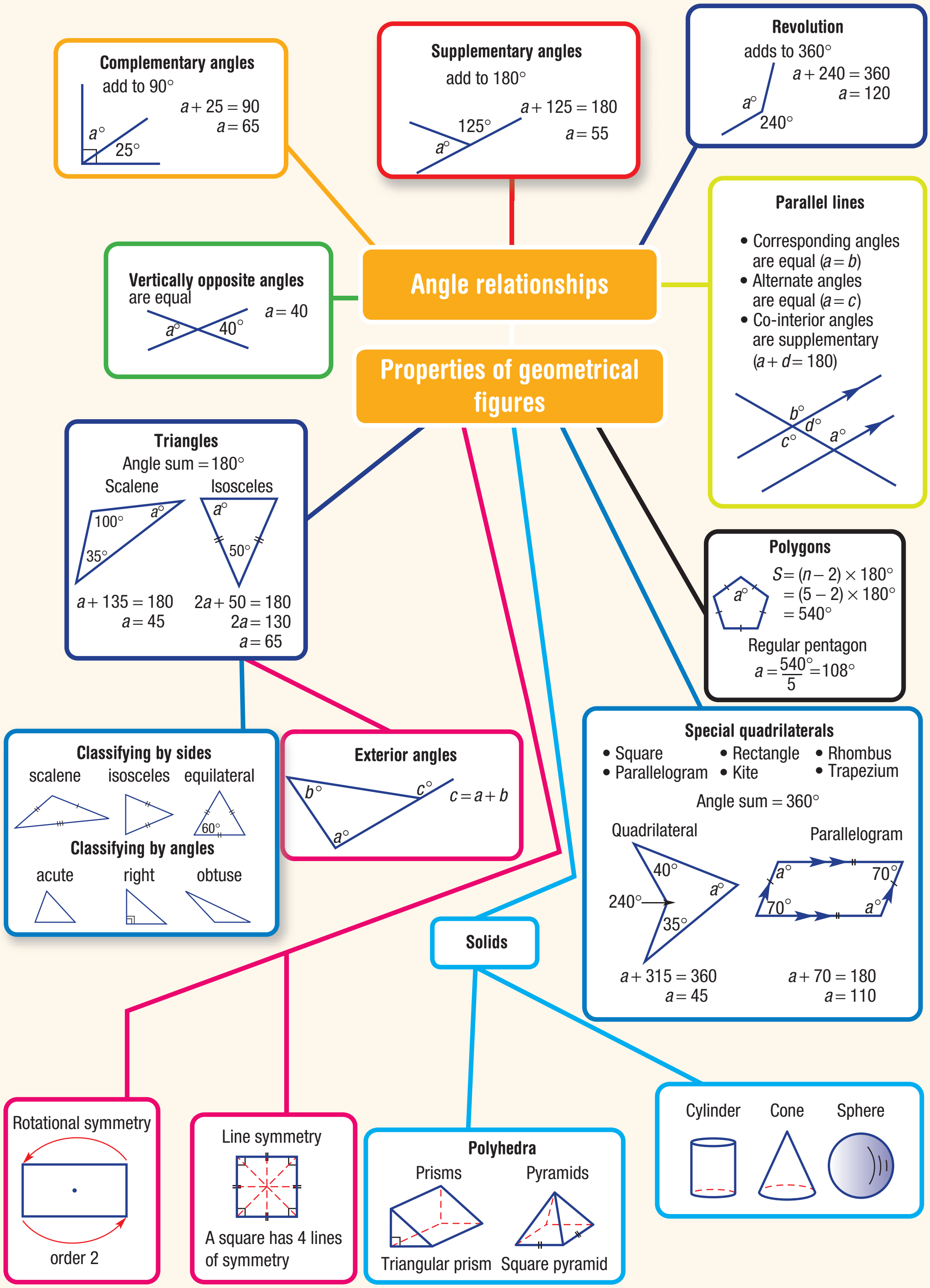
# Chapter summary



## Angle relationships

**Complementary angles**
add to 90°
$a + 25 = 90$
$a = 65$

**Supplementary angles**
add to 180°
$a + 125 = 180$
$a = 55$

**Revolution**
adds to 360°
$a + 240 = 360$
$a = 120$

**Vertically opposite angles**
are equal
$a = 40$

**Parallel lines**

- Corresponding angles are equal ($a = b$)
- Alternate angles are equal ($a = c$)
- Co-interior angles are supplementary ($a + d = 180$)

## Properties of geometrical figures

**Triangles**
Angle sum = 180°

| Scalene | Isosceles |
|---|---|
| $a + 135 = 180$ | $2a + 50 = 180$ |
| $a = 45$ | $2a = 130$ |
| | $a = 65$ |

**Polygons**
$S = (n - 2) \times 180°$
$= (5 - 2) \times 180°$
$= 540°$
Regular pentagon
$a = \frac{540°}{5} = 108°$

**Classifying by sides**
scalene isosceles equilateral

**Classifying by angles**
acute right obtuse

**Exterior angles**
$c = a + b$

**Special quadrilaterals**

- Square
- Rectangle
- Rhombus
- Parallelogram
- Kite
- Trapezium

Angle sum = 360°

| Quadrilateral | Parallelogram |
|---|---|
| $a + 315 = 360$ | $a + 70 = 180$ |
| $a = 45$ | $a = 110$ |

**Solids**

**Polyhedra**
Prisms Pyramids
Triangular prism Square pyramid

Cylinder Cone Sphere

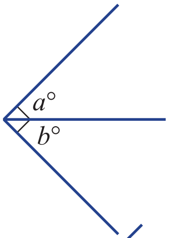
Rotational symmetry
order 2

Line symmetry
A square has 4 lines of symmetry

**Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.**

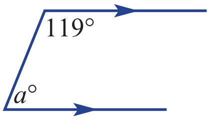
## Multiple-choice questions

**1** What is the name given to two angles that sum to 90°?
**A** Right **B** Supplementary
**C** Revolutionary **D** Complementary
**E** Vertically opposite



**2** Two angles on a straight line add to:
**A** 180° **B** 90° **C** 45°
**D** 270° **E** 360°

**3** The value of $a$ in this diagram is equal to:
**A** 45 **B** 122 **C** 241
**D** 119 **E** 61



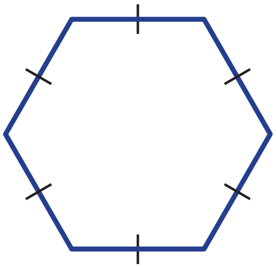
**4** A pair of alternate angles in parallel lines:
**A** are not equal **B** are vertically opposite **C** are equal
**D** are complementary **E** are supplementary.

**5** The rule for the angle sum $S$ of a polygon with $n$ sides is:
**A** $S = n \times 180°$ **B** $S \times n = 180°$ **C** $S = (n - 1) \times 180°$
**D** $S = (n - 2) \times 180°$ **E** $S = (n + 2) \times 180°$

**6** What is the order of rotational symmetry for a parallelogram?
**A** 0 **B** 1 **C** 2 **D** 3 **E** 4

**7** The name given to a ten-sided polygon is:
**A** heptagon **B** tenagon **C** decagon
**D** dodecagon **E** undecagon

**8** The angle sum of a hexagon is:
**A** 720° **B** 540° **C** 900°
**D** 1080° **E** 360°



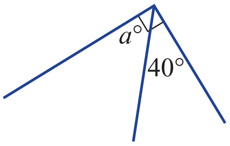
**9** The size of one interior angle of a regular hexagon is:
**A** 135° **B** 180° **C** 120°
**D** 720° **E** 108°

**10** How many edges does a rectangular prism have?
**A** 10 **B** 4 **C** 6
**D** 12 **E** 8

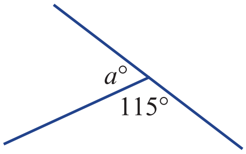
## Short-answer questions

**1** Find the value of $a$ in these diagrams.

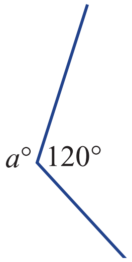
**a**



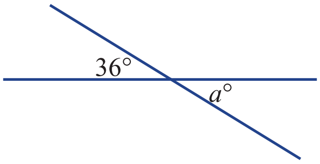
**b**



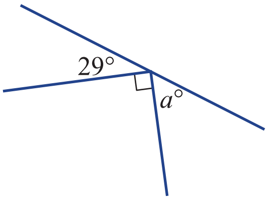
**c**



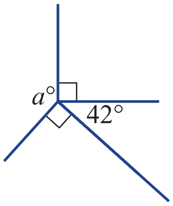
**d**



**e**

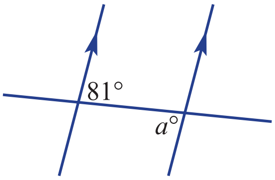


**f**

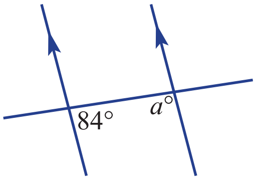


**2** These diagrams include parallel lines. Find the value of $a$.

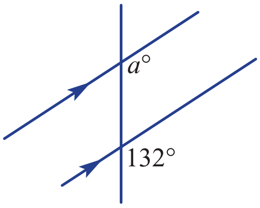
**a**



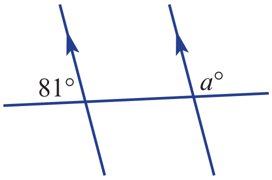
**b**



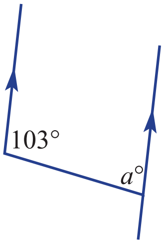
**c**



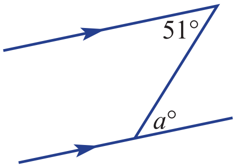
**d**



**e**

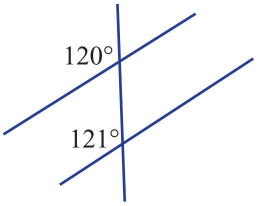


**f**

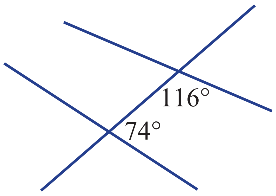


**3** Decide if these diagrams include a pair of parallel lines. Give reasons.

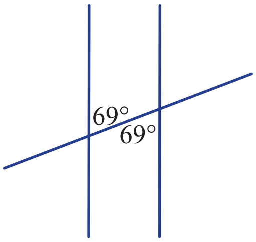
**a**



**b**

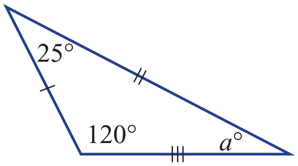


**c**

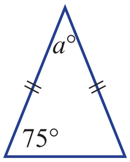


**4** Give a name for each triangle and find the value of $a$.

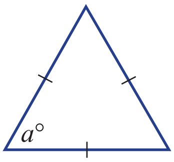
**a**



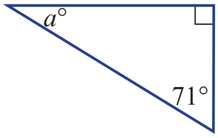
**b**



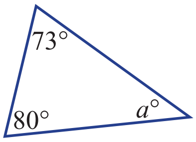
**c**



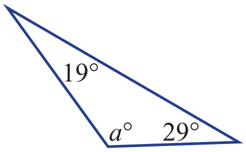
**d**



**e**

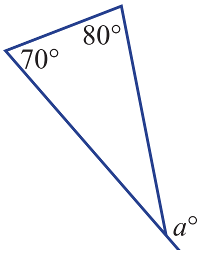


**f**

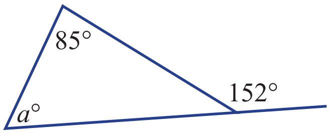


**5** These triangles include exterior angles. Find the value of $a$.

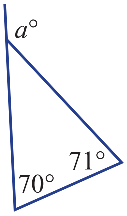
**a**



**b**



**c**



**6** Name the special quadrilateral(s) that definitely have:

**a** all sides equal in length

**b** two pairs of parallel sides

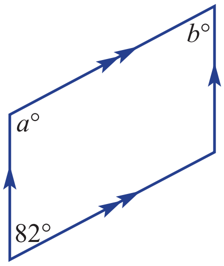
**c** four equal sides

**d** diagonals intersecting at right angles

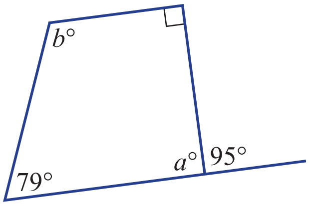
**e** equal length diagonals.

**7** Find the value of $a$ and $b$ in these quadrilaterals.

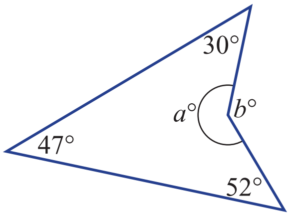
**a**



**b**



**c**



**8** Find the angle sum of these polygons using $S = (n - 2) \times 180°$.

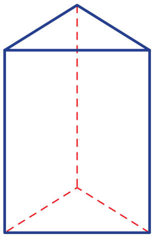
**a** Hexagon **b** Octagon **c** Dodecagon

**9** Find the size of an interior angle of these regular polygons by firstly finding the angle sum.

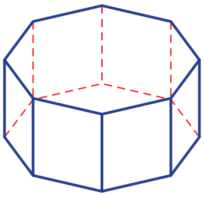
**a** Regular pentagon **b** Regular dodecagon

**10** What type of prism or pyramid are these solids?

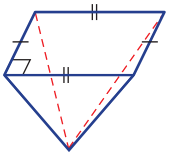
**a**



**b**

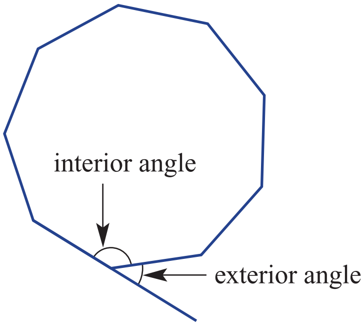


**c**



## Extended-response question

**1** This regular polygon has 9 sides.



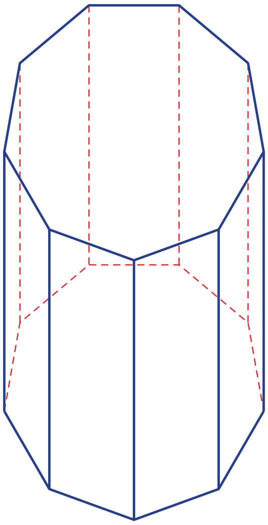
**a** Find the interior angle sum using $S = (n - 2) \times 180°$.

**b** Find the size of each interior angle.

**c** Find the size of each exterior angle.

**d** The polygon is used to form the ends of a prism.
For this prism find the number of:

- **i** faces
- **ii** vertices
- **iii** edges.



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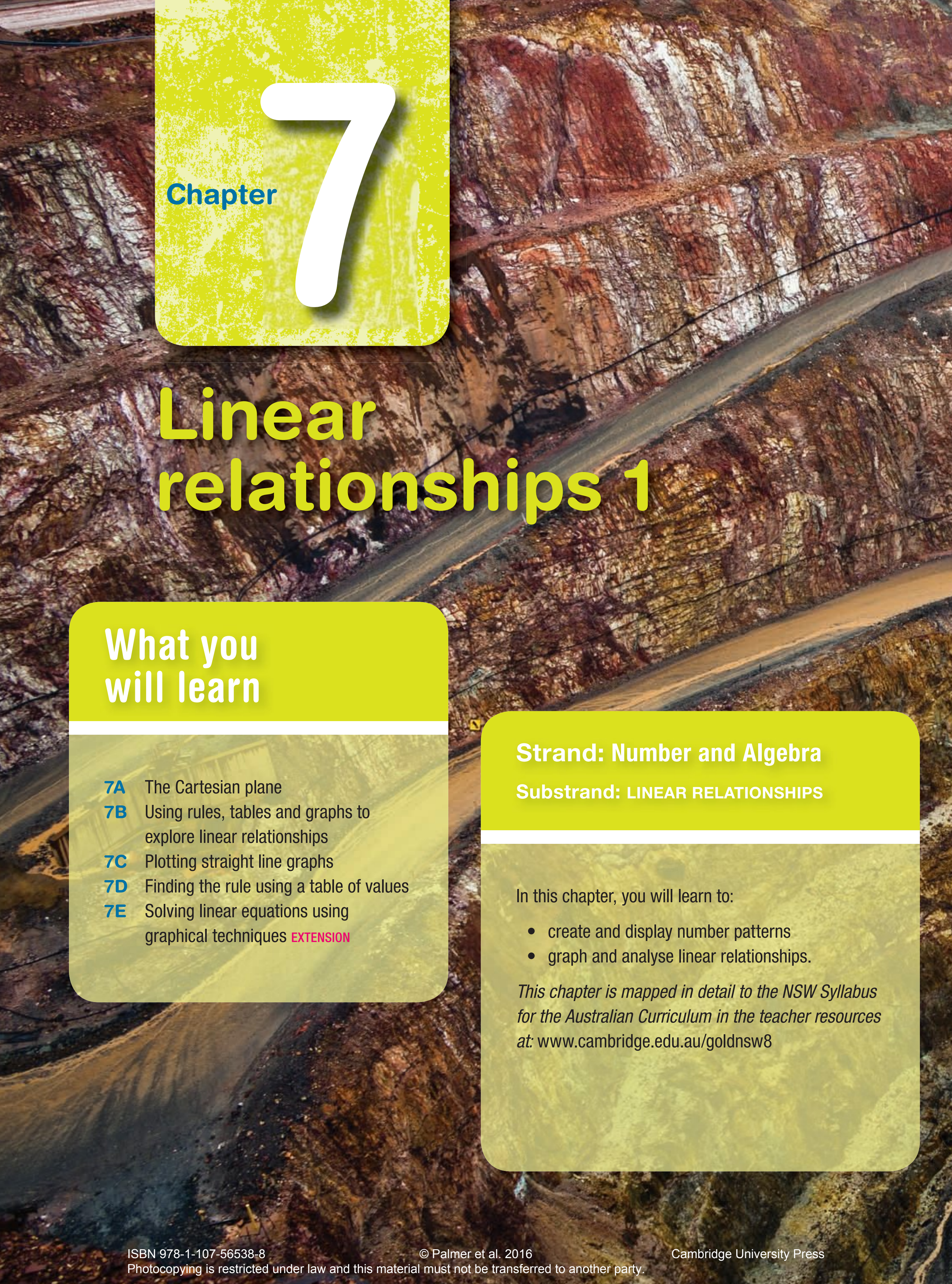

# Chapter 7

# Linear relationships 1

## What you will learn

## Strand: Number and Algebra

**Substrand:** LINEAR RELATIONSHIPS

In this chapter, you will learn to:

- create and display number patterns
- graph and analyse linear relationships.

*This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at:* www.cambridge.edu.au/goldnsw8

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## Mining optimisation

Australian mining companies spend millions of dollars planning and managing their mining operations. The success of a mine depends on many factors.

Through a process called linear programming, many of these factors are represented using linear equations and straight-line graphs. These tell the company the most efficient and cost-effective way to manage all of the given factors. These graphs can save companies millions of dollars.

## Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw8

**Drilling for Gold:**
Building knowledge and skills

**Skillsheets:**
Extra practise of important skills

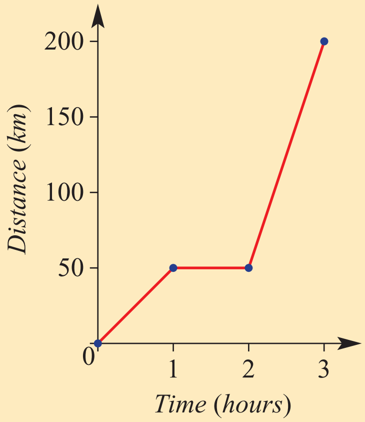
**Literacy activities:**
Mathematical language

**Worksheets:**
Consolidation of the topic

**Chapter Test:**
Preparation for an examination

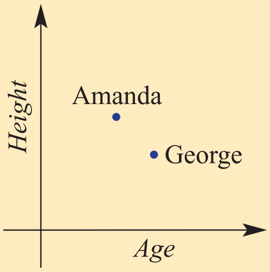
# Pre-test

**1** This graph relates distance and time for a journey in a train.



- **a** How far did the train travel in the:
  - **i** first hour?
  - **ii** second hour?
  - **iii** third hour?
- **b** What was the total distance travelled?
- **c** During which hour was the train at rest?
- **d** During which hour was the train travelling the fastest?

**2** This graph shows the relationship between age and height of two people.



- **a** Who is older?
- **b** Who is taller?

**3** Write the missing numbers.

- **a** –3, –2, __, 0, 1, __, __
- **b** –7, __, –3, –1, 1, __, __
- **c** 12, 7, __, –3, __, __, –18
- **d** –31, __, –13, –4, __, __

**4** The $x$-coordinate in (2, –3) is 2. Write the $x$-coordinate for these points.

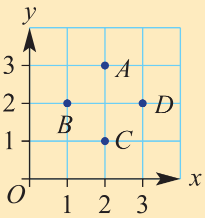
- **a** (1, 2)
- **b** (1, 5)
- **c** (0, –1)
- **d** (–3, 0)

**5** The $y$-coordinate in (2, –3) is –3. Write the $y$-coordinate for these points.

- **a** (1, 6)
- **b** (–4, –1)
- **c** (–3, 0)
- **d** (–4, 2)

**6** The coordinates of the point $A$ on this graph are (2, 3). What are the coordinates of these points?

- **a** $B$
- **b** $C$
- **c** $D$



**7** If $y = x + 4$, find the value of $y$ when:

- **a** $x = 3$
- **b** $x = 0$
- **c** $x = -2$

**8** If $y = 2 \times x - 5$, find the value of $y$ when:

- **a** $x = 2$
- **b** $x = 0$
- **c** $x = -3$

**9** Complete the tables for the given rules.

**a** $y = 2 \times x$

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ |  |  |  |  |  |

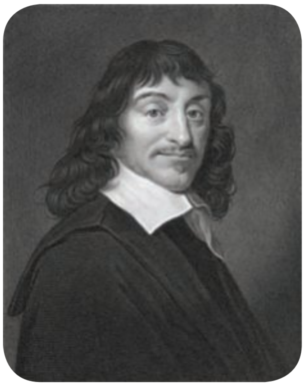
**b** $y = 3 \times x - 4$

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ |  |  |  |  |  |

# 7A The Cartesian plane

The number plane is also called the Cartesian plane after its inventor, René Descartes, who lived in France in the 17th century. The point where these axes meet is called the origin and it provides a reference point for all other points on the plane.

For example, to plot the point (3, 2), start from the origin, move right 3 then up 2.

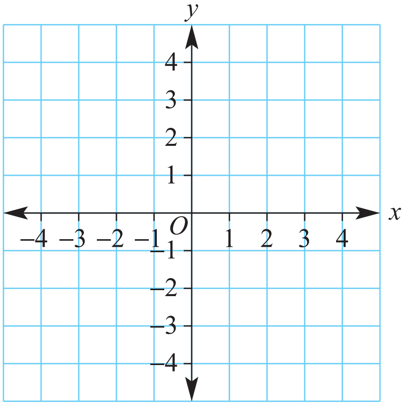


René Descartes, mathematician and philosopher (1596–1650)

## Let's start: Make the shape

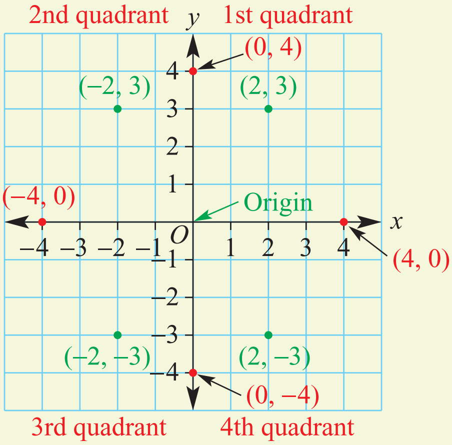
Plot each group of points, then decide what type of shape is formed.

- $A(0, 0)$, $B(3, 1)$, $C(0, 4)$
- $A(-2, 3)$, $B(-2, -1)$, $C(-1, -1)$, $D(-1, 3)$
- $A(-3, -4)$, $B(2, -4)$, $C(0, -1)$, $D(-1, -1)$



## Key ideas

- A **number plane** (or *Cartesian plane*) includes a vertical $y$-axis and a horizontal $x$-axis intersecting at right angles.
  - There are 4 **quadrants** labelled as shown.
- A point on a number plane has **coordinates** $(x, y)$.
  - The $x$-coordinate is listed first, followed by the $y$-coordinate.
- The point (0, 0) is called the **origin**.
- To plot a point such as (2, –3):
  - start from (0, 0)
  - move right 2
  - move down 3.



**Number plane**
A plane on which every point is related to a pair of numbers called coordinates

**Quadrant**
Any of the four sections the number plane is divided up into

**Coordinates**
Numbers or letters used to give a location or position

**Origin**
The point (0, 0) on a graph

## Exercise 7A

Understanding

**1** Complete these sentences.

**a** The $x$-coordinate in (3, –4) is _______.

**b** The $x$-coordinate in (–4, 7) is _______.

**c** The $y$-coordinate in (2, 5) is _______.

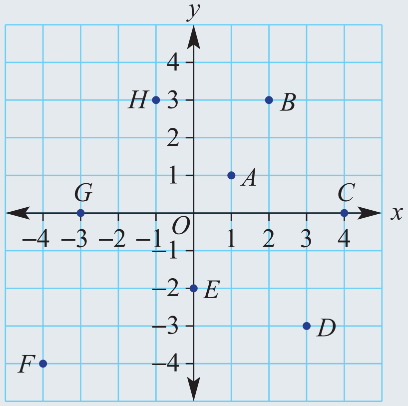
**d** The $y$-coordinate in (–4, –8) is _______.

**e** The coordinates of the origin are _______.

**f** The vertical axis is called the ____-axis.

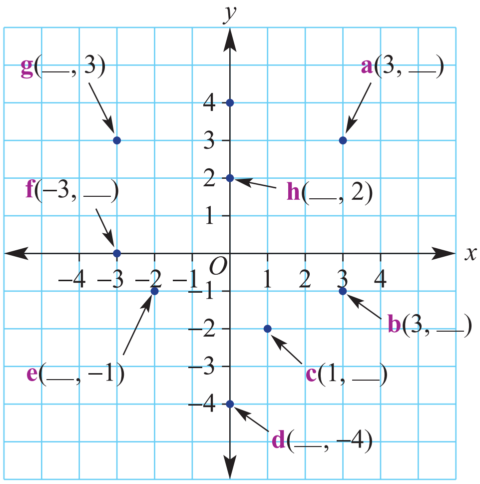
### Example 1 Writing coordinates

Write down the coordinates of the points $A$ to $H$ on this number plane.

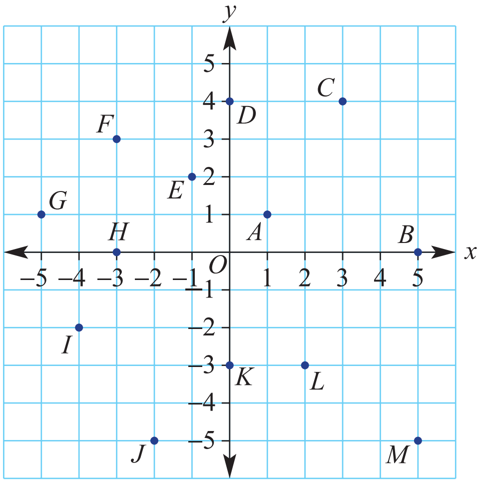


| **Solution** | | **Explanation** |
|---|---|---|
| $A(1, 1)$ | $E(0, -2)$ | First write the $x$-coordinate |
| $B(2, 3)$ | $F(-4, -4)$ | (positive on the right and negative on the left). |
| $C(4, 0)$ | $G(-3, 0)$ | Then write the $y$-coordinate |
| $D(3, -3)$ | $H(-1, 3)$ | (positive above and negative below). |

**2** Write the missing number for the coordinates of the points **a**–**h**.



**3** Write the coordinates of the points labelled $A$ to $M$.



Always start from the origin.
Move:
- left or right then
- up or down.

**Fluency**

## Example 2 Plotting points

**Drilling for Gold 7A1**

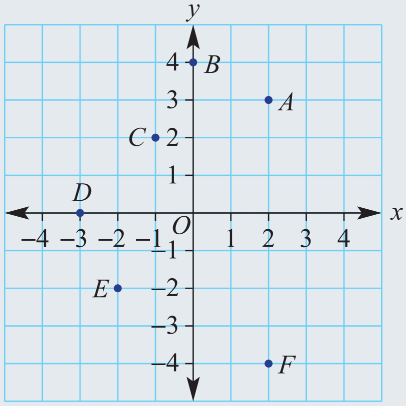
**Drilling for Gold 7A2**

**Drilling for Gold 7A3**

Draw a number plane extending from –4 to 4 on both axes then plot and label these points.

| | | | |
|---|---|---|---|
| **a** | $A(2, 3)$ | **b** | $B(0, 4)$ |
| **c** | $C(-1, 2)$ | **d** | $D(-3, 0)$ |
| **e** | $E(-2, -2)$ | **f** | $F(2, -4)$ |

### Solution



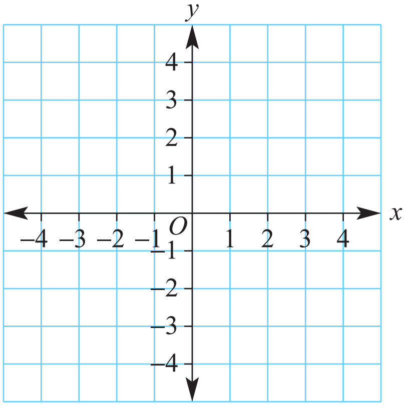
### Explanation

The $x$-coordinate is listed first followed by the $y$-coordinate.

For each point start at the origin $(0, 0)$ and move left or right or up and down to suit both $x$- and $y$-coordinates. For point $C(-1, 2)$, for example, move $1$ to the left and $2$ up.

**4** Draw a number plane extending from –4 to 4 on both axes and then plot and label these points. Or you could use this grid.

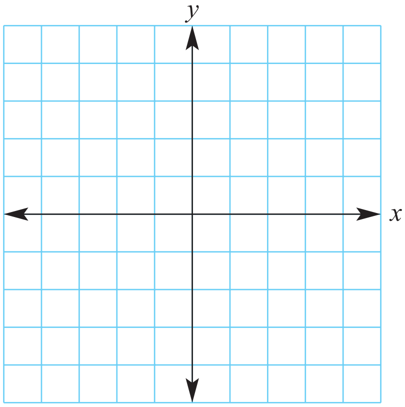
**a** $A(4, 1)$ **b** $B(2, 3)$
**c** $C(0, 1)$ **d** $D(-1, 3)$
**e** $E(-3, 3)$ **f** $F(-2, 0)$
**g** $G(-3, -1)$ **h** $H(-1, -4)$
**i** $I(0, -2)$ **j** $J(0, 0)$
**k** $K(3, -1)$ **l** $L(1, -4)$



First move horizontally for $x$ then vertically for $y$.

**5** Draw a number plane extending from –4 to 4 on both axes and then plot each set of points. What do you notice about each graph?

**a** $(-4, -4)$, $(-3, -3)$, $(-2, -2)$, $(-1, -1)$, $(0, 0)$, $(1, 1)$, $(2, 2)$, $(3, 3)$, $(4, 4)$
**b** $(-2, 4)$, $(-1, 2)$, $(0, 0)$, $(1, -2)$, $(2, -4)$
**c** $(-4, 2)$, $(-2, 1)$, $(0, 0)$, $(2, -1)$, $(4, -2)$

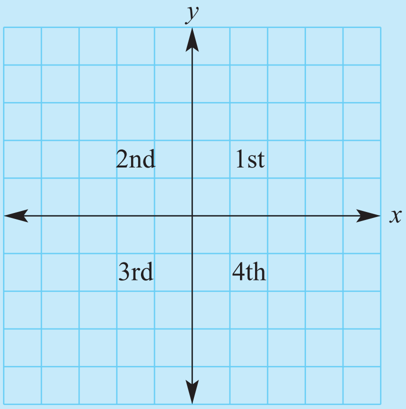


**Problem-solving and Reasoning**

**6** Complete these sentences.

**a** The point $(2, 4)$ is in the _______ quadrant.
**b** The point $(1, -5)$ is in the _______ quadrant.
**c** The point $(-3, 6)$ is in the _______ quadrant.
**d** The point $(-7, -20)$ is in the _______ quadrant.
**e** The quadrant that has positive coordinates for both $x$ and $y$ is the _______ quadrant.
**f** The quadrant that has negative coordinates for both $x$ and $y$ is the _______ quadrant.

Answer as first, second, third or fourth



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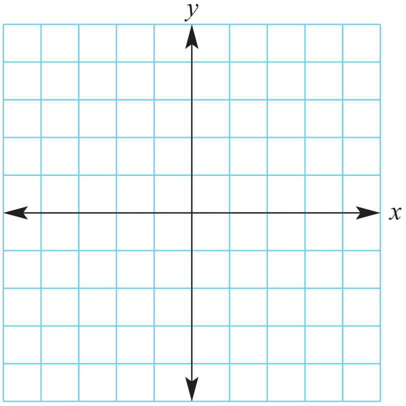
**7** One point in each set is not 'in line' with the other points. Plot the points on this grid then name the point not in line with the others in the same set.

- **a** $A(1, 2)$, $B(2, 4)$, $C(3, 4)$, $D(4, 5)$, $E(5, 6)$
- **b** $A(-5, 3)$, $B(-4, 1)$, $C(-3, 0)$, $D(-2, -3)$, $E(-1, -5)$
- **c** $A(-4, -3)$, $B(-2, -2)$, $C(0, -1)$, $D(2, 0)$, $E(3, 1)$
- **d** $A(6, -4)$, $B(0, -1)$, $C(4, -3)$, $D(3, -2)$, $E(-2, 0)$



**8** Each set of points forms a basic shape. Describe the shape without drawing a graph if you can.

- **a** $A(-2, 4)$, $B(-1, -1)$, $C(3, 0)$
- **b** $A(-3, 1)$, $B(2, 1)$, $C(2, -6)$, $D(-3, -6)$
- **c** $A(-4, 2)$, $B(3, 2)$, $C(4, 0)$, $D(-3, 0)$
- **d** $A(-1, 0)$, $B(1, 3)$, $C(3, 0)$, $D(1, -9)$

**9** A set of points has coordinates $(0, y)$ where $y$ is any number. What does this set of points represent?

## Enrichment: Plotting pictures

**10** Using a scale extending from –5 to 5 on both axes, plot and then join the points for each part. Describe the basic picture formed.

- **a** $(-2, -2)$, $(2, -2)$, $(2, 2)$, $(1, 3)$, $(1, 4)$, $\left(\frac{1}{2}, 4\right)$, $\left(\frac{1}{2}, 3\frac{1}{2}\right)$, $(0, 4)$, $(-2, 2)$, $(-2, -2)$
- **b** $(2, 1)$, $(0, 3)$, $(-1, 3)$, $(-3, 1)$, $(-4, 1)$, $(-5, 2)$, $(-5, -2)$, $(-4, -1)$, $(-3, -1)$, $(-1, -3)$, $(0, -3)$, $(2, -1)$, $(1, 0)$, $(2, 1)$

# 7B Using rules, tables and graphs to explore linear relationships

Consider the following scenario:
- Sophie had \$4 in her money box at the start of this week.
- She is going to add \$3 at the end of every week.

In this scenario, there are two quantities that are changing. They are the:
- number of weeks
- amount of money.

In this chapter, we will investigate the relationship between two changing quantities using:
- patterns (sequences)
- points in a table of values
- a rule, which can be:
  - – a written description, or
  - – an equation
- points and graphs on the Cartesian plane.

Saving money can be an example of a linear relationship. A mathematical equation can predict future savings.

Drilling for Gold 7B1

## Let's start: Sophie's money box

- Using the scenario above, copy this pattern (sequence) and fill in the next five numbers:
  4, 7, 10, 13, __, __, __, __, __
- Copy and complete this table of values.

| **Number of weeks ($W$)** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **Dollars in money box ($D$)** | 4 | 7 | | | | |

- Write down the first three points from the table:
  (0, __), (1, __), (__, __)
- Neatly plot all 6 points from the table onto a number plane
- What do you notice about the pattern of the points on the number plane?

This is an example of a linear relationship, because every time that $W$ increases by 1, $D$ increases by a constant amount (3).

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## Key ideas

- A relationship between two changing quantities is **linear** if a constant change in one quantity produces a constant change in the other.
  e.g. for every week that passes I receive $10 pocket money.
- A **linear relationship** can be represented using a:
  - worded scenario
  - number pattern (sequence)
  - table of values
  - written description of the rule that connects every number in the top row of the table with a number in the bottom row
  - mathematical equation that connects every number in the top row of the table with the number below it
  - series of points on the Cartesian plane that form a straight line.

**Linear** Making or resembling a straight line
**Linear relationship** A situation in which two quantities vary so that their path on a number plane is straight

For example:

**Worded scenario:**

- Sophie had $4 in her money box at the start of this week.
- She is going to add $3 at the end of every week.

**Number pattern (sequence):** 4, 7, 10, 13, 16, …

The numbers in the pattern are called terms, so the first term is 4 and the second term is 7.

**A series of points in a table of values**, with the number pattern in the bottom row:

| **Number of weeks ($W$)** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **Dollars in money box ($D$)** | 4 | 7 | 10 | 13 | 16 |

+3 +3 +3 +3

**A written description of the rule** that connects every number in the top row of the table with a number in the bottom row, such as:

'To find a value for $D$, choose a value for $W$, multiply it by three then add 4.'

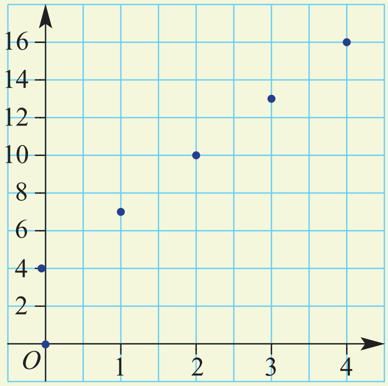
In situations such as these, the pronumerals $W$ and $D$ are often called variables.

A **mathematical equation** that connects every number in the top row of the table with the number below it.

'To find a value for $D$, choose a value for $W$, multiply it by three then add 4' can be written as:

$W \times 3 + 4 = D$ or $3W + 4 = D$ or, more commonly, $D = 3W + 4$

A **series of points on the Cartesian plane** that form a straight line:



## Exercise 7B

**Understanding**

**Drilling for Gold 7B2**

**1** Write the missing values in each table for the given equations.

**a** $y = x + 2$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 2 | | 4 | |

**b** $y = 2 \times x$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | | 2 | 4 | |

**c** $y = x - 7$

| $x$ | 4 | 5 | 6 | 7 |
|---|---|---|---|---|
| $y$ | | –2 | | 0 |

**d** $y = 2 \times x + 4$

| $x$ | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| $y$ | 4 | | 12 | |

**e** $y = 3 \times x - 8$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | –8 | | | 1 |

**f** $y = x \div 2 - 1$

| $x$ | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| $y$ | –1 | | | 2 |

**2** If $y = 2x - 5$, find the value of $y$ for these values of $x$.

**a** $x = 4$
**b** $x = 3$
**c** $x = 2$
**d** $x = 1$
**e** $x = 0$
**f** $x = -1$
**g** $x = -2$
**h** $x = -3$

Remember:
(pos.) × (neg.) = (neg.)
e.g. $2 \times (-1) = -2$

**3** If $y = -3x + 1$, find the value of $y$ for these values of $x$.

**a** $x = 0$
**b** $x = 1$
**c** $x = 4$
**d** $x = 7$
**e** $x = -1$
**f** $x = -3$
**g** $x = -10$
**h** $x = -50$

Remember:
(neg.) × (neg.) = (pos.)
e.g. $-3 \times (-1) = 3$

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Fluency

## Example 3 Constructing tables using positive numbers

The equation connecting the distance travelled ($d$ km) and time ($t$ hours) is $d = 60t$.

**a** Construct a table of values using $t = \{0, 1, 2, 3, 4\}$.
**b** What distance is travelled in the first 3 hours?
**c** How long does it take to travel 90 km?

**Solution** | **Explanation**

**a**

| $t$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $d$ | 0 | 60 | 120 | 180 | 240 |

Substitute each value of $t$ into the rule $d = 60t$.
For example, when $t = 3$, $d = 60 \times 3 = 180$.

**b** 180 km

The value of $d$ at $t = 3$ is 180.

**c** 1.5 hours

60 km is travelled every hour, so 1.5 hours is required for 90 km.

**4** The equation connecting the distance travelled ($d$ km) and time ($t$ hours) is given by $d = 40t$.
- **a** Construct a table of values using $t = \{0, 1, 2, 3, 4\}$.
- **b** What distance is travelled in the first 3 hours?
- **c** How long does it take to travel 80 km?

For every hour, 40 km is travelled.

**5** The equation connecting the volume of a tank ($V$ litres) after $t$ minutes is given by $V = 20t + 1000$.
- **a** Construct a table of values using $t = \{0, 1, 2, 3, 4, 5\}$.
- **b** What is the volume at the end of the 4th minute?
- **c** How long does it take for the volume to reach 1100 litres?

## Example 4 Constructing tables using negative numbers

For the given equation, construct a table of values for $x = \{-2, -1, 0, 1, 2\}$.

**a** $y = 2x - 3$ **b** $y = -x + 4$

**Solution** | **Explanation**

**a**

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | −7 | −5 | −3 | −1 | 1 |

If $y = 2x - 3$ then when $x = -2$, $y = 2 \times (-2) - 3$
$$= -4 - 3$$
$$= -7$$

Repeat for other values of $x$.

**Solution**

**b**

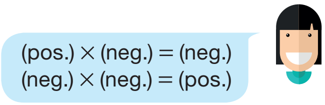
| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | $6$ | $5$ | $4$ | $3$ | $2$ |

**Explanation**

If $y = -x + 4$ then when $x = -2$, $y = -(-2) + 4$
$= 2 + 4$
$= 6$

Repeat for other values of $x$.

**6** For the given equations, complete the given tables.



**a** $y = 3x$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |

**b** $y = x - 2$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |

**c** $y = 2x + 1$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |

**d** $y = 2x - 3$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |

**e** $y = -x + 2$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |

**f** $y = -x - 1$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |

**g** $y = -2x - 1$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |

**h** $y = -4x + 2$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |

**i** $y = -6x - 11$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |

**Problem-solving and Reasoning**

**7** To hire a car costs \$70 per day so the rule for the cost (\$$C$) for $n$ days is $C = 70n$.

**a** What is the cost for:

**i** $n = 2$ days? **ii** $n = 10$ days?

**b** What does it cost to hire the car for 2 weeks?

**c** How long can you hire the car if you have \$280 to spend?

**8** Consider the equation $y = 3x - 4$.

- **a** What is the value of $y$ when:
  - **i** $x = 2$?
  - **ii** $x = 1$?
- **b** What $x$-value gives a $y$-value of:
  - **i** 5?
  - **ii** $-4$?
- **c** What is the smallest whole number for $x$ that makes $y$ positive?
- **d** What is the largest whole number for $x$ that makes $y$ negative?

**9** Lauren wants to buy a new phone at a cost of $794.

She has $50 in her savings account and puts $25 into her account at the end of each week.

Use the equation $t = 25w + 50$, where $t$ is the total amount in savings and $w$ is the number of weeks, to find out the following.

- **a** How much money will Lauren have in total after 10 weeks?
- **b** What is the minimum number of weeks that it will take Lauren to save the $794?
- **c** Write an equation to help Lauren if she started with $50 and saved $30 per week.
- **d** How many fewer weeks will it take Lauren to save $794 if she saves $30 per week?

## Enrichment: The sum shortcut

**10** To sum the first 3 consecutive positive whole number would mean to calculate $1 + 2 + 3$, which equals 6. Note also that $(3 \times 4) \div 2 = 6$.
In a similar way, $1 + 2 + 3 + 4 + 5$ can be calculated as $(5 \times 6) \div 2 = 15$.
In general, the formula is: Sum $= \dfrac{n \times (n+1)}{2}$ so for $n = 5$, Sum $= \dfrac{5 \times 6}{2} = 15$.

- **a** Use the formula Sum $= \dfrac{n \times (n+1)}{2}$ for:
  - **i** $n = 4$
  - **ii** $n = 8$
  - **iii** $n = 10$
- **b** Use the formula to find the sum of:
  - **i** the first six positive whole numbers ($n = 6$)
  - **ii** the first twelve positive whole numbers.
- **c** Use the formula to calculate:
  - **i** $1 + 2 + 3 + \ldots + 7$ $(n = 7)$
  - **ii** $1 + 2 + 3 + \ldots + 20$
  - **iii** $1 + 2 + 3 + \ldots + 100$

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# 7C Plotting straight line graphs

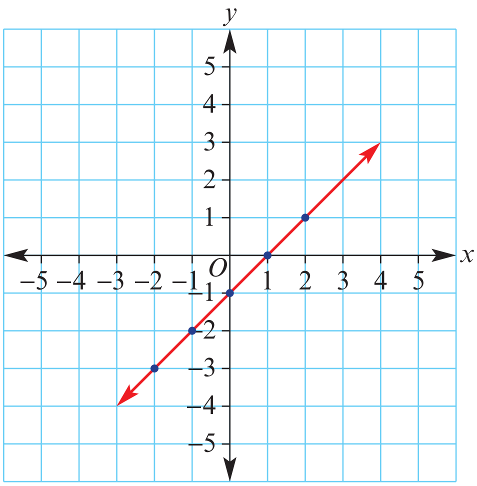
The next step in illustrating the relationship between two changing quantities is to plot points to form a graph. The points are taken from a table of values and plotted on a number plane. If the points form a single straight line, the relationship is said to be linear.

**Equation:** $y = x - 1$

**Table of values:**

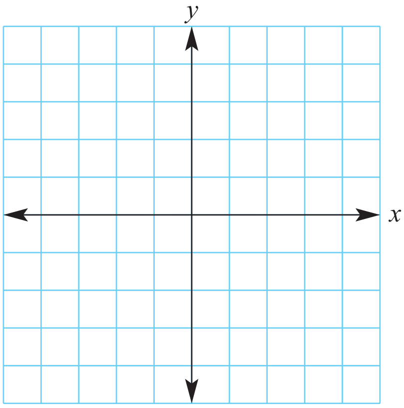
| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $y$ | –1 | 0 | 1 |

**Graph:**



## Let’s start: Which set of points forms a straight line?

Consider the set of points $(x, y)$ from these three tables of values.



**A** $y = 1 \div x$

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | –0.5 | –1 | ☒ | 1 | 0.5 |

**B** $y = x^2$

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 4 | 1 | 0 | 1 | 4 |

**C** $y = 2x$

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | –4 | –2 | 0 | 2 | 4 |

- If each set is plotted on a number plane, which set would form a single straight line?
- What do you notice about the values in the table that gives a straight line graph?

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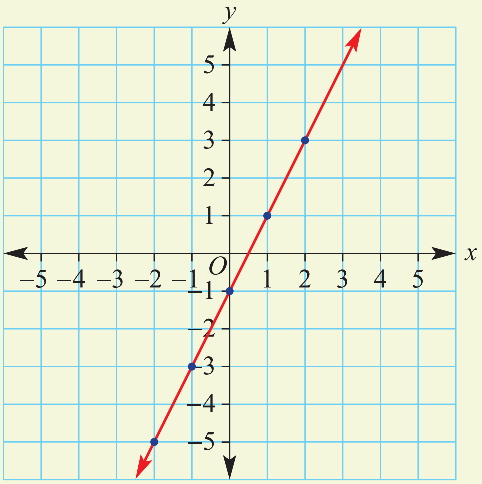
## Key ideas

- A **linear** relationship gives a straight line graph.
  $y = 2x - 1$

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | –5 | –3 | –1 | 1 | 3 |



**Linear** A linear graph in two dimensions is a straight line

- To draw a linear graph using an equation:
  - construct a table of values finding a $y$-coordinate for each given $x$-coordinate
    Substitute each $x$-coordinate into the rule
  - plot the points given in the table on a set of axes
  - draw a line through the points to complete the graph
  - put an arrow on both ends of the line.

## Exercise 7C

Understanding

**1** For the equation $y = 2x + 3$, find the $y$-coordinate for these $x$-coordinates.

**a** 1 **b** 2 **c** 0 **d** –1
**e** –5 **f** –7 **g** 11 **h** –12

**2** Write the missing number in these tables for the given equations.

**a** $y = 2x$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 0 |  | 4 | 6 |

**b** $y = x - 3$

| $x$ | –1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | –4 | –3 |  | –1 |

**c** $y = 5x + 2$

| $x$ | –3 | –2 | –1 | 0 |
|---|---|---|---|---|
| $y$ |  | –8 | –3 | 2 |

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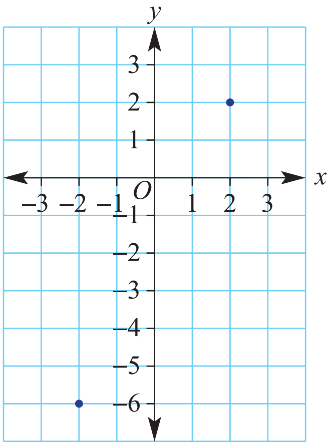
**3** Complete the graph to form a straight line from the given equation and table. Two points have been plotted for you.

$y = 2x - 2$

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | $-6$ | $-4$ | $-2$ | $0$ | $2$ |



For (–1, 4) move 1 left and 4 down. For (0, –2) just move 2 down from the origin (0, 0).

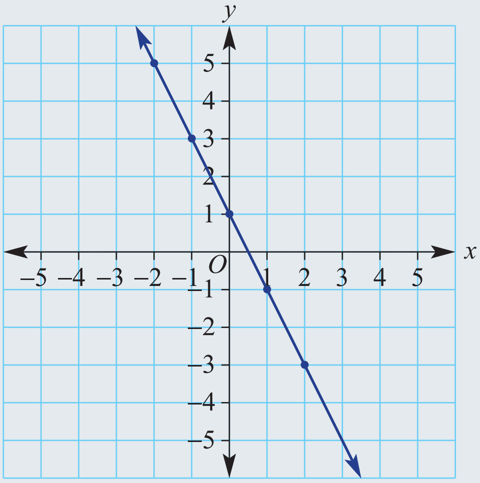
**Fluency**

## Example 5 Plotting a graph from a table

Plot a graph from this table of values.

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | $5$ | $3$ | $1$ | $-1$ | $-3$ |

**Solution**



**Explanation**

Plot the five points (–2, 5), (–1, 3), (0, 1), (1, –1) and (2, –3).

Then join them to form a straight line.

Put an arrow on both ends.

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**4** Plot a graph from these tables of values.

**a**

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 2 | 1 | 0 | –1 | –2 |

**b**

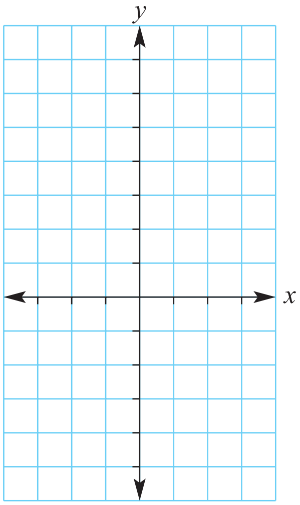
| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | –3 | –2 | –1 | 0 | 1 |

**c**

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |

**d**

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | –5 | –3 | –1 | 1 | 3 | 5 | 7 |



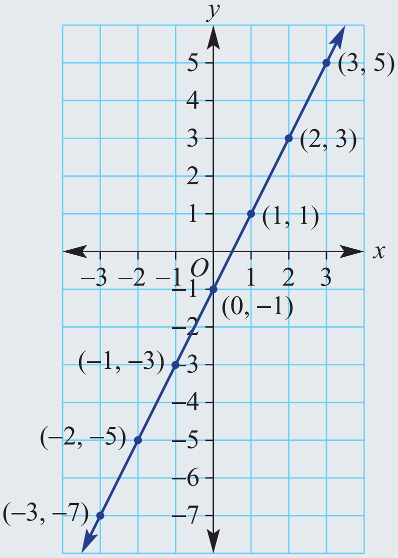
## Example 6 Plotting a graph from a rule

For the rule $y = 2x - 1$, construct a table and draw a graph.

**Drilling for Gold 7C1**

**Solution**

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | –7 | –5 | –3 | –1 | 1 | 3 | 5 |



**Explanation**

Substitute each $x$-coordinate in the table into the rule to find the $y$-coordinate.

Plot each point (–3, –7), (–2, –5)… and join them to form the straight line graph.
Put arrows on both ends.

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Skillsheet 7A

**5** For each equation construct a table then plot and draw a graph. Use a table like the one shown here for each equation.

Substitute each value of $x$ into the rule to find the value of $y$. Then plot this pair on a graph.

| $x$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**a** $y = x + 1$
**b** $y = x - 2$
**c** $y = 2x - 3$
**d** $y = 2x + 1$
**e** $y = -2x + 3$
**f** $y = -3x - 1$
**g** $y = -x$
**h** $y = -x + 4$

**Problem-solving and Reasoning**

**6** Decide if the following tables of values would give a straight line graph.

**a**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | 2 | 4 | 6 | 8 |

**b**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 0 | 3 | 6 | 9 | 12 |

**c**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 2 | 0 | 1 | 5 | 2 |

**d**

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 7 | 0 | 4 | 1 | 6 |

**e**

| $x$ | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y$ | 6 | −2 | 4 | 1 | 8 |

**f**

| $x$ | −2 | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | 2 | 1 | 0 | −1 | −2 | −3 |

**7** The distance a car travelled ($d$ km) is given by the rule $d = 80t$ where $t$ is time in hours.

**a** Complete this table of values.

| $t$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $d$ | 0 | | | |

**b** Plot the points on a graph. Use $t$ for time on the horizontal axis.
**c** How far does the car travel in 4 hours?
**d** How long would it take for the car to travel 400 km?

**8** **a** What is the minimum number of points needed to draw a graph of a straight line?

**b** Draw the graph of these rules by plotting only two points. Use $x = 0$ and $x = 2$.

**i** $y = x \div 2$ **ii** $y = 2x - 1$

**9** The graphs of $y = x$, $y = 3x$ and $y = -2x$ all pass through the origin $(0, 0)$. Explain why.

**10** It is possible to find the intersection point of two graphs by plotting each graph and observing the intersection. For each pair of equation, follow these steps.

**1** Construct a table of values for each equation.
**2** Plot to form two straight line graphs.
**3** Give the coordinates of the intersection point (if any).

**a** $y = x + 2$
$y = -x + 2$

**b** $y = 2x - 3$
$y = -x + 3$

**c** $y = 2x - 2$
$y = 2x + 1$

## Enrichment: Non-linear relationships

**11** **a** **i** Complete this table of values for $y = x^2$.
(Hint: $(-3)^2 = 9$, not $-9$)

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**ii** Plot the points and join them with a smooth curve.

**b** Repeat parts **a** and **b** using $y = x^2 - 4$.

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# 7D Finding the rule using a table of values

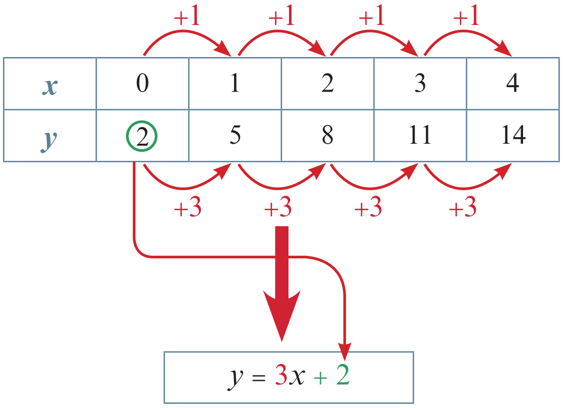
An equation is the best way to represent a linear relationship. In this section you will learn how to use the points in the table of values to write the rule in words and as an equation.

In the table to the right, note that:

- the numbers in the top row are increasing by 1
- the numbers in the bottom row are increasing by 3
- 3 divided by 1 gives 3
- the number in the bottom row under the 0 is $+2$.

These observations make it easy to 'find the rule'.

Look at the red and green numbers!



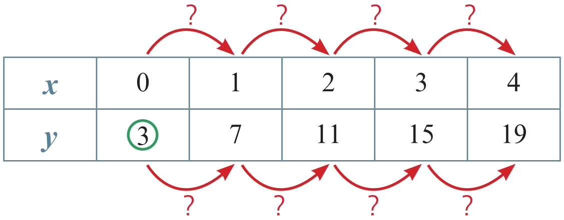
| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 2 | 5 | 8 | 11 | 14 |

$y = 3x + 2$

From table of values to equation

## Let's start: What's my rule?

Consider the table below.



| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 3 | 7 | 11 | 15 | 19 |

- Are the numbers in the top row increasing by 1? If yes, that is great!
- What is the pattern in the bottom row?
- What number is under the 0?
- The rule is one of the following equations. Which one is correct?
  **A** $y = 3x + 3$ **B** $y = 3x + 4$ **C** $y = 4x + 3$ **D** $y = 4x + 4$
- Copy and complete the following sentence.
  To find the value of $y$, choose a value for $x$, multiply by ____________________.

## Key ideas

- It is possible to use the table of values to express 'the rule' as an equation.
  - The equations usually look like these:

    $y = 3x + 2$ $\qquad y = 3x - 1$ $\qquad y = 3x$ $\qquad y = x + 3$

- If the numbers in the top row of the table are in sequence, the following hints make it simple to find the equation.

+1 +1 +1 +1

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | –1 | 3 | 7 | 11 | 15 |

+4 +4 +4 +4

  - The numbers in the top row are increasing by 1.
  - The numbers in the bottom row are increasing by 4.
  - 4 divided by 1 gives 4.
  - The number in the bottom row under the 0 is –1.
  - The equation is $y = 4x - 1$.
  - In words, the rule is 'To find the value of $y$, multiply $x$ by 4 then subtract 1.
- A rule must be true for every pair of numbers in the table of values. In the table above, if any number in the top row is multiplied by 4 then reduced by 1, the result is the number below it, in the bottom row.

## Exercise 7D

**Understanding**

**1** Match the rules **A**, **B** and **C** with the tables **a**, **b** and **c**.

**a**

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 0 | 2 | 4 | 6 |

**b**

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | –3 | –1 | 1 | 3 |

**c**

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 1 | 3 | 5 | 7 |

**A** $y = 2x - 3$ **B** $y = 2x + 1$ **C** $y = 2x$

**2** By how much does $y$ increase for each increase by 1 in $x$? If $y$ is decreasing give a negative answer.

**a**

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 3 | 5 | 7 | 9 |

**b**

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 1 | 0 | –1 | –2 |

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**c**

| | | | | |
|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 |
| $y$ | –4 | –6 | –8 | –10 |

**d**

| | | | | |
|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 |
| $y$ | 3 | 6 | 9 | 12 |

**3** For each of the tables in Question **2**, state the value of $y$ when $x = 0$.

**Fluency**

## Example 7 Finding equations from tables

Find the equations for these tables of values.

**a**

| | | | | |
|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 |
| $y$ | –1 | 2 | 5 | 8 |

**b**

| | | | | |
|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 |
| $y$ | 3 | 4 | 5 | 6 |

**Drilling for Gold**
**7D1**

| Solution | Explanation |
|---|---|
| **a** $y = 3x - 1$ | Top row is increasing by 1.<br>Bottom row is increasing by 3.<br>3 divided by 1 is 3.<br>Number under 0 is –1.<br>So $y = 3x - 1$. |
| **b** $y = x + 3$ | Top row is increasing by 1.<br>Bottom row is increasing by 1.<br>1 divided by 1 is 1.<br>Number under 0 is +3.<br>So $y = 1x + 3$. |

**4** Find the equations for these tables of values.

**a**

| | | | |
|---|---|---|---|
| $x$ | 0 | 1 | 2 |
| $y$ | 1 | 2 | 3 |

**b**

| | | | |
|---|---|---|---|
| $x$ | 0 | 1 | 2 |
| $y$ | 0 | 2 | 4 |

**c**

| | | | |
|---|---|---|---|
| $x$ | 0 | 1 | 2 |
| $y$ | 4 | 6 | 8 |

**d**

| | | | |
|---|---|---|---|
| $x$ | 0 | 1 | 2 |
| $y$ | –1 | 2 | 5 |

**e**

| | | | |
|---|---|---|---|
| $x$ | 0 | 1 | 2 |
| $y$ | 0 | 4 | 8 |

**f**

| | | | |
|---|---|---|---|
| $x$ | 0 | 1 | 2 |
| $y$ | 3 | 6 | 9 |

**5** Repeat Question **4**, but this time write the rule in words, starting with 'To find the value of $y$ ______'.

## Example 8 Finding equations when $y$ is decreasing

Find the equations for these tables of values.

**a**

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 0 | –1 | –2 | –3 |

**b**

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $y$ | 1 | –1 | –3 |

| Solution | Explanation |
|---|---|
| **a** $y = -x$ | Top row is increasing by 1.<br>Bottom row is 'increasing' by –1.<br>–1 divided by 1 is –1.<br>Number under 0 is 0.<br>So $y = -1x + 0$. |
| **b** $y = -2x + 1$ | Top row is increasing by 1.<br>Bottom row is 'increasing' by –2.<br>Number under 0 is +1.<br>So $y = -2x + 1$. |

**6** Find the equations for these tables of values.

**a**

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $y$ | 0 | –1 | –2 |

**b**

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $y$ | –1 | –2 | –3 |

**c**

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $y$ | 1 | 0 | –1 |

**d**

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $y$ | 6 | 4 | 2 |

**e**

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $y$ | 0 | –2 | –4 |

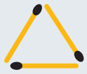
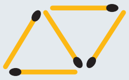
**f**

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $y$ | 1 | –2 | –5 |

**Problem-solving and Reasoning**

## Example 9 Finding equations for patterns

If $x$ = number of triangles and $y$ = number of matchsticks, use a table to help find an equation for this pattern.

| Shape 1 | Shape 2 | Shape 3 | Shape 4 |
|---|---|---|---|
|  |  |  |  |

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**Solution**

| | | | | |
|---|---|---|---|---|
| $x$ | 1 | 2 | 3 | 4 |
| $y$ | 3 | 5 | 7 | 9 |

$y = 2x + 1$

**Explanation**

Extend the table and work backwards to 0.
The number below 0 is 1.

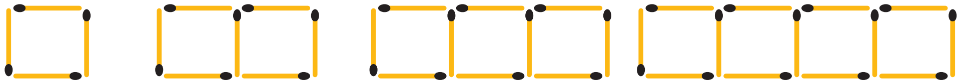
| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 |
| $y$ | 1 | 3 | 5 | 7 | 9 |

$-2$ $+2$ $+2$ $+2$

**7** Write the equations for these matchstick patterns.

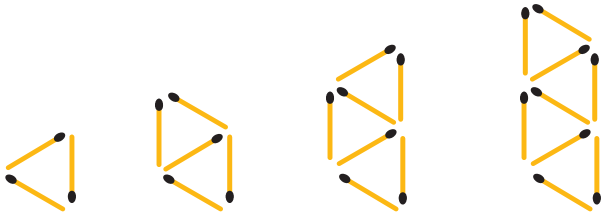
**a** $x$ = number of squares $y$ = number of matchsticks

Shape 1 Shape 2 Shape 3 Shape 4



**b** $x$ = number of triangles $y$ = number of matchsticks

Shape 1 Shape 2 Shape 3 Shape 4



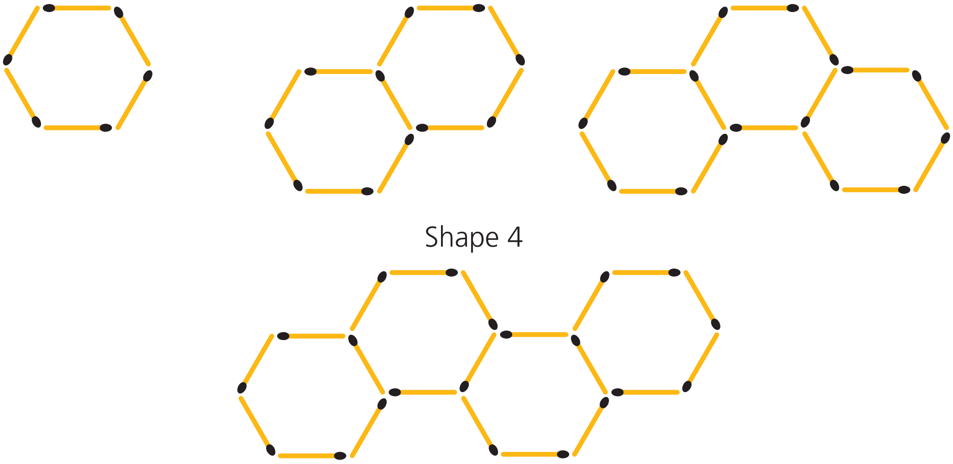
First draw a table and fill it in.

| | | | | |
|---|---|---|---|---|
| $x$ | 1 | 2 | 3 | 4 |
| $y$ | 4 | 7 | | |

**c** $x$ = number of hexagons $y$ = number of matchsticks

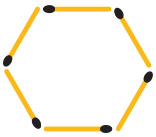
Shape 1 Shape 2 Shape 3

Shape 4

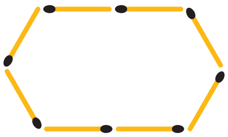


**d** $x$ = number of matchsticks on top row $y$ = number of matchsticks

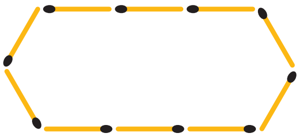
Shape 1



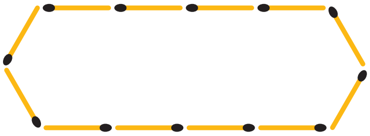
Shape 2



Shape 3



Shape 4



**8** An equation is of the form $y = 3x + \square$. Find the value of $\square$, if a pair of $(x, y)$ is:

**a** $(1, 4)$ **b** $(-1, 0)$ **c** $(-2, 1)$ **d** $(0, 0)$

**9** Look at this table of values.

| | | | | |
|---|---|---|---|---|
| $x$ | 0 | 2 | 4 | 6 |
| $y$ | −2 | 0 | 2 | 4 |

**a** The increase in $y$ for each unit increase in $x$ is not 2. Explain why.

**b** If the pattern is linear, state the increase in $y$ for each increase by 1 in $x$.

**c** Write the equation for the relationship.

**d** Find the equations for these tables.

**i**

| | | | | | |
|---|---|---|---|---|---|
| $x$ | −4 | −2 | 0 | 2 | 4 |
| $y$ | −5 | −1 | 3 | 7 | 11 |

**ii**

| | | | | | |
|---|---|---|---|---|---|
| $x$ | −3 | −1 | 1 | 3 | 5 |
| $y$ | −10 | −4 | 2 | 8 | 14 |

**iii**

| | | | | | |
|---|---|---|---|---|---|
| $x$ | −6 | −3 | 0 | 3 | 6 |
| $y$ | 15 | 9 | 3 | −3 | −9 |

**iv**

| | | | | | |
|---|---|---|---|---|---|
| $x$ | −10 | −8 | −6 | −4 | −2 |
| $y$ | 20 | 12 | 4 | −4 | −12 |

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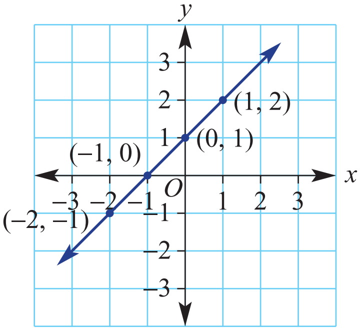
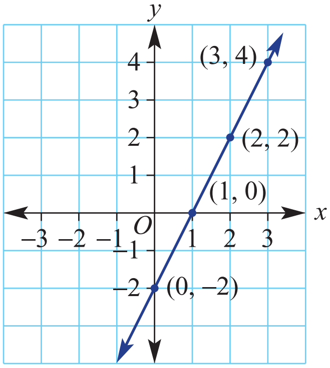
## Enrichment: Equations from graphs

**10** Find the equations for these graphs by first constructing a table of $(x, y)$ values.

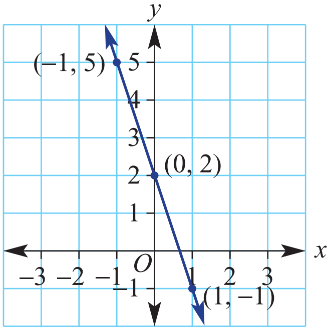
**a**



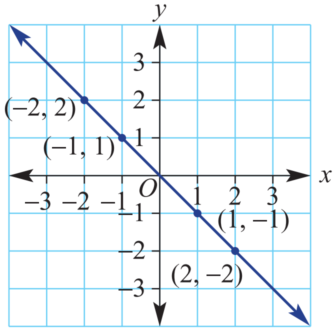
**b**

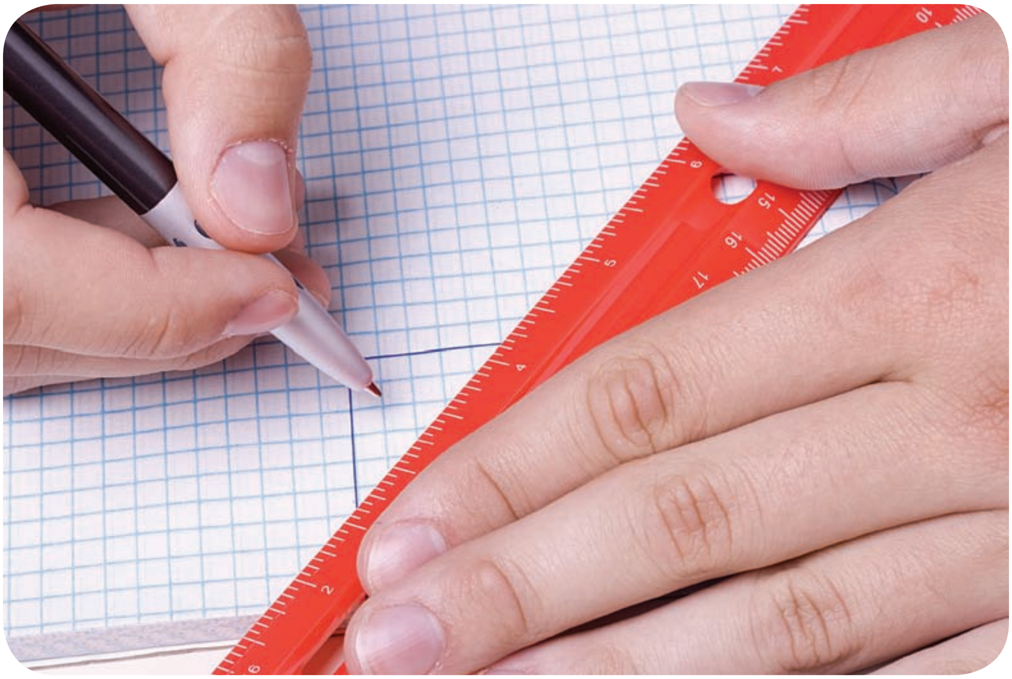


**c**



**d**





# 7E Solving linear equations using graphical techniques

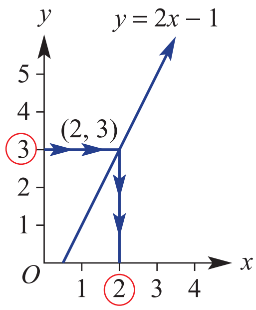
**EXTENSION**

The diagram to the right shows the line $y = 2x - 1$.

- Go to 3 on the $y$-axis.
- Go across to the line $y = 2x - 1$.
- Go down to the $x$-axis, at $x = 2$.

  When $y = 3$, $x = 2$.

  We have used the graph $y = 2x - 1$ to solve $2x - 1 = 3$.



## Let's start: Matching equations and solutions

When a value is substituted into an equation and it makes the equation true (LHS = RHS), then that value is a solution to that equation.

**a** From the lists below, match each equation with a solution. Some equations have more than one solution.

| **Equations** | |
|---|---|
| $2x - 4 = 8$ | $y = x + 4$ |
| $3x + 2 = 11$ | $y = 2x - 5$ |
| $y = 10 - 3x$ | $5x - 3 = 2$ |

| **Possible solutions** | | | |
|---|---|---|---|
| $x = 1$ | $(1, 5)$ | $x = 2$ | $(3, 1)$ |
| $x = -1$ | $x = 6$ | $(2, -1)$ | $(2, 6)$ |
| $(-2, -9)$ | $(-2, 16)$ | $x = 3$ | $(2, 4)$ |

**b** Which two equations have the same solution and what is this solution?

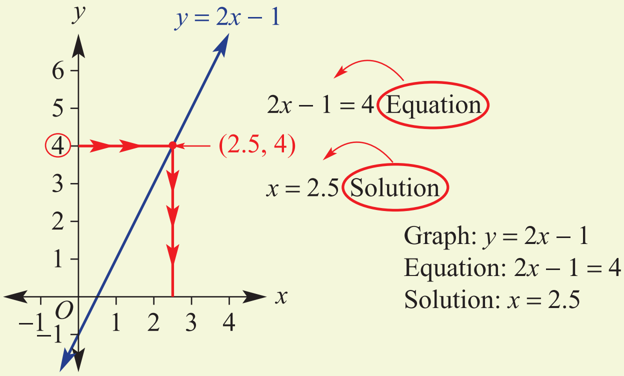
**c** List the equations that have only one solution. What is a common feature of these equations?

**d** List the equations that have more than one solution. What is a common feature of these equations?

## Key ideas

- The $x$-coordinate of each point on the graph of a straight line is a solution to a particular linear equation.
  - A particular linear equation is formed by substituting a chosen $y$-coordinate into a linear relationship.

    E.g. if $y = 2x - 1$ and $y = 4$, then the linear equation is $2x - 1 = 4$.



Graph: $y = 2x - 1$
Equation: $2x - 1 = 4$
Solution: $x = 2.5$

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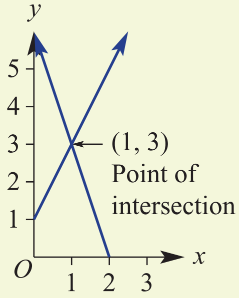
  - The solution to this equation is the $x$-coordinate of the point with the chosen $y$-coordinate.
    E.g. the point $(2.5, 4)$ shows that $x = 2.5$ is the solution to $2x - 1 = 4$.
- The point of intersection of two straight lines is the only solution that satisfies both equations.
  - The point of intersection is the shared point where two straight lines cross each other.
  - This is the only point with coordinates that make both equations true.
    E.g. $(1, 3)$ is the only point that makes both $y = 6 - 3x$ and $y = 2x + 1$ true.



## Exercise 7E

**Understanding**

**1** Use the given rule to complete this table and then plot and join the points to form a straight line. $y = 2x - 1$.

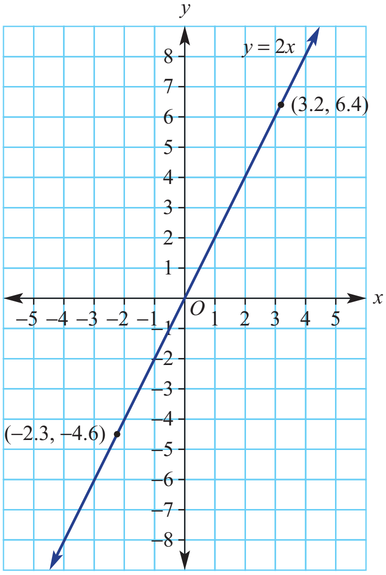
| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |

**2** Substitute each given $y$-coordinate into the rule $y = 2x - 3$, and then solve the equation algebraically to find the $x$-coordinate.

**i** $y = 7$ **ii** $y = -5$

**3** State the coordinates $(x, y)$ of the point on this graph of $y = 2x$ where:

**a** $2x = 4$ (i.e. $y = 4$)
**b** $2x = 6.4$
**c** $2x = -4.6$
**d** $2x = 7$
**e** $2x = -14$
**f** $2x = 2000$
**g** $2x = 62.84$
**h** $2x = -48.602$
**i** $2x =$ *any number* (worded answer)



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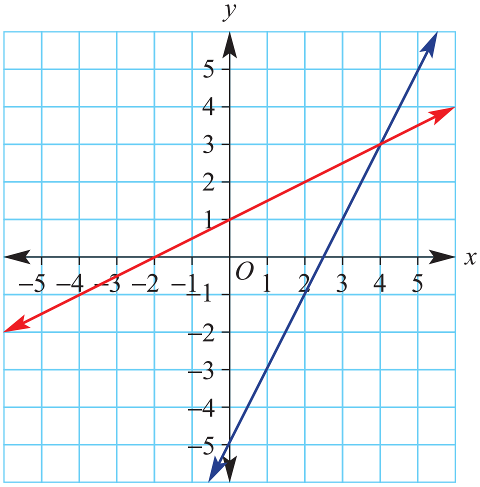
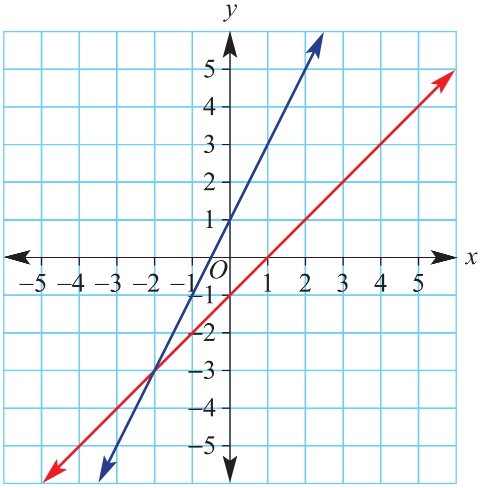
**4** For each of these graphs write down the coordinates of the point of intersection (i.e. the point where the lines cross over each other).

**a**



**b**

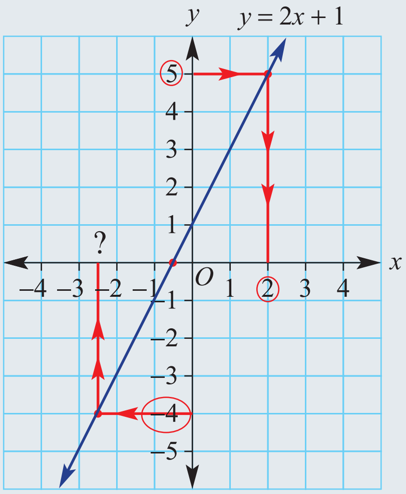


**Fluency**

### Example 10 Using a linear graph to solve an equation

Use the graph of $y = 2x + 1$ shown here to solve each of the following equations.

**a** $2x + 1 = 5$
**b** $2x + 1 = 0$
**c** $2x + 1 = -4$



| **Solution** | **Explanation** |
|---|---|
| **a** $x = 2$ | Locate the point on the line with $y$-coordinate 5. The $x$-coordinate of this point is 2 so $x = 2$ is the solution to $2x + 1 = 5$. |
| **b** $x = -0.5$ | Locate the point on the line with $y$-coordinate 0. The $x$-coordinate of this point is $-0.5$ so $x = -0.5$ is the solution to $2x + 1 = 0$. |
| **c** $x = -2.5$ | Locate the point on the line with $y$-coordinate $-4$. The $x$-coordinate of this point is $-2.5$ so $x = -2.5$ is the solution to $2x + 1 = -4$. |

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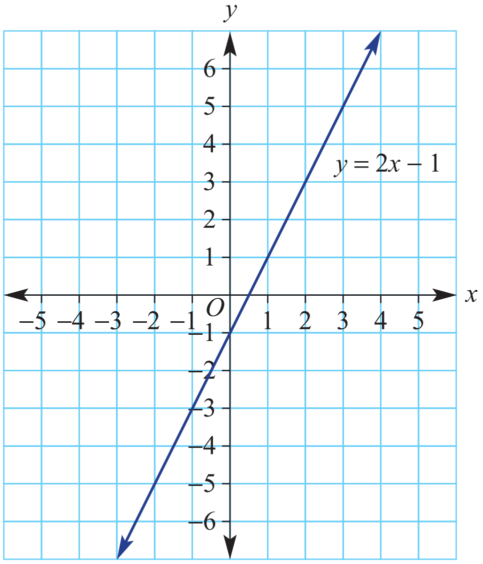
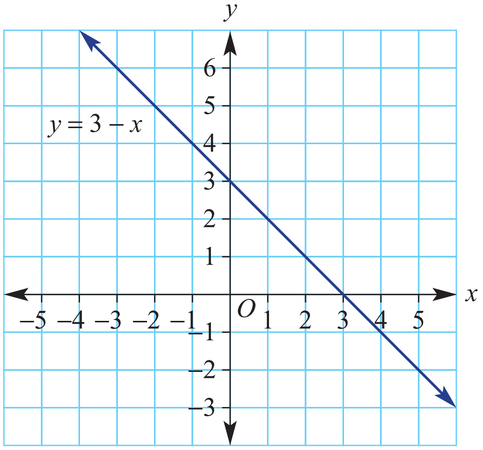
**5** Use the graph of $y = 2x - 1$, shown here, to find the solution to each of these equations.

- **a** $2x - 1 = 3$
- **b** $2x - 1 = 0$
- **c** $2x - 1 = 5$
- **d** $2x - 1 = -6$
- **e** $2x - 1 = -4$



**6** Use the graph of $y = 3 - x$, shown here, to solve each of the following equations.

- **a** $3 - x = 5.5$
- **b** $3 - x = 0$
- **c** $3 - x = 3.5$
- **d** $3 - x = -1$
- **e** $3 - x = -2$



**7** Graph each pair of lines on the same set of axes and read off the point of intersection.

**a** $y = 2x - 1$

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

$y = x + 1$

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

**b** $y = -x$

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

$y = x + 2$

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

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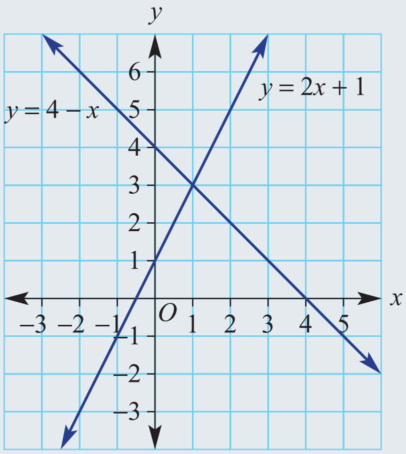
## Example 11 Using the point of intersection of two lines to solve an equation

Use the graph of $y = 4 - x$ and $y = 2x + 1$, shown here, to answer these questions.

- **a** Write two equations that each have $x = -2$ as a solution.
- **b** Write four solutions $(x, y)$ for the line with equation $y = 4 - x$.
- **c** Write four solutions $(x, y)$ for the line with equation $y = 2x + 1$.
- **d** Write the solution $(x, y)$ that is true for both lines and show that it satisfies both line equations.
- **e** Solve the equation $4 - x = 2x + 1$.



| **Solution** | **Explanation** |
|---|---|
| **a** $4 - x = 6$<br>$2x + 1 = -3$ | $(-2, 6)$ is on the line $y = 4 - x$ so $4 - x = 6$ has solution $x = -2$.<br>$(-2, -3)$ is on the line $y = 2x + 1$ so $2x + 1 = -3$ has solution $x = -2$. |
| **b** $(-2, 6)$ $(-1, 5)$ $(1, 3)$ $(4, 0)$ | Many correct answers. Each point on the line $y = 4 - x$ is a solution to the equation for that line. |
| **c** $(-2, -3)$ $(0, 1)$ $(1, 3)$ $(2, 5)$ | Many correct answers. Each point on the line $y = 2x + 1$ is a solution to the equation for that line. |
| **d** $(1, 3)$ $\quad (1, 3)$<br>$y = 4 - x$ $\quad y = 2x + 1$<br>$3 = 4 - 1$ $\quad 3 = 2 \times 1 + 1$<br>$3 = 3$ True $\quad 3 = 3$ True | The point of intersection $(1, 3)$ is the solution that satisfies both equations.<br>Substitute $(1, 3)$ into each equation and show that it makes a true equation (LHS = RHS). |
| **e** $x = 1$ | The solution to $4 - x = 2x + 1$ is the $x$-coordinate at the point of intersection. The value of both rules is equal for this $x$-coordinate. |

8 Use the graph of $y = 5 - 2x$ and $y = x + 2$, shown here, to answer the following questions.

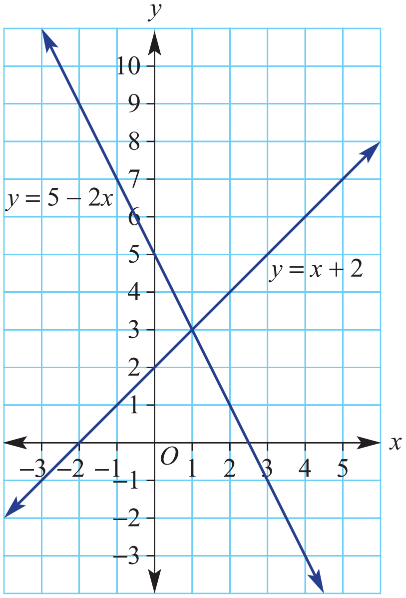
**a** Write two equations that each have $x = -1$ as a solution.

**b** Write four solutions $(x, y)$ for the equation $y = 5 - 2x$.

**c** Write four solutions $(x, y)$ for the equation $y = x + 2$.

**d** Write the solution $(x, y)$ that is true for both lines and show that it satisfies both line equations.

**e** Solve the equation $5 - 2x = x + 2$ from the graph.



9 Jessica and Max had a 10-second running race.

- Max started at the starting line and ran 6 m/s.
- Jessica was given a 10 m head-start and ran at 4 m/s.

**a** Copy and complete this table.

| Time ($t$) in seconds | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Max's distance ($d$) in metres from the starting line | 0 | | | | | | | | | | |
| Jessica's distance ($d$) in metres from the starting line | 10 | | | | | | | | | | |

**b** Plot these points on a distance/time graph and join to form two straight lines labelling them 'Jessica' and 'Max'.

**c** Find the rule linking distance $d$ and time $t$ for Max.

**d** Using the rule for Max's race, write an equation that has the solution:

**i** $t = 3$ **ii** $t = 5$ **iii** $t = 8$

**e** Find the rule linking distance $d$ and time $t$ for Jessica.

**f** Using the rule for Jessica's race, write an equation that has the solution:

**i** $t = 3$ **ii** $t = 5$ **iii** $t = 8$

**g** Write the solution $(t, d)$ that is true for both distance equations and show that it satisfies both equations.

**h** Explain what happened in the race at the point of intersection and for each athlete state the distance from the starting line and time taken.

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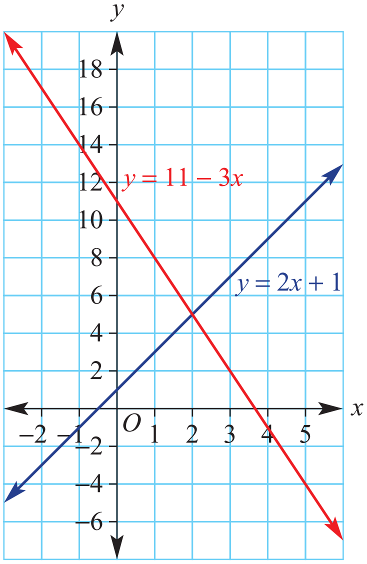
**10** This graph shows two lines with equations $y = 11 - 3x$ and $y = 2x + 1$.



**a** Copy and complete the coordinates of each point that is a solution for the given linear equation.

- **i** $y = 11 - 3x$
  (–2, ?), (–1, ?) (0, ?) (1, ?) (2, ?) (3, ?) (4, ?) (5, ?)
- **ii** $y = 2x + 1$
  (–2, ?), (–1, ?) (0, ?) (1, ?) (2, ?) (3, ?) (4, ?) (5, ?)

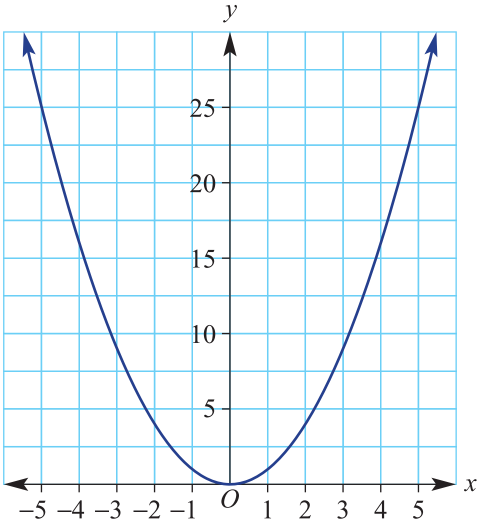
**b** State the coordinates of the point of intersection and show it is a solution to both equations.

**c** Explain why the point of intersection is the only solution that satisfies both equations.

## Enrichment: More than one solution

**11 a** Use this graph of $y = x^2$ to solve the following equations.

- **i** $x^2 = 4$
- **ii** $x^2 = 9$
- **iii** $x^2 = 16$
- **iv** $x^2 = 25$



**b** Explain why there are two solutions to each of the equations in question **a** above.

**c** Give one reason why the graph of $y = x^2$ does *not* give a solution to the equation $x^2 = -9$.

**d** List three more equations of the form $x^2 =$ 'a number' that *can't* be solved from the graph of $y = x^2$.

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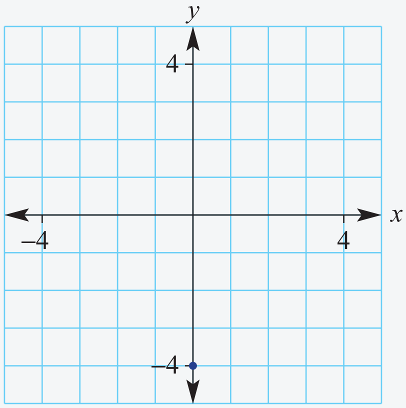
## Puzzles and games

**1** Plot and join the set of points in order to form the picture. What is the picture of?
(4, 1), (2, 1), (1, –1), (0, –1), (0, 1), (–3, 1), (–3, 3), (–2, 2), (3, 2), (4, 1)



**2** Find the equation linking $y$ and $x$.

**a**

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | –15 | –11 | –7 | –3 | 1 |

**b**

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 10 | 9 | 8 | 7 | 6 |

**c**

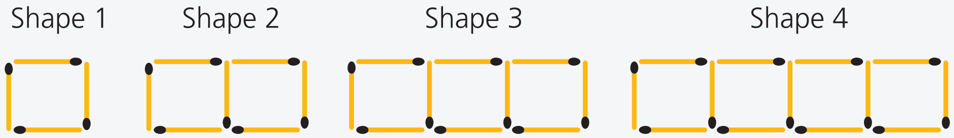
| $x$ | –10 | –9 | –8 | –7 | –6 |
|---|---|---|---|---|---|
| $y$ | –100 | –95 | –90 | –85 | –80 |

**d**

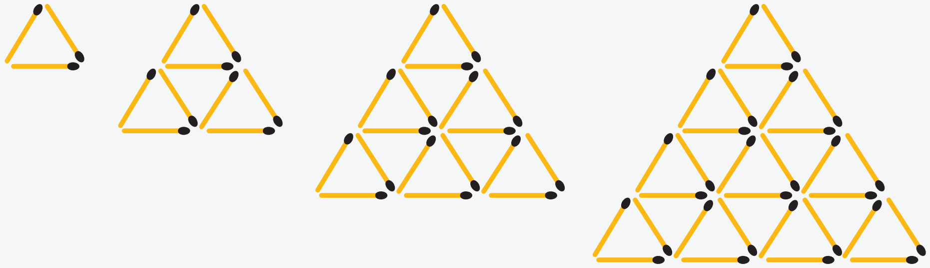
| $x$ | 0 | 3 | 6 | 9 | 12 |
|---|---|---|---|---|---|
| $y$ | –10 | –7 | –4 | –1 | 2 |

**3** How many matchsticks would be needed for the 10th shape in each pattern.

**a**



**b**



**4** A trekker hikes along a track at 3 km per hour. Two hours later, a second trekker sets off on the same track at 5 km per hour. How long is it before the second trekker to catches up with the first?

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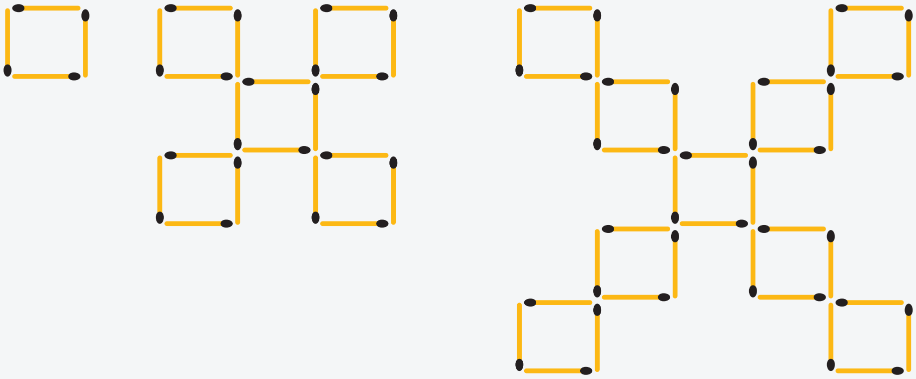
**5** Two cars travel towards each other on a 100 km stretch of road. One car travels at 80 km per hour and the other at 70 km per hour. If they set off at the same time, how long will it be before the cars meet?

**6** Find the number of matchsticks needed in the 100th diagram in the pattern given below. The first three diagrams in the pattern are given.



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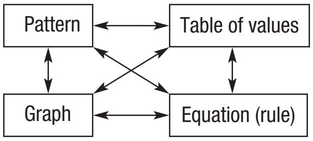
# Chapter summary

## Linear relationships

### Four different representations of linear relationships



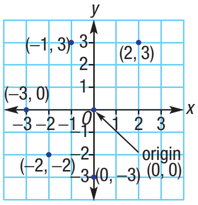
Pattern

Table of values

Graph

Equation (rule)

### Cartesian plane (number plane)



### Equation and table

$y = -2x + 3$

If $x = -2$

$y = -2 \times (-2) + 3$

$= 4 + 3$

$= 7$

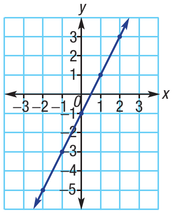
| | | | | | |
|---|---|---|---|---|---|
| $x$ | –2 | –1 | 0 | 1 | 2 |
| $y$ | 7 | 5 | 3 | 1 | –1 |

### Plotting straight line graphs

Rule $y = 2x - 1$

Table

| | | | | | |
|---|---|---|---|---|---|
| $x$ | –2 | –1 | 0 | 1 | 2 |
| $y$ | –5 | –3 | –1 | 1 | 3 |



### Finding the equation from the table

+1 +1 +1 +1

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 |
| $y$ | 3 | 5 | 7 | 9 | 11 |

+2 +2 +2 +2

Top row increases by 1

Bottom row increases by 2

$2 \div 1 = 2$

Number under 0 is 3

Equation is $y = 2x + 3$

### Applications

- Distance increases by 20 km per hour.

| | | | | |
|---|---|---|---|---|
| $t$ | 0 | 1 | 2 | 3 |
| $d$ | 0 | 20 | 40 | 60 |

$d = 20t$

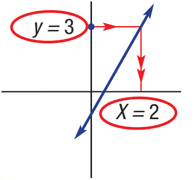
- Volume decreases from 1000 L by 200 L per minute.

| | | | | | | |
|---|---|---|---|---|---|---|
| $t$ | 0 | 1 | 2 | 3 | 4 | 5 |
| $V$ | 1000 | 800 | 600 | 400 | 200 | 0 |

$V = -200t + 1000$

### Solving equations using graphs

Graph of $y = 2x - 1$



Solve $2x - 1 = 3$

- Go to $y = 3$
- Across to line
- Down to x-axis
- $x = 2$ is solution

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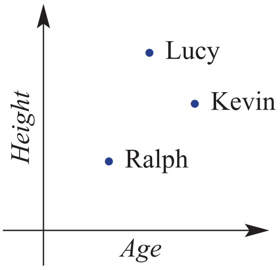
**Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.**

## Multiple-choice questions



**1** This graph shows the relationship between the height and age of 3 people. Who is the tallest person?

**A** Ralph
**B** Lucy
**C** Kevin
**D** Lucy and Ralph together
**E** Kevin and Lucy together

**2** The name of the point (0, 0) on a number (Cartesian) plane is:

**A** $y$-intercept **B** gradient **C** origin
**D** axis **E** $x$-intercept

**3** Which point is not in line with the other points? $A(-2, 3)$, $B(-1, 2)$, $C(0, 0)$, $D(1, 0)$, $E(2, -1)$

**A** $A$ **B** $B$ **C** $C$ **D** $D$ **E** $E$

**4** If $d = 10t$, then the value of $d$ when $t = 6$ is:

**A** 600 **B** 6 **C** 10 **D** 60 **E** 100

**5** The equation for this table of values is:

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $y$ | 1 | 2 | 3 |

**A** $y = x$ **B** $y = x + 1$ **C** $y = -x$ **D** $y = -x - 1$ **E** $y = 1$

**6** Which line passes through the point (3, 2)?

**A** $y = 2x + 1$ **B** $y = 2x - 1$ **C** $y = 2x$
**D** $y = 2x - 4$ **E** $-y = 2x + 4$

**7** The equation for this table of values is:

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $y$ | −3 | −4 | −5 |

**A** $y = -x - 3$ **B** $y = x - 3$ **C** $y = -x + 3$
**D** $y = x + 3$ **E** $-(x - 1)$

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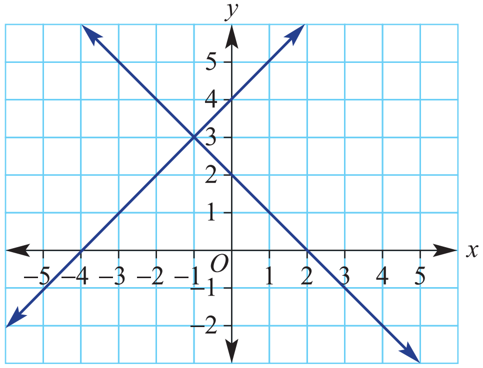
**8** Which point is the point of intersection of the two lines graphed here?

**A** (–4, 0)
**B** (2, 0)
**C** (0, 4)
**D** (–1, 3)
**E** (0, 2)



**9** Which of the lines given below would pass through the $y$-axis at (0, 3)?

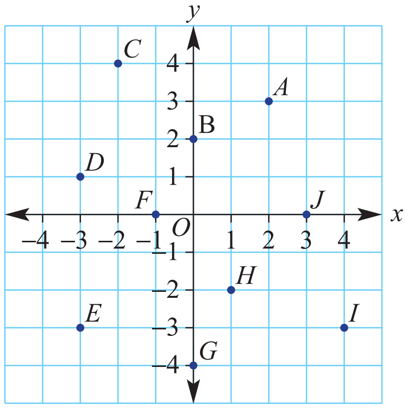
**A** $y = 3x$
**B** $y = x - 3$
**C** $y = 3x + 1$
**D** $y = -3x$
**E** $y = x + 3$

**10** The water level in a dam starts at 300 cm deep and decreases by 5 cm every day for 10 days. The water level after 7 days would be:

**A** 35 cm **B** 275 cm **C** 230 cm **D** 160 cm **E** 265 cm

## Short-answer questions

**1** Write the coordinates of all the points $A$–$J$ in the graph below.



**2** Use the equations to complete the missing values.

**a** $y = x - 1$

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | –3 | | | | |

**b** $y = 2x$

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | | –2 | | | |

**c** $y = 3x + 1$

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | | –2 | | | |

**d** $y = -x + 1$

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | | 2 | | | |

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**3** For each equation, create a table using $x$ values from –3 to 3 and plot to draw a straight line graph.

**a** $y = 2x$
**b** $y = 3x - 1$
**c** $y = 2x + 2$
**d** $y = -x + 1$
**e** $y = -2x + 3$
**f** $y = 3 - x$

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

**4** Write the equations for these tables of values.

**a**

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | –3 | –1 | 1 | 3 | 5 |

**b**

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | –4 | –1 | 2 | 5 | 8 |

**c**

| $x$ | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| $y$ | 6 | 7 | 8 | 9 | 10 |

**d**

| $x$ | –3 | –2 | –1 | 0 | 1 |
|---|---|---|---|---|---|
| $y$ | 4 | 3 | 2 | 1 | 0 |

**e**

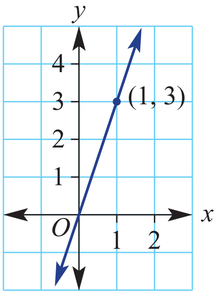
| $x$ | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y$ | 3 | –1 | –5 | –9 | –13 |

**f**

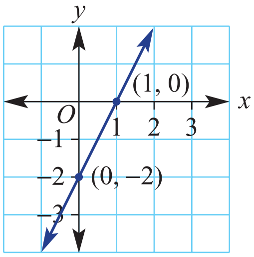
| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $y$ | 8 | 7 | 6 | 5 | 4 |

**5** Find the equation of each of these lines.

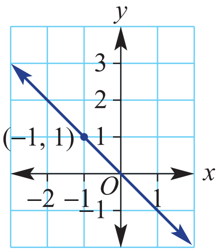
**a**



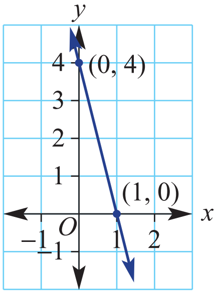
**b**



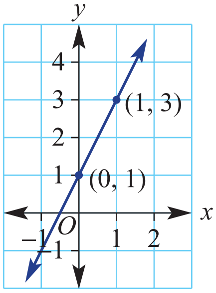
**c**



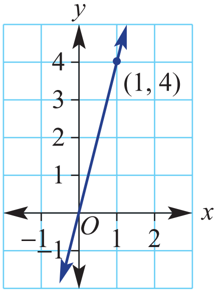
**d**



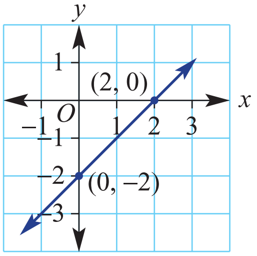
**e**



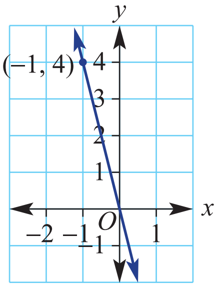
**f**



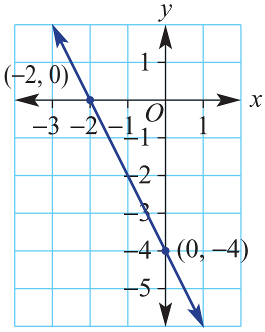
**g**



**h**



**i**



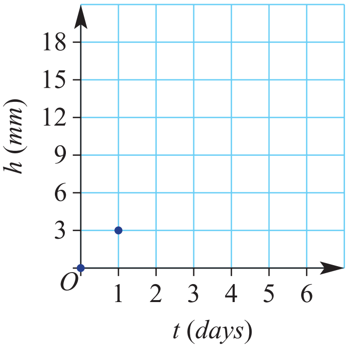
## Extended-response questions

**1** A seed sprouts and the plant grows 3 millimetres per day in height for 6 days.

**a** Complete this table of values using $t$ for time in days and $h$ for height in millimetres.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $h$ | 0 | | | | | | |

**b** Complete this graph using the points from your table.



**c** Find an equation linking $h$ with $t$.

**d** Use your equation to find the height of the plant after 3.5 days.

**e** If the linear pattern continued, what would be the height of the plant after 10 days?

**f** How long will it be before the plant grows to 15 mm in height?

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2 A speed boat at sea is initially 12 km from a distant rock. The boat travels towards the rock at a rate of 2 km per minute. The distance between the boat and the rock will therefore decrease over time.

- **a** Complete this table showing $t$ for time in minutes and $d$ for distance to the rock in kilometres.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $d$ | 12 | 10 | | | | | |

- **b** Draw a graph using the points from your table. Use $t$ on the horizontal axis.
- **c** How long does it take the speed boat to reach the rock?
- **d** Find an equation linking $d$ with $t$.
- **e** Use your equation to find the distance from the rock at the 2.5 minute mark.
- **f** How long does it take for the distance to reduce to 3.5 km?

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# Chapter 8

# Transformations and congruence

## What you will learn

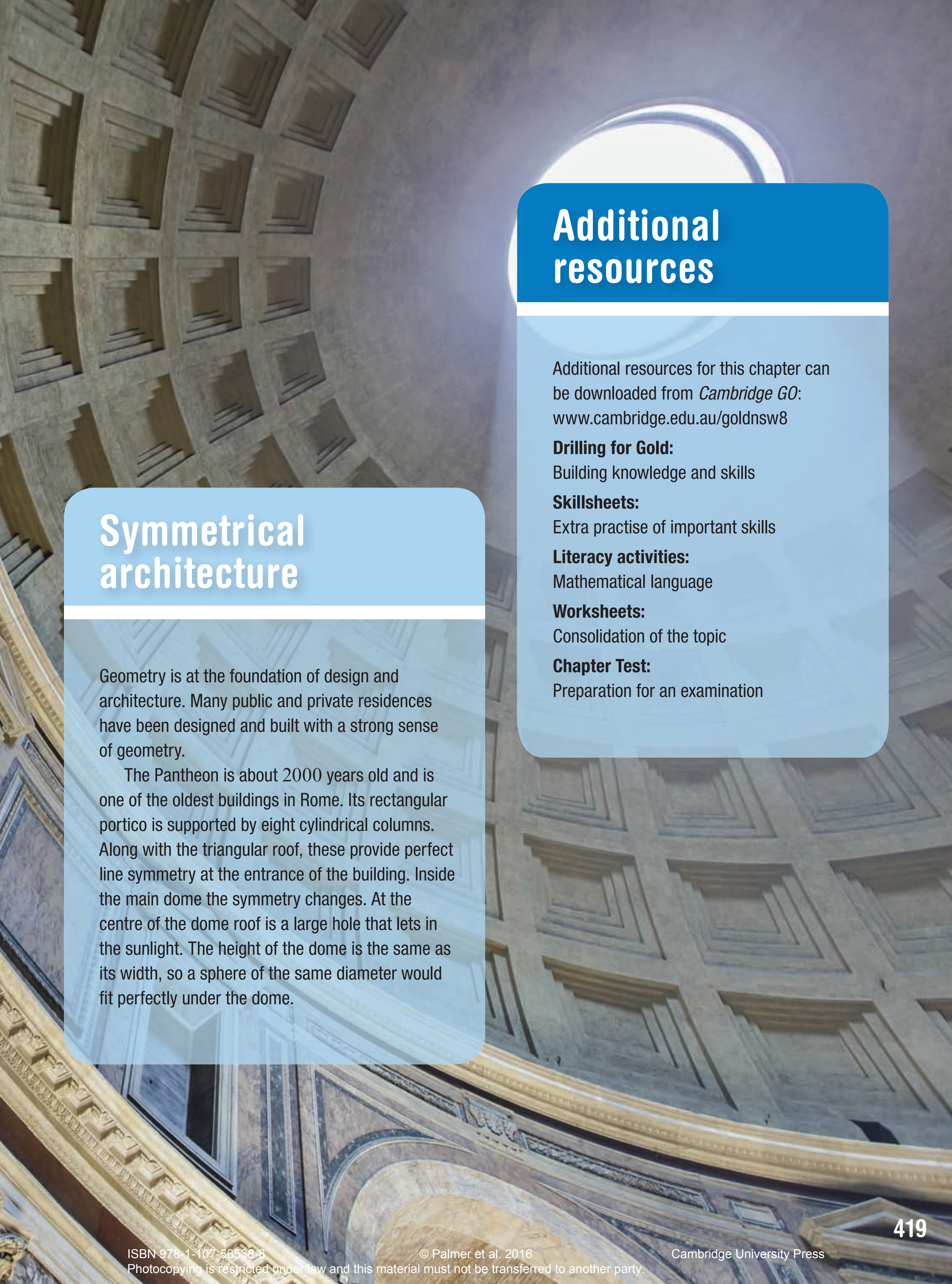
### Strand: Measurement and Geometry Number and Algebra

**Substrand:** LINEAR RELATIONSHIPS, PROPERTIES OF GEOMETRICAL FIGURES

In this chapter, you will learn to:

- create and display number patterns
- graph and analyse linear relationships
- perform transformations on the Cartesian plane
- classify, describe and use the properties of triangles and quadrilaterals, and determine congruent triangles to find unknown side lengths and angles.

*This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw8*



## Symmetrical architecture

Geometry is at the foundation of design and architecture. Many public and private residences have been designed and built with a strong sense of geometry.

The Pantheon is about $2000$ years old and is one of the oldest buildings in Rome. Its rectangular portico is supported by eight cylindrical columns. Along with the triangular roof, these provide perfect line symmetry at the entrance of the building. Inside the main dome the symmetry changes. At the centre of the dome roof is a large hole that lets in the sunlight. The height of the dome is the same as its width, so a sphere of the same diameter would fit perfectly under the dome.

## Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw8

**Drilling for Gold:**
Building knowledge and skills

**Skillsheets:**
Extra practise of important skills

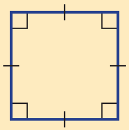
**Literacy activities:**
Mathematical language

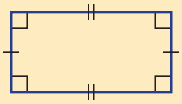
**Worksheets:**
Consolidation of the topic

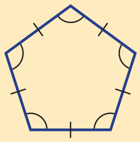
**Chapter Test:**
Preparation for an examination

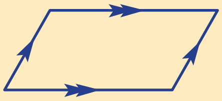
# Pre-test

**1** How many lines (axes) of symmetry are there in these shapes?

**a**

Square

**b**

Rectangle

**c**

Regular pentagon

**d**

Parallelogram

**2** What is the order of rotational symmetry for the shapes in question **1**?

**3** This number plane shows four points $A$, $B$, $C$, $D$.

**a** State the coordinates of the points $A$, $B$, $C$, $D$.

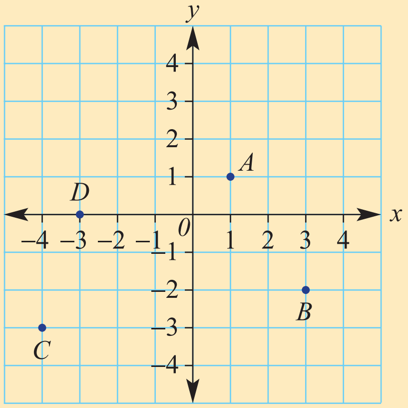
**b** What would be the coordinates of point $A$ if it were shifted:

  **i** left by 1 unit?

  **ii** right by 2 units and 1 unit down?

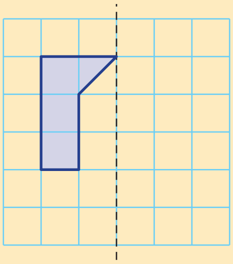
  **iii** left by 5 units and 3 units down?

**c** What would be the coordinates of point $C$ if it were reflected in the $x$-axis?

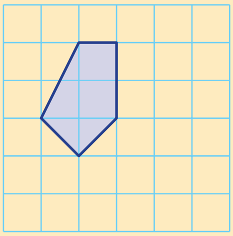


**4** Complete the simple transformations of the given shapes as instructed.

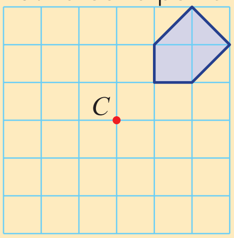
**a** Reflect this shape over the mirror line.



**b** Shift this shape 2 units to the right and 1 unit down.



**c** Rotate this shape 180° around point $C$.



**5** Which of the special quadrilaterals **A**–**F** fit the descriptions **a**–**d**?

**A** Square
**B** Rectangle
**C** Rhombus
**D** Parallelogram
**E** Kite
**F** Trapezium

**a** Opposite sides are of equal length.

**b** It has at least one pair of equal opposite angles.

**c** Diagonals are of equal length.

**d** Diagonals intersect at right angles.

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# 8A Reflection

In this chapter we will investigate three *transformations*:

- **Reflection:** Using a line to create a mirror image
- **Translation:** Shifting points/shapes from one position to another
- **Rotation:** Revolving/turning objects about a point

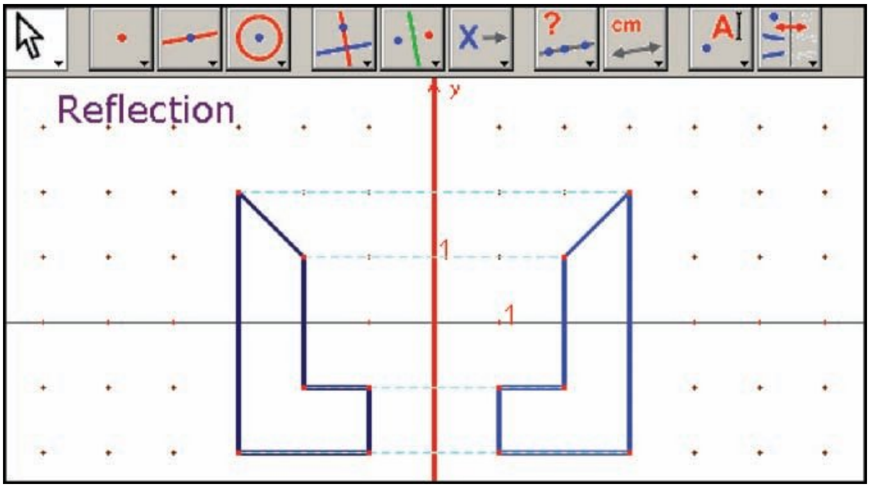
Reflection creates a mirror image.

## Let's start: Visualising the image

This activity can be done:

- by hand on a page of grid paper
- by folding and tracing
- using a website or dynamic geometry software.

- Draw any shape with straight sides.
- Draw a vertical or horizontal mirror line outside the shape.
- Try to draw the reflected image of the shape in the mirror line.
- If dynamic geometry is used, reveal the precise image (the answer) using the reflection tool to check your result.
- For a further challenge, redraw or drag the mirror line so it is not horizontal or vertical. Then try to draw the image.



Dynamic geometry software provides a reflection tool.

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## Key ideas

**Reflection** The result of flipping a geometrical figure across a line

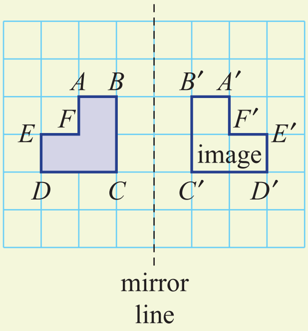
**Transformation** An alteration made to a shape or a graph using the methods of reflection, translation or rotation

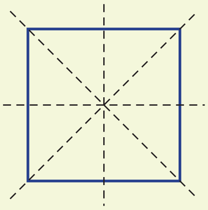
**Image** The result of a reflection

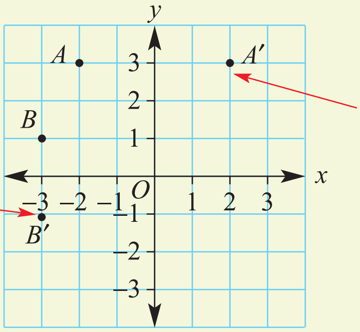
**Mirror line** The line over which a figure is reflected

**Line of symmetry** A line that divides a figure into two identical parts

- **Reflection** is a **transformation** in which the size and shape of the object is unchanged.
- The **image** of a point $A$ is denoted $A'$.
- Each point is reflected at right angles to the **mirror line**.
- The distance from a point $A$ to the mirror line is equal to the distance from the image point $A'$ to the mirror line.
- **Lines of symmetry** are mirror lines that result in an image being reflected onto itself.
  - A square has four lines of symmetry.
- We can use coordinates on the number plane to pinpoint an image after transformation.







$B'(-3, -1)$ is the image of $B(-3, 1)$ after reflection in the $x$-axis.

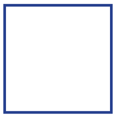
$A'(2, 3)$ is the image of $A(-2, 3)$ after reflection in the $y$-axis.

## Exercise 8A

**Understanding**

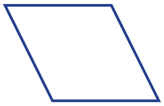
**1** Draw all the lines of symmetry for these shapes.

**a**



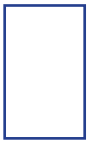
Square

**b**



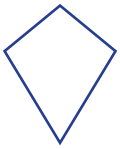
Rhombus

**c**



Rectangle

**d**



Kite

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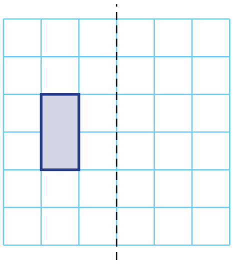
**2** Copy each grid and reflect the shape in the mirror line.

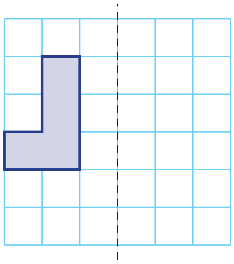
First reflect each vertex (corner point), then join them to form the image.

Drilling for Gold 8A1

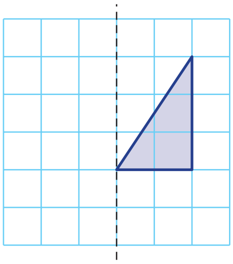
**a**



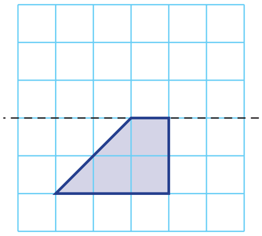
**b**



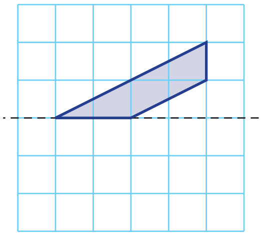
**c**



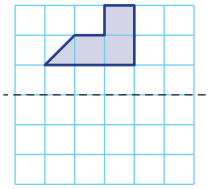
**d**



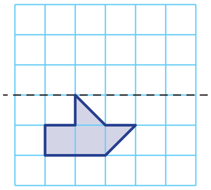
**e**



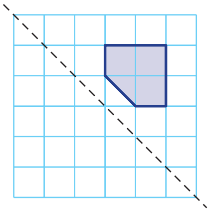
**f**



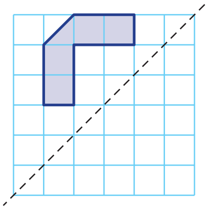
**g**



**h**



**i**

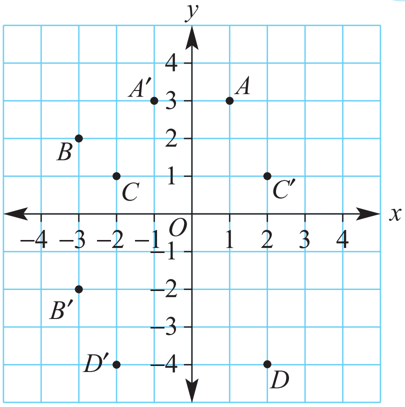


For parts **h** and **i**, turn your page so the mirror line is vertical.

**3** Write the coordinates of each of the points shown on this grid.

| | | | |
|---|---|---|---|
| **a** | $A$ | **b** | $A'$ |
| **c** | $B$ | **d** | $B'$ |
| **e** | $C$ | **f** | $C'$ |
| **g** | $D$ | **h** | $D'$ |

Write the $x$-coordinate first.



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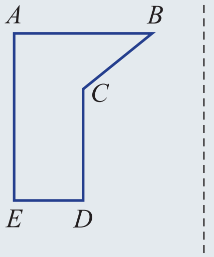
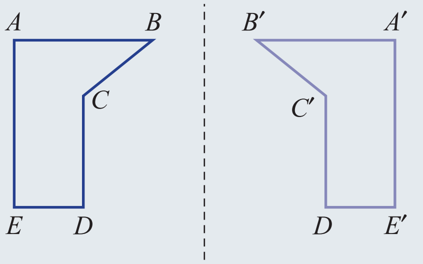
## Example 1 Drawing simple reflected images

Copy the diagram and draw the reflected image over the given mirror line.



**Solution**

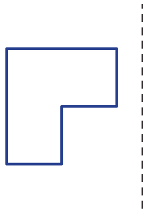


**Explanation**

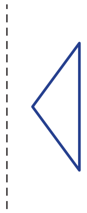
Reflect each vertex point at right angles to the mirror line. Join the image points to form the final image.
Use $A'$ as the image point of $A$.

**4** Copy the diagram and draw the reflected image over the given mirror line.

**a**

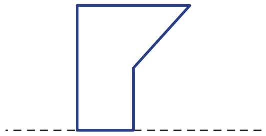


**b**

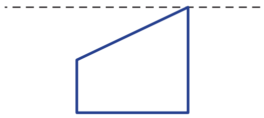


Start by reflecting each vertex point at $90^\circ$ across the mirror line. Then join these points to form the shape.

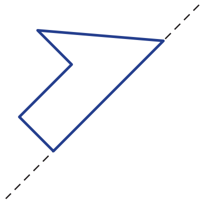
**c**



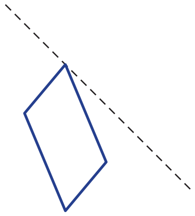
**d**



**e**



**f**



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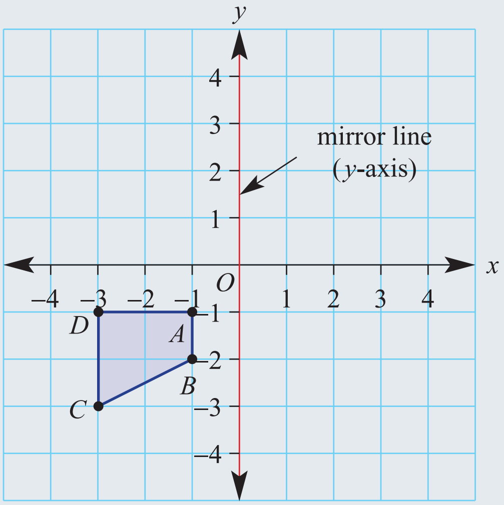
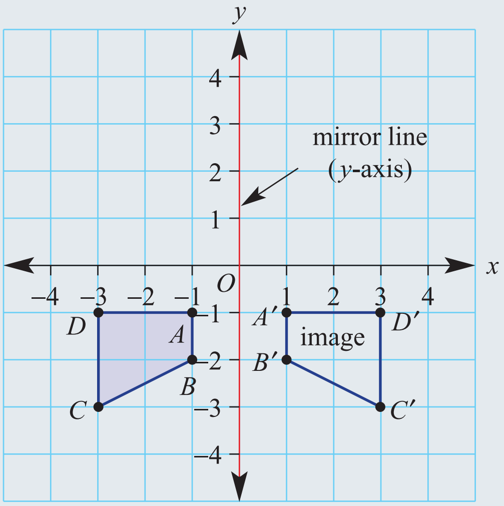
**Fluency**

## Example 2 Drawing reflections on a number plane

Draw the reflected image of this shape and give the coordinates of $A'$, $B'$, $C'$ and $D'$. The $y$-axis is the mirror line.



### Solution



$A' = (1, -1)$, $B' = (1, -2)$
$C' = (3, -3)$, $D' = (3, -1)$

### Explanation

Reflect each vertex $A$, $B$, $C$ and $D$ about the mirror line. The line segment from each point to its image should be at 90° to the mirror line.

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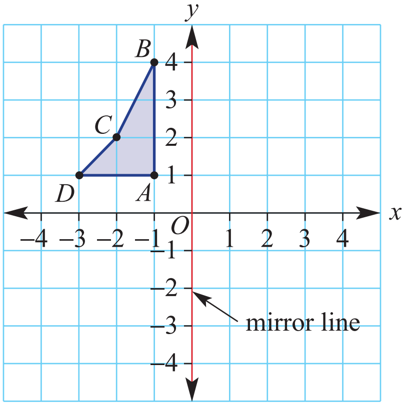
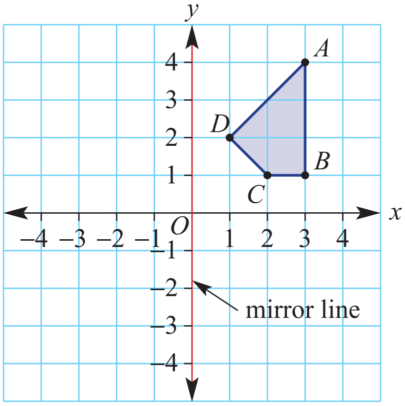
**8A**

**5** Draw the image of each shape and give the coordinates of $A'$, $B'$, $C'$ and $D'$.

**a**



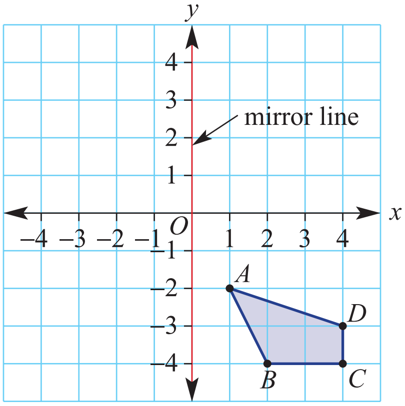
**b**



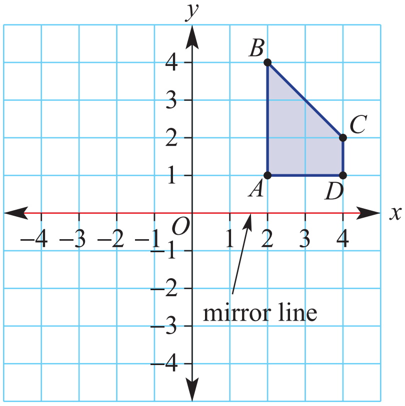
Reflect point $A$ in the mirror line and label the image $A'$. Repeat to locate $B'$, $C'$ and $D'$, then join the image points.

Drilling for Gold 8A2

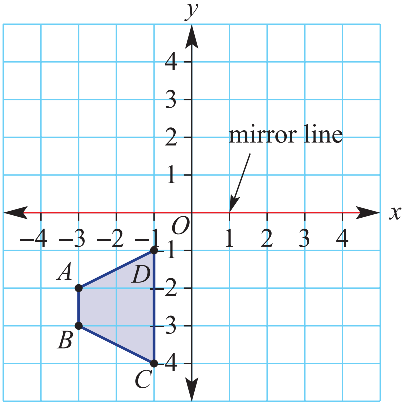
**c**



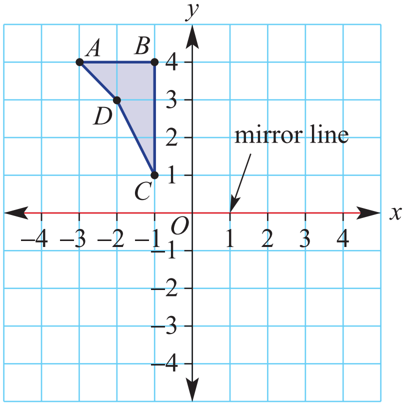
**d**



**e**



**f**



**6** **a** Write the coordinates of the following points after reflection in the $x$-axis.

**i** (2, 5) **ii** (4, 1) **iii** (–3, 2) **iv** (–3, 4)
**v** (0, –4) **vi** (3, 0) **vii** (–2, 0) **viii** (–6, –10)

**b** Which coordinate, $x$ or $y$, changes after reflection in the $x$-axis?

Reflection in the $x$-axis is a vertical transformation.

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**7** **a** Write the coordinates of the following points after reflection in the $y$-axis.

**i** $(3, 2)$ **ii** $(7, 1)$ **iii** $(-2, 4)$ **iv** $(-4, 6)$
**v** $(0, 7)$ **vi** $(-4, 0)$ **vii** $(-4, -6)$ **viii** $(0, -3)$

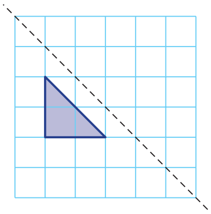
**b** Which coordinate, $x$ or $y$, changes after reflection in the $y$-axis?

Reflection in the $y$-axis is a horizontal transformation.

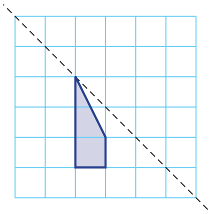
**Problem-solving and Reasoning**

**8** The mirror lines on these grids are at a 45° angle. Draw the reflected image.

**a**

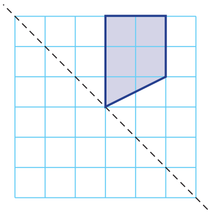


**b**

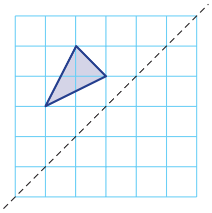


Turn your page so that the mirror line is vertical.

**c**



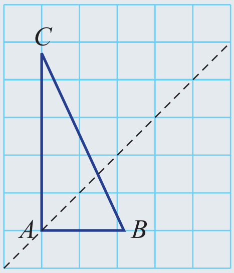
**d**



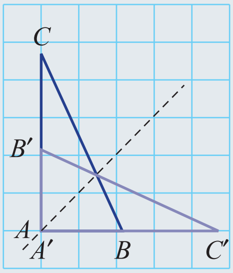
**9** On the number plane, the point $A(-2, 5)$ is reflected in the $x$-axis and this image point is then reflected in the $y$-axis. What are the coordinates of the final image?

## Example 3 Drawing more complex reflected images

Copy and reflect over the mirror line.



### Solution



### Explanation

Reflect points $A$, $B$ and $C$ at right angles to the mirror line to form $A'$, $B'$ and $C'$. Note that $A'$ is in the same position as $A$ as it is on the mirror line. Join the image points to form the image triangle.

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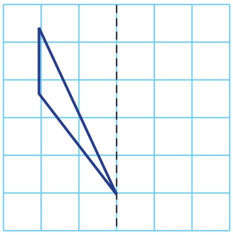
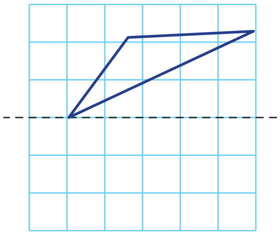
**10** Copy the diagram and draw the reflected image over the given mirror line.

**a**

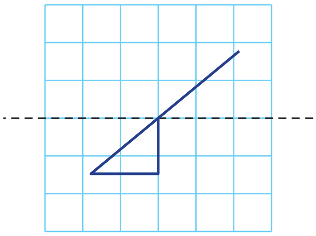


**b**

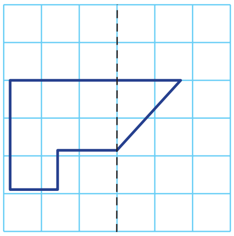


Reflect the vertex points first. Then join the points to finish.

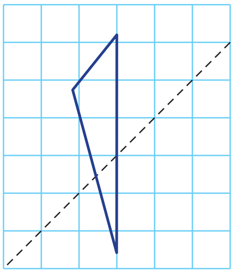
**c**



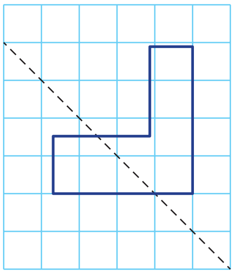
**d**



**e**

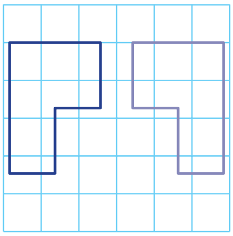


**f**

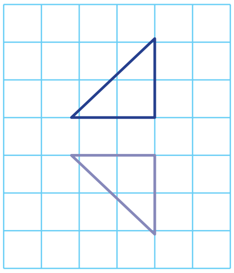


**11** Copy the diagram and accurately locate and draw the mirror line. Alternatively, pencil in the line on this page.

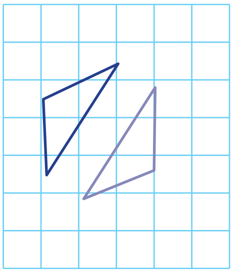
**a**



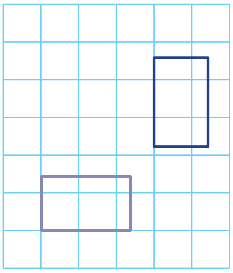
**b**



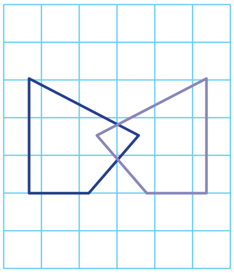
**c**



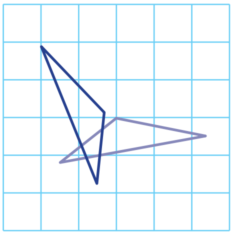
**d**



**e**



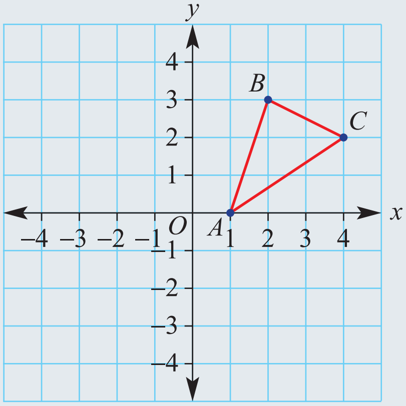
**f**



## Example 4 Using coordinates in reflection

State the coordinates of the vertices $A'$, $B'$ and $C'$ after this triangle is reflected in the given axes.

**a** $x$-axis **b** $y$-axis

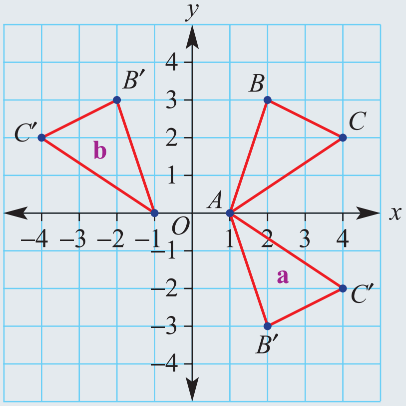


**Solution**

**a** $A' = (1, 0)$
$B' = (2, -3)$
$C' = (4, -2)$

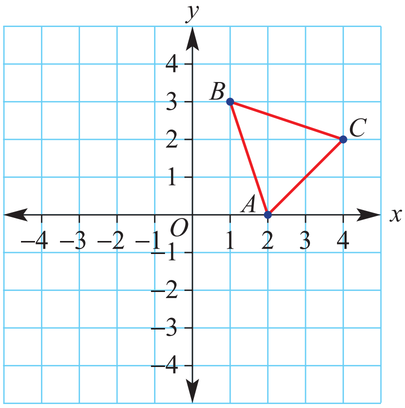
**b** $A' = (-1, 0)$
$B' = (-2, 3)$
$C' = (-4, 2)$

**Explanation**



**12** State the coordinates of the vertices $A'$, $B'$ and $C'$ after the triangle (right) is reflected in the given axes.

**a** $x$-axis
**b** $y$-axis



Pencil in the reflection, then look at the position of the image points. The $x$-axis is the horizontal axis and the $y$-axis is the vertical axis.

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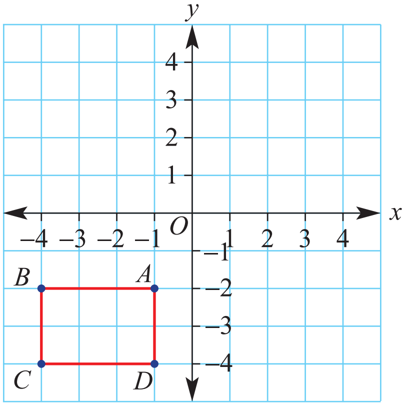
**13** State the coordinates of the vertices $A'$, $B'$, $C'$ and $D'$ after this rectangle is reflected in the given axes.

**a** $x$-axis

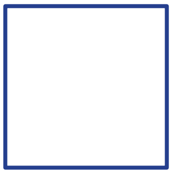
**b** $y$-axis



**14** How many lines of symmetry do these shapes have?

Be careful: Not all diagonals are lines of symmetry.

**a**

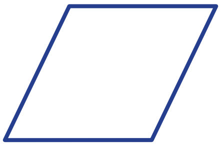


Square

**b**

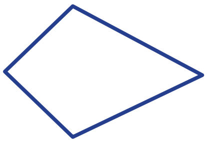
Rectangle

**c**



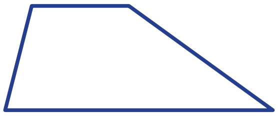
Rhombus

**d**



Kite

**e**

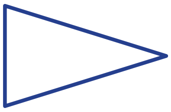


Trapezium

**f**

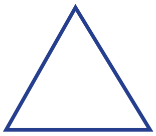
Parallelogram

**g**



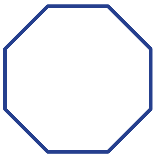
Isosceles triangle

**h**



Equilateral triangle

**i**



Regular octagon

**15** A point is reflected in the $x$-axis then in the $y$-axis and finally in the $x$-axis again. What single reflection could replace all three reflections?

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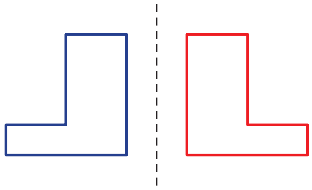
## Enrichment: Computer reflection



**16** Use computer software to construct a shape and a mirror line.

- **a** Reflect your shape in the mirror line.
- **b** Drag the mirror line. What do you notice?
- **c** Drag your original shape. What do you notice?
- **d** Drag the mirror line across the middle of your original shape. What do you notice?

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# 8B Translation

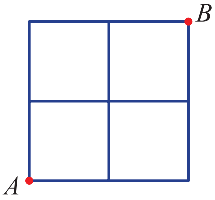

To solve this puzzle, you translate square tiles left, right, up or down.

Translation involves shifting a point or a shape left, right, up or down. The orientation of a shape is unchanged.

## Let's start: City grid

Imagine that a point $A$ on a simple city grid map is your starting point, and point $B$ is your destination.

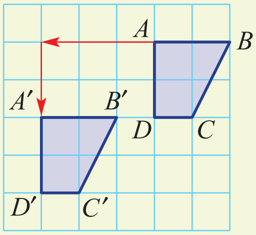
- Describe one simple way of travelling from $A$ to $B$.
- Describe two other ways of travelling from $A$ to $B$.
- How many different ways are there if 'back-tracking' is not allowed?



## Key ideas

**Translation** Moving a shape a certain distance up, down, left or right

- **Translation** is a transformation involving a shift to the left, right, up or down.
  - Describing a translation involves saying how many units a shape is shifted left or right and/or up or down.
- In the diagram, point $A$ is translated to $A'$, where $A'$ is the image of $A$.
- Translation does not change the shape, size or orientation of a shape.



translation 3 units left and 2 units down

## Exercise 8B

Understanding

**1** The point (7, 4) is translated to the point (0, 1).

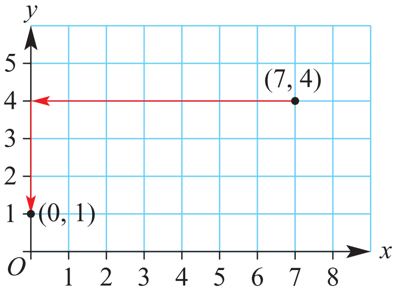
**a** How far left has the point been translated?

**b** How far down has the point been translated?

**c** If the point (0, 1) is translated to (7, 4), how far:

**i** right has the point been translated?

**ii** up has the point been translated?



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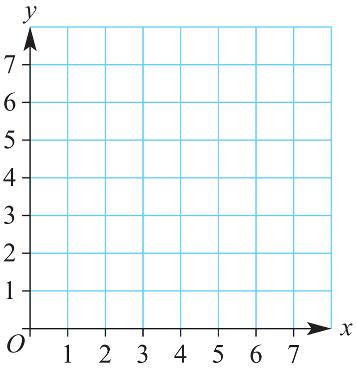
**2** A point is translated to its image. Write the missing word (i.e. left, right, up or down) for each of these.

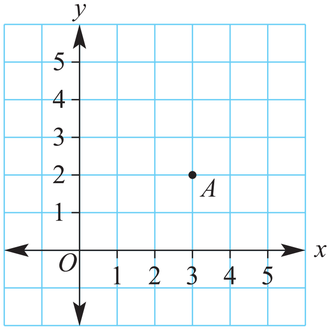
- **a** (1, 1) is translated ________ to the point (1, 3).
- **b** (5, 4) is translated ________ to the point (1, 4).
- **c** (7, 2) is translated ________ to the point (7, 0).
- **d** (3, 0) is translated ________ to the point (3, 1).
- **e** (5, 1) is translated ________ to the point (4, 1).
- **f** (2, 3) is translated ________ to the point (1, 3).
- **g** (0, 2) is translated ________ to the point (5, 2).
- **h** (7, 6) is translated ________ to the point (11, 6).

Pencil each pair of points onto a grid to see the translation.



**3** Point $A$ has coordinates (3, 2). Write the coordinates of the image point $A'$ when point $A$ is translated in each of the following ways.

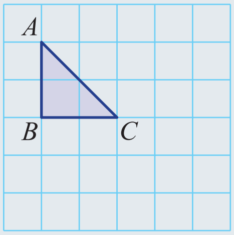
- **a** 1 unit right
- **b** 2 units left
- **c** 3 units up
- **d** 1 unit down
- **e** 1 unit left and 2 units up
- **f** 3 units left and 1 unit down
- **g** 2 units right and 1 unit down
- **h** 0 units left and 2 units down



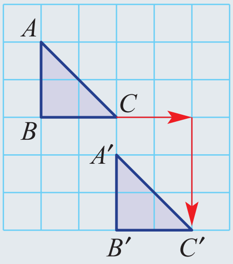
**Fluency**

## Example 5 Translating shapes

Draw the image of the triangle $ABC$ after a translation 2 units to the right and 3 units down.



**Solution**



**Explanation**

Shift each vertex 2 units to the right and 3 units down. Then join the vertices to form the image.

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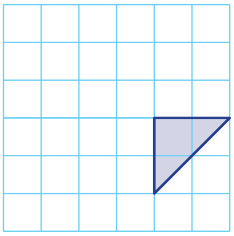
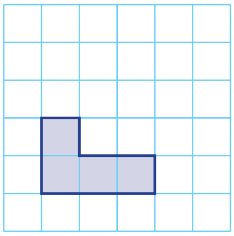
**4** Draw the image of these shapes after each translation.

First translate each corner, then join the points to form the image.

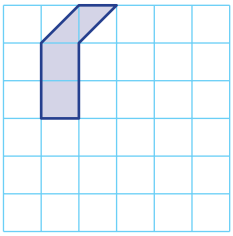
**a** 3 units left and 1 unit up



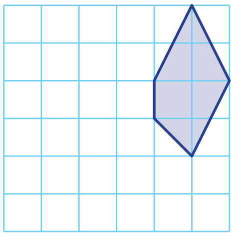
**b** 1 unit right and 2 units up



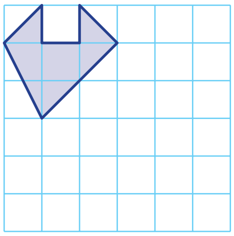
**c** 3 units right and 2 units down



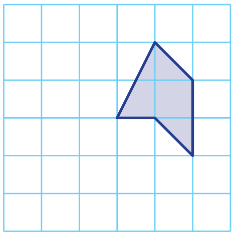
**d** 4 units left and 2 units down



**e** 2 units right and 3 units down



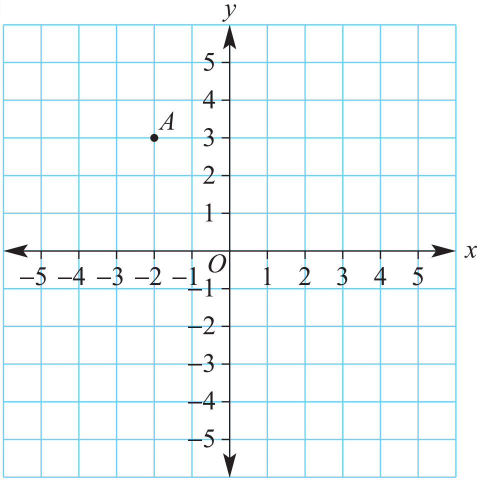
**f** 3 units left and 1 unit down



**5** Point $A$ has coordinates (–2, 3). Write the coordinates of the image point $A'$ when point $A$ is translated in each of the following ways.

**a** 3 units right
**b** 2 units left
**c** 2 units down
**d** 5 units down
**e** 2 units up
**f** 10 units right
**g** 3 units right and 1 unit up
**h** 4 units right and 2 units down
**i** 5 units right and 6 units down
**j** 1 unit left and 2 units down
**k** 3 units left and 1 unit up
**l** 2 units left and 5 units down

Drilling for Gold 8B1



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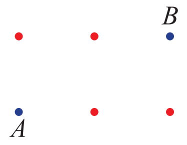
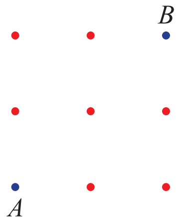
**Problem-solving and Reasoning**

**6** If only horizontal or vertical translations are allowed, how many different paths are there from points $A$ to $B$ on each grid below? No point can be visited more than once (on each path).

**a**

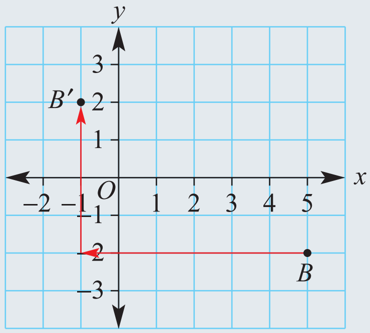


**b**



## Example 6 Describing translations

A point $B(5, -2)$ is translated to $B'(-1, 2)$. Describe the translation.

| **Solution** | **Explanation** |
|---|---|
|  Translation is 6 units left and 4 units up. | Plot the points on a Cartesian plane. Then describe the direction and distance of both translations. |

**7** Describe the translation from each point to its image.

- **a** $A(1, 3)$ is translated to $A'(1, 6)$.
- **b** $B(4, 7)$ is translated to $B'(4, 0)$.
- **c** $C(-1, 3)$ is translated to $C'(-1, -1)$.
- **d** $D(-2, 8)$ is translated to $D'(-2, 10)$.
- **e** $E(4, 3)$ is translated to $E'(-1, 3)$.
- **f** $F(2, -4)$ is translated to $F'(4, -4)$.
- **g** $G(0, 0)$ is translated to $G'(-1, 4)$.
- **h** $H(-1, -1)$ is translated to $H'(2, 5)$.
- **i** $I(-3, 8)$ is translated to $I'(0, 4)$.
- **j** $J(2, -5)$ is translated to $J'(-1, 6)$.
- **k** $K(-10, 2)$ is translated to $K'(2, -1)$.
- **l** $L(6, 10)$ is translated to $L'(-4, -3)$.

Use a number plane:
- −4 to 4 on the $x$-axis
- −4 to 10 on the $y$-axis

Give answers such as ‘4 units right’ or ‘2 units left and 3 units up’.

**8** The point $A$ is translated to its image, $A'$. Describe the translation that takes $A'$ to $A$ (i.e. the reverse translation).

**a** $A(2, 3)$ and $A'(4, 1)$

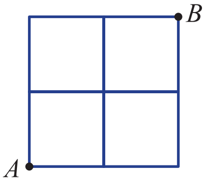
**b** $B(0, 4)$ and $B'(4, 0)$

**c** $C(0, -3)$ and $C'(-1, 2)$

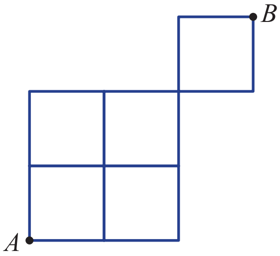
**d** $D(4, 6)$ and $D'(-2, 8)$

**9** If only horizontal and vertical translations are allowed, how many different paths are there from point $A$ to point $B$? No section can be used more than once in each path.

**a**



**b**



## Enrichment: Combined transformations

**10** Write the coordinates of the image point after each sequence of transformations. (Apply each transformation to the image of the previous transformation.)

**a** $(2, 3)$
- Reflection in the $x$-axis
- Reflection in the $y$-axis
- Translation 2 units left and 2 units up

**b** $(-1, 6)$
- Translation 5 units right and 3 units down
- Reflection in the $y$-axis
- Reflection in the $x$-axis

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# 8C Rotation

We know that rotational symmetry involves turning a shape around its centre. Rotation can also involve moving a shape around a centre of rotation that is outside the shape. This type of transformation results in an image that is the same size and shape as the original.

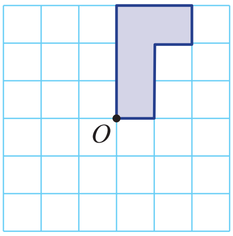
The hands rotate around the centre of the clock.

## Let's start: Rotation on a grid

Look at the shape on the grid. Draw the image after rotating the shape about point $O$ by:

- $180°$
- $90°$ clockwise
- $90°$ anticlockwise

Discuss what method you used to draw each image.

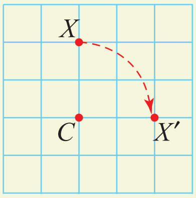


## Key ideas

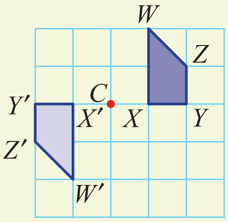
- **Rotation** is a transformation that involves a turn of every point on a shape about a given point.
- A rotation involves a centre point of rotation ($C$) and an angle of rotation, as shown.
  - A pair of compasses can be used to draw each circle, to help find the position of image points.
- Rotation can be clockwise ↷ or anticlockwise ↶.
- Every point on a shape is rotated on a circular arc.
- When a shape is rotated, the orientation changes but the shape and size remain unchanged.
- In the diagram, $A'$ is the image of $A$ after it has been rotated $90°$ clockwise about $C$.

**Rotation** A turn around a point, which may be outside the shape

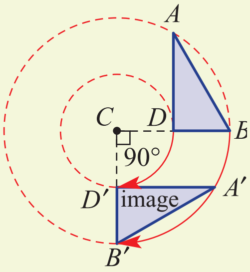
**Centre of rotation** Fixed point about which a figure rotates



$x$ is rotated $90°$ clockwise about $C$



Shape $XYZW$ is rotated $180°$ about $C$



Rotation $90°$ clockwise about $C$

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## Exercise 8C

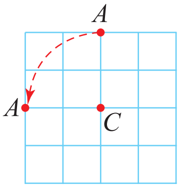
**Understanding**

**Drilling for Gold 8C1**

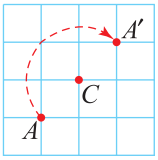
**1** Match each description **a**–**c** with a diagram **A**–**C**.

**a** rotation 90° clockwise **b** rotation 90° anticlockwise **c** rotation 180° clockwise

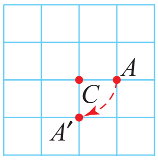
**A**



**B**

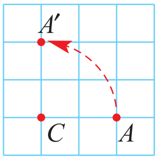


**C**

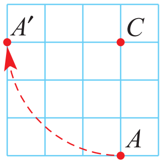


**2** Point $A$ has been rotated to its image point $A'$. For each part state whether the point has been rotated clockwise or anticlockwise and by how many degrees it has been rotated.

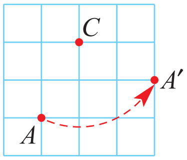
**a**



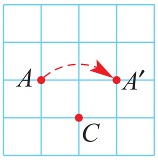
**b**



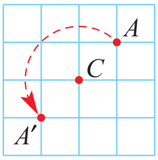
**c**



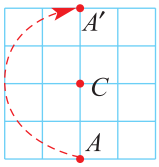
**d**



**e**

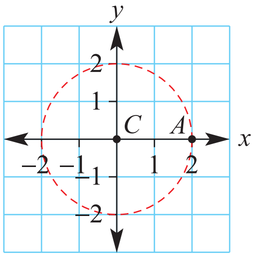


**f**



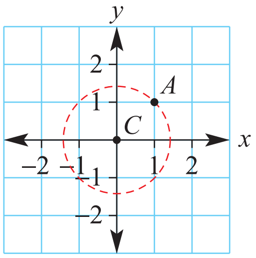
**3** Give the coordinates of the point $A$ and its image $A'$ after rotation about point $C(0, 0)$ by:

**a** 180° clockwise **b** 180° anticlockwise **c** 90° clockwise **d** 90° anticlockwise.



**4** Give the coordinates of the point $A$ and its image $A'$ after rotation about point $C(0, 0)$ by:

**a** 180° clockwise **b** 180° anticlockwise **c** 90° clockwise **d** 90° anticlockwise.



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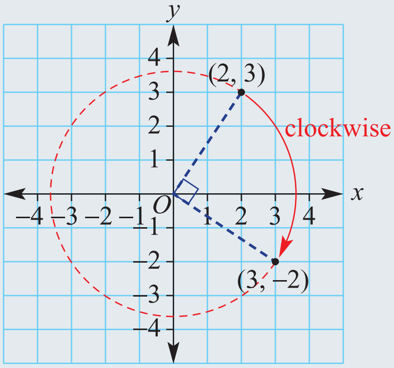
Fluency

## Example 7 Rotating a point

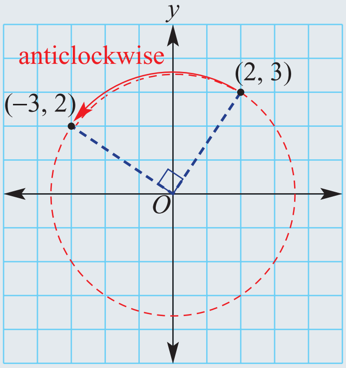
Give the coordinates of the image of the point (2, 3) after each of the following rotations about the origin (0, 0).

**a** 90° clockwise **b** 90° anticlockwise **c** 180°

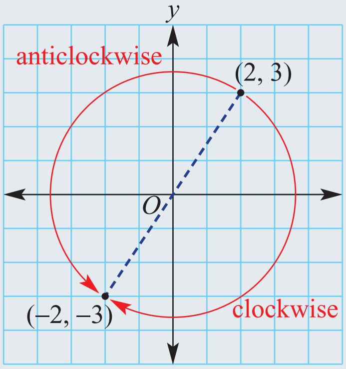
| **Solution** | **Explanation** |
|---|---|
| **a** (3, –2) | Plot the point (2, 3).<br>Use a pair of compasses to draw a circle through the point.<br>Draw a line interval from (2, 3) to the origin.<br>Use a protractor to measure 90° clockwise from the interval.<br>Plot the image point. |



| | |
|---|---|
| **b** (–3, 2) | Use similar steps in the opposite direction. |



| | |
|---|---|
| **c** (–2, –3) | Rotating 180° in either direction gives the same image point. |



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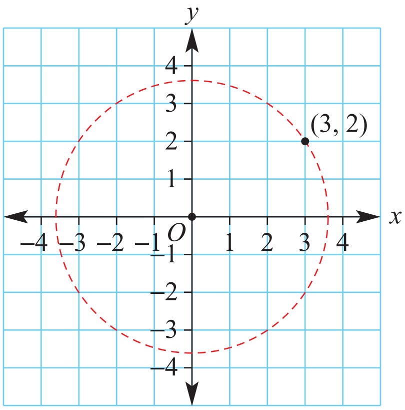
**5** Give the coordinates of each of the following points after rotation about the origin (0, 0) by:

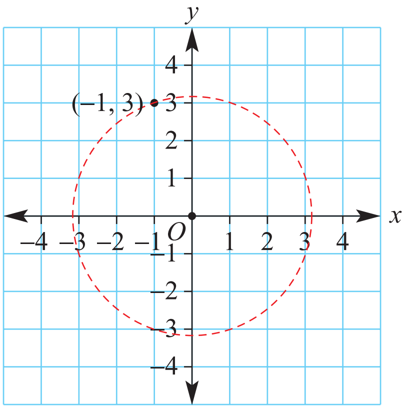
**i** 90° clockwise
**ii** 90° anticlockwise
**iii** 180°

First draw a line interval between the point and (0, 0). Then measure 90° with protractor

**a**

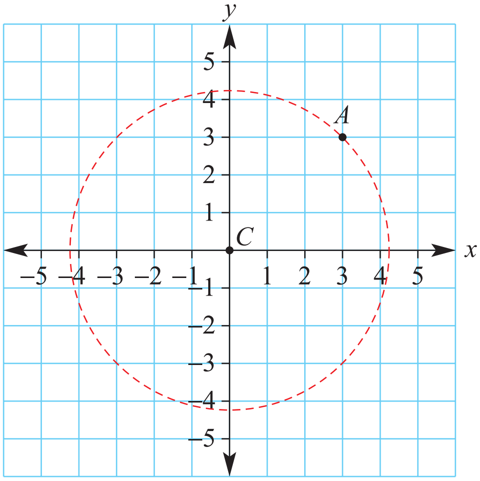


**b**



**6** Give the new coordinates of the image point $A'$ after point $A$ has been rotated around point $C(0, 0)$ by:

**a** 180° clockwise
**b** 90° clockwise
**c** 90° anticlockwise
**d** 270° clockwise
**e** 360° anticlockwise
**f** 180° anticlockwise.



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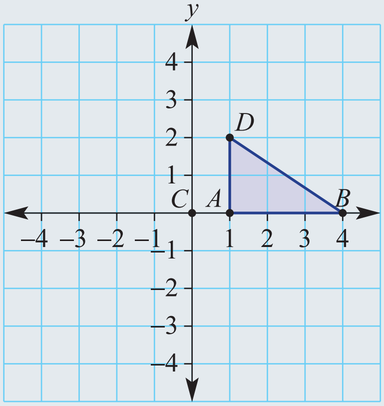
## Example 8 Drawing rotations

**Drilling for Gold 8C2**

Draw the image of this shape and give the coordinates of $A'$, $B'$ and $D'$ after carrying out the following rotations.

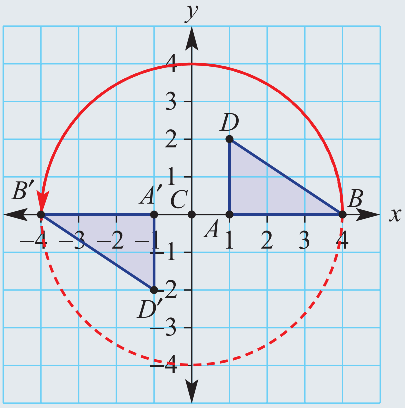
**a** 180° about $C$

**b** 90° anticlockwise about $C$



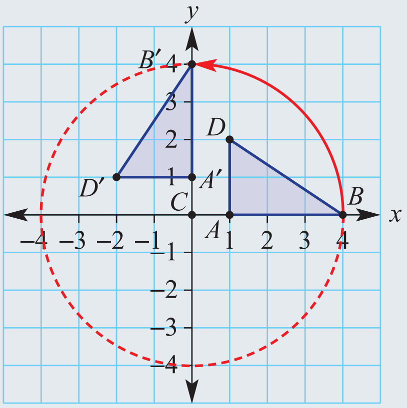
### Solution

**a**



$A' = (-1, 0)$, $B' = (-4, 0)$, $D' = (-1, -2)$

**b**



$A' = (0, 1)$, $B' = (0, 4)$, $D' = (-2, 1)$

### Explanation

Rotate each point on a circular arc around point $C$ by 180° in either direction.

Rotate each point on a circular arc around point $C$ by 90° anticlockwise.

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Skillsheet 8A

**7** Draw the image of each shape and give the coordinates of $A'$, $B'$ and $D'$ after the following rotations.

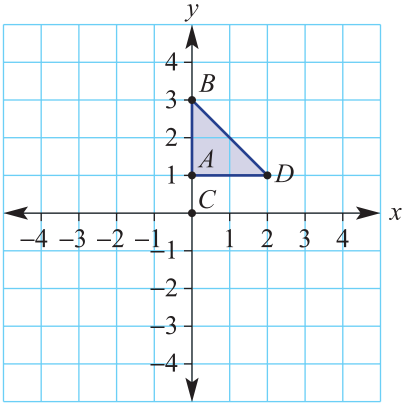
**i** 90° anticlockwise about $C$

**ii** 180° about $C$

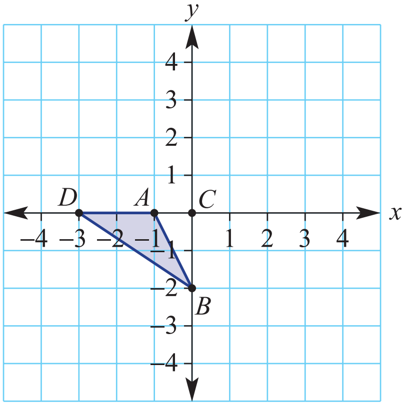
**iii** 90° clockwise about $C$

Rotate the points first, then join to form the shape.

**a**

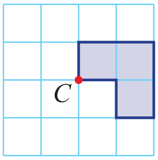


**b**

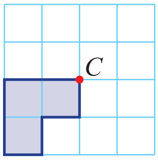


**8** Rotate these shapes about the point $C$ by 90° in the given direction.

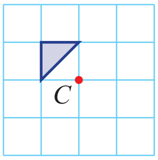
**a** Clockwise



**b** Anticlockwise

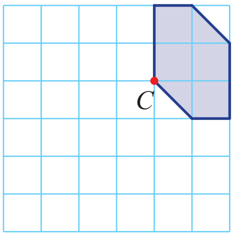


**c** Anticlockwise

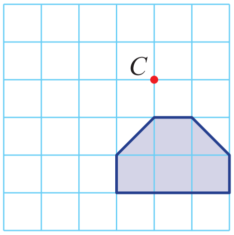


Try just rotating a point or a side first. Then join to form the image shape.

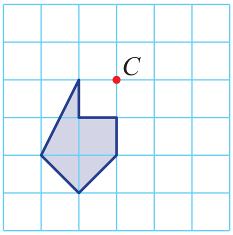
**d** Clockwise



**e** Clockwise

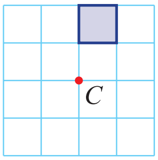


**f** Anticlockwise

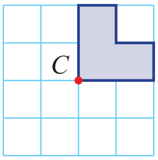


**9** Rotate these shapes about the point $C$ by 180°.

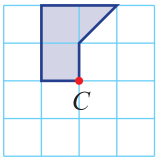
**a**



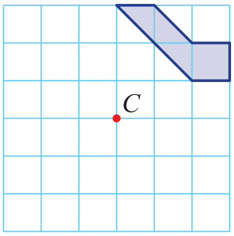
**b**



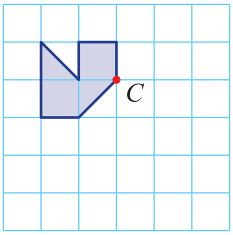
**c**



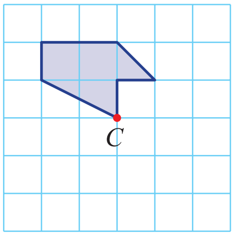
**d**



**e**



**f**



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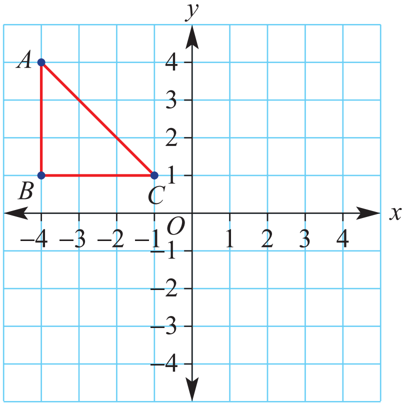
**Problem-solving and Reasoning**

**10** Give the coordinates of the image points $A'$, $B'$ and $C'$.

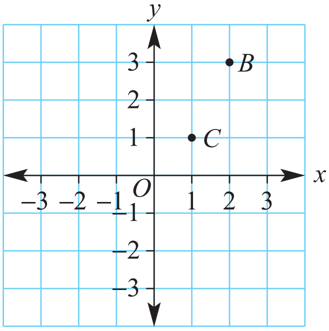
**a** The triangle shown here is rotated $180°$ about $(0, 0)$.

**b** What is the easy way to rotate a point by $180°$ about $(0, 0)$ (without drawing a diagram)?



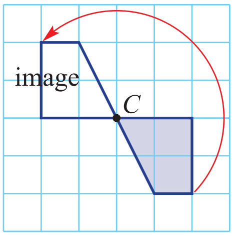
**11** The point $B(2, 3)$ is rotated about the point $C(1, 1)$. State the coordinates of the image point $B'$ for the following rotations.

**a** $180°$ **b** $90°$ clockwise **c** $90°$ anticlockwise

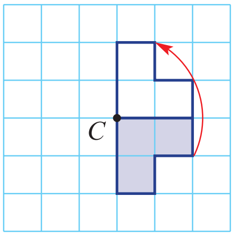


**12** How many degrees has each shape been rotated, and in which direction?

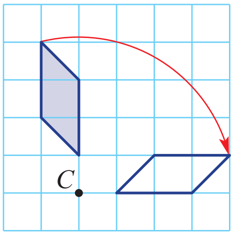
**a**



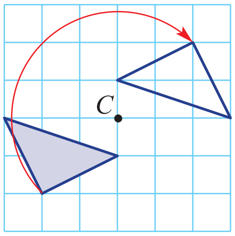
**b**



**c**



**d**



Pick a vertex and identify its image. Draw a line interval joining the vertex to point $C$, and another joining its image to point $C$. Then measure the angle between the two intervals.

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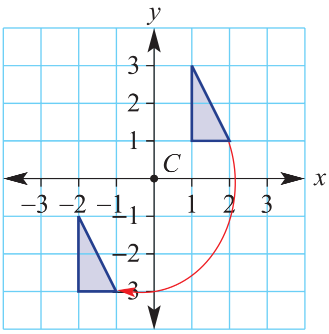
**13** Write the missing number in these sentences.

**a** Rotating a point $180^\circ$ clockwise is the same as rotating a point ______ anticlockwise.

**b** Rotating a point $90^\circ$ anticlockwise is the same as rotating a point ______ clockwise.

**14** Explain what is wrong with this $180^\circ$ rotation about $C(0, 0)$.



## Enrichment: Dynamic geometry exploration

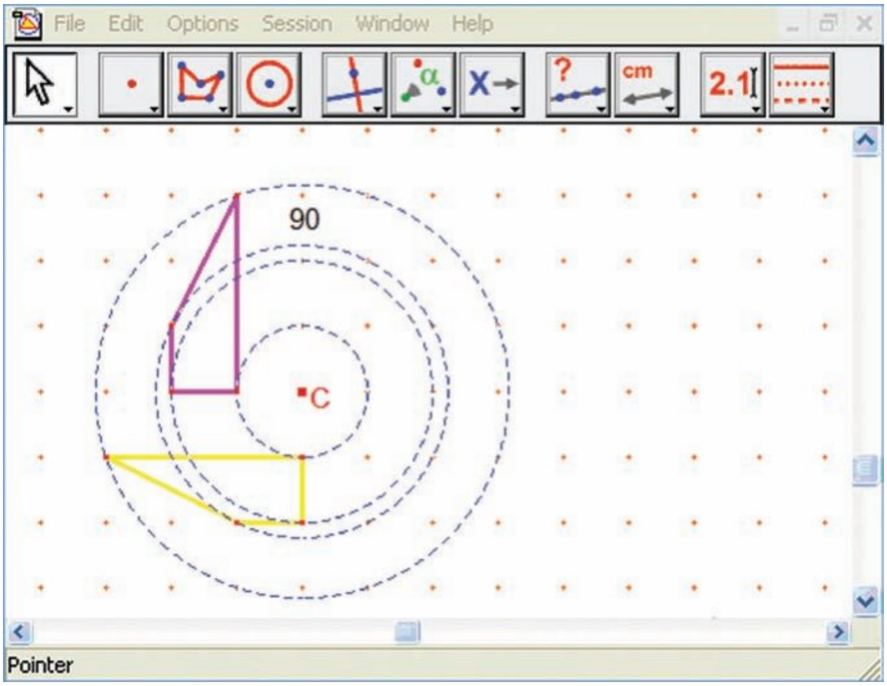
**15** Try rotating shapes using computer geometry software.

**a** On a grid, create any shape using the polygon tool.

**b** Construct a centre of rotation point and a rotating angle (or number).

**c** Use the rotation tool to create the rotated image that has your nominated centre of rotation and angle. You will need to click on the shape, the centre of rotation and your angle.

**d** Drag the vertices of your original shape and observe the changes in the image. Also try changing the angle of rotation.



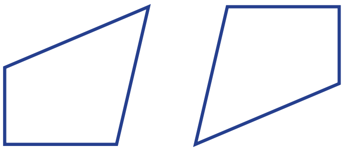
# 8D Congruent figures

If two or more objects are identical in size and shape, we say they are congruent. In the diagram, the green shape is exactly the same shape and size as the other three pieces.

## Let's start: Are they congruent?

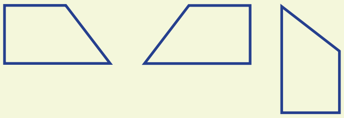
Here are two shapes. To be congruent they need to be exactly the same shape and size.

- Do you think they look congruent? Give reasons.
- What measurements could be taken to help establish whether or not they are congruent?
- Can you just measure angles or do you need to measure lengths as well? Discuss.



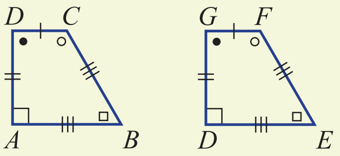
## Key ideas

- A **figure** is a shape, diagram or illustration.
- **Congruent figures** have the same size and shape.



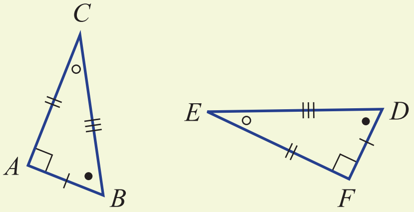
- The image of a figure that is reflected, translated or rotated is congruent to the original figure.
- Corresponding (matching) parts of a figure have the same geometric properties. For example:

In these quadrilaterals:



- Vertex $B$ corresponds to vertex $E$.
- Side $CD$ corresponds to side $FG$.
- Angle $\angle C$ corresponds to $\angle F$.

In these triangles:



- Vertex $C$ corresponds to vertex $E$.
- Side $AB$ corresponds to side $FD$.
- Angle $\angle B$ corresponds to $\angle D$.

- This triangle could be called $\Delta ABC$ or $\Delta ACB$.

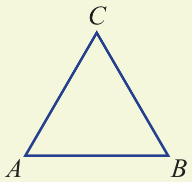


**Figure**
A shape, diagram or illustration

**Congruent figures**
Shapes that are exactly the same size and shape

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## Exercise 8D

**Understanding**

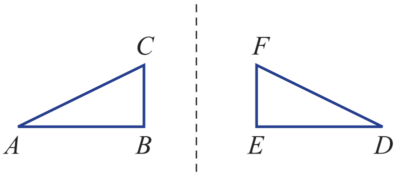
**1** Answer true or false.

- **a** Congruent shapes can be of different size.
- **b** Congruent shapes have equal matching sides.
- **c** Congruent shapes have equal matching angles.
- **d** The image of a shape after reflection is congruent to the original shape.
- **e** The image of a shape after rotation is congruent to the original shape.
- **f** The image of a shape after translation is congruent to the original shape.

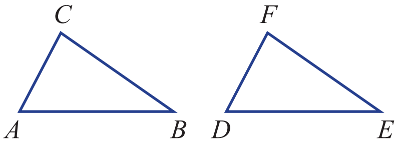
**2** In this diagram $\Delta ABC$ has been reflected to give the image triangle $\Delta DEF$.

- **a** Is $\Delta DEF$ congruent to $\Delta ABC$?
- **b** Name the vertex on $\Delta DEF$ that corresponds to:
  - **i** vertex $A$
  - **ii** vertex $B$
  - **iii** vertex $C$.
- **c** Name the side on $\Delta DEF$ that corresponds to:
  - **i** side $AB$
  - **ii** side $BC$
  - **iii** side $AC$.
- **d** Name the angle in $\Delta DEF$ that corresponds to:
  - **i** $\angle B$
  - **ii** $\angle C$.
  - **iii** $\angle A$

Choose the matching vertex (corner), side or angle on the opposite triangle.

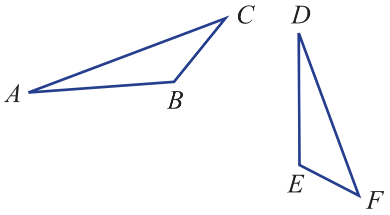


**3** In this diagram $\Delta ABC$ has been translated (shifted) to give the image triangle $\Delta DEF$.



- **a** Is $\Delta DEF$ congruent to $\Delta ABC$?
- **b** Name the vertex on $\Delta DEF$ that corresponds to:
  - **i** vertex $A$
  - **ii** vertex $B$
  - **iii** vertex $C$.
- **c** Name the side on $\Delta DEF$ that corresponds to:
  - **i** side $AB$
  - **ii** side $BC$
  - **iii** side $AC$.
- **d** Name the angle in $\Delta DEF$ that corresponds to:
  - **i** $\angle B$
  - **ii** $\angle C$
  - **iii** $\angle A$.

**4** In this diagram $\Delta ABC$ has been rotated to give the image triangle $\Delta DEF$.



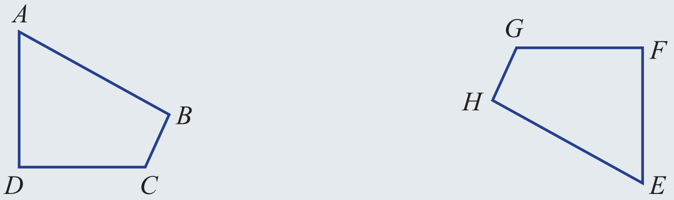
- **a** Is $\Delta DEF$ congruent to $\Delta ABC$?
- **b** Name the vertex on $\Delta DEF$ that corresponds to:
  - **i** vertex $A$
  - **ii** vertex $B$
  - **iii** vertex $C$.
- **c** Name the side on $\Delta DEF$ that corresponds to:
  - **i** side $AB$
  - **ii** side $BC$
  - **iii** side $AC$.
- **d** Name the angle in $\Delta DEF$ that corresponds to:
  - **i** $\angle B$
  - **ii** $\angle C$
  - **iii** $\angle A$.

**Fluency**

### Example 9 Naming corresponding pairs

These two quadrilaterals are congruent. Name the objects in quadrilateral $EFGH$ that correspond to these objects in quadrilateral $ABCD$.

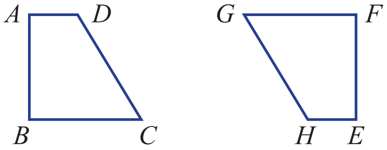
**a** Vertex $C$ **b** Side $AB$ **c** $\angle C$



| **Solution** | **Explanation** |
|---|---|
| **a** Vertex $G$ | $C$ sits opposite $A$ and $\angle A$ is the smallest angle. $G$ sits opposite $E$ and $\angle E$ is also the smallest angle. |
| **b** Side $EH$ | Sides $AB$ and $EH$ are both the longest sides of their respective shapes. $A$ corresponds to $E$ and $B$ corresponds to $H$. |
| **c** $\angle G$ | $\angle C$ and $\angle G$ are both the largest angle in their corresponding quadrilateral. |

**5** These two quadrilaterals are congruent. Name the object in quadrilateral $EFGH$ that corresponds to these objects in quadrilateral $ABCD$.

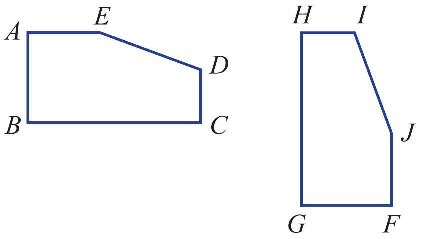
**a** **i** Vertex $A$ **ii** Vertex $D$
**b** **i** Side $AD$ **ii** Side $CD$
**c** **i** $\angle C$ **ii** $\angle A$



Corresponding angles will be equal and corresponding sides are the same length.

**6** These two pentagons are congruent. Name the object in pentagon $FGHIJ$ that corresponds to these objects in pentagon $ABCDE$.

**a** **i** Vertex $A$ **ii** Vertex $D$
**b** **i** Side $AE$ **ii** Side $CD$
**c** **i** $\angle C$ **ii** $\angle E$



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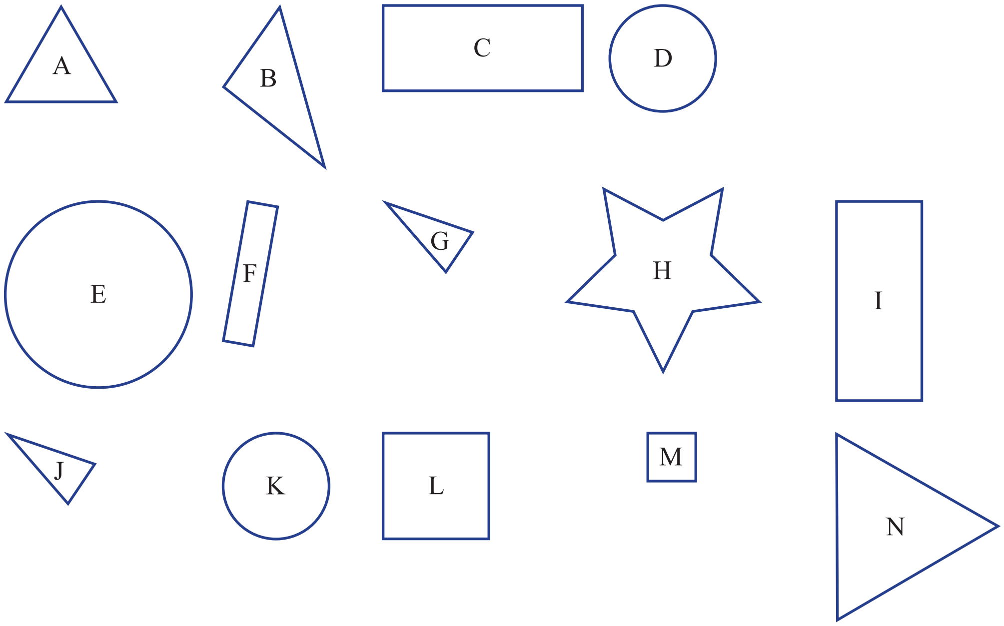
**Problem-solving and Reasoning**

Drilling for Gold 8D1

**7** From all the shapes shown here, find three pairs that are congruent.



**8** These triangles are congruent.

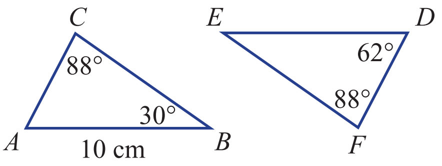
**a** Which side on $\Delta DEF$ corresponds to side $AB$?

**b** Which angle on $\Delta ABC$ corresponds to $\angle E$?

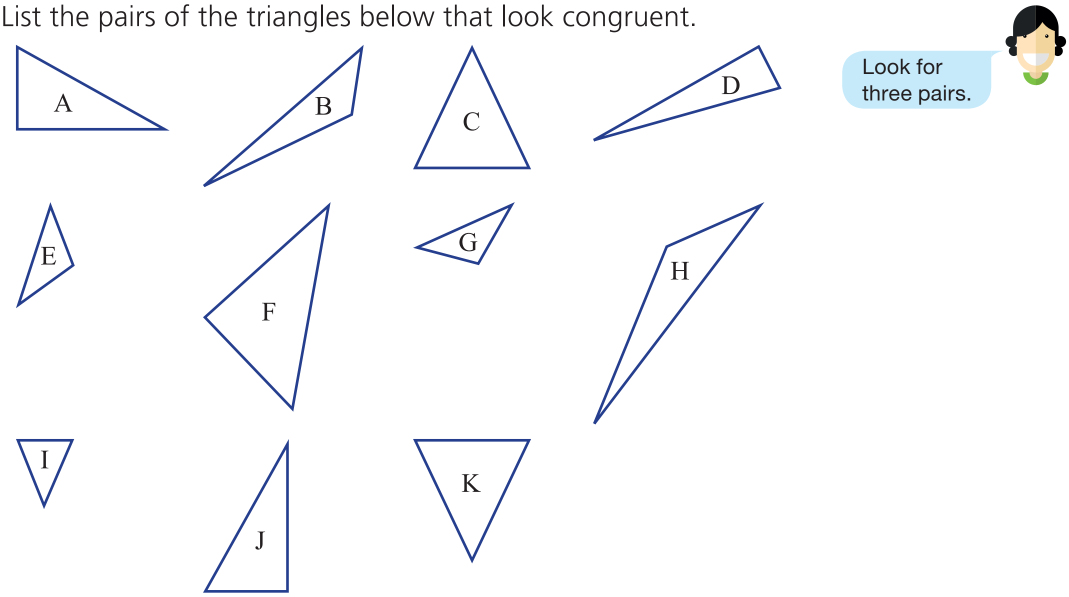
**c** What is the length $DE$?

**d** What is the size of $\angle A$?

**e** What is the size of $\angle E$?



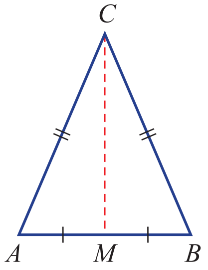
**9** List the pairs of the triangles below that look congruent.


Look for three pairs.

**10** An isosceles triangle is cut as shown, using the midpoint of $AB$.

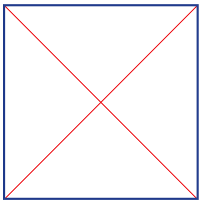
**a** Name the two triangles formed.

**b** Will the two triangles be congruent? Give reasons.

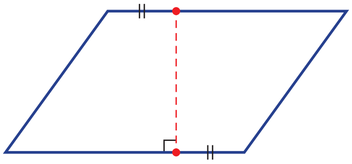


**11** If a parallelogram is cut by either diagonal will the two triangles be congruent?

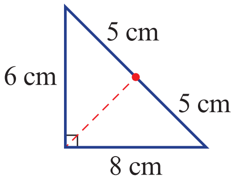
**12** If this square is cut into four triangles, are they congruent?



**13** If this parallelogram is cut as shown, will the two pieces be congruent?



**14** If this triangle is cut as shown, will the two pieces be congruent?



## Enrichment: Combined congruent transformations

Drilling for Gold 8D1

**15** Describe the combination of transformations (reflections, translations and/or rotations) that map each shape to its image under the given conditions. The reflections that are allowed include only those in the $x$- and $y$-axes and rotations will use $(0, 0)$ as its centre.

**a** $A$ to $A'$ with a reflection and then a translation

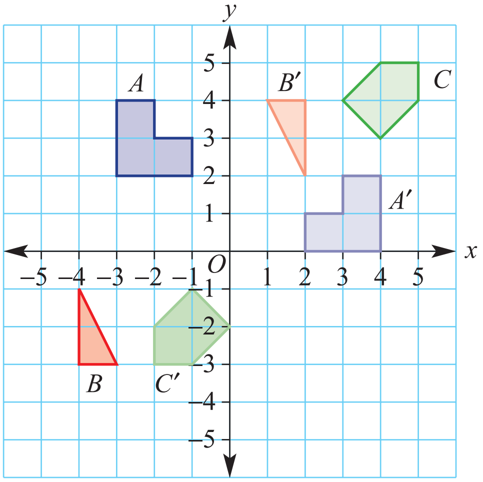
**b** $A$ to $A'$ with a rotation and then a translation

**c** $B$ to $B'$ with a rotation and then a translation

**d** $B$ to $B'$ with 2 reflections and then a translation

**e** $C$ to $C'$ with 2 reflections and then a translation

**f** $C$ to $C'$ with a rotation and then a translation



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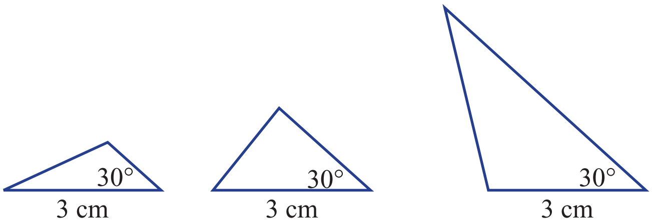
# 8E Congruent triangles

## Let’s start: How much information is enough?

Given one corresponding angle (say 30°) and one corresponding equal side length (say 3 cm), it is clearly not enough information to say two triangles are congruent. This is because more than one triangle can be drawn with the given information.

This wall at Federation Square in Melbourne includes many congruent triangles.



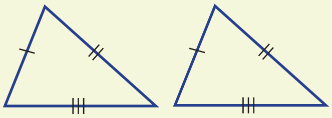
Knowing one corresponding side and one corresponding angle is not enough to say that two triangles will be congruent.

Decide if the following information is enough to determine if two triangles are congruent. If you can draw two non-identical triangles, then there is not enough information. You could use a ruler and a protractor or simply try this by hand-labelling vertices, sides and angles as you go.

- $\Delta ABC$ with $AC = 4$ cm and $\angle C = 40°$
- $\Delta ABC$ with $AB = 5$ cm and $AC = 4$ cm
- $\Delta ABC$ with $AB = 5$ cm, $AC = 4$ cm and $\angle A = 45°$
- $\Delta ABC$ with $AB = 5$ cm, $AC = 4$ cm and $BC = 3$ cm
- $\Delta ABC$ with $AB = 4$ cm, $\angle A = 40°$ and $\angle B = 60°$

## Key ideas

- There are four ‘tests’ that can be used to decide if two triangles are congruent.
- Two triangles are congruent if:
  - the **three sides of a triangle** are respectively equal to the **three sides of another triangle**, **(SSS test)**



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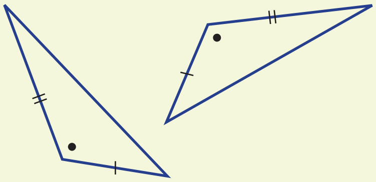
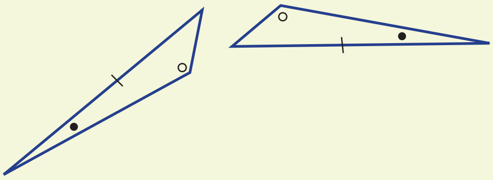
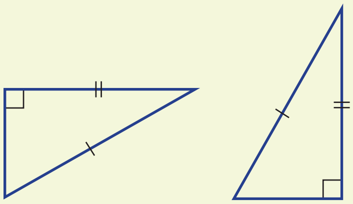
- **two sides and the included angle** of a triangle are respectively equal to **two sides and the included angle of another triangle (SAS test)**



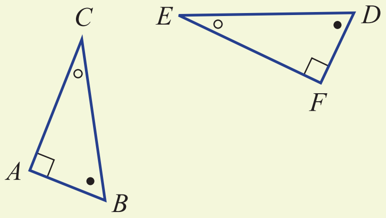
- **two angles and one side** of a triangle are respectively equal to **two angles and the matching side of another triangle (AAS test)**



- the **hypotenuse and a second side of a right-angled triangle** are respectively equal to the **hypotenuse and a second side of another right-angled triangle, (RHS test)**.



- Corresponding (matching) parts of a figure have the same geometric properties.
  - Vertex $C$ corresponds to vertex $E$.
  - Side $AB$ corresponds to side $FD$.
  - $\angle B$ corresponds to $\angle D$.



- A congruence statement can be written using the symbol ≡, e.g. $\Delta ABC \equiv \Delta FDE$.
  - This is read as '$\Delta ABC$ is congruent to $\Delta FDE$'.
  - In a congruence statement, vertices are named in matching order, e.g. $\Delta ABC \equiv \Delta FDE$ not $\Delta ABC \equiv \Delta DEF$ because $B$ matches $D$.

## Exercise 8E

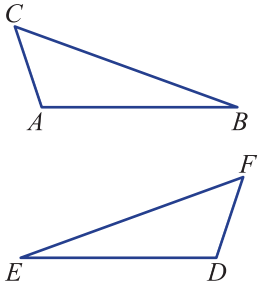
**Understanding**

Drilling for Gold 8E1

**1** Are the following tests for congruent triangles? Answer yes or no.

**a** SSS **b** SAS **c** AAA **d** AAS **e** RHS **f** SSA

**2** Look at this pair of congruent triangles.



**a** Which vertex on $\Delta DEF$ corresponds to (matches) these vertices on $\Delta ABC$?

**i** vertex $C$ **ii** vertex $A$ **iii** vertex $B$

**b** Which angle on $\Delta ABC$ corresponds to these angles on $\Delta DEF$?

**i** $\angle D$ **ii** $\angle F$ **iii** $\angle E$

**c** Which side on $\Delta DEF$ corresponds to these sides on $\Delta ABC$?

**i** $AB$ **ii** $CA$ **iii** $BC$

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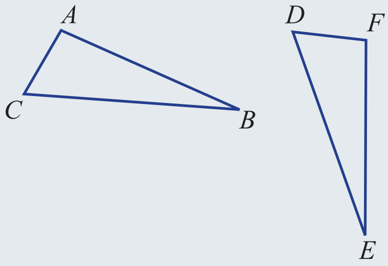
**Fluency**

### Example 10 Writing a congruence statement

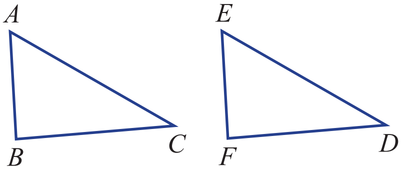
Write a congruence statement for this pair of congruent triangles.



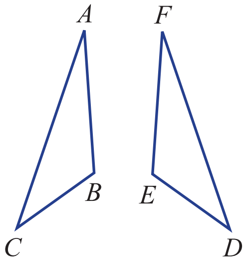
| Solution | Explanation |
|---|---|
| $\Delta ABC \equiv \Delta DFE$ | Given the size of the angles and the side lengths, it appears that $A$ matches $F$, $B$ matches $E$ and $C$ matches $D$. |

**3** Write a congruence statement (e.g. $\Delta ABC \equiv \Delta DEF$) for these pairs of congruent triangles. Try to match vertices.

**a**

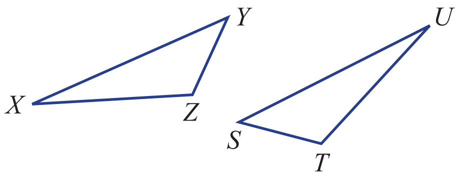


**b**

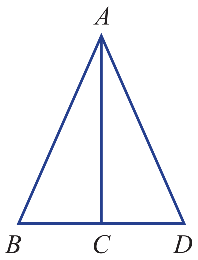


Match vertices that have the same matching angles.

**c**



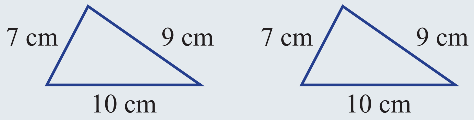
**d**



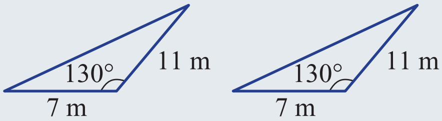
## Example 11 Deciding on a test for congruence

Which of the tests (SSS, SAS, AAS or RHS) would you choose to test the congruence of these pairs of triangles?

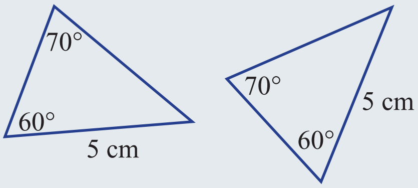
**a**



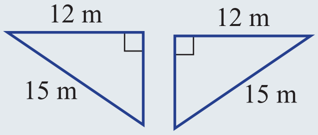
**b**



**c**



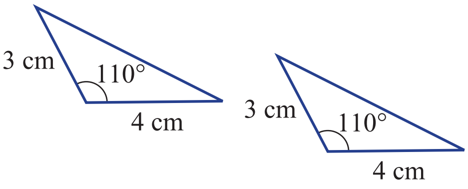
**d**



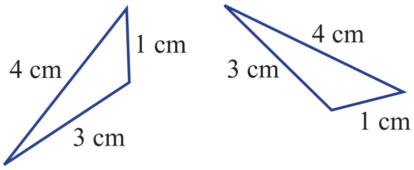
| **Solution** | **Explanation** |
|---|---|
| **a** SSS | There are 3 equal corresponding pairs of sides. |
| **b** SAS | There are 2 equal corresponding pairs of sides and 1 equal angle between them. |
| **c** AAS | There are two equal angles and 1 pair of equal corresponding sides. The side that is 5 cm is adjacent to the 60° angle on both triangles. |
| **d** RHS | There is a pair of right angles with hypotenuses of equal lengths. A second pair of corresponding sides are also of equal length. |

**4** Which of the tests (SSS, SAS, AAS or RHS) would you choose to test the congruence of these triangles?

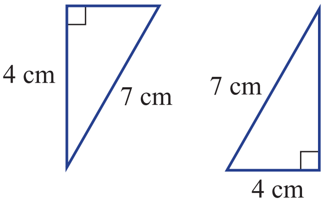
**a**



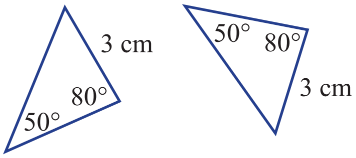
**b**



**c**



**d**



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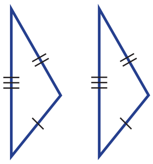
**8E**

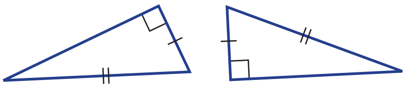
**Skillsheet 8B**

**5** Pick the congruence test (SSS, SAS, AAS or RHS) that matches each pair of congruent triangles.

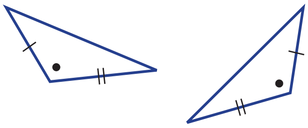
**a**



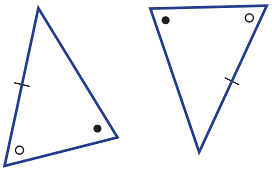
**b**



**c**



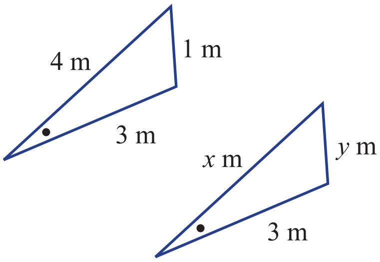
**d**



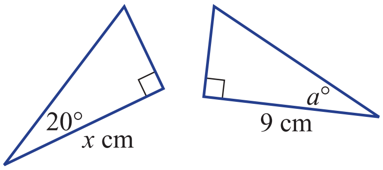
**Problem-solving and Reasoning**

**6** These pairs of triangles are congruent. Find the values of the pronumerals.

**a**

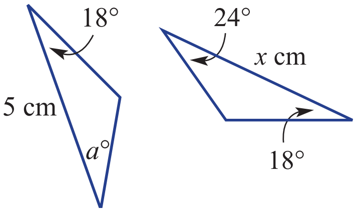


**b**

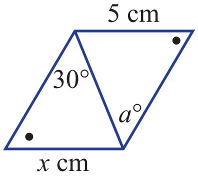


Matching sides will be equal and matching angles will be equal.

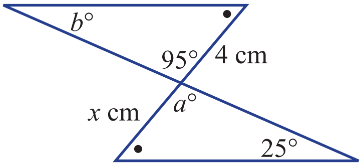
**c**



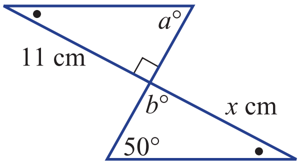
**d**



**e**

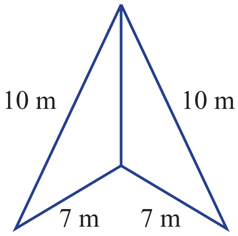


**f**

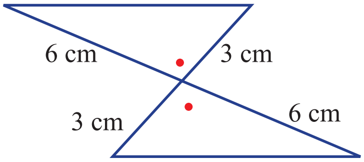


**7** Which of SSS, SAS, AAS or RHS would you choose to say that each pair of triangles is congruent?

**a**

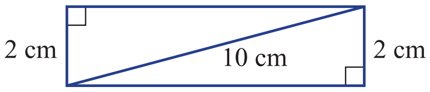


**b**

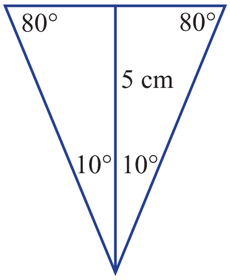


Use the information given in the diagram.

**c**

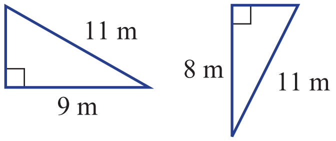


**d**

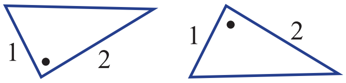


**8** Are these pairs of triangles congruent? If they are, write the test (SSS, SAS, AAS or RHS).

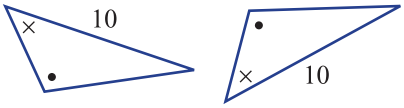
**a**



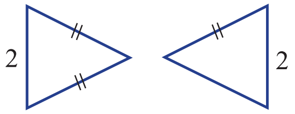
**b**



**c**



**d**

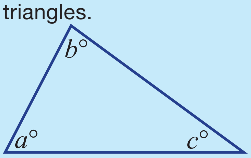


**9** Explain why AAA is not sufficient to prove that two triangles are congruent. Draw diagrams to show your reasoning.
Hint: Draw an equilateral triangle, then double the side lengths.

## Enrichment: Using the triangle angle sum

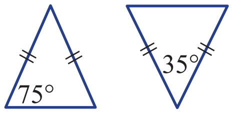
**10** Decide if each pair of triangles is congruent. You may first need to use the angle sum of a triangle to help calculate some of the angles.

First work out all the missing angles in the triangles.

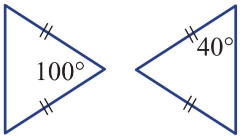


$a + b + c = 180$

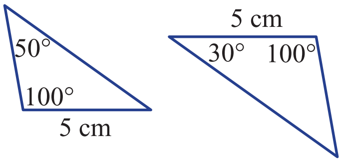
**a**



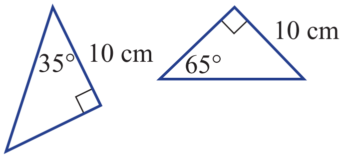
**b**



**c**



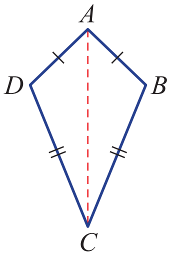
**d**



# 8F Using congruent triangles to establish properties of quadrilaterals

The properties of special quadrilaterals (parallelogram, rhombus, rectangle, square, trapezium and kite) can be examined more closely using congruence. By drawing the diagonals and using the tests for the congruence of triangles, we can verify properties of these special quadrilaterals.

## Let's start: Why is one pair of opposite angles in a kite equal?

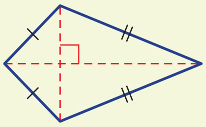


A kite with two pairs of equal length sides can be divided into two triangles, as shown.

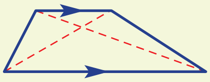
- Are these two triangles congruent?
- Which congruent triangle test (SSS, SAS, AAS, RHS) would be used to confirm this?
- What does this say about $\angle B$ and $\angle D$?

## Key ideas

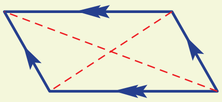
This is a summary of the properties of the special quadrilaterals.



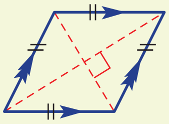
- Kite: A quadrilateral with two pairs of adjacent sides equal
  - Two pairs of adjacent sides of a kite are equal.
  - One diagonal of a kite bisects the other diagonal.
  - One diagonal of a kite bisects the opposite angles.
  - The diagonals of a kite are perpendicular.



- Trapezium: A quadrilateral with at least one pair of parallel sides
  - At least one pair of sides of a trapezium are parallel.



- Parallelogram: A quadrilateral with both pairs of opposite sides parallel
  - The opposite sides of a parallelogram are parallel.
  - The opposite sides of a parallelogram are equal.
  - The opposite angles of a parallelogram are equal.
  - The diagonals of a parallelogram bisect each other.



- Rhombus: A parallelogram with two adjacent sides equal in length
  - The opposite sides of a rhombus are parallel.
  - All sides of a rhombus are equal.
  - The opposite angles of a rhombus are equal.
  - The diagonals of a rhombus bisect the vertex angles.
  - The diagonals of a rhombus bisect each other.
  - The diagonals of a rhombus are perpendicular.

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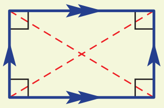
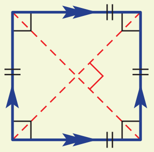
- Rectangle: A parallelogram with a right angle
  - The opposite sides of a rectangle are parallel.
  - The opposite sides of a rectangle are equal.
  - All angles at the vertices of a rectangle are 90°.
  - The diagonals of a rectangle are equal.
  - The diagonals of a rectangle bisect each other.



- Square: A rectangle with two adjacent sides equal
  - Opposite sides of a square are parallel.
  - All sides of a square are equal.
  - All angles at the vertices of a square are 90°.
  - The diagonals of a square bisect the vertex angles.
  - The diagonals of a square bisect each other.
  - The diagonals of a square are perpendicular.



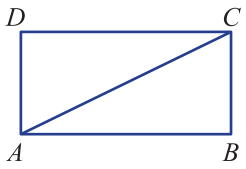
## Exercise 8F

**Understanding**

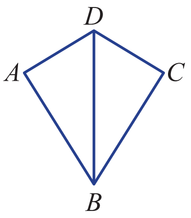
**1** SSS is one test for congruence of triangles. Write down the other three.

**2** Name the side (e.g. $AB$) that is common to both triangles in each diagram.

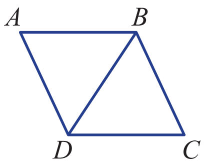
**a**



**b**

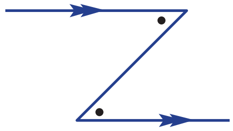


**c**

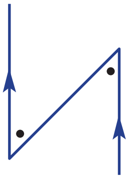


**3** Give the reason why the two marked angles are equal.

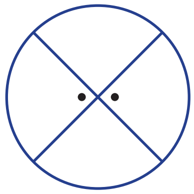
**a**



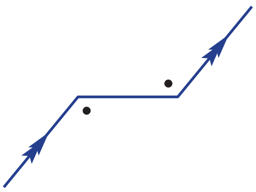
**b**



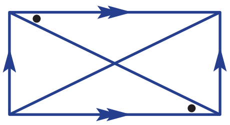
**c**



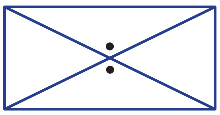
**d**



**e**



**f**



Recall that in parallel lines:
- corresponding angles are equal
- alternate angles are equal
- co-interior angles add to 180°
- vertically opposite angles are equal

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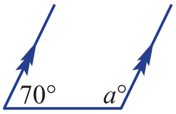
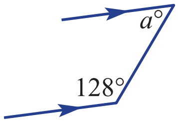
**4** Give the reason why the two marked angles add to 180° and then state the value of $a$.

**a**



**b**



**Fluency**

**Drilling for Gold 8F1**

**5** Which statements are *definitely* true?

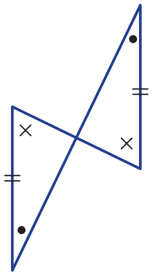
**a** Opposite sides of a parallelogram are parallel.
**b** Opposite sides of a kite are equal.
**c** A trapezium has two pairs of parallel sides.
**d** The diagonals of a rectangle are equal.
**e** The diagonals of a kite are equal.
**f** The diagonals of a parallelogram are equal.
**g** The diagonals of a trapezium are equal.
**h** The diagonals of a rhombus are equal.
**i** The diagonals of a square are equal.
**j** All angles inside a square are 90°.
**k** Opposite angles in a kite are equal.
**l** The diagonals of a parallelogram intersect at right angles.
**m** The diagonals of a rhombus intersect at right angles.
**n** The diagonals of a kite intersect at right angles.
**o** The diagonals of a rhombus bisect each other.
**p** The diagonals of a parallelogram bisect each other.
**q** The diagonals of a rectangle bisect each other.

Use the information in the Key ideas to help.

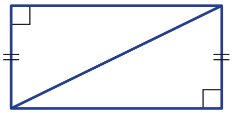
*Bisect* means to cut in half.

**6** Which of the four tests for congruence of triangles would be used to prove that each pair of triangles is congruent? Angles and sides with the same markings are equal.

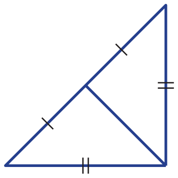
**a**



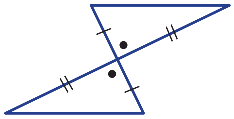
**b**



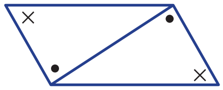
**c**



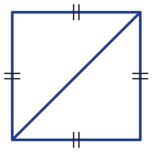
**d**



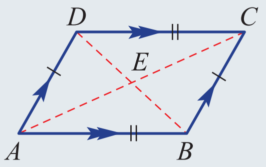
**e**



**f**



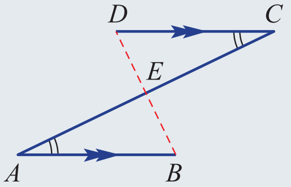
## Example 12 Exploring the diagonals of a parallelogram

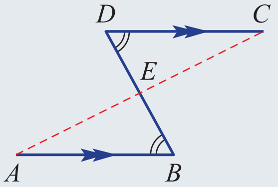


Answer these questions regarding the diagonals of this parallelogram.

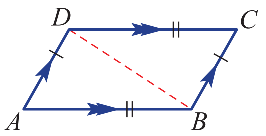
- **a** What can be said about $\angle BAE$ and $\angle DCE$?
- **b** What can be said about $\angle ABE$ and $\angle CDE$?
- **c** Does $AB = DC$?
- **d** Which reason (SSS, SAS, AAS, RHS) explains why $\Delta ABE \equiv \Delta CDE$?
- **e** Why do parallelogram diagonals bisect each other?

| **Solution** | **Explanation** |
|---|---|
| **a** They are equal. | |
| **b** They are equal. | |
| **c** Yes. | |
| **d** AAS. | |
| **e** $\Delta ABE \equiv \Delta CDE$<br>so $BE = DE$ and<br>$AE = CE$<br>because they are corresponding (matching) sides in congruent triangles | The two triangles are congruent using the AAS test.<br>Since $\Delta ABE$ and $\Delta CDE$ are congruent the corresponding sides are equal. |



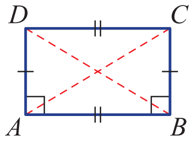


**7** Answer these questions about angles in this parallelogram.



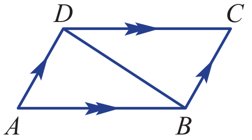
- **a** List two triangles formed by the diagonal.
- **b** What can be said about sides $AB$ and $DC$?
- **c** What can be said about sides $AD$ and $BC$?
- **d** Which side is common to both triangles?
- **e** Which reason (SSS, SAS, AAS, RHS) explains why $\Delta ABD \equiv \Delta CDB$?
- **f** Why is $\angle A = \angle C$?

**8** Answer these questions about diagonals in this rectangle.



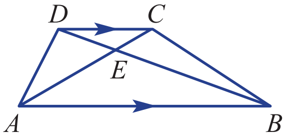
- **a** Locate $\Delta ABD$ and $\Delta BAC$. Is $\angle A = \angle B$?
- **b** Is $AD = BC$?
- **c** Is $AB = CD$?
- **d** Which reason (SSS, SAS, AAS, RHS) explain why $\Delta ABC = \Delta BAD$?
- **e** Why is $AC = BD$?

**9** A parallelogram $ABCD$ has two pairs of parallel sides.

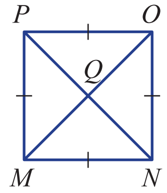


- **a** What can be said about $\angle ABD$ and $\angle CDB$? Give a reason.
- **b** What can be said about $\angle BDA$ and $\angle DBC$? Give a reason.

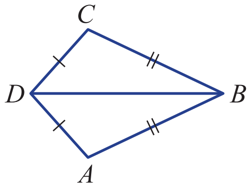
**c** Which side is common to both $\Delta ABD$ and $\Delta CDB$?
**d** Which congruence test would be used to show that $\Delta ABD = \Delta CDB$?
**e** Since $\Delta ABD = \Delta CDB$, what can be said about the opposite sides of a parallelogram?



**10** A trapezium $ABCD$ has one pair of parallel sides.
**a** Which angle is equal to $\angle BAE$?
**b** Which angle is equal to $\angle ABE$?
**c** Explain why $\Delta ABE$ is not congruent to $\Delta CDE$.



**11** For this square assume that $MQ = QO$ and $NQ = PQ$.
**a** Give reasons why $\Delta MNQ = \Delta ONQ$.
**b** Give reasons why $\angle MQN = \angle OQN = 90°$.
**c** Give reasons why $\angle QMN = 45°$.

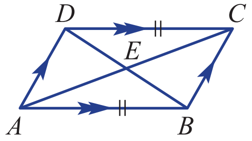


**12** Use the information in this kite to prove these results.
**a** $\Delta ABD = \Delta CBD$
**b** $\angle DAB = \angle DCB$
**c** $\angle ADB = \angle CDB$

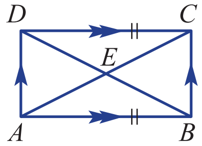
## Enrichment: Writing a formal proof

**13** Prove by giving reasons that the diagonal in these special quadrilaterals bisect each other. Opposite sides are equal, so use $AB = CD$. Complete the proof by following these steps.
Step 1. List the pairs of equal angles in $\Delta ABE$ and $\Delta CDE$ giving reasons why they are equal.
Step 2. List the pairs of equal sides in $\Delta ABE$ and $\Delta CDE$ giving reasons why they are equal.
Step 3. Write $\Delta ABE \equiv \Delta CDE$ and give the reason SSS, SAS, AAS or RHS.
Step 4. State that $BE = DE$ and $AE = CE$ and give a reason.

**a**



**b**



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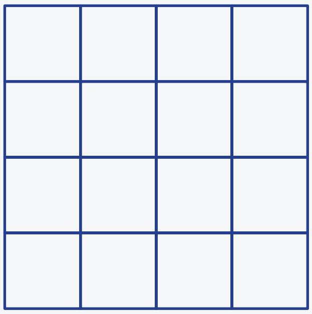
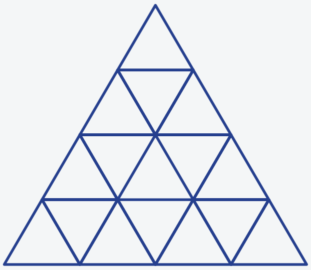
# Puzzles and games

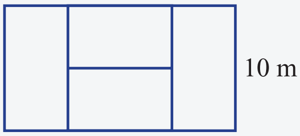
**1** How many squares are there in this diagram?



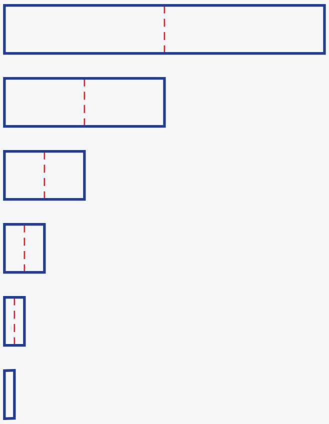
**2** How many triangles are there in this diagram?



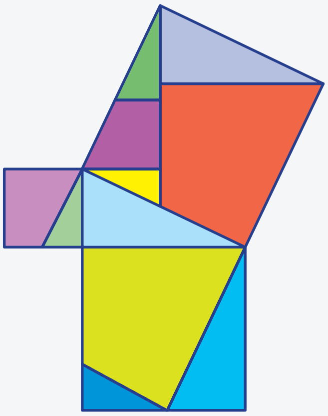
**3** The four rectangles inside this diagram are congruent. What is the perimeter of each rectangle?



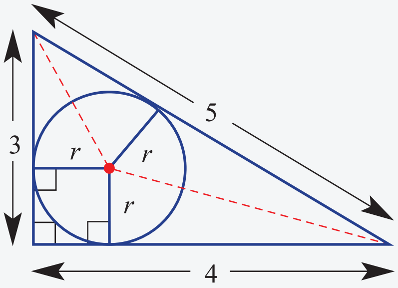
**4** A strip of paper is folded 5 times in one direction only. How many creases will there be in the original strip when it is unfolded?

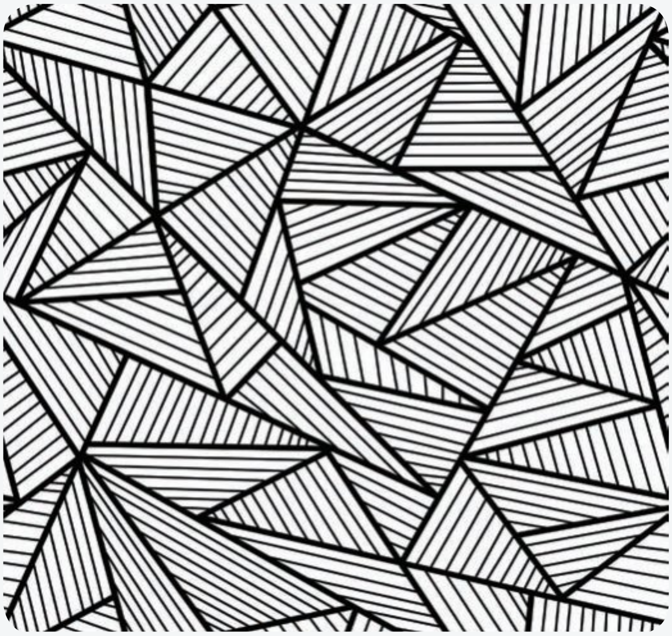


**5** Can you fit the shapes in the two smaller squares into the largest square? Try drawing or constructing the design and then use scissors to cut out each shape.



**6** Use congruent triangles to find the radius $r$ in this diagram.





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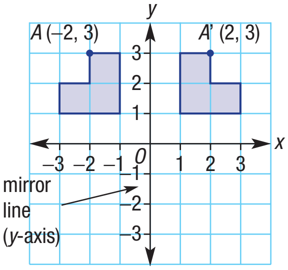
# Chapter summary

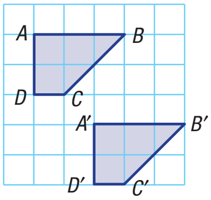
## Transformations and congruence

### Transformations

### Reflection

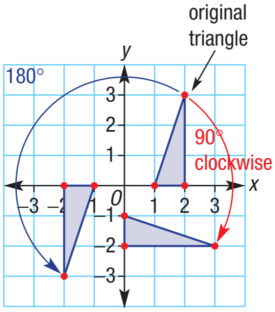


### Translation



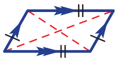
2 units right and 3 units down

### Rotation

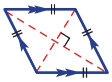


### Quadrilaterals and congruence

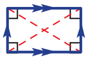
- Parallelogram



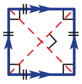
- Rhombus



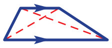
- Rectangle



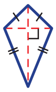
- Square



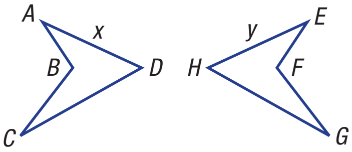
- Trapezium



- Kite



### Congruent figures same size and shape

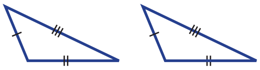


$ABCD \equiv EFGH$ $\quad x = y$

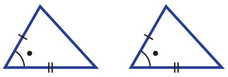
$AB = EF$ $\quad \angle C = \angle G$

### Tests for congruent triangles

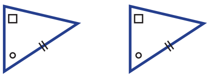
- SSS



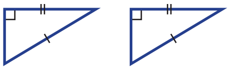
- SAS



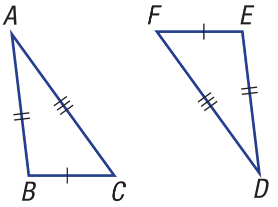
- AAS



- RHS



### Congruent triangles



$\Delta ABC \equiv \Delta DEF$

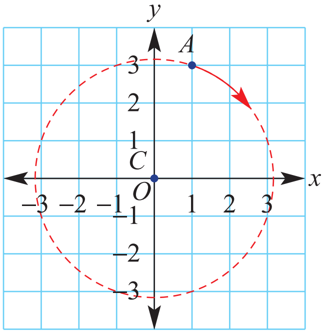
$AB = DE,\ BC = EF,\ AC = DF$

$\angle A = \angle D,\ \angle B = \angle E,\ \angle C = \angle F$

**Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.**

## Multiple-choice questions

Questions **1–5** relate to this diagram.



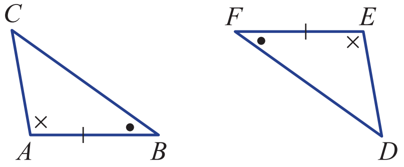
**1** Point $A(1, 3)$ is translated 2 units to the left to $A'$. The coordinates of $A'$ are:
**A** $(3, 3)$ **B** $(–1, 3)$ **C** $(1, 1)$ **D** $(1, 5)$ **E** $(0, 3)$

**2** Point $A(1, 3)$ is reflected across the $x$-axis to $A'$. The coordinates of $A'$ are:
**A** $(–1, 3)$ **B** $(–1, –3)$ **C** $(3, –1)$ **D** $(1, –3)$ **E** $(–3, –1)$

**3** Point $A(1, 3)$ is rotated 180° about $A$ to $A'$. The coordinates of $A'$ are:
**A** $(3, –1)$ **B** $(–3, 1)$ **C** $(–1, –3)$ **D** $(1, –3)$ **E** $(–1, 3)$

**4** Point $A(1, 3)$ is rotated clockwise about $C$ by 90° to $A'$. The coordinates of $A'$ are:
**A** $(3, 0)$ **B** $(3, –1)$ **C** $(–3, 1)$ **D** $(–1, 3)$ **E** $(3, 1)$

Questions **6–8** relate to this pair of congruent triangles.



**5** The transformation in Question **4** is equivalent to:
**A** reflecting $A$ across the $y$-axis then the $x$-axis
**B** translating $A$ 2 units left then 4 units down
**C** translating $A$ 2 units right then 4 units down
**D** rotating $A$ 90° anticlockwise about $C$
**E** none of the above.

**6** The angle on $\Delta DEF$ that corresponds to $\angle A$ is:
**A** $\angle C$ **B** $\angle B$ **C** $\angle F$ **D** $\angle D$ **E** $\angle E$

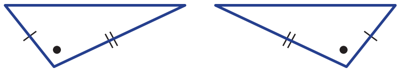
**7** If $AC = 5$ cm, then $ED$ is equal to:
**A** 5 cm **B** 10 cm **C** 2.5 cm **D** 15 cm **E** 1 cm

**8** A congruent statement with ordered vertices for the triangles is:

**A** $\Delta ABC \equiv \Delta FED$
**B** $\Delta ABC \equiv \Delta EDF$
**C** $\Delta ABC \equiv \Delta DFE$
**D** $\Delta ABC \equiv \Delta DEF$
**E** $\Delta ABC \equiv \Delta EFD$

**9** Which of the four tests (SSS, SAS, AAS, RHS) would be chosen to show that these two triangles are congruent?

**A** AAS
**B** RHS
**C** AAA
**D** SAS
**E** SSS



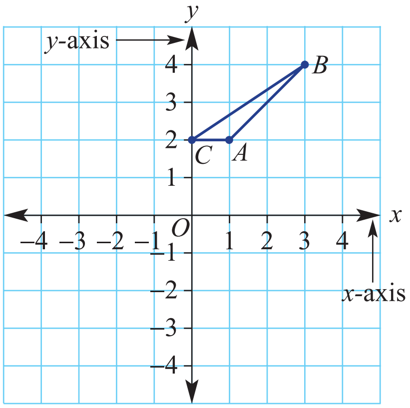
**10** Which of the following definitely has diagonals of equal length?

**A** Trapezium
**B** Parallelogram
**C** Kite
**D** Rhombus
**E** Rectangle

## Short-answer questions

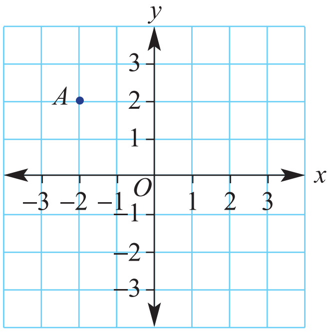
**1** State the coordinates of the image points $A'$, $B'$ and $C'$ when $\Delta ABC$ is reflected in the:

**a** $x$-axis
**b** $y$-axis.



**2** Give the coordinates of the image of the point $A$ if it is translated:

**a** 2 units right and 1 unit up
**b** 4 units right and 3 units down
**c** 1 unit left and 1 unit down
**d** 1 unit left and 4 units down.



**3** What translation will shift:

**a** $A(2, 5)$ to $A'(3, 9)$?
**b** $A(-1, 4)$ to $A'(2, -2)$?
**c** $A(0, 7)$ to $A'(-3, 0)$?
**d** $A(-4, -6)$ to $A'(0, 0)$?

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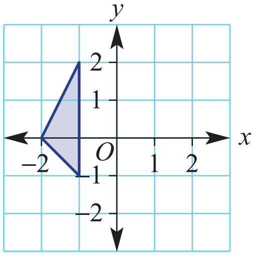
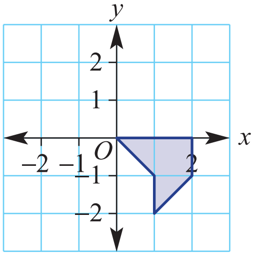
**4** Copy these shapes and draw the image following a translation of:

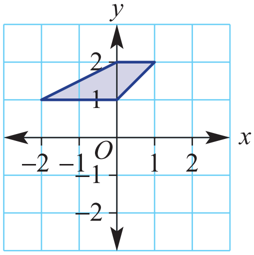
**a** right 3, down 1



**b** left 2, up 2

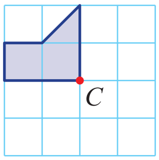


**c** down 3.

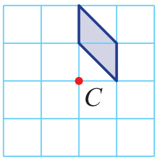


**5** Rotate these shapes about the point $C$ by the given angle.

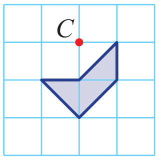
**a** Clockwise 90°



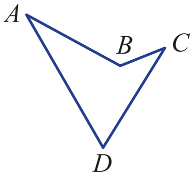
**b** Clockwise 180°

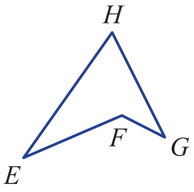


**c** Clockwise 90°



**6** For these congruent quadrilaterals, name the object in quadrilateral $EFGH$ that corresponds to the given object in quadrilateral $ABCD$.



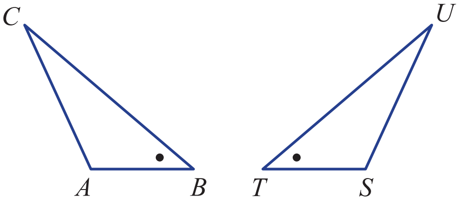


**a** **i** Vertex $B$ **ii** Vertex $C$

**b** **i** Side $AD$ **ii** Side $BC$

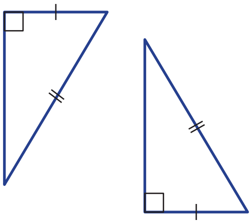
**c** **i** $\angle C$ **ii** $\angle A$

**7** Write a congruence statement for these congruent triangles.

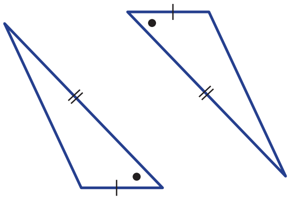


**8** Which of the tests SSS, SAS, AAS or RHS would you choose to explain the congruence of these pairs of triangles? Sides or angles with the same markings are equal.

**a**



**b**



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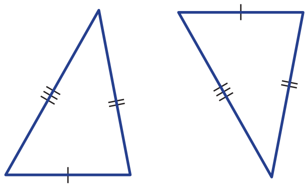
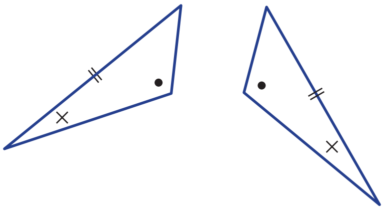
**c**

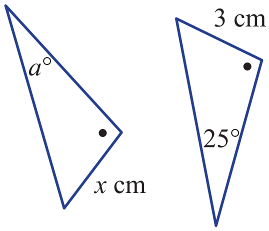


**d**

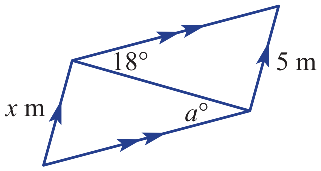


**9** Find the values of the pronumerals for these congruent triangles.

**a**



**b**



**10** This quadrilateral is a parallelogram with 2 pairs of parallel sides. You can assume that $AB = DC$ as shown.

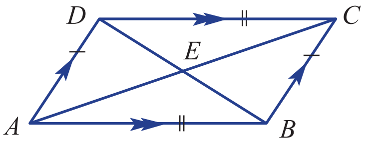
**a** Is $\angle BAE = \angle DCE$? Give a reason.

**b** Is $\angle ABE = \angle CDE$? Give a reason.

**c** Is $AB = DC$?

**d** Which test (SSS, SAS, AAS, RHS) would be used to explain that $\Delta ABE \equiv \Delta CDE$?

**e** Explain why $BD$ and $AC$ bisect each other.



## Extended-response question

**1** The shape on this set of axes is to be transformed by a succession of transformations. The image of the first transformation is used to start the next transformation. For each set of transformations write down the coordinates of the vertices $A'$, $B'$ and $C'$ of the final image.

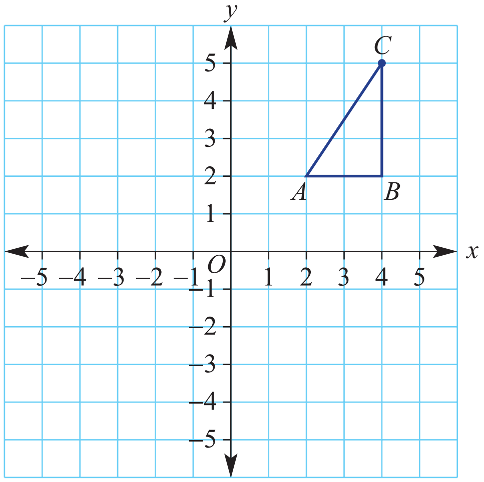
Parts **a** and **b** are to be treated as separate questions.

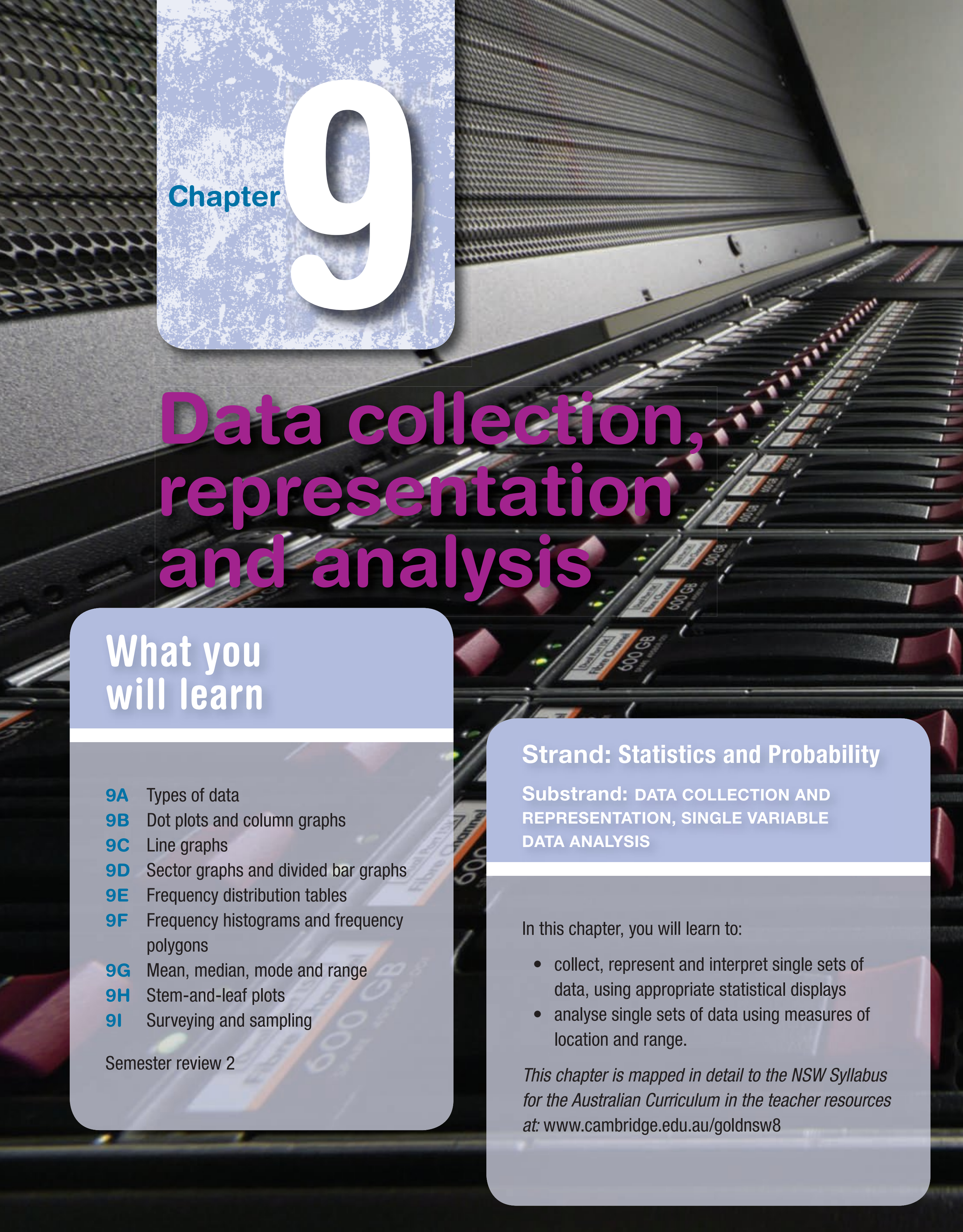
**a** Set 1

- **i** Reflection in the $x$-axis.
- **ii** Translation left 2, up 1.
- **iii** Rotation about (0, 0) by 180°.

**b** Set 2

- **i** Rotation about (0, 0) clockwise by 90°.
- **ii** Reflection in the $y$-axis.
- **iii** Translation right 5, up 3.





## Chapter 9

# Data collection, representation and analysis

## What you will learn

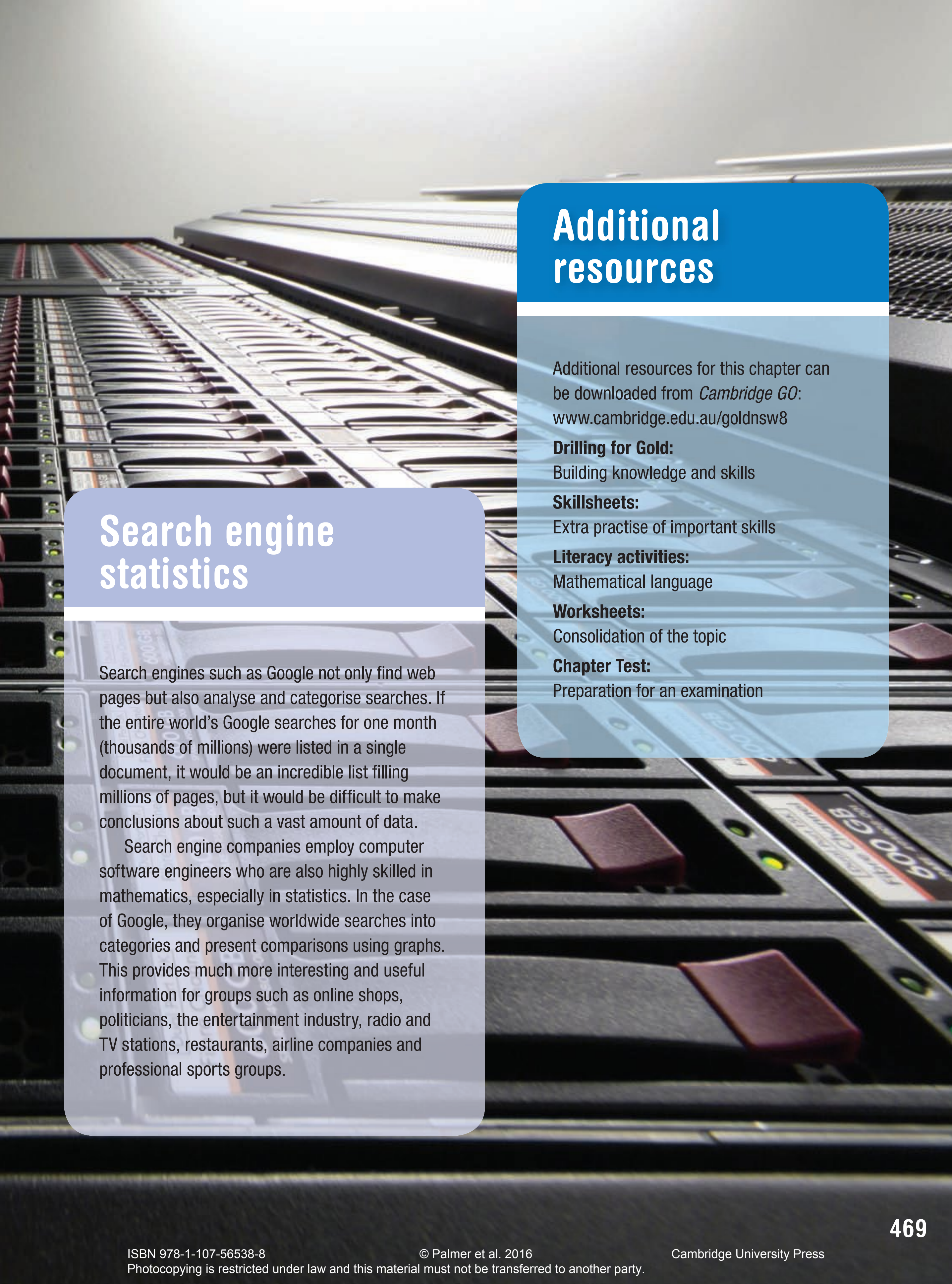
## Strand: Statistics and Probability

### Substrand: DATA COLLECTION AND REPRESENTATION, SINGLE VARIABLE DATA ANALYSIS

In this chapter, you will learn to:

- collect, represent and interpret single sets of data, using appropriate statistical displays
- analyse single sets of data using measures of location and range.

*This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at:* www.cambridge.edu.au/goldnsw8



## Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw8

**Drilling for Gold:**
Building knowledge and skills

**Skillsheets:**
Extra practise of important skills

**Literacy activities:**
Mathematical language

**Worksheets:**
Consolidation of the topic

**Chapter Test:**
Preparation for an examination

## Search engine statistics

Search engines such as Google not only find web pages but also analyse and categorise searches. If the entire world's Google searches for one month (thousands of millions) were listed in a single document, it would be an incredible list filling millions of pages, but it would be difficult to make conclusions about such a vast amount of data.

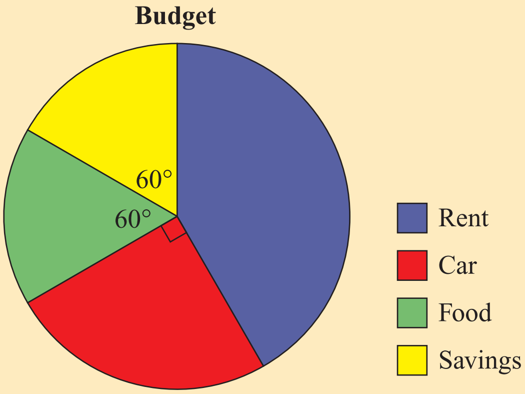
Search engine companies employ computer software engineers who are also highly skilled in mathematics, especially in statistics. In the case of Google, they organise worldwide searches into categories and present comparisons using graphs. This provides much more interesting and useful information for groups such as online shops, politicians, the entertainment industry, radio and TV stations, restaurants, airline companies and professional sports groups.

## Pre-test

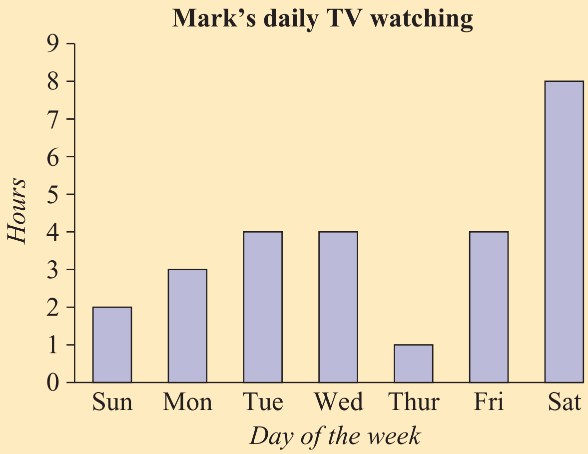
**1** Arrange the following in ascending order.
- **a** 2, 4, 10, 7, 1, 0, 6, 14, 9
- **b** 101, 20, 30.6, 204, 36, 100
- **c** 1.2, 1.9, 2.7, 1.7, 3.5, 3.2

**2** Write down the total and the average (mean) for each of the sets below.
- **a** 4, 6, 8, 10 and 12
- **b** 15, 17, 19, 19 and 24
- **c** 0.6, 0.6, 0.6, 0.7 and 0.8

**3** Use the information in the sector graph to answer the following questions.
- **a** What fraction of the income was spent on food?
- **b** What is the size of the angle for the rent sector?
- **c** If $420 is saved each month, find how much is spent on:
  - **i** food?
  - **ii** the car?



**4**
- **a** How many hours of television were watched on Wednesday?
- **b** How many hours of television were watched on Monday?
- **c** On which day was the most TV watched?
- **d** How many hours of TV were watched over the week shown?
- **e** What fraction of Saturday was spent watching TV?



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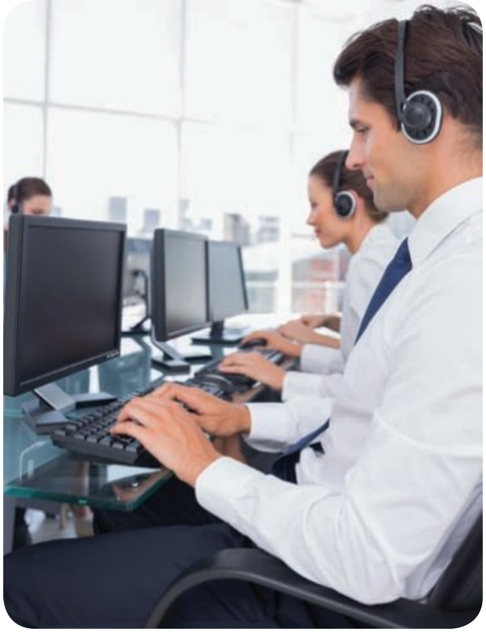
# 9A Types of data

People collect or use data almost every day. Athletes and sports teams look at performance data, customers compare prices at different stores, investors look at daily interest rates, and students compare marks with other students in their class. Companies often collect and analyse data to help produce and promote their products to customers and to make predictions about the future.



## Let's start: Collecting data – Class discussion

Consider, as a class, the following questions and discuss their implications.

- Have you or your family ever been surveyed by a telemarketer at home? What did they want? What time did they call?
- Do you think that telemarketers get accurate data? Why or why not?
- Why do you think companies collect data this way?

## Key ideas

- In statistics, a **variable** is something measurable or observable that is expected to change over time or between individual observations. It can be numerical or categorical.
  - **Numerical (quantitative)**, which can be discrete or continuous:
    - **Discrete numerical** – data that can only be particular numerical values, e.g. the number of TV sets in a house (could be 0, 1, 2, 3 but not values in between such as 1.3125).
    - **Continuous numerical** – data that can take any value in a range. Variables such as heights, weights and temperatures are all continuous. For instance, someone could have a height of 172 cm, 172.4 cm or 172.215 cm (if it can be measured that accurately).
  - **Categorical**, which is usually data such as colours, gender and brands of cars. In a survey, categorical data comes from answers that are given as words (e.g. 'yellow' or 'female') or ratings (e.g. 1 = dislike, 2 = neutral, 3 = like).

**Variable** Something that can be measured or observed, and which is expected to change over time

**Numerical (or quantitative) data** Data that is measured using numbers

**Discrete numerical data** Data that can only take particular numerical values

**Continuous numerical data** Data that can take any numerical value

**Categorical data** Data that can be put into separate and distinct categories like red or blue

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**Primary source** Information collected by the person using it

**Secondary source** Information collected by someone else

**Census** A survey of an entire population

**Sample** A smaller group surveyed or studied to represent an entire population

- Data can be collected from primary or secondary sources.
  - Data from a **primary source** is first-hand information collected from the original source by the person or organisation needing the data, e.g. a survey an individual student conducts or census data collected by the Bureau of Statistics.
  - Data from a **secondary source** has been collected, published and possibly summarised by someone else before we use it. Data collected from newspaper articles, textbooks or internet blogs represents secondary source data.
- Samples and populations
  - When an entire population (e.g. a maths class, a company or a whole country) is surveyed, it is called a **census**.
  - When a subset of the population is surveyed, it is called a **sample**. Samples should be randomly selected and large enough to represent the views of the overall population.

## Exercise 9A

**Understanding**

**1** Match each word (**a**–**f**) to its meaning (**i**–**vi**).

| | | | |
|---|---|---|---|
| **a** | Sample | **i** | Only takes on particular numbers within a range |
| **b** | Categorical | **ii** | A complete set of data |
| **c** | Discrete numerical | **iii** | A smaller group taken from the population |
| **d** | Primary source | **iv** | Data grouped in categories such as ‘male’ and ‘female’ |
| **e** | Continuous numerical | **v** | Data collected first hand |
| **f** | Population | **vi** | Can take on any number in a range |

### Example 1 Classifying variables

Classify the following variables as categorical, discrete numerical or continuous numerical.

**a** The gender of a newborn baby
**b** The length of a newborn baby

| Solution | Explanation |
|---|---|
| **a** categorical | As the answer is ‘male’ or ‘female’, the data is categorical. |
| **b** continuous numerical | Length is a measurement, so all numbers are possible. |

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**2** Classify the following as categorical or numerical.
- **a** The eye colour of each student in your class
- **b** The date of the month each student was born, e.g. the 9th of a month
- **c** The weight of each student when they were born
- **d** The types of aeroplanes landing at Sydney's international airport
- **e** The temperature of each classroom
- **f** The number of students in each classroom period one on Tuesday

**3** Give an example of:
- **a** discrete numerical data
- **b** continuous numerical data
- **c** categorical data.

**Fluency**

**Drilling for Gold 9A1**

**4** Classify the following variables as categorical, continuous numerical or discrete numerical data.
- **a** The number of cars per household
- **b** The weights of packages sent by Australia Post of the 20th of December
- **c** The highest temperature of the ocean each day
- **d** The favourite brand of chocolate of the teachers at your school
- **e** The colours of the cars in the school car park
- **f** The brands of cars in the school car park
- **g** The number of letters in different words on a page
- **h** The number of advertisements in a time period over each of the free-to-air channels
- **i** The length of time spent doing this exercise
- **j** The number of SMS messages sent by an individual yesterday
- **k** The times for the 100 m freestyle event at the world championships over the last 10 years
- **l** The number of Blu-ray discs someone owns

Categorical: Choose a category such as male or female
Continuous: Usually the result of a measurement
Diserete: Usually the result of counting

**5** Is observation or a sample or a census the most appropriate way to collect data on each of the following?
- **a** The arrival times of trains at central station during a day
- **b** The arrival times of trains at central station over the year
- **c** The heights of students in your class
- **d** The heights of all Year 7 students in the school
- **e** The heights of all Year 7 students in NSW
- **f** The number of plastic water bottles sold in a year

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**g** The religion of Australian families

**h** The number of people living in each household in your class

**i** The number of people living in each household in your school

**j** The number of people living in each household in Australia

Census: A survey of all members of the population
Sample: A survey of some members of the population

## Example 2 Collecting data from primary and secondary sources

Decide whether a primary source or a secondary source is suitable for collection of data on each of the following and suggest a method for its collection.

**a** The average income of Australian households

**b** The favourite washing powder or liquid for households in Australia

| Solution | Explanation |
|---|---|
| **a** Primary source by looking at the census data | The population census held every 5 years in Australia is an example of a primary data source collection and will have this information. |
| **b** Secondary data source using the results from a market research agency | A market research agency might collect these results using a random phone survey. Obtaining a primary source would involve conducting the survey yourself but it is unlikely that the sample will be large enough to be suitable. |

**6** Identify whether a primary or secondary source is suitable for the collection of data on the following.

**a** The number of soft drinks bought by the average Australian family in a week

**b** The age of school leavers in far North Queensland

**c** The number of cigarettes consumed by school age students in a day

**d** The highest level of education by the adults in a household

**e** The reading level of students in Year 7 in Australia

Primary: Collect it yourself
Secondary: Collected by someone else

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Problem-solving and Reasoning

**7** Give a reason why someone might have trouble obtaining reliable and representative data using a primary source to find the following.

- **a** The temperature of the Indian Ocean over the course of a year
- **b** The religions of Australian families
- **c** The average income of people India
- **d** Drug use by teenagers within a school
- **e** The level of education of different cultural communities within NSW

**8** When obtaining primary source data you can survey the population or a sample.

- **a** Explain the difference between a ‘population’ and a ‘sample’ when collecting data.
- **b** Give an example situation where you should survey a population rather than a sample.
- **c** Give an example situation where you should survey a sample rather than a population.

**9** A sample should be representative of the population it reports on. For the following surveys, describe who might be left out and how this might introduce a bias.

- **a** A telephone poll with numbers selected from a phone book
- **b** A postal questionnaire
- **c** Door-to-door interviews during the week days
- **d** A *Dolly* magazine poll
- **e** A Facebook survey

**10** Television ratings are determined by surveying a sample of the population.

- **a** Explain why a sample is taken rather than conducting a census.
- **b** What would be a limitation of the survey results if the sample included $50$ people nationwide?
- **c** If a class census was taken on which (if any) television program students watched from $7{:}30–8{:}30$ last night, why might the results be different to the official ratings?
- **d** Research how many people are sampled by Nielsen Television Audience Measurement in order to get an accurate idea of viewing habits.

**11** Australia’s census surveys the entire population every five years.

- **a** Why might Australia not conduct a census every year?
- **b** The census can be filled out on a paper form or using the internet. Given that the data must be collated in a computer eventually, why does the government still allow paper forms to be used?
- **c** Why might a country like India or China conduct their national census every $10$ years?

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## Enrichment: One population, many samples

**12** Find 100 playing cards (or blocks, balls or counters). Ensure they are all the same shape and size, but 50 are red and 50 are black. Place them into a bag or box and mix them up.

**a** Randomly choose four cards, then fill in the first row of the table below.
Place the cards back into the bag.
Repeat this sampling experiment until the table is full.
Compare your results with those of other students.
In your class, how many times were 2 reds and 2 blacks chosen?

| | Sample size is 4 | |
|---|---|---|
| | red | black |
| Sample 1 | | |
| Sample 2 | | |
| Sample 3 | | |
| Sample 4 | | |
| Sample 5 | | |

**b** Randomly choose 10 cards, then fill in the first row of the table below.
Place the cards back in the bag.
Repeat this sampling experiment until the table is full.
In your class, how many times were 5 reds and 5 blacks chosen?

| | Sample size is 10 | |
|---|---|---|
| | red | black |
| Sample 1 | | |
| Sample 2 | | |
| Sample 3 | | |
| Sample 4 | | |
| Sample 5 | | |

**c** In this experiment the 'population' is 100 cards, half of which are black.
Which sample size (4 or 10) generally gave the better estimate of the number of black cards in the bag?

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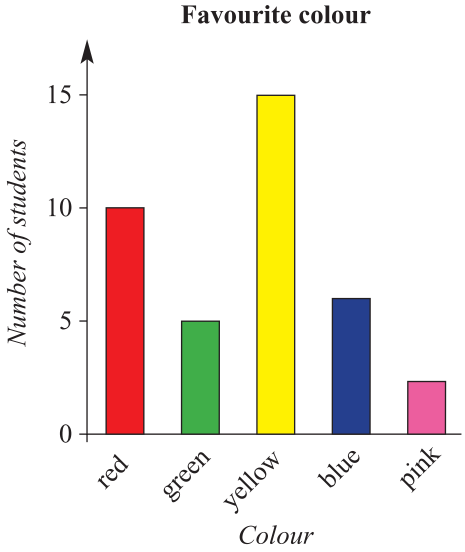
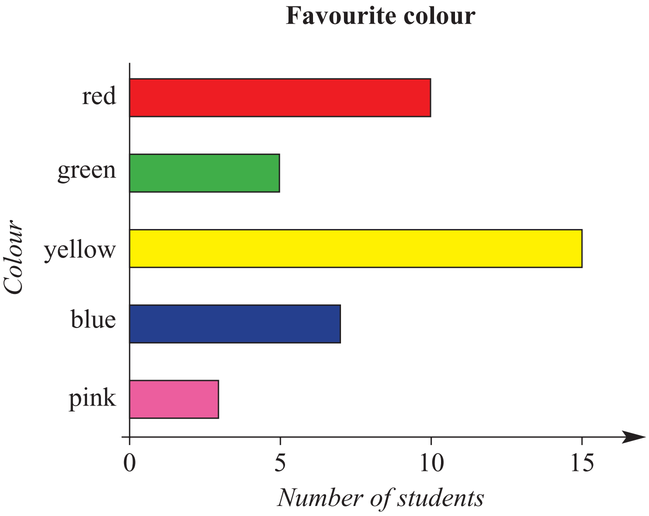
# 9B Dot plots and column graphs

Graphs are a good way to display and summarise data. For example, if students were surveyed on their favourite colours, the results could be shown as a column graph. The 'columns' can be vertical (going up) or horizontal (going across).



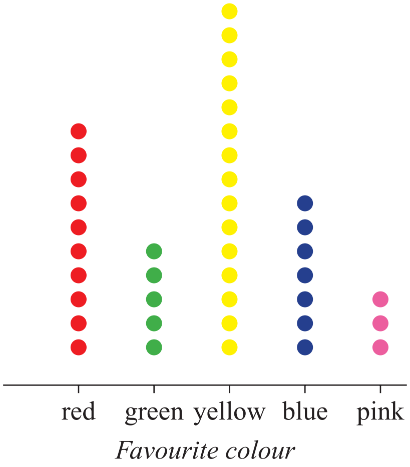


The data could also be shown in a dot plot (below, right).

## Let's start: Favourite colours

Survey the class to determine each student's favourite colour from the possibilities red, green, yellow, blue and pink.

- Draw a column graph or a dot plot to represent the results.
- Compare your graph with those of other students. Describe any differences you notice.



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## Key ideas

**Dot plot** A graph in which each dot represents one data value

**Datum** One piece of data

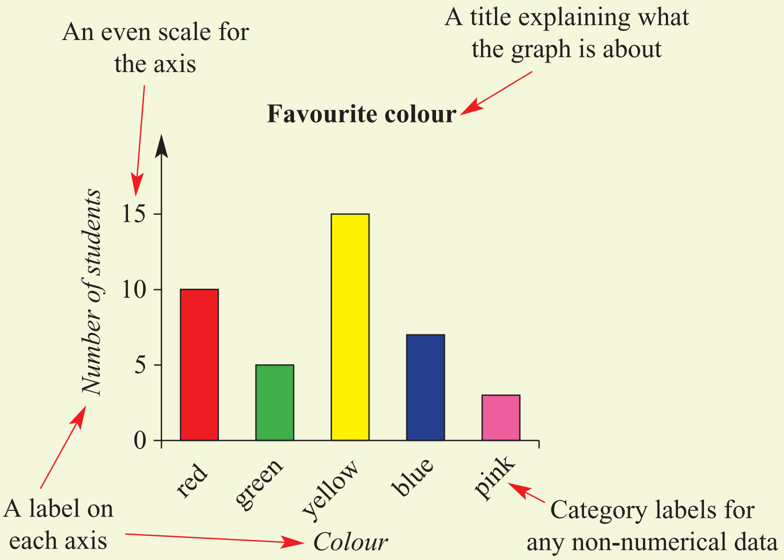
**Outlier** A value that is much larger or much smaller than the rest of the data

**Mode** The most common value

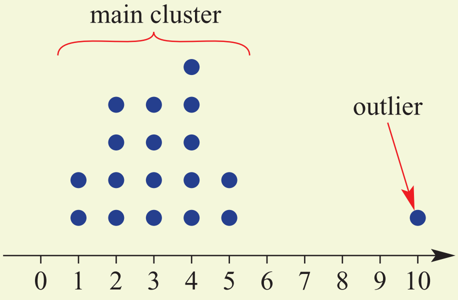
**Median** The middle value if the values are sorted from lowest to highest

Drilling for Gold 9B1

- A **dot plot** can be used to display data, where each dot represents one **datum**.
- A column graph is a good way to show data in different categories, and is useful when more than a few items of data are present.
- Column graphs can be drawn vertically (going up) or horizontally (going across).
- Graphs should have the following features.



- An **outlier** is a value that is noticeably distinct from the main cluster of data values.



- The **mode** is the most common response. It can be seen in the tallest columns. In the graphs above, the modes are yellow and 4.
- The **median** is the middle value if the values are sorted from lowest to highest. If the values are 1, 3, 5, 9, 11, then the median is 5.

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## Exercise 9B

Understanding

**1** Fill in the blanks in the following sentences.

**a** A ________ ________ is a graph that uses dots to represent data.

**b** A graph showing data in different categories as rectangles is called a ________ ________.

**c** An ________ is a value that is noticeably distinct from the main cluster of points.

**2** This column graph shows the height of four boys. Answer true or false to each of the following statements.

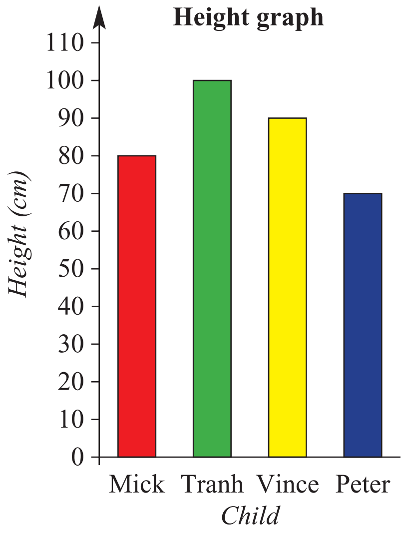
**a** Mick is 80 cm tall.

**b** Vince is taller than Tranh.

**c** Peter is the shortest of the four boys.

**d** Tranh is 100 cm tall.

**e** Mick is the tallest of the four boys.



### Example 3 Interpreting a dot plot

This dot plot represents the results of a survey that asked some children how many pets they have at home.

**a** Use the graph to state how many children have 2 pets.

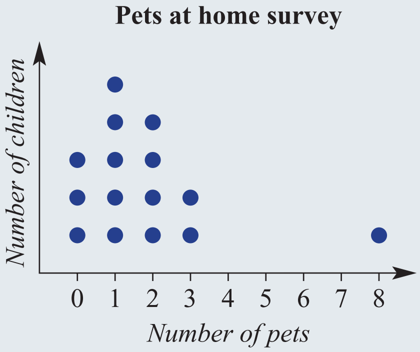
**b** How many children participated in the survey?

**c** What is the range of values?

**d** What is the median number of pets?

**e** What is the outlier?

**f** What is the mode?



| Solution | Explanation |
|---|---|
| **a** 4 children | There are 4 dots in the '2 pets' category, so 4 children have 2 pets. |
| **b** 15 children | The total number of dots is 15. |
| **c** 8 – 0 = 8 | Range = highest – lowest<br>In this case, highest = 8, lowest = 0. |
| **d** 1 pet | Write the values in order: 0, 0, 0, 1, 1, 1, 1, ①, 2, 2, 2, 2, 3, 3, 8<br>Middle value = median = 1 |

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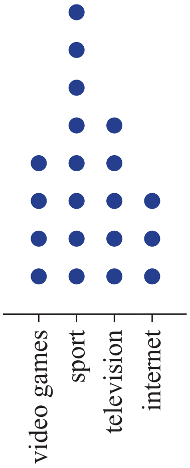
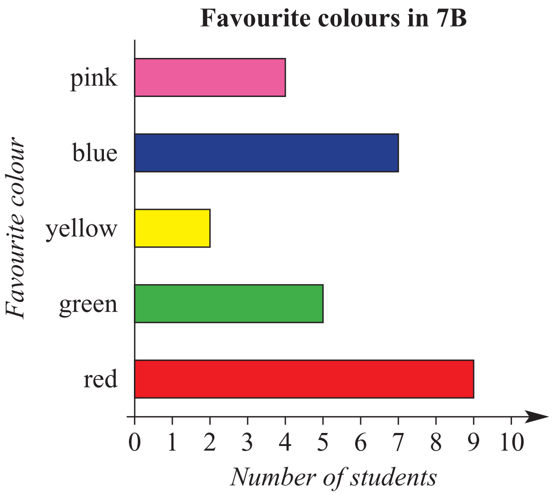
| **Solution** | **Explanation** |
|---|---|
| **e** 8 pets | The main cluster of values is from 0 pets to 3 pets. The dot showing 8 pets is significantly outside this cluster. |
| **f** 1 pet | The most common number of pets is 1 pet. |

**3** The favourite after-school activity of a number of Year 7 students is recorded in this dot plot.
- **a** How many students have chosen television as their favourite activity?
- **b** How many students have chosen surfing the internet as their favourite activity?
- **c** What is the most popular after school activity for this group of students?
- **d** How many students participated in the survey?

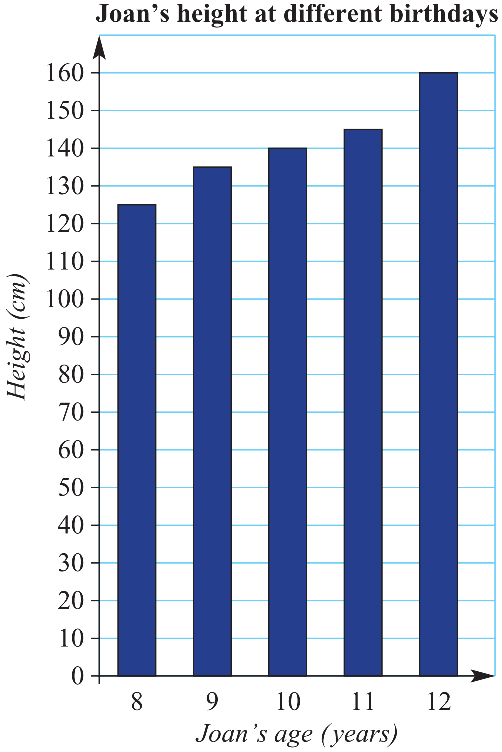


**Fluency**

**4** From a choice of pink, blue, yellow, green or red, each student of Year 7B chose their favourite colour. The results are graphed on the right.
- **a** How many students chose yellow?
- **b** How many students chose blue?
- **c** What is the most popular colour?
- **d** How many students participated in the class survey?



**5** Joan has graphed her height at each of her past five birthdays.

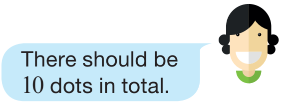


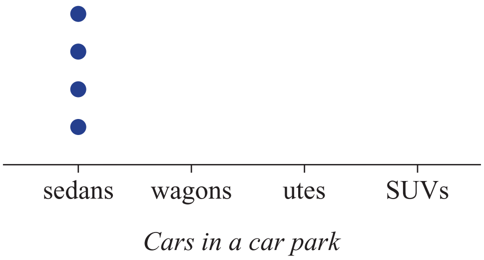
- **a** How tall was Joan on her 9th birthday?
- **b** How much did she grow between her 8th birthday and 9th birthday?
- **c** How much did Joan grow between her 8th and 12th birthdays?
- **d** How old was Joan when she had her biggest growth spurt?

**6** The types of cars parked in a small car park were:

| Sedan | Wagon | Ute | SUV |
|---|---|---|---|
| 4 | 1 | 2 | 3 |

- **a** How many utes were in the car park?
- **b** Copy and complete the dot plot.





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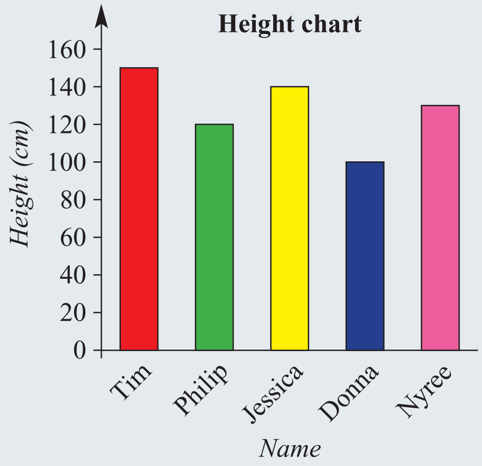
## Example 4 Constructing a column graph

Draw a column graph to represent the following people's heights.

| Name | Tim | Philip | Jessica | Donna | Nyree |
|---|---|---|---|---|---|
| Height (cm) | 150 | 120 | 140 | 100 | 130 |

**Solution**



**Explanation**

First decide which scale goes on the vertical axis.
Maximum height = 150 cm, so axis goes from 0 cm to 160 cm (to allow a bit above the highest value).

Remember to include all the features required, including axes labels and a graph title.

**7** Draw a column graph to represent each of these boys' heights at their birthdays.

The scale on your vertical axis could go 0, 10, 20, … 150.

**a** Mitchell

| Age (years) | Height (cm) |
|---|---|
| 8 | 120 |
| 9 | 125 |
| 10 | 135 |
| 11 | 140 |
| 12 | 145 |

**b** Fatu

| Age (years) | Height (cm) |
|---|---|
| 8 | 125 |
| 9 | 132 |
| 10 | 140 |
| 11 | 147 |
| 12 | 150 |

**8** The ages (in years) of children at a party were: 7, 10, 8, 11, 8, 7, 9, 10, 12, 8.

**a** Represent this as a dot plot.

**b** What is the range of the ages?

Range = largest – smallest

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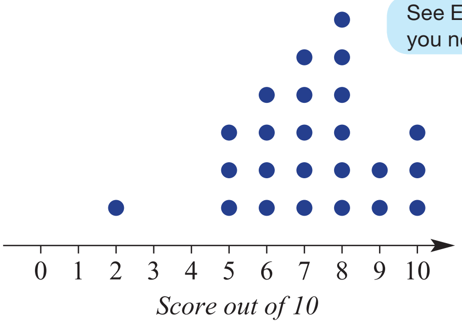
**Problem-solving and Reasoning**

**9** The results of a Year 4 spelling quiz are shown as a dot plot.

- **a** How many students got a score of 6?
- **b** What is the most common score in the class?
- **c** How many students participated in the quiz?
- **d** What is the range of scores achieved?
- **e** What is the median score?
- **f** Identify the outlier.

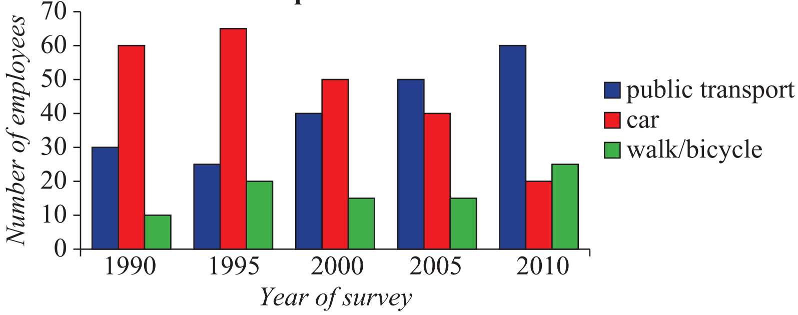
**Spelling quiz results**



See Example 3 if you need help.

**10** Every five years, a company in the city conducts a transport survey of the way people get to work in the mornings. The results are graphed below.

**Transport methods**



- **a** Copy and complete this table to show the data in the graph.
- **b** In which year(s) was public transport the most popular option?
- **c** In which year(s) were more people walking or cycling to work than driving?
- **d** Suggest one reason why the number of people driving to work has decreased.
- **e** What is one other trend that you can see from looking at this graph?

| | 1990 | 1995 | 2000 | 2005 | 2010 |
|---|---|---|---|---|---|
| **Use public transport** | 30 | | | | |
| **Drive a car** | 60 | | | | |
| **Walk or cycle** | 10 | | | | |

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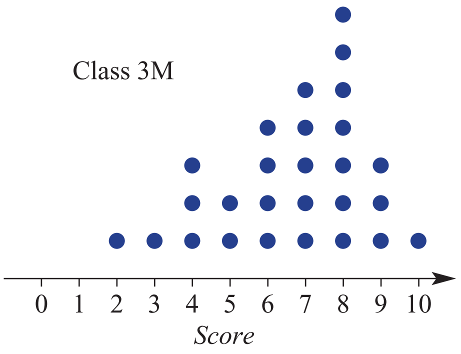
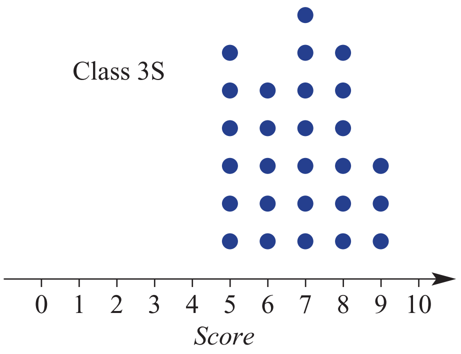
**11** **a** Draw a column graph to show the results of the following survey of the number of boys and girls born at a certain hospital. Put the years on the horizontal axis.

| | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 |
|---|---|---|---|---|---|---|
| **Number of boys born** | 40 | 42 | 58 | 45 | 30 | 42 |
| **Number of girls born** | 50 | 40 | 53 | 41 | 26 | 35 |

**b** During which year(s) were more girls born than boys?
**c** Which year had the smallest number of births?
**d** Which year had the greatest number of births?
**e** During the time of the survey, were more boys or girls born?

**12** Mr Martin and Mrs Stevensson are the two Year 3 teachers at a school. For the latest arithmetic test, they plotted their students' scores on dot plots.





**a** What is the median score for class 3M?
**b** What is the median score for class 3S?
**c** State the range of scores for each class.
**d** Based on this test, which class has a greater spread of arithmetic abilities?
**e** If the two classes competed in an arithmetic competition, where each class is allowed only one representative, which class is more likely to win? Why?

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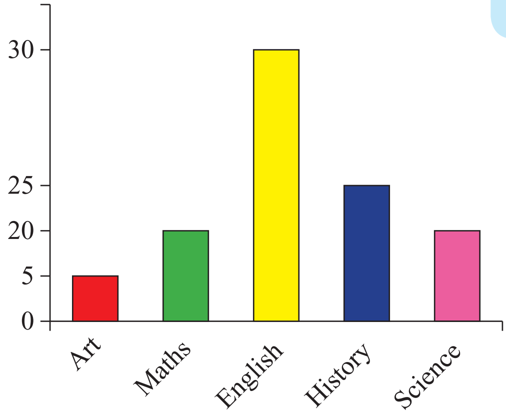
## Enrichment: Misleading graphs

**13** A survey is conducted of students' favourite subjects. Someone has tried to show the results in a column graph.

Check the Key ideas to see what features graphs should show.



**a** What is wrong with the scale on the vertical axis?
**b** Give at least two other problems with this graph.
**c** Redraw the graph with an even scale and appropriate labels.
**d** The original graph makes Maths look twice as popular as Art, based on the column size. According to the survey, how many times more popular is Maths?
**e** The original graph makes English look three times more popular than Maths. From the survey, how many times more popular is English?
**f** Look on the internet for a graph with an uneven scale that makes the graph misleading.

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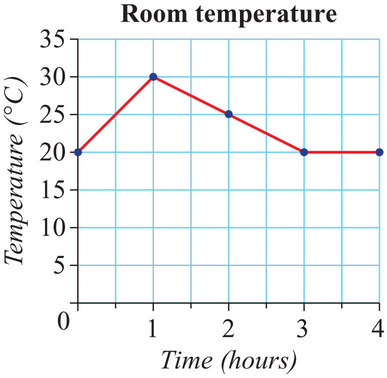
# 9C Line graphs

Line graphs can be used to show quantities that change as time passes.

## Let's start: Room temperature

As an experiment, the temperature in a room is measured hourly over 4 hours. The results are shown in this line graph.

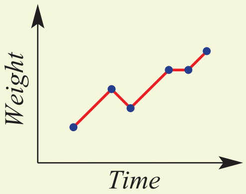
- Describe the temperature changes over the four hours.
- An air conditioner was turned on at some stage. When do you think this happened? Why?
- What was the approximate temperature 90 minutes (1.5 hours) after the experiment started?



## Key ideas

**Line graph** A graph that shows the data as points joined with line segments

- A **line graph** consists of a series of points joined by straight line segments.
- Time is often shown on the horizontal axis. For example:



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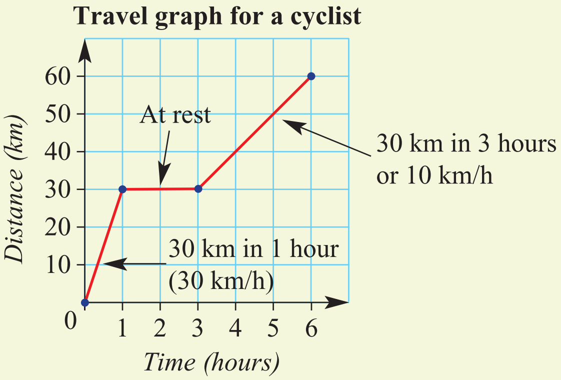
- A common type of line graph is a **travel graph**.
  - Time is shown on the horizontal axis.
  - Distance is shown on the vertical axis.
  - The slope of the line shows the rate at which the distance is changing over time. This rate is called speed.

**Travel graph** A line graph that describes a traveller's position at different times

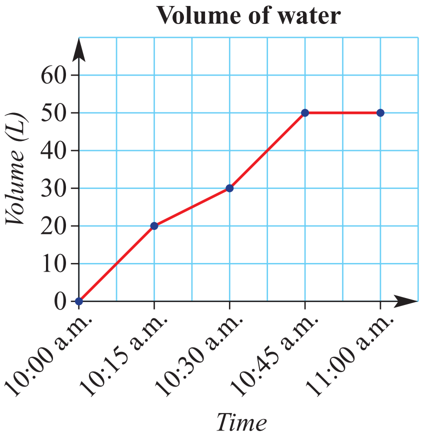


## Exercise 9C

**Understanding**

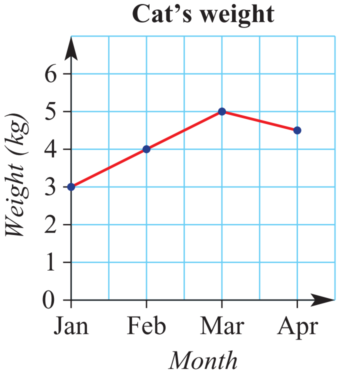
**1** The volume of water running into a tank is measured and graphed. State the volume of water at:

**a** 10:15 a.m.
**b** 10:30 a.m.
**c** 10:45 a.m.
**d** 11:00 a.m.



**2** This line graph shows the weight of a cat over a 3-month period. The cat is weighed at the start of each month.
State the cat's weight at the start of:

**a** January
**b** February
**c** March
**d** April



## Example 5 Drawing a line graph

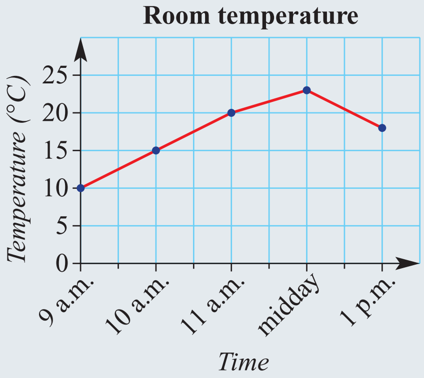
The temperature in a room is noted at hourly intervals.

| Time | 9:00 a.m. | 10:00 a.m. | 11:00 a.m. | midday | 1:00 p.m. |
|---|---|---|---|---|---|
| Temperature (°C) | 10 | 15 | 20 | 23 | 18 |

**a** Draw a line graph of the temperature from 9:00 a.m. until 1:00 p.m.
**b** Use your graph to estimate the room temperature at 12:30 p.m.

### Solution

**a**



**b** About 20°C

### Explanation

- The vertical axis is from 0 to 25. The scale is even (i.e. increasing by 5 each time).
- Dots are placed for each measurement and joined with straight line segments.

Look at the graph halfway between midday and 1 p.m. and form an estimate.

**3** A dog is weighed over a period of 3 months. Draw a line graph of its weight.
January: 5 kg, February: 6 kg, March: 8 kg, April: 7 kg.

Use grid paper to help draw graphs.

**Fluency**

## Example 6 Interpreting a line graph

The graph shows Lillian's height over a 5-year period from birth.

**a** What was her height when she turned 2?

**b** Estimate her height at $2\frac{1}{2}$ years.

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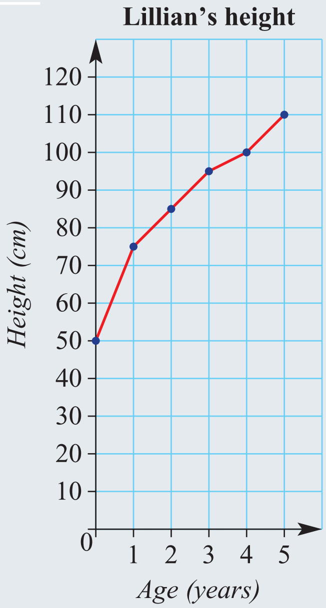
| **Solution** | **Explanation** |
|---|---|
| **a** 85 cm | Read this directly from the graph. |
| **b** About 90 cm | This is halfway between 2 years (85 cm) and 3 years (95 cm). |



**4** Look at the graph of Lillian's height in Example 6.
  **a** What was Lillian's height when she was born?
  **b** What was Lillian's height when she turned 3?
  **c** How much did Lillian grow in the year when she was 2 years old?
  **d** Use the graph to estimate her height at the age of $4\frac{1}{2}$ years.

For part **c**, compare Lillian's height at age 2 and age 3.

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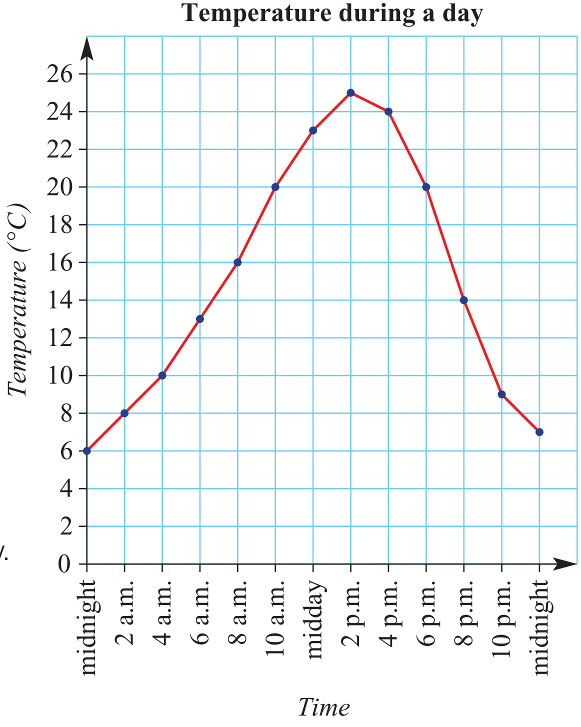
**5** This graph shows the outside temperature over a 24-hour period that starts at midnight.
  **a** What was the temperature at midday?
  **b** When was the hottest time of the day?
  **c** When was the coolest time of the day?
  **d** Use the graph to estimate the temperature at these times of the day.
    **i** 4:00 a.m.
    **ii** 9:00 a.m.
    **iii** 1:00 p.m.
    **iv** 5:00 p.m.



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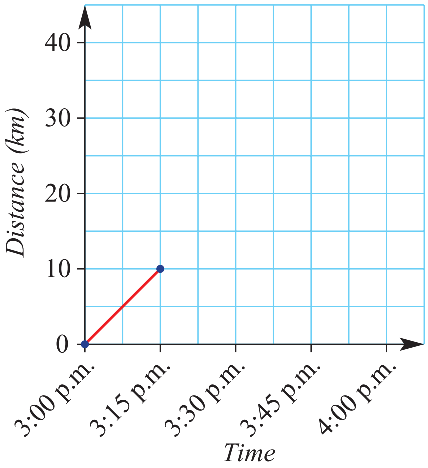
**6** Oliver measures his pet dog's weight over the course of a year. He gets the following results.

| **Month** | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Weight (kg)** | 7 | 7.5 | 8.5 | 9 | 9.5 | 9 | 9.2 | 7.8 | 7.8 | 7.5 | 8.3 | 8.5 |

- **a** Draw a line graph showing this information, making sure the vertical axis has an equal scale from 0 kg to 10 kg.
- **b** Describe any trends or patterns that you see.
- **c** Oliver put his dog on a weight loss diet for a period of 3 months. When do you think the dog started the diet? Justify your answer.

**7** This table shows how far Aisha has driven over the course of an hour. Copy and complete the travel graph.

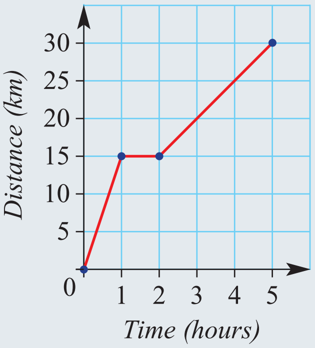
| **Time** | **Distance (km) from home** |
|---|---|
| 3:00 p.m. | 0 |
| 3:15 p.m. | 10 |
| 3:30 p.m. | 15 |
| 3:45 p.m. | 25 |
| 4:00 p.m. | 30 |



## Example 7 Interpreting a travel graph

This travel graph shows the distance travelled by a cyclist over 5 hours.

- **a** How far did the cyclist travel in total?
- **b** How far did the cyclist travel in the first hour?
- **c** What is happening in the second hour?
- **d** When is the cyclist travelling the fastest?
- **e** In the fifth hour, how far does the cyclist travel?



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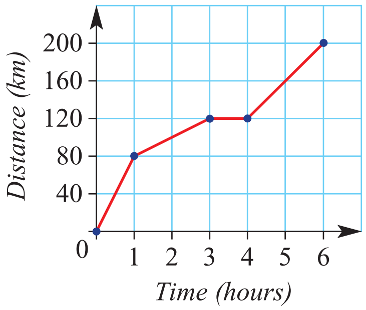
| Solution | Explanation |
|---|---|
| **a** 30 km | The point at the right-hand end of the graph is (5, 30). |
| **b** 15 km | At time = 1 hour, the distance covered is 15 km. |
| **c** At rest | The distance travelled does not increase in the second hour. |
| **d** In the first hour | This is the steepest part of the graph. |
| **e** 5 km | In the last 3 hours, the distance travelled is 15 km, so in 1 hour, 5 km is travelled. |

**8** This travel graph shows the distance travelled by a van over 6 hours.

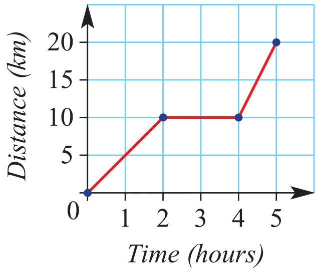
- **a** How far did the van travel in total?
- **b** How far did the van travel in the first hour?
- **c** What is happening in the fourth hour?
- **d** When is the van travelling the fastest?
- **e** In the sixth hour, how far does the van travel?

For part **c**, the fourth hour is from 3 to 4 hours.



**9** This travel graph shows the distance travelled by a cyclist over 5 hours.

- **a** How far did the cyclist ride in total?
- **b** How far did the cyclist ride in the second hour?
- **c** During which hour did the cyclist ride the fastest?
- **d** For how long did the cyclist rest?



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**Problem-solving and Reasoning**

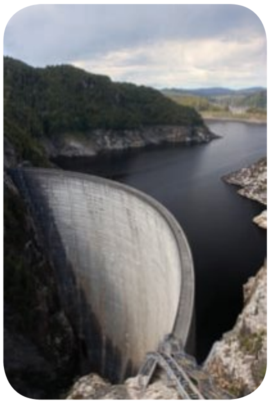
**10** The graph shows water storage levels for a certain city.

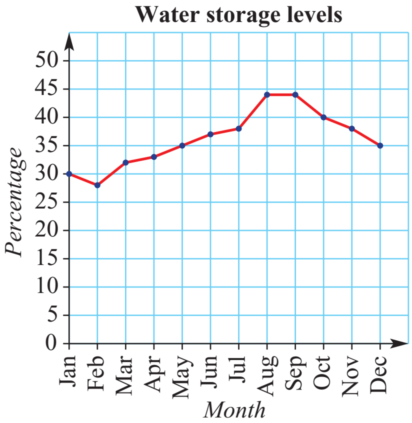
**a** What was the water level at the start of:
  **i** January?
  **ii** May?
  **iii** December?

**b** Which month do you think had the highest rainfall? Why?

**c** What was the maximum water level?

**d** When did the water storage get to its lowest point?





**11** Draw travel graphs to illustrate the following journeys.

**a** A car travels:
- 120 km in the first 2 hours
- 0 km in the third hour
- 60 km in the fourth hour
- 120 km in the fifth hour

**b** A jogger runs:
- 12 km in the first hour
- 6 km in the second hour
- 0 km in the third hour
- at a rate of 6 km/h for 2 hours

When the distance travelled in an hour is 0 km, draw a horizontal line.

## Enrichment: Heating and cooling

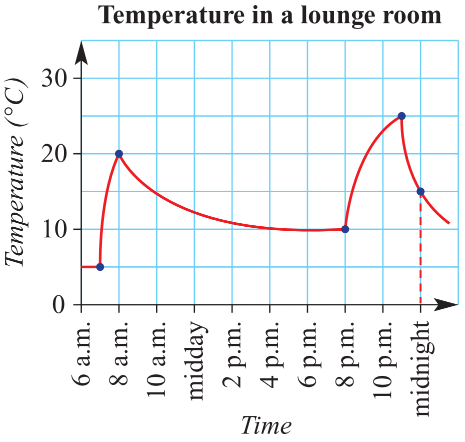
**12** The temperature in a lounge room is measured several times on a particular day. The results are shown in a line graph.

**a** State the room's temperature at:
  **i** 6 a.m.
  **ii** 8 a.m.
  **iii** 10 a.m.
  **iv** 8 p.m.

**b** Twice during the day the heating was switched on. At what times do you think this happened? Explain your reasoning.

**c** When was the heating switched off? Explain your reasoning.

**d** The house has a single occupant, who works during the day. Describe when you think that person is:
  **i** waking up
  **ii** going to work
  **iii** coming home
  **iv** going to bed.



**e** These temperatures were recorded during a cold winter month. Draw a graph that shows what the lounge room temperature might look like during a hot summer month. Assume that the room has an air conditioner, which the person is happy to use when at home.

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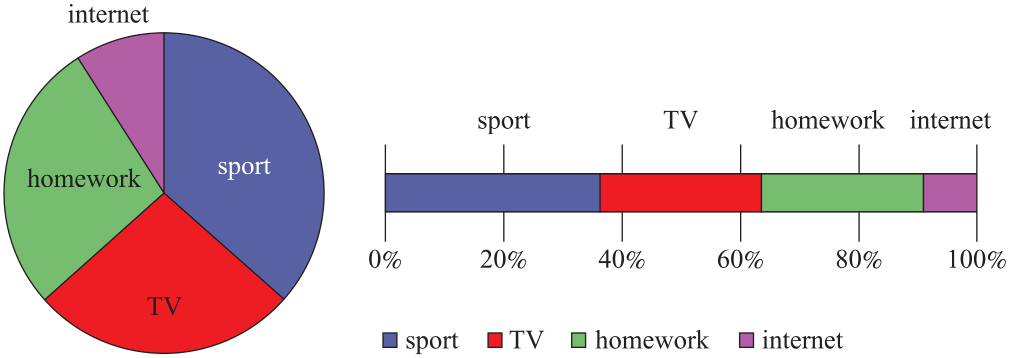
# 9D Sector graphs and divided bar graphs

A sector graph (also called pie chart) consists of a circle divided into different sectors or 'slices of pie'. The size of each sector indicates the proportion occupied by any given item. A divided bar graph is a rectangle divided into different rectangles or 'bars'. The size of each rectangle indicates the proportion of each item. Both types of graphs are suitable for categorical but not numerical data.

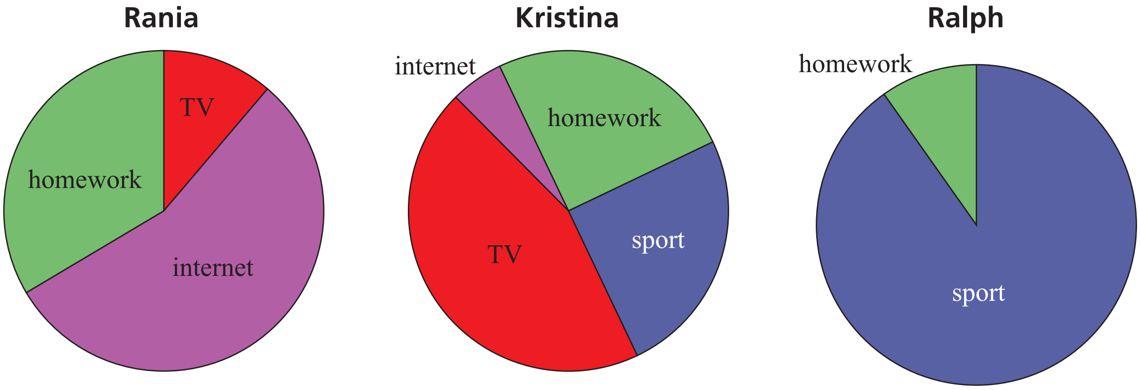
If a student is asked to describe how much time they spend each evening doing different activities, they could present their results as either type of graph.



From both graphs, it is easy to see that the student plays a lot of sport and the least amount of time is spent using the internet.

## Let's start: Student hobbies

Rania, Kristina and Ralph are asked to record how they spend their time after school. They draw the following graphs.



- Based on these graphs alone, describe each student in a few sentences.
- Justify your descriptions based on the graphs.

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## Key ideas

**Sector graph (or pie chart)** A circle divided into different sectors to indicate the proportions of different items

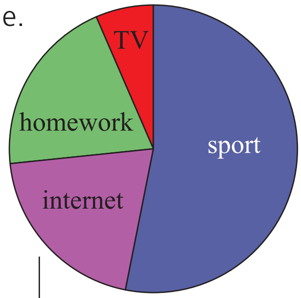
**Divided bar graph** A rectangle divided into different 'bars' to indicate the proportions of different items

- To calculate the size of each section of the graph, divide the value in a given category by the number of data values. This gives the category's proportion or fraction.
- To draw a **sector graph** (also called a **pie chart**), multiply each category's proportion or fraction by 360° and draw a sector of that size.
- To draw a **divided bar graph**, multiply each category's proportion or fraction by the total length of the rectangle and draw a rectangle of that size.

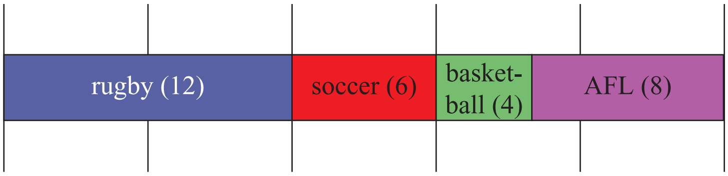
## Exercise 9D

Understanding

**1** Jasna graphs a sector graph of how she spends her leisure time.
  **a** What does Jasna spend the most time doing?
  **b** What does Jasna spend the least time doing?
  **c** Does she spend more or less than half of her time playing sport?



**2** Thirty students are surveyed to find out their favourite sport and their results are graphed below.



  **a** What is the most popular sport for this group of students?
  **b** What is the least popular sport for this group of students?
  **c** What fraction of the students has chosen soccer as their favourite sport?
  **d** What fraction of the students has chosen either rugby or AFL?

Fluency

### Example 8 Drawing a sector graph and a divided bar graph

On a particular Saturday, Sanjay measured the number of hours he spent on different activities.

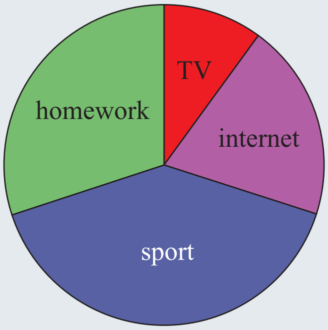
| TV | internet | sport | homework |
|---|---|---|---|
| 1 hour | 2 hours | 4 hours | 3 hours |

Represent the table as a:
  **a** sector graph **b** divided bar graph.

**Solution**

**Explanation**

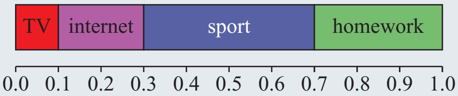
**a**



The total amount of time is $1 + 2 + 4 + 3 = 10$ hours. Then we can calculate the proportions and sector sizes:

| Category | Proportion | Sector size (°) |
|---|---|---|
| TV | $\frac{1}{10} = 0.1 = 10\%$ | $\frac{1}{10} \times 360 = 36$ |
| internet | $\frac{2}{10} = 0.1 = 20\%$ | $\frac{2}{10} \times 360 = 72$ |
| sport | $\frac{4}{10} = 0.4 = 40\%$ | $\frac{4}{10} \times 360 = 144$ |
| homework | $\frac{3}{10} = 0.3 = 30\%$ | $\frac{3}{10} \times 360 = 108$ |

**b**



Using the same proportions calculated above, make sure that each rectangle takes up the correct amount of space. For example, if the total width is 15 cm, then sport occupies $\frac{2}{5} \times 15 = 6$ cm.

**Drilling for Gold 9D1**

**Drilling for Gold 9D2**

**3** A group of passengers arriving at an airport is surveyed to establish which countries they have come from. The results are presented below.

| Country | China | United Kingdom | USA | France |
|---|---|---|---|---|
| No. of passengers | 6 | 5 | 7 | 2 |

**a** What is the total number of passengers who participated in the survey?

**b** What proportion of the passengers surveyed have come from the following countries? Express your answer as a fraction.

**i** China **ii** United Kingdom **iii** USA **iv** France

**c** On a sector graph, determine the angle size of the sector representing:

**i** China **ii** United Kingdom **iii** USA **iv** France

**d** Draw a sector graph showing the information calculated in part **c**.

**4** A group of students in Years 7 and 8 is polled on their favourite colour, and the results are shown at right.

| Colour | Year 7 votes | Year 8 votes |
|---|---|---|
| red | 20 | 10 |
| green | 10 | 4 |
| yellow | 5 | 12 |
| blue | 10 | 6 |
| pink | 15 | 8 |

**a** Draw a sector graph to represent the Year 7 colour preferences.

**b** Draw a different sector graph to represent the Year 8 colour preferences.

**c** Describe two differences between the charts.

**d** Construct a divided bar graph that shows the popularity of each colour across the total number of Years 7 and 8 students combined.

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**5** Consider the following results of a study on supermarket shopping habits.

| **Items** | food | drinks | household items | other |
|---|---|---|---|---|
| **Proportion of money spent** | 50% | 25% | 20% | 5% |

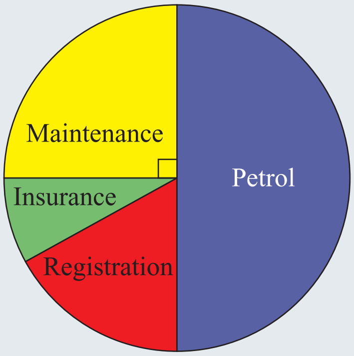
**a** Represent this information in a divided bar graph.
**b** Graph this information as a sector graph.
**c** Use a digital technology (e.g. a spreadsheet) to create each type of graph and describe how your answers to parts **a** and **b** differ from the technology's output.

**Problem-solving and Reasoning**

## Example 9 Interpreting sector graphs

A car owner graphs the amount of money spent per year on car-related expenses.

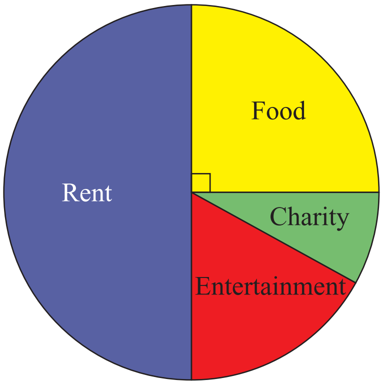
If the car owner spends $3000 per year on petrol, what is the total amount spent on the car each year?



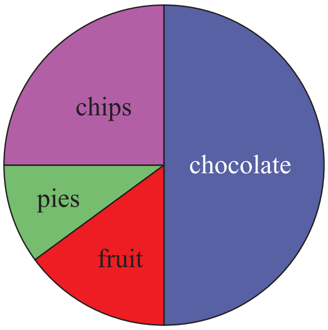
| **Solution** | **Explanation** |
|---|---|
| $$\times 2 \left( \begin{array}{l} 50\% \text{ of expenses} = \$3000 \\ 100\% \text{ of expenses} = \$6000 \end{array} \right) \times 2$$<br>The car owner spends $6000 each year. | Petrol occupies half the chart's area, which is 50%. This is doubled to find the total amount spent. |

**6** This sector graph shows one person's spending in a month.
**a** What percentage of the month's spending was on rent?
**b** What percentage was spent on food?
**c** If the person spent a total of $600 on food in the month, what was their total spending?

$$\times ? \left( \begin{array}{c} ?\% = \$600 \\ 100\% = ? \end{array} \right) \times ?$$



**7** A group of Year 7 students was polled on their favourite foods, and the results are shown in this sector graph.



**a** If 40 students participated in the survey, find how many of them chose:

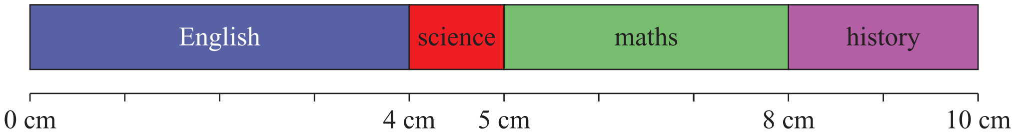
**i** chocolate **ii** chips **iii** fruit **iv** pies

**b** Health experts are worried about what these results mean. They would like fruit to appear more prominently in the sector graph, and to not have the chocolate sector next to the chips. Redraw the sector graph so this is the case.

**c** Another 20 students were surveyed. Ten of these students chose chocolate and the other 10 chose chips. Their results are to be included in the sector graph. Of the four sectors in the graph, state which sector will:

**i** increase in size **ii** decrease in size **iii** stay the same size.

**8** Yakob has asked his friends what is their favourite school subject, and he has created the following divided bar graph from the information.



**a** Calculate the percentage of the whole represented by:

**i** English **ii** maths **iii** history

**b** If Yakob surveyed 30 friends, state how many of them like:

**i** maths best **ii** history best **iii** either English or science best.

**c** Redraw these results as a sector graph.

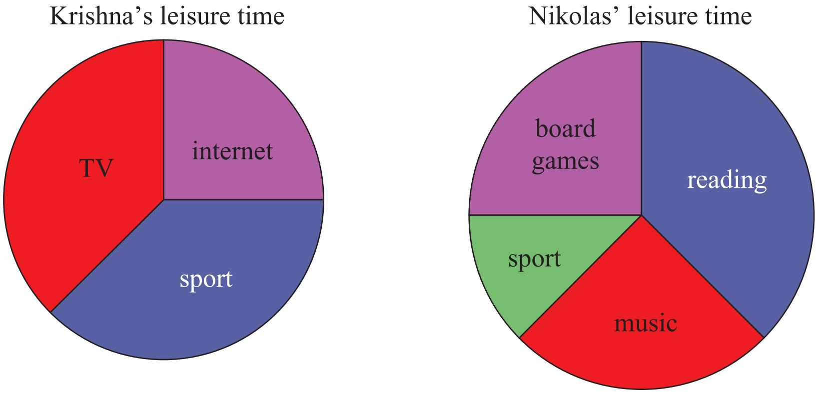
**9** Friends Krishna and Nikolas have each graphed their leisure habits, as shown below.

**a** Which of the two friends spends more of their time playing sport?

**b** Which of the two friends does more intellectual activities in their leisure time?

**c** Krishna has only 2 hours of leisure time each day because he spends the rest of his time doing homework. Nikolas has 8 hours of leisure time each day. How does this affect your answers to parts **a** and **b** above?

**d** Given that Krishna's TV time and sport time are equal, what percentage of his leisure time does he spend watching TV?



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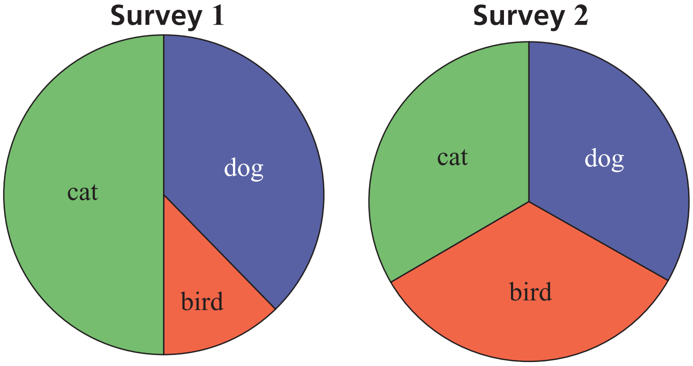
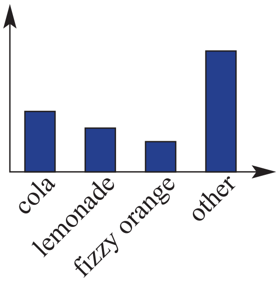
**10** In two surveys, people were asked what is their favourite pet animal.


**Survey 1** **Survey 2**

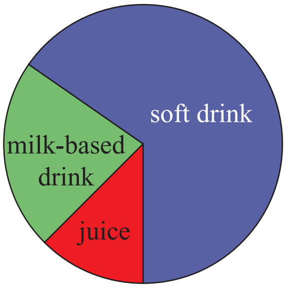
**a** If 16 people participated in survey 1, how many chose a dog?
**b** If 30 people participated in survey 2, how many chose a bird?
**c** Jason claims that 20 people participated in survey 1. Explain clearly why this cannot be true.
**d** Jaimee claims that 40 people participated in survey 2. Explain clearly why this cannot be true.
**e** In actual fact, the same number of people participated for each survey. Given that fewer than 100 people participated, how many participants were there? Give all the possible answers.

**11** Explain why you can use a sector graph for categorical data but you cannot use a line graph for categorical data.

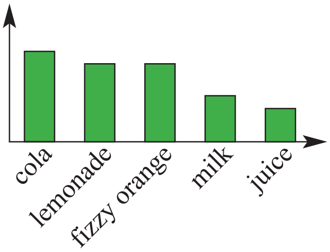
**12** Three different surveys are conducted to establish whether soft drinks should be sold in the school canteen.



Survey 1: Favourite drink



Survey 2: Favourite type of drink



Survey 3: Sugar content per drink

**a** Which survey's graph would be the most likely to be used by someone who wished to show the financial benefit to the cafeteria of selling soft drinks?
**b** Which survey's graph would be the most likely to be used by someone who wanted to show there was not much desire for soft drink?
**c** Which survey's graph would be the most likely to be used by a person wanting to show how unhealthy soft drink is?

## Enrichment: Water footprint

**13** The 'water footprint' of different foods refers to the volume of fresh water that is used to produce the food. The water footprint of some foods is shown in the table below.

| **Food** | bread | cheese | chicken | cucumber | lettuce | milk | potato | rice |
|---|---|---|---|---|---|---|---|---|
| **Footprint (L/kg)** | 1608 | 3178 | 4325 | 353 | 237 | 1800 | 287 | 2497 |

- **a** What type(s) of graph could be used for the data above? Justify your choice(s).
- **b** Choose a suitable type of graph and depict the above numbers graphically.
- **c** How is a food's water footprint related to how sustainable it is to produce?
- **d** Estimate how many litres of water would be used for a chicken burger. Include your estimates of each item's weight.
- **e** Another way to present the data is to say how many grams of each food is made from 1 kilolitre of water. Redraw the table above with a row for 'water efficiency' in g/kL.

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# 9E Frequency distribution tables

A frequency distribution table is a tool used to organise and display data.

The data set on the right could be displayed as follows.

| Score | Tally | Frequency |
|---|---|---|
| 0 | 𝍸 \| | 6 |
| 1 | 𝍸 | 5 |
| 2 | 𝍸 \| | 6 |
| 3 | \|\|\| | 3 |
| | | 20 |

| | | | | |
|---|---|---|---|---|
| 0 | 3 | 1 | 1 | 0 |
| 2 | 2 | 1 | 1 | 2 |
| 0 | 0 | 0 | 3 | 3 |
| 0 | 2 | 2 | 2 | 1 |

A data set

## Let's start: Subject preferences

**Drilling for Gold 9E1**

- Survey your class to find their favourite school subject out of Maths, English and Science.
- Represent your results in a table like the one below.

| Subject | Tally | Frequency |
|---|---|---|
| Maths | | |
| English | | |
| Science | | |
| | | |

- How would you expect the results to differ for different classes at your school, or for different schools?

## Key ideas

**Tally marks** Line strokes used to record data in groups of 5

**Frequency table** A table summarising data by showing all possible scores from lowest to highest in one column, and the frequency of each score in another column

- A **tally** is a tool used for counting as results are gathered. Numbers are written as vertical lines with every 5th number having a cross through a group of lines. e.g. 4 is |||| and 7 is 𝍸 ||.
- A **frequency table** has a column for the scores and another column for the frequency of each score.
  The frequency shows how often each score occurs.
- Frequency distribution tables can be used for listing particular values or ranges of values.

| Number of cars | Frequency |
|---|---|
| 0 | 10 |
| 1 | 12 |
| 2 | 5 |
| 3 | 3 |

| Age | Frequency |
|---|---|
| 0–4 | 7 |
| 5–9 | 12 |
| 10–14 | 10 |
| 15–19 | 11 |

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## Exercise 9E

Understanding

**1** The table below shows survey results for students' favourite colours.

| Colour | Frequency |
|---|---|
| Red | 5 |
| Green | 2 |
| Orange | 7 |
| Blue | 3 |

Are the following true or false?

**a** 5 people chose red as their favourite colour.
**b** 9 people chose orange as their favourite colour.
**c** Blue is the favourite colour of 3 people.
**d** More people chose green than orange as their favourite colour.

**2** Fill in the blanks.

**a** The tally |||| represents the number ____.
**b** The tally 𝍸 || represents the number ____.
**c** The tally ____ represents the number 2.
**d** The tally ____ represents the number 11.

𝍸 = 5

**3** This frequency table is for the size of families in a Year 8 class.

Frequency means 'how many'. So 1 family of 2, 4 families of 3 etc.

| Family size | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| Frequency | 1 | 4 | 10 | 8 | 2 | 1 |

Make a list of all the family sizes for this Year 8 class, starting with 1 family of 2.

**4** This is a list of some students' handspans measured in cm.
19, 18, 20, 17, 22, 19, 22, 20, 24, 18, 20, 19
Copy and complete each of these frequency tables.

Frequency for 17–19: Count how many values were 17, 18 or 19.

**a**

| Handspan (cm) | Frequency |
|---|---|
| 17 | |
| 18 | |
| 19 | |
| 20 | |
| 21 | |
| 22 | |
| 23 | |
| 24 | |

**b**

| Handspan (cm) | Frequency |
|---|---|
| 17–19 | |
| 20–22 | |
| 23–25 | |

**9E**

**Fluency**

## Example 10 Interpreting tallies

**a** The different car colours in a car park are noted. Convert the following tally into a frequency table.

| White | Black | Blue | Red | Yellow |
|---|---|---|---|---|
| \|\|\| | 𝍸 𝍸 \|\|\| | 𝍸 𝍸 𝍸 \|\| | 𝍸 \| | 𝍸 \|\|\|\| |

**b** How many red cars were seen?
**c** What was the total number of cars seen?

| **Solution** | **Explanation** |
|---|---|
| **a** | Each tally is converted into a frequency. For example, black is two groups of 5 plus 3, giving $10 + 3 = 13$. |

| **Colour** | White | Black | Blue | Red | Yellow |
|---|---|---|---|---|---|
| **Frequency** | 3 | 13 | 17 | 6 | 9 |

| | |
|---|---|
| **b** 6 red cars were seen. | This can be read directly from the table. |
| **c** 48 cars seen | $3 + 13 + 17 + 6 + 9 = 48$<br>Add the frequencies to find the total. |

**Skillsheet 9A**

**5** A basketball player's performance in one game is recorded in the following table.

| | **Passes** | **Shots at goal** | **Shots that go in** | **Steals** |
|---|---|---|---|---|
| **Tally** | \|\|\| | 𝍸 𝍸 \|\| | 𝍸 \|\|\| | \|\| |
| **Frequency** | | | | |

**a** Copy and complete the table, filling in the frequency row.
**b** How many shots did the player have at the goal?
**c** How many shots went in?
**d** How many steals did the player have during the game?

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## Example 11 Constructing tables from data

Put the following data into a frequency table: 1, 4, 1, 4, 1, 2, 3, 4, 6, 1, 5, 1, 2, 1.

**Solution**

| Score | Tally | Frequency |
|---|---|---|
| 1 | 𝍸 \| | 6 |
| 2 | \|\| | 2 |
| 3 | \| | 1 |
| 4 | \|\|\| | 3 |
| 5 | \| | 1 |
| 6 | \| | 1 |
| | | 14 |

**Explanation**

Construct the tally as you read through the list.
Then go back and convert the tally to frequencies.

**6** A student surveys her class to ask how many people are in their family. The results are:

6, 3, 3, 2, 4, 5, 4, 5, 8, 5, 4, 8, 6, 7, 6, 5, 8, 4, 7, 6

- **a** Construct a frequency table.
- **b** How many students have exactly 5 people in their family?
- **c** How many students have at least 6 people in their family?

Check that the number of scores in the list equals the total of the frequencies in the table.

Add the frequencies to find the total number surveyed.

**7** Braxton surveys a group of people to find out how much time they spend watching television each week. They give their answers rounded to the nearest hour.

| **Number of hours** | 0–4 | 5–9 | 10–14 | 15–19 | 20–24 | 25+ |
|---|---|---|---|---|---|---|
| **Tally** | 𝍸 \|\|\| | 𝍸 𝍸 \|\| | 𝍸 𝍸 𝍸 | 𝍸 \|\|\|\| | \|\|\|\| | \|\| |

- **a** How many people altogether did Braxton survey?
- **b** How many people spend 15–19 hours per week watching television?
- **c** How many people watch television for less than 5 hours per week?
- **d** How many people watch television for an average of 2 hours *per day* or less?

Less than 5 hours doesn't include 5 hours. So 0–1 and 2–4

**8** The heights of a group of 21 people are shown below, given to the nearest cm.

174 179 161 132 191 196 138 165 151 178 189
147 145 145 139 157 193 146 169 191 145

**a** Copy and complete the frequency table below.

| Height (cm) | Tally | Frequency |
|---|---|---|
| 130–139 | | |
| 140–149 | | |
| 150–159 | | |
| 160–169 | | |
| 170–179 | | |
| 180–189 | | |
| 190+ | | |
| | | |

180 cm or taller means 180–189 and 190+

**b** How many people are in the range 150–159 cm?
**c** How many people are 180 cm or taller?
**d** How many people are between 140 cm and 169 cm tall?

### Problem-solving and Reasoning

**9** A tennis player records the number of double faults they serve per match during one month.

| Double faults | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Frequency | 4 | 2 | 1 | 0 | 2 | 1 |

**a** How many matches did they play in total during the month?
**b** How many times did they serve exactly 1 double fault?

Add the frequencies to find the total number of matches played.

**c** In how many matches did they serve no double faults?
**d** How many double faults did they serve in total during the month?

**10** Five different classes are in the same building in different rooms at the same time. The ages of students in each room are recorded in the frequency table below.

Average = $\frac{\text{sum}}{n}$

| Age (years) | Room A Frequency | Room B Frequency | Room C Frequency | Room D Frequency | Room E Frequency |
|---|---|---|---|---|---|
| 12 | 3 | 2 | 0 | 0 | 0 |
| 13 | 20 | 18 | 1 | 0 | 0 |
| 14 | 2 | 4 | 3 | 0 | 10 |
| 15 | 0 | 0 | 12 | 10 | 11 |
| 16 | 0 | 0 | 12 | 10 | 11 |
| 17 | 0 | 0 | 0 | 1 | 0 |

**a** How many students are in room C?
**b** How many students are in the building?
**c** How many 14-year-olds are in the building?
**d** What is the average (mean) age of students in room B? Answer to 1 decimal place.
**e** Make a frequency table showing age and the number of each age group in the building.

**11** Some exam results are presented in the frequency table below.

| 0–9 | 10–19 | 20–29 | 30–39 | 40–49 | 50–59 | 60–69 | 70–79 | 80–89 | 90–99 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 3 | 1 | 2 | 5 | 8 | 12 | 10 | 2 |

Redraw the table so that the intervals are of width 20 rather than 10 (i.e. so the first column is 0–19, the second is 20–39, and so on).

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## Enrichment: Homework puzzle

**12** Priscilla records the numbers of hours of homework she completes each evening from Monday to Thursday. Her results are shown in this frequency table.

| Number of hours | Frequency |
|---|---|
| 1 | 1 |
| 2 | 1 |
| 3 | 2 |

**a** On how many nights did Priscilla do 3 hours homework?

**b** One possibility is that she worked 3 hours on Monday, 2 hours on Tuesday, 3 hours on Wednesday and 1 hour on Thursday. Copy and complete this table to show other ways her time could have been allocated for the four nights.

| Monday | Tuesday | Wednesday | Thursday |
|---|---|---|---|
| ☐ hours | ☐ hours | ☐ hours | ☐ hours |
| ☐ hours | ☐ hours | ☐ hours | ☐ hours |
| ☐ hours | ☐ hours | ☐ hours | ☐ hours |

**c** Priscilla's brother Joey did homework on all five nights. On two nights he worked for 1 hour, on two nights he worked for 2 hours and on one night he worked for 3 hours. Show three ways that the table below could be filled in to match his description.

| Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|
| ☐ hours | ☐ hours | ☐ hours | ☐ hours | ☐ hours |

**d** Calculate the average hours of homework per night for Priscilla and Joey.

**e** How many hours more homework per week would Joey have to do over 5 nights to make his average per night equal to Priscilla's average?

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# 9F Frequency histograms and frequency polygons

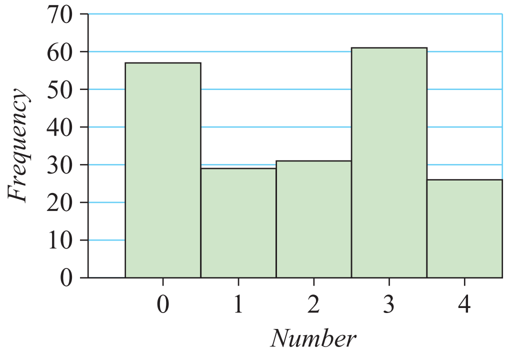
A frequency distribution table can easily be converted into a frequency histogram and a frequency polygon.

For example, the data below is represented as a frequency table and as a histogram.

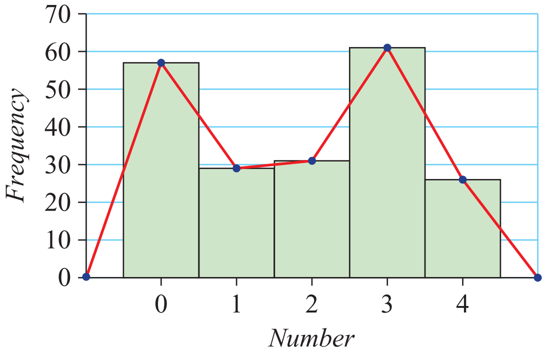
**As a table**

| Number | Frequency |
|---|---|
| 0 | 57 |
| 1 | 29 |
| 2 | 31 |
| 3 | 61 |
| 4 | 26 |

**As a frequency histogram**



**As a frequency polygon**

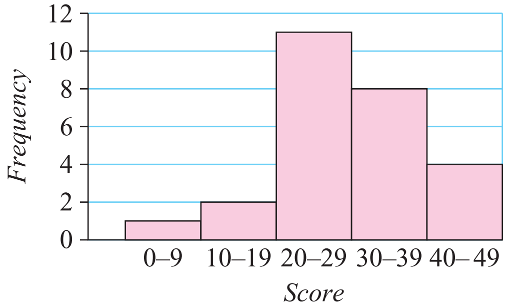


At a glance, you can see from the histogram that 0 and 3 are about twice as common as the other values. This is harder to read straight from the table. A histogram makes comparisons of frequency easier.

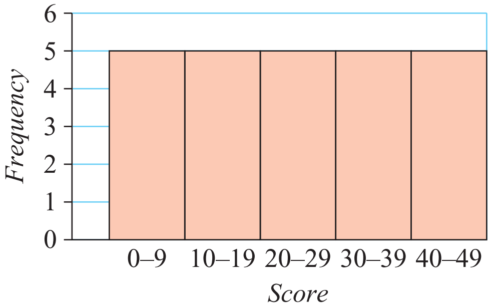
## Let’s start: Test analysis

The results for some end-of-year tests are shown for four different classes in four different histograms.

**Class 1**



**Class 2**



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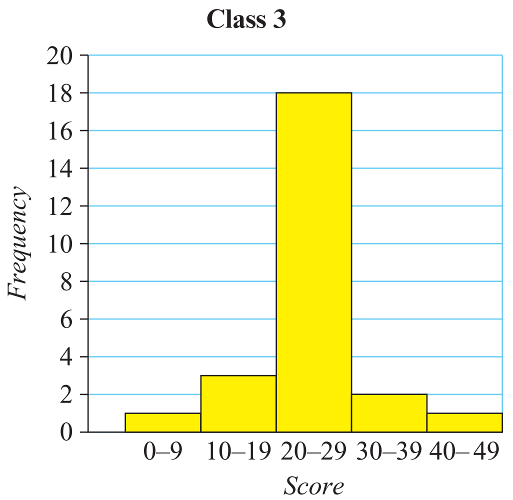
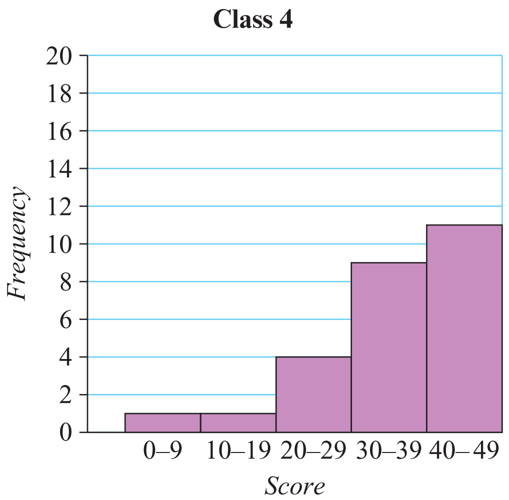




Work with a classmate and discuss the answers to these questions.

**1** Choose which class has results that can be described as:
  - **a** a few low scores, a few high scores and a lot of scores around the middle
  - **b** equal numbers of students getting low, middle and high scores
  - **c** more students getting high scores than low scores
  - **d** more students getting middle scores than either high or low scores.

**2** Which class has the highest average score?

**3** Which class has the highest overall score?

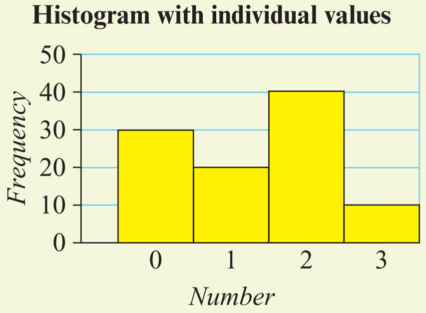
**4** Which class would be the easiest to teach and which would be the hardest, do you think?

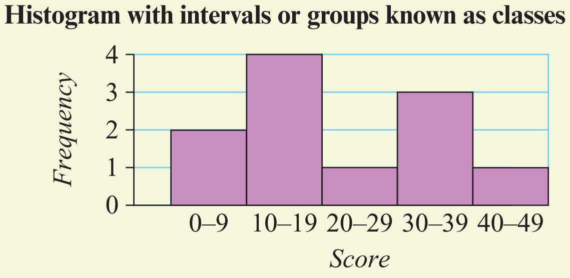
## Key ideas

**Histogram** A special type of column graph for quantitative data with no gaps between the columns

- A frequency **histogram** is a graphical representation of a frequency distribution table. It can be used when the scores are numerical.
- The vertical axis is used to represent the frequency of each score.
- Columns are placed next to one another with no gaps in between.

For example,





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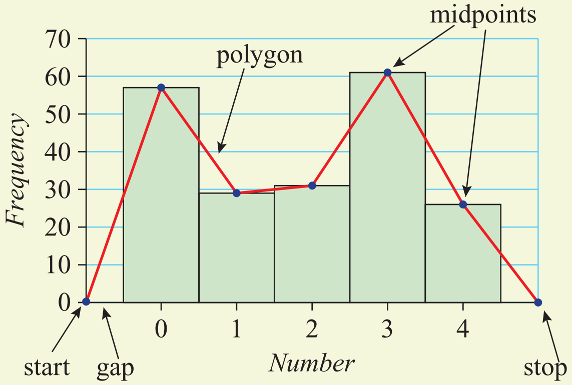
- A half-column-width space is placed between the vertical axis and the first column.
- A frequency polygon begins and ends on the horizontal axis and joins the mid-points of the tops of the columns.



## Exercise 9F

**Understanding**

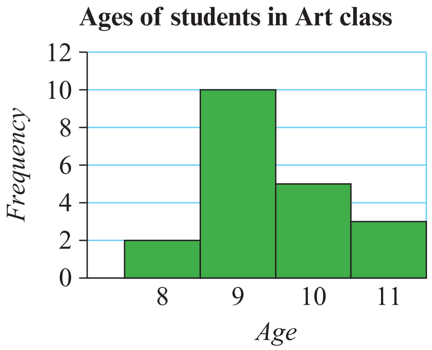
**1** This histogram below shows the ages of people in an Art class.

**a** How many 8-year-olds are in this class?

**b** What is the most common age for students in this class?

**c** What is the age of the oldest person in the class?

Frequency is the number of people.



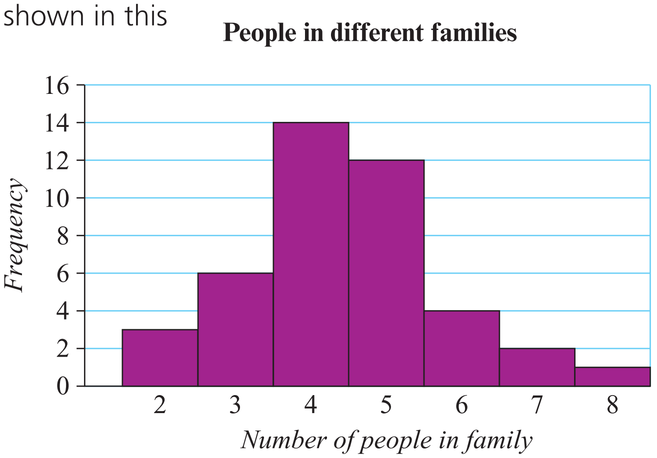
The frequency shows ‘how many’ of each family size.

**2** A survey is conducted of the number of people in different families. The results are shown in this histogram.

**a** What is the most likely number of people in a family, on the basis of this survey?

**b** How many people responding to the survey said they had a family of 6?

**c** What is the least likely number (from 2 to 8) of people in a family, on the basis of this survey?



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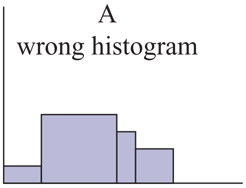
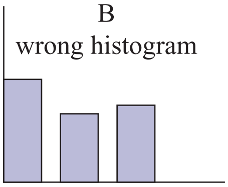
**3** If the columns are each 1 cm wide, how much gap should be left between the vertical axis and the first column?

**4** A student draws two incorrect histograms like this.





**a** What mistake has been made with the columns in histogram A? How could this error be prevented?
**b** What mistake has been made with the columns in histogram B?
**c** What is missing from both histograms?

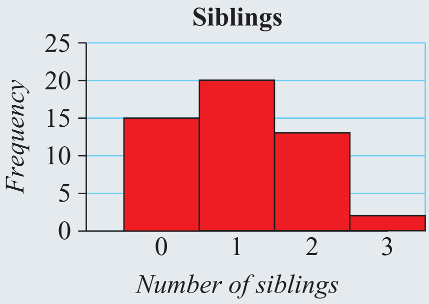
**Fluency**

## Example 12 Constructing histograms from frequency tables with individual labels

Represent this frequency table below as a histogram.

| Number of siblings | Frequency |
|---|---|
| 0 | 15 |
| 1 | 20 |
| 2 | 13 |
| 3 | 2 |

**Solution**



**Explanation**

The scale 0–25 is chosen to fit the highest frequency (20).

Each different number of siblings in the frequency table is given a column in the graph.

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**5** Represent the following frequency tables as histograms.

**a**

| Number | Frequency |
|---|---|
| 0 | 5 |
| 1 | 3 |
| 2 | 5 |
| 3 | 2 |
| 4 | 4 |

**b**

| Number | Frequency |
|---|---|
| 0 | 3 |
| 1 | 9 |
| 2 | 3 |
| 3 | 10 |
| 4 | 7 |


Remember to rule up even scales.

Drilling for Gold 9F1

**c**

| Age | Frequency |
|---|---|
| 12 | 15 |
| 13 | 10 |
| 14 | 25 |
| 15 | 20 |
| 16 | 28 |

**d**

| Number of cars | Frequency |
|---|---|
| 0 | 4 |
| 1 | 5 |
| 2 | 4 |
| 3 | 2 |

**6** For the following sets of data:
- **i** create a frequency table
- **ii** draw a histogram from the frequency table.


Frequency table

| number | tally | frequency |
|---|---|---|

- **a** 1, 2, 5, 5, 3, 4, 4, 4, 5, 5, 5, 1, 3, 4, 1
- **b** 5, 1, 1, 2, 3, 2, 2, 3, 3, 4, 3, 3, 1, 1, 3
- **c** 4, 3, 8, 9, 7, 1, 6, 3, 1, 1, 4, 6, 2, 9, 7, 2, 10, 5, 5, 4
- **d** 60, 52, 60, 59, 56, 57, 54, 53, 58, 56, 58, 60, 51, 52, 59, 59, 52, 60, 50, 52


Frequency shows how many of each number.

## Example 13 Constructing histograms from frequency tables using intervals

Draw the frequency table below as a histogram.

| Number of words in story | Frequency |
|---|---|
| 0–99 | 2 |
| 100–199 | 10 |
| 200–299 | 12 |
| 300–399 | 8 |
| 400–499 | 3 |

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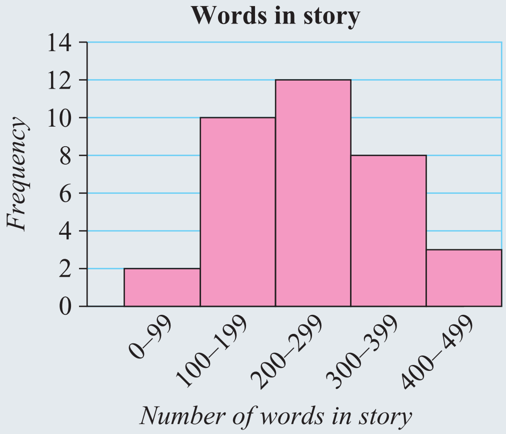
**Solution**



**Explanation**

The scale 0–14 is chosen to fit the highest frequency (12).

The different intervals (0–99 words, 100–199 words etc.) are displayed on the horizontal axis.

**7** Represent the following frequency tables as histograms.

**a**

| Score | Frequency |
|---|---|
| 0–19 | 1 |
| 20–39 | 4 |
| 40–59 | 10 |
| 60–79 | 12 |
| 80–99 | 5 |

**b**

| Age | Frequency |
|---|---|
| 1–5 | 5 |
| 6–10 | 12 |
| 11–15 | 14 |
| 16–20 | 11 |
| 21–25 | 5 |
| 26–30 | 8 |
| 31–35 | 2 |
| 36–40 | 1 |

Frequency is on the vertical axis.

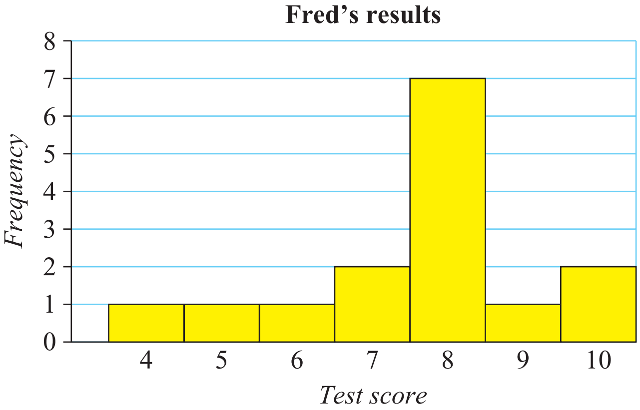
Mark even scales.

**Problem-solving and Reasoning**

**8** Edwin records the results for his spelling tests out of 10. They are 3, 9, 3, 2, 7, 2, 9, 1, 5, 7, 10, 6, 2, 6, 4.

**a** Draw a histogram for his results.

**b** Fred's results are given by the histogram shown at right. Is Edwin a better or a worse speller generally than Fred? Give a reason for your answer.



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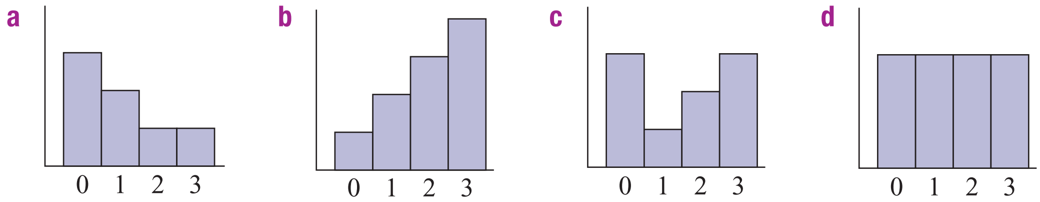
**9** Some tennis players count the number of aces served in different matches. Match up the histograms with the descriptions.

Higher columns means more tennis matches.



**A** Often serves aces
**B** Generally serves 3 aces or 0 aces
**C** Serves a different number of aces in each match
**D** Serves very few aces

0 1 2 3 shows the number of aces per match.

**10** This histogram shows the ages of a group of people in a room.

**a** Which part of this histogram would change if a histogram is drawn for the ages of the same group of people in exactly 12 years' time?

**b** How would this histogram look if it showed the ages of the same group of people exactly 12 years ago?

The frequency shows the number of people of each age.

**11** A car dealership records the number of sales each salesperson makes per day over three weeks.

The frequency shows the number of days that sales were made.

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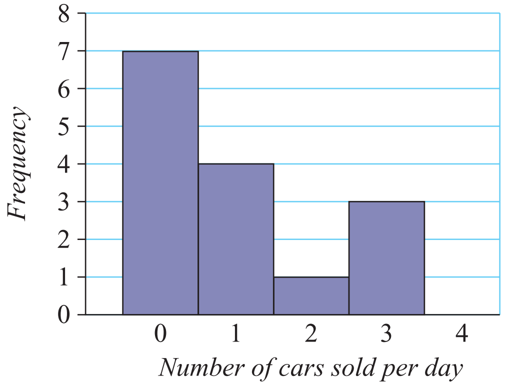
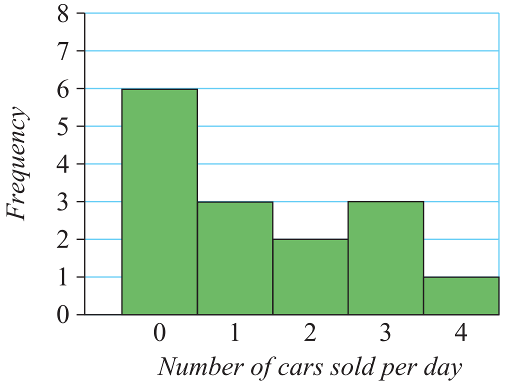
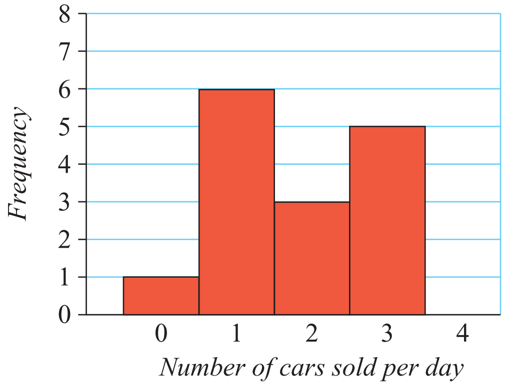


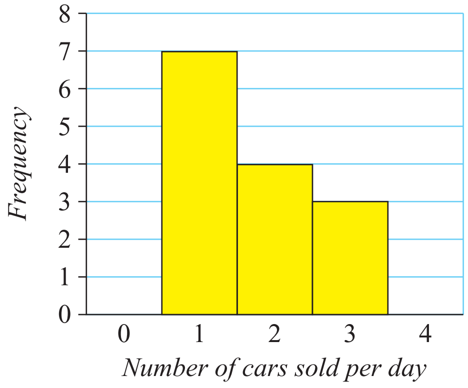
Bill's sales



Marie's sales



Frank's sales



Con's sales

**a** On how many days did Bill not make any sales?

**b** For how many days did Bill sell one car per day?

**c** What is the record for the greatest number of sales in one day and who holds this record?

**d** Which salesperson made at least one sale every day?

**e** Over the whole period, which salesperson made the most sales in total? How many cars did they sell?

**f** Over the whole period, which salesperson made the fewest sales in total? How many cars did they sell?

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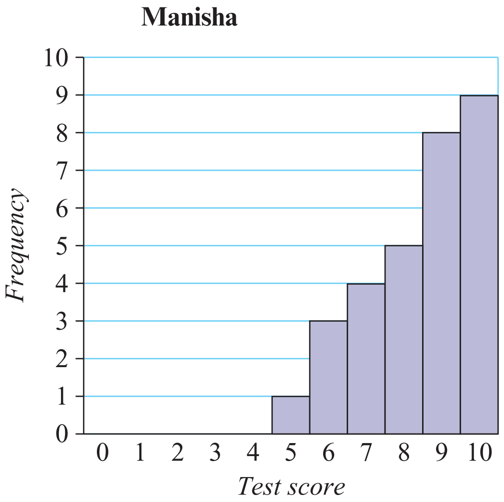
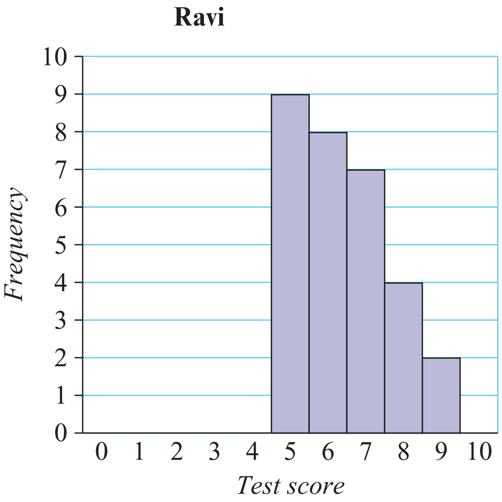
**12** Two students have each graphed a histogram that shows their results for a number of spelling tests. Each test is out of 10 and there has been one test per week for 30 weeks.

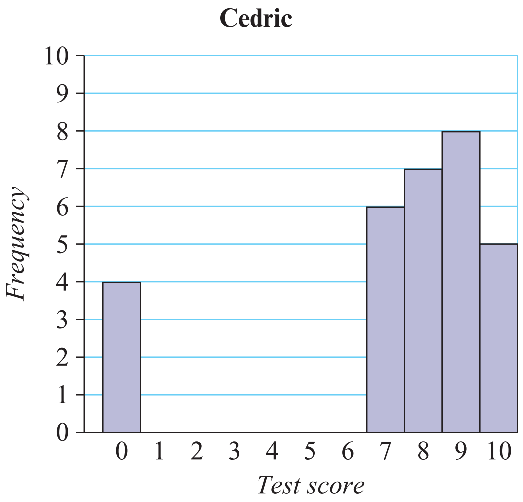

**Manisha**


**Ravi**

**a** Manisha's scores started very high but have got worse during the year. Give an example of a list of scores that Manisha might have received over the 30 weeks.

**b** Ravi's spelling has actually improved consistently over the course of the year. Give an example of a list of the scores he might have received for the 30 weeks.

**c** A third student, Cedric, has the following results. What is a likely explanation for the '0' results?


**Cedric**


The frequency shows the number of tests for each result.

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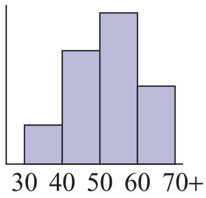
## Enrichment: Heights, weights and ages mix-up

**13** Three students survey different groups of people to find out their heights, weights and ages. Unfortunately they have mixed up all the graphs they obtained.

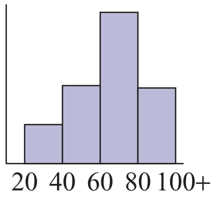
**a** Copy and complete the table below, stating which graph corresponds to which set of data.

| Survey location | Height graph (cm) | Weight graph (kg) | Age graph (years) |
|---|---|---|---|
| Primary school classroom | Graph 4 | | |
| Shopping centre | | | |
| Teachers' common room | | | |

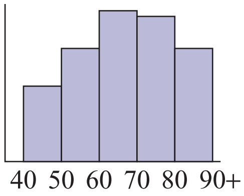
**Graph 1**



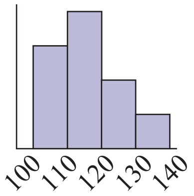
**Graph 2**



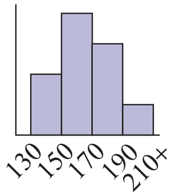
**Graph 3**



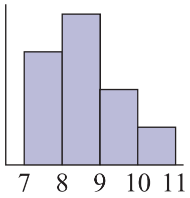
**Graph 4**



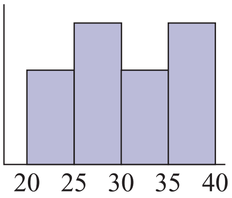
**Graph 5**



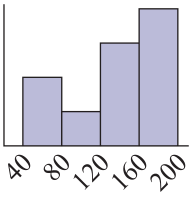
**Graph 6**



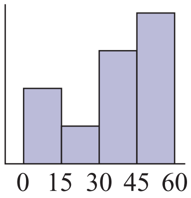
**Graph 7**



**Graph 8**



**Graph 9**



**b** Show with rough sketches how the age graphs would look for:

- **i** people in a retirement village
- **ii** students at a secondary school
- **iii** guests at a 30-year high school reunion.

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# 9G Mean, median, mode and range

It is sometimes useful to summarise a large group of data with a single value. The concept of 'average' is familiar to most people, but more precise mathematical terms to use are 'mean', 'median' and 'mode'.

## Let's start: Family heights

Each New Year, the Green family measure and record their heights. This year their height measurements are:

Georgia 78 cm, Emily 130 cm, Amy 130 cm, Ethan 188 cm, Mrs Green 165 cm, Mr Green 182 cm.

Work with a classmate to help each other to complete these activities.

### Range

1. Who is the shortest and who is the tallest person in the Green family?
2. What is the range (the difference) between the shortest and tallest heights in the Green family?
3. The shortest person in the world is 59.93 cm and the tallest person in the world is 251 cm. What is the current range of heights for all adult humans?
4. The Green family had a snow-skiing holiday. One morning it is $-8^\circ$C and that afternoon it is $5^\circ$C. What was the range of temperature that day?

### Median

1. List the heights of the Green family in ascending (increasing) order.
2. What are the two middle heights?
3. Find the median (middle of these two central heights).
4. If the tallest man in the world, height 251 cm, is added into the Green family heights list, what is the median (middle) height now?
5. By how much has the median height changed by adding the tallest man into the list?
6. Does the median value always have to be one of the scores in the list?

### Mean

1. Add up the total of all the heights of the Green family.
2. Now find the mean height. (mean = total of heights divided by the number of heights)
3. If the tallest man in the world is included with the Green family heights, what is the mean height now?
4. By how much has the mean height changed by including the tallest man into the list?
5. Does the mean value always have to be one of the scores in the list?

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### Mode

1. The Green family has twins. Who are they and what is their height?
2. What is the mode (most common) of the Green family heights?
3. The Pink family has heights: 125 cm, 142 cm, 142 cm, 142 cm, 160 cm and 178 cm. What is the height of the Pink triplets?
4. What is the mode of the Pink family heights?
5. Does the mode value always have to be one of the scores in the list?

## Key ideas

- The **range** of a set of data is given by:
  Range = highest number – lowest number.

  1 5 2 7 5

  lowest (1) highest (7) ⟶ range = 7 – 1 = 6

- The **mean** ($\bar{x}$) of a set of data is given by:

$$\bar{x} = \frac{\text{sum of data values}}{\text{number of data values}}$$

  $1 + 5 + 2 + 7 + 5 = 20 \longrightarrow$ mean = $20 \div 5 = 4$

- The **median** is the middle value if the values are sorted from lowest to highest. If there are two middle values, then add them together and divide by 2.

  1 2 ⑤ 5 7

  middle ⟶ median = 5

  2 3 ⑤ ⑨ 10 12

  $5 + 9 = 14$
  $14 \div 2 = 7$ = median

- The **mode** is the most common value. It is the value that occurs most frequently. We also say that it is the value with the highest frequency.

  1 2 ⑤ ⑤ 7 ⟶ mode = 5

- The range, mean and median can only be calculated for numerical data, but the mode can be calculated for numerical and categorical data.
- Every scientific calculator is able to calculate the mean, using a button labelled $\bar{x}$.

**Range** The difference between the highest data value and the lowest data value

**Mean** The sum of the data values divided by the number of data values

**Median** The middle data value when the data is arranged in order

**Mode** The most frequently occurring value in a set of data

**Outlier** Any value that is much larger or much smaller than the rest of the data in a set

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## Exercise 9G

**Understanding**

**1** Fill in the blanks.
- **a** The most common value in a set of data is called the ___________.
- **b** The sum of all values, divided by the number of values is called the ___________.
- **c** The ___________ can be calculated by finding the middle values of the numbers placed in ascending order.
- **d** The difference between the highest and lowest values is called the ___________.
- **e** A value that is much larger or smaller than the other values is called an ___________.

**2** Use the set of numbers 1, 7, 1, 2, 4.
- **a** Find the sum of these numbers.
- **b** How many numbers are listed?
- **c** Hence find the mean.

$$\text{Mean} = \frac{\text{sum of data values}}{\text{number of data values}}$$

**3** Use the values 5, 2, 1, 7, 9, 4, 6.
- **a** Sort these numbers from smallest to largest.
- **b** What is the middle value in your sorted list?
- **c** What is the median of this set?

The median is the middle value.

**4** Use the set 1, 5, 7, 9, 10, 13.
- **a** State the two middle values.
- **b** Find the sum of the two middle values.
- **c** Divide your answer by 2 to find the median of the set.

**5** Use the set of numbers 1, 3, 2, 8, 5, 6.
- **a** State the largest number.
- **b** State the smallest number.
- **c** Now state the range, by finding the difference of these two values.

**Fluency**

### Example 14 Finding the range

Find the range of the following sets of data.

**a** 1, 5, 2, 3, 8, 12, 4 **b** –6, –20, 7, 12, –24, 19

| Solution | Explanation |
|---|---|
| **a** Range = 12 – 1<br>= 11 | Maximum: 12, minimum: 1<br>Range = maximum – minimum |
| **b** Range = 19 – (–24)<br>= 43 | Maximum: 19, minimum: –24<br>Range = 19 – (–24) = 19 + 24 |

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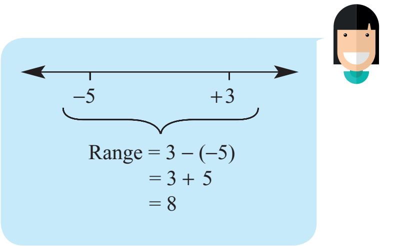
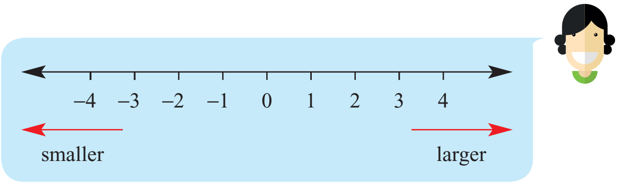
**6** Find the range of the following sets.

**a** 5, 1, 7, 9, 10, 3, 10, 6
**b** 9, 3, 9, 3, 10, 5, 0, 2
**c** 4, 13, 16, 9, 1, 6, 5, 8, 11, 10
**d** 16, 7, 17, 13, 3, 12, 6, 6, 3, 6
**e** –7, 4, 12, –5, –18, –16, 7, 9
**f** 16, –3, –5, –6, 18, –4, 3, –9
**g** 3.5, 6.9, –9.8, –10.0, 6.2, 0.8
**h** –4.6, 2.6, –6.1, 2.6, 0.8, –5.4





## Example 15 Finding the mean and the mode

For the set of numbers 10, 2, 15, 1, 15, 5, 11, 19, 4, 8 find:

**a** the mean **b** the mode

| **Solution** | **Explanation** |
|---|---|
| **a** 10 + 2 + 15 + 1 + 15 + 5 + 11 + 19 + 4 + 8 = 90 | The numbers are added to find the total. |
| Mean = 90 ÷ 10<br>= 9 | The mean is found by dividing the total by the number of items (in this case 10). |
| **a** Mode = 15 | The most common value is 15, so this is the mode. |

**7** For each of the following sets, find the:

**i** mean **ii** mode

**a** 5, 6, 3, 4, 4, 8
**b** 2, 2, 1, 2, 1, 4, 2
**c** 4, 3, 3, 10, 10, 2, 3
**d** –10, –4, 0, 0, –2, 0, –5
**e** 3, 4, 5, –9, 6, –9
**f** 3, –6, 7, –4, –3, 3
**g** 13, 15, 7, 7, 20, 9, 15, 15, 11, 17
**h** 20, 12, 15, 11, 20, 3, 18, 2, 14, 16
**i** 18, 12, 12, 14, 12, 3, 3, 16, 5, 16
**j** 18, 5, 14, 5, 19, 12, 13, 5, 10, 3
**k** –15, –6, –6, 16, 6, 13, 3, 2, 19, –8
**l** –13, –6, –6, –13, –6, 10, –15, 6, 7, 2

Drilling for Gold 9G1

$$\text{Mean} = \frac{\text{sum of data values}}{\text{number of data values}}$$

The mode is the most common score.

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## Example 16 Finding the median

Find the median of:

**a** 16, 18, 1, 13, 14, 2, 11

**b** 7, 9, 12, 3, 15, 10, 19, 3, 19, 1

| **Solution** | **Explanation** |
|---|---|
| **a** Sorted: $\boxed{1, 2, 11}, \textcircled{13}, \boxed{14, 16, 18}$<br>Median = 13 | Sort the numbers from smallest to largest.<br>Split the list into two equal halves.<br>The middle value is 13. |
| **b** Sorted: $\boxed{1, 3, 3, 7}, \textcircled{9}, \textcircled{10}, \boxed{12, 15, 19, 19}$<br>$\text{Median} = \frac{9 + 10}{2} = 9.5$ | Sort the numbers from smallest to largest.<br>Split the list into two equal halves.<br>There are two middle values (9 and 10) so we add them and divide by 2. |

**8** For each of the following sets, calculate the median.

**a** 3, 5, 6, 8, 10

**b** 3, 4, 4, 6, 7

**c** 1, 2, 4, 8, 10, 13, 13

**d** 2, 5, 5, 5, 8, 12, 14

**e** 14, 15, 7, 1, 11, 2, 8, 7, 15

**f** 4, 14, 5, 7, 12, 1, 12, 6, 11

**g** 2, 2, 4, 6, 7, 9

**h** 1, 1, 2, 9, 9, 10

**i** 1, 3, 5, 7, 8, 10, 13, 14

**j** 0, 1, 9, 13, 1, 10, 7, 12, 9, 2

**k** 12, 17, 7, 10, 2, 17, –2, 15, 11, –8

**l** –2, –1, –3, 15, 13, 11, 14, 17, 1, 14

List the scores from smallest to largest.

The median is the middle score.

**9** Bernie writes down how many hours he works each day for one week.

| **Day** | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| **Number of hours** | 8 | 10 | 8 | 7 | 9 |

**a** What is the mean number of hours Bernie worked each day?

**b** What is the median number of hours Bernie worked each day?

**c** What is the mode number of hours Bernie worked each day?

The modal category has the highest frequency.

**10** State the modal category for the following frequency tables.

**a** Colours of cars are noted as they drive past

| **Colour** | Red | Blue | Orange | White | Green | Black |
|---|---|---|---|---|---|---|
| **Frequency** | 21 | 14 | 3 | 42 | 7 | 25 |

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**b** Pizza preferences are noted within a group of teenagers

| Hawaiian | Meat-lovers | Vegetarian | Cheese |
|---|---|---|---|
| 5 | 7 | 4 | 2 |

**c** The favourite day of the week of a group of people

| Day | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| Frequency | 4 | 12 | 41 | 16 | 28 |

**d** The number of gymnasts in different states

| New South Wales | Queensland | South Australia | Tasmania | Victoria | Western Australia |
|---|---|---|---|---|---|
| 152 | 135 | 193 | 86 | 144 | 159 |

### Problem-solving and Reasoning

Drilling for Gold 9G2

**11** Federica is in a dancing competition and each week she is rated out of 10. Her results for one term are shown in the frequency table below.

| Score | 7 | 8 | 9 | 10 |
|---|---|---|---|---|
| Frequency | 3 | 0 | 3 | 4 |

**a** In how many weeks did she get 7 out of 10?
**b** What score did she receive the most often?
**c** List out all the scores.
**d** What is her mean dancing score for the 10 weeks?
**e** What is her median dancing score for the 10 weeks?
**f** What is the range of Federica's dancing scores?

**12** Business A pays wages of \$42 000, \$48 000, \$50 000, \$50 000 and the boss gets \$70 000.

Business B pays wages of \$42 000, \$48 000, \$50 000, \$50 000 and the boss gets \$200 000.

**a** Which group of wages includes an outlier? What is its value?
**b** Find the mean wage of each business.
**c** How much larger is the mean wage of Business B than the mean wage of Business A?
**d** State the median wage of each business.
**e** Has the outlier affected the median wage?
**f** Which measure better shows how much the workers are paid in each business, the mean or median? Give a reason for your answer.

An outlier is a value much larger (or smaller) than the other values.

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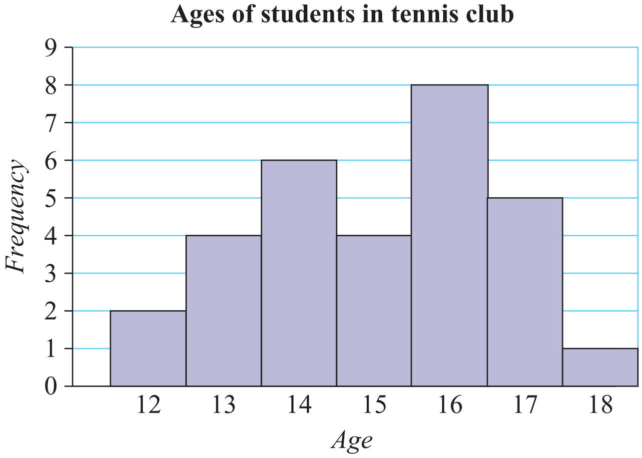
**13** Gary and Nathan compare the number of runs they score in cricket over a number of weeks.

Gary: 17, 19, 17, 8, 11, 20, 5, 13, 15, 15
Nathan: 39, 4, 26, 28, 23, 18, 37, 18, 16, 20

**a** Calculate Gary's range.
**b** Calculate Nathan's range.
**c** Who has the greater range?
**d** Which cricketer is more consistent, on the basis of their ranges only?

**14** The histogram below shows the ages of all students in a school's tennis club.
**a** List all the ages from smallest to largest.
**b** What is the range of the ages in the tennis club?
**c** What is the most common age?
**d** Calculate the mean age correct to 2 decimal places.
**e** Calculate the median age.
**f** Now include the teacher's age of 52 in the list of ages.
  **i** Find the new mean age.
  **ii** Find the new median age.
  **iii** Which measure has changed the most, the mean or the median?



**15** The children playing in a room are aged 3, 7, 8 and 10 years.
**a** What is the mean of these ages?
**b** An adult enters the room and the mean age is doubled. How old is the adult?

You could guess the adult's age and find the mean. Keep adjusting your guess until you get the correct mean.

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9G

## Enrichment: House for sale

**16** The prices of all the houses in School Court are recorded: \$520 000, \$470 000, \$630 000, \$580 000, \$790 000, \$540 000, \$710 000, \$8.4 million, \$660 000.

**a** What is the mean house price in School Court, correct to the nearest dollar?

**b** What is the median house price in School Court?

**c** What effect does having a single \$8.4 million mansion in School Court have on the mean?

**d** What effect does having a single \$8.4 million mansion in School Court have on the median?

**e** Why might 'median house price' be a more useful measure than 'mean house price' when people are looking at living in a particular area?

**f** Search the internet to find the median house price in your suburb.

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# 9H Stem-and-leaf plots

A stem-and-leaf plot is a useful way of presenting numerical data. It allows trends to be spotted easily. Each number is split into a stem (the first digit or digits) and a leaf (the last digit).

|  | Stem | Leaf |
|---|---|---|
| 53 is | 5 | 3 |
| 78 is | 7 | 8 |
| 125 is | 12 | 5 |

## Let's start: Test score analysis

In a class, students' results on a recent test (out of 50) are recorded.

- How many students:
  - achieved a perfect score (i.e. 50)?
  - scored less than 25?
  - achieved a mark in the 40s?
- If there are 100 test results to analyse, would you prefer a list or a stem-and-leaf plot? Why?

*Test results*

| Stem | Leaf |
|---|---|
| 1 | 8 |
| 2 | 7 8 |
| 3 | 2 2 4 5 5 7 9 |
| 4 | 0 1 2 3 3 6 8 8 |
| 5 | 0 0 |

## Key ideas

- A **stem-and-leaf plot** is a way to display numerical data.
- Each number is split into a stem (the first digit or digits) and a leaf (the last digit).
  For example:

|  | Stem | Leaf |
|---|---|---|
| The number 7 is | 0 | 7 |
| The number 31 is | 3 | 1 |
| The number 152 is | 15 | 2 |

- Leaves are aligned vertically, getting bigger as you move away from the stem.

**Stem-and-leaf plot**
A graph that lists numbers in order, grouped in rows

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## Exercise 9H

**Understanding**

**1** Copy and complete.
In a stem-and-leaf plot the first digit(s) of a data value is called the _______________ and the last digit is called the _______________.

**2** The number 52 is entered into a stem-and-leaf plot.
**a** Which digit is the stem? **b** Which digit is the leaf?

**3** What number is represented by the following combinations?
**a** 3|9 **b** 2|7 **c** 13|4

**4** In this stem-and-leaf plot, the smallest number is 35. What is the largest number?

| Stem | Leaf |
|---|---|
| 3 | 5 7 7 9 |
| 4 | 2 8 |
| 5 | 1 7 |

**Fluency**

### Example 17 Interpreting an ordered stem-and-leaf plot

Average daily temperatures are shown for some different countries.

**a** Write out the temperatures as a list.
**b** How many countries' temperatures are represented?
**c** What are the maximum and minimum temperatures?
**d** What is the range of temperatures recorded?
**e** What is the median temperature recorded?

| Stem | Leaf |
|---|---|
| 1 | 3 6 6 |
| 2 | 0 0 1 2 5 5 6 8 9 |
| 3 | 0 2 |

| Solution | Explanation |
|---|---|
| **a** 13, 16, 16, 20, 20, 21, 22, 25, 25, 26, 28, 29, 30, 32 | Each number is converted from a stem and a leaf to a single number.<br>For example, 1\|3 is converted to 13. |
| **b** 14 | The easiest way is to count the number of leaves – each leaf corresponds to one country. |
| **c** minimum = 13<br>maximum = 32 | The first stem and leaf is 1\|3.<br>The last stem and leaf is 3\|2. |
| **d** range = 19 | Range = maximum – minimum = 32 – 13 = 19 |
| **e** median = 23.5 | The middle value is halfway between the numbers 2\|2 and 2\|5, so median = $\frac{1}{2}(22 + 25) = 23.5$ |

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**5** This stem-and-leaf plot shows the ages of people in a group.

| Stem | Leaf |
|---|---|
| 0 | 8 9 |
| 1 | 0 1 3 5 7 8 |
| 2 | 1 4 |

- **a** Write out the ages as a list.
- **b** How many ages are shown?
- **c** Answer true or false to each of the following.
  - **i** The youngest person is aged 10.
  - **ii** Someone in the group is 17 years old.
  - **iii** Nobody listed is aged 20.
  - **iv** The oldest person is aged 4.

**6** For each of the stem-and-leaf plots below, state the range and the median. (See Example 17 parts **d** and **e**.)

**a**

| Stem | Leaf |
|---|---|
| 0 | 9 |
| 1 | 3 5 6 7 7 8 9 |
| 2 | 0 1 9 |

**b**

| Stem | Leaf |
|---|---|
| 1 | 1 4 8 |
| 2 | 1 2 4 4 6 8 |
| 3 | 0 3 4 7 9 |
| 4 | 2 |

**c**

| Stem | Leaf |
|---|---|
| 3 | 1 1 2 3 4 4 8 8 9 |
| 4 | 0 1 1 2 3 5 7 8 |
| 5 | 0 0 0 |

**7** Copy and complete the stem-and-leaf plot for this set of data.
25, 27, 29, 30, 32, 39, 41, 42, 45, 51

| Stem | Leaf |
|---|---|
| 2 | |
| 3 | |
| 4 | |
| 5 | |

Remember to list the leaves in increasing order across each row.

## Example 18 Creating a stem-and-leaf plot

Represent this set of data as a stem-and-leaf plot: 23, 10, 36, 25, 31, 34, 34, 27, 36, 37, 16, 33

| Solution | Explanation |
|---|---|
| Sorted list: 10, 16, 23, 25, 27, 31, 33, 34, 34, 36, 36, 37 | Sort the list in increasing order so that it can be put directly into a stem-and-leaf plot. |

**Solution**

| Stem | Leaf |
|---|---|
| 1 | 0 6 |
| 2 | 3 5 7 |
| 3 | 1 3 4 4 6 6 7 |

**Explanation**

Split each number into a stem and a leaf. Stems are listed in increasing order. Leaves are written vertically, listed in increasing order across each row.

**8** Show each of the following sets of data as a stem-and-leaf plot.

**a** 11, 12, 13, 14, 14, 15, 17, 20, 24, 28, 29, 31, 32, 33, 35

**b** 20, 22, 39, 45, 47, 49, 49, 51, 52, 52, 53, 55, 56, 58, 58

**9** Show each of the following as a stem-and-leaf plot.

**a** 21, 35, 24, 31, 16, 28, 48, 18, 49, 41, 50, 33, 29, 16, 32

**b** 32, 27, 38, 60, 29, 78, 87, 60, 37, 81, 38, 11, 73, 12, 14

**10** Show each of the following sets of data as a stem-and-leaf plot.

**a** 80, 84, 85, 86, 90, 96, 101, 104, 105, 110, 113, 114, 114, 115, 119

**b** 401, 420, 406, 415, 416, 406, 412, 402, 409, 418, 404, 405, 391, 411, 413, 413, 408, 395, 396, 417

Remember, 101 is represented as $10|1$.

**Problem-solving and Reasoning**

Drilling for Gold 9H1

**11** A company recorded the duration (in seconds) that visitors spent on its website's home page.

**a** How many visitors spent less than 20 seconds on the home page?

**b** How many visitors spent more than half a minute?

**c** How many visitors spent between 10 and 30 seconds?

**d** What is the outlier for this stem-and-leaf plot?

| Stem | Leaf |
|---|---|
| 0 | 2 4 6 8 9 |
| 1 | 0 0 1 2 8 |
| 2 | 2 7 9 |
| 3 | |
| 4 | |
| 5 | 8 |

An outlier is a value that is not close to the other values.

**12** A teacher has compiled test scores out of 50 as a stem-and-leaf plot. However, some values are missing, as represented by the letters $a$, $b$, $c$ and $d$.

**a** How many students took the test?

**b** How many students passed the test (i.e. achieved a mark of 25 or higher)?

**c** State the possible values for each of the missing digits $a$ to $d$.

| Stem | Leaf |
|---|---|
| 1 | 5 |
| 2 | 4 5 $a$ 6 7 9 |
| 3 | $b$ 0 1 5 |
| 4 | 2 8 $c$ |
| 5 | $d$ |

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## Enrichment: Back-to-back stem-and-leaf plots

**13** Two radio stations poll their audience to determine their ages.

| Station 1 | Stem | Station 2 |
|---|---|---|
| 0 | 1 | 2 3 3 4 5 6 8 9 |
| 8 7 | 2 | 0 0 1 2 4 5 8 8 |
| 9 7 5 4 3 3 | 3 | 1 1 2 |
| 7 6 5 5 4 4 1 | 4 | 8 |
| 9 3 2 0 | 5 | |

Back-to-back stem-and-leaf plots are used to compare two sets of data.

**a** Find the age difference between the oldest and youngest listener polled for:
- **i** station 1
- **ii** station 2.

**b** One radio station plays modern music that appeals to teenagers. The other plays classical music and broadcasts the news. Which radio station is most likely to be the one that plays classical music and news?

**c** Advertisers want to know the age of the stations' audiences. This lets them target their advertisements more effectively (e.g. to 38 to 58 year olds). Give a 20-year age range for the audience majority who listen to:
- **i** station 1
- **ii** station 2.

# 9I Surveying and sampling

When a sample of people from a population is surveyed, it is hoped that the information given by this smaller group is representative of the larger group of people. Choosing the right sample size and obtaining a representative sample is not easy.

## Let's start: Do you like coming to school each day?

The principal has asked Karen, Tara and Josh to conduct a survey.

He wants to know if the students enjoy coming to school each day.

Karen, Tara and Josh decide to choose a sample of students.

Karen says: 'I am going to survey every student in the school.'

Tara says: 'I am going to survey the first $10$ students who arrive at the front gate tomorrow from 7 a.m.'

Josh says: 'My little brother is in the Year 7 rugby league team. I am going to survey all the students in his squad.'

Consider then discuss the following questions.

- What problems might Karen encounter?
- Will Tara's sample give the principal accurate results?
- What is wrong with the sample that Josh has chosen?

## Key ideas

- A **population** is the entire group in which we are interested. For example, if we want to find the average height of $14$-year-old girls in Australia, then the population is all the $14$-year-old girls in Australia.
- A **sample** is a small group randomly selected from a population. For example, a sample could be $100$ randomly chosen $14$-year-old Australian girls.
- A **survey** is a set of questions used to collect data.
- The accuracy of the survey's conclusion can be affected by:
  - the **sample size** (number of participants or items considered)
  - whether the sample is **representative** of the larger group, or **biased**
  - whether there were any **measurement errors**, which could lead to **outliers** – values that are noticeably different from the other values.

**Population** The entire group is selected

**Sample** A small group randomly selected out of the population

**Survey** A set of questions

**Sample size** The number of participants or items included in the data

**Representative** The sample reflects the entire population

**Biased** The sample does not reflect the entire population. Some members or traits are more or less likely to be included.

**Measurement error** An error made in the collecting or recording of data

**Outlier** A value that is much larger or much smaller than the rest of the data

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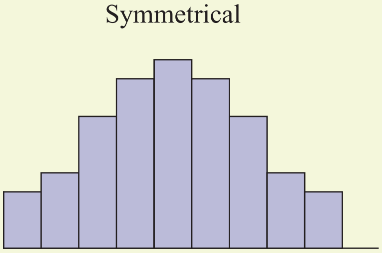
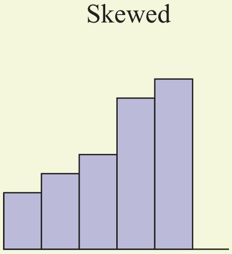
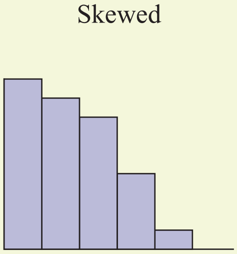
- Data represented as a histogram can be seen as **symmetrical** or **skewed**.







- If a data distribution is symmetrical, the mean and the median are approximately equal.

**Symmetrical** A symmetrical histogram looks basically the same on either side of the vertical centre. The mean and the median are approximately equal.

**Skewed** A skewed histogram is sloped more to the left or right. The mean and the median have different values.

## Exercise 9I

**Understanding**

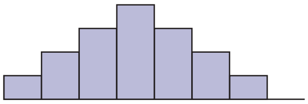
**1** Write down the missing word from each statement.

**a** A __________ is a set of questions.

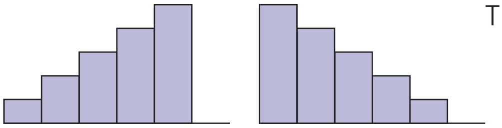
**b** A small group out of a population is called a __________.

**c** A __________ sample doesn't represent the population.

**d** This histogram has a __________ shape.



**e** These histograms have a __________ shape.



**2** Marieko wishes to know the average age of drivers in her city. She could survey 10 of her friends, or survey 1000 randomly selected drivers.

**a** Which of these options would give a more accurate result?

**b** Which would be easier for Marieko to perform?

### Example 19 Calculating population numbers from random sample data

Out of a random sample of 10 Tasmanian devils, there are 7 that have a facial tumour.

**a** What proportion of this sample has facial tumours?

**b** If there are 200 Tasmanian devils in this region, on the basis of this sample, how many would have facial tumours?

**c** If there are 100 Tasmanian devils in this region, how many would not have a facial tumour?

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| Solution | Explanation |
|---|---|
| **a** $\frac{7}{10}$ | 7 have tumours out of a total of 10. |
| **b** $\frac{7}{10} \times 200 = 140$ | The sample proportion × 200. |
| **c** $\frac{3}{10} \times 100 = 30$ | 3 out of 10 don't have a tumour. |

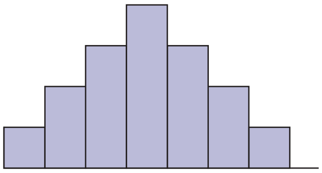
**3** Ajith looks at a random sample of penguins and notes that of the 50 he sees, 20 of them have spots on their bodies.

- **a** What proportion of the sample has spots?
- **b** If there are 5000 penguins in a region, on the basis of this sample how many would have spots on their bodies?
- **c** If there are 500 penguins in a region, how many would not have spots on their bodies?

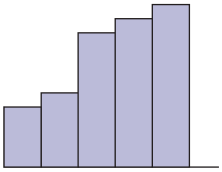
A proportion can be written as a fraction.

**4** Classify the following distributions as symmetrical or skewed.

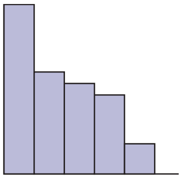
**a**



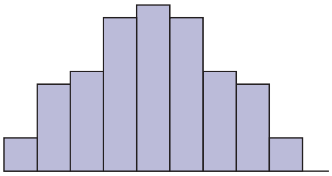
**b**



**c**



**d**



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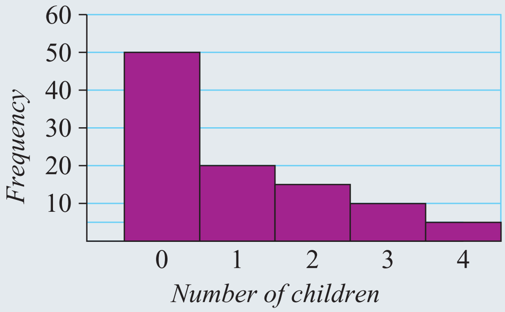
Fluency

## Example 20 Interpreting survey results

A survey is conducted asking 100 randomly selected adults how many children they have. You can assume that this sample is representative of the adult population. The results are shown in this histogram:



**a** Is this distribution symmetrical or skewed?
**b** What proportion of the adult population has two or more children?
**c** In a group of 9000 adults, how many would you expect to have 4 children?
**d** Which of the following methods of conducting the survey could lead to bias? Give a reason why.

| | |
|---|---|
| **Method 1** | Asking people waiting outside a childcare centre |
| **Method 2** | Randomly selecting people at a night club |
| **Method 3** | Choosing 100 adults at random from the national census and noting how many children they claimed to have. |

| **Solution** | **Explanation** |
|---|---|
| **a** Skewed | Many more people have 0 children, so the distribution is not symmetrical. |
| **b** $\frac{3}{10}$ | $15 + 10 + 5 = 30$ adults have two or more children |
| **c** $\frac{1}{20} \times 9000 = 450$ | Proportion $= \frac{30}{100} = \frac{3}{10}$ |
| | In the survey $\frac{5}{100} = \frac{1}{20}$ of the population have four children. |

**d** Method 1 could lead to bias. If someone is waiting outside a childcare centre they are more likely to have at least one child.
Method 2 could lead to bias. If someone is at a night club they are likely to be a younger adult, and so less likely to have a child.

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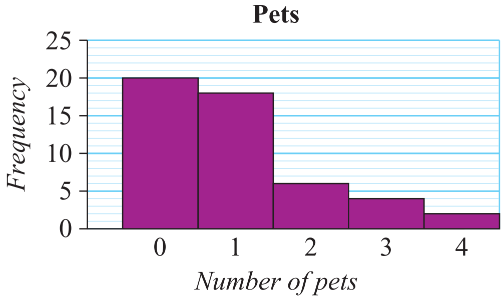
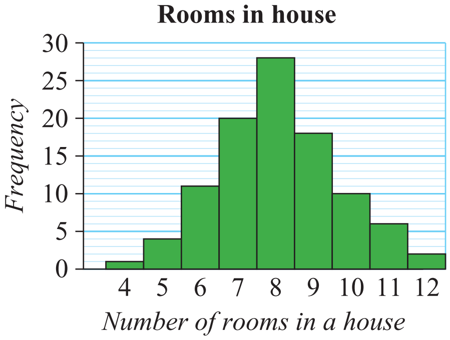
9I

5 A survey is conducted asking 50 people how many pets they own. You can assume it is a representative sample of the population.
The results are shown in this histogram.
- **a** Is the distribution skewed or symmetrical?
- **b** What proportion of people had no pets?
- **c** Of a group of 1000 people, how many of them would you expect to have no pets?
- **d** What proportion of people had 2 or more pets?
- **e** Of a group of 5000 people, how many of them would you expect to have 2 or more pets?
- **f** Why would conducting this survey outside a veterinary clinic cause a bias in the results?

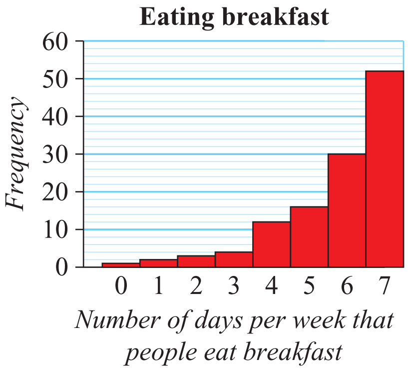
Expected number = proportion × total



6 A survey of 100 randomly selected people, who live in a house, asked how many rooms were in their house. You can assume that it is a representative sample of the population. The results are shown in this histogram.
- **a** Is the distribution skewed or symmetrical?
- **b** What proportion of people live in an 8-room house?
- **c** In a group of 1500 people, how many would you expect to live in an 8-room house?
- **d** What proportion of people live in a house with 5, 6 or 7 rooms?
- **e** In a group of 3000 people, how many would you expect to live in a house with 5, 6 or 7 rooms?
- **f** Why would conducting this survey in a wealthy suburb cause bias in the results?



7 A survey of 120 randomly selected people asked how many days per week each person ate breakfast. You can assume that it is a representative sample of the population. The results are shown in this histogram.
- **a** Is the distribution skewed or symmetrical?
- **b** What proportion of people eat breakfast 7 days a week?
- **c** In a group of 36 000 people, how many would you expect to eat breakfast 7 days a week?
- **d** What proportion of people eat breakfast 4 or 5 days a week?
- **e** In a group of 4800 people, how many would you expect to eat breakfast 4 or 5 days a week?
- **f** Why would conducting this survey on a 6 a.m. suburban train to the city cause bias in the results?



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**8** In a factory producing chocolate bars, a sample of bars is taken and automatically weighed to check whether they are between 50 and 55 grams. The results are shown in a frequency table.

| **Weight (g)** | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 108 |
|---|---|---|---|---|---|---|---|---|
| **Frequency** | 2 | 5 | 10 | 30 | 42 | 27 | 11 | 1 |

Add up the frequencies to find the total number of chocolate bars in the sample.

- **a** Which weight value is an outlier?
- **b** If you leave out the 108 gram result, is this distribution skewed or symmetrical?
- **c** What proportion of chocolate bars are 53 g, 54 g or 55 g?
- **d** In a batch of 800 chocolate bars how many would be expected to be 53 g, 54 g or 55 g?
- **e** What proportion of chocolate bars are less than 52 g?
- **f** In a batch of 2048 chocolate bars, how many would be expected to be less than 52 g?

Drilling for Gold 9I1

Problem-solving and Reasoning

**9** Fred attempts to find a relationship between people's ages and their incomes. He is considering some questions to put in his survey. For each question, decide whether it should be included in the survey, giving a brief explanation.

- **a** What is your current age in years?
- **b** Are you rich?
- **c** Are you old?
- **d** How much money do you have?
- **e** What is your name?
- **f** How much money did you earn in the past year?
- **g** How much money did you receive today?

**10** For each of the following survey questions, give an example of an unsuitable location and time to conduct the survey if you wish to avoid a bias.

- **a** A survey to find the average number of children in a car
- **b** A survey to find how many people are happy with the current prime minister
- **c** A survey to find the proportion of Australians who are vegetarians
- **d** A survey to find the average cost of supermarket groceries

**11** A survey is being conducted to decide how many adults use Mathematics later in life.

- **a** If someone wanted to make it seem that most adults do not use Mathematics, where and when could they conduct the survey?
- **b** If someone wanted to make it seem that most adults use Mathematics a lot, where and when could they conduct the survey?
- **c** How could the survey be conducted to provide less biased results?

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**12** Robert wishes to find out how much time high school students spend on homework.

**a** Give some reasons why surveying just his Maths class might introduce bias.

**b** Why would surveying just the people on his soccer team introduce bias?

**c** Give a reason why surveying 10 students would not be a representative sample.

**d** He decides to choose 50 people from across the whole school. Who should he choose in order to minimise the bias? Justify your answer.

## Enrichment: Design a survey and graph sample results

**13** Task 1

Design survey questions to find out the following information.

- The mean number of siblings of the students in your class
- The mean number of car trips made to school each week by families in your class
- The mean number of computers owned by families in your class

Task 2

Write down how you will choose an unbiased sample of students for your survey.
Run the survey on your chosen sample students. Keep a record of all results.

Task 3

In an Excel spreadsheet, record your results as tables showing the frequency of each answer.
Use Excel to draw a histogram for each table. Comment on whether each set of data is symmetrical or skewed.

Task 4

In each table add a column for the proportions and enter the proportion that each frequency is of the total.
Multiply these proportions by the total number of students in Year 8 at your school to find the expected numbers from your year level.

Task 5

For each set of data, use an Excel spreadsheet to help you to find the expected mean for the students in your year level. Write your conclusions in sentences.

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# Puzzles and games

**1** John Venn was an English mathematician who invented Venn diagrams to make sorting data and probability calculations easier. He was also a fan of cricket. Solve the questions below to find the answer to this question:

*What did John Venn invent that, in 1909, clean bowled (i.e. bowled out) one of our best Australian cricket batsmen four times?*

| |
|---|
| |
| 20 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | |
| 7 | 30 | 6 | 4 | 23 | F | 12 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | |
| 5 | 20 | 15 | 14 | 23 | F | T |

**A** Find the mean of these scores: 26, 25, 13, 24, 12 and 20.

**B** What is the median of these scores: 2, 4, 4, 7, 8, 9, 10?

**C** What is the range of these scores: 13, 18, 5, 7, 16, 3?

**E** True (T) or false (F)? A histogram has no spaces between the columns.

**G** What frequency does this tally represent?
𝍸 𝍸 ||

**H** If 8 students have a cat only and 6 have both a cat and a dog and 9 a dog only, how many have a cat?

**I** If 8 students have a cat only and 6 have both a cat and a dog and 9 a dog only, how many have a cat or a dog or both?

**L** What is the mean of 1, 2, 3, 4, 5, 6, 7?

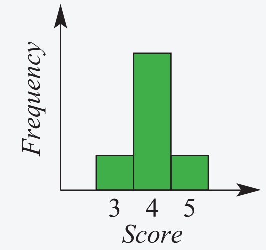
**M** If Amy got 7 marks for 5 tests in a term, which of these numbers is the frequency?

**N** True (T) or false (F)? A skewed histogram has its highest frequency in the middle.

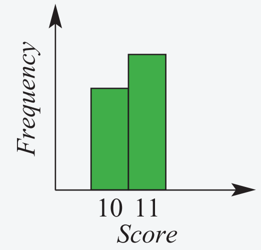
**W** How many of these ages are in the interval 12–15 years?
12, 13, 16, 11, 12, 15, 19, 19, 14, 16

**2** The following histograms are drawn to scale but the frequency scale has been omitted. Determine the median for each one.

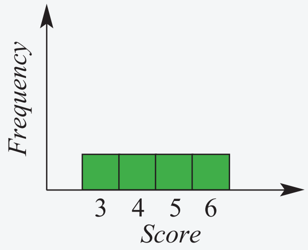
**i**



**ii**



**iii**



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**3** Jayden is an AFL player and notes the number of points he scores in his 22-week season. His results are shown in this table.

| Points | Frequency |
|---|---|
| 0–4 | 3 |
| 5–9 | 11 |
| 10–14 | 6 |
| 15–19 | 1 |
| 20–25 | 1 |

Jayden said he scored a different number of points for each of the 22 games. Is he correct? Use an example to explain your answer.

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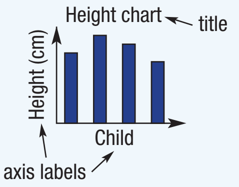
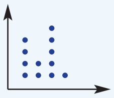
# Chapter summary

### Column graphs

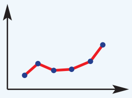


### Dot plots



Each dot represents one piece of data.

### Line graphs



Time is often displayed on the horizontal axis.

### Data can be represented as a graph or plot

### Stem-and-leaf plots

24, 25, 27, 31, 34 can be shown as:

| Stem | Leaf |
|---|---|
| 2 | 4 5 7 |
| 3 | 1 4 |

### Graphs

### Graphs

- Sector graph

- Divided bar graph

### Tables

### Frequency distribution tables

#### Individual scores

| Number of children Score | Number of families |  |
|---|---|---|
|  | Tally | Frequency |
| 0 | \|\|\| | 3 |
| 1 | 卌 | 5 |
| 2 | \|\|\|\| 卌 \| | 11 |
| 3 | 卌 | 5 |
| 4 | 卌\| | 6 |
|  |  | $n = 30$ |

#### Scores in intervals

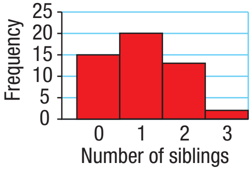
| Age | Frequency |
|---|---|
| 0–4 | 3 |
| 5–9 | 7 |
| 10–14 | 5 |
|  | $n = 15$ |

Drilling for Gold 9R1

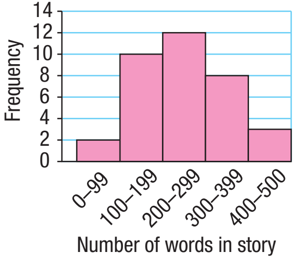
## Data collection, representation and analysis

### Histograms

One column for every score



Grouped data



### Summarising data numerically

Range = 10 – 1
= 9

$$1, 1, 2, 3, \underbrace{4, 4, 4}, \boxed{5}, 5, 6, 7, 7, 8, 8, 10$$

Mode = 4
(most common value)

Median = 5
(middle value)

$$\text{Mean} = \frac{\text{sum of values}}{\text{number of values}}$$

$$= \frac{75}{15} = 5$$

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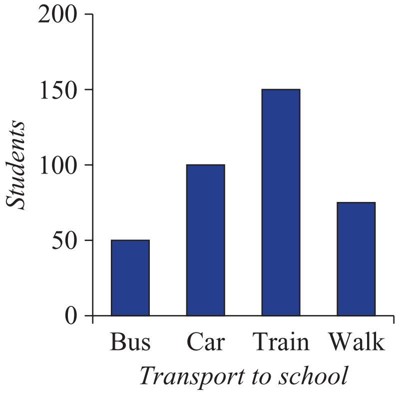
**T** **Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.**

## Multiple-choice questions

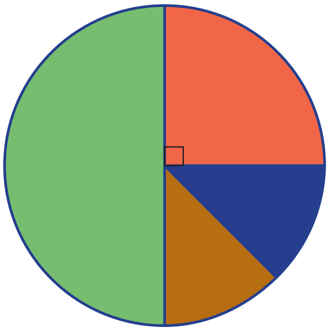
**1** Using the information in the column graph, how many students don't walk to school?

**A** 75 **B** 150 **C** 300 **D** 375 **E** 100



**2** The chocolates in a bag are grouped by colour and the proportions shown in the sector graph. If there are equal numbers of blue and brown chocolates, how many are blue, given that the bag contains 28 green ones?

**A** 112 **B** 7 **C** 56
**D** 14 **E** 28



**3** The table below shows the number of goals scored by a soccer team over a season.

| Goals | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| Tally for number of games | 𝍸 \|\| | 𝍸 | \|\|\| | \|\|\| | \|\| |

The total number of games played by the soccer team is:

**A** 28 **B** 10 **C** 20 **D** 13 **E** 5

**4** For the soccer results in the table in question **3**, the total number of goals scored in the season is:

**A** 28 **B** 10 **C** 4 **D** 20 **E** 13

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**5** Which is the best description of the mode in a set of test scores?

**A** The average of the scores
**B** The score in the middle
**C** The score with the highest frequency
**D** The difference between the highest and lowest score
**E** The lowest score

**6** For the set of data 1, 5, 10, 12, 14, 20, the range is:

**A** 1 **B** 19 **C** 4 **D** 11 **E** 6

Questions **7** and **8** relate to the following information. In a group of 10 students, the number of days each student was absent over a term is recorded: 1, 0, 2, 4, 2, 3, 0, 1, 2, 3

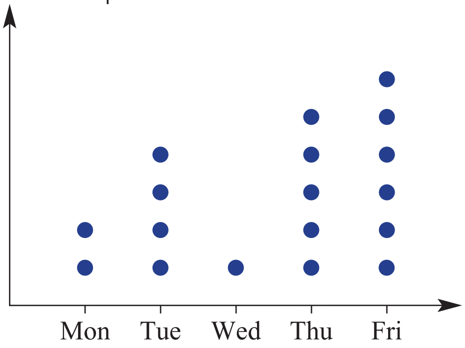
**7** The mode is:

**A** 0 **B** 1 **C** 2 **D** 3 **E** 4

**8** The mean number of days a student was absent is:

**A** 1 **B** 2 **C** 4 **D** 1.8 **E** 18

**9** Students are asked to state their favourite week day. The results are shown in this dot plot.

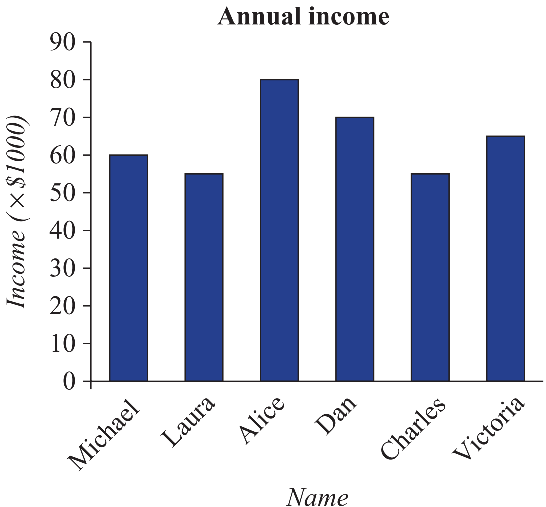


The total number of students asked was:

**A** 2 **B** 5 **C** 12 **D** 18 **E** 20

**10** In the following column graph, the highest income is earned by:

**A** Michael **B** Alice **C** Dan **D** Laura **E** Victoria



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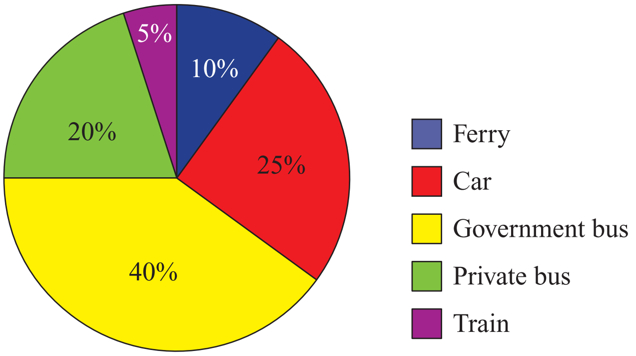
## Short-answer questions

**1** The sector graph shows the type of transport office workers use to get to work every day.



**a** Which type of transport is the most commonly used?
**b** Which type of transport is the least commonly used?
**c** What percentage of office workers did not travel by car?
**d** If 20 000 workers were surveyed, how many people travelled to work each day by train?
**e** The year after this survey was taken, it was found that the number of people using government buses had decreased. Give a reason why this could have occurred.

**2** Some students were asked how many hours of study they did before their half-yearly Maths exam. Their responses are represented in a tally.

| 0 hours | 1 hour | 2 hours | 3 hours | 4 hours |
|---|---|---|---|---|
| \|\| | 𝍸 \| | \|\|\| | \|\|\| | 𝍸 \|\|\| |

**a** How many students are in the class?
**b** Convert the tally above into a frequency table.
**c** Draw a histogram to represent the results of the survey.
**d** What proportion of the class did no study for the exam?
**e** Find the total number of hours of study done by this group.
**f** Calculate the mean number of hours per student in the class spent studying for the exam, giving the answer correct to 1 decimal place.

**3** **a** Rewrite the following data in ascending order:
56 52 61 63 43 44 44 72 70 38 55
60 62 59 68 69 74 84 66 53 71 64
**b** What is the mode?
**c** What is the median for these scores?
**d** Calculate the range.

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**4** The ages of boys in an after-school athletics squad are shown in the table below.
- **a** State the total number of boys in the squad.
- **b** List the ages of all these boys in ascending order.
- **c** Calculate the mean age of the squad, correct to 2 decimal places.
- **d** What is the median age of the boys in the squad?

| Age | Frequency |
|---|---|
| 10 | 2 |
| 11 | 3 |
| 12 | 4 |
| 13 | 8 |
| 14 | 10 |

**5** A group of teenagers were weighed. Their weights were recorded to the nearest kilogram. The results are as follows:

56 64 72 81 84 51 69 69 63 57 59 68 72 73 72 80 78 61 61 70
57 53 54 65 61 80 73 52 64 66 66 56 50 64 60 51 59 69 70 85

- **a** Find the highest and lowest weights and the range.
- **b** Create a grouped frequency distribution table using the groups (intervals) of 50–54, 55–59, 60–64 etc.
- **c** Find the modal group.
- **d** Why is this sample not representative of the whole human population?

**6** **a** Use the data 5, 1, 7, 9, 1, 6, 4, 10, 12, 14, 6, 3 to find the:
- **i** mean
- **ii** median
- **iii** range.

- **b** An extra score of 52 is added into the list in part **a**. Calculate the new median and mean, and state which measure has changed the most.
- **c** What is the name for a score that is much larger than all the other values in a list?

**7** In an attempt to find the average number of hours of homework that a Year 8 student completes, Samantha asks 10 of her friends in Year 8 how much homework they do.
- **a** Explain two ways in which Samantha's sampling is inadequate for representing the population of Years 8s in her school.
- **b** If Samantha wished to convey to her parents that she did more than the average, how could she choose 10 people to bias the results in this way?

**8** The weight in grams of various meat patties at a local burger shop are measured. The results are shown in this stem-and-leaf plot.

| Stem | Leaf |
|---|---|
| 10 | 5 8 |
| 11 | 2 6 6 8 9 |
| 12 | 0 2 4 5 |
| 13 | 1 |

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- **a** What is the weight of the pattie represented as $11|8$?
- **b** What is the weight of the lightest pattie?
- **c** What is the weight of the heaviest pattie?
- **d** Find the range of the weights of the patties.

**9** Consider the data $1, 2, 2, 3, 4, 7, 9, 12$. State the:
**a** range **b** mean **c** median **d** mode.

**10** Consider the data $0, 4, 2, 9, 3, 7, 3, 12$. State the:
**a** range **b** mean **c** median **d** mode.

## Extended-response questions

**1** This table shows the number of rainy days in a certain town over one year.

| Month | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No. of rainy days | 10 | 11 | 3 | 7 | 2 | 0 | 1 | 5 | 6 | 9 | 7 | 5 |

- **a** Show this information in a line graph.
- **b** On how many days of the year did it rain in this town?

**2** At a school camp, a survey was conducted to establish each student's favourite dessert.

| Ice-cream | Yoghurt | Danish pastry | Jelly | Pudding | Cheesecake |
|---|---|---|---|---|---|
| 10 | 5 | 2 | 7 | 4 | 12 |

- **a** How many students participated in the survey?
- **b** What is the most popular dessert?
- **c** If a student is picked at random, what is the probability that jelly is their favourite dessert?
- **d** For each of the following graphs and plots, state whether it would be a reasonable way of presenting the survey's results.
  - **i** Column graph
  - **ii** Line graph
  - **iii** Dot plot
  - **iv** Stem-and-leaf plot
- **e** If the campers attend a school with $800$ students, how many students from the entire school would you expect to choose pudding as their preferred dessert?

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## Chapter 6: Angle relationships and properties of geometrical figures 1

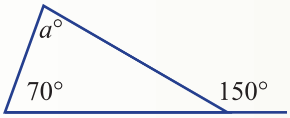
### Multiple-choice questions

**1** The supplement of 80° is:

**A** 10° **B** 100° **C** 280° **D** 20°

**2** In this diagram $a$ equals:

**A** 150 **B** 220 **C** 70 **D** 80

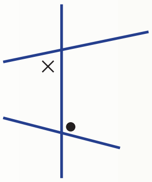


**3** The angle sum of a regular pentagon is:

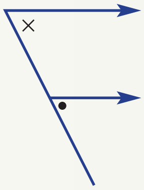
**A** 72° **B** 108° **C** 540° **D** 120°

**4** Which diagram shows equal alternate angles?

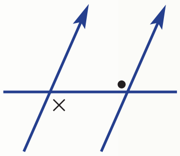
**A**



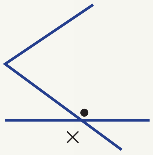
**B**



**C**



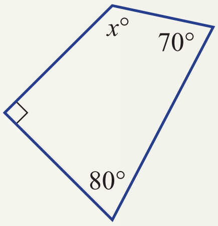
**D**



**5** The value of $x$ in this quadrilateral is:

**A** 150 **B** 240

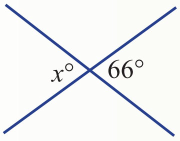
**C** 120 **D** 300



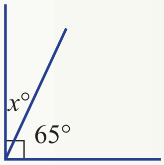
### Short-answer questions

**1** Find the value of $x$.

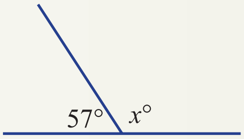
**a**



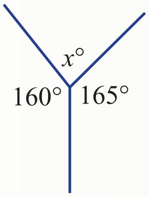
**b**



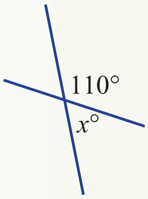
**c**



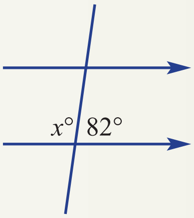
**d**



**e**

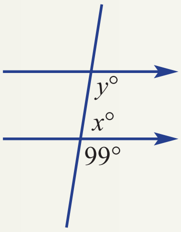


**f**

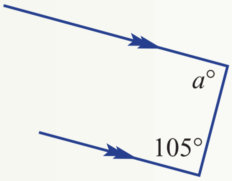


**2** Find the value of each pronumeral.

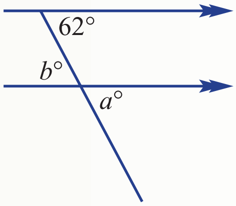
**a**



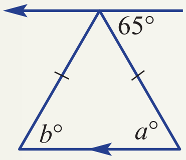
**b**



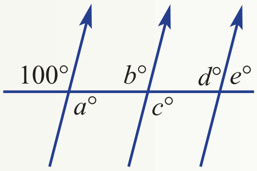
**c**



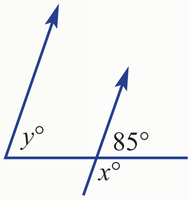
**d**



**e**

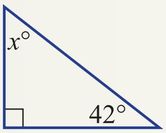


**f**

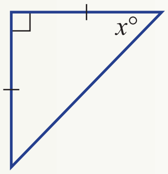


**3** Find the value of the pronumeral in these triangles.

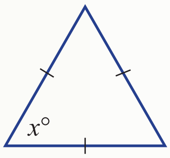
**a**



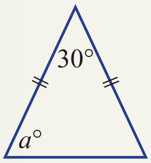
**b**



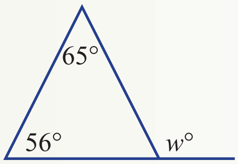
**c**



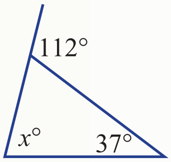
**d**



**e**

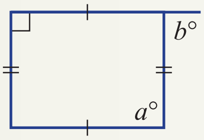


**f**

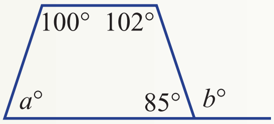


**4** Find the value of $a$ and $b$ in these quadrilaterals.

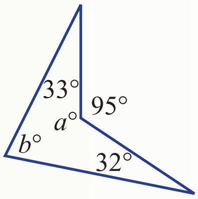
**a**



**b**



**c**

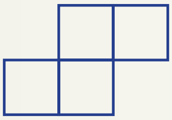


**5** Write down the:

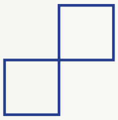
**i** number of lines of symmetry

**ii** order of rotational symmetry.

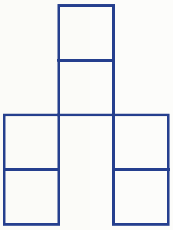
**a**



**b**



**c**



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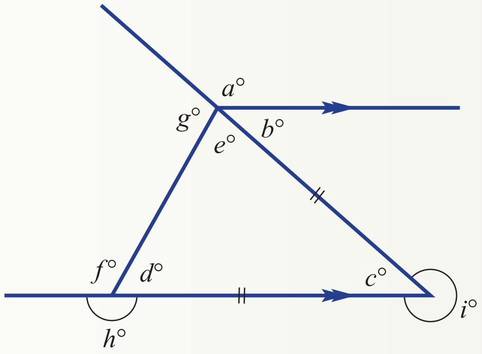
### Extended-response question



If $a = 115$, then find the value of each pronumeral. Write your answers in the order you found them.

Is the order the same for everybody in the class? Discuss any differences and the reasons associated with each.

## Chapter 7: Linear relationships 1

### Multiple-choice questions

**1** The value of $y$ in the rule $y = 2x - 1$ when $x = -1$ is:
**A** 3 **B** 1 **C** –1 **D** –3

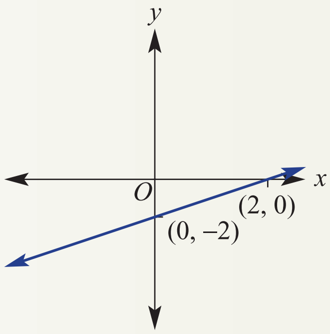
**2** The coordinates of the point 3 units directly above the origin is:
**A** (0, 0) **B** (0, 3) **C** (0, –3) **D** (3, 0)

**3** The equation for the table of values shown is:
**A** $y = 2x$ **B** $y = 2x + 2$
**C** $y = 2(x + 2)$ **D** $y = x + 4$

| $x$ | 0 | 2 | 4 |
|---|---|---|---|
| $y$ | 4 | 8 | 12 |

**4** Which line does not pass through (3, 2)?
**A** $y = x + 1$ **B** $y = x - 1$ **C** $y = 5 - x$
**D** $y = 3x - 7$ **E** $y = 8 - 2x$

**5** Which equation suits the given graph?
**A** $y = 6x - 2$
**B** $y = 3x - 2$
**C** $y = x - 2$
**D** $y = x + 2$



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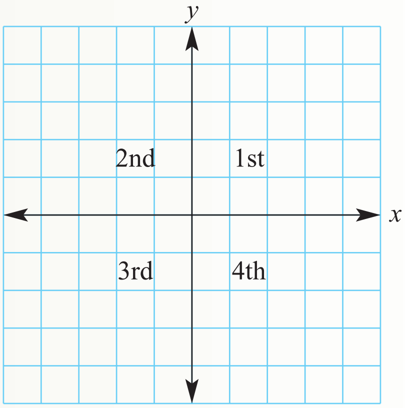
## Short-answer questions



**1** In which quadrant does each point lie?

**a** (5, 1)
**b** (–3, 4)
**c** (–5, –1)
**d** (8, –3)

**2** Complete these tables of values.

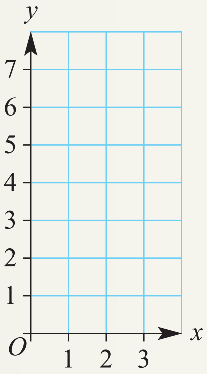
**a** **i** $y = 2x + 1$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ |  |  |  |  |

**ii** $y = 4 - x$

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ |  |  |  |  |

**b** Plot the points from both tables and join to form two graphs.



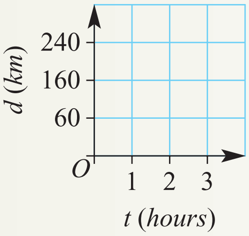
**3** Write the rules in question 2*a* in words, starting with: 'To find a value for $y$, choose value for $x$…'.

**4** The distance a car travels ($d$ km) over $t$ hours is given by $d = 80t$.

**a** Complete this table of values.

| $t$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $d$ |  |  |  |  |

**b** Plot a graph using your table.



**c** How far does the car travel after 3 hours?

**d** How long would it take for the car to travel 320 km?

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**5** For each equation, complete a table of values like the one shown and plot to form a graph.

**a** $y = 2x - 2$ **b** $y = -x + 1$

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

### Extended-response question

The cost ($$C$) of running a coffee shop is given by the rule $C = 400 + 5n$, where $n$ is the number of customers on any given day. The revenue (income) is $$R$ and is given by $R = 13n$.

**a** Complete this table.

| $n$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|---|
| $C$ | | | | | | | |
| $R$ | | | | | | | |

**b** Plot a graph for both $C$ and $R$ on the same set of axes.

**c** What is the 'break even' point for the coffee shop i.e. where does the cost = revenue?

**d** If they are particularly busy on a Saturday, and serve 100 people, calculate the shop's profit (profit = revenue – cost).

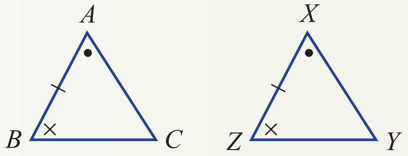
## Chapter 8: Transformations and congruence

### Multiple-choice questions

Questions **1** and **2** relate to $\Delta ABC$ and $\Delta XYZ$

**1** The angle $ABC$ corresponds to:

**A** $\angle XYZ$ **B** $\angle ZYX$ **C** $\angle XZY$
**D** $\angle ZXY$ **E** $\angle YXZ$

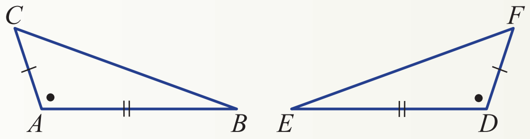


**2** The side $AC$ corresponds to:

**A** $XZ$ **B** $XY$ **C** $ZY$ **D** $BC$

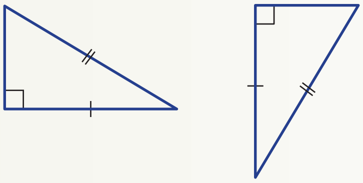
**3** A congruency statement for these triangles is:

**A** $\Delta ABC \equiv \Delta DFE$ **B** $\Delta ABC \equiv \Delta EFD$
**C** $\Delta ABC \equiv \Delta EDF$ **D** $\Delta ABC \equiv \Delta DEF$



**4** Which test is used to show that these triangles are congruent?

**A** SSS
**B** SAS
**C** AAS
**D** RHS



**5** Which of the following is not enough to prove congruency for triangles?

**A** SSS **B** AAS **C** AAA **D** SAS

## Short-answer questions

**1** Which of the following quadrilaterals definitely have equal diagonals?

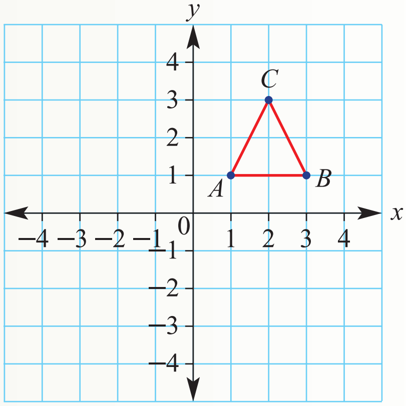
**a** trapezium **b** kite **c** rhombus
**d** rectangle **e** parallelogram **f** square

**2** What translation will shift point $P$ to its image $P'$?

**a** $P(1, 1)$ to $P'(3, 3)$ **b** $P(-1, 4)$ to $P'(-2, 2)$

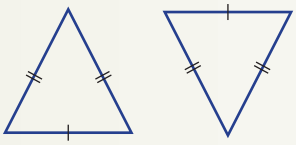
**3** Triangle $ABC$ is on a Cartesian plane as shown. List the coordinates of the image points $A'$, $B'$ and $C'$ after:

**a** a reflection in the $x$-axis
**b** translation of left 4 down 2
**c** a rotation 90° clockwise about (0, 0)
**d** a rotation 180° about (0, 0).

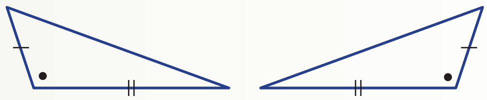


**4** Which congruency test (SSS, SAS, AAS or RHS) would be used to prove the following pairs of triangles are congruent?

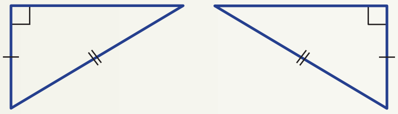
**a**



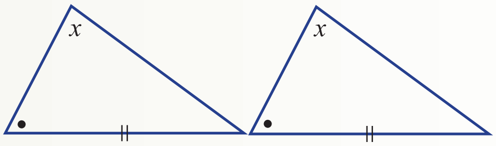
**b**



**c**



**d**



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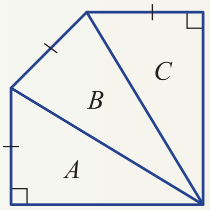
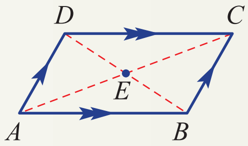
**5** Which two triangles are congruent?



### Extended-response question

For this parallelogram:

**a** why is $\angle BAE = \angle DCE$?
**b** why is $\angle ABE = \angle CDE$?
**c** give the test (SSS, SAS, AAS, RHS) that shows $\Delta ABE \equiv \Delta CDE$.
**d** explain why the diagonals bisect each other.



## Chapter 9: Data collection, representation and analysis

### Multiple-choice questions

**1** For the set of numbers 3, 2, 1, 3, 5, 1, 3, 9, 3, 5, the mode is:
**A** 3 **B** 3.5 **C** 8 **D** 35

**2** Look at the set of numbers 8, 9, 10, 10, 16, 19, 20, 20. Which of the following statements is true?
**A** Median = 13 **B** Mean = 13 **C** Mode = 13 **D** Range = 13

*Questions **3** and **4** relate to the following information.*
A stem-and-leaf plot shows the ages of various people.

| Stem | Leaf |
|---|---|
| 1 | 7 9 |
| 2 | 3 4 6 |
| 3 | 2 7 9 |

**3** The youngest person's age is:
**A** 1 **B** 17 **C** 7 **D** 2 **E** 39

**4** The number of people represented is:
**A** 8 **B** 11 **C** 39 **D** 3 **E** 26

**5** The median of the numbers 2, 4, 7, 9, 11 is:
**A** 7 **B** 7.5 **C** 9 **D** 8 **E** 11

### Short-answer questions

**1** Find the: **i** mean, **ii** median and **iii** range of these data sets.
**a** 10, 15, 11, 14, 14, 16, 18, 12
**b** 1, 8, 7, 29, 36, 57
**c** 1.5, 6, 17.2, 16.4, 8.5, 10.4

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**2** Draw a histogram for the following frequency table.

| Score | Frequency |
|---|---|
| 10 | 2 |
| 11 | 3 |
| 12 | 5 |
| 13 | 1 |

**3** Draw a column graph to represent the following people's ages.

| Name | Sven | Dane | Kelly | Hugo | Frankie |
|---|---|---|---|---|---|
| Age (years) | 20 | 12 | 15 | 22 | 25 |

**4** A Year 8 group was asked how many hours of television they watch in a week. The results are given in the table.

**a** How many students participated in the survey?

**b** How many students watched 11 or 12 hours of television?

**c** What was the most common amount of television watched?

**d** Show this information in a column graph.

| TV watched (hours) | No. of students |
|---|---|
| 8 | 5 |
| 9 | 8 |
| 10 | 14 |
| 11 | 8 |
| 12 | 5 |

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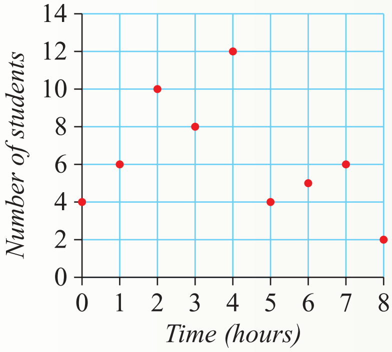
**5** The number of students in the library is recorded hourly, as displayed in the graph.



**a** How many students entered the library when it first opened?
**b** How many students were in the library at 8 hours after opening?
**c** If the library opens at 9:00 a.m., at what time are there the most number of students in the library?
**d** How many students were in the library at 4:00 p.m.?
**e** Why do you think these points have not been joined to make a line graph?

### Extended-response question

Two groups of students have their pulse rates recorded as beats per minute. The results are listed here.

Group A: 65, 70, 82, 81, 67, 74, ☐ 81, 88, 84, 72, 65, 66, 81, 72, 68, 86, 86

Group B: 83, 88, 78, 60, 81, 89, 91, 76, 78, 72, 86, 80, 64, 77, 62, 74, 87, 78

**a** How many students are in group B?
**b** If the median pulse rate for group A is 76, what number belongs in the ☐?
**c** What is the median pulse rate for group B?
**d** Which group has the largest range?

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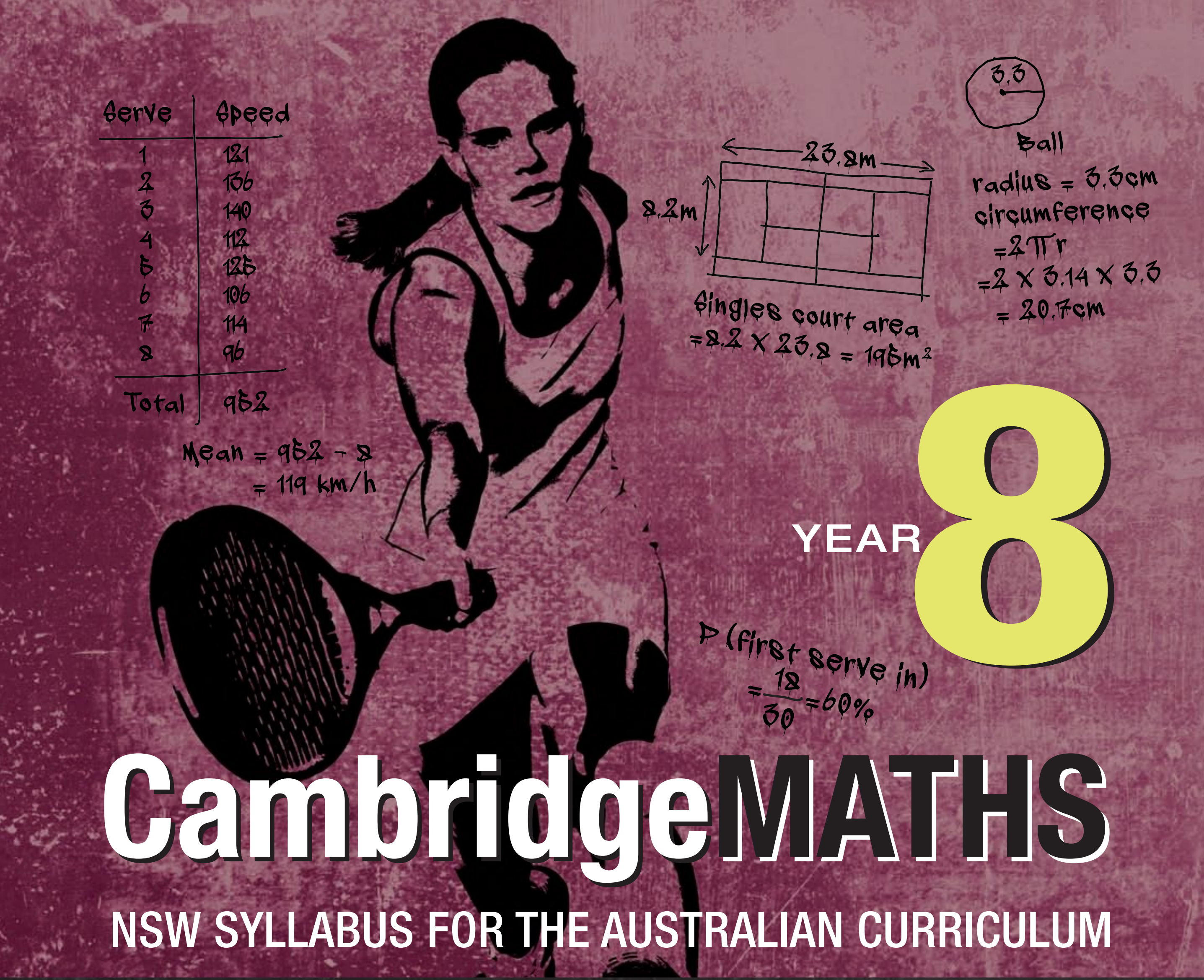

# YEAR 8

# CambridgeMATHS

## NSW SYLLABUS FOR THE AUSTRALIAN CURRICULUM

## APPENDIX 1

## GOLD

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## Appendix 1

# Computation with integers

## What you will learn

- **A1A** Whole number addition and subtraction REVISION
- **A1B** Whole number multiplication and division REVISION
- **A1C** The order of operations REVISION
- **A1D** Number properties REVISION
- **A1E** Divisibility and prime factorisation REVISION
- **A1F** Negative numbers REVISION
- **A1G** Addition and subtraction of integers REVISION
- **A1H** Multiplication and division of integers REVISION

### Strand: Number and Algebra

**Substrand: COMPUTATION WITH INTEGERS**

In this chapter, you will learn to:

- compare, order and calculate with integers, and apply a range of strategies to aid computation.

*This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at:* www.cambridge.edu.au/goldnsw8

## Public key encryption

Most of the world's electronic commercial transactions are encrypted so that important information does not get into the wrong hands. The encryptions use an algorithm that uses prime numbers, division and remainders, equations and the 2300-year-old Euclidean division algorithm to complete the task. If it wasn't for Euclid (about 300 BC) and the prime numbers, today's electronic transactions would not be secure.

## Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw8

**Drilling for Gold:**
Building knowledge and skills

**Skillsheets:**
Extra practise of important skills

**Literacy activities:**
Mathematical language

**Worksheets:**
Consolidation of the topic

**Chapter Test:**
Preparation for an examination

# Pre-test

**1** Put the following terms under the headings of addition (+), subtraction (–), multiplication (×) or division (÷).

**a** sum **b** of **c** and **d** less than
**e** total **f** into **g** more than **h** increase
**i** quotient

**2** Complete these additions.

**a** $12 + 7$ **b** $50 + 19$ **c** $42 + 31$
**d** $146 + 213$ **e** $15 + 19 + 23$ **f** $\begin{array}{r} 123 \\ +\ 39 \\ \hline \end{array}$

**3** Complete these subtractions.

**a** $12 - 8$ **b** $50 - 28$ **c** $47 - 29$
**d** $12 - 6 - 6$ **e** $784 - 163$ **f** $\begin{array}{r} 336 \\ -\,289 \\ \hline \end{array}$

**4** Complete these multiplications.

**a** $9 \times 4$ **b** $5 \times 8$ **c** $12 \times 11$
**d** $15 \times 5$ **e** $\begin{array}{r} 121 \\ \times\ \ 9 \\ \hline \end{array}$ **f** $\begin{array}{r} 338 \\ \times\ 14 \\ \hline \end{array}$

**5** Complete these divisions.

**a** $28 \div 4$ **b** $99 \div 3$ **c** $18 \div 6$
**d** $72 \div 12$ **e** $3\overline{)453}$ **f** $7\overline{)364}$

**6** **a** List the first 5 multiples of 6.
**b** List the first 4 multiples of 9.
**c** What is the lowest common multiple (LCM) of 6 and 9?

**7** **a** List all the factors of 12.
**b** List all the factors of 15.
**c** What is the highest common factor (HCF) of 12 and 15?

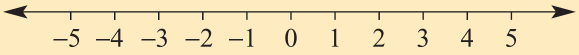
**8** Prime numbers have exactly two factors. Copy these numbers into your workbook and circle the prime numbers. The first prime is circled for you.

1 ② 3 4 5 6 7 8 9 10 11 12 13 14 15

**9** Answer the following as true or false.

**a** $2 + 3 \times 4 = 2 + 12$ **b** $10 - 8 \div 2 = 10 - 4$
**c** $(5 - 2) \times 7 = 3 \times 7$ **d** $9 \times 3 + 5 = 9 \times 8$
**e** $9 \times (3 + 5) = 9 \times 8$ **f** $12 \div 3 \times 4 = 1$

**10** Use this number line to help find the answer.



**a** $2 - 5$ **b** $0 - 3$ **c** $-4 + 6$ **d** $-2 + 7$

# A1A Whole number addition and subtraction

REVISION

The number system that we use today is called the Hindu–Arabic or decimal system. It uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. The value of each digit depends on its place in the number, so, for example, the 4 in 3407 has a place value of 400. Whole numbers include 0 (zero) and the counting (natural) numbers 1, 2, 3, 4, …

## Let's start: Sum and difference

Use a guess-and-check method to try to find a pair of numbers described by these sentences.

- The sum of two numbers is 41 and their difference is 11.
- The sum of two numbers is 41 and their difference is 1.

Describe the meaning of the words 'sum' and 'difference'. Discuss how you found the pair of numbers in each case.

## Key ideas

- You can add in any order.
  e.g. $7 + 5 = 5 + 7$
  $9 + 3 + 1 = 9 + 1 + 3$
  – This is called the **commutative law** for addition.
- You cannot subtract in any order.
  e.g. $7 - 5 \neq 5 - 7$
- If the numbers are large, write addition and subtraction as algorithms as shown.

$$\begin{array}{r} 431 \\ +\ 165 \\ \hline 596 \end{array} \qquad \begin{array}{r} 394 \\ -\ 153 \\ \hline 241 \end{array}$$

**Commutative law**
When adding and multiplying, the order in which two numbers are combined does not matter

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## Exercise A1A

**Understanding**

**1** Match each part **a**–**d** to the working out **I**–**IV**.

**a** The total of 156, 94 and 6

**I** $\begin{array}{r} 2491 \\ +\ 945 \\ \hline \end{array}$

**b** Take 856 away from 2491

**II** 2491 – 856

**c** 945 more than 2491

**III** 156 + 94 + 6

**d** The difference between 945 and 863

**IV** $\begin{array}{r} 945 \\ -\ 863 \\ \hline \end{array}$

**2** Write each of the following as an addition (+) or as a subtraction (–).

**a** 26 plus 17

**b** 43 take away 9

**c** 134 minus 23

**d** 451 add 50

**e** The sum of 19 and 29

**f** The sum of 111 and 236

**g** The difference between 59 and 43

**h** The difference between 339 and 298

**i** 36 more than 8

**j** 142 more than 421

**k** 32 less than 49

**l** 120 less than 251

**3** Copy and complete.

**a**

| + | 2 | 5 | 7 | 10 | 12 |
|---|---|---|---|---|---|
| 5 | | | | | |
| 0 | | | | | |
| 18 | | | | | |
| 58 | | | | | |

**b**

| + | 3 | 9 | | |
|---|---|---|---|---|
| 15 | | | | 30 |
| | | 10 | | |
| 6 | | | 24 | |
| 2 | | | | |

**4** Are these statements true or false?

**a** 15 + 6 = 6 + 15

**b** 29 – 6 = 6 – 29

**c** 95 + 0 = 95

**d** 81 – 81 = 0

**e** 15 + 6 + 4 = 15 + 10

**f** 41 – 6 + 4 = 41 – 10

**Fluency**

### Example 1 Using mental arithmetic

Evaluate this difference and these sums mentally.

**a** 347 – 39 **b** 125 + 127 **c** 28 + 13

| **Solution** | **Explanation** | |
|---|---|---|
| **a** 347 – 39 = 308 | 347 – 39 = 347 – 40 + 1<br>= 307 + 1<br>= 308 | This method is called compensating.<br>e.g. 134 + 29 = 134 + 30 – 1 |

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| Solution | Explanation | |
|---|---|---|
| **b** $125 + 127 = 252$ | $125 + 127 = 2 \times 125 + 2$<br>$= 250 + 2$<br>$= 252$ | This method is called doubling.<br>e.g. $127 = 125 + 2$ |
| **c** $28 + 13 = 41$ | $28 + 13 = 28 + 12 + 1$<br>$= 40 + 1$<br>$= 41$ | This method is called counting on.<br>e.g. $28 + 13 = 28 + 12 + 1$ |

**5** Complete these additions.

**a** $21 + 5$
**b** $3 + 14$
**c** $17 + 13$
**d** $298 + 2$
**e** $35 + 11$
**f** $16 + 19$
**g** $21 + 5$
**h** $6 + 18$

Do these without a calculator or algorithm.

**6** Complete these subtractions.

**a** $5 - 2$
**b** $16 - 4$
**c** $16 - 14$
**d** $21 - 21$
**e** $16 - 3$
**f** $45 - 13$
**g** $52 - 12$
**h** $52 - 14$

**7** Evaluate these sums and differences.

**a** $94 - 62$
**b** $146 + 241$
**c** $1494 - 351$
**d** $36 + 19$
**e** $138 + 25$
**f** $251 - 35$
**g** $99 - 20$
**h** $441 - 50$
**i** $350 + 351$
**j** $115 + 114$
**k** $80 - 41$
**l** $320 - 159$

## Example 2 Using an algorithm

Use an algorithm to find this sum and difference.

**a**
$$\begin{array}{r} 938 \\ +\,217 \\ \hline \end{array}$$

**b**
$$\begin{array}{r} 141 \\ -\;\;86 \\ \hline \end{array}$$

| Solution | Explanation |
|---|---|
| **a** $\begin{array}{r} 9^{1}38 \\ +\,217 \\ \hline 1155 \end{array}$ | $8 + 7 = 15$ (trade the 1 to the tens column)<br>$1 + 3 + 1 = 5$<br>$9 + 2 = 11$ |
| **b** $\begin{array}{r} 1\,{}^{3}\cancel{4}\,{}^{1}1 \\ -\;\;8\;6 \\ \hline 5\;5 \end{array}$ | Traded from the tens column then subtract 6 from 11.<br>Now trade from the hundreds column and then subtract 8 from 13. |

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**8** Use an algorithm to find these sums and differences. Check with a calculator.

Trade the 1 for totals larger than 9 and trade 'ten' for subtraction.

**a**
$$\begin{array}{r} 128 \\ +\ 46 \\ \hline \end{array}$$

**b**
$$\begin{array}{r} 94 \\ +\ 337 \\ \hline \end{array}$$

**c**
$$\begin{array}{r} 9014 \\ +\ 927 \\ +\ 421 \\ \hline \end{array}$$

**d**
$$\begin{array}{r} 814 \\ +\ 1439 \\ +\ 326 \\ \hline \end{array}$$

**e**
$$\begin{array}{r} 94 \\ -\ 36 \\ \hline \end{array}$$

**f**
$$\begin{array}{r} 421 \\ -\ 201 \\ \hline \end{array}$$

**g**
$$\begin{array}{r} 1726 \\ -\ 1699 \\ \hline \end{array}$$

**h**
$$\begin{array}{r} 14072 \\ -\ 328 \\ \hline \end{array}$$

**i**
$$\begin{array}{r} 428 \\ +\ 314 \\ +\ 107 \\ +\ 29 \\ \hline \end{array}$$

**j**
$$\begin{array}{r} 1004 \\ +\ 2407 \\ +\ 9116 \\ +\ 10494 \\ \hline \end{array}$$

**k**
$$\begin{array}{r} 3017 \\ -\ 2942 \\ \hline \end{array}$$

**l**
$$\begin{array}{r} 10024 \\ -\ 936 \\ \hline \end{array}$$

## Problem-solving and Reasoning

**9** A racing bike's odometer shows 21 432 km at the start of a race and 22 110 km at the end of the race. How far was the race? Check with a calculator.

Casey Stoner racing at the Malaysian Grand Prix

**10** Kristian has $246 more than Sally. David has $56 less than Sally. If Sally has $492, how much do Kristian and David have? Check with a calculator.

**11** Callum walks 15 km on Monday and 3 km more each day. How many kilometres does Callum walk on Thursday?

**12** The sum of two numbers is 39 and their difference is 5. What is the larger number?

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## Enrichment: Magic triangles and tricky additions and subtractions

**13 a** Write the digit missing from these sums and differences.

**i**
$$\begin{array}{r} 2\ 3\ 7 \\ +\ \ 4\ \square \\ \hline 2\ 7\ 9 \end{array}$$

**ii**
$$\begin{array}{r} 4\ 9 \\ +3\ 8 \\ \hline 8\ \square \end{array}$$

**iii**
$$\begin{array}{r} 4\ 9\ 3 \\ +2\ 1\ 4 \\ \hline 7\ \square\ 7 \end{array}$$

**iv**
$$\begin{array}{r} 1\ \square\ 4 \\ +3\ 9\ 2 \\ \hline 5\ 5\ 6 \end{array}$$

**v**
$$\begin{array}{r} 3\ 8 \\ -1\ 9 \\ \hline 1\ \square \end{array}$$

**vi**
$$\begin{array}{r} 1\ 2\ 8 \\ -\ \ 8\ \square \\ \hline 3\ 9 \end{array}$$

**vii**
$$\begin{array}{r} 3\ \square\ 4 \\ -1\ 6\ 2 \\ \hline 1\ 4\ 2 \end{array}$$

**viii**
$$\begin{array}{r} 2\ 5\ 1 \\ -1\ \square\ 4 \\ \hline 8\ 7 \end{array}$$

**b** Find the missing digits in these sums and differences.

**i**
$$\begin{array}{r} 2\ 3\ \square \\ +\square\ 9\ 4 \\ \hline 6\ \square\ 1 \end{array}$$

**ii**
$$\begin{array}{r} \square\ 3\ \square \\ +\ \ \square\ 2 \\ \hline 2\ 1\ 9 \end{array}$$

**iii**
$$\begin{array}{r} \square\ 3\ 7 \\ +4\ 9\ \square \\ \hline 7\ \square\ 2 \end{array}$$

**iv**
$$\begin{array}{r} \square\ 3 \\ -2\ 9 \\ \hline 6\ \square \end{array}$$

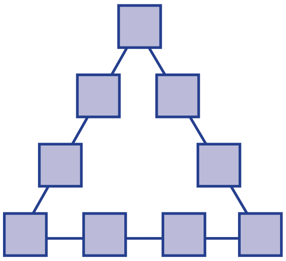
**v**
$$\begin{array}{r} 3\ \square\ 2 \\ -\square\ 3\ \square \\ \hline 1\ 0\ 4 \end{array}$$

**vi**
$$\begin{array}{r} 2\ \square\ \square\ 5 \\ -\ \ 6\ 8\ \square \\ \hline \square\ 3\ 1\ 8 \end{array}$$

**c** The sides of a magic triangle all sum to the same total.

**i** Show how it is possible to arrange all the digits from 1 to 9 so that each side adds to 17.

**ii** Show how it is possible to arrange the same digits to a different total. How many different totals can you find?



# A1B Whole number multiplication and division

REVISION

Multiplying and dividing numbers without a calculator is easy in many situations, such as finding the cost of 9 tickets at $109 each.

A typical large mining truck has a capacity of 140 tonnes.

## Let's start: Multiplication or division?

In solving many problems it is important to know whether multiplication or division should be used. Decide if the following situations require the use of multiplication or division. Discuss them in a group or with a partner.

- The number of cookies 4 people get if a packet of 32 cookies is shared equally between them.
- The cost of paving 30 square metres of courtyard at a cost of $41 per square metre.
- The number of sheets of paper in a shipment of 4000 boxes of 5 reams each (1 ream is 500 sheets).
- The number of hours I can afford a plumber at $75 per hour if I have a fixed budget of $1650.

Make up your own situation that requires the use of multiplication and another for division.

## Key ideas

**Product** The result of multiplication

**Quotient** The result of division

**Remainder** The leftover amount after one number has been divided by another

**Distributive law** Adding numbers *then* multiplying the total gives the same answer as multiplying each number first *then* adding the products

- Another word for multiplication is **product**.
- You need to know your multiplication tables.
- Multiplication can be done:
  - mentally
  - with an algorithm

e.g. $6 \times 5 = 30$

e.g.

$$\begin{array}{rl} 217 & \\ \times \quad 26 & \\ \hline 1302 & \leftarrow 217 \times 6 \\ 4340 & \leftarrow 217 \times 20 \\ \hline 5642 & \leftarrow 1302 + 4340 \end{array}$$

- You can multiply numbers in any order.
  e.g. $6 \times 5 = 30$ and $5 \times 6 = 30$
  - This is the commutative law for multiplication.

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- Using division results in finding a **quotient** and a **remainder**.

e.g. $38 \div 11 = 3$ and 5 remainder $= 3\frac{5}{11}$

dividend divisor quotient

$$\begin{array}{r} 7\,3\,2 \\ 7\overline{)5\,1^{2}2^{1}4} \end{array}$$

- The **distributive law** is helpful when multiplying.

e.g. $5 \times (20 + 3) = 5 \times 20 + 5 \times 3$

## Exercise A1B

**Understanding**

**1** Match each of parts **a**–**e** to the working **i**–**v**.

**a** The product of 9 and 6
**b** 36 divided by 12
**c** 15 lots of 12
**d** The quotient when 15 is divided by 5
**e** Divide 12 into 15

**i** $15 \times 12$
**ii** $15 \div 5$
**iii** $9 \times 6$
**iv** $15 \div 12$
**v** $36 \div 12$

**2** Copy and complete these multiplication grids.

**a**

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | | | | | |
| 4 | | | | | | | |
| 5 | | | | | | | |
| 6 | | | | | | | |
| 7 | | | | | | | |

**b**

| X | 2 | 5 | 7 | 9 |
|---|---|---|---|---|
| | 6 | | | |
| | | 20 | | |
| | | | 63 | |
| | | | | 90 |

**3** Use your knowledge of the multiplication table to answer the following.

**a** $5 \times 8$ **b** $11 \times 9$ **c** $6 \times 7$ **d** $9 \times 8$
**e** $11 \times 6$ **f** $12 \times 11$ **g** $8 \times 4$ **h** $7 \times 9$
**i** $100 \div 10$ **j** $88 \div 8$ **k** $121 \div 11$ **l** $144 \div 12$
**m** $56 \div 7$ **n** $33 \div 3$ **o** $65 \div 5$ **p** $78 \div 6$

You should know these off by heart.

**4** Are these simple equations true or false?

**a** $4 \times 13 = 13 \times 4$
**b** $2 \times 7 \times 9 = 7 \times 9 \times 2$
**c** $6 \div 3 = 3 \div 6$
**d** $60 \div 20 = 30 \div 10$
**e** $14 \div 2 \div 7 = 7 \div 2 \div 14$
**f** $51 \times 7 = (50 \times 7) + (1 \times 7)$
**g** $79 \times 13 = (80 \times 13) - (1 \times 13)$
**h** $93 \div 3 = (90 \div 3) + (3 \div 3)$

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**A1B**

**Fluency**

## Example 3 Using mental strategies for multiplication

Use a mental strategy to evaluate the following.

**a** $5 \times 160$ **b** $7 \times 89$ **c** $5 \times 43 \times 2$

| **Solution** | **Explanation** |
|---|---|
| **a** $5 \times 160 = 800$ | To multiply by $5$ you can multiply by $10$ then halve the result. $160 \times 10 = 1600$, $1600 \div 2 = 800$ |
| **b** $7 \times 89 = 623$ | $89 = 90 - 1 \therefore 7 \times 89 = 7 \times 90 - 7 \times 1 = 630 - 7 = 623$ (this is the distributive law) |
| **c** $5 \times 43 \times 2 = 430$ | $5 \times 43 \times 2 = \underbrace{5 \times 2}_{\text{look for easy pairs}} \times 43$<br>$= 10 \times 43$<br>$= 430$ |

**5** Use a mental strategy to evaluate the following.

**a** $15 \times 3$ **b** $18 \times 4$ **c** $6 \times 5 \times 2$ **d** $7 \times 20$
**e** $16 \times 4$ **f** $99 \times 7$ **g** $79 \times 3$ **h** $42 \times 5$
**i** $5 \times 13 \times 2$ **j** $2 \times 26 \times 5$ **k** $4 \times 35$ **l** $17 \times 4$
**m** $17 \times 1000$ **n** $136 \times 100$ **o** $59 \times 7$ **p** $119 \times 6$
**q** $9 \times 51$ **r** $6 \times 61$ **s** $4 \times 252$ **t** $998 \times 6$

Do these mentally.

## Example 4 Using mental strategies for division

Use a mental strategy to evaluate the following

**a** $464 \div 4$ **b** $480 \div 5 \div 2$

| **Solution** | **Explanation** |
|---|---|
| **a** $464 \div 4 = 116$ | To divide by $4$ you can divide by $2$ twice.<br>$464 \div 4 = 464 \div 2 \div 2$ ($\div\ 2$ is the same as halving the number)<br>$= 232 \div 2$<br>$= 116$ |
| **b** $480 \div 5 \div 2 = 48$ | Dividing by $5$ and then by $2$ is the same as dividing by $10$.<br>$480 \div 10 = 48$ |

**6** Use a mental strategy to evaluate the following.

**a** $64 \div 2$ **b** $64 \div 4$ **c** $640 \div 4$ **d** $492 \div 4$
**e** $185 \div 5$ **f** $1980 \div 5 \div 2$ **g** $128 \div 8$ **h** $252 \div 4$
**i** $123 \div 3$ **j** $508 \div 4$ **k** $96 \div 6$ **l** $1016 \div 8$

Choose one of the mental strategies described above.

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## Example 5 Using multiplication and division setting out

Use an algorithm to evaluate the following.

**a** $412 \times 25$ **b** $974 \div 3$

| **Solution** | **Explanation** |
|---|---|
| **a** $\begin{array}{r} 412 \\ \times\ 25 \\ \hline 2060 \\ 8240 \\ \hline 10300 \end{array}$ | $412 \times 5 = 2060$ and $412 \times 20 = 8240$<br>Add these two products to get the final answer. |
| **b** $\begin{array}{r} 324 \\ 3\overline{)97^{1}4} \end{array}$ Rem 2<br>so $974 \div 3 = 324\frac{2}{3}$ | $9 \div 3 = 3$<br>$7 \div 3 = 2$ and 1 remainder<br>Trade the 1 to the units<br>$14 \div 3 = 4$ and 2 remainder<br>write the remainder as a fraction. |

**7** Use setting out to evaluate the following. Check with a calculator.

**a** $\begin{array}{r} 67 \\ \times\ 9 \\ \hline \end{array}$ **b** $\begin{array}{r} 129 \\ \times\ 4 \\ \hline \end{array}$

**c** $\begin{array}{r} 294 \\ \times\ 13 \\ \hline \end{array}$ **d** $\begin{array}{r} 1004 \\ \times\ 90 \\ \hline \end{array}$

**e** $\begin{array}{r} 690 \\ \times\ 14 \\ \hline \end{array}$ **f** $\begin{array}{r} 96 \\ \times\ 12 \\ \hline \end{array}$

**g** $\begin{array}{r} 58 \\ \times\ 24 \\ \hline \end{array}$ **h** $\begin{array}{r} 163 \\ \times\ 52 \\ \hline \end{array}$

Use the setting out described in Example 5.

**8** Use the short division setting out to evaluate the following, with remainders as fractions check with a calculator.

**a** $3\overline{)85}$ **b** $7\overline{)214}$

**c** $3\overline{)1000}$ **d** $7\overline{)300}$

**e** $6\overline{)15084}$ **f** $3\overline{)1236}$

**g** $5\overline{)2703}$ **h** $2\overline{)10081}$

**Problem-solving and Reasoning**

**9** A university student earns $550 for 20 hours work. What is the student's pay rate per hour?

**10** Packets of biscuits are purchased by a supermarket in boxes of 12. The supermarket orders 220 boxes and sells 89 boxes in one day. How many boxes are left? How many packets of biscuits remain in the supermarket?

**11** Riley buys a fridge, which he can pay for by the following options.

**A** 9 payments of $183

**B** $1559 up front

Which option is cheaper and by how much?

**12** The shovel of a giant excavator can move 6 tonnes of rock in each load. How many loads are needed to shift 750 tonnes of rock?

**13** Tom saves $362 a week. How much will he save in 52 weeks?

A1B

## Enrichment: Maximum tickets

**14** A child's ticket is $7 and an adult's ticket is $12.

**a** Find the cost of 2 adult's and 3 children's tickets.

**b** Find the cost of 1 adult's and 5 children's tickets.

**c** Gen spends exactly $90 to buy children's tickets and adult's tickets. Find the maximum number of tickets that Gen could purchase.

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# A1C The order of operations

**REVISION**

When working with more than one operation, such as multiplication and addition, a particular order needs to be followed.

Let us look at the simple sum of $5 + 4 \times 5 = 25$.

If we did the addition first, then $5 + 4 \times 5 = 9 \times 5 = 45$, but we know that this is not true.

## Let's start: How many?

How many ways can you get $36 - 20 = 16$?

See if you can come up with at least five different statements using the four operations ($+ - \times \div$) and brackets that give the same subtraction above. One example is $9 \times 4 - (24 - 4)$.

## Key ideas

Order of operations

- Deal with the **grouping symbols** or brackets first.
- Do any multiplication ($\times$) and division ($\div$) next, working from left to right.
- Do any addition ($+$) and subtraction ($-$) next, again working from left to right.

NOTE: Within any brackets the order of operations still needs to be followed.

**Grouping symbols**
Parentheses ( ), brackets [ ] and braces { } are used to collect terms and operations together

## Exercise A1C

**Understanding**

**1** Copy each expression into your book. By following the order of operations, underline the operation that needs to be done first.

- **a** $2 + 3 \times 9$
- **b** $10 - 2 \div 2$
- **c** $1 \times 3 + 5$
- **d** $6 \times (9 - 6)$
- **e** $(12 + 6) \div 2$

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**2** Match each part **a–e** to the correct working **I–V**.

- **a** $10 + 7 \times 3$
- **b** $15 - 9 \div 3$
- **c** $(9 - 4) \times 6$
- **d** $(9 - 4) - (10 - 6)$
- **e** $18 \div 9 + 5 \times 2$

- **I** $10 + 21$
- **II** $5 - 4$
- **III** $15 - 3$
- **IV** $2 + 10$
- **V** $5 \times 6$

## Example 6 Two operations

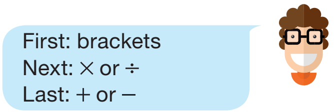
Find the answers to each of the following.

**a** $10 + 5 \times 3$  **b** $18 \div 6 \times 2$  **c** $15 - (7 - 3)$

| **Solution** | **Explanation** |
|---|---|
| **a** $10 + 5 \times 3 = 10 + 15$<br>$= 25$ | Multiplication (×) is done BEFORE addition (+).<br>$5 \times 3 = 15$ |
| **b** $18 \div 6 \times 2 = 3 \times 2$<br>$= 6$ | Division (÷) and multiplication (×) are done as they appear from left to right.<br>$18 \div 6$ is done first then $\times 2$ last. |
| **c** $15 - (7 - 3) = 15 - 4$<br>$= 11$ | Brackets need to be done first $(7 - 3) = 4$.<br>Then do the subtraction $15 - 4$. |

**3** Find the answers to each of the following.

- **a** $12 + 5 \times 2$
- **b** $24 - 6 \times 3$
- **c** $10 \times 2 + 6$
- **d** $15 \div 3 - 2$
- **e** $(9 - 2) \times 4$
- **f** $18 - (12 - 8)$
- **g** $28 \div (2 \times 7)$
- **h** $56 - 5 \times 10$
- **i** $120 + 200 \div 5$
- **j** $88 \times 2 \div 8$
- **k** $12 \div (18 \div 6)$
- **l** $16 - 18 \div 9$
- **m** $55 \div 11 \times 5$
- **n** $55 - 25 \div 5$
- **o** $240 \div 10 \times 2$
- **p** $58 + 100 \div 20$
- **q** $100 - 25 \div 5$
- **r** $(24 - 9) \times 3$



**4** Find the answer to the following by first writing the sentence using numbers and symbols.

- **a** Double the sum of 3 and 7
- **b** Double the quotient of 24 and 8
- **c** The product of 5 and 7 plus 4
- **d** 8 more than the product of 12 and 5
- **e** 10 less than the quotient of 66 and 3
- **f** Triple the difference between 18 and 12

## Example 7 Several steps

Find the answers to each of the following.

**a** $4 \times 5 - 3 \times 2$ **b** $(7 + 2) \times 5 - 6$ **c** $10 + (2 \times (6 - 4))$

| **Solution** | **Explanation** |
|---|---|
| **a** $4 \times 5 - 3 \times 2$<br>$= 20 - 6$<br>$= 14$ | Both sets of multiplication (×) need to be done first.<br>Then do the subtraction (–). |
| **b** $(7 + 2) \times 5 - 6$<br>$= 9 \times 5 - 6$<br>$= 45 - 6$<br>$= 39$ | Do the brackets first $(7 + 2)$.<br>Next do the multiplication $9 \times 5$.<br>Then the subtraction $45 - 6$. |
| **c** $10 + (2 \times (6 - 4))$<br>$= 10 + (2 \times 2)$<br>$= 10 + 4$<br>$= 14$ | Start with the inner most brackets $(6 - 4)$.<br>Finish working with the brackets – we follow the order of operations within the brackets $(2 \times 2)$.<br>Then the addition $10 + 4$. |

**Fluency**

**5** Find the answers to the following.

**a** $2 \times 4 - 4 \div 2$ **b** $13 + 4 \times 5 - 3$ **c** $(14 - 12) \times 4 + 11$

**d** $(12 - 5) \times (6 + 3)$ **e** $5 \times 6 + 12 \times 3$ **f** $25 - 20 \div 5 + 2$

**g** $25 - 20 \div 5 + 2 \times 5$ **h** $(10 + 10) \div (25 - 5)$ **i** $(10 \times 10 + 5) \div 5$

Show steps of working as in the examples.

**6** Simplify. Check with a calculator.

**a** $5 \times 4 + 8 \times 4$ **b** $24 \div 4 \times 6 - 8$ **c** $(15 - 5) \times 8 + 200$

**d** $6 \times 4 - 2 \times 6 + 12$ **e** $96 \div (12 \times 8)$ **f** $5 + (12 \times (23 - 6))$

**g** $1 + 4 + 3 \times (8 - 5)$ **h** $(12 - 5) \times (22 - 12)$ **i** $12 + (18 - (12 - 5))$

Skillsheet A1A

**7** Evaluate:

**a** $56 - 4 \times 6$ **b** $96 \div 4 + 3 \times 6$

**c** $150 - (7 \times (10 - 3 \times 2))$ **d** $(12 \times (13 - 8) \times (24 - 18))$

**Problem-solving and Reasoning**

**8** True or false?

**a** $5 + 9 = 5 + 3 \times 3$ **b** $10 + 2 \times 7 = 12 + 7$ **c** $18 - 6 + 5 = 12 + 5$

**d** $3 \times 5 \times 6 = 15 \times 6$ **e** $120 \div 6 \times 2 = 20 \times 2$ **f** $(5 + 3) \times 9 = 8 \times 9$

**9** Insert brackets into each of the following statements to make it true.

**a** $12 - 8 \times 2 = 8$ **b** $4 \times 5 + 6 = 44$ **c** $16 \div 2 \times 8 = 1$

**d** $6 \times 2 + 6 \times 1 = 48$ **e** $15 \times 4 - 2 = 30$ **f** $1 + 2 + 3 \times 4 = 24$

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**10** Insert operation symbols (+, –, × , ÷) between the numbers to make each of the following statements true.

**a** 5 ___ 4 ___ 9 = 0

**b** 5 ____ 4 ____ 9 = 11

**c** 5 ______ 4 _____ 9 = 41

**11** Write each of the following situations into mathematical symbols and numbers, and then calculate.

**a** Murray receives four dollars from his mum and seven dollars from his dad as pocket money each week for 12 weeks. How much money does he have at the end of the 12 weeks?

**b** A raffle prize consists of $5000 cash and 6 shopping vouchers each worth $500. What is the total value of the raffle prize?

**c** Sally has fifty dollars. She buys four pens at two dollars each and eight exercise books at three dollars each. How much change does Sally get?

**12** Decide if the brackets in each of the following are required or not.

**a** 10 + (9 × 8)

**b** 12 + (3 + 4)

**c** 12 – (3 + 4)

**d** 25 × (3 – 1)

**e** (100 – 4 × 3)

**f** 1 + (2 + 3) × 4 = 21

## Enrichment: Make ten from four

**13** Can you make the first 10 counting numbers (1, 2, 3, 4, 5, 6, 7, 8, 9 and 10) using only the four digits 1, 2, 3 and 4 (once each), brackets and any of the four operations?
For example 1 × 2 × 3 × 4 = 24 (but that is too big!)

# A1D Number properties

**REVISION**

Knowing the properties of numbers helps us with our problem-solving work.
A prime number, for example, has exactly two factors.

| Number | Factors | |
|---|---|---|
| 7 | 1, 7 | Prime |
| 9 | 1, 3, 9 | Composite |

## Let's start: How many?

Write down all the whole numbers from 1 to 20.

- Circle the multiples of 5.
- Highlight the factors of 20.
- Tick the prime numbers.

## Key ideas

- The **multiples** of a number are obtained by multiplying the number by the **counting numbers** 1, 2, 3, …
  e.g. Multiples of 9 include 9, 18, 27, 36, 45, … (think of your 9 times table)
- The **lowest common multiple** (LCM) is the smallest multiple of two or more numbers that is common.
  e.g. Multiples 3 are 3, 6, 9, 12, 15, 18, …
  e.g. Multiples of 5 are 5, 10, 15, 20, 25, …
  The LCM of 3 and 5 is therefore 15, because it is the lowest number that appears in both lists.
- A **factor** of a number has a remainder of zero when divided into the given number.
  e.g. 4 is a factor of 20 since 20 ÷ 4 = 5 with 0 remainder.
- The **highest common factor** (HCF) is the largest factor of two or more numbers that is common.
  - Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.
  - Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36.

  The HCF of 24 and 36 is therefore 12.
- **Prime numbers** have only two factors, the number itself and 1.
  - 2, 13 and 61 are examples of prime numbers.
- **Composite numbers** have more than two factors.
  - 6, 20 and 57 are examples of composite numbers.
- The number 1 is neither prime nor composite.

**Multiple** The multiple of a number is the product of that number and any other whole number

**Counting numbers** The set of whole numbers starting at 1

**Lowest common multiple** The smallest number that two or more numbers divide into without remainder

**Factor** A whole number that will divide into another number exactly

**Highest common factor** The largest number that is a factor of all the given factors

**Prime number** An integer greater than 1 that only has two factors, itself and 1

**Composite number** A number that has at least three factors

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## Exercise A1D

Understanding

**1** Write down the factors of each number.

**a** 4 **b** 6 **c** 12 **d** 15 **e** 20

**2** Write down the next term in each of these multiplication table results.

**a** 2, 4, 6, 8, __ **b** 3, 6, 9, 12, __ **c** 5, 10, 15, 20, 25, __
**d** 7, 14, 21, __ **e** 6, 12, 18, __ **f** 11, 22, 33, 44, __

**3**

| |
|---|
| The factors of 16 are 1, 2, 4, 8, 16. |
| The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24. |
| The factors of 18 are 1, 2, 3, 6, 9, 18. |
| The factors of 30 are 1, 2, 3, 5, 6, 10, 15, 30. |
| The factors of 8 are 1, 2, 4, 8. |

Using the information given in the table, write down the highest common factor (HCF) of each pair of numbers.

**a** 16 and 24 **b** 24 and 30 **c** 18 and 30 **d** 16 and 8
**e** 24 and 18 **f** 8 and 24 **g** 16 and 18 **h** 18 and 8

**4** Use the first six multiples of the numbers given to find the LCM of each pair of numbers.

| Number | Multiples |
|---|---|
| 2 | 2, 4, 6, 8, 10, 12 |
| 4 | 4, 8, 12, 16, 20, 24 |
| 3 | 3, 6, 9, 12, 15, 24 |
| 5 | 5, 10, 15, 20, 25, 30 |
| 6 | 6, 12, 18, 24, 30, 36 |

**a** 2 and 4 **b** 4 and 3 **c** 3 and 6
**d** 4 and 6 **e** 4 and 5 **f** 5 and 6

Fluency

### Example 8 Primes and composites

Decide whether each of the following is a prime number or a composite number.

**a** 29 **b** 117

| Solution | Explanation |
|---|---|
| **a** 29 is a prime number. | 29 has only 2 factors 1 and 29. It is a **prime** number. |
| **b** 117 is a composite number. | 117 has factors 1, 3, 9, 13, 39, 117 |

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**5** Decide whether each of the following numbers is prime or composite.

**a** 7 **b** 12 **c** 27 **d** 69
**e** 105 **f** 28 **g** 15 **h** 11
**i** 31 **j** 37 **k** 49 **l** 99

Primes have exactly two factors, composites have more than two factors.

**6** Copy the following list of the first 30 counting numbers and circle the prime numbers.
1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30.

## Example 9 Finding the LCM

Find the LCM of 6 and 8.

| Solution | Explanation |
|---|---|
| Multiples of 6 are:<br>6, 12, 18, ㉔, 30, … | First, list some multiples of 6 and 8.<br>Continue the lists until there is at least one in common. |
| Multiples of 8 are:<br>8, 16, ㉔, 32, 40, … | |
| The LCM is 24. | Choose the smallest number that is common to both lists. |

**7** Find the LCM of these pairs of numbers.

**a** 2, 3 **b** 5, 9 **c** 8, 12 **d** 4, 8
**e** 25, 50 **f** 4, 18 **g** 8, 60 **h** 12, 20
**i** 5, 7 **j** 10, 15 **k** 4, 12 **l** 12, 18

## Example 10 Finding the HCF

Find the HCF of 36 and 48.

| Solution | Explanation |
|---|---|
| Factors of 36 are:<br>1, 2, 3, 4, 6, 9, ⑫, 18, 36 | First, list factors of 36 and 48. |
| Factors of 48 are:<br>1, 2, 3, 4, 6, 8, ⑫, 16, 24, 48 | |
| The HCF is 12. | Choose the largest number that is common to both lists. |

**8** Find the HCF of these pairs of numbers.

**a** 6, 8 **b** 18, 9 **c** 16, 24 **d** 24, 30
**e** 7, 13 **f** 19, 31 **g** 72, 36 **h** 108, 64
**i** 6, 4 **j** 6, 12 **k** 8, 24 **l** 15, 25

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A1D

**Problem-solving and Reasoning**

**9** Find the:

**a** LCM of 8, 12 and 6
**b** LCM of 7, 3 and 5
**c** HCF of 20, 15 and 10
**d** HCF of 32, 60 and 48.

**10** A teacher has 64 students to divide into equal groups of 3 or more with no remainder. In how many ways can this be done?

**11** Three sets of traffic lights (A, B and C) all turn red at 9.00 am exactly. Light set A turns red every 2 minutes, light set B turns red every 3 minutes and light set C turns red every 5 minutes. How long does it take for all three lights to turn red again at the same time?

**12** Below are the numbers 1 to 100. Copy the grid and highlight all the prime numbers. How many numbers less than 100 are prime numbers?

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

## Enrichment: Goldbach's conjecture and twin primes

**13** Goldbach's conjecture is a famous mathematical statement that says that every even number greater than four can be written as the sum of two prime numbers.

On the next page, the even numbers 4, 6 and 8 have been written as the sum of two primes.

Show how the even numbers 10 to 30 can be written as the sum of two primes. Some can be done in more than one way.

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**4** = 2 + 2

**6** = 3 + 3

**8** = 3 + 5

**10** =

**12** =

**14** =

**16** =

**18** =

**20** =

**22** =

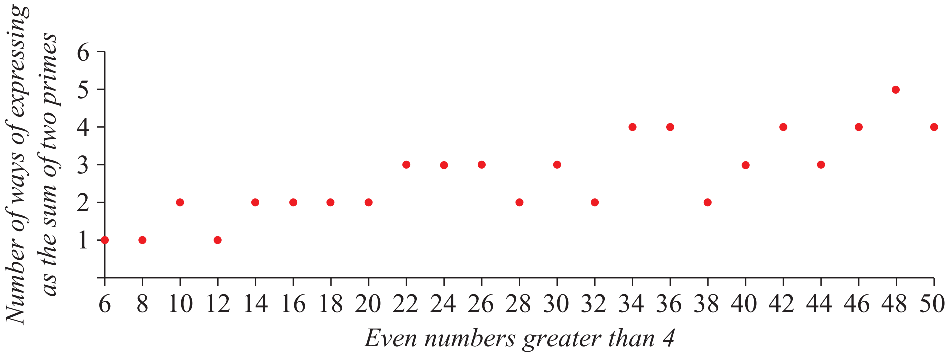
**24** =

**26** =

**28** =

**30** =

The first ten prime numbers.



A graph illustrating Goldbach's conjecture up to and including 50, is obtained by plotting the number of ways of expressing even numbers greater than 4 as the sum of two primes.

**14** Twin primes are pairs of prime numbers that differ by 2. It has been suggested that there are infinitely many twin primes. Use the table of primes you created in Question **12** of this exercise and list the pairs of twin primes less than 100.

# A1E Divisibility and prime factorisation

**REVISION**

Every composite number can be written as a product of prime numbers, e.g. $6 = 3 \times 2$ and $20 = 2 \times 2 \times 5$. Writing numbers as a product of prime numbers can help to simplify expressions and determine other properties of numbers or pairs of numbers.

$4 = 2 \times 2$
$6 = 2 \times 3$
$8 = 2 \times 2 \times 2$
$9 = 3 \times 3$
$10 = 2 \times 5$
$12 = 2 \times 2 \times 3$
$14 = 2 \times 7$
$15 = 3 \times 5$
$16 = 2 \times 2 \times 2 \times 2$

Composite numbers expressed as products of primes

## Let's start: Products of primes

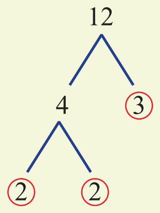
Continue the pattern in the table above to get a product of primes for 18, 20, 21, 22, 24, 25, 26, 27 and 28

## Key ideas

**Prime factorisation** Writing a number as a product of its prime factors

**Factor tree** An illustrated breakdown of a number into its prime factors

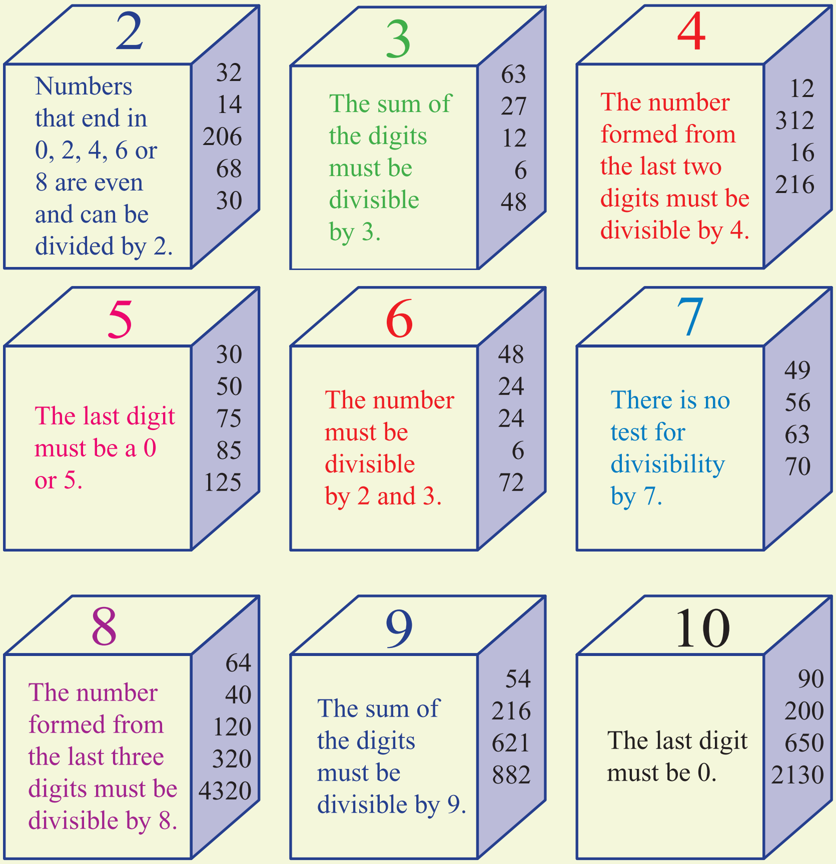
- **Prime factorisation** uses a **factor tree**, or similar, to write a number as a product of its prime factors.
  e.g. $12 = 2 \times 2 \times 3$ or $2^2 \times 3$ (using indices)
- A number is divisible by another number if there is no remainder after the division. For example, 84 is divisible by 4 because $84 \div 21 = 4$ exactly, with no remainder. That is, 4 is a factor of 84.



## ■ Divisibility tests

All numbers are divisible by 1.

**Divisibility test** A way to work out whether a whole number is divisible by another whole number, without actually doing the division



2
Numbers that end in 0, 2, 4, 6 or 8 are even and can be divided by 2.
32 14 206 68 30

3
The sum of the digits must be divisible by 3.
63 27 12 6 48

4
The number formed from the last two digits must be divisible by 4.
12 312 16 216

5
The last digit must be a 0 or 5.
30 50 75 85 125

6
The number must be divisible by 2 and 3.
48 24 24 6 72

7
There is no test for divisibility by 7.
49 56 63 70

8
The number formed from the last three digits must be divisible by 8.
64 40 120 320 4320

9
The sum of the digits must be divisible by 9.
54 216 621 882

10
The last digit must be 0.
90 200 650 2130

## Exercise A1E

**Understanding**

**1** Write down all the factors of these numbers.
**a** 15 **b** 24 **c** 40 **d** 84

**2** Write down the first 10 prime numbers. Note that 1 is not a prime number.

**3** Write using powers.
**a** $3 \times 3 \times 3 \times 3$ **b** $5 \times 5$ **c** $7 \times 7 \times 7 \times 7$
**d** $2 \times 2 \times 3 \times 3 \times 3$ **e** $2 \times 2 \times 5 \times 5$ **f** $2 \times 2 \times 3 \times 3 \times 5$

**4** Evaluate:
**a** $2^2 \times 3$ **b** $2 \times 3^2 \times 5$ **c** $2^3 \times 5 \times 7$ **d** $3^3 \times 7$

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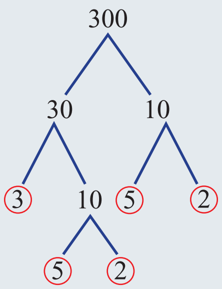
**Fluency**

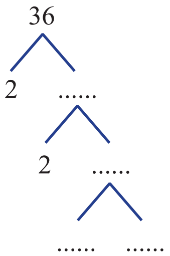
### Example 11 Finding prime factor form

Use a factor tree to write 300 as a product of prime factors.

| **Solution** | **Explanation** |
|---|---|
| <br>$300 = 2 \times 2 \times 3 \times 5 \times 5$<br>$= 2^2 \times 3 \times 5^2$ | First, divide 300 into the product of **any two** factors. Choose the easiest pair $300 = 30 \times 10$.<br>Continue dividing numbers into two factors until the factors are prime.<br>Circle the prime factors.<br>Write the factors in ascending order.<br>Use index notation (powers) to abbreviate your answer. |

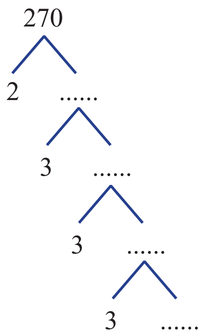
**5** Copy and complete these factor trees to help write the prime factor form of the given numbers.

**a**



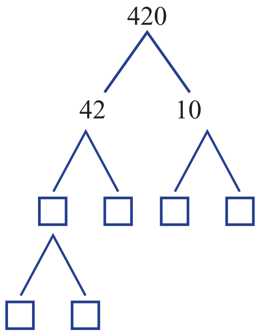
$\therefore\ 36 = 2^2 \times$ ......

**b**



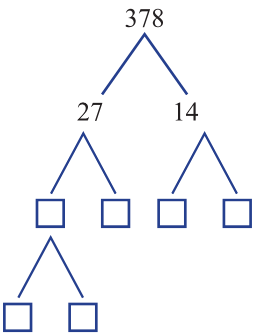
$\therefore\ 270 = 2 \times$ ...... $\times$ ......

**c**



420 = ____

**d**



378 = _______

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**6** Use a factor tree to find the prime factor form of these numbers.

**a** 20 **b** 28
**c** 40 **d** 90
**e** 280 **f** 196
**g** 360 **h** 600

### Example 12 Testing for divisibility

Use divisibility tests to decide if the number 627 is divisible by 2, 3, 4, 5, 6, 8 or 9.

| **Solution** | **Explanation** |
|---|---|
| Not divisible by 2 since 7 is odd. | The last digit needs to be even. |
| Divisible by 3 since 6 + 2 + 7 = 15 and this is divisible by 3. | The sum of all the digits needs to be divisible by 3. |
| Not divisible by 4 as 27 is not divisible by 4. | The number formed from the last two digits needs to be divisible by 4. |
| Not divisible by 5 as the last digit is not a 0 or 5. | The last digit needs to be a 0 or 5. |
| Not divisible by 6 as it is not divisible by 2. | The number needs to be divisible by both 2 and 3. |
| Not divisible by 8 as the last 3 digits together are not divisible by 8. | The number formed from the last three digits needs to be divisible by 8. |
| Not divisible by 9 as 6 + 2 + 7 = 15 is not divisible by 9. | The sum of all the digits needs to be divisible by 9. |

**7** Use divisibility tests to decide if these numbers are divisible by 2, 3, 4, 5, 6, 8 or 9.

**a** 51 **b** 126 **c** 248 **d** 387
**e** 315 **f** 517 **g** 894 **h** 3107


Do the seven tests on each number.

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**Problem-solving and Reasoning**

**8** What is the smallest number that can be divided, without giving a remainder, by all of the following four numbers?

**a** 2, 3, 4 and 6 **b** 2, 6, 8 and 9 **c** 2, 5, 15 and 6

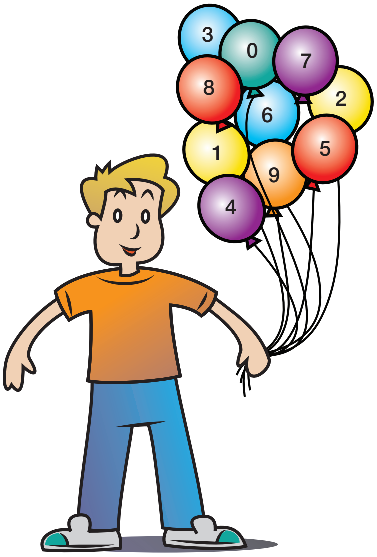
**9** Nana Magoo's two grandchildren love to visit her. Lachlan visits her every 8 days while Bryce visits every 18 days. They both visited her last Monday. How many days will it be before they both visit her on the same day again?

You might like to make a list to help you here!

## Enrichment: Find the missing digit

**10** Use the divisibility rules given to you at the start of this section to find the missing digit for each of the following.

**a** 2 ☐ 6 if the number is divisible by 3 (can you have more than one answer?)
**b** 1 ☐ 35 if the number is divisible by 9.
**c** 4 ☐ 3 if the number is divisible by 3.
**d** 4 ☐ 3 if the number is divisible by 3 and 9.
**e** 276 ☐ if the number is divisible by 2.
**f** 276 ☐ if the number is divisible by 2 and 5.

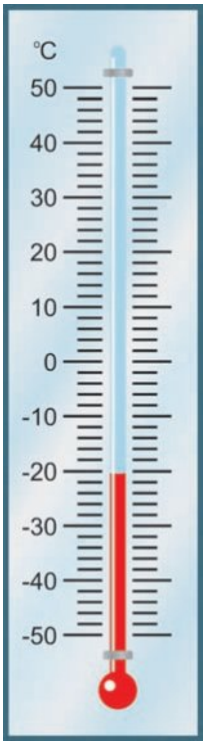


# A1F Negative numbers

REVISION

The Indian mathematician Brahmagupta set out rules for negative numbers in the 7th century.

Today, negative numbers are used in science, engineering and business. They help us describe opposites such as left and right, up and down, profit and loss, and temperatures above and below freezing.

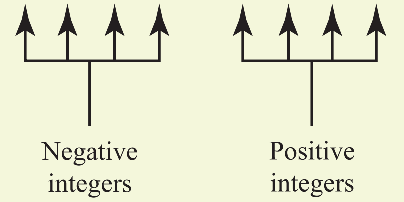


## Let's start: Opposites

Write down the opposite of:
- 6°C above zero
- a profit of $4200
- 150 m above sea level
- the number 6
- 5 – 3.

## Key ideas

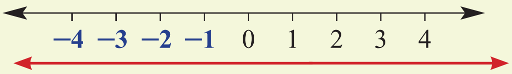
- **Negative numbers** are numbers *less than zero*.
- The **integers** are …, −4, −3, −2, −1, 0, 1, 2, 3, 4…



**Negative numbers** number less than 0

**Integers** The set of positive and negative whole numbers, including zero

These include positive integers (natural numbers), zero and negative integers. These are illustrated clearly on a number line.



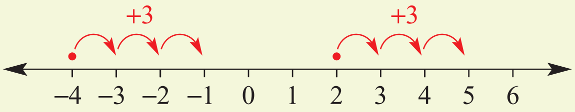
negative numbers

Values decreases as you move to the left along the number line

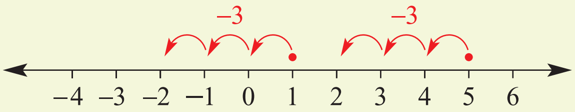
positive numbers

Values increase as you move to the right along the number line

- Adding or subtracting a positive integer can result in a positive or negative number.
  - Adding a positive integer
    e.g. $2 + 3 = 5$
    $-4 + 3 = -1$



  - Subtracting a positive integer
    e.g. $1 - 3 = -2$
    $5 - 3 = 2$



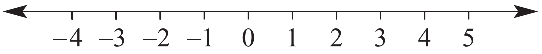
## Exercise A1F

**Understanding**

**1** Write down the number suggested by:

**a** 2 above zero
**b** 5 above zero
**c** 3 below zero
**d** 10 below zero
**e** 1 below zero.

**2** Copy the number line below and mark (with a dot) the integers –3, –1, 1, 3 and 5.



**3** Write the symbol < (less than) or > (greater than) to make these statements true.

**a** 5 ___ –1
**b** –3 ___ 4
**c** –10 ___ 3
**d** –1 ___ –2
**e** –20 ___ –24
**f** –62 ___ –51
**g** 2 ___ –99
**h** –61 ___ 62

**4** What is the final temperature?

**a** 10°C is reduced by 12°C.
**b** 32°C is reduced by 33°C.
**c** –11°C is increased by 2°C.
**d** –4°C is increased by 7°C.

**Fluency**

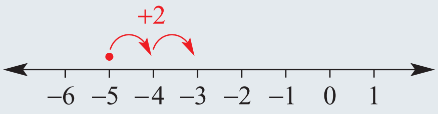
## Example 13 Adding a positive integer

Evaluate the following.

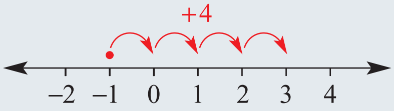
**a** $-5 + 2$ **b** $-1 + 4$

**Solution** **Explanation**

**a** $-5 + 2 = -3$



**b** $-1 + 4 = 3$



**5** Evaluate the following.

**a** $-1 + 2$ **b** $-3 + 7$ **c** $-10 + 11$ **d** $-4 + 12$
**e** $-20 + 35$ **f** $-6 + 4$ **g** $-7 + 2$ **h** $-15 + 8$
**i** $-26 + 19$ **j** $-38 + 24$ **k** $-10 + 15$ **l** $-2 + 9$
**m** $-7 + 3$ **n** $-7 + 7$ **o** $-6 + 9$ **p** $-6 + 1$

Start with the first number and move right on the number line.

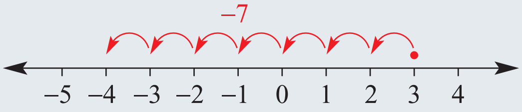
## Example 14 Subtracting a positive integer

Evaluate the following.

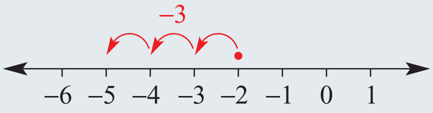
**a** $3 - 7$ **b** $-2 - 3$

**Solution** **Explanation**

**a** $3 - 7 = -4$



**b** $-2 - 3 = -5$



**6** Evaluate the following.

**a** $4 - 5$ **b** $10 - 15$ **c** $0 - 26$ **d** $14 - 31$
**e** $6 - 8$ **f** $10 - 9$ **g** $-4 - 7$ **h** $-11 - 20$
**i** $-14 - 15$ **j** $-10 - 100$ **k** $-11 - 6$ **l** $0 - 12$
**m** $-15 - 5$ **n** $3 - 12$ **o** $8 - 4$ **p** $-8 - 4$

Start with the first number and move left on the number line.

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**7** Evaluate the following.

| | | | |
|---|---|---|---|
| **a** −9 + 6 | **b** −9 − 6 | **c** −12 + 12 | **d** −12 − 12 |
| **e** −7 − 7 | **f** −7 + 0 | **g** 15 − 14 | **h** 15 − 16 |
| **i** −9 − 10 | **j** −9 + 10 | **k** 9 − 15 | **l** −20 + 10 |
| **m** 100 − 101 | **n** −50 − 50 | **o** −5 + 25 | **p** −9 + 40 |

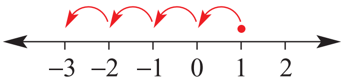
**8** Work from left to right to evaluate the following.

| | |
|---|---|
| **a** −3 + 4 − 8 + 6 | **b** 0 − 10 + 19 − 1 |
| **c** 26 − 38 + 14 − 9 | **d** 9 − 18 + 61 − 53 |

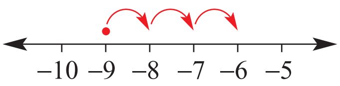
Problem-solving and Reasoning

**9** Write the sum (e.g. −3 + 4 = 1) or difference (e.g. 1 − 5 = −4) to match these number lines.

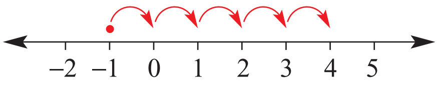
**a**



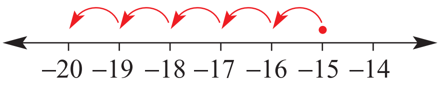
**b**



**c**



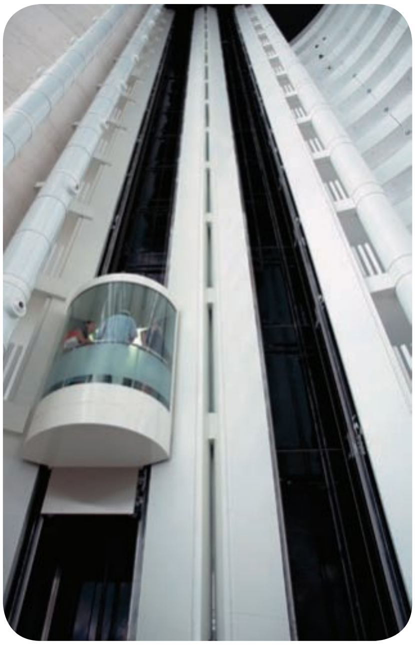
**d**



**10** Write the missing number.

| | |
|---|---|
| **a** −1 + ___ = 5 | **b** ___ + 30 = 26 |
| **c** ___ + 11 = −3 | **d** −32 + ___ = −21 |
| **e** 5 − ___ = −10 | **f** ___ − 17 = −12 |
| **g** ___ − 4 = −7 | **h** −26 − ___ = −38 |

**11** In a high-rise building there are 8 floors above ground level and 6 floors below ground level. A lift starts at the 2nd floor and moves 4 floors up, then 7 floors down before moving down a further 3 floors.
At what floor does the lift finish?



**12** On Monday Milly borrows $35 from a friend. On Tuesday she pays her friend $40. On Friday she borrows $42 and pays back $30 that night. How much does Milly owe her friend then?

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## Enrichment: Earning and spending

**13 a** Complete Suzanne's account for the week shown.
A credit is an addition (+) and a debit is a subtraction (–).

| Spending and earning | Credit (+) | Debit (–) | Balance |
|---|---|---|---|
| opening balance | | | \$500 |
| pays 1 weeks rent of \$375 | | 375 | |
| earns \$80 baby sitting | | | |
| receives \$100 from her parents for her birthday | | | |
| buys a pair of jeans for \$90 | | | |
| buys a top for \$45 | | | |
| pays her monthly mobile phone bill \$49 | | | |
| gives \$25 to charity | | | |

**b** How much would Suzanne need to deposit (credit) into her account so that she can pay the rent for the next week?

**14** Find what positive integer needs to be added or subtracted to each so that the end result is zero.

**a** $-6$ ____ $= 0$

**b** $-8$ ____ $= 0$

**c** $16$ ____ $= 0$

**d** $10 - 7$ ____ $= 0$

**e** $-9 + 7$ ____ $= 0$

**f** $-9 - 7 - 2$ ____ $= 0$

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# A1G Addition and subtraction of integers

**REVISION**

In the diagram to the right:
- the positive numbers add to +6
- the negative numbers add to –6
- all the numbers add to 0.

In this section we will investigate what happens when a negative number is added to (or subtracted from) a number.

| | |
|---|---|
| +2 | –2 |
| +2 | –2 |
| +2 | –2 |

The sum is 0

## Let’s start: Looking at patterns for adding and subtracting negative numbers

Copy and complete.

**a**

| | | |
|---|---|---|
| 6 + 4 | 10 | |
| 6 + 3 | 9 | |
| 6 + 2 | 8 | |
| 6 + 1 | | |
| 6 + 0 | | |
| 6 + (–1) | | → same as 6 – 1 = 5 |
| 6 + (–2) | | → same as 6 ☐ 2 = |
| 6 + (–3) | | → same as 6 ☐ 3 = |
| 6 + (–4) | | → same as 6 ☐ 4 = |

**b**

| | | |
|---|---|---|
| 6 – 4 | 2 | |
| 6 – 3 | 3 | |
| 6 – 2 | 4 | |
| 6 – 1 | | |
| 6 – 0 | | |
| 6 – (–1) | | → same as 6 + 1 = |
| 6 – (–2) | | → same as ☐ |
| 6 – (–3) | | → same as ☐ |
| 6 – (–4) | | → same as ☐ |

## Key ideas

- When adding a negative number:
  $12 \oplus (\ominus 3)$ ‘12 plus negative 3’ is the same as
  $= 12 \ominus 3$ ‘12 minus 3’
  $= 9$
- When subtracting a negative number:
  $12 \ominus (\ominus 3)$ ‘12 minus negative 3’ is the same as
  $= 12 \oplus 3$ ‘12 plus 3’
  $= 15$

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## Exercise A1G

Understanding

**1** −3 and 3 are opposites. Write down the opposites of these numbers.

**a** $-6$ **b** $10$ **c** $38$ **d** $-46$

**e** $-32$ **f** $88$ **g** $673$ **h** $-349$

**2** Write the words 'add' or 'subtract' to suit each sentence.

**a** To add a negative number ____________ its opposite.

**b** To subtract a negative number ____________ its opposite.

**3** Are the following statements true or false?

**a** $5 + (-2) = 5 + 2$ **b** $3 + (-4) = 3 - 4$ **c** $-6 + (-4) = -6 - 4$

**d** $-1 + (-3) = 1 - 3$ **e** $8 - (-3) = 8 + 3$ **f** $2 - (-3) = 2 - 3$

**g** $-3 - (-1) = 3 + 1$ **h** $-7 - (-5) = -7 + 5$ **i** $-6 - (-3) = 6 + 3$

**4** Rewrite each of the following with only a (+) or (−) between the two numbers.

**a** $7 + (-3)$ **b** $10 + (-5)$ **c** $8 - (-1)$

**d** $6 - (-8)$ **e** $15 + (-20)$ **f** $-3 - (-4)$

**g** $-9 - (-9)$ **h** $0 + (-5)$ **i** $18 + (-18)$

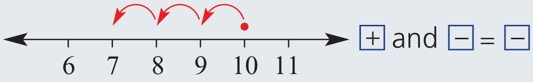
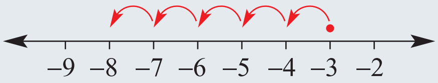
$6 + (-9) = 6 - 9$
$3 - (-7) = 3 + 7$

Fluency

### Example 15 Adding negative numbers

Evaluate the following.

**a** $10 + (-3)$ **b** $-3 + (-5)$

| **Solution** | **Explanation** |
|---|---|
| **a** $10 + (-3) = 10 - 3$<br>$= 7$ | Adding −3 is the same as subtracting 3.<br><br> <br>$\boxed{+}$ and $\boxed{-} = \boxed{-}$ |
| **b** $-3 + (-5) = -3 - 5$<br>$= -8$ | Adding −5 is the same as subtracting 5.<br><br> <br>$\boxed{+}$ and $\boxed{-} = \boxed{-}$ |

**5** Evaluate the following.

**a** $6 + (-2)$ **b** $4 + (-1)$ **c** $7 + (-12)$ **d** $20 + (-5)$

**e** $2 + (-4)$ **f** $26 + (-40)$ **g** $-3 + (-6)$ **h** $-16 + (-5)$

**i** $-18 + (-20)$ **j** $-36 + (-50)$ **k** $-83 + (-22)$ **l** $-120 + (-10)$

**m** $7 + (-8)$ **n** $-9 + (-12)$ **o** $6 + (-12)$ **p** $-6 + (-12)$

**q** $-8 + (-8)$ **r** $5 + (-5)$ **s** $-70 + (-15)$ **t** $-100 + (-6)$

To add a negative, subtract its opposite.

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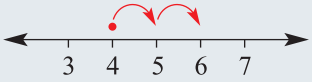
### Example 16 Subtracting negative numbers

Evaluate the following.

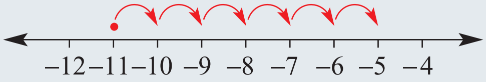
**a** $4 - (-2)$  **b** $-11 - (-6)$

| **Solution** | **Explanation** |
|---|---|
| **a** $4 - (-2) = 4 + 2$<br>$= 6$ | Subtracting –2 is the same as adding 2. |



$\boxed{-}$ and $\boxed{-} = \boxed{+}$

| | |
|---|---|
| **b** $-11 - (-6) = -11 + 6$<br>$= -5$ | Subtracting –6 is the same as adding 6. |



$\boxed{-}$ and $\boxed{-} = \boxed{+}$

**6** Evaluate the following.

**a** $2 - (-3)$
**b** $4 - (-4)$
**c** $15 - (-6)$
**d** $24 - (-14)$
**e** $59 - (-13)$
**f** $147 - (-320)$
**g** $-5 - (-3)$
**h** $-8 - (-10)$
**i** $-13 - (-16)$
**j** $-10 - (-42)$
**k** $-88 - (-31)$
**l** $-125 - (-5)$
**m** $60 - (-5)$
**n** $-60 - (-5)$
**o** $-12 - (-12)$
**p** $-10 - (-18)$
**q** $41 - (-41)$
**r** $48 - (-52)$
**s** $-46 - (-8)$
**t** $-170 - (-12)$

**7** Simplify:

**a** $46 - 50$
**b** $46 + (-50)$
**c** $9 - 12$
**d** $9 + (-12)$
**e** $-8 + 6$
**f** $-8 - (-6)$
**g** $81 - 15$
**h** $81 + (-15)$
**i** $7 + (-7)$

**Problem-solving and Reasoning**

**8** Write down the missing number.

**a** 4 + ___ = 1
**b** 6 + ___ = 0
**c** –2 + ___ = –1
**d** ___ + (–8) = 2
**e** ___ + (–5) = –3
**f** ___ + (–3) = –17
**g** 12 – ___ = 14
**h** 8 – ___ = 12
**i** –1 – ___ = 29
**j** ___ – (–7) = 2
**k** ___ – (–2) = –4
**l** ___ – (–436) = 501

**9** An ice cube is removed from a freezer at –25°C and placed into a glass of juice at 7°C. What is the difference in the two temperatures?

**10** Kelvin owes the bank $450 000. What must he deposit into his account to only owe $270 000?

**11** What must be added or subtracted to each of the following to obtain an answer of zero?

**a** $-6 + \square = 0$  **b** $7 - \square = 0$  **c** $-18 - \square = 0$

**12** If $a = -5$ and $b = -3$. Find the value of:

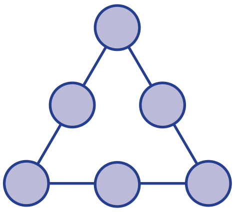
**a** $a + (-3)$
**b** $a - (-2)$
**c** $b - (-4)$
**d** $a + b$
**e** $a - b$
**f** $b - a$

Replace the pronumeral in the statement with the number it represents.
e.g. $a = -2$
then $a + (-5)$
$= -2 + (-5)$
$= -2 - 5$
$= -7$

## Enrichment: Puzzles with negatives

**13** Place the integers from $-3$ to $2$ in this magic triangle so that each side adds to the given number.

**a** $-3$
**b** $0$



**14** A magic square has each row, column and main diagonal adding to the same magic sum. Complete these magic squares.

**a**

| | | |
|---|---|---|
| | | 1 |
| 0 | –2 | –4 |
| | | |

**b**

| | | |
|---|---|---|
| –12 | | |
| | –15 | |
| | –11 | –18 |

# A1H Multiplication and division of integers REVISION

Multiplication is repeated addition.
$2 + 2 + 2 + 2$ is '4 lots of 2'
So $2 + 2 + 2 + 2$ is equal to $4 \times 2$, which is 8.

Similarly,
$(-2) + (-2) + (-2) + (-2)$ is equal to $4 \times (-2)$, which is $-8$.
Therefore (positive) × (negative) gives (negative).

Multiplication is reversible, so
$4 \times (-2) = (-2) \times 4 = -8$
Therefore (negative) × (positive) also gives (negative).

At the moment, the sum of the cards in the diagram is 0.
If 4 of the –2 cards are removed, that could be written as:

$0 - 4 \times -2$, which is the same as $-4 \times -2$.
$= 0 - \ -8$
$= 0 + 8$
$= 8$

Therefore (negative) × (negative) gives (positive).

| | |
|---|---|
| +2 | –2 |
| +2 | –2 |
| +2 | –2 |
| +2 | –2 |

These cards show 4 'lots of' +2 and 4 'lots of' –2

## Let's start: Patterns in times tables

Use the pattern to fill in the blanks.

| The 3 times table | The –3 times table |
|---|---|
| $3 \times 4 =$ | $-3 \times 4 =$ |
| $3 \times 3 = 9$ | $-3 \times 3 =$ |
| $3 \times 2 = 6$ | $-3 \times 2 = -6$ |
| $3 \times 1 = 3$ | $-3 \times 1 = -3$ |
| $3 \times 0 = 0$ | $-3 \times 0 = \ 0$ |
| $3 \times -1 = -3$ | $-3 \times -1 =$ |
| $3 \times -2 =$ | $-3 \times -2 =$ |
| $3 \times -3 =$ | $-3 \times -3 =$ |
| $3 \times -4 =$ | $-3 \times -4 =$ |
| $3 \times -5 =$ | $-3 \times -5 =$ |

## Key ideas

In the table above, the:
- yellow zone shows that **positive** × **positive** = **positive**
- green zone shows that **positive** × **negative** = **negative**
- blue zone shows that **negative** × **positive** = **negative**
- orange zone shows that **negative** × **negative** = **positive**

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In the yellow and orange zones:

**If two numbers have the same sign, their product is positive**

For example, 4 × 3 =12 and (–4) × (–3) = 12

In the green and the blue zones:

**If two numbers have opposite signs, their product is negative**

For example, 4 × (–3) = –12 and (–4) × 3 = –12

The same rules apply for division:

**positive ÷ positive = positive**
**positive ÷ negative = negative**
**negative ÷ positive = negative**
**negative ÷ negative = positive**

## Exercise A1H

**Understanding**

**1** Write the missing numbers in these tables. You should create a pattern in the third column.

**a**

| $x$ | $y$ | $x \times y$ |
|---|---|---|
| 3 | 5 | 15 |
| 2 | 5 | |
| 1 | 5 | |
| 0 | 5 | |
| –1 | 5 | |
| –2 | 5 | |
| –3 | 5 | |

**b**

| $x$ | $y$ | $x \times y$ |
|---|---|---|
| 3 | –5 | –15 |
| 2 | –5 | –10 |
| 1 | –5 | |
| 0 | –5 | |
| –1 | –5 | |
| –2 | –5 | |
| –3 | –5 | |

**2** Write the missing numbers in these sentences. Use the tables in question **1** to help.

**a** 3 × 5 = ___ so 15 ÷ 5 = ___
**b** –3 × 5 = ___ so –15 ÷ 5 = ___
**c** 3 × (–5) = ___ so 15 ÷ (–5) = ___
**d** –3 × (–5) = ___ so 15 ÷ (–5) = ___

**3** Without finding these products decide if the answer would be positive or negative.

**a** 109 × 4
**b** –76 × 5
**c** 15 × (–9)
**d** –6 × (–13)
**e** 89 × 104
**f** –74 × 8
**g** –94 × (–5)
**h** 80 × (–7)
**i** –37 × –3

**4** Without finding these quotients decide if the answer would be positive or negative.

**a** 16 ÷ 2
**b** 24 ÷ (–3)
**c** 78 ÷ (–2)
**d** –56 ÷ 2
**e** –81 ÷ 9
**f** –99 ÷ (–11)

**Fluency**

### Example 17 Finding products

Evaluate the following.

**a** 3 × (–7)
**b** –4 × (–12)

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| **Solution** | **Explanation** |
|---|---|
| **a** $3 \times (-7) = -21$ | The product of two numbers of opposite sign is negative.<br>(pos.) × (neg.) = (neg.) |
| **b** $-4 \times (-12) = 48$ | $-4$ and $-12$ are both negative and so the product will be positive.<br>(neg.) × (neg.) = (pos.) |

**5** Evaluate the following.

- **a** $4 \times (-5)$
- **b** $6 \times (-9)$
- **c** $-4 \times 10$
- **d** $-11 \times 9$
- **e** $-2 \times (-3)$
- **f** $-6 \times 7$
- **g** $-9 \times 8$
- **h** $-11 \times (-9)$
- **i** $20 \times (-2)$
- **j** $-16 \times 4$
- **k** $-5 \times (-7)$
- **l** $8 \times (-4)$
- **m** $-10 \times (-6)$
- **n** $44 \times (-1)$
- **o** $-9 \times (-1)$
- **p** $-5 \times 12$

## Example 18 Finding quotients

Evaluate the following.

**a** $-63 \div 7$

**b** $-121 \div (-11)$

| **Solution** | **Explanation** |
|---|---|
| **a** $-63 \div 7 = -9$ | The two numbers are of opposite sign so the answer will be negative. (neg.) ÷ (pos.) = (neg.) |
| **b** $-121 \div (-11) = 11$ | $-121$ and $-11$ are both negative so the quotient will be positive. (neg.) ÷ (neg.) = (pos.) |

**6** Evaluate the following.

- **a** $-10 \div 2$
- **b** $-38 \div 19$
- **c** $-60 \div 15$
- **d** $-120 \div 4$
- **e** $32 \div (-16)$
- **f** $-6 \div 2$
- **g** $6 \div (-2)$
- **h** $-6 \div (-2)$
- **i** $-12 \div 6$
- **j** $-24 \div (-3)$
- **k** $-45 \div 5$
- **l** $-45 \div (-9)$
- **m** $-66 \div (-6)$
- **n** $-5 \div (-5)$
- **o** $-8 \div 1$
- **p** $-8 \div (-1)$

## Example 19 Order of operations

**a** $-7 + 6 \times (-5)$

**b** $-4 \times 6 \div (-2)$

| **Solution** | **Explanation** |
|---|---|
| **a** $-7 + 6 \times (-5)$<br>$= -7 + (-30)$<br>$= -7 - 30$<br>$= -37$ | Multiplication first<br>$6 \times (-5) = -30$<br>Lastly addition of a negative = subtraction<br>$-7 + (-30) = -7 - 30$ |
| **b** $-4 \times 6 \div (-2)$<br>$= -24 \div (-2)$<br>$= 12$ | Do multiplication and division from left to right<br>$-4 \times 6 = -24$<br>$-24 \div -2 = 12$ |

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**7** Follow the order of operations to find the following.

**a** $10 + (-6) \times 5$ **b** $15 - 3 \times (-2)$
**c** $18 \times (-2) \div 3$ **d** $-9 \times 2 + (-5)$
**e** $45 - 50 \div (-10)$ **f** $9 - 6 \times 3$
**g** $-10 \div (-2) \times (-3)$ **h** $9 \times 3 - 6 \times (-2)$
**i** $18 \div (-3) + 3 \times (-4)$ **j** $-9 \times (-2) + (-10)$

**8** If $(-2)^2 = -2 \times -2 = 4$, find the value of the following.

**a** $(-5)^2$ **b** $(-6)^2$ **c** $(-7)^2$
**d** $(-8)^2$ **e** $(-9)^2$ **f** $(-10)^2$

**9** Write the missing number.

**a** $\_\_\_ \times 3 = -9$ **b** $\_\_\_ \times (-7) = 35$ **c** $\_\_\_ \times (-4) = -28$
**d** $-3 \times \_\_\_ = -18$ **e** $-19 \times \_\_\_ = 57$ **f** $\_\_\_ \div (-9) = 8$
**g** $\_\_\_ \div 6 = -42$ **h** $85 \div \_\_\_ = -17$ **i** $-150 \div \_\_\_ = 5$

**10** Will $(-2)^3$ be positive or negative?

**11** Insert a × sign and/or ÷ sign to make these equations true.

**a** $-2 \_\_\_ 3 \_\_\_ (-6) = 1$ **b** $10 \_\_\_ (-5) \_\_\_ (-2) = 25$
**c** $6 \_\_\_ (-6) \_\_\_ 20 = -20$ **d** $-14 \_\_\_ (-7) \_\_\_ (-2) = -1$

**12** The product of two numbers is –24 and their sum is –5. What are the two numbers?

## Enrichment: Further substitution with integers using brackets

**13** Evaluate these expressions using $a = -2$ and $b = 1$.

**a** $a + b$ **b** $a - b$ **c** $2a - b$ **d** $b - a$
**e** $a - 4b$ **f** $3b - 2a$ **g** $b \times (2 + a)$ **h** $(2b + a) - (b - 2a)$

**14** Evaluate these expressions using $a = -3$ and $b = 5$.

**a** $ab$ **b** $ba$ **c** $a + b$ **d** $a - b$
**e** $b - a$ **f** $3a + 2b$ **g** $(a + b) \times (-2)$ **h** $(a + b) - (a - b)$

**15** Evaluate these expressions using $a = -3$ and $b = 5$.

**a** $a + b^2$ **b** $a^2 - b$ **c** $b^2 - a$ **d** $b^3 + a$
**e** $a^3 - b$ **f** $a^2 - b^2$ **g** $b^3 - a^3$ **h** $(b - a^2)^2$

**16** Evaluate these expressions using $a = -4$ and $b = -3$.

**a** $3a + b$ **b** $b - 2a$ **c** $4b - 7a$
**d** $-2a - 2b$ **e** $4 + a - 3b$ **f** $ab - 4a$
**g** $-2 \times (a - 2b) + 3$ **h** $ab - ba$ **i** $3a + 4b + ab$
**j** $a^2 - b$ **k** $a^2 - b^2$ **l** $b^3 - a^3$

**17** Insert brackets in these statements to make them true.

**a** $-2 + 1 \times 3 = -3$ **b** $-10 \div 3 - (-2) = -2$
**c** $-8 \div (-1) + 5 = -2$ **d** $-1 - 4 \times 2 + (-3) = 5$
**e** $-4 + (-2) \div 10 + (-7) = -2$ **f** $20 + 2 - 8 \times (-3) = 38$
**g** $1 - (-7) \times 3 \times 2 = 44$ **h** $4 + (-5) \div 5 \times (-2) = -6$

# Puzzles and games

**1** Hey, do you know what a wisecracker is?

**A** –6 – 4
**E** 8 – 10
**O** 6 – (–4)
**R** –8 – (–2)
**M** –6 – 7 – 4
**C** 46 + (–6) – 8
**I** –4 + 7 – 10
**S** 20 – 7
**V** 12 + (–3) – 6
**Y** –17 – 6
**K** 16 – (–6)
**T** –13 – 7 – 6 + 8

Complete the sums above to unlock the puzzle code.

| | | | | | |
|---|---|---|---|---|---|
| –10 | | 3 | –2 | –6 | –23 |
| 13 | –17 | –10 | –6 | –18 | |
| 32 | 10 | 10 | 22 | –7 | –2 |

**2** What explosive event was in the year 1000 AD?
Answer the following directed number multiplications and divisions to work out the puzzle code. Write your answer on another sheet of paper.

**K** –3 × 4
**A** –1 × 6
**C** 100 ÷ –5
**L** –8 × –6
**W** 40 ÷ 8 × –2
**H** 4 × –4
**R** $(-10)^2$
**E** 0 × –5
**M** –16 ÷ –8
**I** 24 ÷ 8
**N** 8 ÷ –4
**S** $\frac{36}{6}$
**U** –9 × –7
**G** $\frac{10}{-2}$
**D** –2 × 2
**O** –12 + 5
**P** $(-4)^2$
**V** –5 × –4
**F** $(-3)^2$
**T** –3 × –2 × –4

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| –20 | –16 | 3 | –2 | –6 | | –4 | 0 | 20 | 0 | 48 | –7 | 16 | 6 |
| –5 | 63 | –2 | 16 | –7 | –10 | –4 | 0 | 100 | | –24 | –16 | 3 | 6 |
| 48 | 0 | –6 | –4 | 6 | | –24 | –7 | | –24 | –16 | 0 | | |
| 2 | –6 | –2 | 63 | 9 | –6 | –20 | –24 | 63 | 100 | 0 | | | |
| –7 | 9 | | 9 | 3 | 100 | 0 | –10 | –7 | 100 | –12 | 6 | | |

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# Chapter summary

## Computation with integers

### Whole numbers
0, 1, 2, 3, ...

**Addition and subtraction**

$$\begin{array}{r} 2\,{}^{1}4\,7 \\ +\,1\,0\,8 \\ \hline 3\,5\,5 \end{array} \qquad \begin{array}{r} {}^{8}\cancel{9}\,\overset{13}{\cancel{4}}\,2^{1} \\ -\,3\,6\,8 \\ \hline 5\,7\,4 \end{array}$$

**Multiplication and division**

$$\begin{array}{rl} 167 & \\ \times\,15 & \\ \hline 835 & 5 \times 167 \\ 1670 & 10 \times 167 \\ \hline 2505 & 835 + 1670 \end{array} \qquad \begin{array}{r} 5\,9 \\ 7\overline{)4\,1\,{}^{6}6} \end{array}$$

$\therefore 416 \div 7 = 59\frac{3}{7}$

**Mental strategies**

- $156 + 79 = 156 + 80 - 1 = 235$
- $45 + 47 = 45 + 45 + 2 = 92$
- $3 \times 22 = 3 \times 20 + 3 \times 2 = 66$
- $4 \times 88 = 2 \times 176 = 352$
- $164 \div 4 = 82 \div 2 = 41$
- $297 \div 3 = (300 \div 3) - (3 \div 3) = 99$

**Addition and subtraction**

$4 - (-3) = 4 + 3$
$4 + (-3) = 4 - 3$

**Multiplication**

(pos.) × (pos.) = (pos.)
(neg.) × (neg.) = (pos.)

(pos.) × (neg.) = (neg.)
(neg.) × (pos.) = (neg.)

The same rules apply for division.

**Substitution**

$a = -2,\ b = 5,\ c = -4$

$$\begin{aligned} 2c - ab &= 2 \times (-4) - (-2) \times 5 \\ &= -8 - (-10) \\ &= 2 \end{aligned}$$

Use brackets with negatives.

**Order of operations**

- Brackets, × and ÷ then + and –

$$\begin{aligned} &(3 + 5) \div 2 \times 5 \\ &= 8 \div 2 \times 5 \\ &= 4 \times 5 \\ &= 20 \end{aligned}$$

### Properties

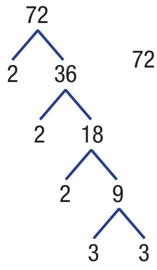
**Primes (2 factors)**

2, 3, 5, 7, 11, 13 ...

**Composite**

more than
2 Factors
4, 6, 8, 12

**Prime factorisation**



$72 = 2^3 \times 3^2$

**Lowest common multiple**

**LCM**

Multiples of 3: 3, 6, 9, **12** ...
Multiples of 4: 4, 8, **12** ...
∴ LCM = **12**

**Highest common factor**

**HCF**

Factors of 8: 1, 2, **4**, 8
Factors of 28: 1, 2, **4**, 7, 14, 28
∴ HCF = **4**

**Divisibility**

- 2 Even number.
- 3 Sum of digits divisible by 3.
- 4 Number from last 2 digits divisible by 4.
- 5 Last digit 0 or 5.
- 6 Divisible by 2 and 3.
- 8 Number from last 3 digits divisible by 8.
- 9 Sum of digits divisible by 9.
- 10 Last digit 0.

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# Chapter review

**Additional consolidation and review material, including literacy activities, worksheets and a chapter test, can be downloaded from *Cambridge GO*.**

## Multiple-choice questions

**1** $400 \div 5 \times 2$ is the same as:
**A** $400 \div 10$ **B** $80 \times 2$ **C** $16$ **D** $400 \div 2 \times 5$

**2** The sum and difference of 97 and 49 are:
**A** 146 and 58 **B** 246 and 48 **C** 136 and 58 **D** 146 and 48

**3** 561 is divisible by:
**A** 5 **B** 2 **C** 3 **D** 9

**4** $89 \times 5$ is the same as:
**A** $90 \times 4$ **B** $90 \times 5 - 1 \times 5$ **C** $89 \times 10 \times 2$ **D** $178 \times 10$

**5** $2 \times 2 \times 2 \times 2 \times 5 \times 5$ is:
**A** $2^4 \times 5^2$ **B** $2 \times 4 + 5 \times 2$ **C** $2^4 + 5^2$ **D** $10^7$

**6** $156 \div 4$ is the same as:
**A** $156 \div 2 \times 2$ **B** $156 \div 2 \div 2$ **C** $312 \div 2$ **D** $156 \times 2 \div 2$

**7** $-24 + 6 \times (-3)$ is equal to:
**A** $6$ **B** $42$ **C** $-42$ **D** $-6$

**8** $-6 + (-4)$ is the same as:
**A** $-6 - 4$ **B** $-6 + 4$ **C** $-4 + 6$ **D** $6 + 4$

**9** What is the smallest number that can be added to 1923 to make the answer divisible by 9?
**A** 1 **B** 2 **C** 3 **D** 4

**10** $(-15)^2$ equals:
**A** $225$ **B** $30$ **C** $-30$ **D** $-225$

## Short-answer questions

**1** Use a mental strategy to evaluate the following.
**a** $324 + 173$
**b** $592 - 180$
**c** $89 + 40$
**d** $135 - 68$
**e** $55 + 57$
**g** $1001 + 998$
**f** $280 - 141$
**h** $10\,000 - 4325$

**2** Use a mental strategy to find these sums and differences.

**a** $\begin{array}{r} 392 \\ +\ 147 \\ \hline \end{array}$ **b** $\begin{array}{r} 1031 \\ +\ 999 \\ \hline \end{array}$ **c** $\begin{array}{r} 147 \\ -86 \\ \hline \end{array}$ **d** $\begin{array}{r} 3970 \\ -\ 896 \\ \hline \end{array}$

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**3** Use a mental strategy for these products and quotients.
**a** $2 \times 17 \times 5$ **b** $3 \times 99$
**c** $8 \times 42$ **d** $141 \times 3$
**e** $164 \div 4$ **f** $357 \div 3$
**g** $618 \div 6$ **h** $1005 \div 5$

**4** Find these products and quotients using setting out.
**a** $\begin{array}{r} 139 \\ \times\ 12 \\ \hline \end{array}$ **b** $\begin{array}{r} 507 \\ \times\ 42 \\ \hline \end{array}$ **c** $3\overline{)843}$ **d** $7\overline{)854}$

**5** Find the remainder when 673 is divided by these numbers.
**a** 5 **b** 3 **c** 7 **d** 9

**6** **a** Find all the factors of 60.
**b** Find all the multiples of 7 between 110 and 150.
**c** Find all the prime numbers between 30 and 60.
**d** Find the LCM of 8 and 6.
**e** Find the HCF of 24 and 30.

**7** Write these numbers in prime factor form. You may wish to use a factor tree.
**a** 36 **b** 84 **c** 198

**8** Use divisibility tests to decide if these numbers are divisible by 2, 3, 4, 5, 6, 8 or 9.
**a** 84 **b** 155 **c** 124 **d** 621

**9** Evaluate:
**a** $-6 + 9$ **b** $-24 + 19$
**c** $5 - 13$ **d** $-7 - 24$
**e** $-62 - 14$ **f** $-194 - 136$
**g** $-111 + 110$ **h** $-328 + 426$

**10** Evaluate:
**a** $5 + (-3)$ **b** $-2 + (-6)$
**c** $-29 + (-35)$ **d** $162 + (-201)$
**e** $10 - (-6)$ **f** $-20 - (-32)$
**g** $-39 - (-19)$ **h** $37 - (-55)$

**11** Evaluate:
**a** $-5 \times 2$ **b** $-11 \times (-8)$
**c** $9 \times (-7)$ **d** $-100 \times (-2)$
**e** $-10 \div (-5)$ **f** $48 \div (-16)$
**g** $-32 \div 8$ **h** $-81 \div (-27)$

**12** Evaluate the following using the order of operations.
**a** $2 + 3 \times (-2)$ **b** $-3 \div (11 + (-8))$
**c** $-2 \times 3 + 10 \div (-5)$ **d** $-20 \div 10 - 4 \times (-7)$

**13** Let $a = -2$, $b = 3$ and $c = -5$ and evaluate these expressions.
**a** $ab + c$ **b** $a^2 - b$ **c** $ac - b$ **d** $a + b + c$

## Extended-response questions

**1** A monthly bank account show deposits as positive numbers and purchases and withdrawals (P + W) as negative numbers.

| Details | P + W | Deposits | Balance |
|---|---|---|---|
| Opening balance | – | – | \$250 |
| Water bill | –\$138 | – | *a* |
| Cash withdrawal | –\$320 | – | *b* |
| Deposit | – | *c* | \$115 |
| Supermarket | *d* | – | –\$160 |
| Deposit | – | 400 | *e* |

**a** Find the values of $a$, $b$, $c$, $d$ and $e$.
**b** If the water bill amount was \$150, what would be the new value for letter $e$?
**c** What would the final deposit need to be if the value for $e$ was \$0? Assume the original water bill amount is \$138 as in the table above.

**2** The weather for a November day is given for different cities around the world.

| | Minimum (°C) | Maximum (°C) |
|---|---|---|
| Amsterdam | 3 | 12 |
| Auckland | 11 | 18 |
| LA | 8 | 14 |
| Hong Kong | 16 | 28 |
| Moscow | 6 | 8 |
| Beijing | –3 | 0 |
| New York | 8 | 10 |
| Paris | 6 | 13 |
| Tel Aviv | 16 | 23 |
| Wollongong | 18 | 22 |

**a** Which city recorded the highest temperature on the day shown in the table?
**b** Which two cities only had a two-degree difference in temperature between minimum and maximum temperature?
**c** Which city had the largest difference in temperature on this November day?
**d** What is the difference in the minimum temperatures of Beijing and Auckland?

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# Appendix 1

## Pre-test

**1** **a** + **b** × **c** +
**d** – **e** + **f** ÷
**g** + **h** + **i** ÷

**2** **a** 19 **b** 69 **c** 73
**d** 359 **e** 57 **f** 162

**3** **a** 4 **b** 22 **c** 18
**d** 0 **e** 621 **f** 47

**4** **a** 36 **b** 40 **c** 132
**d** 75 **e** 1089 **f** 4732

**5** **a** 7 **b** 33 **c** 3
**d** 6 **e** 151 **f** 52

**6** **a** 6, 12, 18, 24, 30 **b** 9, 18, 27, 36 **c** 18

**7** **a** 1, 2, 3, 4, 6, 12 **b** 1, 3, 5, 15 **c** 3

**8** 2, 3, 5, 7, 11, 13

**9** **a** true **b** true **c** true
**d** false **e** true **f** false

**10** **a** –3 **b** –3 **c** 2 **d** 5

## Exercise A1A

**1** **a** III **b** II **c** I **d** IV

**2** **a** 26 + 17 **b** 43 – 9 **c** 134 – 23
**d** 451 + 50 **e** 19 + 29 **f** 111 + 236
**g** 59 – 43 **h** 339 – 298 **i** 8 + 36
**j** 421 + 142 **k** 49 – 32 **l** 251 – 120

**3** **a**

| + | 2 | 5 | 7 | 10 | 12 |
|---|---|---|---|---|---|
| 5 | 7 | 10 | 12 | 15 | 17 |
| 0 | 2 | 5 | 7 | 10 | 12 |
| 18 | 20 | 23 | 25 | 28 | 30 |
| 58 | 60 | 63 | 65 | 68 | 70 |

**b**

| + | 3 | 9 | 18 | 15 |
|---|---|---|---|---|
| 15 | 18 | 24 | 33 | 30 |
| 1 | 4 | 10 | 19 | 16 |
| 6 | 9 | 15 | 24 | 21 |
| 2 | 5 | 11 | 20 | 17 |

**4** **a** true **b** false **c** true
**d** true **e** true **f** false

**5** **a** 26 **b** 17 **c** 30 **d** 300
**e** 46 **f** 35 **g** 26 **h** 24

**6** **a** 3 **b** 12 **c** 2 **d** 0
**e** 13 **f** 32 **g** 40 **h** 38

**7** **a** 32 **b** 387 **c** 1143 **d** 55
**e** 163 **f** 216 **g** 79 **h** 391
**i** 701 **j** 229 **k** 39 **l** 161

**8** **a** 174 **b** 431 **c** 10 362 **d** 2579
**e** 58 **f** 220 **g** 27 **h** 13 744
**i** 878 **j** 23 021 **k** 75 **l** 9088

**9** 678 km

**10** David has $436 and Kristian has $738

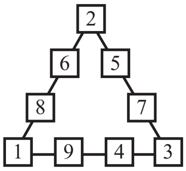
**11** 24 km

**12** 22

**13** **a** **i** 2 **ii** 7 **iii** 0 **iv** 6
**v** 9 **vi** 9 **vii** 0 **viii** 6

**b** Answers given from top row down and from left to right.
**i** 7, 3, 3 **ii** 1, 7, 8 **iii** 2, 5, 3
**iv** 9, 4 **v** 4, 2, 8 **vi** 0, 0, 7, 1

**c** **i**



**ii** 5 totals, 17, 19, 20, 21 and 23

## Exercise A1B

**1** **a** III **b** V **c** I **d** II **e** IV

**2** **a**

| × | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 |

**b**

| × | 2 | 5 | 7 | 9 |
|---|---|---|---|---|
| 3 | 6 | 15 | 21 | 27 |
| 4 | 8 | 20 | 28 | 36 |
| 9 | 18 | 45 | 63 | 81 |
| 10 | 20 | 50 | 70 | 90 |

**3** **a** 40 **b** 99 **c** 42 **d** 72
**e** 66 **f** 132 **g** 32 **h** 63
**i** 10 **j** 11 **k** 11 **l** 12
**m** 8 **n** 11 **o** 13 **p** 13

**4** **a** true **b** true **c** false **d** true
**e** false **f** true **g** true **h** true

**5** **a** 45 **b** 72 **c** 60 **d** 140
**e** 64 **f** 693 **g** 237 **h** 210

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**i** 130 **j** 260 **k** 140 **l** 68
**m** 17 000 **n** 13 600 **o** 413 **p** 714
**q** 459 **r** 366 **s** 1008 **t** 5988

**6 a** 32 **b** 16 **c** 160 **d** 123
**e** 37 **f** 198 **g** 16 **h** 63
**i** 41 **j** 127 **k** 16 **l** 127

**7 a** 603 **b** 516 **c** 3822 **d** 90 360
**e** 9660 **f** 1152 **g** 1392 **h** 8476

**8 a** $28\frac{1}{3}$ **b** $30\frac{4}{7}$ **c** $333\frac{1}{3}$ **d** $42\frac{6}{7}$
**e** 2514 **f** 412 **g** $540\frac{3}{5}$ **h** $5040\frac{1}{2}$

**9** \$27.50 an hour

**10** 131 boxes; 1572 packets

**11** Option B by \$88

**12** 125 loads

**13** \$18 824

**14 a** \$45 **b** \$47
**c** 4 adults and 6 kids = 10 tickets

## Exercise A1C

**1 a** $3 \times 9$ **b** $2 \div 2$ **c** $1 \times 3$
**d** $(9 - 6)$ **e** $(12 + 6)$

**2 a** I **b** III **c** V **d** II **e** IV

**3 a** 22 **b** 6 **c** 26 **d** 3
**e** 28 **f** 14 **g** 2 **h** 6
**i** 160 **j** 22 **k** 4 **l** 14
**m** 25 **n** 50 **o** 48 **p** 63
**q** 95 **r** 45

**4 a** $2 \times (3 + 7) = 20$ **b** $2 \times (24 \div 8) = 6$
**c** $5 \times 7 + 4 = 39$ **d** $12 \times 5 + 8 = 68$
**e** $66 \div 3 - 10 = 12$ **f** $3 \times (18 - 12) = 18$

**5 a** 6 **b** 30 **c** 19
**d** 63 **e** 66 **f** 23
**g** 31 **h** 1 **i** 21

**6 a** 52 **b** 28 **c** 280
**d** 24 **e** 1 **f** 209
**g** 14 **h** 70 **i** 23

**7 a** 32 **b** 42 **c** 122 **d** 360

**8 a** true **b** false **c** true **d** true
**e** true **f** true

**9 a** $(12 - 8) \times 2$ **b** $4 \times (5 + 6)$
**c** $16 \div (2 \times 8)$ **d** $6 \times (2 + 6) \times 1$
**e** $15 \times (4 - 2)$ **f** $(1 + 2 + 3) \times 1 = 24$

**10 a** $5 + 4 - 9$ **b** $5 \times 4 - 9$ **c** $5 + 4 \times 9$

**11 a** $(4 + 7) \times 12 = \$132$
**b** $5000 + 6 \times 500 = \$8000$
**c** $50 - (4 \times 2 + 8 \times 3) = \$18$

**12 a** no **b** no **c** yes
**d** yes **e** no **f** yes

**13** Some suggestions include:

| | |
|---|---|
| 1 | $(3 + 2 - 4) \div 1$ |
| 2 | $1 + 2 + 3 - 4$ |
| 3 | $(1 + 2 \times 3) - 4$ |
| 4 | $(1 + 2) \div 3 \times 4$ |
| 5 | $(1 + 2) \div 3 + 4$ |
| 6 | Change of order gives $4 \times 3 \div 2 \times 1$ |
| 7 | $(4 + 3) \times (2 - 1)$ |
| 8 | $(1 + 3) \times 4 \div 2$ |
| 9 | $1 \times 2 + 3 + 4$ |
| 10 | $1 + 2 + 3 + 4$ |

## Exercise A1D

**1 a** 1, 2, 4 **b** 1, 2, 3, 6 **c** 1, 2, 3, 4, 6, 12
**d** 1, 3, 5, 15 **e** 1, 2, 4, 5, 10, 20

**2 a** 10 **b** 15 **c** 30
**d** 28 **e** 24 **f** 55

**3 a** 8 **b** 6 **c** 6 **d** 8
**e** 6 **f** 8 **g** 2 **h** 2

**4 a** 4 **b** 12 **c** 6
**d** 12 **e** 20 **f** 30

**5 a** prime (P) **b** composite (C) **c** C
**d** C **e** C **f** C
**g** C **h** P **i** P
**j** P **k** C **l** C

**6** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

**7 a** 6 **b** 45 **c** 24 **d** 8
**e** 50 **f** 36 **g** 120 **h** 60
**i** 35 **j** 30 **k** 12 **l** 36

**8 a** 2 **b** 9 **c** 8 **d** 6
**e** 1 **f** 1 **g** 36 **h** 4
**i** 2 **j** 6 **k** 8 **l** 5

**9 a** 24 **b** 105 **c** 5 **d** 4

**10** 4 ways

**11** 30 minutes

**12** 25: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

**13** $10 = 5 + 5$ $12 = 5 + 7$ $14 = 7 + 7$
$16 = 5 + 11$ $18 = 5 + 13$ $20 = 3 + 17$
$22 = 5 + 17$ $24 = 5 + 19$ $26 = 7 + 19$
$28 = 5 + 23$ $30 = 7 + 23$

**14** (3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73)

## Exercise A1E

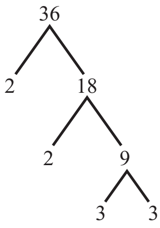
**1 a** 1, 3, 5, 15 **b** 1, 2, 3, 4, 6, 8, 12, 24
**c** 1, 2, 4, 5, 8, 10, 20, 40
**d** 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

**2** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

**3** **a** $3^4$ **b** $5^2$ **c** $7^4$
**d** $2^2 \times 3^3$ **e** $2^2 \times 5^2$ **f** $2^2 \times 3^2 \times 5$

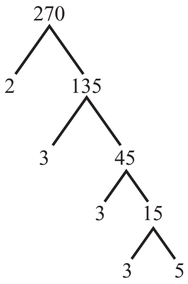
**4** **a** 12 **b** 90 **c** 280 **d** 189

**5** **a**



$\therefore 36 = 2^2 \times 3^2$

**b**



$\therefore 270 = 2 \times 3^3 \times 5$

**c** $420 = 2^2 \times 3 \times 5 \times 7$ **d** $378 = 2 \times 3^3 \times 7$

**6** **a** $2^2 \times 5$ **b** $2^2 \times 7$ **c** $2^3 \times 5$
**d** $2 \times 3^2 \times 5$ **e** $2^3 \times 5 \times 7$ **f** $2^2 \times 7^2$
**g** $2^3 \times 3^2 \times 5$ **h** $2^2 \times 3 \times 5 \times 11$

**7** **a** divisible by 3 **b** divisible by 2, 3, 6, 9
**c** divisible by 2, 4, 8 **d** divisible by 3, 9
**e** divide by 3, 5, 9 **f** none
**g** 2, 3, 6 **h** none

**8** **a** 12 **b** 72 **c** 30

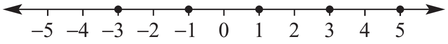
**9** 72 days

**10** **a** 1, 4, 7 **b** 0, 9 **c** 2, 5, 8
**d** 2 **e** 0, 2, 4, 6, 8, **f** 0

## Exercise A1F

**1** **a** 2 **b** 5 **c** –3 **d** –10 **e** –1

**2**



**3** **a** > **b** < **c** < **d** >
**e** > **f** < **g** > **h** <

**4** **a** –2°C **b** –1°C **c** –9°C **d** 3°C

**5** **a** 1 **b** 4 **c** 1 **d** 8
**e** 15 **f** –2 **g** –5 **h** –7
**i** –7 **j** –14 **k** 5 **l** 7
**m** –4 **n** 0 **o** 3 **p** –5

**6** **a** –1 **b** –5 **c** –26 **d** –17
**e** –2 **f** 1 **g** –11 **h** –31
**i** –29 **j** –110 **k** –17 **l** –12
**m** –20 **n** –9 **o** 4 **p** –12

**7** **a** –3 **b** –15 **c** 0 **d** –24
**e** –14 **f** –7 **g** 1 **h** –1
**i** –19 **j** 1 **k** –6 **l** –10
**m** –1 **n** –100 **o** 20 **p** 31

**8** **a** –1 **b** 8 **c** –7 **d** –1

**9** **a** $1 - 4 = -3$ **b** $-9 + 3 = -6$
**c** $-1 + 5 = 4$ **d** $-15 - 5 = -20$

**10** **a** 6 **b** –4 **c** –14 **d** 11
**e** 15 **f** 5 **g** –3 **h** 12

**11** 4 floors below ground level (–4)

**12** $7

**13** **a**

| + | – | Balance |
|---|---|---|
| | | 500 |
| | 375 | 125 |
| 80 | | 205 |
| 100 | | 305 |
| | 90 | 215 |
| | 45 | 170 |
| | 49 | 121 |
| | 25 | 96 |

**b** $279

**14** **a** +6 **b** +8 **c** –16 **d** –3
**e** +2 **f** +18

## Exercise A1G

**1** **a** 6 **b** –10 **c** –38 **d** 46
**e** 32 **f** –88 **g** –673 **h** 349

**2** **a** subtract **b** add

**3** **a** false **b** true **c** true
**d** false **e** true **f** false
**g** false **h** true **i** false

**4** **a** $7 - 3$ **b** $10 - 5$ **c** $8 + 1$
**d** $6 + 8$ **e** $15 - 20$ **f** $-3 + 4$
**g** $-9 + 9$ **h** $0 - 5$ **i** $18 - 18$

**5** **a** 4 **b** 3 **c** –5 **d** 15
**e** –2 **f** –14 **g** –9 **h** –21
**i** –38 **j** –86 **k** –105 **l** –130
**m** –1 **n** –21 **o** –6 **p** –18
**q** –16 **r** 0 **s** –85 **t** –106

**6** **a** 5 **b** 8 **c** 21 **d** 38
**e** 72 **f** 467 **g** –2 **h** 2
**i** 3 **j** 32 **k** –57 **l** –120
**m** 65 **n** –55 **o** 0 **p** 8
**q** 82 **r** 100 **s** –38 **t** –158

**7** **a** –4 **b** –4 **c** –3
**d** –3 **e** –2 **f** –2
**g** 66 **h** 66 **i** 0

**8** **a** –3 **b** –6 **c** 1 **d** 10
**e** 2 **f** –14 **g** –2 **h** –4
**i** –30 **j** –5 **k** –6 **l** 65

**9** 32°

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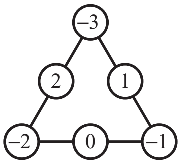
**10** $180 000

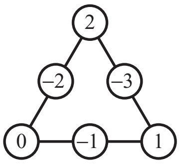
**11 a** 6 **b** 7 **c** −18

**12 a** −8 **b** −3 **c** 1
**d** −8 **e** −2 **f** 2

**13 a**



**b**



**14 a**

| | | |
|---|---|---|
| −1 | −6 | 1 |
| 0 | −2 | −4 |
| −5 | 2 | −3 |

**b**

| | | |
|---|---|---|
| −12 | −19 | −14 |
| −17 | −15 | −13 |
| −16 | −11 | −18 |

## Exercise A1H

**1 a**

| □ | Δ | □ × Δ |
|---|---|---|
| 3 | 5 | 15 |
| 2 | 5 | 10 |
| 1 | 5 | 5 |
| 0 | 5 | 0 |
| −1 | 5 | −5 |
| −2 | 5 | −10 |
| −3 | 5 | −15 |

**b**

| □ | Δ | □ × Δ |
|---|---|---|
| 3 | −5 | −15 |
| 2 | −5 | −10 |
| 1 | −5 | −5 |
| 0 | −5 | 0 |
| −1 | −5 | 5 |
| −2 | −5 | 10 |
| 3 | 5 | 15 |

**2 a** 15, 3 **b** −15, −3
**c** −15, −3 **d** 15, −3

**3 a** + **b** − **c** −
**d** + **e** + **f** −
**g** + **h** − **i** +

**4 a** + **b** − **c** −
**d** − **e** − **f** +

**5 a** −20 **b** −54 **c** −40 **d** −99
**e** 6 **f** −42 **g** −72 **h** 99
**i** −40 **j** −64 **k** 35 **l** −32
**m** 60 **n** −44 **o** 9 **p** −60

**6 a** −5 **b** −2 **c** −4 **d** −30
**e** −2 **f** −3 **g** −3 **h** 3
**i** −2 **j** 8 **k** −9 **l** 5
**m** 11 **n** 1 **o** −8 **p** 8

**7 a** −20 **b** 21 **c** −12 **d** −23
**e** 50 **f** −9 **g** −15 **h** 39
**i** −18 **j** 8

**8 a** 25 **b** 36 **c** 49 **d** 64
**e** 81 **f** 100

**9 a** −3 **b** −5 **c** 7
**d** 6 **e** −3 **f** −72
**g** −252 **h** −5 **i** −30

**10** negative

**11 a** ×, ÷ **b** ×, ÷ **c** ÷, × **d** ÷, ÷

**12** −8 and 3

**13 a** −1 **b** −3 **c** −5 **d** 3
**e** −6 **f** 7 **g** 0 **h** −5

**14 a** −15 **b** −15 **c** 2 **d** −8
**e** 8 **f** 1 **g** −4 **h** 10

**15 a** 22 **b** 4 **c** 28 **d** 122
**e** −32 **f** −16 **g** 152 **h** 16

**16 a** −15 **b** 5 **c** 16 **d** 14
**e** 9 **f** 28 **g** −1 **h** 0
**i** −12 **j** 19 **k** 7 **l** 37

**17 a** $(-2 + 1) \times 3 = -3$
**b** $-10 \div (3 - (-2)) = -2$
**c** $-8 \div (-1 + 5) = -2$
**d** $(-1 - 4) \times (2 + (-3)) = 5$
**e** $(-4 + -2) \div (10 + (-7)) = -2$
**f** $20 + ((2 - 8) \times (-3)) = 38$
**g** $(1 - (-7) \times 3) \times 2 = 44$
**h** $(4 + -5 \div 5) \times (-2) = -6$

## Puzzles and games

**1** See teacher.
**2** See teacher.

## Chapter review

### Multiple-choice questions

**1** B **2** D **3** C **4** B **5** A
**6** B **7** C **8** A **9** C **10** A

### Short-answer questions

**1 a** 497 **b** 412 **c** 129 **d** 67
**e** 112 **f** 139 **g** 1999 **h** 5675

**2 a** 539 **b** 2030 **c** 61 **d** 3074

**3 a** 170 **b** 297 **c** 336 **d** 423
**e** 41 **f** 119 **g** 103 **h** 201

**4 a** 1668 **b** 21 294 **c** 281 **d** 122

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**5** **a** 3 **b** 1 **c** 1 **d** 7

**6** **a** 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
**b** 112, 119, 126, 133, 140, 147
**c** 31, 37, 41, 43, 47, 53, 59 **d** 24 **e** 6

**7** **a** $2^2 \times 3^2$ **b** $2^2 \times 3 \times 7$ **c** $2 \times 3^2 \times 11$

**8** **a** divisible by 2, 3, 4, 6
**b** divisible by 5
**c** divisible by 2, 4
**d** divisible by 3, 9

**9** **a** 3 **b** –5 **c** –8 **d** –31
**e** –76 **f** –330 **g** –1 **h** 98

**10** **a** 2 **b** –8 **c** –64 **d** –39
**e** 16 **f** 12 **g** –20 **h** 92

**11** **a** –10 **b** 88 **c** –63 **d** 200
**e** 2 **f** –3 **g** –4 **h** 3

**12** **a** –4 **b** –1 **c** –8 **d** 26

**13** **a** –11 **b** 1 **c** 7 **d** –4

## Extended-response questions

**1** **a** $a$ = $112, $b$ = –$208, $c$ = $323, $d$ = –$275, $e$ = $240
**b** $228 **c** $160

**2** **a** Hong Kong **b** Moscow, New York
**c** Hong Kong **d** 14°

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# Chapter 1

## Pre-test

**1 a** 32 **b** 14 **c** 12 **d** 30
**2 a** 17 **b** 12 **c** 30 **d** 12
**3 a** 12 **b** 70 **c** 7 **d** 20
**4 a** 24 **b** 12 **c** 66 **d** 600
**5 a** $x + 5$ **b** $m - 7$ **c** $xy$ **d** $\frac{w}{2}$
**6** 25
**7 a**

| | | | | |
|---|---|---|---|---|
| $x$ | 1 | 3 | 11 | 0 |
| $y$ | 13 | 15 | 23 | 12 |

**b**

| | | | | |
|---|---|---|---|---|
| $x$ | 0 | 3 | 7 | 10 |
| $y$ | 3 | 9 | 17 | 23 |

**8 a** 8 **b** 12 **c** 16 **d** 18
**9 a** 2 **b** 4 **c** 6
**10 a** 25 **b** 6 **c** 27 **d** 2

## Exercise 1A

**1 a** Yes **b** Yes **c** No
**d** No **e** Yes **f** Yes
**2 a** $3a$, $2b$, $5c$ **b i** 3 **ii** 2 **iii** 5
**c** $2x + 5y + 8z$. Other answers are possible.
**3 a** $4x$, $6y$, $2z$ **b** $y$
**4 a** 6 **b i** 5 **ii** 7 **iii** 1
**c** $x + 2y + 3z + 4w + 91k$. Other answers are possible.
**5 a** 3 **b** 3 **c** 2
**d** 4 **e** 2 **f** 5
**6 a** 2 **b** 1 **c** 9
**d** –2 **e** –6 **f** –1
**7 a** F **b** C **c** E
**d** D **e** A **f** B
**8 a** $y + 7$ **b** $x - 3$ **c** $a + b$ **d** $4p$
**e** $4 - \frac{q}{2}$ **f** $10 + \frac{r}{3}$ **g** $2(b + c)$ **h** $b + 2c$
**9 a** the sum of 3 and $x$ **b** the sum of $a$ and $b$
**c** double $k$ **d** half $m$
**10 a** the product of 4, $b$ and $c$
**b** double $a$ is added to $b$
**c** $b$ is subtracted from 4 and the result is doubled.
**d** $b$ is doubled and the result is subtracted from 4.
**11 a** $10\ x$ **b** $A + B$ **c** $22 - k$ **d** $50 - x$
**12 a** \$70 **b** $7x$ **c i** $x - 3$ **ii** $7(x - 3)$
**13 a** $2p$ **b** $48p$ **c** $30p + 18(p + 20)$
**14 a i** $4a$ **ii** $7b$ **iii** $5a + 5b$ **iv** $\frac{7a + 7b}{2}$
**b** 7 numbers, 2 Proof by Induction ⇒ total = 9 DVDs

## Exercise 1B

**1 a** 11 **b** 17 **c** 9 **d** 7
**2 a** 11 **b** 12 **c** 3 **d** 3
**3 a** 16 **b** 21 **c** 111 **d** 70
**4** equivalent expressions
**5 a** 15 **b** 8 **c** 20
**6 a** 14 **b** 30 **c** no
**7 a** 30 **b** 37 **c** 16 **d** 58
**8 a** 7 **b** 5 **c** 10 **d** 23
**9 a** 14 **b** 13 **c** 11 **d** 34
**e** 19 **f** 29 **g** 3 **h** 17
**10 a** 8 **b** 2
**11 a** E **b** E **c** N
**d** N **e** E **f** E
**12** $a + a$, $2 \times a$, $a \times 2$, $2a$
**13 a** 8 **b** 3, 4, 5
**14 a** If $a = 3$ and $b = 4$ $3 + 4 = 7$, $3 \times 4 = 12$
**b** $a = 2$ and $b = 2$
**c** Not equal if $a = 10$ $(12 \neq 8)$
**d** No, always 4 apart.
**15 a**

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | 3 | 5 | 2 | 0 | 4 | 2 |
| $y$ | 8 | 7 | 3 | –3 | –2 | 6 |
| $x + y$ | 11 | 12 | 5 | –3 | 2 | 8 |
| $x - 2y$ | –13 | –9 | –4 | 6 | 8 | –10 |
| $xy$ | 24 | 35 | 6 | 0 | –8 | 12 |

**b**

| | | | | | | |
|---|---|---|---|---|---|---|
| $a$ | 10 | 0 | 2 | 12 | 5 | 1 |
| $a + 2$ | 12 | 2 | 4 | 14 | 7 | 3 |
| $2a$ | 20 | 0 | 4 | 24 | 10 | 2 |
| $a^2$ | 100 | 0 | 4 | 144 | 25 | 1 |
| $2 - a$ | –8 | 2 | 0 | –10 | –3 | 1 |
| $\frac{a}{2}$ | 5 | 0 | 1 | 6 | 2.5 | 0.5 |

## Exercise 1C

**1 a** $a$ **b** $a$, $c$ **c** $x$, $y$ **d** $w$, $z$
**2 a** like terms **b** equivalent
**3 a** 21 **b** 21 **c** true
**4 a** 23 **b** 84 **c** false
**5 a** $a$, $b$, $c$ **b** $a$, $b$, $c$ **c** Yes
**6 a** L **b** N **c** L **d** N
**7 a** L **b** N **c** L **d** N
**e** L **f** N **g** L **h** L
**8 a** $5x$ **b** $19a$ **c** $9x$ **d** $7y$
**e** $7xy$ **f** $13uv$ **g** $14ab$ **h** $15pq$
**i** $2x$ **j** $7x$ **k** 0 **l** $5x$
**m** $x$ **n** 0 **o** $-x$ **p** $x$
**9 a** $9f + 12$ **b** $13x + 8y$ **c** $7a + 11b$
**d** $13a + 9b$ **e** $12 + 12x$ **f** $8a + 3b + 3$
**g** $14x + 30y$ **h** $21a + 4$ **i** $5\mathrm{b} + 9c$ **j** $2a + 3b$
**10 a** C **b** A **c** D **d** E **e** B
**11 a** $12x$ **b** $22x$ **c** $12a + 4b$
**12 a** $13c$ **b** $9nc$

**13 a** If $a = 1$, $b = 2$: $4a + 3b = 10$, $7ab = 14$.
Other answers are possible.
**b** Yes, for example if $a = 0$ and $b = 0$.
**c** No. They are equivalent.
**14 a** $5a + 7b + 5a$. Other answers are possible.
**b** 9 ways

## Exercise 1D

**1 a** true **b** true **c** false **d** false **e** true
**2** B
**3** A
**4 a** $3xy$ **b** $5abc$ **c** $12ab^2$ **d** $4ac^3$
**5 a** $63d$ **b** $10a$ **c** $36x$
**d** $24k$ **e** $6q$ **f** $30xy$
**g** $8abcd$ **h** $60abcd$ **i** $48abde$
**6 a** $x^2$ **b** $a^2$ **c** $3d^2$
**d** $10d^2e$ **e** $14x^2y$ **f** $10x^2y$
**g** $8x^2yz$ **h** $8a^2b^2cd$ **i** $48x^2y$
**j** $18a^2b$ **k** $24x^2y^2$ **l** $24a^2b^2$
**7 a** $\frac{k}{4}$ **b** $\frac{x}{5}$ **c** $\frac{2q}{5}$ **d** $\frac{3k}{10}$
**e** $\frac{5}{a}$ **f** $\frac{a}{b}$ **g** $\frac{x}{y}$ **h** $\frac{12}{g}$
**8 a** $\frac{3}{5}$ **b** $\frac{1}{3}$ **c** $\frac{3}{2}$ **d** $\frac{3}{5}$
**9 a** $\frac{1}{2}$ **b** $\frac{x}{2y}$ **c** $\frac{5x}{6}$ **d** $\frac{a}{4}$
**e** $\frac{x}{3}$ **f** $\frac{1}{6x}$ **g** $\frac{4y}{7}$ **h** $\frac{ac}{2}$
**10 a** $8ab$ **b** $24x^2$ **c** $18xy$
**11 a** $11ab$ **b** $24qr$ **c** $2xy$
**12 a** $2y$ **b** $3b$ **c** $28rs$ **d** $8ab^2$
**13 a** no **b** $\frac{2a}{5}$ and $\frac{2}{5} \times a$ **c** $a = 1$ or $a = -1$
**14 a** $16ab$ **b** 2, 5, 6, 1. Others possible
**c** $2a \times 3b + 3a \times 2b + 4a \times b$. Others possible.

## Exercise 1E

**1 a** $4(x + 9)$ **b** $2(x + 10)$ **c** $5(x + 7)$
**2** B, C, D, E, F
**3 a** $3a + 6$ **b** $2x + 2y$
**c** $4p + 4$ **d** $12a + 6b$
**4 a** $3 \times 2 + 3 \times 5$ **b** $3 \times x + 3 \times 2$
**c** $a \times b + a \times c$
**5 a** $4(x + 2) = 4x + 8$ **b** $3(a + 1) = 3a + 3$
**c** $4(k + 7) = 4k + 28$ **d** $3(b + 5) = 3b + 15$
**6 a** $6y + 48$ **b** $7\ell + 28$ **c** $9a + 63$ **d** $2t + 12$
**7 a** $2m - 20$ **b** $8y - 24$ **c** $3e - 21$ **d** $7e - 21$
**8 a** $60g - 70$ **b** $15e - 40$ **c** $35w + 50$
**d** $10u + 25$ **e** $56x - 14$ **f** $27v - 12$
**g** $14q - 28$ **h** $20c - 4v$ **i** $8 + 20x$
**j** $21 + 6y$ **k** $72 - 24x$ **l** $22 - 44k$
**9 a** 20 **b** 6 **c** 10 **d** 14
**10** $2\ell + 2b$
**11 a** $7x + 6$ **b** $2a + 12$ **c** $15b$
**d** $10c + 24$ **e** $2x + 10$ **f** $4x + 5$
**12 a** $5(x + 3) = 5x + 15$ **b** $2(b + 6) = 2b + 12$
**c** $3(z - 4) = 3z - 12$ **d** $7(10 - y) = 70 - 7y$
**13** $2(4a + 12b)$ and $8(a + 3b)$. Others possible.
**14 a** $ab + 4b + 2a + 8$ **b** $xy + 3y + 5x + 15$
**c** $6ac + 15c + 4a + 10$ **d** $20ab + 5b + 12a + 3$

## Exercise 1F

**1 a** 4 **b** 3 **c** 2 **d** 6
**2** 2
**3 a** 6 **b** 5 **c** 20 **d** 2
**4 a** 3 **b** 4 **c** $2b$ **d** $7x$
**5 a** 12 **b** 35 **c** 12, 30
**d** $14a$, $21b$ **e** 7 **f** 3
**g** 2, $q$ **h** 4
**6 a** 5 **b** 4 **c** 9
**d** 7 **e** 3 **f** 6
**7 a** $6x$ **b** $8a$ **c** $3b$
**d** $12y$ **e** $2q$ **f** $4p$
**8 a** $3(x + 2)$ **b** $8(v + 5)$ **c** $5(3x + 7)$
**d** $5(2z + 5)$ **e** $4(10 + w)$ **f** $5(j - 4)$
**g** $3(3b - 5)$ **h** $4(3 - 4f)$ **i** $5(d - 6)$
**j** $5(2x + 1)$ **k** $6(k - 2)$ **l** $2(9p + 10)$
**9 a** $2n(5c + 6)$ **b** $8y(3 + r)$ **c** $2n(7j + 5)$
**d** $4g(6 + 5j)$ **e** $2(5h + 2z)$ **f** $10(3u - 2n)$
**g** $3(7p - 2c)$ **h** $3(4a + 5b)$
**10** For example: length = 2, breadth = $6x + 8$. Other answers are possible.
**11 a** 5 **b** $4a + 12$
**12** $(x + 2)(y + 3)$
**13 a** $6x + 18$ **b** $6(x + 3)$ **c** $x + 3$
**d** $2x + 6$ **e** $3x + 9$

## Exercise 1G

**1 a** \$10 **b** \$12 **c** \$26
**2 a i** 60 min **ii** 150 min **iii** 300 min
**b** B
**3 a** 35 **b** 41 **c** 5
**4 a** $2x + y$ **b** 8
**5 a** $2x + 6$ **b** 24 **c** $3\ x$
**6 a** \$30 **b** $\$(3n)$ **c** \$36
**7 a** \$210 **b** C
**8 a** $5x$ **b** $10x$ **c** $5(x + 3)$ or $5x + 15$
**9 a** $\$(30 + 40x)$ **b** \$350
**10 a** \$50 **b** \$60 **c** \$230
**11 a** \$140 **b** $60 + 80x$ **c i** \$60 **ii** \$80
**12 a** $\$(F + H)$ **b** $\$(F + 2H)$ **c** $\$\left(F + \frac{H}{2}\right)$

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**Exercise 1G cont.**

**13 a** \$(10 + 4*n*) **b** \$(20 + *n*) **c** \$30 **d** deal 1
**e** deal 3 **f i** 3 **ii** 4, 10 **iii** 9

## Exercise 1H

**1 a** $2^2$ **b** $4^2$ **c** $5^2$
**d** $5^3$ **e** $6^4$ **f** $7^3$

**2 a** IV **b** V **c** VI
**d** III **e** II **f** I

**3** $3 \times 3 = 9$
$4 \times 4 = 16$
$5 \times 5 = 25$
$6 \times 6 = 36$
$7 \times 7 = 49$
$8 \times 8 = 64$
$9 \times 9 = 81$
$10 \times 10 = 100$

**4** $3 \times 3 \times 3 = 27$
$4 \times 4 \times 4 = 64$
$5 \times 5 \times 5 = 125$
$6 \times 6 \times 6 = 216$

**5 a** $7^3$ **b** $10^4$ **c** $8^2$
**d** $4^3$ **e** $2^7$ **f** $6^7$
**g** $12^2$ **h** $5^6$ **i** $6^1$

**6 a** $8 \times 8 \times 8 \times 8 \times 8$ **b** $3 \times 3 \times 3 \times 3$
**c** $9 \times 9$ **d** $4 \times 4 \times 4 \times 4$
**e** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
**f** $11 \times 11$

**7 a** 8 **b** 16 **c** 27
**d** 10 000 **e** 125 **f** 1

**8 a** 16 **b** 100 **c** 169
**d** 225 **e** 10 000 **f** 400
**g** 5 **h** 7 **i** 11
**j** 10 **k** 12 **l** 16

**9 a** 8 **b** 64 **c** 343
**d** 125 **e** 216 **f** 1000
**g** 3 **h** 2 **i** 5
**j** 8 **k** 9 **l** 100

**10 a** $3^2$ **b** $2^4$ **c** $2^5$

**11 a** 13 **b** 15 **c** 625
**d** 9 **e** 1331

**12 a** $6^2 \times 7^4$ **b** $5^4 \times 2^2$ **c** $3^2 \times 8^2$
**d** $9^4 \times 11$ **e** $4^3 \times 12^2$ **f** $2^6 \times 3^3$

**13 a** $m^3$ **b** $a^5$ **c** $n^7$
**d** $p^{10}$ **e** $p^3q^2$ **f** $a^4b^2$
**g** $a^2b^4$ **h** $x^4y$

## Exercise 1I

**1** A

**2** C

**3 a** $6^2$ **b** $7^0$ **c** $5^6 \times 5^8$ **d** $6^9 \div 6^3$

**4** C

**5 a** 6 **b** 3 **c** 9 **d** 7
**e** 13 **f** 6 **g** 6 **h** 3
**i** 4 **j** 3 **k** 6 **l** 5

**6 a** $3^6$ **b** $2^5$ **c** $10^4$
**d** $9^{10}$ **e** $4^5$ **f** $2^{12}$
**g** $8^{10}$ **h** $12^{10}$ **i** $16^8$

**7 a** $3^2$ **b** $2^2$ **c** $9^4$
**d** $4^3$ **e** $17^6$ **f** $11^6$

**8 a** 12 **b** 10 **c** 4
**d** 12 **e** 6 **f** 20

**9 a** $7^4$ **b** $2^{20}$ **c** $3^{14}$ **d** $8^8$
**e** $3^8$ **f** $10^{30}$ **g** $9^{14}$ **h** $5^{15}$

**10 a** 1 **b** 1 **c** 1 **d** 1
**e** 1 **f** 8 **g** 7 **h** 10
**i** 1 **j** 2 **k** 6 **l** 3

**11 a** $2^9$ **b** $5^6$ **c** $6^2 = 36$
**d** the same **e** $144 = 12^2$

**12 a** $2^8$ **b** $2^{13}$ **c** $10^3$
**d** $7^{14}$ **e** $6^1 = 6$ **f** $3^9$

**13 a** $a^{11}$ **b** $m^7$ **c** $a^9$ **d** $x^{13}$
**e** $n^{11}$ **f** $m^{14}$ **g** $n^6$ **h** $a^3$
**i** $m^2$ **j** $a^{12}$ **k** $w^9$ **l** $p^4$

**14 a** $5m^7$ **b** $24m^8$ **c** $16m^{10}$
**d** $12a^9$ **e** $21x^7$ **f** $20x^{12}$

## Puzzles and games

**1** $A = 5$, $B = 2$, $C = 7$

**2**

| | | | | |
|---|---|---|---|---|
| $3x$ | + | $\mathbf{4x + 3y + 1}$ | = | $7x + 3y + 1$ |
| + | | + | | + |
| $\mathbf{2y}$ | + | $\mathbf{y}$ | = | $\mathbf{3y}$ |
| = | | | | = |
| $2y + 3x$ | + | $\mathbf{4x + 4y + 1}$ | = | $7x + 6y = 1$ |

**3** 4

**4** $3(2n + 4) - 12$ simplifies to $6n$ → Not a coincidence.

**5 a** 25 **b** 56.25 **c** 0

## Chapter review

### Multiple-choice questions

**1** C **2** A **3** D **4** C **5** E
**6** E **7** D **8** E **9** D **10** A

### Short-answer questions

**1 a** false **b** true **c** true **d** false **e** true

**2 a** 2 **b** 3 **c** 4 **d** 6

**3 a** 10 **b** 8 **c** 4 **d** 0

**4 a** 20 **b** 7 **c** 3 **d** 16

**5 a** 9 **b** 9 **c** 9 **d** 2

**6 a** $16m$ **b** $2a + 5b$ **c** $4y - x + 1$
**d** $7x + 7y$ **e** $9x + xy$ **f** $10m - 6n$

**7 a** $36ab$ **b** $30xy$ **c** $30xyz$

**8 a** $2x$ **b** $3a$ **c** $\frac{z}{5y}$

**9 a** $3x - 12$ **b** $10 + 2x$ **c** $6y + 12$
**d** $20x + 70$ **e** $3x - 15$ **f** $11z - 22$
**g** $12a - 44$ **h** $12b - 6$
**10 a** 4 **b** $7a$
**11 a** $2(x+3)$ **b** $8(3 - 2g)$
**c** $3x(4 + y)$ **d** $7a(1 + 2b)$
**12 a** $5a$ **b** $3p$ **c** $5a + 3b$
**13 a** 70 km **b** $10n$
**14 a** 4 **b** 8 **c** 2 **d** 2 **e** 1
**15 a** $2^3$ **b** $2^{12}$ **c** $2^2$ **d** $2^{35}$

### Extended-response questions

**1 a** $\$(120 + 80n)$
**b** $\$(80 + 100n)$
**c** A costs \$360, B costs \$380.
**d** any more than two hours
**e** $\$(200 + 180n)$
**2 a** $2xy - x^2$ **b** 33 m$^2$
**c** $4x + 2y$ **d** 26 m
**e** Area = $3xy - 3x^2$ Perimeter = $6x + 2y$
**3 a** \$5.12
**b** \$10.23
**c** 27 years

# Chapter 2

## Pre-test

**1 a** 12 **b** 27 **c** 3 **d** 10
**2 a** 8 **b** 2 **c** 5 **d** 15
**3 a** 8 **b** 42 **c** 4 **d** 2
**4 a** $11m$ **b** $a$ **c** $9n$
**d** $10a - 10$ **e** $11x + 2$ **f** $8b + 4$
**5 a** $3m + 12$ **b** $2a + 12$ **c** $3x + 21$ **d** $4k - 24$
**6** 12
**7 a** true **b** false **c** false **d** true
**8 a** $x = 4$ **b** $x = 8$ **c** $m = 4$ **d** $m = 6$
**9 a** $\div 5$ **b** $-2$ **c** $\times 3$ **d** $+3$
**10 a** $p + 10$ **b** $4x$ **c** $2z$ **d** $q - 6$
**11 a** false **b** true **c** true **d** true

## Exercise 2A

**1 a** 19 **b** 15 **c** 16 **d** 6
**e** 11 **f** 13 **g** 35 **h** 1
**2 a** true **b** false **c** true
**d** false **e** true **f** true
**3 a** 8 **b** 12 **c** 15 **d** 45
**4 a** 13 **b** 9 **c** 2 **d** 2
**5 a** 7 **b** 9 **c** 15 **d** 8
**6 a** true **b** false **c** true
**d** true **e** true **f** false
**7 a** true **b** false **c** true
**d** true **e** false **f** true
**8 a** $x = 8$ **b** $x = 3$ **c** $x = 7$
**d** $x = 4$ **e** $x = 1$ **f** $x = 5$
**9 a** $x = 7$ **b** $x = 13$ **c** $u = 7$
**d** $p = 19$ **e** $x = 2$ **f** $k = 11$
**10** C
**11 a** $k + 4 = 20$ **b** $2x + 7 = 10$
**c** $x + \frac{x}{2} = 12$ **d** $h + 30 = 147$
**e** $4c + 6 = 22$ **f** $8c + 2000 = 3600$
**12 a** 7 **b** 42 **c** 13 **d** 26
**13 a** $3.2x = 9.6$ **b** $x = 3$
**14 a** $a = 10$, $b = 6$, $c = 12$, $d = 20$, $e = 2$
**b** $a = 20$, $b = 6$, $c = 24$, $d = 80$, $e = 4$

## Exercise 2B

**1 a** $2 + 3 = 1 + 4$ **b** $x + 3 = 7$ **c** $x + 5 = 2x + 2$
**2 a** $6x = 36$ **b** $2x + 1 = 13$ **c** $x = 6$
**3 a** 12 **b** 25 **c** 12
**4 a** 8 **b** $x = 8$
**5** B
**6 a** $2x = 20$ **b** $2 + q = 10$
**c** $18 = 17 - q$ **d** $12x = 24$
**e** $7p + 6 = 2p + 10$ **f** $3q = 2q$
**7 a** $x = 3$ **b** $q = 7$ **c** $k = 11$
**d** $4x = 20$, $x = 5$
**e** $7p = 28$, $p = 4$ (missing operation $\div 7$)
**f** $10x = 30$, $x = 3$ (missing operation $\div 10$)
**8 a** $a = 3$ **b** $t = 7$ **c** $q = 9$ **d** $k = 9$
**e** $x = 10$ **f** $h = 10$ **g** $\ell = 4$ **h** $g = 9$
**9 a** $h = 3$ **b** $u = 4$ **c** $s = 3$ **d** $w = 8$
**e** $x = 4$ **f** $w = 5$ **g** $a = 2$ **h** $y = 12$
**10 a** $x = 2$ **b** $k = 5$ **c** $x = 42$ **d** $x = 20$
**e** $k = 7$ **f** $x = 30$ **g** $y = 6$ **h** $x = 20$
**11 a** $x = -6$ **b** $a = -3$ **c** $x = -10$ **d** $k = -5$
**e** $k = -4$ **f** $p = -1$ **g** $p = -16$ **h** $x = -5$
**12 a** $p + 8 = 15$, $p = 7$ **b** $3q = 12$, $q = 4$
**c** $2k - 4 = 18$, $k = 11$ **d** $3r + 4 = 34$, $r = 10$
**13 a** $x = 7$, $y = 2$ **b** $x = 2$, $y = 40$ **c** $x = 4$
**14 a** $x = 2$ **b** $x = 2$ **c** $x = 5$
**15 a** $x = 5$
**b** Opposite operations from bottom to top.
**c** For example, $7 - 3x = -8$

## Exercise 2C

**1** B
**2 a** true **b** false **c** false **d** true
**3 a** 8 **b** 5 **c** no
**4 a** 30 **b** 10 **c** $\times 2$, 22 **d** $\times 10$, 70
**5 a** $b = 20$ **b** $g = 20$ **c** $a = 15$ **d** $k = 18$
**e** $x = 35$ **f** $x = 100$ **g** $t = 8$ **h** $t = 8$
**6 a** $\ell = 20$ **b** $w = -10$ **c** $s = -6$ **d** $v = 12$
**e** $m = 14$ **f** $n = 14$ **g** $j = -5$ **h** $f = 20$

**Exercise 2C** cont.

**7** **a** $t = 28$ **b** $h = 2$ **c** $a = 13$
**d** $c = 17$ **e** $s = 10$ **f** $j = 2$
**g** $x = 44$ **h** $x = 22$ **i** $x = 2$

**8** **a** $v = 20$ **b** $x = 20$ **c** $y = 14$
**d** $x = 10$ **e** $p = 7$ **f** $k = 6$

**9** **a** C **b** A **c** B **d** D

**10** **a** $g = 8$ **b** $x = 14$ **c** $k = 15$
**d** $x = 36$ **e** $q = 10$ **f** $x = 27$
**g** $p = 6$ **h** $x = 1$ **i** $r = 20$

**11** **a** $x = 35$ **b** $y = 24$ **c** $p = 14$
**d** $x = 16$ **e** $x = 12$ **f** $k = 11$

**12** **a** 7 **b** 19 **c** 3
**d** 12 **e** 26

**13** **a** $\frac{b}{3} = 40$ **b** \$120

**14** **a** $x = 6$ **b** $x = 3$ **c** $x = 5$
**d** $x = 2$ **e** $x = 8$ **f** $x = 12$

## Exercise 2D

**1** **a** true **b** false **c** false **d** true

**2** **a** $3x + 3$ **b** 5
**c** $4p + 9 = 5$ **d** $22k + 12 = 13$

**3** B

**4** **a** $f = 5$ **b** $y = 3$ **c** $s = 3$
**d** $j = 2$ **e** $t = -2$ **f** $n = -5$
**g** $y = -5$ **h** $t = -4$ **i** $q = -3$

**5** **a** $t = -2$ **b** $c = 2$ **c** $t = 5$
**d** $z = 3$ **e** $t = 3$ **f** $q = -2$
**g** $x = 9$ **h** $w = 9$ **i** $j = -5$

**6** **a** $a = 3$ **b** $g = 2$ **c** $n = -2$ **d** $u = 7$
**e** $h = -5$ **f** $j = -5$ **g** $c = 1$ **h** $n = -1$
**i** $a = -4$ **j** $v = -7$ **k** $c = -3$ **l** $t = 3$
**m** $n = 4$ **n** $n = -3$ **o** $\ell = 2$

**7** **a** $x = \frac{1}{2}$ **b** $k = \frac{2}{3}$ **c** $m = \frac{-3}{2}$
**d** $j = \frac{5}{2}$ **e** $j = \frac{-1}{2}$ **f** $z = \frac{11}{2}$

**8** **a** $2x + 3 = 3x + 1$ so $x = 2$ **b** $z + 9 = 2z$ so $z = 9$
**c** $7y = y + 12$ so $y = 2$ **d** $n + 10 = 3n - 6$ so $n = 8$

**9** **a** $4p + 1.5 = 2p + 4.9$ **b** \$1.70 **c** 11

**10** **a** $x = 5$ **b** $x = 5$
**c** Pronumeral appears on RHS if subtract $3x$.

**11** $x = 8$, $y = 6$, so length = breadth = 29.

**12** **a** No solutions.
**b** Subtract $2x$, then $3 = 7$ (impossible).
**c** $5x + 23 = 5x + 10$. Other answers are possible.

**13** **a** $x = 20$ **b** $x = 17$, $y = 51$, $z = 10$
**c** $a = 60$, $b = 30$, $c = 20$ **d** $b = 10$, $a = 50$

## Exercise 2E

**1** C, D, E and F

**2** **a** 12 **b** 14 **c** 8, 10 **d** 50, 30

**3** **a** C **b** A **c** D **d** B

**4** **a** true **b** false **c** true **d** true

**5** **a**

$$\begin{tikzcd} 3(x+1) = 18 \arrow[d, bend right, "\div 3"'] \arrow[d, bend left, "\div 3"] \\ x + 1 = 6 \arrow[d, bend right, "-1"'] \arrow[d, bend left, "-1"] \\ x = 5 \end{tikzcd}$$

**b** $3(x + 1) = 18$

$$\begin{tikzcd} 3x + 3 = 18 \arrow[d, bend right, "\boxed{-3}"'] \arrow[d, bend left, "\boxed{-3}"] \\ 3x = 15 \arrow[d, bend right, "\boxed{\div 3}"'] \arrow[d, bend left, "\boxed{\div 3}"] \\ x = 5 \end{tikzcd}$$

**6** **a** $x = 5$ **b** $k = 1$ **c** $r = 17$
**d** $u = 6$ **e** $j = 3$ **f** $p = 6$
**g** $m = 4$ **h** $n = 5$ **i** $a = 3$

**7** (same as 6)

**8** **a** $x = 8$ **b** $x = 10$ **c** $r = 10$ **d** $y = 3$
**e** $\ell = 2$ **f** $w = 2$ **g** $c = 2$ **h** $d = 2$
**i** $w = 6$ **j** $p = 4$ **k** $k = 2$ **l** $c = 10$

**9** **a** $2(n + 5)$ **b** B **c** 15

**10** **a** $d + 4$ **b** $2(d + 4)$
**c** $2(d + 4) = 50$ **d** 21

**11** **a** 3 × cost of shirt + 2 × cost of trousers
**b** $s = 37$ **c** \$37 **d** \$57 **e** \$356

**12** **a** $5w + 3(w + 4)$ **b** \$11.50

**13** **a** $x = -6$ **b** $p = -4$ **c** $q = -19$ **d** $r = 4$
**e** $r = 2$ **f** $x = -11$ **g** $k = -10$ **h** $s = 0$

## Exercise 2F

**1** **a** **i** 9 and 9 **ii** 36 and 36
**iii** 1 and 1 **iv** 100 and 100
**b** They are equal

**2** **a** **i** 9 **ii** 49 **iii** 169 **iv** 64
**b** No

**3** **a** 9, 3, −3 **b** 25, 5, −5 **c** 121, 11, −11

**4** **a** ±2 **b** ±7 **c** ±10 **d** ±8
**e** ±1 **f** ±12 **g** ±6 **h** ±11
**i** ±13 **j** ±16 **k** ±30 **l** ±100

**5** **a** ±2.45 **b** ±3.46 **c** ±6.08 **d** ±6.40
**e** ±10.20 **f** ±17.80 **g** ±19.75 **h** ±26.34

**6** **a** 2 **b** 2 **c** 2 **d** 0
**e** 0 **f** 1 **g** 1 **h** 2

**7** 20 m

**8** 4 m

**9** **a** ±2 **b** ±1 **c** ±3 **d** ±2
**e** ±5 **f** 0 **g** ±6 **h** ±10

**10** **a** $x = 0$ is the only number that squares to give 0
**b** $x^2$ is positive for all values of $x$.

**11** **a** $\pm\sqrt{11}$ **b** $\pm\sqrt{17}$ **c** $\pm\sqrt{33}$ **d** $\pm\sqrt{156}$

**12** **a** ±2 **b** ±1 **c** ±3
**d** ±1 **e** ±2 **f** ±5
**g** ±2 **h** ±3 **i** ±6

## Exercise 2G

**1** **a** formula **b** subject **c** area

**2** **a** 11 **b** 6 **c** 7 **d** −28

**3** C

**4** B

**5** A

**6** **a** $A = 19$ **b** $A = 51$ **c** $A = 7$ **d** $A = 407$

**7** **a** **i** 40 **ii** 12 **iii** 30

**b** $x = 11$ **c** 11 units

**8** **a** $a = 2$ **b** $a = 5$ **c** $a = 11$

**9** **a** $y = 10$ **b** $x = 6$ **c** $x = -2$

**10** $m = 5.5$

**11** **a** $A = 60$ **b** $h = 4$ **c** 11

**12** **a** \$23

**b** **i** $161 = 3 + 2d$ **ii** $d = 79$ **iii** 79 km

**13** **a** 92 **b** Yes, if $p = 30$

**14** **a** $A$: \$5.30, $B$: \$6 **b** 12 min

**c** $10\frac{1}{2}$ min **d** 3 min

## Exercise 2H

**1** **a** D **b** C **c** A

**2** **a** D **b** A **c** E

**d** C **e** B

**3** **a** B **b** C **c** A **d** D

**4** **a** $p = 6$ **b** $x = 9$ **c** $k = 4$ **d** $a = 3$

**5** **a** $a = 13$ **b** 13 years old

**6** **a** Let $c$ = cost of one cup. **b** $4c = 14$

**c** $c = 3.5$ **d** \$3.50

**7** **a** Let $t$ = time spent (hours)

**b** $70 + 80t = 310$ **c** $t = 3$ **d** 3 hours

**8** **a** Let $c$ = cost of one chair.

**b** $6c - 200 = 1300$ **c** $c = 250$ **d** \$250

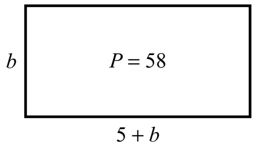
**9** **a** $2(4 + \ell) = 72$ or $8 + 2\ell = 72$

**b** $\ell = 32$ **c** 32 cm

**10** **a** $4x = 24$, $x = 6$ **b** 36 cm$^2$

**11** **a** $a + a + 4 = 40$, $a = 18$ **b** 22 years old

**12** **a**



**b** 12 m **c** 204 m$^2$

**13** **a** $x = 80$ **b** $x = 75$ **c** $x = 30$

**d** $x = 110$ **e** $x = 45$ **f** $x = 65$

## Puzzles and games

**1** □ = 12, Δ = 2, ○ = 9

**2** **a** 88 **b** 6 **c** 13 **d** \$44.44 **e** $33\frac{1}{3}$

**3** **a** 2nd step or 3rd line (can't divide by 0)

**b**

$$\begin{tikzcd} 0 = 1 \arrow[d, bend right, "\times 28"'] \arrow[d, bend left, "\times 28"] \\ 0 = 28 \arrow[d, bend right, "+22"'] \arrow[d, bend left, "+22"] \\ 22 = 50 \end{tikzcd}$$

**4** **a** $4x + 2$ and $4(x + \frac{1}{2})$, $2(x + 4)$ and $4(x + 2)$

**b** $4x + 2 = 4(x + \frac{1}{2})$ **c** $2(x + 4) = 2x + 4$

**5** **a** 65 kg, 62 kg, 55 kg **b** 70 kg, 60 kg, 48 kg

**c** 35 kg, 42 kg, 45 kg, 48 kg

## Chapter review

### Multiple-choice questions

**1** B **2** C **3** D **4** B **5** C

**6** A **7** D **8** E **9** C **10** B

### Short-answer questions

**1** **a** false **b** true **c** true

**2** **a** $m = 4$ **b** $m = 6$ **c** $q = 5$ **d** $z = 50$

**3** **a** $2m + 3 = 27$ **b** $3(n + 4) = 18$

**c** $x + x + 1 = 7$ (or $2x + 1 = 7$)

**4** **a** $x = 4$ **b** $u = 4$ **c** $d = 6$

**d** $b = 7$ **e** $f = 3$ **f** $k = 2$

**5** **a** $3x = 12$ **b** $2b = 14$ **c** $x = 5$

**6** **a** subtract 15 **b** add 5 **c** multiply by 2

**7** **a** $a = 5$ **b** $b = 6$ **c** $n = 16$

**d** $c = 2$ **e** $x = 9$ **f** $x = 2$

**8** **a** $m = 6$ **b** $x = 8$ **c** $k = 30$

**d** $y = 18$ **e** $k = 52$ **f** $x = 32$

**9** **a** $x = 2$ **b** $x = -2$ **c** $x = -2$ **d** $x = 2$

**10** **a** $x = 3$ **b** $x = 2$ **c** $p = 6$

**d** $x = 5$ **e** $x = 10$ **f** $k = 12$

**11** **a** $F = 30$ **b** $m = 4$ **c** $m = 1$

**12** **a** $I = 21$ **b** $M = 3$ **c** $c = 4$

**13** **a** $D$ **b** $m = 1.5$ **c** \$1.50

**14** **a** $x = 3, y = 2$ **b** $x + x + 1 + x + 2 = 39$, $x = 12$

**c** 8.5

### Extended-response questions

**1** **a** $10 + 5n$

**b** **i** $10 + 5n = 55$ **ii** $n = 9$ **iii** 9 rides

**c** \$100 **d** 7 rides

**2** **a** $S = 20 + 0.12n$ **b** 30 times

**c** $Y = 15 + 0.2n$ **d** 25

**e** 63 is the minimum number.

# Chapter 3

## Pre-test

**1** **a** circle **b** square **c** rectangle

**d** parallelogram **e** rhombus

**f** kite **g** triangle **h** trapezium

**2** **a** 30 m **b** 14.5 cm **c** 18 m

**3** **a** 10 **b** 27 **c** 25 **d** 121

**4** **a** 300 cm **b** 200 mm **c** 1800 m **d** 25 cm

**e** 3.5 cm **f** 4.2 km **g** 5 m **h** 0.1 m

**i** 120 s **j** 3000 mL **k** 4 L **l** 3 kg

**5** **a** 6 **b** 9 **c** 4

**6** **a** 30 cm$^2$ **b** 25 cm$^2$ **c** 16 cm$^2$ **d** 20 cm$^2$

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## Exercise 3A

**1** **a** metric **b** centimetres, metres, kilometres
**2** **a** 200 **b** 5200 **c** 78
**d** 8.4 **e** 961 **f** 41.2
**3** **a** 10 **b** 100 **c** 1000
**d** 100 000 **e** 1000 **f** 1 000 000
**4** **a** 10 **b** 10 **c** 2
**5** **a** 30 mm **b** 610 cm **c** 8930 m **d** 300 cm
**e** 2.1 m **f** 32 cm **g** 9.62 km **h** 380 m
**i** 4.8 cm **j** 2 mm **k** 0.042 m **l** 40 cm
**m** 3.7 km **n** 0.6 km **o** 710 m **p** 2 cm
**6** **a** 19 m **b** 44 m **c** 13 cm
**d** 10.4 cm **e** 6.6 m **f** 18 cm
**g** 17.2 mm **h** 34.4 cm
**7** **a** 32 cm **b** 28 km **c** 18 cm
**8** **a** 4.3 mm **b** 2040 cm **c** 23.098 m
**d** 3.42 km **e** 194.3 m **f** 0.01 km
**g** 24.03 mm **h** 0.994 km
**9** **a** 5 **b** 2 **c** 4 **d** 18
**e** 9.5 **f** 6.5
**10** \$2392 **11** 8 min
**12** 240 cm
**13** **a** $P = 2a + b$ **b** $P = 2a + 2b$ **c** $P = 2a + 2b$
**14** **a** 40 cm **b** 17 cm **c** 7.8 cm
**d** 2000 cm **e** 46 cm **f** 17 600 cm

## Exercise 3B

**1** **a** diameter **b** radius **c** circumference
**2** **a** **i** 10 m **ii** 22 cm **iii** 4.6 mm
**b** **i** 6 cm **ii** 15.5 mm **iii** 0.21 m
**3** **a** 3.1 **b** 3.14 **c** 3.142
**4** **a** 15.71 **b** 40.84
**c** 18.85 **d** 232.48
**5** **a** 12.57 mm **b** 113.10 m **c** 245.04 cm
**d** 13.19 m **e** 4.40 km **f** 0.25 cm
**6** **a** 12.57 m **b** 21.99 km **c** 15.71 cm
**d** 13.51 cm **e** 25.95 m **f** 0.13 mm
**7** 251 cm **8** 11.0 m
**9** 176 cm **10** 12 566 m
**11** **a** 64.27 cm **b** 12.34 m **c** 61.70 mm
**12** Svenya and Andre
**13** $d = 2r$, so $2\pi r$ is the same as $\pi d$.
**14** **a** $(6\pi + 24)$ cm **b** $(3\pi + 24)$ cm
**c** $(4\pi + 24)$ cm **d** $(24\pi + 48)$ cm
**e** $(18\pi + 24)$ cm **f** $(48\pi)$ cm

## Exercise 3C

**1** B
**2** **a** **i** 100 **ii** 400
**b** **i** 10 000 **ii** 70 000
**c** **i** 1 000 000 **ii** 5 000 000
**d** **i** 10 000 **ii** 30 000
**3** **a** 7 m, 3 m **b** 8 cm, 6 cm (or other way around)
**c** 2.4 mm, 1.7 mm
**4** **a** 200 mm$^2$ **b** 70 000 cm$^2$ **c** 500 000 m$^2$
**d** 30 000 m$^2$ **e** 34 mm$^2$ **f** 0.07 m$^2$
**g** 30.9 cm$^2$ **h** 4000 m$^2$ **i** 0.2 m$^2$
**j** 0.45 km$^2$ **k** 0.4 ha **l** 32.1 cm$^2$
**m** 32 ha **n** 51 cm$^2$ **o** 4.3 mm$^2$
**p** 0.4802 m$^2$ **q** 1.904 ha **r** 0.2933 ha
**s** 49 m$^2$ **t** 77 000 m$^2$
**5** **a** 9 cm$^2$ **b** 21 m$^2$ **c** 10 cm$^2$
**d** 121 m$^2$ **e** 33 m$^2$ **f** 144 mm$^2$
**6** **a** 42 m$^2$ **b** 39 cm$^2$ **c** 100 cm$^2$
**d** 63 m$^2$ **e** 3 m$^2$ **f** 6 km$^2$
**7** 50 m
**8** 10 cm
**9** **a** 6 m **b** 1.5 cm
**10** **a** 25 m$^2$ **b** 20.25 cm$^2$ **c** 28 cm **d** 52 m
**11** \$48 **12** \$548 100
**13** **a** 70 m$^2$ **b** 54 m$^2$ **c** 140 cm$^2$
**d** 91 cm$^2$ **e** 46 km$^2$ **f** 64 mm$^2$

## Exercise 3D

**1** **a** B **b** D **c** A **d** C
**2** **a** 6 **b** 30 **c** 13.5 **d** 33
**3** **a** 90 **b** height **c** perpendicular
**d** parallel, perpendicular **e** rhombus, kite
**4** **a** 50 m$^2$ **b** 4.5 cm$^2$ **c** 6 m$^2$
**d** 165 m$^2$ **e** 18 cm$^2$ **f** 17.94 m$^2$
**5** **a** 7.5 cm$^2$ **b** 121 km$^2$ **c** 9.61 m$^2$
**d** 4 cm$^2$ **e** 300 mm$^2$ **f** 0.9 mm$^2$
**6** **a** 96 cm$^2$ **b** 32.5 m$^2$ **c** 560 mm$^2$ **d** 13.5 cm$^2$
**7** \$120 **8** 0.27 m$^2$
**9** 2 m **10** \$1160
**11** **a** 6 cm$^2$ **b** 35 m$^2$ **c** 84.5 cm$^2$
**12** No, use formula for parallelogram $A = bh$, as we already know these lengths.
**13** **a**

$$\begin{aligned} A &= \text{length} \times \text{width} \\ &= b \times h \\ &= bh \end{aligned}$$

**b**

$$\begin{aligned} A &= 4 \text{ triangle areas} \\ &= 4 \times \frac{1}{2} \times \text{base} \times \text{height} \\ &= 4 \times \frac{1}{2} \times \frac{1}{2} x \times \frac{1}{2} y \\ &= \frac{1}{2} xy \end{aligned}$$

**c**

$$\begin{aligned} A &= \text{Area (triangle 1)} + \text{Area (triangle 2)} \\ &= \frac{1}{2} \times \text{base}_1 \times \text{height}_1 + \frac{1}{2} \times \text{base}_2 \times \text{height}_2 \\ &= \frac{1}{2} \times a \times h + \frac{1}{2} \times b \times h \\ &= \frac{1}{2} ah + \frac{1}{2} bh \\ &= \frac{1}{2}(a + b)h \end{aligned}$$

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## Exercise 3E

**1 a** $c = 2\pi r$ or $c = \pi d$ **b** $A = \pi r^2$
**2 a** 78.54 **b** 530.93 **c** 30.19 **d** 301.72
**3 a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$
**4 a** 5 m **b** 2.3 mm **c** 3.5 km
**5 a** 28.27 cm$^2$ **b** 113.10 m$^2$ **c** 7.07 mm$^2$
**d** 78.54 km$^2$ **e** 36.32 cm$^2$ **f** 9.08 m$^2$
**6 a** 50.27 cm$^2$ **b** 153.94 km$^2$ **c** 615.75 mm$^2$
**d** 314.16 km$^2$ **e** 38.48 m$^2$ **f** 31 415.93 m$^2$
**7 a** $16\pi$ cm$^2$ **b** $196\pi$ m$^2$ **c** $4\pi$ km$^2$
**8** $225\pi$ cm$^2$
**9** yes, by 1310 cm$^2$
**10** no ($A = 0.79$ km$^2$)
**11 a** 3.14 cm$^2$ **b** 201.06 cm$^2$ **c** 226.98 mm$^2$
**d** 39.27 cm$^2$ **e** 5.09 mm$^2$ **f** 100.53 m$^2$
**12** 78.54 cm$^2$
**13** 80 cm$^2$
**14 a** 34.82 m$^2$ **b** 9.14 m$^2$ **c** 257.08 cm$^2$
**d** 116.38 mm$^2$ **e** 123.61 km$^2$ **f** 53.70 mm$^2$
**g** 50.27 m$^2$ **h** 75.40 mm$^2$ **i** 12.57 cm$^2$

## Exercise 3F

**1 a** length **b** capacity **c** volume **d** area
**e** area **f** area **g** length **h** volume
**i** volume **j** area **k** capacity **l** length
**m** length **n** capacity **o** capacity **p** area
**2 a** 24 **b** 12 **c** 72
**3 a** 1 cm$^3$ contains 1 mL
**b** 1 L contains 1000 mL
**c** A cube with edges 10 cm long has a volume of 1000 cm$^3$ and contains 1000 mL, which is 1 L.
**d** A cube with edges 1 m long has a volume of 1 m$^3$ and contains 1000 L, which is 1 kL.
**4 a** 36 cm$^3$ **b** 20 m$^3$ **c** 27 mm$^3$
**d** 64 m$^3$ **e** 320 mm$^3$ **f** 24 m$^3$
**5 a** 0.4 L **b** 0.7 L **c** 2 kL
**d** 36 kL **e** 4 ML **f** 0.5 ML
**g** 2000 mL **h** 100 mL **i** 6000 kL
**j** 3000 kL **k** 24 000 L **l** 38 000 L
**m** 2 kL **n** 3.5 L **o** 70 L
**p** 2.5 ML **q** 257 mL **r** 9.32 L
**s** 3847 kL **t** 47 kL **u** 5.8 ML
**6 a** 24 L **b** 42 L **c** 27 L
**d** 18 000 L **e** 24 000 L **f** 0.36 L
**7 a** D **b** B **c** F
**d** A **e** C **f** E
**8** 15 days
**9 a i** 60 000 000 L **ii** 60 000 kL **iii** 60 ML
**b** 200 days
**10** 8000 kg
**11** 80 minutes
**12** 9
**13 a** 2500 m$^3$ **b** 2 500 000 L
**14 a** 0.2 mL
**b i** 1 L **ii** 0.6 L **iii** 14.4 L **iv** 5256 L

## Exercise 3G

**1 a** rectangle **b** square **c** triangle
**2 a** 90 cm$^2$ **b** 16 m$^2$ **c** 5 m$^2$
**3 a i** prism **ii** rectangle
**b i** prism **ii** triangle
**c i** not a prism (pyramid)
**d i** not a prism (cone)
**e i** prism **ii** square
**f i** not a prism (truncated pyramid)
**4 a** 44 m$^3$ **b** 20 m$^3$ **c** 352 mm$^3$
**d** 10 cm$^3$ **e** 33 mm$^3$ **f** 110 m$^3$
**5 a** 200 cm$^3$ **b** 15 m$^3$ **c** 980 cm$^3$
**d** 192 cm$^3$ **e** 45 m$^3$ **f** 32 cm$^3$
**6** 40 m$^3$
**7 a** 60 m$^3$ **b** 270 mm$^3$ **c** 60 m$^3$
**d** 24 cm$^3$ **e** 112 m$^3$ **f** 3200 mm$^3$
**8 a** 56 000 L **b** 56 hours
**9 a** 785.40 m$^3$ **b** 12 566.37 mm$^3$ **c** 251.33 cm$^3$
**d** 7696.90 cm$^3$ **e** 461.81 m$^3$ **f** 384.85 m$^3$

## Exercise 3H

**1 a** 60 **b** 7 **c** 24
**d** 120 **e** 4 **f** 31
**2 a** 120 s **b** 3 min **c** 2 h **d** 240 min
**e** 72 h **f** 2 days **g** 5 weeks **h** 280 days
**3 a** 6 h 30 min **b** 10 h 45 min
**c** 16 h 20 min **d** 4 h 30 min
**4 a** 120 s **b** 2 days **c** 3 weeks **d** 180 min
**e** 630 s **f** 4 min **g** 1.5 h **h** 144 h
**i** 3 days **j** 168 h **k** 1440 min **l** 210 min
**5 a** 6:30 p.m. **b** 9:00 a.m. **c** 6:30 p.m.
**d** 4:30 p.m. **e** 5:30 p.m. **f** 11:40 a.m.
**6 a** 1330 h **b** 2015 h **c** 1023 h **d** 2359 h
**e** 6:30 a.m. **f** 1:00 p.m. **g** 2:29 p.m. **h** 7:38 p.m.
**i** 11:51 p.m. **j** 4:26 a.m. **k** 1847 h **l** 0432 h
**7 a** 2:00 p.m. **b** 5:00 a.m.
**c** 1200 hours **d** 1800 hours
**8 a** 10:00 a.m. **b** 9:30 a.m. **c** 9:30 a.m. **d** 8:00 a.m.
**e** 10:00 a.m. **f** 10:00 a.m. **g** 8:00 a.m. **h** 10:00 a.m.
**9 a** 5:30 p.m. **b** 3:30 p.m. **c** 5:30 p.m.
**d** 3:30 p.m. **e** 5:30 p.m. **f** 5:30 p.m.
**g** 5:00 p.m. **h** 7:30 p.m. **i** 4:30 p.m.
**10 a** F **b** D **c** A
**d** E **e** B **f** C
**11 a** 2 h 50 min **b** 6 h 20 min
**c** 2 h 44 min **d** 8 h 50 min
**e** 8 h 19 min **f** 10 h 49 min

**Exercise 3H cont.**

**12** 17 min 28 s
**13** 7 h 28 min
**14** 23 h 15 min
**15** **a** 33c **b** 143c or $1.43
**16** **a** $900 **b** $90 **c** $1.50 **d** 2.5c
**17** **a** 5:30 a.m. **b** 6:30 a.m. **c** 6:30 a.m. **d** 1:30 p.m.
**e** 2:30 p.m. **f** 2:30 a.m. **g** 3:00 p.m. **h** 5:30 p.m.
**18** **a** 10:00 a.m. **b** 12 noon **c** 8:00 p.m. **d** 7:30 p.m.
**e** 7:00 a.m. **f** 5:00 a.m. **g** 1:00 a.m. **h** 10:00 a.m.

## Exercise 3I

**1** **a** 9 **b** 25 **c** 144 **d** 2.25
**e** 20 **f** 58 **g** 157 **h** 369
**2** **a** false **b** true **c** true
**d** true **e** false **f** false
**3** hypotenuse, triangle
**4** **a** $c$ **b** $x$ **c** $u$
**5** **a** No **b** No **c** Yes
**d** Yes **e** Yes **f** No
**g** Yes **h** No **i** No
**6**

| $a$ | $b$ | $c$ | $a^2$ | $b^2$ | $a^2 + b^2$ | $c^2$ |
|---|---|---|---|---|---|---|
| 3 | 4 | 5 | 9 | 16 | 25 | 25 |
| 6 | 8 | 10 | 36 | 64 | 100 | 100 |
| 8 | 15 | 17 | 64 | 225 | 289 | 289 |

**a** $a^2 + b^2$ and $c^2$
**b** **i** 13 **ii** 20
**c** **i** 25 **ii** 110
**7** **a** $3^2 + 4^2 = 5^2$ **b** $8^2 + 15^2 = 17^2$
**c** $9^2 + 12^2 = 15^2$ **d** $5^2 + 12^2 = 13^2$
**e** $9^2 + 40^2 = 41^2$ **f** $2.5^2 + 6^2 = 6.5^2$
**8** **a** $a^2 + b^2 = x^2$ **b** $a^2 + b^2 = d^2$
**c** $d^2 + h^2 = x^2$
**9** **a** no
**b** No, $a^2 + b^2 = c^2$ must be true for a right-angled triangle.
**10** **a** yes **b** no **c** no
**d** yes **e** no **f** yes
**11** {(6, 8, 10), (9, 12, 15), (12, 16, 20), (15, 20, 25), (18, 24, 30), (21, 28, 35), (24, 32, 40), (27, 36, 45), (30, 40, 50), (33, 44, 55), (36, 48, 60), (39, 52, 65), (42, 56, 70), (45, 60, 75), (48, 64, 80), (51, 68, 85), (54, 72, 90), (57, 76, 95)}, {(5, 12, 13), (10, 24, 26), (15, 36, 39), (20, 48, 52), (25, 60, 65), (30, 72, 78), (35, 84, 91)}, {(7, 24, 25), (14, 48, 50), (21, 72, 75)}, {(8, 15, 17), (16, 30, 34), (24, 45, 51), (32, 60, 68), (40, 75, 85)}, {(9, 40, 41), (18, 80, 82)}, {(11, 60, 61)}, {(20, 21, 29), (40, 42, 58), (60, 63, 87)}. {(12, 35, 37), (24, 70, 74)}, {(28, 45, 53)}. {(33, 56, 65)}, {(16, 63, 65)}, {(48, 55, 73)}, {(13, 84, 85)}, {(36, 77, 85)}, {(39, 80, 89)}, {(65, 72, 97)}

## Exercise 3J

**1** **a** yes **b** no **c** no **d** yes
**2** **a** 3.16 **b** 5.10 **c** 8.06 **d** 15.17
**3** **a**
$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 5^2 + 12^2 \\ &= 169 \\ \therefore c &= \sqrt{169} \\ &= 13 \end{aligned}$$
**b**
$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 9^2 + 40^2 \\ &= 1681 \\ \therefore c &= \sqrt{1681} \\ &= 41 \end{aligned}$$
**c**
$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 9^2 + 12^2 \\ &= 225 \\ \therefore c &= \sqrt{225} \\ &= 15 \end{aligned}$$
**4** **a** 5 **b** 25 **c** 41
**d** 20 **e** 45 **f** 61
**5** **a** 9.22 **b** 5.39 **c** 5.66
**d** 3.16 **e** 4.30 **f** 37.22
**6** 3.16 m or 316 cm
**7** 139 cm
**8** 5.5 km
**9** 3.88 cm
**10** **a** 2nd line is incorrect, cannot take the square root of each term.
**b** 2nd line is incorrect, cannot add $3^2 + 4^2$ to get $7^2$.
**c** Last line should say $\therefore\ c = \sqrt{29}$.
**11** **a** $3^2 + 5^2 \neq 7^2$ **b** $5^2 + 8^2 \neq 10^2$ **c** $12^2 + 21^2 \neq 24^2$
**12** **a** 8.61 m **b** 48.59 cm **c** 18.56 cm
**d** 22.25 mm **e** 14.93 m **f** 12.25 m

## Exercise 3K

**1** **a** 4 **b** 7 **c** 3
**d** 4 **e** 8 **f** 20
**g** 3 **h** 5 **i** 25
**2** **a** $a = 12$ **b** $b = 24$
**3** **a** 4 **b** 9 **c** 40
**d** 15 **e** 16 **f** 60
**4** **a** 2.24 **b** 4.58 **c** 11.49
**d** 12.65 **e** 10.72 **f** 86.60
**5** 8.94 m
**6** 12 cm
**7** 12.12 cm
**8** 8.49
**9** **a** Should subtract not add 10.
**b** Should say $a = 5$.
**c** Cannot take the square root of each term.
**10** **a** $\sqrt{24}$ **b** $\sqrt{3}$ **c** $\sqrt{4400}$
**11** **a** 3.54 **b** 7.07 **c** 43.13 **d** 24.04

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**12** **a** $6^2 + 8^2 = 10^2$
**b** It is a multiple of (3, 4, 5).
**c** (9, 12, 15), (12, 16, 20), (15, 20, 25)
**d** (8, 15, 17)
**e** (3, 4, 5), (5, 12, 13), (8, 15, 17), (7, 24, 25), (9, 40, 41), etc.

## Puzzles and games

**1** **a** 240 **b** 56 **c** 50
**2** **a** 12.5 **b** 6.3 **c** 7
**3** 10 cm each side
**4** Yes, 1 L will overflow.
**5** 75.4 cm$^2$
**6** $\frac{1}{2}$
**7** 78.5%
**8** 3 cm

## Chapter review

## Multiple-choice questions

**1** E **2** B **3** A **4** C **5** B
**6** E **7** E **8** B **9** D **10** D

## Short-answer questions

**1** **a** 2000 mm **b** 500 m **c** 0.32 km **d** 40 m
**e** 300 mm$^2$ **f** 0.4 m$^2$ **g** 10 000 m$^2$ **h** 3.5 cm$^2$
**i** 4 L **j** 3000 mm$^3$ **k** 0.4 L **l** 4.3 ML
**2** **a** 13 m **b** 28 cm **c** 25.13 m
**d** 12.57 m **e** 30.6 km **f** 25.8 m
**g** 51.42 mm **h** 48 m **i** 20 cm
**3** **a** 55 cm$^2$ **b** 63 m$^2$ **c** 12 cm$^2$
**d** 9 cm$^2$ **e** 201.06 km$^2$ **f** 136 km$^2$
**g** 64 m$^2$ **h** 20 cm$^2$ **i** 28.27 cm$^2$
**4** **a** 9 L **b** 4.5 L **c** 1000 L
**5** **a** 1 m$^3$ **b** 8000 cm$^3$ **c** 10 m$^3$
**d** 144 cm$^3$ **e** 40 cm$^3$ **f** 6 cm$^3$
**6** **a** $x = 15$ **b** $x = 10$
**7** **a** 10 h 17 min **b** 9:45 p.m. **c** 2331 hours
**8** **a** 6:30 p.m. **b** 6:00 p.m. **c** 6:00 p.m.
**d** 4:30 p.m. **e** 4:30 p.m. **f** 4:30 p.m.
**g** 8:30 p.m. **h** 6:30 p.m. **i** 6:30 p.m.

## Extended-response questions

**1** **a** 160 m$^2$ **b** 56 m **c** 12.57 m$^2$ **d** 147.43 m$^2$
**e** **i** 100 cm$^2$ **ii** 0.01 m$^2$ **f** 14 744 tiles
**g** Some tiles will break and some tiles around the edge of the pond will have pieces cut off and thrown away.
**2** **a** 70 cm$^2$ **b** 50.14 cm$^2$ **c** 74 cm$^2$

# Chapter 4

## Pre-test

**1** **a** mixed numeral **b** proper
**c** improper **d** improper
**2** **a** 4 **b** 8 **c** 20
**3** **a** 3 **b** 9 **c** 3 **d** 8
**4** **a** 100 **b** 1 **c** 4 **d** 1
**e** 30 **f** 60 **g** 100 **h** 4
**5** **a** $\frac{1}{4}$ **b** $\frac{3}{7}$
**6** **a** D **b** C **c** E
**d** A **e** B
**7** **a** $\frac{3}{4}$ **b** 1 **c** $1\frac{2}{3}$
**d** 0.6 **e** 1.2 **f** 3
**8** **a** **i** $\frac{1}{10}$ **ii** 0.1 **b** **i** $\frac{1}{4}$ **ii** 0.25
**c** **i** $\frac{1}{2}$ **ii** 0.5 **d** **i** $\frac{3}{4}$ **ii** 0.75
**9** **a** \$5 **b** \$6.60 **c** 0.8 km **d** 690 m
**10** **a** 10 **b** 18 **c** 90 cents
**11**

| **Fraction** | $\frac{3}{4}$ | $\frac{1}{5}$ | $\frac{3}{20}$ | $\frac{2}{5}$ | $\frac{99}{100}$ | 1 | $\frac{8}{5}$ | 2 |
|---|---|---|---|---|---|---|---|---|
| **Decimal** | 0.75 | 0.2 | 0.15 | 0.4 | 0.99 | 1.0 | 1.6 | 2.0 |
| **Percentage** | 75% | 20% | 15% | 40% | 99% | 100% | 160% | 200% |

## Exercise 4A

**1** **a** $\frac{6}{10}, \frac{9}{15}$ **b** 14, 16, $\frac{32}{56}$ **c** 50, 20, 5, 2 **d** 6, 9, 12
**2** $\frac{10}{15}$
**3** ***b***, ***c*** and ***e***
**4** **a** false **b** true **c** true
**d** true **e** true **f** false
**5** **a** 8 **b** 6 **c** 12 **d** 10
**e** 20 **f** 120 **g** 18 **h** 21
**6** **a** 6 **b** 10 **c** 15 **d** 90
**e** 20 **f** 11 **g** 75 **h** 15
**7** **a** 2 **b** 20 **c** 10 **d** 30
**e** 18 **f** 4 **g** 3 **h** 9
**i** 6 **j** 18 **k** 2 **l** 7
**m** 28 **n** 50 **o** 15 **p** 44
**8** **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{4}{5}$ **d** $\frac{7}{10}$
**e** $\frac{1}{3}$ **f** $\frac{1}{2}$ **g** $\frac{5}{6}$ **h** $\frac{5}{6}$
**i** $\frac{1}{4}$ **j** $\frac{3}{5}$ **k** $\frac{8}{9}$ **l** $\frac{5}{7}$
**m** $\frac{5}{3}$ **n** $\frac{11}{10}$ **o** $\frac{6}{5}$ **p** $\frac{4}{3}$
**9** **a** $\frac{9}{10}$ **b** $1\frac{1}{3}$ **c** $\frac{1}{4}$
**d** $\frac{7}{13}$ **e** $\frac{2}{3}$ **f** $\frac{1}{2}$
**10** **a** $\frac{1}{2}$ or $\frac{2}{4}$ **b** $\frac{1}{3}$ or $\frac{2}{6}$ **c** $\frac{1}{4}, \frac{4}{16}$ or $\frac{2}{8}$ **d** $\frac{2}{3}$ or $\frac{6}{9}$

**Exercise 4A cont.**

**11 a** Mary ate the most (125 grams) **b** $\frac{1}{4}$

**12** Answers vary some include: $\frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{10}{50}, \frac{20}{100}$

**13** $\frac{18}{28}$ as it does not simplify to $\frac{3}{4}$

**14** E.g. $\frac{1}{3}$ becomes $\frac{3}{5}$, which is bigger but what about improper fractions?

## Exercise 4B

**1 a** +, − **b** ×, ÷

**2 a** 20 **b** 9 **c** 50 **d** 24

**3 a** 3, 12 **b** 14, 5
**c** 11, 33 **d** ×, 14, 1, 1

**4 a** $\frac{8}{5}$ **b** $\frac{2}{3}$ **c** $\frac{4}{13}$ **d** $\frac{11}{12}$

**5 a** $\frac{2}{3}$ **b** $\frac{1}{2}$ **c** $\frac{1}{12}$ **d** $\frac{2}{5}$
**e** $\frac{3}{5}$ **f** $\frac{5}{9}$ **g** $1\frac{1}{2}$ **h** $1\frac{6}{7}$
**i** $1\frac{3}{20}$ **j** $1\frac{1}{10}$ **k** $\frac{1}{21}$ **l** $\frac{4}{9}$

**6 a** $4\frac{4}{7}$ **b** $9\frac{3}{5}$ **c** $2\frac{3}{8}$ **d** $1\frac{2}{11}$
**e** $9\frac{1}{2}$ **f** $22\frac{3}{14}$ **g** $3\frac{3}{4}$ **h** $1\frac{17}{30}$

**7 a** $\frac{3}{20}$ **b** $\frac{10}{63}$ **c** $1\frac{17}{25}$ **d** $1\frac{13}{27}$
**e** $\frac{1}{6}$ **f** $\frac{3}{8}$ **g** $\frac{8}{15}$ **h** 5

**8 a** $3\frac{2}{3}$ **b** $1\frac{2}{21}$ **c** 15 **d** 35

**9 a** $\frac{10}{27}$ **b** $\frac{5}{6}$ **c** $\frac{16}{77}$ **d** $1\frac{7}{15}$
**e** $\frac{7}{8}$ **f** 2 **g** $1\frac{1}{3}$ **h** $3\frac{3}{5}$

**10 a** $\frac{33}{35}$ **b** $\frac{48}{125}$ **c** $1\frac{2}{5}$ **d** 3

**11 a** 4 **b** 15 **c** 25 **d** 3

**12 a** $\frac{29}{70}$ **b** $\frac{41}{70}$

**13 a** $\frac{1}{9}$ **b** $6\frac{11}{120}$ **c** $1\frac{9}{56}$ **d** $3\frac{1}{3}$

**14** 60

## Exercise 4C

**1** E

**2** C

**3 a** 37.123 **b** 21.953 **c** 0.0375
**d** 4.218 09 **e** 65.4112 **f** 9.528 135 2

**4 a** true **b** false **c** true
**d** false **e** false **f** false

**5 a** $\frac{3}{10}$ **b** $\frac{3}{100}$ **c** $\frac{3}{1000}$ **d** $1\frac{3}{10}$
**e** $\frac{13}{100}$ **f** $\frac{103}{1000}$ **g** $\frac{13}{1000}$ **h** $\frac{1}{5}$
**i** $\frac{1}{50}$ **j** $\frac{1}{4}$ **k** $\frac{3}{4}$ **l** $\frac{4}{5}$

**6 a** 0.17 **b** 0.301 **c** 0.45 **d** 0.6
**e** 0.67 **f** 0.674 **g** 0.15 **h** 0.79
**i** 0.7 **j** 1.7 **k** 1.18 **l** 0.041

**7 a** 0.6 **b** 0.5 **c** 1.5 **d** 1.4
**e** 0.22 **f** 0.25 **g** 0.75 **h** 0.64

**8** 2.4, 2.3, 2.25, 2.18

**9** A1, B5, C07, P9, BW Theatre, gym

**10** 1st English
2nd Maths
3rd Science

**11** Hint: It is between 10 and 20

**12** Answers vary, one possible is given for each:
**a** 0.7 **b** 0.8 **c** 0.5 **d** 0.6

**13 a**

| | | |
|---|---|---|
| 2.6 | 4.6 | $1\frac{4}{5}$ |
| 2.2 | $\frac{6}{2}$ | 3.8 |
| 4.2 | 1.4 | $3\frac{2}{5}$ |

**b**

| | | | |
|---|---|---|---|
| 0.8 | 1.8 | 1.0 | 3.2 |
| 3.0 | 1.2 | 2.0 | 0.6 |
| 2.8 | 1.4 | 2.2 | 0.4 |
| 0.2 | 2.4 | 1.6 | 2.6 |

## Exercise 4D

**1** B **2** E **3** C **4** B

**5 a** 6.8 **b** 10.5 **c** 21.9 **d** 10.2
**e** 16.3 **f** 13.2 **g** 62.71 **h** 277.99
**i** 23.963 **j** 94.172 **k** 60.71 **l** 6.71

**6 a** 4.4 **b** 6.3 **c** 15.3 **d** 4.1
**e** 6.1 **f** 4 **g** 14.41 **h** 23.12
**i** 84.59 **j** 4.77 **k** 92.1 **l** 80.411

**7 a** 96.1 **b** 961 **c** 15 463
**d** 1.94 **e** 0.194 **f** 2.74
**g** 0.0274 **h** 1600 **i** 3651.73
**j** 81.55 **k** 0.75 **l** 0.038 12
**m** 6 348 000 **n** 0.001 061 5 **o** 30
**p** 0.000 452

**8 a** 5.6 **b** 0.56 **c** 1.5 **d** 0.12
**e** 30.8 **f** 0.36 **g** 0.32 **h** 0.032
**i** 3 **j** 4.9 **k** 8.1 **l** 1.44

**9 a** 12.27 **b** 5.88 **c** 0.0097 **d** 49.65
**e** 66.72 **f** 1228.15 **g** 0.322 655 **h** 3.462

**10 a** 52 **b** 620 **c** 150.6, 75.3
**d** 3, 1530 **e** 4.84, 1.21

**11** 7.12 m

**12 a** \$7.60, \$8.20, \$13.50 **b** \$12.40, \$11.80, \$6.50

**13** Answer comes from the puzzle – ask your teacher if your answer does not make sense.

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## Exercise 4E

**1** **a** T **b** R **c** R **d** T
**e** T **f** R **g** T **h** R

**2** **a** $0.\dot{3}$ or $0.\overline{3}$ **b** $6.\dot{2}\dot{1}$ or $6.\overline{21}$
**c** $8.576\dot{4}$ or $8.576\overline{4}$ **d** $2.1\dot{3}5\dot{6}$ or $2.1\overline{356}$
**e** $11.\dot{2}857\dot{3}$ or $11.\overline{28573}$ **f** $0.00\dot{3}5\dot{2}$ or $0.00\overline{352}$

**3** **a** 4 **b** 9 **c** 7 **d** 6

**4** **a** 5.5 **b** 7.42 **c** 0.4 **d** 2.0

**5** **a** 0.6 **b** 0.75 **c** 0.125 **d** 0.55
**e** 0.5 **f** 0.8 **g** 0.04 **h** 0.18

**6** **a** $0.\dot{3}$ **b** $0.\dot{5}$ **c** $0.8\dot{3}$
**d** $0.\dot{7}$ **e** $0.\overline{428571}$ **f** $0.1\dot{6}$
**g** $1.\dot{3}$ **h** $1.\dot{8}5714\dot{2}$

**7** **a** 0.6 **b** 0.8 **c** 1.5
**d** 8.2 **e** 9.5 **f** 8.3
**g** 1.5 **h** 3.4 **i** 0.3

**8** **a** 0.78 **b** 0.67 **c** 1.48
**d** 0.89 **e** 15.49 **f** 9.04
**g** 9.42 **h** 8.75 **i** 1.79

**9** **a** **i** 8 **ii** 8 **iii** 5
**b** **i** 5.0 **ii** 8.9 **iii** 6.0

**10**

| Fraction | Decimal | 1 dec pl | 2 dec pl |
|---|---|---|---|
| $\frac{1}{3}$ | 0.333… | 0.3 | 0.33 |
| $\frac{2}{3}$ | 0.666… | 0.7 | 0.67 |
| $\frac{5}{6}$ | 0.555… | 0.6 | 0.56 |
| $\frac{1}{6}$ | 0.166… | 0.2 | 0.17 |

**11** **a** Greer by 0.06 of a second
**b** 12.8 for both, as they are the same to 1 decimal place you can't tell who came first.
**c** 12.75,12.76,12.78,12.79,12.80,12.81,12.82,12.84

**12** Answers vary, some include: 3.451, 3.446, 3.450 71, 3.449, 3.451

**13** **a** \$43.15 **b** \$10 **c** **i** \$0.25 **ii** \$520
**d** **i** 15.0 **ii** 3.5
**e** The water is more expensive.

## Exercise 4F

**1** B **2** B **3** C

**4**

| Fraction | Decimal in words | Decimal in figures | Percent in words | Percent in figures |
|---|---|---|---|---|
| $\frac{13}{100}$ | thirteen hundredths | 0.13 | thirteen percent | 13% |
| $\frac{45}{100}=\frac{9}{20}$ | forty-five hundredths | 0.45 | forty-five percent | 45% |
| $\frac{70}{100}=\frac{7}{10}$ | seven tenths | 0.7 | seventy percent | 70% |
| $\frac{99}{100}$ | ninety-nine hundredths | 0.99 | ninety-nine percent | 99% |

**5** **a** $\frac{39}{100}$ **b** $\frac{11}{100}$ **c** $\frac{17}{100}$ **d** $\frac{99}{100}$
**e** $\frac{1}{5}$ **f** $\frac{7}{10}$ **g** $\frac{3}{4}$ **h** $\frac{11}{20}$

**6** **a** 0.39 **b** 0.11 **c** 0.17 **d** 0.99
**e** 0.2 **f** 0.7 **g** 0.75 **h** 0.55
**i** 0.07 **j** 0.01 **k** 0.1 **l** 0.47

**7** **a** 77% **b** 49% **c** 75% **d** 80%
**e** 28% **f** 45% **g** 55% **h** 38%
**i** 94% **j** 70% **k** 120% **l** 150%

**8** **a** 16% **b** 79% **c** 83% **d** 97%
**e** 3% **f** 33% **g** 91% **h** 9%
**i** 12.5% **j** 37.5% **k** 125% **l** 106%

**9** **a** $85\% = 0.85 = \frac{85}{100} = \frac{17}{20}$
**b** $35\% = 0.35 = \frac{35}{100} = \frac{7}{20}$
**c** $80\% = 0.8 = \frac{80}{100} = \frac{4}{5}$
**d** $125\% = 1.25 = \frac{125}{100} = \frac{5}{4}$
**e** $16\frac{2}{3}\% = 0.1\dot{6} = \frac{1}{6}$

**10** **a** 70% **b** $3 \times 20\% = 60\%$
**c** $7 \times 12.5\% = 87.5\%$ **d** 150%

**11** **a** $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}, \frac{1}{512}$
**b**
1
0.5
0.25
0.125
0.0625
0.03125
0.015625
0.0078125
0.00390625
0.001953125
**c** 1.998046875

**12** 65%, 80%

**13**

| Cent per 100 cents | Cents in the dollar | Percentage |
|---|---|---|
| 5 c | \$0.05 | 5% |
| 10 c | \$0.10 | 10% |
| 9 c | \$0.09 | 9% |
| 17 c | \$0.17 | 17% |
| 25 c | \$0.25 | 25% |
| 70 c | \$0.70 | 70% |
| 90 c | \$0.90 | 90% |
| 75 c | \$0.75 | 75% |
| 100 c | \$1.00 | 100% |
| 200 c | \$2 | 200% |

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## Exercise 4G

**1** D

**2** A

**3** **a** half the test correct, 50%
**b** no answers correct, 0%
**c** every answer correct, 100%

**4** **a** 100 **b** 10 **c** 5
**d** 2 **e** 4

**5** **a** 80% **b** 65% **c** 78%
**d** 68% **e** 60% **f** 98%
**g** 70% **h** 40% **i** 75%
**j** 80% **k** 60% **l** 75%

**6** **a** 5% **b** 25% **c** 5%
**d** 25% **e** 4% **f** 2.5%

**7** **a** 56% **b** 75% **c** 86%
**d** 25% **e** 40% **f** 50%

**8** **a** 18 **b** 8 **c** 150 **d** 18
**e** 8 **f** 12 **g** 60 **h** 22
**i** 12.5 **j** 3 **k** 300 **l** 7.2
**m** 3 **n** 198 **o** 720

**9** **a** $24.11 **b** $2345 **c** $0.84 **d** $2000

**10** **a** $75 **b** 100 m **c** 45 kg
**d** 18 minutes **e** 500 mL **f** 15 minutes
**g** $3.25 **h** 16 cents **i** 35 g

**11**

| | |
|---|---|
| $\frac{1}{5}$ | 20% |
| $\frac{3}{20}$ | 15% |
| $\frac{7}{20}$ | 35% |
| $\frac{1}{4}$ | 25% |
| $\frac{1}{20}$ | 5% |

**12** **a** $31 500 **b** $45 000
**c** $13 500 **d** Yes $500 more

**13** **a** 67 c **b** 260 m **c** $36.25
**d** 14 min 24 s (14.4 min) **e** $14 477.40
**f** 24 **g** 101 **h** $50 112

## Exercise 4H

**1** **a** decrease **b** increase **c** decrease
**d** increase **e** increase

**2** **a** 120% **b** 115% **c** 90% **d** 85%

**3** **a** $12 **b** $33.99 **c** $14.50 **d** $225

**4** **a** $440 **b** $264 **c** $275 **d** $840
**e** $505 **f** $1000 **g** $105 **h** $135

**5** **a** $360 **b** $216 **c** $225 **d** $72
**e** $170 **f** $630 **g** $500 **h** $51

**6** **a** $200 **b** $2700 **c** $2300

**7** **a** $480 **b** $127.50 **c** $39

**8** **a** $12 **b** $24 **c** $37.50
**d** $63.75 **e** $97.50 **f** $4.95

**9** **a** $88 **b** $15.40 **c** $210

**10** **a** $90 300 **b** $10.08 **c** $37 600
**d** $81.40 **e** $960 **f** $620 000

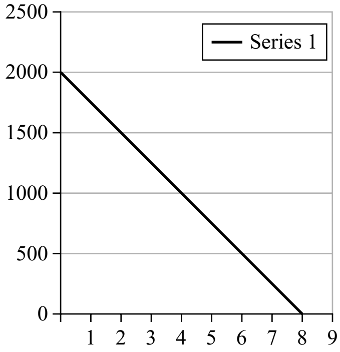
**11** **a** $180
**b** shop 1 = $1620, shop 2 = $1600
**c** 2 as the bike is cheaper
**d** **i** Bikes are the same price so either shop is recommended
**ii** Shop 1 is now cheaper $1980 versus $2000

**12** Lower

**13** **a** **i**

| | |
|---|---|
| 0 | 2000 |
| 1 | 1750 |
| 2 | 1500 |
| 3 | 1250 |
| 4 | 1000 |
| 5 | 750 |
| 6 | 500 |
| 7 | 250 |
| 8 | 0 |

**ii**



**iii** straight line **iv** after 8 years

**b** **i**

| | |
|---|---|
| 3 | $1339.84 |
| 4 | $1172.36 |

**ii** never

## Exercise 4I

**1** **a** $5 **b** $16 **c** $25 **d** $70
**e** $1.50 **f** $8.80 **g** $0.50 **h** $0.25

**2** **a** $10, $110 **b** $5, $55 **c** $15, $165
**d** $0.50, $5.50 **e** $0.10, $1.10 **f** $12, $132

**3** $80

**4** C

**5** **a** $68 **b** $400 **c** $55
**d** $2.80 **e** $35.70 **f** 57c

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**6** **a** $770 **b** $3300 **c** $495
**d** $37.40 **e** $62 370 **f** $5.39

**7** **a** $200 **b** $60 **c** $8000
**d** $110 **e** $100 **f** $0.90

**8** **a** $320 **b** $968 **c** $460
**d** $47.50 **e** 9810 **f** $5.60

**9**

| Pre-GST price | 10% GST | Final cost including the 10% GST |
|---|---|---|
| $599 | $59.90 | $658.90 |
| $680 | $68 | $748 |
| $700 | $70 | $770 |
| $600 | $60 | $660 |
| $789 | $78.90 | $867.90 |
| $892 | $89.20 | $981.20 |
| $645 | $64.50 | $709.50 |
| $87.25 | $8.73 | $95.98 |

**10**

| Superbarn | Gymea fruit market | Xmart |
|---|---|---|
| **a** $16.85<br>**b** 0.27 kg<br>**c** marshmallows – marked with<br>**d** 1.89 – 0.17 = $1.72 | **a** $14.99 per kg<br>**b** 5th July 2011<br>**c** taking the cash amount to the nearest 5 cents<br>**d** $9.85<br>**e** 55 cents<br>**f** 5.6% (1 dec pl) | **a** 8<br>**b** $35<br>**c** all toys<br>**d** $13.55<br>**e** 9.09% |

**11** **a** $24 **b** $240
**c** Dividing the post-GST price by 11 gives the GST included in the price as 11 × 10% gives 110%
**d** As 1.1 × 100 gives 110% so dividing by 1.1 yields 100%

**12** **a** $56 **b** $97 **c** $789 **d** $9.87

**13** Example, if the price is $100 it goes up to $110, but 10% of this is not $10, so it does not go back down to $100.

**14** 1.07

**15** Raw material $110 (includes the 10% GST)
GST on sale = $10
GST credit = $0
Net GST to pay = $10
Production stage $440 (includes the 10% GST)
GST on sale = $40
GST credit = $10
Net GST to pay = $30
Distribution stage $572 (includes the 10% GST)
GST on sale = $52
GST credit = $40
Net GST to pay = $12
Retail stage $943.80 (includes the 10% GST)
GST on sale = $85.80
GST credit = $52
Net GST to pay = $33.80
GST paid by the final consumer = $85.80

## Exercise 4J

**1** **a** profit **b** loss **c** loss **d** profit **e** profit

**2** **a** $7 **b** $28 **c** $3.45 **d** $436

**3** **a** $13 **b** $45 **c** $25.90 **d** $247

**4** D

**5** **a** 80% **b** 30% **c** 25% **d** 20%
**e** $66\frac{2}{3}\%$ **f** $37\frac{1}{2}\%$ **g** 50% **h** 100%

**6** **a** 25% **b** 16% **c** 50% **d** 75%
**e** $33\frac{1}{3}\%$ **f** 10% **g** 20% **h** 10%

**7** **a**

| | | | |
|---|---|---|---|
| 4 | 5 | 1 | 25% |
| 10 | 12 | 2 | 20% |
| 24 | 30 | 6 | 25% |
| 100 | 127 | 27 | 27% |

**b**

| | | | |
|---|---|---|---|
| 10 | 7 | 3 | 30% |
| 16 | 12 | 4 | 25% |
| 50 | 47 | 3 | 6% |
| 100 | 93 | 7 | 7% |

**8** **a** 20% increase **b** $16\frac{2}{3}\%$ decrease
**c** 500% increase **d** 150% increase

**9** **a** 25% increase **b** 20% increase
**c** 140% increase

**10** 20% loss

**11** **a** $36 **b** 75% profit

**12** **a** $320 **b** 80%

**13** **a** $2200 **b** 44% loss **c** $5500

**14** **a**

| | March 2011 | Change in the past 12 months | % Change |
|---|---|---|---|
| NSW | 7 287 600 | 82 100 | 1.1% |
| VIC | 5 605 600 | 81 600 | 1.5% |
| QLD | 4 561 700 | 73 200 | 1.6% |
| SA | 1 654 200 | 13 900 | 0.8% |
| WA | 2 331 500 | 51 000 | 2.2% |
| TAS | 510 200 | 3 200 | 0.6% |
| NT | 229 200 | 900 | 0.4% |
| ACT | 363 800 | 6 400 | 1.8% |
| AUSTRALIA | 22 546 300 | 312 400 | 1.4% |

## Exercise 4K

**1** **a** 8 **b** 25 **c** 100 **d** 50
**2** **a** $6 **b** $30 **c** $300
**3** **a** $80 **b** $800
**4** $4, $400
**5** **a** $900 **b** $800 **c** $1100
**d** $500 **e** $550 **f** $250
**6** $90
**7** **a** $120 **b** $240 **c** $15 **d** $21
**8** $300
**9** **a** $50 **b** $150 **c** $600
**d** $30 **e** $10 **f** $2000
**10** $75
**11** $282
**12** D
**13** **a** No **b** No, it went to $118.80
**c** 9.09% **d** 8 years

## Puzzles and games

**1** Answers vary, some include: 2.6701, 2.666, 2.668, 2.6712…
**2** 10
**3** $\frac{1}{2}$, 50%, 0.5, $\frac{2}{4}$, $\frac{10}{20}$ etc.
**4** **a**

| | | |
|---|---|---|
| $\frac{4}{3}$ | 3 | $\frac{2}{3}$ |
| 1 | $1\frac{2}{3}$ | $\frac{7}{3}$ |
| $2\frac{2}{3}$ | $\frac{1}{3}$ | 2 |

**b**

| | | |
|---|---|---|
| $\frac{5}{3}$ | $\frac{5}{2}$ | $\frac{4}{3}$ |
| $\frac{3}{2}$ | $\frac{11}{6}$ | $2\frac{1}{6}$ |
| $\frac{7}{3}$ | $\frac{7}{6}$ | 2 |

**5** See teacher if your answer to the puzzle does not make sense.

## Chapter review

## Multiple-choice questions

**1** B **2** D **3** D **4** C **5** A
**6** B **7** B **8** A **9** C **10** D

## Short-answer questions

**1** **a** 21 **b** 8 **c** 12
**2** **a** $\frac{5}{9}$ **b** 3 **c** $1\frac{1}{3}$
**3** **a** $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{3}{8}$ **d** $\frac{4}{5}$
**e** $\frac{1}{8}$ **f** $\frac{3}{4}$ **g** $\frac{2}{3}$ **h** $1\frac{3}{10}$
**4** **a** $1\frac{3}{4}$ **b** 4 **c** $6\frac{1}{2}$ **d** $5\frac{1}{5}$
**5** **a** 4 **b** 2 **c** 8 **d** 12
**6** **a** $\frac{1}{6}$ **b** $\frac{1}{10}$ **c** $\frac{7}{20}$ **d** $\frac{1}{3}$
**7** **a** 12 **b** 2 **c** $1\frac{3}{5}$ **d** 2
**8** **a** 0.5 **b** 0.25 **c** 0.6 **d** 0.117
**9** **a** $\frac{3}{5}$ **b** $\frac{3}{25}$ **c** $\frac{1}{25}$ **d** $\frac{19}{20}$
**10** **a** 20 **b** 14.19 **c** 8.2
**d** 4.6 **e** 22.91 **f** 6.18
**11** **a** 6 **b** 0.06 **c** 4.8 **d** 0.048
**e** 0.6 **f** 716.4 **g** 96 **h** 0.42
**12** **a** 40 **b** 6.2 **c** 71.1
**13** **a** 0.667 **b** 3.580 **c** 0.005
**14**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0.1 | 0.01 | 0.05 | 0.5 | 0.25 | 0.75 | $0.\dot{3}$ | 0.125 |
| $\frac{1}{10}$ | $\frac{1}{100}$ | $\frac{1}{20}$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{3}$ | $\frac{1}{8}$ |
| 10% | 1% | 5% | 50% | 25% | 75% | $33\frac{1}{3}\%$ | 12.5% |

**15** **a** $5 **b** $16 **c** 35 g
**16** **a** 87.5% **b** 25% **c** 150% **d** 4%
**17** **a** $616 **b** $3400
**18** $300
**19** $155.20
**20** $88 200

## Extended-response question

**1** **a** 1200 **b** 24 **c** 360
**d** 125 **e** 40

# Chapter 5

## Pre-test

**1** **a** $\frac{2}{4}=\frac{1}{2}$ **b** $\frac{15}{20}=\frac{3}{4}$ **c** $\frac{12}{15}=\frac{4}{5}$
**2** **a** 2 : 5 = 4 : 10 **b** 20 : 28 = 5 : 7 **c** 3 : 2 = 12 : 8
**3** **a** 3 : 4 **b** 4 : 5 **c** 5 : 7
**4** **a** 500 **b** 6000 **c** 5
**d** 8 **e** 1.2 **f** 15
**5** **a** 2 : 1 **b** 9 : 20 **c** 3 : 4
**d** 1 : 3 **e** 4 : 1 **f** 2 : 25
**6** **a** $1.50 **b** $18.00
**7** **a** $9.98 **b** $24.95 **c** $49.90 **d** $2.50
**8** 1.5 km
**9** $30/h
**10** **a** **i** 120 km **ii** 300 km **iii** 30 km
**b** **i** 3 hours **ii** $1\frac{1}{2}$ hours **iii** $\frac{1}{3}$ hour
**c** 6 minutes

## Exercise 5A

**1** **a** 1 : 1 **b** 1 : 5 **c** 4 : 1
**2** **a** 3 : 7 **b** 5 : 4
**3** **a** 9 : 4 **b** 7 : 12 **c** 10 : 75 (or 2 : 15)
**4** **a** 1 : 3 **b** 7 : 15
**5** **a** 5 : 7 **b** 12 **c** 5 : 12
**6** **a** 8 : 3 **b** 3 : 14 **c** 3 : 11 **d** 8 : 6

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**7** **a** 13:7 **b** 11:9

**c** 13:9:11:7 **d** 20:20 or 1:1

**8** **a** 12 **b** 6 **c** 18 **d** 8

**e** 2 **f** 32

**9** **a** 6 **b** 3 **c** 6 **d** 4

**e** 1 **f** 3

**10** **a** 12 **b** 14 **c** 4 **d** 9

**e** 2 **f** 4 **g** 2 **h** 10

**i** 4:6:10 **j** 2:6:8

**11** Answers may vary.

**a** 2:4, 3:6, 5:10 **b** 4:10, 20:50, 200:500

**c** 4:3, 16:12, 40:30 **d** 3:1, 6:2, 18:6

**12** 2:5 and 4:10, 6:12 and 1:2, 7:4 and 70:40

**13** **a** 4:6 **b** 3:5 **c** 4:6 **d** 5:4

**14** **a** 8 boys, 4 girls **b** 4 boys, 8 girls

**c** 9 boys, 3 girls **d** 2 boys, 10 girls

**15** **a** 5 **b** 19

**16** **a** 4:21 **b** 3:7

**17** **a** Answers will vary.

**b** Each ratio length : area simplifies to 1 : breadth

## Exercise 5B

**1** **a** $\frac{1}{2}$ **b** $\frac{3}{5}$ **c** $\frac{1}{3}$ **d** $\frac{3}{7}$ **e** $\frac{8}{5}$

**2** **a** 1:2 **b** 3:5 **c** 1:3 **d** 3:7 **e** 8:5

**3** 3:2

**4** **a** 1:1 **b** 1:2

**5** **a** 1:4 **b** 1:5 **c** 1:6 **d** 1:3

**e** 4:5 **f** 5:8 **g** 3:4 **h** 3:10

**i** 9:7 **j** 2:1 **k** 9:7 **l** 3:1

**m** 3:1 **n** 1:9 **o** 6:11 **p** 2:1

**q** 12:1 **r** 1:6 **s** 8:5 **t** 6:5

**6** **a** 1:2:3 **b** 4:7:11 **c** 7:10:2 **d** 17:7:3

**e** 1:2:3 **f** 2:6:5 **g** 9:14:2 **h** 2:4:7

**7** **a** 2:5 **b** 14:1 **c** 3:25 **d** 1:35

**e** 20:3 **f** 2:25 **g** 50:11 **h** 5:1

**i** 2:5 **j** 1:6 **k** 12:1 **l** 9:1

**m** 1:16 **n** 2:9 **o** 1:7 **p** 14:3

**q** 1:8 **r** 30:1

**8** **a** 1:2 **b** 8:1 **c** 2:1

**d** 7:10 **e** 10:1 **f** 11:13

**g** 3:2 **h** 6:5 **i** 4:9

**9** D **10** B **11** C

**12** **a** 5:5:2:4:3:1:20

**b** 20:20:8:16:12:4:80

**c** **i** 1:4 **ii** 1:1

**13** Andrew did not convert amounts to the same units. Correct ratio is 40:1.

**14** Answers may vary.

**a** 2 hours to 300 minutes, 24 minutes to 1 hour

**b** 4 kilometres to 3000 metres, 2 kilometres to 1500 metres

## Exercise 5C

**1** **a** 10 **b** 6 **c** 14 **d** 9

**2** **a** In the ratio 3:2 the total parts = 2 + 3 = 5

**b** 5 parts = \$30, so 1 part = \$6

**c** Marta gets 3 parts, so Marta gets 3 × \$6 = \$18

**d** Joshua gets 2 parts, Joshua gets 2 × \$6 = \$12

**3** **a** 1:3 **b** 1:1 **c** 2:5 **d** 1:4

**4** **a** 4 **b** 2 **c** 7 **d** 5

**5** **a** \$24 and \$36 **b** \$70 and \$40

**c** \$150 and \$850 **d** 8 kg and 40 kg

**e** 8 kg and 6 kg **f** 150 kg and 210 kg

**g** 24 m and 48 m **h** 15 m and 25 m

**i** 124 m and 31 m

**6** **a** \$100 and \$300 **b** \$160 and \$240

**c** \$150 and \$250 **d** \$180 and \$220

**7** **a** \$40, \$80, \$80 **b** \$50, \$150, \$200

**c** 2 kg, 4 kg, 6 kg **d** 22 kg, 11 kg, 55 kg

**e** 96 kg, 104 kg, 120 kg

**f** \$5000, \$10 000, \$15 000, \$20 000

**8** **a** 60, 540 **b** 200, 100, 300

**c** 100, 250, 250 **d** 240, 140, 160, 60

**9** nitrogen: 500 g, potassium: 625 g, phosphorus: 375 g

**10** 40°, 60°, 80°

**11** **a** 30 **b** 15 **c** 24

**12** 240 students

**13** 120 pages

**14** shirt \$160, jacket \$400

**15** **a** 2 boys and 2 girls were absent or 5 girls and 9 boys.

**b** 3:5

## Exercise 5D

**1** **i** **a** 100 000 mm **b** 100 m **c** 0.1 km

**ii** **a** 0.56 km **b** 56 000 cm

**c** 560 000 mm

**2** **a** 4 cm, 400 cm **b** real car 100 × bigger

**c** 1:100 **d** 100

**3** **a** 60 cm, 30 000 cm **b** real ship 500 × bigger

**c** 1:500

**4** **a** 50 000 **b** 50 000 cm **c** 50 000m

**d** 250 000 mm **e** 250 000 cm

**5** **a** **i** 620 cm **ii** 5 mm

**b** **i** 200 m **ii** 40 m

**c** **i** 16 km **ii** 25 km

**d** **i** 6.4 m **ii** 288 m

**e** **i** 0.3 mm **ii** 8.15 mm

**6** **a** **i** 1 m **ii** 20 m

**b** **i** 20 m **ii** 2 m

**c** **i** 13.5 cm **ii** 7.365 cm

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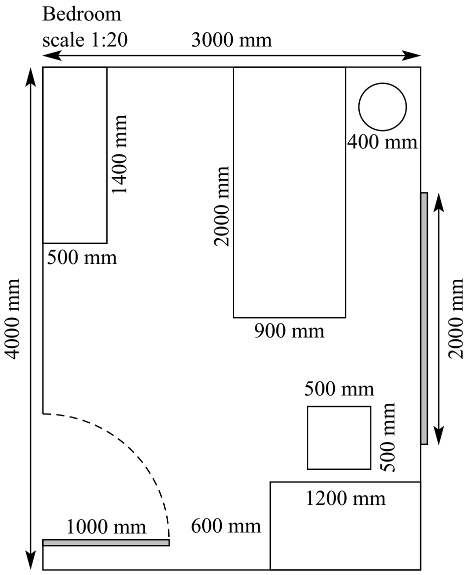
**Exercise 5D** cont.

**d** **i** 20 000 mm **ii** 5 mm
**e** **i** 1.5 m **ii** 0.164 m
**7** **a** 1 : 10 000 **b** 1 : 1000 **c** 1 : 300
**d** 1 : 150 000 **e** 1 : 125 **f** 1 : 200 000
**g** 1 : 100 000 **h** 50 : 1 **i** 10 000 : 1
**8** **a** 1 : 250 **b** 1 : 50 000 **c** 1 : 50 000
**d** 1 : 18 000 **e** 7 : 1 or 1 : $\frac{1}{7}$ **f** 600 : 1 or 1 : $\frac{1}{600}$
**9** **a** 80 m **b** 4.5 cm
**10** 8.5 km
**11** **a** 3.8 m × 2.7 m **b** 5 m × 5 m
**c** 8.3 m × 2.1 m
**12** **a** 2800 km **b** 3300 km **c** 2500 km
**d** 1300 km **e** 3900 km
**13** Note: Different furniture arrangements also correct.



## Exercise 5E

**1** B, C, E, F, H
**2** **a** employee's wage: $15/h
**b** speed of a car: 68 km/h
**c** cost of building new home: \$2100/m$^2$
**d** population growth: 90 people/day
**e** resting heart rate: 64 beats/min
**3** **a** $/kg **b** $/L **c** words per minute
**d** goals/shots on goal **e** kJ/serve **f** L/min
**g** mg/tablet **h** runs/over
**4** **a** 3 days/year **b** 5 goals/game **c** $30/h
**d** $3.50/kg **e** $14 000/acre
**f** 4500 cans/hour **g** 1200 revs/min
**h** 16 mm/day **i** 4 min/km
**j** 0.25 km/min or 250 m/min
**5** **a** 300 km/day **b** $140/year
**c** 6.5 runs/over **d** 7.5 cm/year
**e** 1.5 kg/year **f** dropped 2.5°C/h or –2.5°C/h
**6** **a** 3.8 cm/year **b** 3 cm/year
**7** **a** 3 L/h **b** 7 hours
**8** 158 cm
**9** **a** 1.5 rolls/person **b** $6/person **c** $4/roll
**10** Harvey: 3.75 min/km, Jacques: 3.33 min/km; Jacques
**11** **a** 1200 members/year **b** 12 years
**12** **a** 9 km/L **b** $11.\dot{1}$ L/100 km

## Exercise 5F

**1** **a** 3 hours **b** 5 hours, ×5
**c** ×10, 30 minutes, ×10 **d** ×6, 720 litres, ×6
**2** **a** $12, $60, ×5
**b** ÷5, 30 rotations, ÷5, ×7, 210 rotations, ×7
**3** **a** speed = $\frac{\text{distance}}{\text{time}}$ **b** distance = speed × time
**c** time = $\frac{\text{distance}}{\text{speed}}$
**4** D
**5** **a** 1600 words **b** 50 minutes
**6** **a** 2400 bottles **b** 19 200 bottles
**7** **a** $3.87/100 g **b** Yes ($4.38/100 g)
**c** **i** $57.16 **ii** $5.72/100 g
**8** **a** 10 m/s **b** 7 m/s **c** 60 km/h
**d** 50 km/h **e** 2 km/min, 120 km/h
**f** 0.75 km/min, 45 km/h
**9** **a** 1080 m **b** 4.5 m **c** 36 km **d** 50 km
**10** **a** 8 hours **b** $\frac{1}{2}$ hour or 30 minutes
**c** 11.5 hours **d** 7 seconds
**11** **a** 3750 beats **b** 1380 beats **c** 80 minutes
**12** 2025 km
**13** **a** small $1.25/100 g
medium $1.20/100 g
large $1.10/100 g
**b** 4 large, 1 medium, 1 small, $45.20/100 g
**14** $0.06/100 g
**15** **a** 58.2 km/h **b** 69.4 km/h
**16** **a** 343 m/s **b** 299 792 458 m/s
**c** 0.29 s **d** 0.0003 s **e** 874 030
**f** How many times the speed of sound
(Mach 1 = speed of sound)
**g** 40 320 km/h, 11.2 km/s
**h** 107 208 km/h, 29.78 km/s
**i** 7.7 km/s
**j–l** Answers vary.

## Exercise 5G

**1** Graph i = Journey C; Graph ii = Journey A;
Graph iii = Journey D; Graph iv = Journey B
**2** **a** 240 km **b** 3 hours **c** 80 km/h
**3** **a** $Q$ **b** $P$ **c** $S$ **d** $R$
**e** $S$ **f** $T$ **g** $T$

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**4** **a** A man travelled quickly away from home then stopped for a short time. He then travelled slowly away from home and finally stopped again for a short time.

**b** A boy travelled quickly away from home then stopped for a short time. He then turned around and travelled slowly back to his home.

**c** A girl travelled slowly towards home and part-way back stopped for a short time. She then travelled quickly back to her home.

**5** **a** 80 m **b** 1.5 minutes **c** 160 m
**d** $\frac{1}{2}$ minute **e** 180 m **f** 220 m

**6** **a** segments *b* and *d* **b** 50 s
**c** 20 m **d** 100 s **e** 170 s
**f** 40–60 s and 180 s **g** 160 m
**h** segment *c*, slowest walking speed
**i** segment *e*, fastest walking speed

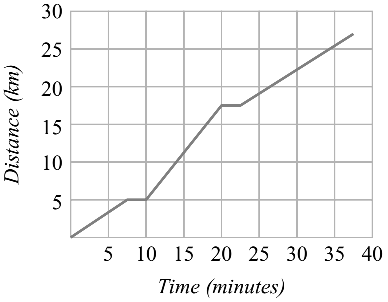
**7** **a** 12:15 p.m. **b** 9.6 km/h **c** 24 km/h
**d** 12:45 p.m., 1:45 p.m. **e** 1.25 hours
**f** 12:15–12:30 p.m. and 2:15 p.m.
**g** 2:45 p.m. **h** 12 km/h **i** 8.5 km/h

**8** **a** 37.5 km/h, 72 km/h, 40 km/h

**b** The three moving sections are over different time intervals and also the whole journey average must include the stops.

**c** 43.8 km/h

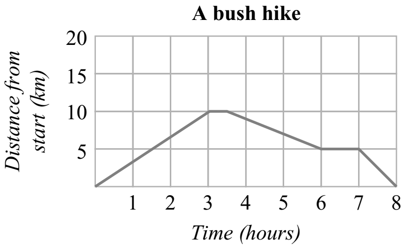
**d**



**9** **a** 2 km/h (uphill) and 5 km/h (downhill).

**b** Many possible stories, for example:
A weekend bushwalk started with a 20 km hike on the first day. The first section of the hike was a 10 km, 3-hour hike to waterfalls where we had a half-hour rest. The hike up the mountain was steep and we progressed slowly needing $2\frac{1}{2}$ hours to cover a distance of 5 km. On the summit of the mountain, we had a one-hour rest. Finally we walked downhill for 5 km taking 1 hour to reach our campsite. On this last downhill section, we achieved our fastest average speed of 5 km/h compared to the initial average speed of 3.3 km/h and the slowest speed of 2 km/h when we were hiking up the mountain.

**c**



**10** **a** True, as these points correspond to the same position on the distance scale.

**b** True, as this point is on a flat line segment.

**c** False, the line segment through A is flatter than the line segment through D.

**d** False, Deanna was riding away from home at C and turned towards home at E.

**e** True as the line segment through F is steeper than the line segment through A.

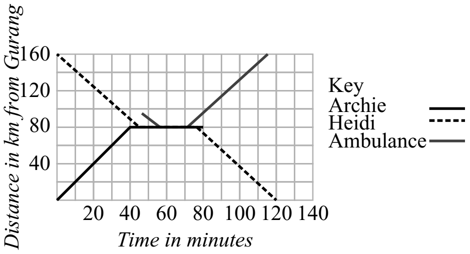
**f** False, as point F corresponds to a smaller distance than point D from home.

**g** True, point F is at a later time on Deanna's return trip home.

**11** Journey A: Isla as she walks from the front and stops twice only.
Journey B: Adam as he has 3 stops and a very fast walk in one section.
Journey C: Conner as he walks to the back of the room twice.
Journey D: Ruby as her walk starts from the back.

**12** Graph A: Incorrect as one rider didn't stop; Graph B: Incorrect as riders did not stop at the same time; Graph C: Incorrect as riders did not stop at the same place; Graph D: correct, Jayden stopped first, Cooper stopped same place same time, both left together.

**13**



## Puzzles and games

**1** **a** toothpicks **b** to rock festivals

**2** **a** 1:1:2:2:2:4:4 **b** **i** 13:3 **ii** 1:3

**3** **a** Hannah 15, Blake 10 **b** Hannah 25, Blake 20
**c** Hannah 55, Blake 50

**4** **a** 2 **b** $3\frac{1}{5}$ **c** $2\frac{2}{3}$

**5** 1 : 3

**6** A flat route (1 h 48 min) faster by 2 minutes.

**7** 9 km/h

**8** because he thought he was a griller.

## Chapter review

### Multiple-choice questions

**1** A **2** C **3** D **4** A **5** B
**6** B **7** C **8** C **9** B **10** D

### Short-answer questions

**1** **a** 1 : 2 **b** 2 : 1 **c** 1 : 3
**2** **a** false **b** false **c** true **d** false
**3** **a** 25 **b** 9 **c** 32 **d** 3
**4** **a** 1 : 4 **b** 3 : 2 **c** 3 : 4 **d** 1 : 8
**e** 3 : 1 **f** 1 : 5 **g** 3 : 2 **h** 2 : 1
**i** 2 : 3 **j** 2 : 1 : 5
**5** **a** 5 : 2 **b** 1 : 3 **c** 2 : 5 **d** 1 : 2
**e** 1 : 5 **f** 1 : 4 **g** 3 : 25 **h** 3 : 10
**6** **a** \$35, \$45 **b** 160 kg, 40 kg
**c** 30 m, 10 m **d** \$340, \$595, \$510
**e** \$60, \$20, \$20
**7** 1.125 L or $1\frac{1}{8}$ L
**8** **a** 600 m **b** 2.4 km
**9** **a** scale ratio = 1 : 200
scale factor = 200
**b** scale ratio = 1 : 250 000
scale factor = 250 000
**10** 50 mm
**11** **a** 5 km/h **b** \$50/h **c** 140 km/day
**12** **a**

$$\begin{tikzcd}\text{7 km uses 1 L} \arrow[d, bend right, "\times 40"'] \arrow[d, bend left, "\times 40"] \\ \text{280 km uses 40 L}\end{tikzcd}$$

**b**

$$\begin{tikzcd}\text{60 words typed in 1 minute} \arrow[d, bend right, "\times 10"'] \arrow[d, bend left, "\times 10"] \\ \text{600 words typed in 10 minutes}\end{tikzcd}$$

**13** **a** **i** 7 km **ii** 294 km
**b** **i** \$5.60 **ii** \$39.20
**14** **a** 75 km/h **b** 1.8 hours **c** 9 km

### Extended-response question

**a** 160 km
**b** 500 km
**c** 11:30 a.m.
**d** 5 hours
**e** 100 km/h
**f** 4:15 p.m.
**g** 82.5 km/h
**h** Harrison's cost \$80.63
Nguyen's cost \$110.86

# Semester review 1

## Algebraic techniques 2 and indices

### Multiple-choice questions

**1** D **2** A **3** B **4** D **5** C

### Short-answer questions

**1** **a** $p + q$ **b** $3p$ **c** $\frac{m^2}{2}$ **d** $\frac{x + y}{2}$
**2** **a** 19 **b** 68 **c** 33 **d** 698
**3** **a** 11 **b** 23 **c** 18
**d** 26 **e** 24 **f** 1
**4** **a** $24k$ **b** $3a$ **c** $a^3$ **d** $\frac{p}{2}$
**e** $7ab + 2$ **f** $x - 1$ **g** $2y$ **h** $2n - 2m$
**5** **a** $xy$ **b** $\frac{10x}{7y}$ **c** $\frac{w}{5}$ **d** $\frac{17a}{5}$
**6** **a** $2x + 10$ **b** $12m - 18$ **c** $4 + 2m$
**7** **a** $6(3a - 2)$ **b** $6m(n + 2)$ **c** $4(2x + 3)$
**8** **a** 7 **b** 144 **c** 5 **d** 6
**e** 9 **f** 5 **g** 14
**9** **a** $2x + 20$ **b** $10x$
**10** **a** $\$(2x)$ **b** $\$(3y)$ **c** $\$(2x + 3y)$

### Extended-response question

**a** \$220
**b** $\$(60 + 80n)$
**c** **i** $\$ (100n)$ **ii** 3 hours

## Equations 2

### Multiple-choice questions

**1** C **2** B **3** D **4** B **5** B

### Short-answer questions

**1** **a** $w = 9$ **b** $m = 7$ **c** $x = 5$
**d** $a = 2$ **e** $w = 13$ **f** $x = 1$
**2** **a** $x = 30$ **b** $q = 10$ **c** $p = 15$ **d** $x = 10$
**e** $r = 15$ **f** $a = 8$
**3** **a** $x = 5$ **b** $k = 10$ **c** $r = 0$ **d** $z = 10$
**4** **a** 20 **b** 2 **c** 7
**5** **a** $x = 40$ **b** $x = 60$ **c** $x = 75$

### Extended-response question

**a** $1500 + 5n$ **b** 100 books
**c** $20n$ **d** 50 books
**e** **i** \$1500 **ii** \$13 500 **iii** 100 books
**f** They make a \$750 loss

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## Measurement and Pythagoras' Theorem

### Multiple-choice questions

**1** C **2** B **3** D **4** B **5** D

### Short-answer questions

**1** **a** 500 cm **b** 180 cm **c** 90 000 $\text{cm}^2$
**d** 180 cm **e** 4000 mL **f** 10 000 $\text{m}^2$
**2** **a** 18.6 **b** 64 **c** 40
**3** **a** **i** 25.13 m **ii** 50.27 $\text{m}^2$
**b** **i** 47.12 cm **ii** 176.71 $\text{cm}^2$
**4** **a** **i** 25.71 m **ii** 39.27 $\text{m}^2$
**b** **i** 17.85 cm **ii** 19.63 $\text{cm}^2$
**5** **a** 30 $\text{m}^2$ **b** 48 $\text{m}^2$ **c** 21 $\text{cm}^2$
**6** **a** 74.088 $\text{m}^3$ **b** 50 $\text{m}^3$ **c** 24 $\text{m}^3$
**7** **a** 1530 **b** 0735
**8** **a** $18^2 + 80^2 = 6724$ $82^2 = 6724$ $18^2 + 80^2 = 82^2$, so the three sides satisfy Pythagoras' Theorem so it must be a right-angled triangle
**b** $\sqrt{2}$ cm **c** $x = 15$

### Extended-response question

**a** 2.5 m **b** 9.82 $\text{m}^2$ **c** 59.82 $\text{m}^2$ **d** 32.86 m

## Fractions, decimals, percentages and financial mathematics

### Multiple-choice questions

**1** D **2** C **3** C **4** B **5** B

### Short-answer questions

**1** **a** 18 **b** 1 **c** 5
**2** **a** $\frac{1}{4}$ **b** $\frac{7}{5}$ **c** $3\frac{1}{4}$
**d** $-\frac{2}{21}$ **e** $\frac{1}{3}$ **f** $\frac{9}{10}$
**3** **a** $\frac{5}{2}$ **b** $\frac{1}{8}$ **c** $\frac{5}{21}$
**4** **a** $\frac{9}{2}$ **b** $\frac{3}{4}$ **c** $\frac{2}{3}$
**5** **a** 6.93 **b** 7.58 **c** 4.03
**d** 6.51 **e** 3854.8 **f** 792
**6** **a** 530 **b** 9600 **c** 0.614
**7**

| **Fraction** | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{5}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{4}{5}$ | $\frac{19}{20}$ | $\frac{99}{100}$ | $\frac{1}{200}$ |
|---|---|---|---|---|---|---|---|---|---|
| **Decimal** | 0.25 | 0.5 | 0.2 | $0.\dot{3}$ | $0.\dot{6}$ | 0.8 | 0.95 | 0.99 | 0.005 |
| **Percentage** | 25% | 50% | 20% | $33.\dot{3}\%$ | $66.\dot{6}\%$ | 80% | 95% | 99% | 0.5% |

**8** **a** 5.6 **b** 11.76 **c** 85.5 m
**d** $1.98 **e** $210 **f** 4250 g
**9** **a** $700 **b** $882
**10** **a** $87 **b** 25%

### Extended-response question

**a** **i** $1600 **ii** $1280 **iii** $1024
**b** 5 years
**c** No. There will always be 80% of the previous value.

## Ratios and rates

### Multiple-choice questions

**1** C **2** B **3** C **4** C **5** D

### Short-answer questions

**1** **a** 2 : 3 **b** 1 : 2 : 3 **c** 6 : 7
**d** 3 : 40 **e** 5 : 1 **f** 3 : 10
**2** **a** 576 cm, 384 cm **b** $1500, $2500
**c** $1.60, $4, $2.40
**3** $7750
**4** 60 000 cm 600 m
**5** **a** 12 mm/day **b** 3 goals/game
**c** 2 cents/g or $0.02/g
**6** $2.27
**7** 85.6 km/h

### Extended-response question

**a** 16.5 km **b** 742.5 km **c** 6.1 L
**d** $35.37 **e** 18 km

# Chapter 6

### Pre-test

**1** **a** segment $AB$ **b** point $A$
**c** angle $ABC$ **d** line $AB$
**2** **a** $\angle ACB$ or $\angle BCA$ **b** $\angle DGF$ or $\angle FGD$
**c** $\angle SQU$ or $\angle UQS$
**3** **a** revolution **b** right **c** acute
**d** obtuse **e** straight **f** reflex
**4** **a** obtuse **b** isosceles **c** acute
**d** scalene **e** equilateral **f** right
**5** square, rectangle, rhombus, parallelogram, kite, trapezium
**6** **a** 60 **b** 130 **c** 140
**7** **a** 130 **b** $(c, e, g)$ **c** $(b, d, f)$
**8** **a** 40 **b** 110

### Exercise 6A

**1** **a** complementary **b** supplementary
**c** perpendicular **d** equal
**2** **a** acute **b** reflex **c** straight
**d** right **e** revolution **f** obtuse

**3 a** complementary **b** supplementary

**c** revolution

**4 a** 40° **b** 110° **c** 220°

**5 a** $\angle AOB$ **b** $\angle BOA$ or $\angle DOE$

**c** $\angle AOB$ or $\angle EOD$ **d** $\angle COB$ or $\angle EOA$

**e** $\angle COB$ or $\angle BOA$ or $\angle DOE$

**f** $\angle COE$ or $\angle AOE$ or $\angle DOB$

**g** $\angle AOD$ or $\angle BOE$

**h** $\angle COA$ or $\angle COD$ **i** $\angle COD$

**6 a** 45 **b** 130 **c** 120

**d** 240 **e** 90 **f** 180

**7 a** $a = 70$ (angles in a right angle)

$b = 270$ (angles in a revolution)

**b** $a = 25$ (angles in a right angle)

$b = 90$ (angles on a straight line)

**c** $a = 128$ (angles on a straight line)

$b = 52$ (vertically opposite angles)

**d** $a = 34$ (angles on a straight line)

$b = 146$ (vertically opposite angles)

**e** $a = 25$ (angles in a right angle)

**f** $a = 40$ (angles on a straight line)

**g** $a = 120$ (angles in a revolution)

**h** $a = 50$ (angles on a straight line)

$b = 90$ (angles on a straight line)

**i** $a = 140$ (angles in a revolution)

**8 a** 135° **b** 225°

**9 a** 40° **b** 72° **c** 120° **d** 200°

**10 a** $x = 45$, $y = 315$ **b** $x = 45$, $y = 135$

**11 a** 60 **b** 135 **c** 35

**d** $a = 110$, $b = 70$ **e** $a = 148$

**f** $a = 90$, $b = 41$, $c = 139$

**12 a** Supplementary angles should add to 180°.

**b** Angles in a revolution should add to 360°.

**c** Angles on straight line should add to 180°.

**13 a i** 180° **ii** 360° **iii** 30° **iv** 90°

**b i** 360° **ii** 180° **iii** 30° **iv** 120°

**14 a** 90° **b** 180° **c** 30° **d** 120°

## Exercise 6B

**1 a** equal **b** supplementary **c** equal

**2 a** $\angle BCH$ **b** $\angle ABE$ **c** $\angle GCB$

**d** $\angle BCH$ **e** $\angle FBC$ **f** $\angle GCB$

**g** $\angle FBC$ **h** $\angle DCG$

**3 a** alternate **b** alternate

**c** co-interior **d** corresponding

**e** corresponding **f** co-interior

**4 a** 80 (corresponding) **b** 120 (corresponding)

**c** 131 (corresponding) **d** 82 (alternate)

**e** 118 (alternate) **f** 78 (alternate)

**g** 100 (co-interior) **h** 129 (co-interior)

**5 a** $a = 58$ (co-interior angles on parallel lines)

$b = 58$ (co-interior angles on parallel lines)

**b** $a = 141$ (co-interior angles on parallel lines)

$b = 141$ (co-interior angles on parallel lines)

**c** $a = 100$ (co-interior angles on parallel lines)

$b = 80$ (co-interior angles on parallel lines)

**d** $a = 62$ (co-interior angles on parallel lines)

$b = 119$ (co-interior angles on parallel lines)

**e** $a = 105$ (co-interior angles on parallel lines)

$b = 64$ (corresponding angles on parallel lines)

**f** $a = 25$ (alternate angles on parallel lines)

$b = 30$ (alternate angles on parallel lines)

**6 a** $a = 110$ (corresponding angles on parallel lines)

$b = 70$ (angles on a straight line)

**b** $a = 120$ (alternate angles on parallel lines)

$b = 60$ (angles on a straight line)

$c = 120$ (corresponding angles on parallel lines)

**c** $a = 74$ (alternate angles on parallel lines)

$b = 106$ (co-interior angles on parallel lines)

$c = 106$ (vertically opposite angles)

**d** $a = 100$ (angles on a straight line)

$b = 100$ (corresponding angles on parallel lines)

**e** $a = 95$ (corresponding angles on parallel lines)

$b = 85$ (angles on a straight line)

**f** $a = 40$ (alternate angles on parallel lines)

$b = 140$ (angles on a straight line)

**7 a** No, the alternate angles are not equal.

**b** Yes, the co-interior angles are supplementary.

**c** No, the corresponding angles are not equal.

**8 a** 250 **b** 320 **c** 52

**d** 40 **e** 31 **f** 63

**9 a** 130° **b** 95° **c** 90°

**10 a** $(90 - a)°$ **b** $a°$ **c** $(90 + a)°$

**d** $(180 - a)°$ **e** $(180 - a)°$ **f** $(360 - a)°$

**g** $(180 - a)°$ **h** $a°$ **i** $(180 - a)°$

**j** $a°$ **k** $a°$ **l** $(180 - 2a)°$

**m** $(2a)°$ **n** $a°$ **o** $(180 - a)°$

## Exercise 6C

**1 a** right-angled triangle **b** isosceles triangle

**c** acute-angled triangle **d** equilateral triangle

**e** obtuse-angled triangle **f** equilateral triangle

**g** isosceles triangle **h** scalene triangle

**2 a** scalene **b** isosceles **c** isosceles

**d** equilateral **e** scalene **f** isosceles

**3 a** right **b** obtuse **c** acute

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**4** Ask someone to check your measurements with ruler and protractor.

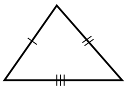
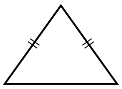
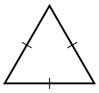
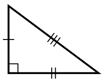
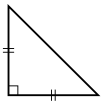
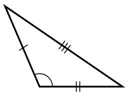
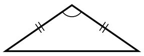
**5** **a** 80 **b** 40 **c** 58
**d** 19 **e** 34 **f** 36

**6** **a** 68 **b** 106 **c** 20
**d** 65 **e** 40 **f** 76

**7** **a** 150 **b** 80 **c** 160
**d** 50 **e** 140 **f** 55

**8** **a** yes **b** no **c** yes **d** yes
**e** yes **f** yes **g** yes

**9**

| Triangles | Scalene | Isosceles | Equilateral |
|---|---|---|---|
| acute |  |  |  |
| right |  |  | ✗ |
| obtuse |  |  | ✗ |

**10** Check measurements with a ruler and protrator.

**11** **a** The sum of the two short sides is less than the long side
**b** The angle sum (210°) is too big
**c** E.g. 91° and 91°, Angle sum is too big.

**12** **a** Isosceles, the two radii are of equal length.
**b** $\angle OAB$, $\angle OBA$ **c** 30° **d** 108° **e** 40°

**13** **a** 60 **b** 231 **c** 18
**d** 91 **e** 65 **f** 60

**14** **a** The angle sum is 360°

## Exercise 6D

**1** **a** non-convex **b** non-convex **c** convex

**2**

| Trapezium | Kite | Parallelogram | Rectangle | Rhombus | Square |
|---|---|---|---|---|---|
| | | YES | YES | YES | YES |
| | | | | YES | YES |
| | | | YES | | YES |
| | | YES | YES | YES | YES |
| | | | YES | | YES |
| | | YES | YES | YES | YES |
| | | | | YES | YES |
| | | | | YES | YES |

**3** **a** 360° **b** equal **c** 2 **d** 90°

**4** **a** 90 **b** 61 **c** 105
**d** 170 **e** 70 **f** 70

**5** **a** $a = 104$, $b = 76$ **b** $a = 72$, $b = 72$
**c** $a = 128$ **d** $a = 50$, $b = 130$
**e** $a = 54$, $b = 54$ **f** $a = 138$, $b = 42$

**6** **a** square, rhombus
**b** square, rectangle, rhombus, parallelogram
**c** rectangle, parallelogram, square, rhombus
**d** square, rhombus, kite
**e** square, rectangle
**f** square, rectangle
**g** square, rectangle, rhombus, parallelogram
**h** square, rectangle, rhombus, parallelogram

**7** **a** 152 **b** 69 **c** 145
**d** 74 **e** 59 **f** 30

**8** **a** true **b** false **c** true
**d** true **e** false **f** true

**9** **a** $a = 100$, $b = 3$, $c = 110$
**b** $a = 2$, $b = 90$
**c** $a = 5$, $b = 70$

**10** It is possible.

## Exercise 6E

**1** **a** heptagon **b** triangle **c** octagon
**d** nonagon **e** dodecagon **f** decagon
**g** quadrilateral **h** undecagon

**2** **a** 6 **b** 4 **c** 10
**d** 7 **e** 5 **f** 12

**3** **a** 720° **b** 1440° **c** 3600°

**4** **a** square **b** equilateral triangle

**5** **a** hexagon **b** octagon **c** pentagon

**6** **a** 540° **b** 1080° **c** 1440°
**d** 720° **e** 1260° **f** 900°

**7** **a** 130 **b** 80 **c** 120
**d** 130 **e** 155 **f** 105

**8** **a** 108° **b** 144° **c** 135°

**9** **a** 108° **b** 128.6° **c** 120°
**d** 144° **e** 135° **f** 147.3°

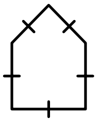
**10** **a** 115 **b** 135 **c** 250

**11** **a** 9 **b** 15 **c** 21 **d** 167

**12** **a** 6 **b** 20 **c** 11

**13** **a** 150° **b** 162°

**14** **a** Any rhombus with no right angles
**b**



## Exercise 6F

**1** **a** yes **b** yes **c** no
**d** no **e** no **f** yes

**2** **a** 4 ways **b** 2 ways **c** 3 ways
**d** 1 way **e** 2 ways **f** 0 ways

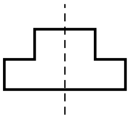
**Exercise 6F cont.**

**3** **a** 4 **b** 2 **c** 3
**d** 1 **e** 2 **f** 2

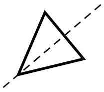
**4** **a** 4 and 4 **b** 2 and 2 **c** 2 and 2
**d** 1 and 1 **e** 1 and 1 **f** 0 and 2
**g** 0 and 2 **h** 4 and 4 **i** 1 and 0

**5** **a** **i** kite **ii** rectangle, rhombus
**iii** square
**b** **i** trapezium, kite
**ii** rectangle, rhombus, parallelogram **iii** square

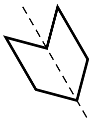
**6** **a**



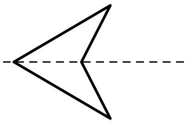
**b**



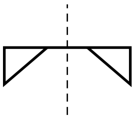
**c**



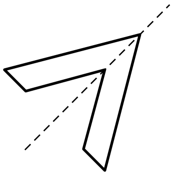
**d**



**e**



**f**



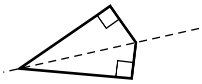
**7**

| | **line** | **rotational** |
|---|---|---|
| **a** | 3 | 3 |
| **b** | 0 | 3 |
| **c** | 0 | 2 |
| **d** | 0 | 4 |
| **e** | 4 | 4 |
| **f** | 3 | 3 |

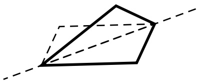
**8** **a** equilateral **b** isosceles **c** scalene

**9** **a** A, B, C, D, E, M, T, U, V, W, Y
**b** H, O, X
**c** H, I, O, S, X, Z

**10** **a**



**b**

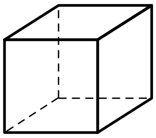


**11** **a** 9 **b** 3 **c** 4
**d** 1 **e** infinite **f** infinite

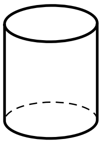
## Exercise 6G

**1** **a** cube **b** rectangular prism **c** cylinder
**d** cone **e** square-based pyramid
**f** triangular-based pyramid

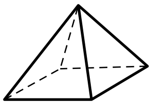
**2** **a**



**b**

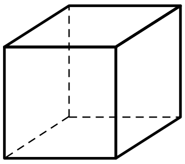


**c**

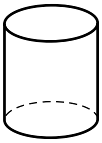


**3** **a** 2 **b** 4 **c** 6

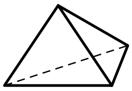
**4** **a**



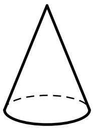
**b**



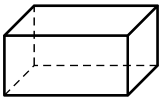
**c**



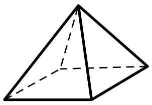
**d**



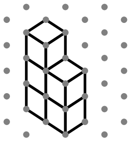
**e**



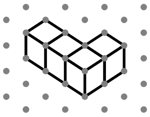
**f**



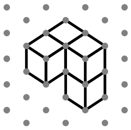
**5** **a**



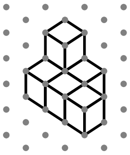
**b**



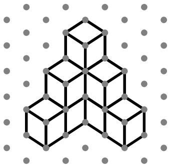
**c**



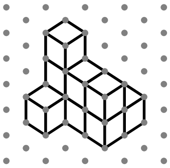
**d**



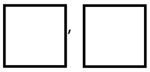
**e**



**f**

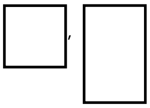


**6** **a**



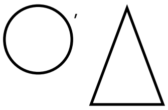
(two squares)

**b**



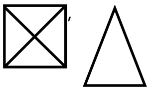
(square and rectangle with the same top side length)

**c**



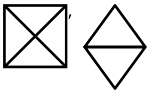
(circle, isosceles triangle)

**d**



(square, isosceles triangle)

**e**



(square, 2 isosceles triangles)

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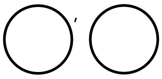
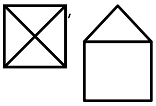
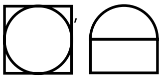
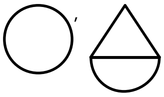
**f**  (2 circles)

**g**  (square, triangle on square)

**h**  (circle in square, semicircle on rectangle)

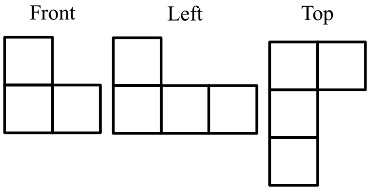
**i**  (circle, triangle on semicircle)

**7** 6

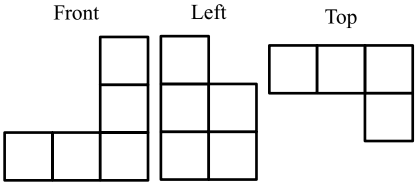
**8** 20

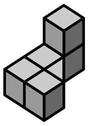
**9** **a** C **b** A **c** B **d** D

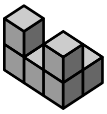
**10** **a** **i**



**ii**



**b** **i** 

**ii** 

## Exercise 6H

**1** **a** 6 **b** circle **c** cube
**d** vertices **e** seven **f** congruent
**g** seven **h** octagonal

**2** cylinder, sphere, cone

**3** **a** 6, 8, 12 **b** 5, 6, 9 **c** 7, 7, 12

**4** A, cube; B, pyramid; F, rectangular prism; G, tetrahedron; H, hexahedron

**5** **a** triangular prism **b** rectangular prism
**c** trapezoidal prism **d** pentagonal prism
**e** hexagonal prism **f** octagonal prism

**6**

| | faces | vertices | edges |
|---|---|---|---|
| **a** | 4 | 4 | 6 |
| **b** | 5 | 5 | 8 |
| **c** | 6 | 6 | 10 |
| **d** | 7 | 7 | 12 |
| **e** | 9 | 9 | 16 |

**7** **a** triangular prism **b** pentagonal prism
**c** rectangular prism

**8** **a** rectangular pyramid **b** heptagonal pyramid
**c** triangular pyramid

**9** **a** triangular prism
**b** octagonal prism
**c** square pyramid

**10** **a** true **b** false **c** true **d** false
**e** false (sphere) **f** true **g** false

**11** A hexagonal prism has a uniform cross-section at both ends and all the way through. It is like a loaf of bread in which every slice is a hexagon of the same size. It is made from two hexagons and six rectangles. A hexagonal pyramid has a hexagonal base and six triangles, which meet at a single point called the apex.

**12** true

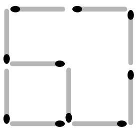
**13** **a**

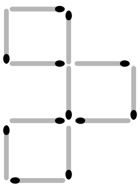
| Solid | Faces ($F$) | Vertices ($V$) | Edges ($E$) | $F + V$ |
|---|---|---|---|---|
| Cube | 6 | 8 | 12 | 14 |
| Square pyramid | 5 | 5 | 8 | 10 |
| Triangular pyramid | 4 | 4 | 6 | 8 |
| Triangular prism | 5 | 6 | 9 | 11 |

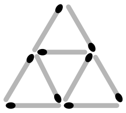
**b** $F + V$ is 2 more than $E$. **c** Yes

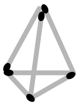
**14** **a** 26 **b** 11 **c** 28

## Puzzles and games

**1** **a** 

**b** 

**2** **a** 

**b** 

**3** GRACE CHISHOLM YOUNG

## Chapter review

### Multiple-choice questions

**1** D **2** A **3** E **4** C **5** D
**6** C **7** C **8** A **9** C **10** D

### Short-answer questions

**1** **a** 50 **b** 65 **c** 240
**d** 36 **e** 61 **f** 138

**2** **a** 81 **b** 96 **c** 132
**d** 99 **e** 77 **f** 51

**3** **a** No – corresponding angles are not equal.
**b** No – co-interior angles do not add to 180°
**c** Yes – alternate angles are equal

**4** **a** scalene or obtuse, 35
**b** isosceles or acute, 30
**c** equilateral or acute, 60
**d** right angle or scalene, 19
**e** acute or scalene, 27
**f** obtuse or scalene, 132

**5** **a** 150 **b** 67 **c** 141

**6** **a** square, rhombus
**b** square, rhombus, parallelogram, rectangle
**c** rhombus, square
**d** square, rhombus, kite
**e** square, rectangle

**7** **a** $a = 98, b = 82$ **b** $a = 85, b = 106$
**c** $a = 231, b = 129$

**8** **a** 720° **b** 1080° **c** 1800°

**9** **a** 108° **b** 150°

**10** **a** triangular prism **b** octagonal prism
**c** rectangular pyramid

### Extended-response question

**1** **a** 1260° **b** 140° **c** 40°
**d** **i** 11 **ii** 18 **iii** 27

# Chapter 7

## Pre-test

**1** **a** **i** 50 km **ii** 0 km **iii** 150 km
**b** 200 km **c** 2nd hour **d** 3rd hour

**2** **a** George **b** Amanda

**3** **a** –1, 2, 3 **b** –5, 3, 5
**c** 2, –8, –13 **d** –22, 5, 14

**4** **a** 1 **b** 1 **c** 0 **d** –3

**5** **a** 6 **b** –1 **c** 0 **d** 2

**6** **a** (1, 2) **b** (2, 1) **c** (3, 2)

**7** **a** 7 **b** 4 **c** 2

**8** **a** –1 **b** –5 **c** –11

**9** **a**

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | –4 | –2 | 0 | 2 | 4 |

**b**

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | –10 | –7 | –4 | –1 | 2 |

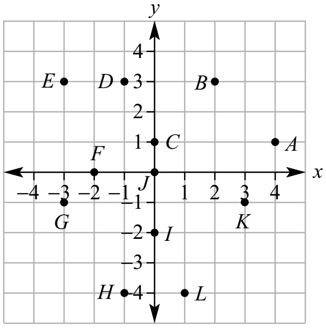
## Exercise 7A

**1** **a** 3 **b** –4 **c** 5
**d** –8 **e** (0, 0) **f** $y$

**2** **a** 3 **b** –1 **c** –2 **d** 0
**e** –2 **f** 0 **g** –3 **h** 0

**3** $A(1, 1)$, $B(5, 0)$, $C(3, 4)$, $D(0, 4)$, $E(-1, 2)$, $F(-3, 3)$, $G(-5, 1)$, $H(-3, 0)$, $I(-4, -2)$, $J(-2, -5)$, $K(0, -3)$, $L(2, -3)$, $M(5, -5)$

**4** **a**



**5** All give straight lines.

**6** **a** first **b** fourth **c** second
**d** third **e** first **f** third

**7** **a** B **b** C **c** E **d** D

**8** **a** triangle **b** rectangle
**c** parallelogram **d** kite

**9** a line on the $y$-axis

**10** **a** house **b** fish

## Exercise 7B

**1** **a** 3, 5 **b** 0, 6 **c** –3, –1
**d** 8, 16 **e** –5, –2 **f** 0, 1

**2** **a** 3 **b** 1 **c** –1 **d** –3
**e** –5 **f** –7 **g** –9 **h** –11

**3** **a** 1 **b** –2 **c** –11 **d** –20
**e** 4 **f** 10 **g** 31 **h** 151

**4** **a**

| $t$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $d$ | 0 | 40 | 80 | 120 | 160 |

**b** 120 km **c** 2 hours

**5**

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $v$ | 1000 | 1020 | 1040 | 1060 | 1080 | 1100 |

**b** 1080 L **c** 5 hours

**6** **a**

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | –6 | –3 | 0 | 3 | 6 |

**b**

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | –4 | –3 | –2 | –1 | 0 |

**c**

| $x$ | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | –3 | –1 | 1 | 3 | 5 |

**d**

| *x* | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| *y* | –7 | –5 | –3 | –1 | 1 |

**e**

| *x* | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| *y* | 4 | 3 | 2 | 1 | 0 |

**f**

| *x* | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| *y* | 1 | 0 | –1 | –2 | –3 |

**g**

| *x* | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| *y* | 3 | 1 | –1 | –3 | –5 |

**h**

| *x* | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| *y* | 10 | 6 | 2 | –2 | –6 |

**i**

| *x* | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| *y* | 1 | –5 | –11 | –17 | –23 |

**7** **a** **i** $140 **ii** $700
**b** $980 **c** 4 days

**8** **a** **i** 2 **ii** –1
**b** **i** 3 **ii** 0
**c** 2 **d** 1

**9** **a** $300 **b** 30 weeks
**c** $S = 30x + 50$ **d** 5 weeks less

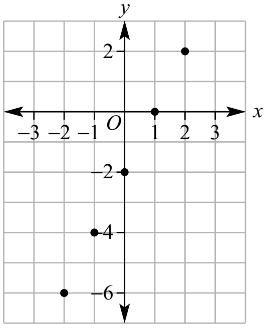
**10** **a** **i** 10 **ii** 36 **iii** 55
**b** **i** 21 **ii** 78
**c** **i** 28 **ii** 210 **iii** 5050

## Exercise 7C

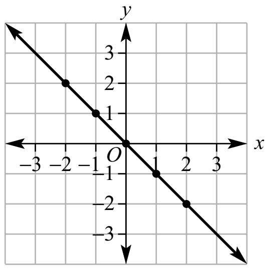
**1** **a** 5 **b** 7 **c** 3
**d** 1 **e** –7 **f** –11
**g** 25 **h** –21

**2** **a** 2 **b** –2 **c** –13

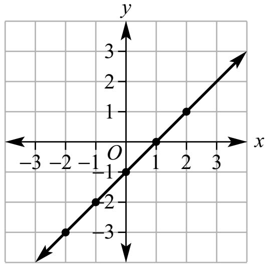
**3**



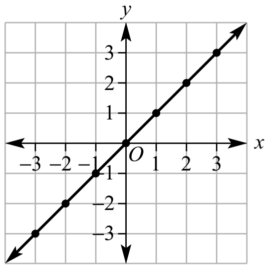
**4** **a**



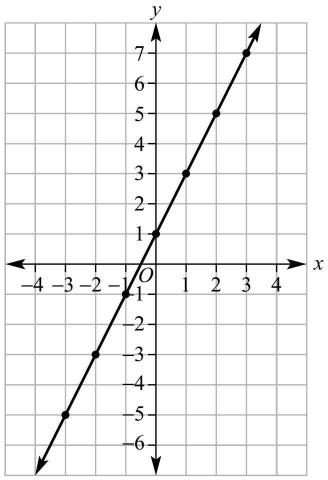
**b**



**c**



**d**



**5** **a**

| *x* | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| *y* | –2 | –1 | 0 | 1 | 2 | 3 | 4 |

**b**

| *x* | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| *y* | –5 | –4 | –3 | –2 | –1 | 0 | 1 |

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**Exercise 7C** cont.

**c**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
| $y$ | –9 | –7 | –5 | –3 | –1 | 1 | 3 |

**d**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
| $y$ | –5 | –3 | –1 | 1 | 3 | 5 | 7 |

**e**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
| $y$ | 9 | 7 | 5 | 3 | 1 | –1 | –3 |

**f**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
| $y$ | 8 | 5 | 2 | –1 | –4 | –7 | –10 |

**g**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
| $y$ | 3 | 2 | 1 | 0 | –1 | –2 | –3 |

**h**

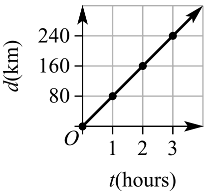
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 |
| $y$ | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

**6** **a** yes **b** yes **c** no
**d** no **e** no **f** yes

**7** **a**

| | | | | |
|---|---|---|---|---|
| $t$ | 0 | 1 | 2 | 3 |
| $d$ | 0 | 80 | 160 | 240 |

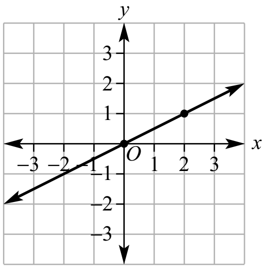
**b**



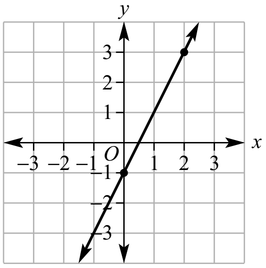
**c** 320 km **d** 5 hours

**8** **a** 2

**b** **i**



**ii**

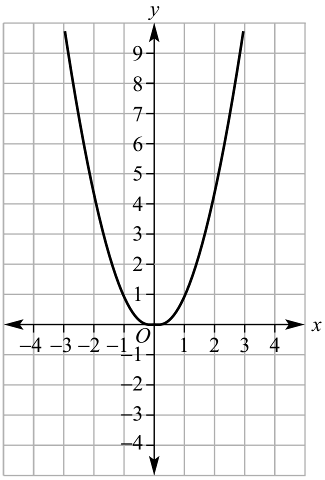


**9** When $x = 0$, $y = 0$ for all rules.

**10** Intersection points are:
**a** (0, 2) **b** (2, 1) **c** No intersection

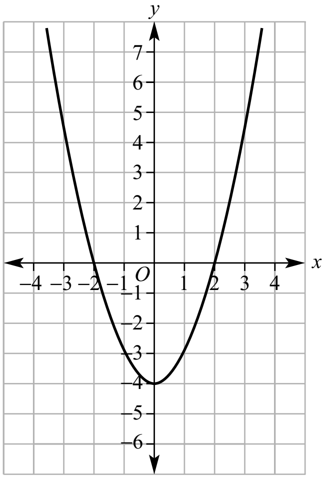
**11** **a** **i** $y = 9, 4, 1, 0, 1, 4, 9$

**ii**



**b** **i** $y = 5, 0, –3, –4, –3, 0, 5$

**ii**



## Exercise 7D

**1** **a** C **b** A **c** B

**2** **a** 2 **b** –1 **c** –2 **d** 3

**3** **a** 3 **b** 1 **c** –4 **d** 3

**4** **a** $y = x + 1$ **b** $y = 2x$ **c** $y = 2x + 4$
**d** $y = 3x – 1$ **e** $y = 4x$ **f** $y = 3x + 3$

**5** **a** To find a value for $y$, choose a value of $x$ then add 1.
**b** To find a value for $y$, choose a value of $x$ then multiply by 2.
**c** To find a value for $y$, choose a value of $x$, multiply by 2 then add 4.
**d** To find a value for $y$, choose a value of $x$, multiply by 3 then subtract 1.

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**e** To find a value for $y$, choose a value of $x$, multiply by 4.

**f** To find a value for $y$, choose a value of $x$, multiply by 3 then add 3.

**6** **a** $y = -x$ **b** $y = -x - 1$
**c** $y = -x + 1$ **d** $y = -2x + 6$
**e** $y = -2x$ **f** $y = -3x + 1$

**7** **a** $y = 3x + 1$ **b** $y = 2x + 1$
**c** $y = 5x + 1$ **d** $y = 2x + 4$

**8** **a** 1 **b** 3 **c** 7 **d** 0

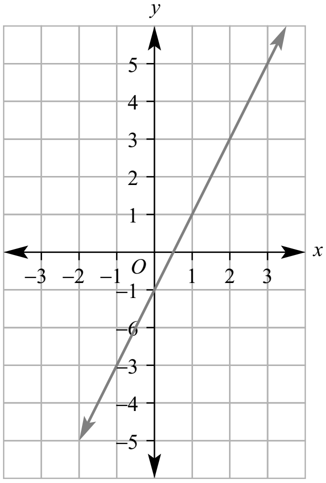
**9** **a** $x$ is not increasing by 1. **b** 1 **c** $y = x - 2$
**d** **i** $y = 2x + 3$ **ii** $y = 3x - 1$
**iii** $y = -2x + 3$ **iv** $y = -4x - 20$

**10** **a** $y = x + 1$ **b** $y = 2x - 2$
**c** $y = -3x + 2$ **d** $y = -x$

## Exercise 7E

**1**

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | −5 | −3 | −1 | 1 | 3 |



**2** **i** $x = 5$ **ii** $x = -1$

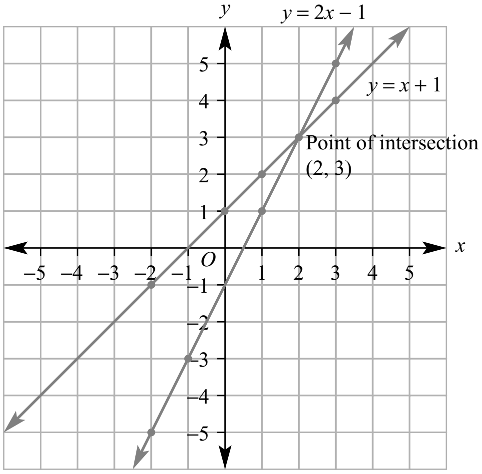
**3** **a** (2, 4) **b** (3.2, 6, 4)
**c** (–2.3, –4, 6) **d** (3.5, 7)
**e** (–7, –14) **f** (1000, 2000)
**g** (31.42, 62.84) **h** (–24.301, –48.602)
**i** $\left(\frac{\text{any number}}{2}, \text{any number}\right)$

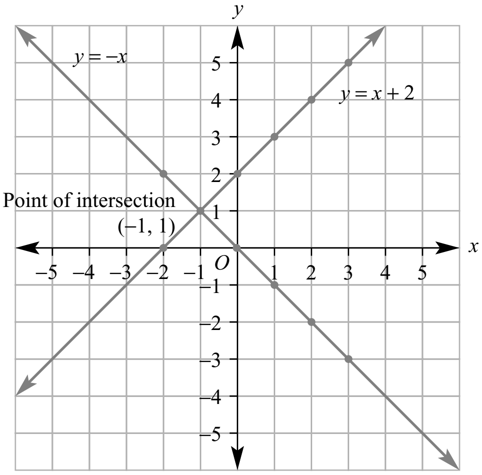
**4** **a** (4, 3)
**b** (–2, –3)

**5** **a** $x = 2$ **b** $x = 0.5$ **c** $x = 3$
**d** $x = -2.5$ **e** $x = -1.5$

**6** **a** $x = -2.5$ **b** $x = 3$ **c** $x = -0.5$
**d** $x = 4$ **e** $x = 5$

**7**





**8** **a** $x + 2 = 1$ and $5 - 2x = 7$

**b** Any point that lies on the line is correct, e.g. (–2, 9) (0, 5) (1, 3) (2, 1)

**c** Any point that lies on the line is correct, e.g. (–2, 0) (0, 2) (1, 3) (3, 5)

**d** (1, 3)

$y = x + 2$ $y = 5 - 2x$
$3 = 1 + 2$ $3 = 5 - 2 \times 1$
$3 = 3$ True $3 = 3$ True

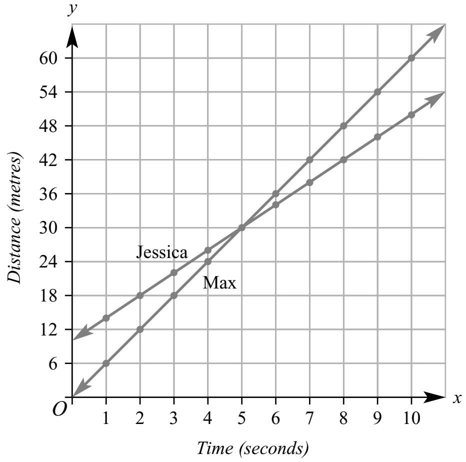
**e** $x = 1$

**9** **a**

| **Time in seconds** | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Max's distance in metres** | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| **Jessica's distance in metres** | 10 | 14 | 18 | 22 | 26 | 30 | 34 | 38 | 42 | 46 | 50 |

**Exercise 7E cont.**

**b**



**c** $d = 6t$

**d i** $6t = 18$ **ii** $6t = 30$ **iii** $6t = 48$

**e** $d = 10 + 4t$

**f i** $10 + 4t = 22$ **ii** $10 + 4t = 30$ **iii** $10 + 4t = 42$

**g** (5, 30) (5, 30)

$d = 6t$ $d = 10 + 4t$

$30 = 6 \times 5$ $30 = 10 + 4 \times 5$

$30 = 30$ True $30 = 30$ True

**h** Max catches up to Jessica. They are both 30 m from the starting line and have each run for 5 seconds.

**10 a i** (−2, 17) (−1, 14) (0, 11) (1, 8) (2, 5) (3, 2) (4, −1) (5, −4)

**ii** (−2, −3) (−1, −1,) (0, 1) (1, 3) (2, 5) (3, 7) (4, 9) (5, 11)

**b** (2, 5) $y = 11 - 3x$ $y = 2x + 1$

$5 = 11 - 3 \times 2$ $5 = 2 \times 2 + 1$

$5 = 11 - 6$ $5 = 4 + 1$

$5 = 5$ True $5 = 5$ True

**c** It is the only shared point.

**11 a i** $x = 2, x = -2$ **ii** $x = 3, x = -3$

**iii** $x = 4, x = -4$ **iv** $x = 5, x = -5$

**b** For each $y$-coordinate there are two different points so two different solutions.

**c** The graph of $y = x^2$ does not include a point where $y = -9$.

**d** Many correct answers all with $x^2$ equal to a negative number, e.g. $x^2 = -5$, $x^2 = -10$, $x^2 = -20$

## Puzzles and games

**1** Plane

**2 a** $y = 4x - 7$ **b** $y = -x + 11$

**c** $y = 5x - 50$ **d** $y = x - 10$

**3 a** 31 **b** 165

**4** 3 hours

**5** 40 min

**6** 1588

## Chapter review

### Multiple-choice questions

**1** B **2** C **3** C **4** D **5** B

**6** D **7** A **8** D **9** E **10** E

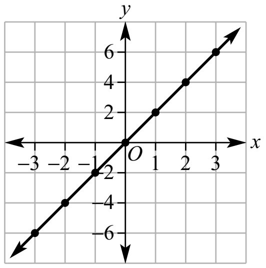
### Short-answer questions

**1** $A(2, 3)$, $B(0, 2)$, $C(-2, 4)$, $D(-3, 1)$, $E(-3, -3)$, $F(-1, 0)$, $G(0, -4)$, $H(1, -2)$, $I(4, -3)$, $J(3, 0)$

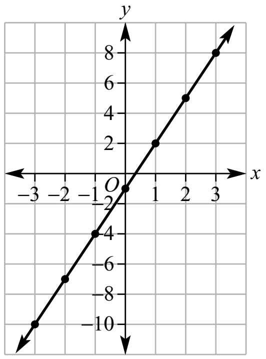
**2 a** −2, −1, 0, 1 **b** −4, 0, 2, 4

**c** −5, 1, 4, 7 **d** 3, 1, 0, −1

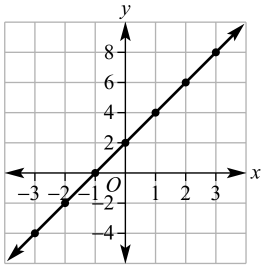
**3 a**



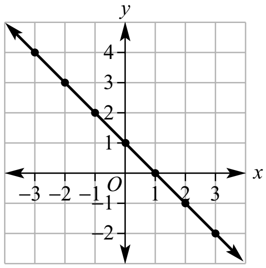
**b**



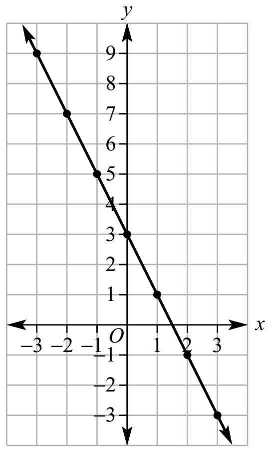
**c**



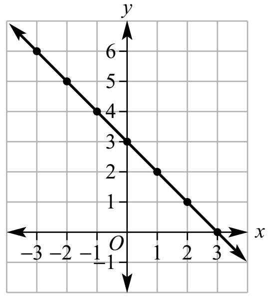
**d**



**e**



**f**



**4** **a** $y = 2x + 1$ **b** $y = 3x + 2$
**c** $y = x + 3$ **d** $y = -x + 1$
**e** $y = -4x - 1$ **f** $y = -x + 8$

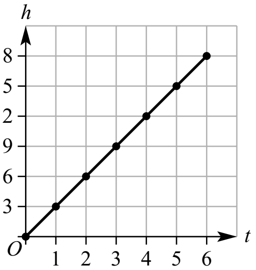
**5** **a** $y = 3x$ **b** $y = 2x - 2$ **c** $y = -x$
**d** $y = -4x + 4$ **e** $y = 2x + 1$ **f** $y = 4x$
**g** $y = x - 2$ **h** $y = -4x$ **i** $y = -2x - 4$

## Extended-response questions

**1** **a**

| | | | | | | |
|---|---|---|---|---|---|---|
| ***t*** | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| ***h*** | 0 | 3 | 6 | 9 | 12 | 15 | 18 |

**b**

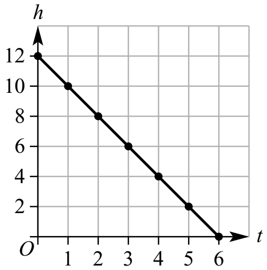


**c** $h = 3t$ **d** 10.5 mm **e** 30 mm **f** 5 days

**2** **a**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| ***t*** | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| ***h*** | 12 | 10 | 8 | 6 | 4 | 2 | 0 |

**b**



**c** 6 minutes **d** $h = -2t + 12$
**e** 7 km **f** 4 minutes 15 seconds

# Chapter 8

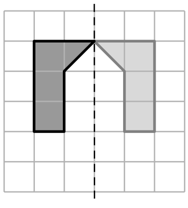
## Pre-test

**1** **a** 4 **b** 2 **c** 5 **d** 0

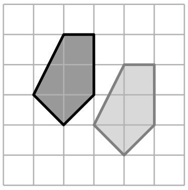
**2** **a** 4 **b** 2 **c** 5 **d** 2

**3** **a** $A$ (1, 1), $B$ (3, −2), $C$ (−4, −3), $D$ (−3, 0)
**b** **i** (0, 1) **ii** (3, 0) **iii** (−4, −2)
**c** (−4, 3)

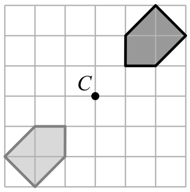
**4** **a**



**b**



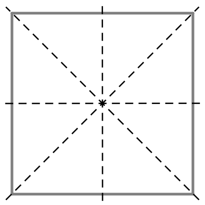
**c**



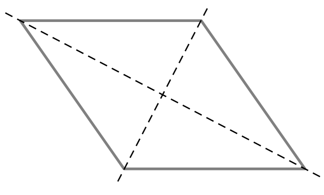
**5** **a** A, B, C, D **b** A, B, C, D, E
**c** A, B **d** A, C, E

## Exercise 8A

**1** **a**

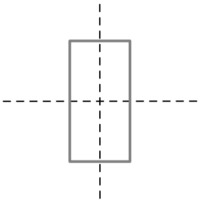


**b**

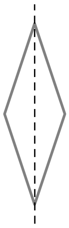


**Exercise 8A** cont.

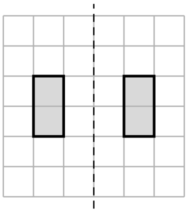
**c**



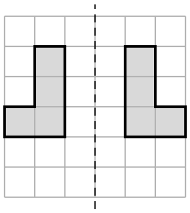
**d**



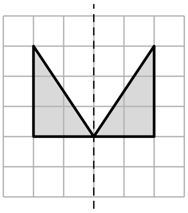
**2 a**



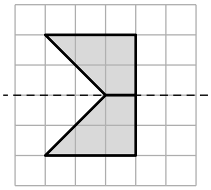
**b**



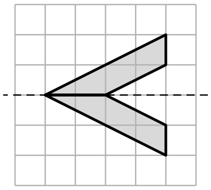
**c**



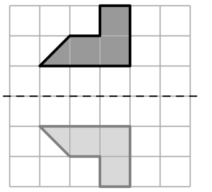
**d**



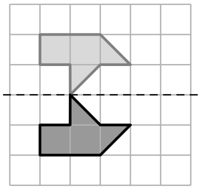
**e**



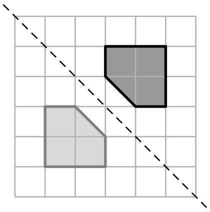
**f**



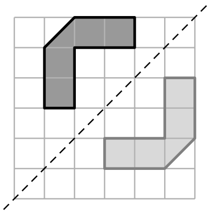
**g**



**h**

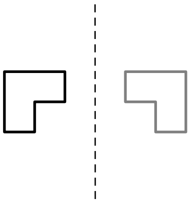


**i**

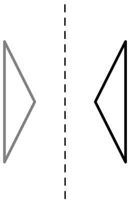


**3 a** (1, 3) **b** (−1, 3) **c** (−3, 2) **d** (−3, −2)
**e** (−2, 1) **f** (2, 1) **g** (2, −4) **h** (−2, −4)

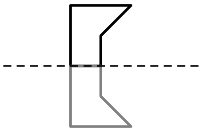
**4 a**



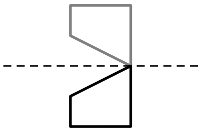
**b**



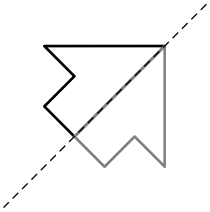
**c**



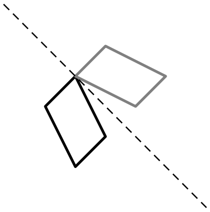
**d**



**e**



**f**

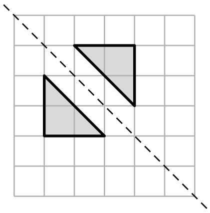


**5 a** $A'(1, 1)$, $B'(1, 4)$, $C'(2, 2)$, $D'(3, 1)$
**b** $A'(-3, 4)$, $B'(-3, 1)$, $C'(-2, 1)$, $D'(-1, 2)$
**c** $A'(-1, -2)$, $B'(-2, -4)$, $C'(-4, -4)$, $D'(-4, -3)$
**d** $A'(2, -1)$, $B'(2, -4)$, $C'(4, -2)$, $D'(4, -1)$
**e** $A'(-3, 2)$, $B'(-3, 3)$, $C'(-1, 4)$, $D'(-1, 1)$
**f** $A'(-3, -4)$, $B'(-1, -4)$, $C'(-1, -1)$, $D'(-2, -3)$

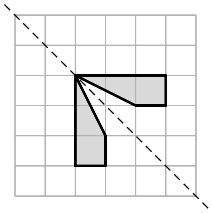
**6 a** **i** (2, −5) **ii** (4, −1)
**iii** (−3, −2) **iv** (−3, −4)
**v** (0, 4) **vi** (3, 0)
**vii** (−2, 0) **viii** (−6, 10)
**b** The $x$ coordinate

**7 a** **i** (−3, 2) **ii** (−7, 1)
**iii** (2, 4) **iv** (4, 6)
**v** (0, 7) **vi** (4, 0)
**vii** (4, −6) **viii** (0, −3)
**b** The $x$ coordinate

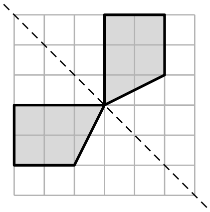
**8 a**



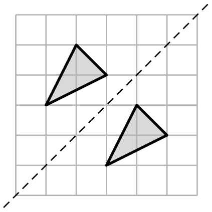
**b**



**c**

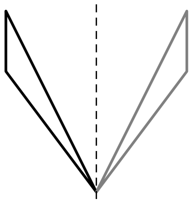


**d**

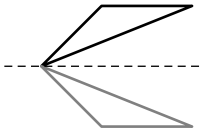


**9** (2, –5)

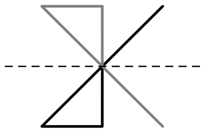
**10 a**



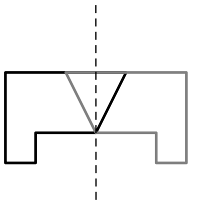
**b**



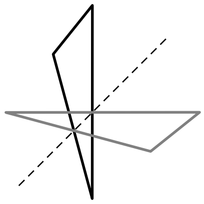
**c**



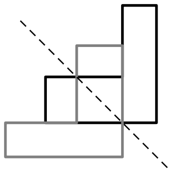
**d**



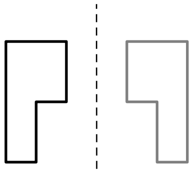
**e**



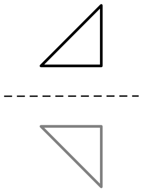
**f**



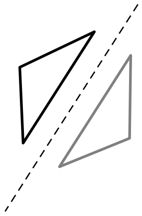
**11 a**



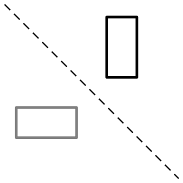
**b**



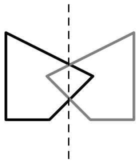
**c**



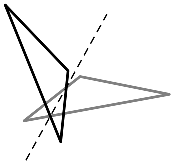
**d**



**e**



**f**



**12 a** $A'$ (2, 0), $B'$(1, –3), $C'$(4, –2)
**b** $A'$(–2, 0), $B'$(–1, 3), $C'$(–4, 2)

**13 a** $A'$(–1, 2), $B'$(–4, 2), $C'$(–4, 4), $D'$(–1, 4)
**b** $A'$(1, –2), $B'$(4, –2), $C'$(4, –4), $D'$(1, –4)

**14 a** 4 **b** 2 **c** 2
**d** 1 **e** 0 **f** 0
**g** 1 **h** 3 **i** 8

**15** Reflection in the $y$-axis.

**16** Computer geometry required.

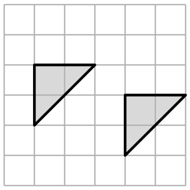
## Exercise 8B

**1 a** 7 units **b** 3 units **c i** 7 units **ii** 3 units

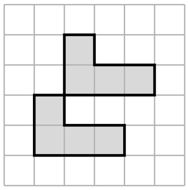
**2 a** up **b** left **c** down **d** up
**e** left **f** left **g** right **h** right

**3 a** (4, 2) **b** (1, 2) **c** (3, 5) **d** (3, 1)
**e** (2, 4) **f** (0, 1) **g** (5, 1) **h** (3, 0)

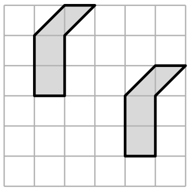
**4 a**



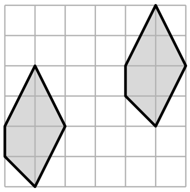
**b**



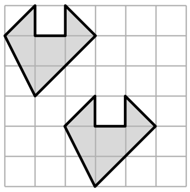
**c**



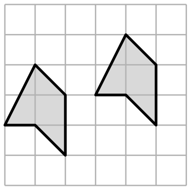
**d**



**e**



**f**



**5 a** (1, 3) **b** (–4, 3)
**c** (–2, 1) **d** (–2, –2)
**e** (–2, 5) **f** (8, 3)
**g** (1, 4) **h** (2, 1)
**i** (3, –3) **j** (–3, 1)
**k** (–5, 4) **l** (–4, –2)

**6 a** 4 **b** 12

**7 a** 3 units up **b** 7 units down
**c** 4 units down **d** 2 units up
**e** 5 units left **f** 2 units right
**g** 1 unit left and 4 units up
**h** 3 units right and 6 units up
**i** 3 units right and 4 units down
**j** 3 units left and 11 units up
**k** 12 units right and 3 units down
**l** 10 units left and 13 units down

**8 a** 2 units left and 2 units up
**b** 4 units left and 4 units up
**c** 1 unit right and 5 units down
**d** 6 units right and 2 units down

**9 a** 14
**b** 28

**10 a** (–4, –1)
**b** (–4, –3)

## Exercise 8C

**1** **a** C **b** A **c** B

**2** **a** anticlockwise, 90° **b** clockwise, 90°
**c** anticlockwise, 90° **d** clockwise, 90°
**e** anticlockwise, 180° **f** clockwise, 180°

**3** **a** (–2, 0) **b** (–2, 0)
**c** (0, –2) **d** (0, 2)

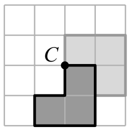
**4** **a** (–1, –1) **b** (–1, –1)
**c** (1, –1) **d** (–1, 1)

**5** **a** (2, –3), (–2, 3), (–3, –2) **b** (3, 1), (–3, –1), (1, –3)

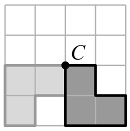
**6** **a** (–3, –3) **b** (3, –3) **c** (–3, 3)
**d** (–3, 3) **e** (3, 3) **f** (–3, –3)

**7** **a** **i** *A*′(–1, 0), *B*′(–3, 0), *D*′(–1, 2), **ii** *A*′(0, –1), *B*′(0, –3), *D*′(–2, –1), **iii** *A*′(1, 0), *B*′(3, 0), *D*′(1, –2)
**b** **i** *A*′(0, –1), *B*′(2, 0), *D*′(0, –3), **ii** *A*′(1, 0), *B*′(0, 2), *D*′(3, 0), **iii** *A*′(0, 1), *B*′(–2, 0), *D*′(0, 3)

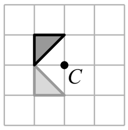
**8** **a**



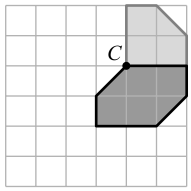
**b**



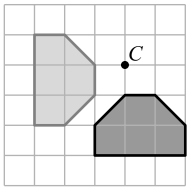
**c**



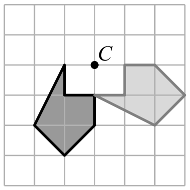
**d**



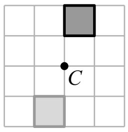
**e**



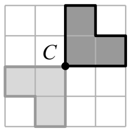
**f**



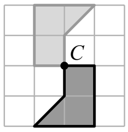
**9** **a**



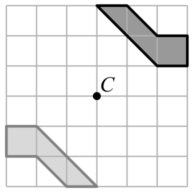
**b**



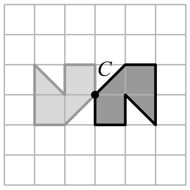
**c**



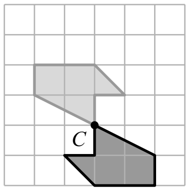
**d**



**e**



**f**



**10** **a** *A*′(4, –4), *B*′(4, –1), *C*′(1,–1)
**b** Change the sign of both coordinates from positive to negative or negative to positive

**11** **a** (0, –1) **b** (3, 0) **c** (–1, 2)

**12** **a** 180° anticlockwise **b** 90° anticlockwise
**c** 90° clockwise **d** 180° clockwise

**13** **a** 180° **b** 270°

**14** The triangle has been shifted, not rotated.

**15** Check with your teacher.

## Exercise 8D

**1** **a** false **b** true **c** true
**d** true **e** true **f** true

**2** **a** yes
**b** **i** *D* **ii** *E* **iii** *F*
**c** **i** *DE* **ii** *EF* **iii** *DF*
**d** **i** ∠*E* **ii** ∠*F* **iii** ∠*D*

**3** **a** yes
**b** **i** *D* **ii** *E* **iii** *F*
**c** **i** *DE* **ii** *EF* **iii** *DF*
**d** **i** ∠*E* **ii** ∠*F* **iii** ∠*D*

**4** **a** yes
**b** **i** *D* **ii** *E* **iii** *F*
**c** **i** *DE* **ii** *EF* **iii** *DF*
**d** **i** ∠*E* **ii** ∠*F* **iii** ∠*D*

**5** **a** **i** *E* **ii** *H*
**b** **i** *EH* **ii** *GH*
**c** **i** ∠*G* **ii** ∠*E*

**6** **a** **i** *F* **ii** *I*
**b** **i** *FJ* **ii** *HI*
**c** **i** ∠*H* **ii** ∠*J*

**7** (J, G), (D, K), (C, I)

**8** **a** *DE* **b** ∠*B* **c** 10 cm
**d** 62° **e** 30°

**9** (A, J), (C, K), (E, G)

**10** **a** Δ*AMC*, Δ*BMC*
**b** Yes, all corresponding sides and angles will be equal.

**11** Yes

**12** Yes

**13** Yes

**14** No

**15** **a** Reflect in the *y*-axis, then translate right 1 and down 2.
**b** Rotate anticlockwise 90°, then translate right 6 and up 3.
**c** Rotate by 180°, then translate left 2 and up 1.
**d** Reflect in the *x*-axis, reflect in the *y*-axis, then translate left 2 and up 1.
**e** Reflect in the *x*-axis, reflect in the *y*-axis, then translate right 3 and up 2.
**f** Rotate by 180°, then translate right 3 and up 2.

## Exercise 8E

**1** **a** yes **b** yes **c** no
**d** yes **e** yes **f** no

**2** **a i** $F$ **ii** $D$ **iii** $E$
**b i** $\angle A$ **ii** $\angle C$ **iii** $\angle B$
**c i** $DE$ **ii** $FD$ **iii** $EF$

**3** **a** $\Delta ABC \equiv \Delta EFD$ **b** $\Delta ABC \equiv \Delta FED$
**c** $\Delta XYZ \equiv \Delta UST$ **d** $\Delta ABC \equiv \Delta ADC$

**4** **a** SAS **b** SSS **c** RHS **d** AAS

**5** **a** SSS **b** RHS **c** SAS **d** AAS

**6** **a** $x = 4$, $y = 1$ **b** $x = 9$, $a = 20$
**c** $x = 5$, $a = 24$ **d** $x = 5$, $a = 30$
**e** $x = 4$, $a = 95$, $b = 25$ **f** $x = 11$, $a = 50$, $b = 90$

**7** **a** SSS **b** SAS **c** RHS **d** AAS

**8** **a** no **b** yes, SAS **c** yes, AAS **d** no

**9** You can draw an infinite number of triangles with the same shape but of different size.

**10** **a** no **b** yes **c** yes **d** no

## Exercise 8F

**1** SAS, AAS and RHS

**2** **a** $AC$ **b** $BD$ **c** $DB$

**3** **a** alternate angles in parallel lines are equal
**b** alternate angles in parallel lines are equal
**c** vertically opposite angles are equal
**d** alternate angles in parallel lines are equal
**e** alternate angles in parallel lines are equal
**f** vertically opposite angles are equal

**4** **a** co-interior angles in parallel lines, $a = 110$
**b** co-interior angles in parallel lines, $a = 52$

**5** $a, d, i, j, m, n, o, p, q$

**6** **a** AAS **b** RHS **c** SSS
**d** SAS **e** AAS **f** SSS

**7** **a** $\Delta ABD$, $\Delta CDB$ **b** equal **c** equal
**d** $BD$ **e** SSS
**f** Corresponding angles in congruent triangles.

**8** **a** yes (90°) **b** yes **c** yes **d** SAS
**e** Corresponding sides in congruent triangles.

**9** **a** equal (alternate angles in parallel lines)
**b** equal (alternate angles in parallel lines)
**c** $BD$ **d** AAS **e** They must be equal.

**10** **a** $\angle DCE$ **b** $\angle CDE$
**c** There are no pairs of equal sides.

**11** **a** SSS (3 equal sides)
**b** They are equal and add to 180° so each must be 90°.
**c** Since $\Delta QMN$ is isosceles and $\angle MQN$ is 90° then $\angle QMN = 45°$.

**12** **a** $AB = CB$, $AD = CD$ and $BD$ is common.
So $\Delta ABD \equiv \Delta CBD$ by SSS.
**b** $\Delta ABD \equiv \Delta CBD$ so $\angle DAB = \angle DCB$
**c** $\Delta ABD \equiv \Delta CBD$ so $\angle ADB = \angle CDB$

**13** **a** $\angle ABE = \angle CDE$ (alternate angles in parallel lines)
$\angle BAE = \angle DCE$ (alternate angles in parallel lines)
$AB = CD$ (given)
$\Delta ABE \equiv \Delta CDE$ ($AAS$)
$BE = DE$ and $AE = CE$ because corresponding sides on congruent triangles are equal.
**b** As per part **a** above.

## Puzzles and games

**1** 30 **2** 27
**3** 30 m **4** 31
**5** Yes, illustrates Pythagoras' Theorem using areas
**6** $(3 - r) + (4 - r) = 5$, so $r = 1$

## Chapter review

### Multiple-choice questions

**1** B **2** D **3** C **4** B **5** C
**6** E **7** A **8** E **9** D **10** E

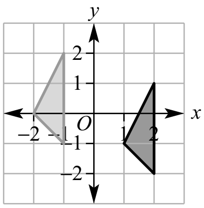
### Short-answer questions

**1** **a** $A'(1, -2)$, $B'(3, -4)$, $C'(0, -2)$
**b** $A'(-1, 2)$, $B'(-3, 4)$, $C'(0, 2)$

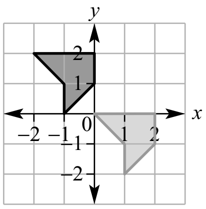
**2** **a** (0, 3) **b** (2, –1) **c** (–3, 1) **d** (–3, –2)

**3** **a** right 1, up 4, **b** right 3, down 6
**c** left 3, down 7 **d** right 4, up 6

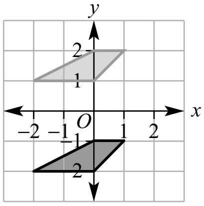
**4** **a**



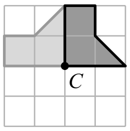
**b**



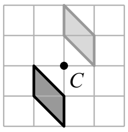
**c**



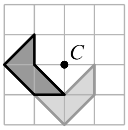
**5** **a**



**b**



**c**



**6** **a i** F **ii** G
**b i** EH **ii** FG
**c i** ∠G **ii** ∠E

**7** $\Delta ABC \equiv \Delta STU$

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**8** **a** RHS **b** SAS **c** SSS **d** AAS

**9** **a** $x = 3$, $a = 25$ **b** $x = 5$, $a = 18$

**10** **a** yes – alternate angles **b** yes – alternate angles

**c** yes – given **d** AAS

**e** $AE = CE$ (matching sides of congruent triangles), $BE = DE$ (matching sides of congruent triangles), therefore $AC$ and $BD$ bisect each other.

### Extended-response question

**1** **a** $A'(0, 1)$, $B'(-2, 1)$, $C'(-2, 4)$

**b** $A'(3, 1)$, $B'(3, -1)$, $C'(0, -1)$

# Chapter 9

### Pre-test

**1** **a** 0, 1, 2, 4, 6, 7, 9, 10, 14

**b** 20, 30.6, 36, 100, 101, 204

**c** 1.2, 1.7, 1.9, 2.7, 3.2, 3.5

**2** **a** Total = 40, average = 8

**b** Total = 94, average = 18.8

**c** Total = 3.3, average = 0.66

**3** **a** $\frac{1}{6}$ **b** 150° **c** **i** \$420 **ii** \$630

**4** **a** 4 **b** 3 **c** Saturday

**d** 26 **e** $\frac{1}{3}$

### Exercise 9A

**1** **a** iii **b** iv **c** i

**d** v **e** vi **f** ii

**2** **a** categorical **b** numerical **c** numerical

**d** categorical **e** numerical **f** numerical

**3** Answers will vary.

**4** **a** discrete numerical **b** continuous numerical

**c** continuous numerical **d** categorical

**e** categorical **f** categorical

**g** discrete numerical **h** discrete numerical

**i** continuous numerical **j** discrete numerical

**k** continuous numerical **l** discrete numerical

**5** **a** observation

**b** sample of days using observation or secondary source records

**c** census of the class

**d** sample

**e** sample

**f** sample using secondary source data

**g** census (every 5 years this question appears on the population census)

**h** census of the class

**i** sample

**j** results from the population census

**6** **a** secondary – a market research company

**b** secondary – department of education data

**c** primary data collection via a sample

**d** secondary source using results from the census

**e** secondary source using NAPLAN results or similar

**7** **a** Proximity to the Indian Ocean makes first hand collection of the data difficult.

**b** Too many people to ask and a sensitive topic means that using the census results as your source would be better.

**c** Extremely large population makes primary data difficult to collect.

**d** Sensitive topic might make student less keen to give honest and reliable answers.

**e** Cultural issues and the different cultural groups that exist in the community makes collection difficult.

**8** **a** Population is the entire group of people but a sample is a selection from within it.

**b** If the population is small enough (e.g. a class) or there is enough time/money to survey the entire population (e.g. national census).

**c** When it is too expensive or difficult to survey the whole population, e.g. television viewing habits of all of NSW.

**9** **a** Excludes people who have only mobile numbers or who are out when phone is rung; could bias towards people who have more free time.

**b** Excludes people who do not respond to these types of mail outs; bias towards people who have more free time.

**c** Excludes working parents; bias towards shift workers or unemployed.

**d** Excludes anyone who does not read this magazine; bias towards girls.

**e** Excludes people who do not use Facebook; bias towards younger people or people with access to technology.

**10** **a** Too expensive and difficult to measure television viewing in millions of households.

**b** Not enough people – results can be misleading.

**c** Programs targeted at youth are more likely to be watched by the students.

**d** Research required.

**11 a** Too expensive and people might refuse to respond if it came too often.

**b** English as a second language can impact the collection of data (simple, unambiguous English is required). Some people from particular cultures may not be keen to share information about themselves.

**c** Some people cannot access digital technologies and they would be excluded from the results.

## Exercise 9B

**1 a** dot plot **b** column graph **c** outlier

**2 a** true **b** false **c** true
**d** true **e** false

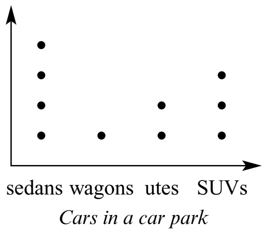
**3 a** 5 **b** 3
**c** sport **d** 20

**4 a** 2 **b** 7
**c** red **d** 27

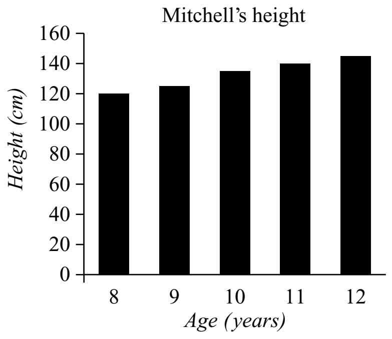
**5 a** 135 cm **b** 10 cm
**c** 35 cm **d** 11 years old

**6 a** 10

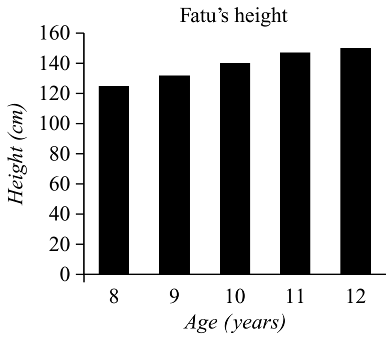
**b**



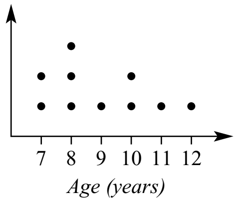
**7 a**



**b**



**8 a**



**b** 5

**9 a** 4 **b** 8 **c** 24
**d** 8 **e** 7 **f** 2

**10 a**

| | 1990 | 1995 | 2000 | 2005 | 2010 |
|---|---|---|---|---|---|
| **Using public transport** | 30 | 25 | 40 | 50 | 60 |
| **Driving a car** | 60 | 65 | 50 | 40 | 20 |
| **Walking or cycling** | 10 | 20 | 15 | 15 | 25 |

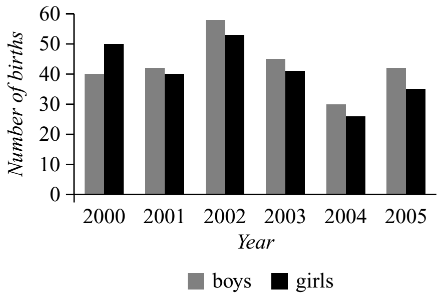
**b** 2005 and 2010

**c** 2010

**d** Environmental concerns; others answers possible.

**e** Public transport usage is increasing; other answers possible.

**11 a**



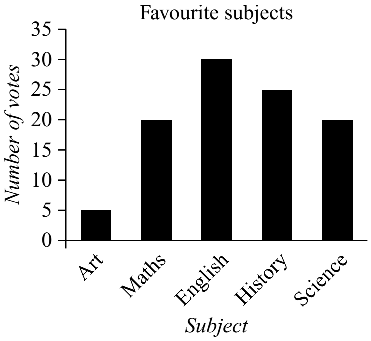
**b** 2000 **c** 2004 **d** 2002 **e** boys

**12 a** 7 **b** 7
**c** 3M: 8, 3S: 4 **d** 3M
**e** 3M because the student who got 10 is in that class.

**13 a** It is unequal.

**b** The axes have no labels and it does not have a title.

**c**



**Exercise 9B** cont.

**d** four times as popular

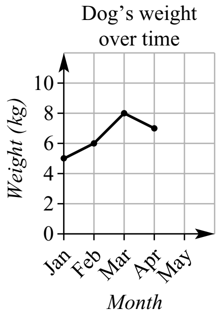
**e** one and a half times as popular

## Exercise 9C

**1** **a** 20 L
**b** 30 L
**c** 50 L
**d** 50 L

**2** **a** 3 kg
**b** 4 kg
**c** 5 kg
**d** 4.5 kg

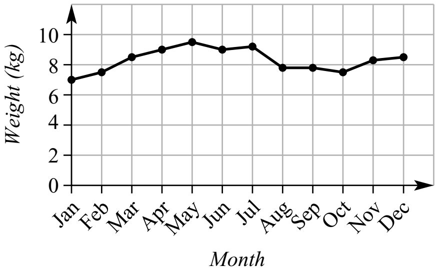
**3**



**4** **a** 50 cm **b** 95 cm
**c** 10 cm **d** 105 cm

**5** **a** 23°C
**b** 2:00 p.m.
**c** 12:00 a.m.
**d** **i** 10°C **ii** 18°C **iii** 24°C **iv** 22°C

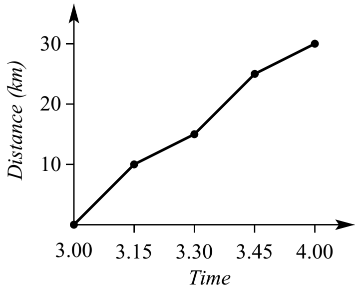
**6** **a**



**b** Weight increases from January until July, then goes down suddenly.

**c** July, as the weight goes down for the next three months.

**7**



**8** **a** 200 km **b** 80 km **c** at rest
**d** in the first hour **e** 40 km

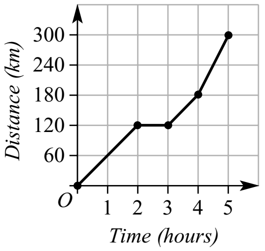
**9** **a** 20 km **b** 5 km
**c** fifth hour **d** 2 hours

**10** **a** **i** 30%
**ii** 35%
**iii** 35%

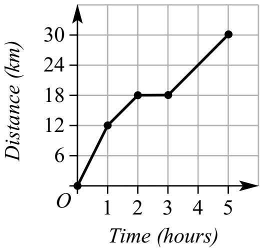
**b** July, because there was the greatest rise in water level. However, at this time of year the levels of consumption and evaporation would be quite low.

**c** 44% **d** Start of February

**11** **a**



**b**



**12** **a** **i** 5°C
**ii** 20°C
**iii** 15°C
**iv** 10°C

**b** at 7:00 a.m. and 8:00 p.m.

**c** at 8:00 a.m. and 11:00 p.m.

**d** **i** around 7:00 a.m. (heater goes on)
**ii** around 8:00 a.m. (turns heater off)
**iii** around 8:00 p.m. (heater put back on)
**iv** around 11:00 p.m. (heater turned off)

**e** Answers will vary.

## Exercise 9D

**1** **a** playing sport **b** watching TV **c** more

**2** **a** rugby
**b** basketball
**c** $\frac{1}{5}$
**d** $\frac{2}{3}$

**3** **a** 20
**b** **i** $\frac{3}{10}$ **ii** $\frac{1}{4}$ **iii** $\frac{7}{20}$ **iv** $\frac{1}{10}$

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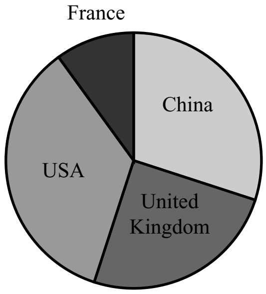
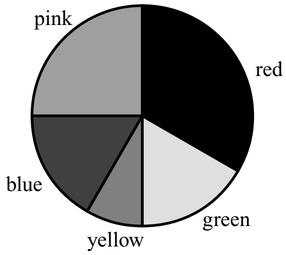
**c** **i** 108° **ii** 90° **iii** 126° **iv** 36°

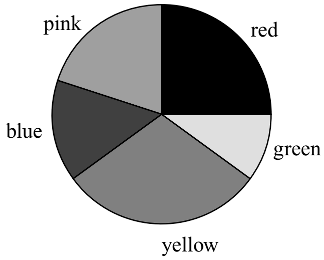
**d**



**4** **a**

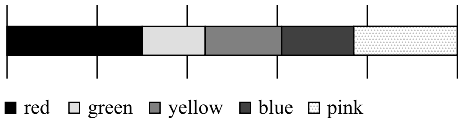


**b**

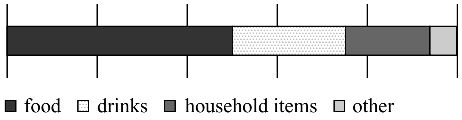


**c** Higher proportion of Year 7s like red;
higher proportion of Year 8s likes yellow.

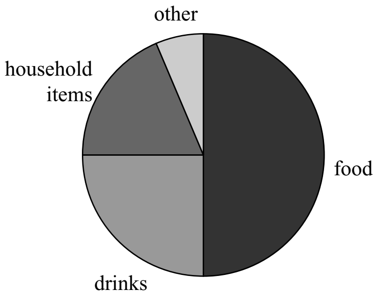
**d**



**5** **a**



**b**

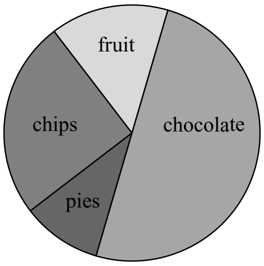


**c** Comparison required.

**6** **a** 50%
**b** 25%
**c** $2400

**7** **a** **i** 20 **ii** 10
**iii** 6 **iv** 4

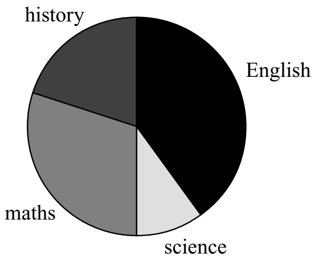
**b**



**c** **i** chips **ii** fruit and pies **iii** chocolate

**8** **a** **i** 40% **ii** 30% **iii** 20%
**b** **i** 9 **ii** 6 **iii** 15

**c**



**9** **a** Krishna
**b** Nikolas
**c** It means Nikolas also spends more time playing sport.
**d** 37.5°

**10** **a** 6
**b** 10
**c** Bird was chosen by $\frac{1}{8}$, which would be 2.5 people.
**d** Each portion is $\frac{1}{3}$, but $\frac{1}{3}$ of 40 is not a whole number.
**e** 24, 48, 72 or 96 people participated in survey.

**11** Need numbers for a meaningful axis but not for labels of each sector.

**12** **a** Survey 2
**b** Survey 1
**c** Survey 3

**13** **a** Column graph – categorical data. (Sector graph is inappropriate as not measuring proportions of a whole)

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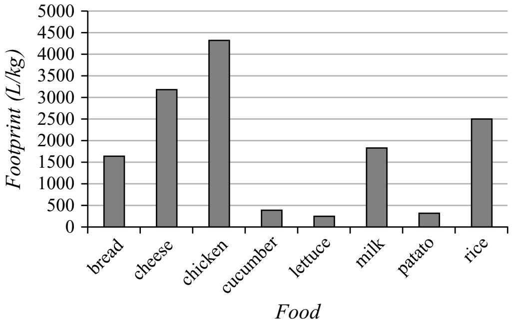
Exercise 9D cont.

**b** Column graph



**c** As water becomes scarcer it is more difficult to produce these foods.

**d** Answers will vary

**e**

| | Bread | Cheese | Chicken | Cucumber | Lettuce | Milk | Potato | Rice |
|---|---|---|---|---|---|---|---|---|
| **Efficiency (g/kL)** | 622 | 315 | 231 | 2833 | 4219 | 556 | 3484 | 400 |

## Exercise 9E

**1** **a** true **b** false **c** true **d** false

**2** **a** 4 **b** 7 **c** || **d** 𝍸 𝍸 I

**3** 3 2, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 7

**4** **a**

| Handspan | Frequency |
|---|---|
| 17 | 1 |
| 18 | 2 |
| 19 | 3 |
| 20 | 3 |
| 21 | 0 |
| 22 | 2 |
| 23 | 0 |
| 24 | 1 |

**b**

| Handspan | Frequency |
|---|---|
| 17–19 | 6 |
| 20–22 | 5 |
| 23–25 | 1 |

**5** **a**

| | Passes | Shots at goal | Shots that go in | Steals |
|---|---|---|---|---|
| **Frequency** | 3 | 12 | 8 | 2 |

**b** 12
**c** 8
**d** 2

**6** **a**

| **People in family** | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| **Tally** | I | II | IIII | IIII | IIII | II | III |
| **Frequency** | 1 | 2 | 4 | 4 | 4 | 2 | 3 |

**b** 4
**c** 9

**7** **a** 50
**b** 9
**c** 8
**d** 35

**8** **a**

| Height (cm) | Tally | Frequency |
|---|---|---|
| 130–139 | III | 3 |
| 140–149 | 𝍸 | 5 |
| 150–159 | II | 2 |
| 160–169 | III | 3 |
| 170–179 | III | 3 |
| 180–189 | I | 1 |
| 190+ | IIII | 4 |

**b** 2 **c** 5 **d** 10

**9** **a** 10 **b** 2 **c** 4 **d** 17

**10** **a** 28 **b** 130
**c** 19 **d** 13.1 years old
**e**

| **Age** | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|
| **Frequency** | 5 | 39 | 19 | 33 | 33 | 1 |

**11**

| **Score** | 0–19 | 20–39 | 40–59 | 60–79 | 80–99 |
|---|---|---|---|---|---|
| **Frequency** | 0 | 4 | 7 | 20 | 12 |

**12** **a** 2
**b** Any arrangement of 3, 3, 2, 1 will be correct.
**c** All arrangements of 3, 2, 2, 1, 1 will be correct.
**d** Priscilla = 2.25 hrs/night, Joey 1.8 hrs/night
**e** 2.25 hours more homework.

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## Exercise 9F

**1** **a** 2 **b** 9 **c** 11 years old

**2** **a** 4 **b** 4 **c** 8

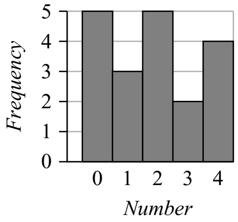
**3** 0.5 cm

**4** **a** The mistake is the columns are not of equal width. Use a ruler to mark an even scale.

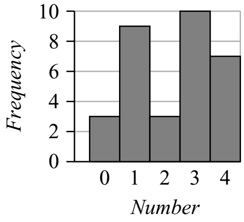
**b** The mistake is that there are gaps between the columns.

**c** Both histograms do not have numbers and words labelling each scale. There is also no gap before the first column.

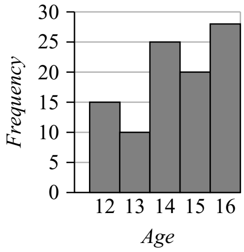
**5** **a**



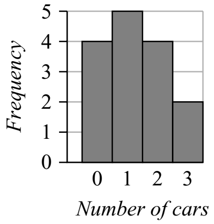
**b**



**c**



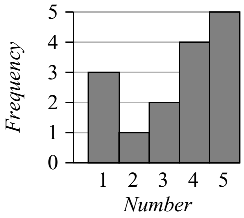
**d**



**6** **a** **i**

| Number | Tally | Frequency |
|---|---|---|
| 1 | III | 3 |
| 2 | I | 1 |
| 3 | II | 2 |
| 4 | IIII | 4 |
| 5 | 𝍸 | 5 |

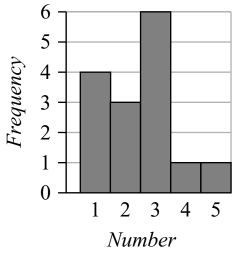
**ii**



**b** **i**

| Number | Tally | Frequency |
|---|---|---|
| 1 | IIII | 4 |
| 2 | III | 3 |
| 3 | 𝍸I | 6 |
| 4 | I | 1 |
| 5 | I | 1 |

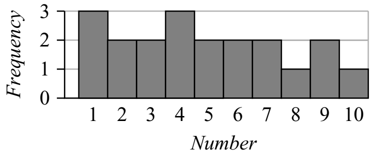
**ii**



**c** **i**

| Number | Tally | Frequency |
|---|---|---|
| 1 | III | 3 |
| 2 | II | 2 |
| 3 | II | 2 |
| 4 | III | 3 |
| 5 | II | 2 |
| 6 | II | 2 |
| 7 | II | 2 |
| 8 | I | 1 |
| 9 | II | 2 |
| 10 | I | 1 |

**ii**



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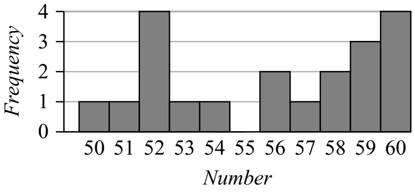
**Exercise 9F cont.**

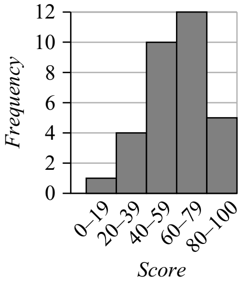
**d i**

| Number | Tally | Frequency |
|---|---|---|
| 50 | I | 1 |
| 51 | I | 1 |
| 52 | IIII | 4 |
| 53 | I | 1 |
| 54 | I | 1 |
| 55 | | 0 |
| 56 | II | 2 |
| 57 | I | 1 |
| 58 | II | 2 |
| 59 | III | 3 |
| 60 | IIII | 4 |

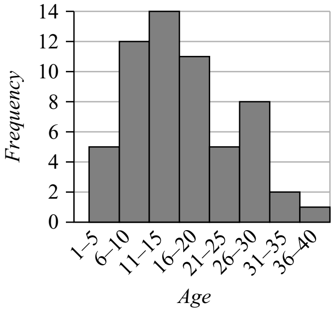
**ii**



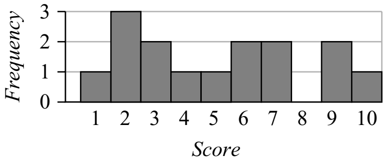
**7 a**



**b**



**8 a**



**b** Edwin is worse than Fred as most of Fred's scores are 8 or higher.

**9 a** D **b** A **c** B **d** C

**10 a** It would look identical but the age labels would start at 22 and go to 26.

**b** It would look just like the right half (12, 13, 14) but with the age axis labelled 0, 1, 2.

**11 a** 7 days **b** 4 days

**c** 4 cars in one day sold by Marie

**d** Con **e** Frank who sold 27 cars

**f** Bill who sold 15 cars

**12 a** 9 weeks of 10, 8 weeks of 9, 5 weeks of 8, 4 weeks of 7, 3 weeks of 6, 1 week of 5 (any list with the higher scores coming first and the lowest scores last is correct).

**b** 9 weeks of 5, then 8 weeks of 6, then 7 weeks of 7, then 4 weeks of 8, then 2 weeks of 9 out of 10

**c** They were absent from the test, or having a very bad day.

**13 a**

| Survey location | Height graph (cm) | Weight graph (kg) | Age graph (years) |
|---|---|---|---|
| Primary school classroom | Graph 4 | Graph 7 | Graph 6 |
| Shopping centre | Graph 8 | Graph 2 | Graph 9 |
| Teachers common room | Graph 5 | Graph 3 | Graph 1 |

**b** Answers will vary.

## Exercise 9G

**1 a** mode **b** mean **c** median
**d** range **e** outlier

**2 a** 15 **b** 5 **c** 3

**3 a** 1, 2, 4, 5, 6, 7, 9 **b** 5 **c** 5

**4 a** 7 and 9 **b** 16 **c** 8

**5 a** 8 **b** 1 **c** 7

**6 a** 9 **b** 10 **c** 15 **d** 14
**e** 30 **f** 27 **g** 16.9 **h** 8.7

**7 a i** 5 **ii** 4 **b i** 2 **ii** 2
**c i** 5 **ii** 3 **d i** –3 **ii** 0
**e i** 0 **ii** –9 **f i** 0 **ii** 3
**g i** 12.9 **ii** 15 **h i** 13.1 **ii** 20
**i i** 11.1 **ii** 12 **j i** 10.4 **ii** 5
**k i** 2.4 **ii** –6 **l i** –3.4 **ii** –6

**8 a** 6 **b** 4 **c** 8 **d** 5
**e** 8 **f** 7 **g** 5 **h** 5.5
**i** 7.5 **j** 8 **k** 10.5 **l** 12

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**9** **a** 8.4 **b** 8 **c** 8

**10** **a** white **b** meat-lovers
**c** Wednesday **d** South Australia

**11** **a** 3 **b** 10 **c** 7, 7, 7, 9, 9, 9, 10, 10, 10, 10
**d** 8.8 **e** 9 **f** 3

**12** **a** Business $B$, \$200 000
**b** Mean $A$ = \$52 000, mean $B$ = \$78 000
**c** \$26 000 larger
**d** \$50 000 for both $A$ and $B$
**e** No
**f** The median, \$50 000 as it is not affected by the outlier.

**13** **a** 15 **b** 35 **c** Nathan **d** Gary

**14** **a** 12, 12, 13, 13, 13, 13, 14, 14, 14, 14, 14, 14, 15, 15, 15, 15, 16, 16, 16, 16, 16, 16, 16, 16, 17, 17, 17, 17, 17, 18
**b** 6 years **c** 16 years old
**d** 15.03 years old
**e** 15
**f** **i** 16.23 years **ii** 15 years
**iii** The mean has increased the most. The median is unchanged.

**15** **a** 7 **b** 42

**16** **a** \$1 477 778
**b** \$630 000
**c** A strong effect – it makes the mean significantly higher.
**d** Only increase the median by a small amount.
**e** Median is not easily distorted by a few very large values.

## Exercise 9H

**1** **a** stem, leaf

**2** **a** 5 **b** 2

**3** **a** 39 **b** 27 **c** 134

**4** 57

**5** **a** 8, 9, 10, 11, 13, 15, 17, 18, 21, 24
**b** 10
**c** **i** false
**ii** true
**iii** true
**iv** false

**6** **a** range = 20, median = 17
**b** range = 31, median = 26
**c** range = 19, median = 40.5

**7**

| Stem | Leaf |
|---|---|
| 2 | 5 7 9 |
| 3 | 0 2 9 |
| 4 | 1 2 5 |
| 5 | 1 |

**8** **a**

| Stem | Leaf |
|---|---|
| 1 | 1 2 3 4 4 5 7 |
| 2 | 0 4 8 9 |
| 3 | 1 2 3 5 |

**b**

| Stem | Leaf |
|---|---|
| 2 | 0 2 |
| 3 | 9 |
| 4 | 5 7 9 9 |
| 5 | 1 2 2 3 5 6 8 8 |

**9** **a**

| Stem | Leaf |
|---|---|
| 1 | 6 6 8 |
| 2 | 1 4 8 9 |
| 3 | 1 2 3 5 |
| 4 | 1 8 9 |
| 5 | 0 |

**b**

| Stem | Leaf |
|---|---|
| 1 | 1 2 4 |
| 2 | 7 9 |
| 3 | 2 7 8 8 |
| 6 | 0 0 |
| 7 | 3 8 |
| 8 | 1 7 |

**10** **a**

| Stem | Leaf |
|---|---|
| 8 | 0 4 5 6 |
| 9 | 0 6 |
| 10 | 1 4 5 |
| 11 | 0 3 4 4 5 9 |

**b**

| Stem | Leaf |
|---|---|
| 39 | 1 5 6 |
| 40 | 1 2 4 5 6 6 8 9 |
| 41 | 1 2 3 3 5 6 7 8 |
| 42 | 0 |

**Exercise 9H** cont.

**11 a** 10 **b** 1
**c** 8 **d** 58
**12 a** 15 **b** 13
**c** $a$ is 5 or 6, $b$ is 0, $c$ is 8 or 9, $d$ is 0.
**13 a i** 49 years **ii** 36 years
**b** radio station 1
**c i** 33 to 53 years **ii** 12 to 32 years

## Exercise 9I

Some answers may vary.

**1 a** survey
**b** sample
**c** biased
**d** symmetrical
**e** skewed
**2 a** surveying 1000 randomly selected people
**b** surveying 10 friends
**3 a** $\frac{2}{5}$ **b** 2000 **c** 300
**4 a** symmetrical
**b** skewed
**c** skewed
**d** symmetrical
**5 a** skewed **b** $\frac{2}{5}$ **c** 400
**d** $\frac{6}{25}$ **e** 1200
**f** More likely that people will have pets if near a vet clinic.
**6 a** symmetrical
**b** $\frac{28}{100} = \frac{7}{25}$ **c** 420
**d** $\frac{35}{100} = \frac{7}{20}$ **e** 1050
**f** In a wealthy suburb the houses are more likely to be larger.
**7 a** skewed **b** $\frac{52}{100} = \frac{13}{30}$ **c** 15 600
**d** $\frac{28}{120} = \frac{7}{30}$ **e** 1120
**f** The people on this train probably start work early and are less likely to eat breakfast.
**8 a** 108 g **b** symmetrical **c** $\frac{5}{8}$
**d** 500 **e** $\frac{17}{128}$ **f** 272
**9 a** Yes, it is required information.
**b** No, it is too vague or personal.
**c** No, it is too vague or personal.
**d** No, it addresses wealth but not income.
**e** No, it is irrelevant.
**f** Yes, it can be used to decide income.
**g** No, if it is not a pay day then results will be distorted.
**10 a** at midday on a Thursday on a major road.
**b** outside a political party office.
**c** in a butcher's shop.
**d** at 11 p.m., when people will buy just a few items.
**11 a** at a professional dance studio in the afternoon.
**b** in a Bank.
**c** Choose a large random sample.
**12 a** Only one year level. Possibly streamed class, so similar work ethic.
**b** Only males would be surveyed, also same age.
**c** Sample size too small.
**d** A range of students in age, gender and results.
**13** Answers vary.

## Puzzles and games

**1** A BOWLING MACHINE
**2** 4, 11, 4.5
**3** No, must have repeated points in 5–9 and 10–14.

## Chapter review

### Multiple-choice questions

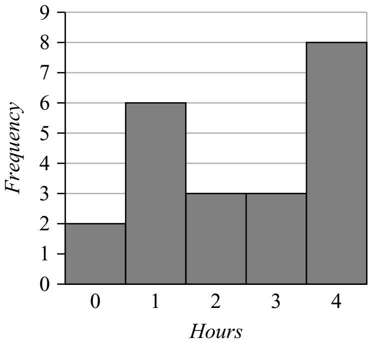
| | | | | |
|---|---|---|---|---|
| **1** C | **2** B | **3** C | **4** A | **5** C |
| **6** B | **7** C | **8** D | **9** D | **10** B |

### Short-answer questions

**1 a** government bus **b** train
**c** 75% **d** 1000
**e** Example: Prices went up for government buses.
**2 a** 22
**b**

| **Hours** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **Frequency** | 2 | 6 | 3 | 3 | 8 |

**c**



**d** $\frac{1}{11}$ **e** 53 hours **f** 2.4
**3 a** 38, 43, 44, 44, 52, 53, 55, 56, 59,60, 61, 62, 63, 64, 66,68,69,70, 71,72, 74,84
**b** 44 **c** 61.5 **d** 46
**4 a** 27
**b** 10, 10, 11, 11, 11, 12, 12, 12, 12, 13, 13, 13, 13, 13, 13,13, 13, 14, 14, 14, 14, 14, 14, 14, 14, 14, 14
**c** 12.78 years **d** 13 years

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**5 a** Lowest: 50 kg, highest: 85 kg, range = 35 kg

**b**

| Weight | Frequency |
|---|---|
| 50–54 | 6 |
| 55–59 | 6 |
| 60–64 | 8 |
| 65–69 | 7 |
| 70–74 | 7 |
| 75–79 | 1 |
| 80–85 | 5 |

**c** 60–64 kg

**d** Only teenagers were chosen, not including children or adults.

**6 a i** 6.5 **ii** 6 **iii** 13

**b** median = 6

mean = 10

The mean changes the most.

**c** Outlier.

**7 a** Not enough people, and her friends might work harder (or less hard) than other students.

**b** She could choose 10 people who worked less hard than her.

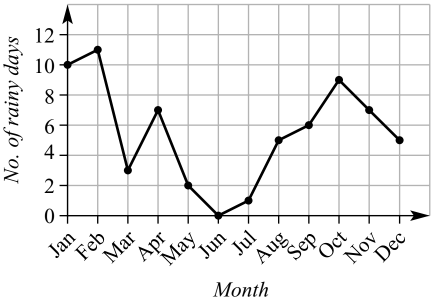
**8 a** 118g **b** 105g **c** 131g **d** 26g

**9 a** 11 **b** 5 **c** 3.5 **d** 2

**10 a** 12 **b** 5 **c** 3.5 **d** 3

### Extended-response questions

**1 a**



**b** 66

**2 a** 40 **b** cheesecake **c** $\frac{7}{40}$

**d i** yes **ii** no **iii** yes **iv** no

**e** 80

# Semester review 2

## Angle relationships and properties of geometrical figures 1

### Multiple-choice questions

**1** B **2** D **3** C **4** C **5** C

### Short-answer questions

**1 a** 66 **b** 25 **c** 123

**d** 35 **e** 70 **f** 98

**2 a** $x = 81, y = 99$ **b** $a = 75$

**c** $a = 62, b = 62$ **d** $a = 65, b = 65$

**e** $a = b = c = d = 100, e = 80$

**f** $x = 95, y = 85$

**3 a** 48 **b** 45 **c** 60

**d** 75 **e** 121 **f** 75

**4 a** $a = b = 90$ **b** $a = 73, b = 95$

**c** $a = 265, b = 30$

**5 a i** 0 **ii** 2

**b i** 2 **ii** 2

**c i** 1 **ii** 0

### Extended-response question

This is one of many possible orders.

$b = 65$

$c = 65$

$d = e = 57.5$

$f = 122.5$

$g = 122.5$

$h = 180$

$i = 295$

## Linear relationships

### Multiple-choice questions

**1** D **2** B **3** C

**4** A **5** C

### Short-answer questions

**1 a** 1st **b** 2nd **c** 3rd **d** 4th

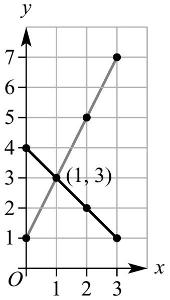
**2 a i**

| | | | | |
|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 |
| $y$ | 1 | 3 | 5 | 7 |

**ii**

| | | | | |
|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 |
| $y$ | 4 | 3 | 2 | 1 |

**b**



**3 a i** To find a value for $y$, choose a value for $x$, then multiply it by 2 and add 1.

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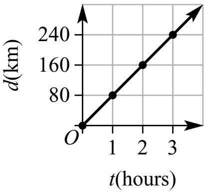
**ii** To find a value for $y$, choose a value for $x$, then subtract it from 4.

**4 a**

| $t$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $d$ | 0 | 80 | 160 | 240 |

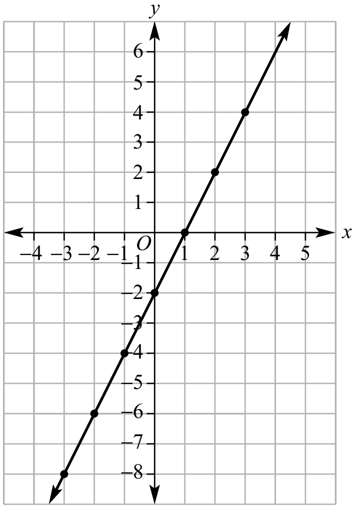
**b**



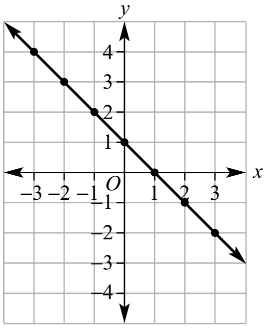
**c** 240 km

**d** 4 hours

**5 a**



**b**

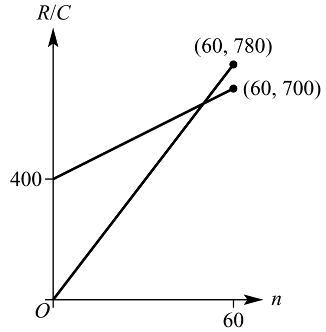


## Extended-response question

**a**

| $n$ | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|---|
| $C$ | 400 | 450 | 500 | 550 | 600 | 650 | 700 |
| $R$ | 0 | 130 | 260 | 390 | 520 | 650 | 780 |

**b**



**c** (50, 650)

**d** \$400

## Transformations and congruence

### Multiple-choice questions

**1** C **2** B **3** D **4** D **5** C

### Short-answer questions

**1** c, d, f

**2 a** right 2, up 2 **b** left 1, down 2

**3 a** $A'(1, -1)$, $B'(3, -1)$, $C'(2, -3)$

**b** $A'(-3, -1)$, $B'(-1, -1)$, $C'(-2, 1)$

**c** $A'(1, -1)$, $B'(1, -3)$, $C'(3, -2)$

**d** $A'(-1, -1)$, $B'(-3, -1)$, $C'(-2, -3)$

**4 a** SSS **b** SAS **c** RHS **d** AAS

**5** $A$, $C$

### Extended-response question

**a** Alternate angles in parallel lines are equal.

**b** Alternate angles in parallel lines are equal.

**c** AAS

**d** $\Delta ABE \cong \Delta CDE$ so $AE = CE$ and $BE = DE$. because they are matching sides in congruent triangles.

## Data collection, representation and analysis

### Multiple-choice questions

**1** A **2** A **3** B **4** A **5** A

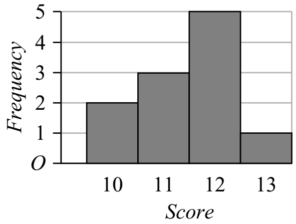
### Short-answer questions

**1 a i** 13.75 **ii** 14 **iii** 8

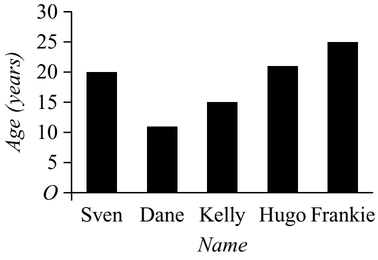
**b i** 23 **ii** 18.5 **iii** 56

**c i** 10 **ii** 9.45 **iii** 15.7

**2**

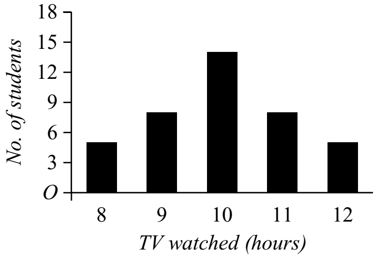


**3**



**4** **a** 40 **b** 13 **c** 10 hours

**d**



**5** **a** 4 students

**b** 2 students

**c** 1:00 p.m.

**d** 6 students

**e** The number of students must be a whole number. Joining the dots would include fractions for the number of students.

## Extended-response question

**a** 18 **b** 78 **c** 78 **d** Group B

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