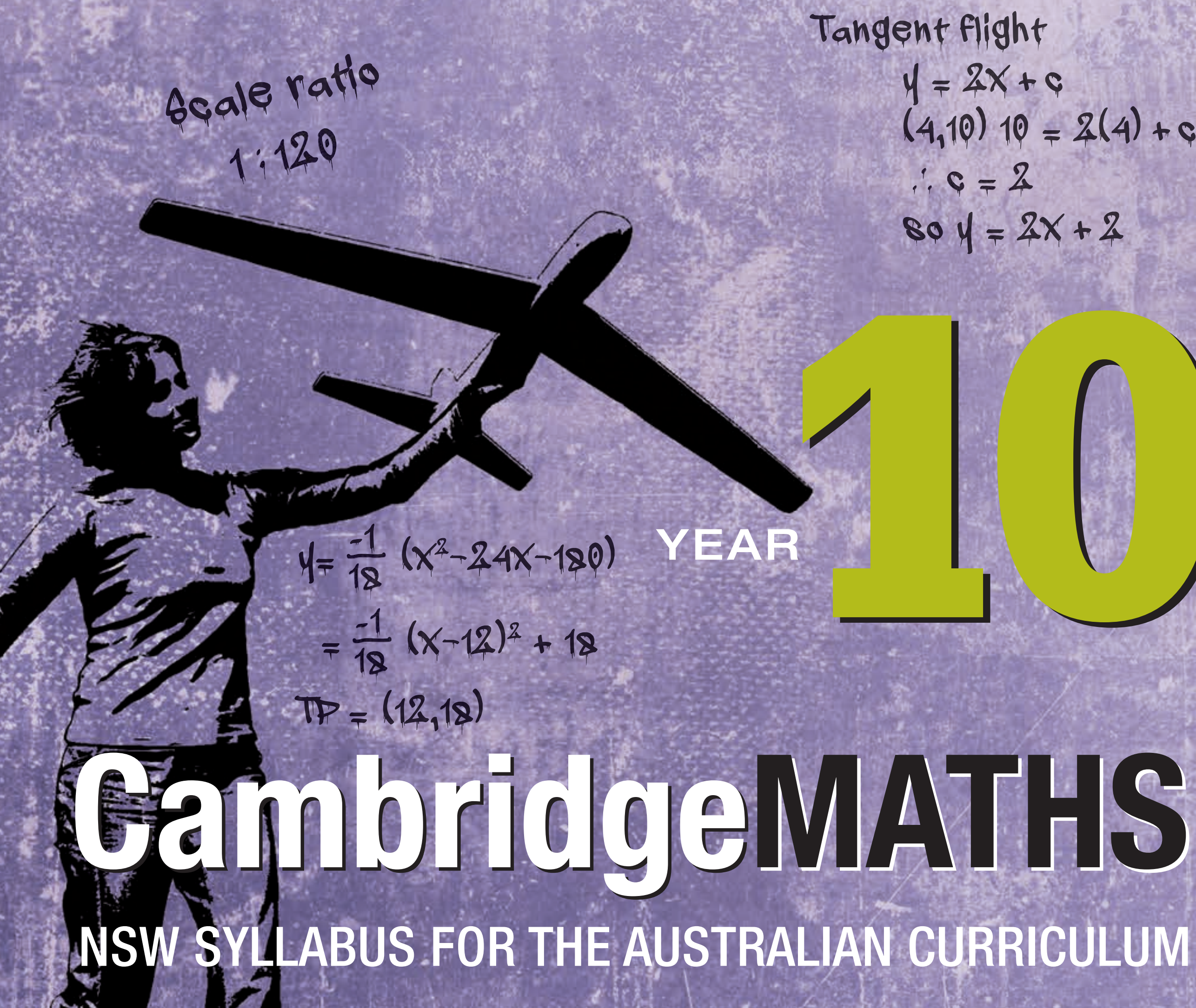

YEAR **10**

CambridgeMATHS

NSW SYLLABUS FOR THE AUSTRALIAN CURRICULUM

STAGE 4/5.1

GOLD

Interactive Textbook included

STUART PALMER | KAREN McDAID | DAVID GREENWOOD
SARA WOOLLEY | JENNY GOODMAN | JENNIFER VAUGHAN

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About the authors

Stuart Palmer was born and educated in New South Wales. He is a high school mathematics teacher with more than 25 years' experience teaching boys and girls from all walks of life in a variety of schools. Stuart has taught all the current NSW Mathematics courses in Stages 4, 5 and 6 many times. He has been a head of department in two schools and is now an educational consultant who conducts professional development workshops for teachers all over NSW and beyond. He also works with pre-service teachers at the University of Sydney.

Karen McDaid is an academic and lecturer in Mathematics Education in the School of Education at Western Sydney University. She has taught mathematics to both primary and high school students and is currently teaching undergraduate students on their way to becoming primary school teachers.

David Greenwood is the Head of Mathematics at Trinity Grammar School in Melbourne and has 20 years' experience teaching mathematics from Years 7 to 12. He has run numerous workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 20 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.

Sara Woolley was born and educated in Tasmania. She completed an Honours degree in Mathematics at the University of Tasmania before completing her education training at the University of Melbourne. She has taught mathematics in Victoria from Years 7 to 12 since 2006 and has a keen interest in the creation of resources that cater for a wide range of ability levels.

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Jenny Goodman has worked for 20 years in comprehensive State and selective high schools in New South Wales and has a keen interest in teaching students of differing ability levels. She was awarded the Jones Medal for Education at the University of Sydney and the Bourke prize for Mathematics. She has written for Cambridge NSW and was involved in the Spectrum and Spectrum Gold series.

Jennifer Vaughan has taught secondary mathematics for more than 30 years in New South Wales, Western Australia, Queensland and New Zealand, and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has taught special needs students and has had extensive experience in developing resources that make mathematical concepts more accessible.

Introduction and guide to this book

Thank you for choosing *CambridgeMATHS Gold*. This book is one component of an integrated package of resources designed for the NSW Syllabus for the Australian Curriculum. *CambridgeMATHS Gold* follows on from the standard CambridgeMATHS series published in 2013–14, and the two series have been structured so that they can be used in the same classroom. Mapping documents that show the relationship between the series can be found on *Cambridge GO*.

Whereas the standard *CambridgeMATHS* books for Years 9 and 10 begin at Stage 5, the *Gold* books for Years 9 and 10 are intended for students who need to consolidate Stage 4 learning prior to progressing to Stage 5. The aim is to develop Understanding and Fluency in core mathematical skills. Clear explanations of concepts, worked examples embedded in each exercise and carefully graded questions contribute to this goal. Problem-solving, Reasoning and Communicating are also developed through carefully constructed activities, exercises and enrichment.

Drilling for Gold 5A3

An important component of *CambridgeMATHS Gold* is a set of worksheets, and exercises in the print book, called ***Drilling for Gold***. These are engaging, innovative, skill-and-drill style tasks that provide the kind of practise and consolidation of the skills required for each topic without adding to the length of the textbook. All of them can be accessed in worksheet form from the online interactive version.

In Years 9 and 10 we have introduced major new activities to prepare students for mathematics in the workplace, the marketplace (consumer maths) and at home, and to improve numeracy. **Keeping in touch with Numeracy**, **Maths@work**, **Maths@home** and **Consumer maths** are their titles.

The relationship between literacy and maths is a major focus of *CambridgeMATHS Gold*. Key words and concepts are defined using student-friendly language; real-world contexts and applications of mathematics help students connect these concepts to everyday life; and literacy skills are built into questions and activities throughout. In the interactive version of this book, definitions are enhanced by audio pronunciation, visual definitions and examples. More information about the interactive version can be found on page xii.

A suite of accompanying resources, including Drilling for Gold and other worksheets can be downloaded from *Cambridge GO*.

Chapter introductions provide real-world context for students.

What you will learn gives an overview of the chapter contents.

A summary of the chapter connects the topic to the NSW Syllabus. Detailed mapping documents to the NSW Syllabus can be found in the teaching program on *Cambridge GO*.

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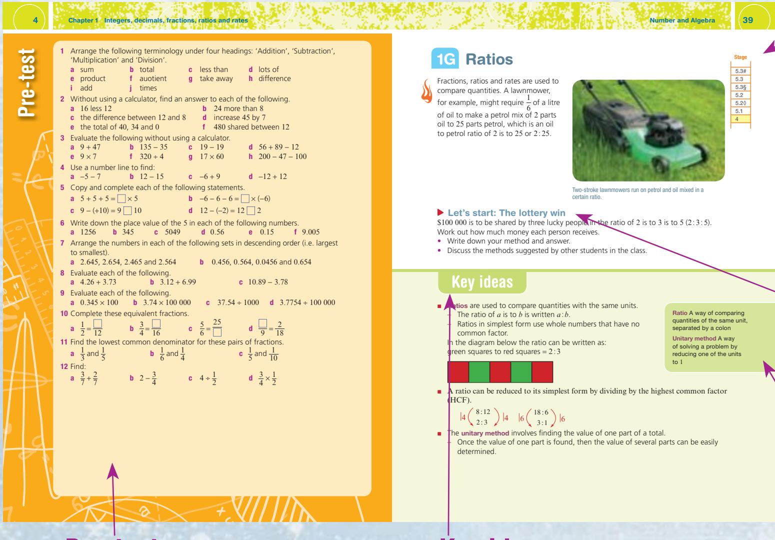


Ladder icons indicate the stage covered by each section (highlighted yellow).

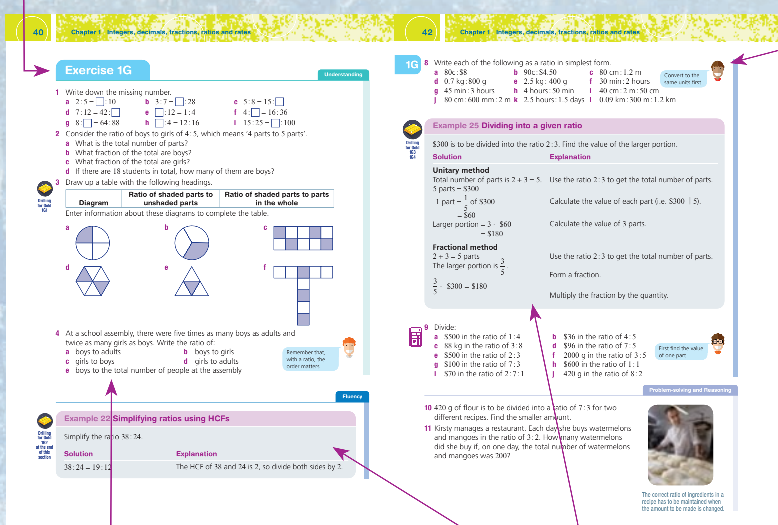
Let's start activities provide an engaging way to begin thinking about the topic.

Important terms in the **Key ideas** contain a simple-language definition.

A **Pre-test** for each chapter establishes prior knowledge.

Key ideas summarises the knowledge and skills for the topic.

Exercises are structured according to the four Working Mathematically strands: **Understanding**, **Fluency**, **Problem-solving and Reasoning**, with **Communicating** present in each of the other three. **Enrichment** questions at the end of the exercise challenge students to reach further.



Hints give advice for tackling questions.

Within each Working Mathematically strand, questions are further carefully graded from easier to challenging.

Examples with worked solutions and explanations are embedded in the exercises immediately before the relevant question(s).

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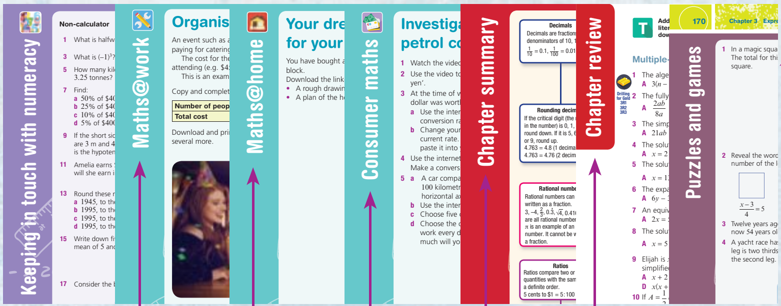


Puzzles and games allow students to have fun with the mathematics contained in the chapter.

Chapter summaries give mind maps of key concepts.

Chapter reviews provide multiple-choice, short-answer and extended-response questions.

Consumer maths activities help students become more informed consumers and citizens.

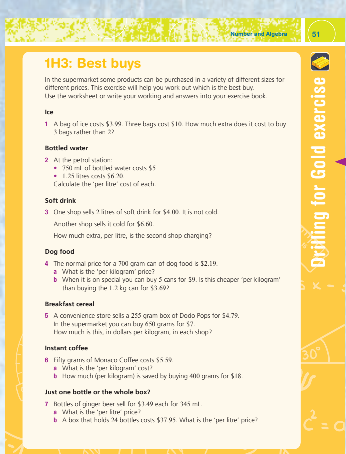
Maths@home activities help students develop life skills in mathematics.

Maths@work activities develop useful vocational mathematics skills, with supporting spreadsheets and videos in the interactive textbook.

Keeping in touch with Numeracy activities reinforce basic number skills through carefully structured alternating calculator and non-calculator questions.

Drilling for Gold exercises in the print book and downloadable worksheets

Drilling for Gold is a collection of engaging and motivating learning resources that provide opportunities for students to repeatedly practise routine mathematical skills. Their purpose is to improve automaticity, fluency and understanding through 'hands-on' resources, games, competitions, puzzles, investigations and sets of closed questions. In years 9 and 10, Drilling for Gold exercises are included in the print textbook, and Drilling for Gold worksheets are referenced in the pages of the textbook via a 'gold' icon and unique reference number. The worksheets can be downloaded from the Interactive Textbook.



1H3: Best buys

In the supermarket some products can be purchased in a variety of different sizes for different prices. This exercise will help you work out which is the best buy.
Use the worksheet or write your working and answers into your exercise book.

Ice

1 A bag of ice costs $3.99. Three bags cost $10. How much extra does it cost to buy 3 bags rather than 2?

Bottled water

2 At the petrol station:
- 750 mL of bottled water costs $5
- 1.25 litres costs $6.20.

Calculate the 'per litre' cost of each.

Soft drink

3 One shop sells 2 litres of soft drink for $4.00. It is not cold.
Another shop sells it cold for $6.60.
How much extra, per litre, is the second shop charging?

Dog food

4 The normal price for a 700 gram can of dog food is $2.19.
a What is the 'per kilogram' price?
b When it is on special you can buy 5 cans for $9. Is this cheaper 'per kilogram' than buying the 1.2 kg can for $3.69?

Breakfast cereal

5 A convenience store sells a 255 gram box of Dodo Pops for $4.79.
In the supermarket you can buy 650 grams for $7.
How much is this, in dollars per kilogram, in each shop?

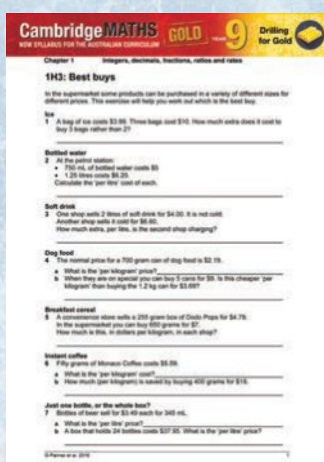
Instant coffee

6 Fifty grams of Monaco Coffee costs $5.59.
a What is the 'per kilogram' cost?
b How much (per kilogram) is saved by buying 400 grams for $18.

Just one bottle or the whole box?

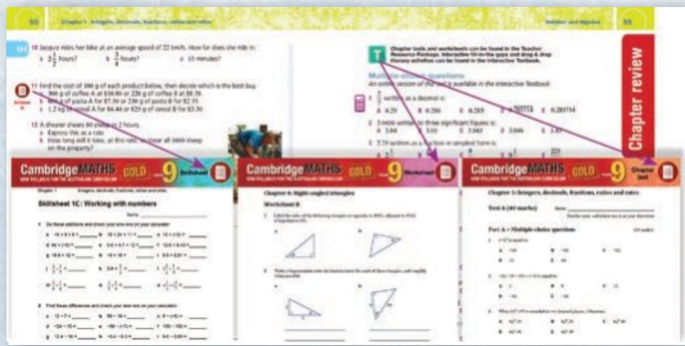
7 Bottles of ginger beer sell for $3.49 each for 345 mL.
a What is the 'per litre' price?
b A box that holds 24 bottles costs $37.95. What is the 'per litre' price?

A Drilling for Gold exercise in the print book.



A Drilling for Gold worksheet from the Interactive Textbook.

Other resources on *Cambridge GO*



- **Skillsheets** provide practice of the key skills learned across the entirety of the chapter.
- A **Chapter test** provides exam-style assessment, with multiple-choice, short-answer and extended-response questions.
- **Worksheets** cover multiple topics within a chapter and can be done in class or completed as homework. (Years 9 and 10 have the same online features.)

About your Interactive Textbook

An interactive digital textbook is included with your print textbook and is an integral part of the *CambridgeMATHS Gold* learning package. As well as being an attractive, easy-to-navigate digital version of the textbook, it contains many features that enhance learning in ways that are not possible with a print book:

- **Videos** provide detailed guidance on developing spreadsheets or other technology to use in a number of work, consumer and life skills settings
- Supporting spreadsheet and technology files are provided
- **Clickable 'enhanced' definitions** containing diagrams, illustrations, examples and audio pronunciation provide instant assistance and revision
- **Roll-over hints** for selected questions are provided within exercises
- **Matching *HOTmaths* lessons** can be accessed by clicking HOTmaths icon
- **Additional teacher resources** can be accessed by clicking the 'T' icon in the chapter review
- **Drilling for Gold** worksheets and **Skillsheets** can be downloaded by clicking on their icons in the margins
- **Fill-the-gap** and **drag-and-drop** literacy activities at the end of each chapter provide a fun way of learning key terms

- Interactive tests provide online versions of the multiple-choice questions from the chapter review
- **Answers** can be conveniently accessed via a button
- **Font size** can be increased or decreased as required
- **Annotations** can be added to allow critical engagement with the textbook.

A more detailed guide to using the Interactive Textbook can be found on *Cambridge GO*.

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1 Financial mathematics

What you will learn

Strand: Number and Algebra

Substrand: FINANCIAL MATHEMATICS

In this chapter you will learn to:

- solve financial problems to do with purchases
- solve financial problems to do with earning, spending and investing money.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw10

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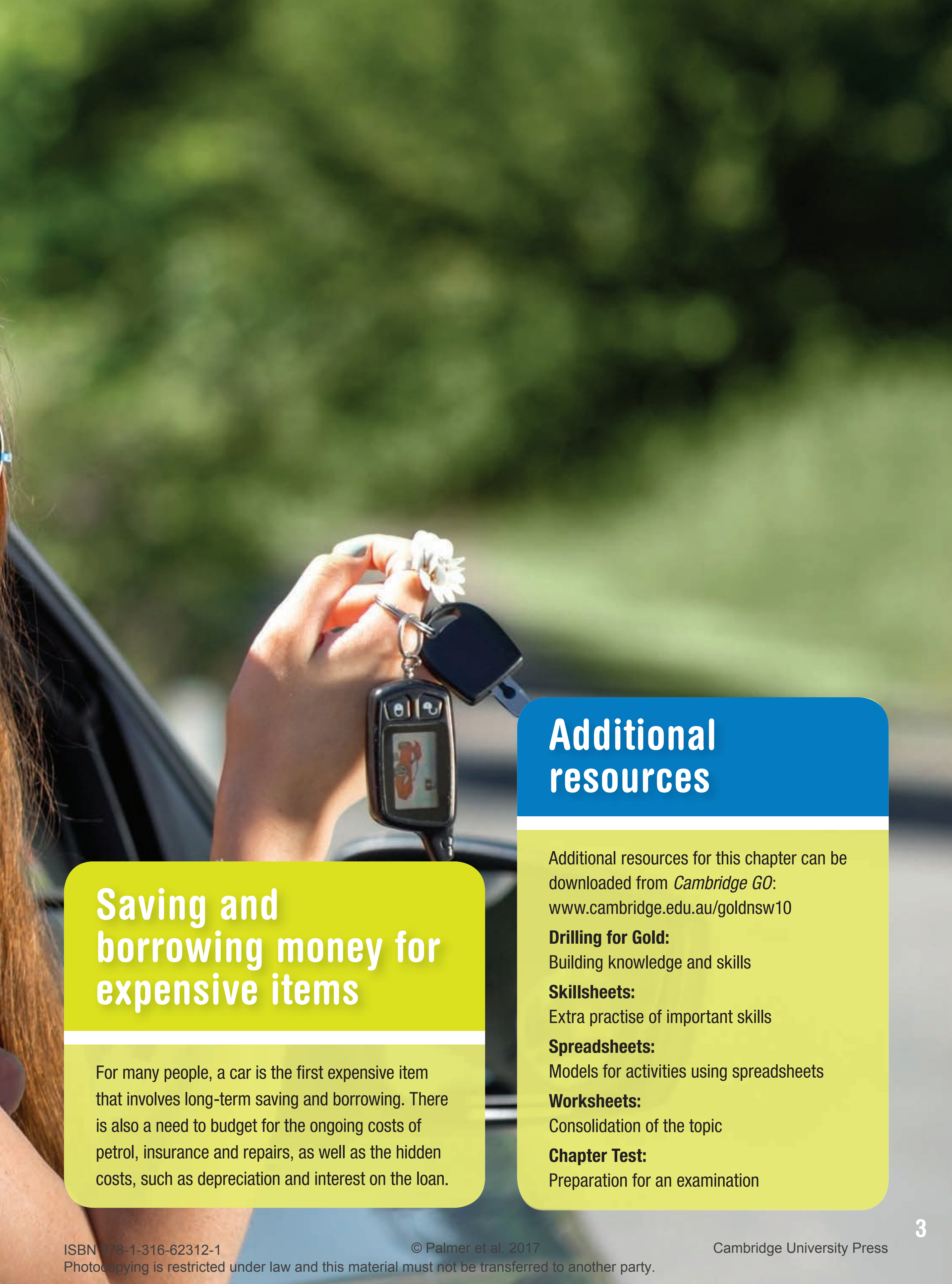

Saving and borrowing money for expensive items

For many people, a car is the first expensive item that involves long-term saving and borrowing. There is also a need to budget for the ongoing costs of petrol, insurance and repairs, as well as the hidden costs, such as depreciation and interest on the loan.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw10

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Spreadsheets:
Models for activities using spreadsheets

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 Find the following totals.
a $15.92 + $27.50 + $56.20
b $134 + $457 + $1021
c $\$457 \times 6$
d $\$56.34 \times 1\frac{1}{2}$
e $87 560 ÷ 52 (to the nearest cent)

2 Express the following fractions as percentages.
a $\frac{1}{2}$
b $\frac{3}{4}$
c $\frac{1}{5}$
d $\frac{17}{25}$
e $\frac{9}{20}$

3 Round the following decimals to 2 decimal places.
a 16.7893
b 7.347
c 45.3444
d 6.8389
e 102.8999

4 Copy and complete the following table.

Gross income ($)	Deductions ($)	Net income ($)
4976	456.72	a
92 411	b	62 839
c	18 472.10	79 431.36

Net income = gross income – deductions

5 Calculate the following annual incomes for each of these people.
a Tom: $1256 per week
b Viviana: $15 600 per month
c Anthony: $1911 per fortnight
d Crystal: $17.90 per hour, for 40 hours per week, for 50 weeks per year

6 Without a calculator, find:
a 10% of $400
b 5% of $5000
c 2% of $100
d 25% of $844
e 20% of $12.80
f 75% of $1000

7 Find the simple interest earned on the following amounts.
a $400 at 5% p.a. for 1 year
b $5000 at 6% p.a. for 1 year
c $800 at 4% p.a. for 2 years

Simple interest $I = PRN$

8 Complete the following table.

Cost price	Deduction	Sale price
$34	$16	a
$460	$137	b
$500	c	$236
d	$45	$67
e	$12.65	$45.27

9 The following amounts include the 10% GST. By dividing each one by 1.1, find the original costs before the GST was added.
a $55
b $61.60
c $605

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1A Review of percentages

Stage

5.2
5.2◊
5.1
4

It is important that we are able to work with percentages in our everyday lives. Banks, retailers and governments use percentages every day to work out fees, prices and interest.

▶ Let's start: Which option should Jamie choose?

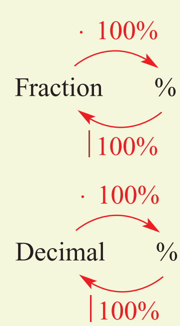
Jamie currently earns $38 460 p.a. (i.e. per year) and is given a choice of two different pay rises. Which should she choose and why?

Choice A	Choice B
Increase of $20 a week	Increase of 2% on p.a. salary

Key ideas

Drilling for Gold 1A1 1A2 at the end of this section

- A **percentage** means 'out of 100'. It can be written using the symbol % or as a fraction or a decimal.
  For example: 75 per cent $= 75\% = \frac{75}{100}$ or $\frac{3}{4} = 0.75$
- To convert a fraction or a decimal to a percentage, multiply by 100% or $\frac{100\%}{1}$.
- To convert a percentage to a fraction, write it with a **denominator** of 100 and simplify.
  For example: $15\% = \frac{15}{100} = \frac{3}{20}$
- To convert a percentage to a decimal, divide by 100%.
  For example: $15\% = 15 \div 100 = 0.15$
- To find a percentage of a quantity, write the percentage as a fraction or a decimal, then multiply by the quantity.
  For example: 35% of $\$600 = \frac{35}{100} \times \$600$



Percentage A convenient way of writing fractions with denominators of 100

Denominator The part of a fraction that sits below the dividing line

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Exercise 1A

Understanding

1 Write the following with denominators of 100.

a $\frac{2}{5}$ **b** $\frac{17}{20}$ **c** $\frac{49}{50}$

d $\frac{7}{25}$ **e** $\frac{9}{10}$

Make sure you have an equivalent fraction: $\frac{2}{5} = \frac{\square}{100}$.

2 Complete the following.

a $7\% = \frac{7}{\square}$ **b** $0.9 = \square\%$ **c** $\frac{3}{5} = \square\%$

Drilling for Gold 1A3 at the end of this section

3 Use mental strategies to find:

a 10% of \$7.50 **b** 20% of \$400 **c** 50% of \$98

d 75% of \$668 **e** 25% of \$412 **f** 2% of \$60

g 5% of \$750 **h** $33\frac{1}{3}\%$ of \$1200 **i** 30% of \$15

$10\% = \frac{1}{10}$
'Of' means times.

Fluency

Example 1 Converting to a percentage without a calculator

Write each of the following as a percentage.

a $\frac{19}{20}$ **b** $\frac{3}{8}$ **c** 0.07

Solution	Explanation
a $\frac{19}{20} = \frac{95}{100}$ (×5 numerator and denominator)	Write using a denominator of 100. Alternatively, multiply the fraction by 100%. $\frac{19}{\cancel{20}_1} \cdot \frac{\cancel{100}^5}{1}\% = 19 \cdot 5\% = 95\%$ Note: With a calculator, enter $\frac{19}{20} \cdot 100$.
b $\frac{3}{8} = \frac{3}{\cancel{8}_2} \cdot \frac{\cancel{100}^{25}}{1}\%$ $= \frac{75}{2}\%$ $= 37.5\%$	Multiply the fraction by 100%. Cancel common factors, then simplify.
c $0.07 = 0.07 \times 100\%$ $= 7\%$	Multiply the decimal by 100%. Alternatively, $0.07 = \frac{7}{100} = 7\%$.

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4 Convert each fraction to a percentage. Check your answers with a calculator.

a $\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $\frac{1}{4}$

d $\frac{1}{10}$ **e** $\frac{1}{100}$ **f** $\frac{7}{25}$

g $\frac{15}{50}$ **h** $\frac{3}{4}$ **i** $\frac{5}{8}$

j $\frac{19}{25}$ **k** $\frac{99}{100}$ **l** $\frac{47}{50}$

First, write using a denominator of 100 or, alternatively, multiply by 100%.

5 Write these decimals as percentages.

a 0.17 **b** 0.73 **c** 0.48 **d** 0.09

e 0.06 **f** 0.13 **g** 1.13 **h** 1.01

i 0.8 **j** 0.9 **k** 0.99 **l** 0.175

To multiply by 100%, move the decimal point two places to the right.

Example 2 Writing percentages as simple fractions without a calculator

Write each of the following percentages as a simple fraction.

a 37% **b** 58% **c** $6\frac{1}{2}\%$

Solution	Explanation
a $37\% = \frac{37}{100}$	Write the percentage with a denominator of 100.
b $58\% = \frac{58}{100}$ $= \frac{29}{50}$	Write the percentage with a denominator of 100. Simplify $\frac{58}{100}$ by cancelling, using the HCF of 58 and 100, which is 2. $\frac{\cancel{58}^{29}}{\cancel{100}_{50}} = \frac{29}{50}$
c $6\frac{1}{2}\% = \frac{6\frac{1}{2}}{100}$ $= \frac{13}{200}$	Write the percentage with a denominator of 100. Double the numerator $\left(6\frac{1}{2}\right)$ and the denominator (100) so that the numerator is a whole number.

6 Write each percentage as a simple fraction. Use a calculator to check your answers.

a 71% **b** 80% **c** 25% **d** 55%

e 40% **f** 88% **g** 15% **h** $16\frac{1}{2}\%$

i $17\frac{1}{2}\%$ **j** $2\frac{1}{2}\%$ **k** $5\frac{1}{2}\%$ **l** $52\frac{1}{2}\%$

Write with a denominator of 100, then simplify if possible.

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1A

Example 3 Writing a percentage as a decimal

Convert these percentages to decimals.

a 93% **b** 7% **c** 30%

Solution	Explanation
a $93\% = 0.93$	Divide the percentage by 100. This is done by moving the decimal point two places to the left. $93 \div 100 = 0.93$
b $7\% = 0.07$	Divide the percentage by 100. $7 \div 100 = 0.07$
c $30\% = 0.3$	Divide the percentage by 100. $30 \div 100 = 0.30$ Write 0.30 as 0.3.

7 Convert to decimals.

a 61% **b** 83% **c** 75% **d** 45%
e 9% **f** 90% **g** 50% **h** 16.5%
i 7.3% **j** 200% **k** 430% **l** 0.5%

Example 4 Finding a percentage of a quantity, with a calculator

Find 42% of $1800.

Solution	Explanation
42% of \$1800 $= 0.42 \times 1800$ $= \$756$	Remember that 'of' means multiply. Write 42% as a decimal or a fraction: $42\% = \frac{42}{100} = 0.42$ Then multiply by the amount. If using a calculator, enter 0.42×1800. Without a calculator: $\frac{42}{\cancel{100}} \cdot 18\cancel{00} = 42 \cdot 18$

8 Use a calculator to find:

a 10% of \$250 **b** 50% of \$300 **c** 75% of \$80
d 12% of \$750 **e** 9% of \$240 **f** 43% of 800 grams
g 90% of \$56 **h** 110% of \$98 **i** $17\frac{1}{2}\%$ of 2000 m

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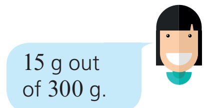
Problem-solving and Reasoning

9 A 300 g jar of spread contains 15 g of saturated fat.

a What fraction of the spread is saturated fat?

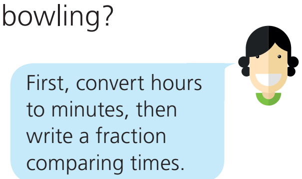
b What percentage of the spread is saturated fat?



10 About 80% of the mass of a human body is water. If Hugo is 85 kg, how many kilograms of water are in his body?

11 Rema spends 12% of the 6.6 hour school day in Maths. How many minutes are spent in the Maths classroom?

12 In a cricket match, Brett spent 35 minutes bowling. His team's total fielding time was $3\frac{1}{2}$ hours. What percentage of the fielding time, correct to 2 decimal places, did Brett spend bowling?



13 Malcolm lost 8 kg and now weighs 64 kg. What percentage of his original weight did he lose?

14 47.9% of a local council's budget is spent on garbage collection. If a rate payer pays $107.50 per quarter in total rate charges, how much do they contribute in a year to garbage collection?

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1A

Enrichment: Australia's statistics

15 Below is data on Australia's population growth, as gathered by the Australian Bureau of Statistics for September 2012. Use this data, or download more recent data from the ABS website.

	Population at end of September quarter 2012 ('000)	Change over previous year ('000)	Change over previous year (%, 1 decimal place)
New South Wales	7314.1	86.0	
Victoria	5649.1	94.8	
Queensland	4584.6	91.4	
South Australia	1658.1	16.4	
Western Australia	2451.4	81.7	
Tasmania	512.2	0.5	
Northern Territory	236.3	4.2	
Australian Capital Territory	376.5	7.4	
Australia	**22 782.3**	**382.4**	

a Calculate the percentage change for each State and Territory shown using the previous year's population, and complete the table.

You will need to calculate the previous year's population; e.g. for NSW, 7314.1 − 86.0.

b What percentage of Australia's overall population, correct to 1 decimal place, is living in:
- **i** NSW?
- **ii** Victoria?
- **iii** WA?

c Use a spreadsheet to draw a sector graph (i.e. a pie chart) showing the populations of the eight States and Territories given in the table. What percentage of the total is represented by each State/Territory?

d In part **c**, what is the angle size of the sector representing Victoria?

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1A1: Missing information

Drilling for Gold exercise

Find the missing piece of information in each of the following.
Complete the worksheet or write the answers in your exercise book.

Guess the missing information in the following equations. Check your answers with a calculator.	Use your calculator to find the missing information in the following equations. Give answers correct to 1 decimal place where necessary.
1 10% of ______ = 12	**13** 17.5% of \$250 = ________
2 10% of ________= \$15	**14** 20% of _______ = 29.4 cm
3 _______ of \$70 = \$7	**15** ______ of \$31 = \$5.27
4 _______ of \$35 = \$7	**16** 17% of 440 = ______
5 15% of 20 = ________	**17** 22% of ______ = \$136.40
6 _______ of 100 = 35	**18** _______ of 180 mL = 99 mL
7 20% of _______ = 10 mL	**19** 12.5% of \$180 = _______
8 20% of _______ = 20	**20** 14% of _______ = 56
9 _______ of \$120 = \$30	**21** 6.5% of 2100 m = ______
10 25% of 180 g = ________	**22** _______ of \$600 = \$48
11 15% of _______ = 9 cm	**23** 20.25% of _______ = 162 m
12 _______ of 80 m = 60 m	**24** 7.5% of _______ = \$37.50

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Drilling for Gold exercise

1A2: Nine ways with percentages

Use the worksheet or write your working and answers in your exercise book.

Question	Non-calculator method	Calculator method
1 Calculate 25% of 40.		
2 Express 25 as a percentage of 40.		
3 25% of a number is 40. What is the number?		
4 Increase 40 by 25%.		
5 Decrease 40 by 25%.		
6 A number is increased by 25% to give 40. What is the number?	Not applicable.	
7 A number is decreased by 25% to give 40. What is the number?	Not applicable.	
8 A number is increased from 25 to 40. What is the percentage increase?		
9 A number is decreased from 40 to 25. What is the percentage decrease?		

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1A3: Percentage of a quantity (mentally)

Use the worksheet, or write the answers in your exercise book.

400	800	600	240	120	20	90	50	440	5000

Choose a number from the above table
Imagine the number is here.
This is 100%.

1 What is 50% of the number?

2 What is 75% of the number?

3 What is 25% of the number?

4 What is 12.5% of the number?

5 What is 10% of the number?

After you have finished with each number, check your answers using a calculator.

6 What is 5% of the number?

7 What is 2.5% of the number?

8 What is 15% of the number?

9 What is 70% of the number?

10 What is 90% of the number?

11 What is 1% of the number?

12 What is 0.5% of the number?

13 What is 3% of the number?

14 What is 3.5% of the number?

15 What is 2% of the number?

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1B Applying percentages

Stage
5.2
5.2◊
5.1
4

There are many applications of percentages. Prices are often increased by a percentage to create a profit or decreased by a percentage when on sale.

▶ Let's start: Discounts

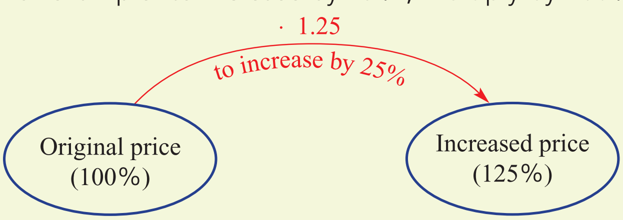
Discuss as a class:

- Which is better: 20% off or a $20 discount?
- If a discount of $20 or 20% off resulted in the same price, what would be the original price?
- Why are percentages used to show discounts rather than a fixed amount?

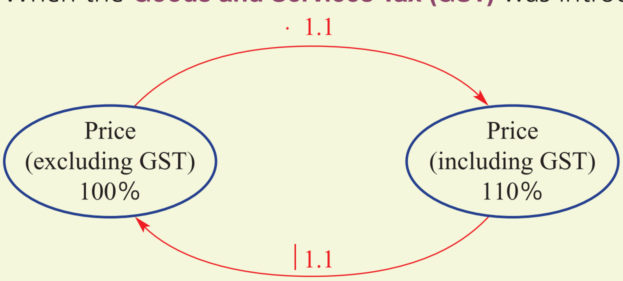
Key ideas

- To increase an amount by a given percentage:
  - Add the percentage increase to 100%.
  - Multiply the amount by this new percentage.

  For example: to increase by 25%, multiply by 100% + 25% = 125% = 1.25.

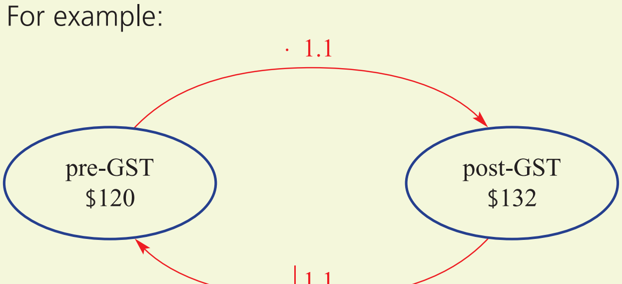


- When the **Goods and Services Tax (GST)** was introduced, prices were increased by 10%.



Goods and Services Tax (GST) In Australia, a 10% government levy included in the purchase price of merchandise.

For example:



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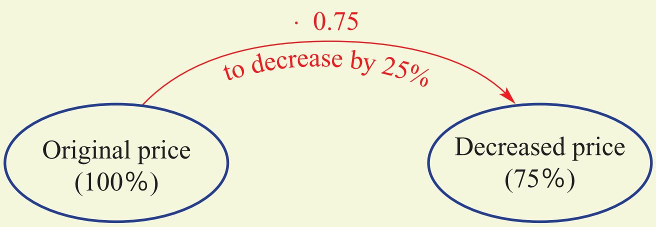
- To decrease an amount by a given percentage:
  - Subtract the percentage from 100%.
  - Multiply the amount by this new percentage.

  For example: to decrease by 25%, multiply by 100% – 25% = 75% = 0.75.



- To find a percentage change, use:

$$\text{Percentage change} = \frac{\text{change in price}}{\text{original price}} \times 100\%$$

- When goods are purchased by a store, the cost to the owner is called the **cost price**.
- The price on display in the shop is the **selling price**.
- **Profit** is the amount of money made on a sale.

  Profit = selling price – cost price

- A **loss** is made when the selling price is less than the cost price.

  Loss = cost price – selling price

- **Mark-up** is the amount added to the cost price to produce the selling price.

  Selling price = cost price + mark-up

- The **percentage profit** or **percentage loss** can be found by dividing the profit or loss by the cost price and multiplying by 100%:

$$\% \text{ Profit / Loss} = \frac{\text{profit / loss}}{\text{cost price}} \times 100\%$$

- **Discount** is the amount by which an item is marked down.
  Discount = % discount × original price

  New price = original price – discount

Profit The amount of money made by selling something for more than its cost

Loss The amount of money lost by selling something for less than its cost

Discount An amount subtracted from a price

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Exercise 1B

Understanding

1 By what percentage do you multiply to increase an amount by:
a 10%? **b** 20%? **c** 50%? **d** 2%? **e** 18%?

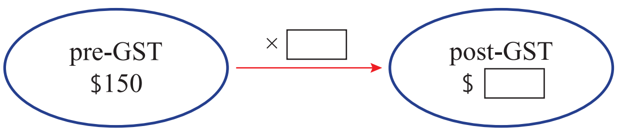
2 By what percentage do you multiply to decrease an amount by:
a 5%? **b** 30%? **c** 15%? **d** 50%? **e** 17%?

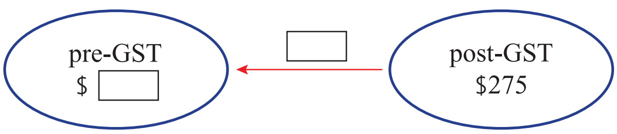
Increase
100% + percentage

Decrease
100% − percentage

3 Decide how much profit or loss is made in each of the following situations.
a cost price = $15 selling price = $20
b cost price = $17.50 selling price = $20
c cost price = $250 selling price = $234
d cost price = $147 selling price = $158
e cost price = $3.40 selling price = $1.20

4 Copy and complete the following, assuming the GST is 10%.

a


b


Fluency

Drilling for Gold 1B1 1B2 at the end of this section

Example 5 Increasing by a given percentage, with a calculator

Increase $370 by 8%.

Solution	Explanation
$\$370 \times 1.08 = \$399.60$	$100\% + 8\% = 108\%$ Write 108% as a decimal (or fraction) and multiply by the amount. Remember that money has 2 decimal places.

5 **a** Increase $90 by 5%. **b** Increase $400 by 10%.
c Increase $55 by 20%. **d** Increase $490 by 8%.
e Increase $50 by 12%. **f** Increase $7000 by 3%.
g Increase $49.50 by 14%. **h** Increase $1.50 by 140%.

To increase by 5%, multiply by 100% + 5% = 1.05.

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Example 6 Decreasing by a given percentage, with a calculator

Decrease \$8900 by 7%.

Solution	Explanation
$\$8900 \times 0.93 = \$8277.00$	$100\% - 7\% = 93\%$ Write 93% as a decimal (or fraction) and multiply by the amount. Remember to put the units in your answer.

6 **a** Decrease \$1500 by 5%.
b Decrease \$400 by 10%.
c Decrease \$470 by 20%.
d Decrease \$80 by 15%.
e Decrease \$550 by 25%.
f Decrease \$49.50 by 5%.
g Decrease \$119.50 by 15%.
h Decrease \$47.10 by 24%.

To decrease by 5%, multiply by $100\% - 5\% = 0.95$.

Example 7 Calculating profits and percentage profit

The cost price for a new car is \$24 780 and it is sold for \$27 600.

a Calculate the profit.

b Calculate the percentage profit, to 2 decimal places.

Solution

a Profit = selling price – cost price
= \$27 600 – \$24 780
= \$2820

Explanation: Write the rule. Substitute the values and evaluate.

b $\text{Percentage profit} = \dfrac{\text{profit}}{\text{cost price}} \times 100\%$ — Write the rule.

$= \dfrac{2820}{24780} \times 100\%$ — Substitute the values and calculate.

$= 11.38\%$ — Round your answer as instructed.

7 Copy and complete the table on profits and percentage profit.

	Cost price	Selling price	Profit	Percentage profit
a	\$10	\$16		
b	\$240	\$300		
c	\$15	\$18		
d	\$250	\$257.50		
e	\$3100	\$5425		
f	\$5.50	\$6.49		

$\text{Percentage profit} = \dfrac{\text{profit}}{\text{cost price}} \times 100\%$

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1B

Example 8 Finding the selling price

A retailer buys some material for $43.60 per roll. She wishes to make a 35% profit.

a What will be the selling price per roll?

b If she sells 13 rolls, what profit will she make?

Solution	Explanation
a Selling price = 135% of $43.60 = 1.35 × $43.60 = $58.86 per roll	For a 35% profit the unit price is 135%. Write 135% as a decimal (1.35) and evaluate.
b Profit per roll = $58.86 – $43.60 = $15.26 Total profit = $15.26 × 13 = $198.38	Profit = selling price – cost price There are 13 rolls at $15.26 profit per roll.

8 A retailer buys some snow globes for $41.80 each. He wishes to make a 25% profit.

a What will be the selling price per snow globe?

b If he sells a box of 25 snow globes, what profit will he make?

9 Ski jackets are delivered to a shop in packs of 50 for $3500. If the shop owner wishes to make a 35% profit:

a What will be the total profit made on a pack?

b What is the profit on each jacket?

10 A second-hand car dealer buys a trade-in car for $1200 and wishes to resell it for a 28% profit. What will be the resale price?

Example 9 Finding the discounted price

A shirt worth $25 is discounted by 15%.

a What is the selling price?

b How much is the saving?

Solution	Explanation
a Selling price = 85% of $25 = 0.85 × 25 = $21.25	15% discount means there must be 85% left (100% – 15%). Convert 85% to 0.85 and multiply by the amount.
b Saving = 15% of $25 = 0.15 × 25 = $3.75 or saving = $25 – $21.25 = $3.75	You save 15% of the original price. Convert 15% to 0.15 and multiply by the original price. Saving = original price – discounted price

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11 Samantha buys a wetsuit from the sports store where she works. Its original price was $79.95. If employees receive a 15% discount:
- **a** What is the selling price?
- **b** How much will Samantha save?

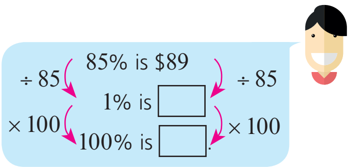
12 A travel agent offers a 12.5% discount on airfares if you travel during May or June. If the normal return fare to London is $2446:
- **a** What is the selling price?
- **b** How much is the saving?

13 A store sells second-hand goods at 40% off the recommended retail price. For a lawn mower valued at $369:
- **a** What is the selling price?
- **b** How much do you save?

Problem-solving and Reasoning

14 A pair of sports shoes is discounted by 47%. If the recommended price is $179:
- **a** What is the amount of the discount?
- **b** What will be the discounted price?

15 Jeans are priced during a sale for $89. If this is a saving of 15% off the selling price, what do the jeans normally sell for?



Drilling for Gold 1B3

16 Discounted tyres are reduced in price by 35%. They now sell for $69 each. Determine:
- **a** the normal price of one tyre
- **b** the saving when you buy one tyre

17 A price tag says '$308, inc. GST'.
- **a** What is the 'pre-GST' price?
- **b** How much GST is included in the price?

18 The local shop purchases a carton of containers for $54. Each container is sold for $4. If the carton has 30 containers, determine:
- **a** the profit per container
- **b** the percentage profit per container, to 2 decimal places
- **c** the overall profit per carton
- **d** the overall percentage profit, to 2 decimal places

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1B

19 A retailer buys a book for $50 and wants to sell it for a 26% profit. The 10% GST must then be added to the cost of the book.

- **a** Calculate the profit on the book.
- **b** How much GST is added to the cost of the book?
- **c** What is the advertised price of the book, including the GST?
- **d** Find the overall percentage increase of the final selling price compared to the $50 cost price.

$$\% \text{ Increase} = \frac{\text{increase}}{\text{cost price}} \cdot 100\%$$

Enrichment: Building a gazebo

20 Christopher designs a gazebo for a new house. He buys the timber from a retailer, who sources it at a wholesale price and then adds a mark-up before selling to Christopher at the retail price. The table below shows the wholesale prices, as well as the mark-up for each type of timber.

- **a** Determine Christopher's overall cost for the material, including the mark-up.
- **b** Determine the profit the retailer made.
- **c** Determine the retailer's overall percentage profit, to 2 decimal places.
- **d** If the retailer pays 27% of their profits in tax, how much tax do they pay on this sale?

Quantity	Description	Cost/unit	Mark-up
6	treated pine posts	$23	20%
11	300 × 50 oregon beams	$75	10%
5	sheet lattice work	$86	15%
2	300 × 25 oregon fascias	$46	12%
8	laserlite sheets	$32	10%

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Drilling for Gold exercise

1B2: Repeated percentage change (with a calculator)

Calculator short-cut for *in*creasing by a percentage

Example: Increase $120 by 5%.
The original amount was 100%, so the new amount will be 100% + 5% = 105%.
To convert to a decimal, 105 divided by 100 gives 1.05.
Calculation: $120 × 1.05 = $126

Calculator short-cut for *de*creasing by a percentage

Example: Decrease $120 by 5%.
The original amount was 100%, so the new amount will be 100% – 5% = 95%.
To convert to a decimal, 95 divided by 100 gives 0.95.
Calculation: $120 × 0.95 = $114

Complete the following questions on the worksheet or in your exercise book. The first one has been done for you as an example.

1. Increase $120 by 5% and then increase the result by 5%.
   $120 × 1.05 × 1.05 = $132.30
2. Decrease $120 by 5% and then decrease the result by 5%.
3. Increase $120 by 5% and then decrease the result by 5%.
4. Decrease $120 by 5% and then increase the result by 5%.
5. Increase $150 by 8% and then increase the result by 8%.
6. Decrease $150 by 8% and then decrease the result by 8%.
7. Increase $150 by 8% and then decrease the result by 8%.
8. Decrease $150 by 8% and then increase the result by 8%.

Calculator short-cut for repeated increase

The population of a town is expected to increase by 4% every year.
The current population is 50 000.

Try this on your calculator:

- Enter 50 000, then enter [=].
- Now enter [×] 1.04.
- Now enter [=] [=] [=] [=] [=] repeatedly and watch the population growing.

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Drilling for Gold exercise

Calculator short-cut for repeated decrease

The population of a town is expected to decrease by 4% every year.
The current population is 50 000.

Try this on your calculator:

- Enter 50 000, then enter [=].
- Now enter [×] 0.96.
- Now enter [=] [=] [=] [=] [=] repeatedly and watch the population falling.

In Questions **9** to **12**, start with a population of 50 000.

9 If the population increases by 4% every year, what will it be at the end of the tenth year?

10 If the population decreases by 4% every year, what will it be at the end of the tenth year?

11 If the population increases by 4% every year, how many years will it take for it to double?

12 If the population decreases by 4% every year, how many years will it take for it to be halved?

13 A car is purchased for $40 000 and loses 15% in value every year. How much will it be worth by the end of the fifth year?

14 If I invest $1000 now and it increases by 4% every year, how much will it be worth by the end of the 40th year?

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Stage

5.2
5.2◊
5.1
4

1C Income

You may have earned money for babysitting or delivering newspapers, or have a part-time job. As you move more into the workforce, it is important that you know how to check that you are being paid the correct amount.

Let's start: Casual teaching versus full-time teaching

When one of your normal teachers is absent from school, your school might employ a casual teacher to take his or her classes for the day.

Teacher A, a casual, was paid $340 for every school day they worked during 2015.
Teacher B, a full-time teacher, was paid about $65 000 during 2015.

- If Teacher A had worked every school day of 2015, how much would they have earned?
- Is this more or less than Teacher B?
- If Teacher A had worked only two days every school week of 2015, how much would they have earned?
- What are some reasons for being a casual teacher compared with a full-time teacher?
- Research the meaning of the following advantages of being a full-time teacher compared with a casual teacher.

Holiday pay	Sick pay	Salary scale	Long service leave
Annual leave loading	Salary packaging	Job security	International teacher exchange

Key ideas

Methods of payment

- Hourly **wages**: You are paid a certain amount per hour worked.
- **Commission**: You are paid a percentage of the total amount of sales. Some people who work for commission are also paid a set weekly amount, called a **retainer**.
- **Piecework**: You are paid according to number of things you make or do.
- **Salary**: You are paid a set amount per year.
- **Fees**: You are paid according to the charges you set; e.g. doctors, lawyers, contractors.

Wages Earnings paid to an employee based on an hourly rate

Commission Earnings of a salesperson based on a percentage of the value of goods or services sold

Salary An employee's fixed agreed yearly income

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Terminology

- **Gross income**: The total amount of money you earn before taxes and other deductions.
- **Deductions**: Money taken from your income before you are paid; e.g. taxation, union fees, superannuation.
- **Net income**: The amount of money you actually receive after the deductions are taken from your gross income.

Net income = gross income – deductions

Gross income
Total income before any deductions (e.g. income tax) are made

Deductions
Amounts of money taken from gross income

Net income
Income remaining after deductions have been made from gross income

Penalty rates
A higher hourly rate for working unsocial hours

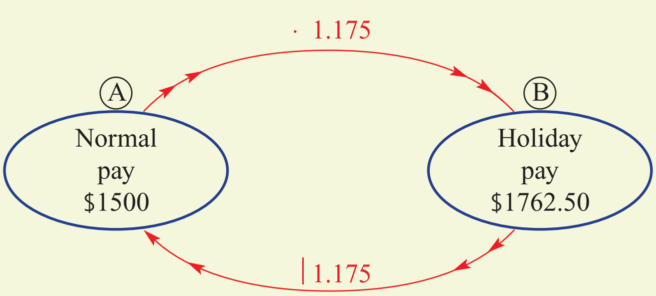
Payments by hourly rate

- If you are paid by the hour you will be paid an amount per hour for your normal working time. Usually, normal working time is 38 hours per week. If you work overtime the rates may be different.
  - Normal: $1.0 \times$ normal rate
  - Time and a half: $1.5 \times$ normal rate
  - Double time: $2.0 \times$ normal rate
- Some people are paid **penalty rates**; e.g. time and a half for working on Saturdays or double time for working on Sundays.

Leave loading

- Some wage and salary earners are paid leave loading. When they are on holidays, they earn their normal pay plus a bonus called leave loading. This is usually 17.5% of their normal pay.

$$\begin{aligned} 100\% + 17.5\% &= 117.5\% \\ &= 1.175 \end{aligned}$$



Leave loading = 17.5% of (A) = \$262.50

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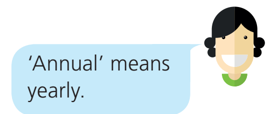
Exercise 1C

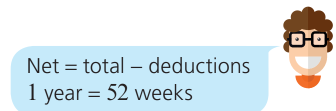
Understanding

1 If Tao earns \$570 for 38 hours' work, calculate his:
  a hourly rate of pay
  b time-and-a-half rate
  c double-time rate
  d annual income, given that he works 52 weeks a year, 38 hours a week



2 Which is better: \$5600 a month or \$67 000 a year?

3 Callum earns \$1090 a week and has annual deductions of \$19 838. What is Callum's net income for the year?



Fluency

Example 10 Finding gross and net income (including overtime)

Pauline is paid \$13.20 per hour at the local stockyard, where she normally works 38 hours per week. She receives time and a half for the next 4 hours worked and double time after that.

a What will be Pauline's gross income if she works 50 hours?
b If Pauline pays \$220 in tax and \$4.75 in union fees, what will be her net income?

Solution	Explanation
a Gross income = 38 × \$13.20	First 38 hours is paid at normal rate.
+ 4 × 1.5 × \$13.20	Overtime rate for next 4 hours: time and a half = 1.5 × normal rate
+ 8 × 2 × \$13.20	Overtime rate for next 8 hours: double time = 2 × normal rate
= \$792	
b Net income = \$792 – (\$220 + \$4.75)	Net income = gross income – deductions
= \$567.25	

4 Copy and complete this table.

	Hourly rate	Normal hours worked	Time and a half hours	Double time hours	Gross income	Deductions	Net income
a	\$15	38	0	0		\$155	
b	\$24	38	2	0		\$220	
c	\$13.15	38	4	1		\$300	
d	\$70	40	2	3		\$510	
e	\$17.55	35	4	6		\$184	

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1C

Example 11 Calculating shift work

Michael is a shift worker and is paid \$21.20 per hour for the morning shift, \$24.68 per hour for the afternoon shift and \$33.56 per hour for the night shift. Each shift is 8 hours. In a given fortnight he works four morning, two afternoon and three night shifts. Calculate his gross income.

Solution	Explanation
Gross income $= 4 \times \$21.20 \times 8$	4 morning shifts at \$21.20 per hour for 8 hours
$+ 2 \times \$24.68 \times 8$	2 afternoon shifts at \$24.68 per hour
$+ 3 \times \$33.56 \times 8$	3 night shifts at \$33.56 per hour
$= \$1878.72$	Gross income, as tax has not been paid.

5 Greg works shifts at a processing plant. In a given rostered fortnight he works:
- three day shifts (\$21.20 per hour)
- four afternoon shifts (\$24.68 per hour)
- four night shifts (\$33.56 per hour)

A fortnight = 2 weeks

a If each shift is 8 hours long, determine Greg's gross income for the fortnight.

b If the answer to part **a** is Greg's average fortnightly income, what will be his gross income for a year (i.e. 52 weeks)?

c If Greg is to be paid monthly, what will be his gross income for a month?

Example 12 Calculating income involving commission

Erika sells memberships to a gym and receives \$225 per week plus 5.5% commission on her sales. Calculate her gross income after a 5-day week.

Day	1	2	3	4	5
Sales (\$)	680	450	925	1200	1375

Solution	Explanation
Total sales $= \$4630$	Determine the total sales by adding the daily sales.
Commission $= 5.5\%$ of $\$4630$	Determine the commission on the total sales at
$= 0.055 \times 4630$	5.5% by multiplying 0.055 by the total sales.
$= \$254.65$	
Gross income $= \$225 + \$254.65$	Gross income is \$225 plus commission.
$= \$479.65$	

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6 A real estate agent receives 2.75% commission on the sale of a house valued at $375 000. Find the commission earned.

Divide by 100 to convert 2.75% to a decimal.

7 A car salesperson earns $500 a month plus 3.5% commission on all sales. In the month of January their sales total $56 000. Calculate:

a their commission for January
b their gross income for January

8 Portia earns an annual retainer of $27 000 plus 2% commission on all sales. Find:

a her weekly base salary before sales
b her commission for a week when her sales totalled $7500
c her gross weekly income for the week mentioned in part **b**.
d her annual gross income if over the year her sales totalled $571 250

Example 13 Calculating holiday pay

Rachel is normally paid $1200 per week. When she is on holidays she is paid 17.5% p.a. leave loading.

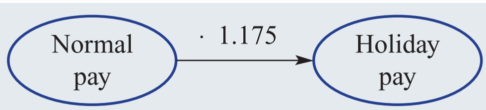
a How much is Rachel's holiday pay for 1 week?
b How much is Rachel's leave loading for 1 week?

Solution

a Holiday pay = $1200 × 1.175
= $1410

b Leave loading = $1410 – $1200
= $210

Explanation



Loading is 17.5% of normal pay.

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1C

9 Ashton is normally paid \$900 per week. When he is on holidays he is paid leave loading.
   a Calculate Ashton's holiday pay for 1 week.
   b Calculate Ashton's leave loading for 1 week.

Leave loading is an extra 17.5% of normal pay.

10 Mary earns \$800 per week. Calculate her holiday pay for 4 weeks, including leave loading.

Problem-solving and Reasoning

11 If Simone receives \$14 370 on the sale of a property worth \$479 000, calculate her rate of commission.

What percentage of \$479 000 is \$14 370?

12 Jonah earns a commission on his sales of fashion items. For goods to the value of \$2000 he receives 6% and for sales over \$2000 he receives 9% on the amount in excess of \$2000. In a given week he sold \$4730 worth of goods. Find the commission earned.

13 Mel is taking her holidays. She receives \$2937.50. This includes her normal pay and her leave loading. How much is the leave loading?

\$2937.50 is 117.5% of Mel's normal pay.

14 Toby earns 1.75% commission on all sales at the electrical goods store where he works. If Toby earns \$35 in commission on the sale of one television, how much did the TV sell for?

1.75% is \$35. Find 1%, then 100%.

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Enrichment: Elmo's payslip

15 Refer to the payslip below to answer the following questions. During 2015, Elmo received 26 of these payslips.

Kuger Incorporated

Employee ID: 75403A — Page: 1
Name: Elmo Clowner — Pay Period: 21/05/2015

Pay method: EFT — Tax status: Gen Exempt
Bank account name: E. Clowner
Bank: Mathsville Credit Union
BSB: 102-196 Account No: 00754031

Payment details this pay:

Amount	Days	Payment description	Rate/Frequency
2777.16	14.00	Normal time	$72 454/annum

Before tax deductions:

This pay	Description
170	Salary sacrifice: car pre-tax deduction

Miscellaneous deductions:

This pay	Description
52.90	Health fund
23.10	Union fees
76.00	

Reconciliation details:

This pay	YTD	Description
2607.15	62 571.60	Taxable gross pay
616.00	14 784.00	Less income tax
76.00	1 824.00	Less miscellaneous deductions
1915.15	45 693.60	

a For what company does Elmo work?
b What is the name of Elmo's bank and what is his account number?
c How much gross pay does Elmo earn in 1 year?
d How often does Elmo get paid?
e How much, per year, does Elmo salary sacrifice?
f How much each week is Elmo's health fund contributions?
g Calculate the union fees for 1 year.
h Using the information on this payslip, calculate Elmo's annual tax and also his annual net income.

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Maths@work

Worksheet 1A

So many ways to earn a living!

There are hundreds of different occupations and careers you could pursue in your working life. Some of these options may not exist at the moment but will in the future.

When it comes to 'pay day', different careers are paid in a variety of ways, such as:

- Casual pay rates, with the possibility of penalty rates
- Wages, with the possibility of overtime
- Salaries
- Commission, either with or without a retainer
- Piecework
- Consignment
- Royalties
- Allowances
- Bonuses

Here is an example to get you started.

For many young people, their first job is working in a fast-food outlet or a shop. There may be some students in your classroom doing this or maybe someone you know. Ask them the following questions, then ask your teacher the same questions.

1 Do you get paid if you are too sick to work?
2 Can you take holidays when it suits you?
3 Do you get paid when you are on holidays?
4 Do you always work the same number of hours every week at the same time and get paid the same amount?
5 Is there opportunity for pay rises and promotions within the company?
6 Do you ever receive a bonus on top of your normal pay?
7 Do you get paid more if you sell more?
8 Do you get paid extra for working on weekends and public holidays, such as time and a half or double time?
9 Do you get long-service leave or holiday leave loading?
10 Does your employer pay for your continuing education, training, courses, conferences or pay you while you study?

Continue with the activity by downloading the worksheet to help you understand the different ways of being paid, and the advantages and disadvantages of each.

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1D The PAYG income tax system

Stage
5.2
5.2◊
5.1
4

It has been said that there are only two sure things in life: death and taxes! The Australian Taxation Office (ATO) collects taxes on behalf of the government to pay for education, hospitals, roads, railways, airports and services, such as the police force and fire brigades.

In Australia, the financial year runs from July 1 to June 30 the following year. People engaged in paid employment are normally paid weekly or fortnightly. Most of them pay some income tax every time they are paid for their work. This is known as the Pay-As-You-Go system (PAYG).

At the end of the financial year (June 30), people who earned an income complete an income tax return to determine if they have paid the correct amount of income tax during the year. If they paid too much they will receive a refund. If they did not pay enough, they will be required to pay more.

The Australian tax system is very complex and the laws change frequently. This section covers the main aspects only.

▶ Let's start: The ATO website

The Australian Taxation Office website has some income tax calculators. Use one to find out how much income tax you would need to pay if your taxable income is:

$5200 per annum (i.e. $100 per week)
$10 400 per annum (i.e. $200 per week)
$15 600 per annum (i.e. $300 per week)
$20 800 per annum (i.e. $400 per week)
$26 000 per annum (i.e. $500 per week)

Does a person earning $1000 per week pay twice as much tax as a person earning $500 per week?

Does a person earning $2000 per week pay twice as much tax as a person earning $1000 per week?

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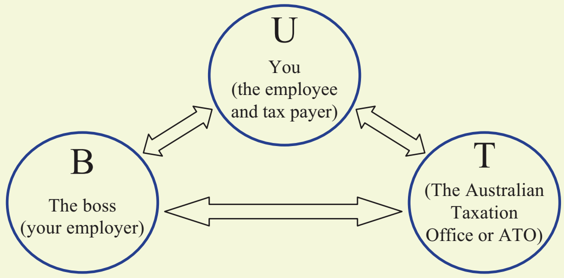
Key ideas



- The PAYG tax system works in the following way.
  - U works for and gets paid by B every week, fortnight or month.
  - B calculates the tax that U should pay for the amount earned by U.
  - B sends that tax to T every time U gets paid.
  - T passes the income tax to the federal government.
  - On June 30, B gives U a **payment summary** to confirm the amount of tax that has been paid to T on behalf of U.
  - Between July 1 and October 31, U completes a **tax return** and sends it to T. Some people pay a registered tax agent to do this for them.
  - On this tax return, U lists the following.
    - *All* forms of income, including interest from investments.
    - Legitimate deductions shown on receipts and invoices, such as work-related expenses and donations.
  - **Taxable income** is calculated using the formula:
    Taxable income = gross income – deductions
  - There are tables and calculators on the ATO website, such as the following. Each row in the table is called a tax bracket.

Taxable income	Tax on this income
0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001 – $180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

    This table can be used to calculate the amount of tax U *should have* paid (i.e. the **tax payable**), as opposed to the tax U *did* pay during the year (i.e. the tax withheld).
  - U may also need to pay the Medicare levy. This is a scheme in which all Australian taxpayers share in the cost of running the medical system. For many people this is currently 1.5% of their taxable income.
  - It is possible that U may have paid too much tax during the year and will receive a **tax refund**.
  - It is also possible that U may have paid too little tax and will receive a letter from T asking for the **tax liability** to be paid.

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Exercise 1D

Understanding

Note: The questions in this exercise relate to the information given in Key ideas, unless stated otherwise.

1 Complete this statement: Taxable income = ________ income minus ___________.

2 Is the following statement true or false?

The highest income earners in Australia pay 45 cents tax for every dollar they earn.

3 Joseph paid no income tax. What could his taxable income have been?

4 Ann's taxable income was $80 000, which puts her at the very top of the middle tax bracket in the tax table. Ben's taxable income was $80 001, which puts him in a higher tax bracket. Ignoring the Medicare levy, how much extra tax did Ben pay compared to Ann?

Fluency

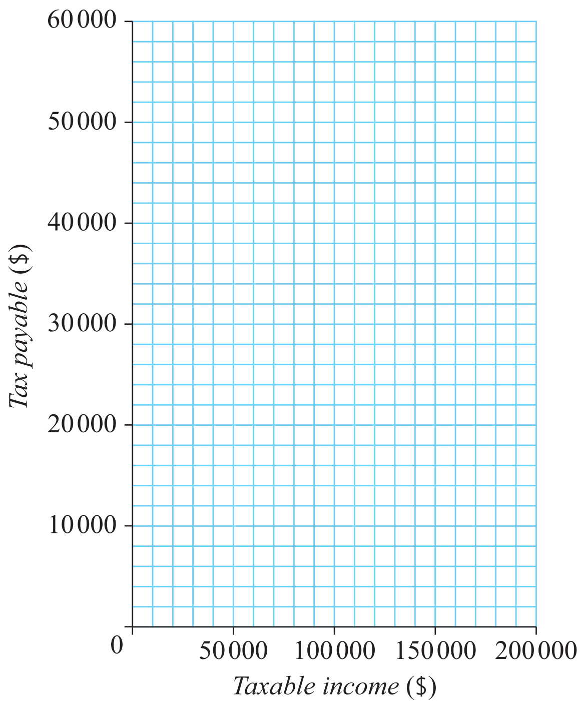
5 Use an online tax calculator on the ATO website to calculate the income tax payable on these taxable incomes.

a $30 000 **b** $60 000 **c** $150 000 **d** $200 000

6 Consider the amount of tax payable for these six people.

Taxable income	$0	$18 200	$37 000	$80 000	$180 000	$200 000
Tax payable	$0	$0	$3572	$17 457	$54 547	$63 547

Make a copy of this set of axes, plot the points and then join the dots with straight-line segments.



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1D **7** Jim worked for three different employers. They each paid him $15 000. Based on your graph in the previous question, how much income tax should Jim have paid?

Example 14 Calculating income tax payable

Richard earned $1050 per week ($54 600 dollars per annum) from his employer and other sources, such as interest on investments. He has receipts for $375 for work-related expenses and donations.

a Calculate Richard's taxable income.

b Use this tax table to calculate Richard's tax payable amount.

Taxable income	Tax on this income
0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18,200
$37 001 – $80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001 – $180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

c Richard must also pay the Medicare levy of 1.5% of his taxable income. How much is the Medicare levy?

d Add the tax payable and the Medicare levy amounts.

e Express the total tax in part **d** as a percentage of Richard's taxable income, to 1 decimal place.

f During the financial year, Richard's employer sent a total of $7797 in tax to the ATO. Has Richard paid too much tax or not enough? Calculate his refund or liability.

Solution	Explanation
a Gross income = $54 600 Deductions = $375 Taxable income = $54 225	Taxable income = gross income – deductions
b Tax payable: $\$3572 + 0.325 \times (\$54\,225 - \$37\,000)$ $= \$9170.13$	Richard is in the middle tax bracket in the table, in which it says: $3572 plus 32.5c for each $1 over $37 000 Note: 32.5 cents is $0.325.
c $\frac{1.5}{100} \cdot 54\,225 = \$813.38$	Medicare levy is 1.5% of the taxable income. Round your answer to the nearest cent.
d $\$9170.13 + \$813.38 = \$9983.51$	This is the total amount of tax that Richard should have paid.
e $\frac{9983.51}{54\,225} \cdot 100\% = 18.4\%$ (to 1 decimal place)	This implies that Richard paid approximately 18.4% tax on every dollar. This is sometimes read as '18.4 cents in the dollar'.

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f Richard paid \$7797 in tax during the year.
He should have paid \$9983.51.
Richard has not paid enough tax.
He must pay another \$2186.51 in tax.

This is known as a shortfall or a liability.
Richard will receive a letter from the ATO requesting payment of the difference.
\$9983.51 – \$7797 = \$2186.51

8 Lee has come to the end of her first financial year employed as a website developer. On June 30 she made the following notes about the financial year.

Gross income from employer	\$58 725
Gross income from casual job	\$7500
Interest on investments	\$75
Donations	\$250
Work-related expenses	\$425
Tax paid during the financial year	\$13 070

Taxable income
= all incomes – deductions

a Calculate Lee's taxable income.
b Use the tax table shown in **Example 14** to calculate Lee's tax payable amount.
c Lee must also pay the Medicare levy of 1.5% of her taxable income. How much is the Medicare levy?
d Add the tax payable and the Medicare levy.
e Express the total tax in part **d** as a percentage of Lee's taxable income, to 1 decimal place.
f Has Lee paid too much tax or not enough? Calculate her refund or liability.

Problem-solving and Reasoning

9 Alec's Medicare levy is \$1312.50. This is 1.5% of his taxable income. What is his taxable income?

10 Tara is saving for an overseas trip. Her taxable income is usually about \$20 000. She estimates that she will need \$5000 for the trip, so she is going to do some extra work to raise the money. How much extra will Tara need to earn in order to save the extra \$5000 after tax?

11 When Saled used the tax table to calculate his income tax payable, it turned out to be \$23 097. What is his taxable income?

Use the tax table given in Example 14 to determine in which tax bracket Saled falls.

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1D

12 Explain the difference between gross income and taxable income.

13 Explain the difference between a tax refund and a tax liability.

14 Gordana looked at the last row of the tax table and said, "It is so unfair that people in that tax bracket must pay 45 cents in every dollar in tax." Explain why Gordana is incorrect.

15 Consider the tax tables for the two consecutive financial years. Note that the amounts listed first in each table is often called the tax-free threshold (i.e. the amount that a person can earn before they must pay tax).

2011/2012	
Taxable income	**Tax on this income**
0 – $6000	Nil
$6001 – $37 000	15c for each $1 over $6000
$37 001 – $80 000	$4650 plus 30c for each $1 over $37 000
$80 001 – $180 000	$17 550 plus 37c for each $1 over $80 000
$180 001 and over	$54 550 plus 45c for each $1 over $180 000
2012/2013	
Taxable income	**Tax on this income**
0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001 – $180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

a There are some significant changes between the financial years 2011/2012 and 2012/2013. Describe three of them.

b The following people had the same taxable income during both financial years. Find the difference and state whether they were advantaged or disadvantaged by the changes, or not affected at all?

i Ali: Taxable income = $5000

ii Xi: Taxable income = $15 000

iii Charlotte: Taxable income = $30 000

iv Diego: Taxable income = $50 000

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16 Below is the 2012/2103 tax table for people who are not residents of Australia but are working in Australia.

Taxable income	Tax on this income
0 – $80 000	32.5c for each $1
$80 001 – $180 000	$26 000 plus 37c for each $1 over $80 000
$180 001 and over	$63 000 plus 45c for each $1 over $180 000

Compare this table to the one in the example for Australian residents.

What difference would it make to the tax paid by these people in 2012/2013 if they were non-residents rather than residents?

a Ali: Taxable income = $5000

b Xi: Taxable income = $15 000

c Charlotte: Taxable income = $30 000

d Diego: Taxable income = $50 000

Enrichment: What are legitimate tax deductions?

17 **a** Choose an occupation or career in which you are interested. Imagine that you are working in that job. During the year you will need to keep receipts for items you have bought that are legitimate work-related expenses. Do some research on the internet and write down some of the things that you will be able to claim as work-related expenses in your chosen occupation.

b **i** Imagine your taxable income is $80 000. What is your tax payable amount?

ii You just found a receipt for a $100 donation to a registered charity. This decreases your taxable income by $100. By how much does it decrease your tax payable amount?

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Keeping in touch with numeracy

Non-calculator

1 When a fair die is rolled, what is the probability that it will show a number greater than 1?

3 In the number 352.823, the digit 8 represents which of the following?
A eight hundred
B eighty
C eight-tenths
D eight-hundredths

5 Convert the following test results into percentages:
a 15 out of 30
b 15 out of 50

7 The square numbers are 1, 4, 9, 16, …. What is the tenth square number?

9 A recipe requires 500 grams of mince to serve four people. How many grams of mince will be required to serve two people?

11 The temperature one morning was –7°C. Later in the day is was 10°C. By how much did the temperature increase?

13 Lucy worked 5.5 hours every day from Monday to Saturday. How many hours did she work?

15 A class contains 12 girls and 8 boys. What percentage of the class are girls?

17 Which of the following shapes might *not* contain a right angle?
square triangle rectangle trapezium

19 Decrease $40 by 20%.

Calculator

2 If a fair die is rolled 240 times, how many times would you expect to roll the number 1?

4 Write the number 'twelve-hundredths' as a decimal and as a simple fraction.

6 Convert the following test results into percentages:
a 15 out of 40
b 15 out of 80

8 The square numbers are 1, 4, 9, 16, …. What is the sum of the first ten square numbers?

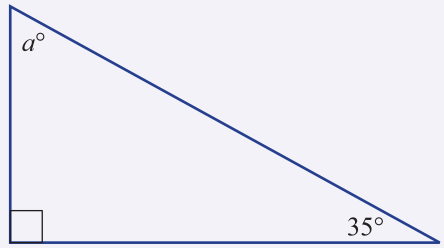
10 A recipe requires 500 grams of mince to serve four people. How many kilograms of mince will be required to serve 10 people?

12 What is the first positive number in the pattern –20, –17, –14, …?

14 In Question **13**, Lucy's normal rate of pay is $12.75 per hour. She is paid time and a half on Saturdays. What was her pay for the week?

16 A class contains 12 girls and 8 boys. If this information is presented in a sector graph, what will be the angle representing girls?

18 Find the value of a.



20 Clara bought a dress on sale for $40. It had been reduced by 20%. What was the original price of the dress?

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1E Simple interest

Stage
- 5.2
- 5.2◊
- 5.1
- 4

If you invest money you will *earn* interest.
If you borrow money you will *pay* interest.
There are two different types of interest:
- simple interest
- compound interest.

▶ Let's start: I want to double my money!

Drilling for Gold 1E1a 1E1

You may choose to download the 'Drilling for Gold' files to assist with this activity.
Sophie has \$100 invested but she would like to have \$200.
Her investment increases every year by 10% of the original investment.
- By how much will it increase in the first year?
- Copy and complete the following table.

Year	0	1	2	3	4	5	6	7	8	9	10
Amount	\$100	\$110	\$120								

- How long will it take to reach her goal?
- How long will it take if her investment increases by 5% every year?
- How long will it take if her investment increases by 7.5% every year?
- How long will it take if her investment increases by 5% every year for the first 5 years, then 6% every year thereafter?

Key ideas

- Simple interested is always calculated using the original amount invested or borrowed.
- The terms needed to understand **simple interest** are:
  - **principal (P)**: The amount of money borrowed or invested.
  - **rate of interest (R)**: The annual (yearly) percentage rate of interest (e.g. 3% p.a.).
  - time periods (N): This is usually the number of years.
  - interest (I): The amount of interest accrued over a given time.
- The formula for calculating simple interest is:

$$I = PRN$$

where I = the amount of simple interest (in \$)
P = the principal (the initial amount borrowed or invested)
R = the interest rate per period, expressed as a decimal
N = the number of periods

Simple interest A type of interest that is paid on a loan or earned on an investment, which is always calculated on the principal amount

Principal (*P*) An amount of money invested in a financial institution or loaned to a person/business

Rate of interest (*R*) The annual percentage rate of interest paid or earned on a loan or investment

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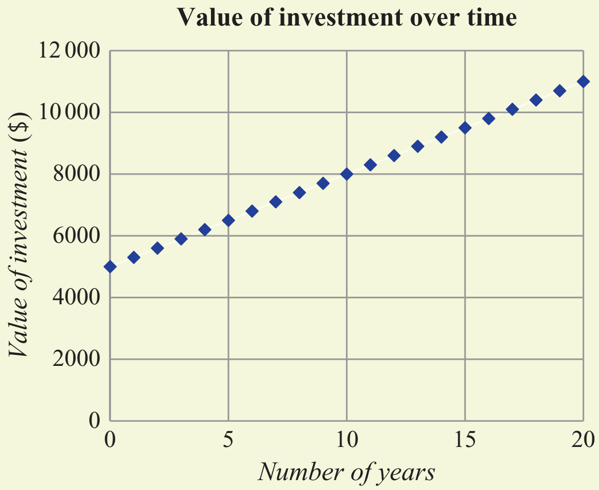
- The graph below shows an investment of $5000 growing with 6% p.a. simple interest, which is a linear relationship.



- In the graph above, the interest earned is $300 per year.
  The points in the graph can be generated on a calculator:
  Enter the number 5000, then press [=].
  Add 300.
  Press [=] [=] [=] [=] etc.

Exercise 1E

Understanding

1 Copy and complete:

a 12 months = ________ year

b $\frac{1}{2}$ year = ________ months

c ______ weeks = 1 year

d $2\frac{1}{2}$ years = ________ months

2 Interest on a loan is fixed at $60 per year. How much interest is due in:

a 2 years? **b** 7 years? **c** 6 months?

3 Simple interest on $7000 is 6% p.a. How much interest is earned in:

a 1 year? **b** 2 years? **c** 1 month?

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Fluency

Example 15 Using the simple interest formula

Use the simple interest formula, $I = PRN$, to find:

a the interest (I) when \$600 is invested at 8% p.a. for 18 months

b the annual interest rate (R) when \$5000 earns \$150 interest in 2 years

Solution	Explanation
a $P = 600$ $R = 8 \div 100 = 0.08$ $N = 18$ months $= \frac{18}{12} = 1.5$ years	Write out the information that you know and the formula. Express the rate as a decimal (or fraction).
$I = PRN$ $= 600 \times 0.08 \times 1.5$ $= 72$ The interest is \$72 in 18 months.	Substitute values into the formula.
b $P = 5000$ $I = 150$ $N = 2$ years	Write down the information known.
$I = PRN$ $150 = 5000 \times R \times 2$ $150 = 10\,000R$ $10\,000R = 150$ $R = 0.015$ (÷ 10 000 on both sides) The simple interest rate is 1.5% per year.	Write down the formula. Substitute the values into the formula and solve the equation to find R. Swap LHS and RHS. Divide both sides by 10 000. Multiply R by 100 to convert it to a percentage.

4 Copy and complete this table of values for I, P, R and N.

	P	R	N	I
a	\$700	5% p.a.	4 years	
b	\$2000	7% p.a.	3 years	
c	\$3500	3% p.a.	22 months	
d	\$750	2.5% p.a.	30 months	
e	\$22 500		3 years	\$2025
f	\$1770		5 years	\$354

Recall:
$I = PRN$
N must be years if rate is p.a.

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1E

Example 16 Calculating repayments with simple interest

Skillsheet 1A

$3000 is borrowed at 12% p.a. simple interest for 2 years.

a What is the total amount owed over the 2 years?

b If repayments of the loan are made monthly, how much would each payment need to be?

Solution	Explanation
a $P = \$3000,\ R = 12 \div 100 = 0.12,\ N = 2$	List the information you know.
$I = PRN$	Write the formula.
$= 3000 \times 0.12 \times 2$ $= \$720$	Substitute the values and evaluate.
Total amount = $3000 + $720 = $3720	Total amount is the original amount *plus* the interest.
b Amount of each payment = 3720 ÷ 24 = $155 per month	2 years = 24 months, so there are 24 payments to be made. Divide the total by 24.

5 $5000 is borrowed at 11% p.a. simple interest for 3 years.

a What is the total amount owed over the 3 years?

b If repayments of the loan are made monthly, how much would each payment need to be?

Calculate the interest first.

6 Under hire purchase, John bought a car for $11 500. He paid no deposit and decided to pay off the loan in 7 years. If the simple interest rate was 6.45% p.a., determine:

a the total interest paid

b the total amount of the repayment

c the payments per month

7 $10 000 is borrowed to buy a second-hand BMW. The interest is calculated at a simple interest rate of 19% p.a. over 4 years.

a What is the total interest on the loan?

b How much is to be repaid?

c What is the monthly repayment on this loan?

Problem-solving and Reasoning

8 Rebecca invests $4000 for 3 years at 5.7% p.a. simple interest, paid yearly.

a How much interest will she receive in the first year?

b What is the total amount of interest Rebecca will receive over the 3 years?

c How much money will Rebecca have after the 3-year investment?

9 How much interest will Giorgio receive if he invests $7000 in stocks at 3.6% p.a. simple interest for 4 years?

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10 An investment of $15000 receives an interest payment over 3 years of $7200. What is the rate of simple interest per annum?

Substitute into the formula $I = PRN$ and solve the remaining equation.

11 Jonathon wishes to invest $3000 at 8% per annum. How long will he need to invest for his total investment to double?

12 Gretta wishes to invest some money for 5 years at 4.5% p.a., paid yearly. If she wishes to receive $3000 in interest payments per year, how much should she invest? Round your answer to the nearest dollar.

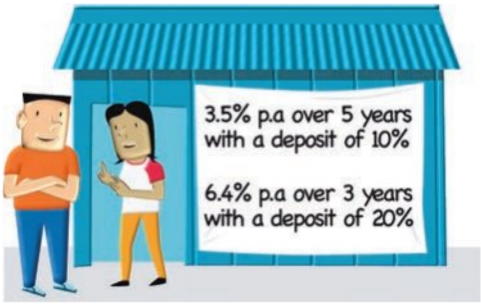
13 Jakob's interest payment on his loan totals $1875. If the interest rate was 5% p.a. and the loan had a life of 5 years, what amount did he borrow?

Enrichment: Which way is best?

14 A shed manufacturer offers the following finance.

- a rate of 3.5% p.a. paid at the end of 5 years with a deposit of 10%
- a rate of 6.4% p.a. repaid over 3 years with a deposit of 20%

Melania and Donald decide to purchase a shed for $12500.



a How much deposit will they need to pay in each case?

b What is the total interest they will pay in each case?

c If they decided to pay per month, what would be their monthly repayment?

d Discuss the benefits of the different types of purchasing methods.

In part **b**, don't forget to take off the deposit before calculating the interest.

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1F Compound interest

Stage

5.2
5.2◊
5.1
4

Simple interest is always calculated using the amount invested or borrowed, so the amount of interest earned or charged is the same every year.

In this section you will see that compound interest on an investment is calculated so that you earn interest on your interest.

Let's start: I want to double my money faster!

Drilling for Gold 1F1a 1F1

You may choose to download the 'Drilling for Gold' files to assist with this activity.

As noted in Section 1E, Sophie has $100 invested but she would like to have $200.

In this scenario, her investment increases every year by 10% of the **value at the end of the previous year**.

- By how much will her investment increase in the first year?
- How much will her investment be worth at the end of the first year?
- By how much will her investment increase in the second year?
- By how much will her investment increase in the third year?
- Copy and complete the following table.

Year	0	1	2	3	4	5	6	7	8	9	10
Amount	$100	$110									

- How long will it take Sophie to reach her goal?
- How long will it take if her investment increases by 5% every year?
- How long will it take if her investment increases by 7.5% every year?
- How long will it take if her investment increases by 5% every year for the first 5 years, then 6% every year thereafter?

Key ideas

Spreadsheet 1F1

- **Compound interest** is calculated on the current value of an investment (i.e. not the original value).
- The amount of simple interest stays the same for each period. In contrast, the amount of compound interest grows because you earn interest on your interest.

Compound interest A type of interest that is paid on a loan or earned on an investment, which is calculated not only on the initial principal, but also on the interest accumulated during the loan/investment period

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- The table below compares simple interest and compound interest on the same investment.

Simple Interest 6% p.a.			
Year	**Opening Balance**	**Interest**	**Closing Balance**
2012	\$5000.00	\$300.00	\$5300.00
2013	\$5300.00	\$300.00	\$5600.00
2014	\$5600.00	\$300.00	\$5900.00
2015	\$5900.00	\$300.00	\$6200.00
2016	\$6200.00	\$300.00	\$6500.00

Compound Interest 6% p.a.			
Year	**Opening Balance**	**Interest**	**Closing Balance**
2012	\$5000.00	\$300.00	\$5300.00
2013	\$5300.00	\$318.00	\$5618.00
2014	\$5618.00	\$337.08	\$5955.08
2015	\$5955.08	\$357.30	\$6312.38
2016	\$6312.38	\$378.74	\$6691.13

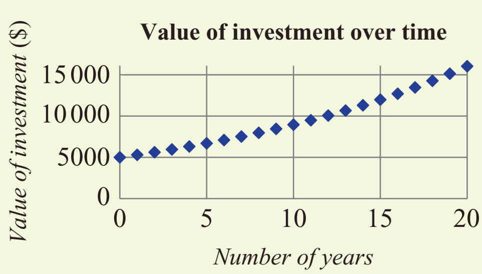
- For compound interest: the numbers in the right-hand table above can be generated on a calculator:

  Enter the number 5000, then press [=].
  100% plus 6% = 106% = 1.06, so multiply by 1.06.
  Press [=] [=] [=] [=] etc.

- The final value also can be calculated as 5000×1.06^5.

- To calculate the amount of compound interest earned, subtract the original value from the final value.
  In the example above:
  Amount of compound interest = \$6691.13 – \$5000 = \$1691.13

- The graph below shows an investment of \$5000 growing as it earns interest of 6% per annum, compounding annually. This is a non-linear relationship. The value of the investment grows exponentially.



- The final value of a compound interest investment can be calculated using the formula

$$A = P(1 + R)^n$$

where A = the final value of the investment
P = the principal (i.e. the amount invested)
R = the interest rate per period, expressed as a decimal
n = the number of compounding periods

- Amount of compound interest = $A - P$

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Exercise 1F

Understanding

1 Consider \$500 invested at 10% p.a., compounded annually.

- **a** How much interest is earned in the first year?
- **b** What is the balance of the account once the first year's interest is added?
- **c** How much interest is earned in the second year?
- **d** What is the balance of the account at the end of the second year?
- **e** Use your calculator to work out $500(1.1)^2$.

For the second year, you need to use \$500 plus the interest from the first year.

2 Find the value of the following, correct to 2 decimal places.

- **a** $\$1000 \times 1.05 \times 1.05$
- **b** $\$1000 \times 1.05^2$
- **c** $\$1000 \times 1.05 \times 1.05 \times 1.05$
- **d** $\$1000 \times 1.05^3$

3 Fill in the missing numbers.

- **a** \$700 invested at 8% p.a., compounded annually for 2 years.

$$A = \square(1.08)^{\square}$$

- **b** \$1000 invested at 15% p.a., compounded annually for 6 years.

$$A = 1000(\square)^6$$

- **c** \$850 invested at 6% p.a., compounded annually for 4 years.

$$A = 850(\square)^{\square}$$

15% as a decimal is 0.15.

Example 17 Converting rates and time periods

For the following, calculate the number of periods and the rates of interest offered per period.

- **a** 6% p.a. over 4 years, paid monthly
- **b** 18% p.a. over 3 years, paid quarterly

Solution	Explanation
a $n = 4 \times 12$ $= 48$	4 years is the same as 48 months since 12 months = 1 year.
$R = 6 \div 12 \div 100$ $= 0.005$	6% p.a. = 6% in 1 year Divide by 12 to find the monthly rate. Divide by 100 to convert the percentage to a decimal.
b $n = 3 \times 4$ $= 12$	There are 4 quarters in 1 year; hence, there are 12 quarters in 3 years.
$R = 18 \div 4 \div 100$ $= 0.045$	Divide by 100 to convert the percentage to a decimal.

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4 For the following, calculate the number of periods (n) and the rates of interest (R) offered per period. (Round the interest rate to 5 decimal places where necessary.)
- a 6% p.a. over 3 years, paid bi-annually
- b 12% p.a. over 5 years, paid monthly
- c 4.5% p.a. over 2 years, paid fortnightly
- d 10.5% p.a. over 3.5 years, paid quarterly
- e 15% p.a. over 8 years, paid quarterly
- f 9.6% p.a. over 10 years, paid monthly

'Bi-annually' means twice a year.
26 fortnights = 1 year

5 By considering an investment of $4000 at 5% p.a., compounded annually, copy and complete the table shown.

Year	Amount ($)	Interest ($)	New amount ($)
1	4000	200	4200
2	4200		
3			
4			
5			

Fluency

Example 18 Compounding annually

Determine the amount after 5 years when $4000 is compounded annually at 8% p.a.

Solution	Explanation
$P = 4000,\ n = 5,\ R = 0.08$	List the values for the terms you know.
$A = P(1 + R)^n$	Write the formula.
$= 4000(1 + 0.08)^5$	Substitute the values.
$= 4000(1.08)^5$	Simplify and evaluate, using a calculator.
$= \$5877.31$	Write your answer to the nearest cent.

Alternative method:
Enter 4000, then [=],
then [×] 1.08, — 100% + 8% = 108% = 1.08
then [=] [=] [=] [=] [=]. — Press [=] five times for 5 years.

6 Determine the amount after 5 years when:
- a $4000 is compounded annually at 5% p.a.
- b $8000 is compounded annually at 8.35% p.a.
- c $6500 is compounded annually at 16% p.a.
- d $6500 is compounded annually at 8% p.a.

$A = P(1 + R)^n$

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1F

7 Determine the amount when \$100 000 is compounded annually at 6% p.a. for:

a 1 year **b** 2 years **c** 3 years
d 5 years **e** 10 years **f** 15 years

Example 19 Compounding monthly

Skillsheet 1B

Tony's investment of \$4000 is compounded at 8.4% p.a. over 5 years. Determine the amount he will have after 5 years if the interest is paid monthly.

Solution	Explanation
$P = 4000$	List the values of the terms you know.
$n = 5 \times 12$ $= 60$	Convert the time in years to the number of periods (in this question, months). 60 months = 5 years.
$R = 8.4 \div 12 \div 100$ $= 0.007$	Convert the rate per year to the rate per period (i.e. months) by dividing by 12. Then divide by 100 to make a decimal.
$A = P(1 + R)^n$	Write the formula.
$= 4000(1 + 0.007)^{60}$	Substitute the values.
$= 4000(1.007)^{60}$ $= \$6078.95$	Simplify and evaluate.

8 Calculate the value of the following investments when interest is compounded monthly.

a \$2000 at 6% p.a. for 2 years
b \$34 000 at 24% p.a. for 4 years
c \$350 at 18% p.a. for 8 years
d \$670 at 6.6% p.a. for $2\frac{1}{2}$ years
e \$250 at 7.2% p.a. for 12 years

Convert years to months and the annual rate to the monthly rate.

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Problem-solving and Reasoning

9 Use your calculator to work out how long it will take for a $100 investment to double when the interest rate is:

a 4% per annum
b 6% per annum
c 8% per annum
d 12% per annum

10 a Calculate the amount of compound interest paid on $8000 at the end of 3 years for each rate below.

i 12% compounded annually
ii 12% compounded bi-annually (i.e. twice a year)
iii 12% compounded monthly
iv 12% compounded weekly
v 12% compounded daily

b What is the interest difference between annual and daily compounding in this case?

Remember:
1 year = 365 days

11 Saffira does the following calculation for a 5-year investment that she is considering: $3000(1.04)^{10}$.

a How much is she considering investing?
b How many times per year will the interest be compounded?
c What is the annual interest rate, as a percentage?
d At the end of the 5-year term, how much interest will she earn?

12 Paula must decide whether to invest her $13 500 for 6 years at 4.2% p.a. compounded monthly or 5.3% compounded bi-annually. Decide which investment would be the best for Paula.

Enrichment: Depreciation

A new car loses 15% in value every year. This is called depreciation. The formula is $A = P(1 - R)^n$.

13 A car worth $20 000 loses 15% in value every year. How much will it be worth at the end of 5 years?

Use $A = P(1 - R)^n$, where $R = 0.15$.

14 Explain why the car discussed in Question 13 will never have a value of $0.

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1G Investments and loans

Stage
5.2
5.2◊
5.1
4

When you invest money, the institution with which you invest (e.g. bank or credit union) pays you interest. However, when you borrow money, the institution from which you borrow charges you interest, so that you must pay back the money you initially borrowed, plus the interest.

Credit cards charge high rates of interest if the full amount owing is not paid off every month.

Let's start: Credit card statements

Refer to Allan's credit card statement below.

- How many days were there between the closing balance and the due date?
- What is the minimum payment due?
- If Allan pays only the minimum, on what balance is the interest charged?
- How much interest is charged if Allan pays $475.23 on 25/5?

Statement Issue Date: 2/5/16		
Date of purchase	**Details**	**Amount**
3/4/16	Opening balance	$314.79
5/4/16	Dean's Jeans	$59.95
16/4/16	Tyre Warehouse	$138.50
22/4/16	Payment made—thank you	–$100.00
27/4/16	Cottonworth's Grocery Store	$58.64
30/4/16	Interest charges	$3.35
2/5/16	Closing balance	$475.23
Percentage rate	**Due date**	**Min. payment**
18.95%	25/5/16	$23.75

Key ideas

- **Loans** (money borrowed) have interest charged to them on the amount owing (i.e. the balance).
- **Repayments** are amounts paid to the bank, usually each month, to repay a loan plus interest within an agreed time period.

Loan Money borrowed and then repaid, usually with interest

Repayment An amount paid to a financial institution at regular intervals to repay a loan, with interest included

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Exercise 1G

Understanding

1 Donna can afford to repay \$220 a month. How much does she repay over:
a 1 year? **b** 18 months? **c** 5 years?

2 Sarah buys a new bed on an 'interest free' offer. No interest is charged if she pays for the bed in 2 years. Sarah's bed costs \$2490 and she pays it back in 20 months in 20 equal instalments. How much is each instalment?

3 A bank pays 0.3% interest on the amount in an account. Determine the interest due on accounts with the following balances.
a \$400 **b** \$570 **c** \$1000 **d** \$29.55

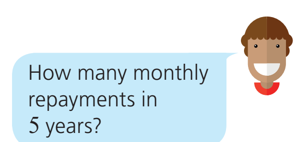
Fluency

Example 20 Repaying a loan

Wendy takes out a personal loan of \$7000 to pay for a holiday. Repayments are made monthly for 3 years at \$275 a month. Find:
a the total cost of Wendy's trip
b the interest charged on the loan

Solution	Explanation
a Total cost = \$275 × 36 = \$9900	3 years = 3 × 12 = 36 months Cost = 36 lots of \$275
b Interest = \$9900 − \$7000 = \$2900	Interest = total paid − amount borrowed

4 Jason has a personal loan of \$10 000. He is repaying the loan over 5 years. The monthly repayment is \$310.
a Calculate the total amount Jason repays over the 5-year loan.
b How much interest is he charged?



5 Robert borrows \$5500 to buy a second-hand car. He repays the loan in 36 equal monthly instalments of \$155.
a Calculate the total cost of the loan.
b How much interest does Robert pay?

6 Alma borrows \$250 000 to buy a house. The repayments are \$1736 a month for 30 years.
a How many repayments does Alma make?
b What is the total amount Alma pays for the house?
c How much interest is paid over the 30 years?

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1G

Example 21 Paying off a loan

Harry buys a $2100 computer on the following terms.

- 20% deposit
- monthly repayments of $90 for 2 years

Find:

a the deposit paid
b the total amount paid for the computer
c the interest charged

Solution	Explanation
a Deposit $= 0.2 \times 2100$ $= \$420$	Find 20% of 2100.
b Repayments $= \$90 \times 24$ $= \$2160$	2 years = 24 months Repay 24 lots of $90.
Total paid $= \$2160 + \$420$ $= \$2580$	Repay = repayments + deposit
c Interest $= \$2580 - \$2100$ $= \$480$	Interest = total paid – original price

7 Jorja buys a car for $12 750 on the terms 20% deposit and 36 monthly repayments of $295.

a Calculate the deposit.
b How much does Jorja owe after the deposit is paid?
c Find the total of all the repayments.
d Find the cost of buying the car on those terms.
e Find the interest Jorja pays on these terms.

Example 22 Calculating interest

An account has a balance of $200. The interest rate is 1.5% per annum.

a Determine the amount of interest to be credited at the end of the month.
b If the bank charges a fixed administration fee of $5 per month and other fees totalling $1.07, what will be the net amount credited or debited to the account at the end of the month?

Solution	Explanation
a Interest $= 1.5\%$ of $\$200 \div 12$ $= 0.015 \times 200 \div 12$ $= \$0.25$	Interest is 1.5% per month. Change 1.5% to a decimal and calculate.
b Net amount $= \$0.25 - \$5 - \$1.07$ $= -\$5.82$ $\therefore$ $5.82 will be debited from the account.	Subtract the deductions from the interest. A negative amount is called a debit.

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8 A savings account has a balance of $300 and interest is credited monthly at 1.5% per annum.

a Determine the amount of interest to be credited each month.

b If the bank charges a fixed administration fee of $3 per month and fees of $0.24, what will be the net amount debited from the account at the end of the month?

9 An investment account has no administration fee. The balances for May–October are shown in the table. If the interest payable on the minimum monthly balance is 4% per annum, how much interest will be added:

May	June	July	August
$4000	$5000	$6000	$7000

a for each separate month?   **b** over the 4-month period?

Problem-solving and Reasoning

10 Supersound offers two deals on a sound system worth $7500.

- Deal A: no deposit, interest free and nothing to pay for 18 months
- Deal B: 15% off for cash

15% off is 85% of the original amount.

a Thomas chooses deal A. Find:

i the deposit he must pay   **ii** the interest charged

iii the total cost if Thomas pays off the system within the 18 months

b Phil chooses deal B. What does Phil pay for the same sound system?

c How much does Phil save by paying cash?

11 Camden Finance Company charges 35% simple interest on all loans.

a Mei borrows $15 000 from Camden Finance over 6 years.

i Calculate the interest on the loan.

ii What is the total amount repaid (i.e. loan + interest)?

iii What is the value of each monthly repayment?

b Lancelle borrows $24 000 from the same company over 10 years.

i Calculate the interest on her loan.   **ii** What is the total amount repaid?

iii What is the value of each monthly instalment?

12 A list of transactions that Suresh made over a 1-month period is shown. The bank calculates interest *daily* at 0.01% and adds the total to the account balance at the end of this period. It has an administrative fee of $7 per month and other fees over this time total $0.35.

In part **b**, interest is calculated on the end-of-the-day balance.

a Copy the table and complete the balance column.

b Determine the amount of interest added over this month.

c Determine the final balance after all calculations have been made.

d Suggest what the regular deposits might be for.

Date	Deposit	Withdrawal	Balance
1 May			$3010
3 May	$490		
5 May		$2300	
17 May	$490		
18 May		$150	
20 May		$50	
25 May		$218	
31 May	$490		

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1G

13 The table below shows the interest and monthly repayments on loans when the simple interest rate is 8.5% p.a.

a Use the table to find the monthly repayments for a loan of:

i \$1500 over 2 years **ii** \$2000 over 3 years **iii** \$1200 over 18 months

b Damien and his wife Lisa can afford monthly repayments of \$60. What is the most they can borrow and on what terms?

	18-month term		24-month term		36-month term	
Loan amount (\$)	Interest (\$)	Monthly payments (\$)	Interest (\$)	Monthly payments (\$)	Interest (\$)	Monthly payments (\$)
1000	127.50	62.64	170.00	48.75	255.00	34.86
1100	140.25	68.90	187.00	53.63	280.50	38.35
1200	153.00	75.17	204.00	58.50	306.00	41.83
1300	165.75	81.43	221.00	63.38	331.50	45.32
1400	178.50	87.69	238.00	68.25	357.00	48.81
1500	191.25	93.96	255.00	73.13	382.50	52.29
1600	204.00	100.22	272.00	78.00	408.00	55.78
1700	216.75	106.49	289.00	82.88	433.50	59.26
1800	229.50	112.75	306.00	87.75	459.00	62.75
1900	242.25	119.01	323.00	92.63	484.50	66.24
2000	255.00	125.28	340.00	97.50	510.00	69.72

14 Part of a credit card statement is shown here.

Understanding your account		
CLOSING BALANCE \$403.80	MINIMUM PAYMENT DUE \$10.00	PAYABLE TO MINIMISE FURTHER INTEREST CHARGES \$403.80
CLOSING BALANCE This is the amount you owe at the end of the statement period.	MINIMUM PAYMENT DUE This is the minimum payment that must be made towards this account.	PAYABLE TO MINIMISE FURTHER INTEREST CHARGES This amount you must pay to minimise interest charges for the next statement period.

a What is the closing balance?

b What is due on the credit card if only the minimum payment is made on the due date?

c This credit card charges 21.9% p.a. interest, calculated daily on the unpaid balances. To find the daily interest, the company multiplies this balance by 0.0006. What does it cost in interest per day when only the minimum payment is made?

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Enrichment: Understanding a loan statement

15 Loans usually involve an establishment fee to set up the loan and an interest rate that is calculated monthly on your balance. You make a monthly or fortnightly payment, which reduces the balance. Bank fees also apply.

Consider the period for the loan statement shown below.

a What is the opening balance for this statement?
b What is the administrative fee charged by the bank for each transaction?
c What is the regular fee charged by the bank for servicing the loan?
d If the term of the loan is 25 years, what will be the total servicing fees charged by the bank?
e What is the regular fortnightly payment made?
f What will be the total fortnightly payments made over the term of the 25-year loan?

Complete Home Loan Transactions – Account number 33164000				
Date	**Transaction description**	**Debits**	**Credits**	**Balance**
	Balance brought forward from previous page			98 822.90 Dr
15 Oct	Repayment/Payment		378.50	
	Administrative fee	0.23		98 444.63 Dr
24 Oct	Interest charged	531.88		98 976.51 Dr
24 Oct	Fee for servicing your loan	8.00		98 984.51 Dr
29 Oct	Repayment/Payment		378.50	
	Administrative fee	0.23		98 606.24 Dr
12 Nov	Repayment/Payment		378.50	
	Administrative fee	0.23		98 227.97 Dr
24 Nov	Interest charged	548.07		98 776.04 Dr
24 Nov	Fee for servicing your loan	8.00		98 784.04 Dr
26 Nov	Repayment/Payment		378.50	
	Administrative fee	0.23		98 405.77 Dr
→	Change in interest rate on 03/12/15 to 06.800% per annum			
10 Dec	Repayment/Payment		378.50	
	Administrative fee	0.23		98 027.50 Dr
24 Dec	Interest charged	543.08		98 570.58 Dr
24 Dec	Fee for servicing your loan	8.00		98 578.58 Dr
24 Dec	Repayment/Payment		378.50	
	Administrative fee	0.23		98 200.31 Dr
31 Dec	Closing balance			98 200.31 Dr

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1H Using spreadsheets for investments, loans and depreciation

Stage

5.2
5.2◊
5.1
4

In this section you will be using spreadsheets to investigate and analyse loans and investments. Download the spreadsheets from 'Drilling for Gold'.

Let's start: Who earns the most?

- Ceanna invests $500 at 8% p.a., compounded monthly over 3 years.
- Huxley invests $500 at 10% p.a., compounded annually over 3 years.
- Loreli invests $500 at 15% p.a. simple interest over 3 years.
  - How much does each person have at the end of the 3 years?
  - Who earned the most?

Key ideas

- A **spreadsheet** is an excellent tool for investigating financial scenarios. It performs repetitive calculations quickly and accurately. It also makes it simple to explore what happens when interest rates change or loan repayments change.

Spreadsheet A table comprised of rows and columns for entering data

- A formula in a spreadsheet always begins with =.
  - For example, the formula =B3 + C3 will find the sum of the numbers in cells B3 and C3.
- The following table shows important things to know about spreadsheets.

Formula	Explanation
=C3/100	The symbol / is used for division, so this formula will divide the number in cell C3 by 100. This could be useful for converting percentages to decimals.
=C3/100*C2	The symbol * is used for multiplication, so this formula will divide C3 by 100 and then multiply the answer by the number in C2. This could be useful for calculating interest.
=C3^2	The symbol ^ is used for powers of 2, 3, 4 etc. This formula will give you the square of the number in cell C3.
=SUM(C4:C100)	This formula will find the sum of all the numbers in a column from C4 down to C100. This could be useful for finding the sum of the numbers in a column, such as the interest received every year for 20 years.

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Exercise 1H

Understanding

Spreadsheet 1H1

Note: Spreadsheets are provided to answer the questions in this exercise.

1 Which earns more on an investment of $100 for 2 years?
 A simple interest calculated at 5% p.a.
 B compound interest calculated at 5% p.a., compounded annually

2 Karen started the spreadsheet below.

	A	B	C	D
1	Year	Opening balance	Interest	Closing balance
2	1	5000		
3	2			
4	3			

 a In cell C2 she typed = 4/100*B2, then she pressed the 'Enter' key. What number should appear in cell C2?
 b Which formula could be used to give the value in cell D2?
  A D2 = B2 + C2
  B = B2 + C2
 c What value should the formula produce in cell D2?
 d The closing balance in Year 1 is also the opening balance in Year 2. What formula can be typed into cell B3 to make this happen?

3 Consider the spreadsheet shown below.

	A	B	C	D
1	Year	Opening balance	Interest	Closing balance
2	1	5000	300	5300
3	2	5300	300	5600

 a How can you tell that this spreadsheet is calculating simple interest, not compounding interest?
 b What is the rate of interest?
 c Fill in the blanks for the formula that could be in cell C2, using a number and a cell reference.

= 6 / ☐ * ☐

 d Write a formula for cell C3 and cell D3.

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1H

Fluency

Example 23 Using a spreadsheet

Find the total amount of the following investments, using technology.

a \$5000 at 5% p.a., compounded annually for 3 years
b \$5000 at 5% p.a. simple interest for 3 years

Solution	Explanation
a \$5788.13 **b** \$5750	Use the spreadsheets (provided) to compare the closing balances for Year 5.

4 **a** Use a spreadsheet to find the closing balance of the following investments.
 i \$6000 at 6% p.a., compounded annually for 3 years
 ii \$6000 at 3% p.a., compounded annually for 5 years
 iii \$6000 at 3.4% p.a., compounded annually for 4 years
 iv \$6000 at 10% p.a., compounded annually for 2 years
 v \$6000 at 5.7% p.a., compounded annually for 5 years
b Which of the investments above yields the most interest?

5 **a** Use a spreadsheet to find the closing balance of the following investments.
 i \$6000 at 6% p.a. simple interest for 3 years
 ii \$6000 at 3% p.a. simple interest for 6 years
 iii \$6000 at 3.4% p.a. simple interest for 7 years
 iv \$6000 at 10% p.a. simple interest for 2 years
 v \$6000 at 5.7% p.a. simple interest for 5 years
b Which of the above investments yields the most interest?

6 Cars depreciate by 15% per annum. Use the spreadsheet to find the value of a \$30 000 car at the end of the:
a first year
b second year
c third year
d tenth year

7 If you borrow \$30 000 at an interest rate of 12% per annum (i.e. 1% per month) and you agree to repay \$1000 per month, how much do you owe at the end of the:
a first month?
b second month?
c third month?
d tenth month?

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Problem-solving and Reasoning

8 **a** When a $30 000 car is depreciating by 15% per annum, how long does it take for the car to lose half of its value?
b When a $20 000 car is depreciating by 15% per annum, how long does it take for the car to lose half of its value?
c Try other values. Do all cars lose half of their value at the same time?

9 Steve is going to borrow $30 000 at an interest rate of 12% per annum (i.e. 1% per month). He agrees to repay $1000 per month.
a How many months will it take him to completely repay the loan and interest?
b How much interest will he pay on the loan?

10 Lauren takes out the same loan as Steve (see Question **9**) but she decides to repay $1200 per month. How much interest will Lauren save compared to Steve?

11 If the interest rate in Question **9** is 15% rather than 12%, how much extra interest will Steve pay on the loan?

Enrichment: $1000 per month versus $500 per fortnight

12 Adam and Bree both borrowed $30 000 at 12% per annum.
Adam's loan requires him to repay $1000 per month. The interest is calculated monthly.
Bree's loan requires her to repay $500 per fortnight. The interest is calculated fortnightly.
The spreadsheet you have been given is designed for Adam's loan. Modify the spreadsheet so it will model Bree's loan.

Spreadsheet 1H2

Puzzles and games

How do you stop a bull charging you? Answer the following problems and match the letters to the answers below to find out.

\$19.47 – \$8.53 E	5% of \$89 T	50% of \$89 I
$12\frac{1}{2}$% of \$100 A	If 5% = \$8.90 then 100% is? S	\$4.48 to the nearest 5 cents R
6% of \$89 W	Increase \$89 by 5%. H	10% of \$76 O
\$15 monthly for 2 years D	$12\frac{1}{2}$% as a decimal K	\$50 – \$49.73 U
Decrease \$89 by 5%. C	\$15.96 + \$12.42 Y	

\$28.38 \$7.60 27c

\$4.45 \$12.50 0.125 \$10.94

\$12.50 \$5.34 \$12.50 \$28.38

\$93.45 \$44.50 \$178

\$84.55 \$4.50 \$10.94 \$360 \$44.50 \$4.45

\$84.55 \$12.50 \$4.50 \$360

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Chapter summary

Financial mathematics

Conversions

Fraction → Percentage: $\times 100\%$

Percentage → Fraction: $\div 100\%$

Decimal → Percentage: $\times 100\%$

Percentage → Decimal: $\div 100\%$

Percentage work

15% of 459:

$= \frac{15}{100} \times 459$

Increase 20 by 8%:

$= 20 \times 108\%$

$= 20 \times 1.08$

Decrease 20 by 8%:

$= 20 \times 92\%$

$= 20 \times 0.92$

Profit = selling price − cost price

% Profit = $\frac{\text{profit}}{\text{cost price}} \times 100\%$

% Discount = $\frac{\text{discount}}{\text{cost price}} \times 100\%$

Percentages

'Out of 100'

79 per cent = 79% = $\frac{79}{100}$

Income

Time and a half = 1.5 × hourly rate

Double time = 2 × hourly rate

Commission is x % of total sales.

Gross income = total of all money earned

Net income = gross income – deductions

Simple (flat rate) interest

$I = PRN$

I = simple interest

P = principal ($ invested)

R = rate per year, as a decimal

N = number of years

Tax

Taxation = money given from wages (income tax) to the Government

Compound interest

$A = P(1 + R)^n$

A = final balance

P = principal ($ invested)

R = rate per time period, as a decimal

n = number of time periods

Loans

Balance owing = amount left to repay

Repayment = money given each month to repay the loan amount and the interest

Spreadsheets can be used to manage money or compare investments.

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Chapter review

Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.

Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

1 28% of $89 is closest to:
A $28.00 **B** $64.08 **C** $113.92
D $2492 **E** $24.92

2 As a percentage, $\frac{21}{60}$ is:
A 21% **B** 3.5% **C** 60%
D 35% **E** 12.6%

3 A price tag says '$55 (GST inc.)'. How much GST is included in the price?
A $62.50 **B** $50 **C** $49.50
D $5 **E** $5.50

4 The gross income for 30 hours at $15.78 per hour is:
A $105.78 **B** $1167.72 **C** $473.40
D $747.60 **E** $74.76

5 If Erin receives $11 496 commission on the sale of a property worth $783 200, her rate of commission, to 1 decimal place, was:
A 21% **B** 1.5% **C** 60%
D 15% **E** 12.6%

6 In a given rostered fortnight, Bill works the following number of 8-hour shifts:
- three day shifts ($21.20 per hour)
- three afternoon shifts ($24.68 per hour)
- five night shifts ($33.56 per hour)

His total income for the fortnight is:
A $305.44 **B** $2914.68 **C** $2000
D $337.68 **E** $2443.52

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Chapter review

7 A shirt is discounted by 26%. What is the price if it was originally \$56?
A \$14.56 **B** \$41.44 **C** \$26.56
D \$13.24 **E** \$35.22

8 A \$5000 loan is repaid by monthly instalments of \$200 for 5 years. The amount of simple interest charged is:
A \$300 **B** \$7000 **C** \$12 000
D \$2400 **E** \$6000

9 The simple interest on \$600 at 5% for 4 years is:
A \$570 **B** \$630 **C** \$120
D \$720 **E** \$30

10 The compound interest on \$4600 at 12% p.a. for 2 years is:
A \$1104 **B** \$5704 **C** \$4600
D \$5770.24 **E** \$1170.24

Short-answer questions

1 Find 15.5% of \$9000.

2 Increase \$968 by 12%.

3 Decrease \$4900 by 7%.

4 The cost price of an item is \$7.60. If the mark-up is 50%, determine:
a the retail price
b the profit made

5 An airfare of \$7000 is discounted 40% when you fly off-peak. What is the discounted price?

6 A couch is discounted to \$375. If this is a 35% discount, find the recommended retail price.

7 Fiona budgets 20% of her income for entertainment. If her yearly income is \$37 000, how much could be spent on entertainment in:
a a year?
b a month?
c a week?

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8 Mariah works a 34-hour week at $27.26 per hour. Her net income is 62% of her wage.
- **a** Work out Mariah's net income.
- **b** If 15% of her net income is spent on clothing, determine the amount Mariah can spend each week.
- **c** If Mariah saves $50 each week, what percentage (to 2 decimal places) of her gross weekly income is this?

9 Milan has the following costs to run his car.

hire purchase payment	$350 per month
registration	$685 per year
insurance	$315 per quarter
servicing	$1700 per year
petrol	$90 per week

- **a** Find the total cost of running his vehicle for 1 year.
- **b** What percentage (to the nearest per cent) of the overall cost to run the car is the cost of the petrol?

10 Tranh works 36 hours at $28.89 per hour. He pays $142.59 in tax and $22.50 in superannuation. Determine:
- **a** his gross wage
- **b** his net pay

11 Lily receives an annual salary of $47 842. Using the tax table shown, calculate the amount of tax she pays over the year.

Taxable income	Tax on this income
0 – $18 200	Nil
$18 201 – $37 000	19c for each $1 over $18 200
$37 001 – $80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001 – $180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

12 Pedro receives 4.5% commission on sales of $790. Determine the amount of his commission.

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Extended-response questions

1 $5000 is invested at 4% p.a., compounding annually for 3 years.
 a What is the value of the investment after the 3 years?
 b How much interest is earned in the 3 years?
 c How much interest is earned on the investment if it is compounded monthly at 4% p.a. for the 3 years?

2 A vehicle worth $7000 is purchased on a finance package. The purchaser pays 15% deposit and $250 per month over 4 years.
 a How much deposit is paid?
 b What is the total acount repaid?
 c How much interest is paid over the term of the loan?

3 Find the interest paid on a $5000 loan under the following conditions.
 a 8% p.a. simple interest over 4 years
 b 7% p.a. simple interest over 3 years and 4 months
 c 4% p.a. compounded annually over 3 years
 d 9.75% p.a. compounded annually over 2 years

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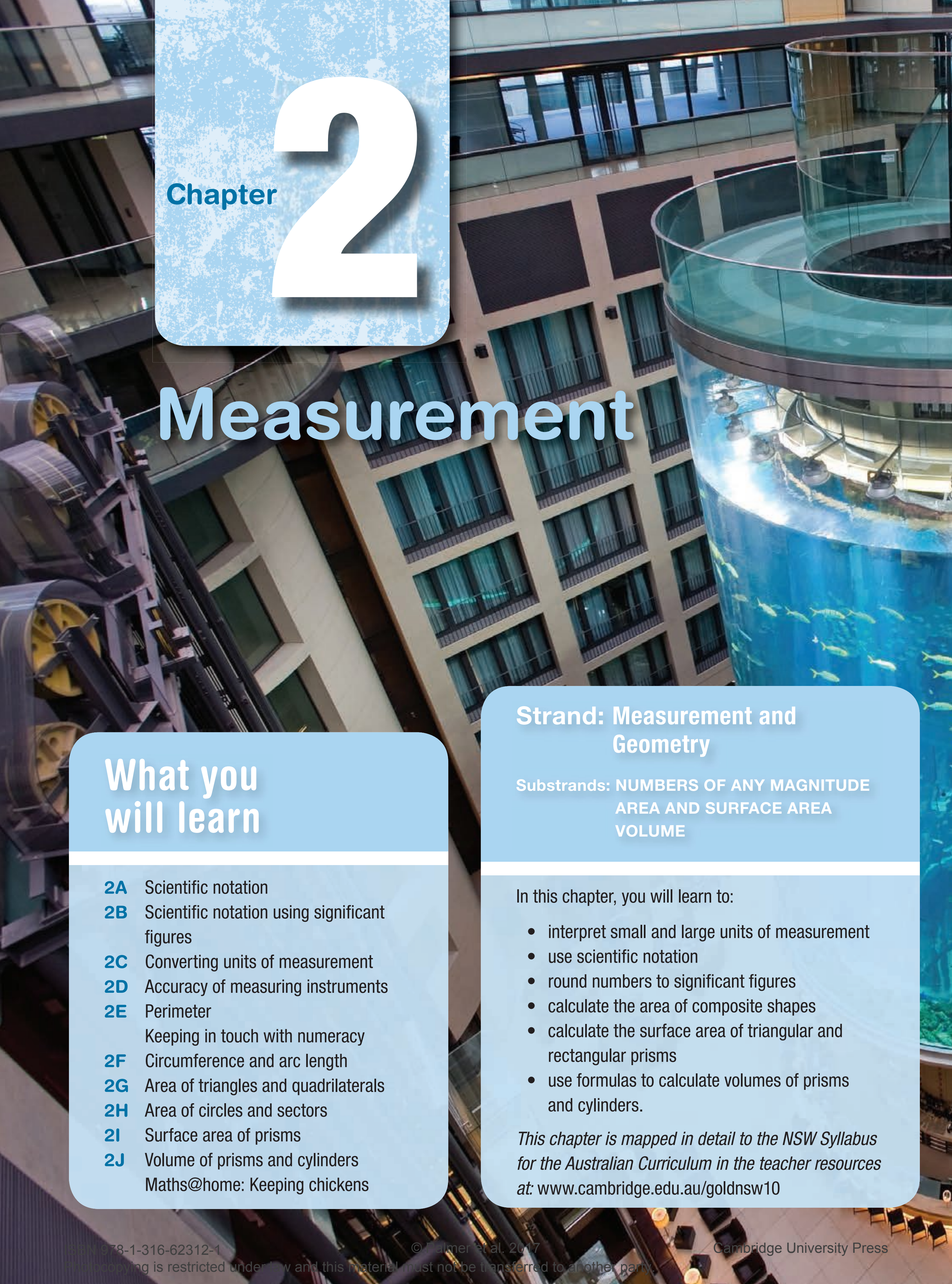

Chapter

2 Measurement

What you will learn

2A Scientific notation
2B Scientific notation using significant figures
2C Converting units of measurement
2D Accuracy of measuring instruments
2E Perimeter
Keeping in touch with numeracy
2F Circumference and arc length
2G Area of triangles and quadrilaterals
2H Area of circles and sectors
2I Surface area of prisms
2J Volume of prisms and cylinders
Maths@home: Keeping chickens

Strand: Measurement and Geometry

Substrands: NUMBERS OF ANY MAGNITUDE
AREA AND SURFACE AREA
VOLUME

In this chapter, you will learn to:

- interpret small and large units of measurement
- use scientific notation
- round numbers to significant figures
- calculate the area of composite shapes
- calculate the surface area of triangular and rectangular prisms
- use formulas to calculate volumes of prisms and cylinders.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw10

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World's largest cylindrical aquarium

Inside the Radisson SAS hotel in Berlin is the world's largest cylindrical aquarium. Some of its measurement facts include:

- Height: 25 m
- Diameter: 11 m
- Volume of sea water: 900 000 L
- Curved surface area: 864 m^2

The transparent casing is made from a special polymer that is very strong and can be made and delivered as one piece. Formulas are used to calculate the amount of polymer needed and the capacity of the cylinder.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO:* www.cambridge.edu.au/goldnsw10

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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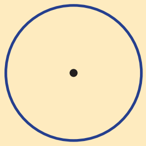
Pre-test

1 Name these shapes.

a


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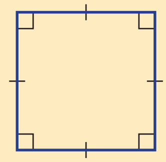
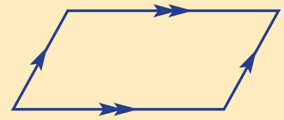


c


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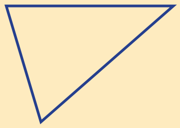

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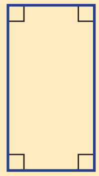
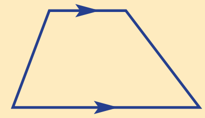
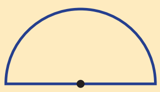
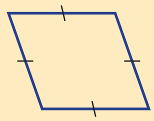


f


g


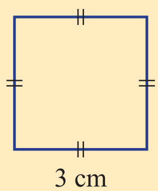
h


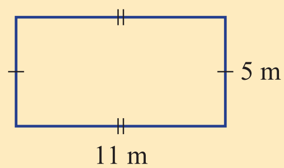
2 Write the missing number.

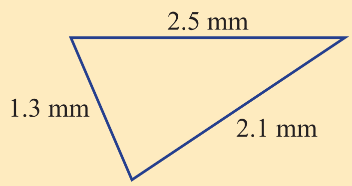
a 1 km = ☐ m **b** 1 m = ☐ cm **c** 1 cm = ☐ mm

d 1 L = ☐ mL **e** 0.5 km = ☐ m **f** 2.5 cm = ☐ mm

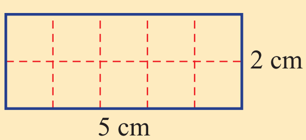
3 Find the perimeter of these shapes.

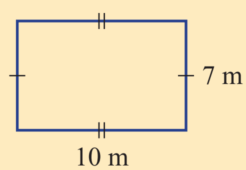
a


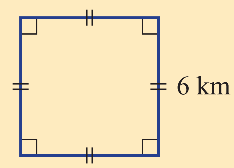
b


c


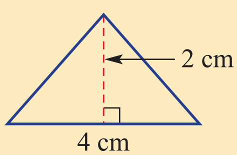
4 Find the area of these shapes.

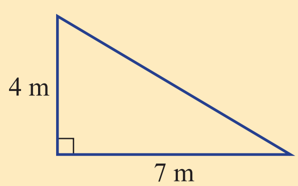
a


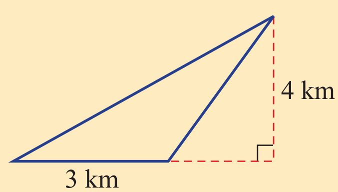
b


c


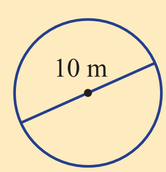
5 Find the area of these triangles using $A = \frac{1}{2}bh$.

a


b


c


6 Use $C = \pi d$ and $A = \pi r^2$ to find the circumference and area of this circle, correct to 2 decimal places.



$d = 10$ m, $r = 5$ m

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2A Scientific notation

Stage
5.2
5.2◊
5.1
4

Sometimes measurement involves very large or very small numbers. The amount of concrete used to build the Hoover Dam in the United States was 3 400 000 m^3 (cubic metres). The mass of a molecule of water is 0.0000000000000000000000299 grams. Numbers like these can be written using powers of 10 with positive or negative indices. This is called scientific notation.

At the time of construction, the Hoover Dam was the largest concrete structure in the world.

▶ Let's start: Power pattern

Drilling for Gold 2A1

Download the 'Drilling for Gold' document and complete the table. When you start with a very large number and then divide by 10 repeatedly, the results get closer and closer to zero but will never make it to zero. Note the use of negative powers for numbers between 0 and 1. You may not have seen negative powers prior to this.

↑↑↑ Multiplying by 10 ↑↑↑ (upwards) ↓↓↓ Dividing by 10 ↓↓↓ (downwards)

Powers of ten	Examples of scientific notation
$10^7 = 10\,000\,000$ = ten million	$\therefore$ 52 million $= 52\,000\,000 = 5.2 \times 10^7$
$10^6 = 1\,000\,000$ = one million	$\therefore$ 5 million $= 5\,000\,000 = 5 \times 10^6$
$10^5 = 100\,000$ = one hundred thousand	$\therefore 520\,000 = 5.2 \times 10^5$ and $600\,000 = 6 \times 10^5$
$10^4 = 10\,000$ = ten thousand	$\therefore 11\,000 = 1.1 \times 10^{\square}$ and $\therefore 10\,000 = 1 \times 10^{\square}$
$10^3 = 1000$ = one thousand	$\therefore 1500 = 1.5 \times 10^{\square}$
$10^2 = 100$ = one hundred	$\therefore 750 = 7.5 \times 10^{\square}$
$10^1 = 10$ = ten	Scientific notation is usually used only for numbers that contain many digits, such as those above, which are very large, and those below, which are close to zero.
$10^0 = 1$ = one	
$10^{-1} = 0.1$ = one tenth	
$10^{-2} = 0.01$ = one hundredth	
$10^{-3} = 0.001$ = one thousandth	$0.007 = 7 \times 10^{-3}$ and $0.0071 = 7.1 \times 10^{-3}$
$10^{-4} = 0.0001$ = one ten thousandth	$0.0007 = 7 \times 10^{\square}$ and $0.00071 = \square.\square \times 10^{\square}$
$10^{-5} = 0.00001$ = one hundred thousandth	$0.0000715 = \square.\square \times 10^{\square}$
$10^{-6} = 0.000001$ = one millionth	$0.000001234 = \square.\square \times 10^{\square}$

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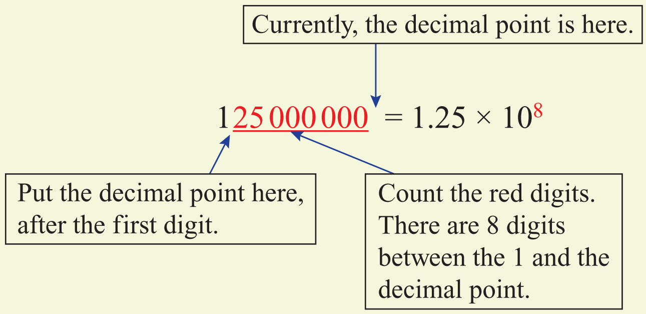
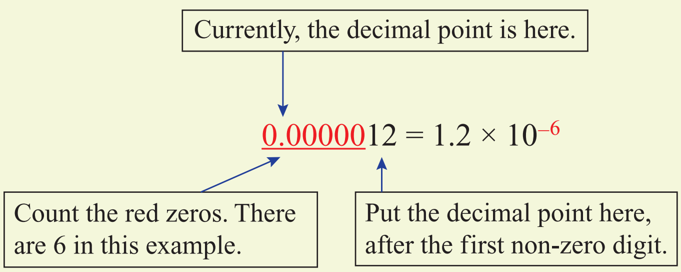
Key ideas

- **Scientific notation** is used to write numbers that contain many digits in a shorter form.
- Very large numbers, such as 125 million, can be written as 1.25×10^8, as follows:



Scientific notation A method used to express very large and very small numbers

- For numbers that are less than 1 and may be very close to zero, negative powers of 10 are used.
  – For example, the number 0.0000012, which is 'twelve millionths', could be written as 1.2×10^{-6}, as follows:



- Sometimes your calculator will display an answer in scientific notation. Try this on your calculator:
  – Enter 12 and then press [=].
  – Divide by 10, then press [=] [=] [=] [=] [=] and so on.
  – When the answer is too long to fit on the screen, your calculator will show you the answer in scientific notation. In this case, it will use negative powers of 10.
- Repeat the process above but multiply by 10 rather than dividing by 10.
- Your calculator might have a button like this [$\times 10^x$] or this [EXP]. This can be used to enter numbers into your calculator.
  – For example, for the number 1.2×10^{-6}, you could enter 1.2 [$\times 10^x$] –6. If the answer fits on the screen it will be displayed as a decimal. If not, it will be displayed in scientific notation.
- On some devices, such as a spreadsheet or mobile phone, the output of a calculation might include the character E or e, which stands for 'exponent'.
  – For example, if you calculate 2 to the power of 60, your calculator might display 1.15292… E+18. This could be written as 1.15×10^{18} (to 2 decimal places).
  It represents a very large number that starts with 115 and is followed by 16 digits.
  $1.15 \times 10^{18} = 1\,150\,000\,000\,000\,000\,000$

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Exercise 2A

Understanding

1 Copy and complete this table. The first row has been done for you.

Scientific notation	Power of 10 expanded	Basic numeral
5×10^3	5×1000	5 000
3×10^4		
2×10^5		
7×10^2		
		70 000
		400 000
		6000
		2 000 000

The power of 10 is equal to the number of zeros.

2 Copy and complete this table. The first row has been done for you.

Scientific notation	Positive power	Fraction	Basic numeral
2×10^{-4}	$\frac{2}{10^4}$	$\frac{2}{10\,000}$	0.0002
3×10^{-2}			
5×10^{-3}			
7×10^{-6}			
			0.009
			0.08
			0.0006
			0.00004

3 Which of the numbers 1000, 10 000 or 100 000 completes each equation?

a $6.2 \times$ ______ $= 62\,000$ **b** $9.41 \times$ ______ $= 9410$
c $1.03 \times$ ______ $= 103\,000$ **d** $3.2 \div$ ______ $= 0.0032$
e $5.16 \div$ ______ $= 0.0000516$ **f** $1.09 \div$ ______ $= 0.000109$

The number of zeros tells you how many places to move the decimal point.

4 If these numbers were written in scientific notation, would positive or negative indices be used?

a 2000 **b** 0.0004 **c** 19300 **d** 0.00101431

Fluency

Example 1 Writing large numbers in scientific notation

Write 4 500 000 in scientific notation.

Solution	Explanation
4 500 000 $= 4.5 \times 10^6$	Place the decimal point after the first digit (4). Count the number of digits after the 4. There are 6.

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2A

Drilling for Gold 2A2 at the end of this section

5 Write the following in scientific notation.

a 40000	**b** 230 000 000 000	**c** 16 000 000 000
d 7 200 000	**e** 3500	**f** 8 800 000
g 52 hundred	**h** 3 million	**i** 21 thousand

Large numbers: use 10 to a positive power.

Example 2 Writing small numbers in scientific notation

Write 0.0000004 in scientific notation.

Solution	Explanation
0.000 000 4	The first non-zero digit is 4.
$= 4 \times 10^{-7}$	Count the number of zeros before the 4.
	There are 7.

6 Write the following in scientific notation.

a 0.000003	**b** 0.0004	**c** 0.00876
d 0.00000000073	**e** 0.00003	**f** 0.000000000125
g 0.00000000809	**h** 0.000000024	**i** 0.0000345

Small numbers: use 10 to a negative power.

7 Write each of the following numbers in scientific notation.

a 6000	**b** 720 000	**c** 324.5	**d** 7869.03
e 8459.12	**f** 0.2	**g** 0.000328	**h** 0.00987
i 0.00001	**j** 460 100 000	**k** 17 467	**l** 128

Place the decimal point after the first non-zero digit.

Example 3 Writing basic numerals from positive powers

Express 9.34×10^6 as a basic numeral.

Solution	Explanation
9.34×10^6	We need 6 digits after the 9, so add 4 zeros.
= 9 340 000	

8 Express each of the following as a basic numeral.

a 5.7×10^4	**b** 3.6×10^6	**c** 4.3×10^8
d 3.21×10^7	**e** 4.23×10^5	**f** 9.04×10^{10}
g 1.97×10^8	**h** 7.09×10^2	**i** 6.357×10^5

Move the decimal point right to increase the place value of each digit.

Example 4 Writing basic numerals from negative powers

Express 4.71×10^{-6} as a basic numeral.

Solution	Explanation
4.71×10^{-6}	We need 6 zeros before the 4.
= 0.000 004	

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Skillsheet 2A

9 Express each of the following as a basic numeral.

a 1.2×10^{-4} **b** 4.6×10^{-6} **c** 8×10^{-10}
d 3.52×10^{-5} **e** 3.678×10^{-1} **f** 1.23×10^{-7}
g 9×10^{-5} **h** 5×10^{-2} **i** 4×10^{-1}

Move the decimal point left to decrease the place value of each digit.

Problem-solving and Reasoning

10 Express each of the following approximate numbers in scientific notation.

a The area of Australia is about 7 700 000 km^2 (square kilometres).
b The circumference of Earth is 40 000 000 m.
c The diameter of a gold atom is 0.0000000001 m.
d The radius of Earth's orbit around the Sun is 150 000 000 km.
e The universal constant of gravitation is 0.0000000000667 N m^2/kg^2.
f The half-life of polonium-214 is 0.00015 seconds.
g Uranium-238 has a half-life of 4 500 000 000 years.

11 Express each of the following numbers as a basic numeral.

a Neptune is approximately 4.6×10^9 km from Earth.
b A population of bacteria contains 8×10^{12} organisms.
c Earth is approximately 3.84×10^5 km from the Moon.
d A fifty-cent coin is approximately 3.8×10^{-3} m thick.
e The diameter of the nucleus of an atom is approximately 1×10^{-14} m.
f The population of a city is 7.2×10^5.


Earth is about 3.84×10^5 km from the Moon.

12 Write the answers to each of these problems in scientific notation.

a Two planets are 2.8×10^8 km and 1.9×10^9 km from their closest sun. What is the difference between these two distances?
b Two particles weigh 2.43×10^{-2} g and 3.04×10^{-3} g. Find the difference in their masses.

Look at the calculator instructions in Key ideas.

Enrichment: NOT scientific notation!

13 The number 47×10^4 is not written in scientific notation, since 47 is not a number between 1 and 10. The following shows how to convert to scientific notation.

$$\begin{aligned} 47 \times 10^4 &= 4.7 \times 10 \times 10^4 \\ &= 4.7 \times 10^5 \end{aligned}$$

Write these numbers in scientific notation.

a 32×10^3 **b** 41×10^5 **c** 0.13×10^5 **d** 0.092×10^3
e 61×10^{-3} **f** 424×10^{-2} **g** 0.02×10^{-3} **h** 0.0004×10^{-2}

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Drilling for Gold exercise

2A2: Convert me!

The left-hand column contains numbers expressed in scientific notation.
The right-hand column contains numbers.
Match the left (**1–12**) column with the right (**A–L**) column by writing 1K, 2E etc. in your exercise book or the worksheet.

1	1.2×10^8
2	1.23×10^8
3	1.2×10^6
4	1.23×10^{10}
5	1.2×10^{12}
6	1.23×10^2
7	1.2×10^{-5}
8	1.23×10^{-8}
9	1.2×10^5
10	1.23×10^{-2}
11	1.2×10^{-10}
12	1.23×10^{-10}

A	1 200 000
B	123
C	0.0000000123
D	120 000 000
E	12 300 000 000
F	0.000012
G	120 000
H	0.000000000123
I	0.00000000012
J	0.0123
K	1 200 000 000 000
L	123 000 000

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2B Scientific notation using significant figures

Stage
5.2
5.2◊
5.1
4

The volume of Earth has been calculated as 1 083 210 000 000 km^3. This can be written in scientific notation as 1.08321×10^{12}.

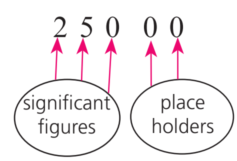
The mass of a single oxygen molecule is known to be 0.000000000000000000000000053 g. This is written in scientific notation as 5.3×10^{-26}.

The accuracy of a measurement of the volume of Earth depends in part on the number of significant figures.

▶ Let's start: How many people in the ground?

The officials at a football match counted the people who entered the stadium, then rounded off to the nearest hundred. They announced, 'The attendance today is 25 000.'

- How many people might have been in the stadium?
- What is the smallest possible number of people?
- What is the largest possible number of people?



In this example, the officials are using 3 significant figures.

Key ideas

- **Significant figures** are counted from left to right, starting at the first non-zero digit.
  – Zeros on the end of a decimal are definitely significant.
  – Zeros on the end of a whole number might be significant.

  For example:

  0.0025 definitely has 2 significant figures.
  0.00250 definitely has 3 significant figures.
  0.205 definitely has 3 significant figures.
  6.0 definitely has 2 significant figures.
  66 definitely has 2 significant figures.
  60 might have 1 or 2 significant figures.
  66 000 might have 2, 3, 4 or 5 significant figures.

- When using scientific notation, the first significant figure sits to the left of the decimal point.
  For example: 3.210×10^4 has 4 significant figures.

Significant figure A digit that indicates how accurate a number is

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Exercise 2B

Understanding

1 a Round each of these numbers to the nearest hundred.
- **i** 267
- **ii** 32 740
- **iii** 18 350

b Round each of these numbers to the nearest tenth.
- **i** 0.063
- **ii** 0.1902
- **iii** 21.04

c Round each of these numbers to the nearest thousand.
- **i** 267 540
- **ii** 38 290
- **iii** 4 060 990

Rounding rules:
Locate the first digit to the right of the required digit.
- Round down (leave it as it is) for a 4 or less.
- Round up (increase by 1) for a 5 or more.

2 Which of these numbers definitely has 2 significant figures?
62 000, 30 500, 420,
0.0071, 0.0805, 201 000

Note that 204 has 3 significant figures but 240 might have 2 or 3 significant figures.

Drilling for Gold 2B1 at the end of this section

3 Copy and complete the tables, rounding each number to the given number of significant figures.

a 57 263

Significant figures	Rounded number
4	
3	57 300
2	
1	

b 4 170 162

Significant figures	Rounded number
5	
4	
3	4 170 000
2	
1	

57 263
6 is the 4th significant figure, so round to the nearest 10 for 4 significant figures.

c 0.0036612

Significant figures	Rounded number
4	
3	
2	
1	0.004

d 24.8706

Significant figures	Rounded number
5	
4	
3	
2	25
1	

24.8706
0 is the 5th significant figure, so round to the nearest thousandth for 5 significant figures.

4 Are the following numbers written in scientific notation with 3 significant figures? (Answer yes or no.)

a 4.21×10^4 **b** 32×10^{-3} **c** 1800×10^6
d 0.04×10^2 **e** 1.89×10^{-10} **f** 9.04×10^{-6}
g 5.56×10^{-14} **h** 0.213×10^2 **i** 26.1×10^{-2}

The digit to the left of the decimal point must be 1, 2, 3, 4, 5, 6, 7, 8 or 9.

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Fluency

Example 5 Stating the number of significant figures

State the number of significant figures given in these numbers.

a 401 **b** 0.005012 **c** 3.2×10^7 **d** 125 000

Solution	Explanation
a 3 significant figures	All the digits are significant.
b 4 significant figures	Start counting at the first non-zero digit (5).
c 2 significant figures	With scientific notation, the first significant figure is to the left of the decimal point.
d 3, 4, 5 or 6 significant figures	The zeros on the end of a whole number may or may not be significant.

5 State the number of significant figures given in these numbers.

a 27 200 **b** 1007 **c** 301 010
d 190 **e** 0.0183 **f** 0.20
g 0.706 **h** 0.00109 **i** 4.21×10^3
j 2.905×10^{-2} **k** 1.07×10^{-6} **l** 5.90×10^5

The zeros on the end of a whole number may or may not be significant.

Example 6 Writing numbers in scientific notation using significant figures

Write these numbers in scientific notation, using 3 significant figures.

a 2 183 000 **b** 0.0019482

Solution	Explanation
a $2\,183\,000 = 2.18 \times 10^6$	Put the decimal point after the first non-zero digit (2). There are 6 digits after the 2. Round the third significant figure down (i.e. leave it as it is) since the following digit (3) is less than 5.
b $0.0019482 = 1.93 \times 10^{-3}$	There are 3 zeros before the 1. Round the third significant figure up to 5 since the following digit (8) is greater than 4.

6 Write these numbers in scientific notation, using 3 significant figures.

a 242 300 **b** 171 325 **c** 2829 **d** 3 247 000
e 0.00034276 **f** 0.006859 **g** 0.01463 **h** 0.001031

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2B

7 Write each number in scientific notation, rounding to the number of significant figures given in the brackets.

a 47 760 (3) **b** 21 610 (2) **c** 4 833 160 (4)

d 2 716 000 (2) **e** 0.0002716 (2) **f** 0.0002796 (2)

g 0.00201 (1) **h** 0.08516 (1) **i** 0.0001010 (1)

First round the number to the required number of significant figures.

Example 7 Using a calculator with scientific notation

Use a calculator to evaluate $3.67 \times 10^5 \times 23.6 \times 10^4$. Leave your answer in scientific notation, correct to 4 significant figures.

Solution	Explanation
$3.67 \times 10^5 \times 23.6 \times 10^4$ $= 8.661 \times 10^{10}$	Use the $\boxed{\times 10^x}$ or $\boxed{\text{EXP}}$ button. Write in scientific notation with 4 significant figures.

8 Use a calculator to evaluate each of the following. Leave your answers in scientific notation, correct to 4 significant figures.

a 4^{-6} **b** 78^{-3} **c** $(-7.3 \times 10^{-4})^{-5}$

d $\frac{3.185}{7 \cdot 10^4}$ **e** $2.13 \times 10^4 \times 9 \times 10^7$ **f** $5.671 \times 10^2 \times 3.518 \times 10^5$

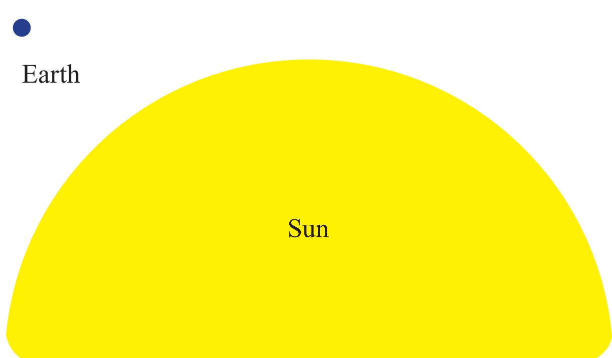
g $9.419 \times 10^5 \times 4.08 \times 10^{-4}$ **h** $2.85 \times 10^{-9} \times 6.33 \times 10^{-3}$ **i** $12\,345^2$

j 87.14^8 **k** $\frac{1.83 \cdot 10^{26}}{4.5 \cdot 10^{22}}$ **l** $\frac{-4.7 \times 10^{-2} \times 6.18 \times 10^7}{3.2 \times 10^6}$

Problem-solving and Reasoning

9 The mass of Earth is approximately 6 000 000 000 000 000 000 000 000 kg. The mass of the Sun is 330 000 times the mass of Earth. Find the mass of the Sun. Express your answer in scientific notation, correct to 3 significant figures.

10 The diameter of Earth is approximately 12 756 000 m. If the Sun's diameter is 109 times that of Earth, compute its diameter in kilometres. Express your answer in scientific notation, correct to 3 significant figures.



The size of the Sun and Earth compared (distance of Earth to Sun is not to scale).

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11 Write these numbers from largest to smallest.
2.41×10^6, 24.2×10^5, 0.239×10^7, 2421×10^3, 0.02×10^8

First write each number in scientific notation.

12 The following output is common on a number of different calculators and computers. Write down the number that you think they represent.

a 4.26E6 **b** 9.1E-3 **c** 5.04EXP11
d 1.931EXP-1 **e** 2.1^{06} **f** 6.14^{-11}

Enrichment: Combining bacteria

13 A flask of type A bacteria contains 5.4×10^{12} cells and a flask of type B bacteria contains 4.6×10^8 cells. The two types of bacteria are combined in the same flask.

Set up a table to show the number of each type of bacteria after every 8 hours.

a How many bacterial cells are there in the flask?
b If type A bacterial cells double every 8 hours and type B bacterial cells triple every 8 hours, how many cells are in the flask after:
  i 8 hours?
  ii 1 day?

Express your answers to part **b** in scientific notation, correct to 3 significant figures.

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Drilling for Gold exercise

2B1: I am significant

Round off the given numbers to the required number of significant figures.
Use the worksheet or write the answers in your exercise book.
The first one in each table has been done for you.

	Round to 4 significant figures			Round to 3 significant figures	
1	54.2783	54.28	1	54.2783	54.3
2	765.432		2	765.432	
3	3.14159		3	3.14159	
4	34.97245		4	34.97245	
5	0.285714		5	0.285714	
6	0.034567		6	0.034567	
7	1487.56		7	1487.56	
8	25 190		8	25 190	
9	105 105		9	105 105	
10	109.999999		10	109.999999	

	Round to 2 significant figures			Round to 1 significant figure	
1	54.2783	54	1	54.2783	50
2	765.432		2	765.432	
3	3.14159		3	3.14159	
4	34.97245		4	34.97245	
5	0.285714		5	0.285714	
6	0.034567		6	0.034567	
7	1487.56		7	1487.56	
8	25 190		8	25 190	
9	105 105		9	105 105	
10	109.999999		10	109.999999	

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2C Converting units of measurement

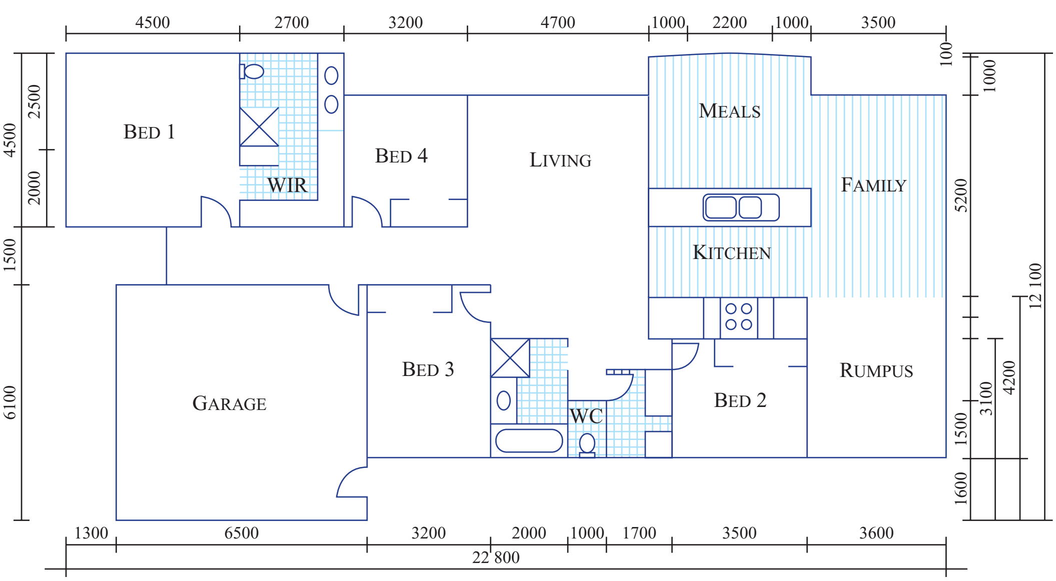
Stage
5.2
5.2◊
5.1
4

Timber is often used in buildings for frames and roof trusses. To minimise costs it is important to order the correct amount of timber. Building plans give measurements in millimetres. Builders often need to convert between different units of measurement.

Building a house also involves many area and volume calculations and conversions.

▶ Let's start: House plans

All homes start from a plan, which is usually designed by an architect and shows most of the basic features and measurements that are needed to build the house. Measurements are given in millimetres.



- How many bedrooms are there?
- What are the dimensions of the master bedroom (BED 1), in millimetres?
- What are the dimensions of the master bedroom, in metres?
- Will the rumpus room fit a pool table that measures 2.5 m × 1.2 m and still have room to play?
- Will 3 cars fit in the garage?

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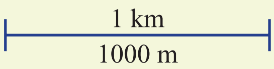
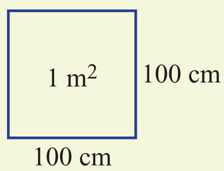
Key ideas

Drilling for Gold 2C1 at the end of this section 2C2 2C3

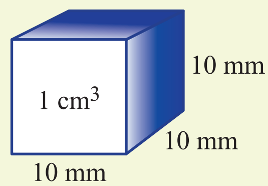
- To convert units, draw an appropriate diagram and use it to find the conversion factor. For example:



1 km = 1000 m

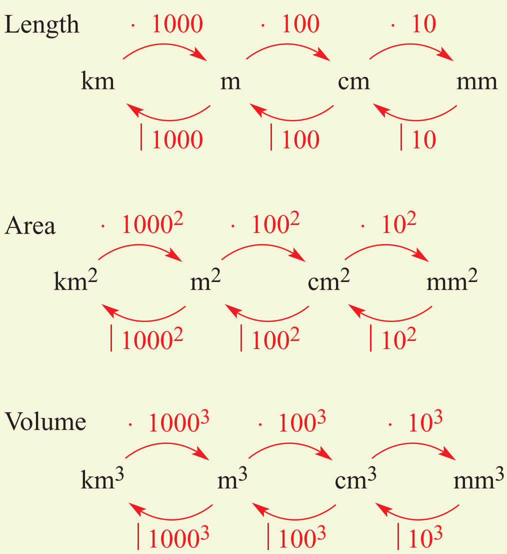


$1\ \text{m}^2 = 100 \times 100$
$= 10\,000\ \text{cm}^2$



$1\ \text{cm}^3 = 10 \times 10 \times 10$
$= 1000\ \text{mm}^3$

- Conversions:



- To multiply by 10, 100, 1000 etc. move the decimal point one place to the right for each zero; e.g. $3.425 \times 100 = 342.5$

- To divide by 10, 100, 1000 etc. move the decimal point one place to the left for each zero; e.g. $4.10 \div 1000 = 0.0041$

- Metric prefixes in everyday use.

Prefix	Symbol	Factor of 10	Standard form	
tera	T	1 000 000 000 000	10^{12}	1 trillion
giga	G	1 000 000 000	10^9	1 billion
mega	M	1 000 000	10^6	1 million
kilo	k	1000	10^3	1 thousand
hecto	h	100	10^2	1 hundred
deca	da	10	10	1 ten
deci	d	0.1	10^{-1}	1 tenth
centi	c	0.01	10^{-2}	1 hundredth
milli	m	0.001	10^{-3}	1 thousandth
micro	μ	0.000001	10^{-6}	1 millionth
nano	n	0.000000001	10^{-9}	1 billionth

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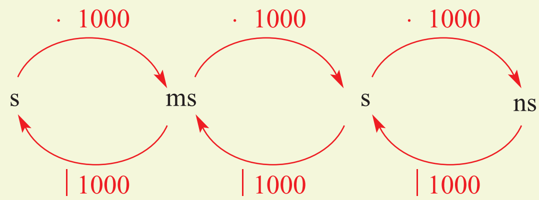
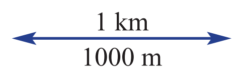
For time conversions, this can be represented on a flow diagram similar to those for length, area and volume.
s – second
ms – millisecond
μs – microsecond
ns – nanosecond



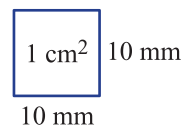
Exercise 2C

Understanding

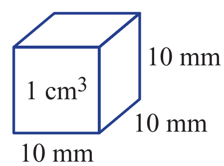
1 Write the missing numbers in these sentences involving length.
 a There are ☐ m in 1 km.
 b There are ☐ mm in 1 cm.
 c There are ☐ cm in 1 m.



2 Write the missing numbers in these sentences involving area units.
 a There are ☐ mm^2 in 1 cm^2.
 b There are ☐ cm^2 in 1 m^2.
 c There are ☐ m^2 in 1 km^2.



3 Write the missing numbers in these sentences involving volume units.
 a There are ☐ mm^3 in 1 cm^3.
 b There are ☐ m^3 in 1 km^3.
 c There are ☐ cm^3 in 1 m^3.

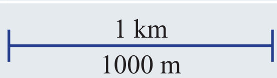
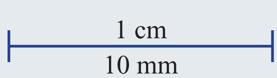


4 Match the units of measurement with the correct abbreviation.
nanosecond milligram millimetre gigalitre millisecond microsecond
 a mm **b** mg **c** GL
 d ms **e** μs **f** ns

Fluency

Example 8 Converting length measurements

Convert these length measurements to the units shown in brackets.
a 8.2 km (m) **b** 45 mm (cm)

Solution	**Explanation**
a $8.2 \text{ km} = 8.2 \times 1000 \text{ m}$ $= 8200 \text{ m}$	 1 km = 1000 m
b $45 \text{ mm} = 45 \div 10 \text{ cm}$ $= 4.5 \text{ cm}$	 1 cm = 10 mm Divide if converting from a smaller unit to a larger unit.

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2C

5 Convert the following measurements of length to the units given in brackets.

a 4.32 cm (mm)
b 327 m (km)
c 834 cm (m)
d 0.096 m (mm)
e 297.5 m (km)
f 0.0127 m (cm)

When converting to a smaller unit, multiply. Otherwise, divide.

Example 9 Converting other units

Convert the following.

a 3 minutes to microseconds
b 4 000 000 000 mg to t

Solution	Explanation
a 3 minutes = 180 s	1 minute = 60 seconds ($3 \times 60 = 180$)
$= 180 \times 10^6$ μs	1 second = 1 000 000 microseconds ($180 \times 1\,000\,000$)
$= 1.8 \times 10^8$ μs	Express the answer in scientific notation.
b 4 000 000 000 mg = 4 000 000 g	mg means milligrams. 1000 mg = 1 gram
= 4000 kg	1000 g = 1 kg
= 4 t	1000 kg = 1 tonne (t)
	Dividing by the conversion factor converts a small unit to a larger unit as there are less of them.

6 Convert the following.

a 7 kg to g
b 7000 m to km
c 15 Mt to t
d 4 kW to W (watts)
e 8900 t to Mt
f 5 ns to s
g 0.6 g to μg
h 600 s to min
i 1285 s to ms
j 680 t to Mt
k 40 000 000 μm to cm
l 8 GB to B (bytes)
m 8500 ms to s
n 3 000 000 000 ns to s
o 9000 mg to g

Use the table in Key ideas.

The *Salmonella* bacterium, which is a common cause of food poisoning, is so small that it is measured in micrometres (μm).

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Example 10 Converting area measurements

Convert these area measurements to the units shown in brackets.

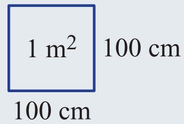
a 930 cm^2 (m^2) **b** 0.4 cm^2 (mm^2)

Solution

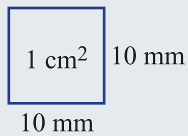
a 930 cm^2 = 930 ÷ 10 000 m^2
= 0.093 m^2

b 0.4 cm^2 = 0.4 × 100 mm^2
= 40 mm^2

Explanation



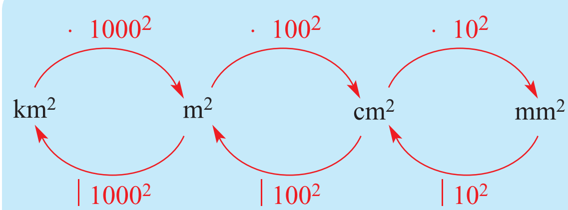
1 m^2 = 100 × 100 cm^2
= 10 000 cm^2



1 cm^2 = 10 × 10 mm^2
= 100 mm^2

7 Convert the following area measurements to the units given in brackets.

a 3000 cm^2 (mm^2)
b 0.5 m^2 (cm^2)
c 5 km^2 (m^2)
d 2 980 000 mm^2 (cm^2)
e 537 cm^2 (mm^2)
f 0.023 m^2 (cm^2)



Example 11 Converting volume measurements

Convert these volume measurements to the units shown in brackets.

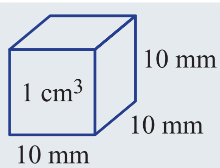
a 3.72 cm^3 (mm^3) **b** 4300 cm^3 (m^3)

Solution

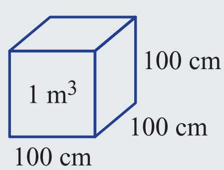
a 3.72 cm^3 = 3.72 × 1000 mm^3
= 3720 mm^3

b 4300 cm^3 = 4300 ÷ 1 000 000 m^3
= 0.0043 m^3

Explanation



1 cm^3 = 10 × 10 × 10 mm^3
= 1000 mm^3



1 m^3 = 100 × 100 × 100 cm^3
= 1 000 000 cm^3

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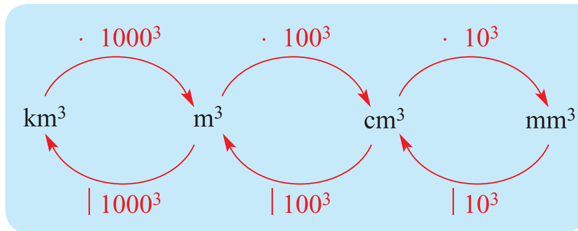
2C

8 Convert these volume measurements to the units given in brackets.

a 2 cm^3 (mm^3)
b 0.2 m^3 (cm^3)
c 5700 mm^3 (cm^3)
d 0.015 km^3 (m^3)
e 28 300 000 m^3 (km^3)
f 762 000 cm^3 (m^3)

Skillsheet 2B

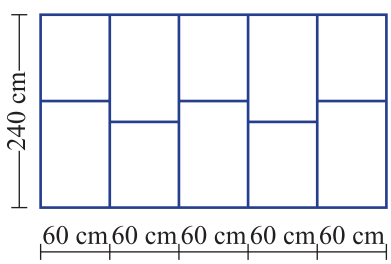


Problem-solving and Reasoning

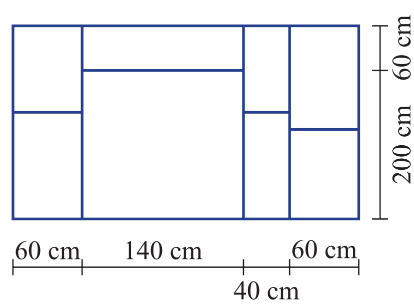
9 An athlete has completed a 5.5 km run. How many metres did the athlete run?

10 Determine the metres of timber needed to construct the following frames.

a



b



11 Find the total sum of the measurements given, expressing your answer in the units given in brackets.

a 10 cm, 18 mm (mm)
b 1.2 m, 19 cm, 83 mm (cm)
c 453 km, 258 m (km)
d 400 mm^2, 11.5 cm^2 (cm^2)
e 0.3 m^2, 251 cm^2 (cm^2)
f 0.00003 km^2, 9 m^2, 37 000 000 cm^2 (m^2)
g 482 000 mm^3, 2.5 cm^3 (mm^3)
h 0.00051 km^3, 27 300 m^3 (m^3)

Convert to the units in brackets. Add up to find the sum.

12 A snail is moving at a rate of 43 mm every minute. How many centimetres will the snail move in 5 minutes?

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13 Why do you think that builders measure using only millimetres?

14 How many 4 kB files can fit onto an 8 GB USB stick?

1 GB = 1 000 000 000 bytes

15 An Olympic sprinter places second in the 100 metres, with the time 10.45 seconds. If this athlete was beaten by 2 milliseconds, what is the winning time for the race?

Enrichment: File size

16 Emily has photos of her recent weekend away stored on her computer. The files have the following sizes: 1.2 MB, 171 KB, 111 KB, 120 KB, 5.1 MB and 2.3 MB. (Note that some computers use KB instead of kB in their information on each file.)

a What is the total size of the photos of her weekend, in kilobytes?

b What is the total, in megabytes?

c Emily wishes to email these photos to her mum. However, her mum's file server can only receive email attachments no bigger than 8 MB. Is it possible for Emily to send all of her photos from the weekend in one email?

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Drilling for Gold exercise

2C1: Units of measurement and their abbreviations

The abbreviations in column 3 are mixed up.
For each unit of measurement, write the correct abbreviation in column 2. Use the worksheet or write the answers in your exercise book.
Don't forget to practise spelling the words in column 1.

Column 1	Column 2	Column 3
Unit of measurement – length	Abbreviation	
1 kilometres		t
2 metres		kg
3 centimetres		cm^2
4 millimetres		mg
Unit of measurement – mass	Abbreviation	
5 tonnes		m^3
6 kilograms		mm^3
7 grams		m^2
8 milligrams		mm
Unit of measurement – volume	Abbreviation	
9 cubic metres		s
10 cubic centimetres		km^2
11 cubic millimetres		km
Unit of measurement – capacity	Abbreviation	
12 kilolitres		mm^2
13 litres		kL
14 millilitres		m
Unit of measurement – time	Abbreviation	
15 years		ha
16 days		h
17 hours		g
18 minutes		L
19 seconds		d
Unit of measurement – area	Abbreviation	
20 square kilometres		cm
21 hectares		mL
22 square metres		min
23 square centimetres		Y
24 square millimetres		cm^3

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Stage
5.2
5.2◊
5.1
4

2D Accuracy of measuring instruments

All measurements are approximate. Errors can come from the equipment being used or the person using the measuring device. The degree or level of accuracy required usually depends on what is being measured and what the information is being used for.

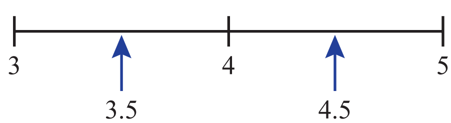
Accuracy is the measure of how true to the 'real' the measure is, whereas **precision** is the ability to obtain the same result over and over again.

▶ Let's start: Limits of accuracy

There is no such thing as an exact measurement. Every measurement is an approximation. Accuracy is limited by the device with which you are measuring.

A measuring device shows the scale below. Objects are measured to the nearest whole number.

If something is measured as 4, the actual measurement could be anything from 3.5 up to (but not including) 4.5, as indicated in the diagram below.



The degree of accuracy is **half a unit (0.5)** each side of the unit of measure.

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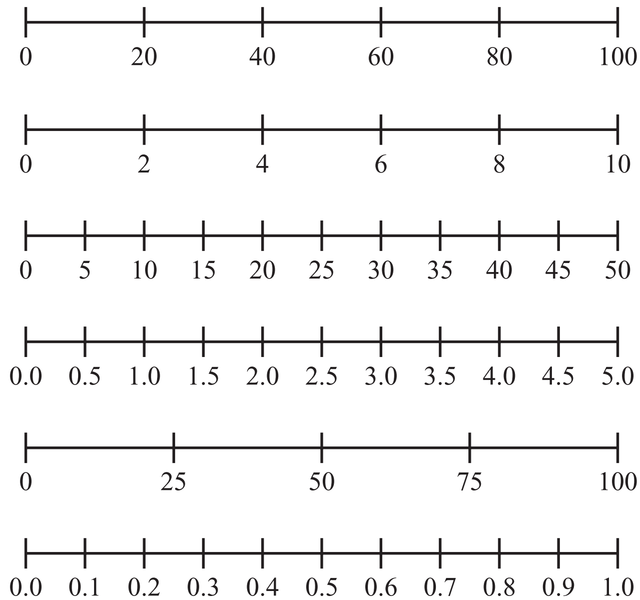
Therefore the limits of accuracy are ± 0.5 because the actual measurement could be half a unit higher or lower than 4.

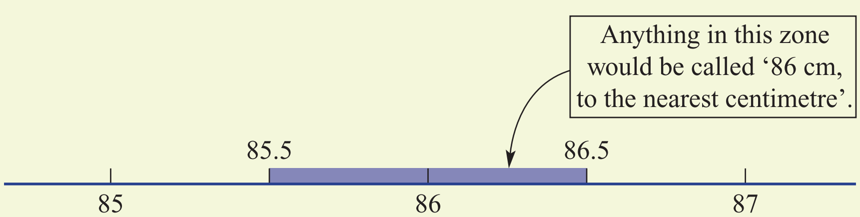
- Explain why the limits of accuracy of this device are ± 5.

- What are the limits of accuracy for devices that show these scales?



Key ideas

- The limits of accuracy tell you what the upper and lower boundaries are for the true measurement.
  - They are $\pm 0.5 \times$ smallest unit of measurement.



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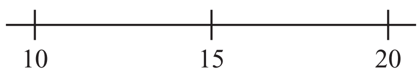
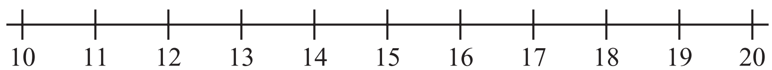
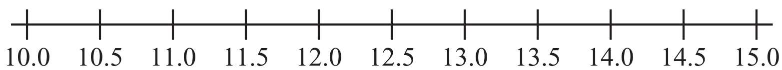
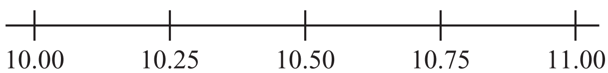
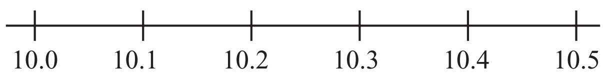
Exercise 2D

Understanding

1 What number is halfway between the two numbers given?

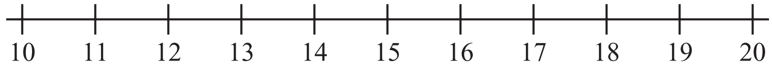
a 20 and 30
b 4 and 5
c 0 and 5
d 4 and 6
e 3 and 3.2
f 3 and 3.5
g 3 and 3.1
h 3 and 3.01

2 Match the scale with the limits of accuracy.

	Measurement scale	Limit of accuracy
a		±0.125
b		±0.25
c		±0.05
d		±0.5
e		±1
f		±2.5

Fluency

3 Copy and complete the following, which refer to the diagram below.



a When using the measurement scale, we are measuring to the nearest ______.
b The limits of accuracy are ± ______.
c If a measurement is quoted as 16, it could be anything from _____ up to, but not including, ______.

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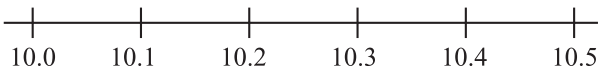
2D

4 Copy and complete the following, which refer to the diagram below.



a When using the measurement scale, we are measuring to the one _____ _____.
b The limits of accuracy are ± _____.
c If a measurement is quoted as 10.4, it could be anything from ______ up to, but not including, ______.

Example 12 Finding lower and upper limits

Give lower and upper limits for these measurements.

a 72 cm
b 86.6 mm

Solution	Explanation
a $72 \pm 0.5 \times 1$ cm $= 72 \pm 0.5$ cm $= 72 - 0.5$ cm to $72 + 0.5$ cm $= 71.5$ cm to 72.5 cm	Unit of measurement is one whole cm. Error $= 0.5 \times 1$ cm This error is subtracted and added to the given measurement to find the limits of accuracy.
b $86.6 \pm 0.5 \times 0.1$ mm $= 86.6 \pm 0.05$ mm $= 86.6 - 0.05$ mm to $86.6 + 0.05$ mm $= 86.55$ mm to 86.65 mm	Unit of measurement is 0.1 mm. Error $= 0.5 \times 0.1$ mm This error is subtracted and added to the given measurement to find the limits of accuracy.

5 Give the lower and upper limits of these measurements.

a 5 m
b 8 cm
c 78 mm
d 5 ns
e 2 km
f 34.2 cm
g 3.9 kg
h 19.4 kg
i 457.9 t
j 18.65 m
k 7.88 km
l 5.05 s

6 Write each of the following as a measurement, given that the lower and upper limits of the measurement are as follows.

a 29.5 m to 30.5 m
b 140 g to 150 g
c 4.55 km to 4.65 km

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Problem-solving and Reasoning

7 Martha writes down the length of her fabric as 150 cm. As Martha does not give her level of accuracy, give the lower and upper limits of the length of her fabric if it was measured correct to the nearest:
 a centimetre
 b 10 centimetres

8 A length of copper pipe is given as 25 cm, correct to the nearest centimetre.
 a What are the limits of accuracy for this measurement?
 b If 10 pieces of pipe, each with a given length of 25 cm, are joined:
  i What is the minimum length that it could be?
  ii What is the maximum length that it could be?

9 The sides of a square are recorded as 9.2 cm, correct to 2 significant figures.
 a What is the minimum length that each side of this square could be?
 b What is the maximum length that each side of this square could be?
 c Find the upper and lower boundaries for this square's perimeter.

10 Johan measures the mass of an object to be 6 kg. Amy says the same object is 5.8 kg and Toby gives his answer as 5.85 kg.
 a Explain how all three people could have different answers for the same measurement.
 b Write down the level of accuracy being used by each person.
 c Are all their answers correct? Discuss.

11 Write down a sentence explaining the need to accurately measure items in our everyday lives and the accuracy that is needed for each of your examples. Give three examples of items that need to be measured correct to the nearest:
 a kilometre
 b millimetre
 c millilitre
 d litre

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2D

Enrichment: Practical measurement

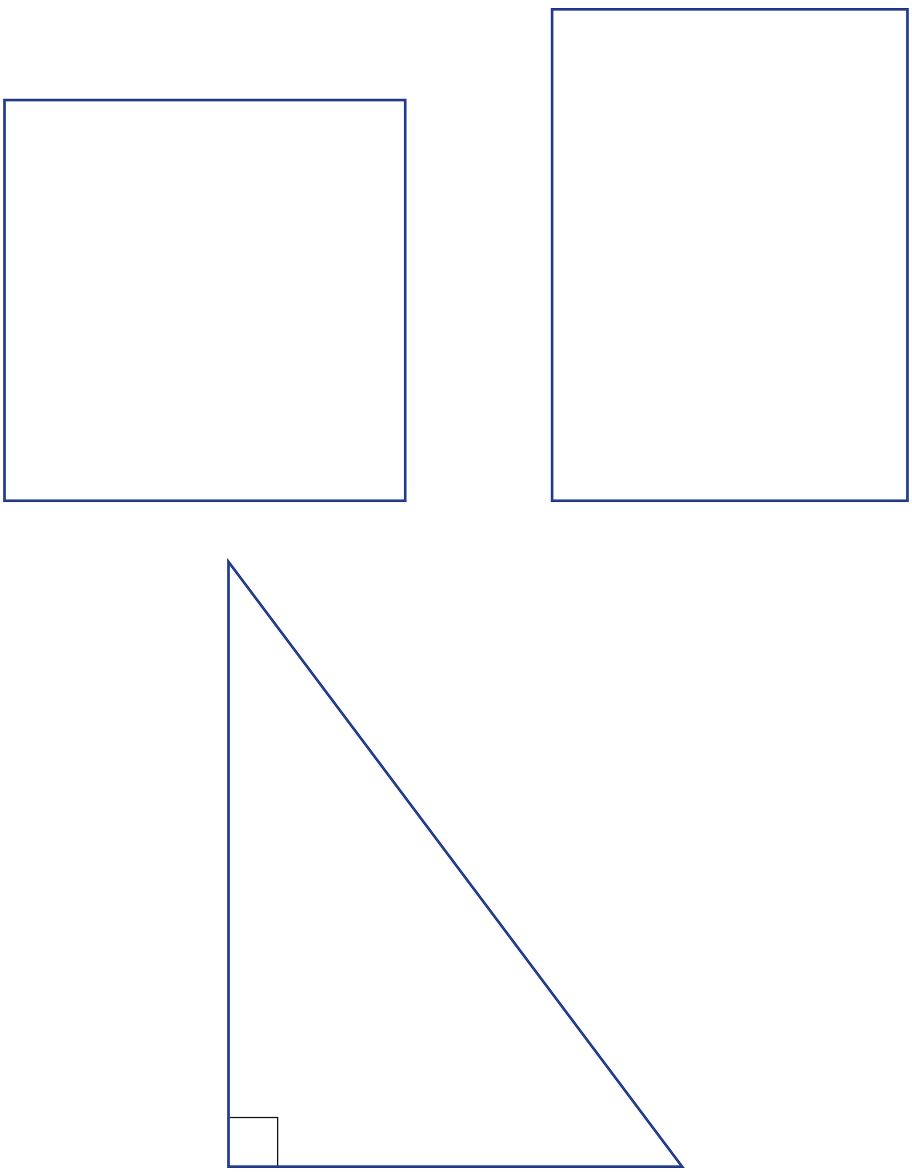
12 a Measure each of the shapes below, correct to the nearest:

i cm **ii** mm

b Use your measurements to find the perimeter and area of each shape.

c After collating your classmates' measurements, find the average perimeter for each shape.

d By how much did the lowest and highest perimeters vary? How can this difference be explained?



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2E Perimeter

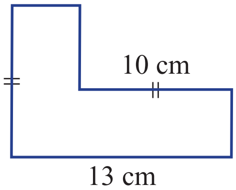
Stage
5.2
5.2◊
5.1
4

Perimeter is a measure of length around the outside of a shape. We calculate perimeter when ordering ceiling cornices for a room or materials for fencing a paddock.

▶ Let's start: L-shaped perimeters

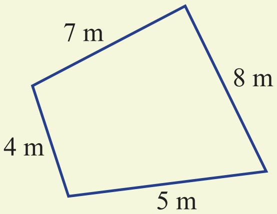
This L-shaped figure includes only right (90°) angles. Only two measurements are given.

- Can you figure out any other side lengths?
- Is it possible to find its perimeter? Why?

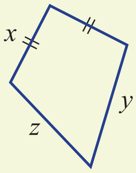


Key ideas

- **Perimeter** is the distance around the outside of a two-dimensional shape.
  – To find the perimeter we add all the lengths of the sides in the same units.
  $P = 4 + 5 + 7 + 8 = 24$ m



  – If two sides of a shape are the same length, they are labelled with the same markings.
  $P = 2x + y + z$



Perimeter The total distance (length) around the outside of a figure

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Exercise 2E

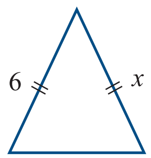
Understanding

1 Write the missing word: The distance around the outside of a shape is called the _____________.

2 Write down the value of x for these shapes.

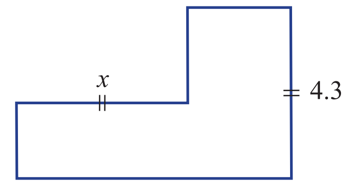
Sides with the same markings are the same length.

a



b

c

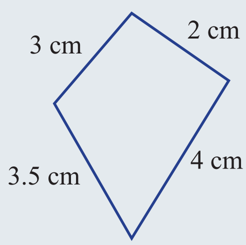


Fluency

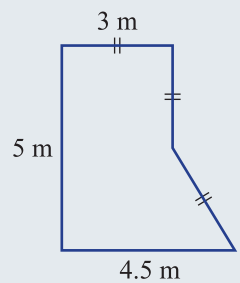
Example 13 Finding perimeters of basic shapes

Find the perimeter of these shapes.

a



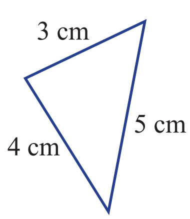
b



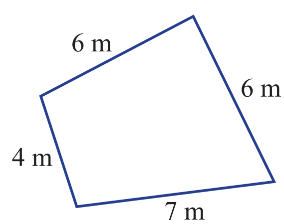
Solution	Explanation
a Perimeter = 3 + 2 + 4 + 3.5 = 12.5 cm	Add all the lengths of the sides together.
b Perimeter = 5 + 4.5 + 3 × 3 = 18.5 m	Three lengths have the same markings and are therefore the same length.

3 Find the perimeter of these shapes.

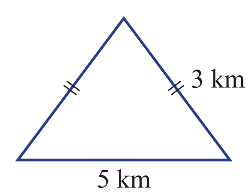
a



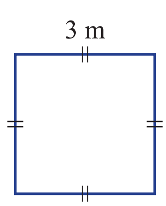
b



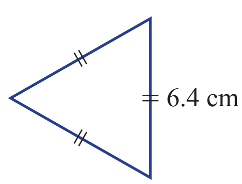
c



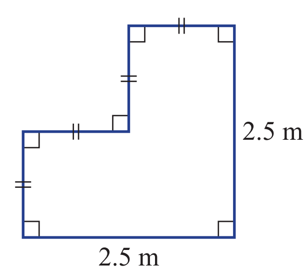
d



e



f



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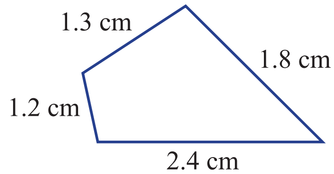
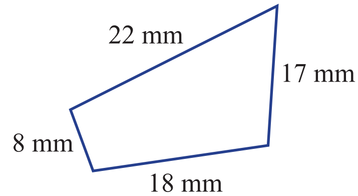
4 Find the perimeter of these shapes.

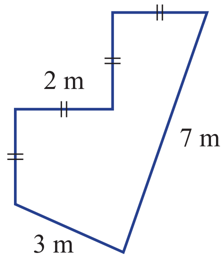
a



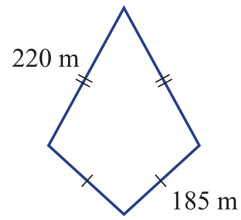
b



c



d



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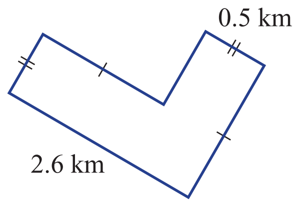
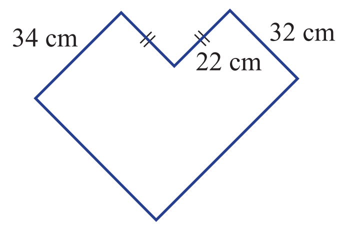


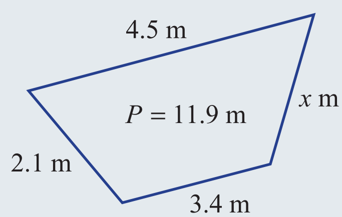
f



Problem-solving and Reasoning

Example 14 Finding a missing side length

Find the value of x for this shape with the given perimeter, P.



Solution	Explanation
$4.5 + 2.1 + 3.4 + x = 11.9$	All the sides add to 11.9 in length.
$10 + x = 11.9$	Simplify.
-10 $x = 1.9$ -10	Subtract 10 from both sides.

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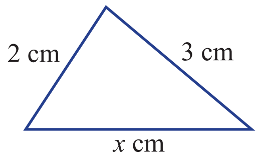
2E

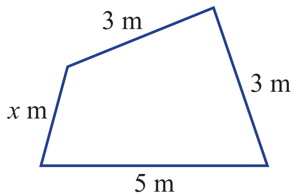
5 Find the value of x for these shapes with the given perimeters.

a



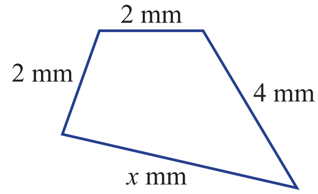
Perimeter = 9 cm

b



Perimeter = 13 m

c

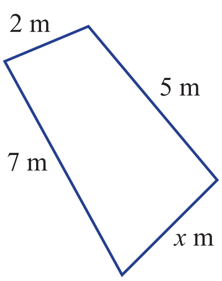


Perimeter = 14 mm

Add up all the sides, then determine the value of x to suit the given perimeters.

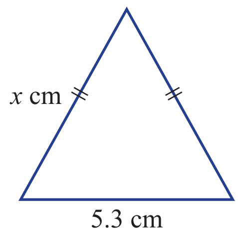
6 Find the value of x for these shapes with the given perimeters.

a



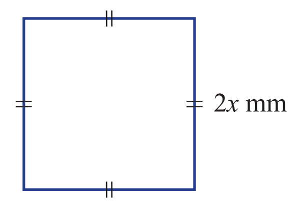
Perimeter = 17 m

b



Perimeter = 22.9 cm

c



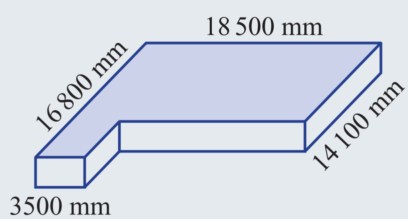
Perimeter = 8 mm

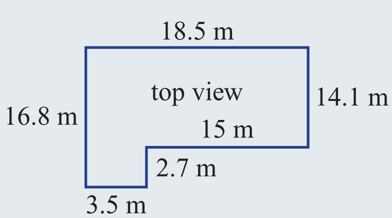
Example 15 Working with concrete slabs

For the concrete slab shown:

a Draw a top view, showing all the measurements in metres.

b Determine the lineal metres of timber needed to surround it.



Solution	Explanation
a 	Convert your measurements and place them all on the diagram. 1 m = 100 × 10 = 1000 mm Add or subtract to find the missing measurements.
b Perimeter = 18.5 + 16.8 + 3.5 + 2.7 + 15 + 14.1 = 70.6 m	Add all the measurements.
The lineal metres of timber needed is 70.6 m.	Write your answer in words.

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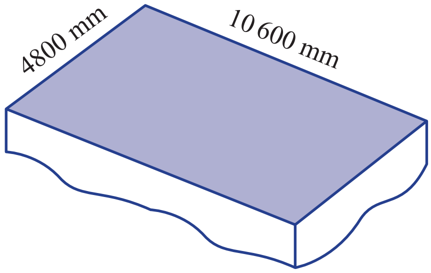
7 For the concrete slabs shown:

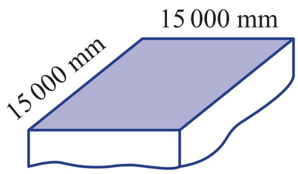
i Draw a top view with the measurements in metres.

ii Determine the lineal metres of timber needed to surround it.

a



b



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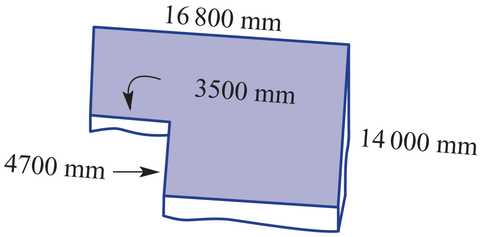
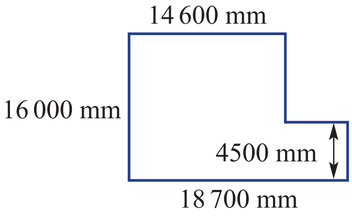
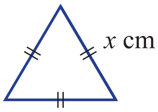


d



8 A rectangular paddock has a perimeter of 100 m. Find the breadth of the paddock if its length is 30 m.

9 The equilateral triangle shown has a perimeter of 45 cm. Find its side lengths.



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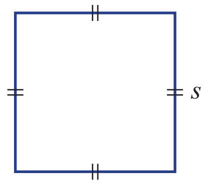
2E

10 Find formulas for the perimeter of these shapes, using the pronumerals given.

a



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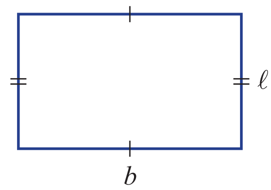

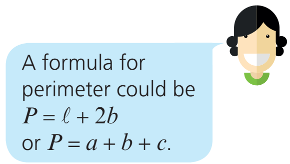

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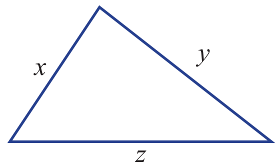

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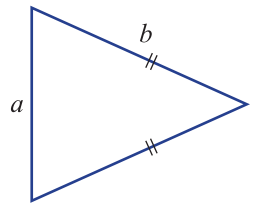

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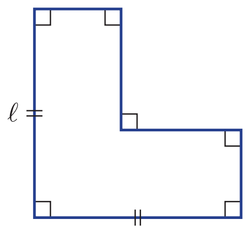

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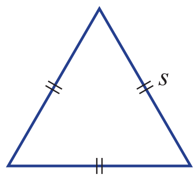

Enrichment: How many different tables?

11 How many rectangles (using whole number lengths) have perimeters between 16 m and 20 m, inclusive?

12 A large dining table is advertised with a perimeter of 12 m. The length and breadth are a whole number of metres (e.g. 1 m, 2 m, …). How many different-sized tables are possible?

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Keeping in touch with numeracy

Non-calculator

1 Subtract 125 from 1000.

3 $97 \times 5 + 3 \times 5 = ?$

5 Write the number 1 234 000 in scientific notation.

7 Give the total number of kilograms for 7 packs of 250 grams of flour.

9 What is the square root of 81?

11 What fraction must be added to one-fifth to make one-half?

13 How many times does 15 need to be added to 40 to make 100?

15 What is the perimeter of an equilateral triangle with sides 8 cm long?

17 Raffle tickets are 50 cents each or 5 for $2. How much, per ticket, do you save when you buy a group of 5 rather than 5 individual tickets?

19 Tennis balls are sold in containers. They are priced as follows:
- 1 for $5
- 6 for $28
- 12 for $40
- 24 for $60

Your club needs 90 containers for the season.
- **a** What is the cheapest way to buy exactly 90 containers?
- **b** What is the cheapest way to buy 90 or more containers? How many containers will be left over at the end of the season?

Calculator

2 $\$1023.45 - \$879.98 = ?$

4 $97 \times 15 + 3 \times 17 = ?$

6 Calculate 2^{50}. Give your answer in scientific notation, correct to 3 significant figures.

8 How many minutes are there in 17.5 hours?

10 What is the square of 81?

12 $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = ?$

14 How many items that cost $12.75 each can be purchased with $100?

16 A rectangle 15 cm long has a perimeter of 75 cm. What is its breadth?

18 Tomato sauce is priced as follows:
- 200 mL for $2.35
- 500 mL for $5.24

Which is the cheaper way to buy it?

20 A holiday resort is offering the following rates:
- Monday to Thursday: $87 per night
- Friday and Saturday: $187
- $550 per week
- **a** If you stay for a week, what is the cost per day?
- **b** What is the daily rate for a weekend?
- **c** What will it cost to stay 6 nights, arriving Friday and leaving Thursday?
- **d** How much extra will it cost to stay one extra night and pay the weekly rate?

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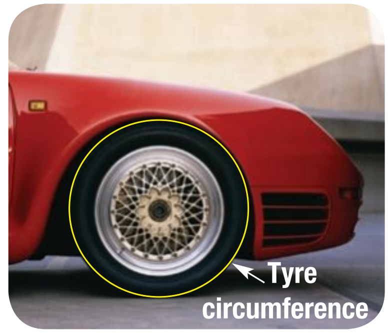
2F Circumference and arc length

Stage

5.2
5.2◊
5.1
4

To find the distance around the outside of a circle (i.e. the circumference) we use the special number called pi (π). Pi is a ratio between the diameter and the circumference of a circle.

One revolution of a wheel moves the vehicle a distance equal to the wheel's circumference.



▶ Let's start: Which value of pi?

Before calculators were invented, people used 3.14 and $\frac{22}{7}$ as approximations for pi. Your calculator quotes pi to be 3.141592654… .

- Using your calculator, find out which is closer to the real value of pi: 3.14 or $\frac{22}{7}$?

NASA uses 16 digits of pi in calculations involving space travel.

- Use the internet to write down the first 16 digits of pi.

The circumference of a circle is the product of pi and the diameter.

$$\text{Circumference} = \text{pi} \times \text{diameter}$$
$$C = \pi \times d$$

Here are 8 different values that could be used for pi.

3	3.1	3.14	3.142	3.1416	3.14159	$\frac{22}{7}$	The value of pi from my calculator.

A large circular water reservoir has a diameter of 100 metres. You need to order material to build a fence around the dam.

- Use the 8 different values of pi given above to see if it makes any difference to your answer.
- If you buy material only in whole metres, how many metres should you buy?

Key ideas

Drilling for Gold 2F1 at the end of this section

- The **radius** is the distance from the centre of a circle to a point on the circumference.
- The **diameter** is the distance across a circle through its centre.
  – Radius = $\frac{1}{2}$ diameter *or* diameter = 2 × radius.
- **Circumference** is the distance around a circle.
  – Circumference = 2π × radius

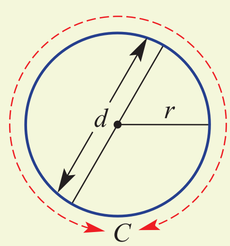
  $$C = 2\pi r$$

  or Circumference = π × diameter

  $$C = \pi d$$

  – π is a special number and can be found on your calculator.

  $\pi \approx 3.14159\ldots$, which is approximately $\frac{22}{7}$.



Radius The distance from the centre of a circle to a point on the circumference

Diameter A line passing through the centre of a circle with its end points on the circumference

Circumference The distance around the outside of a circle; the curved boundary

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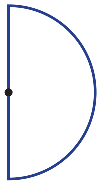
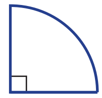
Exercise 2F

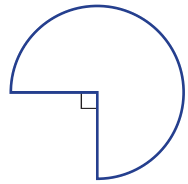
Understanding

1 Write the formula for the circumference of a circle using:
a d for diameter
b r for radius

2 For each of the following, what fraction of a circle is shown?

a


b


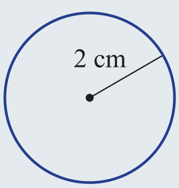
c


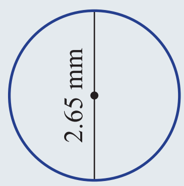
3 **a** What is the diameter of a circle when its radius is 4.3 m?
b What is the radius of a circle when its diameter is 3.6 cm?

Fluency

Example 16 Finding the circumference of a circle

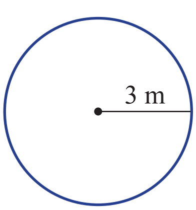
Find the circumference of these circles, to 2 decimal places.

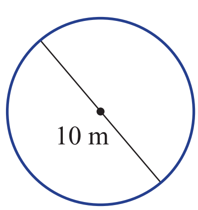
a


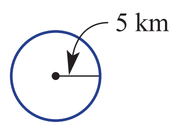
b


Solution	Explanation
a $C = 2\pi r$	Write the formula involving radius.
$= 2\pi(2)$	Substitute $r = 2$.
Circumference = 12.57 cm (to 2 d.p.)	Write your answer to 2 decimal places.
b $C = \pi d$	Write the formula involving diameter.
$= \pi(2.65)$	Substitute $d = 2.65$.
Circumference = 8.33 mm (to 2 d.p.)	Write your answer to 2 decimal places.

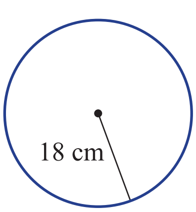
4 Find the circumference of these circles, to 2 decimal places.

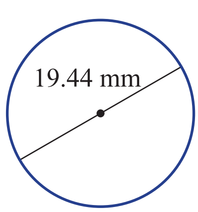
a


b


c


Use $C = 2\pi r$ or $C = \pi d$.

d


e


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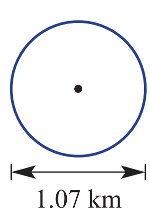

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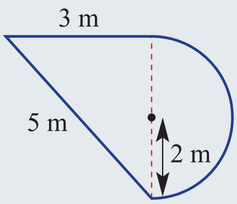
2F

Example 17 Finding perimeters of composite shapes

Find the perimeter of this composite shape, to 2 decimal places.



Solution

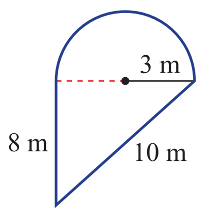
$P = 3 + 5 + \frac{1}{2} \times 2\pi(2)$
$\quad = 8 + 2\pi$
Perimeter = 14.28 m (to 2 d.p.)

Explanation

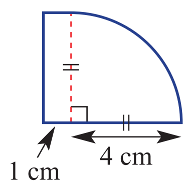
Add all the sides, including half a circle.
Simplify.
Round your answer as instructed.

5 Find the perimeter of these composite shapes, to 2 decimal places.

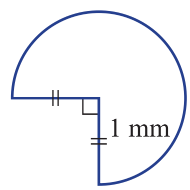
a



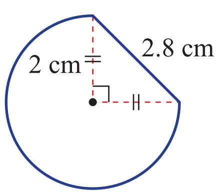
b



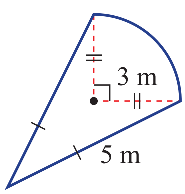
c



d



e



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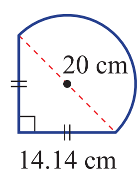

Don't forget to add the straight sides to the fraction $\left(\frac{1}{4}, \frac{1}{2} \text{ or } \frac{3}{4}\right)$ of the circumference.

Problem-solving and Reasoning

6 David wishes to build a circular fish pond. The diameter of the pond is to be 3 m.
a How many linear metres of bricks are needed to surround it? Round your answer to 2 decimal places.
b What is the cost if the bricks are $45 per metre? (Use your answer from part **a**.)

7 The wheels of a bike have a diameter of 50 cm.
a How many metres will the bike travel (to 2 decimal places) after one full turn of the wheels?
b How many kilometres will the bike travel after 1000 full turns of the wheels? (Give your answer correct to 2 decimal places.)

For one revolution, use $C = \pi d$.

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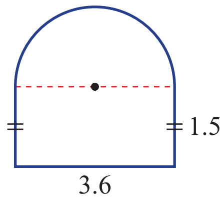
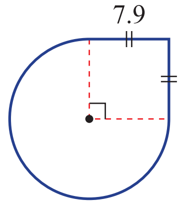
8 What is the minimum number of times a wheel of diameter 1 m needs to spin to cover a distance of 1 km? You will need to find the circumference of the wheel first. Give your answer as a whole number.

9 Find the perimeter of these composite shapes, correct to 2 decimal places.

a

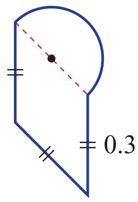


b

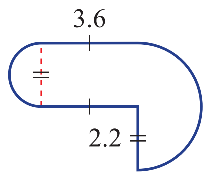


Make sure you know the radius or diameter of the circle you are dealing with.

c



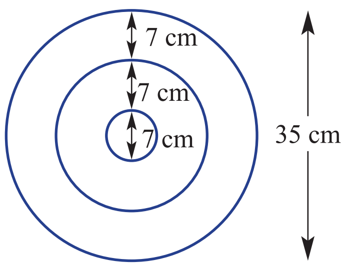
d



Enrichment: Target practice

10 A target is made up of three rings, as shown.

- **a** Find the radius of the smallest ring.
- **b** Find, to 2 decimal places, the circumference of:
  - **i** the smallest ring
  - **ii** the middle ring
  - **iii** the outside ring.



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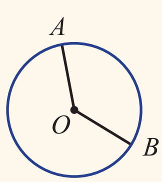
Drilling for Gold exercise

2F1: Circle terminology

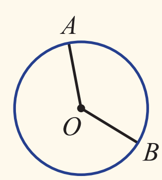
Match the definitions **1**–**8** with the diagrams **A**–**H** by writing 1D, 2A etc. on the worksheet or write the answers in your exercise book.

Definition 1 Part of the circumference of a circle. In the diagram there are two arcs, called AB. The shorter one is called the minor arc and the longer one is called the major arc.	**Definition 2** A plane shape that is perfectly round. All points on the edge of a circle are the same distance from a point called the centre (O).
Definition 3 **1** A line segment (interval) that passes through the centre of a circle with end points on the circumference. AB is a diameter. **2** The length of the line segment AB.	**Definition 4** A region inside a circle bounded by two radii and an arc. The diagram shows sector AOB.
Definition 5 **1** A line segment (interval) with one end point at the centre and the other on the circumference. OA and OB are radii. **2** The length of the line segment OA, which is half the diameter.	**Definition 6** Half a circle. In the diagram, the diameter AB creates two semicircles.
Definition 7 **1** The edge of a circle. Points A and B are on the circumference. **2** The perimeter of a circle.	**Definition 8** A sector that is exactly one-quarter of a circle. The diagram shows quadrant AOB.

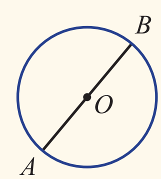
A
circle



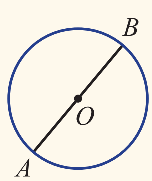
B
circumference



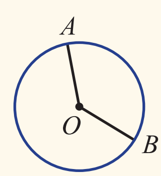
C
diameter



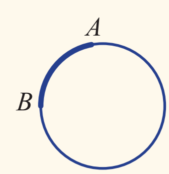
D
semicircle



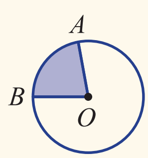
E
radius (plural is radii)



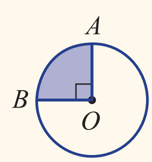
F
arc



G
sector



H
quadrant



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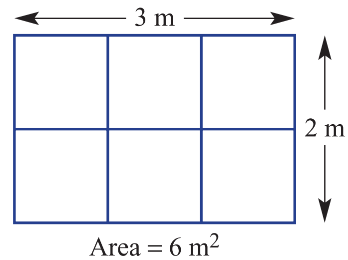
2G Area of triangles and quadrilaterals

In this diagram, a rectangle with side lengths 2 m and 3 m has an area of 6 square metres or 6 m^2. This can be calculated by counting the number of squares (each being a square metre) that make up the rectangle.

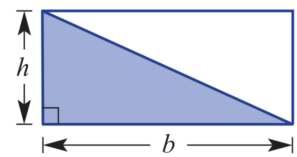
We use formulas to help us quickly calculate the number of square units contained within a shape. For this rectangle, for example, the formula $A = \ell b$ tells us to multiply the length by the breadth to find the area.



▶ Let's start: How does $A = \frac{1}{2}bh$ work for a triangle?

Draw a rectangle with a diagonal.

- How does the shape of the triangle relate to the shape of the rectangle?
- How can you use the formula for a rectangle to help find the area of the triangle (or parts of the triangle)?
- Why is the rule for the area of a triangle given by $A = \frac{1}{2}bh$?

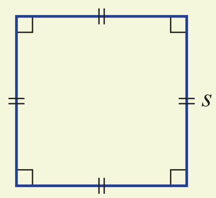


Key ideas

- The **area** of a two-dimensional shape is the number of square units contained within its boundaries.
- Some of the common area formulas are as follows.

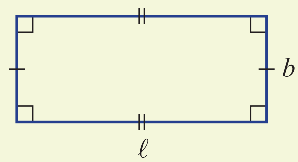
Area The number of square units needed to cover the space inside the boundaries of a 2D shape

Square



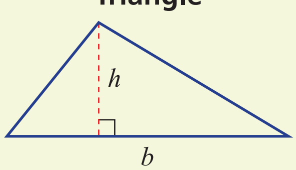
$A = s^2$

Rectangle

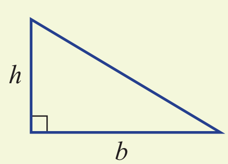


$A = \ell b$

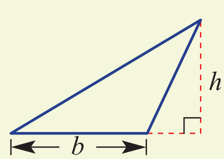
Triangle



$A = \frac{1}{2}bh$



$A = \frac{1}{2}bh$



$A = \frac{1}{2}bh$

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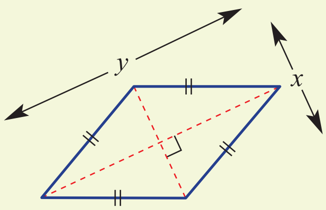
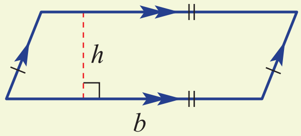
Rhombus



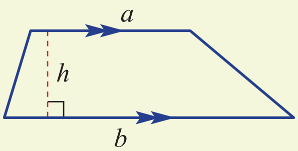
$A = \frac{1}{2}xy$

Parallelogram



$A = bh$

Trapezium



$A = \frac{1}{2}h(a + b)$

- The 'height' in a triangle, parallelogram or trapezium must be perpendicular (i.e. at $90°$) to the base.

Exercise 2G

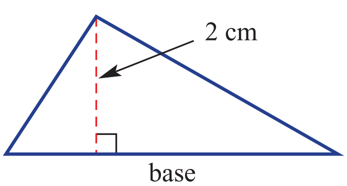
Understanding

1 Match each shape (**a**–**f**) with its area formula (**A**–**F**).

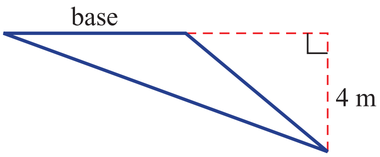
a square	**A** $A = \frac{1}{2}bh$
b rectangle	**B** $A = \ell b$
c rhombus	**C** $A = bh$
d parallelogram	**D** $A = \frac{1}{2}h(a + b)$
e trapezium	**E** $A = s^2$
f triangle	**F** $A = \frac{1}{2}xy$

2 These shapes show the base and a height length. What is the height of each shape?

a



b



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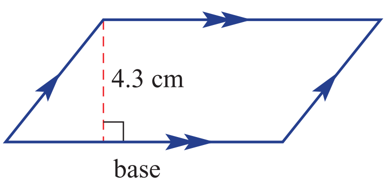

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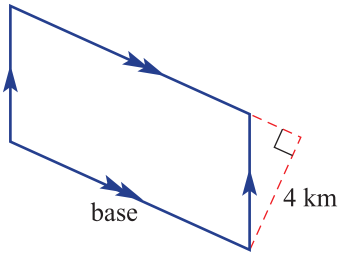

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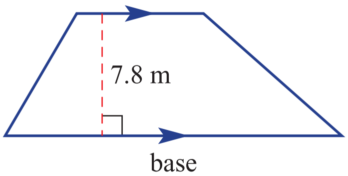
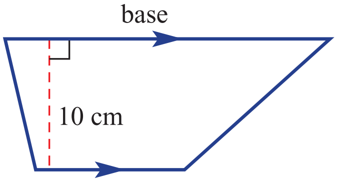


f



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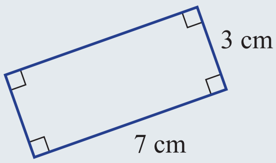
Fluency

Example 18 Using area formulas

Find the area of these basic shapes.

a



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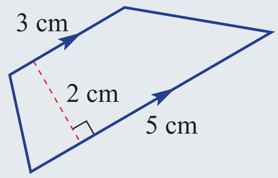
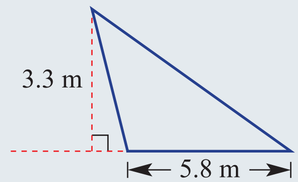


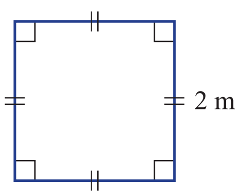
c



Solution	Explanation
a $A = \ell b$ $= 7 \times 3$ Area $= 21\ \text{cm}^2$	Write the formula for a rectangle. Substitute the lengths $\ell = 7$ and $b = 3$. Simplify and add the units.
b $A = \frac{1}{2}h(a + b)$ $= \frac{2}{2}(3 + 5)$ Area $= 8\ \text{cm}^2$	Write the formula for a trapezium. Substitute the lengths $a = 3$, $b = 5$ and $h = 2$. Simplify and add the units.
c $A = \frac{1}{2}bh$ $= \frac{1}{2}(5.8)(3.3)$ Area $= 9.57\ \text{m}^2$	Write the formula for a triangle. Substitute the lengths $b = 5.8$ and $h = 3.3$. Simplify and add the units.

3 Find the area of these basic shapes.

a



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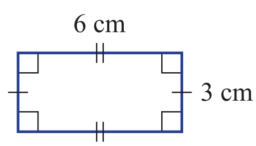

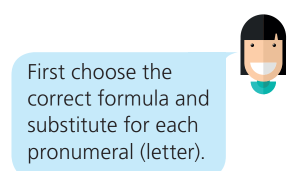

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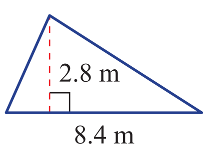

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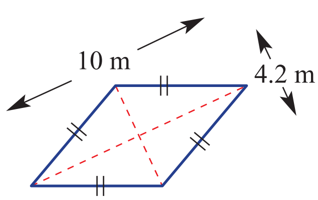

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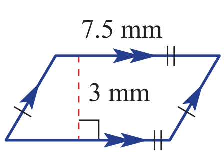

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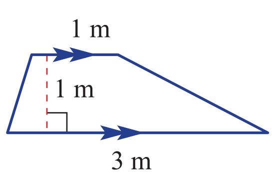

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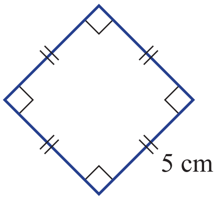
2G

4 Find the area of these basic shapes, rounding to 2 decimal places where necessary.

a


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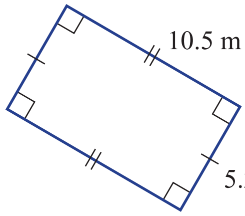

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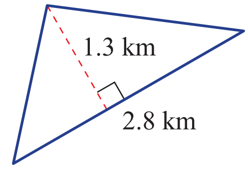

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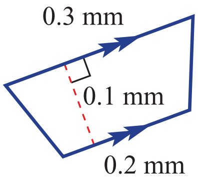

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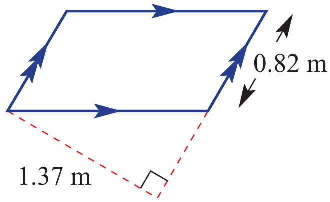

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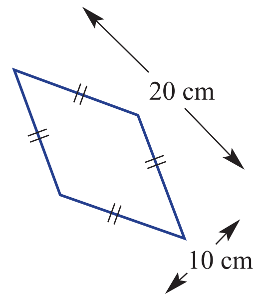


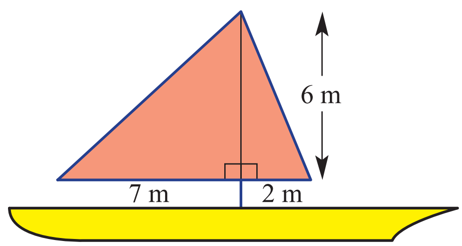
Problem-solving and Reasoning

5 A rectangular table top is 1.2 m long and 80 cm wide. Find the area of the table top using:
a square metres (m^2) b square centimetres (cm^2)

First convert to the units that you wish to work with.

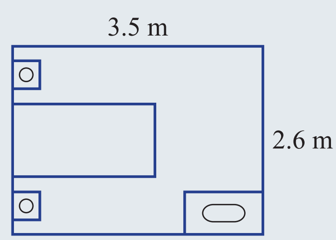
Drilling for Gold 2G1

6 Two triangular sails have side lengths as shown. Find the total area of the two sails.



Example 19 Finding areas of floors

Christine decides to use carpet to cover the floor of her bedroom, shown at right. Determine:
a the area of floor to be covered
b the total cost if the carpet costs \$32 a square metre



Solution	Explanation
a $A = \ell \times b$ $= 3.5 \times 2.6$ Area $= 9.1\ m^2$	The room is a rectangle, so use $A = \ell \times b$ to calculate the total floor space.
b Cost of carpet $= 9.1 \times \$32$ $= \$291.20$	Every square metre of carpet costs \$32.

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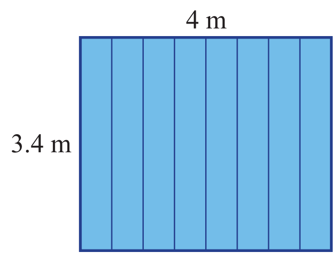
7 Jack's shed is to have a flat metal roof.

a Determine the total area of the roof.

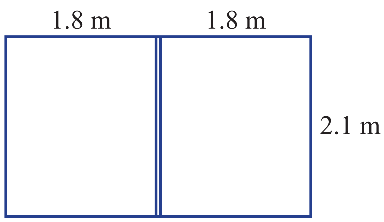
b If the roofing costs \$11 per square metre, how much will it cost in total?



8 A sliding door has two glass panels. Each of these is 2.1 m high with a breadth of 1.8 m.

a How many square metres of glass are needed?

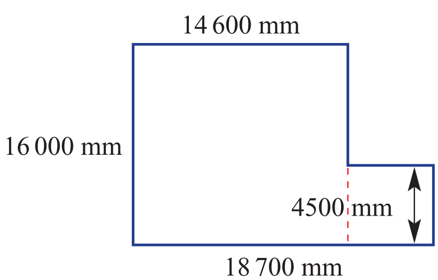
b What is the total cost of the glass if the price is \$65 per square metre?



9 A rectangular window has a whole number measurement for its length and breadth and its area is 24 m^2. Write down the possible lengths and breadths for the window.

10 Determine the area of the houses shown, in square metres (correct to 2 decimal places).

a



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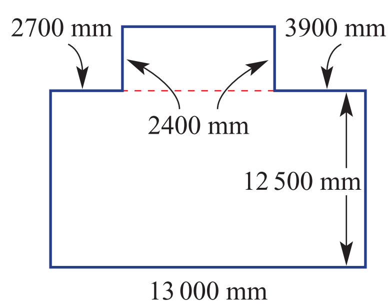
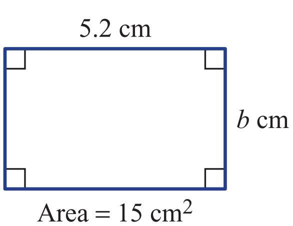


Note that there are 1000 mm in 1 m. Use the red lines to break up shapes into two rectangles.

11 Find the value of the pronumeral in these shapes, rounding to 2 decimal places each time.

a



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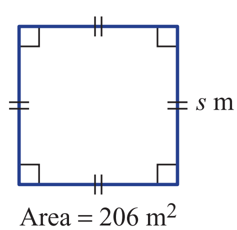

If $x \times 2 = 15$, then $x = \frac{15}{2} = 7.5$.

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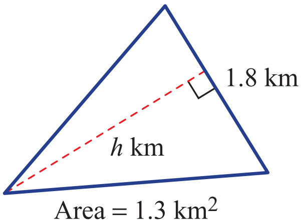
c



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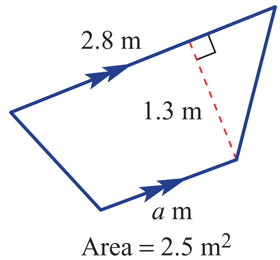

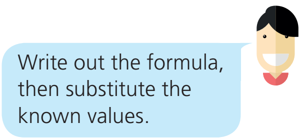

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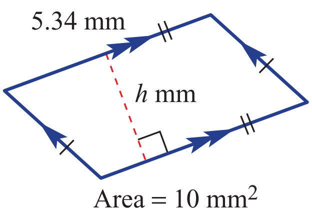

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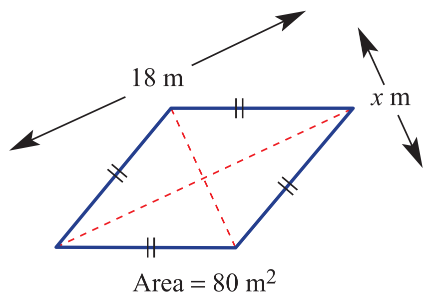
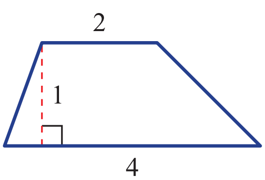


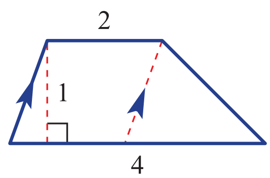
Enrichment: Four ways to find the area of a trapezium

12 Find the area of the trapezium, using each of the suggested methods.

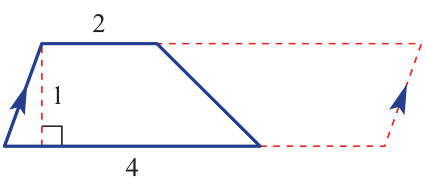
a formula $A = \frac{1}{2}h(a + b)$



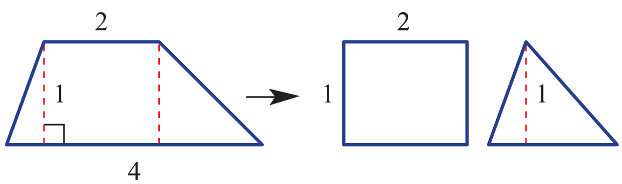
b parallelogram and triangle



c half parallelogram



d rectangle + triangle



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2H Area of circles and sectors

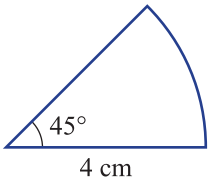
Stage: 5.2, 5.2◊, 5.1, 4

Like its circumference, a circle's area is linked to the special number pi (π). To find the area of a circle we use the formula $A = \pi r^2$.

Let's start: What fraction is that?

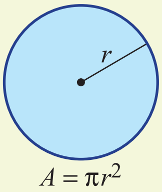
When finding areas of sectors, first we need to decide what fraction of a circle we are dealing with. This sector, for example, has a radius of 4 cm and a 45° angle.

- What fraction of a full circle is shown in this sector?
- How can you use this fraction to help find the area of this sector?
- How would you set out your working?



Key ideas

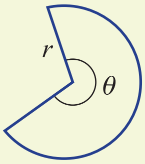
- The formula for finding the area (A) of a circle of radius r is
  $A = \pi r^2$



- If the diameter (d) of the circle is given, determine the radius before calculating the area of the circle: $r = d \div 2$.
- The area of a sector is given by the formula

$$A = \frac{\theta}{360} \times \pi r^2$$

where $\frac{\theta}{360}$ represents the fraction of a full circle.

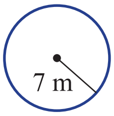


Exercise 2H

Understanding

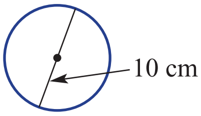
1 Which is the correct working step for the area of this circle?

A $A = \pi(7)$ **B** $A = 2\pi(7)$ **C** $A = \pi(14)^2$
D $A = (\pi 7)^2$ **E** $A = \pi(7)^2$



2 Which is the correct working step for the area of this circle?

A $A = \pi(10)^2$ **B** $A = \pi(10)^2$ **C** $A = \pi(5)^2$
D $A = 2\pi(5)$ **E** $A = 5\pi$



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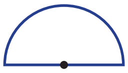
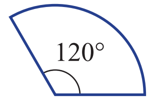
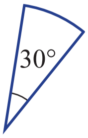
2H

3 What fraction of a circle is shown by these sectors? Simplify your fraction.

a


b


c


d


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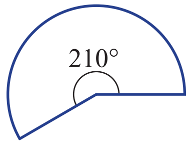

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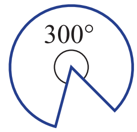
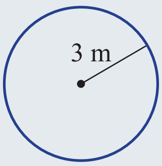


A full circle has 360°.

Fluency

Example 20 Finding areas of circles

Find the area of these circles, correct to 2 decimal places.

a


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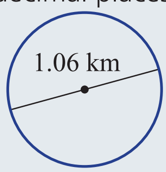
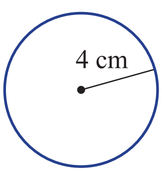


Solution	Explanation
a $A = \pi r^2$	Write the formula.
$= \pi(3)^2$	Substitute $r = 3$.
$= \pi \times 9$	Evaluate $3^2 = 9$, then multiply by π.
Area $= 28.27\ \text{m}^2$ (to 2 d.p.)	Round your answer as required.
b Radius, $r = 1.06 \div 2 = 0.53$ km	Find the radius, given that the diameter is 1.06.
$A = \pi r^2$	Write the formula.
$= \pi(0.53)^2$	Substitute $r = 0.53$.
Area $= 0.88\ \text{km}^2$ (to 2 d.p.)	Write your answer to 2 decimal places, with units.

4 Find the area of these circles, correct to 2 decimal places.

a


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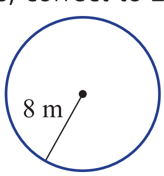

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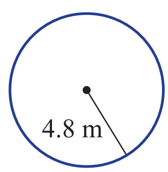
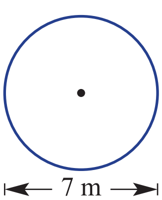


Remember: $r = d \div 2$

d


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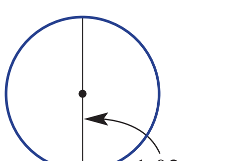

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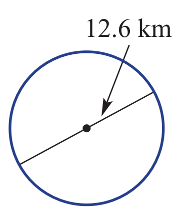

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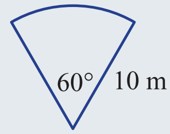
Example 21 Finding areas of sectors

Find the area of this sector, correct to 2 decimal places.

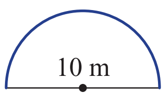


Solution	Explanation
Fraction of circle $= \frac{60}{360} = \frac{1}{6}$	The sector uses 60° out of the 360° in a whole circle.
$A = \frac{1}{6} \times \pi r^2$	Write the formula, including the fraction part.
$= \frac{1}{6} \times \pi(10)^2$	Substitute $r = 10$.
Area $= 52.36$ m^2 (to 2 d.p.)	Write your answer to 2 decimal places.

5 Find the area of these sectors, correct to 2 decimal places.

First determine the fraction of a full circle that you are dealing with.

a



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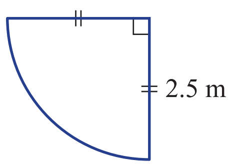

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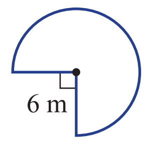

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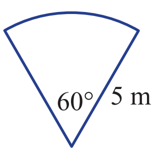

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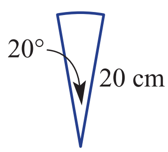

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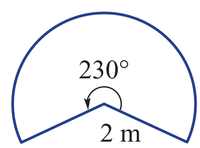

Problem-solving and Reasoning

6 A pizza with diameter 40 cm is divided into eight equal parts. Find the area of each portion, correct to 1 decimal place.

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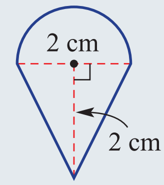
2H

Example 22 Finding areas of composite shapes

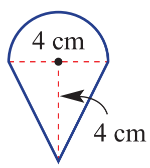
Find the area of this composite shape, correct to 2 decimal places.



Solution	Explanation
$A = \frac{1}{2}\pi r^2 + \frac{1}{2}bh$	The shape is made up of a semicircle and a triangle. Write the formulas for both.
$= \frac{1}{2}\pi(1)^2 + \frac{1}{2}(2)(2)$	Substitute $r = 1$, $b = 2$ and $h = 2$.
$= 1.5707\ldots + 2$	
Area = 3.57 cm² (to 2 d.p.)	Write your answer to 2 decimal places, with units.

7 Find the area of these composite shapes, correct to 2 decimal places.

a



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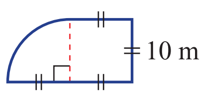

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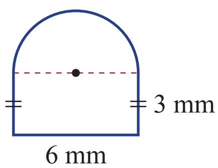
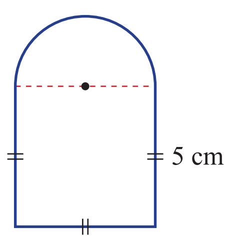


Find the area of each shape within the larger shape, then add them; e.g. triangle + semicircle.

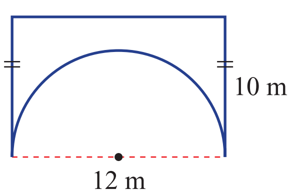
8 A lawn is made up of a semicircle with diameter 6.5 m and a triangle of length 8.2 m, as shown. Find the area of lawn, correct to 2 decimal places.

9 Find the area of these composite shapes, correct to 1 decimal place.

a

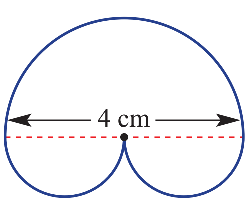


b



Use addition or subtraction, depending on the shape given.

c



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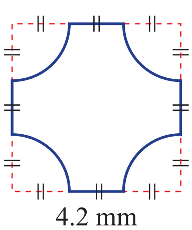

Four equal quarter circles make up the area of *one* full circle.

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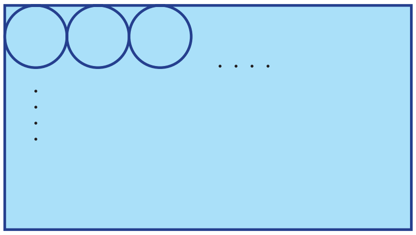
Enrichment: Circular pastries

10 A rectangular piece of pastry is used to create small circular pastry discs for the base of Christmas tarts. The rectangular piece of pastry is 30 cm long, 24 cm in breadth and each circular piece has a diameter of 6 cm.



a How many circular pieces of pastry can be removed from the rectangle?
b Find the total area removed from the original rectangle, correct to 2 decimal places.
c Find the total area of pastry remaining, correct to 2 decimal places.
d If the remaining pastry is collected and re-rolled to the same thickness, how many circular pieces could be cut? Assume that the pastry can be re-rolled many times.

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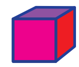
2I Surface area of prisms

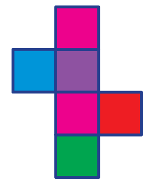
Stage
5.2
5.2◊
5.1
4

The surface of a cube has six squares. The sum of the areas of the squares is called the surface area of the cube.

Cube



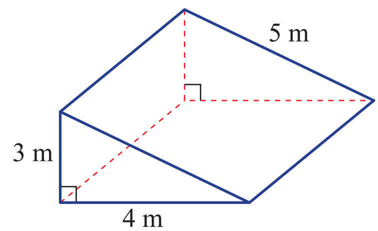
Net of a cube



Let's start: Which net?

The solid shown at right is a triangular prism with a right-angled triangle as its cross-section.

- How many different types of shapes make up its outside surface?
- What is a possible net for the solid? Is there more than one?
- How would you find the surface area of the solid?



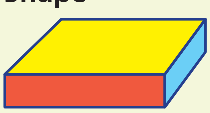
Triangular prism

Key ideas

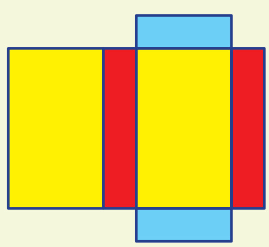
Drilling for Gold 2I1

- To calculate the **surface area (*A*)** of a solid:
  - Draw a net (a two-dimensional drawing including all the surfaces).
  - Determine the area of each shape inside the net.
  - Add the areas of each shape together.

Shape



Net



Surface area (*A*) The total number of square units needed to cover the outside of a solid

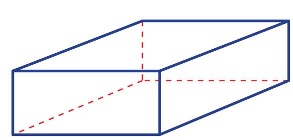
Exercise 2I

Understanding

1 For a rectangular prism, answer the following.

 a How many faces does the prism have?

 b How many *different* rectangles form the surface of the prism?



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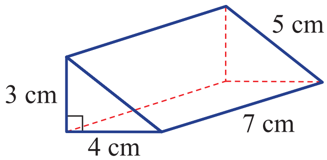
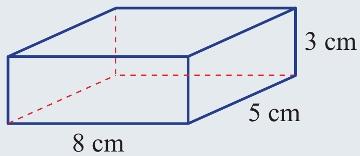
2 For this triangular prism, answer the following.
- **a** What is the area of the largest rectangle?
- **b** What is the area of the smallest rectangle?
- **c** What is the combined area of the two triangles?
- **d** What is the surface area?



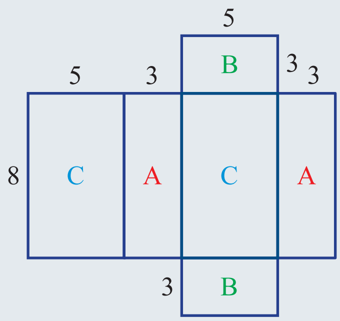
Fluency

Example 23 Finding the surface area of a rectangular prism

Find the surface area (A) of this rectangular prism by first drawing its net.



Solution



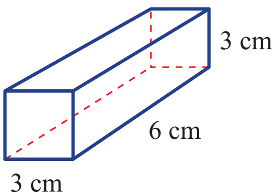
$A = 2 \times \text{area A} + 2 \times \text{area B} + 2 \times \text{area C}$
$\quad = 2 \times (8 \times 3) + 2 \times (5 \times 3) + 2 \times (8 \times 5)$
Surface area $= 158\text{ cm}^2$

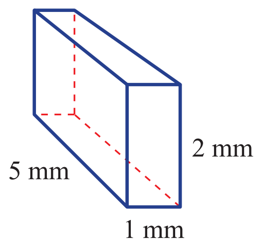
Explanation

Draw the net of the solid, labelling the lengths and shapes of equal areas.
There are six rectangles:
- Two are: 8 by 3.
- Two are: 5 by 3.
- Two are: 8 by 5.

Describe each area.
Substitute the correct lengths.
Simplify and add units.

3 Find the surface area (A) of these rectangular prisms by first drawing their nets.

a


b


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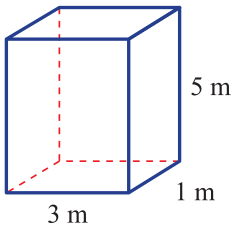

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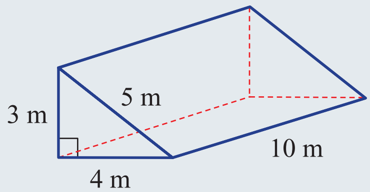
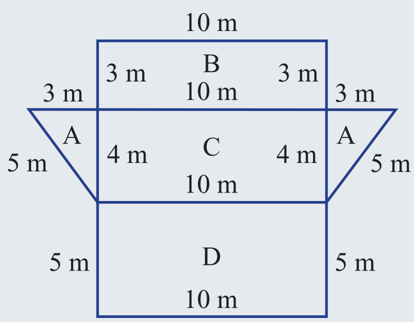
2I

Example 24 Finding the surface area of a triangular prism

Find the surface area of the triangular prism shown.



Solution



$A = 2 \times \text{area A} + \text{area B} + \text{area C} + \text{area D}$

$= 2 \times \left(\frac{1}{2} \times 3 \times 4\right) + (3 \times 10) + (4 \times 10) + (5 \times 10)$

$= 12 + 30 + 40 + 50$

Surface area = $132\ \text{m}^2$

Explanation

Draw a net of the object with all the measurements and label the sections to be calculated.

There are two triangles with the same area and three different rectangles:

- 3 by 10
- 4 by 10
- 5 by 10

Substitute the correct lengths.

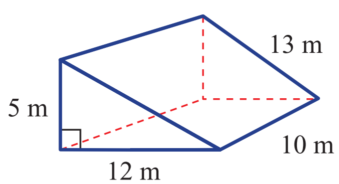
Calculate the area of each shape.

Add the areas together.

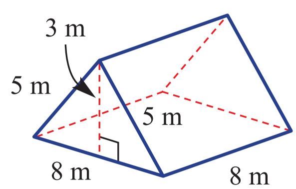
4 Find the surface area of the following prisms.

There are three rectangles and two identical triangles.

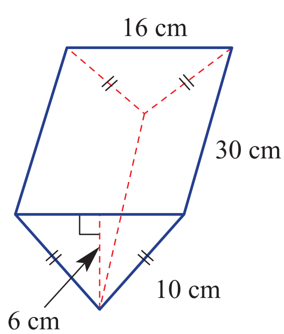
a



b



c



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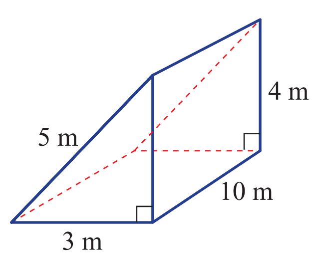

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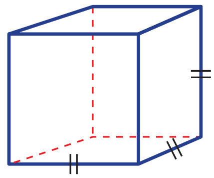
5 Find the surface area of these prisms by first drawing a net.

a


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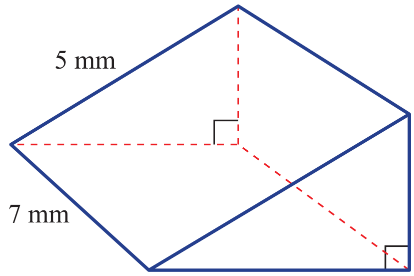
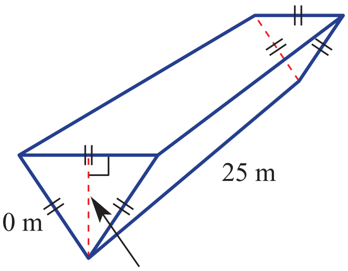


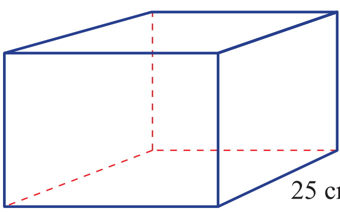
Skillsheet 2C

c


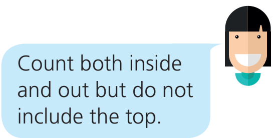
Problem-solving and Reasoning

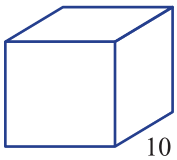
6 A cube with side lengths 8 cm is to be painted. What is the surface area that is to be painted?

7 What is the minimum amount of paper required to wrap a box with dimensions of breadth 25 cm, length 32 cm and height 20 cm?



8 An open-topped box with sides that are 10 cm long is to be covered inside and out with a special material. Find the minimum amount of material required to cover the box.





9 James wishes to paint his bedroom. The ceiling and walls are to be the same colour. If the room measures 3.3 m × 4 m and the ceiling is 2.6 m high, find the amount of paint needed:

a if each litre covers 10 square metres
b if each litre covers 5 square metres.

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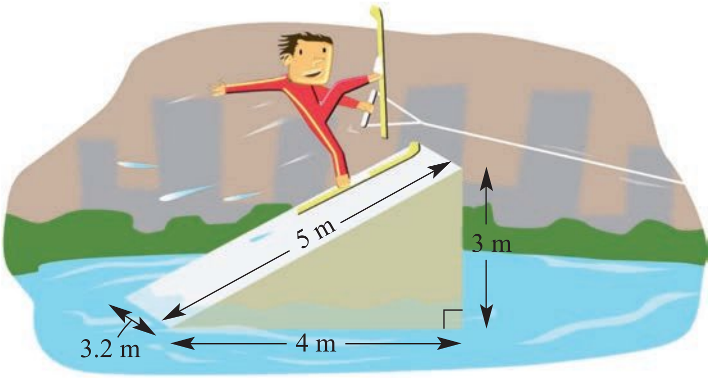
10 A ski ramp in the shape of a triangular prism needs to be painted. The base and sides of the ramp require a fully waterproof paint, which covers 2.5 square metres per litre. The top needs special smooth paint, which covers only 0.7 square metres per litre.

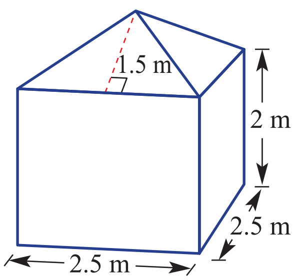


a Determine the amount of each type of paint required. Round your answer to 2 decimal places where necessary.

b If the waterproof paint is \$7 per litre and the special smooth paint is \$20 per litre, calculate the total cost of painting the ramp, to the nearest cent. (Use the exact answers from part **a** to help.)

Enrichment: Will I have enough paint?

11 I have 6 litres of paint and on the tin it says that the coverage is 5.5 m^2 per litre. I wish to paint the four outside walls of a shed and the roof, which has four triangular sections. Will I have enough paint to complete the job?



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2J Volume of prisms and cylinders

Stage
5.2
5.2◊
5.1
4

The volume of a solid is the amount of space within the shape. It is measured in cubic units.

The volume of a solid can be calculated by multiplying the area of the base by the perpendicular height. Consider the rectangular prism on the right.

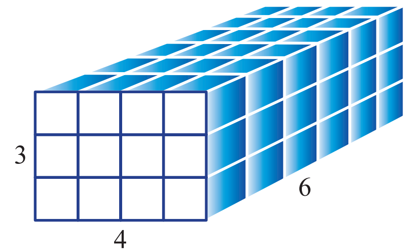
Area of base $= 4 \times 6 = 24$ units^2

Height $= 3$ units

Volume $=$ Area of base $\times$ height

$= 24 \times 3$

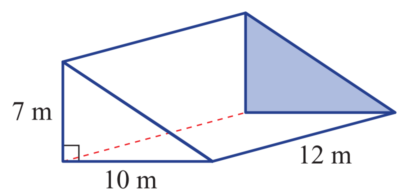
$= 72$ units^3



Let's start: Volume of a triangular prism

This prism has a triangular cross-section.

- What is the area of the cross-section?
- What is the 'height' of the prism?
- How can the formula $V = A \times h$ be applied to this prism, where A is the area of the cross-section?

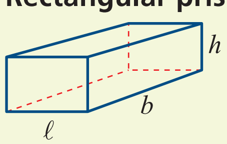


Key ideas

- The **volume** of a solid with a uniform cross-section is given by $V = A \times h$, where:
  - A is the area of the cross-section.
  - h is the perpendicular (at 90°) height.

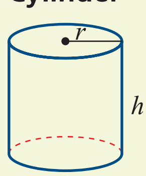
Volume The amount of three-dimensional space within an object

Rectangular prism



$V = A \times h$
$= \ell bh$

Cylinder



$V = A \times h$
$= \pi r^2 h$

- Units for capacity include:
  - 1 L = 1000 mL

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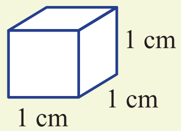
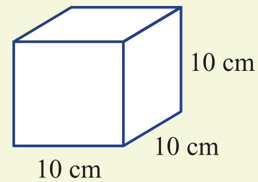
- One cubic centimetre holds one millilitre.



- This cube holds one litre.



- One cubic metre holds 1000 litres.

Exercise 2J

Understanding

1 Match the solid (**a**–**c**) with the volume formula (**A**–**C**).

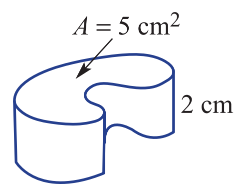
a cylinder
b rectangular prism
c triangular prism

A $V = \ell bh$
B $V = \frac{1}{2}bh \times$ length
C $V = \pi r^2 h$

2 Write the missing number.

a There are __________ mL in one litre.
b One litre fills __________ cubic centimetres.

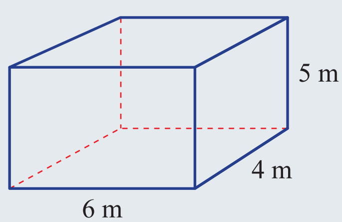
3 The area of the cross-section of this solid is given. Find the solid's volume using $V = A \times h$.



Fluency

Example 25 Finding the volume of a rectangular prism

Find the volume of this rectangular prism.



Solution	Explanation
$V = A \times h$	Write the formula.
$= 6 \times 4 \times 5$	$A = \ell \times b = 6 \times 4$, and $h = 5$.
Volume = 120 m^3	Simplify and include units.

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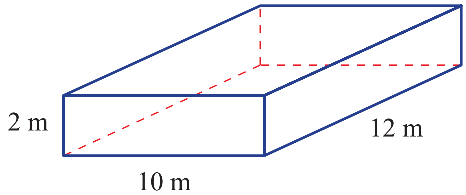
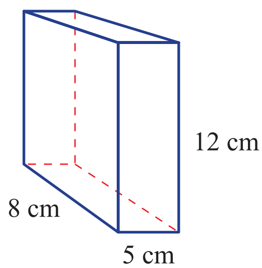
4 Find the volume of these rectangular prisms.

Use $V = \ell bh$.

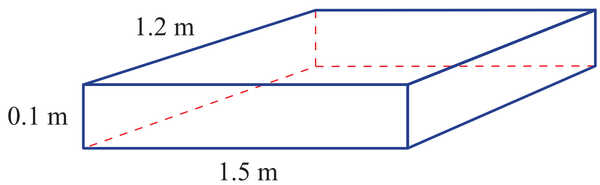
a



b

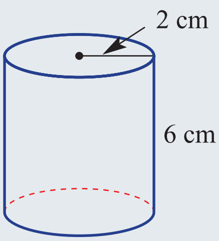


c



Example 26 Finding the volume of a cylinder

Find the volume of this cylinder, correct to 2 decimal places.



Solution	Explanation
$V = A \times h$	Write the formula.
$= \pi r^2 \times h$	The cross-section is a circle.
$= \pi(2)^2 \times 6$	Substitute $r = 2$ and $h = 6$.
Volume $= 75.40\text{ cm}^3$ (to 2 d.p.)	Simplify and write your answer as an approximation, with units.

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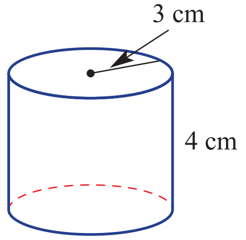
2J

5 Find the volume of these cylinders, correct to 2 decimal places.

$V = \pi r^2 \times h$

a



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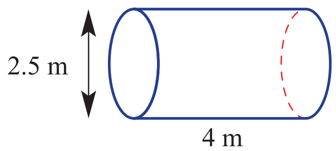
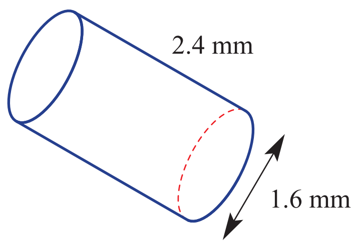


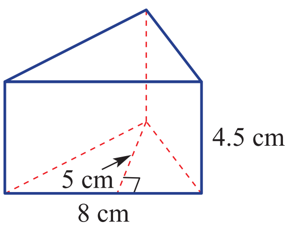
c



6 A triangle with base 8 cm and height 5 cm forms the base of a prism, as shown. If the prism stands 4.5 cm high:

a Find the area of the triangular base.

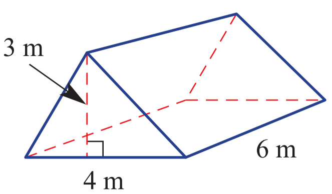
b Find the volume of the prism.



7 Find the volume of these triangular prisms.

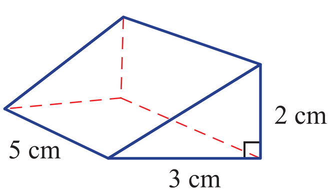
Use $V = A \times h$, where A is the area of a triangle.

a

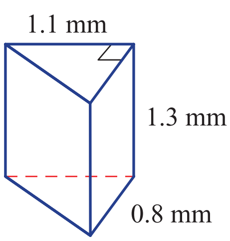


Skillsheet 2D

b



c



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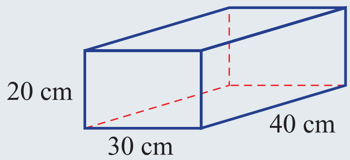
Problem-solving and Reasoning

Example 27 Working with capacity

Drilling for Gold 2J1 2J2 2J3

Find the number of litres of water this container can hold.

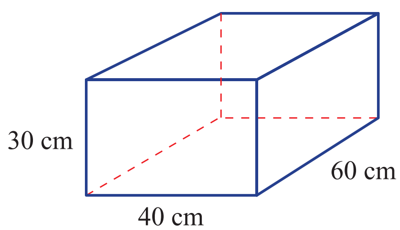


Solution	Explanation
$V = \ell bh$ $= 30 \times 40 \times 20$ Volume = 24 000 cm^3 ∴ Capacity = 24 000 mL = 24 L	First work out the volume in cm^3. Then divide by 1000 to convert to litres, since 1 cm^3 = 1 mL and there are 1000 mL in 1 litre.

8 Find the number of litres of water these containers can hold.

Use 1 L = 1000 cm^3.

a



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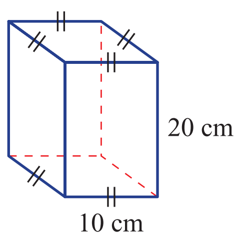

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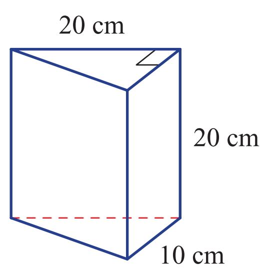

9 A cylindrical drum with a diameter of 25 cm stands on one end and water is filled to a height of 12 cm. Find the volume of water in the drum, in litres, correct to 2 decimal places.

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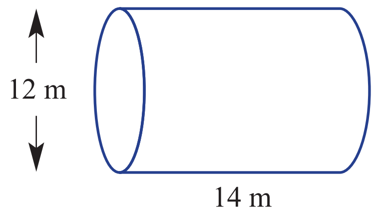
2J

10 Find the volume of these prisms, rounding your answers to 2 decimal places where necessary.

Find the area of the cross-section first.

a



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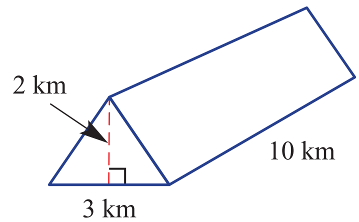

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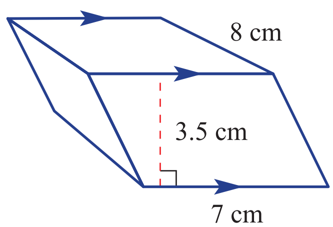
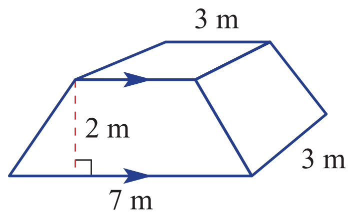
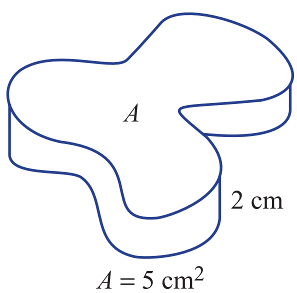


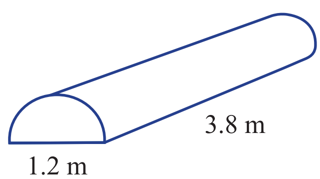
d



e



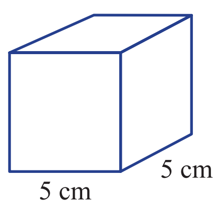
f



11 100 millilitres of water is to be poured into this cube.

a Find the area of the base of the container.

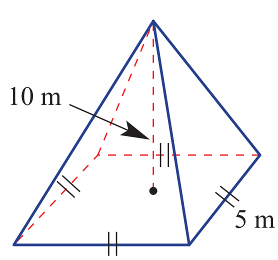
b Find the depth of water in the container.



12 The volume of a pyramid or cone is exactly one-third the volume of the prism with the same base area and height; i.e. $V = \frac{1}{3} \times A \times h$.

Find the volume of these pyramids and cones. Round to 1 decimal place where necessary.

a



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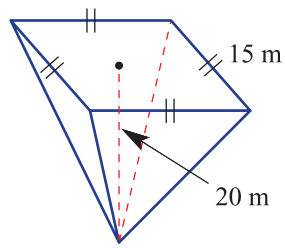

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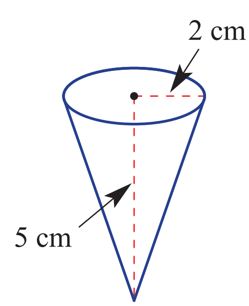

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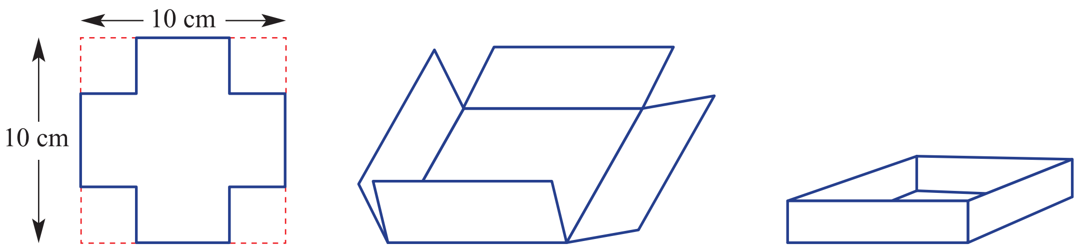
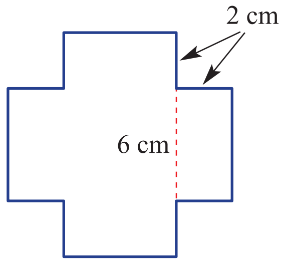
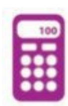
Enrichment: Maximising volume

13 Imagine that a company asks you to make a tray out of a square piece of card, measuring 10 cm by 10 cm, by cutting out four corner squares and folding them to form a tray, as shown.



a What will be the volume of the tray if the side length of the square cut-outs is:
i 1 cm? **ii** 2 cm? **iii** 3 cm?





b Which square cut-out from part **a** gives the largest tray volume?
c Can you find another sized cut-out that gives a larger volume than any of those in part **a**?
d What sized cut-out gives the maximum volume?

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Maths@home

Worksheet 2A

Keeping chickens

Many people, even some city folk, keep chickens on their property. In this activity, we will investigate the mathematics involved in planning for this and maintaining happy and healthy chickens that provide tasty eggs.

We will investigate:

- how many eggs your family eats in a typical week
- how many chickens you would need to meet that demand
- how much space the chickens will require
- how you will keep them safe from cats and foxes, especially at night, by building a low-cost chicken-friendly coop
- other costs associated with keeping and feeding chickens
- how much in food scraps you throw away each week that could be fed to the chickens.

To get started, search the internet for 'keeping chickens in New South Wales' (or 'in Australia') and find answers to the following questions.

1 How many eggs will a chicken lay every day/week/year?
2 How many chickens can you have without a permit?
3 How much does it cost to buy a simple coop and two chickens?
4 What should you feed to chickens?
5 What sort of household scraps can be fed to chickens?

Use this website to make a list of five important things to consider when designing and building accommodation for your chickens:
www.cambridge.edu.au/goldnsw10weblinks

Download the worksheet to complete the activity.

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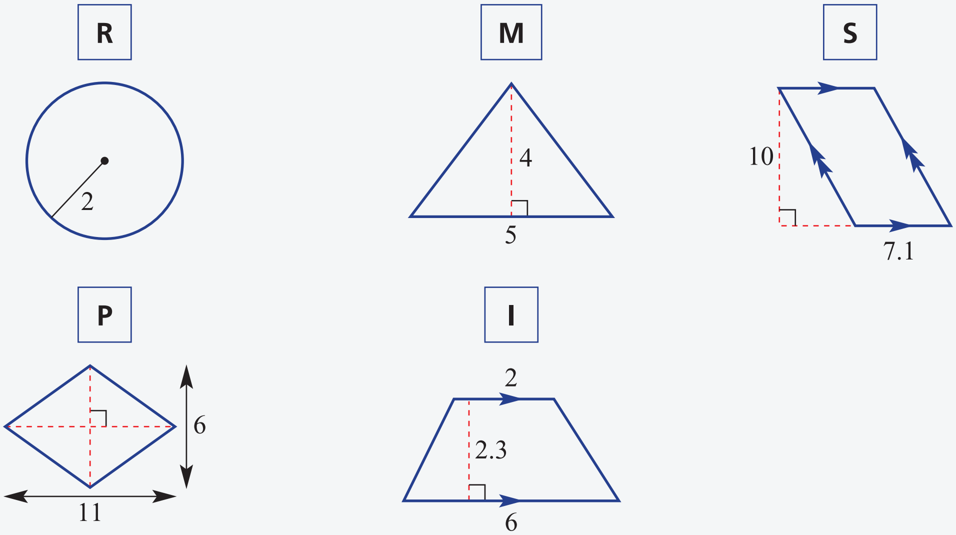
Puzzles and games

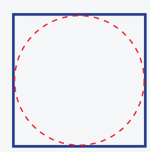
1 'I am the same shape all the way through. What am I?' Find the area of each shape. Match the letters to the answers below to solve the riddle.



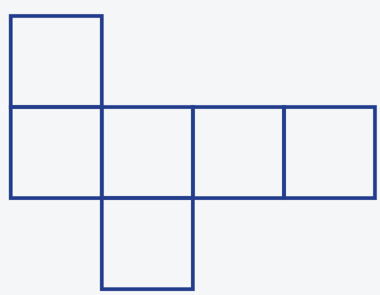
33	12.57	9.2	71	10

2 1 litre of water is poured into a container, which is in the shape of a rectangular prism. The dimensions of the prism are 8 cm by 12 cm by 11 cm. Will the water overflow?

3 A circular piece of pastry is removed from a square sheet with side length 30 cm. What percentage of pastry remains?



4 There are 11 different nets for a cube. Draw them all. Do not count reflections or rotations of the same net. Shown is one example.



5 Give the radius of a circle whose value for the circumference is equal to the value for the area.

6 A cube's surface area is 54 cm^2. What is its volume?

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Chapter summary

Measurement

Scientific notation

In scientific notation, very large or very small numbers are written in the form $a \times 10^m$, where $1 \leq a < 10$.

Large numbers will use positive powers of 10;
e.g. 2 350 000 kg = 2.35×10^6 kg

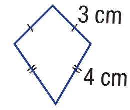
Small numbers will use negative power of 10;
e.g. 0.000000016 = 1.6×10^{-8}

Significant figures

These are counted from left to right, starting at the first non-zero digit.
e.g.
0.00217 has 3 significant figures.
1.07×10^4 has 3 significant figures.
2 403 000 might have 4, 5 or 6 significant figures.

Perimeter

The distance around the outside of a shape.

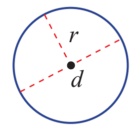


$P = 2 \times 3 + 2 \times 4$
$= 14$ cm

Circumference

The distance around the outside of a circle.

$C = 2\pi r$ or $C = \pi d$



Conversion of units

Length

km $\xrightarrow{\times 1000}$ m $\xrightarrow{\times 100}$ cm $\xrightarrow{\times 10}$ mm

mm $\xrightarrow{\div 10}$ cm $\xrightarrow{\div 100}$ m $\xrightarrow{\div 1000}$ km

Area

$km^2 \xrightarrow{\times 1000^2} m^2 \xrightarrow{\times 100^2} cm^2 \xrightarrow{\times 10^2} mm^2$

$mm^2 \xrightarrow{\div 10^2} cm^2 \xrightarrow{\div 100^2} m^2 \xrightarrow{\div 1000^2} km^2$

Volume

$km^3 \xrightarrow{\times 1000^3} m^3 \xrightarrow{\times 100^3} cm^3 \xrightarrow{\times 10^3} mm^3$

$mm^3 \xrightarrow{\div 10^3} cm^3 \xrightarrow{\div 100^3} m^3 \xrightarrow{\div 1000^3} km^3$

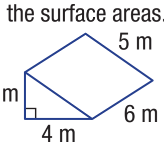
Area of basic shapes

Square: $A = s^2$
Rectangle: $A = \ell b$
Triangle: $A = \frac{1}{2}bh$
Rhombus: $A = \frac{1}{2}xy$
Parallelogram: $A = bh$
Trapezium: $A = \frac{1}{2}h(a + b)$

Surface area

Draw a net and sum the surface areas.

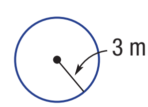
Triangular prism



$A = 2 \times \frac{1}{2} \times 4 \times 3$
$+ 6 \times 4 + 6 \times 3 + 6 \times 5$
Surface area = 84 m^2

Area of a circle

$A = \pi r^2$
$= \pi \times 3^2$
Area = 28.27 m^2 (to 2 d.p.)



Volume

Prism: $V = Ah$

Cylinder: $V = \pi r^2 h$

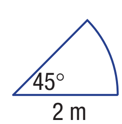
Capacity: 1 L = 1000 mL
1 cm^3 holds 1 mL.

Accuracy

Accuracy of measurement is affected by human error and measuring instruments.
Limits of accuracy give upper and lower boundaries for measurement.
Usually, $\pm$ 0.5 $\times$ smallest unit of measurement.

Area of sectors

$A = \frac{45}{360} \times \pi r^2$
$= \frac{1}{8} \times \pi \times 2^2$
Area = 1.57 m^2 (to 2 d.p.)



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T **Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.**

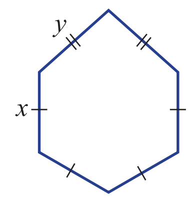
Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

1 The number of metres in a kilometre is:
A 10 **B** 100 **C** 1000 **D** 10 000 **E** 100 000

2 The perimeter of a square with side length 2 cm is:
A 4 cm **B** 8 cm **C** 4 cm^2 **D** 8 cm^2 **E** 16 cm

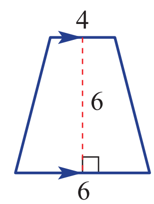
3 The perimeter of the shape shown is given by the formula:
A $x - y$ **B** $2x + y$ **C** $4x + 2y$
D $x - 2y$ **E** $4x + y$



4 A correct expression for the circumference of a circle with diameter 6 cm is:
A $\pi \times 6$ **B** $\pi \times 3$ **C** $2 \times \pi \times 6$ **D** 2×6 **E** $\pi \times 6^2$

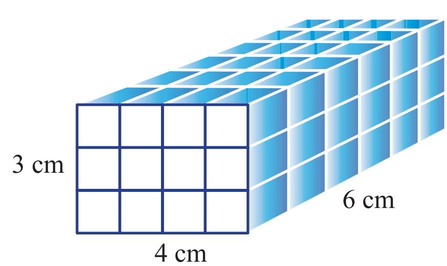
5 The area of a rectangle with side lengths 3 cm and 4 cm is:
A 12 cm^2 **B** 12 cm **C** 7 cm^2 **D** 14 cm **E** 14 cm^2

6 The correct expression for calculating the area of this trapezium is:
A $(6 - 4) \times 6$ **B** $\frac{1}{2} \times 6 \times (6 + 4)$
C $\frac{6}{2} \times (6 \times 4)$ **D** $6 \times 6 - 4$
E $6 \times 6 + 6 \times 4$



7 A sector's centre angle measures 90°. This is equivalent to:
A $\frac{1}{5}$ of a circle **B** $\frac{1}{2}$ of a circle **C** $\frac{3}{4}$ of a circle
D $\frac{2}{3}$ of a circle **E** $\frac{1}{4}$ of a circle

8 The volume of the shape shown is:
A 13 cm^3 **B** 27 cm^3 **C** 72 cm^2
D 72 cm^3 **E** 27 cm^2



9 The volume of a cube with side length 3 cm is:
A 9 cm^3 **B** 27 cm^3 **C** 54 cm^2 **D** 54 cm^3 **E** 27 cm^2

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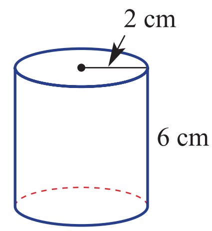
10 The capacity of this cylinder is closest to:

A 12 mL **B** 24 mL **C** 75 mL
D 76 mL **E** 750 mL



Short-answer questions

1 Convert these measurements to the units shown in brackets.

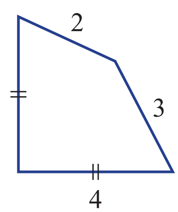
a 5.3 km (m) **b** 27 000 cm^2 (m^2) **c** 0.04 cm^3 (mm^3)
d 1 day (s) **e** 0.125 s (ms) **f** 89 000 000 KB (TB)

2 Give the lower and upper limits for these measurements.

a 6 cm **b** 4.2 kg **c** 15 mL

3 Find the perimeter of these shapes.

a



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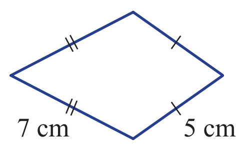

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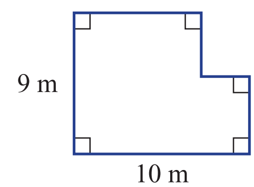
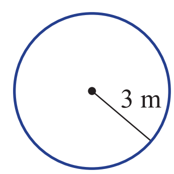


4 For the circle, find, to 2 decimal places:

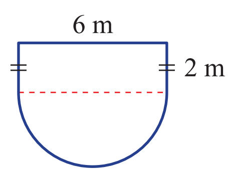
a the circumference
b the area.



5 For these composite shapes, find, to 2 decimal places:

i the perimeter **ii** the area

a



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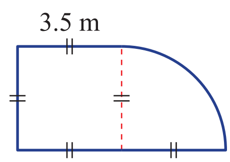

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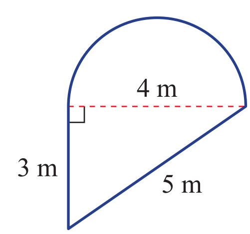
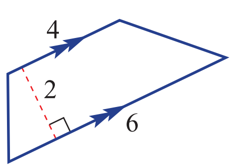


6 Find the area of these shapes.

a



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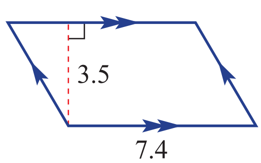

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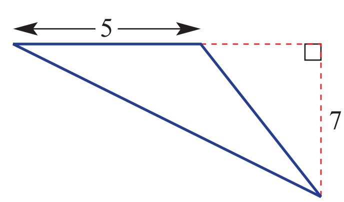

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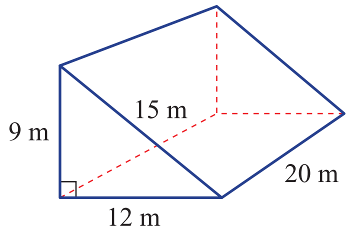
7 Find the surface area of these prisms.

a



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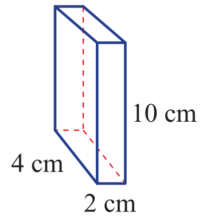
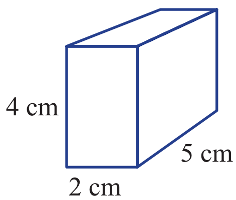


8 **a** Write 232 000 in scientific notation.
b Write 0.000232 in scientific notation.
c Write 4.54×10^6 as a number.
d Write 4.54×10^{-6} as a number.

9 Find the volume of these solids, to 2 decimal places where necessary.

a



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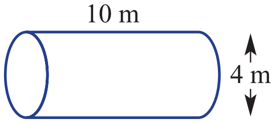

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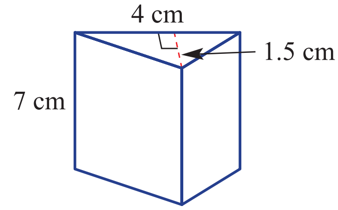
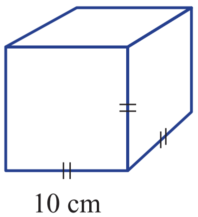
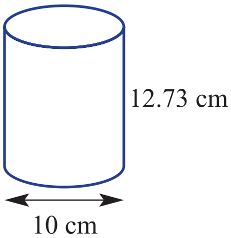


10 These two solids are to be filled with water. Which one holds the most?

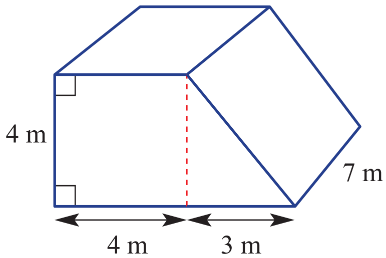




Extended-response questions

1 A cylindrical tank has diameter 8 m and height 2 m.
a Find the volume of the tank, to 2 decimal places.
b Find the capacity of the tank in litres, to 2 decimal places. Note: Every cubic metre holds 1000 litres.

2 A rabbit hutch is to be built in the shape shown.
a Use Pythagoras' theorem to find the slant height of the hutch.
b The hutch will be covered in chicken wire. Determine, in square metres, the amount of chicken wire required. Do not include the base.
c If chicken wire costs \$6 per square metre, find the cost of covering the hutch.
d What is the volume of the hutch?



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Chapter

3

Algebraic expressions and indices

What you will learn

3A Algebraic expressions
3B Simplifying algebraic expressions
3C Expanding algebraic expressions
3D Factorising algebraic expressions
Keeping in touch with numeracy
3E Index notation
3F Index laws for multiplying and dividing
3G The zero index and power of a power
3H Negative indices
Maths@home: Population growth, wage indexation and housing affordability

Strands: Number and Algebra Measurement and Geometry

Substrands: ALGEBRAIC TECHNIQUES
INDICES
NUMBERS OF ANY MAGNITUDE

In this chapter, you will learn to:

- use algebraic expressions with positive-integer and zero indices
- understand the meaning of negative indices for numerical bases
- use positive-integer and zero indices of numerical bases.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw10

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Investment returns

Combined with indices, algebra plays an important role in simplifying calculations in the financial world. Algebra and indices can be used to predict the value of an investment.

Does an average investment return of 15% sound good to you? This return is compounded annually, so a $\$10\,000$ investment would grow to more than $\$40\,000$ after 10 years. This is calculated by multiplying the investment total by 1.15 (to return the original amount plus the 15%) for each year. Using indices, the total investment value after n years would be given by

Value $= 10\,000 \times 1.15^n$.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw10

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 Write algebraic expressions for the following.
- **a** 3 lots of x
- **b** one more than a
- **c** 5 less than $2m$
- **d** 4 times the sum of x and y

2 Find the value of the following when $x = 4$ and $y = 7$.
- **a** $5x$
- **b** $2y + 3$
- **c** $xy - 5$
- **d** $2(x + y)$

3 Decide whether the following pairs of terms are like terms.
- **a** $6x$ and 8
- **b** $3a$ and $7a$
- **c** $4xy$ and $2yx$
- **d** $3x^2$ and $10x$

4 Simplify:
- **a** $3m + 5m$
- **b** $8ab - 3ab$
- **c** $4x + 3y + 2x + 5y$
- **d** $2 \times 4 \times x$
- **e** $5 \times a \times 3 \times b$
- **f** $6y \div 2$

5 Expand:
- **a** $2(x + 5)$
- **b** $3(y - 2)$
- **c** $4(2x - 3)$
- **d** $x(3x + 1)$

6 Write each of the following in index form (e.g. $5 \times 5 \times 5 = 5^3$).
- **a** $7 \times 7 \times 7 \times 7$
- **b** $4 \times 4 \times 4$

7 Evaluate:
- **a** 7^2
- **b** 3^3
- **c** 5^2
- **d** 2^4
- **e** 5^1
- **f** 4^0

8 Write the following as 3 raised to a single power.
- **a** $3^4 \times 3^3$
- **b** $3^7 \div 3^5$
- **c** $(3^2)^5$
- **d** $3^2 \times 3$
- **e** $3^4 \times 3^2$
- **f** $3^3 \times 3^2$

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3A Algebraic expressions

Stage
5.2
5.2◊
5.1
4

Algebra involves the use of pronumerals, which are letters that represent numbers.

If a ticket to an art gallery costs \$12, then the cost for y visitors is given by the expression $\$12 \times y = \$12y$. By substituting values for y we can find the costs for different numbers of visitors. For example, if there are five visitors, then $y = 5$ and $\$12y = \$12 \times 5 = \$60$.

▶ Let's start: Expressions at the gallery

Boris, Alea and Victoria are visiting the art gallery. The three of them combined have \$$c$ between them. Drinks cost \$$d$ and Boris has bought x postcards in the gift shop.
Write expressions for the following.

- the cost of two drinks
- the amount of money each person has if the money is shared equally
- the number of postcards Alea and Victoria bought if Alea bought three more than Boris and Victoria bought five less than twice the number Boris bought.

Key ideas

Drilling for Gold 3A1 3A2

- Algebraic **expressions** are made up of one or more terms connected by addition or subtraction; e.g. $3a + 7b$, $\frac{x}{2} + 3y$, $3x - 4$.
  - A **term** is a group of numbers and pronumerals connected by multiplication and division; e.g. $2x$, $\frac{y}{4}$, $5x^2$.
  - A **constant term** is a number with no attached pronumerals; e.g. 7, –3.
  - The **coefficient** is the number multiplied by the pronumerals in the term; e.g. 3 is the coefficient of y in $3y$.
    –4 is the coefficient of x in $-4x$.
    1 is the coefficient of x^2 in x^2.

  The following expression has three terms.

  $$3x + 2y + 4$$

  3 is the coefficient of x; 2 is the coefficient of y; 4 is the constant term

- Operations
  - The operations for addition and subtraction are written with '+' and '–'.
  - Multiplication is written without the sign; e.g. $3 \times y = 3y$.
  - Division is sometimes written as a fraction; e.g. $y \div 4 = \frac{y}{4}$ or $\frac{1}{4}y$.
- The value of an expression can be found by **substituting** a value for each pronumeral. The order of operations is followed. For example: If $x = 2$ and $y = 3$:

$$\begin{aligned} 4xy - y^2 &= 4 \times 2 \times 3 - 3^2 \\ &= 24 - 9 \\ &= 15 \end{aligned}$$

Expression A group of mathematical terms containing no equals sign

Term A number or pronumeral in an expression

Constant term The part of an equation or expression without any pronumerals

Coefficient A numeral placed before a pronumeral, showing that the pronumeral is multiplied by that factor

Substitute To replace pronumerals with numerical values

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Exercise 3A

Understanding

1 Fill in the missing word(s) in the sentences, using these words:
expression, term, constant term, coefficient

a An algebraic ________ is made up of one or more terms connected by addition and subtraction.
b A term without a pronumeral part is a ________.
c A number multiplied by the pronumerals in a term is a ________.
d Numbers and pronumerals connected by multiplication and division form a/an ________.

2 Decide which mathematical operation (i.e. ×, ÷, +, –) matches each of the following.

a sum **b** less than **c** product
d difference **e** more than **f** quotient

3 Substitute the value 3 for the pronumeral x in the following and then evaluate.

a $x + 4$ **b** $5x$ **c** $8 - x$ **d** x^2 **e** $\frac{18}{x}$

4 Evaluate:

a $2 \times (-3)$ **b** -4×5 **c** $2 - 8$
d $4 - 11$ **e** $7 - (-2)$ **f** $8 - (-10)$
g $-9 + 3$ **h** $-9 + 16$ **i** $-3 - 4$
j $-6 - 7$ **k** $-8 \div 2$ **l** $20 \div (-4)$

Positive × negative = negative. To subtract a negative, add its opposite: $2 - (-3) = 2 + 3$.

Fluency

Example 1 Naming parts of an expression

Consider the expression $\frac{xy}{2} - 4x + 3y^2 - 2$. Determine:

a the number of terms **b** the constant term
c the coefficient of:
i y^2 **ii** x

Solution	Explanation
a 4	There are four terms with different combinations of pronumerals and numbers, separated by + or –.
b –2	The term with no pronumerals is –2. The negative is included.
c i 3	The number multiplied by y^2 in $3y^2$ is 3.
ii –4	The number multiplied by x in $-4x$ is –4. The negative sign belongs to the term that follows.

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5 For these algebraic expressions, determine:
  i the number of terms
  ii the constant term
  iii the coefficient of y

a $4xy + 5y + 8$

b $2xy + \frac{1}{2}y^2 - 3y + 2$

c $2x^2 - 4 + y$

The coefficient is the number multiplied by the pronumerals in each term. The constant term has no pronumerals.

Drilling for Gold 3A3 at the end of this section

Example 2 Writing algebraic expressions

Write algebraic expressions for the following.

a three more than x

b 4 less than 5 times y

c the sum of c and d is divided by 3

d the product of a and the square of b

Solution	Explanation
a $x + 3$	More than means add (+).
b $5y - 4$	Times means multiply ($5 \times y = 5y$) and less than means subtract (–).
c $\frac{c + d}{3}$	Sum c and d first (+), then divide by 3 (÷). Division is written as a fraction.
d ab^2	'Product' means multiply. The square of b is b^2 (i.e. $b \times b$). $a \times b^2 = ab^2$

6 Write an expression for the following.

a two more than x

b four less than y

c the sum of ab and y

d three less than 2 lots of x

e the product of x and 5

f twice m

g three times the value of r

h half of x

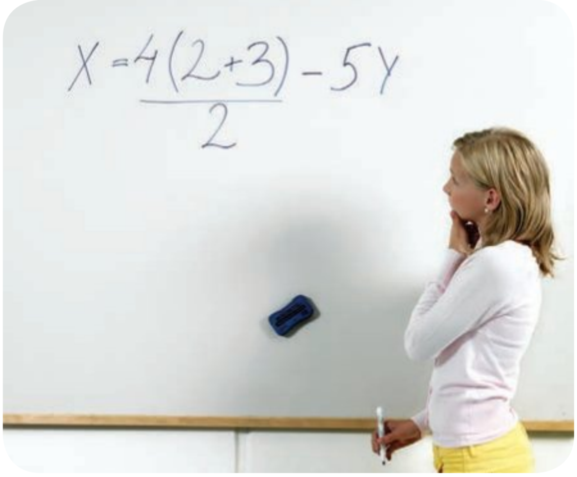
i three-quarters of m

j the quotient of x and y

k the sum of a and b is divided by 4

l the product of the square of x and y

Quotient is ÷. Product is ×. $\frac{1}{2}x = \frac{x}{2}$



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3A

Drilling for Gold 3A4 at the end of this section

Example 3 Substituting values

Find the value of these expressions when $x = 2$, $y = 3$ and $z = -5$.

a $xy + 3y$ **b** $y^2 - \frac{8}{x}$ **c** $2x - yz$

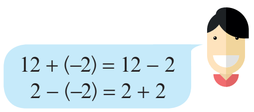
Solution	Explanation
a $xy + 3y = 2 \times 3 + 3 \times 3$ $= 6 + 9$ $= 15$	Substitute for each pronumeral: $x = 2$ and $y = 3$. Recall: $xy = x \times y$ and $3y = 3 \times y$. Simplify, following order of operations, by multiplying first.
b $y^2 - \frac{8}{x} = 3^2 - \frac{8}{2}$ $= 9 - 4$ $= 5$	Substitute $y = 3$ and $x = 2$. Do subtraction last.
c $2x - yz = 2 \times 2 - 3 \times (-5)$ $= 4 - (-15)$ $= 4 + 15$ $= 19$	Substitute for each pronumeral. $3 \times (-5) = -15$ To subtract a negative number, add its opposite.

7 Find the value of these expressions when $a = 4$, $b = -2$ and $c = 3$.

a ac **b** $2a - 5$ **c** $3a - c$ **d** $a^2 - 2c$

e $ac + b$ **f** $3b + a$ **g** $ab + c^2$ **h** $\frac{a}{2} - b$

i $\frac{ac}{b}$ **j** $2a - b$ **k** $a + bc$ **l** $\frac{6bc}{a}$



Problem-solving and Reasoning

8 Write an expression for the following.

a the cost of 5 pencils at x cents each
b the cost of y apples at 35 cents each
c one person's share when \$500 is divided among n people
d the cost of a pizza (\$11) shared between m people
e Paul's age in x years' time if he is 11 years old now

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9 A taxi has a pick-up charge (i.e. flag fall) of $3.40 and charges $2 per km.

a Write an expression for the taxi fare for a trip of d kilometres.

b Use your expression in part **a** to find the cost of a trip that is:

i 10 km **ii** 22 km

The taxi fare has initial cost + cost per km × number of km.

10 Ye thinks of a number, which we will call x.

a Write an expression for each of the following stages.

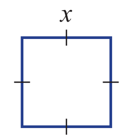
i He doubles the number.

ii He decreases the result by 3.

iii He multiplies the result by 3.

b If $x = 5$, use your answer to part **a iii** to find the final number.

11 A square of side length x is changed into a rectangle by increasing the length by 1 and decreasing the breadth by 1.



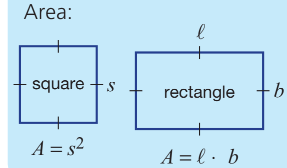
a Write an expression for the new length and breadth of the rectangle.

b Is there any change in the perimeter of the shape?

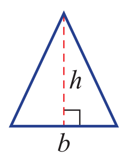
c **i** Write an expression for the area of the rectangle.

ii Use trial and error to determine whether the area of the rectangle is more or less than the original square. By how much?

Perimeter is the sum of the side lengths.
Area:



12 The area of a triangle is given by $\frac{1}{2}bh$.



a If $b = 6$ and $h = 7$, what is the area?

b If the area is 9, what are the possible whole number values for b when h is also a whole number?

Enrichment: Area and perimeter

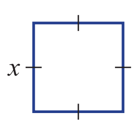
13 For the shapes shown, write an expression for the:

i perimeter

ii area

Perimeter = sum of the side lengths
Area of a rectangle = length × breadth
Area of triangle = $\frac{1}{2}$ × base × height

a



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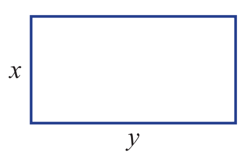

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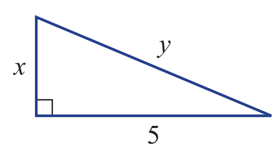

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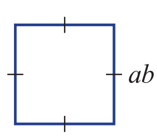

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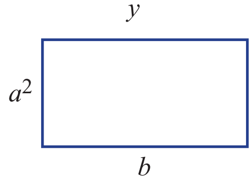

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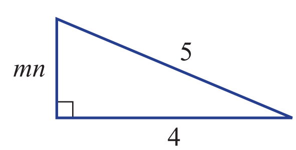

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Drilling for Gold exercise

3A3: Words and expressions match-up

Use the worksheet or write the answers in your exercise book.
Match each of the following expressions (**A**–**T**) with one of the questions (**1**–**20**).

A $a+a$	**B** $a+3$	**C** $\frac{2a}{3}$	**D** $2a+3$
E $3-a$	**F** $\frac{a}{3}$	**G** $\frac{a}{3}-1$	**H** $\frac{3a}{2}$
I $\frac{a}{2}+3$	**J** $2(a+b)$	**K** $2(a+b+c)$	**L** $3-3a$
M $3a+b$	**N** $a-3$	**O** $\frac{3a}{2}$	**P** $a+3$
Q abc	**R** $a+b+c$	**S** $3-2a$	**T** $3-a$

1 The sum of three numbers.

2 Half of a number is tripled.

3 3 more than a number.

4 A number is tripled, then the result is halved.

5 Half a number is added to 3.

6 1 less than one-third of a number.

7 A number added to itself.

8 A number is increased by 3.

9 Double the sum of two numbers.

10 3 less than a number.

11 3 more than twice a number.

12 A number is tripled and then added to another number.

13 A number is subtracted from 3.

14 The number 3 is decreased by a number.

15 A number is divided by 3.

16 The double of a number is subtracted from 3.

17 The product of three numbers.

18 A number is doubled and then divided by 3.

19 The sum of three numbers is doubled.

20 A number is tripled and then subtracted from 3.

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3A4: Substitution skill drill

Drilling for Gold exercise

Your teacher will give you numbers to place in the gaps at the top of the table.
Use the worksheet or write the answers in your exercise book.
Write out the substitution and the value for each expression.

My numbers are: $a =$ ___, $b =$ ___, $c =$ ___

Expression	Substitution	Value
1 $a + 2$		
2 $2a$		
3 $a + b + c$		
4 $a - b$		
5 $b - a$		
6 $a + b + c$		
7 $a - b + c$		
8 $2a + 5$		
9 $2(a + 5)$		
10 $-2(a + 5)$		
11 $3 - 2(a - 2)$		
12 c^2		
13 $a^2 + c^2$		
14 $(a + c)^2$		
15 $a^2 - c^2$		
16 $2b^2$		
17 $(2b)^2$		

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3B Simplifying algebraic expressions

Stage

5.2
5.2◊
5.1
4

Many areas of finance and industry involve complex algebraic expressions. Often these expressions can be made simpler by applying the operations of addition, subtraction, multiplication and division.

Just as we would write 3 + 3 + 3 + 3 as 4 × 3, we write $x + x + x + x$ as $4 \times x$ or $4x$. Similarly, $3x + 2x = 5x$ and $3x - 2x = 1x$ ($1x$ is written as x). By writing a division as a fraction we can also cancel common factors; e.g. $9x \div 3 = \frac{9x}{3} = 3x$.

▶ Let's start: Equivalent expressions

Simplify these expressions and split them into two groups: a $9x$ group and a $12x$ group.

$3x + 6x$	$17x - 5x$	$x + 7x + x$	$4x + 3 + 5x - 3$
$2 \times 6x$	$\frac{24xy}{2y}$	$3x \times 3$	$3x - 2y + 9x + 2y$
$8x + 6x - 2x$	$18x \div 2$	$\frac{9x^2}{x}$	$6x - (-6x)$

Key ideas

- **Like terms** have the exact same pronumeral factors, including powers; e.g. $3x$ and $7x$, and $4x^2y$ and $-3x^2y$.
  - $3xy$ and $2yx$ are like terms because xy is equivalent to yx.
- Expressions in which like terms are added or subtracted can be simplified.
  For example: $5x + 7x = 12x$
  $7ab - 6ab = 1ab = ab$
  But $3x + 2y$ cannot be simplified.
- Like terms are not required when multiplying and dividing.
  - In multiplication, deal with numerals and pronumerals separately.
    For example: $2 \times 8a = 2 \times 8 \times a = 16a$
    $6x \times 3y = 6 \times 3 \times x \times y = 18xy$
- When dividing, write as a fraction and cancel common factors.

  For example: $\frac{\cancel{8}^{4}x}{\cancel{2}^{1}} = 4x$ and $6x^2 \div (3x) = \frac{6x^2}{3x} = \frac{\cancel{6}^{2} \cdot \cancel{x}^{1} \cdot x}{\cancel{3}^{1} \cdot \cancel{x}^{1}} = 2x$

Like terms Terms with the same pronumerals and the same powers

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Exercise 3B

Understanding

1 Are the following sets of terms like terms? Answer yes (Y) or no (N).

a $3x, 2x, -5x$ **b** $2ax, 3xa, -ax$

c $2ax^2, 2ax, 62a^2x$ **d** $-3p^2q, 2pq^2, 4pq$

e $3ax^2y, 2ayx^2, -x^2ay$ **f** $\frac{3}{4}x^2, 2x^2, \frac{x^2}{3}$

2 Simplify the following.

a $8g + 2g$ **b** $3f + 2f$ **c** $12e - 4e$

d $3h - 3h$ **e** $5x + x$ **f** $14st + 3st$

g $7ts - 4ts$ **h** $4ab - ab$ **i** $9xy - 8xy$

Add or subtract the numerals in like terms.

3 Simplify the following.

a $3 \times 2x$ **b** $4 \times 3a$ **c** $2 \times 5m$ **d** $-3 \times 6y$

4 Simplify these fractions by cancelling.

a $\frac{4}{8}$ **b** $\frac{12}{3}$ **c** $\frac{24}{8}$

d $\frac{12}{18}$ **e** $\frac{14}{21}$ **f** $\frac{35}{15}$

g $\frac{27}{36}$ **h** $\frac{18}{45}$ **i** $\frac{20}{24}$

Choose the highest common factor to cancel.

Fluency

Drilling for Gold 3B1 at the end of this section

Example 4 Identifying like terms

Write down the like terms in the following lists.

a $3x, 6a, 2ax, 3a, 5xa$ **b** $-2ax, 3x^2a, 3a, -5x^2a, 3x$

Solution	Explanation
a $6a$ and $3a$	Both terms contain a.
$5xa$ and $2ax$	Both terms contain ax. Note: $x \times a = a \times x$.
b $3x^2a$ and $-5x^2a$	Both terms contain x^2a.

5 Write down the like terms in the following lists.

a $3ac, 2a, 5x, -2ac$ **b** $4pq, 3qp, 2p^2, -4p^2q$

c $7x^2y, -3xy^2, 2xy^2, 4yx^2$ **d** $2r^2, 3rx, -r^2, 4r^2x$

e $-2ab, 5bx, 4ba, 7xa$ **f** $3p^2q, -4pq^2, \frac{1}{2}pq, 4qp^2$

g $\frac{1}{3}lm, 2l^2m, \frac{lm}{4}, 2lm^2$ **h** $x^2y, yx^2, -xy, yx$

Like terms have the same pronumeral factors. $x \times y = y \times x$, so $3xy$ and $5yx$ are like terms.

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3B

Drilling for Gold 3B2 at the end of this section

Example 5 Collecting like terms

Simplify the following.

a $4a + 5a + 3$ **b** $3x + 2y + 5x - 3y$ **c** $5xy + 2xy^2 - 2xy + xy^2$

Solution	Explanation
a $4a + 5a + 3 = 9a + 3$	Collect like terms ($4a$ and $5a$) and add coefficients.
b $3x + 2y + 5x - 3y = 3x + 5x + 2y - 3y$ $= 8x - y$	Collect like terms. Note that $-1y$ is written as $-y$.
c $5xy + 2xy^2 - 2xy + xy^2$ $= 5xy - 2xy + 2xy^2 + xy^2$ $= 3xy + 3xy^2$	Collect like terms. The negative belongs to $2xy$. In xy^2, recall that xy^2 is $1xy^2$.

6 Simplify the following by collecting like terms.

a $4t + 3t + 10$ **b** $5g - g + 1$ **c** $3x - 5 + 4x$
d $4m + 2 - 3m$ **e** $2x + 3y + x$ **f** $3x + 4y - x + 2y$
g $8a + 4b - 3a - 6b$ **h** $2m - 3n - 5m + n$ **i** $3de + 3de^2 + 2de + 4de^2$
j $6kl - 4k^2l - 6k^2l - 3kl$ **k** $3x^2y + 2xy^2 - xy^2 + 4x^2y$ **l** $4fg - 5g^2f + 4fg^2 - fg$

Example 6 Multiplying algebraic terms

Simplify the following.

a $2a \times 7d$ **b** $-3m \times 8mn$

Solution	Explanation
a $2a \times 7d = 2 \times 7 \times a \times d$ $= 14ad$	Multiply coefficients and collect the pronumerals: $2 \times a \times 7 \times d = 2 \times 7 \times a \times d$. Multiplication can be done in any order.
b $-3m \times 8mn = -3 \times 8 \times m \times m \times n$ $= -24m^2n$	Multiply coefficients ($-3 \times 8 = -24$) and pronumerals. Recall: $m \times m$ can be written as m^2.

7 Simplify:

a $3r \times 2s$ **b** $2h \times 3u$ **c** $4w \times 4h$ **d** $2r^2 \times 3s$
e $-2e \times 4s$ **f** $5h \times (-2v)$ **g** $-3c \times (-4m^2)$ **h** $-7f \times (-5l)$
i $2x \times 4xy$ **j** $3ab \times 8a$ **k** $xy \times 3y$ **l** $-2a \times 8ab$
m $-3m^2n \times 4n$ **n** $-5xy^2 \times (-4x)$ **o** $5ab \times 4ab$ **p** $-8xy \times 6xy$

Multiply the numerals and collect the pronumerals.
Recall: $a \times b = ab$

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Drilling for Gold 3B3 3B4 at the end of this section

Example 7 Dividing algebraic terms

Simplify the following.

a $\dfrac{18x}{6}$

b $12a^2b \div (8ab)$

Solution	Explanation
a $\dfrac{\cancel{18}^{3}x}{\cancel{6}_{1}} = 3x$	Cancel the highest common factor of numerals; i.e. 6.
b $12a^2b \div (8ab) = \dfrac{12a^2b}{8ab}$ $= \dfrac{^{3}\cancel{12} \cdot a \cdot \cancel{a}_{1} \cdot \cancel{b}_{1}}{_{2}\cancel{8} \cdot \cancel{a}_{1} \cdot \cancel{b}_{1}}$ $= \dfrac{3a}{2}$	Write division as a fraction. Cancel the highest common factor of 12 and 8, and cancel an a and b.

8 Simplify by cancelling common factors.

a $\dfrac{7x}{14}$ **b** $\dfrac{6a}{2}$ **c** $3a \div 9$

d $2ab \div 8$ **e** $\dfrac{4ab}{2a}$ **f** $\dfrac{15xy}{5y}$

g $4xy \div (8x)$ **h** $28ab \div (35b)$ **i** $\dfrac{8x^2}{20x}$

j $\dfrac{12xy^2}{18y}$ **k** $30a^2b \div (10a)$ **l** $12mn^2 \div (36mn)$

Write each division as a fraction first, where necessary.

Problem-solving and Reasoning

9 A rectangle's length is three times its breadth, x. Write a simplified expression for the rectangle's:

a perimeter

b area

Draw a rectangle and label the breadth x and the length $3x$.

10 Fill in the missing term to make the following true.

a $8x + 4 - \square = 3x + 4$ **b** $3x + 2y - \square + 4y = 3x - 2y$

c $3b \times \square = 12ab$ **d** $4xy \times (\square) = -24x^2y$

e $12xy \div (\square) = 6y$ **f** $\square \div (15ab) = \dfrac{2a}{3}$

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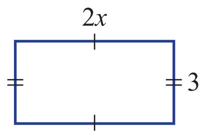
3B

11 Find expressions in simplest form for the perimeter (P) and area (A) of these shapes.

a



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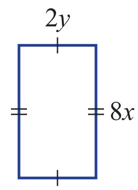

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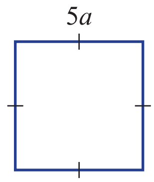
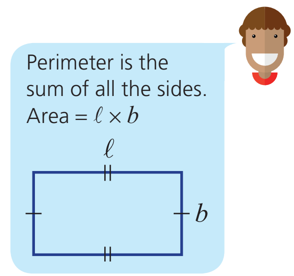
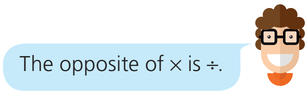
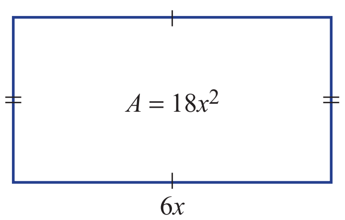




12 A rectangular garden bed has its length given by $6x$ and area $18x^2$. What is the breadth of the garden bed?





Enrichment: Order of operations

13 Simplify the following expressions, using order of operations.

a $4 \times 3x \div 2$

b $2 + 4a \times 2 + 5a \div a$

c $5a \times 2b \div a - 6b$

d $8x^2 \div (4x) + 3 \times 3x$

e $2x \times (4x + 5x) \div 6$

f $5xy - 4x^2y \div (2x) + 3x \times 4y$

g $(5x - x) \times (16xy \div (8y))$

h $9x^2y \div (3y) + 4x \times (-8x)$

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3B1: Like or unlike?

Drilling for Gold exercise

Examine the terms and then decide if they are like terms or not.
Use the worksheet or write the answers in your exercise book.

	Terms		Are they like terms? (Yes or No)
1	a	$2b$	
2	$2a$	2	
3	$2a$	$3a$	
4	$2a$	$2a$	
5	$2x$	$2y$	
6	$2b$	$3b$	
7	$2a$	$2ab$	
8	ab	ba	
9	ab	$2ba$	
10	a	a^2	
11	ab	a^2b	
12	ab^2	a^2b	
13	ab^2	b^2a	
14	abc	$2cab$	
15	$(ab)^2$	ab^2	

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Drilling for Gold exercise

3B2: Skill drill – Adding and subtracting like terms

If you are using the worksheet, circle the equivalent expressions and then highlight the simplest correct answer. If you are writing your answers in your exercise book, write out:

a the equivalent expression(s)

b the simplest correct answer

1 $3x + 2x$ is equivalent to
$2x + 3x$ | $5x$ | $6x$ | $5xx$ | $5x^2$

2 $3x + x$ is equivalent to
$x + 3x$ | $5x$ | $4x$ | $3xx$ | $3x^2$

3 $3x - x$ is equivalent to
$x - 3x$ | 3 | $2x$ | $x + x$

4 $3x - 3x$ is equivalent to
x | $0x$ | 0 | 1

5 $6x - 5x$ is equivalent to
$5x - 6x$ | $-5x + 6x$ | $1x$ | 1 | x

6 $x + x$ is equivalent to
$x + 1$ | $2x$ | xx | $x - x$ | x^2

7 $x - x$ is equivalent to
$x + x$ | $0x$ | 0 | x | 1

8 $4x - 5x$ is equivalent to
$5x - 4x$ | x | $-1x$ | $-x$

9 $x + x - x$ is equivalent to
$2x - x$ | $1x$ | x | $x + 0$

10 $x - x - x$ is equivalent to
x | $-x$ | $-1x$

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3B4: Sum, differences, product, quotients

Consider the expressions $8x$ and $6x$. Look at the six results given below for their sum, differences, product and quotients.
Note the way fractions are used for the quotients.

A $8x + 6x = 14x$ **B** $8x - 6x = 2x$ **C** $6x - 8x = -2x$

D $8x \times 6x = 48x^2$ **E** $8x \mid 6x = \frac{4}{3}$ **F** $6x \mid 8x = \frac{3}{4}$

Write out the sum, differences, product and quotients for the following pairs of expressions. Simplify the answers as much as possible.
Use the worksheet or write the answers in your exercise book.

1 $3x$ and $6x$

2 $3x$ and 3

3 $6x$ and 6

4 $6a$ and $4b$

5 $4a$ and $6b$

6 $2x$ and $5x$

7 x^2 and xy

8 $4x$ and $2x^2$

Drilling for Gold exercise

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3C Expanding algebraic expressions

The expression $2(3 + 4)$ means $2 \times (3 + 4)$. It can be simplified in two ways:

$$\begin{aligned} 2 \times (3 + 4) &= 2 \times 7 \\ &= 14 \end{aligned}$$

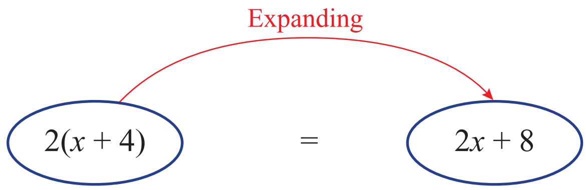
This method *cannot* be used to expand $2(x + 4)$.

OR

$$\begin{aligned} 2 \times (3 + 4) &= 2 \times 3 + 2 \times 4 \\ &= 6 + 8 \\ &= 14 \end{aligned}$$

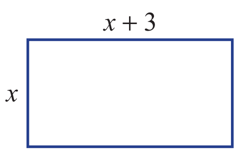
This is called the distributive law.
It can also be applied as follows:

$$\begin{aligned} 2(x + 4) &= 2 \times x + 2 \times 4 \\ &= 2x + 8 \end{aligned}$$



▶ Let's start: Rectangle brackets

A rectangle's length is 3 more than its breadth.



- Write down as many expressions as you can, both with and without brackets, for the rectangle's:
  - perimeter
  - area
- Can you explain why all the expressions for the perimeter are equivalent?
- Can you explain why all the expressions for the area are equivalent?

Key ideas

- The **distributive law** is used to expand and remove brackets.
- The terms inside the brackets are multiplied by the term outside the brackets.

$$a(b + c) = ab + ac \qquad a(b - c) = ab - ac$$

For example: $2(x + 4) = 2 \times x + 2 \times 4 = 2x + 8$ and $2(x - 4) = 2 \times x - 2 \times 4 = 2x - 8$

Distributive law
Adding numbers and then multiplying the total gives the same answer as multiplying each number first and then adding the products

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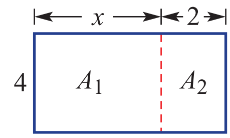
Exercise 3C

Understanding

1 Consider the diagram shown.



- **a** Write an expression for the area A_1.
- **b** Write an expression for the area A_2.
- **c** Add your results from parts **a** and **b** to give the area of the rectangle.
- **d** Write an expression for the total length of the rectangle.
- **e** Using part **d**, write an expression for the area of the rectangle.
- **f** Combine your results to complete this statement: $4(x+2) = \square + \square$.

2 Multiply the following expressions involving negatives.

- **a** $2 \times (-4)$
- **b** $3 \times (-6)$
- **c** $3 \times (-x)$
- **d** $4 \times (-2x)$
- **e** -4×5
- **f** $-2 \times 8x$
- **g** $-5 \times (-3)$
- **h** $-6 \times (-4)$
- **i** $-2x \times (-3)$

negative × positive = negative
negative × negative = positive

3 Complete the following.

- **a** $3(x+4) = 3 \times \square + 3 \times \square$
  $= 3x + \square$
- **b** $2(x-5) = 2 \times \square + \square \times (-5)$
  $= \square - 10$
- **c** $2(4x+3) = 2 \times \square + \square \times 3$
  $= \square + 6$
- **d** $x(x-3) = x \times \square + \square \times (\square)$
  $= \square - \square$

4 Simplify the following.

- **a** $3 \times 2x$
- **b** $4x \times 2y$
- **c** $3x \times 5x$
- **d** $5 + 2x + 4$
- **e** $3x + 9 + 4x - 4$
- **f** $5x + 10 - 2x - 14$

Fluency

Example 8 Expanding expressions with brackets

Expand the following.

- **a** $2(x+5)$
- **b** $3(2x-3)$
- **c** $3y(2x+4y)$

Solution	Explanation
a $2(x+5) = 2 \times x + 2 \times 5$ $= 2x + 10$	Multiply each term inside the brackets by 2.
b $3(2x-3) = 3 \times 2x + 3 \times (-3)$ $= 6x - 9$	Multiply $2x$ and -3 by 3. $3 \times 2x = 3 \times 2 \times x = 6x$
c $3y(2x+4y) = 3y \times 2x + 3y \times 4y$ $= 6xy + 12y^2$	Multiply $2x$ and $4y$ by $3y$. $3y \times 2x = 3 \times 2 \times x \times y$ and $3y \times 4y$. $= 3 \times 4 \times y \times y$ Recall: $y \times y$ is written as y^2.

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3C

5 Expand the following.

a $2(x + 4)$ **b** $3(x + 7)$ **c** $4(y - 3)$
d $5(y - 2)$ **e** $2(3x + 2)$ **f** $4(2x + 5)$
g $3(3a - 4)$ **h** $7(2y - 5)$ **i** $5(2a + b)$
j $3(4a - 3b)$ **k** $2x(x + 5)$ **l** $3x(x - 4)$
m $2a(3a + 2b)$ **n** $2y(3x - 4y)$ **o** $3b(2a - 5b)$

Use the distributive law:

$a(b + c) = a \times b + a \times c$
$= ab + ac$

$a(b - c) = a \times b + a \times (-c)$
$= ab - ac$

Example 9 Expanding expressions with a negative at the front

Expand the following.

a $-3(x - 4)$ **b** $-2x(3x - 2y)$

Solution	Explanation
a $-3(x - 4) = -3 \times x + (-3) \times (-4)$ $= -3x + 12$	Multiply each term inside the brackets by -3. $-3 \times (-4) = +12$ If there is a negative sign outside the brackets, the sign of each term inside the brackets is changed when expanded.
b $-2x(3x - 2y) = -2x \times 3x + (-2x) \times (-2y)$ $= -6x^2 + 4xy$	$-2x \times 3x = -2 \times 3 \times x \times x$ and $-2x \times (-2y) = -2 \times (-2) \times x \times y$

6 Expand the following.

a $-2(x + 3)$ **b** $-5(m + 2)$ **c** $-3(w + 4)$
d $-4(x - 3)$ **e** $-2(m - 7)$ **f** $-7(w - 5)$
g $-(x + y)$ **h** $-(x - y)$ **i** $-2x(3x + 4)$
j $-3x(2x + 5)$ **k** $-4x(2x - 2)$ **l** $-3y(2y - 9)$
m $-2x(3x - 5y)$ **n** $-3x(3x + 2y)$ **o** $-6y(2x + 3y)$

A negative at the front will change the sign of each term inside the brackets when expanded; e.g. $-2(x - 3) = -2x + 6$

Example 10 Simplifying expressions by removing brackets

Expand and simplify the following.

a $8 + 3(2x - 3)$ **b** $3(2x + 2) + 4(x + 4)$

Solution	Explanation
a $8 + 3(2x - 3) = 8 + 6x - 9$ $= 6x - 1$	Expand the brackets: $3 \times 2x + 3 \times (-3)$ $= 6x - 9$ Collect like terms: $8 - 9 = -1$.
b $3(2x + 2) + 4(x + 4) = 6x + 6 + 4x + 16$ $= 10x + 22$	Expand the brackets first. Collect like terms: $6x + 4x = 10x$ and $6 + 16 = 22$.

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Skillsheet 3A

7 Expand and simplify the following.

a $2 + 5(x + 3)$ **b** $3 + 7(x + 2)$ **c** $5 + 2(x - 3)$
d $7 + 2(x + 3)$ **e** $21 + 5(x + 4)$ **f** $4 + 3(2x - 1)$
g $3(x + 2) + 4(x + 3)$ **h** $2(p + 2) + 5(p - 3)$ **i** $4(x - 3) + 2(3x + 4)$
j $3(2s + 3) + 2(s + 2)$ **k** $4(3f + 2) + 2(6f + 2)$ **l** $3(2x - 5) + 2(2x - 4)$

Expand first, then collect like terms.

Problem-solving and Reasoning

8 Fill in the missing term/number to make each statement true.

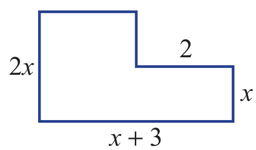
a $\square(x + 4) = 2x + 8$ **b** $\square(2x - 3) = 8x - 12$
c $\square(2x + 3) = 6x^2 + 9x$ **d** $4(\square + 5) = 12x + 20$
e $4y(\square - \square) = 4y^2 - 4y$ **f** $-2x(\square + \square) = -4x^2 - 6xy$

9 Four rectangular rooms in a house have floor side lengths listed below. Find an expression for the area of each floor, in expanded form.

a 2 and $x - 5$ **b** x and $x + 3$
c $2x$ and $x + 4$ **d** $3x$ and $2x - 1$

Area of a rectangle = length × breadth

10 The deck on a house is constructed in the shape shown. Find the area of the deck, in expanded form.



11 Virat earns $\$x$, where x is greater than 18 200, but he does not have to pay tax on the first \$18 200.

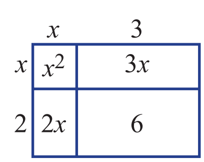
a Write an expression for the amount of money Virat is taxed.
b Virat is taxed 10% of his earnings in part **a**. Write an expanded expression for how much tax he pays.

To find 10% of an amount, multiply by $\frac{10}{100} = 0.1$.

Enrichment: Expanding binomial products

12 A rectangle has dimensions $(x + 2)$ by $(x + 3)$, as shown. The area can be found by summing the individual areas:

$(x + 2)(x + 3) = x^2 + 3x + 2x + 6$
$= x^2 + 5x + 6$



This can be done using the distributive law:

$(x + 2)(x + 3) = x(x + 3) + 2(x + 3)$
$= x^2 + 3x + 2x + 6$
$= x^2 + 5x + 6$

Expand and simplify these binomial products, using this method.

a $(x + 4)(x + 3)$ **b** $(x + 3)(x + 1)$ **c** $(x + 2)(x + 5)$
d $(x + 2)(x - 4)$ **e** $(x + 5)(x - 2)$ **f** $(x + 4)(2x + 3)$
g $(2x + 3)(x - 2)$ **h** $(x - 3)(x + 4)$ **i** $(4x - 2)(x + 5)$

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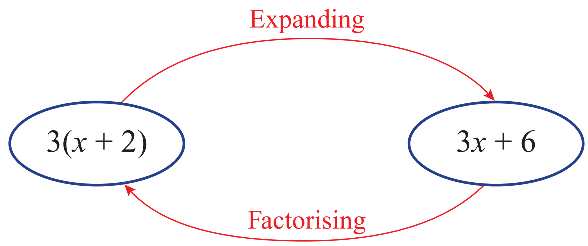
3D Factorising algebraic expressions

Stage: 5.2, 5.2◊, 5.1, 4

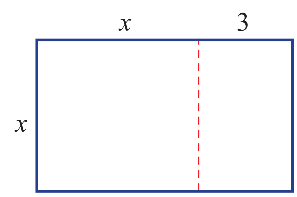
We know that $3(x + 2)$ can be expanded to give $3x + 6$. Similarly, $3x + 6$ can be factorised to give $3(x + 2)$. Factorising is the opposite of expanding.



Let's start: Factorised areas

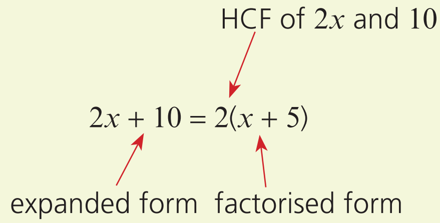
Here is a rectangle of length $(x + 3)$ and breadth x.

- Write an expression for the total area using the given length and breadth.
- Write an expression for the total area by adding up the area of the two smaller regions.
- Are your two expressions equivalent? How could you work from your second expression (expanded) to the first expression (factorised)?



Key ideas

- Factorisation and expansion are opposite processes.
- When **factorising** expressions with common factors, take out the highest common factor (HCF). The HCF could be:
  - a number
    For example: $2x + 10 = 2(x + 5)$
  - a pronumeral
    For example: $x^2 + 5x = x(x + 5)$
  - the product of numbers and variables
    For example: $2x^2 + 10x = 2x(x + 5)$



- A factorised expression can be checked by using expansion.

  For example: $2x(x + 5) = 2x^2 + 10x$

Factorise To write an expression as a product

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Exercise 3D

Understanding

1 Write down the highest common factor (HCF) of these pairs of numbers.

a 8, 12 **b** 10, 20 **c** 5, 60 **d** 24, 30
e 6, 3 **f** 100, 75 **g** 16, 24 **h** 36, 72

2 Write down the missing factor.

a $5 \times$ ___ $= 5x$ **b** $7 \times$ ___ $= 7x$ **c** ___ $\times x = 4x$
d $a \times$ ___ $= ab$ **e** $3a \times$ ___ $= 3ab$ **f** ___ $\times 2b = 6ab$
g $2a \times$ ___ $= 2a^2$ **h** $5a \times$ ___ $= 10a^2$ **i** ___ $\times 3y = -6y^2$
j ___ $\times 12x = -36x^2$ **k** $-3 \times$ ___ $= 6x$ **l** $-2x \times$ ___ $= 20x^2$

3 a Write down the missing factor in each part.

i ___ $(x^2 + 2x) = 6x^2 + 12x$
ii ___ $(2x + 4) = 6x^2 + 12x$
iii ___ $(x + 2) = 6x^2 + 12x$

b Which equation above uses the HCF of $6x^2$ and $12x$?

Expand to check.

4 Consider the expression $4x^2 + 8x$.

a Which of the following factorised forms uses the HCF?

A $2(2x^2 + 4x)$ **B** $4(x^2 + 8x)$ **C** $4x(x + 2)$ **D** $2x(2x + 4)$

b What can be said about the terms inside the brackets once the HCF is removed, which is not the case for the other forms?

Fluency

Example 11 Finding the HCF

Determine the HCF of the following.

a $8a$ and 20 **b** $6a$ and $8ab$ **c** $3x^2$ and $6xy$

Solution	**Explanation**
a HCF of $8a$ and 20 is 4.	a is NOT a common factor.
b HCF of $6a$ and $8ab$ is $2a$.	HCF of 6 and 8 is 2. HCF of a and ab is a.
c HCF of $3x^2$ and $6xy$ is $3x$.	HCF of 3 and 6 is 3. HCF of x^2 and xy is x.

5 Determine the HCF of the following.

a $6x$ and 12 **b** 10 and $15y$ **c** $8a$ and $12b$
d $6x$ and $14xy$ **e** $12a$ and $18a$ **f** $10m$ and 4
g $12y$ and 8 **h** $15t$ and $6s$ **i** 15 and p
j $9x$ and $24xy$ **k** $6n$ and $21mn$ **l** $10y$ and $2y$
m $8x^2$ and $14x$ **n** $4x^2y$ and $18xy$ **o** $5ab^2$ and $15a^2b$

HCF stands for highest common factor.

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3D

Example 12 Factorising expressions

Factorise the following.

a $4x + 12$ b $6a - 15b$ c $40 - 16b$

Solution	Explanation
a $4x + 12 = 4(x + 3)$	4 is the HCF of $4x$ and 12. $4x \div 4 = x$ and $12 \div 4 = 3$. Check your answer by expansion.
b $6a - 15b = 3(2a - 5b)$	HCF is 3. Place 3 in front of the brackets and divide each term by 3.
c $40 - 16b = 8(5 - 2b)$	The HCF of 40 and $16b$ is 8. Place 8 in front of the brackets and divide each term by 8.

6 Factorise the following.

a $7x + 7$ b $3x + 3$ c $4x - 4$ d $5x - 5$
e $4 + 8y$ f $10 + 5a$ g $3 - 9b$ h $6 - 2x$
i $12a + 3b$ j $6m + 6n$ k $10x - 8y$ l $4a - 20b$
m $x^2 + 2x$ n $a^2 - 4a$ o $y^2 - 7y$ p $x - x^2$
q $3p^2 + 3p$ r $8x - 8x^2$ s $4b^2 + 12b$ t $6y - 10y^2$

Always take out the highest common factor (HCF) and check your answer by expanding.

Example 13 Factorising expressions with pronumeral common factors

Factorise the following.

a $8y + 12xy$ b $4x^2 - 10x$

Solution	Explanation
a $8y + 12xy = 4y(2 + 3x)$	HCF of 8 and 12 is 4. HCF of y and xy is y. Place $4y$ in front of the brackets and divide each term by $4y$. Check that $4y(2 + 3x) = 8y + 12xy$.
b $4x^2 - 10x = 2x(2x - 5)$	HCF of $4x^2$ and $10x$ is $2x$. Place $2x$ in front of the brackets and divide each term by $2x$. Recall: $x^2 = x \times x$.

Skillsheet 3B

7 Factorise the following.

a $14x + 21xy$ b $6ab - 15b$ c $32y - 40xy$
d $5x^2 - 5x$ e $x^2 + 7x$ f $2a^2 + 8a$
g $12a^2 + 42ab$ h $9y^2 - 63y$ i $6x^2 + 14x$
j $9x^2 - 6x$ k $16y^2 + 40y$ l $10m - 40m^2$

Place the HCF in front of the brackets and divide each term by the HCF:
$14x + 21xy = 7x(\_\_ + \_\_)$

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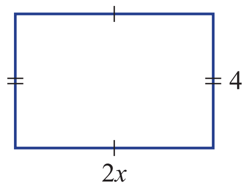
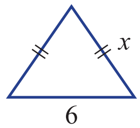
Problem-solving and Reasoning

8 Write the missing number or expression.

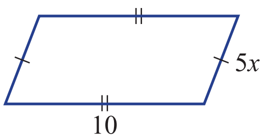
a $3x + 9 = \_\_\_(x + 3)$ **b** $xy + x = x(\_\_\_ + 1)$
c $a^2 - a = \_\_\_(a - 1)$ **d** $5xy + 10x = \_\_\_(y + 2)$
e $-7a - 14 = \_\_\_(a + 2)$ **f** $-24a^2 - 36a = \_\_\_(2a + 3)$

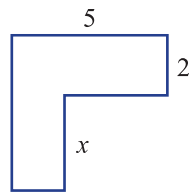
9 Write down the perimeter of these shapes in factorised form.

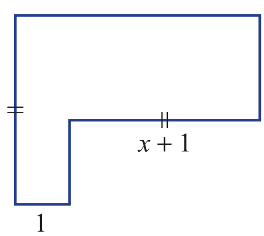
a


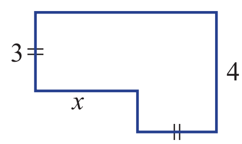
b


First find a simplified expression for the perimeter, then factorise it.

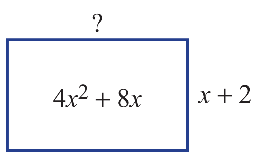
c


d


e


f


10 The expression for the area of a rectangle is $(4x^2 + 8x)$. Find an expression for its breadth when the length is $(x + 2)$.



11 $7 \times 9 + 7 \times 3$ can be evaluated by first factorising to $7(9 + 3)$.
This gives $7 \times 12 = 84$. Use a similar technique to evaluate the following.

a $9 \times 2 + 9 \times 5$ **b** $6 \times 3 + 6 \times 9$ **c** $-2 \times 4 - 2 \times 6$
d $-5 \times 8 - 5 \times 6$ **e** $23 \times 5 - 23 \times 2$ **f** $63 \times 11 - 63 \times 8$

Enrichment: Further factorisation

12 Common factors can also be removed from expressions with more than two terms.
For example: $2x^2 + 6x + 10xy = 2x(x + 3 + 5y)$
Factorise these expressions by taking out the HCF.

a $3a^2 + 9a + 12$ **b** $5z^2 - 10z + zy$ **c** $x^2 - 2xy + x^2y$
d $4by - 2b + 6b^2$ **e** $-12xy - 8yz - 20xyz$ **f** $3ab + 4ab^2 + 6a^2b$

13 You can factorise some expressions by taking out a binomial factor. For example: $3(x - 2) + x(x - 2) = (x - 2)(3 + x)$
Factorise the following by taking out a binomial common factor.

In the example, $(x - 2)$ is the common factor.

a $4(x + 3) + x(x + 3)$ **b** $3(x + 1) + x(x + 1)$ **c** $7(m - 3) + m(m - 3)$
d $x(x - 7) + 2(x - 7)$ **e** $8(a + 4) - a(a + 4)$ **f** $5(x + 1) - x(x + 1)$
g $y(y + 3) - 2(y + 3)$ **h** $a(x + 2) - x(x + 2)$ **i** $t(2t + 5) + 3(2t + 5)$
j $m(5m - 2) + 4(5m - 2)$ **k** $y(4y - 1) - (4y - 1)$ **l** $(7 - 3x) + x(7 - 3x)$

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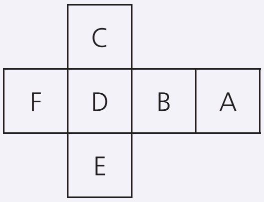
Keeping in touch with numeracy

Non-calculator

1 What is 10% of \$500?

3 If the net below is folded into a cube, what letter will be opposite A?



5 Today is Thursday. What day will it be 20 days from now?

7 $7 + 8 + 9 + 10 + 11 + 12 = ?$

9 $-2 \times -3 \times -4 = ?$

11 Find the value when 3.75 is:
- **a** multiplied by 100
- **b** divided by 1000
- **c** subtracted from 10

13 Given that $\frac{544}{32} = 17$, what is $\frac{5440}{3.2}$?

15 Fifteen people are ordering pizzas. The pizzas are cut into 8 pieces. Every person will get at least 3 pieces. What is the smallest number of pizzas that must be ordered?

17 A cube has edges 5 metres long.
- **a** Find the volume.
- **b** Find the surface area.

19 Lucy is 11 years old and Sofia is 17. When their ages add up to 48, how old will Lucy be?

Calculator

2 What is 11.5% of \$470?

4 The outer surface area of a cube is 3.84 square centimetres. How long is each edge of the cube?

6 Today is January 17. What will be the date 20 days from now?

8 Jess started reading a book from the top of page 7 and stopped at the bottom of page 20. How many pages did she read?

10 The temperature was –20°C. It fell by 3.5°C, then rose by 12.7°C. What is the temperature now?

12 Find the value when 3.75 is:
- **a** multiplied by 2.7
- **b** divided by 1.25
- **c** subtracted from 4.1

14 Pens cost \$1.48 and pencils cost 79 cents. I have \$100 to buy 35 pens and some pencils. How many pencils can I buy?

16 A small circular pizza tray has diameter 15 cm. If the diameter is doubled, by what factor is the area multiplied?

18 For the cube in Question **17**:
- **a** Find the cost of filling the cube with petrol, which costs \$1.19 per litre.
- **b** Find the number of litres required to paint the inside walls and floor of the cube. The paint covers 16 square metres per litre.

20 Anna has half as much money as Blake, who has half as much money as Katya. Together they have \$87.50. How much money does Anna have?

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3E Index notation

Stage: 5.2, 5.2◊, 5.1, 4

When a product has the same number multiplied by itself over and over, index notation can be used to write a simpler expression. For example:

- $5 \times 5 \times 5$ can be written as 5^3
- $x \times x \times x \times x \times x$ can be written as x^5.

Index notation is a way to carry out calculations, such as how much mass is lost over time from ancient stone monuments.

▶ Let's start: Who has the most?

A person offers you one of two prizes.

- Which offer would you take?
- Try to calculate the final amount for prize B.
- How might you use index notation to help calculate the value of prize B?
- How can a calculator help to find the amount for prize B using the power button?

Prize A
$1000 now

Prize B
1 cent, then the amount is doubled every day for 20 days

Key ideas

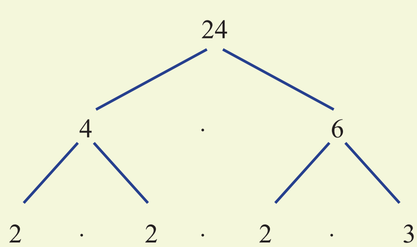
- When a number is multiplied by itself many times, that product can be written using **index form**. For example,

Expanded form: $2 \times 2 \times 2 \times 2 \times 2$ = Index form: $2^5 = 32$ (2 is the base, 5 is the index, 32 is the basic numeral)

$x \times x \times x \times x = x^4$ (x is the base, 4 is the index)

- The **base** is the factor in the product.
- The **index** is the number of times the factor (base number) appears.
  - 2^2 reads '2 to the power of 2' or '2 squared', where $2^2 = 4$.
  - 2^3 reads '2 to the power of 3' or '2 cubed', where $2^3 = 8$.
  - 2^5 reads '2 to the power of 5', where $2^5 = 32$.

  Note that $a^1 = a$. For example: $5^1 = 5$.
  - 3^2 does *not* mean $3 \times 2 = 6$.
- **Prime factorisation** involves writing a number as a product of its prime factors.
  - A prime number has only two factors: 1 and itself.



Prime factor form: $24 = 2 \times 2 \times 2 \times 3$

$= 2^3 \times 3$ (Index form)

Index form A way of writing numbers that are multiplied by themselves

Base A number or pronumeral that is being raised to a power

Index The number of times a factor is repeated under multiplication

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Exercise 3E

Understanding

1 Fill in the missing word(s) in the sentences, using these words:
indices, *index*, *index*, *base*, *prime*, *expanded*, *power*, *prime*, *factor*, *factors*

a The product $3 \times 3 \times 3 \times 3$ is called the ________ or ________ form of 81.
b 3^4 is called the ________ form of 81.
c 3^4 reads '3 to the ________ of 4'.
d In 3^4 the special name for the 3 is the ________ number.
e In 3^4 the special name for the 4 is the ________ number.
f A ________ number has only two factors, itself and 1.
g Prime factorisation involves writing a number as a product of its _______ _______.
h The plural of index is ______.

2 Evaluate the following.
a 5^2 b 2^3 c 3^3 d $(-4)^2$

3 Write the number or pronumeral that is the base in these expressions.
a 3^7 b 6^4 c $(1.2)^5$ d $(-7)^3$
e $\left(\frac{2}{3}\right)^4$ f y^{10} g w^6 h t^2

4 Write the number that is the index in these expressions.
a 4^3 b 10^8 c $(-3)^7$ d $\left(\frac{1}{2}\right)^4$
e x^{11} f $(xy)^{13}$ g $\left(\frac{x}{2}\right)^9$ h $(1.3x)^2$

5 Use a factor tree to write the prime factors of these numbers.
a 6 b 15 c 30 d 77

Fluency

Example 14 Writing in expanded form

Write the following in expanded form.
a 5^4 b a^3 c $(xy)^4$ d $2a^3b^2$

Solution	Explanation
a $5^4 = 5 \times 5 \times 5 \times 5$	Factor 5 appears four times.
b $a^3 = a \times a \times a$	Factor a appears three times.
c $(xy)^4 = xy \times xy \times xy \times xy$	Factor xy appears four times.
d $2a^3b^2 = 2 \times a \times a \times a \times b \times b$	Factor a appears three times and factor b appears twice. Factor 2 appears only once.

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6 Write each of the following in expanded form.

a 4^3	**b** 7^4	**c** 3^5	**d** 5^3
e a^4	**f** b^3	**g** x^3	**h** $(xp)^6$
i $(5a)^4$	**j** $(3y)^3$	**k** $4x^2y^5$	**l** $(pq)^2$
m $-3s^3t^2$	**n** $6x^3y^5$	**o** $5(yz)^6$	**p** $4(ab)^3$

factor → a^5 ← number of times a numeral appears

$a^5 = a \times a \times a \times a \times a$

Example 15 Expanding and evaluating

Write each of the following in expanded form and then evaluate.

a 5^3 **b** $(-2)^5$ **c** $\left(\frac{2}{5}\right)^3$

Solution	Explanation
a $5^3 = 5 \times 5 \times 5$ $= 125$	Write in expanded form with 5 appearing three times, then evaluate.
b $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2)$ $= -32$	Write in expanded form with −2 appearing five times, then evaluate.
c $\left(\frac{2}{5}\right)^3 = \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5}$ $= \frac{8}{125}$	Write in expanded form. Evaluate by multiplying numerators and denominators.

7 Write each of the following in expanded form and then evaluate.

a 6^2	**b** 2^4	**c** 3^5	**d** 12^1
e $(-2)^3$	**f** $(-1)^7$	**g** $(-3)^4$	**h** $(-5)^2$
i $\left(\frac{2}{3}\right)^3$	**j** $\left(\frac{3}{4}\right)^2$	**k** $\left(\frac{1}{6}\right)^3$	**l** $\left(\frac{5}{2}\right)^2$
m $\left(\frac{2}{-3}\right)^3$	**n** $\left(\frac{-3}{4}\right)^4$	**o** $\left(\frac{-1}{4}\right)^2$	**p** $\left(\frac{5}{-2}\right)^5$

$\underbrace{-2 \times (-2)} \times (-2)$
$= 4 \times (-2)$
$= -8$

Example 16 Writing in index form

Write each of the following in index form.

a $5 \times 5 \times 5 \times 5$ **b** $6 \times x \times x \times x \times x$ **c** $4 \times a \times 4 \times a \times 4 \times a$

Solution	Explanation
a $5 \times 5 \times 5 \times 5 = 5^4$	Factor 5 appears 4 times.
b $6 \times x \times x \times x \times x = 6x^4$	Factor x appears 4 times; 6 appears only once.
c $4 \times a \times 4 \times a \times 4 \times a$ $= 4 \times 4 \times 4 \times a \times a \times a$ $= 4^3a^3$	Group the factors of 4 together and the factors of a together. Write in index form.

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3E

8 Write each of the following in index form.

a $3 \times 3 \times 3$
b $8 \times 8 \times 8 \times 8 \times 8 \times 8$
c $y \times y$
d $3 \times x \times x \times x$
e $4 \times c \times c \times c \times c \times c$
f $5 \times 5 \times 5 \times d \times d$
g $x \times x \times y \times y \times y$
h $7 \times b \times 7 \times b \times 7$

The index or power is the number of appearances of a factor.

Example 17 Writing in index form with fractions

Write each of the following in index form.

a $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$

b $\frac{3}{7} \times \frac{3}{7} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$

Solution	Explanation
a $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \left(\frac{3}{4}\right)^3$	The fraction $\frac{3}{4}$ appears 3 times.
b $\frac{3}{7} \times \frac{3}{7} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \left(\frac{3}{7}\right)^2 \times \left(\frac{4}{5}\right)^3$	$\left(\frac{3}{7}\right)$ appears twice and $\left(\frac{4}{5}\right)$ appears three times.

9 Write each of the following in index form.

a $\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$

b $\frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5}$

c $\frac{4}{7} \cdot \frac{4}{7} \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$

d $\frac{7x}{9} \cdot \frac{7x}{9} \cdot \frac{y}{4} \cdot \frac{y}{4} \cdot \frac{y}{4}$

Example 18 Writing in index form with a combination of pronumerals

Write each of the following in index form.

a $8 \times a \times a \times 8 \times b \times b \times a \times b$
b $3a \times 2m \times 3a \times 2m$
c $4am(4am)(4am)$

Solution	Explanation
a $8 \times a \times a \times 8 \times b \times b \times a \times b$ $= 8 \times 8 \times a \times a \times a \times b \times b \times b$ $= 8^2a^3b^3$	Group the numerals and like pronumerals, and write in index form. $64a^3b^3$ and $64(ab)^3$ are alternative answers.
b $3a \times 2m \times 3a \times 2m$ $= 2 \times 2 \times 3 \times 3 \times a \times a \times m \times m$ $= 2^23^2a^2m^2$	Rearrange so that like factors are grouped together, and write in index form. $36a^2m^2$ and $36(am)^2$ are alternative answers.
c $4am(4am)(4am)$ $= 4 \times 4 \times 4 \times a \times a \times a \times m \times m \times m$ $= 4^3a^3m^3$	Rearrange and write in index form. $64a^3m^3$ and $64(am)^3$ are alternative answers.

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10 Write each of the following in index form.

a $3 \times x \times y \times x \times 3 \times x \times 3 \times y$ **b** $3x \times 2y \times 3x \times 2y$
c $4d \times 2e \times 4d \times 2e$ **d** $6by(6by)(6y)$
e $3pq(3pq)(3pq)(3pq)$ **f** $7mn \times 7mn \times mn \times 7$

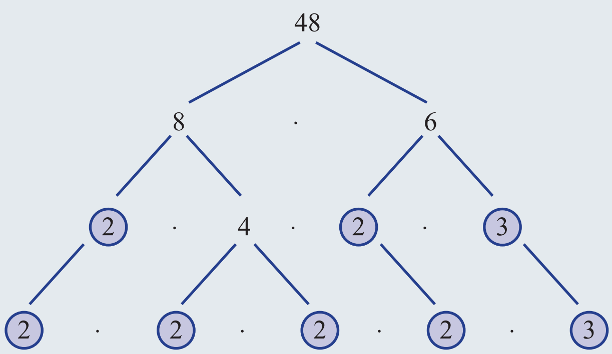
First rearrange the factors with numbers, then form groups of like bases.

Example 19 Finding the prime factor form

Express 48 as a product of prime factors in index form. Prime numbers are divisible only by 1 and themselves.

Solution

Factor tree method



$48 = 2 \times 2 \times 2 \times 2 \times 3$
$= 2^4 \times 3$

Explanation

Choose a pair of factors of 48; for example, 8 and 6.
Choose a pair of factors of 8; i.e. 2 and 4.
Choose a pair of factors of 6; i.e. 2 and 3.
Continue this process until the factors are all prime numbers.

Write the prime factors of 48.
Express in index notation.

Repeated division method

2	48
2	24
2	12
2	6
3	3
	1

Start by dividing by a prime number.
$48 \div 2 = 24$
$24 \div 2 = 12$
$12 \div 2 = 6$
$6 \div 2 = 3$
$3 \div 3 = 1$

$48 = 2 \times 2 \times 2 \times 2 \times 3$
$= 2^4 \times 3$

Write prime factors in ascending order.
Express in index notation.

11 Express each of the following as a product of prime factors in index form.

a 10 **b** 8 **c** 144
d 75 **e** 147 **f** 500

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3E

Problem-solving and Reasoning

12 Copy and fill in the missing numbers or symbols.

a $3 \times 3 \times a \times a \times a = 3^{\square}a^{\square}$

b $\square \times \square \times k \times k \times k = 5^2k^{\square}$

c $\frac{2}{7} \times \frac{2}{7} \times \frac{2}{7} = \left(\frac{2}{7}\right)^{\square}$

d $3p^2q = 3 \times \square \times \square \times q$

e $(abc)^1 = a \square b \square c$

Example 20 Evaluating expressions in index form after substitution

When $a = 5$, $b = 2$ and $c = -5$, evaluate these expressions.

a $(ab)^2$

b $\left(\frac{a}{c}\right)^3$

c $\left(\frac{b}{c}\right)^2$

Solution	Explanation
a $(ab)^2 = (5 \times 2)^2$ $= 10^2$ $= 10 \times 10$ $= 100$	Replace a with 5 and b with 2. Include the × sign. $5 \times 2 = 10$ is done first. Base of 10 is repeated twice.
b $\left(\frac{a}{c}\right)^3 = \left(\frac{5}{-5}\right)^3$ $= (-1)^3$ $= -1 \times (-1) \times (-1)$ $= -1$	Replace a with 5 and c with –5. $5 \div (-5) = -1$ $-1 \times (-1) = +1, \quad +1 \times (-1) = -1$
c $\left(\frac{b}{c}\right)^2 = \left(\frac{2}{-5}\right)^2$ $= \frac{2}{-5} \times \frac{2}{-5}$ $= \frac{4}{25}$	Replace b with 2 and c with –5. Write in expanded form. $2 \times 2 = 4, \quad -5 \times (-5) = 25$

13 When $a = 3$, $b = 2$ and $c = -3$, evaluate these expressions.

a $(ab)^2$ **b** $(bc)^3$ **c** $\left(\frac{a}{c}\right)^4$ **d** $\left(\frac{b}{c}\right)^3$

e $(abc)^1$ **f** $c^2 + ab$ **g** ab^2c **h** c^2ab^3

$abc = a \times b \times c$

14 Find the missing number.

a $3^{\square} = 81$ **b** $2^{\square} = 256$ **c** $\square^3 = 125$ **d** $\square^5 = 32$

e $\square^3 = -64$ **f** $\square^7 = -128$ **g** $\square^3 = \frac{1}{8}$ **h** $\left(\frac{2}{3}\right)^{\square} = \frac{16}{81}$

For part **a**, ask 'How many times is 3 repeated so that the product is 81?'

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Enrichment: Splitting cells

15 Certain bacterial cells divide into two cells every minute. New cells also continue splitting in the same way. So, after each minute, the number of bacteria cells has doubled.

a Copy and complete this table showing the number of bacteria after each minute for 10 minutes.

Time in minutes	Number of bacteria	Number in index form
0	1	2^0
1	$1 \times 2 = 2$	2^1
2	$2 \times 2 = 4$	2^2
3	$2 \times 2 \times 2 = 8$	2^3

b How long will it take for 1 cell to divide into:
i 4 cells? ii 16 cells? iii 64 cells?

c A single cell is set aside to divide for 24 minutes. Use index form to quickly find how many cells there will be after this time.

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3F Index laws for multiplying and dividing

Stage

5.2
5.2◊
5.1
4

When multiplying or dividing numbers with the same base, index laws can be used to simplify the expression.

Consider $5^{18} \times 5^{10}$:

Using expanded form: $5^{18} \times 5^{10} = \underbrace{\overbrace{5 \times 5 \times 5 \times \ldots \times 5}^{\text{18 factors of 5}} \times \overbrace{5 \times 5 \times \ldots \times 5}^{\text{10 factors of 5}}}_{\text{18 + 10 factors of 5}}$

$$= 5^{18+10}$$
$$= 5^{28}$$

So, the total number of factors of 5 is $18 + 10 = 28$.

Also, $5^{18} \div 5^{10} = \dfrac{\overbrace{5 \cdot 5 \cdot \ldots \cdot \not{5} \cdot \not{5} \cdot \not{5} \cdot \not{5}}^{\text{18 factors of 5}}}{\underbrace{\not{5} \cdot \ldots \cdot \not{5} \cdot \not{5}}_{\text{10 factors of 5}}}$

$$= 5^{18-10}$$
$$= 5^{8}$$

So, the total number of factors of 5 is $18 - 10 = 8$.

Let's start: Discovering the index laws

Consider the two expressions $2^3 \times 2^5$ and $6^8 \div 6^6$.

Complete this working.

$$2^3 \times 2^5 = 2 \times \square \times \square \times 2 \times \square \times \square \times \square \times \square$$
$$= 2^{\square}$$

$$6^8 \div 6^6 = \frac{6 \times \square \times \square \times \square \times \square \times \square \times \square \times \square}{6 \times \square \times \square \times \square \times \square \times \square}$$
$$= \frac{6 \cdot 6}{1}$$
$$= 6^{\square}$$

- What do you notice about the given expression and the answer in each case? Can you express this as a rule or law in words?
- Repeat the type of working given above and test your laws on these expressions.
  a $3^2 \times 3^7$ **b** $4^{11} \div 4^8$

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Key ideas

- Index law for multiplication: $a^m \times a^n = a^{m+n}$
  - When multiplying terms with the same base, add the indices.
    For example: $7^3 \times 7^2 = 7^{3+2} = 7^5$
- Index law for division: $a^m \div a^n = \dfrac{a^m}{a^n} = a^{m-n}$
  - When dividing terms with the same base, subtract the indices.
    For example: $8^5 \div 8^3 = \dfrac{8^5}{8^3} = 8^{5-3} = 8^2$

Exercise 3F

Understanding

1 Write the missing words.

a When you multiply two terms with the same _______, you _______ the indices.

b When you divide two terms with the same _______, you _______ the indices.

2 Copy and complete the following to give answers in index form.

a $3^2 \times 3^4 = 3 \times \square \times 3 \times \square \times \square \times \square$
$= 3^{\square}$

b $6^4 \times 6^3 = 6 \times \square \times \square \times \square \times 6 \times \square \times \square$
$= 6^{\square}$

c $k^3 \times k^2 = k \times \square \times \square \times \square \times \square$
$= k^{\square}$

d $m^4 \times m^3 = m \times \square \times \square \times \square \times \square \times \square \times \square$
$= m^{\square}$

e $5^3 \times 5 = \square \times \square \times \square \times \square$
$= \square^{\square}$

f $2 \times 2^4 = \square \times \square \times \square \times \square \times \square$
$= \square^{\square}$

3 Copy and complete the following to give answers in index form.

a $5^5 \div 5^3 = \dfrac{5 \times \square \times \square \times \square \times \square}{5 \times \square \times \square}$
$= 5^{\square}$

b $9^4 \div 9^2 = \dfrac{9 \times \square \times \square \times \square}{9 \times \square}$
$= 9^{\square}$

c $a^6 \div a^2 = \dfrac{a \times \square \times \square \times \square \times \square \times \square}{\square \times \square}$
$= a^{\square}$

d $\dfrac{n^6}{n^3} = \dfrac{n \times \square \times \square \times \square \times \square \times \square}{\square \times \square \times \square}$
$= n^{\square}$

e $x^4 \div x = \dfrac{x \times \square \times \square \times \square}{\square}$
$= x^{\square}$

f $x^4 \div x^3 = \dfrac{x \times \square \times \square \times \square}{\square \times \square \times \square}$
$= x^{\square}$
$= \square$

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3F

4 Copy and complete the following.

a $6^5 \times 6^7 = 6^{\square + \square} = 6^{\square}$ b $a^{13} \times a^2 = a^{\square + \square} = a^{\square}$

c $5^{12} \div 5^4 = 5^{\square - \square} = 5^{\square}$ d $\dfrac{m^{16}}{m^2} = m^{\square - \square} = m^{\square}$

When dividing, subtract the indices.

Fluency

Example 21 Multiplying and dividing with a common numerical base

Simplify the following, giving your answers in index form.

a $3^6 \times 3^4$ b $4^5 \times 4$ c $7^9 \div 7^5$ d $6^8 \div 6$

Solution	Explanation
a $3^6 \times 3^4 = 3^{6+4}$ $= 3^{10}$	$a^m \times a^n = a^{m+n}$ Add indices: $6 + 4 = 10$. The base 3 is unchanged.
b $4^5 \times 4 = 4^{5+1}$ $= 4^6$	$4 = 4^1$ Add indices: $5 + 1 = 6$. The base 4 is unchanged.
c $7^9 \div 7^5 = 7^{9-5}$ $= 7^4$	$a^m \div a^n = a^{m-n}$ Subtract indices: $9 - 5 = 4$. The base 7 is unchanged.
d $6^8 \div 6 = 6^{8-1}$ $= 6^7$	$6 = 6^1$ Subtract indices: $8 - 1 = 7$. The base 6 is unchanged.

5 Simplify the following, giving your answers in index form.

a $2^4 \times 2^3$ b $5^6 \times 5^3$ c $7^2 \times 7^4$ d $8^9 \times 8$

e $3^4 \times 3^4$ f $6^5 \times 6^9$ g $3^7 \div 3^4$ h $6^8 \div 6^3$

i $5^4 \div 5$ j $10^6 \div 10^5$ k $9^9 \div 9^6$ l $(-2)^5 \div (-2)^3$

Remember: $8 = 8^1$.

Example 22 Multiplying with non-numerical bases

Simplify each of the following, using the index law for multiplication.

a $x^4 \times x^5 \times x^2$ b $x^3y^4 \times x^2y$

Solution	Explanation
a $x^4 \times x^5 \times x^2 = x^{4+5+2}$ $= x^{11}$	Add the indices since all terms have base x.
b $x^3y^4 \times x^2y = x^3x^2y^4y$ $= x^{3+2}y^{4+1}$ $= x^5y^5$	Regroup so like indices are together. Add the indices corresponding to each different base. Recall that $y = y^1$.

6 Simplify each of the following.

a $x^2 \times x^4$ b $x \times x^4$ c $b^2 \times b^2$ d $b^2 \times b$

e $x^4 \times x^3$ f $a^6 \times a^3$ g $t^5 \times t^3$ h $y \times y^4$

i $d^2 \times d$ j $y^2 \times y \times y^4$ k $b \times b^5 \times b^2$ l $q^6 \times q^3 \times q^2$

m $a^2m^2 \times a^3m^2$ n $k^3p^2 \times k^2p$ o $x^2y^3 \times x^4y^5$ p $m^5e^3 \times m^2e$

Check that the bases are the same before adding the indices.

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Example 23 Using the index law for division

Simplify $x^{10} \div x^2$, using the index law for division.

Solution	Explanation
$x^{10} \div x^2 = x^{10-2}$ $= x^8$	Subtract the indices: $10 - 2 = 8$. The base x is unchanged.

7 Simplify each of the following.

a $5^7 \div 5^2$ **b** $5^7 \div 5$ **c** $10^8 \div 10^3$ **d** $10^8 \div 10^7$

e $a^6 \div a^4$ **f** $x^5 \div x^2$ **g** $\dfrac{q^{12}}{q^2}$ **h** $\dfrac{d^7}{d^6}$

i $\dfrac{b^{10}}{b^5}$ **j** $\dfrac{d^9}{d^4}$ **k** $\dfrac{a^{14}}{a^7}$ **l** $\dfrac{y^{15}}{y^{14}}$

Recall:
$\dfrac{q^{12}}{q^2} = q^{12} \div q^2$

Example 24 Simplifying expressions using index laws

Simplify each of the following, using the index laws for multiplication or division.

a $3m^4 \times 2m^5$ **b** $12y^7 \div (4y^3)$ **c** $\dfrac{8a^6}{12a^2}$

Solution	Explanation
a $3m^4 \times 2m^5 = 3 \times 2 \times m^4 \times m^5$ $= 6 \times m^{4+5}$ $= 6m^9$	Regroup with numbers first, then like bases together. Multiply the numbers, then add the indices of the base m.
b $12y^7 \div 4y^3 = \dfrac{12y^7}{4y^3}$ $= 3y^{7-3}$ $= 3y^4$	$12 \div 4 = 3$ Subtract the indices.
c $\dfrac{8a^6}{12a^2} = \dfrac{8}{12} \times \dfrac{a^6}{a^2}$ $= \dfrac{2}{3}a^4$ or $\dfrac{2a^4}{3}$	$\dfrac{8}{12} = \dfrac{2}{3}$ in simplest form. Subtract the indices. $6 - 2 = 4$

8 Simplify, using the index laws.

a $2x^2 \times 3x^3$ **b** $2x^4 \times x^2$ **c** $4a \times 2ab$ **d** $2p^2 \times p^3$

e $c^4 \times 3c^4$ **f** $2s^4 \times 3s^7$ **g** $3a^2b^2 \times 4a$ **h** $3a^2b^2 \times 4b$

i $7x^3y^3 \times x^4y^2$ **j** $3x^7y^3 \times x^2y$ **k** $5x^3y^5 \times xy^4$ **l** $xy^4z \times 4xy$

m $3m^3 \times 5m^2$ **n** $4e^4f^2 \times 2e^2f^2$ **o** $5c^4d \times 4c^3d$ **p** $9yz^2 \times 2yz^5$

q $9m^3 \div (3m^2)$ **r** $14x^4 \div (2x)$ **s** $5y^4 \div y^2$ **t** $6a^6 \div (2a^5)$

u $\dfrac{36m^7}{12m^2}$ **v** $\dfrac{5w^2}{25w}$ **w** $\dfrac{4a^4}{20a^3}$ **x** $\dfrac{7x^5}{63x}$

Rearrange first, and group numbers and like bases together.

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3F

Problem-solving and Reasoning

Example 25 Combining index laws

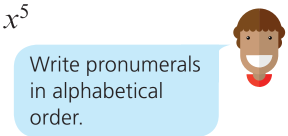
Simplify $x^2 \times x^3 \div x^4$, using the index laws.

Solution	Explanation
$x^2 \times x^3 \div x^4 = x^5 \div x^4$	Add the indices for $x^2 \times x^3$.
$= x$	Subtract the indices for $x^5 \div x^4$.

9 Simplify each of the following.

a $b^5 \times b^2 \div b$ **b** $y^5 \times y^4 \div y^3$ **c** $c^4 \div c \times c^4$ **d** $x^4 \times x^2 \div x^5$

e $\dfrac{t^4 \cdot t^3}{t^6}$ **f** $\dfrac{p^2 \cdot p^7}{p^3}$ **g** $\dfrac{d^5 \cdot d^3}{d^2}$ **h** $\dfrac{x^9 \cdot x^2}{x}$



10 Write the missing number.

a $2^7 \times 2^{\square} = 2^{19}$ **b** $6^{\square} \times 6^3 = 6^{11}$ **c** $11^6 \div 11^{\square} = 11^3$

d $19^{\square} \div 19^2 = 19$ **e** $x^6 \times x^{\square} = x^7$ **f** $a^{\square} \times a^2 = a^{20}$

g $b^{13} \div b^{\square} = b$ **h** $y^{\square} \div y^9 = y^2$ **i** $\square \times x^2 \times 3x^4 = 12x^6$

j $15y^4 \div (\square y^3) = y$ **k** $\square a^9 \div (4a) = \dfrac{a^8}{2}$ **l** $13b^6 \div (\square b^5) = \dfrac{b}{3}$

11 Evaluate without using a calculator. Leave your answers in index form.

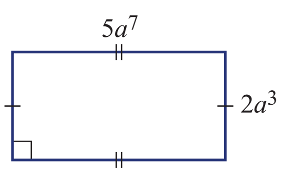
a $7^7 \div 7^5$ **b** $10^6 \div 10^5$ **c** $13^{11} \div 13^9$

d $2^{20} \div 2^{17}$ **e** $101^5 \div 101^4$ **f** $200^{30} \div 200^{28}$

Enrichment: Areas and index notation

12 a Write the area of each of these shapes using index notation.

i



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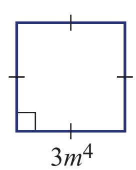

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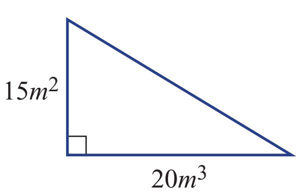

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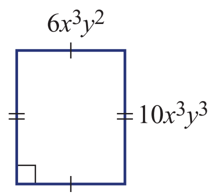
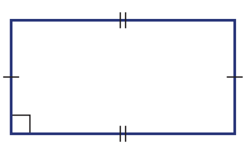
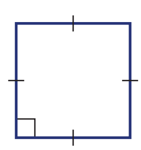


b Find the breadth of each of these shapes using index notation.

i Area $= 24a^3m^2$



ii Area $= 36x^2y^4$



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3G The zero index and power of a power

Stage

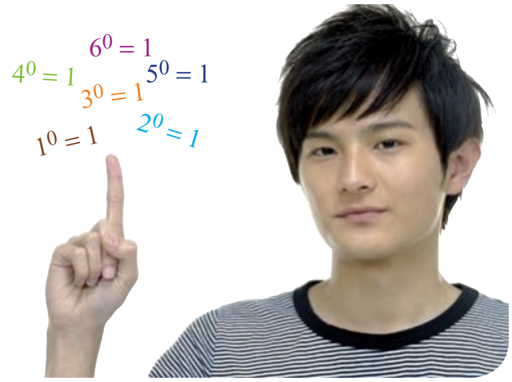
5.2
5.2◊
5.1
4

Sometimes we find that expressions already written in index form are raised to another power, such as $(2^3)^4$ or $(a^2)^5$.

Consider $(a^2)^5$.

$$\begin{aligned}(a^2)^5 &= a^2 \times a^2 \times a^2 \times a^2 \times a^2 \\ &= a \times a \times a \times a \times a \times a \times a \times a \times a \times a \\ &= a^{10}\end{aligned}$$

The power of 0 has a special property.



Any number raised to the power of zero is 1.

Consider $\frac{a^3}{a^3}$.

Simplify using expanded form:

$$\frac{a^3}{a^3} = \frac{\cancel{a}^1 \cdot \cancel{a}^1 \cdot \cancel{a}^1}{\cancel{a}^1 \cdot \cancel{a}^1 \cdot \cancel{a}^1}$$
$$= 1$$

Simplify by subtracting indices:

$$\frac{a^3}{a^3} = a^{3-3}$$
$$= a^0$$

So $a^0 = 1$

▶ Let's start: Power of a power and the zero index

Use the expanded form of 5^3 to simplify $(5^3)^2$, as shown.

$$\begin{aligned}(5^3)^2 &= 5 \times \square \times \square \times 5 \times \square \times \square \\ &= 5^{\square}\end{aligned}$$

- Repeat these steps to also simplify $(3^2)^4$ and $(x^4)^2$.
- What do you notice about the given expression and answer in each case? Can you express this as a law or rule in words?

Now copy and complete this table.

Index form	3^5	3^4	3^3	3^2	3^1	3^0
Basic numeral	243	81				

- What pattern do you notice in the basic numerals?
- What conclusion do you come to regarding 3^0?

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Key ideas

- Index law for power of a power: $(a^m)^n = a^{m \times n} = a^{mn}$
  - When raising a term in index form to another power, retain the base and multiply the indices.
    For example: $(x^2)^3 = x^{2 \times 3} = x^6$
  - A power outside brackets applies only to the expression inside those brackets.
    For example: $5(a^3)^2 = 5a^{3 \times 2} = 5a^6$ but $(5a^3)^2 = 5^2 \times a^6 = 25a^6$.
- The zero index: $a^0 = 1$, where $a \neq 0$
  - Any term except 0 raised to the power of zero is 1.
    For example: $5^0 = 1$, $m^0 = 1$ and $(2a)^0 = 1$.

Exercise 3G

Understanding

1 Write the missing words or numbers in these sentences.

a When raising a term or numbers in index form to another power, _________ the indices.

b Any number (except 0) raised to the power 0 is equal to ____.

Indices means powers.

2 Copy and complete these tables.

a

Index form	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Basic numeral	64	32					

b

Index form	4^5	4^4	4^3	4^2	4^1	4^0
Basic numeral	1024	256				

3 Copy and complete this working.

a $(4^2)^3 = 4^2 \times 4^2 \times 4^2$
$= (4 \cdot \square) \cdot (4 \cdot \square) \cdot (4 \cdot \square)$
$= 4^{\square}$

b $(12^3)^2 = 12^{\square} \times 12^{\square}$
$= (12 \cdot \square \cdot \square) \cdot (12 \cdot \square \cdot \square)$
$= 12^{\square}$

c $(x^4)^2 = x^{\square} \times x^{\square}$
$= (x \cdot \square \cdot \square \cdot \square) \cdot (x \cdot \square \cdot \square \cdot \square)$
$= x^{\square}$

d $(a^2)^3 = a^{\square} \times a^{\square} \times a^{\square}$
$= (a \cdot \square) \cdot (a \cdot \square) \cdot (a \cdot \square)$
$= a^{\square}$

e $(2x^3)^2 = (2x^3) \times (2x^3)$
$= \square x^{\square}$

f $(3x^2)^3 = (3x^2) \times (3x^2) \times (3x^2)$
$= \square x^{\square}$

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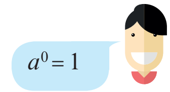
4 Find the value of each of the following.

a 6^0 **b** 21^0 **c** 2^0 **d** 1^0

e $(3.7)^0$ **f** 582^0 **g** $\left(\frac{3}{4}\right)^0$ **h** 2760^0



Fluency

Example 26 Simplifying a power of a power

Simplify each of the following.

a $(x^5)^4$ **b** $3(y^5)^2$

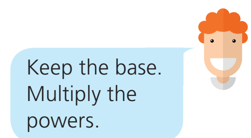
Solution	Explanation
a $(x^5)^4 = x^{5\times 4}$ $= x^{20}$	Keep x as the base and multiply the indices.
b $3(y^5)^2 = 3y^{5\times 2}$ $= 3y^{10}$	Keep y and multiply the indices. The index of 2 is applied only inside the brackets. The 3 is outside the brackets, so it is not raised to the power of 2.

5 Simplify each of the following. Leave your answers in index form.

a $(y^6)^2$ **b** $(m^3)^6$ **c** $(x^2)^5$ **d** $(b^3)^4$

e $(3^2)^3$ **f** $(4^3)^5$ **g** $(3^5)^6$ **h** $(7^5)^2$

i $5(m^8)^2$ **j** $4(q^7)^4$ **k** $-3(c^2)^5$ **l** $2(j^4)^6$



Example 27 The power of a product

Simplify the following.

a $(2s)^4$ **b** $(x^2y^3)^5$

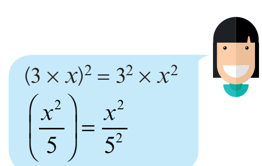
Solution	Explanation
a $(2s)^4 = 2^4 \times s^4$ $= 16s^4$	$(a \times b)^m = a^m \times b^m$ Evaluate: $2^4 = 2 \times 2 \times 2 \times 2$
b $(x^2y^3)^5 = (x^2)^5 \times (y^3)^5$ $= x^{10}y^{15}$	Apply the index 5 to each factor in the brackets. Multiply indices: $(x^2)^5 = x^{2\times 5}$, $(y^3)^5 = y^{3\times 5}$

6 Simplify the following.

a $(3x)^2$ **b** $(4m)^3$ **c** $(5y)^3$

d $(2x^3)^4$ **e** $(x^2y)^5$ **f** $(3a^3)^3$

g $(x^4y^2)^6$ **h** $(a^2b)^3$ **i** $(m^3n^3)^4$



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3G

Example 28 Using the zero power

Apply the zero power rule to evaluate each of the following.

a $(-3)^0$ **b** $3(5x)^0$ **c** $2y^0 - (3y)^0$

Solution	Explanation
a $(-3)^0 = 1$	Any number raised to the power of 0 is 1.
b $3(5x)^0 = 3 \times 1$ $= 3$	Everything in the brackets is to the power of 0, so $(5x)^0$ is 1. The 3 is not to the power of 0.
c $2y^0 - (3y)^0 = 2 \times 1 - 1$ $= 2 - 1$ $= 1$	$2y^0$ has no brackets so the power applies to the y only, so $2y^0 = 2 \times y^0 = 2 \times 1$ and $(3y)^0 = 1$.

7 Evaluate each of the following.

a 5^0 **b** 9^0 **c** $(-6)^0$ **d** $(-3)^0$

e $(4)^0$ **f** $\left(\frac{3}{4}\right)^0$ **g** $\left(-\frac{1}{7}\right)^0$ **h** $(4y)^0$

i $5m^0$ **j** $-3p^0$ **k** $6x^0 - 2x^0$ **l** $-5n^0 - (8n)^0$

m $(3x^4)^0$ **n** $1^0 + 2^0 + 3^0$ **o** $3a^0 + (3a)^0$ **p** $100^0 - a^0$

Any number (except zero) raised to the power 0 equals 1.

Example 29 Combining index laws

Simplify $(x^2)^3 \times (x^3)^5$ by applying the various index laws.

Solution	Explanation
$(x^2)^3 \times (x^3)^5 = x^{2 \times 3} \times x^{3 \times 5}$ $= x^6 \times x^{15}$ $= x^{21}$	Use power of a power to remove brackets first by multiplying indices. Then use index law for multiplication to add indices.

8 Simplify each of the following by combining various index laws.

a $4 \times (4^3)^2$ **b** $(3^4)^2 \times 3$ **c** $x \times (x^0)^5$

d $y^5 \times (y^2)^4$ **e** $b^5 \times (b^3)^3$ **f** $(a^2)^3 \times a^4$

g $(d^3)^4 \times (d^2)^6$ **h** $(y^2)^6 \times (y)^4$ **i** $z^4 \times (z^3)^2 \times (z^5)^3$

First remove brackets by multiplying powers. Remember that $4 = 4^1$.

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Example 30 Combining index laws

Simplify $\frac{(m^3)^4}{m^7}$ by applying index laws.

Solution	Explanation
$\frac{(m^3)^4}{m^7} = \frac{m^{3 \cdot 4}}{m^7}$	Remove brackets by multiplying indices, then simplify using index law for division.
$= \frac{m^{12}}{m^7}$	
$= m^5$	$12 - 7 = 5$

9 Simplify each of the following.

a $\frac{(b^2)^5}{b^4}$ **b** $\frac{(x^4)^3}{x^7}$ **c** $\frac{(y^3)^3}{y^3}$

d $7^8 \div (7^3)^2$ **e** $(4^2)^3 \div 4^5$ **f** $(3^6)^3 \div (3^5)^2$

g $(m^3)^6 \div (m^2)^9$ **h** $(y^5)^3 \div (y^6)^2$ **i** $(h^{11})^2 \div (h^5)^4$

First remove brackets by multiplying the powers.

10 If m and n are positive integers, in how many ways can $(a^m)^n = a^{16}$? Show each possibility.

11 Explain the error made in the following problems, then give the correct answer.

a $(a^4)^5 = a^9$ **b** $3(x^3)^2 = 9x^6$ **c** $(2x)^0 = 2$

Enrichment: Rabbits!

12 There are 100 rabbits on Mt Burrow at the start of the year 2015. The rule for the number of rabbits, N, after t years (from the start of the year 2015) is $N = 100 \times 2^t$.

a Find the number of rabbits at:
 i $t = 2$
 ii $t = 6$
 iii $t = 0$

b Find the number of rabbits at the beginning of:
 i 2018
 ii 2022
 iii 2025

c How many years will it take for the population to first rise to more than 500 000? Give a whole number of years.

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3H Negative indices

Stage
5.2
5.2◊
5.1
4

We know that $2^3 = 8$ and $2^0 = 1$. What about 2^{-1} or 2^{-6}? Numbers written in index form using negative indices also have meaning.

Negative indices are used for microscopic measurement. For example, a human hair is 7×10^{-3} mm wide.

For example, simplify $6^2 \div 6^5$.

Method 1: By subtracting the indices

$$\frac{6^2}{6^5} = 6^{2-5}$$
$$= 6^{-3}$$

Method 2: By cancelling

$$\frac{6^2}{6^5} = \frac{\cancel{6}^1 \cdot \cancel{6}^1}{6 \cdot 6 \cdot 6 \cdot \cancel{6}_1 \cdot \cancel{6}_1}$$
$$= \frac{1}{6^3}$$

So we can see that $6^{-3} = \frac{1}{6^3}$.

Let's start: Continuing the pattern

Explore the use of negative indices by copying and completing this table.

Index form	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
Whole number or fraction	16	8					$\frac{1}{4} = \frac{1}{2^2}$	

÷ 2 ÷ 2 ÷ 2 ÷ 2 ÷ 2 ÷ 2 ÷ 2

- What do you notice about the numbers with negative indices in the top row and the fractions in the second row?
- Can you describe this connection in words?
- What might be another way of writing 2^{-7} or 5^{-4}?

Key ideas

- A base with a negative index can be rewritten as a fraction.

  For example: $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

  $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

- All the index laws also apply to negative indices.

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Exercise 3H

Understanding

1 Write the following using positive indices. For example: $\frac{1}{8} = \frac{1}{2^3}$.

a $\frac{1}{4}$ **b** $\frac{1}{9}$ **c** $\frac{1}{125}$ **d** $\frac{1}{27}$

Write the denominator as a power of a prime number.

2 Copy and complete these tables.

a

Index form	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	81	27					$\frac{1}{9} = \frac{1}{3^2}$	

$\div 3$ $\div 3$ $\div 3$ $\div 3$ $\div 3$ $\div 3$

b

Index form	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Whole number or fraction	10 000							$\frac{1}{1000} = \frac{1}{10^3}$

$\div 10$ $\div 10$ $\div 10$ $\div 10$ $\div 10$ $\div 10$ $\div 10$

3 Copy and complete each of the following.

a $10^{-4} = \frac{1}{10^{\square}}$ **b** $3^{-2} = \frac{1}{3^{\square}}$ **c** $7^{-3} = \frac{1}{7^{\square}}$

d $8^{-6} = \frac{1}{8^{\square}}$ **e** $9^{-4} = \frac{1}{9^{\square}}$ **f** $5^{-4} = \frac{1}{5^{\square}}$

$10^{-4} = \frac{1}{10^4}$

4 True or false?

a $4^{-2} = -8$ **b** $4^{-2} = \frac{1}{8}$ **c** $4^{-2} = \frac{1}{16}$ **d** $4^{-2} = \frac{1}{4^2}$ **e** $4^{-2} = -16$

Fluency

Example 31 Converting to a fraction

Rewrite the following as fractions containing positive indices.

a 3^{-2} **b** 5×4^{-3}

Solution	**Explanation**
a $3^{-2} = \frac{1}{3^2}$	Put 3^2 in the denominator.
b $5 \times 4^{-3} = \frac{5}{1} \times \frac{1}{4^3}$	The 5 does not have a negative power, so it remains unchanged.
$= \frac{5}{4^3}$	Multiply numerators and denominators: $5 \times 1 = 5$, $1 \times 4^3 = 4^3$.

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3H

Skillsheet 3C 3D

5 Write each of the following as fractions with positive indices.

a 5^{-2} b 7^{-4} c 8^{-3} d 3^{-5}
e 9^{-2} f 10^{-3} g 4^{-5} h 2^{-3}

$5^{-2} = \frac{1}{5^2}$.

6 Rewrite the following as fractions with positive indices.

a 3×2^{-4} b 5×4^{-3} c 7×5^{-6} d 2×3^{-4}
e 4×3^{-5} f 9×5^{-2} g 8×7^{-3} h 6×5^{-6}

$5 \times 2^{-3} = \frac{5}{2^3}$ (the 5 is circled, with an arrow labelled "Don't change")

Example 32 Changing to fractions

Rewrite 3×2^{-4} with a positive power and then as a fraction.

Solution	Explanation
$3 \times 2^{-4} = 3 \times \frac{1}{2^4}$	$2^{-4} = \frac{1}{2^4}$
$= \frac{3}{2^4}$	$2^4 = 2 \times 2 \times 2 \times 2 = 16$
$= \frac{3}{16}$	

7 Rewrite each of these with a positive power and then as a fraction. Simplify where possible.

a 6×5^{-2} b 2×3^{-2} c 4×5^{-3} d 6×7^{-1}
e 4×10^{-3} f 2×10^{-1} g 5×2^{-4} h 4×5^{-1}

Remember: $5^1 = 5$.
So $3 \times 5^{-1} = 3 \times \frac{1}{5^1}$
$= \frac{3}{5}$

Example 33 Changing to fractions and decimals

Rewrite 5×10^{-3} with a positive power and then as a fraction and a decimal.

Solution	Explanation
$5 \times 10^{-3} = 5 \times \frac{1}{10^3}$	$10^{-3} = \frac{1}{10^3}$
$= \frac{5}{10^3}$	$10^3 = 10 \times 10 \times 10 = 1000$
$= \frac{5}{1000}$ or $\frac{1}{200}$	5 thousandths $= 0.005$
$= 0.005$	

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8 Rewrite each of the following with a positive power and then as an unsimplified fraction and a decimal.

- **a** 2×10^{-3}
- **b** 5×10^{-2}
- **c** 7×10^{-1}
- **d** 3×10^{-4}
- **e** 5×10^{-4}
- **f** 8×10^{-5}
- **g** 2×10^{-6}
- **h** 4×10^{-8}

$\frac{1}{10} = 0.1$

$\frac{1}{100} = 0.01$

$\frac{1}{1000} = 0.001$

$\frac{1}{10\,000} = 0.0001$

$\frac{1}{100\,000} = 0.00001$

Problem-solving and Reasoning

9 Find the error and correct it.

a
$$\begin{aligned} 3 \times 2^{-3} &= 3^{-3} \times 2^{-3} \\ &= \frac{1}{9} \times \frac{1}{8} \\ &= \frac{1}{72} \end{aligned}$$

b $2x^{-2} = \dfrac{1}{2x^2}$

10 Copy and complete:

- **a** $\dfrac{1}{8} = \dfrac{1}{2^{\square}} = 2^{\square}$
- **b** $\dfrac{1}{9} = \dfrac{1}{3^{\square}} = 3^{\square}$
- **c** $\dfrac{1}{16} = \dfrac{1}{2^{\square}} = 2^{\square}$
- **d** $\dfrac{1}{25} = \dfrac{1}{5^{\square}} = 5^{\square}$

11 Write each of the following numbers as a basic numeral and then arrange them in ascending order.

2.35, 0.007×10^2, 0.0012, 3.22×10^{-1}, 0.4, 35.4×10^{-3}

12 Write each of the following numbers as a basic numeral.

- **a** 3.24×10^2
- **b** 1.725×10^5
- **c** 2.753×10^{-1}
- **d** 1.49×10^{-3}

13 Write each of the following values in scientific notation, using 3 significant figures.

- **a** The population of Australia in 2010 was approximately 22 475 056.
- **b** The area of the USA is 9 629 091 km^2.
- **c** The time taken for light to travel 1 metre (in a vacuum) is 0.00000000333564 seconds.
- **d** The wavelength of ultraviolet light is 0.000000294 m.

Enrichment: Algebraic bases

14 Write each of the following as fractions containing only positive indices.

- **a** a^2b^{-2}
- **b** $a^{-2}b^2$
- **c** $a^{-2}b^{-2}$
- **d** $(ab)^{-2}$

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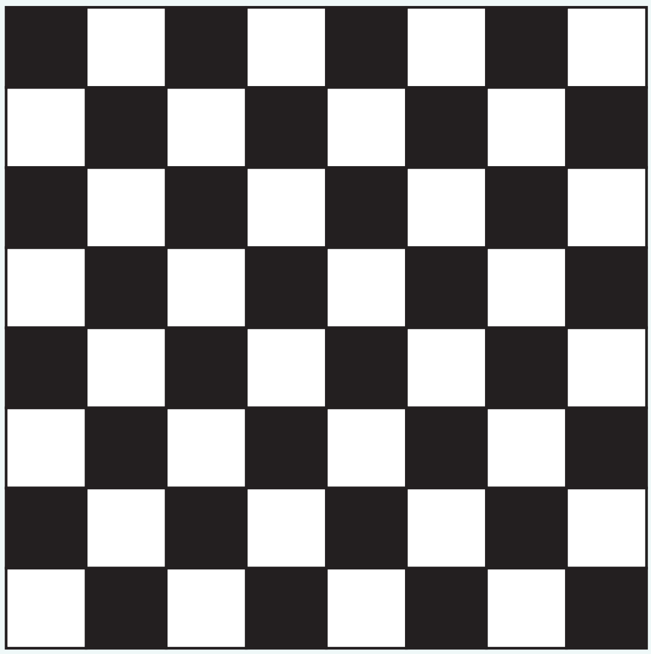
Maths@home

Worksheet 3A

Population growth, wage indexation and housing affordability

It is thought that the game of chess was invented by an ancient Indian mathematician. The king was so pleased with the game that he offered the inventor any reward of his choice: rare jewels, bags of gold or even a large property.
To the king's surprise the Indian mathematician asked for some wheat! He asked for 1 grain for the first square of the chess board, 2 grains for the second square, 4 grains for the third square, 8 grains for the fourth square etc., continuing in this way right up to the 64th square.



1 If one grain of wheat weighs 2×10^{-8} tonnes, what weight of wheat would the inventor have received for the 64th square? Answer in scientific notation, correct to 3 significant figures.

2 How much money would this wheat be worth at the 2016 Australian price of $\$275$ per tonne?

In the attached worksheet, we will use technology to investigate situations like the one above, in which the quantity of something grows in an exponential pattern. Patterns like these occur in many situations, both real and imagined.

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Puzzles and games

1 In this magic square, each row and column adds to a sum that is an algebraic expression. Complete the square to find the sum.

$\frac{4x^2}{2x}$	$-y$	$x + 3y$
$x - 2y$		$2y$

2 Write $3^{n-1} \times 3^{n-1} \times 3^{n-1}$ as a single power of 3.

3 You are offered a choice of two prizes:
- 1 million dollars right now, or
- you can receive 1 cent on the first day of a 30-day month, double your money every day for 30 days and receive the total amount on the 30th day.

Which prize offers the most money?

4 Write $(((2^1)^2)^3)^4$ as a single power of 2.

5 How many zeros are there in 100^{100}, in expanded form?

6 A population of bacteria doubles every 5 minutes. What is this type of growth called? Solve this puzzle to find the answer.

Write the basic numeral for each of the following. Write the letter corresponding to each answer in the boxes below to form a word.

T $\frac{1}{10^3}$ **L** 2.15×10^3 **I** $\frac{9^5}{9^3}$

A 5×10^{-2} **O** 2^{-2} **N** 2^4

P $3^2 \times 2^2 - 3^0$ **X** $5^2 + 2^4 - 3^2 \times 4$ **E** $\frac{4^3}{8}$

8	5	35	0.25	16	8	16	0.001	81	0.05	2150

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Chapter summary

Algebraic expressions

Simplifying expressions

Add/subtract like terms only. Like terms have the same pronumeral factors.

e.g. $3x$ and $7x$, $2xy$ and $4yx$;
not $2x$ and x^2 or $3y$ and $4xy$.

For example:

$3x + 2y - x + 7y = 3x - x + 2y + 7y$
$= 2x + 9y$

Multiply/Divide

$3a \times 2b = 3 \times 2 \times a \times b$
$= 6ab$

$7xy \div 14y = \dfrac{{}^{1}\cancel{7}x\cancel{y}^{1}}{{}_{2}\cancel{14}\cancel{y}_{1}} = \dfrac{x}{2}$

Expanding

$a(b + c) = ab + ac$

$a(b - c) = ab - ac$

e.g. $2(4x + 3) = 8x + 6$

Algebraic expressions

$5xy + 7x - 4y + 3$

4 terms

3 is the constant term.
7 is the coefficient of x.
-4 is the coefficient of y.

Factorising

This is the opposite of expanding.

Factorised form		Expanded form
$2(x + 4)$	=	$2x + 8$

Look for highest common factor of terms

e.g. $3ab + 6a$ HCF is $3a$.
$= 3a(b + 2)$

Indices

Index form

a^m ← index
base

e.g. $2^3 = 2 \times 2 \times 2$
$3^4 = 3 \times 3 \times 3 \times 3$

$a \times b \times a \times b \times b = a^2b^3$

Scientific notation

Used for very large and small numbers

$3\,870\,000 = 3.87 \times 10^6$

$0.00021 = 2.1 \times 10^{-4}$

Negative indices

$2^{-3} = \dfrac{1}{2^3}$

$2 \times 5^{-4} = \dfrac{2}{1} \times \dfrac{1}{5^4}$
$= \dfrac{2}{5^4}$

Zero index

$a^0 = 1$

Any number (except 0) to the power 0 equals 1.

e.g. $3x^0 = 3 \times 1$
$= 3$
$(3x)^0 = 1$

Index laws of multiplication

$a^m \times a^n = a^{m+n}$

When multiplying terms with the same base, add indices.

e.g. $x^3 \times x^5 = x^{3+5}$
$= x^8$

Index laws of division

$a^m \div a^n = a^{m-n}$

When dividing terms with the same base, subtract indices.

e.g. $\dfrac{x^7}{x^4} = x^{7-4}$
$= x^3$

Power of a power

$(a^m)^n = a^{mn}$

To expand brackets, multiply indices.

e.g. $(x^2)^5 = x^{2 \times 5}$
$= x^{10}$

Power of a product

$(ab)^m = a^m b^m$

e.g. $(3x^2)^3 = 3^3(x^2)^3$
$= 27x^6$

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Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.

Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

Drilling for Gold 3R1 3R2

1 The coefficient of x in $3xy - 4x + 7$ is:
A 4 B 7 C -4
D 3 E -1

2 The simplified form of $7ab + 2b - 5ab + b$ is:
A $2ab + 2b^2$ B $2ab + 3b$ C $5ab$
D $2ab + b$ E $12ab + 3b$

3 The expanded form of $2x(3x - 5)$ is:
A $6x^2 - 5$ B $6x - 10$ C $6x^2 - 10x$
D $5x^2 - 10x$ E $-4x$

4 The fully factorised form of $8xy - 24y$ is:
A $4y(2x - 6y)$ B $8(xy - 3y)$ C $8y(x - 24)$
D $8y(x - 3)$ E $8x(y - 24)$

5 $x^2 - 2xy + 2yx$ is equal to:
A xy B x^2 C $x^2 - 4xy$
D 0 E $4xy$

6 $-3ab \times 4b$ is equal to:
A $-7ab^2$ B $-12ab^2$ C $-7a^2b$
D $-12a^2b$ E $12ab^2$

7 $3x^3y \times 2x^5y^3$ is equal to:
A $5x^{15}y^3$ B $6x^{15}y^3$ C $6x^8y^4$
D $5x^8y^4$ E $6x^8y^3$

8 $12a^4 \div (4a^7)$ simplifies to:
A $3a^3$ B $8a^3$ C $3a^{11}$
D $\frac{8}{a^3}$ E $\frac{3}{a^3}$

9 $(2x^4)^3$ can be written as:
A $2x^{12}$ B $2x^7$ C $6x^{12}$
D $8x^{12}$ E $8x^7$

10 $5x^0 - (2x)^0$ is equal to:
A 4 B 0 C 3
D 2 E -1

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Chapter review

Short-answer questions

1 Consider the expression $3xy - 3b + 4x^2 + 5$.
- **a** How many terms are in the expression?
- **b** What is the constant term?
- **c** State the coefficient of:
  - **i** x^2
  - **ii** b

2 Write algebraic expressions for the following.
- **a** 3 more than y
- **b** 5 less than the product of x and y
- **c** the sum of a and b is divided by 4

3 Evaluate the following when $x = 3$, $y = 5$ and $z = -2$.
- **a** $3x + y$
- **b** xyz
- **c** $y^2 - 5z$

4 Simplify the following expressions.
- **a** $4x - 5 + 3x$
- **b** $4a - 5b + 9a + 3b$
- **c** $3xy + xy^2 - 2xy - 4y^2x$
- **d** $3m \times 4n$
- **e** $-2xy \times 7x$
- **f** $\dfrac{8ab}{12a}$

5 Expand the following and collect like terms where necessary.
- **a** $5(2x + 4)$
- **b** $-2(3x - 4y)$
- **c** $3x(2x + 5y)$
- **d** $3 + 4(a + 3)$

6 Factorise the following expressions.
- **a** $16x - 40$
- **b** $10x^2y + 35xy^2$
- **c** $4x^2 - 10x$
- **d** $-2xy - 18x$ (include the common negative)

7 Simplify the following, using the appropriate index laws.
- **a** $3x^5 \times 4x^2$
- **b** $4xy^6 \times 2x^3y^{-2}$
- **c** $\dfrac{b^7}{b^3}$
- **d** $(b^2)^4$
- **e** $(2m^2)^3$
- **f** $3^0 + x^0$

8 Simplify the following, using the zero power.
- **a** 7^0
- **b** $4x^0$
- **c** $5a^0 + (2y)^0$
- **d** $(x^2 + 4y)^0$

9 Write these numbers as a basic numeral.
- **a** 4.25×10^3
- **b** 3.7×10^7
- **c** 2.1×10^{-2}
- **d** 7.25×10^{-5}

10 Convert these numbers to scientific notation, using 3 significant figures.
- **a** 123 574
- **b** 39 452 178
- **c** 0.0000090241
- **d** 0.00045986

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11 Copy and complete:

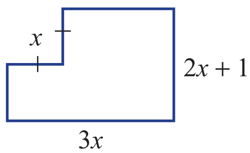
a $3^{-2} = \dfrac{1}{\square}$ **b** $4^{-1} = \dfrac{1}{\square}$ **c** $5^{-3} = \dfrac{1}{\square}$

12 Copy and complete:

a $\dfrac{1}{16} = \dfrac{1}{2^{\square}} = 2^{\square}$ **b** $\dfrac{5}{9} = 5 \cdot \dfrac{1}{\square} = 5 \cdot 3^{\square}$

Extended-response questions

1 A room in a house has the shape and dimensions, in metres, shown. Note: All angles are 90°.

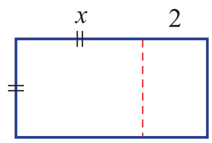


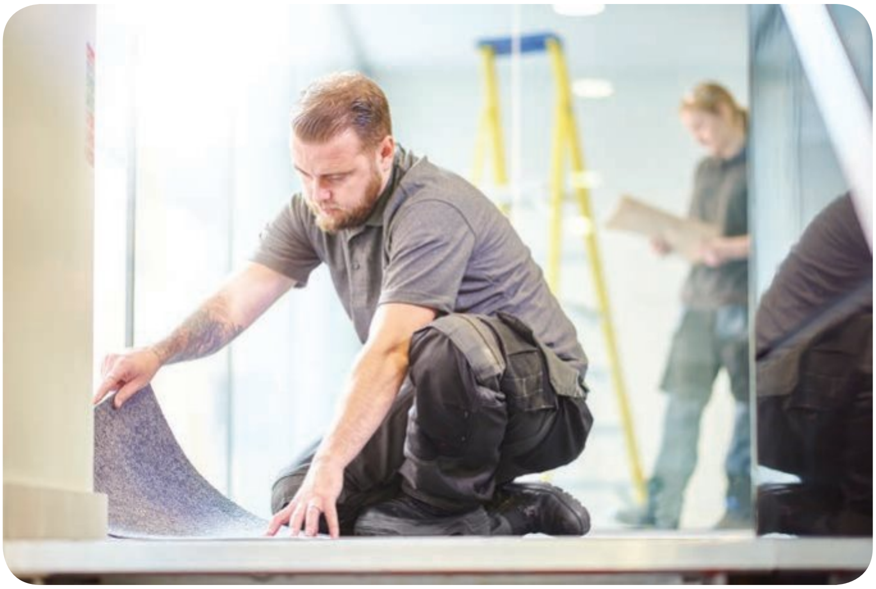
a Find the perimeter of the room, in factorised form.
b If $x = 3$, what is the room's perimeter?

The floor of the room is to be recarpeted.

c Give the area of the floor in terms of x and in expanded form.
d If the carpet costs \$20 per square metre and $x = 3$, what is the cost of laying the carpet?

2 Write two expressions for the area of this rectangle, one with brackets and one without.





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Chapter 4

Probability and statistics

What you will learn

Strand: Statistics and Probability

Substrand: SINGLE VARIABLE DATA ANALYSIS

BIVARIATE DATA ANALYSIS

In this chapter you will learn to:

- estimate probabilities of simple and compound events by calculating relative frequencies
- compare sets of data using statistical displays
- evaluate statistical claims made in the media.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw10

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Census Australia

The Census is run by the Australian Bureau of Statistics every 5 years.

The Census counts all people who spend census night in Australia but it is not used to measure only population growth. It also collects data on people's employment, citizenship, education level, use of public transport and a range of other topics.

The data are used by the Australian Government to allocate funds to education, health and infrastructure.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO:* www.cambridge.edu.au/goldnsw10

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Spreadsheets:
Models for activities using spreadsheets

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 A letter is selected from the word PROBABILITY.

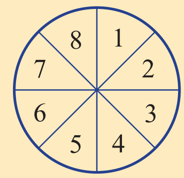
a How many letters are there in total?

b Find the chance (i.e. probability) of selecting:

i the letter R **ii** the letter B **iii** a vowel

iv not a vowel **v** a T or an I **vi** neither a B nor a P

2 A spinning wheel has 8 equal sectors numbered 1 to 8. On one spin of the wheel, find the following probabilities.



a $P(5)$

b $P(\text{even})$

c $P(\text{not even})$

d $P(\text{multiple of } 3)$

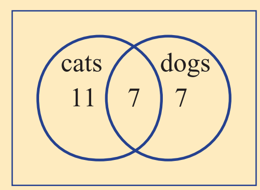
e $P(\text{factor of } 12)$

f $P(\text{odd or a factor of } 12)$

g $P(\text{both odd and a factor of } 12)$

3 Arrange from lowest to highest: $\frac{1}{2}$, 0.4, 1 in 5, 39%, $\frac{3}{4}$, 1, 0, $\frac{9}{10}$, 0.62, 71%.

4 This Venn diagram shows the number of people in a group of 25 who own cats and/or dogs.



a State the number of people from the group who own:

i a dog

ii a cat or a dog (including both)

iii only a cat

b If a person from this group is selected at random, find the probability that they will own:

i a cat

ii a cat and a dog

iii only a dog

5 Drew shoots from the free-throw line on a basketball court. After 80 shots he counts 35 successful throws.

a Estimate the probability that his next throw will be successful.

b Estimate the probability that his next throw will not be successful.

6 Two fair 4-sided dice are rolled and the sum of the two numbers obtained is noted.

a Copy and complete this grid.

b What is the total number of outcomes?

c Find the probability that the total sum is:

i 2 **ii** 4

iii less than 5 **iv** less than or equal to 5

v at most 6 **vi** no more than 3

Roll 2 \ Roll 1	1	2	3	4
1				
2				
3				
4				

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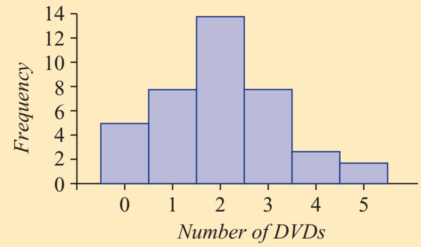
7 Below is a list of statistical tools and a list of diagrams. Match each tool (**a–o**) with the most appropriate diagram (**A–O**).

a frequency distribution table

A



b dot plot

B $\dfrac{\text{sum of all data values}}{\text{number of data values}}$

c stem-and-leaf plot

C 1, 1, 2, 2, 3, 3, ④, 4, 4, 5, 6, 7, 18

d sector graph (i.e. pie chart)

D [5] 8 10 15 20 12 10 [50]

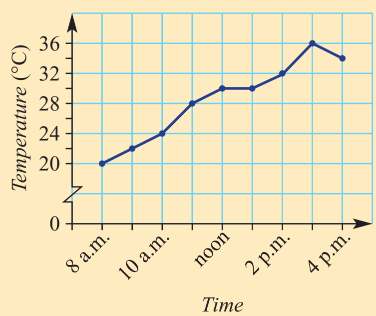
e divided bar graph

E [5] 8 10 15 20 12 10 [50]

50 − 5 = 45

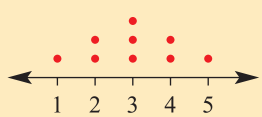
f column graph

F



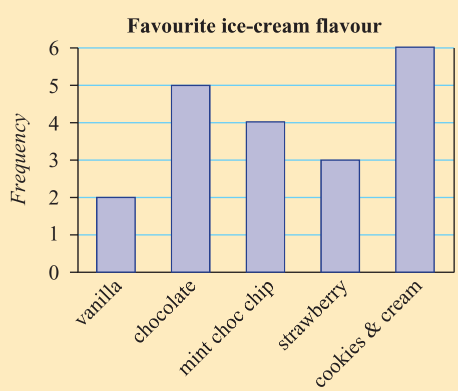
g histogram

G



h time-series graph

H



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Pre-test

i mean

j median

k mode

l minimum and maximum

m range

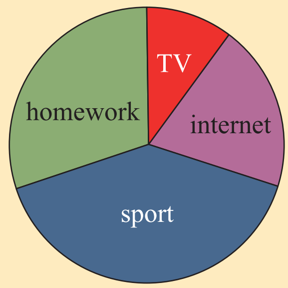
n symmetrical data

o bimodal data

I

Interval	Frequency	Percentage frequency
0–4	3	15
5–9	7	35
10–14	6	30
15–19	4	20
Total	20	100

J

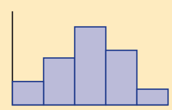


K

L 4 5 3 6 4 3 3

M 3, 15, 12, 9, 12, 15, 6, 8

N



O

Stem	Leaf
9	8
10	2 6
11	1 1 4 9
12	3 6
13	8 9 9
14	0 2 5

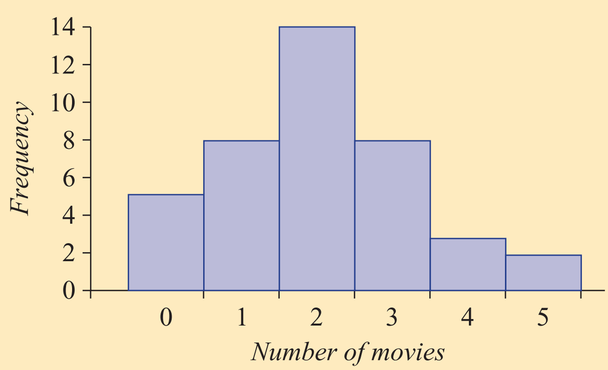
8 The number of movies rented per person from a website is shown in this graph.

a How many customers rented three movies?

b How many customers were surveyed?

c How many movies were rented during the survey?

d How many customers rented fewer than two movies?



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9 This table shows the frequency of scores in a test.

Score	Frequency
0–19	2
20–39	3
40–59	6
60–79	12
80–100	7

a How many scores are in the 40 to less than 60 range?
b How many scores are:
  i at least 60? **ii** less than 80?
c How many scores are there in total?
d What percentage of scores are in the 20 to less than 40 range?

10 For each of these data sets, find:
  i the mean (i.e. by dividing the sum by the number of scores)
  ii the mode (most frequent)
  iii the median (middle value of ordered data)
  iv the range (difference between highest and lowest)
a 38, 41, 41, 47, 58
b 2, 2, 2, 4, 6, 6, 7, 9, 10, 12

11 This stem-and-leaf plot shows the weight, in grams, of some small calculators.
a How many calculators are represented in the plot?
b What is the mode (most frequent)?
c What is the minimum weight and maximum weight?
d Find the range (i.e. maximum value – minimum value).

Stem	Leaf
9	8
10	2 6
11	1 1 4 9
12	3 6
13	8 9 9
14	0 2 5

13 | 6 means 136 grams

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4A Review of probability

Stage

5.2
5.2◊
5.1
4

Probability is the likelihood of particular random events occurring. When rolling a die, we can determine theoretical probabilities because we know the total number of outcomes and the number of favourable outcomes. In other cases, we can use experimental results to estimate the chance that an event will occur. The chance that a particular soccer team will win its next match, for example, could be estimated using results from several previous games.

A soccer team could win, lose or draw the next match it plays, but these three outcomes do not necessarily have the same probability.

▶ Let's start: More or less than a 50% chance

When a coin is tossed once, the chance of tossing heads is 50%. Describe an experiment and outcome that has the following probability.

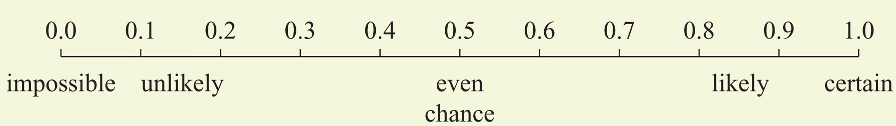
a 0% **b** less than 50% **c** more than 50% **d** 100%

Key ideas

Drilling for Gold 4A1 4A2

- Key terms used in probability are given below.
  - A **chance experiment** is an activity that may produce a variety of different results that occur randomly. Rolling a die is a single-step experiment.
  - A **trial** is a single occurrence of an experiment, such as a single roll of a die.
  - The **sample space** is the list of all possible outcomes from an experiment.
  - An **outcome** is a possible result of an experiment.
  - An **event** is either one outcome or a collection of outcomes.
  - **Equally likely outcomes** are events that have the same chance of occurring.

- In the study of probability, a numerical value based on a scale from 0 to 1 is used to describe levels of **chance**.



Trial One occurrence of an experiment

Sample space All the possible outcomes of an event

Outcome One of the possibilities from a chance experiment

Chance The likelihood of an event happening

Equally likely outcomes Events that have the same chance of occurring

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- The **theoretical probability** of an event in which outcomes are equally likely is calculated as follows.

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

- A probability can be written as a fraction, decimal or percentage.
- **Experimental probability** is calculated in the same way as theoretical probability but uses the results of an experiment:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of trials}}$$

  – If the number of trials is large, the experimental probability should be very close to the theoretical probability.

Exercise 4A

Understanding

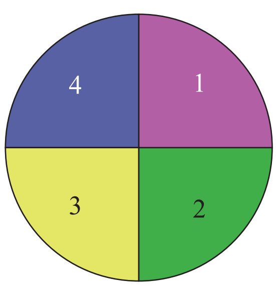
1 Order these events (**A**–**D**) from least likely to most likely.

A the chance that it will rain every day for the next 10 days
B the chance that a member of class is ill on the next school day
C the chance that school is cancelled next year
D the chance that the Sun comes up tomorrow

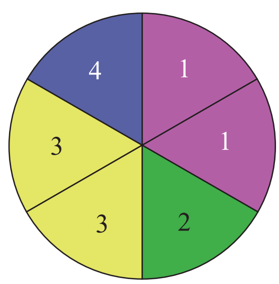
2 For the following spinners, find the probability that the outcome will be a 4.

$$P(4) = \frac{\text{number of 4s}}{\text{total number of sections}}$$

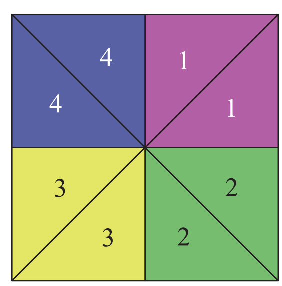
a



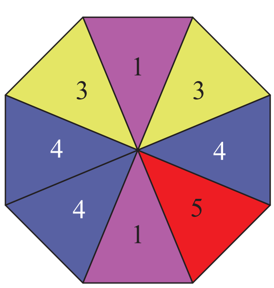
b



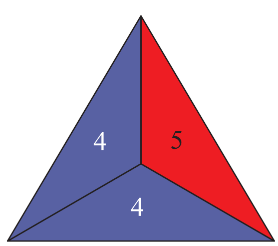
c



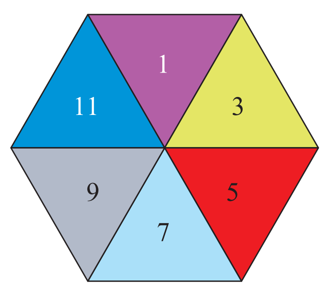
d



e



f



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4A

3 A coin is flipped once.

- **a** How many different outcomes are possible from a single flip of the coin?
- **b** What are the possible outcomes from a single flip of the coin (i.e. list the sample space)?
- **c** Are the possible outcomes equally likely?
- **d** What is the probability of obtaining a tail?
- **e** What is the probability of not obtaining a tail?
- **f** What is the probability of obtaining a tail or a head?

Fluency

Example 1 Calculating simple theoretical probabilities

A letter is chosen randomly from the word TELEVISION.

a How many letters are there in the word TELEVISION?

b Find the probability that the letter is:

i V **ii** E
iii not E **iv** E or V

Solution	Explanation
a 10	The sample space includes 10 letters.
b **i** $P(\text{V}) = \frac{1}{10} = 0.1$	$P(\text{V}) = \frac{\text{number of Vs}}{\text{total number of letters}}$
ii $P(\text{E}) = \frac{2}{10}$	There are 2 Es in the word TELEVISION.
$= \frac{1}{5} = 0.2$	Simplify the fraction.
iii $P(\text{not E}) = \frac{8}{10}$ $= \frac{4}{5} = 0.8$	If there are 2 Es in the word TELEVISION, which has 10 letters, then there must be 8 letters that are not E.
iv $P(\text{E or V}) = \frac{3}{10} = 0.3$	The number of letters that are either E or V is 3.

4 A letter is chosen from the word TEACHER.

- **a** How many letters are there in the word TEACHER?
- **b** Find the probability that the letter is:

  i R **ii** E **iii** not E **iv** R or E

5 A letter is chosen from the word EXPERIMENT. Find the probability that the letter is:

a E **b** a vowel
c not a vowel **d** X or a vowel

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Example 2 Calculating simple experimental probabilities

An experiment involves tossing three coins and counting the number of heads.
Here are the results after running the experiment 100 times.

Number of heads	0	1	2	3
Frequency	11	40	36	13

a How many times did 2 heads occur?
b How many times did fewer than 2 heads occur?
c Find the experimental probability of obtaining:
  i 0 heads **ii** 2 heads
  iii fewer than 2 heads **iv** at least 1 head

Solution	**Explanation**
a 36	From the table you can see that 2 heads has a frequency of 36.
b $11 + 40 = 51$	Fewer than 2 means obtaining 0 heads or 1 head.
c i $P(0\text{ heads}) = \frac{11}{100} = 0.11$	$P(0\text{ heads}) = \frac{\text{number of times 0 heads is observed}}{\text{total number of trials}}$
ii $P(2\text{ heads}) = \frac{36}{100} = 0.36$	$P(2\text{ heads}) = \frac{\text{number of times 2 heads is observed}}{\text{total number of trials}}$
iii $P(\text{fewer than 2 heads}) = \frac{11+40}{100} = \frac{51}{100} = 0.51$	Fewer than 2 heads means to observe 0 or 1 head.
iv $P(\text{at least 1 head}) = \frac{40+36+13}{100} = \frac{89}{100} = 0.89$	At least 1 head means that 1, 2 or 3 heads can be observed.

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4A

6 An experiment involves tossing three coins and counting the number of heads. Here are the results after running the experiment 100 times.

The total number of outcomes is 100.

Number of heads	0	1	2	3
Frequency	9	38	43	10

a How many times did 2 heads occur?
b How many times did fewer than 2 heads occur?
c Find the experimental probability of obtaining:
- **i** 0 heads
- **ii** 2 heads
- **iii** fewer than 2 heads
- **iv** at least 1 head

Skillsheet 4A

7 An experiment involves rolling two dice and counting the number of sixes. Here are the results after running the experiment 100 times.

Number of sixes	0	1	2
Frequency	62	35	3

Find the experimental probability of obtaining:
a 0 sixes **b** 2 sixes **c** fewer than 2 sixes **d** at least 1 six

Problem-solving and Reasoning

8 A 10-sided die, numbered 1 to 10, is rolled once. Find these probabilities.

Prime numbers less than 10 are 2, 3, 5 and 7.

a $P(8)$ **b** $P(\text{odd})$
c $P(\text{even})$ **d** $P(\text{less than } 6)$
e $P(\text{prime})$ **f** $P(3 \text{ or } 8)$
g $P(8, 9 \text{ or } 10)$ **h** $P(\text{at least } 2)$

9 Marcus is a prizewinner in a competition and will be randomly awarded a single prize chosen from a collection of 50 prizes. The type and number of prizes to be handed out are listed below.

Remember that the total number of prizes is 50.

Prize	Car	Holiday	iPad	Blu-ray player
Number	1	4	15	30

Find the probability that Marcus will be awarded the following.
a a car
b an iPad
c a prize that is not a car

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10 Some of the 50 cars inspected at an assembly plant had one or more faults. The results of the inspection were as follows.

Number of faults	0	1	2	3	4
Number of cars	30	12	4	3	1

Find the experimental probability that a car selected from the assembly plant will have:

a 1 fault
b 4 faults
c fewer than 2 faults
d 1 or more faults
e 3 or 4 faults
f at least 2 faults

11 A bag contains red and yellow counters. A counter is drawn from the bag and then replaced. This happens 100 times and 41 of the counters drawn are red.

a How many counters drawn are yellow?
b If there are 10 counters in the bag, how many do you expect are red? Give a reason.
c If there are 20 counters in the bag, how many do you expect are red? Give a reason.

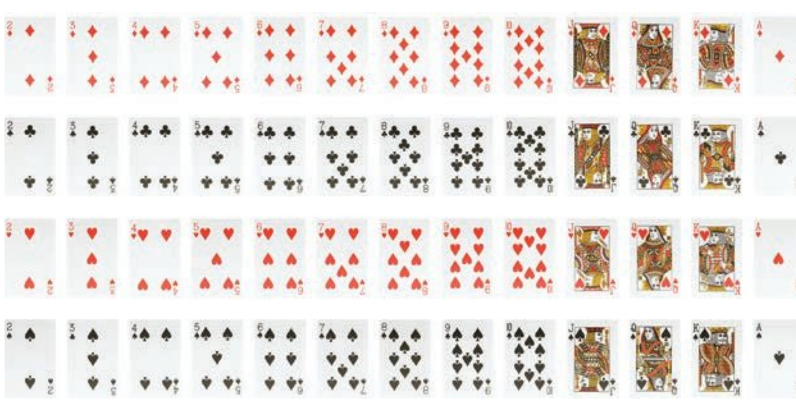
$\frac{41}{100}$ were red.

Enrichment: Cards probability

12 A card is chosen from a standard deck of 52 playing cards that includes 4 aces, 4 kings, 4 queens and 4 jacks. Find the following probabilities.

a P(heart)
b P(king)
c P(king of hearts)
d P(heart or club)
e P(king or jack)
f P(heart or king)
g P(not a king)
h P(neither a heart nor a king)

There are 4 suits in a deck of cards: hearts, diamonds, spades and clubs.



4B Venn diagrams

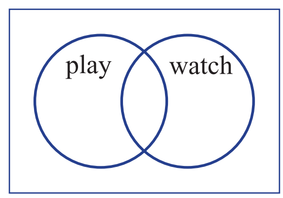
Stage
5.2
5.2◊
5.1
4

Sometimes we need to work with situations where there are overlapping events. A TV network, for example, might be collecting statistics regarding whether or not a person watches cricket and/or tennis or neither over a certain period of time. Venn diagrams are a useful tool when dealing with such data.

▶ Let's start: How many like both?

Of 20 students in a class, 12 people like to play tennis and 15 people like to watch tennis. Two people like neither playing nor watching tennis. Some like to both play and watch tennis.

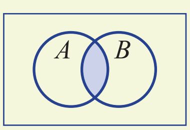
- Represent this information in a Venn diagram.
- How many students like to play and watch tennis?
- How many students like to only watch tennis?
- From the group of 20 students, what would be the probability of selecting a person that likes watching tennis only?



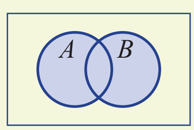
Key ideas

Drilling for Gold 4B1 at the end of this section

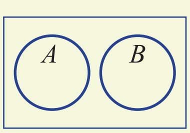
- A **Venn diagram** illustrates how all elements in the sample space are distributed among the events.
  - All elements that belong to both A and B.



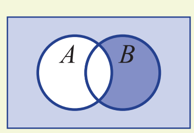
  - All elements that belong to either events A or B.



  - The two sets A and B are **mutually exclusive** when they have no elements in common.



  - For an event A, the complement of A is A' (or 'not A'). $P(A') = 1 - P(A)$



Venn diagram
A diagram using circles to show the relationships between two or more sets of data

Mutually exclusive
Two events that cannot both occur at the same time

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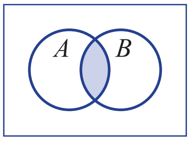
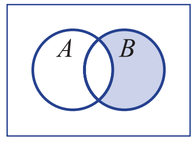
Exercise 4B

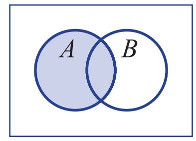
Understanding

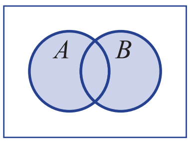
1 Match the descriptions (**a–e**) to the pictures (**A–E**).

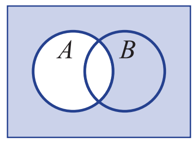
a A or B **b** A **c** not A
d A and B **e** B only

A


B


C


D


E


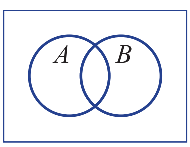
2 Decide whether the events A and B are mutually exclusive.

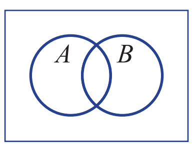
a $A = \{1, 3, 5, 7\}$
$B = \{5, 8, 11, 14\}$

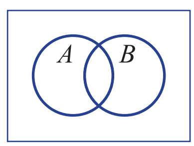
b $A = \{-3, -2, \ldots, 4\}$
$B = \{-11, -10, \ldots, -4\}$

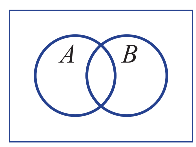
Mutually exclusive events have nothing in common.

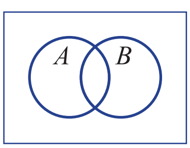
3 Copy these Venn diagrams and shade the region described by each of the following.

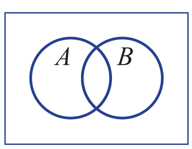
a A


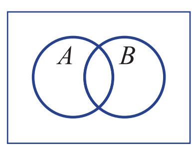
b B


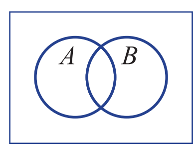
c A and B


d A or B


e A only


f B only


g not A


h neither A nor B


Fluency

Example 3 Listing sets

Consider the given events A and B that involve numbers taken from the first 10 positive integers.

$$A = \{1, 2, 3, 4, 5, 6\} \qquad B = \{1, 3, 7, 8\}$$

a Represent the two events A and B in a Venn diagram.
b List the sets:
  i A and B **ii** A or B
c If a number from the first 10 positive integers is randomly selected, find the probability that the following events occur.
  i A **ii** A and B **iii** A or B
d Are the events A and B mutually excusive? Why or why not?

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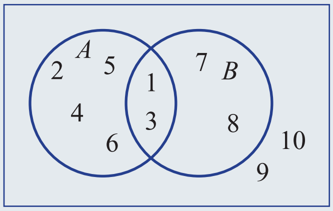
4B

Solution	Explanation
a 	The elements 1 and 3 are common to both sets A and B. The elements 9 and 10 belong to neither set A nor set B.
b **i** A and $B = \{1, 3\}$	1 and 3 are in A and B.
ii A or $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$	These numbers are in either A or B or both.
c **i** $P(A) = \frac{6}{10} = \frac{3}{5}$	There are 6 numbers in A.
ii $P(A \text{ and } B) = \frac{2}{10} = \frac{1}{5}$	A and B contains 2 numbers.
iii $P(A \text{ or } B) = \frac{8}{10} = \frac{4}{5}$	A or B contains 8 numbers.
d The sets A and B are not mutually exclusive since there are numbers inside A and B.	The set A and B contains at least 1 number.

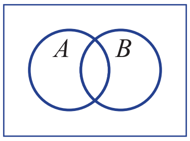
4 Events A and B involve numbers taken from the first 10 positive integers.

$A = \{1, 2, 4, 5, 7, 8, 10\}$ $B = \{2, 3, 5, 6, 8, 9\}$

a Represent events A and B in a Venn diagram, as shown below.

b List the following sets.

i A and B **ii** A or B



c If a number from the first 10 positive integers is randomly selected, find the probability that the following events occur.

i A **ii** A and B **iii** A or B

d Are the events A and B mutually exclusive? Why or why not?

5 The elements of the events A and B described below are numbers taken from the first 10 prime numbers.

$A = \{2, 5, 7, 11, 13\}$ $B = \{2, 3, 13, 17, 19, 23, 29\}$

a Represent events A and B in a Venn diagram.

b List the elements belonging to the following.

i A and B **ii** A or B

c If a number from the first 10 prime numbers is selected, find the probability that these events occur.

i A **ii** B **iii** A and B **iv** A or B

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Example 4 Using Venn diagrams

From a class of 30 students, 12 enjoy cricket (C), 14 enjoy netball (N) and 6 enjoy both cricket and netball.

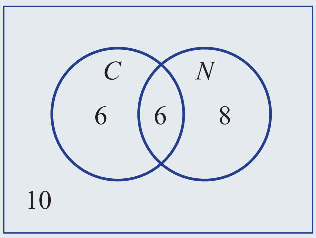
a Illustrate this information in a Venn diagram.

b State the number of students who enjoy:

i netball only **ii** neither cricket nor netball

c Find the probability that a person chosen at random will enjoy:

i netball **ii** netball only **iii** both cricket and netball

Solution	Explanation
a 	First, place the 6 in the intersection (6 enjoy cricket and netball) and then determine the other values according to the given information. The total must be 30, with 12 in the cricket circle and 14 in netball.
b i 8	Includes students in N but not in C.
ii 10	These are the students outside both C and N.
c i $P(N) = \frac{14}{30} = \frac{7}{15}$	14 of the 30 students enjoy netball.
ii $P(N \text{ only}) = \frac{8}{30} = \frac{4}{15}$	8 of the 30 students enjoy netball but not cricket.
iii $P(C \text{ and } N) = \frac{6}{30} = \frac{1}{5}$	6 students like both cricket and netball.

6 From a group of 50 adults, 35 enjoy reading fiction (F), 20 enjoy reading non-fiction (N) and 10 enjoy reading both fiction and non-fiction.

a Illustrate the information in a Venn diagram.

b State the number of people who enjoy:

i fiction only **ii** neither fiction nor non-fiction

c Find the probability that a person chosen at random will enjoy reading:

i non-fiction **ii** non-fiction only **iii** both fiction and non-fiction

First enter the '10' in the intersection, then fill in all the other regions. $35 - 10 = 25$ enjoy fiction only.

7 At a show, 45 children have the choice of riding on the Ferris wheel (F) and/or the Big Dipper (B). Thirty-five of the children wish to ride on the Ferris wheel, 15 children want to ride on the Big Dipper and 10 children want to ride on both.

a Illustrate the information in a Venn diagram.

b State the number of children who want to:

i ride on the Ferris wheel only

ii ride on neither the Ferris wheel nor the Big Dipper

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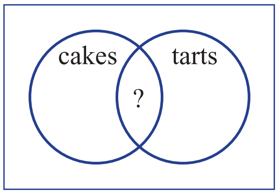
4B

c For a child chosen at random from the group, find the probability that they will want to ride on:
   i the Ferris wheel
   ii both the Ferris wheel and the Big Dipper
   iii the Ferris wheel or the Big Dipper
   iv not the Ferris wheel
   v neither the Ferris wheel nor the Big Dipper

Problem-solving and Reasoning

8 In a group of 12 chefs, all enjoy baking cakes and/or tarts. In fact, 7 enjoy baking cakes and 8 enjoy baking tarts. Find out how many chefs enjoy baking both cakes and tarts.



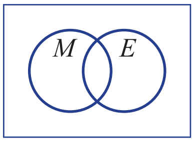
9 In a group of 32 car enthusiasts, all collect either vintage cars or modern sports cars. Of the group, 18 collect vintage cars and 19 collect modern sports cars. How many collect both vintage cars and modern sports cars?

10 Mario and Elisa are choosing a colour to paint the interior walls of their house. They have six colours to choose from: white (w), cream (c), navy (n), sky blue (s), maroon (m) and violet (v).

Mario would be happy with white or cream and Elisa would be happy with cream, navy or sky blue, but they can't decide, so a colour is chosen at random for them.

Let M be the event that Mario will be happy with the colour and let E be the event that Elisa will be happy with the colour.

a Represent the events M and E in a Venn diagram.
b Find the probability that the following events occur.
   i Mario will be happy with the colour choice; i.e. find $P(M)$.
   ii Mario will not be happy with the colour choice.
   iii Both Mario and Elisa will be happy with the colour choice.
   iv Mario or Elisa will be happy with the colour choice.
   v Neither Mario nor Elisa will be happy with the colour choice.



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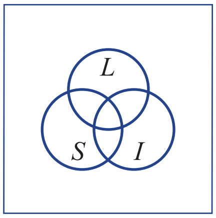
Enrichment: Courier companies

11 Of 15 chosen courier companies, 9 offer a local service (L), 7 offer an interstate service (S) and 6 offer an international service (I). Two companies offer all three services, 3 offer both local and interstate services, 5 offer only local services and 1 offers only an international service.

a Draw a Venn diagram displaying the given information.



b Find the number of courier companies that offer neither local, interstate nor international services.

c If a courier is chosen at random from the 15 initially examined, find the following probabilities.

i $P(L)$
ii $P(L \text{ only})$
iii $P(L \text{ or } S)$
iv $P(L \text{ and } S \text{ only})$

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Drilling for Gold exercise

4B1: Venn diagram alphabet

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

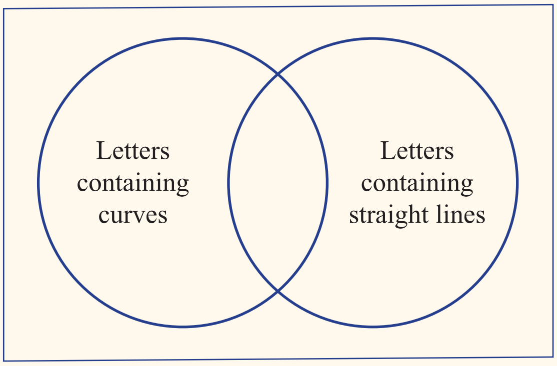
The letters of the alphabet are made up of straight lines, curved lines or a mixture of both straight and curved lines.

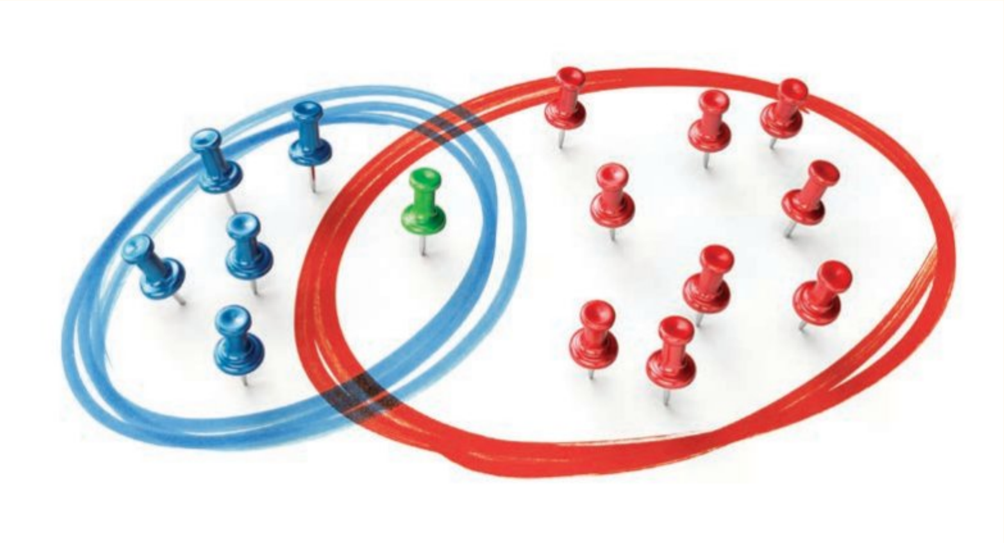
There are four regions in the Venn diagram below: outside the circles, inside the circles and the overlapping section of the circles.

Each of the regions defines the characteristics of a set of objects, items or, in this case, the letters of the alphabet.

Use the worksheet or copy the Venn diagram below into your exercise book and write where each of the letters from the alphabet above should be placed so that the Venn diagram is correct.

Compare your Venn diagram with those of your classmates.





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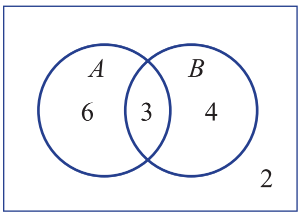
4C Two-way tables

Stage
5.2
5.2◊
5.1
4

Like a Venn diagram, two-way tables are useful tools for the organisation of overlapping events. The totals at the end of each column and row help to find the unknown numbers required to solve various problems.

▶ Let's start: Comparing Venn diagrams with two-way tables

Here is a Venn diagram and an incomplete two-way table.



	A	not A	
B		4	
not B			8
	9		15

- Complete the two-way table.
- Describe what each box in the two-way table means.
- Is it possible to find all the missing numbers in the two-way table without referring to the Venn diagram?

Key ideas

Drilling for Gold 4C1 at the end of this section

- **Two-way tables** use rows and columns to describe the number of elements in different regions of overlapping events.

Two-way tables Tables with columns and rows recording the numbers of items belonging to different categories

Two-way table

	A	not A		
B	4 (A and B)	6 (B only)	10	← Total for B
not B	10 (A only)	3 (Neither A nor B)	13	← Total for not B
	14	9	23	← Total
	↑ Total for A	↑ Total for not A		

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Exercise 4C

Understanding

1 Match the shaded two-way tables (**A–D**) with each description (**a–d**).

a A and B **b** B only **c** A **d** A or B

A

	A	not A	
B			
not B			

B

	A	not A	
B			
not B			

C

	A	not A	
B			
not B			

D

	A	not A	
B			
not B			

2 Look at this two-way table.

	A	not A	
B	4	3	7
not B	6	1	7
	10	4	14

A only means A and not B.

a State the number of elements in these events.

i A and B **ii** A only

iii B only **iv** neither A nor B

v A **vi** B

vii not A **viii** not B

b A or B includes A and B, A only and B only. Find the total number of elements in A or B.

Fluency

Example 5 Using two-way tables

The Venn diagram shows the distribution of elements in two sets, A and B.

a Transfer the information in the Venn diagram to a two-way table.

b Find the number of elements for these regions.

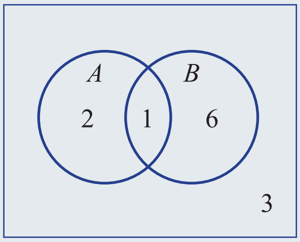
i A and B **ii** B only **iii** A only

iv neither A nor B **v** A **vi** not B

vii A or B

c Find:

i $P(A \text{ and } B)$ **ii** $P(\text{not } A)$ **iii** $P(A \text{ only})$



Solution

a

	A	not A	
B	1	6	7
not B	2	3	5
	3	9	12

Explanation

	A	not A	
B	A and B	B only	Total the row
not B	A only	Neither A nor B	Total the row
	Total the column	Total the column	Overall total

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b	**i**	1	In both A and B.
	ii	6	In B but not A.
	iii	2	In A but not B.
	iv	3	In neither A nor B.
	v	3	Total of A: $2 + 1 = 3$
	vi	5	Total not in B: $2 + 3 = 5$
	vii	$2 + 1 + 6 = 9$	In A only or B only or both (three regions).

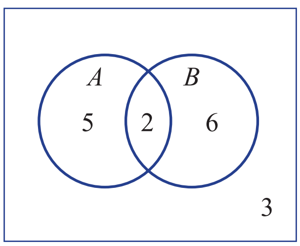
c **i** $P(A \text{ and } B) = \frac{1}{12}$

ii $P(\text{not } A) = \frac{9}{12} = \frac{3}{4}$

iii $P(A \text{ only}) = \frac{2}{12} = \frac{1}{6}$

When calculating probabilities, you will need to divide the number of elements in each set by the number of elements in the sample space, which is 12.

3 The Venn diagram shows the distribution of elements in two sets, A and B.



A two-way table has these headings:

	A	not A	
B			
not B			

a Transfer the information in the Venn diagram to a two-way table.

b Find the number of elements in these regions.

i A and B **ii** B only **iii** A only **iv** neither A nor B

v A **vi** not B **vii** A or B

c Find:

i $P(A \text{ and } B)$ **ii** $P(\text{not } A)$ **iii** $P(A \text{ only})$

4 From a total of 10 people, 5 like oranges (O), 6 like grapes (G) and 4 like both oranges and grapes.

a Draw a Venn diagram for the 10 people.

b Draw a two-way table.

c Find the number of people from the group who like:

i only grapes **ii** oranges

iii oranges and grapes **iv** oranges or grapes

d Find:

i $P(G)$ **ii** $P(O \text{ and } G)$ **iii** $P(O \text{ only})$

iv $P(\text{not } G)$ **v** $P(O \text{ or } G)$

Once you have your Venn diagram, you can transfer the information to the two-way table.

Skillsheet 4B

5 Of 12 people interviewed at a train station, 7 like staying in hotels, 8 like staying in apartments and 4 like staying in hotels and apartments.

a Draw a two-way table for the 12 people.

b Find the number of people interviewed who like:

i only hotels **ii** neither hotels nor apartments

c Find the probability that one of the people interviewed likes:

i hotels or apartments **ii** only apartments

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4C

Problem-solving and Reasoning

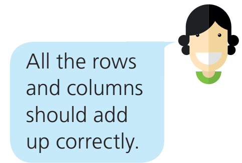
6 Complete the following two-way tables.

a

	A	not A	
B		3	6
not B			
		4	11

b

	A	not A	
B	2	7	
not B			3
	4		



7 In a class of 24 students, 13 like Mathematics, 9 like English and 3 like both.

a Find the probability that a randomly selected student from this class likes both Mathematics and English.

b Find the probability that a randomly selected student from this class likes neither Mathematics nor English.

8 Two sets, A and B, are mutually exclusive.

a Find $P(A \text{ and } B)$.

b Now complete this two-way table.

	A	not A	
B		6	
not B			12
	10		18

9 Of 32 cars at a show, 18 cars have 4WD, 21 are sports cars and 27 have 4WD or are sports cars.

a Find the probability that a randomly selected car from the show is both 4WD and a sports car.

b Find the probability that a randomly selected car from the show is neither 4WD nor a sports car.

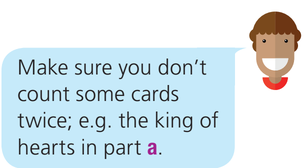
10 A card is selected from a standard deck of 52 playing cards. Find the probability that the card is:

a a heart or a king

b a club or a queen

c a black card or an ace

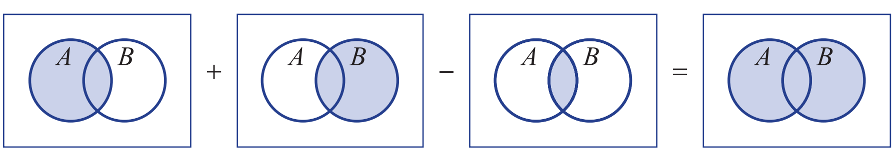
d a red card or a jack



Enrichment: The addition rule

For some of the problems above you will have noticed the following, which is called the addition rule.

A + B − A and B = A or B



11 Use the addition rule to find A or B in these problems.

a Of 20 people at a sports day, 12 people like hurdles (H), 14 like discus (D) and 8 like both hurdles and discus (H and D). How many from the group like hurdles or discus?

b Of 100 households, 84 have wide-screen TVs, 32 have computers and 41 have both. How many of the households have wide-screen TVs or computers?

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Keeping in touch with numeracy

Non-calculator

1 Convert these decimals to simple fractions.
a 0.5 **b** 0.6
c 0.75 **d** 1.25

3 Convert the following test results to percentages.
a 37 out of 50
b 11 out of 20
c 20 out of 25
d 10 out of 30

5 When $a = 6$, write down the value of $2a + 1$.

7 The direction from *A* to *B* is north-west. What is the direction from *B* to *A*?

9 Write down the next three numbers in this sequence: 1, 4, 9, 16, __ , __ , __.

11 Jan walked the dog for 35 minutes, starting from 9:43 a.m. At what time did she finish?

13 The diameter of a circle is 25 metres. What is its radius?

15 How much simple interest is earned on an investment of $100 for 6 years at 6% per annum?

17 Josh is normally paid $10 per hour. How much is he paid for 8 hours at time and a half?

19 One Australian dollar buys 70 US cents. How many US dollars can be purchased with A$50?

Calculator

2 Convert these decimals to simple fractions.
a 0.55 **b** 0.64
c 0.375 **d** 1.0625

4 Convert the following test results to percentages (correct to 1 decimal place).
a 37 out of 51
b 11 out of 19
c 21 out of 24
d 19 out of 35

6 When $a = -6$, write down the value of $2a^2 + 1$.

8 Find the obtuse angle between north and south-west.

10 What is the 15th number in this sequence: 1, 4, 9, 16, ...

12 How many hours and minutes are there from 2:47 a.m. to 2:09 p.m. on the same day?

14 The diameter of a circular table top is 1.7 metres. Use the formula $C = \pi d$ to find the circumference, correct to 2 decimal places.

16 How much simple interest is earned on an investment of $600 for 6.5 years at 5.6% per annum?

18 Tara worked 6 hours at normal time and 6 hours at time and a half. She was paid $330. What is her normal hourly rate of pay?

20 One Australian dollar buys 70 US cents. At this rate, how many Australian dollars are required to buy US$100?

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Drilling for Gold exercise

4C1: Two-way table alphabet

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

The letters of the alphabet may have line symmetry, no line symmetry, rotational symmetry or no rotational symmetry.

For example:

The letter  has line symmetry but does not have rotational symmetry.

There are four regions in the two-way table below.

Each of the regions defines the characteristics of a set of objects, items or, in this case, the letters of the alphabet.

Use the worksheet or copy the two-way table below into your exercise book and write where each of the letters from the alphabet above should be placed so that the two-way table is correct.

Compare your two-way table with those of your classmates.

	Rotational symmetry	**No rotational symmetry**	
Line symmetry			
No line symmetry			

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4D Collecting data

Stage

5.2
5.2◊
5.1
4

A statistician collects, analyses and interprets data. They assist the government, companies and other organisations to make decisions and plan for the future.

Statisticians:

- **Formulate** and **refine** questions for a survey.
- Choose some **subjects** (i.e. people) to complete the survey.
- **Collect** the data.
- **Organise and display** the data using the most appropriate graphs and tables.
- **Analyse** the data.
- **Interpret the data** and draw conclusions.

There are many reports in the media that begin with the words 'A recent study has found that…'. These are usually the result of a survey or investigation that a researcher has conducted to collect information about an important issue, such as unemployment, crime or obesity.

▶ Let's start: Critiquing survey questions

Here is a short survey. It is not very well constructed.

Question 1: How old are you?

Question 2: How much time did you spend sitting in front of the television or a computer yesterday?

Question 3: Some people say that teenagers like you are lazy and spend too much time sitting around when you should be outside exercising. What do you think of that comment?

Have a class discussion about the following.

- What will the answers to Question 1 look like? How could they be displayed?
- What will the answers to Question 2 look like? How could they be displayed?
- Is Question 2 going to give a realistic picture of your normal daily activity?
- How could Question 2 be improved?
- What will the answers to Question 3 look like? How could they be displayed?
- How could Question 3 be improved?

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Key ideas

- Surveys are used to collect statistical data.
- Survey questions need to be constructed carefully so that the person knows exactly what sort of answer to give. They should use simple language and should not be ambiguous.
- Survey questions should not be worded so that they deliberately try to provoke a certain kind of response.
- Surveys should respect the privacy of the people being surveyed.
- If the question contains an option to be chosen from a list, the number of options should be an odd number, so that there is a 'neutral' choice. For example, the options could be:

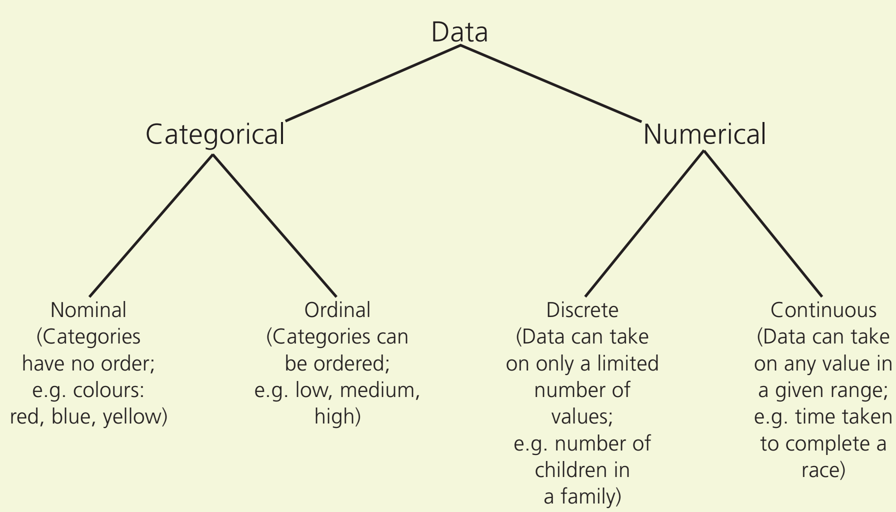
strongly disagree	disagree	unsure	agree	strongly agree

- A **population** is a group of people, animals or objects with something in common. Some examples of populations are:
  - all the people in Australia on Census night
  - all the students in your school
  - all the boys in your Maths class
  - all the tigers in the wild in Sumatra
  - all the cars in Sydney
  - all the wheat farms in NSW

Population The entire group selected

Statistical data Information gathered by observation, survey or measurement

- A *sample* is a group that has been chosen from a population. Sometimes information from a sample is used to describe the whole population, so it is important to choose the sample carefully.
- If information is collected from a sample of a population so that some members are less likely to be included, then the sample is thought to be biased.
- **Statistical data** can be categorised as follows.



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Exercise 4D

Understanding

1 A popular Australian 'current affairs' television show recently investigated the issue of spelling. They suspected that people in their twenties are not as good at spelling as people in their fifties, so they decided to conduct a statistical investigation. They chose a sample of 12 people aged 50–59 years and 12 people aged 20–29 years.

Answer the following questions on paper, then discuss in a small group or as a whole class.

- **a** Do you think that the number of people surveyed is enough?
- **b** How many people in Australia do you think there are aged 20–29 years?
- **c** How many people in Australia do you think there are aged 50–59 years?
- **d** Use the website of the Australian Bureau of Statistics to look up the answers to parts **b** and **c**.
- **e** Do you think it is fair and reasonable to compare the spelling ability of these two groups of people?
- **f** How would you go about comparing the spelling ability of these two groups of people?
- **g** Would you give the two groups the same set of words to spell?
- **h** How could you give the younger people an unfair advantage?
- **i** What sorts of words would you include in a spelling test for the survey?
- **j** How and where would you choose the people to do the spelling test?

2 Match each word (**a**–**h**) with its definition (**A**–**H**).

a population	**A** a group chosen from a population
b census	**B** a tool used to collect statistical data
c sample	**C** the state of being secret
d survey	**D** an element or feature that can vary
e data	**E** all the people or objects in question
f variable	**F** statistics collected from an entire population
g statistics	**G** the practise of collecting and analysing data
h confidentiality	**H** the factual information collected from a survey or other source

3 Match each word (**a**–**f**) with its definition (**A**–**F**).

a numerical	**A** categorical data that have no order
b continuous	**B** data that are numbers
c discrete	**C** numerical data that take on a limited number of values
d categorical	**D** data that can be divided into categories
e ordinal	**E** numerical data that take any value in a given range
f nominal	**F** categorical data that can be ordered

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4D

Fluency

Example 6 Describing types of data

What type of data would the following survey questions generate?

a How many televisions do you have in your home?

b To what type of music do you most like to listen?

Solution	Explanation
a numerical and discrete	The answer to the question is a number with a limited number of values; in this case, a whole number.
b categorical and nominal	The answer is a type of music and these categories have no order.

4 Which one of the following survey questions would generate numerical data?

A What is your favourite colour?

B What type of car does your family own?

C How long does it take for you to travel to school?

D What type of dog do you own?

5 Which one of the following survey questions would generate categorical data?

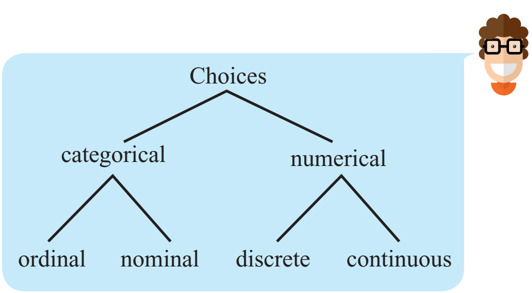
A How many times do you eat at your favourite fast-food place in a typical week?

B How much do you usually spend buying your favourite fast food?

C How many items did you buy last time you went to your favourite fast-food place?

D Which is your favourite fast food?

6 Year 10 students were asked the following questions in a survey. Describe what type of data each question generates.



a How many people under the age of 18 years are there in your immediate family?

b How many letters are there in your first name?

c Which company is the carrier of your mobile telephone calls? Optus/Telstra/Vodafone/Virgin/Other (Please specify.)

d What is your height?

e How would you describe your level of application in Maths? (Choose from very high, high, medium or low.)

7 Every student in Years 7 to 12 votes in the prefect elections. The election process is an example of:

A a population

B continuous data

C a representative sample

D a census

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Problem-solving and Reasoning

8 TV 'ratings' are used to determine the shows that are the most popular. Every week some households are chosen at random and a device is attached to their television. The device keeps track of the shows the households are watching during the week.

The company that chooses the households should always attempt to find:

- **A** a census
- **B** continuous data
- **C** a representative sample
- **D** ungrouped data

9 The principal decides to survey Year 10 students to determine their opinion of mathematics. In order to increase the chance of choosing a representative sample, the principal should:

- **A** Give a survey form to the first 30 Year 10 students who arrive at school.
- **B** Give a survey form to all the students studying the most advanced maths subject.
- **C** Give a survey form to 5 students in every maths class.
- **D** Give a survey form to 20% of the students in every class.

10 Discuss some of the problems with the selection of a survey sample for each given topic.

- **a** A survey at the train station of how Australians get to work.
- **b** An email survey on people's use of computers.
- **c** Phoning people on the electoral roll to determine Australia's favourite sport.

11 Choose a topic in which you are especially interested, such as football, cricket, movies, music, cooking, food, computer games or social media.

Make up a survey about your topic that you could give to the students in your class. It must have *four* questions.
Question 1 must produce data that are categorical and ordinal.
Question 2 must produce data that are categorical and nominal.
Question 3 must produce data that are numerical and discrete.
Question 4 must produce data that are numerical and continuous.

Enrichment: The Australian Census

12 Research the 2011 or 2016 Australian Census on the website of the Australian Bureau of Statistics. Find out something interesting from the results of the Australian Census. Write a short news report or record a 3 minute news report on your computer.

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4E Column graphs and histograms

Stage

Stage
5.2
5.2◊
5.1
4

Data can be collected in a number of ways, including surveys, experiments, recording the performance of a sportsperson or just counting. Sorting the data into a frequency table allows us to make sense of it and draw conclusions from it.

Statistical graphs are an essential part of the analysis and representation of data. By looking at statistical graphs, we can draw conclusions about the numbers or categories in the data set.

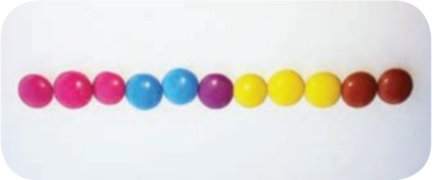
▶ Let's start: A packet of Smarties

Drilling for Gold 4E1a 4E1

Smarties are sold in small packets.

- How many Smarties would you expect to find in one packet?
- Would you expect every packet to contain the same number of Smarties?
- How many different colours would you expect to find in your packet?

Consider these photos of the Smarties found in one packet.



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- Which of the following tools could be used to display and analyse the data produced by the question: What are the colours of the Smarties in your packet?
  A frequency distribution table
  B dot plot
  C stem-and-leaf plot
  D sector graph (i.e. pie chart)
  E divided bar graph
  F column graph
  G histogram
  H mean
  I median
  J mode
  K range
- Open a small packet of Smarties and use some of the tools listed above to analyse its contents. Compare your results with your initial expectations.

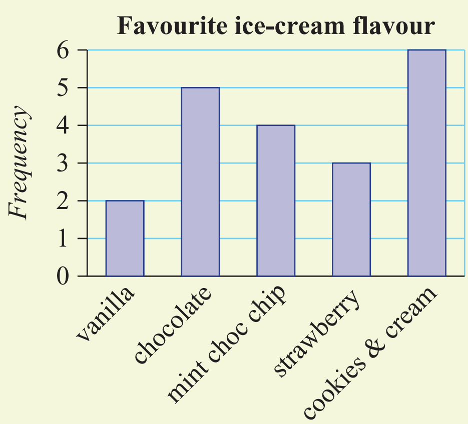
Key ideas

- A **frequency table** displays data by showing the number of values within a set of categories or class intervals. It may include a tally column to help count the data.

Favourite ice-cream flavour	Tally	Frequency
vanilla	||	2
chocolate	~~||||~~	5
mint choc chip	||||	4
strawberry	|||	3
cookies and cream	~~||||~~ |	6

Frequency table A table showing all possible scores in one column and the frequency of each score in another column

- A **column graph** can be used for a single set of categorical or discrete data.



Column graph A graphical representation of a single set of categorical or discrete data, where columns are used to show the frequency of scores

- **Histograms** can be used for grouped discrete or continuous numerical data. The frequency of particular class intervals is recorded.
  – In the following tables, the interval 10–19 includes all numbers from 10 (including 10) to less than 20.

Histogram A special type of column graph with no gaps between the columns; it can represent class intervals

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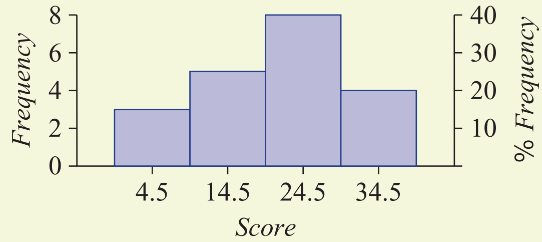
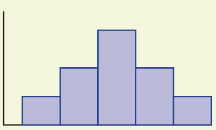
– The percentage frequency is calculated as % Frequency $= \frac{\text{frequency}}{\text{total}} \times 100\%$.

Class interval	Frequency	Percentage frequency
1–9	3	$\frac{3}{20} \times 100 = 15\%$
10–19	5	$\frac{5}{20} \times 100 = 25\%$
20–29	8	40%
30–39	4	20%
Total	20	100%

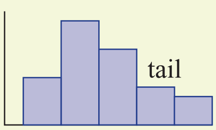


■ Data can be symmetrical or skewed.

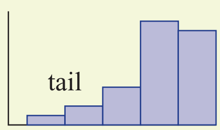
Symmetrical



Positively skewed



Negatively skewed



Exercise 4E

Understanding

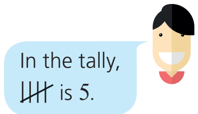
1 Classify each set of data as categorical or numerical.

a 4.7, 3.8, 1.6, 9.2, 4.8
b red, blue, yellow, green, blue, red
c low, medium, high, low, low, medium
d 3 g, 7 g, 8 g, 7 g, 4 g, 1 g, 10 g

2 Complete these frequency tables.

a

Car colour	Tally	Frequency
red	|||	
white	~~||||~~ |	
green	||	
silver	||	
Total		



b

Class interval	Frequency	Percentage frequency
80–84	8	$\frac{8}{50} \times 100 = 16\%$
85–89	23	
90–94	13	
95–100		
Total	50	

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Fluency

Example 7 Constructing a frequency table and column graph

Twenty people checking out of a hotel were surveyed on the level of service provided by the hotel staff. The results were:

poor	first class	poor	average	good
good	average	good	first class	first class
good	good	first class	good	average
average	good	poor	first class	good

a Construct a frequency table to record the data, with headings Category, Tally and Frequency.

b Construct a column graph for the data.

Solution

Explanation

a

Category	Tally	Frequency
Poor	|||	3
Average	||||	4
Good	~~||||~~ |||	8
First class	~~||||~~	5
Total	20	20

Construct a table with the headings Category, Tally, Frequency.

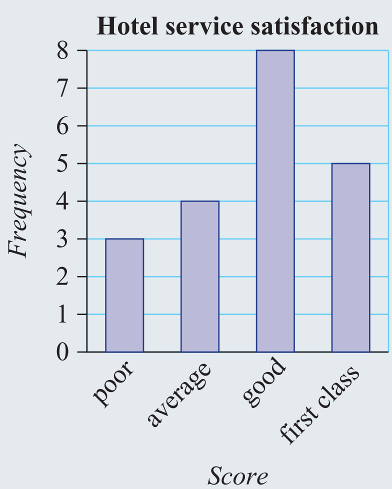
Fill in each category shown in the data. Work through the data in order, recording a tally mark (|) next to the category. It is a good idea to tick the data as you go, to keep track.

On the 5th occurrence of a category, place a diagonal line through the tally marks (~~||||~~). Then start again on the 6th. Do this every five values, as it makes the tally marks easy to count up.

Once all data are recorded, count the tally marks for the frequency.

Check that the frequency total adds up to the number of people surveyed (in this case 20).

b



Draw a set of axes with frequency going up to 8.

For each category, draw a column with height up to its frequency value.

Leave gaps between each column.
Give your graph an appropriate heading.

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4E

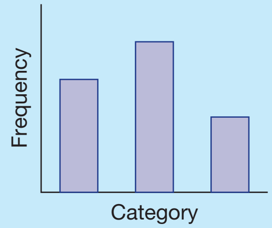
3 For the data below obtained from surveys:

i Copy and complete this frequency table.

Category	Tally	Frequency
⋮	⋮	⋮

ii Construct a column graph for the data and include a heading.

In the column graph leave spaces between each column.



a The results from 10 subjects on a student's school report are:

good	low	good	very low	good
low	good	good	excellent	low

b The favourite sports of a class of students are:

football	football	netball	netball
netball	tennis	football	football
basketball	basketball	tennis	basketball
football	basketball	football	football
tennis	tennis	football	tennis

Example 8 Constructing and analysing a histogram

Twenty people were surveyed to find out how many times they use the internet in a week. The raw data are listed.

$$21, 19, 5, 10, 15, 18, 31, 40, 32, 25$$
$$11, 28, 31, 29, 16, 2, 13, 33, 14, 24$$

a Organise the data into a frequency table, using class intervals of 10. Include a percentage frequency column.

b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.

c Which interval is the most frequent?

d What percentage of people used the internet 20 times or more?

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Solution

a

Class interval	Tally	Frequency	Percentage frequency
0–9	𝍷𝍷	2	10%
10–19	𝍸 𝍷𝍷𝍷	8	40%
20–29	𝍸	5	25%
30–39	𝍷𝍷𝍷𝍷	4	20%
40–49	𝍷	1	5%
Total	20	20	100%

Explanation

Work through the data and place a tally mark in the correct interval each time.

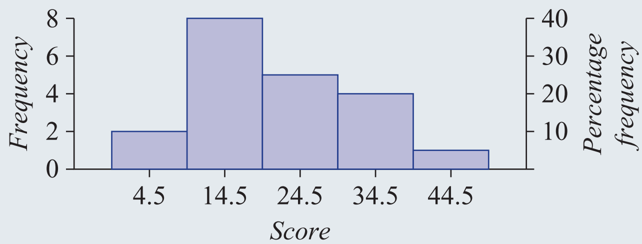
The interval 10–19 includes all numbers from 10 (including 10) to less than 20, so 10 is in this interval but 20 is not.

Count the tally marks to record the frequency.

Add the frequency column to ensure all 20 values have been recorded.

Calculate each percentage frequency by dividing the frequency by the total (i.e. 20) and multiplying by 100%; i.e. $\frac{2}{20} \times 100 = 10$.

b **Number of times the internet is accessed**



Transfer the data from the frequency table to the histogram. Axes scales are evenly spaced and the histogram bar is placed across the boundaries of the class interval. There is no space between the bars.

c The 10–19 interval is the most frequent.

The frequency (8) is highest for this interval. It is the highest bar on the histogram.

d 50% of those surveyed used the internet 20 or more times.

Sum the percentages for the class intervals from 20–49 and above:
$25 + 20 + 5 = 50$

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4E

4 The Maths test results of a class of 25 students were recorded as:

74	65	54	77	85	68	93
59	71	82	57	98	73	66
88	76	92	70	77	65	68
81	79	80	75			

a Organise the data into a frequency table, using class intervals of 10. Include a percentage frequency column.

b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.

c Which interval is the most frequent?

d If an A is awarded for a score of 80 or more, what percentage of the class received an A?

Construct a frequency table like this:

Class interval	Tally	Frequency	Percentage frequency
50–59	|||	3	$\frac{\text{freq.}}{\text{total}} \times 100\%$
60–69			
70–79			
80–89			
90–99			
Total			

5 The number of wins scored this season is given for 20 hockey teams. Here are the raw data.

4, 8, 5, 12, 15, 9, 9, 7, 3, 7
10, 11, 1, 9, 13, 0, 6, 4, 12, 5

a Organise the data into a frequency table using class intervals of 5, starting with 0–4, then 5–9 etc. and include a percentage frequency column.

b Construct a histogram for the data, showing both the frequency and percentage frequency on the one graph.

c Which interval is the most frequent?

d What percentage of teams scored 5 or more wins?

6 This frequency table displays the way in which 40 people travel to and from work.

Type of transport	Frequency	Percentage frequency
Car	16	
Train	6	
Ferry	8	
Walking	5	
Bicycle	2	
Bus	3	
Total	40	

a Copy and complete the table.

b Use the table to find:

i the frequency of people who travel by train

ii the most popular form of transport

iii the percentage of people who travel by car

iv the percentage of people who walk or cycle to work

v the percentage of people who travel by public transport, including trains, buses and ferries

Percentage frequency:
$= \frac{\text{frequency}}{\text{total}} \times 100$

Skillsheet 4C

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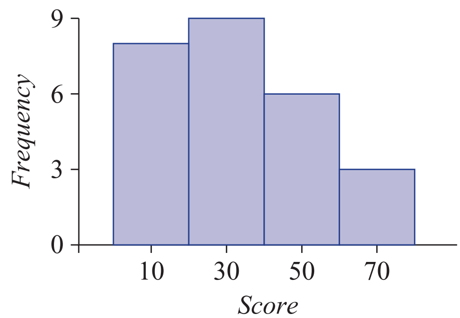
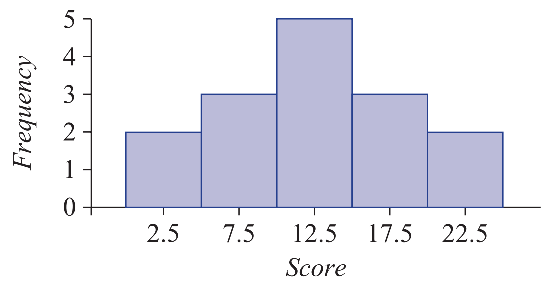
Problem-solving and Reasoning

7 Which of these histograms shows a symmetrical data set and which one shows a skewed data set?

a



b



8 This tally records the number of mice that were weighed and categorised into particular mass intervals for a scientific experiment.

a Construct a table using these column headings: Mass, Frequency and Percentage frequency.
b Find the total number of mice weighed in the experiment.
c State the percentage of mice that were in the 20–24 gram interval.
d Which was the most common weight interval?
e What percentage of mice were in the most common mass interval?
f What percentage of mice had a mass of 15 grams or more?

Mass (grams)	Tally
10–14	|||
15–19	卌 |
20–24	卌 卌 卌 |
25–29	卌 卌 卌 卌 |
30–34	||||

9 A school orchestra contains four musical sections: string, woodwind, brass and percussion. The number of students playing in each section is summarised in this tally.

a Construct and complete a percentage frequency table for the data.
b What is the total number of students in the school orchestra?
c What percentage of students play in the string section?
d What percentage of students do not play in the string section?
e If the number of students in the string section increases by 3, what will be the percentage of students who play in the percussion section? Round your answer to 1 decimal place.

Section	Tally
String	卌 卌 卌 卌 |
Woodwind	卌 |||
Brass	卌 ||
Percussion	||||

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4E

Enrichment: Interpreting further graphical displays

10 The graph shown compares the life expectancy of males and females in 10 different countries. Use the graph to answer the questions that follow.

- **a** Which country has the biggest difference in life expectancy for males and females? Approximately how many years is this difference?
- **b** Which country appears to have the smallest difference in life expectancy between males and females?
- **c** From the information in the graph, write a statement comparing the life expectancy of males and females.
- **d** South Africa is clearly below the other countries. Provide some reasons why you think this may be the case.

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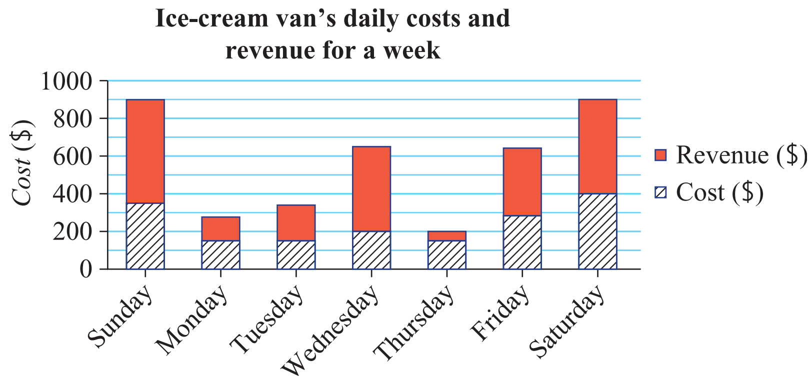
11 This graph shows the amount, in dollars, spent (Cost) on the purchase and storage of ice-cream each day by an ice-cream vendor, and the amount of money made from the daily sales of ice-cream (Revenue) over the course of a week.



a On which particular days was the cost highest for the purchase and storage of ice-cream? Why do you think the vendor chose these days to spend the most?

b Wednesday earned the greatest revenue for any weekday. What factors may have led to this?

c Daily profit is determined by the difference in revenue and cost. Identify:

i on which day the largest profit was made and the amount of profit (in dollars)

ii on which day the vendor suffered the biggest financial loss

d Describe some problems associated with this type of graph.

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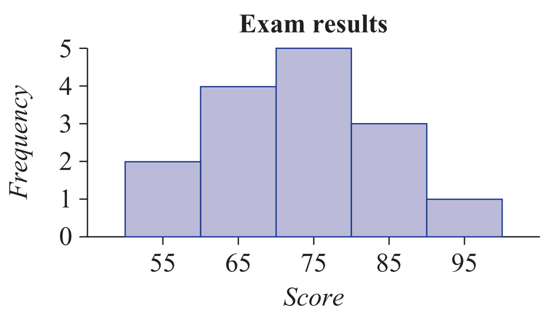
4F Dot plots and stem-and-leaf plots

Stage
5.2
5.2◊
5.1
4

In addition to column graphs, dot plots and stem-and-leaf plots can be used to display categorical or discrete data. They can also display two related sets for comparison. Like a histogram, they help to show how the data are distributed. A stem-and-leaf plot has the advantage of still displaying all the individual data items.

▶ Let's start: Alternate representations

The histogram and stem-and-leaf plot below represent the same set of data. They show the exam scores achieved by a class.



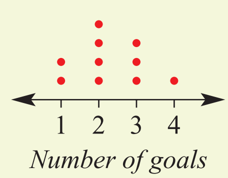
Stem	Leaf
5	1 3
6	5 7 8 8
7	1 2 4 4 6
8	3 4 7
9	6

6|8 means 68

- Describe the similarities in what the two graphs display.
- What information does the stem-and-leaf provide that the histogram does not? What is the advantage of this?
- Which graph do you prefer?
- Discuss any other types of graphs that could be used to present the data.

Key ideas

- A **dot plot** records the frequency of each discrete value in a data set.
  – Each occurrence of the value is marked with a dot.



Dot plot A graph in which each dot represents one score

- A **stem-and-leaf plot** displays each value in the data set using a stem number and a leaf number.
  – The data are displayed in two parts: a stem and a leaf.
  – The 'key' tells you how to interpret the stem and leaf parts.
  – The graph is similar to a histogram with class intervals but the original data values are not lost.
  – The stem-and-leaf plot is ordered to allow for further statistical calculations.

Stem	Leaf
1	0 1 1 5
2	3 7
3	4 4 6
4	2 9

2|3 means 23
↑ key

Stem-and-leaf plot A table that lists numbers in order, grouped in rows

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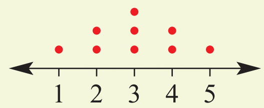
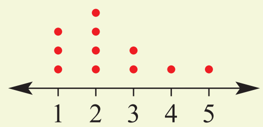
- The shape of each of these graphs gives information about the distribution of the data.
  - A graph that is even either side of the centre is **symmetrical**.



  - A graph that is bunched to one side of the centre is **skewed**.



Symmetrical A distribution of data which is symmetrical on either side of the mean and the median

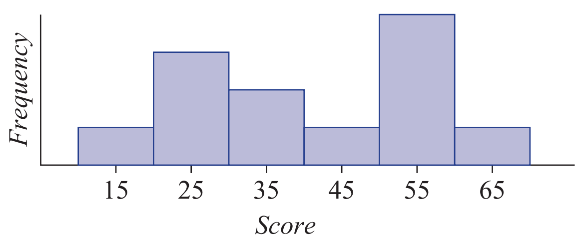
Skewed The shape of the graph of some data that is bunched to one side of the centre

Exercise 4F

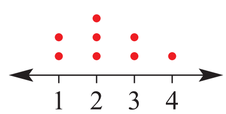
Understanding

1 Name each of these types of graphs.

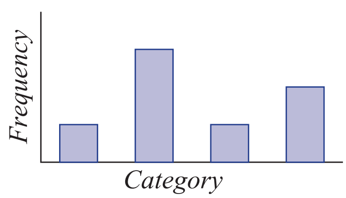
a



b



c



d

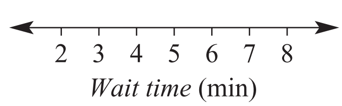
Stem	Leaf
0	1 1 3
1	2 4 7
2	0 2 2 5 8
3	1 3

2|5 means 25

2 A student records the following wait times, in minutes, for his school bus over 4 school weeks.

5 4 2 8 4 2 7 5 3 3
5 4 2 5 4 5 8 7 2 6

Copy and complete this dot plot of the data.



3 List the data shown in these stem-and-leaf plots.

a

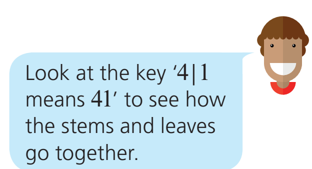
Stem	Leaf
3	2 5
4	1 3 7
5	4 4 6
6	0 2
7	1 1

4|1 means 41

b

Stem	Leaf
0	2 3 7
1	4 4 8 9
2	3 6 6
3	0 5

2|3 means 23



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4F

4 Order this stem-and-leaf plot.

Stem	Leaf
12	7 2 3
10	1 4 8 1
13	9 0 2
11	3 0 3 6

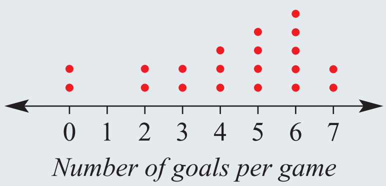
12|2 means 122

Stems and leaves need to be placed in numerical order.

Fluency

Example 9 Interpreting a dot plot

This dot plot shows the number of goals per game scored by a team during the soccer season.

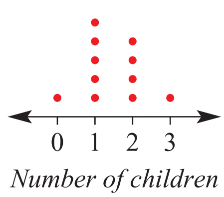


a How many games were played?
b What was the most common number of goals per game?
c How many goals were scored for the season?
d Describe the data in the dot plot.

Solution	Explanation
a There were 20 matches played.	Each dot represents a match. Count the number of dots.
b 6 goals in a game occured most often.	The most common number of goals has the most dots.
c $2 \times 0 + 2 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 6 + 2 \times 7$ $= 0 + 4 + 6 + 12 + 20 + 30 + 14$ $= 86$ goals	Count the number of games (i.e. dots) for each number of goals and multiply by the number of goals. Add these together.
d Two games resulted in no goals but the data were generally skewed towards a higher number of goals.	Consider the shape of the graph; it is bunched towards the 6 end of the goal scale.

5 A number of families were surveyed to find the number of children in each. The results are shown in this dot plot.

a How many families were surveyed?
b What was the most common number of children in a family?
c How many children were there in total?
d Describe the data in the dot plot.



4 families had 2 children (4 dots), so that represents 8 children from these families.

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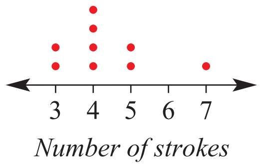
6 This dot plot shows the number of strokes a golfer played, each hole, in his round of golf.
- **a** How many holes did he play?
- **b** How many strokes did he play in the round?
- **c** Describe his round of golf.



Example 10 Constructing a stem-and-leaf plot

For the following set of data:
- **a** Organise the data into an ordered stem-and-leaf plot.
- **b** Describe the distribution of the data as symmetrical or skewed.

22	62	53	44	35	47	51	64	72
32	43	57	64	70	33	51	68	59

Solution

a

Stem	Leaf
2	2
3	2 3 5
4	3 4 7
5	1 1 3 7 9
6	2 4 4 8
7	0 2

5|1 means 51

b The distribution of the data is almost symmetrical.

Explanation

For two-digit numbers, select the tens value as the stem and the units as the leaves.
The data ranges from 22 to 72, so the graph will need stems 2 to 7.
Work through the data and record the leaves in the order of the data.

Stem	Leaf
2	2
3	5 2 3
4	4 7 3
5	3 1 7 1 9
6	2 4 4 8
7	2 0

51 occurs twice, so the leaf 1 is recorded twice in the 5 stem row.
Once data are recorded, redraw and order the leaves from smallest to largest.
Include a key to explain how the stem and leaf go together; i.e. 5|1 means 51.

The shape of the graph is roughly symmetrical (i.e. evenly spread) either side of the centre.

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7 For each of the following sets of data:

i Organise the data into an ordered stem-and-leaf plot.
ii Describe the distribution of the data as symmetrical or skewed.

Remember to include a key such as '4|6 means 46'.

a 46 22 37 15 26 38 52 24
31 20 15 37 21 25 26

b 35 16 23 55 38 44 12 48 21 42
53 36 35 25 40 51 27 31 40 36 32

c 153 121 124 117 125 118
135 137 162 145 147 119
127 149 116 133 160 158

Symmetrical

Stem	Leaf
1	1 2
2	1 2 3
3	1 2 3 4
4	1 2 7
5	3

Skewed

Stem	Leaf
1	2 5 7 8
2	3 4 6 6
3	1 2
4	5

Example 11 Constructing back-to-back stem-and-leaf plots

Two television sales employees sell the following number of televisions each week over a 15-week period.

Employee 1
23 38 35 21 45 27 43 36
19 35 49 20 39 58 18

Employee 2
28 32 37 20 30 45 48 17
32 37 29 17 49 40 46

a Construct an ordered back-to-back stem-and-leaf plot.
b Describe the distribution of each employee's sales.

Solution

a

Employee 1 | **Employee 2**

Leaf	Stem	Leaf
9 8	1	7 7
7 3 1 0	2	0 8 9
9 8 6 5 5	3	0 2 2 7 7
9 5 3	4	0 5 6 8 9
8	5	

3|7 means 37

Explanation

Construct an ordered stem-and-leaf plot with employee 1's sales on the left-hand side and employee 2's sales on the right-hand side.
Include a key.

b Employee 1's sales are almost symmetrical, whereas employee 2's sales are skewed.

Observe the shape of each employee's graph. If appropriate, use the words symmetrical (spread evenly around the centre) or skewed (bunched to one side of the centre).

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8 For the following sets of data:

- **i** Draw a back-to-back stem-and-leaf plot.
- **ii** Comment on the distribution of the two data sets.

a Set 1: 61 38 40 53 48 57 64
39 42 59 46 42 53 43

Set 2: 41 55 64 47 35 63 61
52 60 52 56 47 67 32

b Set 1: 176 164 180 168 185 187 195 166 201
199 171 188 175 192 181 172 187 208

Set 2: 190 174 160 170 186 163 182 171
167 187 171 165 194 182 163 178

Using a key for part **b** may help.
Recall that 17|6 means 176.
Stems will be 16, 17 etc.

Problem-solving and Reasoning

9 Two football players, Nick and Jake, compare their personal tallies of the number of goals scored for their team over a 12-match season. Their tallies are as follows.

Game	1	2	3	4	5	6	7	8	9	10	11	12
Nick	0	2	2	0	3	1	2	1	2	3	0	1
Jake	0	0	4	1	0	5	0	3	1	0	4	0

- **a** Draw a dot plot to display Nick's goal-scoring achievement.
- **b** Draw a dot plot to display Jake's goal-scoring achievement.
- **c** How would you describe Nick's scoring habits?
- **d** How would you describe Jake's scoring habits?

10 This stem-and-leaf plot shows the times, in minutes, that Chris has achieved in the past 14 fun runs he competed in.

Stem	**Leaf**
20	5 7
21	1 2 6
22	2 4 6 8
23	4 5 6
24	3 6

22|4 means 22.4 min

- **a** What is the difference between his slowest and fastest times?
- **b** Just by looking at the stem-and-leaf plot, what would you estimate to be Chris's average time?
- **c** If Chris records another time of 24.9 minutes, how would this affect your answer to part **b**?

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4F

11 The data below show the distances travelled (in km) by students at an inner-city and an outer-suburb school.

Inner city:	3	10	9	14	21	6	Outer suburb:	12	21	18	9	34	19
	1	12	24	1	19	4		24	3	23	41	18	4

a Draw a back-to-back stem-and-leaf plot for the data.
b Comment on the distribution of distances travelled by students for each school.
c Give a practical reason for the distribution of the data.

12 Determine the possible values of the pronumerals in the following ordered stem-and-leaf plots.

a

Stem	Leaf
1	2 4
2	3 6 9 b
a	1 4
4	7 c 8

2|3 means 2.3

b

Stem	Leaf
20	a 1 4
21	2 2 9
22	0 b 5 7
23	1 4

22|7 means 227

The stems and leaves are ordered from smallest to largest. A leaf can appear more than once.

Enrichment: Splitting stems

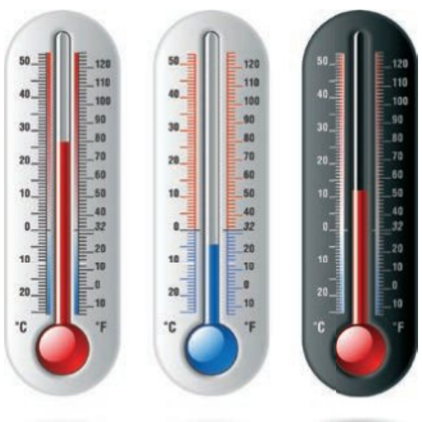
13 The back-to-back stem-and-leaf plot below shows the maximum daily temperature for two cities over a 2-week period.

Maximum temperature

City A Leaf	Stem	City B Leaf
	0	
9 8 8	0*	
4 3 3 1 1 1	1	
8 8 6 6 5	1*	7 9
	2	0 2 2 3 4 4
	2*	5 6 7 7 8
	3	1

1|4 means 14
1*|5 means 15

a Describe the difference between the stems 1 and 1*.
b To which stem would these numbers be allocated?
i 12°C **ii** 5°C
c Why might you use this process of splitting stems, like that used for 1 and 1*?
d Compare and comment on the differences in temperatures between the two cities.
e What might be a reason for these different temperatures?



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4G Using the range and the three measures of centre

Stage

5.2
5.2◊
5.1
4

Summary statistics allow us to describe data using a single numerical value. The mean may be used to describe a student's performance over a series of tests. The median (middle value when data are ordered smallest to largest) is often used when describing the house prices in a suburb, and the mode is the score that appears the most. These are termed *measures of centre.*

Let's start: Mean, median or mode?

The following data represent the number of goals scored by Ellie in each game of a 9-game netball season.

$$24 \quad 18 \quad 25 \quad 16 \quad 3 \quad 23 \quad 27 \quad 19 \quad 25$$

It is known that the figures below represent, in some order, the mean, median and mode.

$$\mathbf{25} \quad \mathbf{20} \quad \mathbf{23}$$

- Without doing any calculations, can you suggest which statistic is which? Explain.
- From the data, what gives an indication that the mean will be less than the median (middle value)?
- Describe how you would calculate the mean, median and mode from the data values.

Key ideas

- The **mean** is calculated by summing all the data values and dividing by the total number of values.

  $$\text{Mean } (\bar{x}) = \frac{\text{sum of all data values}}{\text{number of data values}}$$

  – The mean can be affected by extreme values (outliers) in the data.
- The **mode** is the most commonly occurring value in the data set.
  – A data set can have two modes (called **bimodal**) or no mode at all.
- The **median** is the middle value of a data set when the data are arranged in order.
  – If the data set has an even number of values, then the median is the average of the two middle values. For example:

  2 3 (6) 8 12 — Median = 6

  4 7 8 10 13 17 — $\text{Median} = \frac{8 + 10}{2} = 9$
- The **range** is a measure of how spread out the data are.
  – Range = maximum value – minimum value

Mean A value calculated by dividing the total of a set of numbers by the number of values

Mode The score that appears most often in a set of numbers

Bimodal When a set of data has two modes

Median The middle score when all the numbers in a set are arranged in order

Range The difference between the highest and lowest numbers in a set

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Exercise 4G

Understanding

1 Use the words from the list below to fill in the missing word in these sentences.

mean, median, mode, bimodal, range

a The ________ is the most frequently occurring value in a data set.
b Dividing the sum of all the data values by the total number of values gives the ________.
c The middle value of a data set ordered from smallest to largest is the ________.
d A data set with two most common values is ________.
e A data set has a maximum value of 7 and a minimum value of 2. The ________ is 5.

2 Calculate the following.

a $\dfrac{1+4+5+8+2}{5}$ **b** $\dfrac{3+7+6+2}{4}$ **c** $\dfrac{3.1+2.3+6.4+1.7+2.5}{5}$

3 Circle the middle value(s) of these ordered data sets.

a 2 4 6 7 8 10 11
b 6 9 10 14 17 20

Recall that an even number of data values will have two middle values.

4 Sebastian drinks the following number of cups of coffee each day in a week.

4 5 3 6 4 3 3

a How many cups of coffee does he drink in the week (sum of the data values)?
b How many days are in the week (total number of data values)?
c What is the mean number of cups of coffee Sebastian drinks each day (i.e. part **a** ÷ part **b**)?

Fluency

Drilling for Gold 4G1a 4G1

Example 12 Finding the mean, mode and range

For the following data sets, find:

i the mean **ii** the mode **iii** the range

a 2, 4, 5, 8, 8 **b** 3, 15, 12, 9, 12, 15, 6, 8

Solution	**Explanation**
a i $\text{Mean} = \dfrac{2+4+5+8+8}{5}$ $= \dfrac{27}{5}$ $= 5.4$	$\text{Mean} = \dfrac{\text{sum of all data values}}{\text{number of data values}}$ Add all the data values and divide by the number of values (in this case, 5).
ii The mode is 8.	The mode is the most common value in the data.
iii Range $= 8 - 2$ $= 6$	Range = maximum value − minimum value

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b i Mean $= \dfrac{3 + 15 + 12 + 9 + 12 + 15 + 6 + 8}{8}$

$= \dfrac{80}{8}$

$= 10$

Mean $= \dfrac{\text{sum of all data values}}{\text{number of data values}}$

Add all the data values and divide by the number of values (8).

ii There are two modes, 12 and 15.

The data are bimodal as 12 and 15 are the most common data values.

iii Range $= 15 - 3$

$= 12$

Range = maximum value – minimum value

5 For each of the following data sets, find:

i the mean **ii** the mode **iii** the range

a 2 4 5 8 8

b 5 8 10 15 20 12 10 50

c 55 70 75 50 90 85 50 65 90

d 27 30 28 29 24 12

e 2.0 1.9 2.7 2.9 2.6 1.9 2.7 1.9

f 1.7 1.2 1.4 1.6 2.4 1.3

Recall:

Mean $= \dfrac{\text{sum of data values}}{\text{number of data values}}$

Mode is the most common value.

Range = maximum – minimum

Example 13 Finding the median

Find the median of each data set.

a 4, 7, 12, 2, 9, 15, 1 **b** 16, 20, 8, 5, 21, 14

Solution	Explanation
a 1 2 4 (7) 9 12 15 Median = 7	The data must first be ordered from smallest to largest. The median is the middle value. For an odd number of data values there will be one middle value.
b 5 8 14 16 20 21 Median $= \dfrac{14 + 16}{2}$ $= 15$	Order the data from smallest to largest. For an even number of data values there will be two middle values. The median is the average of these two values (i.e. the value halfway between the two middle numbers).

6 Find the median of each data set.

a 1 4 7 8 12

b 1 2 2 4 4 7 9

c 11 13 6 10 14 13 11

d 62 77 56 78 64 73 79 75 77

e 2 4 4 5 6 8 8 10 12 22

f 1 2 2 3 7 12 12 18

First, make sure that the data are in order. For two middle values, find their average.

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4G

7 Nine people watch the following number of hours of television on a weekend.

4 4 6 6 6 8 9 9 11

Skillsheet 4D

a Find the mean number of hours of television watched.
b Find the median number of hours of television watched.
c Find the range of the television hours watched.
d What is the mode number of hours of television watched?

Problem-solving and Reasoning

8 Eight students compare the amount of pocket money they receive. The data are as follows.

\$12 \$15 \$12 \$24 \$20 \$8 \$50 \$25

a Find the range of pocket money received.
b Find the median amount of pocket money.
c Find the mean amount of pocket money.
d Why is the mean larger than the median?

Example 14 Calculating summary statistics from a stem-and-leaf plot

For the data in this stem-and-leaf plot, find:
a the range
b the mode
c the mean
d the median

Stem	Leaf
2	5 8
3	1 2 2 2 6
4	0 3 3
5	2 6

5|2 means 52

Solution	Explanation
a Minimum value = 25 Maximum value = 56 Range = 56 – 25 = 31	The first data item is the minimum and the last is the maximum. Range = maximum value – minimum value
b Mode = 32	The mode is the most common value. The leaf 2 appears three times with the stem 3.

c Mean

$$= \frac{25 + 28 + 31 + 32 + 32 + 32 + 36 + 40 + 43 + 43 + 52 + 56}{12}$$

$$= \frac{450}{12}$$

$$= 37.5$$

Form each data value from the graph and add them all together. Then divide by the number of data values in the stem-and-leaf plot.

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d Median $= \dfrac{32 + 36}{2}$

$= 34$

There is an even number of data values; i.e. 12. The median will be the average of the middle two values (i.e. the 6th and 7th data values).

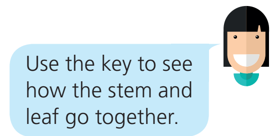
9 For the data in these stem-and-leaf plots, find:

i the range

ii the mode

iii the mean (rounded to 1 decimal place)

iv the median



a

Stem	Leaf
2	1 3 7
3	2 8 9 9
4	4 6

3|2 means 32

b

Stem	Leaf
0	4 4
1	0 2 5 9
2	1 7 8
3	2

2|7 means 27

c

Stem	Leaf
10	1 2 4
11	2 6
12	5

11|6 means 116

d

Stem	Leaf
3	0 0 5
4	2 7
5	1 3 3
6	0 2

3|2 means 3.2

10 This back-to-back stem-and-leaf plot shows the results achieved by two students, Hugh and Mark, on their end-of-year examination in each subject.

Hugh leaf	Stem	Mark leaf
8 8 5	6	4
7 3	7	4 7
5 4 2 1 1	8	2 4 6 8
	9	2 4 5

7|4 means 74%

a For each student, find:

i the mean

ii the median

iii the range

b Compare the performance of the two students using your answers to part **a**.

11 A real estate agent recorded the following amounts for the sale of five houses.

$120 000 $210 000 $280 000 $370 000 $1 700 000

The mean is $536 000 and the median is $280 000.
Which is a better measure of the centre of the five house prices: the mean or the median? Give a reason.

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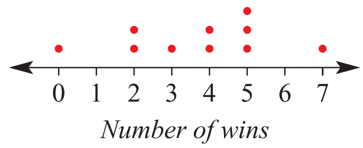
4G

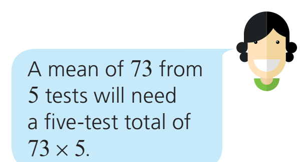
Drilling for Gold 4G2 at the end of this section

12 This dot plot shows the number of wins recorded by a school lacrosse team in the past 10 8-game seasons.



a What is the median number of wins?
b What is the mean number of wins?
c The following season, the team records 3 wins. What effect will this have (i.e. increase/decrease/no change) on the:
  i median?
  ii mean?

13 Catherine achieves the following scores on her first four Maths tests: 64 70 72 74
a What is her mean mark from the Maths tests?
b In the fifth and final test, Catherine is hoping to raise her mean mark to 73. What mark does she need on the last test to achieve this?



Enrichment: Moving run average

14 A moving average is determined by calculating the average of all data values up to a particular time or place in the data set.
Consider a batsman in cricket with the following runs scored from 10 completed innings.

Innings	1	2	3	4	5	6	7	8	9	10
Score	26	38	5	10	52	103	75	21	33	0
Moving average	26	32								

In the table, 26 is the average after 1 inning and 32 is the average after 2 innings.
a Complete the table by calculating the moving average for innings 3–10. Round to the nearest whole number where required.
b Plot the score and moving averages for the batsman on the same set of axes, with the innings number on the horizontal axis. Join the points to form two line graphs.
c Describe the behaviour of the:
  i score graph
  ii moving average graph
d Describe the main difference in the behaviour of the two graphs. Give reasons.

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Drilling for Gold exercise

4G2: Which one has changed? Why or why not?

Use the worksheet or write the answers in your exercise book.

1 Consider this data set.

1	4	5	7	8

a If the 8 is changed to 10, explain why the median will not change.

b If the 8 is changed to 10, what happens to the range? Explain your decision.

c If the 8 is changed to 10 and the 1 is changed to 2, what happens to the mean? Explain your decision.

d If all the data values are increased by 1, which one of the following does *not* change: the mean, the median or the range? Explain your decision.

e If all the data values are changed to 5, which one of the following does *not* change: the mean, the median or the range? Explain your decision.

2 A new data value called *A* is going to be included in the data set.

a Which value of *A* will keep the median at 5?

b When *A* is included, what is the highest possible median? What is the lowest?

c What value of *A* will increase the mean from 5 to 10? Show your calculations.

3 Two new data values (*A* and *B*) will be included in the data set.

a Is it possible for the median to remain unchanged? How?

b If both *A* and *B* are whole numbers greater that 6, what will be the new median?

c When *A* and *B* are included, the mean is 7. Give possible values for *A* and *B*.

d A set of five whole number data values has a mean of 5, a mode of 2, a median of 4 and a range of 8. What are the five values?

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Consumer maths

Worksheet 4A

Lotto, Keno and other gambling activities

Some people say, 'You have to be in it to win it' or 'Someone has to win the jackpot', but in all forms of gambling the probability of winning a large amount of money is very close to zero. Also, the prize you win is not as much as it should be for doing something that was so unlikely to happen!

In this activity you will learn how to use your calculator to calculate the probability of winning some of the gambling activities that are currently available.

For example, in the game called Lotto, 45 balls are placed in a barrel and 6 of them are selected. To enter you choose 6 numbers with values between 1 and 45.

A calculator with a button labelled ${}^{n}C_{r}$ can quickly calculate the number of combinations, where:

- n stands for the total number of objects.
- C stands for combinations.
- r stands for the number of objects to be chosen.

The ${}^{n}C_{r}$ button is usually an alternative or second function of one of the regular buttons. It is activated by pressing a button labelled SHIFT or 2ndF.

The number of different ways in which 6 balls can be selected is calculated by pressing this sequence of buttons:

4, then 5, then SHIFT or 2ndF, then ${}^{n}C_{r}$ then 6, then =.

Try it for yourself. The answer in this case would be written by mathematicians as:

$${}^{45}C_{6} = 8\,145\,060$$

So there are 8 145 060 different sets of 6 numbers that can be chosen.

The chance that a gambler will guess them correctly is $\frac{1}{8\,145\,060}$, which is 0.0000001, correct to 7 decimal places.

If you played this game every day for about 22 300 years, you could expect to win it once.

Use your calculator to perform the following calculations, then download the activity sheet and start calculating!

1 ${}^{10}C_{2}$ **2** ${}^{40}C_{6}$ **3** ${}^{50}C_{8}$

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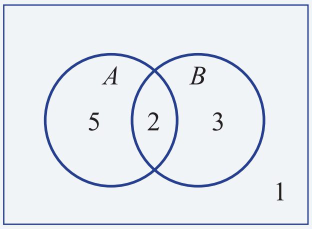
Puzzles and games

1 'I have nothing in common.' Match the answers to the letters in parts **a** and **b** to uncover the code.

___	___	___	___	___	___	___	___
$\frac{5}{14}$	5	2	5	7	10	10	$\frac{7}{11}$

___	___	___	___	___	___	___	___	___
$\frac{5}{11}$	$\frac{3}{14}$	$\frac{1}{2}$	10	5	$\frac{10}{11}$	$\frac{1}{7}$	3	$\frac{5}{11}$

a These questions relate to the Venn diagram shown.



- [T] How many elements are in A and B?
- [L] How many elements are in A or B?
- [V] How many elements are in B only?
- [Y] Find $P(A)$.
- [S] Find $P(A \text{ or } B)$.
- [E] Find $P(A \text{ only})$.

b These questions relate to the two-way table at right.

	P	**not P**	
Q	U	4	9
not Q	2		
		A	14

- [U] What number should be in place of the letter U?
- [A] What number should be in place of the letter A?
- [M] Find P(P and Q).
- [C] Find P(not P).
- [X] Find P(neither P nor Q).
- [I] Find P(P only).

2 *Game for two people:* You will need a bag or pocket and coloured counters.

- One person places 8 counters of 3 different colours in a bag or pocket. The second person must not look!
- The second person then selects a counter from the bag. The colour is noted, then the counter is returned to the bag. This is repeated 100 times.
- Complete this table.

Colour	Tally	Frequency	Guess
Total	100	100	

- Using the experimental results, the second person now tries to guess how many counters of each colour are in the bag.

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Puzzles and games

3 The mean mass of 6 boys is 71 kg. The mean mass of 5 girls is 60 kg. Find the mean mass of all 11 people put together.

4 Sean has a current four-topic average of 78% for Mathematics. What score does he need in the fifth topic to have an overall average of 80%?

5 I am a data set made up of five whole number values. My mode is 2 and both my mean and median are 5. What is my biggest possible range?

6 A single data set has 3 added to every value. Describe the change in:

a the mean
b the median
c the range

7 I am a data set with four whole number values.

- I have a range of 8.
- I have a mode of 3.
- I have a median of 6.

What are my four values?

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Chapter summary

Probability and statistics

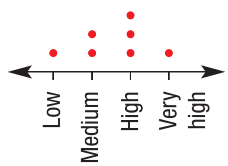
Data

Categorical
- Nominal (e.g. red, blue, ...)
- Ordinal (e.g. low, medium, ...)

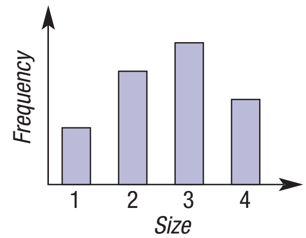
Numerical
- Discrete (e.g. 1, 2, 3, ...)
- Continuous (e.g. 0.31, 0.481, ...)

Graphs for a single set of categorical or discrete data

Dot plot



Column graphs



Stem-and-leaf plot

Stem	Leaf
0	1 6
1	2 7 9
2	3 8
3	4

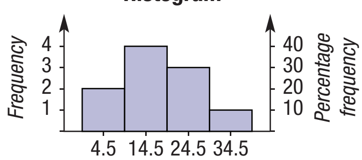
2 | 3 means 23

Frequency tables

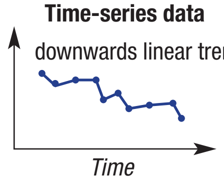
Class interval	Frequency	Percentage frequency
0–9	2	20%
10–19	4	40%
20–29	3	30%
30–39	1	10%
Total	10	100%

$$\text{Percentage frequency} = \frac{\text{frequency}}{\text{total}} \times 100$$

Histogram



Time-series data



Measures of centre

- Mean $(\bar{x}) = \dfrac{\text{sum of all values}}{\text{number of values}}$
- Median = middle value of ordered data

odd number: 1 4 ⑥ 9 12 — median

even number: 1 2 4 6 7 11 — Median $= \dfrac{4+6}{2} = 5$

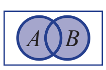
- Mode = most common value

Measure of spread

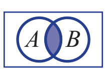
- Range = max – min

Notation

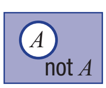
- A or B



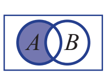
- A and B



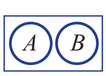
- Complement of A is not A.



- A only



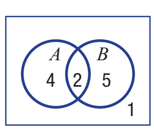
- Mutually exclusive events



Probability

- Sample space is the list of all possible outcomes
- $P(\text{event}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

Venn diagram



Two-way table

	A	not A	
B	2	5	7
not B	4	1	5
	6	6	12

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T **Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.**

Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

1 A letter is chosen from the word SUCCESS. The probability that the letter is not a C is:

A $\frac{2}{7}$ **B** $\frac{3}{5}$ **C** $\frac{5}{7}$ **D** $\frac{4}{7}$ **E** $\frac{3}{7}$

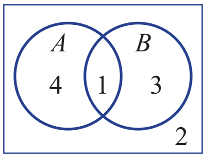
2 The number of manufacturing errors spotted in a car plant on 20 days is given by this table.

Number of errors	0	1	2	3	Total
Frequency	11	6	2	1	20

An estimate of the probability that on the next day no errors will be observed is:

A $\frac{3}{10}$ **B** $\frac{9}{20}$ **C** $\frac{11}{20}$ **D** $\frac{17}{20}$ **E** $\frac{3}{20}$

3 For this Venn diagram, $P(A \text{ or } B)$ is equal to:



A $\frac{4}{5}$ **B** $\frac{1}{2}$ **C** $\frac{5}{8}$

D $\frac{1}{4}$ **E** $\frac{1}{10}$

4 15 people like apples or bananas. Of those 15 people, 10 like apples and 3 like both apples and bananas. How many people from the graph like only apples?

A 5 **B** 3 **C** 13 **D** 7 **E** 10

5 For this two-way table, $P(A \text{ and } B)$ is:

	A	not A	
B		1	3
not B			4
		4	

A $\frac{2}{3}$ **B** $\frac{1}{4}$ **C** $\frac{1}{7}$

D $\frac{1}{3}$ **E** $\frac{2}{7}$

6 What type of data are generated by the survey question: 'What is your favourite sport to play?'

A numerical and discrete
B numerical and continuous
C categorical and continuous
D categorical and nominal
E categorical and ordinal

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Questions **7** and **8** refer to the stem-and-leaf plot below.

7 The minimum score in the data is:
A 4 **B** 0 **C** 24
D 38 **E** 54

Stem	Leaf
2	4 9
3	1 1 7 8
4	2 4 6
5	0 4

4|2 means 42

8 The mode is:
A 3 **B** 31 **C** 4 **D** 38 **E** 30

9 The range and mean of 2, 4, 3, 5, 10 and 6 are:
A range = 8, mean = 5
B range = 4, mean = 5
C range = 8, mean = 4
D range = 2–10, mean = 6
E range = 8, mean = 6

10 The median of 29, 12, 18, 26, 15 and 22 is:
A 18 **B** 22 **C** 20 **D** 17 **E** 26

Short-answer questions

1 A fair 6-sided die is rolled once. Find:
a $P(4)$ **b** $P(\text{even})$ **c** $P(\text{at least }3)$

2 A letter is chosen from the word INTEREST. Find the probability that the letter will be:
a I **b** E **c** a vowel
d not a vowel **e** E or T

3 An engineer inspects 20 houses in a street for cracks. The results are summarised in this table.

Number of cracks	0	1	2	3	4
Frequency	8	5	4	2	1

a From these results, estimate the probability that the next house inspected in the street will have the following number of cracks:
i 0 **ii** 1 **iii** 2 **iv** 3 **v** 4
b Estimate the probability that the next house will have:
i at least 1 crack
ii no more than 2 cracks

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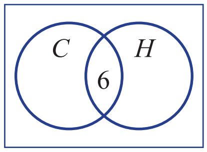
4 Of 36 people surveyed, 18 have an interest in cars (C), 11 have an interest in homewares (H) and 6 have an interest in both cars and homewares.

a Complete this Venn diagram.



b Complete this two-way table.

	C	not C	
H	6		
not H			

c State the number of people surveyed who do not have an interest in either cars or homewares.

d If a person is chosen at random from the group, find the probability that the person will:

i have an interest in cars and homewares

ii have an interest in homewares only

iii not have any interest in cars

5 All 26 birds in an aviary have clipped wings and/or a tag. In total, 18 birds have tags, 14 have clipped wings and 6 have both clipped wings and a tag.

a Find the number of birds that have only clipped wings.

b Find the probability that a bird chosen at random will have a tag only.

6 A group of 16 people was surveyed to find the number of hours of television they watch in a week. The raw data are listed:

6, 5, 11, 13, 24, 8, 1, 12,
7, 6, 14, 10, 9, 16, 8, 3

a Organise the data into a table with class intervals of 5. Start with 0–4, 5–9 etc. Include a tally, frequency and percentage frequency column.

b Construct a histogram for the data, showing both the frequency and percentage frequency on the graph.

c Would you describe the data as symmetrical or skewed?

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7 A basketball team scores the following points per match for a season.
20, 19, 24, 37, 42, 34, 38, 49, 28, 15, 38, 32, 50, 29
a Construct an ordered stem-and-leaf plot for the data.
b Describe the distribution of scores.

8 For the following sets of data, determine:
i the mean ii the range iii the median
a 2, 7, 4, 8, 3, 6, 5
b 10, 55, 67, 24, 11, 16
c 1.7, 1.2, 1.4, 1.6, 2.4, 1.3

9 Thirteen adults compare their ages at a party. They are:
40, 41, 37, 32, 48, 43, 32, 76, 29, 33, 26, 38, 87
a Find the mean age of the adults, to 1 decimal place.
b Find the median age of the adults.
c Why do you think the mean age is larger than the median age?

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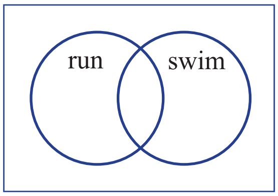
Extended-response questions

1 Of 15 people surveyed to find out if they run or swim for exercise, 6 said they run, 4 said they swim and 3 said they both run and swim.



a How many people surveyed neither run nor swim?
b One of the 15 people is selected at random.
Find the probability that they:
i run or swim ii only swim
c Represent the information in a two-way table.

2 The number of flying foxes taking refuge in a fig tree is recorded over a period of 14 days. The data collected are given here.

73, 50, 36, 82, 15, 24, 73, 57, 65, 86, 51, 32, 21, 39

a Arrange the data in ascending order.
b Find the:
i mean
ii median
iii range
c Describe the distribution. Give two possible reasons why the numbers of flying foxes taking refuge varies so much.

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Chapter 1: Financial mathematics

Multiple-choice questions

1 Nigel earns $1256 a week. Using 52 weeks in a year, his annual income is:
A $24.15 B $32 656 C $65 312 D $15 072 E $12 560

2 Who earns the most?
A Sally: $56 982 p.a.
B Jurek: $1986 per fortnight
C Abdhul: $1095 per week
D Chloe: $32.57 per hour, 38-hour weeks for 44 weeks
E Jordan: $20 000 p.a.

3 Adriana works 35 hours a week, earning $575.75. Her wage for a 38-hour week would be:
A $16.45 B $21 878.50 C $625.10 D $530.30 E $575.75

4 Ayden earns a retainer of $420 per week plus a 2% commission on all sales. What is his fortnightly pay when his sales total $56 000 for the fortnight?
A $2240 B $840 C $1540 D $1960 E $56 420

5 Danisha earns $4700 gross a month. She has annual deductions of $14 100 in tax and $1664 in health insurance. Her net monthly income is:
A $3386.33 B $11 064 C $40 636 D $72 164 E $10 000

Short-answer questions

1 Thuong earns $25.76 an hour as a mechanic. Calculate his:
a time-and-a-half rate
b double-time rate
c weekly wage for 38 hours at normal rate
d weekly wage for 38 hours at normal rate plus 3 hours at time and a half

2 Imogen earns $15.40 an hour on weekdays and double time on the weekends. Calculate her weekly pay if she works 9 a.m. to 3 p.m. Monday to Friday and 9 a.m. till 11:30 a.m. on Saturday.

3 Cara invests 10% of her net annual salary for 1 year into an investment account earning 4% p.a. simple interest for 5 years. Calculate the simple interest earned if her annual net salary is $17 560.

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4 Marina has a taxable income of $42 600. Calculate her income tax if she falls into the following tax bracket.

$3572 plus 32.5c for each $1 over $37 000

5 Darren earns $372 per week plus 1% commission on all sales. Find his weekly income if his sales for the week total $22 500.

6 a A $120 Blu-ray player is discounted by 15%. What is the sale price?
b A $1100 dining table is marked up by 18% of its cost price. What was its cost price, to the nearest dollar?

7 Each fortnight, Raj earns $1430 gross and pays $34.94 in superannuation, $23.40 in union fees and $493.60 in tax.
a What is Raj's annual gross income?
b How much tax does Raj pay each year?
c What is Raj's net annual income?
d What is Raj's net weekly income?

8 Find the final value of an investment of $7000 at 6% p.a., compounded annually for 4 years.

Extended-response question

A tablet computer, with a recommended retail price of $749, is offered for sale in three different ways.

Method A	Method B	Method C
5% discount for cash	3% fee for a credit card payment	20% deposit and then $18.95 per month for 3 years

a Jai buys a tablet for cash. How much does Jai pay?
b Talia buys a tablet using her mother's credit card. How much more does Talia pay for her tablet compared to Jai?
c Georgia wishes to pay for her tablet using method C.
i Calculate the deposit Georgia must pay.
ii What is the final cost of Georgia purchasing the tablet on terms?
iii How much interest does Georgia pay on her purchase?
iv What percentage of the recommended retail price is Georgia's interest? Round your answer to 2 decimal places.

Chapter 2: Measurement

Multiple-choice questions

1 The number of centimetres in 2.8 metres is:
A 0.28 B 28 C 280 D 2.8 E 2800

2 A rectangle has length 7 cm and perimeter 22 cm. Its breadth is:
A 7.5 cm B 15 cm C 14 cm D 8 cm E 4 cm

3 The area of a circle with diameter 10 cm is given by:
A $\pi(10)^2$ cm^2 B $\pi(5)^2$ cm^2 C 10π cm^2 D $5 \times \pi$ cm^2 E 25 cm^2

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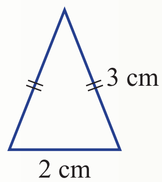
4 In scientific notation, the number 350 000 000 is:
A 3.5×10^8 **B** 3.5×10^7 **C** 35×10^7 **D** 350×10^6 **E** 0.35×10^9

5 The area of the triangular cross-section of a prism is 8 mm^2 and the prism's height is 3 mm. The prism's volume is:
A 48 mm^3 **B** 12 mm^3 **C** 24 mm^2 **D** 24 mm^3 **E** 12 mm^2

Short-answer questions

1 Convert these measurements to the units shown in brackets.
a 0.43 m (cm) **b** 32 000 mm^2 (cm^2) **c** 0.03 m^3 (cm^3) **d** 23 m (mm)
e 8 s (ms) **f** 7.8 s (ns) **g** 8000 t (Mt) **h** 2.3×10^{12} MB (TB)

2 Find the perimeter of each of these shapes.

a


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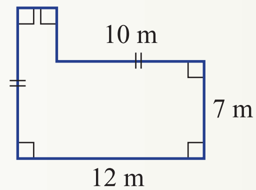

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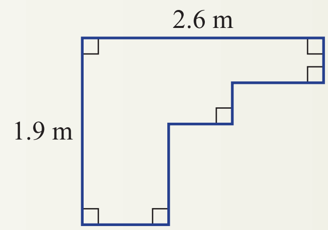
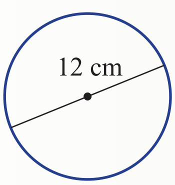


3 For these shapes, find, correct to 2 decimal places:
i the perimeter **ii** the area

a


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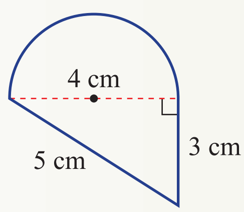

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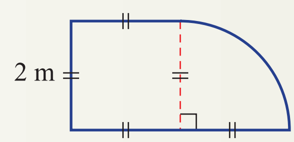
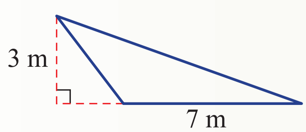


4 Find the area of each of these shapes.

a


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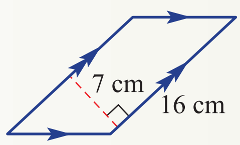

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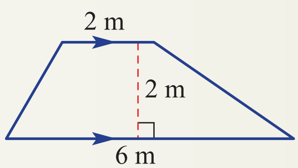
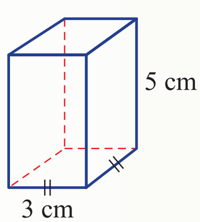


5 For these solids, find:
i the volume **ii** the surface area

a


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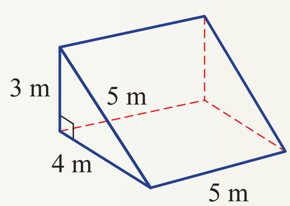

6 Calculate 250^{25}. Write your answer in scientific notation, correct to 3 significant figures.

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7 Give the lower and upper limits of each of the following measurements.

a 7 mL **b** 8.99 g

8 A rectangle has length 4.3 m and breadth 6.8 m.

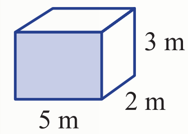
a Between what two values does the true length lie?

b What are the limits of accuracy for the breadth of this rectangle?

c What are the limits of accuracy for the perimeter and area of this rectangle?

Extended-response question

A new zoo enclosure for snakes will have these dimensions.
The front (shaded) is glass.



a What area of glass is required?

b The walls and ceiling need to be painted black. What is the area to be painted?

c The floor is covered in sand 10 cm deep. How many cubic metres of sand are required?

Chapter 3: Algebraic expressions and indices

Multiple-choice questions

1 The expanded and simplified form of $4(2x - 3) - 4$ is:

A $8x - 7$ **B** $6x - 11$ **C** $8x - 16$ **D** $8x - 8$ **E** $6x - 7$

2 The fully factorised form of $4x^2 + 12x$ is:

A $4(x^2 + 3x)$ **B** $4x(x + 12)$ **C** $4(x^2 + 12x)$ **D** $4x(x + 3)$ **E** $2x(x + 6)$

3 4^{-2} can be expressed as:

A $\frac{1}{4^{-2}}$ **B** $\frac{1}{8}$ **C** -16 **D** $\frac{1}{16}$ **E** -8

4 3×10^{-4} written with positive indices is:

A -3×10^4 **B** $\frac{1}{3 \cdot 10^4}$ **C** $\frac{-3}{10^4}$ **D** $\frac{1}{3 \times 10^{-4}}$ **E** $\frac{3}{10^4}$

5 0.00371 in scientific notation is:

A 0.371×10^{-3} **B** 3.7×10^{-2} **C** 3.71×10^{-3}

D 3.71×10^3 **E** 371×10^3

Short-answer questions

1 Simplify the following.

a $2xy + 7x + 5xy - 3x$ **b** $-3a \times 7b$ **c** $\frac{4a^2b}{8ab}$

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2 **a** Expand and simplify the following.

i $-4(x-3)$ **ii** $3x(5x+2)$ **iii** $4(2x+1)+5(x-2)$

b Factorise the following.

i $18-6b$ **ii** $3x^2+6x$ **iii** $8xy-12y$

3 Evaluate the following when $a=3$, $b=-2$ and $c=-4$.

a $2a+b$ **b** abc **c** $2a \div 3b$

d $-3c+b$ **e** a^2+b^2 **f** $c^2 \div b$

g $\frac{a^2+c^2}{2}$ **h** $\frac{a-c}{7}$ **i** $\frac{1}{a}(b+c)$

j a^3-b^3 **k** $2b^3-c$ **l** $\sqrt{a^2+b^2}$

4 Use index laws to simplify the following.

a $2x^2 \times 5x^4$ **b** $4b \times 2ab$ **c** $(2m^4)^3$ **d** $3x^0+(4x)^0$

5 **a** Write the following as basic numerals.

i 4.73×10^5 **ii** 5.21×10^{-3}

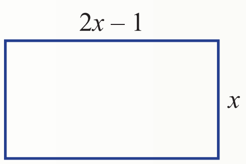
b Convert these to scientific notation, using 3 significant figures.

i 0.000027561 **ii** 8 707 332

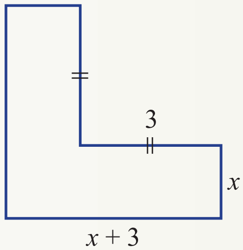
Extended-response question

Find the area of these shapes, in expanded form.

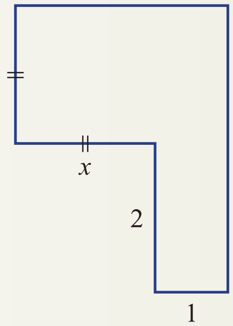
a



b



c



Chapter 4: Probability and statistics

Multiple-choice questions

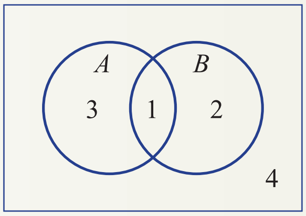
1 A letter is chosen from the word PROBABILITY. What is the probability that it will not be a vowel?

A $\frac{3}{11}$ **B** $\frac{4}{11}$ **C** $\frac{7}{11}$ **D** $\frac{1}{2}$ **E** $\frac{8}{11}$

2 For this Venn diagram, $P(A \text{ or } B)$ is equal to:

A 1 **B** $\frac{1}{6}$ **C** $\frac{1}{10}$

D $\frac{3}{10}$ **E** $\frac{3}{5}$



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3 The number of faults in a computer network over a period of 10 days is recorded in this table.

Number of faults	0	1	2	3
Frequency	1	5	3	1

An estimate for the probability that on the next day there would be at least two faults is:

A $\frac{3}{10}$ B $\frac{1}{5}$ C $\frac{4}{5}$ D $\frac{2}{5}$ E $\frac{1}{10}$

4 The values of a and b in this frequency table are:
A $a = 3, b = 28$
B $a = 4, b = 28$
C $a = 4, b = 19$
D $a = 6, b = 20$
E $a = 3, b = 30$

Colour	Frequency	Percentage frequency (%)
blue	4	16
red	7	b
green	a	12
white	6	24
black	5	20
Total	25	

5 The mean, median and mode of the data set 3, 11, 11, 7, 1, 9 are:
A mean = 7, median = 9, mode = 11
B mean = 6 , median = 9, mode = 11
C mean = 7, median = 8, mode = 11
D mean = 7, median = 11, mode = 8
E mean = 8, median = 7, mode = 11

Short-answer questions

1 A keen bird-watcher records the number of different species of birds in his backyard over a 20-day period.

Number of species	0	1	2	3	4	5	6
Frequency	0	2	3	8	4	2	1

From these results, estimate the probability that on the next day the bird-watcher will observe the following number of species.

a 3 b 2 or 3 c fewer than 5 d at least 2

2 Of 25 students, 18 are wearing jackets, 14 are wearing hats and 10 are wearing both jackets and hats.
a Represent this information in a Venn diagram.
b Represent this information in a two-way table.
c How many students are wearing neither a hat nor a jacket?
d If a student is chosen randomly from the group, find the probability that they will be wearing:
i a hat and not a jacket
ii a hat or a jacket
iii a hat and a jacket

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3 Twenty people were surveyed to find out how many days in the past completed month they had used public transport. The results were as follows.

7, 16, 22, 23, 28, 12, 18, 4, 0, 5, 8, 19, 20, 22, 14, 9, 21, 24, 11, 10

a Organise the data into a frequency table with class intervals of 0–4, 5–9 etc., and include a percentage frequency column.

b Construct a histogram for the data, showing both the frequency and the percentage frequency on the one graph.

c i State the frequency of people surveyed who used public transport on 10 or more days.

ii State the percentage of people surveyed who used public transport on fewer than 15 days.

iii State the most common interval of days for which public transport was used. Can you think of a reason for this?

4 The data set shows the number of video games owned by students in a class.

12 24 36 17 8 24 9 4 15 32 41 26 15 18 7

a Display the data using a stem-and-leaf plot.

b Describe the distribution of the data as symmetrical or skewed.

5 Antonia tossed a fair coin 5 times. It showed heads 5 times. What is the probability of tossing tails on the next toss?

Extended-response question

Farsan’s bank balance over 12 months is recorded below.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Balance ($)	1500	2100	2300	2500	2200	1500	1200	1600	2000	2200	1700	2000

a Plot the time series for the 12 months.

b Describe the way in which the bank balance has changed over the 12 months.

c Between which consecutive months did the biggest change in the bank balance occur?

d What is the overall change in the bank balance over the year?

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Chapter 5

Linear and non-linear relationships

What you will learn

Strand: Number and Algebra

Substrands: LINEAR RELATIONSHIPS
RATIOS AND RATES
NON-LINEAR RELATIONSHIPS

In this chapter you will learn to:

- determine midpoint, gradient and length of an interval
- graph linear relationships
- graph simple non-linear relationships.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw10

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Roller coaster engineering

Engineers apply their knowledge when they design buildings, bridges and roads. When designing a roller coaster ride, an engineer must calculate exactly where support poles need to be placed. The length of each pole and the angle at which it leans are vital measurements that ensure the poles will safely support the massive weights of theme park rides.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw10

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Spreadsheets:
Models for activities using spreadsheets

Videos:
Demonstrations of the use of technology

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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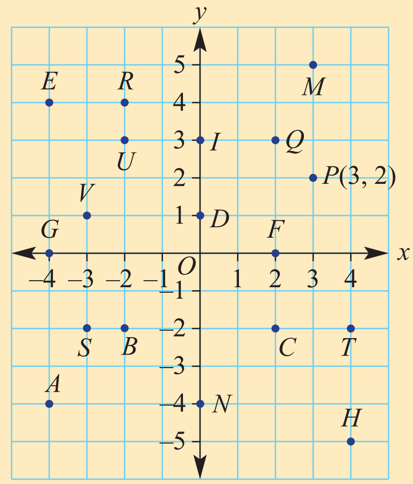
Pre-test

1 The coordinates of P on this number plane are (3, 2). Write down the coordinates of:

a M
b T
c A
d V
e C
f F



2 Name the point with coordinates:

a (–4, 0) **b** (0, 1)
c (–2, –2) **d** (–3, –2)
e (0, –4) **f** (2, 3)

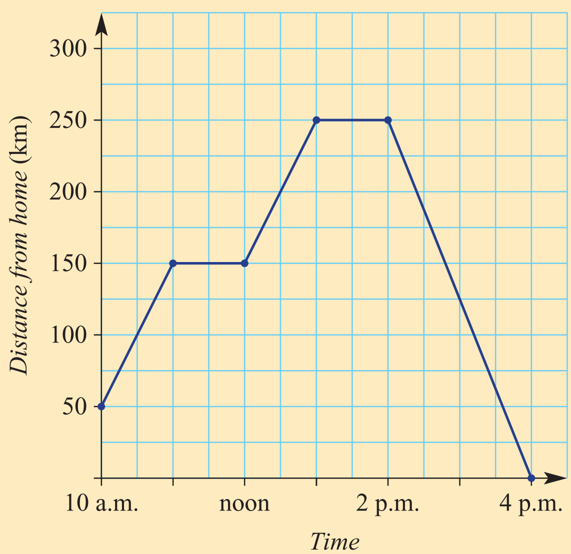
3 Draw up a four-quadrant number plane and plot the following points. What shapes do they form?

a (0, 0), (0, 5), (5, 5), (5, 0)
b (–3, –1), (–3, 1), (4, 0)
c (–2, 3), (–4, 1), (–2, –3), (2, –3), (4, 1), (2, 3)

4 Find the mean of the following pairs.

a 10 and 12 **b** 15 and 23
c 6 and 14 **d** 3 and 4
e –6 and 6 **f** –3 and 1
g 0 and 7 **h** –8 and –10

5 This travel graph shows a journey taken by the Hart family.



a For how many minutes did the family stop on their trip?
b How far had they travelled by 1 p.m., after starting at 10 a.m.?
c What was their speed in the first hour of travel?

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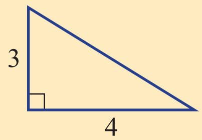
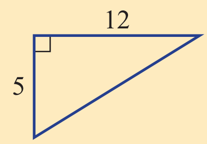
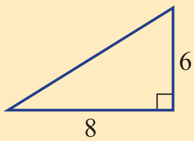
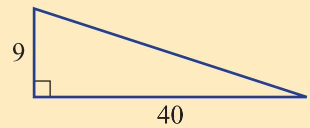
Pre-test

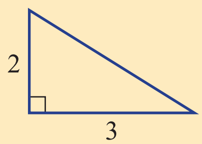
6 Find the length of the hypotenuse in each right-angled triangle. Use $a^2 + b^2 = c^2$. Round to 2 decimal places in parts **e** and **f**.

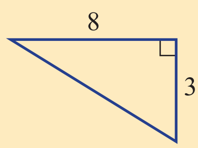
a


b


c


d


e


f


7 Copy and complete the table of values for each rule given.

a $y = x + 3$

x	0	1	2	3
y				

b $y = x - 2$

x	0	1	2	3
y				

c $y = 2x$

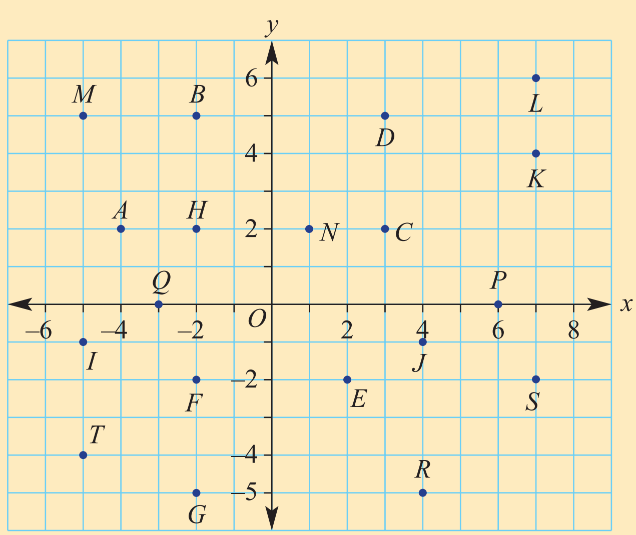
x	0	1	2
y			

d $y = 4 - x$

x	–2	–1	0
y			

8 Use the Cartesian plane to find the following distances.

a OP **b** QP
c MB **d** FS
e BD **f** TM
g AC **h** LS
i AH **j** RJ
k LK **l** BG



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5A Interpreting straight-line graphs

Stage
5.2
5.2◊
5.1
4

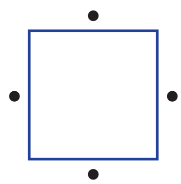
Many hospital patients are given medicine or other fluids through a drip. This is so that the patient receives the liquid in small amounts at a constant rate. This is an example of a linear relationship, which can be represented using a straight-line graph.

A drip like this is able to dispense fluid in very small amounts – less than 1 millilitre.

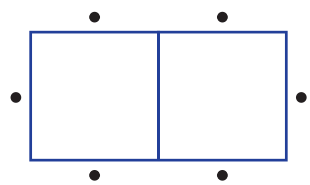
Drilling for Gold 5A1a 5A1

Let's start: Café tables

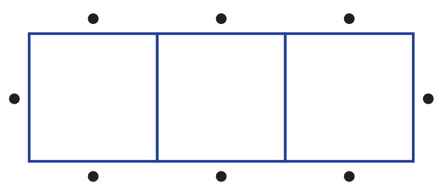
A café has many square tables. Four people can sit around one table, like this.



For a group of six people, two tables can be used like this.



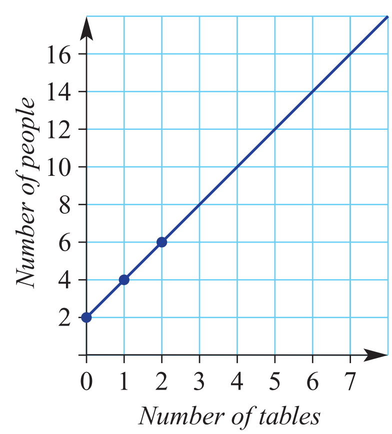
Three tables can accommodate eight people.



- How many people can sit around 4 tables?
- Copy and complete the table below.

Number of tables	1	2	3	4	5	6	7
Number of people	4	6					

- Plot the values in the table onto this graph. Then use a ruler to confirm that the points form a straight line.



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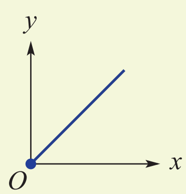
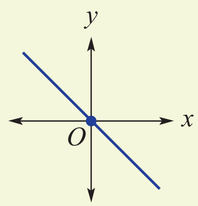
Key ideas

- There are many real-life situations that involve two variables, such as the time spent cycling and the distance travelled.
- A table is often used to show some values that satisfy a relationship. In this example, the values in the top row are increasing by 1 and the numbers in the bottom row are increasing by 5. This is an example of a **linear relationship**.

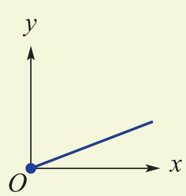
Time	0	1	2	3	4	5
Distance	0	5	10	15	20	25

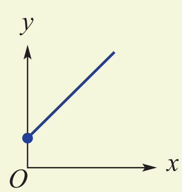
- A graph on the Cartesian plane is used to display all the values that satisfy a relationship between two variables. Linear relationships are always straight-line graphs.
  - Some straight-line graphs pass through the origin (O).



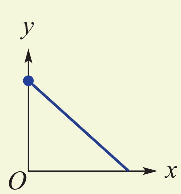


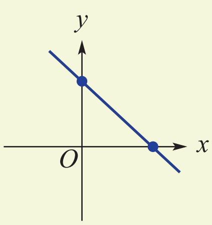
  - Some straight-line graphs indicate that both variables are increasing.





  - Some straight-line graphs indicate that one variable decreases as the other increases.





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Exercise 5A

Understanding

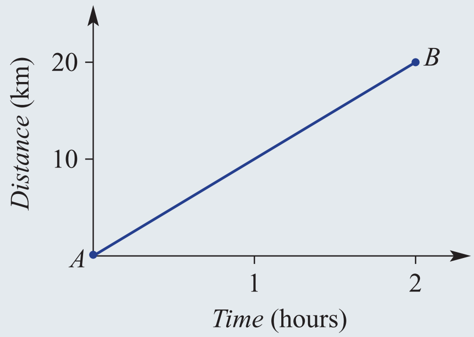
Example 1 Reading information from a graph

The graph shown here shows the journey of a cyclist from one place (A) to another (B).

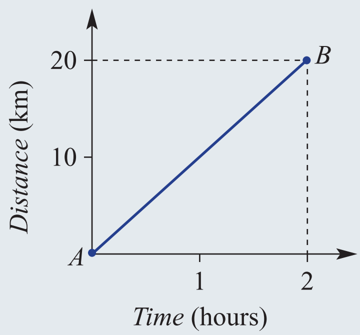
a How far did the cyclist travel?

b How long did it take the cyclist to complete the journey?

c If the cyclist rode from A to B and then halfway back to A, how far was the journey?



Solution	Explanation
a 20 km	Draw an imaginary line from point B to the vertical axis; i.e. 20 km.
b 2 hours	Draw an imaginary line from point B to the horizontal axis; i.e. 2 hours.
c $20 + 10 = 30$ km	Ride 20 km out and 10 km back.

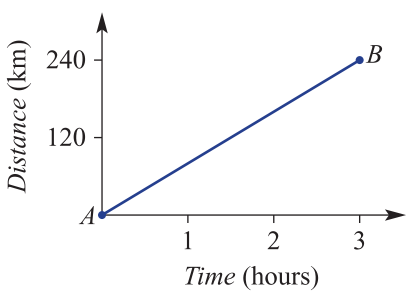


1 This graph shows a car journey from one place (A) to another (B).

a How far did the car travel?

b How long did it take to complete the journey?

c If the car was driven from A to B and then halfway back to A, how far was the journey?



For distance travelled, draw a horizontal line from B to the distance scale.

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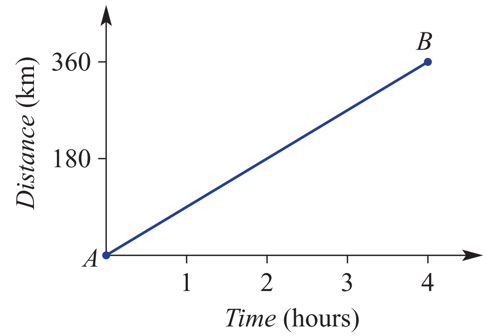
2 This graph shows a motorcycle journey from one place (A) to another (B).

a How far did the motorcycle travel?

b How long did it take to complete the journey?

c If the motorcycle travelled from A to B and then halfway back to A, how far was the journey?

To find the total time taken to go from A to B, look on the time scale that is level with point B on the line.



Fluency

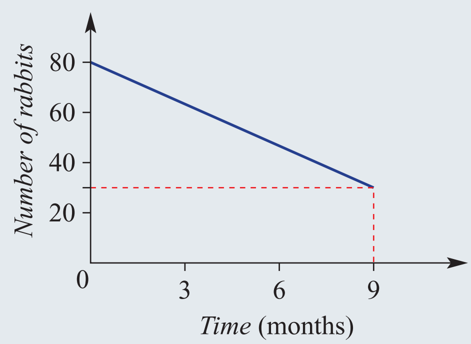
Example 2 Interpreting information from a graph

The number of rabbits in a colony has decreased according to this graph.

a How many rabbits were there in the colony to begin with?

b How many rabbits were there after 9 months?

c How many rabbits disappeared from the colony during the 9-month period?



Solution	Explanation
a 80 rabbits	At $t = 0$ there were 80 rabbits.
b 30 rabbits	Read the number of rabbits from the graph at $t = 9$.
c $80 - 30 = 50$ rabbits	There were 80 rabbits at the start and 30 after 9 months.

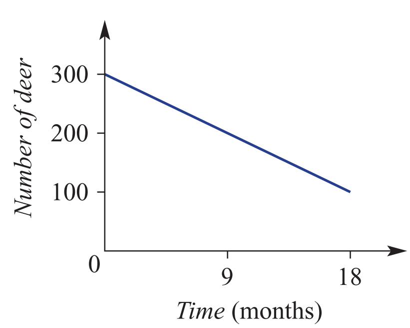
3 The number of deer in a particular forest has decreased over recent months according to the graph shown.

a How many deer were there to begin with?

b How many deer were there after 18 months?

c How many deer disappeared from the colony during the 18-month period?

'To begin with' means time = 0.



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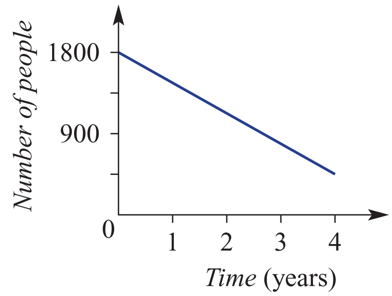
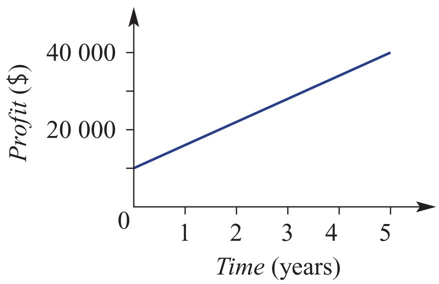
5A

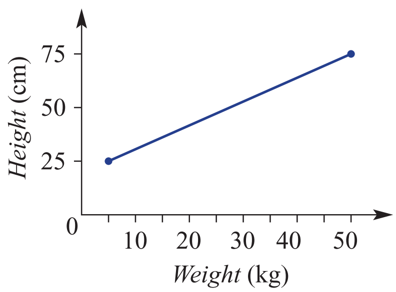
4 The number of people in a small village has decreased over recent years according to the graph shown.
  a How many people were there to begin with?
  b How many people were there after 4 years?
  c How many people disappeared from the village during the 4-year period?



5 This graph shows the profit result for a company over a 5-year period.
  a What is the profit of the company at:
    i the beginning of the 5-year period?
    ii the end of the 5-year period?
  b Has the profit increased or decreased over the 5-year period?
  c How much has the profit increased over the 5 years?



6 A height versus weight graph for a golden retriever dog breed is shown.
  a From the smallest to the largest dog, use the graph to find the total increase in:
    i height ii weight
  b Fill in the missing numbers.
    i The largest weight is ____ times the smallest weight.
    ii The largest height is ____ times the smallest height.



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Problem-solving and Reasoning

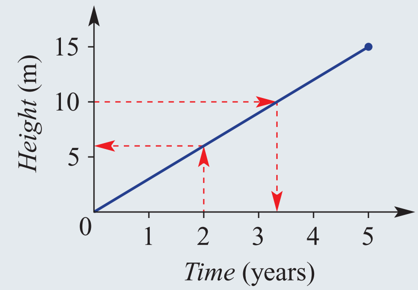
Example 3 Reading values from a graph

This graph shows the growth of a tree over 5 years.

a How many metres has the tree grown over the 5 years?

b Use the graph to find how tall the tree is after 2 years.

c Use the graph to find how long it took for the tree to grow to 10 metres.



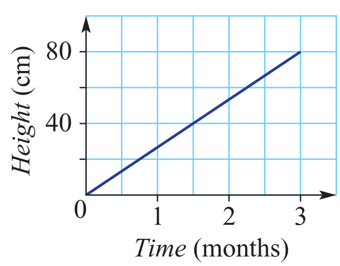
Solution	Explanation
a 15 metres	The end point of the graph is at 15 metres.
b 6 metres	Draw a dotted line at 2 years and read the height.
c 3.3 years	Draw a dotted line at 10 metres and read the time.

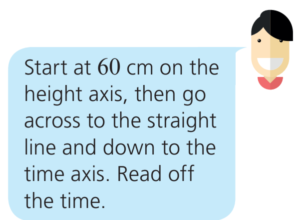
7 The graph below shows the height of a tomato plant over 3 months.

a How many centimetres has the tree grown over 3 months?

b Use the graph to find how tall the tomato plant is after $1\frac{1}{2}$ months.

c Use the graph to find how long it took for the plant to grow to 60 centimetres.



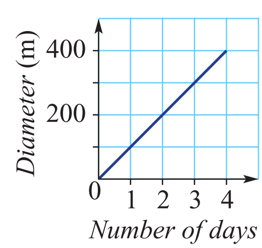


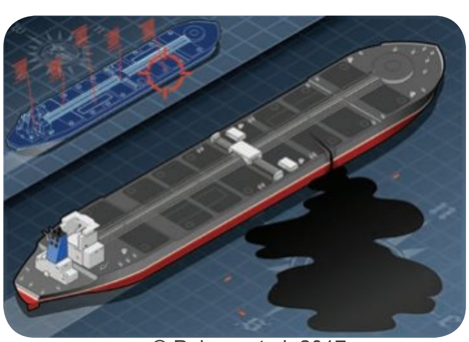
8 The diameter of an oil slick increased every day after an oil tanker hit some rocks. Use the graph to find:

a how wide the oil slick is after 4 days

b how wide the oil slick is after 2.5 days

c how many days it took for the oil slick to reach a diameter of 350 m





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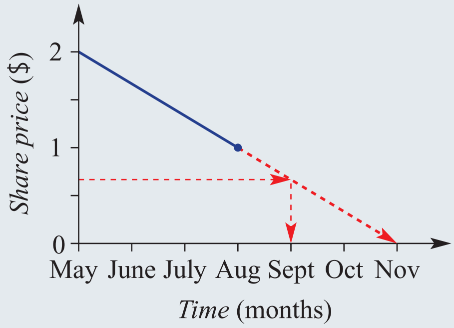
Example 4 Using a graph to make predictions

The value of a company's share price is falling.

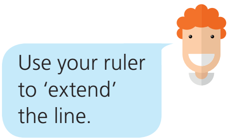
a By the end of August how much has the share price fallen?
b At the end of November what would you estimate the share price to be?
c Near the end of which month would you estimate the share price to be 70 cents?

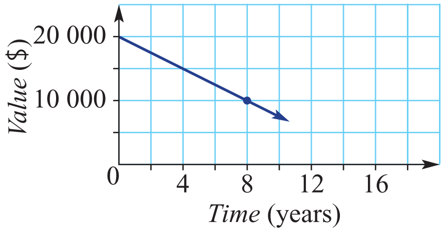


Solution	Explanation
a Price has dropped by $1.	By August the price has changed from $2 to $1.
b $0	Use a ruler to extend your graph (as shown by the dotted line) and read the share price for November.
c September	Move across from 70 cents to the extended line and read the month.

9 The value of a car decreases with time, as shown in the graph below.

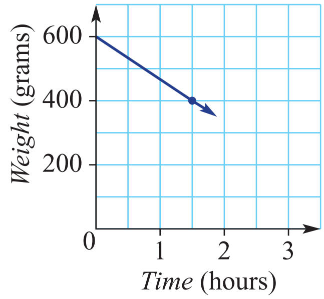
a By the end of 8 years, by how much has the car's value fallen?
b At the end of 16 years, what would you estimate the car's value to be?
c Near the end of which year would you estimate the car's value to be $5000?





10 The weight of a wet sponge is reduced after it is left in the sun to dry.

a The weight of the sponge has been reduced by how many grams over the first 1.5 hours?
b What would you estimate the weight of the sponge to be after 3 hours?
c How many hours would it take for the sponge to weigh 300 g?



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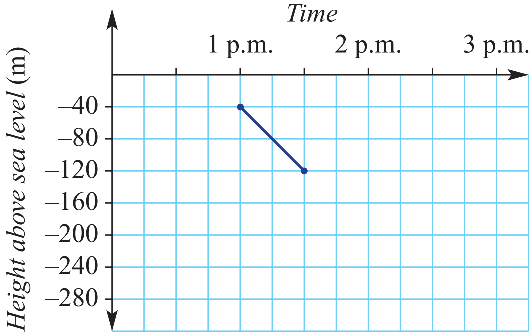
Enrichment: Submarine depth



11 A submarine goes to depths below sea level, as shown in this graph.

a How long did it take for the submarine to drop from 40 m to 120 m below sea level?

b At what time of day was the submarine at:

i –40 m? **ii** –80 m?

iii –60 m? **iv** –120 m?

c What is the submarine's depth at:

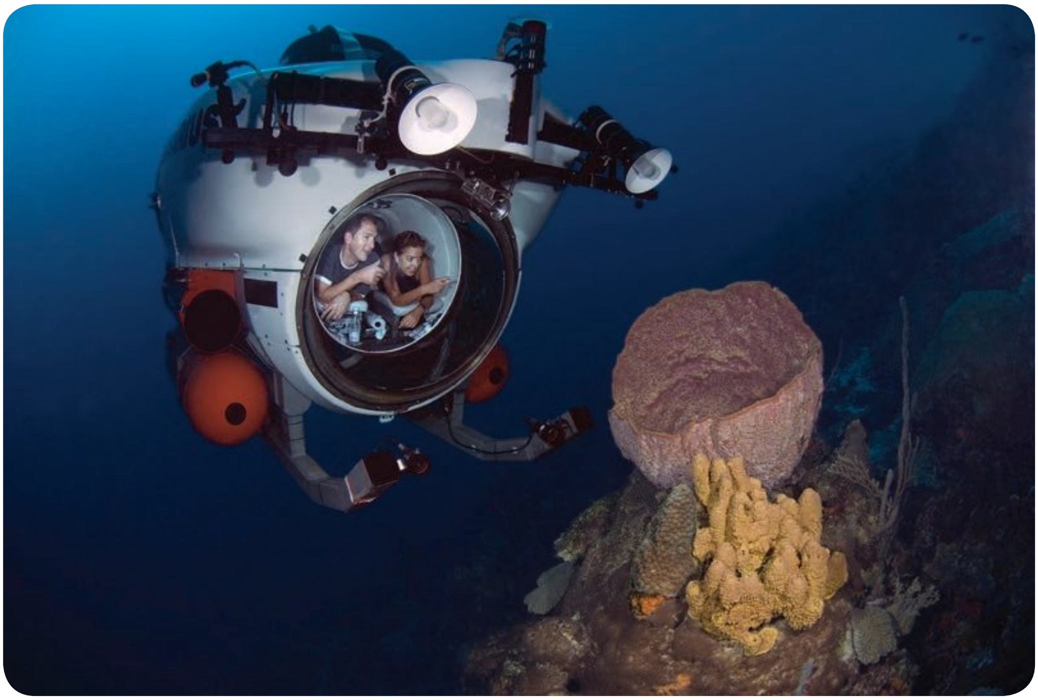
i 1:30 p.m.? **ii** 1:15 p.m.?

d Extend the graph to find the submarine's depth at:

i 12:45 p.m. **ii** 1:45 p.m. **iii** 2:30 p.m.

e Use your extended graph to estimate the time when the submarine was at:

i 0 m **ii** –200 m **iii** –320 m



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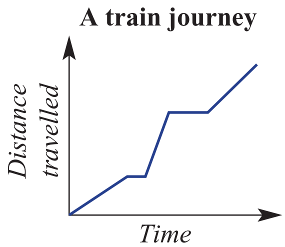
5B Distance–time graphs

Stage

5.2
5.2◊
5.1
4

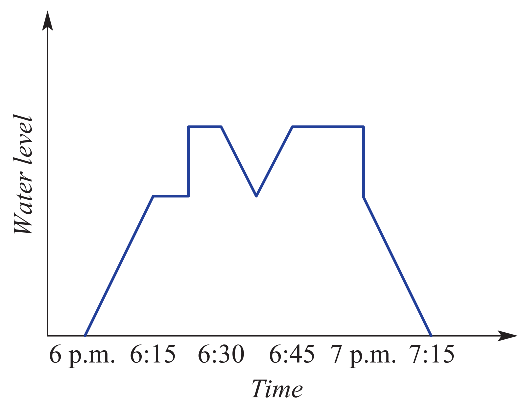
A distance–time graph shows the *distance* travelled on the vertical axis and the *time* on the horizontal axis. Many important features of a journey can be displayed. For example, a train journey could be graphed with a series of sloping line segments showing travel between stations and flat line segments showing when the train is stopped at a station.



Let's start: Sam's bath time

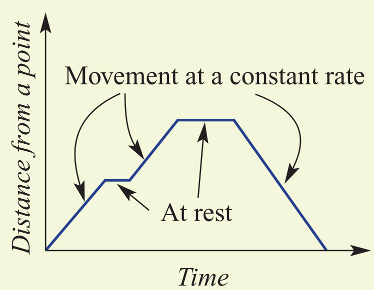
The graph shows the water level in Sam's bath before, during and after he uses it.

- With a partner, write a short description of what the graph might tell us about Sam's bathtime.
- Share your stories with the class.



Key ideas

- Graphs of *distance* versus *time* sometimes consist of **line segments**.
- Each segment shows whether the object is moving or at rest.
- To draw a graph of a journey, use time on the horizontal axis and distance on the vertical axis.



Line segment
A section of a straight line

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Exercise 5B

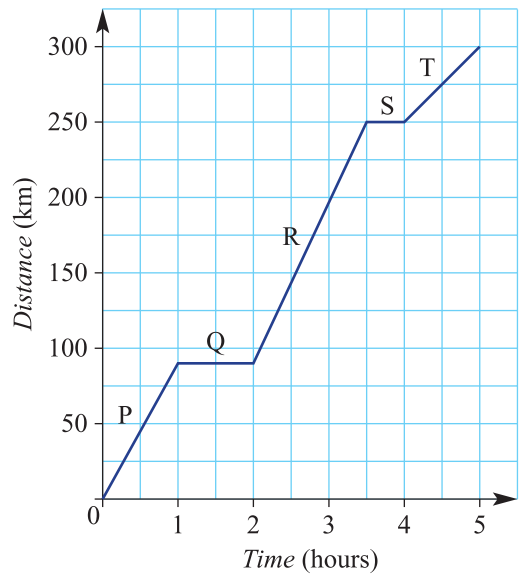
Understanding

1 The Martin family makes a 300 km car journey, which takes 5 hours. The distance–time graph of this journey is shown at right. For each description below, choose the line segment of the graph that matches it. Some segments will have more than one descriptor.

A flat line segment shows that the car is stopped.

- a A half-hour rest break is taken after travelling 250 km.
- b In the first hour the car travels 90 km.
- c The car is at rest for 1 hour, 90 km from the start.
- d The car takes 1.5 hours to travel from 90 km to 250 km.
- e The distance from 250 km to 350 km takes 1 hour.
- f The distance travelled stays constant at 250 km for half an hour.
- g A 1-hour rest break is taken after travelling 90 km.

Distance–time graph of car journey

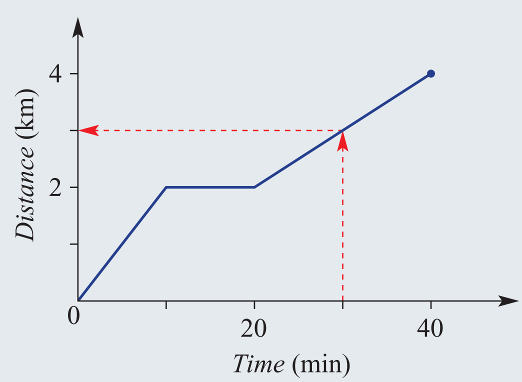


Fluency

Example 5 Interpreting a distance–time graph

This distance–time graph shows a car's journey from home, to school and then to the local shopping centre.

- a What was the total distance travelled?
- b How long was the car resting at the school?
- c What was the total distance travelled after 30 minutes?



Solution	Explanation
a 4 km	Read the distance from the end point of the graph.
b 10 minutes	The rest starts at 10 minutes and finishes at 20 minutes.
c 3 km	Draw a line from 30 minutes and read off the distance.

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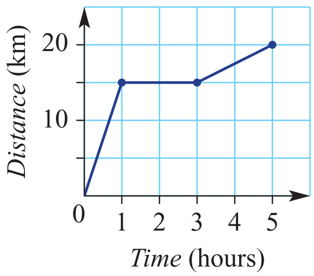
5B

2 A bicycle journey is shown on the distance–time graph below.

a What was the total distance travelled?

b How long was the cyclist at rest?

c How far has the cyclist travelled after 4 hours?



From the end of the line segments, go across to the distance scale. This will show the total distance travelled.

3 The weight of a water container increases while water is poured into it from a tap.

a What is the total weight of the container after:

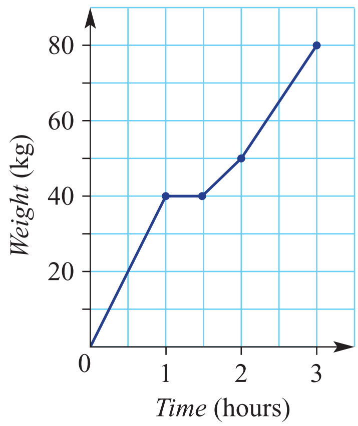
i 1 hour?

ii 2 hours?

iii 3 hours?

b During the 3 hours, how long was the container not actually being filled with water?

c During which hour was the container filling the fastest?



A flat-line segment shows that the weight is not changing, so no water is being poured in at that time.

4 This graph shows a shopper's short walk in a shopping mall.

a What is the total distance the shopper travelled?

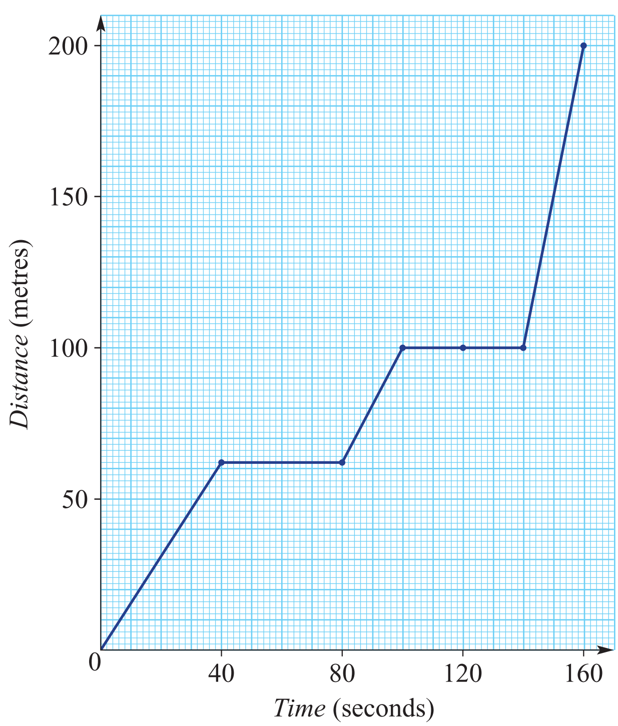
b How long was the shopper not walking?

c What was the total distance the shopper had travelled by the following times?

i 20 seconds

ii 80 seconds

iii 2.5 minutes



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Problem-solving and Reasoning

5 This graph shows the distance of a train from the station over a period of time.

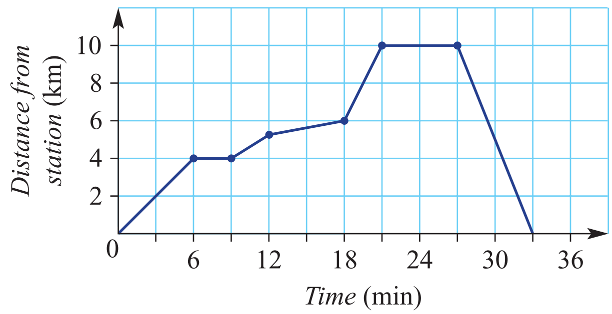
a What was the farthest distance the train travelled from the station?

b What was the total distance travelled?

c After how many minutes did the train begin to return to the station?

d What was the total number of minutes the train was stationary?

Remember to include the return trip in the total distance travelled.

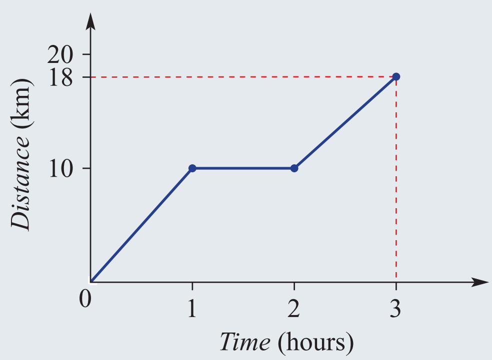


Example 6 Sketching a distance–time graph

Sketch a distance–time graph displaying all of the following information.

- total distance covered is 18 km in 3 hours
- 10 km covered in the first hour
- a 1-hour long rest after the first hour

Solution



Explanation

Draw axes with time on the horizontal (up to 3 hours) and distance on the vertical (up to 18 km).
Start at time zero.
Draw the first hour with 10 km covered.
Draw the rest stop, which lasts for 1 hour.
Draw the remainder of the journey, so that 18 km is completed after 3 hours.

6 Sketch a distance–time graph displaying all of the following information.

- total distance covered is 100 km in 2 hours
- 50 km covered in the first hour
- a half-hour rest stop after the first hour

Draw axes with time on the horizontal (up to 2 hours) and distance on the vertical (up to 100 km).

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5B

7 Sketch a graph to illustrate a journey described by the following.
- total distance covered is 15 m in 40 seconds
- 10 m covered in the first 10 seconds
- a 25-second rest after the first 10 seconds

Always use a ruler to draw line segments.

8 A bus travels 5 km in 6 minutes, stops for 2 minutes, travels 10 km in 8 minutes, stops for another 2 minutes and then completes the journey by travelling 5 km in 4 minutes.

a What was the total distance travelled?

b What was the total time taken?

c Sketch a distance–time graph for the journey.

Find the total time taken to determine the scale for the horizontal axis. Find the total distance travelled to determine the scale for the vertical axis.

Enrichment: Pigeon flight

9 The distance travelled by a pigeon is described by these points.
- a half-hour flight, covering a distance of 18 km
- a 15-minute rest
- another 15-minute flight covering 12 km
- a half-hour rest
- turning and flying 10 km back towards 'home' over the next $\frac{1}{2}$ hour
- a rest for $\frac{1}{4}$ of an hour
- reaching 'home' after another 45-minute flight

a Sketch a graph illustrating the points above, using 'distance' on the vertical axis.

b What was the fastest speed (in km/h) at which the pigeon flew? $\left(\text{Speed} = \frac{\text{distance}}{\text{time}}\right)$

c What was the average speed of the bird during the journey?

$$\left(\text{Average speed} = \frac{\text{total distance}}{\text{total flying time}}\right)$$

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5C Graphing straight lines (part 1)

Stage: 5.2, 5.2◊, 5.1, 4

Some equations have only one solution. The equation $10 = 2x + 4$ has only one value of x that makes it true. That value is $x = 3$.

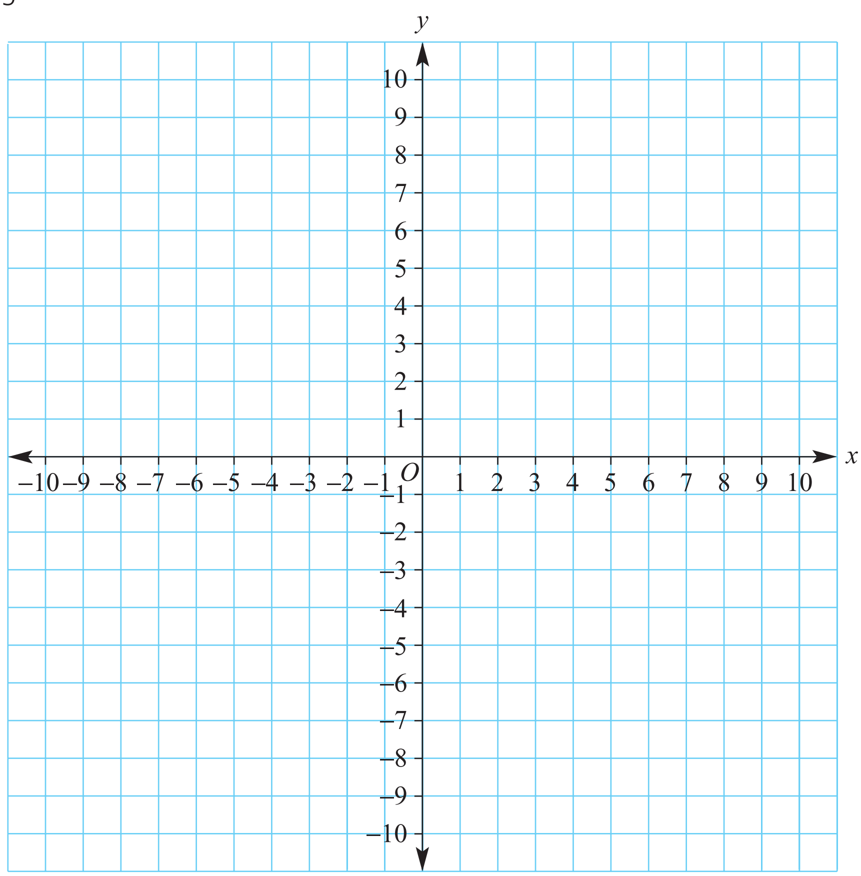
On the other hand, the equation $y = 2x + 4$ has many solutions, such as $x = 5, y = 14$ and $x = 0, y = 4$. When these solutions are graphed on the Cartesian plane they form a straight line. The line shows every possible solution.

These equations have one solution	These equations have many solutions
$10 = 5x$	$y = 5x$
$9 = 2x - 4$	$y = 2x - 4$
$1 = x - 5$	$y = x - 5$
$y = 1$	$y = x$
$x - 5 = 2$	$x - y = 2$

Let's start: One equation, two variables, multiple solutions

Consider the statement, 'The sum of two numbers is 5.'
If the two numbers are called x and y, this statement could be written as $x + y = 5$.

- Write down ten pairs of numbers that have a sum of 5, like this: $4 + 1 = 5$, so $x = 4$ and $y = 1$, which is the point $(4, 1)$.
- Plot the ten points on this Cartesian plane, then use a ruler to join them with a single straight line.



- Look at the ten points another student has chosen. Did they choose the same points as you? Did they draw the same line as you?

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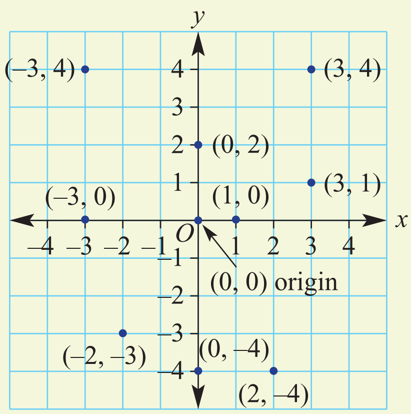
Key ideas

- A **number plane** or **Cartesian plane** includes a vertical y-axis and a horizontal x-axis intersecting at right angles at the origin $O(0, 0)$.



- A point on a number plane has **coordinates** (x, y).
  - The x-coordinate is listed first, followed by the y-coordinate.

- $(x, y) = \left(\text{horizontal units from origin}, \text{vertical units from origin}\right)$

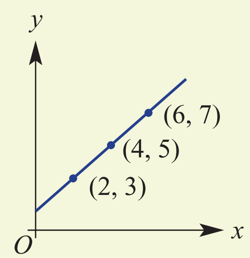
- A rule is an equation connecting two or more variables.

- Some equations describe linear relationships.

- Equations are often written with y as the subject. For example: $y = 2x - 3$ or $y = -x + 7$.

- To graph a linear relationship using a rule:
  - Construct a table of values finding a y-coordinate for each given x-coordinate by substituting each x-coordinate into the rule.
  - Plot the points given in the table on a set of axes.
  - Draw a line through the points to complete the graph.

x	2	4	6
y	3	5	7



- The **point of intersection** of two lines is the point that sits on both lines.

x-coordinate The first coordinate of an ordered pair

y-coordinate The second coordinate of an ordered pair

Point of intersection The point at which two lines cross each other and therefore have the same coordinates

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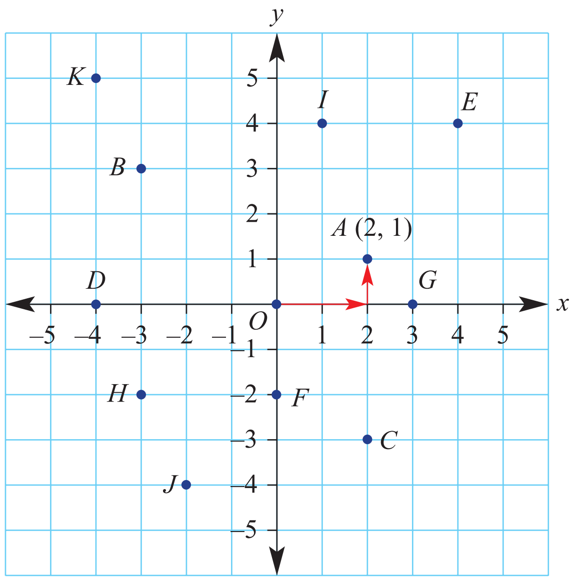
Exercise 5C

Understanding

1 **a** List the coordinates of each point plotted on this number plane.
b Which points are on the x-axis?
c Which points are on the y-axis?
d What are the coordinates of the point called the 'origin'?



$(x, y) = $ (right or left, up or down)

The 'origin' is the point where the x-axis and y-axis meet.

2 The statement, 'One number is 3 less than the other' can be written as $y = x - 3$. Copy and complete the following.
a When $x = 5$, $y =$ __ , which gives the point (,).
b When $x = 3$, $y =$ __ , which gives the point (,).
c When $x = 0$, $y =$ __ , which gives the point (,).
d When $x = -2$, $y =$ __ , which gives the point (,).

3 Write the coordinates for each point listed in this table.

x	–2	–1	0	1	2
y	1	–1	–3	–5	–7

Coordinates are written as (x, y).

x	–2
y	1

} (–2, 1)

Coordinates on a number plane have many applications.

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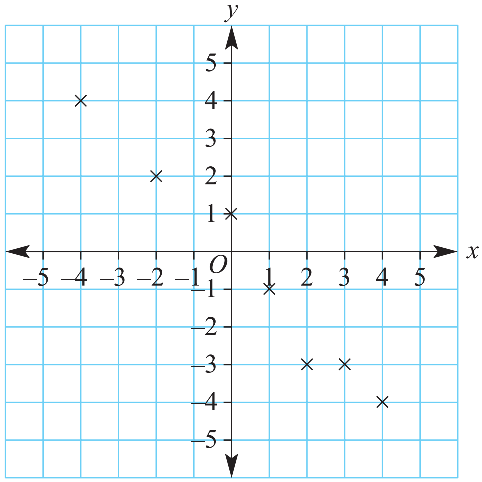
5C

4 Jenna has plotted these points for the rule $y = -x$ and she knows they should all lie in a straight line.

a State the coordinates of any points that are not in line with most of the other points.

b Using the rule $y = -x$, calculate the correct coordinates for these two points.

Place your ruler along the plotted points. Any point not in a straight line needs to be re-calculated.



Fluency

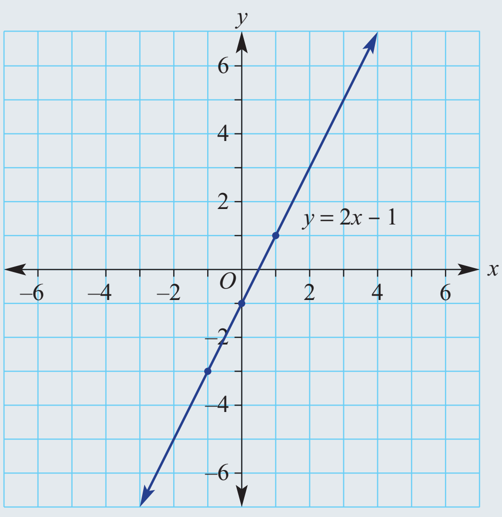
Example 7 Plotting a graph from a rule

Plot the graph of $y = 2x - 1$ by first completing the table of values.

x	-1	0	1
y			

Solution

x	-1	0	1
y	-3	-1	1



Explanation

Substitute each value into the equation:

$x = -1$, $y = 2 \times (-1) - 1 = -3$

$x = 0$, $y = 2 \times 0 - 1 = -1$

$x = 1$, $y = 2 \times 1 - 1 = 1$

The points are:

$(-1, -3)$

$(0, -1)$

$(1, 1)$

Plot the points and draw the line with a ruler.

When labelling axes, put the numbers on the grid lines, not in the spaces.

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Drilling for Gold 5C1 at the end of this section

5 Complete the following tables, then plot the graph of each one on a separate number plane.

a $y = 2x$

x	−1	0	1
y			

b $y = x + 4$

x	0	1	2
y			

When multiplying, same signs make a positive; e.g. $-2 \times (-1) = 2$

c $y = 2x - 3$

x	0	1	2
y			

d $y = -2x$

x	−1	0	1
y			

e $y = x - 4$

x	1	2	3
y			

f $y = 6 - x$

x	0	1	2
y			

6 Complete the following tables, then plot the graph of each pair on the same axes.

a i $y = x + 2$

x	0	2	4
y			

ii $y = -x + 2$

x	0	2	4
y			

For each part, draw line **i** and line **ii** on the same axes.

b i $y = x - 4$

x	0	4	6
y			

ii $y = 4 - x$

x	0	1	2
y			

c i $y = 2 + 3x$

x	−3	0	3
y			

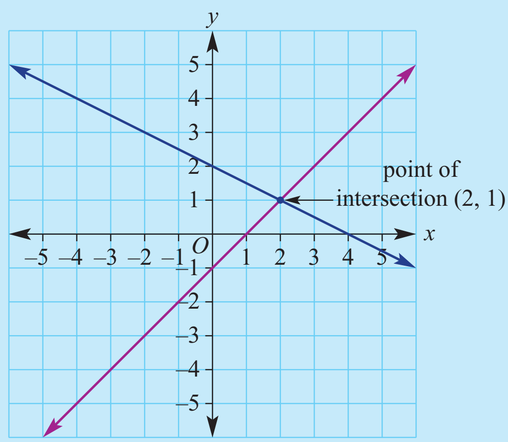
ii $y = 3x - 4$

x	−3	0	3
y			

7 By plotting the graphs of each of the following pairs of lines on the same axes, find the coordinates of the point of intersection. Use a table of values, with x from −2 to 2.

a $y = 2x$ and $y = x$
b $y = x + 3$ and $y = 2x + 2$
c $y = 2 - x$ and $y = 2x + 5$
d $y = 2 - x$ and $y = x + 2$
e $y = 2x - 3$ and $y = x - 4$

The point of intersection of two lines is where they cross each other.
For example:



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5C

Problem-solving and Reasoning

Example 8 Interpreting a graph when given a table of values

Jasmine is organising a school dance. The venue is chosen and the costs are shown in the table.

Number of students (S)	0	100	200	300
Total cost in dollars (C)	500	1000	1500	2000

a Plot a graph of the total cost against the number of students.

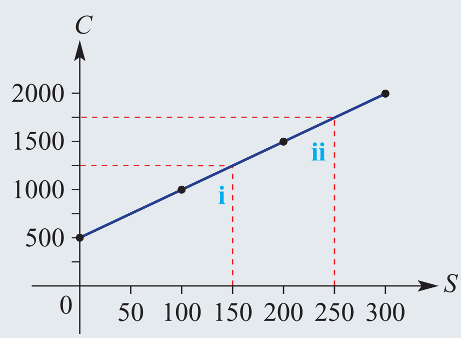
b Use the graph to determine:

i the total cost for 150 students

ii how many students could attend the dance if Jasmine has a budget of \$1750 to spend

Solution

a



Explanation

Construct a set of axes using S between 0 and 300 and C between 0 and 2000.
'Number of students' is placed on the horizontal axis.
Plot each point using the information in the table.

b i The total cost for 150 students is \$1250.

Draw a vertical dotted line at $S = 150$ to meet the graph. Then draw another dotted line horizontally to the C-axis.

ii 250 students could attend the dance for \$1750.

Draw a horizontal dotted line at $C = 1750$ to meet the graph. Then draw a dotted line vertically to the S-axis.

8 A furniture removalist company charges by the hour. Their rates are shown in the table below.

No. of hours (n)	0	1	2	3	4	5
Cost (C)	200	240	280	320	360	400

a Plot a graph of cost against hours.

b Use the graph to determine:

i the total cost for 2.5 hours' work

ii the number of hours the removalist company will work for \$380

Place 'No. of hours' on the horizontal axis.

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9 Olive oil is sold in bulk for \$8 per litre.

No. of litres (L)	1	2	3	4	5
Cost (C)	8	16	24	32	40

a Plot a graph of cost against number of litres.
b Use the graph to determine:
  i the total cost for 3.5 litres of oil
  ii the number of litres of oil you can buy for \$20

Example 9 Constructing a table and graph for interpretation

An electrician charges \$50 for a service call plus \$60 an hour for labour.

a Complete the table of values.

No. of hours (n)	0	1	2	3	4	5
Cost (C)						

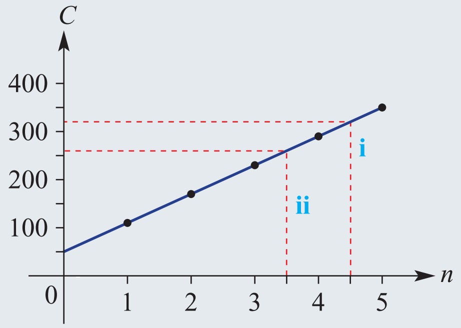
b Plot a graph of cost against number of hours.
c Use the graph to determine:
  i the cost for 4.5 hours' work
  ii how long the electrician will work for \$260

Solution

a

No. of hours (n)	0	1	2	3	4	5
Cost (C)	50	110	170	230	290	350

b



c **i** The cost is \$320.

ii The electrician will work for 3.5 hours.

Explanation

Initial cost (i.e. $n = 0$) is \$50.
Cost for 1 hour = \$50 + \$60 = \$110
Cost for 2 hours = \$50 + 2 × \$60 = \$170
Cost for 3 hours = \$50 + 3 × \$60 = \$230 etc.
Plot the points from the table, using C on the vertical axis and n on the horizontal axis.
Join all the points to form the straight line.

Draw a vertical dotted line at $n = 4.5$ to meet the graph, then draw a line horizontally to the C-axis.

Draw a horizontal dotted line at $C = 260$ to meet the graph, then draw vertically to the n-axis.

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5C

10 A car rental firm charges $200 plus $1 for each kilometre travelled.

a Complete the table of values below.

No. of km (k)	0	100	200	300	400	500
Cost (C)						

b Plot a graph of cost against kilometres.

c Use the graph to determine:

i the cost when you travel 250 km

ii how many kilometres you can travel on a $650 budget

11 Matthew delivers pizza for a fast-food outlet. He is paid $20 a shift plus $3 per delivery.

a Complete the table of values below.

No. of deliveries (d)	0	5	10	15	20
Wages (W)					

b Plot a graph of Matthew's wages against number of deliveries.

c Use the graph to determine:

i Mathew's wages for 12 deliveries

ii the number of deliveries made if Matthew is paid $74

Enrichment: Which mechanic?

12 Two mechanics charge different rates for their labour. Ethan charges $75 for a service call plus $50 per hour. Sherry charges $90 for a service call plus $40 per hour.

a Create a table for each mechanic for up to 5 hours of work.

b Plot a graph for the total charge against the number of hours worked for Ethan and Sherry on the same axes.

c Use the graph to determine:

i the cost of hiring Ethan for 3.5 hours

ii the cost of hiring Sherry for 1.5 hours

iii the number of hours of work if Ethan charges $100

iv the number of hours of work if Sherry charges $260

v the number of hours of work if the cost from Ethan and Sherry is the same

d Write a sentence describing who is cheaper for different hours of work.

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Drilling for Gold exercise

5C1: From equation to table

Use the equation to complete the table of values on the worksheet or copy and complete them in your exercise book.

1 $y = x + 2$

x	0	1	2	3	4
y					

2 $y = 2x$

x	0	1	2	3	4
y					

3 $y = x - 2$

x	0	1	2	3	4
y					

4 $y = 2 - x$

x	0	1	2	3	4
y					

5 $y = 2x + 3$

x	0	1	2	3	4
y					

6 $y = 2 + 3x$

x	0	1	2	3	4
y					

7 $y = 3(x - 2)$

x	0	1	2	3	4
y					

8 $y = 2x - 3$

x	0	1	2	3	4
y					

9 $y = 3x - 2$

x	0	1	2	3	4
y					

10 $y = 3 - 2x$

x	0	1	2	3	4
y					

11 $y = -3x + 2$

x	0	1	2	3	4
y					

12 $y = -x$

x	0	1	2	3	4
y					

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5D Midpoint and length of line segments

Stage

5.2
5.2◊
5.1
4

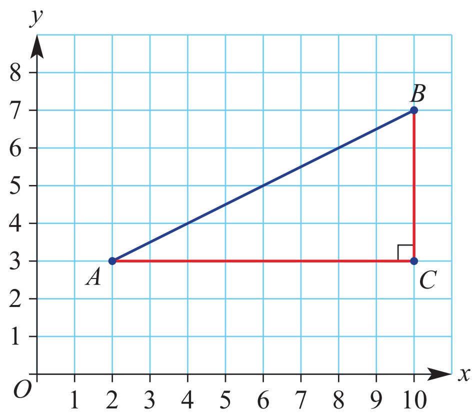
A line extends infinitely in both directions, whereas a line segment (or interval) has two end points. The middle (midpoint) of a line segment can be found by using the coordinates of the end points.

Builders use mathematical calculations to determine the length, midpoint and angle of inclination of wooden beams when constructing the timber frame of a house.

▶ Let's start: Finding a method

This is a graph of the line segment AB. A right-angled $\triangle ABC$ has been drawn so that AB is the hypotenuse (longest side).

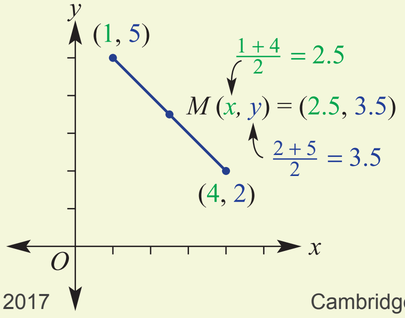
- How long is AC and BC?
- Discuss and explain a method for finding the length of the line segment AB.
- What is the x value of the middle point of the horizontal side of the right-angled triangle?
- What is the y value of the middle point of the vertical side of the right-angled triangle?
- What are the coordinates of the point in the middle of the line segment AB?
- Discuss and explain a method for finding the midpoint of a line segment.



Key ideas

- The **midpoint** (M) of a line segment (interval) is the halfway point between the two end points.
  - $\text{Midpoint} = \left(\begin{matrix}\text{average of two} \\ x \text{ values at} \\ \text{end points}\end{matrix}, \begin{matrix}\text{average of two} \\ y \text{ values at} \\ \text{end points}\end{matrix}\right)$
  - When finding the average, add the values in the numerator before dividing by 2.

Midpoint The point on an interval that is equal in distance from the end points of the interval



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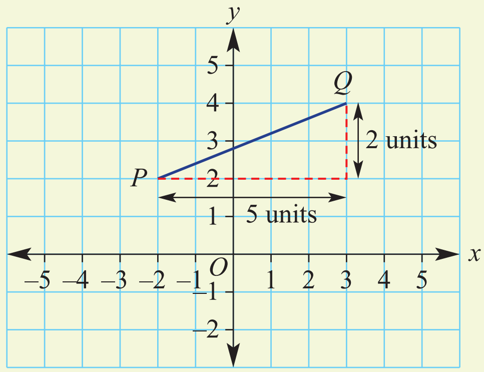
- The length of the line segment PQ is sometimes called the distance PQ.
- The length of a line segment is found using **Pythagoras' theorem**. To find the length of the line segment PQ:
  - Draw a right-angled triangle with the line segment PQ as the hypotenuse (longest side).
  - Count the grid squares to find the length of each smaller side.
  - Apply Pythagoras' theorem.



$$PQ^2 = 5^2 + 2^2$$
$$= 25 + 4$$
$$= 29$$
$$PQ = \sqrt{29} \text{ units}$$

  - $\sqrt{29}$ is the exact length of line segment PQ. It is approximately 5.4 units.

Pythagoras' theorem In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides

Exercise 5D

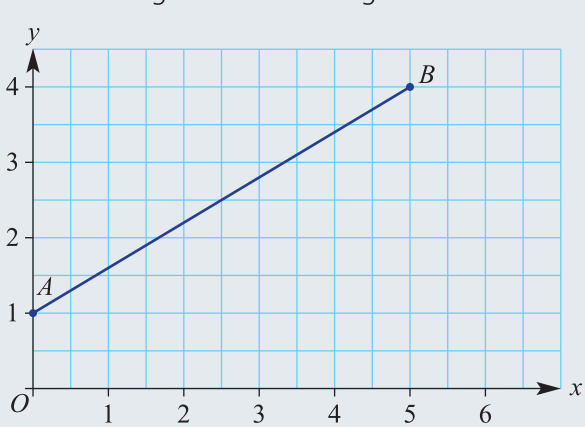
Understanding

1 Consider the points $A(3, 5)$ and $B(7, 3)$.
a What are the x values?
b What is the average of the x values?
c What are the y values?
d What is the average of the y values?
e What is the midpoint of AB?

Fluency

Example 10 Finding the length of a line segment from a graph

Find the length of the line segment between $A(0, 1)$ and $B(5, 4)$.



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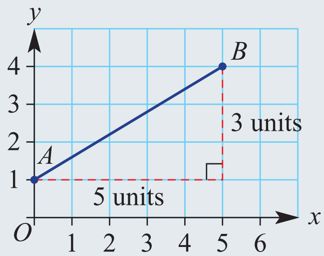
Solution

$AB^2 = 5^2 + 3^2$
$AB^2 = 25 + 9$
$AB^2 = 34$
$AB = \sqrt{34}$ units

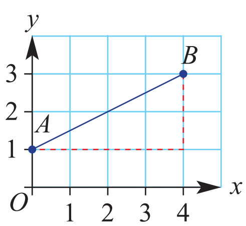
Explanation



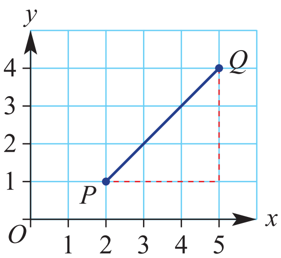
Create a right-angled triangle and use Pythagoras' theorem.
For $AB^2 = 34$, take the square root of both sides to find AB.
$\sqrt{34}$ is the exact answer.

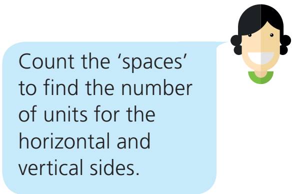
2 Find the length of each of the following line segments. Leave each answer in square root form.

a



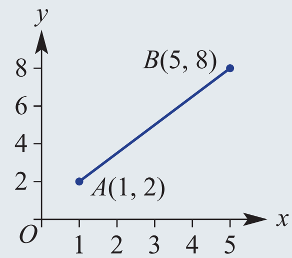
b





Example 11 Finding the midpoint of a line segment from a graph

Find the midpoint of the interval between $A(1, 2)$ and $B(5, 8)$.



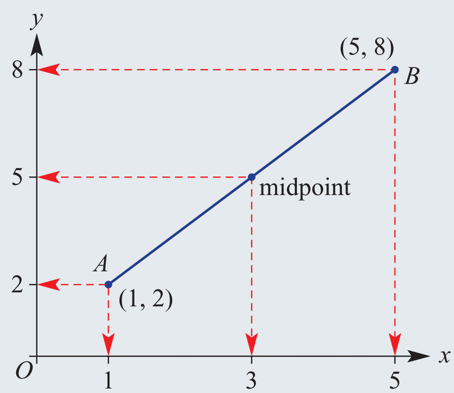
Solution

$$\text{Average of } x \text{ values} = \frac{1+5}{2} = \frac{6}{2} = 3$$

$$\text{Average of } y \text{ values} = \frac{2+8}{2} = \frac{10}{2} = 5$$

Midpoint is (3, 5).

Explanation



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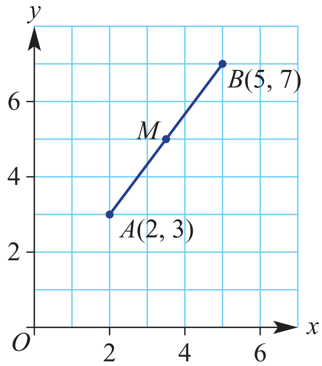
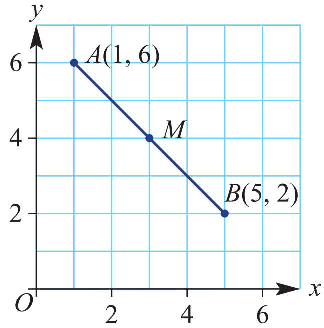
3 Find the midpoint, M, of each of the following intervals.

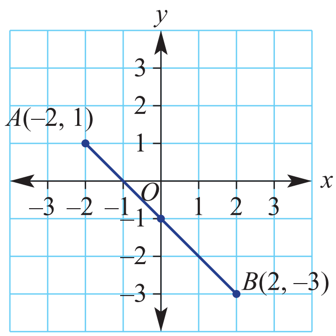
a



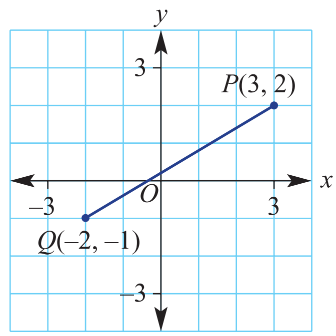
b



c



d

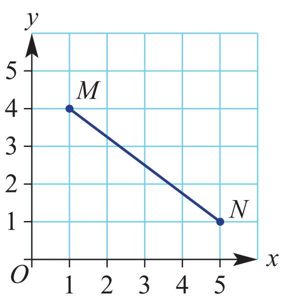


When finding the average, add the numerator values before dividing by 2.

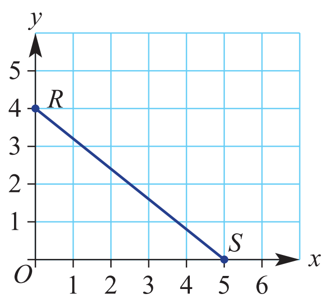
4 Find the length of each of the following line segments.

Video 5D

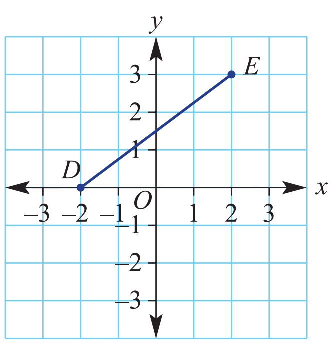
a



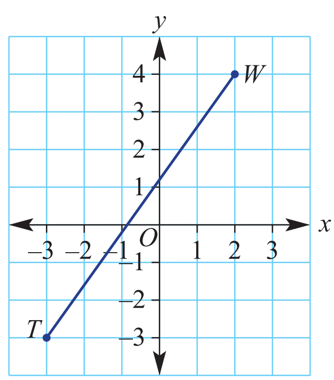
b



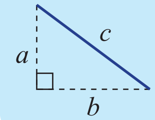
c



d



Use Pythagoras' theorem ($c^2 = a^2 + b^2$) by forming a right-angled triangle:



Write the answer in square root form if it is not a known square root.

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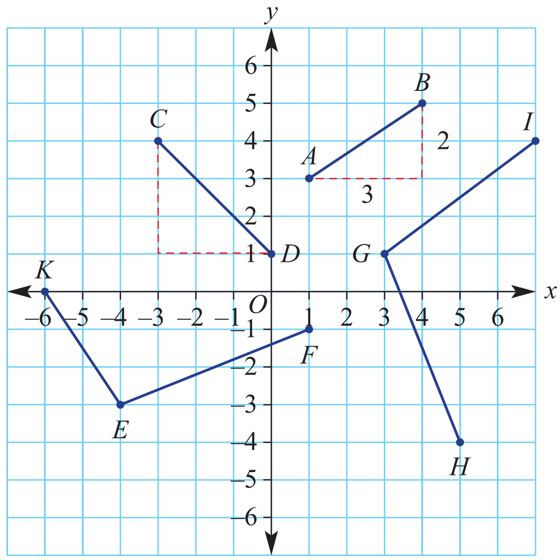
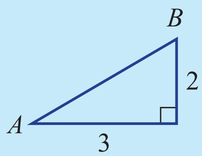
5D

5 Find the length of each line segment on the Cartesian plane shown. Leave your answers in square root form.

a AB **b** CD
c EF **d** GH
e KE **f** GI



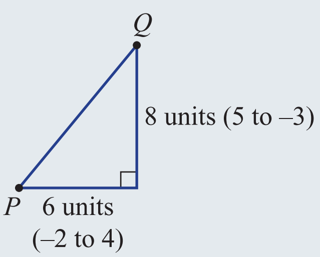
First sketch a right-angled triangle for each line segment, labelling the known sides.



Example 12 Finding the length of a line segment when given the coordinates of the end points

Find the distance between the points P and Q if P is at (–2, –3) and Q is at (4, 5).

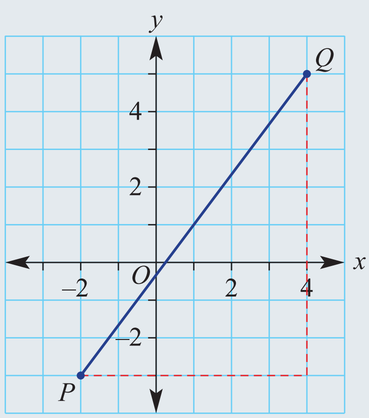
Solution



$PQ^2 = 6^2 + 8^2$
$PQ^2 = 36 + 64$
$PQ^2 = 100$
$PQ = \sqrt{100}$
$PQ = 10$ units

Explanation

Use Pythagoras' theorem to find PQ, the hypotenuse.



If you know the value of the square root, write its value.

6 Plot each of the following pairs of points and find the distance between them, correct to 1 decimal place where necessary.

a (2, 3) and (5, 7) **b** (0, 1) and (6, 9)
c (0, 0) and (–5, 10) **d** (–4, –1) and (0, –5)
e (–3, 0) and (0, 4) **f** (0, –1) and (2, –4)

First rule up axes with x from –5 to 10 and y from –5 to 10.

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7 Find the exact length between these pairs of points.

a (1, 3) and (2, 2)
b (4, 1) and (7, 3)
c (–3, –1) and (0, 4)
d (–2, –3) and (3, 5)
e (–1, 0) and (–6, 1)
f (1, –3) and (4, –2)

Exact length means leave the $\sqrt{\ }$ sign in the answers.

Example 13 Finding the midpoint of a line segment when given the coordinates of the end points

Find the midpoint of the line segment joining $P(-3, 1)$ and $Q(5, -4)$.

Solution	Explanation
$x = \dfrac{-3+5}{2} = \dfrac{2}{2} = 1$	Average the x coordinates. Calculate the numerator before dividing by 2. $-3 + 5 = 2$
$y = \dfrac{1+(-4)}{2} = \dfrac{-3}{2} = -1.5$	Average the y coordinates. Calculate the numerator before dividing by 2. $1 + (-4) = 1 - 4 = -3$
Midpoint is $(1, -1.5)$.	Write the coordinates of the midpoint.

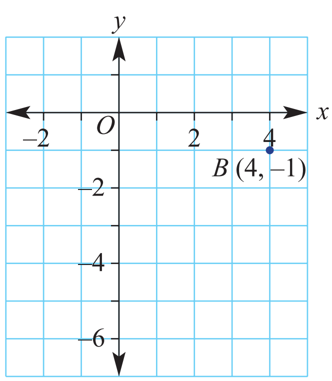
8 Find the midpoint of the line segment that is joining the following points.

a (1, 4) and (3, 6)
b (3, 7) and (5, 9)
c (0, 4) and (6, 6)
d (2, 4) and (3, 5)
e (7, 2) and (5, 3)
f (1, 6) and (4, 2)
g (0, 0) and (–2, –4)
h (–2, –3) and (–4, –5)
i (–3, –1) and (–5, –5)
j (–3, –4) and (5, 6)
k (0, –8) and (–6, 0)
l (3, –4) and (–3, 4)

Check that your answer appears to be halfway between the end points.

Problem-solving and Reasoning

9 Copy the diagram on the right. Mark the point $B(4, -1)$, as shown, then mark the point $M(1, -3)$. Find the coordinates of A if M is the midpoint of the interval AB.



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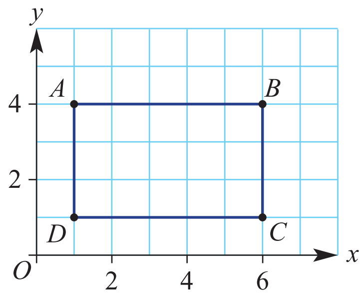
5D

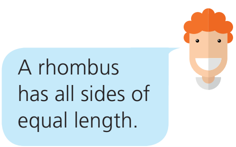
10 Copy the diagram of rectangle $ABCD$.

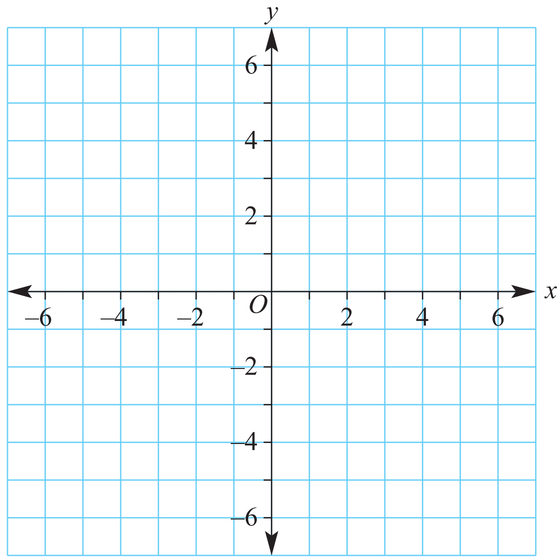
- **a** What are the coordinates of each vertex?
- **b** Find the midpoint of the diagonal AC.
- **c** Find the midpoint of the diagonal BD.
- **d** What does this tell us about the diagonals of a rectangle?



11 Draw up a four-quadrant number plane like the one shown.

- **a** Plot the points $A(-4, 0)$, $B(0, 3)$ and $C(0, -3)$, then form the triangle ABC.
- **b** What is the length of:
  - **i** AB?
  - **ii** AC?
- **c** What type of triangle is ABC?
- **d** Calculate its perimeter and area.
- **e** Write down the coordinates of D such that $ABDC$ is a rhombus.

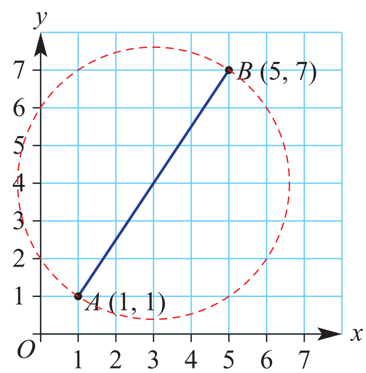




Enrichment: Features of a circle

12 The diameter of a circle is shown on this graph.

- **a** What are the coordinates of X, the centre of the circle? Mark this point on your graph.
- **b** What is the length of the radius XA?
- **c** Find the distance from X to the point $(5, 1)$. How can we tell that $(5, 1)$ lies on the circle?
- **d** Use $C = 2\pi r$ to find the circumference of the circle shown. Round your answer to 1 decimal place.
- **e** Calculate the area of this circle using $A = \pi r^2$, correct to 1 decimal place.



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Keeping in touch with numeracy 3

Non-calculator

1 Solve the following.
a $500 \times 20 = ?$
b $2000 - 15 = ?$
c $2000 \div 40 = ?$

3 What is three-quarters of 24?

5 Convert 0.3 hours to minutes.

7 Which is greatest in value?
A $\frac{1}{2}$ **B** 20% **C** $\frac{3}{10}$ **D** 0.6

9 Solve the following.
a $5 + 5 \times 5 = ?$
b $(5 + 5) \times 5 = ?$
c $8 - 3 \times 2 = ?$
d $7 + 7 \div 7 + 7 \times 7 - 7 = ?$

11 If Tara is paid $12.50 per hour, how much will she be paid for 20 hours?

13 Find the mean and range of this data set:
2, 2, 2, 3, 3, 4, 4, 4, 5, 5

15 One-quarter of a number is one-half. What is the number?

17 The scale on a house plan is 1 : 100. A wall on the plan is 35 mm long. How long is the wall on the house?

19 A packet of 200 screws costs $15. Find the price, in cents, of each screw.

Calculator

2 Buses hold 43 people. How many buses will be needed to take 825 students to the swimming carnival?

4 What is five-eighths of $100?

6 How many minutes are there in 18.75 hours?

8 Arrange these fractions in ascending order:
$\frac{3}{8}$ $\frac{1}{2}$ $\frac{2}{10}$ $\frac{2}{7}$

10 A holiday house costs $500 for the first two nights and $200 for every extra night. Five people stay for six nights and divide the cost equally. How much does each person pay?

12 Which of the following gives the highest annual income: $652 per week or $2850 per month?

14 The mean of this data set is 5.
2, 2, 2, 3, 3, 4, 4, 4, 5, 5, __, __, __
What could be the missing numbers?

16 Three-fifths of a number is 75. What is the number?

18 On a map, the scale indicates that 1 cm represents 50 km. Two towns are 375 km apart. How many centimetres apart should they be on the map?

20 A packet of 25 screws costs $2.36. A packet of 200 costs $15. How much money is saved by buying 200 screws in one packet rather than 200 screws in packets of 25?

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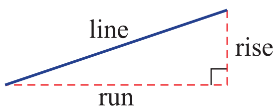
5E Exploring gradient

Stage
5.2
5.2◊
5.1
4

The gradient of a line is a measure of its slope. It is a number that shows the steepness of a line. It is calculated by knowing how far a line rises or falls (called the *rise*) within a certain horizontal distance (called the *run*). The gradient is equal to the *rise* divided by the *run*. The letter *m* is used to represent gradient.

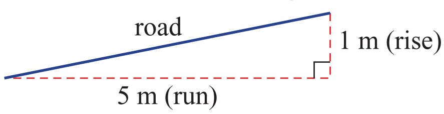


$$\text{Gradient of line} = m = \frac{\text{rise}}{\text{run}}$$

Engineers apply their knowledge of gradients when designing roads, bridges, railway lines and buildings. Some mountain railways have a gradient greater than 1, which is a slope far too steep for a normal train or even a powerful car.

For example, this train takes tourists to the Matterhorn, a mountain in Switzerland. To cope with the very steep slopes it has an extra wheel with teeth, which grips a central notched line.

▶ Let's start: What is the gradient?



A road that rises by 1 m for each 5 m of horizontal distance has a gradient of $\frac{1}{5}$ or 0.2 or 20%.

Trucks would find this gradient very steep.

The gradient is calculated by finding the rise divided by the run.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{1}{5} = 0.2 = 20\%$$

1 Find the gradient for each of these roads.
Give the answer as a decimal and a perentage.
 a Baldwin Street, Dunedin, New Zealand is known as the steepest street in the world. For each 2.86 m of horizontal distance (run), the road rises by 1 m.
 b Gower Street, Toowong, is Brisbane's steepest street. For each 3.2 m of horizontal distance (run), the road rises by 1 m.

2 The Scenic Railway, Katoomba, NSW has a maximum gradient of 122% as it passes through a gorge in the cliff. What is its vertical distance (rise) for each 1 metre of horizontal distance (run)?

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3 Use computer software (dynamic geometry) to produce a set of axes and grid.

- Construct a line segment with end points on the grid. Show the coordinates of the end points.
- Calculate the rise (vertical distance between the end points) and the run (horizontal distance between the end points).
- Calculate the gradient as the *rise* divided by the *run*.
- Now drag the end points and explore the effect on the gradient.
- Can you drag the end points but retain the same gradient value? Explain why this is possible.
- Can you drag the end points so that the gradient is zero or undefined? Describe how this can be achieved.

Key ideas

- **Gradient** is given by the formula

$$m = \frac{\text{rise}}{\text{run}}$$

This value of the gradient can be written as a fraction (which may simplify to a whole number), or as a decimal, percentage or ratio.

Gradient (m) The steepness of a slope

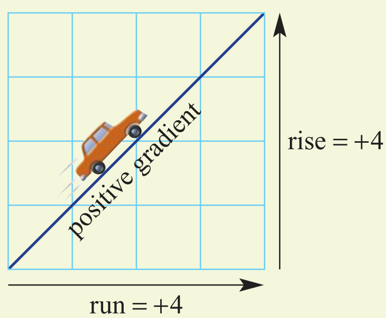
Always move from left to right when considering the rise and the run.

– The horizontal 'run' always goes to the right and is always positive. The vertical 'rise' can go up (positive) or down (negative).

– If the line slopes up from left to right, the rise is positive and the gradient is positive.

e.g. $m = \frac{\text{rise}}{\text{run}} = \frac{+4}{+4} = 1$

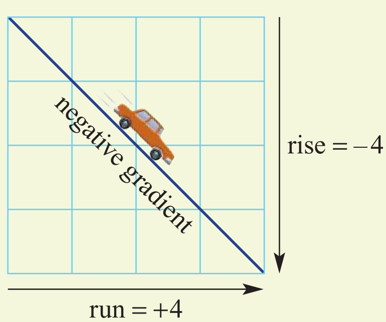
(This could also be written as 100% or 1 : 1)



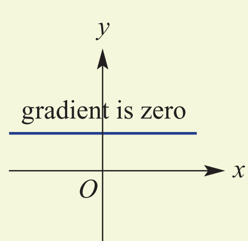
– If the line slopes down from left to right, the rise is considered to be negative and the gradient is negative.

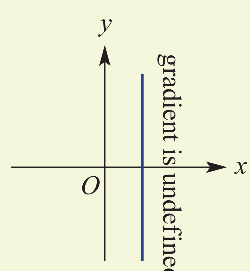
e.g. $m = \frac{\text{rise}}{\text{run}} = \frac{-4}{+4} = -1$

(This could also be written as 100% or 1:1)



- The gradient can also be zero (when a line is horizontal) and undefined (when a line is vertical).





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Exercise 5E

Understanding

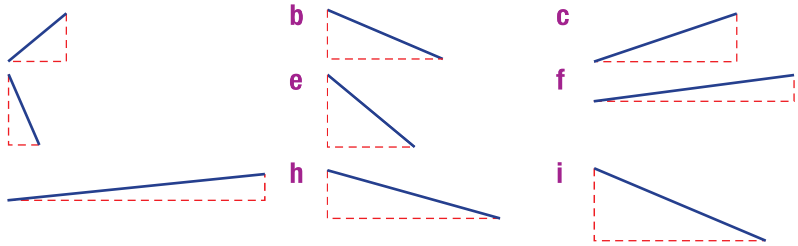
Gradients in the answers are written as fractions or whole numbers, except where specified.

1 Use the words *positive*, *negative*, *zero* or *undefined* to complete each sentence.

- **a** The gradient of a horizontal line is _____________.
- **b** The gradient of the line joining (0, 3) and (5, 0) is _____________.
- **c** The gradient of the line joining (–6, 0) and (1, 1) is _____________.
- **d** The gradient of a vertical line is _____________.

2 Decide whether each of the following lines would have a positive or negative gradient.

a

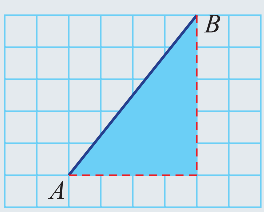


Lines going downhill from left to right have a negative gradient.

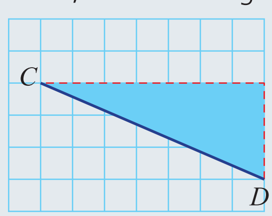
Example 14 Finding the gradient from a grid

Find the gradient of the following line segments, where each grid box equals 1 unit.

a



b

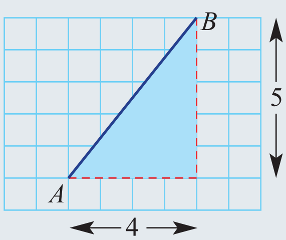


Solution

Explanation

a Gradient of $AB = \frac{\text{rise}}{\text{run}}$

$= \frac{5}{4}$

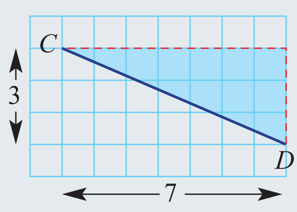


The slope is upwards, therefore the gradient is positive.
The rise is 5 and the run is 4.

b Gradient of $CD = \frac{\text{rise}}{\text{run}}$

$= \frac{-3}{7}$

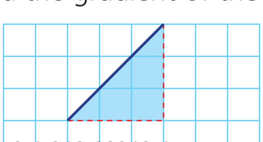
$= -\frac{3}{7}$



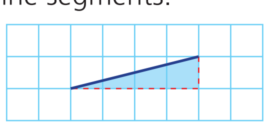
The slope is downwards, therefore the gradient is negative.
The fall is 3, so we write rise = –3, and the run is 7.

3 Find the gradient of the following line segments.

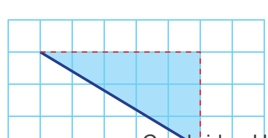
a



b



c



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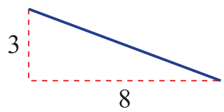
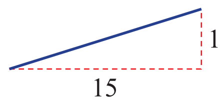
4 Find the gradient of the following.

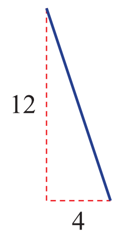
a



b



c



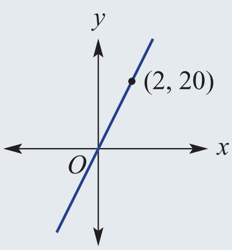
Write the gradient as a fraction or a whole number.

Fluency

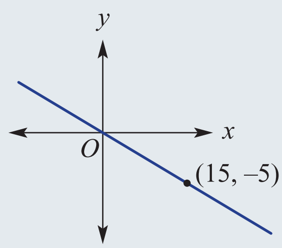
Example 15 Finding the gradient from graphs

Find the gradient of the following lines.

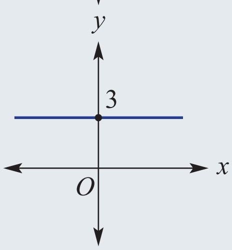
a



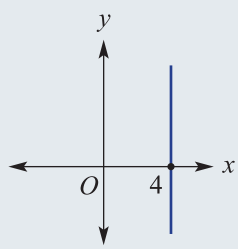
b



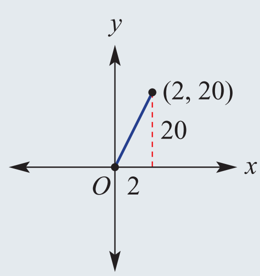
c

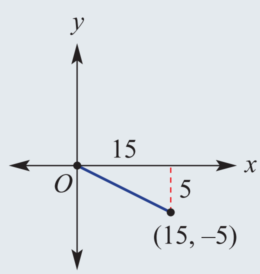


d



Solution	Explanation
a Gradient $= \dfrac{\text{rise}}{\text{run}}$	Write the rule each time.
$= \dfrac{20}{2}$ $= 10$	The rise is 20 and the run is 2 between the two points (0, 0) and (2, 20). Simplify by cancelling.
b Gradient $= \dfrac{\text{rise}}{\text{run}}$ $= \dfrac{-5}{15}$ $= \dfrac{-1}{3}$ $= -\dfrac{1}{3}$	Note this time that, when working from left to right, there will be a slope downwards. The fall is 5 (rise = –5) and the run is 15. Simplify.
c Gradient = 0	Horizontal lines have a zero gradient.
d Gradient is undefined.	Vertical lines have an undefined gradient.





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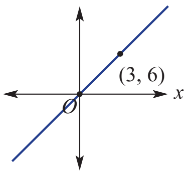
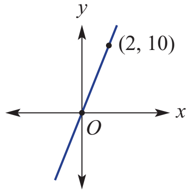
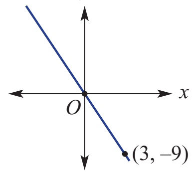
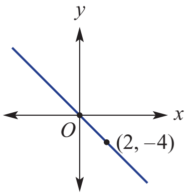
5E

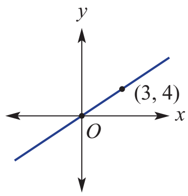
5 Find the gradient of the following lines.

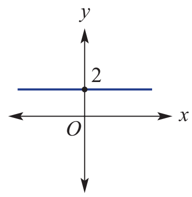
a


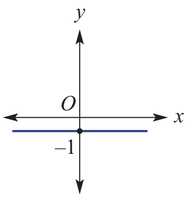
b


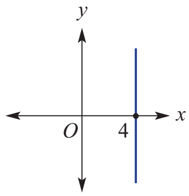
c


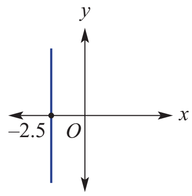
d


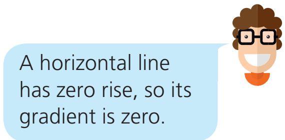
e


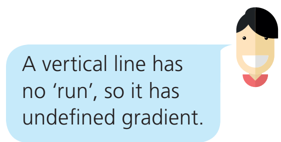
f


g


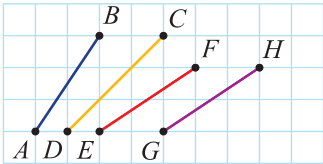
h


i




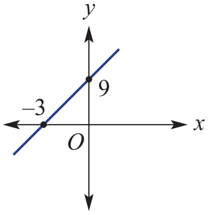


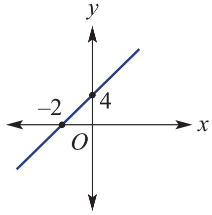
6 Use the grid to find the gradient of the following line segments. Then order the segments from least to steepest gradient.

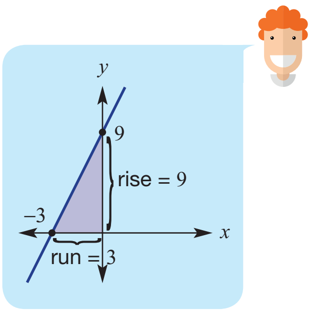


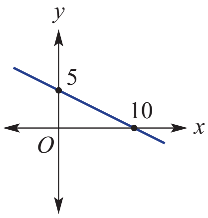
Skillsheet 5A

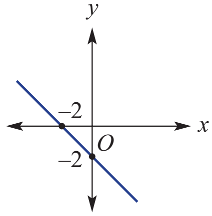
7 Determine the gradient of the following lines.

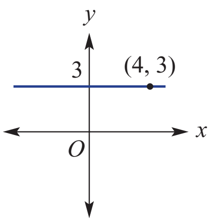
a


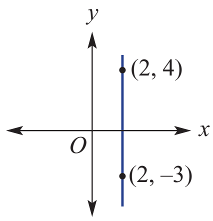
b




c


d


e


f


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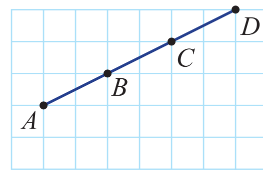
Problem-solving and Reasoning

8 **a** Copy and complete the table below.

Line segment	Rise	Run	Gradient
AB			
AC			
AD			
BC			
BD			
CD			

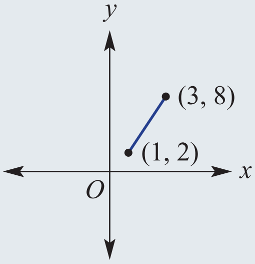


b What do you notice about the gradient between points on the same line?

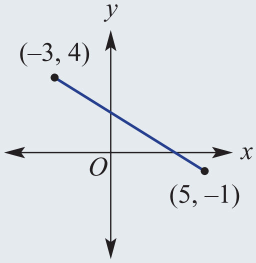
Example 16 Using two points to calculate gradient

Find the gradient of the line segments between the following pairs of points.

a



b



Solution

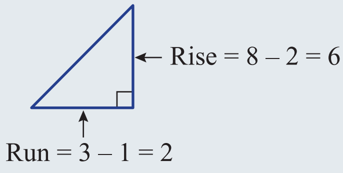
a $m = \frac{\text{rise}}{\text{run}}$

$= \frac{6}{2}$

$= 3$

Explanation

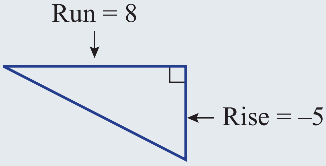
Use the x values to find the run.
Use the y values to find the rise.



b $m = \frac{\text{rise}}{\text{run}}$

$= \frac{-5}{8}$

$= -\frac{5}{8}$



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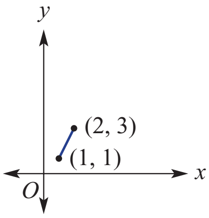
5E

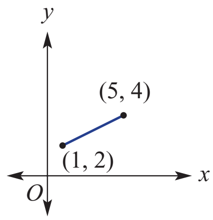
Video 5E

9 Find the gradient between these pairs of points.

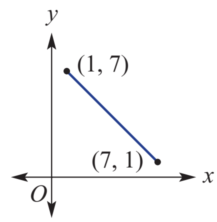
a



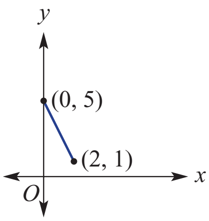
b



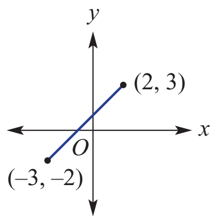
c



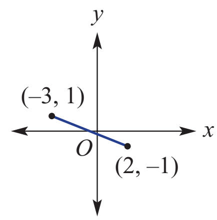
d



e



f



10 Find the gradient between the following pairs of points:

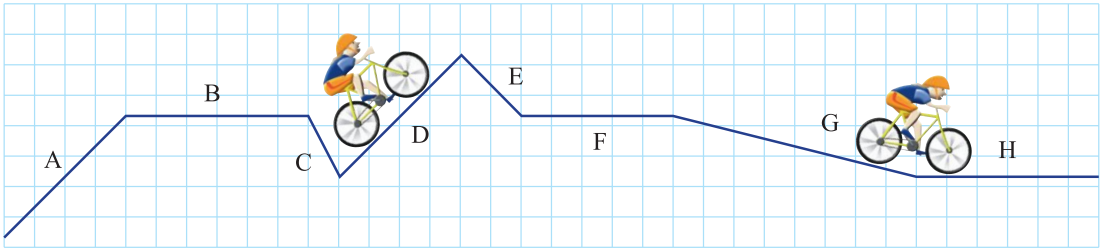
- **a** (1, 3) and (5, 7)
- **b** (–1, –1) and (3, 3)
- **c** (–3, 4) and (2, 1)
- **d** (–6, –1) and (3, –1)
- **e** (1, –4) and (2, 7)
- **f** (–4, –2) and (–1, –1)

11 The first section of the Cairns Skyrail travels from Caravonica terminal at 5 m above sea level to Red Peak terminal, which is 545 m above sea level. This is across a horizontal distance of approximately 1.57 km. What is the overall gradient of this section of the Skyrail? Write your answer as a decimal to 3 decimal places.

> Both distances need to be in the same units.

Enrichment: From Bakersville to Rolland

12 A transversal map for a bike ride from Bakersville to Rolland is shown.



- **a** Which sections, A, B, C, D, E, F, G or H, indicate travelling a positive gradient?
- **b** Which sections indicate travelling a negative gradient?
- **c** Which will be the hardest section to ride?
- **d** Which sections show a zero gradient?
- **e** Which section is the flattest of the downhill rides?
- **f** Design your own travel graph with varying gradients and ask a classmate to find the section with the steepest gradient.

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5F Rates from graphs

Stage: 5.2, 5.2◊, 5.1, 4

The speed or rate at which something changes can be analysed by looking at the gradient (steepness) of a graph.

Graphs of a patient's records provide valuable information to a doctor. For example, from a graph of temperature versus time, the rate of temperature change in °C/minute can be calculated.

Let's start: What's the rate?

Calculate each of these rates.

a \$60 000 for 200 tonnes of wheat = \$___ /t

b Lee travels 840 km in 12 hours = ___ km/h

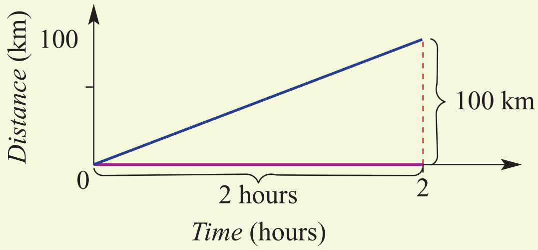
c A foal grows 18 cm in height in 3 months = ___ cm/month

d Petrol costs \$96 for 60 litres = \$___ /L

e Before take-off, a hot-air balloon of volume 6000 m^3 is filled in 60 seconds = ___ m^3/s

Key ideas

- A **rate** compares two quantities. Many graphs show how a quantity changes over *time*.



$$\text{Rate} = \frac{\text{rise}}{\text{run}} = \frac{100}{2} = 50 \text{ km/h}$$

Rate A measure of one quantity against another

- Rate = change in quantity ÷ change in time
  (L, kg, …)   (seconds, hours, …)
- A common rate is speed.
  - Speed = change in distance ÷ change in time
    (cm, km, …)   (seconds, hours, …)
- The gradient of a line gives the rate.
- To determine a rate from a linear graph, calculate the gradient and include the units; e.g. km/h.

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Exercise 5F

Understanding

1 Complete the sentences.

a A rate is found from a linear graph by calculating the ____________ of the line.

b A rate compares ____________ quantities.

c A rate has two ____________.

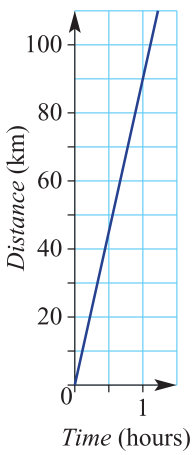
d A speed of 60 kilometres per hour is written as 60 ____________.

e If the rate of filling a bath is 50 litres per minute, this is written as 50 ____________.

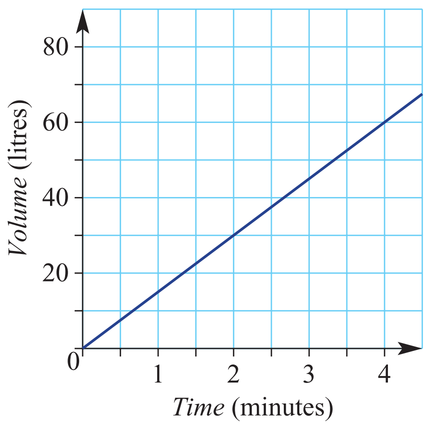
Choose from: km/h, gradient, units, L/min, two.

2 Write down the rate by calculating the gradient of each line graph.

a



b



A rate = gradient with units.
Gradient = $\frac{\text{rise}}{\text{run}}$.

Fluency

Example 17 Calculating speed from a graph

A 4WD vehicle completes a journey, which is described by this graph.

a For the first hour, find:

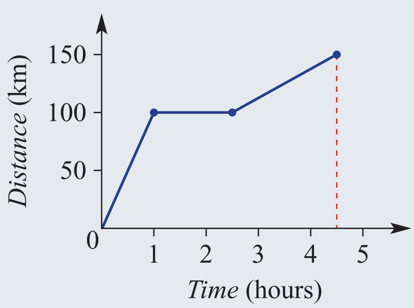
i the total distance travelled

ii the speed

b How fast was the 4WD travelling during:

i the second section?

ii the third section?



Solution	Explanation
a i 100 km	Read the distance at 1 hour.
ii 100 km/1 h = 100 km/h	Speed = distance ÷ time
b i 0 km/1.5 h = 0 km/h	The vehicle is at rest.
ii (150 – 100) km/(4.5 – 2.5) h = 50 km/2 h = 25 km/h	Determine the distance travelled and the amount of time, then apply the rate formula. Speed = distance ÷ time 50 km in 2 hours is $\frac{50}{2} = 25$ km in 1 hour.

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3 A car completes a journey, which is described by this graph.

a For the first hour, find:

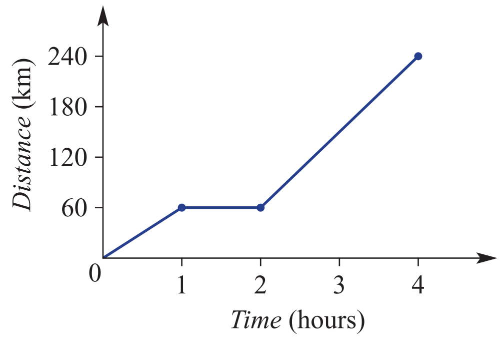
i the total distance travelled

ii the speed

b How fast was the car travelling during:

i the second section?

ii the third section?



4 A cyclist includes a rest stop between two travelling sections.

a For the first hour, find:

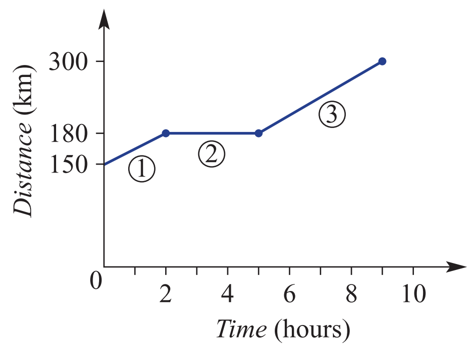
i the total distance travelled

ii the speed

b How fast was the cyclist travelling during:

i the second section?

ii the third section?



Example 18 Calculating the rate of change of volume in L/s

A container is being filled with water from a hose.

a How many litres are filled during:

i the first 10 seconds?

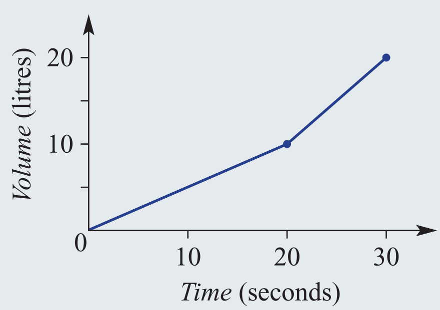
ii the final 10 seconds?

b How fast (i.e. what rate in L/s) is the container being filled:

i during the first 10 seconds?

ii during the final 10 seconds?

iii between the 10- and 20-second marks?



Solution	Explanation
a **i** 5 litres	Read the number of litres after 10 seconds.
ii 10 litres	Read the change in litres from 20 to 30 seconds.
b **i** 5 L/10 s = 0.5 L/s	5 litres is added in the first 10 seconds.
ii 10 L/10 s = 1 L/s	10 litres is added in the final 10 seconds.
iii 5 L/10 s = 0.5 L/s	5 litres is added between 10 and 20 seconds.

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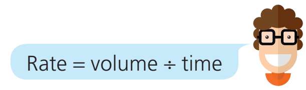
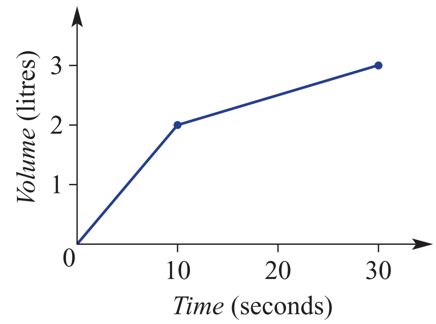
5F

5 A container is being filled.

a How many litres are filled during:
- **i** the first 10 seconds?
- **ii** the final 10 seconds?

b How fast (i.e. what rate in L/s) is the container being filled:
- **i** during the first 10 seconds?
- **ii** during the final 10 seconds?
- **iii** between the 10- and 20-second marks?

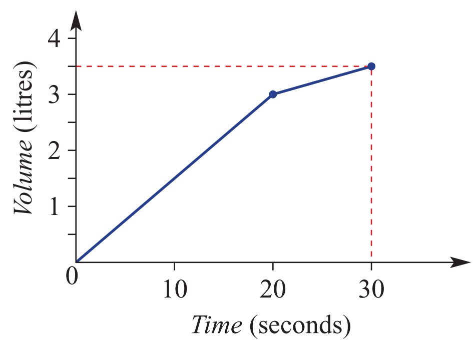




6 A large bottle is being filled.

a How many litres are filled during:
- **i** the first 10 seconds?
- **ii** the final 10 seconds?

b How fast (i.e. what rate in L/s) is the bottle being filled:
- **i** during the first 10 seconds?
- **ii** during the final 10 seconds?
- **iii** between the 10- and 20-second marks?



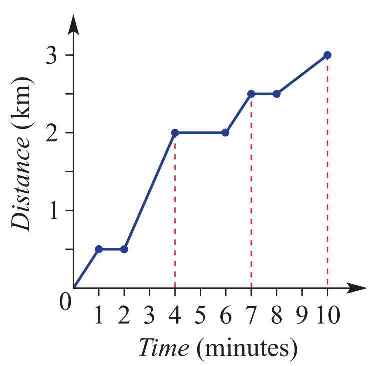
Problem-solving and Reasoning

7 A postal worker stops to deliver mail to three houses along a lane.

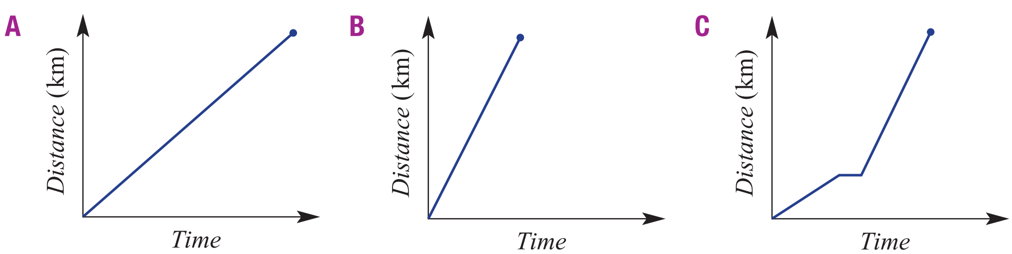
a What is the total length of the lane?

b What is the total time the postal worker spends standing still?

c Find the speed (use km/min) of the postal worker at the following times.
- **i** before the first house
- **ii** between the first and the second house
- **iii** between the second and the third house
- **iv** after the delivery to the third house



8 Three friends, Breanna, Billy and Cianne, travel 5 km from school to the library. Their journeys are displayed in the three graphs below. All three graphs are drawn to the same scale.



a If Breanna walked a short distance before getting picked up by her mum and driven to the library, which graph represents her trip?

b If Cianne arrived at the library last, which graph best represents her journey?

c Which graph represents the fastest journey? Explain your answer.

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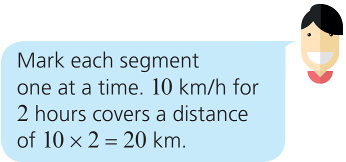
9 **a** Draw your own graph to show the following journey.
- travel 10 km/h for 2 hours
- then rest for 1 hour
- then travel 20 km/h for 2 hours

b Now use your graph to find the total distance travelled.



Enrichment: Sienna's training

10 Sienna is training for a marathon. Her distance–time graph is shown below.

a How many stops did Sienna make?

b How far did she jog between:

i 6 a.m. and 7 a.m.? **ii** 7.30 a.m. and 8.30 a.m.?

c Which sections of the graph have a zero gradient?

d Which sections of the graph have the steepest gradient?

e At what speed did Sienna run in these sections?

i A **ii** C **iii** E **iv** F **v** H

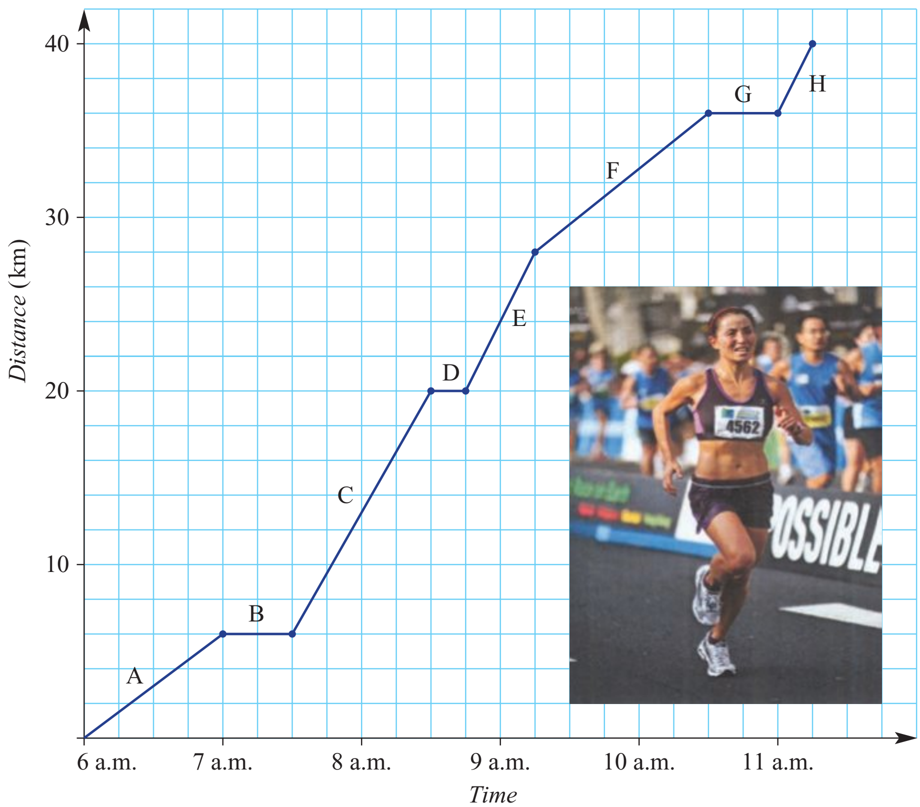
f In which sections is Sienna travelling at the same speed? How does the graph show this?

g How long did the training session last?

h What was the total distance travelled by Sienna during the training session?

i What was her average speed for the entire trip, excluding rest periods?

Sienna's training



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5G Graphing straight lines (part 2)

Stage

5.2
5.2◊
5.1
4

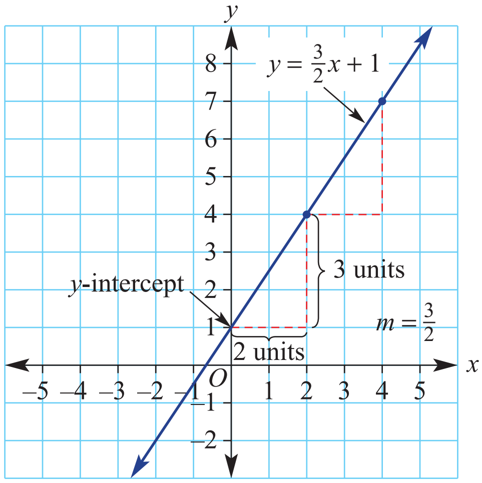
Drilling for Gold 5G1

It is possible to use the equation of a line to predict the position of the line. The y-intercept is the y value of the point where the line cuts the y-axis.

Here is a graph of $y = \frac{3}{2}x + 1$

gradient $m = \frac{3}{2}$

y-intercept = 1



Let's start: Matching lines with equations

Below are some equations of lines and some graphs. Work with a classmate and help each other to match each equation with its correct line graph.

a $y = 2x - 3$

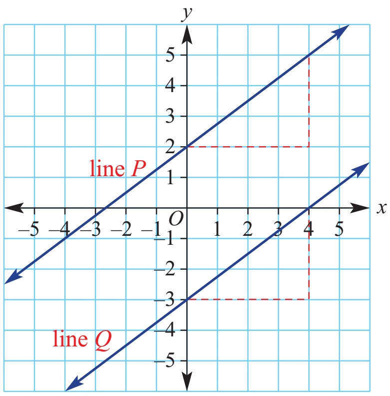
b $y = \frac{3}{4}x + 2$

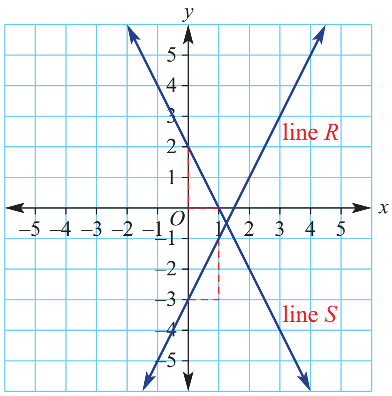
c $y = -\frac{2}{3}x + 1$

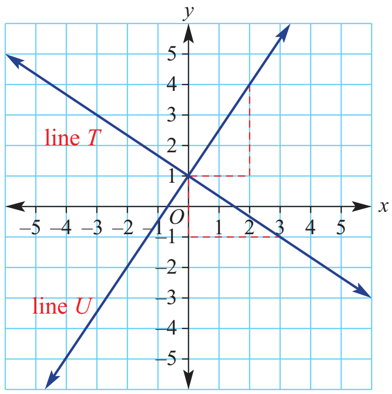
d $y = -2x + 2$

e $y = \frac{3}{4}x - 3$

f $y = \frac{3}{2}x + 1$







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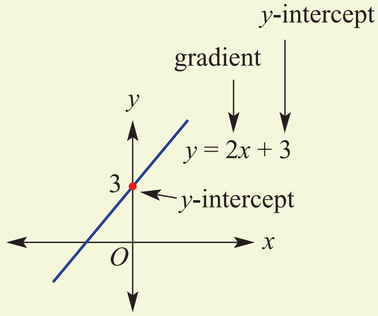
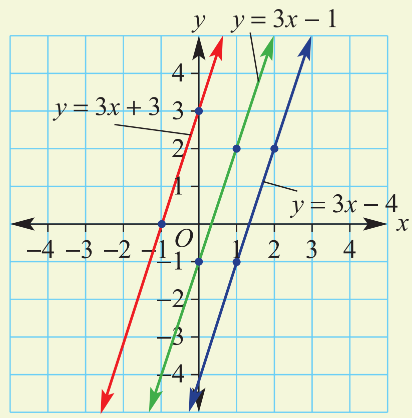
Key ideas

- The gradient or slope is the **coefficient** of x.
- The **y-intercept** is the y value of the point where the line cuts the y-axis.
  For example: In $y = 2x + 3$, the y-intercept is 3.

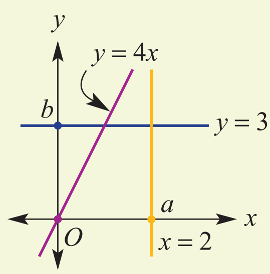


y-intercept The y value of the point at which a line cuts the y-axis

- **Parallel lines** have equal gradient.
  For example, $y = 3x - 1$ and $y = 3x + 8$ have the same gradient of 3.



- Some special lines include:
  - horizontal lines, such as $y = 3$.
  - vertical lines, such as $x = 2$.
  - lines passing through the origin $(0, 0)$, such as $y = 4x$.



- Graphing software is a convenient tool for drawing straight-line graphs. Video 5G shows how it is done.

Video 5G

Exercise 5G

Understanding

1 **a** The line $y = 5$ cuts the y-axis at 5. Is it horizontal or vertical?
b The line $x = 4$ cuts the x-axis at 4. Is it horizontal or vertical?
c Which of these lines is horizontal? (Answer yes or no.)

i $y = x$	**ii** $y = 5x$	**iii** $x = 5$
iv $y = 5$	**v** $x = 0$	**vi** $y = 0$

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Example 19 Reading the gradient and y-intercept from an equation

For the following equations, state the:
i gradient **ii** y-intercept

a $y = 3x + 4$ **b** $y = -\frac{3}{4}x - 7$

Solution	Explanation
a i Gradient is 3.	The coefficient of x is 3.
ii y-intercept is 4.	The constant term is 4.
b i Gradient is $-\frac{3}{4}$.	The coefficient of x is $-\frac{3}{4}$.
ii y-intercept is –7.	The constant term is –7.

2 For the following equations, state the:
i gradient **ii** y-intercept

a $y = 2x + 4$ **b** $y = 6x - 7$ **c** $y = -\frac{2}{3}x + 7$

d $y = -7x - 3$ **e** $y = \frac{3}{5}x - 8$ **f** $y = 9x - 5$

The gradient is the coefficient of x, which is the number multiplied by x. It does not include the x.

3 Which lines are parallel to $y = 2x$?
$y = x + 2$ $y = x - 2$ $y = 2x - 1$ $y = 2x - 5$

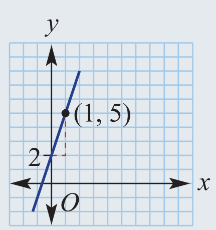
4 Which lines are parallel to $y = x - 3$?
$y = x + 2$ $y = x - 2$ $y = 2x - 1$ $y = 2x - 5$

Fluency

Example 20 Sketching a line using the y-intercept and gradient

Sketch the graph of $y = 3x + 2$ by considering the y-intercept and the gradient.

Solution



Explanation

The constant term is 2. This is the y-intercept.
The coefficient of x is 3 and therefore the gradient is 3 or $\frac{3}{1}$.

Start at the y-intercept 2 and, with the gradient of $\frac{3}{1}$, move 1 unit right (run) and 3 units up (rise) to the point (1, 5). Join the points in a line.

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5 Sketch the graph of the following by considering the y-intercept and the gradient. Use graphing software to check your answer.

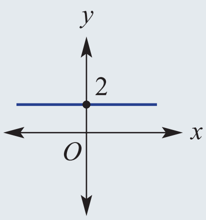
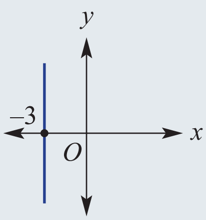
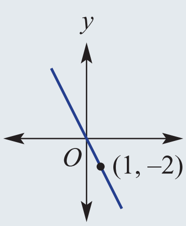
a $y = 2x + 3$
b $y = 3x - 12$
c $y = x + 4$
d $y = -2x + 5$
e $y = -5x - 7$
f $y = -x - 4$

Plot the y-intercept first.
For a line with $m = -2$:
$m = -2 = \frac{-2}{1} = \frac{\text{down } 2}{\text{right } 1}$.
From the y-intercept, go right 1 then down 2 to plot the next point.

Example 21 Sketching special lines

Sketch the graphs of these equations.

a $y = 2$
b $x = -3$
c $y = -2x$

Solution	Explanation
a 	Sketch a horizontal line with a y-intercept at 2.
b 	Sketch a vertical line passing through $(-3, 0)$.
c When $x = 0$, $y = -2 \times (0) = 0$. When $x = 1$, $y = -2 \times 1 = -2$. 	The line passes through the origin $(0, 0)$. Use $x = 1$ to find another point. Sketch the graph passing through $(0, 0)$ and $(1, -2)$.

6 Sketch the following lines.

a $y = 3x$
b $y = 6x$
c $y = -2x$
d $y = 4$
e $y = -2$
f $y = 5$
g $x = 5$
h $x = -2$
i $x = 9$

7 Write the equation of the following lines.

a gradient = 4
y-intercept at 2
b gradient = 3
y-intercept at –2
c gradient = 5
y-intercept at 0
d gradient = –3
y-intercept at 5
e gradient = –4
y-intercept at –3
f gradient = –2
y-intercept at 0

$y = 4x + 2$
gradient y-intercept

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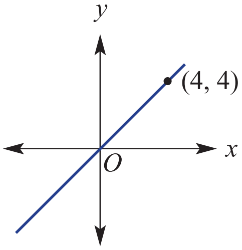
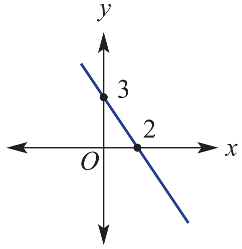
5G

8 Determine the gradient and y-intercept for the following lines.

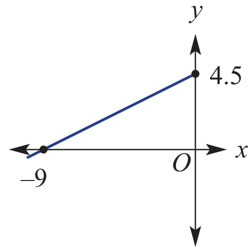
a



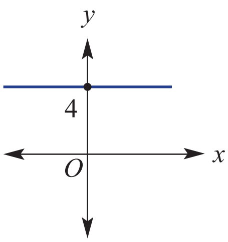
b



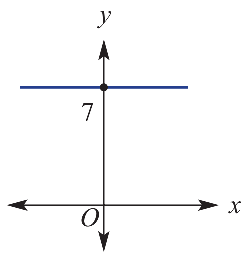
c



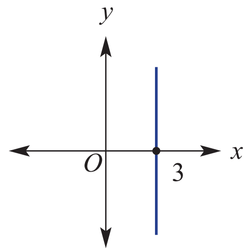
d



e



f



Problem-solving and Reasoning

Drilling for Gold 5G3

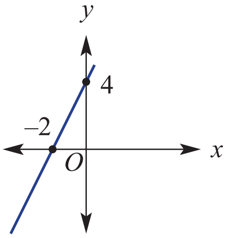
9 Match each of the following equations (**a**–**i**) with one of the sketches (**A**–**I**) shown.

a $y = -\frac{2}{3}x + 2$ **b** $y = -x + 4$ **c** $y = x + 3$

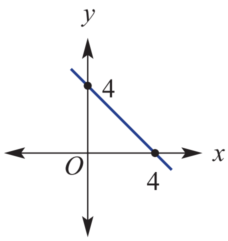
d $y = 2x + 4$ **e** $y = 4$ **f** $y = 7x$

g $y = -3x + 6$ **h** $x = 2$ **i** $y = -3x$

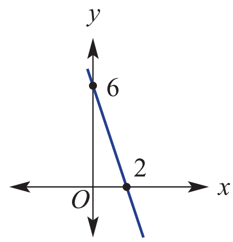
A



B



C



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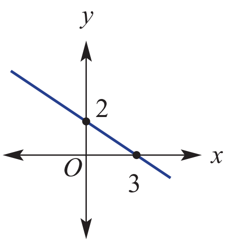
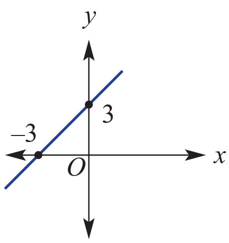


E



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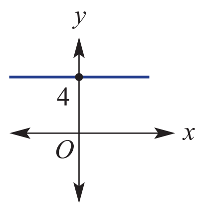

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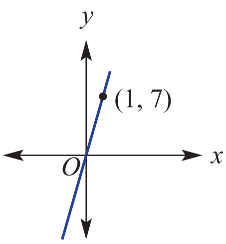
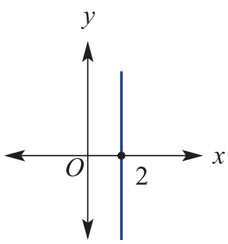
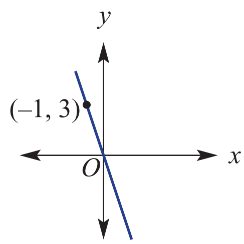


H



I



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10 a Write down three different equations that have a graph with a y-intercept of 5.
b Write down three different equations that have a graph with a y-intercept of -2.
c Write down three different equations that have a graph with a y-intercept of 0.

11 a Write down three different equations that have a graph with a gradient of 3.
b Write down three different equations that have a graph with a gradient of -1.
c Write down three different equations that have a graph with a gradient of 0.
d Write down three different equations that have a graph with an undefined gradient.

12 a Which of the following points lie on the line $y = 2$?
A (2, 3) **B** (1, 2) **C** (5, 2) **D** (−2, −2)
b Which of the following points lie on the line $x = 5$?
A (5, 3) **B** (3, 5) **C** (1, 7) **D** (5, −2)

Example 22 Identifying points on a line

Does the point (3, −4) lie on the line $y = 2x - 7$?

Solution	Explanation
$y = 2x - 7$	Copy the equation and substitute $x = 3$ and $y = -4$.
LHS $= y = -4$; RHS $= 2 \times 3 - 7 = -1$	Compare the LHS and RHS.
LHS ≠ RHS No, (3, −4) is *not* on the line.	So (3, −4) is *not* on the line.

13 a Does the point (3, 2) lie on the line $y = x + 2$?
b Does the point (−2, 0) lie on the line $y = x + 2$?
c Does the point (1, −5) lie on the line $y = 3x + 2$?
d Does the point (2, 2) lie on the line $y = x$?
e Does the line $y - 2x = 0$ pass through the origin?

Substitute the x value into the equation and compare the two y values. When the y values are the same, the point is on the line.

14 Draw each of the following on a number plane and write down the equation of the line.

a

x	0	1	2	3
y	4	5	6	7

b

x	0	1	2	3
y	−1	0	1	2

c

x	−2	0	4	6
y	−1	0	2	3

d

x	−2	0	2	4
y	−3	1	5	9

Use your graph to find the gradient between two points (m) and locate the y-intercept (b). Then use $y = mx + b$.

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5G

Enrichment: Sketching graphs using technology

15 Use technology to sketch a graph of these equations.

a $y = x + 2$ **b** $y = -4x - 3$ **c** $y = \frac{1}{2}x - 1$ **d** $y = 1.5x + 3$

e $y = 2x - 5$ **f** $y = 0.5x + 5$ **g** $y = -0.2x - 3$ **h** $y = 0.1x - 1.4$

16 a On the same set of axes, plot graphs of $y = 2x$, $y = 2x + 1$, $y = 2x + 4$, $y = 2x - 2$ and $y = 2x - 3$, using technology.
Discuss what you see and describe the connection with the given equations.

b On the same set of axes, plot graphs of $y = x - 1$, $y = 2x - 1$, $y = 3x - 1$, $y = \frac{1}{2}x - 1$ and $y = \frac{3}{4}x - 1$, using technology.
Discuss what you see and describe the connection with the given equations.

c On some forms of technology, the equations of families of graphs can be entered using only one line. For example, $y = 2x + 1$, $y = 2x + 2$ and $y = 2x + 3$ can be entered as $y = 2x + \{1, 2, 3\}$ using set brackets. Use this notation to draw the graphs of the rules in parts **a** and **b**.

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Maths@work

Worksheet 5A

Spreadsheet 5A

Real-world linear relationships

Linear relationships can be used to represent many real-world scenarios. For example, when a company is deciding whether or not to put a new product on the market, straight-line graphs can be used to predict the number of items they will need to sell in order to 'break even'. This means that the income from sales will be the same as the set-up costs and the cost of producing that number of items.

In this activity you will use spreadsheets to investigate some business scenarios that can be modelled with straight-line graphs.

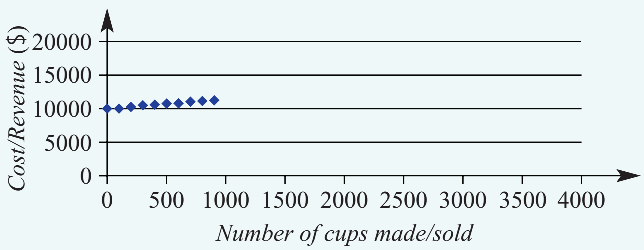
Another cappuccino machine?

A busy café is deciding whether or not to buy a second cappuccino machine. It will cost about $10 000.

A regular cup of coffee costs $1.50 to make, including labour, and is sold for $4 each.

How many regular coffees need to be sold before the new machine starts to make a profit?

This screenshot shows the start of a graph in the spreadsheet that has been set up to work out the answer.



Download the attached worksheet and spreadsheet to see how to do it.

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5H Exploring parabolas

Stage
5.2
5.2◊
5.1
4

Sometimes the relationship between two variables does not produce a straight-line graph. These are called non-linear relationships. The first non-linear graphs we will investigate are called parabolas.

When water comes out of a pipe or hose it forms a shape that looks very much like a parabola.

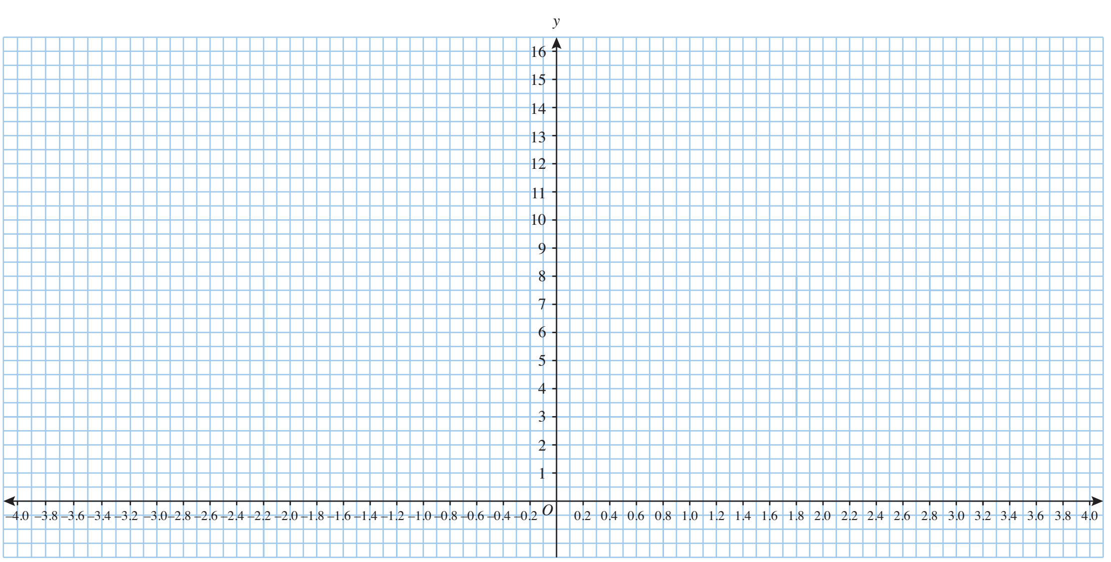
Let's start: Square your number!

Everybody in your class will be given one or more of these numbers.

−4	−3	−2	−1	0	0.1	0.2	0.3	0.4	0.5
0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4
1.5	1.6	1.7	1.8	1.9	2	2.5	3	3.5	4

- Square the number you have been given and write down the result.
- Was your result the same as the number you were given? For which numbers is this true?
- Did your result have a different sign to the number you were given? For which numbers is this true?
- Is it possible for the result to be negative?
- Was your result greater than the number you were given? For which numbers is this true?
- Was your result less than the number you were given? For which numbers is this true?
- Use your number and the result to make a point, such as (3, 9). All students in the class will place a point on a copy of the chart in the 'Drilling for Gold' worksheet 5H1, which looks like this:

Drilling for Gold 5H1



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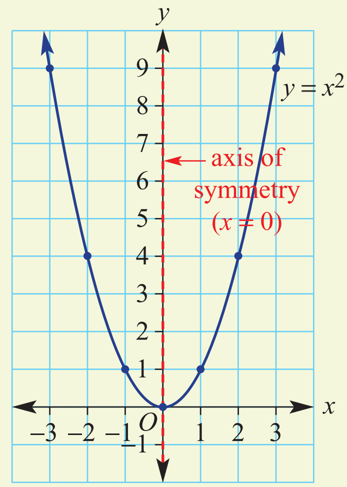
Key ideas

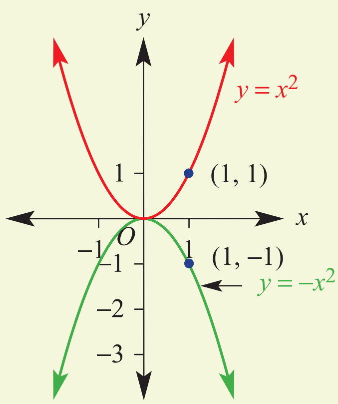
- A **parabola** is the graph of a non-linear relationship.
  The basic parabola has the rule $y = x^2$.
  Key points on $y = x^2$:

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

  – The vertex (or turning point) is (0, 0).
  – It is a minimum turning point.
  – Axis of symmetry is $x = 0$ (the y-axis).



If $y = x^2$ is reflected across the x-axis, the equation becomes $y = -x^2$.

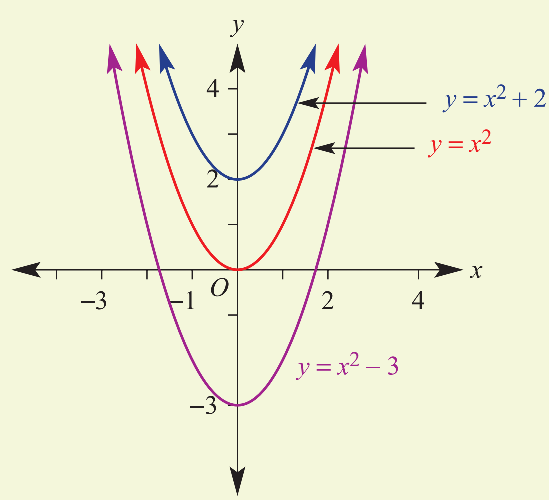


Parabola A smooth U-shaped curve with the basic rule $y = x^2$

Reflection A transformation where a curve is flipped across a line on the number plane

Translation A transformation where a curve is moved a certain distance on the number plane

The parabola $y = x^2$ can be translated up or down the Cartesian plane by adding a positive or negative number to the right-hand side.
– $y = x^2 + 2$ translates $y = x^2$ up by 2 units.
– $y - x^2 - 3$ translates $y = x^2$ down by 3 units.



Drilling for Gold 5H2

- Graphing software is a convenient tool for drawing graphs. 'Drilling for Gold' 5H2 shows how it is done.

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Exercise 5H

Understanding

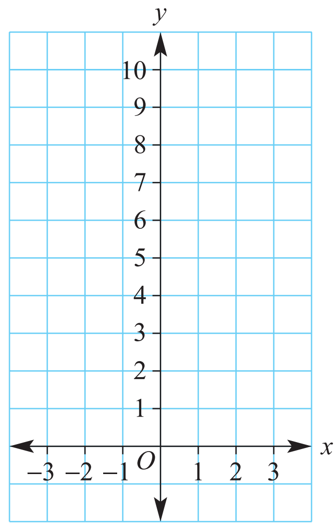
1 Find the square of these numbers.

a 5 **b** –5 **c** 4 **d** –4 **e** 2 **f** –2 **g** 0.5 **h** 0

2 Complete this table to plot the graph of $y = x^2$.

x	–3	–2	–1	0	1	2	3
y	9						

Recall: $(-3)^2 = -3 \times (-3)$
$= 9$



3 Complete this table and draw the parabola of $y = x^2 + 1$ on the same plane as Question **1**.

x	–3	–2	–1	0	1	2	3
y	10						

Fluency

4 Complete the tables below and use them to draw the parabolas.

a $y = x^2 + 2$

x	–3	–2	–1	0	1	2	3
y							

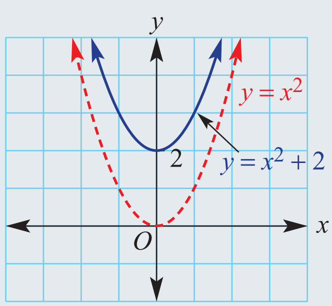
b $y = x^2 - 2$

x	–3	–2	–1	0	1	2	3
y							

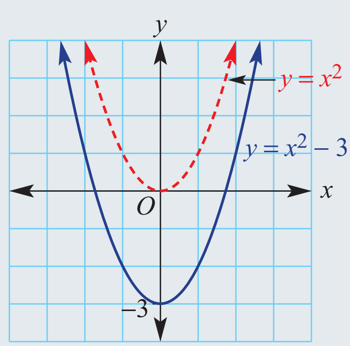
Example 23 Translating vertically

Copy and complete the table for the following graphs.

a



b



	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$
a	$y = x^2 + 2$				
b	$y = x^2 - 3$				

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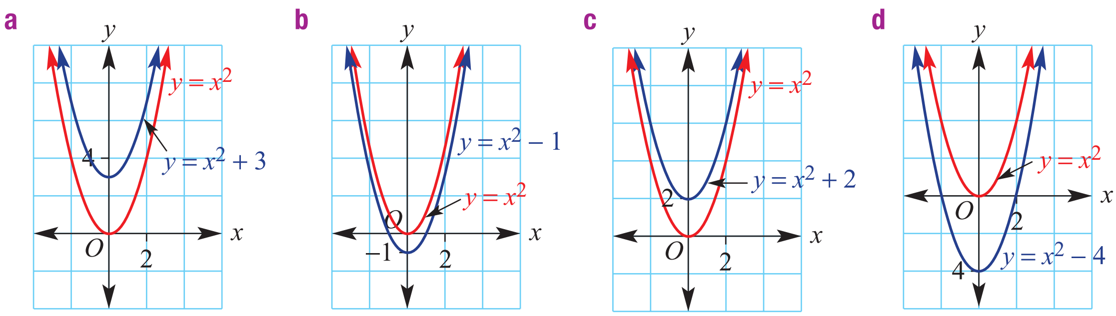
Solution **Explanation**

	Formula	Maximum or minimum	Reflected in the x-axis (yes/no)	Turning point	y value when $x = 1$
a	$y = x^2 + 2$	minimum	no	(0, 2)	3
b	$y = x^2 - 3$	minimum	no	(0, −3)	−2

The effect is to shift up or down; up for $y = x^2 + 2$ and down for $y = x^2 - 3$.

5 Copy and complete the table for the graphs that follow.

	Formula	Turning point	y-intercept ($x = 0$)	y value when $x = 1$
a	$y = x^2 + 3$			
b	$y = x^2 - 1$			
c	$y = x^2 + 2$			
d	$y = x^2 - 4$			



6 Using the table below, change all the y values to negative numbers, then draw the parabola with equation $y = -x^2$.

Skillsheet 5B

x	−3	−2	−1	0	1	2	3
y							

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5H

Problem-solving and Reasoning

7 Write a rule for a parabola with each feature.
  a same shape as $y = x^2$, minimum turning point $(0, 2)$
  b same shape as $y = x^2$, maximum turning point $(0, 0)$
  c same shape as $y = x^2$, minimum turning point $(-1, 0)$
  d same shape as $y = x^2$, minimum turning point $(5, 0)$

What turns $y = x^2$ into a graph with a maximum turning point?

8 a The points $(x, 5)$ lie on a parabola. The equation of the parabola is $y = x^2 - 9$. Find the exact values of x.
  b The points $(x, 0)$ lie on a parabola with the equation $y = 2x^2 - 18$. Find the values of x.
  c Explain why there is no point on the curve $y = x^2 - 9$, which has a y value of -10.

Enrichment: Parabolas with technology

Drilling for Gold 5H2

9 a Using technology, plot the following pairs of graphs on the same set of axes and compare their graphs.
  i $y = x^2$ and $y = 4x^2$
  ii $y = x^2$ and $y = \frac{1}{3}x^2$
  iii $y = x^2$ and $y = 6x^2$
  iv $y = x^2$ and $y = \frac{1}{4}x^2$
  v $y = x^2$ and $y = 8x^2$
  vi $y = x^2$ and $y = \frac{2}{5}x^2$
  b Suggest how the constant a in $y = ax^2$ transforms the graph of $y = x^2$.

10 a Using technology, plot the following sets of graphs on the same set of axes and compare the turning point of each.
  i $y = x^2$, $y = (x + 1)^2$, $y = (x + 2)^2$, $y = (x + 3)^2$
  ii $y = x^2$, $y = (x - 1)^2$, $y = (x - 2)^2$, $y = (x - 3)^2$
  b Explain how the constant h in $y = (x + h)^2$ transforms the graph of $y = x^2$.

11 a Using technology, plot the following sets of graphs on the same set of axes and compare the turning point of each.
  i $y = x^2$, $y = x^2 + 1$, $y = x^2 + 2$, $y = x^2 + 3$
  ii $y = x^2$, $y = x^2 - 1$, $y = x^2 - 3$, $y = x^2 - 5$
  b Explain how the constant k in $y = x^2 + k$ transforms the graph of $y = x^2$.

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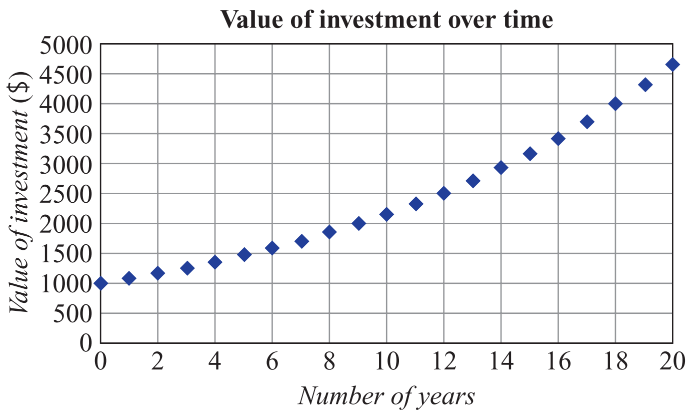
51 Graphs of circles and exponentials

Stage: 5.2, 5.2◊, 5.1, 4

In addition to the parabola, there are two more non-linear relationships we will investigate in this chapter.

Equations such as $y = 2^x$ are called exponential functions, which have graphs like the one on the right.

Equations such as $x^2 + y^2 = 4$ produce a circle with the origin as the centre.



When an investment grows due to compound interest, the relationship between time and money is exponential.

Let's start: Increase *x* by 1 and double the value of *y*

In the table below, the first value of x is 0. The value of y is 1.

As we move from left to right, x will increase by 1 and y will double.

(Arrows above: +1, +1, +1, +1; arrows below: ×2, ×2, ×2, ×2)

x	0	1	2	3	4	5	6	7	8
y	1	2	4						

- Copy and complete the table.
- The pattern of y values can be generated on your calculator by pressing 1 then [=], then × 2, then [=] [=] [=] [=] [=], etc. Try it. What is the biggest number that fits on your screen without being converted to scientific notation?
- If the number 25 appeared in the top row, what number would appear in the bottom row?
- If the number 1 073 741 824 appeared in the bottom row, what number would appear in the top row?
- Which one of the following equations is true for every pair of numbers in the table?

  i $y = x^2$ **ii** $y = x^3$ **iii** $y = 2x$ **iv** $y = 3x$

- Continue the table in the opposite direction (i.e. right to left) to see what happens for negative values of x.

(Arrows above: −1, −1, −1, −1; arrows below: ÷2, ÷2, ÷2, ÷2)

x							0	1	2
y							1	2	4

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Key ideas

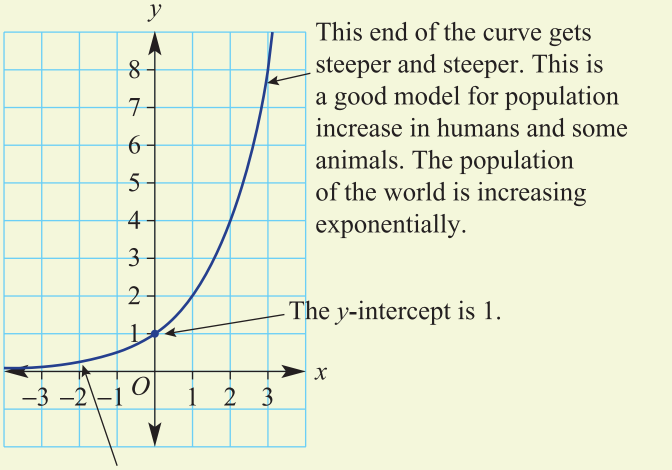
- The simplest **exponential function** is $y = 2^x$.

  To find a value for y, start with the number 2 and raise it to the power of x.

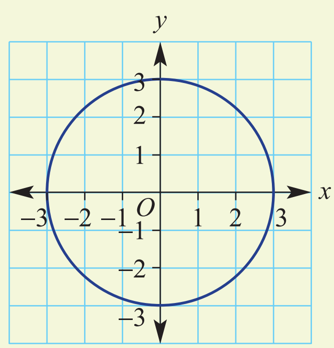
  For example, if $x = 3$, y will be 2 to the power of 3, which is 8.

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

The graph of $y = 2^x$



- The equation $x^2 + y^2 = 9$ is one of the simplest circles to draw on the Cartesian plane.
  - The centre of the circle is $(0, 0)$.
  - The radius of the circle is the square root of 9, which is 3.



- Graphing software is a convenient tool for drawing graphs. 'Drilling for Gold' 5I1 shows how it is done.

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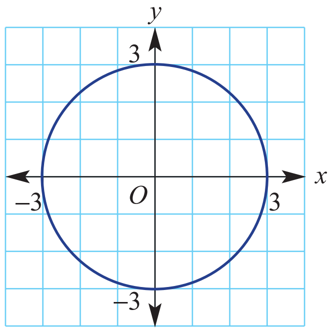
Exercise 5I

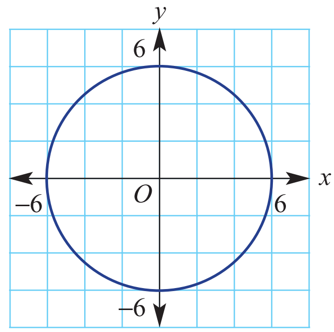
Understanding

1 Write the coordinates of the centre and give the radius of these circles.

a



b



2 A circle has equation $x^2 + y^2 = 16$. Complete these sentences.
a The centre of the circle is ________. b The radius of the circle is ________.

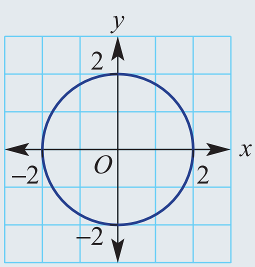
3 Evaluate:

a 2^0 b 2^1 c 2^4 d 3^0 e 3^1
f 3^3 g 4^0 h 4^2 i 5^0 j 5^2

Fluency

Example 24 Sketching a circle

Complete the following for the equation $x^2 + y^2 = 4$.
a State the coordinates of the centre.
b State the radius.
c Sketch a graph, showing intercepts.

Solution	Explanation
a (0, 0)	(0, 0) is the centre for all circles $x^2 + y^2 = r^2$.
b $r = 2$	$x^2 + y^2 = r^2$ so $r^2 = 4$.
c 	Draw a circle with centre (0, 0) and radius 2. Label intercepts.

4 A circle has equation $x^2 + y^2 = 49$. Complete the following.
a State the coordinates of the centre.
b State the radius.
c Sketch a graph, showing intercepts.

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5I

5 Complete the following for the equation $x^2 + y^2 = 25$.

- **a** State the coordinates of the centre.
- **b** State the radius.
- **c** Sketch a graph, showing intercepts.

Example 25 Plotting an exponential graph

For the rule $y = 3^x$:

a Complete this table.

b Plot the points to form its graph.

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{8}$	$\frac{1}{3}$				

Solution

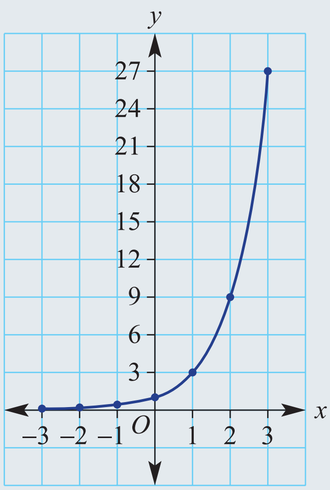
a

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{8}$	$\frac{1}{3}$	1	3	9	27

Explanation

$3^0 = 1,\ 3^1 = 3,\ 3^2 = 9,\ 3^3 = 27.$

b



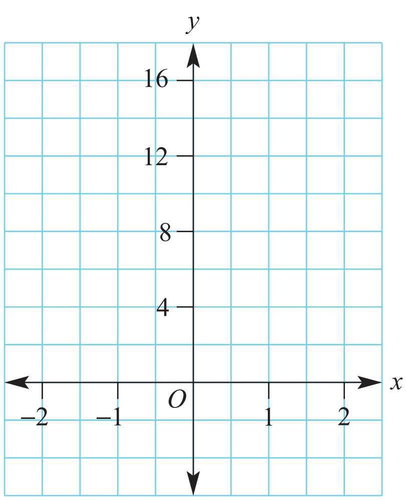
Plot each point and join them to form a smooth curve.

6 Consider the exponential rule $y = 4^x$.

a Complete this table.

x	-2	-1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$			

b Plot the points in the table to form the graph of $y = 4^x$.



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Problem-solving and Reasoning

Drilling for Gold 5I1

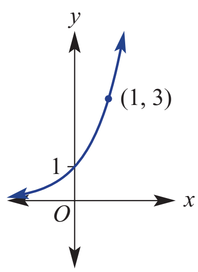
7 **a** Use graphing software to graph the following on the same set of axes.
 i $y = 2^x$
 ii $y = 4^x$
 iii $y = 5^x$
b What do you notice about the y-intercept on each graph?
c What does increasing the base number do to each graph?

8 Give the radius of the circles with these equations.
a $x^2 + y^2 = 36$ **b** $x^2 + y^2 = 81$ **c** $x^2 + y^2 = 144$
d $x^2 + y^2 = 5$ **e** $x^2 + y^2 = 14$ **f** $x^2 + y^2 = 20$

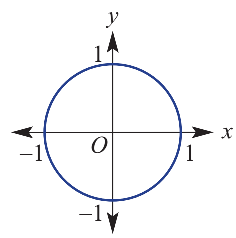
9 Write the equation of a circle with centre $(0, 0)$ and radius 7.

10 Match equations **a–c** with graphs **A–C**.
a $y = -x - 2$ **b** $y = 3^x$ **c** $x^2 + y^2 = 1$

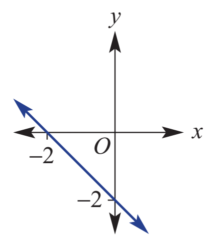
A



B



C



Enrichment: Population growth

11 A study shows that the population of a town is modelled by the rule $P = 2^t$, where t is in years and P is in thousands of people.
a State the number of people in the town at the start of the study (i.e. when $t = 0$).
b State the number of people in the town after:
 i 1 year **ii** 3 years
c When is the town's population expected to reach:
 i 4000 people? **ii** 16 000 people?
d Sketch a graph of P versus t for $t \geq 0$.

If $P = 3$, there are 3000 people.

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Puzzles and games

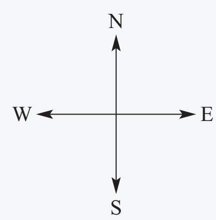
1 Find the words.

- DISTANCE
- GRAPH
- HORIZONTAL
- INCREASE
- RATE
- SEGMENT
- SPEED
- TIME
- VARIABLE

T	F	Z	T	V	M	V	Z	J	H	E	R
O	W	I	M	J	G	R	J	O	A	L	A
T	M	G	T	E	R	K	R	U	T	B	T
E	C	N	A	T	S	I	D	N	E	A	E
S	T	M	P	R	Z	A	E	S	I	I	F
R	P	H	G	O	X	M	E	O	Z	R	E
D	Y	E	N	Z	G	J	U	R	X	A	F
H	G	T	E	E	J	Q	W	G	C	V	H
I	A	L	S	D	R	U	S	G	U	N	A
L	P	G	R	A	P	H	A	P	R	P	I

2 Cooper and Samara are in a cycling orienteering competition.

- From the starting point, Cooper cycles 7 km east, then 3 km south to checkpoint 1. From there, Cooper cycles 5 km east and 8 km north to checkpoint 2.
- Samara cycles 10 km north from the starting point to checkpoint 3.



Use calculations to show that the distance between where Samara and Cooper are now is the same as the direct distance that Cooper is now from the starting point.

3 Lucas and Caroline want to raise money for their school environment club, so they have volunteered to run a strawberry ice-cream stall at their town's annual show. It costs \$200 to hire the stall and they make \$1.25 profit on each ice-cream sold.

a How many ice-creams must be sold to make zero profit (i.e. not a loss)?

b If they make a total profit of \$416.25, how many ice-creams did they sell?

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Chapter summary

Linear relationships

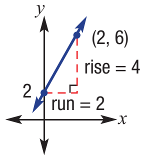
Gradient of a line

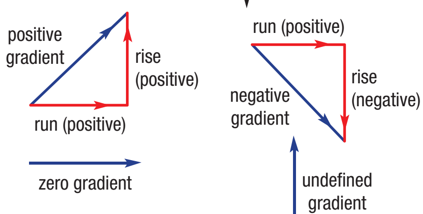
Gradient measures the slope of a line.

Gradient $m = \frac{\text{rise}}{\text{run}}$

$m = \frac{4}{2} = 2$

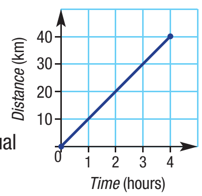
e.g.





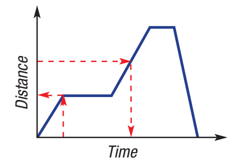
A rate equals the gradient with units.

e.g. Speed = $\frac{40}{4}$ = 10 km/h



If two or more lines have equal gradient they are parallel.

Distance–time graph



- Flat segment means the object is at rest.

Reading a graph:
- Start on given distance; move across to line, then down to time scale (or in reverse).

Equation of a line

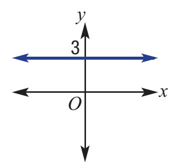
$y = 3x - 2$

gradient is 3 $\quad$ y-intercept is –2

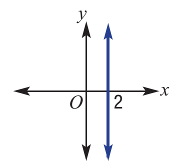
- The rule is a linear equation.
- The graph is made up of points in a straight line.

Special lines

Horizontal lines e.g. $y = 3$



Vertical lines e.g. $x = 2$



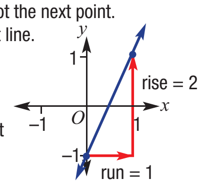
Sketching a line

Plotting straight-line graphs
- Complete a table of values.
- Plot points and join them to form a straight line.

Using the y-intercept and gradient
- Plot the y-intercept.
- Use the gradient to plot the next point.
- Join to form a straight line.

e.g. $y = 2x - 1$

gradient is 2 $\quad$ y-intercept is –1



Midpoint of a line segment

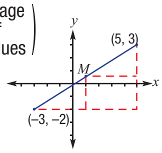
Find the average of the end point coordinates

$M = \left(\begin{matrix}\text{average of} \\ x \text{ values}\end{matrix}, \begin{matrix}\text{average of} \\ y \text{ values}\end{matrix}\right)$

$x = \frac{-3+5}{2} = \frac{2}{2} = 1$

$y = \frac{-2+3}{2} = \frac{1}{2} = 0.5$

$\therefore M = (1, 0.5)$



Length of a line segment

Use Pythagoras' theorem.

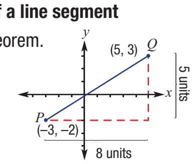
$PQ^2 = 8^2 + 5^2$

$PQ^2 = 64 + 25$

$PQ^2 = 89$

$PQ = \sqrt{89}$ units

$\sqrt{89}$ is an exact length.

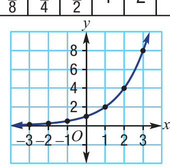


Non-linear relationships

Exponential graphs

$y = 2^x$

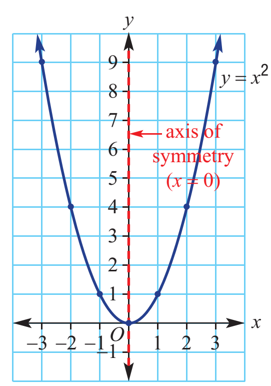
x	–3	–2	–1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



Basic parabola

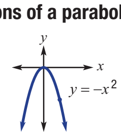
$y = x^2$

x	–3	–2	–1	0	1	2	3
y	9	4	1	0	1	4	9



Transformations of a parabola

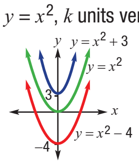
$y = -x^2$ is reflected across the x-axis



$y = x^2 + k$ translates $y = x^2$, k units vertically

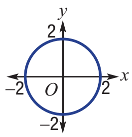
$k > 0$ move up

$k < 0$ move down



Circles

$x^2 + y^2 = r^2$ is a circle centred at (0, 0) or O with radius r.



For example:

$x^2 + y^2 = 4$ is a circle:

- Centre is (0, 0).
- Radius = $\sqrt{4} = 2$

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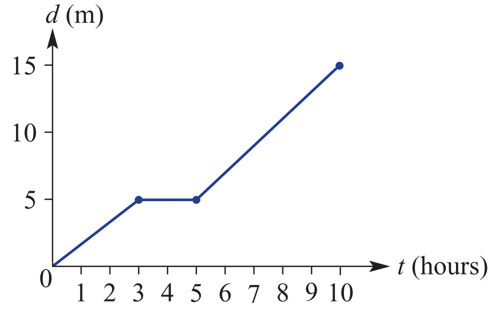
Chapter review

Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.

Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

Questions **1** to **4** refer to the following graph of the movement of a snail.



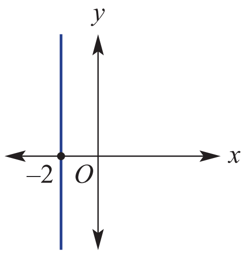
1 The total number of hours the snail was at rest is:
A 2 **B** 4 **C** 5
D 6 **E** 10

2 The distance travelled by the snail in the first 3 hours was:
A 3 m **B** 3 h **C** 7 m
D 4 m **E** 5 m

3 The speed of the snail in the last 5 hours was:
A 5 h **B** 10 m **C** 10 m/h
D 2 m/h **E** 5 m/h

4 The total distance travelled by the snail is:
A 15 m **B** 10 m **C** 5 m **D** 12 m **E** 8 m

5 The equation of the line shown at right is:
A $x = 2$ **B** $x = -2$ **C** $y = 1$
D $y = -2$ **E** $y = -2x$



6 The graph of $y = 10x + 5$ would pass through which of the following points?
A (1, 10) **B** (1, 20) **C** (2, 20)
D (4, 50) **E** (5, 55)

7 The gradient of the line joining (0, 0) and (2, –6) is:
A 2 **B** 3 **C** –3 **D** 6 **E** –6

8 A vertical line has gradient:
A undefined **B** zero **C** positive **D** negative **E** 1

9 A line passes through (–2, 7) and (1, 2). The gradient of the line is:
A –3 **B** $-\frac{5}{3}$ **C** 3 **D** $\frac{5}{3}$ **E** $-\frac{3}{5}$

10 Which of the following equations has a gradient of 2 and a y-intercept of –1?
A $2y + x = 2$ **B** $y - 2x = 1$ **C** $y = -2x + 1$ **D** $y = 2x - 1$ **E** $2x + y = 1$

11 The equation of a circle centred at the origin with radius 4 units is:
A $y = 4x^2$ **B** $x^2 + y^2 = 4$ **C** $x^2 + y^2 = 8$ **D** $y = 4^x$ **E** $x^2 + y^2 = 16$

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12 The graph of $y = 3^x$ has y-intercept with coordinates:

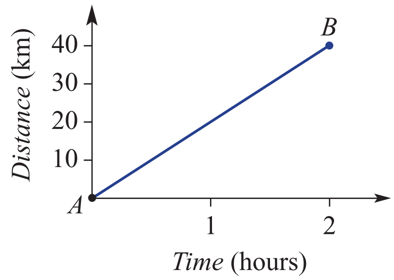
A $(0, 3)$ **B** $(3, 0)$ **C** $(0, 1)$ **D** $(1, 3)$ **E** $\left(0, \frac{1}{3}\right)$

13 Which line is parallel to $y = 4x + 1$?

A $y = 4$ **B** $x = 4$ **C** $y = x + 4$ **D** $y = 4x - 1$ **E** $y = \frac{x}{4}$

Short-answer questions

1 This graph shows the journey of a cyclist from place A to place B.

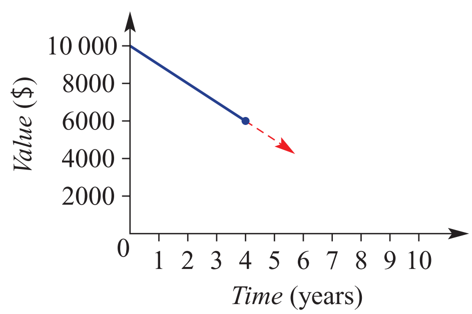


a How far did the cyclist travel?

b How long did it take the cyclist to complete the journey?

c If the cyclist rode from A to B and then halfway back to A, how far was the journey?

2 The value of a poor investment has decreased according to this graph.



a Find the value of the investment after:

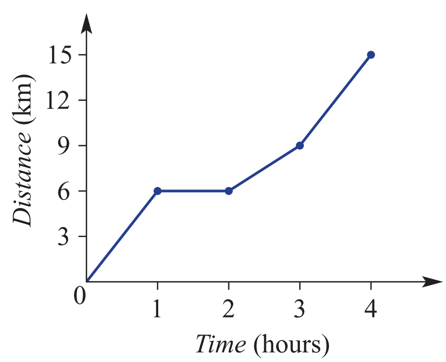
i 4 years **ii** 2 years **iii** 1 year

b Extend the graph and use it to estimate the value of the investment after:

i 8 years **ii** 6 years **iii** 5 years

c After how many years will the investment be valued at \$0?

3 The distance travelled by a walker is described by this graph.



a What is the total distance walked?

b How long was the person actually walking?

c How far had the person walked after:

i 1 hour? **ii** 2 hours?

iii 3 hours? **iv** 4 hours?

d How long did it take to walk a distance of 12 km?

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4 Sketch a graph to show a journey described by:
- a total distance of 60 metres in 15 seconds
- 30 metres covered in the first 6 seconds
- a 5-second rest after the first 6 seconds

5 Caleb delivers pizza orders for a restaurant. He is paid $10 a shift plus $5 per delivery.

a Complete the table of values.

No. of deliveries (d)	0	5	10	15	20
Payment (P)					

b Plot a graph of amount paid against number of deliveries.

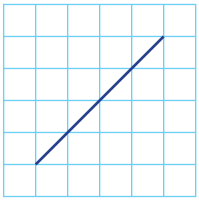
c Use the graph to determine:

i the amount of pay for 12 deliveries

ii the number of deliveries made if Caleb is paid $95

6 Find the gradient of the following lines.

a



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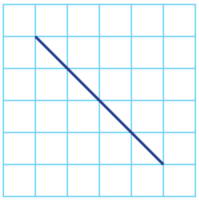

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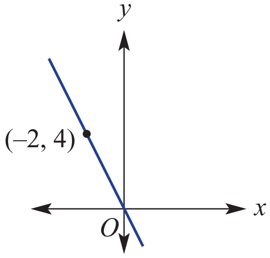
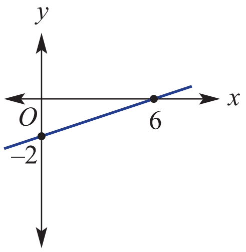


d



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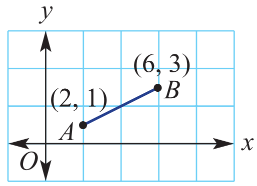
7 Find the midpoint of each line segment.

a

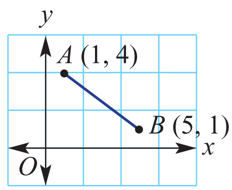


b $P(5, 7)$ to $Q(-1, -2)$

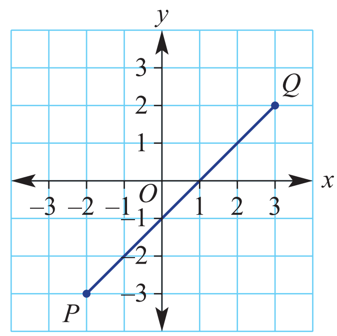
c $G(-3, 8)$ to $H(6, -10)$

8 Find the length of each line segment.

a



b



9 State the gradient and y-intercept of the following lines.

a $y = 3x + 4$

b $y = -2x$

10 Sketch the following lines.

a $y = 2x + 3$

b $y = -4x$

c $y = 2$

d $x = -1$

11 Match each of the linear equations (**a–f**) to the sketches shown (**A–F**).

a $y = 3x - 3$

b $y = 5x$

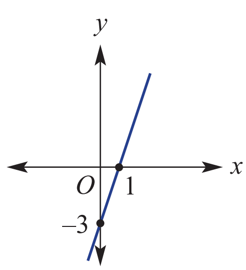
c $5x + 4y = 20$

d $x = 2$

e $-2x + 5y = 10$

f $y = -4$

A



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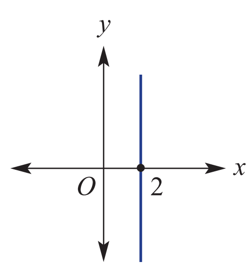
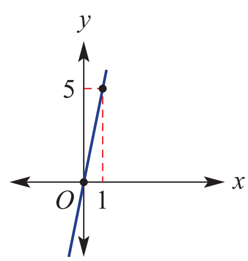
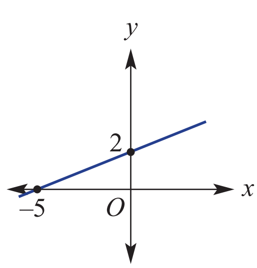


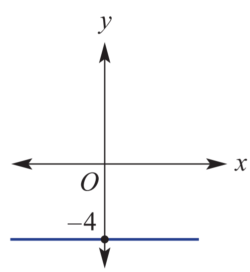
C



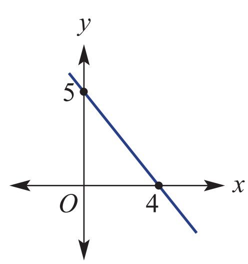
D



E



F



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Chapter review

12 A fruit picker earns \$50 plus \$20 per bin of fruit picked. If the picker earns \$$E$ for n bins picked, complete the following.

a Complete the table of values.

Number of bins (n)	0	1	2	3	4	5	6
Pay (\$$E$)	50	70					

b Sketch a graph for n between 0 and 6.

c Use your rule to find:

i the amount earned after picking 4 bins of fruit

ii the number of bins of fruit picked if \$160 is earned

13 Sketch these circles. Label the centre and axes intercepts.

a $x^2 + y^2 = 25$ b $x^2 + y^2 = 4$

14 Sketch the following graphs, labelling the y-intercepts and the point where $x = 1$.

a $y = 2^x$ b $y = 4^x$

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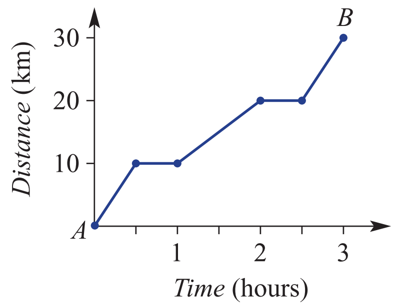
Extended-response questions

1 A courier van picks up goods from two different houses, A and B, as shown on the graph.



a Between houses A and B, find:

i the distance travelled

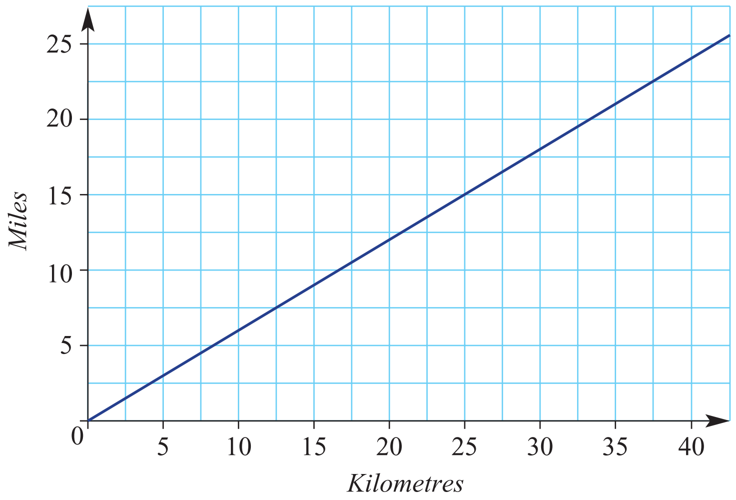
ii the average speed (not including stops)

b How fast was the courier van driving during:

i the first $\frac{1}{2}$ hour? **ii** the second $\frac{1}{2}$ hour?

iii the final $\frac{1}{2}$ hour?

2 This graph shows the direct proportional relationship between miles and kilometres.



a Use the graph to convert 5 miles to kilometres.

b Use the graph to convert 35 kilometres to miles.

c Given that 15 miles is 24.14 km, find the gradient, to 3 decimal places.

d State the conversion rate in miles/km, to 3 decimal places.

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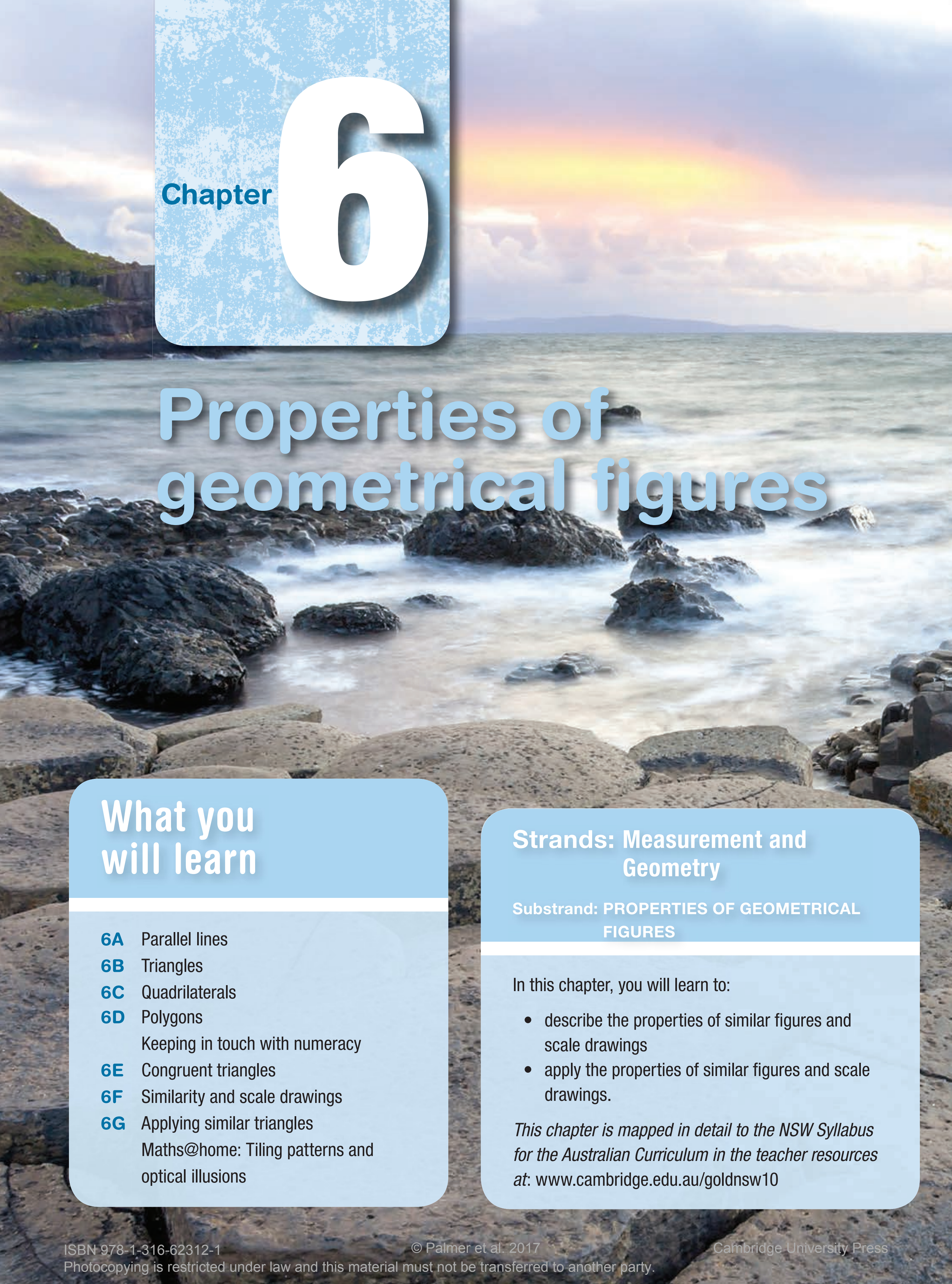

Chapter 6

Properties of geometrical figures

What you will learn

Strands: Measurement and Geometry

Substrand: PROPERTIES OF GEOMETRICAL FIGURES

In this chapter, you will learn to:

- describe the properties of similar figures and scale drawings
- apply the properties of similar figures and scale drawings.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw10

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Geometry meets geology

The Giant's Causeway in Northern Ireland is a rock formation containing approximately $40\,000$ basalt columns. About 50 million years ago lava cooled, solidified and cracked into pillars. Most are hexagonal but some have four, five, seven or eight sides.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw10

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 Write the missing word or number.

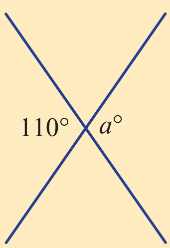
- **a** ________ angles are between 0° and 90°.
- **b** A right angle is ________.
- **c** An obtuse angle is between 90° and ________.
- **d** A 180° angle is called a ________ angle.
- **e** A ________ angle is between 180° and 360°.
- **f** A revolution is ________.
- **g** Complementary angles sum to ________.
- **h** ________ angles sum to 180°.

2 Name the type of triangle with the given properties.

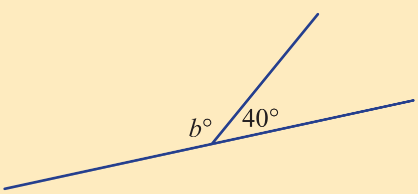
- **a** all sides of different length
- **b** two sides have the same length
- **c** one right angle
- **d** one obtuse angle
- **e** three sides of equal length
- **f** all angles acute

3 Find the values of the pronumerals.

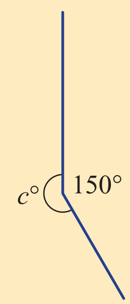
a



b



c



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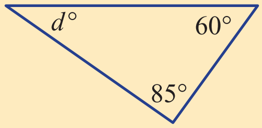
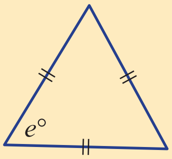


e



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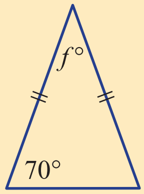
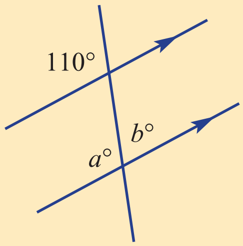


4 Find the value of the pronumerals in these sets of parallel lines.

a



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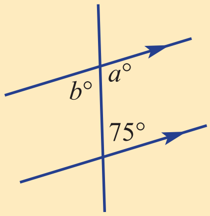
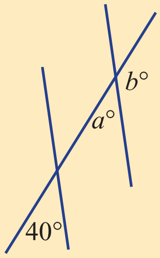


c



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Pre-test

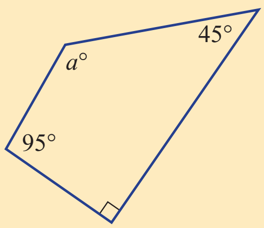
5 The special quadrilaterals are *trapezium*, *kite*, *parallelogram*, *rhombus*, *rectangle* and *square*.

Name the quadrilaterals that definitely have these properties:

a all sides equal and all angles $90°$
b two pairs of parallel sides
c two pairs of parallel sides and all angles $90°$
d two pairs of parallel sides and all sides equal
e one pair of parallel sides
f two pairs of equal length sides and no sides parallel

6 Use the angle sum formula, $S = 180° \times (n - 2)$, to find the angle sum of these polygons and the value of the pronumeral.

a



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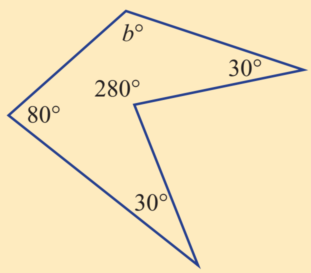

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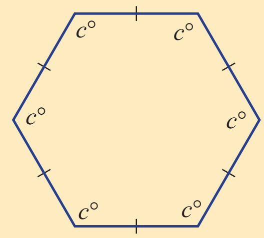
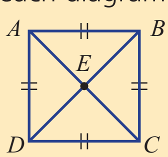


7 In each diagram below, is $\triangle ABC$ definitely congruent (i.e. identical) to $\triangle CDE$?

a



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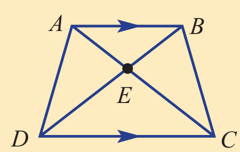

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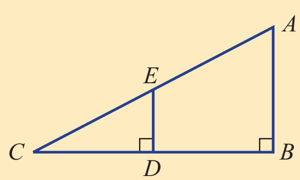

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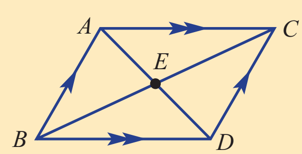
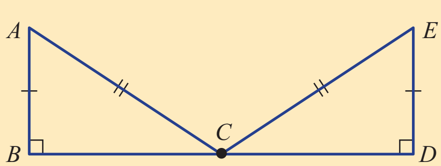
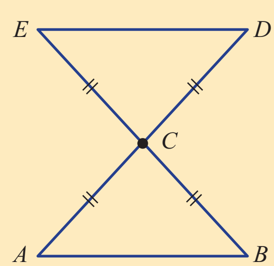


e



f



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6A Parallel lines

Stage
5.2
5.2◊
5.1
4

Parallel lines are everywhere – in buildings, nature and sections of straight railway lines.

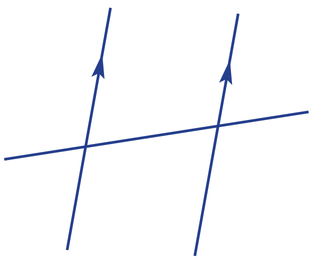
Parallel lines are always the same distance apart and never meet. In diagrams, arrows are used to show that lines are parallel.

▶ Let's start: 2, 4 or 8 different angles

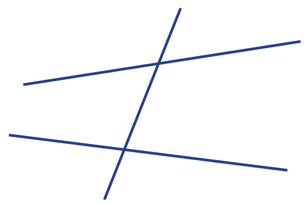
Drilling for Gold 6A1

Diagram **A** and diagram **B** show a pair of lines crossed by a transversal. One pair is parallel and the other is not.

A



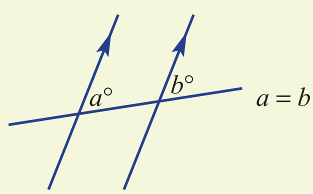
B



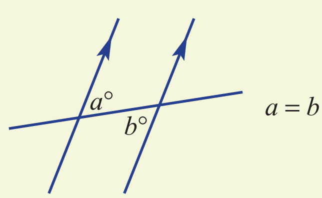
- How many angles of different size are in diagram **A**?
- How many angles of different size are in diagram **B**?
- If only one angle is known in diagram **A**, can you determine all the other angles? Give reasons.

Key ideas

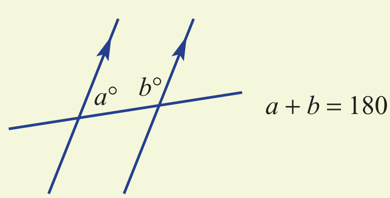
- A **transversal** is a line cutting two or more other lines.
- For parallel lines:
  - **Corresponding angles** are equal.



  - **Alternate angles** are equal.



  - **Cointerior angles** are supplementary.



Transversal A line that cuts two or more lines

Corresponding angles Pairs of angles formed by two lines cut by a transversal

Alternate angles Two angles that lie between two lines on either side of a transversal

Cointerior angles A pair of angles lying between two lines on the same side of a transversal

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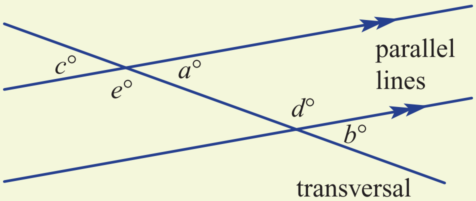


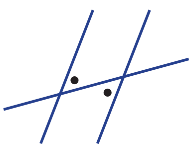
$a = b$	Corresponding angles on parallel lines.
$a = c$	Vertically opposite angles.
$d = e$	Alternate angles on parallel lines.
$a + e = 180$	Angles on a straight line.
$a + d = 180$	Cointerior angles on parallel lines.

Exercise 6A

Understanding

1 Write the missing word or number.
  a Supplementary angles add to ________.
  b Vertically opposite angles are _________.
  c If two lines are parallel and are crossed by a transversal, then:
    i corresponding angles are _______.
    ii alternate angles are _______.
    iii cointerior angles are _________________.

2 For the diagrams below, decide whether the given pair of marked angles are corresponding, alternate, cointerior or vertically opposite.

a


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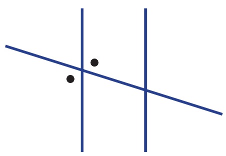
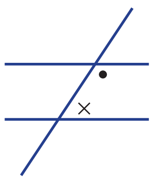
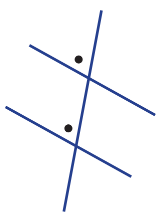


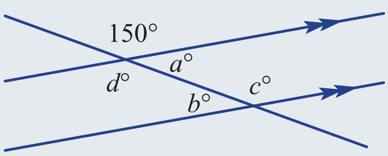
c


d


Fluency

Example 1 Finding angles in parallel lines

Find the values of the pronumerals in this diagram. Write down the reason in each case.



Solution	Explanation
$a = 180 - 150 = 30$ $a°$ and $150°$ are supplementary.	Two angles on a straight line sum to $180°$.
$b = 30$ $b°$ is alternate to $a°$.	Alternate angles are equal in parallel lines.
$c = 150$ $c°$ is corresponding to $150°$ or cointerior to $a°$.	Corresponding angles are equal in parallel lines or cointerior angles are supplementary in parallel lines.
$d = 150$ $d°$ is vertically opposite to $150°$.	Vertically opposite angles are equal.

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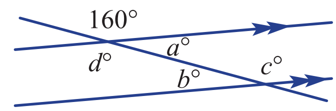
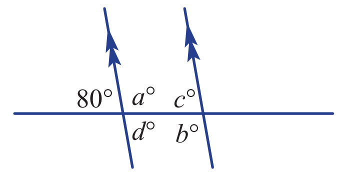
6A

3 Find the values of the pronumerals in this diagram. Write down the reason in each case.

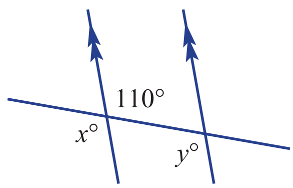


4 Find the values of the pronumerals in this diagram. Write down the reason in each case.

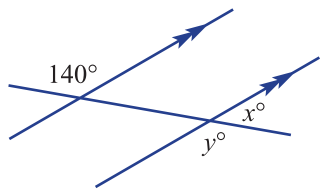


5 Find the value of x and y in these diagrams.

a

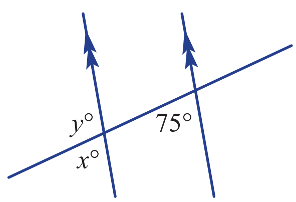


b

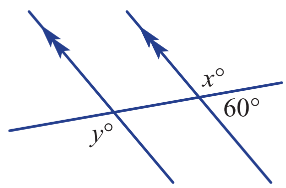


For parallel lines:
Corresponding angles are equal.
Alternate angles are equal.
Cointerior angles add to 180°.
Vertically opposite angles are equal.

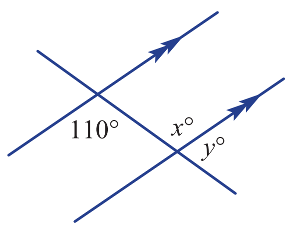
c



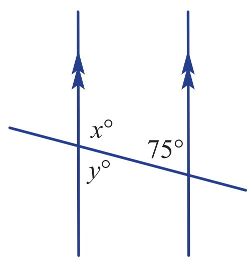
d



e



f

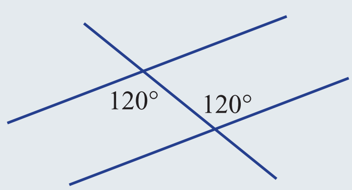


Problem-solving and Reasoning

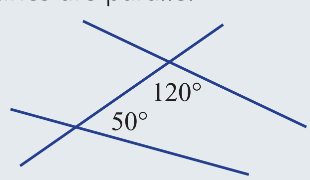
Example 2 Proving that two lines are parallel

Decide, with reasons, whether the given pairs of lines are parallel.

a



b



Solution	Explanation
a Yes, alternate angles are equal.	If alternate angles are equal, then lines are parallel.
b No, cointerior angles are not supplementary.	If lines are parallel, then cointerior angles should add to 180°, but $120° + 50° = 170°$.

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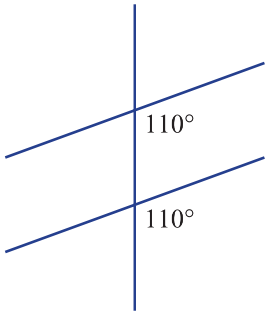
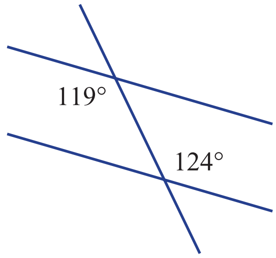
6 Decide, with reasons, whether the given pairs of lines are parallel.

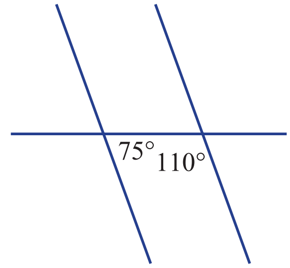
a



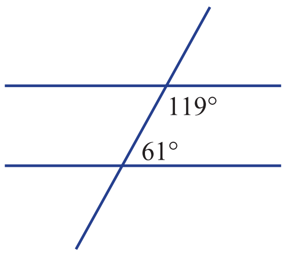
b



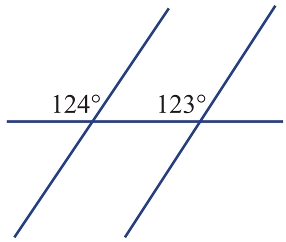
c



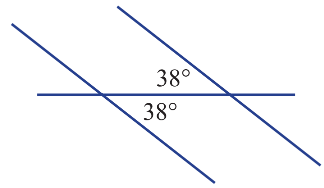
d



e

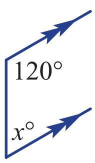


f



7 These diagrams have a pair of parallel lines. Find the unknown value of x.

a



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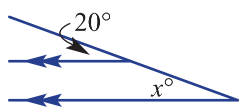

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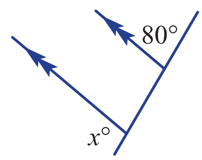

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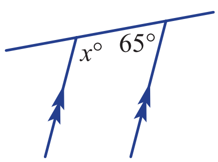

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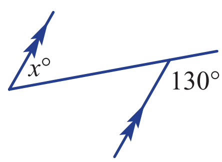

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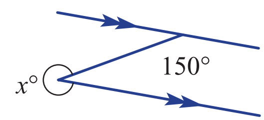
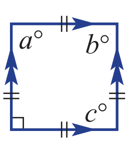


8 These common shapes consist of parallel lines. One internal angle is given. Find the values of the pronumerals.

a



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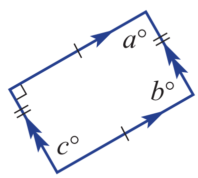

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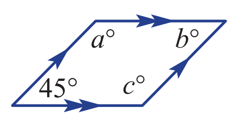
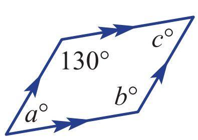
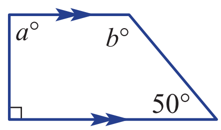


d



e



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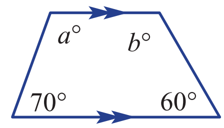

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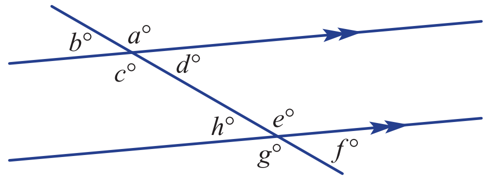
6A

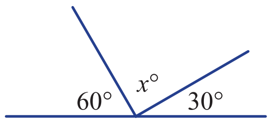
9 For this diagram, list all pairs of angles that are:
- **a** corresponding
- **b** alternate
- **c** cointerior
- **d** vertically opposite



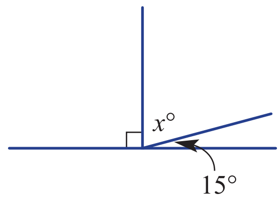
One example for part **a** is (a, e).

10 Find the unknown value of x in each of these cases.

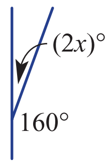
a



b

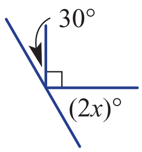


c

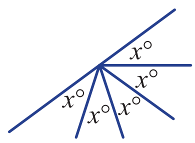


Angles on a straight line add to 180°.

d



e



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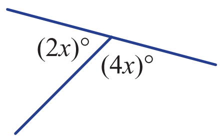
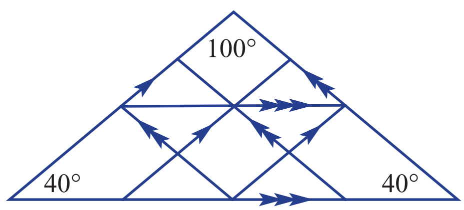


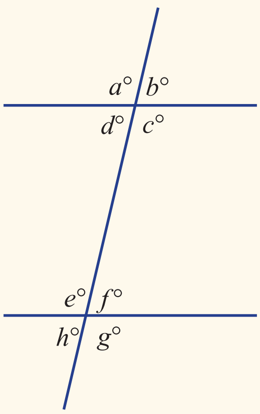
Enrichment: The roof truss

11 This diagram is of a roof truss with three groups of parallel supports. How many of the angles are:
- **a** 100° in size?
- **b** 40° in size?
- **c** 140° in size?



6A2: Corresponding, alternate or cointerior?

The diagram below shows two parallel lines crossed by a transversal.
Study the diagram and then complete questions **1–8** below.
Use the worksheet or write the answers in your exercise book.



Corresponding angles

1 $a =$

2 $b =$

3 $c =$

4 $d =$

Alternate angles

5 $c =$

6 $d =$

Cointerior angles

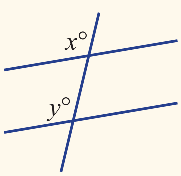
7 $c +$ ___ $= 180$

8 ___ + ___ $= 180$

The diagrams below show two parallel lines crossed by a transversal.
Two angles are marked. Which is true: $x = y$ or $x + y = 180$? Give reasons why.

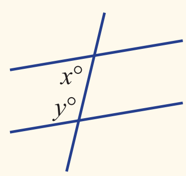
The first two questions have been done for you. Complete the rest.

9



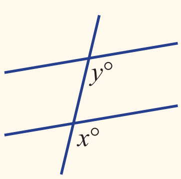
$x = y$ (corresponding angles in parallel lines)

10

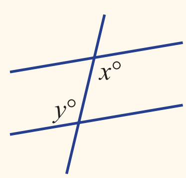


$x + y = 180$ (cointerior angles in parallel lines)

11



12



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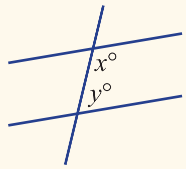
Drilling for Gold exercise

13


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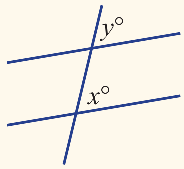

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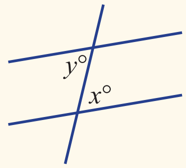

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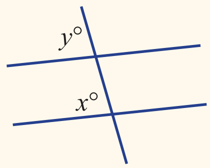

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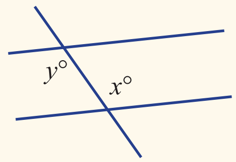

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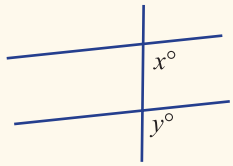

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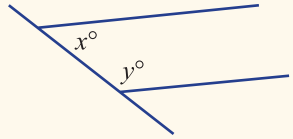

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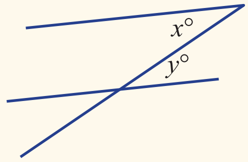

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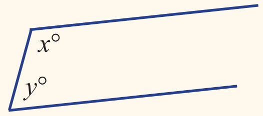

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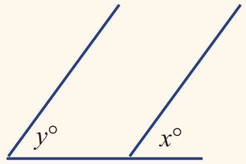

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6B Triangles

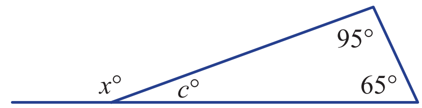
Stage: 5.2, 5.2◊, 5.1, 4

One of the best known and most useful properties of triangles is the internal angle sum (180°). You can check this by measuring and adding up the three internal angles of any triangle.

Let's start: Exterior angle proof

Consider this triangle with exterior angle $x°$.

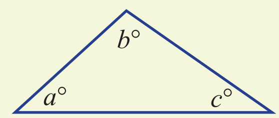
- Use the angle sum of a triangle to find the value of c.
- Now find the value of x.
- What do you notice about $x°$ and the two given angles? Is this true for other triangles? Give examples and reasons.



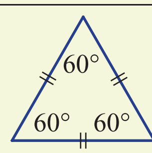
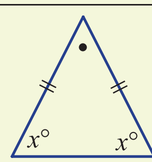
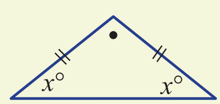
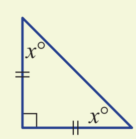
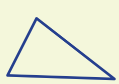
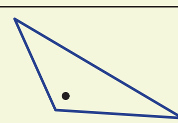
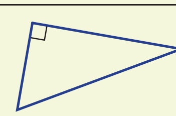
Key ideas

Drilling for Gold 6B1 at end of this section

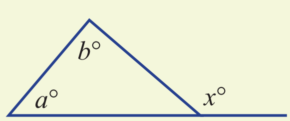
- The sum of all three internal angles of a triangle is 180°.
- Triangles can be classified by their side lengths or their internal angles.



$a + b + c = 180$

		Classified by internal angles		
		Acute-angled triangles (all angles acute, < 90°)	Obtuse-angled triangles (one angle obtuse, > 90°)	Right-angled triangles (one right angle, 90°)
Classified by side lengths	Equilateral triangles (three equal side lengths)		Not possible	Not possible
	Isosceles triangles (two equal side lengths)			
	Scalene triangles (no equal side lengths)			

- The **exterior angle theorem**: The exterior angle is equal to the sum of the two opposite interior angles.



$x = a + b$

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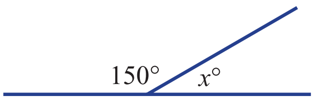
Exercise 6B

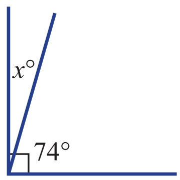
Understanding

1 Choose from the following to match the value of x in these diagrams.
60, 98, 16, 33, 77, 30

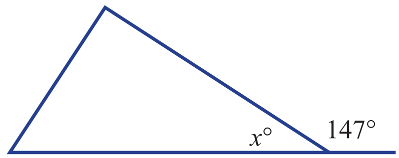
a



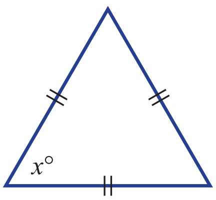
b



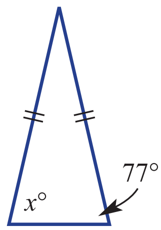
c



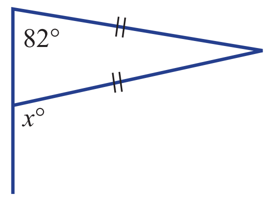
d



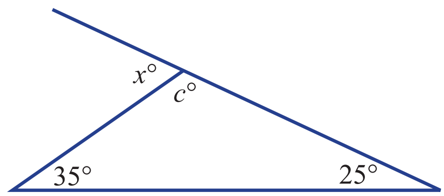
e



f

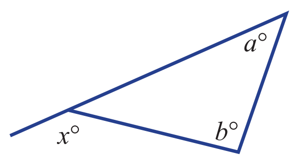


2 The two given interior angles for this triangle are 25° and 35°.



a Use the angle sum (180°) to find the value of c.
b Hence, find the value of x.
c What do you notice about the value of x and the two given interior angles?

3 Choose the correct expression for this exterior angle.

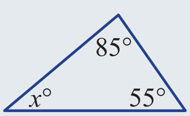


A $a = x + b$ **B** $b = x + a$ **C** $x = a + b$
D $a + b = 180$ **E** $2a + b = 2x$

Fluency

Example 3 Using the angle sum of a triangle

Find the value of the unknown angle (x) in this triangle.



Solution	Explanation
$x + 85 + 55 = 180$	The sum of the three internal angles in a triangle is 180°. Simplify before solving for x.
$x + 140 = 180$ (−140 from both sides) $x = 40$	Solve for x by subtracting 140 from both sides of the equals sign.
∴ The unknown angle is 40°.	

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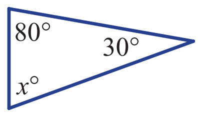
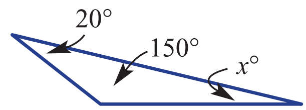
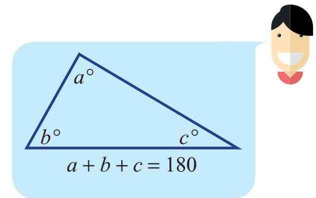
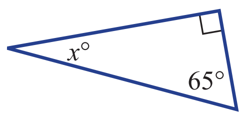
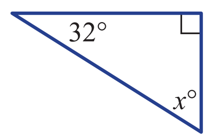
4 Find the value of the unknown angle (x) in these triangles.

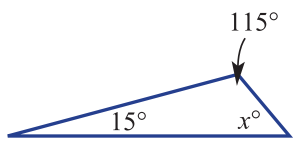
a


b




c


d


e


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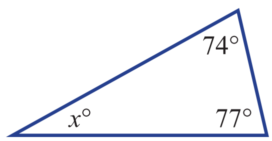
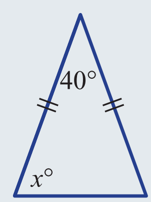
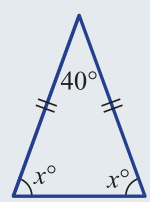


Example 4 Working with an isosceles triangle

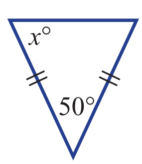
Find the value of x in this isosceles triangle.

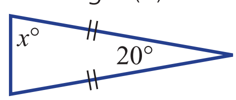


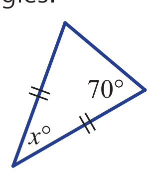
Solution	Explanation
$x + x + 40 = 180$	The triangle is isosceles, therefore the two base angles are equal. Collect like terms.
-40 ($2x + 40 = 180$) -40	Subtract 40 from both sides.
$\div 2$ ($2x = 140$) $\div 2$	Divide both sides by 2.
$x = 70$	
∴ The unknown angle is 70°.	

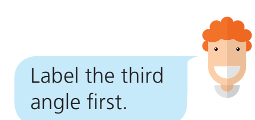


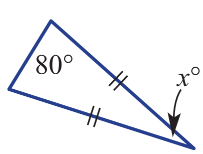
5 Find the value of the unknown angle (x) in these triangles.

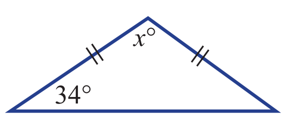
a


b


c




d


e


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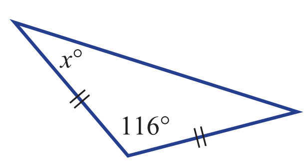

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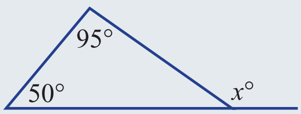
6B

Example 5 Using the exterior angle theorem

Use the exterior angle theorem to find the value of x.

a



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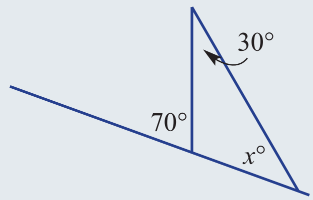
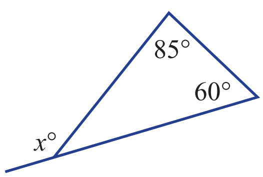


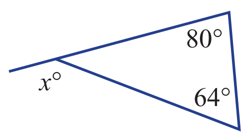
Solution	**Explanation**
a $x = 95 + 50$ $= 145$	The exterior angle $x°$ is the sum of the two opposite interior angles.
b $x + 30 = 70$ $\quad -30$ $x = 40$	The two opposite interior angles are $x°$ and $30°$, and $70°$ is the exterior angle.

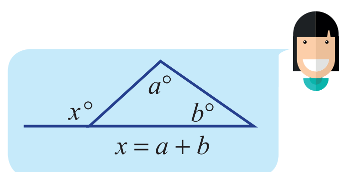
6 Use the exterior angle theorem to find the value of x.

a



b





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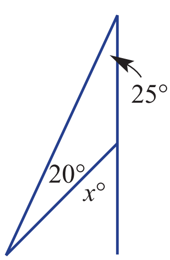

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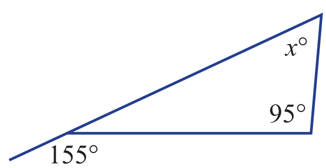
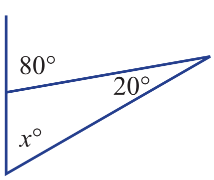


e



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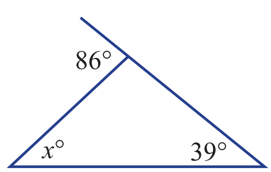

Problem-solving and Reasoning

7 Decide whether the following are possible. If so, make a drawing.

a acute scalene triangle
b acute isosceles triangle
c obtuse equilateral triangle
d acute equilateral triangle
e obtuse isosceles triangle
f obtuse scalene triangle
g right equilateral triangle
h right isosceles triangle

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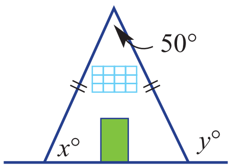
8 An architect draws the cross-section of a new ski lodge, which includes a very steep roof, as shown. The angle at the top is 50°. Find:

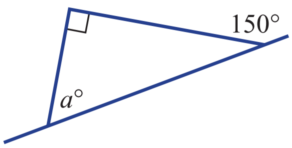
a the acute angle that the roof makes with the floor ($x°$)

b the obtuse angle that the roof makes with the floor ($y°$)

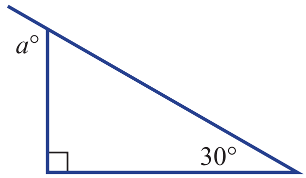


9 Use your knowledge of parallel lines and triangles to find out the value of the pronumerals in these diagrams.

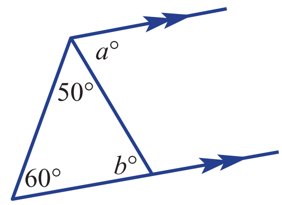
a



b



c

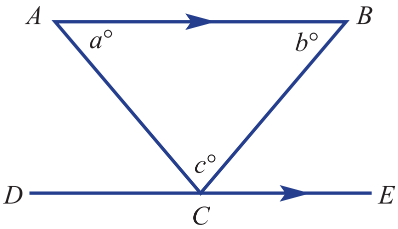


10 For this diagram, AB is parallel to DE.

a What is the size of $\angle ACD$? Use a pronumeral and give a reason.

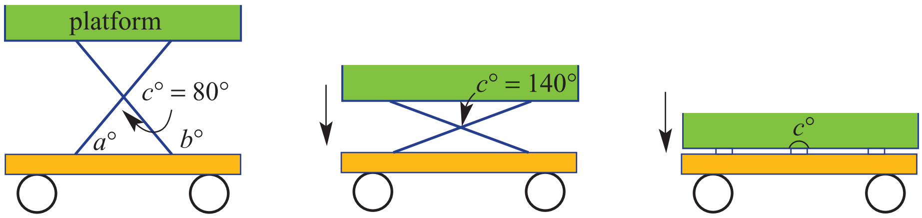
b What is the size of $\angle BCE$? Use a pronumeral and give a reason.

c Since $\angle DCE = 180°$, what does this tell us about a, b and c?



Enrichment: The hydraulic platform

11 A hydraulic platform includes a moveable 'X'-shaped support system, as shown. When the platform is at its highest point, the angle at the centre ($c°$) of the 'X' is $80°$, as shown.



a Find the following when the platform is at its highest position.

i the acute angle that the 'X' makes with the platform ($a°$)

ii the obtuse angle that the 'X' makes with the platform ($b°$)

b The platform now moves down so that the angle at the centre ($c°$) of the 'X' changes from $80°$ to $140°$. At this platform position, find the values of:

i the acute angle that the 'X' makes with the platform ($a°$)

ii the obtuse angle that the 'X' makes with the platform ($b°$)

c The platform now moves down to the base so that the angle at the centre ($c°$) of the 'X' is now $180°$. Find:

i the acute angle that the 'X' makes with the platform ($a°$)

ii the obtuse angle that the 'X' makes with the platform ($b°$)

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Drilling for Gold exercise

6B1: Draw me!

Use the description to draw the seven triangles into your exercise book or onto blank paper.
You may use a ruler, pencil, protractor and a pair of compasses.

	I am a scalene triangle. I have no equal sides.	I am an isosceles triangle. I have two equal sides and two equal angles.	I am an equilateral triangle. I have three equal sides and three equal angles of 60°.
I am an acute-angled triangle. All my angles are less than 90°.	**1** My sides are 4 cm, 5 cm and 6 cm.	**2** My sides are 5 cm, 5 cm and 3 cm.	**3** All my sides are 4 cm.
I am a right-angled triangle. One of my angles is 90°.	**4** My sides are 3 cm, 4 cm and 5 cm.	**5** My short sides are both 4 cm.	This is not possible.
I am an obtuse-angled triangle. One of my angles is between 90° and 180°.	**6** My sides are 3 cm, 6 cm and 8 cm.	**7** My sides are 5 cm, 5 cm and 8 cm.	This is not possible.

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6C Quadrilaterals

Stage

5.2
5.2◊
5.1
4

Quadrilaterals are shapes that have four straight sides and an angle sum of 360°. There are six special quadrilaterals, each with their own special set of properties.

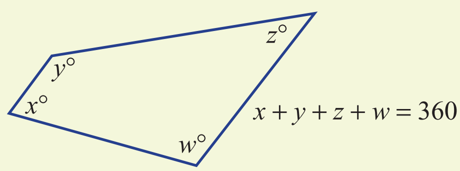
▶ Let's start: Why is a rectangle a parallelogram?

By definition, a parallelogram is a quadrilateral with two pairs of parallel sides.

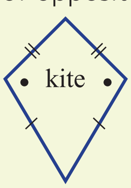
- Using this definition, do you think that a rectangle is also a parallelogram? Why?
- What properties does a rectangle have that a general parallelogram does not?
- What other special shapes are parallelograms? What are their properties?

Key ideas

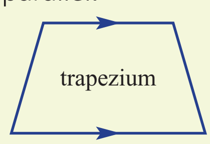
- The sum of the interior angles of any quadrilateral is 360°.



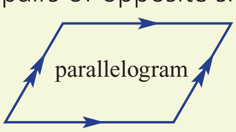
- Formal definitions:
  - A *kite* is a quadrilateral with two pairs of adjacent sides that are equal and one pair of opposite equal angles.



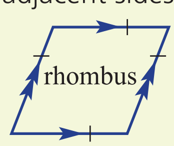
  - A *trapezium* is a quadrilateral with at least one pair of opposite sides that are parallel.



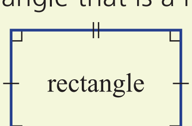
  - A *parallelogram* is a quadrilateral with both pairs of opposite sides that are parallel.



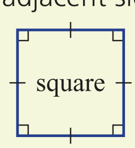
  - A *rhombus* is a parallelogram with two adjacent sides that are equal in length.



  - A *rectangle* is a parallelogram with one angle that is a right angle.



  - A *square* is a rectangle with two adjacent sides that are equal.



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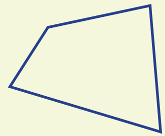
Drilling for Gold 6C1

Quadrilateral

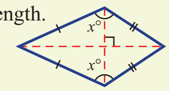


Two pairs of adjacent sides that are equal → Kite

One pair of parallel sides → Trapezium

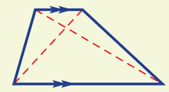
Kite properties

- Two pairs of adjacent sides of a kite are equal in length.
- One diagonal of a kite bisects the other diagonal.
- One diagonal of a kite bisects the opposite angles.
- The diagonals of a kite are perpendicular.
- A kite has one axis of symmetry.



Trapezium properties

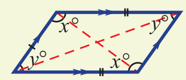
- At least one pair of sides of a trapezium are parallel.



Another pair of parallel sides → Parallelogram

Parallelogram properties

- The opposite sides of a parallelogram are parallel.
- The opposite sides of a parallelogram are of equal length.
- The opposite angles of a parallelogram are equal.
- The diagonals of a parallelogram bisect each other.
- A parallelogram has point symmetry.



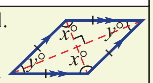
Kite: All sides are equal → Rhombus

Parallelogram: All sides are equal → Rhombus

Parallelogram: One right angle, therefore has four right angles → Rectangle

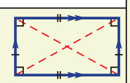
Rhombus properties

- The opposite sides of a rhombus are parallel.
- All sides of a rhombus are equal in length.
- The opposite angles of a rhombus are equal.
- The diagonals of a rhombus bisect the vertex angles.
- The diagonals of a rhombus bisect each other.
- The diagonals of a rhombus are perpendicular.
- A rhombus has two axes of symmetry.
- A rhombus has point symmetry.



Rectangle properties

- The opposite sides of a rectangle are parallel.
- The opposite sides of a rectangle are of equal length.
- All angles at the vertices of a rectangle are 90°.
- The diagonals of a rectangle are equal in length.
- The diagonals of a rectangle bisect each other.
- A rectangle has two axes of symmetry.
- A rectangle has point symmetry.

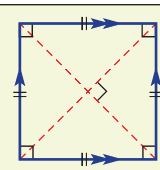


Rhombus: One right angle, therefore has four right angles → Square

Rectangle: All sides are equal → Square

Square properties

- Opposite sides of a square are parallel.
- All sides of a square are of equal length.
- All angles at the vertices of a square are 90°.
- The diagonals of a square are of equal length.
- The diagonals of a square bisect the vertex angles.
- The diagonals of a square are perpendicular.
- A square has four axes of symmetry.
- A square has point symmetry.



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Exercise 6C

Understanding

1 Which special quadrilaterals are parallelograms?

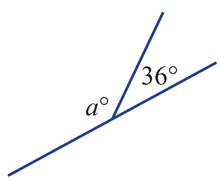
2 List all the quadrilaterals that definitely have the following properties.

a two pairs of parallel sides
b two pairs of equal length sides
c equal opposite angles
d one pair of parallel sides
e one pair of equal angles
f all angles 90°
g equal length diagonals
h diagonals intersecting at right angles

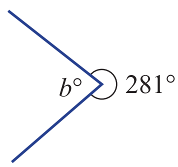
Refer to the Key ideas for help.

3 Find the value of the pronumerals.

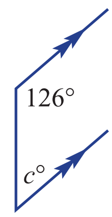
a



b



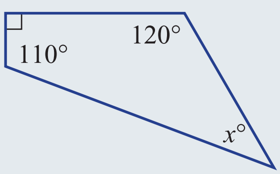
c



Fluency

Example 6 Using the angle sum of a quadrilateral

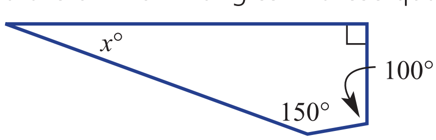
Find the unknown angle in this quadrilateral.



Solution	Explanation
$x + 110 + 120 + 90 = 360$	The sum of the interior angles is 360° in a quadrilateral.
$x + 320 = 360$	Simplify.
-320 $x = 40$ -320	Subtract 320 from both sides.
$\therefore$ The unknown angle is 40°.	

4 Find the unknown angles in these quadrilaterals.

a



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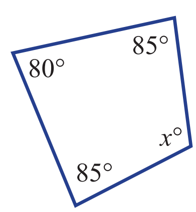
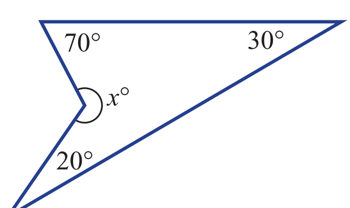
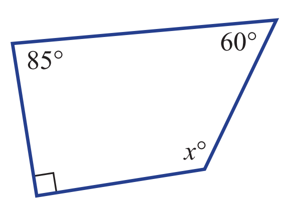


The angle sum of a quadrilateral is 360°.

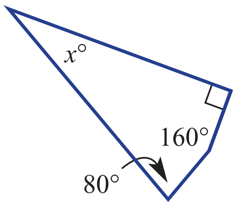
c



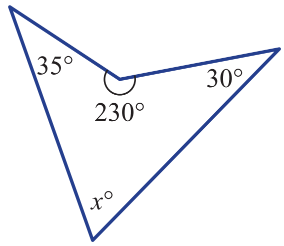
d



e



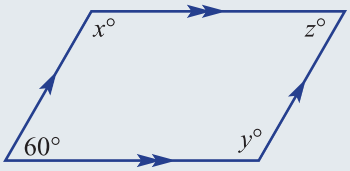
f



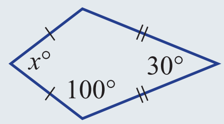
Example 7 Finding angles in special quadrilaterals

Find the value of the pronumerals in these special quadrilaterals, giving reasons.

a



b



Solution

a $x + 60 = 180$ (cointerior angles in parallel lines)

$x = 120$

$\therefore y = 120$ (opposite angles in a parallelogram)

$\therefore z = 60$ (opposite angles in a parallelogram)

b $x + 100 + 100 + 30 = 360$ (angle sum of a quadrilateral)

$x + 230 = 360$

$x = 130$

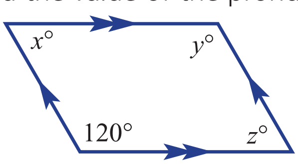
Explanation

a $x°$ and $60°$ are cointerior angles and sum to $180°$.
Subtract 60 from both sides.
$y°$ is opposite and equal to $x°$.
$z°$ is opposite and equal to $60°$.

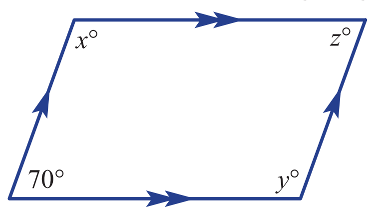
b A kite has a pair of equal, opposite angles, so there are two $100°$ angles.
The total sum is still $360°$.

5 Find the value of the pronumerals in these special quadrilaterals, giving reasons.

a

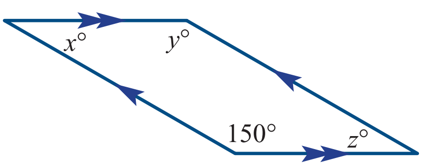


b



Refer to the properties of special quadrilaterals for help.

c



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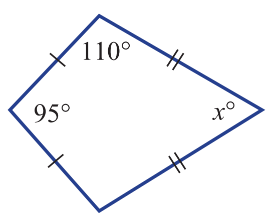

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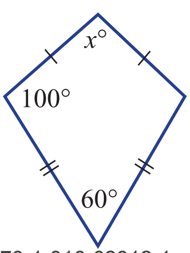

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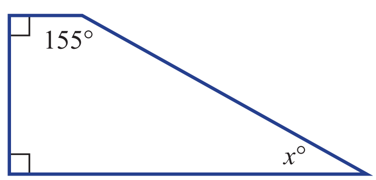

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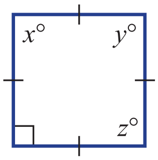
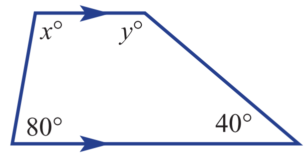
g



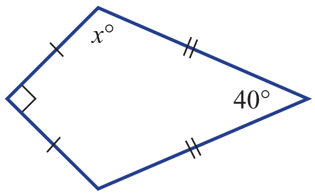
h



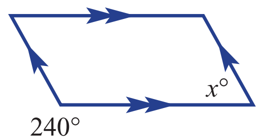
Problem-solving and Reasoning

6 Find the value of the pronumerals in these shapes.

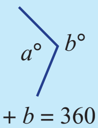
a



b

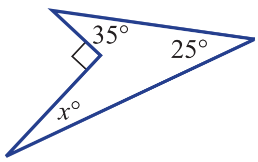


Angles in a revolution add to 360°.

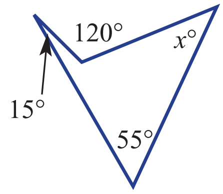


$a + b = 360$

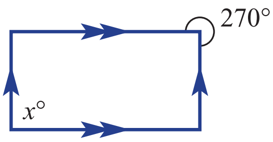
c



d



e



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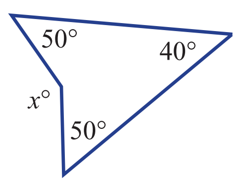


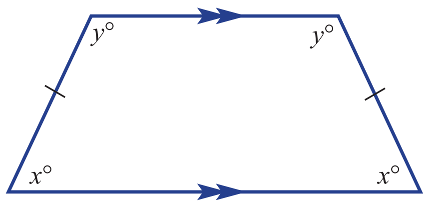
7 This shape is called an isosceles trapezium.

a Why do you think it is called an isosceles trapezium?

b **i** If $x = 60$, find the value of y.

ii If $y = 140$, find the value of x.

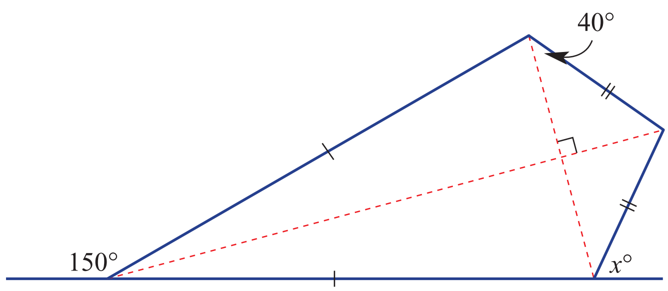
c List the properties of an isosceles trapezium.



8 The floor of a building is in the shape of a kite. Some angles are given in the diagram.

a Draw a copy of just the kite shape, including the diagonals.

b Find the angle that the right-hand wall makes with the ground ($x°$).



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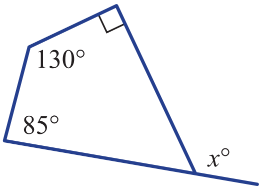
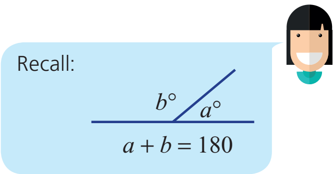
6C

9 These quadrilaterals also include exterior angles.
Find the value of x.

a





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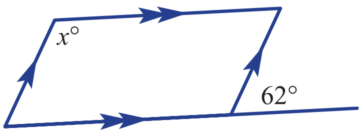
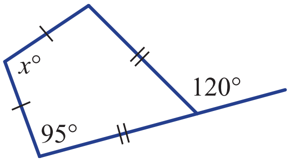


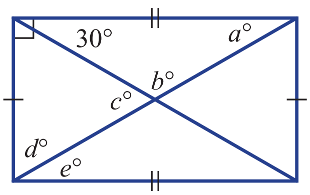
c



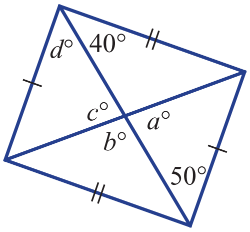
Enrichment: Quadrilaterals and triangles

10 The following shapes combine quadrilaterals with triangles. Find the values of the pronumerals.

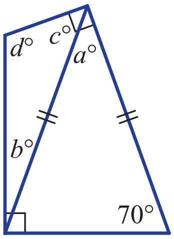
a



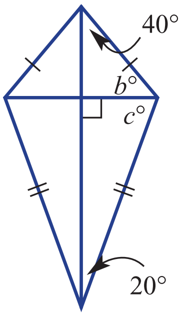
b



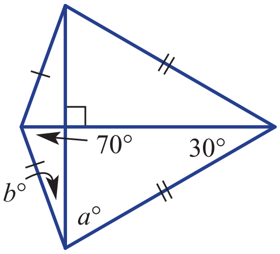
c



d



e



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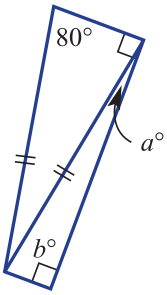

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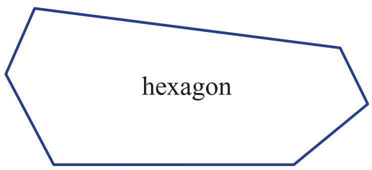
6D Polygons

Stage
5.2
5.2◊
5.1
4

A closed shape with straight sides is called a polygon. Like triangles and quadrilaterals (which are both polygons), they each have a special angle sum.



The Pentagon building in Washington, D.C.

▶ Let's start: Remember the names

From previous years you should remember some of the names for polygons. See if you can remember them by completing this table.

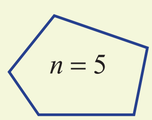
Number of sides	Name
3	
4	
5	
6	
7	heptagon

Number of sides	Name
8	
9	
10	
11	undecagon
12	

Key ideas

Drilling for Gold 6D1

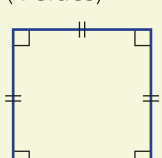
- A **polygon** is a shape with straight sides.
  - They are named by their number of sides.
- The sum of internal angles (S) of a polygon is given by the rule:
  $S = 180° \times (n - 2)$
  where n is the number of sides



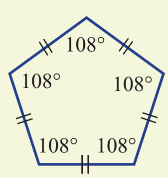
$S = 180° \times (n - 2)$
$S = 180° \times (5 - 2)$
$= 180° \times 3$
$= 540°$

- A **regular polygon** has equal angles and sides of equal length.

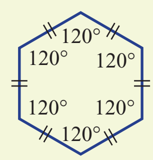
regular quadrilateral (square) (4 sides)



regular pentagon (5 sides)



regular hexagon (6 sides)



Polygon A two-dimensional shape where three or more straight lines are joined together to form a closed figure

Regular polygon A polygon with all sides of equal length and all angles equal

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Exercise 6D

Understanding

1 How many sides do these shapes have?

a quadrilateral **b** octagon **c** decagon **d** heptagon
e nonagon **f** hexagon **g** pentagon **h** dodecagon

2 Use the angle sum rule, $S = 180° \times (n - 2)$, to find the angle sum of these polygons.

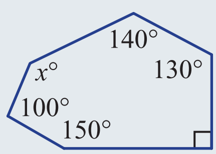
a pentagon ($n = 5$) **b** hexagon ($n = 6$) **c** heptagon ($n = 7$)
d octagon ($n = 8$) **e** nonagon ($n = 9$) **f** decagon ($n = 10$)

3 What is always true about a polygon that is regular?

Fluency

Example 8 Finding and using the angle sum of a polygon

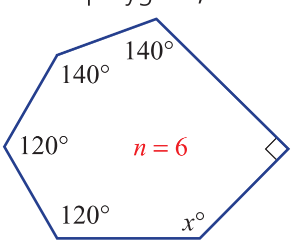
For this polygon, find the angle sum and then the value of x.



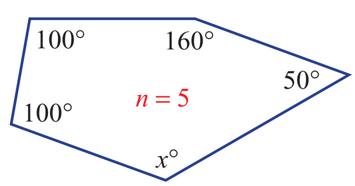
Solution	**Explanation**
$S = 180° \times (n - 2)$ $= 180° \times (6 - 2)$ $= 720°$	Use the angle sum rule first, with $n = 6$ because there are 6 sides. Find the angle sum.
$x + 100 + 150 + 90 + 130 + 140 = 720$ $x + 610 = 720$ $x = 110$	Use the total angle sum to find the value of x. Solve for the value of x.

4 For these polygons, find the angle sum and then find the value of x.

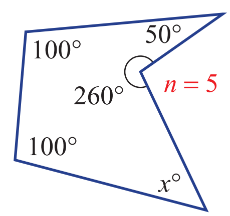
a

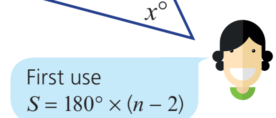


b

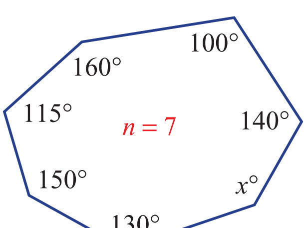


c

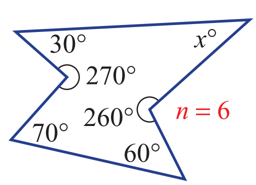




d



e



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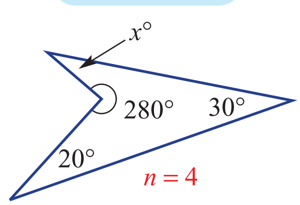

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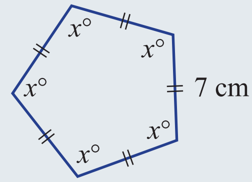
Example 9 Working with regular polygons

Shown here is a regular pentagon with straight edge side lengths of 7 cm.

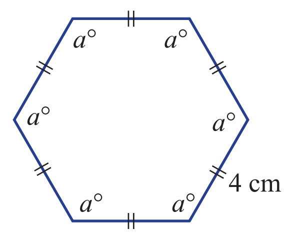
a Find the perimeter of the pentagon.
b Find the total internal angle sum (S).
c Find the size of each interior angle $x°$.

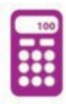


Solution	Explanation
a 35 cm	There are five sides of length 7 cm each.
b $S = 180° \times (n - 2)$ $= 180° \times (5 - 2)$ $= 180° \times 3$ $= 540°$	Write the general rule for the sum of internal angles for a polygon. $n = 5$ because there are five sides. Simplify and evaluate.
c $540° \div 5 = 108°$ $\therefore x = 108$	There are five equally sized angles since it is a regular pentagon.

5 Shown here is a regular hexagon with straight edge side lengths of 4 cm.

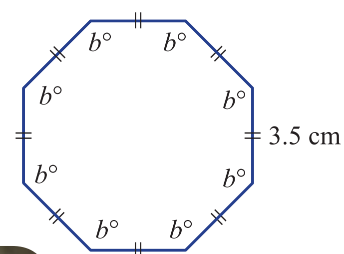
a Find the perimeter of the hexagon.
b Find the total internal angle sum (S).
c Find the size of each interior angle $a°$.





6 Shown here is a regular octagon with straight edge side lengths of 3.5 cm.

a Find the perimeter of the octagon.
b Find the total internal angle sum (S).
c Find the size of each interior angle $b°$.



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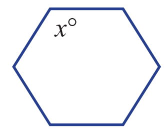
6D

Problem-solving and Reasoning

7 The cross-section of a pencil is a regular hexagon.
   a Find the sum of the interior angle.
   b Find the interior angle ($x°$).

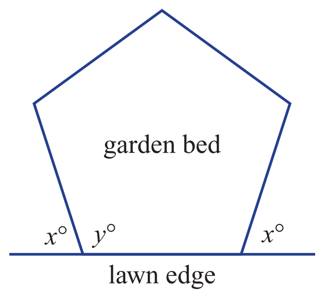


8 Find the total internal angle sum for a polygon with:
   a 11 sides
   b 20 sides

Remember:
$S = 180° \times (n - 2)$

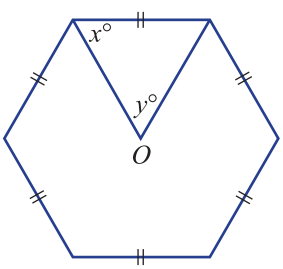
9 Find the size of a single interior angle for a regular polygon with:
   a 10 sides
   b 25 sides

10 A garden bed is to be designed in the shape of a regular pentagon and sits adjacent to a lawn edge, as shown.
   a Find the interior angle for each corner ($y°$).
   b Find the angle the lawn edge makes with the garden bed ($x°$).

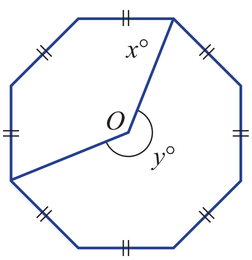


11 For these diagrams, find the values of the unknowns. The shapes are regular.

a



b



The point O is the centre.

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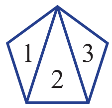
Enrichment: Develop the angle sum rule

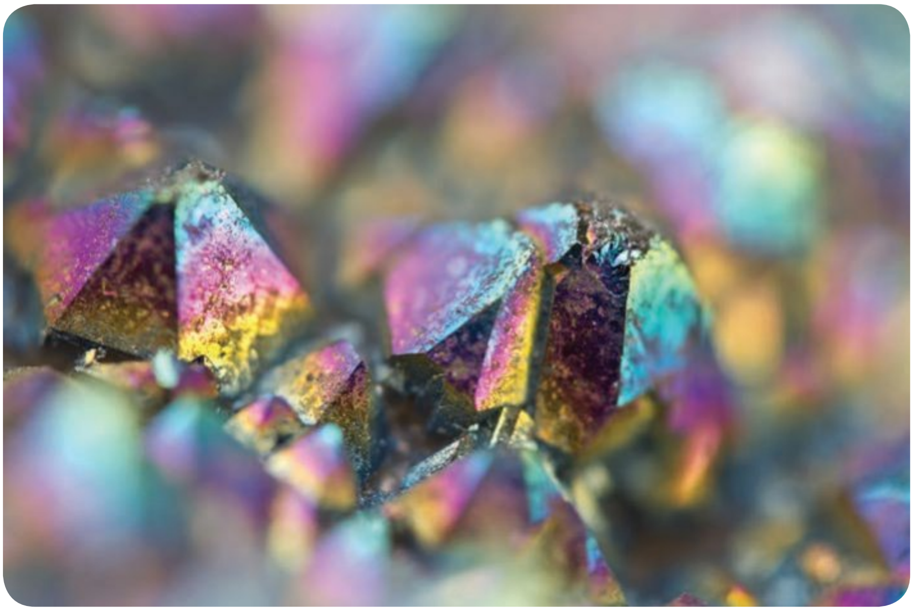
12 a Copy and complete this table. For the diagram, use diagonals to divide the shape into triangles, as shown for the pentagon.

Regular polygon	Number of sides	Diagram	Number of triangles	Interior angle sum (S)	Single interior angle (A)
triangle					
quadrilateral					
pentagon	5		3	$3 \times 180^\circ = 540^\circ$	$540^\circ \div 5 = 108^\circ$
hexagon					
…					
n-gon	n				

b Complete these sentences by writing the rule.

i For a polygon with n sides, the interior angle sum, S, is given by $S =$ _____.

ii For a regular polygon with n sides, a single interior angle, A, is given by $A =$ _____.



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Keeping in touch with numeracy

Non-calculator

1 A fair 6-sided die is numbered from 1 to 6. What is the probability of rolling a 3 or a 4?

3 Reema has $5 more than Peter. In total they have $25.
How much money does Reema have?

5 To evaluate $10 - 3 \times 2$, which operation must you do first?

7 A map has a scale factor of 1 cm : 10 km.
The distance from Penrith to Sydney on the map is 5.7 cm. How far is it in kilometres?

9 Find the mean and range of the following set of numbers.
2, 2, 3, 3, 5

11 Lorna's alarm clock wakes her at 7:35 a.m. It takes her:
- 20 minutes to shower and dress
- 15 minutes to eat breakfast
- 10 minutes to make her lunch
- 5 minutes to brush her teeth.

At what time does Lorna leave for school?

Calculator

2 If you roll a die 225 times, how many times would you expect to roll a 3 or a 4?

4 Goran earns $70.35 more per week than Ahmed.
Ahmed earns $867.50 per week.
How much do they earn in total?

6 Petrol costs 114.6 cents per litre. Stuart filled his tank with 43.7 litres. How much did he pay for the petrol, in dollars and cents?

8 A picture with side lengths of 12.3 cm and 14.2 cm is enlarged by a scale factor of 4.
What are the new side lengths?

10 Find the average mass.
Taylor 45.5 kg
Miguel 57.25 kg
Lauren 46 kg
Trent 54.75 kg

12 Sydney is 2 hours behind Fiji. A plane leaves Sydney at 8 a.m. and flies to Fiji. The flight takes 4 hours and 30 minutes. What time is it in Fiji when the plane arrives?

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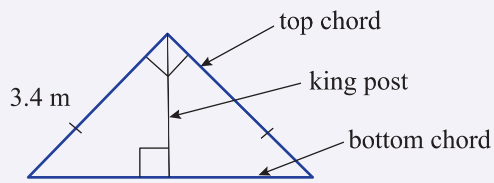
13 What is 10% of \$250?

14 Roberta borrows \$17 500 over 5 years to buy a car. The interest rate on the car loan is 16.5% per annum. If she makes no repayments, how much interest will Roberta pay in the first month?

15 In Pythagoras' theorem $c^2 = a^2 + b^2$, which letter represents the length of the hypotenuse?

16 A builder needs to check his measurements for a roof truss before buying the timber. He knows the measurements for the two top chords but not the length of the bottom chord or the king post.



a What length timber will he need for the bottom chord?
b What length timber will he need for the king post?

17 Estimate (to the nearest dollar) the total cost of the following food items.
Bread \$2.75
Litre of milk \$1.49
Dozen eggs \$4.25
Butter \$2.92

18 Which of these is the best buy?
125 g coffee for \$2.80
200 g coffee for \$4.55
500 g coffee for \$10.95
750 g coffee for \$16.50

19 Write 0.02 as a percentage.

20 Danny's car loan repayments are 17% of his weekly wage.
Danny earns \$935 per week. How much is his car repayment?

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6E Congruent triangles

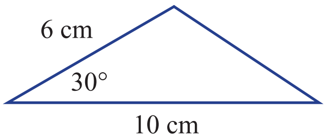
Stage: 5.2, 5.2◊, 5.1, 4

When building structures, it is important to know whether or not objects are identical. The mathematical word used to describe identical objects is **congruence**.

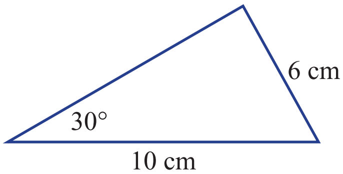
Let's start: Why are AAA and ASS not tests for congruence?

If everybody in your class drew a triangle with sides 5 cm, 6 cm and 7 cm, they would all be congruent. Therefore, SSS is a test for congruence.

- Draw a triangle with angles of 50°, 60° and 70°.
- Is it possible to draw a larger or smaller triangle with these angles?
- Is it possible to draw this triangle in more than one way?

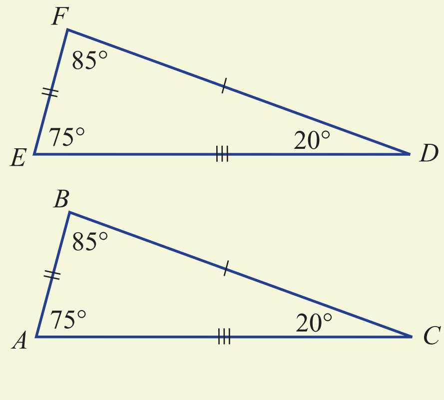


- Draw two different triangles that have these measurements.



Key ideas

- Two triangles are said to be **congruent** when they are exactly the same *size* and *shape*. Corresponding sides and angles will be of the same size, as shown in these triangles.



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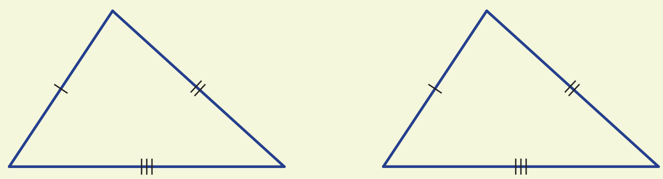
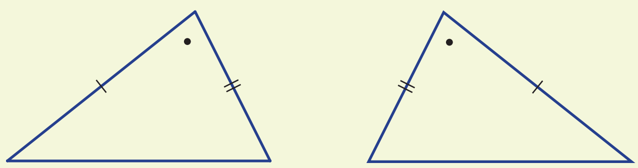
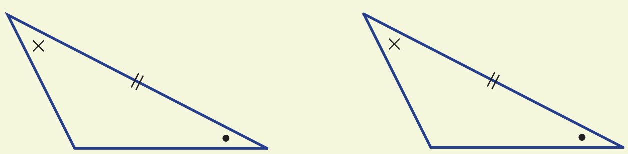
- If triangle ABC is congruent to triangle EFD, we write $\triangle ABC \equiv \triangle EFD$.
  - This is called a congruence statement.
  - Letters are written in matching order.
- Two triangles can be tested for congruence by considering the following necessary conditions.
  1. Three pairs of sides are equal (SSS).



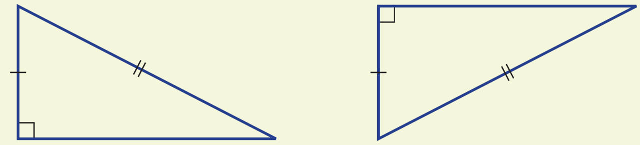
  2. Two corresponding sides and the angle between them are equal (SAS).



  3. Two angles and any corresponding side are equal (AAS).



  4. A right angle, the hypotenuse and one other pair of corresponding sides are equal (RHS).

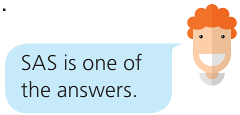


Exercise 6E

Understanding

Drilling for Gold 6E1

1. True or false?
   a. SSA is a test for the congruence of triangles.
   b. AAA is a test for the congruence of triangles.
   c. Two congruent triangles are the same shape and size.
   d. If $\triangle ABC \equiv \triangle DEF$, then triangle ABC is congruent to triangle DEF.
2. Write the four tests for congruence, using their abbreviated names.



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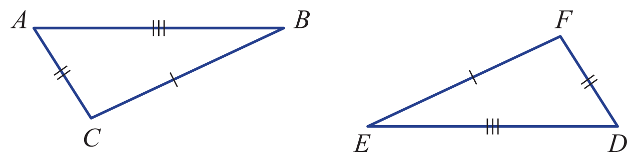
6E

3 Here is a pair of congruent triangles.

a Which point on $\triangle DEF$ corresponds to point B on $\triangle ABC$?

b Which side on $\triangle ABC$ corresponds to side DF on $\triangle DEF$?

c Which angle on $\triangle DEF$ corresponds to $\angle BAC$ on $\triangle ABC$?

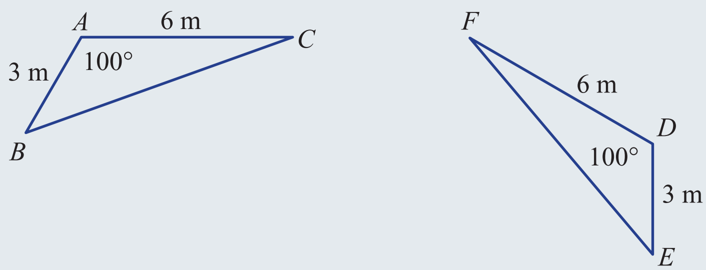


Fluency

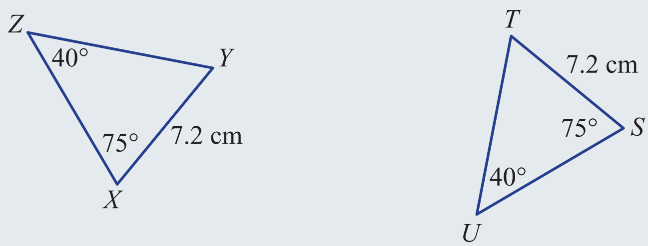
Example 10 Choosing a test for congruence

Write a congruence statement and the test to prove congruence for these pairs of triangles.

a



b



Solution	Explanation
a $\triangle ABC \equiv \triangle DEF$ (SAS)	Write letters in corresponding (matching) order. Two pairs of sides are equal, as well as the angle between.
b $\triangle XYZ \equiv \triangle STU$ (AAS)	X matches S, Y matches T and Z matches U. Two angles and one pair of matching sides are equal.

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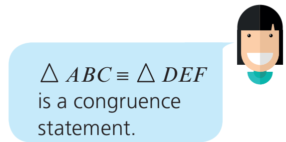
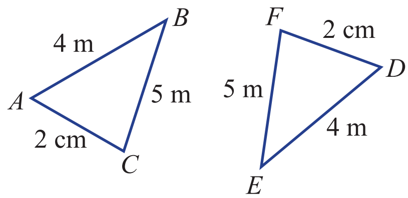
4 Write a congruence statement and the test to prove congruence for these pairs of triangles. Choose from SSS, SAS, AAS or RHS.

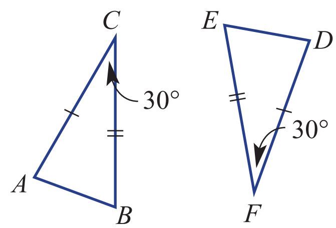


a



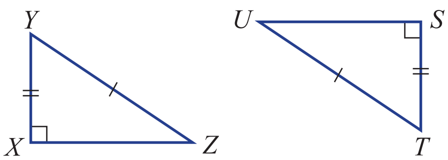
$\triangle ABC \equiv \triangle DEF$ (_____)

b



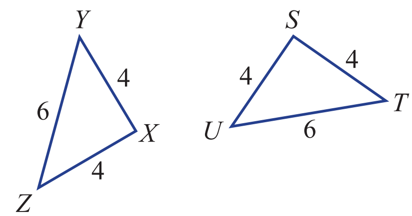
$\triangle ABC \equiv \triangle DEF$ (_____)

c



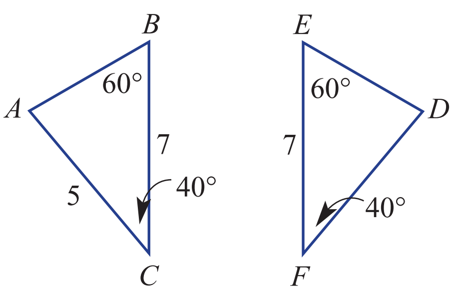
$\triangle XYZ \equiv \triangle STU$ (_____)

d



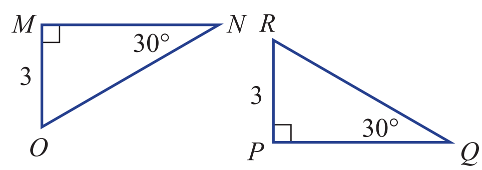
$\triangle XYZ \equiv \triangle STU$ (_____)

e



$\triangle$_____ $\equiv$ $\triangle$_____ (_____)

f

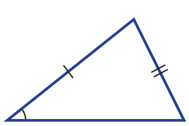


$\triangle$_____ $\equiv$ $\triangle$_____ (_____)

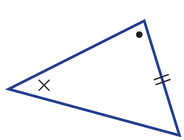
Problem-solving and Reasoning

5 Identify the pairs of congruent triangles from those below.

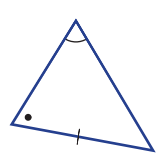
A



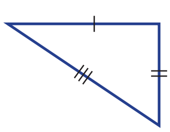
B



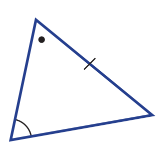
C



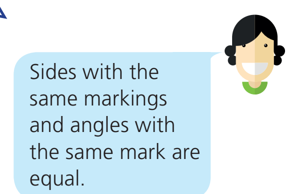
D



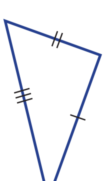
E



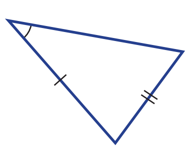
F



G



H



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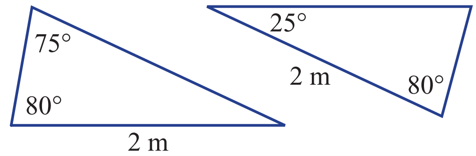
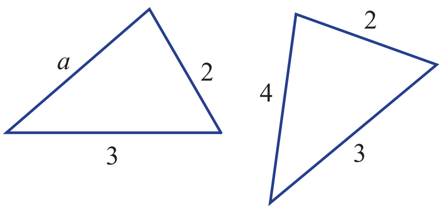
6E

6 Two triangular windows have the given dimensions.
   a Find the missing angle in each triangle.
   b Are the two triangles congruent? Give a reason.

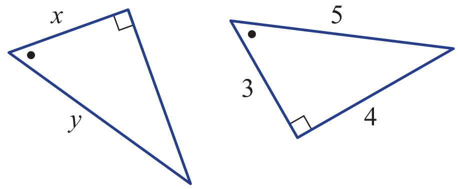


7 For the pairs of congruent triangles, find the values of the pronumerals.

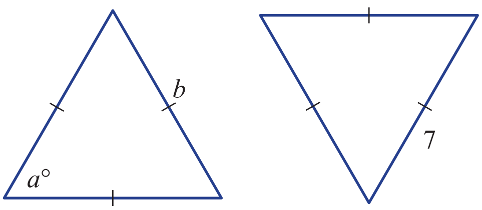
a

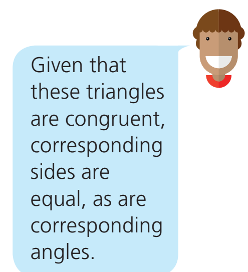


b



c





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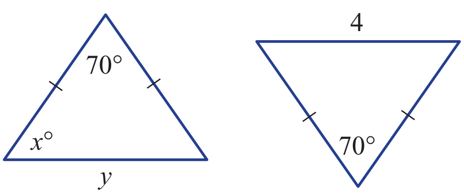
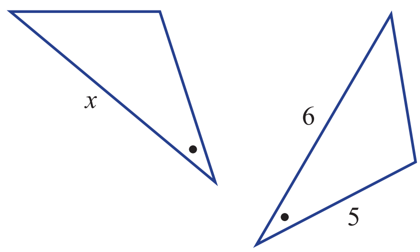


e



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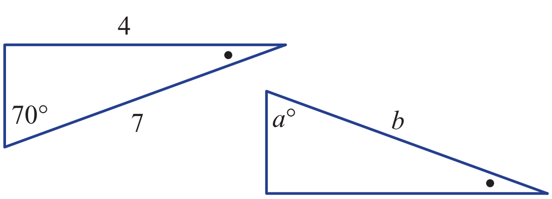
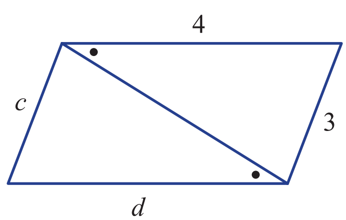
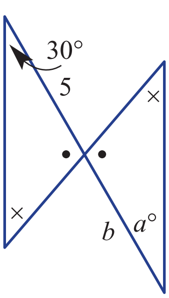


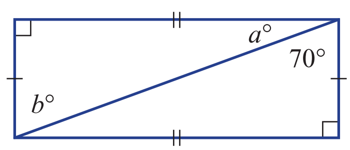
g



h



i



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8 A new garden design includes two triangular lawn areas, as shown.

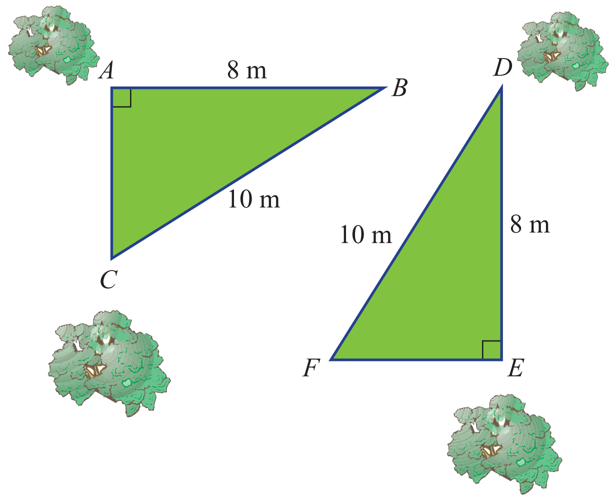
a Which test shows that the two triangular lawn areas are congruent?

b If the length of AC is 6 m, find the length of EF.

c If the angle $ABC = 37°$, find these angles.

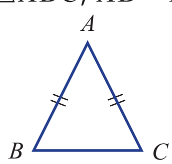
i $\angle EDF$

ii $\angle DFE$

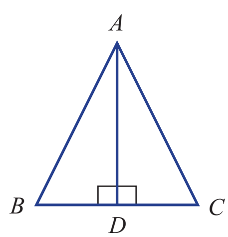


Enrichment: More challenging proofs

9 In $\triangle ABC$, $AB = AC$.



Complete the following proof to show that $\angle B = \angle C$.



Construct $AD \perp BC$.
In $\triangle ABD$ and $\triangle ACD$,
$AB =$ ____ (given) (H)
$\angle ADB = \angle$____ $= 90°$ ($AD \perp BC$) (R)
AD is common (S).

$\therefore \triangle$ ____ $\equiv \triangle$ ____ (RHS)
$\therefore \angle B = \angle C$ (matching angles in congruent triangles)

This proves that when two sides of an isosceles triangle are equal, then the angles opposite the equal sides are also equal.

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6F Similarity and scale drawings

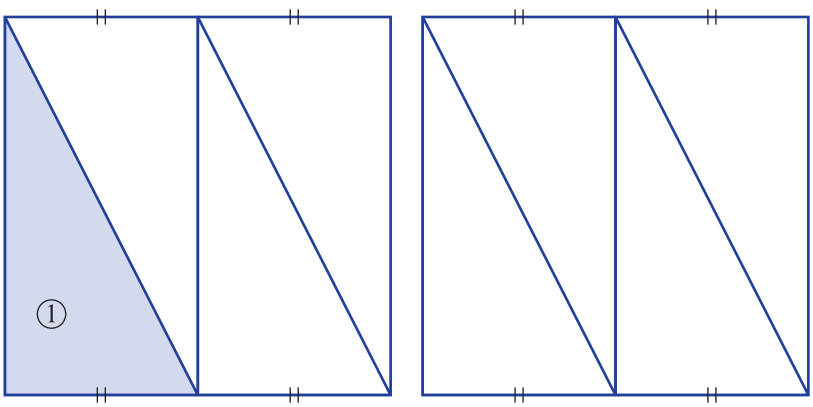
Stage

5.2
5.2◊
5.1
4

When two objects are similar, they are the same shape but of different size. For example, a computer image reproduced on a large screen will show all aspects of the image in the same way except in size. The computer image and screen image are said to be similar figures.

Let's start: Same shape, different size

Drilling for Gold 6F1



- Start with two congruent squares. Cut them into eight congruent triangles, as shown in the diagrams above.
- Use four triangles to make a larger triangle that is similar to triangle 1.
- Try adding a fifth triangle to make another triangle similar to triangle 1.
- It is possible to make a triangle similar to triangle 1 using nine triangles? Work with a partner to create the triangle.
- Can you make a similar triangle using 16 triangles?

Key ideas

Drilling for Gold 6F2

- Two figures are **similar** if one can be enlarged to be congruent to the other.
  - Matching angles are equal.
  - Pairs of matching sides are in the same proportion or ratio.

Similar figures
Figures of the same shape but not the same size

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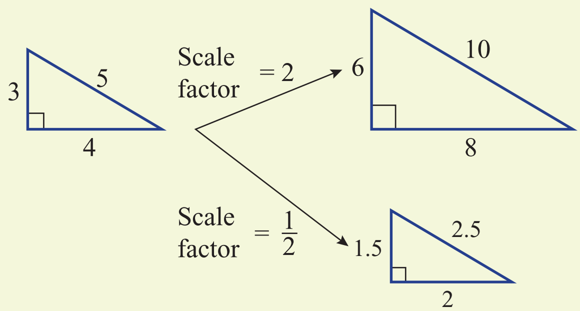
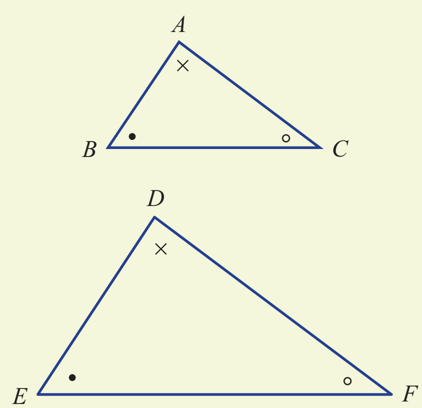
- The **scale factor** $= \dfrac{\text{image length}}{\text{original length}}$

- **Enlargement** is a transformation that involves the increase or decrease in size of an object.
  - The 'shape' of the object is unchanged.
  - Enlargement is done by multiplying all dimensions of a shape by a **scale factor**.



- If $\triangle ABC$ is similar to $\triangle DEF$, then we write $\triangle ABC \,|||\, \triangle DEF$.



- A scale drawing, such as the plan of a house, is smaller to the actual house. The scale factor is usually written as a ratio, such as $1:100$.
  $1:100$ means that 1 mm on the plan represents 100 mm of the house.

Enlargement A transformation that changes the size of figure without changing its shape

Scale factor The number by which you multiply each side length to enlarge or reduce the size of a shape

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Exercise 6F

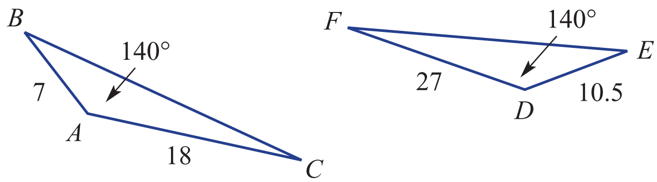
Understanding

1 Two figures are said to be ____________ when they are the same shape, but not necessarily the same size.

2 Use the words *factor*, *equal*, *ratio* and *scale* to fill in the blanks.
In similar figures:

- **a** Matching angles are ____________.
- **b** Matching sides are in ____________.
- **c** The _____ ______ can be used to make the sides of the image longer or shorter than the original.

3 Consider this pair of triangles.

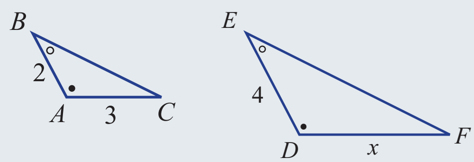


- **a** Work out $\frac{DE}{AB}$.
- **b** Work out $\frac{DF}{AC}$. What do you notice?
- **c** What is the scale factor?

Fluency

Example 11 Using similarity to find unknown values

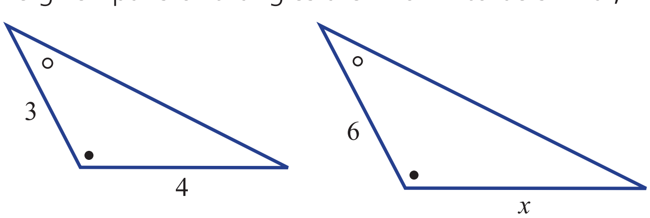
If the given triangles are known to be similar, find the value of x.



Solution	**Explanation**
$\text{Scale factor} = \frac{DE}{AB} = \frac{4}{2} = 2$	First, find the scale factor using a pair of corresponding sides. Divide the larger number by the smaller number.
$x = 3 \times 2$ $\therefore\ x = 6$	Multiply the corresponding length of the smaller triangle using the scale factor.

4 If the given pairs of triangles are known to be similar, find the value of x.

a



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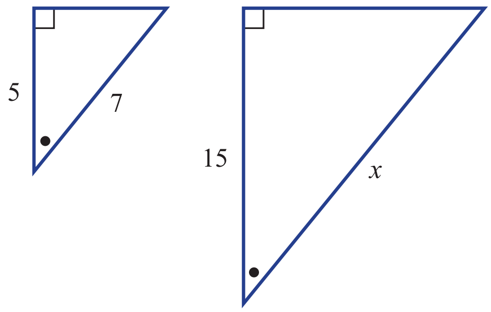
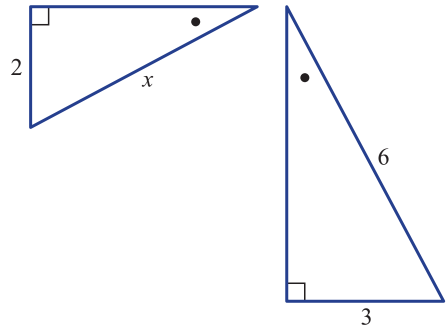
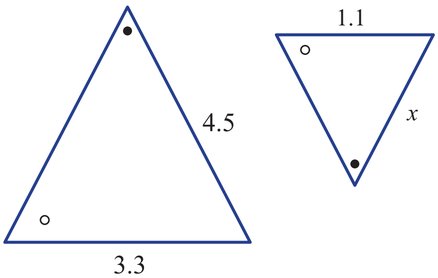
b



c



d



For parts **c** and **d**, use division to find x.

Example 12 Measuring to find actual lengths

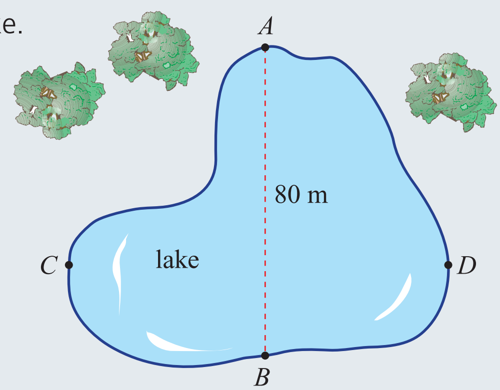
The given diagram is a simple map of a park lake.

a Use a ruler to measure the distance across the lake (AB). (Answer in cm.)

b Find the scale factor between the map and ground distance.

c Use a ruler to measure the map distance across the lake (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the lake (CD). (Answer in m.)



Solution	Explanation
a 4 cm	Check with your ruler.
b $\frac{8000}{4} = 2000$	Using the same units, divide the real distance (80 m = 8000 cm) by the measured distance (4 cm).
c 5 cm	Check with your ruler.
d $5 \times 2000 = 10\,000$ cm $= 100$ m	Multiply the measured distance by the scale factor and convert to metres by dividing by 100.

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6F

Drilling for Gold 6F3

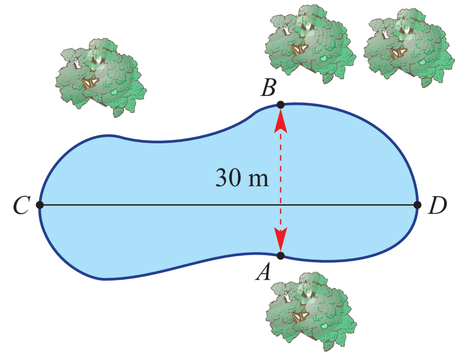
5 The given diagram is a map of a park lake.

a Use a ruler to measure the distance across the lake (AB). (Answer in cm.)

b Find the scale factor between the map and ground distance.

c Use a ruler to find the map distance across the lake (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the lake (CD). (Answer in m.)



6 The given diagram is a map of a children's play area.

a Use a ruler to measure the distance across the children's play area (AB). (Answer in cm.)

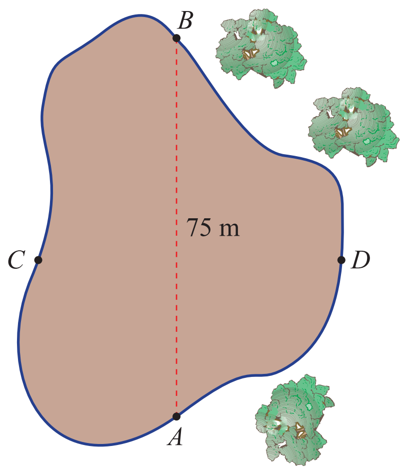
b Find the scale factor between the map and ground distance.

c Use a ruler to find the map distance across the children's play area (CD). (Answer in cm.)

d Use your scale factor to find the real distance across the children's play area (CD). (Answer in m.)

Use the measured distance AB and the actual distance AB to find the scale factor.

Skillsheet 6A



Problem-solving and Reasoning

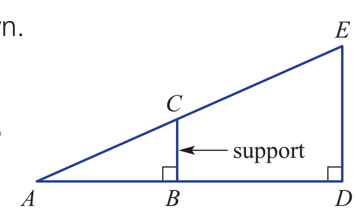
7 A ski ramp has a vertical support, as shown.

a List the two triangles that are similar.

b Why are the two triangles similar?

c If $AB = 4$ m and $AD = 10$ m, find the scale factor.

d If $BC = 1.5$ m, find the height of DE.



List triangles like this: $\triangle STU$.

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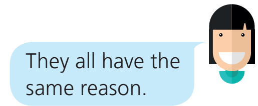
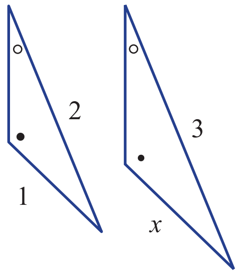
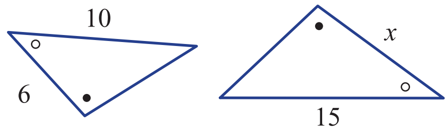
8 The pairs of triangles are similar. Determine the value of x in each case.



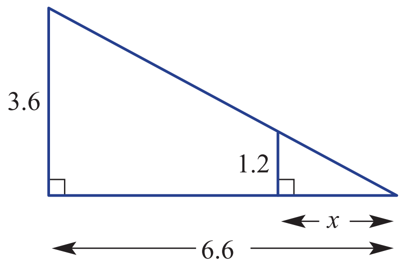
a



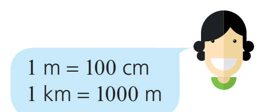
b



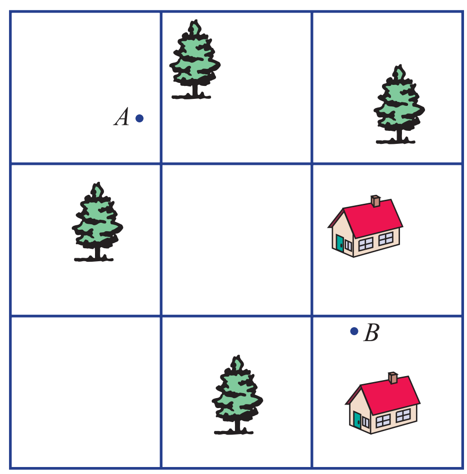
c



9 The given map has a scale factor of $50\,000$ (ratio $1:50\,000$).



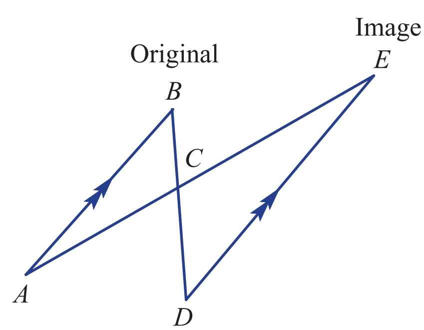
- **a** How far on the ground, in km, is represented by these map distances?
  - **i** 2 cm **ii** 6 cm
- **b** How far on the map, in cm, is represented by these ground distances?
  - **i** 5 km **ii** 0.5 km
- **c** What is the actual ground distance, in km, between the two points A and B?



Enrichment: Triangles in parallel lines

10 In the given diagram, AB is parallel to DE.

- **a** List the three pairs of angles that are equal and give reasons.
- **b** If $AB = 8$ cm and $DE = 12$ cm, find:
  - **i** the scale factor
  - **ii** DC if $BC = 4$ cm
  - **iii** AC if $EC = 9$ cm



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6G Applying similar triangles

Stage: 5.2, 5.2◊, 5.1, 4

Once it is established that two triangles are similar, the scale factor between side lengths can be used to find unknown side lengths.

Similar triangles have many applications in the real world. One application is finding the distance across a deep canyon from one side of the canyon.

The distance across this canyon can be found without having to actually measure it physically.

▶ Let's start: One-two-three similar triangles

Drilling for Gold 6G1

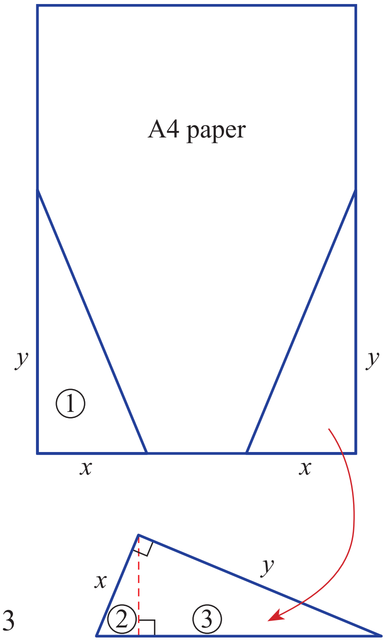
This mathematical experiment requires careful drawing and accurate measurements.

Start with an A4 sheet of paper.

- Use a ruler to carefully draw two congruent triangles, as shown in the diagram at right.

Choose your own values for x and y. Make sure they are different to those chosen by the student next to you.

- Cut your triangles off the sheet.
- Using a protractor, carefully measure the angles in triangle 1 and write them on the shape.
- Carefully measure the sides of triangle 1, in millimetres. Write them on the shape.
- Take the other triangle and rotate it so that the hypotenuse is horizontal, as in the diagram at right. Carefully fold it along a line that is perpendicular to the hypotenuse.
- Cut along that line to divide your triangle into triangle 2 and triangle 3, where triangle 2 is the smaller triangle.
- Measure all the sides and angles of triangle 2 and triangle 3 and write them on the shapes.
- If you have done this carefully, triangles 1, 2 and 3 will contain the same three angles (and therefore they are similar). If this did not happen, check your measurements.
- Use your side lengths to find an approximation for the scale factor between triangle 2 and triangle 3.
- Use your side lengths to find an approximation for the scale factor between triangle 2 and triangle 1.



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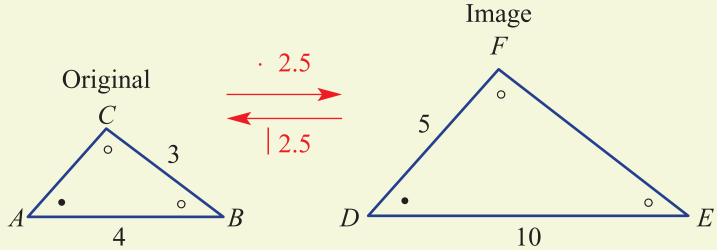
Key ideas

- For similar triangles, the ratio of the corresponding side lengths written as a single number is called the scale factor.

  Scale factor = $\dfrac{\text{image length}}{\text{original length}}$



- Once the scale factor is known, it can be used to find unknown side lengths.

  Scale factor $= \dfrac{DE}{AB} = \dfrac{10}{4} = \dfrac{5}{2} = 2.5$

  ∴ All sides must be multiplied by 2.5.

  $\therefore\ EF = 3 \times 2.5 = 7.5$

AC can be calculated by working backwards (i.e. by dividing by 2.5)

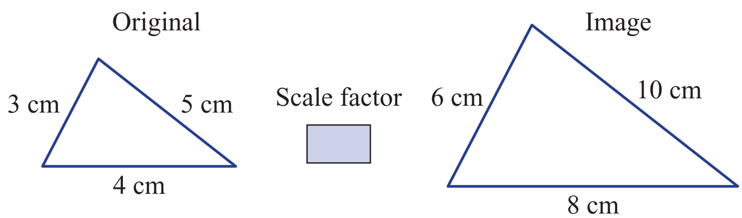
$\therefore\ AC = 5 \div 2.5$
$= 2$

Exercise 6G

Understanding

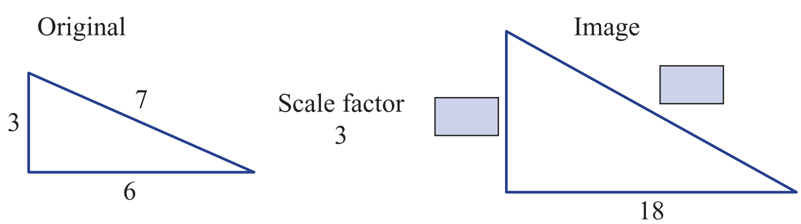
1 Write down the missing numbers that should go in the blue rectangles for these similar triangles.

a

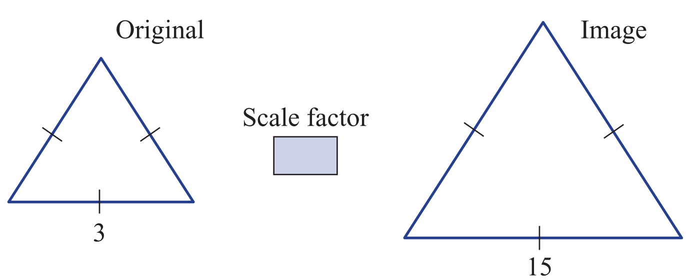


Compare AB and AD for the scale factor.

b



c



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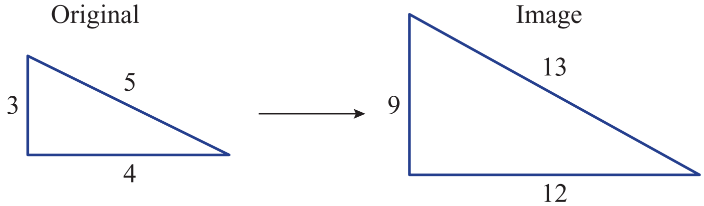
6G

2 For the similar triangles shown:

a Has the same scale factor been applied to all sides of the original triangle?

b Are the triangles similar?



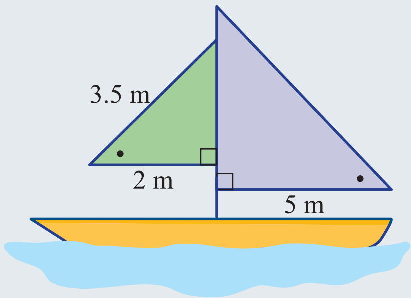
Fluency

Example 13 Applying similar triangles

A home-made raft consists of two similar sails with measurements and angles as shown in this diagram.

a Find the scale factor for the side lengths of the sails.

b Find the length of the longest side of the large sail.

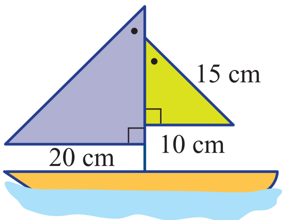


Solution	Explanation
a Scale factor $= \frac{5}{2} = 2.5$	Choose two corresponding sides with known lengths and divide the larger by the smaller.
b Longest side $= 3.5 \times 2.5$ $= 8.75$ m	Multiply the corresponding side on the smaller triangle by the scale factor.

3 A toy yacht consists of two similar sails with measurements and angles as shown in this diagram.

a Find the scale factor for the side lengths of the sails.

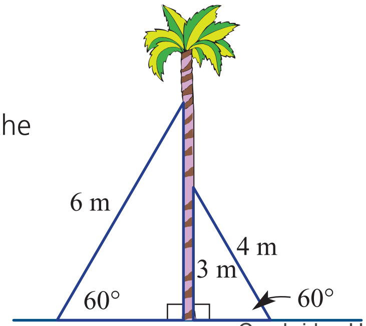
b Find the length of the longest side of the large sail.



4 A tall palm tree is held in place with two cables of length 6 m and 4 m, which create similar triangles, as shown.

a Find the scale factor for the side lengths of the cables.

b Find the height of the point above the ground where the longer cable is attached to the palm tree.



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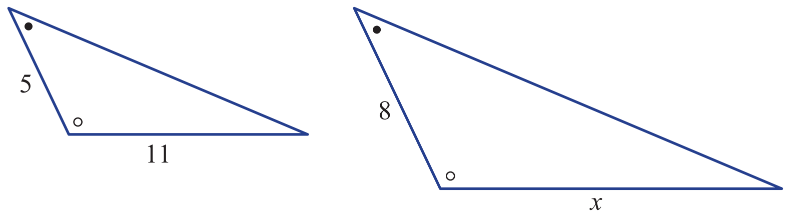
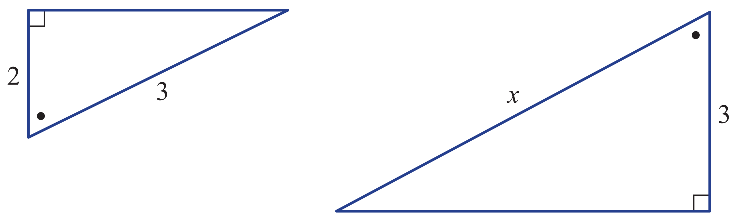
5 These pairs of triangles are known to be similar. By finding the scale factor, find the value of x.

a



b



Problem-solving and Reasoning

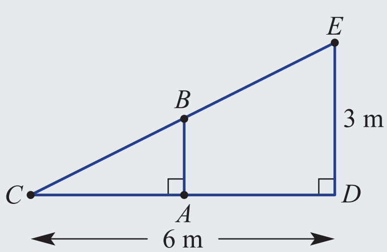
Drilling for Gold 6G2

Example 14 Working with combined triangles

A ramp is supported by a vertical stud, AB, where A is at the centre of CD. It is known that $CD = 6$ m and that the ramp is 3 m high.

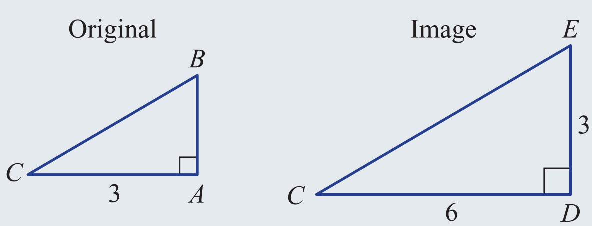
a Draw the triangle separately and include all known lengths.

b Use the scale factor to find the length of the stud AB.



Solution

a



b Scale factor $= \frac{CD}{CA} = \frac{6}{3} = 2$

$\therefore AB = 3 \div 2$
$= 1.5$ m

Explanation

CA is half of CD.

$CD = 6$ m and $CA = 3$ m.

Divide the larger side length, DE, by the scale factor.

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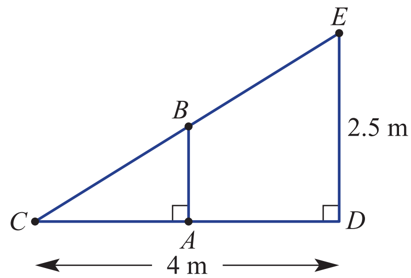
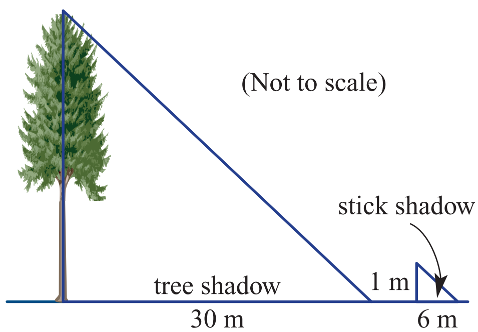
6G

6 A ramp is supported by a vertical stud, AB, where A is at the centre of CD. It is known that $CD = 4$ m and that the ramp is 2.5 m high.
Find the length of the stud AB.



7 A 1 m vertical stick and a tree cast their shadows at a particular time in the day. The shadow lengths are shown in this diagram.
  a Find the scale factor for the side lengths of the triangles.
  b Find the height of the tree.

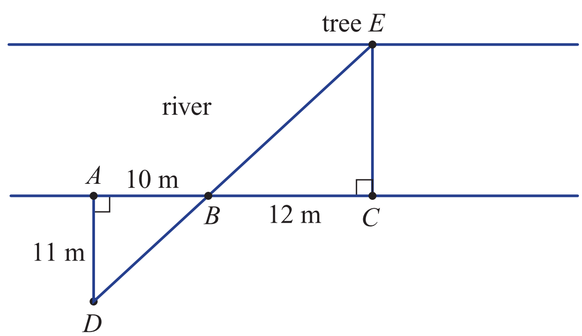


At the same time of day, the angle that the light makes with the ground will be the same.

8 From a place on the river (C), a tree (E) is spotted on the opposite bank. The distances between selected trees A, B, C and D are measured as shown.
  a Find the scale factor.
  b Find the width of the river.

Drilling for Gold 6G3

AB corresponds to CB and AD corresponds to CE.



9 At a particular time of day, Aaron casts a shadow 1.3 m long, whereas Theo, who is 1.75 m tall, casts a shadow 1.2 m long. Find the height of Aaron, to 2 decimal places.

Draw a diagram to find the scale factor.

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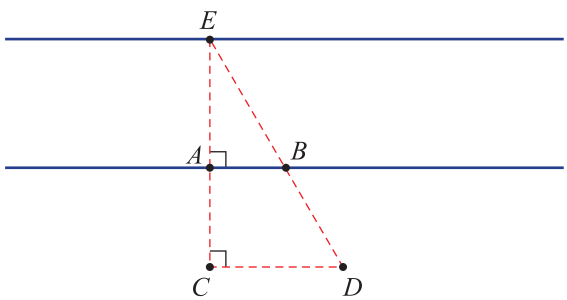
10 Try this activity with a partner but ensure that at least one person knows their height.
   a Go out into the Sun and measure the length of each person's shadow.
   b Use these measurements plus the known height of one person to find the height of the other person.

Enrichment: Gorge challenge

11 Mandy sets up a series of rocks alongside a straight section of a deep gorge. She places rocks A, B, C and D as shown. Rock E sits naturally on the other side of the gorge. She then measures the following distances.
   - $AB = 10$ m
   - $AC = 10$ m
   - $CD = 15$ m



   a What is the scale factor?
   b Use trial and error to find the distance across the gorge from rocks A to E.
   c Can you find instead the length AE by setting up an equation?

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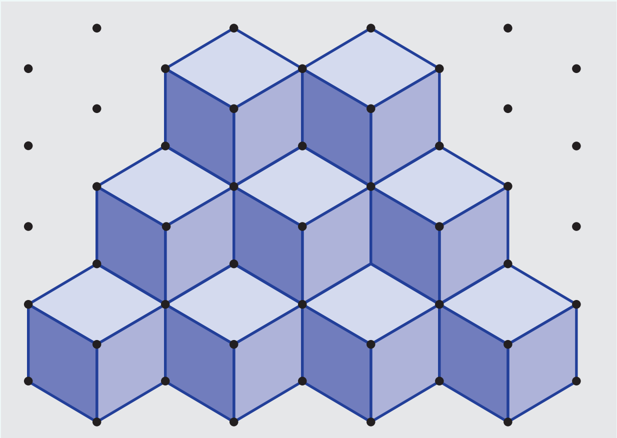
Maths@home

Worksheet 6A

Tiling patterns and optical illusions

In this practical activity you will be using a variety of different shapes to design beautiful geometric tiling patterns that could be used on a floor or wall of a building, using tools such as the one available via www.cambridge.edu.au/goldnsw10weblinks



You will also investigate the amazing tessellations and optical illusions that have been drawn by artists such as M.C. Escher. You might also create some of your own!

Download the worksheet for the activity.

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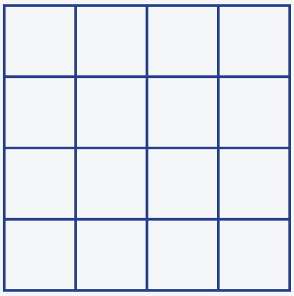
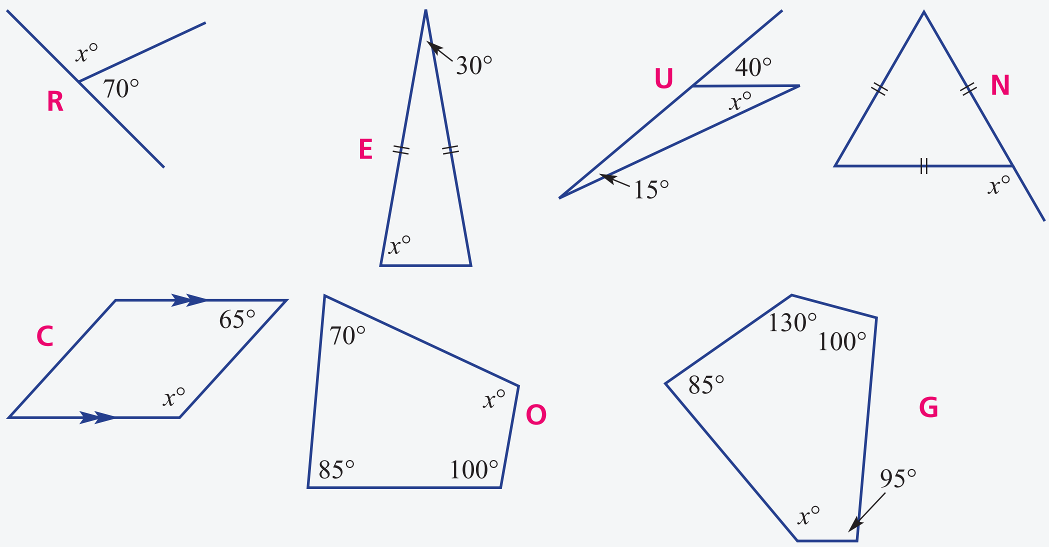
Puzzles and games

1 How many squares can you see in this diagram?



2 'I think of this when I look in the mirror.' Find the value of x in each diagram, then match the letters beside the diagrams to the answers below.

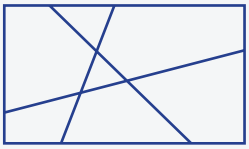


___ 115 ___ 105 ___ 120 ___ 130 ___ 110 ___ 25 ___ 75 ___ 120 ___ 115 ___ 75

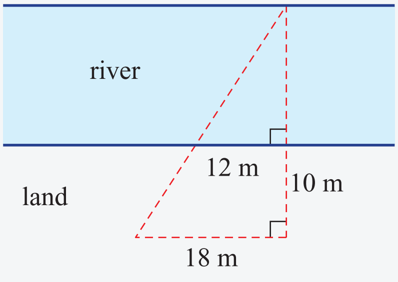
3 This rectangle is subdivided by three straight lines.

a How many regions are formed?

b What is the maximum number of regions formed if four lines are used instead of three?



4 Find the distance across the river.



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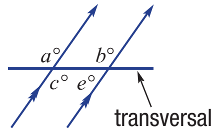
Chapter summary

Drilling for Gold 6R1 6R2 6R3

Properties of geometrical figures

Parallel lines



$a = b$ corresponding
$b = c$ alternate
$c + e = 180$ cointerior
$a = c$ vertically opposite

Congruent triangles

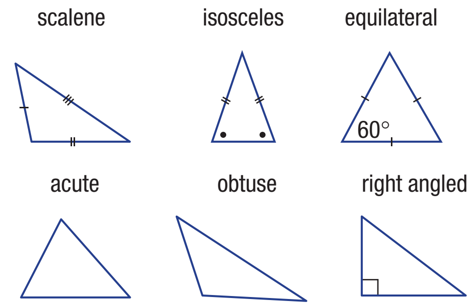
Identical in size and shape

Tests: SSS, SAS, AAS, RHS

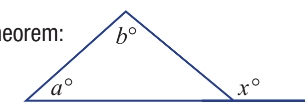
A congruence statement:
$\triangle ABC \equiv \triangle DEF$

Triangles

Angle sum = 180°



Exterior angle theorem:
$x = a + b$



Quadrilaterals

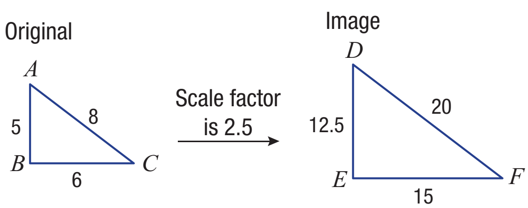
Angle sum = 360°

Special types include:
- parallelogram
- square
- rectangle
- rhombus
- kite
- trapezium

Similar triangles

Identical in shape and sides are in proportion

A similarity statement:
$\triangle ABC \,|||\, \triangle DEF$



$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

Polygons

Interior angle sum $S = 180° \times (n - 2)$

Regular polygons have:
- equal side lengths
- equal angles

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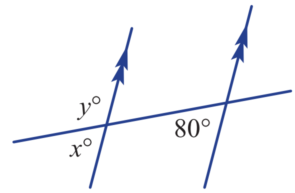
Chapter review

T Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.

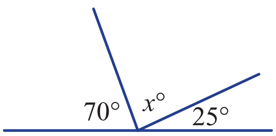
Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

1 The values of x and y in the diagram opposite are:
A 100, 100 **B** 80, 100 **C** 80, 80
D 60, 120 **E** 80, 60

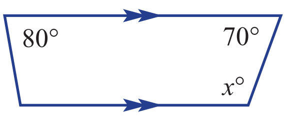


2 The unknown value x in this diagram is:
A 85 **B** 105 **C** 75
D 80 **E** 90



3 A triangle has one angle of 60° and another angle of 70°. The third angle is:
A 60° **B** 30° **C** 40° **D** 50° **E** 70°

4 The value of x in this quadrilateral is:
A 130 **B** 90 **C** 100 **D** 120 **E** 110



5 The sum of the interior angles of a hexagon is:
A 180° **B** 900° **C** 360° **D** 540° **E** 720°

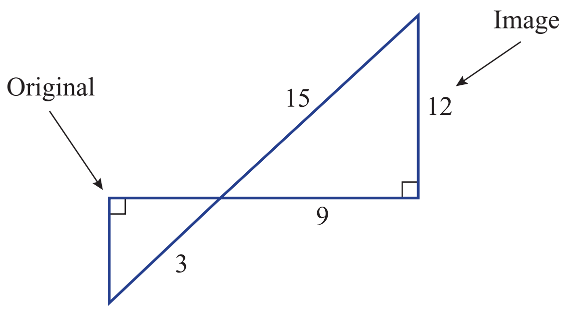
6 Which abbreviated reason is not relevant for proving congruent triangles?
A AAS **B** RHS **C** SSS **D** AAA **E** SAS

7 Two similar triangles have a length ratio of 2 : 3. If one side on the smaller triangle is 5 cm, the length of the corresponding side on the larger triangle is:
A 3 cm **B** 7.5 cm **C** 9 cm **D** 8 cm **E** 6 cm

8 A stick of length 2 m and a tree of unknown height stand vertically in the Sun. The shadow lengths cast by each are 1.5 m and 30 m, respectively. The height of the tree is:
A 40 m **B** 30 m **C** 15 m **D** 20 m **E** 60 m

9 How many sides in a dodecagon?
A 5 **B** 10 **C** 11 **D** 12 **E** 20

10 What is the scale factor of the similar triangles shown?
A $\frac{1}{5}$ **B** $\frac{1}{4}$ **C** 3 **D** 4 **E** 5



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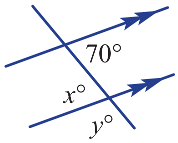
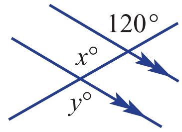
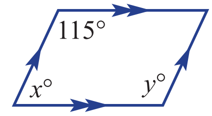
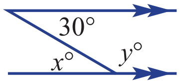
Short-answer questions

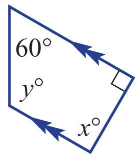
1 Find the value of x and y in these diagrams.

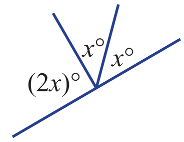
a


b


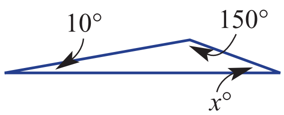
c


d


e


f


2 Find the value of x in these triangles.

a


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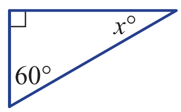

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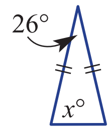

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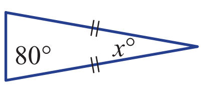

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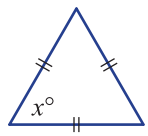

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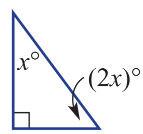
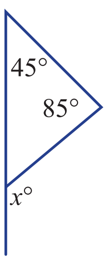


g


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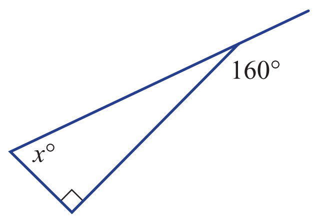
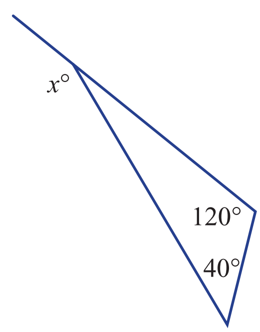


i


3 Which of the special quadrilaterals have:

a two pairs of parallel lines?
b opposite angles equal?
c one pair of equal angles?
d diagonals intersecting at right angles?

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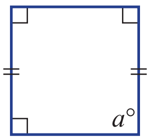
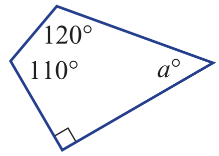
4 Find the values of the pronumerals in these quadrilaterals.

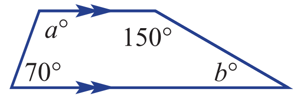
a



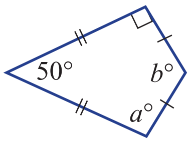
b



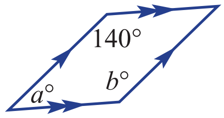
c



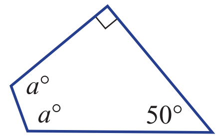
d



e

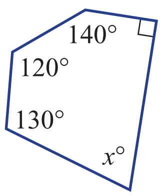


f

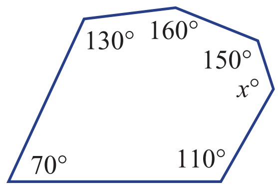


5 Find the value of x by first finding the angle sum. Use $S = 180° \times (n - 2)$.

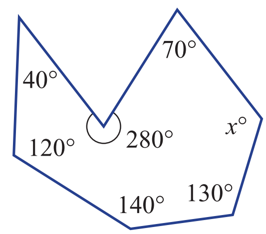
a



b



c



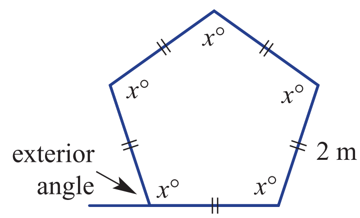
6 Shown here is an example of a regular pentagon with side lengths of 2 m. Find:

a the perimeter of the pentagon

b the total interior angle sum (S)

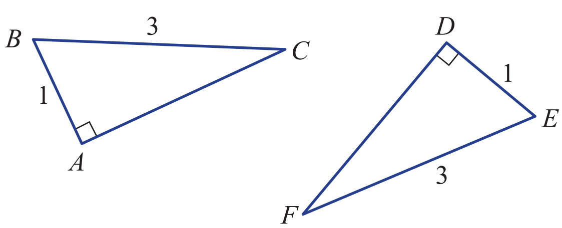
c the size of each interior angle ($x°$)

d the size of each exterior angle

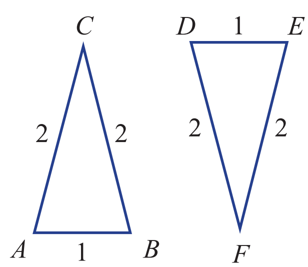


7 Which test could be used to prove that the following pairs of triangles are congruent?

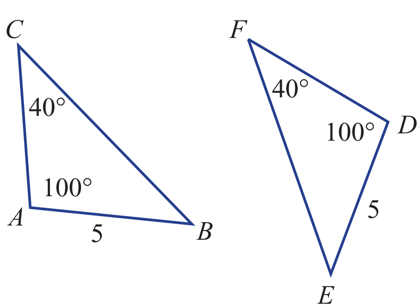
a



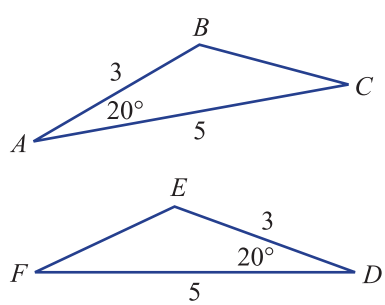
b



c



d



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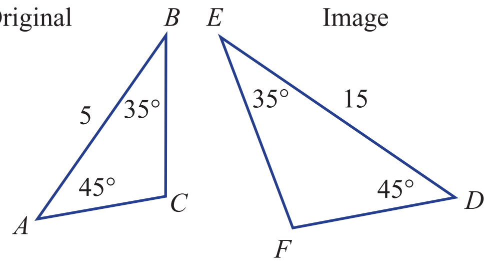
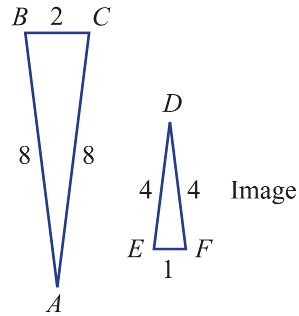
Chapter review

8 What is the scale factor?

a



b



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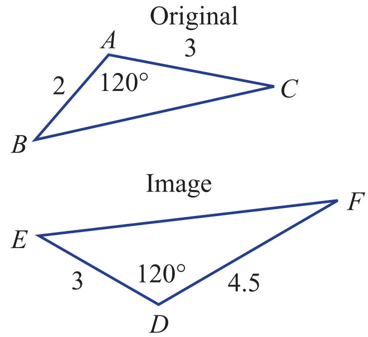
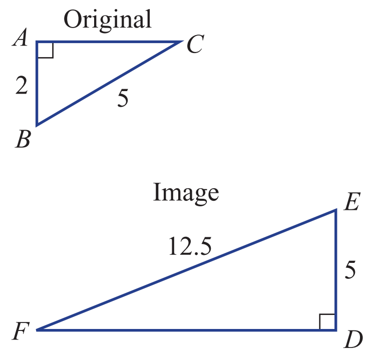


d

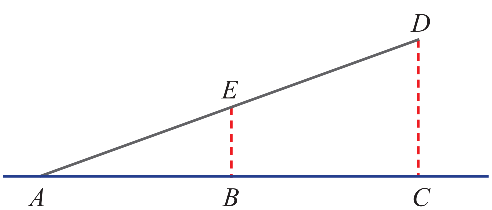


9 A skateboard ramp is supported by two vertical struts, BE (2 m) and CD (5 m).

a Name two triangles that are similar, using the letters A, B, C, D and E.

b Find the scale factor from the smallest to the largest triangle.

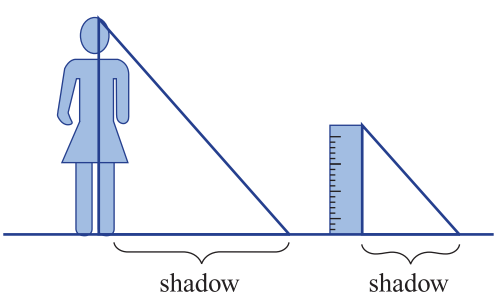
c If the length AB is 3 m, find the horizontal length of the ramp AC.



10 The shadow of Ming standing in the Sun is 1.5 m long, and the shadow of a 30 cm ruler is 24 cm.

a Find the scale factor between the two similar triangles.

b How tall is Ming?



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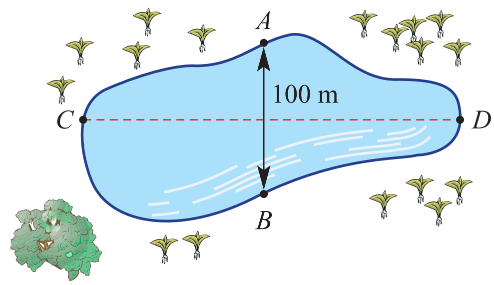
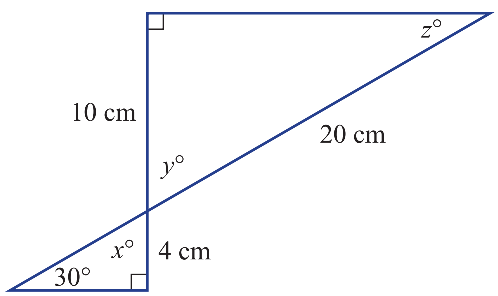
11 The given diagram is a simple map of a swamp in bushland.



a Use a ruler to measure the distance across the swamp (AB). (Answer in cm.)
b Find the scale factor between the map and ground distance.
c Use a ruler to find the map distance across the swamp (CD). (Answer in cm.)
d Use your scale factor to find the real distance across the lake (CD). (Answer in m.)

Extended-response questions

1 A company logo contains two similar triangles, as shown.
a Write down the value of x, y and z.
b Write down the scale factor for length.
c Find the length of the longest side of the smaller triangle.



2 A toy model of a car is 8 cm long and the actual car is 5 m long.
a Write down the length ratio of the toy car to the actual car.
b If the toy car is 4.5 cm wide, what is the width of the actual car?

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Chapter 7

Right-angled triangles

What you will learn

Strand: Measurement and Geometry

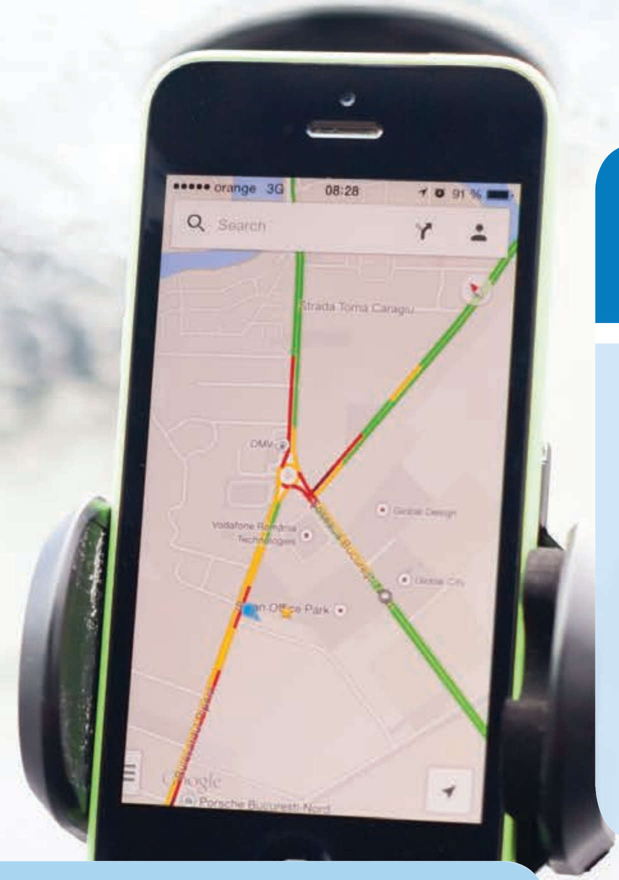
Substrand: RIGHT-ANGLED TRIANGLES

In this chapter, you will learn to:

- apply trigonometry to solve problems in diagrams
- apply trigonometry to solve problems involving angles of elevation and depression.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw10

Pythagoras and position location



Pythagoras was born in 582 BC in Greece. His theorem is still used today in measurement and design.

Trigonometry is the branch of mathematics that relates to right-angled triangles, linking the ratio of sides to angles. Trigonometry has many applications and is used widely. If you have a GPS (global positioning system) in your family's car or if you have a map function on your mobile phone, these use trigonometry to help locate your position.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO:* www.cambridge.edu.au/goldnsw10

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 Round the following decimals, correct to 2 decimal places.

a 15.84312 **b** 164.8731 **c** 0.86602 **d** 0.57735
e 0.173648 **f** 0.7071 **g** 12.99038 **h** 14.301

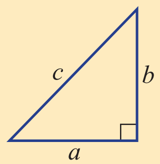
2 Find the value of each of the following.

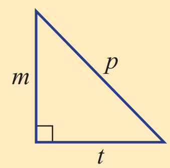
a 5^2 **b** 6.8^2 **c** 19^2
d $9^2 + 12^2$ **e** $3.1^2 + 5.8^2$ **f** $41^2 - 40^2$

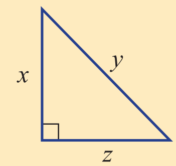
3 Find the following, correct to 1 decimal place.

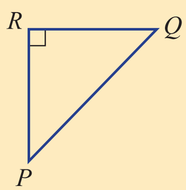
a $\sqrt{8}$ **b** $\sqrt{7}$ **c** $\sqrt{15}$ **d** $\sqrt{10}$
e $\sqrt{12.9}$ **f** $\sqrt{8.915}$ **g** $\sqrt{3.8}$ **h** $\sqrt{200}$

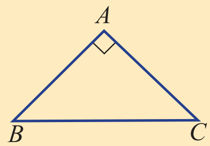
4 Which letter(s) represent the hypotenuse (i.e. the side opposite the right angle) on the following triangles.

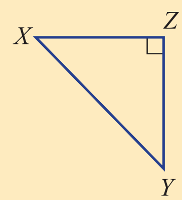
a


b


c


d


e


f


5 Solve for x.

a $3x = 9$ **b** $4x = 16$ **c** $5x = 60$ **d** $\frac{x}{5} = 7$ **e** $\frac{x}{12} = 9$ **f** $\frac{2x}{3} = 6$

6 Solve for m.

a $7m = 25.55$ **b** $9m = 10.8$ **c** $1.5m = 6.6$
d $\frac{m}{1.3} = 4$ **e** $\frac{m}{5.89} = 3.2$ **f** $\frac{m}{5.4} = 1.06$

7 Solve each of the following equations, correct to 1 decimal place.

a $\frac{3}{x} = 5$ **b** $\frac{4}{x} = 17$ **c** $\frac{32}{x} = 15$
d $\frac{3.8}{x} = 9.2$ **e** $\frac{15}{x} = 6.2$ **f** $\frac{29.3}{x} = 3.2$

8 If x is a positive integer, solve:

a $x^2 = 16$ **b** $x^2 = 400$ **c** $x^2 = 5^2 + 12^2$ **d** $x^2 + 3^2 = 5^2$

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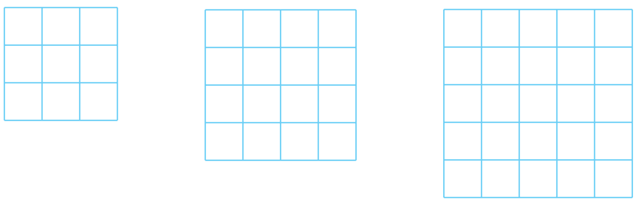
7A Reviewing Pythagoras' theorem

Stage
5.2
5.2◊
5.1
4

The ancient Egyptians knew of the relationship between the numbers 3, 4 and 5 and how they could be used to form a right-angled triangle.

Greek philosopher and mathematician Pythagoras expanded on this idea and the theorem $c^2 = a^2 + b^2$, which we use today, is named after him.

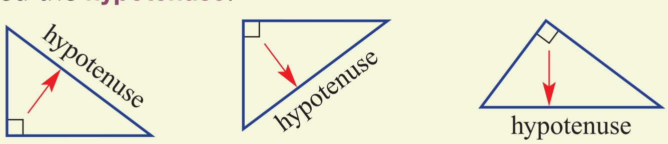
▶ Let's start: Three, four and five



- On square grid paper, construct three squares, as shown above.
- Cut them out and place the middle-sized square on top of the largest square. Then cut the smallest square into nine smaller squares and also place them on the largest square to finish covering it.
- What does this show about the numbers 3, 4 and 5?

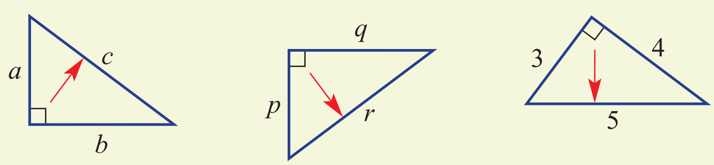
Key ideas

- A right-angled triangle has its longest side opposite the right angle. This side is called the **hypotenuse**.
  For example:



Hypotenuse The longest side of a right-angled triangle (i.e. the side opposite the right angle)

- **Pythagoras' theorem** states:
  The square of the hypotenuse is equal to the sum of the squares on the other two sides.
  For example:



$c^2 = a^2 + b^2$ ↑ hypotenuse

$r^2 = p^2 + q^2$ ↑ hypotenuse

$5^2 = 3^2 + 4^2$ ↑ hypotenuse

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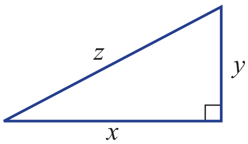
Exercise 7A

Understanding

1 Write the relationship between the sides of these triangles.

a



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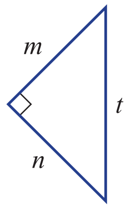

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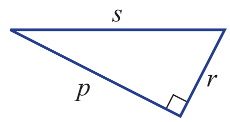
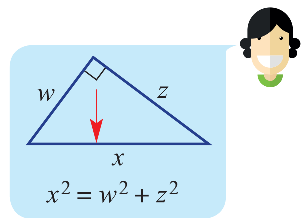




2 Copy and complete.

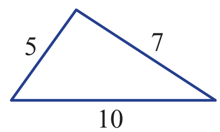
Q: Is this a right-angled triangle?

A: $10^2 = \square$ and $5^2 + 7^2 = \square + \square$

$= \square$

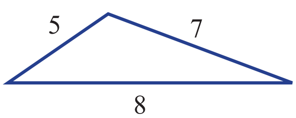
$100 \neq 74$

∴ This is not a right-angled triangle.



3 Using the method in Question **2**, decide if these triangles are right angled or not.

a



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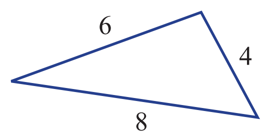

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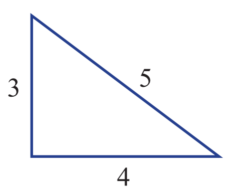

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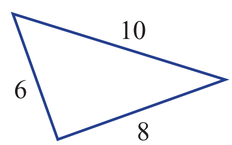

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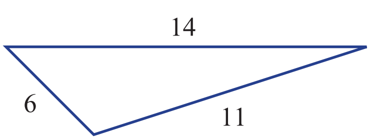

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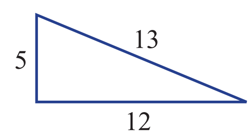
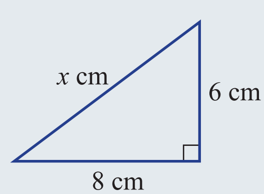


Fluency

Example 1 Finding the length of the hypotenuse

Find the length of the hypotenuse of the triangle shown.



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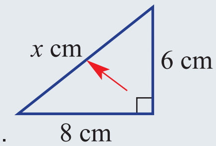
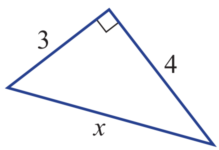
Solution	Explanation
$x^2 = 6^2 + 8^2$ $= 36 + 64$ $= 100$	Write the relationship for the given triangle, using Pythagoras' theorem.
$x = \sqrt{100}$ $= 10$	Take the square root to find x.
$\therefore$ Hypotenuse length = 10 cm.	Write your answer.



4 Find the length of the hypotenuse in these right-angled triangles.

a



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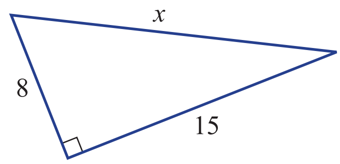

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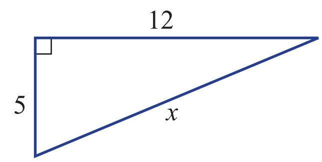

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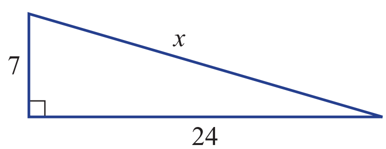

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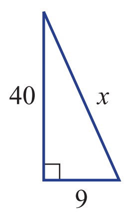

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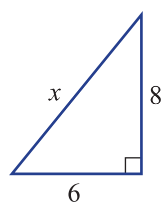
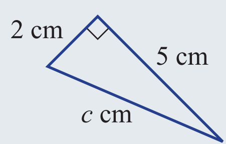
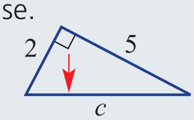


Example 2 Finding the length of the hypotenuse as a decimal

Find the length of the hypotenuse in this triangle, correct to 1 decimal place.



Solution	Explanation
$c^2 = 5^2 + 2^2$	Write the relationship for this triangle, where c is the length of the hypotenuse.
$= 25 + 4$ $= 29$	Simplify.
$c = \sqrt{29}$ $c = 5.38516\ldots$	Take the square root to find c.
$c = 5.4$	Round 5.3(8)516... to 1 decimal place.
$\therefore$ Hypotenuse length = 5.4 cm.	



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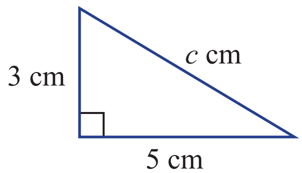
7A

5 Find the length of the hypotenuse in these triangles, correct to 1 decimal place.

a



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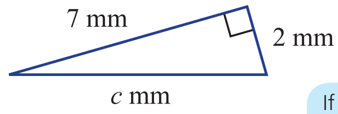

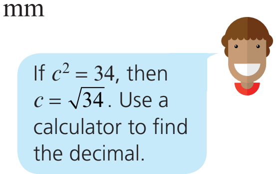

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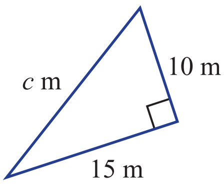

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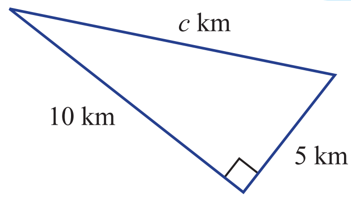

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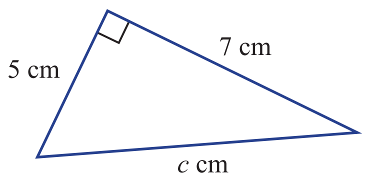

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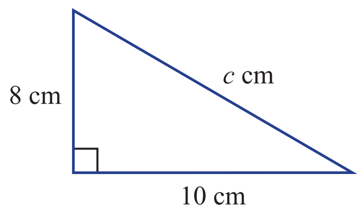
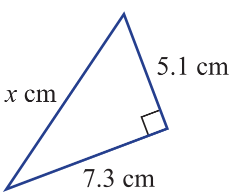


6 Find the value of the hypotenuse in these triangles, correct to 2 decimal places.

a



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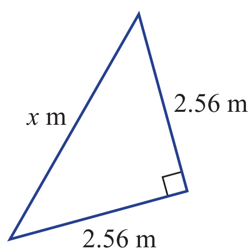

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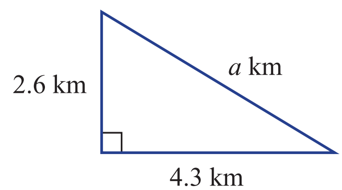
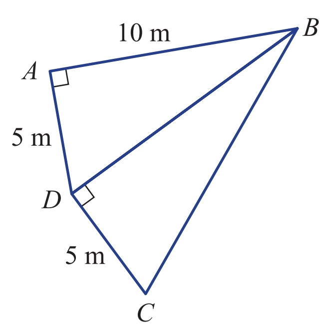


7 For the diagram, find the lengths of:

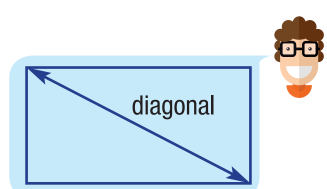
a BD, correct to 2 decimal places

b BC, correct to 1 decimal place



Problem-solving and Reasoning

8 A TV screen is 154 cm long and 96 cm high. Calculate the length of its diagonal, correct to 1 decimal place.



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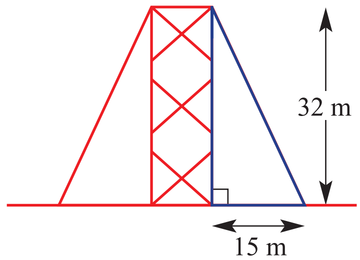
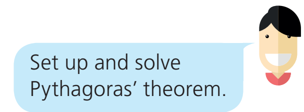
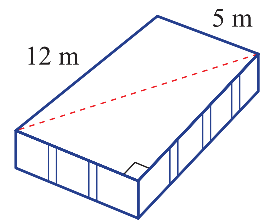
9 A 32 m tower is supported by cables from the top to a position on the ground 15 m from the base of the tower. Determine the length of each cable needed to support the tower, correct to 1 decimal place.

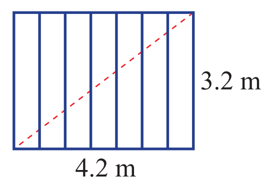




10 A builder uses Pythagoras' theorem to check the corners of his concrete slab. What will be the length of the diagonal if the opposite angle is 90°?

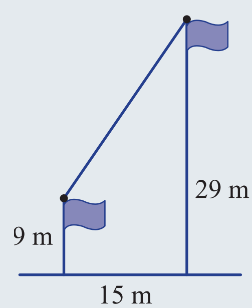


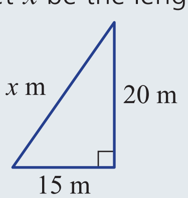
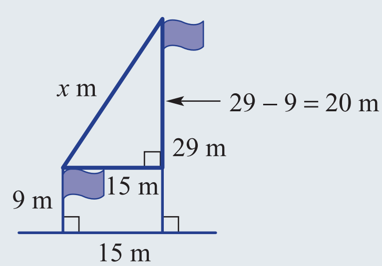
11 Find the length of the diagonal steel brace needed to support a gate of length 4.2 m and width 3.2 m, correct to 2 decimal places.



Example 3 Applying Pythagoras' theorem

Two flagpoles are 15 m apart and a rope links the tops of both poles. Find the length of the rope if one flagpole is 9 m high and the other is 29 m high.



Solution	Explanation
Let x be the length of rope. 	Locate and draw the right-angled triangle, showing all measurements. Introduce a pronumeral for the missing side. 
$x^2 = 15^2 + 20^2$	Write the relationship, using Pythagoras' theorem.
$= 225 + 400$ $= 625$	Simplify.
$x = \sqrt{625}$ $= 25$	Take the square root to find x.
The rope is 25 m long.	Answer the question.

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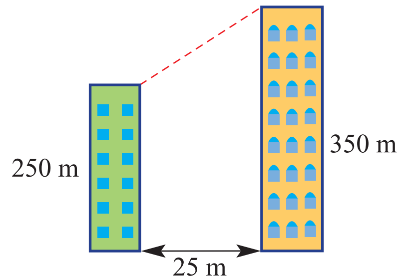
7A

12 Two skyscrapers are 25 m apart and a cable runs from the top of one building to the top of the other. One building is 350 m tall and the other is 250 m tall.

a Determine the difference in the heights of the buildings.

b Draw an appropriate right-angled triangle you could use to find the length of the cable.

c Find the length of the cable, correct to 2 decimal places.

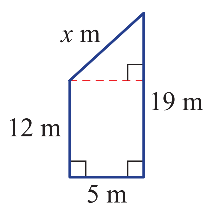


(Not to scale)

13 Find the value of x in each of the following, correct to 1 decimal place where necessary.

Label the two known lengths of each triangle first.

a



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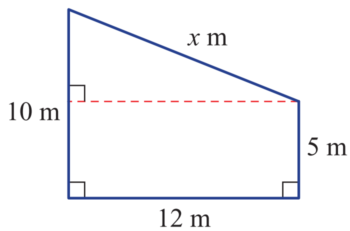

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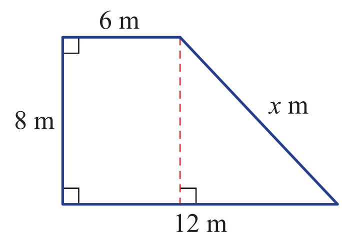
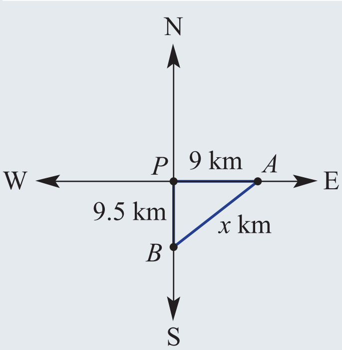
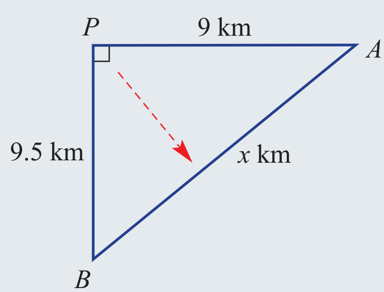
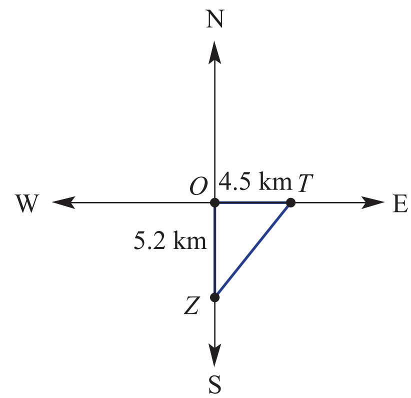


Example 4 Using direction with Pythagoras' theorem

Two hikers leave their camp (P) at the same time. One walks due east for 9 km; the other walks due south for 9.5 km. How far apart are the two hikers at this point? (Give your answer to 1 decimal place.)

Solution	Explanation
	Draw a diagram. Consider $\triangle PAB$. 
$\therefore x^2 = 9^2 + 9.5^2$ $= 171.25$	Write Pythagoras' theorem and evaluate.
$x = \sqrt{171.25}$ $= 13.086\ldots$	Take the square root to find x.
$= 13.1$ (to 1 decimal place)	Round to 1 decimal place.
$\therefore$ The hikers are 13.1 km apart.	Answer the question in words.

14 Travis (T) walks 4.5 km east while Zara (Z) walks 5.2 km south. How far from Travis is Zara? Answer to 1 decimal place.



15 Find the distance between Sui and Kevin if:

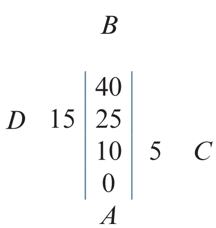
a Sui walks 6 km north from camp O and Kevin walks 8 km west from camp O.

b Sui walks 40 km east from point A and Kevin walks 9 km south from point A.

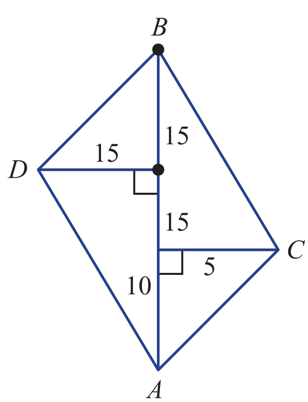
c Kevin walks 15 km north-west from O and Sui walks 8 km south-west also from O.

Enrichment: An offset survey

An offset survey measures distances perpendicular to the baseline offset. A notebook entry is made showing these distances and then perimeters and areas are calculated.



Notebook entry



Field diagram (Not to scale)

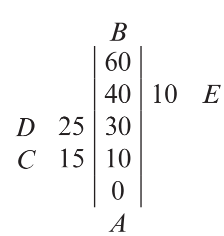
16 a Using the diagrams above, find these lengths, correct to 1 decimal place.

i AC **ii** BC **iii** DB **iv** AD

b Find the perimeter of the field $ACBD$, correct to the nearest metre.

c Find the area of the field.

17 Shown at right is a notebook entry. Draw the field diagram and find the perimeter of the field, to 1 decimal place.



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7B Finding the lengths of the shorter sides

Stage

5.2
5.2◊
5.1
4

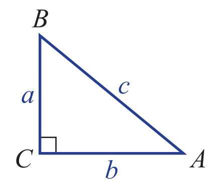
Pythagoras' theorem can be used by everyone, from surveyors who want to find out how tall a mountain is, to astronomers who want to calculate the distance to a star. Carpenters use it to check that their building is accurate and bridge designers use it to create strong, safe bridges. If we know Pythagoras' theorem, we can work out measurements along any side of a right-angled triangle.

Pythagoras' theorem at work in the Sydney Harbour Bridge.

▶ Let's start: Choosing the correct numbers

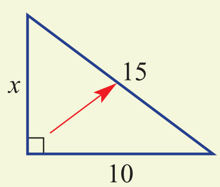
For the triangle ABC, Pythagoras' theorem is written as $c^2 = a^2 + b^2$.
Choose the three numbers from each group that work for $c^2 = a^2 + b^2$.

Group 1: 6, 7, 8, 9, 10
Group 2: 15, 16, 20, 25
Group 3: 9, 10, 12, 15
Group 4: 9, 20, 21, 40, 41



Key ideas

- We can use Pythagoras' theorem to determine the length of one of the shorter sides if we know the length of the hypotenuse and the other side.
  For example:



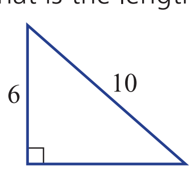
$$15^2 = x^2 + 10^2$$
$$x^2 + 10^2 = 15^2$$
$$x^2 + 100 = 225$$
$$x^2 = 125 \quad (-100 \text{ from both sides})$$
$$x = \sqrt{125}$$

Exercise 7B

Understanding

1 What is the length of the hypotenuse in each of these triangles?

a



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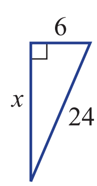

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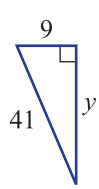

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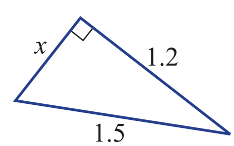

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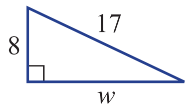
2 Copy and complete the following.

$$\begin{aligned} 10^2 &= w^2 + 6^2 \\ w^2 + 6^2 &= 10^2 \\ -\square \quad w^2 + \square &= \square \quad -\square \\ w^2 &= \square \\ w &= \square \end{aligned}$$

3 Write down Pythagoras' theorem for each of these triangles.

a



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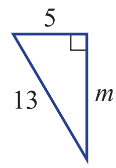

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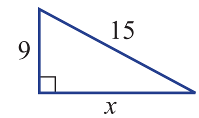
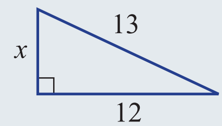


Fluency

Example 5 Calculating a shorter side

Determine the value of x in the triangle shown, using Pythagoras' theorem.



Solution

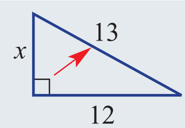
$$\begin{aligned} 13^2 &= x^2 + 12^2 \\ x^2 + 12^2 &= 13^2 \\ -144 \quad x^2 + 144 &= 169 \quad -144 \\ x^2 &= 25 \\ x &= \sqrt{25} \\ \therefore x &= 5 \end{aligned}$$

Explanation

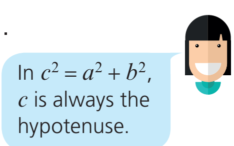
Write the relationship.
Swap the LHS and RHS.

Simplify.

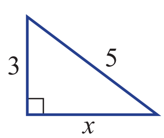
Take the square root to find x.



4 Determine the value of x in these triangles, using Pythagoras' theorem.



a



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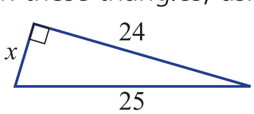

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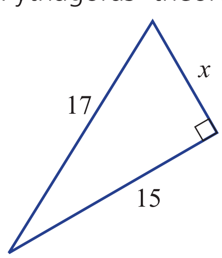

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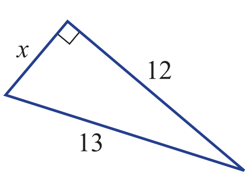

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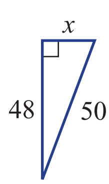

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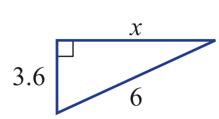

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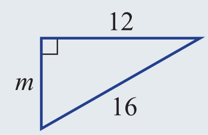
7B

Example 6 Finding a shorter side length as a decimal value

Determine the value of m in the triangle, correct to 1 decimal place.

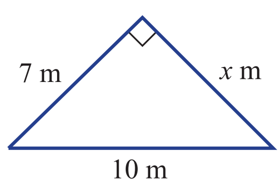


Solution	Explanation
$16^2 = m^2 + 12^2$	Write the relationship for this triangle.
$m^2 + 12^2 = 16^2$	
$-144 \quad m^2 + 144 = 256 \quad -144$	Make m^2 the subject by first swapping the LHS and RHS.
$m^2 = 112$	Simplify, using your calculator.
$m = \sqrt{112}$	Take the square root of both sides to find m.
$m = 10.583\ldots$	
$m = 10.6$	Round your answer to 1 decimal place.

5 Determine the value of x in these triangles, using Pythagoras' theorem. Answer correct to 1 decimal place.

To round to 1 decimal place, look at the 2nd decimal place. If it is 5 or more, round up. If it is 4 or less, round down. For example, 7.1④14… rounds to 7.1.

a



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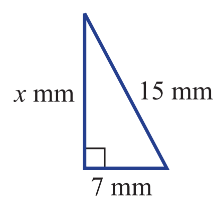

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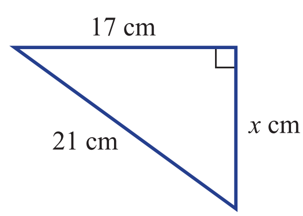

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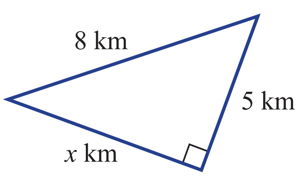

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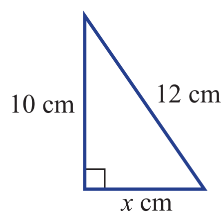

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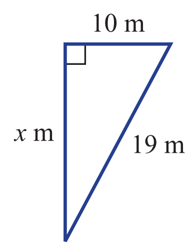
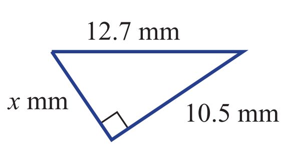


6 Determine the value of x in these triangles, using Pythagoras' theorem. Answer correct to 2 decimal places.

a



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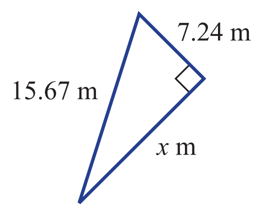

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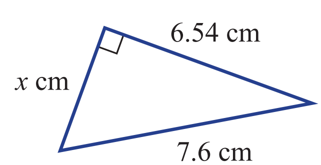
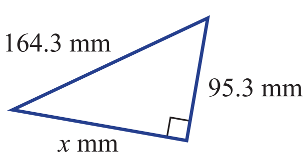


Skillsheet 7A

d



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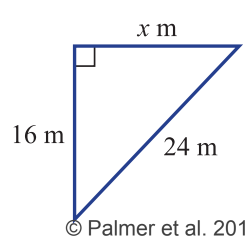

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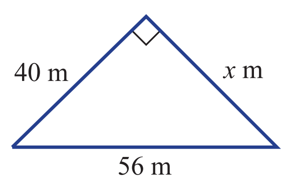

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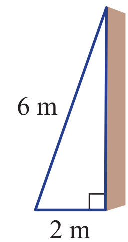
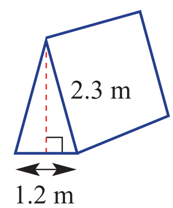
Problem-solving and Reasoning

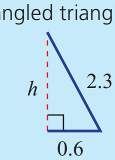
7 A 6 m ladder leans against a wall. If the base of the ladder is 2 m from the wall, determine how high the ladder is up the wall, correct to 2 decimal places.



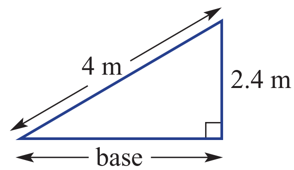
8 A tent has sloping sides of length 2.3 m and a base of 1.2 m. Determine the height of the tent pole, correct to 1 decimal place.



Identify the right-angled triangle.



9 A city council wants to build a skateboard ramp measuring 4 m long and 2.4 m high. What would be the length of the base of the ramp?



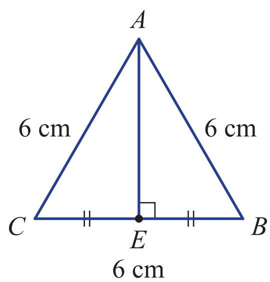
10 Triangle ABC is equilateral. AE is an axis of symmetry.

a Find the length of:

i EB

ii AE, to 1 decimal place

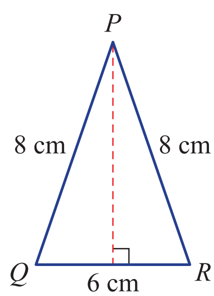
b Find the area of triangle ABC, to 1 decimal place.



An equilateral triangle has 3 equal sides.

Remember: $A = \frac{1}{2}bh$ is the area of a triangle.

11 What is the height of this isosceles triangle, to 1 decimal place?

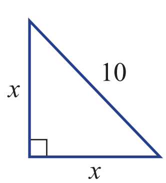


Pythagoras' theorem applies only to right-angled triangles.

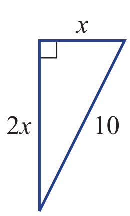
Enrichment: Two unknown sides

12 Find the value of x in each of the following. Give your answer to 1 decimal place.

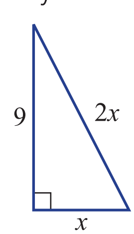
a



b



c



Remember to square the entire side. The square of $2x$ is $(2x)^2$ or $4x^2$. Also, $x^2 + x^2 = 2x^2$.

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Keeping in touch with numeracy

Non-calculator

1 What is $6.39 + 2.11$?

3 What number is 10 less than 5?

5 Convert to decimals:

a $\frac{7}{10}$ **b** $\frac{7}{5}$ **c** $\frac{7}{2}$

7 Solve:
$1 \times 2 \times 3 \times 4 \times 5 = ?$

9 Given that $323 \div 19 = 17$, what is the value of $32.3 \div 19$?

11 Reduce the ratio $8:20$ to simplest form.

13 If the sun rises at 6:17 a.m. and sets at 8:01 p.m., how many hours and minutes are there from sunrise to sunset?

15 The area of a triangle is $60\ \text{cm}^2$. The base is 12 cm. What is the height?

17 A box has 5 square faces but no top. The edges are 6 cm long. Find the total surface area of the inside and outside of the box.

19 US\$1 will buy A\$1.41. How many Australian dollars are required to buy US\$1000?

Calculator

2 Calculate 6.39×2.11, to 1 decimal place.

4 Give the first positive number in this pattern: $-50, -38, -26, \ldots$

6 Convert to decimals:

a $\frac{7}{3}$ **b** $\frac{7}{8}$ **c** $\frac{7}{9}$

8 Calculate the volume of a rectangular prism with edges 1.5 m, 1.3 m and 0.5 m.

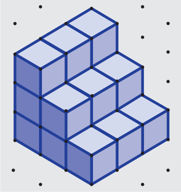
10 Petrol costs 129.9 cents per litre. How many litres can be purchased for \$50? Give your answer correct to 1 decimal place.

12 In a school there are 675 boys and 525 girls. What is the ratio of boys to girls, in simplest form?

14 The time in Los Angeles is 19 hours behind Sydney. The time in Sydney is 10:02 a.m. on Monday. What is the day and time in Los Angeles?

16 The formula for the area of a rhombus is $A = \frac{1}{2}xy$, where x and y are the lengths of the diagonals. If the area is $36\ \text{cm}^2$ and one diagonal is twice as long as the other, find the length of the longer diagonal.

18 Find the surface area of this solid.



20 US\$1 will buy A\$1.41. How many US cents can be purchased with A\$1?

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7C Trigonometric ratios

Stage

5.2
5.2◊
5.1
4

Trigonometry is used to find unknown sides and unknown angles in right-angled triangles.

In right-angled triangles with an acute angle θ, there are three trigonometric ratios:

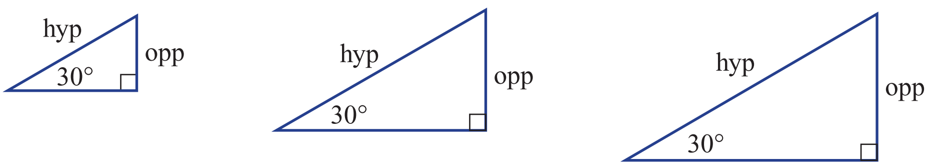
- the sine ratio ($\sin\theta$)
- the cosine ratio ($\cos\theta$)
- the tangent ratio ($\tan\theta$).

Surveyors use trigonometry to calculate accurate lengths.

Let's start: 30°

Drilling for Gold 7C1

- Draw three different right-angled triangles that each have a 30° angle or print the page from 'Drilling for Gold' 7C1.

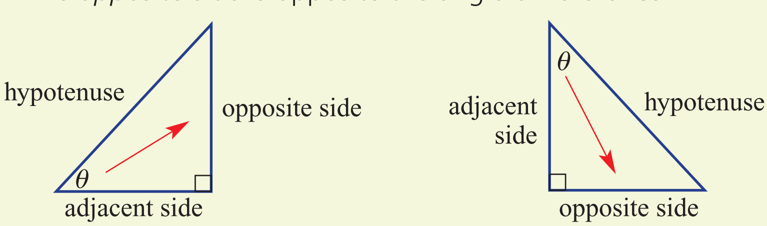


- Measure each side of each triangle and add these measurements to your diagrams.
- The hypotenuse, as we know, is opposite the right angle. The side opposite the 30° is called the opposite side. For each of your three triangles, write down the ratio of the opposite side divided by the hypotenuse. What do you notice?
- Put your calculator in degree mode and enter [sin] [3] [0] [=]. What do you notice?

Key ideas

- Any right-angled triangle has three sides: the hypotenuse, adjacent and opposite.
  - The *hypotenuse* is always opposite the right angle.
  - The *adjacent* side is next to the **angle of reference** (θ).
  - The *opposite* side is opposite the angle of reference.

Angle of reference The angle in a right-angled triangle that is used to determine the opposite side and the adjacent side



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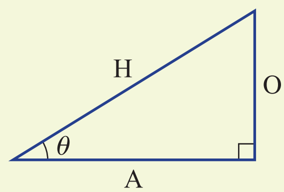
Drilling for Gold 7C2

- For a right-angled triangle with a given angle θ (theta), the three trigonometric ratios of **sine (sin)**, **cosine (cos)** and **tangent (tan)** are given by:
  - sine of angle θ: $\sin\theta = \dfrac{\text{length of opposite side}}{\text{length of the hypotenuse}}$
  - cosine of angle θ: $\cos\theta = \dfrac{\text{length of adjacent side}}{\text{length of the hypotenuse}}$
  - tangent of angle θ: $\tan\theta = \dfrac{\text{length of opposite side}}{\text{length of adjacent side}}$
- When working with right-angled triangles, label each side of the triangle O (opposite), A (adjacent) and H (hypotenuse).



- The three trigonometric ratios are:

$\sin\theta = \dfrac{O}{H}$ $\quad\cos\theta = \dfrac{A}{H}$ $\quad\tan\theta = \dfrac{O}{A}$

We can remember this as **SOH CAH TOA**.

Sine (sin) The ratio of the length of the opposite side to the length of the hypotenuse in a right-angled triangle

Cosine (cos) The ratio of the length of the adjacent side to the length of the hypotenuse in a right-angled triangle

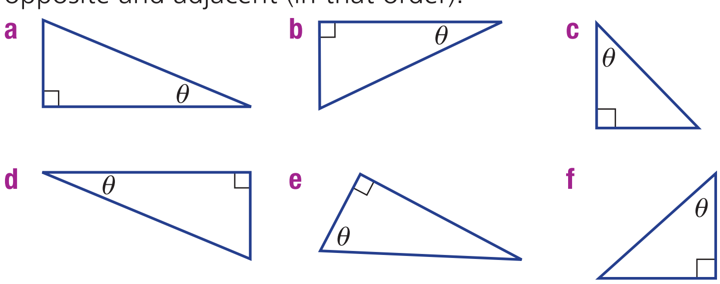
Tangent (tan) The ratio of the length of the opposite side to the length of the adjacent side in a right-angled triangle

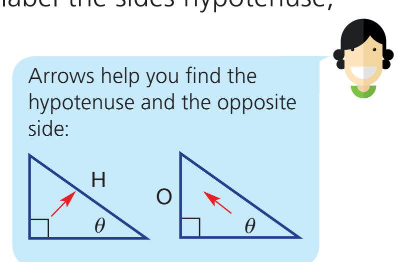
SOH CAH TOA A way of remembering the trigonometric ratios

Exercise 7C

Understanding

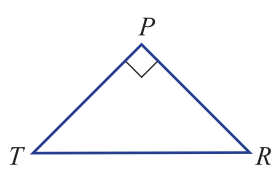
1 By referring to the angles marked, copy each triangle and label the sides hypotenuse, opposite and adjacent (in that order).

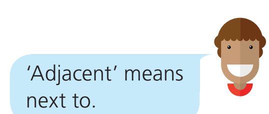




2 Referring to triangle PTR, name the:

a side opposite the angle at T
b side adjacent to the angle at T
c side opposite the angle at R
d side adjacent to the angle at R
e hypotenuse
f angle opposite the side PR





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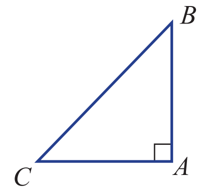
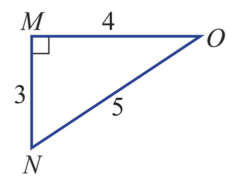
3 Referring to triangle ABC, name the:

- **a** hypotenuse
- **b** side opposite the angle at B
- **c** side opposite the angle at C
- **d** side adjacent to the angle at B



4 In triangle MNO, write the ratio of:

- **a** $\dfrac{\text{the side opposite angle } O}{\text{hypotenuse}}$
- **b** $\dfrac{\text{the side opposite angle } N}{\text{hypotenuse}}$
- **c** $\dfrac{\text{the side adjacent angle } O}{\text{hypotenuse}}$

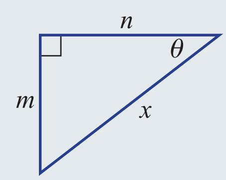


Fluency

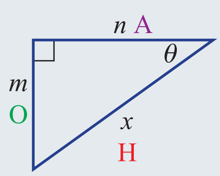
Example 7 Writing trigonometric ratios

Label the sides of the triangle O, A and H and write the ratios for:

a $\sin\theta$ **b** $\cos\theta$ **c** $\tan\theta$



Solution



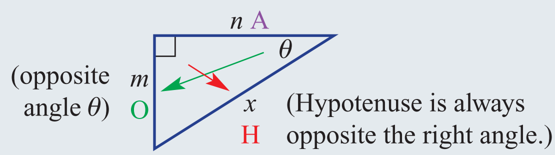
a $\sin\theta = \dfrac{m}{x}$

b $\cos\theta = \dfrac{n}{x}$

c $\tan\theta = \dfrac{m}{n}$

Explanation

Use arrows to label the sides correctly.



SOH CAH TOA

$\sin\theta = \dfrac{\text{O}}{\text{H}} = \dfrac{m}{x}$

$\cos\theta = \dfrac{\text{A}}{\text{H}} = \dfrac{n}{x}$

$\tan\theta = \dfrac{\text{O}}{\text{A}} = \dfrac{m}{n}$

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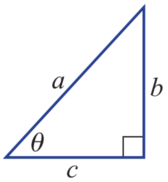
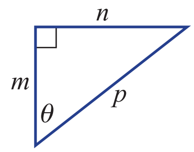
7C

5 For each of the following triangles, write a ratio for:
i sin θ **ii** cos θ **iii** tan θ

a


b


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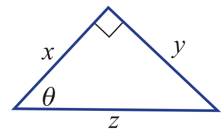

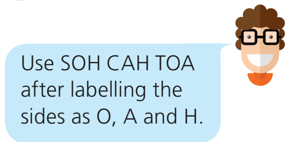

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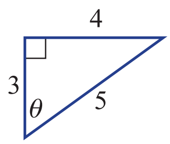

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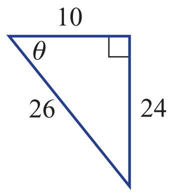

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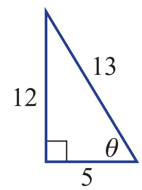
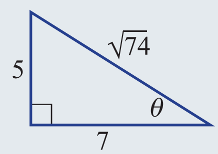


Example 8 Writing a trigonometric ratio

Write down the ratio of cos θ for this triangle.



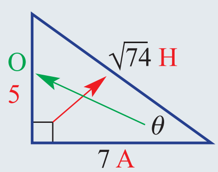
Solution

$\cos\,\theta = \frac{\text{A}}{\text{H}}$

$\cos\,\theta = \frac{7}{\sqrt{74}}$

Explanation

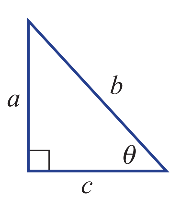
Label the sides of the triangle.



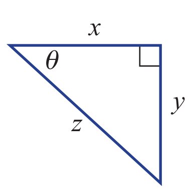
SOH CAH TOA tells us cos θ is $\frac{\text{adjacent}}{\text{hypotenuse}}$.

Substitute the values for the adjacent (A) and hypotenuse (H).

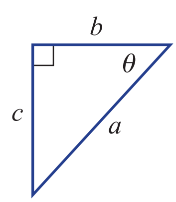
6 Write the trigonometric ratio asked for in each of the following.

a


tan θ =

b


sin θ =

c


cos θ =

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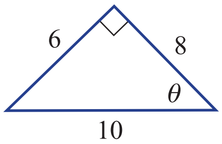
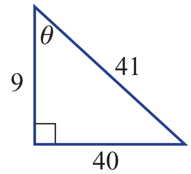
d



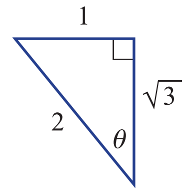
sin θ =

e



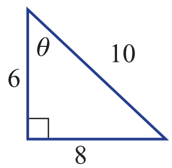
sin θ =

f



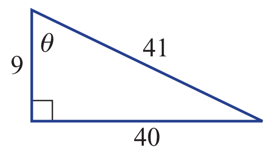
tan θ =

g



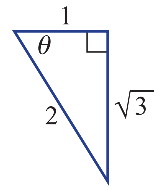
tan θ =

h



cos θ =

i

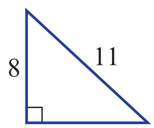


tan θ =

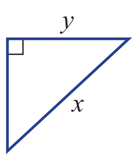
Problem-solving and Reasoning

7 Copy each of these triangles and mark the angle θ that will enable you to write a ratio for sin θ, using the sides given.

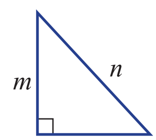
a



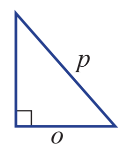
b



c

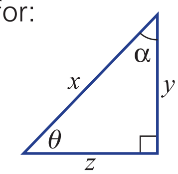


d



8 For the triangle shown on the right, write a ratio for:

a sin θ **b** sin α **c** cos θ
d cos α **e** tan θ **f** tan α

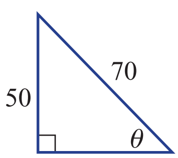


θ and α are letters of the Greek alphabet that are used to mark angles.

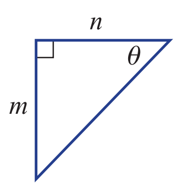
Drilling for Gold 7C3 / 7C4 at the end of this section

9 For each of the triangles below, decide which trigonometric ratio (i.e. sin, cos or tan) you would use for angle θ?

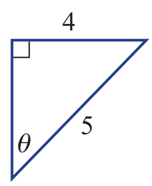
a



b

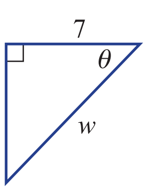


c

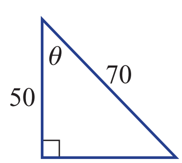


First decide which two sides you have: O, A or H?

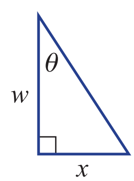
d



e



f



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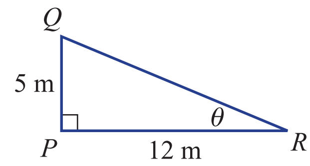
7C

10 Consider triangle PQR.

a Use Pythagoras' theorem to find the length of QR.

b Write down the ratio of sin θ.



11 For a given right-angled triangle, sin $\theta = \frac{1}{2}$.

a Draw up a right-angled triangle and show this information.

b What is the length of the third side? Use Pythagoras' theorem and answer in square root form (e.g. $\sqrt{7}$).

c Find the value of:

i cos θ **ii** tan θ

Enrichment: Relationship between sine and cosine

12 Use your calculator to complete the table, answering to 3 decimal places where necessary.

For most calculators, you enter the values in the same order as they are written; e.g. sin 30° → [sin] 30 = 0.5.

Angle (θ)	sin θ	cos θ
0°		
10°		
20°		
30°		
40°		
45°		
50°		
60°		
70°		
80°		
90°		

a For what angle is sin θ = cos θ?

b Copy and complete the following.

i sin 10° = cos ____°

ii sin 60° = cos ____°

iii sin 90° = cos ____°

c Write down a relationship, in words, between sin and cos.

d Why do you think it's called cosine?

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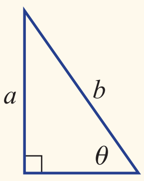
Drilling for Gold exercise

7C3: Which ratio?

Which ratio is the one to use: sin, cos or tan?
Two sides are labelled a and b and one angle is labelled θ.
Which ratio connects the angle with the two sides?
Write sine, cosine or tangent on the worksheet or in your exercise book.

1



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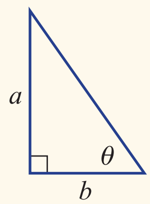

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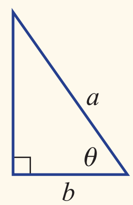

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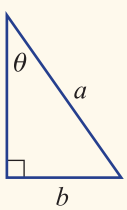

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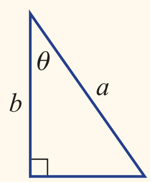

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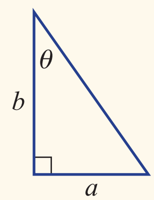
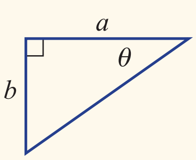


7



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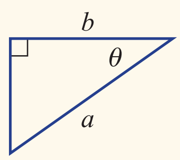

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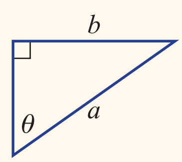

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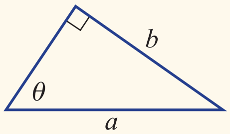

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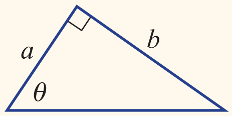

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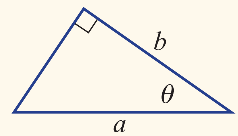

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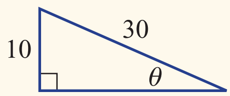
Drilling for Gold exercise

7C4: Choose a ratio and write the statement

Which ratio is the one to use: sin, cos or tan?

Write a statement, such as $\sin\theta = \frac{x}{2}$, on the worksheet or in your exercise book.

1



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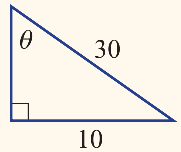

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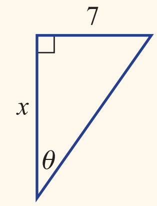

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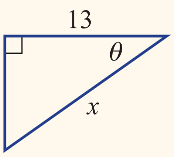

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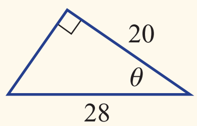

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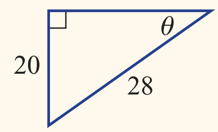

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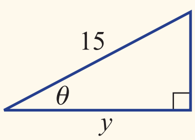

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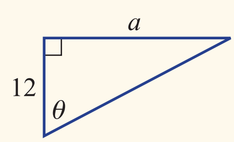

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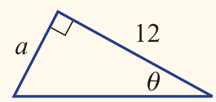

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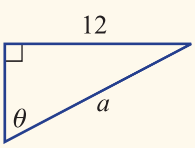

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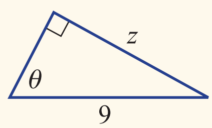

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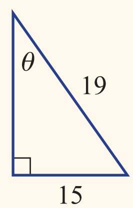

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7D Finding unknown sides

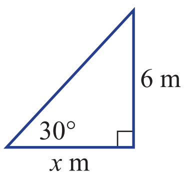
Stage: 5.2, 5.2◊, 5.1, 4

In any right-angled triangle, when given one of the acute angles and a side length, you can find the lengths of the other two sides. This can help builders find the lengths in right-angled triangles if they know an angle and the length of another side.

▶ Let's start: Is it sin, cos or tan?

Out of the six triangles below, only two provide enough information to use the sine ratio. Which two triangles are they?

a



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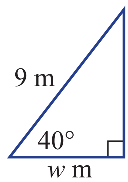

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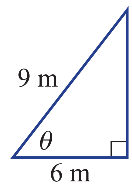

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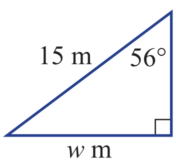

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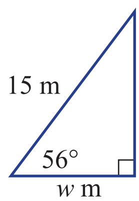

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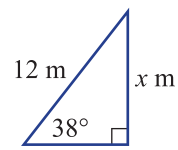


Key ideas

- To find a missing side when given a right-angled triangle with one acute angle and one of the sides:
  - Label the triangle using O (opposite), A (adjacent) and H (hypotenuse).
  - Use SOH CAH TOA to decide on the correct trigonometric ratio.
  - Write down the relationship.
  - Solve the equation, using your calculator, to find the unknown.

$$\sin 40^\circ = \frac{x}{2}$$

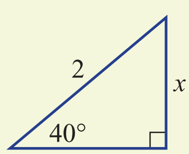
$$\frac{x}{2} = \sin 40^\circ$$

×2 (both sides)

$$x = 2 \cdot \sin 40^\circ$$

$$x = 1.2855\ldots$$

$$x = 1.3 \text{ (to 1 decimal place)}$$



- Always check that your answer is reasonable. The hypotenuse (the longest side) is 2, so x must be less than 2.

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Exercise 7D

Understanding

1 Use a calculator to find the value of each of the following, correct to 4 decimal places.

a $\sin 10^\circ$ **b** $\cos 10^\circ$ **c** $\tan 10^\circ$
d $\tan 30^\circ$ **e** $\cos 40^\circ$ **f** $\sin 70^\circ$

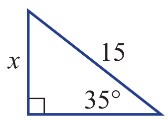
> Locate the sin, cos and tan buttons on your calculator.

2 Evaluate each of the following, correct to 2 decimal places.

a $12 \tan 10^\circ$ **b** $12 \sin 25^\circ$ **c** $18 \tan 60^\circ$
d $56 \sin 56^\circ$ **e** $8 \tan 45^\circ$ **f** $20 \sin 70^\circ$
g $6 \cos 70^\circ$ **h** $5 \cos 15^\circ$ **i** $27.4 \sin 18^\circ$

> On your calculator, enter $12 \tan 10^\circ$ as $12 \times \tan 10$.

3 Decide which of the three trigonometric ratios is suitable for these triangles.

a


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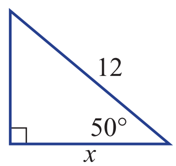
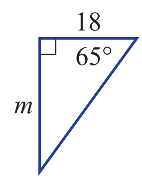
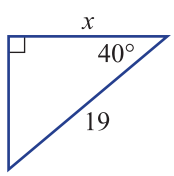


c


> Remember to label the triangle and think SOH CAH TOA. Consider which two sides are involved.

d


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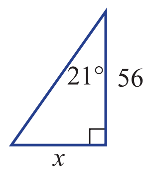

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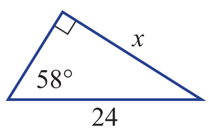

Fluency

Skillsheet 7B

Example 9 Solving a trigonometric equation

Find the value of x, correct to 2 decimal places, for $\cos 30^\circ = \frac{x}{12}$.

Solution	Explanation
$\cos 30^\circ = \frac{x}{12}$	Write the equation.
$\frac{x}{12} = \cos 30^\circ$	Swap the LHS and RHS.
$\times 12$: $x = 12 \cdot \cos 30^\circ$	Multiply both sides by 12. Use your calculator.
$= 10.39230\ldots$	
$= 10.39$ (to 2 decimal places)	Round your answer as required.

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4 Find the value of x in these equations, correct to 2 decimal places.

a $\sin 20° = \dfrac{x}{4}$ **b** $\cos 43° = \dfrac{x}{7}$ **c** $\tan 85° = \dfrac{x}{8}$

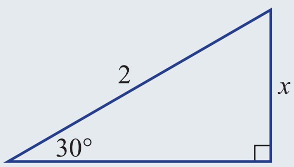
d $\tan 30° = \dfrac{x}{24}$ **e** $\sin 50° = \dfrac{x}{12}$ **f** $\cos 40° = \dfrac{x}{12}$

Drilling for Gold 7D1

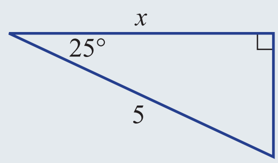
Example 10 Finding a missing side using SOH CAH TOA

Find the value of the unknown length (x) in these triangles. Round your answer to 2 decimal places where necessary.

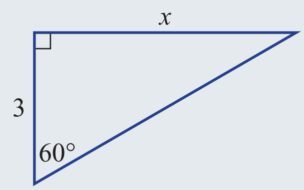
a



b



c



Solution	Explanation

a

$$\sin\theta = \frac{O}{H}$$

$$\sin 30° = \frac{x}{2}$$

$$\frac{x}{2} = \sin 30°$$

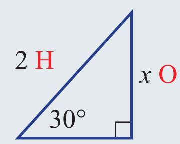
$\times 2$ $\times 2$

$$x = 2 \cdot \sin 30°$$

$$\therefore x = 1$$

Label the triangle and decide on your trigonometric ratio using SOH CAH TOA.

Write the ratio.



Swap the LHS and RHS.

Multiply both sides by 2.

Calculate.

b

$$\cos\theta = \frac{A}{H}$$

$$\cos 25° = \frac{x}{5}$$

$$\frac{x}{5} = \cos 25°$$

$\times 5$ $\times 5$

$$x = 5 \cdot \cos 25$$

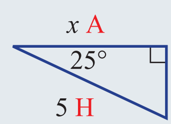
$$x = 4.5315\ldots$$

$$\therefore x = 4.53 \text{ (to 2 decimal places)}$$

Label the triangle.

SOH CAH TOA

Write the ratio.



Swap the LHS and RHS.

Multiply both sides by 5.

Calculate.

Round to 2 decimal places.

c

$$\tan\theta = \frac{O}{A}$$

$$\tan 60° = \frac{x}{3}$$

$$\frac{x}{3} = \tan 60°$$

$\times 3$ $\times 3$

$$x = 3 \cdot \tan 60°$$

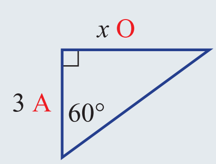
$$x = 5.1961\ldots$$

$$\therefore x = 5.20 \text{ (to 2 decimal places)}$$

Label the triangle.

SOH CAH TOA

Write the ratio.



Swap the LHS and RHS.

Multiply both sides by 3.

Calculate.

Round to 2 decimal places.

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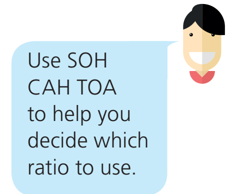
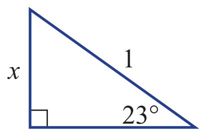
7D

5 For the triangles given below:
  i Copy each one and label the three sides hypotenuse (H), opposite (O) and adjacent (A), in that order.
  ii Decide on a trigonometric ratio.
  iii Find the value of each pronumeral, correct to 2 decimal places.



a


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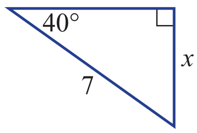

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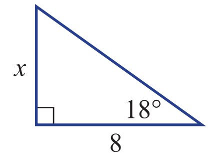

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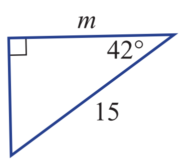
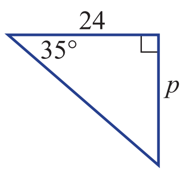
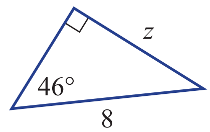
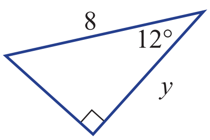
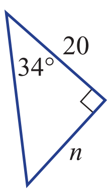
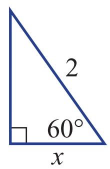


e


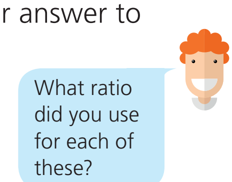
f


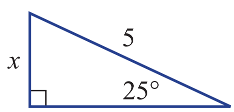
g


h


i


6 Find the value of the unknown length (x) in these triangles. Round your answer to 2 decimal places.



a


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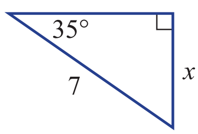

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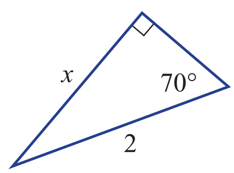
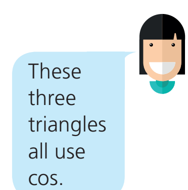
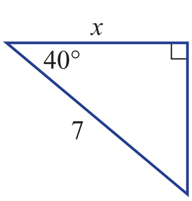


7 Find the value of the unknown length (x) in these triangles. Round your answer to 2 decimal places.



a


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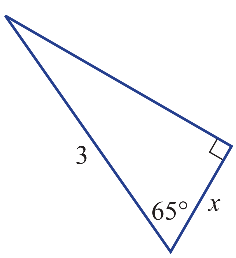

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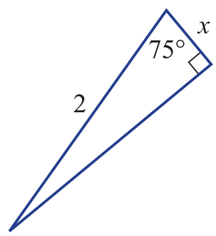

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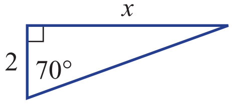
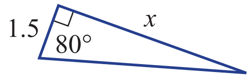
8 Find the value of the unknown length (x) in these triangles. Round your answer to 2 decimal places.

a



b



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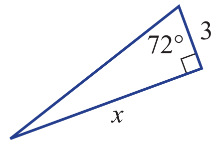
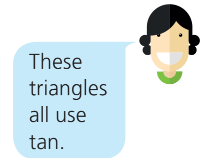
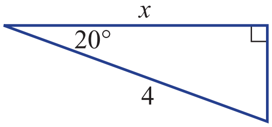




9 Decide whether to use sin, cos or tan, then find the value of x in these triangles. Round to 2 decimal places.

a



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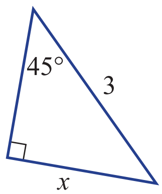

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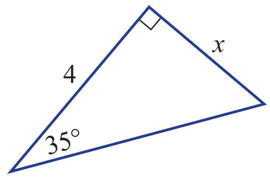

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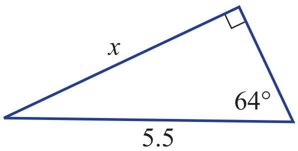
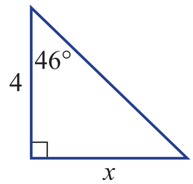


e



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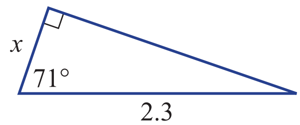
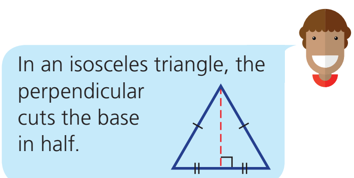
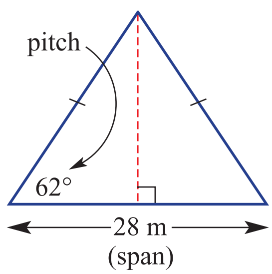


Problem-solving and Reasoning

10 a Find the height of this isosceles triangle, which is similar to a roof truss, to 2 decimal places.

b If the span doubles to 56 m, what is the height of the roof, to 2 decimal places?





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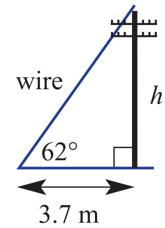
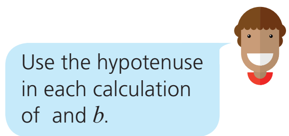
7D

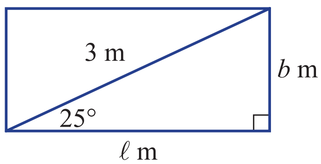
11 The stay wire of a power pole joins the top to the ground. It makes an angle of 62° with the ground. It is fixed to the ground 3.7 m from the bottom of the pole. How high is the pole, correct to 2 decimal places?



12 Find the length and breadth of these rectangles, to 2 decimal places.



a



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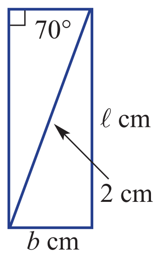

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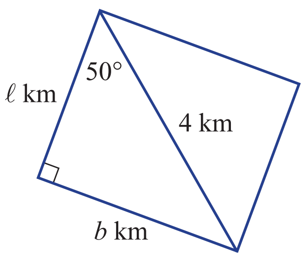


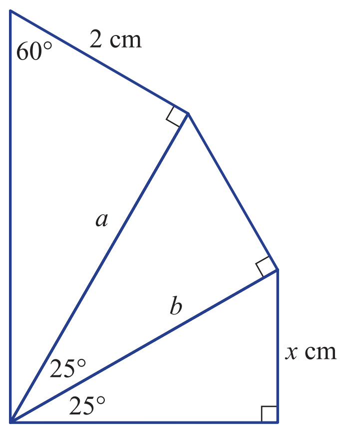
Enrichment: Accuracy and errors

13 Our aim is to find the value of x, correct to 2 decimal places, by first finding the value of a and b.

a Find the value of a, then b and then x, using 1 decimal place for a and b.

b Repeat this process, finding a and b, correct to 3 decimal places each, before finding x.

c Does it make any difference to your final answer for x if you round off the values of a and b during calculations?



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7E Solving for the denominator

Stage
5.2
5.2◊
5.1
4

So far, we have been dealing with equations that have the pronumeral in the numerator. However, sometimes the unknown is in the denominator.


Right-angled triangles can be formed from this house to help find particular lengths.

Let's start: Solving equations with x in the denominator

Consider the equations $\frac{x}{3} = 4$ and $\frac{3}{x} = 4$.

- Do the equations have the same solution?
- What steps are used to solve the equations?
- Now solve $\frac{4}{x} = \sin 30°$ and $\frac{2}{x} = \cos 40°$.

Key ideas

- If the unknown value is in the **denominator**, there are steps to find the unknown. For example:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 55° = \frac{4}{x}$$

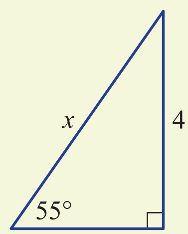
$\times x$ (both sides)

$$x \times \sin 55° = 4$$

$\div \sin 55°$ (both sides)

$$x = \frac{4}{\sin 55°}$$

$$= 4.88 \text{ (to 2 decimal places)}$$



Denominator The part of a fraction that sits below the dividing line

Exercise 7E

Understanding

1 Find the value, correct to 2 decimal places, of:

a $\frac{10}{\tan 30°}$ **b** $12 \div \sin 60°$ **c** $\frac{15}{\tan 8°}$

d $\frac{12.4}{\tan 32°}$ **e** $\frac{15.2}{\sin 38°}$ **f** $\frac{9}{\cos 47°}$

For part **a**, enter $10 \div \tan 30$ into your calculator.

2 Solve these equations for x.

a $\frac{4}{x} = 2$ **b** $\frac{10}{x} = 2$ **c** $\frac{15}{x} = 30$

d $\frac{1.2}{x} = 1.2$ **e** $\frac{0.6}{x} = 6$ **f** $\frac{9}{x} = 90$

To solve these, you will need to complete two steps.

Drilling for Gold 7E1 at the end of this section

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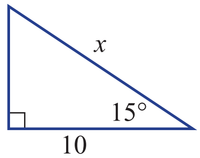
7E

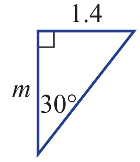
3 For each of these triangles, complete the required trigonometric ratio.

a



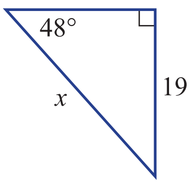
$\cos 15° = \dfrac{\square}{\square}$

b



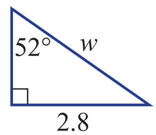
$\tan 30° = \dfrac{\square}{\square}$

c



$\sin 48° = \dfrac{\square}{\square}$

d



$\sin 52° = \dfrac{\square}{\square}$

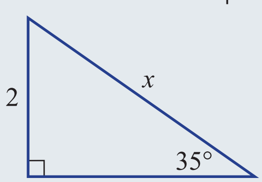
Fluency

Drilling for Gold 7E2

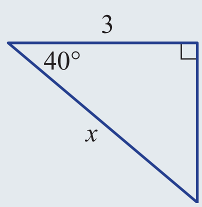
Example 11 Finding the value in the denominator

Find the value of the unknown length (x) in these right-angled triangles. Round your answer to 2 decimal places.

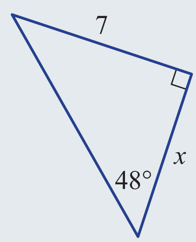
a



b



c



Solution

a

$\sin 35° = \dfrac{2}{x}$

$\times x$ (both sides)

$x \times \sin 35° = 2$

$\div \sin 35°$ (both sides)

$x = \dfrac{2}{\sin 35°}$

$x = 3.48689\ldots$

$\therefore\ x = 3.49$ (to 2 d.p.)

Explanation

Use $\sin\theta = \dfrac{O}{H}$, since we can use the opposite (2) and hypotenuse (x).

Multiply both sides by x.

Divide both sides by $\sin 35°$.

Evaluate and round your answer.

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b

$$\cos 40^\circ = \frac{3}{x}$$

$\times x$ (both sides)

$$x \times \cos 40^\circ = 3$$

$\div \cos 40^\circ$ (both sides)

$$x = \frac{3}{\cos 40^\circ}$$

$$x = 3.9162\ldots$$

$$\therefore\ x = 3.92 \text{ (to 2 d.p.)}$$

Use $\cos\theta = \frac{A}{H}$, as we can use the adjacent (3) and hypotenuse (x).
Multiply both sides by x.
Divide both sides by cos 40°.
Evaluate and round your answer.

c

$$\tan 48^\circ = \frac{7}{x}$$

$\times x$ (both sides)

$$x \times \tan 48^\circ = 7$$

$\div \tan 48^\circ$ (both sides)

$$x = \frac{7}{\tan 48^\circ}$$

$$x = 6.3028\ldots$$

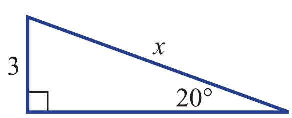
$$\therefore\ x = 6.30 \text{ (to 2 d.p.)}$$

Use $\tan\theta = \frac{O}{A}$, as we can use the adjacent (x) and opposite (7).
Multiply both sides by x.
Divide both sides by tan 48°.
Evaluate and round your answer.

4 Find the value of the unknown length (x) in these right-angled triangles. Round your answer to 2 decimal places.

> In $\sin 20^\circ = \frac{3}{x}$, multiply both sides by x first.

a



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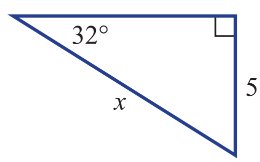

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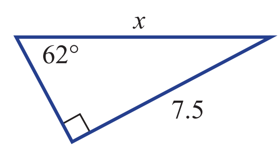
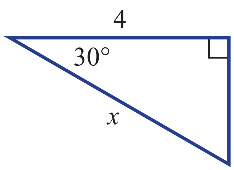


5 Find the value of the unknown length (x) in these right-angled triangles. Round your answer to 2 decimal places.

a



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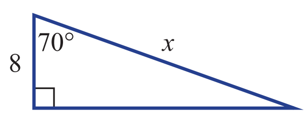

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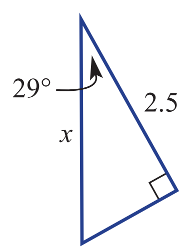
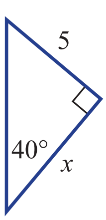


> $\cos\theta = \frac{A}{H}$

6 Find the value of the unknown length (x) in these right-angled triangles. Round your answer to 2 decimal places.

a



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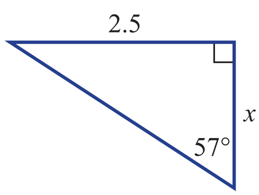

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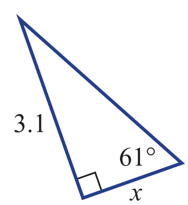

> $\tan\theta = \frac{O}{A}$

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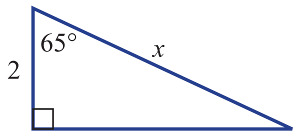
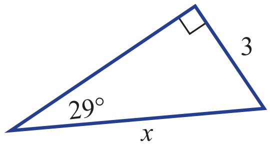
7E

7 By first deciding whether to use sin θ, cos θ or tan θ, find the value of x in these triangles. Round your answer to 2 decimal places.

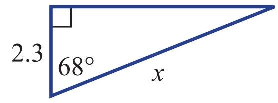
a

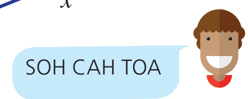


b



c





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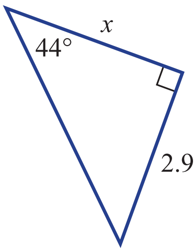
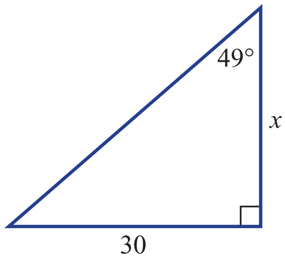


e



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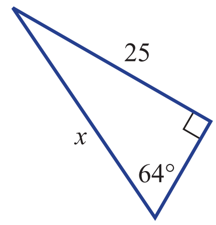
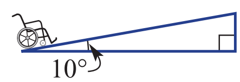
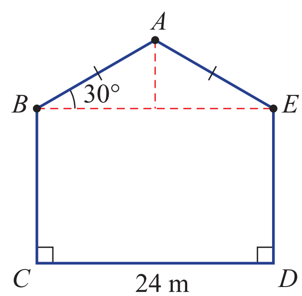


Problem-solving and Reasoning

8 The recommended angle of a wheelchair ramp to the horizontal is approximately 10°. How long is the ramp if the horizontal distance is 2.5 metres? Round your answer to 2 decimal places.

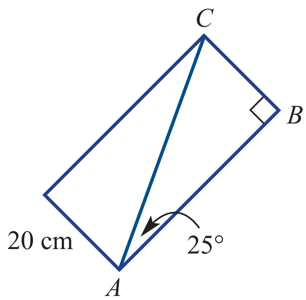


9 The roof of this barn has a pitch of 30°, as shown. Find the length of roof section AB, to 1 decimal place.

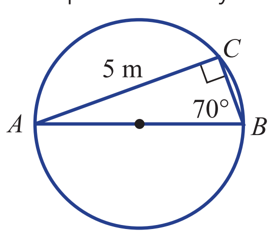


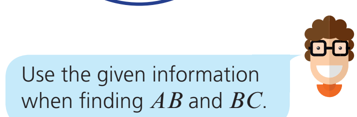
10 Find the length AB and BC in these shapes. Round your answer to 2 decimal places.

a

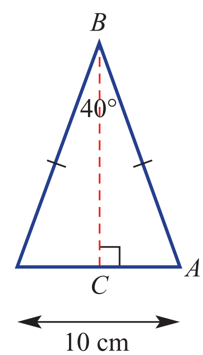


b





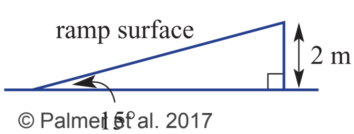
c

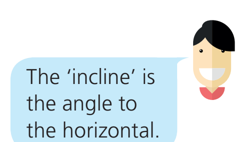


11 The ramp shown has an incline angle of 15° and a height of 2 m. Find, correct to 3 decimal places:

a the base length of the ramp

b the length of the ramp surface





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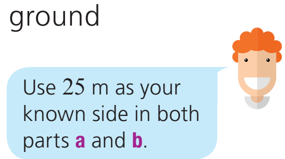
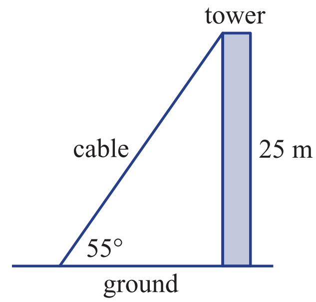
12 For this communications tower, find, correct to 1 decimal place:

a the length of the cable

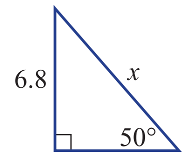
b the distance from the base of the tower to the point where the cable is attached to the ground





Enrichment: Inverting the fractions

Shown below is another way of solving trigonometric equations with x in the denominator.
Find the value of x, to 2 decimal places.



$$\sin 50^\circ = \frac{6.8}{x}$$ Which means $\frac{\sin 50^\circ}{1} = \frac{6.8}{x}$.

$$\frac{1}{\sin 50^\circ} = \frac{x}{6.8}$$ Invert both fractions so x becomes the numerator.

$$\frac{x}{6.8} = \frac{1}{\sin 50^\circ}$$ Swap the LHS and RHS.

$\times 6.8$ (both sides)

$$x = \frac{1}{\sin 50^\circ} \cdot 6.8$$ Multiply both sides by 6.8 to get x on its own.

$$\frac{1}{\sin 50^\circ} \cdot 6.8 = \frac{x}{6.8} \cdot 6.8$$

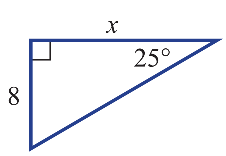
$$x = \frac{6.8}{\sin 50^\circ}$$

$x = 8.87676\ldots$ Use your calculator.

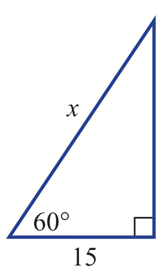
$x = 8.88$ (to 2 decimal places) Round your answer as required.

13 Use the method shown above to find the value of x, to 2 decimal places where necessary, in each of the following.

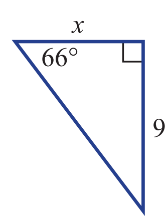
a



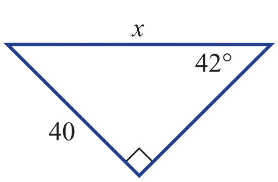
b



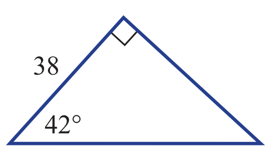
c



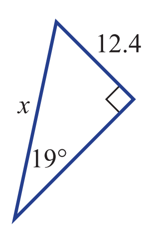
d



e



f



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Drilling for Gold exercise

7E1: Solving an equation with the unknown in the denominator

Study the example, then solve the other equations using the same method.
Use the worksheet or write the answers in your exercise book.

In Questions **10**, **11** and **12**, give your solution correct to 2 decimal places.

1
$$3 = \frac{5}{x}$$
$\times x$ (both sides)
$$3x = 5$$
$\div 3$ (both sides)
$$x = \frac{5}{3}$$

2 $5 = \dfrac{15}{x}$

3 $15 = \dfrac{5}{x}$

4 $6 = \dfrac{18}{x}$

5 $18 = \dfrac{6}{x}$

6 $8 = \dfrac{16}{x}$

7 $\sin 30° = \dfrac{15}{x}$

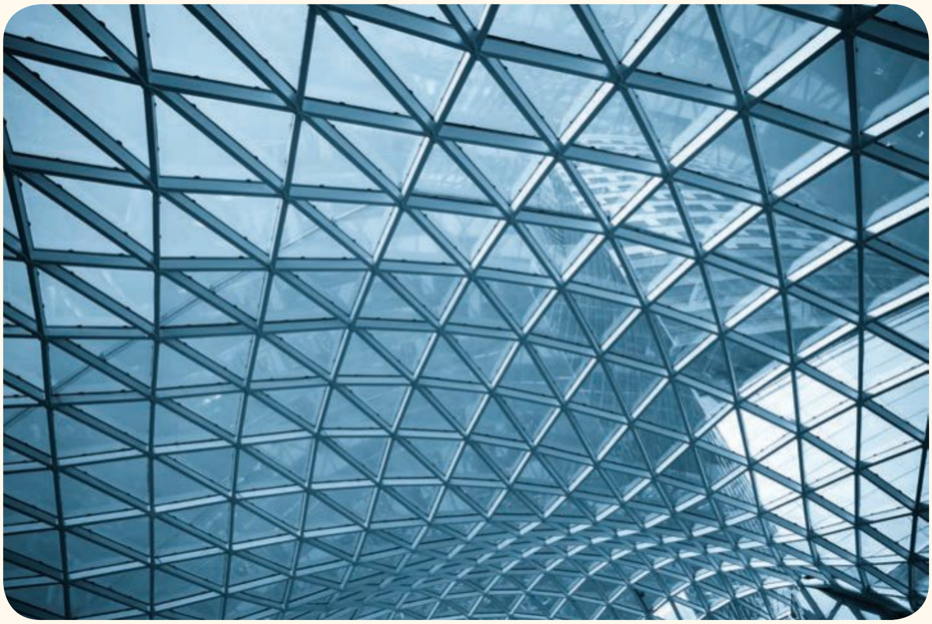
8 $\cos 60° = \dfrac{5}{x}$

9 $\tan 45° = \dfrac{5}{x}$

10 $\sin 35° = \dfrac{15}{x}$

11 $\cos 65° = \dfrac{5}{x}$

12 $\tan 55° = \dfrac{5}{x}$



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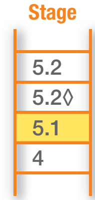
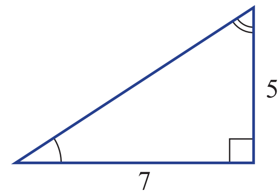
7F Finding unknown angles



When given two or three side lengths of a right-angled triangle, you can find either of the acute angles. Given a statement like $\sin\theta = \frac{5}{7}$, your calculator can given you a value for θ.

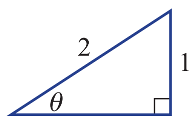


This is enough information to calculate the size of the unknown angles.

▶ Let's start: Knowing the angle

Imagine a triangle that produces $\sin\theta = 0.5$.

- Use your calculator and trial and error to find a value of θ for which $\sin\theta = 0.5$.
- Repeat for $\tan\theta = 1$ and $\cos\theta = \frac{\sqrt{3}}{2}$.
- Do you know of a quicker method, rather than using trial and error?



Key ideas

- There are buttons on your calculator that find unknown angles:
  [$\sin^{-1}$] [$\cos^{-1}$] [$\tan^{-1}$]
- If solving $\sin\theta = \frac{3}{4}$ on your calculator, enter [$\sin^{-1}$] $\left(\frac{3}{4}\right)$ [=] to find θ.

Exercise 7F

Understanding

1 Calculate $\sin 30°$. Now use the [$\sin^{-1}$] button to solve $\sin\theta = 0.5$.

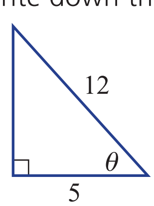
2 Use your calculator to find θ.

a $\sin\theta = \frac{1}{2}$ **b** $\cos\theta = \frac{1}{2}$ **c** $\tan\theta = 1$

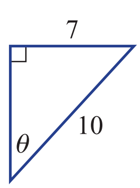
Most calculators use [shift] to access $\sin^{-1}$ or $\cos^{-1}$ or $\tan^{-1}$.

3 Write down the trigonometric ratio for these triangles. Is it sin, cos or tan?

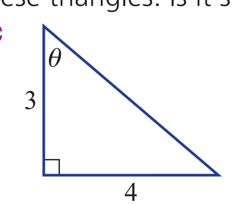
a



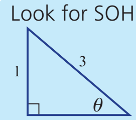
b



c



Look for SOH CAH TOA.



$\sin\theta = \frac{1}{3}$

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7F

Fluency

Skillsheet 7C

Example 12 Finding an unknown angle, correct to the nearest degree

Find the angle θ, correct to the nearest degree, for each of the following.

a $\sin\theta = \frac{2}{3}$ **b** $\cos\theta = \frac{1}{2}$ **c** $\tan\theta = 1.7$

Solution	Explanation
a $\sin\theta = \frac{2}{3}$	Use the $\sin^{-1}$ button on your calculator.
$\theta = 41.8103\ldots°$	Enter $\sin^{-1}\left(\frac{2}{3}\right)$ [=].
$\theta = 42°$ (to the nearest degree)	Round your answer as required.
b $\cos\theta = \frac{1}{2}$	Use the $\cos^{-1}$ button on your calculator.
$\theta = 60°$	Enter $\cos^{-1}\left(\frac{1}{2}\right)$ [=].
c $\tan\theta = 1.7$	Enter $\tan^{-1}(1.7)$ [=].
$\theta = 59.53\ldots°$	
$\theta = 60°$ (to the nearest degree)	Round your answer to the nearest degree.

4 Find the angle θ, to the nearest degree, for the following.

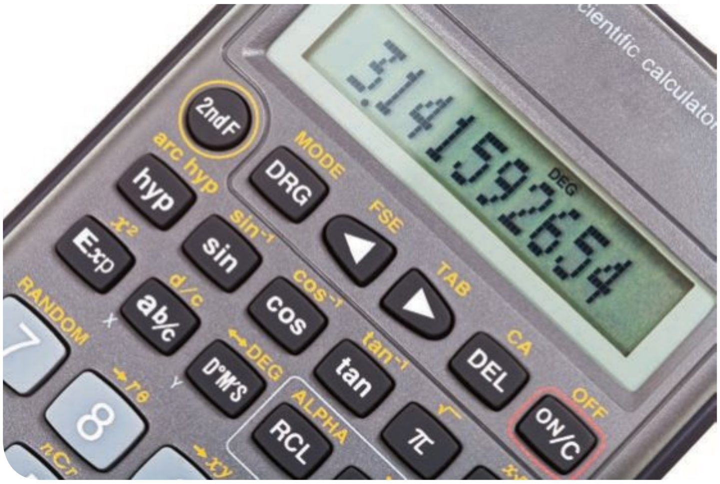
a $\sin\theta = \frac{1}{2}$ **b** $\cos\theta = \frac{3}{5}$ **c** $\sin\theta = \frac{7}{8}$

d $\tan\theta = 1$ **e** $\tan\theta = \frac{7}{8}$ **f** $\sin\theta = \frac{8}{10}$

g $\cos\theta = \frac{2}{3}$ **h** $\sin\theta = \frac{1}{10}$ **i** $\cos\theta = \frac{4}{5}$ **j** $\tan\theta = 6$

k $\cos\theta = \frac{3}{10}$ **l** $\tan\theta = \sqrt{3}$ **m** $\sin\theta = \frac{4}{6}$ **n** $\cos\theta = \frac{4}{6}$

Remember: Use [$\sin^{-1}$], [$\cos^{-1}$] or [$\tan^{-1}$] on the calculator.



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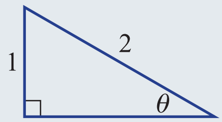
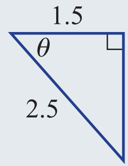
Example 13 Using SOH CAH TOA to find angles

Find θ in the following right-angled triangles, correct to the nearest degree.

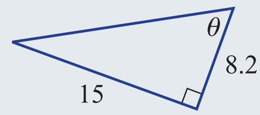
a



b

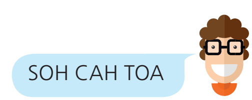


c

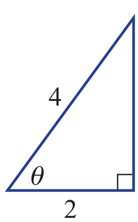


Solution	Explanation
a $\sin\theta = \dfrac{O}{H}$	Use sin θ since we know the **O**pposite and the **H**ypotenuse.
$\sin\theta = \dfrac{1}{2}$	Substitute **O** = 1 and **H** = 2.
$\theta = 30°$	Use your calculator to find $\sin^{-1}\left(\dfrac{1}{2}\right)$.
b $\cos\theta = \dfrac{A}{H}$	Use cos θ since we know the **A**djacent and the **H**ypotenuse.
$\cos\theta = \dfrac{1.5}{2.5}$	Substitute **A** = 1.5 and **H** = 2.5.
$\theta = 53.1301\ldots°$	Use your calculator to find $\cos^{-1}\left(\dfrac{1.5}{2.5}\right)$.
$\theta = 53°$ (to nearest degree)	Round your answer to the nearest degree.
c $\tan\theta = \dfrac{O}{A}$	Use tan θ since we know the **O**pposite and the **A**djacent.
$\tan\theta = \dfrac{15}{8.2}$	Substitute **O** = 15 and **A** = 8.2.
$\theta = 61.336\ldots°$	Use your calculator to find $\tan^{-1}\left(\dfrac{15}{8.2}\right)$.
$\theta = 61°$ (to nearest degree)	Round your answer to the nearest degree.

5 Use one of sin, cos or tan to find θ in these triangles, rounding your answer to the nearest degree.



a



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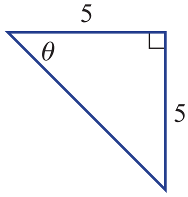

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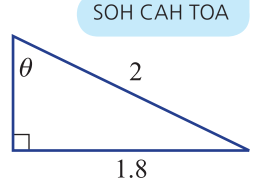

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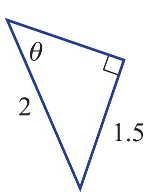

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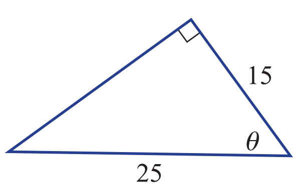

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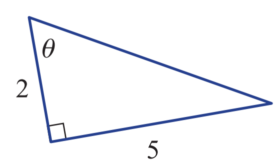

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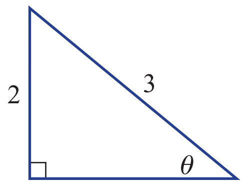

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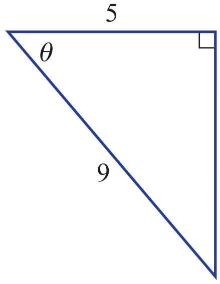

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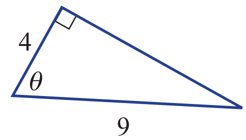

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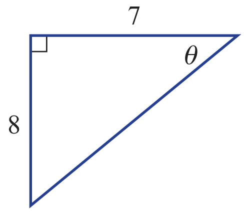

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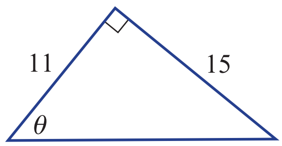

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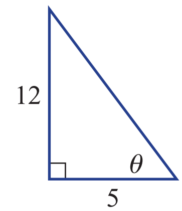
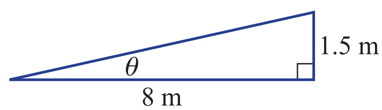
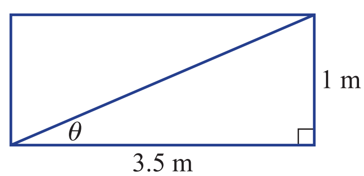


Problem-solving and Reasoning

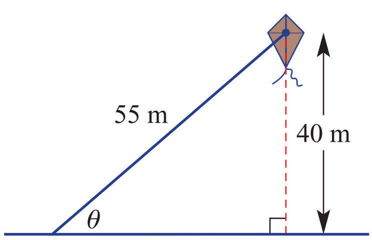
6 A ramp is 8 m long and 1.5 m high. Find the angle that the ramp makes with the ground, correct to the nearest degree.

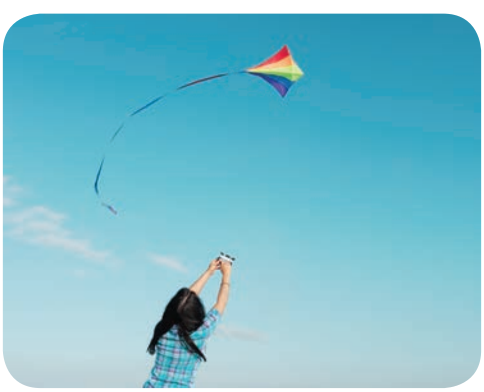


7 A rectangular piece of timber, measuring 1 m wide and 3.5 m long, is to be cut across the diagonal. Find the angle that the cut makes with the long side (correct to the nearest degree).



8 At what angle to the ground is a kite (shown) with height 40 m and string length 55 m? Round your answer to the nearest degree.





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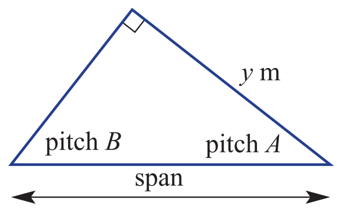
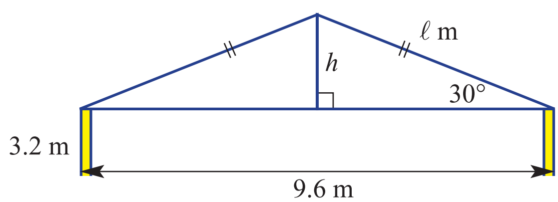
9 A roof is pitched so that the angle at its peak is 90°. If each roof truss spans 10.5 m and distance y is 7.2 m, find the pitch angles A and B, to the nearest degree.



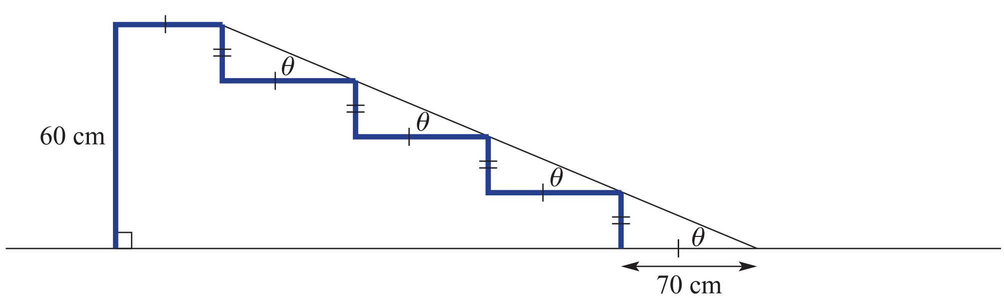
Enrichment: Building construction

10 a Find the length of the slats (ℓ metres) needed along each hypotenuse for this roof cross-section, correct to 2 decimal places.

b Find the height of the highest point of the roof above ground level, correct to 2 decimal places.



11 A ramp is to be constructed to allow disabled access over a set of existing stairs, as shown. (Note that the diagram is not to scale.)



a What angle does the ramp make with the ground, to the nearest degree?

b Government regulations state that the ramp cannot be more than 13° to the horizontal. Does this ramp meet these requirements?

c Use Pythagoras' theorem to find the length of the ramp. Round your answer to 1 decimal place.

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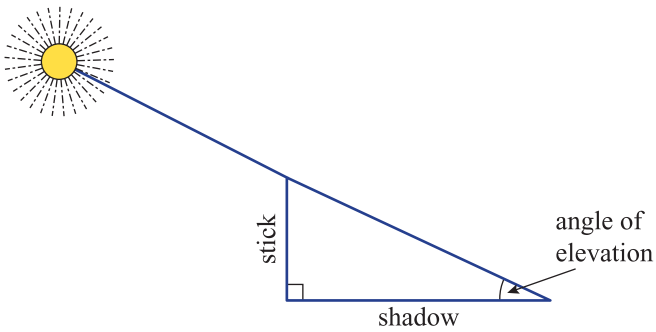
7G Angles of elevation and depression

Stage: 5.2, 5.2◊, 5.1, 4

Many applications of trigonometry involve angles of elevation and angles of depression. These angles are measured up or down from a horizontal level.

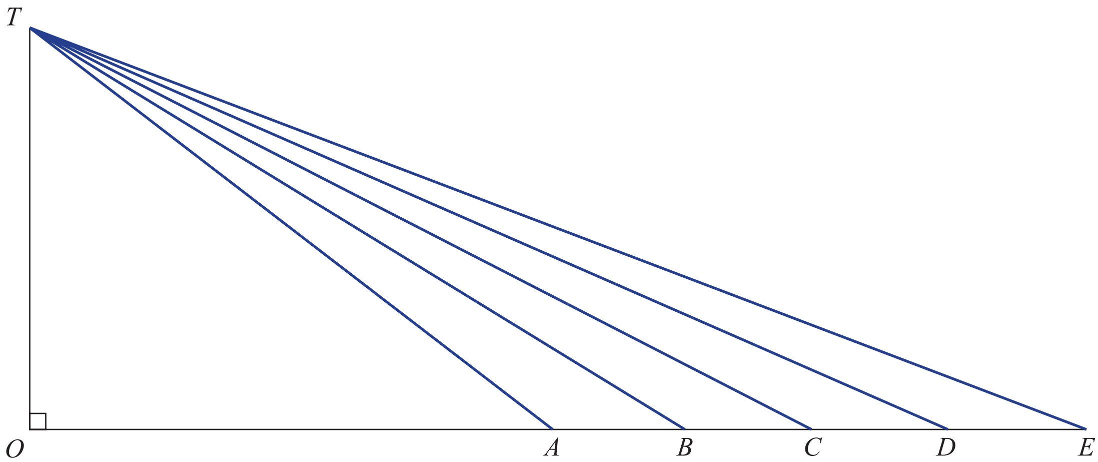


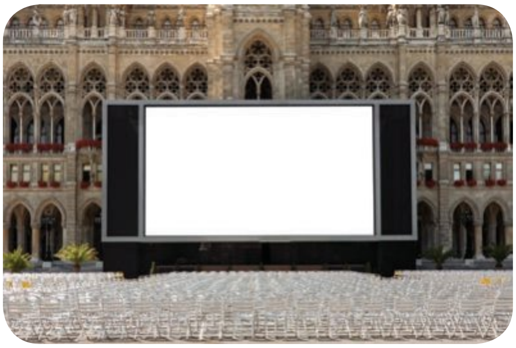
On flat ground, a vertical stick and the shadow it makes can be used to calculate the angle of elevation of the Sun.

▶ Let's start: How close should you sit?

The diagram below shows an outdoor movie screen (OT). The point T is the top of the screen. The points A–E are the five rows of seats in the theatre, from which a person's line of sight is taken. The line OE is the horizontal line.

- Use your protractor to measure the angle of elevation from each point along the horizontal to the top of the movie screen.
- Where should you sit if you wish to have an angle of elevation between 25° and 20° and not be in the first or last row of the theatre?





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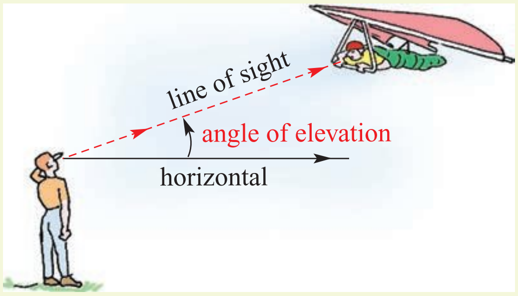
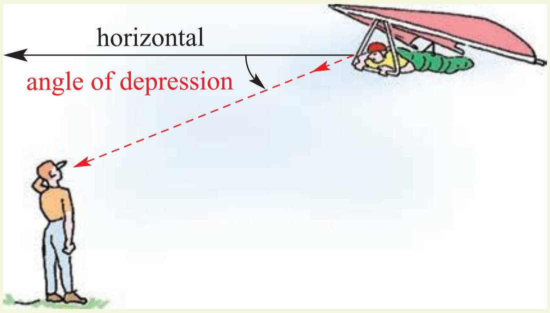
Key ideas

- Looking up to an object forms an **angle of elevation**.

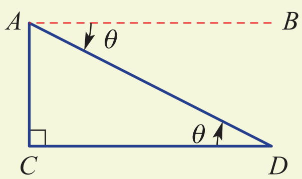


Angle of elevation
The angle of your line of sight from the horizontal when looking up at an object

- Looking down to an object forms an **angle of depression**.



Angle of depression
The angle of your line of sight from the horizontal when looking down at an object

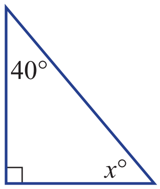
- 

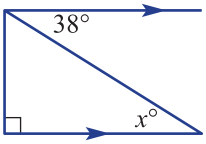
AB is parallel to CD.
$\therefore \angle BAD = \angle ADC$ because they are alternate angles in parallel lines.
$\therefore$ Angle of elevation = Angle of depression

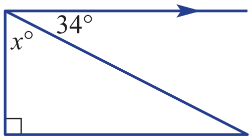
Exercise 7G

Understanding

1 Find the value of x in each triangle.

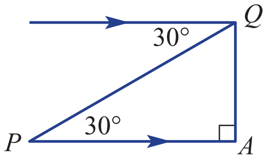
a 

b 

c 

2 In Question **1**, which diagram shows angles of elevation and depression?

3 For this diagram:



a What is the angle of elevation of Q from P?
b What is the angle of depression of P from Q?
c What is the size of $\angle PQA$?

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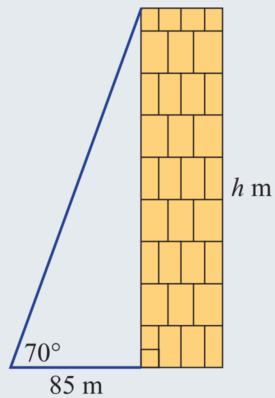
7G

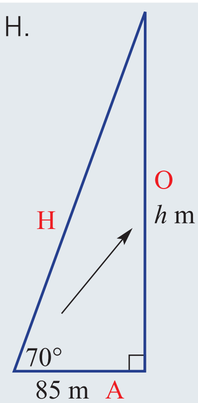
Fluency

Example 14 Using an angle of elevation

To find the height of a tall building, Johal stands 85 m away from its base and measures the angle of elevation to the top of the building as 70°. Find the height of the building, correct to the nearest metre.

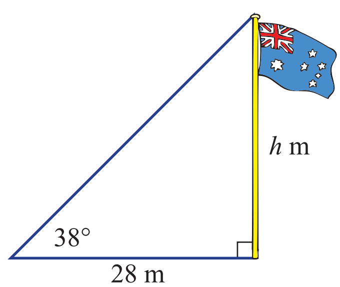


Solution	Explanation
$\tan\theta = \frac{O}{A}$	Label the triangle with O, A, and H.
$\tan 70° = \frac{h}{85}$	Use tan since the opposite and adjacent are given.
$\frac{h}{85} = \tan 70°$ $\times 85$ $\quad$ $\times 85$ $h = 85 \cdot \tan 70°$	Swap the LHS and RHS. Multiply both sides by 85.
$= 233.53558\ldots$	Calculate.
$= 234$ (to the nearest metre) ∴ The building is 234 m tall.	Round to the nearest metre.

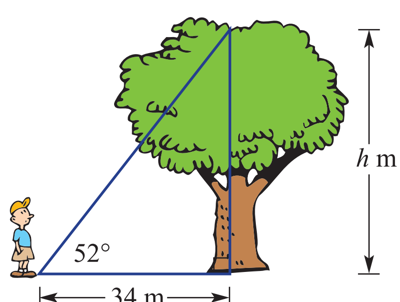


4 Solve the following problems about angles of elevation.

a The angle of elevation to the top of a flagpole from a point 28 m from its base is 38°. How tall is the flagpole, correct to 2 decimal places?



b Alvin is 34 m away from a tree and the angle of elevation to the top of the tree from the ground is 52°. What is the height of the tree, correct to 1 decimal place?



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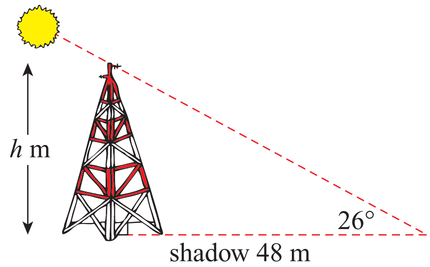
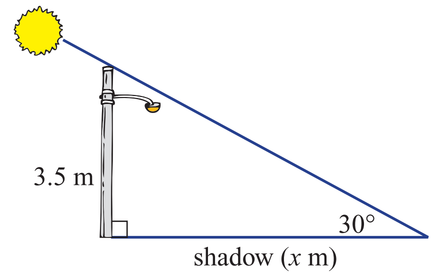
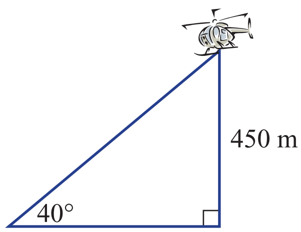
c The Sun's rays shining over a tower make an angle of elevation of 26° and casts a 48 m shadow on the ground. How tall, to 2 decimal places, is the tower?



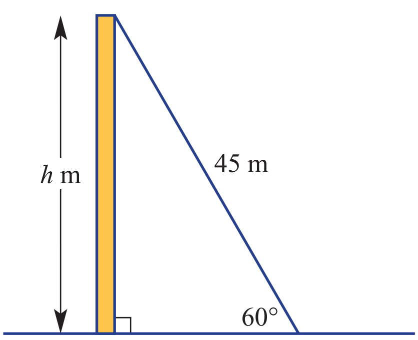
d The Sun makes an angle of elevation of 30° with a lamp post 3.5 m tall. How long is the shadow on the ground, correct to 2 decimal places?



e The altitude of a hovering helicopter is 450 m, and the angle of elevation from the helipad to the helicopter is 40°. Find the horizontal distance from the helicopter to the helipad, correct to 2 decimal places.

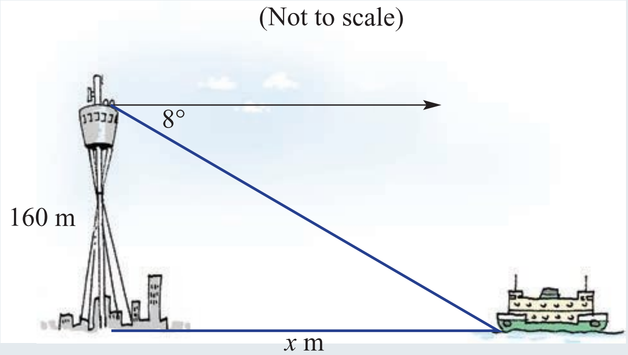


f A cable of length 45 m is anchored from the ground to the top of a communications mast. The angle of elevation of the cable to the top of the mast is 60°. Find the height of the communications mast, correct to 2 decimal places.



Example 15 Using an angle of depression

From the observation room of Centrepoint Tower in Sydney, which has a height of 160 m, the angle of depression of a boat moored at Circular Quay is observed to be 8°. How far from the base of the tower is the boat, correct to the nearest metre?



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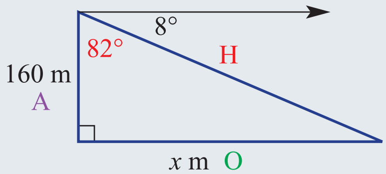
Solution



$$\tan\theta = \frac{O}{A}$$

$$\tan 82° = \frac{x}{160}$$

$$\frac{x}{160} = \tan 82°$$

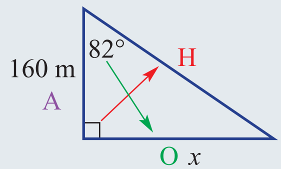
$\times 160$ $\times 160$

$$x = 160 \cdot \tan 82°$$
$$= 1138.459\ldots$$
$$= 1138 \text{ (to the nearest metre)}$$

∴ The boat is about 1138 m from the base of the tower.

Explanation

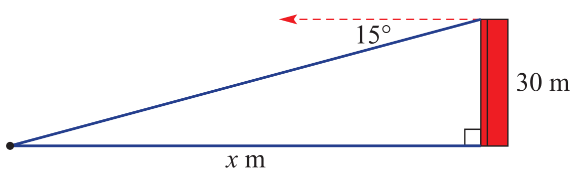
Draw the triangle and find the angle inside the triangle: $90° - 8° = 82°$ (*or* use alternate angles to label the angle of elevation as $8°$).
Use this angle to label the triangle.
Use tan since we have the opposite and adjacent.



Find x by solving the equation.
Round your answer to the nearest metre.

5 Answer these problems relating to angles of depression.

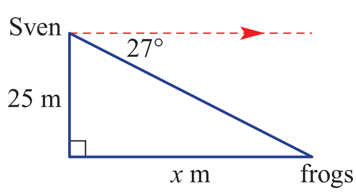
a The angle of depression from the top of a tower 30 m tall to a point x m from its base is $15°$. Find the value of x, correct to 1 decimal place.



Use the angle of depression to mark an angle inside the triangle.

b From a bridge 25 m above a stream, Sven spots two frogs on a lilypad. He estimates the angle of depression to the frogs to be $27°$. How far from the bridge are the frogs, to the nearest metre?

The angle of depression is the angle below the horizontal, looking down at an object.



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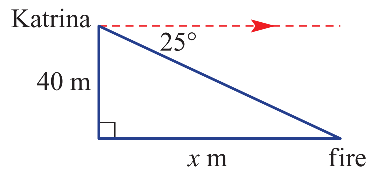
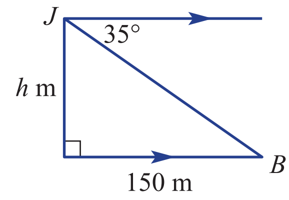
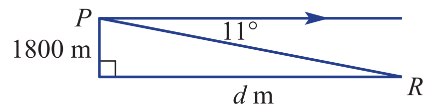
c From a lookout tower, Katrina spots a bushfire at an angle of depression of 25°. If the lookout tower is 40 m high, how far away (to the nearest metre) is the bushfire from the base of the tower?



d From the top of a vertical cliff, Jung spots a boat 150 m out to sea. The angle of depression from Jung to the boat is 35°. How many metres (to the nearest whole number) above sea level is Jung?

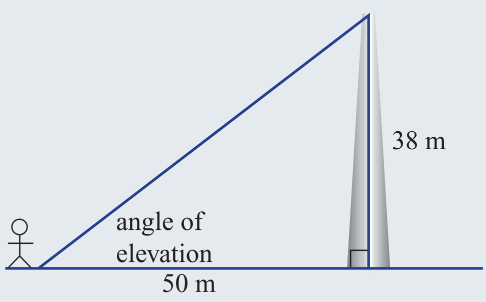


e A plane is flying 1800 m above the ground. At the time the pilots spot the runway, the angle of depression to the edge of the runway is 11°. How far does the plane have to fly to be above the edge of the runway at its current altitude? Give your answer to the nearest whole number.



Example 16 Finding angles of elevation and depression

A person on flat ground is 50 m from the base of a pole. The pole is 38 m high. Calculate the angle of elevation to the top of the pole.



Solution

$\tan\theta = \dfrac{O}{A}$

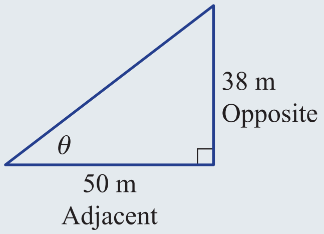
$\tan\theta = \dfrac{38}{50}$

$\theta = 37.2348\ldots°$

$\theta = 37°$ (to the nearest degree)

Angle of elevation is 37°.

Explanation



Aim is to find θ.

Enter $\tan^{-1} = \left(\dfrac{38}{50}\right)$.

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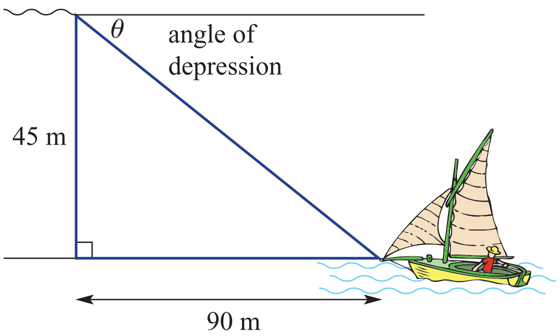
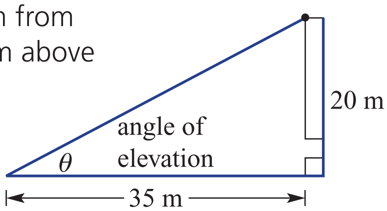
7G

6 Answer these questions about finding angles of elevation and depression. Round all answers to the nearest degree.

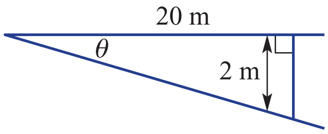
a From the top of a vertical cliff, Jacky spots a boat 90 m out to sea. If the top of the cliff is 45 m above sea level, find the angle of depression from the top of the cliff to the boat.



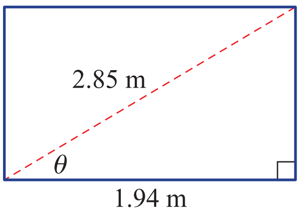
b Find the angle of elevation from a person sitting 35 m from a movie screen to the top of the screen, which is 20 m above the ground.



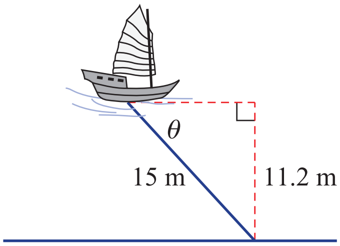
c A person sits 20 m away from a screen that is 2 m below the horizontal viewing level. Find the angle of depression of the person's viewing level to the screen.



d A diagonal cut 2.85 m long is to be made on a piece of plaster board attached to a wall, as shown. The base of the plaster board measures 1.94 m. Find the angle of elevation of the diagonal cut from the base.

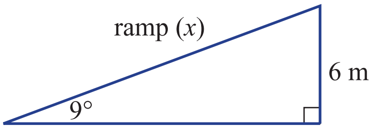


e As shown in the diagram, a 15 m chain with an anchor attached is holding a boat in position against a current. If the water depth is 11.2 m, find the angle of depression from the boat to where the anchor is fixed to the seabed.



Problem-solving and Reasoning

7 A ramp for wheelchairs is constructed to a footbridge that is 6 m high. The angle of elevation is to be 9°. What is the length of the ramp, correct to 2 decimal places?



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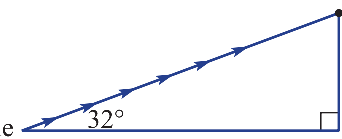
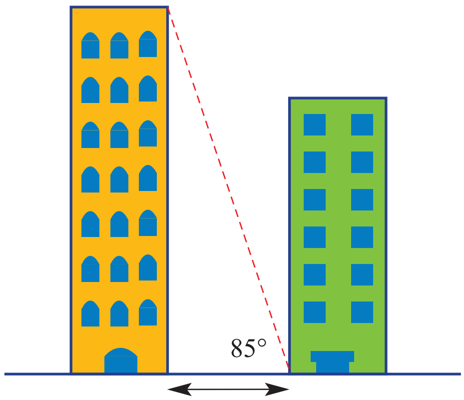
8 A missile is launched at an angle of elevation of 32°. If the target is 2 km away on the horizontal, how far above ground level is the target, correct to 2 decimal places?

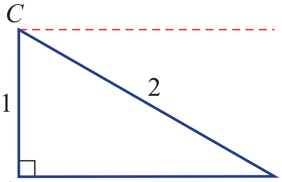


9 The distance between two buildings is 24.5 m. Find the height of the taller building, correct to 2 decimal places, if the angle of elevation from the base of the shorter building to the top of the taller building is 85°.



10 For this triangle:

- **a** Find the angle of elevation from B to C.
- **b** State the angle of depression from C to B.
- **c** Describe the relationship that exists between these two angles.
- **d** Find the length AB, correct to 1 decimal place.

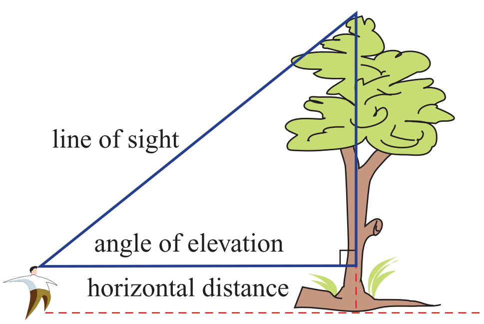


Enrichment: Practical trigonometry – measuring heights

11 It is not always possible or practical to measure the height of an object directly.

Select a building or other structure (e.g. a statue or flagpole) that is standing on flat ground. You must be able to measure right up to the base of the structure.

- **a** Choose a position from which you can see the top of your structure and measure the angle of elevation, θ, from your eye level. (Use a clinometer, if your teacher has one, or simply estimate the angle using a protractor.)



- **b** Measure the distance along the ground (d) from your location to the base of the structure.
- **c** Calculate the height of the structure. *Remember to make an adjustment for the height of your eye level from the ground.*
- **d** Move to another position and repeat the measurements. Calculate the height using your new measurements.
- **e** Is there much difference between the calculated heights? Suggest reasons for any differences.

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Maths@home

Worksheet 7A

Rex's fence

Rex needs a new fence between his house and the next-door neighbours'. The problem is that his land is very steep. He asked a fencer to give him a quote. When the quote arrived it was $9000. Rex decided that he and his son would do it themselves.

In this activity we will investigate the cost of doing this and the geometry and trigonometry involved in building a fence that runs down a hill.
The posts are 2.4 metres apart and the land slopes at $15°$. Between every pair of posts there are three rails.

The fence is made using:

- Square steel fence posts measuring 60 mm × 60 mm × 2.4 m, which are placed in holes dug 0.6 m deep.
- Concrete to hold the posts in place.
- Treated pine fencing rails measuring 75 mm × 50 mm, which are available in a number of lengths. Three rails run between two posts and the fence palings are nailed to the rails.
- Galvanised brackets and screws (1 bracket and 4 screws secures the end of a rail to a post).
- Treated pine fence palings measuring 100 mm × 12 mm × 1.8 m.
- Nails to attach the palings to the rails.

1 Make a sketch of two posts on the sloping ground ($15°$ to the horizontal). Show the three rails between them parallel to the ground. Show the top of the fence (the tops of the palings) running between the two posts. Label your sketch with appropriate dimensions and angles.

2 How would you calculate the length of each rail so that it fits exactly between posts? Remember, the rails slope parallel to the slope ground. Describe the method you would use for the calculation.

3 Carry out your calculation of the length of the rails needed with the help of a calculator.

In this downloadable worksheet you will estimate the cost of building the fence.

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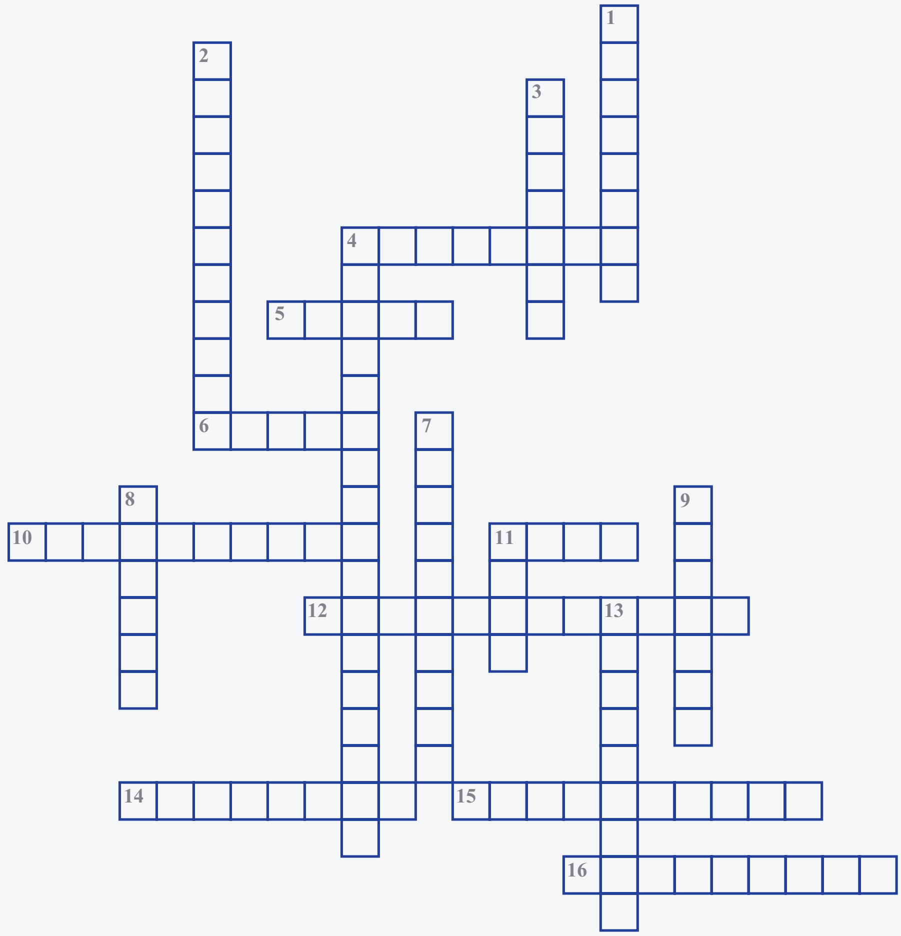
Complete the crossword below using words found in this chapter.



Across

4 The side next to the angle of reference.
5 A measure of rotation.
6 A comparison of two quantities or measurements.
10 The longest side in a right-angled triangle.
11 The edge of a triangle.
12 The study of the relationship between sides and angles in triangles.
14 A number sentence containing an equal sign.
15 90°
16 A memory aid for the trigonometric ratios.

Down

1 The side directly across from the angle of reference.
2 The number in the bottom of a fraction.
3 The __________ ratio.
4 The angle below a line of sight.
7 Greek mathematician.
8 The __________ ratio.
9 A unit used for measuring angles.
11 The __________ ratio.
13 Angle of __________.

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Chapter summary

Drilling for Gold 7R1 7R2 7R3

Right-angled triangles

Pythagoras' theorem

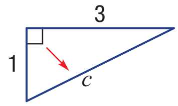
Finding the hypotenuse

$c^2 = 1^2 + 3^2$

$c^2 = 10$

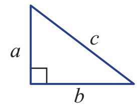
$c = \sqrt{10}$

$= 3.2$ (to 1 decimal place)



Pythagoras' theorem

For the triangle shown, *Pythagoras' theorem* is:



$c^2 = a^2 + b^2$

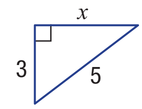
Finding a shorter side

$5^2 = x^2 + 3^2$

$x^2 = 5^2 - 3^2$

$x^2 = 16$

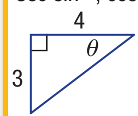
$x = 4$



Trigonometry

Finding angles

Use $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$.



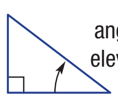
$\tan\theta = \frac{3}{4}$

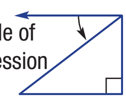
$\theta = 36.869\ldots°$

$\theta = 37°$

(to the nearest degree)

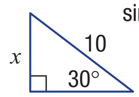
Elevation and depression





Angle of elevation is equal to angle of depression.

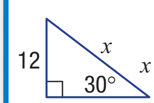
Finding lengths with trigonometry



$\sin 30° = \frac{x}{10}$

$x = 10 \sin 30°$

$x = 5$



$\sin 30° = \frac{12}{x}$

$x \times \sin 30° = 12$

$x = \frac{12}{\sin 30°}$

$x = 24$

Trigonometric ratios

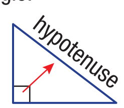
S O H → $\sin\theta = \frac{O}{H}$

C A H → $\cos\theta = \frac{A}{H}$

T O A → $\tan\theta = \frac{O}{A}$

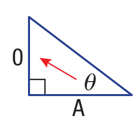
Hypotenuse

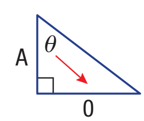
The *hypotenuse* is the longest side in a right-angled triangle. It is opposite the right angle.



Opposite and Adjacent

In a right-angled triangle, the sides marked 'opposite' and 'adjacent' are named according to the angle used.





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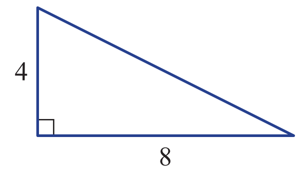
Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.

Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

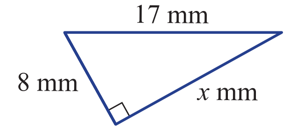
1 The length of the hypotenuse in the triangle shown is closest to:

A 10 **B** 9 **C** 4
D 100 **E** 64



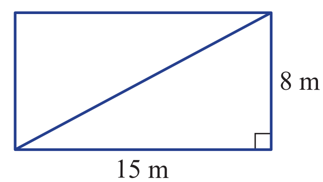
2 The length of the side marked x in the triangle shown is:

A 23 **B** 17 **C** 12
D 19 **E** 15



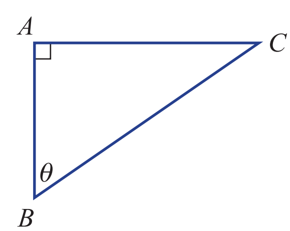
3 For the shape shown to be a rectangle, the length of the diagonal must be:

A 15 m **B** 8 m **C** 17 m
D 23 m **E** 32 m



4 Which side (i.e. AB, AC or BC) is the adjacent to θ in this triangle?

A AC **B** AB **C** BC
D hypotenuse **E** opposite

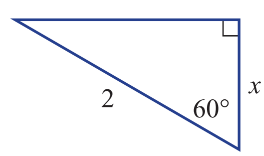


5 The value of cos 21° is closest to:

A –0.55 **B** 0.9 **C** 0.9336 **D** 0.93 **E** 0.934

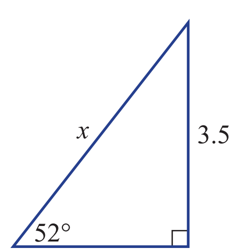
6 The value of x in this triangle is:

A $2 \div \cos 60°$ **B** $2 \div \sin 60°$ **C** $2 \times \cos 60°$
D $2 \times \sin 60°$ **E** $2 \times \tan 60°$



7 The value of x in this triangle is closest to:

A 2.76 **B** 4.48 **C** 5.68
D 4.44 **E** 2.73



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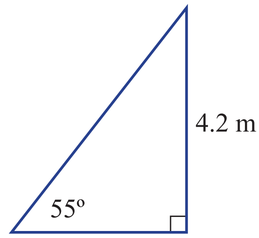
Chapter review

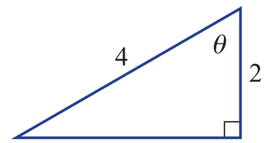
8 A metal brace sits at 55° to the horizontal and reaches 4.2 m up a wall. The distance between the base of the wall and the base of the brace is closest to:

A 6.00 m **B** 2.41 m **C** 7.32 m
D 5.13 m **E** 2.94 m



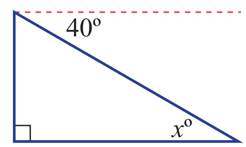
9 The angle θ in this triangle is:

A 60° **B** 30° **C** 26.57°
D 20° **E** none of the above



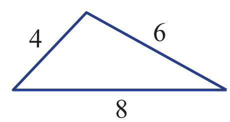
10 In the diagram shown, the angle of depression is 40°. The value of x is:

A 10° **B** 20° **C** 40°
D 50° **E** 140°

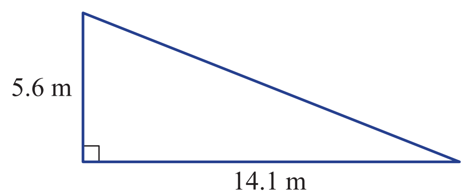


Short-answer questions

1 Determine whether or not the triangle shown contains a right angle.



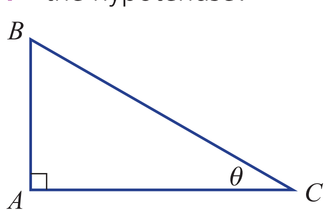
2 Find the length of the hypotenuse, correct to 2 decimal places, in the triangle shown.



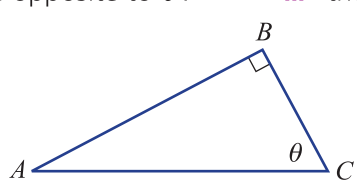
3 Which side (i.e. AB, AC or BC) of these triangles is:

i the hypotenuse? **ii** the opposite to θ? **iii** the adjacent to θ?

a



b



4 Use a calculator to find the value of each of the following, rounding your answer to 2 decimal places.

a sin 35° **b** cos 17° **c** tan 83°

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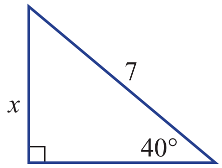
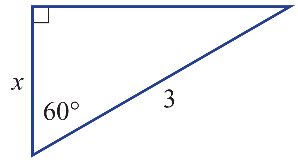
5 Find the value of the unknown length (x) in these triangles. Round your answer to 2 decimal places where necessary.

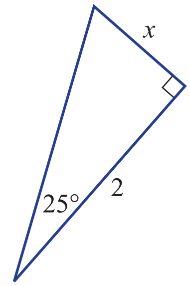
a



b

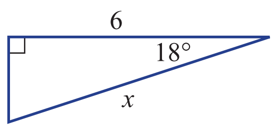


c

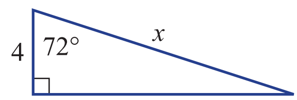


6 Find the value of the unknown length (x) in these right-angled triangles. Round your answer to 2 decimal places.

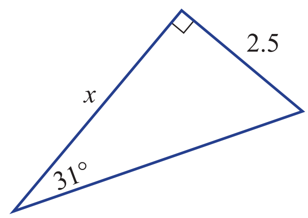
a



b



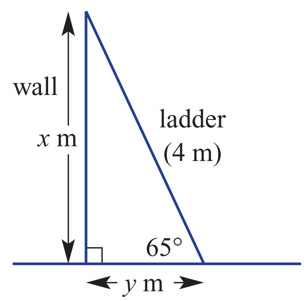
c



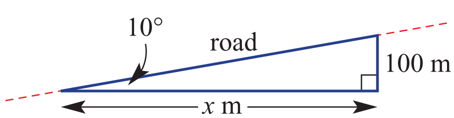
7 A 4 m ladder leans, as shown, against a wall at an angle of $65°$ to the horizontal.

a Find how high up the wall the ladder reaches (x m), correct to 2 decimal places.

b Find how far the bottom of the ladder is from the wall (y m), correct to 2 decimal places.



8 A section of road has a slope of $10°$ and gains 100 m in height. Find the value of x, correct to 2 decimal places.



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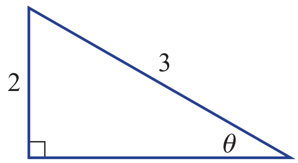
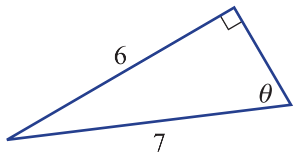
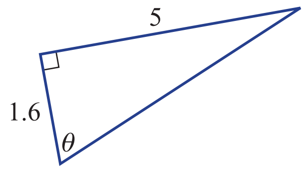
Chapter review

9 Find θ in the following right-angled triangles, correct to the nearest degree.

a


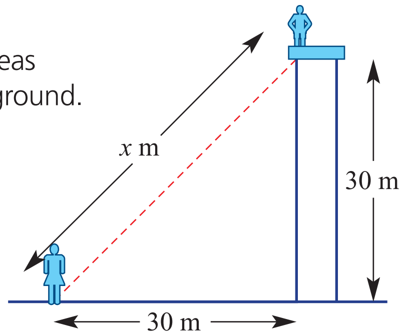
b


c


10 Barney and Mariana view each other from two different places, as shown. Barney is on a viewing platform, whereas Mariana is 30 m from the base of the platform, on the ground. The platform is 30 m above the ground.

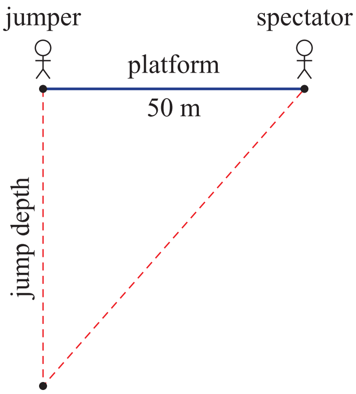
a Find the angle of elevation from Mariana's feet to the base of the platform.

b Find the distance (x m) between Mariana's and Barney, correct to 1 decimal place.



Extended-response questions

1 A spectator is viewing bungee jumping from a point 50 m to the side but level with the jumping platform.



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a The first bungee jumper has a maximum fall of 70 m. Find the angle of depression from the spectator to the bungee jumper at the maximum depth, correct to 2 decimal places.

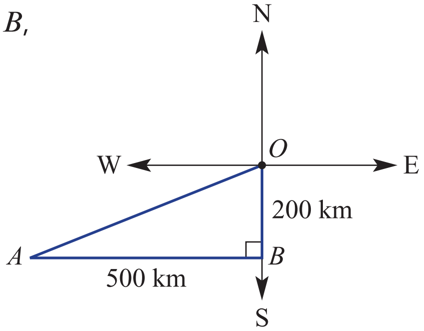
b The second bungee jumper's maximum angle of depression from the spectator is 69°. Find the jumper's maximum depth, correct to 2 decimal places.

c The third jumper wants to do the 'Head Dunk' into the river below. This occurs when the spectator's angle of depression to the river is 75°. Find, correct to the nearest metre, the height of the platform above the river.

2 A military plane flies 200 km from point O to point B, then west 500 km to point A.

a How far is A from O (to the nearest kilometre)?

b What is angle BOA, correct to the nearest degree?



Chapter 8

Equations and formulas

What you will learn

- **8A** Linear equations with pronumerals on one side
- **8B** Equations with brackets, fractions and pronumerals on both sides
- **8C** Using formulas
- Keeping in touch with numeracy
- Maths@work: Some formulas you might meet at work

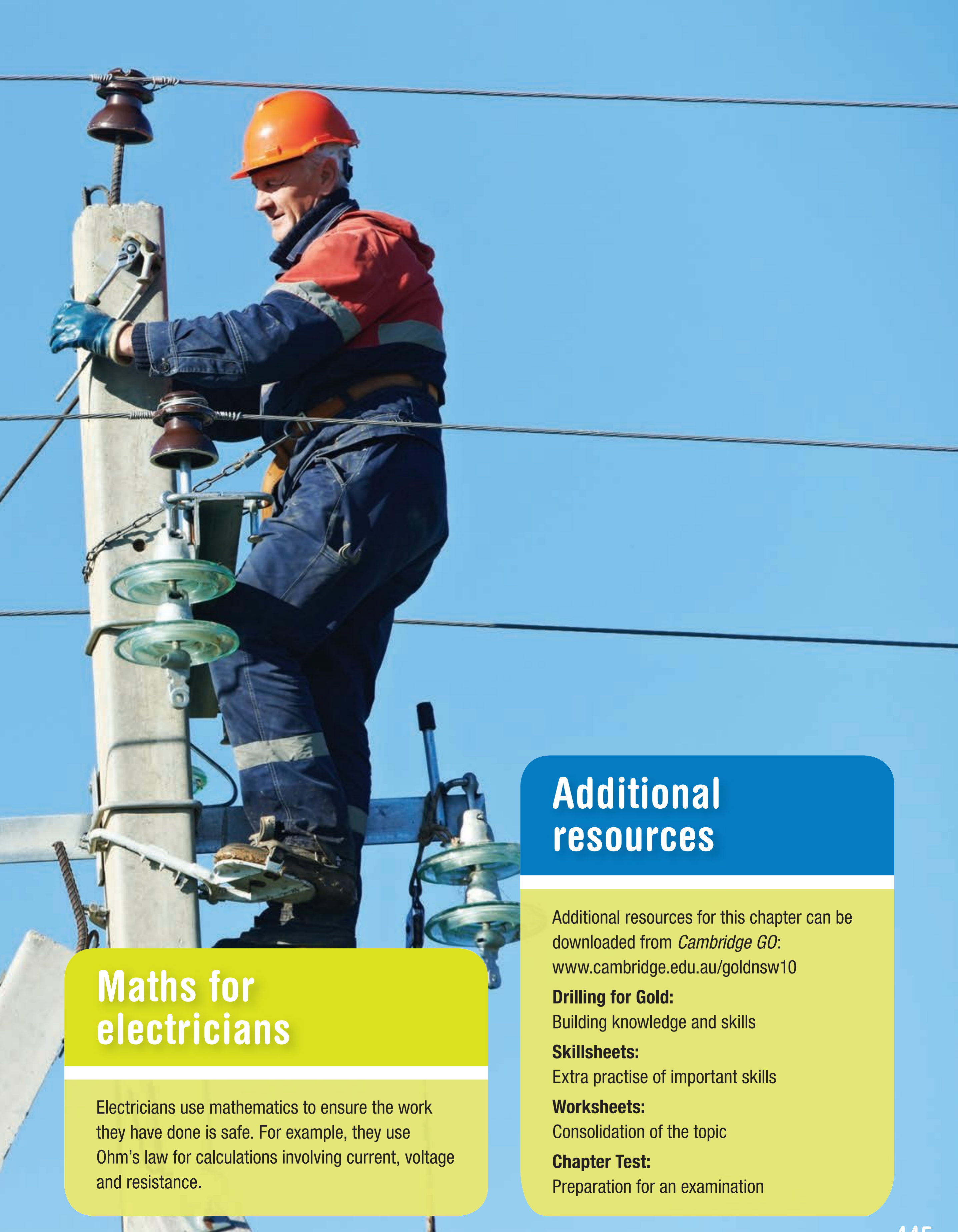
Strand: Number and Algebra

Substrand: EQUATIONS

In this chapter, you will learn to:

- use algebraic techniques to solve simple linear equations.

This chapter is mapped in detail to the NSW Syllabus for the Australian Curriculum in the teacher resources at: www.cambridge.edu.au/goldnsw10



Maths for electricians

Electricians use mathematics to ensure the work they have done is safe. For example, they use Ohm's law for calculations involving current, voltage and resistance.

Additional resources

Additional resources for this chapter can be downloaded from *Cambridge GO*: www.cambridge.edu.au/goldnsw10

Drilling for Gold:
Building knowledge and skills

Skillsheets:
Extra practise of important skills

Worksheets:
Consolidation of the topic

Chapter Test:
Preparation for an examination

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Pre-test

1 If $a = 6$ and $b = -3$, evaluate the following.
- **a** $a + b$
- **b** $a - b$
- **c** ab
- **d** a^2
- **e** b^2
- **f** $3(a + 2b)$

2 If $m = 4$, $n = 7$ and $p = -2$, evaluate the following.
- **a** $m + n + p$
- **b** $4m + p$
- **c** $p(4 - n)$
- **d** $3m + 2n$
- **e** $\frac{8m}{p}$
- **f** $2m^2$

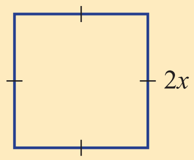
3 Simplify the following.
- **a** $a + 2a$
- **b** $4m - m$
- **c** $6p + 2p$
- **d** $7m - 7m$
- **e** $2m - 7m$
- **f** $8x + y - x$
- **g** $8p + 4p - 3p$
- **h** $7m - 4m + 3m$

4 Simplify the following.
- **a** $5x \times 3$
- **b** $4p \times 4$
- **c** $8x \times 4y$
- **d** $6a \times (-5)$
- **e** $a \times b$
- **f** $6x \div 6$
- **g** $m \div m$
- **h** $6a \div 3$
- **i** $\frac{15a}{5a}$

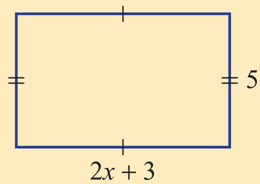
5 Write an expression for each of the following.
- **a** the sum of x and 3
- **b** 6 more than n
- **c** double w
- **d** half of x
- **e** 6 more than double x
- **f** 7 less than x
- **g** 3 more than x is then doubled
- **h** 1 more than triple x

6 Write an expression for the perimeter of the following.

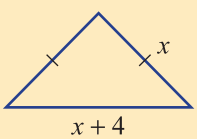
a



b



c



7 Choose the equations from the following.
- **a** $x + 3$
- **b** $3x - 6 = 9$
- **c** $x^2 - 8$
- **d** $2x$
- **e** $3a = 12$
- **f** $x^2 = 100$
- **g** $1 = x - 3$
- **h** $m - m$
- **i** $2p = 0$

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8A Linear equations with pronumerals on one side

Stage
5.2
5.2◊
5.1
4

A cricket batsman will put on socks, then cricket shoes and, finally, pads, in that order. When the game is over, these items are removed in reverse order: first the pads, then the shoes and, finally, the socks. A similar reversal occurs when solving equations.

We can undo the operations around the pronumeral (e.g. x) by applying the opposite operations in the reverse order to how they have been applied to the pronumeral. To keep each equation balanced, we always apply the same operation to both sides of an equation.

For example:

Applying operations to $x = 7$:

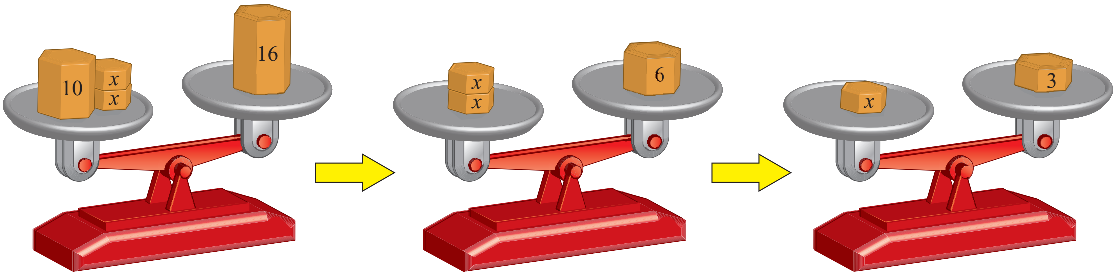
$$x = 7$$
$\times 2$ (both sides)
$$2x = 14$$
$+ 12$ (both sides)
$$2x + 12 = 26$$

Solving an equation:

$$2x + 12 = 26$$
$- 12$ (both sides)
$$2x = 14$$
$\div 2$ (both sides)
$$x = 7$$

▶ Let's start: Keeping it balanced

Three scales are each balanced with various weights on the left and right pans.



Balanced scales 1 → Balanced scales 2 → Balanced scales 3

- What weight has been removed from each side of scales 1 to get to scales 2?
- What has been done to both the left and right sides of scales 2 to get to scales 3?
- What equations are represented in each of the balanced scales shown above?
- What methods can you recall for solving equations?

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Key ideas

Drilling for Gold 8A1

- An **equation** is a mathematical statement that contains an equals sign. The equation will be true only when the left-hand side is equal to the right-hand side.
  For example: $\frac{5x}{6} = -2,\ 3p + 2t = 6$ are equations.
  $6x - 13$ is not an equation.

- A **linear equation** contains a pronumeral (e.g. x) to the power of 1 and no other powers.
  For example: $3x - 5 = 7,\ 4(m - 3) = m + 6$ are linear equations.
  $x^2 = 49$ is not linear.

- Every equation has a left-hand side (LHS) and a right-hand side (RHS).
  For example: $\underbrace{2x + 12}_{\text{LHS}} = \underbrace{26}_{\text{RHS}}$

- To **solve** an equation, undo the operations built around the pronumeral by doing the opposite operation in the reverse order.
  – Always perform the same operation to both sides of an equation so it remains balanced.
  Equations are solved like this:

$$\begin{aligned} 5x + 2 &= 17 \\ -2 \quad 5x &= 15 \quad -2 \\ \div 5 \quad x &= 3 \quad \div 5 \end{aligned}$$

  $x = 3$ is called 'the solution'.

- To 'verify' means to check that the solution is correct by substituting the answer to see if it makes the equation true.
  For example:
  Verify that $x = 3$ is a solution to $5x + 2 = 17$, using substitution.

$$\begin{aligned} \text{LHS} &= 5x + 2 \qquad \text{RHS} = 17 \\ &= 5 \times 3 + 2 \\ &= 17 \end{aligned}$$

  LHS = RHS
  $\therefore\ x = 3$ is a solution.

- Sometimes it is a good idea to swap the LHS and RHS.
  For example: $15 = 2x + 3$ is the same as $2x + 3 = 15$.

Equation A mathematical statement that states that two expressions have the same value

Linear equation An equation whose pronumerals are always to the power of 1 and do not multiply or divide each other

Solve To find the value of an unknown quantity

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Exercise 8A

Understanding

1 Decide whether $x = 2$ is a solution to these equations.

a $x + 3 = 5$ **b** $2x = 7$ **c** $x - 1 = 4$

d $2x - 1 = 10$ **e** $3x + 2 = 8$ **f** $2 - x = 0$

Substitute $x = 2$ to see whether LHS = RHS.

Example 1 Solving one-step equations involving addition

Solve $x + 7 = 12$.

Solution	Explanation
$x + 7 = 12$ (−7 from both sides)	Write the equation.
$x = 5$	Subtract 7 from both sides.
	Simplify.
Verify: LHS $= 5 + 7 = 12$ RHS $= 12$ LHS = RHS	Check that your answer is correct, using substitution.

2 Solve the following.

a $t + 5 = 8$ **b** $m + 4 = 10$ **c** $8 + x = 14$

d $m + 7 = 0$ **e** $x + 3 = 11$ **f** $x + 6 = 2$

g $m + 8 = 40$ **h** $a + 1 = -5$ **i** $16 = m + 1$

$8 + x = 14$ is the same as $x + 8 = 14$. $16 = m + 1$ is the same as $m + 1 = 16$.

Example 2 Solving one-step equations involving subtraction

Solve $x - 9 = 3$.

Solution	Explanation
$x - 9 = 3$ (+9 to both sides)	Write the equation.
$x = 12$	Add 9 to both sides.
	Simplify.
Verify: LHS $= 12 - 9 = 3$ RHS $= 3$ LHS = RHS	Check that your answer is correct, using substitution.

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8A

3 Find the value of x.

a $x - 3 = 3$ **b** $x - 7 = 2$ **c** $x - 8 = 9$

d $x - 3 = 0$ **e** $x - 2 = -8$ **f** $x - 5 = 7$

g $x - 12 = 24$ **h** $x - 50 = 70$ **i** $x - 1 = 100$

Example 3 Solving one-step equations involving multiplication

Solve $3x = 12$.

Solution	Explanation
$3x = 12$	Write the equation.
$\div 3$ (both sides)	Divide both sides by 3.
$x = 4$	Simplify.
Verify: LHS $= 3 \times 4$ $= 12$; RHS $= 12$; LHS = RHS	Check that your answer is correct, using substitution.

4 Solve the following.

a $8p = 24$ **b** $5c = 30$ **c** $27 = 3d$

d $2m = 16$ **e** $5z = 125$ **f** $9w = 81$

g $15p = 15$ **h** $6m = -42$ **i** $-10 = 20p$

$27 = 3d$ is the same as $3d = 27$. Swap LHS & RHS.

Example 4 Solving one-step equations involving division

Solve $\frac{x}{4} = 20$.

Solution	Explanation
$\frac{x}{4} = 20$	Write the equation.
$\times 4$ (both sides)	Multiply both sides by 4.
$x = 80$	Simplify.
Verify: LHS $= \frac{80}{4}$ $= 20$; RHS $= 20$; LHS = RHS	Check that your answer is correct, using substitution.

5 Solve each of the following equations.

a $\frac{x}{5} = 10$ **b** $\frac{m}{3} = 7$ **c** $\frac{a}{6} = -2$

d $\frac{z}{7} = 0$ **e** $\frac{x}{8} = -1$ **f** $\frac{w}{9} = -3$

g $8 = \frac{r}{7}$ **h** $\frac{w}{3} = \frac{1}{2}$ **i** $\frac{1}{4} = \frac{m}{2}$

$3 \cdot \frac{1}{2} = \frac{3}{1} \cdot \frac{1}{2} = \frac{3}{2}$

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Fluency

6 Solve the following equations.

a $x + 9 = 12$ **b** $x + 3 = 12$ **c** $x + 15 = 4$
d $3 = x - 7$ **e** $x - 2 = 12$ **f** $x - 5 = 5$
g $3x = 9$ **h** $4x = 16$ **i** $100 = 2x$
j $\frac{x}{5} = 4$ **k** $\frac{x}{3} = 7$ **l** $1 = \frac{x}{7}$

Carry out the 'opposite' operation to solve for x.

Example 5 Solving two-step equations

Solve $4x + 5 = 17$.

Solution	Explanation
$4x + 5 = 17$	Write the equation.
-5 both sides: $4x = 12$	First, subtract 5 from both sides.
$\div 4$ both sides: $x = \frac{12}{4}$	Then divide both sides by 4.
$x = 3$	Simplify.
Verify: LHS $= 4 \times 3 + 5 = 17$ RHS $= 17$ LHS = RHS	Check your answer, using substitution.

7 Solve the following equations.

a $2x + 5 = 7$ **b** $3x + 2 = 11$ **c** $4x - 3 = 9$
d $6x + 13 = 1$ **e** $8x + 16 = 8$ **f** $10x + 92 = 2$
g $3x - 4 = 8$ **h** $2x - 7 = 9$ **i** $5x - 4 = 36$
j $-10 = 2x - 6$ **k** $-24 = 7x - 3$ **l** $27 = 6x - 3$

In parts **j**, **k** and **l** it is okay to swap the LHS and RHS.

Example 6 Solving two-step equations involving simple fractions

Solve $\frac{x}{5} - 3 = 4$.

Solution	Explanation
$\frac{x}{5} - 3 = 4$	Write the equation.
$+3$ both sides: $\frac{x}{5} = 7$	Add 3 to both sides.
$\times 5$ both sides: $x = 35$	Multiply both sides by 5. Write the answer.
Verify: LHS $= \frac{35}{5} - 3 = 4$ RHS $= 4$ LHS = RHS	Check that your answer is correct, using substitution.

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8A

8 Solve the following equations.

a $\frac{x}{3}+2=5$ **b** $\frac{x}{6}+3=3$ **c** $\frac{x}{7}+4=12$

d $\frac{x}{4}-3=2$ **e** $\frac{x}{5}-4=3$ **f** $\frac{x}{10}-2=7$

g $-6=\frac{x}{8}-2$ **h** $-8=\frac{x}{4}-3$ **i** $10=\frac{x}{2}-1$

When solving equations, the order of steps is important. For $\frac{x}{3}-5$, undo the -5 first, then undo the $\div 3$.

Example 7 Solving more two-step equations

Solve $\frac{x+4}{2}=6$.

Solution	Explanation
$\frac{x+4}{2}=6$	Write the equation.
$\times 2$ $\quad x+4=12 \quad$ $\times 2$	Multiply both sides by 2.
-4 $\quad x=8 \quad$ -4	Subtract 4 from both sides.
Verify: LHS = $\frac{8+4}{2}$ = 6 RHS = 6 LHS = RHS	Check that your answer is correct, using substitution.

9 Solve the following.

a $\frac{m+1}{2}=3$ **b** $\frac{a-1}{3}=2$ **c** $\frac{x+5}{2}=3$

d $\frac{x+5}{3}=2$ **e** $\frac{n-4}{5}=1$ **f** $\frac{m-6}{2}=8$

g $\frac{w+4}{3}=-1$ **h** $\frac{m+3}{5}=2$ **i** $\frac{w-6}{3}=7$

j $2=\frac{a+7}{4}$ **k** $-5=\frac{a-3}{8}$ **l** $0=\frac{m+5}{8}$

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Problem-solving and Reasoning

Example 8 Writing equations from a word problem

For each of the following statements, write an equation and solve for the pronumeral.

a If 7 is subtracted from x, the result is 12.

b If x is divided by 5 and then 6 is added, the result is 10.

c If 4 is subtracted from x and that answer is divided by 2, the result is 9.

Solution	Explanation
a $x - 7 = 12$ $+7$ (both sides) $x = 19$	Start with x and then subtract 7. Solve the equation by adding 7 to both sides. 'The result' means '='.
b $\frac{x}{5} + 6 = 10$ -6 (both sides) $\frac{x}{5} = 4$ $\times 5$ (both sides) $x = 20$	Divide x by 5, then add 6 and make it equal to 10. Solve the equation by subtracting 6 from both sides first. Multiply both sides by 5 Check your answer.
c $\frac{x-4}{2} = 9$ $\times 2$ (both sides) $x - 4 = 18$ $+4$ (both sides) $x = 22$	Subtracting 4 from x gives $x - 4$, and then divide that answer by 2. Solve the equation by multiplying both sides by 2. Then add 4 to both sides.

Drilling for Gold 8A2 at the end of this section

10 For each of the following statements, write an equation and solve for the pronumeral.

a If 4 is added to x, the result is 6.

b If x is added to 12, the result is 8.

c If 5 is subtracted from x, the result is 5.

d If x is divided by 3 and then 2 is added, the result is 8.

e Twice the value of x is added to 3 and the result is 9.

f $(x - 3)$ is divided by 5 and the result is 6.

g 3 times x plus 4 is equal to 16.

5 subtracted from x is $x - 5$.

Drilling for Gold 8A3 at the end of this section

11 Write an equation and solve it for each of the following.

a The perimeter of a square is 52 cm. Determine the length of its side.

b The perimeter of an isosceles triangle is 42 mm. If the equal sides are both 10 mm, determine the length of the other side.

Draw a diagram and choose a pronumeral to represent the unknown side, then write an equation and solve it.

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8A

12 Convert the following into equations, then solve them for the unknown number.

- **a** n is multiplied by 2, then 5 is added. The result is 11.
- **b** Four times a certain number is added to 9 and the result is 29. What is the number?
- **c** Half of a number less 2 equals 12. What is the number?
- **d** A number plus 6 has been divided by 4. The result is 12. What is the number?
- **e** 12 is subtracted from a certain number and the result is divided by 5. If the answer is 14, what is the number?

Choose a pronumeral to represent the unknown number, then write an equation using the pronumeral. $\frac{1}{2}$ of x can be written as $\frac{x}{2}$.

13 Write an equation and solve it for each of these questions.

- **a** The sum of two consecutive numbers is 23. What are the numbers?
- **b** A person is 19 years older than another person. Their age sum is 69. What are their ages?
- **c** Andrew threw the shotput 3 m more than twice the distance that Barry threw it. If Andrew threw the shotput 19 m, how far did Barry throw it?

Consecutive numbers are one number apart; e.g. 3, 4, 5, 6 etc. The next consecutive number after x is $x + 1$.

Enrichment: Modelling with equations

14 A service technician charges \$40 up front and \$60 for each hour that she works. The equation for the total charge, \$$C$, of any job for h hours worked is $C = 40 + 60h$.

- **a** What will a 4-hour job cost?
- **b** If the technician works on a job for 3 days and averages 6 hours per day, what will be the overall cost?
- **c** If a customer is charged \$400, how long did the job take?

15 A petrol tank holds 71 litres. It originally contained 5 litres. The equation for the amount of fuel (V litres) in the tank at time t minutes is $V = 5 + 6t$.

- **a** How long it will take to fill the tank to 23 litres?
- **b** How long it will take to fill the tank?

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8A2: Equations match-up

Drilling for Gold exercise

Consider the equations **A** to **J**.
Match one of these equations to the questions below and solve for n in each case.
The first one has been done for you.
Use the worksheet or write the answers in your exercise book.

A $\frac{n}{2}+4=20$ **B** $n+4=20$ **C** $4n=20$ **D** $\frac{n}{2}-4=20$

E $\frac{n+4}{2}=20$ **F** $4n+4=20$ **G** $\frac{n}{4}=20$ **H** $2n-4=20$

I $2(n+4)=20$ **J** $\frac{n}{4}+2=20$

1 A number is decreased by 4 to give 20.	$n-4=20,\ n=24$
2 4 more than four times a number is 20.	
3 Dividing a number by 4 gives 20.	
4 A number is doubled, then 4 is subtracted to give 20.	
5 A number is increased by 4, then the result is doubled to give 20.	
6 The sum of a number and 4 is 20.	
7 A number is halved, then 4 is added to give 20.	
8 The product of 4 and a number is 20.	
9 A number is increased by 4, then the result is halved to give 20.	
10 A number is increased by 4 to give 20.	
11 4 less than half of a number is 20.	
12 2 more than a quarter of a number is 20.	

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Drilling for Gold exercise

8A3: I can solve problems!

Some mathematical problems can be solved using equations.
Use the following steps.

Step	Instruction
1	Use a pronumeral to stand for the unknown.
2	Write an equation to describe the situation.
3	Solve the equation, either by inspection or systematically.
4	Make sure that you answer the original question and that the solution seems reasonable and realistic.

Example:

My brother is 8 years older than me.

The sum of our ages is 36.

How old are we?

Solution:

1 Let a = my age, in years.

2 Therefore, my brother's age is $a + 8$.

The sum of our ages is 36, so:

3
$$\begin{aligned} a+a+8&=36\\ 2a+8&=36\\ 2a&=28\\ a&=14 \end{aligned}$$

Therefore, I am 14 and my brother is 22.

4 **Check:**

I am 14.

My brother is 22.

$14 + 22 = 36$ ✓

Use this method to solve the following problems on the next page.

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Drilling for Gold exercise

Use the worksheet or write the answers in your exercise book. Answers should be written out in full as complete sentences.

Problem 1
My brother is 8 years younger than me.
The sum of our ages is 50.
How old are we?

Problem 2
In a rectangle, one side is 4 cm longer than the other.
The perimeter is 60 cm.
How long are the sides?

Problem 3
In a rectangle, one side is 4 times longer than the other.
The perimeter is 60 cm.
How long are the sides?

Problem 4
In an isosceles triangle, one side is 3 cm longer than the equal sides.
The perimeter is 60 cm.
How long are the sides?

Problem 5
The equal sides of an isosceles triangle are 12 cm long.
The perimeter is 35 cm.
How long is the other side?

Problem 6
I can buy 3 rulers and 4 pencils for $13.50.
The pencils cost $1.50 each.
What is the price for each ruler?

Problem 7
A 50 m long rope is cut into four pieces.
Three of them are equal but the other is 6 m shorter.
How long are the pieces?

Problem 8
The sum of three consecutive integers is 105.
What are they?

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8B Equations with brackets, fractions and pronumerals on both sides

Stage

5.2
5.2◊
5.1
4

More complex linear equations may have variables on both sides of the equation and/or brackets. Examples are $6x = 2x - 8$ or $5(x + 3) = 12x + 4$.

Brackets can be removed by expanding. Equations with variables on both sides can be solved by collecting variables to one side, using addition or subtraction of a term.

More complex linear equations of this type are used when constructing buildings and in science and engineering.

Let's start: Steps in the wrong order

The steps to solve $8(x+2)=2(3x+12)$ are listed here in the incorrect order.

$$8(x+2)=2(3x+12)$$
$$x=4$$
$$2x+16=24$$
$$8x+16=6x+24$$
$$2x=8$$

- Arrange them in the correct order, working from the question to the solution.
- By considering all the steps in the correct order, write what has happened in each step.

Key ideas

- When solving linear equations:
  1. First, **expand** any brackets.
     In this example, multiply the 3 into the first bracket and the 2 into the second bracket.

$$3(2x - 1) + 2(x + 2) = 22$$
$$6x - 3 + 2x + 4 = 22$$

Expand Remove grouping symbols (such as brackets)

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2 Collect any **like terms** on the LHS and any like terms on the RHS. Collecting like terms on the left side of the example below. $5x - 3x = 2x$ and $-4 - 9 = -13$.

$$5x - 4 - 3x - 9 = x - 5 + 2x + 10$$
$$2x - 13 = 3x + 5$$

Like terms Terms with the same pronumerals and same powers

3 Sometimes it is good to swap the LHS and RHS.
For example: $2x - 13 = 3x + 5$

2 is less than 3
∴ swap LHS and RHS.
$2x - 13 = 3x + 5$
becomes
$3x + 5 = 2x - 13$

4 If an equation has variables on both sides, collect to one side by adding or subtracting one of the terms.
For example, when solving the equation $12x + 7 = 5x + 19$, first subtract $5x$ from both sides:

$$-5x \quad 12x + 7 = 5x + 19 \quad -5x$$
$$7x + 7 = 19$$

5 Start to perform the opposite operation to both sides of the equation.

6 Repeat step 5 until the equation is solved.

7 Verify that the answer is correct, using substitution.

- To solve a word problem using algebra:
  - Read the problem and find out what the question is asking for.
  - Define a variable and write a statement such as: 'Let x be the number of . . .'. The variable is often what you have been asked to find in the question.
  - Write an equation using your defined variable.
  - Solve the equation and check that the solution is reasonable.
  - Answer the question in words.

Exercise 8B

Understanding

1 Expand brackets and collect like terms in each of these expressions.

a $3(x - 1)$	**b** $5(x + 3)$
c $2(x + 2)$	**d** $3(x - 4)$
e $4(2x - 1)$	**f** $2(x + 5) + 3(x + 1)$
g $5(x + 4) + 2(x + 3)$	**h** $6(x + 2) + 3(x - 1)$

The number in front of the bracket needs to be multiplied to both terms inside the bracket.

$-5(2x - 3)$
$= -10x + 15$

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8B

2 For each of these equations, describe what could be done as the first step to collect the terms with x onto one side.

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a $5x = 2x + 12$
b $2x = x - 4$
c $8x = 3x + 25$
d $7x = -x + 8$
e $2x + 11 = 5x - 4$
f $3x + 47 = 8x + 2$
g $7x - 5 = -2x + 13$
h $2x + 3 = -3x + 38$

Add or subtract to remove the term containing x on the RHS.

Fluency

Example 9 Solving equations with brackets

Solve $4(x - 1) = 16$.

Solution

$$4(x - 1) = 16$$
$$4x - 4 = 16 \quad (+4)$$
$$4x = 20 \quad (\div 4)$$
$$x = 5$$

Explanation

Expand the brackets: $4 \times x$ and $4 \times (-1)$.

Add 4 to both sides.
Divide both sides by 4.

3 Solve each of the following equations by first expanding the brackets.

a $3(x + 2) = 9$
b $4(x - 1) = 16$
c $3(x + 5) = 12$
d $4(a - 2) = 12$
e $5(a + 1) = 10$
f $2(x - 10) = 10$
g $6(m - 3) = 6$
h $3(d + 4) = 15$
i $7(a - 8) = 14$
j $20 = 10(a + 2)$
k $15 = 5(3 + x)$
l $0 = 2(a - 3)$

Example 10 Solving equations with two sets of brackets

Solve $3(2x + 4) + 2(3x - 2) = 20$.

Solution

$$3(2x + 4) + 2(3x - 2) = 20$$
$$6x + 12 + 6x - 4 = 20$$
$$12x + 8 = 20 \quad (-8)$$
$$12x = 12 \quad (\div 12)$$
$$x = 1$$

Verify: LHS $= 3 \times 6 + 2 \times 1$
$= 20$

RHS $= 20$
LHS = RHS

Explanation

Use the distributive law to expand each set of brackets.
Collect like terms on the LHS.
Subtract 8 from both sides.
Divide both sides by 12.
Check your answer.

4 Solve the following equations.

a $3(2x + 3) + 2(x + 4) = 25$
b $2(2x + 3) + 4(3x + 1) = 42$
c $2(2x + 3) + 3(4x - 1) = 51$
d $3(2x - 2) + 5(x + 4) = 36$
e $4(2x - 3) + 2(x - 4) = 10$
f $2(3x - 1) + 3(2x - 3) = 13$
g $2(x - 4) + 3(x - 1) = -21$
h $4(2x - 1) + 2(2x - 3) = -22$

Expand each pair of brackets and collect like terms before solving.

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Example 11 Solving equations with variables on both sides

Solve the following for x.

a $7x + 9 = 2x - 11$ **b** $2x + 5 = 5x + 11$

Solution	Explanation
a $7x + 9 = 2x - 11$ ($-2x$ from both sides)	Subtract $2x$ from both sides.
$5x + 9 = -11$ (-9)	Subtract 9 from both sides.
$5x = -20$ ($\div 5$)	Divide both sides by 5.
$x = -4$	
Verify: LHS $= -28 + 9 = -19$, RHS $= -8 - 11 = -19$; LHS = RHS	Check your answer by substituting $x = -4$ in LHS and RHS.
b $2x + 5 = 5x + 11$	$2x + 5 = 5x + 11$ ($2x$ and $5x$ circled)
$5x + 11 = 2x + 5$ ($-2x$)	$2x$ is less than $5x$, so swap LHS and RHS.
$3x + 11 = 5$ (-11)	Collect like terms by subtracting $2x$ from both sides.
$3x = -6$ ($\div 3$)	
$x = -2$	
Verify: LHS $= 2 \times (-2) + 5 = 1$, RHS $= 5 \times (-2) + 11 = 1$; LHS = RHS	Always verify your solution, using substitution.

5 Find the value of x in the following.

a $7x = 2x + 10$ **b** $10x = 9x + 12$ **c** $4x - 12 = 8x$

d $6x = 2x + 80$ **e** $2x = 12 - x$ **f** $2x = 8 + x$

g $3x + 4 = x + 12$ **h** $x - 3 = 4x + 9$ **i** $2x - 9 = x - 10$

j $12 + 4x = 6x - 10$ **k** $9x = 10 - x$ **l** $1 - x = x + 3$

For parts **e**, **k** and **l**, you will need to add x to both sides.

Drilling for Gold 8B2

Example 12 Solving equations with fractions

Solve $\frac{2x+3}{4} = 2$ for x.

Solution	Explanation
$\frac{2x+3}{4} = 2$ ($\times 4$)	Multiply both sides by 4.
$2x + 3 = 8$ (-3)	Subtract 3 from both sides.
$2x = 5$ ($\div 2$)	Divide both sides by 2.
$= 2.5$	

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8B

6 Solve the following equations.

a $\dfrac{x+2}{3}=5$ **b** $\dfrac{x+4}{2}=5$ **c** $\dfrac{x-1}{3}=4$

d $\dfrac{x-5}{3}=2$ **e** $\dfrac{2x+1}{7}=3$ **f** $\dfrac{2x+2}{3}=4$

g $9=\dfrac{5x-3}{3}$ **h** $9=\dfrac{3x-6}{2}$ **i** $-3=\dfrac{5x-2}{4}$

Example 13 Solving equations with more difficult fractions

Solve $\dfrac{3x}{2}-4=2$ for x.

Solution	Explanation
$\dfrac{3x}{2}-4=2$	
$\dfrac{3x}{2}=6$ (+4 to both sides)	Add 4 to both sides.
$3x=12$ (×2 to both sides)	Multiply both sides by 2.
$x=4$ (÷3 to both sides)	Divide both sides by 3.

7 Solve the following equations.

Skillsheet 8A

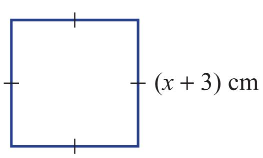
a $\dfrac{x}{3}+1=5$ **b** $\dfrac{x}{3}+1=7$ **c** $\dfrac{x}{4}-5=10$

d $5=\dfrac{3x}{4}-2$ **e** $\dfrac{2x}{5}-3=-1$ **f** $\dfrac{3x}{2}-5=-14$

Problem-solving and Reasoning

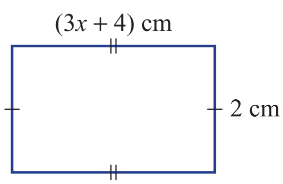
8 For each of these questions, write an equation and solve it for x.

a



Perimeter = 52 cm

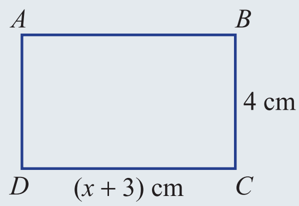
b



Perimeter = 22 cm

Example 14 Solving a word problem

Find the value of x if the area of rectangle $ABCD$ shown is 24 cm^2.



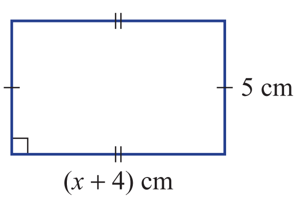
Solution

$$A = \ell \times b$$
$$24 = (x + 3) \times 4$$
$$24 = 4x + 12$$
$$4x + 12 = 24 \quad (-12)$$
$$4x = 12 \quad (\div 4)$$
$$x = 3$$

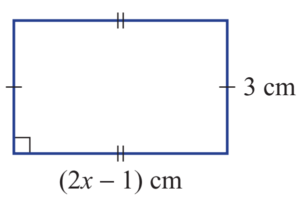
Explanation

Write an equation for area.
Substitute: $\ell = (x + 3)$, $b = 4$, $A = 24$
Expand the brackets: $(x + 3) \times 4 = 4(x + 3)$
Swap LHS and RHS.
Subtract 12 from both sides, then divide both sides by 4.

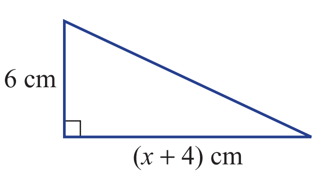
9 **a** Find the value of x if the area is 35 cm^2.



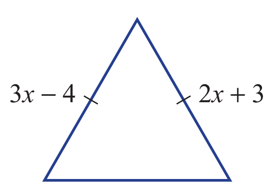
b Find the value of x if the area is 27 cm^2.



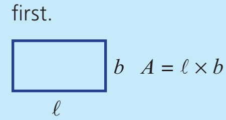
c Find the value of x if the area is 42 cm^2.

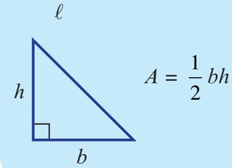


d Find the value of x.



Form the area equation first.





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8B

10 Using x for the unknown number, write down an equation and then solve it to find the number.

a The product of 5 and 1 more than a number is 40.
b The product of 5 and 6 less than a number is –15.
c When 6 less than 3 lots of a number is doubled, the result is 18.
d When 8 more than 2 lots of a number is tripled, the result is 36.
e 10 more than 4 lots of a number is equivalent to 6 lots of the number.
f 5 more than 4 times a number is equivalent to 1 less than 5 times the number.
g 6 more than a doubled number is equivalent to 5 less than 3 lots of the number.

- 'Product' means to multiply.
- 'The product of 5 and 1 more than a number' means $5(x + 1)$.
- '6 less than 3 lots of a number is doubled' will require brackets.
- 'Tripled' means three times a number.
- 'Equivalent' means equal to.

Enrichment: Wedding car

11 Yasmin and Zayne are planning to hire a car for their wedding day. 'Vehicles For You' have the following deal: \$850 hiring fee plus a charge of \$156 per hour. The number of hours must be a whole number.
The equation for the cost (\$$C$) of hiring a car for h hours is $C = 850 + 156h$

a If Yasmin and Zayne have budgeted for the car to cost a maximum of \$2000, find the maximum number of full hours they can hire the car.
b If the car picks up the bride at 1:15 p.m., at what time must the event finish if the cost is to remain within budget?

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8C Using formulas

Stage
5.2
5.2◊
5.1
4

A formula is an equation that relates two or more variables. You can find the value of one of the variables if you are given the value of all other unknowns.

You will already be familiar with many formulas. For example:

- $C = 2\pi r$ is the formula for finding the circumference, C, of a circle given its radius, r.
- $F = \frac{9}{5}C + 32$ is the formula for converting degrees Celsius, C, to degrees Fahrenheit, F.
- $s = \frac{d}{t}$ is the formula for finding the speed, s, given the distance, d, and time, t.

C, F and s are said to be the subjects of the formulas given above.

A metal worker building pipes applies circle, area and volume formulas.

▶ Let's start: Jumbled solution

Problem: The formula for the area of a trapezium is $A = \frac{1}{2}h(a + b)$.

Xavier was asked to find a given that $A = 126$, $b = 10$ and $h = 14$, and to write the explanation beside each step of the solution.

Xavier's solution and explanation are below. His solution is correct but he has jumbled up the steps in the explanation. Copy Xavier's solution and write the correct instruction(s) beside each step.

Solution	Explanation
$A = \frac{1}{2}h(a + b)$	Write the answer.
$126 = \frac{14}{2}(a + 10)$	Subtract 70 from both sides.
$\frac{14}{2}(a + 10) = 126$	Divide both sides by 7.
	Substitute the given values.
	Copy the formula.
$7(a + 10) = 126$	Simplify $\frac{14}{2}$.
$7a + 70 = 126$	
$7a = 56$	Expand the brackets.
$a = 8$	Swap LHS and RHS.

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Key ideas

- The **subject** of a **formula** is a variable that usually sits on its own on the left-hand side. For example, the C in $C = 2\pi r$ is the subject of the formula.
- A variable in a formula can be evaluated by substituting numbers for all other variables.
- A formula can be rearranged to make another variable the subject: $C = 2\pi r$ can be rearranged to give $r = \dfrac{C}{2\pi}$.

Subject The pronumeral or variable that is alone on one side of an equation

Formula A general rule for finding the value of one quantity, given the values of others

Exercise 8C

Understanding

1 State the letter that is the subject in these formulas.

a $I = PRN$ **b** $F = ma$ **c** $V = \dfrac{4}{3}\pi r^3$

d $A = \pi r^2$ **e** $c = \sqrt{a^2 + b^2}$ **f** $P = 2x + 2y$

The subject of a formula is the letter on its own, on the left-hand side.

2 Substitute the given values into each of the following formulas to find the value of each subject. Round the answer to 1 decimal place where appropriate.

a $m = \dfrac{F}{a}$, when $F = 180$ and $a = 3$

b $A = \ell b$, when $\ell = 6$ and $b = 8$

c $A = \dfrac{1}{2}(a + b)h$, when $a = 6$, $b = 12$ and $h = 4$

d $v^2 = u^2 + 2as$, when $u = 6$ and $a = 12$ and $s = 7$

e $m = \sqrt{\dfrac{x}{y}}$, when $x = 56$ and $y = 4$

Copy each formula, substitute the given values and then calculate the answer.

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Fluency

Drilling for Gold 8C1

Example 15 Substituting values and solving equations

If $v = u + at$, find t when $v = 16$, $u = 4$ and $a = 3$.

Solution	Explanation
$v = u + at$	Substitute each value into the formula: $v = 16$, $u = 4$, $a = 3$
$16 = 4 + 3t$	An equation now exists. Solve this equation for t.
$4 + 3t = 16$	Swap LHS and RHS.
$3t = 12$ (−4 from both sides)	Subtract 4 from both sides.
$t = 4$ (÷3 both sides)	Divide both sides by 3.
	Answer with the pronumeral on the LHS.

3 If $v = u + at$, find t when:

- **a** $v = 16,\ u = 8,\ a = 2$
- **b** $v = 20,\ u = 8,\ a = 3$
- **c** $v = 100,\ u = 10,\ a = 9$
- **d** $v = 84,\ u = 4,\ a = 10$

First copy the formula. Then substitute the given values. Solve the remaining equation. Sometimes it is wise to swap the LHS and RHS.

4 If $P = 2(\ell + b)$, find b when:

- **a** $P = 60,\ \ell = 10$
- **b** $P = 48,\ \ell = 6$
- **c** $P = 96,\ \ell = 14$
- **d** $P = 12.4,\ \ell = 3.6$

5 If $V = \ell bh$, find h when:

- **a** $V = 100,\ \ell = 5,\ b = 4$
- **b** $V = 144,\ \ell = 3,\ b = 4$
- **c** $V = 108,\ \ell = 3,\ b = 12$
- **d** $V = 280,\ \ell = 8,\ b = 5$

6 If $A = \frac{1}{2}bh$, find b when:

- **a** $A = 90,\ h = 12$
- **b** $A = 72,\ h = 9$
- **c** $A = 108,\ h = 18$
- **d** $A = 96,\ h = 6$

For $90 = \frac{1}{2} \times b \times 12$:
Swap b and 12.
$90 = \frac{1}{2} \times 12 \times b$
$90 = 6b$
Divide both sides by 6 to solve for b.

7 If $A = \frac{1}{2}h(a + b)$, find h when:

- **a** $A = 20,\ a = 4,\ b = 1$
- **b** $A = 48,\ a = 5,\ b = 7$
- **c** $A = 108,\ a = 9,\ b = 9$
- **d** $A = 196,\ a = 9,\ b = 5$

Undo the division by 2 by multiplying both sides by 2.

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8C

8 $E = mc^2$. Find m when:

a $E = 100$, $c = 5$
b $E = 4000$, $c = 10$
c $E = 72$, $c = 1$
d $E = 144$, $c = 6$

Square the c value before solving the equation.

9 If $V = \pi r^2 h$, find h (to 1 decimal place) when:

a $V = 160$, $r = 3$
b $V = 400$, $r = 5$
c $V = 1460$, $r = 9$
d $V = 314$, $r = 2.5$

For $160 = 9\pi h$, divide both sides by 9π to find h:

$h = \dfrac{160}{9\pi}$

Then evaluate using a calculator.

Problem-solving and Reasoning

10 The formula $F = \dfrac{9C}{5} + 32$ is used to convert temperature from degrees Celsius (°C) (which is used in Australia) to degrees Fahrenheit (°F) (which is used in the USA).

When finding C you will have an equation to solve.

a When it is 30°C in Sydney, what is the temperature in degrees Fahrenheit?
b How many degrees Celsius is 30° Fahrenheit? Answer to 1 decimal place.
c Water boils at 100°C. What is this temperature in degrees Fahrenheit?
d What is 0°F in degrees Celsius? Answer to 1 decimal place.

11 The cost (in dollars) of a taxi is $C = 3 + 1.45d$, where d is the distance travelled, in kilometres.

a What is the cost of a 20 km trip?
b How many kilometres can be travelled for $90?

12 $I = PRN$ calculates interest on an investment. Find:

a P when $I = 60$, $R = 0.08$ and $N = 1$
b N when $I = 125$, $R = 0.05$ and $P = 800$
c R when $I = 337.50$, $P = 1500$ and $N = 3$

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13 The number of tablets a nurse must give a patient is found by using the formula:

$$\text{Tablets} = \frac{\text{strength required}}{\text{tablet strength}}$$

a 750 milligrams of a drug must be administered to a patient. How many 500 milligram tablets should the nurse give the patient?

b If the nurse gives 2.5 of these tablets to another patient, how much of the drug did the patient take?

14 A drip is a way of pumping a liquid drug into a patient's blood. The flow rate of the pump, in millilitres per hour, is calculated using the formula: $\text{Rate} = \frac{\text{volume (mL)}}{\text{time (h)}}$

a A patient needs 300 mL of the drug over 4 hours. Calculate the rate in mL/h which needs to be delivered by the pump.

b A patient received 100 mL of the drug at a rate of 300 mL/h. For how long was the pump running?

Enrichment: Tax time

15 A tax agent charges $680 for an 8-hour day. The agent uses the formula $F = \frac{680x}{8}$ to calculate a fee to a client, in dollars.

a What does the x represent?

b If the fee charged to a client is $637.50, how many hours, to 1 decimal place, did the agent spend working on the client's behalf?

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Keeping in touch with numeracy

Non-calculator

1 What units do these abbreviations represent?
mm, t, ha, m^3, h

3 What is the remainder when 100 is divided by 7?

5 Copy and complete:

a $20\% = \frac{1}{\square} = 0.\square$

b $75\% = \frac{3}{\square} = 0.\square$

7 What number is halfway between 3.75 and 4?

9 A number is divided by 3. The result is increased by 5 to give 13. Use an equation to find the number.

11 The diameter of a water tank is 1.5 metres. What is its radius, in centimetres?

13 Use the formula $V = I \times R$ to find the value of V when $I = 15$ and $R = 4$.

15 A car was bought for \$4000 and then sold for \$5000. What is the percentage profit?

17 Write 127.5 million as a numeral and in scientific notation.

19 Increase 300 by 10%, then decrease the result by 10%.

Calculator

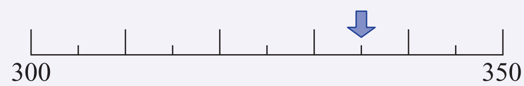
2 How many kilograms are there in 0.05 tonnes?

4 Bread rolls are placed into bags of 6. How many full bags can be created using 1000 rolls?

6 On a very rainy day, only 240 out of 400 students turn up to school. What is the absenteeism rate as:

a a percentage?
b a simple fraction?
c a decimal?

8 What measurement is indicated on the scale?



10 The area of a square is 50 cm^2. Calculate the perimeter of the square, correct to 1 decimal place.

12 The formula for the volume of a cylinder is $V = \pi r^2 h$. Find the volume of a cylinder with diameter 1.5 metres and height 1.5 metres. Give your answer correct to 2 decimal places.

14 Use the formula $V = I \times R$ to find the value of R when $V = 15$ and $I = 4$.

16 A \$30 000 car loses 15% in value every year. What is it worth at the end of 5 years?

18 Light travels at 299 792 458 metres per second. How many kilometres will it travel in 1 week?

20 The price of a shirt is decreased by 15% to \$297.50. What was the original price?

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Maths@work

Worksheet 8A

Some formulas you might meet at work

In some occupations, workers are required to use formulas to perform important calculations. For example, electricians use Ohm's law to decide which electrical wires they should use for different jobs.

Ohm's law can be written in three different ways:

$E = I \times R$ or $I = \frac{E}{R}$ or $R = \frac{E}{I}$

- E is the electromagnetic force, measured in volts.
- I is the intensity or strength of the electric current, measured in amperes.
- R is the resistance, measured in ohms.

In this activity, you will use formulas like this to do real-world calculations.

First, write Ohm's law in the three different ways above but use words in place of the symbols E, I and R.

1 When $I = 20$ and $R = 2$, find the value of E.

2 When $E = 18$ and $R = 3$, find the value of I.

3 When $E = 10$ and $I = 2$, find the value of R.

Download the worksheet, which gives an example to help with Ohm's Law and more formulas to use.

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Puzzles and games

1 The answers to these equations will form a magic square, where each row, column and diagonal will add to the same number. Draw a 4 by 4 square for your answers and check that they do make a magic square.

$x-3=6$	$x+15=10$	$\frac{x}{2}=-2$	$5x=30$
$3x+7=1$	$\frac{x}{4}-8=-7$	$\frac{x+7}{2}=5$	$3(x+4)=x+14$
$\frac{x}{2}-5=-4$	$4x-9=-9$	$x+7=4x+10$	$2(3x-12)-5=1$
$\frac{9-3x}{3}=6$	$-2(3-x)=x+1$	$x-16=-x$	$5x+30-3x=-3x$

2 Write an equation and solve it to help you find the unknown number in these puzzles.

a Three-quarters of a number plus 16 is equal to 64.

b A number is increased by 6, then that answer is doubled and the result is 4 more than triple the number.

c The average of a number and its triple is equal to 58.6.

d In 4 years' time, Ashwin's age will be double the age he was 7 years ago. How old is Ashwin now?

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Chapter summary

Equations, formulas and inequalities

Solving linear equations that have brackets

- Expand all brackets.
- Collect like terms on each side of the equation.
- Collect terms with a pronumeral to one side (usually the LHS).
- Solve for unknown.

e.g. $3(x+2) = 4x + 1$

$3x + 6 = 4x + 1$

$4x + 1 = 3x + 6$

$x + 1 = 6$

$x = 5$

Solving linear equations

Solving involves finding the value that makes an equation true.

e.g. $2x + 5 = 9$

$2x = 4$

$x = 2$

Equations with fractions

e.g.

$\frac{3x}{4} - 2 = 7$ (first $+2$ to both sides)

$\frac{3x}{4} = 9$ ($\times 4$ both sides)

$3x = 36$ ($\div 3$ both sides)

$x = 12$

e.g.

$\frac{2x-5}{3} = 7$ (first $\times 3$ to both sides)

$2x - 5 = 21$ ($+5$ both sides)

$2x = 26$ ($\div 2$ both sides)

$x = 13$

Solving word problems

1. Define variable(s).
2. Set up equation(s).
3. Solve equation(s).
4. Check each answer and write in words.

Formulas

Some common formulas

e.g. $A = \pi r^2$, $C = 2\pi r$

An unknown value can be found by substituting values for the other variables.

A formula can be rearranged to make a different variable the subject.

e.g. $E = mc^2$, find m when $E = 320$ and $c = 4$.

$320 = m \times 4^2$ (Substitute values.)

$320 = 16m$

$16m = 320$ (Swap LHS and RHS.)

$m = 20$ (Divide both sides by 16.)

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Chapter review

Chapter tests and worksheets can be found in the Teacher Resource Package. Interactive fill-in-the gaps and drag & drop literacy activities can be found in the Interactive Textbook.

Multiple-choice questions

An online version of this test is available in the Interactive Textbook.

1 The solution to $x + 7 = 9$ is:
A $x = 16$ **B** $x = -2$ **C** $x = 2$ **D** $x = 1$ **E** $x = -16$

2 To solve the equation $3(2x + 4) - 4(x + 2) = 6$, you would first:
A divide both sides by 12
B expand the brackets
C subtract 6 from both sides
D multiply both sides by 6
E add $4(x + 2)$ to both sides

3 A number is increased by 6 and then doubled. The result is 36. This translates to:
A $6x + 2 = 36$ **B** $2x + 6 = 36$ **C** $2(x + 6) = 36$
D $2(x - 6) = 36$ **E** $x + 12 = 36$

4 If $4a - 6 = 2a$, then a equals:
A -1 **B** 1 **C** 6 **D** 3 **E** -3

5 The solution to $\frac{5x}{9} - 4 = 1$ is:
A $x = 6$ **B** $x = -9$ **C** $x = -5$ **D** $x = 9$ **E** $x = 5$

6 The solution to $3(x - 1) = 12$ is:
A $x = -1$ **B** $x = 2$ **C** $x = 0$ **D** $x = 5$ **E** $x = 4$

Short-answer questions

1 Solve the following.
a $4a = 32$ **b** $\frac{m}{5} = -6$ **c** $1 = 9 + x$
d $x + x = 16$ **e** $9m = 0$ **f** $9 = w - 6$
g $8m = -1.6$ **h** $\frac{w}{4} = 1$ **i** $3 = r - 3$

2 Find the solutions to the following.
a $2m + 7 = 11$ **b** $3w - 6 = 18$ **c** $6 = 1 + \frac{m}{2}$
d $\frac{5w}{4} - 3 = 7$ **e** $\frac{m - 6}{2} = 4$ **f** $1 = \frac{3m + 2}{6}$
g $6a - 9 = 0$ **h** $4 - x = 3$ **i** $9 = 6 + x$

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3 Solve the following by first expanding the brackets.

a $3(m + 1) = 12$ **b** $4(a - 3) = 16$ **c** $30 = 5(2 + x)$
d $4(2x + 1) = 16$ **e** $2(3m - 3) = 9$ **f** $9 = 2(1 + 4x)$

4 Find the value of p in the following.

a $7p = 5p + 8$ **b** $2p = 12 - p$ **c** $5p = 6p + 9$
d $2p + 10 = p + 8$ **e** $3p + 1 = p - 9$ **f** $p - 2 = 4p - 8$

5 **a** For $A = \frac{1}{2}hb$, find b when $A = 24$ and $h = 6$.

b For $V = \ell bh$, find b when $V = 84$, $\ell = 6$ and $h = 4$.

c For $A = \frac{x + y}{2}$, find x when $A = 3.2$ and $y = 4$.

d For $E = mc^2$, find m when $E = 40$ and $c = 2$.

e For $F = \frac{9}{5}C + 32$, find C when $F = 95$.

6 Write an equation for the following and then solve it.

a Six times a number equals 420. What is the number?
b Eight more than a number equals 5. What is the number?
c A number divided by 9 gives 12. What is the number?
d Seven more than three times a number gives 16. What is the number?
e Two lots of the sum of a number and 6 is 18. What is the number?

Extended-response question

For the shape shown:

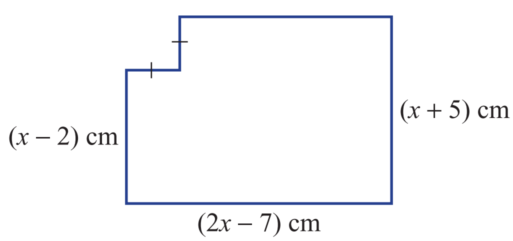
a Determine the equation of its perimeter.

b **i** If the perimeter is 128 cm, determine the value of x.

ii Find the actual side lengths.

c Repeat part **b** for perimeters of:

i 152 cm **ii** 224 cm



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Chapter 5: Linear and non-linear relationships

Multiple-choice questions

1 If a straight line has a gradient of –3 and a y-intercept of 5, its equation is:

A $y = 5$ **B** $y = 3x + 5$ **C** $y = 5x - 3$
D $y = -3x + 5$ **E** $m = -3$

2 The gradient of the line joining (0, 6) and (2, –4) is:

A –2 **B** 6 **C** 5
D $\frac{1}{5}$ **E** –5

3 The midpoint of the line segment between (–3, 8) and (7, 2) has coordinates:

A (4, 10) **B** (2, 5) **C** (2.5, 4.5) **D** (0.5, 9) **E** (–5, 3)

4 The equation and gradient of the vertical line through the point (1, 3) are:

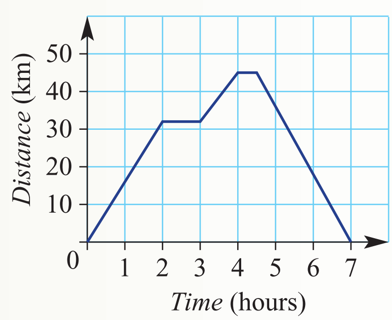
A $x = 1$; gradient undefined **B** $x = 1$; gradient zero
C $y = 3$; gradient positive **D** $y = 3$; gradient negative
E $y = 3$; gradient undefined

5 Which point lies on the line $y = 3x + 2$?

A (2, 0) **B** (1, 1) **C** (2, 2) **D** (5, 1) **E** (0, 2)

Short-answer questions

1 This distance–time graph shows the journey of a cyclist from home to a location and back again.



a How many kilometres had the cyclist travelled after:
  i 1 hour?
  ii 1.5 hours?
  iii 3 hours?

b Calculate the cyclist's speed over the first 2 hours.

c What was the total time in rest breaks?

d What was the cyclist's greatest distance from home?

e How long did the return trip take?

f Calculate the cyclist's speed for the return journey.

g What was the total distance cycled?

2 Copy and complete this table for the rule $y = 2x - 1$, then sketch its graph.

x	–2	–1	0	1	2	3
y						

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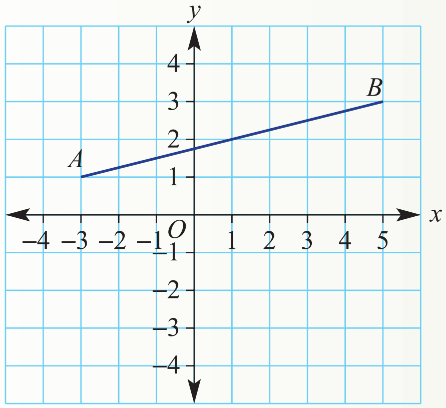
3 For each of the graphs below, find the midpoint of the line segment.

a



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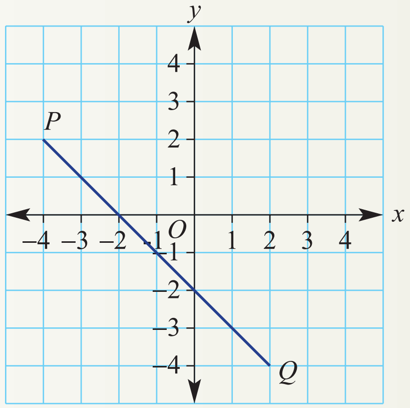


4 Plot and join each pair of points and for the line segment joining these points, find:

i its gradient, m **ii** its length (as a square root)

a $A(3, 2)$ and $B(5, 6)$ **b** $K(1, -3)$ and $L(-2, 6)$

c $P(3, 4)$ and $Q(-1, 9)$ **d** $R(-4, 2)$ and $M(1, 10)$

5 Sketch each of these lines.

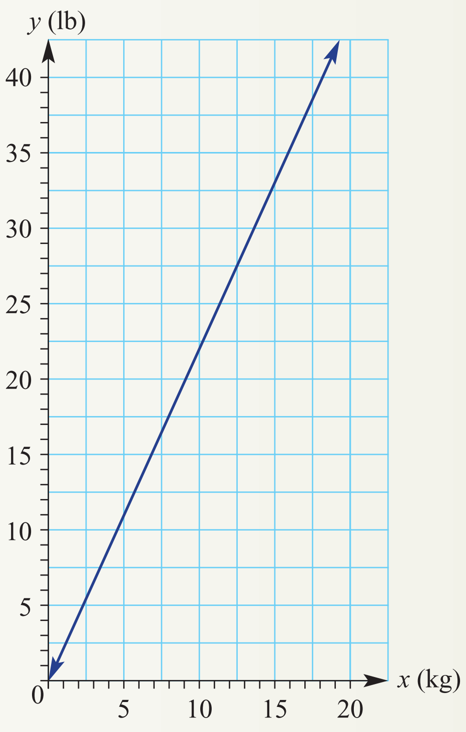
a $y = -3x + 4$ **b** $y = \frac{2}{3}x + 1$ **c** $y = -2$ **d** $x = -3$

6 This graph shows the relationship between weight in kilograms (kg) and weight in pounds (lb).

a Use the graph to convert 6 kg to pounds (lb).

b Use the graph to convert 35 lb to kg.

c Find 20 lbs in kg and use this to find the conversion rate in lb/kg, to 1 decimal place.



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Extended-response question

David and Kaylene travel from Melton to Moorbank. The total distance for the trip is 720 km, and they travel an average of 90 km per hour.

a Complete the table of values below from 0 to 8 hours.

Time (hours)	0	2	4	6	8
Distance from Moorbank (km)	720				

b Plot a graph of the number of kilometres from Moorbank against time.

c David and Kaylene start their trip at 6 a.m. If they decide to stop for breakfast at Albury and Albury is 270 km from Melton, what time would they stop for breakfast?

d If the car they are driving needs refilling every 630 km, for how long could they drive before refilling the car?

e What would be the total driving time if they didn't stop at all?

Chapter 6: Properties of geometrical figures

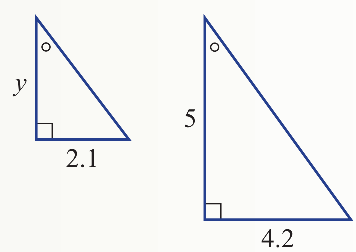
Multiple-choice questions

1 If two lines are parallel, then cointerior angles will:
A be equal **B** sum to 90° **C** sum to 180°
D sum to 360° **E** sum to 270°

2 The sum of the internal angles of a hexagon is:
A 360° **B** 540° **C** 1080° **D** 900° **E** 720°

3 Which of the following is not a test for congruent triangles?
A SSS **B** SAS **C** AAA **D** AAS **E** RHS

4 The scale factor for these similar triangles is:
A 2 **B** 4 **C** 5
D 0.1 **E** 0.4



5 In the diagram in Question **4**, the value of y is:
A 2 **B** 2.9 **C** 3 **D** 2.5 **E** 10

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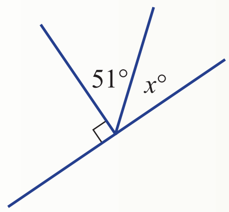
Short-answer questions

1 Find the value of x in these diagrams.

a



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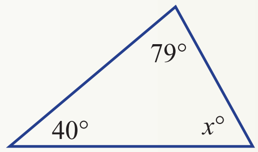

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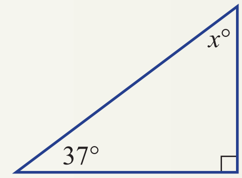

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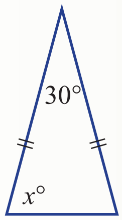

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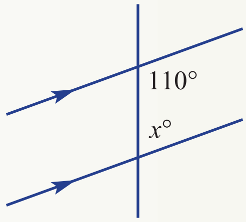

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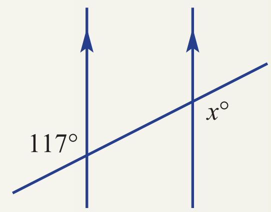

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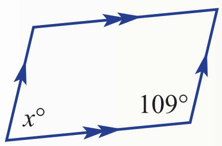
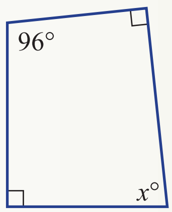
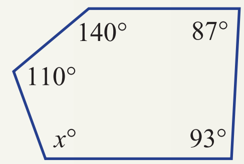


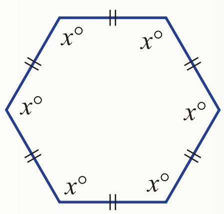
h



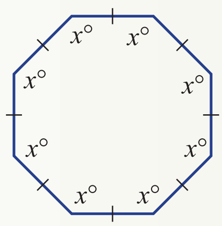
i



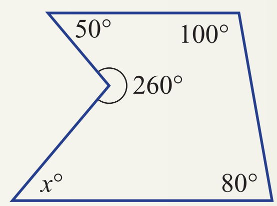
j



k



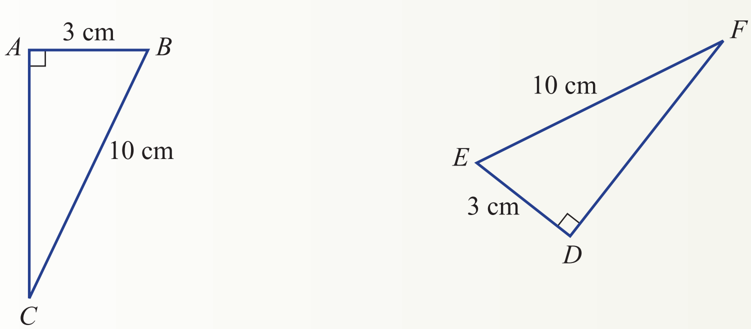
l



2 Write a congruence statement and the test to prove congruence in these pairs of triangles.

a

b



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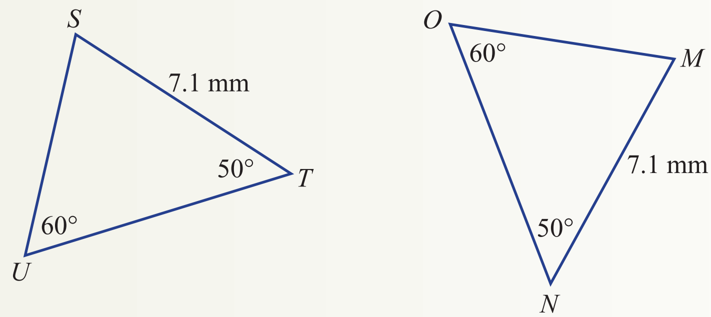
c



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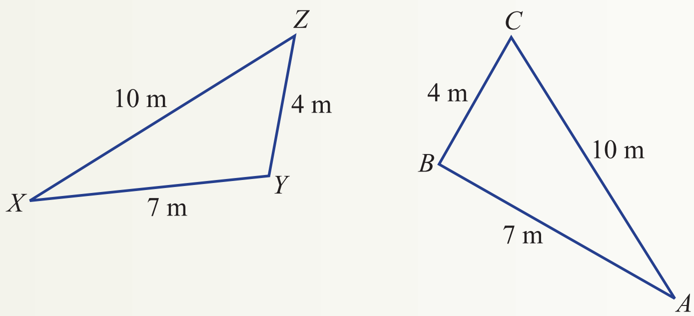
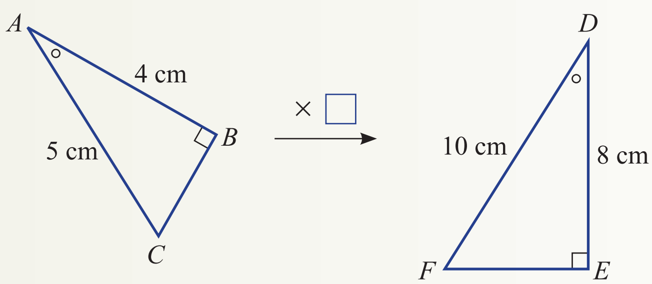


3 Find the scale factor.

a



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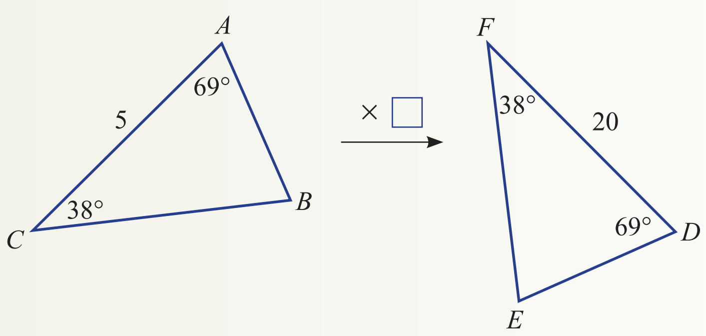

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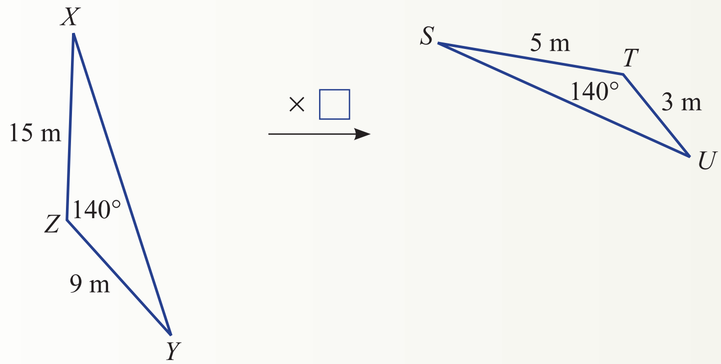
c



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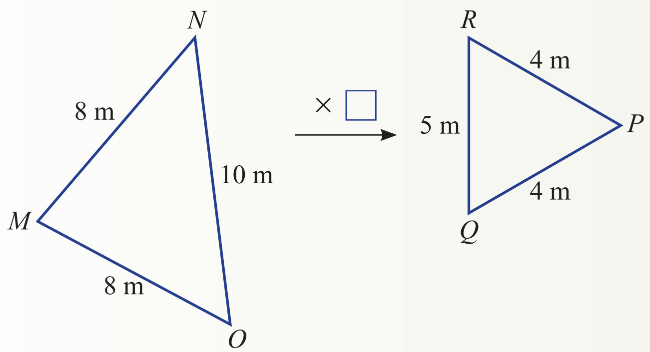
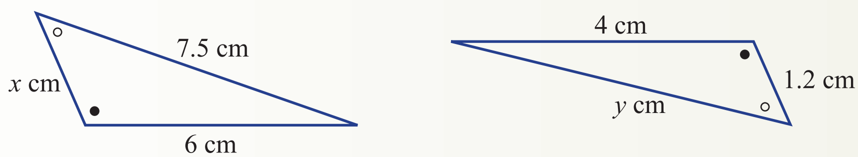


4 The given pair of triangles are known to be similar. Find the scale factor and value of x and y.

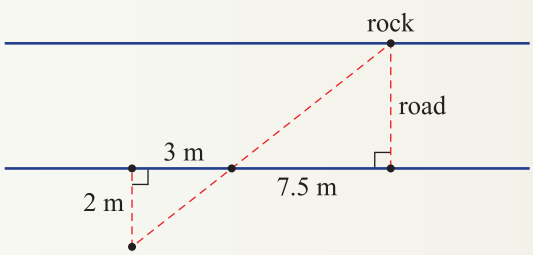


Extended-response question

A chicken wants to know the distance across the road without having to cross it. The chicken places four pebbles in various positions on its own side of the road, as shown. There is a rock on the other side of the road aligned with one of the pebbles. The triangles are similar.

a Find the scale factor.

b What is the distance across the road?



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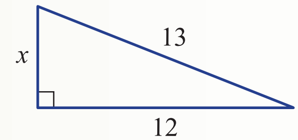
Chapter 7: Right-angled triangles

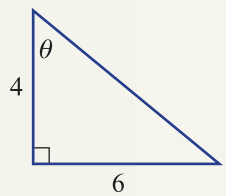
Multiple-choice questions

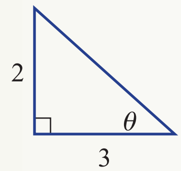
1 The value of x in the triangle shown is:

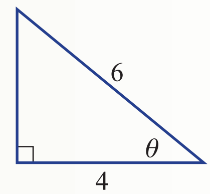
A 1 **B** 11 **C** 4
D 10 **E** 5

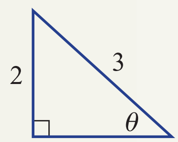


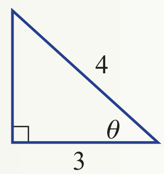
2 In which of the following triangles does $\cos\theta = \frac{2}{3}$?

A


B


C


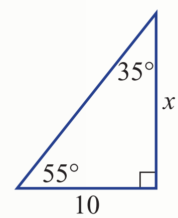
D


E


3 Choose the correct trigonometric statement for the diagram shown.

A $\tan 55° = \frac{x}{10}$ **B** $\tan 35° = \frac{x}{10}$ **C** $\sin 55° = \frac{x}{10}$

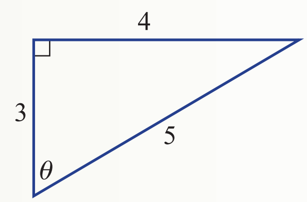
D $\sin 35° = \frac{x}{10}$ **E** $\cos 35° = \frac{x}{10}$



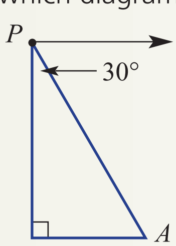
4 For the triangle shown, $\sin\theta$ is equal to:

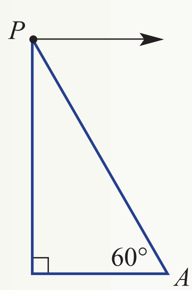
A $\frac{3}{5}$ **B** $\frac{4}{5}$ **C** $\frac{5}{3}$

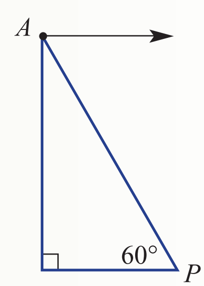
D $\frac{3}{4}$ **E** $\frac{4}{3}$

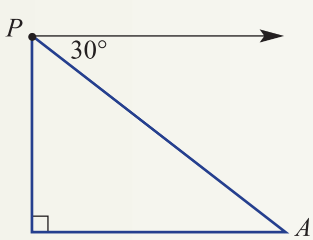


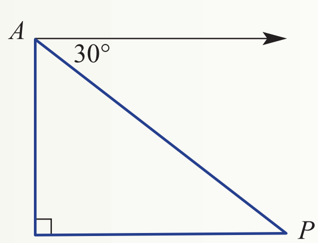
5 In which diagram is the angle of depression of A from P equal to 30°?

A


B


C


D


E


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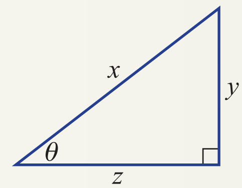
Short-answer questions

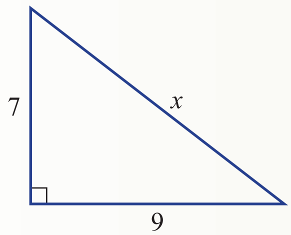
1 Use the triangle shown to help you write a fraction for:

a sin θ **b** cos θ **c** tan θ

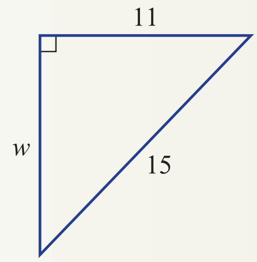


2 Find the value of each pronumeral, correct to 1 decimal place.

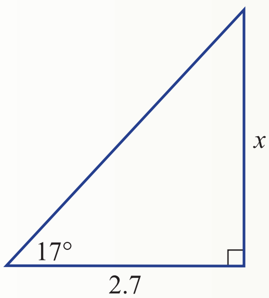
a



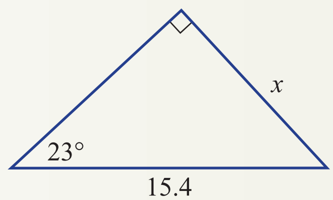
b



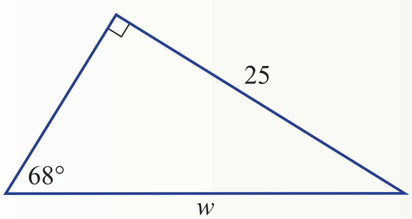
c



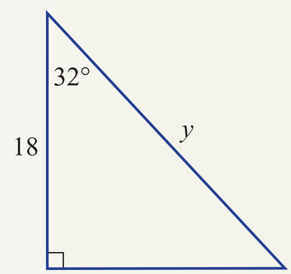
d



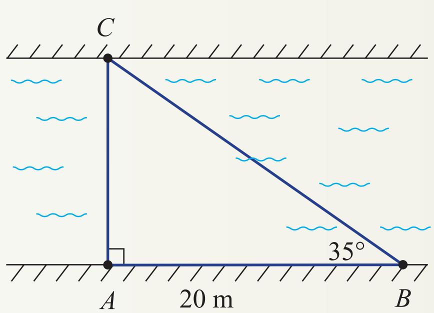
e



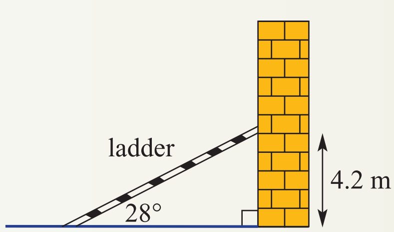
f



3 Kara wants to measure the breadth of a river. She places two markers, A and B, 20 m apart along one side. C is a point directly opposite marker A. Kara measures angle ABC as 35°. How broad is the river, to the nearest metre?



4 A ladder is inclined at an angle of 28° to the ground. If the ladder reaches 4.2 m up the wall, what is the length of the ladder, correct to 2 decimal places?



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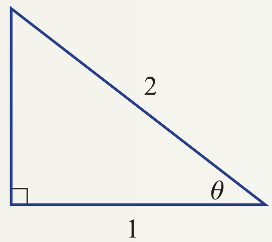
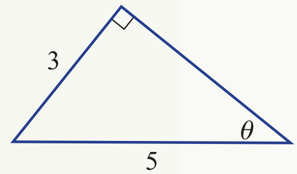
5 Find the angle θ in the following triangles, correct to the nearest degree.

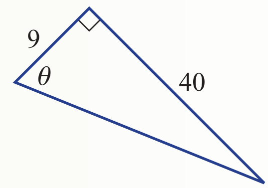
a



b

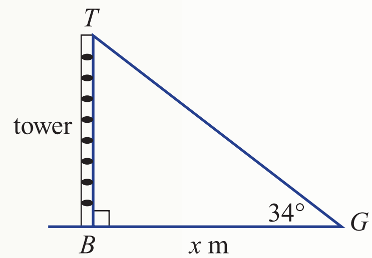


c

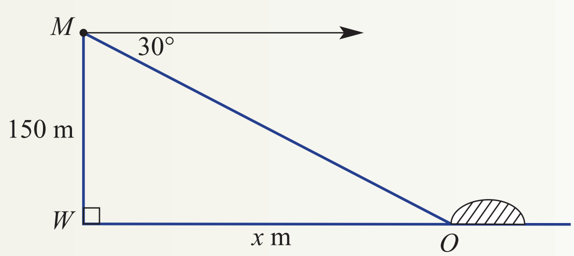


Extended-response questions

1 Justin measures the angles of elevation to the top of a 120 m tower to be 34°. How many metres is Justin from the base of the tower? Round your answer to 1 decimal place.



2 Alessia is sitting on top of a bridge 150 m above the water level of the river. She notices an object floating on the river some distance away. The angle of depression to the object is 30°. Find the value of x, correct to one decimal place.



Chapter 8: Equations and formulas

Multiple-choice questions

1 Which of the following is *not* an equation?

A $x - 3 = 5$ **B** $2x + 4 = 5x - 11$ **C** $y + 7x - 4$
D $y = 3x - 5$ **E** $y = 8$

2 A number is decreased by 8 and then doubled. The result is equal to 24. This can be written as:

A $2x - 8 = 24$ **B** $x - 8 \times 2 = 24$ **C** $x - 8 = 2 \times 24$
D $2(x - 8) = 24$ **E** $\dfrac{x-8}{2} = 24$

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3 The solution to $\frac{x-9}{3}=6$ is:

A $x=27$ **B** $x=45$ **C** $x=9$ **D** $x=11$ **E** $x=3$

4 The solution to $3(x-1)=5x+7$ is:

A $x=-4$ **B** $x=-5$ **C** $x=5$ **D** $x=3$ **E** $x=1$

5 Which equation has $x=2$, $y=5$ as a solution?

A $xy=7$ **B** $x-y=3$ **C** $y=x-3$ **D** $y=3-x$ **E** $x+y=7$

Short-answer questions

1 Solve the following equations.

a $2p+3=7$ **b** $3a-10=2$ **c** $\frac{x}{2}+3=9$ **d** $3=\frac{x-8}{4}$

2 Solve the following equations.

a $2(x-4)=8$

b $3(k-2)+4k=15$

c $m+5=3m-13$

d $\frac{3x+1}{2}=8$

e $\frac{3a-2}{7}=-2$

f $4x+7+3x-12=5x+3$

3 For each of the following statements, write an equation and then solve it for the pronumeral.

a When 5 is subtracted from x, the result is 8.

b When 8 is added to the product of 4 and x, the result is 20.

c When 6 less than 3 lots of x is doubled, the result is 18.

4 Find the value of the unknown in each of the following formulas.

a For $A=\frac{1}{2}bh$, find b when $A=120$ and $h=24$.

b For $I=PRN$, find P when $I=80$, $R=0.05$ and $N=4$.

Extended-response questions

1 **a** Solve the equation $\frac{2x}{3}+5=11$.

b Substitute your solution into both sides to ensure that your solution is correct.

c The equation contains the numbers 2, 3, 5 and 11. Change one of those numbers so that the solution will be $x=12$.

2 Repeat Question **1** using the equation $\frac{2x+5}{3}=11$.

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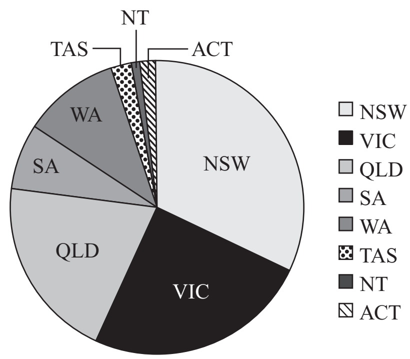
Chapter 1

Pre-test

1 a $99.62 b $1612 c $2742 d $84.51 e $1683.85
2 a 50% b 75% c 20% d 68% e 45%
3 a 16.79 b 7.35 c 45.34 d 6.84 e 102.90
4 a $4519.28 b $29 572 c $97 903.46
5 a $65 312 b $187 200 c $49 686 d $35 800
6 a $40 b $250 c $2 d $211 e $2.56 f $750
7 a $20 b $300 c $64
8 a $18 b $323 c $264 d $112 e $57.92
9 a $50 b $56 c $550

Exercise 1A

1 a $\frac{40}{100}$ b $\frac{85}{100}$ c $\frac{98}{100}$ d $\frac{28}{100}$ e $\frac{90}{100}$
2 a 100 b 90 c 60
3 a $0.75 b $80 c $49 d $501 e $103 f $1.20 g $37.50 h $400 i $4.50
4 a 50% b 20% c 25% d 10% e 1% f 28% g 30% h 75% i $62\frac{1}{2}$% j 76% k 99% l 94%
5 a 17% b 73% c 48% d 9% e 6% f 13% g 113% h 101% i 80% j 90% k 99% l 17.5%
6 a $\frac{71}{100}$ b $\frac{4}{5}$ c $\frac{1}{4}$ d $\frac{11}{20}$ e $\frac{2}{5}$ f $\frac{22}{25}$ g $\frac{3}{20}$ h $\frac{33}{200}$ i $\frac{7}{40}$ j $\frac{1}{40}$ k $\frac{11}{200}$ l $\frac{21}{40}$
7 a 0.61 b 0.83 c 0.75 d 0.45 e 0.09 f 0.9 g 0.5 h 0.165 i 0.073 j 2 k 4.3 l 0.005
8 a $25 b $150 c $60 d $90 e $21.60 f 344 grams g $50.40 h $107.80 i 350 m
9 a $\frac{15}{300} = \frac{1}{20}$ b 5%
10 68 kg
11 47.52 minutes
12 16.67%
13 $11\frac{1}{9}$%
14 $205.97
15 a 1.2, 1.7, 2.0, 1, 3.3, 0.1, 1.8, 2, 1.7
b i 32.1% ii 24.8% iii 10.8%
c NSW – 32%, Vic – 25%, Qld – 20%, SA – 7%, WA – 11%, Tas – 2%, NT – 1%, ACT – 2%
d 90°



Drilling for Gold 1A1

1 120
2 150
3 10%
4 20%
5 3
6 35%
7 50 mL
8 100
9 25%
10 45 g
11 60 cm
12 75%
13 $43.75
14 147 cm
15 17%
16 74.8
17 $620
18 55%
19 $22.50
20 400
21 136.5 m
22 8%
23 800 m
24 $500

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Drilling for Gold 1A2

1 10
2 62.5%
3 160
4 50
5 30
6 32
7 53.33 (2 d.p.)
8 60%
9 37.5%

Drilling for Gold 1A3

Students will check their answers with a calculator.

Exercise 1B

1 a 110% b 120% c 150%
d 102% e 118%
2 a 95% b 70% c 85%
d 50% e 83%
3 a P: $5 b P: $2.50 c Loss: $16
d P: $11 e Loss: $2.20
4 a 1.1, $165 b $250, ÷1.1
5 a $94.50 b $440 c $66 d $529.20
e $56 f $7210 g $56.43 h $3.60
6 a $1425 b $360 c $376 d $68
e $412.50 f $47.03 g $101.58 h $35.80

7

a	$6	60%
b	$60	25%
c	$3	20%
d	$7.50	3%
e	$2325	75%
f	$0.99	18%

8 a $52.25 b $261.25
9 a $1225 b $24.50
10 $1536
11 a $67.96 b $11.99
12 a $2140.25 b $305.75
13 a $221.40 b $147.60
14 a $84.13 b $94.87
15 $104.71
16 a $106.15 b $37.15
17 a $280 b $28
18 a $2.20 b 122.22% c $66 d 122.22%
19 a $13 b $6.30 c $69.30 d 38.6%
20 a $1952.24 b $211.24 c 12.13% d $57.03

Drilling for Gold 1B2

1 $132.30
2 $108.30
3 $119.70
4 $119.70
5 $174.96
6 $126.96
7 $149.04
8 $149.04
9 74 012
10 33 242
11 18 years
12 17 years
13 $17 748.21
14 $4801.02

Exercise 1C

1 a $15 b $22.50 c $30 d $29 640
2 $5600 a month by $200
3 $36 842
4

	Gross income	Net income
a	$570	$415
b	$984	$764
c	$604.90	$304.90
d	$3430	$2920
e	$930.15	$746.15

5 a $2372.48 b $61 684.48 c $5140.37
6 $10 312.50
7 a $1960 b $2460
8 a $519.23 b $150 c $669.23 d $38 425
9 a $1057.50 b $157.50
10 $3760 11 3%
12 $365.70 13 $437.50 14 $2000
15 a Kuger Incorporated
b Mathsville Credit Union, 00754031
c $72 454 d fortnightly e $4420 f $26.45
g $600.60 h $16 016 tax, net = $49 793.90

Maths@work: So many ways to make a living!

Answers will vary.

Exercise 1D

1 Taxable income = gross income minus deductions
2 false
3 Anything from $0 to $18 200 4 37c
5 Answers will vary as the tax rates are changed every financial year.

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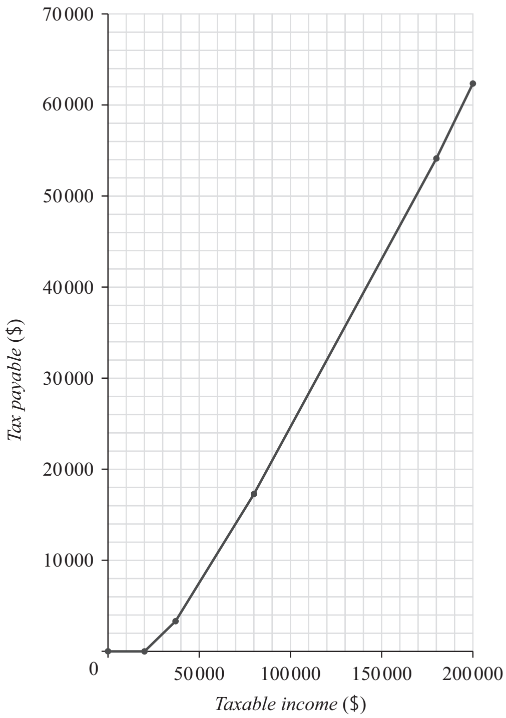
Exercise 1D cont.

6



7 $6172

8 a $65 625 b $12 875.13 c $984.38
d $13 859.51 e 21.1% (to 1 d.p.)
f Not enough paid; owes $789.51

9 $87 500

10 $6172.84

11 $95 000

12 Gross income is the total income earned before tax is deducted. Taxable income is found by subtracting tax deductions from gross income.

13 If a person pays too much tax during the year they will receive a tax refund. If they do not pay enough tax during the year they will have a tax liability to pay.

14 They only pay 45 cents for every dollar over $180 000.

15 a The tax-free threshold has been increased from $6000 to $18 200. In the second tax bracket, the rate has changed from 15c to 19c. In the third tax bracket, the rate has changed from 30c to 32.5c.

b

	2011/2012	2012/2013	
Ali	$0	$0	No change
Xi	$1350	$0	$1350 less tax to pay
Charlotte	$3600	$2242	$1358 less tax to pay
Diego	$8550	$7797	$753 less tax to pay

16

	Resident	Non-resident	
Ali	$0	$1625	Non-residents pay a lot more tax than residents.
Xi	$0	$4875	
Charlotte	$2242	$9750	
Diego	$7797	$16 250	

17 a Answers will vary.
b i $17 547
ii $32.50, so this means that the $100 donation really only cost you $67.50.

Keeping in touch with numeracy

1 $\frac{5}{6}$

2 40

3 C

4 0.12, $\frac{3}{25}$

5 a 50% b 30%

6 a 37.5% b 18.75%

7 100

8 385

9 250

10 1.25

11 17°

12 1

13 33

14 $455.81 (to nearest cent)

15 60%

16 216°

17 triangle, trapezium

18 55

19 $32

20 $50

Exercise 1E

1 a 1 b 6 c 52 d 30

2 a $120 b $420 c $30

3 a $420 b $840 c $35

4 a $140 b $420 c $192.50
d $46.88 e 3% p.a. f 4% p.a.

5 a $6650 b $184.72 per month

6 a $5192.25 b $16 692.25 c $198.72

7 a $7600 b $17 600 c $366.67

8 a $228 b $684 c $4684

9 $1008

10 16% 11 12.5 years 12 $66 667 13 $7500

14 a $1250, $2500 b $1968.75, $1920.00
c $220.31, $331.11 d Answers will vary.

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Exercise 1F

1 **a** \$50 **b** \$550 **c** \$55 **d** \$605 **e** \$605

2 **a** \$1102.50 **b** \$1102.50 **c** \$1157.63 **d** \$1157.63

3 **a** $700(1.08)^2$ **b** $1000(1.15)^6$ **c** $850(1.06)^4$

4 **a** 6, 0.02625 **b** 60, 0.01 **c** 52, 0.00173
d 14, 0.02625 **e** 32, 0.0375 **f** 120, 0.008

5

2	4200	210	4410
3	4410	220.50	4630.50
4	4630.50	231.53	4862.03
5	4862.03	243.10	5105.13

6 **a** \$5105.13 **b** \$11 946.33
c \$13 652.22 **d** \$9550.63

7 **a** \$106 000 **b** \$112 360 **c** \$119 101.60
d \$133 822.56 **e** \$179 084.77 **f** \$239 655.82

8 **a** \$2254.32 **b** \$87 960.39 **c** \$1461.53
d \$789.84 **e** \$591.63

9 **a** 18 years
b 12 years
c 10 years
d 7 years

10 **a** **i** \$3239.42 **ii** \$3348.15 **iii** \$3446.15
iv \$3461.88 **v** \$3465.96
b \$226.54

11 **a** \$3000 **b** twice **c** 8% p.a. **d** \$1440.73

12 5.3% compounded biannually

13 \$8874.11

14 Every year the car has an opening value of A. During the year it loses 15% of A. A is more than 15% of A. Therefore, A – 15% of A will not be zero.

Exercise 1G

1 **a** \$2640 **b** \$3960 **c** \$13 200

2 \$124.50

3 **a** \$1.20 **b** \$1.71 **c** \$3 **d** \$0.09

4 **a** \$18 600 **b** \$8600

5 **a** \$5580 **b** \$80

6 **a** 360 **b** \$624 960 **c** \$374 960

7 **a** \$2550 **b** \$10 200 **c** \$10 620
d \$13 170 **e** \$420

8 **a** \$0.38 **b** \$2.87

9 **a**

May	June	July	August
\$13.33	\$16.67	\$20	\$23.33

b \$73.33

10 **a** **i** \$0 **ii** \$0 **iii** \$7500
b \$6375 **c** \$1125

11 **a** **i** \$5250 **ii** \$20 250 **iii** \$281.25
b **i** \$8400 **ii** \$32 400 **iii** \$270

12 **a**

Date	Deposit	Withdrawal	Balance
1 May			\$3010
3 May	\$490		\$3500
5 May		\$2300	\$1200
17 May	\$490		\$1690
18 May		\$150	\$1540
20 May		\$50	\$1490
25 May		\$218	\$1272
31 May	\$490		\$1762

b \$4.90 **c** \$1759.55 **d** wages

13 **a** **i** \$73.13 **ii** \$69.72 **iii** \$75.17
b \$1700 over 3 years

14 **a** \$403.80 **b** \$393.80 **c** 24 cents a day

15 **a** \$98 822.90 **b** \$0.23 **c** \$8.00
d \$2400 **e** \$378.50 **f** \$246 025

Exercise 1H

1 B

2 **a** 200 **b** B **c** 5200 **d** = D2

3 **a** The interest in Year 1 is equal to the interest in Year 2.
b 6% p.a.
c = 6/100 * B2
d Formula for cell C3 is either = 6/100 * B3 or = C2
Formula for cell D3 is = B3 + C3

4 **a** **i** \$7146.10 **ii** \$6955.64 **iii** \$6858.57
iv \$7260 **v** \$7916.37
b \$6000 at 5.7% p.a. for 5 years

5 **a** **i** \$7080 **ii** \$7080 **iii** \$7428
iv \$7200 **v** \$7710
b 6000 at 5.7% p.a., for 5 years

6 **a** \$25 500 **b** \$21 675
c \$18 423.75 **d** \$5906.23

7 **a** \$29 300 **b** \$28 593
c \$27 878.93 **d** \$22 676.45

8 **a** 4 to 5 years **b** 4 to 5 years **c** yes

9 **a** approx. 35–36 months **b** approx. \$5850

10 Lauren will pay \$4700 in interest, which is approximately \$1150 less than Steve.

11 Steve will pay approximately \$7830 in interest, which is approximately \$1980 more than Question **9**.

12 Check with your teacher

Puzzles and games

You take away his credit card

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Multiple-choice questions

1 E 2 D 3 D 4 C 5 B
6 E 7 B 8 B 9 C 10 E

Short-answer questions

1 $1395
2 $1084.16
3 $4557
4 a $11.40 b $3.80
5 $4200
6 $576.92
7 a $7400 b $616.67 c $142.31
8 a $574.64 b $86.20 c 5.39%
9 a $12 525 b approx. 37%
10 a $1040.04 b $874.95
11 $7095.65
12 $35.55

Extended-response questions

1 a $5624.32 b $624.32 c $636.36
2 a $1050 b $12 000 c $6050
3 a $1600 b $1166.67 c $624.32 d $1022.53

Chapter 2

Pre-test

1 a circle b square
c parallelogram d triangle
e rectangle f trapezium
g semicircle h rhombus
2 a 1000 b 100 c 10
d 1000 e 500 f 25
3 a 12 cm b 32 m c 5.9 mm
4 a 10 cm^2 b 70 m^2 c 36 km^2
5 a 4 cm^2 b 14 m^2 c 6 km^2
6 $C = 31.42$ m
$A = 78.54$ m^2

Exercise 2A

1

Scientific notation	Power of 10 expanded	Basic numeral
5×10^3	5×1000	5000
3×10^4	$3 \times 10\,000$	30 000
2×10^5	$2 \times 100\,000$	200 000
7×10^2	7×100	700
7×10^4	$7 \times 10\,000$	70 000
4×10^5	$4 \times 100\,000$	400 000
6×10^3	6×1000	6000
2×10^6	$2 \times 1\,000\,000$	2 000 000

2

Scientific notation	Positive power	Fraction	Basic numeral
2×10^{-4}	$\frac{2}{10^4}$	$\frac{2}{10\,000}$	0.0002
3×10^{-2}	$\frac{3}{10^2}$	$\frac{3}{100}$	0.03
5×10^{-3}	$\frac{5}{10^3}$	$\frac{5}{1000}$	0.005
7×10^{-6}	$\frac{7}{10^6}$	$\frac{7}{1\,000\,000}$	0.000007
9×10^{-3}	$\frac{9}{10^3}$	$\frac{9}{1000}$	0.009
8×10^{-2}	$\frac{8}{10^2}$	$\frac{8}{100}$	0.08
6×10^{-4}	$\frac{6}{10^4}$	$\frac{6}{10\,000}$	0.0006
4×10^{-5}	$\frac{4}{10^5}$	$\frac{4}{100\,000}$	0.00004

3 a 10 000 b 1000 c 100 000
d 1000 e 100 000 f 10 000
4 a positive b negative c positive d negative
5 a 4×10^4 b 2.3×10^{11} c 1.6×10^{10}
d 7.2×10^6 e 3.5×10^3 f 8.8×10^6
g 5.2×10^3 h 3×10^6 i 2.1×10^4
6 a 3×10^{-6} b 4×10^{-4} c 8.76×10^{-3}
d 7.3×10^{-10} e 3×10^{-5} f 1.25×10^{-10}
g 8.09×10^{-9} h 2.4×10^{-8} i 3.45×10^{-5}
7 a 6×10^3 b 7.2×10^5 c 3.245×10^2
d 7.86903×10^3 e 8.45912×10^3 f 2×10^{-1}
g 3.28×10^{-4} h 9.87×10^{-3} i 1×10^{-5}
j 4.601×10^8 k 1.7467×10^4 l 1.28×10^2
8 a 57 000 b 3 600 000 c 430 000 000
d 32 100 000 e 423 000 f 90 400 000 000
g 197 000 000 h 709 i 635 700
9 a 0.00012 b 0.0000046 c 0.0000000008
d 0.0000352 e 0.3678 f 0.000000123
g 0.00009 h 0.05 i 0.4
10 a 7.7×10^6 km^2 b 4×10^7 m c 1×10^{-10} m
d 1.5×10^8 km e 6.67×10^{-11} N m^2/kg^2
f 1.5×10^{-4} s g 4.5×10^9 years
11 a 4 600 000 000 km
b 8 000 000 000 000 organisms
c 384 000 km

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d 0.0038 m
e 0.00000000000001 m
f 720 000 people

12 a 1.62×10^9 km **b** 2.126×10^{-2} g

13 a 3.2×10^4 **b** 4.1×10^6
c 1.3×10^4 **d** 9.2×10^1
e 6.1×10^{-2} **f** 4.24
g 2×10^{-5} **h** 4×10^{-6}

Drilling for Gold 2A2

1 D
2 L
3 A
4 E
5 K
6 B
7 F
8 C
9 G
10 J
11 I
12 H

Exercise 2B

1 a i 300 **ii** 32 700 **iii** 18 400
b i 0.1 **ii** 0.2 **iii** 21.0
c i 26 8000 **ii** 38 000 **iii** 40 61000

2 0.0071

3 a 57 260, 57 300, 57 000, 60 000
b 4 170 200, 4 170 000, 4 170 000, 4 200 000, 4 000 000
c 0.003661, 0.00366, 0.0037, 0.004
d 24.871, 24.87, 24.9, 25, 20

4 a Yes **b** No **c** No
d No **e** Yes **f** Yes
g Yes **h** No **i** No

5 a 3, 4 or 5 **b** 4 **c** 5 or 6 **d** 2 or 3
e 3 **f** 2 **g** 3 **h** 3
i 3 **j** 4 **k** 3 **l** 3

6 a 2.42×10^5 **b** 1.71×10^5
c 2.83×10^3 **d** 3.25×10^6
e 3.43×10^{-4} **f** 6.86×10^{-3}
g 1.46×10^{-2} **h** 1.03×10^{-3}

7 a 4.78×10^4 **b** 2.2×10^4 **c** 4.833×10^6
d 2.7×10^6 **e** 2.7×10^{-4} **f** 2.8×10^{-4}
g 2×10^{-3} **h** 9×10^{-2} **i** 1×10^{-4}

8 a 2.441×10^{-4} **b** 2.107×10^{-6} **c** -4.824×10^{15}
d 4.550×10^{-5} **e** 1.917×10^{12} **f** 1.995×10^8
g 3.843×10^2 **h** 1.804×10^{-11} **i** 1.524×10^8
j 3.325×10^{15} **k** 4.067×10^3 **l** -9.077×10^{-1}

9 1.98×10^{30} kg

10 1.39×10^6 km

11 2421×10^3, 24.2×10^5, 2.41×10^6, 0.239×10^7, 0.02×10^8

12 a 4.26×10^6 **b** 9.1×10^{-3} **c** 5.04×10^{11}
d 1.931×10^{-1} **e** 2.1×10^6 **f** 6.14×10^{-11}

13 a 5.40046×10^{12}
b i 1.08×10^{13} **ii** 4.32×10^{13}

Drilling for Gold 2B1

	Round 1	Round 2	Round 3	Round 4
2	765.4	765	770	800
3	3.142	3.14	3.1	3
4	34.97	35.0	35	30
5	0.2857	0.286	0.29	0.3
6	0.03457	0.0346	0.035	0.03
7	1488	1490	1500	1000
8	25 190	25 200	25 000	30 000
9	105 100	105 000	110 000	100 000
10	110.0	110	110	100

Exercise 2C

1 a 1000 **b** 10 **c** 100

2 a 100 **b** 10 000 **c** 1 000 000

3 a 1000 **b** 1 000 000 000 **c** 1 000 000

4 a millimetre **b** milligram **c** gigalitre
d millisecond **e** microsecond **f** nanosecond

5 a 43.2 mm **b** 0.327 km **c** 8.34 m
d 96 mm **e** 0.2975 km **f** 1.27 cm

6 a 7000 g **b** 7 km **c** 15 000 000 t
d 4000 W **e** 0.0089 Mt **f** 5×10^{-9} s
g 600 000 μg **h** 10 min **i** 1 285 000 ms
j 0.00068 Mt **k** 4000 cm **l** 8×10^9 bytes
m 8.5 s **n** 3 s **o** 9 g

7 a 300 000 mm^2 **b** 5000 cm^2 **c** 5 000 000 m^2
d 29 800 cm^2 **e** 53 700 mm^2 **f** 230 cm^2

8 a 2000 mm^3 **b** 200 000 cm^3 **c** 5.7 cm^3
d 15 000 000 m^3 **e** 0.0283 km^3 **f** 0.762 m^3

9 5500 m

10 a 23.4 m **b** 22 m

11 a 118 mm **b** 147.3 cm **c** 453.258 km
d 15.5 cm^2 **e** 3251 cm^2 **f** 3739 m^2
g 484 500 mm^3 **h** 537 300 m^3

12 21.5 cm

13 For a high level of accuracy

14 2 million

15 10.448 s

16 a 9.002×10^6 B or 9002 kB
b 9.002 MB
c No, two separate emails will be needed.

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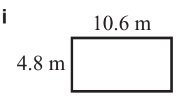
Drilling for Gold 2C1

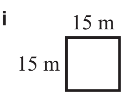
1 km
2 m
3 cm
4 mm
5 t
6 kg
7 g
8 mg
9 m^3
10 cm^3
11 mm^3
12 kL
13 L
14 mL
15 Y
16 d
17 hr
18 min
19 sec
20 km^2
21 ha
22 m^2
23 cm^2
24 mm^2

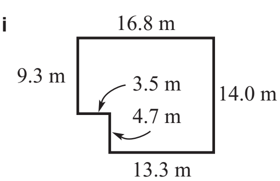
Exercise 2D

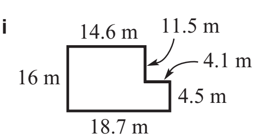
1 **a** 25 **b** 4.5 **c** 2.5 **d** 5
e 3.1 **f** 3.25 **g** 3.05 **h** 3.005
2 **a** ± 1 **b** ± 2.5 **c** ± 0.5
d ± 0.25 **e** ± 0.125 **f** ± 0.05
3 **a** whole number **b** 0.5 **c** 15.5, 16.5
4 **a** decimal place **b** 0.05 **c** 10.35, 10.45
5 **a** 4.5 m to 5.5 m **b** 7.5 cm to 8.5 cm
c 77.5 mm to 78.5 mm **d** 4.5 ns to 5.5 ns
e 1.5 km to 2.5 km **f** 34.15 cm to 34.25 cm
g 3.85 kg to 3.95 kg **h** 19.35 kg to 19.45 kg
i 457.85 t to 457.95 t **j** 18.645 m to 18.655 m
k 7.875 km to 7.885 km **l** 5.045 s to 5.055 s
6 **a** 30 m **b** 145 g **c** 4.6 km
7 **a** 149.5 cm to 150.5 cm **b** 145 cm to 155 cm
8 **a** 24.5 cm to 25.5 cm
b **i** 245 cm **ii** 255 cm
9 **a** 9.15 cm **b** 9.25 cm **c** 36.6 cm to 37 cm
10 **a** If they all choose a different level of accuracy, then they will have different answers. Also, human error plays a part.
b Johan: nearest kg; Amy: nearest 100 g; Toby: nearest 10 g.
c Yes; however, the more decimal places being considered then the more accurate that the measurement will be when used in further calculations, if they are required.
11 **a** distances between towns, cities airplane rides, length of major rivers
b house plans, plumbing plans and building, in general
c mixing chemicals, administering cough mixture to children, matching paint colours, paying for petrol
d filling a swimming pool, describing the fuel tank of a car or plane
12 **a** square: **i** 5.3 cm **ii** 53 mm
rectangle: **i** 6.5 cm by 4.7 cm **ii** 65 mm by 47 mm
triangle: **i** 6 cm, 8 cm, 10 cm **ii** 60 mm, 80 mm, 100 mm
b square: $P = 212$ mm, $A = 2809$ mm^2
rectangle: $P = 224$ mm, $A = 3055$ mm^2
triangle: $P = 240$ mm, $A = 2400$ mm^2
c Answers will vary.
d Answers will vary.

Exercise 2E

1 perimeter
2 **a** 6 **b** 7.1 **c** 4.3
3 **a** 12 cm **b** 23 m **c** 11 km
d 12 m **e** 19.2 cm **f** 10 m
4 **a** 6.7 cm **b** 65 mm **c** 18 m
d 810 m **e** 9.4 km **f** 220 cm
5 **a** $x = 4$ **b** $x = 2$ **c** $x = 6$
6 **a** $x = 3$ **b** $x = 8.8$ **c** $x = 1$
7 **a** **i**


ii 30.8 m
b **i**


ii 60 m
c **i**


ii 61.6 m
d **i**


ii 69.4 m
8 20 m
9 15 cm
10 **a** $P = 4s$ **b** $P = 2\ell + 2b$ **c** $P = x + y + z$
d $P = a + 2b$ **e** $P = 4\ell$ **f** $P = 3s$
11 13
12 3

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Keeping in touch with numeracy

1 875
2 $143.47
3 500
4 1506
5 1.234×10^6
6 1.13×10^{15}
7 1.75
8 1050
9 9
10 6561
11 $\frac{3}{10}$
12 $1\frac{17}{60}$
13 4
14 7
15 24 cm
16 22.5 cm
17 10 cents
18 500 mL
19 **a** $248 (3 lots of 24, 1 lot of 12, 1 lot of 6)
b $240 (4 lots of 24), 6 left over
20 **a** $78.57 (to the nearest cent)
b $93.50 **c** $535 **d** $15

Exercise 2F

1 **a** $C = \pi d$ **b** $C = 2\pi r$
2 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$
3 **a** 8.6 m **b** 1.8 cm
4 **a** 18.85 m **b** 31.42 m **c** 31.42 km
d 113.10 cm **e** 61.07 mm **f** 3.36 km
5 **a** 27.42 m **b** 16.28 cm **c** 6.71 mm
d 12.22 cm **e** 14.71 m **f** 59.70 cm
6 **a** 9.42 m **b** $423.90
7 **a** 1.57 m **b** 1.57 km
8 319 times
9 **a** 12.25 **b** 53.03 **c** 1.37 **d** 19.77
10 **a** 3.5 cm
b **i** 21.99 cm **ii** 65.97 cm **iii** 109.96 cm

Drilling for Gold 2F1

1 F
2 A
3 C
4 G
5 E
6 D
7 B
8 H

Exercise 2G

1 **a** E **b** B **c** F **d** C **e** D **f** A
2 **a** 2 cm **b** 4 m **c** 4.3 cm
d 4 km **e** 7.8 m **f** 10 cm
3 **a** 4 m^2 **b** 18 cm^2 **c** 11.76 m^2
d 21 m^2 **e** 22.5 mm^2 **f** 2 m^2
4 **a** 25 cm^2 **b** 54.6 m^2 **c** 1.82 km^2
d 0.03 mm^2 **e** 1.12 m^2 **f** 100 cm^2
5 **a** 0.96 m^2 **b** 9600 cm^2
6 27 m^2
7 **a** 13.6 m^2 **b** $149.60
8 **a** 7.56 m^2 **b** $491.40
9 1 and 24, 2 and 12, 3 and 8, 4 and 6
10 **a** 252.05 m^2 **b** 177.86 m^2
11 **a** $b = 2.88$ **b** $s = 14.35$ **c** $h = 1.44$
d $a = 1.05$ **e** $h = 1.87$ **f** $x = 8.89$
12 All answers = 3

Exercise 2H

1 E
2 C
3 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{3}$
d $\frac{1}{12}$ **e** $\frac{7}{12}$ **f** $\frac{5}{6}$
4 **a** 50.27 cm^2 **b** 201.06 m^2 **c** 72.38 m^2
d 38.48 m^2 **e** 0.82 mm^2 **f** 124.69 km^2
5 **a** 39.27 m^2 **b** 4.91 m^2 **c** 84.82 m^2
d 13.09 m^2 **e** 69.81 cm^2 **f** 8.03 m^2
6 157.1 cm^2
7 **a** 14.28 cm^2 **b** 178.54 m^2 **c** 32.14 mm^2
8 43.24 m^2
9 **a** 34.8 cm^2 **b** 63.5 m^2
c 9.4 cm^2 **d** 103.3 mm^2
10 **a** 20 **b** 565.49 cm^2
c 154.51 cm^2 **d** 5

Exercise 2I

1 **a** 6 **b** 3
2 **a** 35 cm^2 **b** 21 cm^2
c 12 cm^2 **d** 96 cm^2
3 **a** 90 cm^2 **b** 34 mm^2 **c** 46 m^2
4 **a** 360 m^2 **b** 168 m^2
c 1176 cm^2 **d** 132 m^2
5 **a** 8.64 cm^2 **b** 96 mm^2 **c** 836.6 m^2
6 384 cm^2
7 3880 cm^2
8 1000 cm^2
9 **a** 5.116 L **b** 10.232 L

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Exercise 2I cont.

10 a waterproof 13.76 L; smooth paint 22.86 L
b \$553.46

11 Yes, only 5 L required.

Exercise 2J

1 a C **b** A **c** B

2 a 1000 **b** 1000

3 10 cm^3

4 a 240 m^3 **b** 480 cm^3 **c** 0.18 m^3

5 a 113.10 cm^3 **b** 19.63 m^3 **c** 4.83 mm^3

6 a 20 cm^2 **b** 90 cm^3

7 a 36 m^3 **b** 15 cm^3 **c** 0.572 mm^3

8 a 72 L **b** 2 L **c** 2 L

9 5.89 L

10 a 1583.36 m^3 **b** 30 km^3 **c** 196 cm^3
d 30 m^3 **e** 10 cm^3 **f** 2.15 m^3

11 a 25 cm^2 **b** 4 cm

12 a 83.3 m^3 **b** 1500 m^3 **c** 20.9 cm^3

13 a i 64 cm^3 **ii** 72 cm^3 **iii** 48 cm^3
b 2 cm cut-out **c** yes (close to 1.7 cm)
d cut-out length of $\frac{5}{3}$ cm

Maths@home: Keeping chickens

Answers will vary.

Puzzles and games

1 PRISM **2** no **3** 21.46%
4 11 **5** $r = 2$ **6** 27 cm^3

Multiple-choice questions

1 C **2** B **3** C **4** A **5** A
6 B **7** E **8** D **9** B **10** C

Short-answer questions

1 a 5300 m **b** 2.7 m^2 **c** 40 mm^3
d 86 400 s **e** 125 ms **f** 0.089 TB

2 a 5.5 cm to 6.5 cm **b** 4.15 kg to 4.25 kg
c 14.5 mL to 15.5 mL

3 a 13 **b** 24 cm **c** 38 m

4 a 18.85 m **b** 28.27 m^2

5 a i 19.42 m **ii** 26.14 m^2
b i 19.50 m **ii** 21.87 m^2
c i 14.28 m **ii** 12.28 m^2

6 a 10 **b** 25.9 **c** 17.5

7 a 828 m^2 **b** 136 cm^2

8 a 2.32×10^5 **b** 2.32×10^{-4}
c 4 540 000 **d** 0.00000454

9 a 40 cm^3 **b** 125.66 m^3 **c** 21 cm^3

10 cube

Extended-response questions

1 a 100.53 m^3 **b** 100 530.96 L

2 a 5 m **b** 135 m^2 **c** \$810 **d** 154 m^3

Chapter 3

Pre-test

1 a $3x$ **b** $a + 1$ **c** $2m - 5$ **d** $4(x + y)$

2 a 20 **b** 17 **c** 23 **d** 22

3 a no **b** yes **c** yes **d** no

4 a $8m$ **b** $5ab$ **c** $6x + 8y$ **d** $8x$
e $15ab$ **f** $3y$

5 a $2x + 10$ **b** $3y - 6$ **c** $8x - 12$ **d** $3x^2 + x$

6 a 7^4 **b** 4^3

7 a 49 **b** 27 **c** 25
d 16 **e** 5 **f** 1

8 a 3^7 **b** 3^2 **c** 3^{10}
d 3^3 **e** 3^6 **f** 3^5

Exercise 3A

1 a expression **b** constant term
c coefficient **d** term

2 a + **b** – **c** × **d** – **e** + **f** ÷

3 a 7 **b** 15 **c** 5 **d** 9 **e** 6

4 a –6 **b** –20 **c** –6 **d** –7
e 9 **f** 18 **g** –6 **h** 7
i –7 **j** –13 **k** –4 **l** –5

5 a i 3 **ii** 8 **iii** 5
b i 4 **ii** 2 **iii** –3
c i 3 **ii** –4 **iii** 1

6 a $x + 2$ **b** $y - 4$ **c** $ab + y$ **d** $2x - 3$
e $5x$ **f** $2m$ **g** $3r$ **h** $\frac{1}{2}x$
i $\frac{3}{4}m$ **j** $\frac{x}{y}$ **k** $\frac{a+b}{4}$ **l** x^2y

7 a 12 **b** 3 **c** 9 **d** 10
e 10 **f** –2 **g** 1 **h** 4
i –6 **j** 10 **k** –2 **l** –9

8 a $5x$ cents **b** $35y$ cents **c** $\frac{\$500}{n}$
d $\frac{\$11}{m}$ **e** $11 + x$

9 a $\$(3.40 + 2d)$ **b i** \$23.40 **ii** \$47.40

10 a i $2x$ **ii** $2x - 3$ **iii** $3(2x - 3)$ **b** 21

11 a $x + 1$ and $x - 1$ **b** no
c i $(x + 1)(x - 1)$ **ii** Less by 1 square metre.

12 a 21 sq. units **b** 1, 2, 3, 6, 9, 18

13 a i $4x$ **ii** x^2
b i $2x + 2y$ **ii** xy
c i $x + y + 5$ **ii** $\frac{5x}{2}$

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d i $4ab$ ii a^2b^2
e i $2a^2 + 2b$ ii a^2b
f i $mn + 9$ ii $2mn$

Drilling for Gold 3A3

1 $a + b + c$
2 $\frac{3a}{2}$
3 $a + 3$
4 $\frac{3a}{2}$
5 $\frac{a}{2} + 3$
6 $\frac{a}{3} - 1$
7 $a + a$
8 $a + 3$
9 $2(a + b)$
10 $a - 3$
11 $2a + 3$
12 $3a + b$
13 $3 - a$
14 $3 - a$
15 $\frac{a}{3}$
16 $3 - 2a$
17 abc
18 $\frac{2a}{3}$
19 $3(a + b + c)$
20 $3 - 3a$

Drilling for Gold 3A4

Answers will vary

Exercise 3B

1 a Y b Y c N
d N e Y f Y
2 a $10g$ b $5f$ c $8e$
d 0 e $6x$ f $17st$
g $3ts$ h $3ab$ i xy
3 a $6x$ b $12a$ c $10m$ d $-18y$
4 a $\frac{1}{2}$ b 4 c 3
d $\frac{2}{3}$ e $\frac{2}{3}$ f $\frac{7}{3}$
g $\frac{3}{4}$ h $\frac{2}{5}$ i $\frac{5}{6}$
5 a $3ac$ and $-2ac$ b $4pq$ and $3qp$
c $7xy^2$ and $4yx^2$, $-3xy^2$ and $2xy^2$
d $2r^2$ and $-r^2$ e $-2ab$ and $4ba$
f $3p^2q$ and $4qp^2$ g $\frac{1}{3}\ell m$ and $\frac{\ell m}{4}$
h x^2y and yx^2, $-xy$ and yx

6 a $7t + 10$ b $4g + 1$ c $7x - 5$
d $m + 2$ e $3x + 3y$ f $2x + 6y$
g $5a - 2b$ h $-3m - 2n$ i $5de + 7de^2$
j $3kl - 10k^2l$ k $7x^2y + xy^2$ l $3fg - fg^2$
7 a $6rs$ b $6hu$ c $16wh$ d $6r^2s$
e $-8es$ f $-10hv$ g $12cm^2$ h $35fl$
i $8x^2y$ j $24a^2b$ k $3xy^2$ l $-16a^2b$
m $-12m^2n^2$ n $20x^2y^2$ o $20a^2b^2$ p $-48x^2y^2$
8 a $\frac{x}{2}$ b $3a$ c $\frac{a}{3}$ d $\frac{ab}{4}$
e $2b$ f $3x$ g $\frac{y}{2}$ h $\frac{4a}{5}$
i $\frac{2x}{5}$ j $\frac{2xy}{3}$ k $3ab$ l $\frac{n}{3}$
9 a $8x$ b $3x^2$
10 a $5x$ b $8y$ c $4a$
d $-6x$ e $2x$ f $10a^2b$
11 a $P = 4x + 6$, $A = 6x$ b $P = 4y + 16x$, $A = 16xy$
c $P = 20a$, $A = 25a^2$
12 $3x$
13 a $6x$ b $8a + 7$ c $4b$ d $11x$
e $3x^2$ f $15xy$ g $8x^2$ h $-29x^2$

Drilling for Gold 3B1

1 N
2 N
3 Y
4 Y
5 N
6 Y
7 N
8 Y
9 Y
10 N
11 N
12 N
13 Y
14 Y
15 N

Drilling for Gold 3B2

1 $2x + 3x$, $5x$
2 $x + 3x$, $4x$
3 $2x$, $x + x$
4 $0x$, 0
5 $-5x + 6x$, $1x$, x
6 $2x$
7 $0x$, 0
8 $-1x$, $-x$
9 $2x - x$, $1x$, x, $x + 0$
10 $-x$, $-1x$

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Drilling for Gold 3B4

1 $9x, -3x, 3x, 18x^2, \frac{1}{2}, 2$

2 $3x+3, 3x-3, 3-3x, 9x, x, \frac{1}{x}$

3 $6x+6, 6x-6, 6-6x, 36x, x, \frac{1}{x}$

4 $6a+4b, 6a-4b, 4b-6a, 24ab, \frac{3a}{2b}, \frac{2b}{3a}$

5 $4a+6b, 4a-6b, 6b-4a, 24ab, \frac{2a}{3b}, \frac{3b}{2a}$

6 $7x, -3x, 3x, 10x^2, \frac{2}{5}, \frac{5}{2}$

7 $x^2+xy, x^2-xy, xy-x^2, x^3y, \frac{x}{y}, \frac{y}{x}$

8 $4x+2x^2, 4x-2x^2, 2x^2-4x, 8x^3, \frac{2}{x}, \frac{x}{2}$

Exercise 3C

1 **a** $4x$ **b** 8 **c** $4x+8$
d $x+2$ **e** $4\times(x+2)$ **f** $4x+8$

2 **a** -8 **b** -18 **c** $-3x$ **d** $-8x$ **e** -20
f $-16x$ **g** 15 **h** 24 **i** $6x$

3 **a** $3(x+4)=3\times x+3\times 4$
$=3x+12$
b $2(x-5)=2\times x+2\times(-5)$
$=2x-10$
c $2(4x+3)=2\times 4x+2\times 3$
$=8x+6$
d $x(x-3)=x\times x+x\times(-3)$
$=x^2-3x$

4 **a** $6x$ **b** $8xy$ **c** $15x^2$
d $2x+9$ **e** $7x+5$ **f** $3x-4$

5 **a** $2x+8$ **b** $3x+21$ **c** $4y-12$ **d** $5y-10$
e $6x+4$ **f** $8x+20$ **g** $9a-12$ **h** $14y-35$
i $10a+5b$ **j** $12a-9b$ **k** $2x^2+10x$ **l** $3x^2-12x$
m $6a^2+4ab$ **n** $6xy-8y^2$ **o** $6ab-15b^2$

6 **a** $-2x-6$ **b** $-5m-10$ **c** $-3w-12$
d $-4x+12$ **e** $-2m+14$ **f** $-7w+35$
g $-x-y$ **h** $-x+y$ **i** $-6x^2-8x$
j $-6x^2-15x$ **k** $-8x^2+8x$ **l** $-6y^2+27y$
m $-6x^2+10xy$ **n** $-9x^2-6xy$ **o** $-12xy-18y^2$

7 **a** $5x+17$ **b** $7x+17$ **c** $2x-1$ **d** $2x+13$
e $5x+41$ **f** $1+6x$ **g** $7x+18$ **h** $7p-11$
I $10x-4$ **j** $8s+13$ **k** $24f+12$ **l** $10x-23$

8 **a** 2 **b** 4 **c** $3x$
d $3x$ **e** y, 1 **f** $2x$, $3y$

9 **a** $2x-10$ **b** x^2+3x **c** $2x^2+8x$ **d** $6x^2-3x$

10 $2x^2+4x$

11 **a** $x-18\,200$ **b** $0.1x-1820$

12 **a** $x^2+7x+12$ **b** x^2+4x+3 **c** $x^2+7x+10$
d x^2-2x-8 **e** $x^2+3x-10$ **f** $2x^2+11x+12$
g $2x^2-x-6$ **h** x^2+x-12 **i** $4x^2+18x-10$

Exercise 3D

1 **a** 4 **b** 10 **c** 5 **d** 6
e 3 **f** 25 **g** 8 **h** 36

2 **a** x **b** x **c** 4 **d** b
e b **f** $3a$ **g** a **h** $2a$
i $-2y$ **j** $-3x$ **k** $-2x$ **l** $-10x$

3 **a** **i** 6 **ii** $3x$ **iii** $6x$
b iii

4 **a** C
b There is no common factor inside the brackets.

5 **a** 6 **b** 5 **c** 4
d $2x$ **e** $6a$ **f** 2
g 4 **h** 3 **i** 1
j $3x$ **k** $3n$ **l** $2y$
m $2x$ **n** $2xy$ **o** $5ab$

6 **a** $7(x+1)$ **b** $3(x+1)$ **c** $4(x-1)$
d $5(x-1)$ **e** $4(1+2y)$ **f** $5(2+a)$
g $3(1-3b)$ **h** $2(3-x)$ **i** $3(4a+b)$
j $6(m+n)$ **k** $2(5x-4y)$ **l** $4(a-5b)$
m $x(x+2)$ **n** $a(a-4)$ **o** $y(y-7)$
p $x(1-x)$ **q** $3p(p+1)$ **r** $8x(1-x)$
s $4b(b+3)$ **t** $2y(3-5y)$

7 **a** $7x(2+3y)$ **b** $3b(2a-5)$ **c** $8y(4-5x)$
d $5x(x-1)$ **e** $x(x+7)$ **f** $2a(a+4)$
g $6a(2a+7b)$ **h** $9y(y-7)$ **i** $2x(3x+7)$
j $3x(3x-2)$ **k** $8y(2y+5)$ **l** $10m(1-4m)$

8 **a** 3 **b** y **c** a **d** $5x$ **e** -7 **f** $-12a$

9 **a** $4(x+2)$ **b** $2(x+3)$ **c** $10(x+2)$
d $2(x+7)$ **e** $2(2x+3)$ **f** $2(x+7)$

10 $4x$

11 **a** 63 **b** 72 **c** -20
d -70 **e** 69 **f** 189

12 **a** $3(a^2+3a+4)$ **b** $z(5z-10+y)$
c $x(x-2y+xy)$ **d** $2b(2y-1+3b)$
e $-4y(3x+2z+5xz)$ **f** $ab(3+4b+6a)$

13 **a** $(x+3)(4+x)$ **b** $(x+1)(3+x)$
c $(m-3)(7+m)$ **d** $(x-7)(x+2)$
e $(a+4)(8-a)$ **f** $(x+1)(5-x)$
g $(y+3)(y-2)$ **h** $(x+2)(a-x)$
i $(2t+5)(t+3)$ **j** $(5m-2)(m+4)$
k $(4y-1)(y-1)$ **l** $(7-3x)(1+x)$

Keeping in touch with numeracy

1 \$50
2 \$54.05
3 D
4 0.8 cm
5 Wednesday
6 February 6
7 57
8 14
9 -24
10 -10.8

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11 a 375 **b** 0.00375 **c** 6.25
12 a 10.125 **b** 3 **c** 0.35
13 1700
14 61
15 6
16 4
17 a 125 m^3 **b** 150 m^2
18 a \$148.75 **b** 7.8125 L
19 21
20 \$12.50

Exercise 3E

1 a expanded, factor **b** index
c power **d** base **e** index
f prime **g** prime factors **h** indices
2 a 25 **b** 8 **c** 27 **d** 16
3 a 3 **b** 6 **c** 1.2 **d** –7
e $\frac{2}{3}$ **f** y **g** w **h** t
4 a 3 **b** 8 **c** 7 **d** 4
e 11 **f** 13 **g** 9 **h** 2
5 a 2, 3 **b** 3, 5 **c** 2, 3, 5 **d** 7, 11
6 a $4 \times 4 \times 4$ **b** $7 \times 7 \times 7 \times 7$
c $3 \times 3 \times 3 \times 3 \times 3$ **d** $5 \times 5 \times 5$
e $a \times a \times a \times a$ **f** $b \times b \times b$ **g** $x \times x \times x$
h $xp \times xp \times xp \times xp \times xp \times xp$
i $5a \times 5a \times 5a \times 5a$
j $3y \times 3y \times 3y$
k $4 \times x \times x \times y \times y \times y \times y \times y$
l $pq \times pq$
m $-3 \times s \times s \times s \times t \times t$
n $6 \times x \times x \times x \times y \times y \times y \times y \times y$
o $5 \times y \times z \times y \times z \times y \times z \times y \times z \times y \times z \times y \times z$
p $4 \times a \times b \times a \times b \times a \times b$
7 a 36 **b** 16 **c** 243 **d** 12
e –8 **f** –1 **g** 81 **h** 25
i $\frac{8}{27}$ **j** $\frac{9}{16}$ **k** $\frac{1}{216}$ **l** $\frac{25}{4}$
m $-\frac{8}{27}$ **n** $\frac{81}{256}$ **o** $\frac{1}{16}$ **p** $-\frac{3125}{32}$
8 a 3^3 **b** 8^6
c y^2 **d** $3x^3$
e $4c^5$ **f** 5^3d^2
g x^2y^3 **h** 7^3b^2
9 a $\left(\frac{2}{3}\right)^4$ **b** $\left(\frac{3}{5}\right)^5$ **c** $\left(\frac{4}{7}\right)^2 \times \left(\frac{1}{5}\right)^4$
d $\left(\frac{7x}{9}\right)^2 \times \left(\frac{y}{4}\right)^3$
10 a $3^3x^3y^2$ **b** $(3x)^2(2y)^2$ or $3^22^2x^2y^2$
c $(4d)^2(2e)^2$ or $4^22^2d^2e^2$ **d** $6^3b^2y^3$
e $(3pq)^4$ or $3^4p^4q^4$ **f** $(7mn)^3$ or $7^3m^3n^3$
11 a 2×5 **b** 2^3 **c** $2^4 \times 3^2$
d 3×5^2 **e** 3×7^2 **f** $2^2 \times 5^3$
12 a $3 \times 3 \times a \times a \times a = 3^2 a^3$
b $5 \times 5 \times k \times k \times k = 5^2 k^3$
c $\frac{2}{7} \times \frac{2}{7} \times \frac{2}{7} = \left(\frac{2}{7}\right)^3$
d $3p^2q = 3 \times p \times p \times q$
e $(abc)^1 = a \times b \times c$
13 a 36 **b** –216 **c** 1 **d** $-\frac{8}{27}$
e –18 **f** 15 **g** –36 **h** 216
14 a 4 **b** 8 **c** 5 **d** 2
e –4 **f** –2 **g** $\frac{1}{2}$ **h** 4
15 a

Time (min)	Number of bacteria	Number in index form
0	1	2^0
1	$1 \times 2 = 2$	2^1
2	$2 \times 2 = 4$	2^2
3	$2 \times 2 \times 2 = 8$	2^3
4	$2 \times 2 \times 2 \times 2 = 16$	2^4
5	$2 \times 2 \times 2 \times 2 \times 2 = 32$	2^5
6	$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$	2^6
7	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$	2^7
8	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$	2^8
9	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$	2^9
10	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$	2^{10}

b i 2 min **ii** 4 min **iii** 6 min
c 2^{24} = 16 777 216 cells

Exercise 3F

1 a base, add
b base, subtract
2 a $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$
b $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^7$
c $k \times k \times k \times k \times k = k^5$
d $m \times m \times m \times m \times m \times m \times m = m^7$
e $5 \times 5 \times 5 \times 5 = 5^4$
f $2 \times 2 \times 2 \times 2 \times 2 = 2^5$

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Exercise 3F cont.

3 **a** $\frac{5\times5\times\cancel{5}^1\times\cancel{5}^1\times\cancel{5}^1}{\cancel{5}^1\times\cancel{5}^1\times\cancel{5}^1} = 5^2$

b $\frac{9\times9\times\cancel{9}^1\times\cancel{9}^1}{\cancel{9}^1\times\cancel{9}^1} = 9^2$

c $\frac{a\times a\times a\times a\times\cancel{a}^1\times\cancel{a}^1}{\cancel{a}^1\times\cancel{a}^1} = a^4$

d $\frac{n\times n\times n\times\cancel{n}^1\times\cancel{n}^1\times\cancel{n}^1}{\cancel{n}^1\times\cancel{n}^1\times\cancel{n}^1} = n^3$

e $\frac{x\times x\times x\times\cancel{x}^1}{\cancel{x}^1} = x^3$

f $\frac{x\times\cancel{x}^1\times\cancel{x}^1\times\cancel{x}^1}{\cancel{x}^1\times\cancel{x}^1\times\cancel{x}^1} = x$

4 **a** $6^{5+7} = 6^{12}$ **b** $a^{13+2} = a^{15}$
c $5^{12-4} = 5^8$ **d** $m^{16-2} = m^{14}$

5 **a** 2^7 **b** 5^9 **c** 7^6 **d** 8^{10}
e 3^8 **f** 6^{14} **g** 3^3 **h** 6^5
i 5^3 **j** 10 **k** 9^3 **l** $(-2)^2$

6 **a** x^6 **b** x^5 **c** b^4 **d** b^3
e x^7 **f** a^9 **g** t^8 **h** y^5
i d^3 **j** y^7 **k** b^8 **l** q^{11}
m a^5m^4 **n** k^5p^3 **o** x^6y^8 **p** m^7e^4

7 **a** 5^5 **b** 5^6 **c** 10^5 **d** 10^1
e a^2 **f** x^3 **g** q^{10} **h** d
i b^5 **j** d^5 **k** a^7 **l** y

8 **a** $6x^5$ **b** $2x^6$ **c** $8a^2b$
d $2p^5$ **e** $3e^8$ **f** $6s^{11}$
g $12a^3b^2$ **h** $12a^2b^3$ **i** $7x^7y^5$
j $3x^9y^4$ **k** $5x^4y^9$ **l** $4x^2y^5z$
m $15m^5$ **n** $8e^6f^4$ **o** $20c^7d^2$
p $18y^2z^7$ **q** $3m$ **r** $7x^3$
s $5y^2$ **t** $3a$ **u** $3m^5$
v $\frac{w}{5}$ **w** $\frac{a}{5}$ **x** $\frac{x^4}{9}$

9 **a** b^6 **b** y^6 **c** c^7 **d** x
e t **f** p^6 **g** d^6 **h** x^{10}

10 **a** 12 **b** 8 **c** 3 **d** 3
e 1 **f** 18 **g** 12 **h** 11
i 4 **j** 15 **k** 2 **l** 39

11 **a** 7^2 **b** 10
c 13^2 **d** 2^3
e 101 **f** 200^2

12 **a** **i** $10a^{10}$ **ii** $9m^8$ **iii** $150m^5$ **iv** $60x^6y^5$
b **i** $4am$ **ii** $6xy^2$

Exercise 3G

1 **a** multiply **b** 1

2 **a** 16, 8, 4, 2, 1 **b** 64, 16, 4, 1

3 **a** $4^2\times4^2\times4^2 = 4\times4\times4\times4\times4\times4 = 4^6$
b $12^3\times12^3 = 12\times12\times12\times12\times12\times12 = 12^6$
c $x^4\times x^4 = x\times x\times x\times x\times x\times x\times x\times x = x^8$
d $a^2\times a^2\times a^2 = a\times a\times a\times a\times a\times a = a^6$
e $8x^6$
f $27x^6$

4 **a** 1 **b** 1 **c** 1 **d** 1
e 1 **f** 1 **g** 1 **h** 1

5 **a** y^{12} **b** m^{18} **c** x^{10} **d** b^{12}
e 3^6 **f** 4^{15} **g** 3^{30} **h** 7^{10}
i $5m^{16}$ **j** $4q^{28}$ **k** $-3c^{10}$ **l** $2j^{24}$

6 **a** $9x^2$ **b** $64m^3$ **c** $125y^3$
d $16x^{12}$ **e** $x^{10}y^5$ **f** $27a^9$
g $x^{24}y^{12}$ **h** a^6b^3 **i** $m^{12}n^{12}$

7 **a** 1 **b** 1 **c** 1 **d** 1
e 1 **f** 1 **g** 1 **h** 1
i 5 **j** −3 **k** 4 **l** −6
m 1 **n** 3 **o** 4 **p** 0

8 **a** 4^7 **b** 3^9 **c** x
d y^{13} **e** b^{14} **f** a^{10}
g d^{24} **h** y^{16} **i** z^{25}

9 **a** b^6 **b** x^5 **c** y^6
d 7^2 **e** 4 **f** 3^8
g 1 **h** y^3 **i** h^2

10 5 ways: $(a^{16})^1 = a^{16}$, $(a^1)^{16} = a^{16}$, $(a^2)^8 = a^{16}$, $(a^8)^2 = a^{16}$, $(a^4)^4 = a^{16}$

11 **a** 4×5 not $4+5$, a^{20}
b Power of 2 only applies to x^3, $3x^6$
c Power zero applies to whole bracket, 1

12 **a** **i** 400 **ii** 6400 **iii** 100
b **i** 800 **ii** 12 800 **iii** 102 400
c 13 years

Exercise 3H

1 **a** $\frac{1}{2^2}$ **b** $\frac{1}{3^2}$ **c** $\frac{1}{5^3}$ **d** $\frac{1}{3^3}$

2 **a**

Index form	3^4	3^3	3^2	3^1
Whole number or fraction	81	27	9	3

Index form	3^0	3^{-1}	3^{-2}	3^{-3}
Whole number or fraction	1	$\frac{1}{3}$	$\frac{1}{9} = \frac{1}{3^2}$	$\frac{1}{27} = \frac{1}{3^3}$

b

Index form	10^4	10^3	10^2	10^1
Whole number or fraction	10 000	1000	100	10

Index form	10^0	10^{-1}	10^{-2}	10^{-3}
Whole number or fraction	1	$\frac{1}{10}$	$\frac{1}{100} = \frac{1}{10^2}$	$\frac{1}{1000} = \frac{1}{10^3}$

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3 **a** $10^{-4} = \frac{1}{10^4}$ **b** $3^{-2} = \frac{1}{3^2}$ **c** $7^{-3} = \frac{1}{7^3}$

d $8^{-6} = \frac{1}{8^6}$ **e** $9^{-4} = \frac{1}{9^4}$ **f** $5^{-4} = \frac{1}{5^4}$

4 **a** false **b** false **c** true
d true **e** false

5 **a** $\frac{1}{5^2}$ **b** $\frac{1}{7^4}$ **c** $\frac{1}{8^3}$ **d** $\frac{1}{3^5}$

e $\frac{1}{9^2}$ **f** $\frac{1}{10^3}$ **g** $\frac{1}{4^5}$ **h** $\frac{1}{2^3}$

6 **a** $\frac{3}{2^4}$ **b** $\frac{5}{4^3}$ **c** $\frac{7}{5^6}$ **d** $\frac{2}{3^4}$

e $\frac{4}{3^5}$ **f** $\frac{9}{5^2}$ **g** $\frac{8}{7^3}$ **h** $\frac{6}{5^6}$

7 **a** $\frac{6}{25}$ **b** $\frac{2}{9}$ **c** $\frac{4}{125}$ **d** $\frac{6}{7}$

e $\frac{1}{250}$ **f** $\frac{1}{5}$ **g** $\frac{5}{16}$ **h** $\frac{4}{5}$

8 **a** $\frac{2}{1000} = 0.002$ **b** $\frac{5}{100} = 0.05$

c $\frac{7}{10} = 0.7$ **d** $\frac{3}{10\,000} = 0.0003$

e $\frac{5}{10\,000} = 0.0005$ **f** $\frac{8}{100\,000} = 0.00008$

g $\frac{2}{1\,000\,000} = 0.000002$ **h** $\frac{4}{100\,000\,000} = 0.00000004$

9 **a** $3 \times 2^{-3} = 3 \times \frac{1}{8} = \frac{3}{8}$ **b** $2x^{-2} = 2 \times \frac{1}{x^2} = \frac{2}{x^2}$

10 **a** $\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$ **b** $\frac{1}{9} = \frac{1}{3^2} = 3^{-2}$

c $\frac{1}{16} = \frac{1}{2^4} = 2^{-4}$ **d** $\frac{1}{25} = \frac{1}{5^2} = 5^{-2}$

11 $0.0012, 35.4 \times 10^{-3}, 3.22 \times 10^{-1}, 0.4, 0.007 \times 10^2, 2.35$

12 **a** 324 **b** 172 500 **c** 0.2753 **d** 0.00149

13 **a** 2.25×10^7 **b** 9.63×10^6
c 3.34×10^{-9} **d** 2.94×10^{-7}

14 **a** $\frac{a^2}{b^2}$ **b** $\frac{b^2}{a^2}$

c $\frac{1}{a^2b^2}$ or $\frac{1}{(ab)^2}$ **d** $\frac{1}{a^2b^2}$ or $\frac{1}{(ab)^2}$

Maths@home

1 $1.84 \times 10^{11}\ t$

2 More than 50 trillion dollars ($\doteqdot 5.0729 \times 10^{13}$)

Puzzles and games

1 Magic square sum = $3x + 2y$

$\frac{4x^2}{2x}$	$-y$	$x + 3y$
$4y$	$x + y$	$2x - 3y$
$x - 2y$	$2x + 2y$	$2y$

2 3^{3n-3}

3 1 cent and then double each day

4 2^{24}

5 200

6 EXPONENTIAL

Multiple-choice questions

1 C 2 B 3 C 4 D 5 B
6 B 7 C 8 E 9 D 10 A

Short-answer questions

1 **a** 4 **b** 5 **c** **i** 4 **ii** −3

2 **a** $y + 3$ **b** $xy - 5$ **c** $\frac{a+b}{4}$

3 **a** 14 **b** −30 **c** 35

4 **a** $7x - 5$ **b** $13a - 2b$
c $xy - 3xy^2$ **d** $12mn$
e $-14x^2y$ **f** $\frac{2b}{3}$

5 **a** $10x + 20$ **b** $-6x + 8y$
c $6x^2 + 15xy$ **d** $4a + 15$

6 **a** $8(2x - 5)$ **b** $5xy(2x + 7y)$
c $2x(2x - 5)$ **d** $-2x(y + 9)$

7 **a** $12x^7$ **b** $8x^4y^4$ **c** b^4
d b^8 **e** $8m^6$ **f** 2

8 **a** 1 **b** 4 **c** 6 **d** 1

9 **a** 4250 **b** 37 000 000
c 0.021 **d** 0.0000725

10 **a** 1.24×10^5 **b** 3.95×10^7
c 9.02×10^{-6} **d** 4.60×10^{-4}

11 **a** 9 **b** 4 **c** 125

12 **a** 4, −4 **b** 9, −2

Extended-response questions

1 **a** $2(5x + 1)$ m **b** 32 m **c** $5x^2 + 3x$ **d** \$1080

2 $x^2 + 2x = x(x + 2)$

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Chapter 4

Pre-test

1 **a** 11

b **i** $\frac{1}{11}$ **ii** $\frac{2}{11}$ **iii** $\frac{4}{11}$

iv $\frac{7}{11}$ **v** $\frac{3}{11}$ **vi** $\frac{8}{11}$

2 **a** $\frac{1}{8}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{4}$

e $\frac{5}{8}$ **f** $\frac{7}{8}$ **g** $\frac{1}{4}$

3 0, 1 in 5, 39%, 0.4, $\frac{1}{2}$, 0.62, 71%, $\frac{3}{4}$, $\frac{9}{10}$, 1

4 **a** **i** 14 **ii** 25 **iii** 11

b **i** $\frac{18}{25}$ **ii** $\frac{7}{25}$ **iii** $\frac{7}{25}$

5 **a** $\frac{7}{16}$ **b** $\frac{9}{16}$

6 **a**

		Roll 1			
		1	2	3	4
Roll 2	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

b 16

c **i** $\frac{1}{16}$ **ii** $\frac{3}{16}$ **iii** $\frac{5}{8}$

iv $\frac{7}{8}$ **v** $\frac{13}{16}$ **vi** $\frac{3}{16}$

7 **a** I **b** G **c** O
d J **e** K **f** H
g A **h** F **i** B
j C **k** L **l** D
m E **n** N **o** M

8 **a** 8 **b** 40 **c** 82 **d** 13

9 **a** 6 **b** **i** 19 **ii** 23
c 30 **d** 10%

10 **a** **i** 45 **ii** 41 **iii** 41 **iv** 20
b **i** 6 **ii** 2 **iii** 6 **iv** 10

11 **a** 15 **b** 111 and 139
c 98 and 145 **d** 47

Exercise 4A

1 C, A, B, D

2 **a** $\frac{1}{4}$ **b** $\frac{1}{6}$ **c** $\frac{1}{4}$

d $\frac{3}{8}$ **e** $\frac{2}{3}$ **f** 0

3 **a** 2 **b** {H, T} **c** yes

d $\frac{1}{2}$ **e** $\frac{1}{2}$ **f** 1

4 **a** 7

b **i** $\frac{1}{7}$ **ii** $\frac{2}{7}$ **iii** $\frac{5}{7}$ **iv** $\frac{3}{7}$

5 **a** $\frac{3}{10}$ **b** $\frac{2}{5}$ **c** $\frac{3}{5}$ **d** $\frac{1}{2}$

6 **a** 43 **b** 47
c **i** 0.09 **ii** 0.43 **iii** 0.47 **iv** 0.91

7 **a** 0.62 **b** 0.03 **c** 0.97 **d** 0.38

8 **a** $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$

e $\frac{2}{5}$ **f** $\frac{1}{5}$ **g** $\frac{3}{10}$ **h** $\frac{9}{10}$

9 **a** $\frac{1}{50}$ **b** $\frac{3}{10}$ **c** $\frac{49}{50}$

10 **a** $\frac{6}{25}$ **b** $\frac{1}{50}$ **c** $\frac{21}{25}$

d $\frac{2}{5}$ **e** $\frac{2}{25}$ **f** $\frac{4}{25}$

11 **a** 59

b 4, as $\frac{41}{100}$ of 10 is closest to 4.

c 8, as $\frac{41}{100}$ of 20 is closest to 8.

12 **a** $\frac{1}{4}$ **b** $\frac{1}{13}$ **c** $\frac{1}{52}$ **d** $\frac{1}{2}$

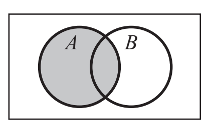
e $\frac{2}{13}$ **f** $\frac{4}{13}$ **g** $\frac{12}{13}$ **h** $\frac{9}{13}$

Exercise 4B

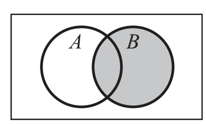
1 **a** D **b** C **c** E **d** A **e** B

2 **a** no **b** yes

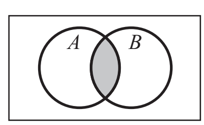
3 **a**



b



c



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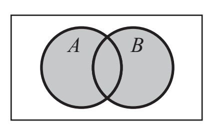

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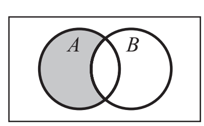

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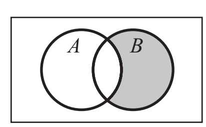
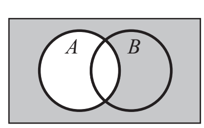
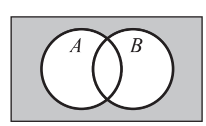


g



h



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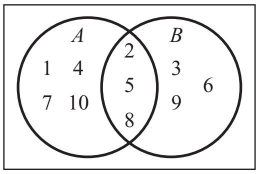
4 a



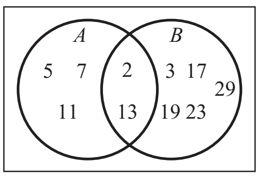
b i {2, 5, 8}

ii {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

c i $\frac{7}{10}$ **ii** $\frac{3}{10}$ **iii** 1

d No, there is at least one number in A and B.

5 a

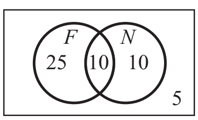


b i {2, 13}

ii {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

c i $\frac{1}{2}$ **ii** $\frac{7}{10}$ **iii** $\frac{1}{5}$ **iv** 1

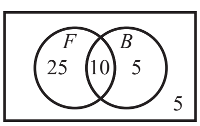
6 a



b i 25 **ii** 5

c i $\frac{2}{5}$ **ii** $\frac{1}{5}$ **iii** $\frac{1}{5}$

7 a



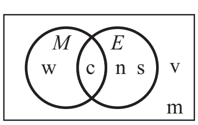
b i 25 **ii** 5

c i $\frac{7}{9}$ **ii** $\frac{2}{9}$ **iii** $\frac{8}{9}$ **iv** $\frac{2}{9}$ **v** $\frac{1}{9}$

8 3

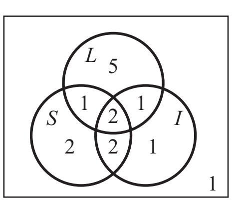
9 5

10 a



b i $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{6}$ **iv** $\frac{2}{3}$ **v** $\frac{1}{3}$

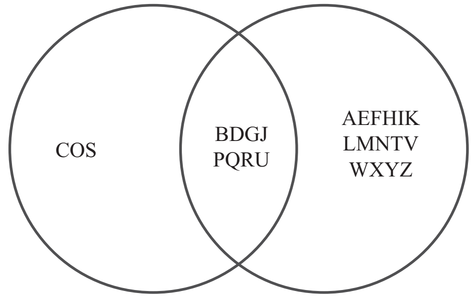
11 a



b 1

c i $\frac{3}{5}$ **ii** $\frac{1}{3}$ **iii** $\frac{13}{15}$ **iv** $\frac{1}{15}$

Drilling for Gold 4B1



Exercise 4C

1 a B **b** A **c** D **d** C

2 a i 4 **ii** 6 **iii** 3 **iv** 1

v 10 **vi** 7 **vii** 4 **viii** 7

b 13

3 a

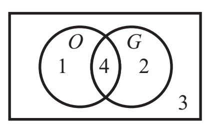
	A	not A	
B	2	6	8
not B	5	3	8
	7	9	16

b i 2 **ii** 6 **iii** 5 **iv** 3

v 7 **vi** 8 **vii** 13

c i $\frac{1}{8}$ **ii** $\frac{9}{16}$ **iii** $\frac{5}{16}$

4 a



b

	O	not O	
G	4	2	6
not G	1	3	4
	5	5	10

c i 2 **ii** 5 **iii** 4 **iv** 7

d i $\frac{3}{5}$ **ii** $\frac{2}{5}$ **iii** $\frac{1}{10}$ **iv** $\frac{2}{5}$ **v** $\frac{7}{10}$

5 a

	A	not A	
H	4	3	7
not H	4	1	5
	8	4	12

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Exercise 4C cont.

b **i** 3 **ii** 1

c **i** $\frac{11}{12}$ **ii** $\frac{1}{3}$

6 **a**

	A	not *A*	
B	3	3	6
not *B*	4	1	5
	7	4	11

b

	A	not *A*	
B	2	7	9
not *B*	2	1	3
	4	8	12

7 **a** $\frac{1}{8}$ **b** $\frac{5}{24}$

8 **a** 0

b

	A	not *A*	
B	0	6	6
not *B*	10	2	12
	10	8	18

9 **a** $\frac{3}{8}$ **b** $\frac{5}{32}$

10 **a** $\frac{4}{13}$ **b** $\frac{4}{13}$ **c** $\frac{7}{13}$ **d** $\frac{7}{13}$

11 **a** 18 **b** 75

Keeping in touch with numeracy

1 **a** $\frac{1}{2}$ **b** $\frac{3}{5}$ **c** $\frac{3}{4}$ **d** $1\frac{1}{4}$

2 **a** $\frac{11}{20}$ **b** $\frac{16}{25}$ **c** $\frac{3}{8}$ **d** $1\frac{1}{16}$

3 **a** 74% **b** 55% **c** 80% **d** $33\frac{1}{3}\%$

4 **a** 72.5% **b** 57.9% **c** 87.5% **d** 54.3%

5 13

6 73

7 south-east

8 135°

9 25, 36, 49

10 225

11 10:18 a.m.

12 11 hours 22 minutes

13 12.5 m

14 5.34 m

15 $36

16 $218.40

17 $120

18 $22 per hour

19 35

20 $142.86

Drilling for Gold 4C1

	Rotational Symmetry	No rotational symmetry	
Line Symmetry	H I O X	A B C D E M T U V W Y	
No line symmetry	N S Z	F G J K L P Q R	

Exercise 4D

1 Check with your teacher.

2 **a** E **b** F **c** A **d** B
e H **f** D **g** G **h** C

3 **a** B **b** E **c** C
d D **e** F **f** A

4 C

5 D

6 **a** numerical and discrete
b numerical and discrete
c categorical and nominal
d numerical and continuous
e categorical and ordinal

7 D

8 C

9 D

10 **a** Carrying out survey at a train station will create a very high proportion of train users in survey's results.
b Survey will reach only those people who use computers.
c Survey will access only people over 18 years of age.

11 Check with your teacher.

12 Check with your teacher.

Exercise 4E

1 **a** numerical **b** categorical (nominal)
c categorical (ordinal) **d** numerical

2 **a**

Car colour	Tally	Frequency
Red	III	3
White	~~IIII~~	5
Green	II	2
Silver	II	2
Total	**12**	**12**

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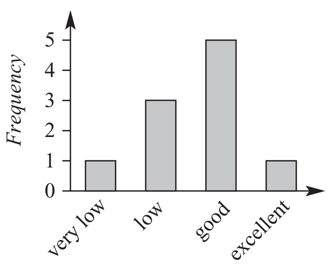
b

Class interval	Frequency	Percentage frequency
80–84	8	16%
85–89	23	46%
90–94	13	26%
95–100	6	12%
Total	**50**	**100%**

3 a i

Application	Tally	Frequency
Very low	/	1
Low	///	3
Good	~~////~~	5
Excellent	/	1
Total	**10**	**10**

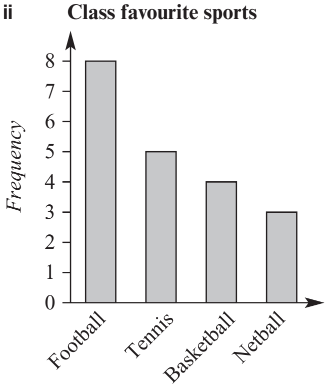
ii **Student level of application**



b i

Favourite sport	Tally	Frequency
Football	~~////~~ ///	8
Tennis	~~////~~	5
Basketball	////	4
Netball	///	3
Total	**20**	**20**

ii **Class favourite sports**

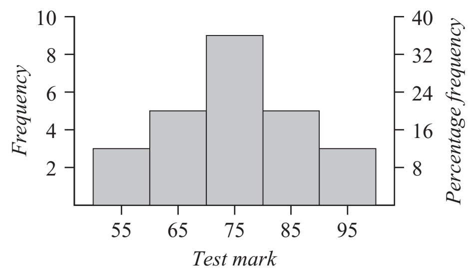


4 a

Class interval	Tally	Frequency	Percentage frequency
50−59	///	3	12%
60−69	~~////~~	5	20%
70−79	~~////~~ ////	9	36%
80−89	~~////~~	5	20%
90−99	///	3	12%
Total	**25**	**25**	**100%**

b

Class test results



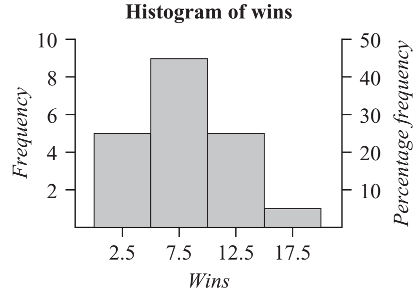
c the 70–79 interval

d 32%

5 a

Class interval	Tally	Frequency	Percentage frequency
0−4	~~////~~	5	25%
5−9	~~////~~ ////	9	45%
10−14	~~////~~	5	25%
15−19	/	1	5%
Total		**20**	**100%**

b **Histogram of wins**



c the 5–9 interval

d 75%

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Exercise 4E cont.

6 a

Type of transport	Frequency	Percentage frequency
Car	16	40%
Train	6	15%
Ferry	8	20%
Walking	5	12.5%
Bicycle	2	5%
Bus	3	7.5%
Total	**40**	**100%**

b **i** 6 **ii** car **iii** 40% **iv** 17.5% **v** 42.5%

7 a skewed

b symmetrical

8 a

Mass	Frequency	Percentage frequency
10–14	3	6%
15–19	6	12%
20–24	16	32%
25–29	21	42%
30–34	4	8%
Total	**50**	**100%**

b 50 **c** 32%

d At least 25 g but less than 30 g.

e 42% **f** 94%

9 a

Section	Frequency	Percentage frequency
Strings	21	52.5%
Woodwind	8	20%
Brass	7	17.5%
Percussion	4	10%
Total	**40**	**100%**

b 40 **c** 52.5% **d** 47.5% **e** 9.3%

10 a Russia; ~14 years

b Pakistan

c In nearly all countries, the female life expectancy is more than that for males.

d Living conditions in some areas; a high prevalence of HIV/AIDS.

11 a Saturday and Sunday; vendor would expect greater sales at the weekend.

b May have been a particularly warm day or a public holiday.

c **i** Wednesday; $250

ii Thursday

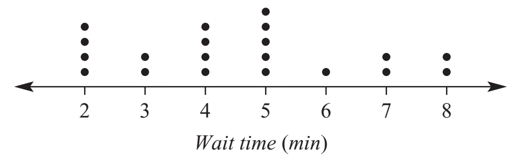
d The graph does not help us to visualise the profit and loss.

Exercise 4F

1 a histogram **b** dot plot

c column graph **d** stem-and-leaf

2



3 a 32, 35, 41, 43, 47, 54, 54, 56, 60, 62, 71, 71

b 2, 3, 7, 14, 14, 18, 19, 23, 26, 26, 30, 35

4

Stem	Leaf
10	1 1 4 8
11	0 3 3 6
12	2 3 7
13	0 2 9

5 a 11 **b** 1 **c** 16

d One family had 3 children and one had 0 childeren, but the data were generally symmetrical.

6 a 9 **b** 39

c Apart from the score of 7, the golfer scored 3, 4 or 5 on every hole.

7 a i

Stem	Leaf
1	5 5
2	0 1 2 4 5 6 6
3	1 7 7 8
4	6
5	2

1|5 means 15

ii skewed

b i

Stem	Leaf
1	2 6
2	1 3 5 7
3	1 2 5 5 6 6 8
4	0 0 2 4 8
5	1 3 5

3|2 means 32

ii symmetrical

c i

Stem	Leaf
11	6 7 8 9
12	1 4 5 7
13	3 5 7
14	5 7 9
15	3 8
16	0 2

13|5 means 135

ii skewed

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8 a i

Set 1 leaf	Stem	Set 2 leaf
9 8	3	2 5
8 6 3 2 2 0	4	1 7 7
9 7 3 3	5	2 2 5 6
4 1	6	0 1 3 4 7

5|2 means 52

ii Set 1 is symmetrical, whereas set 2 is skewed with more data at the higher end.

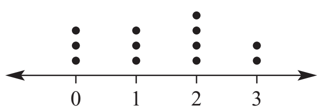
b i

Set 1 leaf	Stem	Set 2 leaf
8 6 4	16	0 3 3 5 7
6 5 2 1	17	0 1 1 4 8
8 7 7 5 1 0	18	2 2 6 7
9 5 2	19	0 4
8 1	20	

19|5 means 195

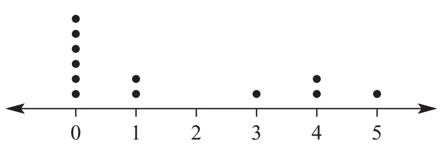
ii Set 1 is symmetrical, whereas set 2 is skewed with most of the data at the lower numbers.

9 a



Nick's goal scoring

b



Jake's goal scoring

c well-spread performance
d irregular performance, skewed

10 a 4.1 min **b** ~ 22.5 min
c This would increase the average time.

11 a

Inner city leaf	Stem	Outer suburb leaf
9 6 4 3 1 1	0	3 4 9
9 4 2 0	1	2 8 8 9
4 1	2	1 3 4
	3	4
	4	1

2|1 means 21 km

b For the inner city, the data are closer together and bunched around the lower distances. The outer-suburb data are more spread out.
c In the outer suburbs, students will be travelling greater distances to their school, whereas at inner-city schools they are more likely to live close to the school.

12 a $a = 3$, $b = 9$, $c = 7$ or 8
b $a = 0$ or 1, $b = 0$, 1, 2, 3, 4 or 5

13 a The stem 1 is allocated the leaves 0–4 (included) and 1* is allocated 5–9 (included).
b i 1 **ii** 0*
c For city B, for example, most temperatures are in the 20s; splitting into 20–24 and 25–29 allows better analysis of the data and still means that a stem-and-leaf plot is an appropriate choice of graph.
d City A experienced cooler weather, with temperatures between 8°C and 18°C. City B had warmer weather and a wider range of temperatures, between 17°C and 31°C.
e The cities may have been experiencing different seasons; maybe winter and summer.

Exercise 4G

1 a mode **b** mean **c** median **d** bimodal **e** range
2 a 4 **b** 4.5 **c** 3.2
3 a 7 **b** 10 and 14
4 a 28 **b** 7 **c** 4
5 a i 5.4 **ii** 8 **iii** 6
b i 16.25 **ii** 10 **iii** 45
c i 70 **ii** 50, 90 **iii** 40
d i 25 **ii** no mode **iii** 18
e i 2.325 **ii** 1.9 **iii** 1
f i 1.6 **ii** no mode **iii** 1.2
6 a 7 **b** 4 **c** 11
d 75 **e** 7 **f** 5
7 a 7 **b** 6 **c** 7 **d** 6
8 a $42 **b** $17.50 **c** $20.75
d Due to the $50 value, which is much larger than the other amounts.
9 a i 25 **ii** 39 **iii** 34.3 **iv** 38
b i 28 **ii** 4 **iii** 17.2 **iv** 17
c i 24 **ii** no mode **iii** 110 **iv** 108
d i 3.2 **ii** 3.0, 5.3 **iii** 4.6 **iv** 4.9
10 a Mark: **i** 83.6 **ii** 85 **iii** 31
Hugh: **i** 76.4 **ii** 79 **iii** 20
b Mark's scores varied more greatly, with a higher range, whereas Hugh's results were more consistent. Mark had the higher mean and median, though, as he had several high scores.
11 The median, since the mean is affected by the one large value ($1 700 000).
12 a 4 **b** 3.7
c i the median is unchanged in this case
ii the mean is decreased
13 a 70 **b** 85

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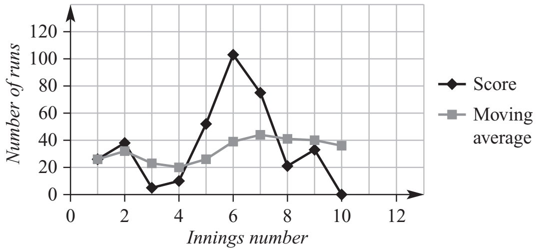
Exercise 4G cont.

14 a

Innings	1	2	3	4	5	6	7	8	9	10
Score	26	38	5	10	52	103	75	21	33	0
Moving average	26	32	23	20	26	39	44	41	40	36

b



c i The score fluctuates wildly.

ii The graph is fairly constant with small increases and decreases.

d The moving average graph follows the trend of the score graph but the fluctuations are much less significant.

Drilling for Gold 4G2

1 a The median remains in the same place

b The range increases by 2, $10 - 1 = 9$

c Mean increases from 5 to 5.6

d Range does not change because the difference between the highest and the lowest is the same.

e The mean and the median remain the same

2 a 5

b Highest 6, lowest 4.5

c 35

3 a Yes, if A and B were both 5.

b 7

c Any two numbers that add to 24

d 2, 2, 4, 7, 10

Consumer maths: Lotto, Keno and other gambling activities

1 45

2 3 838 380

3 536 878 650

Puzzles and games

1 MUTUALLY EXCLUSIVE

2 Results may vary.

3 66 kg **4** 88% **5** 8

6 a larger by 3 **b** larger by 3 **c** no change

7 3, 3, 9, 11

Multiple-choice questions

1 C **2** C **3** A **4** D
5 E **6** D **7** C **8** B
9 A **10** C

Short-answer questions

1 a $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{2}{3}$

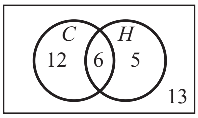
2 a $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$ **d** $\frac{5}{8}$ **e** $\frac{1}{2}$

3 a i $\frac{2}{5}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{5}$

iv $\frac{1}{10}$ **v** $\frac{1}{20}$

b i $\frac{3}{5}$ **ii** $\frac{17}{20}$

4 a



b

	C	not C	
H	6	5	11
not H	12	13	25
	18	18	36

c 13

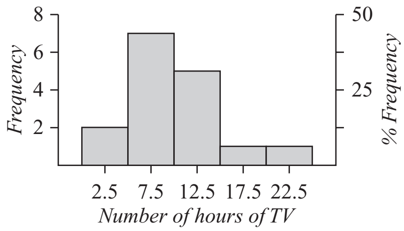
d i $\frac{1}{6}$ **ii** $\frac{5}{36}$ **iii** $\frac{1}{2}$

5 a 8 **b** $\frac{6}{13}$

6 a

Class interval	Tally	Frequency	Percentage frequency
0–4	||	2	12.5%
5–9	~~||||~~ ||	7	43.75%
10–14	~~||||~~	5	31.25%
15–19	|	1	6.25%
20–24	|	1	6.25%
Total		**16**	**100%**

b



c It is skewed.

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7 a

Stem	Leaf
1	5 9
2	0 4 8 9
3	2 4 7 8 8
4	2 9
5	0

3|2 means 32

b The data are symmetrical about scores in the 30s.

8 a i 5 **ii** 6 **iii** 5
b i 30.5 **ii** 57 **iii** 20
c i 1.6 **ii** 1.2 **iii** 1.5

9 a 43.2 years
b 38 years
c The mean is affected by the high ages 76 and 87.

Extended-response questions

1 a 8 **b i** $\frac{7}{15}$ **ii** $\frac{1}{15}$

c

	R	not R	
S	3	1	4
not S	3	8	11
	6	9	15

2 a 15, 21, 24, 32, 36, 39, 50, 51, 57, 65, 73, 73, 82, 86
b i 50.3 **ii** 50.5 **iii** 71
c Changing weather; people may have been scaring them away.

Semester review 1

Financial mathematics

Multiple-choice questions

1 C **2** A **3** C **4** D **5** A

Short-answer questions

1 a \$38.64 **b** \$51.52 **c** \$978.88 **d** \$1094.80
2 \$539
3 \$351.20
4 \$5392
5 \$597
6 a \$102 **b** \$932
7 a \$37 180 **b** \$12 833.60
c \$22 829.56 **d** \$439.03
8 \$8837.34

Extended-response question

a \$711.55 **b** \$59.92
c i \$149.80 **ii** \$832 **iii** \$83 **iv** 11.08%

Measurement

Multiple-choice questions

1 C **2** E **3** B **4** A **5** D

Short-answer questions

1 a 43 cm **b** 320 cm^2
c 30 000 cm^3 **d** 23 000 mm
e 8000 ms **f** 7.8×10^9 ns
g 0.008 Mt **h** 2.3×10^6 TB
2 a 8 cm **b** 44 m **c** 9 m
3 a i 37.70 cm **ii** 113.10 cm^2
b i 14.28 cm **ii** 12.28 cm^2
c i 11.14 m **ii** 7.14 m^2
4 a 10.5 m^2 **b** 112 cm^2 **c** 8 m^2
5 a i 45 cm^3 **ii** 78 m^2
b i 30 m^3 **ii** 72 m^2
6 8.88×10^{59}
7 a 6.5 mL to 7.5 mL **b** 8.985 g to 8.995 g
8 a 4.25 m and 4.35 m **b** 6.75 m to 6.85 m
c Perimeter: 22 m to 22.4 m
Area: 28.6875 m^2 to 29.7975 m^2

Extended-response question

a 15 m^2 **b** 37 m^2 **c** 1

Algebraic expressions and indices

Multiple-choice questions

1 C **2** D **3** D **4** E **5** C

Short-answer questions

1 a $7xy + 4x$ **b** $-21ab$ **c** $\frac{a}{2}$
2 a i $-4x + 12$ **ii** $15x^2 + 6x$ **iii** $13x - 6$
b i $6(3 - b)$ **ii** $3x(x + 2)$ **iii** $4y(2x - 3)$
3 a 4 **b** 24 **c** –1 **d** 10
e 13 **f** –8 **g** 12.5 **h** 1
i –2 **j** 35 **k** –12 **l** $\sqrt{13}$
4 a $10x^6$ **b** $8ab^2$ **c** $8m^{12}$ **d** 4
5 a i 473 000 **ii** 0.00521
b i 2.76×10^{-5} **ii** 8.71×10^6

Extended-response question

a $2x^2 - x$ **b** $x^2 + 6x$ **c** $x^2 + x + 2$

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Probability and statistics

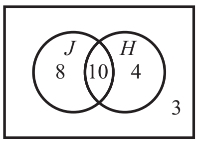
Multiple-choice questions

1 C **2** E **3** D **4** A **5** C

Short-answer questions

1 **a** $\frac{2}{5}$ **b** $\frac{11}{20}$ **c** $\frac{17}{20}$ **d** $\frac{9}{10}$

2 **a**



b

	H	not H	
J	10	8	18
not J	4	3	7
	14	11	25

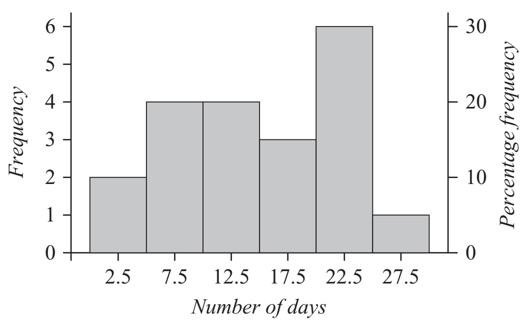
c 3

d **i** $\frac{4}{25}$ **ii** $\frac{22}{25}$ **iii** $\frac{2}{5}$

3 **a**

Class interval	Frequency	Percentage frequency
0–4	2	10%
5–9	4	20%
10–14	4	20%
15–19	3	15%
20– 24	6	30%
25–29	1	5%
Total	**20**	**100%**

b



c **i** 14 **ii** 50%

iii 20–24 days, those that maybe catch public transport to work or school each week day

4 **a**

Stem	Leaf
0	4 7 8 9
1	2 5 5 7 8
2	4 4 6
3	2 6
4	1

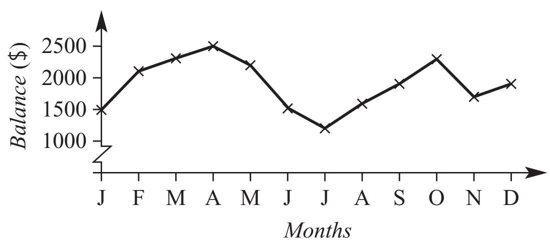
3|6 means 36

b skewed

5 $\frac{1}{2}$

Extended-response question

a



b Balance fluctuated throughout the year but ended up with more money after 12 months.

c May and June

d increase of $500

Chapter 5

Pre-test

1 **a** (3, 5) **b** (4, –2) **c** (–4, –4) **d** (–3, 1) **e** (2, –2) **f** (2, 0)

2 **a** G **b** D **c** B **d** S **e** N **f** Q

3 **a** square **b** isosceles triangle **c** hexagon

4 **a** 11 **b** 19 **c** 10 **d** 3.5 **e** 0 **f** –1 **g** 3.5 **h** –9

5 **a** 120 min **b** 200 km **c** 100 km/h

6 **a** 5 **b** 13 **c** 10 **d** 41 **e** 3.61 **f** 8.54

7 **a** 3, 4, 5, 6 **b** –2, –1, 0, 1 **c** 0, 2, 4 **d** 6, 5, 4

8 **a** 6 **b** 9 **c** 3 **d** 9 **e** 5 **f** 9 **g** 7 **h** 8 **i** 2 **j** 4 **k** 2 **l** 10

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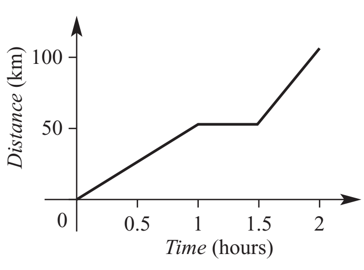
Exercise 5A

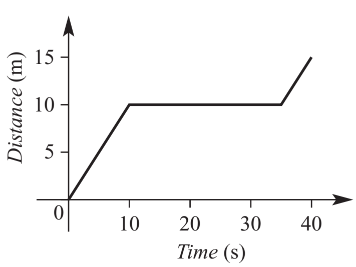
1 **a** 240 km **b** 3 hours **c** 360 km
2 **a** 360 km **b** 4 hours **c** 540 km
3 **a** 300 deer **b** 100 deer **c** 200 deer
4 **a** 1800 people **b** 450 people **c** 1350 people
5 **a** **i** $10 000 **ii** $40 000
b increased **c** $30 000
6 **a** **i** 50 cm **ii** 45 kg
b **i** 10 **ii** 3
7 **a** 80 cm **b** 40 cm **c** approx. $2\frac{1}{4}$ months
8 **a** 400 m **b** approx. 250 m
c approx. $3\frac{1}{2}$ days
9 **a** $10 000 **b** $0 **c** 12 years
10 **a** 200 g **b** 200 g
c $2\frac{1}{4}$ h (2 h 15 min)
11 **a** $\frac{1}{2}$ h (30 min)
b **i** 1 p.m. **ii** 1:15 p.m.
iii approx. 1:08 p.m. **iv** 1:30 p.m.
c **i** –120 m **ii** approx. –80 m
d **i** 0 m **ii** –160 m **iii** –280 m
e **i** 12:45 p.m. **ii** 2 p.m. **iii** 2:45 p.m.

Exercise 5B

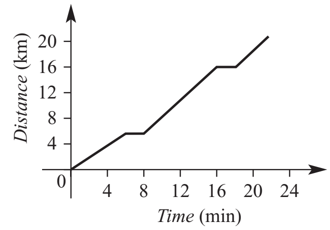
1 **a** S **b** P **c** Q **d** R
e T **f** S **g** Q
2 **a** 20 km **b** 2 h **c** approx. 17 km
3 **a** **i** 40 kg **ii** 50 kg **iii** 80 kg
b $\frac{1}{2}$ h **c** 1st hour
4 **a** 200 m **b** 80 s
c **i** 30 m **ii** 62 m **iii** approx. 150 m
5 **a** 10 km **b** 20 km **c** 27 min **d** 9 min
6



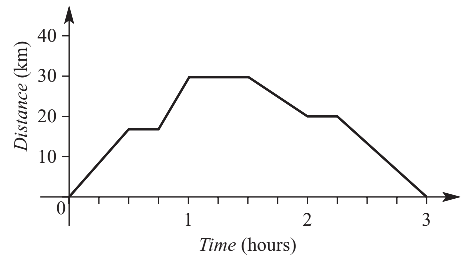
7



8 **a** 20 km **b** 22 min
c



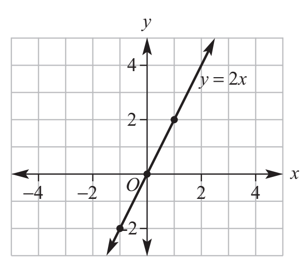
9 **a**



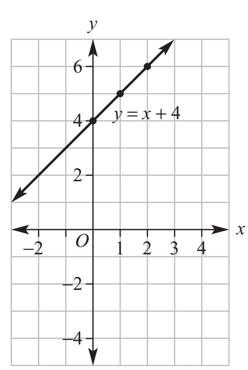
b 48 km/h **c** 20 km/h

Exercise 5C

1 **a** $A(2, 1)$ $B(–3, 3)$ $C(2, –3)$ $D(–4, 0)$ $E(4, 4)$
$F(0, –2)$ $G(3, 0)$ $H(–3, –2)$ $I(1, 4)$ $J(–2, –4)$
$K(–4, 5)$
b D, O, G
c O, F
d (0, 0)
2 **a** 2, (5, 2) **b** 0, (3, 0)
c –3, (0, –3) **d** –5, (–2, –5)
3 (–2, 1) (–1, –1) (0, –3) (1, –5) (2, –7)
4 **a** (0, 1) and (2, –3) are not in line with the other points.
b (0, 0) and (2, –2)
5 **a**



b



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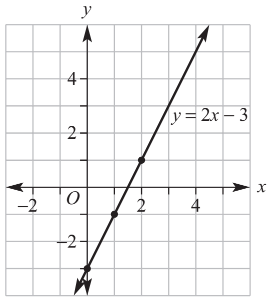
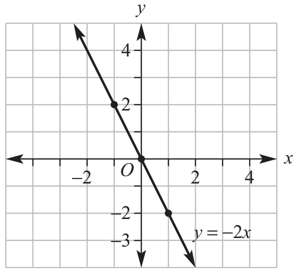
Exercise 5C cont.

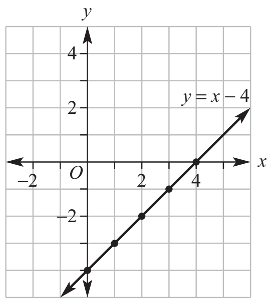
c



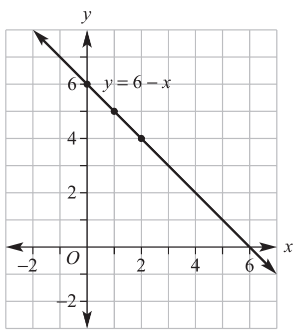
d



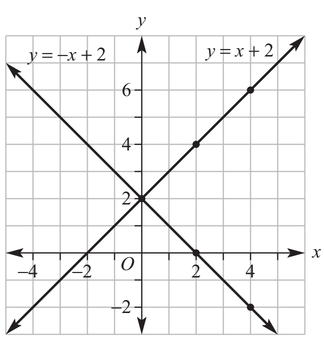
e



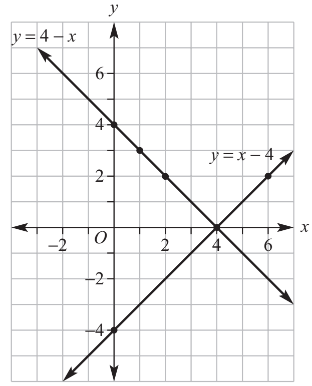
f



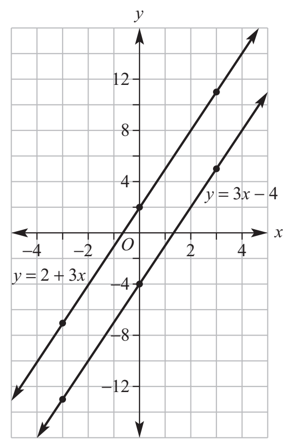
6 a



b

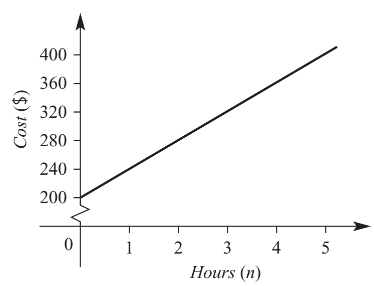


c



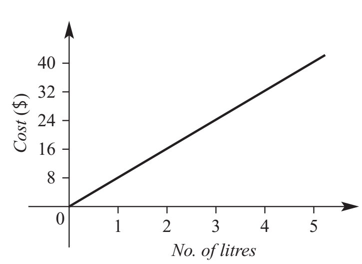
7 a (0, 0) **b** (1, 4) **c** (–1, 3)
d (0, 2) **e** (–1, –5)

8 a



b i \$300 **ii** 4.5 h

9 a



b i \$28 **ii** 2.5 litres

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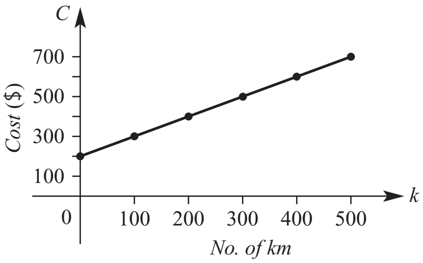
10 a

No. of km (k)	0	100	200	300	400	500
Cost (C)	200	300	400	500	600	700

b

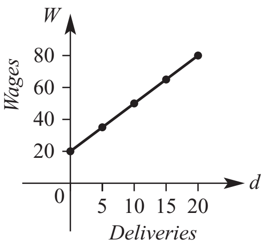


c **i** \$450 **ii** 450 km

11 a

No. of deliveries (d)	0	5	10	15	20
Wages (W)	20	35	50	65	80

b



c **i** \$56 **ii** 18

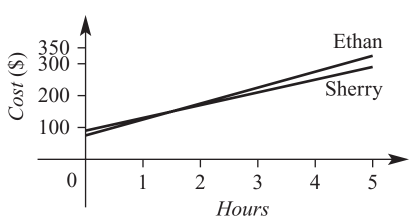
12 a Ethan

No. of hours work	0	1	2	3	4	5
Cost (C)	75	125	175	225	275	325

Sherry

No. of hours work	0	1	2	3	4	5
Cost (C)	90	130	170	210	250	290

b



c **i** \$250 **ii** \$150 **iii** 0.5 hours
iv 4.25 hours **v** 1.5 hours

d Ethan is cheaper only for 1.5 hours or less.

Drilling for Gold 5C1

1 2, 3, 4, 5, 6
2 0, 2, 4, 6, 8
3 –2, –1, 0, 1, 2
4 2, 1, 0, –1, –2
5 3, 5, 7, 9, 11
6 2, 5, 8, 11, 14
7 –6, –3, 0, 3, 6
8 –3, –1, 1, 3, 5
9 –2, 1, 4, 7, 10
10 3, 1, –1, –3, –5
11 2, –1, –4, –7, –10
12 0, –1, –2, –3, –4

Exercise 5D

1 **a** 3 and 7 **b** 5 **c** 5 and 3
d 4 **e** (5, 4)

2 **a** $\sqrt{20}$ **b** $\sqrt{18}$

3 **a** (3.5, 5) **b** (3, 4)
c (0, –1) **d** $\left(\frac{1}{2}, \frac{1}{2}\right)$

4 **a** 5 **b** $\sqrt{41}$ **c** 5 **d** $\sqrt{74}$

5 **a** $\sqrt{13}$ **b** $\sqrt{18}$ **c** $\sqrt{29}$
d $\sqrt{29}$ **e** $\sqrt{13}$ **f** $\sqrt{25} = 5$

6 **a** 5 **b** 10 **c** 11.2
d 5.7 **e** 5 **f** 3.6

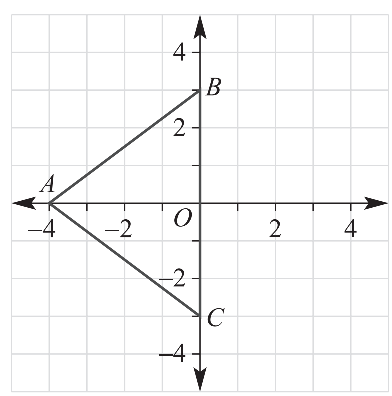
7 **a** $\sqrt{2}$ **b** $\sqrt{13}$ **c** $\sqrt{34}$
d $\sqrt{89}$ **e** $\sqrt{26}$ **f** $\sqrt{10}$

8 **a** (2, 5) **b** (4, 8) **c** (3, 5)
d (2.5, 4.5) **e** (6, 2.5) **f** (2.5, 4)
g (–1, –2) **h** (–3, –4) **i** (–4, –3)
j (1, 1) **k** (–3, –4) **l** (0, 0)

9 (–2, –5)

10 **a** $D(1, 1)$, $A(1, 4)$, $B(6, 4)$, $C(6, 1)$
b (3.5, 2.5) **c** (3.5, 2.5)
d The diagonals of a rectangle bisect (i.e. cut in half) each other.

11 **a**



b **i** 5 **ii** 5 **c** isosceles
d $P = 16$ units, $A = 12$ units^2 **e** (4, 0)

12 **a** (3, 4) **b** $\sqrt{13}$
c $\sqrt{13}$; length of radius
d 22.7 units **e** 40.8 units^2

Keeping in touch with numeracy

1 **a** 10 000 **b** 1985 **c** 50
2 20
3 18

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4 $62.50

5 18 minutes

6 1125

7 D

8 $\frac{2}{10}, \frac{2}{7}, \frac{3}{8}, \frac{1}{2}$

9 **a** 30 **b** 50 **c** 2 **d** 50

10 $260

11 $250

12 $2850 per month

13 3.4 and 3

14 Any three numbers that add to 31

15 2

16 125

17 3500 mm or 3.5 m

18 7.5

19 7.5 cents

20 $3.88

Exercise 5E

Answers are given as fractions or whole numbers, except for question 11 where a decimal is specified.

1 **a** zero **b** negative **c** positive **d** undefined

2 **a** + **b** – **c** + **d** –
e – **f** + **g** + **h** – **i** –

3 **a** 1 **b** $\frac{1}{4}$ **c** $-\frac{3}{5}$

4 **a** $-\frac{3}{8}$ **b** $\frac{1}{15}$ **c** –3

5 **a** 2 **b** 5 **c** –3
d –2 **e** $\frac{4}{3}$ **f** 0
g 0 **h** undefined **i** undefined

6 $EF\ \frac{2}{3}, GH\ \frac{2}{3}, DC\ 1, AB\ \frac{3}{2}$

7 **a** 3 **b** 2 **c** $-\frac{1}{2}$
d –1 **e** 0 **f** undefined

8 **a**

Line segment	Rise	Run	Gradient
AB	1	2	$\frac{1}{2}$
AC	2	4	$\frac{1}{2}$
AD	3	6	$\frac{1}{2}$
BC	1	2	$\frac{1}{2}$
BD	2	4	$\frac{1}{2}$
CD	1	2	$\frac{1}{2}$

b They have the same gradient.

9 **a** 2 **b** $\frac{1}{2}$ **c** –1
d –2 **e** 1 **f** $-\frac{2}{5}$

10 **a** 1 **b** 1 **c** $-\frac{3}{5}$ **d** 0 **e** 11 **f** $\frac{1}{3}$

11 gradient = 0.344

12 **a** A, D **b** C, E, G **c** D
d B, F, H **e** G **f** Answers will vary.

Exercise 5F

1 **a** gradient **b** two **c** units **d** km/h **e** L/min

2 **a** 90 km/h **b** 15 L/min

3 **a** **i** 60 km **ii** 60 km/h
b **i** 0 km/h **ii** 90 km/h

4 **a** **i** 15 km **ii** 15 km/h
b **i** 0 km/h **ii** 30 km/h

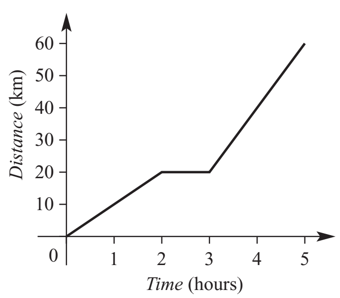
5 **a** **i** 2 L **ii** 0.5 L
b **i** 0.2 L/s **ii** 0.05 L/s **iii** 0.05 L/s

6 **a** **i** 1.5 L **ii** 0.5 L
b **i** 0.15 L/s **ii** 0.05 L/s **iii** 0.15 L/s

7 **a** 3 km **b** 4 min
c **i** 0.5 km/min **ii** 0.75 km/min
iii 0.5 km/min **iv** 0.25 km/min

8 **a** C **b** A **c** B; steepest

9 **a**



b 60 km

10 **a** 3 **b** **i** 6 km **ii** 14 km
c B, D, G **d** E, H
e **i** 6 km/h **ii** 14 km/h **iii** 16 km/h
iv 6.4 km/h **v** 16 km/h
f E and H, same gradient
g $5\frac{1}{4}$ hours **h** 40 km **i** 10 km/h

Exercise 5G

1 **a** horizontal **b** vertical
c **i** N **ii** N **iii** N
iv Y **v** N **vi** Y

2 **a** **i** 2 **ii** 4
b **i** 6 **ii** –7
c **i** $-\frac{2}{3}$ **ii** 7
d **i** –7 **ii** –3

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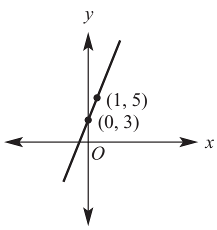
e **i** $\frac{3}{5}$ **ii** -8

f **i** 9 **ii** -5

3 $y = 2x - 1$, $y = 2x - 5$

4 $y = x + 2$, $y = x - 2$

5 **a**



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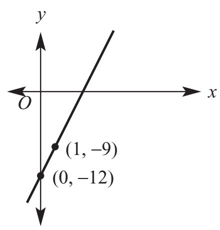
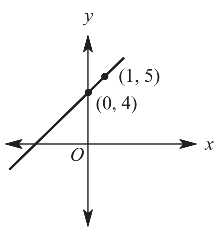
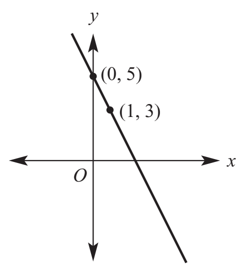


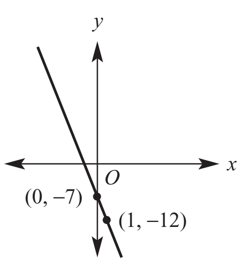
c



d



e



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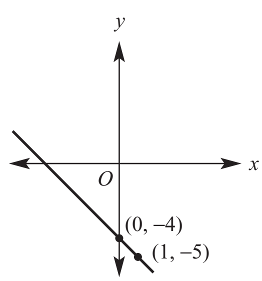
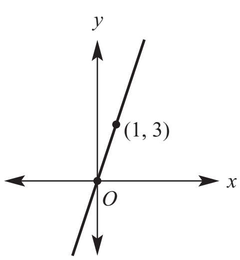


6 **a**



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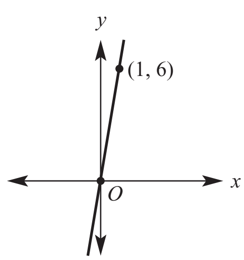

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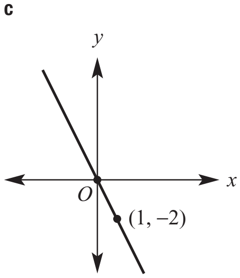

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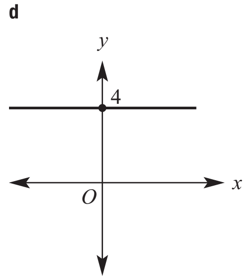

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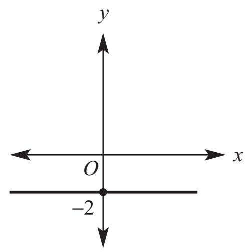

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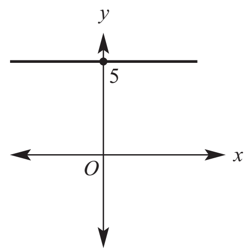

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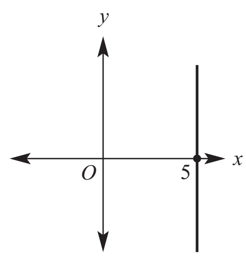
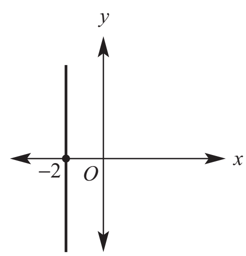
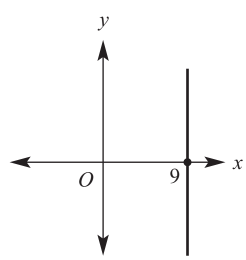


h



i



7 **a** $y = 4x + 2$ **b** $y = 3x - 2$ **c** $y = 5x$

d $y = -3x + 5$ **e** $y = -4x - 3$ **f** $y = -2x$

8 **a** 1, 0 **b** $-\frac{3}{2}$, 3 **c** $\frac{1}{2}$, 4.5

d 0, 4 **e** 0, 7 **f** undefined, none

9 **a** D **b** B **c** E **d** A **e** F

f G **g** C **h** H **i** I

10 **a** $b = 5$ in each equation; e.g. $y = 2x + 5$, $y = -3x + 5$ etc.

b $b = -2$ in each equation; e.g. $y = 7x - 2$, $y = x - 2$ etc.

c $b = 0$ in each equation; e.g. $y = 2x$, $y = -5x$ etc.

11 **a** $m = 3$ in each equation; e.g. $y = 3x - 1$, $y = 3x$, $y = 3x + 4$ etc.

b $m = -1$ in each equation; e.g. $y = -x$, $y = -x + 7$, $y = -x - 3$ etc.

c $m = 0$ in each equation; e.g. $y = 4$, $y = -2$ etc.

d m is undefined in each equation; e.g. $x = -7$ etc.

12 **a** B and C **b** A and D

13 **a** no **b** yes **c** no **d** yes **e** yes

14 **a** $y = x + 4$ **b** $y = x - 1$

c $y = \frac{x}{2}$ **d** $y = 2x + 1$

15 Check with your teacher.

16 **a** They all have the same gradient.

b They all have a y-intercept at -1.

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Maths@work: Real-world linear relationships

Answers will vary.

Exercise 5H

1 **a** 25 **b** 25 **c** 16 **d** 16
e 4 **f** 4 **g** 0.25 **h** 0

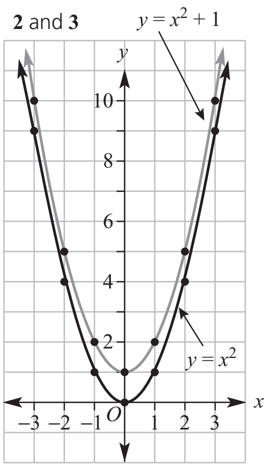
2

x	–3	–2	–1	0	1	2	3
y	9	4	1	0	1	4	9

3

x	–3	–2	–1	0	1	2	3
y	10	5	2	1	2	5	10

2 and **3**

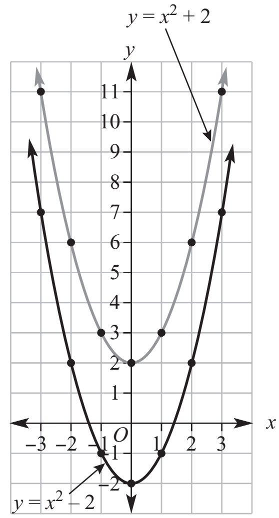


4 **a**

x	–3	–2	–1	0	1	2	3
y	11	6	3	2	3	6	11

b

x	–3	–2	–1	0	1	2	3
y	7	2	–1	–2	–1	2	7

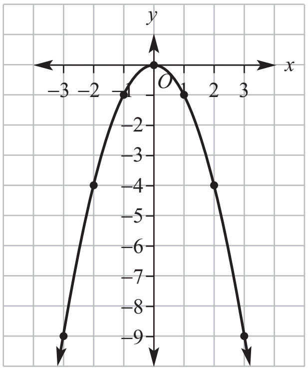


5

	Formula	Turning point	y-intercept ($x = 0$)	y value when $x = 1$
a	$y = x^2 + 3$	(0, 3)	3	$y = 4$
b	$y = x^2 - 1$	(0, –1)	–1	$y = 0$
c	$y = x^2 + 2$	(0, 2)	2	$y = 3$
d	$y = x^2 - 4$	(0, –4)	–4	$y = -3$

6

x	–3	–2	–1	0	1	2	3
y	–9	–4	–1	0	–1	–4	–9



7 **a** $y = x^2 + 2$ **b** $y = -x^2$
c $y = (x + 1)^2$ **d** $y = (x - 5)^2$

8 **a** $\pm\sqrt{14}$ **b** ±3

c
$$\begin{aligned} y &= x^2 - 9 \\ -10 &= x^2 - 9 \\ -1 &= x^2 \\ x^2 &= -1 \end{aligned}$$

This has no solutions; therefore, there is no point on the curve for which the y value is –10.

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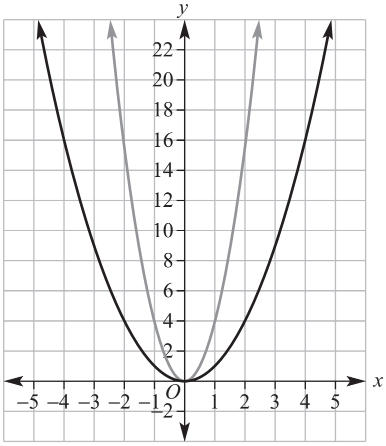
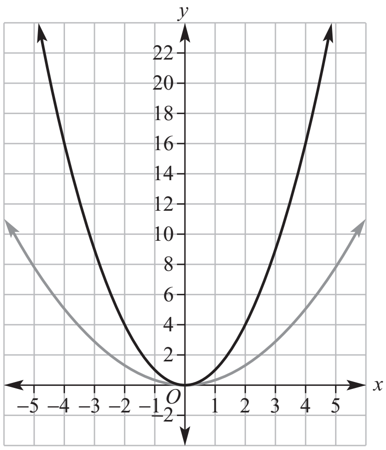
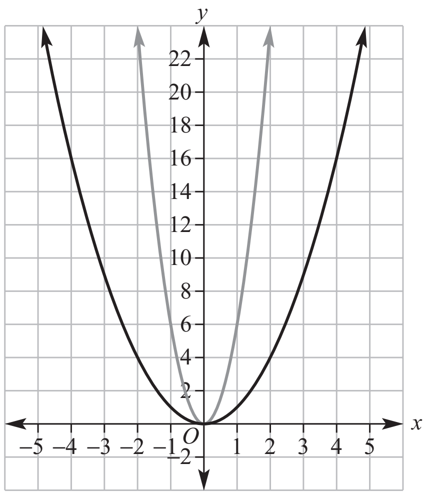
9 a i



ii



iii



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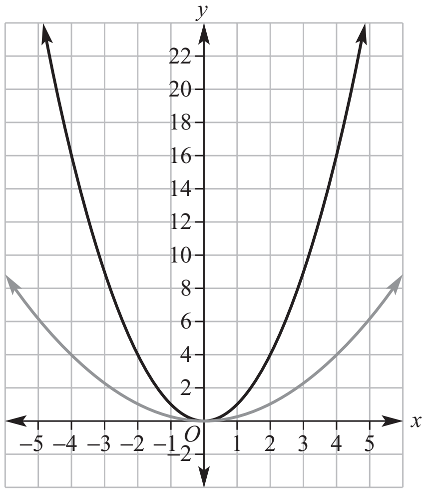
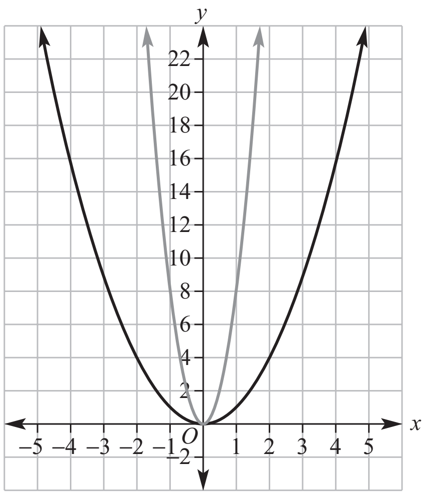
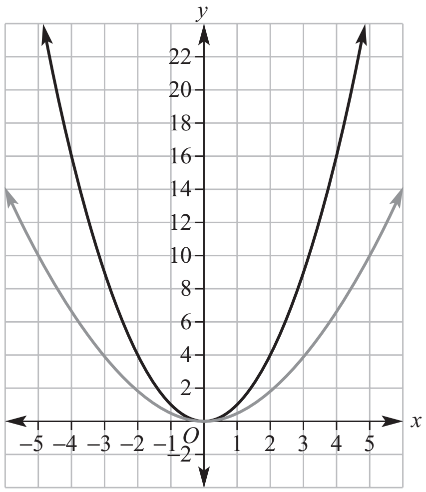


v



vi



b The constant a determines the narrowness of the graph.

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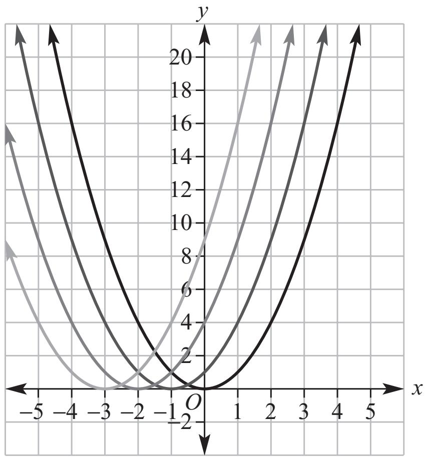
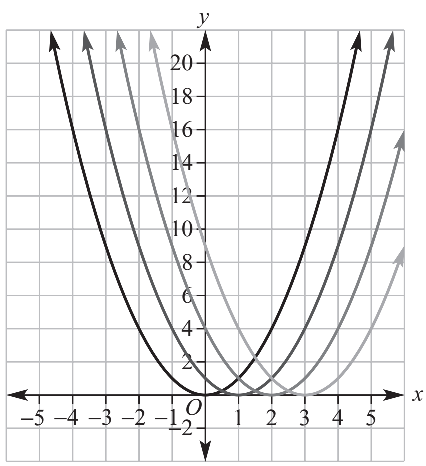
Exercise 5H cont.

10 a i

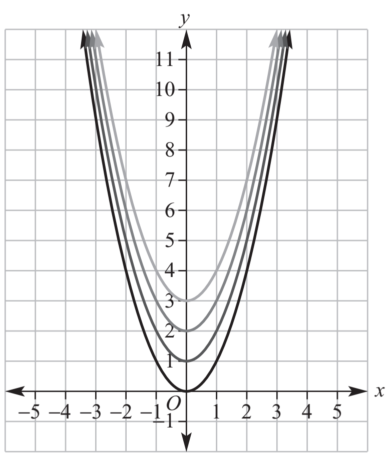


ii

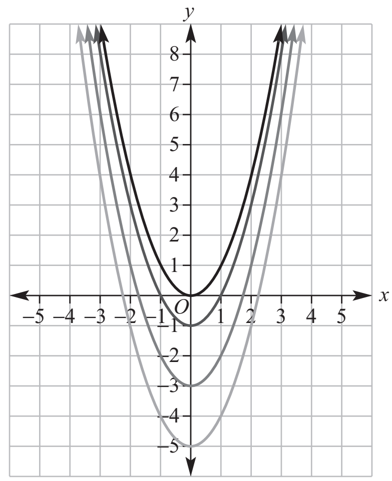


b The constant h determines whether the graph moves left or right from $y = x^2$.

11 a i



ii



b The constant k determines whether the graph moves up or down from $y = x^2$.

Exercise 5I

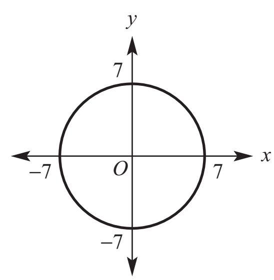
1 **a** (0, 0), $r = 3$ **b** (0, 0), $r = 6$

2 **a** (0, 0) **b** 4

3 **a** 1 **b** 2 **c** 16 **d** 1 **e** 3
f 27 **g** 1 **h** 16 **i** 1 **j** 25

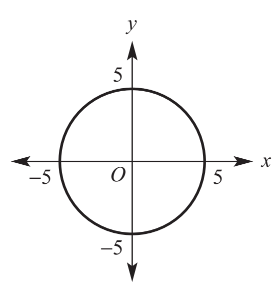
4 **a** (0, 0) **b** $r = 7$

c



5 **a** (0, 0) **b** $r = 5$

c



6 **a**

x	−2	−1	0	1	2
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

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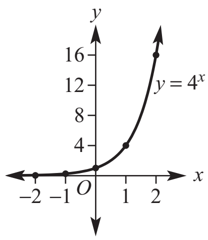
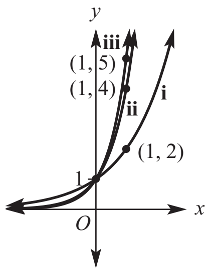
b



7 a



b the same point (0, 1)

c makes it rise more quickly

8 a $r = 6$ **b** $r = 9$ **c** $r = 12$

d $r = \sqrt{5}$ **e** $r = \sqrt{14}$ **f** $r = \sqrt{20}$

9 $x^2 + y^2 = 49$

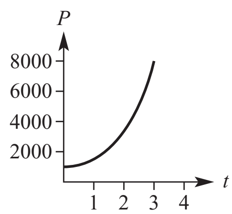
10 a C **b** A **c** B

11 a 1000

b i 2000 **ii** 8000

c i 2 years **ii** 4 years

d



Puzzles and games

1 Check with your teacher.

2 both 13 km apart

3 a 160 ice-creams for zero profit

b 493 ice-creams sold

Multiple-choice questions

1 A **2** E **3** D **4** A **5** B

6 E **7** C **8** A **9** B **10** D

11 E **12** C **13** D

Short-answer questions

1 a 40 km **b** 2 hours **c** 60 km

2 a i \$6000 **ii** \$8000 **iii** \$9000

b i \$2000 **ii** \$4000 **iii** \$5000

c 10 years

3 a 15 km

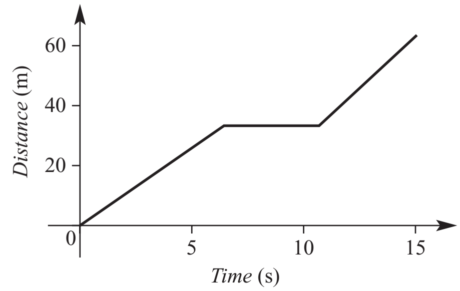
b 3 hours

c i 6 km **ii** 6 km

iii 9 km **iv** 15 km

d $3\frac{1}{2}$ hours

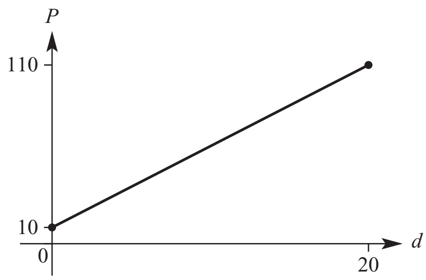
4



5 a

d	0	5	10	15	20
P	10	35	60	85	110

b



c i \$70 **ii** 17

6 a 1 **b** −1 **c** −2 **d** $\frac{1}{3}$

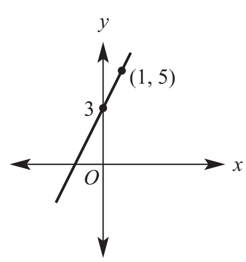
7 a (4, 2) **b** (2, 2.5) **c** (1.5, −1)

8 a $AB = 5$ **b** $PQ = \sqrt{50}$

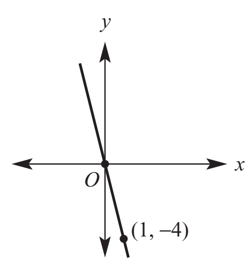
9 a gradient = 3, y-intercept = 4

b gradient = −2, y-intercept = 0

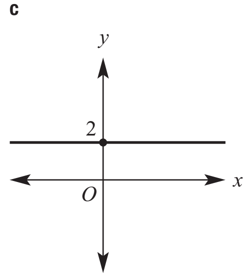
10 a



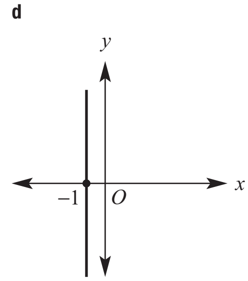
b



c



d



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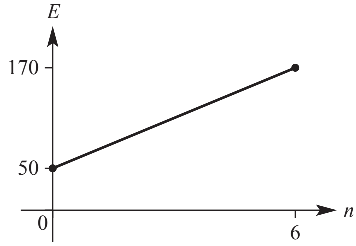
11 **a** A **b** C **c** F **d** B **e** D **f** E

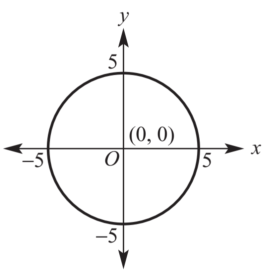
12 **a** 90, 110, 130, 150, 170

b

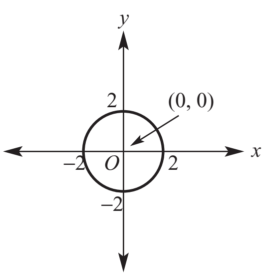


c **i** \$130 **ii** 5.5 bins

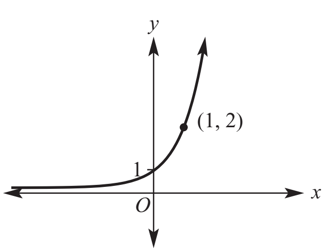
13 **a**



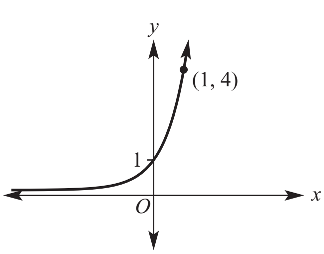
b



14 **a**



b



Extended-response questions

1 **a** **i** 30 km **ii** 15 km/h
b **i** 20 km/h **ii** 0 km/h
iii 20 km/h

2 **a** 8 km **b** 22 miles
c $m = \frac{15}{24.14} = 0.621$ **d** 0.621 miles/km

Chapter 6

Pre-test

1 **a** acute **b** 90° **c** 180° **d** straight
e reflex **f** 360° **g** 90° **h** supplementary

2 **a** scalene **b** isosceles
c right-angled **d** obtuse-angled
e equilateral **f** acute-angled

3 **a** $a = 110$ **b** $b = 140$
c $c = 210$ **d** $d = 35$
e $e = 60$ **f** $f = 40$

4 **a** $a = 110, b = 70$
b $a = 105, b = 75$
c $a = 40, b = 140$

5 **a** square
b parallelogram, incl. square, rectangle and rhombus
c square, rectangle
d square, rhombus
e trapezium
f kite

6 **a** $S = 360°, a = 130$
b $S = 540°, b = 120$
c $S = 720°, c = 120$

7 **a** no **b** no **c** no **d** no **e** yes **f** yes

Exercise 6A

1 **a** 180° **b** equal
c **i** equal **ii** equal **iii** supplementary

2 **a** alternate **b** vertically opposite
c cointerior **d** corresponding

3 $a = 20$ supplementary, $b = 20$ alternate,
$c = 160$ corresponding, $d = 160$ vertically opposite

4 $a = 100$ supplementary, $b = 100$ alternate,
$c = 80$ corresponding, $d = 80$ vertically opposite

5 **a** $x = 110, y = 110$ **b** $x = 40, y = 140$
c $x = 75, y = 105$ **d** $x = 120, y = 120$
e $x = 110, y = 70$ **f** $x = 105, y = 75$

6 **a** Yes, corresponding angles are equal.
b No, alternate angles are not equal.
c No, cointerior angles are not supplementary.

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d Yes, cointerior angles are supplementary.
e No, corresponding angles are not equal.
f Yes, alternate angles are equal.

7 **a** 60 **b** 20 **c** 100
d 115 **e** 50 **f** 330

8 **a** $a = 90, b = 90, c = 90$
b $a = 90, b = 90, c = 90$
c $a = 135, b = 45, c = 135$
d $a = 50, b = 130, c = 50$
e $a = 90, b = 130$
f $a = 110, b = 120$

9 **a** $(a, e), (d, f), (b, h), (c, g)$ **b** $(d, h), (c, e)$
c $(c, h), (d, e)$ **d** $(a, c), (b, d), (e, g), (f, h)$

10 **a** 90 **b** 75 **c** 10 **d** 30 **e** 36 **f** 30

11 **a** 12 **b** 14 **c** 2

Drilling for Gold 6A2

Corresponding angles	Alternate angles	Cointerior angles
1 $a = e$	**5** $c = e$	**7** $c + f = 180$
2 $b = f$	**6** $d = f$	**8** $d + e = 180$
3 $c = g$		
4 $d = h$		

9 $x = y$ (corresponding angles in parallel lines)
10 $x + y = 180$ (cointerior angles in parallel lines)
11 $x = y$ (corresponding angles in parallel lines)
12 $x = y$ (alternate angles in parallel lines)
13 $x + y = 180$ (cointerior angles in parallel lines)
14 $x = y$ (corresponding angles in parallel lines)
15 $x = y$ (alternate angles in parallel lines)
16 $x = y$ (corresponding angles in parallel lines)
17 $x = y$ (alternate angles in parallel lines)
18 $x + y$ (corresponding angles in parallel lines)
19 $x + y = 180$ (cointerior angles in parallel lines)
20 $x = y$ (alternate angles in parallel lines)
21 $x + y = 180$ (cointerior angles in parallel lines)
22 $x = y$ (corresponding angles in parallel lines)

Exercise 6B

1 **a** 30 **b** 16 **c** 33 **d** 60 **e** 77 **f** 98
2 **a** $c = 120$ **b** $x = 60$ **c** $x = 25 + 35$
3 C
4 **a** 70 **b** 10 **c** 25 **d** 58 **e** 50 **f** 29
5 **a** 65 **b** 80 **c** 40 **d** 20 **e** 112 **f** 32
6 **a** 145 **b** 144 **c** 45 **d** 60 **e** 60 **f** 47
7 **a** yes **b** yes **c** no **d** yes
e yes **f** yes **g** no **h** yes
8 **a** 65° **b** 115°
9 **a** $a = 60$ **b** $a = 120$ **c** $a = 70, b = 70$
10 **a** a°, alternate **b** b°, alternate **c** sum to 180°

11 **a** **i** 50° **ii** 130°
b **i** 20° **ii** 160°
c **i** 0° **ii** 180°

Drilling for Gold 6B1

Ask your teacher to check the accuracy of your measurements.

Exercise 6C

1 Parallelograms, incl. squares, rectangles and rhombuses.
2 **a** square, rectangle, rhombus, parallelogram
b rectangle, square, parallelogram, rhombus, kite
c rhombus, square, parallelogram, rectangle
d trapezium **e** kite
f square, rectangle **g** square, rectangle
h square, rhombus, kite
3 **a** $a = 144$ **b** $b = 79$ **c** $c = 54$
4 **a** $x = 20$ **b** $x = 110$ **c** $x = 240$
d $x = 125$ **e** $x = 30$ **f** $x = 65$
5 **a** $x = 60$ (cointerior angles in parallel lines)
$\therefore y = 120$ (opposite angles in a parallelogram)
$z = 60$ (cointerior angles in parallel lines)
b $x = 110$ (cointerior angles in parallel lines)
$y = 110$ (cointerior angles in parallel lines)
$z = 70$ (cointerior angles in parallel lines)
c $x = 30$ (cointerior angles in parallel lines)
$y = 150$ (cointerior angles in parallel lines)
$z = 30$ (cointerior angles in parallel lines)
d $x = 45$ (angle sum of a quadrilateral)
e $x = 100$ (angle sum of a quadrilateral)
f $x = 25$ (angle sum of a quadrilateral)
g $x = y = z = 90$ (angles in a square)
h $x = 100$ (cointerior angles in parallel lines)
$y = 140$ (cointerior angles in parallel lines)
6 **a** 115 **b** 60 **c** 30 **d** 50 **e** 90 **f** 140
7 **a** It has two equal side lengths.
b **i** 120 **ii** 40
c It has two equal side lengths and two pairs of equal angles and one pair of parallel sides.
8 **b** 65°
9 **a** 125 **b** 118 **c** 110
10 **a** $a = 30, b = 120, c = 60, d = 60, e = 30$
b $a = 80, b = 100, c = 80, d = 50$
c $a = 40, b = 20, c = 50, d = 110$
d $b = 50, c = 70$
e $a = 60, b = 20$
f $a = 10, b = 80$

Exercise 6D

1 **a** 4 **b** 8 **c** 10 **d** 7
e 9 **f** 6 **g** 5 **h** 12

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Exercise 6D cont.

2 **a** 540° **b** 720° **c** 900°
d 1080° **e** 1260° **f** 1440°

3 All sides are equal. All angles are equal.

4 **a** 720°, 110 **b** 540°, 130 **c** 540°, 30
d 900°, 105 **e** 720°, 30 **f** 360°, 30

5 **a** 24 cm **b** 720° **c** 120°

6 **a** 28 cm **b** 1080° **c** 135°

7 **a** 720° **b** 120°

8 **a** 1620° **b** 3240°

9 **a** 144° **b** 165.6°

10 **a** 108° **b** 72°

11 **a** $x = 60$, $y = 60$
b $x = 67.5$, $y = 225$

12 **a** *See table at bottom of page.*
b **i** $S = 180° \times (n - 2)$ **ii** $A = \dfrac{180° \times (n-2)}{n}$

Keeping in touch with numeracy

1 $\frac{1}{3}$

2 75

3 $15

4 $1805.35

5 multiplication

6 $50.08

7 57 km

8 49.2 cm, 56.8 cm

9 3 and 3

10 50.875 kg

11 8:25 a.m.

12 2:30 p.m.

13 $25

14 $240.63

15 *c*

16 **a** 4.808 m **b** 2.404 m

17 $11

18 500 g

19 2%

20 $158.95

Exercise 6E

1 **a** false **b** false **c** true **d** true

2 SSS, SAS, AAS, RHS

3 **a** E **b** AC **c** $\angle EDF$

4 **a** $\triangle ABC \equiv \triangle DEF$ (SSS)
b $\triangle ABC \equiv \triangle DEF$ (SAS)
c $\triangle XYZ \equiv \triangle STU$ (RHS)
d $\triangle XYZ \equiv \triangle STU$ (SSS)
e $\triangle ABC \equiv \triangle DEF$ (AAS)
f $\triangle MNO \equiv \triangle PQR$ (AAS)

5 (D, G), (C, E)

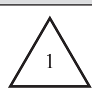
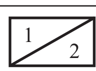
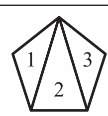
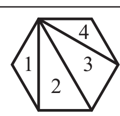
6 **a** 25°, 75° **b** yes, AAS

7 **a** $a = 4$ **b** $x = 3$, $y = 5$
c $a = 60$, $b = 7$ **d** $x = 55$, $y = 4$
e $x = 6$ **f** $a = 70$, $b = 7$
g $c = 3$, $d = 4$ **h** $a = 30$, $b = 5$
i $a = 20$, $b = 70$

8 **a** RHS **b** 6 m
c **i** 37° **ii** 53°

9 AC, ADC, ABD, ACD

12

Polygon	No. of sides	Diagram	No. of triangles	Interior angle sum (S)	Single interior angle (A)
triangle	3		1	180°	60°
quadrilateral	4		2	360°	90°
pentagon	5		3	540°	108°
hexagon	6		4	720°	120°
…					
n-gon	n		$n - 2$	$180° \times (n - 2)$	$\dfrac{180°(n-2)}{n}$

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Exercise 6F

1 similar

2 a equal b ratio c scale factor

3 a 1.5 b 1.5, the same c 1.5

4 a $x = 8$ b $x = 21$
c $x = 4$ d $x = 1.5$

5 a 2 cm b 1500
c 5 cm d 75 m

6 a 5 cm b 1500
c 4 cm d 60 m

7 a $\triangle ABC$, $\triangle ADE$
b $\angle A$ is common and $\angle ABC = \angle ADE$.
c 2.5 d 3.75 m

8 a $x = 1.5$ b $x = 9$ c $x = 2.2$

9 a i 1 km ii 3 km
b i 10 cm ii 1 cm
c 2 km

10 a $\angle BAC = \angle DEC$ (alternate), $\angle ABC = \angle EDC$ (alternate), $\angle ACB = \angle ECD$ (vertically opposite)
b i 1.5 ii $DC = 6$ cm iii $AC = 6$ cm

Exercise 6G

1 a 2 b 9, 21 c 5

2 a no b no

3 a 2 b 30 cm

4 a 1.5 b 4.5 m

5 a $\frac{88}{5} = 17.6$ b $x = 4.5$

6 1.25 m

7 a 5 or $\frac{1}{5}$ b 5 m

8 a $\frac{6}{5} = 1.2$ b 13.2 m

9 1.90 m

10 Answers will vary.

11 a 1.5 b 20 m
c Let $AE = x$
$1.5x = x + 10$
$\therefore x = 20$

Maths@home: Tiling patterns and optical illusions

Answers will vary.

Puzzles and games

1 30

2 CONGRUENCE

3 a 7 b 11

4 20 m

Multiple-choice questions

1 B 2 A 3 D 4 E 5 E
6 D 7 B 8 A 9 D 10 E

Short-answer questions

1 a $x = 70, y = 110$ b $x = 120, y = 120$
c $x = 65, y = 115$ d $x = 30, y = 150$
e $x = 90, y = 120$ f $x = 45$

2 a 20 b 30 c 77 d 20 e 60
f 30 g 130 h 70 i 160

3 a square, rectangle, rhombus, parallelogram
b parallelogram, square, rhombus, rectangle
c kite d square, rhombus, kite

4 a $a = 90$ b $a = 40$ c $a = 110, b = 30$
d $a = 90, b = 130$ e $a = 40, b = 140$
f $a = 110$

5 a $S = 540°, x = 60$ b $S = 720°, x = 100$
c $S = 900°, x = 120$

6 a 10 m b 540° c 108° d 72°

7 a RHS b SSS c AAS d SAS

8 a 3 b $\frac{1}{2}$ c 1.5 d 2.5

9 a $\triangle ABE$, $\triangle ACD$ b 2.5 c 7.5 m

10 a 6.25 b 187.5 cm

11 a 2 cm b 5000 c 5 cm d 250 m

Extended-response questions

1 a 60, 60, 30 b 2.5 c 8 cm

2 a 2 : 125 b 281.25 cm

Chapter 7

Pre-test

1 a 15.84 b 164.87 c 0.87
d 0.58 e 0.17 f 0.71
g 12.99 h 14.30

2 a 25 b 46.24 c 361
d 225 e 43.25 f 81

3 a 2.8 b 2.6 c 3.9
d 3.2 e 3.6 f 3.0
g 1.9 h 14.1

4 a c b p c y
d PQ e BC f XY

5 a $x = 3$ b $x = 4$ c $x = 12$
d $x = 35$ e $x = 108$ f $x = 9$

6 a $m = 3.65$ b $m = 1.2$ c $m = 4.4$
d $m = 5.2$ e $m = 18.848$ f $m = 5.724$

7 a $x = 0.6$ b $x = 0.2$ c $x = 2.1$
d $x = 0.4$ e $x = 2.4$ f $x = 9.2$

8 a $x = 4$ b $x = 20$ c $x = 13$ d $x = 4$

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Exercise 7A

1 **a** $z^2 = x^2 + y^2$
b $t^2 = m^2 + n^2$
c $s^2 = p^2 + r^2$

2 100, 25, 49, 74

3 **a** no **b** no **c** yes **d** yes **e** no **f** yes

4 **a** 5 **b** 17 **c** 13 **d** 25 **e** 41 **f** 10

5 **a** 5.8 cm **b** 7.3 mm **c** 18.0 m
d 11.2 km **e** 8.6 cm **f** 12.8 cm

6 **a** 8.91 cm **b** 3.62 m **c** 5.02 km

7 **a** 11.18 m **b** 12.2 m

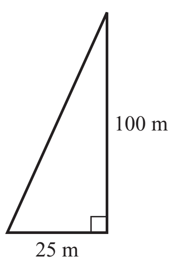
8 181.5 cm

9 35.3 m

10 13 m

11 5.28 m

12 **a** 100 m
b



c 103.08 m

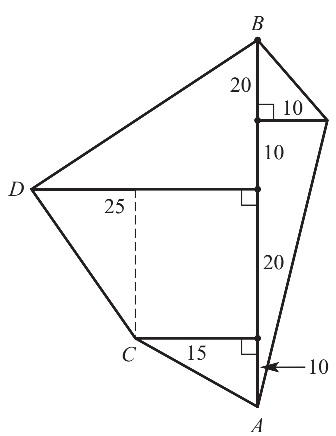
13 **a** 8.6 m **b** 13 m **c** 10 m

14 6.9 km

15 **a** 10 km **b** 41 km **c** 17 km

16 **a** **i** $AC = 11.2$ **ii** $BC = 30.4$
iii $DB = 21.2$ **iv** $AD = 29.2$
b 92 m
c 400 m^2

17 $P = 143.0$ m



Exercise 7B

1 **a** 10 **b** 24 **c** 41 **d** 1.5

2
$$10^2 = w^2 + 6^2$$
$$w^2 + 6^2 = 10^2$$
$$w^2 + 36 = 100 \quad (-36)$$
$$w^2 = 64$$
$$w = 8$$

3 **a** $17^2 = 8^2 + w^2$ **b** $13^2 = m^2 + 5^2$ **c** $15^2 = x^2 + 9^2$

4 **a** 4 **b** 7 **c** 8 **d** 5 **e** 14 **f** 4.8

5 **a** 7.1 **b** 13.3 **c** 12.3 **d** 6.2 **e** 6.6 **f** 16.2

6 **a** 7.14 **b** 13.90 **c** 3.87
d 133.84 **e** 17.89 **f** 39.19

7 5.66 m

8 2.2 m

9 3.2 m

10 **a** **i** 3 cm **ii** 5.2 cm
b 15.6 cm^2

11 7.4 cm

12 **a** 7.1 **b** 4.5 **c** 5.2

Keeping in touch with numeracy

1 8.5

2 13.5

3 –5

4 10

5 **a** 0.7 **b** 1.2 **c** 3.5

6 **a** $2.\dot{3}$ **b** 0.875 **c** $0.\dot{7}$

7 120

8 0.975 m^3

9 1.7

10 38.5 L (to 1 decimal place)

11 2 : 5

12 9 : 7

13 13 h 44 min

14 Sunday 3:02 p.m.

15 10 cm

16 12 cm

17 360 cm^2

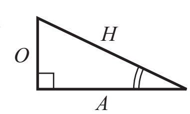
18 48 cm^2

19 A$1410

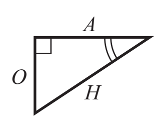
20 US70.9 cents (to 1 decimal place)

Exercise 7C

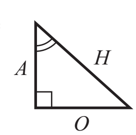
1 **a**



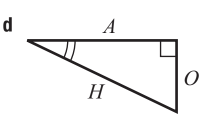
b

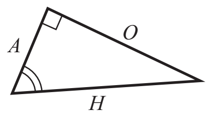


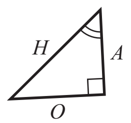
c



d



e 

f 

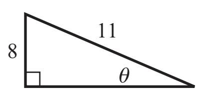
2 **a** PR **b** TP **c** TP
d PR **e** TR **f** $\angle T$

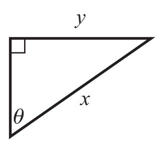
3 **a** BC **b** CA **c** BA **d** BA

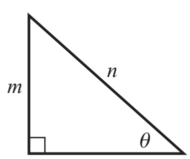
4 **a** $\frac{3}{5}$ **b** $\frac{4}{5}$ **c** $\frac{4}{5}$

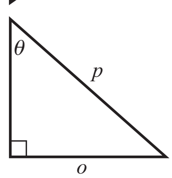
5 **a** **i** $\frac{b}{a}$ **ii** $\frac{c}{a}$ **iii** $\frac{b}{c}$
b **i** $\frac{n}{p}$ **ii** $\frac{m}{p}$ **iii** $\frac{n}{m}$
c **i** $\frac{y}{z}$ **ii** $\frac{x}{z}$ **iii** $\frac{y}{x}$
d **i** $\frac{4}{5}$ **ii** $\frac{3}{5}$ **iii** $\frac{4}{3}$
e **i** $\frac{24}{26} = \frac{12}{13}$ **ii** $\frac{10}{26} = \frac{5}{13}$ **iii** $\frac{24}{10} = \frac{12}{5}$
f **i** $\frac{12}{13}$ **ii** $\frac{5}{13}$ **iii** $\frac{12}{5}$

6 **a** $\frac{a}{c}$ **b** $\frac{y}{z}$ **c** $\frac{b}{a}$
d $\frac{6}{10} = \frac{3}{5}$ **e** $\frac{40}{41}$ **f** $\frac{1}{\sqrt{3}}$
g $\frac{8}{6} = \frac{4}{3}$ **h** $\frac{9}{41}$ **i** $\sqrt{3}$

7 **a** 

b 

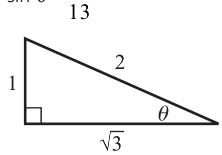
c 

d 

8 **a** $\frac{y}{x}$ **b** $\frac{z}{x}$ **c** $\frac{z}{x}$
d $\frac{y}{x}$ **e** $\frac{y}{z}$ **f** $\frac{z}{y}$

9 **a** $\sin\theta$ **b** $\tan\theta$ **c** $\sin\theta$
d $\cos\theta$ **e** $\cos\theta$ **f** $\tan\theta$

10 **a** $QR = 13$ m
b $\sin\theta = \frac{5}{13}$

11 **a** 

b $\sqrt{3}$
c **i** $\cos\theta = \frac{\sqrt{3}}{2}$ **ii** $\tan\theta = \frac{1}{\sqrt{3}}$

12

Angle (θ)	$\sin\theta$	$\cos\theta$
0°	0	1
10°	0.174	0.985
20°	0.342	0.940
30°	0.5	0.866
40°	0.643	0.766
45°	0.707	0.707
50°	0.766	0.643
60°	0.866	0.5
70°	0.940	0.342
80°	0.985	0.174
90°	1	0

a 45°
b **i** 80 **ii** 30 **iii** 0
c If angles θ and α sum to 90°, then $\sin\theta = \cos\alpha$.
d It's the same as the complement of sine.

Drilling for Gold 7C3

1 sine
2 tangent
3 cosine
4 sine
5 cosine
6 tangent
7 tangent
8 cosine
9 sine
10 sine
11 tangent
12 cosine

Drilling for Gold 7C4

1 $\sin\theta = \frac{10}{30}$
2 $\sin\theta = \frac{10}{30}$
3 $\tan\theta = \frac{7}{x}$
4 $\cos\theta = \frac{13}{x}$
5 $\cos\theta = \frac{20}{28}$
6 $\sin\theta = \frac{20}{28}$
7 $\cos\theta = \frac{y}{15}$
8 $\tan\theta = \frac{a}{12}$
9 $\tan\theta = \frac{a}{12}$

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10 $\sin\theta = \frac{12}{a}$

11 $\sin\theta = \frac{z}{9}$

12 $\sin\theta = \frac{15}{19}$

Exercise 7D

1 **a** 0.1736 **b** 0.9848 **c** 0.1763
d 0.5774 **e** 0.7660 **f** 0.9397

2 **a** 2.12 **b** 5.07 **c** 31.18
d 46.43 **e** 8 **f** 18.79
g 2.05 **h** 4.83 **i** 8.47

3 **a** sin **b** cos **c** tan
d cos **e** tan **f** sin

4 **a** $x = 1.37$ **b** $x = 5.12$ **c** $x = 91.44$
d $x = 13.86$ **e** 9.19 **f** 9.19

5 **a** 0.39 **b** 4.50 **c** 2.60
d 11.15 **e** 16.80 **f** 5.75
g 7.83 **h** 13.49 **i** 1

6 **a** 2.11 **b** 4.02 **c** 1.88

7 **a** 5.36 **b** 1.27 **c** 0.52

8 **a** 5.49 **b** 8.51 **c** 9.23

9 **a** 3.76 **b** 2.12 **c** 2.80
d 4.94 **e** 4.14 **f** 0.75

10 **a** 26.33 m **b** 52.66 m

11 6.96 m

12 **a** $b = 1.27$, $\ell = 2.72$
b $b = 0.68$, $\ell = 1.88$
c $b = 3.06$, $\ell = 2.57$

13 **a** $a = 3.5$, $b = 3.2$, $x = 1.4$
b $a = 3.464$, $b = 3.139$, $x = 1.327$
c It is better not to round off during the process as sometimes it can change the final answer.

Exercise 7E

1 **a** 17.32 **b** 13.86 **c** 106.73
d 19.84 **e** 24.69 **f** 13.20

2 **a** $x = 2$ **b** $x = 5$ **c** $x = \frac{1}{2}$
d $x = 1$ **e** $x = 0.1$ **f** $x = 0.1$

3 **a** $\frac{10}{x}$ **b** $\frac{1.4}{m}$ **c** $\frac{19}{x}$ **d** $\frac{2.8}{w}$

4 **a** 8.77 **b** 9.44 **c** 8.49

5 **a** 4.62 **b** 23.39 **c** 2.86

6 **a** 5.96 **b** 1.62 **c** 1.72

7 **a** 4.73 **b** 6.19 **c** 6.14
d 3.00 **e** 26.08 **f** 27.82

8 2.54 m

9 13.9 m

10 **a** $AB = 42.89$ cm, $BC = 20$ cm
b $AB = 5.32$ m, $BC = 1.82$ m
c $AB = 14.62$ cm, $BC = 13.74$ cm

11 **a** 7.464 m
b 7.727 m

12 **a** 30.5 m
b 17.5 m

13 **a** 17.16 **b** 30
c 4.01 **d** 59.78
e 51.13 **f** 38.09

Drilling for Gold 7E1

1 $x = \frac{5}{3}$

2 $x = 3$

3 $x = \frac{1}{3}$

4 $x = 3$

5 $x = \frac{1}{3}$

6 $x = 2$

7 $x = 30$

8 $x = 10$

9 $x = 5$

10 $x = 26.15$

11 $x = 11.83$

12 $x = 3.50$

Exercise 7F

1 sin 30° = 0.5 and $\theta = 30°$

2 **a** 30° **b** 60° **c** 45°

3 **a** $\cos\theta = \frac{5}{12}$ **b** $\sin\theta = \frac{7}{10}$ **c** $\tan\theta = \frac{4}{3}$

4 **a** 30° **b** 53° **c** 61°
d 45° **e** 41° **f** 53°
g 48° **h** 6° **i** 37°
j 81° **k** 73° **l** 60°
m 42° **n** 48°

5 **a** 60° **b** 45° **c** 64°
d 49° **e** 53° **f** 68°
g 42° **h** 56° **i** 64°
j 49° **k** 54° **l** 67°

6 11°

7 16°

8 47°

9 pitch $A = 47°$, pitch $B = 43°$

10 **a** 5.54 m **b** 5.97 m

11 **a** 12° **b** yes **c** 286.4 cm

Exercise 7G

1 **a** 50 **b** 38 **c** 56

2 diagram b

3 **a** 30° **b** 30° **c** 60°

4 **a** 21.88 m **b** 43.5 m **c** 23.41 m
d 6.06 m **e** 536.29 m **f** 38.97 m

5 **a** 112.0 **b** 49 m **c** 86 m
d 105 m **e** 9260 m

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6 **a** 27° **b** 30° **c** 6°
d 47° **e** 48°

7 38.35 m

8 1.25 km

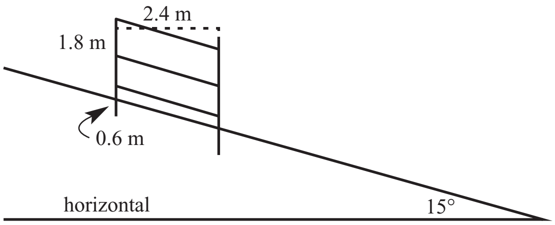
9 280.04 m

10 **a** 30°
b 30°
c equal due to parallel lines
d 1.7

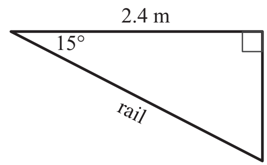
11 Answers will vary.

Maths@work: Rex's fence

1



2



The length of the rail could be calculated using trigonometry in a right-angled triangle.

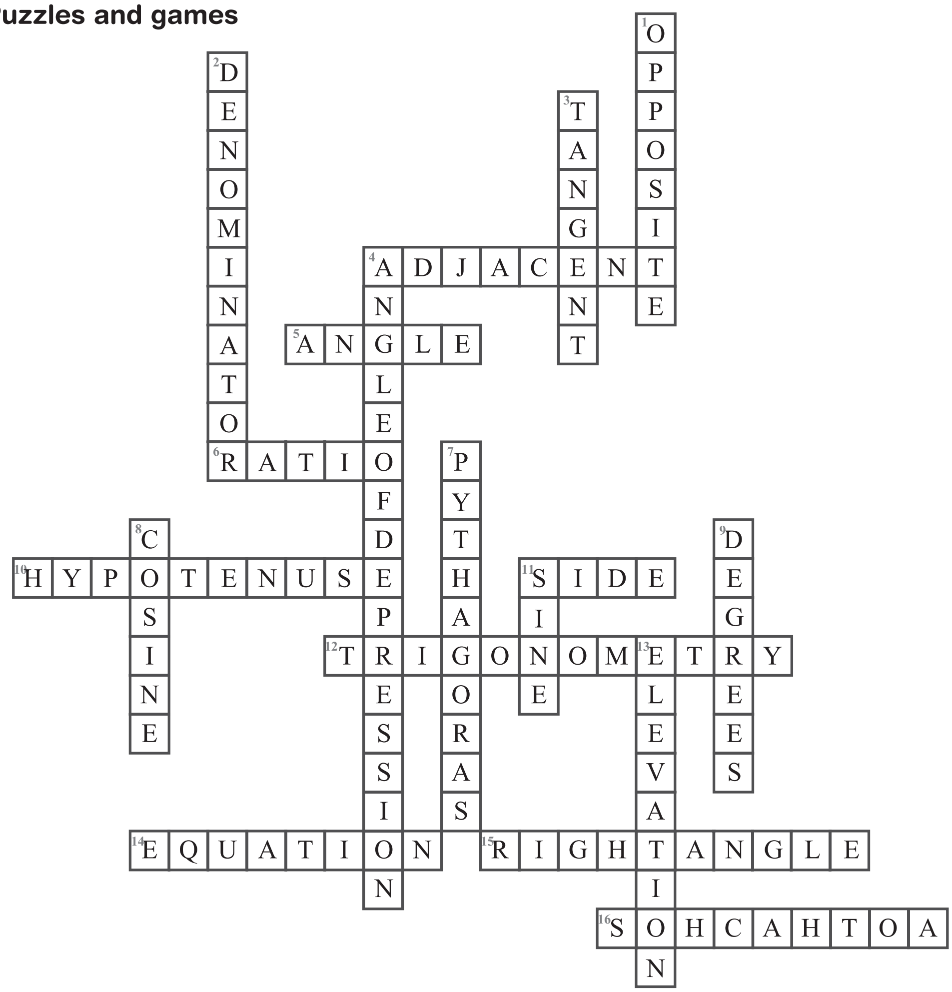
3 $\cos 15° = \frac{2.4}{x}$

$$x = \frac{2.4}{\cos 15°}$$

$$x = 2.4846\ldots$$

The rails are approximately 2.485 m long.

Puzzles and games



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Multiple-choice questions

1 B 2 E 3 C 4 B 5 C
6 C 7 D 8 E 9 A 10 C

Short-answer questions

1 No, $8^2 \neq 4^2 + 6^2$
2 15.17 m
3 a i BC ii AB iii AC
  b i AC ii AB iii BC
4 a 0.57 b 0.96 c 8.14
5 a 4.50 b 1.5 c 0.93
6 a 6.31 b 12.94 c 4.16
7 a 3.63 m b 1.69 m
8 567.13 m
9 a 42° b 59° c 72°
10 a 45° b 42.4 m

Extended-response questions

1 a 54.46°
  b 130.25 m
  c 187 m
2 a 539 km
  b 68°

Chapter 8

Pre-test

1 a 3 b 9
  c –18 d 36
  e 9 f 0
2 a 9 b 14 c 6
  d 26 e –16 f 32
3 a $3a$ b $3m$ c $8p$
  d 0 e $-5m$ f $7x + y$
  g $9p$ h $6m$
4 a $15x$ b $16p$ c $32xy$
  d $-30a$ e ab f x
  g 1 h $2a$ i 3
5 a $x + 3$ b $n + 6$
  c $2w$ d $\frac{x}{2}$
  e $2x + 6$ f $x - 7$
  g $2(x + 3)$ h $3x + 1$
6 a $8x$ b $4x + 16$ c $3x + 4$
7 b, e, f, g, i

Exercise 8A

1 a yes b no c no
  d no e yes f yes
2 a $t = 3$ b $m = 6$ c $x = 6$
  d $m = -7$ e $x = 8$ f $x = -4$
  g $m = 32$ h $a = -6$ i $m = 15$
3 a $x = 6$ b $x = 9$ c $x = 17$
  d $x = 3$ e $x = -6$ f $x = 12$
  g $x = 36$ h $x = 120$ i $x = 101$
4 a $p = 3$ b $c = 6$ c $d = 9$
  d $m = 8$ e $z = 25$ f $w = 9$
  g $p = 1$ h $m = -7$ i $p = -\frac{1}{2}$
5 a $x = 50$ b $m = 21$ c $a = -12$
  d $z = 0$ e $x = -8$ f $w = -27$
  g $r = 56$ h $w = \frac{3}{2}$ i $m = \frac{1}{2}$
6 a $x = 3$ b $x = 9$ c $x = -11$
  d $x = 10$ e $x = 14$ f $x = 10$
  g $x = 3$ h $x = 4$ i $x = 50$
  j $x = 20$ k $x = 21$ l $x = 7$
7 a $x = 1$ b $x = 3$ c $x = 3$
  d $x = -2$ e $x = -1$ f $x = -9$
  g $x = 4$ h $x = 8$ i $x = 8$
  j $x = -2$ k $x = -3$ l $x = 5$
8 a $x = 9$ b $x = 0$ c $x = 56$
  d $x = 20$ e $x = 35$ f $x = 90$
  g $x = -32$ h $x = -20$ i $x = 22$
9 a $m = 5$ b $a = 7$ c $x = 1$
  d $x = 1$ e $n = 9$ f $m = 22$
  g $w = -7$ h $m = 7$ i $w = 27$
  j $a = 1$ k $a = -37$ l $m = -5$
10 a $x + 4 = 6, x = 2$ b $x + 12 = 8, x = -4$
  c $x - 5 = 5, x = 10$ d $\frac{x}{3} + 2 = 8, x = 18$
  e $2x + 3 = 9, x = 3$ f $\frac{x - 3}{5} = 6, x = 33$
  g $3x + 4 = 16, x = 4$
11 a 13 cm b 22 mm
12 a 3 b 5 c 28
  d 42 e 82
13 a 11, 12 b 25, 44 c 8 m
14 a $280 b $1120 c 6 h
15 a 3 min b 11 min

Drilling for Gold 8A2

2 $4n + 4 = 20, n = 4$
3 $\frac{n}{4} = 20, n = 80$
4 $2n - 4 = 20, n = 12$
5 $2(n + 4) = 20, n = 6$
6 $n + 4 = 20, n = 16$
7 $\frac{n}{2} + 4 = 20, n = 32$
8 $4n = 20, n = 5$
9 $\frac{n + 4}{2} = 20, n = 36$
10 $n + 4 = 20, n = 16$
11 $\frac{n}{2} - 4 = 20, n = 48$
12 $\frac{n}{4} + 2 = 20, n = 72$

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Drilling for Gold 8A3

1 21 and 29
2 13 cm and 17 cm
3 6 cm and 24 cm
4 19 cm and 22 cm
5 11 cm
6 $2.50
7 14 m and 8 m
8 34, 35, 36

Exercise 8B

1 **a** $3x - 3$ **b** $5x + 15$ **c** $2x + 4$
d $3x - 12$ **e** $8x - 4$ **f** $5x + 13$
g $7x + 26$ **h** $9x + 9$

2 **a** subtract $2x$ **b** subtract x
c subtract $3x$ **d** add x
e subtract $2x$ **f** subtract $3x$
g add $2x$ **h** add $3x$

3 **a** $x = 1$ **b** $x = 5$ **c** $x = -1$
d $a = 5$ **e** $a = 1$ **f** $x = 15$
g $m = 4$ **h** $d = 1$ **i** $a = 10$
j $a = 0$ **k** $x = 0$ **l** $a = 3$

4 **a** $x = 1$ **b** $x = 2$ **c** $x = 3$
d $x = 2$ **e** $x = 3$ **f** $x = 2$
g $x = -2$ **h** $x = -1$

5 **a** $x = 2$ **b** $x = 12$ **c** $x = -3$
d $x = 20$ **e** $x = 4$ **f** $x = 8$
g $x = 4$ **h** $x = -4$ **i** $x = -1$
j $x = 11$ **k** $x = 1$ **l** $x = -1$

6 **a** $x = 13$ **b** $x = 6$ **c** $x = 13$
d $x = 11$ **e** $x = 10$ **f** $x = 5$
g $x = 6$ **h** $x = 8$ **i** $x = -2$

7 **a** $x = 12$ **b** $x = 18$ **c** $x = 60$
d $x = 9\frac{1}{3}$ **e** $x = 5$ **f** $x = -6$

8 **a** $x = 10$ **b** $x = \frac{5}{3}$

9 **a** $x = 3$ **b** $x = 5$
c $x = 10$ **d** $x = 7$

10 **a** $x = 7$ **b** $x = 3$ **c** $x = 5$ **d** $x = 2$
e $x = 5$ **f** $x = 6$ **g** $x = 11$

11 **a** 7 hours **b** 8 : 15 p.m.

Exercise 8C

1 **a** I **b** F **c** V
d A **e** c **f** P

2 **a** $m = 60$ **b** $A = 48$ **c** $A = 36$
d $v = 14.3$ **e** $m = 3.7$

3 **a** $t = 4$ **b** $t = 4$
c $t = 10$ **d** $t = 8$

4 **a** $b = 20$ **b** $b = 18$
c $b = 34$ **d** $b = 2.6$

5 **a** $h = 5$ **b** $h = 12$ **c** $h = 3$ **d** $h = 7$
6 **a** $b = 15$ **b** $b = 16$ **c** $b = 12$ **d** $b = 32$
7 **a** $h = 8$ **b** $h = 8$ **c** $h = 12$ **d** $h = 28$
8 **a** $m = 4$ **b** $m = 40$ **c** $m = 72$ **d** $m = 4$
9 **a** $h = 5.7$ **b** $h = 5.1$ **c** $h = 5.7$ **d** $h = 16.0$
10 **a** 86°F **b** –1.1°C **c** 212°F **d** –17.8°C
11 **a** $32 **b** 60 km
12 **a** $P = 750$ **b** $t = 3.125$ **c** $R = 0.075$
13 **a** 1.5 tablets **b** 1250 mg
14 **a** 75 mL/h **b** $\frac{1}{3}$ h = 20 min
15 **a** number of hours **b** 7.5 hours

Keeping in touch with numeracy

1 millimetres, tonnes, hectares, cubic metres, hours
2 50
3 2
4 166
5 **a** 5, 2 **b** 4, 75
6 **a** 40% **b** $\frac{2}{5}$ **c** 0.4
7 3.875
8 335
9 24
10 28.3 cm
11 75 cm
12 2.65 m^3
13 $V = 60$
14 $I = 3.75$
15 25%
16 $13 311.16
17 127 500 000, 1.275×10^8
18 1.813×10^{11}
19 297
20 $350

Maths@work: Some formulas you might meet at work

1 40
2 6
3 5

Puzzles and games

1 Each row, column and diagonal adds to 6.

9	–5	–4	6
–2	4	3	1
2	0	–1	5
–3	7	8	–6

2 **a** 64 **b** 8 **c** 29.3 **d** 18 years old

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Multiple-choice questions

1 C **2** B **3** C **4** D **5** D **6** D

Short-answer questions

1 **a** $a = 8$ **b** $m = -30$ **c** $x = -8$
d $x = 8$ **e** $m = 0$ **f** $w = 15$
g $m = -0.2$ **h** $w = 4$ **i** $r = 6$

2 **a** $m = 2$ **b** $w = 8$ **c** $m = 10$
d $w = 8$ **e** $m = 14$ **f** $m = \frac{4}{3} = 1\frac{1}{3}$
g $a = \frac{3}{2} = 1\frac{1}{2}$ **h** $x = 1$ **i** $x = 3$

3 **a** $m = 3$ **b** $a = 7$ **c** $x = 4$
d $x = \frac{3}{2} = 1\frac{1}{2}$ **e** $m = 2\frac{1}{2}$ **f** $x = \frac{7}{8}$

4 **a** $p = 4$ **b** $p = 4$ **c** $p = -9$
d $p = -2$ **e** $p = -5$ **f** $p = 2$

5 **a** $b = 8$ **b** $b = 3.5$ **c** $x = 2.4$
d $m = 10$ **e** $C = 35$

6 **a** $6x = 420$, the number is 70
b $x + 8 = 5$, the number is -3
c $\frac{a}{9} = 12$, the number is 108
d $3x + 7 = 16$, the number is 3
e $2(x + 6) = 18$, the number is 3

Extended-response question

a $P = 6x - 4$
b **i** $x = 22$
ii 20 cm, 27 cm, 37 cm, 7 cm, 30 cm
c **i** $x = 26$; 24 cm, 31 cm, 45 cm, 7 cm, 38 cm
ii $x = 38$; 36 cm, 43 cm, 69 cm, 7 cm, 62 cm

Semester review 2

Linear and non-linear relationships

Multiple-choice questions

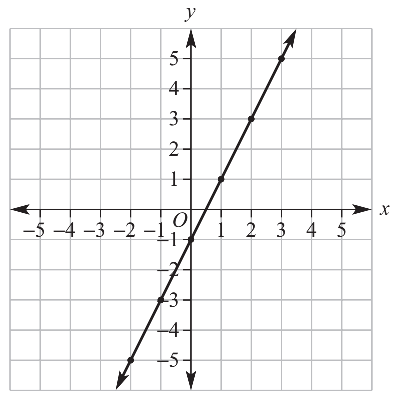
1 D **2** E **3** B **4** A **5** E

Short-answer questions

1 **a** **i** 16 km **ii** 24 km **iii** 32 km
b 16 km/h **c** 1.5 hours
d 45 km **e** 2.5 hours
f 18 km/h **g** 90 km

2

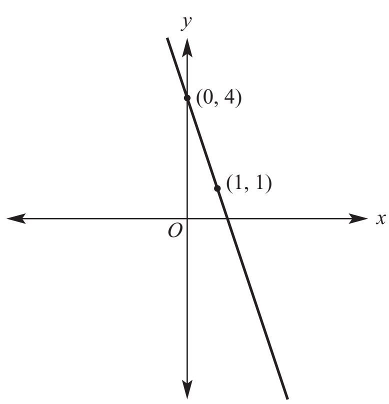
x	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5



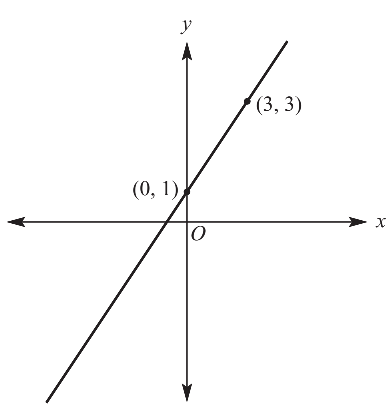
3 **a** AB midpoint (1, 2)
b PQ midpoint (–1, –1)

4 **a** $m = 2$, length = $\sqrt{20}$
b $m = -3$, length = $\sqrt{90}$
c $m = -\frac{5}{4}$, length = $\sqrt{41}$
d $m = \frac{8}{5}$, length = $\sqrt{89}$

5 **a**



b



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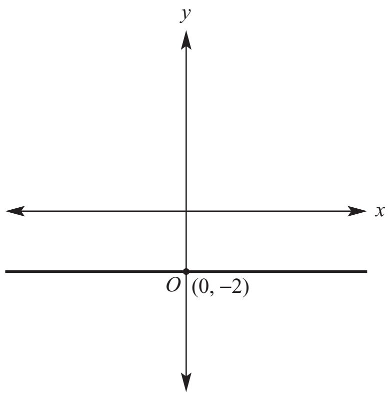
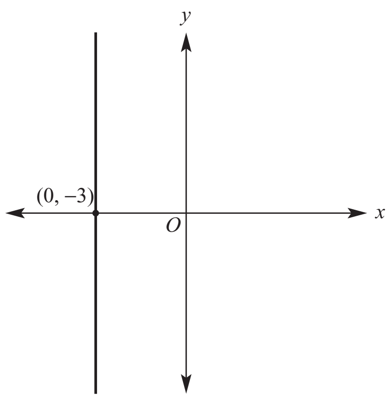
c



d



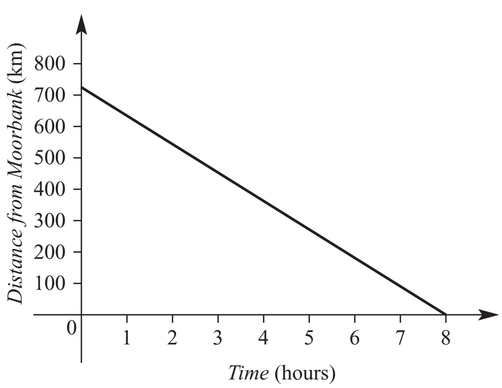
6 **a** 13 lb **b** 16 kg
c 20 lb = 9 kg, rate = 2.2 lb/kg

Extended-response question

a

Time in hours (t)	0	2	4	6	8
Km from Moorbank	720	540	360	180	0

b



c 9 a.m. **d** 7 hours **e** 8 hours

Properties of geometrical figures

Multiple-choice questions

1 C **2** E **3** C **4** A **5** D

Short-answer questions

1 **a** 39 **b** 61 **c** 53 **d** 75
e 70 **f** 117 **g** 71 **h** 84
i 110 **j** 120 **k** 135 **l** 50

2 **a** $\triangle ABC \equiv \triangle DEF$ (SAS)
b $\triangle ABC \equiv \triangle DEF$ (RHS)
c $\triangle STU \equiv \triangle MNO$ (AAS)
d $\triangle XYZ \equiv \triangle ABC$ (SSS)

3 **a** 2 **b** 4 **c** $\frac{1}{3}$ **d** $\frac{1}{2}$

4 $\frac{2}{3}$, $x = 1.8$, $y = 5$

Extended-response question

a 2.5 **b** 5 m

Right-angled triangles

Multiple-choice questions

1 E **2** C **3** A **4** B **5** D

Short-answer questions

1 **a** $\frac{y}{x}$ **b** $\frac{z}{x}$ **c** $\frac{y}{z}$

2 **a** 11.4 **b** 10.2 **c** 0.8
d 6.0 **e** 27.0 **f** 21.2

3 14 m

4 8.95 m

5 **a** 60° **b** 37° **c** 77°

Extended-response questions

1 177.9 m

2 259.8 m

Equations and formulas

Multiple-choice questions

1 C **2** D **3** A
4 B **5** E

Short-answer questions

1 **a** $p = 2$ **b** $a = 4$
c $x = 12$ **d** $x = 20$

2 **a** $x = 8$ **b** $k = 3$

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c $m = 9$ d $x = 5$
e $a = -4$ f $x = 4$

3 a $x - 5 = 8$; $x = 13$
b $4x + 8 = 20$; $x = 3$
c $2(3x - 6) = 18$; $x = 5$

4 a $b = 10$
b $P = 400$

Extended-response questions

1 a $x = 9$
b LHS = 11, RHS = 11
c Answers will vary.

2 a $x = 14$
b LHS = 11, RHS = 11
c Answers will vary.

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