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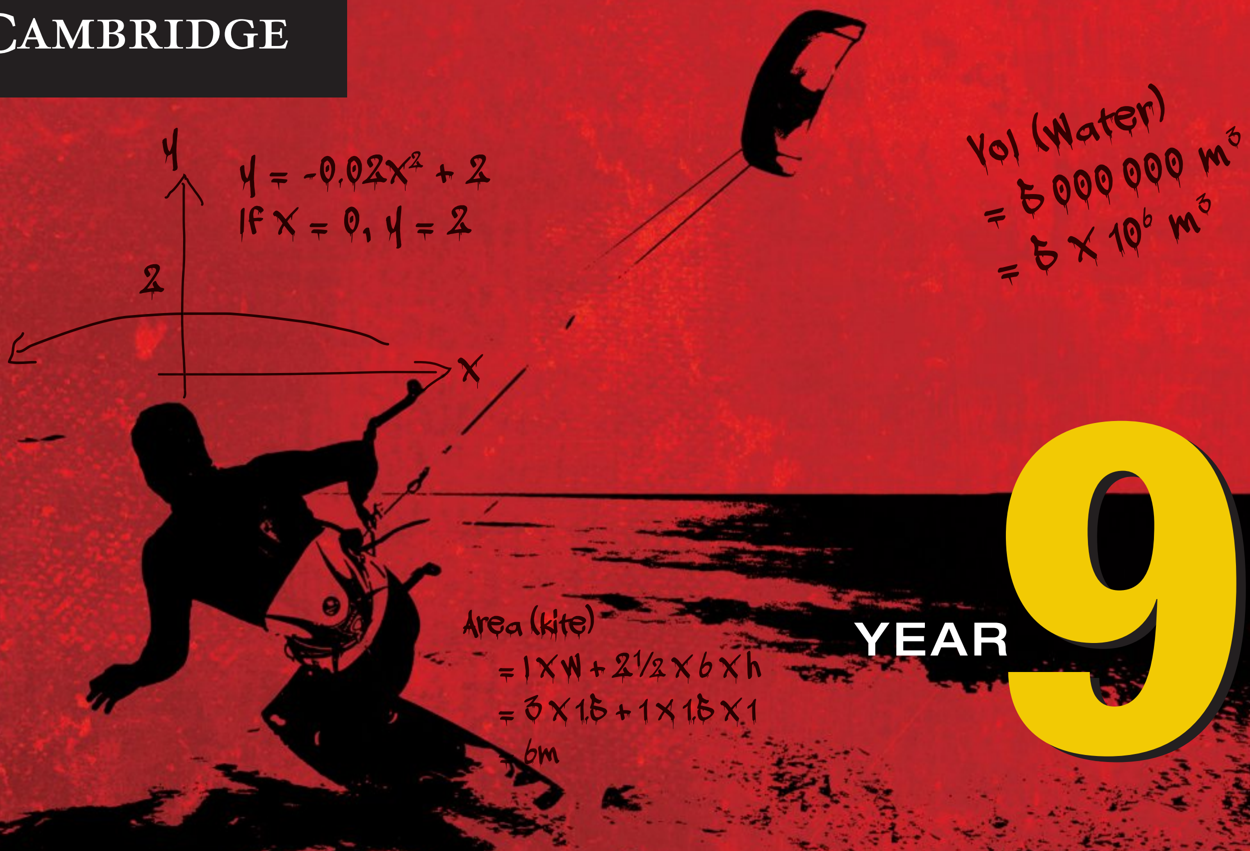


YEAR 9

CambridgeMATHS

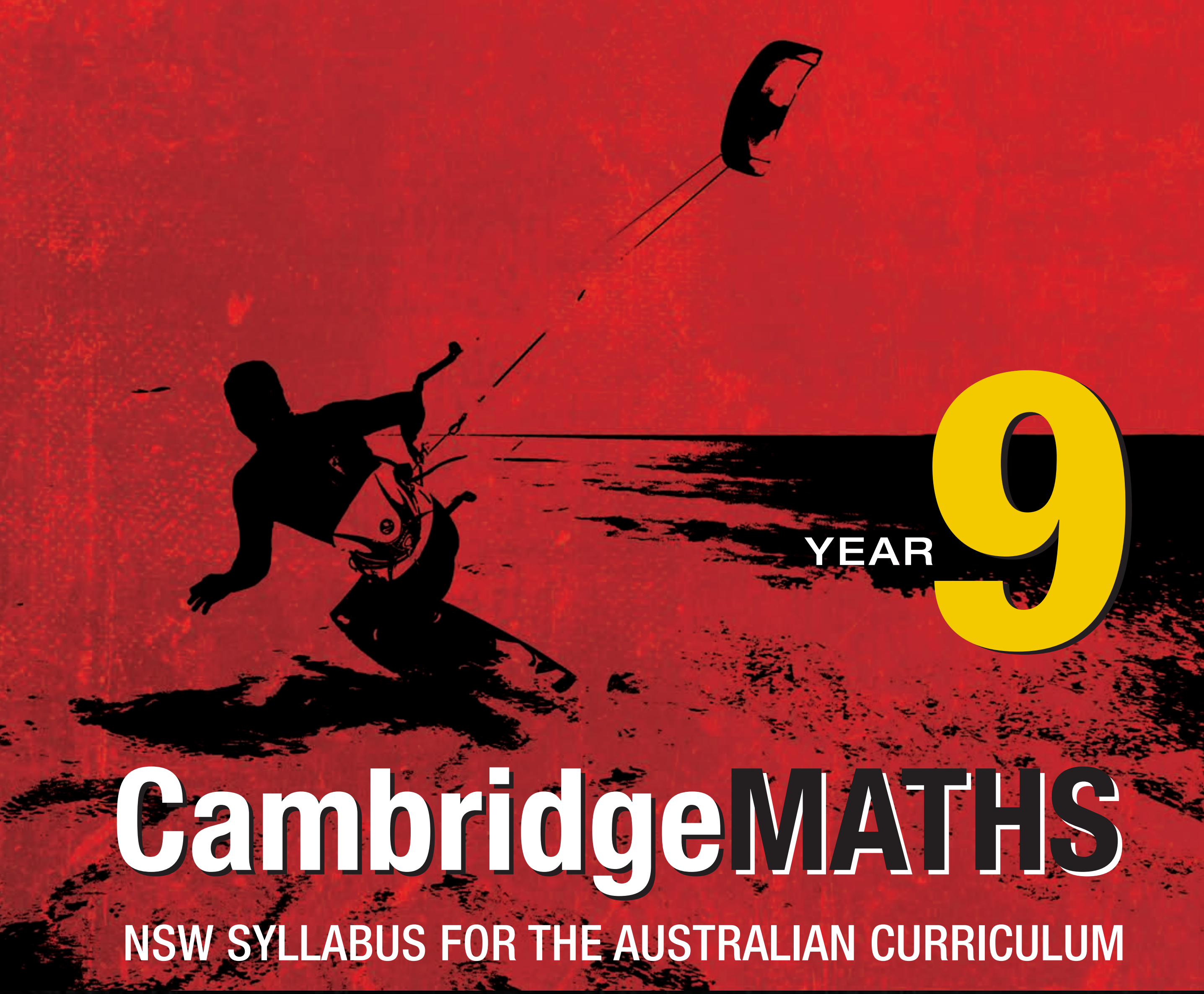
NSW SYLLABUS FOR THE AUSTRALIAN CURRICULUM

STAGE 4/5.1

GOLD

Interactive Textbook included

STUART PALMER | KAREN MCDAID
DAVID GREENWOOD | SARA WOOLLEY
JENNY GOODMAN | JENNIFER VAUGHAN



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Contents

ISBN 978-1-316-61816-5 Stuart Palmer et al. 2016 Cambridge University Press

3 Expressions and equations 116

Number and Algebra

Algebraic techniques
Equations

4 Right-angled triangles 176

Measurement and Geometry

Right-angled triangles (Pythagoras and trigonometry)

ISBN 978-1-316-61816-5 Stuart Palmer et al. 2016 Cambridge University Press

ISBN 978-1-316-61816-5 Stuart Palmer et al. 2016 Cambridge University Press

7 Properties of geometrical figures 346

Measurement and Geometry

Properties of geometrical figures

8 Probability and single variable data analysis 410

Statistics and Probability

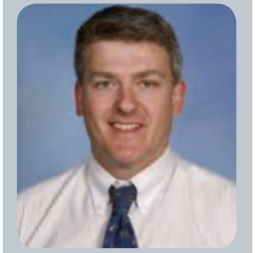
Probability
Single variable data analysis

ISBN 978-1-316-61816-5 Stuart Palmer et al. 2016 Cambridge University Press

About the authors

Stuart Palmer was born and educated in New South Wales. He is a high school mathematics teacher with more than 25 years' experience teaching boys and girls from all walks of life in a variety of schools. Stuart has taught all the current NSW Mathematics courses in Stages 4, 5 and 6 many times. He has been a head of department in two schools and is now an educational consultant who conducts professional development workshops for teachers all over NSW and beyond. He also works with pre-service teachers at the University of Sydney and the University of Western Sydney.

Karen McDaid is an academic and lecturer in Mathematics Education in the School of Education at the Western Sydney University. She has taught mathematics to both primary and high school students and is currently teaching undergraduate students on their way to becoming primary school teachers.



David Greenwood is the head of Mathematics at Trinity Grammar School in Melbourne and has 20 years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 20 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.

Sara Woolley was born and educated in Tasmania. She completed an Honours degree in Mathematics at the University of Tasmania before completing her education training at the University of Melbourne. She has taught mathematics in Victoria from Years 7 to 12 since 2006 and has a keen interest in the creation of resources that cater for a wide range of ability levels.

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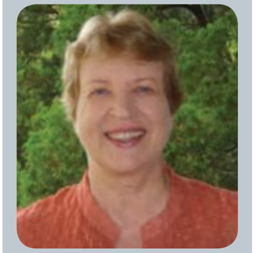
Jenny Goodman has worked for 20 years in comprehensive state and selective high schools in New South Wales and has a keen interest in teaching students of differing ability levels. She was awarded the Jones Medal for Education at the University of Sydney and the Bourke prize for Mathematics. She has written for Cambridge NSW and was involved in the Spectrum and Spectrum Gold series.



Jennifer Vaughan has taught secondary mathematics for more than 30 years in New South Wales, Western Australia, Queensland and New Zealand, and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has taught special needs students and has had extensive experience in developing resources that make mathematical concepts more accessible.

Consultant

Beth Godwin is the author of several Cambridge titles, including Spectrum Maths Gold Year 7 and Year 8. She has presented seminars and workshops to educators on topics, including differentiating the curriculum, behaviour management and developing literacy skills. Currently the principal of Cabramatta High School in New South Wales, Beth has experience in ensuring that the curriculum is accessible to all students.

Introduction and guide to this book

Thank you for choosing *CambridgeMATHS Gold*. This book is one component of an integrated package of resources designed for the NSW Syllabus for the Australian Curriculum. *CambridgeMATHS Gold* follows on from the standard CambridgeMATHS series published in 2013–14, and the two series have been structured so that they can be used in the same classroom. Mapping documents that show the relationship between the series can be found on *Cambridge GO*.

Whereas the standard *CambridgeMATHS* books for Years 9 and 10 begin at Stage 5, the *Gold* books for Years 9 and 10 are intended for students who need to consolidate Stage 4 learning prior to progressing to Stage 5. The aim is to develop Understanding and Fluency in core mathematical skills. Clear explanations of concepts, worked examples embedded in each exercise and carefully graded questions contribute to this goal. Problem-solving, Reasoning and Communicating are also developed through carefully constructed activities, exercises and enrichment.

Drilling for Gold
5A3

An important component of *CambridgeMATHS Gold* is a set of worksheets, and exercises in the print book, called ***Drilling for Gold***. These are engaging, innovative, skill-and-drill style tasks that provide the kind of practise and consolidation of the skills required for each topic without adding to the length of the textbook. All of them can be accessed in worksheet form from the online interactive version.

In Years 9 and 10 we have introduced major new activities to prepare students for mathematics in the workplace, the marketplace (consumer maths) and at home, and to improve numeracy. **Keeping in touch with Numeracy**, **Maths@work**, **Maths@home** and **Consumer maths** are their titles.

The relationship between literacy and maths is a major focus of *CambridgeMATHS Gold*. Key words and concepts are defined using student-friendly language; real-world contexts and applications of mathematics help students connect these concepts to everyday life; and literacy skills are built into questions and activities throughout. In the interactive version of this book, definitions are enhanced by audio pronunciation, visual definitions and examples. More information about the interactive version can be found on page xii.

Chapter 1

Integers, decimals, fractions, ratios and rates

What you will learn

- 1A Adding and subtracting positive and negative integers
  Drilling for Gold exercise
- 1B Multiplying and dividing positive and negative integers
- 1C Decimal places and significant figures
  Drilling for Gold exercise
- 1D Rational numbers and irrational numbers
- 1E Adding and subtracting fractions
  Keeping in touch with numeracy
- 1F Multiplying and dividing fractions
- 1G Ratios
  Drilling for Gold exercise
- 1H Rates
  Drilling for Gold exercise
  Consumer maths: Investigating currency conversions, petrol consumption and other rates

**Strands: Number and Algebra
Measurement and Geometry**

Substrands: COMPUTATION WITH INTEGERS
FRACTIONS, DECIMALS AND PERCENTAGES
RATIOS AND RATES

In this chapter you will learn to:

- compare and order integers and calculate with them, using different strategies
- operate with fractions, decimals and percentages
- operate with ratios and rates, and explore their graphical representation

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw9

Negative numbers in the ancient world

The ancient Babylonians, Hindus and Egyptians were all known for their work with numbers. There is no mention of negative numbers in their writings. The first mention of negative numbers is dated back to only 200 BC in China. The Chinese used abacuses with black rods for negative numbers and red rods for positive numbers.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO:* www.cambridge.edu.au/goldnsw9

Drilling for Gold: Building knowledge and skills

Skillsheets: Extra practise of important skills

Worksheets: Consolidation of the topic

Chapter Test: Preparation for an examination

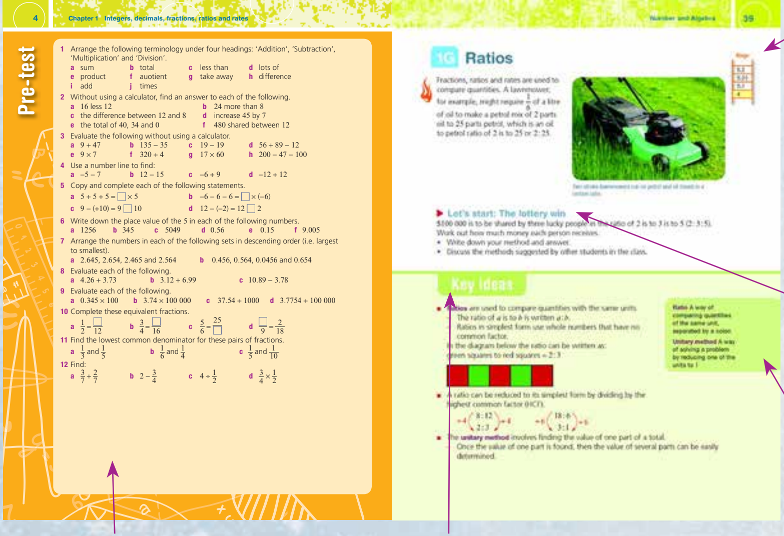
3

A suite of accompanying resources, including Drilling for Gold and other worksheets can be downloaded from *Cambridge GO*.

Chapter introductions provide real-world context for students.

What you will learn gives an overview of the chapter contents.

A summary of the chapter connects the topic to the NSW Syllabus. Detailed mapping documents to the NSW Syllabus can be found in the teaching program on *Cambridge GO*.



Ladder icons indicate the stage covered by each section (highlighted yellow).

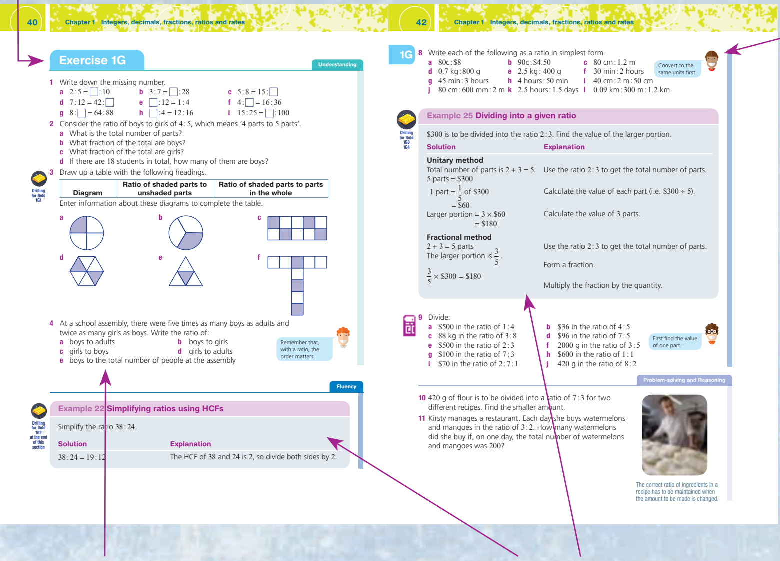
Let's start activities provide an engaging way to begin thinking about the topic.

Important terms in the **Key ideas** contain a simple-language definition.

A **Pre-test** for each chapter establishes prior knowledge.

Key ideas summarises the knowledge and skills for the topic.

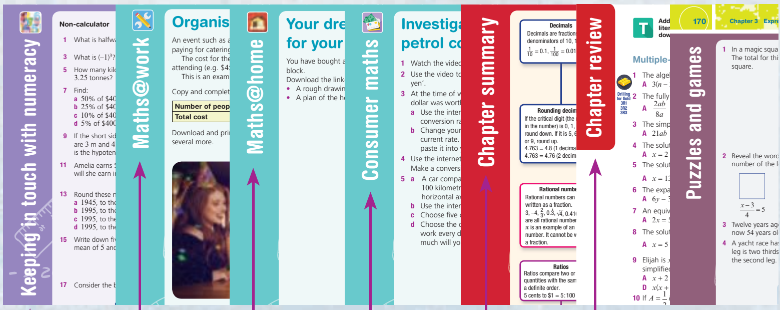
Exercises are structured according to the four Working Mathematically strands: **Understanding**, **Fluency**, **Problem-solving and Reasoning**, with **Communicating** present in each of the other three. **Enrichment** questions at the end of the exercise challenge students to reach further.



Hints give advice for tackling questions.

Within each Working Mathematically strand, questions are further carefully graded from easier to challenging.

Examples with worked solutions and explanations are embedded in the exercises immediately before the relevant question(s).



Puzzles and games allow students to have fun with the mathematics contained in the chapter.

Chapter summaries give mind maps of key concepts.

Chapter reviews provide multiple-choice, short-answer and extended-response questions.

Consumer maths activities help students become more informed consumers and citizens.

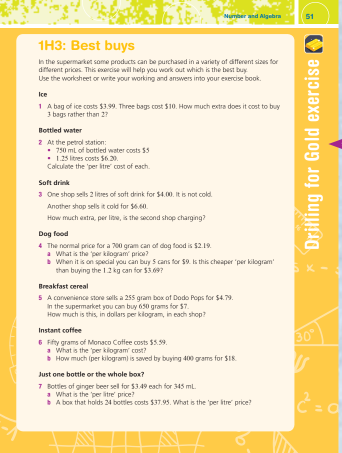
Maths@home activities help students develop life skills in mathematics.

Maths@work activities develop useful vocational mathematics skills, with supporting spreadsheets and videos in the interactive textbook.

Keeping in touch with Numeracy activities reinforce basic number skills through carefully structured alternating calculator and non-calculator questions.

Drilling for Gold exercises in the print book and downloadable worksheets

Drilling for Gold is a collection of engaging and motivating learning resources that provide opportunities for students to repeatedly practise routine mathematical skills. Their purpose is to improve automaticity, fluency and understanding through 'hands-on' resources, games, competitions, puzzles, investigations and sets of closed questions. In years 9 and 10, Drilling for Gold exercises are included in the print textbook, and Drilling for Gold worksheets are referenced in the pages of the textbook via a 'gold' icon and unique reference number. The worksheets can be downloaded from the Interactive Textbook.



1H3: Best buys

In the supermarket some products can be purchased in a variety of different sizes for different prices. This exercise will help you work out which is the best buy.
Use the worksheet or write your working and answers into your exercise book.

Ice

1 A bag of ice costs \$3.99. Three bags cost \$10. How much extra does it cost to buy 3 bags rather than 2?

Bottled water

2 At the petrol station:
- 750 mL of bottled water costs \$5
- 1.25 litres costs \$6.20.

Calculate the 'per litre' cost of each.

Soft drink

3 One shop sells 2 litres of soft drink for \$4.00. It is not cold.

Another shop sells it cold for \$6.60.

How much extra, per litre, is the second shop charging?

Dog food

4 The normal price for a 700 gram can of dog food is \$2.19.
  a What is the 'per kilogram' price?
  b When it is on special you can buy 5 cans for \$9. Is this cheaper 'per kilogram' than buying the 1.2 kg can for \$3.69?

Breakfast cereal

5 A convenience store sells a 255 gram box of Dodo Pops for \$4.79.
In the supermarket you can buy 650 grams for \$7.
How much is this, in dollars per kilogram, in each shop?

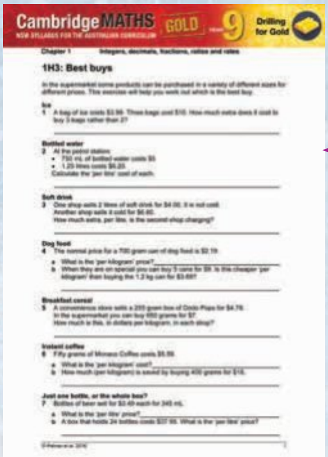
Instant coffee

6 Fifty grams of Monaco Coffee costs \$5.59.
  a What is the 'per kilogram' cost?
  b How much (per kilogram) is saved by buying 400 grams for \$18.

Just one bottle or the whole box?

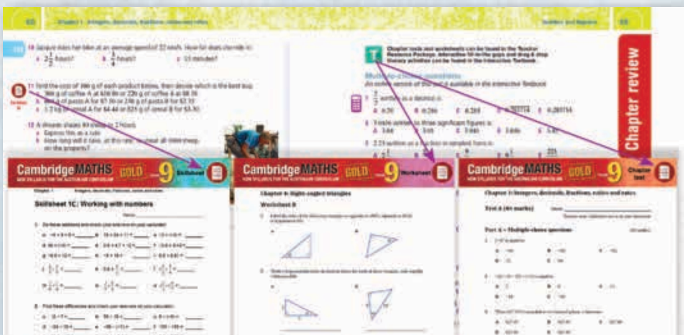
7 Bottles of ginger beer sell for \$3.49 each for 345 mL.
  a What is the 'per litre' price?
  b A box that holds 24 bottles costs \$37.95. What is the 'per litre' price?

A Drilling for Gold exercise in the print book.



A Drilling for Gold worksheet from the interactive textbook.

Other resources on *Cambridge GO*

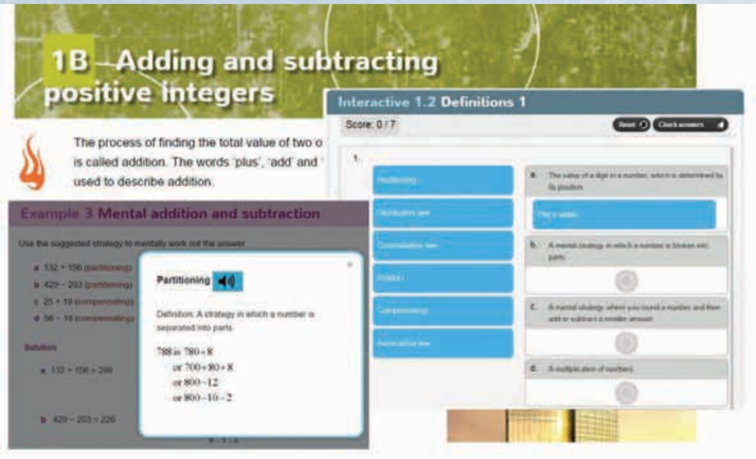


- **Skillsheets** provide practice of the key skills learned across the entirety of the chapter.
- A **Chapter test** provides exam-style assessment, with multiple-choice, short-answer and extended-response questions.
- **Worksheets** cover multiple topics within a chapter and can be done in class or completed as homework.

About your Interactive Textbook

An interactive digital textbook is included with your print textbook and is an integral part of the *CambridgeMATHS Gold* learning package. As well as being an attractive, easy-to-navigate digital version of the textbook, it contains many features that enhance learning in ways that are not possible with a print book:

- **Videos** provide detailed guidance on developing spreadsheets or other technology to use in a number of work, consumer and life skills settings
- Supporting spreadsheet and technology files are provided
- **Clickable 'enhanced' definitions** containing diagrams, illustrations, examples and audio pronunciation provide instant assistance and revision
- **Roll-over hints** for selected questions are provided within exercises
- **Matching *HOTmaths* lessons** can be accessed by clicking HOTmaths icon
- **Additional teacher resources** can be accessed by clicking the 'T' icon in the chapter review
- **Drilling for Gold** worksheets and **Skillsheets** can be downloaded by clicking on their icons in the margins



The process of finding the total value of two o
is called addition. The words 'plus', 'add' and
used to describe addition.

- **Fill-the-gap** and **drag-and-drop** literacy activities at the end of each chapter provide a fun way of learning key terms
- Interactive tests provide online versions of the multiple-choice questions from the chapter review
- **Answers** can be conveniently accessed via a button
- **Font size** can be increased or decreased as required
- **Annotations** can be added to allow critical engagement with the textbook.

A more detailed guide to using the Interactive Textbook can be found on *Cambridge GO*.

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Chapter 1

Integers, decimals, fractions, ratios and rates

What you will learn

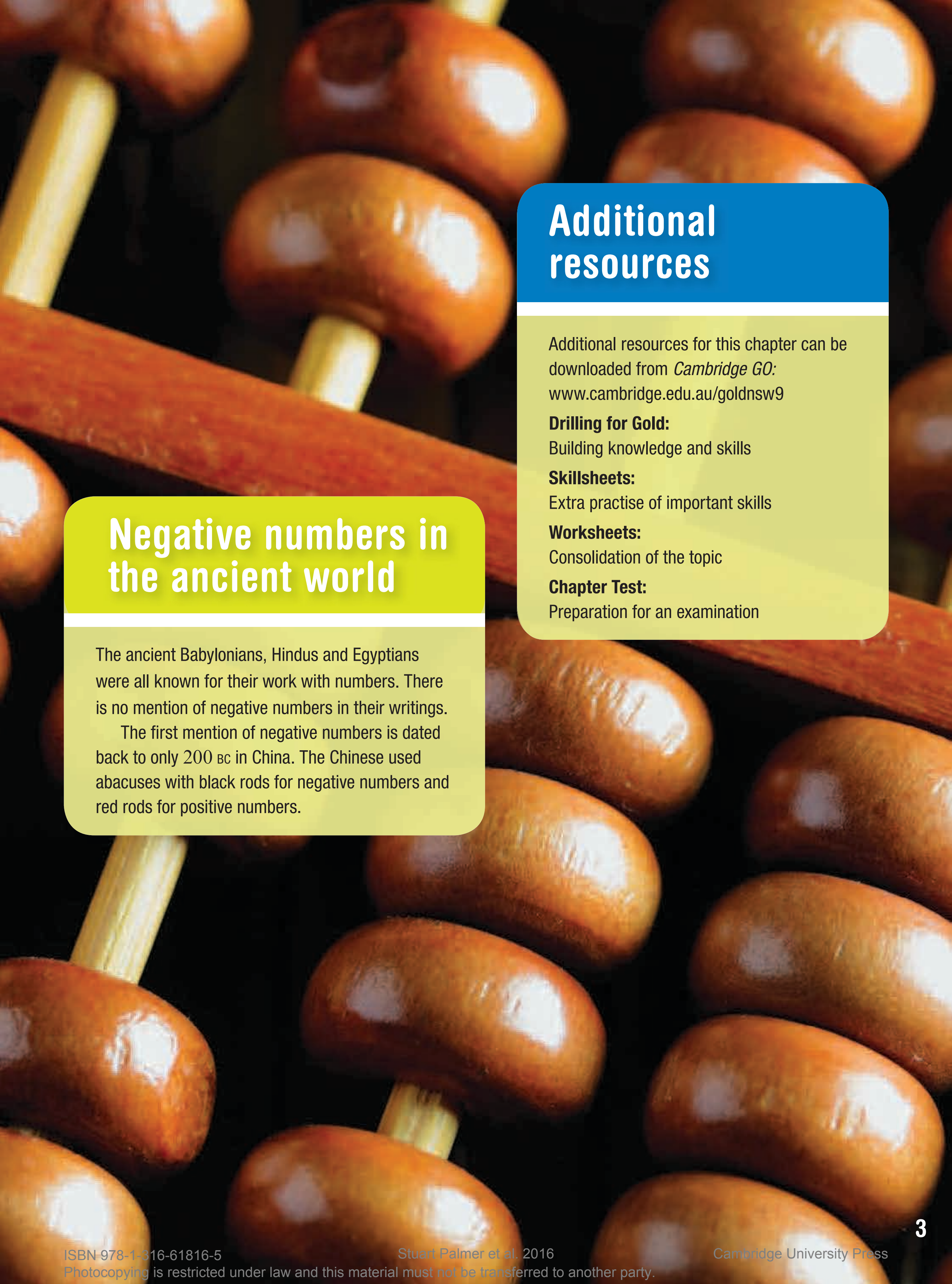
Strands: Number and Algebra
Measurement and Geometry

Substrands: COMPUTATION WITH INTEGERS
FRACTIONS, DECIMALS
AND PERCENTAGES
RATIOS AND RATES

In this chapter you will learn to:

- compare and order integers and calculate with them, using different strategies
- operate with fractions, decimals and percentages
- operate with ratios and rates, and explore their graphical representation.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw9



Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO:* www.cambridge.edu.au/goldnsw9

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

Negative numbers in the ancient world

The ancient Babylonians, Hindus and Egyptians were all known for their work with numbers. There is no mention of negative numbers in their writings.

The first mention of negative numbers is dated back to only 200 BC in China. The Chinese used abacuses with black rods for negative numbers and red rods for positive numbers.

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Pre-test

1 Arrange the following terminology under four headings: ‘Addition’, ‘Subtraction’, ‘Multiplication’ and ‘Division’.

a sum **b** total **c** less than **d** lots of
e product **f** quotient **g** take away **h** difference
i add **j** times

2 Without using a calculator, find an answer to each of the following.

a 16 less 12 **b** 24 more than 8
c the difference between 12 and 8 **d** increase 45 by 7
e the total of 40, 34 and 0 **f** 480 shared between 12

3 Evaluate the following without using a calculator.

a $9 + 47$ **b** $135 - 35$ **c** $19 - 19$
d $56 + 89 - 12$ **e** 9×7 **f** $320 \div 4$
g 17×60 **h** $200 - 47 - 100$

4 Use a number line to find:

a $-5 - 7$ **b** $12 - 15$ **c** $-6 + 9$ **d** $-12 + 12$

5 Copy and complete each of the following statements.

a $5 + 5 + 5 = \square \times 5$ **b** $-6 - 6 - 6 = \square \times (-6)$
c $9 - (+10) = 9 \square 10$ **d** $12 - (-2) = 12 \square 2$

6 Write down the place value of the 5 in each of the following numbers.

a 1256 **b** 345 **c** 5049
d 0.56 **e** 0.15 **f** 9.005

7 Arrange the numbers in each of the following sets in descending order (i.e. largest to smallest).

a 2.645, 2.654, 2.465 and 2.564 **b** 0.456, 0.564, 0.0456 and 0.654

8 Evaluate each of the following.

a $4.26 + 3.73$ **b** $3.12 + 6.99$ **c** $10.89 - 3.78$

9 Evaluate each of the following.

a 0.345×100 **b** $3.74 \times 100\,000$
c $37.54 \div 1000$ **d** $3.7754 \div 100\,000$

10 Complete these equivalent fractions.

a $\frac{1}{2} = \frac{\square}{12}$ **b** $\frac{3}{4} = \frac{\square}{16}$ **c** $\frac{5}{6} = \frac{25}{\square}$ **d** $\frac{\square}{9} = \frac{2}{18}$

11 Find the lowest common denominator for these pairs of fractions.

a $\frac{1}{3}$ and $\frac{1}{5}$ **b** $\frac{1}{6}$ and $\frac{1}{4}$ **c** $\frac{1}{5}$ and $\frac{1}{10}$

12 Find:

a $\frac{3}{7} + \frac{2}{7}$ **b** $2 - \frac{3}{4}$ **c** $4 \div \frac{1}{2}$ **d** $\frac{3}{4} \times \frac{1}{2}$

1A Adding and subtracting positive and negative integers

Stage
5.2
5.2◊
5.1
4

Being able to work with whole numbers is very important. They are used every day for counting, calculating measuring and ordering.

Numbers are used in marketplaces all over the world to describe prices, quantity and sometimes quality.

▶ Let's start: Naming groups

In groups of two or three, use the correct mathematical terms to describe each group of numbers. (Suggestions include: *multiples of*, *factors of*, *integers*, *squares* and *cubes*.)

- 2, 4, 6, 8, …
- 1, 4, 9, 16, …
- 1, 3, 5, 7, 9, …
- 1, 2, 3, 4, 5, 6, …
- –1, –2, –3, –4, –5, …
- 1, 8, 27, 64, …
- 1, 2, 3, 4, 6 and 12

Key ideas

- Integers are positive and negative whole numbers, including zero.
  …, –3, –2, –1, 0, 1, 2, 3, …
- To add a negative number, you subtract its opposite.
  For example: $5 + (-7) = 5 - 7$
  $-6 + (-2) = -6 - 2$
- To subtract a negative number, you add its opposite.
  For example: $5 - (-7) = 5 + 7$
  $-6 - (-2) = -6 + 2$
- Adding or subtracting zero leaves a number unchanged.
  For example: $5 + 0 = 5$
  $5 - 0 = 5$

Negative number A number less than 0

Positive number A number greater than zero; a number to the right of zero on the number line, such as +4, +71, +0.5, +213

Exercise 1A

Understanding

1 Match each of the following sentences (**a**–**e**) to the correct expression (**A**–**E**).

a the sum of 5 and 7
b the total of negative 5 and 7
c the difference between negative 5 and 7
d the sum of 5 and negative 7
e the difference between 5 and negative 7

A $5 + (-7)$
B $5 - (-7)$
C $5 + 7$
D $-5 - 7$
E $-5 + 7$

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2 Match each expression (**a**–**e**) to a number (**A**–**E**).

a $10 + (-7)$
b $-10 - (-7)$
c $10 - (-7)$
d $-10 - 7$
e $-10 + 7 + 3$

A -17
B -3
C 0
D 3
E 17

3 True or false?

a $9 - 7 = -2$
b $18 + 0 = 18$
c $-6 + 6 = 0$
d $9 + 8 = 8 + 9$
e $6 - (-4) = 6 + 4$
f $4 - (-2) = 4 - 2$

Fluency

Drilling for Gold 1A1

Example 1 Using a number line for addition and subtraction of integers

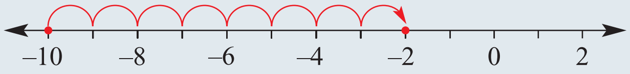
Use a number line to find:

a $-10 + 8$
b $-3 - 5$

Solution

Explanation

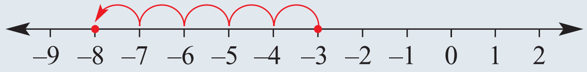
a



$-10 + 8 = -2$

Draw a number line showing −10 on it.
Start at −10 and count up (for addition) 8 places to finish at −2.

b

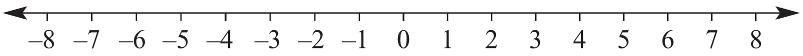


$-3 - 5 = -8$

Draw a number line showing −3 on it.
Count down (for subtraction) 5 places to −8.

Drilling for Gold 1A2 at the end of this section

4 Use the number line below to help you simplify the following.



a $-6 + 4$
b $-6 + 8$
c $0 - 3$
d $-1 + 5$
e $6 - 10$
f $-2 - 3$
g $-1 - 1$
h $-3 + 3$
i $-3 - 3$

−6 + 4 means:
- Start at −6.
- Move 4 to the right.

5 Use a number line to evaluate the following.

a $9 - 6$
b $9 - 7$
c $9 - 8$
d $9 - 9$
e $9 - 10$
f $-9 - 0$
g $-9 + 8$
h $-9 - 8$
i $-6 - 4$
j $-3 + 12$
k $-8 + 6$
l $-1 + 12$

On the number line, move right for addition and left for subtraction.

6 Evaluate the following. Check your answers with a calculator.

a $4 + 8 - 7$
b $5 + 6 + 9$
c $-19 - 1$
d $-9 + 8 + 1$
e $-12 - 3 + 8$
f $8 + 3 - 5$
g $-12 - 12$
h $15 + 5 - 15$
i $-6 - 5 - 4$

Do in order from left to right.

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Example 2 Adding a negative integer

Find 17 + (–12).

Solution	**Explanation**
17 + (–12) = 17 – 12 = 5	Adding a negative is the same as subtraction: 17 + (–12) = 17 – 12

7 Find the value of the following. Use a calculator if necessary.

a 9 + (–5)
b 12 + (–16)
c 3 + (–7)
d 15 + (–24)
e –9 + (–23)
f –13 + (–25)
g –100 + (–89)
h 56 + (–80)
i –9 + (–9)
j 18 + (–18)
k –245 + (–560)
l 98 + (–155)

9 + (–5) = 9 – 5
–9 + (–23) = –9 – 23

Example 3 Subtracting a negative integer

Find –13 – (–9).

Solution	**Explanation**
– 13 – (– 9) = –13 + 9 = –4	Subtracting a negative is the same as addition: – 13 – (– 9) = –13 + 9

8 Evaluate the following. Use a calculator if necessary.

a –5 – (–9)
b –8 – (–6)
c 7 – (–6)
d 2 – (–7)
e 12 – (–12)
f –34 – (–34)
g –35 – (–7)
h –90 – (–9)
i 90 – (–90)
j –68 – (–70)
k –90 – (–87)
l 234 – (–6)

–5 – (–9) = –5 + 9

Problem-solving and Reasoning

9 Calculate:

a –9 – 9 – 9
b –23 – (–8) + 12
c 50 – 46 – (–6)
d 24 – (–8) + (–6)
e –20 – (–5) – (–15)
f –18 – (–6) + (–12)

Work from left to right.

10 Copy and complete each of the following statements.

a ☐ + 6 = 8
b –6 + ☐ = 5
c 12 – ☐ = 15
d –9 + ☐ = –10
e 13 – ☐ = 20
f 10 – ☐ = –1

Try using a number line.

11 What must be added to each of the following to obtain a final result of zero?

a 8
b –7 + 3
c –8 – 5
d –124
e 19
f 0
g 12 – 8 + 18
h –98
i 12 – (–14)

1A

12 The temperature inside a car was measured at 23°C. Fifteen minutes later it had risen by 18°C. The air conditioner was then switched on and the temperature fell by 22°C. What was the temperature inside the car at this time?

Enrichment: Magic squares with integers

13 a Copy and complete this magic square. Each row, column and diagonal add to the same number.

–8		
	–2	–6
		4

What is the sum of the diagonal?

b Arrange these 9 integers into a 3 by 3 magic square.
–13, –10, –7, –4, –1, 2, 5, 8 and 11

c Complete this 4 by 4 magic square.

5			
0	10		6
–1	4	2	8
		7	–4

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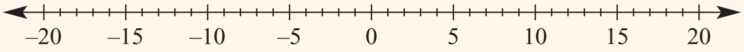
1A2: Number line skill drill

Use the worksheet or draw a number line from –20 to +20 in your exercise book.
For each question, go to the position shown in the first column.
Make the move given in the second column. Write down your new position.



Question	Your current position	Your ‘move’ is
1	+3	+6
2	+3	–6
3	–3	+6
4	–3	–6
5	+3	+2
6	+2	–2
7	+1	–5
8	0	+5
9	0	–5
10	–1	+6
11	–1	–6
12	–2	+5
13	–2	–5
14	–6	+4
15	–6	–4
16	–4	+8
17	–4	–8
18	–5	+16
19	–13	–14
20	–18	+16

Drilling for Gold exercise

1B Multiplying and dividing positive and negative integers

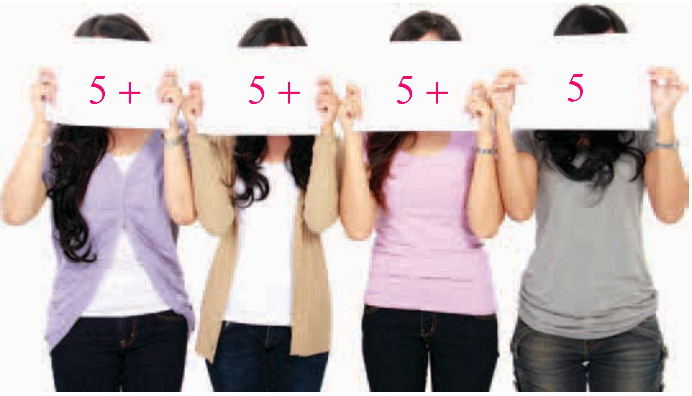
Stage
5.2
5.2◊
5.1
4

Multiplication is repeated addition. In this section we will revise the rules for multiplying and dividing negative integers.

Let's start: Repeated addition

Write each of the following as a multiplication before finding the answer.

- **i** $4 + 4 + 4$
- **ii** $(-4) + (-4) + (-4)$
- **iii** $5 + 5 + 5 + 5$
- **iv** $(-5) + (-5) + (-5) + (-5)$
- **v** $(-7) + (-7) + (-7) + (-7) + (-7)$
- **vi** $(-8) + (-8)$



Use your results from above to answer these divisions.

- **i** $12 \div 4$
- **ii** $-12 \div (-4)$
- **iii** $-12 \div 3$
- **iv** $-20 \div 4$
- **v** $-20 \div (-5)$
- **vi** $20 \div 5$
- **vii** $-35 \div (-7)$
- **viii** $-35 \div 5$
- **ix** $-16 \div (-8)$

What can you conclude about dividing a negative by a:

a positive number? **b** negative number?

Key ideas

- When multiplying two numbers with the same sign, the result is a positive number.

  positive × **positive** = **positive**
  negative × **negative** = **positive**

  (If the signs are the same, the result is positive.)

  For example: $-5 \times (-4) = 20$
- When multiplying two numbers with different signs, the result is a negative number.

  positive × **negative** = **negative**
  negative × **positive** = **negative**

  For example: $-5 \times 4 = -20$
- The same rules apply for division:

  positive ÷ **positive** = **positive**
  negative ÷ **negative** = **positive**
  positive ÷ **negative** = **negative**
  negative ÷ **positive** = **negative**
- On your calculator, use brackets when finding powers of negative numbers.
  For example: $(-13)^2 = 169$ (not -169)

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Exercise 1B

Understanding

1 Without actually finding the answer to any of these questions, decide whether the answer to each would be a positive or a negative number.

a $2 \times (-8)$ **b** $78 \times (-1)$ **c** $56 \div (-2)$

d $-90 \div (-10)$ **e** 8×12 **f** $-18 \times (-9)$

2 Copy and complete this table.

×	–4	–3	–2	–1	0	1	2	3	4
–4									
–3									
–2									
–1									
0									
1									
2									
3									
4									

3 Complete the following statements.

a A negative number multiplied by a negative number gives a __________ number.

b A negative number multiplied by a positive number gives a __________ number.

c A negative number divided by a negative number gives a __________ number.

d A positive number divided by a negative number gives a __________ number.

e A negative number divided by a positive number gives a __________ number.

Fluency

Example 4 Multiplying by a negative number

Find the value of:

a $-6 \times (-7)$ **b** $8 \times (-12)$

Solution	Explanation
a $-6 \times (-7) = 42$	The product of two negative numbers gives a positive answer. $- \times - = +$
b $8 \times (-12) = -96$	The product of a positive number and a negative number results in a negative answer. $+ \times - = -$

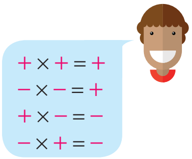
4 Find these products. Use a calculator to check your answers.

a 4×3 **b** -4×3 **c** $4 \times (-3)$

d $-4 \times (-3)$ **e** 5×2 **f** -5×2

g 5×-2 **h** -5×-2 **i** $10 \times (-5)$

j -4×5 **k** $-8 \times (-5)$ **l** $-12 \times (-5)$



Example 5 Finding squares and cubes of negative numbers

Find the value of:

a $(-5)^2$ **b** $(-6)^3$

Solution	Explanation
a $(-5)^2 = 25$	$(-5)^2$ is the same as $-5 \times (-5)$. The product of two negative numbers gives a positive answer. $- \times - = +$
b $(-6)^3 = -216$	$(-6)^3 = -6 \times (-6) \times (-6)$ Multiply the first two numbers together: $-6 \times (-6) = 36$ Now, $36 \times (-6) = -216$

5 Find the answers to each of the following. Use a calculator to check your answers.

a $(-2)^2$ **b** $(-2)^3$ **c** $(-3)^2$

d $(-3)^3$ **e** $(-4)^2$ **f** $(-4)^3$

g $(-5)^2$ **h** $(-5)^3$ **i** $(-10)^3$

$(-6)^2 = -6 \times (-6)$

$(-3)^3 = -3 \times (-3) \times (-3)$

Example 6 Dividing with a negative number

Find the value of:

a $-24 \div 8$ **b** $-45 \div (-3)$

Solution	Explanation
a $-24 \div 8 = -3$	The division of two numbers with different signs gives a negative answer. $- \div + = -$
b $-45 \div (-3) = 15$	The division of two numbers with the same sign gives a positive answer. $- \div - = +$

6 Perform the following divisions. Use a calculator to check your answers.

a $20 \div 4$ **b** $20 \div (-4)$ **c** $-20 \div 4$

d $-20 \div (-4)$ **e** $20 \div (-5)$ **f** $-20 \div 5$

g $20 \div 10$ **h** $-20 \div 10$ **i** $-20 \div (-10)$

$+ \div + = +$

$- \div - = +$

$+ \div - = -$

$- \div + = -$

Example 7 Applying the order of operations

Find the value of $9 \times (-8) + (5 - 12)$.

Solution	Explanation
$9 \times (-8) + (5 - 12)$	Work out the value of the brackets first: $(5 - 12) = (-7)$
$= 9 \times (-8) + (-7)$	Multiplication is next: $9 \times (-8) = -72$
$= -72 - 7$	The addition of a negative is the same as subtraction.
$= -79$	

7 Find the value of these expressions. Check your answers with a calculator.

a $(7 - 3) \times 4$ **b** $(4 + 9) \times (-2)$
c $(6 - 2) \times (-3)$ **d** $-6 + 4 \times (-2)$
e $10 \div (-2) \times 4$ **f** $(-5 - 2) \times 4$
g $6 \times (-3) \times 4$ **h** $-21 \div 7 + (-5)$
i $-45 - 9 \times 5$ **j** $(4 - 6) \times (7 - 12)$
k $(15 - 9) \times (3 - 7)$ **l** $9 + 9 \times (-3)$

The order of operations is:
- Brackets first, following the order of operations within them.
- Then × and ÷, working from left to right.
- Then + and –, working from left to right.

Problem-solving and Reasoning

8 Copy and complete:

a $-5 \times \square = -35$ **b** $\square \div (-4) = 2$ **c** $-10 \times \square = 80$
d $15 \times \square = -30$ **e** $34 \div \square = -34$ **f** $-6 \times \square = -36$
g $\square \div 9 \times (-3) = 3$ **h** $15 \div \square \div 3 = -1$ **i** $-15 \times \square = 45$

9 The sum of two numbers is -3 and their product is -10. What are the two numbers?

10 Explain why $-5 \times 4 \div (-3)$ produces a positive answer.

11 Decide whether each of the following would produce a positive or a negative answer.

a $-6 \times (-4) \times 5$ **b** $-12 \div 4 \times 9 \div (-3)$ **c** $-8 \div (-2) \times 5 \times (6 - 9)$
d $(-3)^4$ **e** $(-1)^{201}$ **f** $-(-2)^5$

Enrichment: Substitutions involving negative integers

12 Evaluate the following by substituting $a = -1$, $b = 2$ and $c = -3$.

a $a^2 - b$ **b** $a - b^2$
c $2c + a$ **d** $b^2 - c^2$
e $a^3 + c^2$ **f** $3b + ac$
g $c - 2ab$ **h** $abc - (ac)^2$

13 Use a calculator to evaluate the following, where $x = -45$ and $y = -15$.

a $x + y$ **b** $x - y$
c xy **d** $x - 3y$
e $x^2 + y^2$ **f** $x^2 - y^2$
g $(x + y)^2$ **h** $3y^2$

1C Decimal places and significant figures

Stage
5.2
5.2◊
5.1
4

Numbers can be rounded in two different ways:

- Using nearest hundred, nearest ten, etc.
- Using significant figures.

Let's start: In the middle

For each number given on the left, decide which of the two numbers on the right it is closest to.

a 84 80 or 90
b 856 800 or 900
c 856 850 or 860
d 0.599 0.5 or 0.6
e 1.2099 1.2 or 1.3
f 1.2099 1.20 or 1.21
g 89 543 89 000 or 90 000
h 0.035 0.03 or 0.04

Key ideas

- **Rounding** involves approximating a decimal number using fewer digits.
- Rounding to **two** decimal places:
  In the following decimals, more than two decimals are given.
  A blue line has been drawn after two decimal places.
  The 'critical digit' is circled.

  If the critical digit is 0, 1, 2, 3 or 4, then round down.
  For example: 185.26|③ = 185.26 (to 2 d.p.)
  185.26|⓪05 = 185.26 (to 2 d.p.)
  185.26|④499 = 185.26 (to 2 d.p.)

  If the critical digit is 5, 6, 7, 8 or 9, then round up.
  For example: 185.26|⑤ = 185.27 (to 2 d.p.)
  185.26|⑥05 = 185.27 (to 2 d.p.)
  185.26|⑨499 = 185.27 (to 2 d.p.)

Ignore all digits to the right of the critical digit.

- The symbols ≑ and ≈ are used to mean 'approximately equal to'.
- To round a number to a required number of **significant figures**:
  - Locate the first *non-zero* digit, counting from left to right.
  - From this first significant digit, count out the number of digits, including zeros.
  - Stop at the required number of digits and round this last digit.
  - Replace any non-significant digits to the left of a decimal point with a zero.

  For example, these numbers are all rounded to **3** significant figures:
  $\underline{2}.\underline{5}\underline{3}$⑨1 ≑ 2.54 0.00$\underline{2}\underline{7}\underline{1}$③ ≑ 0.00271 $\underline{5}\underline{6}\underline{8}$⑧10 ≑ 569 000

Rounding To make an approximation of a number with fewer digits

Approximation A value that is close to the real value

Critical digit The digit to the right of the last decimal place or significant figure to which a number is rounded. The critical digit determines whether rounding should be up or down.

Exercise 1C

Understanding

1 How many decimal places do each of these numbers have?

a 0.46 **b** 1.467 **c** 0.7 **d** 0.08
e 146.1 **f** 47 900 **g** 0.123 **h** 7000.6

2 How many significant figures do each of these numbers have?

a 0.46 **b** 1.467 **c** 0.7 **d** 0.08
e 146.1 **f** 479 **g** 0.123 **h** 4906

Count the digits starting from the first *non-zero* digit. These are the significant figures.

3 Answer each question.

a Is 44 closer to 40 or 50?
b Is 266 closer to 260 or 270?
c Is 7.89 closer to 7.8 or 7.9?
d Is 0.043 closer to 0.04 or 0.05?

4 Choose the correct answer if the first given number is rounded to three significant figures.

a 32 124 is rounded to 321, 3210 or 32 100.
b 431.92 is rounded to 431, 432 or 430.
c 5.8871 is rounded to 5.887, 5.88 or 5.89.
d 0.44322 is rounded to 0.44, 0.443 or 0.44302.
e 0.0019671 is rounded to 0.002, 0.00197 or 0.00196.

Fluency

Drilling for Gold 1C1 at the end of this section

Example 8 Rounding to a number of decimal places

Round each of these to two decimal places.

a 256.1793 **b** 0.04459 **c** 4.8972

Solution	**Explanation**
a $256.1793 \doteqdot 256.18$	$256.\underline{1}\,\underline{7}\mathbf{9}3$ The number after the second decimal place is 9, so round up (i.e. increase the 7 by 1).
b $0.044\,59 \doteqdot 0.04$	$0.\underline{0}\,\underline{4}\mathbf{4}\,59$ The number after the second decimal place is 4, so round down.
c $4.8972 \doteqdot 4.90$	$4.\underline{8}\,\underline{9}72$ The number after the second decimal place is 7, so round up. 89 rounds to 90.

5 Round each of the following numbers to two decimal places.

a 17.962
b 11.082
c 72.986
d 47.859
e 63.925
f 23.807
g 804.5272
h 500.5749
i 821.2749
j 5810.2539
k 1004.9981
l 2649.9974

The critical digit will be the third decimal place.

Drilling for Gold 1C2 at the end of this section

Example 9 Rounding to a number of significant figures

Round each of these numbers to two significant figures.

a 2587 **b** 23 067.453 **c** 0.04059

Solution	Explanation
a 2587 $\doteq$ 2600	**25**87 significant digits The next digit is 8, so round up. Replace last two digits with zeros to maintain place value of thousands.
b 23 067.453 $\doteq$ 23 000	**23**067.453 significant digits The next digit is 0, so round down. Replace the remaining digits, in front of the decimal point, with zeros.
c 0.04059 $\doteq$ 0.041	Locate the first non-zero digit: 0.0**40**59 significant digits The next digit is 5, so round up. No extra zeros are needed, as they are after the decimal point.

6 Round each of these numbers to two significant figures.

a 2436
b 35 057.4
c 0.06049
d 34.024
e 107 892
f 0.00245
g 2.0745
h 0.7070
i 4706
j 59 134
k 0.4567
l 1.0631

Start at the first non-zero digit on the left.

7 Copy and complete this table.

	Number	Rounded to two decimal places	Rounded to two significant figures
a	1.4638		
b	0.0936		
c	23.7124		
d	0.007 83		
e	100.465		

8 Round these numbers to the nearest whole number.

a 6.814 **b** 73.148
c 129.94 **d** 36 200.49

9 Round these numbers to one significant figure.

a 32 000 **b** 194.2
c 0.0492 **d** 0.0006413
e 4793 **f** 890
g 0.89 **h** 0.000304

$48.06 = 50$ to one significant figure.
$4730 = 5000$ to one significant figure.
$0.638 = 0.6$ to one significant figure.

Problem-solving and Reasoning

Example 10 Estimating using significant figures

Estimate the answer to the following. Round each number in the problem to one significant figure. Use your calculator to check how reasonable your answer is.

$$27 + 1329.5 \times 0.0064$$

Solution	Explanation
$27 + 1329.5 \times 0.0064$	
$\approx 30 + 1000 \times 0.006$	Round each number to one significant figure.
$= 30 + 6$	Recall multiplication occurs before the addition.
$= 36$	
The estimated answer is reasonable.	By calculator (to one decimal place): $27 + 1329.5 \times 0.0064 = 35.5$

10 Estimate the answers to the following by rounding each number in the problem to one significant figure. Use a calculator to check how reasonable your answer is.

a $567 + 3126$ **b** $795 - 35.6$
c 97.8×42.2 **d** $965.98 + 5321 - 2763.2$
e $4.23 - 1.92 \times 1.827$ **f** $17.43 - 2.047 \times 8.165$
g $0.0704 + 0.0482$ **h** 0.023×0.98

11 Copy and complete this table of common fractions.

Fraction form	Decimal form	Written to one decimal place	Written to one significant figure
$\frac{1}{3}$			
$\frac{2}{3}$			
$\frac{1}{9}$			
$\frac{3}{4}$			

12 An electronic timer records the time for a running relay between two teams, A and B. Team A's time is 54.283 seconds and team B's time is 53.791 seconds. What would be the difference in the times for teams A and B if the times were written down using:

a one decimal place?
b four significant figures?
c two significant figures?
d one significant figure?

13 A scientific experiment uses very small amounts of magnesium (0.0025 g) and potassium (0.0062 g). Why does it make sense to use two significant figures instead of two decimal places when recording numbers in a situation like this?

14 Should 2.14999 be rounded down or up if it is to be rounded to one decimal place? Give reasons.

Enrichment: *n*th decimal place and π

15 The fraction $\frac{2}{11}$ can be written as 0.18181818, correct to eight decimal places.

a Using the decimal pattern described, find the digit in the:
  i 20th decimal place
  ii 45th decimal place
  iii 1000th decimal place

b Express $\frac{1}{7}$ as a decimal, correct to 13 decimal places.

c Using the decimal pattern from part **b**, find the digit in the:
  i 20th decimal place
  ii 45th decimal place
  iii 1000th decimal place

1C1: Rounding decimals skill drill

Drilling for Gold exercise

Round off to the required number of decimal places.
Use the worksheet or write the answers in your exercise book.

Round 1: Nearest whole number	
1	1.451046
2	34.75016
3	0.95362
4	124.574999
5	31.8181818
6	3.14159
7	4.471235
8	543.286341
9	1975.6245
10	99.919191

Round 2: One decimal place	
1	1.451046
2	34.75016
3	0.95362
4	124.574999
5	31.8181818
6	3.14159
7	4.471235
8	543.286341
9	1975.6245
10	99.919191

Round 3: Two decimal places	
1	1.451046
2	34.75016
3	0.95362
4	124.574999
5	31.8181818
6	3.14159
7	4.471235
8	543.286341
9	1975.6245
10	99.919191

Round 4: Three decimal places	
1	1.451046
2	34.75016
3	0.95362
4	124.574999
5	31.8181818
6	3.14159
7	4.471235
8	543.286341
9	1975.6245
10	99.919191

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Drilling for Gold exercise

1C2: Significant figures skill drill

Round off the given numbers to the required number of significant figures.
The first one in each table has been done for you.
Use the worksheet or write the answers in your exercise book.

	Round 1: Four significant figures			Round 2: Three significant figures	
1	54.2783	54.28	**1**	54.2783	54.3
2	103.478		**2**	103.478	
3	2.84537		**3**	2.84537	
4	27.5016		**4**	27.5016	
5	0.28716		**5**	0.28716	
6	0.028716		**6**	0.028716	
7	1275.964		**7**	1275.964	
8	25 279		**8**	25 279	
9	350 275		**9**	350 275	
10	0.999999		**10**	0.999999	

	Round 3: Two significant figures			Round 4: One significant figure	
1	54.2783	54	**1**	54.2783	50
2	103.478		**2**	103.478	
3	2.84537		**3**	2.84537	
4	27.5016		**4**	27.5016	
5	0.28716		**5**	0.28716	
6	0.028716		**6**	0.028716	
7	1275.964		**7**	1275.964	
8	25 279		**8**	25 279	
9	350 275		**9**	350 275	
10	0.999999		**10**	0.999999	

1D Rational numbers and irrational numbers

Stage
5.2
5.2◊
5.1
4

Rational numbers can be written as a fraction in the form $\frac{a}{b}$, where $b \neq 0$.

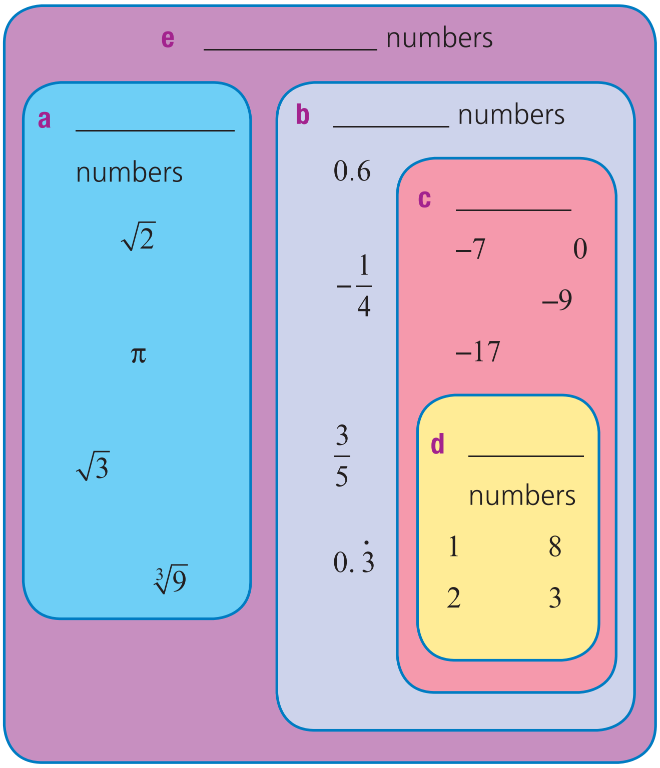
Around 500 BC, Pythagoras and his followers discovered that not all numbers are rational. Pi (π) and $\sqrt{2}$ are examples of these numbers. When written as decimals, they do not terminate or repeat. We call them irrational numbers.

This is $\sqrt{2}$ to 100 decimal places:

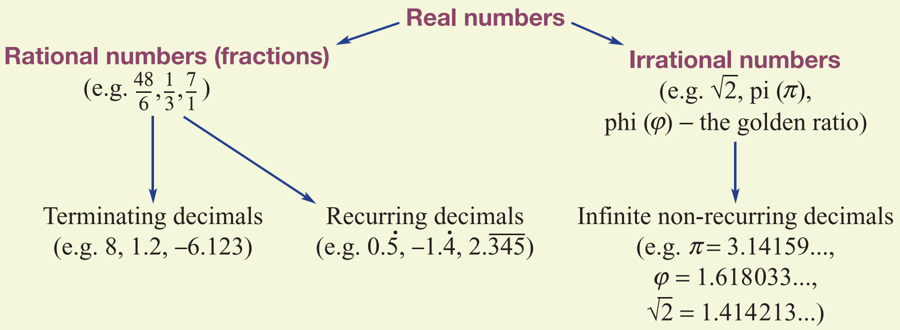
$$\begin{aligned}1.\ &4142135623730950488016887\\&2420969807856967187537694\\&8073176679737990732478462\\&1070388503875343276415727\end{aligned}$$

▶ Let's start: The real number system

Copy the diagram below and insert these words correctly: *rational*, *irrational*, *real*, *integers*, *counting*.



Key ideas



- An infinite or non-terminating decimal is one where the decimal places continue forever.
- Equivalent fractions have the same value.

  For example: $\frac{2}{3} = \frac{6}{9}$
- A fraction can be simplified by dividing the numerator and denominator by their highest common factor (HCF).
- If $\frac{a}{b}$ is a proper fraction, then $a < b$.

  For example: $\frac{2}{7}$
- If $\frac{a}{b}$ is an improper fraction, then $a > b$.

  For example: $\frac{10}{3}$
- A mixed numeral is written as a whole number plus a proper fraction.

  For example: $2\frac{3}{5}$
- Fractions can be compared using a common denominator. This should be the lowest common multiple of both denominators.
- A dot or bar is used to show a pattern in a recurring decimal number.

  For example: $\frac{1}{6} = 1 \div 6 = 0.16666.... = 0.1\dot{6}$

  $\frac{3}{11} = 3 \div 11 = 0.272727.... = 0.\overline{27}$ or $0.\dot{2}\dot{7}$

Real number Any positive or negative number, or zero

Rational number A real number that can be expressed as a fraction

Irrational number A real number that cannot be expressed as a fraction

Recurring decimal A decimal with a pattern of digits that repeats indefinitely

Terminating decimal A decimal that contains a finite number of digits

Numerator The number in a fraction above the vinculum

Denominator The number in a fraction below the vinculum. It is the number of equal parts into which the whole is divided.

Highest common factor (HCF) The largest number that is a factor of all the given factors

Lowest common multiple (LCM) The smallest number that two or more numbers divide into without remainder

Mixed numeral A number with a whole number and a fraction part

Exercise 1D

Understanding

1 Answer the following as true or false.

a $\sqrt{3}$ is an irrational number.

b $\sqrt{4}$ is an irrational number.

c 0.3 is a rational number.

An irrational number as a decimal is never-ending and has no pattern.

2 Which of the following can be written as a fraction?

a $0.\dot{6}$ **b** 1.4 **c** $\sqrt{9}$ **d** $\sqrt{5}$

3 Write each of these in the form $\frac{a}{b}$.

a 0.7 **b** 0.3 **c** $1\frac{1}{2}$ **d** $2\frac{1}{3}$

Fluency

Example 11 Identifying rational numbers

Choose the rational numbers in the following list.

$\sqrt{9}, \sqrt{2}, 0.6, \pi, -\frac{3}{4}$

Solution	Explanation
$\sqrt{9}$, 0.6 and $-\frac{3}{4}$ are rational numbers.	$\sqrt{9} = 3 = \frac{3}{1}$ ∴ rational (a fraction) $0.6 = \frac{6}{10}$ ∴ rational $-\frac{3}{4}$ is also rational. π and $\sqrt{2}$ are infinite non-recurring decimals.

4 Which of the following are rational numbers?

a 0.1 **b** 20% **c** $6\frac{1}{4}$ **d** $\frac{2}{5}$

e $\frac{2}{3}$ **f** 0.001 **g** $\sqrt{64}$ **h** $\sqrt{7}$

i 42 **j** π **k** $0.\dot{2}$ **l** $\sqrt{11}$

Rational numbers can be written as a fraction.

Example 12 Writing fractions as decimals

Write $4\frac{3}{8}$ as a decimal.

Solution	Explanation
$\frac{3}{8} = 3 \div 8$	3 divided by 8 means the number of times 8 goes into 3.
$8 \overline{)\, 3\,.\,{}^{3}0\,{}^{6}0\,{}^{4}0}$ with quotient $0\,.\,3\;\;7\;\;5$	Find a decimal for $\frac{3}{8}$ by dividing 8 into 3 using the short division algorithm.
$\therefore \frac{3}{8} = 0.375$	
$\therefore 4\frac{3}{8} = 4.375$	

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1D

5 Write these fractions as decimals. Check your answers using a calculator.

a $\frac{11}{4}$ **b** $\frac{7}{4}$ **c** $3\frac{2}{5}$ **d** $\frac{15}{8}$

e $2\frac{5}{8}$ **f** $3\frac{4}{5}$ **g** $\frac{1}{8}$ **h** $\frac{7}{8}$

Use short division.
$\frac{11}{4}$ means $11 \div 4$; i.e. $4\overline{)11}$.

Example 13 Recurring decimals from fractions

Write $\frac{5}{13}$ as a recurring decimal.

Solution	**Explanation**
$\frac{5}{13} = 5 \div 13$	5 divided by 13 means the number of times 13 goes into 5.
$\begin{array}{r} 0\,.\,3\ \ 8\ \ 4\ \ 6\ \ 1\ \ 5\ \ 3 \\ 13\overline{)\,5\,.{}^{5}0{}^{11}0{}^{6}0{}^{8}0{}^{2}0{}^{7}0{}^{5}0} \end{array}$	Divide 13 into 5 and continue until the pattern repeats. Add a bar over the repeating pattern.
$\therefore \frac{5}{13} = 0.\overline{384615}$	This can also be written as $0.\dot{3}8461\dot{5}$.

6 Write these fractions as recurring decimals.

a $\frac{3}{11}$ **b** $\frac{7}{9}$ **c** $\frac{9}{7}$ **d** $\frac{5}{12}$

e $\frac{10}{9}$ **f** $3\frac{5}{6}$ **g** $7\frac{4}{15}$ **h** $\frac{29}{11}$

Check using your calculator.

Drilling for Gold 1D1

Example 14 Writing decimals as fractions

Write these decimals as fractions.

a 0.8 **b** 0.24

Solution	**Explanation**
a $0.8 = \frac{8}{10}$	The 8 is in the tenths column.
$= \frac{4}{5}$	Simplify the fraction.
b $0.24 = \frac{24}{100}$ $= \frac{6}{25}$	Write as a fraction using the smallest place value (hundredths), then simplify using the HCF of 4.

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7 Write these decimals as fractions.

a 0.35	**b** 0.06	**c** 3.7	**d** 0.56
e 1.07	**f** 0.075	**g** 3.32	**h** 7.375
i 2.005	**j** 10.044	**k** 6.45	**l** 2.101

Divide by 100 when there are two decimal places.

Example 15 Comparing fractions

Decide which is the larger fraction of the following.

$\frac{4}{5}$ or $\frac{7}{8}$

Solution	**Explanation**
LCM of 5 and 8 is 40. $\underset{\times 8}{\overset{\times 8}{\frac{4}{5} = \frac{32}{40}}}$ and $\underset{\times 5}{\overset{\times 5}{\frac{7}{8} = \frac{35}{40}}}$ $\therefore \frac{4}{5} < \frac{7}{8}$	Find the lowest common multiple (LCM) of 5 and 8. Write each fraction as an equivalent fraction. Compare numerators to determine the larger fraction.

8 Decide which is the larger fraction in the following pairs.

a $\frac{3}{4}, \frac{5}{6}$

b $\frac{13}{20}, \frac{3}{5}$

c $\frac{7}{10}, \frac{8}{15}$

d $\frac{5}{12}, \frac{7}{18}$

First write both fractions, using a common denominator.

Problem-solving and Reasoning

9 Express the following quantities as simplified fractions.

a \$45 out of \$100

b 12 kg out of 80 kg

c 64 baskets out of 90 shots in basketball

10 These sets of fractions form a pattern. Find the next two fractions in the pattern.

a $\frac{1}{3}, \frac{5}{6}, \frac{4}{3}, \frac{\square}{\square}, \frac{\square}{\square}$

b $\frac{6}{5}, \frac{14}{15}, \frac{2}{3}, \frac{\square}{\square}, \frac{\square}{\square}$

c $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{\square}{\square}, \frac{\square}{\square}$

d $\frac{1}{2}, \frac{4}{7}, \frac{9}{14}, \frac{\square}{\square}, \frac{\square}{\square}$

11 A jug has 400 mL of half-strength orange juice. The following amounts of full-strength juice are added to the mix. Find a fraction to describe the strength of the orange drink after the full-strength juice is added.

a 100 mL **b** 50 mL **c** 120 mL **d** 375 mL

12 If x is an integer, determine the values that x can take in the following.

a The fraction $\frac{x}{3}$ is a number between (and not including) 10 and 11.

b The fraction $\frac{x}{7}$ is a number between (and not including) 5 and 8.

c The fraction $\frac{34}{x}$ is a number between 6 and 10.

You may find more than one answer.

Enrichment: What is $0.\dot{9}$?

13 On your calculator, type 0.111... until the screen is full. Press $\boxed{=}$ and then press the button that converts the decimal to a fraction. It should tell you that $0.\dot{1} = \frac{1}{9}$.

a What is $0.\dot{2}$?

b What is $0.\dot{7}$?

c What is $0.\dot{8}$?

d What is $0.\dot{9}$?

1E Adding and subtracting fractions

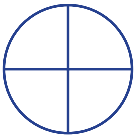
Stage
5.2
5.2◊
5.1
4

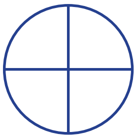
Fractions can be added and subtracted. To add or subtract fractions they need to be the same type; that is, they both need to be eighths, quarters, thirds, tenths etc. They need to have a common denominator.

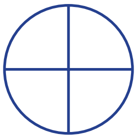
Fractions are all around you and part of everyday life.

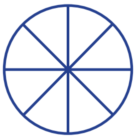
▶ Let's start: Shade the fraction

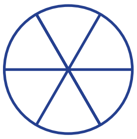
Copy the diagrams and shade the fraction suggested by each of these additions and subtractions.

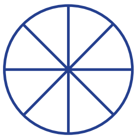
a $\frac{1}{4}+\frac{1}{4}=$ 

b $\frac{1}{4}+\frac{1}{2}=$ 

c $\frac{3}{4}-\frac{1}{2}=$ 

d $\frac{5}{8}+\frac{1}{4}=$ 

e $\frac{1}{2}+\frac{1}{3}=$ 

f $1-\frac{5}{8}=$ 

Key ideas

- Equivalent fractions are created by multiplying or dividing both the numerator and the denominator by the same factor.
  For example:

$$\frac{3}{4} \underset{\times 3}{\overset{\times 3}{=}} \frac{9}{12} \qquad \frac{10}{15} \underset{\div 5}{\overset{\div 5}{=}} \frac{2}{3}$$

Equivalent fractions Fractions that represent the same amount. They can be reduced to the same basic fraction.

- To add or subtract fractions, the denominators need to be the same.
  - If the denominators are the same, add or subtract the numerators, keeping the denominator unchanged.

  For example: $\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$

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– If the denominators are different, find the lowest common denominator (LCD) using equivalent fractions. Then add or subtract the numerators, remembering to simplify your answer where possible.

For example: $\frac{1}{2}+\frac{2}{3}=\frac{3}{6}+\frac{4}{6}$ The LCD of halves and thirds is sixths.

$=\frac{7}{6}$

$=1\frac{1}{6}$

Exercise 1E

Understanding

1 Copy and complete these equivalent fractions.

What you do to the numerator you must also do to the denominator, and vice versa.

a $\frac{3}{4}=\frac{\square}{16}$ **b** $\frac{3}{5}=\frac{\square}{10}$

c $\frac{2}{3}=\frac{\square}{9}$ **d** $\frac{2}{3}=\frac{\square}{12}$

e $\frac{5}{6}=\frac{\square}{18}$ **f** $\frac{3}{7}=\frac{21}{\square}$

g $\frac{4}{5}=\frac{12}{\square}$ **h** $\frac{5}{7}=\frac{25}{\square}$

2 True or false?

a $\frac{7}{9}+\frac{1}{9}=\frac{8}{9}$ **b** $\frac{6}{9}-\frac{1}{3}=\frac{5}{6}$

3 Copy and complete:

a $\frac{3}{10}+\frac{2}{5}$

$=\frac{3}{10}+\frac{\square}{10}$

$=\square$

b $\frac{1}{2}-\frac{3}{8}$

$=\frac{\square}{8}-\frac{3}{8}$

$=\frac{\square}{8}$

c $1\frac{1}{2}+\frac{1}{4}$

$=1\frac{2}{\square}+\frac{1}{4}$

$=1\frac{\square}{\square}$

4 What is the LCD needed for each of these additions?

The LCD is the lowest common multiple of both denominators.

a $\frac{1}{2},\frac{1}{3}$ **b** $\frac{3}{7},\frac{5}{9}$

c $\frac{2}{5},\frac{1}{13}$ **d** $\frac{3}{10},\frac{8}{15}$

e $\frac{1}{2},\frac{1}{4}$ **f** $\frac{9}{11},\frac{4}{33}$

g $\frac{5}{12},\frac{7}{30}$ **h** $\frac{13}{29},\frac{2}{3}$

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Fluency

Example 16 Adding and subtracting with common denominators

Find:

a $\frac{5}{11}+\frac{8}{11}$ **b** $1\frac{3}{8}-\frac{5}{8}$

Solution	**Explanation**
a $\frac{5}{11}+\frac{8}{11}=\frac{13}{11}$ $=1\frac{2}{11}$	Denominators are the same (i.e. elevenths). Keep the denominator and add the numerators. Write as a mixed number.
b $1\frac{3}{8}-\frac{5}{8}=\frac{11}{8}-\frac{5}{8}$ $=\frac{6}{8}$ $=\frac{3}{4}$	Write $1\frac{3}{8}$ as an improper fraction. Denominators are the same (i.e. eighths). Subtract the numerators. Simplify the answer by dividing by the HCF.

5 Find:

a $\frac{2}{5}+\frac{1}{5}$ **b** $\frac{3}{8}+\frac{2}{8}$ **c** $\frac{9}{10}-\frac{2}{10}$ **d** $\frac{2}{7}+\frac{2}{7}$

e $\frac{13}{17}-\frac{8}{17}$ **f** $\frac{3}{5}-\frac{2}{5}$ **g** $\frac{11}{6}-\frac{1}{6}$ **h** $\frac{13}{8}-\frac{3}{8}$

i $\frac{4}{5}+\frac{3}{5}$ **j** $1\frac{1}{5}+\frac{3}{5}$ **k** $2\frac{2}{3}-\frac{1}{3}$ **l** $1\frac{1}{8}-\frac{5}{8}$

m $1\frac{1}{10}-\frac{7}{10}$ **n** $3\frac{1}{4}+1\frac{1}{4}$ **o** $2\frac{3}{10}-\frac{9}{10}$

Writing mixed numbers as improper fractions may help.

Drilling for Gold
1E1
1E2

Example 17 Adding and subtracting with different denominators

Evaluate $\frac{1}{2}+\frac{3}{5}$.

Solution	**Explanation**
$\frac{1}{2}+\frac{3}{5}=\frac{5}{10}+\frac{6}{10}$	The lowest common denominator of 2 and 5 is 10. Rewrite as equivalent fractions using a denominator of 10.
$=\frac{11}{10}$ or $1\frac{1}{10}$	Add the numerators.

6 Evaluate the following. Check your answers with a calculator.

a $\frac{1}{2}+\frac{1}{8}$ **b** $\frac{1}{2}-\frac{1}{6}$ **c** $\frac{3}{10}+\frac{1}{5}$ **d** $\frac{4}{3}-\frac{5}{6}$ **e** $\frac{3}{2}-\frac{3}{5}$

f $\frac{1}{3}+\frac{1}{4}$ **g** $\frac{3}{5}+\frac{1}{10}$ **h** $\frac{3}{4}-\frac{3}{5}$ **i** $\frac{5}{6}-\frac{3}{8}$ **j** $\frac{2}{3}+\frac{1}{2}$

First rewrite with the LCD.

Example 18 Adding and subtracting mixed numbers

Evaluate:

a $1\frac{2}{3}+4\frac{5}{6}$ **b** $3\frac{2}{5}-2\frac{3}{4}$

Solution	**Explanation**
a $1\frac{2}{3}+4\frac{5}{6}=\frac{5}{3}+\frac{29}{6}$ $=\frac{10}{6}+\frac{29}{6}$ $=\frac{39}{6}$ $=\frac{13}{2}$ or $6\frac{1}{2}$	Change each mixed number to an improper fraction. Remember that the lowest common denominator of 3 and 6 is 6. Change $\frac{5}{3}$ to an equivalent fraction with denominator 6, then add the numerators and simplify.
b $3\frac{2}{5}-2\frac{3}{4}=\frac{17}{5}-\frac{11}{4}$ $=\frac{68}{20}-\frac{55}{20}$ $=\frac{13}{20}$	Convert to improper fractions, then rewrite as equivalent fractions with the same denominator. Subtract the numerators.

7 Evaluate the following.

a $1\frac{1}{3}+2\frac{1}{2}$ **b** $1\frac{1}{3}-\frac{3}{4}$ **c** $5\frac{1}{2}-2\frac{7}{10}$

d $2\frac{1}{3}+\frac{1}{4}$ **e** $4\frac{1}{2}+2\frac{1}{4}$ **f** $6\frac{1}{3}-2\frac{1}{2}$

g $1\frac{3}{8}-1\frac{1}{4}$ **h** $2\frac{3}{4}-1\frac{1}{3}$ **i** $4\frac{1}{10}-2\frac{2}{5}$

j $1\frac{7}{12}-\frac{2}{3}$ **k** $3\frac{1}{5}+1\frac{1}{2}$ **l** $2\frac{5}{6}-1\frac{1}{4}$

First rewrite using improper fractions.

8 To remove impurities a mining company filters $3\frac{1}{2}$ tonnes of raw material. If $2\frac{5}{8}$ tonnes are removed, what quantity of material remains?

9 When a material is processed it produces $3\frac{1}{7}$ tonnes of mineral and $2\frac{3}{8}$ tonnes of waste. How many tonnes of raw material were processed?

The concentration (proportion) of the desired mineral within an ore body is vital information in the minerals industry.

10 The ingredients in a punch recipe are $2\frac{1}{4}$ L of apple juice, $1\frac{1}{2}$ L of guava juice and $1\frac{1}{5}$ L of lemonade.

- **a** How much punch is produced?
- **b** If a cup holds 150 mL, how many cups can this punch serve?

11 Clarissa owns $\frac{3}{10}$ of a company, Sally owns another $\frac{1}{4}$ of the company and Keith owns $\frac{2}{5}$ of it. The bank owns the rest. How much of the company does the bank own?

Enrichment: Which fractions do you choose?

12 The six fractions listed below are each used exactly once in the following questions.

- **a** Arrange the six fractions correctly so that each equation produces the required answer. Use each fraction only once.

  $\frac{1}{8}, \frac{5}{12}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$

  i $\square + \square = \frac{7}{8}$ **ii** $\square - \square = \frac{1}{4}$ **iii** $\square - \square = \frac{1}{3}$

- **b** Which fractions in part **a** can be added together to produce an answer of 2?

Keeping in touch with numeracy

Non-calculator

1 Which number is closest to 0:
- **a** 5 or -3?
- **b** -5 or 3?
- **c** -5 or -3?
- **d** 0.1 or 0.01?
- **e** 0.25 or 0.5?
- **f** one or one-quarter?
- **g** three-quarters or two-thirds?

3 Will the answers be positive or negative?
- **a** $-48 + 29$
- **b** $-48 - 29$
- **c** $-29 + 48$
- **d** -48×29
- **e** $-48 \times (-29)$

5 Convert these fractions to decimals.
- **a** one-half
- **b** one-quarter
- **c** three-quarters
- **d** one-third
- **e** two-thirds
- **f** one-tenth
- **g** seven-tenths
- **h** one-fifth
- **i** four-fifths

7 (Multiple-choice question) In words, the number 105 000 is:
- **A** one hundred thousand and five
- **B** one hundred thousand five hundred
- **C** one hundred and five thousand
- **D** one hundred and fifty thousand

9 Round the following numbers, correct to two decimal places.
- **a** 23.463125
- **b** 23.468125
- **c** 0.8144179
- **d** 0.8154179
- **e** 0.0012345
- **f** 0.0092345

Calculator

2 Write down the next three numbers in these patterns.
- **a** 10, 5, 0, ….., ….., …..
- **b** 13, 7, 1, ….., ….., …..
- **c** 25, 16, 7, ….., ….., …..
- **d** -8, -5, -2, ….., ….., …..
- **e** 16, 8, 4, ….., ….., …..
- **f** 3, -6, 12, ….., ….., …..

4 Calculate the value of:
- **a** $-48 + 29$
- **b** $-48 - 29$
- **c** $-29 + 48$
- **d** -48×29
- **e** $-48 \times (-29)$

6 Convert these fractions to decimals.
- **a** one-eighth
- **b** three-eighths
- **c** five-eighths
- **d** seven-eighths
- **e** one-sixteenth
- **f** three-sixteenths
- **g** one-sixth
- **h** five-sixths
- **i** one-twelfth

8 How many seconds are there in a leap year?
- **a** Write down the calculation.
- **b** Write the answer as a number.
- **c** Write the answer in words.

10 Calculate the following, correct to two decimal places.
- **a** five-sevenths
- **b** $\sqrt{10}$
- **c** $\$126.25 \div 12$

11 Round the following numbers, correct to two significant figures.
- **a** 23.463125
- **b** 23.468125
- **c** 0.8144179
- **d** 0.8154179
- **e** 0.0012345
- **f** 0.0089999

13 a What is half of $1.50?
- **b** What is one-quarter of $24?
- **c** What is three-quarters of $24?
- **d** What is $150 divided by 4?

15 Find the remainder when 100 is divided by:
- **a** 2
- **b** 3
- **c** 4
- **d** 5
- **e** 6
- **f** 7
- **g** 8
- **h** 9

17 How many minutes are in 2.5 hours?

19 a A number is increased by 10 and the result is doubled to give 10. What is the number?
- **b** A number is doubled and the result is increased by 10 to give 10. What is the number?

12 Calculate the following, correct to two significant figures.
- **a** five-sevenths
- **b** $\sqrt{10}$
- **c** $126.25 ÷ 12

14 a What is half of $3.80?
- **b** What is one-quarter of $39?
- **c** What is three-quarters of $39?
- **d** What is $150 divided by 12?

16 Find the remainder when 289 is divided by:
- **a** 2
- **b** 3
- **c** 4
- **d** 5
- **e** 6
- **f** 7
- **g** 8
- **h** 9

18 How many minutes are in 12.3 hours?

20 What is the first negative number in the following patterns?
- **a** 100, 80, 60, …
- **b** 100, 85, 70, …
- **c** 100, 92, 84, …
- **d** 100, 87, 74, …

1F Multiplying and dividing fractions

Stage
5.2
5.2◊
5.1
4

Fractions can also be multiplied and divided.

Let's start: How much is left?

Tom, Sarah, Zara and Kimberly all share a strawberry tart. How much is left for Kimberly to eat if:

- Tom eats half the tart
- then Sarah eats half of what's left
- then Zara eats a third of the remaining section?

Key ideas

- To multiply fractions (proper or improper), multiply the numerators together and multiply the denominators together.
  - In general: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

    For example: $\frac{2}{5} \times \frac{3}{7} = \frac{2 \times 3}{5 \times 7}$

    $= \frac{6}{35}$
- To find the reciprocal of a fraction, you simply invert it (i.e. swap the numerator and denominator).
  - 2 is equal to $\frac{2}{1}$, so the reciprocal of 2 is $\frac{1}{2}$
  - The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.
  - $1\frac{1}{5}$ is equal to $\frac{6}{5}$, so the reciprocal of $1\frac{1}{5}$ is $\frac{5}{6}$.
- The word 'of' usually means multiply.

  For example: $\frac{1}{3}$ of $24 = \frac{1}{3} \times 24$
- Cancel common factors between numerators and denominators.

  $$\frac{{}^{2}\cancel{4}}{{}_{1}\cancel{5}} \times \frac{\cancel{10}^{2}}{\cancel{6}_{3}}$$
- To divide a number by a fraction, multiply by its reciprocal.
  - In general: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

    For example: $\frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \times \frac{6}{5}$

    $= \frac{2}{{}_{1}\cancel{3}} \times \frac{\cancel{6}^{2}}{5}$

    $= \frac{4}{5}$

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Exercise 1F

Understanding

1 Simplify the following fractions.

a $\frac{5}{10}$ **b** $\frac{10}{15}$ **c** $\frac{6}{12}$

d $\frac{12}{18}$ **e** $\frac{10}{20}$ **f** $\frac{15}{35}$

g $\frac{9}{27}$ **h** $\frac{12}{20}$ **i** $\frac{16}{24}$

First, find the HCF of the numerator and denominator.

2 Find:

a $\frac{1}{2}$ of 24 **b** $\frac{3}{4}$ of 16 **c** $1\frac{1}{2}$ of 4

d $\frac{1}{3}$ of 12 **e** $\frac{9}{10}$ of 200 **f** $\frac{2}{5}$ of 30

3 Find the reciprocal of:

a $\frac{3}{4}$ **b** $\frac{1}{7}$ **c** $\frac{2}{3}$

d $\frac{5}{8}$ **e** 6 **f** 8

g $1\frac{1}{2}$ **h** $1\frac{2}{3}$ **i** $2\frac{3}{4}$

Whole numbers can be first written with a denominator of 1. $6 = \frac{6}{1}$, $8 = \frac{8}{1}$

4 True or false?

a $1\frac{3}{4} = \frac{7}{4}$ **b** $2\frac{1}{3} = 6$ **c** $\frac{2}{5} \times \frac{5}{2} = 1$

d $4 = \frac{4}{1}$ **e** $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ **f** $\frac{7}{10}$ of $20 = 14$

Fluency

Drilling for Gold 1F1

Example 19 Multiplying with proper fractions

Evaluate the following.

a $\frac{2}{3} \times \frac{5}{7}$ **b** $\frac{4}{5} \times \frac{25}{32}$

Solution	**Explanation**
a $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$	Multiply the numerators together. Multiply the denominators together. $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$
b $\frac{4}{5} \times \frac{25}{32} = \frac{\cancel{4}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{25}^{5}}{\cancel{32}_{8}}$ $= \frac{5}{8}$	Cancel the common factors between the numerators and the denominators. Multiply the numerators and then multiply the denominators. $\frac{1 \times 5}{1 \times 8} = \frac{5}{8}$

5 Evaluate the following. Check your answers with a calculator.

a $\frac{3}{5}\times\frac{1}{4}$ **b** $\frac{2}{3}\times\frac{2}{5}$ **c** $\frac{3}{4}\times\frac{5}{7}$ **d** $\frac{1}{2}\times\frac{1}{3}$ **e** $\frac{1}{4}\times\frac{1}{5}$

f $\frac{1}{9}\times\frac{2}{3}$ **g** $\frac{5}{8}\times\frac{16}{17}$ **h** $\frac{1}{2}\times\frac{6}{7}$ **i** $\frac{2}{3}\times\frac{3}{5}$ **j** $\frac{8}{9}\times\frac{3}{4}$

k $\frac{4}{5}\times\frac{1}{2}$ **l** $\frac{5}{6}\times\frac{24}{25}$ **m** $\frac{6}{7}\times\frac{7}{8}$ **n** $\frac{7}{9}\times\frac{18}{21}$ **o** $\frac{8}{21}\times\frac{7}{12}$

Example 20 Multiplying with mixed numbers

Evaluate $1\frac{2}{3}\times 2\frac{1}{10}$.

Solution

$$1\frac{2}{3}\times 2\frac{1}{10}=\frac{{}^{1}\cancel{5}}{{}_{1}\cancel{3}}\times\frac{\cancel{21}^{7}}{\cancel{10}_{2}}$$
$$=\frac{7}{2}\text{ or }3\frac{1}{2}$$

Explanation

Rewrite as improper fractions.
Cancel common factors between numerators and denominators and then multiply numerators and denominators.

6 Evaluate the following. Check your answers using a calculator.

a $1\frac{1}{2}\times\frac{1}{3}$ **b** $1\frac{2}{3}\times\frac{3}{4}$ **c** $2\frac{1}{3}\times\frac{9}{10}$ **d** $1\frac{1}{2}\times 1\frac{1}{3}$ **e** $2\frac{1}{3}\times 1\frac{4}{5}$

f $\frac{3}{4}\times 1\frac{1}{7}$ **g** $2\frac{1}{2}\times 2\frac{1}{2}$ **h** $4\frac{1}{2}\times\frac{8}{9}$ **i** $1\frac{3}{4}\times 1\frac{2}{3}$ **j** $3\frac{3}{4}\times\frac{8}{5}$

Example 21 Dividing fractions

Evaluate the following.

a $\frac{4}{15}\div\frac{12}{25}$ **b** $1\frac{17}{18}\div 1\frac{1}{27}$

Solution

a $\frac{4}{15}\div\frac{12}{25}=\frac{4}{15}\times\frac{25}{12}$

$$=\frac{{}^{1}\cancel{4}}{{}_{3}\cancel{15}}\times\frac{\cancel{25}^{5}}{\cancel{12}_{3}}$$

$$=\frac{5}{9}$$

Explanation

To divide by $\frac{12}{25}$, we multiply by its reciprocal $\frac{25}{12}$.

Cancel common factors between numerators and denominators, then multiply fractions.

b $1\frac{17}{18} \div 1\frac{1}{27} = \frac{35}{18} \div \frac{28}{27}$ Rewrite mixed numbers as improper fractions.

$= \frac{{}^{5}\cancel{35}}{{}_{2}\cancel{18}} \times \frac{\cancel{27}^{3}}{\cancel{28}_{4}}$ Multiply by the reciprocal of the second fraction.

The reciprocal of $\frac{28}{27}$ is $\frac{27}{28}$.

$= \frac{15}{8}$ or $1\frac{7}{8}$

7 Evaluate the following. (Recall that the reciprocal of 8 is $\frac{1}{8}$.) Check your answers using a calculator.

a $\frac{4}{7} \div \frac{3}{5}$
b $\frac{3}{4} \div \frac{2}{3}$
c $\frac{5}{8} \div \frac{7}{9}$
d $\frac{3}{7} \div \frac{4}{9}$
e $\frac{3}{4} \div \frac{9}{16}$
f $\frac{4}{5} \div \frac{8}{15}$
g $\frac{8}{9} \div \frac{4}{27}$
h $\frac{15}{42} \div \frac{20}{49}$
i $15 \div \frac{5}{6}$
j $6 \div \frac{2}{3}$
k $12 \div \frac{3}{4}$
l $24 \div \frac{3}{8}$
m $\frac{4}{5} \div 8$
n $\frac{3}{4} \div 9$
o $\frac{8}{9} \div 6$
p $14 \div 4\frac{1}{5}$
q $6 \div 1\frac{1}{2}$
r $1\frac{1}{3} \div 8$
s $2\frac{1}{4} \div 1\frac{1}{2}$
t $4\frac{2}{3} \div 5\frac{1}{3}$

Multiply by the reciprocal of the second fraction.

Mixed numbers must become improper fractions first.

Problem-solving and Reasoning

8 Find the following amounts.

a $\frac{1}{2}$ of \$20
b $\frac{1}{2}$ of \$7
c $\frac{1}{2}$ of \$569
d $\frac{1}{3}$ of \$18
e $\frac{1}{10}$ of \$500
f $\frac{3}{4}$ of \$24
g $\frac{4}{5}$ of \$8000
h $\frac{5}{9}$ of \$27
i $\frac{9}{10}$ of \$1

The word 'of' means multiply. \$20 as a fraction is $\frac{20}{1}$.

9 In a $1\frac{1}{2}$ hour maths exam, $\frac{1}{6}$ of that time is allocated as reading time. How long is the reading time?

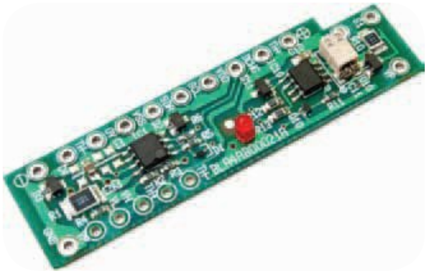
Skillsheet 1A

10 A car's fuel gauge shows that it has $\frac{1}{4}$ of a tank of petrol remaining. The petrol tank holds 64 litres of fuel when full. The car can travel 10 kilometres on 1 litre of petrol. How many kilometres can you travel on the amount of petrol that remains in the tank?

11 Thomas, Ahn and Oscar agree to equally share the job of cleaning the house after a party. They estimate the job would take one person $4\frac{1}{2}$ hours to complete. How many minutes of work should they each contribute?

Enrichment: Electronics and reciprocals

12 Reciprocals are used in the study of electronics. On the calculator, the x^{-1} button can be used to find the reciprocal of a number.



a Use this button to find reciprocals of the following numbers.
  i 2
  ii 5
  iii 6.2
  iv 1.5
  v -1

b If $\frac{1}{6.4} + \frac{1}{7.2} = A$, find the value of $\frac{1}{A}$. (Give your answer to two decimal places.)

c If $\frac{1}{50} + \frac{1}{50} = B$, what is $\frac{1}{B}$?

The techniques in parts **b** and **c** are used to find the total resistance, measured in ohms, Ω, in electronics.

d Investigate, using your calculator, the types of positive numbers for which the reciprocal is bigger than the original number.

e What positive number is its own reciprocal?

f Are there any numbers that do not have a reciprocal?

1G Ratios

Stage
5.2
5.2◊
5.1
4

Fractions, ratios and rates are used to compare quantities. A lawnmower, for example, might require $\frac{1}{6}$ of a litre of oil to make a petrol mix of 2 parts oil to 25 parts petrol, which is an oil to petrol ratio of 2 to 25 or $2:25$.

Two-stroke lawnmowers run on petrol and oil mixed in a certain ratio.

▶ Let's start: The lottery win

\$100 000 is to be shared by three lucky people in the ratio of 2 to 3 to 5 ($2:3:5$). Work out how much money each person receives.

- Write down your method and answer.
- Discuss the methods suggested by other students in the class.

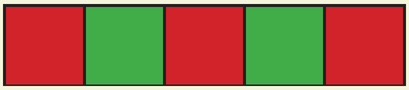
Key ideas

Ratio A way of comparing quantities of the same unit, separated by a colon

Unitary method A way of solving a problem by reducing one of the units to 1

- **Ratios** are used to compare quantities with the same units.
  - The ratio of a is to b is written $a:b$.
  - Ratios in simplest form use whole numbers that have no common factor.

  In the diagram below the ratio can be written as:
  green squares to red squares = $2:3$



- A ratio can be reduced to its simplest form by dividing by the highest common factor (HCF).

$$\begin{tikzcd} 8:12 \arrow[d, bend right, "\div 4"'] \arrow[d, bend left, "\div 4"] \\ 2:3 \end{tikzcd} \qquad \begin{tikzcd} 18:6 \arrow[d, bend right, "\div 6"'] \arrow[d, bend left, "\div 6"] \\ 3:1 \end{tikzcd}$$

- The **unitary method** involves finding the value of one part of a total.
  - Once the value of one part is found, then the value of several parts can be easily determined.

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Exercise 1G

Understanding

1 Write down the missing number.

a $2:5 = \square:10$ **b** $3:7 = \square:28$ **c** $5:8 = 15:\square$

d $7:12 = 42:\square$ **e** $\square:12 = 1:4$ **f** $4:\square = 16:36$

g $8:\square = 64:88$ **h** $\square:4 = 12:16$ **i** $15:25 = \square:100$

2 Consider the ratio of boys to girls of 4 : 5, which means '4 parts to 5 parts'.

a What is the total number of parts?

b What fraction of the total are boys?

c What fraction of the total are girls?

d If there are 18 students in total, how many of them are boys?

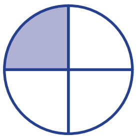
Drilling for Gold 1G1

3 Draw up a table with the following headings.

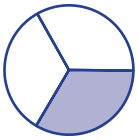
Diagram	Ratio of shaded parts to unshaded parts	Ratio of shaded parts to parts in the whole

Enter information about these diagrams to complete the table.

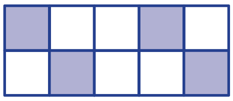
a



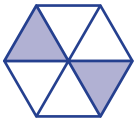
b



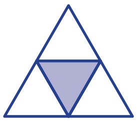
c



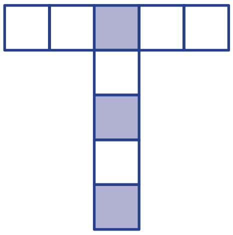
d



e



f



4 At a school assembly, there are five times as many boys as adults and twice as many girls as boys. Write the ratio of:

a boys to adults **b** boys to girls

c girls to boys **d** girls to adults

e boys to the total number of people at the assembly

Remember that with a ratio, the order matters.

Fluency

Drilling for Gold 1G2 at the end of this section

Example 22 Simplifying ratios using HCFs

Simplify the ratio $38:24$.

Solution	**Explanation**
$38:24 = 19:12$	The HCF of 38 and 24 is 2, so divide both sides by 2.

5 Simplify these ratios.

a $6:30$ **b** $8:20$ **c** $40:50$
d $2:20$ **e** $15:18$ **f** $9:27$
g $18:6$ **h** $24:36$ **i** $52:39$
j $144:36$ **k** $48:96$ **l** $30:10$
m $2000:5600$ **n** $3:6:12$ **o** $15:30:10$

Divide the numbers by their HCF.

Example 23 Simplifying ratios involving fractions and decimals

Simplify these ratios.

a $0.2:0.14$ **b** $2\frac{1}{2}:1\frac{1}{3}$

Solution	**Explanation**
a $0.2:0.14 = 20:14$ $= 10:7$	Multiply by 100 to remove all the decimal places, then simplify.
b $2\frac{1}{2}:1\frac{1}{3} = \frac{5}{2}:\frac{4}{3}$	Write as improper fractions, using the same denominator.
$= \frac{15}{6}:\frac{8}{6}$	Find the lowest common denominator (LCD).
$= 15:8$	Multiply both sides by 6 to write as whole numbers.

6 Simplify these ratios.

a $0.1:0.2$ **b** $0.3:4.1$ **c** $0.5:3.2$
d $0.3:0.9$ **e** $0.7:3.5$ **f** $0.4:0.12$
g $8:0.2$ **h** $15:0.01$ **i** $1.6:0.56$

A simplified ratio must include only whole numbers (with no common factors).

7 Simplify the following.

a $\frac{1}{4}:\frac{1}{3}$ **b** $\frac{2}{3}:\frac{1}{4}$ **c** $1\frac{1}{2}:3\frac{1}{3}$
d $2\frac{1}{4}:1\frac{2}{5}$ **e** $\frac{3}{8}:1\frac{3}{4}$ **f** $1\frac{5}{6}:3\frac{1}{4}$

Example 24 Simplifying ratios involving units

Simplify the ratio of 560 metres to 2 kilometres.

Solution	Explanation
$560\text{ m} : 2\text{ km}$	Write using ratio notation.
$= 560\text{ m} : 2000\text{ m}$	Convert the units so that both are the same: $2\text{ km} = 2000\text{ m}$.
$= 560 : 2000$	Cancel the units and divide both numbers by their HCF. HCF of 560 and 2000 is 80.
$= 7 : 25$	

8 Write each of the following as a ratio in simplest form.

a 80c : \$8 **b** 90c : \$4.50 **c** 80 cm : 1.2 m

d 0.7 kg : 800 g **e** 2.5 kg : 400 g **f** 30 min : 2 hours

g 45 min : 3 hours **h** 4 hours : 50 min **i** 40 cm : 2 m : 50 cm

j 80 cm : 600 mm : 2 m **k** 2.5 hours : 1.5 days **l** 0.09 km : 300 m : 1.2 km

Convert to the same units first.

Drilling for Gold 1G3 at the end of this section

Example 25 Dividing according to a given ratio

\$300 is to be divided according to the ratio 2 : 3. Find the value of the larger portion.

Solution	Explanation
Unitary method	
Total number of parts is $2 + 3 = 5$.	Use the ratio 2 : 3 to get the total number of parts.
5 parts = \$300	
1 part $= \frac{1}{5}$ of \$300	Calculate the value of each part (i.e. \$300 ÷ 5).
= \$60	
Larger portion = 3 × \$60	Calculate the value of 3 parts.
= \$180	
Fractional method	
$2 + 3 = 5$ parts	Use the ratio 2 : 3 to get the total number of parts.
The larger portion is $\frac{3}{5}$.	Form a fraction.
$\frac{3}{5} \times \$300 = \$180$	Multiply the fraction by the quantity.

9 Divide:

a \$500 in the ratio of 1 : 4 **b** \$36 in the ratio of 4 : 5

c 88 kg in the ratio of 3 : 8 **d** \$96 in the ratio of 7 : 5

e \$500 in the ratio of 2 : 3 **f** 2000 g in the ratio of 3 : 5

g \$100 in the ratio of 7 : 3 **h** \$600 in the ratio of 1 : 1

i \$70 in the ratio of 2 : 7 : 1 **j** 420 g in the ratio of 8 : 2

First find the value of one part.

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Problem-solving and Reasoning

10 420 g of flour is to be divided into a ratio of 7 : 3 for two different recipes. Find the smaller amount.

11 Kirsty manages a restaurant. Each day she buys watermelons and mangoes in the ratio of 3 : 2. How many watermelons did she buy if, on one day, the total number of watermelons and mangoes was 200?

12 If a prize of $6000 is divided among Georgia, Leanne and Maya in the ratio of 5 : 2 : 3, how much does each girl get?

The correct ratio of ingredients in a recipe has to be maintained when the amount to be made is changed.

13 The dilution ratio for a particular chemical with water is 2 : 3 (chemical to water). If you have 72 litres of chemical, how much water is needed to dilute the chemical?

Skillsheet 1B

If 72 litres is 2 parts, what is one part?

Enrichment: Mixing drinks

14 Four jugs of cordial have a cordial to water ratio as shown, and a given total volume.

Jug	Cordial to water ratio	Total volume
1	1 : 5	600 mL
2	2 : 7	900 mL
3	3 : 5	400 mL
4	2 : 9	330 mL

Concentrations of substances in water or solvents are ratios.

a How much cordial is in:
  i jug 1? **ii** jug 2?
b How much water is in:
  i jug 3? **ii** jug 4?
c Jugs 1 and 2 are mixed together to give 1500 mL of drink.
  i How much cordial is in the drink?
  ii Find the ratio of cordial to water in the drink.
d Find the ratio of cordial to water when the following jugs are mixed.
  i jugs 1 and 3 **ii** jugs 2 and 3 **iii** jugs 2 and 4 **iv** jugs 3 and 4
e Which combination of two jugs gives the strongest cordial to water ratio?

Drilling for Gold exercise

1G2: Simplify me!

Use the worksheet or copy each question into your exercise book.
Write the same thing in both boxes then complete the blanks.
Make sure that your answer is fully simplified.
Question **1** has been completed as an example.

1 $\boxed{\div 5}\begin{array}{c}10:15\\ \downarrow\quad\downarrow\\ 2:3\end{array}\boxed{\div 5}$

2 $\boxed{}\begin{array}{c}6:2\\ \downarrow\quad\downarrow\\ 3:\_\end{array}\boxed{}$

3 $\boxed{}\begin{array}{c}6:6\\ \downarrow\quad\downarrow\\ 1:\_\end{array}\boxed{}$

4 $\boxed{}\begin{array}{c}6:4\\ \downarrow\quad\downarrow\\ \_:2\end{array}\boxed{}$

5 $\boxed{}\begin{array}{c}6:9\\ \downarrow\quad\downarrow\\ 2:\_\end{array}\boxed{}$

6 $\boxed{}\begin{array}{c}6:10\\ \downarrow\quad\downarrow\\ 3:\_\end{array}\boxed{}$

7 $\boxed{}\begin{array}{c}6:12\\ \downarrow\quad\downarrow\\ 1:\_\end{array}\boxed{}$

8 $\boxed{}\begin{array}{c}6:18\\ \downarrow\quad\downarrow\\ 1:\_\end{array}\boxed{}$

9 $\boxed{}\begin{array}{c}12:10\\ \downarrow\quad\downarrow\\ \_:\_\end{array}\boxed{}$

10 $\boxed{}\begin{array}{c}12:9\\ \downarrow\quad\downarrow\\ \_:\_\end{array}\boxed{}$

11 $\boxed{}\begin{array}{c}12:8\\ \downarrow\quad\downarrow\\ \_:\_\end{array}\boxed{}$

12 $\boxed{}\begin{array}{c}12:18\\ \downarrow\quad\downarrow\\ \_:\_\end{array}\boxed{}$

13 $\boxed{}\begin{array}{c}12:36\\ \downarrow\quad\downarrow\\ \_:\_\end{array}\boxed{}$

14 $\boxed{}\begin{array}{c}15:25\\ \downarrow\quad\downarrow\\ \_:\_\end{array}\boxed{}$

15 $\boxed{}\begin{array}{c}15:24\\ \downarrow\quad\downarrow\\ \_:\_\end{array}\boxed{}$

16 $\boxed{}\begin{array}{c}15:30\\ \downarrow\quad\downarrow\\ \_:\_\end{array}\boxed{}$

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1G3: Divided we stand

Amy and Ben have been given \$60 to share between them.
How much will they each receive if they divide it in the ratio given?
Use the worksheet or write your working and answers in your exercise book.
The first question has been done for you.

1 4 : 1 $\frac{4}{5} \times \$60 = \$48,\ \frac{1}{5} \times \$60 = \$12$

2 1 : 2

3 3 : 1

4 3 : 2

5 1 : 5

6 7 : 3

7 1 : 9

8 7 : 8

9 5 : 3

10 7 : 1

For each of the following, draw a line 120 mm long and divide it in the ratio given.
The first one has been done for you.

11 4 : 1 $\frac{4}{5} \times 120 = 96$ mm, $\frac{1}{5} \times 120 = 24$ mm

12 3 : 2

13 3 : 7

14 5 : 7

15 1 : 5

16 7 : 3

17 1 : 9

18 7 : 8

19 5 : 3

20 7 : 1

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1H Rates

Stage
5.2
5.2◊
5.1
4

It is often necessary to compare two quantities with different units. When this occurs it is called a rate.

Rates can be used to describe many things, including speed (m/s, km/h), pay, lap times and prices at the supermarket.

Lap times and speeds in Formula One racing are examples of rates.

Let's start: Fastest animals on Earth

Working in pairs, arrange these animals from fastest to slowest. Then write their speeds in metres per second (m/s).

a zebra 64.37 km/h

b ostrich 70 km/h

c giraffe 52 km/h

d African elephant 40.7 km/h

e lion 80 km/h

f cheetah 120 km/h

g dog 72.4 km/h

h Galapagos tortoise 0.32 km/h

i kangaroo 70.01 km/h

j wildebeest 64.05 km/h

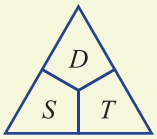
k hyena 65 km/h

l fox 32.03 km/h

Key ideas

- A **rate** compares related quantities with different units.
  - The rate is usually written with one quantity compared to a single unit of the other quantity.
    For example: 50 km in 1 hour or 50 km/h (i.e. 50 km per hour).
- Rates can be used to determine best buys when purchasing products.
- Speed (S) = Distance $(D) \div$ Time (T)

For example: 80 km in 2 hours = 40 km/h



$$S = \frac{D}{T},\quad D = S \times T,\quad T = \frac{D}{S}$$

Rate The number of units of one quantity for each single unit of another quantity

Per For each

Exercise 1H

Understanding

Drilling for Gold 1H1 1H2

1 Complete the following rates.

a Bill travelled 60 km in 1 hour **b** \$84 in 1 hour **c** 600 km in 10 hours

d \$2 for $\frac{1}{2}$ kg **e** Mark earned \$80 in 5 hours

2 George earns \$720 for 8 hours' work. What is his hourly rate of pay?

3 Find the cost of 1 kg if:

a 2 kg costs \$8 **b** 5 kg costs \$15 **c** 4 kg costs \$10

4 If 1 kg costs \$17.50, find the cost of:

a 3 kg **b** 3.5 kg **c** 13.7 kg

Fluency

Example 26 Simplifying rates

Write these rates in simplest form.

a 120 km every 3 hours **b** 5000 revolutions in 2.5 minutes

Solution	**Explanation**
a $\begin{tikzcd}\text{120 km in 3 hours} \arrow[d, bend right, "\div 3"'] \arrow[d, bend left, "\div 3"] \\ \text{40 km in 1 hour}\end{tikzcd}$	Divide by 3 to find km in 1 hour.
b $\begin{tikzcd}\text{5000 rev. in 2.5 min} \arrow[d, bend right, "\div 2.5"'] \arrow[d, bend left, "\div 2.5"] \\ \text{2000 rev. in 1 min}\end{tikzcd}$	Divide by 2.5 to find revolution.

5 Write these as rates in their simplest form.

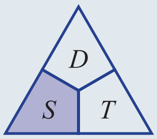
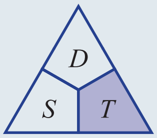
Work out each per one unit.

- **a** \$84 in 3 hours
- **b** \$200 in 4 hours
- **c** 12 kg every 2 minutes
- **d** \$3.50 for $\frac{1}{2}$ kg
- **e** 64 runs in 16 overs
- **f** 623 points in 7 games
- **g** 76 cm in 4 years
- **h** 56 metres in 4 seconds
- **i** 207 heart beats in $2\frac{1}{4}$ minutes
- **j** 180 mL in 22.5 seconds

Example 27 Determining distance, speed and time

A family takes 3 hours to drive 210 km.

- **a** Find their average speed for the trip.
- **b** If they increase their average speed to 90 km/h, how many hours and minutes will the trip take?

Solution	**Explanation**
a Average speed = distance ÷ time = 210 km ÷ 3 hours = 70 km/h	Use the DST triangle and cover the S to provide the formula for finding speed: $S = \frac{D}{T}$ or $D \div T$. Substitute in your values and calculate the speed. (Remember to put units in your answer.) 
b Time = distance ÷ speed = 210 km ÷ 90 km/h = $2\frac{1}{3}$ hours = 2 hours 20 minutes	Use the DST triangle and cover the T to find the formula for time: $T = \frac{D}{S}$ or $D \div S$. Substitute in your values and find the time: $\frac{1}{3}$ hour $= \frac{1}{3} \times 60$ min = 20 minutes 

6 Find the average speed (km/h) of a:

- **a** car travelling 140 km in 2 hours
- **b** bike travelling 60 km in 4 hours
- **c** walker travelling 12 km in 3 hours
- **d** horse galloping 2.7 km in 3 minutes ($\frac{1}{20}$ hour)
- **e** truck travelling 760 km in 9.5 hours

7 Rick drives 1040 km. He averages 80 km/h. How many hours does the trip take?

Example 28 Finding best buys

Drilling for Gold 1H3 at the end of this section

Which is better value: 5 kg of potatoes for \$3.80 or 3 kg for \$2.20?

Solution	Explanation
$$\begin{tikzcd} \text{5 kg for \$3.80} \arrow[d, bend right, "\div 5"'] \arrow[d, bend left, "\div 5"] \\ \text{1 kg for \$0.76} \end{tikzcd}$$	Divide each price by the number of kilograms to find the price per kilogram.
$$\begin{tikzcd} \text{3 kg for \$2.20} \arrow[d, bend right, "\div 3"'] \arrow[d, bend left, "\div 3"] \\ \text{1 kg for \$0.73} \end{tikzcd}$$ 3kg for \$2.20 is better value.	Choose the one that is cheapest for 1 kg.

8 Determine the best buy in each of the following.

- **a** 2 kg of washing powder for \$11.70 or 3 kg for \$16.20
- **b** 1.5 kg of red delicious apples for \$4.80 or 2.2 kg of royal gala apples for \$7.92
- **c** 2.4 litres of orange juice for \$4.20 or 3 litres of orange juice for \$5.40
- **d** 0.7 GB of internet usage for \$14 or 1.5 GB for \$30.90 with different service providers

Problem-solving and Reasoning

Example 29 Using the unitary method

A company claims that one 8 kg bag of fertiliser covers 10 m^2.
How many kilograms would be needed to cover 25 m^2?

Solution	Explanation
$$\begin{tikzcd} \text{8 kg for } 10\ m^2 \arrow[d, bend right, "\div 10"'] \arrow[d, bend left, "\div 10"] \\ \text{0.8 kg for } 1\ m^2 \arrow[d, bend right, "\times 25"'] \arrow[d, bend left, "\times 25"] \\ \text{20 kg for } 25\ m^2 \end{tikzcd}$$	Write the rate. Divide by 10 to find how many kg for 1 m^2. Multiply by 25 to find the kilograms needed for 25 m^2.

9 Consider the following.

- **a** If Zac can type 65 words a minute, how many words can he type in:
  - **i** 2 minutes?
  - **ii** 5 minutes?
  - **iii** 1 hour?
- **b** An alloy is made using 3 grams of copper per 22 grams of gold. Find the gold needed for:
  - **i** 6 grams of copper
  - **ii** 30 grams of copper

10 Jacquie rides her bike at an average speed of 22 km/h. How far does she ride in:

a $2\frac{1}{2}$ hours? **b** $\frac{3}{4}$ hours? **c** 15 minutes?

11 Find the cost of 100 g of each product below, then decide which is the best buy.

a 300 g of coffee A at \$10.80 or 220 g of coffee B at \$8.58.

b 600 g of pasta A for \$7.50 or 250 g of pasta B for \$2.35

c 1.2 kg of cereal A for \$4.44 or 825 g of cereal B for \$3.30

12 A shearer shears 80 sheep in 2 hours.

a Express this as a rate.

b How long will it take, at this rate, to shear all 1000 sheep on the property?

13 Light travels at approximately 300 000 km/s.

a Express this speed as km/h.

b How long does light from the Sun take to travel the 149 million kilometres to Earth?

Skillsheet 1C

Enrichment: Rates of progress

14 A cyclist's progress over a journey of 160 km is recorded. The speed is 20 km/h.

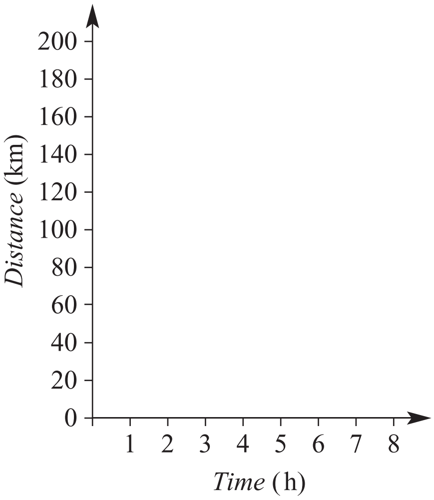
a Copy and complete the table, showing the progress of the cyclist.

Time (in hours)	0	1	2	3	4	5	6	7	8
Distance (in km)	0	20							

b Draw a set of axes like the ones shown and record the information from the table onto the axes.

c What conclusions can you make about the shape of the graph? Why is this the case?

d If the cyclist doubles their speed, on the same set of axes plot a graph that shows the journey for this new speed.



1H3: Best buys

In the supermarket some products can be purchased in a variety of different sizes for different prices. This exercise will help you work out which is the best buy.
Use the worksheet or write your working and answers into your exercise book.

Ice

1 A bag of ice costs $3.99. Three bags cost $10. How much extra does it cost to buy 3 bags rather than 2?

Bottled water

2 At the petrol station:

- 750 mL of bottled water costs $5
- 1.25 litres costs $6.20.

Calculate the 'per litre' cost of each.

Soft drink

3 One shop sells 2 litres of soft drink for $4.00. It is not cold.

Another shop sells it cold for $6.60.

How much extra, per litre, is the second shop charging?

Dog food

4 The normal price for a 700 gram can of dog food is $2.19.

a What is the 'per kilogram' price?

b When it is on special you can buy 5 cans for $9. Is this cheaper 'per kilogram' than buying the 1.2 kg can for $3.69?

Breakfast cereal

5 A convenience store sells a 255 gram box of Dodo Pops for $4.79.
In the supermarket you can buy 650 grams for $7.
How much is this, in dollars per kilogram, in each shop?

Instant coffee

6 Fifty grams of Monaco Coffee costs $5.59.

a What is the 'per kilogram' cost?

b How much (per kilogram) is saved by buying 400 grams for $18.

Just one bottle or the whole box?

7 Bottles of ginger beer sell for $3.49 each for 345 mL.

a What is the 'per litre' price?

b A box that holds 24 bottles costs $37.95. What is the 'per litre' price?

Consumer maths

Video 1A

Investigating currency conversions, petrol consumption and other rates

1 Watch the video: *How to Make a Conversion Spreadsheet and Graph.*

2 Use the video to make the spreadsheet. Call the file 'Australian dollars and Japanese yen'.

3 At the time of writing, one Australian dollar was worth 88.53 Japanese yen.

- **a** Use the internet to find the current conversion rate.
- **b** Change your spreadsheet to the current rate. Take a snapshot and paste it into your exercise book.

88.53 Japanese yen

1 Australian dollar

4 Use the internet to find the number of euros you can buy for 1 Australian dollar. Make a conversion spreadsheet and graph for Australian dollars and euros.

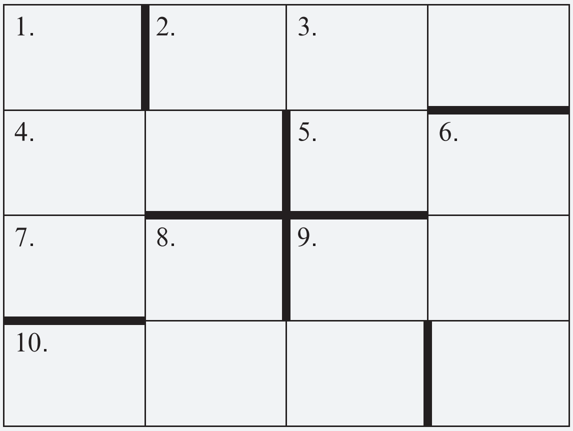
5 A car company claims that their new car uses 5.7 litres of fuel for every 100 kilometres driven.

- **a** Create a spreadsheet and graph. Put kilometres on the horizontal axis and mark from 0 km to 500 km.
- **b** Use the internet to find the current average price of petrol.
- **c** Choose five cars you like. Use the internet to find their fuel consumption.
- **d** Choose the car with the highest fuel consumption rate. If you drive it 50 km to work every day and 50 km home again each day for 5 days every week, how much will you spend on petrol for that purpose every week?

Puzzles and games

1 Can you arrange the first nine counting numbers with operations (+, – , × , ÷) and brackets to give an answer of 100?

2 Complete the number cross below.



1.	2.	3.	
4.		5.	6.
7.	8.	9.	
10.			

Clues

Across

2. The sum of 100, 150 and 3
4. The product of 5 and the first prime number after 6
5. Reverse the digits of the product of 13 and 5
7. One less than the square of 10
9. Half of 6 times 8
10. 2 to the power of 7

Down

1. A palindromic number
2. $5 \times 10 - 5 \times 5$
3. A multiple of 11
6. 639 written correct to two significant figures
8. The next even integer after 90
9. A multiple of 7

3 **a** Triangular numbers are the number of dots required to form triangles, as shown in this table.

i Copy and complete this table.

Number of rows	1	2	3	4	5	6
Diagram						
Number of dots (triangular number)	1	3				

ii Find the 7th and 8th triangular numbers.

b Fibonacci numbers are a sequence of numbers where each number is the sum of the two preceding numbers. The first two numbers in the sequence are 0 and 1.

i Write down the first 10 Fibonacci numbers.

ii If the Fibonacci numbers were to be extended in the negative direction, what would be the first four negative Fibonacci numbers?

Chapter summary

Integers, decimals, fractions, ratios and rates

Decimals

Decimals are fractions with denominators of 10, 100, 1000.

$\frac{1}{10} = 0.1, \frac{1}{100} = 0.01, \frac{1}{1000} = 0.001$

Rounding decimals

If the critical digit (the next one in the number) is 0, 1, 2, 3 or 4, round down. If it is 5, 6, 7, 8 or 9, round up.

4.763 = 4.8 (1 decimal place)

4.763 = 4.76 (2 decimal places)

Rational numbers

Rational numbers can be written as a fraction.

$3, -4, \frac{2}{3}, 0.\dot{3}, \sqrt{4}, 0.416$

are all rational numbers.

π is an example of an irrational number. It cannot be written as a fraction.

Ratios

Ratios compare two or more quantities with the same units in a definite order.

5 cents to \$1 = 5 : 100 = 1 : 20

You can simplify ratios using the HCF.

$$\div 12 \begin{array}{c} 24 : 36 \\ 2 : 3 \end{array} \div 12$$

Rates

Rates compare two quantities with different units.

50 km in 2 hours = 25 km/h

Integers

Integers are the set of positive and negative whole numbers including zero.

$0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots$

Working with negative numbers

- To add a negative number, you subtract.
  $a + (-b) = a - b$
- To subtract a negative number, you add.
  $a - (-b) = a + b$

× and ÷

- Two numbers of the same sign multiply or divide to give a positive answer.
  e.g. $-4 \times (-5) = 20$
  $-10 \div (-2) = 5$
- Two numbers with different signs multiply or divide to give a negative answer.
  e.g. $-4 \times 5 = -20$
  $10 \div (-2) = -5$

Reciprocals

The reciprocal of a proper or improper fraction is the fraction turned upside down (inverted).

The reciprocal of:

6 is $\frac{1}{6}$

$1\frac{1}{2}$ is $\frac{2}{3}$

$\frac{4}{5}$ is $\frac{5}{4}$ or $1\frac{1}{4}$

Fractions

- To add or subtract fractions, use the lowest common denominator.
- To multiply fractions (proper or improper), multiply the numerators and multiply the denominators after cancelling common factors.
  $\frac{8}{9} \times \frac{4}{7} = \frac{8 \times 4}{9 \times 7}$
- To divide by a fraction, multiply by the reciprocal.
  $\frac{3}{2} \div \frac{4}{5} = \frac{3}{2} \times \frac{5}{4}$

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T **Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.**

Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

1 $\frac{2}{7}$ written as a decimal is:

A 0.29 **B** 0.286 **C** $0.28\dot{5}$ **D** $0.\overline{285714}$ **E** $0.28571\dot{4}$

2 3.0456 written to three significant figures is:

A 3.04 **B** 3.05 **C** 3.045 **D** 3.046 **E** 3.45

3 2.25 written as a fraction in simplest form is:

A $2\frac{1}{2}$ **B** $\frac{5}{4}$ **C** $\frac{9}{4}$ **D** $9\frac{1}{4}$ **E** $\frac{225}{100}$

4 $1\frac{1}{2}-\frac{5}{6}$ is equal to:

A $\frac{2}{3}$ **B** $\frac{5}{6}$ **C** $-\frac{1}{2}$ **D** $\frac{2}{6}$ **E** $\frac{1}{2}$

5 $\frac{2}{7}\times\frac{3}{4}$ is equivalent to:

A $\frac{8}{11}$ **B** $\frac{3}{7}$ **C** $\frac{5}{11}$ **D** $\frac{8}{12}$ **E** $\frac{3}{14}$

6 $\frac{3}{4}\div\frac{5}{6}$ is equivalent to:

A $\frac{5}{8}$ **B** 1 **C** 21 **D** $\frac{4}{5}$ **E** $\frac{9}{10}$

7 Simplifying the ratio 50 cm : 4 m gives:

A 50 : 4 **B** 8 : 1 **C** 25 : 2 **D** 1 : 8 **E** 5 : 40

8 Divide $2.50 in the ratio of 4 : 1.

A 50 : 200 **B** 50c **C** $2 : 50c **D** $200 **E** $2

9 A childcare centre requires the ratio of carers to children to be 1 : 5. How many carers are needed for 30 children?

A 5 **B** 6 **C** 150 **D** 30 **E** 36

10 Michael earns $460 in 25 hours. His hourly rate of pay is:

A $11 500/h **B** $0.05/h **C** $18.40/h **D** $18.40/day **E** $5/h

11 36 km in 40 minutes is the same as:

A 40 km/h **B** 36 km/h **C** 54 km/h **D** 90 km/h **E** 56 km/h

12 To mix a particular shade of pink, red paint is mixed with white paint in the ratio 8 to 3. If 20 cans of red paint are used, how many cans of white paint will be needed to mix the correct shade of pink?

A 15 **B** 7.5 **C** 7 **D** 8 **E** 24

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Short-answer questions

1 Evaluate the following.

a $-4-(-12)$ **b** $18-9\times 7$ **c** $-90\times(-3)$

d $(-4)^3$ **e** $-15\div 5\times(-2)$ **f** $(10-12)\times(7-11)$

2 Round these numbers to three significant figures.

a 21.483 **b** 29 130 **c** 0.15271 **d** 0.002414

3 Round these numbers to two decimal places.

a 14.976 83 **b** 0.7149871498… **c** 1.999

4 Write these fractions as decimals.

a $2\frac{1}{8}$ **b** $\frac{5}{6}$ **c** $\frac{13}{7}$

5 Write these decimals as fractions.

a 0.75 **b** 1.6 **c** 2.55

6 What is the reciprocal of:

a $\frac{3}{4}$? **b** $\frac{1}{5}$? **c** 8? **d** $1\frac{2}{7}$?

7 Simplify the following. Check your answers with a calculator.

a $\frac{5}{6}-\frac{1}{3}$ **b** $1\frac{1}{2}+\frac{2}{3}$ **c** $\frac{13}{8}-\frac{4}{3}$ **d** $2\frac{1}{4}-1\frac{1}{3}$

e $\frac{3}{8}\times\frac{4}{5}$ **f** $3\frac{1}{2}\times\frac{4}{7}$ **g** $5\div\frac{4}{3}$ **h** $\frac{6}{7}\div 2$

8 Simplify these ratios.

a $30:12$ **b** $15:30$ **c** $7\frac{1}{2}:1\frac{2}{5}$

9 Divide 80 into the given ratio.

a $5:3$ **b** $5:11$ **c** $1:2:5$

10 Dry dog food can be bought from store A for \$18 for 8 kg or from store B for \$14.19 for 5.5 kg.

a Determine the cost per kilogram at each store and state which is the best buy.

b Determine, to the nearest whole number, how many grams you get per dollar at each store.

11 Share $660 in the ratio of 2 : 3 : 5.

12 Complete these rates.

a $96 in 3 hours = $____ /h

b 14 g in 2 minutes = ____ g/min

c 32 m in 10 seconds = ____ m/min

d $7.40 in 30 minutes = $____ /h

13 Write the following speeds in km/h.

a 780 km in 5 hours

b 90 km in $2\frac{1}{2}$ hours

14 Andrew and Claire share their lottery win in the ratio of 3 : 2. If Andrew got $9000, how much did Claire get?

Extended-response questions

1 A class of 28 students has a ratio of boys to girls of 3 : 4.

a How many boys are in the class?

b Two boys leave and are replaced by two girls. How many boys and girls are now in the class?

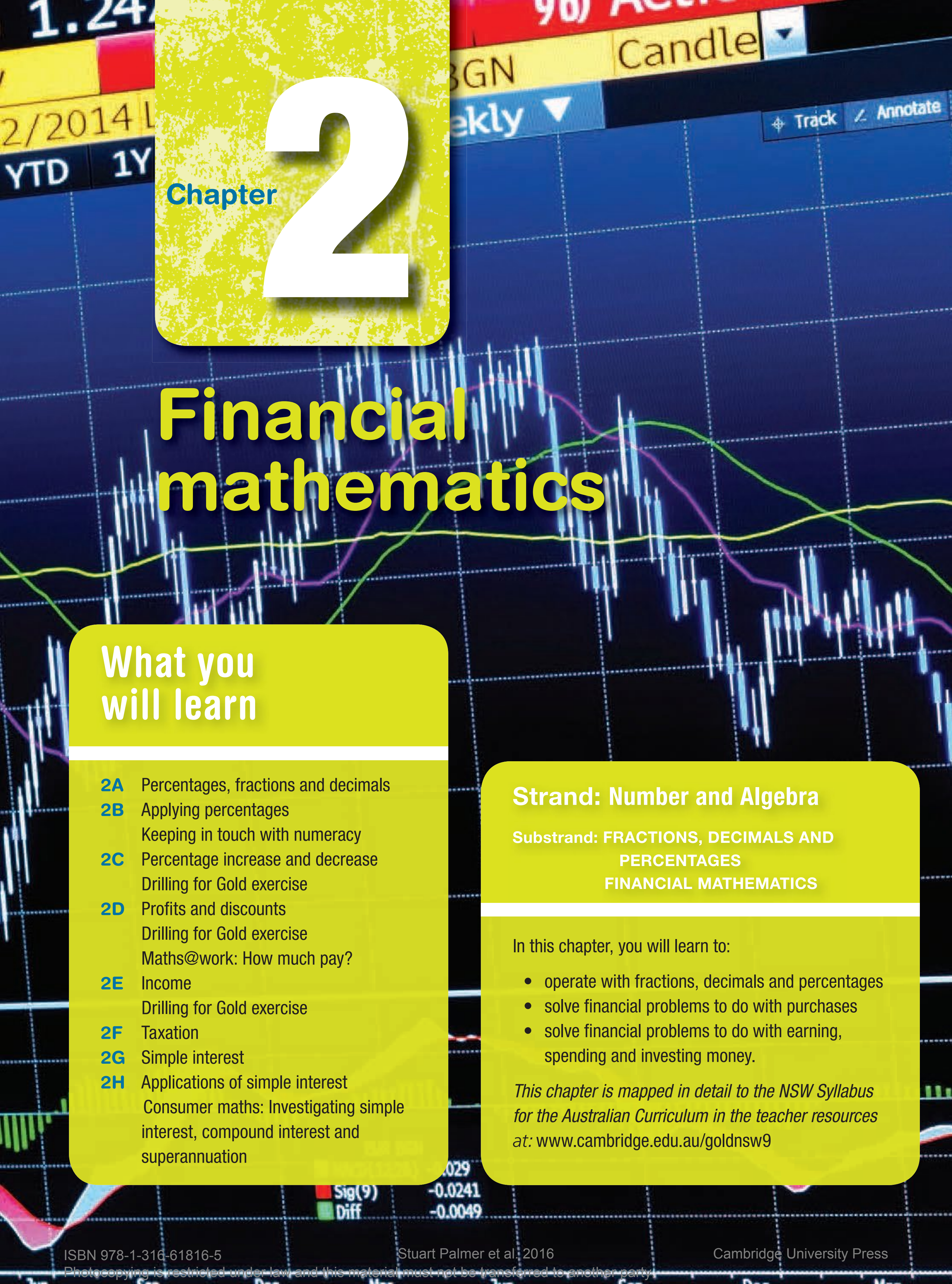
c What is the new ratio of boys to girls?

2 A family plans to drive 856 km.

a At what speed should they travel, on average, if they hope to take only 10 hours?

b The family left home at 6 a.m. and arrived at 8 p.m. How long did the trip take?

c Calculate their average speed, correct to one decimal place.



Chapter 2

Financial mathematics

What you will learn

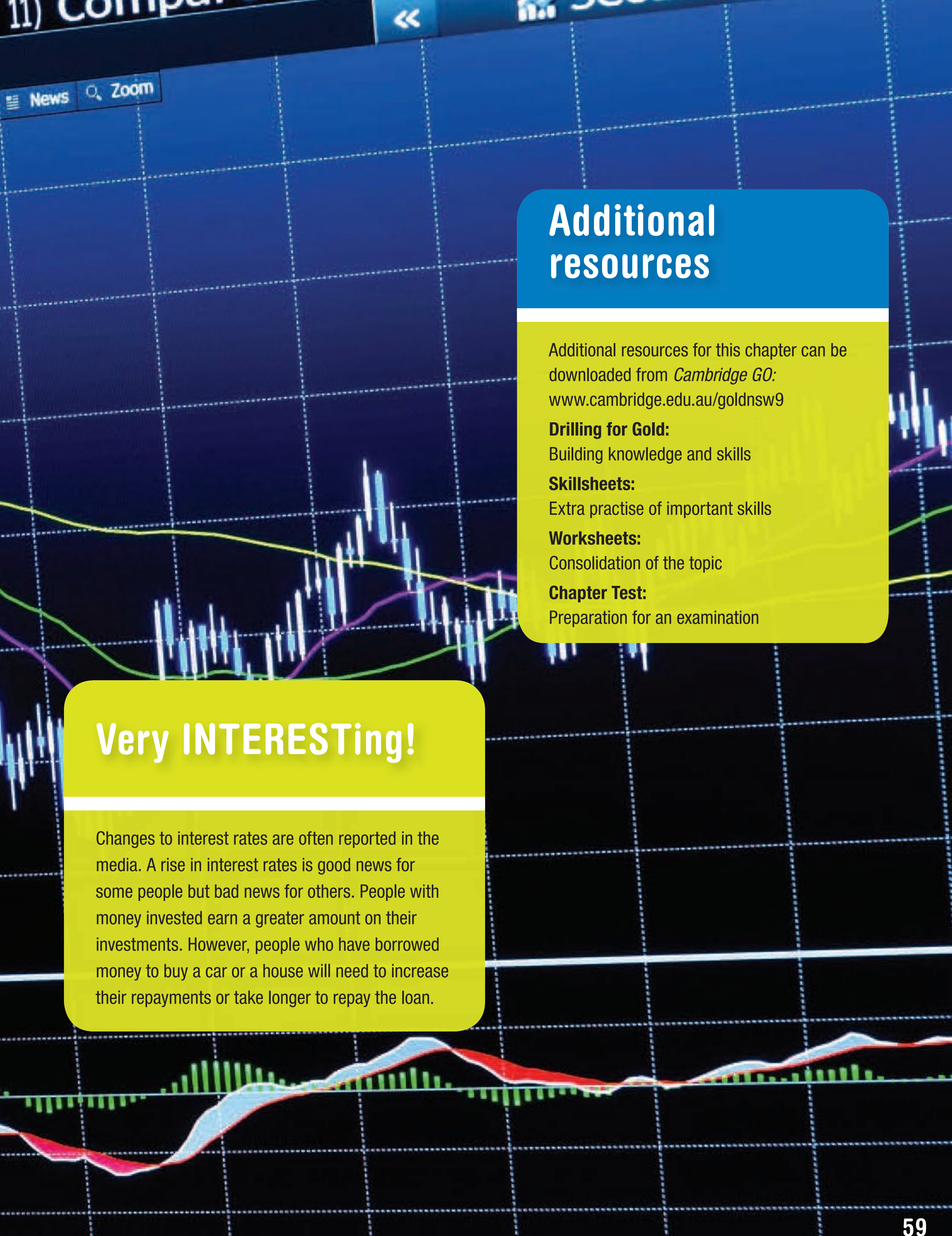
Strand: Number and Algebra

Substrand: FRACTIONS, DECIMALS AND PERCENTAGES
FINANCIAL MATHEMATICS

In this chapter, you will learn to:

- operate with fractions, decimals and percentages
- solve financial problems to do with purchases
- solve financial problems to do with earning, spending and investing money.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw9



Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO:* www.cambridge.edu.au/goldnsw9

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

Very INTERESTing!

Changes to interest rates are often reported in the media. A rise in interest rates is good news for some people but bad news for others. People with money invested earn a greater amount on their investments. However, people who have borrowed money to buy a car or a house will need to increase their repayments or take longer to repay the loan.

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Pre-test

1 Which is larger:

a $\frac{1}{2}$ or 55%? **b** $\frac{3}{4}$ or 70%? **c** $\frac{1}{4}$ or 0.4?

2 Copy and complete:

a $61\% = \frac{\Box}{100}$ **b** $9\% = \frac{\Box}{100}$ **c** $37\% = \frac{\Box}{100}$

d $121\% = \frac{\Box}{100}$ **e** $1\% = \frac{\Box}{100}$ **f** $75\% = \frac{\Box}{100} = \frac{3}{\Box}$

3 Copy and complete the following table.

Fraction	Decimal	Percentage
$\frac{1}{100}$		
	0.1	
	0.25	
		50%
$\frac{3}{4}$		

4 Write down 10% of these amounts.

a 100 g **b** 70 km **c** \$450 **d** \$8000 **e** \$5 **f** 90c

5 Without using a calculator, find:

a 25% of \$400 **b** 75% of \$80 **c** 50% of \$3

d 10% of \$678 **e** 1% of \$600 **f** 20% of \$80

6 Complete the following.

a 1% of 60 = 60 divided by $\Box$ **b** 10% of 50 = 50 divided by $\Box$

c 5% of 100 = 100 divided by $\Box$ **d** $33\frac{1}{3}\%$ of 963 = 963 divided by $\Box$

e 25% of 88 = 88 divided by $\Box$ **f** 50% of 120 = 120 divided by $\Box$

7 Copy and complete:

a 1 week = $\Box$ days **b** 1 year = $\Box$ days

c 1 year = $\Box$ weeks **d** 1 year = $\Box$ months

8 How many hours are there from:

a 5 a.m. to 7 p.m.? **b** 9 a.m. to 3 p.m.? **c** 8:30 a.m. to 9 p.m.?

2A Percentages, fractions and decimals

Stage
5.2
5.2◊
5.1
4

Percentages are used in our daily lives. Some examples are loans, credit cards, discounts and profits.

We know from our previous work on percentages that they represent a fraction with a denominator of 100.

▶ Let's start: Ordering with percentages

Ten different values are given in the table below.
With a classmate, write the ten numbers on cards and arrange them in ascending (i.e. increasing) order.

$\frac{1}{2}$	0.8	0.05	15%	$\frac{7}{10}$	0.9	9%	$\frac{3}{5}$	$\frac{1}{3}$	0.3

Key ideas

Drilling for Gold 2A1

- The table below contains some commonly used fractions, decimals and percentages. These should be memorised.

Fraction	Decimal	Percentage	Therefore
1	1	100%	$3 = 300\%$
$\frac{1}{2}$	0.5	50%	$1.5 = 150\%$
$\frac{1}{3}$	0.333... or $0.\dot{3}$	$33\frac{1}{3}\%$	$\frac{2}{3} = 0.\dot{6} = 66\frac{2}{3}\%$
$\frac{1}{4}$	0.25	25%	$\frac{3}{4} = 0.75 = 75\%$
$\frac{1}{5}$	0.2	20%	$\frac{3}{5} = 0.6 = 60\%$
$\frac{1}{10}$	0.1	10%	$\frac{7}{10} = 0.7 = 70\%$
$\frac{1}{100}$	0.01	1%	$\frac{21}{100} = 0.21 = 21\%$

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Drilling for Gold 2A2

- There are six **conversions** involving fractions, decimals and percentages. See Drilling for Gold 2A2 for more detail.

Conversion Changing (converting) a value into the equivalent value expressed in a different form

Conversion	Without a calculator	With a calculator
decimal to fraction $0.6 = \frac{6}{10} = \frac{3}{5}$	Use tenths, hundredths, etc. Simplify the fraction.	Type the decimal. Press =. Press the button that converts fractions to decimals.
fraction to decimal $\frac{13}{50} = \frac{26}{100} = 0.26$ $\frac{5}{8} = 5 \div 8 = 0.625$	Convert fraction to tenths or hundredths or Divide numerator by denominator.	Type the fraction. Press =. Press the button that converts fractions to decimals.
decimal to percentage $0.24 \times 100 = 24$ $\therefore\ 0.24 = 24\%$	Multiply by 100. Add % symbol. (The decimal point appears to move two places to the right.)	Multiply by 100. Add % symbol.
percentage to decimal $24 \div 100 = 0.24$ $\therefore\ 24\% = 0.24$	Remove % symbol. Divide by 100. (The decimal point appears to move two places to the left.)	Remove % symbol. Divide by 100.
fraction to percentage $\frac{13}{50} = \frac{26}{100} = 26\%$ $\frac{5}{8} = 5 \div 8 = 0.625$ $0.625 \times 100 = 62.5$ $\therefore\ \frac{5}{8} = 62.5\%$	Convert the fraction to hundredths. Add % symbol or Divide numerator by denominator. Multiply by 100. Add % symbol.	Multiply by 100. Add % symbol.
percentage to fraction $60\% = \frac{60}{100} = \frac{3}{5}$	Remove % symbol. Write as 'hundredths'. Simplify fraction.	Remove % symbol. Divide by 100. Press the button that converts fractions to decimals.

- Expressing one quantity as a percentage of another (percentage composition):
  - Form a fraction, then convert to a percentage.

For example: 4 goals from 10 attempts $= \frac{4}{10} = \frac{40}{100} = 40\%$

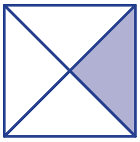
Exercise 2A

Understanding

Drilling for Gold 2A3

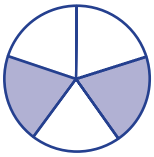
1 What percentage of each of the following diagrams has been shaded?

a



b

c



2 What percentage of each of the following diagrams is not shaded?

a

b

3 Copy and complete this table of common percentages.

%	10%			50%			
Fraction			$\frac{1}{4}$			$\frac{1}{3}$	$\frac{2}{3}$
Decimal		0.2			0.75		

4 Scott scored 30 out of 50 on a maths quiz and Abigail scored 80% on the same test. Who scored the highest mark?

Fluency

Example 1 Rewriting percentages and fractions, without a calculator

a Write $\frac{12}{25}$ as a percentage.

b Write 8% as a simple fraction.

Solution	**Explanation**
a $\frac{12}{25} = \frac{12}{25} \times 100\%$ $= 48\%$	Multiply the fraction by 100, add % symbol. $\frac{12}{25} \times 100\% = \frac{12}{\cancel{25}_{1}} \times \frac{\cancel{100}^{4}}{1}$
b $8\% = \frac{8}{100}$ $= \frac{2}{25}$	Write the percentage as a fraction, using a denominator of 100. Simplify $\frac{8}{100}$ by dividing the numerator and the denominator by the HCF (4). $\underset{\div 4}{\overset{\div 4}{\frac{8}{100} = \frac{2}{25}}}$

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Drilling for Gold 2A4

5 Express the following fractions as percentages. Use a calculator if necessary.

a $\frac{1}{5}$ **b** $\frac{4}{5}$ **c** $\frac{8}{10}$ **d** $\frac{3}{10}$

e $\frac{1}{4}$ **f** $\frac{1}{8}$ **g** $\frac{3}{4}$ **h** $\frac{12}{20}$

i $\frac{14}{25}$ **j** $\frac{7}{20}$ **k** $\frac{9}{100}$ **l** $\frac{3}{40}$

Multiply by 100 and add % symbol.

6 Express the following percentages as simplified fractions. Use a calculator if necessary.

a 19% **b** 23% **c** 99% **d** 5%

e 22% **f** 45% **g** 74% **h** 75%

i 2.5% **j** 17.25% **k** 1% **l** 125%

Remove % symbol. Write as hundredths; e.g. $19\% = \frac{19}{100}$

Example 2 Converting between percentages and decimals

a Write 0.45 as a percentage.

b Write 25% as a decimal.

Solution	Explanation
a $0.45 = 0.45 \times 100\%$ $= 45\%$	Multiply by 100. Add % symbol.
b $25\% = 25 \div 100$ $= 0.25$	Remove the % symbol. Divide by 100.

7 Express the following decimals as percentages.

a 0.78 **b** 0.95 **c** 0.65 **d** 0.48

e 0.75 **f** 1.42 **g** 0.07 **h** 0.3

i 0.03 **j** 1.04 **k** 0.12 **l** 0.1225

The decimal point appears to move two places to the right.

8 Express the following percentages as decimals.

a 12% **b** 83% **c** 57% **d** 88%

e 99% **f** 100% **g** 120% **h** 5%

Example 3 Writing a quantity as a percentage, with a calculator

Write 50c out of $2.50 as a percentage.

Solution	Explanation
$\frac{50}{250} \times 100 = 20$	Convert to the same units ($2.50 = 250c). Form a fraction. Multiply by 100.
$\therefore$ 50c out of $2.50 = 20%	Express as a percentage.

9 In each of the following cases, express the first quantity as a percentage of the second.

a 5 g out of 200 g **b** 40c out of $4
c 10 km out of 200 km **d** 3 seconds out of 1 minute
e 200 m out of 1 km **f** 100 mL out of $\frac{1}{2}$ L
g 200c out of $1 **h** 45 marks out of a possible 60 marks

Make a fraction.
Multiply by 100.
Add % symbol.

Problem-solving and Reasoning

10 Every student in Year 9 chose their favourite sport. Copy and complete the table.

Sport	Number of students who chose sport	Fraction of the total	Percentage of the total
Swimming	44		
Golf	12		
Volleyball	58		
Cricket	36		
TOTAL			

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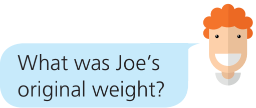
11 Toni scored 31 out of 50 in a test. Express this as a percentage.

12 Bad weather stopped a cricket game for 35 minutes of a scheduled $3\frac{1}{2}$ hour match. What percentage of the scheduled time was lost?

13 Joe lost 4 kg and now weighs 60 kg. What percentage of his original weight did he lose?

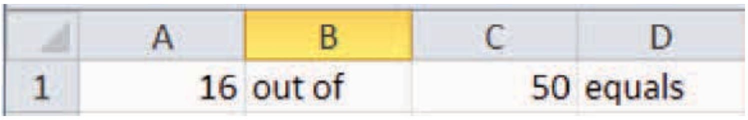


14 A company claims that the apple pies it makes are 97% fat free. If the nutritional information on the side of the pack states that total fat is 7 grams of the 250 gram pie, is the claim correct?

Enrichment: Spreadsheet percentages

15 A spreadsheet is useful for doing a calculation many times with different numbers. On a computer, open a blank spreadsheet.

Type the following into the first four cells of row 1:



	A	B	C	D
1	16	out of	50	equals

Click on cell E1 and type =A1/C1*100, then press the ENTER button.

In cell F1, type the words 'per cent'.

a Explain what the formula in cell E1 does and the meaning of the symbols used.

b What happens when you change the number in cell A1 from 16 to 17?

c Change the numbers in cells A1 and C1 to show that 12 out of 40 is 30 per cent.

d Use your spreadsheet to convert the following test marks to percentages. Give your answers correct to one decimal place if necessary.

- **i** 12 out of 60
- **ii** 12 out of 50
- **iii** 12 out of 40
- **iv** 12 out of 20
- **v** 17 out of 40
- **vi** 17 out of 30
- **vii** 21.5 out of 50
- **viii** 21 out of 47

2B Applying percentages

Stage
5.2
5.2◊
5.1
4

Percentages are seen in news stories and advertisements. For example:

- 90% of dentists prefer this toothbrush.
- There is a 45% chance of rain.
- A swing of 5% is expected in the next election.

Let's start: Newspaper challenge

In pairs, go through today's newspaper and find articles and advertisements that use percentages. Choose two and explain to the class how percentages are used in the items you choose.

Key ideas

- To find a percentage of a quantity (without a calculator):
  - Write the percentage as a fraction or decimal.
  - Multiply by the quantity.
- Calculator method:
  3% of 200 = $3 \div 100 \times 200 = 6$
- Mental strategies:

50%	25%	10%	5%	1%	75%
$\div 2$	$\div 2 \div 2$	$\div 10$	$\div 10 \div 2$	$\div 100$	$\div 2 \div 2 \times 3$

- To find the original amount when given a percentage, you can use the unitary method or solve an equation.

  For example: *3% of an amount is 15. What is the original amount?*

Unitary method

$$\begin{tikzcd} \text{3\% is 15} \arrow[d, bend right, "\div 3"'] \arrow[d, bend left, "\div 3"] \\ \text{1\% is 5} \arrow[d, bend right, "\times 100"'] \arrow[d, bend left, "\times 100"] \\ \text{100\% is 500} \end{tikzcd}$$

∴ The original amount is 500.

Equation method

3% of amount is 15.

$$\begin{tikzcd} 0.03 \times A = 15 \arrow[d, bend right, "\div 0.03"'] \arrow[d, bend left, "\div 0.03"] \\ A = 300 \end{tikzcd}$$

∴ The original amount is 500.

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Exercise 2B

Understanding

Drilling for Gold 2B1

1 Find these percentage amounts mentally.

a 10% of \$40 **b** 5% of \$40

c 25% of 8 kg **d** 75% of 8 kg

e 1% of \$800 **f** 2% of 140 seconds

Use the mental strategies given in the Key ideas.

2 If 1% of an amount is \$4, what is 100% of the amount?

3 True or false: 34% of 568 = 0.34×568?

Fluency

Example 4 Finding a percentage of a quantity

Find 15% of \$35.

Solution	**Explanation**
$15\%\text{ of }\$35 = \frac{{}^{3}\cancel{15}}{{}_{20}\cancel{100}} \times \$35$ $= \$5.25$	Write the percentage as a fraction out of 100 and multiply by \$35.
or $0.15 \times \$35 = \$5.25$ (using a calculator)	Or: Write 15% ÷ 0.15. Note: ‘of’ means to multiply.

4 Find the following amounts. Use a calculator if necessary.

a 10% of 20 **b** 5% of 200 **c** 20% of 40

d 15% of 50 **e** 8% of 720 **f** 5% of 680

g 15% of 8200 **h** 70% of 60 **i** 90% of 500

j 75% of 44 **k** 99% of 200 **l** 3% of 50

10% of 20 = one-tenth of 20

5 Use a calculator to find:

a 10% of \$360 **b** 50% of \$420 **c** 75% of 64 kg

d 12.5% of 240 km **e** 37.5% of 40 apples **f** 87.5% of 400 m

g $33\frac{1}{3}\%$ of 750 people **h** $66\frac{2}{3}\%$ of 300 cars **i** $8\frac{3}{4}\%$ of \$560

$75\%\text{ of }64 = 75 \div 100 \times 64$

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Drilling for Gold 2B2

Example 5 Finding the original amount

Determine the original amount if 5% of the amount is $45.

Solution	Explanation
Unitary method: $\div 5 \quad 5\% \text{ of the amount} = \$45 \quad \div 5$ $\times 100 \quad 1\% \text{ of the amount} = \$9 \quad \times 100$ $100\% \text{ of the amount} = \$900$ So the original amount is $900.	To use the unitary method, find the value of 1 part or 1%, then multiply by 100 to find 100%.
Equation method: 5% of amount is 45. $\div 0.05 \quad 0.05 \times A = 45 \quad \div 0.05$ $A = \$900$	Convert the information to an equation, letting A represent the amount. Solve the equation by dividing both sides by 0.05.

6 Determine the original amount if:

a 10% of the amount is $12

b 6% of the amount is $42

c 3% of the amount is $9

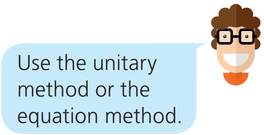
d 40% of the amount is $2.80

e 90% of the amount is $0.18

f 6% of the amount is $27

g 12% of the amount is $96

h 15% of the amount is $54

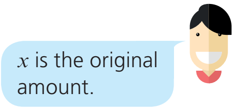


7 Determine the value of x in the following if:

a 10% of x is $54 **b** 15% of x is $90

c 25% of x is $127 **d** 18% of x is $225

e 105% of x is $126 **f** 110% of x is $44



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Problem-solving and Reasoning

$33\frac{1}{3}\% = \frac{1}{3}$

8 Without a calculator, evaluate the following.

a 10% of \$58 **b** 5% of \$84 **c** 1% of \$46

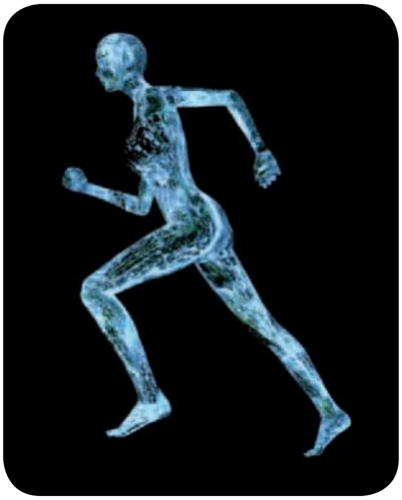
d $2\frac{1}{2}\%$ of \$20 **e** $33\frac{1}{3}\%$ of \$132 **f** $66\frac{2}{3}\%$ of \$60

9 If $\frac{1}{3}$ of $96 = 32$, what is $66\frac{2}{3}\%$ of 96?

10 If 10% of \$800 is \$80, explain how you can use this result to find:

a 1% of \$800 **b** 5% of \$800 **c** $2\frac{1}{2}\%$ of \$800

11 About 80% of the mass of the human body is water. If Carla weighs 60 kg, how many kilograms of water make up her body weight?



12 In a class of 25 students, 40% have been overseas. How many students have not been overseas?

13 Explain why 10% of 24 = 24% of 10.

14 10% of 1 day is the same as x hours and y minutes. What is the value of x and y?

Enrichment: More than 100%

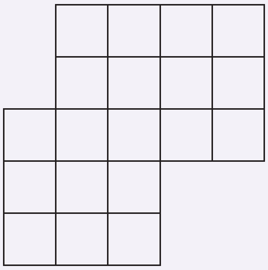
15 a Find 120% of 60.

b Determine the value of x if 165% of $x = 1.5$.

c Write 2.80 as a percentage.

d Write 325% as a fraction.

e \$2000 in a bank account increases to \$5000 over a period of time. By how much has the amount increased as a percentage?

Non-calculator	**Calculator**
1 What is the sum of 14, 15 and 16?	**2** What is the sum of the whole numbers from 14 to 20, inclusive?
3 **a** What is the perimeter of this shape? **b** What is the area of this shape if every square represents 1 square centimetre? 	**4** **a** What is the area of the floor in a square room with 3.1 metre sides? **b** What is the area of a rectangle with sides 2.75 metres and 3.14 metres? **c** Use the formula $A = \pi r^2$ to find the area of a circle with radius 12 metres. Give your answer correct to two decimal places. **d** Use the formula $C = 2\pi r$ to find the circumference of a circle with radius 12 metres. Give your answer correct to two decimal places.
5 Convert the following to percentages. **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{5}$ **d** $\frac{4}{5}$	**6** Convert the following to percentages. **a** $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{17}{40}$ **d** $\frac{53}{60}$
7 How many hours and minutes are there from 7:30 a.m. to 9:45 a.m.?	**8** How many hours and minutes are there from 7:39 a.m. to 11:17 a.m.?
9 What is the best estimate for $12\,600 \div 30$? **A** 4 **B** 40 **C** 400 **D** 4000	**10** A prize of $12 600 is divided evenly between 30 people. How much does each person receive?
11 **a** What is 10% of $240? **b** What is 5% of $240? **c** What is 15% of $240? **d** What is 25% of $240? **e** What is 1% of $240?	**12** What is 17.5% of $240?
13 How much change from $10 should you receive if you buy four $1.20 pens?	**14** How much change from $100 should you receive if you buy four books for $12.75 each and two bags for $19.50 each?
15 How many chocolates costing 70c each can be purchased for $10?	**16** Some chocolates cost 70 cents each. What is the cost of 175 chocolates?
17 The value of $\frac{1}{2}+\frac{3}{4}$ is not $\frac{4}{6}$. Explain why the correct answer is greater than 1. What is the correct value?	**18** Find the sum of $\frac{3}{8}$ and $\frac{5}{12}$.
19 Divide $20 in the ratio 3 : 2.	**20** Divide $180 in the ratio 7 : 2.

2C Percentage increase and decrease

Stage
5.2
5.2◊
5.1
4

A worker may be given a pay increase of 5%. During a sale, prices may be decreased by 5%.

Let's start: The quicker method

Nicky and Mila are asked to increase $250 by 15%.

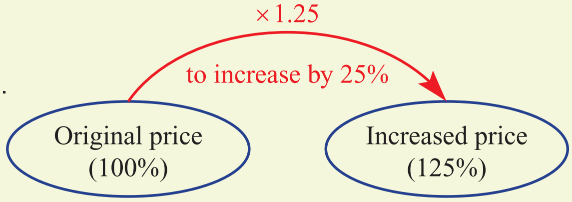
Nicky's method	**Mila's method**
15% of \$250 = \$37.50	100% + 15% = 115% = 1.15
\$250 + \$37.50 = \$287.50	250 × 1.15 = \$287.50

- Which of the two methods do you prefer? Why?
- Consider if the question is altered so that $250 is *decreased* by 15%. How will Nicky's and Mila's methods work in this case?

Key ideas

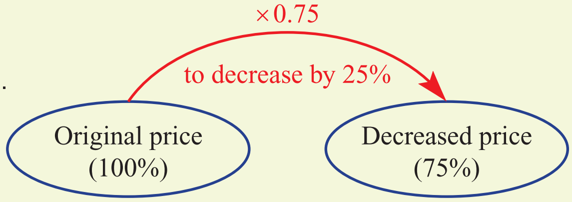
- To increase an amount by a given percentage:
  - Add the percentage increase to 100%.
  - Multiply the amount by this new percentage.

  For example, to increase by 25%, multiply by 100% + 25% = 125% = 1.25.



- To decrease an amount by a given percentage:
  - Subtract the percentage from 100%.
  - Multiply the amount by this new percentage.

  For example, to decrease by 25%, multiply by 100% – 25% = 75% = 0.75.



- To find a percentage change, use:

$$\text{Percentage change} = \frac{\text{change in price}}{\text{original price}} \times 100\%$$

For example, if a price changes from \$40 to \$50:

$$\text{Percentage change} = \frac{10}{40} \times 100\% = 25\%$$

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Exercise 2C

Understanding

1 Write the missing number.
- **a** To increase a number by 25%, multiply by ________.
- **b** To increase a number by 26%, multiply by ________.
- **c** To increase a number by 12%, multiply by ________.
- **d** To increase a number by 21%, multiply by ________.
- **e** To increase a number by 20%, multiply by ________.
- **f** To increase a number by 5%, multiply by ________.
- **g** To decrease a number by 25%, multiply by ________.
- **h** To decrease a number by 10%, multiply by ________.
- **i** To decrease a number by 5%, multiply by ________.

2 The price of a watch increases from $120 to $150. What is the price increase?

3 A person's weight decreases from 108 kg to 96 kg. What is the weight decrease?

4 **a** Write down 10% of 820 metres.
- **b** Increase 820 metres by 10%.
- **c** Decrease 820 metres by 10%.
- **d** Multiply 820 metres by 1.1. What do you notice?
- **e** Multiply 820 metres by 0.9. What do you notice?

Fluency

Example 6 Increasing by a percentage

Increase $70 by 15%.

Solution	**Explanation**
$100\% + 15\% = 115\%$ $= 1.15$ $\div 100$	First add 15% to 100%.
$70 × 1.15 = $80.50, by calculator	Multiply by 1.15 to give $70 plus the increase in one step.

5 Complete the following, using a calculator if necessary.
- **a** Increase 56 by 10%.
- **b** Increase 980 by 20%.
- **c** Increase 100 by 12%.
- **d** Increase 890 by 5%.
- **e** Increase 180 by 15%.
- **f** Increase 450 by 20%.
- **g** Increase 8 by 50%.
- **h** Increase 98 by 100%.
- **i** Increase 30 by 5%.

Example 7 Decreasing by a percentage

Decrease $5.20 by 40%.

Solution	**Explanation**
$100\% - 40\% = 60\%$ $= 0.6$ $\div 100$	First subtract the 40% from 100% to find the percentage remaining.
$5.20 × 0.6 = $3.12, by calculator	Multiply by 60% = 0.6 to get the result.

6 Complete the following, using a calculator if necessary.

a Decrease 80 by 5%. **b** Decrease 600 by 10%. **c** Decrease 45 by 50%.
d Decrease 700 by 12%. **e** Decrease 8000 by 8%. **f** Decrease 450 by 25%.
g Decrease 68 by 75%. **h** Decrease 9000 by 1%. **i** Decrease 7000 by 100%.

Example 8 Finding a percentage change

a The price of a mobile phone increases from \$250 to \$280. Find the percentage increase.

b The population of a town decreases from 3220 to 2985. Find the percentage decrease and round to one decimal place.

Solution	**Explanation**
a Increase = \$280 – \$250 = \$30	First find the actual increase.
Percentage increase = $\dfrac{30}{250} \times 100\%$ = 12%, by calculator	Divide the increase by the original amount and multiply by 100%.
b Decrease = 3220 – 2985 = 235	First find the actual decrease.
Percentage decrease = $\dfrac{235}{3220} \times 100\%$ = 7.3% (to 1 d.p.), by calculator	Divide the decrease by the original population and multiply by 100%. Round as indicated.

7 Copy and complete the tables, showing percentage change. Round to one decimal place where necessary.

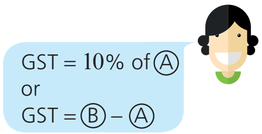
a

Original amount	New amount	Increase	Percentage increase
40	60		
12	16		
100	125		
24	30		
88	100		
48	92		
200	250		

b

Original amount	New amount	Decrease	Percentage decrease
90	81		
100	78		
20	15		
24	18		
150	50		
9	8.3		
3	2.5		

8 When the Goods and Services Tax (GST) was introduced, all prices increased by 10%.



Copy and complete the table.

	Old price (excluding GST)	New price (including GST)	GST amount
a	$40		
b	$120		
c		$55	
d		$154	
e			$7

2C

Problem-solving and Reasoning

9 The price of a computer is decreased by 15% in a sale. What is the sale price, if the original price was $\$2100$?

10 Plumbers on a salary of $\$82\,000$ are given a 3% pay increase. Find their new annual salary.

11 A car manufacturer intends to increase sales by 14.7% next year. If the company sold $21\,390$ new cars this year, how many does it expect to sell next year?

12 The length of a bike sprint race is increased from 800 m to 1200 m. Find the percentage increase.

$$\% \text{ Increase} = \frac{\text{increase}}{\text{original amount}} \times 100\%$$

Drilling for Gold 2C1

13 The number of people on a bus decreased from 25 to 18 after one stop. Find the percentage decrease in the number of people on the bus.

14 After a price increase of 20%, the cost of entry to a museum rose to $\$25.80$. Find the original price.

15 The total price of an item, including GST (at 10%), is $\$120$. How much GST is paid, to the nearest cent?

16 An investor starts with $\$1000$.

a After a bad day the initial investment is reduced by 10%. Find the balance at the end of the day.

b The next day is better and the balance is increased by 10%. Find the balance at the end of the second day.

c The initial amount decreased by 10% on the first day and increased by 10% on the second day. Explain why the balance on the second day didn't return to $\$1000$.

Enrichment: Repeated increase and decrease

Drilling for Gold 2C2 at the end of this section

17 If the cost of an $80 pair of shoes is increased twice, by 10% and then 15%, the final price will be:

$$\$80 \times 1.10 \times 1.15 = \$101.20$$

Use a similar technique to find the final price of these items. Round your answers to the nearest cent.

a skis starting at $450 and increasing by 20% and 10%

b a computer starting at $2750 and decreasing by 6% and then 11%

18 If an amount is increased by the same percentage each time, powers can be used.

For example, 50 kg increased by 12% three times would increase to:

$$\begin{aligned} &50\text{ kg} \times 1.12 \times 1.12 \times 1.12 \\ &= 50\text{ kg} \times (1.12)^3 \\ &= 70.25\text{ kg (to two decimal places)} \end{aligned}$$

You have a power/index key on your calculator.

Use a similar technique to find the final value in these situations. Round your answers to two decimal places.

a An $80 000 car decreases by 5% every year for 4 years.

b The mass of a 60 gram rat increases at a rate of 10% every month for 3 months.

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2C2: Repeated percentage change (with calculator)

Calculator shortcut for increasing by a percentage

Example: Increase \$120 by 15%.

The original amount was 100%, so the new amount will be 100% + 15% = 115%.

To convert to a decimal, 115 divided by 100 gives 1.15.

Calculation: $\$120 \times 1.15 = \$138$

Calculator shortcut for decreasing by a percentage

Example: Decrease \$120 by 15%.

The original amount was 100%, so the new amount will be 100% – 15% = 85%.

To convert to a decimal, 85 divided by 100 gives 0.85.

Calculation: $\$120 \times 0.85 = \$102$

Use the worksheet or write your working and answers in your exercise book.
The first one has been done as an example.

1 Increase \$120 by 15%, then increase the result by 15%,

$\$120 \times 1.15 \times 1.15 = \$158.70$

2 Decrease \$120 by 15%, then decrease the result by 15%,

3 Increase \$120 by 15%, then decrease the result by 15%,

4 Decrease \$120 by 15%, then increase the result by 15%,

5 Increase \$150 by 10%, then increase the result by 10%,

6 Decrease \$150 by 10%, then decrease the result by 10%,

7 Increase \$150 by 10%, then decrease the result by 10%,

8 Decrease \$150 by 10%, then increase the result by 10%.

Calculator shortcut for repeated increase

The population of a town is expected to increase by 12% every year.

The current population is 8000.

Try this on your calculator:

- Press 8000, then press =.
- Now press × 1.12.
- Now press = = = = = repeatedly and watch the population grow.

Calculator shortcut for repeated decrease

The population of a town is expected to decrease by 12% every year.

The current population is 8000.

Try this on your calculator:

- Press 8000, then press =.
- Now press × 0.88.
- Now press = = = = = repeatedly and watch the population fall.

In Questions **9** *to* **12***, start with a population of 8000 in each question.*

9 If the population increases by 12% every year, what will it be at the end of the tenth year?

10 If the population decreases by 12% every year, what will it be at the end of the tenth year?

11 If the population increases by 12% every year, how many years will it take for the population to double?

12 If the population decreases by 12% every year, how many years will it take for the population to be halved?

13 A computer is purchased for $2000 and loses 30% in value every year. How much is it worth by the end of the fifth year?

14 If I invest $1000 now and it increases by 6% every year, how much is it worth by the end of the 40th year?

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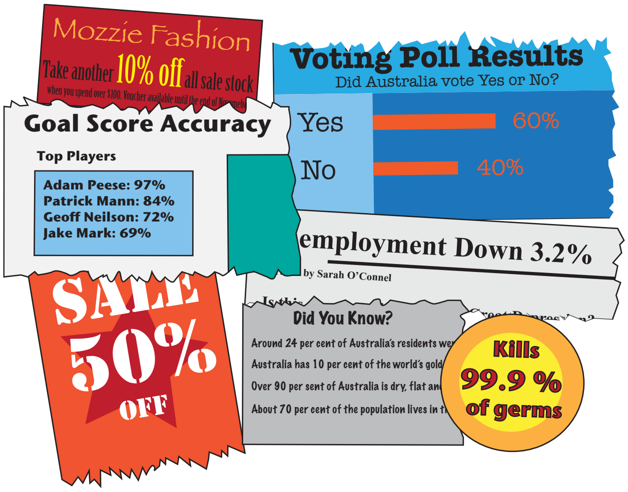
2D Profits and discounts

Stage
5.2
5.2◊
5.1
4

Sales, profits, losses, commissions, discounts and taxation are often expressed and calculated using percentages.



Examples of percentages used in the media

Let's start: The best discount

Two book shops are selling a $100 book at a discounted price.

- Shop A discounts the price by 25%.
- Shop B reduces the price by 15%, then reduces that price by 10%.

Which shop offers the bigger discount?

Key ideas

- **Profit** is the amount of money made on a sale.
  Profit = selling price – cost price
- A **loss** is made when the selling price is less than the cost price.
  Loss = cost price – selling price
- **Mark-up** is the amount added to the cost price to produce the selling price.
  Selling price = cost price + mark-up
- The **percentage profit** or **percentage loss** can be found by dividing the profit or loss by the cost price and multiplying by 100%.

$$\% \text{ Profit/Loss} = \frac{\text{profit/loss}}{\text{cost price}} \times 100\%$$

- **Discount** is the amount by which an item is marked down.
  Discount = % discount × original price
  New price = original price – discount

Profit The amount of money made by selling something for more than its cost

Loss The amount of money lost by selling something for less than its cost

Mark-up The amount added to the cost price to find the selling price (usually expressed as a percentage of the cost price). Selling price = cost price + mark-up.

Discount An amount subtracted from a price

Exercise 2D

Understanding

1 Copy and complete the table of profits and losses.

Cost price ($)	7	18	24.80	7.30	460.95	3250
Selling price ($)	10	15.50	26.20	11.80	395	4430
Profit/Loss ($)						

2 Copy and complete the table of mark-ups.

Cost price ($)	30.95	99.95	199.95			18 000
Mark-up ($)	10	80	395.95	700	16 700	
Selling price ($)				1499.95	35 499	26 995

3 Copy and complete the table of discounts.

Original price ($)	100	49.95	29.95			2215
New price ($)	72	40.90	22.70	176	299.95	
Discount ($)				23	45.55	178

4 The following percentage discounts are given on the price of various products. State the percentage of the original price that each product is selling for.

a 10% off **b** 20% off **c** 15% off **d** 8% off

Fluency

Example 9 Calculating selling price from mark-up

An electrical store marks up all entertainment systems by 30%.
If the cost price of one entertainment system is $8000, what will be its selling price?

Solution	**Explanation**
$100\% + 30\% = 130\%$ $= 1.3$ $\div 100$	Add 30% to 100%.
$8000 \times 1.3 = \$10\,400$	Multiply the cost price by 1.3.

5 Copy and complete this table by calculating the mark-up amount and selling price.

Item	**Cost price**	**Mark-up**	**Mark-up amount**	**Selling price**
Jeans	$60	28%		
Toaster	$40	80%		
Car	$22 000	45%		
Can of drink	$1.20	140%		
Loaf of bread	$1.80	85%		
Handbag	$80	70%		
Computer	$320	35%		

Example 10 Finding the discount amount

An electrical store advertises a 15% discount on all equipment. Find the sale price on a projection system that has a marked price of \$18 000.

Solution	**Explanation**
$100\% - 15\% = 85\%$ $= 0.85$ $\div 100$	The price is going down by 15%, so subtract 15% from 100%.
$\$18\,000 \times 0.85 = \$15\,300$	Multiply the original price by 0.85.

6 Copy and complete the table by writing in the missing values.

Item	Cost price	Discount	Amount of discount	Selling price
Camera	\$900	15%		
Car	\$24 000	20%		
Bike	\$600	25%		
Shoes	\$195	30%		
Blu-ray player	\$245	50%		
Electric razor	\$129	20%		
Lawnmower	\$880	5%		

Example 11 Determining profit

A manufacturer produces an item for \$400 and sells it for \$540.

a Determine the profit made.

b Express this profit as a percentage of the cost price.

Solution	**Explanation**
a Profit = \$540 – \$400 = \$140	Profit = selling price – cost price
b $\%\ \text{Profit} = \dfrac{140}{400} \times 100\%$ $= 35\%$	$\%\ \text{Profit} = \dfrac{\text{profit}}{\text{cost price}} \times 100\%$

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7 Find the missing values in these tables. Round to two decimal places where necessary.

a

Cost price (\$)	Selling price (\$)	Profit (\$)	Profit (%)
10	15		
24	30		
100	150		
250	255		
17.50	20		

b

Cost price (\$)	Selling price (\$)	Loss (\$)	Loss (%)
10	8		
16	12		
100	80		
34	19		
95	80.75		

Example 12 Calculating sale saving

A toy store discounts a toy by 10% in a sale. If the sale price was \$10.80, what was the original price?

Solution	Explanation
$$\begin{array}{rrcll} \div 90 & 90\% & = & \$10.80 & \div 90 \\ & 1\% & = & 0.12 & \\ \times 100 & 100\% & = & \$12 & \times 100 \end{array}$$	The discount factor = 100% – 10% = 90%. Thus, \$10.80 is 90% of the original price. Use the unitary method to find 1%. Multiply by 100 to find the original amount.
The original price was \$12.	Write the answer in words.
Alternative method: $100\% - 10\% = 90\% = 0.9$ $$\begin{array}{rrcll} \div 0.9 & 0.9 \times P & = & \$10.80 & \div 0.9 \\ & P & = & \$12 & \end{array}$$	The price given is 90% of the original price. Form an equation. Solve the equation by dividing both sides by 0.9.

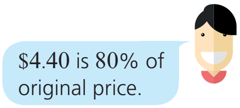
Drilling for Gold 2D1 at the end of this section

8 Find the original price (i.e. cost price) of these items:

a a coffee mug that was discounted by 20% and sold for \$4.40

b a pair of shoes that sold for \$250 after a mark-up of 25%

c a concert ticket that was marked up by 100% and sold for \$250



2D

Problem-solving and Reasoning

9 A manufacturer produces and sells items for the prices shown.

- **i** Determine the profit made.
- **ii** Express this profit as a percentage of the cost price.

a cost price $10, selling price $12
b cost price $20, selling price $25
c cost price $120, selling price $136.80
d cost price $1400, selling price $3850

10 It used to take 20 hours to fly from Sydney to Los Angeles. It now takes 12 hours. What is the percentage decrease in travel time?

11 Lenny marks up all computers in his store by 12.5%. If a computer cost him $890, what will be the selling price of the computer?

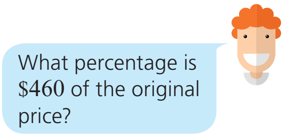
12 A used-car dealer purchases a vehicle for $13 000 and sells it for $18 500. Determine the percentage mark-up on the vehicle, to one decimal place.

13 A store marks up a $500 computer by 80%.

a What is the dollar amount of the mark-up?
b What is the retail price of the computer after the store's mark-up?
c The store offers the computer on sale for a discount of 15%. What is the price now?

14 A refrigerator is discounted by 25%. If Paula pays $460 for it, what was the original price? Round your answer to the nearest cent.

Skillsheet 2A



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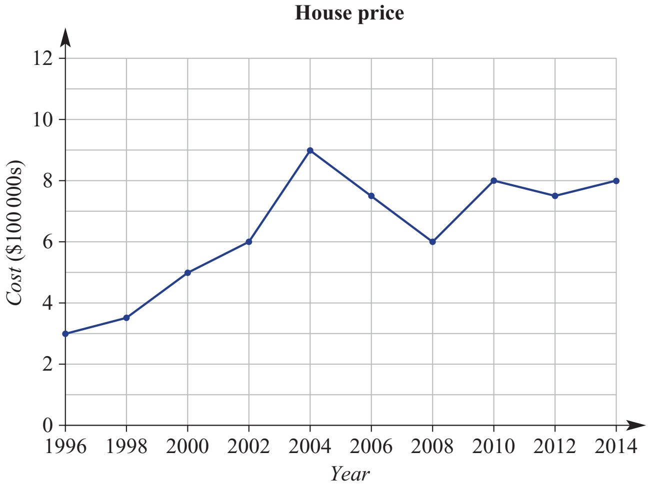
Enrichment: House value

15 The graph shows the changes in the value of a particular house in southern Sydney.



a How much was the house worth in 1998?

b Find the percentage increase, to one decimal place, in the value of the house from 1998 to:

i 2002 **ii** 2004 **iii** 2006

c The owners wish to sell in 2016 for 1 million dollars. What percentage increase is needed in the 2 years from 2014 to 2016 so the owners can sell for that price?

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Drilling for Gold exercise

2D1: Missing information

Find the missing piece of information in each of the following.
Use the worksheet or write your working and answers in your exercise book.

Guess the missing information in the following equations. Check your answers with a calculator	Use your calculator to find the missing information in the following equations
1 10% of ______ = 6	**13** 23.5% of $350 = ________
2 10% of ________ = $5	**14** 12% of _______ = 29.4 cm
3 _______ of $35 = $3.50	**15** ______ of $41 = $3.69
4 _______ of $35 = $1.75	**16** 15% of 538 = ______
5 5% of 20 = ________	**17** 22% of ______ = $136.40
6 _______ of 100 = 25	**18** _______ of 320 mL = 144 mL
7 20% of _______ = 5 mL	**19** 17% of $63 = _______
8 20% of _______ = 40	**20** 24% of _______ = 240
9 _______ of $120 = $60	**21** 16.5% of 4300 m = ______
10 25% of 120 g = ________	**22** _______ of $300 = $47.25
11 15% of _______ = 3 cm	**23** 10.25% of _______ = 51.25 m
12 _______ of 200 m = 30 m	**24** 12.5% of _______ = 8.65625

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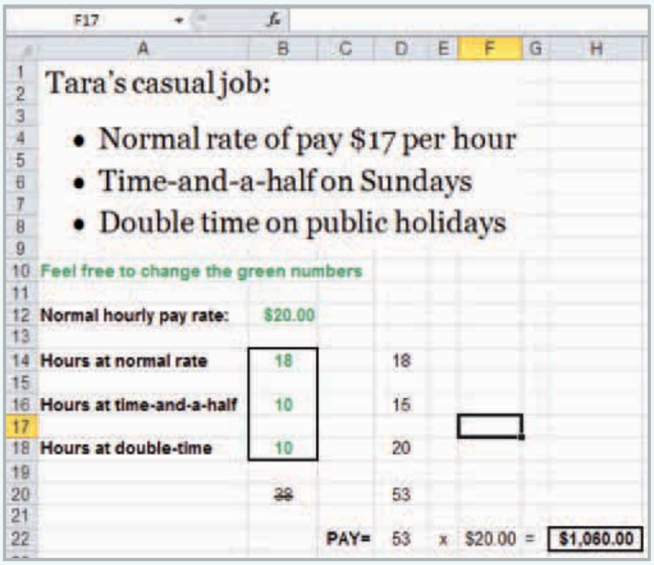
How much pay?

Maths@work

Video 2A

Watch the video to learn how to create a spreadsheet you could use in the future. It will check that your employer has paid you the correct amount of money.

When it is completed it will look like this:



Tara's casual job:

- Normal rate of pay $17 per hour
- Time-and-a-half on Sundays
- Double time on public holidays

Feel free to change the green numbers

Normal hourly pay rate:	$20.00		
Hours at normal rate	18	18	
Hours at time-and-a-half	10	15	
Hours at double-time	10	20	
	38	53	
	PAY=	53	x $20.00 = $1,060.00

Enter the values above into your spreadsheet to make sure it is working correctly.

When you are sure that your spreadsheet is working correctly, use it to complete the table below.

	Employee	Normal rate (per hour)	Hours worked			Pay
			Normal	Time-and-a-half	Double time	
1	Ann	$10	12	5	0	
2	Bailey	$12	12	5	4	
3	Con	$15	26	3.5	4.25	
4	Demi	$17.50	4	4	0	
5	Erik	$63.81	4	4	4	
6	Fiona		6	6	6	$1174.50
7	Gary	$40		8	8	$1200
8	Harry	$40	8		8	$1200
9	Ian	$40	8	8		$1200
Note: There are many possible answers for Question **10**. Write down three of them.						
10	Juan	$40				$800

2E Income

Stage
5.2
5.2◊
5.1
4

There are many different ways to earn money. Your income can be calculated and paid in different ways, such as a salary, a wage or a commission.

Let's start: Types of income

Drilling for Gold 2E1 at the end of this section

As a class, write down one example of a job that earns each of the following types of income.

- salary
- commission
- piecework
- wage (i.e. hourly rate of pay)
- royalties

In the newspaper classifieds or online, find an advertisement for each of the income types above.

Key ideas

- Workers who earn a **wage** are paid a fixed rate per hour. Extra hours are paid at a higher rate called **overtime**. This can be:
  - **time and a half**: Pay is 1.5 times the usual hourly rate.
  - **double time**: Pay is twice the usual hourly rate.
- Workers who earn a **salary** are paid a fixed amount per year, say, \$95 000. This is often paid monthly or fortnightly.
  - 12 months in a year and approximately 52 weeks in a year = 26 fortnights
- People who sell things may be paid a percentage of their sales. This is a **commission**. They may also be paid a set weekly amount called a **retainer**.

Wage Earnings paid to an employee based on an hourly rate

Overtime Hours worked extra to normal working hours

Salary A fixed agreed yearly amount that an employee earns

Income Money that 'comes in' (i.e. money you earn rather than money you spend). In most cases this will be the same as gross income.

Commission Earnings of a salesperson based on a percentage of the value of goods sold

Retainer A set amount a company pays to 'retain' (keep) the services of an employee who is paid on commission even if they make no sales or do no work during that time period

Exercise 2E

Understanding

1 Adam earns \$12.70 an hour. How much does he earn for:
  a 2 hours of work?
  b 8 hours of work?
  c 10 hours of work?
  d 38 hours of work?

2 Sela's hourly rate of pay is \$24. Calculate her overtime rate at:
  a time and a half
  b double time

3 William earns a salary of $136 875 each year. Approximately how much is this each:

- **a** month?
- **b** week (to the nearest cent)?
- **c** day?

1 year is:
- 12 months
- 52 week (approx.)
- 365 days.

4 Find the commission earned on the given sales figures if the percentage commission is 20%.

- **a** $1000
- **b** $280
- **c** $4500
- **d** $725.50

Find 20% of the amounts.

Fluency

Example 13 Comparing wages and salaries

Ken earns an annual salary of $59 735 and works a 38-hour week. His wife Brooke works part time and earns $21.80 per hour.

- **a** How much does Ken earn per week?
- **b** Who has the higher hourly rate of pay?
- **c** If Brooke works 18 hours per week, what is her yearly income?

	Solution	Explanation
a	Weekly rate = $59 735 ÷ 52	$59 735 pay in a year.
	= $1148.75	There are approximately 52 weeks in a year.
	∴ Ken earns $1148.75 per week.	Divide by 52 to find the weekly wage.
b	Brooke: $21.80/h	
	Ken: $1148.75 ÷ 38	Ken works 38 hours in a week.
	= $30.23/h	Hourly rate = weekly rate ÷ number of hours
		Round to the nearest cent.
	∴ Ken is paid more per hour.	Compare hourly rates.
c	In 1 week: $21.80 × 18	Weekly income = hourly rate × number of hours
	= $392.40	
	Yearly income = $392.4 × 52	Multiply by 52 weeks to get yearly income.
	= $20 404.80	

5 Talib earns $16 697.20 per year.

- **a** How much does he earn each week?
- **b** Each week he works 38 hours. Calculate his hourly rate of pay.
- **c** Find his fortnightly pay.
- **d** Find his monthly pay.

A fortnight is 2 weeks.

6 Caitlyn earns $790 each week. How much does she earn each:

- **a** year?
- **b** month?
- **c** hour, if she works 40 hours each week?

7 Copy and complete the table.

Employee	Hourly rate	Hours worked	Income
Rhys	\$10.40	8	
Sofia	\$15.50	$8\frac{1}{2}$	
Ceanna	\$9.70	15	
David	\$24.30	38	
Edward	\$17.85	42	
Francis	\$30	27	
Marius	\$15.20	7.25	

Example 14 Calculating overtime

Georgio works some weekends and late nights. His hourly rate of pay is \$16.

a Calculate Georgio's time-and-a-half rate of pay per hour.
b Calculate Georgio's double-time rate of pay per hour.
c Calculate Georgio's weekly wage for a week when he works 18 hours at his normal rate, 2 hours at time and a half, and 1 hour at double time.

Solution	**Explanation**
a Time and a half = \$16 × 1.5 = \$24	Time and a half is 1.5 times the hourly rate.
b Double time = \$16 × 2 = \$32	Double time is 2 times the hourly rate.
c	Find the sum of the:
\$16 × 18 = \$288	• normal hourly rate (\$16) multiplied by the number of hours worked at the normal rate (18 hours)
\$24 × 2 = \$48	• time-and-a-half hourly rate (\$24) multiplied by the number of hours worked at that rate (2 hours)
\$32 × 1 = \$32 Total wage = \$368	• double-time hourly rate (\$32) multiplied by the number of hours worked at that rate (1 hour)
Alternatively, 18 hours + (2 hours × 1.5) + (1 hour × 2) = 18 + 3 + 2 = 23	Calculate the number of 'normal' hours.
Wage = \$16 × 23 = \$368	Multiply by the hourly rate.

8 A job has a normal hourly rate of $18.40 per hour. Calculate the pay, including overtime, when the following hours are worked.

For time and a half: ×1.5
For double time: ×2

- **a** 3 hours at the normal rate and 4 hours at time and a half
- **b** 4 hours at the normal rate and 6 hours at time and a half
- **c** 14 hours at the normal rate and 3 hours at double time
- **d** 20 hours at the normal rate and 5 hours at double time
- **e** 10 hours at the normal rate, 8 hours at time and a half and 3 hours at double time
- **f** 34 hours at the normal rate, 4 hours at time and a half and 2 hours at double time

Example 15 Calculating commission

A sales representative is paid a retainer of $1500 per month. She also receives a commission of 1.25% on the value of goods she sells. If she sells goods worth $5600 during the month, calculate her earnings for that month.

Solution	**Explanation**
Commission = 1.25% of $5600	Calculate the commission on sales.
= 0.0125 × $5600 = $70	Change the percentage to a decimal and evaluate.
Earnings = $1500 + $70 = $1570	Earnings = retainer + commission

9 Copy and complete the table.

Employee	Weekly retainer	Rate	Sales	Commission earned	Weekly wage
Adina	$0	12%	$7000		
Byron	$160	8%	$600		
Cindy	$300	5%	$680		
Deanne	$260	5%	$40 000		
Elizabeth	$500	8%	$5600		
Faruq	$900	2%	$110 000		
Leo	$1000	1.5%	$45 000		

Problem-solving and Reasoning

10 Jim, a part-time gardener, earned $261 in a week. If he worked 12 hours during normal working hours and 4 hours overtime at time and a half, what was his hourly rate of pay?

4 hours time and a half is equivalent to 4 × 1.5 = 6 hours at normal hourly rate.

11 Ellen earned $329.40 in a week. She worked 10 hours during the week, 6 hours on Saturday at time and a half and 4 hours on Sunday at double time. What is her hourly rate of pay?

12 During a particular week, Amy made $800 in book sales. If she earns $450 per week plus 5% commission, how much did she earn in that week?

13 Jason sold $84 000 worth of campervans in a month. If he earns $650 per month plus 4% commission on sales, how much did he earn that month?

14 Stephen earns an hourly rate of $18.00 for the first 38 hours, time and a half for the next 3 hours and double time for each extra hour above that. Calculate his earnings if he works 44 hours in a week.

15 Calculate how many hours at the standard hourly rate the following working hours are the same as:

- **a** 3 hours and 2 hours at double time
- **b** 6 hours and 8 hours at time and a half
- **c** 15 hours and 12 hours at time and a half

Enrichment: Pay slips

16 Employees at a fast-food restaurant are paid $7.21 per hour for working Monday to Friday up until 7 p.m. and time and a half after 7 p.m. They earn time and a half on Saturdays and double time on Sundays. They are given an unpaid 30-minute meal break for any shift over 5 hours.

Donna's shifts for the week are given below.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
–	4:30 p.m.–7 p.m.	4:30 p.m.–7:30 p.m.	5 p.m.–10 p.m.	5 p.m.–8 p.m.	10 a.m.–6:30 p.m.	10 a.m.–1:30 p.m.

- **a** How much did Donna earn, to the nearest cent?
- **b** The restaurant decides on a new workplace agreement; it gets rid of all overtime and creates a flat hourly rate of $8.15 per hour. How much worse off will Donna be for the week above?
- **c** How many extra hours a week does Donna need to work to make up the extra income? Answer in a whole number of hours.

2E1: Forms of income

Drilling for Gold exercise

People earn income in a variety of ways.
In this activity you will investigate:

- various ways to earn money
- the systems used to calculate income for workers.

Here are some occupations to consider for the following task.

1 real estate agent
2 casual relief teacher (employed to replace an absent teacher)
3 songwriter/author/artist
4 police officer/ambulance officer/customs officer/prison officer
5 car salesperson
6 shop assistant
7 actor/model/musician
8 soldier/sailor
9 registered nurse
10 waiter/bar attendant
11 tradesperson (builder, carpenter, electrician, plumber, mechanic, etc.)
12 flight attendant
13 bus driver
14 truck driver
15 restaurant/hotel owner
16 baker/chef/cook
17 professional sportsperson
18 doctor/dentist/physiotherapist/veterinarian
19 personal trainer/fitness instructor
20 fruit and vegetable picker
21 furniture removalist
22 hairdresser
23 cleaner
24 receptionist/secretary/personal assistant
25 farm manager
26 tutor/instructor (music, maths, skiing, etc.)
27 park ranger
28 miner
29 jackaroo/jillaroo
30 farmer

You may use the internet to assist with answering the questions on the next page.
For best results, use Australian websites.

Drilling for Gold exercise

Use the worksheet or write your working and answers in your exercise book.

1 Some workers are paid a wage or a salary. What does this mean? Explain the difference between a wage and a salary.

2 Some workers do piecework or are employed as casuals. What does this mean? Explain the difference between piecework and casual work.

3 Some workers are paid by commission. Some of them also receive a retainer. What does this mean? Explain the difference between a retainer and a commission.

4 Some people are paid by consignment or royalties. What does this mean? Explain the difference between consignment and royalties.

5 Some people are self-employed and work as a sole trader, small business owner, entrepreneur, freelance consultant, property developer or investor. Explain the differences.

6 Some people, such as the long-term unemployed, receive social service benefits and allowances from the federal government.

7 Using the list on the previous page, list the numbers of all the occupations in which workers are usually paid:

- Wage or salary:
- Casual work/piecework:
- Commission, retainer:
- Consignment, royalties:
- A share of the profits made by the company:

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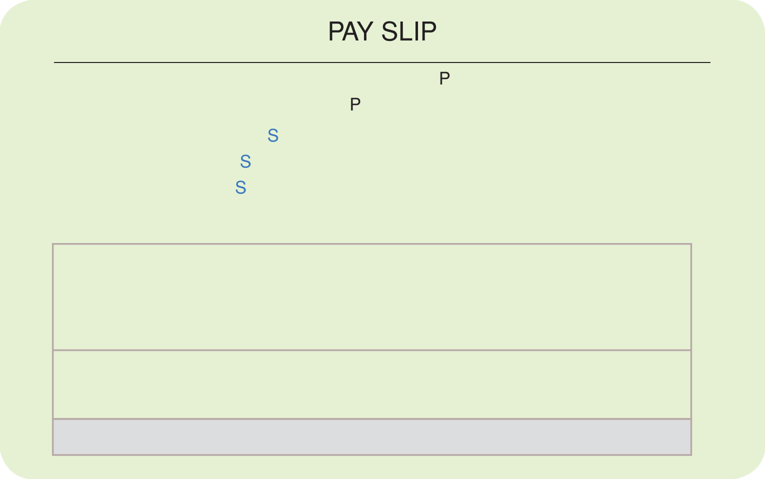
2F Taxation

Stage
5.2
5.2◊
5.1
4

All wage and salary earners have some deductions taken out of their pay. Deductions usually include income tax. Tax is paid to the government. The government uses it to pay for roads, schools, hospitals, universities and police.



PAY SLIP

P
P
S
S
S

Let's start: Deductions from pay

What types of deductions can you and your classmates think of that might be taken out of someone's pay?

Research the meaning of superannuation and the Medicare levy.

Key ideas

- **Gross income** = the total of all income
- **Net income** = gross income minus deductions
- **Taxable income** = gross income minus worked-related expenses and donations to charity
- **Income tax** is paid to the government. It is based on a person's taxable income.
- **Income tax** is calculated according to the current tax table, which is available through the Australian Tax Office.

These are the tax rates for 2015–2016, as obtained from the ATO website. Answers to questions on tax in this section are based on this table, but actual rates may change each year.

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001–$180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

Gross income Total income before any deductions are made

Net income Income that remains after deductions have been made

Taxable income Gross income less certain items allowed by the tax office

Income tax An amount paid to the government by people earning an income

Tax bracket Each row in the table is called a tax bracket

Exercise 2F

Understanding

1 Copy and complete the table by inserting the net income amounts.

Gross income –	Deductions =	Net income
$5600	$450	$5150
$87 000	$28 000	
$50 000	$6700	
$890	$76	
$84 650	$24 790	

2 Find Hoang's gross weekly income if he earns:
- $1200 a week as a teacher
- $60 an hour for tutoring, when he tutors for 3 hours a week
- $25 interest on his bank account per week.

3 John has a gross income of $45 000 and a net income of $20 000. How much did his deductions come to?

4 Pamela pays 31.5% of her gross income in tax. How much tax does she pay on a gross income of $85 790?

Fluency

Example 16 Calculating tax to find net income

Liam has an annual salary of $52 800. He pays $968 in tax monthly.

a Calculate Liam's net income each month.

b What percentage of Liam's monthly pay is being paid to the government by his employer for taxes?

c If the taxation rate for Liam's salary changes to 19% with the first $18 200 tax free, calculate Liam's net income for the year.

Solution	Explanation
a Monthly pay = $52 800 ÷ 12 = $4400	Calculate gross income per month.
∴ Net monthly income = $4400 – $968 = $3432	Net income = gross income – taxation
b % Tax = $\frac{968}{4400} \times 100\%$ = 22%	Calculate what fraction $968 is of the monthly income $4400. Multiply by 100% to convert to a percentage.

c Salary for tax purposes = $52 800 – $18 200 = $34 600	The first $18 200 is not taxed.
Tax amount = 19% of $34 600 = 0.19 × $34 600 = $6574, by calculator	Calculate tax amount on $34 600. Convert percentage to a decimal and evaluate.
∴ Net income = $52 800 – $6574 = $46 226	Net income = gross income – tax amount

5 For each of the following, find the:
- **i** annual net income
- **ii** percentage of gross income paid as tax. Round to one decimal place where necessary.

a gross annual income = $48 241, tax withdrawn = $8206
b gross annual income = $67 487, tax withdrawn = $13 581.20
c gross monthly income = $4041, tax withdrawn = $606.15
d gross monthly income = $3219, tax withdrawn = $714.62

6 Narendra earns $1400 per week and pays 27% of his annual income in tax.
a Calculate the amount of income tax that Narendra pays in one year.
b Find Narendra's annual net income.

Example 17 Using the tax table

Taxable income	Tax on this income
0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001–$180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

Use the tax table above to find the income tax for an income of $48 000.

Solution	Explanation
Tax = $3572 + 0.325 × ($48 000 – $37 000) = $7147	Find the tax bracket in which the amount ($48 000) lies ($37 001–$80 000). Write down the values in this bracket, remembering that 32.5c in the dollar is 32.5% or 0.325. Subtract 37 000 from 48 000 to find the amount of income that the 32.5% applies to. Use your calculator to find the answer.

7 Use the tax table in Example 17 to find the income tax payable on each of these incomes.

a $6000	**b** $10 000	**c** $30 000	**d** $50 000
e $80 000	**f** $129 000	**g** $156 000	**h** $200 000

Find the right tax bracket first.

Skillsheet 2B

Problem-solving and Reasoning

8 Mel has a net annual income of $53 246 after 21% of her income is withdrawn for tax purposes. What is her gross income?

What percentage does $53 246 represent to the gross income?

9 Carlos earns $50 000 each year as a shop clerk and an extra $1200 each year for doing gardening on the weekends.

a Calculate his annual gross salary.

b Use his gross salary to find his income tax, using the tax table in Example 17.

c What is his annual net income, after tax?

d What is his approximate fortnightly net income, to the nearest cent, after tax?

10 Fatima earns $450 a week. Using the tax table in Example 17, determine how much tax is deducted from her pay each week.

Use Fatima's annual salary to find the annual tax amount first.

11 Ayden pays $5000 in tax a year.

a Which tax bracket does Ayden fall into?

b Work backwards from this amount to work out Ayden's taxable income. Answer to the nearest dollar.

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Enrichment: Other types of deductions and taxable income

People on low incomes pay less tax. Therefore, people try to lower their taxable incomes by claiming allowable tax deductions.

Work-related expenses such as uniforms, stationery and job-related travel expenses are all examples of allowable tax deductions. Note that they are not the same as the deductions mentioned earlier in the chapters (amounts deducted from pay).

The income tax is therefore calculated on a person's taxable income.

Taxable income = gross income – allowable tax deductions

Use the tax table from Example 17 to compare the following jobs.

12 An accountant earns $3120 a fortnight, with no allowable work-related expenses.
- **a** What is the accountant's taxable income?
- **b** How much tax does the accountant owe in a year?
- **c** What is the accountant's annual take-home pay?

13 A solicitor earns $3120 a fortnight with allowable work-related expenses totalling $2560 a year.
- **a** What is the solicitor's taxable income?
- **b** How much tax does the solicitor owe in a year?

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2G Simple interest

Stage
5.2
5.2◊
5.1
4

Interest is charged when a person borrows money. The interest is an extra amount that must be paid back, on top of the borrowed amount.

Interest is earned when a person or institution invests money. Simple interest is usually charged or earned each year.

Let's start: Developing the rule

\$5000 is invested with a bank and 5% simple interest is paid to the investor every year. In the table on the right, the amount of interest paid is shown for year 1, and the amount of total interest is shown for years 1 and 2.

- Copy and complete the table.
- How much interest would the investor earn in 10 years' time?

Year	Interest paid that year	Total interest
0	\$0	\$0
1	$\frac{5}{100} \times \$5000 = \$250$	$1 \times \$250 = \$250$
2		$2 \times \$250 = \$500$
3		
4		
5		

Key ideas

- To compute **simple interest**, we apply the formula:

  $I = PRN$

  I is the amount of simple interest (in \$).
  P is the **principal** amount; the money borrowed or invested (in \$).
  R is the interest rate per time period (expressed as a fraction or decimal).
  N is the number of time periods.

- The total amount (\$$A$) equals the principal plus interest.

  $A = P + I$

- p.a. means 'per annum' or 'per year'.

Simple interest rate The percentage rate per year that is paid on a loan or investment amount

Principal An amount of money invested or borrowed

Per annum (p.a.) Per year

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Exercise 2G

Understanding

1 Complete these conversions.
- **a** 1 year = ☐ months
- **b** 36 months = ☐ years
- **c** 18 months = ☐ years
- **d** 52 months = ☐ years

2 \$12 000 is invested at 6% p.a. for 42 months.
- **a** What is the principal amount?
- **b** What is the interest rate?
- **c** What is the time period in years?
- **d** How much interest is earned each year?

3 Use the rule $I = PRN$ to find the simple interest (I) earned in these financial situations.
- **a** $P = \$10\,000,\ R = 10\%,\ N = 3$
- **b** $P = \$6000,\ R = 12\%,\ N = 5$
- **c** $P = \$5200,\ R = 4\%,\ N = 2$
- **d** $P = \$3500,\ R = 6\%,\ N = 1.5$

I = interest
P = principal
R = interest rate, expressed as a decimal
N = time (years)

4 Joanna will earn \$560 p.a. in simple interest from an investment. How much will she earn from the investment in:
- **a** 2 years' time?
- **b** 5 years' time?
- **c** 10 years' time?

Fluency

Example 18 Using the simple interest formula

Calculate the simple interest earned if the principal is \$1000, the rate is 5% p.a. and the time is 3 years.

Solution	Explanation
$P = 1000,\ R = 5\%,\ N = 3$	List the information given.
$I = PRN$	Write the formula. Note: $R = 5\% = 0.05$ as a decimal.
$= 1000 \times 0.05 \times 3$	Substitute the given values and evaluate.
$= 150$	
Interest = \$150	

5 Find the simple interest earned when:
- **a** \$5000 is invested at 6% p.a. for 1 year
- **b** \$5000 is invested at 6% p.a. for 3 years
- **c** \$8000 is invested at 4% p.a. for 5 years
- **d** \$15 000 is invested at 3% p.a. for 7 years
- **e** \$7250 is invested at 5.5% p.a. for 3 years

Remember to write the rate as a decimal.

6 Anwar invests \$15 000 at a rate of 6% p.a. for 3 years. Calculate the simple interest and the amount available at the end of 3 years.

Amount = principal + interest

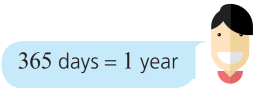
Example 19 Using other time periods

Calculate the simple interest earned when \$7000 is invested at 6.25% p.a. for 18 months.

Solution	**Explanation**
$P = 7000,\ R = 6.25\% = 0.0625$	List the information given.
$N = 18 \text{ months} = \frac{18}{12}$ or 1.5 years	Convert 18 months to years by dividing by 12.
$I = PRN$ $I = 7000 \times 0.0625 \times 1.5$ $= 656.25$ Interest = \$656.25	Write the formula. Substitute in the values and evaluate.

7 Calculate the simple interest earned when:

- **a** \$500 is invested at 7% p.a. for 18 months
- **b** \$1000 is invested at 5% p.a. for 24 months
- **c** \$2000 is invested at 4% p.a. for 6 months
- **d** \$4700 is invested at 4.5% p.a. for 15 months (Round to the nearest cent.)
- **e** \$50 000 is invested at 3.75% p.a. for 200 days (Round to the nearest cent.)



Example 20 Calculating the final balance

Allan and Rachel plan to invest some money for their child Kaylan. They invest \$4000 for 30 months with a bank that pays 4.5% p.a. Calculate the simple interest earned and the amount available at the end of the 30 months.

Solution	**Explanation**
$P = 4000,\ R = 4.5\% = 0.045,\ N = 2.5$	N is written in years since interest rate is per annum. $30 \div 12 = 2.5$ years
$I = PRN$ $= 4000 \times 0.045 \times 2.5$ $= 450$	Write the formula. Substitute and evaluate.
Interest = \$450	
Total amount = \$4000 + \$450 = \$4450	Total amount = principal + interest

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8 Collette invests $22 000 at a rate of 4% p.a. for 27 months. Calculate the simple interest and the amount available at the end of 27 months.

9 Copy and complete the table.

	Principal, P ($)	Annual interest rate, R	Time period, N	Interest, I	Final balance, $A = P + I$
a	7000	3%	4 years		
b	1500	7%	8 years		
c	40 000	2.5%	18 months		
d	70 000	3.25%	2 years		
e	2000	4%	30 months		

10 A finance company charges 14% p.a. simple interest. If Lynne borrows $2000 to be repaid over 2 years, calculate her total repayment.

Problem-solving and Reasoning

11 Marcus borrows $20 000 to buy a car. He is charged simple interest at 18% p.a. for a period of 5 years. What is the total amount that Marcus will have paid at the end of the loan period?

Marcus has to pay back the amount borrowed plus the interest.

12 Wendy wins $5000 during a chess tournament. She wishes to invest her winnings, and has the two choices given below. Which one gives her the greater total at the end of the time?
Choice 1: 8.5% p.a. simple interest for 4 years
Choice 2: 8% p.a. simple interest for 54 months

Skillsheet 2C

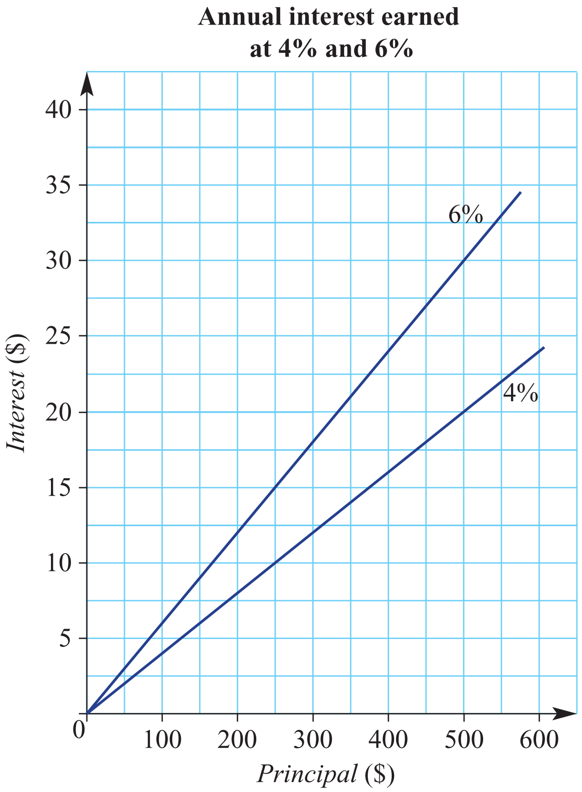
2G

Enrichment: Simple interest graphs

13 The graph below shows the annual interest earned on investments earning interests of 4% p.a. and 6% p.a. Use the graph to answer the following.

a Find the annual interest earned on an investment of:
  - **i** \$300 at 4% p.a.
  - **ii** \$520 at 6% p.a.
  - **iii** \$250 at 4% p.a.

b What investments would earn annual interest of (to the nearest \$5):
  - **i** \$20 at 6% p.a.?
  - **ii** \$20 at 4% p.a.?
  - **iii** \$14 at 6% p.a.?



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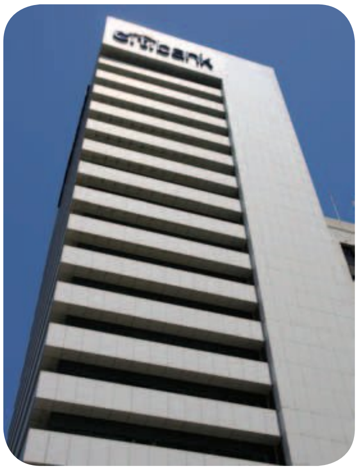
2H Applications of simple interest

Stage
5.2
5.2◊
5.1
4

People use financial calculations to make decisions about investments and loans.



▶ Let's start: Where do I invest?

Bank A: \$4000 invested at 5% p.a. for 8 years
Bank B: \$5000 invested at 8% p.a. for 4 years
Bank C: \$8000 invested at 4% p.a. for 5 years

Calculate the simple interest earned on each investment option. What do you notice?

Which bank would you choose and why?

Key ideas

- The simple interest formula, $I = PRN$, contains four variables:
  I is amount of interest.
  P is principal (i.e. amount invested).
  R is interest rate, expressed as a decimal.
  N is number of time periods.
  If values are given for three of these, then they can be used to find the fourth.
- To repay a loan, you must repay the amount borrowed (the principal) and the interest.
- **Repayments** are the amount of money, usually the same amount each time period, required to repay a loan.

Repayment An amount paid to a bank or financial institution at regular intervals to repay a loan with interest included

Exercise 2H

Understanding

1 If Phil earns \$10 in simple interest in 1 year, how long would it take him to earn:
a \$20? **b** \$40? **c** \$500? **d** \$25?

2 How many years does it take for \$100 to earn \$50 in interest if the simple interest rate is 10% p.a.?

3 A loan of \$4000 has interest of \$500 added to it. Calculate the size of each of the 24 repayments needed to repay the loan.

4 How much money do you need to invest to earn \$100 in interest in 1 year if the simple interest rate is 5%?

Require 5% of amount to be \$100.

Fluency

Example 21 Determining the investment period

Remy invests $2500 at 8% p.a. simple interest, for a period of time, to produce $50 interest. For how long did she invest the money?

Solution	**Explanation**
$I = 50,\ P = 2500,\ R = 8\% = 0.08$	List the information given.
$I = PRN$	Write the formula.
$50 = 2500 \times 0.08 \times N$ $50 = 200 \times N$	Substitute the known information and simplify.
$\div 200 \quad 200 \times N = 50 \quad \div 200$ $N = 0.25$	Solve the remaining equation for N by dividing both sides by 200.
Time = 0.25 years = 0.25 × 12 months = 3 months	Convert decimal time to months where appropriate.

5 Alvi invests $5000 at 8% p.a. simple interest and wants to earn $1200 in interest. For how many years should Alvi invest his money?

6 Sam earns $288 interest on his $1600 investment. If the interest was calculated at 4% p.a., for how many years did Sam invest the money?

7 $8000 earns $600 interest at 5% p.a. over how many months?

Example 22 Determining the interest rate

Bank East advertises $450 interest a year on an investment of $7500. Calculate the simple interest rate for this investment.

Solution	**Explanation**
$I = 450,\ P = 7500,\ N = 1,\ R = ?$	List the information given.
$I = PRN$	Write the formula.
$450 = 7500 \times R \times 1$	Substitute the known values and evaluate R.
$\div 7500 \quad 7500 \times R = 450 \quad \div 7500$ $R = 0.06$	
$R = 6\%$ Interest rate is 6% p.a.	Write R as a percentage by multiplying by 100%.

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8 Find the annual simple interest rate needed for each of the following situations.

a \$4000 earns \$500 in 2 years
b \$500 earns \$120 in 12 years
c \$18 000 earns \$3510 in 3 years
d \$950 earns \$470.25 in 9 years

Example 23 Calculating repayments

A bank offers you a loan of \$24 000 at 16% p.a. simple interest, requiring that the loan be repaid in equal monthly instalments over 5 years.

a How much interest is charged on the loan?
b What is the total amount of the loan and the interest?
c Calculate the size of each repayment.

Solution	Explanation
a $I = PRN$ $P = 24\,000,\ R = 0.16,\ N = 5$ $\therefore\ I = 24\,000 \times 0.16 \times 5$ $= \$19\,200$	Write down the simple interest formula and list the information. Substitute the known values and evaluate.
b Total $= \$24\,000 + \$19\,200$ $= \$43\,200$	Total = principal + interest
c Repayments $= \dfrac{43\,200}{60}$ $= \$720$ Repayments come to \$720 a month.	Divide the total amount by the number of months ($5 \times 12 = 60$).

9 Copy and complete this table and find the monthly repayment for each loan.

Amount borrowed	Annual simple interest rate	Number of years	Interest	Total amount to be repaid	Monthly repayment
\$5000	21%	5			
\$14 000	15%	5			
\$10 000	6%	4			
\$55 000	8%	10			
\$250 000	7%	30			

Problem-solving and Reasoning

10 Calculate the principal amount that earns \$500 simple interest over 3 years at a rate of 8% p.a. Round your answer to the nearest cent.

11 Charlotte borrows \$9000 to buy a car. The loan must be repaid over 5 years at 12% p.a. simple interest. Calculate the:

a total amount to be repaid
b monthly repayment amount if the repayments are spread equally over the 5 years

12 If \$5000 grows to \$11 000 over 12 years, find the simple interest rate.

13 An investor invests $\$P$ and wants to double this amount of money.

- **a** How much interest must be earned to double this initial amount?
- **b** What simple interest rate is required to double the initial amount in 8 years?
- **c** If the simple interest rate is 5% p.a.:
  - **i** How many years will it take to double the investment?
  - **ii** How many years will it take to triple the investment amount?
  - **iii** How do the investment periods in parts **i** and **ii** compare?

Use the simple interest formula with $\$P$ and the interest of $\$I$.

Enrichment: Compound interest

With simple interest, the principal and the interest earned each year remain the same for the period of the investment.

However, with compound interest, each time the interest is calculated it is added to the principal to give a new value. This means that the next time the interest is calculated on a larger amount.

In the following questions you will be asked to do repeated applications of simple interest to find the final compounded amount.

In compound interest, the balance grows faster as time passes.

14 \$500 is invested for 4 years at 10% p.a. interest, compounded annually.

- **a** Copy and complete the following table to find the final value of the investment at the end of this time and the total.

Time (years)	Amount (A)	Interest (I)	New amount ($A + I$)
1	500	$500 \times 0.1 = 50$	$500 + 50 = 550$
2	550	$550 \times 0.1 =$	
3			
4			

- **b** How much interest did the investment earn over the 4 years?

15 a Copy and complete the following table to find the final value of an investment of \$4500 compounded at 5% p.a. annually for 5 years.

Time (years)	Amount (A)	Interest (I)	New amount ($A + I$)
1	4500	225	4725
2	4725		
3			
4			
5			

- **b** Type 4500×1.05^5 into your calculator. What do you notice about this answer?
- **c** Can you explain how the answers to part **a** and part **b** relate?

Investigation simple interest, compound interest and superannuation

Watch the videos:

- *How to Make a Simple Interest Spreadsheet and Graph*
- *How to Make a Compound Interest Spreadsheet and Graph*
- *How to Make a Superannuation Spreadsheet.*

Create the spreadsheets and save them.
Use them to answer the following questions.

1 If you invest \$100 now at 5% per annum and it compounds annually, how much will your investment be worth when you retire from work at the age of 70?

2 What is the result if the interest rate in Question **1** is:
- **a** 2%?
- **b** 8%?
- **c** 12%?

3 How much interest is earned when \$1000 is invested for 10 years at 6% per annum:
- **a** if simple interest is used?
- **b** if the interest compounds every year?

4 If you invest \$100, compounding annually, find how many years it takes for your investment to double in value when the interest rate is:
- **a** 2%
- **b** 3%
- **c** 4%
- **d** 6%

5 In Question **4a–d**, if you multiply the interest rate by the number of years, what do you notice?

Video 2B

Video 2C

Video 2D

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Puzzles and games

1 The answers to the clues are hidden in the wordfind. Can you find all 16 words?

S	A	G	R	O	S	S	W	H	K	L	O	M
A	P	E	R	C	E	N	T	A	G	E	C	O
L	A	R	C	O	M	M	I	S	S	I	O	N
A	S	E	R	V	I	N	T	E	R	E	S	T
R	T	Y	H	E	T	S	L	O	S	S	T	H
Y	E	F	O	R	T	N	I	G	H	T	I	L
W	S	M	O	T	A	X	A	T	I	O	N	Y
E	I	O	L	I	D	I	D	N	P	Y	I	E
K	M	N	Q	M	O	N	T	R	N	I	S	C
E	P	Y	U	E	D	I	S	C	O	U	N	T
M	L	R	E	P	A	Y	M	E	N	T	A	H
D	E	D	U	C	T	I	O	N	S	I	X	L

a A fixed annual income

b A percentage of the value of goods sold, which you earn as an income

c Working longer than normal working hours

d Money from your income given to the government

e 2 weeks

f The total of all income

g Money taken from total pay

h Yearly

i 12 times a year

j Flat-rate interest

k Meaning 'out of 100'

l Money given to repay a loan

m Money earned on an investment

n An item offered for a sale price has had this happen

o The original price of an item

p You incur this when you sell an item for less than you paid for it

2 Find out the four classical elements of the world by calculating the simple interest. Match the letter beside each question to its corresponding answer in each grid.

E = \$600 at 6% p.a. for 1 year

H = \$796 at 5% p.a. for 4 months

R = \$12 500 at $6\frac{1}{4}\%$ p.a. for 2 years

A = \$7000 at 5% p.a. for 3 years

I = \$1000 at 1% p.a. for 100 years

F = \$576.50 at 19% p.a. for 18 months

W = \$36 000 at 2% p.a. for 5 years

T = \$550 at 10% p.a. for 6 months

\$36	\$1050	\$1562.50	\$27.50	\$13.27

\$1050	\$1000	\$1562.50

\$164.30	\$1000	\$1562.50	\$36

\$3600	\$1050	\$27.50	\$36	\$1562.50

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Chapter summary

Drilling for Gold 2R1

Percentages and decimals

$$\begin{tikzcd}\text{Percentage} \arrow[d, bend left, "\div 100\%"] \\ \text{Decimal} \arrow[u, bend left, "\times 100\%"]\end{tikzcd}$$

49% = 0.49

Percentage composition

3 g out of 50 g

$= \frac{3}{50} \times 100\%$

= 6%

Percentages and fractions

$$\begin{tikzcd}\text{Percentage} \arrow[r, bend left, "\div 100\%"] & \text{Fraction} \arrow[l, bend left, "\times 100\%"]\end{tikzcd}$$

$19\% = \frac{19}{100}$

Percentage

Means 'out of 100'

$7\% = \frac{7}{100} = 0.07$

Finding a percentage

$17\% \text{ of } 30 = \frac{17}{100} \times 30$

$= 0.17 \times 30$

$= 5.1$

Percentage increase and decrease

Increase	*Decrease*
20 by 6%	20 by 5%
20 × 1.06 = 21.2	20 × 0.95 = 19

$\text{Percentage change} = \frac{\text{change}}{\text{original}} \times 100\%$

Financial mathematics

Applications of percentages

- Percentage profit/loss

  $= \frac{\text{Profit/loss}}{\text{Cost price}} \times 100\%$
- Mark-up and discount
- Commission/tax

Income and tax

Employees can be paid:

Wage: hourly rate with
overtime at time and
a half = 1.5 × hourly rate
double time = 2 × hourly rate

Salary: annual amount

Commission: % of sales

Net income = **gross** income – deductions

Tax = money taken from pay and given to the government

Simple interest

$I = PRN$

where I = amount of interest
P = amount borrowed or invested
R = interest rate (as a fraction or decimal)
N = number of time periods

If $2000 is invested at 5% p.a. for 4 years:

$I = PRN = \$2000 \times 0.05 \times 4$

$= \$400$

Drilling for Gold exercise

2R1: Nine ways with percentages

Use the worksheet or write your working and answers in your exercise book.

Question	Non-calculator method	Calculator method
1 Calculate 15% of 60.		
2 Express 15 as a percentage of 60.		
3 15% of a number is 60. What is the number?		
4 Increase 60 by 15%.		
5 Decrease 60 by 15%.		
6 A number is increased by 15% to give 60. What is the number?	Not applicable.	
7 A number is decreased by 15% to give 60. What is the number?	Not applicable.	
8 A number is increased from 15 to 60. What is the percentage increase?		
9 A number is decreased from 60 to 15. What is the percentage decrease?		

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Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.

Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

1 2.8% as a decimal is:
A 2.8 **B** 0.28 **C** 0.028 **D** 0.0028 **E** 280

2 Fifty cents as a percentage of $2 is:
A 4% **B** 40% **C** 25% **D** $2\frac{1}{2}\%$ **E** 400%

3 12.5% as a simple fraction is:
A $\frac{12}{100}$ **B** $\frac{1}{8}$ **C** $\frac{3}{25}$ **D** 0.125 **E** 12.5

4 $33\frac{1}{3}\%$ of $660 is the same as:
A $\$660 \div 2$ **B** $\$660 \times 0.3$ **C** $\$660 \times 0.03$
D $\$660 \div 3$ **E** $\$660 \div \frac{1}{3}$

5 15% of $1600 is equal to:
A 24 **B** 150 **C** $240 **D** $24 **E** 240

6 If 110% of a number is 528, then the number is:
A 475.2 **B** 52.8 **C** 480 **D** 580.8 **E** 475

7 9670 increased by 12% is:
A 9682 **B** 9658 **C** 10830.4 **D** 1160.4 **E** 8509.6

8 Jane is paid $7.80 per hour. During a week, she works 12 hours at this rate plus 4 hours on a public holiday, for which she gets paid time and a half. Her pay that week is:
A $140.40 **B** $124.80 **C** $109.20 **D** $156 **E** $62.40

9 Simon earns a weekly retainer of $370 plus 12% commission of any sales he makes. If he makes $2700 worth of sales in a particular week, he will earn:
A $595 **B** $652 **C** $694 **D** $738.40 **E** $649.60

10 $1200 is increased by 10% for 2 years with simple interest. The total balance at the end of the 2 years is:
A $252 **B** $1452 **C** $1450 **D** $240 **E** $1440

Short-answer questions

1 Copy and complete the table on the right.

Decimal	Fraction	Percentage
0.5		
	$\frac{1}{3}$	
		25%
	$\frac{3}{4}$	
1.5		
		200%

2 Find:
- **a** 25% of \$310
- **b** 110% of 1.5

3 Determine the original amount if:
- **a** 20% of the amount is 30
- **b** 72% of the amount is 18

4
- **a** Increase 45 by 60%.
- **b** Decrease 1.8 by 35%.
- **c** Find the percentage change when \$150 is reduced by \$30.

5 The mass of a cat increased by 12% to 14 kg over a 12-month period. What was the cat's previous mass?

6 A dining table with cost price \$1800 is to be marked up by 15% before selling. What is its selling price?

7 Determine the discount given on a \$15 000 car if it is discounted by 12%.

8 The cost price of an item is \$150. If it is sold for \$175:
- **a** Determine the profit made.
- **b** Express the profit as a percentage of the cost price.

9 Determine the hourly rate of pay for a person:
- **a** with an annual salary of \$36 062, working a 38-hour week
- **b** who earns \$429, working 18 hours at the hourly rate and 8 hours at time and a half

10 Josephine's monthly income is \$5270. However, 20% of this is deducted to pay government taxes. What is her net yearly income?

11 Find the simple interest earned on $1500 invested at 7% p.a. for 5 years.

12 $60 simple interest is earned on an investment of $200 for 6 years. What is the interest rate per year?

Extended-response questions

1 Adelaide buys a dress at cost price from her friend Tila. Adelaide paid $420 for the dress, which is normally marked up by 55%.

- **a** How much did she save?
- **b** What is the normal selling price of the dress?
- **c** If Tila gets a commission of 15%:
  - **i** How much commission did she earn?
  - **ii** How much commission did Tila lose by selling the dress at cost price rather than the normal selling price?

2 Blake works a 38-hour week for $975.84.

- **a** Calculate his hourly rate of pay.

Overtime is calculated at time and a half.

- **b** What is Blake's overtime rate of pay?
- **c** How much does Blake earn for 4 hours of overtime?
- **d** How many hours of overtime did Blake work in a week if his wage for that week was $1226.22?
- **e** If Blake usually works the amount of overtime in part **d** in the 52 weeks of the year he works, and he pays 27% of his pay in tax, what is his net annual income?
- **f** If Blake invests 10% of his net income for one year in an account earning 8% p.a. simple interest for 18 months, how much extra income will he have earned?

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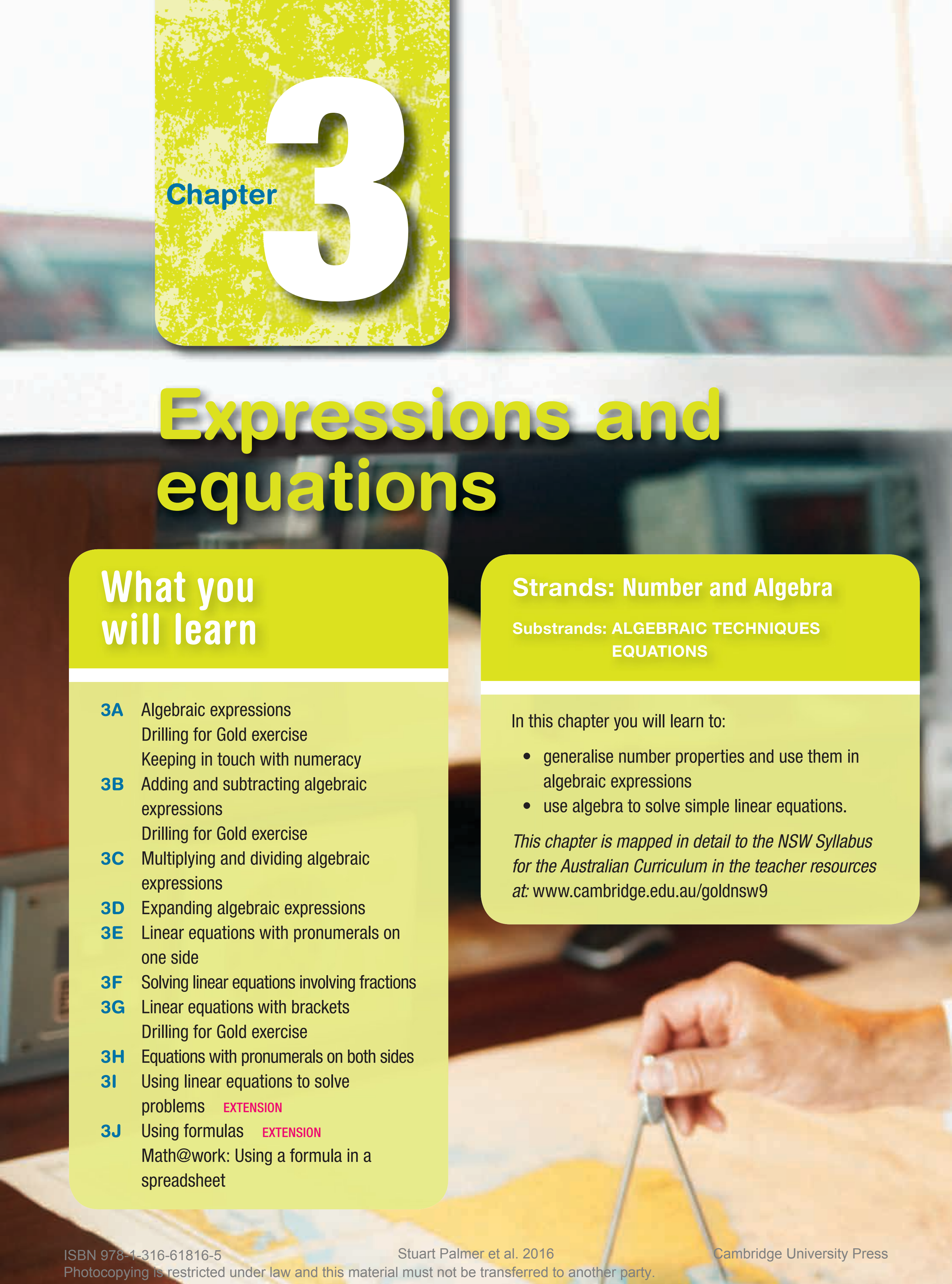


Chapter 3

Expressions and equations

What you will learn

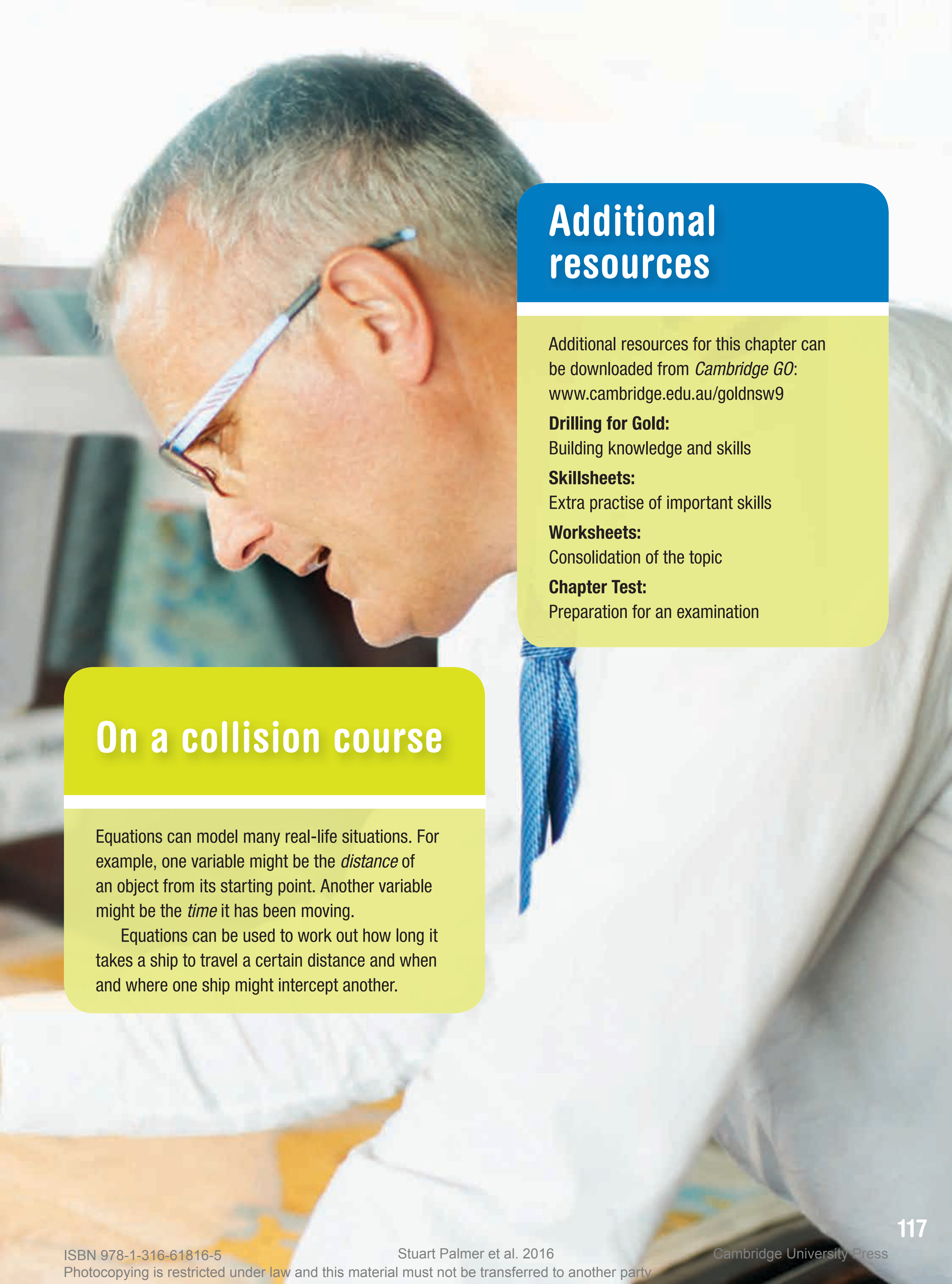
Strands: Number and Algebra

Substrands: ALGEBRAIC TECHNIQUES
EQUATIONS

In this chapter you will learn to:

- generalise number properties and use them in algebraic expressions
- use algebra to solve simple linear equations.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw9



On a collision course

Equations can model many real-life situations. For example, one variable might be the *distance* of an object from its starting point. Another variable might be the *time* it has been moving.

Equations can be used to work out how long it takes a ship to travel a certain distance and when and where one ship might intercept another.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw9

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

Pre-test

1 Write in simplified form.

a $3 \times x$ **b** $4 \times a \times b$

c $2 \times 5 \times x$ **d** $3 \times b \times 7$

2 When $x = 3$, find the value of:

a $x + 5$ **b** $10 - x$

c $4x$ **d** $18 \div x$

e $2x + 3$ **f** $2(x + 3)$

3 Write algebraic expressions for:

a 3 more than x

b the product of a and b

4 Calculate:

a -6×3 **b** $-8 \times (-5)$

c $-2 + 5$ **d** $-8 + 3$

e $7 - 11$ **f** $-2 - 6$

5 Simplify:

a $2x + 5x$ **b** $7y - 2y$

c $3x + x$ **d** $3x - x$

6 Simplify the following.

a $3 \times 2a$ **b** $3a \times 2b$

c $3a \times 2a$ **d** $3a \times 2ab$

7 Expand the following.

a $2(x + 3)$ **b** $3(a - 5)$ **c** $4(2x + 1)$

8 Evaluate the following when $a = 3$ and $b = -2$.

a $2a - 5$ **b** $ab + 4$

c $\dfrac{9}{a} - b$ **d** $2a(b + 1)$

9 Find the value of a that makes the following true.

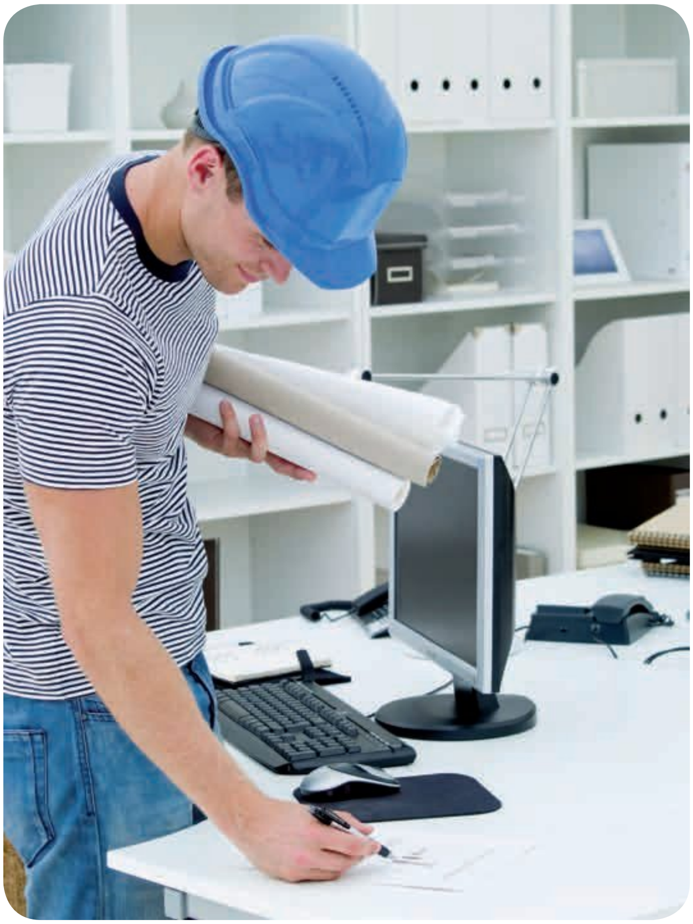
a $a + 4 = 13$ **b** $a - 3 = 7$

c $2a + 1 = 9$ **d** $\dfrac{a-1}{4} = 5$

3A Algebraic expressions

Stage
5.2
5.2◊
5.1
4

Algebra is central to the study of mathematics and is used to solve problems in everyday situations. Algebra involves working with unknown values. Pronumerals are used to show these unknown values.



Variables representing unknown quantities are used in a wide range of jobs and occupations.

▶ Let's start: How many eggs?

In this example, the pronumeral e represents the number of eggs in a basket. Write every expression in the table next to one of the sentences:

$2e + 2$	$\frac{1}{2}e$	$e \div 2$
$e + 2$		$e + e + 2$
$2 \times e$	$e - 1 - 1$	0
$e - e$	$e + e$	$\frac{e}{2}$
$e \times 2$	$2(e + 2)$	
$2e + 4$	$e - 2$	$e + 1 + 1$
$e + 1 - 1$	$2 + e$	$2e$

1 Two eggs are added.
2 Two eggs are removed.
3 The number of eggs is doubled.
4 The number of eggs is halved.
5 The number of eggs is doubled and then two are added.
6 Two eggs are added and then that number is doubled.
7 All the eggs are removed.
8 One egg is added and then removed.

Key ideas

- In algebra, letters are used to represent numbers. These letters are called **pronumerals** or variables.
- In algebra, the multiplication and division signs are generally not shown.
  For example: $3 \times x$ is written as $3x$ and $x \div 4$ is written as $\frac{x}{4}$.
- An **expression** is a combination of numbers and pronumerals.

$$\text{term} \rightarrow \overbrace{\boxed{5x} + \frac{4}{\boxed{y}}}^{\text{expression}} \nwarrow \text{pronumeral}$$

Drilling for Gold 3A1 3A2

- A **term** is a combination of numbers and variables connected by only multiplication and division. The expression above contains two terms: $5x$ and $\frac{4}{y}$.
- **Coefficients** are the numbers being multiplied by pronumerals.
  For example, in $3x + 4y$:
  3 is the coefficient of x.
  -4 is the coefficient of y.
- **Constant terms** consist of a number only. For example:
  $4x \boxed{-2} \leftarrow -2$ is a constant term. (The sign must be included.)
- Expressions can be evaluated by substituting a number for a pronumeral.
  For example: When $a = 2$, then
  $$\begin{aligned} a + 6 &= 2 + 6 \\ &= 8 \text{ (substitute)} \end{aligned}$$
- Order of operations should be followed when evaluating expressions:
  1. Brackets
  2. Powers
  3. Multiplication and division, from left to right
  4. Addition and subtraction, from left to right

Pronumeral A letter or symbol used to represent a number (also called a variable)

Expression A group of mathematical terms containing no equals sign

Term One of the components of an expression

Coefficient A numeral placed before a pronumeral

Constant term The part of an expression without any pronumerals

Order of operations The sequence in which computations are done

Substituting Replacing pronumerals with numbers

Variable A pronumeral that represents more than one numerical value

Exercise 3A

Understanding

1. Use the words from the list below to complete the following.
   expression, *coefficient*, *constant term*, *terms*, *pronumeral*
   a $3x - 2y + 7$ is an example of an algebraic ______________.
   b In $2a + b - 3$, -3 is the ______________.
   c In $4m + 3$, m is called a ______________.
   d In $3 - 2y + 4z$, 4 is the ______________ of z.
   e In the expression $3s + 7t$, $3s$ and $7t$ are the ______________.

2 Match an item in the left column (**a**–**e**) with an item in the right column (**A**–**E**).

a product
b sum
c difference
d quotient
e x^2

A division
B subtraction
C multiplication
D the square of x
E addition

3 Write the following in simplified form.

a $7 \times y$ **b** $-2 \times x$ **c** $1 \times b$
d $a \times b$ **e** $y \div 2$ **f** $x \div y$

4 Substitute $x = 2$ into the following and evaluate.

a $x + 3$ **b** $10 - x$ **c** $7x$ **d** $\frac{8}{x}$

Substitute means to replace the pronumeral with the given numeral.

Fluency

Example 1 Identifying parts of algebraic expressions

Consider the expression $5a + 2ab + 7 - 4b$.

a How many terms are in the expression?
b What is the coefficient of:
  i a?
  ii b?
c What is the constant term?

Solution	**Explanation**
a There are four terms.	The terms in the expression are separated by + or –.
b **i** 5 is the coefficient of a. **ii** –4 is the coefficient of b.	$5a$ is $5 \times a$; 5 is the number being multiplied by a. $-4b$ is $-4 \times b$; the negative sign is included.
c 7 is the constant term.	7 is the number with no pronumeral part.

5 Complete each of the parts **i**–**iii** for the algebraic expressions **a**–**d**.

  i How many terms are in the expression?
  ii What is the coefficient of y?
  iii What is the constant term?

a $5x + 2y + 3$
b $2x - 3y$
c $3xy + 7y - 4$
d $2x^2 - 1 + 4x + \frac{y}{2}$

Remember, the coefficient of a term includes the sign before it; in $3 - 2x$, -2 is the coefficient of x.

Example 2 Writing algebraic expressions for worded scenarios

Write an algebraic expression for the following.

a the number of tickets needed for 3 boys and r girls

b the cost in dollars of P pies at \$3 each

c a \$300 prize shared equally among m friends

Solution	**Explanation**
a $3 + r$	3 tickets plus the number of girls, r.
b $3P$	Two pies would cost $\$3 \times 2 = \$6$. The cost is $3 \times$ the number of pies, so P pies costs $3 \times P$.
c $\dfrac{\$300}{m}$	\$300 shared between three people is \$300 divided into 3 parts, so \$300 divided into m parts is $\$300 \div m = \dfrac{\$300}{m}$.

6 Write an algebraic expression for the following.

a The number of tickets required for:
  - **i** 4 boys and r girls
  - **ii** t boys and 2 girls
  - **iii** x boys, y girls and z adults

b The cost in dollars of:
  - **i** P pies at \$6 each
  - **ii** 10 pies at \$$n$ each
  - **iii** D drinks at \$2 each
  - **iv** P pies at \$5 and D drinks at \$2

c The number of grams of lollies for one child if 500 g of lollies is shared equally among C children.

For part **b i**:

1 pie costs	$\$6 \times 1 = \$6$
2 pies cost	$\$6 \times 2 = \$12$
5 pies cost	$\$6 \times 5 = \$30$
P pies cost	....

Drilling for Gold 3A3 at the end of this section

Example 3 Converting words to expressions

Write an algebraic expression for the following.

a five less than x

b three more than twice x

c the sum of a and b is divided by 4

d the square of the sum of x and y

Solution	**Explanation**
a $x - 5$	5 less than x is 5 subtracted from x.
b $2x + 3$	Twice x $(2 \times x)$ plus 3.
c $\dfrac{a+b}{4}$	The sum of a and b is done first $(a + b)$ and the result is then divided by 4.
d $(x + y)^2$	The sum of x and y is done first and then the result is squared. (Recall that the square of a is $a^2 = a \times a$.)

7 Write an algebraic expression for each of the following.

a the sum of 2 and x
b the sum of ab and y
c 5 less than x
d 7 subtracted from $2y$
e the product of x and 3
f three times the value of p
g four more than twice x
h the sum of x and y is divided by 5
i 10 less than the product of 4 and x
j 3 lots of x subtracted from 1
k the sum of 3 and y is divided by 2
l half of 1 more than x
m the square of the sum of m and n
n the sum of the squares of m and n

Sum (+)
Product (×)
Twice (×2)

Drilling for Gold 3A4 at the end of this section

Example 4 Substituting values into expressions

Evaluate these expressions when $a = 5$, $b = 6$ and $c = -3$.

a $3a + b$ **b** $2a - (b + c)$ **c** $a^2 - bc$

Solution	Explanation
a $3a + b = 3 \times 5 + 6$ $= 15 + 6$ $= 21$	Substitute the value 5 for a and the value 6 for b. Recall that $3a = 3 \times a$.
b $2a - (b + c) = 2 \times 5 - (6 + (-3))$ $= 10 - 3$ $= 7$	Substitute the values for a, b and c. Apply order of operations.
c $a^2 - bc = (5)^2 - 6 \times (-3)$ $= 25 - (-18)$ $= 25 + 18$ $= 43$	Evaluate powers before the other operations: $5^2 = 5 \times 5$ A positive (6) times a negative (–3) gives a negative (–18).

8 Evaluate these expressions when $a = 4$, $b = -3$ and $c = 8$.

a $3a + c$ **b** $ac - 7$ **c** $2c - (a + b)$

d $a^2 - 2c$ **e** $\dfrac{a+c}{2}$ **f** $b + 2(c - a)$

g $2c - ab$ **h** abc **i** $b - c + a^2$

When adding or subtracting a negative number:
$8 + (-3) = 8 - 3 = 5$
$8 - (-3) = 8 + 3 = 11$

Problem-solving and Reasoning

9 A rectangular garden bed is 12 m long and 5 m wide.

a Find the area of the garden bed.

b The length is increased by x m and the breadth is decreased by y m. Find the new length and breadth of the garden.

Draw a diagram to help.
Area of rectangle = $\ell \times b$

10 Jelena earns $(10 + 8t)$ dollars for each shift that she works, where t is the number of hours worked in the shift.

a How much does Jelena earn when $t = 2$?

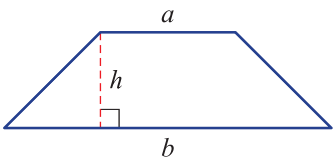
b How much does Jelena earn from a 5-hour shift?

c Will she earn \$100 from a 10-hour shift?

d How many whole hours must she work to earn more than \$100?

11 The expression for the area of a trapezium is $\frac{1}{2}h(a+b)$, where a, b and h represent the lengths shown.

- **a** Find the area of the trapezium with $a = 5$, $b = 7$ and $h = 3$.
- **b** A trapezium has $h = 4$ and area 12. If a and b are whole numbers, what possible values can the variable a have?



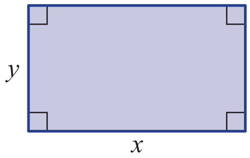
12 Christina earns $\$A$ from selling 20 glasses of lemonade. Write an expression for the cost, in dollars, of:

- **a** one glass of lemonade
- **b** three glasses of lemonade
- **c** n glasses of lemonade

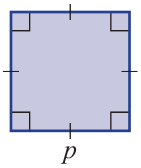
13 For each of these shapes, write an expression for the:

- **i** perimeter
- **ii** area

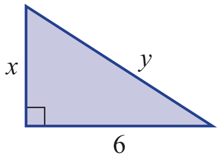
a



b



c



Perimeter is the distance around the outside of the shape.
Area of a rectangle is length × breadth.
Area of a triangle is $\frac{1}{2}$ × base × height.

Enrichment: Should there be brackets?

14 Brackets are used to ensure that the mathematics inside the brackets occurs first.

- **a** By substituting values for x, decide whether the following are the same.
  - **i** $2(x + 1)$ and $2x + 1$
  - **ii** $3+\frac{x}{2}$ and $\frac{3+x}{2}$
- **b** Write down two different expressions that could describe each of the following.
  - **i** 3 lots of x plus 1
  - **ii** the sum of 5 and x divided by 3
  - **iii** half of x plus y
- **c** Write a clearly worded statement for the following expressions.
  - **i** $4(x + 2)$
  - **ii** $3+\frac{x}{2}$
  - **iii** $\frac{1}{3}(m+n)$
  - **iv** $5x + 7$
  - **v** $\frac{x+y}{2}$
  - **vi** $\frac{1}{2}a+b$

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Drilling for Gold exercise

3A3: Words and expressions match-up

Use the worksheet or write the answers in your exercise book.

Match each of the following expressions **a–t** with one of the questions **1–20**.

a $\frac{3x}{2}$
b $\frac{2x}{3}$
c $x - 2$
d $x + 2$
e $2x$
f $2 - x$
g $2x + 3$
h $\frac{x}{2}$
i $\frac{x}{3}$
j $x + x$
k $2x + y$
l $2(x + y)$
m $x + 3$
n $x - 3$
o xy
p $\frac{x}{3} - 1$
q $\frac{x}{2} + 2$
r $3x$
s $3 - x$
t $x + y$

1 The sum of two numbers.
2 Two more than a number.
3 Half a number is added to two.
4 A number added to itself.
5 Double the sum of two numbers.
6 Three more than twice a number.
7 A number is decreased by three.
8 A number is divided by 3.
9 The product of two numbers.
10 A number is doubled.
11 Half of a number.
12 A number is tripled.
13 One less than one-third of a number.
14 A number is increased by 3.
15 Two less than a number.
16 A number is doubled, then added to another number.
17 The number two is decreased by a number.
18 A number is subtracted from 3.
19 A number is doubled and then divided by three.
20 A number is tripled and then the result is halved.

3A4: Substitution skill drill

Your teacher will give you numbers to place in the box at the top of the table. Using your numbers, complete the substitution column and the value column in the worksheet or write your working and answers in your exercise book.

My numbers are: $a =$ __, $b =$ __, $c =$ __

	Expression	Substitution	Value
1	$a + 5$		
2	$5a$		
3	$a + b$		
4	$a - 2$		
5	$2 - a$		
6	$a + b + c$		
7	$a + b - c$		
8	$2a + 3b + 4c$		
9	$2(a + 5)$		
10	$-2(a + 5)$		
11	$3 - 2(a - 2)$		
12	b^2		
13	$a^2 + b^2$		
14	$(a + b)^2$		
15	$a^2 - b^2$		
16	$3b^2$		
17	$(3b)^2$		

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Keeping in touch with numeracy

Non-calculator

1 The product of 0.2 and 0.3 is not 0.6. Copy and complete:

$$0.2 \times 0.3 = \frac{\_}{10} \times \frac{\_}{10} = \frac{\_\_}{100} = 0.\_$$

3 **a** $3 \times 5 + 2 \times 3$
b $(3 \times 5 + 2) \times 3$
c $3 \times (5 + 2) \times 3$
d $3 \times (5 + 2 \times 3)$

5 Lucy's normal rate of pay is $20 per hour. Find her pay earned when she works 5 hours at time and a half.

7 The reflex angle between the hands on this clock is:
A 180° **B** 225°
C 270° **D** 300°

9 How many square tiles with sides of 5 cm will be needed for 1 square metre?

11 What number is halfway between –2 and 10?

13 What is the remainder when 125 is divided by 3?

15 Calculate the simple interest earned when $100 is invested at 5% per annum for 5 years.

17 Increase $500 by 5%.

19 What is the percentage profit when a shirt is bought for $80 and sold for $120?

Calculator

2 Find the cost of 49 litres of petrol at $1.29 per litre.

4 Find the average of: 45, 47, 51, 55, 56 and 60.

6 Sophie is paid $467.50 for working 17 hours. What is her hourly rate of pay?

8 What is the reflex angle between the hands of a clock at the following times?
a 7:00
b 8:00
c 10:00
d 11:00

10 A particular tile costs $27.50 per square metre. What will it cost to tile a rectangular floor measuring 2.5 metres long and 3 metres wide?

12 What number is halfway between 37 and 95?

14 What is the remainder when 1257 is divided by 7?

16 Calculate the simple interest earned when $500 000 is invested at 5.5% per annum for 5 years.

18 Increase $500 by 5%, then decrease the result by 5%.

20 An apartment was bought for $500 000 and later sold for $650 000. What is the percentage increase in value?

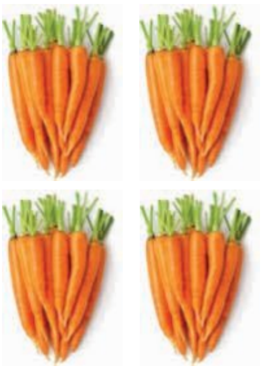
3B Adding and subtracting algebraic expressions

Stage
5.2
5.2◊
5.1
4

Just as $2 + 2 + 2 + 2 = 4 \times 2$, so $x + x + x + x = 4 \times x$ or $4x$. We say that the expression $x + x + x + x$ is simplified to $4x$; i.e. 4 lots of x.

Similarly, $3x + 5x$ is 3 lots of x plus 5 lots of x, which equals 8 lots of x, which equals $8x$.

These expressions have like terms and can be simplified.



$x + x + x + x = 4x$

Let's start: Two groups

Within each of the four sets of terms below, copy like terms into groups on either side of the dividing line. The first set has been started for you.

Set 1
$3x$ $7y$ $2x$ $8x$
$\cancel{4x}$ y $5x$ $2y$ $\cancel{3y}$
$3x$ | $7y$

Set 2
$4ab$ $2a$ $5ab$
$7a$ ab $6ab$

Set 3
2 $5y$ 7 1
$3y$ $2y$ 8 25

Set 4
$3ab$ $4a^2b$ $2ab$ $10ab$
$5a^2b$ $12ab$ $9a^2b$

- Describe how you classified each set into two groups.
- What do the terms in each group have in common?

Key ideas

- $x \times 4$ and $4 \times x$ can be written as $4x$, which means '4 lots of x'.
- $x \times 1$ and $1 \times x$ and $1x$ can be written as x.
- $x \times 0$ and $0 \times x$ and $0x$ can be written as 0.
- **Like terms** contain the same pronumeral(s).
  For example: $5x$ and $7x$ are like terms. $3ab$ and $-8ab$ are like terms.
  - Since $a \times b = b \times a$, then ab and ba are also like terms.
- Pronumerals are usually written in alphabetical order; e.g. xy rather than yx.
- Collect like terms to form a single term by adding or subtracting coefficients.
  For example: $4x + 7x = 11x$ $10ab - 2ab = 8ab$
- Unlike terms do not contain the same pronumerals.
  For example: $5x$, x^2, xy, $2xy^2$ and 3 are all unlike terms.

Like terms Two or more terms which contain the same pronumerals and indices

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Exercise 3B

Understanding

1 Use the words from the list below to complete the following sentences.

1, pronumeral, coefficients, like

a $4a$ and $7a$ are an example of ____________ terms.

b To add like terms, we add the ____________.

c Like terms have the same ____________.

d The coefficient of y in $2x + y$ is ______________.

Drilling for Gold 3B1 at the end of this section

2 Decide whether the following pairs of terms are like terms. (Answer yes or no.)

a $4ab$ and $3ab$ **b** $2x$ and $7xy$ **c** 5 and $4m$

d $3t$ and $-6tw$ **e** $7yz$ and yz **f** $2mn$ and $9nm$

g $5a$ and a **h** $3x^2y$ and $7xy^2$ **i** $4ab^2$ and $-3b^2a$

Like terms must have exactly the same pronumeral factors, including powers.

3 Complete the following.

a $2 - 5 =$ ____ **b** $3 - 4 =$ ____ **c** $1 - 6 =$ ____

d $-5 + 2 =$ ____ **e** $-8 + 3 =$ ____ **f** $-4 + 1 =$ ____

g $-2 + 5 =$ ____ **h** $-4 + 11 =$ ____ **i** $-1 + 4 =$ ____

j $-3 - 5 =$ ____ **k** $-1 - 4 =$ ____ **l** $-2 - 9 =$ ____

4 Which of the following represent the same expression as $w - x + y - z$?

a $w - z + y - x$ **b** $w - y + x - z$ **c** $y + w - z - x$

d $-x + w - z + y$ **e** $y - w - x + z$ **f** $y - z - x + w$

Make sure the correct pronumerals are being added or subtracted.

Fluency

Example 5 Identifying like terms

Choose the pair(s) of like terms from the following sets.

a $3x, 3, 4xy, 5x, y$ **b** $4b, 4bc, b, 2abc, -7bc, 2bc^2$

Solution	Explanation
a $3x$ and $5x$ are like terms. The rest are not.	$3x$ and $5x$ contain the same pronumeral.
b $4b$ and b are like terms. $4bc$ and $-7bc$ are like terms.	$4b$ and b have the same pronumeral. $4bc$ and $-7bc$ have the same pronumeral. $2bc^2$ does not because c is raised to the power of 2. All other terms have different pronumerals.

5 Choose the pair(s) of like terms from the following sets.

a $4y, 5, 3xy, 2y, 7xy$ **b** $3x, 3y, 3, xy, 7x$

c $7ab, 2a, -3ab, b^2, 5a$ **d** $2a^2, 4a, 3ab, a, -3a^2$

e $2xy, 3x^2y, -5x, 7yx^2$ **f** $5ab^2, 3ab, 2b^2, 4ab^2, 7ba$

st and ts are like terms since $s \times t = t \times s$.

Drilling for Gold 3B2 at the end of this section

Example 6 Collecting like terms

Simplify the following by collecting like terms.

a $4a + 7a$ **b** $3x + 4 - 2x$

Solution	**Explanation**
a $4a + 7a = 11a$	Since $4a$ and $7a$ are like terms, they can be simplified to one term by adding their coefficients. $4 + 7 = 11$.
b $3x + 4 - 2x = 3x - 2x + 4$ $= x + 4$	Collect like terms ($3x$ and $-2x$). The sign belongs to the term that follows. Combine their coefficients $3 - 2 = 1$. Recall that $1x$ is written as x.

6 Simplify the following by collecting like terms.

a $3a + 7a$ **b** $4n + 3n$ **c** $12y - 4y$

d $5x + 2x + 4x$ **e** $6ab - 2ab$ **f** $7mn + 2mn - mn$

g $4y + 3y + 8$ **h** $7x + 5 - 4x$ **i** $5m + 2 - m$

In order to add or subtract coefficients, the terms must be like terms.

Example 7 Combining like terms

Simplify the following by collecting like terms.

a $3x + 2y + 4x + 7y$ **b** $3xy + 4x + xy - 6x$ **c** $8ab^2 - 9ab - 5ab^2 + 3ba$

Solution	**Explanation**
a $3x + 2y + 4x + 7y = 3x + 4x + 2y + 7y$ $= 7x + 9y$	Collect the like terms ($3x$ and $4x$ and $2y$ and $7y$) and combine their coefficients ($3 + 4 = 7$ and $2 + 7 = 9$).
b $3xy + 4x + xy - 6x = 3xy + xy + 4x - 6x$ $= 4xy - 2x$	Collect the like terms ($3xy$ and xy and $4x$ and $-6x$).Combine their coefficients. (Recall that $xy = 1xy$.) $3 + 1 = 4$ and $4 - 6 = -2$.
c $8ab^2 - 9ab - 5ab^2 + 3ba$ $= 8ab^2 - 5ab^2 - 9ab + 3ab$ $= 3ab^2 - 6ab$	Collect like terms in ab^2 and ab. Remember that $ba = ab$ and the + or – sign belongs to the term that follows; i.e. $-5ab^2$. Combine coefficients: $8 - 5 = 3$ and $-9 + 3 = -6$.

7 Simplify the following by collecting like terms.

a $2a + 4b + 3a + 5b$ **b** $4x + 3y + 2x + 2y$ **c** $xy + 8x + 4xy - 4x$

d $6t + 5 - 2t + 1$ **e** $5x + 1 + 6x + 3$ **f** $3mn + 4 + 4nm - 5$

g $4ab + 2a + ab - 3a$ **h** $3st - 8ts + 2st + 3ts$ **i** $8ac - 6c + 2ac - 3c$

j $5xy^2 - 2xy^2$ **k** $8m^2n - 6nm^2 + m^2n$ **l** $2x^2y - 4xy + 5yx^2$

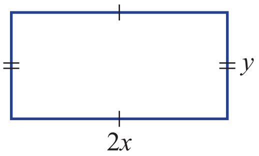
Problem-solving and Reasoning

8 A farmer has x pigs and y chickens.

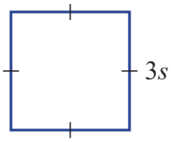
- **a** Write an expression for the total number of heads.
- **b** Write an expression for the total number of legs.

9 Write simplified expressions for the perimeter of the following shapes.

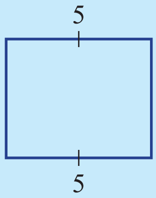
a



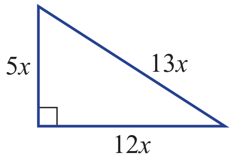
b



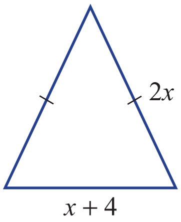
Sides with matching dash marks are equal in length.



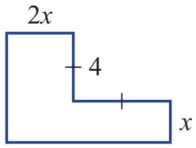
c



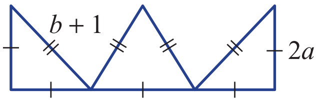
d



e



f



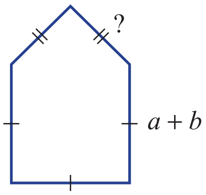
10 A rectangular pool's length is three times its breadth, x metres. Write an expression for its perimeter.

11 Fill in the missing term to make each of the following correct.

- **a** $11t + 3 - \square = 7t + 3$
- **b** $\square + 3y - 2x = 7x + 3y$
- **c** $3x + 4y - \square + 5y = x + 9y$
- **d** $4a + 7b - \square - 2b = 5b - 3a$
- **e** $3mn - \square + nm + 4n = 4mn - n$
- **f** $2pq + 2p - \square - 5pq = 2p - 7pq$
- **g** $3x^2y - 11xy + 3yx^2 - \square = 6x^2y - 13xy$
- **h** $4b^2 - 3b + 2b^2 + \square = 6b^2$

Enrichment: Around the edge

12 Consider the shape shown.



- **a** If the unknown sides (?) have length $2b$, what is the perimeter in simplified form?
- **b** If $a = 3$ and $b = 2$ and the perimeter is 23 units, what is the length of the unknown sides?
- **c** If the perimeter is $(5a + 3b)$, what is the length of each unknown side?
- **d** If the perimeter is $(7a + b)$, what is the length of each unknown side?

Drilling for Gold exercise

3B1: Like or unlike?

Examine the terms and then decide if they are like terms or not.
Complete the worksheet or write your answers in your exercise book.

	The terms		Are they like terms? (yes or no)
1	x	$2x$	
2	$2x$	2	
3	$2x$	$3x$	
4	$2x$	$2x$	
5	$2x$	$2y$	
6	$2x$	$3y$	
7	$2x$	$2xy$	
8	xy	yx	
9	xy	$2yx$	
10	x	x^2	
11	xy	x^2y	
12	xy^2	x^2y	
13	xy^2	y^2x	
14	xyz	$2yxz$	
15	$(xy)^2$	xy^2	

Drilling for Gold exercise

3B2: Skill drill – Adding and subtracting like terms

If you are using the worksheet, circle the equivalent expressions and then highlight the simplest correct answer.
If you are writing your answers in your exercise book, write out:

i the equivalent expression
ii the simplest correct answer.

1 $3x + 2x$ is equivalent to:
$2x + 3x$ $5x$ $6x$ $5xx$ $5x^2$

2 $3x + x$ is equivalent to:
$x + 3x$ $5x$ $4x$ $3xx$ $3x^2$

3 $3x - x$ is equivalent to:
$x - 3x$ 3 $2x$ $x + x$

4 $3x - 3x$ is equivalent to:
x $0x$ 0 1

5 $6x - 5x$ is equivalent to:
$5x - 6x$ $-5x + 6x$ $1x$ 1 x

6 $x + x$ is equivalent to:
$x + 1$ $2x$ xx $x - x$ x^2

7 $x - x$ is equivalent to:
$x + x$ $0x$ 0 x 1

8 $4x - 5x$ is equivalent to:
$5x - 4x$ x $-1x$ $-x$

9 $x + x - x$ is equivalent to:
$2x - x$ $1x$ x $x + 0$

10 $x - x - x$ is equivalent to:
x $-x$ $-1x$

3C Multiplying and dividing algebraic expressions

Stage
5.2
5.2◊
5.1
4

We have seen that $3x + 3x$ can be simplified to $6x$.
$3x + 3x$ can also be written as $2 \times 3x$. Thus, $2 \times 3x = 6x$.

So $4 \times 5a = 4 \times 5 \times a = 20a$. Any two or more terms can be simplified using multiplication.

A single term such as $2 \times 5 \times x \div 10$ can also be simplified using multiplication and division, so $2 \times 5 \times x \div 10 = \frac{10x}{10} = x$, since $\frac{10}{10}$ simplifies to 1 by division.

▶ Let's start: Are they equivalent?

These expressions can be separated into two groups. Group them so that the expressions in each group are equivalent.

$4x$ $\quad 2x - y$ $\quad 6x - x - x$ $\quad 10x - y - 8x$

$\frac{24x}{6}$ $\quad y + 2x - y + 2x$ $\quad 2 \times x - 1 \times y$ $\quad -y + 2x$

$\frac{1}{2} \times 8x$ $\quad 2 \times 2x$ $\quad \frac{8x^2}{2x}$ $\quad -1 \times y + \frac{2x^2}{x}$

Key ideas

- The symbols for multiplication (×) and division (÷) are usually not shown in simplified algebraic terms.
  For example: $5 \times a \times b$ is written as $5ab$.
  $-7 \times x \div y^2$ is written as $-\frac{7x}{y^2}$.
- Order does not matter in multiplication; i.e. $2 \times 3 = 3 \times 2$, $a \times 2 \times b = 2 \times a \times b$.
- Algebraic terms are multiplied by multiplying coefficients with the pronumeral parts.
  For example: $4a \times 3b = 4 \times 3 \times a \times b$
  $= 12ab$
- The factors of a number are the set of numbers that divide evenly into it.
  For example: The factors of 6 are 1, 2, 3 and 6.
- When dividing algebraic expressions, common factors can be cancelled. Cancel the highest common factor of the numerals and then the pronumerals.

For example: $\frac{{}_1\cancel{7}x}{{}_2\cancel{14}} = \frac{x}{2}$ $\qquad \frac{{}_2\cancel{8}x\cancel{y}_1}{{}_3\cancel{12}\cancel{y}_1} = \frac{2x}{3}$ $\qquad \frac{a^2b}{a} = \frac{{}_1\cancel{a} \times a \times b}{{}_1\cancel{a}} = ab$

Factor A whole number that will divide into another number without a remainder

Exercise 3C

Understanding

1 Complete the simplified form of the following.

a $7 \times x = \_\_\_\_$ **b** $a \times b = \_\_\_\_$

c $3 \times 2 \times m = \_\_\_\_m$ **d** $8 \times 3 \times y = 24\_\_\_\_$

e $6 \times 7 \times b = \_\_\_\_$ **f** $4 \times x \times 5 = \_\_\_\_$

g $a \times a = \_\_\_\_$ **h** $9 \times y \times y = 9\_\_\_\_$

2 Fill in the missing word: *positive* or *negative*.

a A positive number multiplied by a negative number is ________________.

b A negative number multiplied by a ________________ number is positive.

c A negative number divided by a positive number is ________________.

3 State the highest common factor of these pairs of numbers.

a 4 and 10 **b** 6 and 18 **c** 12 and 36

d 9 and 21 **e** 14 and 35 **f** 25 and 60

4 Simplify these fractions.

a $\frac{4}{6}$ **b** $\frac{20}{8}$

c $\frac{30}{10}$ **d** $\frac{8}{40}$

e $\frac{-4}{14}$ **f** $\frac{-5}{100}$

g $\frac{-27}{36}$ **h** $\frac{-72}{16}$

Divide numerator and denominator by their highest common factor.

÷2

$$\frac{6}{10} = \frac{3}{5}$$

÷2

Fluency

Example 8 Multiplying algebraic terms

Simplify the following.

a $3 \times 2b$ **b** $-2a \times 3b$

Solution	**Explanation**
a $3 \times 2b = 6b$	Multiply the coefficients.
b $-2a \times 3b = -6ab$	Recall that multiplication can occur in any order; i.e. $2 \times 3 \times 4 = 2 \times 4 \times 3$ etc. Multiply the coefficients and simplify.

5 Simplify the following.

a $5 \times 2m$ **b** $2 \times 6b$ **c** $3 \times 5p$

d $3x \times 2$ **e** $3p \times 6r$ **f** $4m \times 4n$

g $-2x \times 7y$ **h** $5m \times (-3n)$ **i** $-4c \times 3d$

j $2a \times 3b \times 5$ **k** $-4r \times 3 \times 2s$ **l** $5j \times (-4) \times 2k$

Example 9 Dividing algebraic terms

Simplify the following.

a $\frac{4x}{8}$ **b** $10ab \div (15b)$

Solution	**Explanation**
a $\frac{{}^{1}\cancel{4}x}{{}_{2}\cancel{8}} = \frac{x}{2}$	Deal with numerals and pronumerals separately, cancelling any common factors. The highest common factor of 4 and 8 is 4.
b $10ab \div (15b) = \frac{10ab}{15b}$ $= \frac{{}^{2}\cancel{10} \times a \times \cancel{b}^{1}}{{}_{3}\cancel{15} \times \cancel{b}_{1}}$ $= \frac{2a}{3}$	Write division as a fraction first. Write the numerator and denominator as a product and cancel the common factors 5 and b.

6 Simplify the following by cancelling.

Drilling for Gold 3C1

a $\frac{8b}{2}$ **b** $\frac{2a}{6}$ **c** $\frac{4ab}{6}$

d $\frac{3mn}{6n}$ **e** $\frac{5xy}{20y}$ **f** $\frac{10st}{6t}$

g $\frac{3xy}{xy}$ **h** $\frac{27pq}{6p}$ **i** $2x \div 4$

j $12ab \div (2a)$ **k** $7mn \div (3n)$ **l** $8y \div (20xy)$

Make sure they are in fraction form before cancelling; e.g. $2x \div 4 = \frac{2x}{4}$.

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Example 10 Multiplying and dividing with powers of 2

Simplify the following.

a $6x \times 5xy$

b $\frac{12a^2b}{3ab}$

Solution

a $6x \times 5xy = 6 \times 5 \times x \times x \times y$
$= 30x^2y$

b $\frac{12a^2b}{3ab} = \frac{{}^{4}\cancel{12} \times {}^{1}\cancel{a} \times a \times \cancel{b}^{1}}{{}^{1}\cancel{3} \times \cancel{a}^{1} \times \cancel{b}^{1}}$
$= 4a$

Explanation

Multiply the coefficients and the pronumerals. Recall that $x \times x$ is written as x^2 (x squared).

Write the numerator and denominator as products, with $a^2 = a \times a$. Cancel common factor (3) for numerals, then cancel common factors for pronumerals.

7 Simplify the following.

a $4n \times 6n$

b $-3q \times q$

c $5s \times 2s$

d $7a \times 3ab$

e $5mn \times (-3n)$

f $-3gh \times (-6h)$

g $\frac{24ab^2}{8ab}$

h $\frac{25x^2y}{5xy}$

i $\frac{9m^2n}{18mn}$

j $\frac{2xy}{8xy^2}$

k $\frac{6a^2b}{10a}$

l $\frac{45p^2q^2}{15pq}$

Deal with numerals and pronumerals separately.

$\frac{\cancel{24}^{3}}{\cancel{8}_{1}}$ and $\frac{ab^2}{ab} = \frac{\cancel{a}_{1} \times b \times \cancel{b}_{1}}{\cancel{a}_{1} \times \cancel{b}_{1}} = b$

Problem-solving and Reasoning

8 Simplify the following by first writing in fraction form.

a $4 \times x \div y$

b $3 \times m \div 9$

c $4 \times a \div (2b)$

d $5x \times 4 \div (2y)$

e $6 \times 4mn \div (3m)$

f $8a \times 5b \div (8a)$

g $10m \times 4n \div (8mn)$

h $4x \times 3xy \div (2x)$

i $3pq \times p \div (6q)$

j $5ab^2 \times 4 \div (10b)$

Apply one operation at a time from left to right in these; e.g.

$4 \times x \div y = 4x \div y$
$= \frac{4x}{y}$

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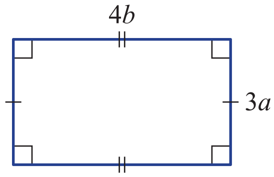
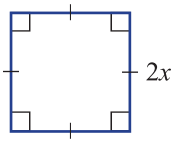
3C

9 Write a simplified expression for the area of the following shapes.

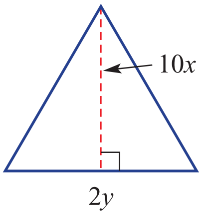
a



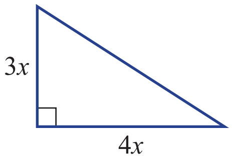
b



c



d



Area of a triangle
$= \frac{1}{2} \times \text{base} \times \text{height}$

10 A bag of lollies contains x jelly beans. Jean buys six bags of lollies to share equally among her three grandchildren. Write a simplified expression for how many jelly beans each grandchild receives.

Skillsheet 3A

11 Fill in the missing term to make each of the following correct.

a $3m \times \square = 12mn$

b $4x \times \square = 28xy$

c $-2xy \times \square = -18xy^2$

d $6a \times \square = 30a^2b$

e $\frac{\square}{7} = 2x$

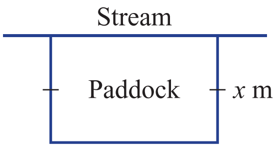
f $\frac{15}{\square} = \frac{3}{2y}$

g $\frac{\square}{18ab} = \frac{2}{3b}$

h $\frac{\square}{4x} = 2xy$

Enrichment: Rectangular paddocks

12 The length of a rectangular paddock is three times its breadth. Its breadth is x metres. It is bordered on one side by a stream.



a Write a simplified expression for the:
  i area of the paddock
  ii length of fencing needed for the three sides of the paddock

b When $x = 50$, use your answers to part **a** to give the:
  i area of the paddock
  ii length of fencing material required

13 The area of another rectangular paddock is given by the expression $4ab^2$. Find the breadth of the paddock when the length is:

a ab

b $2b$

c $4ab$

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3D Expanding algebraic expressions

Stage
5.2
5.2◊
5.1
4

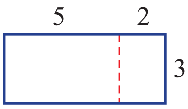
The expression 2(3 + 4) means 2 × (3 + 4). It can be simplified in two ways:

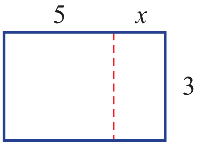
$$2 \times (3+4) = 2 \times 7 = 14 \quad \text{OR} \quad 2 \times (3+4) = 2 \times 3 + 2 \times 4 = 6 + 8 = 14$$

Let's start: Rectangular distributions

This rectangle has the dimensions as shown. Given that the area of a rectangle is length × breadth:

- Describe two different ways to find the area of the rectangle.
- Substitute x for 2 and rewrite the expressions.





Examples of the distributive law can often be found in everyday tasks.

Key ideas

- The **distributive law** is used to expand and remove brackets.
  - A term on the outside of the brackets is multiplied by each term inside the brackets.

    $$a(b+c) = ab + ac \quad \text{or} \quad a(b-c) = ab - ac$$

    For example:

    $$2(x+4) = 2 \times x + 2 \times 4 = 2x + 8$$

  - If the number in front of the bracket is negative, the sign of each of the terms inside the brackets will change when expanded.
    For example: $-2(x-3) = -2x + 6$ because $-2 \times x = -2x$ and $-2 \times (-3) = 6$.

Distributive law Adding numbers and then multiplying the total gives the same answer as multiplying each number first and then adding the product

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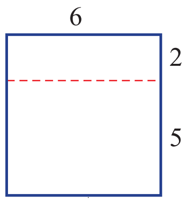
Exercise 3D

Understanding

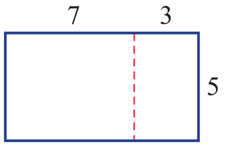
1 State a second way to find the area of these rectangles.

a



1 $A = 6 \times (5 + 2)$
2 $A =$

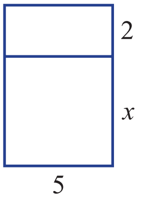
b



1 $A = 5 \times 3 + 5 \times 7$
2 $A =$

Area of a rectangle is length × breadth.

2 This diagram shows two joined rectangles with the given dimensions.



- **a** Write an expression for the area of the:
  - **i** larger rectangle
  - **ii** smaller rectangle
- **b** Use your answers from part **a** to find the combined area of both rectangles.
- **c** Write an expression for the vertical side.
- **d** Use your answer from part **c** to find the combined area of both rectangles. You will need brackets.
- **e** Complete this statement: $5(x + 2) = \underline{\quad\quad} + \underline{\quad\quad}$

3 Substitute $x = 3$ and $x = -2$ into each expression. Which two expressions are equivalent?

a $2(x + 5)$ **b** $2x + 5$ **c** $2x + 10$

4 Complete the following.

a $3(x + 5) = \underline{\quad} \times x + \underline{\quad} \times 5$
$= \underline{\quad} + 15$

b $5(x + 1) = \underline{\quad} \times \underline{\quad} + 5 \times \underline{\quad}$
$= \underline{\quad} + \underline{\quad}$

c $2(x - 7) = 2 \times \underline{\quad} + 2 \times \underline{\quad}$
$= \underline{\quad} - 14$

$$a(b + c) = a \times b + a \times c$$
$$= ab + ac$$
$$a(b - c) = a \times b + a \times (-c)$$
$$= ab - ac$$

5 a Substitute the value $x = 5$ into these expressions.

- **i** $-2(x + 3)$
- **ii** $-2x + 6$

b Do your answers from part **a** suggest that $-2(x + 3) = -2x + 6$? When expanded, what should $-2(x + 3)$ equal?

c Substitute the value $x = 10$ into these expressions.

- **i** $-3(x - 1)$
- **ii** $-3x - 3$

d Do your answers from part **c** suggest that $-3(x - 1) = -3x - 3$? When expanded, what should $-3(x - 1)$ equal?

Example 11 Expanding simple expressions with brackets

Expand these expressions.

a $3(x + 4)$ **b** $5(x - 11)$

Solution	**Explanation**
a $3(x + 4) = 3x + 12$	The 3 on the outside is multiplied by each term inside the brackets; i.e. $3(x + 4) = 3 \times x + 3 \times 4$.
b $5(x - 11) = 5x - 55$	$5 \times x = 5x$ and $5 \times (-11) = -55$.

6 Expand these expressions.

a $2(x + 3)$ **b** $5(x + 12)$ **c** $2(x + 7)$
d $7(x + 9)$ **e** $3(x - 7)$ **f** $6(x - 11)$
g $5(x - 9)$ **h** $10(x - 1)$ **i** $3(2 + x)$
j $7(3 + x)$ **k** $4(7 - x)$ **l** $5(8 - x)$

Remember that the number in front of the brackets is multiplied by each term inside the brackets when expanding:

$2(x + 3) = \ldots$

Example 12 Expanding expressions with a negative number in front

Expand these expressions.

a $-2(x + 5)$ **b** $-3(x - 4)$

Solution	**Explanation**
a $-2(x + 5) = -2x - 10$	-2 is multiplied by each term inside the brackets. $-2 \times x = -2x$ and $-2 \times 5 = -10$.
b $-3(x - 4) = -3x + 12$	Multiplying by a negative changes the sign of each term in the brackets. $-3 \times x = -3x$ and $-3 \times (-4) = +12$.

7 Expand these expressions.

a $-3(x + 2)$ **b** $-2(x + 11)$ **c** $-5(x - 3)$ **d** $-6(x - 6)$
e $-4(2 - x)$ **f** $-13(3 + x)$ **g** $-8(9 + x)$ **h** $-300(1 - x)$

Example 13 Expanding brackets and simplifying

Expand these expressions.

a $4(x + 3y)$ **b** $2x(4x - 3)$

Solution	**Explanation**
a $4(x + 3y) = 4x + 12y$	Multiply each term inside the brackets by 4. $4 \times x = 4x$ and $4 \times 3 \times y = 12y$.
b $2x(4x - 3) = 8x^2 - 6x$	Each term inside the brackets is multiplied by $2x$. $2x \times 4x = 2 \times 4 \times x \times x = 8x^2$ and $2 \times (-3) \times x = -6x$.

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8 Expand these expressions.

- **a** $2(a + b)$
- **b** $5(a + 2b)$
- **c** $3(2m + y)$
- **d** $8(2x - 5)$
- **e** $-3(4x + 5)$
- **f** $4x(x - 2y)$
- **g** $t(2t - 3)$
- **h** $a(3a + 4)$
- **i** $d(2d - 5)$
- **j** $2b(3b - 5)$
- **k** $2x(4x + 1)$
- **l** $5y(1 - 3y)$

$x \times x$ is written as x^2.

Example 14 Simplifying by removing brackets

Expand and then collect like terms.

- **a** $4(x + 5) - 10$
- **b** $12 + 3(x - 4)$

Solution	**Explanation**
a $4(x + 5) - 10 = 4x + 20 - 10$ $= 4x + 10$	Expand brackets first: $4(x + 5) = 4 \times x + 4 \times 5$. Then collect like terms and simplify.
b $12 + 3(x - 4) = 12 + 3x - 12$ $= 0 + 3x$ $= 3x$	Expand brackets: $12 + 3(x - 4)$. Collect like terms.

9 Expand and then collect like terms.

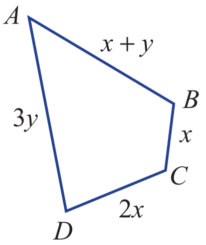
- **a** $2(x + 4) + 3$
- **b** $6(x + 3) + 4$
- **c** $5(x + 2) - 4$
- **d** $3(x + 4) - 2$
- **e** $3 + 4(x - 2)$
- **f** $7 + 2(x - 3)$
- **g** $2 + 3(x - 2)$
- **h** $1 + 5(x + 4)$
- **i** $7 + 2(x + 3)$

Expand the brackets first, then collect like terms.

Problem-solving and Reasoning

10 The diagram shows the route taken by a sales representative who travels from A to D via B and C.

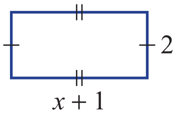
- **a** If the sales representative then returns directly to A, write an expression (in simplest form) for the total distance travelled.
- **b** If $y = x + 1$, write an expression for the total distance the sales representative travels in terms of x only. Simplify your expression.
- **c** When $y = x + 1$, how much would the distance have been reduced by (in terms of x) if the sales representative had travelled directly from A to D and straight back to A?



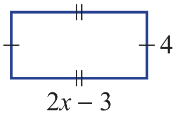
Subtract the distance from A to D and back again from the total distance.

11 In the following diagrams, all angles are right angles. Express the area in a simplified expression with no brackets.

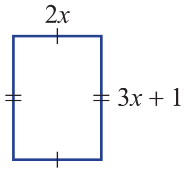
a



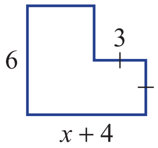
b



c



d



Remember to include brackets when multiplying length by breadth.

12 Murray has \$60 in savings. One Saturday, he does some odd jobs for a neighbour and earns $\$x$, which he adds to his savings. Murray's parents think he's doing a good job of saving money, so they decide to double his savings. Write an expanded expression for the amount of savings Murray has now.

13 Identify the errors in these expressions, then write out the correct expansion.

a $2(x + 6) = 2x + 6$

b $x(x - 4) = 2x - 4x$

c $-3(x + 4) = -3x + 12$

d $-7(x - 7) = -7x - 49$

e $5(x + 2) + 4 = 5x + 6$

f $5 - 2(x - 7) = -9 - 2x$

Watch out for the negative numbers.

Enrichment: Pairs of brackets

14 Expand each pair of brackets first, then collect like terms.

a $2(x + 3) + 3(x + 2)$

b $2(x + 4) + 2(x - 1)$

c $3(2x + 4) + 5(x - 1)$

d $4(3x + 2) + 5(x - 3)$

e $-2(x + 2) + 3(x - 1)$

f $2(4x - 3) - 2(3x - 1)$

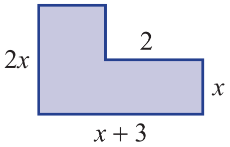
g $x(x + 2) + 2x(x + 4)$

h $3(x - 1) + x(x + 2)$

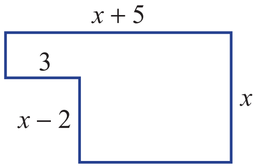
i $x(x + 2) - 4(x + 1)$

15 Use the technique in Question **15** to find the area of these shapes in expanded form.

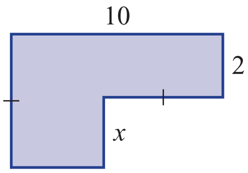
a



b



c



Split these shapes into rectangles and sum or subtract areas.

3E Linear equations with pronumerals on one side

Stage
5.2
5.2◊
5.1
4

A mathematical statement containing an equals sign is called an equation. $5 = 10 \div 2$, and $x^2 + 1 = 10$ are examples of equations.

$4x - 1 = 6$, and $5x = 2x + 1$ are called linear equations.

Equations are solved by finding the value of the pronumeral that makes the equation true. This can be done by inspection for very simple linear equations (e.g. if $3x = 15$, then $x = 5$ since $3 \times 5 = 15$). More complex linear equations can be solved through a series of steps whereby each step produces an equivalent equation.

▶ Let's start: Is $x = 4$ a solution?

Which of the equations in the table have $x = 4$ as the solution?

$5x = 20$	$2x - 1 = 5$	$14 = 10 - x$
$x + 9 = 13$	$14 = 3x + 2$	$-4x = 16$

- How did you find the equations for which $x = 4$ is a solution?
- The equations you have listed should be equivalent equations to $x = 4$. Starting from $x = 4$, describe what you would do to produce each equivalent equation; for example, 'add 1 to both sides'.

Key ideas

Drilling for Gold 3E1 3E2

- An **equation** is a mathematical statement containing an equals sign. For example: $3x + 1 = 7$.
- **Equivalent equations** are created by:
  - adding a number to or subtracting a number from both sides of the equation
    $$\begin{array}{rrcll} -4\,\big(& 5x + 4 & = & 14 & \big)\,-4 \\ & 5x & = & 10 & \end{array}$$
  - multiplying or dividing both sides of the equation by the same number (not including 0)
    $$\begin{array}{rrcll} \div 5\,\big(& 5x & = & 10 & \big)\,\div 5 \\ & x & = & 2 & \end{array}$$
  - swapping the left-hand side (LHS) with the right-hand side (RHS). For example: $13 = 2x + 1$ is equivalent to $2x + 1 = 13$.
- Linear equations can be solved by creating equivalent equations, then using opposite operations.
- The solution to an equation can be checked by substituting the solution into the original equation and checking that both sides are equal.

Equation: $5x + 4 = 14$
Possible solution: $x = 2$

LHS	RHS
$= 5x + 4$ $= 5 \times 2 + 4$ $= 14$	$= 14$

$\therefore\ x = 2$ *is* a solution.

Equation A statement that two expressions have the same value

Equivalent equations Two or more equations that have the same solution

Solution The value of an unknown that makes an equation true

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Exercise 3E

Understanding

1 Which equations have a solution of $x = 3$?

a $x + 5 = 9$	**b** $x - 1 = 2$	**c** $4 - x = 1$
d $1 - x = -4$	**e** $2x + 4 = 10$	**f** $3x - 2 = 7$
g $2 = 7 - 2x$	**h** $-5 = 4 - 3x$	**i** $1 = -x + 2$

If $x = 3$ is a solution, the left-hand side must equal the value that is on the right-hand side of the = sign.

Drilling for Gold 3E3

2 What value of x makes these equations true?

a $x + 4 = 10$	**b** $x - 3 = 5$
c $2 = x + 5$	**d** $2x = 8$
e $20 = 5x$	**f** $-3x = 9$

What value plus 4 equals 10?

3 Obtain an equivalent equation by completing the step given in brackets.

a $2x + 5 = 11$ (Subtract 5 from both sides.)

b $3x - 4 = 17$ (Add 4 to both sides.)

c $4x = 36$ (Divide both sides by 4.)

$$\begin{array}{ccc} & 2x+5=11 & \\ -5 \swarrow & & \searrow -5 \\ & 2x = \ldots & \end{array}$$

Fluency

Drilling for Gold 3E4

4 State the single operation that has been performed to obtain these pairs of equivalent equations; for example, 'add 3 to both sides'.

a $x + 9 = 11$	**b** $2x - 1 = 5$	**c** $3x = 27$	**d** $2x = 7$
$x = 2$	$2x = 6$	$x = 9$	$x = \frac{7}{2}$

Example 15 Solving simple linear equations

Solve each of the following equations and check your solution.

a $2x + 5 = 9$ **b** $11 = 3x - 7$

Solution | **Explanation**

a

$$\begin{array}{ccc} & 2x+5=9 & \\ -5 \downarrow & & \downarrow -5 \\ & 2x=4 & \\ \div 2 \downarrow & & \downarrow \div 2 \\ & x=2 & \end{array}$$

Subtract 5 from both sides to undo the +5.

Divide both sides by 2 since $2x = 2 \times x$ and $\frac{{}^{1}\cancel{2} \times x}{\cancel{2}^{1}} = x$.

Check:

$$\begin{array}{l|l} \text{LHS} = 2x + 5 & \text{RHS} = 9 \checkmark \\ \quad = 2 \times 2 + 5 & \\ \quad = 4 + 5 & \\ \quad = 9 \checkmark & \end{array}$$

Check your solution by substituting $x = 2$ into the original equation to check that the LHS equals the RHS.

b $11 = 3x - 7$

$$\begin{array}{ccc} & 3x-7=11 & \\ +7 \downarrow & & \downarrow +7 \\ & 3x=18 & \\ \div 2 \downarrow & & \downarrow \div 2 \\ & x=6 & \end{array}$$

Swap the LHS and RHS.

Add 7 to both sides to undo the -7, since $-7 + 7 = 0$.

$3x = 3 \times x$, so divide both sides by 3.

Check:

$$\begin{array}{l|l} \text{LHS} = 11 \checkmark & \text{RHS} = 3x - 7 \\ & \quad = 3 \times 6 - 7 \\ & \quad = 18 - 7 \\ & \quad = 11 \checkmark \end{array}$$

Check your solution using substitution.

5 Solve each of the following equations. Check your solution.

a $2x + 7 = 15$ **b** $5a + 6 = 11$ **c** $16 = 3m + 4$
d $2x + 4 = 12$ **e** $2n + 13 = 17$ **f** $-7 = 3x + 5$
g $5b - 3 = 12$ **h** $3y - 2 = 19$ **i** $20 = 7a - 8$
j $4b - 7 = 25$ **k** $2x - 4 = -6$ **l** $-15 = 10y - 35$

In parts **c**, **f**, **i** and **l** swap the LHS & RHS.

Example 16 Solving equations with negative coefficients

Solve the following equations. Check your solution.

a $9 - 2x = 15$ **b** $7 - 4x = 10$

Solution

Explanation

a

$$\begin{array}{rrcll} -9 & 9 - 2x &=& 15 & -9 \\ \div -2 & -2x &=& 6 & \div -2 \\ & x &=& -3 & \end{array}$$

Check:

$$\begin{array}{rl|l} \text{LHS} &= 9 - 2x & \text{RHS} = 15 \checkmark \\ &= 9 - 2 \times (-3) & \\ &= 9 + 6 & \\ &= 15 \checkmark & \end{array}$$

Subtract 9 from both sides, leaving $-2x$ on the LHS.

Divide both sides by -2; recall that a positive number divided by a negative number is negative.

Check your solution using substitution.

b

$$\begin{array}{rrcll} -7 & 7 - 4x &=& 10 & -7 \\ \div -4 & -4x &=& 3 & \div -4 \\ & x &=& -\dfrac{3}{4} & \end{array}$$

Check:

$$\begin{array}{rl|l} \text{LHS} &= 7 - 4x & \text{RHS} = 10 \checkmark \\ &= 7 - 4 \times \left(-\dfrac{3}{4}\right) & \\ &= 7 + 3 & \\ &= 10 \checkmark & \end{array}$$

Subtract 7 from both sides.

Divide both sides by -4. Leave your answer in fraction form; i.e. $3 \div (-4) = -\dfrac{3}{4}$.

Check your solution using substitution.

6 Solve each of the following equations. Check your solution.

a $12 - 2x = 18$ **b** $2 - 7x = 9$
c $15 - 5x = 25$ **d** $19 = 3 - 2x$
e $2 - 5x = -8$ **f** $24 - 7x = 10$
g $14 - 8x = -2$ **h** $-21 = 3 - 4x$

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7 Solve these equations. Leave your answer in fraction form.

a $2x + 5 = 8$
b $5x - 1 = 2$
c $7 = 11x - 3$
d $6 - 2x = 5$
e $4 - 3x = 5$
f $4 = 8 - 5x$

If $5x = 3$, $x = \frac{3}{5}$ (divide both sides by 5).
The answer can be left as a fraction.

Problem-solving and Reasoning

8 For each of the following, write an equation and solve it to find the unknown value. Use x as the unknown value.

a If 8 is added to a certain number, the result is 34.
b Seven less than a certain number is 21.
c I think of a number, double it and add 4. The result is 10.
d I think of a number, triple it and subtract 4. The result is 11.
e Four less than three times a number is 20.
f I start with the number 25 and subtract twice a certain number. The result is 7.

Form the algebraic expression (e.g. $x + 8$) and set it equal to the result to make the equation.

9 Describe the error made in each of these incorrect solutions.

a
$$\begin{aligned} 3x + 2 &= 8 \\ 3x &= 10 \\ x &= \frac{10}{3} \end{aligned}$$

b
$$\begin{aligned} 2x - 1 &= 5 \\ 2x &= 6 \\ x &= 12 \end{aligned}$$

c
$$\begin{aligned} 5 - x &= 12 \\ x &= 7 \end{aligned}$$

d
$$\begin{aligned} 2x + 3 &= 8 \\ x + 3 &= 4 \\ x &= 1 \end{aligned}$$

10 Solve these equations.

a $-3 + 4x = 17$
b $-5 - 2x = 13$
c $-10 - 3x = -4$

Enrichment: More than one solution

11 These equations have exactly two solutions. Can you guess both?

a $x^2 = 9$
b $x^2 = x$
c $x^2 = 2x$

12 Find three solutions for the equation $x^3 - x = 0$.

They are whole numbers between –5 and 5.

3F Solving linear equations involving fractions

Stage
5.2
5.2◊
5.1
4

The division symbol is usually not seen in equations. Division is usually expressed as fractions. For example:

'Half of a number is 18' could be written as $n \div 2 = 18$

but is usually written as $\frac{n}{2} = 18$.

Let's start: Fractions of x

Consider these problems involving fractions.

- If half of x is 8, what is x?
- If a third of x is 5, what is x?
- If three-quarters of x is 30, what is x?
  - State what you did to x to find its value each time.
  - Write an equation to represent each case.
  - Discuss how your method for finding the answer relates to the fraction in the equation.

Key ideas

- Pronumerals with a fraction coefficient can be expressed in fraction form.
  For example: 'One-third of a number' can be written as $x \div 3$ or $\frac{1}{3}x$ or $\frac{x}{3}$.
- Equations involving fractions can be solved by creating a series of simpler equivalent equations.
- At some stage, both sides of the equation will be multiplied by the number in the denominator of the fraction. The order of when this happens is important.

  For example:

$$\begin{array}{rcl} & \frac{x}{5}+3=7 & \\ -3 & & -3 \\ & \frac{x}{5}=4 & \\ \times 5 & & \times 5 \\ & x=20 & \end{array} \qquad \Bigg| \qquad \begin{array}{rcl} & \frac{x+3}{5}=7 & \\ \times 5 & & \times 5 \\ & x+3=35 & \\ -3 & & -3 \\ & x=32 & \end{array}$$

- It is always a good idea to check your solution using substitution.
  For example:
  Equation: $\frac{x}{5}+3=7$
  Possible solution: $x = 20$

$$\begin{array}{l|l} \text{LHS} = \frac{x}{5}+3 & \text{RHS} = 7 \checkmark \\ = \frac{20}{5}+3 & \\ = 7 \checkmark & \end{array}$$

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Exercise 3F

Understanding

1 Write down the value of x that is the solution to these equations. No written working is required.

a $\dfrac{x}{2} = 10$ **b** $\dfrac{x}{4} = 6$ **c** $\dfrac{x+1}{3} = 4$ **d** $\dfrac{x}{5} - 2 = 1$

2 Which one of the following is an equivalent equation to $\dfrac{x}{4} + 1 = 8$?

A $x + 1 = 32$ **B** $x = 36$ **C** $\dfrac{x}{4} = 7$ **D** $\dfrac{x+1}{4} = 2$

3 Match each equation in the first column with its equivalent equation in the second column and state the single operation that generates the equivalent equation (for example, multiply both sides by 5).

a $\dfrac{x}{3} - 1 = 2$	**A** $x + 2 = 3$
b $\dfrac{x}{4} = 2$	**B** $x - 1 = 6$
c $\dfrac{x+2}{3} = 1$	**C** $x = 8$
d $\dfrac{x-1}{2} = 3$	**D** $\dfrac{x}{3} = 3$

Think about which step you would apply first to produce a simple equivalent equation.

Fluency

Drilling for Gold 3F1

Example 17 Solving linear equations with fractional coefficients

Solve each of the following equations.

a $\dfrac{2x}{3} = 8$ **b** $\dfrac{x}{4} - 3 = 7$

Solution	**Explanation**
a $\begin{array}{rrcll} +3 & \dfrac{2x}{3} & = & 8 & \times 3 \\ \div 2 & 2x & = & 24 & \div 2 \\ & x & = & 12 & \end{array}$	Multiply both sides by 3 to remove the fraction on the LHS. Then divide both sides by 2.
Check: $\text{LHS} = \dfrac{2\times 12}{3}$ $= \dfrac{24}{3}$ $= 8$ ✓ RHS = 8 ✓	Check your solution using substitution.
b $\begin{array}{rrcll} +3 & \dfrac{x}{4} - 3 & = & 7 & +3 \\ \times 4 & \dfrac{x}{4} & = & 10 & \times 4 \\ & x & = & 40 & \end{array}$	Add 3 to both sides. Multiply both sides by 4.
Check: $\text{LHS} = \dfrac{40}{4} - 3$ $= 10 - 3$ $= 7$ ✓ RHS = 7 ✓	Check your solution using substitution.

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4 Solve each of the following equations.

a $\frac{x}{5}=3$ **b** $\frac{y}{2}=7$ **c** $\frac{2y}{5}=4$ **d** $6=\frac{3x}{2}$

e $\frac{x}{4}+3=5$ **f** $\frac{x}{2}+4=5$ **g** $\frac{b}{3}+5=9$ **h** $2=5+\frac{t}{2}$

i $\frac{a}{3}+4=2$ **j** $\frac{y}{5}-4=2$ **k** $\frac{x}{3}-7=-2$ **l** $-1=\frac{s}{2}-3$

m $\frac{x}{4}-5=-2$ **n** $\frac{m}{4}-2=-3$ **o** $\frac{y}{5}+4=-3$ **p** $-2=\frac{x}{3}+5$

In part **e**, the first step is to subtract 3 from both sides.

Example 18 Solving simple fractional equations

Solve the equation $\frac{x+4}{6}=2$.

Solution	**Explanation**
$\frac{x+4}{6}=2$ $\times 6$ $\quad$ $\times 6$ $x+4=12$ -4 $\quad$ -4 $x=8$	Multiply both sides by 6 first since all of $(x+4)$ is divided by 6. Subtract 4 from both sides and check the answer.
Check: $\text{LHS}=\frac{8+4}{6}$ $\quad$ RHS = 2 ✓ $=\frac{12}{6}$ $=2$ ✓	Check your solution using substitution.

5 Solve each of the following equations. Check your answers.

a $\frac{x+1}{3}=4$ **b** $\frac{x+4}{2}=5$ **c** $\frac{y+4}{3}=2$

d $3=\frac{b+6}{2}$ **e** $\frac{y-2}{3}=6$ **f** $\frac{t-4}{5}=7$

g $\frac{k-1}{7}=8$ **h** $7=\frac{x-7}{9}$ **i** $\frac{x+2}{4}=-2$

j $\frac{b+3}{7}=-3$ **k** $\frac{y-3}{5}=-5$ **l** $-7=\frac{a-2}{2}$

Think carefully about the order of the steps here.

6 Solve this mixed selection of equations.

a $2t+1=7$ **b** $\frac{m}{4}-2=8$ **c** $9=5-2y$

d $\frac{x-4}{5}=3$ **e** $3+\frac{y}{6}=2$ **f** $-8=4t-5$

Problem-solving and Reasoning

7 For each of the following, write an equation and solve it to find the unknown value. Use x as the unknown value.

a If a certain number is divided by 3, the result is 12.
b A certain number is doubled, then divided by 5. The result is 4.
c I think of a number, halve it and subtract 4. The result is 10.
d I think of a number, add 3 and divide this by 4. The result is 6.
e A number is multiplied by 7 and the product is divided by 3. The final result is 8.
f 5 is added to a third of a certain number. The result is 16.

x halved is $\frac{x}{2}$.
A third of x is $\frac{x}{3}$.

8 A bag of chocolate eggs containing x eggs is shared equally between India and her two brothers. After India eats 2, she has 5 left. How many eggs were in the bag?

Start with an expression for the number of eggs India first receives.

9 Describe the error made in each of these incorrect solutions. Then write out the correct solution.

a
$$\frac{x+2}{3}=7$$
$$\frac{x}{3}=5$$
$$x=15$$

b
$$\frac{x}{3}-4=2$$
$$x-4=6$$
$$x=10$$

10 Solve these equations, which involve more than two steps.

a $\frac{2x}{3}-1=7$ **b** $\frac{3x}{4}-2=7$ **c** $1=3+\frac{x}{2}$

d $\frac{3b-8}{2}=5$ **e** $\frac{2x+2}{3}=4$ **f** $9=\frac{7m-8}{3}$

g $\frac{5-y}{3}=2$ **h** $\frac{4-2t}{6}=3$ **i** $5=1-\frac{4x}{3}$

Treat these just like a two-step equation but add an extra step.

Enrichment: Two fractions and two pronumerals

11 Find a value for x and a different value for y that make these equations true. There are many correct solutions.

a $\frac{x}{2}+\frac{y}{2}=2$ **b** $\frac{x}{3}+\frac{y}{3}=10$ **c** $\frac{x}{2}-\frac{y}{2}=1$ **d** $\frac{x}{2}-\frac{y}{3}=1$

3G Linear equations with brackets

Stage
5.2
5.2◊
5.1
4

We know that $3(x + 2)$ is equivalent to $3x + 6$. Likewise, the equation $3(x + 2) = 10$ has the same solutions as $3x + 6 = 10$.

Let's start: To expand or not to expand?

The steps to solve two problems involving brackets are listed here in the incorrect order.

1 $3(x + 4) = 14$

$x = \frac{2}{3}$

$3x + 12 = 14$

$3x = 2$

2 $3(x + 2) = 12$

$x = 2$

$x + 2 = 4$

- Arrange the steps in the correct order, working from top to bottom.
- By considering all the steps in the correct order, explain what has happened in each step.
- Apply method 1 to the second equation.

Solving mathematical problems (like many other procedures and puzzles) requires steps to be done in the correct order.

Key ideas

- Equations with brackets can be solved by first expanding the brackets and then solving the remaining equation.
  For example: $3(x + 1) = 2$ becomes $3x + 3 = 2$.
- Sometimes it is wise to swap the LHS and RHS.
  For example: $12 = 3(x + 1)$ becomes $3(x + 1) = 12$.
- You can check if a solution is correct by substituting the solution.
  For example:
  Equation: $2(x + 3) = 18$
  Possible solution: $x = 5$
  Check: LHS $= 2(x + 3)$ | RHS $= 18$ ✘
  $= 2 \times (5 + 3)$
  $= 2 \times 8$
  $= 16$ ✘
  $\therefore\ x = 5$ is *not* a solution because LHS ≠ RHS.

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Exercise 3G

Understanding

1 Complete the missing parts.

a $3(x+7) = 3\times\square + \square\times 7$
$= 3x + \square$

b $4(2x-1) = \square\times 2x + 4\times\square$
$= \square - 4$

2 Expand the brackets.

a $4(x+3)$ **b** $5(x+2)$ **c** $7(x-6)$
d $3(2x-5)$ **e** $4(3x+4)$ **f** $3(1-2x)$

3 Which equations have $x = 2$ as a solution?.

a $3(x+1) = 3$ **b** $3(x+1) = 6$ **c** $3(x+1) = 9$
d $5(x+6) = 40$ **e** $4(x-3) = 4$ **f** $2(x-2) = 2$

4 Which equations are showing a correct solution?

a $2(x+3) = 20$, $x = 7$
b $4(x+3) = 20$, $x = 2$
c $4(x-3) = 20$, $x = 7$

Fluency

5 Fill in the boxes.

a
$$\begin{aligned} 2(x+3) &= 20 \\ 2x + \square &= 20 \\ -6 \qquad 2x &= \square \qquad -6 \\ \div 2 \qquad x &= \square \qquad \div 2 \end{aligned}$$

b
$$\begin{aligned} 5(x-2) &= 13 \\ 5x - \square &= 13 \\ +10 \qquad 5x &= \square \qquad +10 \\ \div 5 \qquad x &= \square \qquad \div 5 \end{aligned}$$

Example 19 Solving equations with brackets

Solve: $2(3x-4) = 11$.

Solution	**Explanation**
$2(3x-4) = 11$	Expand the brackets:
$+8 \quad 6x - 8 = 11 \quad +8$	$2(3x-4) = 2\times 3x + 2\times(-4)$
$\div 6 \quad 6x = 19 \quad \div 6$	Solve the remaining equation by adding 8 to both sides, then dividing both sides by 6.
$x = \frac{19}{6}$ or $3\frac{1}{6}$	Leave your answer in fraction form.
Check:	
$\text{LHS} = 2\left(3\times\frac{19}{6}-4\right)$ | RHS = 11 ✓	Check your solution using substitution.
$= 2\times\frac{11}{2}$	
$= 11$ ✓	

6 Solve each of the following equations by first expanding the brackets.

a $2(x + 3) = 11$ **b** $5(a + 3) = 18$ **c** $3(m + 4) = 31$
d $5(y - 7) = 12$ **e** $4(p - 5) = 15$ **f** $2(k - 5) = 9$
g $4(5 - b) = 21$ **h** $13 = 2(1 - m)$ **i** $5(3 - x) = 19$
j $8 = 7(2a + 1)$ **k** $30 = 4(3x - 2)$ **l** $0 = 3(3n - 2)$

In equations like **h** and **j**, begin by swapping the LHS and RHS.

7 Match the equations (**a**–**e**) with their descriptions (**A**–**E**).

a $3(x + 2) = 8$

b $2(x + 3) = 8$

c $3(x - 2) = 8$

d $2(x - 3) = 8$

e $2(3 - x) = 8$

A A number is increased by 3.
The result is doubled to give 8.

B A number is decreased by 2.
The result is tripled to give 8.

C A number is subtracted from 3.
The result is doubled to give 8.

D Three is subtracted from a number.
The result is doubled to give 8.

E A number is increased by 2.
The result is tripled to give 8.

Skillsheet 3B

Problem-solving and Reasoning

Drilling for Gold 3G1 at the end of this section

8 Using x for the unknown number, write down an equation and then solve it to find the number.

a Three times 1 more than a number is 4.
b Twice 2 less than a number is 19.
c The product of 2 and 3 more than a number is 7.
d The product of 3 and 4 less than a number is 8.
e When 2 less than 3 lots of a number is doubled, the result is 5.
f When 5 more than 2 lots of a number is tripled, the result is 10.

You will need to use brackets when setting up the expressions in these equations.

9 Since Rema started her job, her original hourly wage ($\$x$) has been tripled, then decreased by \$6. Her pay is now to be doubled so that she earns \$18 an hour. What was her original hourly wage?

Tripling Rema's pay of $\$x$ is $3 \times x = 3x$.

10 Consider the equation $3(x - 2) = 9$.

a Solve the equation by first dividing both sides by 3.
b Solve the equation by first expanding the brackets.
c Which of the two methods above is preferable and why?

You may recall this idea from the 'Let's start' activity at the beginning of the section.

Enrichment: Change the equation

11 The solution to these equations is not $x = 5$. You may change one number or symbol so that the solution is $x = 5$.

a $4(x + 2) = 17$ **b** $6(x - 7) = -10$ **c** $2(7 - x) = 0$

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3G1: Equations match-up

Match each equation (**a**–**h**) to questions **1**–**12** below.
Write the answers in your exercise book or in the spaces provided in the worksheet. The first one has been done for you.

a $n - 5 = 20$ **b** $n + 5 = 20$ **c** $5 - n = 20$ **d** $\frac{n}{2} + 5 = 20$

e $5n = 20$ **f** $\frac{n}{5} = 20$ **g** $2n + 5 = 20$ **h** $2(n + 5) = 20$

1 A $n + 5 = 20$ number is increased by 5 to give 20. $n + 5 = 20$

2 Five times a number is 20.

3 Dividing a number by 5 gives 20.

4 A number is doubled and then 5 is added to give 20.

5 A number is increased by 5 and then the result is doubled to give 20.

6 What number can be subtracted from 5 to give 20?

7 The sum of a number and 5 is 20.

8 A number is halved, then 5 is added to give 20.

9 The product of 5 and a number is 20.

10 One-fifth of a number is 20.

11 A number is decreased by 5 to give 20.

12 Five more than half of a number is 20.

Drilling for Gold exercise

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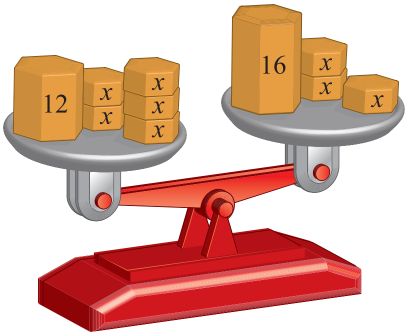
3H Equations with pronumerals on both sides

Stage
5.2
5.2◊
5.1
4

Some linear equations have pronumerals on both sides of the equation.
In the diagram:

- 12 could be subtracted from both sides.
- $3x$ could be subtracted from both sides.



This equation has numerals on both sides and pronumerals on both sides.

▶ Let's start: Which method? Same solution?

There are two methods to solve $5x + 12 = 3x + 16$.

Method 1

Start by subtracting $3x$ from both sides.

$$\begin{array}{rcl} -3x & 5x + 12 = 3x + 16 & -3x \\ -12 & \square + 12 = 16 & -12 \\ \div 2 & \square = \square & \div 2 \\ & x = \square & \end{array}$$

Method 2

Start by subtracting 12 from both sides.

$$\begin{array}{rcl} -12 & 5x + 12 = 3x + 16 & -12 \\ -3x & 5x = 3x + \square & -3x \\ \div 2 & \square = \square & \div 2 \\ & x = \square & \end{array}$$

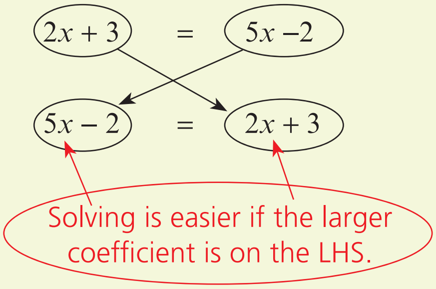
Are both of these correct? Check by substitution.

Key ideas

- If an equation has pronumerals and numbers on both sides, add or subtract one of the terms.
For example:

$$\begin{array}{rcl} -2x & 6x + 4 = 2x - 3 & -2x \\ & 4x + 4 = -3 & \end{array} \qquad \text{OR} \qquad \begin{array}{rcl} -4 & 6x + 4 = 2x - 3 & -4 \\ & 6x = 2x - 7 & \end{array}$$

- At any time, the LHS (left-hand side) and RHS (right-hand side) can be swapped.
For example:



$$2x + 3 = 5x - 2$$

$$5x - 2 = 2x + 3$$

Solving is easier if the larger coefficient is on the LHS.

Exercise 3H

Understanding

1 In each of the following equations, swap the LHS and RHS. Does this make the equation easier to solve?

a $5x = 2x + 3$ **b** $3x - 5 = 7x + 3$

c $6x + 1 = 2x + 5$ **d** $1 - 2x = 3x - 9$

The x coefficient is the numeral multiplied by the x.

2 Copy and complete the following.

a
$$\begin{array}{rcl} & 7x = 3x + 4 & \\ -3x & \downarrow & -3x \\ & \boxed{} = 4 & \\ \div 4 & \downarrow & \div 4 \\ & x = \boxed{} & \end{array}$$

b
$$\begin{array}{rcl} & 9x + 2 = 5x + 10 & \\ -5x & \downarrow & -5x \\ & \boxed{} + 2 = 10 & \\ -2 & \downarrow & -2 \\ & \boxed{} = \boxed{} & \\ \div \boxed{} & \downarrow & \div \boxed{} \\ & x = & \end{array}$$

3 Copy and complete the following.

a
$$\begin{array}{rcl} & 4x + 2 = 8 - 2x & \\ +2x & \downarrow & +2x \\ & \boxed{} + 2 = 8 & \\ -2 & \downarrow & -2 \\ & \boxed{} = \boxed{} & \\ \div \boxed{} & \downarrow & \div \boxed{} \\ & x = \boxed{} & \end{array}$$

b
$$\begin{array}{rcl} & 4x + 2 = 8 - 2x & \\ -2 & \downarrow & -2 \\ & \boxed{} = \boxed{} - 2x & \\ +2x & \downarrow & +2x \\ & \boxed{} = \boxed{} & \\ \div \boxed{} & \downarrow & \div \boxed{} \\ & x = \boxed{} & \end{array}$$

4 Copy and complete the table to solve the equation $2x + 4 = 5x - 2$ in two different ways.

Method 1 steps	Results	Method 2 steps	Results
Swap the LHS and RHS		Swap the LHS and RHS	
Subtract $2x$ from both sides		Add 2 to both sides	
Add 2 to both sides		Subtract $2x$ from both sides	
Divide both sides by 3		Divide both sides by 3	

Fluency

5 Are these solutions true or false?

	Equation	Possible solution
a	$3x = x + 6$	$x = 6$
b	$4x - 3 = 2x + 1$	$x = 2$
c	$x - 3 = 2x + 3$	$x = -6$
d	$4x + 2 = 8 - 2x$	$x = 1$
e	$3x - 1 = 3 - x$	$x = 1$

Use substitution.

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Example 20 Solving equations with variables on both sides

Solve these equations with variables on each side.

a $4x = 2x + 10$ **b** $3x = 16 - 5x$

Solution	Explanation
a $\begin{array}{rrcll} -2x & 4x &=& 2x + 10 & -2x \\ \div 2 & 2x &=& 10 & \div 2 \\ & x &=& 5 & \end{array}$	Collect x terms on LHS (since the x coefficient is larger on that side) by subtracting $2x$ from both sides: $4x - 2x = 2x$. Solve by dividing both sides by 2.
Check: LHS $= 4 \times 5$ $= 20$ ✓ | RHS $= 2 \times 5 + 10$ $= 20$ ✓	Check your solution using substitution.
b $\begin{array}{rrcll} +5x & 3x &=& 16 - 5x & +5x \\ \div 8 & 8x &=& 16 & \div 8 \\ & x &=& 2 & \end{array}$	Collect x terms by adding $5x$ to both sides (collect on LHS since $3x$ coefficient is positive). Divide both sides by 8.
Check: LHS $= 3 \times 2$ $= 6$ ✓ | RHS $= 16 - 5 \times 2$ $= 6$ ✓	Check your solution using substitution.

6 Solve these equations by first collecting the pronumeral on one side.

a $7x = 5x + 8$
b $8x = 3x + 15$
c $9x = 5x + 12$
d $10x = 9x + 7$
e $4x = x - 9$
f $6x = 2x - 8$
g $3x = 10 - 2x$
h $x = 12 - 2x$
i $x = 12 - x$
j $4x = 14 - 3x$

Remember: It is easier to collect on the side that will keep the x coefficient positive.

Example 21 Solving equations with pronumerals and numerals on both sides

Solve each of the following equations.

a $5x + 2 = 3x + 6$ **b** $3 - 2x = 5x - 4$

Solution	Explanation
a $\begin{array}{rrcll} -3x & 5x + 2 &=& 3x + 6 & -3x \\ -2 & 2x + 2 &=& 6 & -2 \\ \div 2 & 2x &=& 4 & \div 2 \\ & \therefore\ x &=& 2 & \end{array}$	Collect x terms on one side by subtracting $3x$ from both sides: $5x - 3x = 2x$. Collect on the side with the larger x term to keep it positive. Solve the remaining equation.
Check: LHS $= 5 \times 2 + 2$ $= 12$ ✓ | RHS $= 3 \times 2 + 6$ $= 12$ ✓	Check your solution using substitution.
b $\begin{array}{rrcll} +2x & 3 - 2x &=& 5x - 4 & +2x \\ +4 & 3 &=& 7x - 4 & -2 \\ \div 7 & 7 &=& 7x & \div 7 \\ & 1 &=& x & \\ & \therefore\ x &=& 1 & \end{array}$	Add $2x$ to both sides to collect x terms on the side with the positive x coefficient. Solve the remaining equation by adding 4 to both sides and dividing both sides by 7. Swap the LHS and RHS.

Alternative method:

$$\begin{aligned} 3-2x &= 5x-4 \\ +2x \quad 5x-4 &= 3-2x \quad +2x \\ 7x-4 &= 3 \\ +4 \quad 7x &= 7 \quad +4 \\ \div 7 \quad x &= 1 \quad \div 7 \end{aligned}$$

Swap LHS and RHS.
Add $2x$ to both sides.
Add 4 to both sides.
Divide both sides by 7.
Don't forget to check your solution using substitution.

7 Solve each of the following equations.

a $5x-3=4x+5$ **b** $9a+3=8a+6$ **c** $3m-8=2m$
d $12x-3=10x+5$ **e** $8x+4=12x-16$ **f** $3x+7=8x-8$
g $2x+6=9-x$ **h** $3y+6=14-y$ **i** $5m-18=15-6m$
j $4-7x=2x-23$ **k** $8-2b=4b+14$ **l** $3-4m=3m+24$

Swap LHS and RHS?

Problem-solving and Reasoning

Skillsheet 3C

8 Using x for the unknown number, write down an equation and then solve it to find the number. The first one has been started for you.

a Twice a number is equal to 4 less than 3 times a number. ($2x=3x-4$)
b 4 more than 2 lots of a number is equal to 5 times the number.
c 10 more than 3 lots of a number is equivalent to 5 lots of the number.
d 2 more than 3 times the number is equivalent to 6 less than 5 times the number.
e 1 less than a doubled number is equivalent to 5 more than 3 lots of the number.
f 4 more than 2 lots of a number is equivalent to the number subtracted from 13.

9 Mardy is x years old. He correctly tells his mother that she is 6 years older than 3 times his age. His mother replies that she is also 10 years younger than 5 times his age. How old is Mardy?

Build the equation step by step. You should get two expressions for the mother's age.

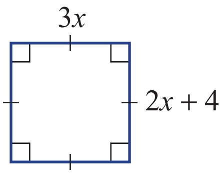
10 Consider the equation $3x+1=5x-7$.

a Solve the equation by first subtracting $3x$ from both sides.
b Solve the equation by first subtracting $5x$ from both sides.
c Which method above do you prefer and why? Describe the differences.

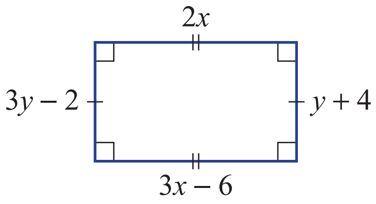
Enrichment: Matching sides

11 Use the properties of these basic shapes to find their perimeters.

a

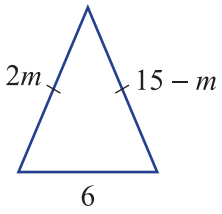


b

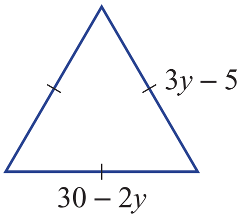


Find the value of each pronumeral by first solving an equation.

c



d



3I Using linear equations to solve problems

EXTENSION

Stage
5.2
5.2◊
5.1
4

Many problems can be solved using equations. Often problems are expressed only in words. Reading and understanding the problem, defining a pronumeral and writing an equation are all important steps in solving the problem.

Let's start

A 30 metre rope breaks into two pieces.
One is 8 metres longer than the other.
How long are the pieces?

- In how many different ways can your classmates solve this problem?
- Can you solve it using an equation?

Key ideas

To solve a problem, follow these steps.

1 Define pronumerals to stand for unknown numbers.	**1** Let x = Julian's age.
2 Write an equation to describe the situation.	**2** $x + x + 4 = 32$
3 Solve the equation by inspection or systematically.	**3** $\begin{array}{rrcll} -4 & 2x+4 &=& 32 & -4 \\ \div 2 & 2x &=& 28 & \div 2 \\ & x &=& 14 & \end{array}$
4 Make sure you answer the original question, including the correct units (e.g. dollars, years, cm).	**4** Julian is 14 years old and his sister is 18.

Exercise 3I EXTENSION

Understanding

1 Write an equation for the following scenario.
The mass of a bag is x kg.
Another bag weighs twice as much.
Together they weigh 30 kg.

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2 Match the written scenario (**a**–**d**) with its possible equation (**A**–**D**).

a 7 is 3 more than a number	**A** $3x + 1 = 16$
b 2 less than a number is 10	**B** $\frac{a}{2} = 3$
c 16 is 1 more than 3 lots of a number	**C** $n + 3 = 7$
d half of a number is 3	**D** $y - 2 = 10$

3 For each of the following examples, make x the unknown number and set up an equation to represent the problem. There is no need to solve the equation.

a Three less than a certain number is equal to 5.

b Seven is added to two lots of a number and the result is 9.

c 2 more than a certain number is divided by 3. The result is 4.

d 3 is added to a certain number, which has been halved. The result is 6.

4 The sum of the ages of Sam and his brother Bernard is 34. If Sam is 4 years older than Bernard, fill in the following to find their ages.

Let x be the ☐ of Bernard.

The age of Sam is ☐.

The sum of their ages is 34.

$\therefore x +$ ☐ $= 34$

☐ $+ 4 = 34$

$2x =$ ☐

$x =$ ☐

$\therefore$ Bernard is _______ years old and Sam is ________ years old.

Start by underlining the key information.

Fluency

Example 22 Turning a word problem into an equation

Drilling for Gold 3I1

Five less than three times a certain number is 13. Write an equation and solve it to find the number.

Solution	Explanation
Let x be the number. $3x - 5 = 13$ $+5$ $+5$ $3x = 18$ $\div 3$ $\div 3$ $x = 6$	Define the unknown as a pronumeral. Interpret the wording bit by bit to construct the equation. Three times the number is $3x$, 5 less than this is $3x - 5$ and this equals 13. Solve the equation by adding 5 to both sides, then dividing both sides by 3.
Check: $3 \times 6 - 5 = 13$ ✓	Check that the solution is reasonable.
The number is 6.	Write the answer in words.

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5 Write an equation and solve it to find the unknown number in the following.

a 4 less than 2 times a number is 10.

b When a certain number is doubled, it results in a number that is 5 more than the original number.

c I think of a number, divide it by 3 and add 5. The result is 12.

d I think of a number, take away 2 and multiply the result by 4. This gives 24.

e 3 less than a certain number is 9 less than 4 times the number.

Example 23 Applying algebra to a word problem

A bicycle shop hires out bikes. It charges an initial fee of $10 for hiring a bike and then a charge of $8 per hour. Leah returns her bike and is charged $42. For how many hours did she hire the bike?

Solution	**Explanation**
Let h be the number of hours of hire.	Define the unknown value as a pronumeral.
$\begin{aligned} -10 \quad 10 + 8h &= 42 \quad -10 \\ \div 8 \quad 8h &= 32 \quad \div 8 \\ \therefore h &= 4 \end{aligned}$	Write an equation from the information – the cost is $10 plus $8 per hour ($8 \times h$), which equals $42. Solve the equation for h.
Check: $\$10 + \$8 \times 4 = \$42$ ✓	Check that the answer is reasonable.
∴ Leah had the bike for 4 hours.	Answer the question in words.

6 Toby rents a car for a total cost of $290. The rental company charges $40 per day, plus a hiring fee of $50.

a Define a pronumeral for the number of days Toby rents the car.

b Write an equation in terms of your pronumeral in part **a** to represent the problem.

c Solve the equation in part **b** to find the unknown value.

d For how many days does Toby rent the car?

What will you define your pronumeral as?

7 A jeweller earns a weekly amount of $200 plus $10 per item she sells. If in 1 week she earned $680, how many items did she sell?

Example 24 Solving word problems with equations

Ed and Michael made 254 runs between them in a cricket match. If Michael made 68 more runs than Ed, how many runs did each of them make?

Solution	**Explanation**
Let the number of runs for Ed be r. Number of runs Michael made is $r + 68$. $r + (r + 68) = 254$ $2r + 68 = 254$ -68 / -68 $2r = 186$ $\div 2$ / $\div 2$ $r = 93$ Ed made 93 runs and Michael made $93 + 68 = 161$ runs.	Define the unknown value as a pronumeral. Write all other unknown values in terms of r. Michael made 68 more runs than Ed: $r + 68$. Write an equation: Number of runs for Ed + Number of runs for Michael = 254. Collect like terms, then solve the equation. Answer the question in words and check that it is reasonable.

8 Leonie and Emma scored 28 goals between them in a netball match. Leonie scored 8 more goals than Emma.

- **a** Define a pronumeral for the number of goals scored by Emma.
- **b** Write the number of goals scored by Leonie in terms of the pronumeral in part **a**.
- **c** Write an equation in terms of your variable to represent the scenario.
- **d** Solve the equation in part **c** to find the unknown value.
- **e** How many goals did each of them score?

9 A rectangle is four times as long as it is wide and its perimeter is 560 cm.

- **a** Define a pronumeral for the unknown breadth.
- **b** Write an expression for the length in terms of your pronumeral in part **a**.
- **c** Write an equation involving your pronumeral to represent the problem.
- **d** Solve the equation in part **c**.
- **e** What is the length and breadth of the rectangle?

Draw a rectangle to help.

10 A prize of \$1000 is divided between Adele and Benita so that Adele receives \$280 more than Benita. How much does each person receive?

Problem-solving and Reasoning

11 Andrew, Brenda and Camille all work part-time. Camille earns \$20 more than Andrew's weekly wage. Brenda earns \$30 less than twice Andrew's weekly wage. If their total combined wage is \$400, find how much each of these employees earns.

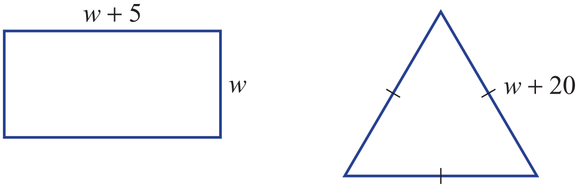
12 Huang walked a certain distance and then ran twice as far as he walked. He then caught a bus for the last 2 km. If he travelled a total of 32 km, find how far Huang walked and ran.

Use the steps in Key ideas.

13 Kate is three times as old as her son. If Kate is 30 years older than her son, what are their ages?

14 Two paddocks in the shapes shown below are to be fenced with wire. If the same total amount of wire is used for each paddock, what are the side lengths of each paddock, in metres?

Solve for w first.



15 Consecutive integers can be represented algebraically as x, $x + 1$, $x + 2$ etc.
- **a** Find three consecutive numbers that add to 84.
- **b** **i** Write three consecutive even numbers starting with x.
  - **ii** Find three consecutive even numbers that add to 18.

Make sure that you are still setting up an equation.

Enrichment: Profit and revenue

16 Tedco produces a teddy bear that sells for \$24. Each teddy bear costs the company \$8 to manufacture and there is an initial start-up cost of \$7200.
- **a** Write a rule for the total cost, $\$T$, of producing x teddy bears.
- **b** If the cost of a particular production run is \$9600, how many teddy bears are manufactured in that run?
- **c** If x teddy bears are sold, write a rule for the revenue, $\$R$, received by the company.
- **d** How many teddy bears were sold if the revenue was \$8400?
- **e** If the company wants to make a profit of \$54 000, how many teddy bears does it need to sell?

Revenue is the total amount of money collected.
Profit is the amount left over from the revenue after the costs have been taken out.
Profit = revenue – costs

3J Using formulas

EXTENSION

Stage

5.2
5.2◊
5.1
4

A formula is an equation that relates two or more variables. The following are some examples of formulas.

- $A = \pi r^2$ is the formula for finding the area, A, of a circle, given its radius, r.
- $V = \ell bh$ is the formula for finding the volume of a rectangular prism, given its length, ℓ, breadth, b, and height, h.
- $F = \frac{9}{5}C + 32$ is the formula for converting degrees Celsius, C, to degrees Fahrenheit, F.

We can find the amount of water in a fish tank if we know the tank's dimensions.

A, V and F are said to be the subjects of the formulas given above.

Let's start: What are these formulas?

Search the internet to find out:

- What each formula represents.
- The meaning of each letter in the formula.

$E = mc^2$	$v = u + at$	$V = \frac{4}{3}\pi r^3$

Key ideas

- The **subject** of a **formula** is a pronumeral that usually sits on its own on the LHS. For example: In $A = \ell b$, A is the subject of the formula.
- A pronumeral in a formula can be evaluated by substituting numbers for all other pronumerals.
  - For example:

$$\begin{aligned} &\text{If } p = 2x + 6y, \text{ where } x = 3 \text{ and } y = 5,\\ &\text{then } p = 2 \times 3 + 6 \times 5\\ &\qquad p = 6 + 30\\ &\qquad p = 36 \end{aligned}$$

Subject The pronumeral that is alone on one side of the equation

Formula A rule for finding the value of one quantity when given the values of others

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Exercise 3J EXTENSION

Understanding

1 Match each formula (**a**–**e**) with its description (**A**–**E**).

a $A = \ell b$

b $A = \frac{1}{2}bh$

c $A = s^2$

d $C = 2\pi r$

e $A = \pi r^2$

A circumference of a circle

B area of a rectangle

C area of a triangle

D area of a circle

E area of a square

2 Write down the letter that is the subject of these formulas.

a $A = \frac{1}{2}bh$ **b** $D = b^2 - 4ac$

c $M = \frac{a+b}{2}$ **d** $A = \pi r^2$

In $V = \ell bh$, V is the subject.

3 The formula for the perimeter of a rectangle is $P = 2\ell + 2b$.

a What does ℓ represent?

b What does b represent?

c Why does the number 2 appear twice in this formula?

d Could the formula be written as $P = 2(\ell + b)$?

Fluency

4 Using the formula $P = 2(\ell + b)$, find the value of P when:

a $\ell = 8$, $b = 6$ **b** $\ell = 7.5$, $b = 2.5$

5 Using the formula $v = u + at$, find the value of v when:

a $u = 2$, $a = 3$, $t = 4$ **b** $u = 10$, $a = 10$, $t = 10$

6 Using the formula $m = \frac{\text{rise}}{\text{run}}$, find the value of m when:

a run = 3, rise = 6 **b** run = 4, rise = –2

Example 25 Substituting values into formulas

Substitute the given values into the formula to work out the subject.

a $A = \frac{1}{2}h(a+b)$, when $a = 3$, $b = 7$ and $h = 5$. **b** $E = \frac{1}{2}mv^2$, when $m = 4$ and $v = 5$.

Solution	**Explanation**
a $A = \frac{1}{2}h(a+b)$ $A = \frac{1}{2}\times 5\times(3+7)$ $= \frac{1}{2}\times 5\times 10$ $= 25$	Substitute $a = 3$, $b = 7$ and $h = 5$. Work out the sum in the brackets first.

b $E = \frac{1}{2}mv^2$

$E = \frac{1}{2} \times 4 \times 5^2$

$= \frac{1}{2} \times 4 \times 25$

$= 50$

Substitute $m = 4$ and $v = 5$.
$5^2 = 5 \times 5 = 25$

7 Substitute the given values into each of the following formulas to work out the subject. Round your answers to two decimal places where appropriate.

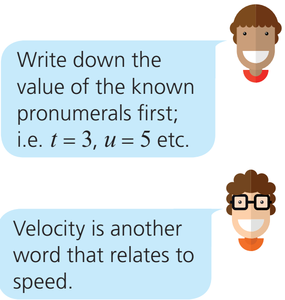
$V = \frac{4}{3}\pi r^3$ is the formula for the volume of a sphere with radius r.

- **a** $A = bh$, when $b = 3$ and $h = 7$
- **b** $F = ma$, when $m = 4$ and $a = 6$
- **c** $m = \frac{a+b}{4}$, when $a = 14$ and $b = -6$
- **d** $t = \frac{d}{v}$, when $d = 18$ and $v = 3$
- **e** $A = \pi r^2$, when $\pi = 3.14$ and $r = 12$
- **f** $V = \frac{4}{3}\pi r^3$, when $\pi = 3.14$ and $r = 2$
- **g** $c = \sqrt{a^2 + b^2}$, when $a = 12$ and $b = 22$
- **h** $Q = \sqrt{2gh}$, when $g = 9.8$ and $h = 11.4$
- **i** $I = \frac{MR^2}{2}$, when $M = 12.2$ and $R = 6.4$
- **j** $x = ut + \frac{1}{2}at^2$, when $u = 0$, $t = 4$ and $a = 10$

Problem-solving and Reasoning

8 The formula $s = \frac{d}{t}$ gives the speed s km/h of a car that has travelled a distance of d km in t hours. Find the speed of a car that has travelled 400 km in 4.5 hours. Round your answers to two decimal places.

9 The velocity, v m/s, of an object is described by the rule $v = u + at$, where u is the initial velocity in m/s, a is the acceleration in m/s^2 and t is the time in seconds.
Find the velocity after 3 seconds if the initial velocity is 5 m/s and the acceleration is 10 m/s^2.



10 The volume of water (V litres) in a tank is given by $V = 4000 - 0.1t$, where t is the time, in seconds, after a tap is turned on.

a Over time, does the water volume increase or decrease according to the formula?

b Find the volume after 2 minutes. (Note: t is in seconds.)

11 The formula $F = \frac{9}{5}C + 32$ converts degrees Celsius, C, to degrees Fahrenheit, F.

Find what each of the following temperatures is in degrees Fahrenheit.

a 100°C **b** 38°C

$\frac{9}{5}C$ can also be written as $\frac{9C}{5}$.

Enrichment: Basketball formulas

12 The formula $T = 3x + 2y + f$ can be used to calculate the total number of points made in a basketball game, where:

x = number of three-point goals
y = number of two-point goals
f = number of free throws made
T = total number of points

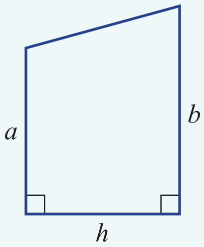
a Find the total number of points for a game in which 12 three-point goals, 15 two-point goals and 7 free throws were made.

b Find the number of three-point goals made if the total number of points was 36, with 5 two-point goals made and 5 free throws made.

Using a formula in a spreadsheet

In some occupations, employees do the same calculation many times, using different numbers.
It is a good idea to use a spreadsheet for this purpose.
The video shows you how to make a spreadsheet that calculates the area of a triangle.
Make the spreadsheet described in the video.
Save it with the filename 'Area of Triangle'.
Make a copy of the file and call it 'Area of trapezium'.
Change the file so that it calculates the area of a trapezium, using the information below.

Video 3A



The area of a trapezium is given by the formula

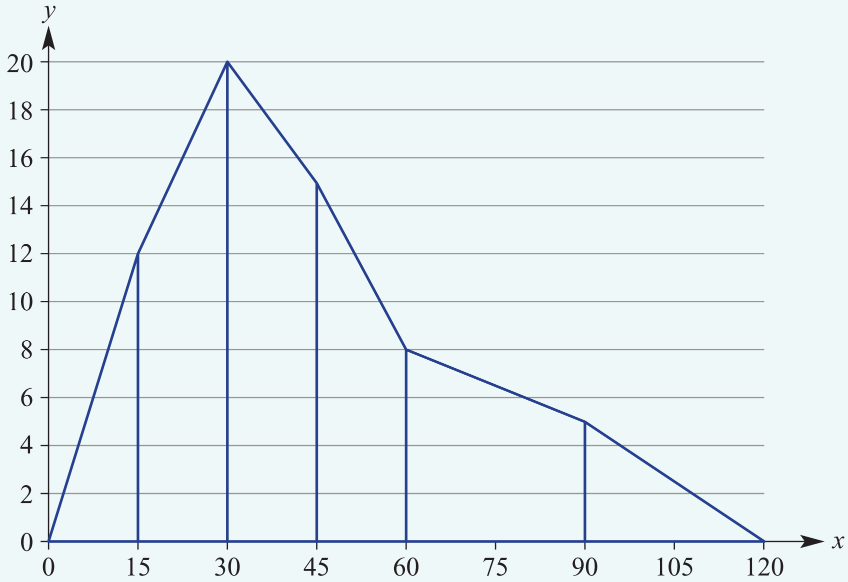
$$A = \frac{1}{2}h(a+b)$$

where:

- h is the perpendicular height
- a and b are the parallel sides.

Use your files to calculate the amount of grass that is needed to cover this block of land.
All measurements are in metres.

x	0	15	30	45	60	90	120
y	0	12	20	15	8	5	0



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Puzzles and games

1 In a magic square, all the rows, columns and main diagonals add to the same total. The total for this magic square is 15. Find the value of x, writing a number in every square.

$2x - 2$	$3x$	$x - 1$
x		

2 Reveal the word below by solving each equation for x. The value of x gives the number of the letter in the alphabet; e.g. $x = 1$ would be A, $x = 2$ would be B etc.

$\dfrac{x-3}{4} = 5$ ☐

$6x = 4x + 30$ ☐

$2(x - 10) = 16$ ☐

$1 - 2x = -7$ ☐

3 Twelve years ago Eric's father was seven times as old as Eric was. If Eric's father is now 54 years old, how old is Eric now?

4 A yacht race has 4 legs. The second leg is half the length of the first leg. The third leg is two-thirds of the length of the second leg. The fourth leg is twice the length of the second leg. If the total distance is 153 km, find the length of each leg.

5 A group of office workers had some prize money to distribute among themselves. When all but one took \$9 each, the last person only received \$5. When they all took \$8 each, there was \$12 left over. How much had they won?

Chapter summary

Expressions and equations

Substitution

The process of replacing a pronumeral with a given value.

e.g. If $x = 3$, $2x + 4 = 2 \times (3) + 4$
$= 6 + 4$
$= 10$

e.g. If $x = 2$, $y = -4$,
then $xy = 2 \times (-4)$
$= -8$

Expanding brackets

e.g. $2(x + 3) = 2x + 6$

$-3(4 - 2x) = -12 + 6x$

Expressions

e.g. $\boxed{2x}\enclose{circle}{+3}y + 5$

term

coefficient

constant

e.g. 2 more than 3 lots of m is $3m + 2$.
3 less than x is $x - 3$.

Solving word problems

1. Define a pronumeral for the unknown.
2. Set up an equation.
3. Solve the equation.
4. Check the answer and write in words.

Addition/subtraction

Only like terms can be combined under addition or subtraction.
Like terms have the same pronumeral factors; i.e. $4xy$ and $2xy$.

e.g. $5a + 2 + 3a$
$= 5a + 3a + 2$
$= 8a + 2$

e.g. $3x + 2y - x + 4y$
$= 3x - x + 2y + 4y$
$= 2x + 6y$

Solving linear equations

The solution of a linear equation is the value that makes the equation true.

e.g.
-5 $\quad 2x + 5 = 9 \quad$ -5
$\div 2$ $\quad 2x = 4 \quad$ $\div 2$
$x = 2$

e.g.
$-2x$ $\quad 5x - 5 = 2x + 7 \quad$ $-2x$
$+5$ $\quad 3x - 5 = 7 \quad$ $+5$
$\div 3$ $\quad 3x = 12 \quad$ $\div 3$
$x = 4$

Multiplication/division

In simplified form the multiplication sign (×) and division sign (÷) are not shown.
That is, $2 \times x = 2x$ and $x \div 2 = \frac{x}{2}$.

Multiplying:
$2a \times 3b = 6ab$

Dividing:
$4x \div 2 = \frac{\overset{2}{\cancel{4}}x}{\underset{1}{\cancel{2}}} = 2x$

$\frac{6ab^2}{9b} = \frac{\overset{2}{\cancel{6}} \times a \times b \times \overset{1}{\cancel{b}}}{\underset{3}{\cancel{9}} \times \underset{1}{\cancel{b}}}$
$= \frac{2ab}{3}$

Formulas

Some common formulas are:
$A = \ell b$, $C = 2\pi r$

An unknown value can be found by substituting values for the other pronumerals.

$A = \ell b$
When $\ell = 3$ and $b = 5$:
$A = 3 \times 5$
$A = 15$

Equations involving fractions

e.g. $\frac{x}{3} + 1 = 5$
$\frac{x}{3} = 4$ (subtract 1)
$x = 12$ (multiply by 3)

e.g. $\frac{x - 2}{4} = 1$
$x - 2 = 4$ (multiply by 4)
$x = 6$ (add 2)

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Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.

Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

Drilling for Gold
3R1
3R2
3R3

1 The algebraic expression that represents 2 less than 3 lots of n is:

A $3(n-2)$ **B** $2-3n$ **C** $3n-2$
D $3+n-2$ **E** n

2 The fully simplified form of $2ab \div (8a)$ is:

A $\frac{2ab}{8a}$ **B** $\frac{4}{b}$ **C** $\frac{b}{4}$
D $4b$ **E** $4a$

3 The simplified form of $6ab + 14a - 2ab + 3a$ is:

A $21ab$ **B** $4ab + 11a$ **C** $ab + 7a$
D $4ab + 17a$ **E** $4 + 17a$

4 The solution of the equation $2x - 3 = 7$ is:

A $x = 2$ **B** $x = -5$ **C** $x = 5$
D $x = 6.5$ **E** $x = 0.5$

5 The solution of $\frac{x}{3} - 1 = 4$ is:

A $x = 13$ **B** $x = 7$ **C** $x = 9$
D $x = 15$ **E** $x = \frac{5}{3}$

6 The expanded form of $3(2y - 1)$ is:

A $6y - 3$ **B** $5y - 1$ **C** $6y - 1$
D $6y + 2$ **E** $3y - 3$

7 An equivalent equation to $5x = 2x + 9$ is:

A $2x = 5$ **B** $3x = 9$ **C** $0 = 3x + 9$
D $7x = 9$ **E** $x = 9$

8 The solution of the equation $2(x - 3) = 7$ is:

A $x = 5$ **B** $x = \frac{13}{2}$ **C** $x = 2$
D $x = \frac{1}{2}$ **E** $x = \frac{7}{3}$

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9 Elijah is x years old. His sister is 2 years older. The sum of their ages is 22. A simplified equation to represent this is:

A $x + 2 = 22$ **B** $2x + 2 = 22$ **C** $2(x + 2) = 22$

D $x(x + 2) = 22$ **E** $2x = 22$

10 If $A = \frac{1}{2}bh$ with $b = 10$ and $h = 40$, then the value of A is:

A 5 **B** 10 **C** 100

D 200 **E** 400

Short-answer questions

1 Write algebraic expressions to represent the following.

- **a** the sum of x and y
- **b** the product of m and 7
- **c** the cost of 3 movie tickets at m dollars each
- **d** 3 less than n, all divided by 4

2 Evaluate the following when $x = 2$, $y = -1$ and $z = 5$.

a $xz + 1$ **b** $4x + y$ **c** $x - 2yz$

d $x(y + z)$ **e** $x^2 - 3z$

3 Simplify the following.

a $2 \times 4n$ **b** $3x \times 2y$

c $8a \div 2$ **d** $\frac{4x^2y}{12x}$

4 Simplify by collecting like terms.

a $2b + 4b + b$ **b** $6x + 3 - 2x$

c $4p - 3q - p + 5q$ **d** $3mn + 2m - 6mn + n$

5 Expand and simplify the following.

a $2(x + 7)$ **b** $3(2x + 5)$ **c** $2x(3x - 4)$

d $-2a(5 - 4a)$ **e** $4(x + 2) + 5$ **f** $3(x - 2) - 1$

6 Solve the following linear equations for x.

a $5x + 6 = 51$ **b** $7x - 4 = 10$

c $7 = 4 - x$ **d** $21 = 3 - 2x$

7 Solve these linear equations involving fractions.

a $\frac{2x}{5} = 4$ **b** $\frac{x}{2} - 1 = 3$

c $7 = \frac{x+2}{4}$ **d** $4 = \frac{1}{3}x + 2$

8 Write an equation to represent each of the following and then solve it for the pronumeral.

a A number, n, is doubled and increased by 3 to give 21.

b A number of lollies, l, is decreased by 5 and then shared equally among three friends so that they each get 7.

c 5 less than the result of Antonia's age, x, divided by 4 is 0.

9 Solve the following linear equations by first expanding the brackets.

a $2(x + 4) = 18$ **b** $3(2x - 3) = 2$

10 Solve these equations with pronumerals on both sides.

a $8x = 2x + 24$ **b** $5x - 2 = 3x + 2$

c $3 - 4x = 7x - 8$ **d** $10x + 20 = 7x + 5$

11 Nick makes an initial bid of $\$x$ in an auction for a signed cricket bat. By the end of the auction he has paid $\$550$, which is $\$30$ more than twice his initial bid, x. Set up and solve an equation to determine Nick's initial bid.

12 Find the value of the unknown in each of the following formulas.

a $E = \sqrt{PR}$ when $P = 90$ and $R = 40$

b $v = u + at$ when $v = 20$, $u = 10$, $t = 2$

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Extended-response questions

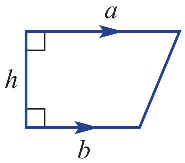
1 Julie hires a jumping castle for her daughter's birthday party. It costs $\$60$ for the set-up, plus $\$25$ for each hour that it is hired.

- **a** What is the cost for:
  - **i** 1 hour of hire?
  - **ii** 2 hours of hire?
- **b** Julie is charged $\$210$ for the hire of the jumping castle.
  - **i** Define a pronumeral to represent the number of hours for which Julie hired the castle.
  - **ii** Set up an equation using your pronumeral.
  - **iii** Solve the equation to find the number of hours.

2 A new backyard deck is being designed in the shape of the trapezium shown.

The area of a trapezium is given by $A = \frac{1}{2}h(a + b)$.

Currently the dimensions are set so that $a = 12$ m and $b = 8$ m.



- **a** If $h = 10$, find the value of A.
- **b** It is decided to increase h to 12 m and increase A to 150 m^2. If b is fixed at 8 m, find the value of a that gives the required area.

Chapter 4

Right-angled triangles

What you will learn

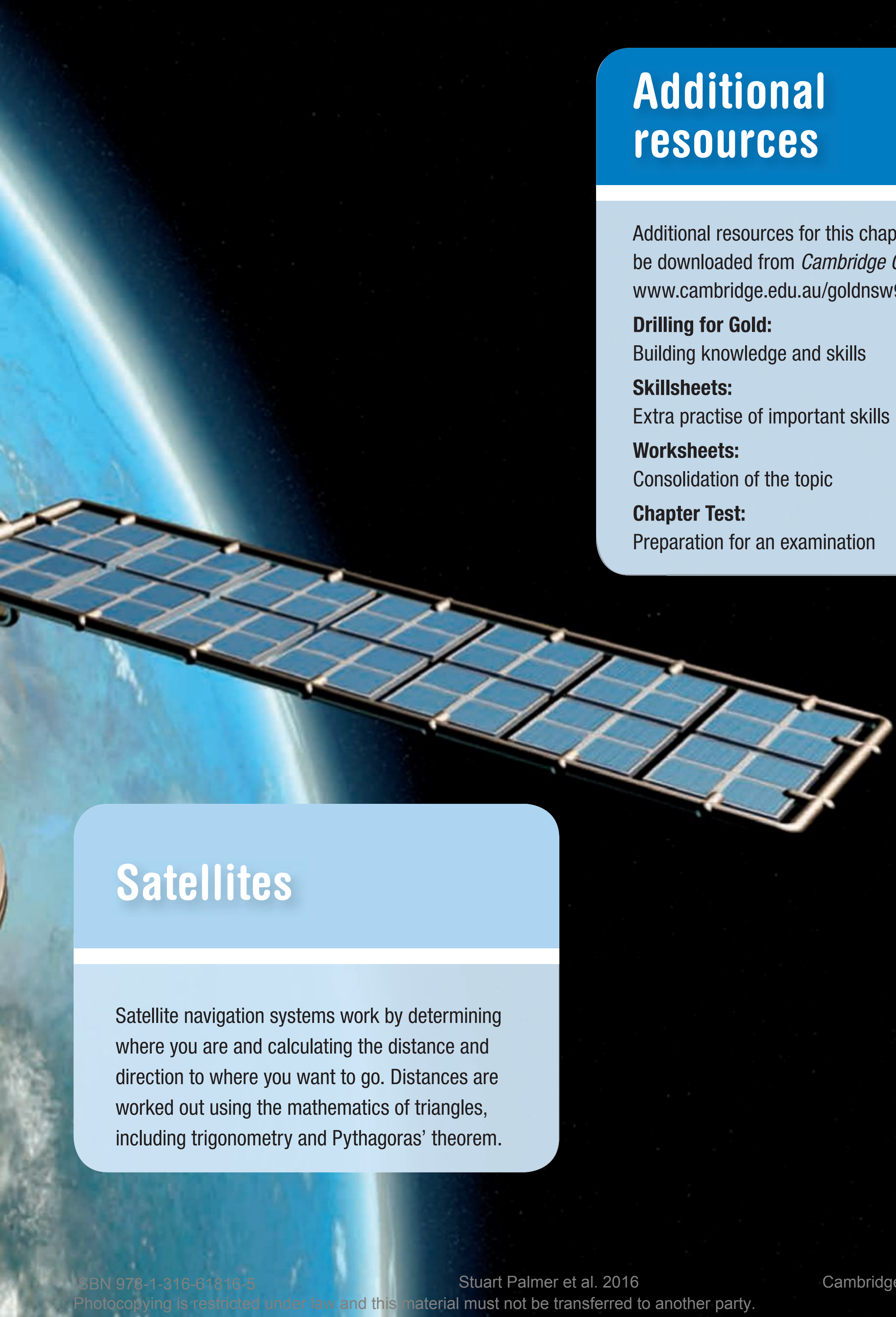
Strand: Measurement and Geometry

Substrand: RIGHT-ANGLED TRIANGLES (PYTHAGORAS AND TRIGONOMETRY)

In this chapter you will learn to:

- apply Pythagoras' theorem to calculate lengths of sides of right-angled triangles and solve related problems
- apply trigonometry to solve problems on angles of elevation and depression in diagrams.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw9



Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw9

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

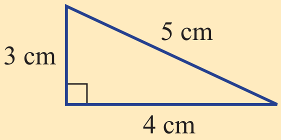
Satellites

Satellite navigation systems work by determining where you are and calculating the distance and direction to where you want to go. Distances are worked out using the mathematics of triangles, including trigonometry and Pythagoras' theorem.

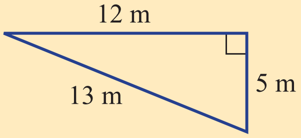
Pre-test

1 What is the length of the longest side given on these triangles?

a



b



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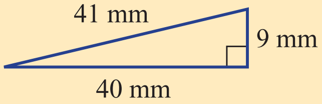


2 Calculate each of the following.

a 3^2 **b** 7^2 **c** $2^2 + 4^2$ **d** $9^2 - 6^2$

3 Round off each number, correct to one decimal place.

a 0.45678 **b** 0.34569 **c** 0.84562 **d** 0.27997

4 Round off each number, correct to two decimal places.

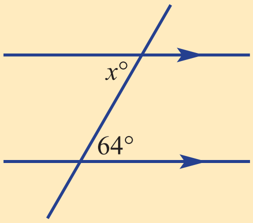
a 4.234 **b** 5.678 **c** 76.895 **d** 23.899

5 Use a calculator to calculate each of the following, correct to two decimal places.

a $\sqrt{35}$ **b** $\sqrt{236}$ **c** $\sqrt{23.6}$ **d** $\sqrt{65.4}$

6 Find the value of x in these diagrams.

a



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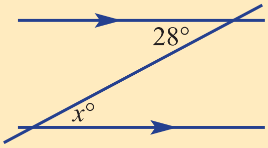

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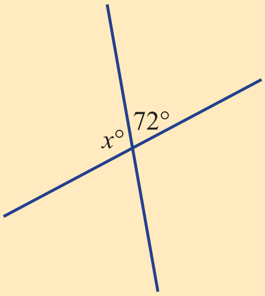

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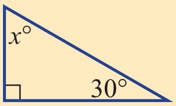

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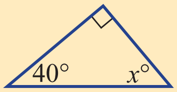

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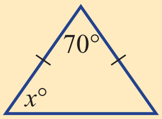

7 Solve each of the following equations to find the value of x.

a $3x = 6$ **b** $4x = 12$ **c** $5x = 60$ **d** $8x = 48$

e $\frac{x}{3} = 4$ **f** $\frac{x}{5} = 6$ **g** $\frac{x}{7} = 4$ **h** $\frac{x}{13} = 10$

8 Write a positive number in the box.

a $\square^2 = 4$ **b** $\square^2 = 16$ **c** $\square^2 = 3^2 + 4^2$

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4A Exploring Pythagoras' theorem

Stage
5.2
5.2◊
5.1
4

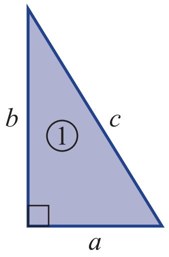
Pythagoras was born in Greece in the 6th century BC. He travelled to Egypt and Persia. His students and followers became known as the Brotherhood of Pythagoreans.

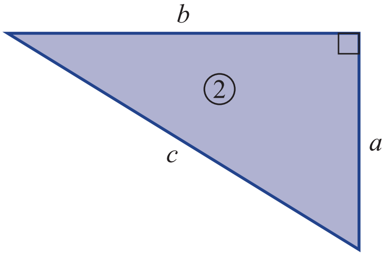
The Pythagoreans discovered the existence of irrational numbers such as $\sqrt{2}$, which cannot be written down as a fraction or terminating decimal.

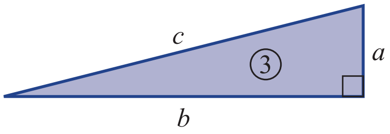
A painting of the Pythagorean brotherhood in ancient Greece

Let's start: Discovering Pythagoras' theorem

Use a ruler to measure the sides of these right-angled triangles to the nearest milimetre. Then copy and complete the table.



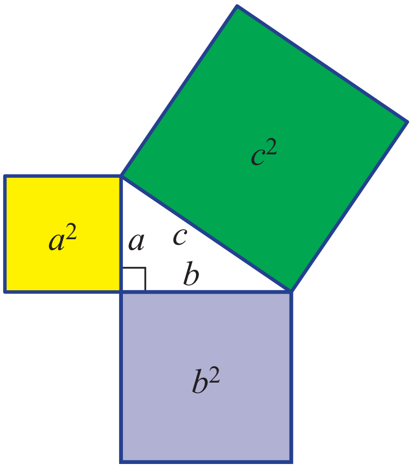




	a	b	c	a^2	b^2	c^2
1						
2						
3						

- What do you notice about the sum of a^2 and b^2?
- Can you write down this relationship as an equation?
- Explain how you might use this relationship to calculate the value of c if it was unknown.

- Look at the diagram on the right. Research how you can cut the two smaller squares (a^2 and b^2) to fit the pieces into the largest square (c^2).



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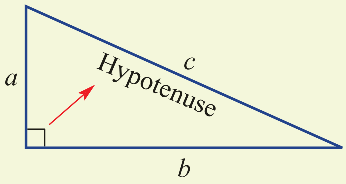
Key ideas

- The **hypotenuse**
  - It is the longest side of a right-angled triangle.
  - It is opposite the right angle.



- **Pythagoras' theorem**
  - The square of the hypotenuse is equal to the sum of the squares of the other two shorter sides.
    $a^2 + b^2 = c^2$ or $c^2 = a^2 + b^2$
- If you know the length of all three sides, you can decide whether or not a triangle is right angled.

Hypotenuse The longest side of a right-angled triangle

Pythagoras' theorem In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides

Exercise 4A

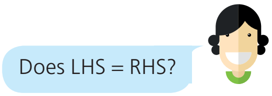
Understanding

1 Calculate these squares and sums of squares.

a 3^2	**b** 5^2	**c** 12^2	**d** 1.5^2
e $2^2 + 4^2$	**f** $3^2 + 7^2$	**g** $6^2 + 11^2$	**h** $12^2 + 15^2$

2 Decide if these equations are true or false.

a $2^2 + 3^2 = 4^2$	**b** $6^2 + 8^2 = 10^2$	**c** $7^2 + 24^2 = 25^2$
d $5^2 - 3^2 = 4^2$	**e** $6^2 - 3^2 = 2^2$	**f** $10^2 - 5^2 = 5^2$

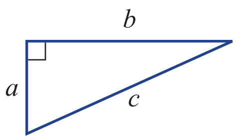


3 Write the missing words in this sentence.

The ____________ is the longest side of a right-angled ____________.

4 Which letter marks the length of the hypotenuse in these triangles?

a



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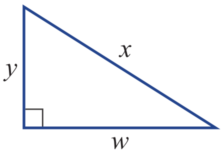

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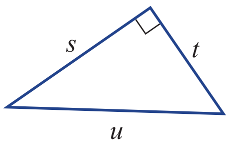

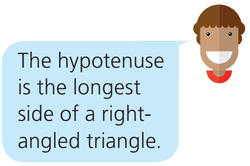

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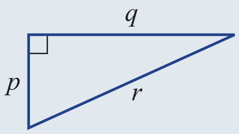
Fluency

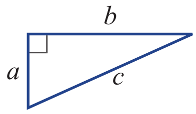
Example 1 Writing Pythagoras' theorem

Write down Pythagoras' theorem for this triangle.



Solution	Explanation
$p^2 + q^2 = r^2$ or $r^2 = p^2 + q^2$	r is the length of the hypotenuse, and p and q are the lengths of the two shorter sides.

5 Write down Pythagoras' theorem for these triangles.

a


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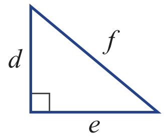
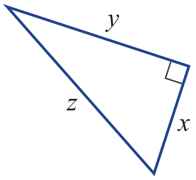
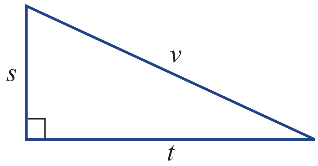
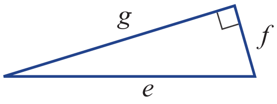
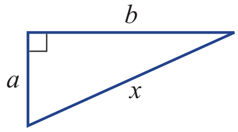


c


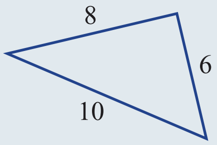
d


e


f


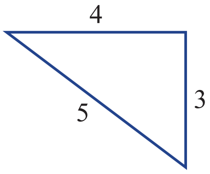
Example 2 Proving that a triangle is right angled

Prove that this triangle is right angled.



Solution	Explanation
$6^2 + 8^2 = 36 + 64 \quad \vert \quad 10^2 = 100$ $= 100$ $6^2 + 8^2 = 10^2$ $\therefore$ This triangle is right angled.	Find the sum of the squares of the two shorter sides and square the longest side.

6 Prove that these triangles are right angled.

a


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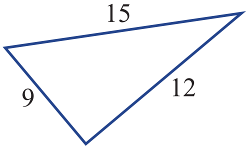

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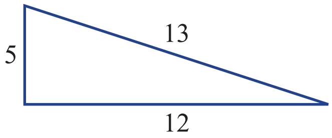
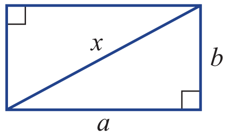


4A

Problem-solving and Reasoning

7 Write down Pythagoras' theorem using the letters given in these diagrams.

a



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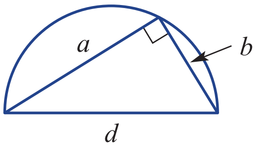
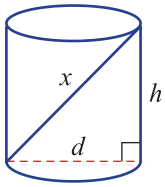
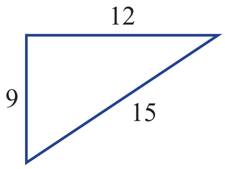


c



8 If $a^2 + b^2 = c^2$, we know that the triangle must have a right angle. Which of these triangles must have a right angle?

a



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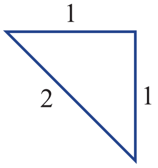

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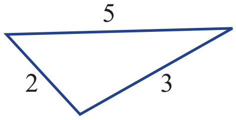

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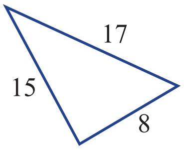

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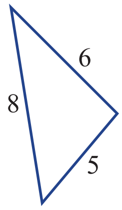

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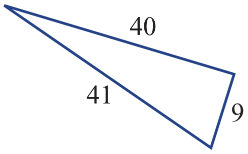

9 A cable connects the top of a 30 m mast to a point on the ground. The cable is 40 m long and connects to a point 20 m from the base of the mast. Do you think the triangle formed by the mast and the cable is right angled?

10 Complete these Pythagorean triads.

a $3^2 + 4^2 = \square^2$ $\therefore (3, 4, \square)$ is a Pythagorean triad.

b $5^2 + 12^2 = \square^2$ $\therefore (5, 12, \square)$ is a Pythagorean triad.

c $6^2 + 8^2 = \square^2$ $\therefore (6, 8, \square)$ is a Pythagorean triad.

Enrichment: Families of triads

11 3, 4, 5 is a Pythagorean triad. Try this:

$$(3, 4, 5) \xrightarrow{\times 2} (6, 8, 10)$$
$$(3, 4, 5) \xrightarrow{\times 3} (9, 12, 15)$$
$$(3, 4, 5) \xrightarrow{\times 4} (\square, \square, \square)$$
$$(3, 4, 5) \xrightarrow{\times \square} (\square, \square, \square)$$

Are these Pythagorean triads?

Is it square?

When you are building something, it is very important that it is 'square'. Builders use the word 'square' to mean meeting at right angles. This means that:

- The walls of a room should meet at right angles.
- The angle between the floor and a wall is 90°.
- The opening for a doorway should have four right angles.
- The door is a rectangle.

Builders check to see that things are 'square' while they are building.
The video shows one way to do this.
Watch the video and then try it out.

Maths@work

Video 4A

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4B Finding the length of the hypotenuse

Stage
5.2
5.2◊
5.1
4

If you know the lengths of the two shorter sides, you can use Pythagoras' theorem to find the length of the hypotenuse.

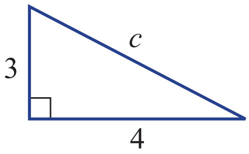
Let's start: Correct substitution

Here are three applications of Pythagoras' theorem for the given triangle.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 3^2 + 4^2 \\ &= 25 \\ \therefore\ c &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 4^2 + 3^2 \\ &= 25 \\ \therefore\ c &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 5^2 + 4^2 \\ &= 41 \\ \therefore\ c &= \sqrt{41} \end{aligned}$$

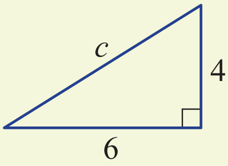


- Which one is incorrect? Explain.
- Why are the other two sets of working correct? Are they identical?

Key ideas

- Using Pythagoras' theorem to find the length of the hypotenuse:
  - $c^2 = a^2 + b^2$
  - Substitute the shorter lengths for a and b.
  - Find the value of $a^2 + b^2$.
  - Take the square root.
  - This is called the exact value.
  - Calculate the square root of 52 to find the value of c.
  - Round to one decimal place.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 4^2 + 6^2 \\ c^2 &= 16 + 36 \\ c^2 &= 52 \\ \therefore\ c &= \sqrt{52} \\ c &= 7.211\ldots \\ c &= 7.2 \text{ (to 1 d.p.)} \end{aligned}$$



- Lengths can be expressed with exact values using **surds**. $\sqrt{2}$, $\sqrt{28}$ and $2\sqrt{3}$ are examples of surds.
- When expressed as a decimal, a surd is an infinite non-recurring decimal with no pattern; e.g. $\sqrt{2} = 1.4142135623\ldots$

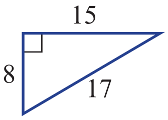
Surd An irrational number expressed as a square root.

Exercise 4B

Understanding

1 Write down the length of the hypotenuse in these right-angled triangles.

a



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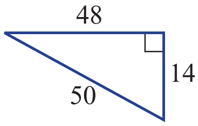
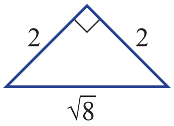


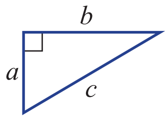
c



2 Write down Pythagoras' theorem using the given pronumerals for these right-angled triangles.

Write a statement such as $u^2 = s^2 + t^2$.

a



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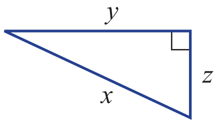

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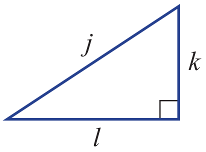


3 Evaluate the following, rounding to two decimal places in parts **g** and **h**.

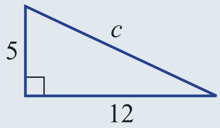
a 9^2 **b** 3.2^2 **c** $3^2 + 2^2$ **d** $9^2 + 5^2$

e $\sqrt{36}$ **f** $\sqrt{64+36}$ **g** $\sqrt{24}$ **h** $\sqrt{3^2+2^2}$

Fluency

Example 3 Finding the length of the hypotenuse

Find the length of the hypotenuse in this right-angled triangle.



Solution	Explanation
$c^2 = a^2 + b^2$ $c^2 = 5^2 + 12^2$	Write the rule and substitute the lengths of the two shorter sides.
$c^2 = 169$	
$\therefore\ c = \sqrt{169}$	If $c^2 = 169$, then $c = \sqrt{169} = 13$.
$c = 13$	

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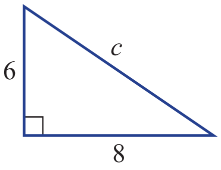
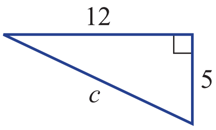
4 Find the length of the hypotenuse in each of the following right-angled triangles.

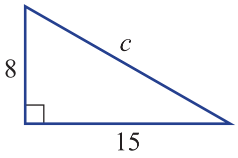
a



b

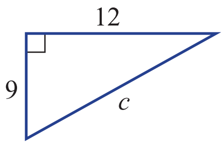


c

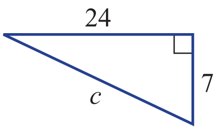


Substitute the two shorter sides for a and b in the equation $c^2 = a^2 + b^2$.

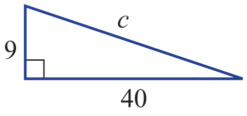
d



e



f



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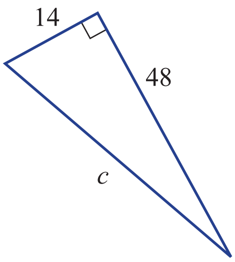
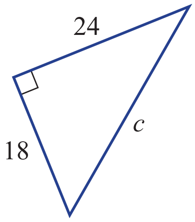
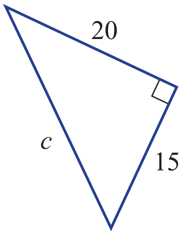


h

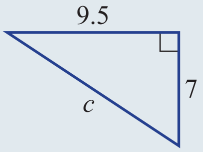


i



Example 4 Using rounding

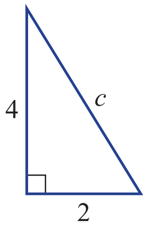
Find the length of the hypotenuse in this right-angled triangle, rounding to two decimal places.



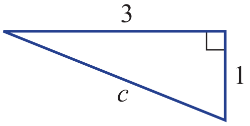
Solution	Explanation
$c^2 = a^2 + b^2$ $c^2 = 7^2 + 9.5^2$	The order for a and b does not matter because $7^2 + 9.5^2 = 9.5^2 + 7^2$.
$c^2 = 139.25$	
$\therefore\ c = \sqrt{139.25}$ $c = 11.8004\ldots$ $c = 11.80$ (to 2 d.p.)	$\sqrt{139.25} = 11.8004\ldots$ and the third decimal place is zero, so round down.

5 Find the length of the hypotenuse in each of these right-angled triangles, correct to two decimal places.

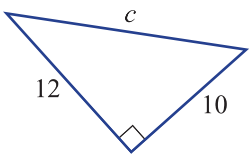
a



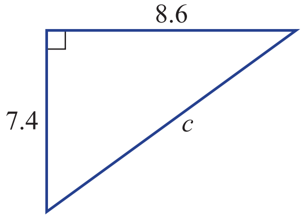
b



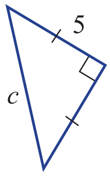
c



d



e



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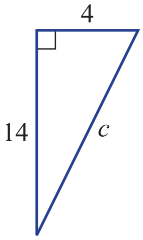
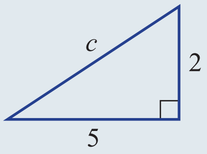


Use a calculator to find the value of c and round your answer to two decimal places.

Example 5 Finding the length of the hypotenuse using exact values

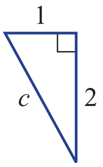
Find the length of the hypotenuse in this right-angled triangle, leaving your answer as an exact value.



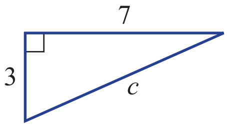
Solution	Explanation
$c^2 = a^2 + b^2$ $c^2 = 5^2 + 2^2$	Apply Pythagoras’ theorem to find the value of c.
$c^2 = 29$ $\therefore\ c = \sqrt{29}$	Express the answer using a surd, which is an exact value.

6 Find the length of the hypotenuse in these triangles, leaving your answer as an exact value.

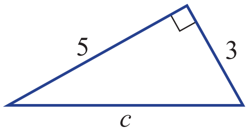
a



b

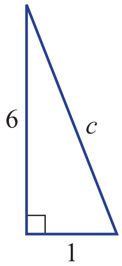


c

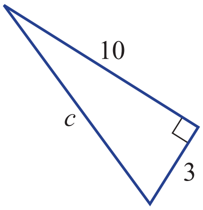


Leave your answer as a surd; e.g. $\sqrt{3}$ or $\sqrt{26}$.

d



e



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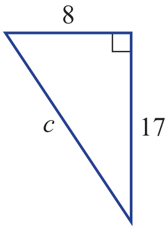
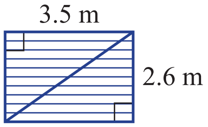
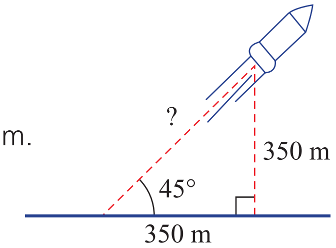


Problem-solving and Reasoning

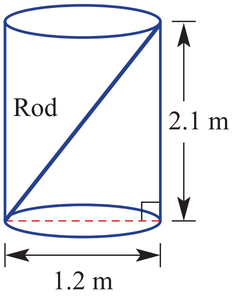
7 Find the length of the diagonal steel brace required to support a wall of length 3.5 m and height 2.6 m. Give your answer correct to one decimal place.



8 A miniature rocket blasts off at an angle of 45° and, after a few seconds, reaches a height of 350 m above the ground. At this point it has also covered a horizontal distance of 350 m. How far has the rocket travelled, to the nearest metre?

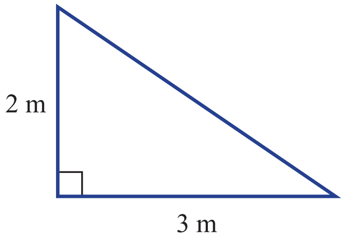


9 Find the length of the longest rod, as shown, that will fit inside a cylinder of height 2.1 m and with a circular end surface of 1.2 m diameter. Give your answer correct to one decimal place.

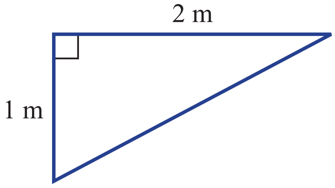


10 For each of these triangles, first calculate the length of the hypotenuse and then find the perimeter, correct to two decimal places.

a



b



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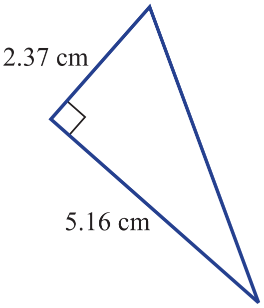

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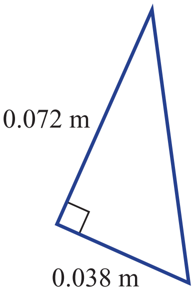

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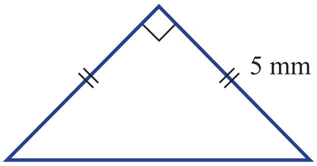

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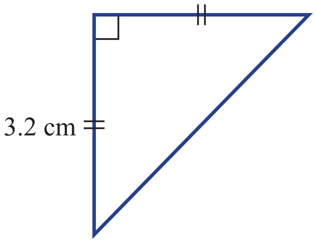
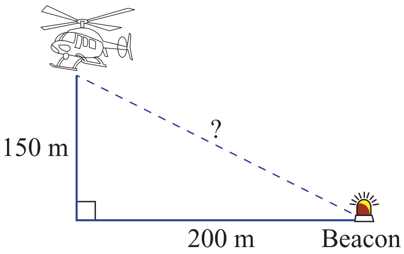
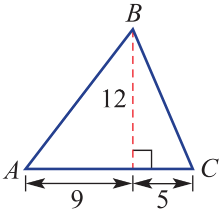


11 A helicopter hovers at a height of 150 m above the ground and is a horizontal distance of 200 m from a beacon on the ground. Find the distance from the helicopter to the beacon.



12 Find the perimeter of this triangle.



You will need to find AB and BC first.

13 One way to check whether a four-sided figure is a rectangle is to make sure that both its diagonals are the same length. What should be the length of the diagonals if a rectangle has side lengths 3 m and 5 m? Answer to two decimal places.

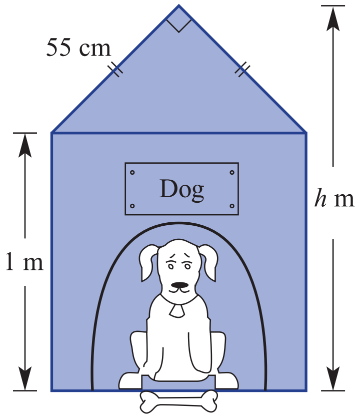
Enrichment: The dog kennel

14 A dog kennel has the dimensions shown in the diagram. Give your answers to the following, correct to two decimal places.

a What is the breadth of the kennel?

b What is the total height, h m, of the kennel?

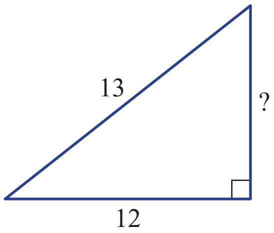
c If the sloping height of the roof is to be reduced from 55 cm to 50 cm, what difference will this make to the total height of the kennel? (Assume that the breadth is the same as in part **a**.)

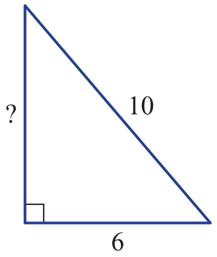


4C Finding the lengths of the shorter sides

Stage
5.2
5.2◊
5.1
4

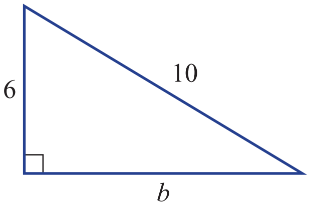
In a right-angled triangle, any two sides can be used to calculate the length of the third side.





Let's start: True or false?

Below are some mathematical statements relating to a right-angled triangle with hypotenuse 10 and the two shorter sides 6 and b.
Some of these mathematical statements are true and some are false. Sort them into true and false groups.

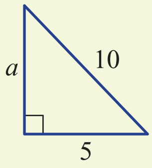


$10^2 = 6^2 + b^2$	$b^2 + 10^2 = 6^2$	$b + 6 = 10$	$10^2 + 6^2 = b^2$
$b^2 = 6^2 + 10^2$	$6^2 + b^2 = 10^2$	$b^2 + 6^2 = 10^2$	$10^2 = b^2 + 6^2$

Key ideas

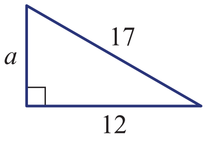
- Using Pythagoras' theorem to find a shorter side:
  - Use $a^2 + b^2 = c^2$.
  - Substitute the known lengths for b and c.
  - Solve the equation.
  - This is called the exact value.
  - Calculate the square root to find the value of a.
  - Round to one decimal place.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 5^2 &= 10^2 \\ -25\quad a^2 + 25 &= 100 \quad -25 \\ a^2 &= 75 \\ \therefore a &= \sqrt{75} \\ a &= 8.66\ldots \\ a &= 8.7 \text{ (to 1 d.p.)} \end{aligned}$$



Exercise 4C

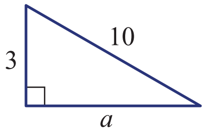
Understanding



1 For the diagram shown, are the following true or false?

a $a^2 + 12^2 = 17^2$ **b** $a^2 = 12^2 + 17^2$

c $12^2 + 17^2 = a^2$



2 For the diagram shown, are the following true or false?

a $a^2 = 3^2 + 10^2$ **b** $a^2 + 10^2 = 3^2$

c $a^2 + 3^2 = 10^2$

3 Find the positive value of a.

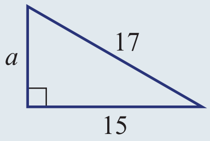
a $a^2 = 144$ **b** $a^2 = 400$ **c** $a^2 + 9 = 25$

d $a^2 + 49 = 625$ **e** $a^2 + 36 = 100$ **f** $a^2 + 15^2 = 17^2$

Fluency

Example 6 Finding a shorter side

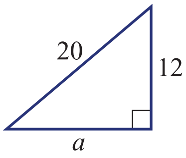
Find the value of the pronumeral.



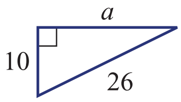
Solution	**Explanation**
$a^2 + b^2 = c^2$ $a^2 + 15^2 = 17^2$	Write the rule. Substitute the known sides. Square 15 and 17.
$-225 \left(\begin{aligned} a^2 + 225 &= 289 \\ a^2 &= 64 \end{aligned} \right) -225$	Subtract 225 from both sides.
$a = \sqrt{64}$ $a = 8$	Calculate the square root.

4 Find the value of the pronumeral.

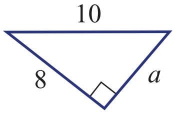
a



b

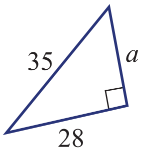


c

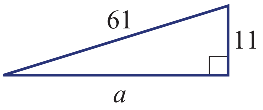


Substitute for a, b and c, then solve.

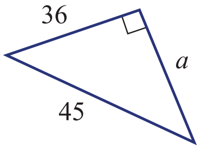
d



e

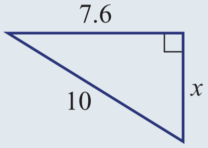


f



Example 7 Estimating with a decimal

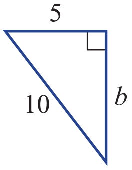
Find the value of x, rounding to two decimal places.



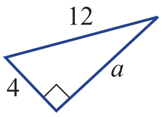
Solution	Explanation
$a^2 + b^2 = c^2$	Write the rule.
$x^2 + 7.6^2 = 10^2$	Substitute the known sides.
$-57.76 \; x^2 + 57.76 = 100 \; -57.76$ $x^2 = 42.24$	Subtract 57.76 from both sides.
$\therefore \; x = \sqrt{42.24}$	Calculate the square root.
$x = 6.499\ldots$	Round to two decimal places.
$x = 6.50$ (to 2 d.p.)	

5 Find the value of the pronumeral. Express your answers correct to two decimal places.

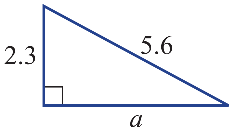
a



b



c



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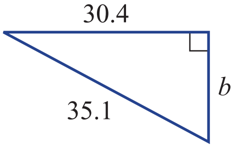
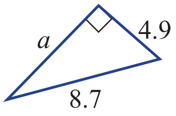


e



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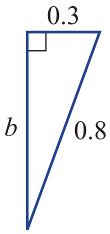
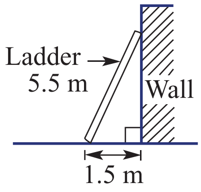


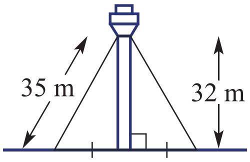
Use your calculator to get your answer, then round to two decimal places.

Problem-solving and Reasoning

6 The base of a ladder leaning against a wall is 1.5 m from the base of the wall. The ladder is 5.5 m long. Find how high the top of the ladder is above the ground, correct to one decimal place.



7 A 32 m communication tower is supported by 35 m cables stretching from the top of the tower to a position at ground level. Find the distance from the base of the tower to the point where the cable reaches the ground, correct to one decimal place.



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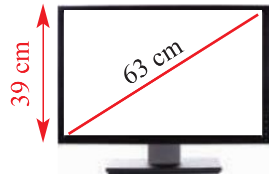
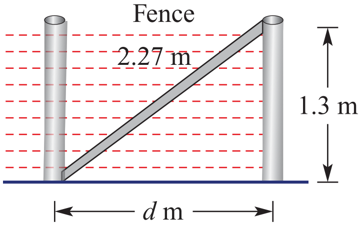
8 If a television has a screen size of 63 cm, it means that the diagonal length of the screen is 63 cm. If the vertical height of the screen is 39 cm, find how wide the screen is, to the nearest centimetre.



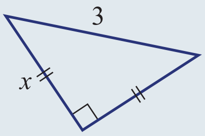
9 A 1.3 m vertical fence post is supported by a 2.27 m bar, as shown in the diagram. Find the distance (d metres) from the base of the post to where the support enters the ground. Give your answer correct to two decimal places.



Skillsheet 4A

Example 8 Using exact values

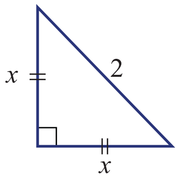
Find the value of x, giving your answer as a surd.



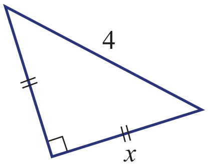
Solution	Explanation
$a^2 + b^2 = c^2$	Write the rule.
$x^2 + x^2 = 3^2$	Two sides are of length x.
$2x^2 = 9$	Add like terms.
$\div 2 \quad x^2 = \frac{9}{2} \quad \div 2$	Divide both sides by 2.
$\therefore\ x = \sqrt{\frac{9}{2}}$	Take the square root. This is the exact value.

10 Find the exact value of x.

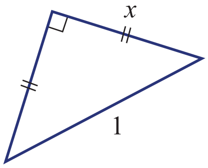
a



b



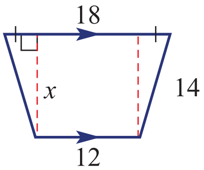
c



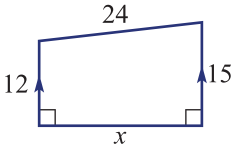
Enrichment: First find a missing length

11 Find the exact value of x.

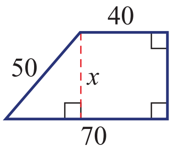
a



b



c



4D Using Pythagoras' theorem to solve two-dimensional problems

Stage
5.2
5.2◊
5.1
4

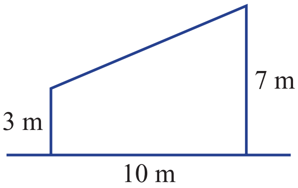
To apply Pythagoras' theorem to solve a problem, look for a right-angled triangle. When two sides of the right-angled triangle are known, the length of the third side can be found.

The length of each cable can be calculated using Pythagoras' theorem.

Let's start: But where is the right-angled triangle?

A shed with two walls of length 3 m and 7 m has a sloping roof. The walls are 10 m apart.

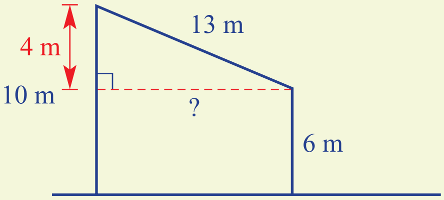
- Can you identify a right-angled triangle?
- What two side lengths on the right triangle do you know?
- Show how Pythagoras' theorem can be used to find the unknown side.
- How would you put the answer in words?



Key ideas

Drilling for Gold 4D1 at the end of this this section

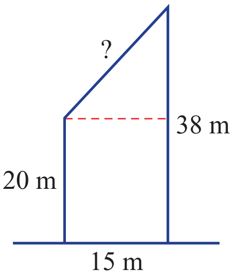
- To solve problems using Pythagoras' theorem:
  - Look for a right-angled triangle.
  - Label the sides with their lengths or with a letter (i.e. pronumeral) if the length is unknown.
  - Use Pythagoras' theorem to find the unknown side.
  - Solve the problem by making any further calculations.
  - Check that the answer is reasonable (remembering that the hypotenuse is always the longest side).
  - Answer the question in a worded statement.



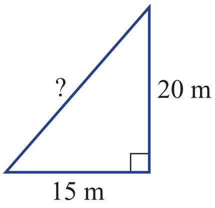
Exercise 4D

Understanding

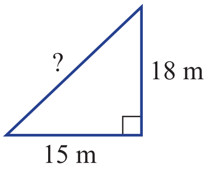
1 Two vertical posts are connected at the top by a wire. Which of the following triangles could be used to help find the cable length?



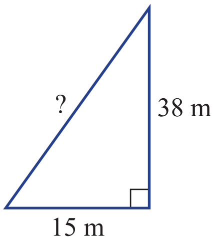
A



B



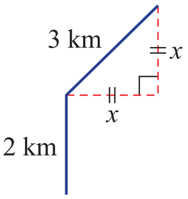
C



2 Match each problem (**a**–**c**) with both a diagram (**A**–**C**) and its solution (**I**–**III**).

a Two trees stand 20 m apart. They are 32 m and 47 m tall. What is the distance between the tops of the two trees?

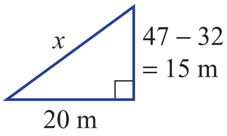
A



I The kite is flying at a height of 30.59 m.

b A kite is flying with a kite string of length 35 m. Its horizontal distance from its anchor point is 17 m. How high is the kite flying?

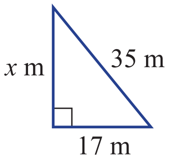
B



II The man has walked a total of $2 + 2.12 = 4.12$ km north from his starting point.

c A man walks due north for 2 km, then north-east for 3 km. How far north is he from his starting point?

C



III The distance between the top of the two trees is 25 m.

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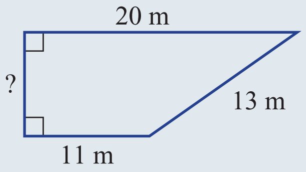
Example 9 Identifying right-angled triangles in a diagram



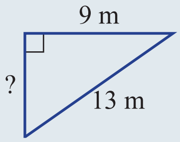
An area of lawn is in the shape of a trapezium, as shown.

a Draw a right-angled triangle to help find the unknown side length.

b Find the unknown side length, correct to two decimal places.

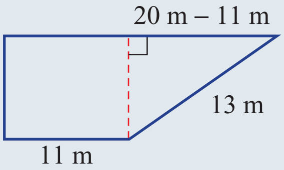
Solution

a



Explanation

The top side of the triangle is 20 m − 11 m = 9 m.



b

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 9^2 &= 13^2 \\ -81 \ \ a^2 + 81 &= 169 \ \ -81 \\ a^2 &= 88 \\ \therefore a &= \sqrt{88} \\ &= 9.3808\ldots \end{aligned}$$

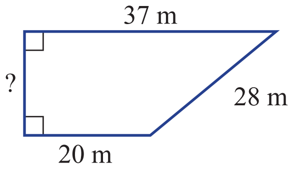
The unknown length is 9.38 m (to two decimal places).

Use Pythagoras' theorem with the hypotenuse as 13 m.
Solve for a and round as required.

3 A paved area is in the shape of a trapezium, as shown.

a Draw a right-angled triangle to help find the unknown side length.

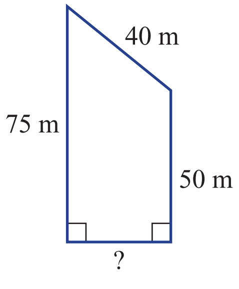
b Find the unknown side length, correct to two decimal places.



4 The side of a sculpture is in the shape of a trapezium, as shown.

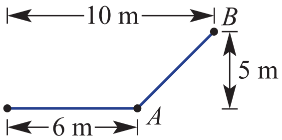
a Draw a right-angled triangle to help find the unknown side length.

b Find the unknown length, correct to two decimal places.

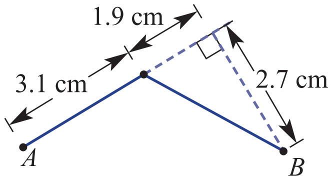


5 Find the direct distance between the points A and B in each of the following, correct to one decimal place.

a

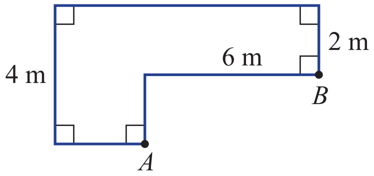


b

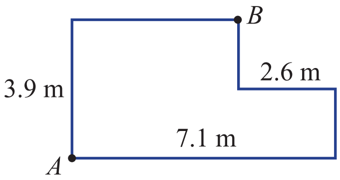


Draw in the right-angled triangle that will help to find the distance AB.

c

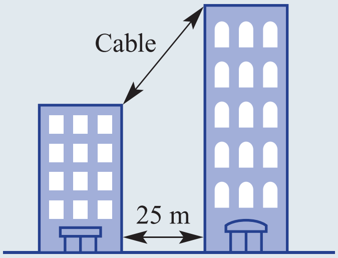


d

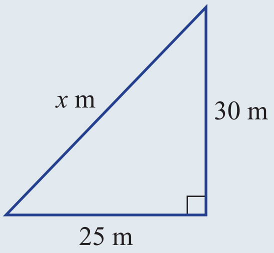


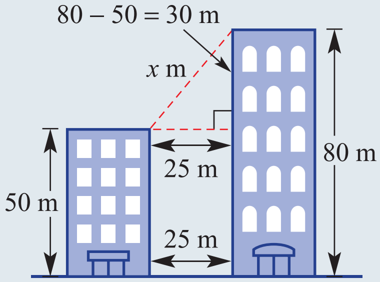
Example 10 Applying Pythagoras' theorem

Two skyscrapers are located 25 m apart. A cable links the tops of the two buildings. Find the length of the cable if the buildings are 50 m and 80 m in height. Give your answer correct to two decimal places.



Solution	**Explanation**
Let x m be the length of the cable.	Draw a right-angled triangle and label the measurements and pronumerals.





$$\begin{aligned} c^2 &= a^2 + b^2 \\ x^2 &= 25^2 + 30^2 \\ x^2 &= 625 + 900 \\ x^2 &= 1525 \\ \therefore\ x &= \sqrt{1525} \\ &= 39.05\ldots \end{aligned}$$

Set up an equation using Pythagoras' theorem and solve for x.

∴ The cable is 39.05 m long (to two decimal places).

Answer the question in words.

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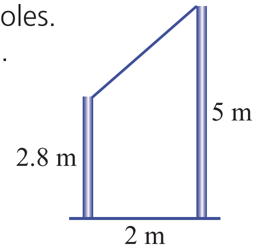
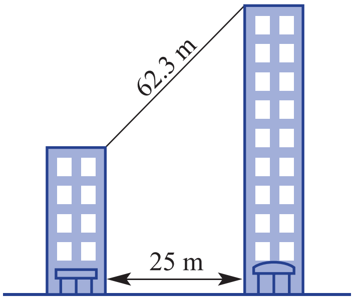
6 Two poles are located 2 m apart. A wire links the tops of the two poles. Find the length of the wire if the poles are 2.8 m and 5 m in height. Give your answer correct to one decimal place.

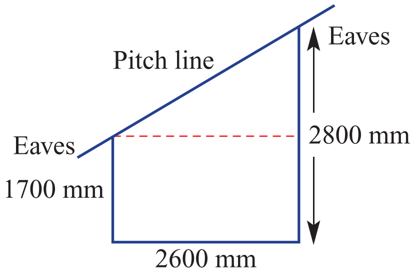


Problem-solving and Reasoning

7 Two skyscrapers are located 25 m apart and a cable of length 62.3 m links the tops of the two buildings. If the taller building is 200 metres tall, what is the height of the shorter building? Give your answer correct to one decimal place.



8 A garage is to be built with a skillion roof (a roof with a single slope). The measurements are given in the diagram. Calculate the pitch line length, to the nearest millimetre. Allow 500 mm for each of the eaves.



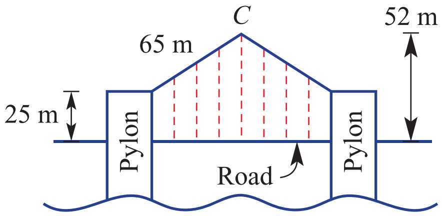
9 Two bushwalkers are standing on different sides of a mountain. One of them is at a height of 2120 m and the other is at a height of 1650 m. The horizontal distance between them is 950 m. Find the direct distance between the two bushwalkers. Give your answer correct to the nearest metre.

You will need to find the hypotenuse length of a right-angled triangle.

Enrichment: The suspension bridge

10 A suspension bridge is built with two vertical pylons and two straight beams of equal length. They extend from the top of the pylons to meet at a point, C, above the centre of the bridge, as shown in the diagram.

- **a** Calculate the vertical height of the point C above the tops of the pylons.
- **b** Calculate the distance between the pylons; that is, the length of the span of the bridge, correct to one decimal place.

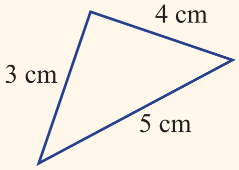
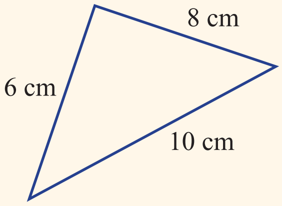
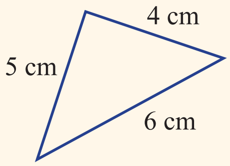
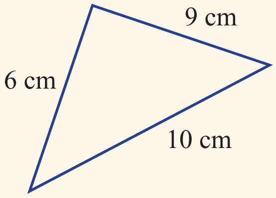


Drilling for Gold exercise

4D1: I can do three things with Pythagoras' theorem

Study examples **A** to **D** in the tables below and on the next page. For each example there is a matching question for you to answer following the solution in the example. Complete your answer in the worksheet or write it out with your working in your exercise book.

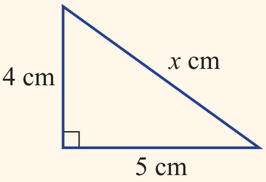
If I know the length of all three sides, I can decide whether or not the triangle is right-angled.

Example A Is this a right-angled triangle? 	**Question 1:** Is this a right-angled triangle? 
The two shorter sides: $3^2 = 9$ $4^2 = 16$ $9 + 16 = 25$ The longest side: $5^2 = 25$ Do the two shorter sides equal the longest side? Yes: $3^2 + 4^2 = 5$ These sides satisfy Pythagoras' theorem. Therefore this triangle is right-angled.	
Example B Show that this triangle is not right-angled. 	**Question 2:** Show that this triangle is not right-angled. 
The two shorter sides: $4^2 = 16$ $5^2 = 25$ $16 + 25 = 41$ The longest side: $6^2 = 36$ Do the two shorter sides equal the longest side? No: $4^2 + 5^2 \neq 6^2$ These sides do not satisfy Pythagoras' theorem. Therefore, this triangle is not right-angled.	

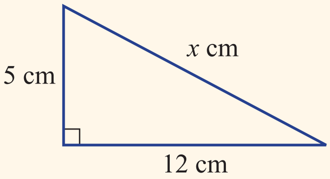
Drilling for Gold exercise

If I know the triangle is right-angled and I know the length of the two shorter sides, I can calculate the length of the hypotenuse.

Example C
Find the length of the hypotenuse.



Question 3:
Find the length of the hypotenuse.



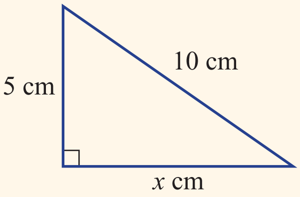
Using Pythagoras' theorem,

$$c^2 = a^2 + b^2$$
$$x^2 = 4^2 + 5^2$$
$$x^2 = 16 + 25$$
$$x^2 = 41$$
$$x = \sqrt{41}$$
$$x = 6.403...$$
$$x = 6.4 \text{ (to one d.p.)}$$

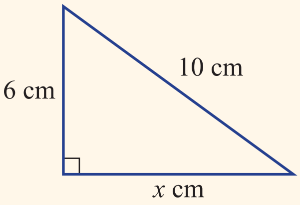
The hypotenuse is exactly $\sqrt{41}$ cm long.
This is approximately 6.4 cm.

If I know the triangle is right-angled and I know the lengths of the hypotenuse and one of the short sides, I can calculate the length of the other side.

Example D
Find the length of the unknown side.



Question 4:
Find the length of the unknown side.



Using Pythagoras' theorem,

$$a^2 + b^2 = c^2$$
$$x^2 + 5^2 = 10^2$$
$$x^2 + 25 = 100$$
$$x^2 = 100 - 25$$
$$x^2 = 75$$
$$x = \sqrt{75}$$
$$x = 8.660 \ldots$$
$$x = 8.7 \text{ (to one d.p.)}$$

The unknown side is exactly $\sqrt{75}$ cm long.

This is approximately 8.7 cm.

Keeping in touch with numeracy

Non-calculator

1 **a** $100 - 15 = \boxed{}$
b $100 - 2 \times 15 = \boxed{}$
c $100 - 3 \times 15 = \boxed{}$
d $100 - 4 \times 15 = \boxed{}$

3 **a** 450 cm = ____ mm
b 4500 g = ____ kg
c 3.25 hours = ______ minutes
d 4.5 hectares = ________ sq. metres

5 Zane started work at 9 a.m. and finished at 5 p.m. He had a 1-hour lunch break. For how many hours did Zane work?

7 Write down three different whole numbers that have a sum of zero.

9 Three consecutive integers have a sum of 30. What are they?

11 Roslyn has 45 20c coins. How many dollars is this?

Calculator

2 **a** How many lengths of 170 mm can be cut from a piece of timber that is 2400 mm long?
b How much timber will be left over?

4 Pipes are sold in lengths of 5 m. How many will be required for a pipeline that is 3.75 km long?

6 Calculate the time duration, in hours and minutes, from:
a 1:15 a.m. to 7:30 a.m.
b 1:15 a.m. to noon
c 9:37 a.m. to 1:15 p.m.
d 9:47 p.m. to 3:15 a.m.

8 Write down a pair of numbers for which the sum is 12 and the product is 32.

10 Use an equation to find three consecutive integers that have a sum of 204.

12 How much money is this, in total?

Coin	Number
$2	60
$1	55
50c	42
20c	38
10c	45
5c	23

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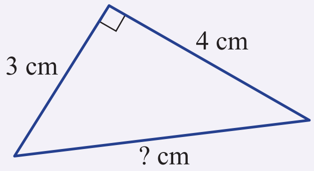
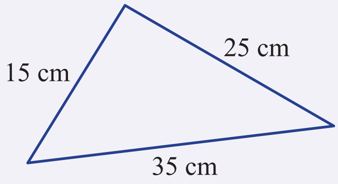
Non-calculator

13 Find the length of the hypotenuse.



Calculator

14 Use Pythagoras' theorem to prove that this is not a right-angled triangle.



15 One Australian dollar will buy 86.9 Japanese yen. How many yen can be bought for 100 Australian dollars?

16 One Australian dollar will buy 86.9 Japanese yen. How much is 100 yen in Australian dollars?

17 Grant drove 400 km and used 50 L of petrol. Express the rate in:

a kilometres per litre

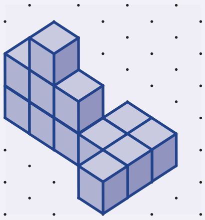
b litres per 100 km

18 Chloe drove 275 km and used 47.5 L of petrol. Giving your answer to one decimal place, express the rate in:

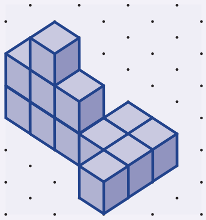
a kilometres per litre

b litres per 100 km

19 How many cubes are in this solid, given that all cubes are visible?



20 How many cubes would need to be added to this solid to make a cube?



4E Introducing the trigonometric ratios

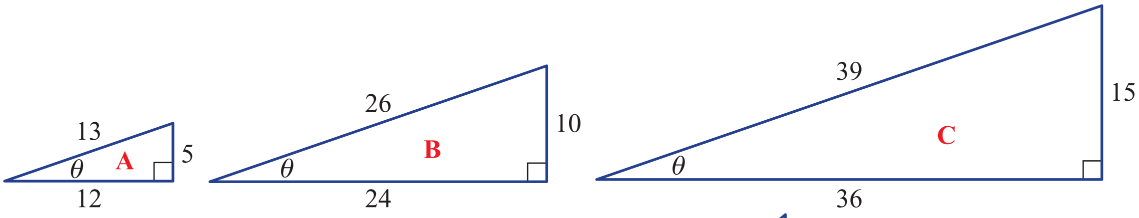
Stage
5.2
5.1
4

Trigonometry deals with the relationship between the side lengths and angles in triangles. Trigonometry dates back to the ancient Egyptian and Babylonian civilisations when trigonometry was used in the building of pyramids and in the study of astronomy. In the first century AD, Claudius Ptolemy developed tables of values linking the sides and angles of a triangle. The Greek letter theta, θ, is used to represent an unknown angle.

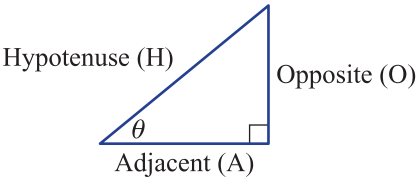
A basic form of trigonometry was used in the building of pyramids in ancient Egypt.

Let's start: What is the sine ratio?

The triangles below are similar. They contain the same angles but triangles B and C are enlargements of triangle A.



Copy and complete the following:



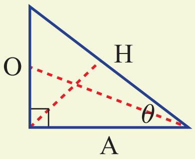
- In triangle A, the ratio $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{\square}{\square}$ (in simplest form)
- In triangle B, the ratio $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{\square}{\square} = \frac{\square}{\square}$ (in simplest form)
- In triangle C, the ratio $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{\square}{\square} = \frac{\square}{\square}$ (in simplest form)

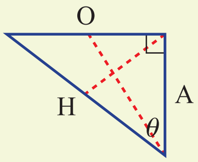
$\therefore$ When a right-angled triangle is enlarged, the angles stay the same and the sine ratio stays the same.

Key ideas

Drilling for Gold 4E1

- If a right-angled triangle has another angle, θ, then the:
  - longest side opposite the right angle is called the hypotenuse (H)
  - side opposite θ is called the **opposite** (O)
  - remaining side is called the **adjacent** (A).

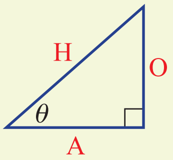




- For a right-angled triangle with a given angle θ, the three ratios **sine** (sin), **cosine** (cos) and **tangent** (tan) are given by the:
  - sine ratio: $\sin\theta = \frac{\text{length of the opposite side}}{\text{length of the hypotenuse}}$
  - cosine ratio: $\cos\theta = \frac{\text{length of the adjacent side}}{\text{length of the hypotenuse}}$
  - tangent ratio: $\tan\theta = \frac{\text{length of the opposite side}}{\text{length of the adjacent side}}$
- In this topic, the Greek letter θ (pronounced theta) is often used to label an unknown angle.
- The letters **SOHCAHTOA** are useful when trying to memorise the three ratios.

SOH CAH TOA

$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$ $\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ $\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$



Opposite (side) In a right-angled triangle, the side opposite a particular angle

Adjacent (side) In a right-angled triangle, the side adjacent to (next to) a particular angle

Sine The ratio of the length of the opposite side to the length of the hypotenuse

Cosine The ratio of the length of the adjacent side to the length of the hypotenuse

Tangent The ratio of the length of the opposite side to the length of the adjacent side

Exercise 4E

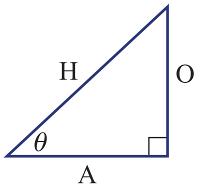
Understanding

1 Write the missing word in these sentences.

a H stands for the word ______________.

b O stands for the word ______________.

c A stands for the word ______________.

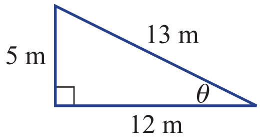


2 For this triangle, what is the length of the:

a hypotenuse?

b side opposite θ?

c side adjacent to θ?



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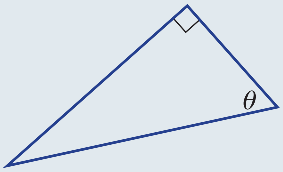
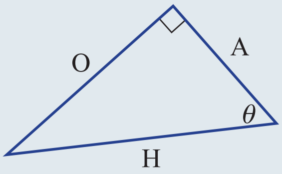
Example 11 Labelling the sides of triangles

Copy this triangle and label the sides as opposite to θ (O), adjacent to θ (A) or hypotenuse (H).

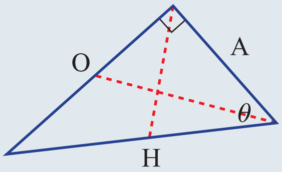


Solution



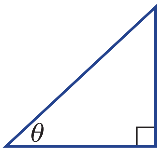
Explanation

Draw the triangle and label the side opposite the right angle as hypotenuse (H), the side opposite the angle θ as opposite (O) and the remaining side next to the angle θ as adjacent (A).

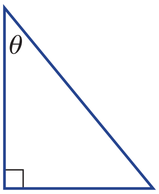


3 Copy each of these triangles and label the sides as opposite to θ (O), adjacent to θ (A) or hypotenuse (H).

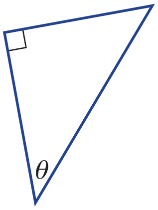
a



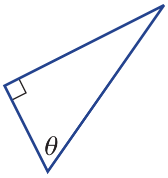
b



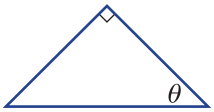
c



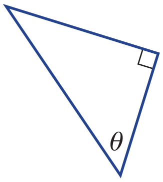
d



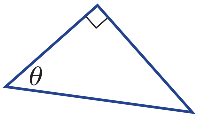
e



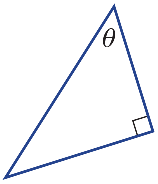
f



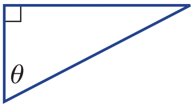
g



h



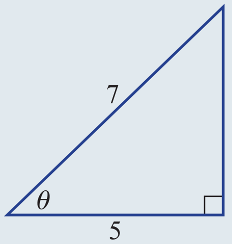
i



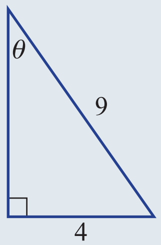
Example 12 Writing trigonometric ratios

Write a trigonometric ratio (in fraction form) for each of the following triangles.

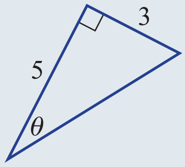
a



b

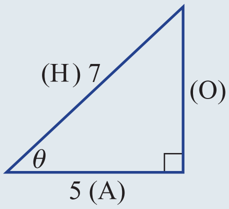


c



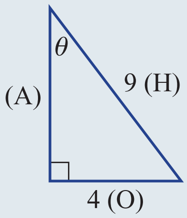
Solution **Explanation**

a $\cos \theta = \frac{\text{A}}{\text{H}} = \frac{5}{7}$



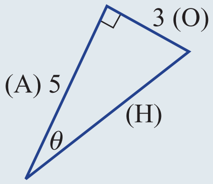
Side length 7 is opposite the right angle, so it is the hypotenuse (H). Side length 5 is adjacent to angle θ, so it is the adjacent (A).
Use cos θ since we have A and H.

b $\sin \theta = \frac{\text{O}}{\text{H}} = \frac{4}{9}$



Side length 9 is opposite the right angle, so it is the hypotenuse (H). Side length 4 is opposite angle θ, so it is the opposite (O).
Use sin θ since we have O and H.

c $\tan \theta = \frac{\text{O}}{\text{A}} = \frac{3}{5}$

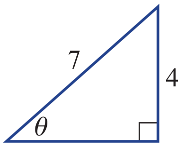


Side length 5 is the adjacent side to angle θ, so it is the adjacent (A). Side length 3 is opposite angle θ, so it is the opposite (O).
Use tan θ since we have O and A.

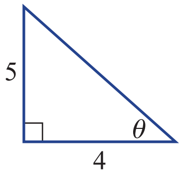
Drilling for Gold 4E2

4 Write a trigonometric ratio (in fraction form) for each of the following triangles and simplify where possible.

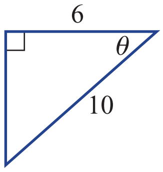
a



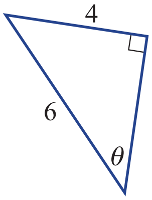
b



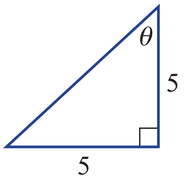
c



d



e



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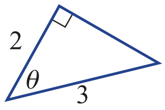
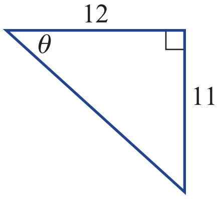
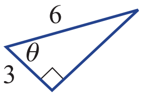


Use sin θ if you have O and H; use cos θ if you have A and H; and use tan θ if you have O and A.

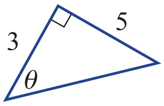
g



h

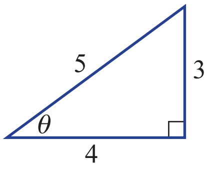


i

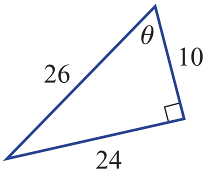


5 For each of these triangles, write a ratio (in simplified fraction form) for sin θ, cos θ and tan θ.

a



b



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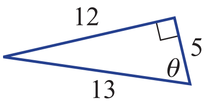
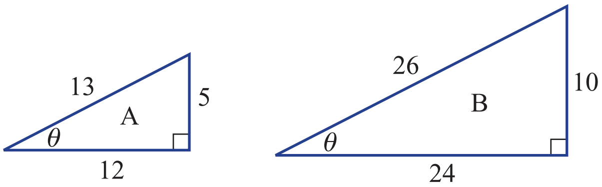


6 Here are two similar triangles, A and B.



a **i** Write the ratio sin θ (as a fraction) for triangle A.

  ii Write the ratio sin θ (as a fraction) for triangle B.

  iii What do you notice about your two answers?

b **i** Write the ratio cos θ (as a fraction) for triangle A.

  ii Write the ratio cos θ (as a fraction) for triangle B.

  iii What do you notice about your two answers?

c **i** Write the ratio tan θ (as a fraction) for triangle A.

  ii Write the ratio tan θ (as a fraction) for triangle B.

  iii What do you notice about your two answers?

Simplify your fractions from the larger triangle to help see the connection.

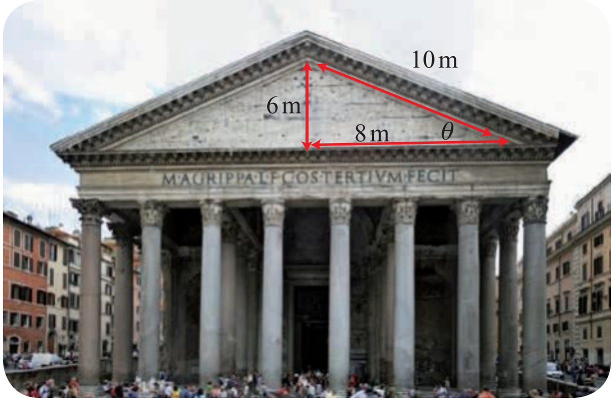
Problem-solving and Reasoning

7 The facade of a Roman temple has the given measurements. Write down the simplified ratio for:

a sin θ

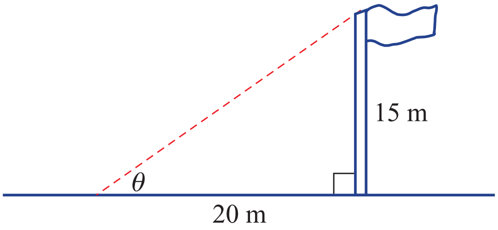
b cos θ

c tan θ



The Pantheon is a Roman temple that was built in 126 AD.

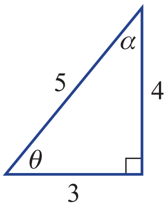
8 A vertical flag pole casts a shadow 20 m long. If the pole is 15 m high, find the ratio for tan θ.



Decide which side is the opposite to θ and which side is adjacent to θ.

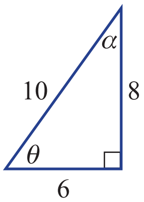
9 For the triangle shown, state the length of the side that corresponds to the:

- **a** hypotenuse
- **b** side opposite angle θ
- **c** side opposite angle α
- **d** side adjacent to angle θ
- **e** side adjacent to angle α



10 For the triangle shown, write a ratio (in fraction form) for:

- **a** sin θ
- **b** sin α
- **c** cos θ
- **d** tan α
- **e** cos α
- **f** tan θ



Skillsheet 4B

11 a Draw a right-angled triangle and mark one of the angles as θ. Mark the length of the opposite side as 15 units and the length of the hypotenuse as 17 units.

b Using Pythagoras' theorem, find the length of the adjacent side.

c Determine the ratios for sin θ, cos θ and tan θ.

Use $a^2 + b^2 = c^2$ for part **b** with $c = 17$ and $b = 15$.

Enrichment: The complementary connection

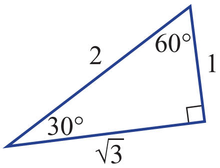
12 This triangle has angles 90°, 60° and 30° and side lengths 1, 2 and $\sqrt{3}$.

a Write a ratio for:

- **i** sin 30°
- **ii** cos 30°
- **iii** tan 30°
- **iv** sin 60°
- **v** cos 60°
- **vi** tan 60°

b What do you notice about the following pairs of ratios?

- **i** cos 30° and sin 60°
- **ii** sin 30° and cos 60°

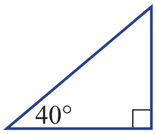


13 a Measure all the side lengths of this triangle, to the nearest millimetre.

b Use your measurements from part **a** to find an approximate ratio for:

- **i** cos 40°
- **ii** sin 40°
- **iii** tan 40°
- **iv** sin 50°
- **v** tan 50°
- **vi** cos 50°

c Do you notice anything about the trigonometric ratios for 40° and 50°?



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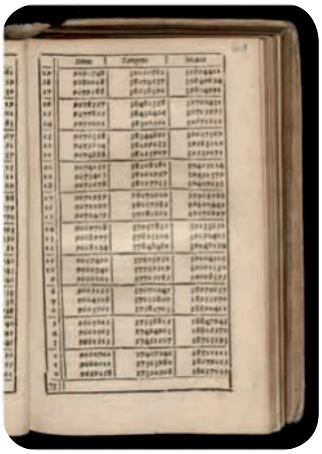
4F Finding unknown sides

Stage
5.2
5.2◊
5.1
4

Since ancient times, mathematicians have attempted to tabulate the three trigonometric ratios for varying angles. Here are the ratios for some angles in a right-angled triangle, correct to three decimal places.

Angle (θ)	sin θ	cos θ	tan θ
$0°$	0	1	0
$15°$	0.259	0.966	0.268
$30°$	0.5	0.866	0.577
$45°$	0.707	0.707	1
$60°$	0.866	0.5	1.732
$75°$	0.966	0.259	3.732
$90°$	1	0	undefined



Trigonometric tables in a 400-year-old book

We can now evaluate these values to a high degree of accuracy using calculators. They can be used to help solve problems involving triangles with unknown side lengths.

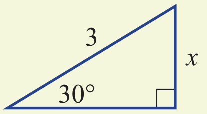
Let's start: Calculator start up

Some CAS calculators can produce accurate values of sin θ, cos θ and tan θ.

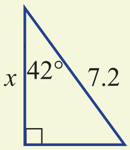
- Ensure that your calculator is in degree mode.
- Find the value of the following, correct to three decimal places:
  - sin $50°$
  - cos $81°$
  - tan $36°$
- Use trial and error to find (to the nearest degree) an angle, θ, which satisfies these conditions:
  - sin $\theta = 0.454$
  - cos $\theta = 0.588$
  - tan $\theta = 9.514$

Key ideas

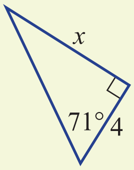
- If θ is in degrees, the ratios for sin θ, cos θ and tan θ can be found accurately using a calculator. For example, sin $35° = 0.5735\ldots$
- If the angles and one side length of a right-angled triangle are known, then the other side lengths can be found using the sin θ, cos θ or tan θ ratios.



$$\sin 30° = \frac{x}{3}$$
$$\therefore x = 3 \times \sin 30°$$
$$= 1.5$$



$$\cos 42° = \frac{x}{7.2}$$
$$\therefore x = 7.2 \times \cos 42°$$
$$= 5.35 \text{ (to 2 d.p.)}$$



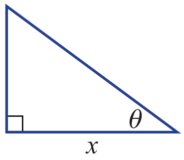
$$\tan 71° = \frac{x}{4}$$
$$\therefore x = 4 \times \tan 71°$$
$$= 11.62 \text{ (to 2 d.p.)}$$

Exercise 4F

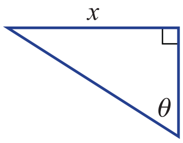
Understanding

1 For the marked angle θ, decide if x represents the length of the opposite (O), adjacent (A) or hypotenuse (H) side.

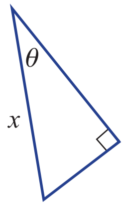
a



b



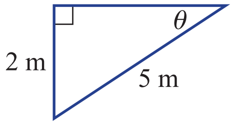
c



Drilling for Gold 4F1

2 Which two sides (choose a pair from O, A and H) are given in these triangles?

a



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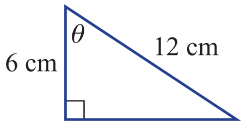

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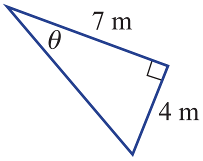
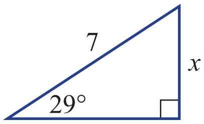


Choose either O, A or O, H or A, H.

3 Decide whether you would use $\sin\theta = \frac{\text{O}}{\text{H}}$, $\cos\theta = \frac{\text{A}}{\text{H}}$ or $\tan\theta = \frac{\text{O}}{\text{A}}$ to help find the value of x in these triangles.

Do not find the value of x; just state which ratio would be used.

a



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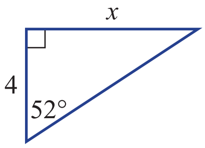

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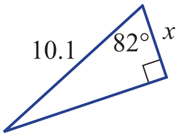


Fluency

Skillsheet 4C

Example 13 Using a calculator

Use a calculator to evaluate the following, correct to two decimal places.

a sin 50° **b** cos 16° **c** tan 77°

Solution	**Explanation**
a $\sin 50° = 0.766044\ldots$ $= 0.77$ (to 2 d.p.)	The third decimal place is greater than 4, so round up.
b $\cos 16° = 0.9612\ldots$ $= 0.96$ (to 2 d.p.)	The third decimal place is less than 5, so round down.
c $\tan 77° = 4.3314\ldots$ $= 4.33$ (to 2 d.p.)	

4 Use a calculator to evaluate the following, correct to two decimal places.

a sin 20° **b** cos 37° **c** tan 64° **d** sin 47°
e cos 84° **f** tan 14° **g** sin 27° **h** cos 76°

Drilling for Gold 4F2

Example 14 Solving for x in the numerator of a trigonometric ratio

Find the value of x in the equation $\cos 20^\circ = \frac{x}{3}$, correct to two decimal places.

Solution	Explanation
$\cos 20^\circ = \frac{x}{3}$	Write the statement.
$\times 3 \quad \frac{x}{3} = \cos 20^\circ \quad \times 3$	Swap LHS and RHS.
$x = 3 \times \cos 20^\circ$	Multiply both sides by 3.
$x = 2.8190\ldots$	Calculate.
$x = 2.82$ (to 2 d.p.)	Round off.

5 Find the value of x, correct to two decimal places.

a $\sin 50^\circ = \frac{x}{4}$

b $\tan 81^\circ = \frac{x}{3}$

c $\cos 33^\circ = \frac{x}{6}$

d $\cos 75^\circ = \frac{x}{3.5}$

e $\sin 24^\circ = \frac{x}{4.2}$

f $\tan 42^\circ = \frac{x}{10}$

g $\frac{x}{7.1} = \tan 18.4^\circ$

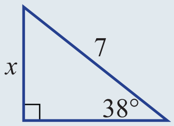
h $\frac{x}{5.3} = \sin 64^\circ$

i $\frac{x}{12.6} = \cos 52^\circ$

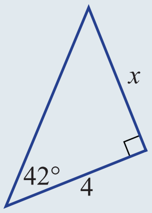
Example 15 Finding side lengths

For each triangle, find the value of x, correct to two decimal places.

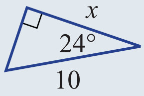
a



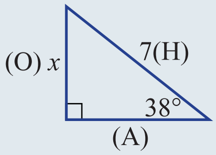
b



c



Solution	Explanation
a $\sin 38^\circ = \frac{\text{O}}{\text{H}}$ $\sin 38^\circ = \frac{x}{7}$	Since the opposite side (O) and the hypotenuse (H) are involved, the sin θ ratio must be used.
$\times 7 \quad \frac{x}{7} = \sin 38^\circ \quad \times 7$	Swap LHS and RHS.
$x = 7 \times \sin 38^\circ$	Multiply both sides by 7.
$= 4.3096\ldots$	Calculate.
$= 4.31$ (to 2 d.p.)	Round off.



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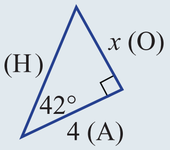
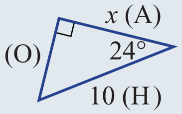
b $\tan 42^\circ = \frac{O}{A}$	Since the opposite side (O) and the adjacent side (A) are involved, the tan θ ratio must be used.
$\tan 42^\circ = \frac{x}{4}$	
×4 $\frac{x}{4} = \tan 42^\circ$ ×4	Swap LHS and RHS.
$x = 4 \times \tan 42^\circ$	Multiply both sides by 4.
$= 3.6016\ldots$	Calculate.
$= 3.60$ (to 2 d.p.)	Round off.



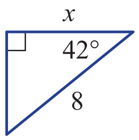
c $\cos 24^\circ = \frac{A}{H}$	Since the adjacent side (A) and the hypotenuse (H) are involved, the cos θ ratio must be used.
$\cos 24^\circ = \frac{x}{10}$	
×10 $\frac{x}{10} = \cos 24^\circ$ ×10	Swap LHS and RHS.
$x = 10 \times \cos 24^\circ$	Multiply both sides by 10.
$= 9.135\ldots$	Calculate.
$= 9.14$ (to 2 d.p.)	Round off.



6 Find the value of x, correct to two decimal places.

Drilling for Gold 4F3

a



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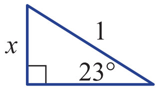
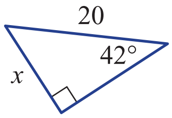
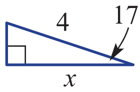


c



First decide which two sides you have, including the unknown (O, A or H). Then choose the correct ratio (sin θ, cos θ or tan θ).

d



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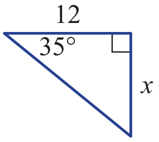

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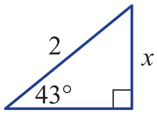

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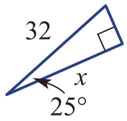

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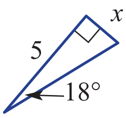

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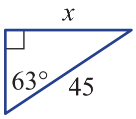

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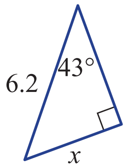

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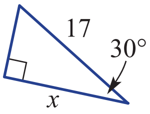

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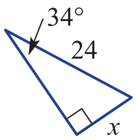
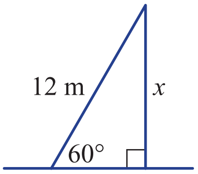
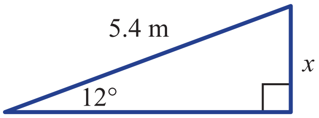


Problem-solving and Reasoning

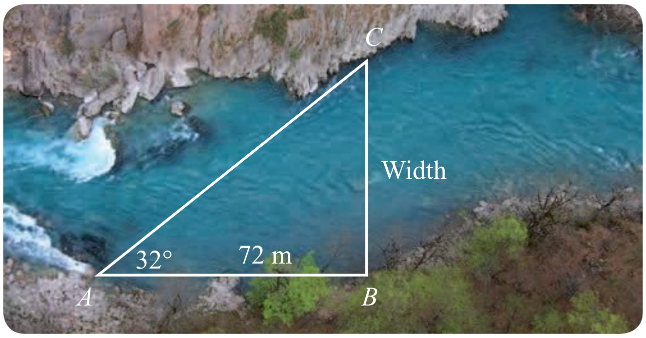
7 A pole is supported by a 12 m cable that makes a 60° angle with the ground. How tall is the pole, correct to two decimal places?



8 Emily walks 5.4 m up a ramp that is inclined at 12° to the horizontal. How high (correct to two decimal places) is she above her starting point?

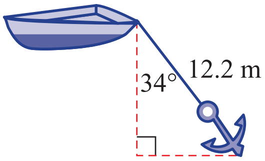


9 Kane wants to measure the width of a river at a certain point. He places two markers, A and B, 72 m apart along the bank. C is a point directly opposite marker B. Kane measures angle CAB to be 32°. Find the width of the river, correct to two decimal places.



The 'width' is opposite (**O**) the 32° angle and the 72 m length is adjacent (**A**) to the 32° angle.

10 One end of a 12.2 m rope is tied to a boat. The other end is tied to an anchor, which is holding the boat steady in the water. If the anchor is making an angle of 34° to the vertical, how deep is the water? Give your answer correct to two decimal places.



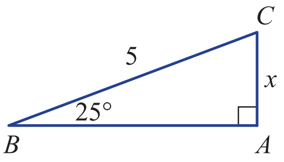
The depth length is adjacent (**A**) to the 34° angle.

11 For this right-angled triangle:

a Find the angle $\angle C$.

b Calculate the value of x, correct to three decimal places, using the sine ratio.

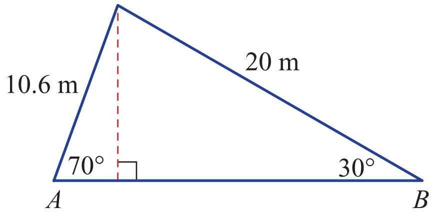
c Calculate the value of x, correct to three decimal places, using the cosine ratio.



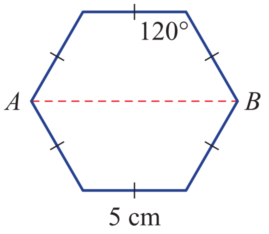
Enrichment: Combining multiple trigonometric calculations

12 Find the length AB in these diagrams. Round to two decimal places where necessary.

a



b



4G Solving for the denominator

Stage
5.2
5.2◊
5.1
4

So far we have constructed trigonometric ratios using a pronumeral that has always appeared in the **numerator**. For example, sin 40° = $\frac{x}{5}$.

When the pronumeral appears in the **denominator**, there are a number of algebraic steps that can be taken to find the solution.

Let's start: Unknown in denominator

Drilling for Gold 4G1

In the equation $10 = \frac{4}{x}$, the unknown is in the denominator.

Read the instructions and fill in the blanks.

- Write the equation.
- Multiply both sides by x.
- Divide both sides by 10.
- Simplify the answer.

$$10 = \frac{4}{x}$$

$\times x$ $\quad$ $\times x$

$$\boxed{} = \boxed{}$$

$\div 10$ $\quad$ $\div 10$

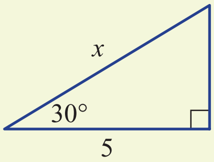
$$\boxed{} = \frac{\boxed{}}{\boxed{}}$$

$$x = \frac{\boxed{}}{\boxed{}} = 0.\boxed{}$$

Key ideas

- If the unknown value of a trigonometric ratio is in the **denominator**, you need to rearrange the equation.

  For example, for the triangle shown:



  - Multiply both sides by x (removes the fraction).
  - Divide both sides by cos 30° (cos 30° is just a number).

$$\cos 30^\circ = \frac{5}{x}$$

$\times x$ $\quad$ $\times x$

$$x \times \cos 30^\circ = 5$$

$\div \cos 30^\circ$ $\quad$ $\div \cos 30^\circ$

$$x = \frac{5}{\cos 30^\circ}$$

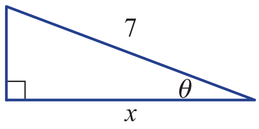
Denominator The bottom part of a fraction

Exercise 4G

Understanding

1 Which trigonometric ratio (i.e. sin θ, cos θ or tan θ) would be used to find the value of x in these triangles?

a



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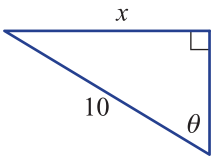

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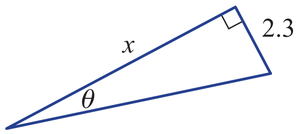


2 Solve these simple equations for x.

a $\frac{4}{x} = 2$ **b** $\frac{20}{x} = 4$ **c** $\frac{15}{x} = 5$ **d** $25 = \frac{100}{x}$

For part **a**, first multiply by x, then divide by 2.

3 Which of these equations has x in the denominator of the fraction?

a $\frac{x}{2} = \sin 40^\circ$ **b** $\frac{3}{x} = \cos 70^\circ$ **c** $\frac{7.1}{x} = \tan 34^\circ$ **d** $\cos 47^\circ = \frac{x}{5}$

Fluency

Example 16 Solving for x in the denominator

Solve for x in the equation $\cos 35^\circ = \frac{2}{x}$, correct to two decimal places.

Solution	**Explanation**
$\cos 35^\circ = \frac{2}{x}$	
$\times x$ $\quad x\cos 35^\circ = 2 \quad$ $\times x$	Multiply both sides of the equation by x.
$\div \cos 35^\circ$ $\quad x = \frac{2}{\cos 35^\circ} \quad$ $\div \cos 35^\circ$	Divide both sides of the equation by cos 35°.
$= 2.4415\ldots$ $= 2.44$ (to 2 d.p.)	Evaluate and round off to two decimal places.

4 Find the value of x, correct to two decimal places.

a $\cos 43^\circ = \frac{3}{x}$ **b** $\sin 36^\circ = \frac{4}{x}$ **c** $\tan 9^\circ = \frac{6}{x}$

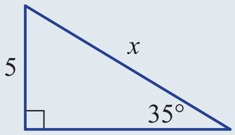
d $\tan 64^\circ = \frac{2}{x}$ **e** $\cos 67^\circ = \frac{5}{x}$ **f** $\sin 12^\circ = \frac{3}{x}$

g $\sin 38.3^\circ = \frac{5.9}{x}$ **h** $\frac{45}{x} = \tan 21^\circ$ **i** $\frac{18.7}{x} = \cos 32^\circ$

First multiply both sides by x.

Example 17 Finding side lengths

Find the value of x, correct to two decimal places.



Solution

$$\sin 35° = \frac{O}{H}$$

$$\sin 35° = \frac{5}{x}$$

× x × x

$$x \sin 35° = 5$$

÷ sin 35° ÷ sin 35°

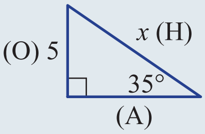
$$x = \frac{5}{\sin 35°}$$

$$= 8.717\ldots$$

$$= 8.72 \text{ (to 2 d.p.)}$$

Explanation

Since the opposite side (O) is given and we require the hypotenuse (H), use sin θ.



Multiply both sides of the equation by x, then divide both sides of the equation by sin 35°.

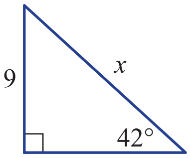
Evaluate on a calculator and round off to two decimal places.

5 Find the value of x, correct to two decimal places, using the sine, cosine or tangent ratios.

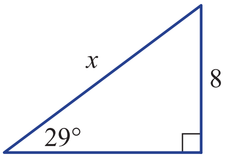
Drilling for Gold 4G2

Are you going to use $\sin \theta = \frac{O}{H}$, $\cos \theta = \frac{A}{H}$ or $\tan \theta = \frac{O}{A}$?

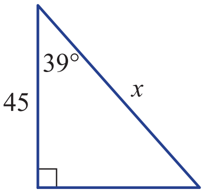
a



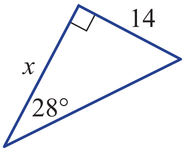
b



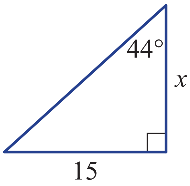
c



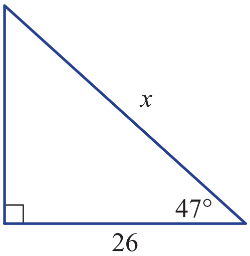
d



e



f



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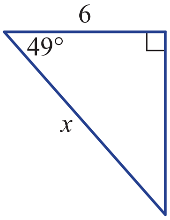
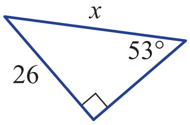


h



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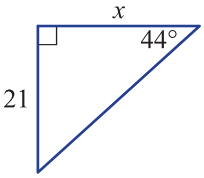

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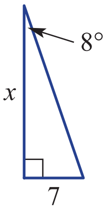

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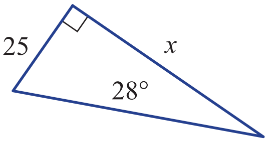
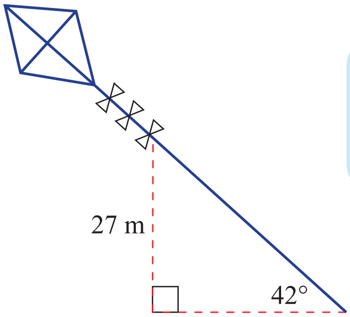


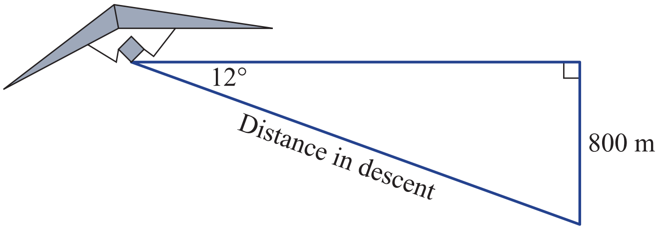
Problem-solving and Reasoning

6 A kite is flying at a height of 27 m above the anchor point. If the string is inclined at 42° to the horizontal, find the length of the string, correct to the nearest metre.



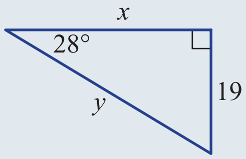
First decide if you are going to use sin θ (SOH), cos θ (CAH) or tan θ (TOA).

7 A hang glider flying at a height of 800 m descends at an angle of 12° to the horizontal. How far (to the nearest metre) has it travelled in descending to the ground?

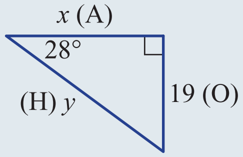


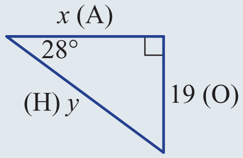
Example 18 Finding two values on one triangle

Find the value of x and y, correct to two decimal places.



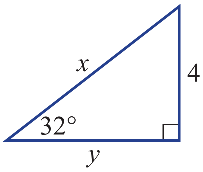
Solution	Explanation
$\tan 28^\circ = \frac{O}{A}$ $\tan 28^\circ = \frac{19}{x}$	Since the opposite side (O) is given and the adjacent (A) is required, use tan θ.
$x \tan 28^\circ = 19$	Multiply both sides of the equation by x.
$x = \frac{19}{\tan 28^\circ}$ $= 35.73$ (to 2 d.p.)	Divide both sides of the equation by tan 28° and round the answer to two decimal places.
$\sin 28^\circ = \frac{19}{y}$ $y \sin 28^\circ = 19$ $y = \frac{19}{\sin 28^\circ}$ $= 40.47$ (to 2 d.p.)	Use the opposite side (19) as it is an exact length, so use $\sin\theta = \frac{O}{H}$. Alternatively, y can be found by Pythagoras' theorem.





8 Find the value of each pronumeral, correct to one decimal place.

a



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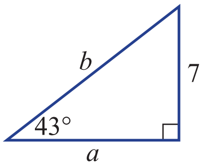

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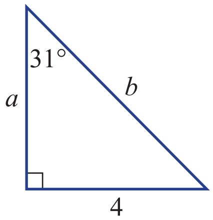

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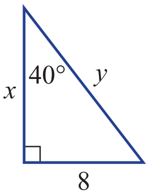

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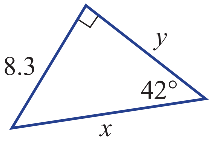

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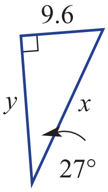
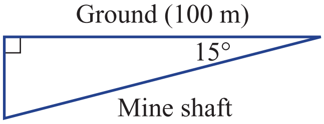


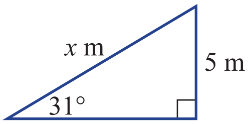
9 A mine shaft is dug at an angle of 15° to the horizontal. The horizontal length of the mine is 100 m. Answer the questions to the nearest metre.

a How far below ground level is the end of the shaft?

b How long is the mine shaft?



10 When calculating the value of x for this triangle, correct to two decimal places, two students come up with these answers.



Student A: $x = \dfrac{5}{\sin 31°}$

$= \dfrac{5}{0.52} = 9.62$

Student B: $x = \dfrac{5}{\sin 31°}$

$= 9.71$

a Student B is more accurate. Why?

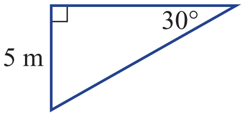
b What advice would you give to the student whose answer is not accurate?

Skillsheet 4D

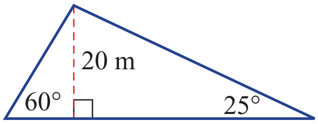
Enrichment: Trigonometry and perimeters

11 Find the perimeter of these triangles, correct to one decimal place. You will first need to find the lengths of all the sides.

a



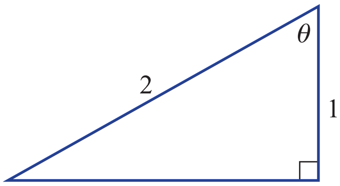
b



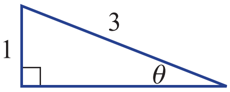
4H Finding unknown angles

Stage
5.2
5.2◊
5.1
4

If you know two sides of a right-angled triangle, you can use trigonometry to calculate the unknown angles.



Let's start: Trial and error can be slow



We know that for this triangle $\sin\,\theta = \frac{1}{3}$.

- Guess the angle θ.
- For your guess, use a calculator to see if $\sin\,\theta = \frac{1}{3}$ $(= 0.333...)$.
- Update your guess and use your calculator to check once again.
- Repeat this trial-and-error process until you think you have the angle θ, correct to the nearest degree.

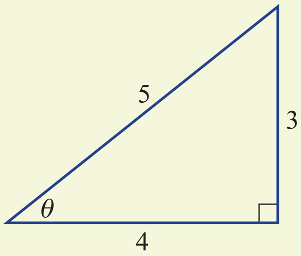
Key ideas

- Your calculator has functions for solving an equation such as $\sin\,\theta = \frac{3}{5}$.

  '$\sin^{-1}$' is written above the 'sin' button.

  You must learn how to use this function.

- Use your calculator to check the following solutions:
  - $\sin\,\theta = \frac{3}{5}$ means $\theta = 37°$ (to nearest degree)
  - $\cos\,\theta = \frac{4}{5}$ means $\theta = 37°$ (to nearest degree)
  - $\tan\,\theta = \frac{3}{4}$ means $\theta = 37°$ (to nearest degree)



- When finding an unknown angle, you can express your answer to the nearest degree.

$$\sin\theta = \frac{3}{5}$$

$$\theta = 36.869\ldots \text{ (by calculator)}$$

$$\theta = 37° \text{ (to nearest degree)}$$

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Exercise 4H

Understanding

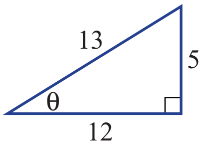
1 True or false?

a If cos $\theta = \frac{1}{2}$ then $\theta = 60°$.

b If tan $\theta = 1$ then $\theta = 45°$.

c If sin $\theta = \frac{1}{2}$ then $\theta = 60°$.

2 Refer to the diagram to complete the following:



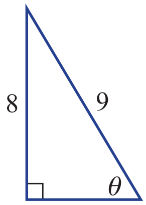
a sin $\theta = \frac{\square}{\square}$

b cos $\theta = \frac{\square}{\square}$

c tan $\theta = \frac{\square}{\square}$

3 Which trigonometric ratio should be used to solve for θ?

a



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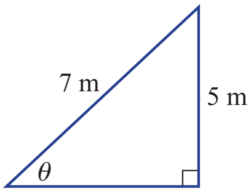
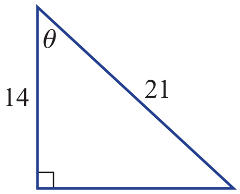


Choose sin θ if using O and H, cos θ if using A and H, and tan θ if using O and A.

c



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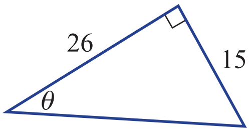


Fluency

Example 19 Calculating the angle

Find the value of θ to the nearest degree when sin $\theta = 0.3907$.

Solution	Explanation
sin $\theta = 0.3907$	Use the $\sin^{-1}$ key on your calculator.
$\theta = 22.998\ldots$	Enter $\sin^{-1}(0.3907)$.
$\theta = 23°$ (to nearest degree)	Round off to the nearest whole number.

4 Find the value of θ, to the nearest degree.

a sin $\theta = 0.5$
b cos $\theta = 0.5$
c tan $\theta = 1$
d cos $\theta = 0.8660$
e sin $\theta = 0.7071$
f tan $\theta = 0.5774$
g sin $\theta = 1$
h tan $\theta = 1.192$
i cos $\theta = 0$
j cos $\theta = 0.5736$
k cos $\theta = 1$
l sin $\theta = 0.9397$

For part **a**, enter $\sin^{-1}(0.5)$ on your calculator.

Example 20 Using inverse trigonometric ratios with fractions

Find the value of θ when $\tan\theta = \frac{1}{2}$. Give your solution to the nearest degree.

Solution	**Explanation**
$\tan\theta = \frac{1}{2}$	
$\theta = 26.56505\ldots$	Use the $\tan^{-1}$ key on your calculator. Enter $\tan^{-1}\left(\frac{1}{2}\right)$.
$\theta = 27°$ (to nearest degree)	Round the answer to the nearest degree.

5 Find the angle θ, correct to the nearest degree.

a $\sin\theta = \frac{4}{7}$ **b** $\sin\theta = \frac{1}{3}$ **c** $\sin\theta = \frac{9}{10}$

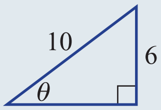
d $\cos\theta = \frac{1}{4}$ **e** $\cos\theta = \frac{4}{5}$ **f** $\cos\theta = \frac{7}{9}$

g $\tan\theta = \frac{3}{5}$ **h** $\tan\theta = \frac{8}{5}$ **i** $\tan\theta = 12$

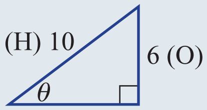
For part **a**, enter $\sin^{-1}\left(\frac{4}{7}\right)$ on your calculator.

Example 21 Finding an angle

Find the value of θ, to the nearest degree.



Solution	**Explanation**
$\sin\theta = \frac{\text{O}}{\text{H}}$ $\sin\theta = \frac{6}{10}$	Since the opposite side (O) and the hypotenuse (H) are given, use $\sin\theta$.
$\theta = 36.869\ldots$	Use the $\sin^{-1}$ key on your calculator. Enter $\sin^{-1}\left(\frac{6}{10}\right)$.
$\theta = 37°$ (to nearest degree)	Round off.

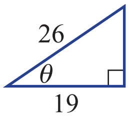


Skillsheet 4E

6 Find the value of θ, to the nearest degree.

First set up an equation (like those in Question **5**), then use a calculator to find θ.

a



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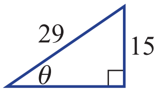

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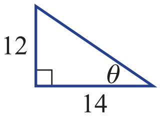

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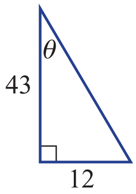

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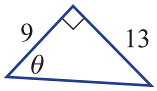

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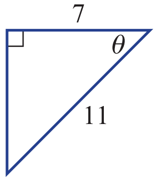

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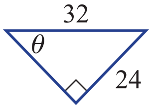

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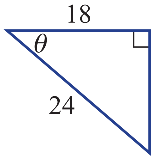

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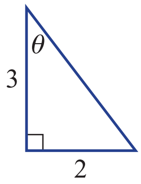
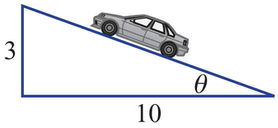


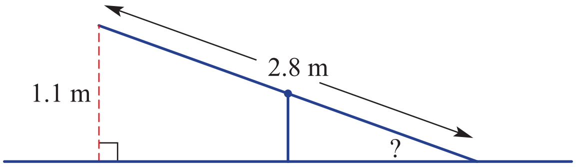
Problem-solving and Reasoning

7 A road rises at a grade of 3 in 10. Find the angle (to the nearest degree) the road makes with the horizontal.



8 When a 2.8 m long see-saw is at its maximum height, it is 1.1 m off the ground. What angle, to the nearest degree, does the see-saw make with the ground?

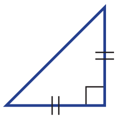
The 1.1 m length is opposite the angle and the 2.8 m length forms the hypotenuse.



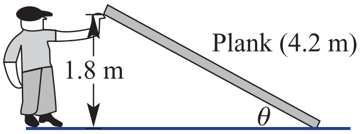
9 Consider this triangle.

a Calculate tan 45° on your calculator.

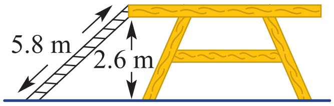
b Explain why tan 45° is exactly 1.



10 Aaron holds one end of a plank of wood 1.8 m above the ground. The other end of the plank rests on the ground. The plank of wood is 4.2 m long. Find the angle that the plank makes with the ground (θ), correct to the nearest degree.



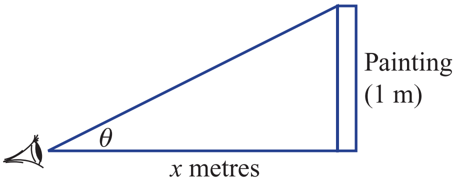
11 A ladder has a length of 5.8 m. The ladder is resting against the top of a platform that is 2.6 m high. Find the angle the ladder makes with the ground, correct to the nearest degree.



12 For what value of θ is sin θ = cos θ? Choose a value of θ between 0° and 90°.

Enrichment: Viewing angle

13 Calvin has trouble with his eyesight but every Sunday he goes to view his favourite painting at the gallery. His eye level is at the same level as the base of the painting and the painting is 1 metre tall.



Answer the following to the nearest degree for angles and to two decimal places for lengths.

a If $x = 3$, find the viewing angle θ.

b If $x = 2$, find the viewing angle θ.

c If Calvin can stand no closer than 1 metre to the painting, what is his largest viewing angle?

d When the viewing angle is 10°, Calvin has trouble seeing the painting. How far is he from the painting at this viewing angle?

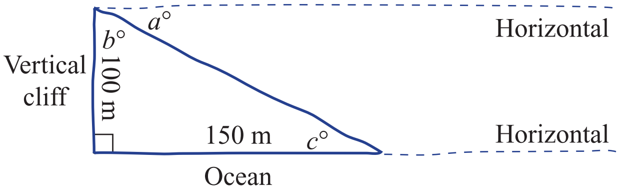
4I Using trigonometry to solve problems

Stage
5.2
5.2◊
5.1
4

In many practical problems, trigonometry can be used to find unknown lengths and angles. Two special angles are called the angle of elevation and the angle of depression.

▶ Let's start: Draw and measure

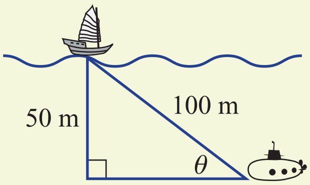
Use a scale where 1 millimetre represents 1 metre to draw a neat, accurate scale drawing of this diagram.



Use a protractor to estimate the values of a, b and c. Do a and c have the same value? Do you have the same values for a, b and c as other people in your class?

Key ideas

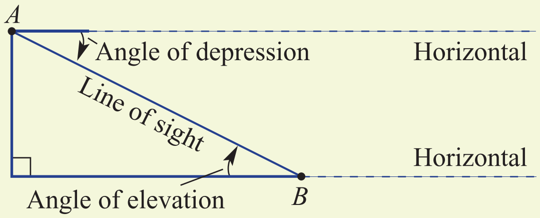
- To solve problems involving trigonometry:
  - Draw a diagram and label the key information.
  - Identify and draw the appropriate right-angled triangles.
  - Use trigonometry to find the unknown measurements.
  - Check that your answer is reasonable.
  - Express your answer in words.



Angle of elevation
The angle of your line of sight from the horizontal when looking up at an object

Angle of depression
The angle of your line of sight from the horizontal when looking down at an object

- **Angles of elevation** and **depression** are always measured from the horizontal.

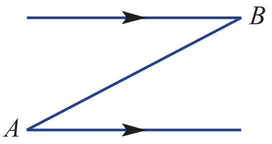


- The horizontal lines are parallel. Therefore, the angle of depression and the angle of elevation are equal alternate angles.

Exercise 4I

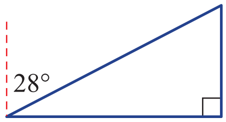
Understanding

1 Draw this diagram and complete these tasks.
- **a** Mark in the:
  - **i** angle of elevation (θ) from A to B
  - **ii** angle of depression (α) from B to A
- **b** Is $\theta = \alpha$ in your diagram? Why?

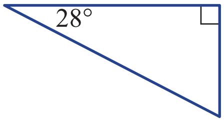


2 Choose the diagram (i.e. **A**, **B** or **C**) that matches the situation where a boy views a kite at an angle of elevation of 28°.

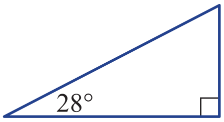
A



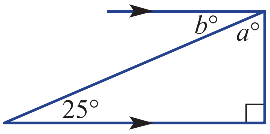
B



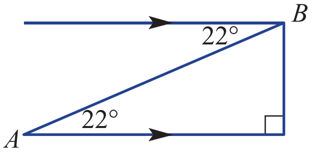
C



3 Without measuring, find the values of the pronumerals in this diagram.



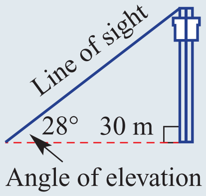
4 For this diagram, what is the:
- **a** angle of elevation from A to B?
- **b** angle of depression from B to A?



Fluency

Example 22 Using angles of elevation

The angle of elevation to the top of a tower from a point on the ground 30 m away from the base of the tower is 28°. Find the height of the tower, to the nearest metre.



Solution

Let the height of the tower be h m.

$$\tan 28^\circ = \frac{\text{O}}{\text{A}}$$

$$\tan 28^\circ = \frac{h}{30}$$

$$\times 30 \quad \frac{h}{30} = \tan 28^\circ \quad \times 30$$

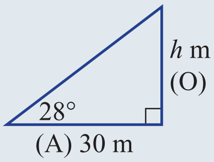
$$h = 30 \times \tan 28^\circ$$

$$= 15.95\ldots$$

The height is 16 m, to the nearest metre.

Explanation

Since the opposite side (O) is required and the adjacent (A) is given, use tan θ.

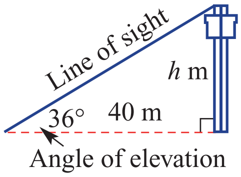


Swap LHS and RHS and multiply both sides by 30.

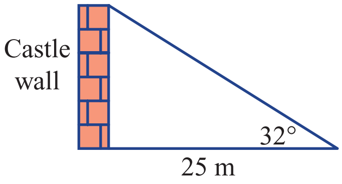
Calculate and round off.
Write the answer in words.

4I

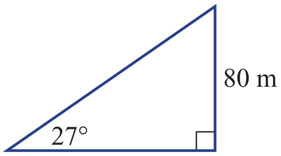
5 The angle of elevation to the top of a tower from a point on the ground 40 m from the base of the tower is 36°. Find the height of the tower, to the nearest metre.



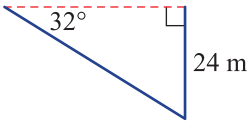
6 The angle of elevation to the top of a castle wall from a point on the ground 25 m from the base of the castle wall is 32°. Find the height of the castle wall, to the nearest metre.



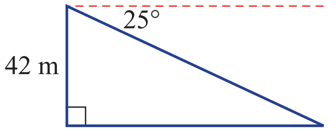
7 From a point on the ground, Lisa measures the angle of elevation of an 80 m tower to be 27°. Find how far Lisa is from the base of the tower, correct to the nearest metre.



8 From a pedestrian overpass, Chris spots a landmark at an angle of depression of 32°. How far away (to the nearest metre) is the landmark from the base of the 24 m high overpass?

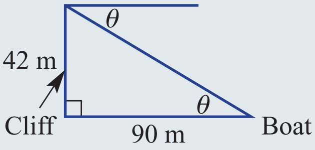


9 From a lookout tower, Jordan spots a bushfire at an angle of depression of 25°. If the lookout tower is 42 m high, how far away (to the nearest metre) is the bushfire from the base of the tower?



Example 23 Finding an angle of depression

From the top of a vertical cliff, Andrea spots a boat out at sea. If the top of the cliff is 42 m above sea level and the boat is 90 m away from the base of the cliff, find Andrea's angle of depression to the boat, to the nearest degree.

Solution	**Explanation**
	Draw a diagram and label all the given measurements. Use alternate angles in parallel lines to mark θ inside the triangle.
$\tan\theta = \frac{O}{A}$	Since the opposite (O) and adjacent sides (A) are given, use tan θ.
$\tan\theta = \frac{42}{90}$	
$\theta = 25.0168\ldots$	Use the $\tan^{-1}$ key on your calculator. Enter $\tan^{-1}\left(\frac{42}{90}\right)$.
The angle of depression is 25°, to the nearest degree.	Round off to the nearest degree. Express the answer in words.

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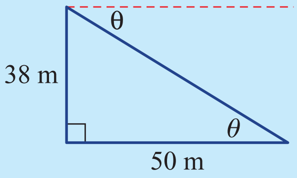
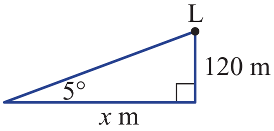
10 From the top of a vertical cliff, Josh spots a swimmer out at sea. If the top of the cliff is 38 m above sea level and the swimmer is 50 m away from the base of the cliff, find the angle of depression from Josh to the swimmer, to the nearest degree.

Start with a diagram like this.

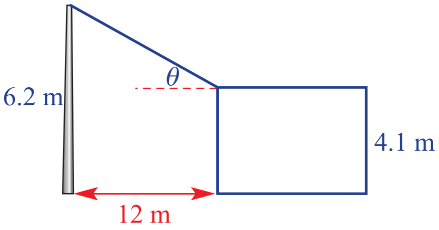


Problem-solving and Reasoning

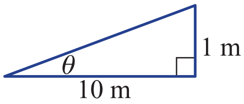
11 From a boat, the angle of elevation of a lighthouse is 5°. The light is 120 m above sea level. How far is the boat from the coast, to the nearest metre?



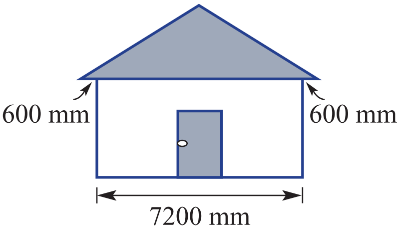
12 A power line is stretched from a pole to the top of a house. The house is 4.1 m high and the power pole is 6.2 m tall. The horizontal distance between the house and the power pole is 12 m. Find the angle of elevation of the top of the power pole from the top of the house, to the nearest degree.



13 A road has a steady gradient of 1 in 10. It rises 1 m for every 10 m across. What angle does the road make with the horizontal? Give your answer to the nearest degree.

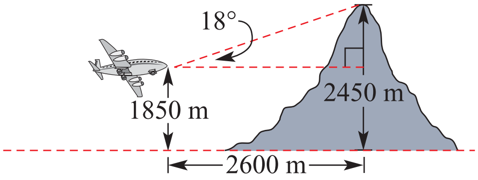


14 A house is to be built using the design shown on the right. The eaves are 600 mm and the house is 7200 mm wide, excluding the eaves. Calculate the length (to the nearest mm) of a sloping edge of the roof, which is pitched at 25° to the horizontal.



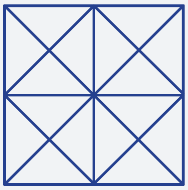
Enrichment: Plane trigonometry

15 A plane flying at 1850 m starts to climb at an angle of 18° to the horizontal when the pilot sees a mountain peak 2450 m high, 2600 m away from her in a horizontal direction. Will the pilot clear the mountain?



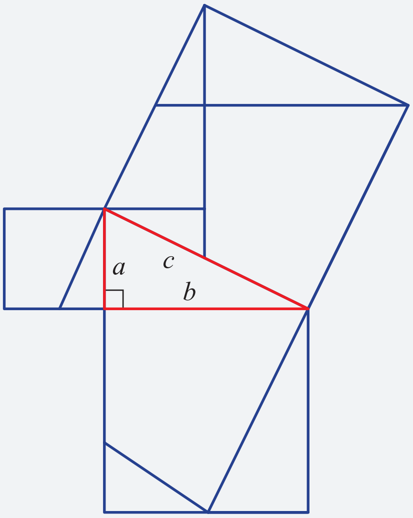
Puzzles and games

1 How many different right-angled triangles are there in this diagram?



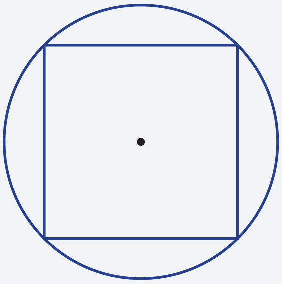
2 Look at this right-angled triangle and the squares drawn on each side. Each square is divided into smaller sections.

- **a** Can you see how the parts of the two smaller squares would fit into the larger square?
- **b** What is the area of each square if the side lengths of the right-angled triangle are a, b and c, as marked?
- **c** What do the answers to parts **a** and **b** suggest about the relationship between a, b and c?



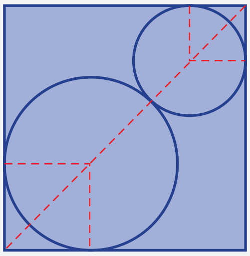
3 Imagine trying to cut the largest square from a circle of a certain size and calculating the side length of the square.

- **a** If the circle has a diameter of 2 cm, can you find a good position to draw the diameter that also helps to form a right-angled triangle?
- **b** Can you determine the side length of the largest square if the circle has a diameter of 2 cm?
- **c** What percentage of the area of a circle does the largest square occupy?



4 Which is a better fit: a square peg in a round hole or a round peg in a square hole? Use area calculations and percentages to investigate.

5 Two circles of radius 10 cm and 15 cm, respectively, are placed inside a square. Find the perimeter of the square, to the nearest centimetre. Hint: First find the diagonal length of the square using the diagram on the right.



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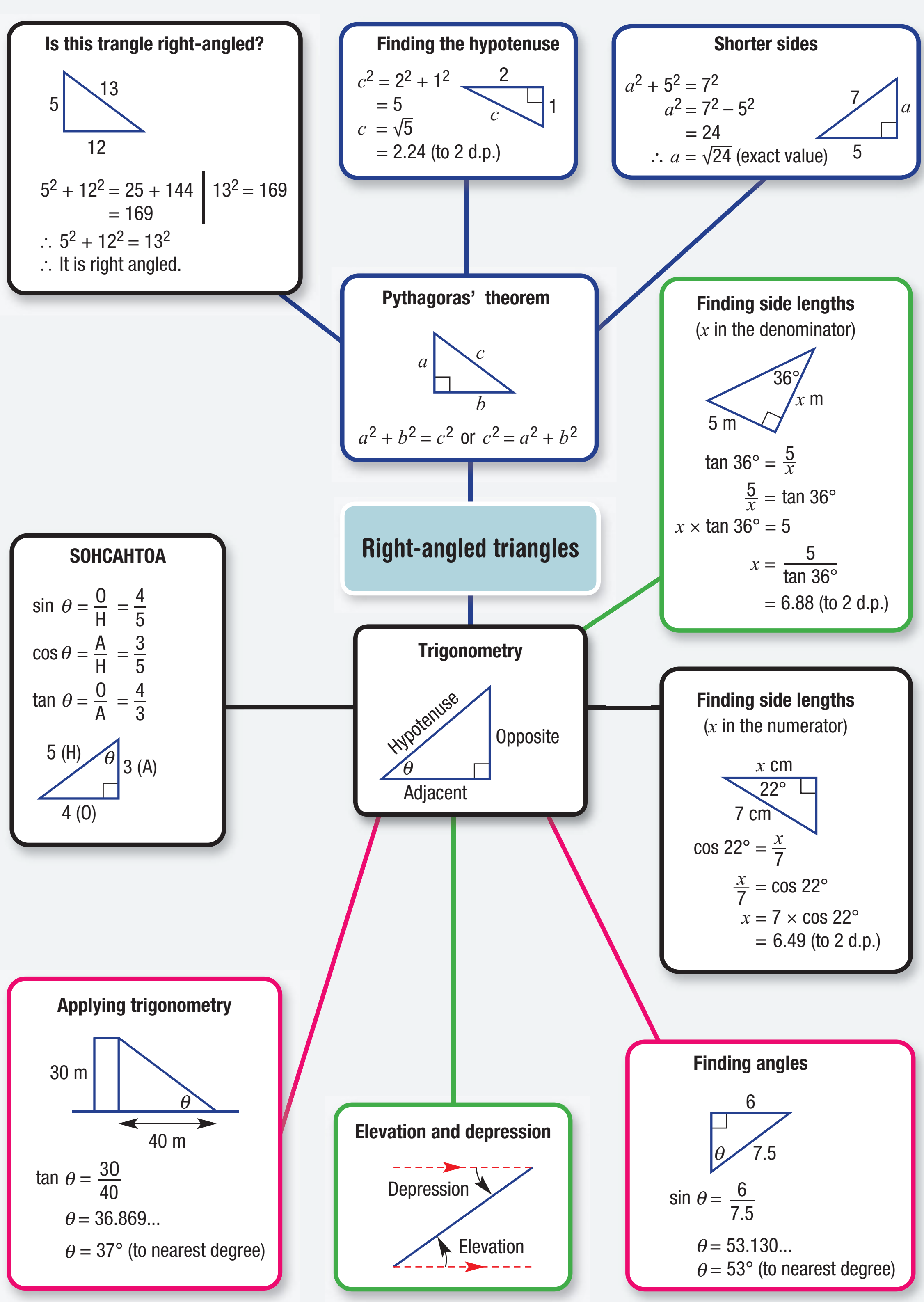
Chapter summary



Is this trangle right-angled?

$5^2 + 12^2 = 25 + 144$ $\quad$ $13^2 = 169$

$= 169$

$\therefore\ 5^2 + 12^2 = 13^2$

$\therefore$ It is right angled.

Finding the hypotenuse

$c^2 = 2^2 + 1^2$

$= 5$

$c = \sqrt{5}$

$= 2.24$ (to 2 d.p.)

Shorter sides

$a^2 + 5^2 = 7^2$

$a^2 = 7^2 - 5^2$

$= 24$

$\therefore\ a = \sqrt{24}$ (exact value)

Pythagoras' theorem

$a^2 + b^2 = c^2$ or $c^2 = a^2 + b^2$

Finding side lengths

(x in the denominator)

$\tan 36° = \frac{5}{x}$

$\frac{5}{x} = \tan 36°$

$x \times \tan 36° = 5$

$x = \frac{5}{\tan 36°}$

$= 6.88$ (to 2 d.p.)

SOHCAHTOA

$\sin\theta = \frac{\text{O}}{\text{H}} = \frac{4}{5}$

$\cos\theta = \frac{\text{A}}{\text{H}} = \frac{3}{5}$

$\tan\theta = \frac{\text{O}}{\text{A}} = \frac{4}{3}$

Trigonometry

Finding side lengths

(x in the numerator)

$\cos 22° = \frac{x}{7}$

$\frac{x}{7} = \cos 22°$

$x = 7 \times \cos 22°$

$= 6.49$ (to 2 d.p.)

Applying trigonometry

$\tan\theta = \frac{30}{40}$

$\theta = 36.869\ldots$

$\theta = 37°$ (to nearest degree)

Elevation and depression

Finding angles

$\sin\theta = \frac{6}{7.5}$

$\theta = 53.130\ldots$

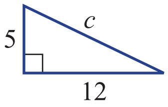
$\theta = 53°$ (to nearest degree)

Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.

Multiple-choice questions

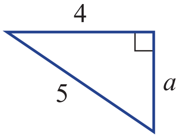
An online version of this test is available in the Interactive Textbook.

1 For the right-angled triangle shown, which is a correct statement?



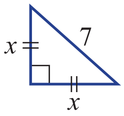
A $c^2 = 5^2 + 12^2$ **B** $c^2 = 5^2 - 12^2$ **C** $c^2 = 12^2 - 5^2$
D $c^2 = 5^2 \times 12^2$ **E** $(5 + 12)^2$

2 Which of the following is true for this triangle?



A $a = 5 - 4$ **B** $a^2 + 4^2 = 5^2$ **C** $a^2 = 5^2 + 4^2$
D $a = \sqrt{10}$ **E** $4^2 + 5^2 = a^2$

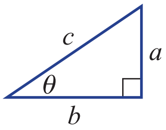
3 For the right-angled triangle shown:



A $2x^2 = 49$ **B** $7x^2 = 2$ **C** $2x^2 = 7$
D $x^2 + 7^2 = x^2$ **E** $x^2 = \frac{2}{7}$

Drilling for Gold
4R1
4R2

4 For the triangle shown:

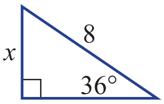


A $\sin\theta = \frac{a}{b}$ **B** $\sin\theta = \frac{c}{a}$ **C** $\sin\theta = \frac{a}{c}$
D $\sin\theta = \frac{b}{c}$ **E** $\sin\theta = \frac{c}{b}$

5 The value of cos 46°, correct to four decimal places, is:

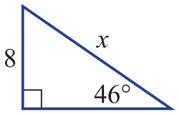
A 0.7193 **B** 0.6947 **C** 0.594
D 0.6532 **E** 1.0355

6 In the diagram, the value of x, correct to two decimal places, is:



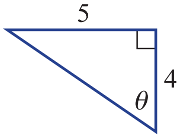
A 40 **B** 13.61 **C** 4.70
D 9.89 **E** 6.47

7 The value of x in the triangle is given by:



A 8 sin 46° **B** 8 cos 46° **C** $\frac{8}{\cos 46°}$
D $\frac{8}{\sin 46°}$ **E** $\frac{\cos 46°}{8}$

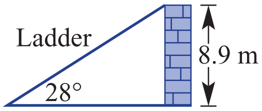
8 Which equation could be used to find the value of θ?



A $\cos\theta = \frac{4}{5}$ **B** $\cos\theta = \frac{5}{4}$ **C** $\tan\theta = \frac{5}{4}$
D $\sin\theta = \frac{4}{5}$ **E** $\tan\theta = \frac{4}{5}$

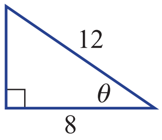
9 A ladder is inclined at an angle of 28° to the horizontal. If the ladder reaches 8.9 m up the wall, the length of the ladder, correct to the nearest metre, is:

A 19 m **B** 4 m **C** 2 m
D 10 m **E** 24 m



10 The value of θ in the diagram, correct to the nearest degree, is:

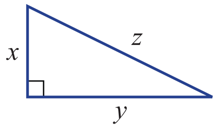
A 1° **B** 48° **C** 48.2°
D 48.18° **E** 49°



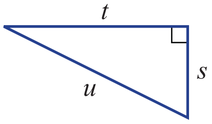
Short-answer questions

1 Write down Pythagoras' theorem for these triangles, using the given pronumerals.

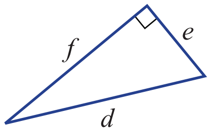
a



b

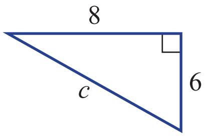


c

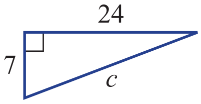


2 Find the length of the hypotenuse in these triangles. Round to two decimal places where appropriate.

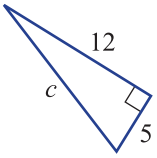
a



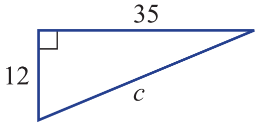
b



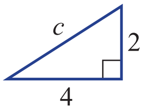
c



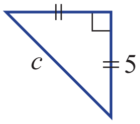
d



e

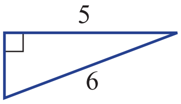


f

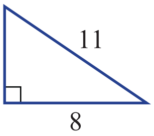


3 Find the length of the unknown side in these right-angled triangles. Round to two decimal places.

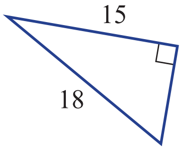
a



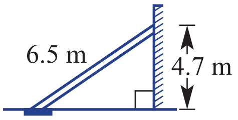
b



c



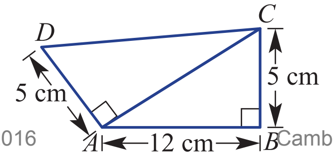
4 A steel support beam of length 6.5 m is connected to a wall at a height of 4.7 m from the ground. Find the distance (to two decimal places) between the base of the building and the point where the beam is joined to the ground.



5 For this double triangle, find the length:

a AC

b CD (correct to two decimal places)



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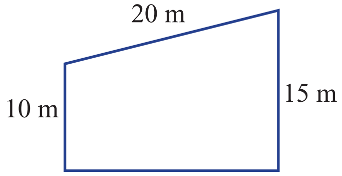
6 Two different cafés on opposite sides of an atrium in a shopping centre are, respectively, 10 m and 15 m above the ground floor. If the cafés are linked by a 20 m escalator, find the horizontal distance (to the nearest metre) across the atrium, between the two cafés.

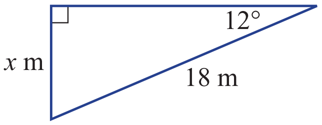


7 Find the value of each of the following, correct to two decimal places.

a sin 40° **b** tan 66° **c** cos 44°

8 Which ratio (i.e. $\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$ or $\tan\theta = \frac{O}{A}$) would be used to find the value of the unknown in these triangles?

a



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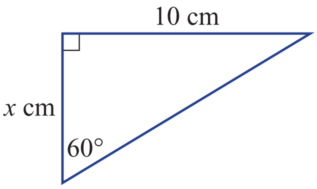

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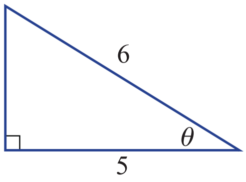
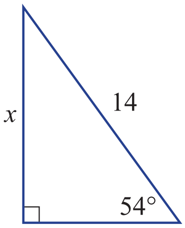


9 Find the value of each pronumeral, correct to two decimal places where necessary. Use $\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$ or $\tan\theta = \frac{O}{A}$.

a



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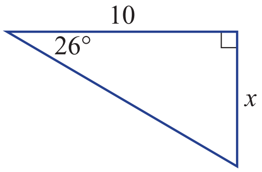

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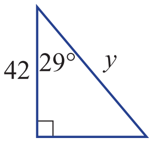

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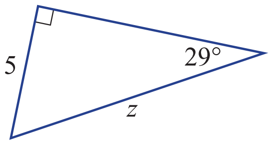

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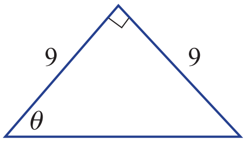

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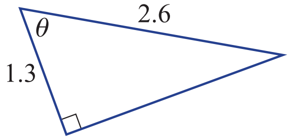
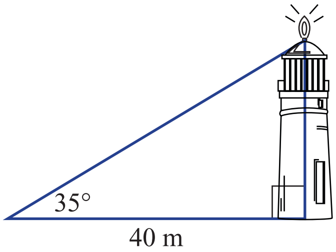
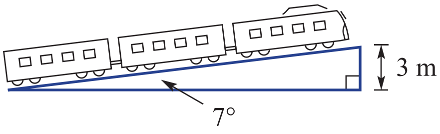


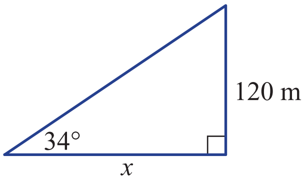
10 The angle of elevation of the top of a lighthouse from a point on the ground 40 m from its base is 35°. Find the height of the lighthouse, correct to two decimal places.



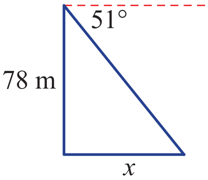
11 A train travels up a slope, making an angle of 7° with the horizontal. When the train is at a height of 3 m above its starting point, find the distance it has travelled up the slope, to the nearest metre.



12 From a point on flat ground, Ahbed measures the angle of elevation of a 120 m tower to be 34°. How far from the base of the tower is Ahbed, correct to two decimal places?



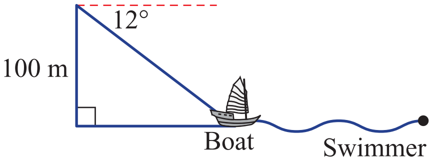
13 Aisha spots a car at an angle of depression of 51° from the roof of a skyscraper. If the skyscraper is 78 m high, how far away is the car from the base of the skyscraper, correct to one decimal place?



Extended-response questions

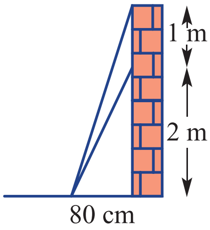
1 From the top of a 100 m cliff, Skevi sees a boat out at sea at an angle of depression of 12°.

- **a** Find how far out to sea the boat is, to the nearest metre.
- **b** A swimmer is 2 km away from the base of the cliff and in line with the boat. What is the angle of depression to the swimmer, to the nearest degree?
- **c** How far away is the boat from the swimmer, to the nearest metre?



2 An extension ladder is initially placed so that it reaches 2 m up a wall. The foot of the ladder is 80 cm from the base of the wall.

- **a** Find the length of the ladder, to two decimal places, in its original position.
- **b** Without the foot of the ladder being moved, the ladder is extended so that it reaches 1 m further up the wall. How far (to two decimal places) has the ladder been extended?
- **c** The ladder is placed so that its foot is now 20 cm closer to the base of the wall. How far up the wall can the ladder length found in part **b** reach? Round your answer to two decimal places.



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Chapter 1: Integers, decimals, fractions, ratios and rates

Multiple-choice questions

1 $7 \times (-5) + 3 \times (-4)$ is equal to:
A 23 **B** –47 **C** 47 **D** –23 **E** 420

2 $45.5 \div 8.5$, when rounded to one significant figure, gives:
A 4.9 **B** 5.35 **C** 5.4 **D** 5 **E** 4.95

3 $450 is divided in the ratio 4 : 5. The value of the smaller portion is:
A $210 **B** $250 **C** $90 **D** $200 **E** $220

4 Harrison runs 4.5 km in 20 minutes. His speed is:
A 1.5 km/h **B** 1.5 km/min **C** 13.5 km/h **D** 90 km/h **E** 22.5 km/h

5 $\frac{7}{10000} + \frac{13}{100}$ is equal to:
A 0.713 **B** 0.0713 **C** 0.1307 **D** 0.83 **E** 7.13

Short-answer questions

1 Evaluate the following.
a $\frac{13}{10} - \frac{3}{10}$ **b** $\frac{3}{4} + \frac{1}{4}$ **c** $2\frac{1}{4} + \frac{1}{2}$ **d** $2\frac{5}{9} - 1\frac{1}{3}$

2 Evaluate, then check with a calculator.
a $\frac{9}{10} \times \frac{5}{12}$ **b** $3\frac{3}{4} \div 2\frac{1}{12}$

3 Round the following to three decimal places.
a 4.1255 **b** 21.00241 **c** 0.0096

4 Divide $800 into the following ratios.
a 1 : 4 **b** 2 : 3 **c** 7 : 1

5 Write these rates and ratios in simplest form.
a Prize money is shared between two people in the ratio 60 : 36.
b Jodie travels 165 km in 3 hours.
c 30 mL of rain falls in $1\frac{1}{4}$ hours.

6 A car averages 68 km/h on a journey. How far will it travel at this speed for 2 hours and 40 minutes? Round your answer to one decimal place.

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Extended-response question

Richard walks, on average, 6 kilometres an hour. Phillip walks at an average speed of 8 km/h.

a How far does Richard walk in 4 hours?

b Phillip starts at 8 a.m.

  i If he stops at 9:45 a.m., how far has he walked?

  ii What time will Phillip finish his walk if he walks twice as far as Richard?

Chapter 2: Financial mathematics

Multiple-choice questions

1 $2\frac{1}{2}\%$ is the same as:

A 0.25 **B** $\frac{1}{40}$ **C** $\frac{25}{100}$ **D** 2.5 **E** $\frac{5}{2}$

2 If the cost price is \$36 and the selling price is \$28.80, the percentage loss is:

A 7.2% **B** 80% **C** 92.8% **D** 20% **E** 2%

3 The simple interest earned on \$660 invested at 5% p.a. for $1\frac{1}{2}$ years is:

A \$49.50 **B** \$40.50 **C** \$594 **D** \$33 **E** \$709.50

4 A book that costs \$27 is discounted by 15%. The new price is:

A \$20.25 **B** \$31.05 **C** \$4.05 **D** \$22.95 **E** \$25.20

5 Anna is paid a normal rate of \$12.10 per hour. In a week she works 6 hours at the normal rate, 2 hours at time and a half and 3 hours at double time. How much does she earn?

A \$181.50 **B** \$145.20 **C** \$193.60 **D** \$175.45 **E** \$163.35

Short-answer questions

1 Convert each of the following to a percentage.

a 0.6 **b** $\frac{5}{16}$ **c** 2 kg out of 20 kg **d** 75c out of \$3

2 Find:

a 10% of \$96 **b** 85% of \$900 **c** $5\frac{1}{2}\%$ of \$200

3 **a** Increase 80 m by 5%. **b** Decrease \$54 by 6%.

4 Giuliana spends 20% of a day reading a novel. How many hours and minutes does she spend reading?

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5 Jamal earns a weekly retainer of \$400 plus 6% of the sales he makes. If he sells \$8200 worth of goods, how much will he earn for the week?

6 Sean invests \$800 at 6% p.a. for 18 months.
  a Calculate the simple interest earned.
  b What is the final balance on Sean's account after the interest has been paid?

7 How long does \$600 need to be invested at 8% p.a. simple interest to earn \$120 in interest?

Extended-response question

Mikayla plans to trial a simple interest plan. Before investing her money, she increases the amount in her account by 20% to \$21 000.

a What was the original amount in Mikayla's account?
b Mikayla invests the \$21 000 for 4 years at a simple interest rate of 3% p.a. How much does she have in her account at the end of the 4 years?
c Mikayla continues with the plan in part **b** and, after a certain number of years, has earned \$5670 interest. For how many years has she had the money invested?
d What percentage increase does this interest represent on Mikayla's initial investment?

Chapter 3: Expressions and equations

Multiple-choice questions

1 The expression that represents 3 more than half a certain number, n, is:
A $\frac{1}{2}n+1.5$ **B** $2n+3$ **C** $\frac{3}{2}n$ **D** $\frac{n}{2}+3$ **E** $\frac{n+3}{2}$

2 The simplified form of $5ab + 3a + 2ab - a$ is:
A $9ab$ **B** $10ab + 3a$ **C** $13ab - a$ **D** $7ab + 3$ **E** $7ab + 2a$

3 The expanded form of $-2(3m - 4)$ is:
A $-6m + 8$ **B** $-6m + 4$ **C** $-6m - 8$ **D** $-5m - 6$ **E** $5m + 8$

4 The solution to $\frac{d}{4} - 7 = 2$ is:
A $d = -20$ **B** $d = 15$ **C** $d = 36$ **D** $d = 1$ **E** $d = 30$

5 The formula $m = \sqrt{\frac{b-1}{a}}$, with $b = 9$ and $a = 2$, gives m equals:
A 3 **B** $\frac{3}{2}$ **C** 2 **D** $\sqrt{3}$ **E** $\frac{\sqrt{3}}{2}$

Short-answer questions

1 Simplify the following.
  a $2x + 6y - 4x + y$ **b** $-3m \times 5n$ **c** $\frac{6xy}{18x}$

2 Solve the following equations.
  a $3x + 7 = 25$ **b** $\frac{x-1}{4} = 2$ **c** $4(2m + 3) = 15$ **d** $5a - 8 = 3a - 2$

3 Noah receives m dollars pocket money per week. His older brother Jake gets \$1 more than twice Noah's amount. If Jake receives \$15:

a Write an equation to represent the problem, using the variable m.

b Solve the equation in part **a** to determine how much Noah receives each week.

4 The formula $S = \frac{n}{2}(a + \ell)$ gives the sum, S, of a sequence of n integers with first term a and last term ℓ. Find the sum of the sequence of 10 terms 2, 5, 8, . . ., 29.

Extended-response question

Reese and his brother Daniel play basketball. In the last game, Reese scored 12 more points than Daniel. Between them they scored 38 points. Let p be the number of points scored by Daniel.

a Write an expression for the number of points scored by Reese.

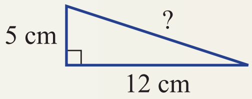
b Write an equation involving the unknown, p, to represent the problem.

c Solve the equation to find the number of points scored by Reese and Daniel.

Chapter 4: Right-angled triangles

Multiple-choice questions

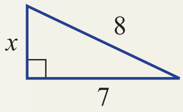
1 The length of the hypotenuse in this triangle is:



A 11 cm **B** 13 cm **C** 14.8 cm

D 169 cm **E** $\frac{1}{\sqrt{119}}$ cm

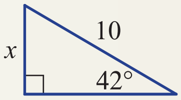
2 The value of x in the triangle shown is:



A 3.9 **B** $\sqrt{113}$ **C** $\sqrt{57}$

D $\sqrt{15}$ **E** 2.7

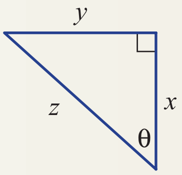
3 The value of x in the triangle shown is closest to:



A 6.7 **B** 7.4 **C** 14.9

D 13.5 **E** 9.0

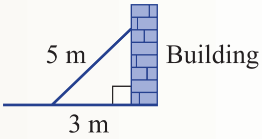
4 A correct ratio for this triangle is:



A $\tan\theta = \frac{x}{y}$ **B** $\cos\theta = \frac{y}{z}$ **C** $\tan\theta = \frac{y}{x}$

D $\sin\theta = \frac{y}{x}$ **E** $\cos\theta = \frac{x}{y}$

5 A 5 m plank of wood is leaning up against the side of a building, as shown. If the wood touches the ground 3 m from the base of the building, the angle the wood makes with the building is closest to:

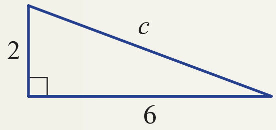


A 36.9° **B** 59° **C** 53.1° **D** 31° **E** 41.4°

Short-answer questions

1 Find the value of each pronumeral, correct to one decimal place.

a



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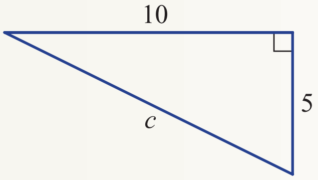

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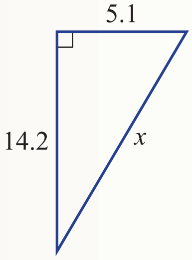

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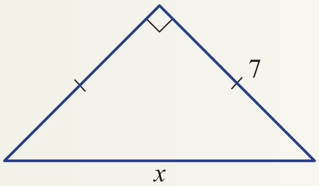

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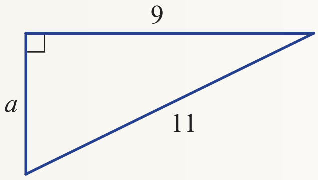

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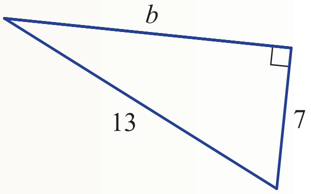
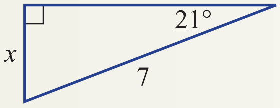


2 Find the value of each pronumeral, correct to one decimal place.

a



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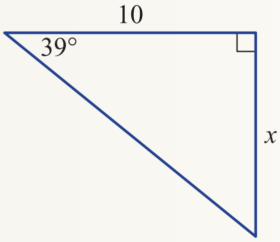

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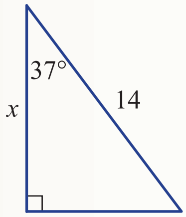

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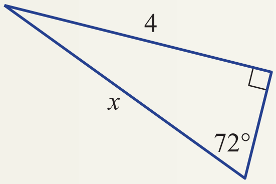

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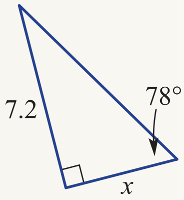

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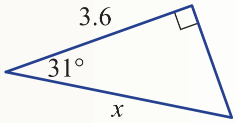

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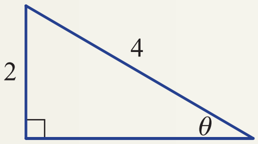

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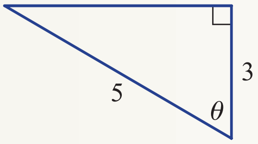

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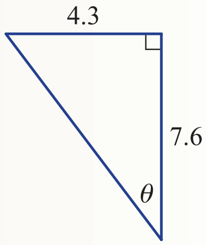
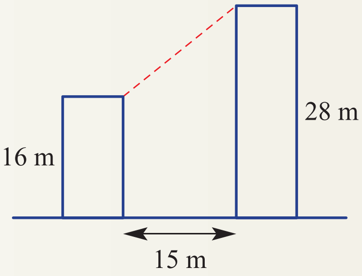


3 A wire is to be connected from the edge of the top of a 28 m tall building to the edge of the top of a 16 m tall building. The buildings are 15 m apart.

a What length of wire is required? Round your answer to two decimal places.

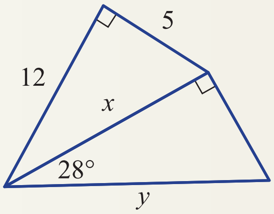
b What is the angle of depression from the top of the taller building to the top of the smaller building? Round your answer to one decimal place.



4 For this diagram, use:

a Pythagoras' theorem to find the value of x

b trigonometry to find the value of y, correct to two decimal places

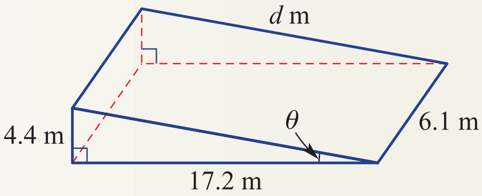


Extended-response question

A skateboard ramp is constructed as shown.

a Calculate the distance, d metres, up the ramp, correct to two decimal places. Use Pythagoras' theorem.

b What is the angle of inclination, θ, between the ramp and the ground, correct to one decimal place?



c If the skateboarder rides from one corner of the ramp diagonally to the other corner, what distance would be travelled? Round your answer to one decimal place.

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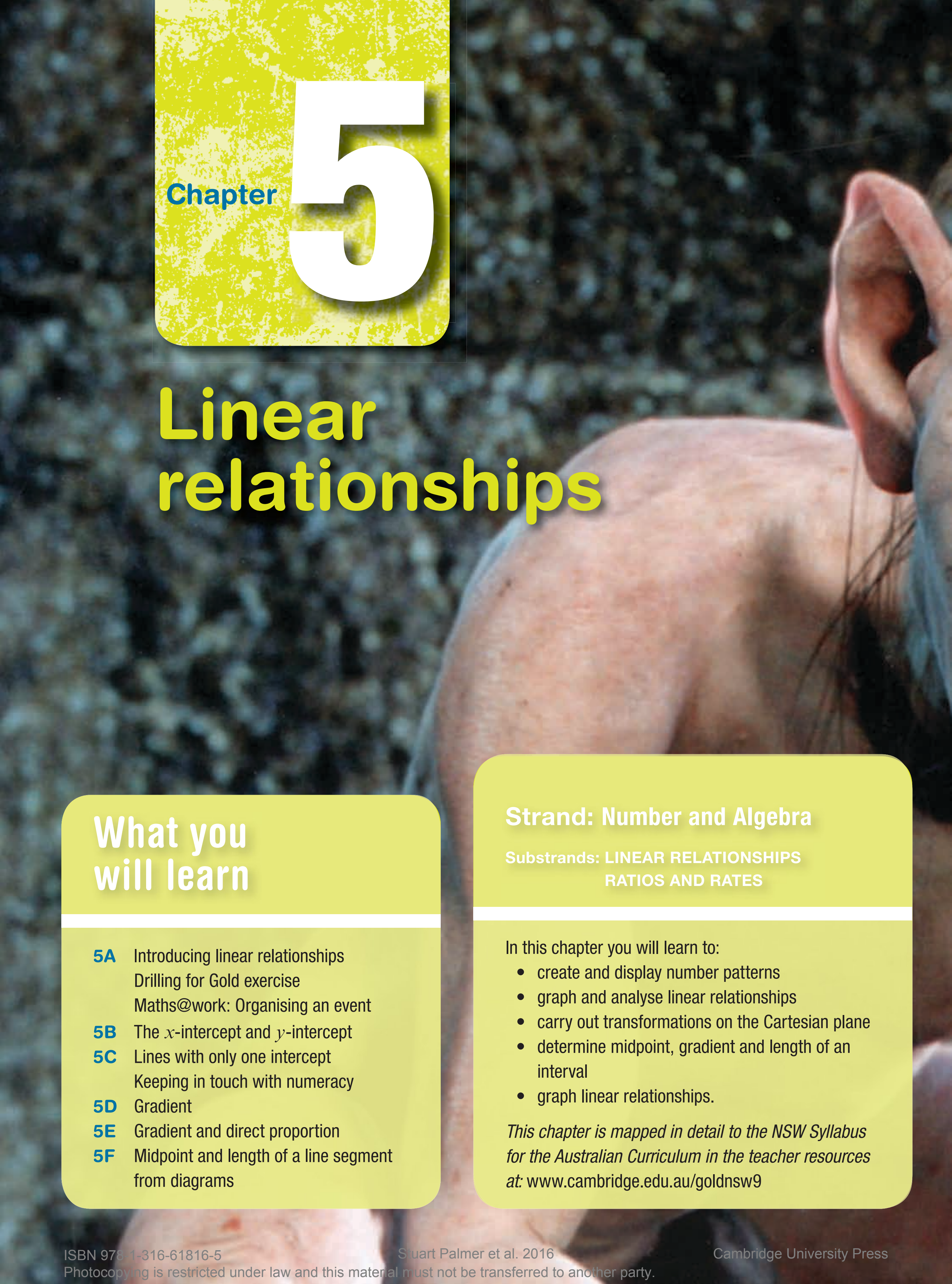

Chapter 5

Linear relationships

What you will learn

Strand: Number and Algebra

Substrands: LINEAR RELATIONSHIPS
RATIOS AND RATES

In this chapter you will learn to:

- create and display number patterns
- graph and analyse linear relationships
- carry out transformations on the Cartesian plane
- determine midpoint, gradient and length of an interval
- graph linear relationships.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw9

Computer-generated imagery (CGI)

Movies and computer games include many scenes and characters that are generated by computer, such as the character Gollum in *The Hobbit* and the *Lord of the Rings* trilogy – one of the most successful CGI characters in a movie.

The fundamentals of CGI and computer graphics are based on computer programming, including linear algebra, which is the focus of this chapter.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO:* www.cambridge.edu.au/goldnsw9

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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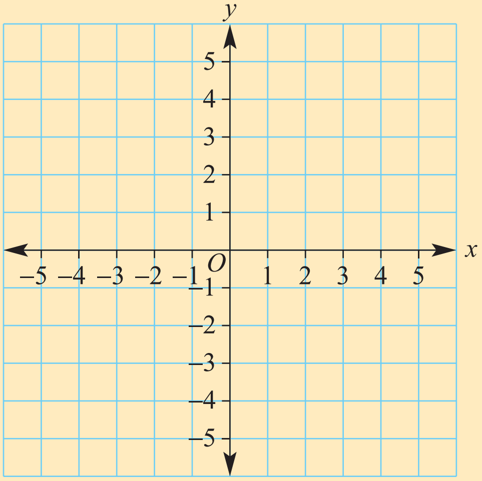
Pre-test

1 Plot and label the following points on a Cartesian plane.

a $(3, 1)$ **b** $(-2, 4)$ **c** $(-3, -3)$ **d** $(5, -3)$



2 Given $y = x - 2$, find the value of y when:

a $x = 3$ **b** $x = 1$ **c** $x = -1$ **d** $x = -2$

3 Solve the following equations for x.

a $3x = 15$ **b** $-2x = 4$ **c** $-x = 3$

d $0 = x - 4$ **e** $0 = 3x + 6$ **f** $0 = 2x - 8$

4 In each of the following, find the value of y when $x = 0$.

a $y = 2x + 3$ **b** $y = 3x - 4$ **c** $x + 2y = 8$ **d** $3x - 4y = 12$

5 Copy and complete the tables of values.

a $y = x + 3$

x	-2	-1	0	1	2
y					

b $y = 2x - 1$

x	-2	-1	0	1	2
y					

6 When $x = 2$ and $y = 1$, decide whether the following equations are true or false.

a $y = 2x + 1$ **b** $y = 3x - 5$ **c** $y = -2x + 5$ **d** $5x - 2y = 6$

7 Simplify:

a $\frac{4+10}{2}$ **b** $\frac{2+11}{2}$ **c** $\frac{-1+7}{2}$ **d** $\frac{-3+(-7)}{2}$

8 Find the vertical distance between the following pairs of points.

a $(3, 2)$ and $(3, 7)$ **b** $(-1, 1)$ and $(-1, -3)$

c $(2, 3)$ and $(2, -2)$ **d** $(1, -4)$ and $(1, -1)$

9 Find the horizontal distance between the following pairs of points.

a $(1, 3)$ and $(5, 3)$ **b** $(-1, 2)$ and $(4, 2)$

c $(-2, -3)$ and $(5, -3)$ **d** $(-4, 1)$ and $(-1, 1)$

5A Introducing linear relationships

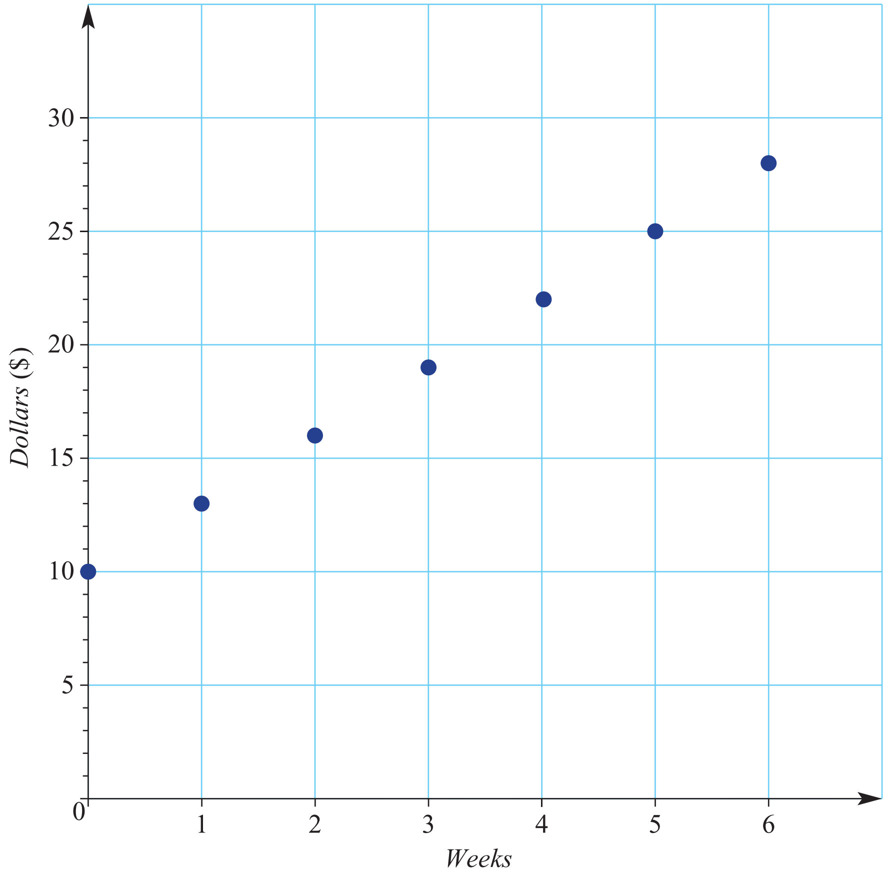
Stage
5.2
5.2◊
5.1
4

In the table below, \$3 is added every week. This is an example of a linear relationship because the points form a straight line.

Weeks	0	1	2	3	4	5	6
Dollars	10	13	16	19	22	25	28

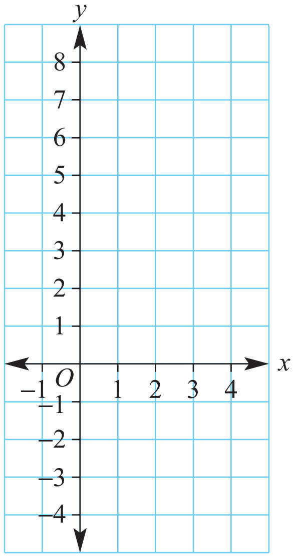
Drilling for Gold 5A1

Spreadsheet 5A1



Let's start: Which equation is linear?

Here are three equations and their tables of values. Copy and complete the tables of values and plot the points on grid paper with the axes shown.
(Use a different colour for each set of points.)



1 $y = 2^x - 1$

x	0	1	2	3
y				

2 $y = x^2 - 1$

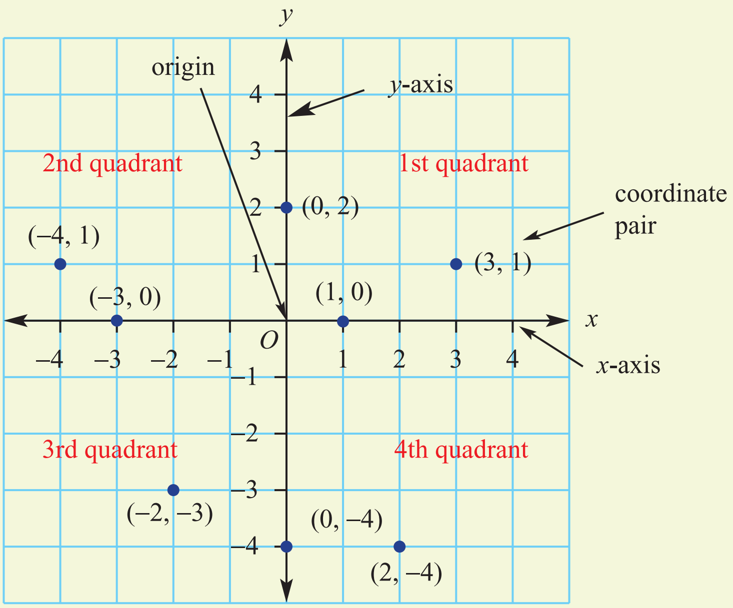
x	0	1	2	3
y				

3 $y = 3x - 4$

x	0	1	2	3
y				

Key ideas

- The **Cartesian plane** (or number plane) consists of two axes that divide the number plane into four quadrants.
  - The ***x*-axis** is the horizontal axis.
  - The ***y*-axis** is the vertical axis.
  - The x-axis and y-axis intersect at the origin $(0, 0)$ at right angles.
  - A point is positioned on a Cartesian plane using the coordinate pair (x, y). x describes the horizontal position of the point from the origin. y describes the vertical position of the point from the origin.



- In the coordinate pair (2, –4), 2 is the x-coordinate and –4 is the y-coordinate. The point is 2 units to the right of the origin (horizontal direction) and 4 units below the origin (vertical direction).
- A **linear relationship** is a set of ordered pairs (x, y) that give a straight line.
- Linear relationships have rules that may be of the form:
  - $y = mx + b$; e.g. $y = 2x + 1$
  - $ax + by = d$; e.g. $2x - 3y = 4$
  - $ax + by + c = 0$; e.g. $2x - 3y - 4 = 0$
- Each point that is on the line fits the rule for the linear relation. For example, in the point (2, 3), $x = 2$, $y = 3$. This point lies on the line $y = 4x - 5$ because $3 = 4 \times 2 - 5$.

Cartesian plane A plane on which every point is related to a pair of numbers called coordinates

***x*-axis** The horizontal axis of the Cartesian plane

***y*-axis** The vertical axis of the Cartesian plane

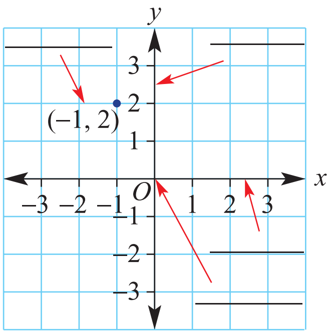
Linear relationship A set of ordered pairs that give a straight line

Exercise 5A

Understanding

1 **a** For the Cartesian plane shown, label the following parts.

- **i** x-axis
- **ii** origin
- **iii** y-axis
- **iv** coordinates



b For the point (3, −1):

- **i** 3 is the ______________.
- **ii** −1 is the ______________.
- **iii** From the origin, the point is 3 units to the _________ and 1 unit ______________.

2 Which one of the following rules is linear?

A $y = x^2 + 4$ **B** $y = \frac{2}{x}$ **C** $y = 2x - 1$ **D** $y = 3^x$

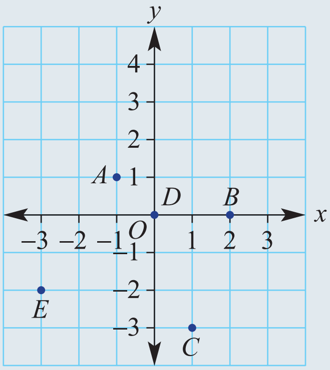
Example 1 Using the Cartesian plane

Using the grid shown:

a Plot the points with coordinates:

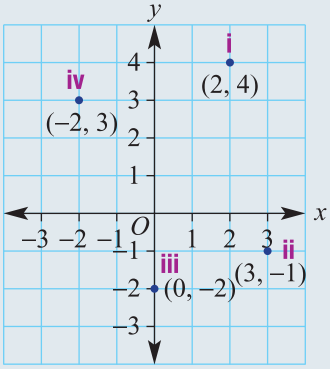
i (2, 4) **ii** (3, −1) **iii** (0, −2) **iv** (−2, 3)

b State the coordinates of the points marked A–E.



Solution

a



Explanation

From the origin, start with the x-coordinate, then the y-coordinate.

- **i** (2, 4) is 2 units right of the origin and 4 units above.
- **ii** (3, −1) is 3 units right of the origin and 1 unit below.
- **iii** For (0, −2), the x coordinate is 0, so there is no move left or right, so the point is 2 units below the origin.
- **iv** (−2, 3) is 2 units left and 3 units above the origin.

b $A(-1, 1)$
$B(2, 0)$
$C(1, -3)$
$D(0, 0)$
$E(-3, -2)$

Each pair is written with the x-coordinate first, followed by the y-coordinate (x, y). To find the x-coordinate, start at the origin and move horizontally along the x-axis until you are in line with the point. To find the y-coordinate, start at the origin and move vertically along the y-axis until you are in line with the point.

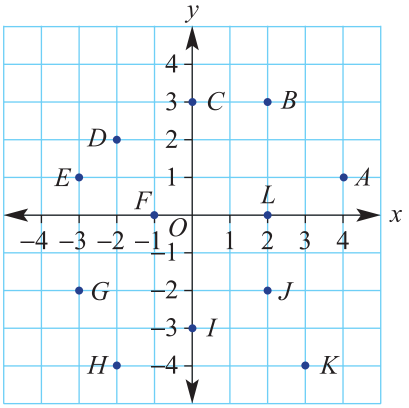
3 Refer to the Cartesian plane shown.

a Draw this set of axes (without A–L) and plot the following points.

i $(3, 1)$ **ii** $(-4, -2)$

iii $(0, 2)$ **iv** $(-1, 2)$

b Give the coordinates of all the points A–L.



4 For the rule $y = 3x - 4$, find the value of y for these x values.

a $x = 2$ **b** $x = 1$ **c** $x = 0$ **d** $x = -1$

Substitute the x value into the rule to find the y value.

5 Which pair of coordinates fits the rule $y = 3x$?

A $(3, 1)$ **B** $(3, 6)$ **C** $(2, 6)$ **D** $(-1, 3)$

Fluency

Example 2 Filling in tables from rules

Complete the tables for the given rules.

a $y = 2x - 1$

x	-2	-1	0	1	2	3
y						

b $y = -3x + 2$

x	-2	-1	0	1	2
y					

Solution

a

x	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5

Explanation

Substitute each x value in the table into the rule $y = 2x - 1$ to find its corresponding y value.

For $x = -2$, $y = 2 \times (-2) - 1$
$= -4 - 1$
$= -5$

For $x = 1$, $y = 2 \times 1 - 1$
$= 2 - 1$
$= 1$

b

x	-2	-1	0	1	2
y	8	5	2	-1	-4

Substitute each x value into the rule $y = -3x + 2$.

Recall that negative × negative = positive.

For $x = -2$, $y = -3 \times (-2) + 2$
$= 6 + 2$
$= 8$

For $x = 0$, $y = -3 \times 0 + 2$
$= 0 + 2$
$= 2$

Drilling for Gold 5A2 at the end of this section

6 Copy and complete the tables for the given rules.

a $y = x + 2$

x	−2	−1	0	1	2	3
y						

b $y = 2x + 3$

x	−2	−1	0	1	2
y					

A negative × positive is negative.

A negative × negative is positive.

c $y = -2x$

x	−2	−1	0	1	2
y					

d $y = -3x - 1$

x	−2	−1	0	1	2	3
y						

Drilling for Gold 5A3

Skillsheet 5A

Example 3 Plotting points to graph straight lines

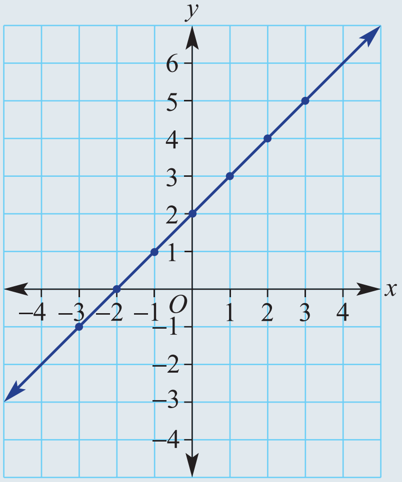
Construct a table of values from $x = -3$ to $x = 3$ and plot a graph for these linear relations.

a $y = x + 2$ **b** $y = -2x + 2$

Solution

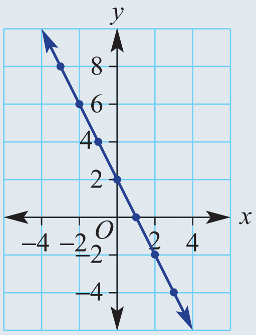
a

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5



b

x	−3	−2	−1	0	1	2	3
y	8	6	4	2	0	−2	−4



Explanation

Use $-3 \leq x \leq 3$ to construct a table of x values, where the lowest x value is −3 and the highest x value is 3.

Substitute each value of x into the rule $y = x + 2$.

The coordinates of the points are read from the table; i.e. (−3, −1), (−2, 0) etc.

Draw a set of axes with an appropriate scale to show the points from the table.

Plot each point and join them to form a straight line.

Extend the line to show it continues in either direction.

Substitute each value of x into the rule $y = -2x + 2$.

Plot each point and join them to form a straight line.
Extend the line beyond the plotted points.

7 Construct a table of values from $x = -3$ to $x = 3$ and plot a graph for these linear relationships.

a $y = x - 1$ **b** $y = x + 3$ **c** $y = 2x - 3$
d $y = -3x$ **e** $y = -2x - 1$ **f** $y = -x + 4$

Problem-solving and Reasoning

Example 4 Deciding whether a point is on a line

Decide whether the point (3, 7) is on the line with the given rule.

a $y = 3x - 4$ **b** $y = 2x + 1$

Solution	Explanation
a $y = 3x - 4$ Substitute $x = 3$: $y = 3 \times 3 - 4$ $= 5$ (not 7) $\therefore$ The point (3, 7) is not on the line.	Find the value of y on the graph of the rule for $x = 3$. To fit the rule, the y value needs to be 5, not 7, so (3, 7) is not on the line.
b $y = 2x + 1$ Substitute $x = 3$: $y = 2 \times 3 + 1$ $= 7$ $\therefore$ The point (3, 7) is on the line.	By substituting $x = 3$ into the rule for the line, we find $y = 7$. Since (3, 7) fits the rule, it is on the line.

8 Decide whether the point (2, 8) is on the line with the given rule.

a $y = 2x + 6$
b $y = 3x + 2$
c $y = -x + 6$

When $x = 2$, does $y = 8$?

9 Decide whether the point (–1, 4) is on the line with the given rule.

a $y = x + 5$
b $y = -2x + 2$
c $y = 4x$

10 Find a rule in the form $y = mx + b$ (e.g. $y = 2x - 1$) that matches these tables of values.

Drilling for Gold 5A4 at the end of this section

It might help to look for a pattern in the y values to find the connection between the x and y values.

a

x	0	1	2	3	4
y	2	3	4	5	6

b

x	0	1	2	3
y	0	2	4	6

c

x	–2	–1	0	1	2	3
y	–3	–1	1	3	5	7

d

x	–2	–1	0	1	2	3
y	–7	–4	–1	2	5	8

11 The table below shows the recorded height, y, of a seedling x days after it sprouts. Find a linear rule in the form $y = \ldots$ that matches the data in the table.

x (day)	1	2	3	4
y (height)	3	6	9	12

Experimental data can lead to a linear rule, such as that used to model the growth of a seedling.

Drilling for Gold 5A5

12 By considering the relationship between the coordinates, match these rules (**i–iv**) to the graphs (**a–d**).

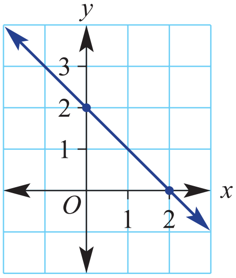
i $y = 2x + 1$

ii $y = -x - 1$

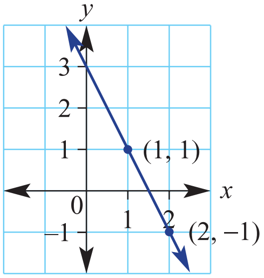
iii $y = -2x + 3$

iv $x + y = 2$

a

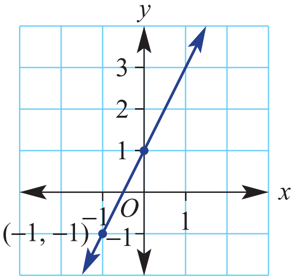


b

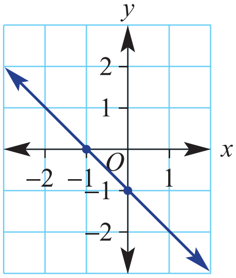


Each rule shows the relationship between the x- and y-coordinates for each point on the line. In **i**, each y-coordinate has to be 1 more than twice its x-coordinate.

c



d



Enrichment: Advanced rule finding

13 Find the linear rule linking x and y in these tables.

a

x	−1	0	1	2	3
y	5	7	9	11	13

b

x	−2	−1	0	1	2
y	22	21	20	19	18

c

x	1	3	5	7	9
y	1	5	9	13	17

d

x	6	7	8	9	10
y	4	4.5	5	5.5	6

e

x	0	2	4	6	8
y	4	−2	−8	−14	−20

f

x	−5	−4	−3	−2	−1
y	−21	−16	−11	−6	−1

5A2: From equation to table

Use the equation to complete the table of values in the worksheet or write the answers in your exercise book.

1 $y = x + 3$

x	0	1	2	3	4
y					

2 $y = 3x$

x	0	1	2	3	4
y					

3 $y = x - 3$

x	0	1	2	3	4
y					

4 $y = 3 - x$

x	0	1	2	3	4
y					

5 $y = 3x + 2$

x	0	1	2	3	4
y					

6 $y = 3 + 2x$

x	0	1	2	3	4
y					

7 $y = 2(x + 3)$

x	0	1	2	3	4
y					

8 $y = 3x - 2$

x	0	1	2	3	4
y					

9 $y = 2x - 3$

x	0	1	2	3	4
y					

10 $y = 2 - 3x$

x	0	1	2	3	4
y					

11 $y = 2x + 3$

x	0	1	2	3	4
y					

12 $y = -2\,x$

x	0	1	2	3	4
y					

5A4: From table to equation

Drilling for Gold exercise

Use the values in the table to find the equation of the line. Write the answers in the worksheet or in your exercise book.

1

x	0	1	2	3	4
y	3	4	5	6	7

$y =$

2

x	0	1	2	3	4
y	–2	–1	0	1	2

$y =$

3

x	0	1	2	3	4
y	0	2	4	6	8

$y =$

4

x	0	1	2	3	4
y	0	5	10	15	20

$y =$

5

x	0	1	2	3	4
y	0	–2	–4	–6	–8

$y =$

6

x	0	1	2	3	4
y	0	1	2	3	4

$y =$

7

x	0	1	2	3	4
y	2	4	6	8	10

$y =$

8

x	0	1	2	3	4
y	1	3	5	7	9

$y =$

9

x	0	1	2	3	4
y	–3	–1	1	3	5

$y =$

10

x	0	1	2	3	4
y	–2	0	2	4	6

$y =$

11

x	0	1	2	3	4
y	4	7	10	13	16

$y =$

12

x	0	1	2	3	4
y	3	7	11	15	19

$y =$

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Maths@work

Worksheet 5A

Organising an event

An event such as an 18th birthday party might involve hiring a venue and a DJ and paying for catering. The venue and DJ will probably be a fixed cost (e.g. $800).

The cost for the catering will depend on the number of people who will be attending (e.g. $45 per head).

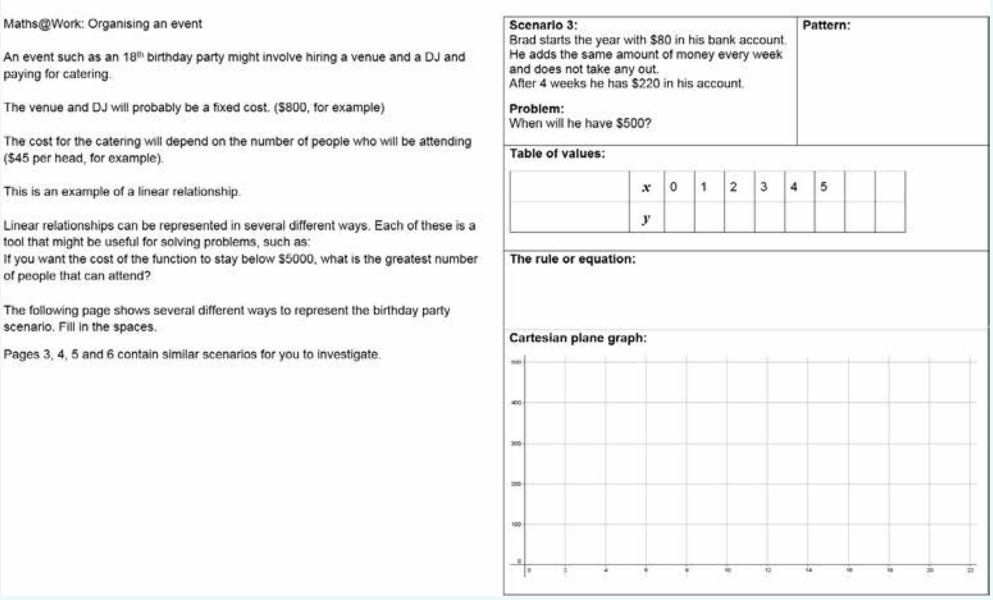
This is an example of a linear relationship.

Copy and complete this table of values.

Number of people	x	0	1	2	3	4	5	6
Total cost	y	800	845					

Download and print the worksheet this activity.

A screenshot from an activity in the worksheet



Maths@Work: Organising an event

An event such as an 18th birthday party might involve hiring a venue and a DJ and paying for catering.

The venue and DJ will probably be a fixed cost. ($800, for example)

The cost for the catering will depend on the number of people who will be attending ($45 per head, for example).

This is an example of a linear relationship.

Linear relationships can be represented in several different ways. Each of these is a tool that might be useful for solving problems, such as:
If you want the cost of the function to stay below $5000, what is the greatest number of people that can attend?

The following page shows several different ways to represent the birthday party scenario. Fill in the spaces.

Pages 3, 4, 5 and 6 contain similar scenarios for you to investigate.

Scenario 3:
Brad starts the year with $80 in his bank account.
He adds the same amount of money every week and does not take any out.
After 4 weeks he has $220 in his account.

Problem:
When will he have $500?

Pattern:

Table of values:

	x	0	1	2	3	4	5		
	y								

The rule or equation:

Cartesian plane graph:

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5B The x-intercept and y-intercept

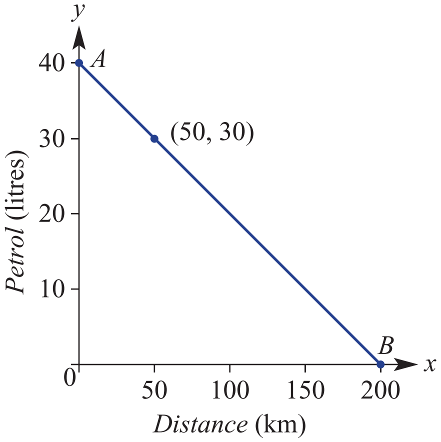
Stage

5.2
5.2◊
5.1
4

When a linear relationship is a model for a real situation, each point gives a piece of information.

For example, the following graph for the linear relationship $y = -0.2x + 40$ models how much petrol is in a tank.

- The y-axis shows the amount of petrol in the tank.
- The x-axis shows the number of kilometres travelled.
- The coordinate pair (50, 30) shows that after travelling 50 km, 30 litres of petrol are left in the tank.
- Point A shows us how much petrol was in the tank before the car has gone anywhere. The point is (0, 40). The y-intercept is 40.
- We also want to know when the car will run out of petrol. This point is (200, 0). This is where the graph cuts the x-axis. The x-intercept is 200.

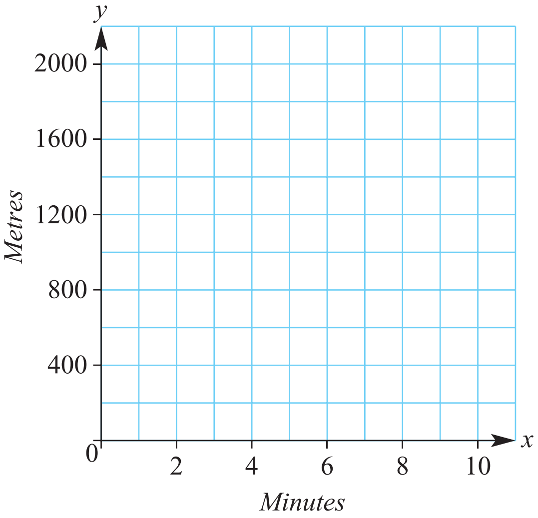


Let's start: Are we there yet?

This table shows the distance (y metres) a student is from his home (at x minutes) when he rides his bike home from school.

x	0	2	4	6	8	10
y	2000	1600	1200	800	400	0

- Copy the axes shown and plot these points to form a line.
- How far is school from home?
- How long does it take to get home?
- Circle the two points above on the graph.
- Which point is the x-intercept?
- Which point is the y-intercept?



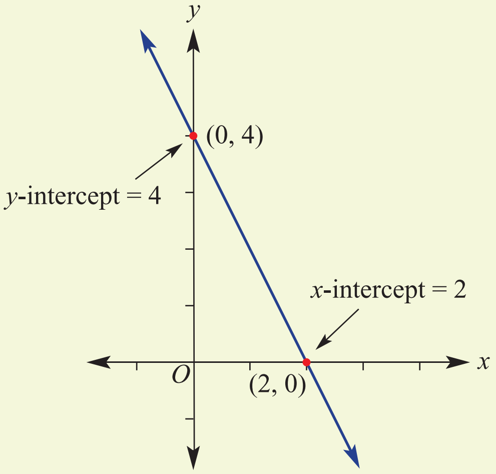
Key ideas

- The ***y*-intercept** is the y value of the point where the line cuts (i.e. intersects) the y-axis.
- The ***x*-intercept** is the x value of the point where the line cuts (i.e. intersects) the x-axis.

x-intercept

x	–2	–1	0	1	2	3
y	8	6	4	2	0	–2

y-intercept



***y*-intercept** The y-value of the point where a line or curve plotted on a cartesian plane cuts the y-axis. This will be the point which has an x-value of zero.

***x*-intercept** The x-value of the point where a line or curve plotted on a cartesian plane cuts the x-axis. This will be the point which has an y-value of zero.

Exercise 5B

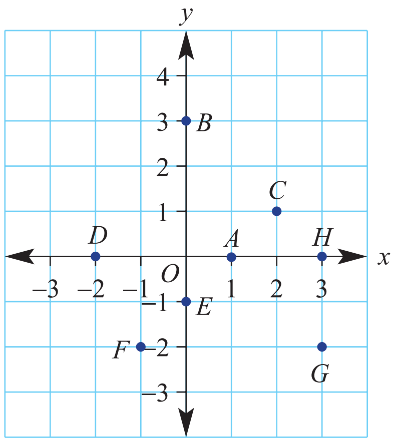
Understanding

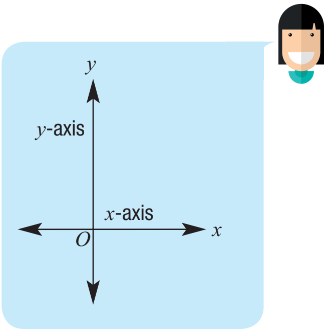
1 Plot the points $A(3, 0)$ and $B(0, 2)$ on a Cartesian plane. Draw a line through A and B. What is the:

a x-intercept? **b** y-intercept?

2 For the Cartesian plane shown, give the coordinates of each point that is on the:

a x-axis **b** y-axis





3 Simplify the following to find the value of y.

- **a** $y = 2 \times 0 + 1$
- **b** $y = 4 \times 0 - 2$
- **c** $y = -2 \times 0 - 3$

Any number multiplied by zero is zero.

4 Solve these equations.

- **a** $4x = 8$
- **b** $2y = 6$
- **c** $-3y = 15$
- **d** $x - 4 = 0$
- **e** $2x - 6 = 0$
- **f** $-3x + 9 = 0$

Fluency

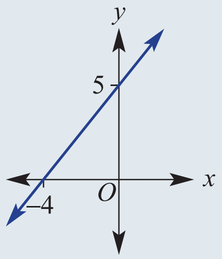
Example 5 Reading off the x-intercept and y-intercept

Read off the x-intercepts and y-intercepts from this table and graph.

a

x	-2	-1	0	1	2	3
y	12	9	6	3	0	-3

b



Solution

Explanation

a The x-intercept is 2.
The y-intercept is 6.

x-intercept

x	0	②
y	⑥	0

y-intercept

b The x-intercept is –4.
The y-intercept is 5.

The line cuts the horizontal axis at –4.
The line cuts the vertical axis at 5.

5B

5 Read off the x- and y-intercepts from these tables and graphs.

a

x	-3	-2	-1	0	1	2	3
y	4	3	2	1	0	1	2

b

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5

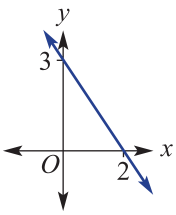
c

x	-1	0	1	2	3	4
y	10	8	6	4	2	0

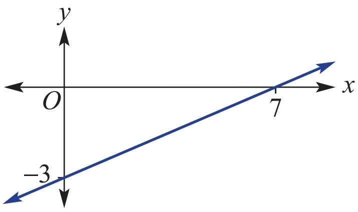
d

x	-5	-4	-3	-2	-1	0
y	0	2	4	6	8	10

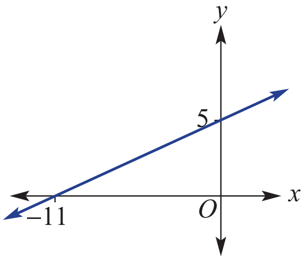
e



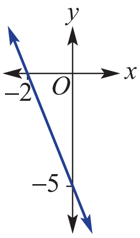
f



g



h



Video 5B

6 Use graphing software to graph these lines. The video in the interactive textbook shows how to do this. Write down the x-intercept and y-intercept.

a $y = x + 6$ **b** $y = x - 4$ **c** $y = 2x - 8$ **d** $y = 3x + 6$
e $y = -2x - 8$ **f** $y = -4x + 12$ **g** $x + 2y = 7$ **h** $2x - y = 4$

7 Which of these lines have a y-intercept of 5?

a $y = x + 3$ **b** $y = x + 4$ **c** $y = x + 5$ **d** $y = 2x + 5$
e $y = 2x + 6$ **f** $y = 2x + 7$ **g** $2x + y = 5$ **h** $x - 3y = 5$

8 Use graphing software to find the x-intercept and y-intercept for each of these linear relationships involving fractions.

a $y = \frac{x-4}{2}$ **b** $y = \frac{1}{2}x - 2$ **c** $y = \frac{x}{2}$
d $y = \frac{x}{2} - 4$ **e** $y = \frac{2x}{3}$ **f** $y = \frac{2x}{3} - 6$

Recall that another way of writing $\frac{1}{2}x$ is $\frac{x}{2}$.

Problem-solving and Reasoning

Drilling for Gold 5B1

9 The height, h, in metres, of a lift above ground after t seconds is given by $h = 100 - 5t$.

a What height does the lift start at (i.e. when $t = 0$)?
b How long does it take for the lift to reach the ground (i.e. when $h = 0$)?
c How many metres does the lift descend every second?

Use the sheet in the Drilling for Gold 5B2.

Drilling for Gold 5B2

10 Water is being drained from a fish tank. The amount of water in the tank, V litres, after t minutes is given by the rule $V = 80 - 4t$.

- **a** How long will it be until the tank is empty?
- **b** How much water was in the tank to begin with (i.e. when $t = 0$)?
- **c** How many litres are drained every minute?

Use the Drilling for Gold 5B3.

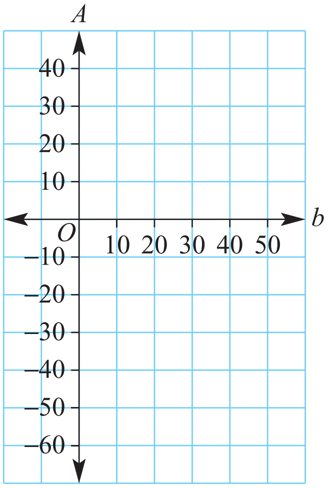
11 The line $y = 3x$ has both its x-intercept and y-intercept with coordinates (0, 0). Can you explain how this is the case, and what it means for the graph of $y = 3x$?

Enrichment: Making money

Drilling for Gold 5B3

12 Priya makes badges to sell at the market. The rule for the amount of money, A dollars, she makes from the sale of b badges is given by $A = 2b - 60$.

- **a** What is the value of A if Priya sells no badges (i.e. $b = 0$)?
- **b** Can you explain your answer to part **a**?
- **c** How many badges must Priya sell to cover her initial costs (i.e. $A = 0$)?
- **d** What happens when Priya sells more badges than in your answer to part **c**?
- **e** Use the information from your answers above to draw a graph for the rule on the axes shown. The A-axis is the y-axis and the b-axis is the x-axis.



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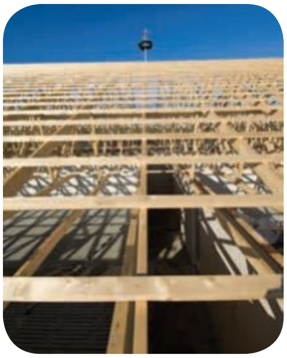
5C Lines with only one intercept

Stage

5.2
5.2◊
5.1
4

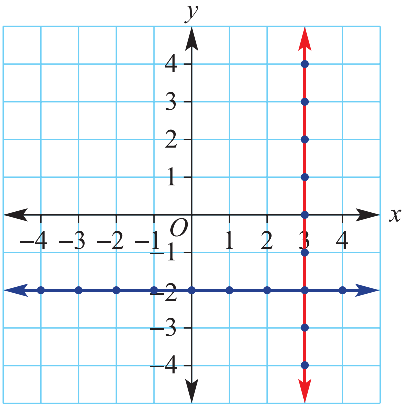
In this section we will investigate:
- vertical lines
- horizontal lines
- lines that pass through the origin.



Let's start: What rule satisfies all points?

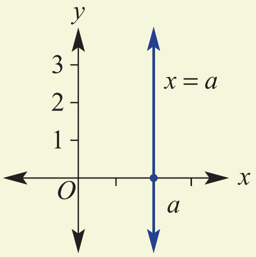
Here is one vertical (red) and one horizontal (blue) line.

- For the vertical line shown, write down the coordinates of all the points shown as dots.
- What is true for every coordinate pair?
- Can you think of a simple equation that describes every point on this line?
- For the horizontal line shown, write down the coordinates of all the points shown as dots.
- What is true for every coordinate pair?
- Can you think of a simple equation that describes every point on this line?

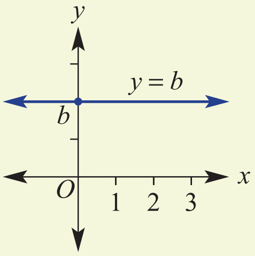


Key ideas

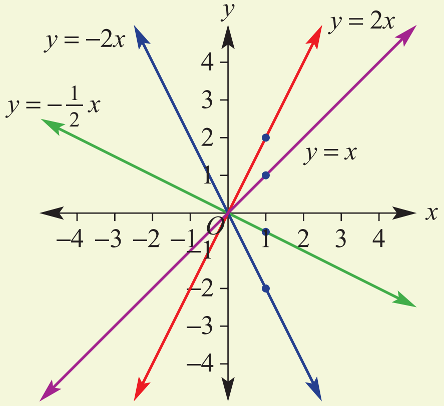
- Vertical lines:
  For example: $x = 2$, $x = -1$, $x = 0$.
  - Are parallel to the y-axis.
  - The x-coordinate is the same for every point on the line.
  - The equation is of the form $x = a$, where a is a constant.
  - The x-intercept is a.
  - There is no y-intercept.



- Horizontal lines:
  For example: $y = 2$, $y = -1$, $y = 0$.
  - Are parallel to the x-axis.
  - The y-coordinate is the same for every point on the line.
  - The equation is of the form $y = b$, where b is a constant.
  - The y-intercept is b.
  - There is no x-intercept.



- Lines through the origin $(0, 0)$:
  - The y-intercept is 0.
  - The x-intercept is 0.
  - $y = 2x$ passes through $(0, 0)$, but $y = 2x + 1$ does not.
  - $y = 2x$ runs 'uphill' but $y = -2x$ runs 'down hill'.



Exercise 5C

Understanding

1 Copy and complete:

a The line $y = 3$ hits the y-axis at ________.
Is it horizontal or vertical? It is ________.
It is parallel to the _______ -axis.

b The line $x = 2$ hits the x-axis at ________.
Is it horizontal or vertical? It is ________.
It is parallel to the _______ -axis.

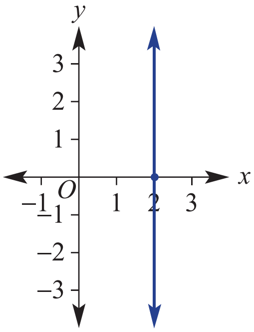
2 Classify the following lines as either *horizontal* or *vertical*.

a $y = 3$ **b** $y = -1$ **c** $x = 2$
d $y = 0$ **e** $x = -5$ **f** $x = 0$

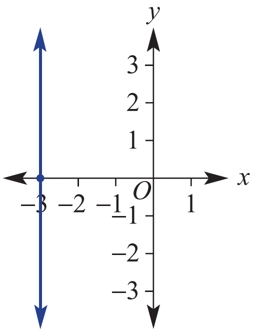
$y = 3$ hits y-axis at 3.
$x = 2$ hits x-axis at 2.

3 What is the x-intercept for each of these graphs?

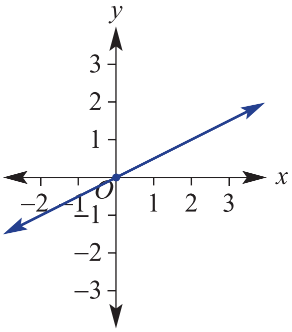
a



b

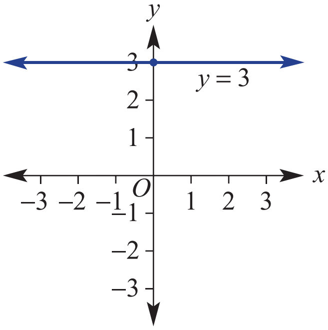


c

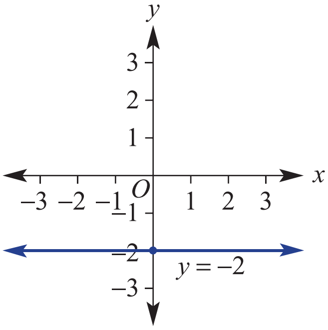


4 What is the y-coordinate of each point on these graphs?

a



b



5 List which of the following rules will have graphs that pass through $(0, 0)$.

a $y = 4$ **b** $y = 2x$ **c** $y = 3x + 2$

d $x = 2$ **e** $y = -x$ **f** $y = \frac{1}{2}x$

Equations such as $y = 5x$ pass through the origin.

6 Find the value of y if $x = 1$, using these rules.

a $y = 5x$ **b** $y = 3x$ **c** $y = -4x$ **d** $y = 5$

Fluency

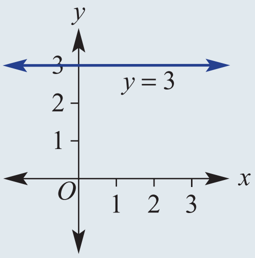
Example 6 Graphing vertical and horizontal lines

Sketch the graph of the following vertical and horizontal lines.

a $y = 3$ **b** $x = -4$ **c** $y = 0$

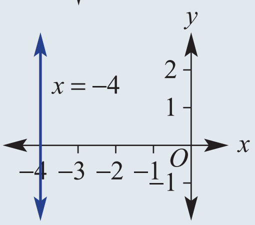
Solution | **Explanation**

a



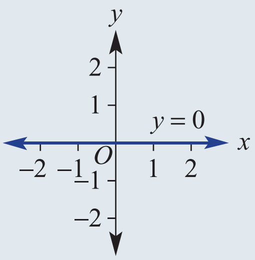
$y = 3$ hits the y-axis at 3.
Sketch a horizontal line through all points where $y = 3$.

b



$x = -4$ hits the x-axis at -4, so the x-intercept is -4.
Sketch a vertical line through all points where $x = -4$.

c



$y = 0$ hits the y-axis at 0.
Sketch a horizontal line through all points where $y = 0$.
This line is the x-axis.

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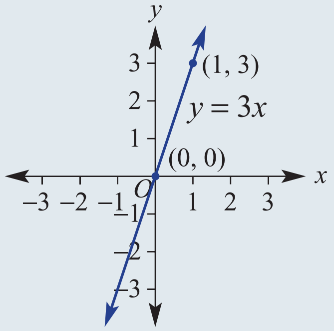
Skillsheet 5B

7 Sketch the graphs of the following vertical and horizontal lines.

a $x = 2$ **b** $x = 5$ **c** $y = 4$ **d** $y = 1$
e $x = -3$ **f** $x = -2$ **g** $y = -1$ **h** $y = -3$

Example 7 Sketching lines that pass through the origin

Sketch the graph of $y = 3x$.

Solution	**Explanation**
$y = 3x$ The x- and y-intercepts are 0.	The equation is of the form $y = mx$; i.e. when you substitute $x = 0$, $y = 3 \times 0 = 0$, giving $(0, 0)$ as both the x- and y-intercepts.
Another point (let $x = 1$): $y = 3 \times (1)$ $y = 3$	Two points are required to generate the straight line. Find another point by substituting $x = 1$.
Another point is at $(1, 3)$.	Any other x value could be used but the calculation is simplest for $x = 1$.
	Plot and label both points and sketch the graph by joining the points in a straight line.

8 Sketch the graphs of the following linear relationships, which pass through the origin.

a $y = 2x$ **b** $y = 5x$ **c** $y = 4x$
d $y = x$ **e** $y = -4x$ **f** $y = -3x$
g $y = -2x$ **h** $y = -x$ **i** $y = \frac{1}{2}x$

Substitute $x = 1$ to obtain a second point.

Drilling for Gold 5C1

9 Sketch the graphs of these special lines all on the same set of axes and label them with their equations.

a $x = -2$ **b** $y = -3$ **c** $y = 2$
d $x = 4$ **e** $y = 4x$ **f** $y = -\frac{1}{2}x$
g $y = -1.5x$ **h** $x = 0.5$ **i** $x = 0$
j $y = 0$ **k** $y = 2x$ **l** $y = 1.5x$

Use the Cartesian plane in Drilling for Gold 5C1.

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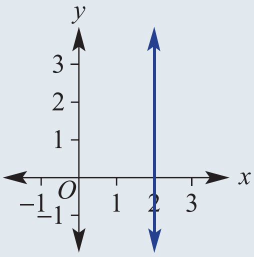
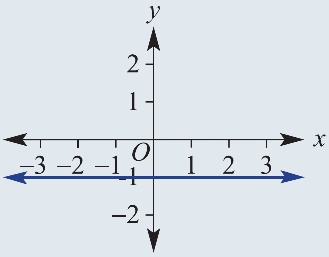
Example 8 Finding the equation of horizontal and vertical lines

Give the equations of the following vertical and horizontal lines.

a



b

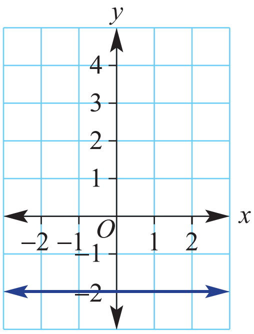


Solution	Explanation
a $x = 2$	The line hits the x-axis at 2.
b $y = -1$	The line hits the y-axis at -1.

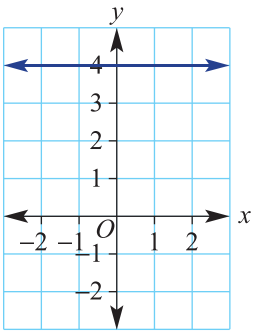
10 Give the equations of the following vertical and horizontal lines.

A horizontal line has the form $y = b$. A vertical line has the form $x = a$.

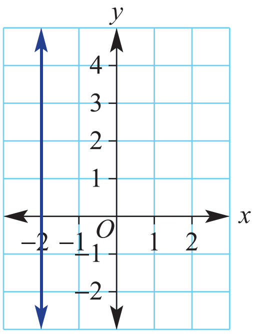
a



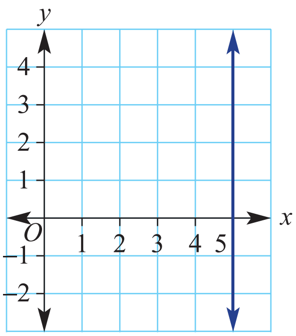
b



c



d



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Problem-solving and Reasoning

11 Find the equation of the straight line that is:

- **a** parallel to the x-axis and passes through the point (1, 3)
- **b** parallel to the y-axis and passes through the point (5, 4)
- **c** parallel to the y-axis and passes through the point (–2, 4)
- **d** parallel to the x-axis and passes through the point (0, 0)

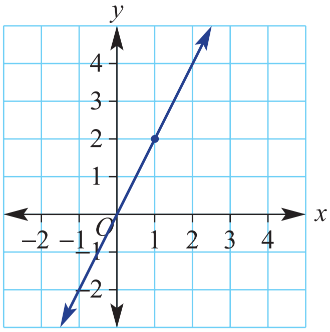
Use a rough sketch to decide whether the line is horizontal or vertical, then use the point.

12 If, in a picture, the surface of the sea is represented by the x-axis, state the equation of the following paths.

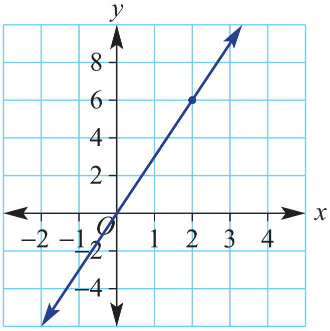
- **a** A plane flies horizontally at 250 m above sea level. One unit is 1 metre.
- **b** A submarine travels horizontally 45 m below sea level. One unit is 1 metre.

13 Choose the equation of these graphs from the list.

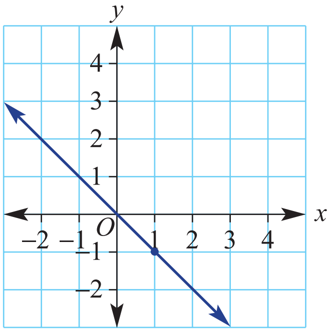
a



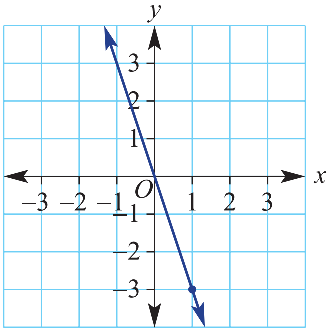
b



c



d



$y = 4x$
$y = 3x$
$y = 2x$
$y = x$
$y = -x$
$y = -2x$
$y = -3x$
$y = -4x$

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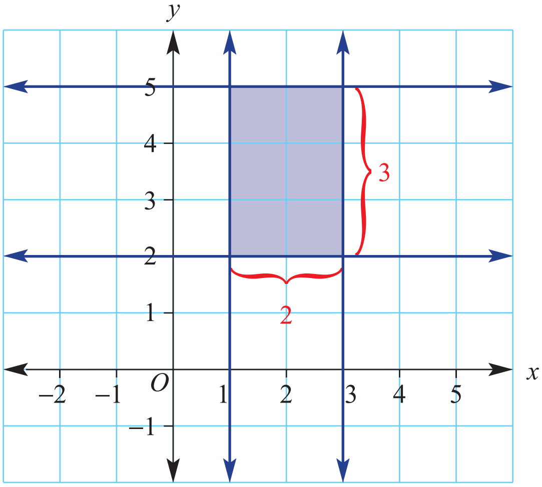
Enrichment: Rectangular areas

The four lines $x = 1$, $x = 3$, $y = 2$ and $y = 5$ are drawn on the one set of axes to form a rectangle, as shown (shaded).

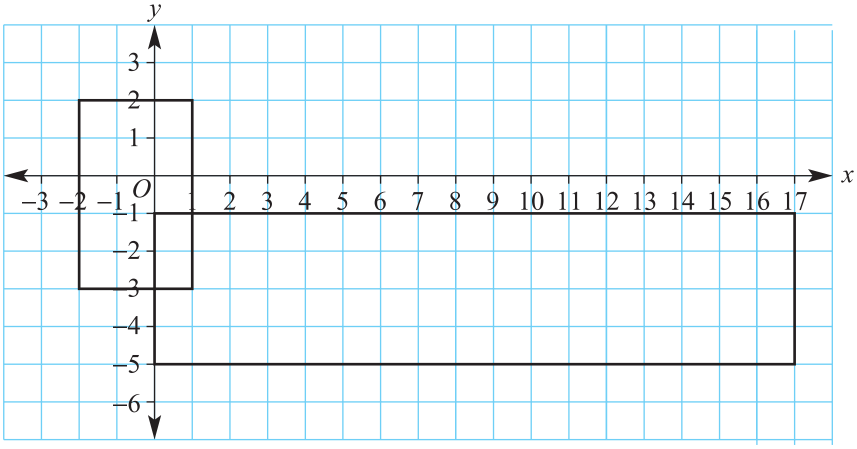
The area of this rectangle is length × breadth $= 3 \times 2 = 6$ square units.



14 Copy the axes below on to graph paper. Plot both sets of four lines below, using a different colour for each set. Find the areas of the rectangles contained within each set of four lines.

a $x = 1$, $x = -2$, $y = -3$, $y = 2$

b $x = 0$, $x = 17$, $y = -5$, $y = -1$



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Keeping in touch with numeracy

Non-calculator

1 Convert these decimals to simplified fractions:
- **a** 0.5
- **b** 0.25
- **c** 0.75
- **d** 0.4
- **e** 0.8

3 Convert these test results to percentages:
- **a** 3 out of 10
- **b** 3 out of 50
- **c** 3 out of 100
- **d** 3 out of 20
- **e** 3 out of 25

5 Which is the best estimate for 5105×23: 10 000 or 100 000 or 1 000 000?

7 Find the total mass, in kilograms, of: 5 kg, 1.75 kg, 200 g, 50 g, 5 g

9 How many 50-cent pieces are required to make $12.50?

11 Find the value of:
- **a** 0.75×10
- **b** $0.75 \times 10\,000$
- **c** $0.75 \div 100$

13 In a class the ratio of boys to girls is 4 : 3. There are 12 girls. How many boys are in the class?

15 Calculate the volume of a rectangular prism with sides 4 cm, 5 cm and 6 cm.

17 How far can a car doing 90 km/h travel in 3 hours?

19 The chance of rain today is 15%. What is the chance that it won't rain?

Calculator

2 Convert these decimals to simplified fractions:
- **a** 0.45
- **b** 0.125
- **c** 0.375
- **d** 1.14
- **e** 4.95

4 Convert these test results to percentages:
- **a** 3 out of 15
- **b** 3 out of 40
- **c** 3 out of 150
- **d** 3 out of 200
- **e** 3 out of 250

6 How many $500 items can be purchased for one million dollars?

8 Find the mass, in kilograms, of 15 items that each weigh 75 grams.

10 Calculate the change from $50 when 12 items are purchased costing $1.75 each.

12 Find the total length, in metres, of 25 steps of 75 cm.

14 Faye and Ben decide to split a $100 restaurant bill in the ratio 5 : 3. How much should Faye pay?

16 Use the formula $V = \pi r^2 h$ to calculate the volume of a tank with radius 1.5 m and height 2.1 m. Give your answer correct to two decimal places.

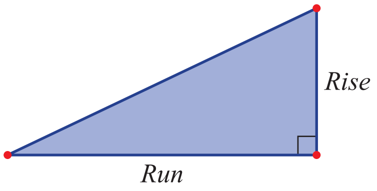
18 How long, in hours and minutes, will it take a car doing 80 km/h to travel 300 km?

20 If a die is rolled 90 times, how many times would you expect to roll a 1?

5D Gradient

Stage
5.2
5.2◊
5.1
4

The gradient of a line is a measure of its slope. It is a number that describes the steepness of a line. It is calculated by considering how far a line rises or falls between two points within a given horizontal distance. The horizontal distance between two points is called the *run*. The vertical distance is called the *rise*.



Gradients are used to describe the steepness of this rollercoaster track.

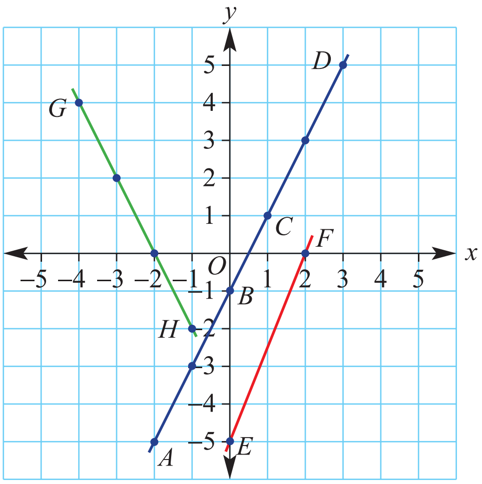
Let's start: Which line is the steepest?

Study the three lines shown.

- Calculate the rise and run (working from left to right) and also the fraction $\frac{\text{rise}}{\text{run}}$ for these segments.

i AB **ii** BC **iii** BD
iv EF **v** GH

- What do you notice about the fractions $\left(\frac{\text{rise}}{\text{run}}\right)$ for parts **i**, **ii** and **iii**?
- How does the $\frac{\text{rise}}{\text{run}}$ for EF compare with the $\frac{\text{rise}}{\text{run}}$ for parts **i**, **ii** and **iii**? Which of the two lines is the steepest?
- Your $\frac{\text{rise}}{\text{run}}$ for GH should be negative. Why?
- Discuss whether or not GH is steeper than AD.



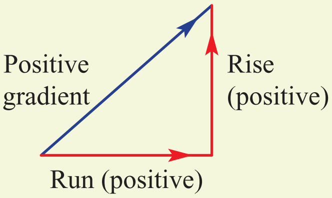
Video 5D

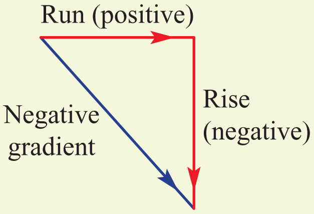
Use computer software (i.e. dynamic geometry) to produce a set of axes and grid. The video in the interactive textbook shows how.

- Construct a line segment with end points on the grid. Show the coordinates of the end points.
- Calculate the rise (vertical distance between the end points) and the run (horizontal distance between the end points).
- Calculate the gradient as the *rise* divided by the *run*.
- Now drag the end points and explore the effect on the gradient.
- Can you drag the end points but retain the same gradient value? Explain why this is possible.
- Can you drag the end points so that the gradient is zero or undefined? Describe how this can be achieved.

Key ideas

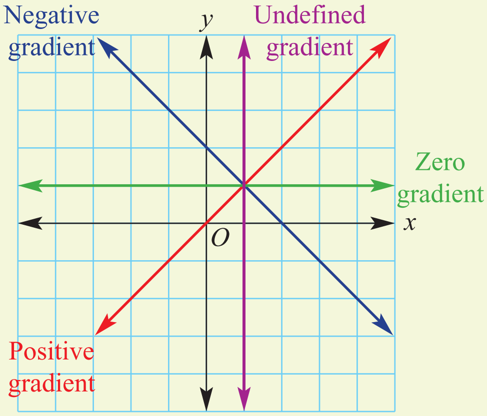
- The **gradient** of a line is a measure of its steepness or slope.
- The gradient of a line can be found using the formula $m = \frac{\text{rise}}{\text{run}}$.





Gradient The steepness of a straight line or interval

- We work from left to right, so the run is always positive.
- The gradient can be positive, negative, zero or undefined.
  - The gradient is positive if the graph increases from left to right.
  - The gradient is negative if the graph decreases from left to right.
  - The gradient is zero if the line is horizontal.
  - The gradient is undefined if the line is vertical.

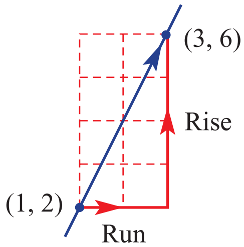


Exercise 5D

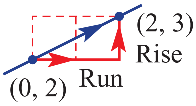
Understanding

1 For these lines, calculate the value for:
i run **ii** rise

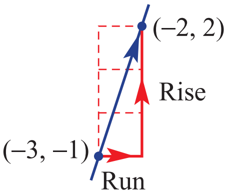
a



b

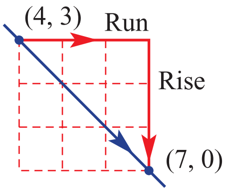


c

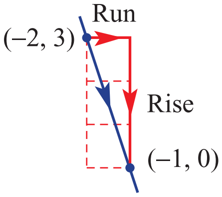


Remember that the rise is negative if the line slopes downward from left to right.

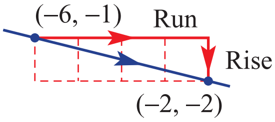
d



e



f



2 Use the word *positive*, *negative*, *zero* or *undefined* to complete each sentence.

- **a** The gradient of a horizontal line is ____________.
- **b** The gradient of the line joining (0, 3) with (5, 0) is ____________.
- **c** The gradient of the line joining (–6, 0) with (1, 1) is ____________.
- **d** The gradient of a vertical line is ____________.

3 Simplify these fractions as much as possible.

a $\frac{6}{3}$ **b** $\frac{-8}{2}$ **c** $\frac{4}{4}$

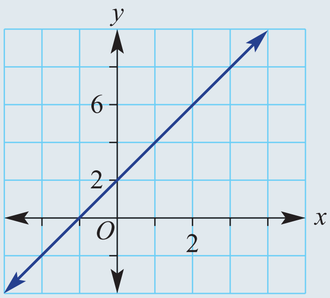
d $\frac{2}{4}$ **e** $\frac{-6}{4}$ **f** $\frac{-10}{6}$

Fluency

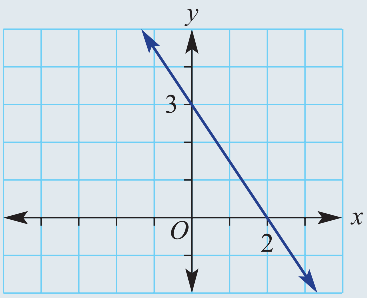
Example 9 Calculating the gradient of a line

For each graph, state whether the gradient is positive, negative, zero or undefined. Then find the gradient, where possible.

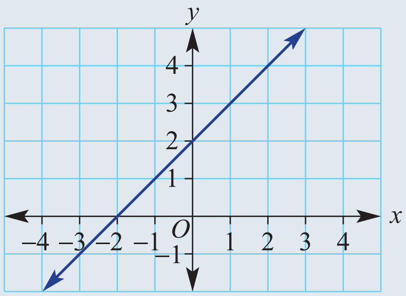
a



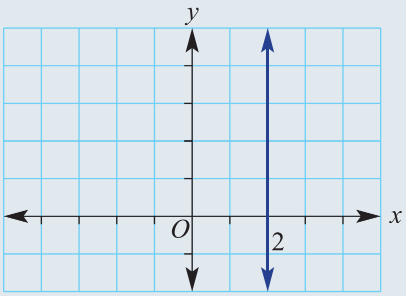
b



c



d

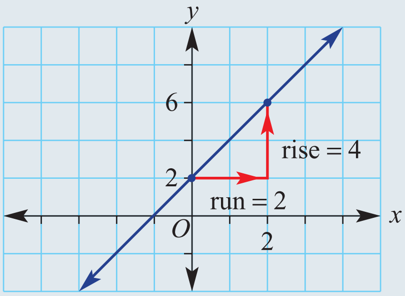


Solution

Explanation

a The gradient is positive.

By inspection, the gradient is positive since the graph increases from left to right.



$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

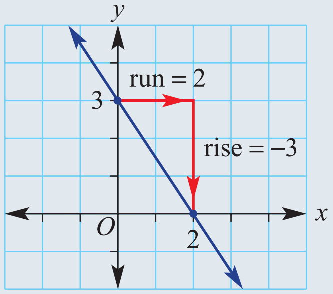
$$= \frac{4}{2}$$

Select any two points and create a right-angled triangle between them to determine the rise and run.

$$= 2$$

Substitute rise = 4 and run = 2, and simplify.

b The gradient is negative.



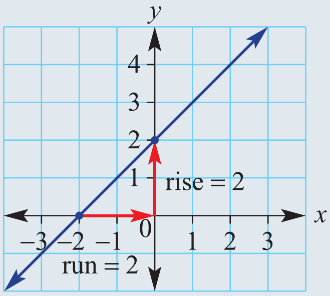
$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{-3}{2}$$

$$= -\frac{3}{2}$$

By inspection, the gradient is negative since y values decrease from left to right.
Create a right-angled triangle between two points.
Rise = –3 since it 'falls' 3 units, and run = 2.
It is best to leave the fraction as improper rather than a mixed numeral.

c The gradient is positive.



$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{2}{2}$$

$$= 1$$

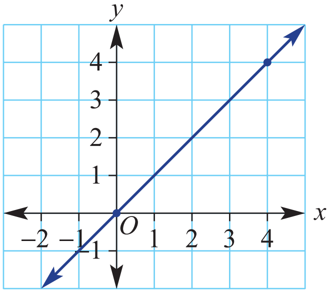
A rising graph indicates a positive gradient.
For this triangle, rise = 2. The run is from left to right, from –2 to 0, so the run is +2.

d The gradient is undefined.

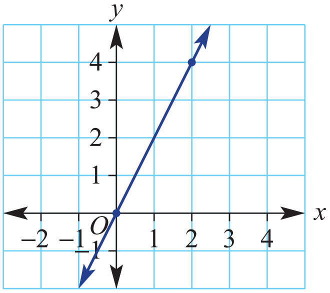
The line is vertical.

4 For each graph, state whether the gradient is positive, negative, zero or undefined. Then find the gradient where possible.

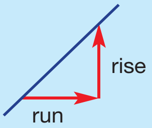
a



b

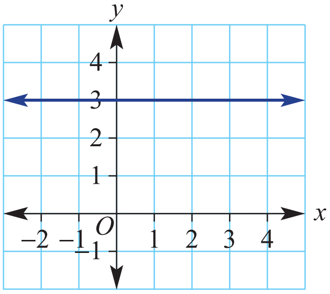


Create a right-angled triangle to find the rise and the run.

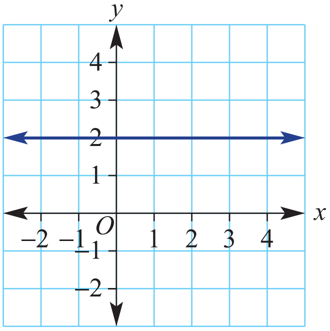


$\text{Gradient} = \frac{\text{rise}}{\text{run}}$

c



d



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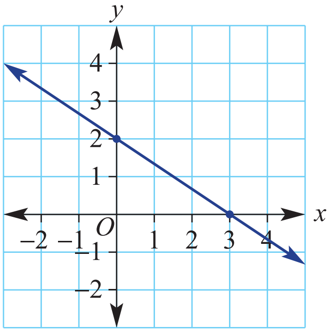
e



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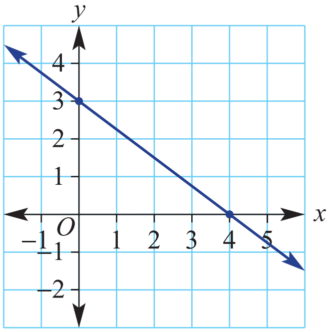

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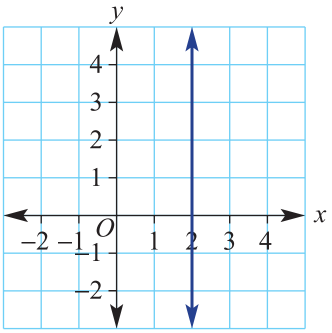

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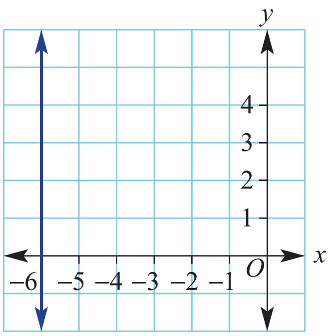
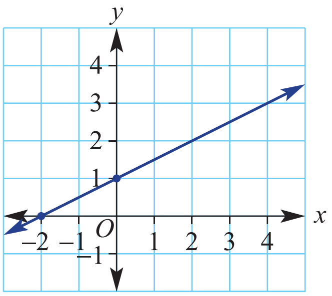


i



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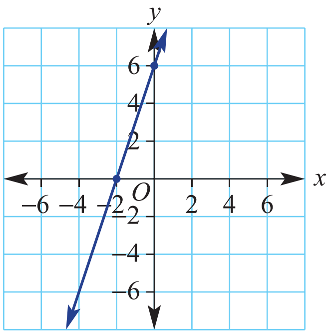
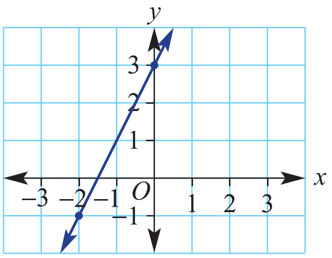


k



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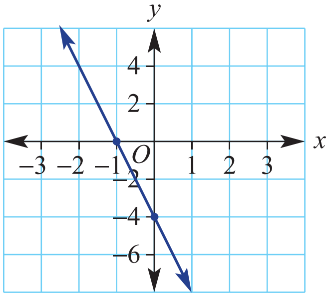

In part **k**, from −1 to 3 is a rise of 4.

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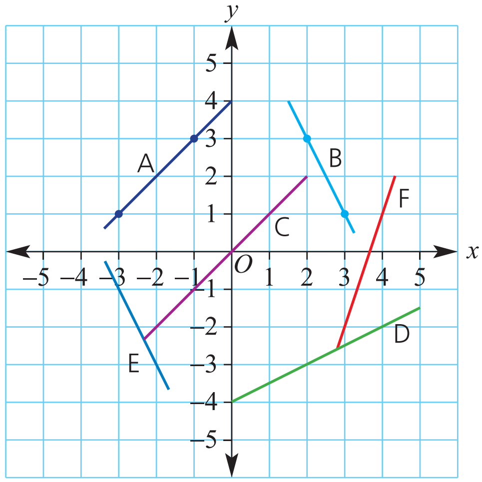
5 Find the gradient of each line A–F on this graph and grid.



Pick two known points on each line to find the gradient between them. A and B have been done for you.

Example 10 Finding the gradient between two points

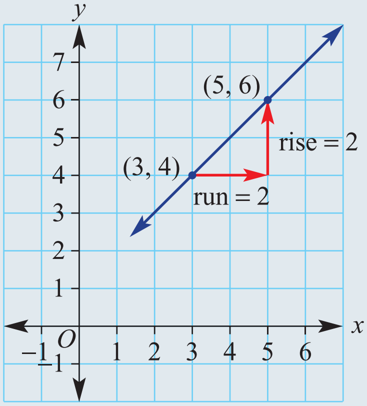
Use a graph to find the gradient (m) of the line joining the given points.

a $A(3, 4)$ and $B(5, 6)$ **b** $A(-3, 6)$ and $B(1, -3)$

Solution

Explanation

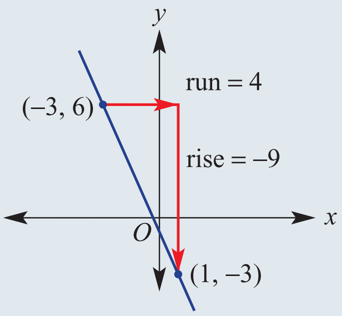
a



$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{2}{2}$$
$$= 1$$

Plot the two points on a set of axes and construct a right-angled triangle between them.
Observe from the graph that the gradient is positive.
Calculate the gradient by finding the rise and run.

b



$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-9}{4} \text{ or } -\frac{9}{4} \text{ or } -2.25$$

Plot points A and B and join in a line. Observe that the gradient is negative.
From left to right, -3 to 1, run $= 4$.
Rise from 6 to -3 is a ‘fall’ of 9 units, so rise $= -9$.
Note: Leave the fraction as a simplified improper fraction, not a mixed numeral.

5D

6 Find the gradients of the lines joining the following pairs of points.

a $A(2, 3)$ and $B(3, 5)$
b $C(0, 8)$ and $D(2, 6)$
c $E(0, 4)$ and $F(2, 0)$
d $G(2, 1)$ and $H(5, 4)$
e $A(1, 5)$ and $B(2, 7)$
f $C(-2, 4)$ and $D(1, -2)$
g $E(-3, 4)$ and $F(2, -1)$
h $G(-1, 5)$ and $H(1, 6)$
i $A(-4, -2)$ and $B(-2, 1)$
j $C(1, 1)$ and $D(3, -4)$
k $E(3, 2)$ and $F(0, 1)$
l $G(-1, 1)$ and $H(-3, -4)$

Plot the points first.

Problem-solving and Reasoning

Skillsheet 5C

7 Find the gradients, using $\frac{\text{rise}}{\text{run}}$, corresponding to the following slopes.

a A road falls 10 m for every 200 horizontal metres.
b A cliff rises 35 metres for every 2 metres horizontally.
c A plane descends 2 km for every 10 horizontal kilometres.
d A submarine ascends 150 m for every 20 horizontal metres.

The horizontal distance is the run.

8 A firecracker ascends with a gradient of 2. How far, horizontally, has the cracker travelled after rising 80 m?

9 Michelle runs up a sand dune that has a gradient of $\frac{1}{3}$. How far, horizontally, has Michelle moved after rising 30 m?

10 Find the missing number.

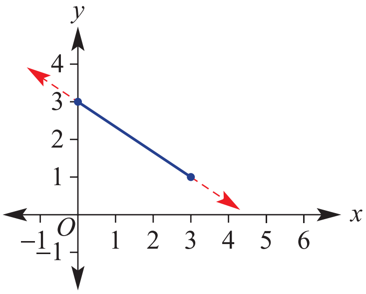
a The gradient joining the points $(0, 2)$ and $(1, ?)$ is 4.
b The gradient joining the points $(?, 5)$ and $(3, 9)$ is 2.
c The gradient joining the points $(-3, 4)$ and $(?, 1)$ is -1.
d The gradient joining the points $(-4, ?)$ and $(-2, -12)$ is -4.

A gradient of $4 = \frac{4}{1}$; a rise of 4 units for every 1 unit to the right.

Enrichment: Where does it hit?

11 The line here has gradient $\frac{-2}{3}$, which means that it falls 2 units for every 3 units across. The y-intercept is 3.

a Use the gradient to find the y-coordinate on the line where:
  i $x = 6$
  ii $x = 9$
b What will be the coordinates of the x-intercept?
c Give the x-intercept when the gradient is changed to:
  i $\frac{-1}{2}$
  ii -2
  iii $\frac{-5}{4}$



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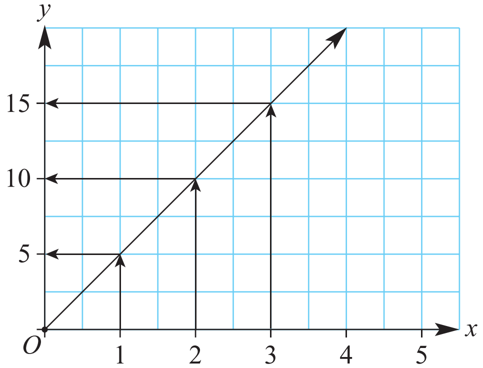
5E Gradient and direct proportion

Stage
5.2
5.2◊
5.1
4

The table below and graph opposite are showing that:

- x starts from O.
- y starts from O.
- Every time x increases by 1, y increases by 5.

x	0	1	2	3
y	0	5	10	15

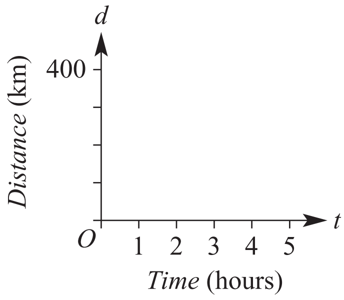


In situations like this we say 'y is in direct proportion to x' or 'y is directly proportional to x'.

Let's start: Average speed

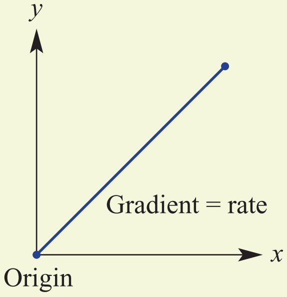
Over 5 hours, Sandy travels 400 km in a car.

- Draw a graph of distance versus time for the journey, assuming a constant speed.
- Where does your graph intersect the axes and why?
- What is Sandy's average speed for the trip?
- Find the gradient of your graph. What do you notice?
- Find a rule linking distance (d) and time (t).



Key ideas

- A **rate** is the change in one variable compared with another (often time). For example, 60 km/h (60 kilometres per hour) is a rate. It states a 60 km change in distance for each hour in time that passes.
- If two variables are **directly proportional**, the:
  - rate of change is constant
  - graph is a straight line passing through the origin
  - rule is of the form $y = mx$
  - gradient (m) of the graph equals the rate of change.



Rate The number of units of one quantity for each single unit of another quantity

Direct proportion The relationship between two quantities that increase or decrease at the same rate

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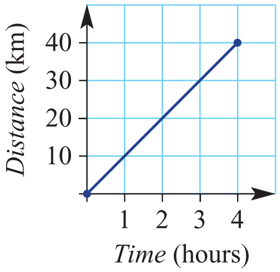
Exercise 5E

Understanding

1 Write 'Yes' or 'No' to state which of the following are rates.
a \$120 **b** 12 cm/s **c** 150 mL/min **d** 80 km

2 This graph shows how far Caroline travels on her bike over 4 hours.

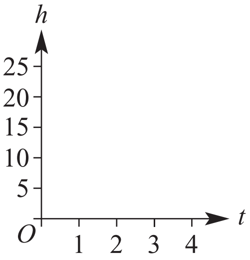


a How far did Caroline travel every hour?
b State how far Caroline has travelled after:
i 1 hour **ii** 2 hours **iii** 3 hours
c Write down the speed of the bike in km/h (rate of change of distance over time).
d Find the gradient of the graph.
e What do you notice about your answers from parts **c** and **d**?

3 A plant grows 5 mm every day. The rule linking the height of a plant to time is given by $h = 5t$, where h is in millimetres and t is in days.
a Find the height of the plant after 3 days (i.e. when $t = 3$).
b Find the time for the plant to reach:
i 30 mm **ii** 100 mm
c Copy and complete this table.

t	0	1	2	3	4
h					

d Copy and complete this graph.



e Find the gradient of the graph.

t represents time.
h represents height.

Example 11 Simplifying rates

Write the following rates in simplest form.

a 80 metres in 10 seconds **b** 160 heartbeats in 2 minutes

Solution	**Explanation**
a $\div 10 \left(\begin{array}{c} 80\text{ m in } 10\text{ s} \\ 8\text{ m in } 1\text{ s} \end{array} \right) \div 10$	Convert to metres per 1 second by dividing by 10.
b $\div 2 \left(\begin{array}{c} 160\text{ in } 2\text{ min} \\ 80\text{ in } 1\text{ min} \end{array} \right) \div 2$	Divide by 2 to get 80 heartbeats in 1 minute.

4 Write these rates in simplest form.

- **a** 120 km in 2 hours
- **b** 180 m in 20 seconds
- **c** 400 L in 80 min
- **d** $30 for 20 litres
- **e** $900 in 30 hours
- **f** 15°C in 5 min

Simplest form is per 1 unit; e.g. 50 km in 1 hour, 50 km/h.

Example 12 Forming rules

Write a rule linking the variables A dollars and t hours if $60 is earned for 5 hours of work.

Solution	**Explanation**
$\div 5 \left(\begin{array}{c} \$60\text{ for } 5\text{ hours} \\ \$12\text{ for } 1\text{ hour} \end{array} \right) \div 5$	Divide $60 by 5 hours.
Rate is $12/h.	
$A = 12 \times t$ $A = 12t$	The amount earned, A, is $12 for each hour worked. t hours of work earns $12 \times t$ dollars.

5 Write down a rule linking the given variables.

- **a** I travel 720 km in 12 hours. Use d for distance and t for time.
- **b** The profit is $10 000 for 500 tonnes. Use P for profit and t for the number of tonnes.
- **c** The cost of petrol is $120 for 80 litres. Use C for cost and n for the number of litres.
- **d** A calf grows 12 cm in 6 months. Use g for growth height and t for time.

Establish the rate first.

Example 13 Exploring direct proportion

Drilling for Gold 5E1

Water is poured into an empty tank. It takes 3 hours to fill the tank with 6000 litres.

a What is the rate at which water is poured into the tank?

b Draw a graph of volume (V litres) vs time (t hours) from $t = 0$ to $t = 3$.

c Find the:

- **i** gradient of your graph
- **ii** rule for V

d Use your rule to find the:

- **i** volume after 1.5 hours
- **ii** time it takes to fill 5000 litres

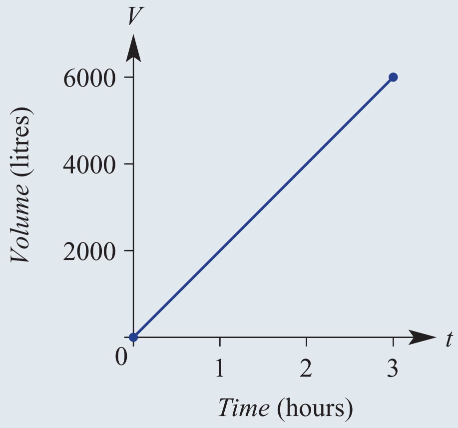
Solution **Explanation**

a

$$\div 3 \left(\begin{array}{l} \text{6000 L per 3 hours} \\ \text{2000 L per 1 hour} \end{array} \right) \div 3$$

6000 L in 3 hours = 2000 L/h

b



Draw axes with volume on the vertical axis and time on the horizontal axis because volume depends on time.

Plot the two end points: (0, 0), since the tank starts empty, and (3, 6000).
Join these points with a straight line.

c **i** Gradient $= \dfrac{6000}{3} = 2000$

Using the end points, rise = 6000, run = 3. Note that the gradient is the same as the rate.

ii $V = 2000t$

2000 L are filled for each hour.

d **i**

$$\begin{aligned} V &= 2000t \\ &= 2000 \times 1.5 \\ &= 3000 \end{aligned}$$

Substitute $t = 1.5$ into your rule.

∴ 3000 L after 1.5 hours

ii

$$\begin{aligned} V &= 2000t \\ 5000 &= 2000t \\ \div 2000 \Big(\quad 2000t &= 5000 \quad \Big) \div 2000 \\ t &= 2.5 \end{aligned}$$

Substitute $V = 5000$ into the rule.
Swap LHS and RHS.

Divide both sides by 2000.

∴ It takes $2\frac{1}{2}$ hours to fill 5000 L.

6 A 300 litre fish tank takes 3 hours to fill, using a hose.

a What is the rate at which water is poured into the tank?

b Draw a graph of volume (V litres) vs time (t hours) using $0 \le t \le 3$.

c Find the:

i gradient of your graph **ii** rule for V

d Use your rule to find the:

i volume after 1.5 hours **ii** time it takes to fill 2000 litres

300 litres in 3 hours; how many litres in 1 hour?

Place time on the horizontal axis.

7 A solar-powered car travels 100 km in 4 hours.

a What is the rate of change of distance over time (i.e. speed)?

b Draw a graph of distance (d km) vs time (t hours) using $0 \le t \le 4$.

c Find the:

i gradient of your graph

ii rule for d

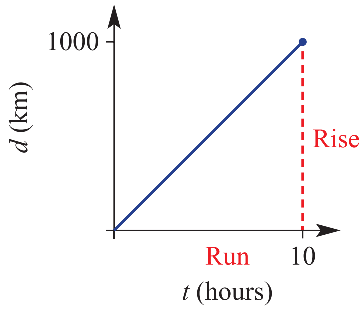
d Use your rule to find the:

i distance after 2.5 hours

ii time it takes to travel 40 km

8 Use the gradient to find the rate of change of distance over time (i.e. speed) for each of these graphs. Use the units given on each graph.

a



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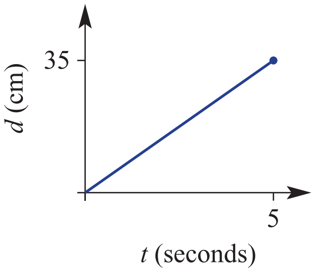

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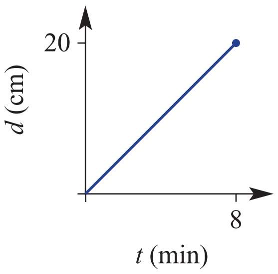

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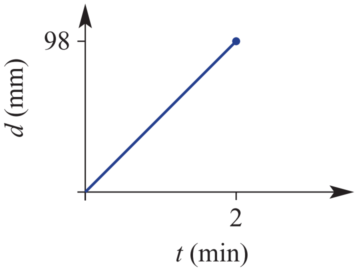


Problem-solving and Reasoning

9 Who is travelling the fastest?
- Mick runs 720 m in 2 minutes.
- Simone rides 550 m in 1 minute.
- Udhav jogs 2000 m in 5 minutes.

To compare rates, they will need to be in the same units.

10 Which animal is travelling the slowest?
- A leopard runs 400 m in 30 seconds.
- A jaguar runs 2700 m in 3 minutes.
- A panther runs 60 km in 1 hour and 20 minutes.

11 A car's trip computer says that the fuel economy for a trip is 8.5 L per 100 km.

a How many litres would be used for 120 km?

b How many litres would be used for 850 km?

c How many kilometres could be travelled if the capacity of the car's petrol tank is 68 L?

Consider how many litres are used in 1 kilometre.

12 The area of a particular rectangle is given by $A = \ell b$. If its length is fixed at 12 cm but its breadth, b cm, can vary:

a Write a rule for the area of this rectangle.

b Draw a graph of A against b for $0 \leq b \leq 4$.

c Decide if the area of this rectangle is directly proportional to its breadth. Explain.

Equations of the form $y = mx$ show that y is directly proportional to x.

Enrichment: Rate challenge

13 Hose A can fill a bucket in 2 minutes and hose B can fill the same bucket in 4 minutes.

a What fraction of the bucket does hose A fill in 1 minute?

b What fraction of the bucket does hose B fill in 1 minute?

c If both hoses are used at the same time, what fraction of the bucket could they fill in 1 minute?

d If both hoses are used at the same time, how long would it take to fill the bucket?

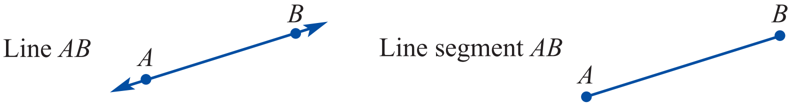
14 Vonda can vacuum an office building in 2 hours and her husband Kris can vacuum the same office building in 3 hours. How long would it take to vacuum the office building if they both vacuum at the same time?

5F Midpoint and length of a line segment from diagrams

Stage

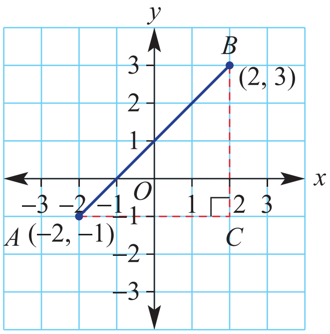
5.2
5.2◊
5.1
4

A line continues on forever. A line segment (or line interval) has end points. This means it has a defined length and therefore must have a midpoint. Both the midpoint and length can be found by using the coordinates of the end points.



▶ Let's start: Choosing a method

This graph shows a line segment between the points at *A*(–2, –1) and *B*(2, 3).



- What is the horizontal distance, *AC*, between the two points?
- What is the vertical distance, *BC*, between the two points?
- Discuss and explain a method for finding the length of a line segment, using the right-angled triangle formed.
- What is the *x*-coordinate of the point halfway along the line segment?
- What is the *y*-coordinate of the point halfway along the line segment?
- Discuss and explain a method for finding the midpoint of a line segment.

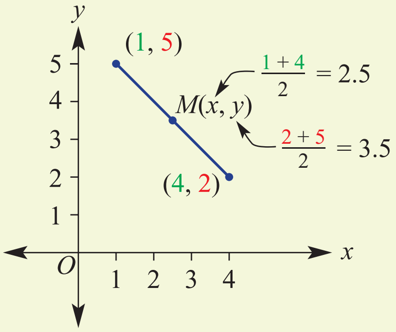
Video 5F

Using graphing software or dynamic geometry software, produce a line segment like the one shown above.
Label the coordinates of the end points and the midpoint.
Also find the length of the line segment.
Now drag one or both of the end points to a new position.

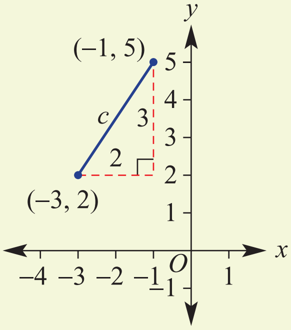
- Describe how the coordinates of the midpoint relate to the coordinates of the end points. Is this true for all positions of the end points that you choose?
- Now use your software to calculate the vertical distance and the horizontal distance between the two end points. Then square these lengths. Describe these squared lengths compared to the square of the length of the line segment. Is this true for all positions of the end points that you choose?

Key ideas

- The **midpoint** (M) of a **line segment** is the halfway point between the two end points.
  - The x-coordinate is the average (mean) of the x-coordinates of the two end points.
  - The y-coordinate is the average (mean) of the y-coordinates of the two end points.



- The length of a line segment (or line interval) is found using Pythagoras' theorem.
  - The line segment is the hypotenuse (longest side) of a right-angled triangle.
  - The coordinates can be used to find the horizontal distance between them, and the vertical distance.



Horizontal distance from -3 to -1 is 2.
Vertical distance from 2 to 5 is 3.
Pythagoras' theorem:

$$c^2 = 2^2 + 3^2$$
$$= 13$$
$$\therefore c = \sqrt{13}$$

Midpoint The point on a line segment that is an equal distance from each of the endpoints of the segment

Line segment A section of a straight line

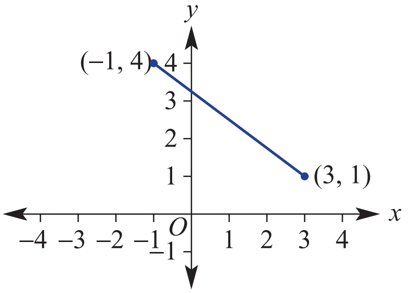
Exercise 5F

Understanding

1 Copy and complete the calculations to find the midpoint of the line segment.

$$M\left(\frac{-1+\square}{2}, \frac{4+\square}{2}\right)$$

$$M(\square, \square)$$



2 Find the number that is halfway between these pairs of numbers.

a 1, 7 **b** 5, 11 **c** −2, 4 **d** −6, 0 **e** 4, 7

3 Evaluate c in the following, correct to two decimal places, given $c > 0$.

a $c^2 = 1^2 + 2^2$ **b** $c^2 = 5^2 + 7^2$ **c** $c^2 = 10^2 + 2^2$

If $c^2 = 7$, $c = \sqrt{7}$

4 **a** Find the horizontal distance between the points:

i (1, 4) and (4, 6) **ii** (−2, 3) and (2, −1)

b Find the vertical distance between the points:

i (−2, 3) and (0, 5) **ii** (2, 5) and (6, −2)

For horizontal distance, consider the x-coordinates. For vertical distance, consider the y-coordinates.

Fluency

Example 14 Finding a midpoint

Find the midpoint $M(x, y)$ of the line segment joining these pairs of points.

a (1, 0) and (5, 4)

b (−3, −2) and (5, 3)

Solution

a

$$M = \left(\frac{1+5}{2}, \frac{0+4}{2}\right)$$

$$M = \left(\frac{6}{2}, \frac{4}{2}\right)$$

$$\therefore M = (3, 2)$$

b

$$M = \left(\frac{-3+5}{2}, \frac{-2+3}{2}\right)$$

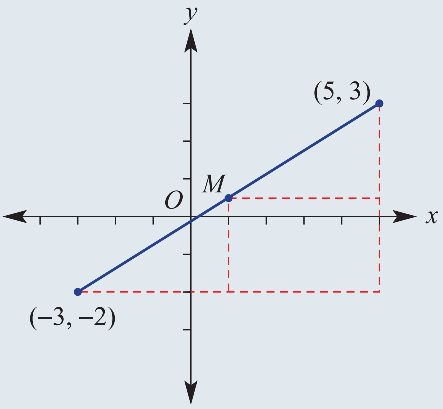
$$M = \left(\frac{2}{2}, \frac{1}{2}\right)$$

$$\therefore M = (1, 0.5)$$

Explanation

Find the average (i.e. mean) of the x-coordinates and y-coordinates for both points.

By plotting the points to form the line segment, you can check that the coordinates you find for the midpoint appear to be the halfway point of the line segment.



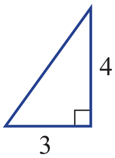
5 Find the midpoints, $M(x, y)$, of the line segments joining these pairs of points.

a $(0, 0)$ and $(6, 6)$
b $(0, 0)$ and $(4, 4)$
c $(0, 2)$ and $(2, 8)$
d $(3, 0)$ and $(5, 2)$
e $(-2, 0)$ and $(0, 6)$
f $(-4, -2)$ and $(2, 0)$
g $(1, 3)$ and $(2, 0)$
h $(-1, 5)$ and $(6, -1)$
i $(-3, 7)$ and $(4, -1)$
j $(-2, -4)$ and $(-1, -1)$

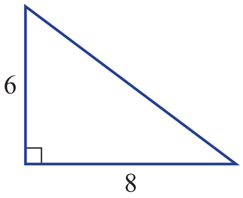
Check that your coordinates for the midpoint appear to lie halfway along the line segment.

6 Use Pythagoras' theorem to find the length of the hypotenuse in these right-angled triangles. Round to one decimal place where required.

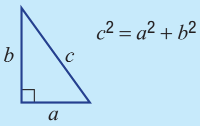
a



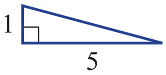
b



Pythagoras' theorem:



c



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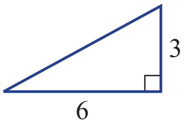
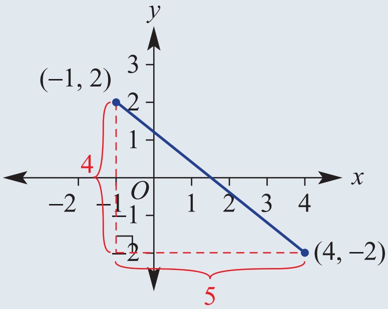


Example 15 Finding the length of a segment

Find the length of the segment joining $(-1, 2)$ and $(4, -2)$, correct to two decimal places.

Solution	Explanation
	Plot the points and form the line segment to visualise the problem. Form the right-angled triangle. Use the coordinates to find the lengths of the horizontal and vertical sides.
Horizontal length = 5 Vertical length = 4 $\therefore c^2 = 5^2 + 4^2$ $= 25 + 16$ $= 41$	Horizontal length: From -1 to 4 is 5 units. Vertical length: From 2 to -2 is 4 units. Apply Pythagoras' theorem: $c^2 = a^2 + b^2$.
$\therefore c = \sqrt{41}$	From $c^2 = 41$, take the square root of both sides.
$\therefore$ Length = 6.40 (to 2 d.p.)	Round to two decimal places on a calculator, as required.

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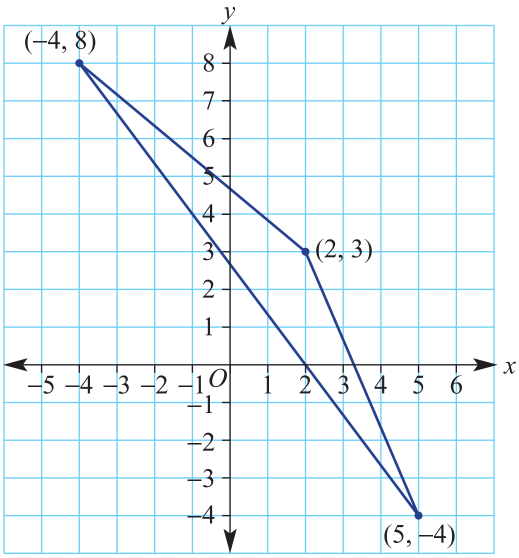
7 Find the lengths of the segments joining these pairs of points, correct to two decimal places.

a (1, 1) and (2, 6)
b (1, 2) and (3, 4)
c (0, 2) and (5, 0)
d (–2, 0) and (0, –4)
e (–1, 3) and (2, 1)
f (–2, –2) and (0, 0)
g (–1, 7) and (3, –1)
h (–4, –1) and (2, 3)
i (–3, –4) and (3, –1)

Problem-solving and Reasoning

8 The diagram shows the coordinates of fence posts on a property. To reinforce the fence, posts are to be placed halfway between the current posts. At what coordinates will these posts be placed?



9 Find the missing coordinates in this table if M is the midpoint of points A and B. Use the scaffold below.

A	B	M
(4, 2)		(6, 1)
	(0, –1)	(–3, 2)

$$M\left(\frac{4+\square}{2},\frac{2+\square}{2}\right)$$

$M(6, 1)$

$$M\left(\frac{0+\square}{2},\frac{-1+\square}{2}\right)$$

$M(-3, 2)$

10 A circle has its centre at (2, 1). Find the coordinates of the end point of a diameter if the other end point has these coordinates.

a (7, 1)
b (3, 6)
c (–4, –1)

The diameter of a circle is shown: ⌀
It passes through the centre.

11 Plot the points to find the perimeter of these shapes, correct to one decimal place.

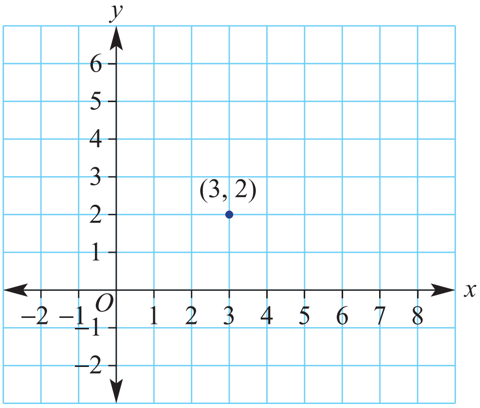
a a triangle with vertices (–2, 0), (–2, 5) and (1, 3)

b a trapezium with vertices (–6, –2), (1, –2), (0, 4) and (–5, 4)

12 For the point (3, 2) shown:

a List four points that are 3 units from this point and add them to the graph.

b Describe the shape that would be formed by all the points that are 3 units from (3, 2).



Enrichment: Division by ratio

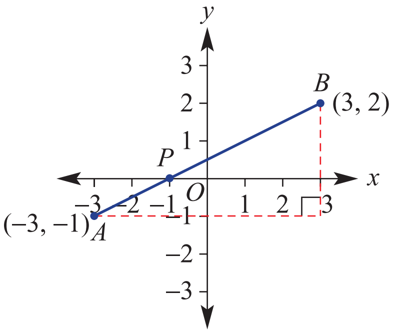
13 From left to right, this line segment shows the point $P(-1, 0)$, which divides the segment in the ratio 1 : 2.

a What fraction of the horizontal distance between the end points is P from A?

b What fraction of the vertical distance between the end points is P from A?

c Find the coordinates of point P on the segment AB if it divides the segment in the ratio:

i 2 : 1 **ii** 1 : 5 **iii** 5 : 1



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1 Matches are arranged by a student so that the first three diagrams in the pattern are:

How many matches are in the 50th diagram of the pattern?

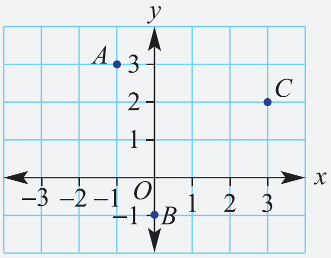
2 The first three shapes in a pattern made with matches are:

How many matches make up the 100th shape?

3 A tank with 520 L of water begins to leak at a rate of 2 L per day. At the same time, a second tank is being filled at a rate of 1 L per hour, starting at 0 L. How long does it take for the tanks to hold the same amount of water?

4 The points (–1, 4), (4, 6), (2, 7) and (–3, 5) are the vertices of a parallelogram. Find the midpoints of its diagonals. What do you notice?

5 Prove that the triangle with vertices at the points $A(-1, 3)$, $B(0, -1)$ and $C(3, 2)$ is isosceles.



6 James takes 3 days to paint a house. Rashid takes 4 days to paint a house. Lucia takes 5 days to paint a house. How long would it take to paint a house (to the nearest hour) if all three of them worked together?

Chapter summary

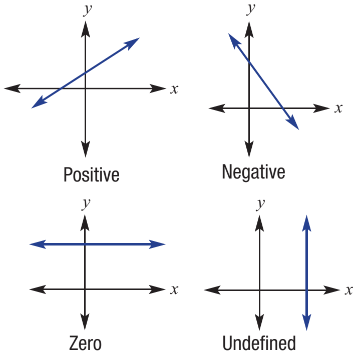
Linear relationships

Direct proportion

If two variables are directly proportional, their rule is of the form $y = mx$.
The gradient of the graph represents the rate of change of y with respect to x; e.g. for a car travelling at 60 km/h for t hours, the distance is $d = 60t$.

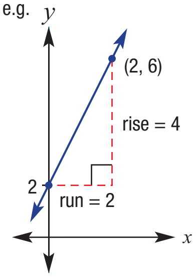
Gradient

This measures the slope of a line.



Gradient, $m = \frac{\text{rise}}{\text{run}}$

e.g.



$m = \frac{4}{2} = 2$

Midpoint and length of a line segment

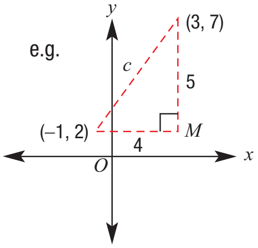
Midpoint is halfway between segment end points.
e.g. Midpoint of segment joining (–1, 2) and (3, 6):

$$M = \left(\frac{-1+3}{2}, \frac{2+6}{2}\right)$$

Length of segment

Use Pythagoras' theorem $c^2 = a^2 + b^2$.

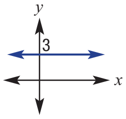
e.g.



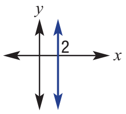
$c^2 = 4^2 + 5^2$
$c = \sqrt{41}$
$c = 6.4$ (to 1 d. p.)

Lines with only one intercept

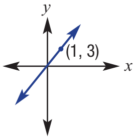
Horizontal line $y = b$
e.g. $y = 3$



Vertical line $x = a$
e.g. $x = 2$

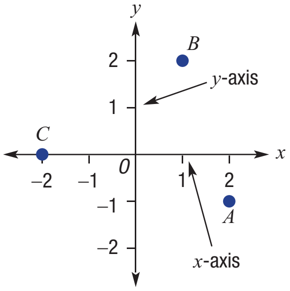


$y = mx$ passes through the origin. Substitute $x = 1$ to find another point; e.g. $y = 3x$.



Coordinates and the Cartesian plane

Each point on the plane is a coordinate pair (x, y).



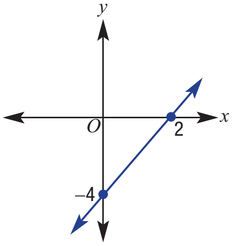
(0, 0) is the origin.
A has coordinates (2, –1).
B has coordinates (1, 2).
C has coordinates (–2, 0).

A linear relationship is made up of points (x, y) that form a straight line when plotted, e.g. $y = 2x - 4$

x–intercept

x	–2	–1	0	1	2
y	–8	–6	–4	–2	0

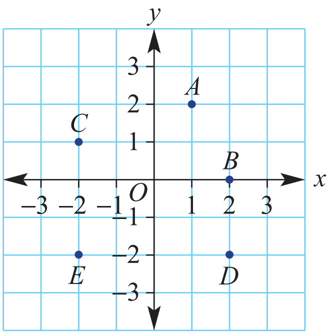
y–intercept



Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.

Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

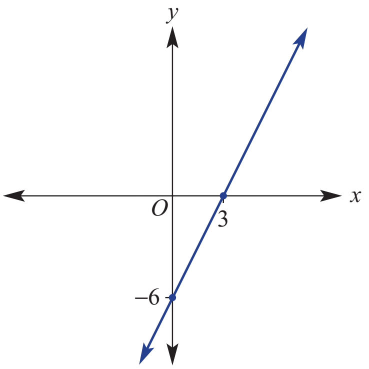


1 The point(s) on the Cartesian plane shown with an x-coordinate of 2 are:
A B only **B** A only **C** C, D and E
D B and D **E** E and D

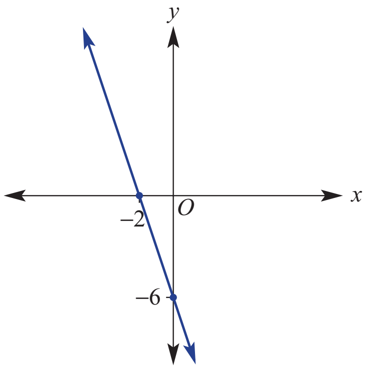
2 The y-intercept of the graph of $y = 2x + 6$ is:
A 1 **B** 2 **C** 6
D 8 **E** 12

3 The graph of $y = 3x - 6$ is represented by:

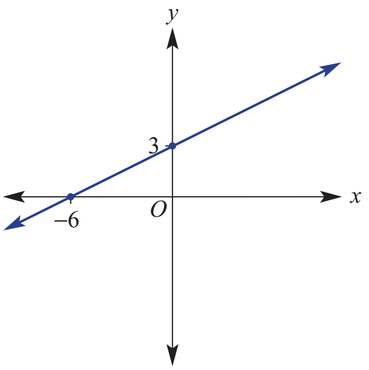
A



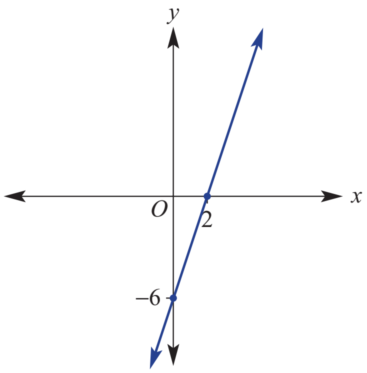
B



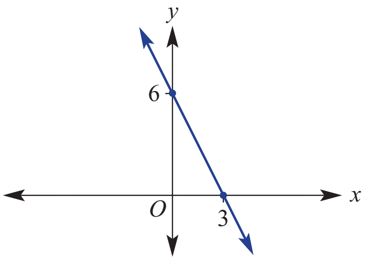
C



D



E



4 The point that is not on the straight line $y = 2x - 1$ is:

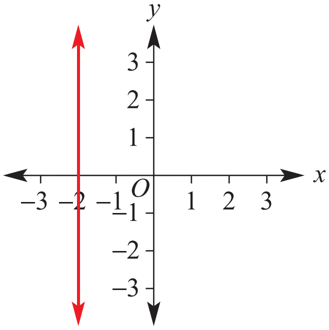
A $(1, 1)$
B $(0, -1)$
C $(3, 5)$
D $(-2, 7)$
E $(-1, -3)$

5 The gradient of the line joining the points $(1, -2)$ and $(5, 6)$ is:

A 2
B 1
C 3
D -1
E $\frac{1}{3}$

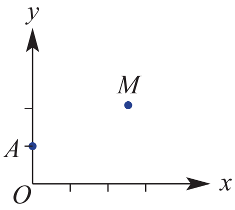
6 The graph shown has equation:

A $y = x - 2$
B $y = -2x$
C $y = -2$
D $x = -2$
E $x + y = -2$



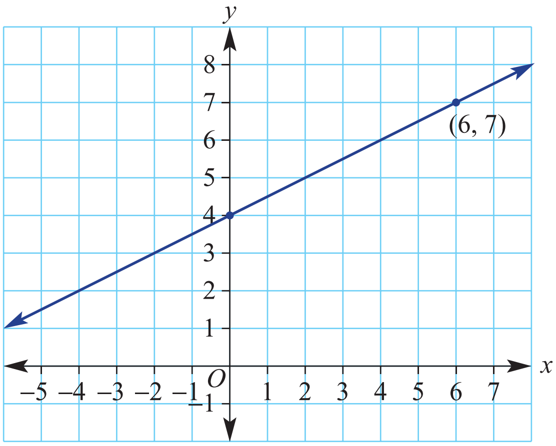
7 $M(3, 2)$ is the midpoint of $A(0, 1)$ and B.
The coordinates of B are:

A $(3, 6)$
B $(6, 3)$
C $(4, 5)$
D $(5, 4)$
E $(6, 4)$



8 The equation of the graph shown is:

A $y = x + 4$
B $y = \frac{1}{2}x + 4$
C $y = 2x + 4$
D $y = -2x + 4$
E $y = -x + 4$



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9 The length of the line segment joining the points $A(1, 2)$ and $B(4, 6)$ is:

A 5
B $\frac{4}{3}$
C 7
D $\sqrt{5}$
E $\sqrt{2}$

10 A phone call costs a 20c connection fee plus 60c per minute. A rule to represent the cost, C cents, of a call lasting n minutes is:

A $C = 20n + 60$
B $n = 60C + 20$
C $C = 60n + 20$
D $n = 20C + 60$
E $20C + 60n = 1$

Short-answer questions

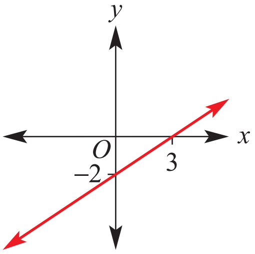
1 Construct a table of values and plot a graph for $y = 2x - 3$.

2 Read off the x- and y-intercepts from the table and graph.

a

x	-2	-1	0	1	2
y	0	2	4	6	8

b

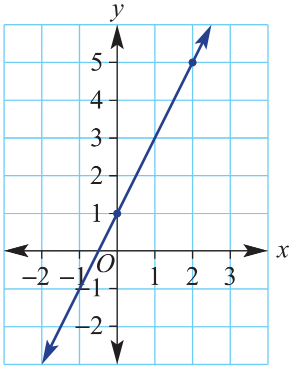


3 Sketch the following lines with one intercept and classify their gradient as positive, negative, zero or undefined.

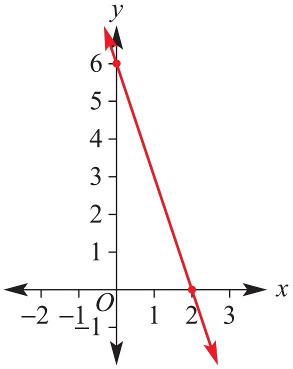
a $y = 3$
b $y = -2$
c $x = -4$
d $x = 5$
e $y = 3x$
f $y = -2x$

4 Find the gradients of the following lines.

a



b



5 Find the gradients of the lines passing through these points.

a $(3, 1)$ and $(5, 5)$ **b** $(2, 5)$ and $(4, 3)$

c $(-3, -2)$ and $(1, 6)$ **d** $(-2, 6)$ and $(1, -4)$

6 An empty swimming pool is being filled with water by a hose. It takes 4 hours to fill 8000 L.

a What is the rate at which water is poured into the pool?

b Draw a graph of volume (V litres) vs time (t hours) from $t = 0$ to $t = 4$.

7 For the line segment joining the following pairs of points, find the:

i midpoint

ii length (to two decimal places)

a $(2, 4)$ and $(6, 8)$ **b** $(5, 2)$ and $(10, 7)$

c $(-2, 1)$ and $(2, 7)$ **d** $(-5, 7)$ and $(-1, -2)$

Extended-response questions

1 Justin is hiring a clown for his son's birthday party. The clown charges a booking fee of $100, as well as $50 per hour at the party.

a Copy and complete the table.

Hours (t)	0	1	2	3	4	5
Cost (C)						

b Write a rule for the cost (C dollars) of hiring the clown for t hours.

c Sketch a graph of cost (C) vs time (t).

d How much will it cost to hire the clown for 4 hours?

e At the end of the party, Justin writes a cheque for $225. For how long was the clown at the party?

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2 Luca runs a marathon at a constant speed. The distance, d km, he has remaining in the marathon t hours after the start is given by $d = 42 - 14t$.

a Copy and complete the table.

Hours (t)	0	1	2	3
Distance remaining (d)				

b Sketch the graph.
c What is the distance of the marathon?
d How long does Luca take to run the marathon?
e Find the gradient of the graph.
f At what speed is Luca running?

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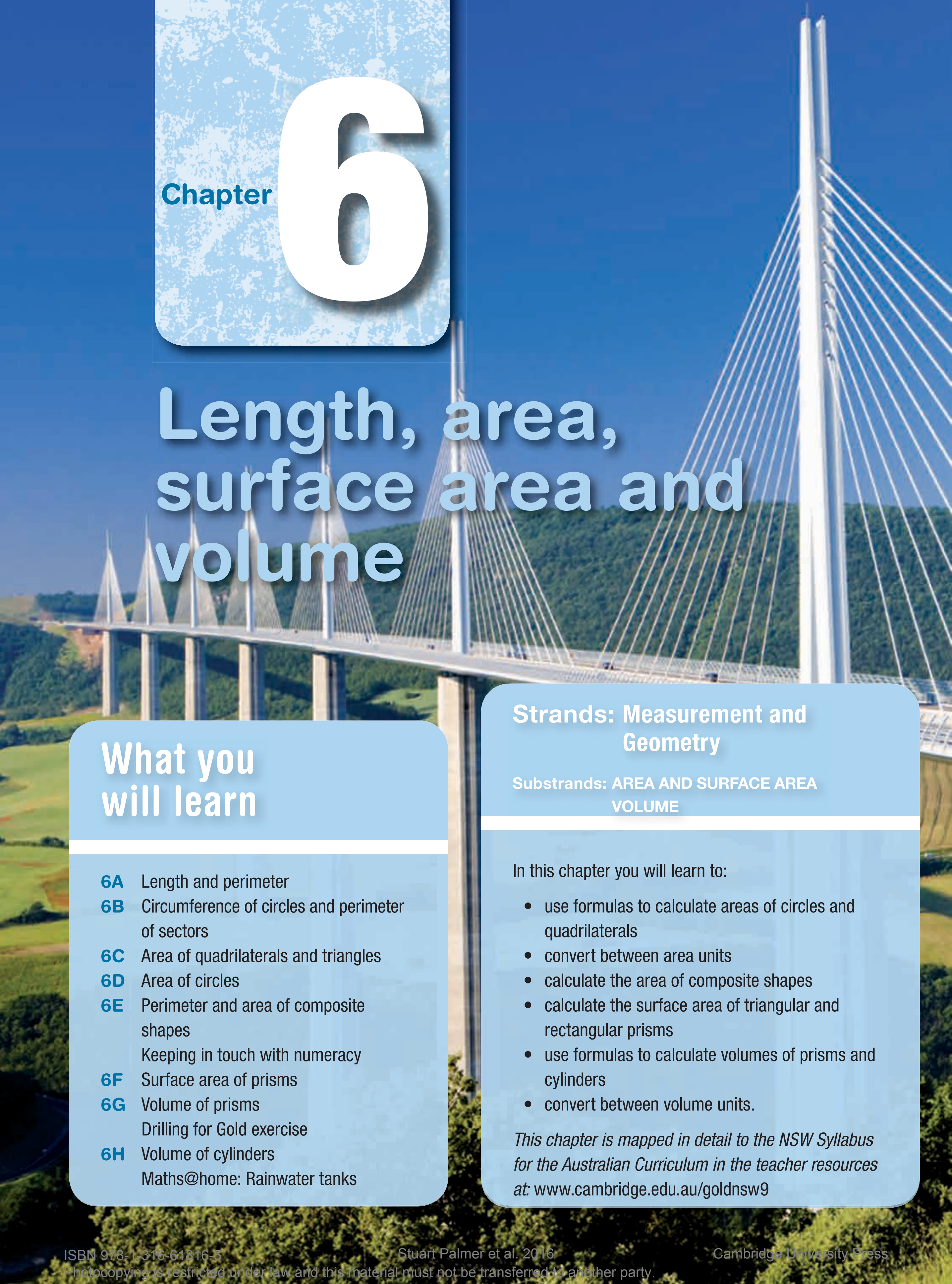

Chapter 6

Length, area, surface area and volume

What you will learn

Strands: Measurement and Geometry

Substrands: AREA AND SURFACE AREA
VOLUME

In this chapter you will learn to:

- use formulas to calculate areas of circles and quadrilaterals
- convert between area units
- calculate the area of composite shapes
- calculate the surface area of triangular and rectangular prisms
- use formulas to calculate volumes of prisms and cylinders
- convert between volume units.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw9

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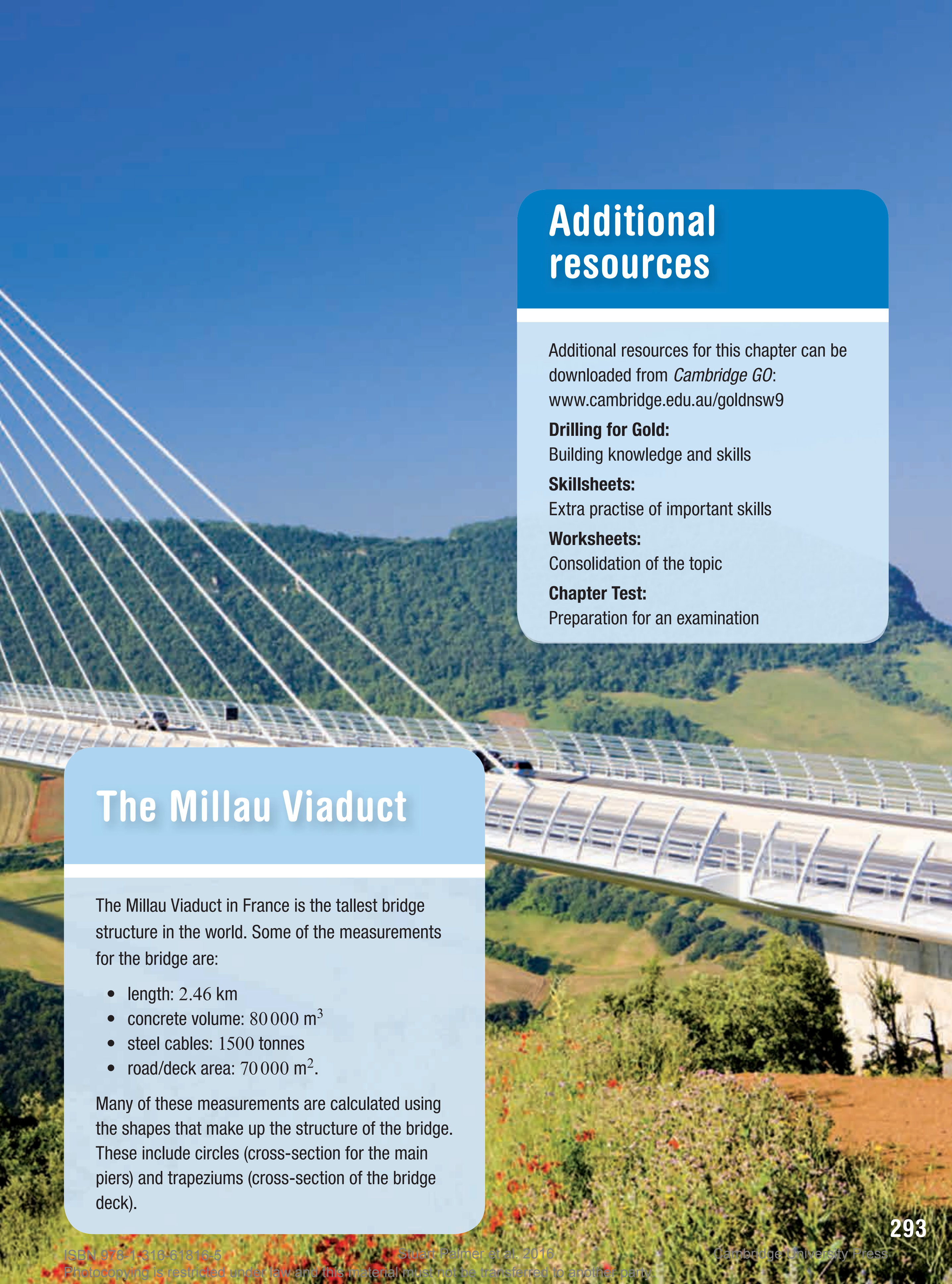

The Millau Viaduct

The Millau Viaduct in France is the tallest bridge structure in the world. Some of the measurements for the bridge are:

- length: 2.46 km
- concrete volume: 80 000 m^3
- steel cables: 1500 tonnes
- road/deck area: 70 000 m^2.

Many of these measurements are calculated using the shapes that make up the structure of the bridge. These include circles (cross-section for the main piers) and trapeziums (cross-section of the bridge deck).

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw9

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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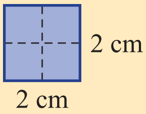
Pre-test

1 Count the number of squares in these shapes to find the areas.

a



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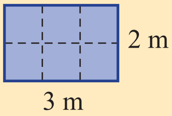
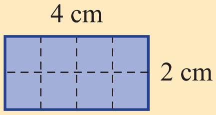


c

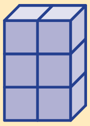


2 Find the perimeter of the shapes in Question **1**.

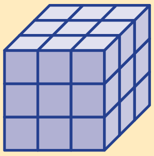
3 Count the number of cubes in these solids to find the volumes.

a

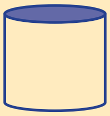
b



c



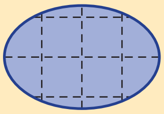
4 **a** Name the solid shown on the right.
b What is the name of the shape that is shaded on top of the solid?



5 Without using a calculator, find the value of:

a 2.3×10 **b** 0.048×1000 **c** $270 \div 100$
d $52\,134 \div 10\,000$ **e** $0.0005 \times 100\,000$ **f** $72\,160 \div 1000$

6 Estimate the area (i.e. number of squares) in this ellipse.

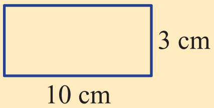


7 Convert the following to the units shown in the brackets.
Remember: 1 km = 1000 m, 1 m = 100 cm and 1 cm = 10 mm.

a 3 cm (mm) **b** 20 m (cm) **c** 1.6 km (m) **d** 23 mm (cm)
e 3167 m (km) **f** 72 cm (m) **g** 20 000 m (km) **h** 0.03 km (m)

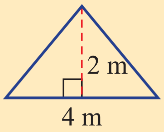
8 Find the area of these basic shapes, using the given formulas.

a



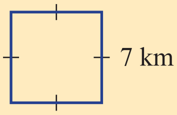
$A = \ell \times b$

b



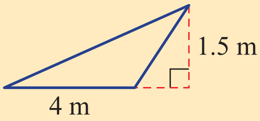
$A = \frac{1}{2}bh$

c



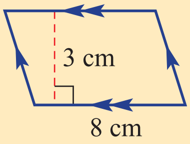
$A = s^2$

d



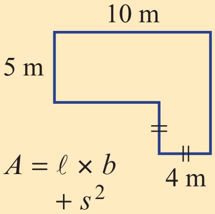
$A = \frac{1}{2}bh$

e



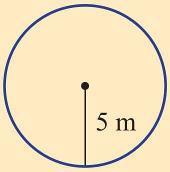
$A = b \times h$

f



$A = \ell \times b$
$+ s^2$

9 Find the circumference ($C = 2\pi r$) and area ($A = \pi r^2$) of this circle, rounding to two decimal places.



6A Length and perimeter

Stage
5.2
5.2◊
5.1
4

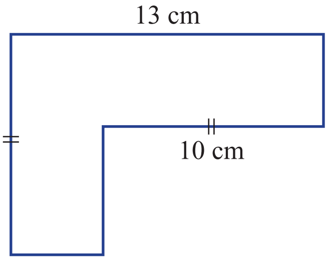
Measurement is the branch of mathematics that includes length, perimeter, circumference, surface area, volume and capacity.

We use measurement when we design buildings, water our gardens, control satellites, fill our cars with petrol or participate in school athletics days.

Let's start: Not enough information?

All the angles at each vertex in this shape are 90° and the two given lengths are 10 cm and 13 cm.

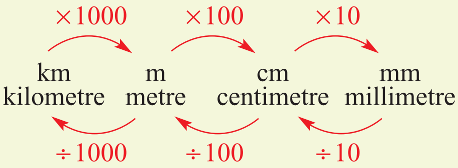
- Is there enough information to find the perimeter of the shape?
- If there is enough information, find the perimeter and discuss your method. If not, what information needs to be provided?



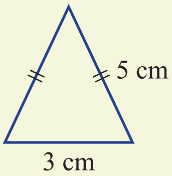
Key ideas

Drilling for Gold
6A1
6A2

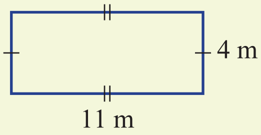
- To convert between metric units of length, multiply or divide by the appropriate power of 10.



- **Perimeter** is the distance around a closed shape.
  - Sides with the same markings are of equal length.



$P = 2 \times 5 + 3$
$= 13$ cm



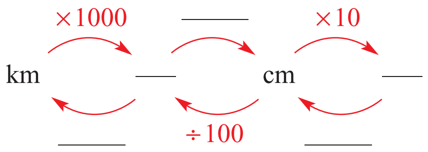
$P = 2 \times 11 + 2 \times 4$
$= 30$ m

Perimeter The total distance (length) around the outside of a figure

Exercise 6A

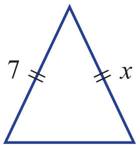
Understanding

1 Fill in the gaps on this flow chart.

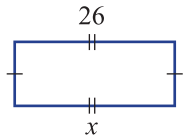


2 Write down the value of x in these diagrams.

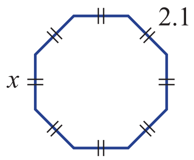
a



b



c



Drilling for Gold 6A3

Example 1 Converting units of length

Convert to the units shown in the brackets.

a 5.41 cm (mm) **b** 3200 m (km)

Solution	**Explanation**
a $5.41 \text{ cm} = 5.41 \times 10$ $= 54.1 \text{ mm}$	1 cm = 10 mm and you are moving to a smaller unit, so multiply. The decimal point appears to move one place to the right.
b $3200 \text{ m} = 3200 \div 1000$ $= 3.2 \text{ km}$	1 km = 1000 m and you are moving to a larger unit, so divide. The decimal point appears to move three places to the left.

3 Convert the following length measurements to the units given in the brackets.

a 5 cm (mm)
b 41 cm (mm)
c 2.8 m (cm)
d 0.4 m (cm)
e 4.6 km (m)
f 0.9 km (m)
g 521 mm (cm)
h 36 mm (cm)
i 240 cm (m)
j 83.7 cm (m)
k 7000 m (km)
l 2170 m (km)

Multiply when changing to a smaller unit and divide when changing to a larger unit.

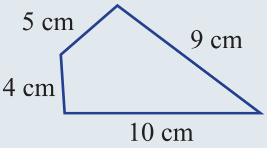
4 A steel beam is 8.25 m long and 22.5 mm wide. Write down the length and the width of the beam, in centimetres.

Fluency

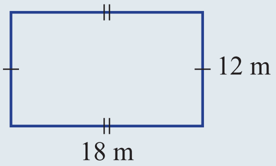
Example 2 Finding perimeters of simple shapes

Find the perimeter of each of the following shapes.

a



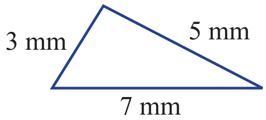
b



Solution	**Explanation**
a $P = 10 + 9 + 5 + 4$ $= 28$ cm	To find the perimeter, add all the side lengths together.
b $P = 2 \times 12 + 2 \times 18$ $= 24 + 36$ $= 60$ m	There are two lengths of 12 m and two lengths of 18 m.

5 Find the perimeter of each of the following shapes.

a



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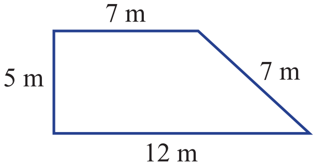
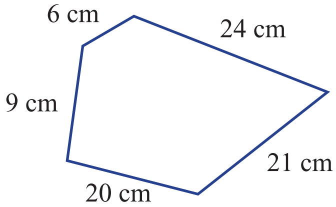
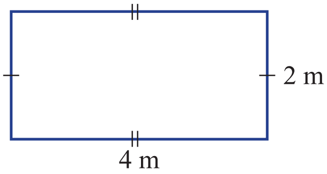


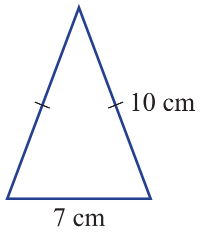
c



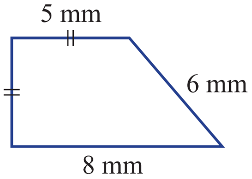
d



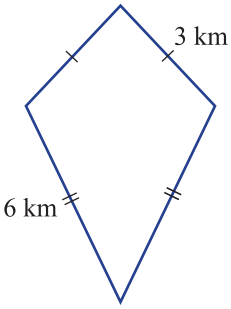
e



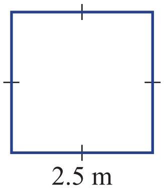
f



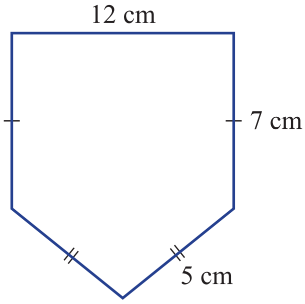
g



h

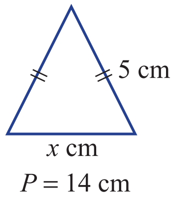


i

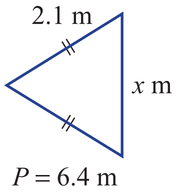


6 Find the unknown side length in these shapes with the given perimeters.

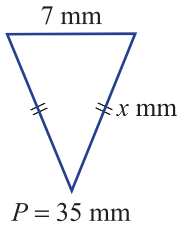
a



b



c

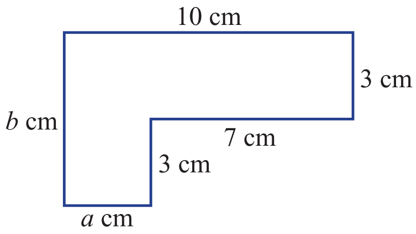


Use trial and error or an equation to find the value of x; e.g. $5 + 5 + x = 14$

Problem-solving and Reasoning

7 Write down the values of the pronumerals in these shapes.

a



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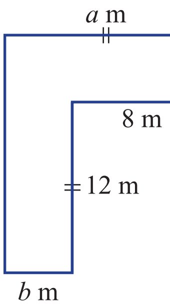
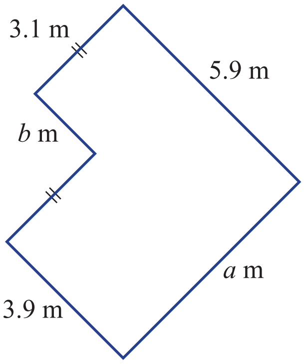


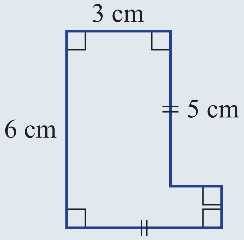
c



Look at the sides opposite the pronumerals.

Example 3 Finding perimeters of composite shapes

Find the perimeter of this composite shape.



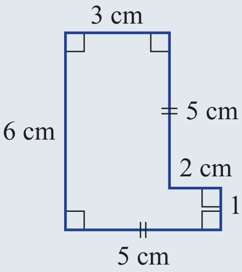
Solution

$$\begin{aligned} P &= (2 \times 5) + 6 + 3 + 2 + 1 \\ &= 22 \text{ cm} \end{aligned}$$

Explanation

Missing sides are:
5 cm − 3 cm = 2 cm
6 cm − 5 cm = 1 cm

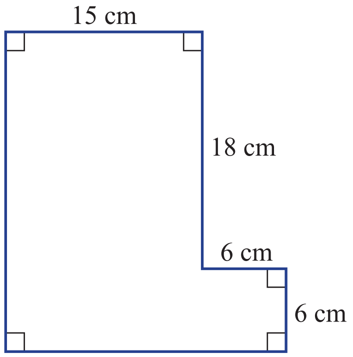
Alternatively,
$2 \times 6 + 2 \times 5 = 22$ cm



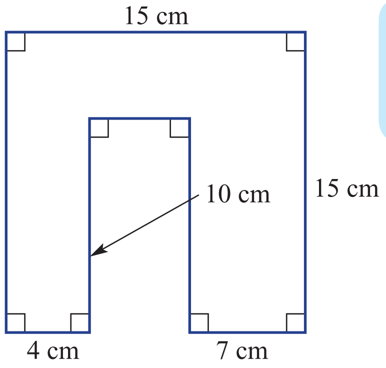
8 Find the perimeter of each of the following composite shapes.

First label all the missing side lengths, then find the perimeter.

a



b



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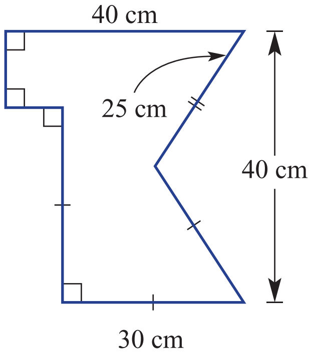

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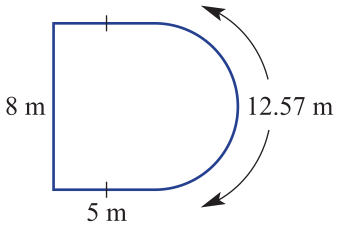

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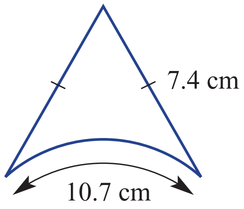

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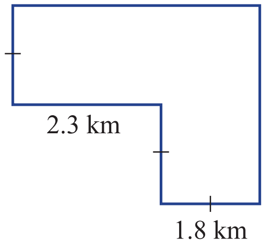


9 A lion cage is made up of five straight fence sections. Three sections are 20 m in length and the other two sections are 15.5 m and 32.5 m. Find the perimeter of the cage.

10 Convert the following measurements to the units given in the brackets.

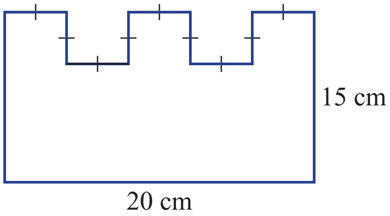
a 8 m (mm) **b** 110 000 mm (m) **c** 0.00001 km (cm)

d 0.02 m (mm) **e** 28 400 cm (km) **f** 62 743 000 mm (km)

In part **a**, first convert to cm, then to mm.

11 Find the perimeters of these shapes. All angles are right angles.

a



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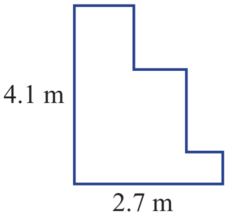

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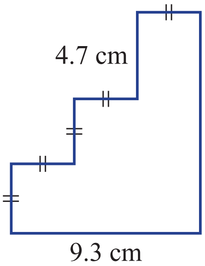


Enrichment: Picture framing

12 A photo 12 cm wide and 20 cm long is surrounded with a picture frame 3 cm thick. Find the outside perimeter of the framed picture.

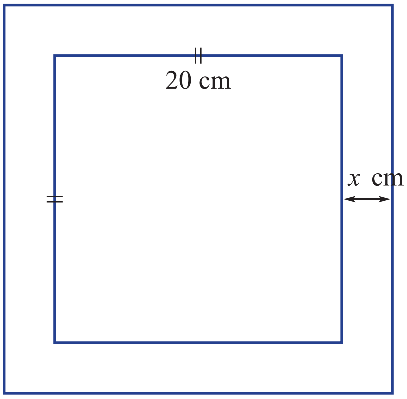
13 A square picture of side lengths 20 cm is inside a frame of breadth x cm.

a Find the perimeter of the framed picture when:

  i $x = 2$ **ii** $x = 3$ **iii** $x = 5$

b Write a rule for the perimeter, P, of the framed picture in terms of x.

c Use your rule to find the perimeter when $x = 3.7$.



ISBN 978-1-316-61816-5 Stuart Palmer et al. 2016 Cambridge University Press

6B Circumference of circles and perimeter of sectors

Stage
5.2
5.2◊
5.1
4

The Egyptians, Babylonians and ancient Indians found a number that links a circle's diameter with its circumference.

Today we call this number pi (π) and know it to be 3.14159, correct to five decimal places.

An exact value of pi cannot be written down as a decimal, as it has an infinite number of decimal places with no pattern.

Some people believe that the ancient Egyptians used $\frac{22}{7}$ for pi.

Drilling for Gold
6B1
6B2
GeoGebra
6B3

▶ Let's start: Discovering pi

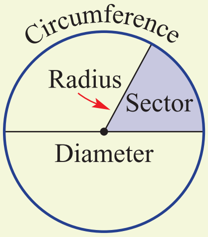
People are still trying to discover more and more decimal places of pi. Use the internet to find the current world record.

Key ideas

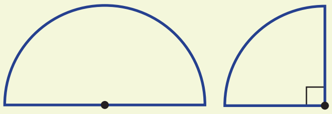
- Features of a circle:
  - **Diameter** (d) is the distance across the centre.
  - **Radius** (r) is the distance from the centre to the circumference. The radius is half of the diameter.
  - **Circumference** of a circle is:

  $C = 2\pi r$ or $C = \pi d$

  Use the π button on your calculator.



- Special circle sectors:
  - A half circle is called a semicircle.
  - A quarter circle is called a quadrant.



Diameter A line passing through the centre of a circle with its end points on the circumference

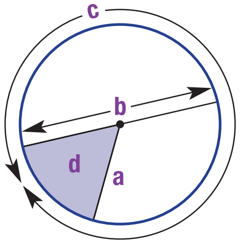
Radius The distance from the centre of a circle to its outside edge

Circumference The curved boundary of a circle

Exercise 6B

Understanding

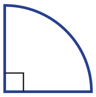
1 Name features **a–d** shown in the circle at right.

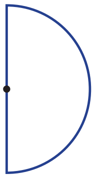


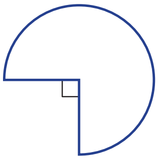
2 **a** What is the radius of a circle if its diameter is 5.6 cm?
b What is the diameter of a circle if its radius is 48 mm?

3 Write down the rule for the circumference of a circle using:
a r, the radius **b** d, the diameter

4 Determine the fraction of a circle shown in these sectors.
Write the fraction in simplest form.

a


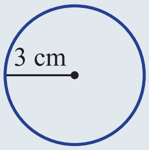
b


c


Fluency

Example 4 Finding the circumference of a circle

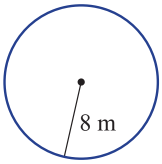
Find the circumference of this circle, correct to two decimal places.



Solution	Explanation
$C = 2\pi r$	Use the formula $C = 2\pi r$ or $C = \pi d$ and
$= 2 \times \pi \times 3$	substitute $r = 3$ (or $d = 6$).
$= 6\pi$	6π is the exact answer.
$= 18.85$ cm (to 2 d.p.)	This is a decimal approximation.

5 Find the circumferences of these circles, correct to two decimal places. Use a calculator for the value of π.

Write the rule $C = 2\pi r$, then substitute the radius length.

a


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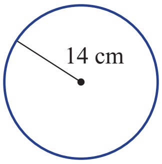

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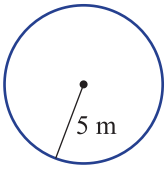
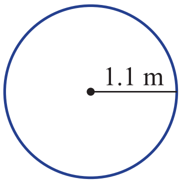


d


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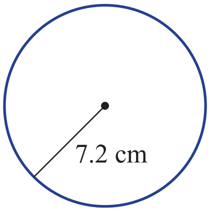

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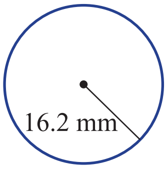
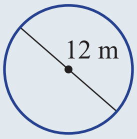


Example 5 Finding the circumference using the diameter

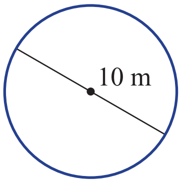
Find the circumference of this circle, correct to two decimal places.



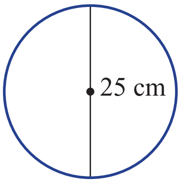
Solution	**Explanation**
$C = \pi d$ $= \pi \times 12$ $= 37.70$ m (to 2 d.p.)	Write the formula. $C = \pi d$ is preferred since d is given. Substitute $d = 12$ and multiply by π. Use a calculator and round your answer. Note: 37.6991 rounds to 37.70 for two decimal places.

6 Find the circumferences of these circles, correct to two decimal places.

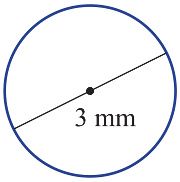
a



b

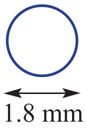


c



Use $C = 2\pi r$ or $C = \pi d$.

d



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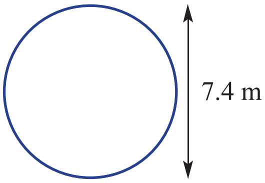

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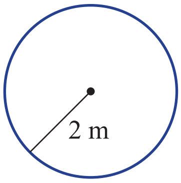

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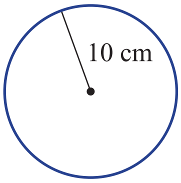

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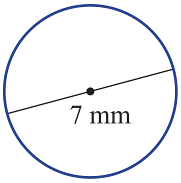
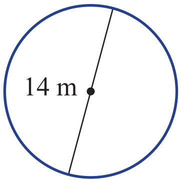
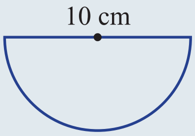


i



Example 6 Working with a semicircle

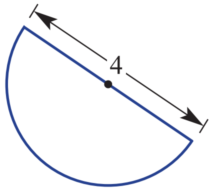
Find the perimeter of this semicircle, correct to two decimal places.



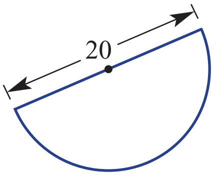
Solution	**Explanation**
$P = \frac{1}{2} \times \pi d + 10$ $= \frac{1}{2} \times \pi \times 10 + 10$ $= 25.71$ cm (to 2 d.p.)	The perimeter consists of half the circumference of a circle (with diameter 10 cm) plus the 10 cm diameter across the top.

7 Find the perimeters of these sectors, correct to two decimal places.

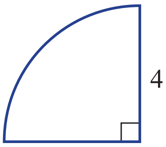
a



b

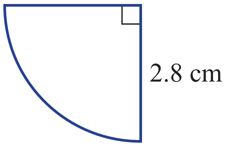


c



Decide what fraction of the circumference you want and don't forget to add the straight sides.

d



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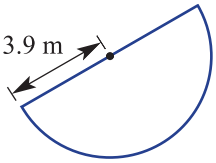

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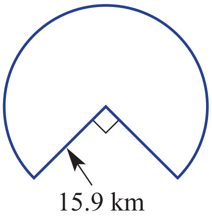


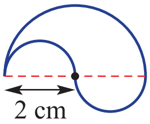
Problem-solving and Reasoning

8 Find the distance around the outside of a circular pool of radius 4.5 m, correct to two decimal places.

9 Find the length of string required to surround the circular trunk of a tree that has a diameter of 1.3 m, correct to one decimal place.

10 Give the perimeters of these shapes, correct to two decimal places.

a



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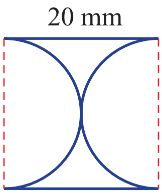

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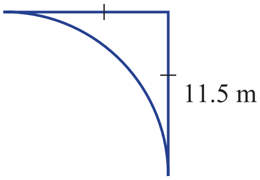
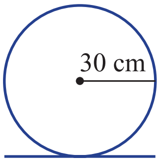


Two semicircles of the same size make a full circle.

Enrichment: The rolling wheel

11 A wheel of radius 30 cm is rolled in a straight line.



- **a** Find the circumference of the wheel, correct to two decimal places.
- **b** How far, correct to two decimal places, has the wheel rolled after completing:
  - **i** 2 rotations?
  - **ii** 10.5 rotations?
- **c** Can you find how many rotations would be required to cover at least 1 km in length? Round your answer to the nearest whole number.

6C Area of quadrilaterals and triangles

Stage
5.2
5.2◊
5.1
4

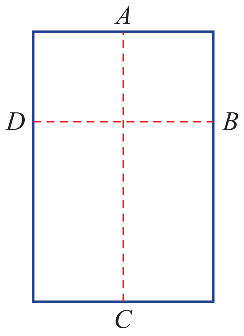
In Year 8 you learnt how to use the formula for the area of a rhombus, which is also the formula for the area of a kite.

In *Let's start* we are going to use a rectangle to explain why the formula for kites and rhombuses works.

Let's start

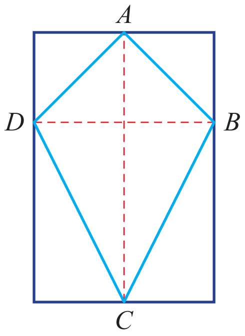
There is a simple way to make a kite from a rectangular piece of paper.

1. Make a fold line down the middle, from top to bottom, along the axis of symmetry (AC).
2. Make another fold line from one side to the other but not across the middle (DB).
3. The dashed lines in the diagram will be the diagonals of the kite.
4. Join A to B, B to C, C to D and D to A to make the kite.
5. Cut off the four triangles outside the kite.
6. Use the four triangles to make another kite identical to $ABCD$.



If the original piece of paper had sides called x and y:

- What is the area of the rectangular piece of paper?
- What is the area of the kite $ABCD$?



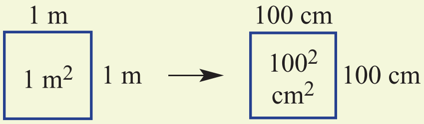
Key ideas

Drilling for Gold 6C1

- Conversion of area units:

$$\begin{tikzcd}
\text{km}^2 \arrow[r, bend left, "\times 1000^2"] & \text{m}^2 \arrow[l, bend left, "\div 1000^2"] \arrow[r, bend left, "\times 100^2"] & \text{cm}^2 \arrow[l, bend left, "\div 100^2"] \arrow[r, bend left, "\times 10^2"] & \text{mm}^2 \arrow[l, bend left, "\div 10^2"]
\end{tikzcd}$$

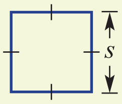
$$10^2 = 10 \times 10 = 100$$
$$100^2 = 100 \times 100 = 10\,000$$
$$1000^2 = 1000 \times 1000 = 1\,000\,000$$



$\therefore$ 1 square metre = 10 000 square centimetres

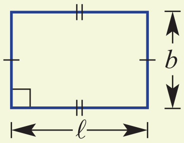
- The area of a two-dimensional shape is a measure of the space enclosed within its boundaries.

Square



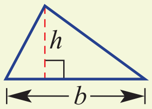
$A = s^2$

Rectangle



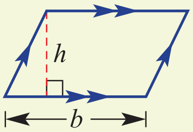
$A = \ell b$

Triangle



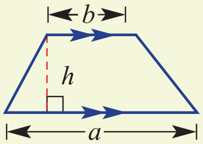
$A = \frac{1}{2}bh$

Parallelogram, rhombus



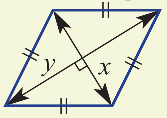
$A = bh$

Trapezium



$A = \frac{1}{2}h(a + b)$

Rhombus, square, kite



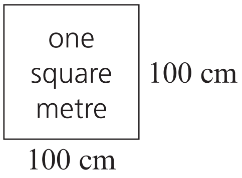
$A = \frac{1}{2}xy$

Exercise 6C

Understanding

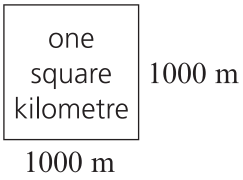
1 Copy and complete:

a



One square metre = ______________ square centimetres

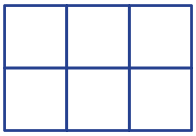
b



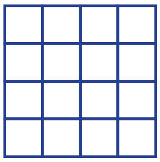
One square kilometre = ______________ square metres

2 Count the number of squares to find the area of these shapes. Each square in each shape represents 1 square unit.

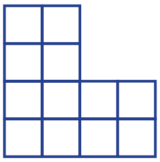
a



b

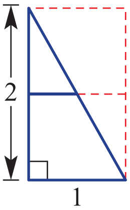


c

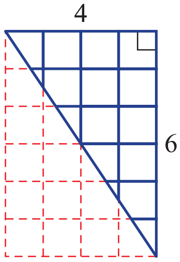


For parts **d** and **e**, note that each triangle is half of a rectangle.

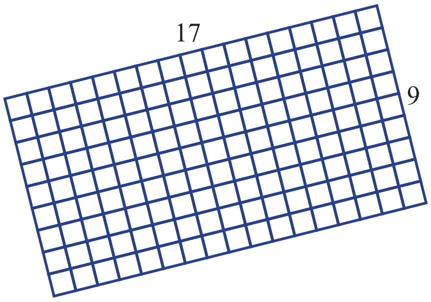
d



e



f



3 Name the shape that has the given area formula.

a $A = \ell b$

b $A = \frac{1}{2}xy$ (3 shapes)

c $A = \frac{1}{2}bh$

d $A = \frac{1}{2}h(a + b)$

e $A = bh$

f $A = s^2$

square	rectangle
rhombus	triangle
trapezium	kite
parallelogram	

Fluency

Example 7 Converting units of area

Convert the following area measurements to the units given in the brackets.

a 859 mm² (cm²)

b 2.37 m² (cm²)

Solution

Explanation

a $859 \text{ mm}^2 = 859 \div 10^2 \text{ cm}^2$
$= 8.59 \text{ cm}^2$

$$\mathrm{cm}^2 \xleftarrow[\div 10^2]{} \mathrm{mm}^2$$

b $2.37 \text{ m}^2 = 2.37 \times 100^2 \text{ cm}^2$
$= 23\ 700 \text{ cm}^2$

$$\mathrm{m}^2 \xrightarrow{\times 100^2} \mathrm{cm}^2$$

4 Convert the following area measurements to the units given in the brackets.

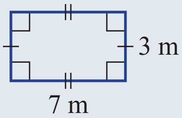
- **a** 2 cm² (mm²)
- **b** 0.4 cm² (mm²)
- **c** 500 mm² (cm²)
- **d** 310 mm² (cm²)
- **e** 2.1 m² (cm²)
- **f** 0.2 m² (cm²)
- **g** 210 000 cm² (m²)
- **h** 3700 cm² (m²)
- **i** 0.001 km² (m²)
- **j** 4.3 km² (m²)
- **k** 3 200 000 m² (km²)
- **l** 39 400 m² (km²)

Multiply or divide by: 10^2
100^2
or 1000^2.

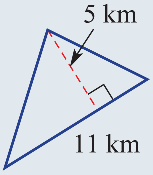
Example 8 Finding areas of rectangles, triangles and parallelograms

Find the area of each of the following plane figures.

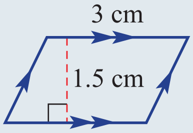
a



b



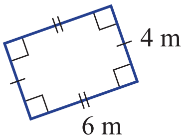
c



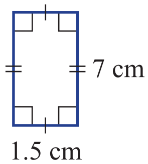
Solution	Explanation
a Area $= \ell b$ $= 7 \times 3$ $= 21\text{ m}^2$	Use the area formula for a rectangle. Substitute $\ell = 7$ and $b = 3$. Include the correct units.
b Area $= \frac{1}{2}bh$ $= \frac{1}{2} \times 11 \times 5$ $= 27.5\text{ km}^2$	Use the area formula for a triangle. Substitute $b = 11$ and $h = 5$, where h is the height perpendicular (at 90°) to the base.
c Area $= bh$ $= 3 \times 1.5$ $= 4.5\text{ cm}^2$	Use the area formula for a parallelogram. Multiply the base length by the perpendicular height.

5 Find the area of each of the following plane figures.

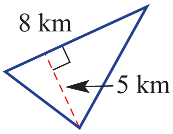
a



b

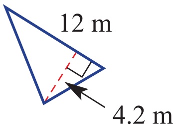


c

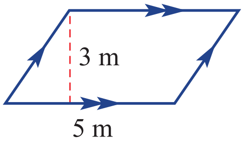


Choose from:
$A = \ell b$
$A = bh$
$A = \frac{1}{2}bh$

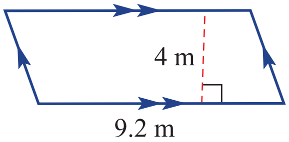
d



e



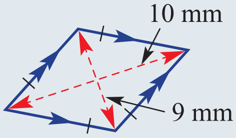
f



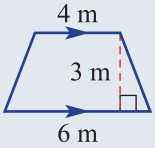
Example 9 Finding areas of rhombuses and trapeziums

Find the area of each of the following plane figures.

a



b



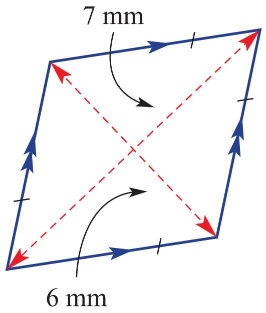
Solution	**Explanation**
a Area $= \frac{1}{2}xy$	Use the area formula for a rhombus.
$= \frac{1}{2} \times 10 \times 9$	x and y are the lengths of the diagonals.
$= 45$ mm^2	$\frac{1}{2} \times 10 \times 9 = 5 \times 9 = 45$
b Area $= \frac{1}{2}h(a + b)$	Use the area formula for a trapezium.
$= \frac{1}{2} \times 3(4 + 6)$	Substitute $a = 4$, $b = 6$ and $h = 3$.
$= 15$ m^2	Include the correct units.

6 Find the area of each of the following plane figures.

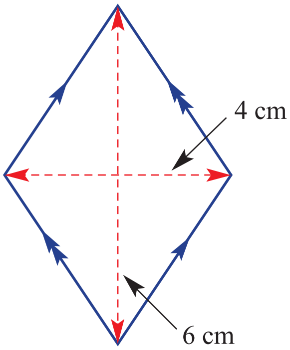
Drilling for Gold 6C2

Choose from:
$A = \frac{1}{2}xy$
$A = \frac{1}{2}h(a + b)$

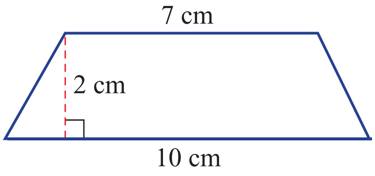
a



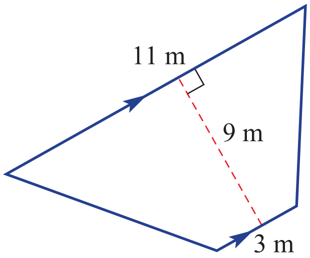
b



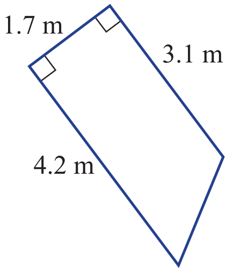
c



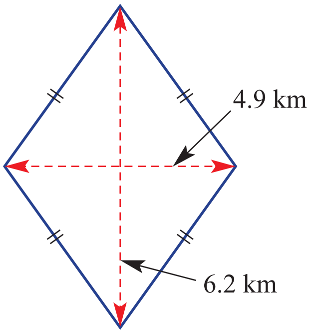
d



e



f



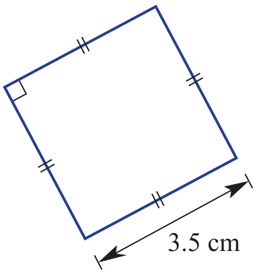
Problem-solving and Reasoning

7 A piece of land has an area of half a square kilometre (0.5 km^2). How many square metres (m^2) is this?

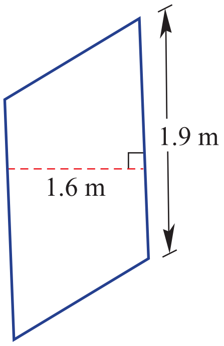
8 A rectangular park covers an area of 175 000 m^2. Give the area of the park, in km^2.

9 Find the area of the following.

a

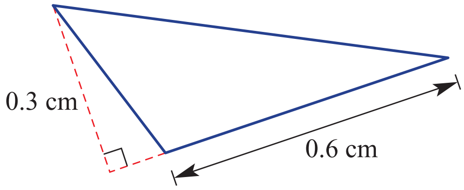


b

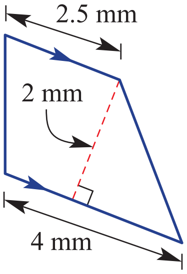


Choose from:
$A = s^2$
$A = bh$
$A = \frac{1}{2}bh$
$A = \frac{1}{2}h(a + b)$

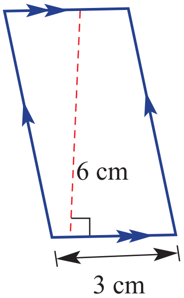
c



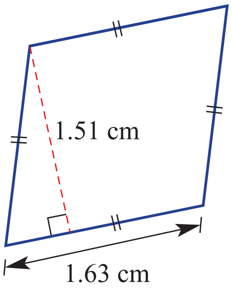
d



e



f

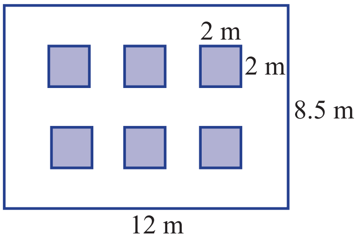


10 An old picture frame that was once square now leans to one side to form a rhombus. If the distances between pairs of opposite corners are 85 cm and 1.2 m, find the area inside the frame, in m^2.

11 Convert the following measurements to the units given in the brackets.

a $1.5\ \text{km}^2$ (cm^2) **b** $0.000005\ \text{m}^2$ (mm^2) **c** $75\,000\ \text{mm}^2$ (m^2)

Enrichment: Windows



12 Six square windows of side length 2 m are placed into a 12 m wide by 8.5 m high wall, as shown. The windows are positioned so that the vertical spacing between the windows and the wall edges are equal. The horizontal spacings are also equal.

- **a** **i** Find the horizontal distance between the windows.
  - **ii** Find the vertical distance between the windows.
- **b** Find the area of the wall, not including the window spaces.
- **c** If the wall included 3 rows of 4 windows (instead of 2 rows of 3), would it be possible to space all the windows so that the horizontal and vertical spacings are the same? (Note: Horizontal spacing doesn't have to be the same as vertical spacing.)

6D Area of circles

Stage
5.2
5.2◊
5.1
4

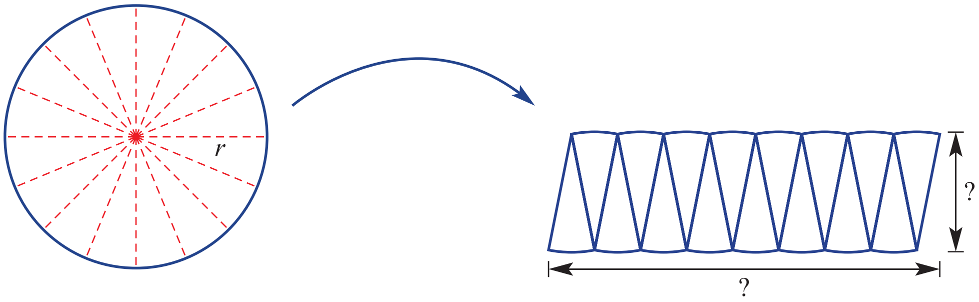
We know that the circumference of a circle is connected to the diameter by the special number pi (π).

The area of a circle is also connected to pi.

The area of this circular stage is the product of pi and the square of its radius.

▶ Let's start: How does a circle become a rectangle?

Consider a circle cut into equal sectors, as shown, then rearranged to form a rectangular-style shape.

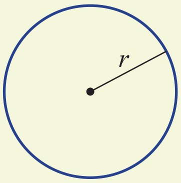


- Compared to the circle, what is the height of the rectangle close to?
- Compared to the circle, what is the length of the rectangle close to?
- What does this say about the area of the rectangle and, hence, the area of a circle?
- How could the rectangle be improved?

Key ideas

- The formula for the area of a circle is:

$$A = \pi r^2$$



- The radius (r) is half the diameter (d).

$d = 10$ cm

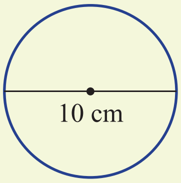
$\therefore\ r = 5$ cm

$\therefore\ A = \pi \times 5^2$

$\therefore$ Area $= 25\pi$ cm^2 (exact value)

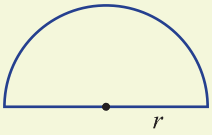
$= 78.539\ldots$

$= 78.54$ cm^2 (to two d.p.)



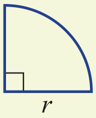
- A half circle is called a semicircle.

$$A = \frac{1}{2}\pi r^2$$



- A quarter circle is called a quadrant.

$$A = \frac{1}{4}\pi r^2$$

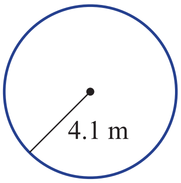


Exercise 6D

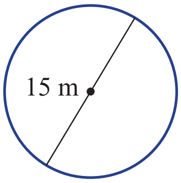
Understanding

1 What is the radius of these circles?

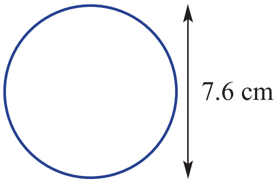
a



b



c

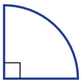


2 Which is the correct formula for the area of a circle?

A $A = \pi d^2$ **B** $A = \frac{1}{2}\pi r^2$ **C** $A = \pi r^2$ **D** $A = (\pi r)^2$

3 What fraction of a full circle is shown by each of these sectors?

a



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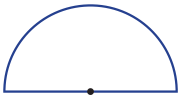
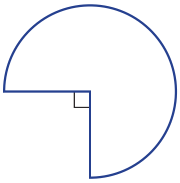


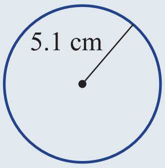
c



Fluency

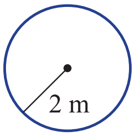
Example 10 Finding areas of circles

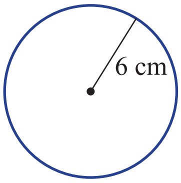
Find the area of this circle, correct to two decimal places.

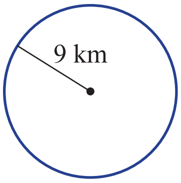


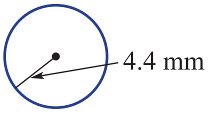
Solution	Explanation
$A = \pi r^2$ $= \pi \times (5.1)^2$ $= 81.7128\ldots$	Write the rule and substitute $r = 5.1$.
$= 81.71\ \text{cm}^2$ (to 2 d.p.)	81.7128 rounds to 81.71 because the third decimal place is 2.

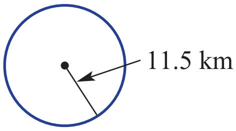
4 Find the area of each of these circles, correct to two decimal places.

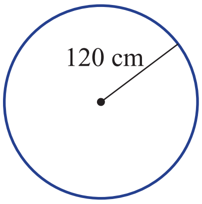
a


b


c


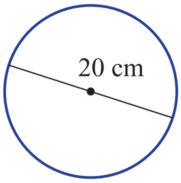
d


e


f


Substitute the radius into $A = \pi r^2$.

5 Find the area of each of these circles, correct to two decimal places.

a


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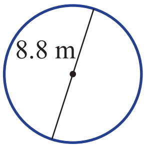
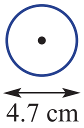


c


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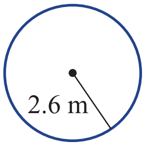
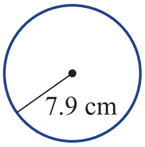


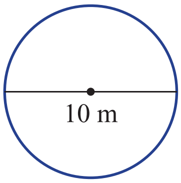
e


f


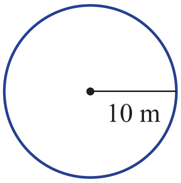
First work out the radius.

6 Find the exact area.

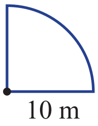
a



b



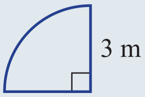
c



Problem-solving and Reasoning

Example 11 Finding areas of quadrants

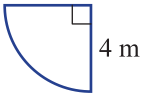
Find the area of this quadrant, correct to two decimal places.



Solution	**Explanation**
$A = \frac{1}{4} \times \pi r^2$ $= \frac{1}{4} \times \pi (3)^2$ $= 7.0685\ldots$ $= 7.07\ \text{m}^2$ (to 2 d.p.)	A quadrant has one-quarter of the area of a full circle. Substitute $r = 3$ and evaluate.

7 Find the areas of these sectors, correct to two decimal places.

a



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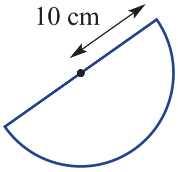

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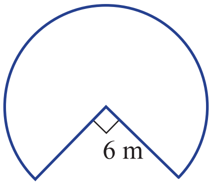

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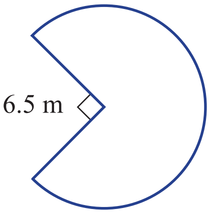
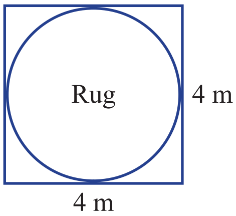


8 A pizza of radius 15 cm is divided into quarters. Find the area of each quarter, correct to the nearest square centimetre.

Skillsheet 6A

9 A circular rug touches the edges of a square room of side length 4 m.



a What is the radius of the rug?

b Find the area of the rug, correct to two decimal places.

c Find the area not covered by the rug, correct to two decimal places.

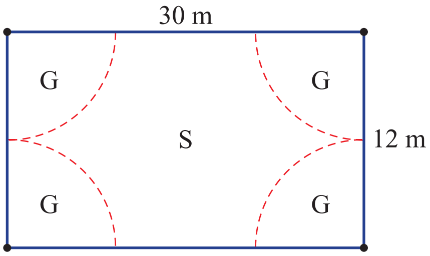
10 A pizza shop is considering increasing the diameter of its family pizza tray from 32 cm to 34 cm. How much bigger in area is the new tray?

11 You can rearrange $A = \pi r^2$ to give $r = \sqrt{\frac{A}{\pi}}$. Use this new rule to find the radius of a circle for these areas. Round to one decimal place where necessary.

a 10 cm^2 **b** 117.8 m^2 **c** 4π km^2

Enrichment: Tennis lights

12 A tennis court area is lit by four corner lights. The area close to each light is considered to be good (G), and the remaining area is satisfactory (S). What percentage of the area is 'good'? Round your answer to the nearest per cent.



6E Perimeter and area of composite shapes

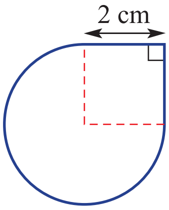
Stage
5.2
5.2◊
5.1
4

Composite shapes are made from simple shapes. You can find the perimeters and areas of composite shapes by starting with simple shapes for which we have known formulas.

Let's start: A fraction of a circle plus a square

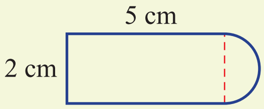
This diagram shows a fraction of a circle plus a square.

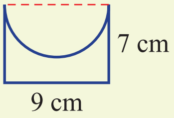
- How could you find the area and perimeter of the shape?
- Write down a full solution for finding the area and perimeter of the shape.
- See if your teacher or another student can easily follow your solution.



Key ideas

- Composite shapes are made up of more than one basic shape.
- Addition and/or subtraction can be used to find areas and perimeters of composite shapes.
  - Use addition.
  - Use subtraction (for area).



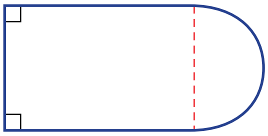


Exercise 6E

Understanding

1 Name the two different shapes that make up each of these composite shapes; e.g. square and semicircle.

a



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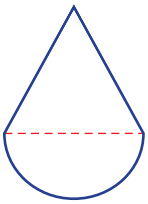

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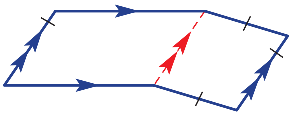
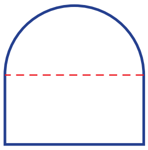


2 What fraction of a circle is used in each of these shapes?

a



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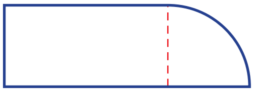

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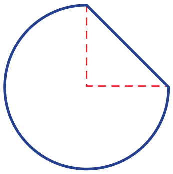

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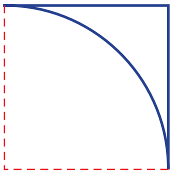


Choose from $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$.

3 This composite shape includes a square and a quadrant (i.e. $\frac{1}{4}$ circle).

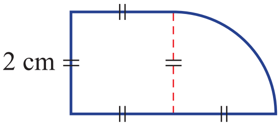
a Find the area of the square.

b Find the area of the quadrant, correct to two decimal places.

c Find the total area, correct to two decimal places.

d Find the perimeter of the entire shape, correct to two decimal places.

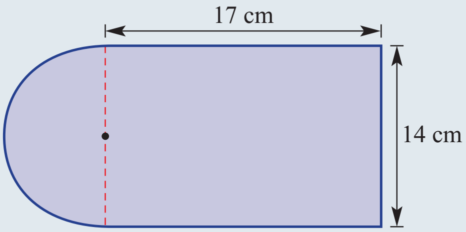
For a full circle:
$A = \pi r^2$
$C = 2\pi r$

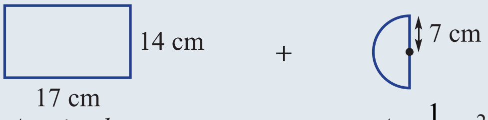


Fluency

Example 12 Finding perimeters and areas of composite shapes

Find the perimeter and area of this composite shape, rounding answers to two decimal places.



Solution	**Explanation**
$P = 3$ straight sides + curved side	
$P = 2 \times \ell + b + \frac{1}{2} \times 2\pi r$	3 straight sides ⊐ + semicircle arc (
$= 2 \times 17 + 14 + \frac{1}{2} \times 2\pi \times 7$	Recall that $C = 2\pi r$. Substitute $\ell = 17$, $b = 14$ and $r = 7$.
$= 69.99$ cm (to 2 d.p.)	Calculate and round to two decimal places.
Rectangles: $A = \ell b$ $A = 17 \times 14$ Area $= 238$ cm^2	The total area consists of a rectangle plus a semicircle.
Semicircle: $A = \frac{1}{2}\pi r^2$ $A = \frac{1}{2} \times \pi \times 7^2$ Area $= 76.969\ldots$ cm^2	 $A = \ell \times b$ $A = \frac{1}{2}\pi r^2$
Total area $= 238 + 76.969\ldots$ $= 314.97$ cm^2 (to 2 d.p.)	Round to two decimal places.

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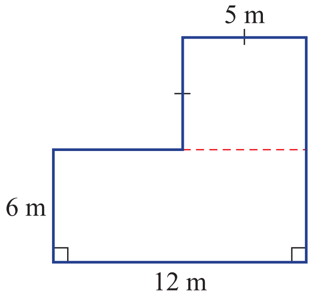
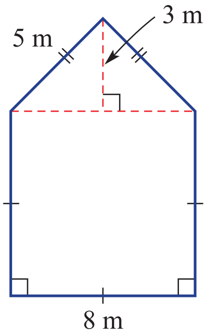
4 Find the perimeter and the area of each of these composite shapes, rounding answers to two decimal places where necessary.

a

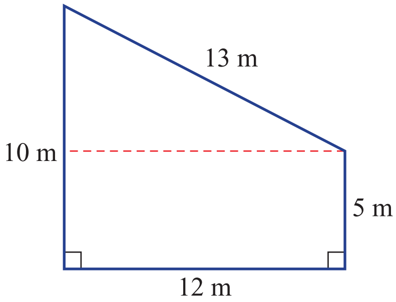


b



First decide which two shapes you will be dealing with.

c



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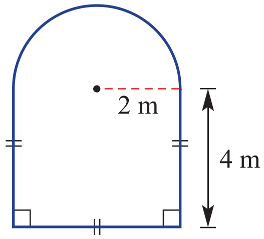

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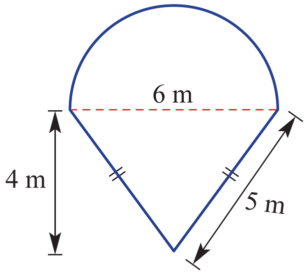

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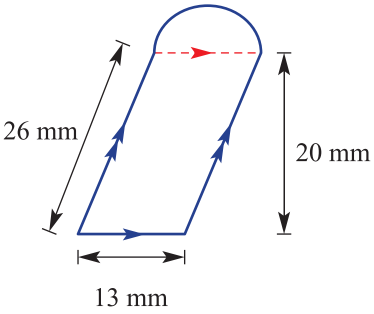
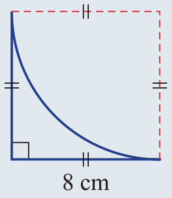


Example 13 Finding more perimeters and areas

This shape is a square with a quadrant (quarter circle) subtracted. Find its perimeter and area, correct to two decimal places.

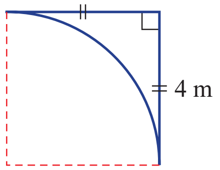


Solution	**Explanation**
$P = 2 \times \ell + \frac{1}{4} \times 2\pi r$ $= 2 \times 8 + \frac{1}{4} \times 2\pi \times 8$ $= 28.57$ cm (to 2 d.p.)	Includes 2 sides and a quarter of the circumference of a circle of radius 8 cm.
$A = s^2 - \frac{1}{4} \times \pi r^2$ $= 8^2 - \frac{1}{4} \times \pi \times 8^2$ $= 13.73$ cm^2 (to 2 d.p.)	Includes a square; subtract a quarter of the area of a circle of radius 8 cm.

5 Find the perimeters and areas of these shapes, correct to two decimal places.

Use subtraction to find the area.

a



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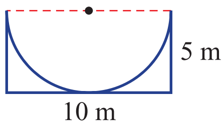

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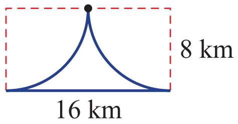
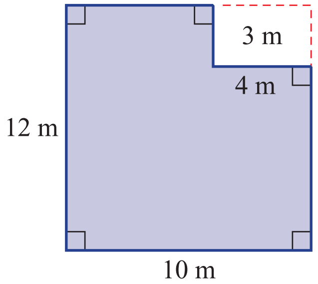


6 Find the area of the shaded part of each of these composite shapes.

Use subtraction in each case.

a



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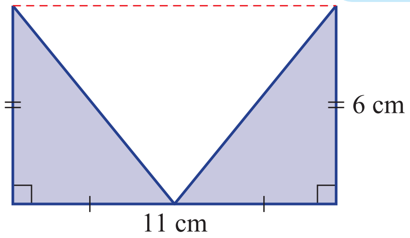

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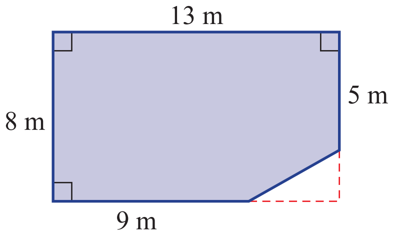

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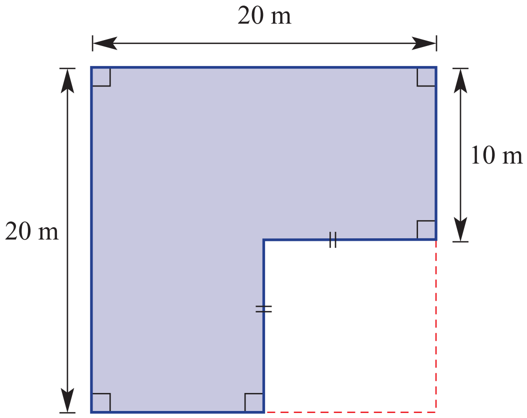

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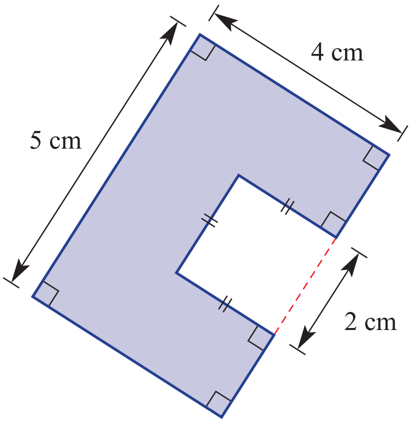

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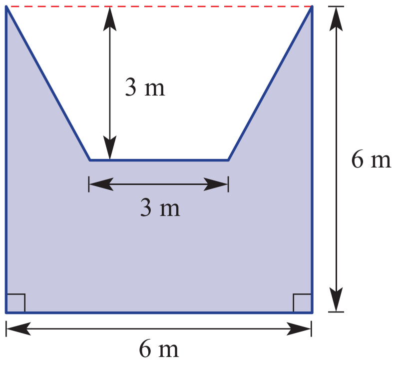

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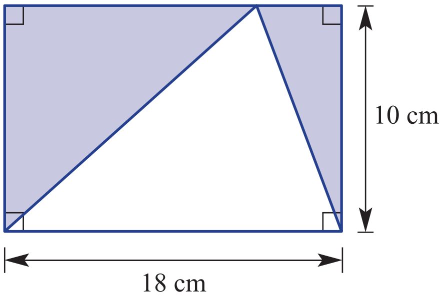
Problem-solving and Reasoning

7 An area of lawn is made up of a rectangle measuring 10 m by 15 m and a semicircle of radius 5 m. Find the total area of lawn, correct to two decimal places.

8 Twenty circular pieces of pastry, each of diameter 4 cm, are cut from a rectangular layer of pastry 20 cm long and 16 cm wide. What is the area, correct to two decimal places, of pastry remaining after the 20 pieces are removed?

9 Find the area of the shaded region of each of the following shapes by subtracting the area of the clear shape from the total area. Round to two decimal places where necessary.

a



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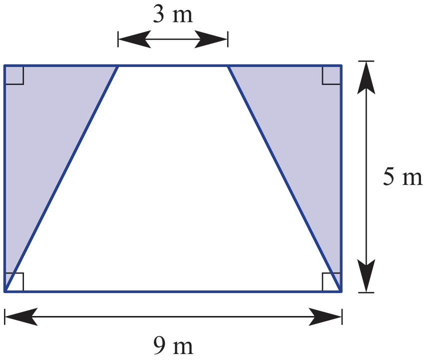

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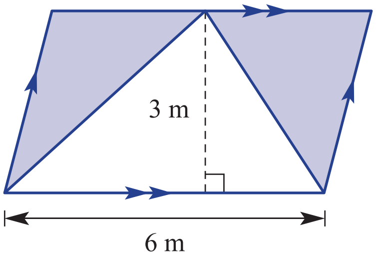

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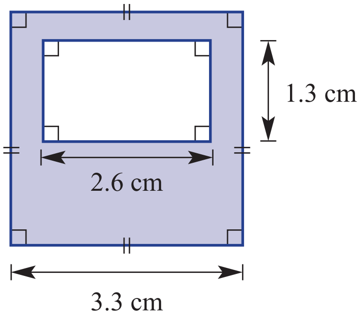

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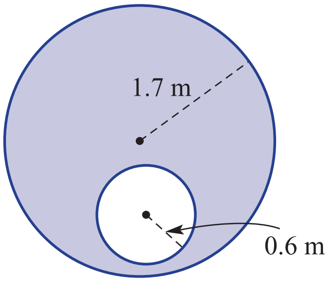

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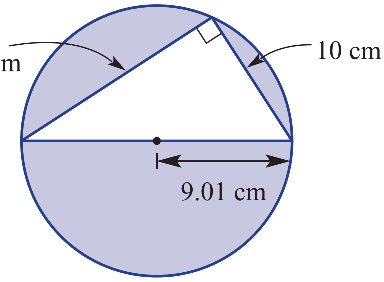
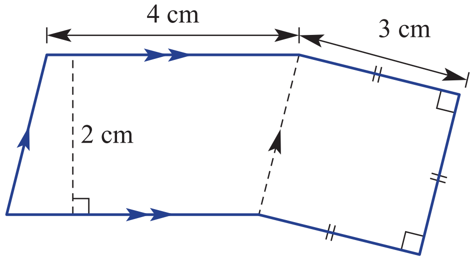


10 Find the area of each of the following composite shapes.

a



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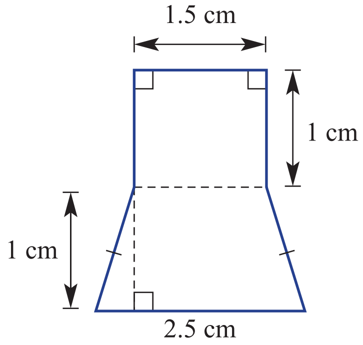

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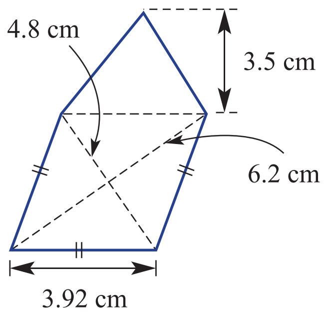
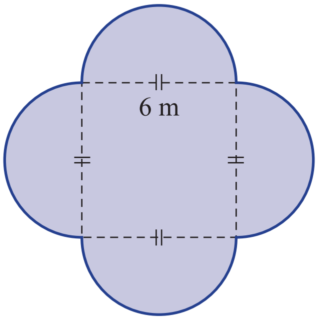


Enrichment: Circular challenges

11 Find the perimeter and the area of each of the following composite shapes, correct to two decimal places where necessary.

a



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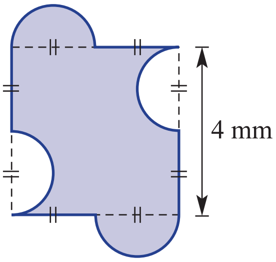

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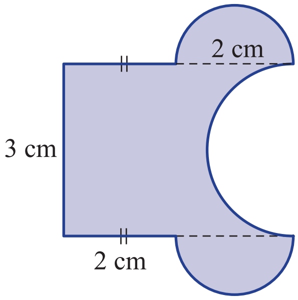

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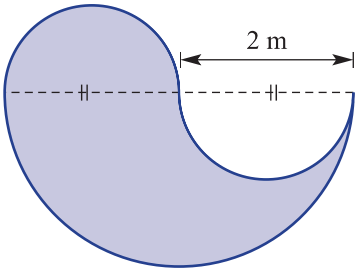

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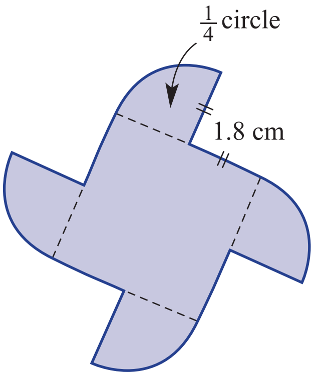

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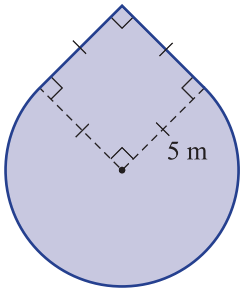

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Keeping in touch with numeracy

Non-calculator

1 What is halfway between $\frac{1}{4}$ and $\frac{1}{2}$?

3 What is $(-1)^3$?

5 How many kilograms are there in 3.25 tonnes?

7 Find:
- **a** 50% of \$400
- **b** 25% of \$400
- **c** 10% of \$400
- **d** 5% of \$400

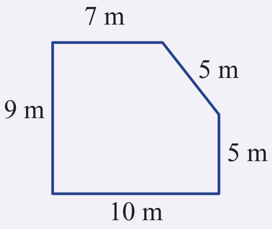
9 If the short sides of a right-angled triangle are 3 m and 4 m, how long is the hypotenuse?

11 Amelia earns \$60 per week. How much will she earn in a year?

13 Round these numbers, as indicated:
- **a** 1945, to the nearest ten
- **b** 1995, to the nearest ten
- **c** 1995, to the nearest hundred
- **d** 1995, to the nearest thousand

15 Write down five numbers that have a mean of 5 and a range of 5.

17 Consider the block of land in the diagram.
- **a** Find the perimeter.
- **b** Find the area.



19 Copy and complete:
- **a** If all the side lengths in Question **17** are doubled, the perimeter will be multiplied by ______.
- **b** If all the side lengths in Question **17** are doubled, the area will be multiplied by ______.

Calculator

2 What is halfway between $\frac{1}{3}$ and $\frac{1}{2}$?

4 What is the cube root of 1728?

6 How many minutes are there in 1 day?

8 Calculate, to the nearest 5c:
- **a** 51% of \$417
- **b** 27% of \$417
- **c** 13% of \$417
- **d** 5.3% of \$417

10 What is the exact length of the diagonal of a square with sides 5 cm?

12 Yasmin is on a salary of \$56 000 per annum. How much should she be paid each week?

14 Karen is laying pavers in her rectangular courtyard, which is 5.5 m by 4.5 m. She needs 10.9 pavers per square metre. There are 160 pavers on a pallet. How many pallets will she need?

16 Steve played nine holes of golf. His scores were: 6, 6, 5, 5, 7, 10, 6, 7 and 3.
- **a** What was his total score?
- **b** What was his mean score?
- **c** What was the range?

18 The owners of the block in Question **17** need to build a new post-and-wire perimeter fence. They need to buy a post for every corner. Along the sides they need the posts to be no more than 3 metres apart. The posts cost \$5 each and the wire costs \$7.50 per metre. How much will this cost?

20 Find the cost of laying new turf (i.e. grass) on the block of land in Question **17** if the turf costs \$9.25 per square metre.

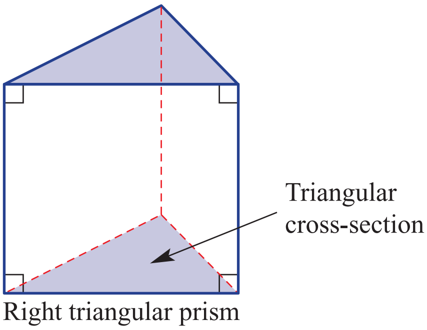
6F Surface area of prisms

Stage

5.2
5.2◊
5.1
4

Three-dimensional objects or solids have faces that form the surface area. Nets are very helpful for determining the number and shapes of the surfaces of a three-dimensional object.

For this section we will deal with right prisms. A right prism has a uniform cross-section with two identical ends and the remaining sides are rectangles.



The Flatiron Building in New York City is the shape of a triangular prism.

Let's start: Building prisms

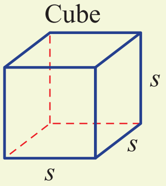
Drilling for Gold 6F1

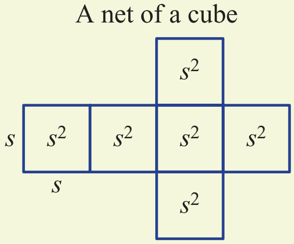
Use the 'Drilling for Gold' document to:

- Print and cut out some nets of prisms.
- Fold them into 3D solids.
- Calculate the surface area.

Key ideas

- The **surface area** of a solid is the sum of the areas of all the surfaces.
- A **net** is a two-dimensional illustration of all the surfaces of a solid.





The formula for the surface area of a cube is: $A = 6s^2$

Surface area The number of square units needed to cover the outside of a solid

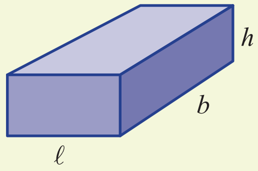
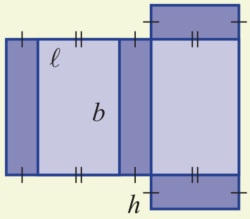
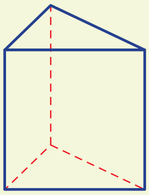
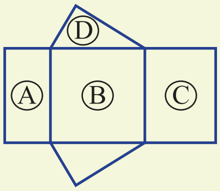
Net A diagram showing how the plane faces of a solid are joined to each other

ISBN 978-1-316-61816-5 Stuart Palmer et al. 2016 Cambridge University Press

- A **right prism** is a solid with a uniform cross-section and with remaining sides as rectangles.
  - Prisms are named by the shape of their cross-section.

Right prism
A solid with a uniform cross-section, and remaining sides are rectangles

- The nets for a rectangular prism and a triangular prism are shown here.

Solid	Net	Surface area
Rectangular prism  	 	$A = 2(\ell b + bh + \ell h)$
Triangular prism 	 	A = area A + area B + area C + 2 × area D

Exercise 6F

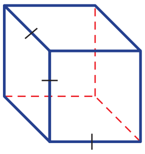
Understanding

1 How many faces do the following solids have?

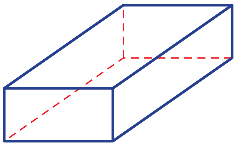
a rectangular prism **b** cube **c** triangular prism

2 Draw a net for these prisms and name each solid.

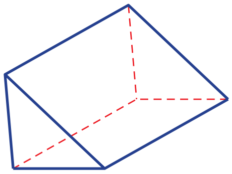
a



b

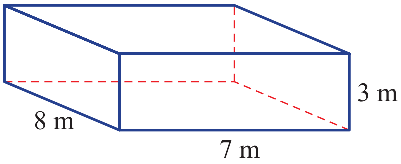


c



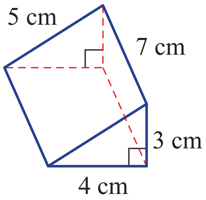
3 Copy and complete the working to find the surface area of these solids.

a



$A = 2\times(\_\_\times\_\_+\_\_\times\_\_+\_\_\times\_\_)$

b

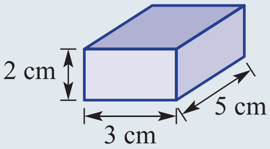


$A = \_\_\times\_\_+\_\_\times\_\_+\_\_\times\_\_+2\times\left(\frac{1}{2}\times\_\_\times\_\_\right)$

Fluency

Example 14 Finding the surface area of a rectangular prism

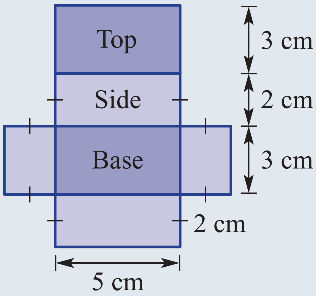
Find the surface area of this rectangular prism.



Solution

$$\begin{aligned} A &= 2\times(3\times5+2\times5+2\times3)\\ &= 2\times31\\ &= 62\text{ cm}^2 \end{aligned}$$

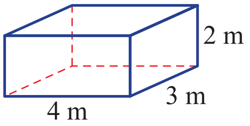
Explanation



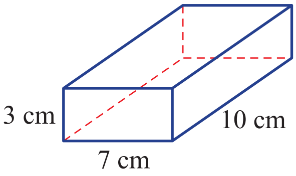
4 Find the surface area of each of the following rectangular prisms. Draw a net of each solid to help you.

Add up all the pairs of rectangles using $A = \ell b$.

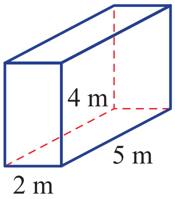
a



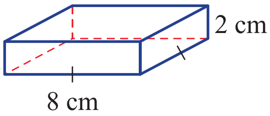
b



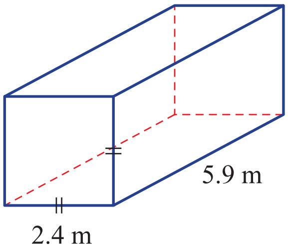
c



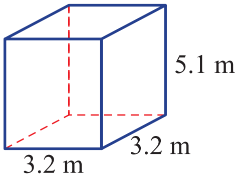
d



e

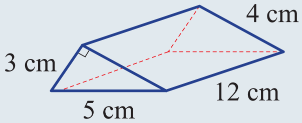


f



Example 15 Finding the surface area of a right triangular prism

Find the surface area of this triangular prism.

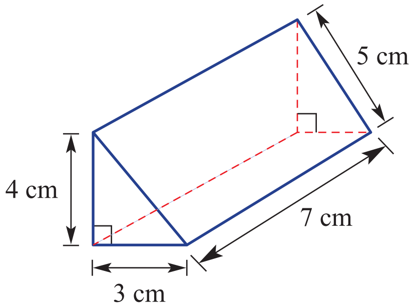


Solution	Explanation
$A = 12 \times 5 + 12 \times 4 + 12 \times 3 + 2 \times (\frac{1}{2} \times 4 \times 3)$ $= 60 + 48 + 36 + 12$ $= 156 \text{ cm}^2$	The surface is made up of three rectangles and two identical triangles. Use 3 cm and 4 cm for the base and height, respectively, of the triangles.

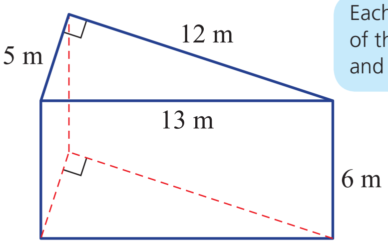
5 Find the surface area of each of the following triangular prisms.

Each one consists of three rectangles and two triangles.

a

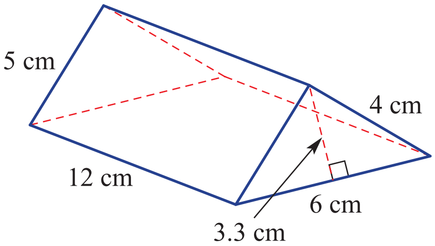


b

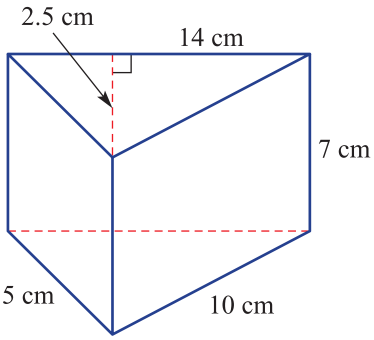


Skillsheet 6B

c



d



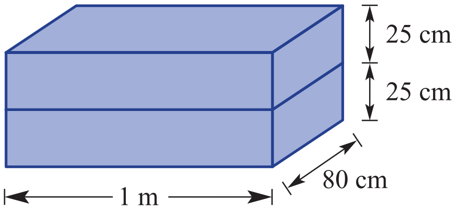
6 Find the surface area of a cube with side length 1 metre.

Problem-solving and Reasoning

7 A rectangular box is to be covered in material. How much material is required to cover the entire exterior of the box if it has the dimensions 1.3 m, 1.5 m and 1.9 m?

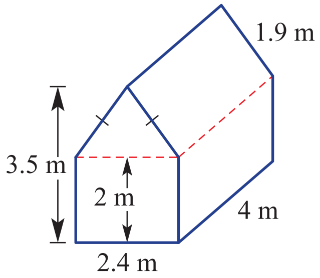
8 Two wooden boxes, both with dimensions 80 cm, 1 m and 25 cm, are placed on the ground, one on top of the other, as shown. The entire outside surface, including the underside of the bottom box, is then painted. Find the area of the painted surface, in cm^2.

Only include the outside surfaces that you could paint.

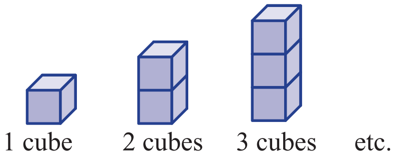


9 The four walls and roof of a barn are to be painted.

- **a** Find the surface area of the barn, not including the floor.
- **b** If 1 litre of paint covers 10 m^2, find how many litres are required to complete the job.



10 Cubes of side length 1 unit are stacked as shown.



- **a** Complete this table.

Number of cubes (n)	1	2	3	4	5	6	7	8	9
Surface area (A)									

- **b** Can you find the rule for the surface area (A) for n cubes stacked in this way? Write down the rule for A in terms of n.
- **c** Use your rule to find the surface area if there are 100 cubes.

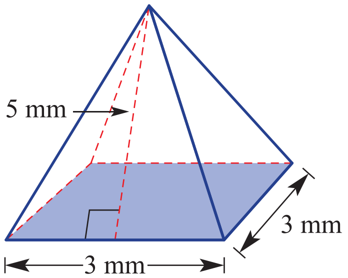
An example of a rule is $A = 2n + 5$, but this is not the rule for this question.

Enrichment: Pyramids

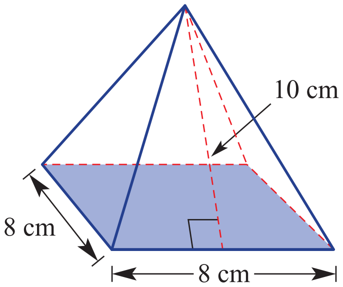
11 Pyramids consist of a base and a number of triangular faces. The surface area can be calculated by adding all the face areas, similar to that of prisms. Remember that the area of a triangle is given by $A = \frac{1}{2}bh$.

Find the surface area of each of these pyramids. Draw a net of each solid to help you.

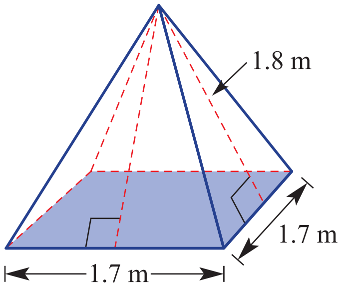
a



b



c



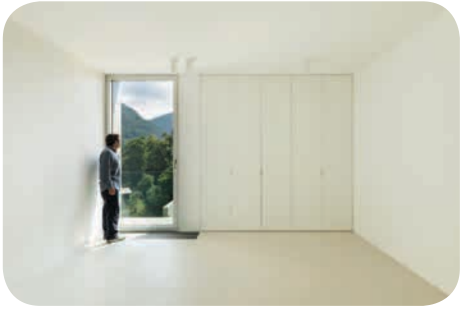
6G Volume of prisms

Stage
5.2
5.2◊
5.1
4

We use cubic units to describe the amount of space inside a three-dimensional object, such as:

- cubic kilometres for the volume of the Earth
- cubic metres for the space inside a room
- cubic centimetres for the volume of space occupied by this book
- cubic millimetres for the volume of metal in a pin.

Units for capacity (millilitres, litres, kilolitres and megalitres) are used for liquids and gases.

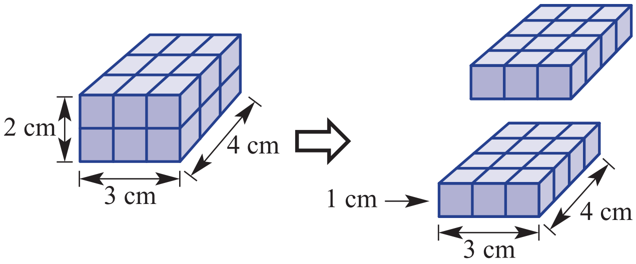


Let's start: Why length × breadth × height?

For most people, the first thing that comes to mind when dealing with volume is length × breadth × height. This rule only applies to finding the volume of rectangular prisms.

Let's look at a rectangular prism split into two layers.

- How many cubes sit on one layer?
- What is the area of the base? What do you notice?
- What is the height and how many layers are there?
- Why is the volume rule given by $V = \ell bh$ in this case?



Key ideas

Drilling for Gold 6G1

- Common metric units for **volume** include cubic kilometres (km^3), cubic metres (m^3), cubic centimetres (cm^3) and cubic millimetres (mm^3).

$$1000^3 = 1\,000\,000\,000$$
$$100^3 = 1\,000\,000$$
$$10^3 = 1000$$

$$\begin{tikzcd} \text{km}^3 \arrow[r, bend left, "\times 1000^3"] & \text{m}^3 \arrow[l, bend left, "\div 1000^3"] \arrow[r, bend left, "\times 100^3"] & \text{cm}^3 \arrow[l, bend left, "\div 100^3"] \arrow[r, bend left, "\times 10^3"] & \text{mm}^3 \arrow[l, bend left, "\div 10^3"] \end{tikzcd}$$

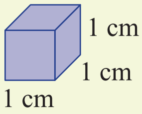
- For **capacity**, common units include:
  - megalitres (ML) 1 ML = 1000 kL
  - kilolitres (kL) 1 kL = 1000 L
  - litres (L) 1 L = 1000 mL
  - millilitres (mL)

$$\begin{tikzcd} \text{ML} \arrow[r, bend left, "\times 1000"] & \text{kL} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{L} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{mL} \arrow[l, bend left, "\div 1000"] \end{tikzcd}$$

Volume The amount of three-dimensional space inside an object

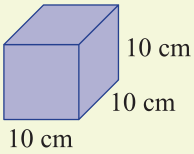
Capacity The amount of liquid a container can hold

- A cubic centimetre holds 1 mL of liquid.



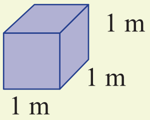
1 cm^3 holds 1 mL.

- This container holds 1 litre of liquid.



1000 cm^3 holds 1 L.

- A cubic metre holds 1000 litres or 1 kilolitre.

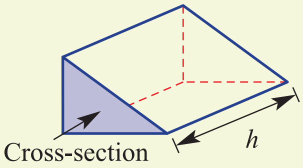


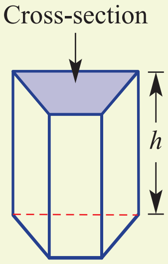
1 m^3 holds 1000 L or 1 kL.

- Volume of solids with a uniform **cross-section** is equal to area of cross-section (A) × height (h).

$V = Ah$

Cross-section The plane figure formed when you slice a solid figure parallel to one of its surfaces

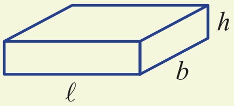




- The 'height' is the length of the edge that runs perpendicular to the cross-section in any solid.

- Volume of a rectangular prism:

$V = \ell bh$

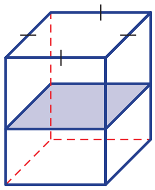


Exercise 6G

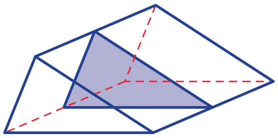
Understanding

1 What is the name given to the shape of the shaded cross-section of each of the following solids?

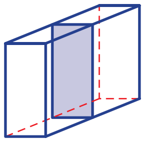
a



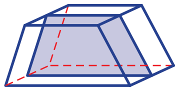
b



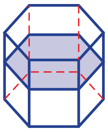
c



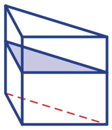
d



e



f



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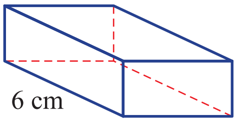
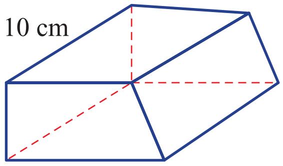
2 Draw the cross-sectional shape for each of these prisms and state the given 'height' (perpendicular to the cross-section).

'Perpendicular' means 'at a right angle (90°)'.

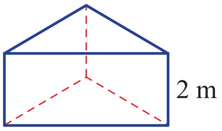
a



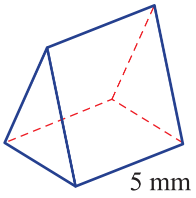
b



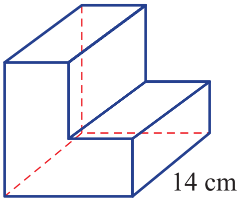
c



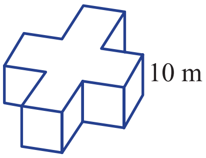
d



e



f



3 Write the missing number.

a The number of mm in 1 cm is _______________.

b The number of mm^2 in 1 cm^2 is _______________.

c The number of mm^3 in 1 cm^3 is _______________.

d There are _______________ cm^3 in 1 m^3.

e There are _______________ m^3 in 1 km^3.

f There are _______________ mL in 1 L.

g There are _______________ L in 1 kL.

h There are _______________ cm^3 in 1 mL.

Choose from:
1 000 000
100
10
1000
1
1 000 000 000

Fluency

Example 16 Converting units of volume

Convert the following volume measurements to the units given in the brackets.

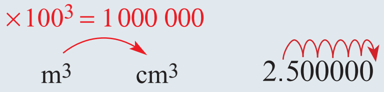
a 2.5 m^3 (cm^3) **b** 458 mm^3 (cm^3)

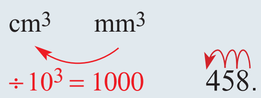
Solution

a $2.5\ m^3 = 2.5 \times 100^3\ cm^3$
$= 2\,500\,000\ cm^3$

b $458\ mm^3 = 458 \div 10^3\ cm^3$
$= 0.458\ cm^3$

Explanation





4 Convert the following volume measurements to the units given in brackets.

a 3 cm^3 (mm^3) **b** 0.3 cm^3 (mm^3) **c** 2000 mm^3 (cm^3)

d 0.001 m^3 (cm^3) **e** 8.7 m^3 (cm^3) **f** 5900 cm^3 (m^3)

g 0.00001 km^3 (m^3) **h** 21 700 m^3 (km^3) **i** 430 000 cm^3 (m^3)

$1\ km^3 = 1000^3\ m^3$
$1\ m^3 = 100^3\ cm^3$
$1\ cm^3 = 10^3\ mm^3$

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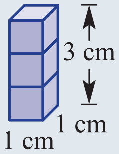
5 Convert these units of capacity to the units given in brackets.

a 3 L (mL)
b 0.2 kL (L)
c 3500 mL (L)
d 0.021 L (mL)
e 37 000 L (kL)
f 42 900 kL (ML)
g 2 cm^3 (mL)
h 2 L (cm^3)
i 1 m^3 (L)

1 ML = 1000 kL
1 kL = 1000 L
1 L = 1000 mL

Example 17 Finding the volume of a rectangular prism

Find the volume of this rectangular prism.

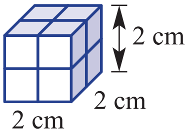


Solution	**Explanation**
Volume = ℓbh $= 1 \times 1 \times 3$ $= 3\ \text{cm}^3$	The solid is a rectangular prism. Length = 1 cm, breadth = 1 cm and height = 3 cm.

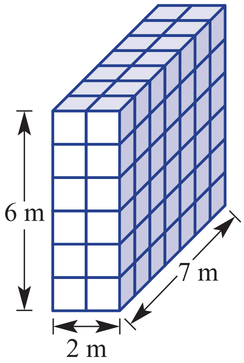
Drilling for Gold 6G2 at the end of this section

6 Find the volume of these three-dimensional rectangular prisms.

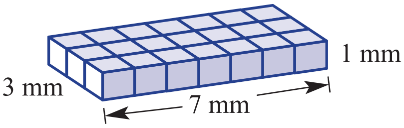
a



b

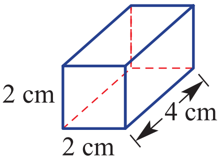


c

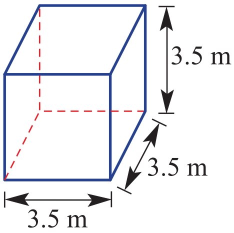


7 Find the volume of each of these rectangular prisms.

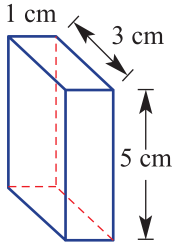
a



b



c



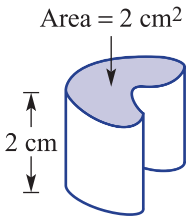
Drilling for Gold 6G3

Skillsheet 6C

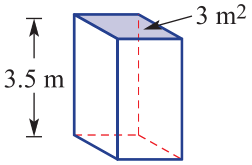
8 Find the volume of each of these three-dimensional objects. The cross-sectional area has been given.

Simply use $V = Ah$, since the area of the cross-section is given.

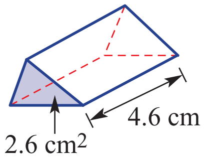
a



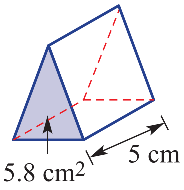
b



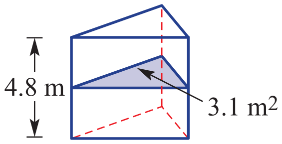
c



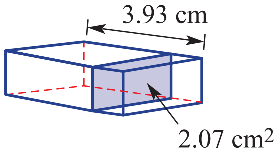
d



e

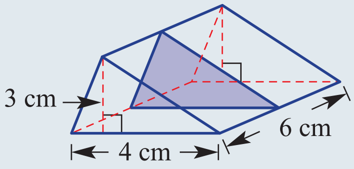


f



Example 18 Finding the volume of a triangular prism

Find the volume of this triangular prism.



Solution

Area of cross-section $= \frac{1}{2}bh$

$= \frac{1}{2} \times 4 \times 3$

$= 6$ cm^2

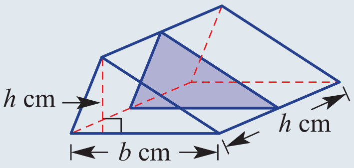
$V = Ah$

$= 6 \times 6$

$= 36$ cm^3

Explanation

The cross-section is a triangle. Note: h = height of triangle.

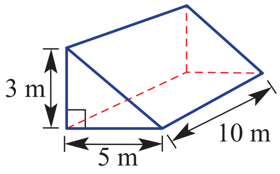


Multiply the area of the cross-section by the height of the prism.

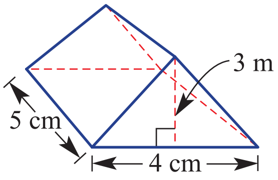
9 Find the volumes of these prisms.

First find the area of the triangular cross-section.

a



b



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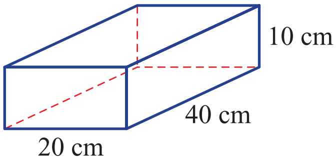
Problem-solving and Reasoning

10 A brick is 10 cm wide, 20 cm long and 8 cm high. How much space (volume) would five of these bricks occupy?

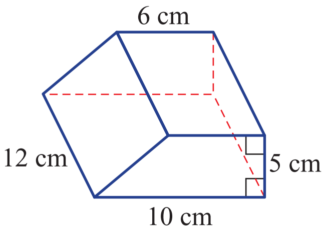
11 25 L of water is poured into a rectangular fish tank that is 50 cm long, 20 cm wide and 20 cm high. Will it overflow?

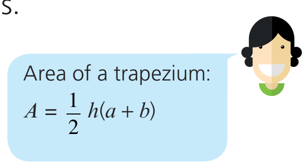
12 Find the volumes of these solids, converting your answers to litres.

a



b

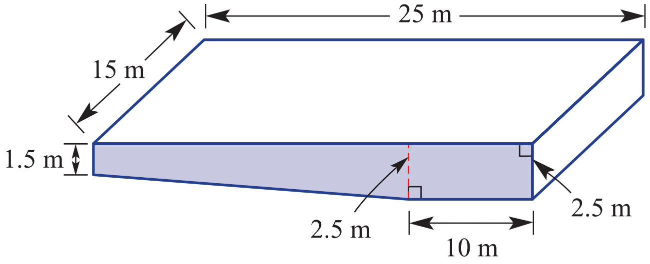




13 This diagram is a sketch of a new 25 m swimming pool to be installed in a school sports complex.

a Find the area of one side of the pool (shaded).

b Find the volume of the pool, in litres. Use $1\ \text{m}^3 = 1000$ L.



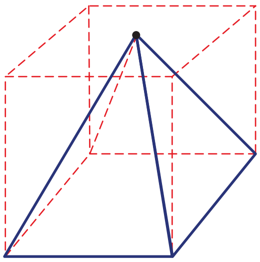
Enrichment: Volume of a pyramid

14 Someone tells you that the volume of a pyramid is half of the volume of a rectangular prism with the same base. Do you think this is true?

a Make an educated guess as to what fraction of the prism's volume is the pyramid's volume.

b Use the internet to find the actual answer to part **a**.

c Draw some pyramids and find their volume, using the results from part **b**.

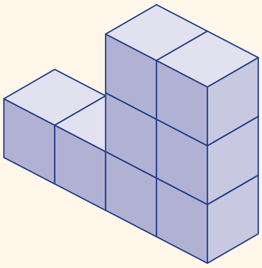


Drilling for Gold exercise

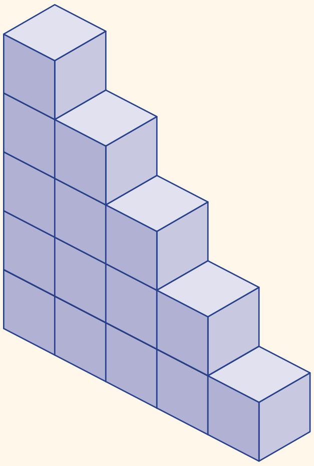
6G2: How many cubes?

How many cubes are in each diagram?
Write your answers on the worksheet or in your exercise book.

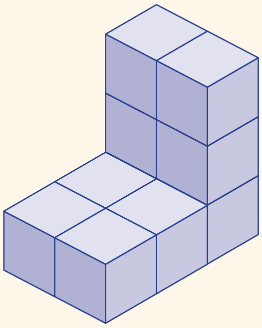
1



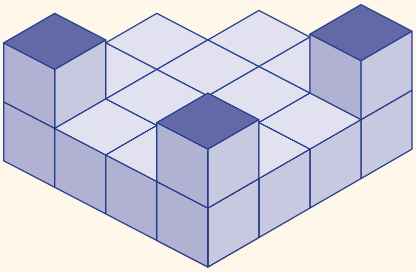
2



3



4



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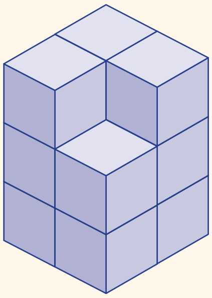
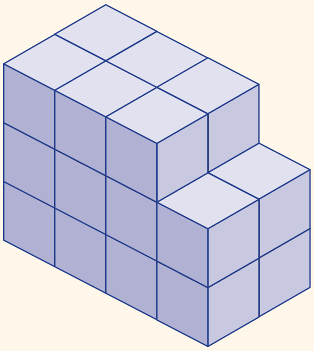
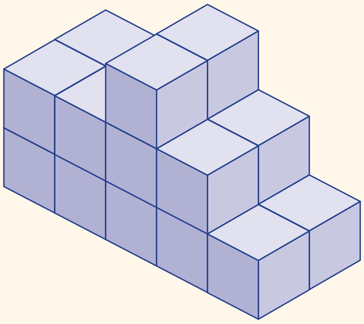


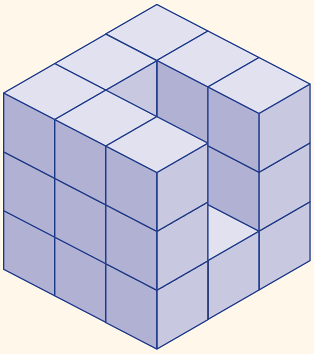
6



7



8



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6H Volume of cylinders

Stage
5.2
5.2◊
5.1
4

A cylinder has uniform cross-section (a circle), so its volume can be calculated in a similar way to that of a prism. Cylindrical objects are often used to store gases and liquids, so working out the volume of a cylinder is an important measurement calculation.


The volume of liquid in this tanker can be estimated using the volume formula for a cylinder.

Let's start: Five glasses

Drilling for Gold 6H1

Find five cylindrical drinking glasses that are roughly the same size, but some are tall and thin and others are short and wide.

Label them randomly A, B, C, D, E.

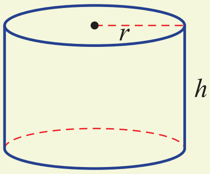
Everybody in your class is required to order the five letters from smallest volume to largest.

Key ideas

- The formula for the volume of a cylinder is:

$$V = \pi r^2 h$$

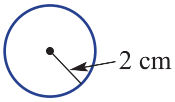
where r is the radius of the circular ends and
h is the length or distance between the circular ends.

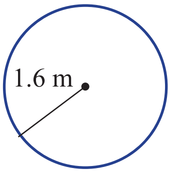


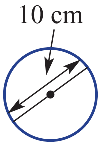
Exercise 6H

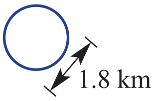
Understanding

1 Find the area of each of these circles, correct to two decimal places.

a


b


c


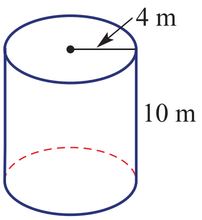
d


2 Convert the following to the units given in the brackets. Remember: 1 L = 1000 cm^3 and 1 m^3 = 1000 L.

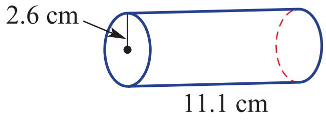
a 2000 cm^3 (L)
b 4.3 cm^3 (mL)
c 3.7 L (cm^3)
d 1 m^3 (L)
e 38 000 L (m^3)
f 0.0002 m^3 (mL)

3 State the radius (r) and the height (h) of these cylinders.

a

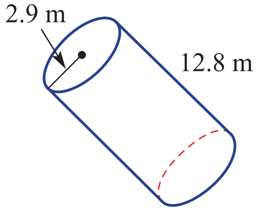


b

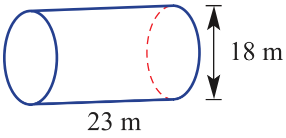


The height is perpendicular (at 90°) to the circular ends.

c

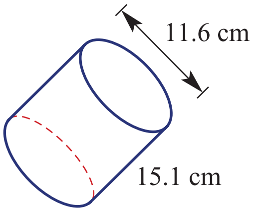


d

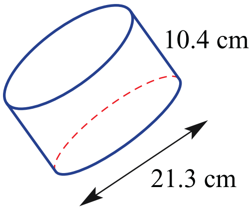


Skillsheet 6D

e



f

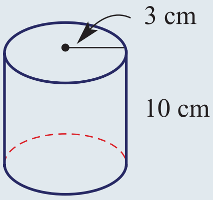


Fluency

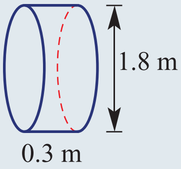
Example 19 Finding the volume of a cylinder

Find the volumes of these cylinders, correct to two decimal places.

a



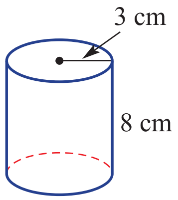
b



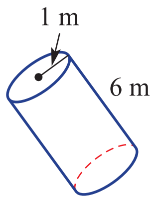
Solution	Explanation
a $V = \pi r^2 h$ $= \pi \times (3)^2 \times 10$ $= 90\pi$ cm^3 $= 282.74$ cm^3 (to 2 d.p.)	Substitute $r = 3$ and $h = 10$ into the formula.
b $V = \pi r^2 h$ $= \pi \times (0.9)^2 \times 0.3$ $= 0.7634\ldots$ $= 0.76$ m^3 (to 2 d.p.)	The diameter is 1.8 m so $r = 0.9$. The distance between the ends is 0.3 m, so $h = 0.3$.

4 Find the volumes of these cylinders, correct to two decimal places.

a

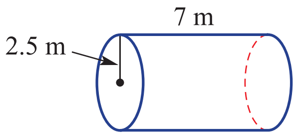


b

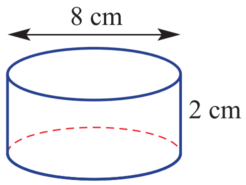


In parts **d**, **e** and **f**, divide the diameter by 2 to get the radius.

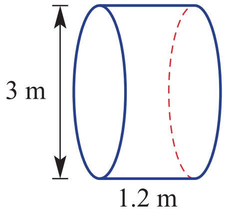
c



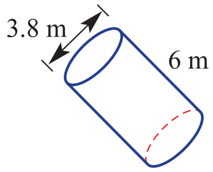
d



e



f



Problem-solving and Reasoning

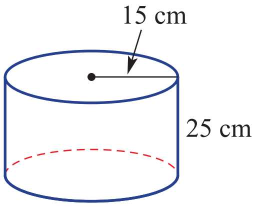
Example 20 Finding the capacity of a cylinder

Find the capacity, in litres, of a cylinder with radius 30 cm and height 90 cm. Round your answer to the nearest litre.

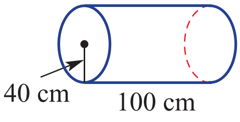
Solution	**Explanation**
$V = \pi r^2 h$	
$= \pi \times (30)^2 \times 90$	Substitute $r = 30$ and $h = 90$.
$= 81\,000\pi$	
Volume $= 254\,469.004\ldots$ cm^3	
Capacity $= 254\,469.004\ldots$ mL	Every cubic cm holds 1 mL.
$= 254.469\ldots$ L	There are 1000 cm^3 in 1 L, so divide by 1000.
$= 254$ L (to nearest litre)	

5 Find the capacity, in litres, of each of these cylinders. Round to the nearest litre.

a

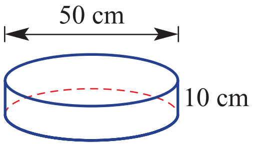


b

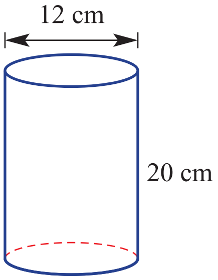


First work out the volume in cm^3. Every 1000 cm^3 holds 1 litre.

c



d



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6 A cylindrical storage drum has a radius of 0.5 m and a height of 2 m.

a Find its volume, in m^3, correct to three decimal places.

b Find its capacity, in L, correct to the nearest litre (1 m^3 = 1000 L).

7 Which has a greater capacity: a 10 cm by 10 cm by 10 cm cube or a cylinder with radius 6 cm and height 10 cm?

8 A cylindrical water tank has a radius of 2 m and a height of 2 m.

a Find its volume, in m^3, rounded to three decimal places.

b Find its capacity, in L, rounded to the nearest litre.

9 How many litres of gas can a tanker carry if its tank is cylindrical with a 2 m diameter and is 12 m in length? Round to the nearest litre.

1 m^3 = 1000 L

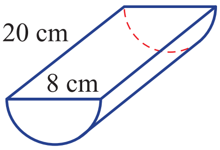
10 Draw a cylinder with its circumference equal to its height. Try to draw it to scale.

Enrichment: Cylinder portions

11 Find the volumes of these cylindrical portions, correct to two decimal places.

First determine what fraction of a cylinder is involved. In part **f**, subtract the 'missing' volume.

a



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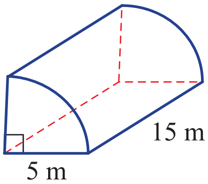
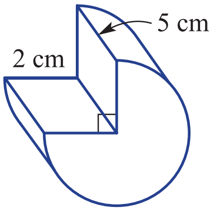


c



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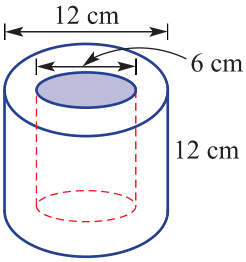


Rainwater tanks

Maths@work

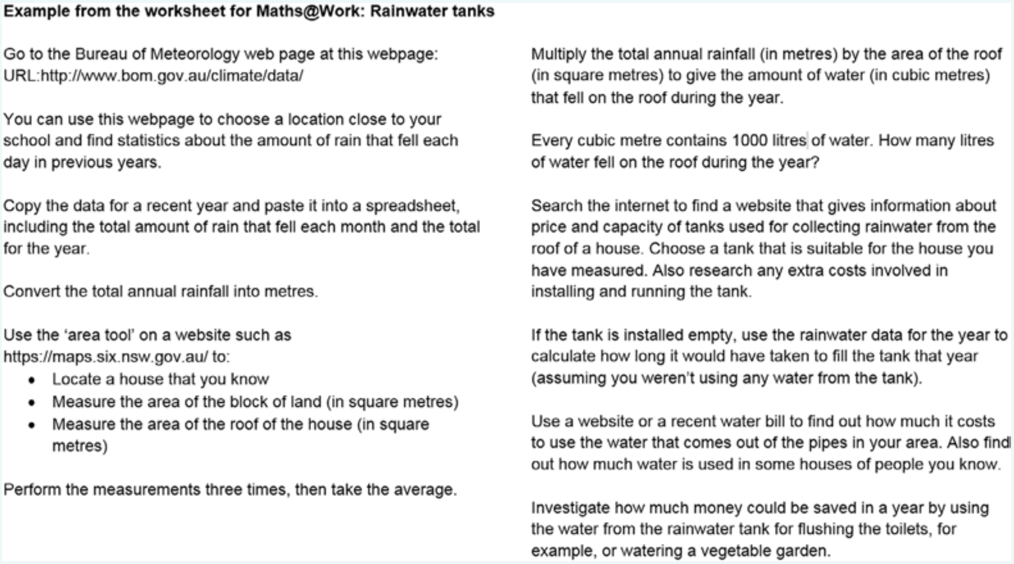
Worksheet 6A

In this activity you will investigate the feasibility of installing and connecting a rainwater tank to a building, possibly your house or your school. The intention is that you will be using the rainwater for various purposes, such as watering a vegetable patch, fighting bushfires or for flushing toilets.

You will:

- Use a website to collect data about the frequency and amount of rain in the area of the building.
- Research the cost and capacity of tanks that would be appropriate for the site.
- Research the costs involved with installing and maintaining the tank.
- Estimate how long it would take to fill your tank.
- Make predictions about how long your tank will provide water for the intended use.

Download the worksheet from the interactive textbook and start planning!



Example from the worksheet for Maths@Work: Rainwater tanks

Go to the Bureau of Meteorology web page at this webpage:
URL:http://www.bom.gov.au/climate/data/

You can use this webpage to choose a location close to your school and find statistics about the amount of rain that fell each day in previous years.

Copy the data for a recent year and paste it into a spreadsheet, including the total amount of rain that fell each month and the total for the year.

Convert the total annual rainfall into metres.

Use the 'area tool' on a website such as https://maps.six.nsw.gov.au/ to:

- Locate a house that you know
- Measure the area of the block of land (in square metres)
- Measure the area of the roof of the house (in square metres)

Perform the measurements three times, then take the average.

Multiply the total annual rainfall (in metres) by the area of the roof (in square metres) to give the amount of water (in cubic metres) that fell on the roof during the year.

Every cubic metre contains 1000 litres of water. How many litres of water fell on the roof during the year?

Search the internet to find a website that gives information about price and capacity of tanks used for collecting rainwater from the roof of a house. Choose a tank that is suitable for the house you have measured. Also research any extra costs involved in installing and running the tank.

If the tank is installed empty, use the rainwater data for the year to calculate how long it would have taken to fill the tank that year (assuming you weren't using any water from the tank).

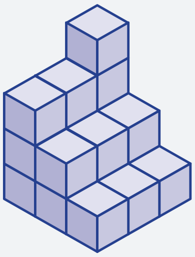
Use a website or a recent water bill to find out how much it costs to use the water that comes out of the pipes in your area. Also find out how much water is used in some houses of people you know.

Investigate how much money could be saved in a year by using the water from the rainwater tank for flushing the toilets, for example, or watering a vegetable garden.

Puzzles and games

1 These towers are made by stacking cubes. How many cubes are needed in each case?

a



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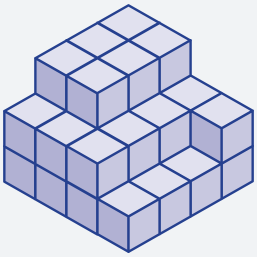
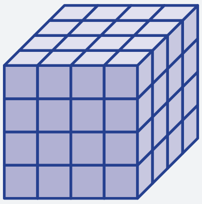
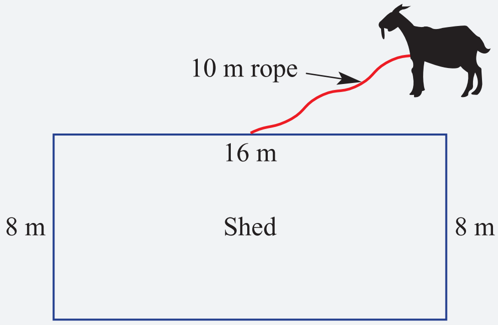


2 A large cube of side length 4 cm is painted and then cut into 64 single 1 cm cubes. How many 1 cm cubes are not painted on any face?

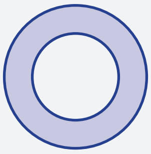


3 To the nearest metre, how far will a wheel of diameter 1 m travel after 100 revolutions?

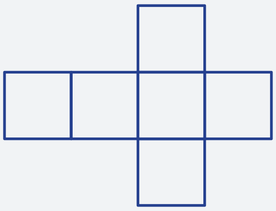
4 A goat is tethered to the centre of one side of a shed with a 10 m length of rope. In what area of grass can the goat graze?



5 A circle of radius 10 cm has a hole cut out of its centre to form a ring. Find the radius of the hole if the remaining area is 50% of the original area. Round your answer to one decimal place.



6 Here is one net for a cube. How many different nets are possible? Do not count nets that can be rotated or reflected to give another net.



7 A factory has a flat roof that has an area of 100 m^2. All the water that collects on the roof is fed into a rainwater tank. If there is 1 mm of rainfall, how many litres of water go into the tank?

Chapter summary

Length, area, surface area and volume

Length and perimeter

Circle circumference

$C = 2\pi r$ or $C = \pi d$

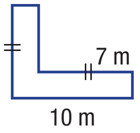
Units of length

×1000 ×100 ×10

km m cm mm

÷1000 ÷100 ÷10

Perimeter



$P = 2 \times 10 + 2 \times 7$

$= 34$ m

Volume

Units of volume

$\times 1000^3$ $\times 100^3$ $\times 10^3$

km^3 m^3 cm^3 mm^3

$\div 1000^3$ $\div 100^3$ $\div 10^3$

1 L = 1000 mL = 1000 cm^3

Prisms and cylinders

$V = Ah$

A = area of cross-section

h = height

$V = \ell bh$

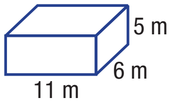
$= 11 \times 6 \times 5$

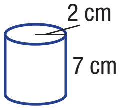
$= 330$ m^3

$V = \pi r^2 h$

$= \pi(2)^2 \times 7$

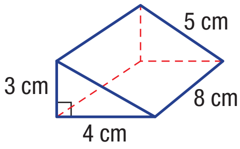
$= 87.96$ cm^3





Surface area

Prisms



A = 2 identical triangles + 3 rectangles

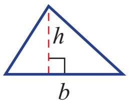
$= 2 \times \frac{1}{2} \times 4 \times 3 + 8 \times 5 + 8 \times 4 + 8 \times 3$

$= 108$ cm^2

Area

Triangle

$A = \frac{1}{2} bh$



Quadrilaterals

Square $A = s^2$

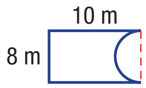
Rectangle $A = \ell b$

Parallelogram $A = bh$

Trapezium $A = \frac{1}{2}h(a + b)$

Rhombus or kite $A = \frac{1}{2}xy$

Composite shapes



$A = \ell b - \frac{1}{2}\pi r^2$

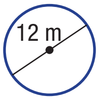
$= 10 \times 8 - \frac{1}{2}\pi \times 4^2$

$= 54.87$ m^2

$P = 2 \times 10 + 8 + \frac{1}{2} \times 2\pi(4)$

$= 40.57$ m

Circle



$A = \pi r^2$

$A = \pi \times 6^2$

$= 113.10$ m^2

$d = 12$ m

$\therefore r = 6$ m

Units of area

$\times 1000^2$ $\times 100^2$ $\times 10^2$

km^2 m^2 cm^2 mm^2

$\div 1000^2$ $\div 100^2$ $\div 10^2$

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Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.

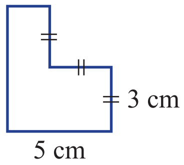
Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

1 If the area of a square field is 25 km^2, the length of one of its sides is:
A 10 km **B** 5 km **C** 4 km **D** 50 km **E** 25 km

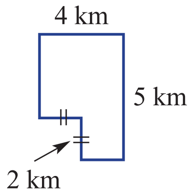
2 If 1 square metre is 10 000 square centimetres, then 2.7 square metres is:
A 270 cm^2 **B** 0.0027 km^2 **C** 27 000 cm^2 **D** 2700 mm^2 **E** 27 cm^2

3 The perimeter of this shape is:
A 21 cm
B 14 cm
C 24 cm
D 20 cm
E 22 cm

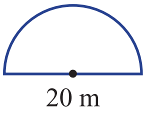


4 A parallelogram has area 10 m^2 and base 5 m. Using $A = bh$, its perpendicular height is:
A 50 m **B** 2 m **C** 50 m^2 **D** 2 m^2 **E** 0.5 m

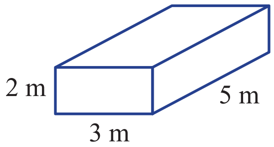
5 This shape is a rectangle with a square removed. The shape's area is:
A 16 km^2
B 12 km^2
C 20 km^2
D 24 km^2
E 6 km^2



6 A semicircular goal area has diameter 20 m. Its perimeter, correct to the nearest metre, is:
A 41 m
B 36 m
C 83 m
D 51 m
E 52 m



7 The surface area of the rectangular prism shown is:
A 30 m^2
B 62 m^2
C 31 m^2
D 60 m^2
E 100 m^2



8 If the perimeter of a rectangle is 20 m, the largest possible area is:
A 5 m^2 **B** 15 m^2 **C** 25 m^2 **D** 35 m^2 **E** 45 m^2

9 A prism's cross-sectional area is 100 m^2. The volume is 6500 m^3. The prism's height is:
A 0.65 m **B** 650 000 m **C** 65 m **D** 6.5 m **E** 650 m

10 The volume of a cylinder with radius 3 cm and height 10 cm is closest to:
A 188 cm^2 **B** 251 cm **C** 141 cm^3 **D** 94 cm^3 **E** 283 cm^3

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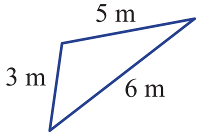
Short-answer questions

1 Convert the following measurements to the units given in brackets.

a 3.8 m (cm)
b 1.27 km (m)
c 48 mm (cm)
d 273 mm^2 (cm^2)
e 5.2 m^2 (cm^2)
f 0.01 m^3 (cm^3)
g 53 100 mm^3 (cm^3)
h 3100 mL (L)
i 0.043 L (mL)
j 2.83 kL (L)
k 4000 cm^3 (L)
l 1 m^3 (L)

2 Find the perimeter of each of the following shapes.

a



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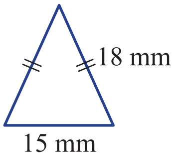
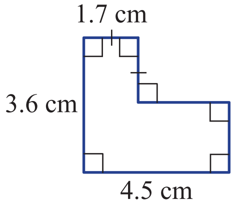
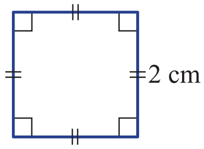


c



3 Find the area of each of the following plane figures.

a



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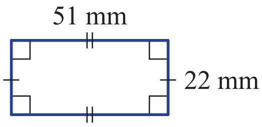

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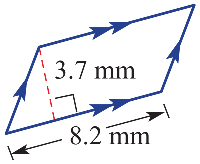

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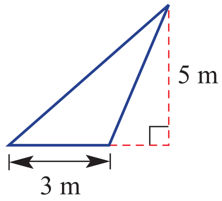
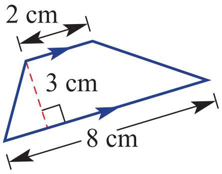
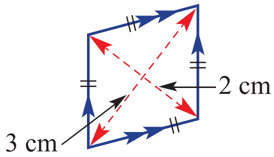


e

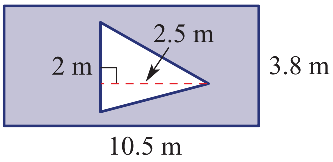


f



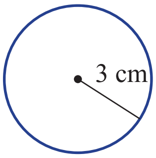
4 Inside a rectangular lawn area of length 10.5 m and breadth 3.8 m, a new garden bed is to be constructed. The garden bed is to be the shape of a triangle with base 2 m and height 2.5 m. Find the area of the:

a garden bed
b lawn remaining around the garden bed



5 Find the area and circumference/perimeter of each of the following shapes, correct to two decimal places.

a



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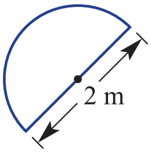

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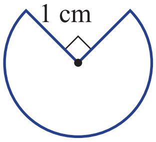
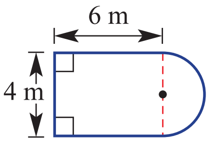


6 Find the perimeter and area of each of the following composite shapes, correct to two decimal places.

a



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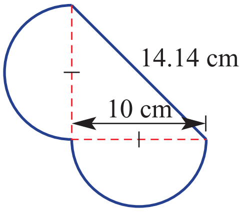
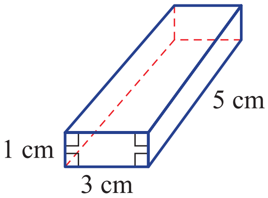


7 Find the surface area of each of the following solid objects.

a



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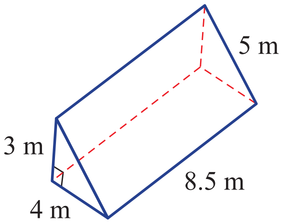
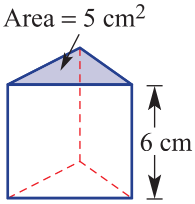


8 Find the volume of each of these solid objects, rounding to two decimal places where necessary.

a



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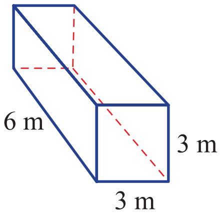

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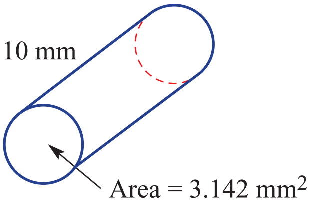

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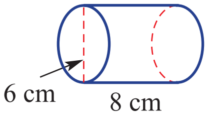
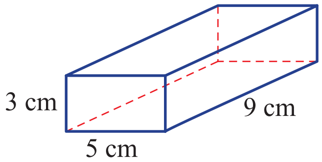


Extended-response questions

1 A kindergarten teacher collects some blocks of wood for a painting activity. Each block is a rectangular prism, as shown.



a Find the volume of each block.

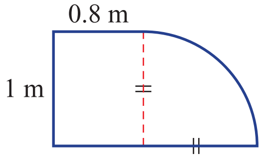
b For each block, find the total outside area to be painted.

c If the paint costs $2.50 per 100 cm^2, find the cost of painting 10 blocks.

d Another wood block is a cylinder with radius 2.2 cm and height 9 cm.

- **i** Which block has the greatest volume?
- **ii** By how much? (Give your answer to one decimal place.)

2 An office receives five new desks with a bench shape made up of a rectangle and quadrant, as shown. The edge of the bench is lined with a rubber strip at a cost of $2.50 per metre.



a Find the length of the rubber edging strip for one desk, correct to two decimal places.

b Find the total cost of the rubber strip for the five desks. Round your answer to the nearest dollar.

The manufacturer claims that the desk top area space is more than 1.5 m^2.

c Is the manufacturer's claim correct?

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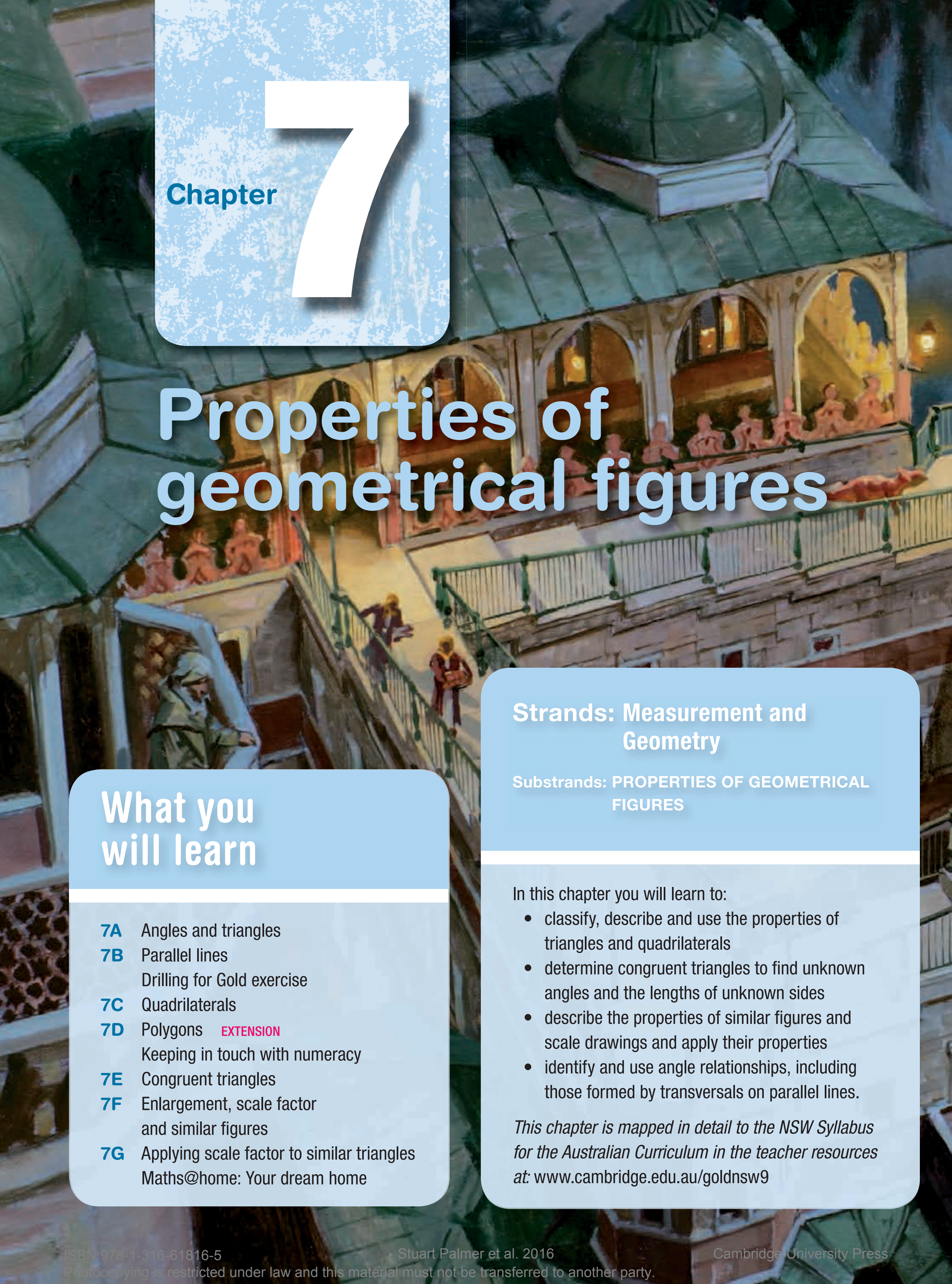

Chapter 7

Properties of geometrical figures

What you will learn

Strands: Measurement and Geometry

Substrands: PROPERTIES OF GEOMETRICAL FIGURES

In this chapter you will learn to:

- classify, describe and use the properties of triangles and quadrilaterals
- determine congruent triangles to find unknown angles and the lengths of unknown sides
- describe the properties of similar figures and scale drawings and apply their properties
- identify and use angle relationships, including those formed by transversals on parallel lines.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw9

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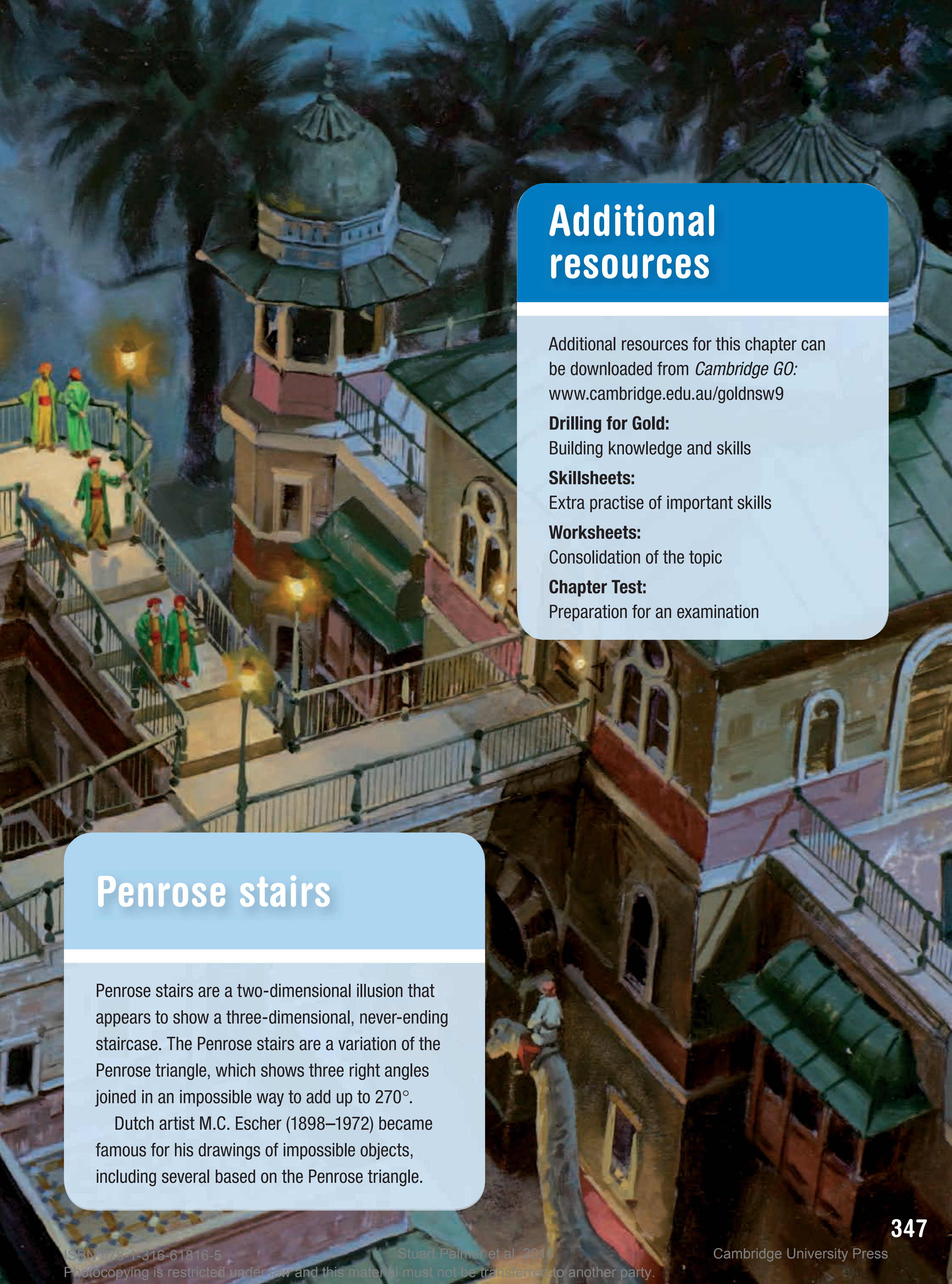

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO:* www.cambridge.edu.au/goldnsw9

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

Penrose stairs

Penrose stairs are a two-dimensional illusion that appears to show a three-dimensional, never-ending staircase. The Penrose stairs are a variation of the Penrose triangle, which shows three right angles joined in an impossible way to add up to 270°.

Dutch artist M.C. Escher (1898–1972) became famous for his drawings of impossible objects, including several based on the Penrose triangle.

Pre-test

1 Name the following types of angles. Choose from *acute*, *right*, *obtuse*, *straight*, *reflex* or *revolution*.

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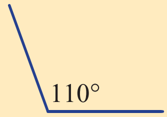

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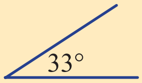

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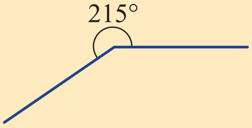

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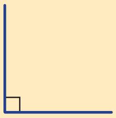

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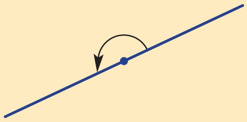

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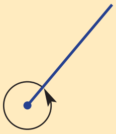
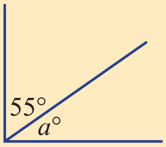


2 Find the value of a.

a


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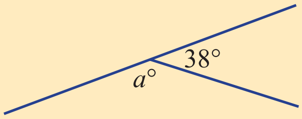

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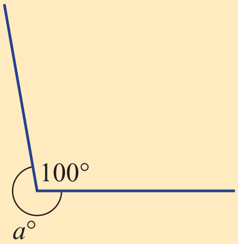
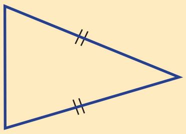


3 Identify the following triangles as equilateral, isosceles or scalene.

a


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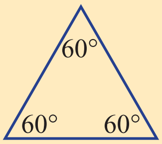
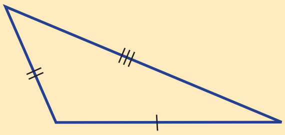
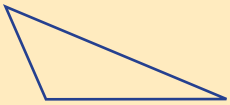


c


4 Identify the following as right, acute or obtuse triangles.

a


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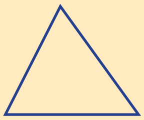

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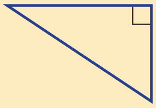
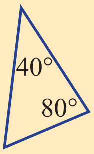


5 Find any missing angles in these triangles.

a


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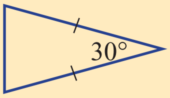

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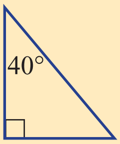


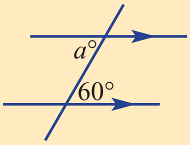
6 Of the six special quadrilaterals (i.e. square, rectangle, parallelogram, rhombus, kite and trapezium), which ones definitely have the following properties?

a two pairs of sides of equal length

b two pairs of parallel sides

c equal length diagonals

7 Find the value of the pronumerals in each of the following.

a


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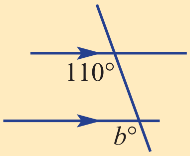

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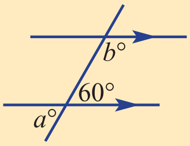

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7A Angles and triangles

Stage
5.2
5.2◊
5.1
4

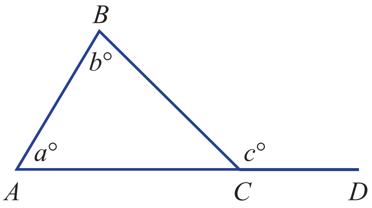
When looking at a building or structure, we often see triangles being used for their strength and rigidity. Lines meeting at a point and triangles have special properties that will be revised in this section.

Engineers have used many triangles in the design of cranes to make them strong and rigid but relatively lightweight.

▶ Let's start: Exterior angle discovery

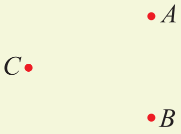
Here is a triangle with one side extended to form the exterior angle $\angle BCD$ (c°).

- If $a = 50$ and $b = 85$, find $\angle ACB$. Then find $\angle BCD$. What do you notice?
- Repeat for $a = 60$ and $b = 95$. What do you notice?
- What is the relationship between a, b and c?

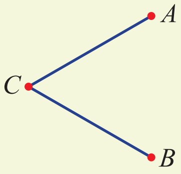


Key ideas

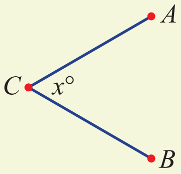
- A **point** represents a position.
  - It is shown using a dot and labelled with an upper case letter.
  - The diagram shows points A, B and C.



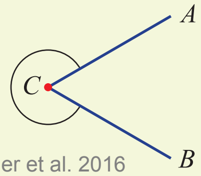
- This diagram shows **intervals** AC and CB. These are sometimes called *line segments*.
  - AC and CB form two **angles**. One is *acute* and one is *reflex*.
  - C is called the **vertex**. The plural is **vertices**.



- This diagram shows acute angle ACB. It can be written as: $\angle C$ or $\angle ACB$ or $\angle BCA$ or $A\hat{C}B$ or $B\hat{C}A$
  - CA and CB are sometimes called **arms**.
  - The pronumeral x represents the number of degrees in the angle.



- This diagram shows reflex angle $\angle ACB$.



Point A position in space, marked with a dot and named with a capital letter

Interval A section of a line with two end points

Vertex A point from which two lines, or 'arms', extend in different directions

Angle A measure of the space between two lines, usually measured in degrees

Arm One of two line intervals joined at a vertex to form an angle

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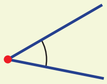
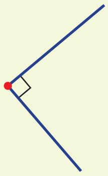
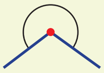
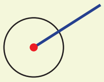
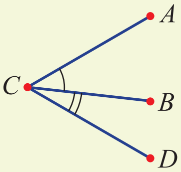
Type of angle	Size of angle	Diagram
acute	greater than 0° but less than 90°	
right	exactly 90°	
obtuse	greater than 90° but less than 180°	
straight	exactly 180°	
reflex	greater than 180° but less than 360°	
revolution	exactly 360°	

- This diagram shows two angles sharing a vertex and an arm. They are called **adjacent** angles.

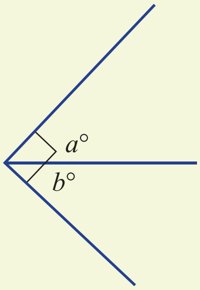


Adjacent Two angles that are next to each other; they share a common arm and vertex

Complementary angles Two angles with a sum of 90°. Each angle is the complement of the other.

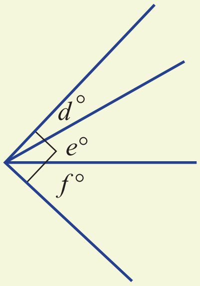
- This diagram shows two angles in a right angle. They are *adjacent complementary angles*. $a°$ is the **complement** of $b°$.

$$a + b = 90$$

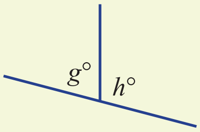


- It is possible to have three or more angles in a right angle. They are not complementary.

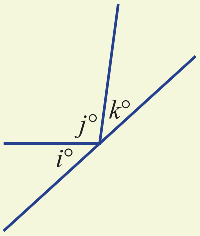
$$d + e + f = 90$$



- This diagram shows two angles on a straight line. They are *adjacent supplementary* angles. $g°$ is the **supplement** of $h°$.
  $$g + h = 180$$



- It is possible to have three or more angles on a straight line. They are not supplementary.
  $$i + j + k = 180$$

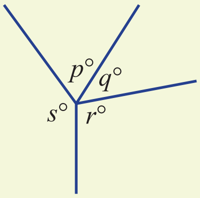


Supplementary angles Two angles with a sum of $180°$. Each angle is the supplement of the other.

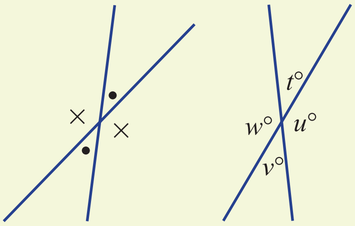
Vertically opposite A pair of angles (always equal) that are opposite each other across a common vertex

Perpendicular At right angles $(90°)$ to each other

- This diagram shows *angles at a point* and *angles in a revolution*.
  $$p + q + r + s = 360$$

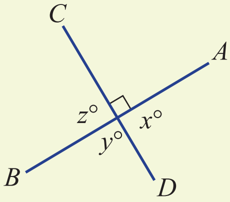


- When two straight lines meet they form two pairs of **vertically opposite angles**. Vertically opposite angles are equal.
  $$t = v$$
  $$u = w$$

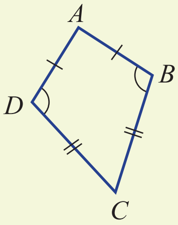


Drilling for Gold 7A2 7A3

- If one of the four angles in vertically opposite angles is a right angle, then all four angles are right angles.
  - AB and CD are **perpendicular lines**. This is written as $AB \perp CD$.
  $$x = 90$$
  $$y = 90$$
  $$z = 90$$



- The markings in this diagram indicate that:
  - $AB = AD$
  - $BC = CD$
  - $\angle ABC = \angle ADC$



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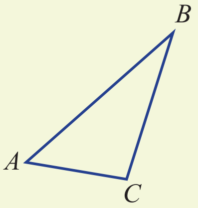
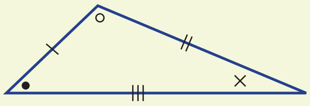
- A *triangle* has:
  - 3 sides: AB, BC and AC
  - 3 vertices (the plural of vertex): A, B and C
  - 3 interior angles.



This triangle could be called $\triangle ABC$ or $\triangle ACB$.

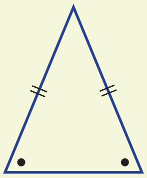
- Triangles classified by side lengths
  - Sides with the same number of dashes are of equal length.

Scalene



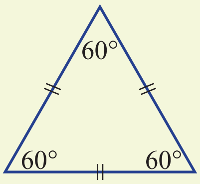
3 unequal sides
3 unequal angles

Isosceles



2 equal sides
2 equal angles
The equal sides are opposite the equal angles.

Equilateral



3 equal sides
3 equal 60° angles

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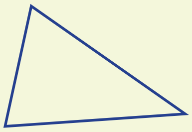
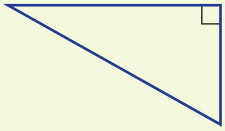
- Triangles classified by interior angles

Acute



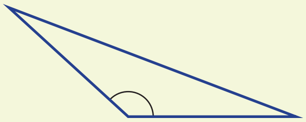
All angles acute

Right



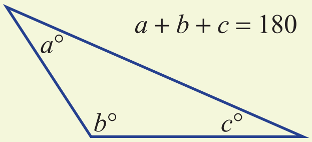
One right angle

Obtuse

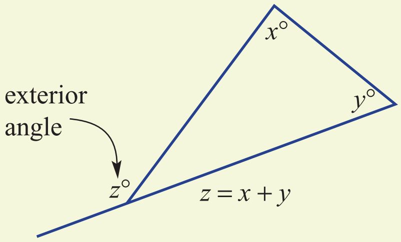


One obtuse angle

- The sum of the interior angles of a triangle is 180°.



- An exterior angle of a triangle is formed by extending one of the sides.
- The exterior angle of a triangle is equal to the sum of the two opposite interior angles.



Scalene triangle A triangle where all sides are different lengths and all angles are different

Isosceles triangle A triangle where two sides have equal lengths and two angles are equal. The equal angles are the ones joining each same-length side to the third side.

Equilateral triangle A triangle where all angles and all sides are equal (all angles are 60°)

Acute-angled triangle A triangle where one of the interior angles is an acute angle (less than 90°)

Right-angled triangle A triangle where one of the interior angles is a right angle (90° exactly)

Obtuse-angled triangle A triangle where one of the interior angles is obtuse (more than 90° but less than 180°)

Exercise 7A

Understanding

1 Choose a word or number from the table to complete each sentence.

180°	revolution	acute	obtuse	180°
supplementary	180°	equal	90°	right

a A 90° angle is called a _______ angle.
b A _______ angle is called a straight angle.
c A 360° angle is called a _______.
d _______ angles are between 90° and 180°.
e _______ angles are between 0° and 90°.
f Reflex angles are between _____ and 360°.
g Complementary angles sum to _______.
h ____________________ angles sum to 180°.
i Angles in a triangle sum to ______.
j Vertically opposite angles are _______.

2 Match the triangles in the table with the descriptions below.

equilateral triangle	right-angled triangle	scalene triangle
obtuse-angled triangle	isosceles triangle	acute-angled triangle

a two equal sides
b one obtuse angle
c all angles 60°
d all angles acute
e all sides of different length
f one right angle

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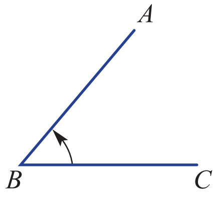
3 Estimate the size of each of the following angles and use your protractor to determine an accurate measurement.

a



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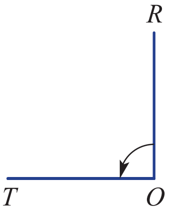

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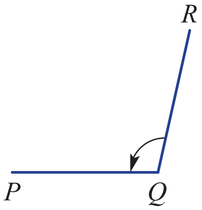

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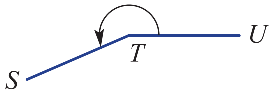
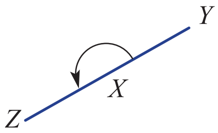


e



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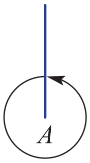


Fluency

Example 1 Finding supplementary and complementary angles

For the angle 47°, determine the:

a supplement **b** complement

Solution	**Explanation**
a $180° - 47° = 133°$	Supplementary angles sum to 180°.
b $90° - 47° = 43°$	Complementary angles sum to 90°.

4 For each of the following angles determine the:

i supplement

ii complement

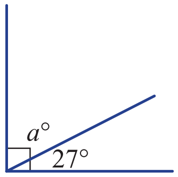
a 55° **b** 31° **c** 74° **d** 10° **e** 89°

f 22° **g** 38° **h** 65° **i** 47° **j** 77°

- Supplementary angles add to 180°.
- Complementary angles add to 90°.

5 Find the value of a.

a



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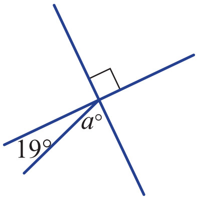

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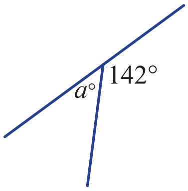
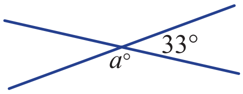


A straight angle is 180°. A revolution is 360°.

d



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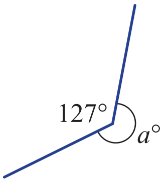

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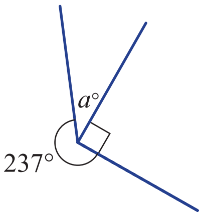
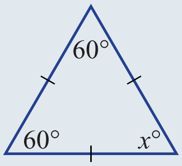


Example 2 Finding unknown angles in triangles

Name the types of triangles shown here and determine the values of the pronumerals.

a



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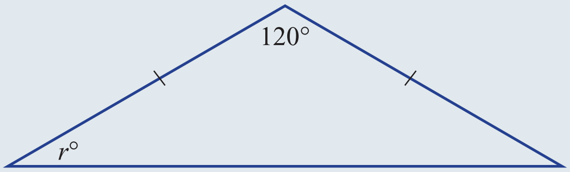

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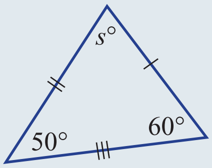
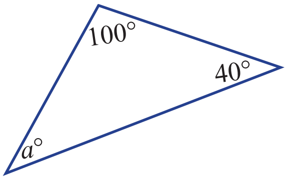


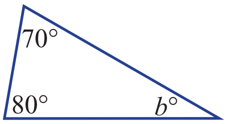
Solution	Explanation
a Equilateral triangle $x = 60$	All sides are equal, therefore all angles are equal.
b Obtuse isosceles triangle $2r + 120 = 180$ $2r = 60$ $r = 30$	One angle is more than 90° and two sides are equal. Angles in a triangle add to 180° and since it is isosceles, the two base angles are equal. Subtract 120° from both sides and then divide both sides by 2.
c Acute scalene triangle $s + 50 + 60 = 180$ $s + 110 = 180$ $s = 70$	All angles are less than 90° and all sides are of different length. Angles in a triangle add to 180°. Simplify and solve for s.

6 Name the types of triangles shown here. What are the values of the pronumerals?

a



b



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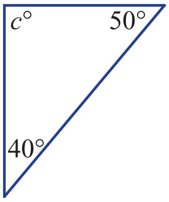

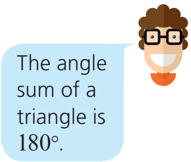

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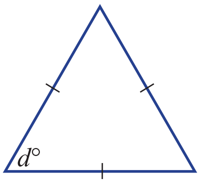

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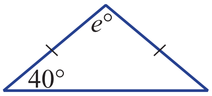

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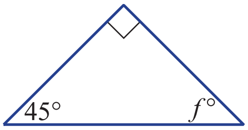

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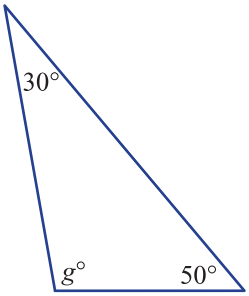

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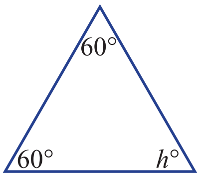

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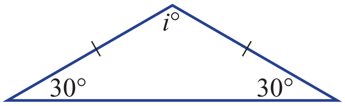

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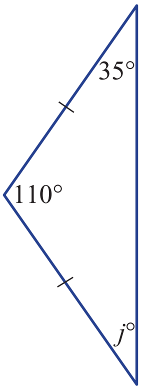

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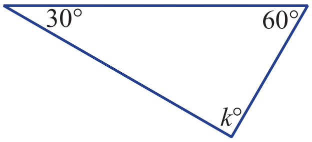

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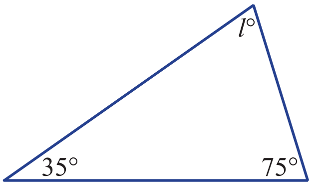

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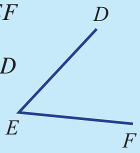
Problem-solving and Reasoning

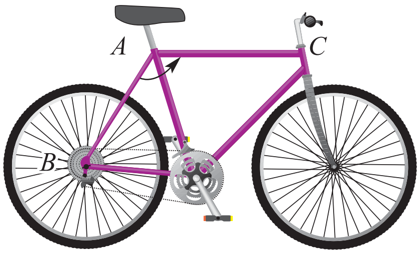
7 For each diagram:

- i Name the angle shown (e.g. $\angle ABC$).
- ii State the type of angle.
- iii Estimate the size of the angle.
- iv Measure the angle using a protractor.

When naming an angle, put the letter at the vertex in the middle.
$\angle DEF$
or
$\angle FED$



a



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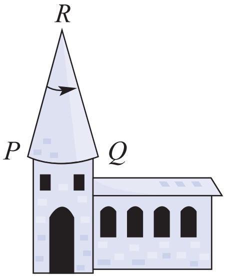

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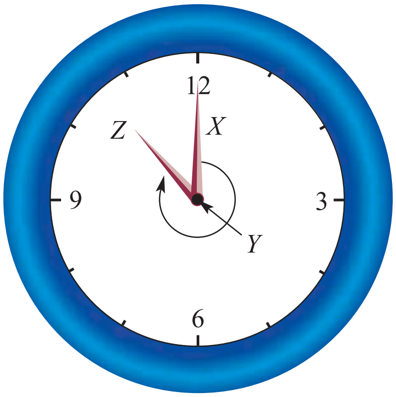

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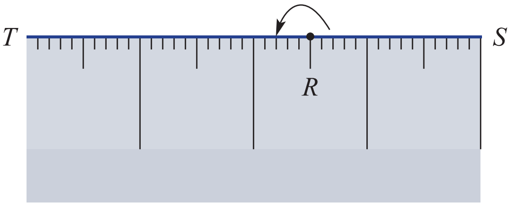

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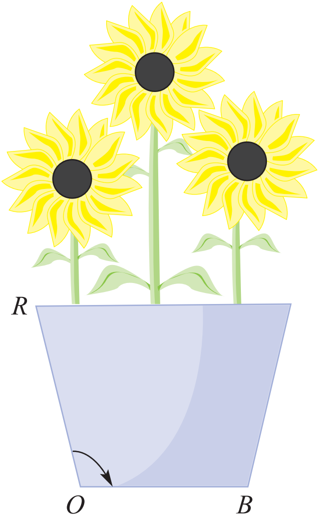

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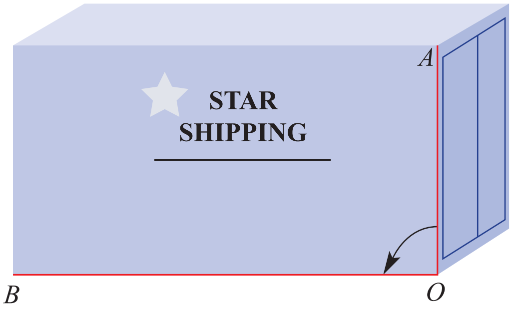

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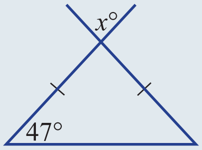
Example 3 Finding exterior angles

Find the value of each pronumeral. Give reasons for your answers.

a



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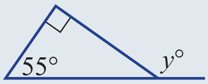


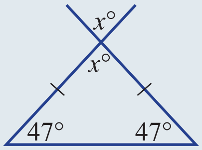
Solution

a $x + 47 + 47 = 180$ (angle sum)

$x + 94 = 180$

$x = 86$

Explanation



Note the isosceles triangle and vertically opposite angles and mark in those angles.

Angles in a triangle add to 180° and vertically opposite angles are equal.

Simplify and solve for x.

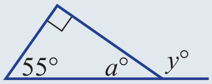
b Let a be the unknown angle.

$a + 90 + 55 = 180$ (angle sum)

$a = 35$

$y + 35 = 180$

$y = 145$

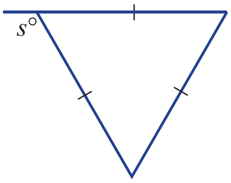


Angles in a triangle sum to 180°.

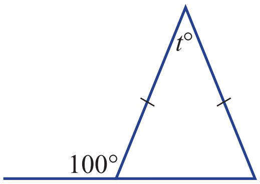
Angles in a straight line are supplementary (i.e. sum to 180°).

8 Find the value of each pronumeral.

a

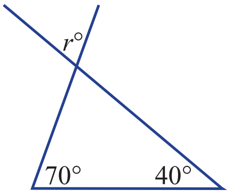


b



If you can't find the value of the pronumeral straight away, try finding one of the other missing angles first.

c



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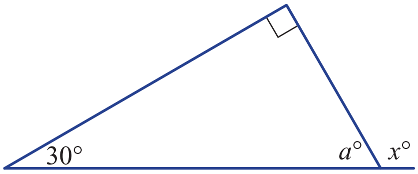

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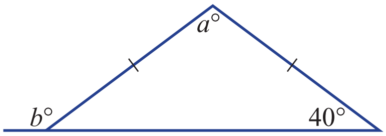

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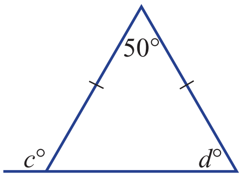

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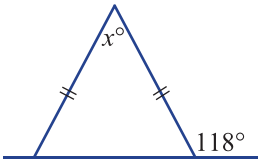

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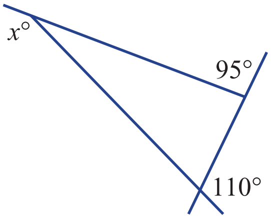
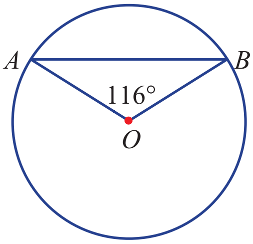


9 Explain why $\angle OAB$ is 32° in this circle if O is the centre of the circle.



Enrichment: Clock geometry

10 Calculate how many degrees the minute hand of a clock rotates in:

a 1 hour **b** $\frac{1}{4}$ of an hour **c** 10 minutes **d** 15 minutes

e 72 minutes **f** 1 minute **g** 2 hours **h** 1 day

11 Find the smallest angle between the hour and minute hands at these times. Remember to consider how the hour hand moves between each whole number. For example, at 9:30, the hour hand is halfway between the 9 and 10.

a 3 p.m. **b** 5 a.m. **c** 6:30 p.m.

d 11:30 p.m. **e** 3:45 a.m. **f** 1:20 a.m.

g 4:55 a.m. **h** 2:42 a.m. **i** 9:27 a.m.

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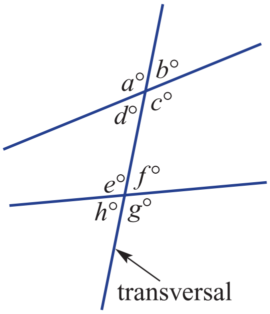
7B Parallel lines

Stage
5.2
5.2◊
5.1
4

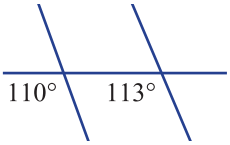
When two lines are crossed by a third line, called a transversal, eight angles are formed. If the two original lines are parallel, then every pair of angles is either equal or supplementary.



▶ Let's start: Are they parallel?

Here are three diagrams that show a transversal crossing two other lines.

Drilling for Gold 7B1

1


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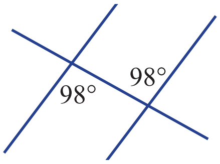

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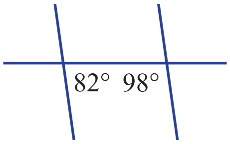
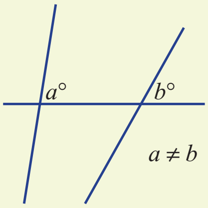
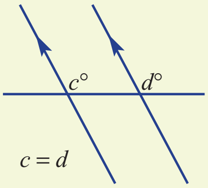
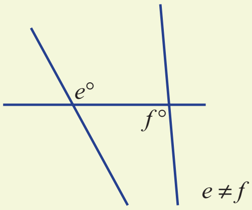
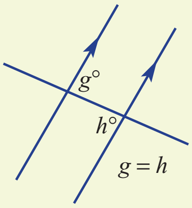
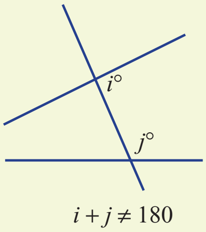
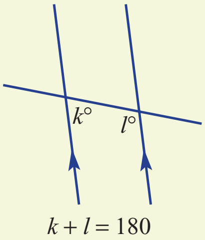


Decide whether each diagram contains a pair of parallel lines. Give reasons for your answer.

Key ideas

- Parallel lines never meet.
  - Arrows indicate that lines are parallel.
- A **transversal** is a line crossing two or more other lines.

	Non-parallel lines	**Parallel lines**
Corresponding angles – If lines are parallel, corresponding angles are equal.		
Alternate angles – If lines are parallel, alternate angles are equal.		
Cointerior angles – If lines are parallel, cointerior angles are supplementary (i.e. sum to 180°).		

Transversal A line that cuts two or more lines

Corresponding angles Pairs of angles formed by two lines cut by a transversal. If the lines are parallel, corresponding angles are equal.

Alternate angles Two angles that lie between two lines on either side of a transversal. If the lines are parallel, alternate angles are equal.

Cointerior angles A pair of angles lying between two lines on the same side of a transversal.

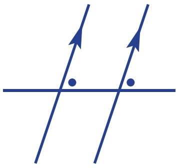
Exercise 7B

Understanding

Drilling for Gold 7B2 at the end of this section

1 Decide whether the pair of marked angles are corresponding, alternate or cointerior.

a



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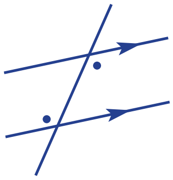

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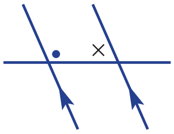

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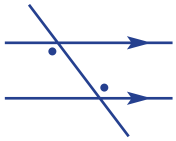

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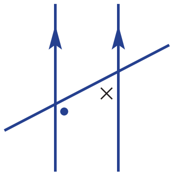

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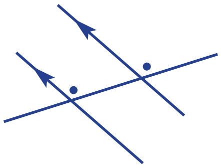


2 Use the word *equal* or *supplementary* to complete these sentences.

a When two lines are parallel, corresponding angles are __________.

b When two lines are parallel, alternate angles are __________.

c When two lines are parallel, cointerior angles are __________.

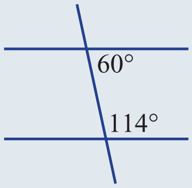
Supplementary angles add to 180°.

Fluency

Example 4 Deciding whether lines are parallel

Decide whether each diagram contains a pair of parallel lines. Give reasons.

a



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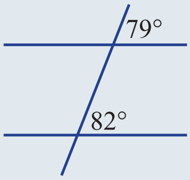

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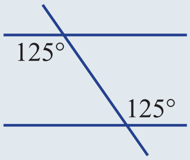
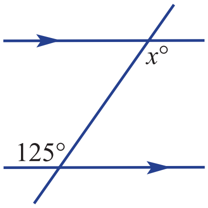


Solution	**Explanation**
a No. The two cointerior angles do not add to 180°.	$60^\circ + 114^\circ = 174^\circ \neq 180^\circ$
b No. The two corresponding angles are not equal.	$79^\circ \neq 82^\circ$
c Yes. The two alternate angles are equal.	When alternate angles are equal, the lines are parallel.

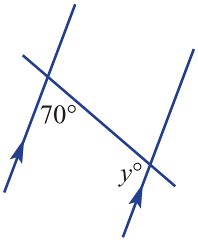
3 Find the values of the pronumerals in these pairs of parallel lines. Give a reason for each answer.

For part **a**, $x = 125$.
Reason: Alternate angles in parallel lines.

a



b



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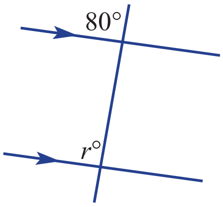

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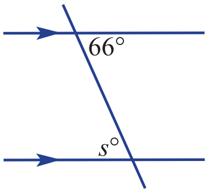
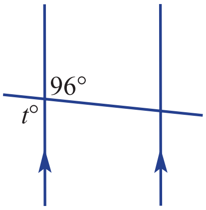


e



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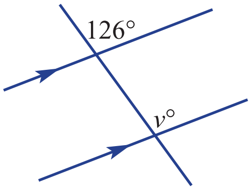

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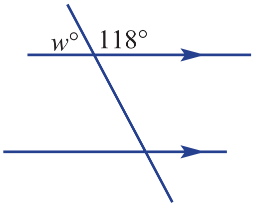

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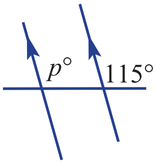
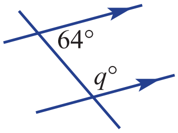


i

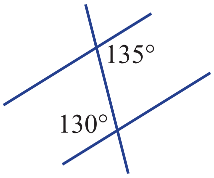


4 Decide if each diagram contains a pair of parallel lines. Give reasons.

The reasons why lines could be parallel include:

1. Corresponding angles are equal.
2. Alternate angles are equal.
3. Cointerior angles add to 180° (are supplementary).

a



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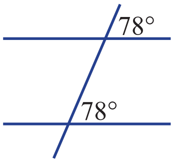

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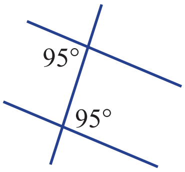

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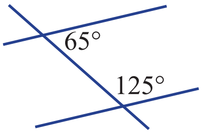

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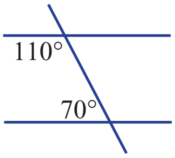

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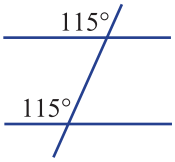

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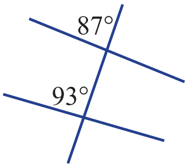

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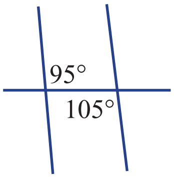

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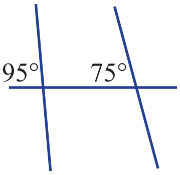
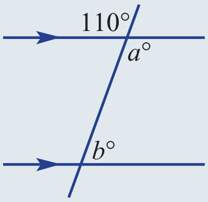


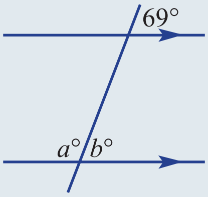
Example 5 Finding angles in parallel lines

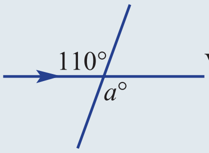
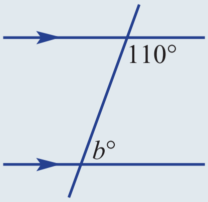
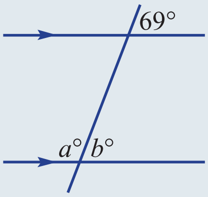
Find the value of each of the pronumerals. Give reasons for your answers.

a



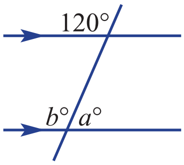
b



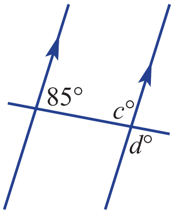
Solution	**Explanation**
a $a = 110$ (vertically opposite angles)	 Vertically opposite angles are equal.
$b + 110 = 180$ (cointerior angles in parallel lines) $b = 70$	 Cointerior angles add to 180° in parallel lines.
b $b = 69$ (corresponding angles in parallel lines)	 Corresponding angles are equal in parallel lines.
$a + 69 = 180$ (angles on a straight line) $a = 111$	Angles on a straight line add to 180°, so $a + b = 180$.

5 Find the value of each pronumeral. Give reasons for your answers.

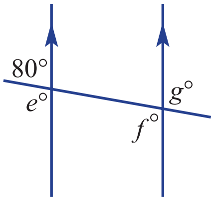
a



b

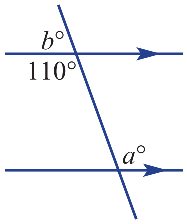


c

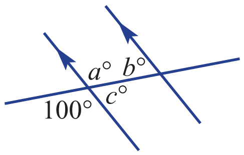


ISBN 978-1-316-61816-5 Stuart Palmer et al. 2016 Cambridge University Press

d



e



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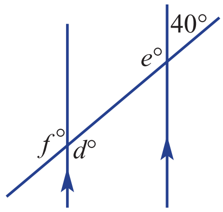
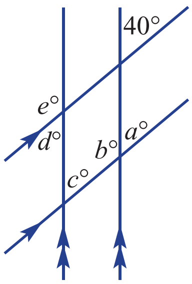


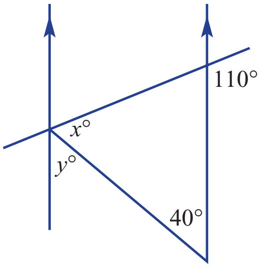
Problem-solving and Reasoning

6 Two pairs of straight train tracks meet at 40°, as shown on the right. Find the value of a, b, c, d and e.



7 Find the value of each pronumeral.

a



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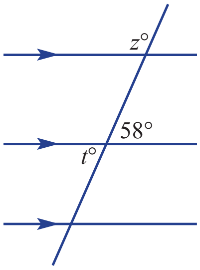
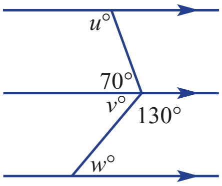
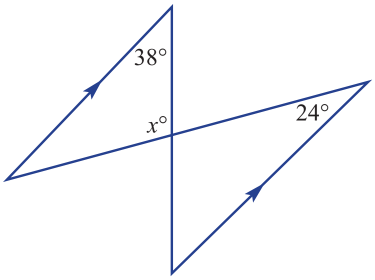


Look for pairs of angles inside parallel lines. Mark in all known missing angles to help find the value of the pronumeral.

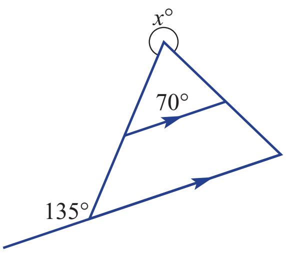
c



d



e



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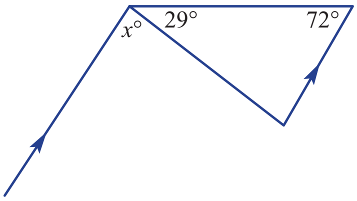
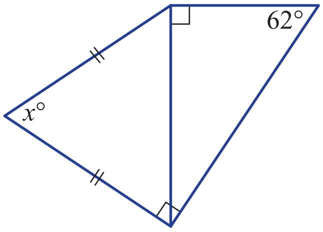


8 Find the value of x.

a



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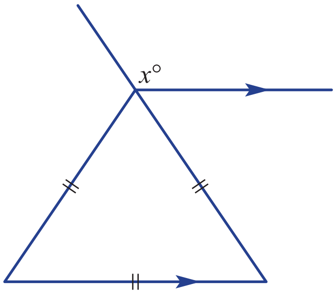

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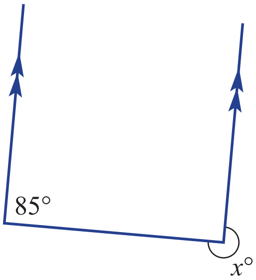
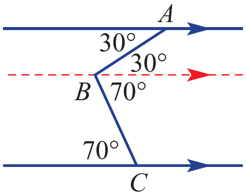
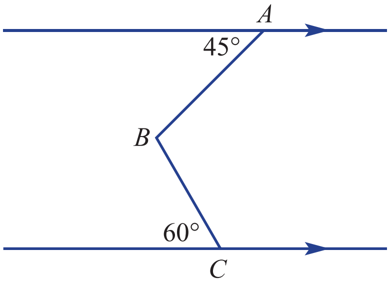


Enrichment: Add a new line to help

9 Sometimes you can add a third parallel line to a diagram to help you find an angle. For example, to find $\angle ABC$ in this diagram, you can draw a parallel line through B, then find the two alternate angles (30° and 70°). So $\angle ABC = 30° + 70° = 100°$. Add a third parallel line to help find $\angle ABC$ in these diagrams.



a



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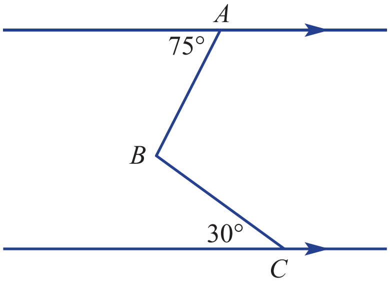

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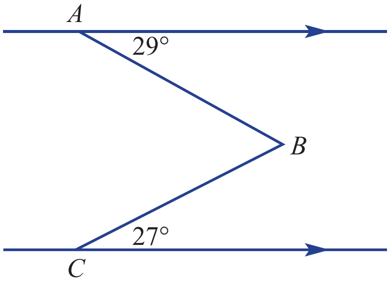

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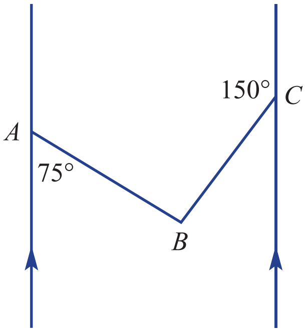

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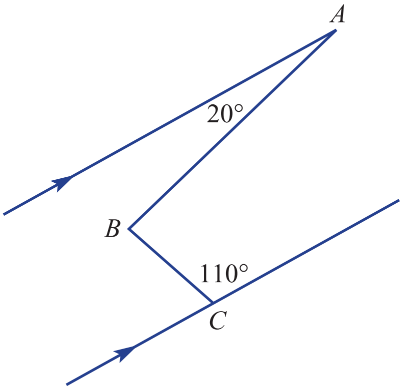

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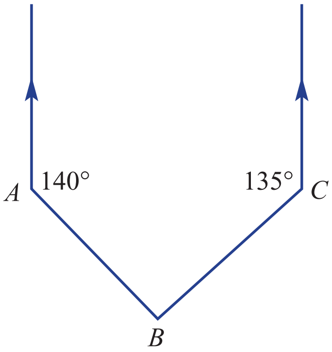
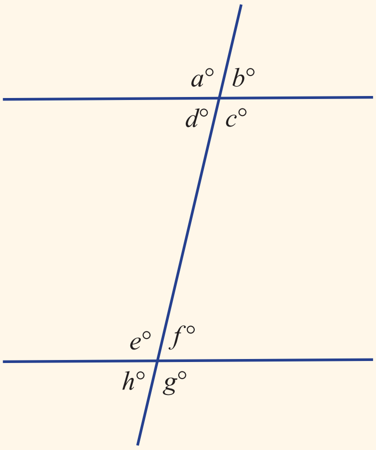


7B2: Corresponding, alternate or cointerior?

The diagram below shows two parallel lines crossed by a transversal. Study the diagram, then answer questions **1–8** in the table below. Write your answers on the worksheet or in your exercise book.



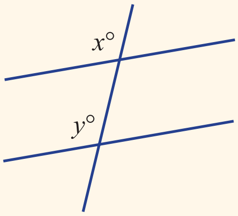
Corresponding angles	Alternate angles	Cointerior angles
1 $a =$	**5** $c =$	**7** $c +$ ___ $= 180°$
2 $b =$	**6** $d =$	**8** __ + __ $= 180°$
3 $c =$		
4 $d =$		

The diagrams below show two parallel lines crossed by a transversal. Two angles are marked. For each diagram, write which of these statements is true:

$x = y$ or $x + y = 180$

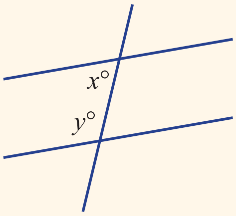
and then give the reason (in brackets). The first two questions have been done for you. Complete the rest on the worksheet or in your exercise book.

9



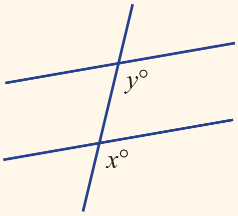
$x = y$ (corresponding angles in parallel lines)

10



$x + y = 180$ (cointerior angles in parallel lines)

11



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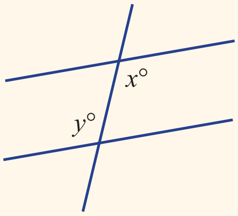

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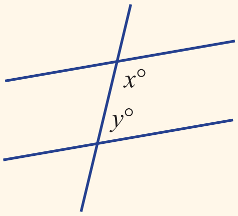

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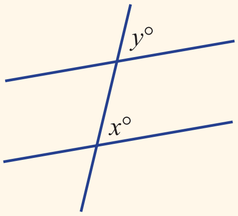

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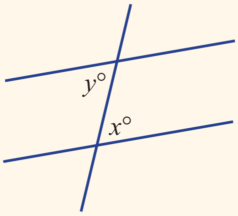

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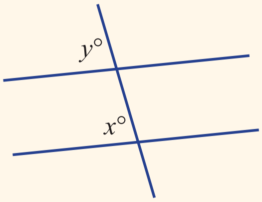

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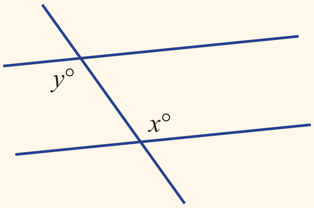

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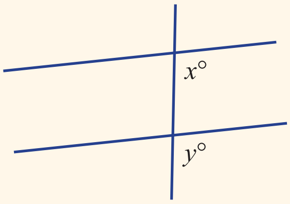

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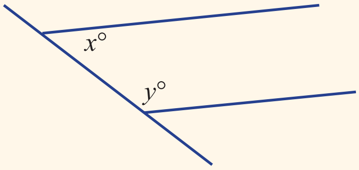

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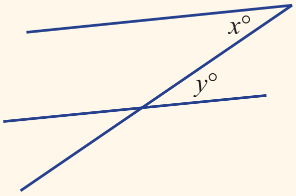
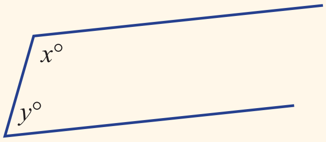


21



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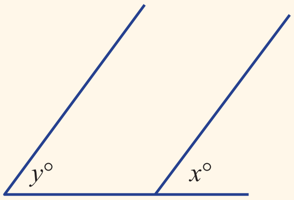
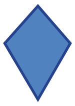
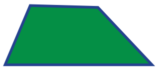
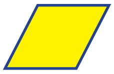


7C Quadrilaterals

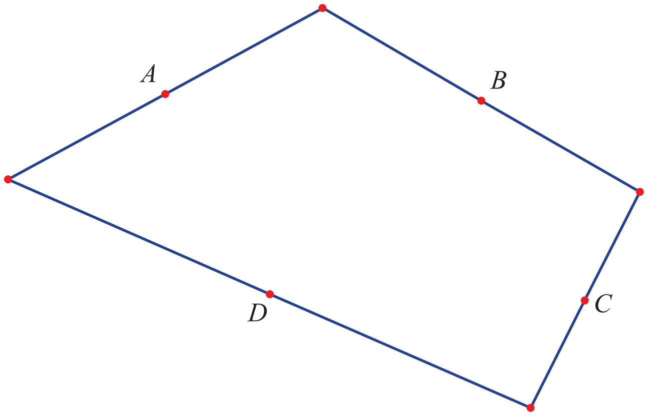
Stage
5.2
5.2◊
5.1
4

In this section we will revise the properties of the six special quadrilaterals. An example of each is shown at right.

Special quadrilaterals	
kite 	trapezium 
rhombus 	parallelogram 
square 	rectangle 

Let's start: Random quadrilateral

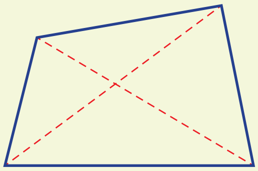
- On a sheet of paper, draw a random quadrilateral with sides more than 5 cm long.
- Use a ruler to carefully find the midpoint of each side.



- Join A to B, B to C, C to D and D to A.
- What type of quadrilateral is $ABCD$?
- Compare yours to others in your class.

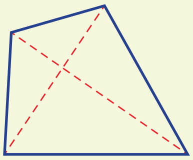
Key ideas

- Every quadrilateral has two diagonals.
  - In some quadrilaterals the diagonals bisect each other (i.e. cut each other in half).



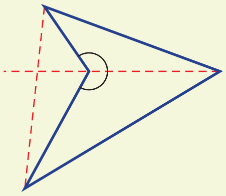
- **Quadrilaterals** can be convex or non-convex.
  - Convex quadrilaterals have all vertices pointing outwards.
  - Non-convex quadrilaterals have one vertex pointing inwards.
  - Both diagonals of convex quadrilaterals lie inside the figure.

Convex



All interior angles less than 180°

Non-convex

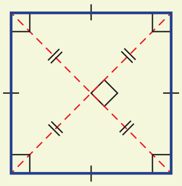


One reflex interior angle

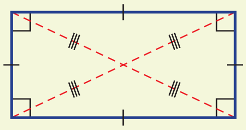
Drilling for Gold 7C1

- Special quadrilaterals

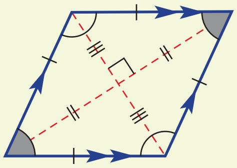
Square



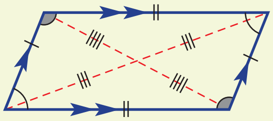
Rectangle



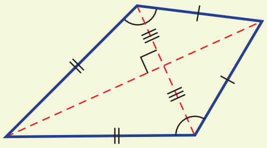
Rhombus



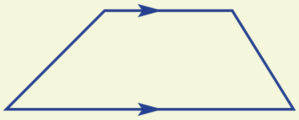
Parallelogram



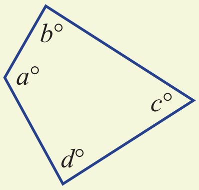
Kite



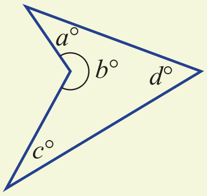
Trapezium



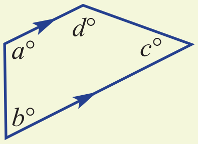
- The angle sum of any quadrilateral is 360°.



$$a + b + c + d = 360$$



- Quadrilaterals with parallel sides contain two pairs of cointerior angles.



$$c + d = 180$$
$$a + b = 180$$

Quadrilateral A two-dimensional shape with four sides of any length

Square A quadrilateral with all sides equal, both pairs of opposite sides parallel, and all angles are right angles

Convex A polygon with all vertices pointing outward

Kite A quadrilateral with two pairs of adjacent sides equal

Non-convex A polygon with at least one vertex pointing inwards

Parallelogram A quadrilateral with both pairs of opposite sides parallel

Rectangle A quadrilateral with both pairs of opposite sides equal and parallel and all angles are right angles

Rhombus A quadrilateral with both pairs of opposite sides parallel and all sides equal

Trapezium A quadrilateral with at least one pair parallel sides

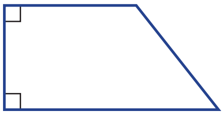
Exercise 7C

Understanding

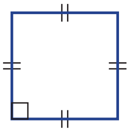
1 **a** Write down the names of the six special quadrilaterals.

b Based on the information in the diagrams below, choose the best name for each of these shapes. *Hint*: All six have different names.

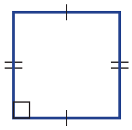
i



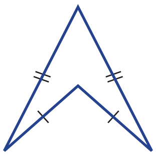
ii



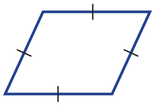
iii



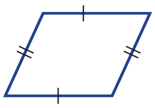
iv



v



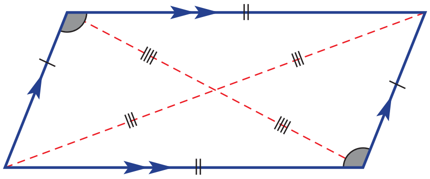
vi



2 Using the diagram of a parallelogram given below, which of the following sentences are *definitely* true.

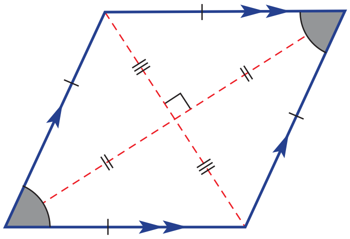
a All sides are equal in length.
b Opposite sides are equal.
c Opposite sides are parallel.
d All angles are 90°.
e Opposite angles are equal.
f The diagonals are equal in length.
g The diagonals bisect each other.
h The diagonals meet at right angles.

Parallelogram



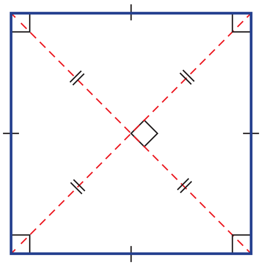
3 Repeat Question **2**, using this diagram of a rhombus.

Rhombus



4 Repeat Question **2**, using this diagram of a square.

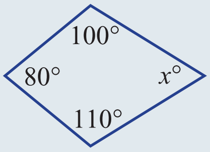
Square



Fluency

Example 6 Finding angles in quadrilaterals

Find the value of the pronumeral in this quadrilateral. Give a reason.

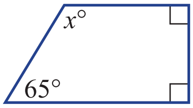


Solution	Explanation
$x + 80 + 100 + 110 = 360$ (angle sum of $x + 290 = 360$ a quadrilateral) $x = 70$	The angles in a quadrilateral add to 360°. Simplify and solve for x.

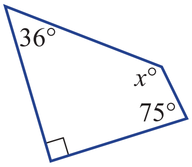
5 Find the value of the pronumeral x in these quadrilaterals.

Use the angle sum of a quadrilateral, which is 360°.

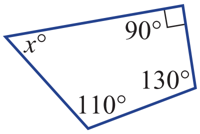
a



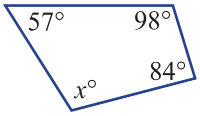
b



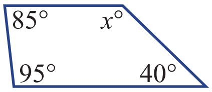
c



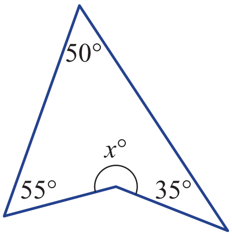
d



e



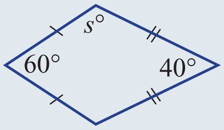
f



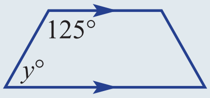
Example 7 Finding angles in special quadrilaterals

Find the value of the pronumeral in this kite and trapezium. Give a reason.

a



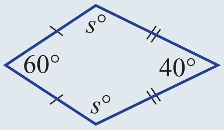
b



Solution

a $2s + 60 + 40 = 360$ (angle sum of a
$2s + 100 = 360$ quadrilateral)
$2s = 260$
$s = 130$

Explanation



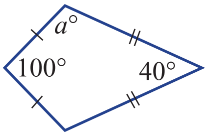
The angles in a quadrilateral add to 360° and the opposite angles (s°) are equal in a kite.

b $y + 125 = 180$ (cointerior angles in
$y = 55$ parallel lines)

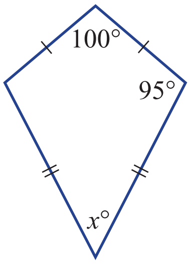
Cointerior angles inside parallel lines are supplementary (i.e. add to 180°).

6 Find the value of the pronumerals.

a

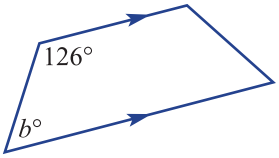


b

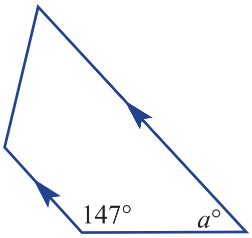


Remember: A kite has one pair of equal opposite angles.
Cointerior angles inside parallel lines are supplementary (i.e. add to 180°).

c

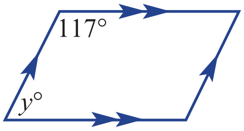


d



Skillsheet 7A

e



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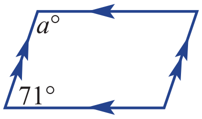


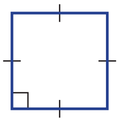
Problem-solving and Reasoning

7 On grid paper, draw a trapezium that has:

- **a** no equal sides
- **b** exactly two equal sides
- **c** exactly three equal sides
- **d** four equal sides

8 A shopping centre's floor is a quadrilateral with one 120° angle and two 90° angles. What is the size of the fourth angle?

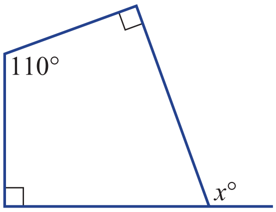
9 Consider the shape shown opposite.



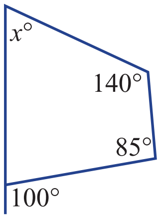
a Is it a quadrilateral?
b Is it a kite?
c Is it a trapezium?
d Is it a parallelogram?
e Is it a rhombus?
f Is it a rectangle?
g Is it a square?
h What is the best name for this shape?

10 These quadrilaterals include exterior angles. Find the value of x in each.

a



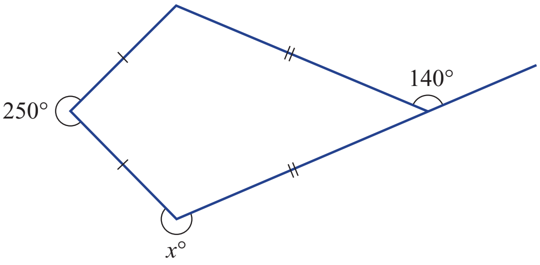
b



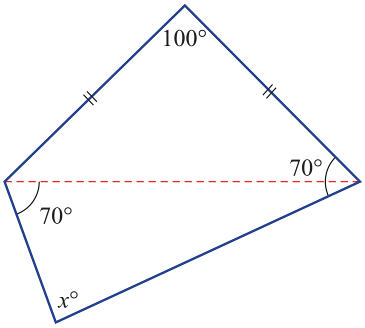
Enrichment: Quad x challenge

11 Find the value of x in these diagrams.

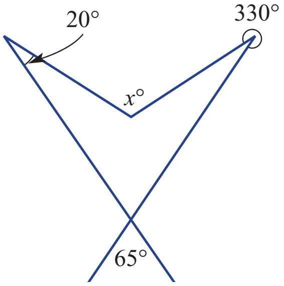
a



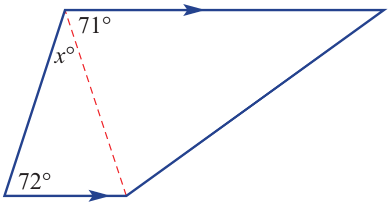
b



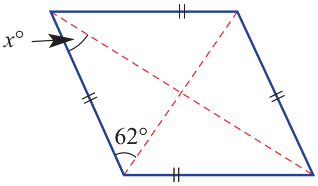
c



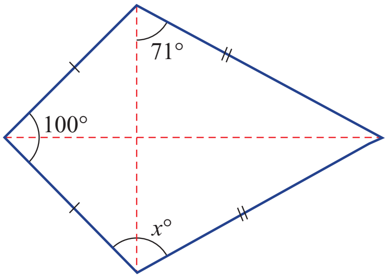
d



e



f



7D Polygons

EXTENSION

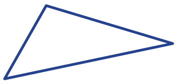
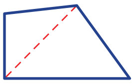
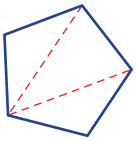
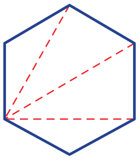
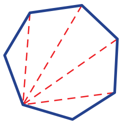
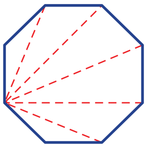
Stage
5.2
5.2◊
5.1
4

Closed two-dimensional shapes with straight sides are called polygons. They are classified by their number of sides. We have already seen triangles (3 sides) and quadrilaterals (4 sides), which have been classified using their special properties.

▶ Let's start: Developing the rule

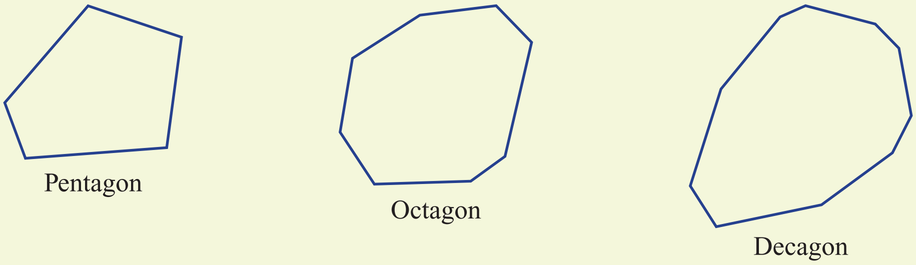
Drilling for Gold 7D1

The following procedure uses the fact that the angle sum of a triangle is 180°. Copy and complete the table and try writing the general rule in the final row on the next page.

Shape	Number of sides	Number of triangles	Angle sum
Triangle 	3	1	$1 \times 180^\circ = 180^\circ$
Quadrilateral 	4	2	$\_\_\_\_ \times 180^\circ = \_\_\_\_$
Pentagon 	5		
Hexagon 	6		
Heptagon 	7		
Octagon 	8		
n-sided polygon	n		$(\_\_\_\_) \times 180^\circ$

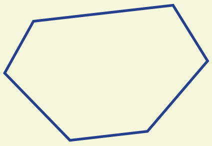
Key ideas

- **Polygons** are shapes with straight sides.



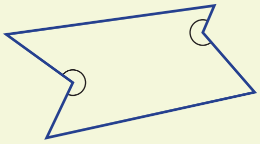
- Polygons can be convex or non-convex.
  - Convex polygons have all vertices pointing outwards.
  - Non-convex polygons have at least one vertex pointing inwards.

Convex



This means all interior angles are less than 180°.

Non-convex

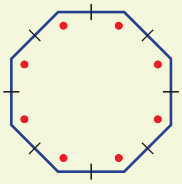


This means there is one or more reflex interior angles.

- **Polygons** are named according to their number of sides.

Number of sides	Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon

- The angle sum, S, of a polygon with n sides is given by the rule $S = (n - 2) \times 180°$.
- A **regular polygon** has sides of equal length and equal interior angles.



A regular octagon

Polygon A plane figure bounded by line segments

Regular polygon A polygon with all sides of equal length and all angles equal

Decagon A ten-sided polygon

Octagon An eight-sided polygon (with eight angles)

Pentagon A five-sided polygon (with five angles)

Heptagon A seven-sided polygon (with seven angles)

Hexagon A six-sided polygon (with six angles)

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Exercise 7D EXTENSION

Understanding

1 Write the missing word/rule from the table to complete the sentences.

regular	$(n-2)\times 180°$
non-convex	hexagon

a A __________ has 6 sides.
b A __________ polygon has sides of equal length and equal angles.
c A __________ polygon has at least one reflex angle.
d The rule for the angle sum of a polygon is $S =$ __________.

2 How many sides do these polygons have?
a pentagon
b heptagon
c quadrilateral
d octagon
e nonagon
f dodecagon

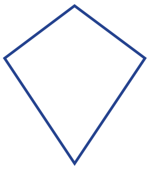
Refer to the Key ideas for help.

3 Use $S = (n - 2) \times 180°$ to find the angle sum, S, of these polygons.
a hexagon
b octagon
c pentagon
d heptagon

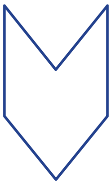
n is the number of sides of the polygon.

4 Name each of these shapes as convex or non-convex, and write their polygon name.

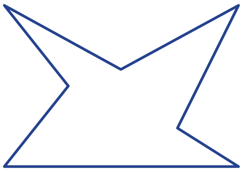
a



b



c

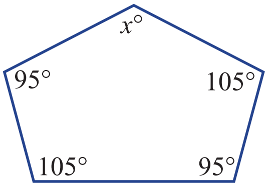


A convex polygon has all angles less than 180°.

The faces of a rhombicosidodecahedron are polygons – 20 triangles, 30 quadrilaterals and 12 pentagons.

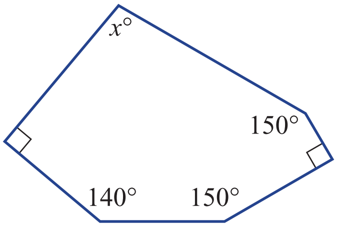
5 Find the value of x, using the given angle sum.

a



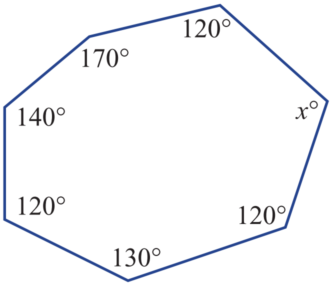
Angle sum = 540°

b



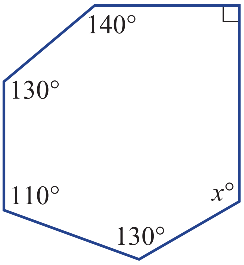
Angle sum = 720°

c



Angle sum = 900°

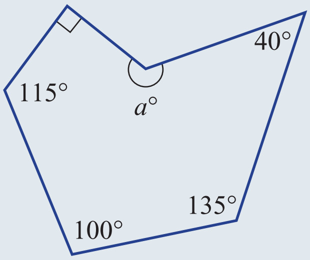
d



Angle sum = 720°

Example 8 Finding angles in polygons

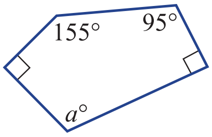
Find the angle sum using $S = (n - 2) \times 180°$, then find the value of a.



Solution	**Explanation**
$n = 6$ and $S = (n - 2) \times 180°$ $= (6 - 2) \times 180°$ $= 720°$	The shape is a hexagon with 6 sides, so $n = 6$. $180 \times 4 = 720$
$a + 90 + 115 + 100 + 135 + 40 = 720$ $a + 480 = 720$ $a = 240$	The sum of all angles is 720°. Simplify and solve for a.

6 For each polygon, find the angle sum using $S = 180\,(n - 2)$. Then find the value of a.

a



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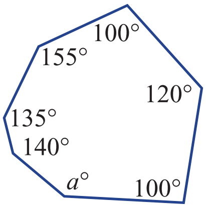

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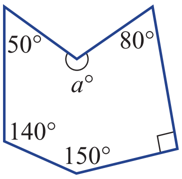

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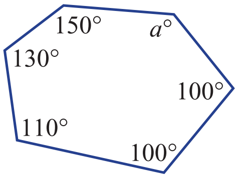

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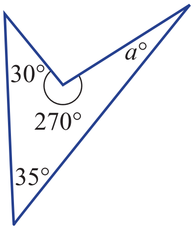

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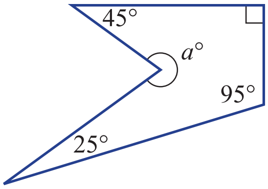
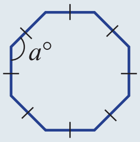


Example 9 Working with regular polygons

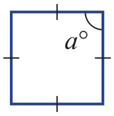
Find the size of an interior angle of a regular octagon.



Solution	**Explanation**
$n = 8$ and $S = (n - 2) \times 180°$	The regular octagon has 8 sides, so use $n = 8$.
$= (n - 2) \times 180°$	
$= 1080°$	
$\therefore\ a = 1080 \div 8$	Each interior angle in a regular polygon is equal, so divide by 8 to get a single angle.
$= 135$	

7 Find the size of an interior angle of these regular polygons. Round your answer to two decimal places in part **e**.

a



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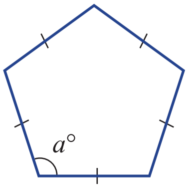

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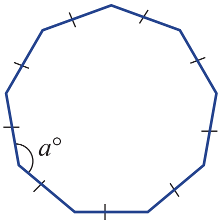

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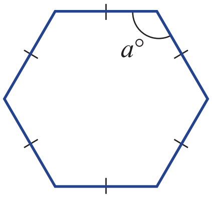

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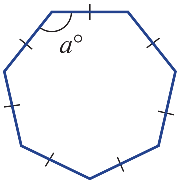

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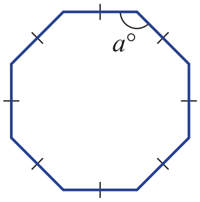


Problem-solving and Reasoning

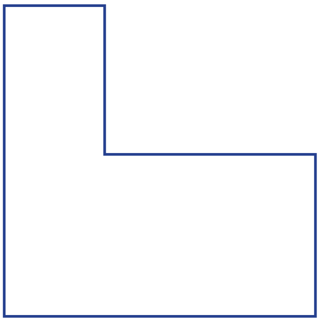
8 Calculate the number of sides when a polygon has the given angle sum.
Suggestion: Use the rule $S = (n - 2) \times 180^\circ$.

a 2520° **b** 4140° **c** $18\,000^\circ$

9 Decide whether the following are possible. If so, make a drawing.

a A hexagon with all angles equal but not all sides equal.
b A hexagon with all sides equal but not all angles equal.

10 This is a hexagon with 'square corners' (i.e. every interior angle is 90° or 270°).



a Draw an octagon with 'square corners'.
b Draw a decagon with 'square corners'.
c Draw a dodecagon with 'square corners'.

Enrichment: Maximum reflex

11 Recall that a non-convex polygon has at least one reflex interior angle.

a Use drawings to decide whether the following are possible.
  i a quadrilateral with 2 reflex angles
  ii a pentagon with 2 reflex angles
  iii a hexagon with 4 reflex angles

b What is the maximum number of interior reflex angles possible for these polygons?
  i quadrilateral
  ii pentagon
  iii octagon

c Write a rule for the maximum number of interior reflex angles for a polygon with n sides.

Keeping in touch with numeracy

Non-calculator

1 Units of measurement:

t	mm	ha	m^3
kL	L/100km	$/L	$/hour

From the table, choose the most suitable unit of measurement for measuring each of the following.

a water in a pool
b length of a desk
c area of a small farm
d petrol consumption
e pay rate
f mass of a car
g space inside a room
h cost of petrol

3 Find the sum of 8, 9, 10, 11 and 12.

5 Which is bigger: $\frac{2}{3}$ or $\frac{3}{5}$?

7 How many 250 mL cups can be filled from a 10 litre bottle?

9 Two of the angles in a triangle are 50° degrees and 60°. How big is the other angle?

11 If 10 apples cost $8, find the cost of 25 apples.

13 If I run 1 km every 5 minutes, how far could I run in 2 hours?

15 A building plan has a scale of 1 : 100. Therefore, 1 mm represents 100 mm. A square bedroom has sides of 30 mm on the plan. Find the floor area of the room, in square metres.

17 Increase $25 by 10%.

19 A 25-metre rope is cut into two pieces. One is 7 metres longer than the other. How long is the shorter piece?

Calculator

2 Perform the following conversions.

a minutes to 3.75 hours
b tonnes to 3750 kilograms
c square metres to 3.75 hectares
d millimetres to 3.75 metres
e litres to 375 mL

4 Find the sum of the first five multiples of 5.

6 Which is bigger: $\frac{5}{12}$ or 40%?

8 A 10 litre bottle is leaking 50 mL per hour. How many litres will be left after 1 day?

10 In a parallelogram, one of the angles is 37°. How big are the other three angles?

12 Which represents the best value: 375 mL for $3.50 or 1 litre for $10

14 A marathon runner covered 42.2 km in 2 hours 18 minutes. Convert this speed to metres per minute.

16 A building plan has a scale of 1 : 100. Therefore, 1 mm represents 100 mm. A hallway is 3.5 metres long. How long should it be on the plan?

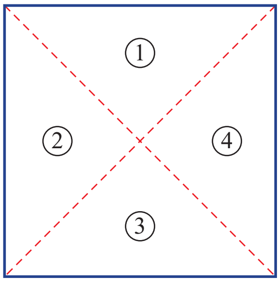
18 Greg bought an item for $25 and sold it for $35. What is the percentage profit?

20 A 25-metre rope is cut so that one piece is nine times longer than the other. How long is the longer piece?

7E Congruent triangles

Stage
5.2
5.2◊
5.1
4

Two shapes are congruent when they are identical. Their matching angles are equal and their matching sides have the same length.



The diagonals divide a square into four congruent triangles.

Let's start: Constructing congruent triangles

Drilling for Gold 7E1

For this task you will need a ruler, pencil and protractor. (You might also use compasses.) Divide these constructions up equally among the members of the class. Each group is to draw one of the following triangles, with the given properties.

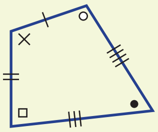
1 triangle ABC with $AB = 8$ cm, $AC = 5$ cm and $BC = 4$ cm
2 triangle DEF with $DE = 7$ cm, $DF = 6$ cm and $\angle EDF = 40^\circ$
3 triangle GHI with $GH = 6$ cm, $\angle IGH = 50^\circ$ and $\angle IHG = 50^\circ$
4 triangle JKL with $\angle JKL = 90^\circ$, $JL = 5$ cm and $KL = 4$ cm

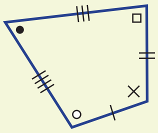
- Compare all triangles with the vertices ABC. What do you notice? What does this say about two triangles that have three pairs of equal side lengths?
- Compare all triangles with the vertices DEF. What do you notice? What does this say about two triangles that have two pairs of equal side lengths and the included angles equal?
- Compare all triangles with the vertices GHI. What do you notice? What does this say about two triangles that have two equal corresponding angles and one corresponding side of equal length?
- Compare all triangles with the vertices JKL. What do you notice? What does this say about two triangles that have one right angle, the hypotenuse and one other corresponding side of equal length?

Key ideas

Drilling for Gold 7E2 7E3

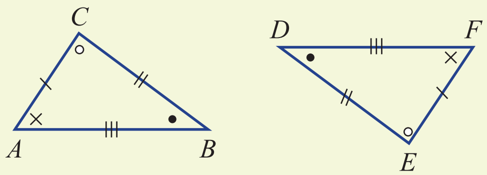
■ **Congruent figures** have the same shape and size. They are identical.





Congruent figures Figures that are exactly the same size and shape

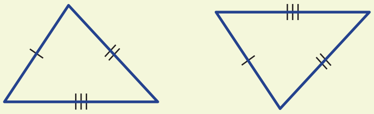
- If triangle ABC ($\triangle ABC$) is congruent to triangle DEF ($\triangle DEF$), we write $\triangle ABC \equiv \triangle FDE$. This is called a congruence statement.



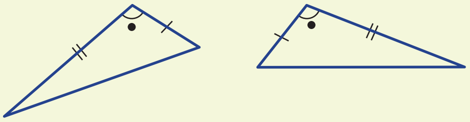
Matching sides	Matching angles
$AB = FD$	$\angle A = \angle F$
$BC = DE$	$\angle B = \angle D$
$AC = FE$	$\angle C = \angle E$

Matching sides
Sides that are in the same position in two or more shapes

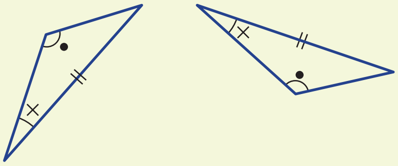
- **Matching sides** are opposite matching angles.
- Tests for triangle congruence:
  - Side, Side, Side (SSS)
    Three pairs of matching sides are equal.



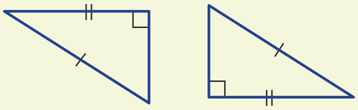
  - Side, Angle, Side (SAS)
    Two pairs of matching sides and the included angle are equal.



  - Angle, Angle, Side (AAS)
    Two angles and any pair of matching sides are equal.



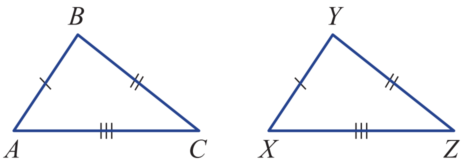
  - Right angle, Hypotenuse, Side (RHS)
    A right angle, the hypotenuse and one other pair of matching sides are equal.



Exercise 7E

Understanding

1 These two triangles are congruent.



a Name the vertex on $\triangle XYZ$ that matches:
i B **ii** A **iii** C

b Name the side on $\triangle XYZ$ that matches:
i AB **ii** AC **iii** BC

c Name the angle in $\triangle ABC$ that matches:
i $\angle X$ **ii** $\angle Y$ **iii** $\angle Z$

2 Use the Key ideas to complete these sentences.

a Congruent figures are exactly the same shape and __________.

b If triangle ABC is congruent to triangle STU, then we write $\triangle ABC \equiv$ _______.

c The short names of the four congruence tests for triangles are SSS, _____, _____ and ______.

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3 Write a congruence statement if:
- **a** triangle ABC is congruent to triangle FGH
- **b** triangle DEF is congruent to triangle STU

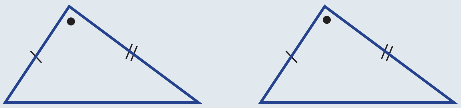
Example: $\triangle DEF \equiv \triangle GHI$

Fluency

Example 10 Choosing a congruence test

Which congruence test (i.e. SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent?

a



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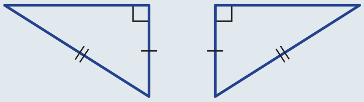

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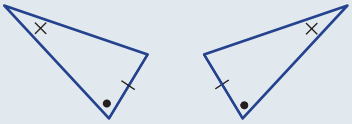

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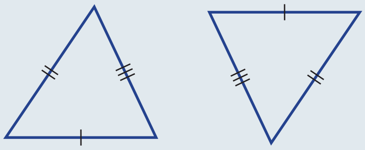
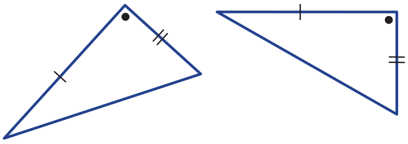


Solution	**Explanation**
a SAS	Two pairs of matching sides and the included angle are equal.
b RHS	A right angle, hypotenuse and one pair of matching sides are equal.
c AAS	Two angles and a pair of matching sides are equal.
d SSS	Three pairs of matching sides are equal.

4 Which congruence test (i.e. SSS, SAS, AAS or RHS) would be used to show that these pairs of triangles are congruent?

a



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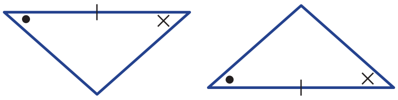

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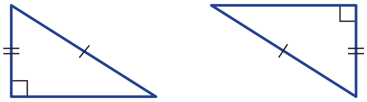

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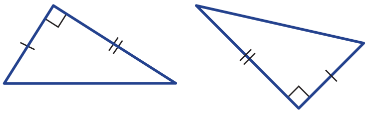

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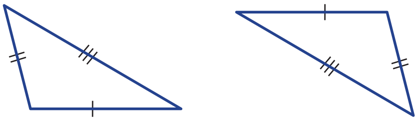

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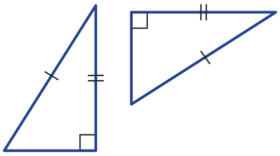

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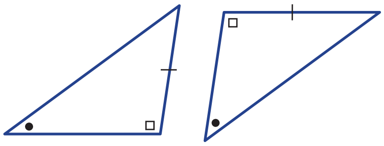

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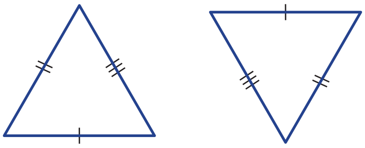
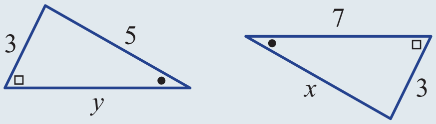


Example 11 Finding missing side lengths and angles using congruence

Find the values of the pronumerals in these pairs of congruent triangles.

a



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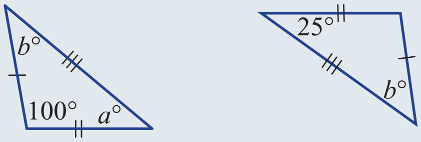
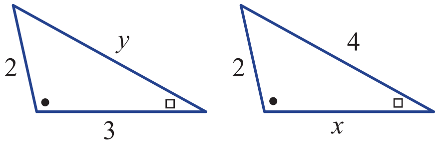


Solution	**Explanation**
a $x = 5$	The side of length x and the side of length 5 are in matching positions (i.e. opposite the ▫).
$y = 7$	The longest side on both triangles must be equal.
b $a = 25$	The angle marked $a°$ matches the 25° angle in the other triangle.
$b = 180 - 100 - 25$ $= 55$	The sum of three angles in a triangle is 180°.

5 Find the values of the pronumerals in these pairs of congruent triangles.

a



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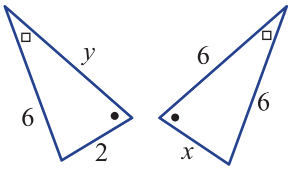

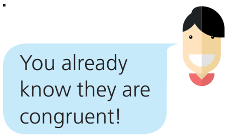

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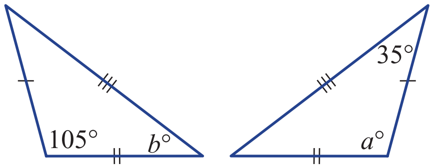

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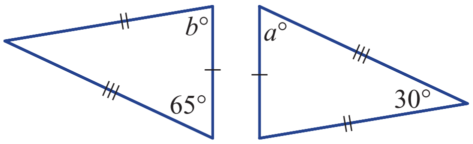

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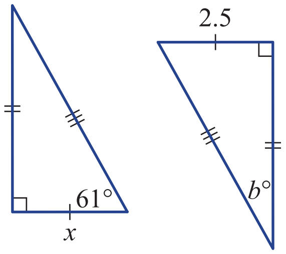

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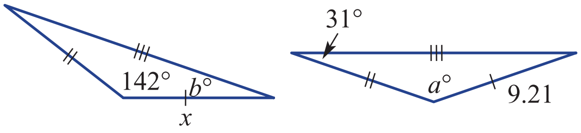
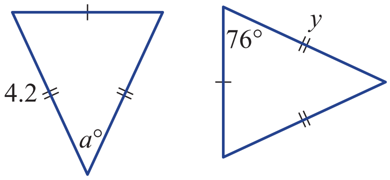
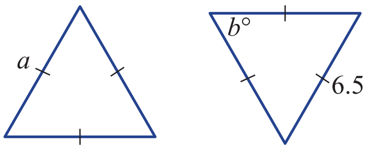


g

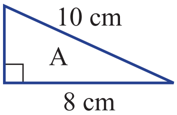


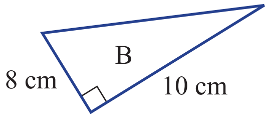
h

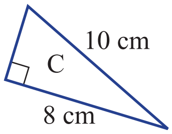


Problem-solving and Reasoning

6 Measurements were taken for three matching triangular tiles and are shown here.







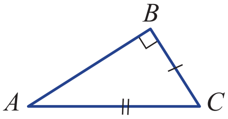
a Which two triangles are congruent?

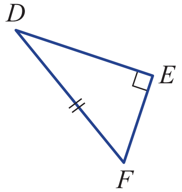
b What reason (i.e. SSS, SAS, AAS or RHS) is used to explain their congruence?

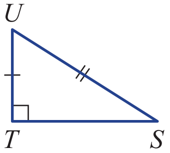
7 For each set of three triangles, choose the two that are congruent. Give a reason (i.e. SSS, SAS, AAS or RHS) and write a congruence statement (e.g. $\triangle ABC \equiv \triangle FGH$).

Write matching vertices in the same order.

a







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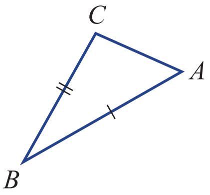

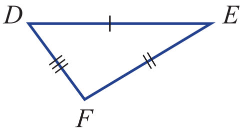

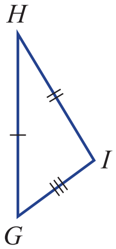

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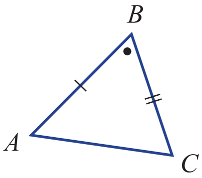
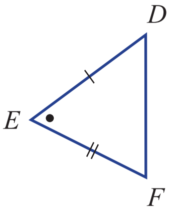
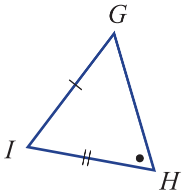
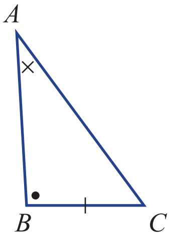
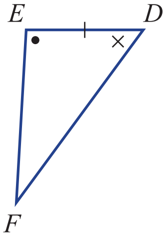


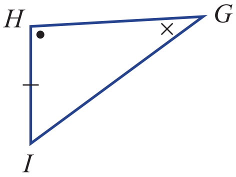




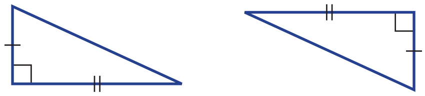
d







8 Explain why RHS is not the first reason you would use to explain why these two triangles are congruent.



Drilling for Gold 7E4

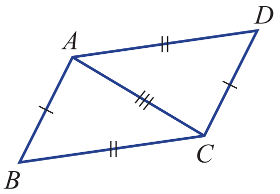
9 Are all triangles with three pairs of equal matching angles congruent? Explain why or why not.

Draw examples to justify your decision.

10 $ABCD$ is a parallelogram.

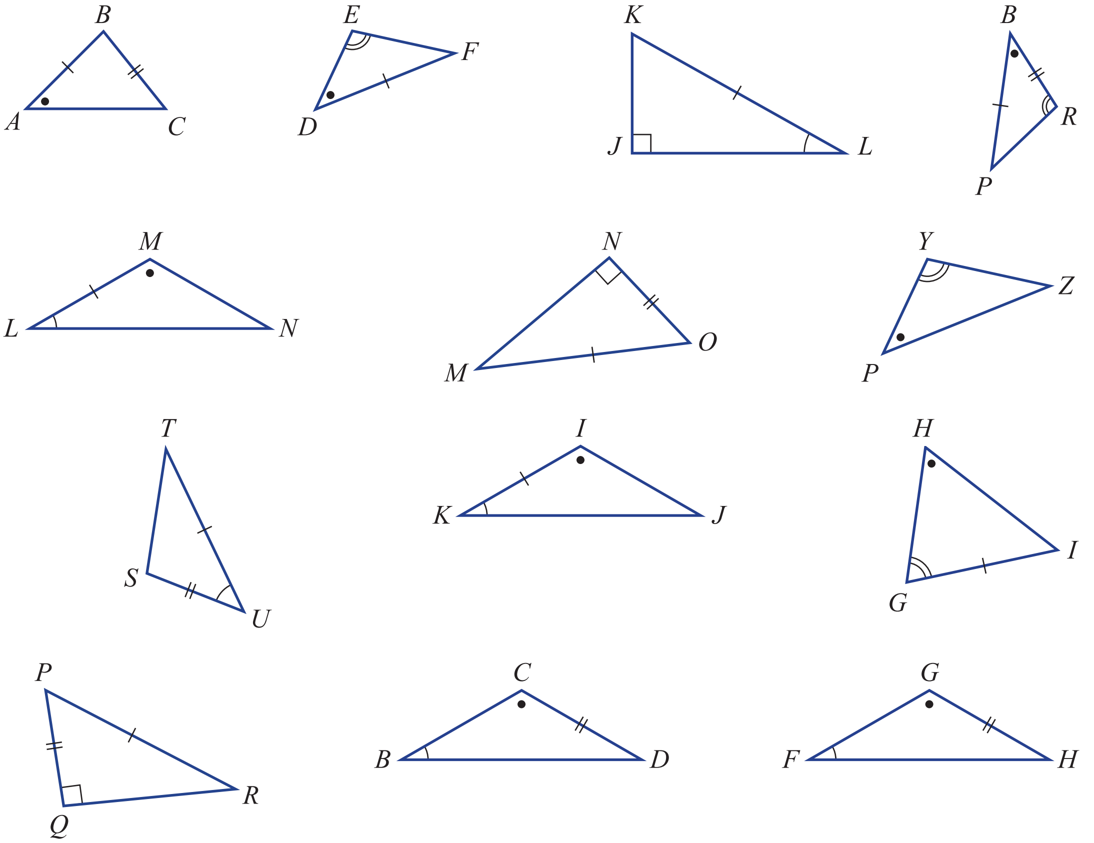
a Give the reason why $\triangle ABC \equiv \triangle CDA$.

b What does this say about $\angle B$ and $\angle D$?



Enrichment: Fishing for congruence

11 Identify all pairs of congruent triangles from those below. Angles with the same mark are equal.

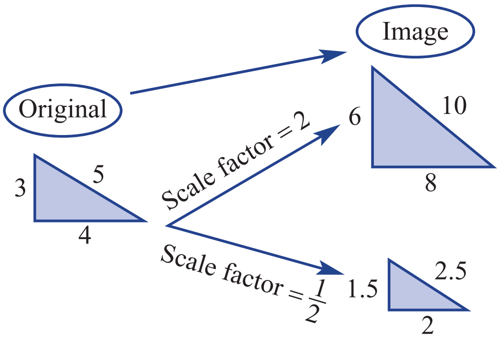


ISBN 978-1-316-61816-5

7F Enlargement, scale factor and similar figures

Stage
5.2
5.2◊
5.1
4

Similar figures have the same shape but not necessarily the same size. Architects and artists use scale factors to create small images and models of larger objects.



Let's start: Enlarging an image (the old-fashioned way)

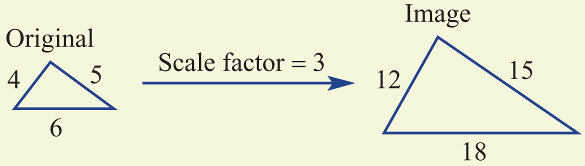
Drilling for Gold 7F1

The image below shows a map of NSW and ACT with a grid overlay.

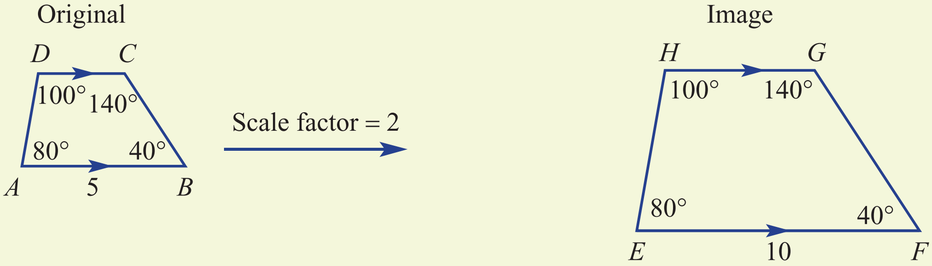
Download the 'Drilling for Gold' activity to print a sheet of the larger grid. Enlarge the map of New South Wales onto the larger grid and indicate the position of all the towns on your larger map. Your teacher may have a prize for the most accurate enlargement.

Key ideas

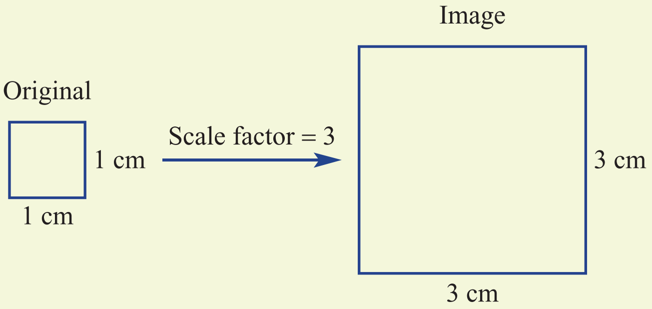
- **Enlargement** is a transformation that involves the increase or decrease in size of an object.
  - The 'shape' of the object is unchanged.
  - Enlargement is done by multiplying all dimensions of a shape by a **scale factor**.



- The scale factor $= \dfrac{\text{image length}}{\text{original length}}$
- If the scale factor is greater than 1, the image will be larger than the original. If the scale factor is between 0 and 1, the image will be smaller. If the scale factor is 1, the image will be congruent.
- Two figures are **similar** if one can be enlarged to be congruent to the other.
  - Matching angles are equal.
  - Pairs of matching sides are in the same proportion or ratio.
- The symbol ||| is used to describe similarity.
  For example: $ABCD \ ||| \ EFGH$
  - The letters are usually written in matching order.
  - Scale factor $= \dfrac{EF}{AB} = \dfrac{10}{5} = 2$, so all the sides of the image are twice the length of the original shape.



- All squares are similar.
- All circles are similar.
- All equilateral triangles are similar.



Enlargement A transformation that changes the size of a figure without changing its shape

Scale factor The number by which you multiply each side length to enlarge or reduce the size of a shape

Similar figures Figures of the same shape but not the same size

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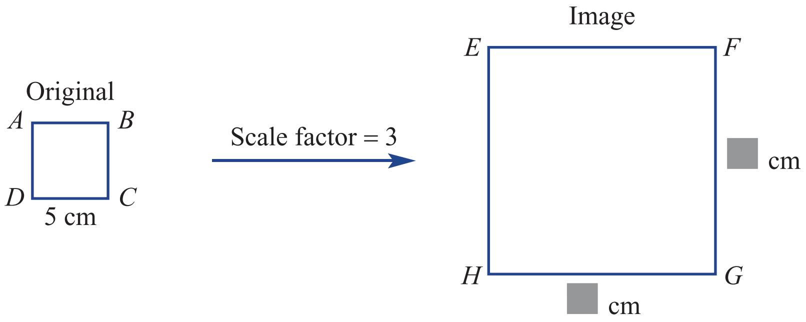
Exercise 7F

Understanding

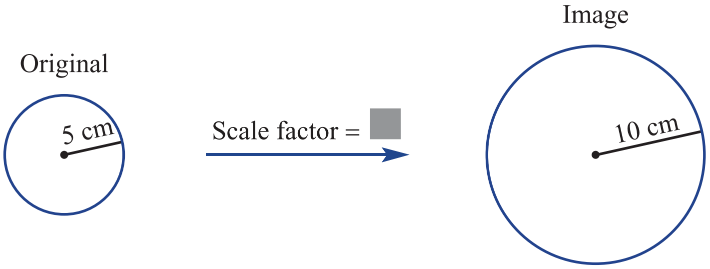
Drilling for Gold 7F2

1 Write the missing information, represented by the grey square (■) in your exercise book or in the Drilling for Gold worksheet.
Note: Diagrams are not to scale.

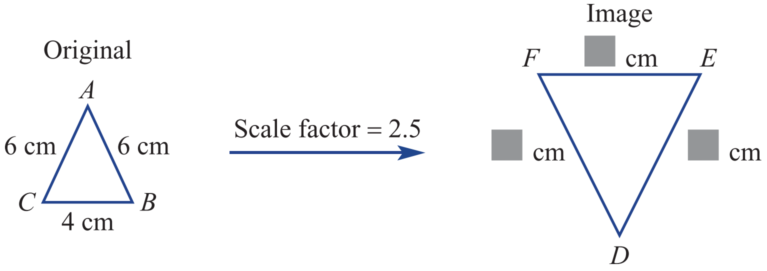
a *ABCD* is a square.



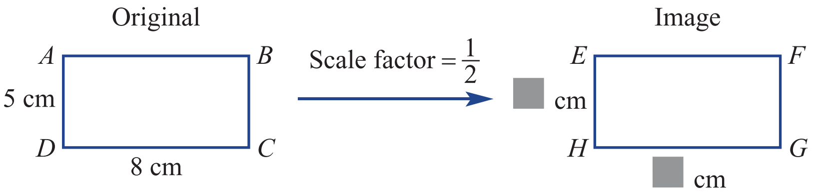
b



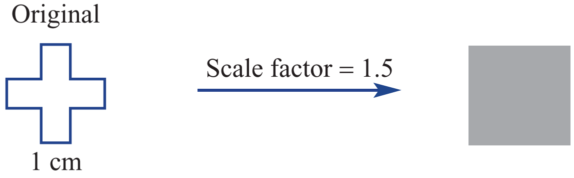
c



d *ABCD* is a rectangle.



e

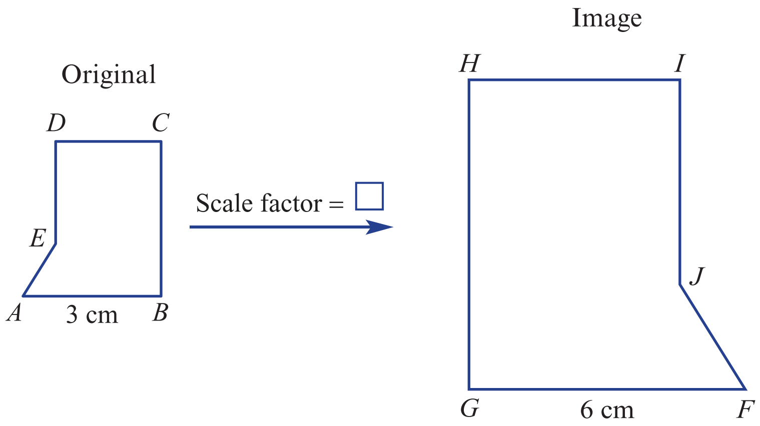


All sides are 1 cm.
All angles are 90°.

2 The two figures below are similar.

- **a** Name the angle in the larger figure that matches to $\angle A$.
- **b** Name the angle in the smaller figure that matches to $\angle I$.
- **c** Name the side in the larger figure that matches to BC.
- **d** Name the side in the smaller figure that matches to FJ.
- **e** Use FG and AB to find the scale factor.

For part **e**, use Scale factor $= \frac{\text{image length}}{\text{original length}}$

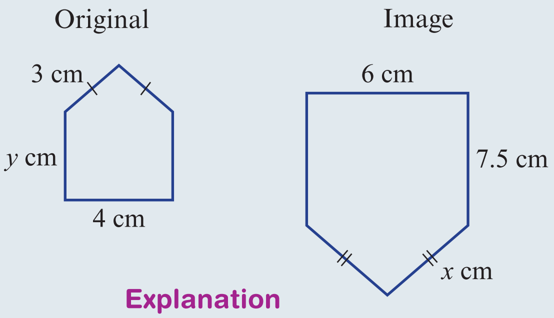


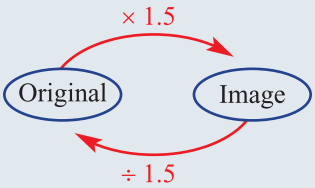
Fluency

Example 12 Using the scale factor

These figures are similar.

- **a** Find a scale factor.
- **b** Find the value of x.
- **c** Find the value of y.



Solution	**Explanation**
a $\text{Scale factor} = \frac{\text{image}}{\text{original}}$ $= \frac{6}{4}$ $= 1.5 \text{ or } \frac{3}{2}$	Choose two matching sides and use $\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$ 
b 3 cm on original = 3 cm × 1.5 on image $\therefore\ x = 4.5$	Multiply by the scale factor to get the length of the side of the larger image.
c 7.5 cm on image = 7.5 ÷ 1.5 on original $\therefore\ y = 5$	Divide by the scale factor to get the length of the side of the smaller image.

7F

Drilling for Gold 7F3

3 Draw a rectangle with base 6 cm and height 4 cm. Use the following scale factors to draw similar rectangles.

a 3 **b** 2 **c** 1.5
d 1.25 **e** 0.5 **f** 0.75

4 Answer true or false.

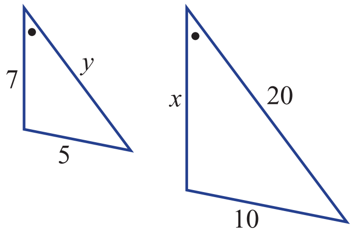
a All squares are similar.
b All rectangles are similar.
c All equilateral triangles are similar.
d All isosceles triangles are similar.
e All circles are similar.

5 Each of the pairs of figures shown here are similar. For each pair find:

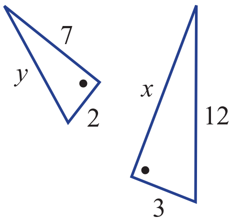
i a scale factor
ii the value of x
iii the value of y

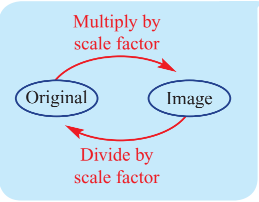
Scale factor = $\frac{\text{image}}{\text{original}}$
Make sure you choose a matching pair of sides.

a

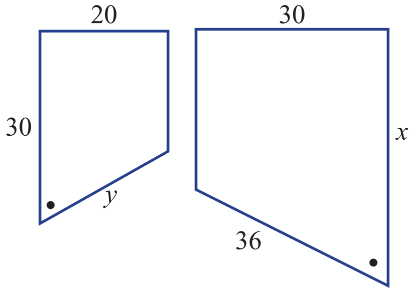


b

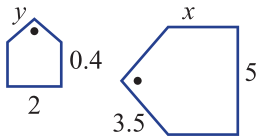




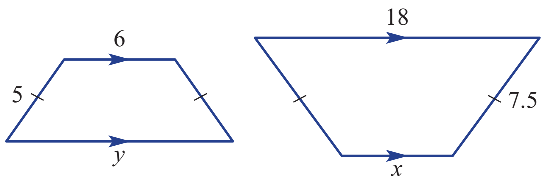
c



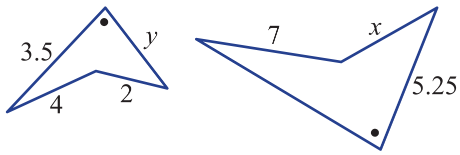
d



e



f



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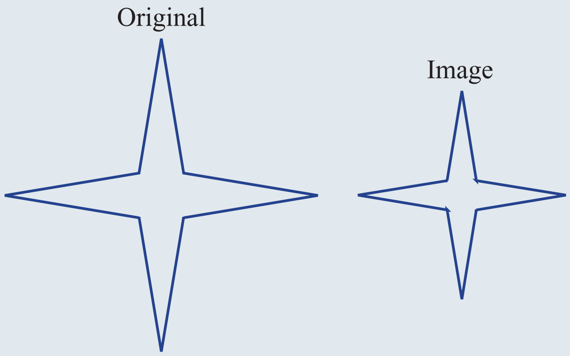
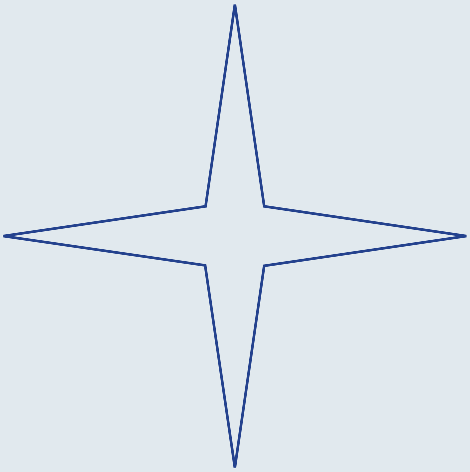
Example 13 Measuring lengths to estimate a scale factor

a Measure the original and the image to estimate the scale factor.



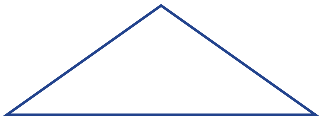
b On grid paper, use a ruler and a protractor to make another image of the original, using a scale factor of 1.5.

Solution	**Explanation**
a Each side of the original is approximately 18 mm. Each side of the image is approximately 12 mm. $\text{Scale factor} = \frac{\text{image}}{\text{original}}$ $\approx \frac{12}{18}$ ≈ 0.7 (to 1 d.p.)	Measure a line segment in the original, in mm. Measure the matching line segment in the image. $\text{Scale factor} = \frac{\text{image}}{\text{original}}$
b 	The edges of the original are approximately 18 mm. $18 \times 1.5 = 27$ The edges of the image will be 27 mm. The angles will not change in size.

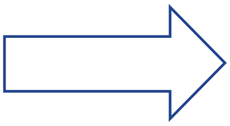
Drilling for Gold 7F4

6 Use a ruler, calculator and grid paper to enlarge these shapes, using a scale factor of 1.5.

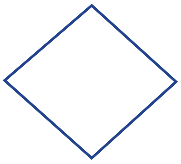
a



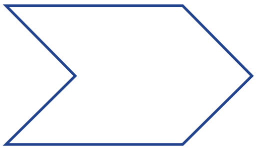
b



c



d

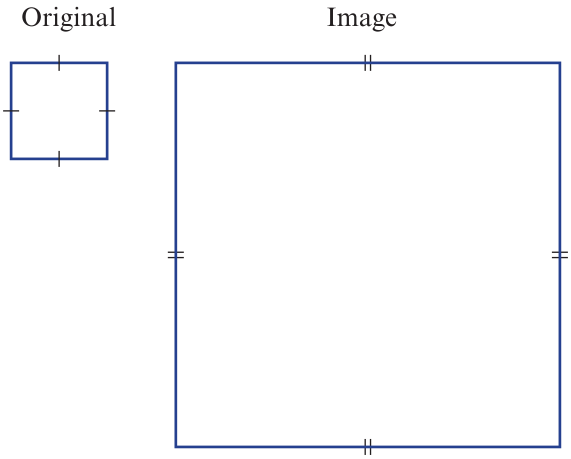


7 Repeat Question **6**, using a scale factor of 0.5.

Problem-solving and Reasoning

8 A square is enlarged by a scale factor of 4.

- **a** Are the internal angles the same for both the original and the image?
- **b** If the side length of the original square is 2 cm, what will be the side length of the image square?
- **c** If the side length of the image square is 100 m, what will be the side length of the original square?



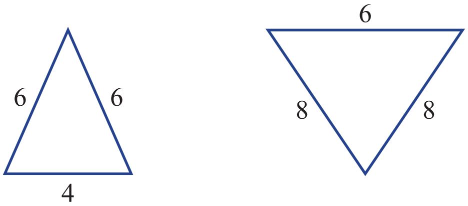
9 A photo of a sunflower is enlarged by a scale factor of 2. If the larger flower has a diameter of 7.4 cm, what is the diameter of the smaller flower?

10 An A4 sheet of paper is 210 mm by 297 mm. An A3 sheet of paper is 297 mm by 420 mm. What is the scale factor, correct to two decimal places?

11 Explain why:

- **a** any two squares are similar
- **b** any two equilateral triangles are similar
- **c** any two rectangles are not necessarily similar
- **d** any two isosceles triangles are not necessarily similar
- **e** a scale factor of 1 gives congruent shapes

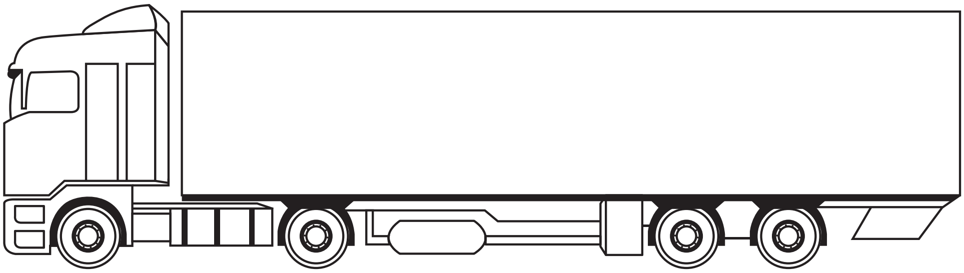
12 **a** Explain why a 12 cm by 8 cm rectangle is not similiar to a 16 cm by 12 cm rectangle.
b Are these isosceles triangles similar?



Enrichment: Actual lengths from drawings and maps

13 The container of this truck is 12.5 metres long.

a Convert 12.5 metres to millimetres.
b Measure the length of the container on the truck in the drawing, in millimetres.
c If the drawing is the original, what scale factor will be used to build the real truck?
d Measure the height of the container on the truck in the drawing, in millimetres.
e Use the scale factor to estimate the height of the container on the real truck, in millimetres.



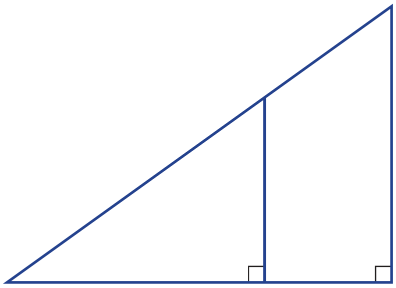
14 A map has a scale ratio of 1 : 50 000.

a What length on the ground is represented by 2 cm on the map?
b What length on the map is represented by 12 km on the ground?

7G Applying scale factor to similar triangles

Stage
5.2
5.2◊
5.1
4

Similar triangles can be used in many mathematical and practical problems. If two triangles are known to be similar, then the properties of similar triangles can be used to find missing lengths or unknown angles. The approximate height of a tall object or the width of a projected image can be found using similar triangles.



This diagram shows two similar triangles.

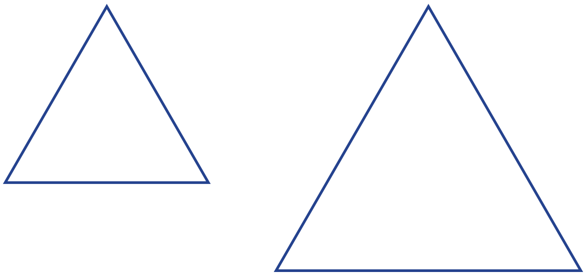
Let's start: Why is sine of 30° exactly 0.5?

When you were doing trigonometry, you may have noticed that your calculator usually gives long, non-repeating decimals. For example, calculate these:

- sin 28°
- sin 29°
- sin 31°

Drilling for Gold 7G1

Now try sin 30°. It is exactly 0.5. The following experiment will explain why this is true.Use the 'Drilling for Gold' worksheet or trace the triangles (right) and follow steps:



1. Choose a triangle.
2. Fold it down the middle along the axis of symmetry.
3. Cut it in half and throw half away.
4. Measure the three angles and write them on your triangle.
5. Measure the three sides and write them on your triangle.
6. Divide the side opposite the 30° angle by the hypotenuse.
7. Repeat the experiment using another equilateral triangle.

- Copy and complete:
  In any right-angled triangle containing an angle of 30°, the ___________ side is exactly _______ of the _____________. Therefore, sine of 30° is exactly ___.

Key ideas

- To apply similarity in practical problems involving triangles that you know to be similar:
  - Calculate the scale factor.
  - Use the scale factor to find the value of any unknowns.

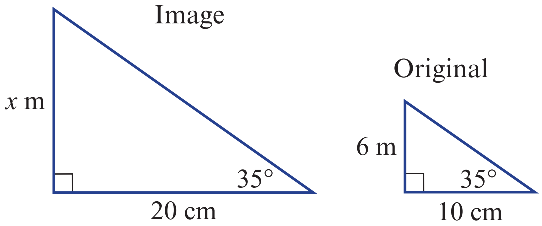
Exercise 7G

Understanding

1 Here are two similar triangles.

a What is the scale factor if the smaller triangle is enlarged to the size of the larger triangle?

b Find the value of x.



2 Use the words in the table to complete the following sentences.

similar	similar	triangle	shape/size

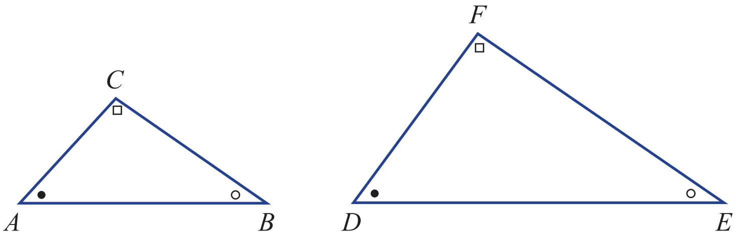
a The symbol $|||$ means 'is ______ to'.

b Similar figures have the same ________ but are not necessarily the same ________.

c $\triangle ABC$ is a ________.

d If $\triangle ABC \; ||| \; \triangle DEF$, then $\triangle ABC$ is ________ to $\triangle DEF$.

3 These two triangles are similar.



a Which vertex on $\triangle DEF$ matches vertex B on $\triangle ABC$?

b Which vertex on $\triangle ABC$ matches vertex F on $\triangle DEF$?

c Which side on $\triangle DEF$ matches side AC on $\triangle ABC$?

d Which side on $\triangle ABC$ matches side EF on $\triangle DEF$?

e Which angle on $\triangle ABC$ matches $\angle D$ on $\triangle DEF$?

f Which angle on $\triangle DEF$ matches $\angle B$ on $\triangle ABC$?

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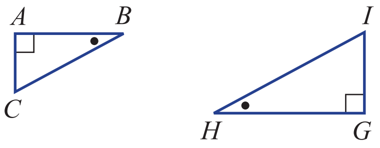
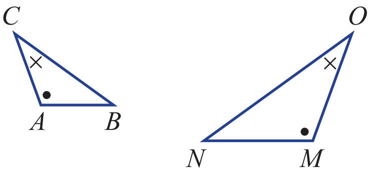
Fluency

4 Write similarity statements for these pairs of similar triangles. Write letters in matching order.

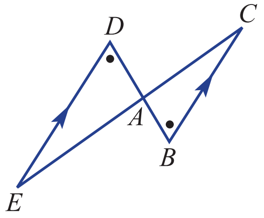
a



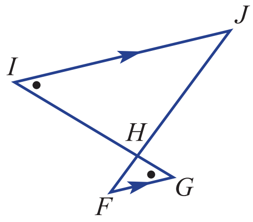
b



c

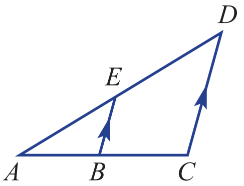


d

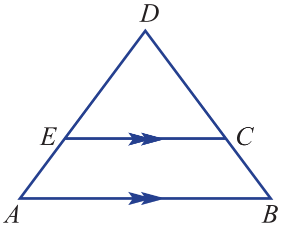


Here is an example of a similarity statement:
$\triangle ABC \,|||\, \triangle DEF$

e

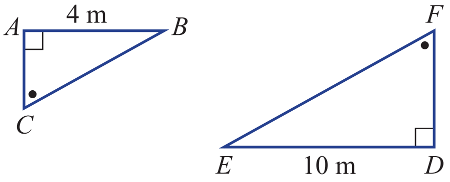


f



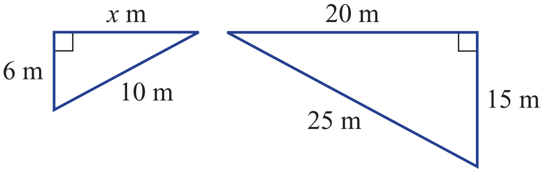
For parts **e** and **f**, use angles in parallel lines.

5 What is the scale factor on this pair of similar triangles that enlarges $\triangle ABC$ to $\triangle DEF$?



6 For this pair of similar triangles:

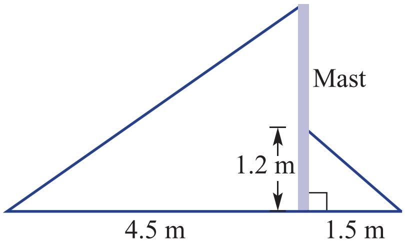
a Find the scale factor.
b Find the value of x.



7 Two similar triangles are created by cables supporting a yacht's mast.

a Find a scale factor for the two triangles.

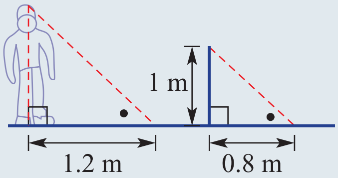
b Find the height of the mast.



Example 14 Applying similarity

Chris' shadow is 1.2 m long and a 1 m vertical stick has a shadow that is 0.8 m long.

Determine Chris' height.

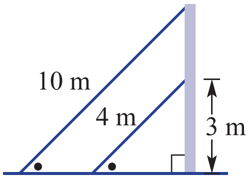


Solution	Explanation
Scale factor $= \dfrac{1.2}{0.8} = 1.5$	First, find the scale factor.
$\therefore$ Chris' height $= 1 \times 1.5$ $= 1.5$ m	Multiply the height of the stick by the scale factor to find Chris' height.

8 A tree's shadow is 20 m long and a 2 m vertical stick has a shadow that is 1 m long. Find the height of the tree.

Construct a diagram as in Example 14.

9 Two cables support a steel pole at the same angle, as shown. The two cables are 4 m and 10 m in length. The shorter cable reaches 3 m up the pole. Find the height of the pole.



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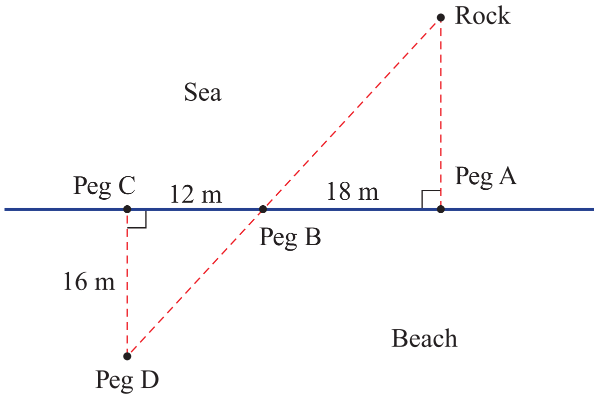
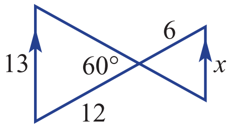
10 Ali is at the beach and decides to estimate how far an exposed rock is from the seashore. He places four pegs in the sand, as shown, and measures the distance between them. How far is the rock from the beach?



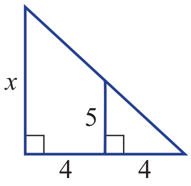
Problem-solving and Reasoning

11 For each pair of similar triangles, find the value of x.

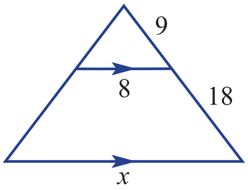
a



b

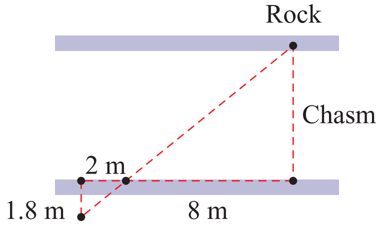


c



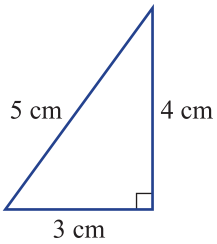
Consider angle properties in parallel lines.

12 A deep chasm has a large rock sitting on its side, as shown. Find the width of the chasm.



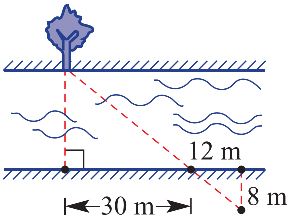
Don't forget to find the scale factor.

13 If the triangle below was enlarged using a whole-number scale factor, what is the biggest scale factor that would still fit on this page?



Skillsheet 7B

14 Joanne wishes to determine the width of the river shown, without crossing it. She places four pegs as shown. Calculate the river's width.



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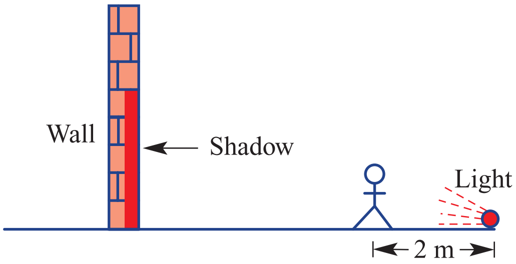
15 A person 1.8 m tall stands in front of a light that sits on the floor.
The person casts a shadow on the wall behind them.



a How tall will the shadow be if the distance between the wall and the light is:
  i 4 m?
  ii 10 m?
  iii 3 m?

b How tall will the shadow be if the distance between the wall and the person is:
  i 4 m?
  ii 5 m?

c Find the distance from the wall to the person if the shadow is:
  i 5.4 m
  ii 7.2 m

Enrichment: Enlargements of photographs

16 When you take some photos to a shop to be 'enlarged' there are several options, which are sometimes expressed using inches. The standard size is 4 inches by 6 inches, which is usually called a '4 by 6'.
Here are some other sizes that are sold in Australia.

5 by 7	6 by 8	8 by 10
8 by 12	11 by 14	12 by 16
16 by 20	20 by 24	20 by 30

a '4 by 6' can be simplified to the ratio 2 : 3. Express the other nine sizes as simplified ratios.
b A '4 by 6' is similar to the '8 by 12'. The scale factor is 2. What other size is similar to the '4 by 6'? What is the scale factor?
c What size is similar to the '6 by 8'? What is the scale factor?
d What size is similar to the '8 by 10'? What is the scale factor?

Maths@home

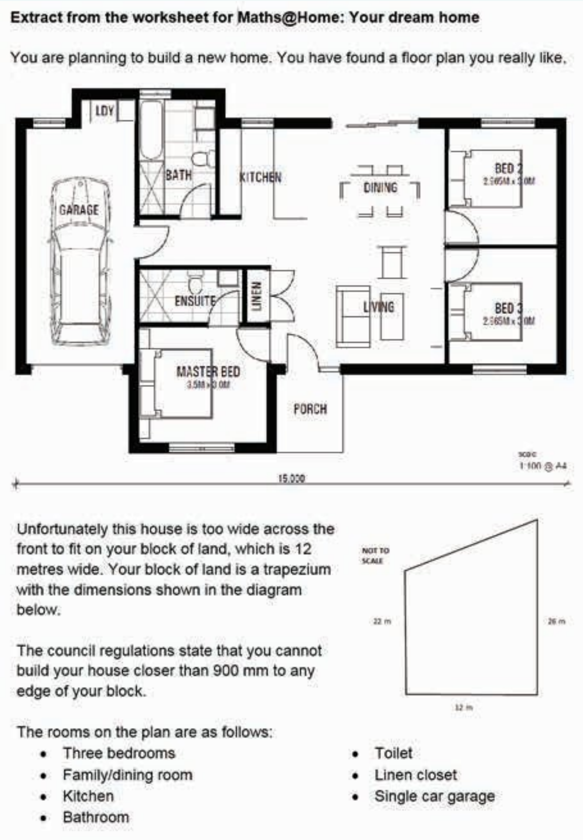
Worksheet 7A

Your dream home (but it's too big for your block of land!)

You have bought a block of land and found a plan for your dream home but it is too big for the block.

Download the worksheet from the interactive textbook, which includes:

- A rough drawing of the block of land, which you are required to draw to scale.
- A plan of the house, which needs to be modified so it will fit on the block.



Extract from the worksheet for Maths@Home: Your dream home

You are planning to build a new home. You have found a floor plan you really like.

Unfortunately this house is too wide across the front to fit on your block of land, which is 12 metres wide. Your block of land is a trapezium with the dimensions shown in the diagram below.

The council regulations state that you cannot build your house closer than 900 mm to any edge of your block.

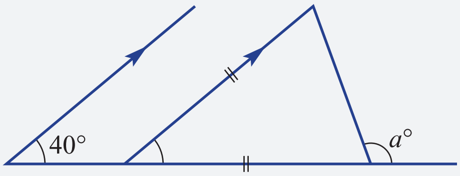
The rooms on the plan are as follows:

- Three bedrooms
- Family/dining room
- Kitchen
- Bathroom
- Toilet
- Linen closet
- Single car garage

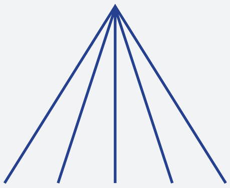
Puzzles and games

1 Use 12 match sticks to make 6 equilateral triangles.

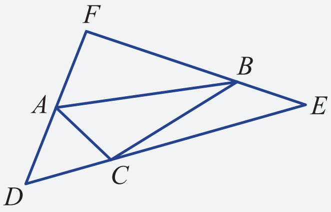
2 Find the value of a in the diagram on the right.



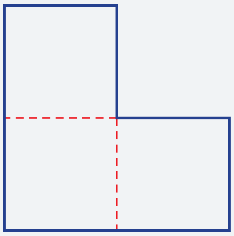
3 How many acute angles are there in this diagram?



4 Explore (using dynamic geometry software) where the points A, B and C should be on the sides of acute-angled $\triangle DEF$ so that the perimeter of $\triangle ABC$ is a minimum.



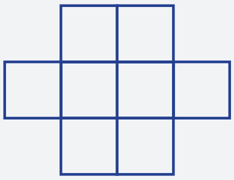
5 Draw this shape on square grid paper.



Use vertical and horizontal cuts.

Divide the shape into four congruent (i.e. identical) shapes.

6 Write the numbers 1 to 8 into the squares, so that the squares with consecutive numbers do not touch (neither edges nor corners). For example, the square with 2 cannot touch squares with 3 or 4.



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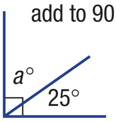
Chapter summary

Angles in a right angle

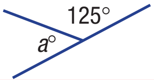
add to 90°



$a + 25 = 90$
$a = 65$

Angles on a straight line

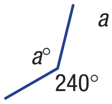
add to 180°



$a + 125 = 180$
$a = 55$

Angles in a revolution

add to 360°

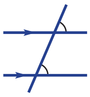


$a + 240 = 360$
$a = 120$

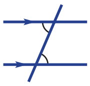
Parallel lines and transversal

When the lines are parallel:

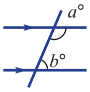
- Corresponding angles are equal.



- Alternate angles are equal.

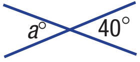


- Cointerior angles are supplementary. $a + b = 180$



Vertically opposite angles

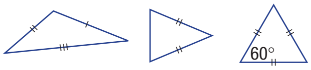
are equal



$a = 40$

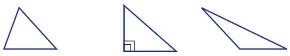
Classifying by sides

scalene isosceles equilateral



Classifying by angles

acute right obtuse

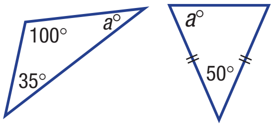


The complement of 50° is 40°.
The supplement of 50° is 130°.

Triangles

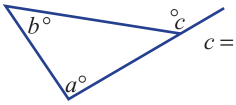
Angle sum = 180°

Scalene Isosceles



$a + 135 = 180$	$2a + 50 = 180$
$a = 45$	$2a = 130$
	$a = 65$

Exterior angles



$c = a + b$

Angles

Acute - between 0° and 90°
Right - 90°
Obtuse - between 90° and 180°
Straight - 180°
Reflex - between 180° and 360°
Revolution - 360°

Properties of geometrical figures

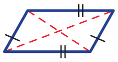
Polygons

The sum of the interior angles in a polygon with n sides is
$S = (n - 2) \times 180^\circ$
In a regular polygon:

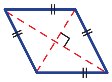
- All sides are equal.
- All angles are equal.

Special quadrilaterals

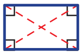
- Parallelogram



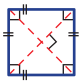
- Rhombus



- Rectangle

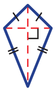


- Square



- Trapezium

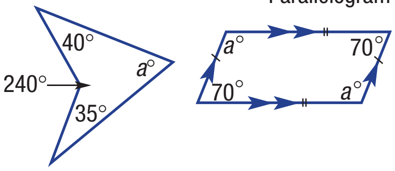
- Kite



Quadrilaterals

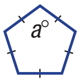
Angle sum = 360°

Quadrilateral Parallelogram



$a + 315 = 360$	$a + 70 = 180$
$a = 45$	$a = 110$

Pentagon



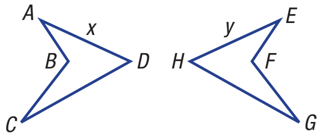
$S = (n - 2) \times 180^\circ$
$= (5 - 2) \times 180^\circ$
$= 540^\circ$

Regular pentagon
$a = \frac{540^\circ}{5} = 108^\circ$

Chapter summary

Congruent figures and similar figures

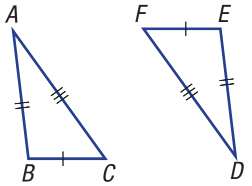
Congruent figures (same shape, same size)



$ABCD \equiv EFGH$ $x = y$

$AB = EF$ $\angle C = \angle G$

Congruent triangles



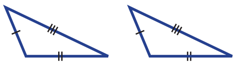
$\triangle ABC \equiv \triangle DEF$

$AB = DE$, $BC = EF$, $AC = DF$

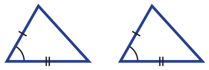
$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

Tests for congruent triangles

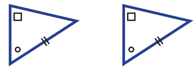
- SSS



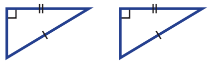
- SAS



- AAS



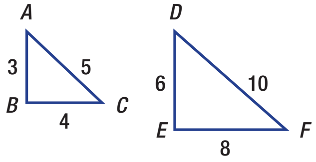
- RHS



Similar figures (same shape, different size)

One can be enlarged to be congruent to the other.

Scale factor



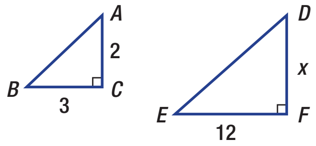
Scale factor = 2

$\triangle ABC\ |||\ \triangle DEF$

- Matching angles are equal.
- Matching sides are in ratio.

Find unknown sides

$\triangle ABC\ |||\ \triangle DEF$



$\times 4$ $2:3$ $\times 4$

$x:12$

$x = 8$

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Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.

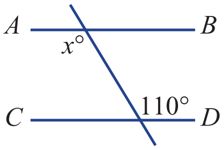
Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

Drilling for Gold 7R1

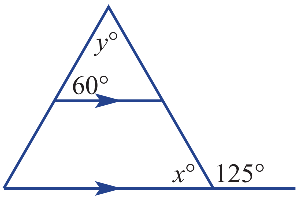
1 The supplement of 55° is:
A 35° **B** 55° **C** 95°
D 125° **E** 305°

2 AB is parallel to CD. The value of x is:
A 110 **B** 70
C 20 **D** 130
E 120



3 If two angles in a triangle are 30° and 80°, then the third angle is:
A 110° **B** 70°
C 90° **D** 80°
E 30°

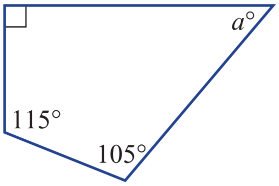
4 The values of x and y in the diagram are:
A $x = 35, y = 85$
B $x = 45, y = 45$
C $x = 50, y = 60$
D $x = 55, y = 65$
E $x = 65, y = 55$



5 Using $S = (n - 2) \times 180°$, the sum of the interior angles in a hexagon is:
A 1078° **B** 360°
C 720° **D** 900°
E 540°

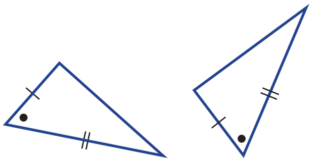
6 The quadrilateral that might have no parallel sides is a:
A kite **B** trapezium
C parallelogram **D** rhombus
E square

7 The value of a in the diagram is:
A 45
B 310
C 75
D 60
E 50



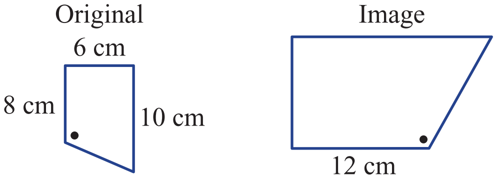
8 The abbreviated reason for congruence in the two triangles shown is:

A AA
B SAS
C SSS
D AAS
E RHS



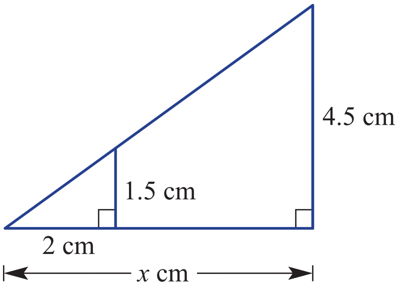
9 The scale factor in the two similar figures shown is:

A $\frac{2}{3}$
B 2
C 1.2
D 1.5
E 0.5



10 The value of x in these similar triangles is:

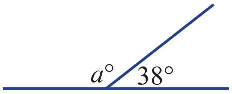
A 6
B 9
C 10
D 8
E 7.5



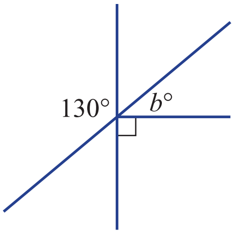
Short-answer questions

1 Find the value of the pronumerals.

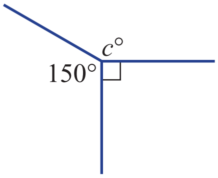
a



b

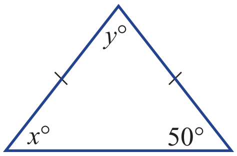


c

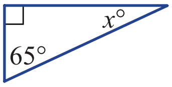


2 Name the following triangles and find the value of the pronumerals.

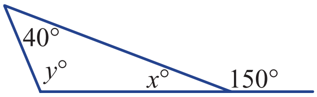
a



b

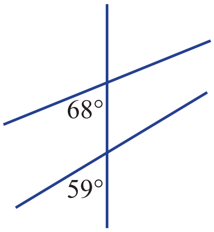


c

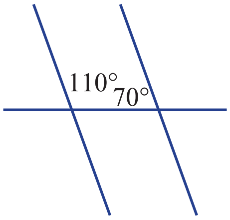


3 Decide whether the following contain a pair of parallel lines. Give a reason.

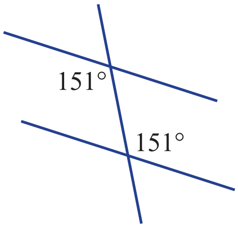
a



b

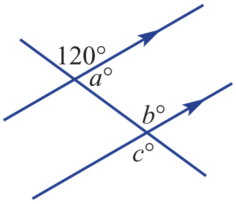


c



4 State the values of the pronumerals.

a



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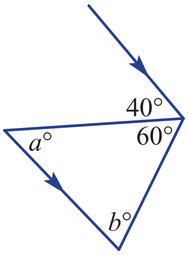


5 Which of the six 'special' quadrilaterals *definitely* have:

a diagonals of equal length?

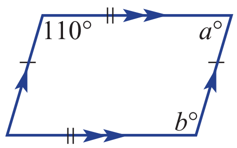
b diagonals intersecting at right angles?

c equal opposite sides?

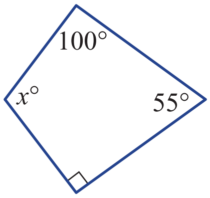
kite	trapezium	rectangle
square	rhombus	parallelogram

6 Find the value of each pronumeral in the following quadrilaterals.

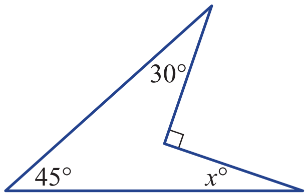
a



b

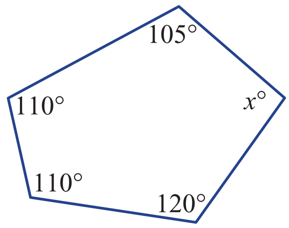


c

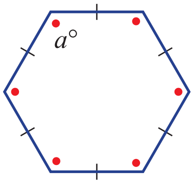


7 Find the value of the pronumerals in these polygons. Use $S = (n - 2) \times 180°$ for the angle sum.

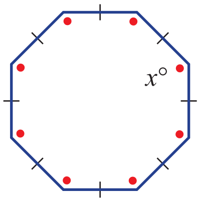
a



b

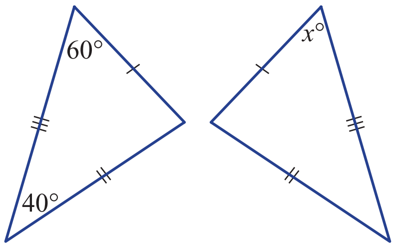


c



8 Determine whether each pair of triangles is congruent. If congruent, give the abbreviated reason (i.e. SSS, SAS, AAS or RHS) and state the value of any pronumerals.

a



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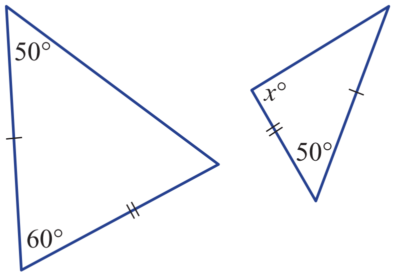

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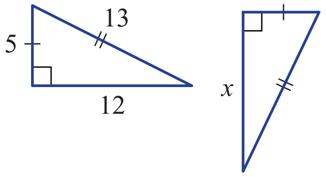
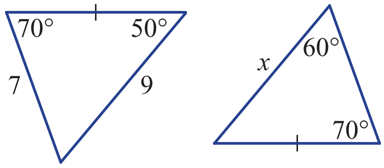


d



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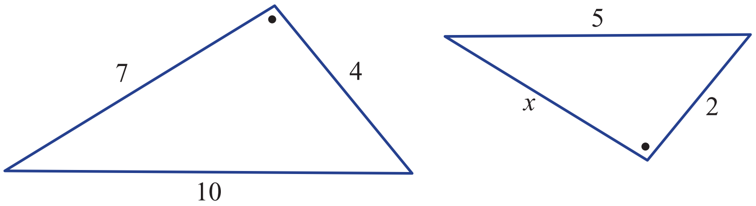
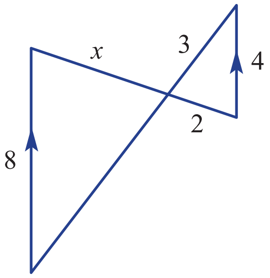
9 For the following pairs of similar triangles, find the value of x.

a



b



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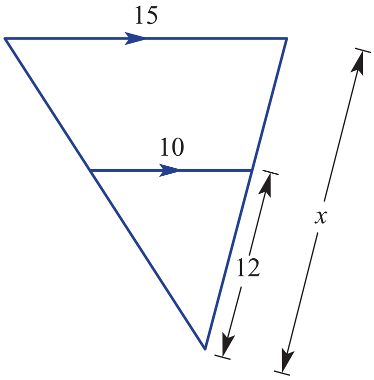
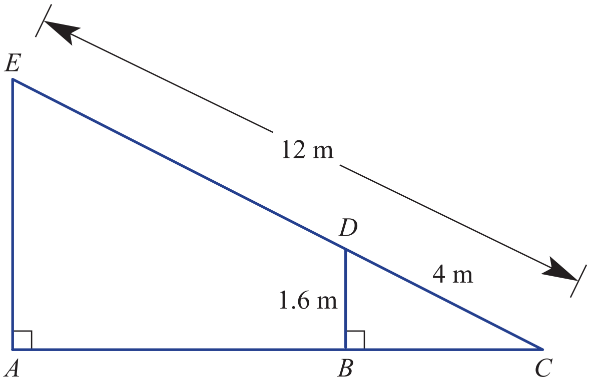
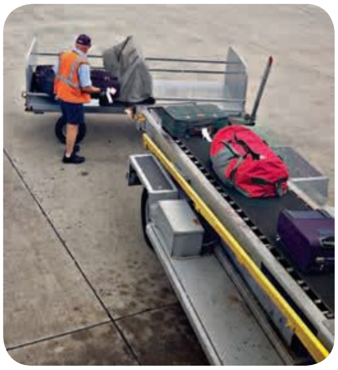


10 A conveyor belt loading luggage onto a plane is 12 m long. A vertical support 1.6 m high is placed under the belt. It is 4 m along the conveyor belt, as shown in the diagram.

a Find a scale factor for the two similar triangles.

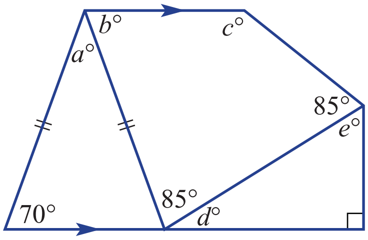
b Find the height (AE) of the luggage door above the ground.





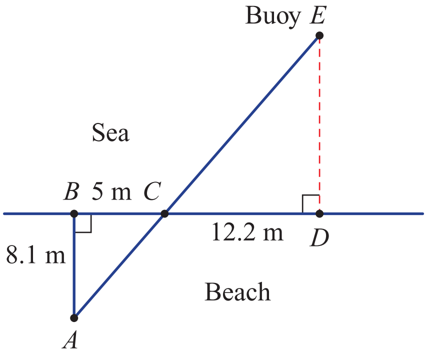
Extended-response questions

1 In this diagram, there are two triangles and one quadrilateral.

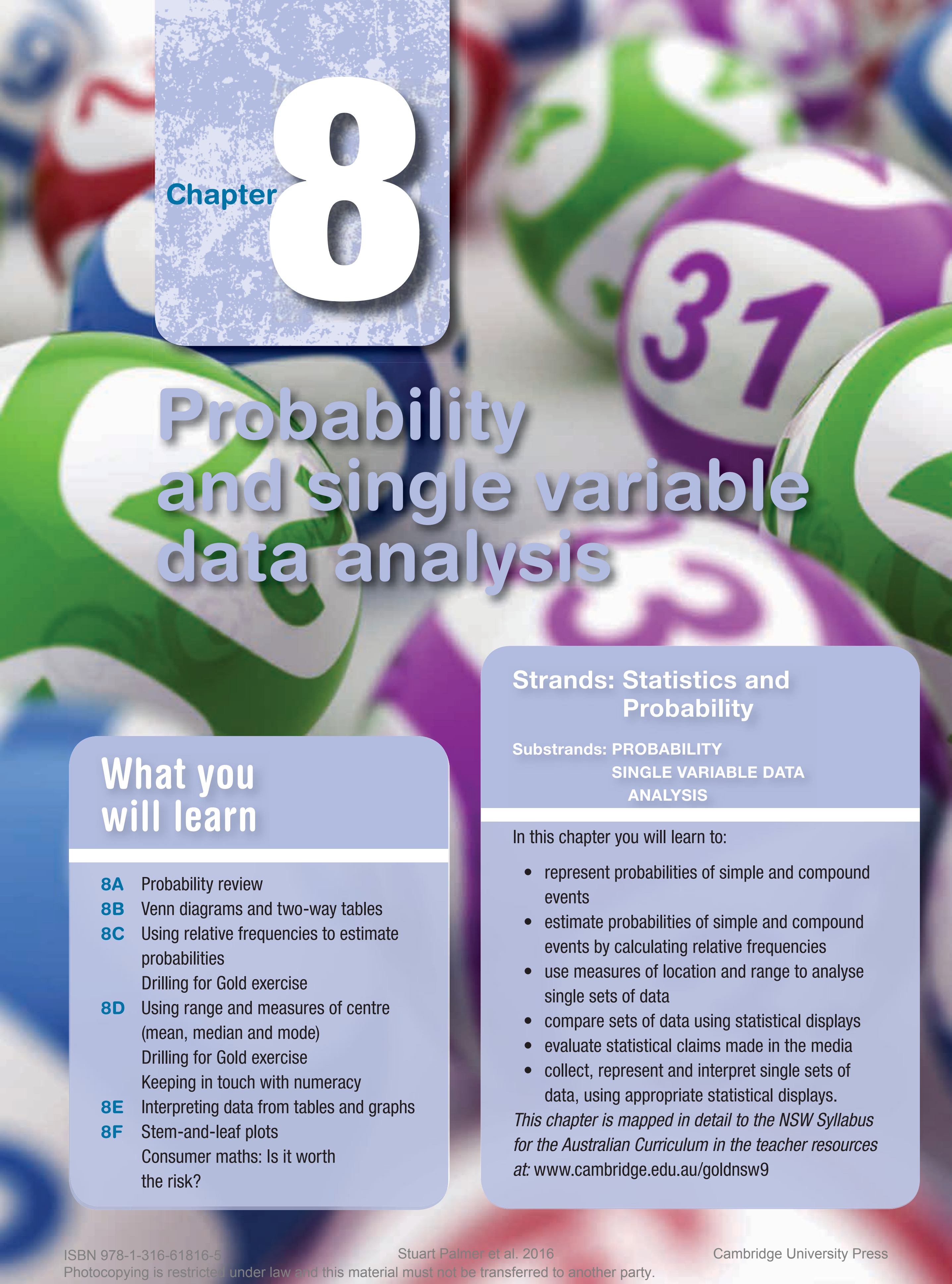


a Name the types of triangles.
b Find the values of the pronumerals, in alphabetical order. Give a reason at each step.

2 A buoy (E) is floating in the sea at some unknown distance from the beach, as shown. The points A, B, C and D are measured and marked out on the beach, as shown. Triangles ABC and EDC are similar.



a Name the angle that is vertically opposite to $\angle ACB$.
b What is the scale factor?
c Find the distance from the buoy to the beach (ED), to one decimal place.



Chapter 8

Probability and single variable data analysis

What you will learn

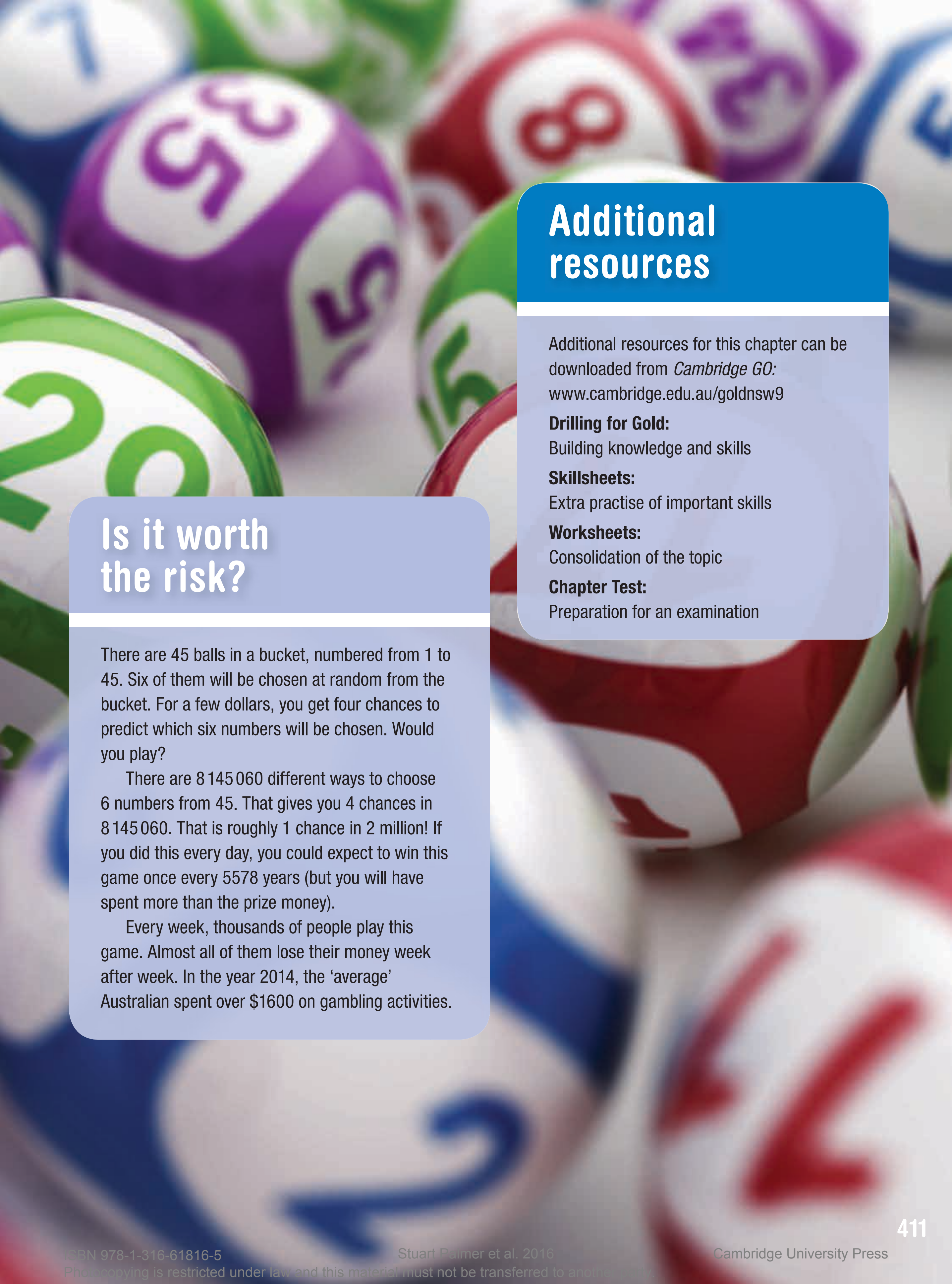
Strands: Statistics and Probability

Substrands: PROBABILITY
SINGLE VARIABLE DATA ANALYSIS

In this chapter you will learn to:

- represent probabilities of simple and compound events
- estimate probabilities of simple and compound events by calculating relative frequencies
- use measures of location and range to analyse single sets of data
- compare sets of data using statistical displays
- evaluate statistical claims made in the media
- collect, represent and interpret single sets of data, using appropriate statistical displays.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw9



Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO:* www.cambridge.edu.au/goldnsw9

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

Is it worth the risk?

There are 45 balls in a bucket, numbered from 1 to 45. Six of them will be chosen at random from the bucket. For a few dollars, you get four chances to predict which six numbers will be chosen. Would you play?

There are 8 145 060 different ways to choose 6 numbers from 45. That gives you 4 chances in 8 145 060. That is roughly 1 chance in 2 million! If you did this every day, you could expect to win this game once every 5578 years (but you will have spent more than the prize money).

Every week, thousands of people play this game. Almost all of them lose their money week after week. In the year 2014, the 'average' Australian spent over $1600 on gambling activities.

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Pre-test

1 Write the following as decimals.

a $\frac{1}{10}$ **b** $\frac{2}{8}$ **c** 30% **d** 85% **e** 23.7%

2 Express the following in simplest form. Use a calculator when necessary.

a $\frac{4}{8}$ **b** $\frac{7}{21}$ **c** $\frac{20}{30}$ **d** $\frac{100}{100}$ **e** $\frac{0}{4}$

f $\frac{12}{144}$ **g** $\frac{36}{72}$ **h** $\frac{36}{58}$ **i** $\frac{72}{108}$ **j** $\frac{2}{7}$

3 A 6 sided die is rolled.

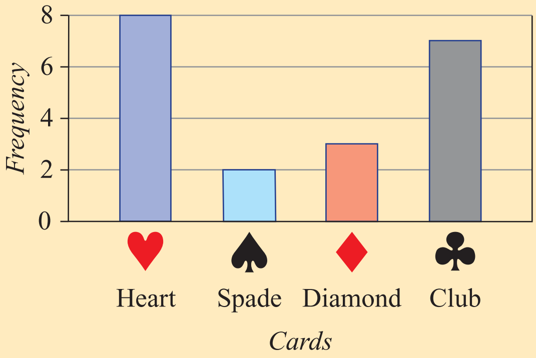
- **a** List the possible outcomes.
- **b** How many of the outcomes are:
  - **i** even? **ii** less than 3? **iii** less than or equal to 3?
  - **iv** at least 2? **v** not a 6? **vi** not odd?

4 One number is selected from the group of the first 10 positive integers {1, 2, . . . , 10}. How many of the numbers are:

a odd? **b** less than 8? **c** greater than or equal to 5?
d no more than 7? **e** prime? **f** not prime?

5 Several cards were randomly selected from a standard deck of playing cards. The suit of each card was noted, the card was replaced and the deck was shuffled. The frequency of each suit is shown in the column graph.

- **a** How many times was a heart selected?
- **b** How many trials took place?
- **c** In what fraction of the trials was a diamond selected?



6 Consider this data set: 1, 2, 3, 5, 5, 7, 8, 10, 13.

- **a** Find the mean (by finding the sum and dividing by 9).
- **b** Find the median (i.e. the middle value).
- **c** Find the mode (i.e. the most common value).
- **d** Find the range (i.e. the difference between the highest and lowest value).
- **e** If one of the numbers is randomly chosen from this data set, find the probability that it is a:
  - **i** 5
  - **ii** number that is not 5
  - **iii** number that is no more than 5

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8A Probability review

Stage

5.2
5.2◊
5.1
4

Probability is the study of chance. The probability of an event occurring is a number between 0 and 1. An impossible event has a probability of 0. An event that is certain to occur has a probability of 1.

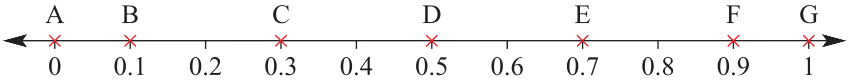
We can use probability to find the likelihood of rolling a total of 7 with two dice.

Let's start: Events and probabilities

As a class group, write down and discuss at least three events that have the following chance of occurring.

- no chance
- very low chance
- medium to low chance
- even (50–50) chance
- medium to high chance
- very high chance
- certainty

For each decimal marked on the number line below, choose the probability description from the list above that best matches it.



Key ideas

- There is a large amount of terminology in this topic. The following two pages include most of the key words and phrases. They are repeated in the Drilling for Gold worksheet so that they can be glued into your book.

Probability terminology

Drilling for Gold 8A1

Terminology	Example	Definition
chance experiment	rolling a fair 6-sided die	A chance experiment is an activity which may produce a variety of different results that occur randomly. The example given is a single-step experiment.
trials	rolling a die 50 times	When an experiment is performed one or more times, each occurrence is called a trial. The example given indicates 50 trials of a single-step experiment.
outcome	rolling a 5	An outcome is one of the possible results of a chance experiment.
equally likely outcomes	rolling a 5 rolling a 6	Equally likely outcomes are two or more results that have the same chance of occurring.
sample space	{1, 2, 3, 4, 5, 6}	The sample space is the set of all possible outcomes of an experiment. It is usually written inside braces, as shown in the example.
event	e.g. 1: rolling a 2 e.g. 2: rolling an even number	An event is either one outcome or a collection of outcomes. It is a subset of the sample space.
compound event	rolling an even number (e.g. 2 or 4 or 6)	A compound event is a collection of two or more outcomes from the sample space of a chance experiment.
mutually exclusive events	rolling a 5 rolling an even number	Two or more events are mutually exclusive if they share no outcomes.
non-mutually exclusive events	rolling a 5 rolling an odd number	Events are non-mutually exclusive if they share one or more outcomes. In the example given, the outcome 5 is shared.
complementary events	rolling a 2 or 3 rolling a 1, 4, 5 or 6	If *all* the outcomes in the sample space are divided into two events, they are complementary events.
complement	Rolling 2, 3, 4 or 5 is an event. Rolling a 1 or 6 is the complement.	If an experiment was performed and an event did not occur, then the complement definitely occurred.

Terminology	Example	Definition
favourable outcome(s)	In some games you must roll a 6 before you can start moving your pieces.	Outcomes are favourable when they are part of some desired event.
theoretical probability or likelihood or chance	The probability of rolling an odd number is written as: $P(\text{odd}) = \frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$ Probabilities can be expressed as fractions, decimals and percentages.	Theoretical probability is the actual chance or likelihood that an event will occur when an experiment takes place. $P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ Probabilities range from 0 to 1 or 0% to 100%.
experimental probability	A die was rolled 600 times and showed a 5 on 99 occasions. The experimental probability of rolling a 5 on this die is: $P(5) = \frac{99}{600} = 0.165 = 16.5\%$	Sometimes it is difficult or impossible to calculate a theoretical probability, so an estimate can be found using a large number of trials. This is called the experimental probability. If the number of trials is large, then the experimental probability should be very close to the theoretical.
certain	rolling a number below 7	The probability is 100% or 1.
likely	rolling a number below 6	
even chance	rolling a 1, 2 or 3	The probability is 50% or 0.5 or $\frac{1}{2}$.
unlikely	rolling a 2	
impossible	rolling a 7	The probability is 0% or 0.
the sum of all probabilities in an experiment	$P(1) = \frac{1}{6}$ $P(2) = \frac{1}{6}$ $P(3) = \frac{1}{6}$ $P(4) = \frac{1}{6}$ $P(5)\frac{1}{6}$ $P(6)\frac{1}{6}$ $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{6}{6}=1$	The sum of the probablities of all the outcomes of a chance experiment is 1.
the sum of the probabilities of an event and its complement	$P(\text{rolling } 1 \text{ or } 6) = \frac{2}{6}$ $P(\text{rolling } 2, 3, 4 \text{ or } 5) = \frac{4}{6}$ $\frac{2}{6}+\frac{4}{6}+\frac{6}{6}=1$	The sum of the probabilities of an event and its complement is 1. $P(\text{event}) + P(\text{complementary event}) = 1$

Exercise 8A

Understanding

Drilling for Gold 8A2

1 Use the words in the table below to complete each statement.

even	impossible, certain	probability
complement	sample space	event

a The ________ ________ is the list of all possible outcomes in an experiment.
b An ______________ is some of the outcomes from an experiment.
c P stands for ________.
d An ________ event has a probability of 0; an event that is ________ to occur has a probability of 1.
e An event with probability of 0.5 has an ________ or 50–50 chance of occurring.
f The ________ of an event, A, is the event where A does not occur.

2 Jim believes that there is a 1 in 4 chance that the flower on his prized rose will bloom tomorrow.

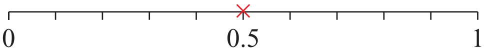
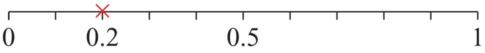
a Write the chance '1 in 4' as a:
  i fraction
  ii decimal
  iii percentage

b Draw a number line from 0 to 1 and mark the level of chance described by Jim.

'1 in 4' means $\frac{1}{4}$.

3 Copy and complete this table.

Percentage ÷ 100 = decimal.

	Percentage	Decimal	Fraction	Number line
a	50%	0.5	$\frac{1}{2}$	
b	25%			
c			$\frac{3}{4}$	
d				
e		0.6		
f			$\frac{17}{20}$	

4 Ten people make the following estimates of the chance that they will get a salary bonus this year.

$0.7,\quad \frac{2}{5},\quad 0.9,\quad \frac{1}{3},\quad 2 \text{ in } 3,\quad \frac{3}{7},\quad 1 \text{ in } 4,\quad 0.28,\quad \frac{2}{9},\quad 0.15$

Order their estimates from lowest to highest.

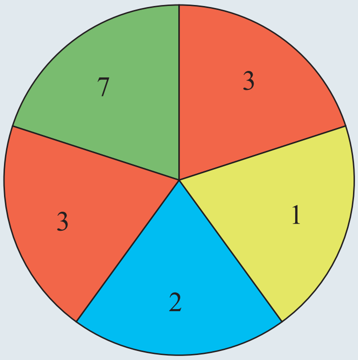
First write each estimate as a decimal.

Fluency

Example 1 Finding probabilities of events

This spinner has five equally divided sections.

- **a** List the sample space, using the given numbers.
- **b** Find $P(3)$.
- **c** Find $P(\text{not a } 3)$.
- **d** Find $P(\text{a } 3 \text{ or a } 7)$.
- **e** Find $P(\text{a number that is at least a } 3)$.



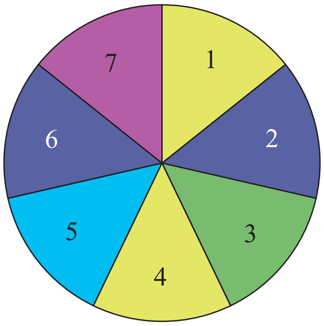
Solution	**Explanation**
a $\{1, 2, 3, 3, 7\}$	Use braces, { }, and list all the possible outcomes in any order. Include 3 twice to represent its increased chance.
b $P(3) = \frac{2}{5}$ or 0.4	$P(3) = \frac{\text{number of sections labelled 3}}{\text{number of equal sections}}$
c $P(\text{not a } 3) = 1 - P(3)$ $= 1 - \frac{2}{5}$ or $1 - 0.4$ $= \frac{3}{5}$ or 0.6	'Not a 3' is the complementary event of obtaining a 3. Alternatively, count the number of sectors that are not 3.
d $P(\text{a } 3 \text{ or a } 7) = \frac{2}{5} + \frac{1}{5}$ $= \frac{3}{5}$	There are two 3s and one 7 in the five sections.
e $P(\text{at least a } 3) = \frac{3}{5}$	Three of the sections have the numbers 3 or 7, which are at least 3; i.e. 3 or more.

5 The spinners below have equally divided sections. Complete the following for each spinner. Write all probabilities as simplified fractions.

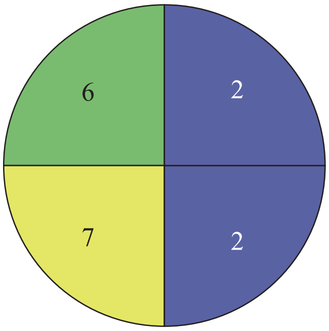
- **i** List the sample space, using the given numbers.
- **ii** Find $P(2)$.
- **iii** Find $P(\text{not a } 2)$.
- **iv** Find $P(\text{a } 2 \text{ or a } 3)$.
- **v** Find $P(\text{a number that is at least a } 2)$.

P(event) means 'find the probability of the event'. 'At least 2' means '2 or more'.

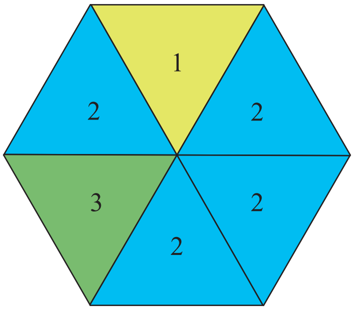
a



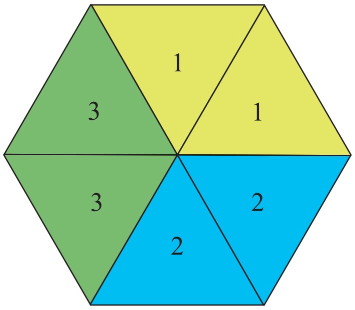
b



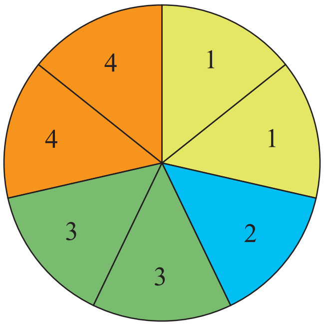
c



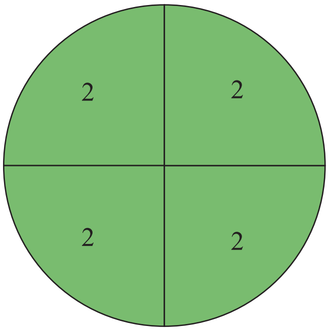
d



e



f



6 Find the probability of obtaining a blue ball when a ball is selected at random from a box that contains:

- **a** 4 blue balls and 4 red balls
- **b** 3 blue balls and 5 red balls
- **c** 1 blue ball, 3 red balls and 2 white balls
- **d** 8 blue balls, 15 black balls and 9 green balls
- **e** 15 blue balls only
- **f** 5 yellow balls and 2 green balls

$$P(\text{blue}) = \frac{\text{number of blue balls}}{\text{total number of balls}}$$

7 Find the probability of *not* selecting a blue ball when a ball is selected at random from a box containing the balls described in Question **6**, parts **a** to **f**, on the previous page.

$P(\text{not blue}) = 1 - P(\text{blue})$

8 If a swimming pool has eight lanes and each of eight swimmers has an equal chance of being placed in lane 1, find the probability that a particular swimmer:

- **a** will swim in lane 1
- **b** will not swim in lane 1

Example 2 Choosing letters from a word

A letter is randomly chosen from the word PROBABILITY. Find the following probabilities.

- **a** $P(\text{L})$
- **b** $P(\text{not L})$
- **c** $P(\text{vowel})$
- **d** $P(\text{consonant})$
- **e** $P(\text{vowel or B})$
- **f** $P(\text{vowel or consonant})$

Solution	**Explanation**
a $P(\text{L}) = \frac{1}{11}$	One of the 11 letters is an L.
b $P(\text{not L}) = 1 - \frac{1}{11}$ $= \frac{10}{11}$	The event 'not L' is the complement of the event 'selecting an L'. Complementary events sum to 1. $P(\text{not L}) = 1 - P(\text{L})$.
c $P(\text{vowel}) = \frac{4}{11}$	The vowels of the alphabet are A, E, I, O and U. There are 4 vowels in PROBABILITY: O, A and two letter Is.
d $P(\text{consonant}) = 1 - \frac{4}{11}$ $= \frac{7}{11}$	The events 'vowel' and 'consonant' are complementary. Alternatively, the other 7 letters are consonants; thus $\frac{7}{11}$.
e $P(\text{vowel or B}) = \frac{6}{11}$	There are 4 vowels and 2 letter Bs.
f $P(\text{vowel or consonant}) = 1$	This event includes all possible outcomes since a letter is either a vowel or a consonant.

8A

Skillsheet 8A

9 A letter is chosen at random from the word ALPHABET. Find the following probabilities.

a $P(\text{L})$ **b** $P(\text{A})$
c $P(\text{A or L})$ **d** $P(\text{vowel})$
e $P(\text{consonant})$ **f** $P(\text{vowel or consonant})$
g $P(\text{Z})$ **h** $P(\text{A or Z})$
i $P(\text{not A})$ **j** $P(\text{letter from the first half of the alphabet})$

Recall that vowels are A, E, I, O and U.

Problem-solving and Reasoning

10 The school captain is to be chosen at random from four candidates. Two are girls (Hayley and Alisa) and two are boys (Rocco and Stuart).

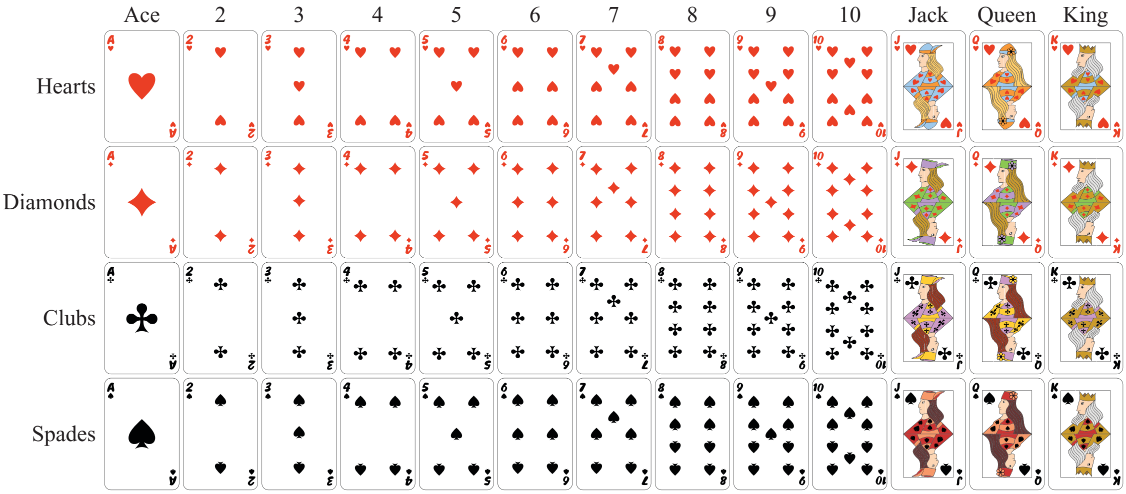
a List the sample space.
b Find the probability that the school captain will be:
  i Hayley
  ii male
  iii neither Stuart nor Alisa

The sample space is a list of the possible students.

11 A card is drawn at random from a deck of 52 playing cards. Find the probability that the selected card will be:

a the queen of diamonds **b** an ace
c a red king **d** a red card
e a jack or a queen **f** any card except a 2
g neither a jack nor a black queen **h** not a black ace

A deck of 52 cards is shown below.



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12 A 6 sided die is rolled and the uppermost face is observed and recorded. Find the following probabilities.

a $P(6)$ **b** $P(3)$
c $P(\text{not } 3)$ **d** $P(1 \text{ or } 2)$
e $P(\text{number less than } 5)$ **f** $P(\text{even number or odd number})$
g $P(\text{square number})$ **h** $P(\text{not a prime number})$

'Less than 5' means that 5 is not included.
1 is not prime.
Square numbers are $1^2 = 1$, $2^2 = 4$, $3^2 = 9$ etc.

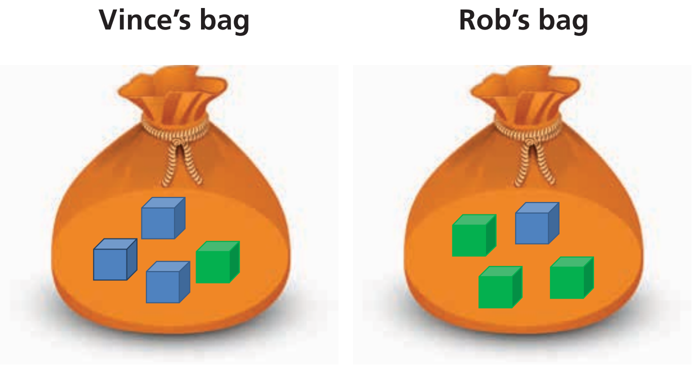
13 Design a spinner that would give the following outcomes.

a 50% chance of green, $\frac{1}{4}$ chance of red, 12.5% chance of yellow, $\frac{1}{8}$ chance of blue

b an equal chance of green and red, 60% chance of blue

Enrichment: Same-same

14 Vince and Rob each have a bag.
In Vince's bag there are 3 blue cubes and 1 green cube
In Rob's bag there are 3 green cubes and 1 blue cube.



Vince's bag **Rob's bag**

They both randomly choose a cube from their bag. They show the cubes to each other.
If the colours are the same, Vince wins a point.
If the colours are different, Rob wins a point.
They return their cubes to their bags.

a If Vince and Rob play this game 100 times, what do you think will happen? Choose from option 1, 2 or 3.
Option 1: The score will be even or close to it (i.e. this is a fair game).
Option 2: Vince will be ahead.
Option 3: Rob will be ahead.
(Record the votes from all the people in your class.)

b Play the game 20 times with a partner.

c Combine your results with the rest of the class to find the experimental probability of each player winning this game.

Drilling for Gold 8A3

d Download 'Drilling for Gold' 8A3 to calculate the theoretical probability of each player winning the game. How close was your prediction?

8B Venn diagrams and two-way tables

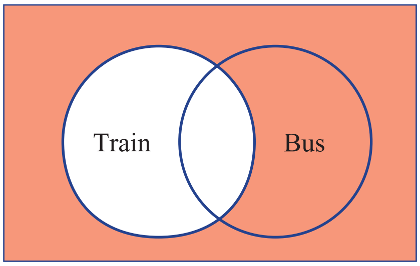
Stage
5.2
5.2◊
5.1
4

When the results of an experiment involve overlapping categories, it can be very helpful to organise the information into a Venn diagram or a two-way table. Probabilities can be easily calculated from these types of diagrams.

▶ Let's start: Solving puzzles

Work with a partner to find the answer to each puzzle. For puzzles 1–4, draw some Venn diagrams, like the ones shown, to help you. For puzzle 5, use a two-way table.

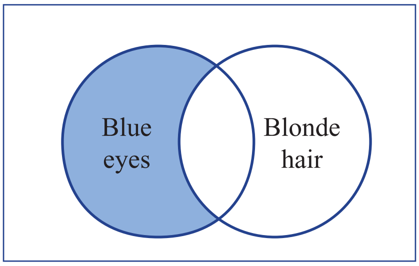
Puzzle 1



12 students travel to school by bus only and 10 students don't travel by either bus or train.

- How many students don't travel by train?

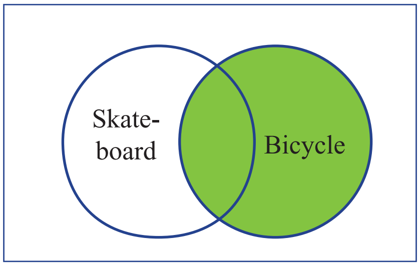
Puzzle 2



14 students in total have blue eyes, and of these students 5 have both blue eyes and blonde hair.

- How many have blue eyes but not blonde hair?

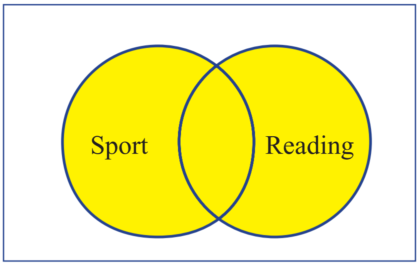
Puzzle 3



11 students own a bicycle only and 4 students own both a skateboard and a bicycle.

- How many students in total own a bicycle?

Puzzle 4



12 students in total like sport, 14 students in total like reading and 9 students like both sport and reading.

- How many students altogether like either sport or reading or both?

Puzzle 5

A survey of 40 students found that a total of 22 play basketball, 9 play both volleyball and basketball and 6 do not play either basketball or volleyball.

- How many basketball players don't play volleyball?
- How many students in total don't play volleyball?
- How many students in total do play volleyball?

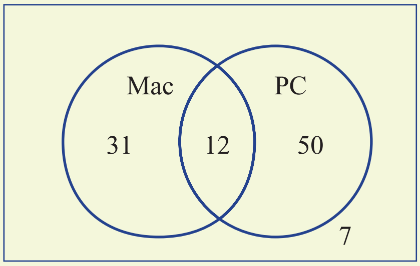
	Basketball	**Not basketball**	**Total**
Volleyball			
Not volleyball			
Total			

Key ideas

Drilling for Gold 8B1 8B2

Venn diagrams and **two-way tables** help to organise outcomes into different categories. This example shows the types of computers owned by 100 people.

Venn diagram



Two-way table

	Mac	**No Mac**	**Total**
PC	12	50	62
No PC	31	7	38
Total	43	57	100

These diagrams show, for example, that:
- 12 people own both a Mac and a PC.
- 62 people own a PC.
- 57 people do not own a Mac.
- $P(\text{Mac}) = \frac{43}{100}$
- $P(\text{only Mac}) = \frac{31}{100}$
- $P(\text{Mac or PC}) = \frac{93}{100}$
- $P(\text{Mac and PC}) = \frac{12}{100} = \frac{3}{25}$

Venn diagram A diagram used to categorise a group into subgroups

Two-way table A tool for organising data into four categories

ISBN 978-1-316-61816-5 Stuart Palmer et al. 2016 Cambridge University Press

Exercise 8B

Understanding

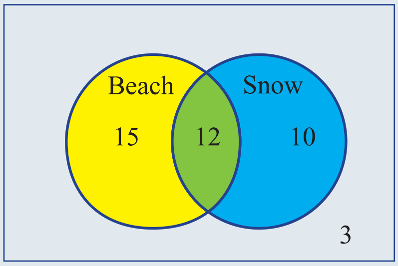
Example 3 Understanding a Venn diagram

This Venn diagram shows the number of people who enjoy holidays at the beach and at the snow.

a How many people are represented in this Venn diagram?

b How many people enjoy holidays:
  - **i** only at the beach?
  - **ii** at the beach (in total)?
  - **iii** both at the beach and at the snow?
  - **iv** neither at the beach nor at the snow?

c How many people do not enjoy holidays at the:
  - **i** beach?
  - **ii** snow?



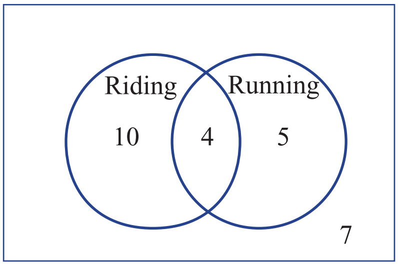
Solution	**Explanation**
a 40 in total	Add all the numbers in the Venn diagram. 15 + 12 +10 + 3 = 40
b i 15 enjoy the beach only	There are 15 people in the beach circle who do not appear in the overlapping section.
ii 27 enjoy the beach	Add the numbers in the beach circle. 15 + 12 = 27
iii 12 enjoy the beach and the snow	12 people are in the overlapping section of the beach and snow circles.
iv 3 don't enjoy either	There are 3 people outside the circles.
c i 13 don't enjoy the beach	Add all numbers outside the beach circle. 10 + 3 = 13
ii 18 don't enjoy the snow	Add all numbers outside the snow circle. 15 + 3 = 18

1 This Venn diagram shows the number of people who enjoy riding and running.

a How many people in total are represented by this Venn diagram?

b How many people enjoy:
  - **i** riding only? **ii** riding (in total)?
  - **iii** running only? **iv** running (in total)?
  - **v** both riding and running?
  - **vi** neither riding nor running?
  - **vii** riding or running?

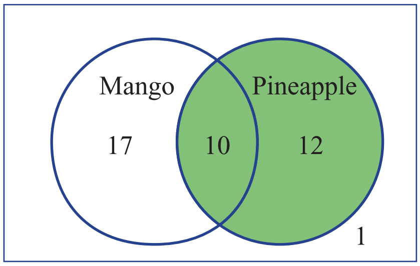
c How many people do not enjoy:
  - **i** riding? **ii** running?



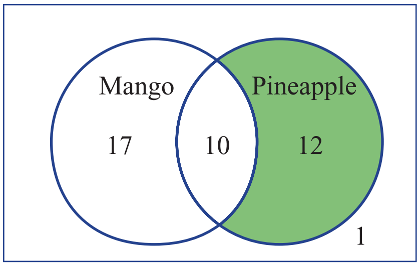
'Riding only' does not include the overlapping section.

2 Each statement is about the shaded area in the Venn diagram. State the missing number in each statement.

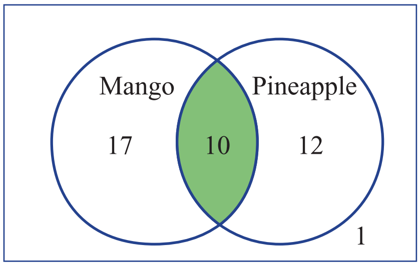
a ________ people in total like pineapple.



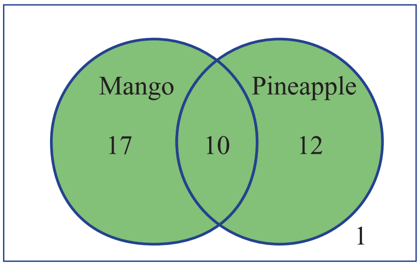
b ________ people like pineapple only.



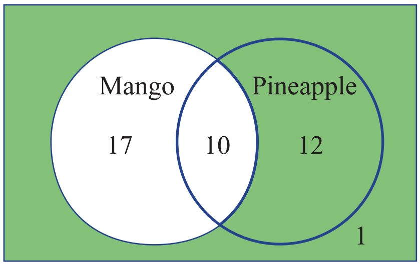
c ________ people like both pineapple and mango.



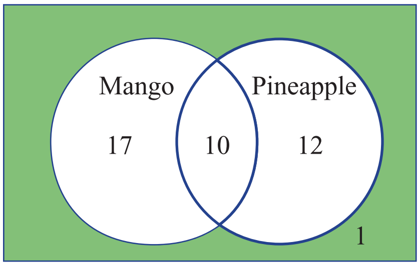
d ________ people like pineapple or mango or both.



e ________ people don't like mango.



f ________ person likes neither mango nor pineapple.

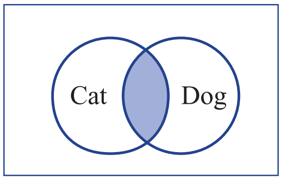


3 Match the diagrams **A–D** with the given description.

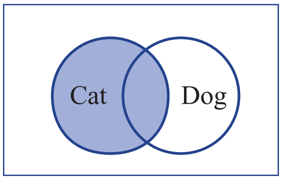
a own a cat
b own a cat only
c own both a cat and a dog
d own a cat or a dog or both

'Both a cat and a dog' is where the circles overlap.

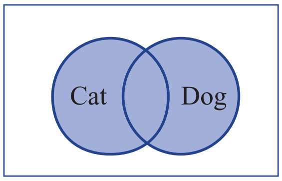
A



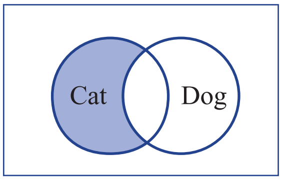
B



C



D



Example 4 Completing a two-way table

Copy and complete this two-way table.

	A	Not A	Total
B	7	6	
Not B		2	
Total	17		

Solution

	A	Not A	Total
B	7	6	13
Not B	10	2	12
Total	17	8	25

Explanation

Start with a row or column that has 2 numbers.
'B' row: $7 + 6 = 13$
'A' column: $7 + 10 = 17$
'Not B' row: $10 + 2 = 12$
'Not A' column: $6 + 2 = 8$

Check that the sum of the row totals = the sum of the column totals.
Sum of column totals = $17 + 8 = 25$
Sum of row totals = $13 + 12 = 25$

4 Copy and complete these two-way tables.

a

	A	Not A	Total
B	7	8	
Not B		1	
Total	10		

b

	A	Not A	Total
B	2		7
Not B		4	
Total			20

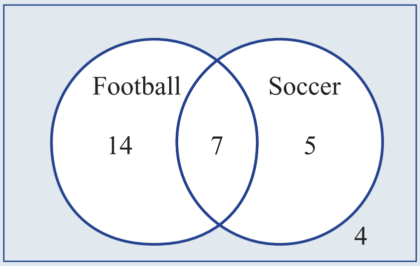
Start with the column or row that has two numbers in it.

Fluency

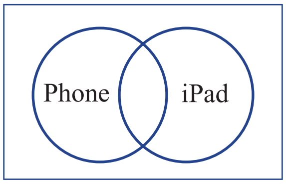
Example 5 Constructing a Venn diagram

A survey of 30 people found that 21 like Australian Rules football and 12 like soccer. Also, 7 people like both football and soccer and 4 like neither football nor soccer.

- **a** Construct a Venn diagram of the survey results.
- **b** How many people:
  - **i** like football or soccer?
  - **ii** do not like soccer?
  - **iii** like only football?
- **c** If one of the 30 people was randomly selected, find:
  - **i** P(like football and soccer)
  - **ii** P(like neither football nor soccer)
  - **iii** P(like only soccer)

Solution	Explanation
a 	Place the appropriate number in each category. Place the 7 in the overlap first, then ensure that: • the total that like football is 21 ($14 + 7 = 21$). • the total that like soccer is 12 ($5 + 7 = 12$). The 4 people that like neither are placed outside the circles.
b **i** 26 like football or soccer or both	The total number of people who like football, soccer or both is $14 + 7 + 5 = 26$.
ii $30 - 12 = 18$ don't like soccer	12 like soccer, so 18 do not.
iii 14 like football but not soccer	21 like football but 7 of these also like soccer.
c **i** P(like football and soccer) $= \frac{7}{30}$	7 out of 30 people like football and soccer.
ii P(like neither football nor soccer) $= \frac{4}{30}$ $= \frac{2}{15}$	The 4 people who like neither football nor soccer sit outside both categories.
iii P(like soccer only) $= \frac{5}{30}$ $= \frac{1}{6}$	5 people like soccer but not football.

5 In a class of 30 students, 22 carried a phone and 9 carried an iPad. Three carried both a phone and an iPad and 2 students carried neither.



- **a** Copy and complete this Venn diagram.
- **b** How many people:
  - **i** carried a phone or an iPad (includes carrying both)?
  - **ii** do not carry an iPad?
  - **iii** carry only an iPad?
- **c** If one of the 30 people was selected at random, find the following probabilities.
  - **i** P(carry a phone and an iPad)
  - **ii** P(carry neither a phone nor an iPad)
  - **iii** P(carry only a phone)

Start by writing a number in the overlapping section. '22 with a phone' is the total for the phone circle, including the overlap.

6 Match each diagram (**A**–**F**) with the correct statement (**a**–**f**).

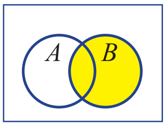
a not A **b** A or B

c A and B **d** B

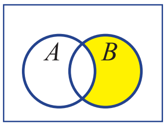
e B only **f** neither A nor B

'A or B' means 'either A or B or both'.
'A and B' means 'both A and B'.

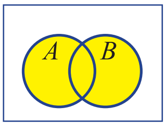
A



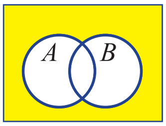
B



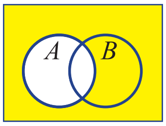
C



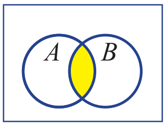
D



E



F



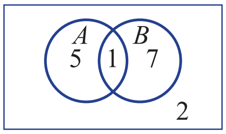
7 For each Venn diagram, find the following probabilities.

i $P(A)$ **ii** $P(A \text{ only})$

iii $P(\text{not } B)$ **iv** $P(A \text{ and } B)$

v $P(A \text{ or } B)$ **vi** $P(\text{neither } A \text{ nor } B)$

a



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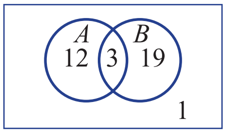

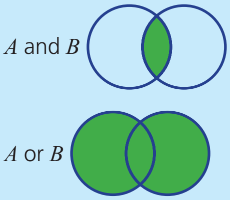

First calculate the total number in each sample.

Example 6 Constructing a two-way table

At a car yard, 24 cars are tested for fuel economy. Eighteen of the cars run on petrol, 8 cars run on gas and 3 cars can run on both petrol and gas.

a Illustrate the situation using a two-way table.

b How many of the cars:

i do not run on gas? **ii** run on neither petrol nor gas?

c Find the probability that a randomly selected car:

i runs on gas **ii** runs on gas only **iii** runs on gas or petrol

Solution

a

	Gas	**Not gas**	**Total**
Petrol	3	15	18
Not petrol	5	1	6
Total	8	16	24

b **i** 16
ii 1

Explanation

Set up a table as shown and enter the numbers (in black) from the given information.
Fill in the remaining numbers (in red), ensuring that each column and row adds to the correct total.

The total at the base of the 'Not gas' column is 16.
The number at the intersection of the 'Not gas' column and the 'Not petrol' row is 1.

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c **i** $P(\text{gas}) = \frac{8}{24}$
$= \frac{1}{3}$

Of the 24 cars, 8 cars run on gas.

ii $P(\text{only gas}) = \frac{5}{24}$

Of the 8 cars that run on gas, 5 of them do not also run on petrol.

iii $P(\text{gas or petrol}) = \frac{15+5+3}{24}$
$= \frac{23}{24}$

Of the 24 cars, some run on petrol only (15), some run on gas only (5) and some run on gas and petrol (3).

8 Of 50 desserts served at a restaurant one evening, 25 were served with ice-cream, 21 were served with cream and 5 were served with both cream and ice-cream.

- **a** Copy and complete this two-way table.
- **b** How many of the desserts:
  - **i** did not have cream?
  - **ii** had neither cream nor ice-cream?
- **c** Find the probability that a chosen dessert:
  - **i** had cream
  - **ii** had cream only
  - **iii** had cream or ice-cream

25 is the total for the ice-cream row.

	Cream	**Not cream**	**Total**
Ice-cream			
Not ice-cream			
Total			

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9 Find the following probabilities using each of the given tables. First copy and complete each two-way table.

i $P(A)$ **ii** $P(\text{not } A)$ **iii** $P(A \text{ and } B)$
iv $P(A \text{ or } B)$ **v** $P(B \text{ only})$ **vi** $P(\text{neither } A \text{ nor } B)$

'Neither A nor B' is the same as 'Not A and not B'.

a

	A	**Not *A***	**Total**
B	3	1	
Not *B*	2		4
Total			

b

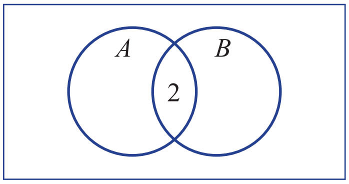
	A	**Not *A***	**Total**
B		4	15
Not *B*	6		
Total			26

Problem-solving and Reasoning

10 For each two-way table, fill in the missing numbers and then transfer the information to a Venn diagram.

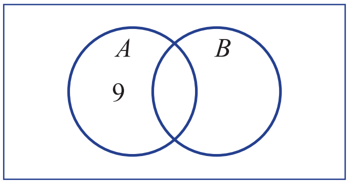
a

	A	**Not *A***	**Total**
B	2		8
Not *B*			
Total		7	12



b

	A	**Not *A***	**Total**
B		4	
Not *B*	9		13
Total	12		



11 In a group of 17 people, 13 rented their house, 6 rented a car and 3 did not rent either a car or their house.

a Draw a Venn diagram, showing circles for 'Rents house' and 'Rents car'.
b How many people rented both a car and their house?
c What is the probability that a person chosen randomly rented a car only?

14 people will be in the circles, but $13 + 6 = 19$. How many must be in the overlap?

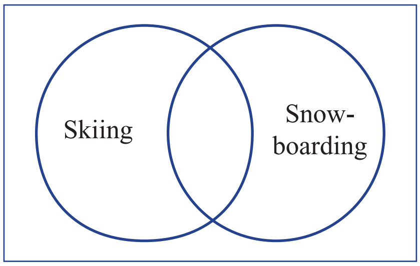
12 One hundred citizens were surveyed regarding their use of water in their garden. Of these, 23 said they used tank water, 48 said they used tap water and 41 said they did not use water on their garden at all.

a Copy and complete this two-way table.
b How many people surveyed used both tank and tap water in their garden?
c What is the probability that a person chosen randomly uses tap water only?
d What is the probability that a person chosen randomly uses tap water or tank water?

	Tank water	**Not tank water**	**Total**
Tap water			
Not tap water			
Total			

13 All members of a ski club enjoy skiing and/or snowboarding. Seven enjoy only snowboarding, 16 enjoy skiing and 4 enjoy both snowboarding and skiing.

a Copy and complete this Venn diagram.



For part **d**, what fraction of skiers also like snowboarding?

b How many people are in the club?

c If a person from the club is randomly selected, what is the probability of choosing a snowboarder?

d If a person is randomly selected out of the group that likes skiing, what is the probability of choosing a snowboarder?

e What is the probability of choosing a skier out of the group that likes snowboarding?

14 Of a group of 30 cats, 24 like either tinned or dry food or both, 10 like only dry food and 5 like both tinned and dry food.

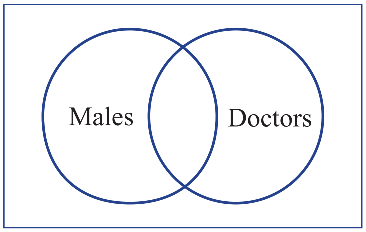
a Find the probability that a selected cat likes only tinned food.

b Out of the group of cats that eat dry food, what is the probability of selecting a cat that also likes tinned food?

Use a Venn diagram to record the information.

Enrichment: Numbers challenge

15 One hundred people were surveyed and it was found that 55 were males and 30 were doctors. The number of male doctors was 17. Copy and complete this Venn diagram and then determine the number of people in each question below.



a the number who are neither male nor a doctor

b the number who are not males

c the number who are not doctors

d the number who are male but not a doctor

e the number who are a doctor but not male

8C Using relative frequencies to estimate probabilities

Stage
5.2
5.2◊
5.1
4

The theoretical probability of tossing 'heads' is 50%. In some real-life scenarios it is impossible to calculate the theoretical probability, so relative frequencies are used.

Tossing a coin 100 times does not mean it will come up heads 50 times.

Let's start: Heads or tails?

Ella tossed a fair coin 10 times and recorded heads 8 out of the 10 tosses.

Jonah tossed the same coin 100 times and recorded heads 49 out of the 100 tosses.

- According to Ella's data, what is the experimental probability of her tossing a head?
- According to Jonah's data, what is the experimental probability of him tossing a head?
- Which of the two data sets is more reliable? Why?
- Ella said that she had a higher chance of rolling a head than Jonah. Is she right?

Key ideas

- The results of repeated trials of a probability experiment can be used to generate **relative frequencies**, which are sometimes called experimental probabilities.
- **Experimental probability** is calculated using the results of an experiment or survey.

$$\text{Experimental probability} = \frac{\text{number of times the outcome occurs}}{\text{total number of trials in the experiment}}$$

- The **expected number of occurrences** = probability × number of trials

$$E = P \times n$$

Relative frequency The proportion (or fraction) of times a particular outcome occurs, written as a fraction, a decimal or a percentage

Experimental probability Probability based on measuring the outcomes of trials

Expected number of occurrences The expected number of favourable outcomes from an experiment

Exercise 8C

Understanding

1 Insert the word *experimental, theoretical or expected* to complete each statement.

a ________________ probability is calculated from the results of an experiment or survey.

b ________________ probability is calculated from the sample space of an event.

c When a coin is tossed, there are 2 possible equally likely outcomes. So the ________________ probability of obtaining a head is $\frac{1}{2}$.

d Jayce tosses a coin 20 times and obtains ‘heads’ 12 times. So the ________________ probability of obtaining a head is $\frac{12}{20} = \frac{3}{5}$.

e A survey finds that 3 out of every 5 students have a sister. Out of 1000 students, the ________________ number of students with a sister will be $\frac{3}{5} \times 1000 = 600$.

2 Write the experimental probability from each set of results listed here.

a Out of 20 people surveyed, 18 preferred cereal for breakfast.

b 50 students were surveyed and it was found that 40 had pets.

c There were 25 boys and 20 girls at the school bus stop one afternoon. If one student is selected from those at the bus stop, state the experimental probability that the student is a boy.

In part **c**, first find the total number of students at the bus stop.

Fluency

Example 7 Using experimental probability

Kris plays rugby and has a record of kicking a goal from a penalty kick 4 times out of every 7 attempts.

a State the experimental probability of Kris achieving a penalty goal.

b Calculate the expected number of goals from 28 penalty kicks that Kris takes.

Solution	Explanation
a $P(\text{goal}) = \frac{4}{7}$	Experimentally, we know that Kris can kick 4 goals out of every 7 kicks.
b Expected number $= \frac{4}{7} \times 28 = 16$ goals	Expected number = probability × number of trials

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3 Ashleigh has found that she can shoot a basketball through the hoop 4 times out of 10 from the '3-point' area.

- **a** State the experimental probability for Ashleigh to shoot a basketball through the hoop from the 3-point area.
- **b** Calculate the expected number of times the ball would go through the hoop from 100 shots Ashleigh makes from the 3-point area.

4 The experimental probability of Jess hitting a bullseye on a dartboard is 0.05 (or $\frac{5}{100}$). How many bullseyes would you expect Jess to get if he threw the following number of darts?

- **a** 100 darts
- **b** 200 darts
- **c** 1000 darts
- **d** 80 darts

Expected number = probability × number of trials

Using probability, we can predict the chance of a dart hitting the bullseye.

5 A bus company surveyed a random selection of 90 people in one suburb. They found that 35 of these people regularly used a bus service from that suburb to the centre of the city.

- **a** State the experimental probability for a person in that suburb to regularly use a bus service from that suburb to the city.
- **b** If there are 2700 residents in that suburb, find the expected number who would regularly use a bus service from that suburb to the city.

Example 8 Finding the experimental probability

A box contains an unknown number of coloured balls and a ball is drawn from the box and then replaced. The procedure is carried out 100 times and the colour of the ball drawn is recorded each time. Twenty-five red balls are recorded.

- **a** Find the experimental probability for selecting a red ball.
- **b** Find the expected number of red balls if the box contains 500 balls in total.

Solution	**Explanation**
a $P(\text{red ball}) = \frac{25}{100}$ $= 0.25$	$P(\text{red ball}) = \frac{\text{number of red balls drawn}}{\text{total number of balls drawn}}$ There are 25 red balls and 100 balls in total.
b Expected number of red balls in 500 $= 0.25 \times 500$ $= 125$	Expected number of occurrences = probability × number of trials

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Drilling for Gold 8C1

6 A bag contains an unknown number of counters, and a counter is selected from the bag and then replaced. The procedure is carried out 100 times and the colour of the counter is recorded each time. Sixty of the counters drawn were blue.

Experimental probability

$$= \frac{\text{number of blue counters}}{\text{total number of counters selected}}$$

- **a** Find the experimental probability of selecting a blue counter.
- **b** Find the expected number of blue counters if the bag contained:
  - **i** 100 counters
  - **ii** 200 counters
  - **iii** 600 counters

7 In an experiment involving 200 people chosen at random, 175 people said that they owned a home computer.

- **a** Calculate the experimental probability of choosing a person who owns a home computer.
- **b** Find the expected number of people who would own a home computer in the following group sizes.
  - **i** 400 people
  - **ii** 5000 people
  - **iii** 40 people

Expected number
= probability
× number of trials

Problem-solving and Reasoning

8 By calculating the experimental probability, estimate the chance that each of the following events will occur.

- **a** Nat will walk to work today, given that she walked to work five times in the past 75 working days.
- **b** Mike will win the next game of cards if, in the past 80 games, he has won 32.
- **c** Brett will hit the bullseye on the dartboard with his next attempt if, in the past 120 attempts, he was successful 22 times.

9 This table shows the results of three different surveys of people in Perth about their use of public transport (PT).

Survey	**Number who use PT**	**Survey size**	**Experimental probability**
A	2	10	$\frac{2}{10} = 0.2$
B	5	20	________
C	30	100	________

A large survey size makes experimental probability calculations more accurate.

- **a** What are the two missing numbers in the experimental probability list?
- **b** Which survey should be used to estimate the probability that a person uses public transport and why?

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10 The results of tossing a drawing pin and observing how many times the pin lands with the spike pointing up are shown in the table. Results are recorded at different stages of the experiment.

Number of throws	Frequency (spike up)	Experimental probability
1	1	1.00
5	2	0.40
10	5	0.50
20	9	0.45
50	18	0.36
100	41	0.41

Experimental probability shows that the chance of a drawing pin landing spike up or spike down is not 50–50.

Which experimental probability would you choose to best represent the probability that the pin will land spike up? Why?

11 A 6-sided die is rolled 120 times. How many times would you expect the following events to occur?

a rolling a 6

b rolling a 1 or 2

c rolling a number less than 4

d rolling a number which is at least 5

Use the theoretical probability to calculate the expected numbers.

Drilling for Gold 8C2 at the end of this section

12 The colour of cars driving along a highway was noted over a short period of time and summarised in this frequency table.

Colour	white	silver	blue	green
Frequency	7	4	5	4

a How many cars had their colour recorded?

b Find the experimental probability that a car's colour is:

- **i** blue **ii** white

c If the colour of 100 cars was recorded, find the expected number of:

- **i** blue cars
- **ii** green cars
- **iii** blue or green cars

13 A spinner is divided into three regions not necessarily of equal size. The regions are numbered 1, 2 and 3 and the spinner is spun 50 times. The table shows the results.

Number	1	2	3
Frequency	26	11	13

The angles of the spinner regions are multiples of 90°.

a Find the experimental probability of obtaining:
  i a 1
  ii at least a 2
  iii a 1 or a 3

b Based on these results, how many 3s would you expect if the spinner is spun 300 times?

c In fact, the spinner is divided up using simple and common fractions. Draw and label how you think the spinner regions are divided up.

Enrichment: Don't roll a 6!

14 The aim of this game is to roll 1, 2, 3, 4 and 5, without rolling a 6.

Lauren played the game once.
She rolled 2, then 2, then 5, then 1 and then 6.
She lost because she did not roll a 3 or a 4 before she rolled a 6.

Steve rolled 5, then 1, then 5, then 4, then 1, then 2 and then 3.
He won because he rolled 1, 2, 3, 4 and 5 and did not roll a 6.

a Play the game ten times and fill in the blanks.
I won the game __ times. I lost the game __ times.

b Combine your results with the rest of the class to estimate the probability of winning this game.

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8C2: Subtracto bingo

This class exercise requires a Subtracto Bingo card (shown below) for each student and one pair of dice for the whole class. The card can be printed out with the worksheet or copied into exercise books.

<table>
<tr><td colspan="12">Round 1</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="12">Round 2</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="12">Round 3</td></tr>
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<tr><td colspan="12">Round 4</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="12">Round 5</td></tr>
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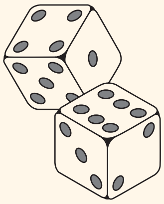

1 Play a Subtracto Bingo game

To start Round 1, choose 12 numbers and write them in the first row on your card.
The only numbers available for use are 0, 1, 2, 3, 4, 5.
You can use each number as many times as you wish or not at all.
Choose who will be 'game master'.
The game master rolls two dice and calculates the difference between them.
The game master calls out the numbers and writes down the difference.
If the difference is 3, for example, all students who wrote a '3' on their Subtracto Bingo card can cross off one of the 3s on their card (i.e. NOT all of the 3s).
The first student to cross off all of their numbers calls out "Subtracto Bingo!".
The winner writes the winning numbers on the whiteboard for all to see.
Repeat for Rounds 2, 3, 4 and 5.

2 Reasoning and communicating activity

Think about which numbers would make the ideal choice for one round of a Subtracto Bingo card. That is, choose numbers that are most likely to win. Write down your choice of numbers. Explain why they are a good choice.

8D Using range and measures of centre (mean, median and mode)

Stage
5.2
5.2◊
5.1
4

Statistics involves collecting and summarising data. It also involves drawing conclusions and making predictions. For example, the type and amount of product stocked on supermarket shelves is determined by the sales statistics and other measures, such as average cost and price range.

▶ Let's start: Mean, median or mode!

Joshua scored the following scores out of 100 on five Maths tests.

$$51, \quad 62, \quad 98, \quad 55, \quad 55$$

- Recall and discuss the meanings of the words mean, median and mode.
- Which one of the three measures of centre (i.e. mean, median or mode) would Joshua prefer to use as an 'average score' when telling his parents the results of his Maths tests?
- Why is the mean greater than the median in this case?

Key ideas

- The **range** of a set of data is given by:

  Range = highest number – lowest number

  $$\underset{\substack{\uparrow\\\text{lowest}}}{1} \quad 5 \quad 2 \quad \underset{\substack{\uparrow\\\text{highest}}}{7} \quad 5 \longrightarrow \text{Range} = 7 - 1 = 6$$

- The **mean** ($\bar{x}$) of a set of data is given by:

  $$\bar{x} = \frac{\text{sum of data values}}{\text{number of data values}}$$

  $$1 + 5 + 2 + 7 + 5 = 20 \longrightarrow \text{Mean} = 20 \div 5 = 4$$

- The **median** is the middle value when the values are sorted from lowest to highest. If there are two middle values, then add them together and divide by 2.

  $$1 \quad 2 \quad \underset{\substack{\uparrow\\\text{middle}}}{\textcircled{5}} \quad 5 \quad 7 \longrightarrow \text{Median} = 5$$

  $$2 \quad 3 \quad \underset{\uparrow}{\textcircled{5}} \quad \underset{\uparrow}{\textcircled{9}} \quad 10 \quad 12$$

  $5 + 9 = 14$
  $14 \div 2 = 7 = \text{Median}$

Range A measure of the amount of spread in a data set

Mean The average of the data values

Median The middle data value when the data are arranged in order

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- The **mode** is the most common value. It is the value that occurs most frequently. We also say that it is the value with the highest frequency.

  1 2 ⑤ ⑤ 7 ⟶ Mode = 5

- The range, mean and median can be calculated only for numerical data, but the mode can be calculated for numerical and categorical data.
- Every scientific calculator is able to calculate the mean, using a button labelled $\overline{x}$.

Mode The most frequently occurring value in a set of data

Outlier Any value that is much larger or much smaller than the rest of the data in a set

Exercise 8D

Understanding

1 **a** To calculate the _______, you add up all the values and divide by the number of values.

b Find the mean of these data sets. (Round each answer to one decimal place.)

i 3 6 8 10 12
ii 6 8 4 12 9 11 16 13
iii 9 12 1 5 7 10 5 7 13 15
iv 32 35 31 37 42

First find the sum of the scores, then divide by the number of scores.

$\text{Mean} = \frac{39}{5}$

2 **a** The mode is the most ___________________ value.

b State the mode of these data sets.

i 1 2 2 2 4 5 6
ii 1 4 8 8 9 10 10 10 12
iii 1 5 7 7 8 9 13
iv red, red, white, blue, red, red, blue

3 **a** The median is the _________ value when the scores are listed in order.

b Find the median of these data sets.

i 1 3 5 6 8
ii 4 5 7 9 10 11
iii 5 6 1 2 8 9 3
iv 16 12 15 11 10 13 17 20

Order the data sets in **iii** and **iv**.

4 **a** The _________ is the difference between the highest and lowest values.

b Find the range of these data sets.

i 3 6 7 12 15 24
ii 5 12 14 18 23 27
iii 22 24 26 27 30 35
iv 8 14 5 22 27 32 16

Range = highest value – lowest value

5 **a** An _________ is a score that is much larger or smaller than the rest of the data.

b For each of these data sets, if there is an outlier, state its value.

i 2 4 5 4 6 2 3 36
ii 21 25 3 27 28 24 29 30
iii 13 15 10 12 17 20 19 22
iv 24 21 100 23 25 27 30 25 25

Fluency

Example 9 Finding measures of centre and the range

For the given data sets, find the:

i mean **ii** median **iii** mode **iv** range

a 5 2 4 10 6 1 2 9 6
b 17 13 26 15 9 10

Solution	**Explanation**
a **i** Mean $= \frac{45}{9}$ $= 5$	$5 + 2 + 4 + 10 + 6 + 1 + 2 + 9 + 6 = 45$ Find the sum of all the numbers and divide by the number of values.
ii $1\ \ 2\ \ 2\ \ 4\ \ \textcircled{5}\ \ 6\ \ 6\ \ 9\ \ 10$ Median = 5	First, order the data. The median is the middle value.
iii Mode = 2 and 6	The data set is bimodal since there are two numbers with the highest frequency.
iv Range = 10 – 1 = 9	The range is the highest score minus the lowest score.
b **i** Mean $= \frac{90}{6}$ $= 15$	$17 + 13 + 26 + 15 + 9 + 10 = 90$ There are 6 values.
ii $9\ \ 10\ \ \underbrace{13\ \ 15}_{14}\ \ 17\ \ 26$ Median $= \frac{13+15}{2}$ $= 14$	First, order the data. Since there are two values in the middle, find the mean of them.
iii No mode	None of the values is repeated so there is no mode.
iv Range = 26 – 9 = 17	The highest score is 26 and the lowest score is 9.

Drilling for Gold 8D1

Spreadsheet 8D1

6 For the given data sets, find the:

i mean **ii** median
iii mode **iv** range

The data must be in order to locate the middle value (i.e. median).

a 7 2 3 8 5 9 8
b 6 13 5 4 16 10 3 5 10
c 12 9 2 5 8 7 2 3
d 10 17 5 16 4 14
e 3.5 2.1 4.0 8.3 2.1
f 0.7 3 2.9 10.4 6 7.2 1.3 8.5
g 6 0 −3 8 2 −3 9 5
h 3 −7 2 3 −2 −3 4

7 These data sets include an outlier. Write down the outlier and then calculate the mean and the median. Include the outlier in your calculations.

An outlier is much larger or smaller than the other scores.

a 5 7 7 8 12 33
b 2 40 42 45 47
c 1.3 1.1 1.0 1.7 1.5 1.6 −1.1 1.5
d −58 −60 −59 −4 −64

8 Decide whether the following data sets are bimodal.

Bimodal means that there are two modes.

a 2 7 9 5 6 2 8 7 4
b 1 6 2 3 3 1 5 4 1 9
c 10 15 12 11 18 13 9 16 17
d 23 25 26 23 19 24 28 26 27

9 This ordered data set shows the number of fish Daniel caught in the 12 weekends that he went fishing during the year.

1 2 3 4 4 5 6 7 7 9 11 13

a Find the range.
b Find the median.
c Find the mean.
d Find the mode.

Problem-solving and Reasoning

10 In three running races, Paula recorded the times 25.1 seconds, 24.8 seconds and 24.1 seconds.

a What is the mean time of the races? Round your answer to two decimal places.
b Find the median time.

11 This is a data set of six house prices in Darwin.

$324 000 $289 000 $431 000 $295 000 $385 000 $1 700 000

To find the median, list the prices in order, then find the middle value.

a Find the mean (to the nearest dollar) and the median house price.
b Which price would be considered the outlier?
c If the outlier is removed from the data set, by how much will the median change?
d If the outlier is removed from the data set, by how much will the mean change, to the nearest dollar?

Example 10 Finding a data value for a required mean

Drilling for Gold 8D2 at the end of this section

The hours a shop assistant works in eight weeks are: $8, 9, 12, 10, 10, 8, 5, 10$.

a Calculate the mean for this set of data.

b What score needs to be added to this data set to make the mean equal to 10?

Solution	**Explanation**
a Mean $= \dfrac{72}{8}$ $= 9$	$8 + 9 + 12 + 10 + 10 + 8 + 5 + 10 = 72$ Sum of the 8 data values is 72.
b Let a be the new score.	$72 + a$ is the total of the new data and $8 + 1$ is the new total number of scores.
Require $\dfrac{72+a}{8+1} = 10$	Set this equal to the required mean of 10.
$\dfrac{72+a}{9} = 10$	Solve for a.
$72 + a = 90$ $a = 18$	9 scores have a mean of 10, so the sum of the scores $= 9 \times 10 = 90$.
The new score would need to be 18.	Write the answer.

12 A netball player scored the following number of goals in her 10 most recent games.

$15 \quad 14 \quad 16 \quad 14 \quad 15 \quad 12 \quad 16 \quad 17 \quad 16 \quad 15$

a What is her mean score?

b What does she need to score in the next game for the mean of her scores to be 16?

13 Stevie obtained the following scores on her first five Maths tests: $92 \ \ 89 \ \ 94 \ \ 82 \ \ 93$

a What is her mean test score?

b If there is one more test left to complete, and she wants to achieve an average of at least 85, what is the lowest score Stevie can obtain for her final test?

Skillsheet 8B

Enrichment: Aiming for an A

14 A school gives grades in Mathematics each semester according to this table. Raj has scored the following results for four topics this semester, and has one topic to go: $75, 68, 85, 79$.

a What is Raj's mean score so far?

b What grade will Raj get for the semester if his fifth score is:

i 50? **ii** 68? **iii** 94?

c Find the maximum average score Raj can receive for the semester. Is it possible for him to get an A+?

d Find the least score that Raj needs in his fifth topic for him to receive an average of:

i B+ **ii** A

Average score	Grade
90–100	A+
80–89.9	A
70–79.9	B+
60–69.9	B
50–59.9	C+
0–49.9	C

Drilling for Gold exercise

8D2: Which ones changed? Why or why not?

Complete the worksheet or write your answers in your exercise book.

1 Consider this data set:

2	4	5	7	9

- **a** If the 9 is changed to 10, explain why the median will not change.
- **b** If the 9 is changed to 10, explain why the mean will increase.
- **c** If the 9 is changed to 10, explain why the range will increase.
- **d** If the 9 is changed to 10 and the 2 is changed to 1, what happens to the range? Explain your decision.
- **e** If the 9 is changed to 10 and the 2 is changed to 1, what happens to the mean? Explain your decision.
- **f** If the 9 is changed to 10 and the 2 is changed to 1, what happens to the median? Explain your decision.
- **g** If each data value is increased by 1, which one of the following does *not* change – the mean, the median or the range? Explain your decision.
- **h** If each data value is changed to 5, which one of the following does *not* change – the mean, the median or the range? Explain your decision.

2 Consider this data set, which had a median of 5 and a mean of 5 before a new data value called A is added to the data set.

4	4	5	6	6	A

- **a** Which value of A will keep the median at 5?
- **b** When A is included, what is the highest possible median? What is the lowest?
- **c** What value of A will increase the mean from 5 to 10? Show your calculations.
- **d** Write down two different values for A that will increase the range from 2 to 6.

3 Consider the same data set as that in Question **2** but with two new data values (A and B) added to the data set.

4	4	5	6	6	A	B

- **a** Is it possible for the median to remain unchanged? How? Give three possible values for A and B that will result in the median being unchanged.
- **b** If both A and B are greater than 6, what will be the new median?
- **c** When A and B are included, the mean is 10. Give possible values for A and B.

Keeping in touch with numeracy

Non-calculator

1 Arrange these numbers from lowest to highest: 5, –3, –6, 0.25

3 Four items weigh 250 g, 1 kg, 30 g and 2 kg. What is their total mass, in grams?

5 In a picture, a building is 5 cm tall. The actual building is 15 m tall. What is the scale factor?

A 3 **B** 30
C 300 **D** 3000

7 A car was going in a north-easterly direction. It made a 90° turn to the right. In what direction is it now going?

A NW **B** SE **C** SW

9 Which of the following shapes could have no parallel sides?

A kite
B trapezium
C rhombus
D parallelogram

11 Which of these is the longest distance?

A 0.4506 km **B** 456 m
C 4560 cm **D** 45 060 mm

13 A square paddock has an area of 5000 m^2. The side is approximately:

A 50 m **B** 70 m
C 1250 m **D** 2500 m

15 The surface area of a cube is 294 cm^2. How long is an edge of the cube?

17 Tom played 9 holes of golf. His scores were: 3, 4, 4, 5, 5, 6, 6, 6, 6.
Find the mode and the median.

19 For the data given in Question **17**, if a 6 is changed to 7, which two of the following measures will change?

mean median mode range

Calculator

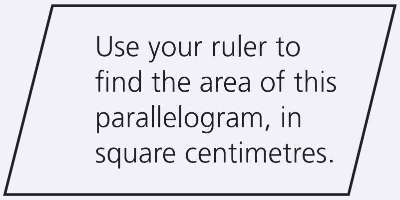
2 The sum of 5, –3, –6 and ☐ is 10. What is ☐?

4 One litre of water weighs 1 kilogram. What is the mass of the water in 5 bottles that each hold 750 mL?

6 A square with sides 5 cm is reduced in size using a scale factor of 0.75. What will be the new side length?

8 What is the smallest angle between east and north-west?

10



Use your ruler to find the area of this parallelogram, in square centimetres.

12 a How many 45 cm lengths can be cut from a piece of wire that is 5 m long?
b How much will be left over?

14 A square paddock has an area of 5000 m^2. Calculate the perimeter, correct to one decimal place.

16 Find the surface area of a rectangular prism with edges 5 cm, 6 cm and 7 cm.

18 a Calculate the mean of the data given in Question **17**.
b Change one score so that the mean increases by 1.

20 Tom is going to play another 9 holes of golf. He wants his mean for the 18 holes to be 4.5 or less. What is the highest total he can score for these 9 holes?

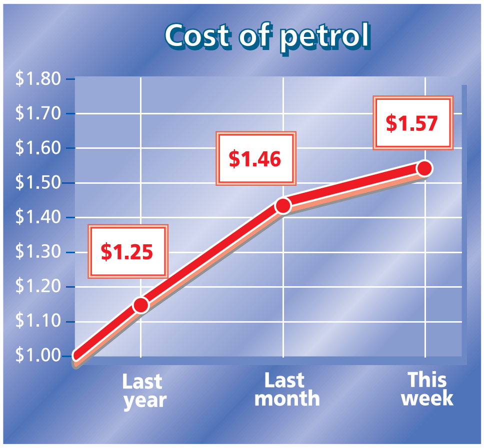
8E Interpreting data from tables and graphs

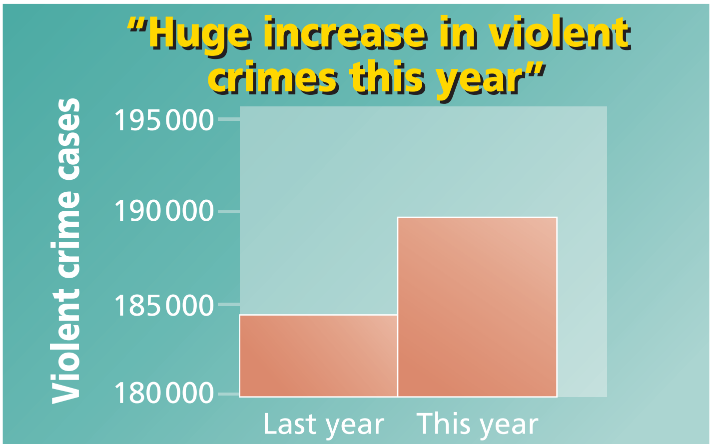
Stage
5.2
5.2◊
5.1
4

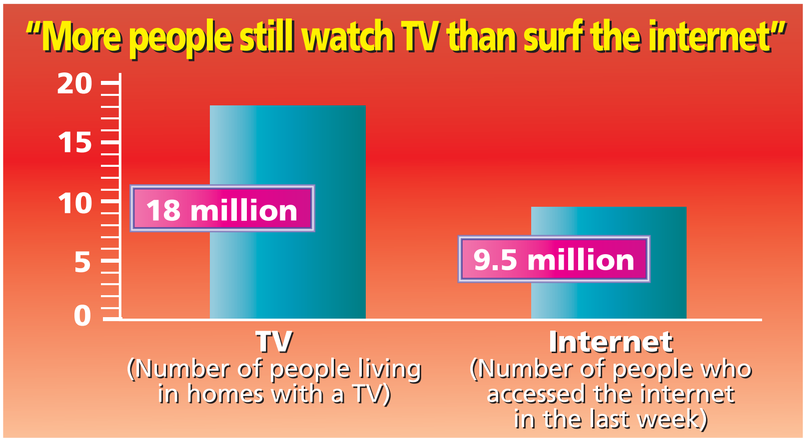
In our everyday lives, it is important to be able to understand many forms of information. Data that are presented in a table or a graph are much easier to interpret than a long list of data. The headings in a table add detail and graphs give a visual comparison between values or categories.

Let's start: Misleading?

Data can be presented in tables, lists or graphs. The headings and labels used on a graph assist us in knowing what the data represent. However, sometimes graphs can be misleading. Look at the graphs below and discuss why each one of them is misleading.







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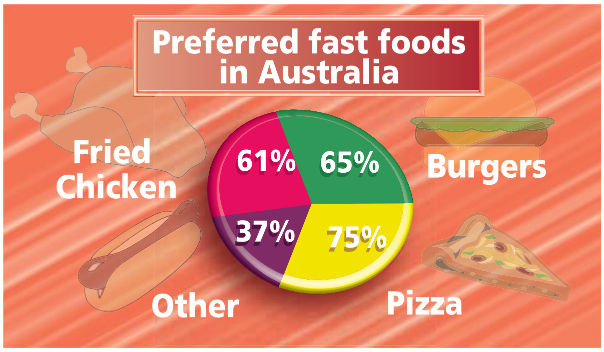
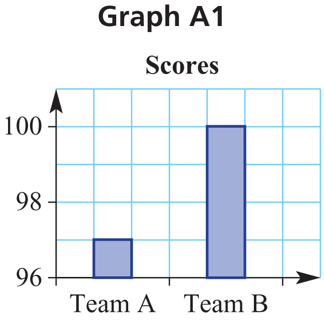
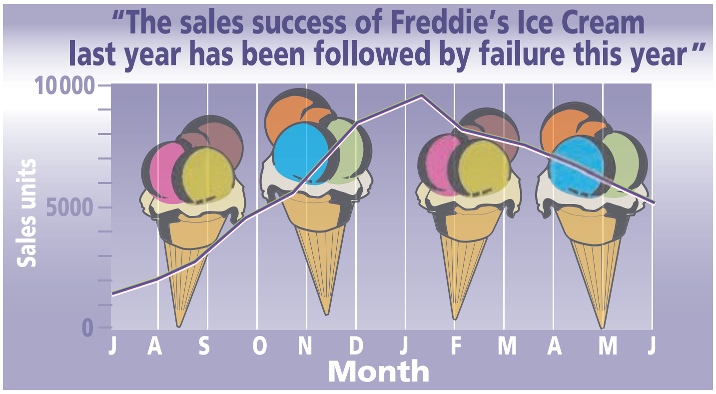


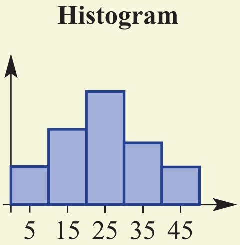


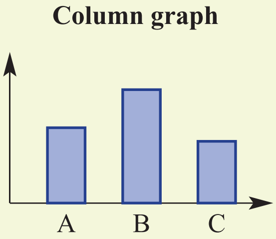


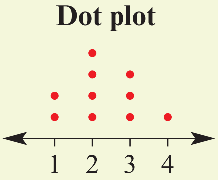
Key ideas

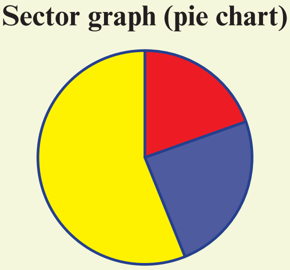
Drilling for Gold 8E1
Spreadsheet 8E1

- Graphs can be presented in different forms. Some common examples are:









- Misleading graphs give a false impression about data.
  - When only part of the scale is shown, the difference between results may be exaggerated.
  - If one column has a different size or colour, it may appear to have greater value than the other columns.

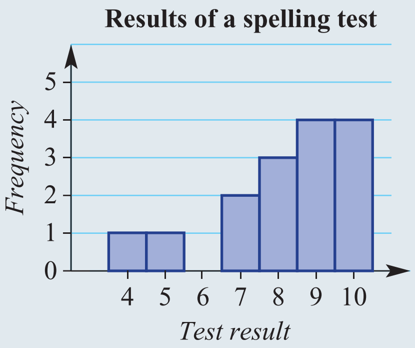
Exercise 8E

Understanding

Example 11 Interpreting histograms

This histogram shows the results of a spelling test out of 10.

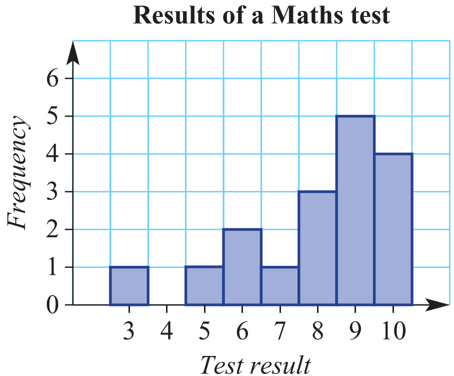
- **a** How many results are shown in this histogram?
- **b** List the results in ascending order.
- **c** Calculate the mean.
- **d** Calculate the median.
- **e** What is the range of results from this test?
- **f** Are the data skewed or symmetrical?
- **g** What proportion of results is greater than or equal to 7?


Results of a spelling test

Solution	**Explanation**
a 15	The frequency shows how many times each score occurred. Add the frequency values to find the total number of scores: $1+1+2+3+4+4=15$.
b 4, 5, 7, 7, 8, 8, 8, 9, 9, 9, 9, 10, 10, 10, 10	1 lot of 4, 1 lot of 5, 2 lots of 7, 3 lots of 8, 4 lots of 9, 4 lots of 10.
c $\text{Mean} = \frac{123}{15}$ $= 8.2$	$4+5+7+7+8+8+8+9+9+9+9+10+10+10+10=123$
d Median = 9	The 8th score is the middle score.
e Range = $10-4$ $= 6$	The range is the highest score minus the lowest score.
f Data are skewed.	Results are bunched to the higher scores. The graph is not symmetrical.
g $\frac{13}{15}$	13 scores are greater than or equal to 7 out of a total of 15 scores.

1 This histogram shows the results of a Maths test out of 10.

- **a** How many results are shown in this histogram?
- **b** List the results in ascending order.
- **c** Calculate the mean.
- **d** Calculate the median.
- **e** What is the range of results from this test?
- **f** Are the data skewed or symmetrical?
- **g** What proportion (fraction) of results is greater than or equal to 8?


Results of a Maths test

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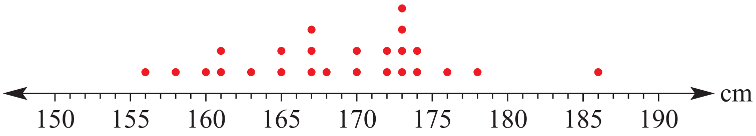
2 This dot plot shows the heights, in cm, of students in a Year 9 class.

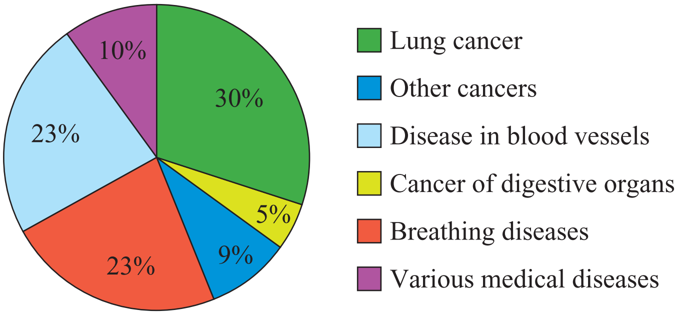


Each dot represents a student.

a How many students have their heights recorded on this dot plot?
b What is the range of heights?
c What is the mode of these heights?
d What is the median height for this class?
e What is the mean height for this class?
f What value is the outlier?

3 This pie chart shows the proportions of deaths in Australia from diseases caused by smoking.

Deaths from smoking-related diseases



Number of deaths

$= \frac{\text{percentage}}{100} \times \text{total deaths}$

Sector angle = percentage × 360°

Around 15 000 Australians in total die from smoking-related illnesses each year.

a What is the total percentage of smoking deaths caused by cancer?
b How many Australian smokers die of smoking-related cancer in a year?
c How many Australian smokers die of smoking-related cancer each day? Round your answer to the whole number.
d How many Australian smokers die from various breathing diseases each day? Round your answer to one decimal place.
e Calculate the sector angle for lung cancer.

4 The following table shows tide times and heights in December for Yamba, which is on the NSW north coast. The tide heights (in metres) shown are red for low tide and blue for high tide. The times are in 24-hour time.

1520 in 24-hour time is 3:20 p.m. in 12-hour time.

Saturday 17		***Sunday 18***		***Monday 19***		***Tuesday 20***		***Wednesday 21***	
Time	**Height**	**Time**	**Height**	**Time**	**Height**	**Time**	**Height**	**Time**	**Height**
0038	1.14	0145	1.18	0255	1.26	0401	1.37	0502	1.49
0619	0.45	0729	0.49	0847	0.50	1008	0.46	1123	0.38
1243	1.42	1340	1.34	1445	1.26	1556	1.21	1703	1.18
1925	0.29	2018	0.29	2115	0.28	2211	0.26	2307	0.22

- **a** How high is the second high tide on Saturday 17 December?
- **b** What time is the first low tide on Monday 19 December? Write your answer in both 24-hour time and 12-hour time.
- **c** How much later in the morning is the low tide on Tuesday 20 December than the low tide on Monday 19 December?
- **d** What is the difference in height between the two high tides on Wednesday 21 December?
- **e** How long is it between the two high tides on Sunday 18 December?

Fluency

5 The following is a table of life expectancy estimates for people in various countries.

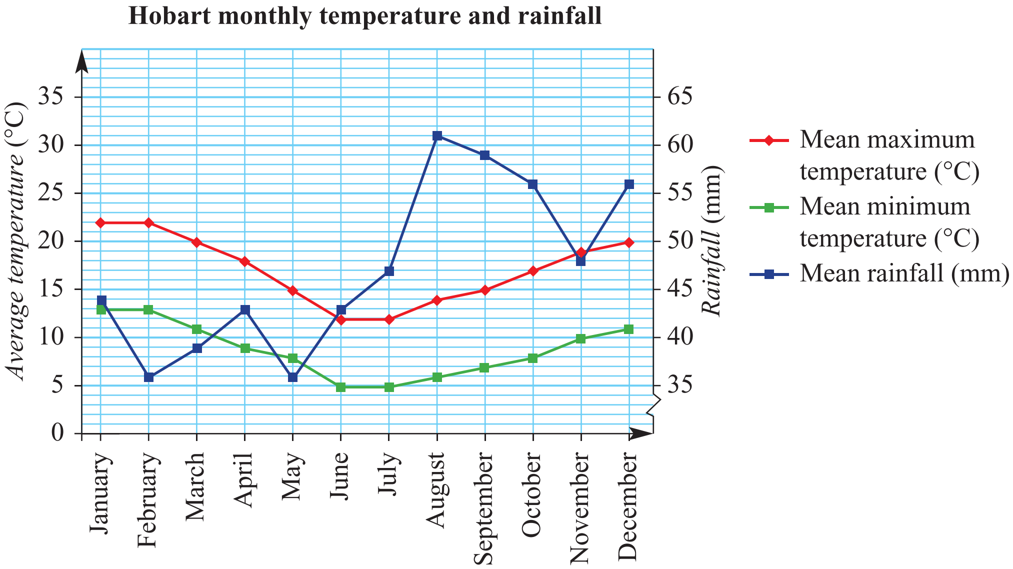
- **a** How many years more do Australian males expect to live than Mozambiquan males?
- **b** How many years more do Australian females expect to live than Papua New Guinean females?
- **c** If the overall world average of life expectancy is 67 years, how far above the average is Japan's overall average life expectancy?
- **d** Find the increase in life expectancy from Indonesian males to Australian males. Now calculate this increase as a percentage of Indonesian male life expectancy. Round to one decimal place.

Country	**Life expectancy (in years)**		
	Overall average	**Male**	**Female**
Japan	83	79	86
Switzerland	82	80	84
Australia	81	79	84
Malaysia	74	72	77
Vietnam	74	72	77
Indonesia	71	69	73
Papua New Guinea	57	55	60
South Africa	49	49	50
Mozambique	39	38	39

Drilling for Gold 8E2

6 The line graphs below show the mean monthly minimum and maximum temperatures and the monthly rainfall, in mm, for Hobart.

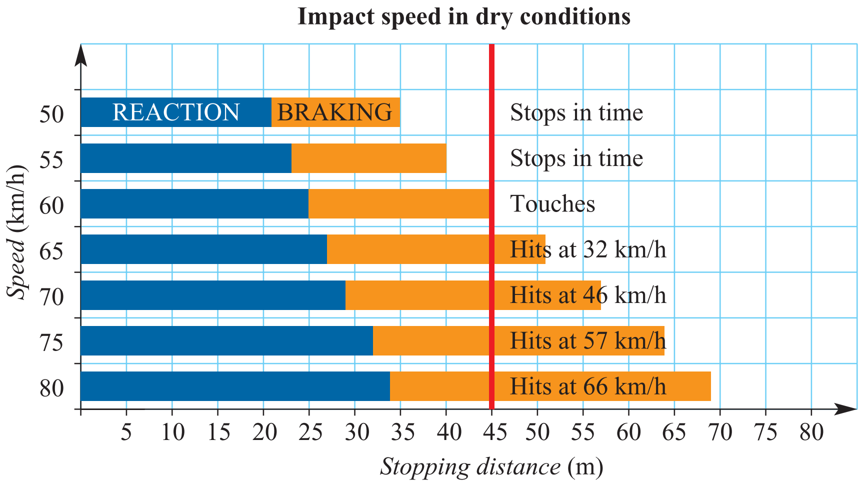
The temperature values are read from the scale on the left and the rainfall values are read from the scale on the right. For example, in January the mean maximum temperature is 22°C and the rainfall is 44 mm.



a What is the mean maximum temperature in April?
b What is the rainfall in April?
c What is the mean minimum temperature in July?
d What is the rainfall in July?
e During what months is the mean maximum temperature greater than 19°C?
f During what months is the rainfall less than 40 mm per month?
g Which are the wettest two months in Hobart?
h What is the temperature range in December?

7 The following graph shows stopping distances (reaction time and distance travelled while braking) for cars when driving at various speeds on a dry road.

Reaction time is shown by the blue bar, braking time is shown by the orange bar.



a The red line on this graph represents an object or person in front of the braking car. How far in front of the car is the object or person at the start of the driver's reaction time?

b How many metres are travelled during *reaction time* when driving at 60 km/h?

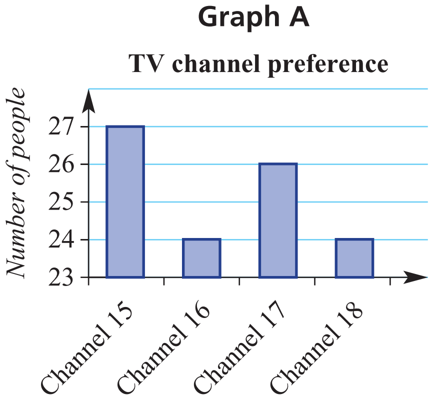
c How many metres are travelled during *braking time* when driving at 60 km/h?

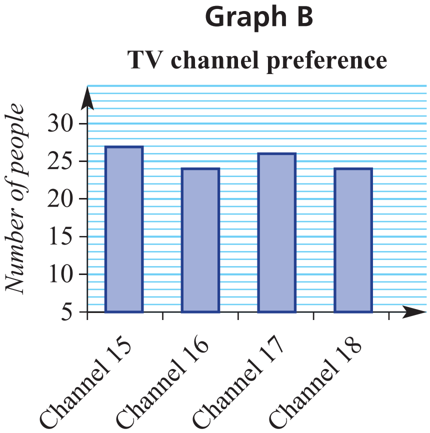
d How much distance is needed, overall, to stop when driving at 80 km/h?

e By how much does the braking distance increase when driving at 80 km/h compared to 50 km/h?

Problem-solving and Reasoning

8 Here are two column graphs, each showing the same results of a survey that asked people which TV channel they preferred.





a From graph A, write down how many viewers preferred each channel.

b Which graph could be titled 'Channel 15 is clearly most popular'?

c Which graph could be titled 'All TV channels have similar popularity'?

d What is the difference between the two graphs?

e Which graph is misleading and why?

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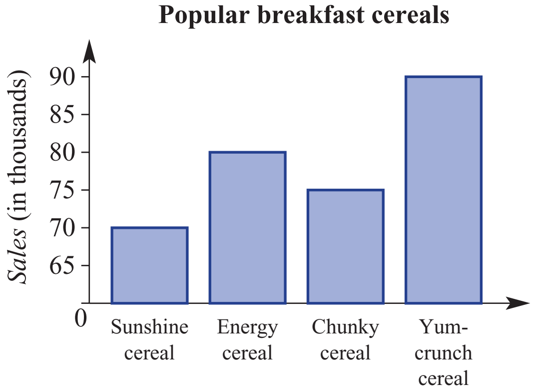
9 This column graph shows the number of sales for one month of four popular breakfast cereals. The height of each column represents the sales.

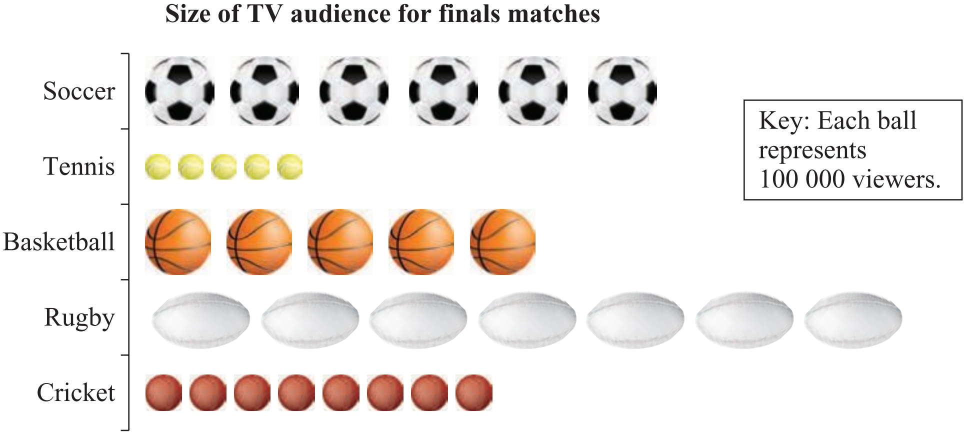
a List the breakfast cereals in order of sales.

b The sales team at Yum-crunch stated that they have twice the sales of Chunky cereal. What is wrong with their claim?

c Draw the graph so that the information is represented correctly.



10 This pictograph shows the number of TV viewers for the final matches of various sports.



a Which sport *appears* to have had the greatest TV audience?

b List the sports in order according to their *length* on the pictograph.

c Using the key, determine the audience sizes for the rugby and soccer finals.

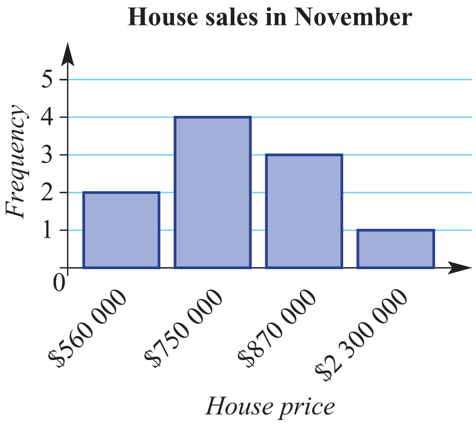
d Which sport actually had the largest TV audience? What was the size of its audience?

e List the sports in order according to the audience size calculated using the key.

f In what way is the graph above misleading?

g How should a pictograph be drawn so that it is not misleading?

11 This histogram shows the price of some houses that were sold during one November in a Sydney suburb.



a How many house sales are shown in this histogram?

b List the price of each house in ascending order.

c Find the mean house price.

d Find the median house price.

e A newspaper headline reads 'House prices now average over $900 000'. Is this 'average' referring to the mean or median price?

f What proportion of house prices are less than the mean value?

g Do you think the mean or the median would better represent the 'average' house price? Give a reason for your answer.

Enrichment: Viewing distance for TV

12 a Draw a line graph showing the minimum and maximum viewing distances for each TV screen size. Include a key.

TV screen size (length of diagonal in inches)	Maximum viewing distance (cm)	Minimum viewing distance (cm)
26	200	100
30	220	120
34	260	130
42	320	160
47	360	180
50	380	200
55	420	220
60	470	230
65	490	250

b If a lounge chair is 1.5 m in front of the TV, which sizes of TV would be suitable?

c If a lounge chair is 3 m in front of the TV, which sizes of TV would be suitable?

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8F Stem-and-leaf plots

Stage
5.2
5.2◊
5.1
4

Stem-and-leaf plots are commonly used to display a single numerical data set or two related data sets. All the data are included on a stem-and-leaf plot. The median and mode can be read easily from a stem-and-leaf plot because all the data sit in order.

Let’s start: Ships vs Chops

At a school, Ms Ships’ class and Mr Chops’ class sit the same exam. The scores are displayed using this back-to-back stem-and-leaf plot. Discuss the following.

- Which class has the most students?
- What are the lowest and highest scores from each class?
- What are the median scores from each class?
- Which class could be described as symmetrical and which as skewed?
- Which class has the better results?

Leaf Ms Ships’ class	**Stem**	**Leaf** Mr Chops’ class
3 1	5	0 1 1 3 5 7
8 8 7 5	6	2 3 5 5 7 9 9
6 4 4 2 1	7	8 9 9
7 4 3	8	0 3
6	9	1

7 | 8 means 78

Digits representing the stem give the tens, and digits representing the leaves give the units.

Key ideas

- A **stem-and-leaf plot** is a way to display numerical data.
- Each number is split into a stem (the first digit or digits) and a leaf (the last digit).
  For example:

	Stem	Leaf
The number 7 is	0	7
The number 31 is	3	1
The number 152 is	15	2

Stem-and-leaf plot A graph that lists numbers in order, grouped in rows

- Leaves are aligned vertically, getting bigger as you move away from the stem.

- **Back-to-back stem-and-leaf plots** can be used to compare two sets of data. The stem is drawn in the middle, with the leaves on either side.

Scores for the last 30 AFL games

Winning scores		Losing scores
	7	4 5 8 8 9
1	8	0 0 3 3 6 7
8 7 5	9	1 2 3 6
8 4 4 1	10	3 9
9 5 0	11	1
1	12	

Lowest winning score was 81

Highest losing score was 111

10 | 9 means 109

Back-to-back stem-and-leaf plot A visual representation of two data sets

Skewed data A distribution of data which is sloped more to one side than the other. The mean and the median have different values

Symmetrical data A distribution of data which is symmetrical either side of the mean and the median

- Symmetrical data will produce a graph that is symmetrical about the centre. The 'winning scores' are symmetrical.
- Skewed data will produce a graph that lacks symmetry. The 'losing scores' are skewed.

Scores for the last 30 AFL games

Winning scores		Losing scores
	7	4 5 8 8 9
1	8	0 0 3 3 6 7
8 7 5	9	1 2 3 6
8 4 4 1	10	3 9
9 5 0	11	1
1	12	

'Shape' of the data is symmetrical

'Shape' of the data is skewed, not symmetrical

10 | 9 means 109

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Exercise 8F

Understanding

1 List the numbers in these stem-and-leaf plots.

a

Stem	Leaf
3	5 7
4	1 3 8

4 | 1 means 41

b

Stem	Leaf
5	2
6	0 1 7
7	3 5

7 | 3 means 73

The same stem goes with each leaf along each row.

2 Rewrite this stem-and-leaf plot with the leaves in order from smallest to largest.

Stem	Leaf
5	8 6 1 7
6	7 3 0 2
7	3 5 1 4

3 This stem-and-leaf plot shows the number of minutes Alexis spoke on her phone for a number of calls.

Count the leaves to find the number of scores.

Stem	Leaf
0	8
1	5 9
2	1 1 3 7
3	4 5

2 | 1 means 21 minutes

a How many calls are represented by the stem and leaf plot?

b What is the length of the:

- **i** shortest phone call?
- **ii** longest phone call?

c What is the mode (the most common call time)?

d What is the median call time (middle value)?

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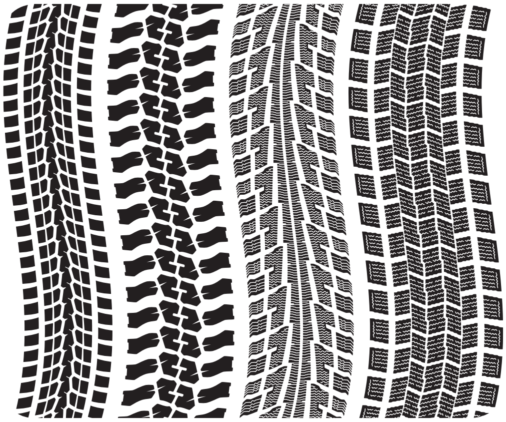
4 This back-to-back stem-and-leaf plot shows the thickness of tyre tread on a selection of cars from the city and country.

06 is just written as 6.

City		Country
8 7 3 1 0 0 0	0	6 8 8 9
8 6 3 1 0	1	0 4 5 5 6 9
1	2	3 4 4

1 | 4 means 14 mm

- **a** How many car tyres were tested altogether?
- **b** What is the smallest tyre tread thickness in the:
  - **i** city?
  - **ii** country?
- **c** What is the largest tyre tread thickness in the:
  - **i** city?
  - **ii** country?
- **d** Find the median tyre tread thickness for tyres in the:
  - **i** city
  - **ii** country
- **e** Is the distribution of tread thickness for city cars more symmetrical or skewed?
- **f** Is the distribution of tread thickness for country cars more symmetrical or skewed?



Skewed distributions have only a few scores at one end.

Fluency

Example 12 Constructing and using a stem-and-leaf plot

Consider the following set of data.

3 25 41 37 20 33 48 33 46 1 41 75 14 24
57 23 34 30 23 41 63 10 58 44 1 68 52 10

- **a** Organise the data into an ordered stem-and-leaf plot.
- **b** Find the median.
- **c** Find the mode.
- **d** Describe the symmetry/skewness of the data.

Solution **Explanation**

a

Stem	Leaf
0	1 1 3
1	0 0 4
2	0 3 3 4 5
3	0 3 3 4 7
4	1 1 1 4 6 8
5	2 7 8
6	3 8
7	5

3 | 4 means 34

The minimum is 1 and the maximum is 75, so stems range from 0 to 70.

Place leaves in order from smallest to largest. Some numbers appear more than once; e.g. two instances of 1 means that the leaf 1 appears twice: 1, 1, ...

b Median $= \frac{33+34}{2}$

$= 33.5$

There are 28 data values. The median is the average of the two middle values (the 14th and 15th values).

c Mode is 41.

The most common value is 41.

d Data are approximately symmetrical.

The distribution of numbers is approximately symmetrical about the stem containing the median.

Drilling for Gold 8F1

5 For each of the following data sets:

i Organise the data into an ordered stem-and-leaf plot.
ii Find the median.
iii Find the mode.
iv Describe the symmetry/skewness of the data.

a 41 33 28 24 19 32 54 35 26 28
19 23 32 26 28

b 31 33 23 35 15 23 48 50 35 42 45 15 21
45 51 31 34 23 42 50 26 30 45 37 39

c 167 159 159 193 161 164 167 157 158 175 177 185
177 202 185 187 159 189 167 159 173 198 200

Draft version: Enter the leaves in the same order as in the question.
Final version: Rewrite the stem-and-leaf plot with leaves in order. Remember to include the key.

In part **c**, use 16 as the stem for 167.

6 The number of vacant rooms in a motel each week over a 20-week period is shown below.

12 8 11 10 21 12 6 11 12 16 14 22 5 15 20 6 17 8 14 9

a Draw a stem-and-leaf plot of this data set.
b In how many weeks were there fewer than 12 vacant rooms?
c Find the median number of vacant rooms.

Example 13 Constructing back-to-back stem-and-leaf plots

A shop owner has two jeans shops. The daily sales in each shop over a 16-day period are monitored and recorded as follows.

Shop A
3 12 12 13 14 14 15 15 21 22 24 24 24 26 27 28

Shop B
4 6 6 7 7 8 9 9 10 12 13 14 14 16 17 27

a Draw a back-to-back stem-and-leaf plot.
b Compare and comment on differences between the sales made by the two shops.

Solution

a

Shop A		Shop B
3	0	4 6 6 7 7 8 9 9
5 5 4 4 3 2 2	1	0 2 3 4 4 6 7
8 7 6 4 4 4 2 1	2	7

1 | 3 means 13

b Shop A sales are generally between 12 and 28, with one low value of 3. Shop B sales are generally between 4 and 17, with one high value of 27. Shop A has many more high values than shop B. Shop B has more low values than shop A.

Explanation

The data for each shop are already ordered. Stems are in intervals of 10. Record leaf digits for shop A on the left and shop B on the right, ordered from the middle to the outside.

Look at both sides of the plot for the highest and lowest values and whether there are a few or many of the small and large numbers.

7 For each of the following sets of data:
- **i** Draw a back-to-back stem-and-leaf plot.
- **ii** State the smallest and largest value in each set and compare the numbers of small and large values in each set.

Order the leaves with the smallest on the inside and largest on the outside. State whether each set has a few or many of the small numbers and large numbers.

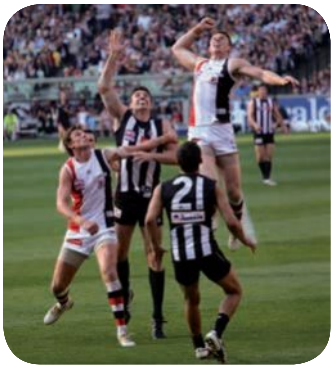
a Set A: 46 32 40 43 45 47 53 54 40 54 33 48 39 43

Set B: 48 49 31 40 43 47 48 41 49 51 44 46 53 44

b Set A: 1 43 24 26 48 50 2 2 36 11 16 37 41 3 36
6 8 9 10 17 22 10 11 17 29 30 35 4 23 23

Set B: 9 18 19 19 20 21 23 24 27 28 31 37 37 38 39 39 39
40 41 41 43 44 44 45 47 50 50 51 53 53 54 54 55 56

8 **a** Draw a back-to-back stem-and-leaf plot for the final scores of St Kilda and Collingwood in the 24 AFL games given here.



St Kilda:	126	68	78	90	87	118
	88	125	111	117	82	82
	80	66	84	138	109	113
	122	80	94	83	106	68
Collingwood:	104	80	127	88	103	95
	78	118	89	82	103	115
	98	77	119	91	71	70
	63	89	103	97	72	68

b In what percentage of their games did each team score more than 100 points?

c Comment on the symmetry of the distribution of the scores for each team.

d Which team has scores that are more consistent? Which team has more higher scores?

Percentage = fraction × 100

Problem-solving and Reasoning

9 This stem-and-leaf plot shows the time taken, in seconds, by Helena to run 100 m in her past 25 races.

a Find Helena's median time.

b What is the difference between the slowest and fastest time?

c If in her 26th race, Helena time is 14.8 seconds and this is added to the stem-and-leaf plot, will her median time change? If so, by how much?

Stem	Leaf
14	9
15	4 5 6 6 7 7 7 8 9
16	0 0 1 1 2 2 3 4 4 5 5 5 7 7
17	2

14 | 9 means 14.9 seconds

10 Find the median if all the data in each back-to-back stem-and-leaf plot is combined.

a

5	3	8 9
9 7 7 1	4	0 2 2 3 6 8
8 6 5 2 2	5	3 3 7 9
7 4 0	6	1 4

4 | 2 means 42

First combine each back-to-back plot into just one stem-and-leaf plot.

b

3	16	0 3 3 6 7 9
9 6 6 1	17	0 1 1 4 8 8
8 7 5 5 4 0	18	2 2 6 7
2	19	0 1

16 | 3 means 163

Enrichment: Birth weights and smoking

11 The back-to-back stem-and-leaf plot below shows the birth weight, in kilograms, of babies born to mothers who do or don't smoke.

Birth weight of babies

Smoking mothers		Non-smoking mothers
4 3 2 2	2	4
9 9 8 7 6 6 5 5	2*	8 9
4 3 2 1 1 1 0 0 0	3	0 0 1 2 2 3
6 5 5	3*	5 5 5 6 6 7 7 8
1	4	
	4*	5 5 6

2 | 4 means 2.4 kg
2* | 5 means 2.5 kg

- **a** What percentage of babies born to smoking mothers have a birth weight of less than 3 kg?
- **b** What percentage of babies born to non-smoking mothers have a birth weight of less than 3 kg?
- **c** Compare and comment on the differences between the birth weights of babies born to mothers who smoke and those born to mothers who don't smoke.

Do babies born to mothers who smoke weigh more or less than those born to mothers who don't smoke?

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Consumer maths

Is it worth the risk?

It is well known that some people become addicted to poker machines and lose huge amounts of money. In some cases, people steal money to fund their gambling addiction.

Here are some facts from the Australian Government's Problem Gambling website: http://cambridge.edu.au/redirect/?id=6641:

- Australians spent more than $19 billion on gambling in 2008–09; around $12 billion of which was spent playing the pokies.
- One in six people who regularly play the pokies has a serious addiction.
- Young people (18–24 year olds) spend more money on poker machines than any other age group. Many adult gamblers report having developed gambling problems during their teenage years.
- Three-quarters of problem gamblers have problems with poker machines. It's even higher for women – in 9 out of 10 cases poker machines are identified as the cause of problems for women.

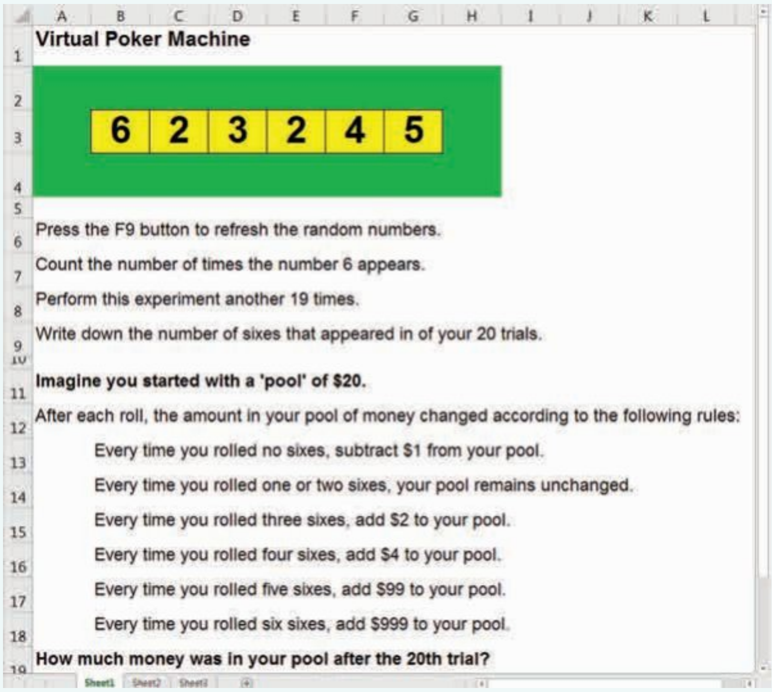
But did you know that poker machines are designed by mathematicians?

In this activity you will see how it is done! It is actually quite simple to design a machine that offers big prizes to gamblers but slowly robs them of their hard-earned savings.

Spreadsheet 8F1

You will begin this activity with 20 virtual dollars. You will spend them $1 at a time in a simple virtual poker machine (i.e. an Excel spreadsheet). You will keep track to see how much you win or lose when you put your $20 through the machine. You will then compare your results with many other people who have done the same experiment.

In the interactive textbook, click on the spreadsheet image below to open the virtual poker machine.



Virtual Poker Machine

Press the F9 button to refresh the random numbers.

Count the number of times the number 6 appears.

Perform this experiment another 19 times.

Write down the number of sixes that appeared in of your 20 trials.

Imagine you started with a 'pool' of $20.

After each roll, the amount in your pool of money changed according to the following rules:

Every time you rolled no sixes, subtract $1 from your pool.

Every time you rolled one or two sixes, your pool remains unchanged.

Every time you rolled three sixes, add $2 to your pool.

Every time you rolled four sixes, add $4 to your pool.

Every time you rolled five sixes, add $99 to your pool.

Every time you rolled six sixes, add $999 to your pool.

How much money was in your pool after the 20th trial?

Puzzles and games

1 One coin says to another coin: *How does my face appear?*
Answer these questions for each scenario, then match the letters in blue to the answers green to find the answer to the riddle.

2 coins are tossed 12 times	1 die is tossed 12 times	Choosing one letter from the word PUZZLES
P(HH) = **D**	P(5) = **A**	P(Z) = **M**
P(HT or TH) = **I**	P(greater than 2) = **N**	P(consonant) = **O**
Expected number of TT = **R**	Expected number of 6s = **S**	
Expected number of one or more tails = **T**		

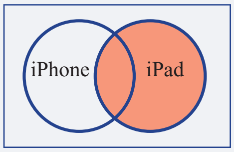
☐	☐'	☐
$\frac{1}{2}$	9	2

☐	☐	☐	☐	☐	☐
3	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{5}{7}$	$\frac{2}{7}$

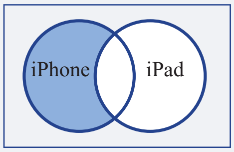
2 Match each description (**1**–**5**) with the most suitable diagram (**A**–**E**). Note: There is only one description for each diagram.

1 Don't own an iPhone
2 Own an iPhone only
3 Own both an iPad and an iPhone
4 Own an iPad or an iPhone or both
5 Own an iPad

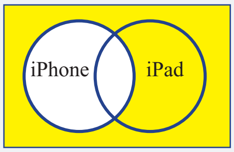
A



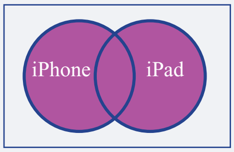
B



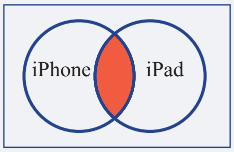
C



D



E



3 In a class of 28, each student owns a cat or a dog or both. If 18 students own cats and 16 students own dogs, how many students own both a cat and a dog?

4 Write down the set of five positive integers that has a mean of 5, a mode of 8 and a range of 6.

5 A 6-sided die and a 10-sided die are rolled simultaneously. What total sums have the highest chance of occurring? (A total sum means the sum of the two uppermost faces.)

Chapter summary

Probability and single variable data analysis

Probability review

The sample space is the list of all possible outcomes of an experiment. For all possible outcomes:

$$P(\text{event}) = \frac{\text{number of outcomes where event occurs}}{\text{total number of outcomes}}$$

e.g. Roll a normal 6-sided die.

$P(>4) = \frac{2}{6} = \frac{1}{3}$

$P(\text{not } A) = 1 - P(A)$

Experimental probability

This is calculated from results of an experiment or survey. Relative frequency is used to estimate theoretical probability.

$$\text{Experimental probability} = \frac{\text{number of times event occurs}}{\text{total number of trials}}$$

Expected number of occurrences = probability × number of trials

Summarising data: Measures of centre

Mode is the most common value (there can be more than one). Two modes means data set is bimodal.

Mean is the average of the data values.

$$\text{Mean } (\bar{x}) = \frac{\text{sum of data values}}{\text{number of values}}$$

Median is the middle value of data set that is ordered.

$2 \quad 4 \quad ⑦ \quad 10 \quad 12$

Median

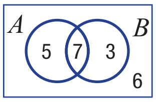
$2 \quad 4 \quad \underbrace{6 \quad 10}_{8} \quad 15 \quad 18$

$\text{Median} = \frac{6+10}{2}$

An outlier is a value that is not near the rest of the data.

Venn diagrams and two-way tables

These organise data from two or more categories



Venn diagram

	A	Not A	Totals
B	7	3	10
Not B	5	6	11
Totals	12	9	21

i.e. 6 in neither category, 7 in both categories

$P(A) = \frac{12}{21} = \frac{4}{7}$ $\quad P(A \text{ and } B) = \frac{7}{21} = \frac{1}{3}$

$P(B \text{ only}) = \frac{3}{21} = \frac{1}{7}$ $\quad P(A \text{ or } B \text{ or both}) = \frac{15}{21} = \frac{5}{7}$

Stem-and-leaf plots

These display all the data values using a stem and a leaf. An ordered back-to-back stem-and-leaf plot:

Leaf	Stem	Leaf
9 8 7 2	1	0 3
7 4 3 3	2	2 2 4
5 2 1	3	3 6 7 8
7	4	4 5 9
0	5	0 7

skewed

symmetrical

3 | 5 means 35

key

Measure of spread

Range = maximum value − minimum value

Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.

Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

1 A letter is randomly chosen from the word XYLOPHONE. The probability that it is an O is:

A $\frac{1}{8}$ **B** $\frac{2}{9}$ **C** $\frac{1}{4}$ **D** $\frac{1}{9}$ **E** $\frac{1}{3}$

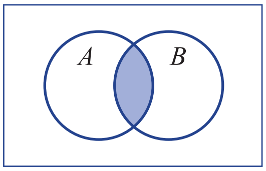
2 The values of x and y in the two-way table are:

A $x = 12, y = 8$ **B** $x = 12, y = 11$
C $x = 16, y = 4$ **D** $x = 10, y = 1$
E $x = 14, y = 6$

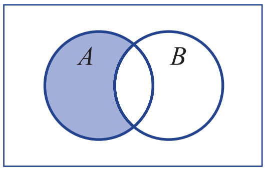
	A	Not A	Total
B		5	9
Not B	8	y	
Total	x		25

3 **a** Which shaded region represents both A and B?
b Which shaded region represents A only?
c Which shaded region represents A or B or both?
d Which shaded region represents the complement of A?

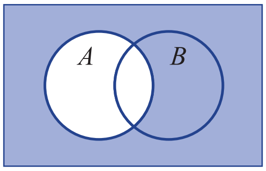
A



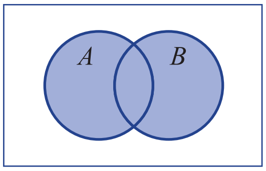
B



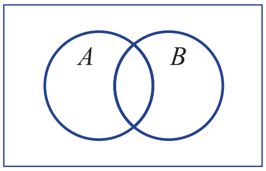
C



D



E



4 From rolling a biased die, a class finds an experimental probability of 0.3 of rolling a 5. From 500 rolls of the die, the expected number of 5s would be:

A 300 **B** 167 **C** 180 **D** 150 **E** 210

5 **a** The median of the data in this stem-and-leaf plot is:

A 74 **B** 71 **C** 86
D 65 **E** 70

b The range of the data in the stem-and-leaf plot is:

A 3 **B** 8
C 33 **D** 86
E 14

Stem	Leaf
5	3 5 8
6	1 4 7
7	0 2 4 7 9
8	2 6 6

7|4 means 74

6 If Jacob achieved scores of 12, 9, 7 and 12 on his last four language vocabulary tests, what score must he get on the fifth test to have a mean of 11?
A 16 **B** 14 **C** 11 **D** 13 **E** 15

Short-answer questions

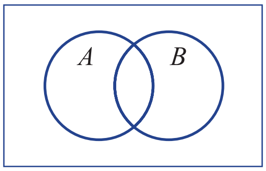
1 Determine the probability of each of the following.
- **a** rolling more than 2 on a normal 6-sided die
- **b** selecting a vowel from the word EDUCATION
- **c** selecting a pink or white jelly bean from a packet containing 4 pink, 2 white and 4 black jelly beans

2 From a survey of 50 people, 30 have the newspaper delivered, 25 read it online, 10 do both and 5 do neither.
- **a** Construct a Venn diagram for the survey results.
- **b** How many people surveyed only read the newspaper online?
- **c** If one of the 50 people surveyed were randomly selected, find:
  - **i** P(have paper delivered and read it online)
  - **ii** P(don't have it delivered)
  - **iii** P(only read it online)

3 **a** Copy and complete this two-way table.

	A	**Not** A	**Total**
B		16	
Not B	8		20
Total	17		

b Convert the information into a Venn diagram, as shown.



c Find the following:
- **i** P(not B)
- **ii** P(both A and B)
- **iii** number of A only
- **iv** number in either A or B or both A and B

4 A quality controller records the frequency of the types of chocolates from a sample of 120 off its production line.

Centre	soft	hard	nut
Frequency	50	22	48

- **a** What is the experimental probability of randomly selecting a nut centre?
- **b** In a box of 24 chocolates, how many would be expected to have a non-soft centre?

5 Claudia records the number of emails she receives each weekday for 2 weeks as follows.

30 31 33 23 29 31 21 15 24 23

Find the:

a mean **b** median **c** mode **d** range

6 Two mobile phone salespeople are both aiming to be promoted to assistant store manager.
The best salesperson over a 15-week period will achieve the promotion.
The number of mobile phones they sold each week is recorded below.

Employee 1	21	34	40	38	46	36	23	51	35	25	39	19	35	53	45
Employee 2	37	32	29	41	24	17	28	20	37	48	42	38	17	40	45

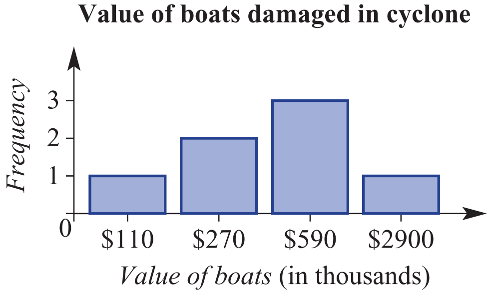
a Draw an ordered back-to-back stem-and-leaf plot for the data.

b For each employee, find the:
- **i** median number of sales
- **ii** mean number of sales

c By comparing the two sets of data, state, with reasons, who you think should get the promotion.

d Describe each employee's data as approximately symmetrical or skewed.

7 This column graph shows the value of some of the boats that were badly damaged in cyclone Yasi in North Queensland in 2011.



a Find the mean and median boat value.

b Do you think the mean or the median would better represent the 'average' of these boat values? Give a reason for your answer.

Extended-response questions

1 Forty-five people were surveyed as they walked through a market as to whether they bought a sausage and/or a drink from the sausage sizzle. Twenty-five people bought a sausage, 30 people bought a drink and 15 bought both.

- **i** Construct a Venn diagram to represent this information.
- **ii** How many people surveyed bought neither a drink nor a sausage?
- **iii** How many people surveyed bought a sausage only?
- **iv** If a person is randomly selected from the 45, what is the probability that they bought a drink but not a sausage?
- **v** Find P(didn't buy a sausage).

2 The delay time (in minutes) of the flight departure of the same evening flight of two rival airlines was recorded over 30 consecutive days. The data are shown below.

Airline A

2	11	6	14	18	1	7	4	12	14	9	2	13	4	19
13	17	3	52	24	19	12	14	0	7	13	18	1	23	8

Airline B

6	12	9	22	2	15	10	5	10	19	5	12	7	11	18
21	15	10	4	10	7	18	1	18	8	25	4	22	19	26

a Copy and complete this ordered back-to-back stem-and-leaf plot for the data. Use two lines for each stem, starting with '0' for leaves 0–4, and then '0*' for leaves 5–9 etc.

Airline A		Airline B
	0	
	0*	
	1	
	1*	
	2	
	2*	
	3	
	3*	
	4	
	4*	
	5	

1 | 2 means 12   1* | 5 means 15

b Do the data for airline A appear to have any outliers (i.e. numbers not near the majority of data elements)?

c By removing any outliers listed in part **b**, find the following for each airline, rounding to one decimal place where necessary.

- **i** the median
- **ii** the mean

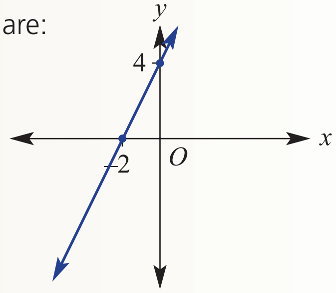
d Airline A reports that half its flights for that month had a delay time of less than 10 minutes. Is this claim correct? Explain.

Chapter 5: Linear relationships

Multiple-choice questions

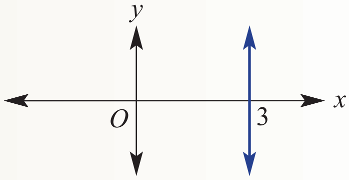
1 The points at which this line meets the x-axis and y-axis are:

A $(-2, 4)$ and $(4, -2)$
B $(0, 4)$ and $(0, -2)$
C $(-2, 0)$ and $(0, 4)$
D $(4, 0)$ and $(0, -2)$
E $(2, 0)$ and $(0, -4)$



2 The graph shown has equation:

A $y = 3x$
B $y = 3$
C $y = x + 3$
D $x = 3$
E $x + y = 3$



3 The line passing through the points $(-3, -1)$ and $(1, 1)$ has gradient:

A $\frac{1}{2}$ **B** -3 **C** 1 **D** 2 **E** -2

4 Which equation is correct for this table?

x	0	1	2	3	4
y	4	7	10	13	16

A $y = x + 4$ **B** $y = x + 6$ **C** $y = 4x + 3$
D $y = 3x + 4$ **E** $y = 4x$

5 The midpoint and length (to one decimal place) of the line segment joining the points $(-2, 1)$ and $(4, 6)$ are:

A $(1, 3.5)$ and 7.8 **B** $(3, 5)$ and 5.4 **C** $(3, 3.5)$ and 6.1
D $(1, 3.5)$ and 3.3 **E** $(3, 3.5)$ and 3.6

Short-answer questions

1 Sketch the following linear graphs, labelling x- and y-intercepts.

a $y = 2x - 6$ **b** $y = 4x$ **c** $y = -2$

2 Find the gradient of each of the following.

a The line passing through the points $(-1, 2)$ and $(2, 4)$.
b The line passing through the points $(-2, 5)$ and $(1, -4)$.

3 John had \$200 in his money box, but he has been spending \$15 every week.

- **a** Fill in the next 5 numbers in this pattern: 200, 185, 170, ……, ……, ……, ……, ……
- **b** Copy the table and fill in the missing numbers.

x	0	1	2	3	4	5	6	7	8
y	200	185							

- **c** Plot the points onto a Cartesian plane.
- **d** What is the correct equation for this relationship?
  - **A** $y = 200x$
  - **B** $y = 200 - x$
  - **C** $y = 200x - 15$
  - **D** $y = 200 - 15x$
  - **E** $y = 15 - 200x$
- **e** After Week 8, John decided to stop spending and started to add \$10 every week to his money box. How long will it take him to get back to \$200?

Extended-response question

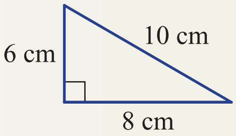
Doug works as a labourer. He is digging a trench and has 180 kg of soil to remove. He has taken 3 hours to remove 36 kg.

- **a** What is the rate at which he is removing the soil?
- **b** If he maintains this rate, write a rule for the amount of soil, S (kg), remaining after t hours.
- **c** Draw a graph of your rule.
- **d** How long will it take to remove all of the soil?
- **e** Doug is paid \$40 for the job plus \$25 per hour worked.
  - **i** Write a rule for his pay, P dollars, for working h hours.
  - **ii** How much will he be paid to remove all the soil?

Chapter 6: Length, area, surface area and volume

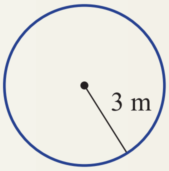
Multiple-choice questions

1 The perimeter and area of this triangle, respectively, are:



- **A** 48 cm, 24 m^2
- **B** 68 cm, 80 cm^2
- **C** 24 cm, 24 cm^2
- **D** 24 cm, 12 cm^3
- **E** 24 cm, 48 cm^2

2 The circumference and area for this circle, correct to two decimal places, respectively, are:



- **A** 37.70 m, 18.85 m^2
- **B** 9.42 m, 56.55 m^2
- **C** 18.85 m, 28.27 m^2
- **D** 18.85 m, 18.85 m^2
- **E** 9.42 m, 28.27 m^2

3 One square metre is equal to:

- **A** 1 cm^2
- **B** 10 cm^2
- **C** 100 cm^2
- **D** 1000 cm^2
- **E** 10 000 cm^2

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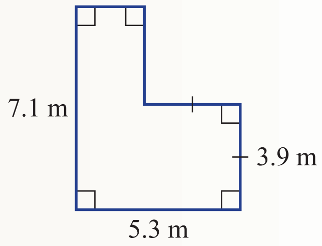
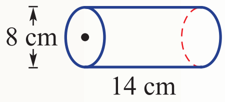
4 The perimeter and area of the figure shown, respectively, are:



- **A** 20.2 m, 22.42 m^2
- **B** 24.8 m, 22.42 m^2
- **C** 24.8 m, 25.15 m^2
- **D** 20.2 m, 25.15 m^2
- **E** 21.6 m, 24.63 m^2

5 The volume of the cylinder shown is closest to:

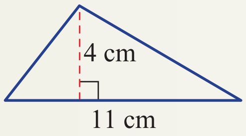


- **A** 703.7 cm^3
- **B** 351.9 cm^3
- **C** 2814.9 cm^3
- **D** 452.4 cm^3
- **E** 1105.8 cm^3

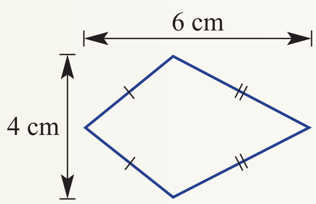
Short-answer questions

1 Find the area.

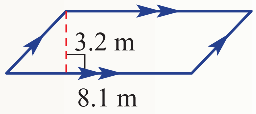
a



b

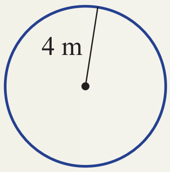


c

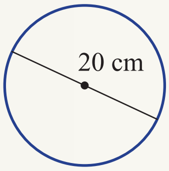


2 Find the circumferences and areas of these circles, correct to two decimal places.

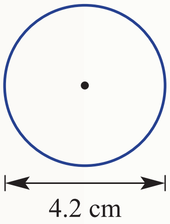
a



b

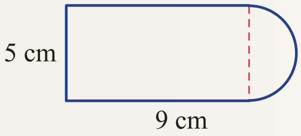


c

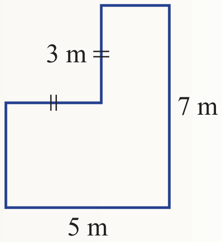


3 Find the area of each of the figures below. Round to two decimal places where necessary.

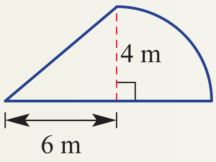
a



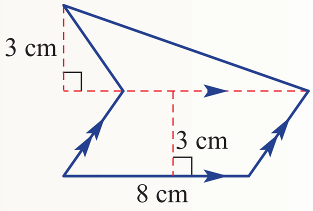
b



c

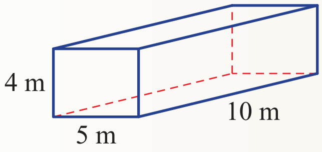


d

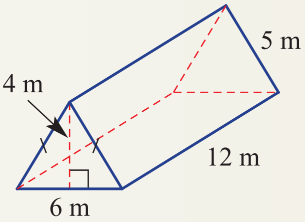


4 Find the surface areas of these solid objects. Round to two decimal places where necessary.

a

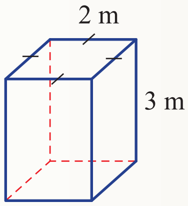


b

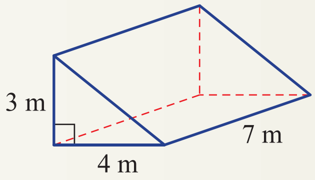


5 Find the volumes of these solids. Round to two decimal places for part **c**.

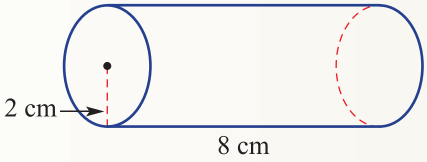
a



b



c



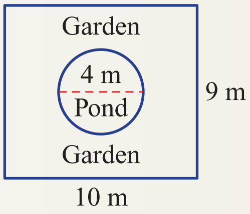
Extended-response question

A pond is to be built inside a rectangular garden, as shown.

a Find the area of the pond, correct to two decimal places.

b What is the area of the garden, not including the pond?

c The pond is to be surrounded by a low wall that costs $50 per metre. What will it cost to build this wall? Round your answer to the nearest dollar.



Chapter 7: Properties of geometrical figures

Multiple-choice questions

1 The supplementary angle to 55° is:

A 55° **B** 35° **C** 125° **D** 135° **E** 70°

2 A quadrilateral which has four equal sides and opposite sides parallel is definitely a:

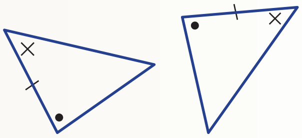
A parallelogram **B** rhombus **C** rectangle
D trapezium **E** kite

3 The size of the interior angle in a regular pentagon with angle sum 540° is:

A 108° **B** 120° **C** 96° **D** 28° **E** 115°

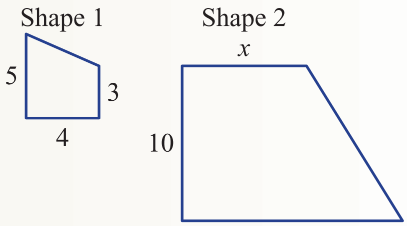
4 The test that proves congruence in these two triangles is:

A SAS **B** RHS **C** AAA
D SSS **E** AAS



5 What is the scale factor that enlarges shape 1 to shape 2 in these similar figures, and what is the value of x?

A 2 and $x = 8$
B 2.5 and $x = 7.5$
C 3.33 and $x = 13.33$
D 2.5 and $x = 12.5$
E 2 and $x = 6$



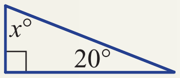
Short-answer questions

1 State the interior angle sums for these shapes.

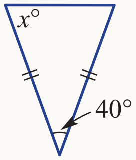
a triangle **b** quadrilateral **c** pentagon
d heptagon **e** octagon **f** decagon

2 Find the value of each pronumeral in the following.

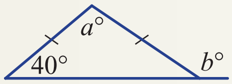
a



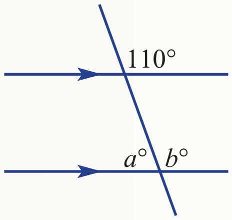
b



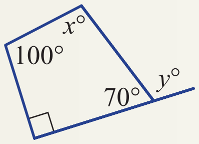
c



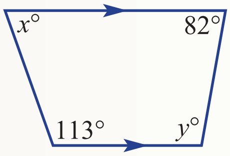
d



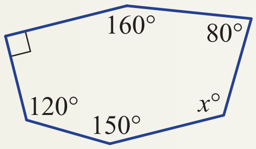
e



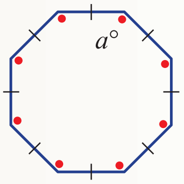
f



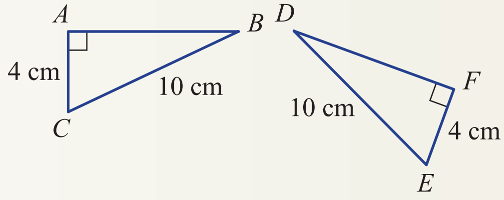
g



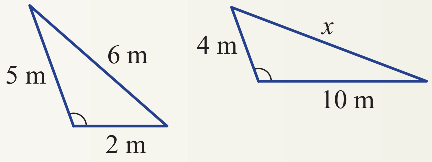
h



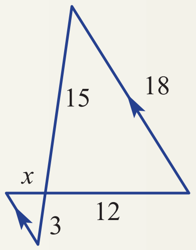
3 The two given triangles are similar.
- **a** Are they congruent? Give a reason.
- **b** Complete the congruence statement: $\triangle ABC \equiv \triangle\_\_\_$
- **c** Which side on $\triangle ABC$ corresponds to side DE on $\triangle DEF$?



4 For the given pair of similar triangles:
- **a** Find the scale factor.
- **b** Find the value of x.



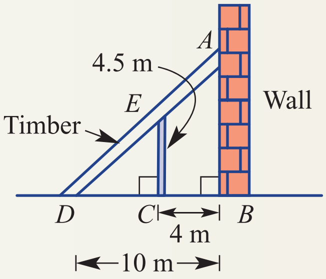
5 For this pair of similar triangles:
- **a** Find the scale factor.
- **b** Find the value of x.



Extended-response question

A vertical wall is being supported by a piece of timber that touches the ground 10 m from the base of the wall. A vertical metal support 4.5 m high is placed under the timber support 4 m from the wall.

- **a** Name the similar triangles.
- **b** What is the length DC?
- **c** What is the scale factor for the two triangles formed by the timber and support?
- **d** Find how far the timber reaches up the wall.
- **e** How far above the ground is the point halfway along the timber support?



Chapter 8: Probability and single variable data analysis

Multiple-choice questions

1 The probability of rolling a number less than 5 on a normal 6-sided die is:

A $\frac{1}{3}$ **B** 4 **C** $\frac{1}{2}$ **D** $\frac{2}{3}$ **E** 3

2 From the two-way table, P(both A and B) is:

A $\frac{1}{5}$ **B** 4 **C** $\frac{9}{20}$ **D** $\frac{1}{4}$ **E** 16

	A	Not A	Total
B		7	
Not B	5		
Total		11	20

3 In a bag of 40 marbles, 28 are blue.

i The probability of selecting a blue marble is:

A 0.28 **B** 0.4 **C** 0.7 **D** 0.54 **E** 0.75

ii If a marble is selected 50 times *with replacement*, the number of blue marbles you would expect to select is:

A 28 **B** 38 **C** 70 **D** 35 **E** 40

4 The median, mean and range of the data set

12 3 1 6 10 1 5 18 11 15

are, respectively:

A 5.5, 8.2, 1–18 **B** 8, 8.2, 17 **C** 5.5, 8, 17 **D** 8, 8.2, 1–18

E 8, 74.5, 18

5 The median of the data set in the stem-and-leaf plot is:

A 55 **B** 67 **C** 7

D 69 **E** 4

Stem	Leaf
5	1 3 5 5 5 5
6	2 4 5 6 7 7 9
7	1 3 4 8 9
8	3 4
9	1 2 6

5 | 1 means 51

Short-answer questions

1 In a survey of 30 people, 18 people drink coffee during the day, 14 people drink tea and 8 people drink both. Let C be the set of people who drink coffee and T the set of people who drink tea.

a Construct a Venn diagram for the survey results.

b Find the number who:

- **i** drink either coffee or tea or both
- **ii** do not drink tea

c If one of the 30 people surveyed was randomly selected, find:

- **i** P(drinks neither coffee nor tea)
- **ii** P(C only)

2 A coin and a 4-sided die are tossed.
The table opposite shows the sample space.

a How many outcomes are in the sample space?

b What is the probability of tossing a 'T, 4'?

c What is the probability of tossing tails and an odd number?

H, 1	T, 1
H, 2	T, 2
H, 3	T, 3
H, 4	T, 4

3 The data below show the number of aces served by a player in each of their grand slam tennis matches for the year.

15 22 11 17 25 25 12 31 26 18 32 11 25 32 13 10

- **a** Construct a stem-and-leaf plot for the data.
- **b** From the stem-and-leaf plot, find the mode and median number of aces.
- **c** Is the data set symmetrical or skewed?

4 The frequency table shows the number of visitors, in intervals of 50, to a theme park each day in April.

- **a** Copy and complete the frequency table shown. Round to one decimal place where necessary.
- **b** **i** On how many days were there fewer than 100 visitors?
  - **ii** What percentage of days had between 50 and 199 visitors?

Class interval	Class centre	Frequency	Relative frequency (%)
0–49	24.5	2	
50–99	54.5	4	
100–149	124.5	5	
150−199	174.5	9	
200−249	224.5		
250–299	274.5	3	
Total		30	

Extended-response question

A game at the school fair involves randomly selecting a green ball and a red ball, each numbered 1, 2 or 3. The outcomes are listed in the table.

		Red		
		1	2	3
Green	1	(1, 1)	(1, 2)	(1, 3)
	2	(2, 1)	(2, 2)	(2, 3)
	3	(3, 1)	(3, 2)	(3, 3)

- **a** What is the probability of getting an odd and an even number?
- **b** Participants win $1 when they draw each ball showing the same number. What is the probability of winning $1?
- **c** The ages of those playing the game in the first hour are recorded and are shown below.

  12 16 7 24 28 9 11 17 18 18 37 9 40 16 32 42 14

  - **i** Approximately 50% of the participants are below what age?
  - **ii** If the data are used as a model for the 120 participants throughout the day, how many would be expected to be aged less than 30?

Chapter 1

Pre-test

1 Addition: **a** sum **b** total **i** add
Subtraction: **c** less than **g** take away **h** difference,
Multiplication: **d** lots of **e** product **j** times
Division: **f** quotient

2 **a** 4 **b** 32 **c** 4
d 52 **e** 74 **f** 40

3 **a** 56 **b** 100 **c** 0 **d** 133
e 63 **f** 80 **g** 1020 **h** 53

4 **a** –12 **b** –3 **c** 3 **d** 0

5 **a** 3 **b** 3 **c** – **d** +

6 **a** 50 **b** 5 **c** 5000
d $\frac{5}{10}$ **e** $\frac{5}{100}$ **f** $\frac{5}{1000}$

7 **a** 2.654, 2.645, 2.564, 2.465
b 0.654, 0.564, 0.456, 0.0456

8 **a** 7.99 **b** 10.11 **c** 7.11

9 **a** 34.5 **b** 374 000 **c** 0.03754 **d** 0.00003754

10 **a** 6 **b** 12 **c** 30 **d** 1

11 **a** 15 **b** 12 **c** 10

12 **a** $\frac{5}{7}$ **b** $1\frac{1}{4}$ **c** 8 **d** $\frac{3}{8}$

Exercise 1A

1 **a** C **b** E **c** D **d** A **e** B

2 **a** D **b** B **c** E **d** A **e** C

3 **a** False **b** True **c** True
d True **e** True **f** False

4 **a** –2 **b** 2 **c** –3 **d** 4 **e** –4
f –5 **g** –2 **h** 0 **i** –6

5 **a** 3 **b** 2 **c** 1 **d** 0 **e** –1
f –9 **g** –1 **h** –17 **i** –10 **j** 9
k –2 **l** 11

6 **a** 5 **b** 20 **c** –20 **d** 0 **e** –7
f 6 **g** –24 **h** 5 **i** –15

7 **a** 4 **b** –4 **c** –4 **d** –9 **e** –32
f –38 **g** –189 **h** –24 **i** –18 **j** 0
k –805 **l** –57

8 **a** 4 **b** –2 **c** 13 **d** 9 **e** 24
f 0 **g** –28 **h** –81 **i** 180 **j** 2
k –3 **l** 240

9 **a** –27 **b** –3 **c** 10 **d** 26 **e** 0
f –24

10 **a** 2 **b** 11 **c** –3 **d** –1 **e** –7
f 11

11 **a** –8 **b** 4 **c** 13 **d** 124 **e** –19
f 0 **g** –22 **h** 98 **i** –26

12 19°C

13 **a**

–8	6	–4
2	–2	–6
0	–10	4

b Example answer (other answers are possible):

2	–13	8
5	–1	–7
–10	11	–4

c

5	–2	7	3
0	10	–3	6
–1	4	2	8
9	1	7	–4

Drilling for Gold 1A2

1 9
2 –3
3 3
4 –9
5 5
6 0
7 –4
8 5
9 –5
10 5
11 –7
12 3
13 –7
14 –2
15 –10
16 4
17 –12
18 11
19 –27
20 –2

Exercise 1B

1 **a** negative **b** negative **c** negative
d positive **e** positive **f** positive

2

×	–4	–3	–2	–1	0	1	2	3	4
–4	16	12	8	4	0	–4	–8	–12	–16
–3	12	9	6	3	0	–3	–6	–9	–12
–2	8	6	4	2	0	–2	–4	–6	–8
–1	4	3	2	1	0	–1	–2	–3	–4
0	0	0	0	0	0	0	0	0	0
1	–4	–3	–2	–1	0	1	2	3	4
2	–8	–6	–4	–2	0	2	4	6	8
3	–12	–9	–6	–3	0	3	6	9	12
4	–16	–12	–8	–4	0	4	8	12	16

3 **a** positive **b** negative **c** positive
d negative **e** negative

4 **a** 12 **b** –12 **c** –12 **d** 12 **e** 10
f –10 **g** –10 **h** 10 **i** –50 **j** –20
k 40 **l** 60

5 **a** 4 **b** –8 **c** 9 **d** –27 **e** 16
f –64 **g** 25 **h** –125 **i** –1000

6 **a** 5 **b** –5 **c** –5 **d** 5 **e** –4
f –4 **g** 2 **h** –2 **i** 2

7 **a** 16 **b** –26 **c** –12 **d** –14 **e** –20
f –28 **g** –72 **h** –8 **i** –90 **j** 10
k –24 **l** –18

8 **a** 7 **b** –8 **c** –8 **d** –2 **e** –1
f 6 **g** –9 **h** –5 **i** –3

9 –5 and 2

10 Two negatives with × and ÷ produce a positive.

11 **a** + **b** + **c** – **d** + **e** – **f** +

12 **a** –1 **b** –5 **c** –7 **d** –5
e 8 **f** 9 **g** 1 **h** –3

13 **a** –60 **b** –30 **c** 675 **d** 0
e 2250 **f** 1800 **g** 3600 **h** 675

Exercise 1C

1 **a** 2 **b** 3 **c** 1 **d** 2
e 1 **f** 0 **g** 3 **h** 1

2 **a** 2 **b** 4 **c** 1 **d** 1
e 4 **f** 3 **g** 3 **h** 4

3 **a** 40 **b** 270 **c** 7.9 **d** 0.04

4 **a** 32 100 **b** 432 **c** 5.89
d 0.443 **e** 0.00197

5 **a** 17.96 **b** 11.08 **c** 72.99 **d** 47.86
e 63.93 **f** 23.81 **g** 804.53 **h** 500.57
i 821.27 **j** 5810.25 **k** 1005.00 **l** 2650.00

6 **a** 2400 **b** 35 000 **c** 0.060 **d** 34
e 110 000 **f** 0.0025 **g** 2.1 **h** 0.71
i 4700 **j** 59 000 **k** 0.46 **l** 1.1

7 **a** 1.46, 1.5 **b** 0.09, 0.094
c 23.71, 24 **d** 0.01, 0.0078
e 100.47, 100

8 **a** 7 **b** 73 **c** 130 **d** 36 200

9 **a** 30 000 **b** 200 **c** 0.05 **d** 0.0006
e 5000 **f** 900 **g** 0.9 **h** 0.0003

10 **a** 3600, 3693 **b** 760, 759.4
c 4000, 4127.16 **d** 3000, 3523.78
e 0, 0.72216 **f** 4, 0.716245
g 0.12, 0.1186 **h** 0.02, 0.02254

11

$\frac{1}{3}$	$0.\dot{3}$	0.3	0.3
$\frac{2}{3}$	$0.\dot{6}$	0.7	0.7
$\frac{1}{9}$	$0.\dot{1}$	0.1	0.1
$\frac{3}{4}$	0.75	0.8	0.8

12 **a** A: 54.3, B: 53.8, 0.5 **b** A: 54.28, B: 53.79, 0.49
c A: 54, B: 54, 0 **d** A: 50, B: 50, 0

13 Magnesium in this case would be zero if rounded to two decimal places rather than two significant figures.

14 2.14999 is closer to 2.1, correct to one decimal place, ∴ round down.

15 **a** **i** 8 **ii** 1 **iii** 8 **b** 0.1428571428571
c **i** 4 **ii** 2 **iii** 8

Drilling for Gold 1C1

	Round			
	1	2	3	4
1	1	1.5	1.45	1.451
2	35	34.8	34.75	34.750
3	1	1.0	0.95	0.954
4	125	124.6	124.57	124.575
5	32	31.8	31.82	31.818
6	3	3.1	3.14	3.142
7	4	4.5	4.47	4.471
8	543	543.3	543.29	543.286
9	1976	1975.6	1975.62	1975.625
10	100	99.9	99.92	99.919

Drilling for Gold 1C2

	Round			
	1	2	3	4
1	54.28	54.3	54	50
2	103.5	103	100	100
3	2.845	2.85	2.8	3
4	27.50	27.5	28	30
5	0.2872	0.287	0.29	0.3
6	0.02872	0.0287	0.029	0.03
7	1276	1280	1300	1000
8	25 280	25 300	25 000	30 000
9	350 300	350 000	350 000	400 000
10	1.000	1.00	1.0	1

Exercise 1D

1 **a** True **b** False **c** True

2 **a** Yes **b** Yes **c** Yes **d** No

3 **a** $\frac{7}{10}$ **b** $\frac{3}{10}$ **c** $\frac{3}{2}$ **d** $\frac{7}{3}$

4 **a, b, c, d, e, f, g, i, k**

5 **a** 2.75 **b** 1.75 **c** 3.4 **d** 1.875
e 2.625 **f** 3.8 **g** 0.125 **h** 0.875

6 **a** $0.\overline{27}$ **b** $0.\dot{7}$ **c** $1.\overline{285714}$ **d** $0.41\dot{6}$
e $1.\dot{1}$ **f** $3.8\dot{3}$ **g** $7.2\dot{6}$ **h** $2.\overline{63}$

7 **a** $\frac{7}{20}$ **b** $\frac{3}{50}$ **c** $3\frac{7}{10}$ **d** $\frac{14}{25}$
e $1\frac{7}{100}$ **f** $\frac{3}{40}$ **g** $3\frac{8}{25}$ **h** $7\frac{3}{8}$
i $2\frac{1}{200}$ **j** $10\frac{11}{250}$ **k** $6\frac{9}{20}$ **l** $2\frac{101}{1000}$

8 **a** $\frac{5}{6}$ **b** $\frac{13}{20}$ **c** $\frac{7}{10}$ **d** $\frac{5}{12}$

9 **a** $\frac{9}{20}$ **b** $\frac{3}{20}$ **c** $\frac{32}{45}$

10 **a** $\frac{11}{6}, \frac{7}{3}$ **b** $\frac{2}{5}, \frac{2}{15}$ **c** $\frac{11}{12}, 1$ **d** $\frac{5}{7}, \frac{11}{14}$

11 **a** $\frac{3}{5}$ **b** $\frac{5}{9}$ **c** $\frac{8}{13}$ **d** $\frac{23}{31}$

12 **a** 31, 32 **b** 36, 37, 38, . . ., 55 **c** 4, 5

13 **a** $\frac{2}{9}$ **b** $\frac{7}{9}$ **c** $\frac{8}{9}$ **d** 1

Exercise 1E

1 **a** 12 **b** 6 **c** 6 **d** 8
e 15 **f** 49 **g** 15 **h** 35

2 **a** True **b** False

3 **a** $\frac{3}{10}+\frac{4}{10}=\frac{7}{10}$ **b** $\frac{4}{8}-\frac{3}{8}=\frac{1}{8}$
c $1\frac{2}{4}+\frac{1}{4}=1\frac{3}{4}$

4 **a** 6 **b** 63 **c** 65 **d** 30
e 4 **f** 33 **g** 60 **h** 87

5 **a** $\frac{3}{5}$ **b** $\frac{5}{8}$ **c** $\frac{7}{10}$ **d** $\frac{4}{7}$
e $\frac{5}{17}$ **f** $\frac{1}{5}$ **g** $\frac{10}{6}=1\frac{2}{3}$ **h** $\frac{10}{8}=1\frac{1}{4}$
i $\frac{7}{5}=1\frac{2}{5}$ **j** $1\frac{4}{5}$ **k** $2\frac{1}{3}$ **l** $\frac{1}{2}$
m $\frac{2}{5}$ **n** $4\frac{1}{2}$ **o** $1\frac{2}{5}$

6 **a** $\frac{5}{8}$ **b** $\frac{1}{3}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$
e $\frac{9}{10}$ **f** $\frac{7}{12}$ **g** $\frac{7}{10}$ **h** $\frac{3}{20}$
i $\frac{11}{24}$ **j** $1\frac{1}{6}$

7 **a** $3\frac{5}{6}$ **b** $\frac{7}{12}$ **c** $2\frac{4}{5}$ **d** $2\frac{7}{12}$
e $6\frac{3}{4}$ **f** $3\frac{5}{6}$ **g** $\frac{1}{8}$ **h** $1\frac{5}{12}$
i $1\frac{7}{10}$ **j** $\frac{11}{12}$ **k** $4\frac{7}{10}$ **l** $1\frac{7}{12}$

8 $\frac{7}{8}$ tonnes

9 $5\frac{29}{56}$ tonnes

10 **a** $4\frac{19}{20}$ litres **b** 33

11 $\frac{1}{20}$

12 **a** **i** $\frac{1}{8}+\frac{3}{4}$ **ii** $\frac{2}{3}-\frac{5}{12}$ **iii** $\frac{5}{6}-\frac{1}{2}$
b $\frac{1}{2}+\frac{2}{3}+\frac{5}{6}=2$

Keeping in touch with numeracy

1 **a** –3 **b** 3 **c** –3 **d** 0.01 **e** 0.25
f one-quarter **g** two-thirds

2 **a** –5, –10, –15 **b** –5, –11, –17
c –2, –11, –20 **d** 1, 4, 7
e 2, 1, 0.5 **f** –24, 48, –96

3 **a** negative **b** negative **c** positive
d negative **e** positive

4 **a** –19 **b** –77 **c** 19 **d** –1392 **e** 1392

5 **a** 0.5 **b** 0.25 **c** 0.75 **d** $0.\dot{3}$ **e** $0.\dot{6}$
f 0.1 **g** 0.7 **h** 0.2 **i** 0.8

6 **a** 0.125 **b** 0.375 **c** 0.625 **d** 0.875 **e** 0.0625
f 0.1875 **g** $0.1\dot{6}$ **h** $0.8\dot{3}$ **l** $0.08\dot{3}$

7 C

8 **a** 60 × 60 × 24 × 366 **b** 31 622 400
c Thirty-one million, six hundred and twenty-two thousand, four hundred

9 **a** 23.46 **b** 23.47 **c** 0.81
d 0.82 **e** 0 **f** 0.01

10 **a** 0.71 **b** 3.16 **c** $10.52

11 **a** 23 **b** 23 **c** 0.81 **d** 0.82 **e** 0.0012 **f** 0.0090

12 **a** 0.71 **b** 3.2 **c** $11

13 **a** $0.75 **b** $6 **c** $18 **d** $37.50

14 **a** $1.90 **b** $9.75 **c** $29.25 **d** $12.50

15 **a** 0 **b** 1 **c** 0 **d** 0
e 4 **f** 2 **g** 4 **h** 1

16 **a** 1 **b** 1 **c** 1 **d** 4
e 1 **f** 2 **g** 1 **h** 1

17 150

18 738

19 **a** –5 **b** 0

20 **a** –20 **b** –5 **c** –4 **d** –4

Exercise 1F

1 **a** $\frac{1}{2}$ **b** $\frac{2}{3}$ **c** $\frac{1}{2}$ **d** $\frac{2}{3}$ **e** $\frac{1}{2}$
f $\frac{3}{7}$ **g** $\frac{1}{3}$ **h** $\frac{3}{5}$ **i** $\frac{2}{3}$

2 **a** 12 **b** 12 **c** 6 **d** 4 **e** 180 **f** 12

3 **a** $\frac{4}{3}$ **b** 7 **c** $\frac{3}{2}$ **d** $\frac{8}{5}$ **e** $\frac{1}{6}$
f $\frac{1}{8}$ **g** $\frac{2}{3}$ **h** $\frac{3}{5}$ **i** $\frac{4}{11}$

4 **a** True **b** False **c** True **d** True
e True **f** True

5 **a** $\frac{3}{20}$ **b** $\frac{4}{15}$ **c** $\frac{15}{28}$ **d** $\frac{1}{6}$ **e** $\frac{1}{20}$
f $\frac{2}{27}$ **g** $\frac{10}{17}$ **h** $\frac{3}{7}$ **i** $\frac{2}{5}$ **j** $\frac{2}{3}$
k $\frac{2}{5}$ **l** $\frac{4}{5}$ **m** $\frac{3}{4}$ **n** $\frac{2}{3}$ **o** $\frac{2}{9}$

6 **a** $\frac{1}{2}$ **b** $1\frac{1}{4}$ **c** $2\frac{1}{10}$ **d** 2 **e** $4\frac{1}{5}$
f $\frac{6}{7}$ **g** $6\frac{1}{4}$ **h** 4 **i** $2\frac{11}{12}$ **j** 6

7 **a** $\frac{20}{21}$ **b** $1\frac{1}{8}$ **c** $\frac{45}{56}$ **d** $\frac{27}{28}$ **e** $1\frac{1}{3}$
f $1\frac{1}{2}$ **g** 6 **h** $\frac{7}{8}$ **i** 18 **j** 9
k 16 **l** 64 **m** $\frac{1}{10}$ **n** $\frac{1}{12}$ **o** $\frac{4}{27}$
p $3\frac{1}{3}$ **q** 4 **r** $\frac{1}{6}$ **s** $1\frac{1}{2}$ **t** $\frac{7}{8}$

8 **a** \$10 **b** \$3.50 **c** \$284.50 **d** \$6 **e** \$50
f \$18 **g** \$6400 **h** \$15 **i** 90c

9 $\frac{3}{12}$ hour = 15 minutes

10 160 km

11 90 minutes each

12 **a** **i** 0.5 **ii** 0.2 **iii** 0.16129… **iv** $0.\dot{6}$ **v** –1.
b 3.39 **c** 25
d Numbers greater than zero but less than 1.
e 1 **f** 0

Exercise 1G

1 **a** 4 **b** 12 **c** 24 **d** 72 **e** 3
f 9 **g** 11 **h** 3 **i** 60

2 **a** 9 **b** $\frac{4}{9}$ **c** $\frac{5}{9}$ **d** 8

3

a	1 : 3	1 : 4
b	1 : 2	1 : 3
c	2 : 3	2 : 5
d	1 : 2	1 : 3
e	1 : 3	1 : 4
f	1 : 2	1 : 3

4 **a** 5 : 1 **b** 1 : 2 **c** 2 : 1 **d** 10 : 1 **e** 5 : 16

5 **a** 1 : 5 **b** 2 : 5 **c** 4 : 5 **d** 1 : 10
e 5 : 6 **f** 1 : 3 **g** 3 : 1 **h** 2 : 3
i 4 : 3 **j** 4 : 1 **k** 1 : 2 **l** 3 : 1
m 5 : 14 **n** 1 : 2 : 4 **o** 3 : 6 : 2

6 **a** 1 : 2 **b** 3 : 41 **c** 5 : 32 **d** 1 : 3 **e** 1 : 5
f 10 : 3 **g** 40 : 1 **h** 1500 : 1 **i** 20 : 7

7 **a** 3 : 4 **b** 8 : 3 **c** 9 : 20
d 45 : 28 **e** 3 : 14 **f** 22 : 39

8 **a** 1 : 10 **b** 1 : 5 **c** 2 : 3 **d** 7 : 8
e 25 : 4 **f** 1 : 4 **g** 1 : 4 **h** 24 : 5
i 4 : 20 : 5 **j** 4 : 3 : 10 **k** 5 : 72 **l** 3 : 10 : 40

9 **a** \$100, \$400 **b** \$16, \$20
c 24 kg, 64 kg **d** \$56, \$40
e \$200, \$300 **f** 750 g, 1250 g
g \$70, \$30 **h** \$300, \$300
i \$14, \$49, \$7 **j** 336 g, 84 g

10 126 g

11 120

12 \$3000, \$1200, \$1800 respectively

13 108 L

14 **a** **i** 100 mL **ii** 200 mL **b** **i** 250 mL **ii** 270 mL
c **i** 300 mL **ii** 1 : 4
d **i** 1 : 3 **ii** 7 : 19 **iii** 26 : 97 **iv** 21 : 52
e Jugs 3 and 4

Drilling for Gold 1G2

1 2 : 3 **2** 3 : 1 **3** 1 : 1 **4** 3 : 2
5 2 : 3 **6** 3 : 5 **7** 1 : 2 **8** 1 : 3
9 6 : 5 **10** 4 : 3 **11** 3 : 2 **12** 2 : 3
13 1 : 3 **14** 3 : 5 **15** 5 : 8 **16** 1 : 2

Drilling for Gold 1G3

1 \$48, \$12
2 \$20, \$40
3 \$45, \$15
4 \$36, \$24
5 \$10, \$50
6 \$42, \$18
7 \$6, \$54
8 \$28, \$32
9 \$37.50, \$22.50
10 \$52.50, \$7.50
11 96 mm, 24 mm
12 72 mm, 48 mm
13 36 mm, 84 mm
14 50 mm, 70 mm
15 20 mm, 100 mm
16 84 mm, 36 mm
17 12 mm, 108 mm

Exercise 1G cont.

18 56 mm, 64 mm

19 75 mm, 45 mm

20 105 mm, 15 mm

Exercise 1H

1 **a** 60 km/h **b** \$84/h **c** 60 km/h
d \$4/kg **e** \$16/h

2 \$90/h

3 **a** \$4 **b** \$3 **c** \$2.50

4 **a** \$52.50 **b** \$61.25 **c** \$239.75

5 **a** \$28/h **b** \$50/h
c 6 kg/min **d** \$7/kg
e 4 runs/over **f** 89 pts/game
g 19 cm/year **h** 14 m/s
i 92 beats/min **j** 8 mL/s

6 **a** 70 km/h **b** 15 km/h **c** 4 km/h
d 54 km/h **e** 80 km/h

7 13 hours

8 **a** 3 kg deal **b** Red delicious **c** 2.4 L **d** 0.7 GB

9 **a** **i** 130 words **ii** 325 words **iii** 3900 words
b **i** 44 g **ii** 220 g

10 **a** 55 km **b** $16\frac{1}{2}$ km **c** $5\frac{1}{2}$ km

11 **a** Coffee A \$3.60, coffee B \$3.90. Therefore, coffee A is the best buy.
b Pasta A \$1.25, pasta B \$0.94. Therefore, pasta B is the best buy.
c Cereal A \$0.37, cereal B \$0.40. Therefore, cereal A is the best buy.

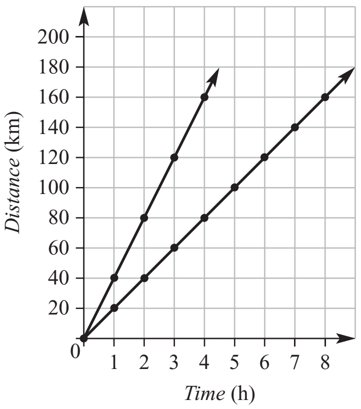
12 **a** 40 sheep/h **b** 25 hours

13 **a** 1 080 000 000 km/h **b** $8\frac{5}{18}$ min

14 **a**

Time (in hours)	0	1	2	3	4	5	6	7	8
Distance (in km)	0	20	40	60	80	100	120	140	160

b, d



c Straight line, speed was constant

Consumer maths

Answers will vary.

Drilling for Gold 1H3

1 \$2.02 **2** \$6.67, \$4.96

3 \$1.30 **4** **a** \$3.13 **b** yes

5 \$18.78, \$10.77

6 **a** \$111.80 **b** \$66.80

7 **a** \$10.12 **b** \$4.58

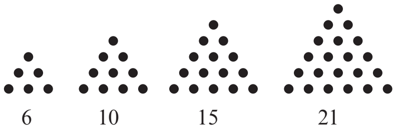
Puzzles and games

1 $1 + 2 + 3 + 4 + 5 + 6 + 7 + (8 \times 9) = 100$

2

1. **9**	2. **2**	3. **5**	**3**
4. **3**	**5**	5. **5**	6. **6**
7. **9**	8. **9**	9. **2**	**4**
10. **1**	**2**	**8**	**0**

3 **a** **i**



ii 28, 36

b **i** 0, 1, 1, 2, 3, 5, 8, 13, 21, 34
ii –21, 13, –8, 5, –3, 2, –1, 1, 0, 1, …
∴ –21, –8, –3, –1

Multiple-choice questions

1 D **2** B **3** C **4** A
5 E **6** E **7** D **8** C
9 B **10** C **11** C **12** B

Short-answer questions

1 **a** 8 **b** –45 **c** 270
d –64 **e** 6 **f** 8

2 **a** 21.5 **b** 29 100 **c** 0.153 **d** 0.00241

3 **a** 14.98 **b** 0.71 **c** 2.00

4 **a** 2.125 **b** $0.8\dot{3}$ **c** $1.\overline{857142}$

5 **a** $\frac{3}{4}$ **b** $1\frac{3}{5}$ **c** $2\frac{11}{20}$

6 **a** $\frac{4}{3}$ **b** 5 **c** $\frac{1}{8}$ **d** $\frac{7}{9}$

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7 a $\frac{1}{2}$ **b** $2\frac{1}{6}$ **c** $\frac{7}{24}$ **d** $\frac{11}{12}$
e $\frac{3}{10}$ **f** 2 **g** $3\frac{3}{4}$ **h** $\frac{3}{7}$
8 a 5 : 2 **b** 1 : 2 **c** 75 : 14
9 a 50, 30 **b** 25, 55 **c** 10, 20, 50
10 a Store A \$2.25/kg, store B \$2.58/kg. Store A is the best buy.
b Store A 444 g/\$, store B 388 g/\$
11 \$132, \$198, \$330
12 a 32 **b** 7 **c** 192 **d** 14.80
13 a 156 km/h **b** 36 km/h
14 \$6000
15 36 m
16 Doubles the perimeter.

Extended-response questions

1 a 12 **b** 10 boys, 18 girls **c** 5 : 9
2 a 85.6 km/h **b** 14 hours **c** 61.1 km/h

Chapter 2

Pre-test

1 a 55% **b** $\frac{3}{4}$ **c** 0.4
2 a 61 **b** 9 **c** 37 **d** 121
e 1 **f** 75, 4
3

Fraction	Decimal	Percentage
$\frac{1}{100}$	0.01	1%
$\frac{1}{10}$	0.1	10%
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%

4 a 10 g **b** 7 km **c** \$45
d \$800 **e** 50c **f** 9c
5 a \$100 **b** \$60 **c** \$1.50
d \$67.80 **e** \$6 **f** \$16
6 a 100 **b** 10 **c** 20 **d** 3 **e** 4 **f** 2
7 a 7 **b** 365 or 366 **c** 52 **d** 12
8 a 14 **b** 6 **c** 12.5

Exercise 2A

1 a 25% **b** 50% **c** 40%
2 a 75% **b** 70%
3

%	10%	20%	25%	50%	75%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$
Fraction	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$
Decimal	0.1	0.2	0.25	0.5	0.75	$0.\dot{3}$	$0.\dot{6}$

4 Abigail because $80\% > \frac{30}{50} = 60\%$.
5 a 20% **b** 80% **c** 80% **d** 30%
e 25% **f** $12\frac{1}{2}\%$ **g** 75% **h** 60%
i 56% **j** 35% **k** 9% **l** $7\frac{1}{2}\%$
6 a $\frac{19}{100}$ **b** $\frac{23}{100}$ **c** $\frac{99}{100}$ **d** $\frac{1}{20}$
e $\frac{11}{50}$ **f** $\frac{9}{20}$ **g** $\frac{37}{50}$ **h** $\frac{3}{4}$
i $\frac{1}{40}$ **j** $\frac{69}{400}$ **k** $\frac{1}{100}$ **l** $1\frac{1}{4}$
7 a 78% **b** 95% **c** 65% **d** 48%
e 75% **f** 142% **g** 7% **h** 30%
i 3% **j** 104% **k** 12% **l** 12.25%
8 a 0.12 **b** 0.83 **c** 0.57 **d** 0.88
e 0.99 **f** 1.0 **g** 1.2 **h** 0.05
9 a 2.5% **b** 10% **c** 5% **d** 5%
e 20% **f** 20% **g** 200% **h** 75%
10

Sport	Number of students who chose sport	Fraction of the total	Percentage of the total
Swimming	44	$\frac{22}{75}$	$29\frac{1}{3}\%$
Golf	12	$\frac{2}{25}$	8%
Volleyball	58	$\frac{29}{75}$	$38\frac{2}{3}\%$
Cricket	36	$\frac{6}{25}$	24%
TOTAL	150	1	100%

11 62% 12 $16\frac{2}{3}\%$
13 $6\frac{1}{4}\%$ **14** Yes, 2.8% fat content
15 a Answers will vary.
b The number in cell E1 changes.
d i 20% **ii** 24% **iii** 30% **iv** 60%
v 42.5% **vi** 56.7% **vii** 43% **viii** 44.7%

Exercise 2B

1 **a** $4 **b** $2 **c** 2 kg **d** 6 kg
e $8 **f** 2.8 seconds
2 $400
3 True
4 **a** 2 **b** 10 **c** 8 **d** 7.5
e 57.6 **f** 34 **g** 1230 **h** 42
i 450 **j** 33 **k** 198 **l** 1.5
5 **a** $36 **b** $210 **c** 48 kg
d 30 km **e** 15 apples **f** 350 m
g 250 people **h** 200 cars **i** $49
6 **a** $120 **b** $700 **c** $300 **d** $7
e $0.20 **f** $450 **g** $800 **h** $360
7 **a** $540 **b** $600 **c** $508
d $1250 **e** $120 **f** $40
8 **a** $5.80 **b** $4.20 **c** $0.46
d $0.50 **e** $44 **f** $40
9 64
10 **a** Divide by 10.
b Divide by 2.
c Divide by 2 and then 2 again (or just 4).
11 48 kg
12 15 students
13 $\frac{10}{100}\times 24=\frac{24}{100}\times 10$ as $\frac{24\times 10}{100}$, can multiply in any order
14 $x=2$, $y=24$
15 **a** 72 **b** $\frac{10}{11}$ **c** 280% **d** $3\frac{1}{4}$ **e** 150%

Keeping in touch with numeracy

1 45
2 119
3 **a** 20 cm **b** 19 cm^2
4 **a** 9.61 m^2 **b** 8.635 m^2 **c** 452.39 m^2 **d** 75.40 m
5 **a** 0.5 **b** 0.25 **c** 0.2 **d** 0.8
6 **a** 12.5% **b** 37.5% **c** 42.5% **d** $88\frac{1}{3}\%$
7 2 hours 15 minutes
8 3 hours 38 minutes
9 C
10 $420
11 **a** $24 **b** $12 **c** $36 **d** $60 **e** $2.40
12 $42
13 $5.20
14 $10
15 14
16 $122.50
17 The first fraction is one-half and the second fraction is more than one-half, so the total must be more than one whole. Correct value is $1\frac{1}{4}$.
18 $\frac{19}{24}$
19 $12, $8
20 $140, $40

Exercise 2C

1 **a** 1.25 **b** 1.26 **c** 1.12 **d** 1.21
e 1.2 **f** 1.05 **g** 0.75 **h** 0.9
i 0.95
2 $30
3 12 kg
4 **a** 82 m **b** 902 m **c** 738 m
d 902 m, same as increasing by 10%
e 738 m, same as decreasing by 10%
5 **a** 61.6 **b** 1176 **c** 112 **d** 934.5 **e** 207
f 540 **g** 12 **h** 196 **i** 31.50
6 **a** 76 **b** 540 **c** 22.5 **d** 616 **e** 7360
f 337.5 **g** 17 **h** 8910 **i** 0
7 **a**

Original amount	New amount	Increase	Percentage change
40	60	20	50%
12	16	4	$33\frac{1}{3}\%$
100	125	25	25%
24	30	6	25%
88	100	12	13.6%
48	92	44	91.7%
200	250	50	25%

b

Original amount	New amount	Decrease	Percentage change
90	81	9	10%
100	78	22	22%
20	15	5	25%
24	18	6	25%
150	50	100	$66\frac{2}{3}\%$
9	8.3	0.7	7.8%
3	2.5	0.5	$16\frac{2}{3}\%$

8 **a** $44, $4 **b** $132, $12 **c** $50, $5
d $140, $14 **e** $70, $77
9 $1785
10 $84 460
11 24 534 cars
12 50%

13 28%
14 $21.50
15 $10.91
16 **a** $900 **b** $990
c As 10% of 1000 = 100 but 10% of 900 = 90.
17 **a** $594 **b** $2300.65
18 **a** $65 160.50 **b** 79.86 g

Drilling for Gold 2C2

2 $86.70
3 $117.30
4 $117.30
5 $181.50
6 $121.50
7 $148.50
8 $148.50
9 24 847
10 22 28
11 Between 6 and 7 years
12 Between 5 and 6 years
13 $336.14
14 $10 285.72

Exercise 2D

1 $3 profit, $2.50 loss, $1.40 profit, $4.50 profit, $65.95 loss, $1180 profit
2 40.95, 179.95, 595.90, 799.95, 18 799, 8995
3 28, 9.05, 7.25, 199, 345.50, 2037
4 **a** 90% **b** 80% **c** 85% **d** 92%
5

Item	Cost price	% Mark-up	Mark-up amount	Selling price
Jeans	$60	28%	$16.80	$76.80
Toaster	$40	80%	$32	$72
Car	$22 000	45%	$9900	$31 900
Can of drink	$1.20	140%	$1.68	$2.88
Loaf of bread	$1.80	85%	$1.53	$3.33
Handbag	$80	70%	$56	$136
Tablet	$320	35%	$112	$432

6

Item	Cost price	% Discount	Discounted amount	Selling price
Camera	$900	15%	$135	$765
Car	$24 000	20%	$4800	$19 200
Bike	$600	25%	$150	$450
Shoes	$195	30%	$58.50	$136.50
Blu-ray player	$245	50%	$122.50	$122.50
Electric razor	$129	20%	$25.80	$103.20
Lawnmower	$880	5%	$44	$836

7 **a**

Cost price ($)	Selling price ($)	Profit ($)	Profit (%)
10	15	5	50%
24	30	6	25%
100	150	50	50%
250	255	5	2%
17.50	20	2.50	14.29%

b

Cost price ($)	Selling price ($)	Loss ($)	Loss (%)
10	8	2	20%
16	12	4	25%
100	80	20	20%
34	19	15	44.12%
95	80.75	14.25	15%

8 **a** $5.50 **b** $200 **c** $125
9 **a** **i** $2 **ii** 20% **b** **i** $5 **ii** 25%
c **i** $16.80 **ii** 14% **d** **i** $2450 **ii** 175%
10 40%
11 $1001.25
12 42.3%
13 **a** $400 **b** $900 **c** $765
14 $613.33
15 **a** $350 000
b **i** 71.4% **ii** 157.1% **iii** 114.3%
c 25%

Drilling for Gold 2D1

1 $60
2 $50
3 10%
4 5%
5 1
6 25%
7 25 mL
8 200
9 50%
10 30 g
11 20 cm
12 15%
13 $82.25
14 245 cm
15 9%
16 80.7
17 $620
18 45%
19 $10.71
20 1000

Exercise 2D cont.

21 709.5 m
22 15.75%
23 500 m
24 8.65625

Maths@work: How much pay?

1 $195 **2** $330 **3** $596.25 **4** $175
5 $1148.58 **6** $43.50 **7** 2 **8** 4
9 5 **10** 10, 4, 2, for example

Exercise 2E

1 **a** $25.40 **b** $101.60 **c** $127 **d** $482.60
2 **a** $36 **b** $48
3 **a** $11 406.25 **b** $2632.21 **c** $375
4 **a** $200 **b** $56
c $900 **d** $145.10
5 **a** $321.10 **b** $8.45
c $642.20 **d** $1391.43
6 **a** $41 080 **b** $3423.33 **c** $19.75
7

Employee	Hourly rate	Hours worked	Income
Rhys	$10.40	8	$83.20
Sofia	$15.50	$8\frac{1}{2}$	$131.75
Ceanna	$9.70	15	$145.50
David	$24.30	38	$923.40
Edward	$17.85	42	$749.70
Francis	$30	27	$810
Marius	$15.20	7.25	$110.20

8 **a** $165.60 **b** $239.20 **c** $368
d $552 **e** $515.12 **f** $809.60
9

Employee	Weekly retainer	Rate of commission	Commission earned	Weekly wage
Adina	$0	12% on $7000	$840	$840
Byron	$160	8% on $600	$48	$208
Cindy	$300	5% on $680	$34	$334
Deanne	$260	5% on $40 000	$2000	$2260
Elizabeth	$500	8% on $5600	$448	$948
Faruq	$900	2% on $110 000	$2200	$3100
Leo	$1000	1.5% on $45 000	$675	$1675

10 $14.50 per hour
11 $12.20 per hour
12 $490
13 $4010
14 $873
15 **a** 7 **b** 18 **c** 33
16 **a** $250.55 **b** $46.80 **c** 6 hours

Drilling for Gold 2E1

Answers will vary.

Exercise 2F

1

Gross income	Deductions	Net income
$5600	$450	$5150
$87 000	$28 000	$59 000
$50 000	$6700	$43 300
$890	$76	$814
$84 650	$24 790	$59 860

2 $1405
3 $25 000
4 $27 023.85
5 **a** **i** $40 035 **ii** 17.0%
b **i** $53 905.80 **ii** 20.1%
c **i** $3434.85 **ii** 15%
d **i** $30 052.56 **ii** 22.2%
6 **a** $19 656 **b** $53 144
7 **a** $0 **b** $0 **c** $2242 **d** $7797
e $17 547 **f** $35 677 **g** $45 667 **h** $63 547
8 $67 400
9 **a** $51 200 **b** $8187 **c** $43 013 **d** $1654.35
10 $19
11 **a** $37 001–$80 000 **b** $41 394
12 **a** $81 120 **b** $17 961.40
c $63 158.60
13 **a** $78 560 **b** $17 079
c $61 481
14 $32.26

Exercise 2G

1 **a** 12 **b** 3 **c** 1.5 **d** $4\frac{1}{3}$
2 **a** $12 000 **b** 6% p.a.
c 3.5 years **d** $720
3 **a** $3000 **b** $3600 **c** $416 **d** $315
4 **a** $1120 **b** $2800 **c** $5600
5 **a** $300 **b** $900 **c** $1600
d $3150 **e** $1196.25
6 $2700, $17 700
7 **a** $52.50 **b** $100 **c** $40
d $264.38 **e** $1027.40
8 $1980, $23 980
9

a	$840	$7840
b	$840	$2340
c	$1500	$41 500
d	$4550	$74 550
e	$200	$2200

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10 $2560

11 $38 000

12 Choice 2

13 **a** **i** $12 **ii** $31 **iii** $10

b **i** $335 **ii** $500 **iii** $235

Exercise 2H

1 **a** 2 years **b** 4 years

c 50 years **d** $2\frac{1}{2}$ years

2 5 years

3 $187.50

4 $2000

5 3 years

6 $4\frac{1}{2}$ years

7 18 months

8 **a** 6.25% **b** 2% **c** 6.5% **d** 5.5%

9

Interest	Total amount to be repaid	Monthly repayment
$5250	$10 250	$170.83
$10 500	$24 500	$408.33
$2400	$12 400	$258.33
$44 000	$99 000	$825
$525 000	$775 000	$2152.78

10 $2083.33

11 **a** $14 400 **b** $240

12 10%

13 **a** $$P$

b 12.5%

c **i** 20 years **ii** 40 years

iii The period in part **ii** is double the period in part **i**.

14 **a** Final amount = $732.05 **b** $232.05

15 **a** $5743.27

b $5743.27, same as final answer in part **a**.

c Increases the amount by 5% five times, giving the same result.

Consumer maths

1 Answers will vary.

2 Answers wil vary.

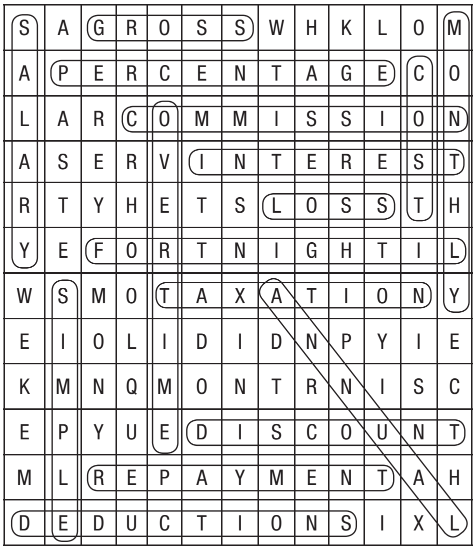
3 **a** $600 **b** $790.85

4 **a** 36 **b** 24 **c** 18 **d** 12

5 You get 72.

Puzzles and games

1



S	A	G	R	O	S	S	W	H	K	L	O	M
A	P	E	R	C	E	N	T	A	G	E	C	O
L	A	R	C	O	M	M	I	S	S	I	O	N
A	S	E	R	V	I	N	T	E	R	E	S	T
R	T	Y	H	E	T	S	L	O	S	S	T	H
Y	E	F	O	R	T	N	I	G	H	T	I	L
W	S	M	O	T	A	X	A	T	I	O	N	Y
E	I	O	L	I	D	I	D	N	P	Y	I	E
K	M	N	Q	M	O	N	T	R	N	I	S	C
E	P	Y	U	E	D	I	S	C	O	U	N	T
M	L	R	E	P	A	Y	M	E	N	T	A	H
D	E	D	U	C	T	I	O	N	S	I	X	L

2 EARTH, AIR, FIRE, WATER

Drilling for Gold 2R1

1 9 **2** 25% **3** 400

4 69 **5** 51 **6** 52.17 (2 d.p.)

7 70.59 (2 d.p.) **8** 300% **9** 75%

Multiple-choice questions

1 C **2** C **3** B **4** D

5 C **6** C **7** C **8** A

9 C **10** E

Short-answer questions

1

Decimal	Fraction	Percentage
0.5	$\frac{1}{2}$	50%
$0.\dot{3}$	$\frac{1}{3}$	$33\frac{1}{3}\%$
0.25	$\frac{1}{4}$	25%
0.75	$\frac{3}{4}$	75%
1.5	$1\frac{1}{2}$	150%
2	2	200%

2 **a** $77.50 **b** 1.65

3 **a** 150 **b** 25

4 **a** 72 **b** 1.17 **c** 20%

5 12.5 kg

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Exercise 2H cont.

6 $2070
7 $1800
8 **a** \$25 **b** $16\frac{2}{3}\%$
9 **a** $18.25 **b** $14.30
10 $50 592
11 $525
12 5%

Extended-response questions

1 **a** $231 **b** $651 **c** **i** $63 **ii** $34.65
2 **a** $25.68 **b** $38.52 **c** $154.08
d 6.5 **e** $46 547.31 **f** $558.57

Chapter 3

Pre-test

1 **a** $3x$ **b** $4ab$ **c** $10x$ **d** $21b$
2 **a** 8 **b** 7 **c** 12
d 6 **e** 9 **f** 12
3 **a** $x+3$ **b** ab
4 **a** –18 **b** 40 **c** 3
d –5 **e** –4 **f** –8
5 **a** $7x$ **b** $5y$ **c** $4x$ **d** $2x$
6 **a** $6a$ **b** $6ab$ **c** $6a^2$ **d** $6a^2b$
7 **a** $2x+6$ **b** $3a-15$ **c** $8x+4$
8 **a** 1 **b** –2 **c** 5 **d** –6
9 **a** 9 **b** 10 **c** 4 **d** 21

Exercise 3A

1 **a** expression **b** constant term **c** pronumeral
d coefficient **e** terms
2 **a** C **b** E **c** B **d** A **e** D
3 **a** $7y$ **b** $-2x$ **c** b
d ab **e** $\frac{y}{2}$ **f** $\frac{x}{y}$
4 **a** 5 **b** 8 **c** 14 **d** 4
5 **a** **i** 3 **ii** 2 **iii** 3
b **i** 2 **ii** –3 **iii** 0
c **i** 3 **ii** 7 **iii** –4
d **i** 4 **ii** $\frac{1}{2}$ **iii** –1
6 **a** **i** $4+r$ **ii** $t+2$ **iii** $x+y+z$
b **i** $6P$ **ii** $10n$ **iii** $2D$ **iv** $5P+2D$
c $\frac{500}{C}$
7 **a** $2+x$ **b** $ab+y$ **c** $x-5$ **d** $2y-7$
e $3x$ **f** $3p$ **g** $2x+4$ **h** $\frac{x+y}{5}$
i $4x-10$ **j** $1-3x$ **k** $\frac{3+y}{2}$ **l** $\frac{1}{2}(x+1)$
m $(m+n)^2$ **n** m^2+n^2

8 **a** 20 **b** 25 **c** 15
d 0 **e** 6 **f** 5
g 28 **h** –96 **i** 5
9 **a** 60 m^2 **b** Length = $12+x$, breadth = $5-y$
10 **a** \$26 **b** \$50 **c** No **d** 12 hours
11 **a** 18 square units **b** 1, 2, 3, 4, 5
12 **a** $\frac{A}{20}$ **b** $\frac{3A}{20}$ **c** $\frac{nA}{20}$
13 **a** **i** $P=2x+2y$ **ii** $A=xy$
b **i** $P=4p$ **ii** $A=p^2$
c **i** $P=6+x+y$ **ii** $A=3x$
14 **a** **i** No **ii** No
b **i** $3x+1$ or $3(x+1)$
ii $5+\frac{x}{3}$ or $\frac{5+x}{3}$
iii $\frac{1}{2}(x+y)$ or $\frac{1}{2}x+y$
c Answers may vary.
i The sum of 2 and x is multiplied by 4.
ii 3 is added to a half of x.
iii A third of the sum of m and n.
iv 7 more than 5 lots of x.
v The sum of x and y is divided by 2.
vi The sum of b and a half of a.

Drilling for Gold 3A3

1 $x+y$ **2** $x+2$ **3** $\frac{x}{2}+2$ **4** $x+x$
5 $2(x+y)$ **6** $2x+3$ **7** $x-3$ **8** $\frac{x}{3}$
9 xy **10** $2x$ **11** $\frac{x}{2}$ **12** $3x$
13 $\frac{x}{3}-1$ **14** $x+3$ **15** $x-2$ **16** $2x+y$
17 $2-x$ **18** $3-x$ **19** $\frac{2x}{3}$ **20** $\frac{3x}{2}$

Drilling for Gold 3A4

Answers will vary.

Keeping in touch with numeracy

1 0.06 **2** $63.21
3 **a** 21 **b** 51 **c** 63 **d** 33
4 $52\frac{1}{3}$ **5** \$150 **6** \$27.50 **7** C
8 **a** 210° **b** 240° **c** 300° **d** 330°
9 400 **10** $206.25
11 4 **12** 66
13 2 **14** 4
15 $25 **16** $137 500
17 $525 **18** $498.75
19 50% **20** 30%

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Exercise 3B

1 **a** like **b** coefficients
c pronumeral **d** 1

2 **a** Yes **b** No **c** No
d No **e** Yes **f** Yes
g Yes **h** No **i** Yes

3 **a** –3 **b** –1 **c** –5 **d** –3
e –5 **f** –3 **g** 3 **h** 7
i 3 **j** –8 **k** –5 **l** –11

4 a, c, d, f

5 **a** $4y$ and $2y$, $3xy$ and $7xy$ **b** $3x$ and $7x$
c $7ab$ and $-3ab$, $2a$ and $5a$
d $2a^2$ and $-3a^2$, $4a$ and a **e** $3x^2y$ and $7yx^2$
f $5ab^2$ and $4ab^2$, $3ab$ and $7ba$

6 **a** $10a$ **b** $7n$ **c** $8y$ **d** $11x$
e $4ab$ **f** $8mn$ **g** $7y + 8$ **h** $3x + 5$
i $4m + 2$

7 **a** $5a + 9b$ **b** $6x + 5y$ **c** $5xy + 4x$ **d** $4t + 6$
e $11x + 4$ **f** $7mn - 1$ **g** $5ab - a$ **h** 0
i $10ac - 9c$ **j** $3xy^2$ **k** $3m^2n$ **l** $7x^2y - 4xy$

8 **a** $x + y$ **b** $4x + 2y$

9 **a** $4x + 2y$ **b** $12s$ **c** $30x$
d $5x + 4$ **e** $6x + 16$ **f** $10a + 4b + 4$

10 $8x$ metres

11 **a** $4t$ **b** $9x$ **c** $2x$ **d** $7a$
e $5n$ **f** $4pq$ **g** $2xy$ **h** $3b$

12 **a** $3a + 7b$ **b** 4 units each
c a **d** $2a - b$

Drilling for Gold 3B1

1 Yes **2** No **3** Yes **4** Yes
5 No **6** No **7** No **8** Yes
9 Yes **10** No **11** No **12** No
13 Yes **14** Yes **15** No

Drilling for Gold 3B2

1 $2x + 3x$, $5x$ **2** $x + 3x$, $4x$
3 $2x$, $x + x$ **4** $0x$, 0
5 $-5x + 6x$, $1x$, x **6** $2x$
7 $0x$, 0 **8** $-1x$, $-x$
9 $2x - x$, $1x$, x, $x + 0$ **10** $-x$, $-1x$

Exercise 3C

1 **a** $7x$ **b** ab **c** 6 **d** y
e $42b$ **f** $20x$ **g** a^2 **h** y^2

2 **a** negative **b** negative **c** negative

3 **a** 2 **b** 6 **c** 12
d 3 **e** 7 **f** 5

4 **a** $\frac{2}{3}$ **b** $\frac{5}{2}$ **c** 3 **d** $\frac{1}{5}$
e $\frac{-2}{7}$ **f** $\frac{-1}{20}$ **g** $\frac{-3}{4}$ **h** $\frac{-9}{2}$

5 **a** $10m$ **b** $12b$ **c** $15p$ **d** $6x$
e $18pr$ **f** $16mn$ **g** $-14xy$ **h** $-15mn$
i $-12cd$ **j** $30ab$ **k** $-24rs$ **l** $-40jk$

6 **a** $4b$ **b** $\frac{a}{3}$ **c** $\frac{2ab}{3}$ **d** $\frac{m}{2}$
e $\frac{x}{4}$ **f** $\frac{5s}{3}$ **g** 3 **h** $\frac{9q}{2}$
i $\frac{x}{2}$ **j** $6b$ **k** $\frac{7m}{3}$ **l** $\frac{2}{5x}$

7 **a** $24n^2$ **b** $-3q^2$ **c** $10s^2$ **d** $21a^2b$
e $-15mn^2$ **f** $18gh^2$ **g** $3b$ **h** $5x$
i $\frac{m}{2}$ **j** $\frac{1}{4y}$ **k** $\frac{3ab}{5}$ **l** $3pq$

8 **a** $\frac{4x}{y}$ **b** $\frac{m}{3}$ **c** $\frac{2a}{b}$ **d** $\frac{10x}{y}$
e $8n$ **f** $5b$ **g** 5 **h** $6xy$
i $\frac{p^2}{2}$ **j** $2ab$

9 **a** $12ab$ **b** $4x^2$ **c** $10xy$ **d** $6x^2$

10 $2x$

11 **a** $4n$ **b** $7y$ **c** $9y$ **d** $5ab$
e $14x$ **f** $10y$ **g** $12a$ **h** $8x^2y$

12 **a** **i** $3x^2$ m^2 **ii** $5x$ metres
b **i** 7500 m^2 **ii** 250 m

13 **a** $4b$ **b** $2ab$ **c** b

Exercise 3D

1 **a** $A = 6 \times 5 + 6 \times 2$ **b** $A = 5 \times (3 + 7)$

2 **a** **i** $5x$ **ii** 10
b $5x + 10$ **c** $x + 2$ **d** $5(x + 2)$ **e** $5x + 10$

3 **a** and **c** give the same values.

4 **a** $3(x+5) = \underline{3} \times x + \underline{3} \times 5$ **b** $5(x+1) = \underline{5} \times \underline{x} + 5 \times \underline{1}$
$= \underline{3x} + 15$ $= \underline{5x} + \underline{5}$
c $2(x-7) = 2 \times \underline{x} + 2 \times (\underline{-7})$
$= \underline{2x} - 14$

5 **a** **i** –16 **ii** –4 **b** No, $-2x - 6$
c **i** –27 **ii** –33 **d** No, $-3x + 3$

6 **a** $2x + 6$ **b** $5x + 60$ **c** $2x + 14$ **d** $7x + 63$
e $3x - 21$ **f** $6x - 66$ **g** $5x - 45$ **h** $10x - 10$
i $6 + 3x$ **j** $21 + 7x$ **k** $28 - 4x$ **l** $40 - 5x$

7 **a** $-3x - 6$ **b** $-2x - 22$ **c** $-5x + 15$ **d** $-6x + 36$
e $-8 + 4x$ **f** $-39 - 13x$ **g** $-72 - 8x$ **h** $-300 + 300x$

8 **a** $2a + 2b$ **b** $5a + 10b$ **c** $6m + 3y$ **d** $16x - 40$
e $-12x - 15$ **f** $4x^2 - 8xy$ **g** $2t^2 - 3t$ **h** $3a^2 + 4a$
i $2d^2 - 5d$ **j** $6b^2 - 10b$ **k** $8x^2 + 2x$ **l** $5y - 15y^2$

Exercise 3D cont.

9 a $2x + 11$ **b** $6x + 22$ **c** $5x + 6$
d $3x + 10$ **e** $4x - 5$ **f** $2x + 1$
g $-4 + 3x$ **h** $21 + 5x$ **i** $13 + 2x$
10 a $4x + 4y$ **b** $8x + 4$ **c** $2x - 2$
11 a $2x + 2$ **b** $8x - 12$ **c** $6x^2 + 2x$ **d** $6x + 15$
12 $\$(2x + 120)$
13 a $2x + 12$ **b** $x^2 - 4x$ **c** $-3x - 12$ **d** $-7x + 49$
e $5x + 10 + 4 = 5x + 14$ **f** $5 - 2x + 14 = 19 - 2x$
14 a $5x + 12$ **b** $4x + 6$ **c** $11x + 7$ **d** $17x - 7$
e $x - 7$ **f** $2x - 4$ **g** $3x^2 + 10x$ **h** $5x - 3 + x^2$
i $x^2 + 6x + 4$
15 a $2x^2 + 4x$ **b** $x^2 + 2x + 6$ **c** $20 + 8x - x^2$

Exercise 3E

1 b, c, e, f, h
2 a 6 **b** 8 **c** –3
d 4 **e** 4 **f** –3
3 a $2x = 6$ **b** $3x = 21$ **c** $x = 9$
4 a Subtract 9 from both sides
b Add 1 to both sides
c Divide both sides by 3
d Divide both sides by 2
5 a $x = 4$ **b** $a = 1$ **c** $m = 4$ **d** $x = 4$
e $n = 2$ **f** $x = -4$ **g** $b = 3$ **h** $y = 7$
i $a = 4$ **j** $b = 8$ **k** $x = -1$ **l** $y = 2$
6 a $x = -3$ **b** $x = -1$ **c** $x = -2$ **d** $x = -8$
e $x = 2$ **f** $x = 2$ **g** $x = 2$ **h** $x = 6$
7 a $x = \frac{3}{2}$ **b** $x = \frac{3}{5}$ **c** $x = \frac{10}{11}$ **d** $x = \frac{1}{2}$
e $x = \frac{-1}{3}$ **f** $x = \frac{4}{5}$
8 a $x + 8 = 34, x = 26$ **b** $x - 7 = 21, x = 28$
c $2x + 4 = 10, x = 3$ **d** $3x - 4 = 11, x = 5$
e $3x - 4 = 20, x = 8$ **f** $25 - 2x = 7, x = 9$
9 a Added 2 to the right-hand side rather than subtracting 2.
b Multiplied the right-hand side by 2 in the last step rather than dividing by 2.
c The negative was dropped, $-x = 7$ so $x = -7$.
d Needed to subtract 3 from both sides before dividing both sides by 2.
10 a $x = 5$ **b** $x = -9$ **c** $x = -2$
11 a $x = -3, x = 3$ **b** $x = 0, x = 1$
c $x = 0, x = 2$
12 $x = -1, x = 0, x = 1$

Exercise 3F

1 a 20 **b** 24 **c** 11 **d** 15
2 C
3 a D, add 1 to both sides
b C, multiply both sides by 4
c A, multiply both sides by 3
d B, multiply both sides by 2
4 a $x = 15$ **b** $y = 14$ **c** $y = 10$ **d** $x = 4$
e $x = 8$ **f** $x = 2$ **g** $b = 12$ **h** $t = -6$
i $a = -6$ **j** $y = 30$ **k** $x = 15$ **l** $s = 4$
m $x = 12$ **n** $m = -4$ **o** $y = -35$ **p** $x = -21$
5 a $x = 11$ **b** $x = 6$ **c** $y = 2$ **d** $b = 0$
e $y = 20$ **f** $t = 39$ **g** $k = 57$ **h** $x = 70$
i $x = -10$ **j** $b = -24$ **k** $y = -22$ **l** $a = -12$
6 a $t = 3$ **b** $m = 40$ **c** $y = -2$ **d** $x = 19$
e $y = -6$ **f** $t = \frac{-3}{4}$
7 a $\frac{x}{3} = 12, x = 36$ **b** $\frac{2x}{5} = 4, x = 10$
c $\frac{x}{2} - 4 = 10, x = 28$ **d** $\frac{x + 3}{4} = 6, x = 21$
e $\frac{7x}{3} = 8, x = \frac{24}{7}$ **f** $\frac{x}{3} + 5 = 16, x = 33$
8 21
9 a Should have ×3 before −2, $x + 2 = 21, x = 19$
b Should have +4 before ×3, $\frac{x}{3} = 6, x = 18$
10 a $x = 12$ **b** $x = 12$ **c** $x = -4$ **d** $b = 6$
e $x = 5$ **f** $m = 5$ **g** $y = -1$ **h** $t = -7$
i $x = -3$
11 Answers will vary.

Exercise 3G

1 a $3(x + 7) = 3 \times \underline{x} + \underline{3} \times 7$
$= 3x + \underline{21}$
b $4(2x - 1) = \underline{4} \times 2x + 4 \times (\underline{-1})$
$= \underline{8x} - 4$
2 a $4x + 12$ **b** $5x + 10$ **c** $7x - 42$
d $6x - 15$ **e** $12x + 16$ **f** $3 - 6x$
3 c and d
4 a and b
5 a 6, 14, 7 **b** 10, 23, $4\frac{3}{5}$
6 a $x = \frac{5}{2}$ or $2\frac{1}{2}$ **b** $a = \frac{3}{5}$
c $m = \frac{19}{3}$ or $6\frac{1}{3}$ **d** $y = \frac{47}{5}$ or $9\frac{2}{5}$
e $p = \frac{35}{4}$ or $8\frac{3}{4}$ **f** $k = \frac{19}{2}$ or $9\frac{1}{2}$
g $b = -\frac{1}{4}$ **h** $m = -\frac{11}{2}$ or $-5\frac{1}{2}$
i $x = -\frac{4}{5}$ **j** $a = \frac{1}{14}$ **k** $x = \frac{19}{6}$ or $3\frac{1}{6}$
l $n = \frac{2}{3}$
7 a E **b** A **c** B **d** D **e** C

8 **a** $3(x+1)=4,\ x=\frac{1}{3}$ **b** $2(x-2)=19,\ x=11.5$
c $2(x+3)=7,\ x=\frac{1}{2}$ **d** $3(x-4)=8,\ x=6\frac{2}{3}$
e $2(3x-2)=5,\ x=1.5$ **f** $3(2x+5)=10,\ x=\frac{-5}{6}$
9 \$5/hour
10 **a** $x=5$ **b** $x=5$
c Dividing both sides by 3 is faster because 9 ÷ 3 is a whole number.
11 Answers will vary.

Drilling for Gold 3G1

1 $n+5=20$ **2** $5n=20$ **3** $\frac{n}{5}=20$
4 $2n+5=20$ **5** $2(n+5)=20$ **6** $5-n=20$
7 $n+5=20$ **8** $\frac{n}{2}+5=20$ **9** $5n=20$
10 $\frac{n}{5}=20$ **11** $n-5=20$ **12** $\frac{n}{2}+5=20$

Exercise 3H

1 **a** $2x+3=5x$ No **b** $7x+3=3x-5$ Yes
c $2x+5=6x+1$ No **d** $3x-9=1-2x$ Yes
2 **a** $x=1$ **b** $x=2$
3 **a** $x=1$ **b** $x=1$
4 The solution is $x=4$ by both methods.
5 **a** False **b** True **c** True **d** True **e** True
6 **a** $x=4$ **b** $x=3$ **c** $x=3$ **d** $x=7$
e $x=-3$ **f** $x=-2$ **g** $x=2$ **h** $x=4$
i $x=6$ **j** $x=2$
7 **a** $x=8$ **b** $a=3$ **c** $m=8$ **d** $x=4$
e $x=5$ **f** $x=3$ **g** $x=1$ **h** $y=2$
i $m=3$ **j** $x=3$ **k** $b=-1$ **l** $m=-3$
8 **a** $2x=3x-4,\ x=4$ **b** $2x+4=5x,\ x=1\frac{1}{3}$
c $3x+10=5x,\ x=5$ **d** $3x+2=5x-6,\ x=4$
e $2x-1=3x+5,\ x=-6$ **f** $2x+4=13-x,\ x=3$
9 Mardy is 8 years old.
10 **a** $x=4$ **b** $x=4$
c Method A: don’t have to deal with negatives.
Final step in method a: divide both sides by a positive number.
Final step in method B: divide both sides by a negative number.
11 **a** 48 **b** 38 **c** 26 **d** 48

Exercise 3I

1 $x+2x=30$ or $3x=30$
2 **a** C **b** D **c** A **d** B
3 **a** $x-3=5$ **b** $2x+7=9$ **c** $\frac{x+2}{3}=4$ **d** $\frac{x}{2}+3=6$
4 age, $x+4$, $x+4$, $2x$, 30, 15, Bernard is 15 and Sam is 19.
5 **a** 7 **b** 5 **c** 21 **d** 8 **e** 2
6 **a** Let d be the number of days the car is rented for.
b $50+40d=290$ **c** $d=6$
d The car was rented for 6 days.
7 48 items
8 **a** Let e be the number of goals for Emma.
b $e+8$ **c** $e+e+8=28$ **d** $e=10$
e Emma scored 10 goals, Leonie scored 18 goals.
9 **a** Let b be the breadth in centimetres. **b** Length = $4b$
c $2b+2(4b)=560$ **d** $b=56$
e Length = 224 cm, breadth = 56 cm
10 Benita \$360, Adele \$640
11 Andrew \$102.50, Brenda \$175, Camille \$122.50
12 Walked 10 km, ran 20 km
13 15 and 45
14 Rectangle $\ell=55$ m, $w=50$ m. Triangle side = 70 m
15 **a** 27, 28, 29 **b** **i** $x,\ x+2,\ x+4$ **ii** 4, 6, 8
16 **a** $T=8x+7200$ **b** 300
c $R=24x$ **d** $x=350$ **e** 3825

Exercise 3J

1 **a** B **b** C **c** E
d A **e** D
2 **a** A **b** D **c** M **d** A
3 **a** The length of the rectangle.
b The breadth of the rectangle.
c Because there are two sides for length and two sides for breadth.
d Yes
4 **a** 28 **b** 20
5 **a** 14 **b** 110
6 **a** 2 **b** $-\frac{1}{2}$
7 **a** 21 **b** 24 **c** 2 **d** 6
e 452.16 **f** 33.49 **g** 25.06 **h** 14.95
i 249.86 **j** 80
8 88.89 km/h
9 35 m/s
10 **a** Decrease **b** 3988 L
11 **a** 212°F **b** 100.4°F
12 **a** 73 **b** 7

Maths@work: Using a formula in a spreadsheet

Area = 1035 m^2

Puzzles and games

1 $x=3$

4	9	2
3	5	7
8	1	6

Exercise 3J cont.

2 WORD

3 Eric is 18 years old now.

4 1st leg = 54 km, 2nd leg = 27 km, 3rd leg = 18 km, 4th leg = 54 km

5 \$140

Multiple-choice questions

1 C **2** C **3** D **4** C **5** D
6 A **7** B **8** B **9** B **10** D

Short-answer questions

1 **a** $x + y$ **b** $7m$ **c** $\$3m$ **d** $\frac{n-3}{4}$
2 **a** 11 **b** 7 **c** 12 **d** 8 **e** −11
3 **a** $8n$ **b** $6xy$ **c** $4a$ **d** $\frac{xy}{3}$
4 **a** $7b$ **b** $4x + 3$ **c** $3p + 2q$
d $2m - 3mn + n$
5 **a** $2x + 14$ **b** $6x + 15$ **c** $6x^2 - 8x$
d $-10a + 8a^2$ **e** $4x + 13$ **f** $3x - 7$
6 **a** $x = 9$ **b** $x = 2$ **c** $x = -3$ **d** $x = -9$
7 **a** $x = 10$ **b** $x = 8$ **c** $x = 26$ **d** $x = 6$
8 **a** $2n + 3 = 21$, $n = 9$ **b** $\frac{\ell - 5}{3} = 7$, $\ell = 26$
c $\frac{x}{4} - 5 = 0$, $x = 20$
9 **a** $x = 5$ **b** $x = \frac{11}{6}$ or $1\frac{5}{6}$
10 **a** $x = 4$ **b** $x = 2$ **c** $x = 1$ **d** $x = -5$
11 \$260
12 **a** 60 **b** 5

Extended-response questions

1 **a** **i** \$85 **ii** \$110
b **i** Let h be the number of hours of hire.
ii $60 + 25h = 210$ **iii** 6 hours
2 **a** $A = 100\,\text{m}^2$ **b** 17 m

Chapter 4

Pre-test

1 **a** 5 cm **b** 13 m **c** 41 mm
2 **a** 9 **b** 49 **c** 20 **d** 45
3 **a** 0.5 **b** 0.3 **c** 0.8 **d** 0.3
4 **a** 4.23 **b** 5.68 **c** 76.90 **d** 23.90
5 **a** 5.92 **b** 15.36 **c** 4.86 **d** 8.09
6 **a** $x = 64$ **b** $x = 28$ **c** $x = 108$
d $x = 60$ **e** $x = 50$ **f** $x = 55$
7 **a** $x = 2$ **b** $x = 3$ **c** $x = 12$ **d** $x = 6$
e $x = 12$ **f** $x = 30$ **g** $x = 28$ **h** $x = 130$
8 **a** 2 **b** 4 **c** 5

Exercise 4A

1 **a** 9 **b** 25 **c** 144 **d** 2.25
e 20 **f** 58 **g** 157 **h** 369
2 **a** False **b** True **c** True
d True **e** False **f** False
3 hypotenuse, triangle
4 **a** c **b** x **c** u
5 **a** $a^2 + b^2 = c^2$ **b** $d^2 + e^2 = f^2$ **c** $x^2 + y^2 = z^2$
d $s^2 + t^2 = v^2$ **e** $f^2 + g^2 = e^2$ **f** $a^2 + b^2 = x^2$
6 **a** $3^2 + 4^2 = 25$, $5^2 = 25$
b $9^2 + 12^2 = 225$, $15^2 = 225$
d $5^2 + 12^2 = 169$, $13^2 = 169$
7 **a** $a^2 + b^2 = x^2$ **b** $a^2 + b^2 = d^2$ **c** $d^2 + h^2 = x^2$
8 a, d, f
9 No, $20^2 + 30^2 \neq 40^2$.
10 **a** 5 **b** 13 **c** 10
11 Yes

Exercise 4B

1 **a** 17 **b** 50 **c** $\sqrt{8}$
2 **a** $c^2 = a^2 + b^2$ **b** $x^2 = y^2 + z^2$ **c** $j^2 = k^2 + \ell^2$
3 **a** 81 **b** 10.24 **c** 13 **d** 106
e 6 **f** 10 **g** 4.90 **h** 3.61
4 **a** $c = 10$ **b** $c = 13$ **c** $c = 17$ **d** $c = 15$
e $c = 25$ **f** $c = 41$ **g** $c = 50$ **h** $c = 30$
i $c = 25$
5 **a** 4.47 **b** 3.16 **c** 15.62 **d** 11.35
e 7.07 **f** 14.56
6 **a** $\sqrt{5}$ **b** $\sqrt{58}$ **c** $\sqrt{34}$ **d** $\sqrt{37}$
e $\sqrt{109}$ **f** $\sqrt{353}$
7 4.4 m
8 495 m
9 2.4 m
10 **a** 8.61 m **b** 5.24 m **c** 13.21 cm
d 0.19 m **e** 17.07 mm **f** 10.93 cm
11 250 m
12 42 units
13 5.83 m
14 **a** 77.78 cm **b** 1.39 m **c** Reduce by 8 cm

Exercise 4C

1 **a** True **b** False **c** False
2 **a** False **b** False **c** True
3 **a** 12 **b** 20 **c** 4
d 24 **e** 8 **f** 8
4 **a** 16 **b** 24 **c** 6 **d** 21 **e** 60 **f** 27
5 **a** 8.66 **b** 11.31 **c** 5.11
d 17.55 **e** 7.19 **f** 0.74
6 5.3 m
7 14.2 m

8 49 cm

9 1.86 m

10 **a** $\sqrt{2}$ **b** $\sqrt{8}$ **c** $\sqrt{\frac{1}{2}}$

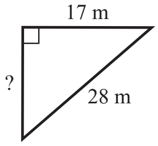
11 **a** $\sqrt{187}$ **b** $\sqrt{567}$ **c** 40

Exercise 4D

1 B

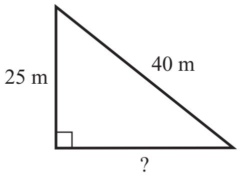
2 **a** B, III **b** C, I **c** A, II

3 **a**



b 22.25 m

4 **a**



b 31.22 m

5 **a** 6.4 m **b** 5.7 cm **c** 6.3 m **d** 6.0 m

6 3.0 m

7 142.9 m

8 3823 mm

9 1060 m

10 **a** 27 m **b** 118.3 m

Drilling for Gold 4D1

1 Yes

2 $6^2 + 9^2 = 117$

$10^2 = 100$

$\therefore 6^2 + 9^2 \neq 10^2$

3 13 cm

4 8 cm

Keeping in touch with numeracy

1 **a** 85 **b** 70 **c** 55 **d** 40

2 **a** 14 **b** 20 mm

3 **a** 4500 **b** 4.5 **c** 195 **d** 45 000

4 750

5 7

6 **a** 6 h 15 min **b** 10 h 45 min

c 3 h 38 min **d** 5 h 28 min

7 Answers will vary.

8 4 and 8

9 9, 10 and 11

10 67, 68, 69

11 9

12 $209.25

13 5 cm

14 Proof required

15 8690

16 $1.15 (nearest cent)

17 **a** 8 km/L **b** 12.5 L/100 km

18 **a** 5.8 km/L **b** 17.3 L/100 km

19 12

20 113

Exercise 4E

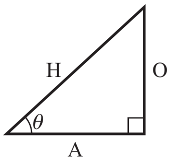
1 **a** Hypotenuse

b Opposite

c Adjacent

2 **a** 13 m **b** 5 m **c** 12 m

3 **a**



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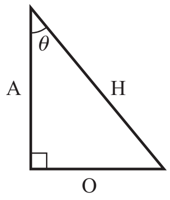

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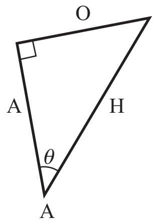

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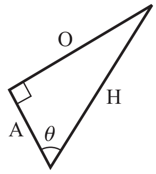

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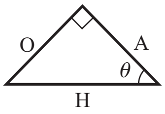

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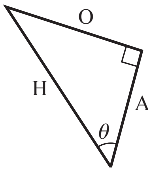

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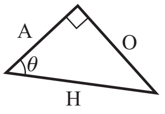

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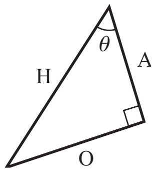
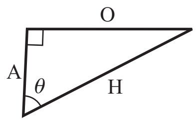


i



4 **a** $\sin\theta = \frac{4}{7}$ **b** $\tan\theta = \frac{5}{4}$ **c** $\cos\theta = \frac{3}{5}$ **d** $\sin\theta = \frac{2}{3}$

e $\tan\theta = 1$ **f** $\cos\theta = \frac{2}{3}$ **g** $\tan\theta = \frac{11}{12}$ **h** $\cos\theta = \frac{1}{2}$

i $\tan\theta = \frac{5}{3}$

5 **a** $\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$

b $\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{5}{13}$ $\tan\theta = \frac{12}{5}$

c $\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{5}{13}$ $\tan\theta = \frac{12}{5}$

Exercise 4E cont.

6 a i $\frac{5}{13}$ **ii** $\frac{5}{13}$ **iii** They are the same.

b i $\frac{12}{13}$ **ii** $\frac{12}{13}$ **iii** They are the same.

c i $\frac{5}{12}$ **ii** $\frac{5}{12}$ **iii** They are the same.

7 a $\frac{3}{5}$ **b** $\frac{4}{5}$ **c** $\frac{3}{4}$

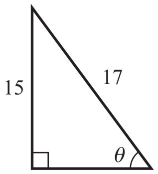
8 $\frac{3}{4}$

9 a 5 **b** 4 **c** 3 **d** 3 **e** 4

10 a $\frac{4}{5}$ **b** $\frac{3}{5}$ **c** $\frac{3}{5}$

d $\frac{3}{4}$ **e** $\frac{4}{5}$ **f** $\frac{4}{3}$

11 a



b 8 **c** $\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$

12 a i $\frac{1}{2}$ **ii** $\frac{\sqrt{3}}{2}$ **iii** $\frac{1}{\sqrt{3}}$

iv $\frac{\sqrt{3}}{2}$ **v** $\frac{1}{2}$ **vi** $\sqrt{3}$

b i They are equal. **ii** They are equal.

13 *Answers are approximate*

b i 0.766 **ii** 0.643 **iii** 0.839

iv 0.766 **v** 1.192 **vi** 0.643

c $\sin 40° = \cos 50°$, $\sin 50° = \cos 40°$, $\tan 50° = \frac{1}{\tan 40°}$

$\tan 40° = \frac{1}{\tan 50°}$

Exercise 4F

1 a A **b** O **c** H

2 a O, H **b** A, H **c** O, A

3 a sin **b** tan **c** cos

4 a 0.34 **b** 0.80 **c** 2.05 **d** 0.73

e 0.10 **f** 0.25 **g** 0.45 **h** 0.24

5 a 3.06 **b** 18.94 **c** 5.03

d 0.91 **e** 1.71 **f** 9.00

g 2.36 **h** 4.76 **i** 7.76

6 a 5.95 **b** 0.39 **c** 13.38 **d** 3.83

e 8.40 **f** 1.36 **g** 29.00 **h** 1.62

i 40.10 **j** 4.23 **k** 14.72 **l** 13.42

7 10.39 m

8 1.12 m

9 44.99 m

10 10.11 m

11 a 65° **b** 2.113 **c** 2.113

12 a 20.95 m **b** 10 cm

Exercise 4G

1 a $\cos\theta$ **b** $\sin\theta$ **c** $\tan\theta$

2 a $x = 2$ **b** $x = 5$

c $x = 3$ **d** $x = 4$

3 b, c

4 a 4.10 **b** 6.81 **c** 37.88

d 0.98 **e** 12.80 **f** 14.43

g 9.52 **h** 117.23 **i** 22.05

5 a 13.45 **b** 16.50 **c** 57.90

d 26.33 **e** 15.53 **f** 38.12

g 9.15 **h** 32.56 **i** 21.75

j 49.81 **k** 47.02

6 40 m

7 3848 m

8 a $x = 7.5$, $y = 6.4$

b $a = 7.5$, $b = 10.3$

c $a = 6.7$, $b = 7.8$

d $x = 9.5$, $y = 12.4$

e $x = 12.4$, $y = 9.2$

f $x = 21.1$, $y = 18.8$

9 a 27 m **b** 104 m

10 a Student B's answer is more correct because they did not use an approximation in their working out.

b Student A used 0.52, which is not exact. Do the calculation in one go. Don't round off and do further calculations.

11 a 23.7 m **b** 124.9 m

Exercise 4H

1 a True **b** True **c** False

2 a $\frac{5}{13}$ **b** $\frac{12}{13}$ **c** $\frac{5}{12}$

3 a sin **b** sin **c** cos **d** tan

4 a 30° **b** 60° **c** 45° **d** 30°

e 45° **f** 30° **g** 90° **h** 50°

i 90° **j** 55° **k** 0° **l** 70°

5 a 35° **b** 19° **c** 64° **d** 76°

e 37° **f** 39° **g** 31° **h** 58°

i 85°

6 a 43° **b** 31° **c** 41°

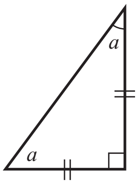
d 16° **e** 55° **f** 50°

g 49° **h** 41° **i** 34°

7 17°

8 23°

9 a $\tan 45° = 1$



b This triangle is isosceles; therefore, 2 angles are equal $a + a = 90$.

$\therefore a = 45°$

$\tan 45° = \frac{\text{O}}{\text{A}}$

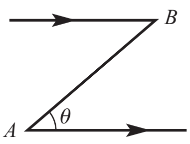
The O and A are equal.

$\therefore \tan 45° = 1$

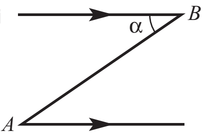
10 25°
11 27°
12 45°
13 **a** 18° **b** 27° **c** 45° **d** 5.67 m

Exercise 4I

1 **a** **i**



ii



b Yes, $\theta = \alpha$, alternate angles are equal on parallel lines.
2 C
3 $a = 65$, $b = 25$
4 **a** 22° **b** 22°
5 29 m
6 16 m
7 157 m
8 38 m
9 90 m
10 37°
11 1372 m
12 10°
13 6°
14 4634 mm
15 Yes, by 244.8 m

Puzzles and games

1 44
2 **b** a^2, b^2, c^2 **c** $a^2 + b^2 = c^2$
3 **a** A diagonal of the square **b** $\sqrt{2}$ cm **c** 63.7%
4 Round peg, square hole
5 171 cm

Multiple-choice questions

1 A **2** B **3** A **4** C **5** B
6 C **7** D **8** C **9** A **10** B

Short-answer questions

1 **a** $x^2 + y^2 = z^2$ **b** $s^2 + t^2 = u^2$ **c** $e^2 + f^2 = d^2$
2 **a** 10 **b** 25 **c** 13
d 37 **e** 4.47 **f** 7.07
3 **a** 3.32 **b** 7.55 **c** 9.95
4 4.49 m
5 **a** 13 cm **b** 13.93 cm **6** 19 m
7 **a** 0.64 **b** 2.25 **c** 0.72
8 **a** $\sin \theta$ **b** $\tan \theta$ **c** $\cos \theta$
9 **a** 11.33 **b** 4.88 **c** 48.02
d 10.31 **e** 45° **f** 60°
10 28.01 m
11 25 m
12 177.91 m
13 63.2 m

Extended-response questions

1 **a** 470 m **b** 3° **c** 1530 m
2 **a** 2.15 m **b** 0.95 m **c** 3.05 m

Semester review 1

Integers, decimals, fractions, ratios and rates

Multiple-choice questions

1 B **2** D **3** D **4** C **5** C

Short-answer questions

1 **a** $\frac{10}{10} = 1$ **b** 1
c $2\frac{3}{4}$ **d** $1\frac{2}{9}$
2 **a** $\frac{3}{8}$ **b** $1\frac{4}{5}$
3 **a** 4.126 **b** 21.002 **c** 0.010
4 **a** \$160, \$640 **b** \$320, \$480 **c** \$700, \$100
5 **a** 5 : 3 **b** 55 km/h **c** 24 mL/h
6 181.3 km

Extended-response question

a 24 km **b** **i** 14 km **ii** 2 p.m.

Financial mathematics

Multiple-choice questions

1 B **2** D **3** A **4** D **5** A

Short-answer questions

1 **a** 60% **b** 31.25% **c** 10% **d** 25%
2 **a** \$9.60 **b** \$765 **c** \$11
3 **a** 84 m **b** \$50.76
4 4 h 48 min
5 \$892
6 **a** \$72 **b** \$872
7 $2\frac{1}{2}$ years

Extended-response question

a \$17 500 **b** \$23 520 **c** 9 years **d** 27%

Expressions and equations

Multiple-choice questions

1 D **2** E **3** A **4** C **5** C

Short-answer questions

1 **a** $-2x + 7y$ **b** $-15mn$ **c** $\frac{y}{3}$

2 **a** $x = 6$ **b** $x = 9$ **c** $m = \frac{3}{8}$ **d** $a = 3$

3 **a** $2m + 1 = 15$ **b** \$7

4 155

Extended-response question

a $p + 12$

b $p + (p + 12) = 38$; i.e. $2p + 12 = 38$

c Daniel scored 13 points and Reese 25 points

Right-angled triangles

Multiple-choice questions

1 B **2** D **3** A **4** C **5** A

Short-answer questions

1 **a** 6.3 **b** 11.2 **c** 15.1
d 9.9 **e** 6.3 **f** 11.0

2 **a** 2.5 **b** 8.1 **c** 11.2
d 4.2 **e** 1.5 **f** 4.2
g 30° **h** 53.1° **i** 29.5°

3 **a** 19.21 m **b** 38.7°

4 **a** $x = 13$ **b** $y = 14.72$

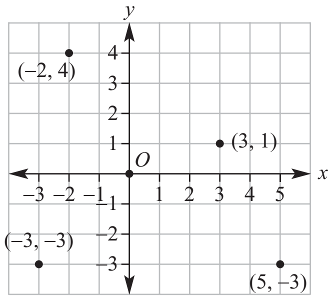
Extended-response question

a 17.75 m **b** 14.3° **c** 18.8 m

Chapter 5

Pre-test

1



2 **a** 1 **b** −1 **c** −3 **d** −4

3 **a** $x = 5$ **b** $x = -2$ **c** $x = -3$
d $x = 4$ **e** $x = -2$ **f** $x = 4$

4 **a** 3 **b** −4 **c** 4 **d** −3

5 **a**

x	−2	−1	0	1	2
y	1	2	3	4	5

b

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

6 **a** False **b** True **c** True **d** False

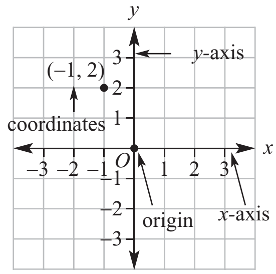
7 **a** 7 **b** 6.5 **c** 3 **d** −5

8 **a** 5 **b** 4 **c** 5 **d** 3

9 **a** 4 **b** 5 **c** 7 **d** 3

Exercise 5A

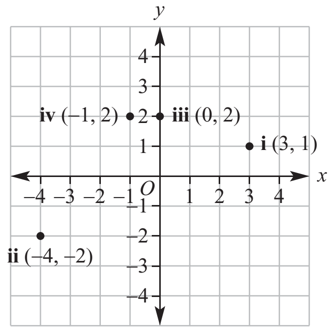
1 **a**



b **i** *x*-coordinate **ii** *y*-coordinate
iii right, down

2 C

3 **a**



b $A(4, 1)$, $B(2, 3)$, $C(0, 3)$, $D(-2, 2)$, $E(-3, 1)$, $F(-1, 0)$, $G(-3, -2)$, $H(-2, -4)$, $I(0, -3)$, $J(2, -2)$, $K(3, -4)$, $L(2, 0)$

4 **a** 2 **b** −1 **c** −4 **d** −7

5 C

6 **a**

x	−2	−1	0	1	2	3
y	0	1	2	3	4	5

b

x	−2	−1	0	1	2
y	−1	1	3	5	7

c

x	−2	−1	0	1	2
y	4	2	0	−2	−4

d

x	−2	−1	0	1	2	3
y	5	2	−1	−4	−7	−10

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7 a

x	–3	–2	–1	0	1	2	3
y	–4	–3	–2	–1	0	1	2

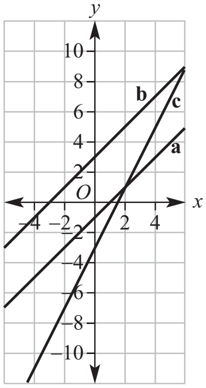
b

x	–3	–2	–1	0	1	2	3
y	0	1	2	3	4	5	6

c

x	–3	–2	–1	0	1	2	3
y	–9	–7	–5	–3	–1	1	3

a $y = x - 1$ **b** $y = x + 3$ **c** $y = 2x - 3$



d

x	–3	–2	–1	0	1	2	3
y	9	6	3	0	–3	–6	–9

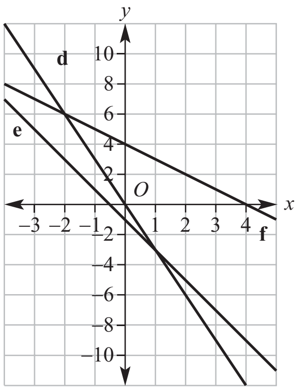
e

x	–3	–2	–1	0	1	2	3
y	5	3	1	–1	–3	–5	–7

f

x	–3	–2	–1	0	1	2	3
y	7	6	5	4	3	2	1

d $y = -3x$ **e** $y = -2x - 1$ **f** $y = -x + 4$



8 a No **b** Yes **c** No

9 a Yes **b** Yes **c** No

10 a $y = x + 2$ **b** $y = 2x$ **c** $y = 2x + 1$ **d** $y = 3x - 1$

11 $y = 3x$

12 a iv **b** iii **c** i **d** ii

13 a $y = 2x + 7$ **b** $y = 20 - x$ **c** $y = 2x - 1$

d $y = \frac{1}{2}x + 1$ **e** $y = -3x + 4$ **f** $y = 5x + 4$

Exercise 5B

1 a 3 **b** 2

2 a $A(1, 0)$, $D(-2, 0)$, $H(3, 0)$ **b** $B(0, 3)$, $E(0, -1)$

3 a $y = 1$ **b** $y = -2$ **c** $y = -3$

4 a $x = 2$ **b** $y = 3$ **c** $y = -5$

d $x = 4$ **e** $x = 3$ **f** $x = 3$

5 a 1 and 1 **b** –2 and 2 **c** 4 and 8

d –5 and 10 **e** 2 and 3 **f** 7 and –3

g –11 and 5 **h** –2 and –5

6

	x-intercept	y-intercept
a	–6	6
b	4	–4
c	4	–8
d	–2	6
e	–4	–8
f	3	12
g	7	3.5
h	2	–4

7 c, d, g, h

8 a 4 and –2 **b** 4 and –2 **c** 0 and 0

d 8 and –4 **e** 0 and 0 **f** 9 and –6

9 a 100 m **b** 20 s **c** 5 m

10 a 20 minutes **b** 80 litres **c** 4 litres

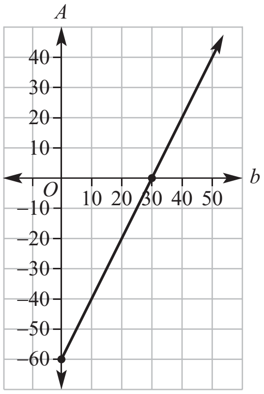
11 When you substitute $x = 0$ into the rule, $y = 3 \times (0) = 0$. It means the graph passes through the origin.

12 a $A = -60$ **b** Priya makes a loss of $60.

c 30 badges

d Priya then starts to make a profit from her stall.

e



Exercise 5C

1 a 3, horizontal, x **b** 2, vertical, y

2 a Horizontal **b** Horizontal **c** Vertical

d Horizontal **e** Vertical **f** Vertical

3 a 2 **b** –3 **c** 0

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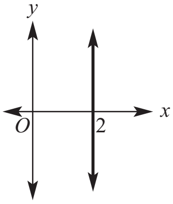
Exercise 5C cont.

4 **a** 3 **b** –2

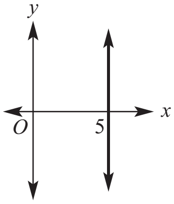
5 b, e, f

6 **a** 5 **b** 3 **c** –4 **d** 5

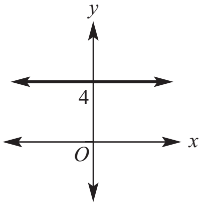
7 **a**



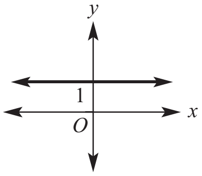
b



c



d



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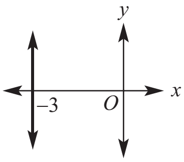

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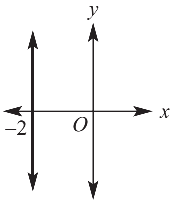

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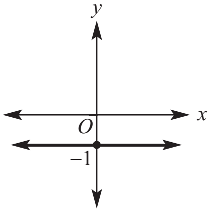

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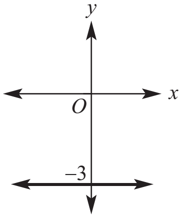
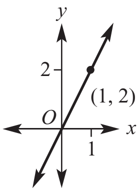
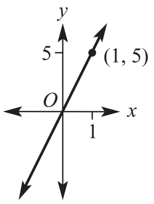


8 **a**



b



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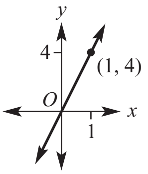

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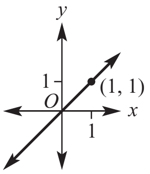
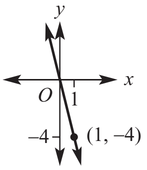


e



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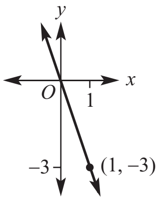

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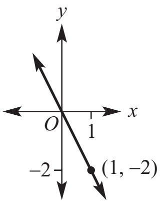

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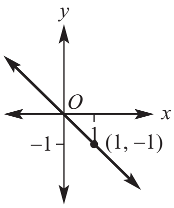
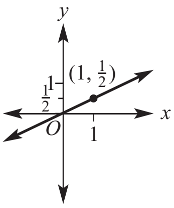
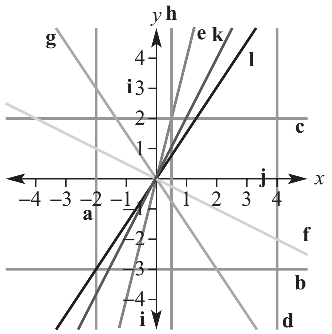


i



9



10 **a** $y = -2$ **b** $y = 4$ **c** $x = -2$
d $x = 5$

11 **a** $y = 3$ **b** $x = 5$ **c** $x = -2$ **d** $y = 0$

12 **a** $y = 250$ **b** $y = -45$

13 **a** $y = 2x$ **b** $y = 3x$ **c** $y = -x$ **d** $y = -3x$

14 **a** $A = 15$ square units **b** $A = 68$ square units

Keeping in touch with numeracy

1 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$ **d** $\frac{2}{5}$ **e** $\frac{4}{5}$

2 **a** $\frac{9}{20}$ **b** $\frac{1}{8}$ **c** $\frac{3}{8}$ **d** $1\frac{7}{50}$ **e** $4\frac{19}{20}$

3 **a** 30% **b** 6% **c** 3% **d** 15% **e** 12%

4 **a** 20% **b** 7.5% **c** 2% **d** 1.5% **e** 1.2%

5 100 000

6 2000

7 7.005 kg

8 1.125 kg

9 25

10 $29

11 **a** 7.5 **b** 7500 **c** 0.0075

12 18.75 m

13 16

14 $62.50

15 120 cm^3

16 14.84 m^3

17 270 km
18 3 h 45 min
19 85%
20 15

Exercise 5D

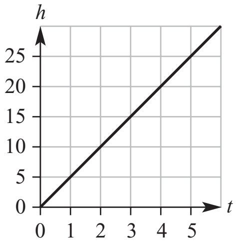
1 **a** **i** 2 **ii** 4 **b** **i** 2 **ii** 1
c **i** 1 **ii** 3 **d** **i** 3 **ii** –3
e **i** 1 **ii** –3 **f** **i** 4 **ii** –1
2 **a** Zero **b** Negative **c** Positive **d** Undefined
3 **a** 2 **b** –4 **c** 1
d $\frac{1}{2}$ **e** $-\frac{3}{2}$ **f** $-\frac{5}{3}$
4 **a** Positive, 1 **b** Positive, 2 **c** Zero
d Zero **e** Negative, $\frac{-2}{3}$ **f** Negative $\frac{-3}{4}$
g Undefined **h** Undefined **i** Positive, $\frac{1}{2}$
j Positive, 3 **k** Positive, 2 **l** Negative, –4
5 A 1 B –2 C 1
D $\frac{1}{2}$ E –2 F 3
6 **a** 2 **b** –1 **c** –2 **d** 1
e 2 **f** –2 **g** –1 **h** $\frac{1}{2}$
i $\frac{3}{2}$ **j** $-\frac{5}{2}$ **k** $\frac{1}{3}$ **l** $\frac{5}{2}$
7 **a** $-\frac{1}{20}$ **b** 17.5 or $\frac{35}{2}$
c $-\frac{1}{5}$ **d** 7.5 or $\frac{15}{2}$
8 40 metres
9 90 metres
10 **a** 6 **b** 1 **c** 0 **d** –4
11 **a** **i** –1 **ii** –3 **b** (4.5, 0)
c **i** (6, 0) **ii** (1.5, 0) **iii** (2.4, 0)

Exercise 5E

1 **a** No **b** Yes **c** Yes **d** No
2 **a** 10 km each hour
b **i** 10 km **ii** 20 km **iii** 30 km
c 10 km/h **d** 10 **e** They are the same.
3 **a** 15 mm **b** **i** 6 days **ii** 20 days
c

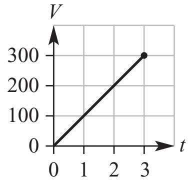
t	0	1	2	3	4
h	0	5	10	15	20

d

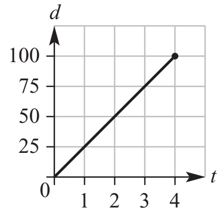


e 5

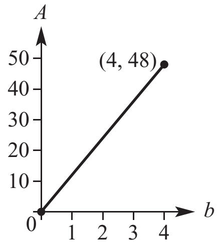
4 **a** 60 km/h **b** 9 m/s **c** 5 L/min
d \$1.50/L **e** \$30/h **f** 3°C/min
5 **a** $d = 60t$ **b** $P = 20t$ **c** $C = 1.5n$ **d** $g = 2t$
6 **a** 100 L/h
b



c **i** 100 **ii** $V = 100t$
d **i** 150 L **ii** 20 hours
7 **a** 25 km/h
b



c **i** 25 **ii** $d = 25t$
d **i** 62.5 km **ii** 1.6 hours
8 **a** 100 km/h **b** 7 cm/s
c 2.5 cm/min **d** 49 mm/min
9 Simone
10 The panther
11 **a** 10.2 L **b** 72.25 L **c** 800 km
12 **a** $A = 12b$
b



c Yes, the rule for the graph is in the form $y = mx$.
13 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$
d 1 min 20 s or $1\frac{1}{3}$ min
14 1.2 hours or 1 hour 12 minutes

Exercise 5F

1 (1, 2.5)
2 **a** 4 **b** 8 **c** 1 **d** –3
e 5.5
3 **a** 2.24 **b** 8.60 **c** 10.20
4 **a** **i** 3 **ii** 4 **b** **i** 2 **ii** 7
5 **a** (3, 3) **b** (2, 2) **c** (1, 5) **d** (4, 1)
e (–1, 3) **f** (–1, –1) **g** (1.5, 1.5) **h** (2.5, 2)
i (0.5, 3) **j** (–1.5, –2.5)

Exercise 5F cont.

6 **a** 5 **b** 10 **c** 5.1 **d** 6.7

7 **a** 5.10 **b** 2.83 **c** 5.39 **d** 4.47
e 3.61 **f** 2.83 **g** 8.94 **h** 7.21
i 6.71

8 (0.5, 2), (–1, 5.5), (3.5, –0.5)

9 $B(8, 0)$, $A(-6, 5)$

10 **a** (–3, 1) **b** (1, –4) **c** (8, 3)

11 **a** 12.8 **b** 24.2

12 **a** (3, 5), (3, –1), (0, 2), (6, 2) are the obvious points.
b A circle

13 **a** $\frac{1}{3}$ **b** $\frac{1}{3}$
c **i** (1, 1) **ii** (–2, –0.5) **iii** (2, 1.5)

Puzzles and games

1 101 **2** 602 **3** 20 days

4 (0.5, 5.5), diagonals intersect at their midpoint

5 Length AB = length $AC = \sqrt{17}$

6 31 hours

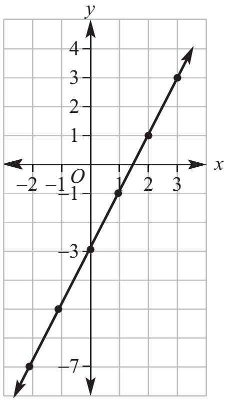
Multiple-choice questions

1 D **2** C **3** D **4** D **5** A
6 D **7** B **8** B **9** A **10** C

Short-answer questions

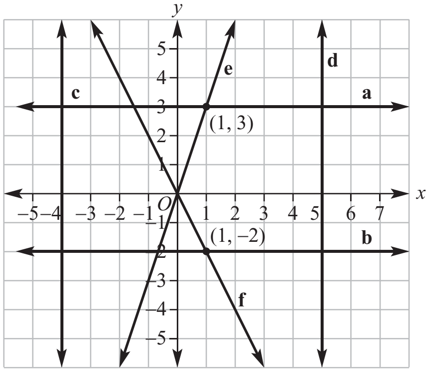
1

x	–2	–1	0	1	2	3
y	–7	–5	–3	–1	1	3



2 **a** –2 and 4 **b** 3 and –2

3



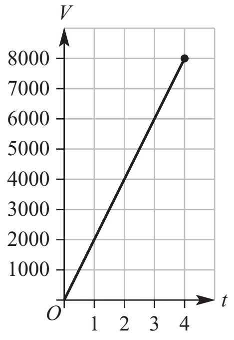
a zero **b** zero **c** undefined
d undefined **e** positive **f** negative

4 **a** 2 **b** –3

5 **a** 2 **b** –1 **c** 2 **d** $-\frac{10}{3}$

6 **a** 2000 L/h

b



7 **a** $M(4, 6)$, 5.66
b $M(7.5, 4.5)$, 7.07
c $M(0, 4)$, 7.21
d $M(-3, 2.5)$, 9.85

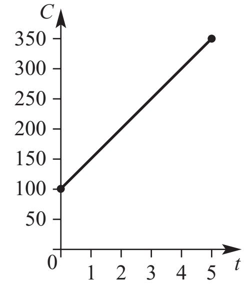
Extended-response questions

1 **a**

Hours (t)	0	1	2	3	4	5
Cost (c)	100	150	200	250	300	350

b $C = 50t + 100$

c

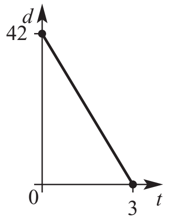


d \$300 **e** $2\frac{1}{2}$ hours

2 **a**

Hours (t)	0	1	2	3
Distance remaining (d)	42	28	14	0

b



c 42 km **d** 3 hours **e** –14 **f** 14 km/h

Chapter 6

Pre-test

1 **a** 4 cm^2 **b** 6 m^2 **c** 8 cm^2

2 **a** 8 cm **b** 12 m **c** 18 cm

3 **a** 3 **b** 6 **c** 27

4 **a** Cylinder **b** Circle

5 **a** 23 **b** 48 **c** 2.7

d 5.2134 **e** 50 **f** 72.16

6 7

7 **a** 30 mm **b** 2000 cm **c** 1600 m **d** 2.3 cm

e 3.167 km **f** 0.72 m **g** 20 km **h** 30 m

8 **a** 30 cm^2 **b** 4 m^2 **c** 49 km^2

d 3 m^2 **e** 24 cm^2 **f** 66 m^2

9 $C = 31.42$ m, $A = 78.54$ m^2

Exercise 6A

1

$$\begin{tikzcd} \text{km} \arrow[r, bend left, "\times 1000"] & \text{m} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 100"] & \text{cm} \arrow[l, bend left, "\div 100"] \arrow[r, bend left, "\times 10"] & \text{mm} \arrow[l, bend left, "\div 10"] \end{tikzcd}$$

2 **a** 7 **b** 26 **c** 2.1

3 **a** 50 mm **b** 410 mm **c** 280 cm **d** 40 cm

e 4600 m **f** 900 m **g** 52.1 cm **h** 3.6 cm

i 2.4 m **j** 0.837 m **k** 7 km **l** 2.17 km

4 825 cm, 2.25 cm

5 **a** 15 mm **b** 31 m **c** 80 cm

d 12 m **e** 27 cm **f** 24 mm

g 18 km **h** 10 m **i** 36 cm

6 **a** $x = 4$ **b** $x = 2.2$ **c** $x = 14$

7 **a** $a = 3, b = 6$ **b** $a = 12, b = 4$

c $a = 6.2, b = 2$

8 **a** 90 cm **b** 80 cm **c** 175 cm

d 30.57 m **e** 25.5 cm **f** 15.4 km

9 108 m

10 **a** 8000 mm **b** 110 m **c** 1 cm

d 20 mm **e** 0.284 km **f** 62.743 km

11 **a** 86 cm **b** 13.6 m **c** 40.4 cm

12 88 cm

13 **a** **i** 96 cm **ii** 104 cm **iii** 120 cm

b $P = 4\,(20 + 2x) \qquad \therefore P = 8x + 80$

c 109.6 cm

Exercise 6B

1 **a** Radius

b Diameter

c Circumference

d Sector

2 **a** 2.8 cm **b** 96 mm

3 **a** $C = 2\pi r$ **b** $C = \pi d$

4 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$

5 **a** 50.27 m **b** 87.96 cm **c** 31.42 m

d 6.91 m **e** 45.24 cm **f** 101.79 mm

6 **a** 31.42 m **b** 78.54 cm **c** 9.42 mm

d 5.65 mm **e** 23.25 m **f** 12.57 m

g 62.83 cm **h** 21.99 cm **i** 43.98 cm

7 **a** 10.28 **b** 51.42 **c** 14.28

d 10.00 cm **e** 20.05 m **f** 106.73 km

8 28.27 m

9 4.1 m

10 **a** 12.57 cm **b** 102.83 mm **c** 41.06 m

11 **a** 188.50 cm **b** **i** 376.99 cm **ii** 1979.20 cm

c 531

Exercise 6C

1 **a** 10 000 **b** 1 000 000

2 **a** 6 **b** 16 **c** 12 **d** 1 **e** 12 **f** 153

3 **a** Rectangle **b** Rhombus, square, kite **c** Triangle

d Trapezium **e** Parallelogram **f** Square

4 **a** 200 mm^2 **b** 40 mm^2 **c** 5 cm^2

d 3.1 cm^2 **e** 21 000 cm^2 **f** 2000 cm^2

g 21 m^2 **h** 0.37 m^2 **i** 1000 m^2

j 4 300 000 m^2 **k** 3.2 km^2 **l** 0.0394 km^2

5 **a** 24 m^2 **b** 10.5 cm^2 **c** 20 km^2

d 25.2 m^2 **e** 15 m^2 **f** 36.8 m^2

6 **a** 21 mm^2 **b** 12 cm^2 **c** 17 cm^2

d 63 m^2 **e** 6.205 m^2 **f** 15.19 km^2

7 500 000 m^2

8 0.175 km^2

9 **a** 12.25 cm^2 **b** 3.04 m^2 **c** 0.09 cm^2

d 6.5 mm^2 **e** 18 cm^2 **f** 2.4613 cm^2

10 0.51 m^2

11 **a** 1.5×10^{10} (15 000 000 000) cm^2

b 5 mm^2 **c** 0.075 m^2

12 **a** **i** 1.5 m **ii** 1.5 m

b 78 m^2 **c** Yes

Exercise 6D

1 **a** 4.1 m **b** 7.5 m **c** 3.8 cm

2 C

3 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$

4 **a** 12.57 m^2 **b** 113.10 cm^2

c 254.47 km^2 **d** 60.82 mm^2

e 415.48 km^2 **f** 45 238.93 cm^2

5 **a** 314.16 cm^2 **b** 60.82 m^2 **c** 17.35 cm^2

d 21.24 m^2 **e** 216.42 km^2 **c** 196.07 cm^2

6 **a** (25π) m^2 **b** (100π) m^2 **c** (25π) m^2

7 **a** 12.57 m^2 **b** 157.08 cm^2

c 84.82 m^2 **d** 99.55 m^2

Exercise 6D cont.

8 177 cm^2

9 **a** 2 m **b** 12.57 m^2 **c** 3.43 m^2

10 414.69 cm^2

11 **a** 1.8 cm **b** 6.1 m **c** 2 km

12 31%

Exercise 6E

1 **a** Semicircle and rectangle **b** Triangle and semicircle
c Rhombus and parallelogram

2 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$ **d** $\frac{1}{4}$

3 **a** 4 cm^2 **b** 3.14 cm^2 **c** 7.14 cm^2 **d** 11.14 cm

4 **a** 46 m, 97 m^2 **b** 34 m, 76 m^2
c 40 m, 90 m^2 **d** 18.28 m, 22.28 m^2
e 19.42 m, 26.14 m^2 **f** 85.42 mm, 326.37 mm^2

5 **a** $P = 14.28$ m, $A = 3.43$ m^2
b $P = 35.71$ m, $A = 10.73$ m^2
c $P = 41.13$ km, $A = 27.47$ km^2

6 **a** 108 m^2 **b** 33 cm^2 **c** 98 m^2
d 300 m^2 **e** 16 cm^2 **f** 22.5 m^2

7 189.27 m^2

8 68.67 cm^2

9 **a** 90 cm^2 **b** 15 m^2
c 9 m^2 **d** 7.51 cm^2
e 7.95 m^2 **f** 180.03 cm^2

10 **a** 17 cm^2 **b** 3.5 cm^2 **c** 21.74 cm^2

11 **a** 37.70 m, 92.55 m^2 **b** 20.57 mm, 16 mm^2
c 18.00 cm, 11.61 cm^2 **d** 12.57 m, 6.28 m^2
e 25.71 cm, 23.14 cm^2 **f** 33.56 m, 83.90 m^2

Keeping in touch with numeracy

1 $\frac{3}{8}$

2 $\frac{5}{12}$

3 –1

4 12

5 3250

6 1440

7 **a** \$200 **b** \$100 **c** \$40 **d** \$20

8 **a** \$212.65 **b** \$112.60 **c** \$54.20 **d** \$22.10

9 5 m

10 $\sqrt{50}$ cm

11 \$3120

12 \$1076.92

13 **a** 1950 **b** 2000 **c** 2000 **d** 2000

14 2

15 Answers will vary.

16 **a** 55 **b** $6.\dot{1}$ **c** 7

17 **a** 36 m **b** 84 m^2

18 \$340

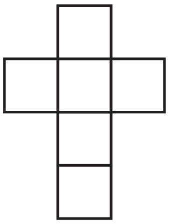
19 **a** 2 **b** 4

20 \$777

Exercise 6F

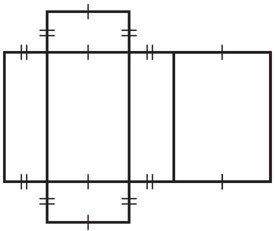
1 **a** 6 **b** 6 **c** 5

2 **a**



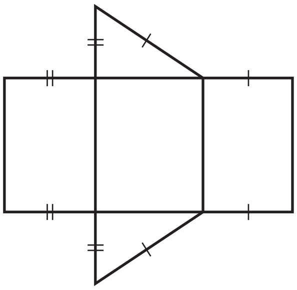
Cube

b



Rectangular prism

c



Triangular prism

3 **a** $A = 2 \times (8 \times 7 + 8 \times 3 + 7 \times 3) = 202$ m^2
b $A = 5 \times 7 + 4 \times 7 + 3 \times 7 + 2 \times (\frac{1}{2} \times 4 \times 3) = 96$ cm^2

4 **a** 52 m^2 **b** 242 cm^2 **c** 76 m^2
d 192 cm^2 **e** 68.16 m^2 **f** 85.76 m^2

5 **a** 96 cm^2 **b** 240 m^2
c 199.8 cm^2 **d** 238 cm^2

6 6 m^2

7 14.54 m^2

8 34 000 cm^2

9 **a** 44.4 m^2 **b** 4.44 L

10 **a** [6, 10, 14, 18, 22, 26, 30, 34, 38]
b $A = 4n + 2$
c 402

11 **a** 39 mm^2 **b** 224 cm^2 **c** 9.01 m^2

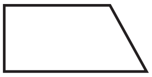
Exercise 6G

1 **a** Square **b** Triangle **c** Rectangle
d Trapezium **e** Hexagon **f** Triangle

2 **a**

, 6 cm

b

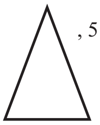


, 10 cm

c

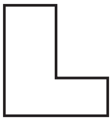
, 2 m

d



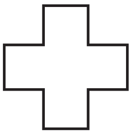
, 5 mm

e



, 14 cm

f



, 10 m

3 **a** 10 **b** 100 **c** 1000 **d** 1 000 000
e 1 000 000 000 **f** 1000 **g** 1000
h 1

4 **a** 3000 mm^3 **b** 300 mm^3 **c** 2 cm^3
d 1000 cm^3 **e** 8 700 000 cm^3 **f** 0.0059 m^3
g 10 000 m^3 **h** 0.000 021 7 km^3 **i** 0.43 m^3

5 **a** 3000 mL **b** 200 L **c** 3.5 L
d 21 mL **e** 37 kL **f** 42.9 ML
g 2 mL **h** 2000 cm^3 **i** 1000 L

6 **a** 8 cm^3 **b** 84 m^3 **c** 21 mm^3

7 **a** 16 cm^3 **b** 42.875 m^3 **c** 15 cm^3

8 **a** 4 cm^3 **b** 10.5 m^3 **c** 11.96 cm^3
d 29 cm^3 **e** 14.88 m^3 **f** 8.1351 cm^3

9 **a** 75 m^3 **b** 30 cm^3

10 8000 cm^3

11 Yes, the tank only holds 20 L.

12 **a** 8 L **b** 0.48 L

13 **a** 55 m^2 **b** 825 000 L

14 **b** $\frac{1}{3}$

Drilling for Gold 6G2

1 8 **2** 15 **3** 10 **4** 18
5 11 **6** 22 **7** 20 **8** 23

Exercise 6H

1 **a** 12.57 cm^2 **b** 8.04 m^2 **c** 78.54 cm^2 **d** 2.54 km^2

2 **a** 2 L **b** 4.3 mL **c** 3700 cm^3
d 1000 L **e** 38 m^3 **f** 200 mL

3 **a** $r = 4$ m, $h = 10$ m **b** $r = 2.6$ cm, $h = 11.1$ cm
c $r = 2.9$ m, $h = 12.8$ m **d** $r = 9$ m, $h = 23$ m
e $r = 5.8$ cm, $h = 15.1$ cm **f** $r = 10.65$ cm, $h = 10.4$ cm

4 **a** 226.19 cm^3 **b** 18.85 m^3
c 137.44 m^3 **d** 100.53 cm^3
e 8.48 m^3 **f** 68.05 m^3

5 **a** 18 L **b** 503 L **c** 20 L **d** 2 L

6 **a** 1.571 m^3 **b** 1571 L

7 Cylinder by 131 cm^3

8 **a** 25.133 m^3 **b** 25 133 L

9 37 699 L

10 A number of answers. Require $h = 2\pi r$.

11 **a** 502.65 cm^3 **b** 294.52 m^3
c 47.12 cm^3 **d** 1017.88 cm^3

Puzzles and games

1 **a** 19 **b** 35
2 8 **3** 314 m
4 163.4 m^2 **5** 7.1 cm
6 11 **7** 100 L

Multiple-choice questions

1 B **2** C **3** E **4** B **5** A
6 D **7** B **8** C **9** C **10** E

Short-answer questions

1 **a** 380 cm **b** 1270 m **c** 4.8 cm
d 2.73 cm^2 **e** 52 000 cm^2 **f** 10 000 cm^3
g 53.1 cm^3 **h** 3.1 L **i** 43 mL
j 2830 L **k** 4 L **l** 1000 L

2 **a** 14 m **b** 51 mm **c** 16.2 cm

3 **a** 4 cm^2 **b** 1122 mm^2 **c** 30.34 mm^2
d 7.5 m^2 **e** 15 cm^2 **f** 3 cm^2

4 **a** 2.5 m^2 **b** 37.4 m^2

5 **a** $A = 28.27$ cm^2, $P = 18.85$ cm
b $A = 1.57$ m^2, $P = 5.14$ m
c $A = 2.36$ cm^2, $P = 6.71$ cm

6 **a** $P = 22.28$ m, $A = 30.28$ m^2
b $P = 45.56$ cm, $A = 128.54$ cm^2

7 **a** 46 cm^2 **b** 114 m^2

8 **a** 30 cm^3 **b** 54 m^3 **c** 31.42 mm^3 **d** 226.19 cm^3

Extended-response questions

1 **a** 135 cm^3 **b** 174 cm^2 **c** $43.50
d **i** cylinder **ii** 1.8 cm^3

2 **a** 5.17 m
b $65
c 1.59 m^2; claim is correct.

Chapter 7

Pre-test

1 **a** Obtuse **b** Acute **c** Reflex **d** Right
e Straight **f** Revolution

2 **a** 35 **b** 142 **c** 260

3 **a** Isosceles **b** Equilateral **c** Scalene

4 **a** Obtuse **b** Acute **c** Right

5 **a** 60° **b** 75° each **c** 50°

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6 **a** Parallelogram, rectangle, kite
b Parallelogram, rectangle, square, rhombus
c Square, rectangle
7 **a** $a = 60$ **b** $b = 110$ **c** $a = 60, b = 120$

Exercise 7A

1 **a** Right **b** 180° **c** Revolution **d** Obtuse
e Acute **f** 180° **g** 90°
h Supplementary **i** 180° **j** Equal
2 **a** Isosceles triangle **b** Obtuse-angled triangle
c Equilateral triangle **d** Acute-angled triangle
e Scalene triangle **f** Right-angled triangle
3 **a** 50° **b** 90° **c** 101°
d 202° **e** 180° **f** 360°
4 **a** **i** 125° **ii** 35°
b **i** 149° **ii** 59°
c **i** 106° **ii** 16°
d **i** 170° **ii** 80°
e **i** 91° **ii** 1°
f **i** 158° **ii** 68°
g **i** 142° **ii** 52°
h **i** 115° **ii** 25°
i **i** 133° **ii** 43°
j **i** 103° **ii** 13°
5 **a** $a = 63$ **b** $a = 71$ **c** $a = 38$
d $a = 147$ **e** $a = 233$ **f** $a = 33$
6 **a** Obtuse isosceles, $a = 40$ **b** Acute scalene, $b = 30$
c Right-angled scalene, $c = 90$
d Equilateral, $d = 60$ **e** Obtuse isosceles, $e = 100$
f Right-angled isosceles, $f = 45$
g Obtuse scalene, $g = 100$ **h** Equilateral, $h = 60$
i Obtuse isosceles, $i = 120$ **j** Obtuse isosceles, $j = 35$
k Right-angled scalene, $k = 90$
l Acute scalene, $\ell = 70$
7 **a** **i** $\angle BAC$ **ii** Obtuse **iv** 120°
b **i** $\angle PRQ$ **ii** Acute **iv** 30°
c **i** $\angle XYZ$ **ii** Reflex **iv** 310°
d **i** $\angle SRT$ **ii** Straight angle **iv** 180°
e **i** $\angle ROB$ **ii** Obtuse **iv** 103°
f **i** $\angle AOB$ **ii** Right **iv** 90°
8 **a** $s = 120$ **b** $t = 20$
c $r = 70$ **d** $a = 60, x = 120$
e $a = 100, b = 140$ **f** $c = 115, d = 65$
g $x = 56$ **h** $x = 155$
9 $AO = BO$ (equal radii)
$\therefore \angle BAO = \angle OBA$ (equal angles are opposite equal sides)
$\therefore \angle OAB = (180 - 116) \div 2 = 32°$
10 **a** 360° **b** 90° **c** 60° **d** 90°
e 432° **f** 6° **g** 720° **h** 8640°
11 **a** 90° **b** 150° **c** 15°
d 165° **e** 157.5° **f** 80°
g 177.5° **h** 171° **i** 121.5°

Exercise 7B

1 **a** Corresponding **b** Alternate **c** Cointerior
d Alternate **e** Cointerior **f** Corresponding
2 **a** Equal **b** Equal **c** Supplementary
3 **a** $x = 125$, alternate angles in || lines
b $y = 110$, cointerior angles in || lines
c $r = 80$, corresponding angles in || lines
d $s = 66$, alternate angles in || lines
e $t = 96$, vertically opposite
f $v = 126$, corresponding angles in || lines
g $w = 62$, angles on a straight line
h $p = 115$, corresponding angles in || lines
i $q = 116$, cointerior angles in || lines
4 **a** No, alternate angles are not equal.
b Yes, corresponding angles are equal.
c Yes, alternate angles are equal.
d No, cointerior angles don't add to 180°.
e Yes, cointerior angles add to 180°.
f Yes, corresponding angles are equal.
g No, corresponding angles are not equal.
h No, alternate angles are not equal.
i No, corresponding angles do not add to 180°.
5 **a** $a = 60, b = 120$ **b** $c = 95, d = 95$
c $e = 100, f = 100, g = 100$
d $a = 110, b = 70$
e $a = 100, b = 80, c = 80$
f $e = 140, f = 140, d = 140$
6 $a = 40, b = 140, c = 40, d = 40, e = 140$
7 **a** $x = 70, y = 40$ **b** $t = 58, z = 122$
c $u = 110, v = 50, w = 50$ **d** $x = 118$
e $x = 295$ **f** $x = 79$
8 **a** 56 **b** 120 **c** 265
9 **a** 105° **b** 105° **c** 56°
d 105° **e** 90° **f** 85°

Drilling for Gold 7B2

Corresponding angles	Alternate angles	Cointerior angles
1 $a = e$	**5** $c = e$	**7** $c + f = 180°$
2 $b = f$	**6** $d = f$	**8** $d + e = 180°$
3 $c = g$		
4 $d = h$		

11 $x = y$ (corresponding angles in parallel lines)
12 $x = y$ (alternate angles in parallel lines)
13 $x + y = 180$ (cointerior angles in parallel lines)

14 $x = y$ (corresponding angles in parallel lines)
15 $x = y$ (alternate angles in parallel lines)
16 $x = y$ (corresponding angles in parallel lines)
17 $x = y$ (alternate angles in parallel lines)
18 $x = y$ (corresponding angles in parallel lines)
19 $x + y = 180$ (cointerior angles in parallel lines)
20 $x = y$ (alternate angles in parallel lines)
21 $x + y = 180$ (cointerior angles in parallel lines)
22 $x = y$ (corresponding angles in parallel lines)

Exercise 7C

1 **a** Parallelogram, rhombus, rectangle, square, kite, trapezium
  b **i** Trapezium **ii** Square
  iii Rectangle **iv** Kite
  v Rhombus **vi** Parallelogram
2 b, c, e, g
3 a, b, c, e, g, h
4 a, b, c, d, e, f, g, h
5 **a** 115 **b** 159 **c** 30
  d 121 **e** 140 **f** 220
6 **a** $a = 110$ **b** $x = 70$ **c** $b = 54$
  d $a = 33$ **e** $y = 63$ **f** $a = 109$
7 Answers will vary.
8 60°
9 **a** Yes **b** Yes **c** Yes **d** Yes
  e Yes **f** Yes **g** Yes **h** Square
10 **a** 110 **b** 55
11 **a** 255 **b** 80 **c** 115 **d** 37 **e** 28 **f** 111

Exercise 7D

1 **a** Hexagon **b** Regular
  c Non-convex **d** $S = (n - 2) \times 180°$
2 **a** 5 **b** 7 **c** 4 **d** 8 **e** 9 **f** 12
3 **a** 720° **b** 1080° **c** 540° **d** 900°
4 **a** Convex quadrilateral
  b Non-convex hexagon
  c Non-convex heptagon
5 **a** 140 **b** 100 **c** 100 **d** 120
6 **a** 110 **b** 150 **c** 210
  d 130 **e** 25 **f** 285
7 **a** 90° **b** 108° **c** 140°
  d 120° **e** 128.57° **f** 135°
8 **a** 16 **b** 25 **c** 102
9 **a** Yes **b** Yes
10 Answers will vary.
11 **a** **i** No **ii** Yes **iii** No
  b **i** One **ii** Two **iii** Five **c** $(n - 3)$

Keeping in touch with numeracy

1 **a** kL **b** mm **c** ha **d** L/100km
  e \$/hour **f** t **g** m^3 **h** \$/L
2 **a** 225 **b** 3.75 **c** 37 500
  d 3750 **e** 0.375
3 50
4 75
5 $\frac{2}{3}$
6 $\frac{5}{12}$
7 40
8 8.8
9 70°
10 37°, 143° and 143°
11 \$20
12 375 mL for \$3.50
13 24 km
14 305.8 m/min (to 1 d.p.)
15 9 m^2
16 35 mm
17 \$27.50
18 40%
19 9 m
20 22.5 m

Exercise 7E

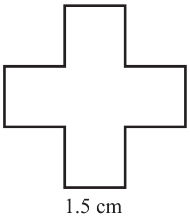
1 **a** **i** Y **ii** X **iii** Z
  b **i** XY **ii** XZ **iii** YZ
  c **i** $\angle A$ **ii** $\angle B$ **iii** $\angle C$
2 **a** size **b** $\triangle STU$ **c** SAS, RHS, AAS
3 **a** $\triangle ABC \equiv \triangle FGH$ **b** $\triangle DEF \equiv \triangle STU$
4 **a** SAS **b** AAS **c** RHS **d** SAS
  e SSS **f** RHS **g** AAS **h** SSS
5 **a** $x = 3, y = 4$ **b** $x = 2, y = 6$
  c $a = 105, b = 40$ **d** $a = 65, b = 85$
  e $x = 2.5, b = 29$ **f** $a = 142, x = 9.21, b = 7$
  g $y = 4.2, a = 28$ **h** $a = 6.5, b = 60$
6 **a** A and C **b** RHS
7 **a** $\triangle ABC \equiv \triangle STU$, RHS **b** $\triangle DEF \equiv \triangle GHI$, SSS
  c $\triangle ABC \equiv \triangle DEF$, SAS **d** $\triangle ABC \equiv \triangle GHI$, AAS
8 It is SAS; the hypotenuse is not given.
9 No – they can all be different sizes, one might have all sides 2 cm and another all sides 5 cm.
10 **a** SSS **b** Equal
11 $\triangle PBR \equiv \triangle FDE$
  $\triangle LMN \equiv \triangle KIJ$
  $\triangle FGH \equiv \triangle BCD$
  $\triangle MNO \equiv \triangle RQP$

Exercise 7F

1 **a** 15, 15
b 2
c 10, 15, 15
d 2.5, 4

2 **a** $\angle F$ **b** $\angle D$ **c** GH
d AE **e** 2

3 **a** 18 cm by 12 cm
b 12 cm by 8 cm
c 9 cm by 6 cm
d 7.5 cm by 5 cm
e 3 cm by 2 cm
f 4.5 cm by 3 cm



(all sides are 1.5 cm, all angles are 90°)

4 **a** true **b** false **c** true **d** false **e** true

5 **a** **i** 2 **ii** 14 **iii** 10
b **i** $1\frac{1}{2}$ **ii** 10.5 **iii** 8
c **i** $1\frac{1}{2}$ **ii** 45 **iii** 24
d **i** 2.5 **ii** 1 **iii** 1.4
e **i** 1.5 **ii** 9 **iii** 12
f **i** 1.75 **ii** 3.5 **iii** 3

6 All sides 1.5 times longer.

7 All sides half as long as original.

8 **a** Yes **b** 8 cm **c** 25 m

9 3.7 cm

10 1.41; Note: On a copier, to enlarge A3 to A4 the setting is 141%

11 **a** All angles of any square equal 90°, with only one side length.
b All angles in any equilateral triangle equal 60° with only one side length.
c The length and width might be multiplied by different numbers.
d Two isosceles triangles do not have to have the same size equal angles.
e Multiplying side lengths by 1 does not change their length, so the shape does not get bigger or smaller.

12 **a** From 8 to 12 the scale factor is 1.5. From 12 to 16 it is not 1.5
b No

13 **a** 12 500 mm **b** 100 mm **c** 125 **d** 24 mm **e** 3000 mm

14 **a** 100 000 cm = 1 km **b** 24 cm

Exercise 7G

1 **a** 2 **b** 12

2 **a** similar **b** shape, size
c triangle **d** similar

3 **a** E **b** C **c** DF
d BC **e** $\angle A$ **f** $\angle E$

4 **a** $\triangle ABC \mathrel{|||} \triangle GHI$ **b** $\triangle ABC \mathrel{|||} \triangle MNO$
c $\triangle ABC \mathrel{|||} \triangle ADE$ **d** $\triangle HFG \mathrel{|||} \triangle HJI$
e $\triangle ADC \mathrel{|||} \triangle AEB$ **f** $\triangle ABD \mathrel{|||} \triangle ECD$

5 2.5

6 **a** 2.5 **b** 8

7 **a** 3 **b** 3.6 m

8 40 m

9 7.5 m

10 24 m

11 **a** 6.5 **b** 10 **c** 24

12 7.2 m

13 6

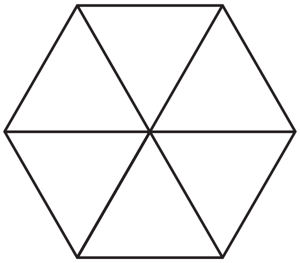
14 20 m

15 **a** **i** 3.6 m **ii** 9 m **iii** 2.7 m
b **i** 5.4 m **ii** 6.3 m **c** **i** 4 m **ii** 6 m

16 **a** 5 : 7, 3 : 4, 4 : 5, 2 : 3, 11 : 14, 3 : 4, 4 : 5, 5 : 6, 2 : 3
b 20 by 30; scale factor is 5
c 12 by 16; scale factor is 2
d 16 by 20; scale factor is 2

Puzzles and games

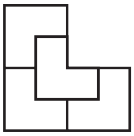
1



2 110 **3** 10

4 Base of altitudes (an altitude is a line through a vertex that meets the opposite side at right angles)

5



6

	5	3	
2	8	1	7
	6	4	

Multiple-choice questions

1 D **2** A **3** B **4** D **5** C
6 A **7** E **8** B **9** D **10** A

Short-answer questions

1 **a** $a = 142$ **b** $b = 40$ **c** $c = 120$

2 **a** Acute-angled isosceles, $x = 50$, $y = 80$
b Right angled scalene, $x = 25$
c Obtuse-angled scalene, $x = 30$, $y = 110$

3 **a** No – corresponding angles are not equal.
b Yes – cointerior angles are supplementary.
c Yes – alternate angles are equal.

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4 a $a = 60, b = 120, c = 120$
  b $a = 40, b = 80$
5 a Square, rectangle **b** Square, rhombus, kite
  c Parallelogram, rectangle, rhombus, square
6 a $a = 70, b = 110$ **b** $x = 115$ **c** $x = 15$
7 a $x = 95$ **b** $a = 120$ **c** $x = 135$
8 a SSS, $x = 60$ **b** Not congruent
  c RHS, $x = 12$ **d** AAS, $x = 9$
9 a 3.5 **b** 4 **c** 18
10 a 3 **b** 4.8 m

Extended-response questions

1 a Isosceles, right angled
  b $a = 40$, angle sum in isosceles triangle
    $b = 70$, angles in parallel lines
    $c = 120$, angle sum of a quadrilateral
    $d = 25$, angles on a straight line
    $e = 65$, angle sum of a triangle
2 a $\angle ECD$
  b 2.44
  c 19.8 m

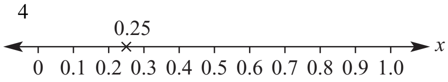
Chapter 8

Pre-test

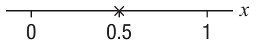
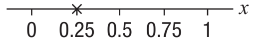
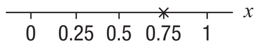
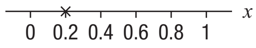
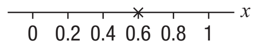
1 a 0.1 **b** 0.25 **c** 0.3 **d** 0.85 **e** 0.237
2 a $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{2}{3}$ **d** 1
  e 0 **f** $\frac{1}{12}$ **g** $\frac{1}{2}$ **h** $\frac{18}{29}$
  i $\frac{2}{3}$ **j** $\frac{2}{7}$
3 a 1, 2, 3, 4, 5 or 6
  b i 3 **ii** 2 **iii** 3 **iv** 5 **v** 5 **vi** 3
4 a 5 **b** 7 **c** 6 **d** 7 **e** 4 **f** 6
5 a 8 **b** 20 **c** $\frac{3}{20}$
6 a 6 **b** 5 **c** 5 **d** 12
  e i $\frac{2}{9}$ **ii** $\frac{7}{9}$ **iii** $\frac{5}{9}$

Exercise 8A

1 a sample space **b** event
  c probability **d** impossible, certain
  e even **f** complement
2 a i $\frac{1}{4}$ **ii** 0.25 **iii** 25%
  b



3

	Percentage	Decimal	Fraction	Number line
a	50%	0.5	$\frac{1}{2}$	
b	25%	0.25	$\frac{1}{4}$	
c	75%	0.75	$\frac{3}{4}$	
d	20%	0.2	$\frac{1}{5}$	
e	60%	0.6	$\frac{3}{5}$	
f	85%	0.85	$\frac{17}{20}$	

4 0.15, $\frac{2}{9}$, 1 in 4, 0.28, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, 2 in 3, 0.7, 0.9
5 a i {1, 2, 3, 4, 5, 6, 7}
    ii $\frac{1}{7}$ **iii** $\frac{6}{7}$
    iv $\frac{2}{7}$ **v** $\frac{6}{7}$
  b i {2, 2, 6, 7} **ii** $\frac{1}{2}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{2}$ **v** 1
  c i {1, 2, 2, 2, 2, 3} **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$
    iv $\frac{5}{6}$ **v** $\frac{5}{6}$
  d i {1, 1, 2, 2, 3, 3} **ii** $\frac{1}{3}$ **iii** $\frac{2}{3}$
    iv $\frac{2}{3}$ **v** $\frac{2}{3}$
  e i {1, 1, 2, 3, 3, 4, 4} **ii** $\frac{1}{7}$ **iii** $\frac{6}{7}$
    iv $\frac{3}{7}$ **v** $\frac{5}{7}$
  f i {2, 2, 2, 2} **ii** 1 **iii** 0 **iv** 1 **v** 1
6 a $\frac{1}{2}$ **b** $\frac{3}{8}$
  c $\frac{1}{6}$ **d** $\frac{1}{4}$
  e 1 **f** 0
7 a $\frac{1}{2}$ **b** $\frac{5}{8}$ **c** $\frac{5}{6}$ **d** $\frac{3}{4}$
  e 0 **f** 1
8 a $\frac{1}{8}$ **b** $\frac{7}{8}$
9 a $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$ **d** $\frac{3}{8}$ **e** $\frac{5}{8}$
  f 1 **g** 0 **h** $\frac{1}{4}$ **i** $\frac{3}{4}$ **j** $\frac{3}{4}$

Exercise 8A cont.

10 **a** {H, A, R, S} **b** **i** $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{1}{2}$

11 **a** $\frac{1}{52}$ **b** $\frac{1}{13}$ **c** $\frac{1}{26}$ **d** $\frac{1}{2}$

e $\frac{2}{13}$ **f** $\frac{12}{13}$ **g** $\frac{23}{26}$ **h** $\frac{25}{26}$

12 **a** $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$ **d** $\frac{1}{3}$

e $\frac{2}{3}$ **f** 1 **g** $\frac{1}{3}$ **h** $\frac{1}{2}$

13 Answers will vary.

14 Answers will vary.

Exercise 8B

1 **a** 26

b **i** 10 **ii** 14 **iii** 5 **iv** 9

v 4 **vi** 7 **vii** 19

c **i** 12 **ii** 17

2 **a** 22 **b** 12 **c** 10

d 39 **e** 13 **f** 1

3 **a** B **b** D **c** A **d** C

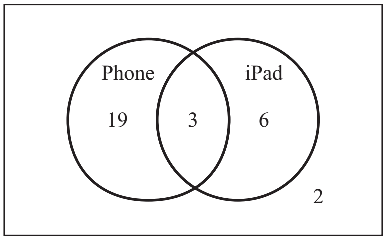
4 **a**

	A	Not *A*	Total
B	7	8	15
Not *B*	3	1	4
Total	10	9	19

b

	A	Not *A*	Total
B	2	5	7
Not *B*	9	4	13
Total	11	9	20

5 **a**



b **i** 28 **ii** 21 **iii** 6

c **i** $\frac{1}{10}$ **ii** $\frac{1}{15}$ **iii** $\frac{19}{30}$

6 **a** E **b** C **c** F **d** A

e B **f** D

7 **a** **i** $\frac{2}{5}$ **ii** $\frac{1}{3}$ **iii** $\frac{7}{15}$ **iv** $\frac{1}{15}$ **v** $\frac{13}{15}$ **vi** $\frac{2}{15}$

b **i** $\frac{3}{7}$ **ii** $\frac{12}{35}$ **iii** $\frac{13}{35}$ **iv** $\frac{3}{35}$ **v** $\frac{34}{35}$ **vi** $\frac{1}{35}$

8 **a**

	Cream	Not cream	Total
Ice-cream	5	20	25
Not ice-cream	16	9	25
Total	21	29	50

b **i** 29 **ii** 9

c **i** $\frac{21}{50}$ **ii** $\frac{8}{25}$ **iii** $\frac{41}{50}$

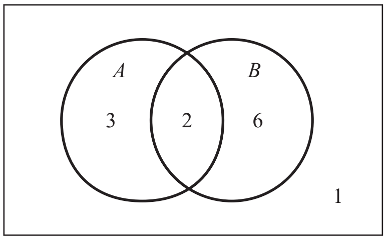
9 **a** **i** $\frac{5}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{3}{8}$ **iv** $\frac{3}{4}$ **v** $\frac{1}{8}$ **vi** $\frac{1}{4}$

b **i** $\frac{17}{26}$ **ii** $\frac{9}{26}$ **iii** $\frac{11}{26}$

iv $\frac{21}{26}$ **v** $\frac{2}{13}$ **vi** $\frac{5}{26}$

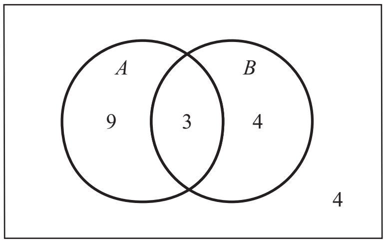
10 **a**

	A	Not *A*	Total
B	2	6	8
Not *B*	3	1	4
Total	5	7	12

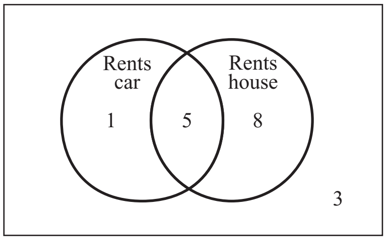


b

	A	Not *A*	Total
B	3	4	7
Not *B*	9	4	13
Total	12	8	20



11 **a**



b 5 **c** $\frac{1}{17}$

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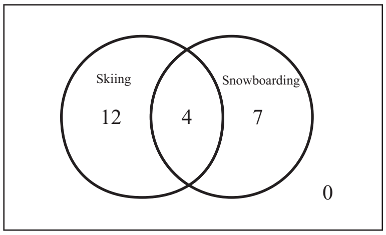
12 **a**

	Tank water	Not tank water	Total
Tap water	12	36	48
Not tap water	11	41	52
Total	23	77	100

b 12 **c** $\frac{9}{25}$ **d** $\frac{59}{100}$

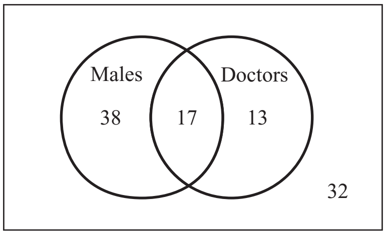
13 **a**



b 23 **c** $\frac{11}{23}$ **d** $\frac{4}{16}=\frac{1}{4}$ **e** $\frac{4}{11}$

14 **a** $\frac{3}{10}$ **b** $\frac{1}{3}$

15



a 32 **b** 45 **c** 70 **d** 38 **e** 13

Exercise 8C

1 **a** experimental **b** theoretical **c** theoretical
d experimental **e** expected

2 **a** $\frac{9}{10}$ **b** $\frac{4}{5}$ **c** $\frac{5}{9}$

3 **a** $\frac{2}{5}$ **b** 40

4 **a** 5 **b** 10 **c** 50 **d** 4

5 **a** $\frac{35}{90}=\frac{7}{18}$ **b** 1050

6 **a** 0.6
b **i** 60 **ii** 120 **iii** 360

7 **a** $\frac{7}{8}$
b **i** 350 **ii** 4375 **iii** 35

8 **a** $\frac{1}{15}$ **b** $\frac{2}{5}$ **c** $\frac{11}{60}$

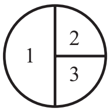
9 **a** B: $\frac{5}{20}=0.25$ C: $\frac{30}{100}=0.3$
b C because it is a larger sample size

10 0.41, from the 100 throws as the more times an experiment is carried out the closer the experimental probability becomes to the actual/theoretical probability.

11 **a** 20 **b** 40 **c** 60 **d** 40

12 **a** 20
b **i** $\frac{1}{4}$ **ii** $\frac{7}{20}$ **c** **i** 25 **ii** 20 **iii** 45

13 **a** **i** 0.52 **ii** 0.48 **iii** 0.78
b 78
c $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$



14 **a** Answers will vary.
b Answers will vary.

Exercise 8D

1 **a** mean
b **i** 7.8 **ii** 9.9 **iii** 8.4 **iv** 35.4

2 **a** common (or frequent)
b **i** 2 **ii** 10 **iii** 7 **iv** red

3 **a** middle
b **i** 5 **ii** 8 **iii** 5 **iv** 14

4 **a** range
b **i** 21 **ii** 22 **iii** 13 **iv** 27

5 **a** outlier
b **i** 36 **ii** 3 **iii** No outlier **iv** 100

6

	Mean	Median	Mode	Range
a	6	7	8	7
b	8	6	5, 10	13
c	6	6	2	10
d	11	12	none	13
e	4	3.5	2.1	6.2
f	5	4.5	none	9.7
g	3	3.5	–3	12
h	0	2	3	11

7 **a** Outlier = 33, mean = 12, median = 7.5
b Outlier = 2, mean = 35.2, median = 42
c Outlier = –1.1, mean = 1.075, median = 1.4
d Outlier = –4, mean = –49, median = –59

8 **a** Yes **b** No **c** No **d** Yes

9 **a** 12 **b** 5.5 **c** 6 **d** Bimodal; 4, 7

10 **a** 24.67 s **b** 24.8 s

Exercise 8D cont.

11 **a** Mean = $570 667, median = $354 500
  b $1 700 000
  c ($354 500 and $324 000) drops $30 500
  d ($570 667 and $344 800) drops $225 867

12 **a** 15 **b** 26

13 **a** 90 **b** 60

14 **a** 76.75 **b** **i** 71.4, B+ **ii** 75, B+ **iii** 80.2, A
  c 81.4; he cannot get an A+.
  d **i** 43 **ii** 93

Drilling for Gold 8D2

Activity 1

1 **a** The 5 will still be the middle of the data set.
  b The sum of the data values will increase by 1 and the number of scores is still 5, so the mean will increase.
  c The range will change from 9 – 2 = 7 to 10 – 2 = 8.
  d The range, which was 9 – 2 = 7, will change to 10 – 1 = 9.
  e The mean will not change because the sum of the data values will not change.
  f Nothing; the middle data value will still be 5.
  g The range will not change because 9 – 2 = 7 and 10 – 3 = 7.
  h The median will not change; the median of 5, 5, 5, 5, 5, is 5.

Drilling for Gold 8D2

Activity 2

2 **a** 5 **b** 5.5, 4.5 **c** 35 **d** 0, 10

3 **a** Yes; 4 and 6, 3 and 6, 3 and 7
  b 6
  c Any two numbers that add to 45.

Keeping in touch with numeracy

1 –6, –3, 0.25, 5

2 14

3 3280 g

4 3.75 kg or 3750 g

5 300

6 3.75 cm

7 SE

8 135°

9 A

10 Approx. 11.25 cm^2

11 B

12 **a** 11 **b** 5 cm

13 B

14 282.8 m

15 7 cm

16 214 cm^2

17 6 and 5

18 **a** 5 **b** change 3 to 12, for example

19 mean and range

20 36

Exercise 8E

1 **a** 17
  b 3, 5, 6, 6, 7, 8, 8, 8, 9, 9, 9, 9, 9, 10, 10, 10, 10
  c 8 **d** 9 **e** 7 **f** Skewed
  g $\frac{12}{17}$

2 **a** 25 **b** 30 cm **c** 173 cm
  d 170 cm **e** 168.88 cm **f** 186 cm

3 **a** 44% **b** 6600/year **c** 18/day
  d 9.5/day **e** 108°

4 **a** 1.42 m **b** 0847, 8:47 a.m.
  c 1 hour 21 minutes **d** 31 cm or 0.31 m
  e 11 hours 55 minutes

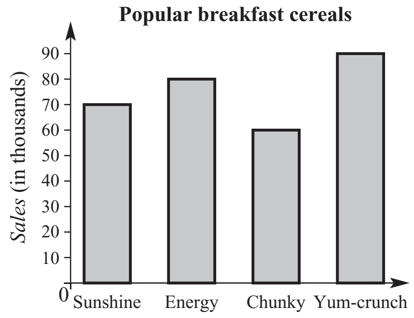
5 **a** 41 years **b** 24 years **c** 16 years
  d 10 years, 14.5%

6 **a** 18°C **b** 43 mm **c** 5°C **d** 47 mm
  e January, February, March, December
  f February, March, May
  g August, September **h** 9°C

7 **a** 45 m **b** 25 m **c** 20 m
  d 69 m **e** 35 m – 14 m = 21 m more

8 **a** Channel 15, 27; channel 16, 24; channel 17, 26; channel 18, 24
  b Graph A
  c Graph B
  d The scale on graph A starts at 23 but the scale on graph starts at 0.
  e Graph A is misleading as the scale expands the difference in column heights.

9 **a** Sunshine, Chunky, Energy, Yum-crunch
  b The graph jumps from zero to 60, which makes Yum-crunch look like it has double the sales of Chunky.
  c



10 **a** Rugby
  b Rugby, soccer, basketball, cricket, tennis
  c Rugby: 700 000; Soccer: 600 000
  d Cricket, 800 000
  e Cricket, rugby, soccer, basketball and tennis equal
  f The length of each row is misleading because the ball sizes are not equal.
  g Each picture should be an equal size.

11 a 10

b \$560 000, \$560 000, \$750 000, \$750 000, \$750 000, \$750 000, \$870 000, \$870 000, \$870 000, \$2 300 000

c \$903 000

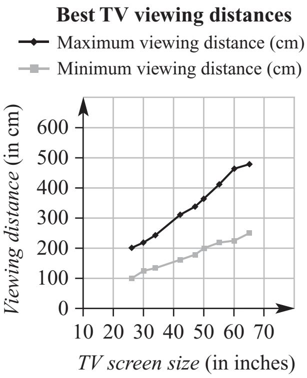
d \$750 000

e Mean

f 9 out of 10 house prices are less than the mean of \$903 000.

g The median is a better measure of 'average' as it is the middle value. The mean is increased by one very large value.

12 a



b 26 inch, 30 inch, 34 inch

c 42 inch, 47 inch, 50 inch, 55 inch, 60 inch, 65 inch

Exercise 8F

1 a 35, 37, 41, 43, 48 **b** 52, 60, 61, 67, 73, 75

2

Stem	Leaf
5	1 6 7 8
6	0 2 3 7
7	1 3 4 5

3 a 9 **b i** 8 min **ii** 35 min

c 21 min

d 21 min

4 a 26 **b i** 0 mm **ii** 6 mm

c i 21 mm **ii** 24 mm

d i 8 mm **ii** 15 mm

e Skewed

f Symmetrical

5 a i

Stem	Leaf
1	9 9
2	3 4 6 6 8 8 8
3	2 2 3 5
4	1
5	4

2|3 means 23

ii 28 **iii** 28 **iv** Skewed

b i

Stem	Leaf
1	5 5
2	1 3 3 3 6
3	0 1 1 3 4 5 5 7 9
4	2 2 5 5 5 8
5	0 0 1

4|2 means 42

ii 35 **iii** 23, 45 **iv** Almost symmetrical

c i

Stem	Leaf
15	7 8 9 9 9 9
16	1 4 7 7 7
17	3 5 7 7
18	5 5 7 9
19	3 8
20	0 2

17|7 means 177

ii 173 **iii** 159 **iv** Skewed

6 a

Stem	Leaf
0	5 6 6 8 8 9
1	0 1 1 2 2 2 4 4 5 6 7
2	0 1 2

1|2 means 12

b 9 **c** 12

7 a i

Set A		Set B
9 3 2	3	1
8 7 6 5 3 3 0 0	4	0 1 3 4 4 6 7 8 8 9 9
4 4 3	5	1 3

3|2 means 32

ii Set A has values spread between 32 and 54, whereas set B has most of its values between 40 and 53 with an outlier at 31.

b i

Set A		Set B
9 8 6 4 3 2 2 1	0	9
7 7 6 1 1 0 0	1	8 9 9
9 6 4 3 3 2 2	2	0 1 3 4 7 8
7 6 6 5 0	3	1 7 7 8 9 9 9
8 3 1	4	0 1 1 3 4 4 5 7
0	5	0 0 1 3 3 4 4 5 6

2|1 means 21

ii Set A has values between 1 and 50 with fewer large numbers, whereas set B has values between 9 and 56 with many large numbers.

Exercise 8F cont.

8 a

Collingwood		St Kilda
8 3	6	6 8 8
8 7 2 1 0	7	8
9 9 8 2 0	8	0 0 2 2 3 4 7 8
8 7 5 1	9	0 4
4 3 3 3	10	6 9
9 8 5	11	1 3 7 8
7	12	2 5 6
	13	8

10|6 means 106

b Collingwood $33\frac{1}{3}$%, St Kilda $41\frac{2}{3}$%

c Collingwood data are almost symmetrical. St Kilda data are not symmetrical.

d Collingwood results seem to be consistent. St Kilda has groups of similar scores and, although less consistent, has higher scores.

9 a 16.1 s **b** 2.3 s **c** Yes, 0.05 s slower

10 a 52 **b** 178

11 a 48% **b** 15%

c In general, birth weights of babies of mothers who smoke are lower.

Puzzles and games

1 IT'S RANDOM
2 1-C, 2-B, 3-E, 4-D, 5-A
3 6 own both a cat and a dog
4 2, 3, 4, 8, 8
5 7, 8, 9, 10 and 11

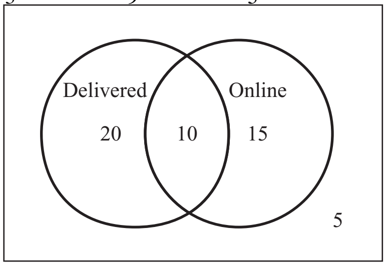
Multiple-choice questions

1 B
2 A
3 **a** A **b** B **c** D **d** C
4 D
5 **a** B **b** C
6 E

Short-answer questions

1 a $\frac{2}{3}$ **b** $\frac{5}{9}$ **c** $\frac{3}{5}$

2 a

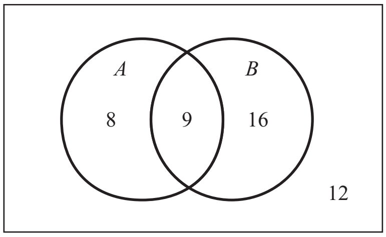


b 15 **c i** $\frac{1}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{3}{10}$

3 a

	A	Not *A*	Total
B	9	16	25
Not *B*	8	12	20
Total	17	28	45

b



c i $\frac{4}{9}$ **ii** $\frac{1}{5}$ **iii** 8 **iv** 33

4 a $\frac{48}{120} = \frac{2}{5}$ **b** 14

5 a 26 **b** 26.5 **c** 23, 31 **d** 18

6 a

Employee 1		Employee 2
9	1	7 7
5 3 1	2	0 4 8 9
9 8 6 5 5 4	3	2 7 7 8
6 5 0	4	0 1 2 5 8
3 1	5	

2|4 means 24

b i Employee 1, 36; employee 2, 37

ii Employee 1, 36; employee 2, 33

c Employee 1, they have a higher mean and more sales at the high end.

d Employee 1 symmetrical, employee 2 skewed

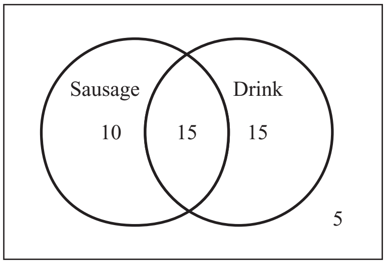
7 a Mean = $760 000

Median = $590 000

b The median is better because the mean has been inflated by the outlier of $2 900 000.

Extended-response questions

1 i



ii 5 **iii** 10 **iv** $\frac{1}{3}$ **v** $\frac{4}{9}$

2 **a**

Airline A		Airline B
4 4 3 2 2 1 1 0	0	1 2 4 4
9 8 7 7 6	0*	5 5 6 7 7 8 9
4 4 4 3 3 3 2	1	0 0 0 0 1 2 2 2 1
9 9 8 8 7	1*	5 5 8 8 8 9 9
4 3	2	1 2 2
	2*	5 6
2	5	

1|2 means 12 1*|5 means 15

b Yes, 52 min

c **i** Airline A, 12
Airline B, 10.5
ii Airline A, 10.6
Airline B, 12.4

d No, the median time is 12 minutes, so at least half the flights have delay 12 minutes or more.

Semester review 2

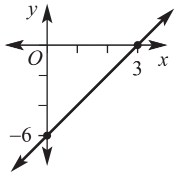
Linear relationships

Multiple-choice questions

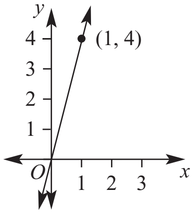
1 C **2** D **3** A **4** D **5** A

Short-answer questions

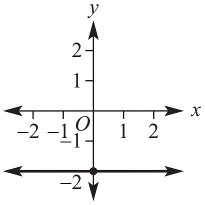
1 **a**



b



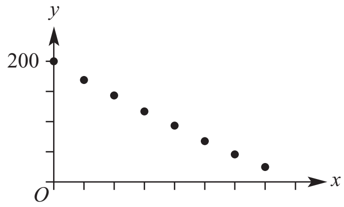
c



2 **a** $\frac{2}{3}$ **b** −3 **c** −2 **d** $\frac{4}{3}$

3 **a** 155, 140, 125, 110, 95
b 155, 140, 125, 110, 95, 80

c

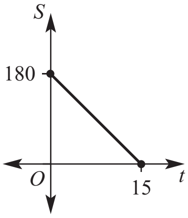


d D **e** Another 12 weeks

Extended-response question

a 12 kg/h **b** $S = 180 - 12t$

c



d 15 hours **e** **i** $P = 40 + 25h$ **ii** $415

Length, area, surface area and volume

Multiple-choice questions

1 C **2** C **3** E **4** C **5** A

Short-answer questions

1 **a** 22 cm^2 **b** 12 cm^2 **c** 25.92 m^2
2 **a** 25.13 m, 50.27 m^2 **b** 62.83 cm, 314.16 cm^2
c 13.19 cm, 13.85 cm^2
3 **a** 54.82 cm^2 **b** 26 m^2 **c** 24.57 m^2 **d** 36 cm^2
4 **a** 220 m^2 **b** 216 m^2
5 **a** 12 m^3 **b** 42 m^3 **c** 100.53 cm^3

Extended-response question

a 12.57 m^2
b 77.43 m^2
c $628

Properties of geometrical figures

Multiple-choice questions

1 C **2** B **3** A
4 E **5** B

Short-answer questions

1 **a** 180° **b** 360° **c** 540°
d 900° **e** 1080° **f** 1440°

2 **a** $x = 70$ **b** $x = 70$
c $a = 100$, $b = 140$ **d** $a = 70$, $b = 110$
e $x = 100$, $y = 110$ **f** $x = 67$, $y = 98$
g $x = 120$ **h** $a = 135$

3 **a** Yes, RHS
b FDE
c BC

4 **a** 2 **b** 12

5 **a** 5 or 0.2 **b** 2.4

Extended-response question

a $\triangle DEC \,|||\, \triangle DAB$ **b** 6 m **c** $\frac{5}{3} = 1.\dot{6}$
d 7.5 m **e** 3.75 m

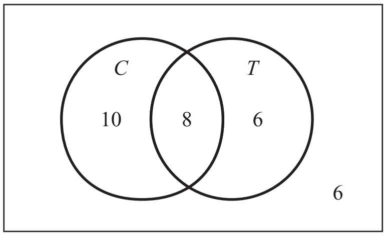
Probability and single variable data analysis

Multiple-choice questions

1 D **2** A **3** **i** C **ii** D
4 B **5** B

Short-answer questions

1 **a**



b **i** 24 **ii** 16
c **i** $\frac{1}{5}$ **ii** $\frac{1}{3}$

2 **a** 8 **b** $\frac{1}{8}$ **c** $\frac{1}{4}$

3 **a**

Stem	Leaf
1	0 1 1 2 3 5 7 8
2	2 5 5 5 6
3	1 2 2

1|3 means 13 aces

b Mode = 25, median = 20
c Skewed

4 **a**

Class interval	Frequency	Relative frequency (%)
0–49	2	6.7
50–99	4	13.3
100–149	5	16.7
150–199	9	30
200–249	7	23.3
250–299	3	10
Total	30	100

b **i** 6 **ii** 60%

Extended-response question

a $\frac{4}{9}$ **b** $\frac{1}{3}$ **c** **i** 17 years **ii** 92

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Index

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