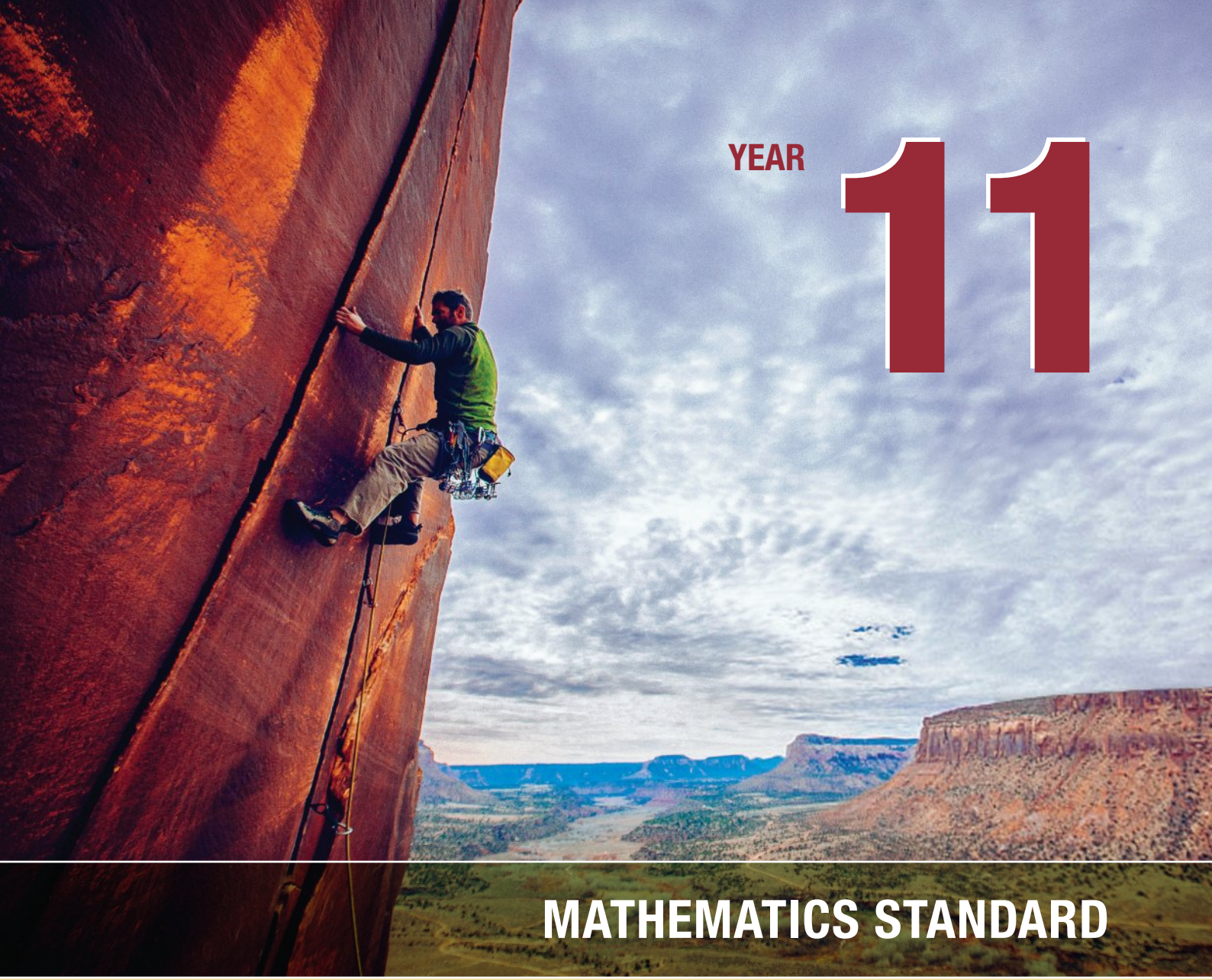

YEAR 11

MATHEMATICS STANDARD

CambridgeMATHS

STAGE 6

GK POWERS

Covers the November 2017 Syllabus Update

INCLUDES INTERACTIVE TEXTBOOK POWERED BY CAMBRIDGE HOTMATHS

CAMBRIDGE UNIVERSITY PRESS

Mathematics Standard Year 11 ISBN 978-1-108-43463-8

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About the author

Greg Powers is currently the Head of Mathematics at Cabramatta High School and the coordinator of the Mathematics Head Teacher Western Network. He is an experienced classroom teacher, having taught for over 30 years in a range of different schools. Greg has been a senior marker for the HSC, educational consultant for the Metropolitan South West Region and presented at numerous MANSW inservices. He has also enjoyed several curriculum roles with the Department of Education and Training. Greg is an experienced author who has written numerous texts on mathematics and technology.

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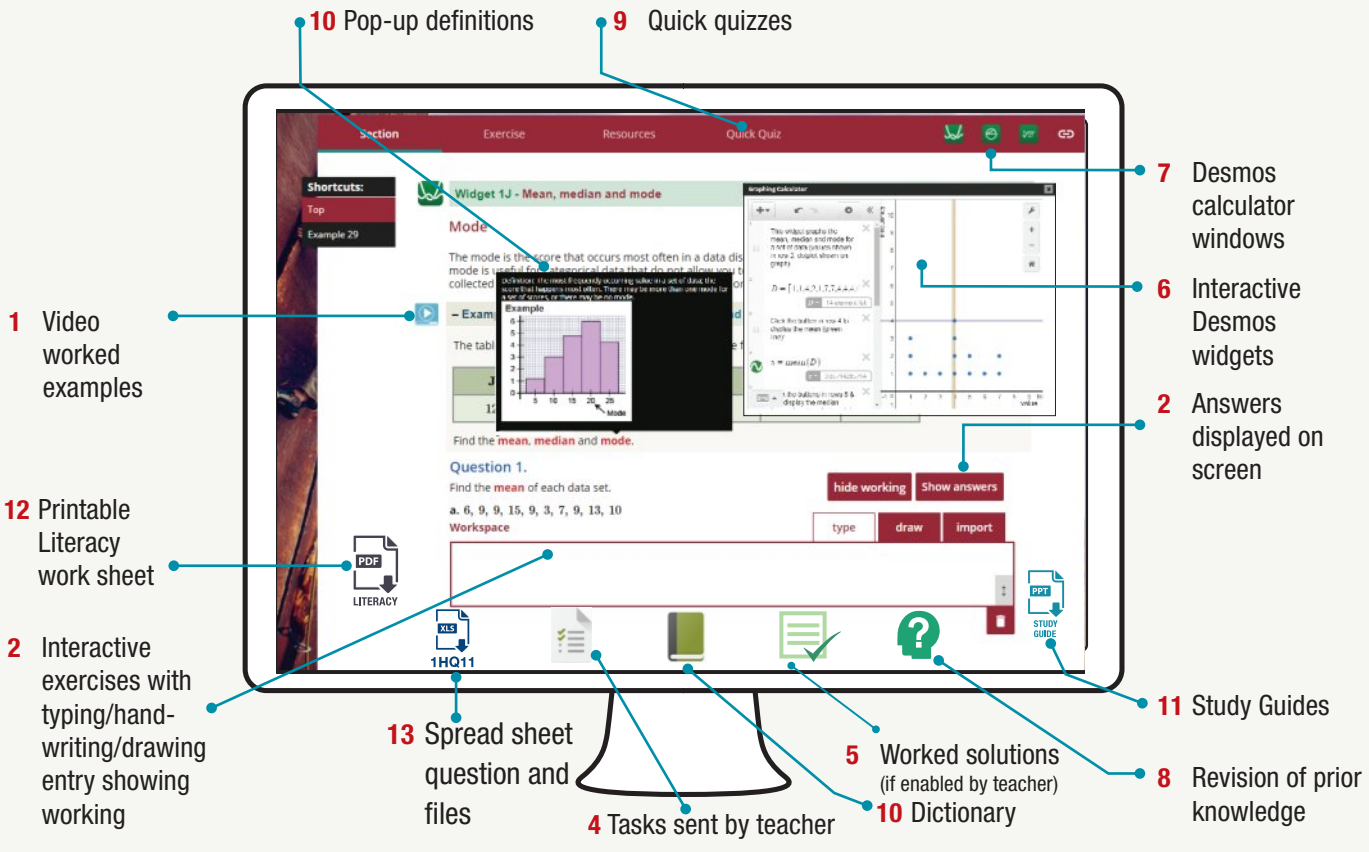

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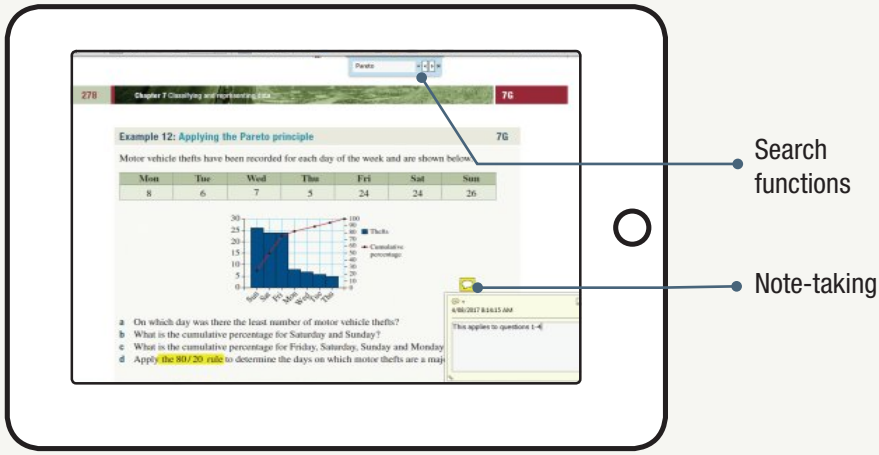

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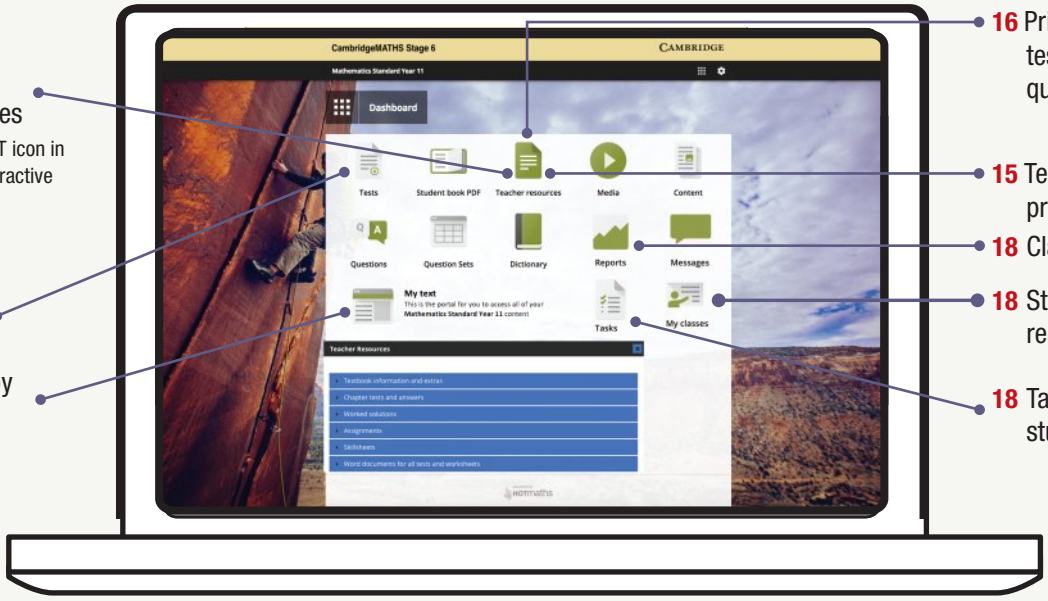

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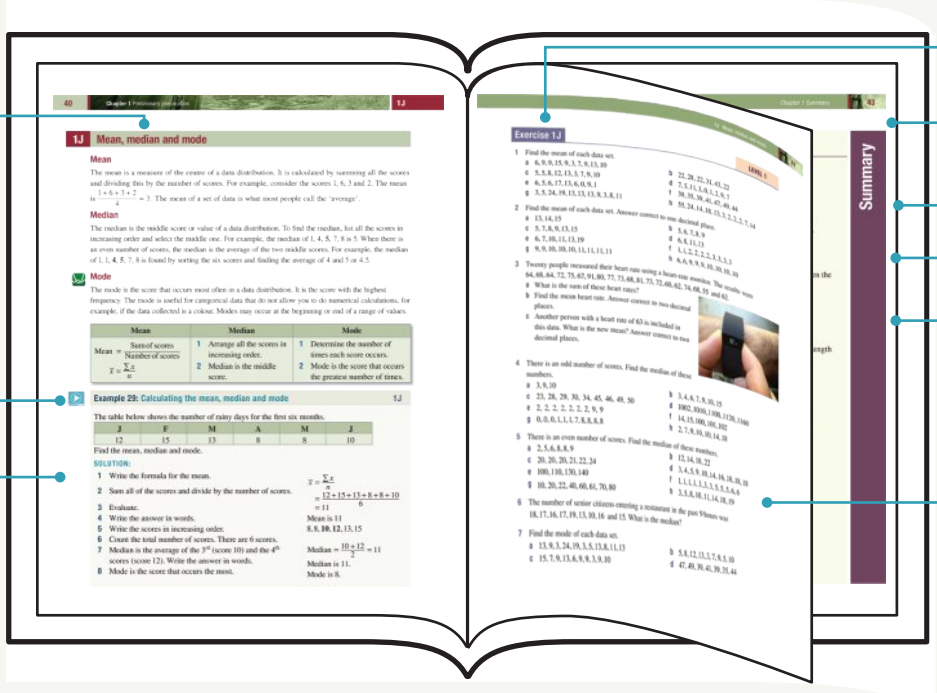

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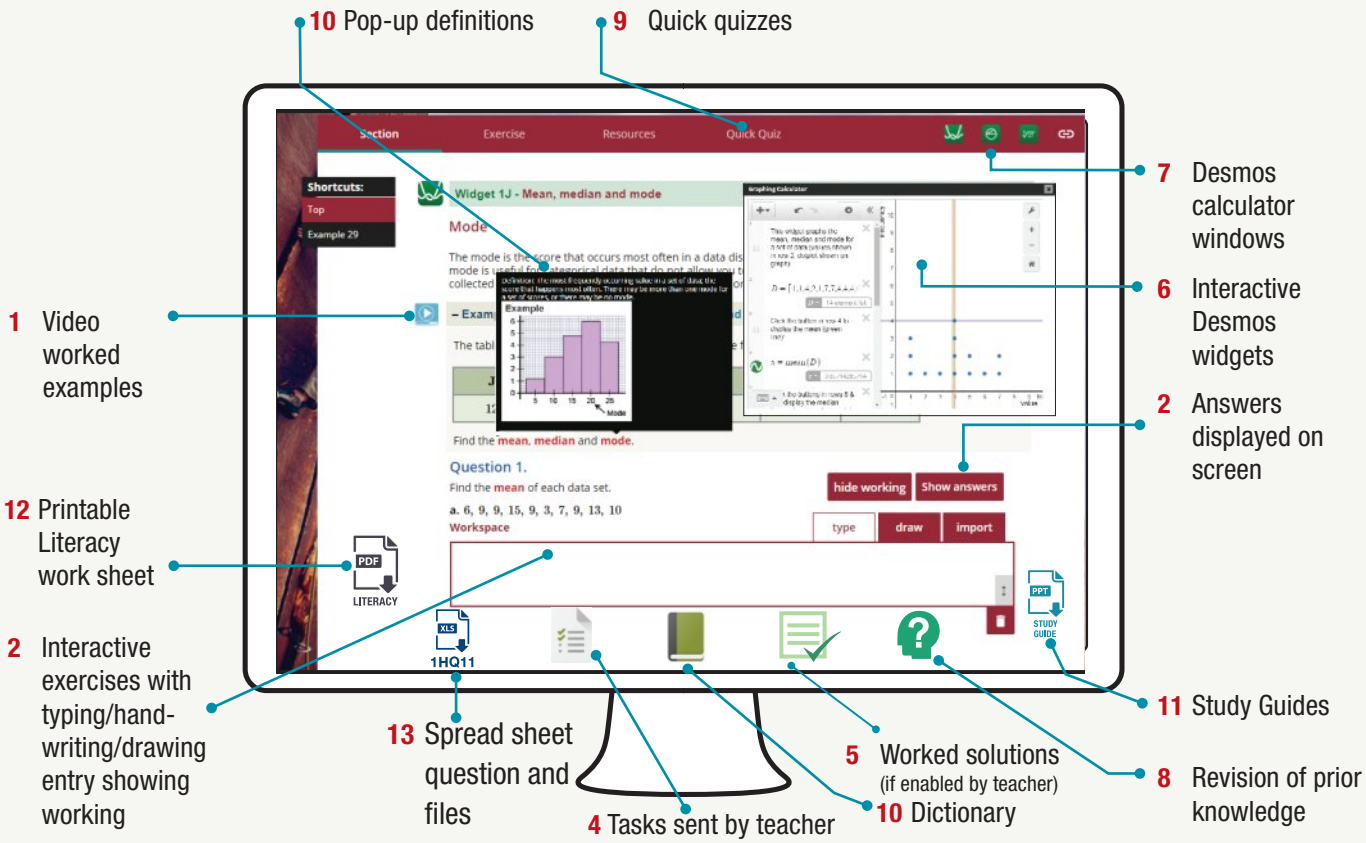

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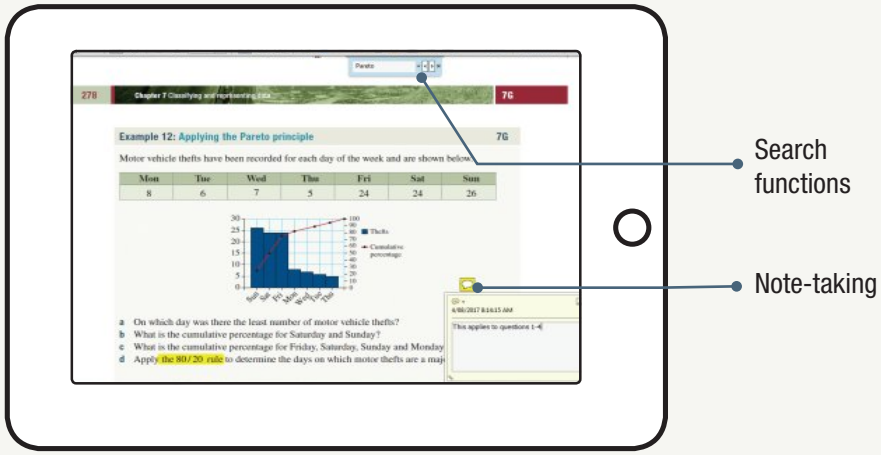

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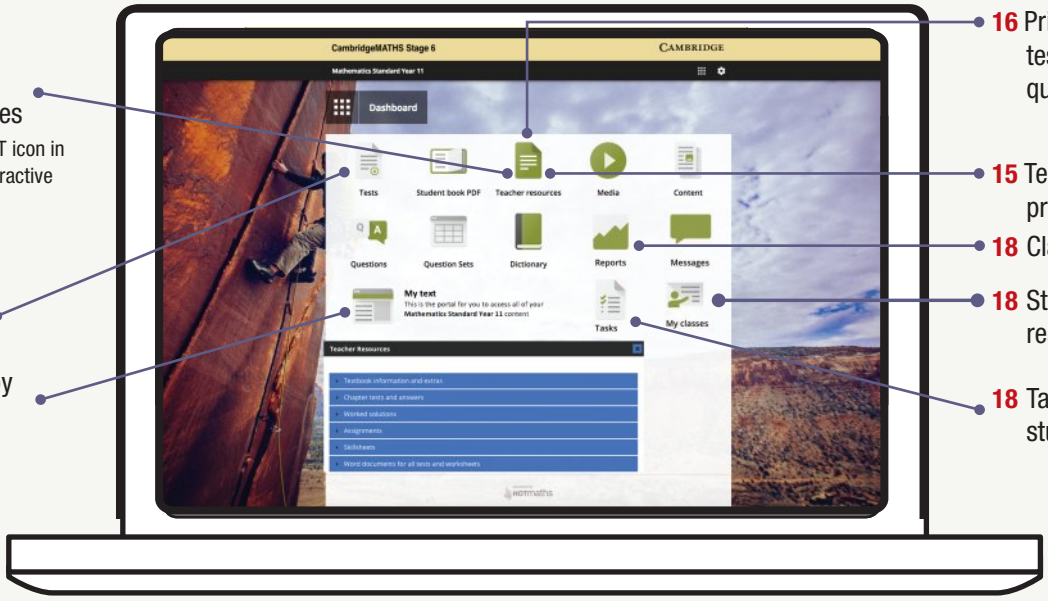

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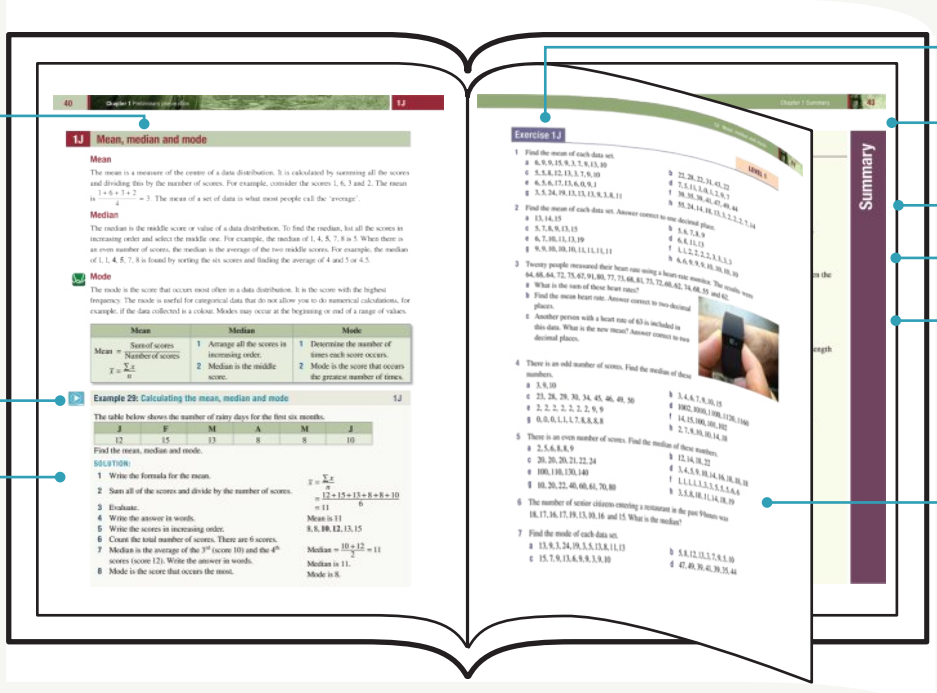

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1 Preliminary preparation

Syllabus prerequisite: Preliminary preparation

This topic provides the basic knowledge, skills and understandings required in Year 11.

Outcomes

- Simplify an algebraic expression.
- Establish and apply index laws in algebraic form.
- Multiply and divide algebraic terms.
- Find unknown sides using Pythagoras' theorem.
- Calculate the perimeter of simple figures.
- Calculate the area of composite shapes.
- Calculate simple interest.
- Calculate the future value, compound interest and present value.
- Create frequency tables to organise data.
- Calculate the median and mean.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Widgets
- Literacy worksheet
- Spreadsheets
- Quick Quiz
- Study guide
- Solutions (enabled by teacher)

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

In the Interactive Textbook you can take a test of prior knowledge required for this chapter, and depending on your score you may be directed to revision from the previous years' work.

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1A Simplifying algebraic expressions

A pronumeral (letter) represents a number. It may stand for an unknown value or series of values that change. For example, in the equation $x + 5 = 8$, x is a pronumeral that represents a value. Its value can be determined because we know $3 + 5 = 8$, so $x = 3$. Terms that have exactly the same pronumerals such as $2a$ and $5a$ are called like terms. Only like terms can be added and subtracted. It involves adding and subtracting the coefficients. Adding and subtracting like terms simplifies the algebraic expression. It is often called collecting the like terms.

ADDING AND SUBTRACTING LIKE TERMS

1. Find the like terms or the terms that have exactly the same pronumerals.
2. Only like terms can be added or subtracted, unlike terms cannot.
3. Add or subtract the coefficients or numbers before the pronumeral of the like terms.

Example 1: Adding and subtracting like terms 1A

Simplify $5x^2 + 6x - 3x^2 - x$.

SOLUTION:

1. Group the like terms together.
2. Add and subtract coefficients of like terms.

$$5x^2 + 6x - 3x^2 - x = 5x^2 - 3x^2 + 6x - x$$
$$= 2x^2 + 5x$$

ADDING AND SUBTRACTING ALGEBRAIC FRACTIONS

An algebraic fraction contains pronumerals.

1. Find a common denominator (preferably the lowest) if denominators are different.
2. Express each fraction as an equivalent fraction with the common denominator.
3. Simplify the numerator by adding or subtracting the like terms.

Example 2: Adding and subtracting algebraic fractions 1A

Write $\frac{2a}{3} - \frac{a}{6}$ as a single fraction.

SOLUTION:

1. The lowest common denominator of 3 and 6 is 6.
2. Write the equivalent fractions by multiplying top and bottom of the first fraction by 2 ($3 \times 2 = 6$).
3. Subtract the like terms in the numerator and simplify.

$$\frac{2a}{3} - \frac{a}{6}$$
$$= \frac{2a \times 2}{3 \times 2} - \frac{a}{6} = \frac{4a}{6} - \frac{a}{6}$$
$$= \frac{3a}{6} = \frac{a}{2}$$

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Exercise 1A

LEVEL 1

1 Simplify by collecting like terms.

a $6g + 2g$ **b** $5x + 12x$ **c** $8d - 7d$

d $5x^2 - x^2$ **e** $c^2 + 8c^2$ **f** $2r^3 - 5r^3$

g $-m + (-2m)$ **h** $21t + (-13t)$ **i** $9v^2 - (-3v^2)$

Example 1

2 Simplify by collecting like terms.

a $3w + 2 + 4w$ **b** $2q + 3q - 8$ **c** $1 + 4a + 11a$

d $5x^2 + 4y - 2x^2$ **e** $4b^2 + 6a - 5a$ **f** $n^3 + 2m - 4n^3$

g $3x^2 + 2y - 15x^2 - 3y$ **h** $5a^2 + b + 3a^2 - 2b$ **i** $5z^2 + y - z^2 + 8y$

3 Simplify by collecting like terms.

a $7u^2 - 4u^4 - u^2 + 9$ **b** $6q^3 + q^2 - 4q^2 + q^3$ **c** $5b + 9b^2 - 5b - 8b^2$

d $2x^2 + 6x^3 - 3x^3 - x^2$ **e** $p^2 + 3p + p^2 - p$ **f** $w + w^2 - 3w + w^2$

g $3c + c^2 + 2 + c$ **h** $5 - k^2 + k^3 - 2$ **i** $4y - 5 - 3y - 2y^2$

Example 2

4 Express each as a single fraction.

a $\frac{5}{7} + \frac{y}{7}$ **b** $\frac{3}{4} - \frac{x}{4}$ **c** $\frac{2w}{5} + \frac{4}{5}$ **d** $\frac{8}{9} - \frac{5t}{9}$

5 Add or subtract these algebraic fractions.

a $\frac{m}{5} + \frac{m}{5}$ **b** $\frac{3y}{7} - \frac{2y}{7}$ **c** $\frac{2x}{3} + \frac{3x}{3}$ **d** $\frac{3b}{11} - \frac{b}{11}$

e $\frac{d}{3} + \frac{2d}{3}$ **f** $\frac{7n}{13} - \frac{2n}{13}$ **g** $\frac{7b}{2} + \frac{4b}{2}$ **h** $\frac{5w}{6} - \frac{4w}{6}$

6 Add or subtract these algebraic fractions.

a $\frac{x}{2} + \frac{x}{4}$ **b** $\frac{7y}{8} - \frac{y}{4}$ **c** $\frac{4m}{2} + \frac{m}{10}$ **d** $\frac{r}{2} - \frac{r}{8}$

e $\frac{5x}{14} + \frac{x}{2}$ **f** $\frac{11b}{6} - \frac{2b}{3}$ **g** $\frac{h}{20} + \frac{h}{10}$ **h** $\frac{2p}{4} - \frac{p}{12}$

7 Add or subtract these algebraic fractions.

a $\frac{2k}{3} + \frac{k}{4} - \frac{k}{12}$ **b** $\frac{5x}{6} - \frac{x}{8} + \frac{5x}{24}$ **c** $\frac{y}{2} - \frac{3y}{10} + \frac{y}{5}$ **d** $\frac{5c}{6} - \frac{c}{2} + \frac{4c}{3}$

LEVEL 2

8 Copy and add like terms where possible to complete the table.

+	x^2	$3x^2 - 2x$	$x^2 + x$
$3x^2$			
$6x$			
$x^2 - 2x$			
$-2x$			

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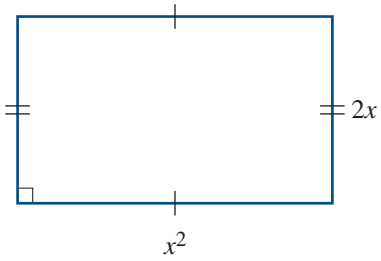
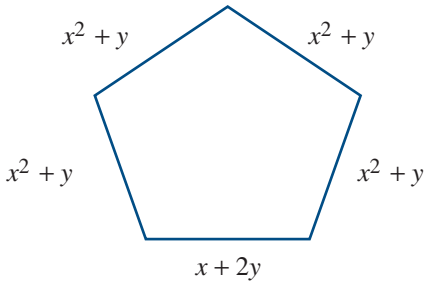
9 The perimeter of a rectangle is the distance around the outside of the rectangle. The rectangle below has a length of x^2 and a breadth of $2x$. Write an expression for the perimeter of this rectangle by collecting like terms.



10 Ava went shopping. She spent $\$4y^2$ on a jacket, $\$3y$ on a pair of shoes, and $\$2y^2$ on a skirt. If she had \$500, write an expression in simplified form for how many dollars she has left.

11 A pentagon has four sides of length $x^2 + y$ and one side of length $x + 2y$. The pentagon is shown below. Write an expression in simplified form for the perimeter of this pentagon by collecting like terms.



12 Add or subtract these algebraic fractions.

a $\frac{k}{3} + \frac{k}{4}$ **b** $\frac{5b}{6} - \frac{b}{8}$ **c** $\frac{3r}{2} + \frac{4r}{3}$ **d** $\frac{6r}{7} - \frac{3r}{10}$

e $\frac{3n}{5} + \frac{n}{6}$ **f** $\frac{5d}{12} - \frac{d}{8}$ **g** $\frac{4a}{15} + \frac{a}{10}$ **h** $\frac{3y}{4} - \frac{y}{11}$

LEVEL 3

13 Add or subtract these algebraic fractions.

a $\frac{m}{6} + \frac{m}{2} + \frac{m}{4}$ **b** $\frac{4x}{5} + \frac{x}{2} - \frac{x}{4}$ **c** $\frac{9m}{10} - \frac{m}{6} + \frac{2m}{3}$ **d** $\frac{h}{3} + \frac{h}{4} - \frac{h}{7}$

14 Add or subtract these algebraic fractions.

a $3 + \frac{y}{7}$ **b** $5 - \frac{x}{2}$ **c** $8m + \frac{m}{7}$ **d** $y - \frac{2y}{9}$

e $4y + \frac{2y-1}{3}$ **f** $3x - \frac{x+2}{4}$ **g** $\frac{2a+1}{2} + \frac{3a+2}{3}$ **h** $\frac{5w+1}{4} - \frac{6w+2}{5}$

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1B Index laws

Index form or index notation is used to write expressions in a shorter way, such as $a \times a = a^2$. When a number or pronumeral is expressed in index form, the index or power represents the number of times the multiplication occurs. That is,

$a^n = a \times a \times a \times \ldots \times a \times a$ (n factors)

Index laws shown below are rules for working with algebraic terms expressed in index form.

INDEX LAWS

1 $a^m \times a^n = a^{m+n}$ – Multiplying terms in index form (same base), add the indices.

2 $a^m \div a^n = a^{m-n}$ – Dividing terms in index form (same base), subtract the indices.

or $\frac{a^m}{a^n} = a^{m-n}$

3 $(a^m)^n = a^{mn}$ – A term in index form raised to a power, multiply the indices.

4 $a^0 = 1$ – A term in index form raised to the power of zero is one.

Multiplying and dividing using the index laws

Algebraic terms are multiplied and divided to form a single algebraic expression. This requires the algebraic expression to be expressed in expanded form. If the algebraic terms contain fractions, it is easier to first cancel any common factors in the numerator and denominator. The coefficients are then multiplied and divided, and finally the pronumerals are multiplied and divided. The index laws shown above are often applied to make the calculations easier.

MULTIPLYING AND DIVIDING ALGEBRAIC TERMS

1 Write in expanded form.
2 If the algebraic term is a fraction, cancel any common factors.
3 Multiply and divide the coefficients.
4 Multiply and divide the pronumerals. Use the index laws if required.
5 Write the coefficient before the pronumerals.
6 Write the pronumerals in alphabetical order and express in index form.

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Example 3: Multiplying algebraic terms using the index laws 1B

Simplify the following.

a $5y^2 \times 4y^3$ **b** $ab^3 \times (-2a^5b)$

SOLUTION:

1 Write in expanded form.
2 Multiply the coefficients ($5 \times 4 = 20$).
3 Multiply the pronumerals using the index laws ($y^2 \times y^3 = y^5$).
4 Express answer using index notation.

a
$$5y^2 \times 4y^3$$
$$= 5 \times y^2 \times 4 \times y^3$$
$$= 5 \times 4 \times y^2 \times y^3$$
$$= 20y^5$$

5 Write in expanded form.
6 Multiply the coefficients ($1 \times -2 = -2$).
7 Multiply the pronumerals using the index laws ($a \times a^5 = a^6$, $b^3 \times b = b^4$).
8 Write the pronumerals in alphabetical order.
9 Express answer using index notation.

b
$$ab^3 \times (-2a^5b)$$
$$= a \times b^3 \times -2 \times a^5 \times b$$
$$= -2 \times a \times a^5 \times b^3 \times b$$
$$= -2a^6b^4$$

Example 4: Dividing algebraic terms using index laws 1B

Simplify the following.

a $10d^{12} \div 5d^3$ **b** $\dfrac{21m^2n}{7m}$

SOLUTION:

1 Write in expanded form.
2 Divide the coefficients ($10 \div 5 = 2$).
3 Divide the pronumerals using the index laws ($d^{12} \div d^3 = d^9$).
4 Express answer using index notation.

a
$$10d^{12} \div 5d^3 = 10 \div 5 \times d^{12} \div d^3$$
$$= 2 \times d^{12} \div d^3$$
$$= 2d^{12-3}$$
$$= 2d^9$$

5 Write in expanded form.
6 Divide the coefficients ($21 \div 7 = 3$).
7 Cancel the pronumeral (m) in both the numerator and denominator ($m^2 \div m = m$).
8 Write the coefficient before the pronumerals.
9 Write the pronumerals in alphabetical order.

b
$$\frac{21m^2n}{7m} = \frac{21 \times m \times m \times n}{7 \times m}$$
$$= \frac{\cancel{7}_1 \times 3 \times m \times \cancel{m}^1 \times n}{\cancel{7}_1 \times \cancel{m}^1}$$
$$= 3 \times m \times n$$
$$= 3mn$$

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Example 5: Applying the index laws 1B

Simplify the following.

a $2x^0 + 5$ **b** $(6pq^3)^2$

SOLUTION:

1. Write in expanded form.
2. Replace x^0 with 1 ($a^0 = 1$).
3. Evaluate.
4. Write in expanded form.
5. Raise each term to the power of 2. Use the index law $(a^m)^n = a^{mn}$ to simplify each algebraic expression.
6. Write the pronumerals in alphabetical order.

a $2x^0 + 5 = 2 \times x^0 + 5$
$= 2 \times 1 + 5$
$= 7$

b $(6pq^3)^2 = (6 \times p^1 \times q^3)^2$
$= 6^2 \times p^2 \times q^6$
$= 36p^2q^6$

Example 6: Simplifying algebraic terms with fractions 1B

Simplify $\frac{x^4}{6y^2} \times \frac{4xy^5}{10}$

SOLUTION:

1. Write fractions in expanded form.
2. Determine any common factors in the numerator and the denominator.
3. Cancel out the common factors (2 is a common factor of 4 and 6, 2 is a common factor of 4 and 10).
4. Cancel the pronumeral (y^2) in both the numerator and denominator as it is a common factor ($y^5 \div y^2 = y^3$).
5. Multiply the numerators together ($x^4 \times x = x^5$).
6. Multiply the denominators together.
7. Express answer using index form.
8. Write the coefficient before the pronumerals. However it is acceptable to leave answer as $\frac{x^5y^3}{15}$.

$\frac{x^4}{6y^2} \times \frac{4xy^5}{10} = \frac{x^4}{6 \times y^2} \times \frac{4 \times x \times y^5}{10}$

$= \frac{x^4}{\cancel{2}_1 \times 3 \times y^2} \times \frac{\cancel{2}^1 \times \cancel{2}^1 \times x \times y^5}{\cancel{2}_1 \times 5}$

$= \frac{x^4}{3 \times \cancel{y}^2} \times \frac{x \times y^{\cancel{5}^3}}{5}$

$= \frac{x^4 \times x \times y^3}{3 \times 5}$

$= \frac{x^5 \times y^3}{3 \times 5} = \frac{x^5 \times y^3}{15}$

$= \frac{x^5y^3}{15}$ (or $\frac{1}{15}x^5y^3$)

Exercise 1B

LEVEL 1

1 Multiply these algebraic terms.

a $5w \times 3$ **b** $6 \times 4p$ **c** $4 \times 6f \times 5$

d $x \times 8y$ **e** $3m \times (-3n)$ **f** $-7r \times 5k \times 3$

g $2h \times 6g \times 3$ **h** $7n \times n \times 3$ **i** $30a \times 5a$

Example 3

2 Simplify each of the following using the index laws.

a $m^3 \times m^4$ **b** $y^6 \times 3y^2$ **c** $2z^3 \times 4z^2$

d $24y^2 \times 2y^2$ **e** $3a^2 \times 2a^5$ **f** $4w^{10} \times 2w^5$

g $2st^3 \times 7s^5t^4$ **h** $-pq^2 \times (-5p^2q^3)$ **i** $6c^5d^2 \times (-2c^3d^8)$

3 Find the product of these terms.

a $5a^2$, a^3 and $2a^4$

b x^4, $3x^7$ and $(-4x)$

c $6y^3$, $2y^2$ and $3y^4$

d mn^2, m^2n^4 and mn

e $(-7ab^2)$, $(-2ab^3)$ and a^2

f $2cde$, c^2d^4 and $4d^3e$

Example 4a

4 Divide these algebraic terms.

a $16w \div 4$ **b** $18c^2 \div 2$ **c** $24s^2 \div (-3)$

d $44b \div 11b$ **e** $(-20gh) \div 4g$ **f** $15xy^2 \div 5xy$

g $4a^3 \div 4a^2$ **h** $24m^2 \div 12m$ **i** $(-28z^3) \div 7z$

5 Simplify each of the following using the index laws.

a $y^7 \div y^2$ **b** $12b^8 \div b^2$ **c** $15x^5 \div (-5x^4)$

d $20m^7 \div 10m^3$ **e** $30t^{12} \div (-10t^3)$ **f** $(-15b^6) \div 15b^2$

g $6d^{15} \div 9d^2$ **h** $16q^4 \div 12q^2$ **i** $21e^5 \div 14e^3$

6 Simplify.

a $3w^5 \times 4w \div 2w^2$ **b** $10y^5 \div 2y^3 \times y^8$ **c** $x^6 \times 9x \div 3x^4$

d $7a^6 \times (-2a) \div a^3$ **e** $(-20\ c^{10}) \times 4c^5 \div 10c$ **f** $7e^3 \times 2e^2 \times -4e$

Example 4b

7 Simplify these algebraic terms.

a $\frac{14a}{7}$ **b** $\frac{20t}{5}$ **c** $\frac{16z}{2z}$

d $\frac{(-5ab)}{30}$ **e** $\frac{14n^6}{14n^2}$ **f** $\frac{(-4x^{10}y)}{(-24x^3)}$

g $\frac{6y^7}{(-48y^3)}$ **h** $\frac{14d^5e}{2e^2}$ **i** $\frac{3x^4y}{6x^2y^3}$

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Example 5b **8** Remove the grouping symbol and express each answer in simplest form.

a $(x^4)^3$ **b** $(s^3)^2$ **c** $(-3y)^2$

d $(4n^3)^3$ **e** $(-a^4b^2)^5$ **f** $(5r^2t)^4$

g $(a^2b)^3 \times (a^3b^4)^2$ **h** $(-6ef^7)^4 \times (e^2f)^3$ **i** $(6ab^2)^3 \times (2a^4b^6)^2$

Example 5a **9** Express in simplest index form.

a y^0 **b** 2^0 **c** $5 \times a^0$

d $3m^0 + 4$ **e** ab^0 **f** c^0b^0

g $-5 \times 2x^0$ **h** $(2p^3)^0$ **i** $7e^0 \times f^0$

10 Simplify each of the following using the index laws.

a $8q^2 \times 2q^3$ **b** $4u^6 \times (-2u)$ **c** $a^4 \times 3a \times 8a^3$

d $28x^8 \div 4x^4$ **e** $34m^{10} \div 17m^3$ **f** $4d^{10} \div (-4d^2)$

g $(5g^4)^3$ **h** $(c^5d^2)^4$ **i** $(-2x^3y)^6$

j $5n^0 \times 4$ **k** $(-9a^5)^0$ **l** $a^0 \times 3b^0$

11 Copy and multiply the algebraic terms to complete the table.

$\times$	x^2	$(-3x^4)$	$4xy^3$
$2x^2$			
$(-5x^3y^5)$			
$7x^5y^8$			

12 Copy and divide the algebraic terms to complete the table.

$\div$	$(-2a^2)$	ab^2	$3a^2b^2$
$6a^4$			
$(-3a^5b^2)$			
$12a^6b^4$			

Example 6 **13** Simplify.

a $\frac{8y}{3} \times 4y$ **b** $6d \times \frac{2d}{5}$ **c** $\frac{5w}{9} \times 2w$

d $\frac{5x}{4} \times \frac{12}{x}$ **e** $\frac{2}{a} \times \frac{3a}{6}$ **f** $\frac{5w}{7} \times \frac{2w}{9}$

g $\frac{10a}{b} \times \frac{3b}{4}$ **h** $\frac{m}{9n} \times \frac{6mn}{15}$ **i** $\frac{d}{e} \times \frac{c}{3d}$

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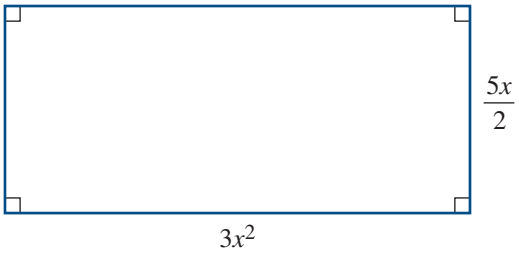
LEVEL 2

14 Simplify each of the following using the index laws.

a $12m^{12} \times 3m^3 \div 9m^2$ **b** $4x^5 \div 2x \times (-7x^7)$ **c** $9y^6 \times (15y^6 \div 3y^2)$

15 Express $\dfrac{8x^3y^2}{16x^2y^3}$ in simplest form.

16 The plane shape below is a rectangle with a length of $3x^2$ and a breadth of $\dfrac{5x}{2}$. Write an expression in simplest form for the area of this rectangle.



17 Simplify.

a $\dfrac{4x^3}{3} \times 6x$ **b** $5a^3 \times \dfrac{3a^3}{6}$ **c** $\dfrac{7v^6}{8} \times 2v^2$

d $\dfrac{4y^4}{3x^2} \times \dfrac{15x^6}{y^3}$ **e** $\dfrac{4n^3}{8n} \times \dfrac{2m^7}{m^4}$ **f** $\dfrac{6e^3}{11f} \times \dfrac{22e^4}{9f^3}$

g $\dfrac{20a^2}{b^5} \times \dfrac{5b^6}{4}$ **h** $\dfrac{m^7}{15n} \times \dfrac{3m^2n}{9}$ **i** $\dfrac{d^4}{e^4} \times \dfrac{c^2e}{7d}$

LEVEL 3

18 Simplify.

a $\dfrac{15h^2}{3k} \times \dfrac{3k^3}{4}$ **b** $\dfrac{21v^2}{15uv} \times \dfrac{5u^3}{7}$ **c** $\dfrac{18a^2}{12b} \times \dfrac{6b^3}{9a} \times \dfrac{a}{2}$

d $\dfrac{9m^3 \times 6mn^2}{3m^2n}$ **e** $\dfrac{3(m+1)}{8m} \times \dfrac{4m^3}{2(m+1)}$ **f** $\dfrac{7y^4}{10(y-2)} \times \dfrac{5(y-2)}{21y^6}$

19 Simplify these algebraic expressions.

a $\dfrac{18x^5}{3yz^3} \times \dfrac{3y^3z}{6xy}$ **b** $\dfrac{2m^5n^4}{4m^2p^3} \times \dfrac{16p^5}{8mn^3}$ **c** $\dfrac{28a^4}{10bc} \times \dfrac{5b^3c^3}{7ab^2}$

d $\dfrac{6d^3}{9c} \times \dfrac{c}{2} \times \dfrac{18c^2}{12d}$ **e** $\dfrac{8w^5 \times (-6wu^2)}{2w^5u \times u^5}$ **f** $\dfrac{16c^7 \times 24ed^{12}}{12c^5d \times 2e^3d^6}$

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1C Expanding algebraic expressions

Grouping symbols in algebraic expressions indicate the order of operations.

The two most commonly used grouping symbols are parentheses ( ) and brackets [ ]. They are removed by using order of operations or the distributive law. This is illustrated below.

Using order of operations

$$\begin{aligned} 2 \times (3+1) &= 2 \times 4 \\ &= 8 \end{aligned}$$

Using distributive law

$$\begin{aligned} 2 \times (3+1) &= 2 \times 3 + 2 \times 1 \\ &= 6 + 2 \\ &= 8 \end{aligned}$$

To expand an algebraic expression using the distributive law, multiply the numbers or terms inside the grouping symbols by the number or term outside the grouping symbols. The resulting algebraic expression is simplified by collecting the like terms.

EXPANDING ALGEBRAIC EXPRESSIONS

1. Multiply the number or term outside the grouping symbol by the
   - **a** first term inside the grouping symbol
   - **b** second term inside the grouping symbol.
2. Simplify and collect like terms if required.

$$a(b+c) = a \times b + a \times c$$
$$= ab + ac$$

$$a(b-c) = a \times b - a \times c$$
$$= ab - ac$$

Make sure you remember to multiply all the terms inside the grouping symbol by the number or term outside the grouping symbols.

Notice that in the numerical example of the distributive law at the top of the page, $2 \times (3 + 1) = 2 \times 3 + 2 \times 1$ is the same as $a(b + c) = a \times b + a \times c$, with $a = 2$, $b = 3$ and $c = 1$.

Example 7: Expanding algebraic expressions 1C

Expand $5(2y - 3)$.

SOLUTION:

1. Write in expanded form.
2. Multiply the first term inside the parenthesis ($2y$) by the number outside the parenthesis (5).
3. Multiply the second term inside the parenthesis (-3) by the number outside the parenthesis (5).
4. Write in simplest form.

$$5(2y - 3) = 5 \times (2y - 3)$$
$$\begin{aligned} 5 \times (2y - 3) &= 5 \times 2y + 5 \times (-3) \\ &= 10y - 15 \end{aligned}$$

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Example 8: Expanding algebraic expressions 1C

Expand $-(m-5)$.

SOLUTION:

1. Multiply the first term inside the parenthesis (m) by the number outside the parenthesis (-1).
2. Multiply the second term inside the parenthesis (-5) by the number outside the parenthesis (-1).
3. Write in simplest form.

$$\begin{aligned} -(m-5) &= -1 \times (m-5) \\ &= -1 \times m - 1 \times -5 \\ &= -m + 5 \end{aligned}$$

Example 9: Expanding and simplifying algebraic expressions 1C

Remove the grouping symbols for $2(3x+4)+3(x-1)$ and simplify if possible.

SOLUTION:

1. Write the expanded form.
2. Multiply the first term inside the parenthesis ($3x$) by the number outside the parenthesis (2).
3. Multiply the second term inside the parenthesis ($+4$) by the number outside the parenthesis (2).
4. Repeat the first two steps for the second parenthesis.
5. Simplify by collecting the like terms.

$$\begin{aligned} &2(3x+4)+3(x-1) \\ &= 2 \times (3x+4) + 3 \times (x-1) \\ &= 2 \times 3x + 2 \times 4 + 3(x-1) \\ &= 6x + 8 + 3(x-1) \\ &= 6x + 8 + 3 \times x + 3 \times -1 \\ &= 6x + 8 + 3x - 3 \\ &= 9x + 5 \end{aligned}$$

Example 10: Expanding and simplifying algebraic expressions 1C

Expand $y(5y+1) - y(y-6)$ and simplify if possible.

SOLUTION:

1. Write in expanded form.
2. Multiply the first term inside the parenthesis ($5y$) by the term outside the parenthesis (y).
3. Multiply the second term inside the parenthesis ($+1$) by the term outside the parenthesis (y).
4. Repeat the first two steps for the second parenthesis.
5. Simplify by collecting the like terms.

$$\begin{aligned} &y(5y+1) - y(y-6) \\ &= y \times (5y+1) - y \times (y-6) \\ &= y \times 5y + y \times 1 - y(y-6) \\ &= 5y^2 + y - y \times y - y \times -6 \\ &= 5y^2 + y - y^2 + 6y \\ &= 4y^2 + 7y \end{aligned}$$

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Exercise 1C

LEVEL 1

1 Ryan was required to remove the grouping symbols. This was his solution.

$$3(3x - 2) = 9x - 2$$

Where is the error in Ryan's working?

Example 7

2 Expand each of the following.

a $3(a + 2)$ **b** $2(d + 1)$ **c** $7(b - 2)$
d $2(3x + 4)$ **e** $2(5x - 7)$ **f** $4(9b + 1)$
g $4(5 + 2t)$ **h** $6(1 - 2w)$ **i** $5(3 + 9d)$
j $8(5e - 2d)$ **k** $5(4a + 9b)$ **l** $7(2h + 8g)$

Example 8

3 Expand each of the following.

a $-4(x + 3)$ **b** $-3(y + 5)$ **c** $-(b + 8)$
d $-7(k - 2)$ **e** $-6(w - 1)$ **f** $-2(x - 13)$
g $-2(4 + 2q)$ **h** $-5(3 - 4r)$ **i** $-7(8 - 2s)$

4 Expand each of the following.

a $y(y + 1)$ **b** $v(v + 4)$ **c** $n(n + 10)$
d $x(2x - 3)$ **e** $e(3e + 5)$ **f** $d(6d - 2)$
g $z(7e + 3f)$ **h** $a(2b - 3c)$ **i** $c(d + 4e)$

5 Remove the grouping symbols and simplify if possible.

a $2(g + 1) + 4g$ **b** $7(s + 2) + s$ **c** $3(y - 9) - 2y$
d $5x - 4(x - 2)$ **e** $6z + 2(z - 1)$ **f** $3q - 7(q - 5)$

6 Remove the grouping symbols and simplify if possible.

a $4(x - 1) + 2x + 5$ **b** $7(3y - 2) + 4y - 2$ **c** $2(5b + 2) - b - 8$
d $4r + 17 + 5(r - 3)$ **e** $2n - 8 + 3(n + 2)$ **f** $5q + 2 - (q + 9)$

7 Expand each of the following.

a $4(y + 3)$ **b** $2(c + 1)$ **c** $5(a - 3)$
d $-3(4 + 3e^2)$ **e** $4(1 - 2b^2)$ **f** $-6(3 + 4g^2)$

8 Expand each of the following.

a $k(k + 3)$ **b** $b(2b - 6)$ **c** $y(5 - 3y)$
d $x(4x - 1)$ **e** $-a(b + 4c)$ **f** $p(5q - 3r)$

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LEVEL 2

Example 9

9 Simplify the following.

a $2(x+1)+5(x-1)$ **b** $3(y+2)+2(y+1)$
c $5(a+2)+3(a+4)$ **d** $8(c-3)+5(c+3)$
e $6(s+6)+2(2s-1)$ **f** $5(h+7)+2(2h-7)$
g $4(3x-1)-2(x-2)$ **h** $9(z+5)-7(z-2)$
i $5(2c-4)-3(c+7)$ **j** $5(5g-1)-4(g-2)$
k $7(2u-3)-(u-3)$ **l** $-(4d-1)-3(d-3)$

10 Taylah was required to remove the grouping symbols and simplify. This was her solution.

$$\begin{aligned} 6(3x-2)-2(x+3) &= 18x-12-2x+6 \\ &= 16x-6 \end{aligned}$$

Where is the error in Taylah's working?

Example 10

11 Remove the grouping symbols and simplify if possible.

a $x(x-5)+x(x+2)$ **b** $b(b+3)+b(b+1)$
c $y(y-3)+y(y+8)$ **d** $g(2g+3)-g(g+3)$
e $v(v-7)+v(6v+4)$ **f** $b(5b-1)-b(2+4b)$
g $2u(u-2)+u(u+9)$ **h** $4n(n-6)-n(n+1)$
i $3d(d+7)+d(2d+5)$ **j** $e(e+2)-7e(e-9)$
k $6k(k-3)+k(k+3)$ **l** $t(5-3t)+7t(2-t)$

12 Expand and simplify the algebraic expression $2ab(ab-3)-ab(ab-1)$.

LEVEL 3

13 Simplify the following.

a $x^2(2x+3)-2(x+1)$ **b** $a^2(a+2)-4(a+3)$
c $y^2(5y+2)-3(y+7)$ **d** $b(b+7)-b^2(3b+2)$
e $z(3z-1)+z^2(z-5)$ **f** $e(7-e)-e^2(2e+6)$
g $x(x^2+7)-x(x^2+2)$ **h** $a(2a^2-1)+a(a^2+4)$
i $v(2-v^2)-v(1-v^2)$ **j** $a^2(a+b)-b(a+b)$
k $x^2(x^2+y)-x(x+3y)$ **l** $y^2(y+4z)-y(z^2+1)$

14 Remove the grouping symbols.

a $m^2(5m^4+1)$ **b** $2x^4(x^2-3x+4)$ **c** $-a^4(2+3a^4-a^8)$

15 Expand and simplify the following algebraic expressions.

a $5a^4(4a^2-2)+a^3(a^3+7)$ **b** $4x^2(3x^3-2)-3x^2(x^3+9)$ **c** $(n+4r)n^2-(7n+r)n^2$

16 Simplify the following algebraic expressions by cancelling factors.

a $\dfrac{7b^3(b+4)}{3(b+4)} \times \dfrac{12b}{14b^2}$ **b** $\dfrac{5r^6}{2(r-1)} \times \dfrac{3(r-1)}{10r^2}$ **c** $\dfrac{6(a-3)}{24b^6} \times \dfrac{6b^5}{12(a-3)}$

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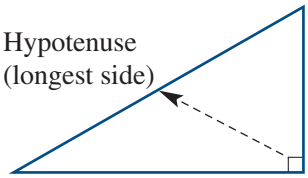
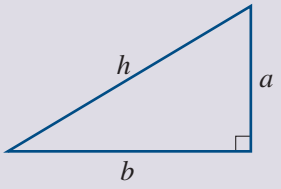
1D Pythagoras' theorem

Pythagoras' theorem links the sides of a right-angled triangle. In a right-angled triangle the side opposite the right angle is called the hypotenuse. The hypotenuse is always the longest side.



PYTHAGORAS' THEOREM

Pythagoras' theorem states that the square of the hypotenuse is equal to the sum of the squares of the other two sides.

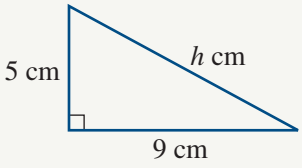


$(\text{Hypotenuse})^2 = (\text{side})^2 + (\text{other side})^2$ $\qquad h^2 = a^2 + b^2$

Pythagoras' theorem is used to find a missing side of a right-angled triangle if two of the sides are given. It can also be used to prove that a triangle is right angled.

Example 11: Finding the length of the hypotenuse 1D

Find the length of the hypotenuse, correct to two decimal places.

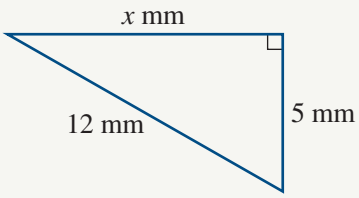


SOLUTION:

1	Write Pythagoras' theorem.	$h^2 = a^2 + b^2$
2	Substitute the length of the sides.	$= 5^2 + 9^2$
3	Calculate the value of h^2.	$h = \sqrt{5^2 + 9^2}$
4	Take the square root to find h.	$h = \sqrt{106}$
5	Express answer correct to two decimal places.	≈ 10.30 cm

Example 12: Finding the length of a shorter side 1D

What is the value of x correct to one decimal place?



SOLUTION:

1	Write Pythagoras' theorem.	$h^2 = a^2 + b^2$
2	Substitute the length of the sides.	$12^2 = x^2 + 5^2$
3	Make x^2 the subject.	$x^2 = 12^2 - 5^2$
4	Take the square root to find x.	$x = \sqrt{12^2 - 5^2}$
5	Express answer correct to one decimal place.	≈ 10.9 mm

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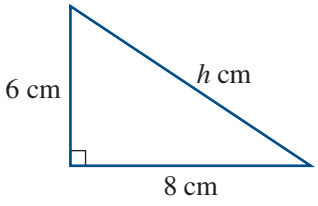
Exercise 1D

LEVEL 1

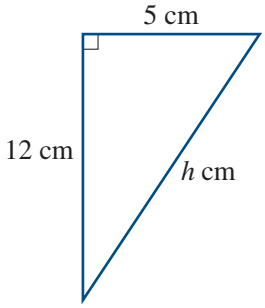
Example 11

1 Find the length of the hypotenuse, correct to one decimal place.

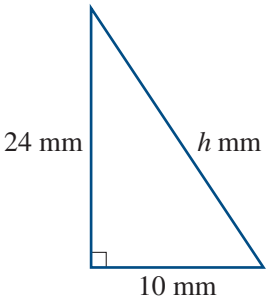
a



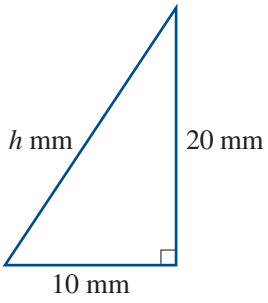
b



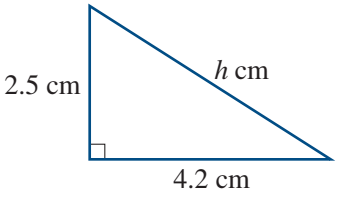
c



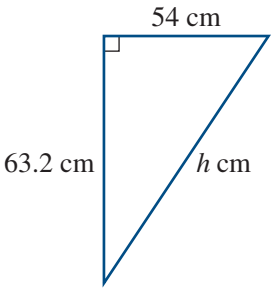
d



e



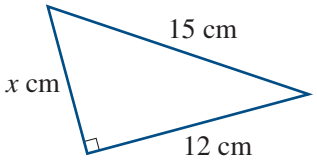
f



Example 12

2 Find the value of x, correct to two decimal places.

a



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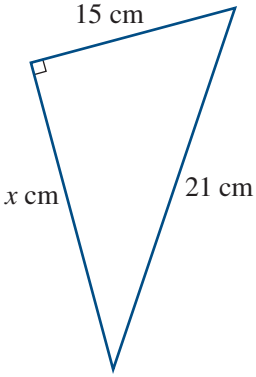
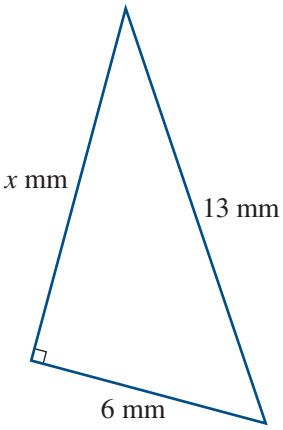
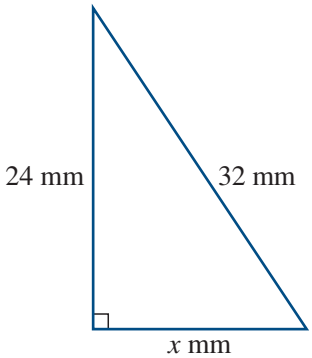


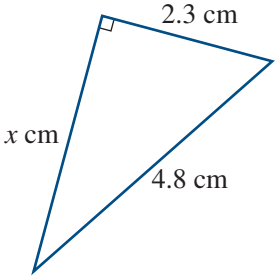
c



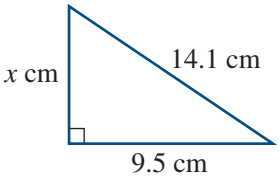
d



e



f



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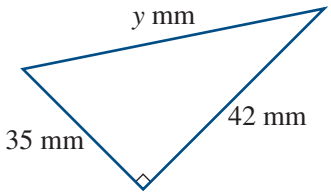
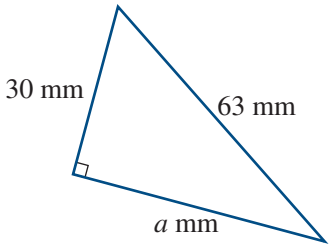
LEVEL 2

3 Calculate the length of the side marked with the pronumeral. (Answer to the nearest millimetre.)

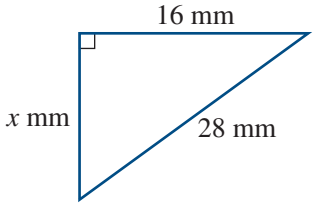
a



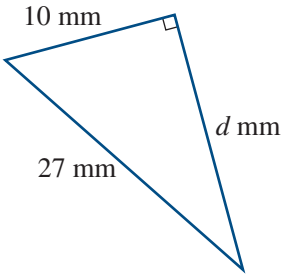
b



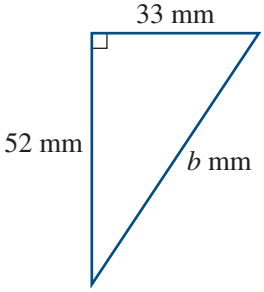
c



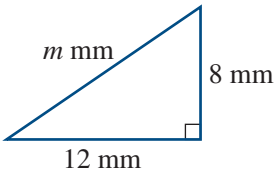
d



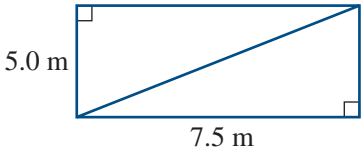
e



f



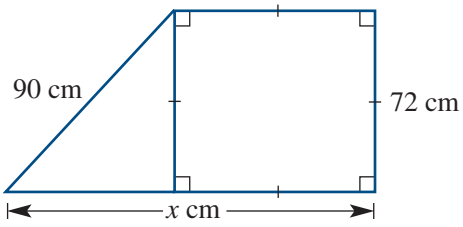
4 Find the length of the diagonal of a rectangle with dimensions 7.5 metres by 5.0 metres. Give the answer correct to one decimal place.



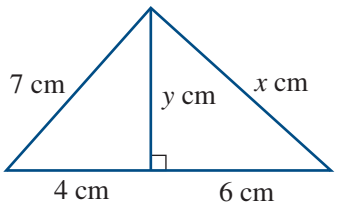
LEVEL 3

5 Find the value of the pronumerals, correct to two decimal places.

a

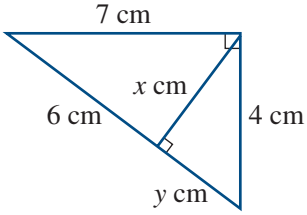


b

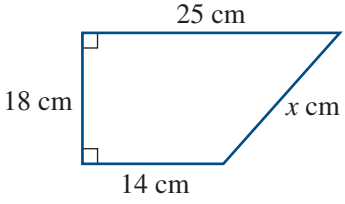


6 Calculate the length of the side marked with a pronumeral, correct to one decimal place.

a



b



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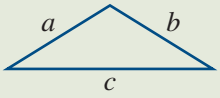
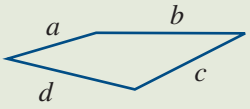
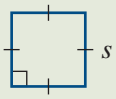
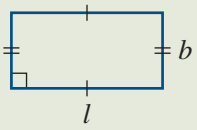
1E Perimeter

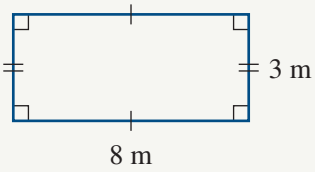
Perimeter is the total length of the outside edges of a shape. It is the distance of the boundary.

Perimeter formulae

Name	Shape	Perimeter
Triangle		$P = a + b + c$
Quadrilateral		$P = a + b + c + d$
Square		$P = 4s$
Rectangle		$P = 2(l + b)$
Circle		Circumference $C = 2\pi r$ $C = \pi d$

Example 13: Finding the perimeter of a rectangle 1E

Find the perimeter of the following rectangle.



SOLUTION:

1	The shape is a rectangle, so use the formula $P = 2(l + b)$.	$P = 2(l + b)$
2	Substitute the values for l and b ($l = 8$ and $b = 3$).	$= 2 \times (8 + 3)$
3	Evaluate.	$= 22$ m
4	Write the answer in words.	Perimeter of the rectangle is 22 m.

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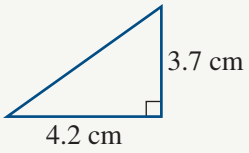
Example 14: Finding the perimeter of a triangle 1E

Find the perimeter of the triangle.
Answer correct to one decimal place.

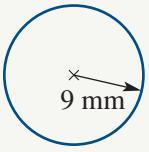


SOLUTION:

1	Find the length of the hypotenuse or h.	
2	Write Pythagoras' theorem.	$h^2 = a^2 + b^2$
3	Substitute the length of the sides.	$h^2 = 4.2^2 + 3.7^2$
4	Evaluate the value of h.	$h = \sqrt{4.2^2 + 3.7^2} \approx 5.6$ cm
5	Add the lengths of sides to find the perimeter.	$P = 3.7 + 4.2 + 5.6$ $= 13.5$ cm
6	Express answer correct to one decimal place.	
7	Write the answer in words.	Perimeter of the triangle is 13.5 cm.

Example 15: Finding the circumference of a circle 1E

Find the perimeter of a circle with a radius of 9 mm.
Answer correct to two decimal places.

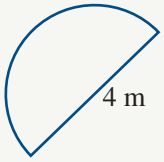


SOLUTION:

1	The shape is a circle, so use the formula $C = 2\pi r$.	$C = 2\pi r$
2	Substitute the value for r ($r = 9$).	$= 2 \times \pi \times 9$
3	Evaluate.	≈ 56.55 mm
4	Write the answer in words.	Perimeter of the circle is 56.55 mm.

Example 16: Finding the perimeter of a semicircle 1E

Find the perimeter of a semicircle with a diameter of 4 m.
Answer correct to two decimal places.



SOLUTION:

1	The shape is a semicircle, so use the formula $C = \pi d \div 2$.	$C = \frac{\pi d}{2}$
2	Substitute the value for d ($d = 4$) to find the curved distance.	$= \frac{\pi \times 4}{2}$
3	Evaluate.	≈ 6.28 m
4	Add the curved distance to the diameter.	$P = 6.28 + 4$
5	Evaluate.	$= 10.28$ m
6	Write the answer in words.	Perimeter is 10.28 m.

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Perimeter of composite shapes

A composite shape is made up of two or more plane shapes. The perimeter of a composite shape is calculated by adding the lengths that make up the boundary of the shape.

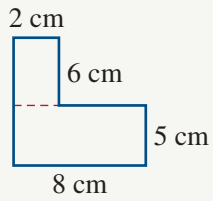
PERIMETER OF A COMPOSITE SHAPE

1. Sides with the same markings are of equal length.
2. Unknown side lengths are determined by using the given lengths of the other sides.
3. Pythagoras' theorem is used to find unknown side lengths involving a right triangle.
4. Lengths that are part of a circle are found using $C = 2\pi r$.
5. Add the lengths that make up the boundary of the shape to calculate the perimeter.

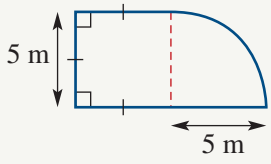
Example 17: Finding the perimeter of composite shapes 1E

Find the perimeter of each of these shapes.

a



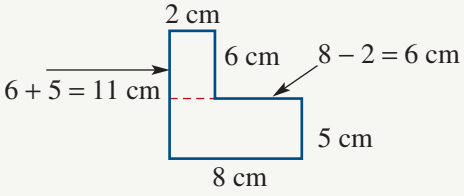
b



SOLUTION:

1. Find the unknown side lengths using the measurements given in the question.

a



2. Add the lengths of all the edges to find the perimeter.
3. Evaluate.

$$P = 2 + 6 + 6 + 5 + 8 + 11$$
$$= 38 \text{ cm}$$

4. Write the answer in words.

Perimeter is 38 cm.

5. Use the formula $C = 2\pi r \div 4$ for the curved length.
6. Substitute the value for r ($r = 5$).

b

$$C = \frac{2\pi r}{4}$$
$$= \frac{2 \times \pi \times 5}{4} \approx 7.85 \text{ m}$$

7. Evaluate.
8. Add the curved length to other edges.
9. Evaluate.

$$P = 7.85 + 5 + 5 + 5 + 5$$
$$= 27.85 \text{ m}$$

10. Write the answer in words.

Perimeter is 27.85 m.

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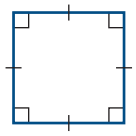
Exercise 1E

LEVEL 1

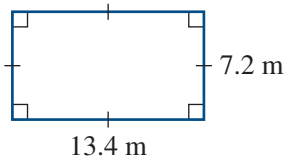
Example 13

1 Find the perimeter of each quadrilateral. Answer correct to one decimal place.

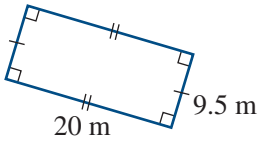
a



b



c

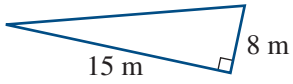


2 Find the perimeter of a square with a side length of 12.3 m. Answer correct to one decimal place.

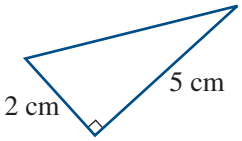
Example 14

3 Find the perimeter of each right triangle. Answer correct to one decimal place.

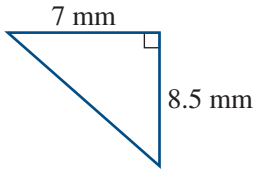
a



b



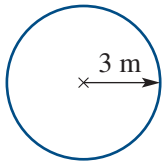
c



4 Find the perimeter of a right triangle with a base of 10.25 cm and a height of 15.15 cm. Answer correct to two decimal places.

5 Find the perimeter of each circle. Answer correct to one decimal place.

a



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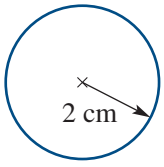
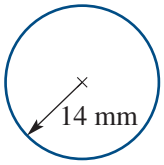


c



6 Find the circumference of each circle. Answer correct to one decimal place.

a Radius of 4 cm
b Radius of 19 m
c Radius of 34 mm
d Diameter of 50 mm
e Diameter of 22 m
f Diameter of 6 cm

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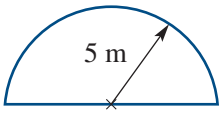
LEVEL 2

Example 16

7 Find the perimeter of each semicircle. Answer correct to one decimal place.

a



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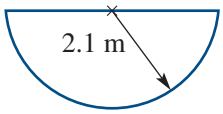
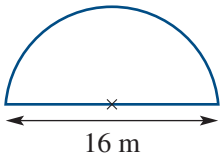
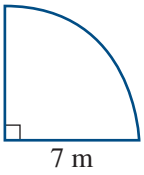


c



8 Find the perimeter of each shape. Answer correct to two decimal places.

a



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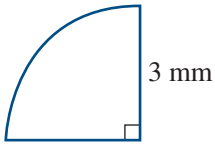
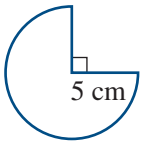


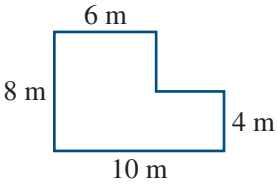
c



Example 17

9 Find the perimeter of each composite shape. Answer to the nearest whole number.

a



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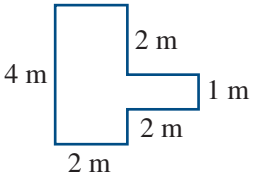

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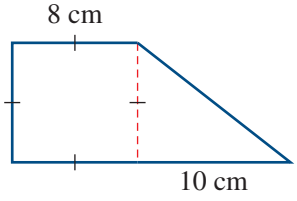
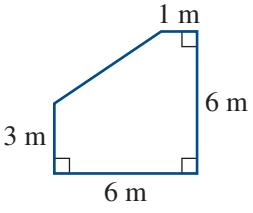
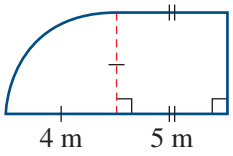


d



e



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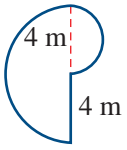


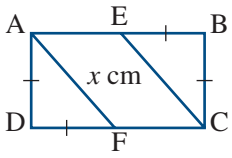
LEVEL 3

10 An annulus is a shape, like a doughnut or a tyre, between two circles with the same centre. Find the perimeter of an annulus if the inner diameter is 3 cm and the outer diameter is 6 cm. Answer correct to the nearest centimetre.

11 A rectangle ABCD has a length of $AB = 12$ cm and a width of $BC = 6$ cm. Answer correct to two decimal places.

a Find the value of x.

b Calculate the perimeter of quadrilateral $AECF$.



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1F Area

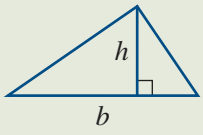
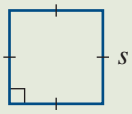
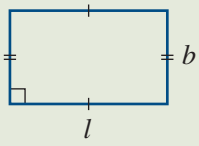
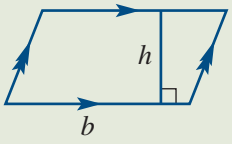
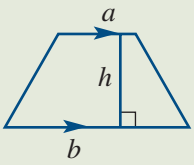
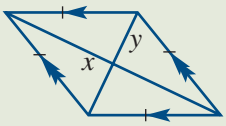
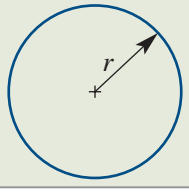
The area of a shape is the amount of surface enclosed by the boundaries of the shape. It is the number of squares that fit inside the shape. When calculating area, the answer will be in square units.

$100\ \text{mm}^2 = 1\ \text{cm}^2$ $\quad 10\,000\ \text{cm}^2 = 1\ \text{m}^2$

$10\,000\ \text{m}^2 = 1\ \text{ha}$ $\quad 1\,000\,000\ \text{m}^2 = 1\ \text{km}^2$

To calculate the area of the most common shapes, we use a formula. These formulae are listed below.

Area formulae

Name	Shape	Area
Triangle		$A = \frac{1}{2}bh$
Square		$A = s^2$
Rectangle		$A = lb$
Parallelogram		$A = bh$
Trapezium		$A = \frac{1}{2}(a+b)h$
Rhombus		$A = \frac{1}{2}xy$
Circle		$A = \pi r^2$

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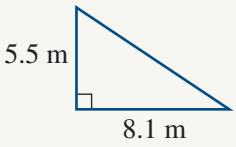
Example 18: Finding the area of a triangle 1F

Find the area of the triangle.



SOLUTION:

1 The shape is a triangle, so use the formula $A = \frac{1}{2}bh$.

$A = \frac{1}{2}bh$

2 Substitute the values for b and h ($b = 8.1$ and $h = 5.5$).

$= \frac{1}{2} \times 8.1 \times 5.5$

3 Evaluate.

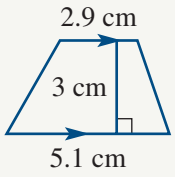
$= 22.275 \text{ m}^2$

4 Write the answer using the correct units.

The area of the triangle is 22.275 m^2.

Example 19: Finding the area of a trapezium 1F

Find the area of the quadrilateral.



SOLUTION:

1 The shape is a trapezium, so use the formula $A = \frac{1}{2}(a + b)h$.

$A = \frac{1}{2}(a + b)h$

2 Substitute the values for a, b and h.

$= \frac{1}{2}(2.9 + 5.1)3$

3 Evaluate.

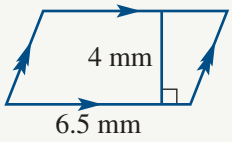
$= 12 \text{ cm}^2$

4 Write the answer using the correct units.

The area of the shape is 12 cm^2.

Example 20: Finding the area of a parallelogram 1F

Find the area of the following quadrilateral.



SOLUTION:

1 The shape is a parallelogram, so use the formula $A = bh$.

$A = bh$

2 Substitute the values for b and h ($b = 6.5$ and $h = 4$).

$= 6.5 \times 4$

3 Evaluate.

$= 26 \text{ mm}^2$

4 Write the answer using the correct units.

The area of the shape is 26 mm^2.

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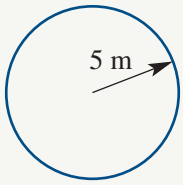
Example 21: Finding the area of a circle 1F

Find the area of a circle with a radius of 5 metres.
Give your answer correct to one decimal place.



SOLUTION:

1	The shape is a circle, so use the formula $A = \pi r^2$.	$A = \pi r^2$
2	Substitute the value for r ($r = 5$).	$= \pi \times 5^2$
3	Evaluate correct to one decimal place.	$\approx 78.5\ \text{m}^2$
4	Write the answer using the correct units.	The area of the circle is $78.5\ \text{m}^2$.

Area of composite shapes

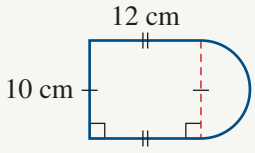
A composite shape is made up of two or more plane shapes. The area of a composite shape is calculated by adding or subtracting the areas of simple shapes.

AREA OF COMPOSITE SHAPES

- Composite shapes are made up of more than one simple shape.
- Area of a composite shape can be found by adding or subtracting the areas of simple shapes.

Example 22: Finding the area of a composite shape 1F

Find the area of the composite shape. Answer correct to one decimal place.



SOLUTION:

1	Divide the shape into a rectangle and a semicircle.	
2	Use the formula $A = lb$ for the rectangle.	$A = lb$
3	Substitute.	$= 12 \times 10$
4	Evaluate.	$= 120\ \text{cm}^2$
5	Use the formula $A = \frac{1}{2}\pi r^2$ for the semicircle.	$A = \frac{1}{2}\pi r^2$
6	Substitute.	$= \frac{1}{2} \times \pi \times 5^2$
7	Evaluate.	$\approx 39.3\ \text{cm}^2$
8	Add the area of the rectangle to the semicircle.	$A = 120 + 39.3$
9	Evaluate.	$= 159.3\ \text{cm}^2$
10	Write the answer using the correct units.	The area of the shape is $159.3\ \text{cm}^2$.

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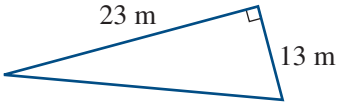
Exercise 1F

LEVEL 1

Example 18

1 Find the area of each triangle. Answer correct to one decimal place where necessary.

a



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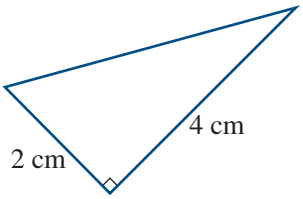
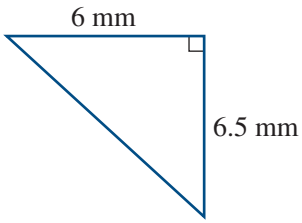
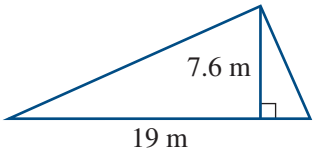


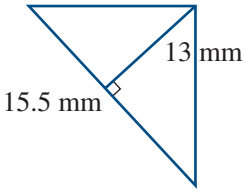
c



d



e



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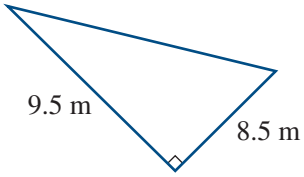
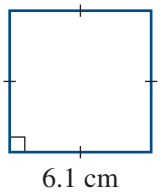


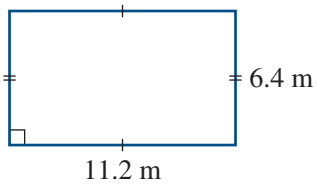
2 Find the area of each shape. Answer correct to one decimal place where necessary.

Example 19

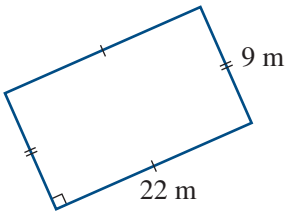
a



b

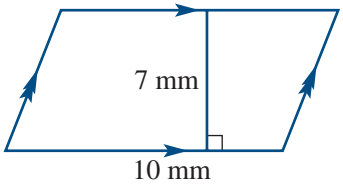


c

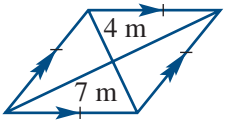


Example 20

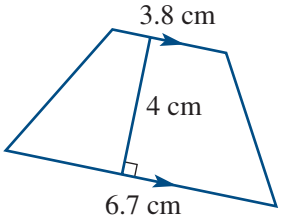
d



e



f



3 Find the area of a triangle with a base of 8.25 cm and a height of 10.15 cm. Answer correct to the nearest square centimetre.

4 Find the area of a square with a side length of 105.1 m. Answer correct to the nearest square metre.

Example 21

5 Find the area of a circle with a radius of 7 cm. Answer correct to the nearest square centimetre.

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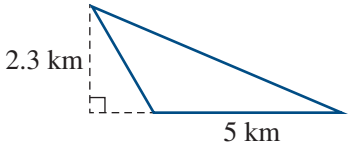
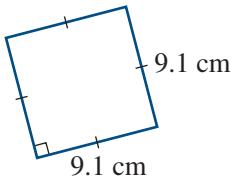
LEVEL 2

6 Find the area of each shape. Answer correct to one decimal place where necessary.

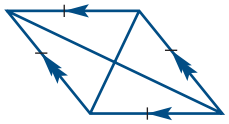
a



b

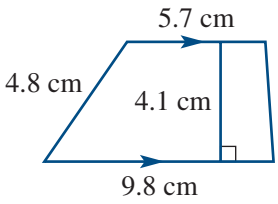


c

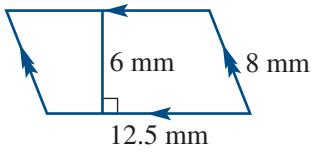


Diagonals are
4.4 mm and 6.8 mm

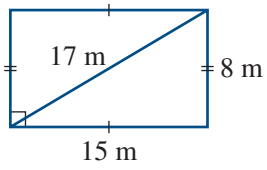
d



e

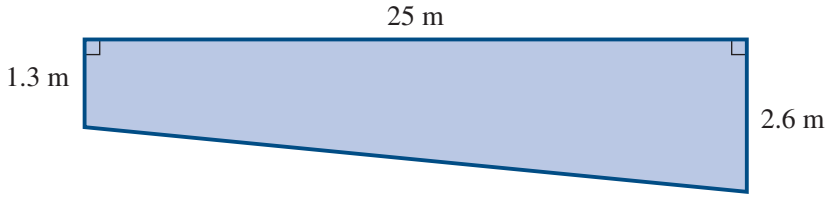


f



7 Jasmine is planning to build a circular pond. The radius of the pond is 1.5 m. What is the area of the pond, correct to the nearest square metre?

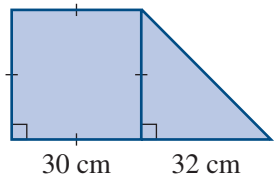
8 A 25 m swimming pool increases in depth from 1.3 m at the shallow end to 2.6 m at the deep end. Calculate the area of one side wall of the pool. Answer correct to the nearest square metre.



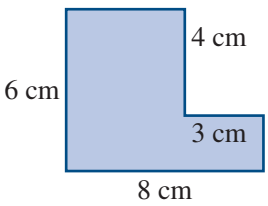
9 Philip wants to tile a rectangular area measuring 2.5 m by 3 m in his backyard. The tiles he wishes to use are 50 cm by 50 cm. How many tiles will he need? Give your answer as a whole number.

10 Find the area of each composite shape.

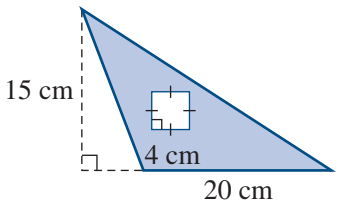
a



b



c



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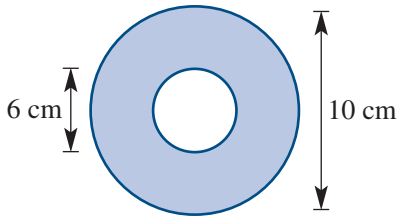
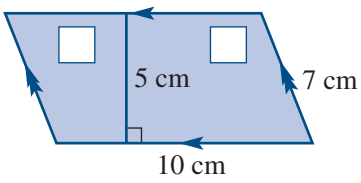
11 An annulus consists of two circles with the same centre. Find the area of an annulus if the inner diameter is 6 cm and the outer diameter is 10 cm. Answer correct to the nearest square centimetre.



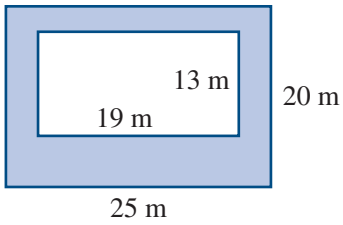
12 A metal parallelogram has two identical squares removed from its shape. The two squares have a side length of 2 cm. Find the shaded area. Answer correct to the nearest square centimetre.



13 A lawn is to be laid around a rectangular garden bed.

a What is the amount of lawn required?

b Find the cost of the new lawn if the required turf costs $20 per square metre.



LEVEL 3

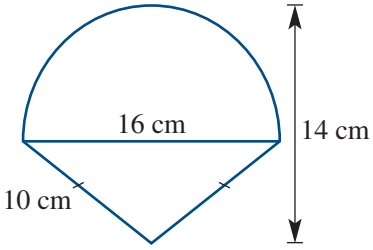
Example 22

14 A light fitting has the shape of half of a sphere mounted on an upside-down cone. Its cross–section is shown opposite.

a What is the radius of the semicircle?

b What is the height of the triangle?

c Calculate the area of the cross-section. Answer correct to one decimal place.

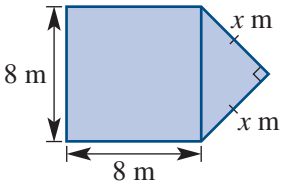


15 What is the area of a quadrant if it has a radius of 8 mm? Answer correct to two decimal places.

16 Decking for a house consists of a square and a triangle. The square has a side length of 8 metres and the triangle is isosceles.

a Use Pythagoras' theorem to find the value of x.

b Calculate the area of the shaded region.



17 A metal worker cut circles with a diameter of 2 cm from a rectangular sheet of tin 4 cm by 8 cm.

a What is the area of the rectangular sheet?

b How many circles can be cut from the rectangular sheet?

c What is the area of the remaining metal after the circles have been removed from the rectangular sheet? Answer correct to two decimal places.

1G Simple interest

Interest is the amount paid for borrowing money or the amount earned for lending money or investing. There are different ways of calculating interest. Simple interest (or flat interest) is a fixed percentage of the amount invested or borrowed and is calculated on the original amount. For example, if we invest \$100 in a bank account that pays interest at the rate of 5% per annum (per year) we would receive \$5 each year. That is,

$$\text{Interest} = \$100 \times \frac{5}{100} = \$5$$

This amount of interest would be paid each year. Simple interest is always calculated on the initial amount, or the principal.

An investment in the context of interest is money that you put into a bank, or other financial institution, in return for interest payments. It is like a loan that you make to the bank.

FORMULA FOR CALCULATING SIMPLE INTEREST

$I = Prn$

I – Interest (simple or flat) earned for the use of money, paid by borrowers to lenders

P – Principal is the initial amount of money borrowed, lent or invested

r – Rate of simple interest per time period expressed as a decimal, e.g. 5% = 0.05

n – Number of time periods (days, weeks, months or years)

Example 23: Finding simple interest 1G

Calculate the amount of simple interest paid on an investment of \$12 000 at 10% simple interest per annum for 3 years.

SOLUTION:

1	Write the simple interest formula.	$I = Prn$
2	Substitute $P = 12\,000$, $r = 0.10$ and $n = 3$ into the formula.	$= 12\,000 \times 0.10 \times 3$
3	Evaluate.	$= \$3600$
4	Write the answer in words.	Simple interest is \$3600.

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Amount owed or future value

The interest is added to the principal to determine the amount owed on a loan or the future value of an investment.

FORMULA FOR AMOUNT OWED OR FUTURE VALUE

$A = P + I$

A – Amount owed or future value

I – Interest (simple or flat) earned

P – Principal is the initial quantity of money borrowed, loaned or invested

Example 24: Calculating the amount owed 1G

Find the amount owed on a loan of \$50 000 at 7% per annum simple interest at the end of two years and six months.

SOLUTION:

1. Write the simple interest formula.
2. Substitute $P = 50\,000$, $r = 0.07$ and $n = 2.5$ into the formula.
3. Evaluate.
4. Write the amount owed formula.
5. Substitute $P = 50\,000$ and $I = 8750$ into the formula.
6. Evaluate.
7. Write the answer in words.

$I = Prn$
$= 50\,000 \times 0.07 \times 2.5$
$= \$8750$
$A = P + I$
$= 50\,000 + 8750$
$= \$58\,750$
Amount owed is \$58 750.

Example 25: Calculating value of an investment 1G

Joel plans to make an investment of \$200 000 at 7½% p.a. simple interest for 2 years. What is the total value of his investment at the end of 2 years?

SOLUTION:

1. Write the simple interest formula.
2. Substitute $P = 200\,000$, $r = 0.075$ and $n = 2$ into the formula.
3. Evaluate.
4. Write the amount owed formula.
5. Substitute $P = 200\,000$ and $I = 30\,000$ into the formula.
6. Evaluate.
7. Write answer in words.

$I = Prn$
$= \$200\,000 \times 0.075 \times 2$
$= \$30\,000$
$A = P + I$
$= \$200\,000 + \$30\,000$
$= \$230\,000$
Total value is \$230 000.

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Exercise 1G

LEVEL 1

Example 23

1 Calculate the amount of simple interest for each of the following.

a Principal = \$15 000, Interest rate = 13% p.a., Time period = 3 years

b Principal = \$2000, Interest rate = $6\frac{1}{2}$% p.a., Time period = 7 years

c Principal = \$200 000, Interest rate = $9\frac{1}{4}$% p.a., Time period = 2 years

d Principal = \$3600, Interest rate = 9% p.a., Time period = $3\frac{1}{2}$ years

e Principal = \$40 000, Interest rate = 7.25% p.a., Time period = $5\frac{1}{4}$ years

Example 24

2 Calculate the amount owed for each of the following.

a Principal = \$500, Simple interest rate = 5% p.a., Time period = 4 years

b Principal = \$900, Simple interest rate = 3% p.a., Time period = 7 years

c Principal = \$4000, Simple interest rate = $8\frac{1}{2}$% p.a., Time period = 3 years

d Principal = \$6900, Simple interest rate = 10% p.a., Time period = $4\frac{1}{2}$ years

e Principal = \$10 000, Simple interest rate = 6.75% p.a., Time period = $2\frac{1}{4}$ years

3 The simple interest rate is given as 4.8% per annum.

a What is the interest rate per quarter?

b What is the interest rate per month?

c What is the interest rate per six months?

d What is the interest rate per nine months?

4 Calculate the amount of simple interest for each of the following.

a Principal = \$800, Interest rate = 12% p.a., Time period = 1 month

b Principal = \$1600, Interest rate = 18% p.a., Time period = 6 months

c Principal = \$60 000, Interest rate = 9.6% p.a., Time period = 3 months

d Principal = \$20 000, Interest rate = 6% p.a., Time period = 9 months

5 Andrew takes a loan of \$30 000 for a period of 6 years, at a simple interest rate of 14% per annum. Find the amount owing at the end of 6 years.

6 A loan of \$1800 is taken out at a simple interest rate of 15.5% per annum. How much interest is owing after 3 months?

Example 25

7 A sum of \$100 000 was invested in a fixed–term account for 4 years.

a Calculate the simple interest earned if the rate of interest is 5.5% per annum.

b Find the value of the investment at the end of 4 years.

8 Joshua invested \$1200 at 8% per annum. What is the simple interest earned between 30 September and 1 January?

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LEVEL 2

9 Sophie decides to buy a car for $28 000. She has saved $7000 for the deposit and takes out a loan over two years for the balance. The flat rate of interest charged is 12% per annum. What is the total amount of interest to be paid?

10 Domenico has borrowed $24 000 to buy furniture. He wishes to repay the loan over four years. Calculate the simple interest on the following rates of interest.

a 8% per annum for the entire period

b 9% per annum after a 6-month interest-free period

c 10% per annum after a 12-month interest-free period

XLS 01GQ11

11 Create the spreadsheet shown.

IF =A5*B5*C5

	A	B	C	D	E	F	G
1	**Cambridge Mathematics Standard Year 11 Exercise 1G Question 11**						
2	Worksheet to calculate simple interest						
3							
4	*Principal*	*Rate*	*Time (yr)*	*Interest*	*Amount*		
5	$500	5.00%	4.0	=A5*B5*C5	$600		
6	$15,000	13.00%	3.00	$5,850	$20,850		
7	$2,000	6.25%	7.00	$875	$2,875		
8	$200,000	9.25%	2.00	$37,000	$237,000		
9	$3,600	9.00%	3.50	$1,134	$4,734		
10	$400,000	7.25%	5.25	$152,250	$552,250		
11	$800	10.00%	0.50	$40	$840		
12	$20,000	11.50%	0.75	$1,725	$21,725		
13							

a Cell D5 has a formula that calculates the simple interest. Enter this formula.

b The formula for cell E5 is '=A5 + D5'. Fill down the contents of E6 to E12 using this formula.

12 Isabelle buys a TV for $1400. She pays it off monthly over 2 years at an interest rate of 11.5% per annum flat. How much per month will she pay?

13 Riley wants to earn $4000 a year in interest. How much must he invest if the simple interest rate is 10% p.a.?

LEVEL 3

14 Samira invests $16 000 for $2\frac{1}{2}$ years. What is the minimum rate of simple interest needed for her to earn $3000?

15 Gurrumul pays back $20 000 on a $15 000 loan at a flat interest rate of 10%. What is the term of the loan?

16 Harry borrowed $300 000 at a flat interest rate of 8.5% per annum. This rate was fixed for 2 years. He pays back the interest only over this period.

a How much interest is to be paid over the 2 years?

b After paying the fixed rate of interest for the first year, Harry finds the bank will decrease the flat interest rate to 7.5% if he pays a charge of $1000. How much will he save by changing to the lower interest rate for the last year?

1H Compound interest

Compound interest is calculated on the principal at the start, and then at each time period, on the principal plus interest. It calculates interest on the interest. For example, if \$100 is invested in a savings account at a compound interest rate of 10% per annum:

First year –	Interest = $\$100 \times 0.10 \times 1$	= \$10
	Amount owed = $\$100 + \$10$	= \$110
Second year –	Interest = $\$110 \times 0.10 \times 1$	= \$11
	Amount owed = $\$110 + \$11$	= \$121
Third year –	Interest = $\$121 \times 0.10 \times 1$	= \$12.10
	Amount owed = $\$121 + \$12.10$	= \$133.10

These calculations show that the interest earned increased each year. In the first year it was \$10, the second year \$11 and the third year \$12.10.

COMPOUND INTEREST FORMULA

$A = P(1 + r)^n$

A – Amount (final balance) or future value of the loan

P – Principal is the initial amount of money borrowed or present value of the loan

r – Rate of interest per compounding time period expressed as a decimal

n – Number of compounding time periods

Total compound interest earned or paid

The total compound interest is calculated by subtracting the principal from the final balance i.e. the amount of money at the end.

INTEREST EARNED OR OWED

$I = A - P$

A – Amount of money or final balance

I – Interest (compound) earned or paid

P – Principal is the initial amount of money

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Example 26: Finding the compound interest 1H

Paige invests $5000 over 5 years at a compound interest rate of 6.5% p.a. Calculate:

a the amount of the investment after 5 years, correct to the nearest cent.

b the interest earned after 5 years, correct to the nearest cent.

SOLUTION:

1	Write the compound interest formula.	**a** $A = P(1 + r)^n$
2	Substitute $P = 5000$, $r = 0.065$ and $n = 5$ into the formula.	$= 5000(1 + 0.065)^5$
3	Evaluate.	$= 6850.433317$ $\approx \$6850.43$
4	Write answer in words.	Amount of investment interest earned is $6850.43.
5	Write the formula.	**b** $I = A - P$
6	Substitute $A = 6850.43$ and $P = 5000$ into the formula.	$= 6850.43 - 5000$
7	Evaluate.	$= \$1850.43$
8	Write in words.	Interest earned is $1850.43.

Example 27: Finding the compound interest 1H

James borrowed $50 000 for 4 years at 11% p.a. interest compounding monthly. Calculate:

a the amount owed after 4 years, correct to the nearest cent

b the interest owed after 4 years, correct to the nearest cent.

SOLUTION:

1	Write the compound interest formula.	**a** $A = P(1 + r)^n$
2	Calculate the number of time periods (4 years × 12 months) and the interest rate per time period.	$n = 4 \times 12$ $r = 0.11 \div 12$
3	Substitute $P = 50\,000$, $r = \frac{0.11}{2}$ and $n = 48$ into the formula.	$A = 50\,000\left(1 + \frac{0.11}{12}\right)^{48}$
4	Evaluate.	$\approx \$77\,479.90$
5	Write answer in words.	Amount owed is $77 479.90.
6	Write the interest owed formula.	**b** $I = A - P$
7	Substitute $P = 50\,000$ and $A = 77\,479.90$ into the formula.	$= \$77\,479.90 - \$50\,000$
8	Evaluate.	$= \$27\,479.90$
9	Write in words.	Interest owed is $27 479.90.

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Exercise 1H

LEVEL 1

Example 26

1 Calculate the amount owed, to the nearest cent, for each of the following loans.

a Principal = \$800, Compound interest rate = 4% p.a., Time period = 3 years

b Principal = \$9000, Compound interest rate = $6\frac{1}{2}$% p.a., Time period = 4 years

c Principal = \$12 000, Compound interest rate = 11% p.a., Time period = $2\frac{1}{2}$ years

d Principal = \$22 000, Compound interest rate = 5.5% p.a., Time period = $4\frac{1}{4}$ years

Example 27

2 Calculate the amount of compound interest to the nearest cent for each of the following.

a Principal = \$25 000, Interest rate = 7% p.a., Time period = 5 years

b Principal = \$300 000, Interest rate = $10\frac{1}{4}$% p.a., Time period = 3 years

c Principal = \$6500, Interest rate = 13% p.a., Time period = $1\frac{1}{2}$ years

d Principal = \$80 000, Interest rate = 8.25% p.a., Time period = $3\frac{1}{4}$ years

3 Amy is investing \$20 000. What sum of money will she receive if she invests for 4 years at 8% p.a. compound interest? Answer to the nearest cent.

4 Use the formula $A = P(1+r)^n$ to calculate the value of an investment of \$10 000 over a period of 2 years with an interest rate of 0.8% compounding monthly. Answer to the nearest cent.

5 Ryan invested \$20 000 for 5 years at 12% p.a. interest compounding monthly. What is the amount of interest earned in the first year? Answer to the nearest cent.

6 Find the amount of money in a bank account after 6 years if an initial amount of \$4000 earns 8% p.a. compound interest, paid quarterly. Answer to the nearest cent.

7 Christopher invested \$13 500 over 7 years at 6.2% p.a. interest compounding quarterly. Calculate:

a the value of the investment after 7 years to the nearest cent.

b the compound interest earned to the nearest cent.

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LEVEL 2

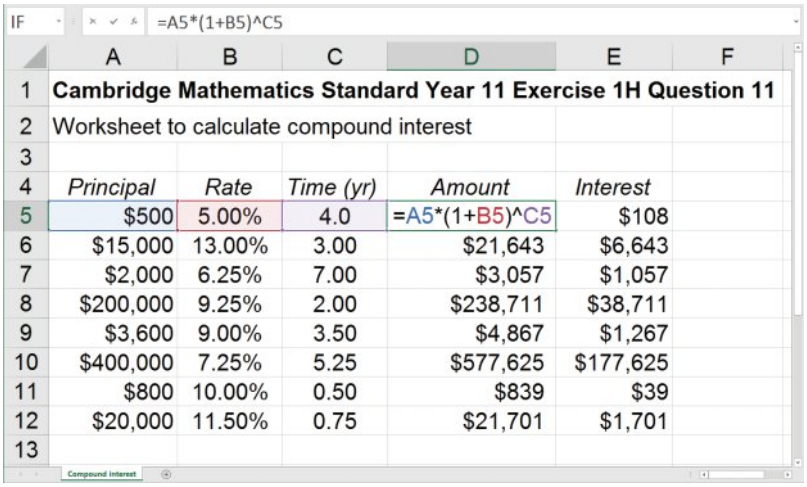
8 What sum of money would Bailey need to invest to accumulate a total of $50 000 at the end of 4 years at 6% p.a. compound interest? Answer to the nearest cent.

9 Calculate the amount that must be invested at 9.3% p.a. interest compounding annually to have $70 000 at the end of 3 years. Answer to the nearest cent.

10 What sum of money needs to be invested to accumulate to a total of $100 000 in 10 years at 7.25% p.a. compound interest? Answer to the nearest cent.

11 Create the spreadsheet below.

01HQ11



IF =A5*(1+B5)^C5

	A	B	C	D	E	F
1	**Cambridge Mathematics Standard Year 11 Exercise 1H Question 11**					
2	Worksheet to calculate compound interest					
3						
4	*Principal*	*Rate*	*Time (yr)*	*Amount*	*Interest*	
5	$500	5.00%	4.0	=A5*(1+B5)^C5	$108	
6	$15,000	13.00%	3.00	$21,643	$6,643	
7	$2,000	6.25%	7.00	$3,057	$1,057	
8	$200,000	9.25%	2.00	$238,711	$38,711	
9	$3,600	9.00%	3.50	$4,867	$1,267	
10	$400,000	7.25%	5.25	$577,625	$177,625	
11	$800	10.00%	0.50	$839	$39	
12	$20,000	11.50%	0.75	$21,701	$1,701	
13						

Compound interest

a Cell D5 has a formula that calculates the compound interest. Enter this formula.

b The formula for cell E5 is '= D5 – A5'. Fill down the contents of E6 to E12 using this formula.

LEVEL 3

12 How much more interest is earned on a $40 000 investment if the interest at 6% p.a. is compounded annually over 6 years, compared with the simple interest at 6% p.a. earned over the same time?

13 Hamish has $50 000 to invest for two years. Which is the better investment and by how much?

Investment 1
Simple
interest rate
4% p.a.

Investment 2
Compound
interest rate
4% p.a.

11 Frequency tables

A frequency table is a listing of the outcomes and how often (its frequency) each outcome occurs. The outcomes may also be called 'scores'. The tally of the frequency and the final count are listed in separate columns. When you create a frequency table from a set of data, you should include a tally column to record your count of how often each score occurs. The tally column may be omitted in a frequency table given to you that has already been created. A frequency table is also called a frequency distribution.

Lowest score →

Score	Tally	Frequency
17	𝍸	1
18	𝍸 𝍸	6
19	𝍸 𝍸	5
20	𝍸 𝍸 𝍸	7
21	𝍸	3

← Lowest frequency

← Highest frequency

Highest score →

FREQUENCY TABLE

1. Scores or outcomes are listed in the first column in ascending order.
2. Tally column records the count of the number of times the score occurred (groups of 5s).
3. Frequency column is the total count of each outcome.

Example 28: Constructing a frequency table 11

The temperatures for 39 days are shown below. Construct a frequency table.

19 20 18 23 27 25 26 27 28 27 25 24 24 19 25 22 21 28 26 26
22 20 25 20 22 24 24 22 21 24 25 26 25 27 21 23 23 22 25

SOLUTION:

1. Draw a table with three columns and label them score, tally and frequency.
2. List the temperatures in the score column from the lowest (18) to the highest (28).
3. Record a mark in the tally column for each temperature.
4. Count the tally marks and write the total in the frequency column.
5. Add the frequency column to find the total number of scores. This should match the total number of temperatures (39).

Score	Tally	Frequency
18	𝍸	1
19	𝍸	2
20	𝍸	3
21	𝍸	3
22	𝍸 𝍸	5
23	𝍸	3
24	𝍸 𝍸	5
25	𝍸 𝍸 𝍸	7
26	𝍸 𝍸	4
27	𝍸 𝍸	4
28	𝍸	2
	Total	39

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Exercise 1I

LEVEL 1

Note: include a tally column, where you are asked to create a frequency table in this exercise.

Example 28

1 The ages, in years, of players in a football team are recorded in a frequency table.

a Copy and complete the table.

b What was the most common age?

c How many players are in the team?

Score	Tally	Frequency
20	𝍸	
21	𝍸 |	
22		6
23	𝍸	
24		2

2 The number of times a fire engine is called out on a given day was recorded in a frequency table.

a Copy and complete the table.

b What was the most common number of calls?

c On how many days was the fire engine called out four times?

d On how many days was the fire engine called out fewer than three times?

Number of calls	Tally	Frequency
0		7
1	𝍸 𝍸 |	
2		8
3	𝍸	
4		2
5	|	

Example 28

3 The number of brothers and sisters for 30 students is recorded below. Construct a frequency table for this data.

2 1 1 5 3 4 1 0 2 1 5 2 6 1 0

3 1 0 4 3 3 4 2 3 0 2 0 1 0 2

4 The shoe sizes of 20 seventeen year olds are recorded below. Construct a frequency table.

11 7 8 10 12 10 8 7 8 11

9 9 8 10 9 8 9 8 7 8

Example 28

5 The assessment result for 30 students is recorded below. Construct a frequency table for this data.

96 97 97 95 92 94 97 98 91 97 95 91 96 97 94

92 92 98 94 93 93 94 95 93 92 91 98 96 92 91

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LEVEL 2

6 A die was rolled and the results listed below.

1	1	5	2	2	5	1	3	6	1	4	3	1	5	5	4	6
3	3	4	2	1	1	6	5	4	2	6	6	1	4	4	1	6
4	1	6	5	2	4	6	2	4	2	2	3	6	4	2	4	1

a Construct a frequency table.
b How many times was the die rolled?
c How many results are higher than 2?
d What was the most common number rolled?
e Do you think the die is biased? Give a reason for your answer.

Example 28

7 David recorded the following times, in seconds, for the 50 m freestyle.

32	34	37	35	35	37	34	33	38	34	36	33	34	37	37	37	38
33	33	36	35	34	34	38	32	36	32	38	38	34	36	36	34	38
32	34	38	37	35	36	38	35	32	35	35	33	38	36	35	36	32

a Construct a frequency table.
b How many times were recorded?
c How many times were below 35 seconds?
d What was the most common time for the 50 m freestyle?
e What percentage of times are 37 seconds or more?

LEVEL 3

8 Count the number of letters in each word of the paragraph below.
A frequency table is a listing of the outcomes and how often (frequency) each outcome occurs. The outcomes are often listed under a heading called 'score'. The tally of the frequency and the final count are listed in separate columns. Frequency tables are also called a frequency distribution.

a Create a frequency table for the length of words used in the above paragraph.
b Using the frequency table, what is the most frequent word length in the English language?
c Comment on the fairness of the conclusion made in part **b**.

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1J Mean, median and mode

Mean

The mean is a measure of the centre of a data distribution. It is calculated by summing all the scores and dividing this by the number of scores. For example, consider the scores 1, 6, 3 and 2. The mean is $\frac{1+6+3+2}{4} = 3$. The mean of a set of data is what most people call the 'average'.

Median

The median is the middle score or value of a data distribution. To find the median, list all the scores in increasing order and select the middle one. For example, the median of 1, 4, **5**, 7, 8 is 5. When there is an even number of scores, the median is the average of the two middle scores. For example, the median of 1, 1, **4**, **5**, 7, 8 is found by sorting the six scores and finding the average of 4 and 5 or 4.5.

Mode

The mode is the score that occurs most often in a data distribution. It is the score with the highest frequency. The mode is useful for categorical data that do not allow you to do numerical calculations, for example, if the data collected is a colour. Modes may occur at the beginning or end of a range of values.

Mean	Median	Mode
Mean $= \frac{\text{Sum of scores}}{\text{Number of scores}}$ $\bar{x} = \frac{\sum x}{n}$	1 Arrange all the scores in increasing order. 2 Median is the middle score.	1 Determine the number of times each score occurs. 2 Mode is the score that occurs the greatest number of times.

Example 29: Calculating the mean, median and mode 1J

The table below shows the number of rainy days for the first six months.

J	F	M	A	M	J
12	15	13	8	8	10

Find the mean, median and mode.

SOLUTION:

1 Write the formula for the mean.

2 Sum all of the scores and divide by the number of scores.

3 Evaluate.

$$\bar{x} = \frac{\sum x}{n} = \frac{12+15+13+8+8+10}{6} = 11$$

4 Write the answer in words.

Mean is 11.

5 Write the scores in increasing order.

8, 8, **10**, **12**, 13, 15

6 Count the total number of scores. There are 6 scores.

7 Median is the average of the 3rd (score 10) and the 4th scores (score 12). Write the answer in words.

$$\text{Median} = \frac{10+12}{2} = 11$$

Median is 11.

8 Mode is the score that occurs the most.

Mode is 8.

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Exercise 1J

LEVEL 1

1 Find the mean of each data set.

a 6, 9, 9, 15, 9, 3, 7, 9, 13, 10
b 22, 28, 22, 31, 43, 22
c 5, 5, 8, 12, 13, 3, 7, 9, 10
d 7, 5, 11, 3, 0, 1, 2, 9, 7
e 6, 5, 6, 17, 13, 6, 0, 9, 1
f 39, 35, 39, 41, 47, 49, 44
g 3, 5, 24, 19, 13, 13, 13, 9, 3, 8, 11
h 55, 24, 14, 18, 13, 3, 2, 2, 2, 7, 14

2 Find the mean of each data set. Answer correct to one decimal place.

a 13, 14, 15
b 5, 6, 7, 8, 9
c 5, 7, 8, 9, 13, 15
d 6, 8, 11, 13
e 6, 7, 10, 11, 13, 19
f 1, 1, 2, 2, 2, 2, 3, 3, 3, 3
g 9, 9, 10, 10, 10, 11, 11, 11, 11
h 6, 6, 9, 9, 9, 10, 10, 10, 10

3 Twenty people measured their heart rate using a heart-rate monitor. The results were 64, 68, 64, 72, 75, 67, 91, 80, 77, 73, 68, 81, 73, 72, 60, 62, 74, 68, 55 and 62.

a What is the sum of these heart rates?
b Find the mean heart rate. Answer correct to two decimal places.
c Another person with a heart rate of 63 is included in this data. What is the new mean? Answer correct to two decimal places.

4 There is an odd number of scores. Find the median of these numbers.

a 3, 9, 10
b 3, 4, 6, 7, 9, 10, 15
c 23, 28, 29, 30, 34, 45, 46, 49, 50
d 1002, 1010, 1100, 1120, 1160
e 2, 2, 2, 2, 2, 2, 2, 9, 9
f 14, 15, 100, 101, 102
g 0, 0, 0, 1, 1, 1, 7, 8, 8, 8, 8
h 2, 7, 9, 10, 10, 14, 18

5 There is an even number of scores. Find the median of these numbers.

a 2, 5, 6, 8, 8, 9
b 12, 14, 18, 22
c 20, 20, 20, 21, 22, 24
d 3, 4, 5, 9, 10, 14, 16, 18, 18, 18
e 100, 110, 130, 140
f 1, 1, 1, 1, 3, 3, 3, 5, 5, 5, 6, 6
g 10, 20, 22, 40, 60, 61, 70, 80
h 3, 5, 8, 10, 11, 14, 18, 19

6 The number of senior citizens entering a restaurant in the past 9 hours was 18, 17, 16, 17, 19, 13, 10, 16 and 15. What is the median?

7 Find the mode of each data set.

a 13, 9, 3, 24, 19, 3, 5, 13, 8, 11, 13
b 5, 8, 12, 13, 3, 7, 9, 5, 10
c 15, 7, 9, 13, 6, 9, 9, 3, 9, 10
d 47, 49, 39, 41, 39, 35, 44

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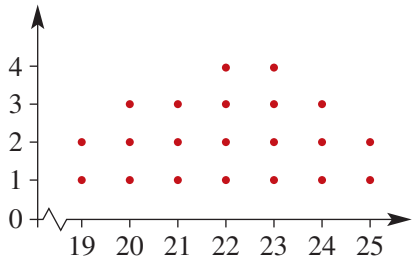
LEVEL 2

8 Use the stem-and-leaf plot opposite to answer these questions.

Stem	Leaf
0	7 8
1	2 3 5 9
2	1 2 6 7 9 9
3	0 1 7
4	2 7

- **a** How many scores are there?
- **b** What is the lowest score?
- **c** What is the median?
- **d** Remove 42 from the data. What is the new median?

9 Use the dot plot opposite to answer these questions.



- **a** What is the highest score?
- **b** What is the lowest score?
- **c** Calculate the median.
- **d** How many scores would need to be added to make the median 23?
- **e** What is the median if a single score of 26 is included in the data?

10 The table opposite shows the ages of players in the local football team.

Age	Frequency
13	3
14	5
15	5
16	2

- **a** What is the age of the oldest player?
- **b** What is the age of the youngest player?
- **c** What is the range of ages?
- **d** What is the median age?

11 Nine students were surveyed on the number of hours they slept last night. Their answers were 10, 8, 7, 7, 4, 7, 8, 9 and 11.

- **a** Find the mean number of hours slept.
- **b** Find the mode number of hours slept.
- **c** A tenth student was then surveyed and the mean changed to 7.5. What was the number of hours slept by the tenth student?

LEVEL 3

12 The mean height of five basketball players at the start of the game is 1.92 m. During the game a player who is 1.80 m tall is injured and replaced by a player who is 1.98 m tall. What is the mean height of the five players now? Answer correct to two decimal places.

13 A score was added to the set of scores: 15, 18, 20, 22, 24 and 26. The new mean is equal to 20. What score was added?

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PPT STUDY GUIDE

Key ideas and chapter summary

Simplifying algebraic expressions

1. Find a common denominator if the denominators are different.
2. Express each fraction with the common denominator.
3. Simplify the numerator by adding or subtracting the like terms.

Index laws

$a^m \times a^n = a^{m+n}$ Multiplying terms in index form, add the indices.

$a^m \div a^n = a^{m-n}$ Dividing terms in index form, subtract the indices.

$(a^m)^n = a^{mn}$ Index form raised to a power, multiply the indices.

$a^0 = 1$ A term in index form raised to the power of zero is one.

Expanding algebraic expressions

1. Multiply the term outside the grouping symbol by the first term then the second term inside the grouping symbol.
2. Simplify and collect like terms if required.

Pythagoras' theorem

$(\text{Hypotenuse})^2 = (\text{side})^2 + (\text{other side})^2$

Perimeter

Perimeter is the total length of the outside edges of a shape. It is the length of the boundary.

Area

Triangle	$A = \frac{1}{2}bh$	Square	$A = s^2$
Rectangle	$A = lb$	Parallelogram	$A = bh$
Trapezium	$A = \frac{1}{2}(a+b)h$	Rhombus	$A = \frac{1}{2}xy$

Simple interest

$I = Prn$ $A = P + I$

Compound interest

$A = P(1+r)^n$ $I = A - P$

Frequency table

1. Scores are listed in ascending order.
2. Tally column records the number of times the score occurred.
3. Frequency column is a count of each outcome or score.

Mean

$$\text{Mean} = \frac{\text{Sum of scores}}{\text{Number of scores}}$$

Median

The median is the middle score or value.

Mode

The mode is the score that occurs the most.

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Review

Multiple-choice questions

1 Simplify $2a^2b - 5ba^2 + 13ab$.

A $7a^2b$ **B** $10a^2b$ **C** $3a^2b + 13ab$ **D** $-3a^2b + 13ab$

2 Simplify $4x^2 \times 3x^2$.

A $7x$ **B** $7x^4$ **C** $12x^2$ **D** $12x^4$

3 Expand $-4(2p + 3q)$.

A $-8p - 12q$ **B** $-8p + 12q$ **C** $-8p - 3q$ **D** $-8p + 3q$

4 What is the length of the hypotenuse if the two other sides are 12 cm and 16 cm?

A 400 cm **B** 20 cm **C** 28 cm **D** 7.46 cm

5 What is the perimeter of a quadrant with a radius of 5 mm?

A 3.9 mm **B** 7.9 mm **C** 13.9 mm **D** 17.9 mm

6 What is the area of a triangle with a base of 5 m and a perpendicular height of 8 m?

A 13 m^2 **B** 20 m^2 **C** 40 m^2 **D** 80 m^2

7 What is the simple interest on \$500 at 8% p.a. for 4 years?

A \$40 **B** \$160 **C** \$660 **D** \$1600

8 James borrows \$3000 at 10% p.a. interest compounding annually. What is the amount owed after 2 years? (Answer to the nearest dollar.)

A \$3030 **B** \$3060 **C** \$3600 **D** \$3630

9 The frequency table shows the results of a test. What is the most common score?

A 3
B 4
C 5
D 6

Score	Tally	Frequency
4	||	3
5	𝍸 |	6
4	||||	4

10 What is the median of 12, 20, 9, 4, 16, 11 and 13?

A 4 **B** 11 **C** 12 **D** 13

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Short-answer questions

1 Simplify by collecting like terms.

a $ab^3 + 5b^3a$ **b** $14x^2 - 2x - 6x^2$ **c** $6m + 2m^2 - m + 7m$

2 Add or subtract these algebraic fractions.

a $\frac{2x}{3} + \frac{5x}{3}$ **b** $\frac{7b}{3} + \frac{2b}{15}$ **c** $\frac{5y}{4} - \frac{y}{2}$

3 Simplify each of the following.

a $5m^4 \times 3m$ **b** $36y^8 \div 3y^2$ **c** $24v^4 \div 16v^3$

d $2n^0 + 5$ **e** $(x^4)^3$ **f** $(4x^2y^3)^2$

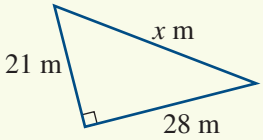
4 Remove the grouping symbols and simplify if required.

a $2(x - y^2)$ **b** $6(w + 6m)$ **c** $-4(6a^2 - 2b^2)$

d $-(8a - 3b)$ **e** $3(y - 5) + 6(y + 6)$ **f** $c^3(c^2 + 4d)$

5 Find the value of x correct to two decimal places.

a



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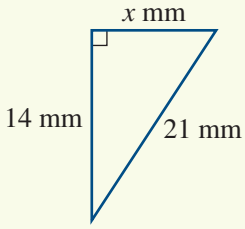

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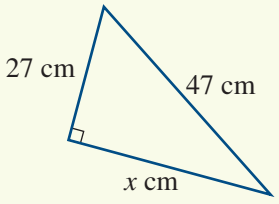
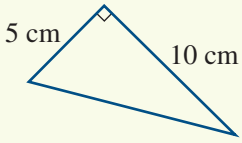


6 Find the perimeter of each shape. Answer correct to one decimal place.

a



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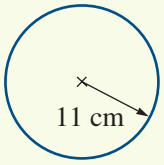

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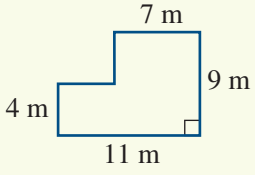

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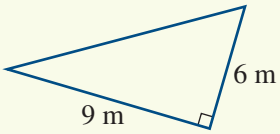
Review

7 Find the area of each shape. Answer correct to one decimal place.

a



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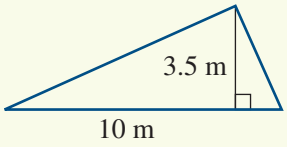

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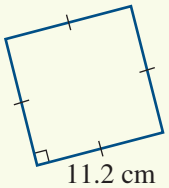

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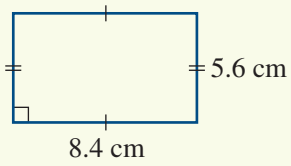

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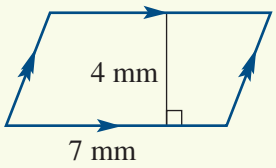

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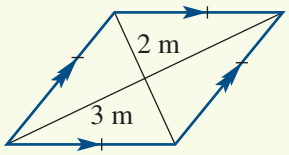

8 What is the simple interest on $1250 at a flat rate of 8% p.a. over 3 years?

9 Riley is investing $120 000 with a superannuation fund. How much will he receive if the money is invested for 4 years at the following rules? Answer to the nearest dollar.

a 3% p.a. compound interest

b 6% p.a. compound interest

10 Arrange these scores in order and find the median.

a 18, 11, 5, 14, 11, 10, 11, 22, 6

b 37, 46, 43, 37, 58, 37

11 Find the mean of each data set.

a 11, 13, 9, 9, 12, 16, 17, 7, 14

b 14, 11, 12, 13, 18, 16, 22, 20, 18

Extended-response questions

12 Zoe wants to earn $9000 a year in interest. How much must she invest if the simple interest rate is 14% p.a.? Answer to the nearest dollar.

13 The time (in hours) spent completing an assessment task is listed in the table.

a Copy and complete the table.

b How many students spent greater than 13 hours?

c What percentage of students spent less than 9 hours?

Class	Frequency
4–8	5
9–13	
14–18	8
19–23	4
	23

PDF LITERACY

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2 Earning and managing money

Syllabus topic — F1.2 Earning and managing money

This topic will develop your skills in managing earnings, wages and taxation.

Outcomes

- Calculate payments from an annual salary.
- Calculate wages from an hourly rate.
- Calculate wages involving overtime rates and allowances.
- Calculate annual leave loading and bonuses.
- Calculate earnings based on commission, piecework and royalties.
- Identify allowable tax deductions for different scenarios.
- Calculate the taxable income.
- Calculate net pay after deductions are made from gross pay.
- Calculate the Medicare levy.
- Determine the PAYG tax payable or refund owing using current tax scales.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Widgets
- Literacy worksheet
- Spreadsheets
- Quick Quiz
- Study guide
- Solutions (enabled by teacher)

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

In the Interactive Textbook you can take a test of prior knowledge required for this chapter, and depending on your score you may be directed to revision from the previous years' work.

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2A Salary and wages

Comparing a salary to a wage

Most people are paid an annual salary or a weekly wage.

	Salary	Wage
Description	A payment for a year's work, which is then divided into equal monthly, fortnightly or weekly payments.	A payment for a week's work, calculated on an hourly basis.
Advantages	• Permanent employment • Superannuation, sick and holiday pay	• Permanent employment • Superannuation, sick and holiday pay • Overtime payments for extra work
Disadvantages	• No overtime for extra work • Hours are fixed	• Lower incentive to work hard each hour • Hours are fixed
Examples	People who are paid a salary include teachers and nurses.	People who are paid a wage include shop assistants, factory workers and mechanics.

CONVERTING A SALARY OR WAGE TO ANOTHER TIME PERIOD

Divide the annual salary by the number of time periods in a year.
Divide the weekly wage by the number of work days or hours in a week.

1 year = 52 weeks 1 year = 26 fortnights 1 year = 12 months

Example 1: Calculating from a salary 2A

Wei earns a salary of $65 208 per annum. He is paid fortnightly. How much does he receive each fortnight? Assume there are 52 weeks in the year.

SOLUTION:

1 Write the quantity to be found.
2 Divide salary by 26 fortnights in a year.
3 Evaluate and write the answer in words.

Fortnightly pay = 65 208 ÷ 26
= $2508.00
Wei is paid $2508 per fortnight.

Example 2: Calculating a wage 2A

Jasmine is paid at a rate of $1098 for a 40-hour week. How much does Jasmine earn per hour?

SOLUTION:

1 Write the quantity to be found.
2 Divide amount by number of hours worked.
3 Evaluate and answer correct to two decimal places.

Wage per hour = 1098 ÷ 40
= $27.45
Jasmine earns $27.45 per hour.

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Exercise 2A

LEVEL 1

Example 1

1 Layla earns a salary of $92 648. Write, to the nearest dollar, her salary as amounts per:
- **a** week
- **b** fortnight
- **c** month.

2 The annual salary for four people is shown in the table below. Calculate their weekly and fortnightly payments. (Answer correct to the nearest dollar.)

Name	Salary	Week	Fortnight
a Aiko	$57 640		
b Blake	$78 484		
c Chloe	$107 800		
d David	$44 240		

3 What is Zachary's fortnightly income if he earns a salary of $43 056?

4 Find the annual salary for the following people.
- **a** Amber earns $580 per week.
- **b** Jun earns $1520 per fortnight.
- **c** Samuel earns $3268 per month.
- **d** Ava earns $2418 per week.

5 Harrison is a civil engineer who earns a salary of $1500 per week.
- **a** How much does he receive per fortnight?
- **b** How much does he receive per year?

6 What is Yasmeen's annual salary if her salary per fortnight is $1610?

7 Dylan receives a weekly salary payment of $1560. What is his annual salary?

8 Stephanie is paid $1898 per fortnight and Tahlia $3821 per month. Calculate each person's equivalent annual income. Who earns more per week and by how much?

9 Laura is paid $1235 per fortnight and Ebony $2459 per month. Who receives the higher annual salary and by how much?

10 Tran is paid $1898 per week and Jake $8330 per month. Calculate each person's equivalent annual income. What is the difference between their annual salaries?

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11 Arana works as a labourer and is paid $25.50 an hour. How much does he earn for working the following hours?

a 35 hours **b** 37 hours
c 40 hours **d** 42 hours

12 Lily earns $29.75 an hour. If she works 6 hours each day during the week and 4 hours a day during the weekend, find her weekly wage.

Example 2

13 Determine the wage for a 37-hour week for each of the following hourly rates.

a $12.00 **b** $9.50
c $23.20 **d** $13.83

14 Determine the income for a year (52 weeks) for each of the following hourly rates. Assume 40 hours of work per week.

a $7.59 **b** $15.25
c $18.78 **d** $11.89

15 Suchitra works at the local supermarket. She gets paid $22.50 per hour. Her time card is shown below. She is paid for her breaks, so they are not shown.

Day	In	Out
Monday	9:00 a.m.	5:00 p.m.
Tuesday	9:00 a.m.	6:00 p.m.
Wednesday	8:30 a.m.	5:30 p.m.
Thursday	9:00 a.m.	4:30 p.m.
Friday	9:00 a.m.	4:00 p.m.

a How many hours did Suchitra work this week?
b Find her weekly wage.

16 Grace earns $525 in a week. If her hourly rate of pay is $12.50, how many hours does she work in the week?

17 Kim is a plumber who earned $477 for a day's work. He is paid $53 per hour. How many hours did Kim work on this day?

18 Na is a hairdresser who earns $24.20 per hour. She works an 8-hour day.

a How much does Na earn per day?
b How much does Na earn per week? Assume she works 5 days a week.
c How much does Na earn per fortnight?
d How much does Na earn per year? Assume 52 weeks in the year.

19 Alyssa is paid $36.90 per hour and Connor $320 per day. Alyssa works a 9-hour day. Who earns more per day and by how much?

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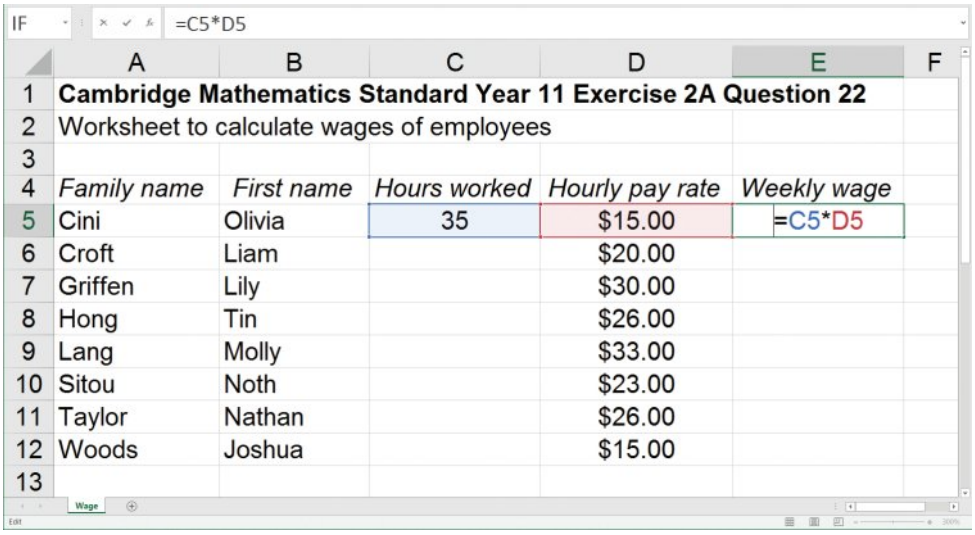
LEVEL 2

20 Feng is retiring and will receive 7.6 times the average of his salary over the past three years. In the past three years he was paid $84 640, $83 248 and $82 960. Find the amount of his payout.

21 Liam's salary is currently $76 000. He will receive salary increases as follows: 5% increase from 1 July and then a 5% increase from 1 January. What will be his new salary from 1 January?

XLS 02AQ22

22 Create the spreadsheet below.



	A	B	C	D	E	F
1	**Cambridge Mathematics Standard Year 11 Exercise 2A Question 22**					
2	Worksheet to calculate wages of employees					
3						
4	*Family name*	*First name*	*Hours worked*	*Hourly pay rate*	*Weekly wage*	
5	Cini	Olivia	35	$15.00	=C5*D5	
6	Croft	Liam		$20.00		
7	Griffen	Lily		$30.00		
8	Hong	Tin		$26.00		
9	Lang	Molly		$33.00		
10	Sitou	Noth		$23.00		
11	Taylor	Nathan		$26.00		
12	Woods	Joshua		$15.00		
13						

a Cell E5 has a formula that multiplies cells C5 to D5. Enter this formula.

b Enter the hours worked for the following employees:

Liam – 20
Lily – 26
Tin – 38
Molly – 40
Noth – 37.5
Nathan – 42
Joshua – 38.5

c Fill down the contents of E5 to E12.

d Edit the hourly pay rate of Olivia Cini to $16.50. Observe the change in E5.

LEVEL 3

23 Isabelle earns $85 324 per annum. Isabelle calculated her weekly salary by dividing her annual salary by 12 to determine her monthly payment and then divided this result by 4 to determine her weekly payment. What answer did Isabelle get? What is the correct answer, and what is wrong with Isabelle's calculation?

24 Hika earns $8 per hour and Ebony earns $9 per hour. Last week they both earned at least $150. What is the least number of hours that Hika could have worked last week?

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2B Annual leave loading and bonuses

Annual leave loading

Annual leave loading is a payment calculated as a fixed percentage of the normal pay (usually 17.5%) over a fixed number of weeks. It is usually paid at the beginning of the annual holidays to meet the increased expenses of a holiday.

Holiday loading = 17.5 × Normal weekly pay × Number of weeks leave

Example 3: Finding the annual leave loading 2B

Thomas works a 40-hour week at a rate of $18.50 per hour. He receives 17.5% of 4 weeks normal pay as holiday loading. What is Thomas's pay for the holiday?

SOLUTION:

1	Write the quantity (4 weeks pay) to be found.	4 weeks pay
2	Multiply pay rate by number of hours worked per week by number of weeks (4).	$= (18.50 \times 40 \times 4)$
3	Evaluate.	$= \$2960$
4	Write the quantity (loading) to be found.	Loading = 17.5% of $2960
5	Multiply 0.175 (17.5%) by 4 weeks pay (2960).	$= 0.175 \times 2960$
6	Evaluate.	$= \$518$
7	Write the quantity (holiday pay) to be found.	Holiday pay = $2960 + $518
8	Add the 4 weeks pay (2960) and the loading (518).	
9	Evaluate.	$= \$3478$
10	Write your answer in words.	Thomas's holiday pay is $3478.

Bonus

A bonus is an extra payment or gift earned as reward for achieving a goal. It is paid in addition to the normal income. Bonuses are an incentive for employees to work harder. For example, an employee may receive a bonus of 5% of their annual salary, or a flat payment of $1000.

Example 4: Calculating a bonus 2B

Amber's employer has decided to reward all employees with a bonus. The bonus awarded is 5% of their annual salary. What is Amber's bonus if her annual salary is $68 560?

SOLUTION:

1	Write the quantity (bonus) to be found.	Bonus = 5% of $68 560
2	Multiply bonus percentage (5%) by annual salary ($68 560).	$= 0.05 \times 68\,560$
3	Evaluate.	$= \$3428$
4	Write the answer in words.	Amber receives a bonus of $3428.

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Exercise 2B

LEVEL 1

Example 3

1 A business pays 17.5% holiday loading on 4 weeks normal pay. Calculate the amount of holiday loading for these employees.

- **a** Nicholas earns $6240 normal pay for 4 weeks.
- **b** Kumar earns $5130 normal pay for 4 weeks.
- **c** Samantha earns $5320 per fortnight.
- **d** Hamza earns $2760 per fortnight.
- **e** Bilal earns $1680 per week.

2 The local government pays its employees 17.5% holiday loading on 4 weeks normal pay. Calculate the amount of holiday loading for these employees.

- **a** Paige earns an annual salary of $105 560. (Assume 52 weeks in a year.)
- **b** Jack earns an annual salary of $58 760. (Assume 52 weeks in a year.)
- **c** Riley earns $32 per hour and works a 35-hour week.
- **d** A'ishah earns $41.50 per hour and works a 37-hour week.

3 Laura works a 37-hour week at a rate of $20.50 per hour. When she takes her 4 weeks annual leave, she is paid a loading of 17.5%. What is Laura's holiday pay when she takes her leave?

4 Ethan is paid $660 per week. He receives a holiday leave loading of 17.5% for three weeks holiday pay. What is his total holiday pay?

Example 4

5 A bonus is awarded as a percentage of a person's annual salary. The percentage awarded depends on the person's achievements. Calculate the following bonuses.

- **a** 6% of $48 360
- **b** 3% of $96 540
- **c** 2% of $103 290
- **d** 4.5% of $65 420
- **e** 2.5% of $88 580
- **f** 1.25% of $164 400

6 Grace received a bonus of 12% of her weekly wage. What was Grace's bonus if her weekly wage is $1850?

7 Patrick's boss has decided to reward all employees with a bonus. The bonus awarded is 7.75% of their annual salary. What is Patrick's bonus if his annual salary is $74 980?

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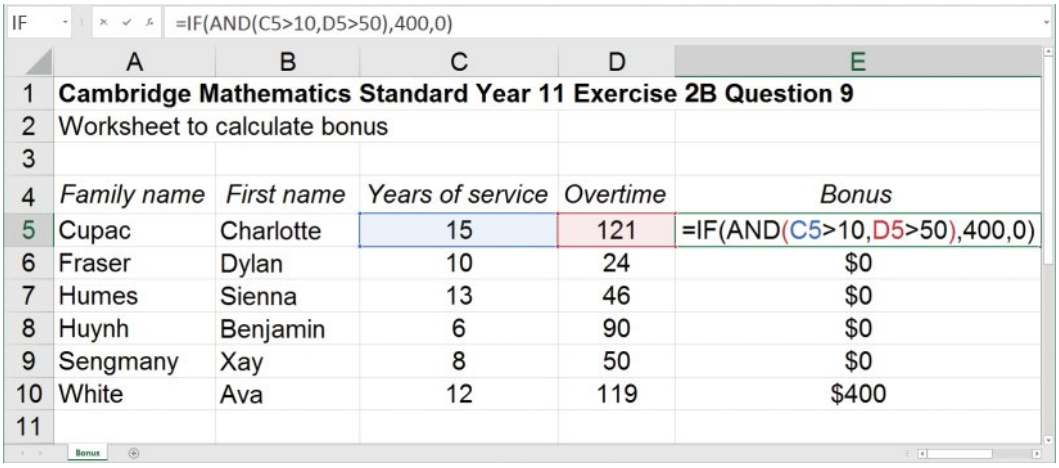
LEVEL 2

8 Chen receives 17.5% of 4 weeks normal pay as leave loading. If Chen's leave loading was $379.40, what was his normal weekly pay?

9 Create the spreadsheet below.

02BQ9



IF =IF(AND(C5>10,D5>50),400,0)

	A	B	C	D	E
1	**Cambridge Mathematics Standard Year 11 Exercise 2B Question 9**				
2	Worksheet to calculate bonus				
3					
4	*Family name*	*First name*	*Years of service*	*Overtime*	*Bonus*
5	Cupac	Charlotte	15	121	=IF(AND(C5>10,D5>50),400,0)
6	Fraser	Dylan	10	24	$0
7	Humes	Sienna	13	46	$0
8	Huynh	Benjamin	6	90	$0
9	Sengmany	Xay	8	50	$0
10	White	Ava	12	119	$400
11					

Bonus

a The formula for cell E5 is '= IF(AND(C5 > 10, D5 > 50), 400, 0)'. It is the formula that calculates a $400 bonus if the employer has more than 10 years of service and more than 50 hours of overtime. Fill down the contents of E6 to E10 using this formula.
b Edit the overtime amount for Sienna Humes to 52. Observe the changes in E7.
c Edit the years of service for Ava White to 10. Observe the changes in E10.
d Edit the overtime amount for Dylan Fraser to 60. Observe the changes in E6.
e Edit the years of service for Xay Sengmany to 20. Observe the changes in E9.
f Edit the overtime amount for Benjamin Huynh to 40. Observe the changes in E8.

LEVEL 3

10 Jim receives holiday loading of 17.5% of 4 weeks pay. His loading was $996.80.
a Find his normal weekly pay.
b Find his normal hourly pay rate if he usually works a 40 hour week.

11 Chloe's annual salary is $72 800.
a Calculate her weekly wage.
b Holiday loading is calculated at 17.5% of four weeks pay. Calculate Chloe's holiday loading.
c Chloe's employer is increasing her annual salary by 1%. What is Chloe's new annual salary?
d The increase in Chloe's annual salary is compensation for removing holiday loading. Explain why Chloe is worse off financially with the 1% increase.

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2C Overtime and special allowances

Overtime

Overtime rates apply when employees work beyond the normal working days. Payment for overtime is usually more than the normal pay rate. For example, a person whose normal pay rate is \$10 an hour would receive \$20 ($\$10 \times 2$) an hour if they where paid overtime at double time. Another common rate is time-and-a-half. It is the normal pay rate multiplied by 1.5. Here a person would receive \$15 ($\$10 \times 1.5$) an hour.

OVERTIME RATES

Time-and-a-half rate	– normal pay rate $\times 1.5$
Double time rate	– normal pay rate $\times 2$

Example 5: Calculating wages involving overtime 2C

John works for a building construction company. Find John's wage during a week in which he works 40 hours at the normal rate of \$16 an hour, 3 hours at time-and-a-half rates and 1 hour at double time rates.

SOLUTION:

1	Write the quantity to be found.	Wage $= (40 \times 16)$	normal pay
2	Normal wage is 40 multiplied by \$16.		
3	Payment for time-and-a-half is 3 multiplied by \$16 multiplied by 1.5.	$+ (3 \times 16 \times 1.5)$	time-and-a-half pay
4	Payment for double time is 1 multiplied by \$16 multiplied by 2.	$+ (1 \times 16 \times 2)$	double time pay
		$= \$744.00$	
5	Evaluate and write your answer in words.	John's wage that week is \$744.	

Special allowances

Employees receive an allowance if they work under difficult or dangerous conditions such as wet weather, extreme temperatures, confined spaces or isolated areas. Allowances are also paid when an employee has an expense related to their type of work, such as work uniform, meals, travel or tools.

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Casual work

Casual work involves a set amount paid for each hour's work. It can be paid weekly or fortnightly.

Advantages

- Working hours are flexible
- Pay rate is often higher

Disadvantages

- No sick leave or holiday pay
- May lose job when not needed
- May not include superannuation

Example 6: Calculating casual pay 2C

Milan is employed on a casual basis for a fast-food company. His rate of pay is $15 per hour plus time-and-a-half on Saturday and double time on Sunday.

Last week Milan worked from 10:30 a.m. until 2:30 p.m. on Thursday, from 9:30 a.m. until 2 p.m. on Saturday, and from 12 noon until 4 p.m. on Sunday. How much did Milan earn last week?

SOLUTION:

1	Write the quantity to be found.	Wage $= (4 \times 15)$	normal pay
2	Normal wage is 4 hours (Thursday 10:30 a.m. until 2:30 p.m.) multiplied by $15.		
3	Payment for time-and-a-half is 4.5 hours (Saturday 9:30 a.m. until 2 p.m.) multiplied by $15 multiplied by 1.5.	$+ (4.5 \times 15 \times 1.5)$	time-and-a-half pay
4	Payment for double time is 4 hours (Sunday 12 noon until 4 p.m.) multiplied by $15 multiplied by 2.	$+ (4 \times 15 \times 2)$	double time pay
5	Evaluate and write using correct units.	= $281.25	
6	Write your answer in words.	Milan's wage is $281.25.	

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Exercise 2C

LEVEL 1

1 Calculate the payment for working 4 hours overtime at time-and-a-half given the following normal pay rates.

a $18.00 **b** $39.50 **c** $63.20 **d** $43.83

2 Calculate the payment for working 3 hours overtime at double time given the following normal pay rates.

a $37.99 **b** $19.05 **c** $48.78 **d** $61.79

Example 5

3 Ibrahim earns $32.50 an hour as a driver. He works 38 hours a week at normal time and 5 hours a week at double time. Find his weekly wage. Answer correct to the nearest cent.

4 Mei is a casual employee who worked 8 hours at normal time and 2 hours at time-and-a-half. Her normal rate of pay is $12.30 per hour. What is her pay for the hours she worked?

5 Oliver earns $23.80 an hour. He earns normal time during week days and time-and-a-half on weekends. Last week he worked 34 hours during the week and 6 hours during the weekend. Find his weekly wage.

6 George works in a take-away food store. He gets paid $18.60 per hour for a standard 35-hour week. Additional hours are paid at double time. His time card is shown below.

DAY	IN	OUT
Monday	8:30 a.m.	4:30 p.m.
Tuesday	9:00 a.m.	6:00 p.m.
Wednesday	8:45 a.m.	5:45 p.m.
Thursday	9:00 a.m.	6:30 p.m.
Friday	10:00 a.m.	8:00 p.m.

a How many hours did George work this week?

b Find his weekly wage.

7 Dave works for 5 hours at double time. He earns $98.00. Find his normal hourly rate.

8 Ella works 3 hours at time-and-a-half and earns $72.00. Find her normal hourly rate.

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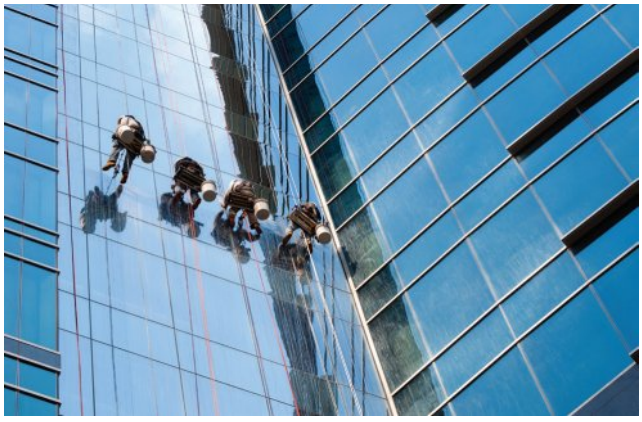
9 Zahid is paid a set wage of $774.72 for a 36-hour week, plus time-and-a-half for overtime. In one particular week he worked 43 hours. What were Zahid's earnings that week?

10 Samantha is paid a set wage of $962.50 for a 35-hour week, plus double time for overtime. In one particular week she worked 40 hours. What are Samantha's earnings?

11 A window washer is paid $22.50 per hour and a height allowance of $55 per day. In one week he works 9 hours each week day.

a Calculate the amount earned each week day.

b Calculate his total weekly earnings for five days of work.



12 Anna works in a factory and is paid $18.54 per hour. When she operates the oven, she is paid a temperature allowance of $4.22 per hour in addition to her normal rate. Find her weekly pay when she works a total of 42 hours including 10 hours working the oven.

13 Scott is a painter who is paid a normal rate of $36.80 per hour plus a height allowance of $21 per day. In one week Scott works 9 hours per day for 5 days on a tall building. Calculate his total earnings.

14 Kathy is a scientist who is working in a remote part of Australia. She earns a salary of $86840 plus a weekly allowance of $124.80 for working under extreme and isolated conditions. Calculate Kathy's fortnightly pay.

15 Chris is a soldier and is paid $27 per hour plus an additional allowance of $12.50 per hour for disarming explosives. What is his total weekly pay if he works from 6 a.m. to 2 p.m. for 7 days a week on explosives?

16 A miner earns a wage of $46.20 per hour plus an allowance of $28.20 per hour for working in cramped spaces. The miner worked a 10-hour day for 5 days in a small shaft. What was his weekly pay?

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LEVEL 2

Example 6

17 Vien is employed on a casual basis. His rate of pay is shown below. Last week Vien worked from 11:30 a.m. until 3:30 p.m. on Thursday, from 8:30 a.m. till 2:00 p.m. on Saturday, and from 12 noon till 6:00 p.m. on Sunday. How much did Vien earn last week?

Rate of pay	
Weekdays	$18.60 per hour
Saturday	Time-and-a-half
Sunday	Double time

18 A mechanic's industrial award allows for normal rates for the first 7 hours on any day. It provides for overtime payment at the rate of time-and-a-half for the first 2 hours and double time thereafter. Find a mechanic's wage for a 12-hour day if the normal pay rate is $42.50 an hour.

19 Kaimi's timesheet is shown opposite. She gets paid $12.80 per hour during the week, time-and-a-half for Saturdays and double time for Sundays. Kaimi is not paid for meal breaks.

Day	In	Out	Meal break
Monday	8:30 a.m.	5:30 p.m.	1 hour
Tuesday	8:30 a.m.	3:00 p.m.	1 hour
Wednesday	8:30 a.m.	5:30 p.m.	1 hour
Thursday	8:30 a.m.	9:00 p.m.	3 hour
Friday	3:00 p.m.	7:00 p.m.	No break
Saturday	8:00 a.m.	4:00 p.m.	No break
Sunday	10:00 a.m.	3:00 p.m.	30 minutes

a How much did Kaimi earn at the normal rate of pay during this week?

b How much did Kaimi earn from working at penalty rates during this week?

c What percentage of her pay did Kaimi earn by working at penalty rates?

LEVEL 3

20 Connor works a 35-hour week and is paid $18.25 per hour. Any overtime is paid at time-and-a-half. Connor wants to work enough overtime to earn at least $800 each week. What is the minimum number of hours of overtime that Connor will need to work?

21 Max works in a shop and earns $21.60 per hour at the normal rate. Each week he works 15 hours at the normal rate and 4 hours at time-and-a-half.

a Calculate Max's weekly wage.

b Max aims to increase his weekly wage to $540 by working extra hours at the normal rate. How many extra hours must Max work?

c Max's rate of pay increased by 5%. What is his new hourly rate for normal hours?

d What will be Max's new weekly wage, assuming he maintains the extra working hours?

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2D Commission

Commission is usually a percentage of the value of the goods sold. People such as real estate agents and salespersons are paid a commission.

Advantages
- Higher sales increase the income
- May receive a small payment (retainer) plus the commission

Disadvantages
- Income may vary each week
- Competition for customers is usually high

COMMISSION

Commission = Percentage of the value of the goods sold

Example 7: Finding the commission 2D

Zoë sold a house for $650 000. Find the commission from the sale if her rate of commission was 1.25%.

SOLUTION:

1 Write the quantity (commission) to be found.
2 Multiply 1.25% by $650 000.
3 Evaluate and write using correct units.
4 Write your answer in words.

Commission = 1.25% of $650 000
= 0.0125 × 650 000
= $8125
Commission earned is $8125.

Example 8: Finding the commission 2D

An electrical goods salesman is paid $570.50 a week plus 4% commission on all sales over $5000 a week. Find his earnings in a week in which his sales amounted to $6800.

SOLUTION:

1 Commission on sales of over $5000 is $1800.
2 Write the quantity (earnings) to be found.
3 Add weekly payment and commission of 4% on $1800
4 Evaluate and write using correct units.
5 Write the answer in words.

Sales = 6800 – 5000
= 1800

Earnings = 570.50 + (4% of $1800)
= 570.50 + (0.04 × 1800)
= $642.50
Earnings were $642.50.

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Exercise 2D

LEVEL 1

Example 7

1 Tama earns a commission of 4% on the sale price. What is the commission on the following sales?
- **a** $8820
- **b** $16 740
- **c** $34 220

2 Michelle Tran is a real estate agent. She earns 2% on all sales. Calculate Michelle's commission on these sales.
- **a** $456 000
- **b** $420 000
- **c** $285 500
- **d** $590 700

3 Olivia sold a car valued at $54 000. Calculate Olivia's commission from the sale if her rate of commission is 3%.

Example 8

4 Sophie earns a weekly retainer of $355 plus a commission of 10% on sales. What are Sophie's total earnings for each week if she made the following sales?
- **a** $760
- **b** $2870
- **c** $12 850

5 Chris earns $240 per week plus 25% commission on sales. Calculate Chris's weekly earnings if he made sales of $2880.

6 Ora is a salesperson for a cosmetics company. She is paid $500 per week and a commission of 3% on sales in excess of $800.
- **a** What does Ora earn in a week in which she makes sales of $1200?
- **b** What does Ora earn in a week in which she makes sales of $600?

7 A real estate agent charges a commission of 5% for the first $20 000 of the sale price and 2.5% for the balance of the sale price. Copy and complete the following table.

Sale price	5% commission on $20 000	2.5% commission on balance
a $150 000		
b $200 000		
c $250 000		
d $300 000		

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LEVEL 2

8 Jade is a real estate agent and is paid an annual salary of $18 000 plus a commission of 2.5% on all sales. She is also paid a car allowance of $50 per week. What was Jade's total yearly income if she sold $1 200 000 worth of property?

9 The commission that a real estate agent charges for selling a property is based on the selling price, as shown in the table.

What is the commission charged on properties with the following selling prices?

a $100 000
b $150 000
c $200 000

Selling price	Commission
First $20 000	5%
Next $120 000	3%
Thereafter	1%

10 Harry is a salesperson. He earns a basic wage of $300 per week and receives commission on all sales. Last week he sold $20 000 worth of goods and earned $700. What was Harry's rate of commission?

LEVEL 3

11 Caitlin and her assistant, Holly, sell perfume. Caitlin earns 20% commission on her own sales, as well as 5% commission on Holly's sales. What was Caitlin's commission last month when she made sales of $1800 and Holly made sales of $2000?

12 A real estate agency charges a commission for selling a property. The commission is based on the selling price shown in the table. Bailey is paid $180 per week by the real estate agent plus 5% of the commission received by the real estate agency. This week, Bailey sold one property for $290 000 and one for $600 000. He sold no properties in the previous week.

Commission rates	
Up to $300 000	4%
$300 000 and over	5%

a What is the commission paid to the real estate agency for the $290 000 sale?
b What is the commission paid to the real estate agency for both properties?
c Calculate Bailey's pay for this week.
d What is Bailey's average weekly income for the two-week period?

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2E Piecework, royalties and income from government

Piecework

Piecework is a fixed payment for work completed. People who are employed to complete a particular task, such as an electrician installing lights, are earning piecework.

Advantages
- Incentive to work hard. Income increases with more work completed
- Often flexible hours and work place

Disadvantages
- No permanent employment
- No sick leave or holiday pay
- May not include superannuation

PIECEWORK

Piecework = Number of units of work × Amount paid per unit

Example 9: Calculating a piecework payment 2E

Noah is a tiler and charges $47 per square metre to lay tiles. How much will he earn for laying tiles in a room whose area is 14 square metres?

SOLUTION:

1	Write the quantity (earnings) to be found.	Earnings = 14 × $47
2	Multiply number of square metres (14) by the charge ($47).	
3	Evaluate and write using correct units.	= $658
4	Write the answer in words.	Noah earns $658.

Royalties

A royalty is a payment for the use of intellectual property such as a book or song. It is calculated as a percentage of the revenue or profit received from its use. People such as creative artists and authors receive a royalty.

Advantages
- Incentive to work hard. Income increases with a better product
- Flexible hours and work place

Disadvantages
- Income varies according to sales
- No superannuation, sick leave or holiday pay

ROYALTY

Royalty = Percentage of the goods sold or profit received

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Example 10: Calculating a royalty 2E

Hiroshi is an author and is paid a royalty of 12% of books sold. Find his royalties if there were 2480 books sold at $67.50 each.

SOLUTION:

1	Write the quantity (royalty) to be found.	Royalty
2	Multiply 12% by the total sales or $2480 \times \$67.50$	$= 12\%$ of $(2480 \times \$67.50)$ $= 0.12 \times 2480 \times 67.50$
3	Evaluate and write using correct units.	$= \$20\,088$
4	Write the answer in words.	Hiroshi earns $20 088 in royalties.

Income from the government

Some people receive a pension, allowance or benefit from the government. For example, the age pension is payable for a person who has reached 65 years of age (male). The requirements for receiving these incomes may change according to the priorities of the current government.

Example 11: Calculating an income from the government 2E

Youth Allowance helps people studying, undertaking training or in an apprenticeship.

Status	Allowance per fortnight
Under 18, at home	$194.50
Under 18, away from home	$355.40
18 and over, away from home	$355.40
18 and over, at home	$233.90

How much Youth Allowance does Ryan receive in a year if he is over 18 and living at home while studying?

SOLUTION:

1	Write the quantity (allowance) to be found.	Allowance $= \$233.90$ per fortnight
2	Multiply allowance per fortnight ($233.90) by 26.	$= \$233.90 \times 26$
3	Evaluate and write using correct units.	$= \$6081.40$
4	Write the answer in words.	Ryan receives $6081.40.

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Exercise 2E

LEVEL 1

Example 9

1 A dry cleaner charges $9 to clean a dress. How much do they earn by dry cleaning:
a 250 dresses? **b** 430 dresses? **c** 320 dresses?

2 Sabika is an artist who makes $180 for each large portrait and $100 for each small portrait. How much will she earn if she sells 13 large and 28 small portraits?

3 Angus works part-time by addressing envelopes at home and is paid $23 per 100 envelopes completed, plus $40 to deliver them to the office. What is his pay for delivering 2000 addressed envelopes?

4 Emilio earns a royalty of 24% on net sales from writing a fiction book. There were $18 640 net sales in the last financial year. What is Emilio's royalty payment?

Example 10

5 Calculate the royalties on the following sales.
a 3590 books sold at $45.60 with a 8% royalty payment
b 18 432 DVDs sold at $20 with a 10% royalty payment
c 4805 computer games sold at $65.40 with a 5% royalty payment

Example 11

6 Austudy provides financial help for people aged 25 or older who are studying full-time.

Status	Fortnightly payment
Single, no children	$355.40
Single, with children	$465.60
Partnered, with children	$390.20
Partnered, no children	$355.40

a How much does Madison receive in a year if she is single with a child and studying full-time? Madison is 29 years old.
b How much does Oscar receive in a year if he is partnered with no children and studying full-time? Oscar is 35 years old.

7 A childcare benefit is available to support parents in the workforce. The rate per fortnight is shown below.

No. of children	Fortnightly pay
1	$337.00
2	$704.34
3	$1099.26

Calculate the yearly payment for:
a one child **b** two children **c** three children

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LEVEL 2

8 Tahlia receives $19.40 for delivering 200 brochures. She receives an additional $30 per day when delivering in wet weather. How much does she receive for delivering:

- **a** 600 brochures on a clear day?
- **b** 1000 brochures on a clear day?
- **c** 800 brochures on a wet day?
- **d** 1400 brochures on a wet day?

9 Tiki works in a factory that makes key rings. Each key ring completed earns him $0.34. Tiki also receives an additional $25 if he works on the weekend. How much does he earn for making:

- **a** 420 key rings on Friday?
- **b** 460 key rings on Wednesday?
- **c** 380 key rings on Saturday?
- **d** 230 key rings on Sunday?

10 A doctor charges each patient $33.50 for a consultation. She works for 6 hours a day and usually sees 5 patients per hour.

- **a** How much money does the doctor receive each day?
- **b** The doctor also has costs of $410 per day. What is the profit for the day?

11 Austudy is reduced by 50 cents for every dollar between $62 and $250 of fortnightly income. Tyler is 28 years of age, partnered and has one child. He is studying full-time but earning $126 per fortnight in a part-time job. What will be Tyler's fortnightly Austudy payment? Use the Austudy table on the previous page.

LEVEL 3

12 Anthony writes crime novels. He has just received the half-yearly statement of sales of his latest novel. He has been informed that 20 000 copies were printed and there are 8760 left in stock. Anthony receives 15% of the retail price as royalties.

- **a** How many copies of his latest novel were sold?
- **b** What is Anthony's royalty if the retail price of his latest novel is $24.95?
- **c** What is Anthony's royalty if the retail price of $24.95 was discounted by 10%?

13 The maximum Youth Allowance is reduced by $1 for every $4 that the youth's parents' income is over $31 400. By how much is Charlotte's Youth Allowance reduced if her parents earn a combined income of $34 728?

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2F Allowable tax deductions

Allowable tax deductions are amounts allowed by the Australian Taxation Office (ATO) to be deducted from gross income to calculate a lower taxable income. Details of allowable tax deductions are given on the ATO website (www.ato.gov.au).

Allowable tax deductions include:

- work-related expenses – costs incurred while performing your job
- self-education expenses – costs of education related to your work
- travel expenses – costs of travel directly connected with your work
- car expenses – costs of using your car related to your work
- clothing expenses – costs of work clothing and laundry
- tools – cost of work tools
- gifts and donations – gifts made to an eligible organisation.

Example 12: Calculating allowable tax deductions 2F

Riley works as an information technology consultant. He is entitled to the following tax deductions:

- equipment costs of $1260
- car expenses of $1060
- professional learning of $985
- union fees of $650
- charity donations of $250
- tax agent fee of $212

What is Riley's total allowable tax deduction?

SOLUTION:

1. Write the quantity (tax deduction) to be calculated.
2. Add all the allowable tax deductions.
3. Evaluate and write using correct units.
4. Write the answer in words.

$$\text{Tax deduction} = 1260 + 1060 + 985 + 650 + 250 + 212$$
$$= \$4417$$

Riley has an allowable tax deduction of $4417.

Example 13: Calculating allowable tax deductions 2F

Ava has used her own car for a total of 7900 km on work-related travel this financial year. Calculate her tax deduction if she is entitled to claim 70 cents per kilometre.

SOLUTION:

1. Write the quantity (tax deduction) to be calculated.
2. Multiply the kilometres travelled by the rate per kilometre.
3. Evaluate and write using correct units.
4. Write the answer in words.

$$\text{Tax deduction} = 7900 \times 0.70$$
$$= \$5530$$

Ava has an allowable tax deduction of $5530.

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Exercise 2F

LEVEL 1

1 Stephanie works as a waitress and is entitled to an allowance for the cost of her work clothing and laundry. Her clothing expenses are listed below. Calculate the total cost for each item of clothing and her allowable tax deduction for clothing.

Work clothing	Quantity	Unit cost	Total cost
Blue shirt	3	$55.00	a
Black trouser	3	$110.00	b
Belt	1	$45.00	c
Tie	2	$34.00	d
Dry-cleaning	4	$32.00	e

Example 12

2 Zara is a childcare worker who is entitled to the following tax deductions:

- $420 for union fees
- charity donations of $160
- stationery costs of $46
- tax agent fee of $125
- self-education fee of $840

What is Zara's total tax deduction?

3 Chris is entitled to the following tax deductions: training courses of $1460, motor vehicle expenses of $1420, stationery costs of $760, union fees of $480, charity donations of $310 and accountant fee of $184. What is Chris's total allowable tax deduction?

Example 13

4 Car expense deductions can be claimed using the cents per kilometre method. Under this method, individuals can claim a tax deduction based on the number of business kilometres travelled in the financial year. This is shown in the table below.

Maximum kilometres allowed per car	Tax deduction rate
5000	66 cents per kilometre

a Calculate the allowable tax deduction if the following business kilometres were travelled.

i 3560 km **ii** 1280 km **iii** 4580 km
iv 2340 km **v** 3105 km **vi** 6000 km

b The car expenses tax deduction rate has decreased to 60 cents per kilometre. Calculate the allowable tax deduction if the following business kilometres were travelled.

i 4350 km **ii** 1289 km **iii** 5500 km
iv 4000 km **v** 80 km **vi** 694 km

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LEVEL 2

5 Harrison buys a new van costing $42 560 for his business. He is entitled to claim a tax deduction of 12% of the cost of the vehicle if the motor vehicle travels more than 5000 business kilometres in a year. What is Harrison's allowable tax deduction for the van if he travelled 16 230 kilometres for business?

6 Joel is a wheat farmer. He has capital equipment on the farm worth $240 000. The ATO allows a tax deduction for the depreciation of capital equipment based on a percentage of the current value. What is the tax deduction using the following rates of depreciation?

a 10% p.a. **b** 20% p.a.
c 30% p.a. **d** 40% p.a.

7 Syed has an investment property that contains furnishings valued at $12 600. The furnishings are an allowable tax deduction with a rate of depreciation of 15% p.a. How much can be claimed for depreciation over the year?

8 Xiang is a teacher who bought a $2350 laptop for school use. The laptop is an allowable deduction with a rate of depreciation of 33% p.a. of the current value. How much can be claimed for depreciation in the:

a first year? **b** second year? **c** third year?

LEVEL 3

9 Dylan is the owner of a newsagency in a shopping centre. He pays rent of $860 per week, has an electricity bill of $280 per quarter and a telephone bill of $110 per month. These expenses are work related so he is entitled to a tax deduction. What is Dylan's total allowable tax deduction?

10 Chelsea has a small office in her home to run a business. The office in her home is 8% of the area of the house. The tax office allows 8% of the household bills as a tax deduction if it is a work-related expense. Calculate the allowable tax deduction for the financial year on the following household bills.

a Electricity bill of $360 per quarter
b Telephone bill of $70 per month
c House insurance of $684 per year
d Rent of $440 per fortnight

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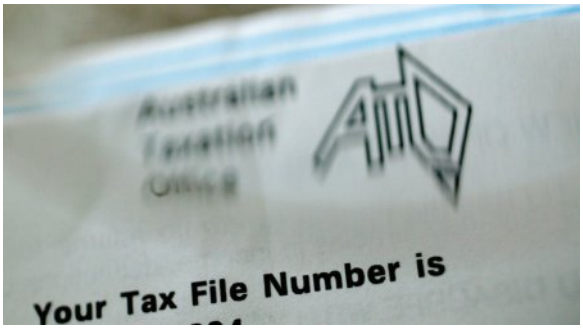
2G Taxable income

Each year people who earn an income are required to complete a tax return on a paper form or digitally, online. A tax return states a person's income, the amount of tax paid and any allowable tax deductions. Most taxpayers have PAYG (pay as you go) tax deducted from their wage or salary throughout the year. The PAYG tax can be greater or less than the required amount of tax to be paid.



Tax is calculated on the taxable income. The taxable income is the gross income minus any allowable deductions. The gross income is the total amount of money earned from all sources. It includes interest, profits from shares or any payment received throughout the year.

TAXABLE INCOME

Taxable income = Gross income – Allowable tax deductions

Example 14: Calculating taxable income 2G

Anthony is a businessman who earns a gross salary of $93 250 per year. His accountant completed his tax return and calculated $2890 in allowable tax deductions.

What is Anthony's taxable income?

SOLUTION:

1 Write the quantity (taxable income) to be calculated.
2 Subtract the deductions from the gross income.
3 Evaluate and write using the correct units.

Taxable income = 93 250 – 2890
= $90 360

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Example 15: Calculating taxable income 2G

Emily is a journalist with a gross annual salary of $87 620. She also made $5680 from her share portfolio and received $7320 in royalties. If Emily has tax deductions totaling $6472, calculate her taxable income.

SOLUTION:

1 Calculate the gross income by adding all income.
2 Write the quantity (taxable income) to be calculated.
3 Subtract the deductions from the gross income.
4 Evaluate.

$$\text{Gross income} = 87\,620 + 5680 + 7320$$
$$= \$100\,620$$
$$\text{Taxable income} = 100\,620 - 6472$$
$$= \$94\,148$$

Example 16: Calculating taxable income 2G

Nicole is a scientist who earns a gross weekly pay of $1624. She has allowable tax deductions of $8 per week for dry-cleaning, $60 for work-related travel per year, $460 per year for union fees and she made donations to charities of $620 throughout the year.

a What is Nicole's gross yearly salary?
b What is Nicole's total allowable tax deduction?
c Calculate Nicole's taxable income.

SOLUTION:

1 Write the quantity (salary) to be calculated.
2 Multiply the weekly pay by 52.
3 Evaluate.

a $\text{Salary} = 1624 \times 52$
$= \$84\,448$

4 Write the quantity (tax deduction) to be calculated.
5 Add all the allowable deductions.
6 Evaluate.

b $\text{Tax deduction} = (8 \times 52) + 60 + 460 + 620$
$= \$1556$

7 Write the quantity (taxable income) to be calculated.
8 Subtract the deductions from the gross income.
9 Evaluate.

c $\text{Taxable income} = 84\,448 - 1556$
$= \$82\,892$

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Exercise 2G

LEVEL 1

Example 14

1 Benjamin has a gross income of $84 000. What is Benjamin's taxable income given the following allowable tax deductions?

a $5120 **b** $9571 **c** $4720
d $24 104 **e** $8205 **f** $17 594
g $12 520 **h** $23 890 **i** $34 560

Example 15

2 Chris earns a gross salary of $67 840 per year. His tax deductions total $3462. Calculate Chris's taxable income.

3 Jessica earned a gross income of $75 480 in the last financial year.

Allowable tax deductions	Amount
Work-related expenses	$1260
Self-education expenses	$680
Travel expenses	$940
Clothing expenses	$320

a The table above is a summary of her allowable tax deductions. What is her total allowable deduction?
b Calculate Jessica's taxable income in the last financial year.

Example 16

4 Eliza earned $88 784 from her employer in the last financial year. She also earned bank interest of $380. Eliza spent $240 on books, $520 on stationery and $380 on a printer, all of which are needed for her work.

a What is Eliza's gross income?
b What are Eliza's total allowable tax deductions?
c Calculate Eliza's taxable income.

5 Daniel is a police officer who earned a gross income of $63 620. He claimed a tax deduction for a utility belt ($160), a pair of safety glasses ($390), bulletproof vest ($1240) and handcuffs ($420).

a What is Daniel's total allowable tax deduction?
b Calculate Daniel's taxable income.

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LEVEL 2

6 Nicholas earns a gross weekly pay of $1120. He has tax deductions of $1460.

a What is Nicholas's gross yearly salary?

b What is Nicholas's taxable income?

7 Ulani is a real estate salesperson who earns a commission of 2% on all sales. During the year Ulani sold real estate to the value of $3 232 100.

a What is Ulani's gross annual income?

b Ulani has calculated her tax deductions to be $4320. What is her taxable income?

8 Isabelle works for a travel agency and earns a gross fortnightly pay of $2780. She pays PAYG tax of $602 per fortnight and has tax deductions of $7 per week for dry-cleaning, $80 for work-related travel per year and $380 per year for union fees.

a What is Isabelle's gross yearly salary?

b How much tax is deducted each week?

c What is Isabelle's total allowable tax deduction?

d Calculate Isabelle's taxable income.

LEVEL 3

9 Ahmed is a tradesman who receives a yearly gross salary of $92 200. He also works part-time at TAFE for a wage of $135 per week. Ahmed received $360 in share dividends.

a What is Ahmed's gross annual income?

b Ahmed is entitled to tax deduction for travelling between his two places of employment. Ahmed has calculated that he travelled 340 km and the allowable deduction is 66 c/km. What is Ahmed's travel expense?

c In addition to the above travel expenses Ahmed is entitled to the following tax deductions: $530 for union fees, tax agent fee of $180, charity donations of $280 and equipment costs of $750. Calculate Ahmed's total allowable deduction.

d What is Ahmed's taxable income?

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2H Gross pay, net pay and deductions

Gross pay is the total of an employee's pay, including all allowances, overtime pay, commissions and bonuses. It is the amount of money before any deductions are made. The amount remaining after deductions have been subtracted is called the net pay or 'take-home pay'.

Deductions from pay are a regular amount of money subtracted from a person's wage or salary, not to be confused with tax deductions. People have many different deductions subtracted from their gross pay such as:

- income tax – a charge that funds the government's operations
- superannuation – an investment for retirement. An employer must contribute 9% of the employee's wages into a superannuation fund
- health insurance – private insurance to cover medical and dental costs
- union fee – payment for union membership.

GROSS PAY, NET PAY AND DEDUCTIONS

Net pay = Gross pay – Deductions

Example 17: Calculating net pay 2H

Laura is a nurse who receives a gross weekly wage of $2345. She has the following deductions taken from her pay:

- Income tax – $861
- Health fund payments – $48.25
- Superannuation – $67.95

What is Laura's net pay?

SOLUTION:

1	Write the quantity (net pay) to be found.	Net pay
2	Write the formula for net pay.	= Gross pay – deductions
3	Substitute the values for gross pay and deductions.	= 2345 – (861 + 48.25 + 67.95)
4	Evaluate and write using correct units.	= $1367.80
5	Write the answer in words.	Laura's net pay is $1367.80.

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Example 18: Reading information from a pay slip 2H

Oscar received the following pay slip. What amount is added or deducted this pay for:

a gross pay?
b net pay?
c superannuation?
d PAYG tax?

EAGLE ENTERPRISES
Employee: Oscar Vargas No: 006914

	Hours	Rate	Amount	This Pay	Year to Date
Ordinary time	26.00	$25.00	$650.00		
Annual holiday	0.0	$25.99	$ 0.0		
Total gross earnings				$ 650.00	$1300.00
PAYG tax				$ 100.00	$ 200.00
Social club				$ 2.00	$ 4.00
HECS repayments				$ 13.00	$ 26.00
Superannuation				$ 35.00	$ 70.00
Less post-tax deductions				$ 50.00	$ 100.00
Net pay				$ 450.00	$ 900.00
Direct credit to account: 00000000					
Total payments				$ 450.00	$ 900.00

SOLUTION:

1 Read the value for gross earnings.
2 Read the value for net pay.
3 Read the value for superannuation.
4 Read the value for PAYG tax.

a Gross pay is $650.00.
b Net pay is $450.00.
c Superannuation is $35.00.
d PAYG tax is $100.00.

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Exercise 2H

LEVEL 1

Example 17

1 Calculate the weekly net pay for these people.

a Isabella receives a gross pay of $1386 and has total deductions of $875.

b Kim-Ly receives a gross pay of $985 and has total deductions of $265.

c Christopher receives a gross pay of $715 and has total deductions of $222.

2 Calculate the weekly net earnings for these people.

a Daniel receives a gross weekly wage of $1056 and has deductions of $294.75 for income tax, $28.80 for superannuation and $325.05 for loan repayments.

b Hannah receives a gross weekly wage of $3042 and has deductions of $1068 for income tax, health fund payments for $50.85, superannuation for $53.55 and savings for $450.

c Kapil receives a gross weekly wage of $2274. He has deductions of $768 for income tax, $28.95 for health insurance, $49.02 for superannuation, $15.30 for life insurance and $450 for loan repayments.

Example 18

3 Jack's annual gross pay is $48 750. The deductions are $9150 for income tax, $1462 for health insurance and $5280 for superannuation.

a What are Jack's total deductions?

b What is Jack's annual net pay?

4 Calculate the weekly gross pay for these people.

a Aaron receives a net weekly pay of $1245 and has deductions of $374.15 for income tax, $45.60 for superannuation and $25.20 for union membership.

b Nani receives a net weekly pay of $2645 and has deductions of $1068 for income tax, $53.95 for health fund payments and $83.75 for superannuation.

c Ivan receives a net weekly pay of $2511. He has deductions of $913 for income tax, $31.95 for health insurance, $59.46 for superannuation, $18.20 for life insurance and $470 for loan repayments.

5 Harry's net pay is $57 908. His deductions are $12 580 for income tax, $2087 for health insurance and $6910 for superannuation. What is Harry's gross pay?

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6 Joshua is on a working holiday picking fruit. He is paid per piece of fruit he picks. He picked fruit on a farm as follows:

Monday – 170
Tuesday – 130
Wednesday – 145
Thursday – 210
Friday –190

a What is his gross salary at $0.55 per piece of fruit?
b What is his net salary if he has total deductions of $121?

7 Charlotte owns an investment property that is rented out for $320 per week. She pays the real estate agent a fee of 3% for managing the property.

a How much does she pay the real estate agent each week?
b How much does Charlotte receive each week from the investment property?
c What is the net income received by Charlotte from this property over the year?

8 Nicholas receives a yearly gross salary of $74 568. He pays 18% of his weekly gross salary in income tax. He contributes 9% of his weekly gross salary to his superannuation fund and has $155 in miscellaneous deductions each week.

a What is his gross weekly pay?
b How much income tax is deducted each week?
c How much superannuation is he contributing each week?
d What is the total amount of deductions made each week?
e What is his net weekly pay each week?

9 Lakshmi receives a fortnightly pay of $2240. She pays 15% of her weekly gross salary in income tax. She contributes 9% of her weekly gross salary to her superannuation fund and has $95 in miscellaneous deductions each week.

a What is her gross weekly pay?
b How much income tax is deducted each week?
c How much superannuation is she contributing each week?
d What is the total amount of deductions made each week?
e What is her net weekly pay each week?

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LEVEL 2

10 Charlie is a building worker who receives $48.50 per hour for a 38-hour week. In addition he receives an allowance of $3.50 per hour for work on a multistorey development. Charlie is currently working on six-storey apartment block. Each week he has deducted from his pay a superannuation contribution of 9% of his gross pay and union fees of $28.45. Because he only started working late in the financial year, he doesn't yet have to pay tax.

- **a** What is his gross weekly pay this week?
- **b** How much superannuation is he contributing each week?
- **c** What is his net weekly pay this week?

11 Amina receives a gross fortnightly salary of $2703 and has deductions of $891.75 for income tax, $54.30 for health fund payments, $753 for car loan payments and $14.55 for union subscription.

- **a** What is Amina's net income each fortnight?
- **b** What percentage of her gross income is deducted for income tax? (Answer correct to one decimal place.)

LEVEL 3

12 Liam received a gross fortnightly salary of $3795. His pay deductions were $937.20 for income tax, $215.25 for superannuation, $21.45 for union fees and $201 for a home loan repayment.

- **a** What is his net income each fortnight?
- **b** What was his weekly net pay?
- **c** What percentage of his gross income was deducted for income tax? (Answer correct to one decimal place.)
- **d** If Liam's loan repayment increased by 10%, what would be his new fortnightly net pay?

13 Jane normally works 37 hours a week at $54 per hour. In one particular week she worked 42 hours and received overtime at the rate of time-and-a-half. Her deductions for the week were income tax $602.20, medical fund $49.60, superannuation $74.40 and motor vehicle repayment $417.40.

- **a** What was Jane's gross weekly wage?
- **b** What was her net income for the week?
- **c** What percentage of her gross income is spent on the motor vehicle repayment? Answer correct to the nearest per cent.

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2I Calculating tax

The amount of taxation varies according to the amount of money you earn. A tax return must be lodged with the ATO each year. It is statement of the income earned and the tax paid during the financial year from 1 July to 30 June. The ATO publishes a tax pack to assist people in completing their tax return. The Tax Pack provides information about the current income tax rates.

Personal income tax rates

The personal income tax rates are regularly changed to take into account government policy and inflation. The tax rates for 2017–18 are listed in the table below. (Note: these may not be the current tax rates.)

Taxable income	Tax payable
0 – $18 200	Nil
$18 201 – $37 000	Nil + 19 cents for each $1 over $18 200
$37 001 – $87 000	$3572 + 32.5 cents for each $1 over $37 000
$87 001 – $180 000	$19 822 + 37 cents for each $1 over $87 000
$180 001 and over	$54 232 + 45 cents for each $1 over $180 000

The tax payable is dependent on the taxable income. If the taxable income is $18 200 or less then there is no tax payable. The tax rates then increase progressively. It starts out at 19 cents for every $1, for amounts between $18 201 and $37 000.The highest tax rate is 45 cents for every $1 over $180 000. Most people have PAYG tax deducted from their wage or salary throughout the year. This PAYG tax can be greater or less than the required amount of tax to be paid. If a person pays more throughout the year than they are required to pay, they will receive a tax refund. If they pay less than the required amount of tax throughout the year, they will have tax owing.

TAX REFUND	TAX OWING
Tax refund = Tax paid – Tax payable	Tax owing = Tax payable – Tax paid

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Example 19: Calculating the tax payable 2I

Manjula has a taxable income of $25 000. How much tax will she have to pay? If Manjula has paid $1650 in PAYG tax, what will be her refund?

SOLUTION:

1	Look at the income tax rates table on page 79.	Tax payable $= \text{Nil} + (25\,000 - 18\,200) \times 0.19$
2	Taxable income of $25 000 is between $18 201 and $37 000 (row 2).	$= \$1292$
3	Read the information in the tax payable column for this range (row 2).	
4	Write an expression for tax payable. The word 'over' implies 'more than'.	
5	Evaluate using correct units.	
6	Write the answer in words.	Manjula needs to pay $1292 in tax.
7	Find the tax refund by subtracting the tax payable from the tax paid.	Tax refund $= \$1650 - \$1292$
		$= \$358$
8	Evaluate.	
9	Write the answer in words.	Manjula's refund will be $358.

Example 20: Calculating a tax refund 2I

Joel has a taxable income of $200 000 and has paid $65 300 in tax instalments.

a Calculate the amount of tax payable by Joel. Use the tax rates on page 79.

b Will Joel receive a refund or will he have to pay more tax? Justify your answer.

c What percentage of his income is paid as tax? Answer correct to one decimal place.

SOLUTION:

1	Look at the income tax rates.	**a** Tax $= 54\,232 + (200\,000 - 180\,000) \times 0.45$
2	Taxable income of $200 000 is between $180 000 and over.	$= \$63\,232$
3	Write an expression for tax payable.	
4	Evaluate and write answer in words.	Joel is required to pay $63 232 in tax.
5	Find the tax refund by subtracting the tax paid from the tax payable.	**b** Tax refund $= \$65\,300 - \$63\,232$
		$= \$2068$
6	Evaluate.	The ATO owes Joel $2068.
7	Write answer in words.	
8	Express the tax paid as a fraction of the taxable income.	**c** $\dfrac{\text{Tax paid}}{\text{Taxable income}} = \dfrac{\$63\,232}{\$200\,000}$
9	Multiply the fraction by 100 to convert it to a percentage.	$\text{Percentage} = \dfrac{63\,232}{200\,000} \times 100$
		$= 31.616$
10	Evaluate.	
11	Write as a decimal to one decimal place.	$\approx 31.6\%$

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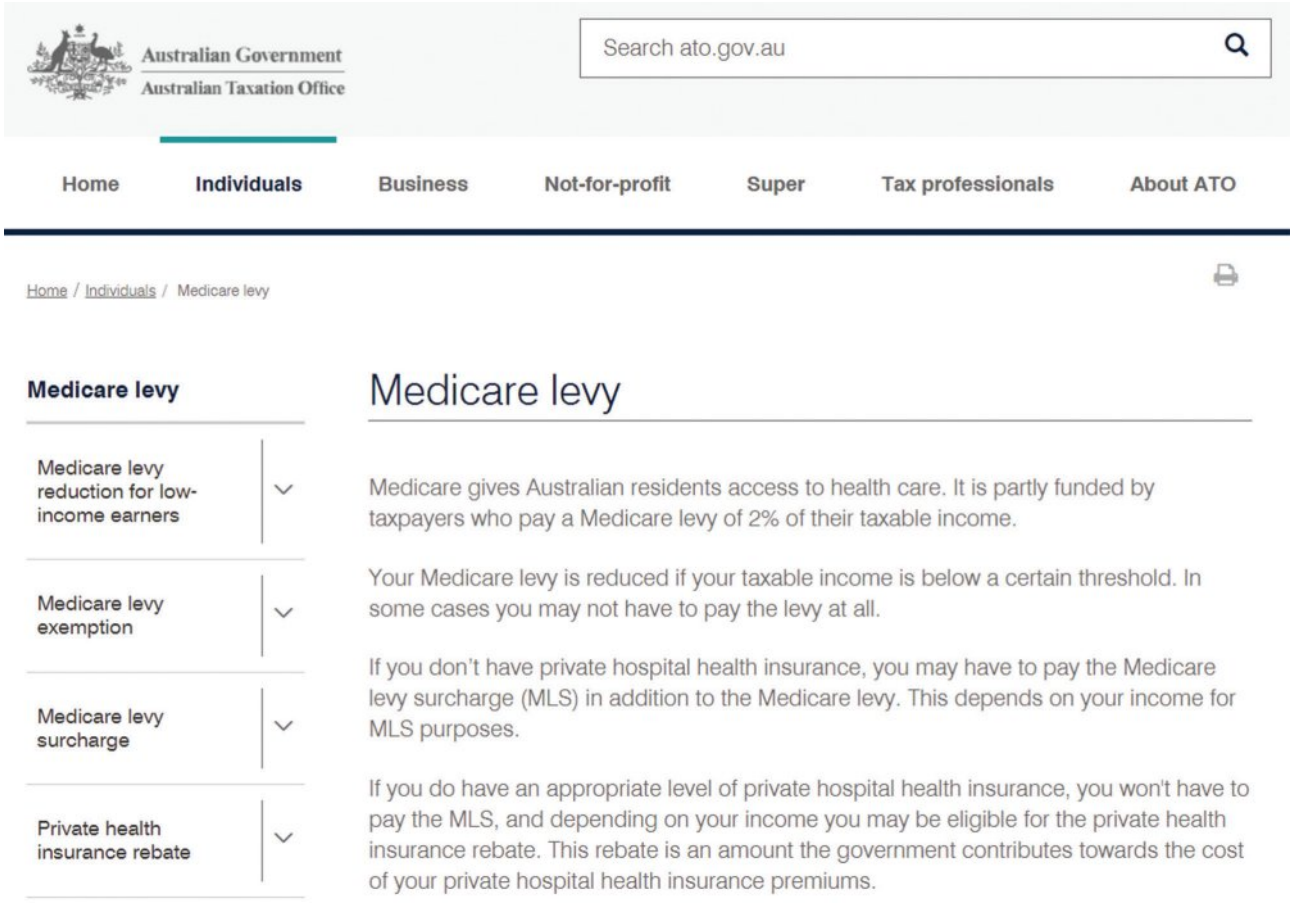
Medicare levy

Medicare levy is an additional charge to support Australia's universal health care system. It ensures that all Australians have access to free or low-cost medical and hospital care. The Medicare levy is calculated at a rate of 2% of taxable income



MEDICARE LEVY

Additional charge for health services. It is calculated at 2% of the taxable income.

Example 21: Calculating the Medicare levy 2I

The Medicare levy is 2% of the taxable income. What is the Medicare levy if the taxable income is $90 600?

SOLUTION:

1	Write the Medicare rate of the taxable income.	Medicare levy = 2% of $90 600
2	Express as a decimal 0.02.	$= 0.02 \times 90\,600$
3	Evaluate.	= $1812
4	Write answer using correct units and in words.	Medicare levy payable is $1812.

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Exercise 2I

LEVEL 1

1 The table below shows the personal income tax rates for 2017–18.

Taxable income	Tax payable	Tax bracket
0–$18 200	Nil	A
$18 201–$37 000	Nil + 19 cents for each $1 over $18 200	B
$37 001–$87 000	$3572 + 32.5 cents for each $1 over $37 000	C
$87 001–$180 000	$19 822 + 37 cents for each $1 over $87 000	D
$180 001 and over	$54 232 + 45 cents for each $1 over $180 000	E

Which tax bracket (A, B, C, D or E) from the table applies to these taxable incomes?

a $24 000 **b** $2500 **c** $75 000
d $122 500 **e** $230 000 **f** $80 000
g $37 001 **h** $180 000 **i** $4500

2 Calculate the tax payable on the following taxable incomes by completing the tax payable expression.

a $21 000 Tax payable = Nil + (21 000 – 18 200) × 0.19 =
b $32 500 Tax payable = Nil + (32 500 – 18 200) × 0.19 =
c $75 000 Tax payable = 3572 + (75 000 – 37 000) × 0.325 =
d $122 600 Tax payable = 19822 + (122 600 – 87 000) × 0.37 =
e $230 000 Tax payable = 54 232 + (230 000 – 180 000) × 0.45 =
f $80 000 Tax payable = 3572 + (80 000 – 37 000) × 0.325 =

3 Ava's taxable income was $28 000. The first $18 200 was tax-free and the balance was taxed at a marginal rate of 19%. Calculate the amount of tax payable.

4 Tyler's taxable income was $111 000. The tax payable on the first $87 000 is $19 822 and the balance was taxed at a marginal rate of 37%. Calculate the amount of tax payable.

Example 19

5 Use the tax table in question 1 to calculate the tax payable on the following taxable incomes.

a $18 201
b $37 001
c $87 001

Example 21

6 The Medicare levy is 2% of the taxable income. What is the Medicare levy on the following taxable incomes? Answer correct to the nearest cent.

a $23 000 **b** $88 541 **c** $40 600
d $46 906 **e** $67 800 **f** $200 592
g $170 300 **h** $15 790 **i** $90 640

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LEVEL 2

Use the tax table on page 82 to answers questions 7 to 11.

7 What is the tax payable on the following taxable incomes?

a $20 890 **b** $73 966 **c** $37 814

d $115 900 **e** $196 430 **f** $53 410

Example 20

8 Calculate the tax refund or tax owing for the following.

a Taxable income of $94 850 with tax instalments paid of $24 678

b Taxable income of $19 864 with tax instalments paid of $220

9 Ebony's gross income is $46 660 with allowable tax deductions of $4120.

a What is Ebony's taxable income?

b Calculate the amount of tax due.

c Ebony has paid $95 per week in tax. How much does Ebony owe in tax?

10 Charlie's gross income is $155 730. His allowable tax deductions are $8230.

a What is Charlie's taxable income?

b Calculate the amount of tax due.

c Charlie has paid $860 per week in tax. How much tax has been paid for the year?

d Will Charlie receive a refund or will he have to pay more tax? Justify your answer.

11 Ruby is a dentist with a taxable income of $145 684.

a Find the tax payable on this amount.

b What percentage of her income is paid as tax? Answer correct to one decimal place.

12 Liam has a taxable income of $56 400. He is required to pay $9877 in tax plus a Medicare levy of 2% of his taxable income.

a How much is Liam's Medicare levy?

b Calculate the total amount of tax due, including the Medicare levy.

13 Mia works for a superannuation fund and received a taxable income of $124 800.

a Medicare levy is 2% of the taxable income. How much is Mia's Medicare levy?

b Calculate the total amount of tax due, including the Medicare levy.

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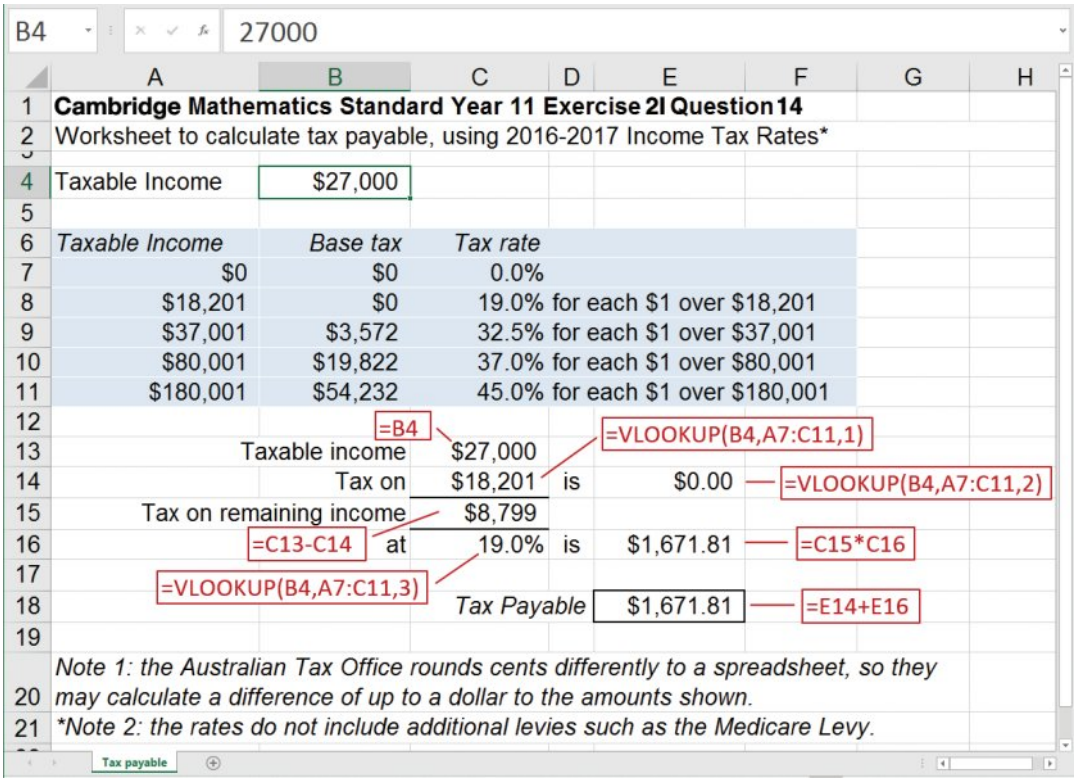
LEVEL 3

02IQ14

14 Create the spreadsheet below.



B4 | 27000

	A	B	C	D	E	F	G	H
1	**Cambridge Mathematics Standard Year 11 Exercise 2I Question 14**							
2	Worksheet to calculate tax payable, using 2016-2017 Income Tax Rates*							
4	Taxable Income	$27,000						
5								
6	*Taxable Income*	*Base tax*	*Tax rate*					
7	$0	$0	0.0%					
8	$18,201	$0	19.0% for each $1 over $18,201					
9	$37,001	$3,572	32.5% for each $1 over $37,001					
10	$80,001	$19,822	37.0% for each $1 over $80,001					
11	$180,001	$54,232	45.0% for each $1 over $180,001					
12			=B4					
13		Taxable income	$27,000		=VLOOKUP(B4,A7:C11,1)			
14		Tax on	$18,201	is	$0.00	=VLOOKUP(B4,A7:C11,2)		
15		Tax on remaining income	$8,799					
16		=C13-C14	at 19.0%	is	$1,671.81	=C15*C16		
17								
18		=VLOOKUP(B4,A7:C11,3)		*Tax Payable*	$1,671.81	=E14+E16		
19								
20	*Note 1: the Australian Tax Office rounds cents differently to a spreadsheet, so they may calculate a difference of up to a dollar to the amounts shown.*							
21	*Note 2: the rates do not include additional levies such as the Medicare Levy.*							

Tax payable

a Formulae have been entered into cells C13, C14, C15, C16, E14, E16 and E18. These formulae are shown above. Enter all the formulae.

b Change the taxable income (cell C4) to the following amounts.

i $50 000 **ii** $10 000 **iii** $200 000

15 Emma received an income of $75 420 from her main job for the last financial year and paid $18 680 in tax instalments. In addition, Emma earned an income of $6890 from a part-time job and paid tax of $2980.

a How much tax has Emma paid for the year?

b Calculate the amount of tax payable by Emma.

c Will Emma receive a refund or will she have to pay more tax? Justify your answer.

d What percentage of her income is paid as tax? Answer correct to one decimal place.

16 Alexander earns an income of $42 000. He also has $15 000 in a bank account that earns a flat rate of interest at 8.5% p.a. Alexander has to pay tax on his total income.

a How much interest does Alexander earn from his bank account this year?

b Calculate the tax payable.

17 Oscar is a coach driver who paid a Medicare levy of $1193.40. What was Oscar's taxable income if the rate of the Medicare levy is 2% of the taxable income? Answer correct to the nearest dollar.

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PPT STUDY GUIDE

Key ideas and chapter summary

Salary and wages

Salary – payment for a year's work, which is then divided into equal monthly, fortnightly or weekly payments

Wage – payment for work that is calculated on an hourly basis

Annual leave loading and bonuses

Annual leave loading – payment for going on holidays

Holiday loading = 17.5% × normal weekly pay × weeks leave

Bonus – extra payment or gift earned as a reward

Overtime and other allowances

Overtime – work beyond the normal working day

Casual rate – set amount paid for each hour's work

Time-and-a-half rate = normal rate × 1.5

Double time rate = normal rate × 2

Allowance – payment for difficult or dangerous conditions

Commission

Commission – percentage of the value of the goods sold

Retainer – small payment in addition to the commission

Piecework, royalties and government

Piecework – payment for work completed

Piecework = Number of units of work × Amount paid per unit

Royalty – percentage of the goods sold or profit received

Government income – pension, allowance or benefit

Allowable tax deductions

Deductions allowed by the Australian Taxation Office; include work-related, self-education, travel, car, clothing, tools and donations

Taxable income

Tax – calculated on the taxable income

Gross income – the total amount of money earned

Taxable income = Gross income – Allowable tax deductions

Gross pay, net pay and deductions

Gross pay – total of the employee's pay including allowances, overtime pay, commissions and bonuses

Deductions – regular amount of money subtracted from a person's wage or salary such as income tax

Net pay = Gross pay – Deductions

Calculating tax

Personal income tax tables have an increasing rate of tax.

Tax refund = Tax paid – Tax payable

Tax owing = Tax payable – Tax paid

Medicare levy

Additional charge for health services.

Medicare is calculated at 2% of the taxable income.

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Review

Multiple-choice questions

1 Alyssa receives a salary of $85 640. How much does she receive each fortnight?
A $3293.84 **B** $3293.85 **C** $1646.92 **D** $1646.93

2 Hine works a 38-hour week at a rate of $26.00 per hour. She receives 17.5% of 4 weeks normal pay as holiday loading. What is Hine's holiday loading?
A $172.90 **B** $691.60 **C** $3952.00 **D** $4643.60

3 Christopher receives a normal hourly rate of $22.60. What is his pay when he works 8 hours at normal rate and 3 hours at time-and-a-half?
A $180.80 **B** $248.60 **C** $282.50 **D** $316.40

4 Qiang works for $15.60 per hour. He works for 6 hours on Thursday and 8 hours on Friday. On Saturday he is paid at time-and-half for his work. Last week Qiang earned $335.40 in total for his work on these three days. How many hours did he work on Saturday?
A 5 hours **B** 6.5 hours **C** 7.5 hours **D** 11.25 hours

5 Taylah earns a weekly retainer of $425 plus a commission of 8% on sales. What was her weekly earnings in a week in which she made sales of $8620?
A $34.00 **B** $459.00 **C** $689.60 **D** $1114.60

6 Alex is landscape gardener and entitled to the following tax deductions: union fees of $484, donations of $720, equipment cost of $860, and car expenses of $1455. What is Alex's total allowable tax deduction?
A $484 **B** $860 **C** $1455 **D** $3519

7 Stephanie has an allowable tax deduction of $4690. What is her taxable income if her gross annual salary is $43 720
A $39 030 **B** $43 720 **C** $48 410 **D** $90 620

8 Charlotte is a librarian who has a taxable income of $69 410. The tax payable on the first $37 000 is $3572 and the balance was taxed at a rate of 32.5%. How much does Charlotte have to pay in tax, to the nearest dollar?
A $3572 **B** $14 105 **C** $22 558 **D** $37 000

9 William has a taxable income of $53 684. What is his Medicare levy? (Assume the Medicare levy is calculated at a rate of 2% of the taxable income.)
A $107.37 **B** $1073.68 **C** $2684.20 **D** $52 610.32

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Short-answer questions

1 Jake earns $96 470.40 per annum and works on average 48 hours per week.

a What is his average weekly wage?

b Calculate the Jake's hourly rate of pay.

2 Carlo's employer has decided to reward all employees with a bonus. The bonus awarded is 6.25% of their annual salary. What is Carlo's bonus if his annual salary is $85 940?

3 The public service provides all employees with a 17.5% holiday loading on four weeks normal wages. Lucy works a 37-hour week for the public service in Canberra. She is paid a normal hourly rate of $32.40.

a How much will Lucy receive in holiday loading?

b Calculate the total amount of pay that Lucy will receive for her holidays

4 Alex works for a fast food company and is paid $13.50 per hour for a 35-hour week. He gets time-and-a-half pay for overtime worked on weekdays and double time for working on weekends. Last week he worked a normal 35-hour week plus 3 hours of overtime during the week and 4 hours of overtime on the weekend. What was his wage last week?

5 Chelsea is a real estate agent and is paid the following commission on her sales: 3% on the first $45 000, then 2% for the next $90 000 and 1.5% thereafter.

a What is Chelsea's commission if she sells a property for $240 000?

b How much would the owner of the property receive from the sale?

6 Patrick is a comedian who makes $120 for a short performance and $260 for a long performance. How much will he earn if he completes 11 short and 12 long performances?

7 Bailey is paid a royalty of 11.3% on the net sales of his book. The net sales of his book in the last financial year were $278 420.

a What was Bailey's royalty payment in the last financial year?

b Net sales this financial year are expected to decrease by 15%. What is the expected royalty payment for this financial year?

8 The maximum Youth Allowance is reduced by $1 for every $4 that the youth's parents income is over $31 400. By how much is Hannah's Youth Allowance reduced if her parents earn a combined income of $35 624?

9 Ali works as a builder. His annual union fees are $278.20. Ali has his union fees deducted from his weekly pay. What is Ali's weekly union deduction?

10 Sienna owns a clothing store and is entitled to the following tax deductions: $570 for union fees, tax agent fee of $375, charity donations of $390, information technology costs of $3910 and printing costs of $528. What is Sienna's total tax deduction?

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Review

11 What is the taxable income if the gross salary is $82 390 and the allowable tax deduction is $4870?

12 Ryan is a small business owner who bought a $3850 computer for business use. The computer is an allowable tax deduction with a rate of depreciation of 33% p.a.

- **a** How much can he claim for depreciation in the first year?
- **b** What is the depreciated value of the computer after the first year?

13 James works for a model agency and earns a gross fortnightly pay of $3720. He pays PAYG tax of $986 per fortnight and has tax deductions of $15 per week for dry-cleaning, and yearly deductions of $1450 for work-related travel and $1000 for charities.

- **a** What is James's gross yearly salary?
- **b** How much tax is deducted each week?
- **c** What is James's total allowable tax deduction?
- **d** Calculate James's taxable income.

14 Ruby has a taxable income of $134 830. Calculate the tax payable on this amount. Use the personal income tax rates on page 79.

15 Cooper is a retail store manager. After he received a $2300 annual pay rise, his salary become $73 450. How much of the $2300 pay rise was he required to pay in tax? Use the personal income tax rates on page 79.

Extended-response questions

16 Eliza is a casual employee and is paid a weekday rate of $18 per hour, a Saturday rate of time-and-a-half for Saturdays and a Sunday rate of double time.

This is her time card for last week.

Day	In	Out	Unpaid break
Friday	8:00 a.m.	3:30 p.m.	30 minutes
Saturday	9:00 a.m.	5:00 p.m.	1 hour
Sunday	9:00 a.m.	3:00 p.m.	30 minutes

- **a** Calculate Eliza's gross pay for last week.
- **b** Eliza's employer withholds 20% of her weekly earnings as PAYG tax. Calculate Eliza's PAYG tax for last week.

17 Nathan has a taxable income is $106 770. He has paid $980 per fortnight in tax.

- **a** How much tax has Nathan paid for the year?
- **b** Calculate the amount of tax payable by Nathan. Use the tax rates on page 79.
- **c** Will Nathan receive a refund or will he have to pay more tax? Justify your answer.
- **d** What percentage of his income is paid as tax? Answer correct to one decimal place.

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3 Formulae and equations

Syllabus topic — MS-A1 Formulae and equations

This topic provides a solid foundation in algebraic skills such as finding solutions to a variety of equations in work-related and everyday contexts. Students develop awareness of the applicability of algebra in their approach to everyday life.

Outcomes

- Evaluate an algebraic expression after substitution.
- Use the formula to calculate distance, speed and time.
- Use a formula to calculate stopping distance.
- Develop and solve linear equations.
- Solve linear equations derived from substituting values into a formula.
- Calculate and interpret blood alcohol content (BAC).
- Calculate required medication dosages for children and adults.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Solutions (enabled by teacher)
- Literacy worksheet
- Widgets
- Quick Quiz
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

In the Interactive Textbook you can take a test of prior knowledge required for this chapter, and depending on your score you may be directed to revision from the previous years' work.

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3A Substitution

Substitution involves replacing pronumerals in an algebraic expression with numbers. The resulting numerical expression is evaluated and expressed to the specified level of accuracy.

SUBSTITUTION OF VALUES

1. Write the algebraic expression.
2. Replace the variables in the expression with the numbers given in the question.
3. Evaluate using the calculator.
4. Write the answer to the specified level of accuracy and correct units if necessary.

Example 1: Substituting values into linear expressions 3A

Evaluate $3a - 4b + c$, given $a = 2$, $b = 5$ and $c = -10$.

SOLUTION:

1. Write the algebraic expression, and substitute the values for a, b and c.
2. Evaluate.

$$\begin{aligned} 3a - 4b + c &= 3 \times 2 - 4 \times 5 + -10 \\ &= 6 - 20 - 10 \\ &= -24 \end{aligned}$$

Example 2: Substituting values into non-linear expressions 3A

Evaluate the following, given $a = 2$.

a $3a^3$ **b** $\sqrt{(2a+5)}$

SOLUTION:

1. Write the algebraic expression, and substitute the value for a.
2. Evaluate.
3. Write the algebraic expression, and substitute the value for a.
4. Evaluate.

a
$$\begin{aligned} 3a^3 &= 3 \times 2^3 \\ &= 3 \times 8 \\ &= 24 \end{aligned}$$

b
$$\begin{aligned} \sqrt{(2a+5)} &= \sqrt{2 \times 2 + 5} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

Example 3: Substituting values into non-linear expressions 3A

Evaluate $a^2 + 5b^3 - c$, given $a = 4$, $b = 2$ and $c = -1$.

SOLUTION:

1. Write the algebraic expression, and substitute the values for a, b and c.
2. Evaluate.

$$\begin{aligned} a^2 + 5b^3 - c &= 4^2 + 5 \times 2^3 - (-1) \\ &= 16 + 40 + 1 \\ &= 57 \end{aligned}$$

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Exercise 3A

LEVEL 1

Example 1

1 Evaluate these expressions, given $a = 3$, $b = 4$ and $c = 8$.

a $5a + b$ **b** $a + 4b$ **c** $a - b + c$

d $a^2 + b^2$ **e** $4b \times (-2a)$ **f** $\sqrt{2ab}$

g $\frac{abc}{2}$ **h** $b^2 \div c$ **i** $\frac{2ab}{c}$

2 Find the value of $3a + 2b$ when:

a $a = 5$ and $b = 5$ **b** $a = 6$ and $b = -4$ **c** $a = 0$ and $b = 0$

d $a = -7$ and $b = -2$ **e** $a = \frac{2}{3}$ and $b = 1$ **f** $a = 2$ and $b = \frac{1}{2}$

Example 2a

3 Calculate the value of $e^2 - 3$ when:

a $e = 1$ **b** $e = 3$ **c** $e = 10$

d $e = 2$ **e** $e = -1$ **f** $e = -2$

g $e = \frac{1}{2}$ **h** $e = 3.1$ **i** $e = \frac{1}{5}$

4 Determine the value of $2r^2 + r + 1$ when:

a $r = 2$ **b** $r = 5$ **c** $r = -7$

d $r = -0.2$ **e** $r = \frac{2}{3}$ **f** $r = -\frac{3}{4}$

5 Calculate the value of mc^2 when:

a $m = 4$ and $c = 2$ **b** $m = 2$ and $c = 7$ **c** $m = -3$ and $c = -10$

d $m = 8$ and $c = 0.5$ **e** $m = -0.2$ and $c = 15$ **f** $m = 6.25$ and $c = -1.5$

g $m = \frac{7}{10}$ and $c = \frac{1}{5}$ **h** $m = -\frac{3}{8}$ and $c = 0.75$ **i** $m = \frac{7}{9}$ and $c = 0.3$

Example 2b

6 Find $\sqrt{\frac{2a}{b}}$ when:

a $a = 6$ and $b = 3$ **b** $a = 50$ and $b = 4$ **c** $a = -1$ and $b = -0.5$

d $a = 18$ and $b = 4$ **e** $a = -0.25$ and $b = -0.5$ **f** $a = 8$ and $b = 0.25$

g $a = \frac{1}{4}$ and $b = 8$ **h** $a = \frac{7}{8}$ and $b = \frac{4}{7}$ **i** $a = -\frac{1}{10}$ and $b = -125$

7 Determine the value of $\sqrt[3]{6c + d}$ when:

a $c = 1$ and $d = 2$ **b** $c = 10$ and $d = 4$

c $c = 4$ and $d = 3$ **d** $c = 0.5$ and $d = -2$

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LEVEL 2

Example 3

8 Evaluate these expressions, given $x = 7$, $y = -5$ and $z = 21$.

a $x^2 + z^2 + y$ **b** $y^3 - 4x$

c $\sqrt{4x + z - 1}$ **d** $\sqrt{z + 4y^2}$

e $\dfrac{3xy^2}{z}$ **f** $\dfrac{6y^2}{5zx}$

9 The area of an annulus is evaluated using $\pi(R^2 - r^2)$ where R is the radius of the outer circle and r the radius of the inner circle. Find the area of an annulus if R is 8 cm and r is 4 cm. Answer correct to one decimal place.

10 Determine the value of $\frac{2}{3}p^2q$, given that $p = 4$ and $q = 6$.

11 Evaluate $\sqrt[3]{2y + 3}$, given that $y = 12$.

12 Evaluate $\sqrt[3]{x^2 + y}$, given that $x = 3$ and $y = 4$. Answer correct to two decimal places.

13 Find the value of $2\pi\sqrt{\dfrac{l}{g}}$ when $l = 2.6$ and $g = 9.8$. Give your answer correct to two decimal places.

14 Find the value of $\sqrt{u^2 + 2as}$ when $u = 6$, $a = 7$ and $s = 2$.

LEVEL 3

15 Find the value of $\dfrac{1}{2\pi fc}$ when $f = 10$ and $c = 2$. Give your answer correct to three decimal places.

16 Find the value of $\dfrac{3Rr}{R + r}$ when $R = 8.2$ and $r = 4.9$. Give your answer correct to two decimal places.

17 What is the value of $\dfrac{yA}{(y + 12)}$ when $y = 9$ and $A = 15$? Give your answer correct to the nearest whole number.

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3B Distance, speed and time

Speed is a rate that compares the distance travelled to the time taken. The speed of a car is measured in kilometres per hour (km/h). The speedometer in a car measures the instantaneous speed of the car. A speedometer is not totally accurate but has a tolerance of 5%. GPS devices are also capable of showing speed readings. Most cars also have an odometer to indicate the distance travelled by the vehicle.

DISTANCE, SPEED AND TIME

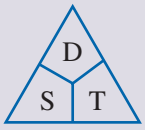
$S = \frac{D}{T}$ or $T = \frac{D}{S}$ or $D = S \times T$

D – Distance

S – Speed

T – Time

Road sign on the right is used to remember the formulae. Cover the required quantity to determine the formula.



Example 4: Finding the distance, speed and time 3B

a Find the distance travelled by a car whose average speed is 65 km/h if the journey lasts 5 hours. (Answer correct to the nearest kilometre.)

b How long will it take a vehicle to travel 150 km at a speed of 60 km/h?

SOLUTION:

1 Write the formula.
2 Substitute 65 for S and 5 for T into the formula.
3 Evaluate and express answer correct to the nearest kilometre.
4 Write the formula.
5 Substitute 150 for D and 60 for S into the formula.
6 Evaluate and give the correct unit.

a $D = S \times T$
$= 65 \times 5$
$= 325$ km

b $T = \frac{D}{S}$
$= \frac{150}{60}$
$= 2.5$ h

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Stopping distance

The stopping distance is the distance a vehicle travels from the time a driver sees an event occurring to the time the vehicle is brought to a stop. It is calculated by adding the reaction distance and the braking distance. Reaction distance (or thinking distance) is the distance travelled by the vehicle from the time that a hazard, such as a pedestrian stepping into the road, first occurs, to when the driver first commences braking. The reaction time averages 0.75 seconds for a fit and alert driver. The braking distance is affected by the road surface (wet, slippery, uneven or unsealed), the slope of the road (uphill or downhill), the weight of the vehicle and the condition of the brakes and tyres.

STOPPING DISTANCE

Stopping distance = Reaction distance + Braking distance

$$d = \frac{5Vt}{18} + \frac{V^2}{170} \text{ (formula is an approximation using average conditions)}$$

d – stopping distance in metres
V – velocity or speed of the motor vehicle in km/h
t – reaction time in seconds

Example 5: Calculating the stopping distance 3B

Trevor was driving at a speed of 45 km/h and he had a reaction time of 0.75 seconds when a hazard occurred. Calculate the stopping distance using the formula $d = \frac{5Vt}{18} + \frac{V^2}{170}$.
Answer correct to the nearest whole metre.

SOLUTION:

1 Write the stopping distance formula.
2 Substitute $V = 45$ and $t = 0.75$ into the formula.
3 Evaluate.
4 Express the answer as required.
5 Write the answer in words.

$$d = \frac{5Vt}{18} + \frac{V^2}{170}$$
$$= \frac{5 \times 45 \times 0.75}{18} + \frac{45^2}{170}$$
$$= 21.28676471$$
$$\approx 21 \text{ m}$$

Stopping distance is 21 m.

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Exercise 3B

LEVEL 1

1 Find the average speed (in km/h) of a vehicle that travels:

a 180 km in 2 hours

b 485 km in 5 hours

c 360 km in 4.5 hours

d 21 km in $\frac{1}{4}$ hour

e 240 km in $2\frac{1}{2}$ hours

f 16 km in 20 minutes

Example 4a

2 Find the distance travelled by a car whose average speed is 56 km/h over the following journey times. (Answer correct to the nearest kilometre.)

a 3 hours

b 7 hours

c 2.6 hours

d $1\frac{1}{4}$ hours

e $3\frac{1}{2}$ hours

f $2\frac{3}{4}$ hours

Example 4b

3 How long will it take a vehicle to travel the following journeys? (Answer correct to the nearest hour.)

a 160 km at a speed of 80 km/h

b 150 km at a speed of 60 km/h

c 120 km at a speed of 48 km/h

d 225 km at a speed of 45 km/h

e 240 km at a speed of 40 km/h

f 556 km at a speed of 69.5 km/h

4 The Melbourne Formula 1 track is 5.303 km in length. The track record is 1 minute and 24 seconds. What is the average speed (km/h) for the lap record? Answer correct to two decimal places.

5 Caitlin lives in Wollongong and travels to Sydney daily.
The car trip requires her to travel at different speeds. Most often she travels 30 kilometres at 60 km/h and 40 kilometres at 100 km/h.

a What is the total distance of the trip?

b How long (in hours) does the trip take?

c What is her average speed (in km/h) when travelling to Sydney? (Answer correct to two decimal places.)

6 Mohammad drives his car to work 3 days a week. The length of the trip is 48 km. The trip took 43 minutes on Monday, 50 minutes on Tuesday and 42 minutes on Wednesday.

a Calculate the average time taken to travel to work.

b What is the average speed (in km/h) for the three trips?

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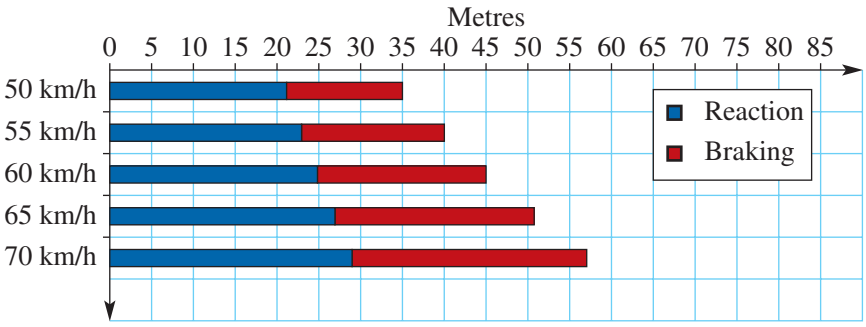
7 The graph below shows the reaction distance and the braking distance.



A car is travelling at 60 km/h.

a What is the reaction distance?

b What is the braking distance?

c What is the stopping distance?

8 What is the stopping distance for each of the following?

a Reaction distance of 25 metres and braking distance of 22 metres

b Reaction distance of 19 metres and braking distance of 30 metres

9 Michael is driving with a reaction time of 0.75 seconds. Calculate the stopping distance (to the nearest metre) for each of the following speeds. Use the formula $d = \frac{5Vt}{18} + \frac{V^2}{170}$.

a 30 km/h **b** 50 km/h **c** 70 km/h

d 90 km/h **e** 110 km/h **f** 130 km/h

10 Sarah was driving her car at 40 km/h through a school zone (reaction time is 0.50 seconds). A school student ran onto the road 12 metres in front of her.

a Do you think Sarah was able to stop without running over the child? Give a reason for your answer.

b What would have happened if Sarah had been driving her car at 60 km/h? Explain your answer.

Example 5

11 Oliver travels to work on the freeway. His reaction time is 0.60 seconds. Oliver usually drives at the speed limit of 110 km/h.

a What is the stopping distance on the freeway? Use the formula $d = \frac{5Vt}{18} + \frac{V^2}{170}$.

b Determine a safe distance (to the nearest metre) between cars that are travelling at 110 km/h on the freeway based on the NSW Roads and Maritime advice to keep three seconds behind the car in front.

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LEVEL 2

12 Find the average speed (in km/h) of a vehicle that travels the following journeys. (Answer correct to the nearest whole number.)

a 500 km in 6 hours and 10 minutes
b 64 km in 1 hour and 30 seconds
c 36 000 m in 45 minutes
d 320 m in 10 seconds

13 Find the distance travelled by a car whose average speed is 68 km/h if the journey lasts the following times. (Answer correct to the nearest kilometre.)

a 30 minutes
b 2 minutes
c 1 hour and 20 minutes
d 4 hours 10 seconds

14 How long will it take a vehicle to travel the following journeys? (Answer correct to the nearest minute.)

a 450 km at a speed of 82 km/h
b 50 km at a speed of 60 km/h
c 250 km at a speed of 49 km/h
d 580 000 m at a speed of 62 km/h
e 24 000 m at a speed of 72 km/h
f 100 km at a speed of 1 km/min

15 The land speed record is 20.4 km/min.

a Express this speed in km/h.
b How far does this vehicle travel in 5 minutes?
c How far does this vehicle travel in 1 second?
d How long would it take for this vehicle to travel from Sydney to Brisbane (982 km)? Answer to the nearest minute.

16 The Bathurst 1000 motor race has a lap record of 2 minutes and 12.339 seconds. The length of the lap is 6.213 km.

a What is the average speed (to nearest km/h) for the lap record?
b How long is the race if the winning car travels the 161 laps at the average speed for the lap record? Answer to the nearest minute.

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17 'If you double your speed you need to double your reaction distance.'

a Use $d = \frac{5Vt}{18}$ to complete the table.
Assume a reaction time of 0.75 seconds.

b Do you agree with the above statement? Give a reason for your answer.

Speed (km/h)	Reaction distance (m)
10	
20	
40	
80	

18 'If you double your speed you need to quadruple your braking distance.'

a Use $d = \frac{V^2}{170}$ to complete the table.

b Do you agree with the above statement? Give a reason for your answer.

Speed (km/h)	Braking distance (m)
10	
20	
40	
80	

LEVEL 3

19 Tahnee is driving at a speed of 30 km/h.

a Write the formula $d = \frac{5Vt}{18} + \frac{V^2}{170}$ with t as the subject.

b Find the value of t when $d = 10$ metres. Answer correct to one decimal place.

c Find the value of t when $d = 20$ metres. Answer correct to one decimal place.

d Find the value of t when $d = 30$ metres. Answer correct to one decimal place.

20 Liam is driving at a speed of 60 km/h.

a Use the formula $d = \frac{5Vt}{18}$ to complete the table.

b What effect does increasing the reaction time have on the stopping distance? Use the calculations in the table to justify your answer.

Reaction time (s)	Reaction distance (m)
0.50	
1.00	
1.50	

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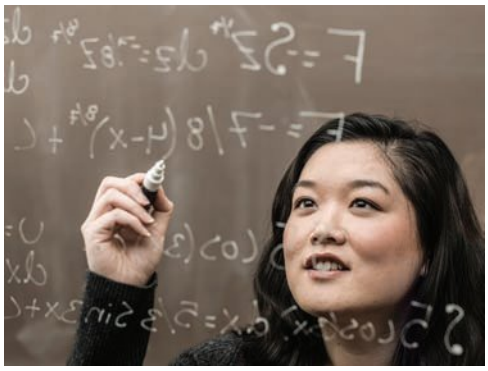
3C Linear equations



An equation is a mathematical statement that says that two things are equal. It has an equals sign, for example $x + 4 = 7$. In linear equations all the variables are raised to the power of 1. The process of finding the unknown value for the variable is called solving the equation. When solving an equation, look to perform the opposite operation:

- $+$ is opposite to $-$
- $\times$ is opposite to $\div$

When solving an equation, the equation must be kept balanced on either side of the equals sign. The same operation needs to be done on both sides of the equals sign to keep the balance. The goal when solving an equation is to get the pronumeral by itself on one side of the equation.

SOLVING AN EQUATION

1. Look to perform the opposite operation ($+$ is opposite to $-$, $\times$ is opposite to $\div$).
2. Add or subtract the same number to both sides of the equation.
3. Multiply or divide both sides of the equation by the same number.
4. To solve two-step or three-step equations, repeat the steps 1 to 3 as required. It is often easier to first add or subtract the same number to both sides of the equation.

When a solution has been reached it can be checked. The solution of the equation must satisfy the equation. Always check your solution by substituting your answer into the original equation. The left-hand side of the equation must equal the right-hand side.

Example 6: Solving a two-step linear equation 3C

Solve the equation $3x + 5 = -2$.

SOLUTION:

1. Write the equation.
2. The opposite operation to adding by 5 is subtracting by 5. Subtract 5 from both sides of the equation.
3. The opposite operation to multiplying by 3 is dividing by 3. Divide both sides of the equation by 3.
4. Express as a mixed number $\left(-2\frac{1}{3}\right)$.
5. Check that the solution is correct by substituting $x = -2\frac{1}{3}$ into the original equation.

$$3x + 5 = -2$$
$$3x + 5 - 5 = -2 - 5$$
$$3x = -7$$
$$\frac{3x}{3} = \frac{-7}{3}$$
$$x = -2\frac{1}{3}$$

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Example 7: Solving a linear equation with fractions 3C

Solve the equation $\frac{y}{3} - \frac{y}{7} = 2$. Express your answer as a simple fraction.

SOLUTION:

1 Write the equation.
$$\frac{y}{3} - \frac{y}{7} = 2$$

2 To simplify the fractions find the lowest common denominator. The smallest number that is divisible by 3 and 7 is 21.

3 Multiply both sides of the equation (all the terms) by 21.
$$21 \times \left(\frac{y}{3}\right) - \left(\frac{y}{7}\right) \times 21 = 2 \times 21$$

4 Cancel out the common factors (21 divided by 3 is 7 and 21 divided by 7 is 3).

5 Write the equation without a fraction ($7y - 3y = 42$).
$$7y - 3y = 42$$

6 Subtract the like terms ($7y - 3y = 4y$).
$$4y = 42$$

7 The opposite operation to multiplying by 4 is dividing by 4. Divide both sides of the equation by 4.
$$\frac{4y}{4} = \frac{42}{4}$$

8 Express as a mixed number in simplest form ($10\frac{1}{2}$).
$$y = 10\frac{1}{2}$$

9 Check that the solution is correct by substituting $y = 10\frac{1}{2}$ into the original equation.

Example 8: Solving a four-step linear equation 3C

Solve the equation $\frac{b}{2} + 5 = 3b + 7$.

SOLUTION:

1 Write the equation.
$$\frac{b}{2} + 5 = 3b + 7$$

2 To simplify the fraction find the lowest common denominator. The only denominator is 2.

3 Multiply both sides of the equation (all the terms) by 2.
$$\left(\frac{b}{2}\right) \times 2 + 5 \times 2 = 3b \times 2 + 7 \times 2$$

4 Cancel out the common factor (2 divided by 2 is 1).
$$b + 10 = 6b + 14$$

5 Write the equation without a fraction.

6 The opposite operation to adding by $6b$ is subtracting by $6b$. Subtract $6b$ from both sides of the equation.
$$\overset{-6b}{b} + 10 = \overset{-6b}{6b} + 14$$

7 The opposite operation to adding 10 is subtracting 10. Subtract 10 from both sides of the equation.
$$\overset{-10}{-5b + 10} = \overset{-10}{14}$$
$$-5b = 4$$

8 The opposite operation to multiplying by -5 is dividing by -5. Divide both sides of the equation by -5.
$$\frac{-5b}{-5} = \frac{4}{-5}$$

9 Express as a fraction in simplest form.
$$b = -\frac{4}{5}$$

10 Check that the solution is correct.

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Exercise 3C

LEVEL 1

1 Solve the following linear equations.

a $a + 7 = 15$ **b** $y + 12 = 23$ **c** $x + 4 = 14$

d $6 + g = 13$ **e** $7 + w = 3$ **f** $11 + z = -2$

g $b - 5 = 4$ **h** $m - 9 = 9$ **i** $c - 6 = 15$

j $12 - q = 5$ **k** $8 - h = 2$ **l** $-3 = n - (-2)$

2 Solve the following linear equations.

a $5n = 15$ **b** $6x = 30$ **c** $3c = -12$

d $2b = 15$ **e** $4m = -21$ **f** $-25 = 6b$

g $\frac{s}{2} = 8$ **h** $\frac{y}{5} = 6$ **i** $\frac{a}{11} = -7$

j $12 = \frac{g}{2}$ **k** $\frac{c}{-15} = -3$ **l** $3 = \frac{k}{-3}$

3 Eliza was required to solve the following equation on the blackboard. This was her solution.

$$\begin{aligned} 6y + 5 &= 7 \\ 6y &= 12 \\ y &= 2 \end{aligned}$$

a Where is the error in Eliza's working?

b What is the correct solution to the equation?

c Check your solution by substituting your answer into the equation.

Example 6

4 Solve the following linear equations. All solutions are integers.

a $5w + 4 = 29$ **b** $2t + 6 = 12$ **c** $7x - 6 = 22$

d $5y - 10 = -15$ **e** $9m - 2 = -11$ **f** $12 = 3a - 9$

g $5 + 2e = 13$ **h** $4 + 3b = 7$ **i** $32 = 17 - 3k$

j $16 = 10 + 2w$ **k** $70 - 10d = 80$ **l** $98 = 38 - 10z$

5 Solve the following linear equations. Express your answer as a simplest fraction.

a $3q + 6 = 19$ **b** $4r + 2 = 27$ **c** $2x + 11 = 14$

d $31 = 4 + 4a$ **e** $18 = 17 + 5m$ **f** $12 = 29 + 2k$

g $2c - 7 = 6$ **h** $5e - 1 = 15$ **i** $6p - 4 = 11$

j $28 = 2r + 19$ **k** $37 = 3w - 7$ **l** $16 = 9c + 8$

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6 Solve these equations.

a $4d = 1\frac{2}{3}$ **b** $3y = 4\frac{1}{5}$ **c** $7a = 4\frac{2}{3}$

d $\frac{x}{3} = 3\frac{2}{9}$ **e** $\frac{b}{2} = 3\frac{3}{5}$ **f** $\frac{n}{5} = 7\frac{1}{4}$

7 Solve these equations.

a $\frac{2z}{5} = 4$ **b** $\frac{6c}{2} = 12$ **c** $\frac{2m}{9} = 8$

d $7 = \frac{9x}{5}$ **e** $14 = \frac{5y}{6}$ **f** $15 = \frac{3a}{4}$

8 Twenty is added to a number divided by four and the result is 27.

a Write an equation using a to represent the number.

b Solve the equation for a.

9 Solve these equations.

a $\frac{g}{2} + 6 = 14$ **b** $5 + \frac{y}{3} = 17$ **c** $3 = \frac{c}{5} - 7$

d $8 = 1 - \frac{x}{7}$ **e** $7 = 1 + \frac{3b}{2}$ **f** $6 = \frac{2w}{5} - 9$

10 Solve these equations.

a $4(m + 3) = 20$ **b** $3(t - 3) = 3$ **c** $2(e + 10) = 12$

d $-2(3x + 1) = 6$ **e** $-5(d - 3) = -15$ **f** $(w + 5)7 = 28$

g $\frac{1}{5}(3 + 2a) = 2$ **h** $\frac{1}{6}(7 - 5g) = -4$ **i** $\frac{1}{3}(9 - 3r) = -2$

11 Solve these equations.

a $\frac{a - 4}{2} = 6$ **b** $\frac{3z - 2}{5} = 7$ **c** $\frac{5 + 2r}{3} = 2$

d $\frac{12 + 6x}{5} = -1$ **e** $\frac{b + 2}{8} = \frac{1}{2}$ **f** $\frac{3w + 2}{2} = \frac{3}{4}$

LEVEL 2

Example 8

12 Solve the following linear equations.

a $4d + 4 = 3d + 9$ **b** $5x + 3 = 4x - 5$ **c** $7a + 2 = 4a - 10$

d $10y - 4 = 2 + 4y$ **e** $8m + 2 = (-17) + 3m$ **f** $7c - 4 = 7 + c$

g $7 + s = 8s - (-9)$ **h** $13 + 2z = 6z + 3$ **i** $1 + 5b = 8 - 2b$

j $-5 + 2x = -3 + 8x$ **k** $4 - (-2a) = 5a + 1$ **l** $2w - 7 + 4w = 8$

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13 Solve these equations.

a $\frac{x}{3}+3=2x+10$ **b** $8+\frac{w}{2}=3w-10$ **c** $6y-4=\frac{y}{2}-5$

d $\frac{e}{5}+4=10+2e$ **e** $10m+2=1+\frac{m}{2}$ **f** $7=\frac{b}{3}-5b+2$

g $2=5+2r-\frac{r}{5}$ **h** $\frac{4t}{3}+3t-5=0$ **i** $5n=\frac{2n}{7}-6+n$

14 Maya bought three dresses when she went shopping, each costing the same price, $\$c$. Maya also paid \$15 for parking. The total cost for the day was \$261.

a Using c as the cost of one dress, write an equation showing the cost of the three dresses and parking.

b Use the equation to find the cost of each dress.

15 If 15 is subtracted from two times a certain number, the result is five times the number. Find the number.

Example 7

16 Solve these equations.

a $\frac{v}{3}+\frac{v}{4}=7$ **b** $\frac{a}{5}+\frac{a}{2}=10$ **c** $\frac{x}{4}+\frac{x}{8}=5$

d $\frac{m}{5}-\frac{m}{15}=7$ **e** $\frac{b}{6}-\frac{b}{7}=2$ **f** $\frac{g}{6}-\frac{g}{8}=4$

g $\frac{4y}{2}-\frac{y}{3}=6$ **h** $\frac{4c}{3}-\frac{c}{4}=3$ **i** $\frac{h}{2}-\frac{2h}{5}=4$

j $\frac{n}{8}=\frac{n}{7}-1$ **k** $\frac{3p}{10}=\frac{p}{2}+4$ **l** $\frac{5z}{2}=\frac{2z}{5}+1$

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17 Solve these equations.

a $\frac{4}{x} = 12$ **b** $-6 = \frac{5}{a}$ **c** $\frac{2}{y} + 1 = 6$

d $7 + \frac{4}{d} = 3$ **e** $5 - \frac{8}{c} = -2$ **f** $4 + \frac{-2}{m} = 0$

18 Solve these equations.

a $\frac{3}{b+1} = 5$ **b** $7 = \frac{8}{1+y}$ **c** $\frac{4}{y-2} - 4 = 0$

d $\frac{2x}{x+1} = 3$ **e** $\frac{8c}{4c-2} = 1$ **f** $11 = \frac{3m}{4+m}$

19 Six more than twice a certain number is two more than the number itself.

a Write an equation using x to represent the number.

b Solve the equation for x.

20 My uncle is 8 years older than I, and 6 years ago he was double my age.

a Write an equation using x to represent my age.

b Solve the equation for x.

LEVEL 3

21 Solve these equations.

a $3(d+7) = -2(d-9)$ **b** $4(r+2)+1 = 3(r-3)$

c $3(5x-1) - 6(x-5) = 11$ **d** $6(2a-5) - 2(3+a) = 20$

e $4(4m+1) - (m-5) = 3$ **f** $3(y-2) = 10 - 2(3y-1)$

22 Solve these equations.

a $\frac{a+2}{3} = 3a - 4$ **b** $2m - 5 = \frac{m+1}{2}$ **c** $\frac{y+1}{4} - (y-1) = 2$

d $\frac{b+5}{3} = \frac{b+8}{2}$ **e** $\frac{3x+2}{5} = \frac{2x+4}{2}$ **f** $\frac{4c-2}{5} = \frac{2c+1}{3}$

g $\frac{n+1}{6} + \frac{n+1}{2} = 7$ **h** $\frac{k+4}{3} - \frac{2k-3}{2} = -3$ **i** $\frac{3p-2}{5} - \frac{2p+1}{4} = 3$

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3D Solving equations after substitution

Formula

A formula is a mathematical relationship between two or more variables. For example:

- $a^2 = b^2 + c^2$ is Pythagoras' formula for relating the sides of a right-angled triangle; a, b and c are the variables
- $A = \pi r^2$ is a formula for relating the area and radius of a circle. A and r are the variables.

By substituting all the known variables into a formula, we are able to find the value of an unknown variable. If the unknown is not the subject of the equation, solve the equation.

USING A FORMULA

1. Write the formula.
2. Replace the variables in the formula with the numbers given in the question.
3. If the unknown is not the subject of the equation, solve the equation.
4. Evaluate using the calculator.
5. Write the answer to the specified level of accuracy and correct units if necessary.

Example 9: Substituting values 3D

A traffic cone has a volume of 4000 cubic centimetres and a height of 40 centimetres. Use the formula $r = \sqrt{\frac{3V}{\pi h}}$ to find the radius of the base of the cone. Answer correct to two decimal places.

SOLUTION:

1. Write the formula.

$$r = \sqrt{\frac{3V}{\pi h}}$$

2. Substitute 4000 for V and 40 for h into the formula.
3. Evaluate.
4. Express the answer correct to two decimal places using correct units.

$$= \sqrt{\frac{3 \times 4000}{\pi \times 40}}$$
$$= 9.772050238$$
$$\approx 9.77\,\text{cm}$$

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Exercise 3D

LEVEL 1

1 If $A = 4\pi r^2$ find the value of A (correct to one decimal place) when:

a $r = 3$ **b** $r = 8$ **c** $r = 24$

2 If $A = \pi ab$ find the value of A (correct to two decimal places) when:

a $a = 10, b = 5$ **b** $a = 22, b = 15$ **c** $a = 8, b = 3$

3 If $I = Prn$ find the value of I (correct to one decimal place) when:

a $P = 10, r = 0.2, n = 3$ **b** $P = 45, r = 0.1, n = 4$ **c** $P = 84, r = 0.3, n = 2$

4 If $A = P(1 + r)^n$ find the value of A (correct to two decimal places) when:

a $P = 9, r = 0.05, n = 5$ **b** $P = 6, r = 0.04, n = 3$ **c** $P = 8, r = 0.03, n = 8$

5 Find the value of S (correct to one decimal place) in the formula $S = V_0 - Dn$, if:

a $V_0 = 25, D = 2.5$ and $n = 2$

b $V_0 = 17.5, D = 5$ and $n = 3$

6 Use the formula $V = \frac{4}{3}\pi r^3$ to find the value of V (correct to the nearest whole number) when:

a $r = 6$ **b** $r = 5.2$ **c** $r = 7.1$

7 If $z = \frac{x - \overline{x}}{s}$ find the value of z when:

a $x = 12, \overline{x} = 8$ and $s = 2$ **b** $x = 24, \overline{x} = 4.5$ and $s = 4$

Example 9

8 The cost of hiring a rotary hoe is given by the rule $C = 25t + 900$ where C is the total cost in dollars and t is the number of hours for which the rotary hoe is hired. Find the cost of hiring a rotary hoe for:

a 6 hours **b** 8.5 hours **c** 24 hours

9 The formula used to convert temperature from degrees Fahrenheit to degrees Celsius is $C = \frac{5}{9}(F - 32)$. Use this formula to convert the following temperatures to degrees Celsius. Answer correct to the nearest whole number.

a 60°F **b** 80°F **c** 100°F

10 The circumference of a circle is given by the formula $C = 2\pi r$ where C is the circumference and r is the radius. Evaluate the radius if the circumference is 26 cm. Answer correct to two decimal places.

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11 A satellite is travelling at a speed of 22 500 km/h.
Use the formula $S = \frac{D}{T}$ to calculate the distance travelled by the satellite in:
a 24 hours **b** 1 week **c** 1 year (assuming exactly 52 weeks in a year)

12 Ohm's law is used to calculate the current (I), potential difference (V) and resistance (R). It is given by the formula $I = \frac{V}{R}$. Find the value of V, in volts, given that $I = 0.5$ amps and $R = 18$ ohms.

13 Use the formula $S = \frac{n}{2}(a + l)$ to evaluate n when $S = 720$ and $a = 21$ and $l = 3$.

14 The area of a trapezium is calculated using the formula $A = \frac{1}{2}(a + b)h$ where the parallel sides are a and b, and height is h. Find the height of the trapezium when the area of the trapezium is 45 cm^2 and its parallel sides are 12 cm and 8 cm.

15 Use the formula $d = 5t^2$ to evaluate t when $d = 300$. Answer correct to two decimal places.

16 Pythagoras' theorem $a^2 = b^2 + c^2$ relates the sides of a right-angled triangle. Determine the value of b when $c = 20$ mm and $a = 35$ mm. Answer in mm correct to two decimal places.

17 Use the formula $P = 3r - sr^2$ to evaluate s, given that $P = 48$ and $r = -2$.

18 If $m = 2n^3 - 3$, what is the value of n when $m = 13$?

19 If $R = \frac{1}{R_1} + \frac{1}{R_2}$, what is the value of R_1, given that $R = 1.15$ and $R_2 = 2.30$. Answer correct to two decimal places.

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LEVEL 2

20 The future value of an annuity is given by the formula $FV = M\left[\dfrac{(1+r)^n - 1}{r}\right]$. Find the future value (FV) when $M = 2600, n = 8$ and $r = 0.075$. Answer correct to the nearest cent.

21 The present value of an annuity is given by the formula $PV = M\left[\dfrac{(1+r)^n - 1}{r \times (1+r)^n}\right]$. Find the present value (PV), given $M = 900, n = 7$ and $r = 0.061$. Answer correct to the nearest dollar.

22 Use the formula $S = \frac{n}{2}[2a + (n-1)d]$ to evaluate a, given $S = 112, n = 8$ and $d = 3$.

23 The entrance to the Louvre is a square pyramid with base length of x, volume V and perpendicular height h. The base length is given by the formula $x = \sqrt{\dfrac{3V}{h}}$. Find the value of h, given that $V = 8200$ and $x = 35$. Answer correct to the nearest whole number.

24 Find the value of c in the formula $c = \sqrt{a^2 + b^2 - 2ab\cos C}$ when $a = 5, b = 6$ and $\cos C = 0.8$. Answer correct to one decimal place.

LEVEL 3

25 Use the formula $t = \sqrt{\dfrac{2(s-ut)}{a}}$ to find the value of s, given $u = 2, a = 5$ and $t = 15$. Answer correct to two decimal places.

26 Find the value of A in the formula $r = \sqrt{\dfrac{360A}{\pi\theta}}$ when $r = 2$ and $\theta = 20$. Answer correct to one decimal place.

27 The radius r of a sphere with volume V is given by the formula $r = \sqrt[3]{\dfrac{3V}{4\pi}}$. What is the volume of a sphere of radius 4 cm? Answer correct to the nearest whole number.

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3E Changing the subject of the formula

When an equation has a pronumeral with no numbers on the left-hand side of the equals sign, this pronumeral is the subject of the equation.

CHANGING THE SUBJECT OF THE FORMULA

Move the pronumerals and numbers, other than the pronumeral you want as the subject, to the right-hand side of the equation.
To move any term or number:

1. Look to perform the opposite operation (+ is opposite to −, × is opposite to ÷).
2. Add or subtract the same term or number to both sides of the equation.
3. Multiply or divide both sides of the equation by the same number.

Example 10: Changing the subject of the formula 3E

The total cost of a child's birthday party is given by the formula:

$$C = 40n + 75$$

where $C(\$)$ is the total cost and n is the number of children attending.

Make n the subject of the equation.

SOLUTION:

1. Write the formula.
2. The opposite operation to adding 75 is subtracting 75. Subtract 75 from both sides of the equation.
3. Rearrange the equation with the subject on the left-hand side.
4. The opposite operation to multiplying by 40 is dividing by 40. Divide both sides of the equation by 40.

$$C = 40n + 75$$
$$\overset{-75}{C} = \overset{-75}{40n + 75}$$
$$C - 75 = 40n$$
$$40n = C - 75$$
$$\frac{40n}{40} = \frac{C - 75}{40}$$
$$n = \frac{C - 75}{40}$$

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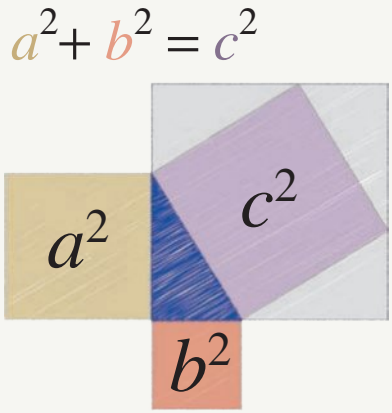
Example 11: Changing the subject of the formula 3E

Pythagoras' theorem is given by the formula $c^2 = a^2 + b^2$ where a, b and c are the sides of a right-angled triangle.

a Make a the subject of the formula.

b What is the length of a given $c = 7$ and $b = 5$? Answer correct to one decimal place.



SOLUTION:

1 Write the formula.

2 The opposite operation to adding b^2 is subtracting b^2. Subtract b^2 from both sides of the equation.

3 Rearrange the equation to make a the subject.

4 The opposite operation to squaring is to take the square root.

5 Substitute the 7 for c and 5 for b into the formula.

6 Evaluate.

7 Express the answer correct to one decimal place.

a
$$c^2 = a^2 + b^2$$
$$c^2 - b^2 = a^2$$
$$a^2 = c^2 - b^2$$
$$a = \sqrt{c^2 - b^2}$$

b
$$a = \sqrt{c^2 - b^2}$$
$$= \sqrt{7^2 - 5^2}$$
$$= \sqrt{24}$$
$$= 4.89897949$$
$$\approx 4.9$$

Example 12: Changing the subject of the formula 3E

Make r the subject of the compound interest formula $A = P(1 + r)^n$.

SOLUTION:

1 Write the formula.

2 The opposite operation to multiplying by P is dividing by P. Divide both sides of the equation by P.

3 The opposite operation of raising to the power of n is taking the nth root. Take the nth root of both sides of the equation.

4 The opposite operation to adding 1 is subtracting 1. Subtract 1 from both sides of the equation.

5 Rearrange the equation with the subject on the left-hand side.

$$A = P(1 + r)^n$$
$$\frac{A}{P} = (1 + r)^n$$
$$\sqrt[n]{\frac{A}{P}} = \sqrt[n]{(1 + r)^n}$$
$$= 1 + r$$
$$\sqrt[n]{\frac{A}{P}} = 1 + r$$
$$r = \sqrt[n]{\frac{A}{P}} - 1$$

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Exercise 3E

LEVEL 1

1 Make x the subject of the following formulae.

a $y = x + 3$ **b** $y = 5x$ **c** $y = x - 7$

d $y = 2x + 9$ **e** $y = 4 - x$ **f** $y = -7 + 3x$

2 Make x the subject of the following formulae.

a $y = \frac{x}{8}$ **b** $y = \frac{x}{2} + 1$ **c** $y = \frac{x}{3} - 1$

d $y = 7 - \frac{x}{6}$ **e** $y = 5 + \frac{x}{5}$ **f** $y = -2 - \frac{x}{3}$

3 The distance a car travels is given by the formula $d = st$ where s is the speed and t is the time.

a Make t the subject of the formula.

b Make s the subject of the formula.

c Use the formula to find the speed of a car that has travelled a distance of 112 km in a time of 1.75 hours.

4 The area of a rectangle A with a length l and breadth b is given by the formula $A = lb$.

a Make l the subject of the formula.

b Make b the subject of the formula.

c Find the breadth of a rectangle with an area of 16 cm^2 and length of 2.5 cm.

d Find the length of a rectangle with an area of 42 mm^2 and breadth of 16 mm.

5 The force F on an object with mass m and acceleration a is given by the formula $F = ma$.

a Make m the subject of the formula.

b Make a the subject of the formula.

c Use the formula to find the acceleration when $m = 2.5$ and $F = 10$.

d Use the formula to find the mass when $a = 16$ and $F = 72$.

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Example 10

6 Make b the subject of the following formulae.

a $a = b + c$
b $a = 6b - c$
c $c = a - b$
d $a = 2c + 6b$
e $c = -4a + 7b$
f $b + 3c = 8a$
g $a + 2b + c = 0$
h $5b - a - c = 0$
i $0 = a + 4c - 9b$

7 The formula $y = mx + b$ represents a straight line.

a Make b the subject of the formula.
b Make m the subject of the formula.
c Make x the subject of the formula.
d Find x when $y = 24, b = 18$ and $m = 2$.
e Find m when $y = 12, b = 4$ and $x = 2$.

8 The perimeter of a rectangle is given by the formula $P = 2(l + b)$ where l is the length and b the breadth.

a Make length the subject of the formula.
b What is the length if the perimeter of a rectangle is 68 m and the breadth is 9 m?
c Make breadth the subject of the formula.

9 A supervisor of a business uses the formula $w = 35(a + 1.5b + 2c)$ to calculate the wages where a, b and c are hours worked.

a Make a the subject of the formula.
b Make b the subject of the formula.
c Make c the subject of the formula.
d If $a = 40, b = 3$ and $w = 1925$, what is the value of c?

10 The area of an ellipse is $A = \pi ab$ where a is the length of the semi-major axis and b is the length of the semi-minor axis.

a Make b the subject of the formula.
b Calculate the length of the semi-minor axis if the area of the ellipse is $26\,\text{cm}^2$ and the length of the semi-major axis is 4 cm. Answer correct to two decimal places.

Example 11

11 Make x the subject of the following formulae.

a $y = x^2$
b $y = 3x^2 + 4$
c $y = 5 - 4x^2$
d $y = \sqrt{x}$
e $y = 7\sqrt{x}$
f $y = 2\sqrt{x} + 7$

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LEVEL 2

12 Write the formula $4m = 5n + 2p^2$ with p as the subject.

13 The volume of a cylinder is evaluated using $V = \pi r^2 h$ where h is the height and r is the radius.

- **a** Write the formula with h as the subject.
- **b** Write the formula with r as the subject.
- **c** Calculate the radius of a cylinder if the volume of the cylinder is $54\,\text{cm}^3$ and the height is 6 cm. Answer correct to two decimal places.

14 Rearrange the formula for the surface area of a sphere $A = 4\pi r^2$ to make r the subject.

15 The body mass index is $B = \dfrac{m}{h^2}$ where m is the mass (kg) and h is the height (m).

- **a** Make m the subject of the formula.
- **b** Make h the subject of the formula.
- **c** Find the height, correct to two decimal places, when:
  - **i** $B = 20$ and $m = 66\,\text{kg}$
  - **ii** $B = 30$ and $m = 102\,\text{kg}$

16 Write the formula $FV = M\left[\dfrac{(1+r)^n - 1}{r}\right]$ with M as the subject.

17 The time taken in seconds, T, for a pendulum of length (metres) L to make a single swing is $T = \sqrt{\dfrac{L}{9.8}}$.

- **a** Write the formula with L as the subject.
- **b** What is the length of the pendulum if it takes 4 seconds for a single swing?

18 Make $\cos C$ the subject in the formula $c^2 = a^2 + b^2 - 2ab\cos C$.

LEVEL 3

Example 12

19 The volume of a sphere is given by the formula $V = \dfrac{4}{3}\pi r^3$ where r is the radius.

- **a** Write the formula with r as the subject.
- **b** What is the radius in metres of a spherical balloon with a volume of $8\,\text{m}^3$? Answer in metres correct to two decimal places.

20 Write the formula $Z = \sqrt{R_2 + (2\pi fL)^2}$ with L as the subject.

21 Make A the subject of the formula $r = \sqrt[n]{\dfrac{A}{P}} - 1$.

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3F Blood alcohol content

Blood alcohol content (BAC) is a measure of the amount of alcohol in your blood. The measurement is the number of grams of alcohol in 100 millilitres of blood. For example, a BAC 0.05 means 0.05 g or 50 mg of alcohol in every 100 mL of blood. BAC is influenced by the number of standard drinks consumed in a given amount of time and a person's mass. Other factors that affect BAC include gender, fitness, health and liver function.

BLOOD ALCOHOL CONTENT (BAC)

Blood alcohol can be estimated using a BAC formula.

$$BAC_{\text{Male}} = \frac{(10N - 7.5H)}{6.8M} \text{ or } BAC_{\text{Female}} = \frac{(10N - 7.5H)}{5.5M}$$

BAC – Blood alcohol content
N – Number of standard drinks consumed
H – Hours drinking
M – Mass in kilograms

Example 13: Calculating the BAC 3F

Josip is 82 kg and has consumed 7 standard drinks in the past two hours. He was stopped by police for a random breath test. What would be an estimate of Josip's BAC using the formula above? Answer correct to three decimal places.

SOLUTION:

1. Write the formula.
2. Substitute the 7 for N, 2 for H and 82 for M into the formula.
3. Evaluate.
4. Express the answer correct to three decimal places.
5. Write the answer in words.

$$BAC_{\text{Male}} = \frac{(10N - 7.5H)}{6.8M}$$
$$= \frac{(10 \times 7 - 7.5 \times 2)}{(6.8 \times 82)}$$
$$= 0.098637$$
$$\approx 0.099$$

Josip's BAC is estimated to be 0.099.

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NSW has three blood alcohol limits: zero, 0.02 and 0.05. Zero BAC applies to all learner and provisional licence holders. This means you cannot drink at all and then drive, as you will be over the limit and likely to lose your licence. 0.02 applies to drivers of heavy vehicles, vehicles with dangerous loads, and drivers of public vehicles such as buses and taxis. The BAC is measured with a breathalyser or by analysing a sample of blood.

HOURS TO WAIT BEFORE DRIVING

The number of hours you should wait before driving can be estimated from your BAC by a formula.

$$\text{Number of hours} = \frac{\text{Blood alcohol content}}{0.015}$$

Example 14: Using BAC tables and a formula to estimate hours to wait before driving 3F

The table below shows the estimated BAC resulting from having 1 to 6 standard drinks, for five different body weights (kg).

	Body weight (kg)				
Drinks	**45**	**55**	**65**	**75**	**85**
1	0.008	0.007	0.006	0.005	0.004
2	0.041	0.033	0.028	0.025	0.022
3	0.074	0.060	0.051	0.044	0.039
4	0.106	0.087	0.074	0.064	0.056
5	0.139	0.114	0.096	0.083	0.074
6	0.172	0.140	0.119	0.103	0.091

Terry weighs 65 kg and consumes four standard drinks in an hour. Calculate the number of hours to wait before driving. (Answer to the nearest hour.)

SOLUTION:

1. Write the formula.
2. From the table, Terry's BAC is 0.074. Substitute 0.074 into the formula.
3. Evaluate.
4. Write the answer correct to the nearest hour.
5. Write the answer in words.

$$\text{Number of hours} = \frac{BAC}{0.015}$$
$$= \frac{0.074}{0.015}$$
$$= 4.933\,33$$
$$\approx 5$$

Terry waits 5 hours to drive.

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Exercise 3F

LEVEL 1

1 Calculate the BAC for the following females. Answer correct to two decimal places.

- **a** Sarah is 48 kg and has consumed 4 standard drinks in the past 2 hours.
- **b** Sienna is 59 kg and has consumed 3 standard drinks in the past hour.
- **c** Alyssa is 81 kg and has consumed 6 standard drinks in the past 2 hours.
- **d** Kayla is 65 kg and has consumed 8 standard drinks in the past 6 hours.
- **e** Tahlia is 71 kg and has consumed 13 standard drinks in the past 3 hours.
- **f** Mia is 55 kg and has consumed 9 standard drinks in the past 5 hours.

Example 13

2 Calculate the BAC for the following males. Answer correct to two decimal places.

- **a** Wei is 53 kg and has consumed 3 standard drinks in the past 3 hours.
- **b** Riley is 64 kg and has consumed 5 standard drinks in the past hour.
- **c** Thomas is 98 kg and has consumed 2 standard drinks in the past 2 hours.
- **d** Zachary is 47 kg and has consumed 10 standard drinks in the past 5 hours.
- **e** Charlie is 85 kg and has consumed 12 standard drinks in the past 4 hours.
- **f** Jacob is 104 kg and has consumed 7 standard drinks in the past 6 hours.

3 James and Olivia are twins and both weigh 73 kg. At a party they each consume 6 standard drinks in two hours.

- **a** What is James's BAC? Answer correct to two decimal places.
- **b** What is Olivia's BAC? Answer correct to two decimal places.
- **c** How long does James need to wait before he drives home?

4 Calculate the time to wait before driving. Answer to the nearest minute.

- **a** BAC of 0.056
- **b** BAC of 0.123
- **c** BAC of 0.087
- **d** BAC of 0.153
- **e** BAC of 0.092
- **f** BAC of 0.172

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Example 14 **5** The table below shows the BAC after one hour for a range of drinks and body weights.

	Body weight (kg)							
Drinks	**45**	**55**	**65**	**75**	**85**	**95**	**105**	**115**
1	0.008	0.007	0.006	0.005	0.004	0.004	0.004	0.003
2	0.041	0.033	0.028	0.025	0.022	0.019	0.018	0.016
3	0.074	0.060	0.051	0.044	0.039	0.035	0.032	0.029
4	0.106	0.087	0.074	0.064	0.056	0.050	0.046	0.042
5	0.139	0.114	0.096	0.083	0.074	0.066	0.060	0.054
6	0.172	0.140	0.119	0.103	0.091	0.081	0.074	0.067

Calculate the time to wait before driving. (Answer to the nearest minute.)

a Joshua weighs 85 kg and consumes 5 standard drinks in an hour.
b Mitchell weighs 115 kg and consumes 3 standard drinks in an hour.
c Harrison weighs 45 kg and consumes 6 standard drinks in an hour.
d Cooper weighs 65 kg and consumes 2 standard drinks in an hour.
e Zachary weighs 95 kg and consumes 4 standard drinks in an hour.
f Aarav weighs 75 kg and consumes 1 standard drink in an hour.

6 Use the above table to construct two separate column graphs for the following body weights. Make the number of drinks the horizontal axis and the BAC the vertical axis.

a Body weight of 45 kg
b Body weight of 115 kg

7 The formula for calculating 'standard drinks' is $S = V \times A \times 0.789$ where S is the number of standard drinks, V is the volume of drink in litres and A is the percentage of alcohol. How many standard drinks are in each of the following drinks? Answer correct to one decimal place.

a 345 mL bottle of full strength beer at 5.2% alcohol
b 750 mL bottle of champagne at 13.5% alcohol
c 150 mL glass of white wine at 12.5% alcohol
d Mixed drink with a 30 mL of brandy at 38% alcohol
e 360 mL can of light beer at 2.1% alcohol

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LEVEL 2

8 Find the value of BAC using the formula Number of hours $= \frac{BAC}{0.015}$, given the number of hours to wait before driving.

a Number of hours to wait before driving is 5 (Answer correct to three decimal places.)
b Number of hours to wait before driving is 3 (Answer correct to three decimal places.)

LEVEL 3

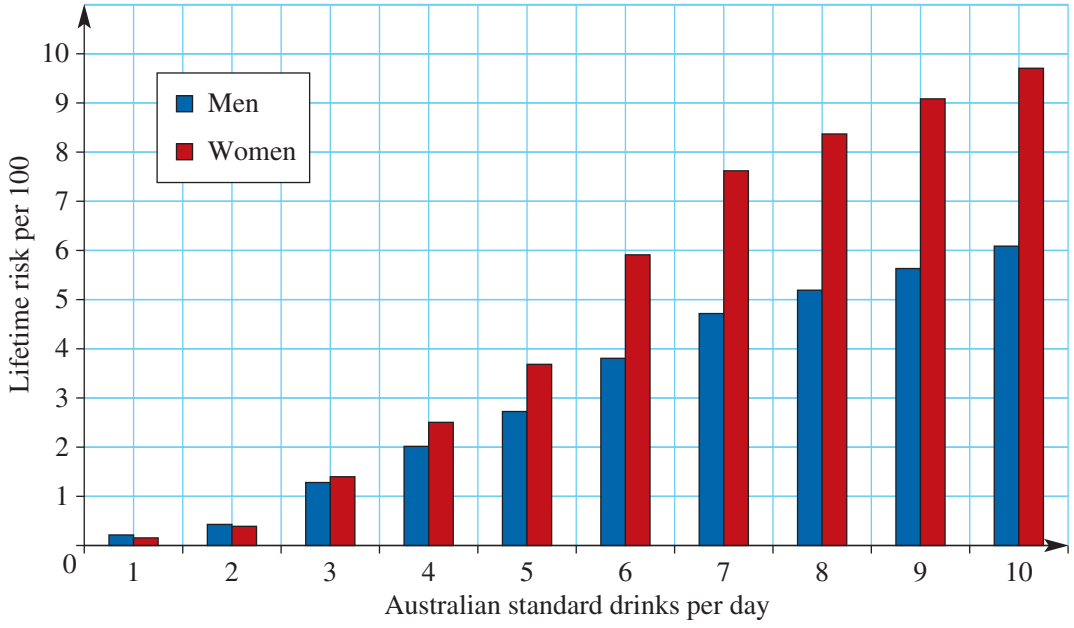
9 Find the value of H in the formula $BAC_{\text{Male}} = \frac{(10N - 7.5H)}{6.8M}$ given:

a $BAC_{\text{Male}} = 0.066$, $M = 60$ and $N = 5$. (Answer correct to the nearest minute.)
b $BAC_{\text{Male}} = 0.050$, $M = 79$ and $N = 7$. (Answer correct to the nearest minute.)

10 Find the value of N in the formula $BAC_{\text{Female}} = \frac{(10N - 7.5H)}{5.5M}$ if:

a $BAC_{\text{Female}} = 0.066$, $M = 48$ and $H = 2$ (Answer correct to one decimal place.)
b $BAC_{\text{Female}} = 0.120$, $M = 57$ and $H = 4$ (Answer correct to one decimal place.)

11 The graph below relates the lifetime risk of death to the number of standard drinks consumed per day.



a What is the lifetime risk for a female and a male who consume 7 drinks per day?
b Why is the effect of alcohol greater on a female than on a male?

3G Medication

Australians have access to free or low-cost medicines. When a medicine is prescribed, the amount per dose, the frequency of dosage and the dosages for different medication types are calculated. Medications involve concentrations that are expressed as mass per volume, for example 5 g/10 mL. A concentration is a measure of how much of a given substance is mixed with another substance.

Example 15: Converting a concentration 3G

The concentration of a medicine is given as 100 mg per 50 mL. What is the dosage rate for this medicine in g/mL?

SOLUTION:

1	Write the rate as a fraction. The numerator is 100 mg and the denominator is 50 mL.	$100\,\text{mg}/50\,\text{mL} = \frac{100\,\text{mg}}{50\,\text{mL}}$
2	Divide the numerator by the denominator.	$= 2\,\text{mg/mL}$
3	Multiply by 0.001 to convert milligrams to grams.	$= 2 \times 0.001\,\text{g/mL}$
4	Evaluate.	$= 0.002\,\text{g/mL}$
5	Write the answer using the correct units.	The concentration is 0.002 g/mL.

Example 16: Using a formula for medication 3G

This formula is used to change the dosage given in tablet form to the equivalent concentration in liquid form.

$$\text{Volume required (mL)} = \frac{\text{Strength required (mg)}}{\text{Stock strength (mg)}} \times \text{Volume of stock (mL)}$$

Joseph is prescribed 1000 mg of a mild painkiller. The medication available contains 100 mg in 5 mL. How much medication should Joseph be given?

SOLUTION:

1	Write the formula.	$\text{Volume required} = \frac{1000}{100} \times 5$
2	Substitute the strength required (1000), the stock strength (100) and the volume of stock (5) into the formula.	
3	Evaluate.	$= 50\,\text{mL}$
4	Write the answer in words.	The medication required is 50 mL.

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Formula for children and infants

There are three formulae used to calculate the required dosages for children and infants.

Name	Purpose	Formula
Fried's rule	Dosage for children 1–2 years	$\text{Dosage} = \dfrac{\text{Age of infant (months)} \times \text{Adult dose}}{150}$
Young's rule	Dosage for children 1–12 years	$\text{Dosage} = \dfrac{\text{Age of child (years)} \times \text{Adult dose}}{\text{Age of child (years)} + 12}$
Clark's rule	Dosage for children General	$\text{Dosage} = \dfrac{\text{Weight (kg)} \times \text{Adult dose}}{70}$

Example 17: Calculating dosages for children 3G

Young's rule is used to prescribe medicine for infants. The formula is:

$$D = \frac{yA}{y+12}$$

The variable D represents the infant's dosage (milligrams), y represents the infant's age (years) and A is the adult dosage (milligrams).

What is the dosage for a 3-year-old child if the adult dosage is 50 mg?

SOLUTION:

1 Write the formula.

$$D = \frac{yA}{y+12}$$

2 Substitute the values for y (3) and A (50) into the formula.

3 Evaluate.

$$= \frac{3 \times 50}{3+12} = \frac{150}{15}$$

$$= 10 \text{ mg}$$

4 Express the answer using the correct units.

5 Write the answer in words.

The child's dosage is 10 mg.

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Exercise 3G

LEVEL 1

1 There are 250 capsules in a container labelled as containing 30 g of a drug. How many milligrams of the drug is in each capsule?

2 A medicine bottle has a capacity of 0.75 L.
- **a** How many tablespoons (15 mL) of medicine does the bottle contain?
- **b** How many teaspoons (5 mL) of medicine does the bottle contain?
- **c** The dosage required is 20 mL, three times a day. For how many days will the bottle last?

Example 15

3 Use the rate provided to answer the following questions.
- **a** The concentration of a drug is 25 mg/50 mL. How many mg are there in 10 mL?
- **b** The concentration of a drug is 100 mg/5 mL. How many mg are there in 2 mL?
- **c** The concentration of a drug is 18 g/100 mL. How many mg are there in 10 mL?

4 Use Fried's rule to find the required infant dose.
- **a** Mikayla aged 6 months, if the adult dose is 15 mL
- **b** Flynn aged 12 months, if the adult dose is 10 mL
- **c** Lara aged 15 months, if the adult dose is 20 mL
- **d** Toby aged 2 years, if the adult dose is 10 mL

Example 17

5 Use Young's rule to find the required dose for these children. Answer to the nearest millilitre.
- **a** Christian who is 4 years old, if the adult dose is 10 mL
- **b** Indiana who is 8 years old, if the adult dose is 15 mL
- **c** Gemma who is 3 years old, if the adult dose is 10 mL
- **d** Alika who is 72 months old, if the adult dose is 30 mL

6 Use Clark's rule to find the required dose for these children. Answer to the nearest millilitre.
- **a** Harper who weighs 40 kg, if the adult dose is 20 mL
- **b** Aaron who weighs 30 kg, if the adult dose is 10 mL
- **c** Eve who weighs 10 kg, if the adult dose is 15 mL
- **d** Tyson who weighs 25 kg, if the adult dose is 20 mL

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LEVEL 2

7 The medical equipment shown inserts a fluid drip by drip into the blood. Use this formula for the following questions:

$$\text{Drip rate} = \frac{\text{Volume (mL)}}{\text{Time (h)}}$$

a Leo receives 1.5 litres of fluid over 8 hours. What is the required drip rate in mL/h?

b Aanya receives 2 litres of fluid over 5 hours. What is the required drip rate in mL/h?

Example 16

8 Use this formula for the following questions:

$$\text{Volume} = \frac{\text{Strength required (mg)}}{\text{Stock strength (mg)}}$$

where 'stock strength' is the strength of the medicine available.

a Alicia is prescribed 750 mg of medication. Tablets, each of 500 mg, are available. How many tablets should Alicia receive?

b Hugo is prescribed 450 mg of medication. Tablets, each of 200 mg, are available. How many tablets should Hugo receive?

9 Use this formula for the following questions:

$$\text{Volume required (mL)} = \frac{\text{Strength required (mg)}}{\text{Stock strength (mg)}} \times \text{Volume of stock (mL)}$$

a Eden is prescribed 50 mg of a mild painkiller. The medication available contains 20 mg in 5 mL. How much medication should be given to Eden?

b Sofia is prescribed 40 mg of a mild painkiller. The medication available contains 50 mg in 15 mL. How much medication should be given to Sofia?

LEVEL 3

10 Alex buys a 200 mL bottle of cough mixture.

a The recommended dose for a 3-year-old child is 5 mL. What is the adult dose using Young's rule?

b How many adult doses are contained in the bottle?

c It is recommended the cough mixture be taken at most 4 times a day. How many days will the cough mixture last at this rate for an adult?

11 The drip rate is the total volume divided by total time. If Claire needed 300 mL of fluid to go through in 2 hours, calculate the drip rate (1 drop = 0.05 mL) in the following units:

a mL per hour

b mL per minute

c drops per hour

d drops per minute

PPT STUDY GUIDE

Key ideas and chapter summary

Substitution

1 Write the algebraic expression.
2 Replace the variables in the expression with the numbers.
3 Evaluate using the calculator.
4 Write the answer to the specified level of accuracy.

Distance, speed and time

$S = \frac{D}{T}$ or $T = \frac{D}{S}$ or $D = S \times T$

D – Distance, S – Speed
T – Time

Stopping distance = Reaction-time distance + Braking distance

Linear equations

1 Look to perform the opposite operation.
2 Add or subtract the same number from both sides.
3 Multiply or divide both sides of the equation by the same number.
4 To solve two- or three-step equations repeat the above steps.

Solving equations after substitution

1 Write the formula.
2 Replace the variables in the formula with the numbers.
3 Solve the equation if the unknown is not the subject.
4 Evaluate using the calculator.
5 Write the answer to the specified level of accuracy.

Changing the subject of the formula

Move the pronumerals and numbers, other than the pronumeral you want as the subject, to the right-hand side of the equation.

Blood alcohol content

$BAC_{\text{Male}} = \frac{(10N - 7.5H)}{6.8M}$ or

$BAC_{\text{Female}} = \frac{(10N - 7.5H)}{5.5M}$

BAC – Blood alcohol content
N – Number of standard drinks
H – Hours drinking
M – Mass in kilograms

Number of hours $= \frac{BAC}{0.015}$

BAC – Blood alcohol content

Medication

Medications involve concentrations expressed as mass/volume such as 5 g/10 mL.

Fried's rule, Young's rule and Clark's rule are used to calculate the required dosages for children and infants.

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Review

Multiple-choice questions

1 Evaluate $\sqrt{x^2 + y}$ if $x = 5$ and $y = 1$. Answer correct to two decimal places.

A 5.00 **B** 5.09
C 5.10 **D** 6.00

2 How long will it take a vehicle to travel 342 km at a speed of 70 km/h?

A 0.20 h **B** 2.394 h
C 4.89 h **D** 272 h

3 What is the solution to $\frac{x}{3} = -8$?

A $x = -24$ **B** $x = -\frac{8}{3}$
C $x = \frac{8}{3}$ **D** $x = 24$

4 Solve the equation $11v - 8 = 27$.

A $v = \frac{11}{35}$ **B** $v = \frac{11}{15}$
C $v = 1\frac{8}{11}$ **D** $v = 3\frac{2}{11}$

5 If $p = \sqrt{q^2 + r^2}$ find the value of p when $q = 8$ and $r = 6$.

A $\sqrt{14}$ **B** $\sqrt{28}$
C 10 **D** 100

6 Use the formula $A = \frac{bh}{2}$ to find the base, b, of a triangle when A is 14 and h is 4.

A 1.75 **B** 7
C 28 **D** 112

7 Make w the subject of the equation $u = -5w + v$.

A $w = \frac{1}{5}u - v$ **B** $w = -\frac{1}{5}u - v$
C $w = \frac{u - v}{5}$ **D** $w = \frac{v - u}{5}$

8 Layla is 61 kg and has consumed 5 standard drinks in the past four hours.

Calculate Layla's blood alcohol content using the formula $BAC_{\text{Female}} = \frac{(10N - 7.5H)}{5.5M}$.

A 0.007 **B** 0.048
C 0.059 **D** 0.060

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Short-answer questions

1 Evaluate P if $P = 2l + 2b$ and:

a $l = 12$ and $b = 10$

b $l = 30$ and $b = 21$

2 The circumference of a circle is given by the formula $C = 2\pi r$ where C is the circumference and r is the radius. Evaluate the circumference of a circle with radius 14 cm. Answer correct to two decimal places.

3 Ihorangi travels 51 km to work and 51 km from work each day.

a How many kilometres does he travel to and from work in a 5-day working week?

b Ihorangi drives a car with a fuel consumption of 7.5 L/100 km to and from work each week. How many litres of petrol does Ihorangi use travelling to and from work?

c What is Ihorangi's weekly petrol bill for work if petrol costs $1.52 per litre?

4 A car is travelling at 70 km/h and the reaction time of the driver is 0.50 seconds.

a Find the stopping distance of this car using the formula $d = \frac{5Vt}{18} + \frac{V^2}{170}$.

b How much further would it take to stop this car if the reaction time of the driver was 2 seconds?

5 Solve the following linear equations.

a $e - 2 = 6$ **b** $6b = 27$ **c** $2 + w = 11$

d $2n + 5 = 17$ **e** $9 = 3a - 6$ **f** $8q + 4 = 36$

g $\frac{h}{7} = 14$ **h** $d + 1\frac{2}{5} = 2\frac{1}{10}$ **i** $7y = 3\frac{1}{2}$

6 Solve the following linear equations.

a $7p + 2 = 6p - 6$ **b** $3 - 2s = 3s + 12$ **c** $1 + p = 2p - 1$

d $\frac{d}{2} + 5 = 4d + 2$ **e** $5(2y + 5) = 15$ **f** $9(1 - 3a) = 21$

7 Solve these equations.

a $\frac{2c + 5}{3} = 6$ **b** $\frac{x}{2} + \frac{x}{6} = 1$ **c** $\frac{1}{3}(5u + 7) = 4$

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Review

8 Use the formula $A = \sqrt[3]{\frac{4B}{2\pi}}$ to find the value of A (correct to two decimal places) when:

a $B = 5$ **b** $B = 20$ **c** $B = 150$

9 The kinetic energy K (joules) of a particle of mass m kg with a velocity of v m/s, is given by the formula $K = \frac{1}{2}mv^2$.

a Find m when $K = 5.2$ joules and $v = 2$ m/s.

b Find v when $K = 6.4$ joules and $m = 1.3$ kg. Answer correct to three decimal places.

10 Make $\sin C$ the subject of the formula $A = \frac{1}{2}ab\sin C$.

11 The formula $v^2 = u^2 + 2as$ is used to relate acceleration, velocity and distance.

a Make s the subject of the formula.

b Make u the subject of the formula.

12 Levi weighs 74 kg and consumes five standard drinks in an hour.

a Calculate Levi's blood alcohol content: $(BAC_{\text{Male}} = \frac{(10N - 7.5H)}{6.8M})$.

b What is the number of hours Levi will need to wait before driving?
(Number of hours $= \frac{BAC}{0.015}$).

Extended-response questions

13 Anna and Bridget live 33 km apart. They cycle to meet each other. Anna travels at 10 km/h and Bridget travels at 12 km/h.

a How long until they meet each other?

b What distance have they each travelled?

14 A landscaper charges \$120 up front and \$65 for each hour, h, that he works.

a Write a linear equation for the total charge, C, of any job.

b How much would a 4-hour job cost?

PDF LITERACY

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4 Measurement and energy

Syllabus topic — M1.1 Practicalities of measurement
M1.3 Units of energy and mass

This topic provides students with the opportunity to appreciate inherent errors in measurements and to become competent in solving practical problems involving energy.

Outcomes

- Review the use of different units of measurement.
- Calculate the absolute and percentage error in a measurement.
- Use standard form and standard prefixes in the context of measurement.
- Express numbers to a certain number of significant figures.
- Use units of energy and mass related to food and nutrition.
- Use units of energy to describe consumption of electricity.
- Investigate common appliances in terms of energy consumption.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Solutions (enabled by teacher)
- Literacy worksheet
- Widgets
- Quick Quiz
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

In the Interactive Textbook you can take a test of prior knowledge required for this chapter, and depending on your score you may be directed to revision from the previous years' work.

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4A Units of measurement

Measurement is used to determine the size of a quantity. It usually involves using a measuring instrument. For example, to measure length, instruments that can be used include the rule, tape measure, caliper, micrometer, odometer, laser range finder, and GPS. There are a number of systems of measurement that define their units of measurement. We use the SI metric system.

SI units

The SI ('Systeme International', or International System of Units of Measurement), is based on multiples of 10. It is a version of the metric system that allows easy multiplication when converting between related units. Units shown in black are SI and those in red (below) are non-SI units approved for everyday or specialised use alongside SI units.

Quantity	Name of unit	Symbol	Value
Length	Metre	m	Base unit
	Millimetre	mm	1000 mm = 1 m
	Centimetre	cm	100 cm = 1 m
	Kilometre	km	1 km = 1000 m
	Nautical mile	nm	1 nm = 1852 m
Area	Square metre	m^2	Base unit
	Square centimetre	cm^2	10 000 cm^2 = 1 m^2
	Hectare	ha	1 ha = 10 000 m^2
Volume	Cubic metre	m^3	Base unit
	Cubic centimetre	cm^3	1 000 000 cm^3 = 1 m^3
	Litre	L	1 L = 1000 cm^3
	Millilitre	mL	1000 mL = 1 L
	Kilolitre	kL	1 kL = 1000 L
Mass	Kilogram	kg	Base unit
	Gram	g	1000 g = 1 kg
	Milligram	mg	1000 mg = 1 g
	Tonne	t	1 t = 1000 kg

Mass

Mass is a measure of the amount of matter in a body. There is a difference between mass and weight. Weight is the measure of the amount of force acting on the mass due to gravity. However if you're on the surface of the Earth and not moving, mass and weight can be considered to be the same in everyday contexts. If you change your location with respect to gravity, mass will remain unchanged, but weight will not.

Converting between SI units of the same type

The SI prefix to the unit indicates the conversion between units of the same type. It indicates a multiple of 10. Common prefixes are tera (10^{12}), giga (10^9), mega (10^6), kilo (10^3), centi (10^{-2}), milli (10^{-3}), micro (10^{-6}) and nano (10^{-9}).

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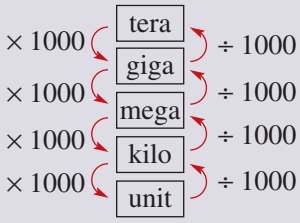
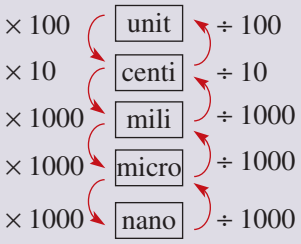
LENGTH, MASS AND VOLUME

To change from one unit to another, multiply or divide by a power of 10.





Example 1: Converting units of length 4A

Complete the following.

a 35 cm = ☐ mm **b** 4500 m = ☐ km

SOLUTION:

1 To change cm to mm multiply by 10.

a 35 cm = 35 × 10 mm
= 350 mm

2 To change m to km divide by 1000.

b 4500 m = 4500 ÷ 1000 km
= 4.5 km

Example 2: Converting units of mass 4A

Complete the following.

a 5.2 kg = ☐ g **b** 3850 kg = ☐ t

SOLUTION:

1 To change kilograms to grams multiply by 1000.

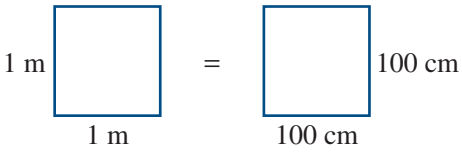
a 5.2 kg = 5.2 × 1000 = 5200 g

2 To change kilograms to tonnes divide by 1000.

b 3850 kg = 3850 ÷ 1000 = 3.85 t

Converting area and volume units

To convert area units, change the side length units and compare the values for area.

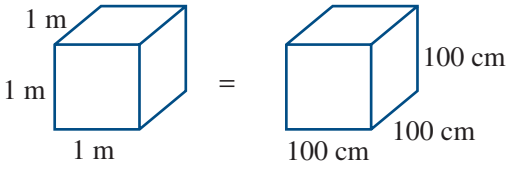


$1\,m^2 = 100 \times 100 = 10\,000\ cm^2$

$1\,m^2 = 10\,000\ cm^2$

or $1\,cm^2 = \frac{1}{10\,000}\,m^2$

To convert volume units, change the side length units and compare the values for volume.



$1\,m^3 = 100 \times 100 \times 100 = 1\,000\,000\ cm^3$

$1\,m^3 = 1\,000\,000\ cm^3$

or $1\,cm^3 = \frac{1}{1\,000\,000}\,m^3$

Exercise 4A

LEVEL 1

Example 1

1 Complete the following.

a 5 cm = ☐ mm
b 78 m = ☐ cm
c 2 km = ☐ m
d 890 m = ☐ cm
e 57 cm = ☐ mm
f 6 km = ☐ cm
g 9400 m = ☐ km
h 600 mm = ☐ cm
i 8100 cm = ☐ m
j 49 000 cm = ☐ km
k 22 000 m = ☐ km
l 51 mm = ☐ cm

Example 2

2 Complete the following.

a 3 t = ☐ kg
b 45 kg = ☐ g
c 76 t = ☐ kg
d 8100 kg = ☐ g
e 4 t = ☐ g
f 0.52 t = ☐ kg
g 6800 g = ☐ kg
h 9 300 000 g = ☐ t
i 45 000 000 g = ☐ t
j 300 kg = ☐ t
k 2300 g = ☐ kg
l 60 000 g = ☐ kg

3 Complete the following.

a 2 L = ☐ mL
b 12 kL = ☐ L
c 9 kL = ☐ mL
d 7800 kL = ☐ L
e 50 L = ☐ mL
f 300 kL = ☐ mL
g 6100 L = ☐ kL
h 400 mL = ☐ L
i 210 000 mL = ☐ kL
j 80 mL = ☐ L
k 79 000 mL = ☐ kL
l 8 000 000 mL = ☐ kL

4 What unit of length is most appropriate to measure each of the following?

a Length of a pen
b Height of a building
c Thickness of a credit card
d Distance from Sydney to Newcastle
e Height of a person
f Length of a football field

5 What unit of mass is most appropriate to measure each of the following?

a Weight of an elephant
b Mass of a mug
c Bag of onions
d Weight of a baby
e Mass of a truck
f Mass of a teaspoon of sugar

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6 There are 20 litres of a chemical stored in a container.

a What amount of chemical remains if 750 mL is removed from the container? Answer in litres.

b How many containers are required to hold a kilolitre of the chemical?

7 The length of the Murray River is 2575 km. The length of the Hawkesbury River is 80 000 m. What is the difference in their lengths? Answer in metres.

8 Christopher bought 3 kg of sultanas. What mass of sultanas remains if he ate 800 grams? Answer in kilograms.

9 A truck is loaded with 3000 bricks, each of mass 4 kg. Find the total mass (in *t*) of the bricks.

10 The mass of a box of cereal is 375 g. What is the total mass (in kg) of 15 boxes of cereal?

11 Find the total mass (in g) of five items of mass 250 mg, 1100 mg, 0.7 g, 5.95 g, and 15.4 g.

12 An empty plastic box has mass 750 g. It is packed with 8 bottles of drink, each of mass 1.25 kg. Calculate the total mass (in g) of the packed box.

13 There are three tonnes of grain in a truck. What is the mass if another 68 kg of grain is added to the truck? Answer in kilograms.

14 Find the total mass (in kg) of the following contents of a shopping trolley:

muesli, 600 g	instant coffee, 250 g	frozen peas, 1 kg
rice, 3 kg	self-raising flour, 2 kg	strawberry jam, 400 g

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LEVEL 2

15 The distance from John's home to Bankstown is 15.9 km

- **a** John travels to Bankstown and back 5 times each week. What is the total distance travelled?
- **b** John's car can travel 8 km on 1 litre of petrol. How many litres of petrol will John use each week travelling to Bankstown and back?

16 A cyclist travels to and from work over a 1200-metre long bridge. Calculate the distance travelled in a week if the cyclist works for 5 days. Answer in kilometres.

17 Madison travels 32 km to work each day. Her car uses 1 litre of petrol to travel 8 km.

- **a** How many litres of petrol will she use to get to work?
- **b** How many litres of petrol will she use for 5 days of work, including return travel?

18 Arrange 500 m, 0.005 km, 5000 cm and 5 000 000 mm in:

- **a** ascending order (smallest to largest)
- **b** descending order (largest to smallest)

19 Complete the following.

- **a** $1\text{ km}^2 = \square\text{ m}^2$
- **b** $1\text{ m}^2 = \square\text{ mm}^2$
- **c** $1\text{ cm}^2 = \square\text{ mm}^2$
- **d** $1000\text{ cm}^2 = \square\text{ m}^2$
- **e** $2000\text{ mm}^2 = \square\text{ cm}^2$
- **f** $5000\text{ m}^2 = \square\text{ km}^2$
- **g** $3.9\text{ m}^2 = \square\text{ cm}^2$
- **h** $310\text{ km}^2 = \square\text{ m}^2$
- **i** $4.7\text{ m}^2 = \square\text{ mm}^2$
- **j** $74\,300\text{m}^2 = \square\text{ km}^2$
- **k** $6500\text{ mm}^2 = \square\text{ cm}^2$
- **l** $4000\text{ cm}^2 = \square\text{ m}^2$

20 The area of a field is 80 000 square metres. Convert the area units to the following.

- **a** Square kilometres
- **b** Hectares

LEVEL 3

21 Jackson swims 30 lengths of a 50-metre pool.

- **a** How many kilometres does he cover?
- **b** If his goal is 4 kilometres, how many more lengths must he swim?

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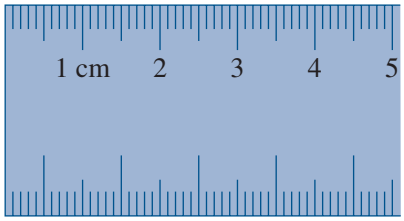
4B Measurement errors

There are varying degrees of instrument error and measurement uncertainty when measuring. Every time a measurement is repeated, with a sensitive instrument, a slightly different result will be obtained. The possible sources of errors include mistakes in reading the scale, parallax error and calibration error. The accuracy of a measurement is improved by making multiple measurements of the same quantity with the same instrument.

Accuracy in measurements

The smallest unit on the measuring instrument is called the precision or limit of reading. For example, a 30 cm ruler with a scale for millimetres has a precision of 1 mm. The accuracy of a measurement is restricted to plus or minus half $\left(\pm\frac{1}{2}\right)$ of the precision. For example, if the measurement on the ruler is 10 mm then the range of errors is 10 ± 0.5 mm. Here the upper bound is $10 + 0.5$ mm or 10.5 mm and the lower bound is $10 - 0.5$ mm or 9.5 mm.



Every measurement is an approximation and has an error. The absolute error is the difference between the actual value and the measured value indicated by the instrument. The maximum value for an absolute error is $\frac{1}{2}$ of the precision.

PRECISION	ABSOLUTE ERROR	UPPER BOUND	LOWER BOUND
Smallest unit on measuring instrument or limit of reading	Measured value – Actual value $\frac{1}{2} \times$ precision	Measurement + Absolute error	Measurement – Absolute error

Relative error gives an indication of how good a measurement is relative to the size of the quantity being measured. The relative error of a measurement is calculated by dividing the precision by the actual measurement. For example, the relative error for the above measurement is $\left(\frac{0.5}{10}\right) = 0.05$. The relative error is often expressed as a percentage and called the 'percentage error'. For example, the percentage error for the above measurement is $\left(\frac{0.5}{10}\right) \times 100 = 5\%$.

RELATIVE ERROR	PERCENTAGE ERROR
$\frac{\text{Absolute error}}{\text{Measurement}}$	$\frac{\text{Absolute error}}{\text{Measurement}} \times 100\%$

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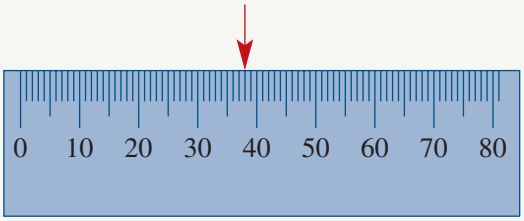
Example 3: Finding the measurement errors 4B

a What is the length indicated by the arrow on this ruler?



b What is the precision or limit of reading?
c What is the upper and lower bound for each measurement?
d Find the relative error. Answer correct to three decimal places.
e Find the percentage error. Answer correct to one decimal place.

SOLUTION:

1 The arrow is pointing to 38 mm.

a Length is 38 mm.

2 Precision is the smallest unit on the ruler (millimetre).

b Precision is 1 mm.

3 Calculate half the precision.

c $\frac{1}{2} \times \text{precision} = \frac{1}{2} \times 1$
$= 0.5\,\text{mm}$

4 Lower bound is the measured value minus $\frac{1}{2}$ the precision.

$\text{Lower bound} = 38 - 0.5 = 37.5\,\text{mm}$

5 Upper bound is the measured value plus $\frac{1}{2}$ the precision.

$\text{Upper bound} = 38 + 0.5 = 38.5\,\text{mm}$

6 Write the formula for relative error.

d $\text{Relative error} = \frac{\text{Absolute error}}{\text{Measurement}}$

7 Substitute the values for absolute error and the measurement.

$= \frac{0.5}{38}$

8 Evaluate correct to three decimal places.

≈ 0.013

9 Write the formula for percentage error.

e $\text{Percentage error} = \frac{\text{Absolute error}}{\text{Measurement}} \times 100\%$

10 Substitute the values for absolute error and the measurement.

$= \frac{0.5}{38} \times 100\%$

11 Evaluate.

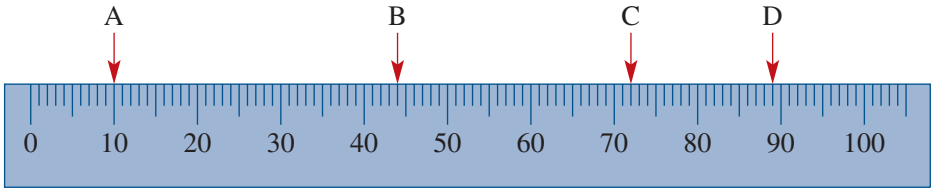
$\approx 1.3\%$

Exercise 4B

LEVEL 1

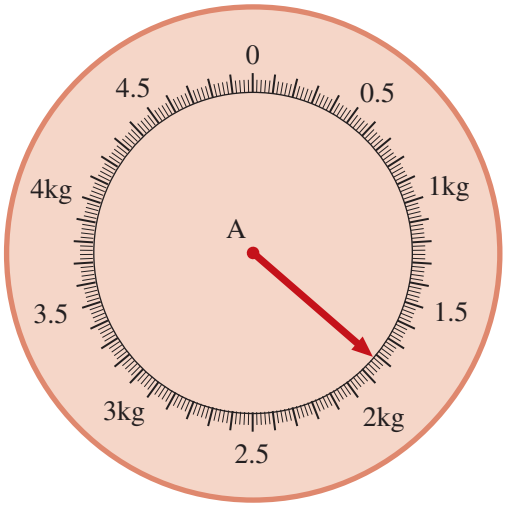
Example 3

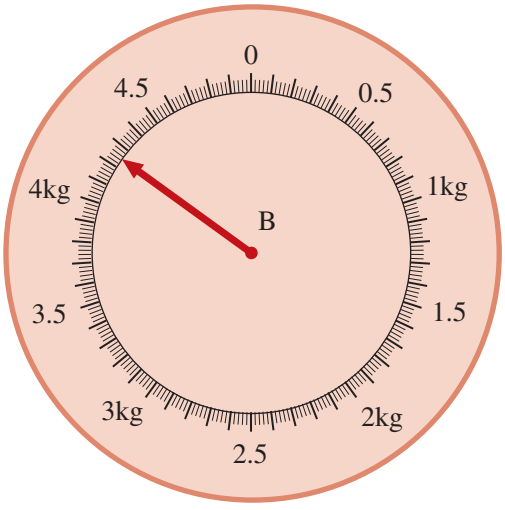
1 Four measurements of length are shown on the ruler below.



a What length is indicated by each letter? Answer to the nearest millimetre.
b What is the precision or limit of reading?
c What the largest possible absolute error?
d What is the upper and lower bound for each measurement?
e Calculate the relative error, correct to three decimal places, for each measurement.
f Calculate the percentage error, correct to one decimal place, for each measurement.

2 Two measurements of mass are shown on the scales below.





a What mass is indicated by A and B?
b What is the precision or limit of reading?
c What the largest possible absolute error?
d What is the upper and lower bound for each measurement?
e Calculate the relative error, correct to three decimal places, for each measurement.
f Calculate the percentage error, correct to one decimal place, for each measurement.

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LEVEL 2

3 A dishwasher has a mass of exactly 49.6 kg. Abbey measured the mass of the dishwasher as 50 kg to the nearest kilogram.

- **a** Find the absolute error.
- **b** Find the relative error. Answer correct to three decimal places.
- **c** Find the percentage error. Answer correct to two decimal places.

4 An iPhone has a mass of exactly 251 g. Vivaan measured the mass of the iPhone as 235 g to the nearest gram.

- **a** Find the absolute error.
- **b** Find the relative error. Answer correct to three decimal places.
- **c** Find the percentage error correct to two decimal places.

5 An LCD screen has a mass of exactly 2.71 kg. Saliha measured the mass of the screen as 3 kg to the nearest kilogram.

- **a** Find the absolute error.
- **b** Find the relative error. Answer correct to three decimal places.
- **c** Find the percentage error correct to three decimal places.

6 A measurement was taken of a skid mark at the scene of a car accident. The actual length of the skid mark was 25.15 metres; however, it was measured as 25 metres.

- **a** What is the absolute error?
- **b** Find the relative error. Answer correct to three decimal places.
- **c** Find the percentage error. Answer correct to one decimal place.

LEVEL 3

7 The length of a building at school is exactly 56 m. Cooper measured the length of the building to be 56.3 m and Filip measured the building at 55.8 m.

- **a** What is the absolute error for Cooper's measurement?
- **b** What is the absolute error for Filip's measurement?
- **c** Compare the relative error for both measurements. Answer correct to four decimal places.
- **d** Compare the percentage error for both measurements. Answer correct to three decimal places.

4C Standard form and significant figures

Standard form

Standard form or scientific notation is used to write very large or very small numbers more conveniently. It consists of a number between 1 and 10 multiplied by a power of 10. For example, the number 4 100 000 is expressed in scientific notation as 4.1×10^6. The power of 10 indicates the number of tens multiplied together. For example:

$$\begin{aligned} 4.1 \times 10^6 &= 4.1 \times (10 \times 10 \times 10 \times 10 \times 10 \times 10) \\ &= 4\,100\,000 \end{aligned}$$

When writing numbers in scientific notation, it is useful to remember that large numbers have a positive power of 10 and small numbers have a negative power of 10.

WRITING NUMBERS IN SCIENTIFIC NOTATION

1. Find the first two non-zero digits.
2. Place a decimal point between these two digits. This is the number between 1 and 10.
3. Count the digits between the new and old decimal point. This is the power of 10.
4. Power of 10 is positive for larger numbers and negative for small numbers.

Example 4: Expressing a number in standard form 4C

The land surface of the Earth is approximately 153 400 000 square kilometres. Express this area more conveniently by using scientific notation.

SOLUTION:

1 The first two non-zero digits are 1 and 5.	1.534
2 Place the decimal point between these numbers.	
3 Count the digits from the old decimal point (end of the number) to the position of the new decimal point.	**1.53 400 000** – eight digits
4 Large number indicates the power of 10 is positive.	Power of 10 is +8 or 8
5 Write in standard form.	$153\,400\,000 = 1.534 \times 10^8$

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Significant figures

Significant figures are used to specify the accuracy of a number. They are often used to round a number. Significant figures are the digits that carry meaning and contribute to the accuracy of the number. This includes all the digits except the zeros at the start of a number and zeros at the end of a number without a decimal point. These zeros are regarded as placeholders and only indicate the size of the number. Consider the following examples.

- 51.340 has five significant figures: 5, 1, 3, 4 and 0.
- 0.00871 has three significant figures: 8, 7 and 1.
- 56091 has five significant figures: 5, 6, 0, 9 and 1.

The significant figures in a number not containing a decimal point can sometimes be unclear. For example, the number 8000 may be correct to one, two, three or four significant figures. To prevent this problem, the last significant figure of a number can be underlined. For example, the number 8$\underline{0}$00 has two significant figures. If the digit is not underlined the context of the problem is a guide to the accuracy of the number.

WRITING NUMBERS TO SIGNIFICANT FIGURES

1. Write the number in standard form.
2. Count the digits in the number to determine its accuracy (ignore zeros at the end, except after a decimal point).
3. Round the number to the required number of significant figures.

Example 5: Writing numbers to significant figures 4C

Write these numbers correct to the number of significant figures indicated.

a 153 400 000 (three significant figures)

b 0.000 657 (two significant figures)

SOLUTION:

1. Write in standard form.
2. Count the digits in the number.
3. Round to three significant figures.
4. Write the answer in standard form correct to three significant figures.
5. Write in standard form.
6. Count the digits in the number.
7. Round to two significant figures.
8. Write the answer in standard form correct to two significant figures.

a $153\,400\,000 = 1.534 \times 10^8$
1.534 has 4 digits
1.53 rounded to 3 sig. fig.
$153\,400\,000 = 1.53 \times 10^8$

b $0.000\,657 = 6.57 \times 10^{-4}$
6.57 has 3 digits
6.6 rounded to 2 sig. fig.
$0.000\,657 = 6.6 \times 10^{-4}$

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Exercise 4C

LEVEL 1

Example 4

1 Write these numbers in standard form.

a 7600
b 1 700 000 000
c 590 000
d 6 800 000
e 35 000
f 310 000 000
g 77 100 000
h 523 000 000 000
i 95 400 000 000
j 540

2 Write these numbers in standard form.

a 0.000 56
b 0.000 068 7
c 0.000 000 812
d 0.0043
e 0.000 058
f 0.000 003 12
g 0.26
h 0.092
i 0.000 000 000 167
j 0.000 06

3 A microsecond is one millionth of a second. Write 5 microseconds in standard form.

4 Sharks existed 410 million years ago.

a Write this number in standard form.
b Express this number correct to one significant figure.

5 Write each of the following as a basic numeral.

a 1.12×10^5
b 5.34×10^8
c 5.2×10^3
d 8.678×10^7
e 2.4×10^2
f 7.8×10^9
g 3.9×10^6
h 2.8×10^1
i 6.4×10^4
j 3.5×10^4

6 Write each of the following as a basic numeral.

a 3.5×10^{-4}
b 7.9×10^{-6}
c 1.63×10^{-7}
d 5.81×10^{-3}
e 4.9×10^{-2}
f 9.8×10^{-1}
g 4.12×10^{-8}
h 6.33×10^{-5}
i 3.0×10^{-9}
j 7.134×10^{-9}

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7 Convert a measurement of 5.81×10^{-3} grams into kilograms. Express your answer in standard form.

8 Evaluate the following and express your answer in standard form.

a $(2.5 \times 10^{3}) \times (5.9 \times 10^{6})$

b $(4.7 \times 10^{5}) \times (6.3 \times 10^{2})$

c $(7.1 \times 10^{-5}) \times (4.2 \times 10^{-2})$

d $(3.0 \times 10^{-4}) \times (6.2 \times 10^{-5})$

9 Evaluate the following and express your answer in standard form.

a $\dfrac{9.1 \times 10^{5}}{2.8 \times 10^{-2}}$

b $\dfrac{7.2 \times 10^{7}}{4.8 \times 10^{-3}}$

c $\dfrac{4.8 \times 10^{-4}}{3.2 \times 10^{-5}}$

Example 5

10 Write these numbers correct to the number of significant figures indicated.

a 1 561 231 (2 sig. fig.)

b 3677 720 (4 sig. fig.)

c 789 001 (5 sig. fig.)

d 3 300 000 (1 sig. fig.)

e 777 777 (3 sig. fig.)

f 3 194 729 (5 sig. fig.)

g 821 076 (4 sig. fig.)

h 7091 (1 sig. fig.)

i 49 172 (2 sig. fig.)

11 Write these numbers correct to the number of significant figures indicated.

a 0.0035 (1 sig. fig.)

b 0.191 785 (4 sig. fig.)

c 0.001 592 (3 sig. fig.)

d 0.111 222 33 (6 sig. fig.)

e 0.000 0271 (1 sig. fig.)

f 0.019 832 6 (5 sig. fig.)

g 0.008 12 (2 sig. fig.)

h 0.092 71 (3 sig. fig.)

i 0.000 419 (2 sig. fig.)

12 A bacterium has a radius of 0.000 015 765 m. Express this length correct to two significant figures.

13 Convert a measurement of 2654 kilograms into centigrams. Express your answer correct to two significant figures.

14 Convert a measurement of 4 239 810 milligrams into grams. Express your answer correct to four significant figures.

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LEVEL 2

15 If $y = \frac{1}{2}x^2$, find the value of y when:

a $x = 2.4 \times 10^3$ **b** $x = 9.8 \times 10^{-3}$

16 The arc length of a circle is $l = \frac{\theta}{360°} 2\pi r$ where θ is the angle at the centre and r is the radius of the circle. Use this formula to calculate the arc length of a circle when $\theta = 30°$ and $r = 7.4 \times 10^8$. Answer in standard form correct to one significant figure.

17 Given that $V = \frac{r}{\sqrt{h}}$ find the value of r in standard form when:

a $V = 5 \times 10^4$ and $h = 9 \times 10^6$ **b** $V = 6 \times 10^{-7}$ and $h = 4 \times 10^2$

18 Use the formula $E = md^2$ to find d correct to three significant figures, given that:

a $m = 0.08$ and $E = 5.5 \times 10^9$ **b** $m = 2.7 \times 10^3$ and $E = 1.6 \times 10^4$

19 Find x, given $x^3 = 2.7 \times 10^{12}$. Answer correct to four significant figures.

20 Light travels at 300000 kilometres per second. Convert this measure to metres per second and express this speed in standard form.

LEVEL 3

21 Use the formula $E = 3p - q$ to evaluate E, given that $p = 7.5 \times 10^5$ and $q = 2.5 \times 10^4$. Answer in standard form correct to one significant figure.

22 The volume of a cylinder is $V = \pi r^2 h$ where r is the radius and h is the height of the cylinder. Use this formula to calculate the volume of the cylinder when $r = 5.6 \times 10^4$ and $h = 2.8 \times 10^3$. Answer in standard form correct to three significant figures.

23 The Earth is 1.496×10^8 km from the Sun. Calculate the distance travelled by the Earth in a year, using the formula $C = 2\pi r$. Answer in standard form correct to two significant figures.

4D Food and nutrition

Food provides our bodies with energy, and nutrients for growth and repair. Food energy is a form of chemical energy, and is measured in kilojoules. A kilojoule is 1000 joules. The common unit for food energy used to be the 'calorie' (cal), or the 'Calorie' (Cal), which is actually a kilocalorie (kcal) (1000 cal = 1 kcal = 1 Cal). The SI unit 'kilojoule' (kJ) is now used internationally (1 calorie = 4.1868 kilojoules). If you consume a lot of food with a high kilojoule rating you may be getting more energy than you need. The excess energy is stored as fat.

The healthy eating pyramid on the right shows how food is placed into groups. It suggests the amount of each food category that a person should eat each day, with the more of the food groups at the bottom of the pyramid being eaten than the foods at the top.

FOOD ENERGY

Food energy is measured in kilojoules (kJ). 1000 cal = 1 kcal = 1 Cal = 4.1868 kJ

Example 6: Using units of energy related to food and nutrition 4D

The number of kilojoules your body requires each day depends on your age, gender and life style. Answer the following questions using the table.

Age	Men kJ per day	Women kJ per day
18–35	12 500	9000
36–70	10 000	8000

a Jenny is 18 years old. How many kilojoules does she need each day?

b Mitchell is 25 years old and works out at the gym for 2 hours. A gym workout uses 2500 kJ per hour. How many kilojoules does he need each day?

SOLUTION:

1 Read table using women aged 18–35.
2 Calculate the energy used in the gym workout. Multiply 2500 (per hour) by 2 (2 h)
3 Add the energy used in the gym workout to the daily kilojoules needed for a man aged 18–35.

a Jenny needs 9000 kJ

b Gym workout = $2 \times 2500 = 5000$ kJ
Energy = $12\,500 + 5000$
= 17 500 kJ

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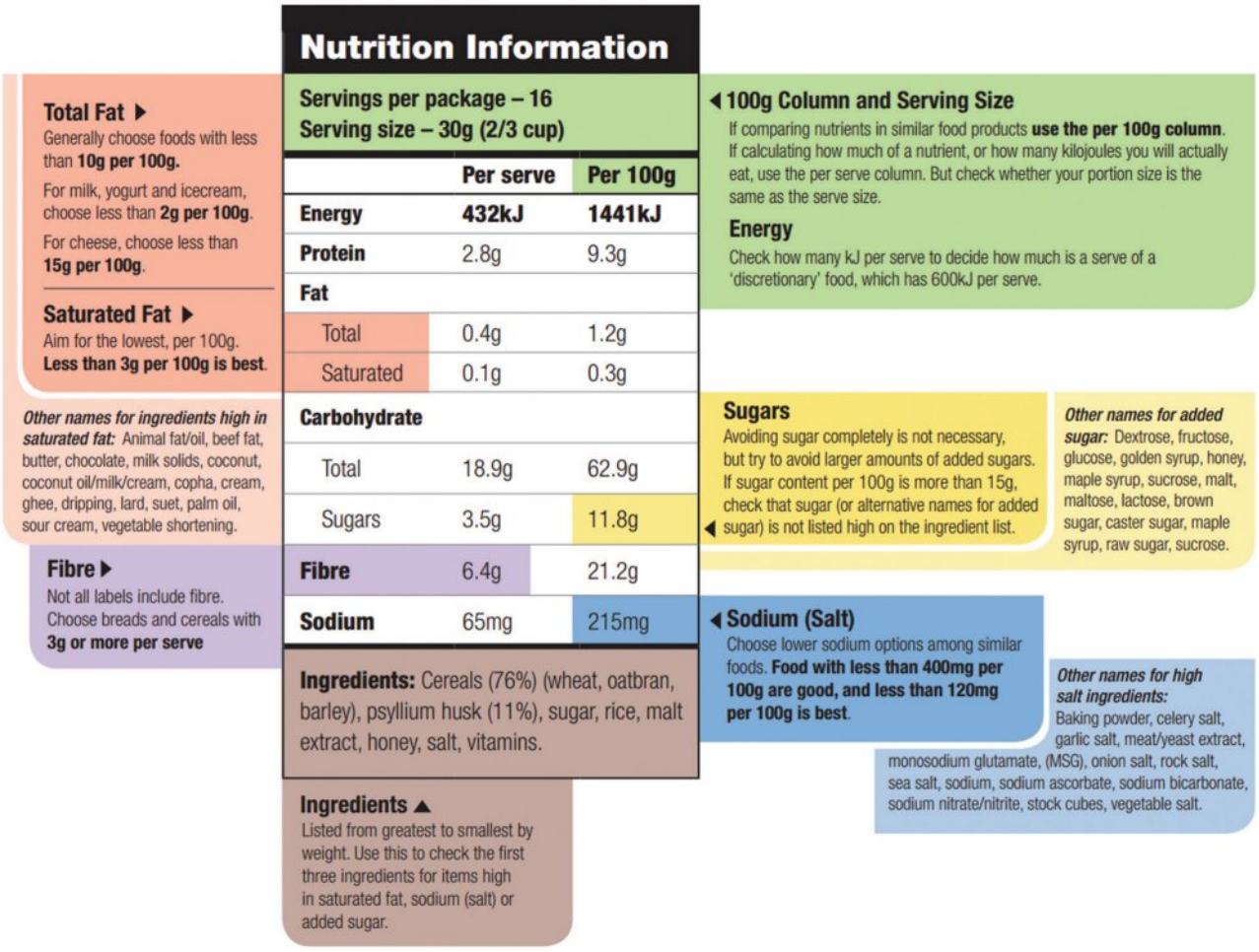
Food labels

Food manufactures are required to label the energy content of their products, to help consumers control their energy intake. The energy available from the food is usually given on labels for 100 g, for a typical serving size and/or the entire pack contents. The nutrition information panel (eatforhealth.gov.au) below provides a few tips on understanding the food label and shopping for healthy food.



www.eatforhealth.gov.au

Example 7: Using units of energy related to food and nutrition 4D

Use the above food label to answer the following questions.

a What is the serving size?
b How many kilojoules per 100 g?
c What is the amount of fibre per serve?
d How much saturated fat per serve?

SOLUTION:

1 The top of the label contains information about the serving size.
2 Energy is measured in kilojoules and the per 100 g column is located on the right-hand side.
3 Fibre per serve is highlighted in purple in the label.
4 Saturated fat per serve is highlighted in salmon in the label.

a Serving size is 30 g.
b 1441 kJ per 100 g
c Fibre per serve is 6.4 g
d Saturated fat per serve is 0.1 g

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Exercise 4D

LEVEL 1

1 Complete (use 1 Calorie = 4.1868 kilojoules). Answer to the nearest whole number.

a 450 Cal = ☐ kJ **b** 1800 Cal = ☐ kJ **c** 15 200 Cal = ☐ kJ

d 7600 kJ = ☐ Cal **e** 370 kJ = ☐ Cal **f** 1250 kJ = ☐ Cal

Example 6

2 The table below shows the approximate energy value of some common foods.

Food	kJ	Food	kJ
Bacon (1 rasher)	960	Milk (small glass)	777
Banana	294	Orange juice (1 glass)	231
Butter (for 1 slice of bread)	105	Potato chips (1 serve)	1512
Cornflakes (1 serve)	441	Toast (1 slice)	420
Egg (fried)	550	Soft drink (1 can)	525

Find the number of kilojoules in following breakfast meals.

a 3 pieces of toast, butter for toast, 1 serve of cornflakes, milk

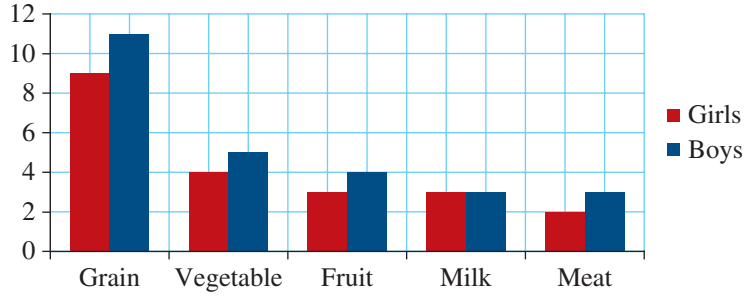
b 2 fried eggs, 2 rashers of bacon, 1 glass of orange juice

c 1 serve of cornflakes, 1 banana, milk, 1 glass of orange juice

d 1 fried egg, 1 rasher of bacon, 1 piece of toast, butter for toast

e 1 serve of cornflakes, milk, 1 glass of orange juice, 2 fried eggs

3 The column graph below shows the recommended daily food servings for teenagers.



a How many servings of vegetables are recommended for a girl?

b How many servings of meat are recommended for a boy?

c Which food groups for girls have three servings of food recommended?

d Which food group for boys has four servings of food recommended?

e For which food group is it recommended that boys and girls have the same number of servings?

f Find the difference in the number of servings of grain recommended for girls and for boys.

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LEVEL 2

Example 7

4 The food label on the right is provided on a 200 g box of cream biscuits.

	per biscuit	per 100 g
Energy	95 kJ	2090 kJ
Protein	0.5 g	11.0 g
Fat, total	1.2 g	26.4 g
– saturated	0.6 g	13.2 g
Carbohydrate, total	2.4 g	52.8 g
– sugars	0.4 g	8.8 g
Fibre	0.3 g	6.6 g
Sodium	25 mg	550 mg
Potassium	10 mg	220 mg

a How many grams of fat are in the 200 g box?

b What is the energy value of:

- **i** one biscuit?
- **ii** five biscuits?
- **iii** the box of biscuits?

c What is the energy value of:

- **i** 100 g of biscuits?
- **ii** 20 g of biscuits?
- **iii** 1 g of biscuits?

d How many kilojoules in 4 boxes of biscuits?

e Harrison is on a diet of 7030 kJ. How many biscuits can he consume?

f The recommended dietary intake of protein is approximately 50 g per day. What percentage of the recommended dietary intake comes from eating one biscuit?

g The recommended dietary intake of sodium is approximately 1600 mg per day. What percentage of the recommended dietary intake comes from eating one biscuit?

h How many biscuits need to be eaten to provide 90 g of protein?

i What percentage of the total carbohydrate in each biscuit is from sugars?

j What percentage of each box of biscuits is fat?

LEVEL 3

5 The table below shows the recommended daily food servings.

Grain	Vegetable	Fruit	Milk	Meat
9	4	3	3	2

a How many servings are recommended for each day?

b Construct a sector graph to represent this data.

c What percentage of the recommended daily food servings is grain? Answer to the nearest whole number.

d What percentage of the recommended daily food servings is fruit and vegetables? Answer to the nearest whole number.

e Cindy needs 8700 kilojoules of energy in her diet. How many kilojoules should she get from grain? Assume that every serving has the same number of kilojoules. Answer to the nearest whole number.

f William needs 10 500 kilojoules of energy in his diet. How many kilojoules should he get from fruit and vegetables? Assume that every serving has the same number of kilojoules. Answer to the nearest whole number.

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4E Energy consumption

Energy is the capacity to do work. Energy exists in numerous forms, such as heat. The joule (symbol J) is the unit of energy used by the International System of Units (SI). Heat energy, such as that produced by burning natural gas in the home, is usually measured in megajoules (one million joules), symbol MJ.

Power

Power is the rate at which energy is generated or consumed. The watt is the SI unit of power and is equal to one joule per second. The symbol for the watt is W. As for the joule, the standard SI prefixes such as milli, kilo, mega, giga and tera are then added and commonly used to measure power.

Name	Symbol	Meaning	Value	Example
Milliwatt	mW	One thousandth of a watt	10^{-3} W	Small laser pointer
Watt	W	One watt	10^{0} W	Smartphone making a call
Kilowatt	kW	Thousand watts	10^{3} W	Electric heater
Megawatt	MW	Million watts	10^{6} W	Large diesel generator
Gigawatt	GW	Billion watts	10^{9} W	Very large power station
Terawatt	TW	Trillion watts	10^{12} W	Worldwide nuclear power

Electrical energy

The joule is not a practical unit for measuring electrical energy in most settings. It is more helpful to think of electrical energy in terms of the power drawn by an electrical device, and the length of time the device is in use. For example, when a device with a power rating of 100 W is turned on for one hour, the amount of energy used is 100 watt-hours (Wh). This is the same amount of energy a 50 W device would use in 2 hours.

The kilowatt-hour (kWh) is commonly used to measure electrical energy in household electricity meters. It represents the amount of electrical energy when a 1000 W power load is drawn for one hour. It is the result of multiplying power in kilowatts and time in hours. Note the conversion 1 kWh = 3.6 MJ.

Consumption is a rate expressed as an amount over time. Electrical energy consumption can be expressed in units of energy (such as kilowatt-hours or megawatt-hours, or megajoules) consumed per unit of time. The average annual energy use per household in Australia is 11 MWh/y or 3 MJ/y, which also equates to about eight tonnes of CO_2 emissions.

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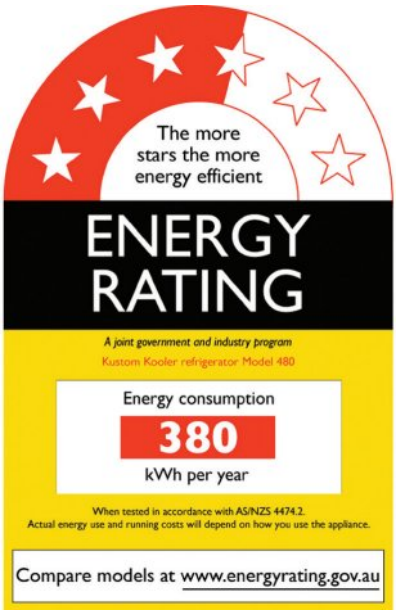
ENERGY CONSUMPTION

Energy consumption is the amount of energy consumed per unit of time.

Energy rating of appliances

In Australia an energy rating label is provided for various appliances. It allows consumers to compare the energy efficiency of similar products. Energy rating labels all have a simple star rating. The more stars on the label, the more energy efficient the appliance. The energy consumption figure is the number in the red box. It indicates the amount of electricity (kWh) the appliance typically uses in a year. The lower the number the less the appliance will cost to run. The running cost of the appliance is calculated by multiplying the energy consumption figure by the electricity price rate. Electricity suppliers usually give prices per kilowatt-hour.



Example 8: Calculating the cost of running appliances 4E

Determine the cost of running the following appliances given the average rate for electricity is \$0.17 per kWh.

a A dishwasher with an energy consumption of 670 kWh per year

b A 2.4 kW fan heater for five hours

c A 20 W LED light bulb for a year

SOLUTION:

1 Determine the cost by multiplying the energy consumption figure by the electricity price rate.

2 Determine the energy (kWh) by multiplying the power rating (kW) by the hours used.

3 Determine the cost by multiplying the energy by the electricity price rate.

4 Determine the energy (kWh) by multiplying the power rating (kW) by the hours used.

5 Determine the cost by multiplying the energy by the electricity price rate.

a $\text{Cost} = 670 \times 0.17$
$= \$113.90$

b $\text{Electricity} = 2.4 \times 5$
$= 12 \text{ kWh}$

$\text{Cost} = 12 \times 0.17$
$= \$2.04$

c $\text{Electricity} = 0.02 \times 24 \times 365$
$= 175.2 \text{ kWh}$

$\text{Cost} = 175.2 \times 0.17$
$= \$29.78$

Exercise 4E

LEVEL 1

1 Answer the following questions using this electricity account.

NSW

Electricity account

NSW Sales Pty Limited ABN

TAX INVOICE

ACCOUNT NUMBER
7034 8751 3763 0

TOTAL AMOUNT DUE
(Including GST)
$349.22

DUE DATE
10 Oct 2017

ISSUE DATE: 21 SEP

ELECTRICITY SERVICE ADDRESS
27 EAST STREET

NEXT SCHEDULED READING
14 Dec

TOTAL LAST BILL	WE RECEIVED	CURRENT CHARGES	GST
$316.68	$316.68	$317.48	$31.74

ACCOUNT SUMMARY	23 Jun -15 Sep	
Peak Use	2208 kWh	$ 281.11
Other Charges		$ 36.37
Total or Current Charges (See over for details)		$ 317.48
GST (refer rear of Bill for calculation)		$ 31.74
Total or Current Charges (Inclusive of GST)		$ 349.22
TOTAL AMOUNT DUE		**$ 349.22**

NSW Government funded energy rebates - you may be entitled to a pensioner or life support rebate.

For all your heating, cooling, cooking and hot water needs call your local NSW Energy Shop

Phone Numbers
Accounts/Connections 00000 000
NSW Customer Service)
Power Failur 24hrs 000 000
(Local Distributor)
Hot Water Emergency 000 000
Home Services (see over) 000 000

NSW

How to pay your bill - see over

ACCOUNT NUMBER TYPE
7034 8751 3763 0

TOTAL AMOUNT DUE
(Including GST)
$349.22

DUE DATE
10 Oct 2017

a When was this electricity account issued?

b What is the total amount due?

c What is the due date?

d How much GST was charged?

e What is the charge for the peak use of electricity?

f How much has the bill increased from the last bill?

g Express the increase as a percentage of the last bill. Answer correct to two decimal places.

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Example 8

2 A washing machine has an energy consumption figure of 206 kWh per year. Calculate the cost of running the washing machine for a year using the following electricity price rates.

a \$0.2185 / kWh
b \$0.2419 / kWh
c \$0.2653 / kWh

3 Leo uses a 800 W microwave oven for a total of 20 hours per week.

a How much energy does the microwave oven use per week?
b What is the cost of using the microwave oven for a week if the electricity is charged at a rate of \$0.2367 per kWh?

4 The Bailey family has a television set with a 200 W rating.

a How many kilowatt-hours of electricity does it use in a week if it is switched on for an average of 9 hours per day?
b What is the cost of running the TV for a week if electricity is \$0.2259 per kWh?

5 Samantha uses a hairdryer with a rating of 1.5 kW for a total of 8 hours. Her electricity is charged at a rate of 22.81 cents per kWh.

a How many kilowatt-hours were used by the hairdryer?
b What is the cost of using the hairdryer? Answer to the nearest cent.

6 Darcy uses an iron with a rating of 500 W for an average of 8 hours per week. Find the cost of the electricity used to do the weekly ironing if it is charged at a rate of 25.67 cents per kWh.

7 Bian received her natural gas account.

Amount used	Charges	Cost
First 5500 MJ	2.491 cents per MJ	
Next 13900 MJ	1.499 cents per MJ	
Supply fee	\$40.00	

a Calculate the cost of the first 5500 MJ. Answer to the nearest cent.
b Calculate the cost of the next 13900 MJ. Answer to the nearest cent.
c What is the total charge? Answer to the nearest cent.

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LEVEL 2

8 The power ratings of various electrical appliances are shown in the table.

Electrical appliance	Power rating
Dishwasher	1600 W
Hair dryer	600 W
Light bulb	75 W
Refrigerator	150 W
Television	300 W
Toaster	1000 W

a How much energy is used by the television for 5 hours? Answer in kilowatt-hours.
b How much energy is used by the toaster for 30 minutes? Answer in watt-hours.
c For how many hours was the dishwasher used if its cost of operation was $11.76? The cost of electricity is $0.2450 per kilowatt-hour.
d For how many hours was the hair dryer used if its cost of operation was $1.95? The cost of electricity is 26 cents per kilowatt-hour.

LEVEL 3

9 An energy company's charges for gas over a 3-month period are shown in the table.

Usage charge	First 4000 MJ	$0.02590 per MJ
	Additional MJ over 4000	$0.01573 per MJ

a Paige used 3580 MJ of gas in this period. What is the cost of this gas? Answer to the nearest cent.
b Muhammad used 4250 MJ of gas in this period. What is the cost of this gas? Answer to the nearest cent.
c What percentage of Muhammad's gas usage was charged at the lower rate? Answer correct to one decimal place.
d Jesse received a gas bill for $93.24. How much gas did he use in this period?
e Ellie has decided to reduce her energy bills. She has a target of $50 for gas. What is the maximum number of MJ she can use in this period?
f Aaron used 5620 MJ of gas in this period. The gas charges are increasing by 5% in next quarter. However, Aaron has purchased a new gas heater with a 5-star rating that will reduce his consumption by 800 MJ. Calculate Aaron's expected bill next quarter.

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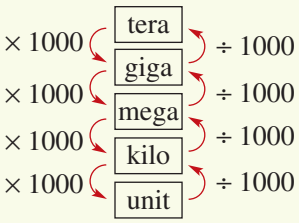
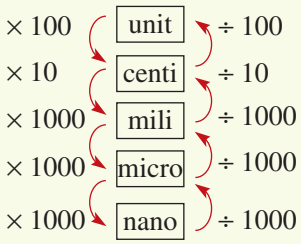
Summary

PPT STUDY GUIDE

Key ideas and chapter summary

Units of measurement





$10\,000\ \text{cm}^2 = 1\ \text{m}^2$

$1\ \text{ha} = 10\,000\ \text{m}^2$

$1\,000\,000\ \text{cm}^3 = 1\ \text{m}^3$

Measurement errors

Precision or limit of reading – smallest unit on the measuring instrument

Absolute error – the difference between actual value and measured value,

$$\text{or } \frac{1}{2} \times \text{precision}$$

Upper bound = Measurement + Absolute error

Lower bound = Measurement – Absolute error

$$\text{Percentage error} = \frac{\text{Absolute error}}{\text{Measurement}} \times 100\%$$

Writing numbers in standard form

1. Find the first two non-zero digits.
2. Place a decimal point between these two digits.
3. Power of 10 is number of the digits between the new and the old decimal point. (Small number – negative value, large number – positive value)

Food and nutrition

Food energy is measured in kilojoules. If you consume a lot of food with a high kilojoule rating you may be getting more energy than you need. The excess energy is stored as fat.

1 Calorie (Cal) = 4.1868 kilojoules (kJ).

The energy available from the food is usually given on labels for 100 g, for a typical serving size and/or the entire pack contents.

Energy consumption

Energy is the capacity to do work. The joule (J) is the basic SI unit of energy.

Power is the rate of consuming or generating energy. Its SI unit is the watt (W) which is equal to one joule per second.

Electricity meters measure electrical energy in kilowatt-hours, kWh, which is the energy consumed by running a 1 kW appliance for one hour.
1 kWh = 3.6 MJ

Running cost of the appliance is calculated by multiplying the energy consumption by the electricity price in cents per kilowatt-hour.

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Review

Multiple-choice questions

1 Convert 7.5 metres to millimetres

A 0.0075 mm **B** 75 mm **C** 750 mm **D** 7500 mm

2 Convert 0.85 g to kilograms.

A 0.00085 kg **B** 0.0085 kg **C** 85 kg **D** 850 kg

3 How many square millimetres are in a square centimetre?

A 10 **B** 100 **C** 1000 **D** 10 000

4 6.2 m^2 is the same as:

A 620 cm^2 **B** 0.0062 km^2 **C** 62000 cm^2 **D** 6200 mm^2

5 What is the absolute error if the precision is 0.1 cm?

A 0.005 cm **B** 0.05 cm **C** 0.5 cm **D** 5 cm

6 Write 4500000 in standard form.

A 4.5×10^{-6} **B** 4.5×10^{-5} **C** 4.5×10^{5} **D** 4.5×10^{6}

7 Express 0.0655 correct to two significant figures.

A 0.06 **B** 0.07 **C** 0.065 **D** 0.066

8 What is the internationally accepted unit to measure food energy?

A Calorie **B** Gram **C** Kilowatt **D** Kilojoule

9 Flynn uses a 1.2-kilowatt dishwasher for a total of 5 hours. He is charged at a rate of 25.72 cents per kilowatt-hour. What is the cost of using the dishwasher?

A \$0.31 **B** \$1.29 **C** \$1.54 **D** \$6.00

10 How much energy does a 600-watt hair dryer use running for 5 hours?

A 1.2 kWh **B** 3 kWh **C** 120 kWh **D** 3000 kWh

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Short-answer questions

1 There are six tonnes of iron ore on a train. What is the mass (in tonnes) if another 246 kg of iron ore is added to the train?

2 Complete the following

a $500\,\text{cm}^2 = \square\,\text{m}^2$ **b** $4000\,\text{cm}^2 = \square\,\text{mm}^2$ **c** $3\,\text{km}^2 = \square\,\text{m}^2$

3 A field has a perimeter of exactly 400 m. Lily measured the field to be 401.2 m, using a long tape marked in 0.1 m intervals.

a Calculate the limit of reading.

b What is the absolute error for Lily's measurement?

c What is the percentage error for Lily's measurement? Answer correct to three decimal places.

4 Write as a basic numeral.

a 4.8×10^6 **b** 6.25×10^{-4} **c** 35×12^4

5 Write these numbers in standard form

a 50 800 **b** 0.0036 **c** 381 000 000

6 Evaluate the following and express your answer in standard form.

a $(7.2 \times 10^5) \times (2.1 \times 10^4)$ **b** $\dfrac{4.6 \times 10^4}{2.3 \times 10^{-2}}$ **c** $\dfrac{5.8 \times 10^{-5}}{2.9 \times 10^4} \times (3.5 \times 10^6)$

7 Convert a measurement of 3580 tonnes into milligrams. Express your answer in standard form correct to two significant figures.

8 Find the value of 45×15^4 and express your answer in standard form correct to two significant figures.

9 The table below shows foods and the energy in kilojoules in a normal size serving.

Food	KJ	Food	KJ
Chips	1425	Apple pie	1380
Fish	340	Ice cream	810
Roast lamb	1064	Soft drink	372

Find the number of kilojoules in following meals.

a Fish and chips, ice cream and soft drink

b Roast lamb, chips, apple pie

c Ice cream, apple pie, soft drink

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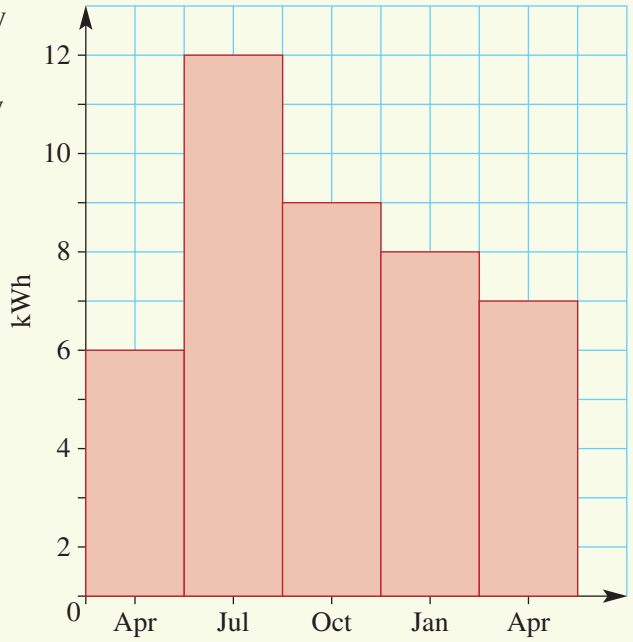
Review

10 The graph on the right is part of an electricity account issued to a customer.

- **a** How many times per year is the electricity meter read?
- **b** What unit is used to measure electrical energy?
- **c** What is the energy consumption in the January quarter?
- **d** What is the energy consumption in the July quarter?
- **e** Which quarter used 6 kWh of electricity?
- **f** Which quarter used 9 kWh of electricity?
- **g** Which quarter had the greatest usage of electricity?



Extended-response question

11 The power rating for electrical appliances is shown in the table.

- **a** How much energy does a laptop computer use for 12 hours?
- **b** How much energy does a microwave use for $\frac{1}{2}$ hour? Answer in watt-hours.
- **c** For how many hours was the vacuum cleaner used, if the cost of operating the vacuum cleaner was \$6.36? The price of electricity is \$0.2650 per kilowatt-hour.

Electrical appliance	Power rating
Electric shaver	15 W
Hair dryer	1000 W
Iron	900 W
Laptop computer	50 W
Microwave	1200 W
Vacuum cleaner	600 W

PDF LITERACY

5 Relative frequency and probability

Syllabus topic — MS-S2 Relative frequency and probability

This topic will develop your awareness of the broad range of applications of probability concepts in everyday life and their use in decision-making.

Outcomes

- Understand and use the language associated with probability.
- Understand and apply the definition of probability.
- Calculate probabilities using fractions, decimals and percentages.
- Use tables or tree diagrams to determine the outcomes for a multistage event.
- Demonstrate the range of possible probabilities.
- Identify and use the complement of an event.
- Calculate and use relative frequencies to estimate probabilities.
- Calculate the expected frequency of future events.
- Predict by calculation the number of people of each blood type in a population.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Widgets
- Literacy worksheet
- Spreadsheets
- Quick Quiz
- Study guide
- Solutions (enabled by teacher)

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

In the Interactive Textbook you can take a test of prior knowledge required for this chapter, and depending on your score you may be directed to revision from the previous years' work.

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5A Language of probability

Probability language is used to specify the chances of an event occurring. For example:

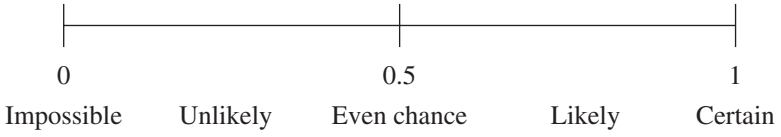
'The chance of rain next week is 50%.'

'Connor has no chance of playing football tomorrow.'

'Winning a race in the next month is very likely.'

'Amber has an even chance of passing this test.'

Events that must occur are *certain* and events that have no chance of occurring are *impossible*. When an event has an equal chance of occurring or not occurring, it is an *even* chance or 50–50 or a 50% chance. Events that have better than an even chance are called likely, often, probable, sure or expected. Conversely, events that have less than an even chance are called unlikely, doubtful, improbable, rarely or unexpected.



PROBABILITY

- Probability is the chance of something happening.
- Certain events have a probability of 1 and impossible events have a probability of 0.

Example 1: Using the language of probability 5A

Ying has a pear, an apple and an orange. She randomly selects one piece of fruit. Describe the chance of the following events using the words 'certain', 'likely', 'even', 'unlikely' or 'impossible'.

a Ying selects a banana.

b Ying selects an orange.

c Ying selects a piece of fruit.

d Ying selects a pear or an apple.

SOLUTION:

1 No chance of selecting a banana.

2 One chance out of three of selecting the orange.

3 Selecting a piece of fruit must happen.

4 Two chances out of three of selecting the pear or the apple.

a Impossible

b Unlikely

c Certain

d Likely

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Sample space

Sample space is the set of all possible outcomes or possible results of an experiment. For example, if the experiment is tossing a coin, then the sample space is a head and a tail. When a die is tossed, the sample space is the numbers $1, 2, 3, 4, 5$ and 6. Each outcome or data value is an element of the sample space. The sample space is usually listed between curly brackets { }.

SAMPLE SPACE

Sample space is the set of all possible outcomes.
Each outcome or data value is an element of the sample space.

Example 2: Identifying the sample space 5A

a Daniel is choosing a day of the week to start his holiday. List the sample space.
b How many elements in the sample space?

SOLUTION:

1 Each day of the week is a possible outcome – Monday, Tuesday, Wednesday, Thursday, Friday, Saturday and Sunday.
2 List each element between the curly brackets.
3 Count the elements in the sample space.

a Sample space $= \{M, T, W, T, F, S, S\}$

b There are 7 elements in the sample space.

Example 3: Identifying the sample space 5A

Two unbiased coins are tossed.
a What is the sample space?
b How many possible outcomes are there?

SOLUTION:

1 Each coin could land on a head or a tail.
2 If the first coin was a head, then the second coin could be a head or a tail.
3 If the first coin was a tail, then the second coin could be a head or a tail.
4 Count the elements in the sample space.

a First coin – H or T.
Second coin – H or T.

Sample space $= \{HH, HT, TH, TT\}$

b Number of possible outcomes is 4.

Exercise 5A

LEVEL 1

1 State whether the probability of each event is impossible or certain.

- **a** Christmas Day occurring on the 25th December
- **b** Winning a raffle without buying a ticket
- **c** Rolling the number 7 on a standard six-sided die
- **d** Friday being the day after Thursday
- **e** Selecting the letter 'A' from letters in the word 'SCHOOL'
- **f** The sum of 1 and 2 being 3
- **g** Choosing an even number from the numbers 2, 4, 6 and 8
- **h** Winning a car race without a car
- **i** Obtaining a head or a tail when a coin is tossed
- **j** Night following day

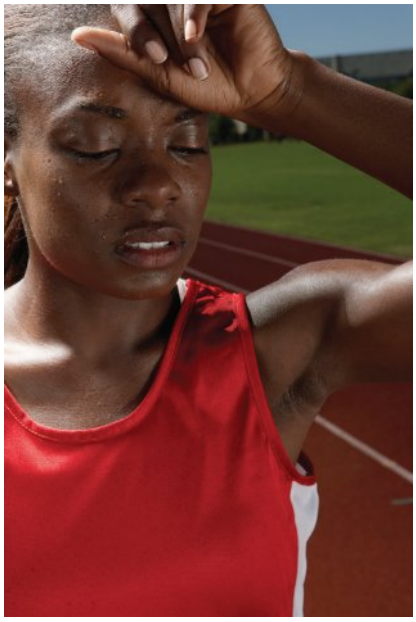
Example 1

2 Describe the chance of the following events using the words 'certain', 'likely', 'even', 'unlikely' or 'impossible'.

- **a** There being 31 days in January
- **b** Tossing a coin and getting a tail
- **c** Randomly choosing a person who has never watched television
- **d** Winning first division Lotto
- **e** Choosing a red ball from a bag containing an equal number of red and white balls
- **f** Snow on Mount Kosciuszko during a cold day in winter
- **g** Newborn baby being a girl
- **h** Drawing a heart from a normal pack of cards
- **i** Tossing a coin and turning up a head
- **j** Australia participating in the next cricket World Cup

3 Aliya has a better than even chance of winning the marathon.

- **a** What word could you use to describe this probability?
- **b** Aliya is sick with a cold. What word could be used to describe her chances now?



4 Two coins are tossed. Describe in words the probability of the following outcomes.

- **a** Two heads
- **b** Head and a tail
- **c** Two tails

5 A standard six-sided die is rolled. Describe in words the probability of the following outcomes.

- **a** Odd number
- **b** Number between 1 and 6
- **c** Number less than 10
- **d** Number 3

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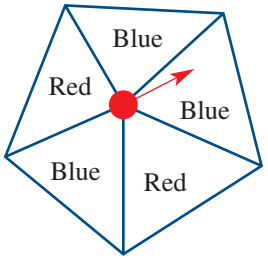
Example 2

6 List the sample space for the following events.

- **a** Tossing a $2 coin
- **b** Selecting a ball from a box containing two red and three white balls
- **c** Choosing a letter from the word 'AUSSIE'
- **d** Drawing a card from cards labelled from 1 to 5
- **e** The sex of a newborn baby
- **f** Selecting the outcome of a football match

7 The arrow on this regular pentagon is spun once. The result is recorded as a blue or a red.



- **a** What is the sample space?
- **b** How many elements are in the sample space?
- **c** Describe in words the probability of a red.
- **d** Describe in words the probability of a blue.

Example 3

8 A fair die is rolled.

- **a** List the sample space.
- **b** How many elements in the sample space?

9 There are 10 counters each labelled with the letters from A to J.

- **a** A counter is selected at random. List all the possible outcomes.
- **b** Describe in words the probability of selecting a vowel.

10 A Rugby Union match is being played between Australia and South Africa.

- **a** List the sample space for the possible outcomes of the match.
- **b** How many elements in the sample space?
- **c** Is each outcome equally likely? Why?

11 Jacob is learning to play chess. He has a match against the Australian champion. There are three outcomes for the match – win, draw or lose.

- **a** What is the sample space?
- **b** Are these events equally likely? Why?

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LEVEL 2

12 Chloe is a clay target shooter. Describe in words the following probabilities.

- **a** Hits the target 50% of the time
- **b** Hits the target 10% of the time
- **c** Hits the target 25% of the time
- **d** Hits the target 90% of the time
- **e** Hits the target 75% of the time

13 The probability of six outcomes is 0, 0.25, 0.50, 0.75, 0.90 and 1. Assign one of these numbers to the following word descriptions.

- **a** Even chance
- **b** Likely
- **c** Certain
- **d** Very likely
- **e** Impossible
- **f** Unlikely

14 A bag contains four $10 notes, two $20 notes and three $50 notes. Two notes are selected from the bag without replacement.

- **a** List the sample space for the first note.
- **b** How many distinct elements are in the sample space of the first note?
- **c** Assume the first note chosen was a $10 note. List the sample space of the second note.
- **d** Assume the first note chosen was a $20 note. List the sample space of the second note.
- **e** Assume the first note chosen was a $50 note. List the sample space of the second note.

LEVEL 3

15 A fair coin and a standard die are thrown. Describe in the words the probability of the following outcomes.

- **a** Head and a number less than 10
- **b** Head and a number greater than 10
- **c** Tail and a 1
- **d** Tail or head and an even number

16 'Six students enter a swimming race. The chance of a particular student winning is $\frac{1}{6}$.' Is this statement true or false? Give reasons to support your opinion.

17 Laura has an 80% chance of winning a tennis tournament. The other two players Mia and Emma are equally likely to win.

- **a** What is the sample space?
- **b** Are the outcomes equally likely? Why?
- **c** What is the probability that Mia wins the tournament?

5B Definition of probability

Probability is the chance of something happening. To accurately calculate the probability a more formal definition is used. When a random experiment is performed, the outcome or result is called the 'event'. For example, tossing a coin is an experiment and a head is the event. The event is denoted by the letter E and $P(E)$ refers to the probability of event E. The probability of the event is calculated by dividing the number of favourable outcomes by the total number of outcomes. It is expressed using fractions, decimals and percentages.

PROBABILITY

$$\text{Probability (Event)} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(E) = \frac{n(E)}{n(S)}$$

Example 4: Calculating the probability 5B

A coin is chosen at random from 7 one dollar coins and 3 two dollar coins. Calculate the probability that the coin is a:

a one dollar coin

b two dollar coin.

SOLUTION:

1 Write the formula for probability.
2 Number of favourable outcomes (or \$1 coins) is 7. The total number of outcomes or coins is 10.
3 Substitute into the formula.
4 Simplify the fraction if possible.
5 Express as a decimal or percentage if required.

a $P(\$1) = \dfrac{n(\$1)}{n(S)}$

$= \dfrac{7}{10}$

$= 0.7$ or

$= 70\%$

6 Write the formula for probability.
7 Number of favourable outcomes (or \$2 coins) is 3. The total number of outcomes or coins is 10.
8 Substitute into the formula.
9 Simplify the fraction if possible.
10 Express as a decimal or percentage if required.

b $P(\$2) = \dfrac{n(\$2)}{n(S)}$

$= \dfrac{3}{10}$

$= 0.3$ or

$= 30\%$

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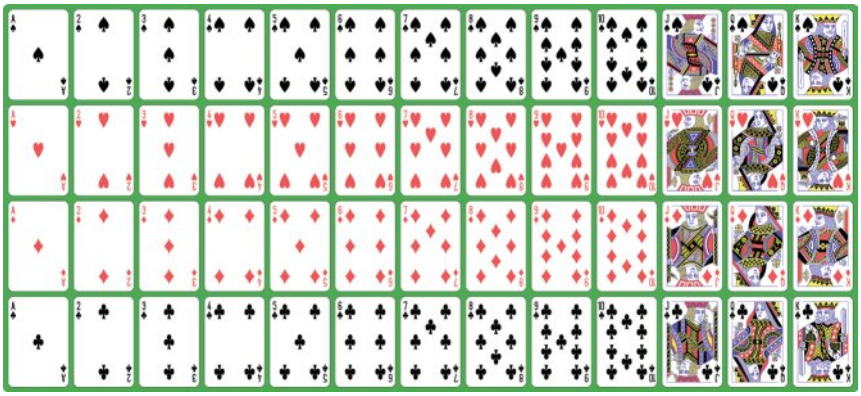
Equally likely outcomes

Equally likely outcomes occur when there is no obvious reason for one outcome to occur more often than any other. For example, when selecting a ball at random from a bag containing a red, a blue and a white ball, each of the balls is equally likely to be chosen.

Winning a bike race is an example of an event for which the outcomes are not equally likely. Some riders have more talent and some riders are better prepared. If one person is a better rider, their chance of winning the race is greater.

A deck of playing cards

A normal deck of playing cards has 52 cards. There are four suits called clubs, spades, hearts and diamonds. In each suit there are 13 cards from ace to king. There are 3 picture cards in each suit (jack, queen and king).



Example 5: Calculating the probability from playing cards 5B

What is the probability of choosing the following cards from a normal pack of cards?

a Red four **b** Diamond **c** Picture card

SOLUTION:

1 Write the formula for probability.
2 Number of favourable outcomes (or red 4s) is 2. The total number of outcomes is 52.
3 Substitute into the formula.
4 Simplify the fraction.

a $P(\text{Red 4}) = \dfrac{n(\text{Red 4})}{n(S)}$

$= \dfrac{2}{52}$

$= \dfrac{1}{26}$

5 Write the formula for probability.
6 Number of favourable outcomes (or diamonds) is 13. The total number of outcomes is 52.
7 Substitute into the formula.
8 Simplify the fraction.

b $P(\text{Diamond}) = \dfrac{n(\text{Diamond})}{n(S)}$

$= \dfrac{13}{52}$

$= \dfrac{1}{4}$

9 Write the formula for probability.
10 Number of favourable outcomes (or picture cards) is 12. The total number of outcomes is 52.
11 Substitute into the formula.
12 Simplify the fraction.

c $P(\text{Picture}) = \dfrac{n(\text{Picture})}{n(S)}$

$= \dfrac{12}{52}$

$= \dfrac{3}{13}$

Exercise 5B

LEVEL 1

Example 4

1 What is the probability of the following experiments?

a A card dealt from a normal deck of cards is a diamond.
b A day selected at random from the week is a weekend.
c A head results when a coin is tossed.
d A letter from the alphabet is a vowel.
e A two results when a die is rolled.
f A six is chosen from $\{2, 4, 6, 8, 10\}$.

2 A bag contains 5 blue and 3 red balls. Find the probability of selecting the following at random.

a A blue ball **b** A red ball **c** Not a red ball

3 Aaron chooses one ball at random from his golf bag. The table below shows the type and number of golf balls in his bag.

Type of golf ball	Number
B51 Impact	3
Maxfli	4
Pinnacle	13

Find the probability of Aaron choosing:

a a Maxfli **b** a Pinnacle
c a B51 Impact **d** not a Maxfli

4 An unbiased coin is tossed three times. On the first two tosses the result is tails. What is the probability that the result of the third toss will be a tail?

5 In Amber Ave there are 3 high school students, 4 primary school students and 5 preschool students. One student from Amber Ave is chosen at random. What is the probability that a primary school student is chosen?

6 A box contains 3 blue, 4 green and 2 white counters. Find the probability of drawing at random a counter that is:

a blue **b** green **c** white **d** not blue

Example 5

7 A card is chosen at random from a standard deck of 52 playing cards. Find the probability of choosing:

a the seven of clubs **b** a spade **c** a red card
d a red picture card **e** a nine **f** the six of hearts
g an even number **h** a picture card **i** a black ace

8 The weather on a particular day is described as either wet or dry. Therefore there is an even chance of a wet day. Do you agree with this statement? Give a reason.

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LEVEL 2

9 A die with 16 faces marked 1 to 16 is rolled. Find the probability that the number is:

a an odd number
b neither a 1 nor a 2
c a multiple of 3
d greater than 12
e less than or equal to 15
f a square number

10 A wheel contains eight evenly spaced sectors numbered 1 to 8. The wheel is spun until it stops at a number. Given that the wheel is equally likely to stop at any number, find the probability that the wheel stops at:

a a 7
b a number greater than 5
c an odd number
d a 1 or 2
e a number less than 10
f a number divisible by 3

11 Lucy is dealt four cards from a normal deck: two aces and two kings. What is the probability that the next card is:

a another ace?
b another king?
c not an ace?
d not a king?

LEVEL 3

12 Two cards are drawn at random from a normal deck of cards. What is the probability that the second card is:

a a two if the first card was a two?
b an ace if the first card was an ace?
c the six of clubs if the first card was a 10?
d a two if the first card was a king?
e a diamond if the first card was a diamond?
f a picture card if the first card was a picture card?

13 A four-digit number is formed from the digits 2, 3, 4 and 5 without replacement. What is the probability that the number:

a starts with the digit 4?
b is greater than 3000?
c ends with a 2 or a 3?
d is 2345?

14 If six students enter a swimming race, then the chance of any particular student winning must be $\frac{1}{6}$. Is this statement true or false? Give reasons to support your opinion.

5C Tables and tree diagrams

Tables and tree diagrams are used to find the sample space for a multistage event. They are orderly methods of determining all the possible outcomes. A multistage event consists of two or more events. For example, tossing a coin and throwing a die or selecting three cards from a pack of cards are multistage events.

Tables

A table is an arrangement of information in rows and columns. The table shows all the possible outcomes for tossing two coins. There are two events: tossing the first coin and tossing the second coin. The outcomes of the first event are listed down the first column (Head or Tail). The outcomes of the second event are listed across the top row (Head or Tail). Each cell in the table is an outcome. There are four possible outcomes.
Sample space = {HH, HT, TH, TT}

	Head	Tail
Head	HH	HT
Tail	TH	TT

TABLES

A table is an arrangement of information in rows and columns.

Each cell in the table is a data value or an outcome.

Example 6: Using a table for a multistage event 5C

Two red cards (R1, R2) and one black card (B1) are placed in a box. Two cards are selected at random with replacement.

a Construct a table to list the sample space.

b What is the probability of selecting R1 R1 or R2 R2?

SOLUTION:

1. List the outcomes of the first event (first card) down the first column. There are three outcomes: R1, R2 and B1.
2. List the outcomes of the second event (second card) across the top row. There are three outcomes: R1, R2 and B1.
3. Write the outcome in each cell using the intersection of the row and column.
4. Write the formula for probability.
5. Substitute into the formula.

a

	R1	R2	B1
R1	R1 R1	R1 R2	R1 B1
R2	R2 R1	R2 R2	R2 B1
B1	B1 R1	B1 R2	B1 B1

b $P(\text{R1R1 or R2R2}) = \dfrac{n(\text{R1R1 or R2R2})}{n(S)}$

$= \dfrac{2}{9}$

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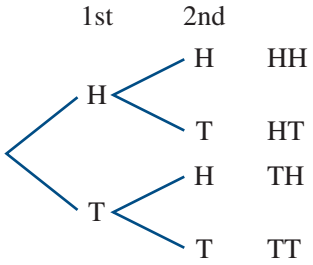
Tree diagrams

A tree diagram is a diagram that details the outcomes of a multistage experiment. It shows each event as a branch of the tree. The tree diagram below shows all the possible outcomes for tossing two coins. The outcomes of the first event are listed (H or T) with two branches. The outcomes of the second event are listed (H or T) with two branches on each of the outcomes from the first event. The sample space is HH, HT, TH and TT.



TREE DIAGRAMS

- Draw a tree diagram with each event as a new branch of the tree.
- Always draw large clear tree diagrams and list the sample space on the right-hand side.

Example 7: Drawing a tree diagram 5C

A coin is tossed and a die is rolled.

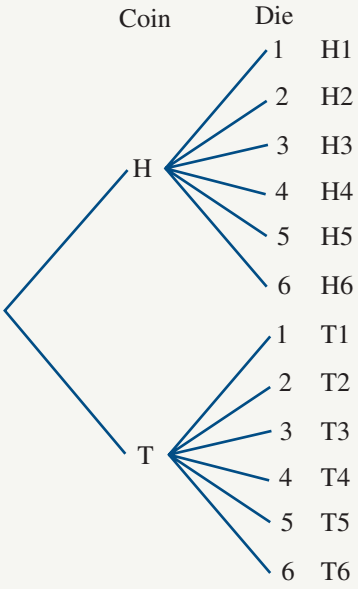
a Construct a tree diagram of these two events to show the sample space.

b What is the probability of a head and a 1?

SOLUTION:

1. Draw branches for first event – tossing a coin.
2. Tossing a coin has two outcomes (head or tail) so there are two branches.
3. Draw branches for the second event – rolling a die.
4. Rolling a die has six outcomes (1, 2, 3, 4, 5 or 6) so there are six branches. Draw six branches for each of the two outcomes from the first event.
5. Use the branches of the tree to list the sample space. Write the outcomes down the right-hand side (sample space).
6. Write the formula for probability.
7. Number of favourable outcomes (H1) is 1. The total number of outcomes is 12.
8. Substitute into the formula and simplify the fraction.

a



b $P(\text{H1}) = \dfrac{n(\text{H1})}{n(S)}$

$= \dfrac{1}{12}$

Exercise 5C

LEVEL 1

1 Emily and Bailey are planning to have two children.

	Boy	Girl
Boy		
Girl		

a Use a table to list the number of elements in the sample space. Consider the sex of each child as an event.

b What is the total number of outcomes?

2 Two fair dice are thrown and their sum recorded.

a Use a table to list all possible outcomes.

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

b How many elements are in the sample space?

Example 6

3 A menu has three entrees (E1, E2 and E3) and four mains (M1, M2, M3 and M4).

a Use a table to list all possible outcomes.

	M1	M2	M3	M4
E1				
E2				
E3				

b Verify the total number of outcomes by using the fundamental counting principle.

4 Three people (A, B and C) applied for a manager's position and two people (D and E) applied for an assistant manager's position.

a Use a table to list the all the possible outcomes.

b What is the total number of outcomes?

5 One bag contains two discs labelled 'X' and 'Y'. A second bag contains four discs labelled 'D', 'E', 'F' and 'G'. A disc is chosen from each bag at random. Use a table to determine the number of elements in the sample space.

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LEVEL 2

6 Three yellow cards (Y1, Y2, Y3) and one green card (G1) are placed in a box. Two cards are selected at random with replacement.

a Use a table to list the number of elements in the sample space.

b Find the probability of the following selections:

i Y1 Y2 **ii** Two green **iii** Two yellow

	Y1	Y2	Y3	G1
Y1				
Y2				
Y3				
G1				

7 Three cards (king, queen and jack) are placed face down on a table. One card is selected at random and the result recorded. This card is returned to the table. A second card is then selected at random.

a Use a table to list the elements in the sample space.

b Find the probability of the following selections:

i KJ **ii** Two kings **iii** No kings

Example 7

8 Two coins are tossed and the results recorded.

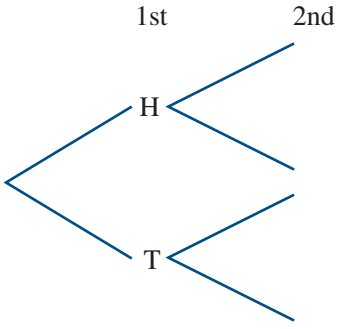
a List the sample space by completing this tree diagram.

b Find the probability of the following results:

i Two tails

ii Head then a tail

iii One head and one tail



9 A survey has two questions whose answers are 'Yes' or 'No'. Construct a tree diagram to list the sample space.

10 There are three questions in a True or False test.

a Fill in the remainder of the partially drawn tree diagram and list the sample space.

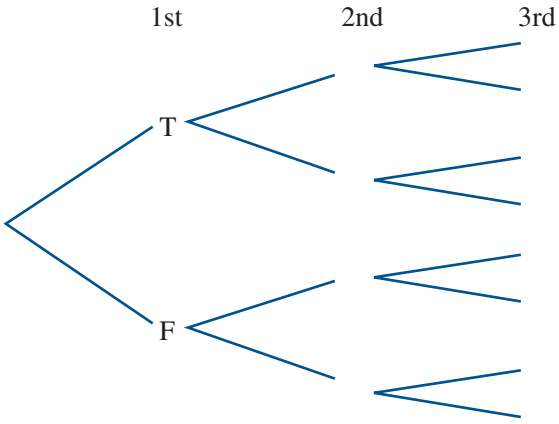
b How many elements in the sample space?

c Find the probability of the following random selections:

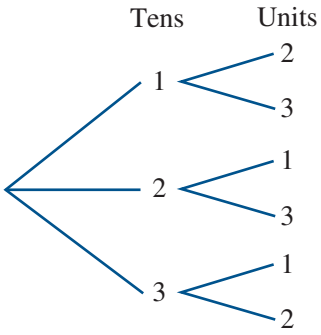
i All of the selections are false.

ii Two true selections and one false selection.

iii At least one of the selections is false.



11 A two-digit number is formed using the digits 1, 2 and 3. The same number cannot be used twice. The first digit chosen is the tens digit and the second digit chosen is the units digit.



a List the sample space from the tree diagram.

b What is the probability of choosing 23?

12 A spinner has an equal number of red and green sections. This spinner is spun twice.

a Use a tree diagram to list the total possible outcomes.

b What is the probability of the same colour when the spinner is spun twice?

13 Ebony tosses a coin and spins a spinner, which has a red, an amber and a green section. Each colour on the spinner is equally likely.

a Use a tree diagram to list the sample space.

b What is the probability of a head on the coin and either a red or green section on the spinner?

14 Four cards (ace, king, queen and jack) are placed face down on a table. One card is selected at random and the result recorded. This card is not returned to the table. A second card is then selected at random. Use a tree diagram to list the total possible outcomes.

LEVEL 3

15 There are four candidates for the positions of leader and deputy leader. The four candidates are Angus, Bridget, Connor and Danielle.

a Construct a tree diagram with the leader as the first event and the deputy leader as the second event. Use a tree diagram to list the sample space.

b What is the probability of Bridget being the leader or deputy leader?

16 A two-digit number is formed using the digits 3, 5 and 7. The same number can be used twice. The first digit chosen is the tens digit and the second digit chosen is the units digit.

a Construct a tree diagram to list the sample space.

b What is the probability of forming a number less than 70?

17 The number of laptops sold in the past two weeks is shown below.

	Brand X	Brand Y	Total
Week 1	80	240	320
Week 2	50	150	200
Total	130	390	520

a Of all brand X laptops sold, what fraction was sold in week 2?

b What percentage of all sales in week 1 was brand Y?

c During the two weeks, what was the probability that any laptop sold would be brand X?

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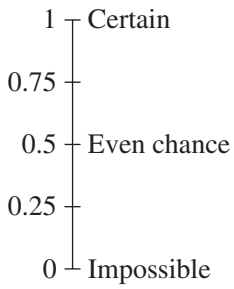
5D Range of probabilities

The probability of an event that is impossible is 0 and the probability of an event that is certain is 1. Probability is always within this range, or from 0 to 1. It is not possible to have the probability of an event as 2. The range of probability is expressed as $0 \leq P(E) \leq 1$ or $P(E) \geq 0$ and $P(E) \leq 1$. It is also important to realise that the probability of every event in an experiment will sum to 1.

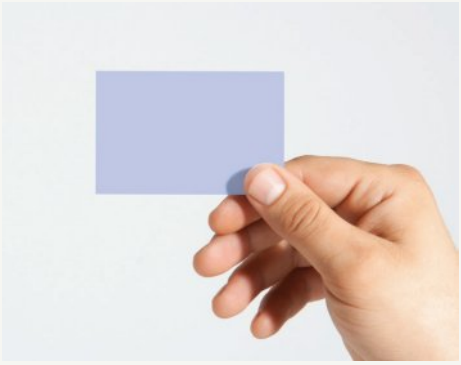


RANGE OF PROBABILITY

Probability of an event is between 0 and 1 or $0 \leq P(E) \leq 1$.
$P(A) + P(B) + \ldots = 1$
$A, B, \ldots$ are all the possible outcomes or events.

Example 8: Using the range of probability 5D

A box contains red, yellow and blue cards. The probability of selecting a red card is $\frac{3}{5}$ and the probability of selecting a yellow card is $\frac{1}{10}$. What is the probability of selecting a blue card?



SOLUTION:

1. Write the formula for the range of probability.
2. Substitute into the formula the probabilities of the other events $\left(P(R) = \frac{3}{5} \text{ and } P(Y) = \frac{1}{10}\right)$.
3. Solve the equation by making $P(B)$ the subject of the equation.
4. Simplify the fraction if possible.
5. Write the answer in words.

$P(R) + P(Y) + P(B) = 1$

$\frac{3}{5} + \frac{1}{10} + P(B) = 1$

$P(B) = 1 - \frac{3}{5} - \frac{1}{10}$

$= \frac{3}{10}$

Probability of a blue card is $\frac{3}{10}$.

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Exercise 5D

LEVEL 1

Example 8

1 A hat contains tickets labelled as 'A', 'B' and 'C'. The probability of selecting ticket A is $\frac{3}{10}$ and the probability of selecting ticket B is $\frac{7}{15}$.

a What is the value of $P(\text{A})$?

b What is the value of $P(\text{B})$?

c What is the probability of selecting a ticket with the letter 'D'?

d What is the probability of selecting tickets A, B or C?

e What is the probability of selecting ticket C?

2 A bag contains black, yellow and white cards. The probability of drawing a black card is 57% and the probability of drawing a yellow card is 8%. What is the value of the following, expressed as a fraction in simplest form?

a $P(\text{Black})$ **b** $P(\text{Yellow})$ **c** $P(\text{White})$

3 In a particular event the probability of Jun winning a gold medal is $\frac{3}{8}$ and a silver medal is $\frac{1}{4}$. There is no bronze medal.

a What are Jun's chances of winning a gold or a silver medal?

b What are Jun's chances of not winning any medals?

4 Some picture cards from a deck of cards are placed face down on the table. The probability of drawing a king is 0.25 and a queen is 0.60. What is the value of the following expressed as a decimal?

a $P(\text{King})$ **b** $P(\text{Jack})$

c $P(\text{Jack}) + P(\text{King})$ **d** $P(\text{King}) + P(\text{Queen}) + P(\text{Jack})$

5 There are four outcomes of an experiment. Three of the outcomes have probabilities of 20%, 25% and 40% respectively. What is the probability of the fourth outcome?

6 A biased die is rolled. The probability of obtaining an even number is 0.4 and the probability of a 1 or a 3 is 0.3. Find the value of the following probabilities.

a $P(1, 2, 3, 4, 5, 6)$ **b** $P\big(2, 4, 5, 6\big)$ **c** $P(\text{Odd})$

LEVEL 2

7 A disc is chosen at random from a bag containing five different colours: black, green, pink, red and white. If $P(\text{B}) = \frac{1}{5}$, $P(\text{G}) = \frac{2}{13}$, $P(\text{P}) = \frac{2}{9}$ and $P(\text{R}) = \frac{1}{6}$, find the probability of the following outcomes.

a Black or green disc
b Pink or red disc
c Black or red disc
d Black, green or pink disc
e Black, green or red disc
f Black, green, pink or red disc

8 A card is chosen at random from some playing cards. The probability of a spade is 0.24, the probability of a club is 0.27 and the probability of a heart is 0.23. Find the probability of the following outcomes.

a Black card
b Red card
c Club or a heart
d Spade or a heart
e Diamond
f Diamond or a club

9 Julia and Natasha are playing a game in which a standard six-sided die is rolled. Julia wins if an even number is rolled. Natasha wins if a number greater than three is rolled. What is the probability that the number rolled is neither even nor greater than three?

10 A bag contains white, green and red marbles. The probability of selecting a white marble is $\frac{2}{7}$ and the probability of selecting a green marble is $\frac{1}{8}$. What is the probability of selecting a red marble?

LEVEL 3

11 The numbers 1 to 20 are written on separate cards. One card is chosen at random. What is the probability that the card chosen is a prime number or is divisible by 3?

12 One letter is selected at random from a word containing the letters T, A, M, P, R. It is given that $P(T) = \frac{1}{5}$, $P(A) = \frac{2}{5}$, $P(M) = \frac{1}{10}$ and $P(P) = \frac{1}{10}$.

a Find the probability of the following outcomes.
i Letters T or A
ii Letters T or P
iii Letters M or P
iv Letters A, M or P
v Letter T, A, M or P
vi Letter R

b The word contains 10 letters. From the letters T, A, M, P, R how many of the following letters are in the word?
i T
ii A
iii M
iv P
v R

c What is the word? (Hint: an Australian place)

5E Complementary events

The complement of an event E is the event not including E. For example, when throwing a die the complement of 2 are the events 1, 3, 4, 5 and 6. The complement of an event E is denoted by $\overline{E}$. An event and its complement represent all the possible outcomes and are certain to occur. Hence the probability of an event and its complement will sum to be 1.

COMPLEMENTARY EVENTS

$P(E) + P(\overline{E}) = 1$ or $P(\overline{E}) = 1 - P(E)$
E – Event or outcome
$\overline{E}$ – Complement of event E or the outcomes not including event E

Example 9: Using the complementary event 5E

Lisa selects a card at random from a normal pack. Find the probability of obtaining the following outcomes.

a Not a 10
b Not a black jack (i.e. not a jack of clubs or spades)
c Not a picture card

SOLUTION:

1 Write the formula for the complement.
2 Substitute into the formula the probability for a 10 $\left(\text{or } P(10) = \frac{4}{52}\right)$.
3 Evaluate.
4 Simplify the fraction.

a $P(\overline{10}) = 1 - P(10)$
$= 1 - \frac{4}{52}$
$= \frac{48}{52}$
$= \frac{12}{13}$

5 Write the formula for the complement.
6 Substitute into the formula the probability for a black jack $\left(\text{or } P(\text{black jack}) = \frac{2}{52}\right)$.
7 Evaluate.
8 Simplify the fraction.

b $P(\overline{\text{black jack}}) = 1 - P(\text{black jack})$
$= 1 - \frac{2}{52}$
$= \frac{50}{52}$
$= \frac{25}{26}$

9 Write the formula for the complement.
10 Substitute into the formula the probability for a picture card $\left(\text{or } P(\text{picture}) = \frac{12}{52}\right)$.
11 Evaluate.
12 Simplify the fraction.

c $P(\overline{\text{picture}}) = 1 - P(\text{picture})$
$= 1 - \frac{12}{52}$
$= \frac{40}{52}$
$= \frac{10}{13}$

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Exercise 5E

LEVEL 1

1 What is the event that is the complement of the following events?

a Selecting a black card from a normal pack of cards

b Winning first prize in Lotto

c Throwing an even number when a die is rolled

d Obtaining a tail when a coin is tossed

e Drawing a spade from a normal pack of playing cards

f Choosing a green ball from a bag containing a blue, a red and a green ball

2 Find the value of $P(\overline{E})$, given the following information about event E.

a $P(E) = \frac{1}{5}$ **b** $P(E) = 0.9$ **c** $P(E) = 62\%$ **d** $P(E) = 1:4$

e $P(E) = \frac{3}{11}$ **f** $P(E) = 0.45$ **g** $P(E) = 37.5\%$ **h** $P(E) = 3:7$

3 The chances of the Sydney Swans winning the premiership are given as 29%. What are the chances that the Sydney Swans will not win the premiership?

a Express your answer as a decimal.

b Express your answer as a fraction.

Example 9

4 The probability of obtaining a 3 on a biased die is 0.6. What is the probability of not obtaining a 3?

5 The probability of a rainy day in March is $\frac{11}{15}$. What is the probability that a particular day in March does not have rain?

6 The probability of drawing a red marble from a bag is $\frac{5}{8}$. What is the probability of not drawing a red marble? Express your answer as a:

a fraction

b decimal

c percentage

LEVEL 2

7 A ball is chosen at random from a bag containing four different colours: brown, orange, purple and yellow. If $P(O) = \frac{2}{11}$, $P(P) = \frac{2}{9}$ and $P(Y) = \frac{1}{4}$, find the probability of the following outcomes.

a Not a yellow ball
b Not an orange ball
c Not a purple ball
d Orange or a purple ball
e Yellow or a purple ball
f Not a brown ball
g A brown ball
h Not an orange or a yellow ball

8 Samuel selects a card at random from a normal pack. Find the probability of obtaining the following outcomes.

a Not a queen
b Not a red ace

9 What is the probability that a person selected at random will:

a not be born on Saturday?
b not be born on a weekend?

LEVEL 3

10 A 12-sided die has faces marked 1 to 12. The die is biased. If $P(8) = 0.1$, $P(2) = 0.15$ and $P(\overline{3}) = 0.91$, find:

a $P(\overline{8})$
b $P(\overline{2})$
c $P(3)$
d $P(8) + P(\overline{8})$
e $P(2) + P(\overline{2})$
f $P(3) + P(\overline{3})$
g $P(2) + P(8)$
h $P(\overline{2}$ or $\overline{8})$

11 One card is selected at random from a non-standard pack of playing cards. If $P(\text{ace}) = 8\%$, $P(\text{king}) = 7\%$ and $P(\text{queen}) = 10\%$, find the probability of the following outcomes.

a Not an ace
b Not a king
c Not a queen
d King or a queen
e Ace or a queen
f Not an ace, king or queen

12 The probability of selecting a card labelled 'T' from 32 cards is given as $P(T) = \frac{3}{16}$.

a What is the probability of not selecting a card labelled with a 'T'?
b How many of the 32 cards were labelled with a 'T'?

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5F Relative frequency

Relative frequency is calculated when an experiment is performed. The frequency of an event is the number of times the event occurred in the experiment. Relative frequency is the frequency of the event divided by the total number of frequencies. It is also known as experimental probability, as it estimates the chances of something happening or the probability of an event. Relative frequency is expressed using fractions, decimals and percentages.

RELATIVE FREQUENCY

Relative frequency is an estimate for the probability of an event.

$$\text{Relative frequency} = \frac{\text{Frequency of the data value}}{\text{Total number of data values}} = \frac{f}{n}$$

Example 10: Finding the relative frequency 5F

An experiment of tossing two coins was completed and the number of heads recorded in the frequency table shown.

Number of heads	Frequency	Relative frequency
0	100	
1	192	
2	108	

Find the relative frequency of obtaining the following number of heads:

a 0 **b** 1 **c** 2

SOLUTION:

1 Add the frequency column to determine the total number of frequencies.
2 Write the formula for relative frequency.
3 Substitute the frequency and total number of frequencies into the formula.
4 Simplify the fraction if possible or express as a decimal.
5 Write answer in words.

a $n = 100 + 192 + 108$
$= 400$

$$\text{Rel. freq.} = \frac{f}{n} = \frac{100}{400}$$
$$= \frac{1}{4}$$
$$= 0.25 \text{ or } 25\%$$

Relative frequency of 0 heads is 0.25.

6 Write the formula for relative frequency.
7 Substitute the frequency and total number of frequencies into the formula.
8 Simplify the fraction if possible or express as a decimal.
9 Write answer in words.

b $$\text{Rel. freq.} = \frac{f}{n} = \frac{192}{400}$$
$$= \frac{12}{25} \text{ or } 0.48 \text{ or } 48\%$$

Relative frequency of 1 head is 0.48.

10 Write the formula for relative frequency.
11 Substitute the frequency and total number of frequencies into the formula.
12 Simplify the fraction if possible or express as a decimal.
13 Write answer in words.

c $$\text{Rel. freq.} = \frac{f}{n} = \frac{108}{400}$$
$$= \frac{27}{100}$$
$$= 0.27 \text{ or } 27\%$$

Relative frequency of 2 heads is 0.27.

Simulation

A simulation is a mathematical model that represents a real experiment or situation. Simulations may involve calculators, computers, coins, dice or tables. Simulations are a useful tool when the real experiment is time consuming or costly.

SIMULATION

A simulation is a mathematical model that represents a real experiment or situation.

Example 11: Performing a simulation 5F

Perform a simulation to determine the results of 50 cricket matches played in a season. There are three outcomes for a match – win, draw or lose. The probability of winning a match is 30%, drawing a match is 50% and losing a match is 20%.

a Assign 10 playing cards numbered from 1 to 10 to the outcomes.

b Perform the simulation by selecting a card at random. Repeat the simulation 50 times as there are 50 matches in a season.

c Construct a frequency table that includes a relative frequency column to record the results of the simulation.

SOLUTION:

1 Find 30% of 10 to determine the number of cards assigned for a win.
2 Find 50% of 10 to determine the number of cards assigned for a draw.
3 Find 20% of 10 to determine the number of cards assigned for a loss.
4 Construct a frequency table with three columns: Score, Frequency and Relative frequency.
5 List the assigned outcomes in the score column.
6 Select a card at random from the 10 cards. Record the result. Replace the card.
7 Repeat the simulation and record the total for each outcome in the frequency column.
8 Calculate the relative frequency of each outcome by dividing the frequency of the outcome by the total frequencies.

a Assign 1, 2 and 3 to win (30%)

Assign 4, 5, 6, 7 and 8 to draw (50%)

Assign 9 and 10 to lose (20%)

b and **c**

Score	Frequency	Relative frequency
Win (1, 2, 3)	13	$\frac{13}{50}$
Draw (4, 5, 6, 7, 8)	27	$\frac{27}{50}$
Lose (9, 10)	10	$\frac{10}{50} = \frac{1}{5}$
Total	50	1

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Exercise 5F

LEVEL 1

Example 10

1 The frequency table shows the outcomes of an experiment. What is the relative frequency for the following outcomes? Express as a fraction in simplest form.

a A **b** B
c C **d** D

Letter	Frequency	Relative frequency
A	12	
B	9	
C	15	
D	6	

2 The frequency table shows the outcomes of an experiment. What is the relative frequency for the following outcomes? Answer correct to three decimal places.

a HH **b** HT
c TH **d** TT

Outcome	Frequency	Relative frequency
HH	8	
HT	20	
TH	28	
TT	12	

3 The frequency table shows the outcomes of an experiment. What is the relative frequency for the following outcomes? Answer as a percentage correct to one decimal place.

a Black **b** Yellow
c Red **d** Blue
e Green **f** White

Colour	Frequency	Relative frequency
Black	105	
Yellow	210	
Red	145	
Blue	170	
Green	215	
White	155	

4 The frequency table shows the outcomes of an experiment. What is the relative frequency for the following outcomes? Answer as a percentage, correct to the nearest whole number.

a 30 **b** 31
c 32 **d** 33
e 34

Score	Frequency	Relative frequency
30	4	
31	6	
32	2	
33	3	
34	5	

5 Calculate the relative frequency for each of these numbers if the total frequency is 48. Write your answer as a fraction in simplest terms.

a 16 **b** 40
c 24 **d** 6

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6 Calculate the relative frequency for each of these numbers if the total frequency is 40. Write your answer as a percentage.

a 4 **b** 30 **c** 15 **d** 32

Example 11

7 Perform a simulation to determine the gender (M or F) of a newly born baby.

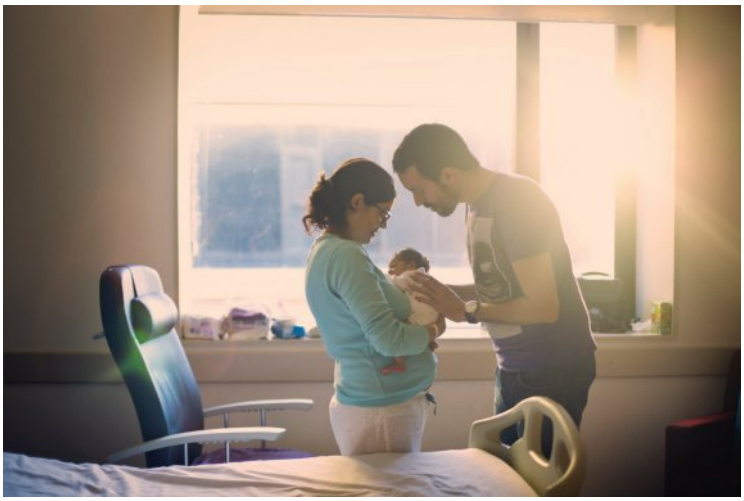
a Use the Ran# function on a calculator or toss a coin for this simulation.

b Complete 100 trials of this simulation.

c Use a frequency table to record the results of this simulation.

d Calculate the relative frequency of each outcome.

e What is the probability of a female baby?



8 Perform a simulation to determine the winner of an election. There are four candidates (A, B, C and D) and each candidate is equally likely to win.

a Use the Ran# function on a calculator or toss two coins for this simulation. Assign each candidate a result obtained from the simulation.

b Complete 148 trials of this simulation.

c Use a frequency table to record the results of this simulation.

d Calculate the relative frequency of each outcome.

e What is the probability of candidate A winning the election?

9 Perform a simulation to answer this question: 'If families only stopped having children as soon as the first girl was born, would there be more girls than boys?'

a Limit the number of children to a maximum of six. List the sample space.

b Use the Ran# function on a calculator or roll a die for this simulation.

c Complete 120 trials of this simulation.

d Use a frequency table to record the results of this simulation.

e Calculate the relative frequency of each outcome.

f How many girls were born in this simulation?

g How many boys were born in this simulation?

h What is your answer to the above question using the simulation results?

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LEVEL 2

10 A retail store sold 512 televisions last year, of which 32 were faulty and returned to the store. What is the relative frequency of a faulty television last year? Answer as a percentage, correct to two decimal places.

11 A pistol shooter at the Olympic Games hits the target 24 out of 25 attempts. What is the relative frequency of the shooter hitting the target? Give the answer as a decimal, correct to two decimal places.

12 The birth statistics in a local community were 142 girls and 126 boys. What is the relative frequency for a girl? Answer as a fraction in lowest terms.

13 In an experiment a die was thrown 120 times. The frequency of each data value is shown in the frequency table.

a How many times would you expect to obtain each of the data values?

b Do you think that the die is fair? Give a reason for your answer.

Number	Frequency	Relative frequency
1	30	
2	16	
3	14	
4	16	
5	16	
6	28	

14 Create the spreadsheet below using the frequency table in question **4**.

XLS 05FQ14

VL... =B5/$B$10

	A	B	C	D
1	**Cambridge Mathematics Standard Year 11 Exercise 5F Question 14**			
2	Worksheet to calculate relative frequencies			
3				
4	*Score*	*Frequency*	*Relative frequency*	*Relative frequency %*
5	30	4	=B5/$B$10	20%
6	31	6	0.30	30%
7	32	2	0.10	10%
8	33	3	0.15	15%
9	34	5	0.25	25%
10		20		
11				

Relative frequency

a Cell B10 has a formula that adds cells B5 to B9. Enter this formula.

b The formula for cell C5 is '= B5/$B$10'. It is the formula for relative frequency. Fill down the contents for C6 to C9 using this formula.

c Cells D5 to D9 have the same formula as cells C5 to C9. Enter this formula and format the cells to a percentage.

LEVEL 3

15 Last year it rained on 140 days out of 365.

a Estimate the probability of it raining on any one day next year.

b How could your estimate be improved?

16 A frequency distribution table is shown below.

Score	Frequency	Relative frequency
3	x	0.20
4	6	0.30
5	5	0.25
6	5	y

a What is the value of x?

b What is the value of y?

c What is the total number of scores?

17 Perform an experiment by rolling a die 120 times.

a Use a frequency table to record the results of the experiment.

b Calculate the relative frequency of each outcome.

c What result would you have predicted for each outcome?

d Compare your results to those of the other students in your class.

18 Perform an experiment by dropping a drawing pin 100 times. Record whether it landed point up or point down.

a Use a frequency table to record the results of the experiment.

b Calculate the relative frequency of each outcome.

c What result would you have predicted for each outcome?

d Compare your results to those of the other students in your class.

19 Perform an experiment by tossing two coins 80 times.

a Use a frequency table to record the results of the experiment.

b Calculate the relative frequency of each outcome.

c What result would you have predicted for each outcome?

d Compare your results to those of the other students in your class.

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5G Expected frequency

The expected frequency is the number of times that a particular event should occur. It may not equal the actual results. For example, when a coin is tossed the probability of getting a head is $\frac{1}{2}$. Hence, if a coin is tossed 100 times, the expected number of heads is 50 or $\frac{1}{2} \times 100$. Clearly, if a coin is tossed 100 times it may not result in exactly 50 heads. However, the larger the number of trials the closer the expected frequency will be to the actual results.

EXPECTED FREQUENCY

Expected frequency is the number of times that a particular event should occur.

Expected frequency $= n \times p = np$

n – number of times the experiment is repeated

p – probability of the event

The expected frequency may not be a whole number. It is an estimate of what to expect. For example, when a die is tossed, the probability of getting a six is $\frac{1}{6}$. Hence, if a die is tossed 100 times the expected number of sixes is $\frac{1}{6} \times 100$ or $16\frac{2}{3}$. Clearly, it is not possible to have $\frac{2}{3}$ of an outcome. However, the expectation is that the number of outcomes will be a whole number close to $16\frac{2}{3}$.

Example 12: Finding the expected frequency 5G

Two coins are tossed 120 times and the results recorded.

a What is the expected frequency for two heads?

b What is the expected frequency for a head and a tail?

SOLUTION:

1 Calculate the probability of two heads.
2 Number of favourable outcomes is 1 (HH). The total number of outcomes is 4 (HH, HT, TH, TT).
3 Write the formula for expected frequency.
4 Substitute into the formula.
5 Evaluate.

a $P(\text{HH}) = \frac{1}{4}$

Expected frequency

$= np$

$= \frac{1}{4} \times 120$

$= 30$

6 Calculate the probability for a head and a tail.
7 Number of favourable outcomes is 2 (HT, TH). The total number of outcomes is 4 (HH, HT, TH, TT).
8 Write the formula for expected frequency.
9 Substitute into the formula.
10 Evaluate.

b $P(\text{HT or TH}) = \frac{2}{4} = \frac{1}{2}$

Expected frequency

$= np$

$= \frac{1}{2} \times 120$

$= 60$

Exercise 5G

LEVEL 1

Example 12

1 The probability of a red traffic light at an intersection is $\frac{1}{3}$. How many red traffic lights are expected on a trip that passes through 54 intersections?

2 The probability of a person living in a certain community of developing melanoma is four out of nine. There are 1404 people living in this community. What is the expected number of people who will develop melanoma?

3 Andrew and Caitlin are planning to have five children. A genetic counsellor has calculated they have a 40% chance of having a child with green eyes. How many of Andrew and Caitlin's children are expected to have green eyes?

4 Amira is a goal shooter for her netball team. The probability that she scores a goal is 88%. This year she had 225 attempts at goal.

a How many goals would you expect Amira to have scored this year?

b How many goals would you expect Amira to have missed this year?

5 Akira is a professional golfer who has a 78% chance of breaking par. He plays 150 games of golf in a year. How many times would you expect Akira to break par in a year?

6 The probability of a worker in an industrial plant having an accident is 0.12. The industrial plant employs 175 workers. What is the expected number of accidents?

7 A die is tossed 480 times and the results recorded.

a What is the probability of throwing a 4?

b How many 4s are expected?

c What is the probability of throwing an odd number?

d How many odd numbers are expected?

e What is the probability of throwing a number greater than 2?

f How many numbers greater than 2 are expected?

g What is the probability of throwing a number divisible by 3?

h How many numbers divisible by 3 are expected?

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LEVEL 2

8 Five cards (ace, king, queen, jack and 10) are placed face down on a table. One card is selected at random and replaced. A second card is then selected at random. This experiment is repeated 200 times.

a What is the probability of selecting two aces?

b How many double aces are expected?

c What is the probability of selecting an ace followed by a king?

d How many aces then kings are expected?

e What is the probability of exactly one of the cards being a 10?

f How many single 10s are expected?

9 A three-digit number is selected from cards labelled 3, 4 and 5. The first card selected is the hundreds digit, the second card is the tens digit and the third card is the units digit. The cards are selected without replacement. This selection is repeated 30 times.

a What is the probability the number starts with the digit 3?

b How many numbers starting with the digit 3 are expected?

c What is the probability the number is 453?

d How many times is the number 453 expected?

e What is the probability the number ends with a 4 or a 5?

f How many numbers ending with a 4 or 5 are expected?

10 A bag contains 6 yellow discs and 5 red discs. Two discs are drawn in succession from the bag. The first disc is not replaced before the second disc is drawn. This process is repeated 352 times.

a How many of the first discs are expected to be yellow discs?

b How many of the first discs are expected to be red discs?

c How many double yellow discs are expected?

d How many double red discs are expected?

e How many are expected to have a first disc yellow and a second disc red?

f How many are expected to have a first disc red and a second disc yellow?

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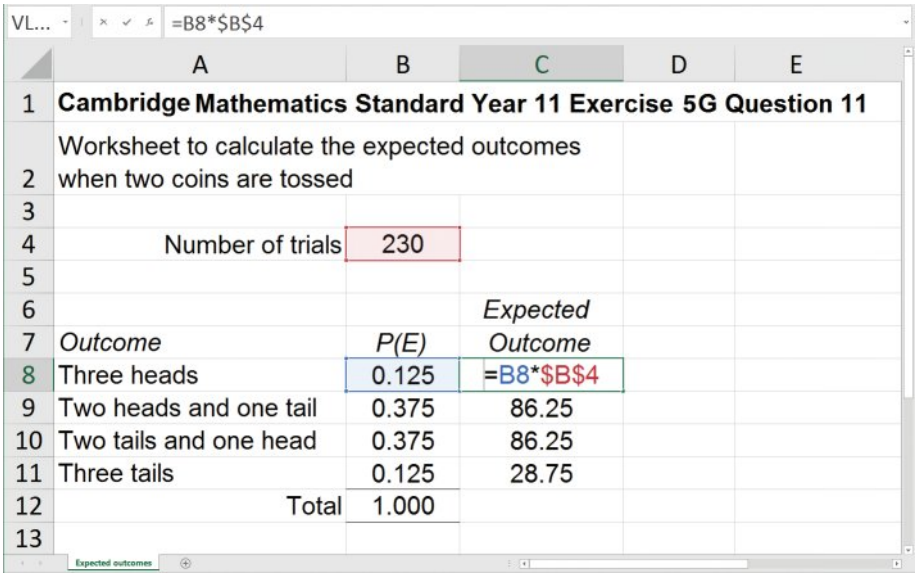
11 Create the spreadsheet below.

XLS
05GQ11



	A	B	C	D	E
1	**Cambridge Mathematics Standard Year 11 Exercise 5G Question 11**				
2	Worksheet to calculate the expected outcomes when two coins are tossed				
3					
4	Number of trials	230			
5					
6			*Expected*		
7	*Outcome*	*P(E)*	*Outcome*		
8	Three heads	0.125	=B8*$B$4		
9	Two heads and one tail	0.375	86.25		
10	Two tails and one head	0.375	86.25		
11	Three tails	0.125	28.75		
12	Total	1.000			
13					

a Cell C8 has a formula that multiplies cells B8 and B4. Enter this formula.

b Fill down the contents of C8 to C11 using the formula in cell C8.

c Change the number of trials from 230 to 800. Observe the change in C8:C11.

12 There are 240 families with three children.

a How many of these families are expected to have three boys?

b How many of these families are expected to have exactly one boy?

c How many of these families are expected to have exactly two boys?

d How many of these families are expected to have no boys?

LEVEL 3

13 Two cards are selected at random from a normal playing pack with replacement. This experiment is repeated 2704 times with the cards being replaced each time.

a What is the expected number of times the result would be two spades?

b What is the expected number of times the result would be two aces?

c What is the expected number of times the result would be two picture cards?

d What is the expected number of times the result would be two cards with a number less than 9?

14 Two dice are tossed one after the other onto a table. This event is repeated 144 times.

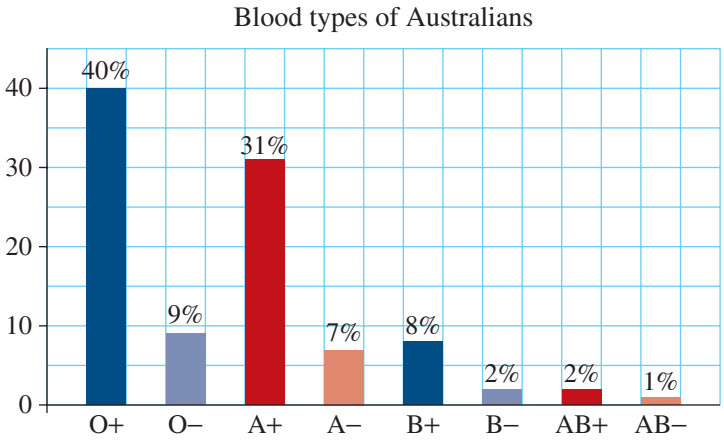
a On how many occasions would you expect the result to be a 6 then a 1?

b On how many occasions would you expect the result to be two 3s?

c On how many occasions would you expect the result to be two odd numbers?

5H Blood types

A person's blood type is described by the appropriate letter (A, B, AB or O) and whether or not their blood is Rh positive or Rh negative. The column graph opposite shows the percentage of blood type frequency in Australia (Source: Australian Red Cross Blood Service). Blood is vital to life, and for many people blood donors are their lifeline. Most of the blood donated is used to treat people with cancer and other serious illnesses.



Example 13: Calculating the number of people of a particular blood type 5H

The table opposite shows the number of males and females living in NSW. Use the above column graph and this table to answer these questions.

NSW population	
Males	3.80 million
Females	3.90 million
Total	7.70 million

a How many males in NSW have blood type B+?
b How many females in NSW have blood type O–?
c How many people in NSW have blood type A+?

SOLUTION:

1 Read the percentage of blood type B+ in the column graph (8%).
2 Read the male population of NSW in the table.
3 Multiply the percentage by the population and evaluate.
4 Read the percentage of blood type O– in the column graph (9%).
5 Read the female population of NSW.
6 Multiply the percentage by the population and evaluate.
7 Read the percentage of blood type A+ in the column graph (31%).
8 Read the population of NSW from the table.
9 Multiply the percentage by the population and evaluate.

a 8% of the Australian population is B+.

8% of 3.80 million $= 0.08 \times 3\,800\,000$
$= 304\,000$

b 9% of the Australian population is O–.

9% of 3.90 million $= 0.09 \times 3\,900\,000$
$= 351\,000$

c 31% of the Australian population is A +.

31% of 7.70 million $= 0.31 \times 7\,700\,000$
$= 2\,387\,000$

Exercise 5H

LEVEL 1

1 A blood compatibility chart is shown below (Source: Australian Red Cross Blood Service).

Patient's blood type \ Donor's blood type	O−	O+	B−	B+	A−	A+	AB−	AB+
AB+	✓	✓	✓	✓	✓	✓	✓	✓
AB−	✓		✓		✓		✓	
A+	✓	✓			✓	✓		
A−	✓				✓			
B+	✓	✓	✓	✓				
B−	✓		✓					
O+	✓	✓						
O−	✓							

a List the donor blood types that are compatible with a patient with the following blood type.

i A−
ii B−
iii AB−
iv O−
v A+
vi B+
vii AB+
viii O+

b Which donor blood type is compatible with every patient's blood type?

c Which donor blood types are compatible with 4 patients' blood types?

d Which patient's blood type is compatible with every donor's blood type?

e Which patient's blood type is only compatible with O−?

f Which patient's blood type is compatible with B+?

2 The table shows the age proportion of blood donors in low-income and high-income countries.

a Which age group from the high-income countries has the greatest proportion of blood donors?

Age groups (years)	High-income countries	Low-income countries
<18	8%	2%
18–24	20%	42.5%
25–44	33%	37%
45–64	34%	18%
>65	5%	0.5%

b Which age group from the low-income countries has the greatest proportion?

c What percentage of blood donors from high-income countries is given by people aged less than 18 years?

d Why is the least proportion of blood donors people aged over 65 years?

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LEVEL 2

3 Australia's blood donation rate is 1 in 30 and 27 250 donations are required every week.
- a Express Australia's donation rate as a percentage.
- b Australia's population is 24.6 million. How many people are blood donors?
- c NSW's population is 7.7 million. How many people are blood donors?
- d Sydney's population is 5.25 million. How many people are blood donors?
- e How many donations are required every year?

4 The table below shows the use of 27 250 blood donations every week in Australia.

Use of blood	
Cancer and blood disease	34%
Other causes of anaemia	19%
Surgical (heart surgery, burns)	18%
Medical problems (kidney, etc.)	13%
Orthopaedic (fractures, etc.)	10%
Obstetrics (pregnant women)	4%
Trauma (road accidents, etc.)	2%

- a How many blood donations are used for cancer and blood disease each week?
- b How many blood donations are used for surgical procedures each week?
- c How many blood donations are used for obstetrics each week?
- d How many blood donations are used for trauma patients each week?

LEVEL 3

5 In Australia there are 550 686 blood donors making 1.3 million blood donations per year.
- a What is the average number of blood donations per blood donor? Answer correct to one decimal place.
- b Australia's population is predicted to increase by 2% next year. How many people are expected to give blood if the donation rate remains the same?
- c The demand for blood products is predicted to double in the next 10 years. How many blood donors are required if the donation rate remains the same?
- d Each donation can be separated into red cells, platelets and plasma, and given to three different patients, leading to the saying 'one donation saves three lives'. If this is true, how many lives are saved by blood donors each year? Does this number seem believable? If not, why not?

PPT STUDY GUIDE

Key ideas and chapter summary

Language of probability

Probability is the chance of something happening.

Sample space is the set of all possible outcomes.

Each outcome is an element of the sample space.

Equally likely outcomes have an equal chance of occurring.

Multistage event consists of two or more events.

Definition of probability

$$\text{Probability (Event)} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} \text{ or } P(E) = \frac{n(E)}{n(S)}$$

Tables and tree diagrams

A table is an arrangement of information in rows and columns.

A tree diagram is a diagram that details the outcomes of a multistage event.

Range of probability

Probability of an event that is impossible is 0 and certain is 1.

Probability of an event is between 0 and 1 or $0 \leq P(E) \leq 1$.

$P(A) + P(B) + \ldots = 1$

$A, B, \ldots$ are all the possible outcomes or events.

Complementary events

Complement of an event E is the event not including E.

Probability of an event and its complement will sum to 1.

$P(E) + P(\overline{E}) = 1$ or $P(\overline{E}) = 1 - P(E)$

E – Event or outcome

$\overline{E}$ – Complement of event E or the outcomes not including event E

Relative frequency

Relative frequency is an estimate for the probability of an event.

$$\text{Relative frequency} = \frac{\text{Frequency of a data value}}{\text{Number of data values}} = \frac{f}{n}$$

Expected frequency

Expected frequency is the number of times that a particular event should occur.

Expected frequency $= n \times p = np$

n – number of times the experiment is repeated

p – probability of the event

Expected value

Expected value = Sum all these results $[P(E) \times \text{Outcome}]$

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Review

Multiple-choice questions

1 An event has the probability of occurring equal to 0. What word can you use to describe this probability?

A Certain. **B** Impossible. **C** Likely **D** Unlikely.

2 How many elements are there in the sample space when two cards are selected without replacement from cards labelled 1 to 7?

A 7 **B** 14 **C** 42 **D** 49

3 A three-digit number is formed from the digits 5, 7, 8 and 9. What is the probability that the number will be odd?

A 0.25 **B** 0.50 **C** 0.75 **D** 0.80

4 One card is selected from cards labelled 1, 2, 3, 4 and 5. What is the probability of an even number or a number divisible by 5?

A 10% **B** 50% **C** 60% **D** 100%

5 A bag contains black, white and grey balls. The probability of selecting a black ball is 0.3 and a grey ball is 0.6. What is the probability of selecting a white ball?

A 0.1 **B** 0.36 **C** 0.63 **D** 0.9

6 A letter is chosen at random from the word 'NEWCASTLE'. What is the probability that the letter will *not* be a vowel?

A $\frac{1}{9}$ **B** $\frac{2}{9}$ **C** $\frac{1}{3}$ **D** $\frac{2}{3}$

7 The frequency of an event is 6 and the total number of frequencies is 20. What is the relative frequency?

A 14% **B** 26% **C** 30% **D** 70%

8 Two unbiased coins are tossed 100 times. Which calculation illustrates the expected number of times you would get a tail and a head?

A $\frac{1}{4} \times 100$ **B** $\frac{1}{2} \times 100$ **C** $\frac{1}{3} \times 100$ **D** $\frac{1}{4} \times 200$

9 A card is selected at random from a normal pack of playing cards. The card is replaced each time. What is the expected number of times you would select a king from 260 trials?

A 5 **B** 20 **C** 52 **D** 65

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Short-answer questions

1 Describe each of these events as either impossible, likely, unlikely or certain.

a Throwing an 8 when a standard die is thrown
b A student travelling to school on the wrong bus
c A piece of gold being found in your backyard

2 List the sample space of the following events.

a Selecting a card from cards labelled with an even number from 1 to 10
b A spinner with each section labelled with a vowel
c Choosing at random a ball from a bag containing three white and four blue balls

3 A paper bag contains three green, four brown and five yellow beads. To win a game, Zoe needs to draw two green beads from the bag in two draws. How many elements are in the sample space?

4 A raffle ticket is drawn from a box containing 50 raffle tickets numbered from 1 to 50. Find the probability of the following outcomes.

a The number 50
b An even number
c A number less than 20
d A number greater than 30
e A number divisible by 5
f A square number

5 What is the probability of choosing a black card from a standard deck of cards?

6 Four kings are taken from a standard deck of cards and placed face down on a table. One card is selected at random. What is the probability of selecting:

a the king of clubs?
b a black king?
c a picture card?

7 There are four girls, three boys and two adults in a house. If one person is chosen at random, what is the probability that the person:

a is a girl?
b is a boy?
c is a girl or a boy?

8 An eight-sided die has the numbers 1 to 8 on it. What is the probability of rolling the following outcomes?

a The number 2
b Either a 3 or a 5
c The number 9
d A number divisible by 3
e An odd number
f A prime number

9 There are five students in a group and their names are Adam, Sarah, Max, Hayley and David. If one name is chosen at random, what is the probability of selecting a name:

a with five letters?
b with the letter ‘a’?
c with one vowel?

10 A fair coin is tossed three times. The probability of throwing three tails is 0.125, two tails is 0.375 and one tail is 0.375. What is the probability of the following outcomes?

a No tails
b Three or two tails
c At least one tail
d Not throwing a head
e Not throwing two tails
f Throwing one head

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Review

11 There are three outcomes of a rugby league game: win, lose or draw. If $P(\text{Win}) = \frac{5}{7}$ and $P(\text{Lose}) = \frac{1}{5}$, find the probability of the following.

a Winning or losing the match
b Drawing the match
c Not winning the match
d Not losing the match

12 Caitlin selects a card at random from a standard pack of cards. Find the probability of obtaining the following outcomes.

a Not an ace **b** Not a heart **c** Not a red six

13 A class frequency table shows the scores in a test. What is the relative frequency for the following outcomes? Answer correct to two decimal places.

a 50–59 **b** 60–69
c 70–79 **d** 80–89

Score	Frequency	Relative frequency
50–59	5	
60–69	6	
70–79	8	
80–89	6	

14 Last year Oscar bought a packet of biscuits every week and found 30 of these packets contained broken biscuits. What is the relative frequency of this event? Answer as a decimal correct to two decimal places.

15 The probability of a couple having a baby with red hair is $33\frac{1}{3}\%$. If the couple have six children, how many children with red hair are expected?

16 A probability of a dog catching heartworm is $\frac{3}{8}$. If there are 896 dogs in the local community, how many of them would you expect to catch heartworm?

Extended-response question

17 A lie detector was used to indicate the guilt or innocence of 200 suspects.

	Accurate	Not accurate	Total
True statements	95	10	105
False statements	70	25	95
Total	165	35	200

a Find the probability a person selected at random, with an accurate test, made a true statement?
b Find the probability a person selected at random, with a false statement, has an accurate test?

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6 Perimeter, area and volume

Syllabus topic — M1.2 Perimeter, area and volume

This topic will develop your skills to competently solve problems involving perimeter, area, volume and capacity.

Outcomes

- Calculate the area of circles and sectors of a circle.
- Calculate the area of composite figures.
- Apply the trapezoidal rule to estimate area.
- Calculate the surface area of right prisms.
- Calculate the surface area of cylinders and spheres.
- Calculate the surface area of pyramids and cones.
- Calculate the volume of right prisms.
- Calculate the volume of cylinders and spheres.
- Calculate the volume of pyramids and cones.
- Calculate the surface area and volume of composite solids.
- Relate capacity to volume.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Widgets
- Literacy worksheet
- Spreadsheets
- Quick Quiz
- Study guide
- Solutions (enabled by teacher)

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

In the Interactive Textbook you can take a test of prior knowledge required for this chapter, and depending on your score you may be directed to revision from the previous years' work.

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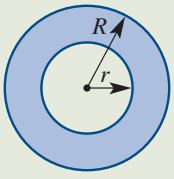
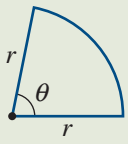
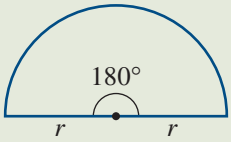
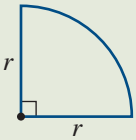
6A Area of circles and sectors

The area of a circle is calculated using the formula $A = \pi r^2$ where r is the radius of the circle. An annulus is the area between a large and a small circle with the same centre.

The area of an annulus is calculated using the formula $A = \pi(R^2 - r^2)$ where R is the radius of the large circle and r is the radius of the small circle.

A sector is part of a circle between two radii and an arc. The area of a sector is calculated using the formula $A = \frac{\theta}{360}\pi r^2$ where θ is the angle formed at the centre of the circle and r is the radius of the circle. The area of a semicircle $\left(A = \frac{1}{2}\pi r^2\right)$ and the area of a quadrant $\left(A = \frac{1}{4}\pi r^2\right)$ are special cases of the area of a sector formula.

Area formulae

Name	Shape	Area
Circle		$A = \pi r^2$
Annulus		$A = \pi(R^2 - r^2)$
Sector		$A = \frac{\theta}{360}\pi r^2$
Semicircle		$A = \frac{1}{2}\pi r^2$
Quadrant		$A = \frac{1}{4}\pi r^2$

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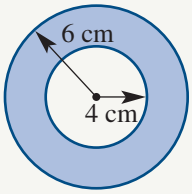
Example 1: Finding the area of an annulus 6A

Thomas draws two concentric circles whose radii are 4 cm and 6 cm. What is the area of the annulus formed, to the nearest square centimetre?

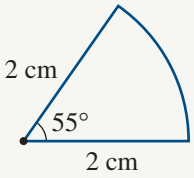
SOLUTION:

1	Draw a diagram to represent the information in the question.	
2	The shape is an annulus, so use the formula $A = \pi(R^2 - r^2)$.	$A = \pi(R^2 - r^2)$
3	Substitute the values for R ($R = 6$) and r ($r = 4$).	$= \pi \times (6^2 - 4^2)$
4	Evaluate.	$= 62.83185307$
5	Express the answer correct to the nearest whole number.	
6	Give your answer with the correct units.	$\approx 63 \text{ cm}^2$
7	Write the answer in words.	The area of the shape is 63 cm^2.

Example 2: Finding the area of a sector 6A

Find the area of a sector with an angle at the centre of 55° and radius 2 cm. Write your answer correct to one decimal place.

SOLUTION:

1	Draw a diagram to represent the information in the question.	
2	The shape is a sector, so use the formula $A = \frac{\theta}{360}\pi r^2$.	$A = \frac{\theta}{360}\pi r^2$
3	Substitute the values for θ ($\theta = 55$) and r ($r = 2$).	$= \frac{55}{360} \times \pi \times 2^2$
4	Evaluate.	$= 1.919862177$
5	Express the answer correct to one decimal place.	
6	Give your answer with the correct units.	$\approx 1.9 \text{ cm}^2$
7	Write the answer in words.	The area of the shape is 1.9 cm^2.

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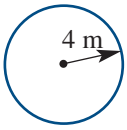
Exercise 6A

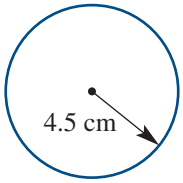
LEVEL 1

1 Find the area of each circle. Answer correct to one decimal place.

a



b



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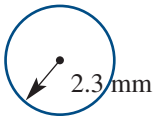

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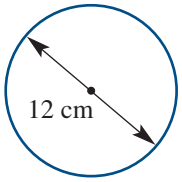
e



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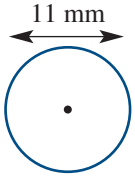


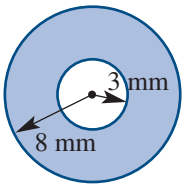
2 What is the area of the following circles? Answer correct to two decimal places.

a Radius of 25 mm
b Radius of 3 cm
c Radius of 10 m
d Diameter of 18 m
e Diameter of 98 cm
f Diameter of 2.8 mm

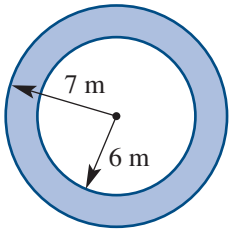
Example 1

3 Find the area of each annulus. Answer correct to one decimal place.

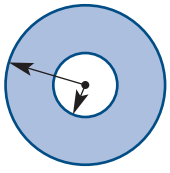
a



b



c Outer radius 3.2 m
Inner radius 0.6 m



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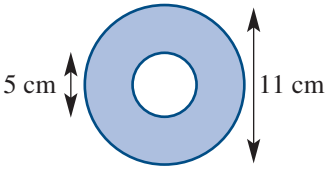
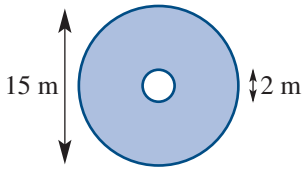


e



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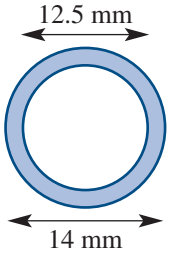

4 What is the area of each annulus? Answer correct to two decimal places.

a Outer radius of 7 m and inner radius of 4 m
b Outer radius of 5.6 cm and inner radius of 2.1 cm
c Outer diameter of 12 cm and inner diameter of 8 cm
d Outer diameter of 44 mm and inner diameter of 38 mm

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LEVEL 2

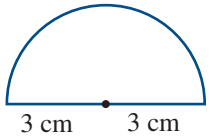
5 What is the area of an annulus with circles of radii 17 cm and 8.5 cm? Answer in square centimetres correct to one decimal place.

6 An annulus consists of two circles with the same centre. Find the area of an annulus that has an inner diameter of 6 cm and an outer diameter of 10 cm. Answer correct to the nearest square centimetre.

Example 2

7 Find the area of each sector. Answer correct to one decimal place.

a



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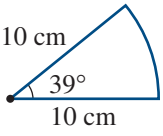

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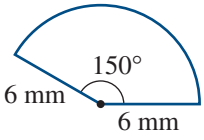

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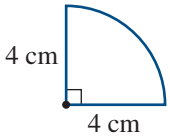

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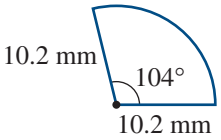

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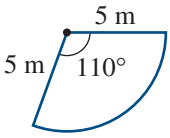
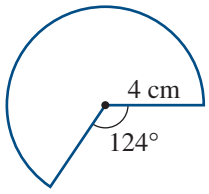


8 Find the area of each sector. Answer correct to two decimal places.

a



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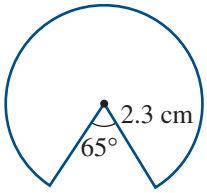

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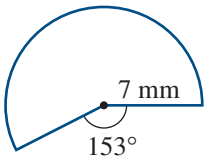

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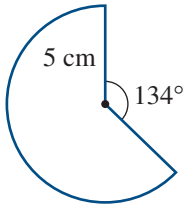

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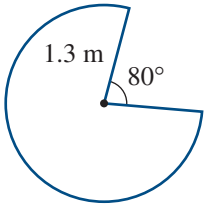

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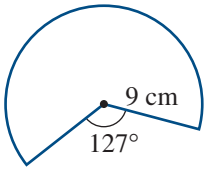
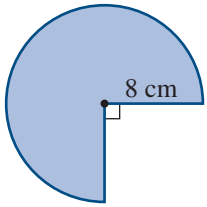


9 Calculate the area of the sector shown opposite. Express your answer correct to the nearest square centimetre.



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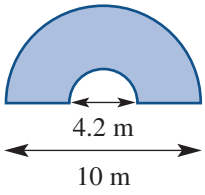
LEVEL 3

10 The cross-section of a piece of conduit is an annulus. It has an outer diameter of 60 mm and an inner diameter of 50 mm.

a Find the area of the cross-section to the nearest square millimetre.

b What is the increase in cross-sectional area if the outer diameter is increased to 65 mm? Answer correct to the nearest square millimetre.

11 An annulus with an inner diameter of 4.2 metres and an outer diameter of 10 metres is divided into two equal parts. What is the area of each part? Answer correct to three significant figures.



12 The area of a circular pond is 5.3 square metres. What is the radius of the pond? Answer correct to one decimal place.

13 A traffic roundabout has a circular garden in the centre and two lanes for traffic encircling the garden. The diameter of the garden is 16 metres and each lane is 3 metres wide. Each lane is to be resurfaced. Calculate the area to be resurfaced. Answer in square metres to the nearest whole number.

14 An annulus has an area of 392.70 square metres. Calculate the inner radius of the annulus if the outer radius is 15 metres. Answer correct to the nearest whole number.

15 Find the angle at the centre of these sectors to the nearest degree.

a Area of 104.72 cm^2 and a radius of 10 cm

b Area of 139.63 m^2 and a radius of 40 m

16 An area to be landscaped is a sector with a radius of 19 metres and an angle at the centre of 145°. The area is to be covered with turf at $12.50 per square metre and then top-dressed with soil at $2.30 per square metre. Find the total cost of the landscaping to the nearest dollar.

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6B Area of composite shapes

A composite shape is made up of two or more plane shapes. The area of a composite shape is calculated by adding or subtracting the areas of each plane shape. In some questions it may be necessary to determine the length of a missing side.

AREA OF COMPOSITE SHAPES

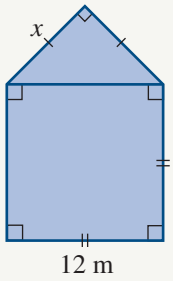
1. Divide the composite shape into two or more plane shapes.
2. Calculate the area of each plane shape.
3. Add or subtract the areas of the plane shapes.
4. Write the answer to the specified level of accuracy with the correct units.

Example 3: Finding the area of a composite shape 6B

A concrete area in the shopping centre consists of a square and a triangle. The square has a side length of 12 metres and the triangle is isosceles.

a Use Pythagoras' theorem to find the value of x.

b Calculate the area of the shaded region.



SOLUTION:

1	Write Pythagoras' theorem.	**a** $a^2 + b^2 = c^2$
2	Substitute the length of the sides.	$x^2 + x^2 = 12^2$
3	Make x^2 the subject.	$2x^2 = 12^2$
		$x^2 = 72$
4	Take the square root to find x.	$x = \sqrt{72}$
5	Divide the shaded region into a square and a triangle.	**b** $A = s^2$ (Square)
6	Use the formula $A = s^2$ to find the area of the square.	
7	Substitute the value for s ($s = 12$) into the formula.	$= 12^2$
8	Evaluate. Give your answer using the correct units.	$= 144\,\text{m}^2$
9	Use the formula $A = \frac{1}{2}bh$ to find the area of the triangle.	$A = \frac{1}{2}bh$ (Triangle)
10	Substitute the values for b and h ($b = \sqrt{72}$, $h = \sqrt{72}$).	$= \frac{1}{2} \times \sqrt{72} \times \sqrt{72}$
11	Evaluate. Give your answer using the correct units.	$= 36\,\text{m}^2$
12	Add the areas of the two plane shapes to determine the area of the shaded region.	Shaded area = Area of square + Area of triangle
13	Express using the correct units.	$= 144 + 36$ $= 180\,\text{m}^2$
14	Write the answer in words.	Area of shaded region is 180 m^2.

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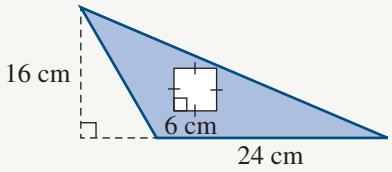
Example 4: Finding the area of a composite shape 6B

A square with a side length of 6 cm is cut from a triangle with a base of 24 cm and a height of 16 cm. What is the shaded area?



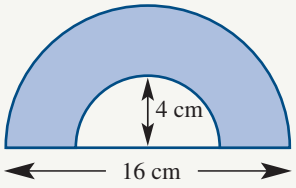
SOLUTION:

1	The shape is a triangle, so use the formula $A = \frac{1}{2}bh$.	$A = \frac{1}{2}bh$ (Triangle)
2	Substitute the values for b and h ($b = 24$ and $h = 16$).	$= \frac{1}{2} \times 24 \times 16$
3	Evaluate and express using correct units.	$= 192 \text{ cm}^2$
4	The shape is a square, so use the formula $A = s^2$.	$A = s^2$ (Square)
5	Substitute the value for s ($s = 6$).	$= 6^2$
6	Evaluate and express using the correct units.	$= 36 \text{ cm}^2$
7	Subtract the area of the square from the area of the triangle to determine the area of the shaded region.	Shaded area = Area of triangle − Area of square $= 192 - 36$
8	Express using the correct units.	$= 156 \text{ cm}^2$
9	Write the answer in words.	Shaded area is 156 cm^2.

Example 5: Finding the area of a composite shape 6B

The diagram shows a semicircle cut out of a semicircle.

a What is the area of the smaller semicircle?
b What is the area of the larger semicircle?
c What is the shaded area correct to one decimal place?



SOLUTION:

1 The shape is a semicircle, so use the formula $A = \frac{1}{2}\pi r^2$.
2 Substitute the value for r ($r = 4$).
3 Evaluate and express using correct units. Unless otherwise instructed leave as an exact answer (8π).
4 The shape is a semicircle, so use the formula $A = \frac{1}{2}\pi r^2$.
5 Substitute the value for r ($r = 8$).
6 Evaluate and express using the correct units.
7 Subtract the areas of the two plane shapes to determine the area of the shaded region.
8 Write the answer correct to one decimal place.
9 Express using the correct units.
10 Write the answer in words.

a $A = \frac{1}{2}\pi r^2$ (Semicircle)
$= \frac{1}{2} \times \pi \times 4^2$
$= 8\pi \text{ cm}^2$

b $A = \frac{1}{2}\pi r^2$ (Semicircle)
$= \frac{1}{2} \times \pi \times 8^2$
$= 32\pi \text{ cm}^2$

c Shaded area $= 32\pi - 8\pi$
$= 75.39822369\ldots$
$\approx 75.4 \text{ cm}^2$

Shaded area is about 75.4 cm^2.

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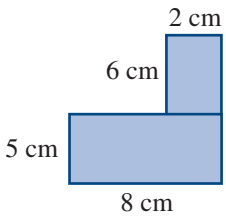
Exercise 6B

LEVEL 1

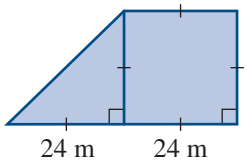
Example 3

1 Find the area of each composite shape. Answer correct to the nearest whole number.

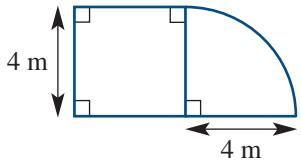
a



b



c



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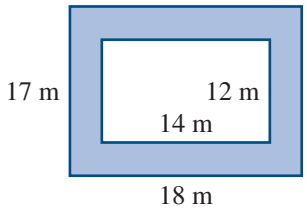

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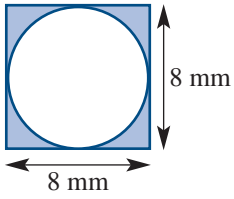

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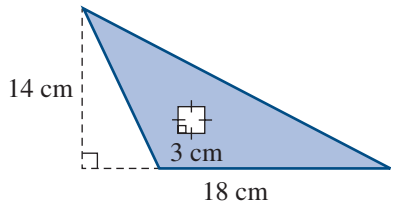
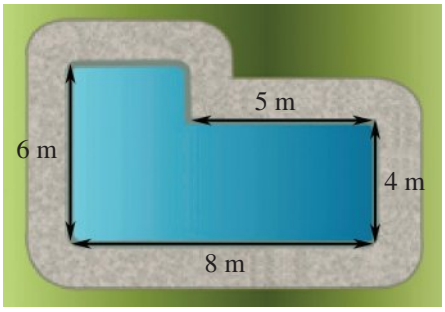
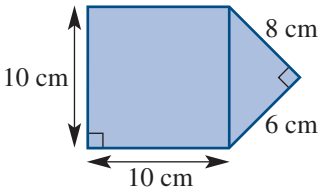


2 A swimming pool is a composite shape. It is an L-shape consisting of two rectangles. The dimensions of the L-shape are shown opposite. What is the area of the swimming pool?

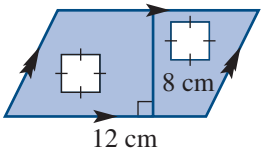


3 A drawing consists of a square and a triangle. The square has a side length of 10 cm and the triangle has sides of length 6 cm and 8 cm. What is the area of the drawing? Answer correct to one decimal place.

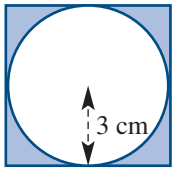


Example 4

4 A piece of cardboard consists of a parallelogram with two identical squares removed from its shape. The two squares have a side length of 2 cm. Find the shaded area. Answer correct to the nearest square centimetre.

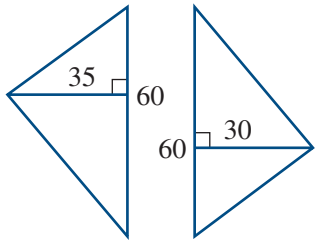


5 A circle is removed from a square as shown in the diagram. The square has a side length of 6 cm and the radius of the circle is 3 cm. What is the shaded area? Answer correct to one decimal place.



6 Find the area of the following irregular shaped blocks of land. Units are metres.

a



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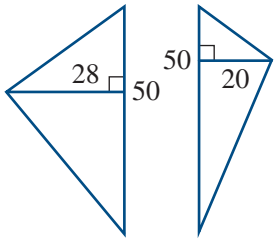

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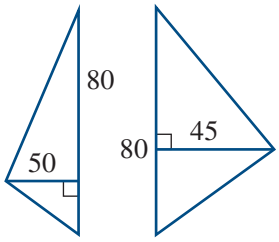

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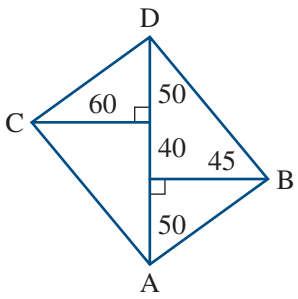

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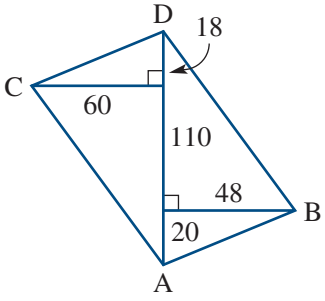

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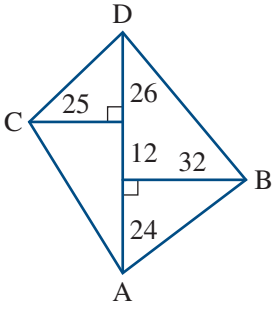
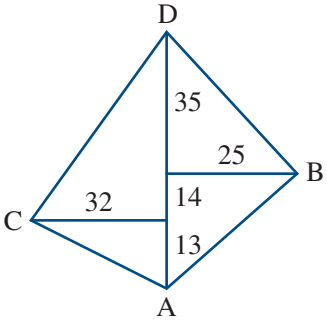


7 The diagram on the right shows a block of land that has been surveyed. All measurements are in metres.

a Find the area of the quadrilateral ABCD. Answer correct to one decimal place.

b What is the length of AB? Answer correct to the nearest metre.



8 The diagram on the right shows a block of land that has been surveyed. All measurements are in metres. Answer correct to one decimal place.

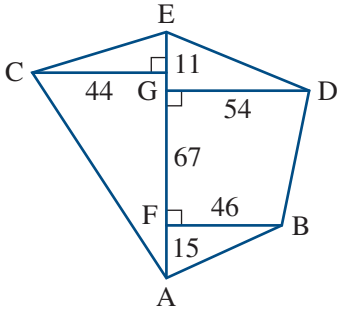
a Find the area of the triangle ABF.

b Find the area of the triangle ACE.

c Find the area of the triangle DGE.

d Find the area of the trapezium BFGD.

e What is the total area of the block of land?



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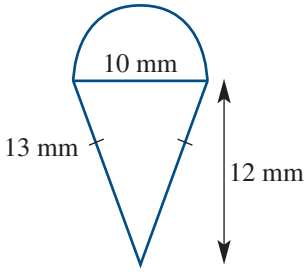
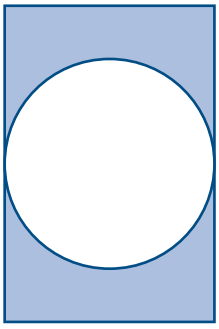
LEVEL 2

9 A composite plane shape consists of a semicircle and an isosceles triangle.



- **a** What is the radius of the semicircle?
- **b** What is the area of the semicircle? Answer correct to one decimal place.
- **c** Calculate the area of the shape. Answer correct to one decimal place.

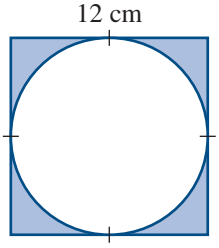
10 A photo can be displayed inside a circle using a rectangular picture frame whose dimensions are 18 cm by 10 cm. The circle has a diameter of 10 cm.



- **a** What is the area of the rectangular picture frame?
- **b** What is the area of the circle? Answer correct to two decimal places.
- **c** What is the area remaining after the circle is removed from the rectangular picture frame? Answer correct to two decimal places.

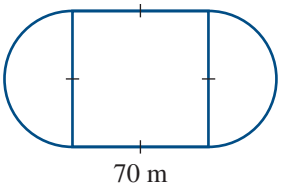
Example 5

11 A circle is cut from a square with a side length of 12 cm.



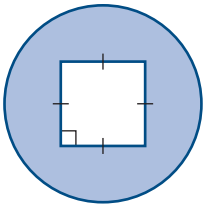
- **a** Determine the area of the circle. Answer correct to two decimal places.
- **b** What is the shaded area after the circle has been removed? Answer correct to two decimal places.

12 A sports ground consists of a square and two semicircles.



- **a** What is the radius of the semicircular ends?
- **b** Calculate the area of the sports ground in square metres correct to one decimal place.

13 A circle of diameter 10 cm surrounds a square with a side length of 5 cm. What is the shaded area between the square and the circle? Answer correct to two decimal places.



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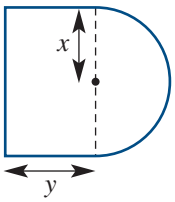
LEVEL 3

14 Lauren cuts circles with a radius of 4 cm from a rectangular piece of cardboard 8 cm by 16 cm.

- **a** What is the area of the rectangular piece of cardboard?
- **b** How many circles can be cut from the piece of cardboard?
- **c** What is the area of the remaining cardboard after the circles have been cut? Answer correct to two decimal places.

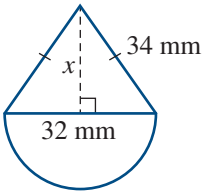
15 Aaron has decided to landscape a new lawn. The lawn is in the shape of a rectangle with a semicircle on one end.



- **a** Explain why the expression for the area of the lawn is $\frac{1}{2}\pi x^2 + 2xy$.
- **b** Find the area of lawn if $x = 4$ metres and $y = 5$ metres. Answer correct to the nearest square metre.

16 Oliver wants to pave a rectangular area measuring 3.0 m by 2.5 m in his backyard. The pavers he wishes to use are 50 cm by 50 cm. How many pavers will he need? Answer correct to the nearest whole number.

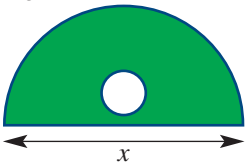
17 Saanvi is using the composite shape on the right as a design element.



- **a** What is the area of the semicircle? Answer correct to one decimal place.
- **b** Find the height of the triangle.
- **c** What is the area of the composite shape? Answer correct to one decimal place.

18 Samantha bought a block of land that is a composite shape. It consists of a square with an equilateral triangle on top. The side length of the square is 40 metres. What is the area of the block of land? Answer correct to the nearest square metre.

19 Aaron is using the composite shape drawn below as the basis for a garden. The composite shape is a semicircle with a circle of radius y removed from the centre.



- **a** Find an expression for the area of the semicircle.
- **b** What is the shaded area of the composite shape?
- **c** Find the area of garden if $x = 6$ metres and $y = 0.5$ metres. Answer correct to the nearest square metre.

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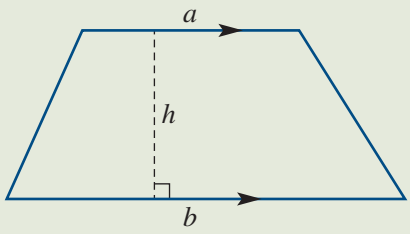
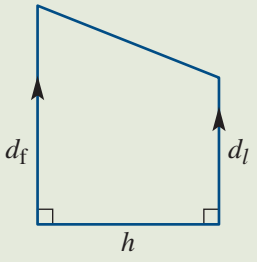
6C Trapezoidal rule

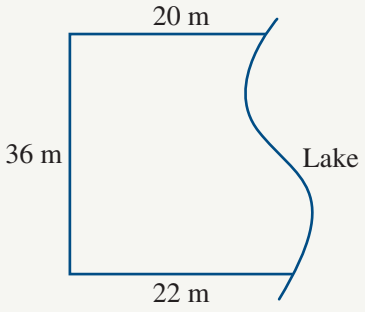
The trapezoidal rule is the area formula for a trapezium with different names for the variables. The variable h is the height or the distance between the parallel sides, while the variables d_f and d_l are the distances of the first and last parallel sides.

Area of trapezium	$A = \frac{1}{2}(a+b)h$ $= \frac{h}{2}(a+b)$	
Trapezoidal rule	$A = \frac{h}{2}(d_f + d_\ell)$	

The trapezoidal rule is used to estimate the area of a shape with an irregular boundary, such as a lake.

Example 6: Using the trapezoidal rule to estimate the area 6C

A lake forms a boundary for a block of land as shown. Apply the trapezoidal rule to approximate the area of the block. Answer to the nearest square metre



SOLUTION:

1	Write the trapezoidal rule $A = \frac{h}{2}(d_f + d_l)$.	$A = \frac{h}{2}(d_f + d_l)$
2	Write the values for h, d_f and d_l.	$h = 36$, $d_f = 20$, and $d_l = 22$
3	Substitute the values for h, d_f and d_l and evaluate.	$A = \frac{36}{2}(20 + 22)$
4	Write the answer using the correct units.	$= 756 \text{ m}^2$

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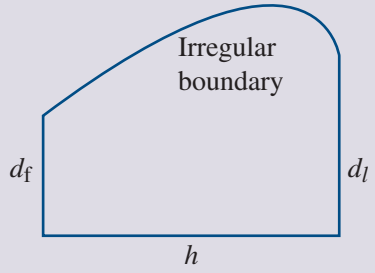
TRAPEZOIDAL RULE

$A = \frac{h}{2}(d_f + d_l)$

A – Area of shape
h – Height or width between the parallel sides
d_f – Distance along first parallel side
d_l – Distance along last parallel side

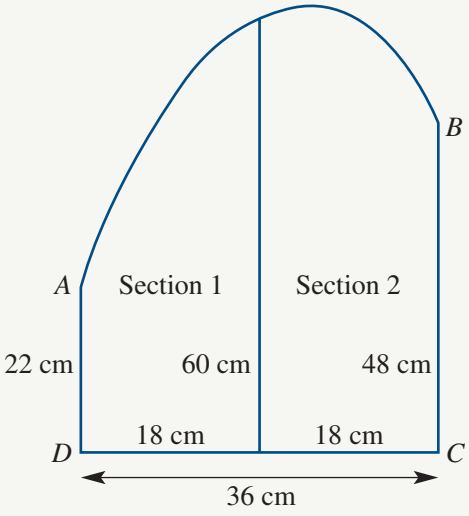


Trapezoidal rule with two applications

If the trapezoidal rule is applied more than once it provides a better estimate for the area of the shape.

Example 7: Using the trapezoidal rule to estimate the area 6C

Use the trapezoidal rule twice to estimate the area of $ABCD$ to the nearest square centimetre.



SOLUTION:

1. Divide the area $ABCD$ into two sections at 60. Label each section.
2. Write the trapezoidal rule twice for each section: $A = \frac{h}{2}(d_f + d_l)$.
3. Substitute the values for h, d_f, and d_l.
4. Evaluate.
5. Write the answer using the correct units.

Section 1: $h = 18$, $d_f = 22$, and $d_l = 60$

Section 2: $h = 18$, $d_f = 60$, and $d_l = 48$

$$A = \frac{h}{2}(d_f + d_l) + \frac{h}{2}(d_f + d_l)$$
$$= \frac{18}{2}(22 + 60) + \frac{18}{2}(60 + 48)$$
$$= 1710 \text{ cm}^2$$

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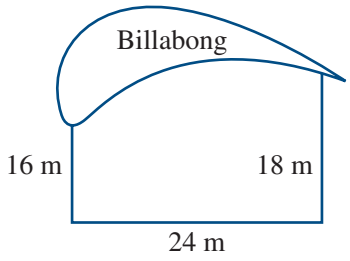
Exercise 6C

LEVEL 1

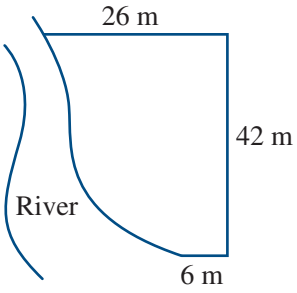
Example 6

1 Estimate the area by applying trapezoidal rule once for the following irregular fields. Answer correct to the nearest square metre.

a



b



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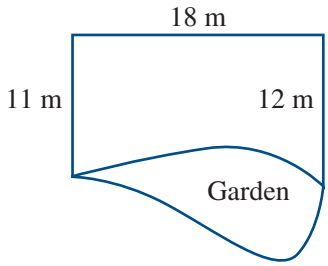
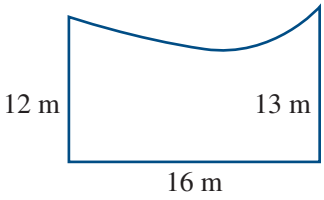


d



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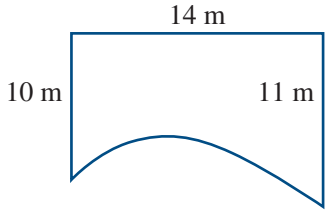

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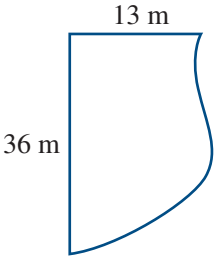
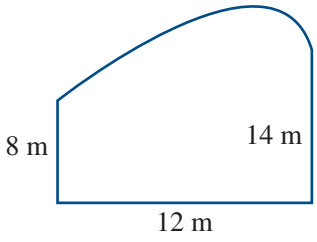
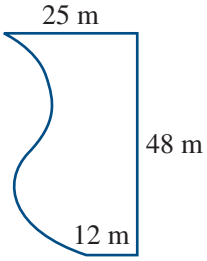


2 The following irregular shapes are the cross-sections for two swimming pools with a uniform depth of 2 metres. Calculate the volume of the swimming pools by applying the trapezoidal rule to estimate the area of the base.

a



b



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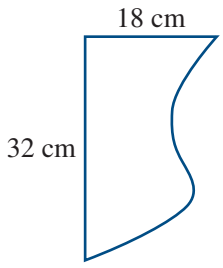
LEVEL 2

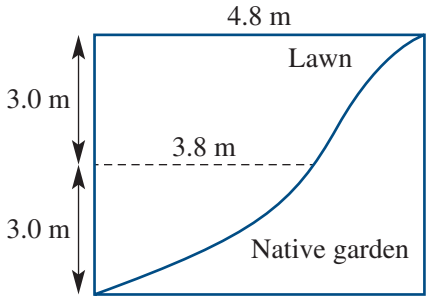
3 The cross-section of a piece of wood is shown opposite. Use the trapezoidal rule to estimate the area of the piece. Answer to the nearest square centimetre.



4 A portion of land 6 m by 4.8 m is divided into lawn and a native garden.

a Find the area of the lawn by applying the trapezoidal rule twice. Answer correct to one decimal place.

b What is the area of the native garden? Answer correct to one decimal place.

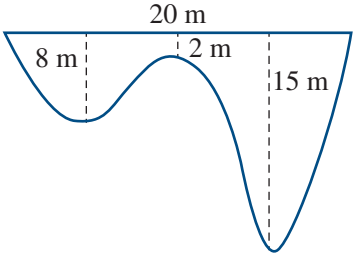


Example 7

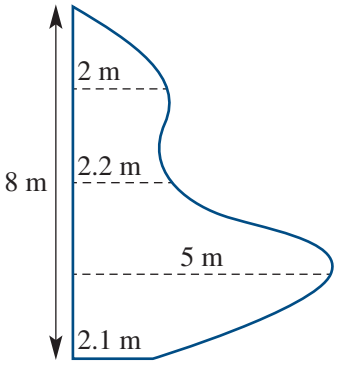
5 Connor took three measurements at 6 m intervals across an irregular-shaped field. The measurements were 10 m, 12 m and 8 m. Use two applications of the trapezoidal rule to calculate the area of the field. Answer correct to the nearest square metre.

LEVEL 3

6 The diagram opposite shows a vertical cross-section of the ocean. It is divided into four vertical strips of equal width by the dashed lines. Estimate the area of the cross-section by applying the trapezoidal rule four times. Answer correct to the nearest square metre.



7 A uniform cross-section of a sculpture is shown opposite. It is divided into four horizontal strips of equal width by the dashed lines. Apply the trapezoidal rule four times to estimate the area of the cross-section. Answer correct to the nearest square metre.



8 A curved road has five offsets to a straight boundary line with distances of 50 m, 60 m, 30 m, 40 m and 30 m. The offsets are 20 m apart. Use four applications of the trapezoidal rule to calculate the area of the field. Answer correct to the nearest square metre.

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6D Surface area of right prisms

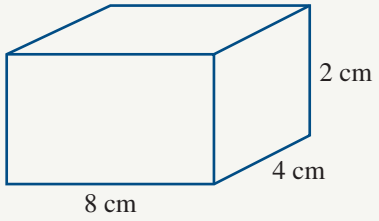
To find the surface area of a prism, it is often useful to draw the net of the solid to ensure that all the sides have been added. A net is a plane figure that shows all the surfaces of a solid. When a net is folded it forms the solid. The net of a prism is identified by each polygon that represents a surface. The surface area (SA) of a solid is the sum of the area of each surface or polygon of the solid. It is common for the area of some of the surfaces to be equal. This makes the calculations easier.

FINDING THE SURFACE AREA OF RIGHT PRISMS

1. Visualise the surfaces of the solid. If necessary draw the net of the solid.
2. Write a formula for the surface area, using the net as a guide.
3. Substitute the values into the formula for the surface area.
4. Use your calculator to find the surface area.
5. Write the answer to the specified level of accuracy with the correct units.

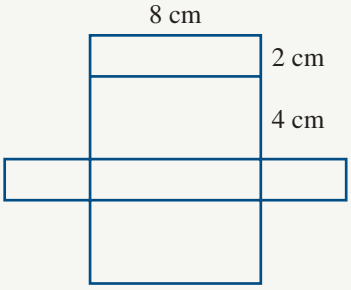
Example 8: Finding the surface area of a rectangular prism 6D

Find the surface area of this rectangular prism.



SOLUTION:

1. Draw a net of the rectangular prism.



2. Write down a formula for surface area.
3. Substitute the values into the formula.
4. Evaluate.
5. Give answer to the correct units.
6. Write the answer in words.

$$SA = (2 \times l \times b) + (2 \times b \times h) + (2 \times l \times h)$$
$$SA = (2 \times 8 \times 4) + (2 \times 4 \times 2) + (2 \times 8 \times 2)$$
$$= 112$$
$$= 112 \text{ cm}^2$$

Surface area is 112 cm^2.

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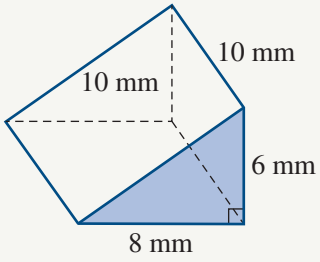
Example 9: Finding the surface area of a triangular prism 6D

Find the surface area of this triangular prism.



SOLUTION:

1. A triangular prism has two triangular faces and three rectangular faces.
2. The triangular face has a base of 8 mm and a height of 6 mm. Write the formula for the area of a triangle.
3. Substitute the values into the area of triangle formula.
4. Evaluate.
5. Write an expression for surface area of the prisms using the area of all the faces.
6. Evaluate and write answer to the correct units.
7. Write the answer in words.

Triangular faces

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 8 \times 6$

$= 24 \text{ mm}^2$

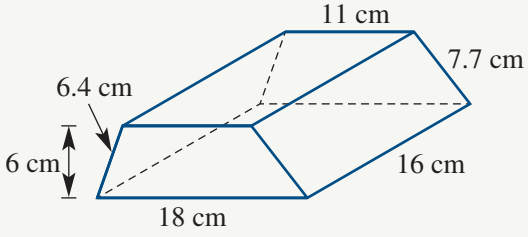
$SA = (2 \times 24) + (10 \times 10) + (10 \times 6) + (10 \times 8)$

$= 288 \text{ mm}^2$

Surface area is 288 mm^2.

Example 10: Finding the surface area of a trapezoidal prism 6D

Find the surface area of this trapezoidal prism.



SOLUTION:

1. A trapezoidal prism has two trapezoidal faces and four rectangular faces.
2. The trapezoidal face has a perpendicular height of 6 cm and two parallel sides of 11 cm and 18 cm.
3. Substitute the values into the formula.
4. Evaluate.
5. Write down an expression for surface area, using the area of all the faces.
6. Evaluate and write answer to the correct units.
7. Write the answer in words.

Trapezoidal faces

$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (18 + 11) \times 6$

$= 87 \text{ cm}^2$

$SA = (2 \times 87) + (11 \times 16) + (7.7 \times 16) + (18 \times 16) + (6.4 \times 16)$

$= 863.6 \text{ cm}^2$

Surface area is 863.6 cm^2.

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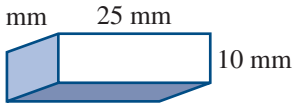
Exercise 6D

LEVEL 1

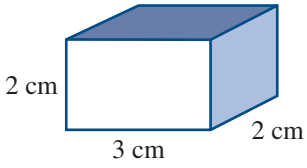
Example 8

1 Find the surface area of the following rectangular prisms.

a



b



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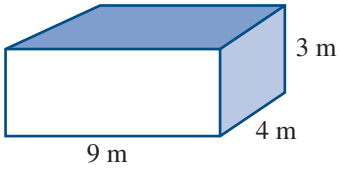

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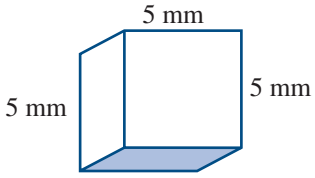

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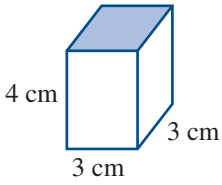

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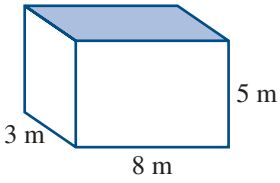


2 Find the surface area of rectangular prisms with the following dimensions.

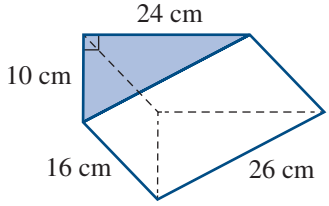
a Length 7 m, breadth 4 m, height 2 m

b Length 10 cm, breadth 9 cm, height 6 cm

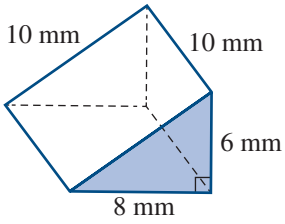
Example 9

3 Find the surface area of the following triangular prisms.

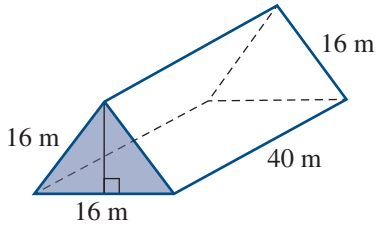
a



b

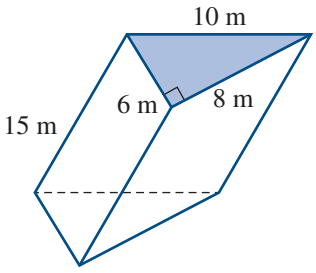


c



Area of base is 110.8 sq m

d



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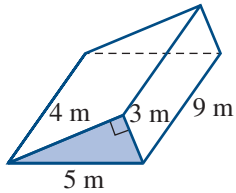
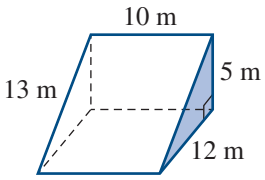


f



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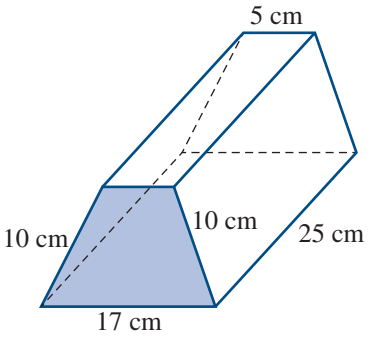
LEVEL 2

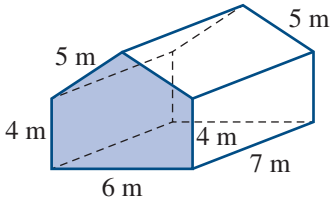
Example 10

4 Find the surface area of the following right prisms.

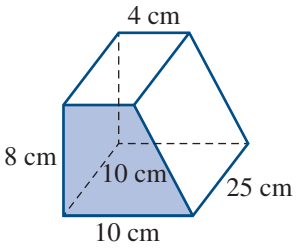
a Area of base is $88\ cm^2$



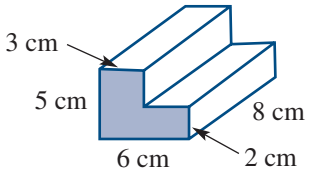
b Area of base is $36\ m^2$



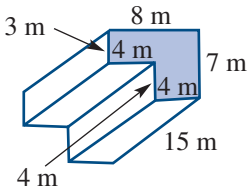
c Area of base is $56\ cm^2$



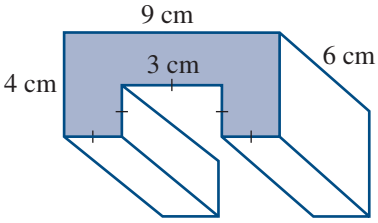
d



e



f



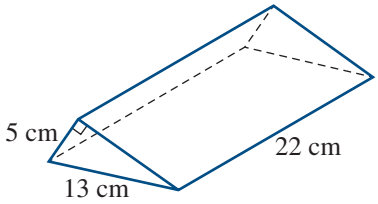
5 A bathroom is $3.5\ m$ long and $2.5\ m$ wide and its walls are to be tiled to a height of $1.5\ m$.

a Find the area of the walls to be tiled, deducting $3\ m^2$ for the door and window.

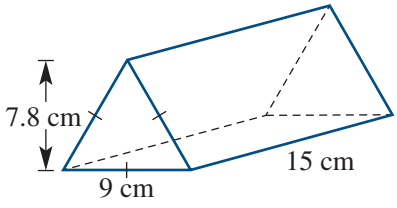
b If the tiles cost \$150 per m^2 and the tiler charges \$120 per m^2 to lay them, find the total cost for tiling the bathroom walls.

6 Find the surface area of the following right prisms.

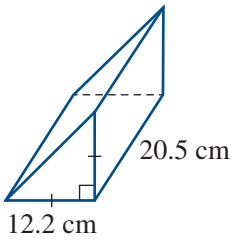
a



b



c



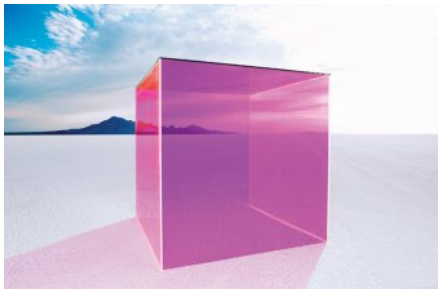
LEVEL 3

7 Find the surface area of a right triangular prism if the triangular base has a perpendicular height of $12\ m$ and a base of $18\ m$. The height of $12\ m$ and base of $18\ m$ form the two short sides of the triangular base. The triangular prism has a length of $20\ m$.

8 The surface area of a cube is $162.24\ cm^2$.

a How many faces make up the cube?

b Find the area of one face of the cube.



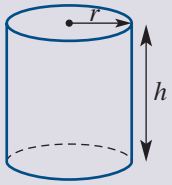
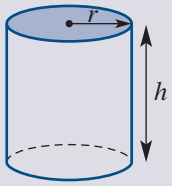
9 What is the height of a rectangular prism that has a surface area of $352\ cm^2$, given that the length of the rectangular prism is $12\ cm$ and its breadth is $4\ cm$?

6E Surface area of cylinders and spheres

Cylinders

The surface area (SA) of a cylinder is the sum of the area of each surface of the solid. To calculate the surface area of cylinders and spheres we use the formulae below.

SURFACE AREA OF A CYLINDER

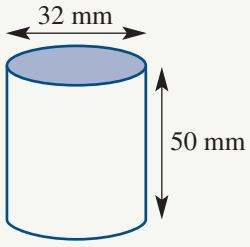
Open cylinder		$SA = 2\pi rh$ Curved surface $= 2\pi rh$
Closed cylinder		$SA = 2\pi r^2 + 2\pi rh$ Area of top $= \pi r^2$ Area of bottom $= \pi r^2$ Curved surface $= 2\pi rh$

Example 11: Finding the surface area of a cylinder 6E

A closed cylinder has a diameter of 32 mm and a height of 50 mm.

a Find the area of the curved surface.

b Find the surface area of this cylinder. Answer in square millimetres correct to two decimal places.



SOLUTION:

1 Write down a formula for surface area of an open cylinder.
2 Substitute the values for r and h into the formula.
3 Write the answer correct to two decimal places.
4 Express using correct units.
5 Write the answer in words.
6 Write down a formula for the surface area of a closed cylinder.
7 Substitute the values for r and h into the formula.
8 Write the answer correct to two decimal places.
9 Express using correct units.
10 Write the answer in words.

a $SA = 2\pi rh$
$= 2 \times \pi \times 16 \times 50$
$= 5026.5482...$
$\approx 5026.55 \text{ mm}^2$

Area of the curved surface is 5026.55 mm^2.

b $SA = 2\pi r^2 + 2\pi rh$
$= 2 \times \pi \times 16^2 + 2 \times \pi \times 16 \times 50$
$= 6635.043684$
$\approx 6635.04 \text{ mm}^2$

Surface area of cylinder is 6635.04 mm^2.

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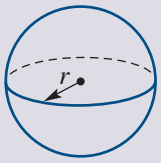
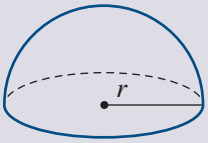
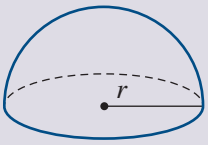
Surface area of a sphere and hemisphere

The surface area (SA) of a sphere is calculated using the formula listed below. The surface area of an open hemisphere is half that of the sphere. For a closed hemisphere, add the area of the base, which is a circle.

SURFACE AREA OF A SPHERE AND HEMISPHERE

Sphere		$SA = 4\pi r^2$
Open hemisphere		$SA = 2\pi r^2$
Closed hemisphere		$SA = 3\pi r^2$ Area of curved surface $= 2\pi r^2$ Area of base $= \pi r^2$

Example 12: Finding the surface area of a sphere 6E

What is the total surface area of one dozen tennis balls? Each ball has a radius of 3.5 cm. Answer correct to the nearest square centimetre.

SOLUTION:

1	Write down a formula for the surface area of a sphere.	$SA = 4\pi r^2$
2	Substitute the values for r into the formula.	$= 4 \times \pi \times 3.5^2$
3	Evaluate and express using correct units.	$\approx 153.93804 \text{ cm}^2$
4	Multiply the surface area of one ball by 12.	$SA \times 12 \text{ balls} = 153.93804 \times 12$
5	Evaluate.	$= 1847.25648$
6	Write the answer correct to the nearest whole number and express using correct units.	$\approx 1847 \text{ cm}^2$
7	Write the answer in words.	Surface area of 12 tennis balls is 1847 cm^2.

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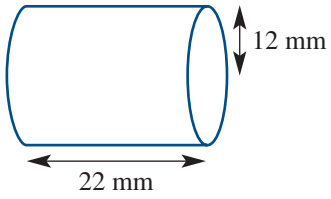
Exercise 6E

LEVEL 1

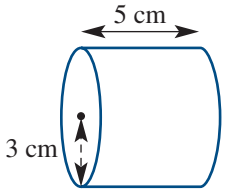
Example 11

1 Find the surface area of these open cylinders. Answer to the nearest whole number.

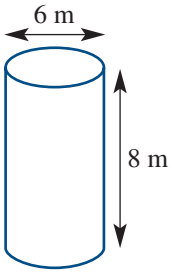
a



b

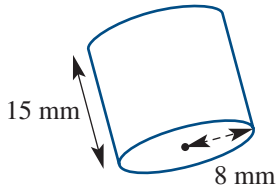


c

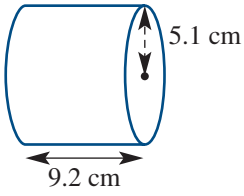


2 Find the surface area of these closed cylinders. Answer correct to one decimal place.

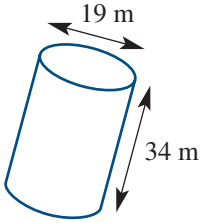
a



b



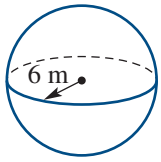
c



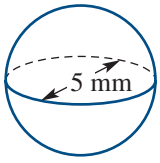
Example 12

3 Find the surface area of these spheres. Answer correct to two decimal places.

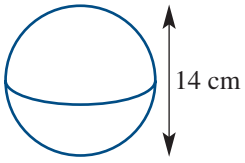
a



b

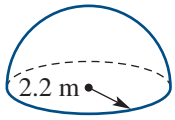


c

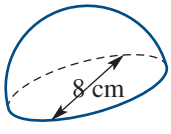


4 Find the surface area of these open hemispheres. Answer correct to two decimal places.

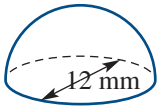
a



b

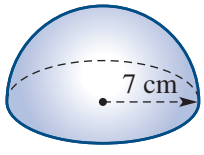


c

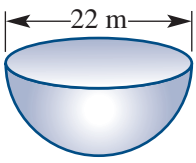


5 Find the surface area of these closed hemispheres. Answer correct to two decimal places.

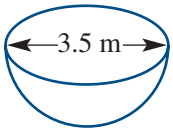
a



b



c



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LEVEL 2

6 A can of soup has a 9 cm diameter and is 14 cm high.

- **a** What is the radius of the can?
- **b** Find the area of a label that wraps around the can. Answer correct to one decimal place.
- **c** What is the surface area of the can including the top and bottom of the can? Answer correct to one decimal place.

7 The outer surface of a spherical ball is made from plastic. The diameter of the ball is 0.5 metres and the cost of plastic is $24 per square metre.

- **a** What is the outer surface area of the ball? Answer correct to two decimal places.
- **b** What is the cost of the plastic for one ball to the nearest cent?
- **c** What is the cost of the plastic for a larger ball whose diameter is twice the size?

LEVEL 3

XLS 06EQ8

8 Create the spreadsheet below.

IF =(2*PI()*A6*B6)

	A	B	C	D	E
1	**Cambridge Mathematics Standard Year 11 Exercise 6E Question 8**				
2	Worksheet to calculate the surface area of a cylinder				
3					
4			*Open cylinder*	*Closed cylinder*	
5	*Radius (r)*	*Height (h)*	*Surface area (SA)*	*Surface area (SA)*	
6	1	2	=(2*PI()*A6*B6)	18.85	
7	1	3	18.85	28.27	
8	1	4	25.13	37.70	
9	1	5	31.42	47.12	
10	1	6	37.70	56.55	
11	1	7	43.98	65.97	
12	1	8	50.27	75.40	
13					

Surface area

- **a** Cell C6 has a formula that calculates the surface area of an open cylinder. Enter this formula.
- **b** Fill down the contents of C6 to C12 using the formula for C6.
- **c** Cell D6 has a formula that calculates the surface area of a closed cylinder. Enter this formula.
- **d** Fill down the contents of D6 to D12 using the formula for D6.
- **e** Change the radius of each cylinder from 1 to 5.
- **f** Change the height of the tallest cylinder from 8 to 30.

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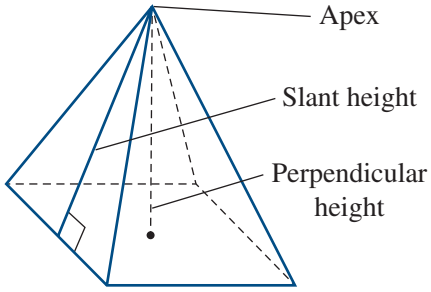
6F Surface area of pyramids and cones

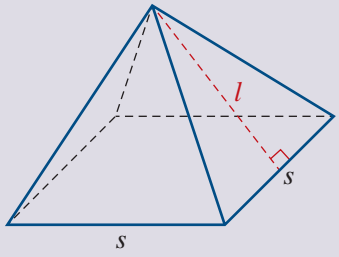
Surface area of a square pyramid

The surface area of a pyramid is found by finding the area of each face. For pyramids, the surface area is calculated by adding the area of the base and the area of the triangular faces. To find the area of the triangular face requires the slant height. The slant height of a right pyramid is the distance from the apex to the base, along a triangular face.



SURFACE AREA OF A SQUARE PYRAMID

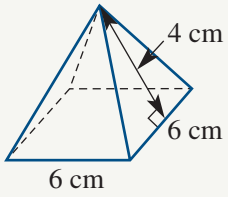
Square pyramid has a square base and 4 triangular faces with equal area.
s – side length of the base
l – slant height of triangular face



$$SA = s^2 + 4 \times \left(\frac{1}{2}bh\right)$$
$$= s^2 + 4 \times \left(\frac{1}{2}sl\right)$$

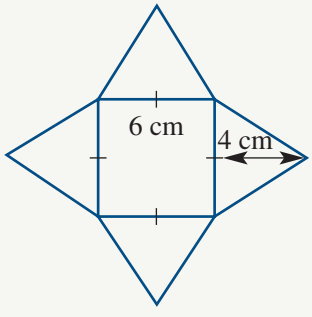
Example 13: Finding the surface area of a square pyramid 6F

A square pyramid has a base with side length is 6 cm and a slant height of 4 cm. Find the surface area of this square pyramid.



SOLUTION:

1 Draw a net of the square pyramid.



2 Write down a formula for surface area.

$$SA = s^2 + 4 \times \left(\frac{1}{2}sl\right)$$

3 Substitute the values into the formula.

$$= (6^2) + 4 \times \left(\frac{1}{2} \times 6 \times 4\right)$$

4 Evaluate.

5 Express answer in the correct units.

$$= 84 \text{ cm}^2$$

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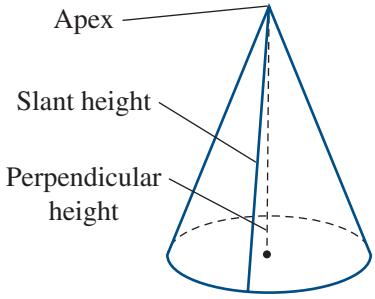
Surface area of a cone

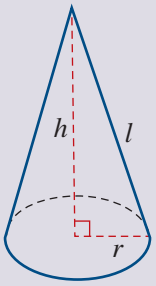
A cone has a flat circular base and a curved surface. The area of the circular base is πr^2 and the area of the curved surface is πrl where r is the radius of the circular base and l is the slant height.



SURFACE AREA OF A CONE

A cone has a flat circular base and a curved surface.

r – radius of the circular base
l – slant height of curved surface
h – perpendicular height of cone



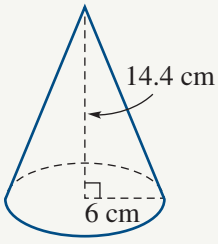
$$SA = \pi r^2 + \pi rl$$

Example 14: Finding the surface area of a cone 6F

A cone has a radius of 6 cm and a perpendicular height of 14.4 cm.

a Find the slant height of the cone. Answer correct to one decimal place.

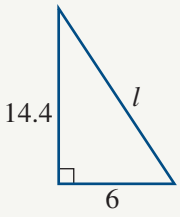
b Find the surface area of the cone. Answer correct to the nearest square centimetre.



SOLUTION:

1 Draw the right triangle and label the radius, perpendicular height and slant height.

a



2 Write Pythagoras' theorem.

3 Substitute the lengths of the sides.

4 Take the square root to find l.

$$\begin{aligned} l^2 &= h^2 + r^2 \\ &= 14.4^2 + 6^2 \\ l &= \sqrt{14.4^2 + 6^2} \\ &\approx 15.6 \end{aligned}$$

5 Write down a formula for surface area.

6 Substitute the values into the formula.

7 Evaluate.

8 Express correct to the nearest square centimetre.

b

$$\begin{aligned} SA &= \pi r^2 + \pi rl \\ &= \pi \times 6^2 + \pi \times 6 \times 15.6 \\ &= 407.1504... \\ &\approx 407 \text{ cm}^2 \end{aligned}$$

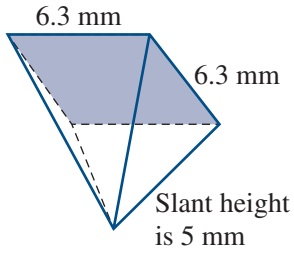
Exercise 6F

LEVEL 1

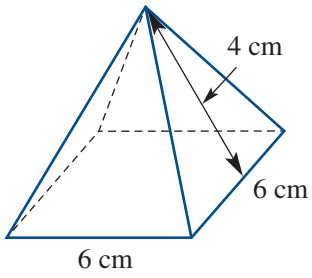
Example 13

1 Find the surface area of the following square pyramids. Answer correct to one decimal place where necessary.

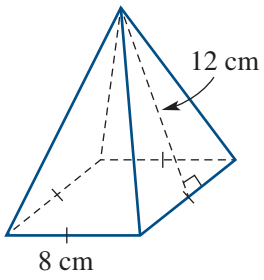
a



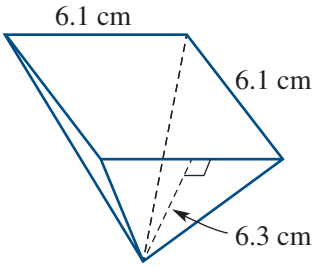
b



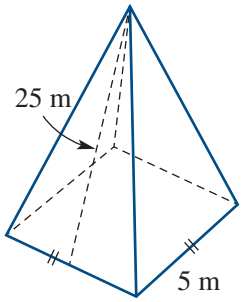
c



d



e



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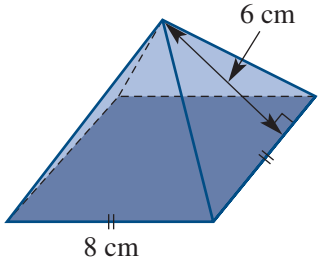
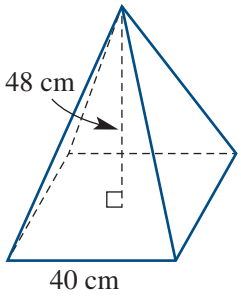
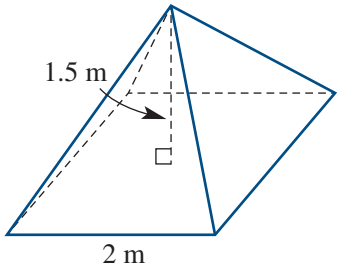


2 Find the surface area of the following square pyramids. Answer correct to one decimal place where necessary.

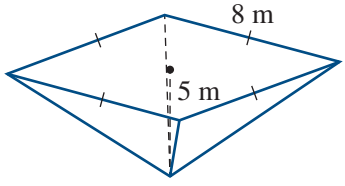
a



b



c



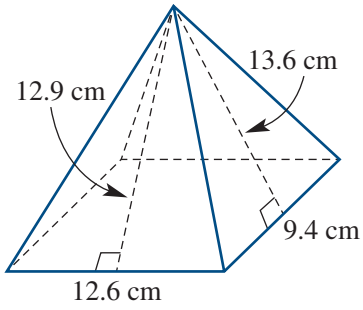
3 A rectangular pyramid has base length of 12.6 cm and breadth of 9.4 cm The slant height to the apex from the length is 12.9 cm and from the breadth is 13.6 cm.

a What is the area of the rectangular base?

b What is the area of the triangular face that has a base of 12.6 cm and height of 12.9 cm?

c What is the area of the triangular face that has a base of 9.4 cm and height of 13.6 cm?

d What is the surface area of the rectangular pyramid?



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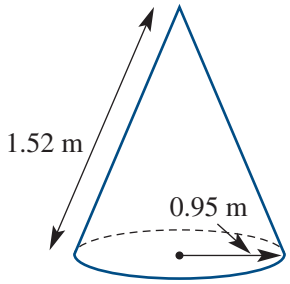
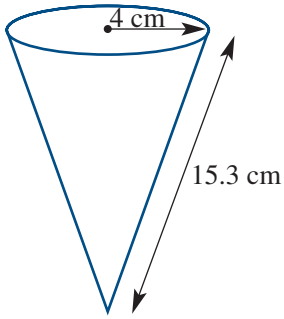
4 Find the surface area of the following cones. Answer correct to one decimal place.

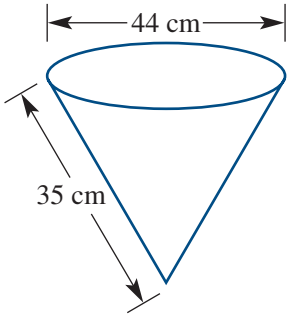
a



b

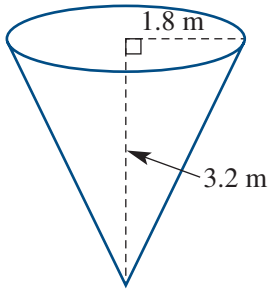


c

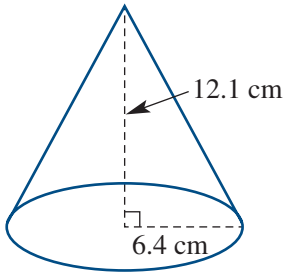


5 Find the surface area of the following cones. Round values used in calculations to one decimal place. Answer correct to the nearest whole number.

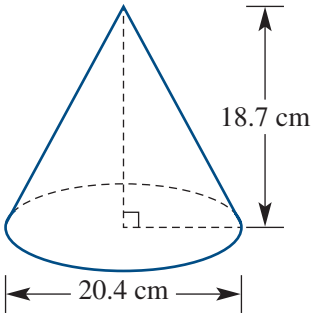
a



b



c



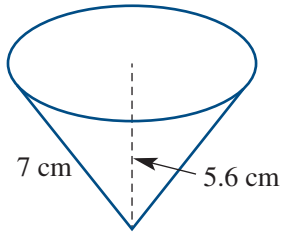
6 A cone has a perpendicular height of 5.6 cm and slant height of 7 cm. Answer these questions correct to one decimal place.

a What is the radius of the cone?

b What is the area of the circular base?

c What is the area of the curved surface?

d What is the surface area of the cone?



7 Find the area of the curved surface for the following cones. Answer correct to the nearest whole number.

a Cone with a radius of 9 cm and a slant height of 14 cm

b Cone with a radius of 7.4 cm and a slant height of 11.5 cm

8 Find the surface area of the following cones. Answer correct to the nearest whole number.

a Cone with a radius of 1.7 cm and a slant height of 2.1 cm

b Cone with a radius of 6.1 cm and a slant height of 7.5 cm

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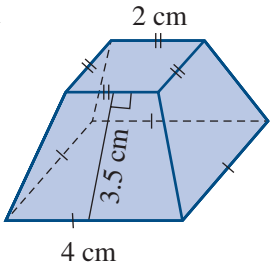
LEVEL 2

9 The solid opposite is called a frustum. It is the portion of a square pyramid that remains after the upper part has been cut off by a plane parallel to its base. What is the surface area of the frustum? Answer correct to one decimal place.



10 Find the surface area of a rectangular pyramid with a base length of 7 cm, base breadth of 5 cm and perpendicular height of 10 cm. Answer correct to 2 significant figures.

11 The curved surface area of a cone is 150 cm^2 and the radius of the base is 5 cm. Give answers correct to one decimal place.

- **a** What is the slant height?
- **b** What is the perpendicular height?
- **c** What is the surface area of the cone?

LEVEL 3

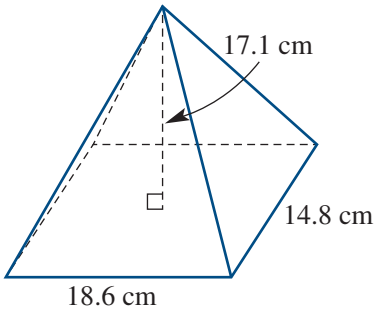
12 A cone with a radius of 5 cm has a surface area of 200π cm^2. What is the perpendicular height of the cone? Answer correct to one decimal place.

13 A cone with a base area of 150 cm^2 has a total surface area of 2000 cm^3.

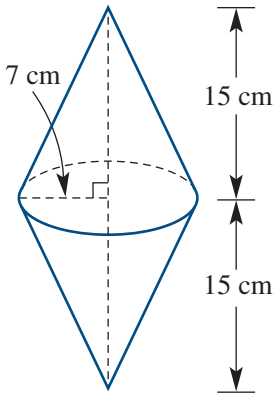
- **a** What is the radius of the base? Answer correct to two decimal places.
- **b** What is the slant height? Answer correct to two decimal places.
- **c** What is the perpendicular height? Answer correct to nearest whole number.

14 Find the surface area of the following solids. Answer correct to the nearest square centimetre.

a



b



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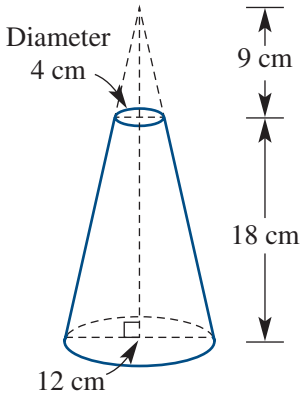

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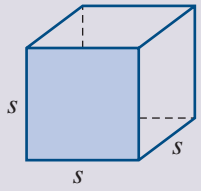
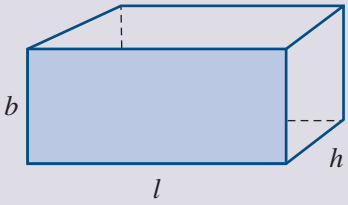
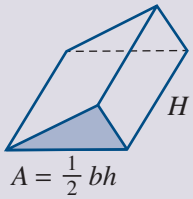
6G Volume of right prisms

Volume is the amount of space occupied by a three-dimensional object. It is measured by counting the number of cubes that fit inside the solid. When calculating volume, the answer will be in cubic units.

$1000\ \text{mm}^3 = 1\ \text{cm}^3$
$1000\,000\ \text{cm}^3 = 1\ \text{m}^3$
$1000\,000\,000\ \text{m}^3 = 1\ \text{km}^3$

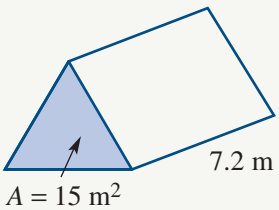
To calculate the volume of the most common solids, we use a formula. Some of these formulae are listed below. The volume of a prism is found by using its cross-sectional area. Prisms are three-dimensional objects that have a uniform cross-section along their entire length.

VOLUME OF RIGHT PRISMS

Cube		$V = Ah$ $= (s^2) \times s$ $= s^3$
Rectangular prism		$V = Ah$ $= lb \times h$ $= lbh$
Triangular prism		$V = Ah$ $= \left(\frac{1}{2}bh\right) \times H$

Example 15: Finding the volume of a right prism 6G

The base area of a triangular prism is $15\ \text{m}^2$ and the height is 7.2 m. What is the volume of the triangular prism?



SOLUTION:

1. Use the volume formula for a right prism $V = Ah$.
2. Substitute the values into the formula.
3. Evaluate.
4. Write answer to the correct units.

$V = Ah$
$= 15 \times 7.2$
$= 108\ \text{m}^3$

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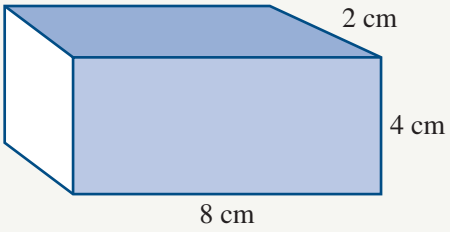
Example 16: Finding the volume of a rectangular prism 6G

A rectangular prism has a length of 8 cm, a breadth of 2 cm and a height of 4 cm. Find the volume of this rectangular prism. Answer in cubic centimetres.



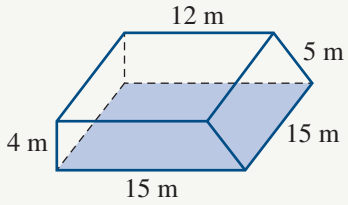
SOLUTION:

1. Use the volume formula for a right prism $V = Ah$.
2. Determine the shape of the base and the formula to calculate the area of the base $A = lb$.
3. Substitute the values into the formula.
4. Evaluate.
5. Give the answer to the correct units.

$$V = Ah$$
$$= lbh$$
$$= 8 \times 2 \times 4$$
$$= 64 \text{ cm}^3$$

Example 17: Finding the volume of a trapezoidal prism 6G

The dimensions of a trapezoidal prism are shown opposite. Find the volume of this prism.



SOLUTION:

1. Find the area of the trapezoidal side (front face in the diagram.)
2. Use the area formula for a trapezium $A = \frac{1}{2}(a+b)h$.
3. Substitute the values into the formula.
4. Evaluate.
5. Write answer to the correct units.
6. Use the volume formula for a right prism $V = Ah$.
7. Substitute the values into the formula.
8. Evaluate.
9. Write answer to the correct units.

$$A = \frac{1}{2}(a+b)h$$
$$= \frac{1}{2} \times (12+15) \times 4$$
$$= 54 \text{ m}^2$$
$$V = Ah$$
$$= 54 \times 15$$
$$= 810 \text{ m}^3$$

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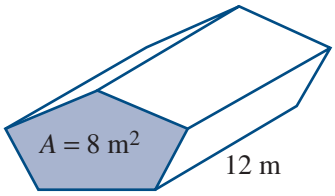
Exercise 6G

LEVEL 1

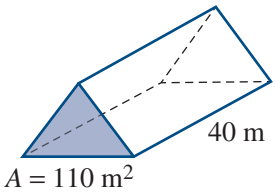
Example 15

1 Find the volume of the following prisms where A is the area of the base.

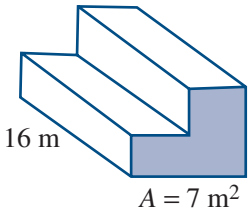
a



b



c



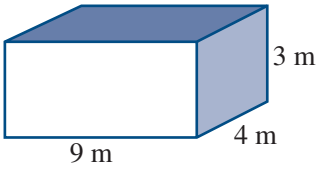
2 What is the volume of a rectangular prism with a base area of 15 mm^2 and a height of 11 mm?

3 Find the volume of a triangular prism with a height of 15 m and a base area of 50 m^2.

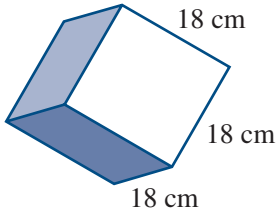
Example 16

4 Find the volume of the following solids. Answer to the nearest whole number.

a



b



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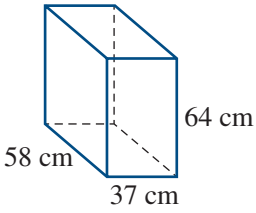
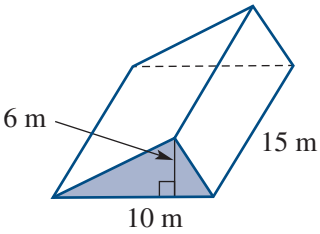


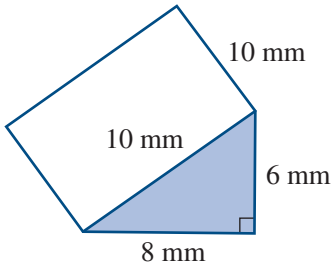
5 What is the volume of a rectangular prism with dimensions 4.5 cm by 6.5 cm by 10.5 cm? Answer correct to one decimal place.

6 Find the volume of the following triangular prisms, to the nearest whole numbers.

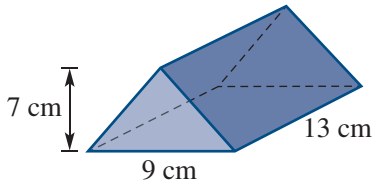
a



b



c



7 A triangular prism has a base length of 12 cm and base height of 20 cm. What is the volume of the prism if the height of the prism is 10 cm? Answer correct to one decimal place.

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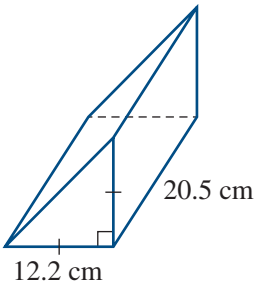
LEVEL 2

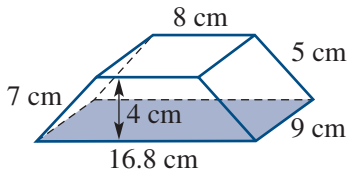
Example 17

8 Find the volume of the following solids. Answer to the nearest whole number.

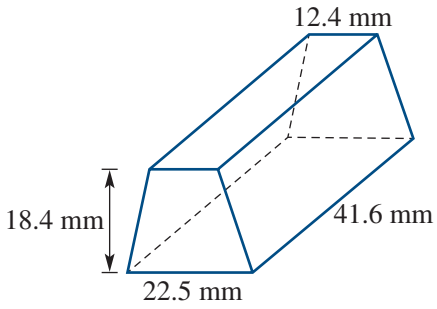
a



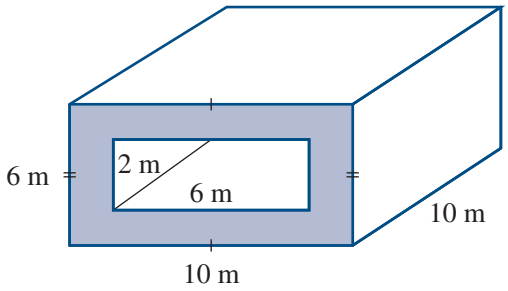
b



c



9 A hollow container is in the shape of a rectangular prism as shown.



a What would be the volume of the container if it were solid?

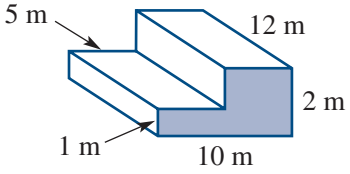
b What is the area of the shaded base?

c What is volume of the hollow container?

10 A step is shown opposite.

a What is the area of the shaded base?

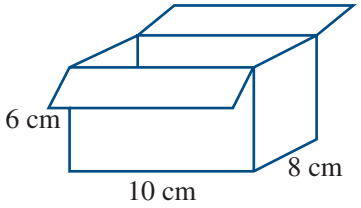
b Determine the volume of the step.



LEVEL 3

11 Find the volume of an equilateral triangular prism with side lengths 3 cm and a depth of 10 cm. Answer correct to three decimal places.

12 A vase with a volume of 200 cm^3 is packed into the cardboard box shown below. The space around the vase is filled with foam to protect the vase from breaking. The parcel is sealed and posted.



a What is the volume of the foam?

b What is the area of cardboard on the surface of the box?

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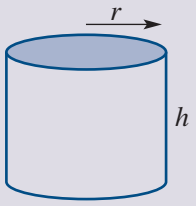
6H Volume of cylinders and spheres

Cylinders

A cylinder is a right prism whose cross-section is a circle. Its volume is found using $V = Ah$ where $A = \pi r^2$. The volume of a cylinder is the product of π, the square of the radius and the height.

VOLUME OF A CYLINDER

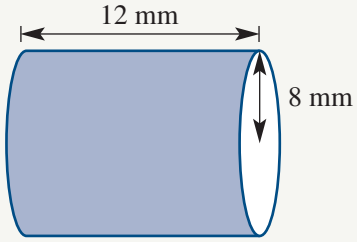
Cylinder



$$\begin{aligned} V &= Ah \\ &= (\pi r^2) \times h \\ &= \pi r^2 h \end{aligned}$$

Example 18: Finding the volume of a cylinder 6H

A cylinder has a radius of 8 mm and a height of 12 mm. Find the volume of the cylinder. Answer correct to two decimal places.



SOLUTION:

1. Use the volume formula for a cylinder $V = \pi r^2 h$.
2. Substitute the $r = 8$ and $h = 12$ into the formula.
3. Evaluate.
4. Write the answer correct number of decimal places and with correct units.

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 8^2 \times 12 \\ &= 2412.74\,3158 \\ &\approx 2412.74 \text{ mm}^3 \end{aligned}$$

Example 19: Finding the volume of a annulus prism 6H

The diameter of a DVD is 12 cm, and the diameter of the hole in its centre is 1.5 cm. Find its volume in cubic centimetres, correct to two decimal places, given that the DVD is 0.12 cm thick.

SOLUTION:

1. Use the area formula for an annulus $A = \pi(R^2 - r^2)$.
2. Substitute the $R = 6$ and $r = 0.75$ into the formula.
3. Evaluate.
4. Use the volume formula for a right prism $V = Ah$.
5. Substitute the $A = 111.33...$ and $h = 0.12$ into the formula.
6. Evaluate.
7. Write answer correct to two decimal places.

$$\begin{aligned} A &= \pi(R^2 - r^2) \\ &= \pi(6^2 - 0.75^2) \\ &= 111.33018\ldots \text{ cm}^2 \\ V &= Ah \\ &= 111.33018\ldots \times 0.12 \\ &= 13.35962\ldots \\ &\approx 13.36 \text{ cm}^3 \end{aligned}$$

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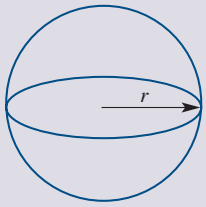
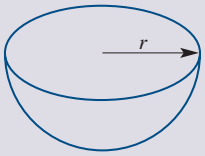
Volume of a sphere

The volume of a sphere is four-thirds the product of π and the cube of the radius. The volume of a hemisphere is half the volume of the related sphere or four-thirds the product of π and the cube of the radius.

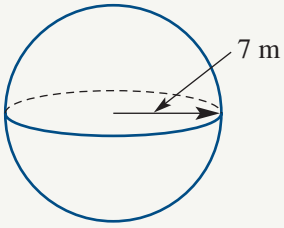
VOLUME OF SPHERE

Sphere		$V = \frac{4}{3}\pi r^3$
Hemisphere		$V = \frac{1}{2} \times \frac{4}{3}\pi r^3$ $= \frac{2}{3}\pi r^3$

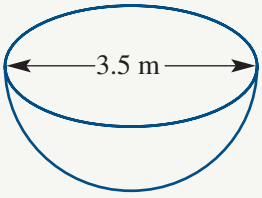
Example 20: Finding the volume of a sphere and a hemisphere 6H

Find the volume of these solids. Answer correct to one decimal place.

a



b



SOLUTION:

1 Use the volume formula for a sphere $V = \frac{4}{3}\pi r^3$.

2 Substitute $r = 7$ into the formula.

3 Evaluate.

4 Write the answer correct to one decimal place and with correct units.

a

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \pi \times 7^3$$
$$= 1436.75504$$
$$\approx 1436.8 \text{ m}^3$$

5 Use the volume formula for a hemisphere.

6 Find r and substitute the value into the formula.

7 Evaluate.

8 Write the answer correct to one decimal place and with correct units.

b

$$V = \frac{1}{2} \times \left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$$
$$r = 3.5 \div 2 = 1.75 \text{ m}$$
$$V = \frac{2}{3} \times \pi \times 1.75^3$$
$$= 11.2246\ldots$$
$$\approx 11.2 \text{ m}^3$$

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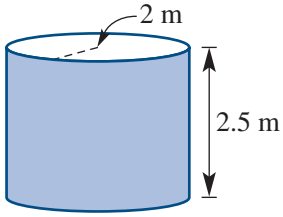
Exercise 6H

LEVEL 1

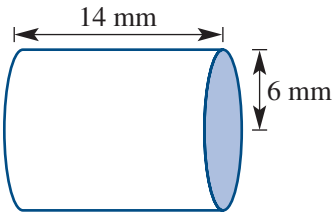
Example 18

1 Find the volume of the following cylinders. Answer to the nearest whole number.

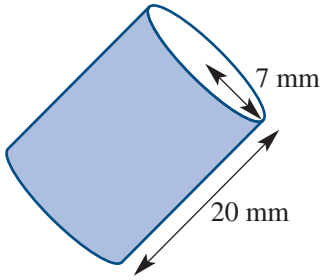
a



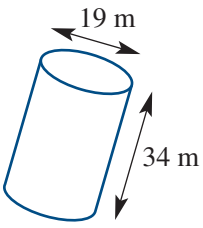
b



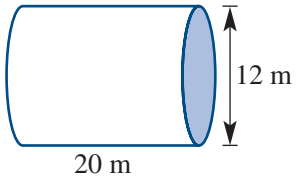
c



d



e



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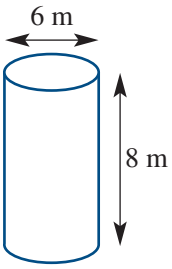
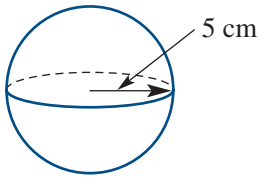


2 A closed cylindrical plastic container is 20 cm high and its circular end surfaces each have a radius of 5 cm. What is its volume, correct to two decimal places?

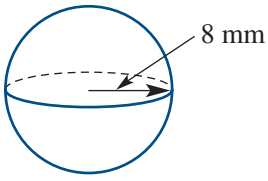
Example 20

3 Find the volume of the following solids. Answer to the nearest whole number.

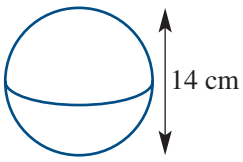
a



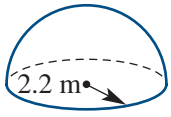
b



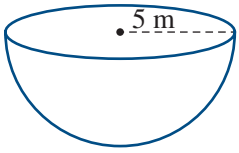
c



d



e



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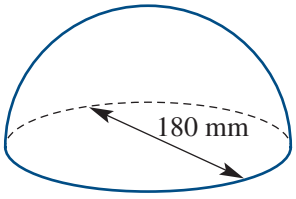

4 Find the volume of a sphere with a diameter of 12 mm. Answer correct to the nearest whole number.

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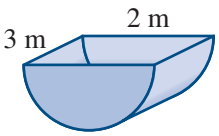
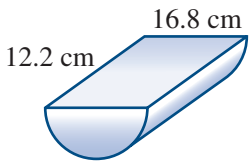
LEVEL 2

5 Find the volume of the following solids. Answer correct to one decimal place.

a



b



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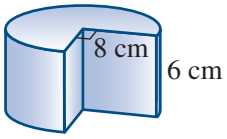
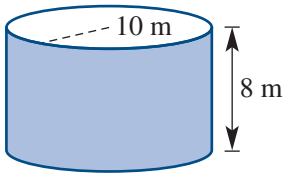


6 A water tank is in the shape of a closed cylinder with a radius of 10 m and height of 8 m.



a What is the area of the top circular face of the water tank? Leave your answer in terms of π.

b Determine the volume of the water tank. Leave your answer in terms of π.

7 A cylindrical wheat hopper is 8 m across and 10 m high.

a Calculate its volume correct to one decimal place.

b Grain pours out at 48 000 cm^3 per second. How long would it take a full hopper to empty? Answer correct to the nearest second.

8 The circumference of Earth at the equator is about 40 075 km.

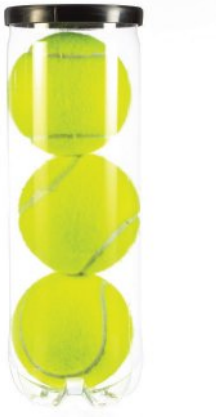
a Use the formula $C = 2\pi r$ to find the radius of Earth correct to the nearest kilometre.

b Use this radius to find the volume of Earth. Express your answer in standard notation using three significant figures.

LEVEL 3

9 A cylinder has its radius doubled. How many times is its volume increased?

10 A can of tennis balls contains three balls squeezed in with no room for the balls to move. The container has a height of 21 cm. Answer these questions correct to one decimal place.



a What is the radius of a tennis ball?

b Calculate the volume of a ball.

c Calculate the volume of the container.

d What volume of the container is not occupied by the balls?

e Express this unused space as a percentage of the total container.

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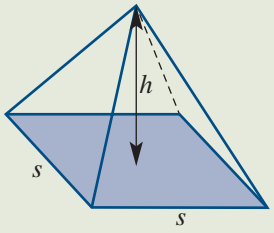
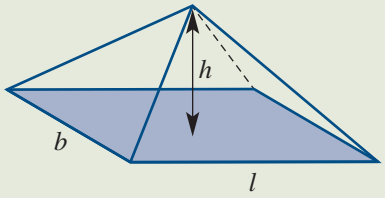
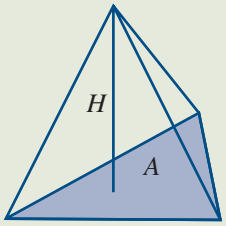
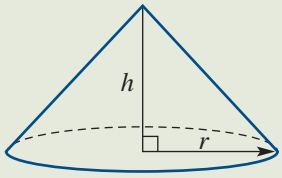
6I Volume of pyramids and cones

A pyramid fits exactly inside a prism and occupies one third of the volume of the prism. Therefore the formula for a pyramid will be $V = \frac{1}{3}Ah$ where A is the area of the base and h is the height of the pyramid. Likewise, the volume of a cone is one third the volume of a cylinder.

Volume formulae

Name	Solid	Volume
Square pyramid		$V = \frac{1}{3}Ah$ $= \frac{1}{3} \times s^2 \times h$
Rectangular pyramid		$V = \frac{1}{3}Ah$ $= \frac{1}{3} \times lb \times h$
Triangular pyramid		$V = \frac{1}{3}AH$ $= \frac{1}{3} \times \frac{1}{2}bh \times H$
Cone		$V = \frac{1}{3}Ah$ $= \frac{1}{3} \times \pi r^2 \times h$

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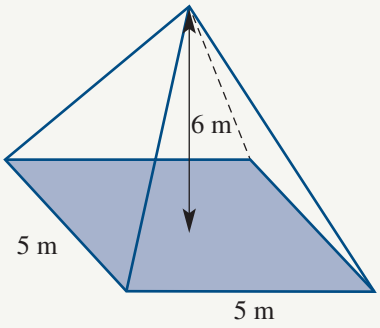
Example 21: Finding the volume of a square pyramid 6I

Find the volume of this square pyramid of height 6 m and base 5 m. Give your answer correct to the nearest cubic metre.



SOLUTION:

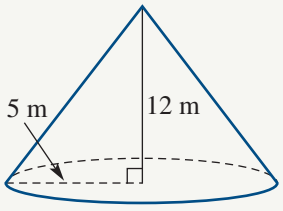
1. Use the volume formula for a pyramid $V = \frac{1}{3}Ah$.
2. Determine the shape of the base and the formula to calculate the area of the base: $A = s^2$.
3. Substitute the values into the formula.
4. Evaluate.
5. Give the answer to the correct units.
6. Write the answer in words.

$$V = \frac{1}{3}Ah$$
$$= \frac{1}{3}s^2h$$
$$= \frac{1}{3} \times 5^2 \times 6$$
$$= 50 \text{ m}^3$$

Volume of the pyramid is 50 m^3.

Example 22: Finding the volume of a cone 6I

Find the volume of this cone. Answer correct to three significant figures.



SOLUTION:

1. Use the volume formula for a cone $V = \frac{1}{3}Ah$.
2. Determine the shape of the base and the formula to calculate the area of the base: $A = \pi r^2$.
3. Substitute the values for r and h into the formula.
4. Evaluate.
5. Express the answer in the correct units.
6. Write the answer in words.

$$V = \frac{1}{3}Ah$$
$$= \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \pi \times 5^2 \times 12$$
$$= 314.1592654...$$
$$\approx 314 \text{ m}^3$$

Volume of the cone is 314 m^3.

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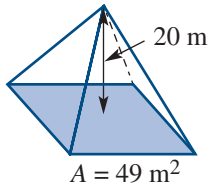
Exercise 6I

LEVEL 1

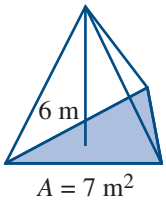
Example 21

1 Find the volume of the following pyramids. Answer to the nearest whole number.

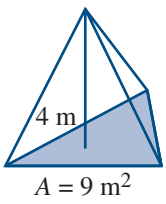
a



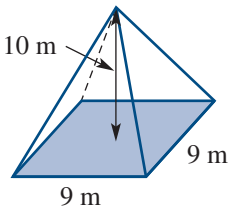
b



c



d



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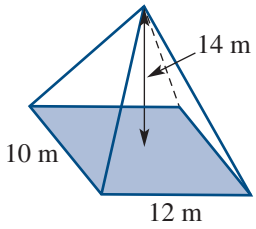

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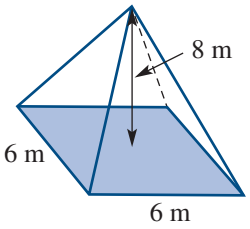


2 Find the volume of the solids with the following dimensions. Give the answers correct to the nearest whole number.

a Rectangular pyramid with base area 24 m^2 and height 20 m.

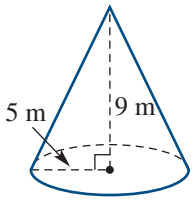
b Triangular pyramid with base area 120 mm^2 and height 50 mm.

3 A square pyramid has a base side length of 3 cm and a height of 4 cm. What is its volume? Answer correct to the nearest cubic centimetre.

Example 22

4 Find the volume of these cones to the nearest whole number.

a



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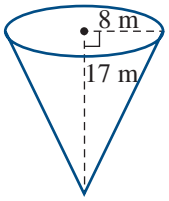

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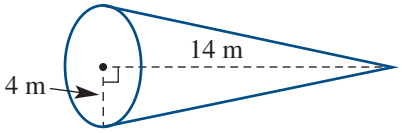
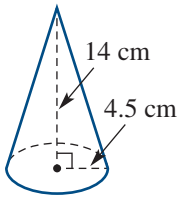


5 Find the volume of a cone with base radius 4.5 cm and height 14 cm. Answer correct to two decimal places.



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LEVEL 2

6 The Red Pyramid is the third tallest pyramid ever built. It has a square base with a side length of 220 m and a height of 104 m.

a What is the area of the square base?

b Determine the volume of the Red Pyramid. Answer correct to two decimal places.

7 An ice-cream cone measures 4.6 cm in diameter and is 9 cm high. Calculate the volume of the cone. Answer correct to two decimal places.

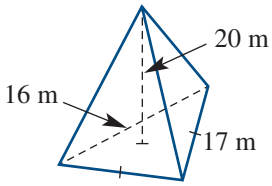
8 A triangular pyramid has a base with a height of 15 cm and base length of 5 cm.

a What is the area of the base of the pyramid?

b What is the volume of the triangular pyramid if the height of the pyramid is 12 cm?

LEVEL 3

9 A triangular pyramid has a base with sides 16 m, 17 m and 17 m. The height of the pyramid is 20 m. What is the volume of the triangular pyramid? Answer correct to two decimal places.

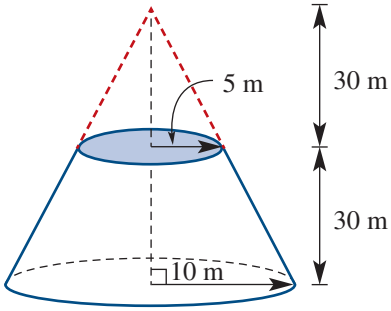


10 A frustum is a portion of a solid. The frustum shown opposite is formed from a cone.

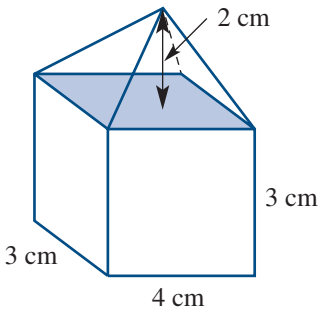
a What is the volume of the missing top cone? Give answers correct to the nearest cubic metre.

b What was the volume of the entire cone?

c What is the volume of the frustum?



11 Find the volume of the composite solid shown opposite. This solid consists of the rectangular prism and a rectangular pyramid. Answer correct to the nearest cubic centimetre.



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6J Surface area and volume of composite solids

Surface area of composite solids

The surface area of a composite solid is calculated in the same way as for right prisms and pyramids. Visualise the surfaces of the solid, calculate the area of each face and then add these areas together.

SURFACE AREA OF A COMPOSITE SOLID

1. Visualise the surfaces of the solid.
2. Write a formula and calculate the surface area of each face.
3. Add the area of each surface.
4. Check that all surfaces have been counted.
5. Write the answer to the specified level of accuracy with the correct units.

Example 23: Finding the surface area of a composite solid 6J

The red cube is a sculpture in New York consisting of a cube with a cylindrical hole removed from the centre. The side length of the cube is 8.53 m and the radius of the cylindrical hole is 1.42 m.

a What is the surface area of the face with the cylindrical hole, correct to two decimal places?

b What is the surface area of the red cube? Include only the outer red surfaces. Answer correct to two decimal places.

SOLUTION:

1. Required face consists of a square with a circle removed, so use the formula for a square and a circle.
2. Substitute the values into formula.
3. Evaluate.
4. Composite solid has two faces with cylindrical holes and four faces that are squares.
5. Write down an expression for surface area using the area of all the faces.
6. Evaluate.
7. Write answer in the correct units.

a $A = s^2 - \pi r^2$

$= 8.53^2 - \pi \times 1.42^2$

$= 66.4261\ldots \approx 66.43\ \text{m}^2$

b $SA = (2 \times 66.43) + (4 \times 8.53^2)$

$= 423.895\ldots$

$\approx 423.90\ \text{m}^2$

Surface area is $423.90\ \text{m}^2$.

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Volume of composite solids

To calculate the volume of most common solids we use a formula. Some of these formulae are listed earlier in this chapter. Composite solids are made up of two or more common solids. The volume of a composite solid is calculated by adding or subtracting the volumes of each common solid.

VOLUME OF COMPOSITE SOLIDS

1. Divide the composite solid into two or more common solids.
2. Calculate the volume of each common solid.
3. Add or subtract the volumes of the common solids.
4. Write the answer to the specified level of accuracy with the correct units.

Example 24: Finding the volume of a composite solid 6J

A grain silo consists of a cylinder with a cone on top, as shown. The diameter of the cylinder is 16 m and its height is 20 m. The diameter of the base of the cone is the same as that of the cylinder, and its height is 2 m. What is the volume of the silo correct to one decimal place?

SOLUTION:

1. Use the volume formula for a cone $V = \frac{1}{3}\pi r^2 h$.
2. Substitute $r = 8$ and $h = 2$ into the formula.
3. Evaluate.
4. Use the volume formula for a cylinder $V = \pi r^2 h$.
5. Substitute $r = 8$ and $h = 20$ into the formula.
6. Evaluate.
7. Add the volume of the cone to the volume of the cylinder.
8. Evaluate.
9. Write the answer correct to one decimal place and with correct units.

$$
\begin{aligned}
V &= \frac{1}{3}\pi r^2 h \text{ (Cone)} \\
&= \frac{1}{3} \times \pi \times 8^2 \times 2 \\
&= 134.0412866\ldots \text{ m}^3 \\
V &= \pi r^2 h \text{ (Cylinder)} \\
&= \pi \times 8^2 \times 20 \\
&= 4021.238597 \text{ m}^3 \\
V &= \text{Cone} + \text{Cylinder} \\
&= 134.0412\ldots + 4021.238\ldots \\
&\approx 4155.3 \text{ m}^3
\end{aligned}
$$

Volume of silo is 4155.3 m^3.

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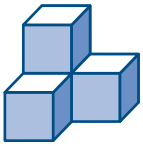
Exercise 6J

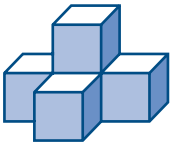
LEVEL 1

1 Each of the following solids is made up of 1 cm cubes. What is the surface area and volume of each solid?

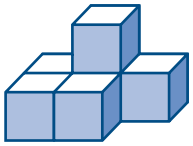
a



b

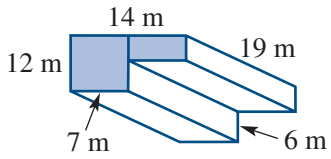


c

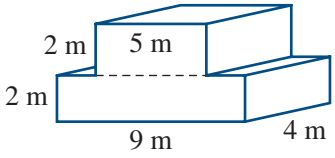


2 What is the surface area and volume of each solid? Answer to the nearest whole number.

a



b



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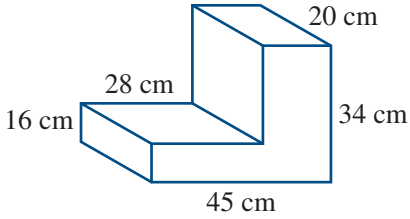
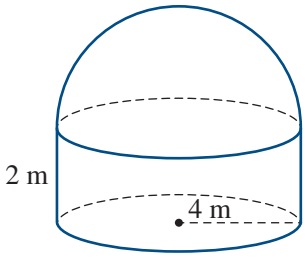
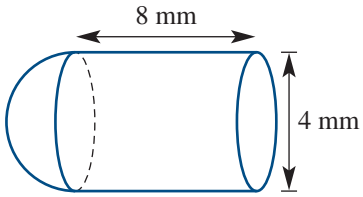


3 What is the surface area and volume of each solid? Answer to the nearest whole number.

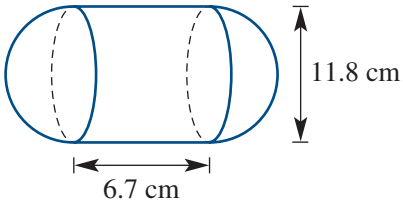
a



b



c



4 A hemisphere of ice cream is placed on a waffle cone. The waffle cone has a radius of 3.5 cm, a perpendicular height of 12 cm and a slant height of 12.5 cm.
Answer the following questions correct to one decimal place.

a What is the (outside) surface area of the waffle cone?

b What is the surface area of the hemisphere of ice cream?

c What is the total surface area of the waffle cone with the ice cream?

d What is the volume of the waffle cone?

e What is the volume of the hemisphere of ice cream?

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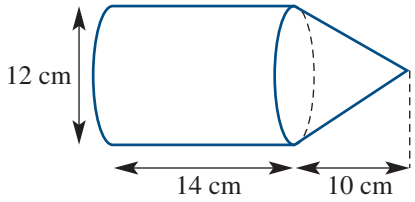
LEVEL 2

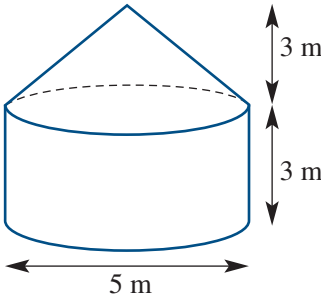
Example 24

5 What is the surface area and volume of each solid? Answer to the nearest whole number.

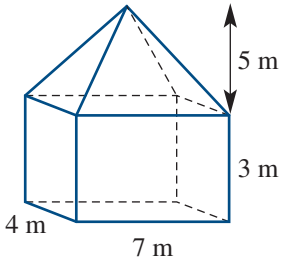
a



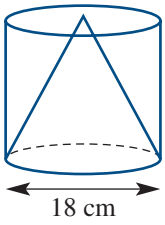
b



c

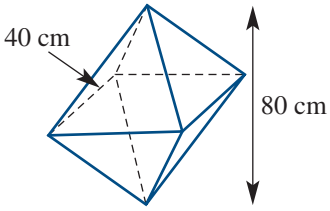


6 A cone sits exactly inside a cylinder as shown. The height of the cone is 16 cm.



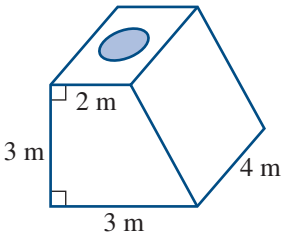
- **a** What is the volume of the cylinder to the nearest cubic centimetre?
- **b** What is the volume of the cylinder not taken up by the cone? Answer to the nearest cubic centimetre.

7 A composite solid consists of two square pyramids with their bases touching, as shown. The side length of the base is 40 cm and the height of the solid is 80 cm. Find the volume of the solid correct to the nearest cubic centimetre.



LEVEL 3

8 A trapezoidal prism has a cylindrical hole of radius of 0.5 m cut out as shown.



- **a** What is the volume of the cylindrical hole? Answer correct to two decimal places.
- **b** What is the area of the trapezoidal base or front face?
- **c** What is the volume of the trapezoidal prism after the hole is cut? Answer correct to two decimal places.

9 The first of two hot water tanks is a square prism with a side length of 80 centimetres and a height of 1.9 metres. The second hot water tank is a cylinder with a diameter of 80 centimetres and a height of 1.9 metres.

- **a** What is the volume of each tank? Answer in cubic metres correct to three decimal places.
- **b** What is the surface area of each tank? Answer to the nearest square metre.

10 A square pyramid has a side length of 200 cm and a perpendicular height of 180 cm. A frustum is formed by removing the top of the square pyramid with a side length of 100 cm. Find the volume of the frustum to the nearest cubic centimetre.

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6K Capacity

The capacity of a container is the amount of liquid it can hold. Some solids have both a volume and a capacity. For example, a can of soft drink is a cylinder that has a volume ($V = \pi r^2 h$) and a capacity (360 mL). The base unit for capacity is the litre (L). Three commonly used units for capacity are the megalitre (ML), kilolitre (kL) and millilitre (mL).

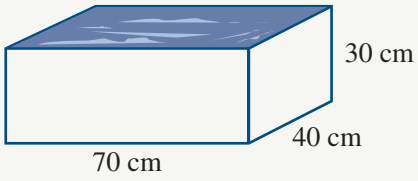
Capacity		
1 ML = 1000 kL 1 ML = 1000 000 L 1 kL = 1000 L 1 L = 1000 mL	$1\text{ cm}^3 = 1\text{ mL}$ $1\text{ cm}^3 = 0.001\text{ L}$ $1000\text{ cm}^3 = 1\text{ L}$	$1\text{ m}^3 = 1\,000\,000\text{ cm}^3$ $1\text{ m}^3 = 1\,000\,000\text{ mL}$ $1\text{ m}^3 = 1000\text{ L}$ $1\text{ m}^3 = 1\text{ kL}$

Example 25: Finding the capacity 6K

The container shown is filled with water.

a Find the volume of the container in cubic centimetres.

b Find the capacity of the container in litres.



SOLUTION:

1 Use the volume formula for a right prism $V = Ah$.

2 Determine the shape of the base and the formula to calculate the area of the base $A = lb$.

3 Substitute the values into the formula.

4 Evaluate.

5 Give answer to the correct units.

6 To change cm^3 to L multiply by 0.001 ($1\text{ cm}^3 = 0.001\text{ L}$).

7 Alternative method is to convert to mL ($1\text{ cm}^3 = 1\text{ mL}$).

a $V = Ah$

$= lbh$

$= 70 \times 40 \times 30$

$= 84\,000\text{ cm}^3$

b Capacity $= 84\,000 \times 0.001\text{ L}$

$= 84\text{ L}$

Capacity $= 84\,000 \times 1\text{ mL}$

$= 84\,000\text{ mL}$

$= 84\text{ L}$

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Exercise 6K

LEVEL 1

1 A can of soft drink has a capacity of 375 mL. How many cans of soft drink would it take to fill a 1.2 L bottle? How much would remain?

2 A medicine bottle has a capacity of 0.3 L.

a What is the capacity in millilitres?

b How many tablespoons (15 mL) does the bottle contain?

c How many teaspoons (5 mL) does the bottle contain?

d The correct dosage is 10 mL, 3 times a day. How many doses does the bottle contain?

3 Complete the following.

a $4\text{ cm}^3 = \square\text{ mL}$ **b** $2000\text{ cm}^3 = \square\text{ L}$ **c** $70\text{ cm}^3 = \square\text{ mL}$

d $34\,000\text{ cm}^3 = \square\text{ L}$ **e** $900\text{ cm}^3 = \square\text{ mL}$ **f** $500\text{ cm}^3 = \square\text{ L}$

g $43\text{ m}^3 = \square\text{ kL}$ **h** $30\text{ m}^3 = \square\text{ L}$ **i** $103\text{ m}^3 = \square\text{ kL}$

j $7\text{ m}^3 = \square\text{ L}$ **k** $5\text{ m}^3 = \square\text{ kL}$ **l** $8\text{ m}^3 = \square\text{ mL}$

Example 25

4 What is the capacity of a rectangular prism whose base area is 20 cm^2 and height is 10 cm? Answer correct to the nearest millilitre.

5 Find the capacity of a triangular prism with a height of 18 m and a base area of 40 m^2. Answer in litres, correct to two significant figures.

6 Find the capacity of a rectangular pyramid whose base area is 12 cm^2 and height is 15 cm. Answer correct to the nearest millilitre.

7 Find the capacity of a cylindrical plastic container 16 cm high and with circular end surfaces of radius 8 cm. Answer correct to the nearest litre.

8 Find the capacity of a cube whose side length is 75 mm. Answer in millilitres, correct to two decimal places.

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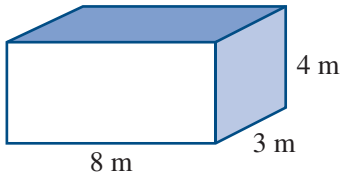
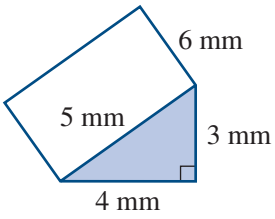
LEVEL 2

9 Find the capacity of the following solids in millilitres, correct to two decimal places.

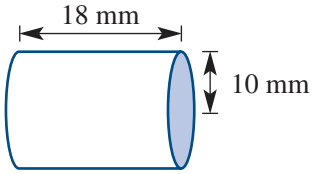
a



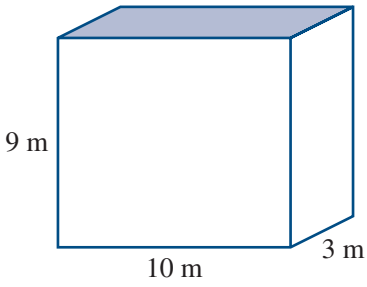
b



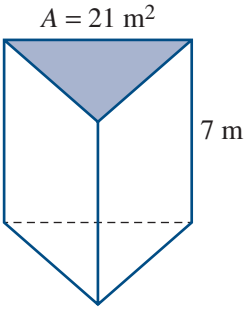
c



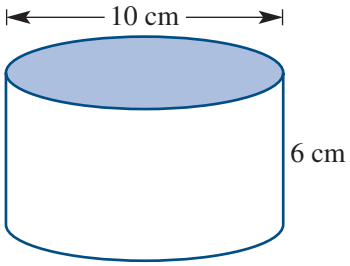
d



e



f

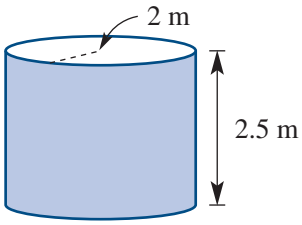


10 A water tank is the shape of a cylinder with a radius of 2 m and height of 2.5 m.

a What is the area of the top circular face of the water tank? Answer correct to one decimal place.

b Determine the volume of the water tank in cubic metres. Answer correct to one decimal place.

c What is the capacity of the tank, to the nearest kilolitre?

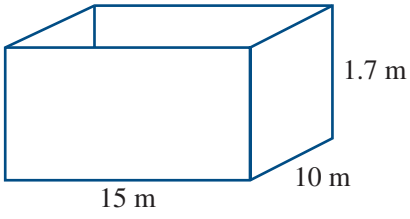


LEVEL 3

11 A swimming pool is the shape of a rectangular prism as shown. The swimming pool is filled to 25 cm from the top.

a What is the volume of water in cubic metres?

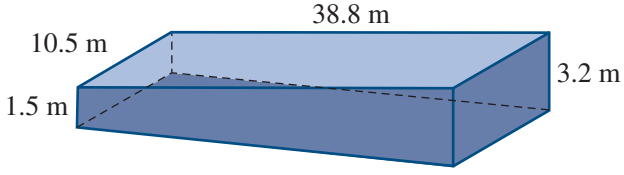
b How much water does the swimming pool contain, to the nearest kilolitre?



12 A swimming pool is the shape of a trapezoidal prism as shown opposite.

a Find the volume of the pool in cubic metres.

b What is the capacity of the pool in kilolitres?



c In hot weather the level of the pool dropped by 75 cm. The pool was originally full. What volume of water, in litres, evaporated?

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PPT STUDY GUIDE

Key ideas and chapter summary

Area of circles and sectors

Circle	$A = \pi r^2$	Semicircle	$A = \frac{1}{2}\pi r^2$
Annulus	$A = \pi(R^2 - r^2)$	Sector	$A = \frac{\theta}{360}\pi r^2$

Area of composite shapes

1 Divide the composite shape into two or more plane shapes.
2 Add or subtract the areas of the plane shapes.

Trapezoidal rule

$A = \frac{h}{2}(d_f + d_l)$

A – Area of shape
h – Height or distance between the parallel sides
d_f – Distance of first parallel side
d_l – Distance of last parallel side

Surface area of prisms, cylinders, spheres, pyramids and cones

Right prisms	Sum of the area of each surface.
Open cylinder	$SA = 2\pi rh$
Closed cylinder	$SA = 2\pi r^2 + 2\pi rh$
Sphere	$SA = 4\pi r^2$
Square pyramid	$SA = s^2 + 4 \times \left(\frac{1}{2}sl\right)$
Cone	$SA = \pi r^2 + \pi rl$

Volume of prisms, cylinders, spheres, pyramids and cones

Cube	$V = Ah = (s^2) \times s = s^3$
Rectangular prism	$V = Ah = lb \times h = lbh$
Triangular prism	$V = AH = (\frac{1}{2}bh) \times H$
Cylinder	$V = Ah = (\pi r^2) \times h = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Square pyramid	$V = \frac{1}{3}Ah = \frac{1}{3} \times s^2 \times h$
Rectangular pyramid	$V = \frac{1}{3}Ah = \frac{1}{3} \times lb \times h$
Cone	$V = \frac{1}{3}Ah = \frac{1}{3} \times \pi r^2 \times h$

Surface area of composite solids

1 Visualise the surfaces of the solid.
2 Calculate the area of each face and add them together.

Volume of composite solids

1 Divide the composite solid into two or more composite solids.
2 Add or subtract the volume of the common solids.

Capacity

The amount of liquid a container can hold. Base unit is the litre.

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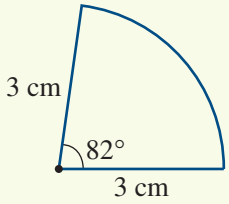
Review

Multiple-choice questions

1 What is the area of the sector?



A 1.6 cm^2 **B** 1.6 cm^2
C 6.4 cm^2 **D** 28.3 cm^2

2 A composite shape consists of a square with a side length of 6 mm and a circle of radius 2 mm removed from the centre of the square. What is the area of the shape?

A 13 mm^2 **B** 23 mm^2 **C** 36 mm^2 **D** 49 mm^2

3 What is the surface area of a rectangular prism with a length of 2.5 m, a breadth of 3 m and a height of 2.8 m?

A 8.3 m^2 **B** 21 m^2 **C** 29 m^2 **D** 45.8 m^2

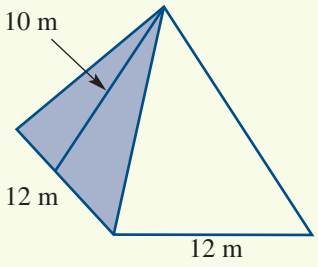
4 What is the surface area of a sphere with a diameter of 16 mm?

A 804 mm^2 **B** 2145 mm^2 **C** 3217 mm^2 **D** 17 157 mm^2

5 What is the volume of a rectangular prism with length 2.5 m, breadth 3 m and height 2.8 m?

A 17.5 m^3 **B** 63 m^3 **C** 12 m^3 **D** 21 m^3

6 What is the surface area of the square pyramid?



A 144 m^2 **B** 384 m^2
C 624 m^2 **D** 1440 m^2

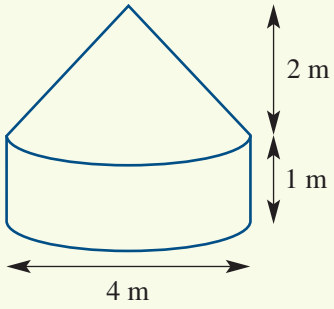
7 What is the volume of the square pyramid?

A 120 m^3 **B** 384 m^3
C 480 cm^3 **D** 1440 cm^3

8 What is the volume of a hemisphere with a radius of 10 m?

A 261 m^3 **B** 526 m^3 **C** 2094 m^3 **D** 4189 m^3

9 What is the volume of the composite solid?



A 12.6 m^3 **B** 12.6 m^3
C 20.9 m^3 **D** 83.8 m^3

10 A cubic water tank has a side length of 6 m. What is the capacity of the tank?

A 36 kL **B** 216 kL **C** 360 kL **D** 216 000 kL

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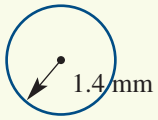
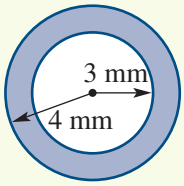
Short-answer questions

1 Find the area of each shape. Answer correct to one decimal place.

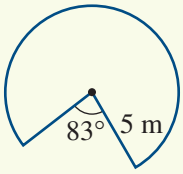
a



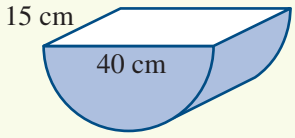
b



c



2 A semicircular trough has a diameter of 40 cm and a width of 15 cm.

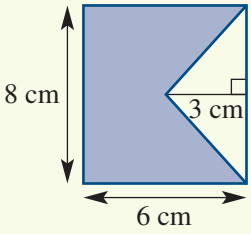


a What is the cross-sectional area of the trough? Answer to the nearest square centimetre.

b What is the volume of water for the trough? Answer to the nearest cubic centimetre.

c How many litres of water will fill the trough? Answer correct to one decimal place.

3 The diagram opposite shows a composite shape.



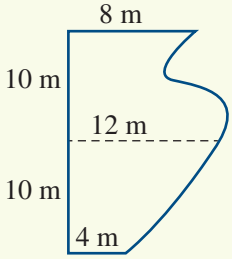
a What is the area of the rectangle?

b What is the area of the triangle?

c What is the area of the composite shape?

4 Find the surface area of a square prism with a side length of 7 cm and height of 6 cm.

5 The diagram opposite shows a vertical cross-section of an ornament. Estimate the area of the cross-section by applying the trapezoidal rule twice. Answer correct to the nearest square metre.



6 A label covers the curved surface of a cylinder. The label does not overlap. The cylinder has a radius of 8 cm and a height of 12 cm. What is the area of the label? Answer correct to four significant figures.

7 A car hose is an open cylinder with a radius of 3 cm and a length of 14 cm. Calculate the area of the outer surface, correct to two decimal places.

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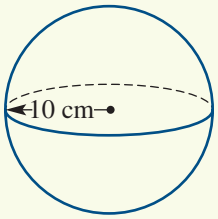
Review

8 A sphere has a radius of 10 cm.

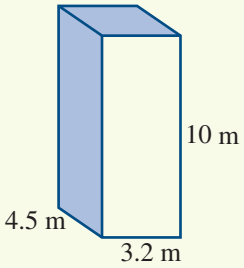
a What is the surface area of the sphere? Answer correct to the nearest whole number.

b What is the volume of the sphere? Answer correct to the nearest whole number.

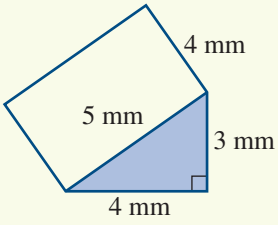


9 Find the volume of the following solids. Answer to the nearest whole number.

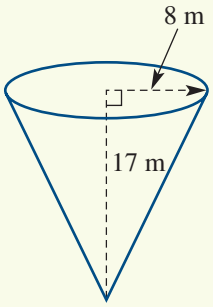
a



b



c

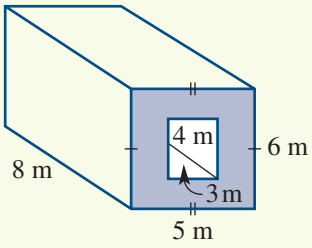


10 A composite solid is shown opposite.

a What is the volume of the outer rectangular prism?

b What is the volume of the inner rectangular prism?

c What is the volume of the composite solid?



11 Find the capacity of a triangular prism with a height of 50 cm and a base area of 120 cm^2. Answer in litres.

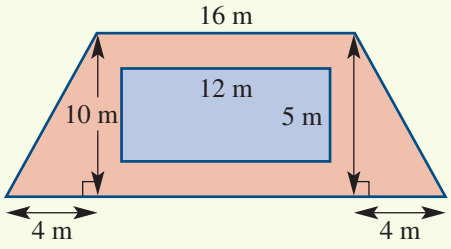
Extended-response questions

12 A pool with dimensions 12 m by 5 m is surrounded by timber decking in the shape of a trapezium.

a What is the area of the timber decking?

b The pool has a constant depth of 2 m. What is the volume of the pool?

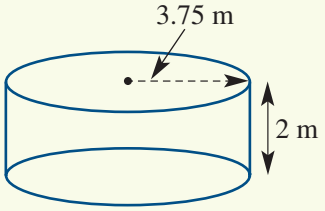
c The interior of the pool is to be painted blue. What is the surface area to be painted?



13 A cylindrical water tank has a radius of 3.75 metres and is 2 metres deep.

a What is the volume of the cylindrical tank? Answer correct to the nearest cubic metre.

b The tank provides water for the sprinklers to water the garden. Sprinklers use water at a rate of 5000 litres per hour. How long will it take the sprinklers to empty a full tank?



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Practice Paper 1

Section I

Attempt Questions 1–15 (15 marks)
Allow about 20 minutes for this section

1 Which of the following is the highest pay?

A \$1442.22 per week
B \$2884.68 per fortnight
C \$6247.50 per month
D \$75 000 per annum

2 What is the number of significant figures in the number 0.00206?

A 2
B 3
C 4
D 5

3 Which of the following is the correct simplification of $11x^4 - 7x^4$?

A 4
B $4x$
C $4x^4$
D $4x^8$

4 Stella sells shoes for a retail store and receives wages of \$1875 per month plus 4% commission on all her sales. What were her sales in a month in which she received a total pay of \$1953?

A \$878.00
B \$1946.88
C \$1950.00
D \$2190.12

5 Simplify $6 - 4(2x - 1)$.

A $4x - 2$
B $4x + 6$
C $7 - 8x$
D $10 - 8x$

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6 Arrange the numbers 4.8×10^{-2}, 4.0×10^{-1}, and 5.6×10^{-2} in ascending order.

A $4.8 \times 10^{-2}, 5.6 \times 10^{-2}, 4.0 \times 10^{-1}$
B $4.0 \times 10^{-1}, 4.8 \times 10^{-2}, 5.6 \times 10^{-2}$
C $5.6 \times 10^{-2}, 4.8 \times 10^{-2}, 4.0 \times 10^{-1}$
D $4.0 \times 10^{-1}, 5.6 \times 10^{-2}, 4.8 \times 10^{-2}$

7 What is the value of $2x - x^2$ if $x = -3$?

A -15
B -3
C 3
D 15

8 A wage sheet of a mobile phone business shows one employee's details.

Employee	Rate per hour	Normal hours	Overtime ($\times 2$)	Wage
Terry Brown	\$20.00	30	x	\$840

Terry worked some overtime at double-time rate but it is missing from the wage sheet. Using the information on the wage sheet, how many hours of overtime did Terry work?

A 4
B 5
C 6
D 8

9 The circumference of a bicycle wheel is 220 cm. How many revolutions will the wheel make if it travels a distance of 6.6 km?

A 30
B 33.3
C 300
D 3000

10 Find the value of m, correct to one decimal place, given $k = 24$ and the formula $m = \sqrt{\frac{k}{5}}$.

A 1.0
B 2.2
C 2.4
D 4.8

11 Find the value of x given $a = 32, y = 2$ and the formula $a = 2xy^2$.

A 3
B 4
C 5
D 6

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12 Tennis balls are sold in a box of 5 yellow balls or a box of x white balls. A tennis coach needs 400 balls and purchases 20 boxes of yellow balls and a certain number of white balls. Which of the following expressions describes the number of boxes of white balls purchased?

A $\frac{400-x}{5}$

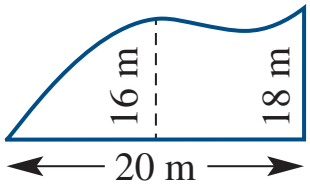
B $300x$

C $300+x$

D $\frac{300}{x}$

13 The irregular shape shown below has a width of 20 metres. The vertical dashed line divides it into two parts of equal width. Two applications of the trapezoidal rule were used to estimate the area.

Which expression is correct?



A $\frac{10}{2}(16+18)$

B $\frac{10}{2}(32+18)$

C $\frac{20}{2}(16+18)$

D $\frac{20}{2}(32+18)$

14 What is the solution to the equation $9x-8=27$?

A $x=\frac{9}{35}$

B $x=\frac{7}{3}$

C $x=1\frac{8}{9}$

D $x=3\frac{8}{9}$

15 A book was bought for $\$(x+10)$ and sold for $\$(x-20)$. Which of the following statements is true?

A There was a profit of $\$(x-30)$.

B There was a loss of \$30.

C There was a profit of $\$(x+10)$.

D There was a loss of $\$(x-30)$.

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Section II

Attempt Questions 16–18 (45 marks)
Allow about 70 minutes for this section
All necessary working should be shown for every question.

Question 16 (15 marks) **Marks**

a A communication company pays overtime at a rate of time-and-a-half for the first 4 hours overtime and double-time thereafter. Natalie is employed as a personal assistant. During a normal week she works 35 hours at $27.80 an hour.

i How much did Natalie earn in a week in which she worked 42 hours? **2**

ii Natalie receives annual leave loading of $17\frac{1}{2}\%$ of 4 weeks basic pay. **2**

1 What is the value of Natalie's leave loading?

2 Calculate the total amount Natalie is paid for her 4 weeks annual leave.

b

Taxable income	Tax payable
0– $18 200	Nil
$18 201–$37 000	Nil +19 cents for each $1 over $18 200
$37 001–$87 000	$3572 + 32.5 cents for each $1 over $37 000
$87 001–$180 000	$19 822+37cents for each $1 over $87 000
$180 001and over	$54 232+45cents for each $1 over $180 000

Daniel earns a gross income of $63 500 during the financial year.
He has allowable deductions of $4500.

i What is Daniel's taxable income? **1**

ii Calculate the tax payable on Daniel's income. **2**

iii Daniel must pay 2% of his taxable income for the Medicare Levy. Calculate how much Daniel pays in Medicare Levy. **1**

iv What is Daniel's total tax payable including the Levy? **1**

v Daniel has paid $12 255 in tax during the financial year. Determine whether Daniel receives a refund or whether he is required to pay more tax, and determine this amount. **2**

c Simplify.

i $1 - 2(5a + 3)$ **ii** $45x^6y^5 \div 9xy^5$ **4**

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Question 17 (15 marks) **Marks**

a Blake plays cards with a normal deck and draws a card from the deck.

i What are the chances of drawing a 3 or a black card? **1**

ii What are the chances of not drawing an ace? **1**

b An ornament is the shape of a cone with a 4.5 cm diameter and a height of 9 cm. Find the volume of the ornament. Answer correct to two decimal places. **2**

c Solve the equation $\frac{2x-5}{3}=1$. **2**

d A cylindrical aluminium can has a diameter of 22 cm and a height of 13 cm.

i What is the total surface area of the cylindrical can? Answer correct to one decimal place. **2**

ii The curved surface of the cylindrical can is cut from an aluminium sheet 100 cm by 70 cm. How many curved surfaces can be cut? **1**

e Solve these equations.

i $11=x-4$ **1**

ii $8x-2=12x$ **1**

f Light travels at a speed of 2.9979×10^8 metres per second. How many kilometres does light travel in one hour? Answer in scientific notation correct to three significant figures. **2**

g In a 100 m freestyle swimming race the winning time was 48.92 seconds. What is the percentage error (to three decimal places) in this time? **2**

Question 18 (15 marks) **Marks**

a The volume of a solid is given by the formula $V=\frac{4}{3}\pi r^3$ where r is the radius.

i Write the formula with r as the subject. **2**

ii What is the radius in metres of the solid, given it has a volume of $2\,m^3$? Answer correct to two decimal places. **1**

b Henry's room measures 5850 mm by 4950 mm and needs carpeting.

i The cost of the carpet is \$90 per m^2, and a tradesperson charges \$40 per m^2 to lay the carpet. What is the cost to have the room carpeted? **2**

ii Henry's room has a ceiling height of 2800 mm. He is considering buying heater A, B or C as shown in the table below. **2**

Heater A	Rooms up to 70 m^3
Heater B	Rooms up to 80 m^3
Heater C	Rooms up to 90 m^3

Determine the most suitable heater and give a reason for your answer.

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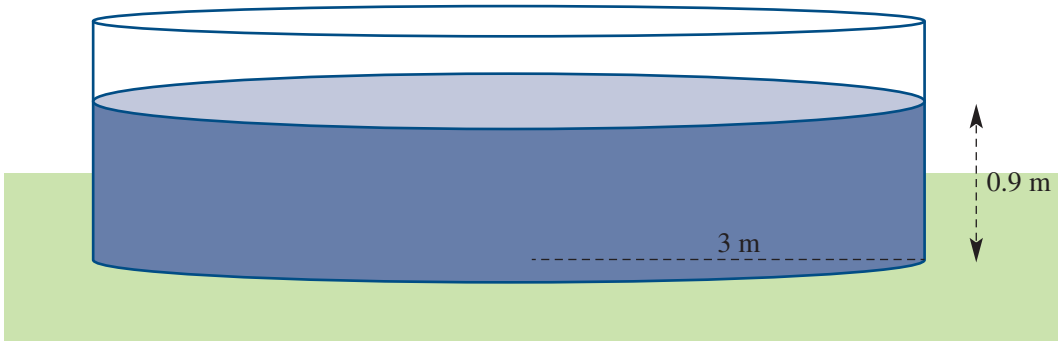
c An above-ground swimming pool is the shape of a cylinder. It has a radius of 3 metres and contains water to a uniform depth of 0.9 metres.

i What is the volume of water in the pool, in cubic metres (two decimal places)? **2**

ii How many litres of water are in the pool? Answer correct to the nearest litre. ($1\,m^3 = 1000\,L$) **1**



d What is the value (two decimal places) of $\sqrt{\frac{a+3b}{4b}}$ if $a = 6.4$ and $b = 2.1$? **2**

e Make x the subject of the equation $y = mx + b$. **1**

f The number of 'standard drinks' in various glasses of wine is shown. **2**

Number of standard drinks			
White wine		**Red wine**	
Small glass	Large glass	Small glass	Large glass
0.8	1.3	1.0	1.5

Jane weighs 66 kg and drinks two small glasses of white wine and three large glasses of red wine between 7 pm and midnight.

What would be her blood alcohol content (BAC) estimate at midnight?

Answer correct to two significant places.

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7 Classifying and representing data

Syllabus topic — S1.1 Classifying and representing data

This topic involves the planning and management of data collection, the classification of the data and then the representation of the data in tables and graphs.

Outcomes

- Describe the distinguishing features of a population and a sample.
- Investigate data collection methods for both samples and populations.
- Classify data as numerical or categorical.
- Class categorical data as nominal or ordinal.
- Classify numerical data as discrete or continuous.
- Organise and display data in dot plots and stem-and-leaf plots.
- Create frequency tables to organise ungrouped and grouped data.
- Calculate the cumulative frequency from a grouped frequency table.
- Draw frequency and cumulative frequency graphs.
- Organise and display data in Pareto charts.
- Construct and interpret tables and graphs related to motor vehicles and water.
- Calculate the fuel consumption and running costs of a vehicle.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Widgets
- Literacy worksheet
- Spreadsheets
- Quick Quiz
- Study guide
- Solutions (enabled by teacher)

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

In the Interactive Textbook you can take a test of prior knowledge required for this chapter, and depending on your score you may be directed to revision from the previous years' work.

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7A Data collection

Data collection involves deciding what data to collect, locating it and collecting it. Data comes from either primary or secondary sources.

- Primary sources – interviewing people, conducting questionnaires or observing a system in operation
- Secondary sources – data collected or created by someone else, such as information gathered from newspapers, books and the internet

It is important that procedures are in place to ensure the collection of data is accurate, up-to-date, relevant and secure. If the data collected comes from unreliable sources or is inaccurate, the information gained from it will be incorrect.

Data collection methods are used for both a census and a sample survey.

- A census is a survey of every person in a population. For example, all the students in your school are regarded as the population. A census can be very expensive and time-consuming, if the population is large.
- A sample is only part of a population. For example, a sample of the school population is the students in your class. Estimates are made about the population based on the sample. Samples are cheaper than censuses, but are not as accurate. Bias can also be an issue. A sample must be large enough to give a good representation of the population, but small enough to be manageable. There are many different types of sampling including a random sample, stratified sample and systematic sample.

Random sample

A random sample occurs when all members of the population have an equal chance of being selected. For example, six students are selected at random from the entire school population. Lotto is another good example of random sampling. A sample of 6 numbers is chosen from 40 numbers. Random samples are simple and easy to use for small populations. However, for large populations, it is possible to miss out on a particular group.

Stratified sample

A stratified sample occurs when categories or strata of a population are chosen and then members from each category are randomly selected. For example, one student is selected from each year 7, 8, 9, 10, 11 and 12. Each year group is a category in a stratified sample. Some other common types of categories are age, sex, religion or marital status. A stratified sample is useful when the categories are simple and easy to determine. However, care needs to be taken when selecting categories to avoid any bias in the data.

Systematic sample

Systematic sampling occurs when the population is divided into a structured sample size. For example, the students in the school population are put in alphabetical order and the 100th student, 200th student,

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300th student, … are selected. A systematic sample is often used by a manufacturer to ensure the machines are working correctly. Here the manufacturer might test a machine every 30 minutes or check the 50th item on a production line. Systematic sampling results in a gap between each selection.

Self-selected sample

In a self-selected sample, members of the population volunteer themselves. For example, six students from the entire school population offer to complete a questionnaire. A self-selected sample often occurs on the internet. There is the potential for self-selected sampling to be biased as only people who are motivated and have the time volunteer themselves.

RANDOM SAMPLE	STRATIFIED SAMPLE	SYSTEMATIC SAMPLE	SELF-SELECTED SAMPLE
Members of the population have an equal chance of being selected.	Categories of a population are chosen. Members then are randomly selected from each category.	Population is divided into a structured sample size. Members are then selected in a certain order from this structure.	Members of the population volunteer themselves.

Example 1: Distinguishing sample types 7A

A retirement village has 63 residents, 42 women and 21 men. Decide whether each sample of resident would be random, stratified, systematic or self-selected.

a Every seventh resident
b Six of the women and three of the men
c Nine names picked from a hat containing the names of the residents.
d Residents sorted into alphabetical order and each ninth resident selected.
e Residents are divided into four age groups (51–60, 61–70, 71–80, 81–90) and two residents selected from each age group

SOLUTION:

1 The population has been divided into a structured sample size – 7th, 14th, 21st, … 63rd.
2 The population has been divided into categories – women and the men.
3 Sample is taken at random.
4 The population is divided into alphabetical order and a structured sample size taken.
5 The population has been divided into four age group categories and then randomly selected.

a Systematic sample
b Stratified sample
c Random sample
d Systematic sample
e Stratified sample

Note: Capture-recapture is covered in the interactive textbook.

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Exercise 7A

LEVEL 1

1 State whether a census or a sample is the most appropriate way to collect this data.

a Information on the shopping experience of people in the city

b John collecting the heights of his best friends

c The travelling habits of the Jones family to work

d Australians watching the grand final

e Number of people eating toast for breakfast

f Length of time every AAA battery lasts

g Number of people entering a gym between 5 p.m. and 6 p.m.

h Holly collecting the length of time students in her class spend on the internet

i The world's reaction to climate change

j Shop manager's reaction to a drop in sales.

Example 1

2 State whether the sample is random, stratified, systematic or self-selected.

a A police officer breathalyses every tenth person.

b Each person is given a raffle ticket and the tickets are drawn out of a hat.

c Each person leaving a bus was given a survey form. We used the data from the people who sent it back.

d A business has 240 married and 120 unmarried employees. A sample was chosen to include 10 of the married and 5 of the unmarried employees.

e Students were sorted into alphabetical order and each third student selected.

f Individuals were randomly selected using their tax file number.

g Every 12th jogger was selected from an alphabetical list.

h Ten cards were randomly selected from a normal deck of cards.

i Ten girls and ten boys were randomly selected from a concert audience.

j Ten people who arrive at a shopping centre each day completed the survey.

3 Michael uses a random sample to survey 10% of the local community. In the local community there are 810 males and 920 females. How many people does Michael need to survey?

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4 Amelia plans to conduct a random sample to survey the netball players in the local association. There are 2850 players in the local association and she plans to survey 171 players. What percentage of the population is her sample?

5 Paige uses a stratified sample to survey 5% of her school population. At the school there are 80 teachers and 1160 students.

a How many teachers should complete the survey?

b How many students should complete the survey?

c How many people should complete the survey?

6 Tyler uses a stratified sample to survey 25% of the members of his swimming club. He uses their sex as a category and selects a random group of female and male swimmers. There are 88 female swimmers and 112 male swimmers in his club.

a How many swimmers are in the entire population?

b How many female swimmers are in the sample?

c How many male swimmers are in the sample?

7 Osman uses a stratified sample to survey 7.5% of his chat room friends. He uses marital status as a category and selects a random group of married and unmarried friends. There are 200 married and 240 unmarried friends in his chat room.

a How many friends are in the entire population?

b How many married friends are in the sample?

c How many unmarried friends are in the sample?

8 Taylia uses a stratified sample to survey 20% of the senior students from her school. There are 205 Year 11 students and 180 Year 12 students. How many students should Taylia choose from Year 12?

9 Ming uses a stratified sample to survey $12\frac{1}{2}\%$ of the junior students from his school. There are 88 Year 7, 120 Year 8, 104 Year 9 and 128 Year 10 students.

a How many students are in the entire population?

b How many students should Ming choose in following years?

i Year 7 **ii** Year 8 **iii** Year 9 **iv** Year 10

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LEVEL 2

10 A survey was conducted in a school on whether Australia should remain a constitutional monarchy or become a republic. The results are shown below.

	Male	Female	Total
Republic	51	79	130
Monarchy	23	47	70
Total	74	126	200

a How many males surveyed did not think Australia should change to a republic?
b What percentage of people favour changing to a republic?
c This survey is not a good random sample of all Australians. Why?

11 A sample of 30 students is taken from a primary school that has an enrolment of 420 students from kindergarten to Year 6. The sampling is designed so that the proportion of each year of the sample matches the population. There are 4 students from Year 1 in the sample. How many Year 1 students are there in the school population?

12 A store has 400 employees of which 208 are female and 192 are male. The store intends to survey 25 of its employees. A stratified survey is to be conducted.
a How many females should be surveyed?
b How many males should be surveyed?

LEVEL 3

13 Identify any possible issues with each of the following survey questions.
a Do you like the government's new policy? Yes/No
b Alan is a lazy boss who should be forced to pay his diligent workers more money. Agree or disagree?

14 Kayla surveyed a group of 15 people at the Tamworth country music festival on their music preferences. She used this data to draw conclusions for the entire population of NSW.
a Do you think her conclusions will be accurate? Give a reason.
b What would be a more appropriate method of sampling music preferences?

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7B Classification of data

There are many types of data that can be collected. For example, if you ask six friends how many pets they own, they might give you the following data: 1, 0, 2, 4, 1, 15. However not all data are numbers. For example, if you also record the gender of each of your friends, you get the following data: male, male, female, female, female, male. Data is divided into two broad classifications: categorical and numerical.

Categorical data

Categorical data represents characteristics such as a person's gender, marital status, address or music they like. Categorical data can take on numerical values (such as '0' for unsatisfactory and '1' for satisfactory), but those numbers don't have mathematical meaning.

Categorical data is further classified as nominal or ordinal.

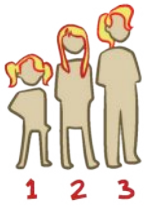
- Nominal data uses a name or label that does not indicate order. For example, a student's gender could be classified as an 'F" for female and an 'M' for male.
- Ordinal data uses a name or label that does indicate order. For example, the quality of work could be classified as an 'A' for excellent, 'B' for good and 'C' for satisfactory. It shows a sequence A, B and C.

Categorical data has no quantity or amount associated with each category.

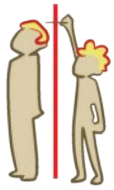
Numerical data

Numerical data indicates a quantity and is used to perform calculations. For example, if we asked each student in the class their height, we would expect to get a variety of answers. However each answer is a number.

Numerical data is further classified as discrete or continuous.



- Discrete data is data that can only take exact numerical values. For example, the number of sisters will give rise to numbers such as 0, 1 or 2. Counting a quantity often results in discrete data.
- Continuous data is data that can take any numerical value (depending on the degree of accuracy). For example, a student's height will give rise to numbers such as 171.2 cm and 173.5 cm. Measuring a quantity often results in continuous data.



CLASSIFICATION OF DATA

1 Categorical data – data is classified by the name of the category it belongs to.
   - a Nominal data – name does not indicate order.
   - b Ordinal data – name does indicate order.

2 Numerical data – data indicates a quantity and is used to perform calculations.
   - a Discrete data – data can only take exact numerical values.
   - b Continuous data – data can take any numerical value.

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Example 2: Classifying data as categorical or numerical 7B

Classify the data from these situations as categorical or numerical.

a The heart rate of a group of personal trainers
b The most watched television show in Australia
c The number of people living in Smith Ave
d The reasons for people travelling to work by train

SOLUTION:

1 The heart rate, such as 70 beats per minute, can be measured and results in a number.
2 A television show, such as the news, does not result in a number.
3 The number of people living in Smith Ave, such as 27, can be counted and results in a number.
4 The reason for travelling to work by train, such as it is cheaper, does not result in a number.

a The heart rate is numerical data.
b A television show is categorical data.
c The number of people living in Smith Ave is numerical data.
d The reasons for travelling to work is categorical data.

Example 3: Classifying data as a nominal or ordinal 7B

Classify the following numerical data as nominal or ordinal.

a School year level
b Internet use at home

SOLUTION:

1 Year level such as Year 11 indicates order but has no mathematical meaning.
2 Internet use such as email is a label that does not indicate any order.

a Year level is ordinal data.
b How the internet is used at home is nominal data.

Example 4: Classifying data as a discrete or continuous 7B

Classify the following numerical data as discrete or continuous.

a The number of pets in your family
b The perimeter of the school

SOLUTION:

1 The number of pets can be counted and is exact.
2 The perimeter of the school is a measurement of distance and assumes a value.

a The number of pets is discrete data.
b The perimeter of the school is continuous data.

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Exercise 7B

LEVEL 1

Example 2

1 Classify the data from these situations as categorical or numerical.

- **a** The favourite colours of Jenny's friends
- **b** The number of people travelling in a car
- **c** The weight of each student in the year in kilograms
- **d** People rating their doctor on personal service (high, medium or low)
- **e** The number of students in each class
- **f** The IQ of a group of students
- **g** Responses to a survey question (agree or disagree)
- **h** A person's lucky number
- **i** A female's favourite mobile phone
- **j** The distance from Sydney to Wollongong
- **k** The cost of bread at the supermarket
- **l** The community's preferred leader
- **m** The number of computers in the school

Example 4

2 Classify the following numerical data as discrete or continuous.

- **a** The price paid for a can of soft drink
- **b** The number of people at a concert
- **c** The time between trains
- **d** The number of pages in the newspaper
- **e** The amount of water used in the past month
- **f** The number of people in your immediate family
- **g** The numbers drawn in this week's lotto
- **h** The length of the cricket pitch
- **i** The distance measured for the long jump at the world championships
- **j** The score achieved from a quiz consisting of 10 questions
- **k** The height of the tallest person in the world

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3 State whether the following is categorical, discrete or continuous data.

- **a** The heights of members of a football team
- **b** The distance to drive to the train station
- **c** The different types of ice creams
- **d** The quality of food in a restaurant
- **e** The eye colour of a group of people
- **f** The number of pets in a household
- **g** The time to swim 50 metres
- **h** The number of goals scored in the first match of the season
- **i** Today's most fashionable style of dress
- **j** The number of computers in the building
- **k** Replies given to a questionnaire (Yes or No)
- **l** The perimeter of Joel's block of land
- **m** The width of the Anzac Bridge
- **n** The number of people killed on the roads due to speed
- **o** The stopping distance for a car travelling at 60 km/h
- **p** The most popular type of car sold in the past 12 months.

Example 3

4 A hospital measures the weight and length of every new baby.

- **a** Classify the data as numerical or categorical.
- **b** If the data is numerical, is this data discrete or continuous? If the data is categorical, is it nominal or ordinal?

5 A coffee shop is conducting a survey on the drinking habits of its customers. One of the questions was: 'How many cups of coffee do you drink each day?'

- **a** State whether the data is numerical or categorical.
- **b** If the data is numerical, is this data discrete or continuous? If the data is categorical, is it nominal or ordinal?

6 The government collected data on its latest policy proposal. The people surveyed answered 10 questions and were given three choices for each question: Agree, Disagree or Not sure. What type of data has the government collected?

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LEVEL 2

7 In a survey customers were asked to rate the service they received by selecting one of the following:

outstanding, excellent, good, satisfactory or needs improvement

Describe the type of data that would result from this question.

8 A teacher marks a class's assessment task and awards a mark out of 100 for each student. Describe the type of data that that has been collected by the teacher.

9 A marketing poll was conducted that asked about a person's employment status:

unemployed, receiving education, part-time job or full-time job

Describe the type of data that would result from this poll.

10 Emma is planning to build a new restaurant. She conducted a survey of the community. One of the questions asked was 'How far in kilometres would you be prepared to travel to get to a good restaurant?' The options given were:

5 km, 10 km, 20 km, 50 km

Describe the type of data that would result from this question.

11 The local community recorded the amount of rainfall each day for the past 3 months. Some of the data is shown below.

Rainfall	
23rd April	10 mm
24th April	0 mm
25th April	25 mm

Describe the data collected by the local community.

LEVEL 3

12 The police department collected data on fatal crashes. One of the questions it asked was: 'What was the age of the driver involved in a fatal crash?'

a Describe the type of data collected by the police department.

b The question was modified to give the interviewee six choices:

17–24 years, 25–34 years, 35–44 years, 45–54 years, 55–64 years, 65+ years

Describe the type of data that would result from these choices.

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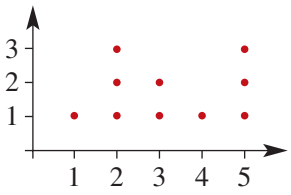
7C Dot plots and stem-and-leaf plots

Dot plots

The simplest display of numerical data is a dot plot. A dot plot consists of a number line with each data point marked by a dot. When several data points have the same value, the points are stacked on top of each other. Dot plots are a great way of displaying fairly small data sets in which the data takes a limited number of values.



DOT PLOT

It is a number line with each data point marked by a dot. When several data points have the same value, the points are stacked on top of each other.

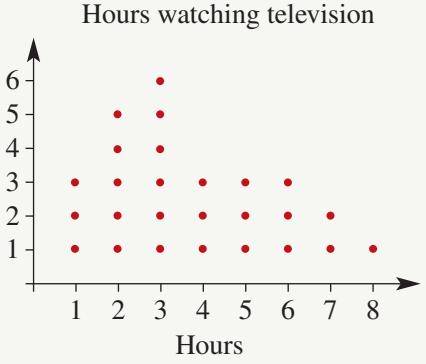
Example 5: Constructing a dot plot 7C

The number of hours spent watching television on the weekend is shown below. Construct a dot plot.

3	4	3	2	7	6	2	2	3	7	3	5	2
3	4	5	6	8	1	6	1	2	3	4	1	5

SOLUTION:

1. Draw a number line, scaled to all the data values. Label the line with the variable being displayed.
2. The vertical axis indicates the frequency of the data value. It may be omitted.
3. Plot each data value by marking in a dot above the corresponding value on the number line.
4. Count the number of dots and check that it matches the number of data values.



Stem-and-leaf plots

A stem-and-leaf plot or stem plot is used to present a small (less than 50 values) numerical data set. The tens digit of the data values becomes the 'stem' and is written in numerical order down the page. The 'units' digits become the 'leaves' and are written in numerical order across the page.

STEM-AND-LEAF PLOT

Stem	Leaf
0	57
1	22557
2	125679

BACK-TO-BACK STEM-AND-LEAF PLOT

Leaf	Stem	Leaf
9	0	0
8532	1	12559
98442	2	4578

Example 6: Constructing a stem-and-leaf plot 7C

Use the following data set to construct a stem-and-leaf plot.

24 7 19 19 15 18 27 28 11 19 29 30 9
6 25 25 26 5 6 6 10 10 28 29 14

SOLUTION:

1. The data set has values from 5 to 30. This requires stems 0, 1, 2 and 3. Write these down from smallest to largest, followed by a vertical line.
2. Attach the leaves. The first data value is '24'. It has a stem of '2' and a leaf of '4'. Opposite the 2 in the stem, write the number 4. Complete all the values.
3. Rewrite the leaves so that they are in increasing order.

0	566679
1	001458999
2	455678899
3	0

Example 7: Constructing a back-to-back stem-and-leaf plot 7C

Use the following data set to construct a back-to-back stem-and-leaf plot.

Girls	28	24	24	31	34	26	27	12	18	13	15	6	29	30	22
Boys	19	27	21	25	35	28	29	13	11	30	31	32	25	16	9

SOLUTION:

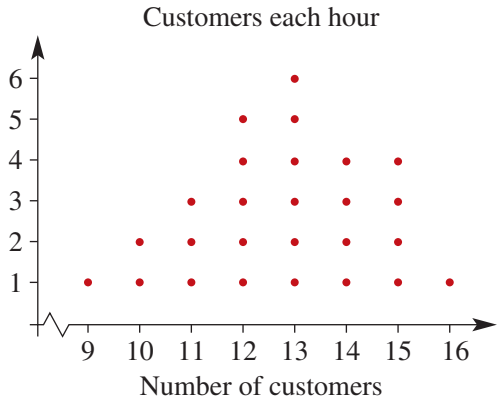
1. The data set has values from 6 to 35. This requires stems 0, 1, 2 and 3. Write these down from smallest to largest, followed by two vertical lines.
2. Attach the leaves for the girls. The first data value is '28'. It has a stem of '2' and a leaf of '8'.
3. Attach the leaves for the boys. The first data value is '19'. It has a stem of '1' and a leaf of '9'.
4. Rewrite the leaves so that they are in increasing order.

Girls		Boys
6	0	9
8532	1	1369
9876442	2	155789
410	3	0125

Exercise 7C

LEVEL 1

1 The dot plot represents the number of customers per hour.



a What is the highest number of customers?
b What is the most common number of customers per hour?
c What are the least common numbers of customers?
d Calculate the total number of customers.

Example 5

2 A group of 26 people were asked how many times in the last week they had shopped at a particular supermarket. Their responses were as follows:

1	3	3	2	1	1	4	5	4	2	1	3	4
6	2	1	1	1	3	3	2	2	1	5	6	1

a Construct a dot plot of this data.
b How many people were at the supermarket four times last week?
c What is the difference between the highest and lowest number of visits to the supermarket?

3 The goals scored in each match are listed below.

4	4	3	3	2	2	2
2	3	4	2	1	2	1
3	1	5	1	2	1	
1	3	2	4	2	1	

a Construct a dot plot for this data set.
b How many matches resulted in 3 goals?
c In how many matches were 4 or 5 goals scored?

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LEVEL 2

4 The stem-and-leaf plot represents the results achieved by students in a test.

0	6
1	2358
2	2446789
3	014

a What is the highest score in this test?
b Which score occurred the most number of times?
c What is the difference between the highest and lowest scores?
d How many students completed the test?
e What is the median for this data set?

5 An investigator recorded the life of 24 similar batteries in a toy. Her results (in hours) were:

41 25 37 46 17 4 33 31 28 34 19 26

40 24 31 27 30 22 33 20 21 27 30 26

a Make a stem-and-leaf plot of these times.
b How many of the batteries lasted for more than 25 hours?
c What is the median for this data set?

LEVEL 3

Example 7

6 The following table shows the number of nights spent away from home in the past year by a group of 15 Australian tourists and by a group of 15 New Zealand tourists.

AUS	21	5	8	7	17	3	15	14	3	11	5	4	11	6	4
NZ	19	6	23	32	17	29	23	22	12	28	26	5	22	14	14

a Construct a back-to-back stem-and-leaf plot of these data sets.
b Compare the number of nights spent away by Australian and New Zealand tourists in terms of shape, centre and spread.

7 The ages of patients admitted to a particular hospital during one week are given below.

Male	72	56	57	77	63	71	57	54	63	72	59	56	57	67	75
Female	61	55	58	78	65	68	71	78	79	72	73	64	68	66	69

a Construct a back-to-back stem-and-leaf plot of these data sets.
b Compare the ages at admission to the hospital for male and female patients in terms of shape, centre and spread.

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7D Grouped frequency tables

Data with a large range of values are often grouped into a small number of convenient intervals, called class intervals. When choosing class intervals ensure:

- Every data value is in an interval.
- Intervals do not overlap.
- No gaps exist between the intervals.

The choice of intervals can vary depending on that data. Recommendations:

- A division of 5 to 15 groups is preferred.
- Choose an interval that is easy to read such as 5 units, 10 units, etc.
- Start the interval with an appropriate exact value. For example, choose intervals such as $0–9, 10–19, \ldots$ rather than $1–10, 11–20, \ldots$

GROUPED FREQUENCY TABLE

1. Classes or groups are listed in the first column in ascending order.
2. Class centre column is obtained by finding the middle of the class endpoints.
3. Tally column shows the number of times a score occurs in a class (groups of 5s).
4. Frequency column is total count of scores in each class.

Example 8: Constructing a grouped frequency table 7D

Twenty-six people were asked to record how many cups of coffee they drank in a particular week. The results are listed below. Draw a frequency table to show the data.

0	33	6	14	0	32	0	25	10	0	2	9	23
0	34	5	17	3	0	23	1	32	0	8	0	2

SOLUTION:

1. Draw a table with 4 columns and label them class, class centre, tally and frequency.
2. The data ranges from 0 to 34. A class interval width of 5 results in 7 classes.
3. Calculate the class centre (middle of each class).
4. Record a mark in the tally column for each data value in the class interval.
5. Count the tally marks in the frequency column.
6. Add the frequency column to find the total number of scores.

Class	Class centre	Tally	Frequency
0–4	2	𝍸 𝍸 ||	12
5–9	7	||||	4
10–14	12	||	2
15–19	17	|	1
20–24	22	||	2
25–29	27	|	1
30–34	32	||||	4
		Total	26

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Exercise 7D

LEVEL 1

1 The ages of the people living in Matilda Rd was recorded.

- **a** Copy and complete the table opposite.
- **b** How many people are younger than 20?
- **c** Which class occurred the most number of times?
- **d** How many people are living in this road?

Class	Class centre	Frequency
5–19		10
20–34		8
35–49		6
50–64		4
	Total	28

2 A grouped frequency table is shown.

- **a** Copy and complete the table.
- **b** How many class intervals have been used?
- **c** Which class occurred the least?
- **d** What is the total number of scores?

Class	Class centre	Tally	Frequency
3–7		𝍸	4
	10	𝍸 |	6
13–17	15		2
18–22	20	𝍸 ||	
	25	|	1
28–32			3
33–37	35	𝍸 |||	
	40	𝍸	5

Example 8

3 The heights of the 30 players in a netball club are recorded below. Construct a grouped frequency table using class intervals (170–174,175–179,180–184,185–189, …).

174	184	183	179	180	181	189	188
194	185	178	173	183	188	183	182
194	189	184	182	183	189	193	
184	189	194	188	192	190	180	

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LEVEL 2

4 A drink vending machine records the number of bottles sold each day.

13	22	26	34	21	33	32	31	33	26	30	33	37	27	36
28	35	24	31	3	35	39	26	13	34	21	29	36	24	33
25	9	39	19	29	36	38	29	38	38	37	38	31	37	37
28	40	34	29	18	35	22	20	35	25	31	39	39	18	36
38	35	8	29	35	20	34	30	37	33	27	32	32	36	16
39	30	14	29	20	22	12	24	17	21	18	17	38	28	25

a Decide on appropriate classes for a frequency table.
b Calculate the class centres for these classes.
c Construct a grouped frequency table using these class intervals.

5 The players' scores after the second round of a golf tournament are recorded below.

162	163	175	161	166	163	167	151	150	176
159	173	162	155	149	171	181	163	154	165
145	177	184	171	154	166	168	158	136	156
161	162	169	162	160	150	174	176	146	137

a Decide on appropriate classes for a frequency table.
b Calculate the class centres for these classes.
c Construct a grouped frequency table using these class intervals.

LEVEL 3

6 Jordan surveyed his friends to check the number of emails they saved on their computers. These are the numbers he recorded:
22, 9, 51, 6, 30, 18, 30, 4, 10, 5, 19, 23, 37, 17, 18, 12, 10, 24, 28, 25, 60, 45, 19, 17, 11, 8, 16, 1, 24, 3

a Decide on appropriate classes for a frequency table.
b Calculate the class centres for these classes.
c Construct a grouped frequency table using these class intervals.
d What percentage of friends had fewer than 20 emails saved on their computer?

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7E Cumulative frequency

Cumulative frequency is the frequency of the score plus the frequency of all the scores less than that score. A cumulative frequency column is often inserted next to the frequency column in a frequency table.

Score	Frequency	Cumulative frequency
18	1	1
19	5	6
20	3	9
21	7	16

CUMULATIVE FREQUENCY

The frequency of the score plus the frequency of all the scores less than that score

Example 9: Calculating the cumulative frequency 7E

The frequency table opposite shows the temperatures for 17 days. Complete the cumulative frequency column.

Score	Frequency	Cumulative frequency
18	1	
19	2	
20	3	
21	3	
22	5	
23	3	

SOLUTION:

1. The lowest score is 18 and it has a frequency of 1. Cumulative frequency of 18 is 1.
2. Add the frequency of the next score to the cumulative frequency of the previous score. Cumulative frequency of 19 = 2 + 1 = 3.
3. Repeat this process.

Score	Frequency	Cumulative frequency
18	1	1
19	2	1 + 2 = 3
20	3	3 + 3 = 6
21	3	6 + 3 = 9
22	5	9 + 5 = 14
23	3	14 + 3 = 17

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Exercise 7E

LEVEL 1

Example 9

1 The frequency table shows the results of a test.

a Copy and complete the table.
b How many students scored 8?
c How many students scored more than 5?
d How many students scored less than 6?
e How many students completed the test?

Score	Frequency	Cumulative frequency
4	4	
5	6	
6	7	
7	10	
8	5	

2 The number of times an ambulance was called out each day is recorded in a frequency table.

a Copy and complete the table.
b On how many days was the ambulance called out 21 times?
c On how many days was the ambulance called out fewer than 25 times?
d On how many days was the ambulance called out more than 23 times?

Number of calls	Frequency	Cumulative frequency
20	4	
21	3	
22	10	
23	12	
24	6	
25	5	

3 The results of a survey are listed below.

13	11	11	10	10	10	9	10	9	11	9	10	12	11	13
10	9	12	11	13	10	11	10	10	10	13	8	8	10	9

a Copy and complete the table.
b How many people completed the survey?
c How many people scored 10 in the survey?
d What is the difference between the highest and lowest scores?
e Which score had the highest frequency?

Score	Tally	Frequency	Cumulative frequency
8			
9			
10			
11			
12			
13			

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LEVEL 2

4 The ages of children at the local park are shown below.

9	8	4	6	10	10	8	6	5	4	5	6	8	4	9
10	5	8	8	9	10	4	6	10	9	9	7	7	6	5

a Construct a frequency table to represent this data; include a tally column.
b Add a cumulative frequency column.
c How many children are there altogether?
d How many children are 8 years old?
e How many children are 6 years old?
f How many children are older than 5?
g How many children are younger than 9?
h What are the most common ages?
i What is the least common age?
j What fraction are 7 years old?
k What fraction are 4 years old?
l What percentage are older than 5?
m What percentage are younger than 6?

LEVEL 3

5 Blake recorded his time (in seconds) to run the 400-metres hurdles throughout the year.

61	62	62	63	64	62	66	64	63	63	62	61	62	62	63
64	63	65	63	63	62	61	62	63	62	61	61	64	64	64
63	63	64	65	64	63	63	62	61	62	62	63	64	63	62

a Construct a frequency table to represent this data; include a tally column.
b Add a cumulative frequency column to the table.
c How many times were recorded?
d How many times did Blake run the 400 metres in 61 seconds?
e How many times are less than or equal to 63?
f What fraction of his 400-metre times is 62?
g What percentage of his 400-metre times is 65? Answer correct to two decimal places.
h What percentage of his times are less than or equal to 64? Answer correct to two decimal places.

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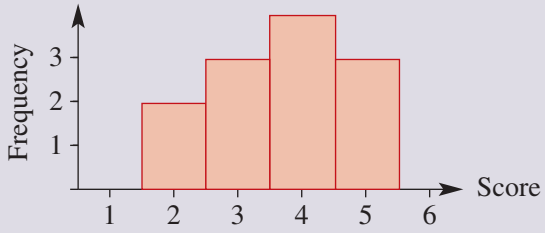
7F Frequency and cumulative frequency graphs

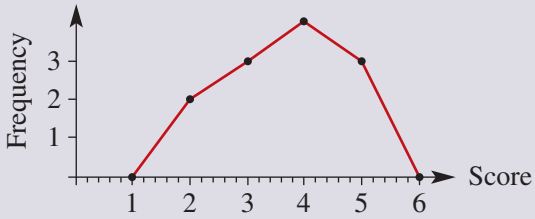
Frequency histogram and polygon

A frequency histogram is a graph of a frequency table in which equal intervals of the scores (or classes) are marked on the horizontal axis and the frequencies associated with these intervals are indicated by vertical rectangles. A frequency polygon is a line graph of the frequency table and can be constructed by joining the midpoints at the tops of the rectangles of a frequency histogram.

FREQUENCY HISTOGRAM



FREQUENCY POLYGON



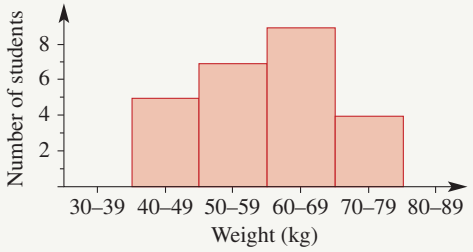
Example 10: Constructing a grouped frequency histogram 7F

The weights of 25 students were recorded and displayed in a grouped frequency table. Construct a grouped frequency histogram and polygon.

Class	Class centre	Frequency
30–39	34.5	0
40–49	44.5	5
50–59	54.5	7
60–69	64.5	9
70–79	74.5	4
80–89	84.5	0

SOLUTION:

1. Draw the horizontal axis with each class (or class centre) the same distance apart.
2. Draw a vertical axis using a scale that will cater for the lowest to highest frequency.
3. Label the horizontal and vertical axes.

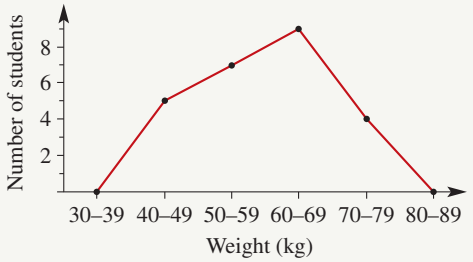


Frequency histogram:

4. Draw a rectangle for each class to the matching frequency. The class is in the centre of the rectangle.

Frequency polygon:

5. Draw a line for each class to the matching frequency.



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Cumulative frequency graphs

A cumulative frequency histogram is constructed using equal intervals of the scores (or classes) on the horizontal axis and the cumulative frequencies associated with these intervals indicated by vertical rectangles. A cumulative frequency polygon or ogive is a line graph constructed by joining the top right-hand corner of the rectangles of a cumulative frequency histogram.

Example 11: Constructing cumulative frequency graphs 7F

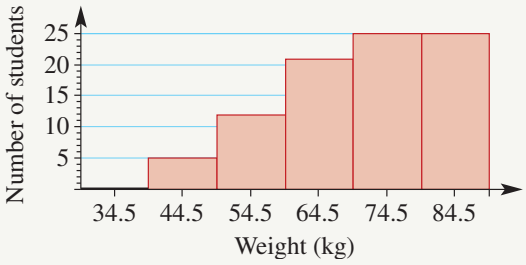
a Construct a cumulative frequency histogram and polygon or ogive.
b Estimate the median using the ogive.

Class	Class centre	Frequency	Cumulative frequency
30–39	34.5	0	0
40–49	44.5	5	5
50–59	54.5	7	12
60–69	64.5	9	21
70–79	74.5	4	25
80–89	84.5	0	25

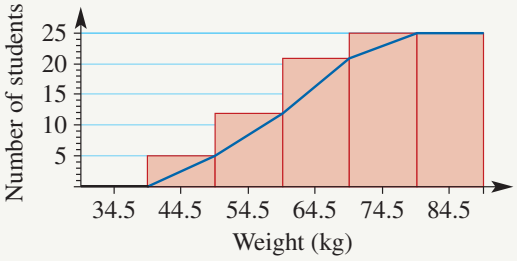
SOLUTION:

1 Draw the horizontal axis with each class (or class centre) the same distance apart.
2 Draw a vertical axis using a scale that will cater for the lowest to highest frequency.
3 Draw a rectangle for each class to the matching cumulative frequency. The class is in the centre of the rectangle.
4 Draw a line for each class to the top right-hand corner of the rectangle.
5 There are 25 students, so the median student is 12.5.
6 Draw a horizontal line from 12.5 until it intersects the ogive. Draw a vertical line from this point to the horizontal axis.
7 Estimate the median value.

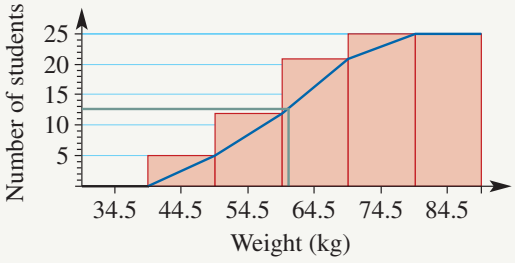
a Cumulative frequency histogram



Cumulative frequency polygon



b



The median is about 60.

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Exercise 7F

LEVEL 1

Example 10

1 The frequency table shows the scores for a mathematics quiz. Use this data to construct a:

a frequency histogram

b frequency polygon.

Score	Tally	Frequency
4	𝍸	3
5	𝍸	5
6	𝍸 |	7
7	𝍸 𝍸	10
8	𝍸	3

2 The numbers of brothers and sisters reported by each of 30 students is as follows.

3	2	2	6	4	5	2	1	3	2	6	3	7	2	1
4	2	1	5	4	4	5	3	4	1	3	1	2	1	3

a Construct a frequency table for this data.

b Use the frequency table to construct a frequency histogram.

c Use the frequency table to construct a frequency polygon.

3 The number of magazines purchased in a month by 20 different people was recorded as:

6	5	8	4	4	5	4	7	5	6
7	4	5	5	4	7	6	5	4	4

a Construct a frequency table for the data.

b Use the frequency table to construct a frequency histogram.

c Use the frequency table to construct a frequency polygon.

4 The maximum temperatures for several capital cities around the world are listed.

22	23	23	24	18	18	19	19	19	20	21	20
21	20	21	17	24	23	17	22	24	24	17	17

a Construct a frequency table for this data.

b Construct a frequency histogram for this data.

c Construct a frequency polygon for this data.

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LEVEL 2

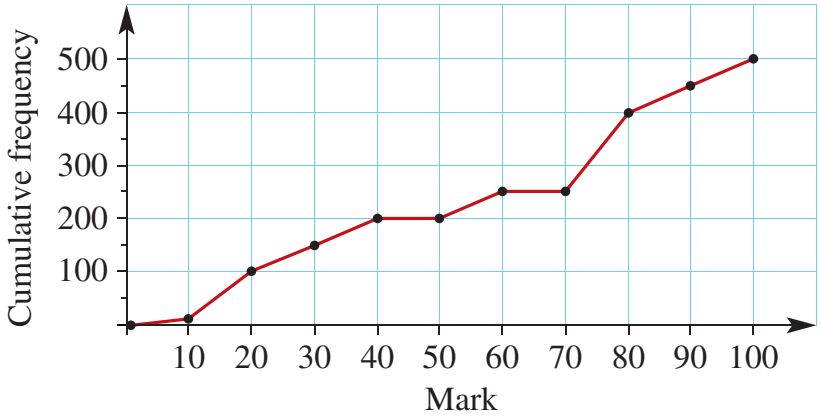
Example 11

5 Jade owns five clothing stores that sell jackets. She recorded the total number of jackets sold each day for the month of April. This data is shown below.

61	66	67	67	60	63	67	63
64	62	65	65	67	66	61	62
65	61	63	63	67	67	60	
66	64	67	62	65	65	67	

a Construct a frequency table with a cumulative frequency column.
b Construct a cumulative frequency histogram for this data.
c Construct a cumulative frequency polygon for this data.
d On how many days was the number of jackets sold less than or equal to 63?
e On how many days was the number of jackets sold less than or equal to 66?

6 The marks for a university exam are shown in the cumulative frequency polygon.



a What was the frequency of 10?
b What was the frequency of 20?
c What was the frequency of 30?
d What was the frequency of 40?
e What was the frequency of 50?
f What was the frequency of 60?
g What was the frequency of 70?
h What was the frequency of 80?
i What was the frequency of 90?
j What was the frequency of 100?
k How many students completed this university exam?
l Construct a frequency table from the cumulative frequency polygon.

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LEVEL 3

7 The number of words in each of the first 30 sentences of a book were recorded.

22	20	22	23	23	24	24	25	25	21	21	20	20	22	26
22	20	24	26	20	21	21	23	22	23	22	20	20	21	21

a Construct a frequency table with a cumulative frequency column.
b Use the frequency table to construct a frequency histogram.
c Use the frequency table to construct a frequency polygon.
d Construct a cumulative frequency histogram and polygon for this data.
e Use the cumulative frequency graphs to estimate the median.
f Use the cumulative frequency graphs to estimate the first and third quartile.
g Use the cumulative frequency graph to estimate the interquartile range.

8 The percentage of female births, correct to the nearest whole number, is shown below. These birth percentages have been taken from 30 different hospitals.

38	56	57	59	58	60	43	52	49	61
47	38	41	50	51	55	45	50	49	53
54	48	51	43	55	53	42	42	44	46

a Decide on appropriate classes for a frequency table.
b Construct a grouped frequency table using these class intervals.
c Add a cumulative frequency column.
d Construct a frequency histogram for this data.
e Construct a frequency polygon for this data.
f Construct a cumulative frequency histogram and polygon for this data.
g Use the cumulative frequency graphs to estimate the median.
h Use the cumulative frequency graphs to estimate the upper and lower quartile.
i Use the cumulative frequency graph to estimate the interquartile range.

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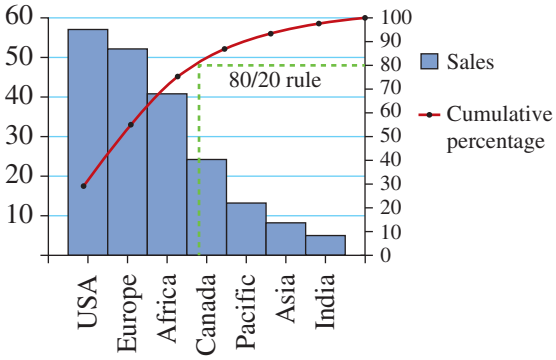
7G Pareto charts

A Pareto chart (named after Vilfredo Pareto) combines a frequency histogram and cumulative frequency line graph. The histogram is plotted in decreasing order of relative frequency from left to right with the vertical scale on the left-hand side. The cumulative frequency line graph is superimposed on the histogram, with the vertical scale on the right-hand side.

Pareto principle

A Pareto chart highlights the Pareto principle which states that 20% of the invested input is responsible for 80% of the results obtained. It is also referred to as the 'Pareto rule' or the '80/20' rule. The Pareto principle is applied in a wide range of areas such as manufacturing, management and human resources. Another common use for the Pareto principle is time management, as most people tend to evenly spread their time and not focus on the most important tasks.



A table of projected sales for a business and the corresponding Pareto chart is shown below.

Region	Sales	Cumulative frequency	Cumulative percentage
USA	57	57	28
Europe	52	109	54
Africa	41	150	74
Canada	24	174	86
Pacific	16	190	94
Asia	8	198	98
India	5	203	100

PARETO CHART

To create a Pareto chart follow these steps.

1. Draw the horizontal axis and place each category on the horizontal axis in order of frequency. The category with the highest frequency is on the left-hand side.
2. Draw the frequency histogram with an appropriate vertical axis on the left-hand side.
3. Find the cumulative frequency and cumulative percentages for each category.
4. Draw a cumulative percentage line graph with the vertical axis on the right-hand side.
5. Construct a legend for both the histogram and the line graph.

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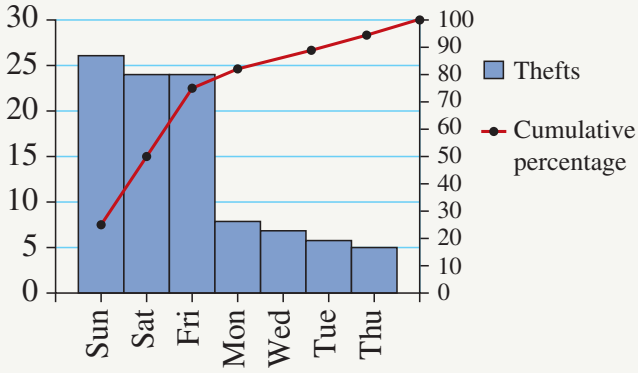
Example 12: Applying the Pareto principle 7G

Motor vehicle thefts have been recorded for each day of the week and are shown below.

Day	Thefts	Cumulative frequency	Cumulative percentage
Sun	26	26	26
Sat	24	50	50
Fri	24	74	74
Mon	8	82	82
Wed	7	89	89
Tue	6	95	95
Thu	5	100	100



a On which day was there the least number of motor vehicle thefts?
b What is the cumulative percentage for Saturday and Sunday?
c What is the cumulative percentage for Friday, Saturday, Sunday and Monday?
d Apply the 80 / 20 rule to determine the days on which motor thefts are a major concern.

SOLUTION:

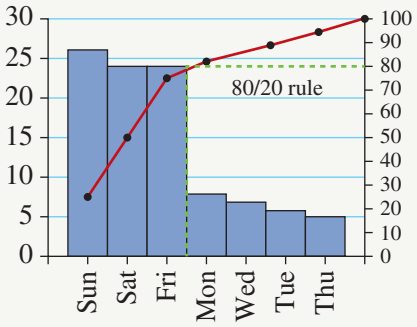
1 The day (category) with the smallest frequency is located on the right-hand side.
2 Read the cumulative percentage for Saturday from the table.
3 Read the cumulative percentage for Monday from the table.
4 Draw a horizontal line from the 80 cumulative percentage until it intersects the line graph. Draw a vertical line from this point of intersection. The most significant categories are located on the left-hand side.

a Thursday – 5 thefts

b Cumulative percentage is 50.

c Cumulative percentage is 82.

d Sunday, Saturday and Friday.



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Exercise 7G

LEVEL 1

1 The orders received by a business for five months are shown below.

Month	Frequency	Cumulative frequency	Cumulative percentage
Apr	25	25	
May	9		68
Mar	8		
Feb	5	47	
Jan	3		100

a Copy and complete the table above.
b How many categories will be on the horizontal axis of a Pareto chart?
c What is an appropriate scale for the vertical axis on the left-hand side of a Pareto chart?
d What is an appropriate scale for the vertical axis on the right-hand side of a Pareto chart?
e Draw a Pareto chart from your frequency table.
f Determine which are the most significant months by applying the Pareto principle.

2 The results of a survey of major health issues are shown below.

Health issue	Frequency	Cumulative frequency	Cumulative percentage
Obesity	56		
Mental health	13	69	
Physical activity	12		81
Smoking	7		
Alcohol	4	92	
Nutrition	4		
Injury	3		99
Environment	1		

a Copy and complete the table above.
b How many categories will be on the horizontal axis of a Pareto chart?
c What is an appropriate scale for the vertical axis on the left-hand side of a Pareto chart?
d What is an appropriate scale for the vertical axis on the right-hand side of a Pareto chart?
e Draw a Pareto chart from your frequency table.
f Determine which are the most significant health issues by applying the Pareto principle.

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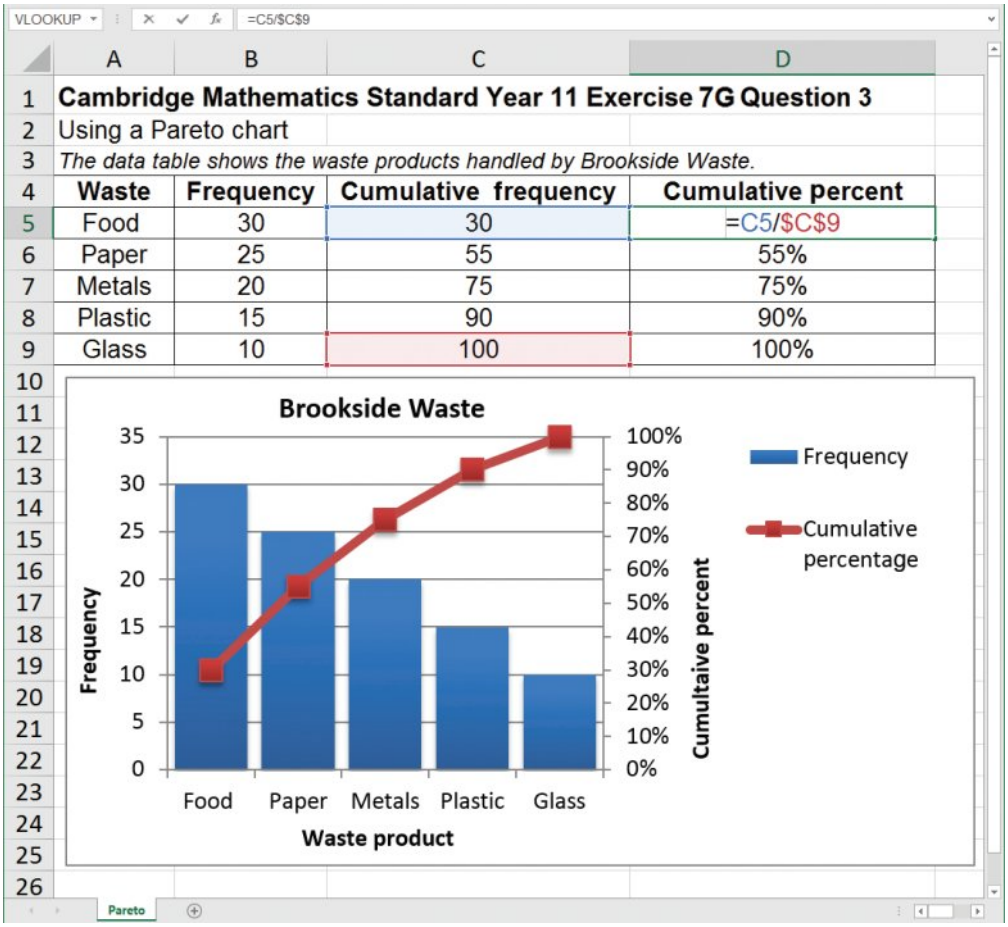
LEVEL 2

07GQ3

3 Create the spreadsheet and chart below.



a Enter the formula '= C5/$C$9 * 100' into cell D5.

b Fill down the contents of D5 to D9.

c Use the Pareto principle to determine which waste products require the most attention.

LEVEL 3

4 The results of a survey showing the preferred sport of 80 students as a percentage are shown below.

Tennis	Netball	Football	Swimming	Golf	Cricket
5	10	20	45	15	8

a Construct a frequency table with columns for sport, frequency, cumulative frequency and cumulative percentage.

b Draw a Pareto chart to represent the above data.

c Use the Pareto principle to determine which sports require the most attention.

7H Motor vehicle statistics

Since there have been motor vehicles, there have been motor vehicle accidents that result in death, injury and damage. Governments collect, present and interpret data on road incidents to try to reduce the problem. There are many factors that may cause a road accident, such as poor driving, speeding, alcohol, fatigue, bad road design and lack of vehicle maintenance.

Example 13: Reading a table of motor vehicle accident statistics 7H

The table below shows the number of road accidents involving fatigue in the last four months of the year.

	Killed	Injured
September	112	1488
October	197	4365
November	89	2019
December	134	3487

a How many deaths occurred in road accidents during October involving fatigue?
b Which month had the least number of injuries in road accidents involving fatigue?
c How many deaths in road accidents involving fatigue occurred in the four months?
d What is the percentage increase in number of injuries involving fatigue from November to December?

SOLUTION:

1 Read the value in the intersection of October and killed.

a 197 deaths

2 Lowest value in the injured column is 1488.

b September

3 Add the deaths for each month.

c Deaths $= 112 + 197 + 89 + 134$
$= 532$

4 Find the increase in the number of injuries (subtract 2109 from 3487).

d Increase $= 3487 - 2019$
$= 1468$

5 Express the increase in the injuries in December over the injuries for November. Multiply this fraction by 100 to find the percentage increase.

$$\text{Percentage} = \frac{1468}{2019} \times 100$$
$$= 72.709...$$
$$\approx 73\%$$

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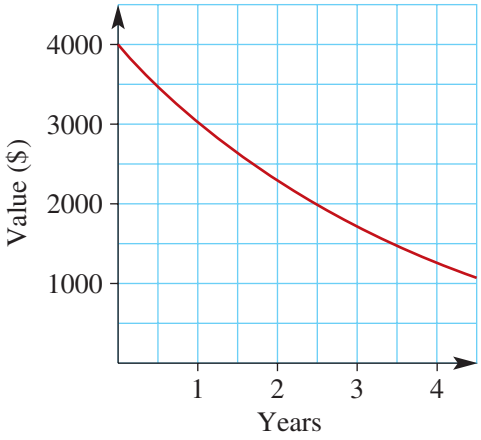
Exercise 7H

LEVEL 1

1 The decrease in the price of a used car over four years is shown in the graph opposite.

- **a** What is the initial value of the used car?
- **b** How much did the used car decrease in price during the first year?
- **c** When is the value of the used car $2000?
- **d** When is the value of the used car $1500?
- **e** What is the value of the used car after 4 years?
- **f** What is the value of the used car after $1\frac{1}{2}$ years?



Example 13

2 The table below shows the running costs for fuel, tyres and servicing as cents per km for four brands of motor vehicle.

	Brand A	Brand B	Brand C	Brand D
Fuel	5.06	6.90	9.69	8.99
Tyres	1.03	1.18	0.88	1.28
Service	2.51	3.73	3.02	3.88

- **a** What is the cost of service for the Brand D vehicle?
- **b** Which of the cars has the best fuel economy?
- **c** Harry has driven his Brand C vehicle 7580 kilometres this year. What is the fuel cost of his vehicle for the year?
- **d** Calculate the difference in service costs between Brand A and Brand B, when both cars are driven 15 000 km in a year.
- **e** What is the difference in tyre costs between Brand D and Brand A, when they are both driven 100 000 km?

3 The table below shows the petrol used at different speeds to cover the same distance.

Speed	50 km/h	70 km/h	90 km/h	110 km/h
Litres	34	38	43	49

- **a** How much petrol would you save by travelling at 50 km/h instead of 70 km/h?
- **b** How much petrol would you save by travelling at 70 km/h instead of 110 km/h?
- **c** What is the difference in cost of travelling at 50 km/h instead of 90 km/h? Assume the petrol costs are $1.45 per litre.

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LEVEL 2

4 The table opposite shows the number of road accidents involving speed that caused an injury in the first five months of the year. Find the following summary statistics.

Month	Accidents
January	2814
February	1652
March	1786
April	1589
May	2182

- **a** How many accidents involved speed in the first five months?
- **b** What percentage of these road accidents occurred in January?
- **c** What fraction of these road accidents occurred in March?
- **d** What is the mean number of accidents per month for these five months?
- **e** Which month had the highest number of accidents? Can you suggest a reason for this month having the most accidents?

5 The speed (in km/h) of some motor vehicles travelling through an intersection was 42, 36, 36, 44, 30, 34, 38, 36 and 39.

- **a** What is the fastest speed?
- **b** What is the slowest speed?
- **c** What is the mean, correct to the nearest whole number?
- **d** What is the mode?
- **e** Find the median.
- **f** What is the range?

6 The frequency table below shows the number of motorbikes passing through a checkpoint each hour for the past 24 hours.

Motorbikes (x)	Frequency (f)	Frequency × Score (fx)
11	4	
12	7	
13	6	
14	5	

- **a** In how many hours did 13 motorbikes pass through the checkpoint?
- **b** What was the greatest number of motorbikes passing through a checkpoint in an hour?
- **c** How many motorbikes passed through the checkpoint?
- **d** Find the mean of this data. Answer correct to one decimal place.
- **e** What is the median of this data?
- **f** What is the mode of this data?

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LEVEL 3

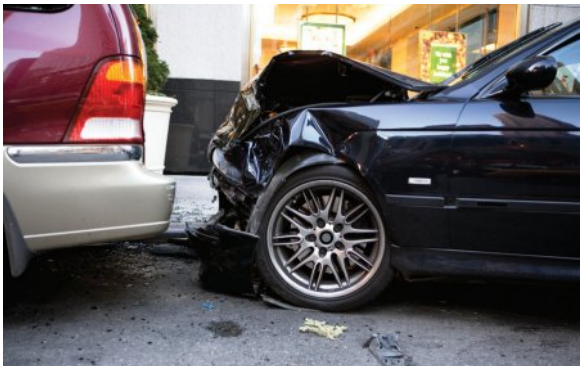
7 The grouped frequency table below shows the ages of drivers involved in fatal road accidents during the past year.

Class	Class centre (x)	Frequency (f)	$f \times x$
20–29		85	
30–39		72	
40–49		71	
50–59		55	
60–69		36	

a Copy and complete the grouped frequency table.
b How many fatal road accidents occurred in the past year?
c Find the mean of this data to the nearest whole number.
d What percentage of road accidents had a driver younger than 30? Answer correct to two decimal places.
e What percentage of road accidents had a driver older than 49? Answer correct to two decimal places.

8 A local community was concerned about the number of accidents at a particular intersection. The number of accidents at the intersection in the past 13 days is recorded below.

0 0 6 1 0 3 0 2 0 3 0 1 0



a Find the mean, median and mode of this data.
b Which is the better measure for the centre for the data? Explain your answer.

9 A motor vehicle is bought for $32 000. It depreciates at 16% per annum and is expected to be used for 5 years.

a How much does the motor vehicle depreciate in the first year?
b Copy and complete the following depreciation table for the first five years.
c Graph the value in dollars against the age in years.

Year	Current value	Depreciation	Depreciated value
1			
2			
3			
4			
5			

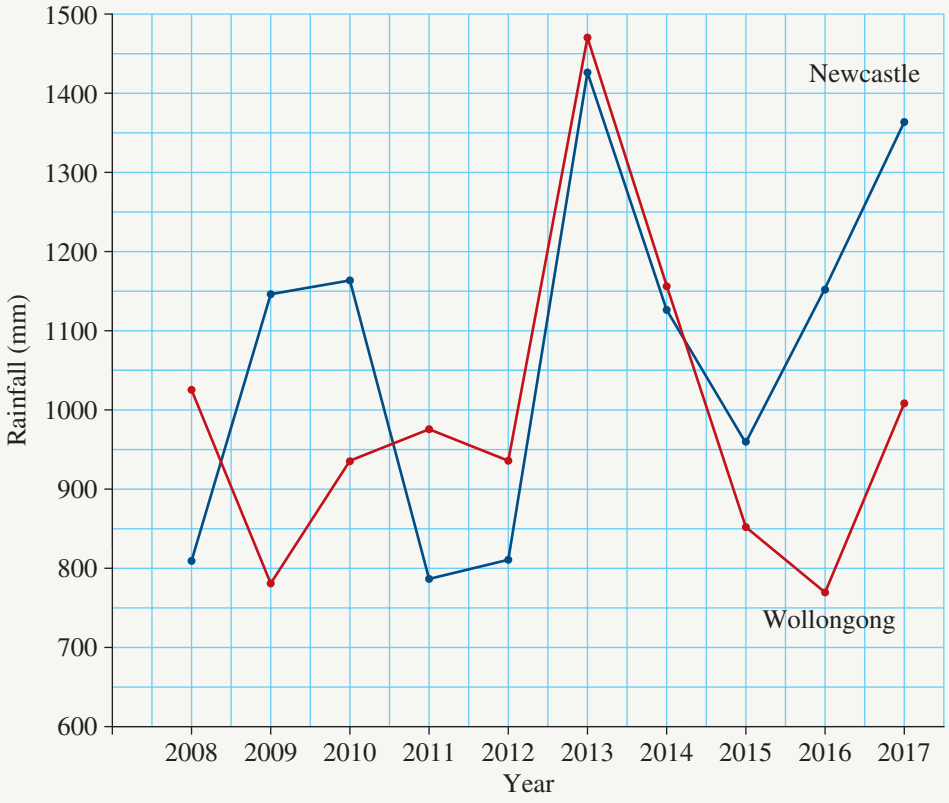
7I Water usage

Rainfall

Rainfall changes are occurring due to climate change as a result of the increased concentration of carbon dioxide in the atmosphere. We are also experiencing more droughts and floods. Australia's annual mean rainfall has increased slightly over the last century.

Example 14: Interpreting rainfall graphs 7I

The graph shows the annual rainfall data for Newcastle (—) and Wollongong (—) over 10 years.



a What was annual rainfall for Newcastle in 2015?
b Which was the wettest year during this 10-year period for Wollongong?
c Which year has the largest difference in annual rainfall?

SOLUTION:

1 Read the values at 2015 for Newcastle.
2 Read the year when Wollongong had the highest rainfall.
3 Find the largest gap between the two line graphs. Read this year.

a Newcastle's rainfall in 2015 was 960 mm.
b Wollongong's wettest year was 2013.
c Largest difference occurred in 2016.

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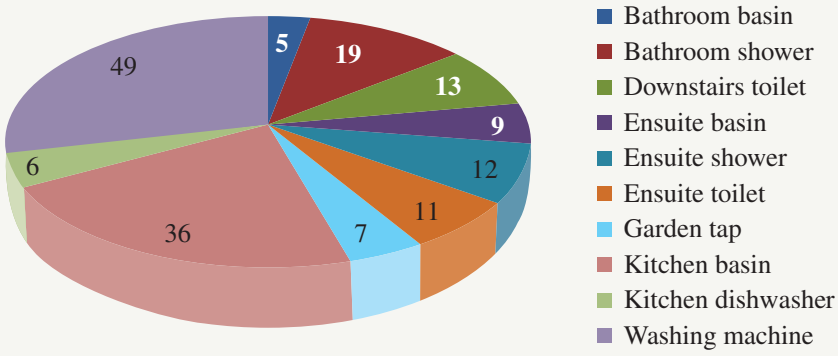
Household water usage

The amount of water that is available for local, state and national needs will vary from year to year. In recent years governments have imposed restrictions on our water use, to help conserve supplies in times of drought.

Example 15: Interpret data about a household's water usage 7I

The pie chart shows the annual water usage in kilolitres for a family.



a How many kilolitres of water were used by this family in a year?

b What percentage of the total water use is in operating the washing machine?

c A dual-flush toilet saves 60% of the water used in a conventional toilet. How many kilolitres per year would be saved by replacing the toilets with dual-flush toilets?

SOLUTION:

1 Read the values for each sector.
2 Add all the sector values to find the total.
3 Divide the amount of water used by the washing machine by the total water use and multiply this fraction by 100.
4 Evaluate.
5 Add the water use in the downstairs and ensuite toilets.
6 Change 60% to a decimal and multiply it by the water use (24).
7 Evaluate.
8 Write the answer in words.

a Water usage $= 36 + 6 + 49 + \ldots$
$= 167$ kL

b Washing machine $= \frac{49}{167} \times 100$
$\approx 29.34\%$

c Water usage $= 13 + 11 = 24$ kL
Water saving $= 60\%$ of 24 kL
$= 0.60 \times 24$
$= 14.4$ kL

Two dual-flush toilets save 14.4 kL a year.

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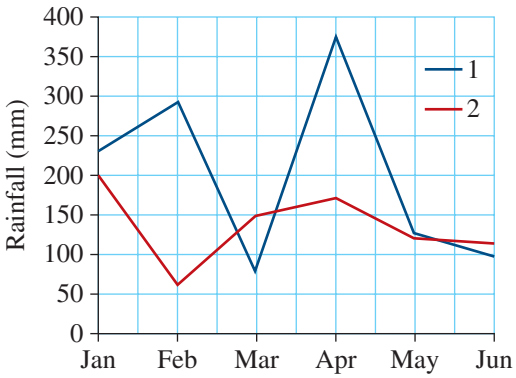
Exercise 7I

LEVEL 1

Example 14

1 The rainfall (January to June) for two years at Byron Bay is shown in the graph.



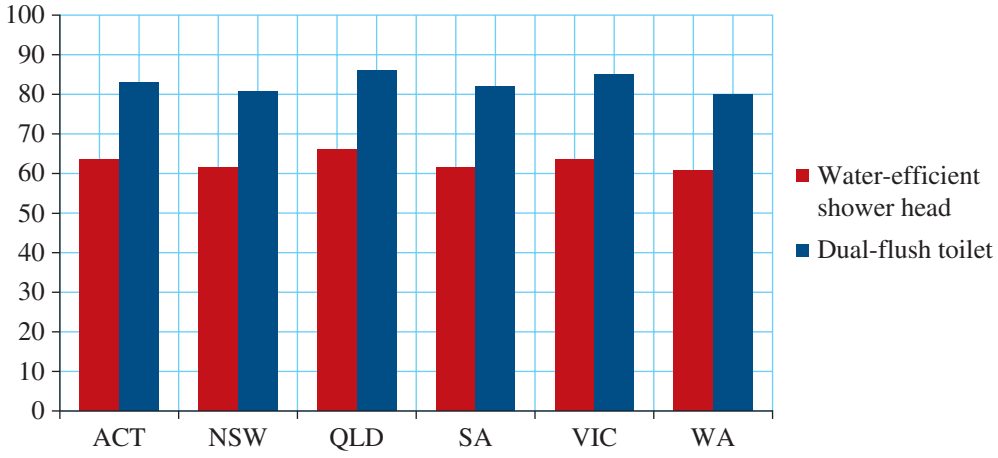
- **a** What was the rainfall in year 1 for April?
- **b** Which month had the lowest rainfall for both years?
- **c** Which year had the lower rainfall for January to June?
- **d** In which month was there the biggest difference in rainfall between year 1 and year 2?

2 The table shows the capacity and available water storage (%) in seven dams.

Dam	Capacity (ML)	Available water (%)
Avon	146 700	80
Cataract	97 190	78
Cordeaux	93 640	93
Nepean	67 730	82
Prospect	33 330	91
Warragamba	2 027 000	99
Woronora	71 790	87

- **a** Which dam has the highest capacity?
- **b** Which dam is closest to full capacity, as a percentage?
- **c** How much water is available in the Avon dam?
- **d** How much water is available in the Woronora dam?
- **e** Calculate the total capacity for these seven dams.
- **f** What is the average capacity for these seven dams?
- **g** Construct a line graph showing the available water by percentage.

3 The column graph below shows the percentage of households with water saving devices.



- **a** Which state has the highest percentage of households with dual-flush toilets?
- **b** Which state has the lowest percentage of households with water-efficient shower heads?
- **c** Which state has the highest percentage of households with both water-saving devices?
- **d** Which state has the lowest percentage of households with both water-saving devices?

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LEVEL 2

4 The table shows Dylan's average water use in one day.

a How much water is Dylan using each day?

b How much water would Dylan use in a year?

c What percentage of his water usage is in taking a shower?

d Construct a column chart for this data.

Water usage	L/day
Shower	51
Toilet	32
Tap	22
Washing machine	40
Other	5

5 The table shows the annual rainfall (mm) at Prospect for the past 10 years.

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
January	76.9	69.5	83.0	109.9	45.1	83.2	22.4	34.0	68.5	140.6

a What was the wettest year in this 10-year period?

b What was the driest year in this 10-year period?

c Find the mean and median rainfall for January.

d What is the range in rainfall for January?

e Construct a line graph to illustrate the changes in rainfall over time.

f Briefly comment on any trends that can be seen from the graph.

LEVEL 3

6 The table below shows the percentage of household water used in different locations.

Location	ACT	NSW	QLD	SA	VIC	WA
Bathroom	20	25	20	15	26	18
Kitchen	11	10	8	10	5	8
Laundry	10	16	10	13	15	14
Toilet	19	24	12	12	20	10
Outdoors	45	25	50	50	34	50

a Construct a column chart with each location as a data series.

b Construct a pie chart for NSW with each location as a sector.

c Which state uses the highest percentage of household water in the toilet? Why?

d Which states use the highest percentage of household water outdoors? Why?

e Which location uses the smallest percentage of household water? Why?

f Which location uses the highest percentage of household water? Why?

g Outline any differences between NSW and the other states and territories.

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PPT STUDY GUIDE

Key ideas and chapter summary

Categorical (Category)	Nominal	Name does not indicate order
	Ordinal	Name does indicate order
Numerical (Numbers)	Discrete	Data can only take exact numerical values such as 0 or 1
	Continuous	Data that can take any numerical value such as 71.25

Random sample
Members of the population have an equal chance of being selected.

Stratified sample
Categories of a population are chosen such as male/female. The members are randomly selected from each category.

Systematic sample
Population is divided into a structured sample size. The members are orderly selected from this structure, such as each 3rd person in order.

Self-selected sample
Members of the population volunteer themselves.

Dot plot
A number line with each data point marked by a dot.

Stem-and-leaf plot
Used to present a small (less than 50 values) numerical data set. The 'tens' digit of the data values becomes the 'stem' and is written in numerical order down the page. The 'units' digits become the 'leaves' and are written in numerical order across the page.

Grouped frequency table
1. Classes are listed in ascending order.
2. Tally column records the number of times the score occurred.
3. Frequency column is a count of each outcome or class.

Cumulative frequency
The frequency of the score plus the frequency of all the scores less than that score.

Frequency and cumulative frequency graphs
1. Frequency histogram is a column graph that uses the score as the horizontal axis and frequency as the vertical axis.
2. Cumulative frequency histogram is a column graph with the score as the horizontal axis and cumulative frequency as the vertical axis.
3. Frequency polygon is a line graph of a frequency table. It can be constructed by joining the mid-points of the bars of the histogram.
4. Cumulative frequency polygon or ogive is a line graph constructed by joining the top right-hand corner of the rectangles in a cumulative frequency histogram.

Pareto charts
A Pareto chart combines a frequency histogram and cumulative frequency line graph.

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Review

Multiple-choice questions

1 Sorting people in alphabetical order and selecting every fifth person. How would you describe this type of sample?

A Self-selected **B** Random **C** Stratified **D** Systematic

2 Sample designed to include four boys and four girls randomly selected from members of a surf club. How would you describe this type of sample?

A Self-selected **B** Random **C** Stratified **D** Systematic

3 'The number of rooms in your house'. What is the classification for this data?

A Categorical **B** Continuous **C** Discrete **D** Text

4 The female and male smoking rates, expressed as percentage are shown below.

Female		Male
9 7 7 6 5	1	7 9
8 6 5 5 6 2 1 0	2	2 2 4 4 5 6 7 7
	3	0 0 1 1 6 6 9

What is the lowest female smoking rate?

A 5% **B** 15% **C** 17% **D** 19%

5 What is the frequency of 22 in this cumulative frequency table?

A 5 **B** 8
C 13 **D** 22

6 What is the cumulative frequency of 23?

A 7 **B** 13
C 16 **D** 20

Score	Frequency	Cumulative frequency
21	5	5
22		13
23	7	
24	3	23
	Total	

7 What is vertical axis title on the right-hand side of a Pareto chart?

A Cumulative percentage **B** Cumulative frequency
C Frequency **D** Category

8 Jesse's water usage in one day was 60 L shower, 45 L toilet, 30 L tap and 15 L washing machine. What percentage of Jesse's water usage is for the washing machine?

A 10% **B** 15% **C** 30% **D** 150%

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Review

Short-answer questions

1 Classify the data from these situations as numerical or categorical.

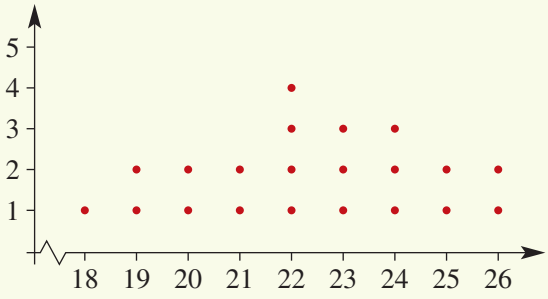
- **a** Most popular student in the class
- **b** Ava's favourite beach in Australia
- **c** The call cost on a mobile phone
- **d** Blake's school high jump record
- **e** The amount of annual leave
- **f** The hair colour of the students in your class
- **g** The number of websites accessed today
- **h** The digital download time for a 4 MB file
- **i** The average age of the people living in NSW

2 The stem-and-leaf plot on the right represents the results achieved by students in a test.

0	9
1	1249
2	135788
3	0235

- **a** What is the highest score?
- **b** What is the difference between the highest and lowest scores?
- **c** What is the median?

3 Use the dot plot to answer these questions.



- **a** What is the lowest score?
- **b** What is the difference between the highest and lowest scores?
- **c** What is the mode?
- **d** What is the median?

4 The times (in hours) students spent completing an assessment task are listed in the table.

Class	Frequency	Cumulative frequency
4–8	5	
9–13	6	
14–18	8	
19–23	4	

- **a** Copy and complete the table.
- **b** How many students completed the assessment task?
- **c** How many students spent greater than 13 hours?
- **d** What percentage of students spent less than 14 hours?

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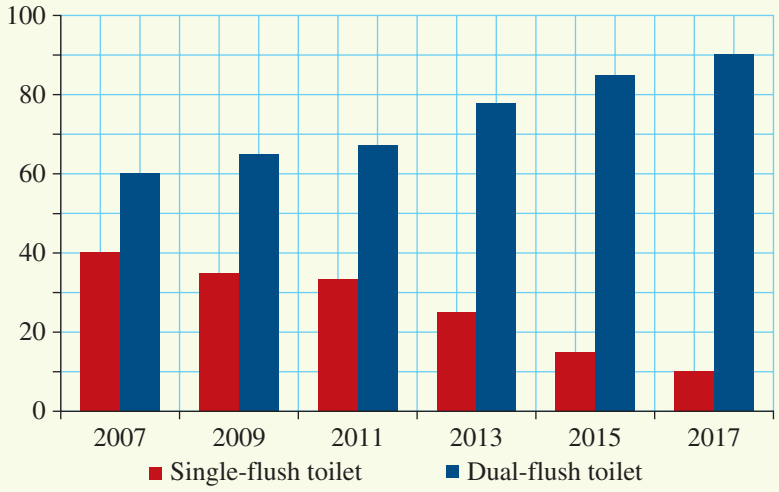
Review

5 The percentage of households using single- and dual-flush toilets is shown below.



a What percentage of households used a dual-flush toilet in 2007?
b In which year was the percentage of single-flush toilets half the percentage of dual-flush toilets?
c What is the trend shown by this column graph?
d There were 14.2 million toilets in 2017. How many of these toilets were dual flush?

Extended-response question

6 Alyssa recorded the following times (in minutes) running a cross-country course.

51	53	57	55	58	57	53	55	54	53	56	55	53	57	51	57	54
58	55	56	51	53	53	54	52	56	52	54	54	53	56	56	53	54
52	53	54	57	58	56	54	55	52	55	55	58	54	56	55	56	52

a Construct a frequency table using a tally column.
b How many times were recorded?
c How many times were below 55 minutes?
d Add a cumulative frequency column.
e What is the Alyssa's fastest time?
f What is the mode?
g What is the difference between Alyssa's fastest and slowest times?
h What is the median?
i Construct a frequency histogram.
j Construct a cumulative frequency histogram and a cumulative frequency polygon.

PDF LITERACY

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8 Linear relationships

Syllabus topic — MS-A2 Linear relationships

This topic involves the graphing and interpretation of practical linear and direct variation relationships. Students develop fluency in the graphical approach to linear modelling and its representativeness in common aspects of their life.

Outcomes

- Construct straight-line graphs.
- Determine and interpret the gradient and intercepts of a straight-line graph.
- Use and interpret graphs of the form $y = mx + c$.
- Construct and analyse a linear model to solve practical situations.
- Determine a direct variation relationship and solve problems.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Solutions (enabled by teacher)
- Literacy worksheet
- Widgets
- Quick Quiz
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

In the Interactive Textbook you can take a test of prior knowledge required for this chapter, and depending on your score you may be directed to revision from the previous years' work.

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8A Graphing linear functions

A linear function makes a straight line when graphed on a number plane. A linear function, for example $y = 3x$, has two variables y and x. When a number is substituted for a variable, such as $x = 2$, then this variable is called the independent variable. The dependent variable depends on the number substituted for the independent variable. That is, when $x = 2$ (independent) then $y = 3 \times 2$ or 6 (dependent).

To graph a linear function, construct a table of values with the independent variable as the first row and the dependent variable as the second row. Plot these points on the number plane with the independent variable on the horizontal axis and the dependent variable as the vertical axis. Join the points to make a straight line.

DRAWING A STRAIGHT-LINE GRAPH

1. Construct a table of values with the independent variable as the first row and the dependent variable as the second row.
2. Draw a number plane with the independent variable on the horizontal axis and the dependent variable as the vertical axis. Plot the points.
3. Join the points to make a straight line.

Example 1: Drawing a straight-line graph 8A

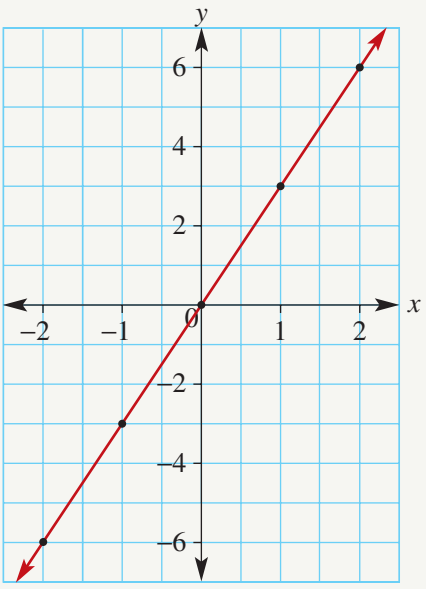
Draw the graph of $y = 3x$.

SOLUTION:

1. Draw a table of values for x and y.
2. Let $x = -2, -1, 0, 1$ and 2. Find y using the linear function $y = 3x$.

x	-2	-1	0	1	2
y	-6	-3	0	3	6

3. Draw a number plane with x as the horizontal axis and y as the vertical axis.
4. Plot the points $(-2, -6)$, $(-1, -3)$, $(0, 0)$, $(1, 3)$ and $(2, 6)$.
5. Join the points to make a straight line.



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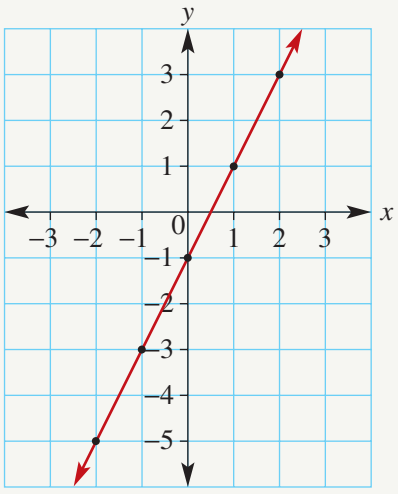
Example 2: Drawing a straight-line graph 8A

Draw the graph of $y = 2x - 1$.

SOLUTION:

1. Draw a table of values for x and y.
2. Let $x = -2, -1, 0, 1$ and 2. Find y using the linear function $y = 2x - 1$.
3. Draw a number plane with x as the horizontal axis and y as the vertical axis.
4. Plot the points $(-2, -5)$, $(-1, -3)$, $(0, -1)$, $(1, 1)$ and $(2, 3)$.
5. Join the points to make a straight line.

x	-2	-1	0	1	2
y	-5	-3	-1	1	3



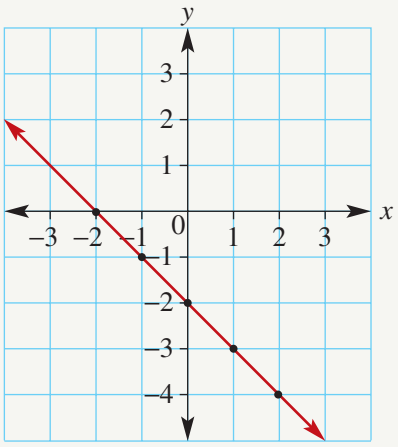
Example 3: Drawing a straight-line graph 8A

Draw the graph of $y = -x - 2$

SOLUTION:

1. Draw a table of values for x and y.
2. Let $x = -2, -1, 0, 1$ and 2. Find y using the linear function $y = -x - 2$.
3. Draw a number plane with x as the horizontal axis and y as the vertical axis.
4. Plot the points $(-2, 0)$, $(-1, -1)$, $(0, -2)$, $(1, -3)$ and $(2, -4)$.
5. Join the points to make a straight line.

x	-2	-1	0	1	2
y	0	-1	-2	-3	-4



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Exercise 8A

LEVEL 1

1 Plot the following points on number plane and join them to form a straight line.

a

x	−2	−1	0	1	2
y	2	1	0	−1	−2

b

x	−2	−1	0	1	2
y	−3	−1	1	3	5

c

x	−2	−1	0	1	2
y	0	1	2	3	4

d

x	−2	−1	0	1	2
y	3	2	1	0	−1

Example 1

2 Complete the following table of values for each linear function.

a $y = 2x$

x	−2	−1	0	1	2
y					

b $y = -2x$

x	−2	−1	0	1	2
y					

c $y = x + 1$

x	−2	−1	0	1	2
y					

d $y = x - 1$

x	−2	−1	0	1	2
y					

Example 2

e $y = 2x + 3$

x	−2	−1	0	1	2
y					

f $y = -x + 2$

x	−2	−1	0	1	2
y					

Example 3

3 Draw the graph of these linear functions by first drawing a table of values.

a $y = x$ **b** $y = 2x + 2$ **c** $y = -x + 3$

d $y = -x - 1$ **e** $y = 3x - 1$ **f** $y = -2x + 3$

4 **a** Complete the following tables of values.

i $b = 3a + 4$

a	−2	−1	0	1	2
b					

ii $q = -p + 1$

p	−2	−1	0	1	2
q					

b Graph $b = 3a + 4$.

c Graph $q = -p + 1$.

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LEVEL 2

5 Chocolates are sold for $12 per kg. The table below shows weight against cost.

Weight (w)	1	2	3	4	5
Cost (c)	12	24	36	48	60

a Which is the dependent variable?
b Which is the independent variable?
c Draw a graph of weight against cost.
d Use the graph to find c if w is 1.5.

6 Soraya conducted a science experiment and presented the results in a table.

Mass (m)	3	6	9	12	15
Time (t)	2.2	3.7	5.2	6.7	8.2

a Draw a graph of mass against time.
b Use the graph to find t if m is 10.

7 Dallas travels 4 km in 2 hours.
a Write a linear equation in the form $d = mt$ to describe this situation.
b Complete the table below and draw the graph of d against t.

Time (t)	0	2	4	6	8
Distance (d)					

8 Answer true or false to the following questions.
a Does the point $(3, 0)$ lie on the line $y = 3x$?
b Does the line $y = x + 7$ pass through the point $(-1, 6)$?
c The point $(1, 4)$ lies on the line $x + 2y = 9$.
d The line $2x - 3y = 0$ passes through the point $(0, 0)$.
e The point $(1, -1)$ lies on the line $4x - y + 1 = 0$.
f The line $3x - 4y + 1 = 0$ passes through the point $(2, 0)$.

LEVEL 3

9 Draw the graph of these linear functions by first drawing a table of values.
a $y + 2 = 3x$
b $x + y - 4 = 0$
c $y + \frac{1}{2}x = 1$
d $4x - y + 2 = 0$
e $\frac{1}{3}x - y = 1$
f $4 - y = 3x$

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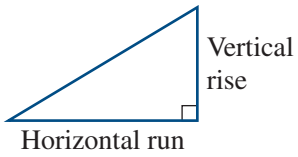
8B Gradient and intercept

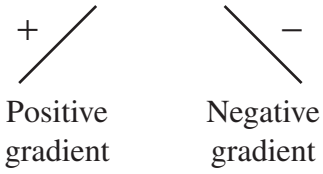
Gradient

The gradient of a line is the slope or steepness of the line. It is calculated by dividing the vertical rise by the horizontal run. The larger the gradient, the steeper the slope. The letter m is often used to indicate gradient.



$$\text{Gradient (or } m) = \frac{\text{Vertical rise}}{\text{Horizontal run}}$$

Positive gradients are lines that go up to the right or are increasing. Conversely, negative gradients are lines that go down to the right or are decreasing.

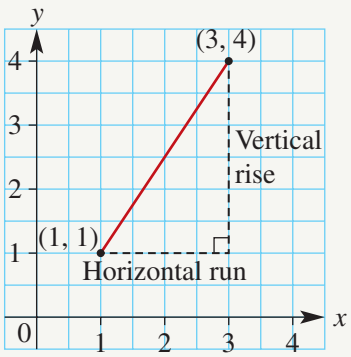


Example 4: Finding the gradient of a line 8B

Find the gradient of a line through the points (1, 1) and (3, 4).

SOLUTION:

1. Draw a number plane with x as the horizontal axis and y as the vertical axis.
2. Plot the points (1, 1) and (3, 4).
3. Draw a line between the two points.
4. Construct a right-angled triangle by drawing a vertical and a horizontal line.
5. The line is positive as it slopes towards the right.
6. Determine the vertical rise $(4 - 1 = 3)$.
7. Determine the horizontal run $(3 - 1 = 2)$.
8. Substitute 3 for the vertical rise and 2 for the horizontal run into the formula.



$$\text{Gradient or } m = \frac{\text{Vertical rise}}{\text{Horizontal run}}$$

$$= +\frac{3}{2}$$

Intercept

The intercept of a line is where the line cuts the axis. The intercept on the vertical axis is called the y-intercept and is denoted by the letter c. The intercept on the horizontal axis is called the x-intercept and is denoted by the letter a.

GRADIENT	INTERCEPT
Gradient of a line is the slope of the line. $\text{Gradient (or } m) = \dfrac{\text{Vertical rise}}{\text{Horizontal run}}$	The intercept of a line is where the line cuts the axis. Vertical intercept is often denoted by c. Previously in this course, b was used.

Example 5: Finding the gradient and vertical intercept 8B

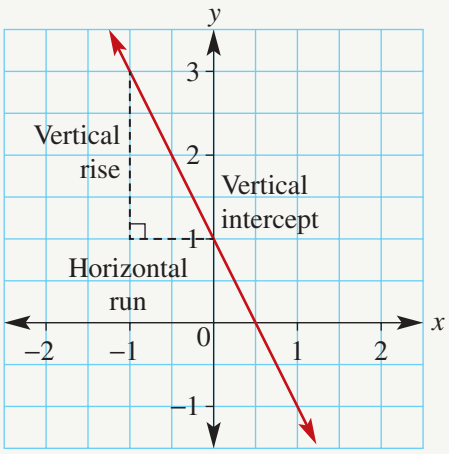
Find the gradient and vertical intercept for the line $y = -2x + 1$.

SOLUTION:

1 Draw a table of values for x and y.
2 Let $x = -1, 0$ and 1. Find y using the linear function $y = -2x + 1$.

x	-1	0	1
y	3	1	-1

3 Draw a number plane with x as the horizontal axis and y as the vertical axis.
4 Plot the points $(-1, 3), (0, 1)$ and $(1, -1)$.
5 Draw a line between these points.
6 Construct a right-angled triangle by drawing a vertical and a horizontal line.



7 The line is negative as it slopes to the left.
8 Determine the vertical rise $(3 - 1 = 2)$.
9 Determine the horizontal run $(-1 - 0 = -1)$.
10 Substitute 2 for the vertical rise and -1 for the horizontal run into the formula.
11 Evaluate.

$$\text{Gradient or } m = \frac{\text{Vertical rise}}{\text{Horizontal run}}$$
$$= -\frac{2}{1}$$
$$= -2$$

12 The line cuts the vertical axis at 1. Intercept on the vertical axis is 1.

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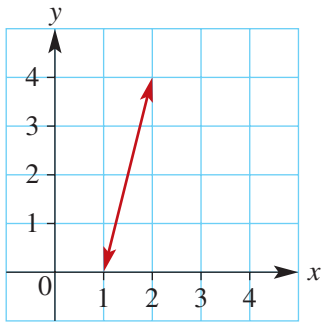
Exercise 8B

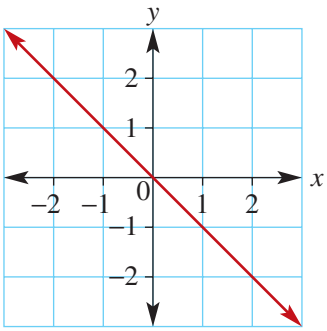
LEVEL 1

1 Find the gradient of the following lines.

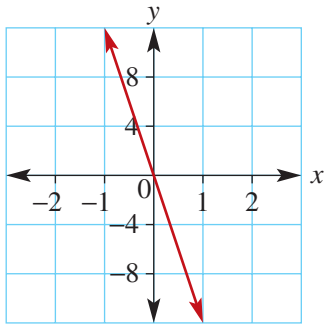
a



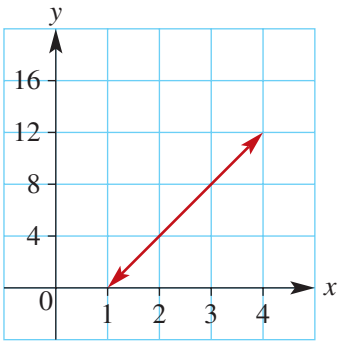
b



c



d



Example 4

2 What is the gradient of the line that joins these points?

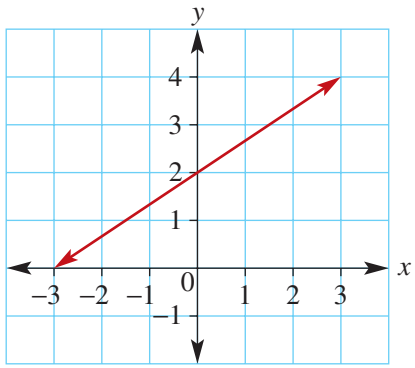
a (0, 1) and (2, 5)

b (1, 3) and (2, – 2)

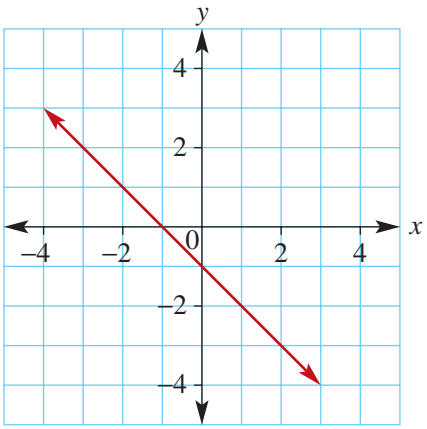
c (2, – 1) and (4, – 2)

3 What is the intercept on the vertical axis for the following lines?

a



b



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LEVEL 2

4 Plot the following points on a number plane and join them to form a straight line. Determine the gradient and y-intercept for each line.

a

x	0	1	2	3	4
y	2	4	6	8	10

b

x	−2	−1	0	1	2
y	3	1	−1	−3	−5

c

x	0	2	4	6	8
y	−1	1	3	5	7

d

x	0	3	6	9	12
y	0	1	2	3	4

Example 5

5 Draw a graph of these linear functions and find the gradient and y-intercept.

a $y = x + 3$ **b** $y = -x + 1$ **c** $y = \frac{1}{2}x + 1$

d $y = -\frac{2}{3}x - 3$ **e** $y + 3 = 4x$ **f** $2x - y = 0$

6 The distance (d) a train travels in kilometres is calculated using the formula $d = 150t$ where (t) is the time taken in hours.

a Construct a table of values using 0, 1, 2, 3 and 4 as values for t. Calculate the distance (d).

b Draw the graph of the distance (d) against the time (t).

c What is the gradient of the graph?

d What is the intercept on the vertical axis?

LEVEL 3

7 Meat is sold for $16 per kilogram.

a Construct a table of values using 0, 1, 2, 3 and 4 as values for the number of kilograms (n). Calculate the cost (c) of the meat.

b Draw the graph of the cost (c) against the number of kilograms (n).

c What is the gradient of the graph?

d What is the intercept on the vertical axis?

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8C Gradient–intercept formula

When the equation of a straight line is written in the form $y = mx + c$ it is called the gradient–intercept formula. The gradient is m, the coefficient of x, and the y-intercept is c, the constant term. The independent variable in the formula is x and the dependent variable in the formula is y.

The gradient–intercept formula is useful in modelling relationships in many practical situations. However, the variables are often changed to reflect the situation. For example, the formula $c = 25n + 100$ has c as the cost of the event ($) and n as the number of guests. These letters are the dependent and independent variables.

GRADIENT–INTERCEPT FORMULA

Linear equation $-y = mx + c$

m – Slope or gradient of the line (vertical rise over the horizontal run)

c – y-intercept where the line cuts the y-axis or vertical axis

Example 6: Finding the gradient and y-intercept from the equation 8C

Write down the gradient and y-intercept for each of the following equations.

a $y = -2 + 5x$ **b** $y = 8 - x$ **c** $y = 6x$ **d** $y - 3x = 4$

SOLUTION:

1 Write the equation and rearrange to gradient–intercept form.
2 Gradient is the coefficient of x.
3 y-intercept is the constant term.

a $y = -2 + 5x$
$y = 5x - 2$
Gradient is 5
y-intercept is -2

4 Write the equation in gradient–intercept form.
5 Gradient is the coefficient of x.
6 y-intercept is the constant term.

b $y = 8 - x$
$y = -1x + 8$
Gradient is -1
y-intercept is 8

7 Write the equation in gradient–intercept form.
8 Gradient is the coefficient of x.
9 y-intercept is the constant term.

c $y = 6x$
$y = 6x + 0$
Gradient is 6
y-intercept is 0

10 Write the equation in gradient–intercept form.
11 Gradient is the coefficient of x.
12 y-intercept is the constant term.

d $y - 3x = 4$
$y = 3x + 4$
Gradient is 3
y-intercept is 4

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Sketching a linear function using the gradient–intercept formula

Sketching a straight-line graph requires at least two points. When an equation is written in gradient–intercept form, one point on the graph is immediately available: the y-intercept. A second point can be quickly calculated by using the gradient or by substituting a suitable value of x into the equation.

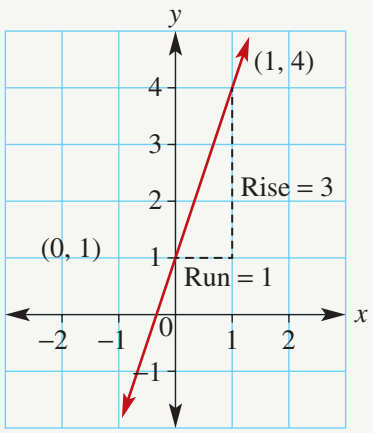
Example 7: Sketching a straight-line graph from its equation 8C

Draw the graph of $y = 3x + 1$.

SOLUTION:

1. Write the equation in gradient–intercept form.
2. Gradient is the coefficient of x or 3.
3. y-intercept is the constant term or 1.
4. Mark the y-intercept on the y-axis at $(0, 1)$.
5. Gradient of 3 (or $\frac{3}{1}$) indicates a vertical rise of 3 and a horizontal run of 1.
6. Start at the y-intercept $(0, 1)$ and draw a horizontal line, 1 unit in length. Then draw a vertical line, 3 units in length.
7. The resulting point $(1, 4)$ is a point on the required line.
8. Join the points $(0, 1)$ and $(1, 4)$ to make the straight line.

$y = 3x + 1$
Gradient is 3
y-intercept is 1



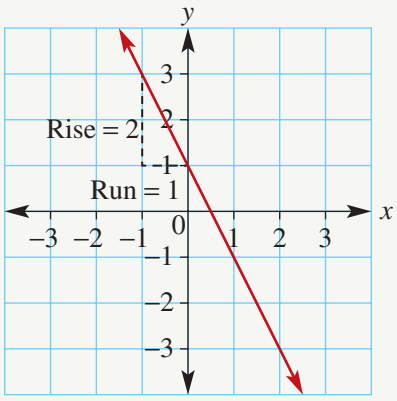
Example 8: Sketching a straight-line graph from its equation 8C

Draw the graph of $y = -2x + 1$.

SOLUTION:

1. Write the equation in gradient–intercept form.
2. Gradient is the coefficient of x or -2.
3. y-intercept is the constant term or 1.
4. Mark the y-intercept on the y-axis at $(0, 1)$.
5. Gradient of -2 (or $-\frac{2}{1}$) indicates a vertical rise of 2 and a horizontal run of 1 to the left.
6. Start at the y-intercept $(0, 1)$ and draw a horizontal line, 1 unit in length. Then draw a vertical line, 2 units in length.
7. The resulting point $(-1, 3)$ is a point on the required line.
8. Join the points $(0, 1)$ and $(-1, 3)$ to make the straight line.

$y = -2x + 1$
Gradient is -2
y-intercept is 1



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Exercise 8C

LEVEL 1

Example 6

1 State the gradient and y-intercept for each of the following equations.

a $y = 4x + 2$

b $y = 3x - 7$

c $y = 5x + 0.4$

d $y = 1.5x - 2$

e $y = \frac{1}{2}x + 3$

f $y = 5 - 3x$

g $y = x$

h $y = 2 + 5x$

2 Write the equation of a line that has the following gradient and y-intercept.

a Gradient = 3 and y-intercept = 2

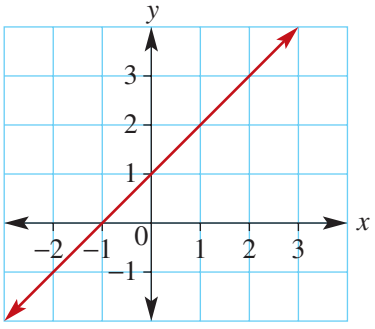
b Gradient = -2 and y-intercept = 10

c Gradient = -4 and y-intercept = -1

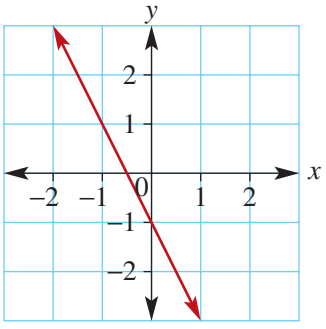
d Gradient = 0.5 and y-intercept = 1

3 Find the equation of the following line graphs.

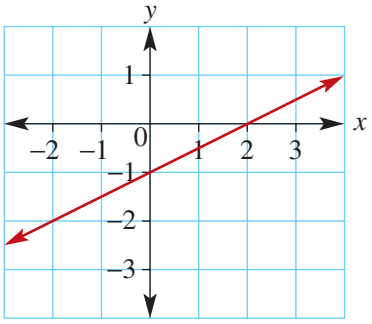
a



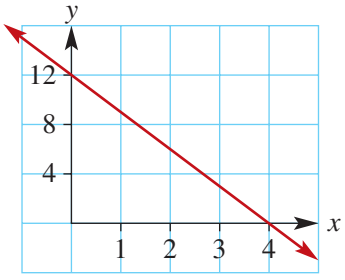
b



c



d



Example 7

4 A straight line has the equation $y = 2x + 3$.

a What are the gradient and the y-intercept?

b Sketch the straight line on a number plane using the gradient and y-intercept.

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LEVEL 2

5 It is known that y varies directly with x. When $x = 4$, $y = 12$.

a Write a linear equation in the form $y = mx$ to describe this situation.

b Draw the graph of y against x.

6 Kalina's pay (p) is directly proportional to the number of hours (h) she works. For an 8-hour day she receives $168.

a Write a linear equation in the form $p = mh$ to describe this situation.

b Draw the graph of p against h.

7 A bike is travelling at constant speed. It travels 350 km in 7 hours.

a Write a linear equation in the form $d = mt$ to describe this situation.

b Draw the graph of d against t.

Example 8

8 Sketch the graphs of the following equations on the same number plane.

a $y = 2x$ **b** $y = 2x + 1$

c $y = 2x + 2$ **d** $y = -x$

e $y = -x - 1$ **f** $y = -x - 2$

g What do you notice about these graphs?

9 Sketch the graphs of the following equations on the same number plane.

a $y = x + 1$ **b** $y = 3x + 1$

c $y = \frac{1}{2}x + 1$ **d** $y = -x - 2$

e $y = -2x - 2$ **f** $y = -3x - 2$

g What do you notice about these graphs?

LEVEL 3

10 Sketch the graphs of the following equations using the gradient–intercept formula.

a $y = \frac{2}{3}x + 2$ **b** $y = 0.25x - 3$ **c** $y = 2 - \frac{1}{3}x$

d $y = -0.5x - 3$ **e** $y + x = 5$ **f** $4x + y = 8$

g $2x + y + 6 = 0$ **h** $x + 4y = 0$ **i** $3x - y = -3$

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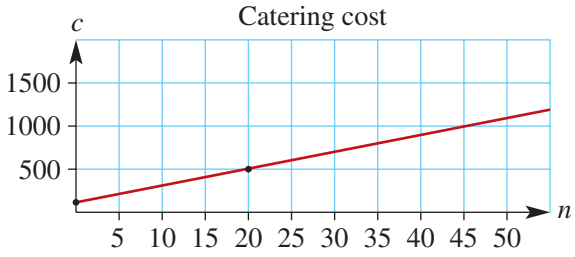
8D Linear models

Linear modelling occurs when a practical situation is described mathematically using a linear function. For example, the gradient–intercept form of a straight-line graph can sometimes be used to model catering costs. A catering company charges a base amount of $100 plus a rate of $25 per guest. Using this information, we can write down a linear equation to model the cost of the event.

Let c be the cost of the event ($) and n be the number of guests. We can then write $c = 25n + 100$.
Note: The number of guests (n) must be greater than zero and a whole number.

The graph of this linear model has been drawn opposite. There are two important features of this linear model:

1. Gradient is the rate per guest or $25.
2. The c-intercept is the base amount or $100.

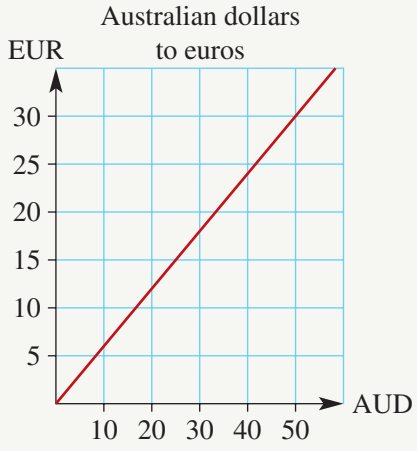


Example 9: Using graphs to make currency conversions 8D

The graph below is used to convert Australian dollars to euros. Use the graph to convert:

a 50 Australian dollars to euros.

b 15 euros to Australian dollars.



SOLUTION:

1 Read from the graph (when AUD = 50, EUR = 30).

2 Read from the graph (when EUR = 15, AUD = 25).

a 30 euros

b 25 Australian dollars

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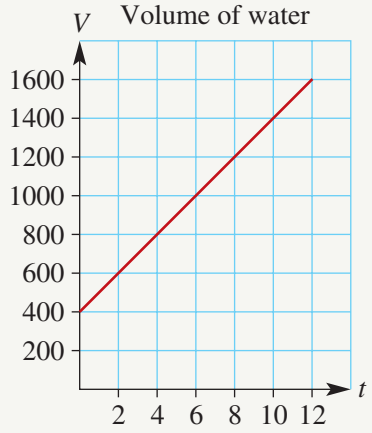
Example 10: Interpreting linear models 8D

Water is pumped into a partially full tank. The graph gives the volume of water V (in litres) after t minutes.



a How much water is in the tank at the start?
b How much water is in the tank after 10 minutes?
c The tank holds 1600 L. How long does it take to fill?
d Find the equation of the straight line in terms of V and t.
e Use the equation to calculate the volume of water in the tank after 7 minutes.
f How many litres are pumped into the tank each minute?

SOLUTION:

1 Read from the graph (when $t = 0$, $V = 300$).
2 Read from the graph (when $t = 10$, $V = 1400$).
3 Read from the graph (when $V = 1600$, $t = 12$).
4 Find the gradient by choosing two suitable points such as (0,400) and (12,1600).
5 Calculate the gradient (m) between these points using the gradient formula.
6 Determine the vertical intercept (400).
7 Substitute the gradient and y-intercept into the gradient–intercept form $y = mx + c$.
8 Use the appropriate variables (V for y, t for x).
9 Substitute $t = 7$ into the equation.
10 Evaluate.
11 Check the answer using the graph.
12 The rate at which water is pumped into the tank is the gradient of the graph ($m = 100$).

a 400 L
b 1400 L
c 12 minutes
d $m = \dfrac{\text{Rise}}{\text{Run}}$
$= \dfrac{1600 - 400}{12 - 0}$
$= 100$
$b = 400$
$y = mx + c$
$V = 100t + 400$
e $V = 100t + 400$
$= 100 \times 7 + 400$
$= 1100$ L
f 100 L/min

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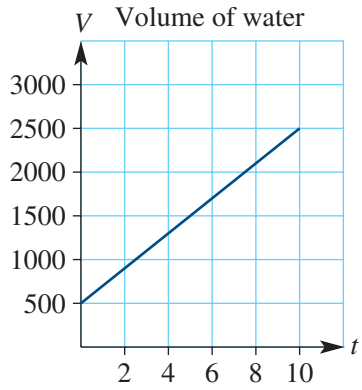
Exercise 8D

LEVEL 1

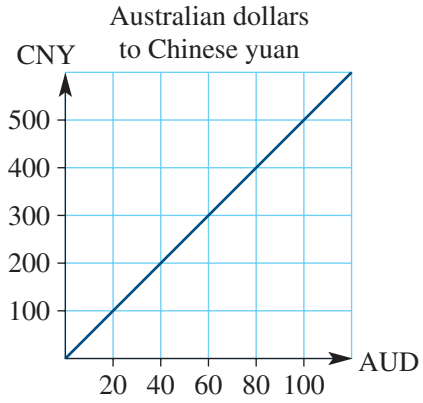
Example 10 **1** Water is pumped into a partially full tank. The graph gives the volume of water V (in litres) after t minutes.

a How much water is in the tank at the start?
b How much water is in the tank after 5 minutes?
c How much water is in the tank after 8 minutes?
d The tank holds 2500 L. How long does it take to fill?
e Use the graph to calculate the volume of water in the tank after 7 minutes.



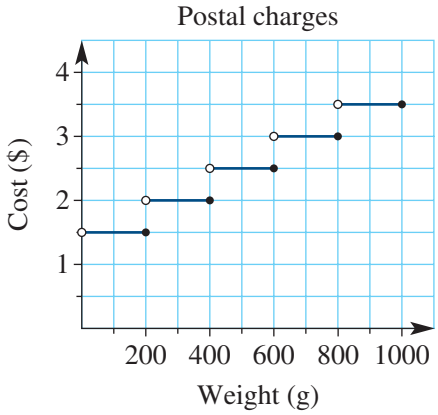
Example 9 **2** The conversion graph opposite is used to convert Australian dollars to Chinese yuan. Use the graph to convert:

a 80 Australian dollars to yuan
b 50 Australian dollars to yuan
c 100 yuan to Australian dollars
d 350 yuan to Australian dollars
e What is the gradient of the conversion graph?



3 Mailing costs are charged according to the weight of the parcel. Use the step graph to determine the postal charges for the following parcels.

a 50 g
b 900 g
c 200 g
d 800 g



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LEVEL 2

4 A new piece of equipment is purchased by a business for $120 000. Its value is depreciated each month using the graph opposite.

a What was the value of the equipment after 32 months?

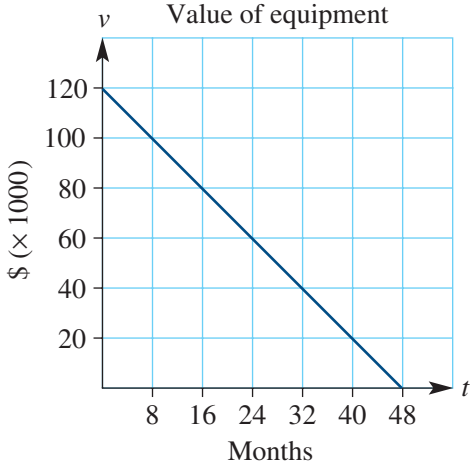
b What was the value of the equipment after one year?

c When does the line predict the equipment will have no value?

d Find the equation of the straight line in terms of v and t.

e Use the equation to calculate the value of the equipment after 2 months.

f By how much does the equipment depreciate in value each month?



5 The amount of money transacted through ATMs has increased with the number of ATMs available. The graph shows this increase.

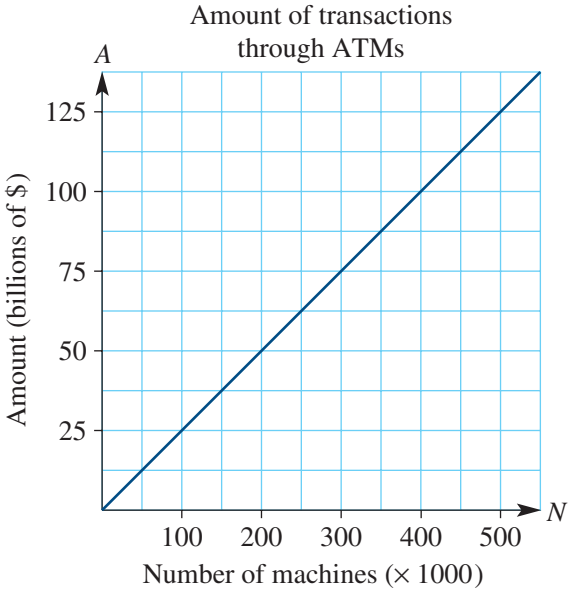
a What was the amount of money transacted through ATMs when there were 500 000 machines?

b How many ATM machines resulted in transactions of $75 billion?

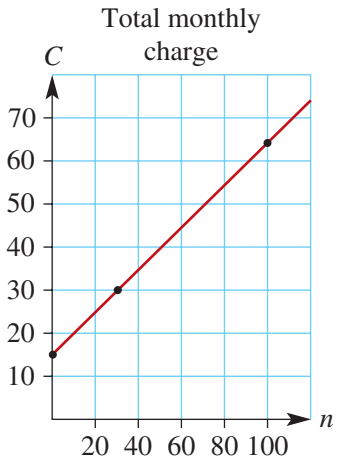
c Find the equation of the line in terms of amount of money transacted, A, and the number of ATMs, N.

d Use the equation to calculate the amount of money transacted when there were 350 000 machines.

e Use the equation to predict how much money will be transacted through ATM machines when there are 1 000 000 machines.



6 A phone company charges a monthly service fee, plus the cost of calls. The graph gives the total monthly charge, C dollars, for making n calls. This includes the service fee.



- **a** How much is the monthly service fee?
- **b** How much does the company charge if you make 20 calls a month?
- **c** How many calls are made if the total monthly charge is $30?
- **d** Find the equation of the line in terms of total monthly charge (C) and the number of calls (n).

LEVEL 3

7 A company charges the following parking fees:
$10 for up to 3 hours, $15 for 3–6 hours and $20 over 6 hours.

- **a** Draw a step graph to illustrate the parking fees, with the Time (h) on the horizontal axis and Cost ($) on the vertical axis.
- **b** What is the cost to park for 4 hours? Use the step graph.
- **c** Liam arrived in the parking area at 10 : 30 a.m. and left at 1 : 00 p.m. How much did he pay for parking?
- **d** Ruby arrived in the parking area at 5 : 15 p.m. and left at 11 : 15 p.m. How much did she pay for parking?

8 Tomas converted 100 Australian dollars to 40 British pounds.

- **a** Draw a conversion graph with Australian dollars on the horizontal axis and British pounds on the vertical axis.
- **b** How many British pounds is 40 Australian dollars? Use the conversion graph.
- **c** How many Australian dollars is 10 British pounds? Use the conversion graph.
- **d** Find the gradient and vertical intercept for the conversion graph.
- **e** Write an equation that relates Australian dollars (AUD) to British pounds (GBP).

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8E Direct variation

Direct variation (or direct proportion) occurs when one variable depends directly on another variable. One quantity increases or decreases at the same rate as another quantity increases or decreases. For example, a person's wage depends directly on the number of hours worked. The more hours worked the greater the wage.

Direct variation problems involve a constant of variation (k). It is the rate at which the quantities vary. For example, the hourly rate of pay is the constant of variation when determining a person's wage. To solve a direct variation problem, write an equation relating the two variables and use the information given to calculate the value of k.

SOLVING A DIRECT VARIATION PROBLEM

1. Write an equation relating the two variables. (k is the constant of variation.)
   If y is directly proportional to x the equation is $y = kx$.
2. Solve the equation for k by substituting values for x and y.
3. Write the equation with the solution for k (step 2) and solve the problem by substituting a value for either x or y.

Example 11: Solving a direct variation problem 8E

The cost of a photocopier (C, in dollars) varies directly with its speed (s, in pages per minute). A photocopier with a speed of 50 pages per minute costs $700.

a What is the constant of variation (k)?
b Write an equation connecting C and s.
c How much does a photocopier with a speed of 60 pages per minute cost?

SOLUTION:

1 Cost is directly proportional to the speed.	**a** $C = ks$
2 Use the formula $y = kx$ by replacing the y with C and the x with s.	
3 Substitute 700 for C and 50 for s into the formula.	$700 = k \times 50$ $k = \frac{700}{50}$
4 Evaluate.	$= 14$
5 Replace the k with 14 in the formula.	**b** $C = 14s$
6 Write the formula using the value for the proportionality constant ($k = 14$).	**c** $C = 14s$
7 Substitute 60 for s into the formula.	$= 14 \times 60$
8 Evaluate.	$= \$840$
9 Write the answer in words.	Photocopier costs $840.

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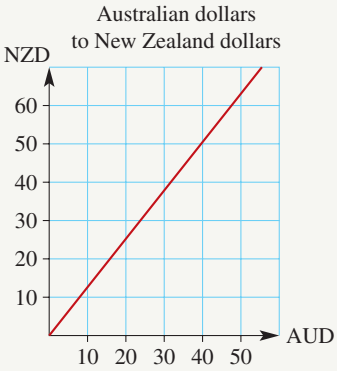
Example 12: Using linear models 8E

The graph below is used to convert Australian dollars to New Zealand dollars. Use the graph to convert:

a 40 Australian dollars to New Zealand dollars.

b 25 NZD to AUD.



SOLUTION:

1 Read from the graph (when AUD = 40, NZD = 50).

2 Read from the graph (when NZD = 25, AUD = 20).

a 50 NZD

b 20 AUD

Example 13: Graphing a linear function from a table of values 8E

The table below shows the cost of postage (c) as a function of the weight of the parcel (w).

Weight (w) kg	1	2	3	4	5
Cost (c) $	1.2	2.4	3.6	4.8	6.0

a Draw a graph of cost (c) against the weight of the parcel (w).

b Use the graph to determine the cost of a parcel if the weight is 2.5 kg

SOLUTION:

1 Draw a number plane with the weight of parcel (w) as the horizontal axis and the cost of postage (c) as the vertical axis.

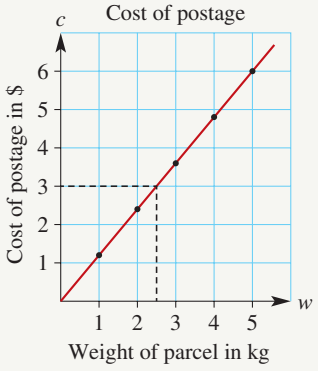
2 Plot the points (1, 1.2) (2, 2.4) (3, 3.6) (4, 4.8) and (5, 6.0).

3 Join the points to make a straight line.

4 Find 2.5 kg on the horizontal axis and draw a vertical line. Where this line intersects the graph draw a horizontal line to the vertical axis.

5 Write the answer in words.

a



b 2.5 kg would cost $3.

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Exercise 8E

LEVEL 1

Example 11

1 y varies directly with x and $y = 24$ when $x = 8$.

a Write an equation connecting y and x, using k as the constant of variation.

b Calculate the constant of variation.

c What is y when x is 4?

d What is x when y is 15?

2 y varies directly with x and $y = 20$ when $x = 10$.

a Write an equation connecting y and x, using k as the constant of variation.

b Calculate the constant of variation.

c What is y when x is 8?

d What is x when y is 12?

3 It is known that y varies directly with x. When $x = 12$ then $y = 3$.

a Write an equation connecting y and x, using k as the constant of variation.

b Calculate the constant of variation.

c What is y when x is 24?

d What is x when y is 4?

4 It is known that a varies directly with b. If $a = 84$ then $b = 56$.

a Write a linear equation connecting a and b, using k as the constant of variation.

b Calculate the constant of variation.

c Find the value of a when the value of b is 22.

d Find the value of b when the value of a is 36.

5 Oscar's pay (p) is directly proportional to the number of hours (h) he works. For a 9-hour day he receives \$193.50.

a Write a linear equation to describe this situation.

b Calculate the constant of variation.

c What is Oscar's pay if he works for 11 hours?

d What is Oscar's pay if he works for 6.5 hours?

e How many hours does Oscar have to work to earn \$365.50?

f How many hours does Oscar have to work to earn \$752.50?

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LEVEL 2

Example 13

6 The cost (c) of apples is \$2.50 per kilogram and is determined by the formula $c = 2.5w$ where (w) is the weight of the apples.

- **a** Construct a table of values for the weight against cost. Use 0, 1, 2, 3 and 4 for w.
- **b** Draw the graph of the cost (c) against the weight (w).
- **c** How many kilograms of apples can be purchased for \$15?

7 The relationship for the age of a computer (t in years) to its current value (v in \$100) is $v = -5t + 30$.

- **a** Construct a table of values for the age against current value ($t = 0, 1, 2, 3, 4$).
- **b** Draw the graph of the age (t) against current value (v).
- **c** What is the initial cost of the computer?
- **d** What will be the value of the computer after two years?
- **e** When will the computer be half its initial cost?

LEVEL 3

8 The cost of hiring a taxi is \$3 flag fall and \$2 per kilometre travelled.

- **a** Construct a table of values for kilometres travelled (d) and cost of a taxi (C). Use 0, 10, 20, 30 and 40 as values for d.
- **b** Draw the graph of kilometres travelled (d) against cost of taxi (C).

9 Emily was born on Jack's tenth birthday.

- **a** Construct a table of values using of 0, 1, 2, 3 and 4 as values for Emily's age (E) and calculating Jack's age (J).
- **b** Draw the graph of Emily's age (E) against Jack's age (J).

Example 12

10 One Australian dollar (AUD) was converted for 1.20 New Zealand dollars (NZD).

- **a** Construct a table of values using of 0, 10, 20, 30 and 40 as values for AUD and calculate the NZD using the above conversion.
- **b** Draw the graph of the AUD against NZD.

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PPT STUDY GUIDE

Key ideas and chapter summary

Graphing linear functions

1. Construct a table of values with the independent variable as the first row and the dependent variable as the second row.
2. Draw a number plane with the independent variable on the horizontal axis and the dependent variable on the vertical axis. Plot the points.
3. Join the points to make a straight line.

Gradient and intercept

Gradient of a line is the slope of the line.

$$\text{Gradient (or } m) = \frac{\text{Vertical rise}}{\text{Horizontal run}}$$

The intercept of a line is where the line cuts the axis.

Gradient–intercept formula

Linear equation in the form $y = mx + c$.

m – Slope or gradient of the line.

c – y-intercept.

Sketching a straight-line graph requires at least two points. When an equation is written in gradient–intercept form, one point on the graph is immediately available: the y-intercept. A second point can be calculated using the gradient.

Linear models

Linear modelling occurs when a practical situation is described mathematically using a linear function.

Direct variation

1. Write an equation relating the two variables, using k is the constant of variation.

   When y is directly proportional to x the equation is $y = kx$.
2. Solve the equation for k by substituting values for x and y.
3. Write the equation with the solution for k (step 2) and solve the problem by substituting a value for either x or y.

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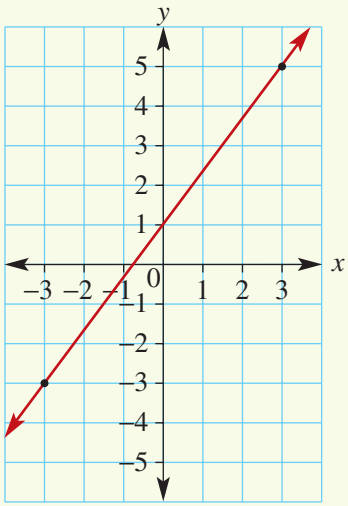
Review

Multiple-choice questions

1 What is the gradient of this line?



A $\frac{2}{3}$ **B** $\frac{3}{4}$

C $\frac{4}{3}$ **D** $\frac{8}{5}$

2 What is the y-intercept of this line?

A -2 **B** -1

C 1 **D** 2

3 A straight line has the equation of $y = -3x + 1$.
What is the y-intercept?

A -3 **B** -1

C $+1$ **D** 3

4 What is the constant of variation if a varies directly with b and when $a = 32$ then $b = 16$?

A -2 **B** -0.5

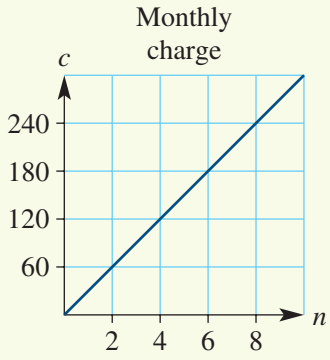
C 0.5 **D** 2

5 The relationship of the age of a fax machine (t in years) to its current value (v) is $v = -40t + 150$. What is the value of the fax machine after two years?

A 70 **B** 108

C 150 **D** 230

6 What is the equation of the line shown in the graph?



A $c = n$ **B** $c = n + 30$

C $c = 30n$ **D** $c = 8n + 240$

7 Using the graph in the previous question, what is the charge for 12 months?

A 24 **B** 36

C 240 **D** 360

8 The cost (C) of a mobile phone plan for a certain number of months (t) is given by $C = 50t + 10$. What is the cost of the phone plan for 10 months?

A \$50 **B** \$60

C \$510 **D** \$600

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Short-answer questions

1 Complete the following table of values for each linear function.

a $y = x + 3$

x	-2	-1	0	1	2
y					

b $y = -2x - 1$

x	-2	-1	0	1	2
y					

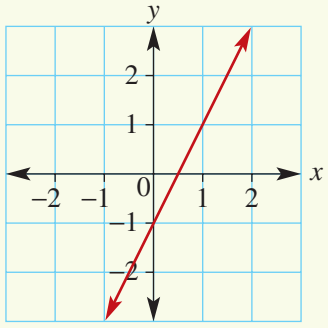
2 Draw the graphs for the two linear functions in question 1.

3 What is the gradient of the line that joins these points?

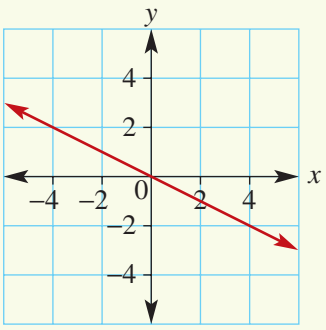
a $(1, 5)$ and $(3, 7)$ **b** $(-2, 1)$ and $(0, 4)$ **c** $(-3, -1)$ and $(2, -11)$

4 Find the equation of the following line graphs.

a



b



5 Draw a graph of these linear functions and find the gradient and y-intercept.

a $y = x + 1$ **b** $y = -2x + 5$ **c** $y = 3x - 2$

6 The circumference of a circle (C) varies directly the diameter of the circle (d) using the formula $C = \pi d$.

a What is the constant of variation?

b What is the diameter (to nearest centimetre) if the circumference of a circle is 6 cm?

7 An internet access plan charges an excess fee of \$8 per GB.

Data (d) GB	1	2	3	4	5	6
Cost (c) \$	8	16	24	32	40	48

a Which is the dependent variable?

b Which is the independent variable

c Draw a graph of data against cost.

d Use the graph to find d if c is 20.

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Review

8 One Australian dollar (AUD) was converted to 0.90 Japanese yen (JPY).

a Construct a table of values using of 0, 10, 20, 30 and 40 as values for AUD and calculate the JPY using the above conversion.

b Draw the graph of the AUD against JPY.

Extended-response questions

9 A motor vehicle is purchased by a business for $30 000. Its value is then depreciated each month using the graph shown.

a What was the value of the motor vehicle after 24 months?

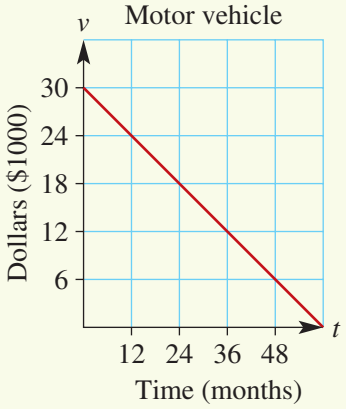
b What was the value of the motor vehicle after one year?

c Find the equation of the straight-line in terms of v and t.

d Use the equation to determine the value of the motor vehicle after 6 months.

e When does the line indicate that the motor vehicle will have no value?

f By how much does the motor vehicle depreciate in value each month?



10 The table below shows the speed v (in km/s) of a rocket at time t seconds.

Time (t)	1	2	3	4	5
Speed (v)	2	3	4.5	6	7.5

a Find a line of best fit for the speed (v), against the time (t).

b Use the line to determine the rocket's speed when $t = 6$ seconds.

c Extend the straight line to predict the time when the rocket's speed is 12 km/s.

PDF LITERACY

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9 Interest and depreciation

Syllabus topic — F1.1 Interest and depreciation

This topic will develop your skills in calculating and graphing simple interest. It presents a variety of applications involving simple interest, including straight-line depreciation.

Outcomes

- Calculate simple interest for different rates and periods.
- Apply percentage increase or decrease in various contexts.
- Calculate GST in absolute and percentage terms.
- Compare simple interest graphs for different rates and periods.
- Calculate the depreciation of an asset using straight-line method.
- Calculate and graph compound interest as a repeated application of simple interest.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Widgets
- Literacy worksheet
- Spreadsheets
- Quick Quiz
- Study guide
- Solutions (enabled by teacher)

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

In the Interactive Textbook you can take a test of prior knowledge required for this chapter, and depending on your score you may be directed to revision from the previous years' work.

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9A Simple interest

Interest is paid for borrowing money. There are different ways of calculating interest. Flat-rate loans use simple interest. Simple interest (or flat interest) is a fixed percentage of the amount borrowed and is calculated on the original amount. For example, if you borrow \$10 000 from a bank at a simple interest rate of 6% per annum (per year) you would be required to pay \$600 each year. That is,

$$\text{Interest} = \$10\,000 \times 0.06 \left(\text{or} \frac{6}{100}\right) = \$600$$

Flat-rate loans are calculated on the initial amount borrowed or the principal. The amount owed on the loan is calculated by adding the interest to the principal.

SIMPLE INTEREST

$I = Prn$ $A = P + I$

I – Interest (simple or flat) to be paid for borrowing the money

P – Principal is the initial amount of money borrowed

r – Rate of simple interest per period, expressed as a decimal

n – Number of time periods

A – Amount owed or total to be paid

Example 1: Calculating simple interest 9A

Abbey applied for a flat-rate loan of \$40 000 at 9% per annum simple interest. She plans to repay the loan after two years and six months.

a How much interest will be paid?

b What is the total owing at the end of two years and six months?

SOLUTION:

1 Write the simple interest formula.

2 Substitute $P = 40\,000, r = 0.09$ and $n = 2.5$ into the formula.

3 Evaluate.

4 Write the answer in words.

5 Write the amount owed formula.

6 Substitute $P = 40\,000$ and $I = 9000$ into the formula.

7 Evaluate.

8 Write the answer in words.

a $I = Prn$

$= 40\,000 \times 0.09 \times 2.5$

$= \$9000$

Simple interest owed is \$9000.

b $A = P + I$

$= 40\,000 + 9000$

$= 49\,000$

Amount owed is \$49 000.

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Example 2: Calculating the simple interest paid to a bank account 9A

The table opposite shows the entries in Shane's bank account. If the bank pays interest at a rate of 3% per annum on the minimum monthly balance, find the interest payable for the month of May correct to the nearest cent.

Date	Transaction	Debit	Credit	Total
30 Mar	Pay		500.00	500.00
4 May	Cash	60.00		440.00
16 May	Cash		100.00	540.00
1 Jun				540.00

SOLUTION:

1. Determine the minimum monthly balance for May.
2. Write the simple interest formula.
3. Substitute $P = \$440.00$, $r = 0.03$, and $n = \frac{1}{12}(1\text{month})$ into the formula.
4. Evaluate.
5. Write answer in words.

The minimum balance in the account for May was \$440.00

$I = Prn$

$= \$440 \times 0.03 \times \frac{1}{12}$

$= \$1.10$

Interest payable is \$1.10

Example 3: Finding the principal for a simple interest loan 9A

Noah applied for a simple interest car loan with an interest rate of 9% p.a. He was told the total simple interest would be \$6300 for $3\frac{1}{2}$ years. What was the principal?

SOLUTION:

1. Write the simple interest formula.
2. Substitute $I = 6300$, $r = 0.09$ and $n = 3.5$ into the formula.
3. Make P the subject of the formula by dividing both sides by (0.09×3.5).
4. Evaluate.
5. Write the answer in words.

$I = Prn$

$6300 = P \times 0.09 \times 3.5$

$P = \frac{6300}{(0.09 \times 3.5)}$

$= \$20\,000$

Principal is \$20 000.

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Loan repayments

A loan repayment is the amount of money to be paid at regular intervals over the time period. The interval is often fortnightly or monthly.

LOAN REPAYMENTS

Loan repayment = Total to be paid ÷ Number of repayments

Example 4: Calculating a loan repayment on a simple interest loan 9A

Jessica wishes to buy a lounge suite priced at $2750. She chooses to buy it on terms by paying a 10% deposit and borrowing the balance. Interest is charged at 11.5% p.a. on the amount borrowed. Jessica makes fortnightly repayments over 3 years. Calculate her fortnightly repayments.

SOLUTION:

1 Calculate the deposit by multiplying 10% or 0.10 by $2750.
2 Calculate the balance by subtracting the deposit ($275) from the cost price ($2750).
3 Write the simple interest formula.
4 Substitute $P = 2475, r = 0.115$ and $n = 3$ into the formula.
5 Evaluate.
6 Write the loan repayment formula.
7 Calculate the total to be paid by adding the balance ($2475) and the interest ($853.875).
8 Calculate the number of repayments by multiplying the fortnights in a year (26) by the number of years (3).
9 Evaluate correct to two decimal places.
10 Write the answer in words.

$$\begin{aligned}
\text{Deposit} &= 10\% \text{ of } \$2750 \\
&= 0.10 \times 2750 \\
&= \$275 \\
\text{Balance} &= 2750 - 275 \\
&= \$2475 \\
I &= Prn \\
&= 2475 \times 0.115 \times 3 \\
&= \$853.875 \\
\text{Repayment} &= \frac{\text{Total to be paid}}{\text{Number of repayments}} \\
&= \frac{\text{Principal + Interest}}{\text{Number of repayments}} \\
&= \frac{(2475 + 853.875)}{(3 \times 26)} \\
&= 42.67788462 \\
&\approx 42.68
\end{aligned}$$

Fortnightly repayments are $42.68.

Exercise 9A

LEVEL 1

Example 1a **1** Calculate the amount of simple interest for each of the following loans.

a Principal = \$25 000, interest rate = 11% p.a., time period = 4 years

b Principal = \$400 000, interest rate = $8\frac{1}{4}$% p.a., time period = 5 years

c Principal = \$560 000, interest rate = 6.75% p.a., time period = 15 years

d Principal = \$7400, interest rate = 7% p.a., time period = 18 months

e Principal = \$80 000, interest rate = 9.25% p.a., time period = 30 months

Example 1b **2** Calculate the amount owed for each of the following loans.

a Principal = \$800, simple interest rate = 6% p.a., time period = 3 years

b Principal = \$5200, simple interest rate = 16% p.a., time period = $7\frac{1}{2}$ years

c Principal = \$12 500, simple interest rate = 11.4% p.a., time period = 4.5 years

d Principal = \$6000, simple interest rate = $4\frac{1}{2}$% p.a., time period = 6 months

e Principal = \$40 000, simple interest rate = 7.75% p.a., time period = 42 months

3 A sum of \$170 000 was borrowed for 3 years.

a Find the simple interest owed if the rate of interest is 6.5% per annum.

b What is the amount owed at the end of 3 years?

4 Hayley intends to borrow \$2700 to build a driveway for her new house. She is offered a flat-rate loan with a simple interest rate of 14.5% per annum. How much interest will be owed after 3 months? Answer correct to the nearest cent.

Example 2 **5** The table shows the entries in Julia's bank account. If the bank pays interest at a rate of 4% per annum on the minimum monthly balance, find the interest payable for the month of March correct to the nearest cent.

Date	Transaction	Debit	Credit	Total
28 Feb	Pay		500.00	800.00
3 Mar	Cash		150.00	650.00
18 Mar	Cash		150.00	500.00
1 Apr				500.00

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LEVEL 2

6 Create the spreadsheet below.

XLS 9AQ6

IF =A5*B5*C5

	A	B	C	D	E	F
1	**Cambridge Mathematics Standard Year 11 Exercise 9A Question 6**					
2	Worksheet to calculate the amount owed on a flat-rate loan					
3						
4	*Principal (P)*	*Interest rate (r)*	*Time (n)*	*Interest (I)*	*Amount (A)*	
5	$100,000	8%	20	=A5*B5*C5	$260,000	
6	$200,000	8%	20	$320,000	$520,000	
7	$300,000	8%	20	$480,000	$780,000	
8	$400,000	8%	20	$640,000	$1,040,000	
9						

a Cell D5 has a formula that calculates the simple interest. Enter this formula.
b Fill down the contents of D6 to D8 using the formula for D5.
c Cell E5 has a formula that calculates the amount owed. Enter this formula.
d Fill down the contents of E6 to E8 using the formula for E5.
e Change the interest rate from 8% to 10%.
f Change the time period from 20 years to 15 years.

7 Kim buys a television for $1800. He pays it off monthly over 2 years at a flat interest rate of 12.5% per annum.
a How many months will it take Kim to pay for the television?
b What is the interest charged for the 2 years?
c How much per month will he pay? Give your answer to the nearest cent.

8 Mitchell approached a bank for a business loan of $22 000. The interest rate is 10.5% p.a. flat. He decides to repay the loan over a period of 4 years.
a What is the principal?
b What is the rate of interest?
c What will be the amount of interest charged over that period?
d What will be the monthly repayment? Give the answer correct to the nearest cent.

Example 4 **9** Jordan decides to buy a car for $23 000.
He has saved $9000 for the deposit and takes out a simple interest loan over 2 years for the balance. The interest charged is 13% per annum.
a What is the balance?
b What is the total amount of interest to be paid?
c What will be his monthly repayment? Answer correct to the nearest cent.

LEVEL 3

10 Mia borrowed $400 000 at a flat rate of interest of 8.5% per annum. This rate was fixed for 2 years on the principal. She pays back the interest only over this period.

a How much interest is to be paid over the 2 years?

b After paying the fixed rate of interest for the first year, Mia finds the bank will decrease the flat interest rate to 7.5% if she pays a charge of $2000. How much will she save by changing to the lower interest rate for the last year?

11 Cooper plans to borrow money to purchase a car and considers the fortnightly repayment guide shown in the table. He decides to borrow $19 000 and pay back the loan in fortnightly instalments over 2 years. What is the flat rate of interest per annum on this loan, correct to two decimal places?

Amount borrowed	Fortnightly repayments		
	1 year	2 years	3 years
$18 000	$755	$427	$305
$18 500	$783	$429	$307
$19 000	$804	$431	$309

12 A truck is advertised at $36 000. It can be bought on terms for a 20% deposit and repayments of $276 per week for 3 years. Assume there are 52 weeks in the year.

a What is the deposit?

b Calculate the total cost of the truck if bought on these terms.

c What is the total interest paid?

d What is the flat interest rate for the loan, correct to one decimal place?

13 A painting was purchased on terms with a 20% deposit and the balance to be paid at $370 per month for 2 years. Determine the flat rate of interest charged on the painting given that it has a cash price of $7500.

14 Grace takes a loan of $30 000 over 60 months for a swimming pool. The repayment rate is $677.50 per month.

a How much will Grace pay back altogether?

b What is the flat interest rate per annum for the loan, correct to one decimal place?

c Grace would like to increase the loan to $40 000 to landscape the pool. What would be her monthly repayment, assuming the same time period and flat interest rate? Answer correct to the nearest cent.

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9B Simple interest graphs

When graphing simple interest, make the horizontal axis the time period and the vertical axis the interest earned. Simple interest will increase by a constant amount each time period. This will result in a straight-line graph.

SIMPLE INTEREST GRAPHS

1. Construct a table of values for I and n using the simple interest formula.
2. Draw a number plane with n the horizontal axis and I the vertical axis. Plot the points.
3. Join the points to make a straight line.

Example 5: Constructing a simple interest graph 9B

Draw a graph showing the amount of simple interest earned over a period of 4 years if \$1000 is invested at 6% p.a. Use the graph to estimate the interest earned after 8 years.

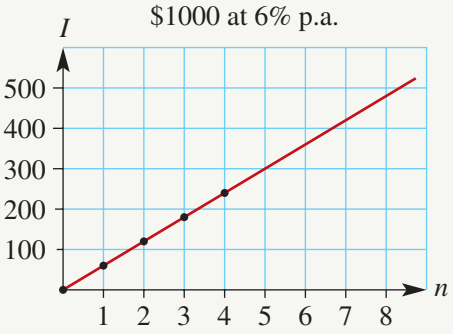
SOLUTION:

1. Write the simple interest formula.
2. Substitute $P = 1000$, $r = 0.06$ and n into the formula.
3. Draw a table of values for I and n.
4. Let $n = 0, 1, 2, 3$ and 4. Find the interest (I) using $I = 60n$.
5. Draw a number plane with n as the horizontal axis and I as the vertical axis.
6. Plot the points $(0, 0), (1, 60), (2, 120), (3, 180)$ and $(4, 240)$.
7. Extend the line to estimate the value of I when $n = 8$.
8. Read the graph to estimate I. ($I = 480$ when $n = 8$).
9. Write the answer in words.

$$I = Prn$$
$$= 1000 \times 0.06 \times n$$
$$= 60n$$

n	0	1	2	3	4
I	0	60	120	180	240

Simple interest on \$1000 at 6% p.a.



Interest after 8 years is approximately \$480.

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Exercise 9B

LEVEL 1

Example 5

1 Luke invested $1000 at 2% per annum simple interest for 3 years.

a Simplify the simple interest formula ($I = Prn$) by substituting values for the principal and the interest rate.

b Use this formula to complete the following table of values.

n	0	1	2	3	4
I					

c Draw a number plane with n as the horizontal axis and I as the vertical axis.

d Plot the points from the table of values. Join the points to make a straight line.

2 Nicholas invested $1000 at 7% per annum simple interest for 4 years.

a Simplify the simple interest formula ($I = Prn$) by substituting values for the principal and the interest rate.

b Use this formula to complete the following table of values.

n	0	1	2	3	4
I					

c Draw a number plane with n as the horizontal axis and I as the vertical axis.

d Plot the points from the table of values. Join the points to make a straight line.

e Use the graph to find the interest after $2\frac{1}{2}$ years.

f Extend the graph to find the interest after 6 years.

g Find the time when the interest is $210.

3 Melissa invested $600 at 5% per annum simple interest for 5 years.

a Simplify the simple interest formula ($I = Prn$) by substituting values for the principal and the interest rate.

b Use this formula to complete the following table of values.

n	0	1	2	3	4	5
I						

c Draw a number plane with n as the horizontal axis and I as the vertical axis.

d Plot the points from the table of values. Join the points to make a straight line.

e Use the graph to find the interest after $3\frac{1}{2}$ years.

f Extend the graph to find the interest after 6 years.

g Find the time when the interest is $360.

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LEVEL 2

4 Draw a graph showing the amount of simple interest earned over a period of 4 years if $1000 is invested at 4% p.a. Use the graph to estimate the interest earned after 6 years.

5 Chloe is comparing three different interest rates for a possible investment.

a Draw a graph to represent the interest earned over 5 years on:

i $5000 invested at 5% per annum simple interest

ii $5000 invested at 7% per annum simple interest

iii $5000 invested at 9% per annum simple interest.

b How much does each investment earn after $2\frac{1}{2}$ years?

c How much does each investment earn after 5 years?

d Find the time for each investment to earn $1000 in interest.

6 Mick is comparing three different interest rates for a possible investment.

a Draw a graph to represent the interest earned for 6 months on:

i $100 000 invested at 6% p.a. simple interest

ii $100 000 invested at 9% p.a. simple interest

iii $100 000 invested at 12% p.a. simple interest.

b How much does each investment earn after 1 month?

c How much does each investment earn after 6 months?

d Find the time for each investment to earn $2000 in interest.

LEVEL 3

7 The table below gives details for a fixed-term deposit.

Time period	Simple interest rate per annum
Less than 3 months	6.5%
3 to 6 months	7.0%
6 to 12 months	7.5%
12 to 24 months	8.1%
24 to 48 months	8.3%

Chris has $50 000 to invest in a fixed-term deposit. Draw a separate graph to represent the interest earned after 12 months given these investments.

a Fixed-term deposit for 3 months. Assume Chris reinvests his $50 000 at the end of every 3 months.

b Fixed-term deposit for 6 months. Assume Chris reinvests his $50 000 at the end of the first 6 months.

c Fixed-term deposit for 12 months

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9C Percentage increase or decrease

Percentage change involves increasing or decreasing a quantity as a percentage of the original amount of the quantity.

PERCENTAGE INCREASE	PERCENTAGE DECREASE
1 Add the % increase to 100%.	1 Subtract the % decrease from 100%.
2 Multiply the above percentage by the amount.	2 Multiply the above percentage by the amount.

Example 6: Calculating the percentage change 9C

The retail price of a toaster is $36 and is to be increased by 5%. What is the new price?

SOLUTION:

1	Add the 5% increase to 100%.	$100\% + 5\% = 105\%$
2	Write the quantity (new price) to be found.	New price = 105% of $36
3	Multiply the new percentage (105%) by the amount.	$= 1.05 \times 36$
4	Evaluate and write using correct units.	= $37.80
5	Write the answer in words.	New price is $37.80.

Example 7: Calculating consecutive percentage changes 9C

Increase $75 by 20% and then decrease the result by 20%.

SOLUTION:

1	Add the 20% increase to 100%.	$100\% + 20\% = 120\%$
2	Write the quantity (new price) to be found.	New price = 120% of $75
3	Multiply the new percentage (120%) by the amount.	$= 1.20 \times 75$
4	Evaluate and write using correct units.	= $90
5	Subtract the 20% decrease from 100%.	$100\% - 20\% = 80\%$
6	Write the quantity (new price) to be found.	New price = 80% of $90
7	Multiply the new percentage (80%) by the amount.	$= 0.80 \times 90$
8	Evaluate and write using correct units.	= $72
9	Write the answer in words.	New price is $72.

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Exercise 9C

LEVEL 1

Example 6

1 What is the amount of the increase for each of the following?

a Increase of 10% on $48
b Increase of 30% on $120
c Increase of 15% on $66
d Increase of 25% on $88
e Increase of 40% on $1340
f Increase of 36% on $196
g Increase of 4.5% on $150
h Increase of $\frac{1}{2}$% on $24

2 What is the amount of the decrease for each of the following?

a Decrease of 20% on $110
b Decrease of 60% on $260
c Decrease of 35% on $320
d Decrease of 75% on $1096
e Decrease of 6% on $50
f Decrease of 32% on $36
g Decrease of 12.5% on $640
h Decrease of $1\frac{1}{4}$% on $56

3 The David Jones clearance sale has a discount of 30% off the retail price of all clothing. Find the amount saved on the following items.

a Men's shirt with a retail price of $80
b Pair of jeans with a retail price of $66
c Ladies jacket with a retail price of $450
d Boy's shorts with a retail price of $22
e Jumper with a retail price of $124
f Girl's skirt with a retail price of $50

4 A manager has decided to award a salary increase of 6% to all employees. Find the new salary of employees currently earning following amounts.

a $46 240
b $94 860
c $124 280
d $64 980

5 Aya has a card that entitles her to a 2.5% discount at the store where she works. How much will she pay for the following items?

a Vase marked at $190
b Cutlery marked at $240
c Painting marked at $560
d Pot marked at $70

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LEVEL 2

6 A used car is priced at $18 600 and offered for sale at a discount of 15%.
- **a** What is the discounted price of the car?
- **b** The car dealer decides to reduce the price of this car by another 15%. What is the new price of the car?

Example 7

7 Find the results after these repeated percentage changes.
- **a** Increase $100 by 20% and then decrease the result by 20%.
- **b** Increase $280 by 10% and then increase the result by 5%.
- **c** Decrease $32 by 50% and then increase the result by 25%.
- **d** Decrease $1400 by 5% and then decrease the result by 5%.
- **e** Increase $960 by 15% and then decrease the result by 10%.
- **f** Decrease $72 by 12.5% and then increase the result by $33\frac{1}{3}$%.

8 An electronic store offered a $30 discount on a piece of software marked at $120.
What percentage discount has been offered?

LEVEL 3

9 The cost price of a sound system is $480.
Retail stores have offered a range of successive discounts. Calculate the final price of the sound system at the following stores.
- **a** Store A: Increase of 10% and then a decrease of 5%
- **b** Store B: Increase of 40% and then a decrease of 50%
- **c** Store C: Increase of 25% and then a decrease of 15%
- **d** Store D: Increase of 30% and then a decrease of 60%

10 The price of a clock has been reduced from $200 to $180.
- **a** What percentage discount has been applied?
- **b** Two months later the price of the clock was increased by the same percentage discount. What is new price of the clock?

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9D Calculating GST

GST

The Australian Government collects a tax when people purchase goods and services. The tax is called the GST (Goods and Services Tax) and is 10% of the pre-tax price of the item. There are exceptions for basic food items and some medical expenses.

GST

To calculate the GST, find 10% of the pre-tax price.
To calculate the total cost of an item, add the GST to the pre-tax price. Alternatively, find 110% of the pre-tax price.
To calculate the pre-tax price given the total cost of an item, divide the total cost by 110%.

Example 8: Finding the GST 9D

John bought a ticket with a pre-tax price of $142 to see a concert at the Sydney Olympic Stadium. He was also required to pay a 10% GST.

a How much GST is payable?
b What was the total cost of his ticket including the GST?
c What was the pre-tax price of a ticket if the final price of the ticket was $149.60?

SOLUTION:

1 Write the quantity (GST) to be calculated.
2 Multiply 0.10 by 142.
3 Evaluate and write using correct units.
4 Write the quantity (total cost) to be calculated.
5 Add the GST to the cost of the ticket.
6 Evaluate and write using correct units.
7 Write the quantity (pre-tax price) to be calculated.
8 Divide the final price by 1.10.
9 Evaluate and write using correct units.

a GST = 10% of $142
$= 0.10 \times 142$
= $14.20

b Total cost = $142 + $14.20
= $156.20

c Pre-tax price = $149.60 ÷ 110%
= $149.60 ÷ 1.10
= $136

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VAT

In some countries the GST is called the VAT (Value Added Tax). The rate of the VAT ranges from 2% to 25%. The calculations for the VAT are similar to the calculations for the GST except the rate of taxation is different.

VAT

- To calculate the VAT, find the VAT rate of the pre-tax price.
- To calculate the total cost of an item, add the VAT to the pre-tax price. Alternatively, find 100% + VAT rate of the pre-tax price.
- To calculate the pre-tax price given the total cost of an item, divide the total cost by 100% + VAT rate.

Example 9: Finding the VAT 9D

Singapore has VAT levied at 5%. Olivia bought a microwave in Singapore for $275 plus a VAT of 5%.

a How much VAT is payable?
b What was the total cost of her microwave including the VAT?
c What was the pre-tax price of another microwave if its final price was $672?

SOLUTION:

1 Write the quantity (VAT) to be calculated.
2 Multiply 0.05 by 275.
3 Evaluate and write using correct units.
4 Write the quantity (total cost) to be calculated.
5 Add the VAT to the cost of the microwave.
6 Evaluate and write using correct units.
7 Write the quantity (pre-tax price) to be calculated.
8 Divide the final price by 1.05.
9 Evaluate and write using correct units.

a VAT = 5% of $275
= 0.05 × 275
= $13.75

b Total cost = $275 + $13.75
= $288.75

c Pre-tax price = $672 ÷ 105%
= $672 ÷ 1.05
= $640

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Exercise 9D

LEVEL 1

Example 8a **1** Calculate the GST payable on each of the following items. The GST rate is 10%.

a Football at $36
b Shoes at $140
c Dinner at $170
d Bucket at $3.20
e Dress at $490
f Book at $42
g Belt at $42.90
h Ring at $2600
i Camera at $370

Example 8b **2** Blake received a $620 bill for electrical work and was required to pay a 10% GST.

a How much GST is payable?
b What was the total cost of the electrical work including the GST?

3 Isabelle received an invoice for her gym membership of $780. In addition she was required to pay a 10% GST.

a How much GST is payable?
b What was the total cost of her gym membership including the GST?

Example 9a **4** Great Britain has VAT of 17.5% on clothing. How much VAT is payable on the following items?

a Football jumper with a pre-tax price of £150
b Football boots with a pre-tax price of £80
c Football shorts with a pre-tax price of £20
d Football socks with a pre-tax price of £8

5 What is the VAT payable in the following countries on a car with a pre-tax price of 42 000? Answer correct to the nearest whole number.

a Argentina – 21% VAT
b Canada – 7% VAT
c China – 17% VAT
d India – 12.5% VAT
e Russia – 18% VAT
f Singapore – 5% VAT
g South Africa – 14% VAT
h Switzerland – 6.5% VAT

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LEVEL 2

Example 8c

6 The cost of the following items includes a 10% GST. What was the pre-GST price?

a Pen at $17.60 **b** Calculator at $24.20 **c** Chair at $99

d DVD at $38.50 **e** Plant at $15.40 **f** Watch at $198

Example 9c

7 New Zealand has VAT levied at 12.5%.

a Ata bought a jacket in New Zealand for $480 plus the VAT. What price did she pay for the jacket?

b A second jacket costs $390 including the VAT. What was the price of the jacket before VAT was added? Answer correct to the nearest dollar.

8 Create the spreadsheet below.

XLS 09DQ8

IF =B5*0.1

	A	B	C	D	E	F
1	**Cambridge Mathematics Standard Year 11 Exercise 9D Question 8**					
2	Worksheet to calculate GST					
3						
4	*Item*	*Pre-tax price*	*GST*	*Total price*		
5	Basketball	$50.00	=B5*0.1	$55.00		
6	Cricket bat	$180.00	$18.00	$198.00		
7	Dumbbell	$55.00	$5.50	$60.50		
8	Golf shoes	$120.00	$12.00	$132.00		
9	Hockey stick	$160.00	$16.00	$176.00		
10	Tennis racquet	$250.00	$25.00	$275.00		
11						

a Cell C5 has a formula that calculates a 10% GST. Enter this formula.

b The formula in cell D5 adds the cost price and the GST. Enter this formula. Fill down the contents of D6 to D10 using this formula.

LEVEL 3

9 After the 10% GST was added, the price of a mobile phone was $362. What was the price without GST? Answer correct to the nearest cent.

10 What was the original cost of a notebook computer that has a GST-included price of $1850? The rate of GST is 10%. Answer correct to the nearest cent.

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9E Straight-line depreciation

Straight-line depreciation occurs when the value of the item decreases by the same amount each period. For example, if you buy a car for $20 000 and it depreciates by $2000 each year, the value of the car after one year is $20 000 – $2000 or $18 000. After the second year the value of the car is $20 000 – $2000 – $2000 or $16 000.

STRAIGHT-LINE DEPRECIATION

$S = V_0 - Dn$

S – Salvage value or current value of an item; also referred to as the book value

V_0 – Purchase price of the item; value of the item when $n = 0$

D – Depreciated amount per time period

n – Number of time periods

Example 10: Calculating the straight-line depreciation 9E

Molly pays $14 500 for a used car. It depreciates $1100 each year. How much will it be worth after three years?

SOLUTION:

1	Write the straight-line depreciation formula.	$S = V_0 - Dn$
2	Substitute $V_0 = 14500$, $D = 1100$ and $n = 3$ into the formula. Evaluate.	$= 14500 - 1100 \times 3$ $= \$11200$
3	Write the answer in words.	The value of the car is $11200.

Example 11: Calculating the salvage value 9E

A new car is purchased for $25 800. After 4 years its salvage value is $15 160. What is the annual amount of depreciation, if the amount of depreciation is constant?

SOLUTION:

1	Write the straight-line depreciation formula.	$S = V_0 - Dn$
2	Substitute $V_0 = 25800$, $S = 15160$ and $n = 4$ into the formula.	$15160 = 25800 - D \times 4$ $D = \frac{25800 - 15160}{4}$
3	Evaluate.	$= \$2660$
4	Write the answer in words.	Annual depreciation is $2660.

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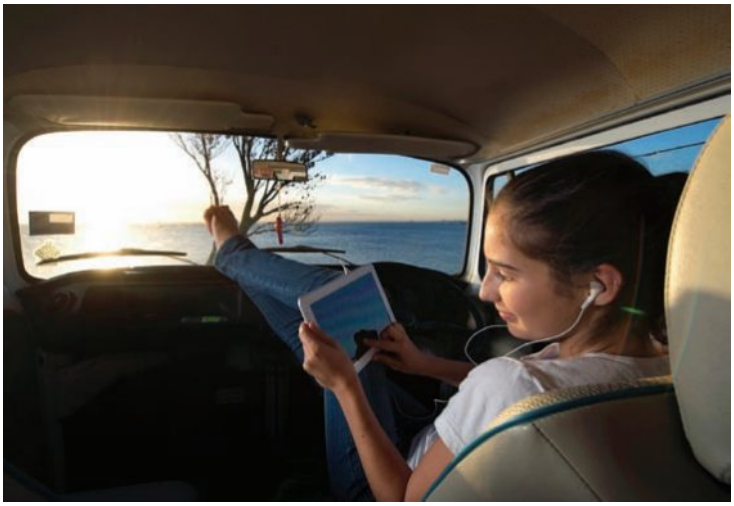
Exercise 9E

LEVEL 1

1 Mia bought a used car for $15200. She estimates that her car will depreciate in value by $3040 each year.

- **a** What is the loss in value (depreciation) during the first year?
- **b** What is the value of the car at the end of the first year?
- **c** What is the loss in value (depreciation) during the second year?
- **d** What is the value of the car at the end of the second year?
- **e** What is the loss in value (depreciation) during the third year?
- **f** What is the value of the car at the end of the third year?



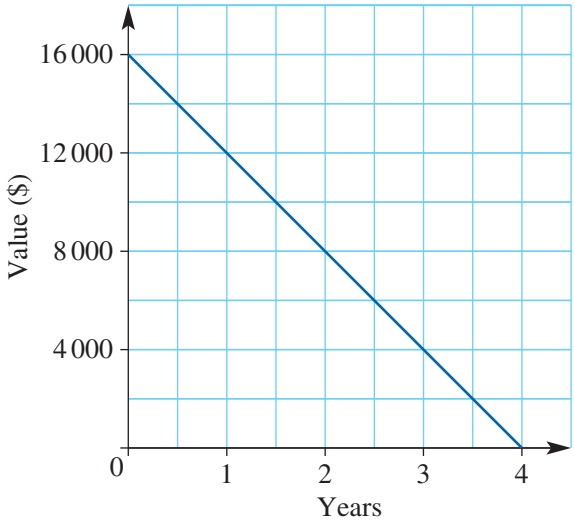
Example 10 **2** Harrison pays $9500 for a motor bike. It depreciates $850 each year. What will be the value of the bike after:

a three years? **b** five years? **c** seven years? **d** nine years?

Example 11 **3** Patrick buys a car for $55500 and it is depreciated at a rate of 10% of its purchase price each year. What is the salvage value of the car after four years?

4 The graph shows the depreciation of a car over four years.

- **a** What is the initial value?
- **b** How much did the car depreciate each year?
- **c** What is the value of the car after 3 years?
- **d** When was the car worth $8000?
- **e** What is the value of the car after $3\frac{1}{2}$ years?
- **f** What is the value of the car after 6 months?



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LEVEL 2

5 Ryan bought a commercial van three years ago. It has a salvage value of $36000 and depreciated $4650 each year. How much did Ryan pay for the van?

6 Lucy bought a used car four years ago. It has a salvage value of $16400 and depreciated $1250 each year. How much did Lucy pay for the used car?

7 Ethan has a car worth $9220. It depreciates by $420 each year.
- **a** When will the car be worth $5440?
- **b** When will the car be worth $3340?

8 A ute is purchased for $18600. After two years it has depreciated to $14800 using the straight line method of depreciation.
- **a** When will the ute be worth $3400?
- **b** When will the ute be worth $1500?

9 A truck is purchased new for $64000. After 3 years its market value is $44800.
- **a** What is the annual amount of depreciation, if the amount of depreciation is constant?
- **b** Determine the book value of the truck after 7 years.

10 Grace bought an SUV costing $38000. It is expected that the SUV will have an effective life of 10 years and then be sold for $14000. Assume the SUV depreciated by the same amount each year. What is the annual depreciation?

11 A utility van is purchased new for $24000. After 3 years its book value is $15000. What is the annual amount of depreciation, if the amount of depreciation is constant?

LEVEL 3

12 A caravan is bought for $82000. It is expected to be used for 4 years and then sold for $50000. Assume the caravan depreciates by the same amount each year.
- **a** How much does the caravan depreciate each year?
- **b** What is the total amount of depreciation for 4 years?
- **c** Copy and complete the following depreciation table for the first four years.

Year	Current value	Depreciation	Depreciated value
1			
2			
3			
4			

- **d** Graph the value in dollars against the age in years.

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PPT STUDY GUIDE

Key ideas and chapter summary

Simple interest

$I = Prn$ $A = P + I$

I – Interest (simple or flat) is the amount earned for the use of the money

P – Principal is the initial amount of money borrowed

r – Rate of simple interest per period expressed as a decimal

n – Number of time periods

A – Amount or final balance

Loan repayments

Loan repayment = Total to be paid ÷ Number of repayments

Simple interest graphs

1. Construct a table of values for I and n using $I = Prn$.
2. Draw a number plane with n the horizontal axis, I the vertical axis. Plot the points and join them to make a straight line.

Percentage increase

1. Add the % increase.
2. Multiply the new percentage by the amount.

Percentage decrease

1. Subtract the % decrease from 100%.
2. Multiply the new percentage by the amount.

Calculating GST

To calculate the GST, find 10% of the pre-tax price.

To calculate the total cost of an item, add the GST to the pre-tax price.

To calculate the pre-tax price given the total cost of an item, divide the total cost by 110%

Calculating Value Added Tax (VAT)

To calculate the VAT, find the VAT rate of the pre-tax price.

To calculate the total cost of an item, add the VAT to the pre-tax price.

To calculate the pre-tax price given the total cost of an item, divide the total cost by (100% + VAT rate)

Straight-line depreciation

$S = V_0 - Dn$

S – Salvage value or current value

V_0 – Purchase price of the item

D – Depreciated amount per time period

n – Number of time periods

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Review

Multiple-choice questions

1 What is the flat-rate interest on $1400 at 7% p.a. for 3 years?

A $98 **B** $294 **C** $498 **D** $1694

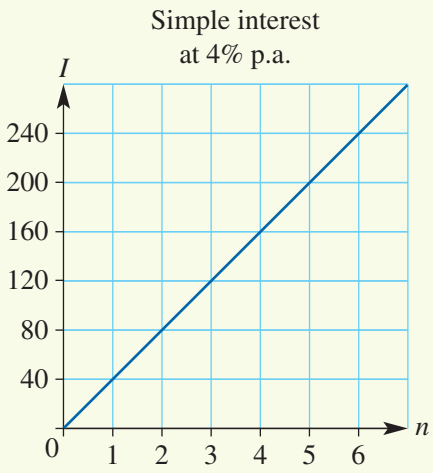
2 Eve invested $800 for 2 years at a simple interest rate of 4% per annum. What is the total amount of interest earned by the investment?

A $32 **B** $64 **C** $160 **D** $320

3 David wants to earn $9000 a year in interest. How much must he invest if the simple interest rate is 15% p.a.? Answer to the nearest dollar.

A $1350 **B** $10 350 **C** $60 000 **D** $600 000

4 Use the graph to find the interest after $3\frac{1}{2}$ years.



A $120 **B** $140

C $160 **D** $240

5 What was the amount of the investment shown in the graph?

A $40 **B** $100

C $240 **D** $1000

6 What is the new price when $80 is increased by 20% then decreased by 20%?

A $51.20 **B** $115.20

C $80.00 **D** $76.80

7 An electrician quoted $300, excluding GST, to complete a job. A GST of 10% is added to the job. What is the total cost of the job?

A $303 **B** $30 **C** $310 **D** $330

8 A house owner receives an electricity bill for $598, before a GST of 10% is added. How much is the GST?

A $5.98 **B** $538.20 **C** $592.02 **D** $59.80

9 After 16% VAT was added, the price of a DVD player was $278. What was the price without VAT? Answer to the nearest cent.

A $44.48 **B** $239.67 **C** $239.66 **D** $322.48

10 Abdul purchased a used car for $7500 and it depreciated by $700 each year. What is its depreciated value after 3 years?

A $5400 **B** $4700 **C** $6100 **D** $6800

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Review

Short-answer questions

1 William takes out a flat-rate loan of $60 000 for a period of 5 years, at a simple interest rate of 12% per annum. Find the amount owing at the end of 5 years.

2 Amet would like to purchase a $2000 TV from an electronics shop. However, to buy the TV he has applied for a flat rate loan over 2 years at 15% p.a. How much does Amet pay altogether for the TV?

3 Ethan borrowed $1800 at 6% per annum. What is the simple interest owed between 30 June and 1 September?

4 Ruby borrows $36 000 for $3\frac{1}{2}$ years. What is the rate of simple interest needed for her to owe $8820 in interest?

5 Chloe has paid $49 500 interest on a $220 000 loan at a flat interest rate of 10%. What was the term of the loan?

6 Adam bought a $500 tablet on hire-purchase plan, consisting of a deposit, then monthly payments. He paid $50 deposit and monthly instalments of $25 for two years. What is the simple interest rate charged per annum? Answer correct to one decimal place. (Hint: If he paid a $50 deposit, consider what the value of the principal was.)

7 The minimum monthly balances for three consecutive months are:

$360.00 $525.75 $718.40

How much interest is earned over the three-month period if interest is calculated on the minimum monthly balance at a rate of 3.5% per annum?

8 Nicholas invested $1000 at 5% per annum simple interest for 5 years.

a Simplify the simple interest formula ($I = Prn$) by substituting values for the principal and the interest rate.

b Use this formula to complete the following table of values.

n	0	1	2	3	4	5
I						

c Draw a number plane with n as the horizontal axis and I as the vertical axis.

d Plot the points from the table of values. Join the points to make a straight line.

e Extend the line to find the amount of interest after $5\frac{1}{2}$ years.

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Review

9 Jill has a shareholder card that entitles her to a 5% discount at a supermarket. How much will she pay for the following items? Answer to the nearest cent.

a Breakfast cereal at $7.60
b Milk at $4.90
c Coffee at $14.20
d Cheese at $8.40

10 An electrician is buying a light fitting for $144 at a hardware store. He receives a clearance discount of 15% then a trade discount of 10%. How much does the electrician pay for the light fitting?

11 Ibrahim received a bill of $286 for internet access. In addition he paid a 10% GST.

a How much GST is payable?
b What was the total cost of internet access including the GST?

12 A caravan is purchased for $12 986. After 3 years it has a salvage value of $6020.

a What is the annual amount of depreciation, if the amount of depreciation is constant?
b Determine the book value of the caravan after 5 years.

Extended-response questions

13 Jackson wants to buy a new computer for $3500. He considers buying the computer on a no-deposit leasing arrangement with terms of 14.4% per annum simple interest over three years with equal monthly repayments. How much would Jackson's monthly repayment be under this proposal? Answer correct to the nearest cent.

14 While shopping on Monday, Madeleine found a dress priced at $350. On Wednesday, Madeleine noticed the dress was marked down by 20%.

a What is the marked-down price of the dress?
b When Madeleine enquired about the dress, she was offered a further $40 discount off the marked-down price. She bought the dress. What was Madeleine's saving as a percentage of the original price of the dress? Answer correct to one decimal place.

15 Audrey is a florist who has just purchased a second-hand delivery van for $23 250 with the intention of replacing it after three years.

a Calculate the trade-in value of the van in three years time given that it depreciates at the rate of $3750 per annum.
b The new van Audrey is planning to buy in three years time is worth $44 360 today. Given that the inflation rate is 2.8% per annum, how much will this van cost in three years time?
c How much extra money will Audrey need to be able to buy the new van in three years time if she sells her second-hand van at the calculated trade-in value?

PDF LITERACY

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10 Exploring and describing data

Syllabus topic — S1.2 Exploring and describing data arising from a single continuous variable

This topic will develop your skills in calculating summary statistics for single data sets and their use in the interpretation of data.

Outcomes

- Calculate and interpret the measures of central tendency: mean, mode and median.
- Calculate and interpret measures of spread: range, quantiles, interquartile range and standard deviation.
- Investigate and describe the effect of outliers on summary statistics.
- Identify outliers and investigate their effect on the mean and the median.
- Describe, compare and interpret datasets in graphical and numerical form.
- Construct and compare parallel box-and-whisker plot using a five-number summary.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Widgets
- Literacy worksheet
- Spreadsheets
- Quick Quiz
- Study guide
- Solutions (enabled by teacher)

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

In the Interactive Textbook you can take a test of prior knowledge required for this chapter, and depending on your score you may be directed to revision from the previous years' work.

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10A Measures of central tendency

The mean is calculated by summing all the scores and dividing by the number of scores.

There are two notations for the mean depending on whether the data is a population or a sample. The population mean is denoted by the symbol μ and the sample mean is denoted by the symbol $\bar{x}$.

To calculate the mean from a grouped frequency table, the 'class centre' is used instead of the score (x). The 'frequency × score' (fx) column becomes the 'frequency × class centre' column. The same formula for the mean is used. The mean calculated from grouped data is an approximation.

MEAN

$$\text{Mean} = \frac{\text{Sum of scores}}{\text{Number of scores}} \text{ or } \bar{x} = \frac{\sum x}{n} \text{ or } \bar{x} = \frac{\sum fx}{\sum f} \text{ (for grouped data)}$$

μ – Population mean $\bar{x}$ – Sample mean

Example 1: Finding the mean from a grouped frequency table 10A

The points scored by a rugby team are shown in the grouped frequency table. The data represent all the team's matches for the past two seasons.

What is the mean number of points scored over the two seasons?

Class	Class centre (x)	Frequency (f)	$f \times x$
0–5	3	2	
6–10	8	9	
11–15	13	11	
16–20	18	13	
21–25	23	10	
26–30	28	5	

SOLUTION:

1. Complete the fx column by multiplying the class centre (x) by the frequency (f).
2. Sum the f column ($\sum f = 50$).
3. Sum the fx column ($\sum fx = 825$).
4. Write the formula $\bar{x} = \frac{\sum fx}{\sum f}$.

x	f	fx
3	2	6
8	9	72
13	11	143
18	13	234
23	10	230
28	5	140
Total	50	825

5. Substitute the values for $\sum fx$ and $\sum f$.
6. Evaluate.

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{825}{50}$$
$$= 16.5$$

7. Write the answer in words.

Mean number of points is 16.5.

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Median

The median is the middle score or value. To find the median, list all the scores in increasing order and select the middle one. For example, the median of 1, 4, **5**, 7, 8 is 5. When there is an even number of scores, the median is the average of the two middle scores. For example, the median of 1, 1, **4**, **5**, 7, 8 is found by sorting the six scores and finding the average of 4 and 5 or 4.5.

MEDIAN

1. Arrange all the scores in increasing order.
2. Count the total number of scores. This is represented by the letter n.
3. Odd number of scores then the median is $\frac{n+1}{2}$ score.
4. Even number of scores then the median is the average of the $\frac{n}{2}$ and the $\frac{n+1}{2}$ score.

Example 2: Calculating the median 10A

The data below shows the change in temperature of a sick child.

37 38 39 38 40 40

a Sort the data in ascending order.

b Calculate the median.

SOLUTION:

1. Write the scores in increasing order.
2. Count the total number of scores ($n = 6$).
3. There is an even number of scores, so the median is the average of the 3rd (score 38) and the 4th scores (score 39).
4. The average or midpoint of 38 and 39 is 38.5.
5. Write the answer in words.

a 37, 38, 38, 39, 40, 40

b $\frac{n}{2} = \frac{6}{2}, \frac{n}{2}+1 = \frac{6}{2}+1$

$= 3 \qquad = 4$

$$\text{Median} = \frac{38+39}{2}$$
$$= 38.5$$

Median is 38.5.

Mode

The mode is the score that occurs the most. It is the score with the highest frequency. The mode is useful for categorical data that do not allow numerical calculations, such as when the data collected are colours. Modes may occur at the beginning or end of a range of values; therefore, conclusions based only on the mode may be inaccurate. It is common for data to have several modes. For example, if there are two modes the data is referred to as bimodal. When data is grouped into classes, the class that occurs the most is called the modal class.

MODE

1. Determine the number of times each score occurs.
2. Mode is the score that occurs the most number of times. If two or more scores occur the same number of times they are both regarded as the mode.

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Exercise 10A

LEVEL 1

Example 1

1 The grouped frequency table shows the number of people by age at a Gold Coast resort.

Class	Class centre (x)	Frequency (f)	$f \times x$
25–29		25	
30–34		21	
35–39		35	
40–44		21	
45–49		26	
50–54		23	
55–59		32	

a Copy and complete the table by finding the class centre and fx column.

b How many people stayed at the Gold Coast resort?

c Find the mean of this data. Answer correct to the nearest whole number.

2 Find the mean of the data in the following table. Answer correct to one decimal place.

Class	30–39	40–49	50–59	60–69	70–79	80–89	90–99
Class centre							
Frequency	9	4	6	8	10	6	7

3 The grouped frequency table shows the weights of randomly selected motor vehicles. All motor vehicle weights are in kilograms.

Class	Class centre (x)	Frequency (f)	$f \times x$
2000–2004		9	
2005–2009		15	
2010–2014		18	
2015–2019		10	
2020–2024		8	
2025–2029		6	
2030–2034		3	
2035–2039		1	

a Copy and complete the table by finding the class centre and the fx column.

b How many motor vehicles were weighed?

c Find the mean of this data.

Example 2

4 The number of hours Amy works per week was 32, 41, 36, 35, 41, 58, 56, 61 and 72.

a What is the median number of hours worked?

b What is the mean number of hours worked?

c What measure of location (median or mean) will not change if 72 is deleted?

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LEVEL 2

5 The number of people entering a competition over a number of weeks was 19, 16, 10, 15, 11, 17, 26, 23 and 16.

- **a** Calculate the mean, median and mode for this data.
- **b** What is the effect on the mean, median and mode of adding 5 to each score?
- **c** Calculate the mean, median and mode with these extra people entering the competition.

6 A score was added to this set of scores: 13, 18, 20, 20, 22 and 26. The new mean is now 19. What score was added?

7 Ten adults were surveyed on the number of hours they slept last night. Their answers were 8, 10, 9, 5, 5, 6, 9, 8, 4 and 12.

- **a** Find the mean number of hours slept.
- **b** Another adult was then surveyed and the mean changed to 8. How many hours did this adult sleep?

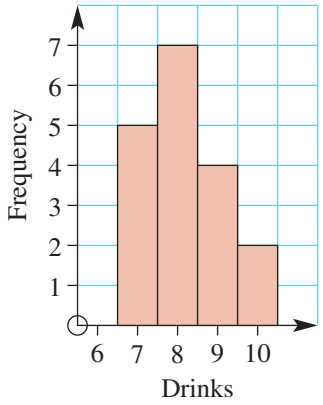
8 The table on the right shows the ages of students at an acting academy.

- **a** What is the median age?
- **b** What is the mean age? Answer correct to one decimal place.
- **c** Complete the cumulative frequency column.
- **d** The two oldest students leave the academy. What is the new median?

Age	Frequency	Cumulative frequency.
18	2	
19	5	
20	6	
21	2	

9 The frequency histogram shows the number of drinks sold each day.

- **a** What is the maximum number?
- **b** What is the minimum number?
- **c** Construct a frequency table.
- **d** Calculate the median.
- **e** The next day there were 14 drinks sold. Does the median change?



10 The mean distance travelled by five friends on the weekend was calculated as 60 km. One friend who claimed to have travelled 45 km did not include a journey of 22 km, so the total distance travelled by this person was actually 67 km. What was the mean distance of the five friends now? Answer correct to one decimal place.

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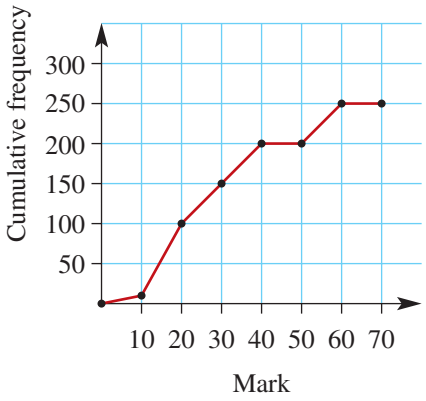
LEVEL 3

11 The marks for a Mathematics exam are shown in the cumulative frequency polygon.



- **a** What is the frequency of 70?
- **b** What is the frequency of 30?
- **c** How many students completed the exam?
- **d** Estimate the median using the graph.

12 The number of mobile calls made by a phone company is shown below.

80	84	87	70	68	83	85	86	70	90
68	90	78	83	75	69	82	89	89	82
80	85	90	74	74	70	90	80	75	75
70	90	83	68	79	79	82	79	91	80

- **a** Decide on appropriate classes for a grouped frequency table.
- **b** Calculate the class centres for these classes.
- **c** Construct a grouped frequency table using these class intervals.
- **d** How many pieces of data have been collected?
- **e** Using the class centres, find the mean of this data. Answer correct to one decimal place.
- **f** One score entered above was found to be incorrect. A score of 86 should have been 68. What is the new mean of this data? Answer correct to one decimal place.

13 The number of plants sold in a nursery each day is shown below.

21	20	20	24	21	23	21	21	23	22	22	22	20	20	26
22	23	20	22	26	21	24	24	25	25	22	21	20	20	23

- **a** Construct a frequency table with a cumulative frequency column.
- **b** Construct a cumulative frequency histogram and polygon for this data.
- **c** Use the cumulative frequency graphs to estimate the median.

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10B Measures of spread

A measure of the spread is calculated to determine whether most of the values are clustered together or stretched out. The range, interquartile range and standard deviation are all measures of spread.

Range

The range is the difference between the highest and lowest scores. It is a simple way of measuring the spread of the data.

RANGE

Range = Highest score – Lowest score

Example 3: Calculating the range 10B

The assessment results for two different tasks are shown below. Find the range for Task A and Task B.

Task A	3	5	10	13	14	14	17	22	24	24	27	27	28	33	38
	40	40	41	43	44	45	45	46	50	52	52	55	55	58	95
Task B	10	15	19	20	24	27	31	31	35	38	40	49	51	51	54
	55	58	62	62	68	68	71	72	76	78	79	79	86	88	90

SOLUTION:

1 Write the formula for range.
2 For Task A, substitute the highest score (95) and the lowest score (3).
3 Evaluate.
4 Write the formula for range.
5 For Task B, substitute the highest score (90) and the lowest score (10).
6 Evaluate.

Task A
Range = Highest – Lowest
= 95 – 3
= 92

Task B
Range = Highest – Lowest
= 90 – 10
= 80

Note: The range for Task A is not a good indicator of the spread as it is affected by the score of 95 (outlier).

Quantiles

Quantiles are a set of values that divide an ordered dataset into equal groups. Examples include quartiles, deciles and percentiles.

QUARTILES	DECILES	PERCENTILES
4 equal parts	10 equal parts	100 equal parts
1st quartile – 25% of data (Q_1)	1st decile – 10% of data	1st percentile – 1% of data
2nd quartile – 50% of data (Q_2)	5th decile – 50% of data	50th percentile – 50% of data
3rd quartile – 75% of data (Q_3)	9th decile – 90% of data	90th percentile – 90% of data.

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Interquartile range

The interquartile range does not rely on the extreme values like the range. The data is arranged in increasing order and divided into 4 equal parts or quartiles. The interquartile range (IQR) is the difference between the first quartile and the third quartile. The first quartile cuts off the lowest 25% $\left(\frac{1}{4}\right)$ of scores and the third quartile cuts off the lowest 75% $\left(\frac{3}{4}\right)$ of scores.

INTERQUARTILE RANGE

IQR = Third quartile (Q_3) – First quartile (Q_1)
$= Q_3 - Q_1$

1. Arrange the data in increasing order.
2. Divide the data into two equal-sized groups. If n is odd, omit the median.
3. Find Q_1, the median of the first group.
4. Find Q_3, the median of the second group.
5. Calculate the interquartile range (IQR) by subtracting Q_1 from Q_3.

Example 4: Calculating the interquartile range 10B

The assessment results for two different tasks are shown below.
Find the interquartile range for Task A and Task B.
What is shown by these results?

Task A	3	5	10	13	14	14	17	**22**	24	24	27	27	28	33	38
	40	40	41	43	44	45	45	**46**	50	52	52	55	55	58	95
Task B	10	15	19	20	24	27	31	**31**	35	38	40	49	51	51	54
	55	58	62	62	68	68	71	**72**	76	78	79	79	86	88	90

SOLUTION:

1. Arrange the data in increasing order.
2. Divide the data into two equal-sized groups. There are 30 scores in total, hence 15 scores in each group.
3. Write the formula for interquartile range.
4. For Task A, the median of the first group is 22 (or Q_1) and the median of the second group is 46 (or Q_3).
5. Substitute into the formula and evaluate.
6. For Task B, the median of the first group is 31 (or Q_1) and the median of the second group is 72 (or Q_3).
7. Substitute into the formula and evaluate.
8. Interpret the result and answer the question.

Task A

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 46 - 22 \\ &= 24 \end{aligned}$$

Task B

$$\begin{aligned} IQR &= Q_3 - Q_1 \\ &= 72 - 31 \\ &= 41 \end{aligned}$$

Task A is more consistent than Task B (lower IQR).

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Standard deviation

The standard deviation is a measure of the spread of data about the mean. There are two slightly different methods for calculating the standard deviation. The population standard deviation (σ_n or σ_x) is a better measure when we have the data for the entire population. However, when a sample is taken from a large population, the sample standard deviation (σ_{n-1} or S_x) is a better measure.

STANDARD DEVIATION

Standard deviation measures the spread of data about the mean.
σ_n or σ_x – Population standard deviation S_x – Sample standard deviation

Example 5: Calculating the standard deviation 10B

Find the population and sample standard deviation for these assessment tasks (Example 4).

Task A	3	5	10	13	14	14	17	22	24	24	27	27	28	33	38
	40	40	41	43	44	45	45	46	50	52	52	55	55	58	95
Task B	10	15	19	20	24	27	31	31	35	38	40	49	51	51	54
	55	58	62	62	68	68	71	72	76	78	79	79	86	88	90

SOLUTION:

1 Enter the **statistics** mode of the calculator.
2 Clear memory and enter the data into the calculator.
3 Select the σ_x and S_x key to view the results.

Task A: $\sigma_x = 19.25$, $S_x = 19.58$
Task B: $\sigma_x = 23.30$, $S_x = 23.69$

Example 6: Calculating the standard deviation 10B

The frequency table below shows the number of customers at the local vegetable market for the past 22 days.

Score (x)	Frequency (f)
15	3
16	4
17	6
18	4
19	5

Find the population and sample standard deviation.
Answer correct to two decimal places.

SOLUTION:

1 Enter the **statistics** mode of the scientific calculator.
2 Clear memory and enter the data.
3 Select the σ_x and S_x key to view the results.

$\sigma_x = 1.34$, $S_x = 1.37$

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Exercise 10B

LEVEL 1

Example 3

1 Find the range of each data set.

a 4, 23, 6, 6, 9, 14, 32, 4, 4
b 14, 13, 13, 12, 23, 18, 13, 13, 15
c 12, 5, 0, 8, 5, 6, 7, 16, 0
d 12, 21, 18, 12, 33, 12
e 6, 33, 6, 1, 3, 6, 31, 11, 3, 7
f 5, 5, 5, 10, 7, 10, 15

2 Aki scored the following marks in a spelling test:
4, 5, 5, 6, 7, 7, 8, 8, 9 and 9

a What is the first quartile?
b What is the third quartile?
c What is the second decile?
d What is the seventh decile?

3 The number of calls for help each hour after a storm are recorded below:

8	12	5	11	10	12
7	9	11	10	9	9
7	2	5	9	6	

a What is the range?
b What is the first quartile?
c What is the third quartile?
d What is the interquartile range?

Example 4

4 Find the interquartile range for each data set.

a 13, 19, 8, 28, 16, 27, 11, 17
b 62, 51, 53, 64, 65, 53, 56, 61
c 29, 24, 30, 22, 32, 29, 24, 26, 25, 28
d 58, 59, 58, 34, 29, 29, 37, 57, 31, 39
e 31, 34, 38, 39, 43, 57, 57, 61, 41
f 59, 44, 40, 41, 46, 50, 46, 53, 46

5 The systolic blood pressure for a sample of 20 people is listed below:

203	124	180	210	105	148	161	131	192	125
159	106	170	138	100	120	109	144	190	193

a What is the range?
b What is the first quartile?
c What is the third quartile?
d What is the interquartile range?

Example 5

6 Find the population standard deviation of each data set (correct to one decimal place).

a 5.3, 6.6, 4.4, 8.8, 3.3, 10.1
b 98, 112, 7, 4, 0, 100, 1
c 7, 7, 7, 7, 7, 7, 7, 7
d 130, 145, 145, 132, 124, 135, 156

7 Find the sample standard deviation of each data set (correct to one decimal place).

a 7.5, 8.9, 1.0, 3.4, 1.3
b 3, 0, 11, 7, 5, 1, 2, 9, 7
c 9, 3, 6, 9, 13, 9, 15, 10, 7, 9
d 9, 10, 100, 5, 100, 7, 9

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8 The number of kilometres Stephanie walked each day while on her trek are 22, 26, 34, 28, 36, 30, 29, 31 and 28.

a What is the population standard deviation, correct to two decimal places?

b The longest and the shortest distances were not included in this data. The longest distance was 44 km and the shortest distance was 17 km. What is the new sample standard deviation? (Answer correct to two decimal places.)

9 Find the sample standard deviation for each of the following tables. Answer correct to one decimal place.

a

Score	2	3	4	5	6	7	8	9
Frequency	6	6	5	5	4	4	3	3

b

Score	24	25	26	27	28	29	30	31
Frequency	1	3	5	7	7	5	3	1

c

Score	55	56	57	58	59	60	61	62
Frequency	5	6	9	10	11	8	7	4

10 A grouped frequency table is shown below.

Class	Class centre (x)	Frequency (f)
10–29		5
30–49		10
50–69		11
70–89		4

a What is the population standard deviation? Answer correct to one decimal place.

b What is the sample standard deviation? Answer correct to one decimal place.

11 A railway attendant recorded the delay times of seven trains during peak hour: 0, 60, 35, 120, 10, 40 and 80. All times were measured in seconds.

a What is the range for these times?

b What is the sample standard deviation for these times, correct to two decimal places?

c Calculate the interquartile range for these times.

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LEVEL 2

12 Joshua has been ill and only scored 36% in a recent English test. However, the results of his class were very good, with every other student scoring above 70%. Joshua's teacher has decided not to include Joshua's results in the class statistics. What effect would this have on the mean and sample standard deviation?

13 The results for Molly and Harry in their tests this term are listed below.

Molly	10	19	16	15	7	15	11	3	9	16
Harry	18	16	17	8	14	15	13	10	13	16

a What is the mean and population standard deviation for Molly's results?
b What is the mean and population standard deviation for Harry's results?
c Which student had the more consistent results throughout the term? Give a reason.

LEVEL 3

14 Charlie's median score for cricket is 32 and the interquartile range of his scores is 5. Thomas's median score for cricket is 30 and the interquartile range of his scores is 10.

a Which player is the more consistent? Explain your answer.
b Which player is more likely to make the higher score? Explain your answer.

15 The table below shows Amber's results for two Mathematics tests as well as the class mean and standard deviation.

Topic	Amber's result	Class mean	Class standard deviation
Data	78	70	7
Algebra	83	74	10

Which is the better result for Amber compared to her class? Justify your answer.

16 Twelve teenagers were selected at random from a concert audience and their ages were recorded:

14 16 15 16 17 19 17 16 15 15 15 18

a Calculate the mean of their ages. Answer correct to two decimal places.
b What is the population standard deviation, correct to one decimal place?
c Explain the meaning of standard deviation.

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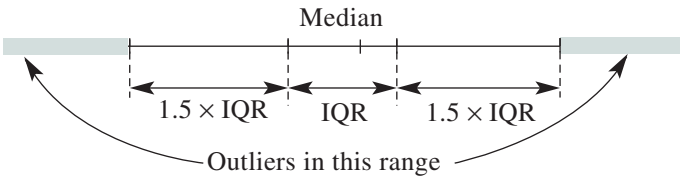
10C Outliers

An outlier is a score that is separated from the majority of the data. It is sometimes difficult to decide whether or not a score should be classified as an outlier. We define an outlier as any value more than 1.5 interquartile ranges above the third quartile ($Q_3 + 1.5 \times IQR$), or more than 1.5 interquartile ranges below the first quartile ($Q_1 - 1.5 \times IQR$).



Outliers have little effect on the mean, median and mode for large sets of data. However, in small data sets, the presence of an outlier will have a large effect on the mean, a smaller effect on the median and usually no effect on the mode.

OUTLIER

An outlier is a score that is separated from the majority of the data.
Use $Q_1 - 1.5 \times IQR$ and $Q_3 + 1.5 \times IQR$ as criteria to determine an outlier.

Example 7: Determining the effect of an outlier 10C

Task A	5	7	12	15	16	16	19	**20**	26	26	29	29	29	35	40
	40	42	43	45	46	47	47	**48**	52	54	54	57	57	60	91

a What is the outlier in Task A?

b What is the effect of the outlier on the mean, median and mode?

SOLUTION:

1 Looking at the data it appears that 91 is an outlier.
2 To check assumptions, use interquartile range.
3 Lower limit is first quartile minus 1.5 times the IQR.
4 Evaluate.
5 Upper limit is third quartile plus 1.5 times the IQR.
6 Evaluate.
7 Determine if any values are greater than 90.

a IQR = 48 – 20 = 28
Lower limit = 20 – (1.5 × 28)
= –22
No values

Upper limit = 48 + (1.5 × 28)
= 90
The outlier is 91.

8 Enter the data into the calculator in statistics mode.
9 Select $\bar{x}$ for the mean. Clear the outlier and select $\bar{x}$.
10 Mode is the score with the highest frequency (29).
11 Median of the 30 scores is the average of the 15th and 16th scores (40 and 40).
12 Median of the 29 scores (without outlier) is the 15th score (40).
13 State the effect of the outlier on the mean, median and mode.

b $\bar{x}$ = 36.9 (with outlier)
$\bar{x}$ = 35.0 (without outlier)
Mode = 29 (with outlier)
Mode = 29 (without outlier)
Median = 40 (with outlier)
Median = 40 (without outlier)

Outlier has increased the mean but has had no effect on the median or the mode.

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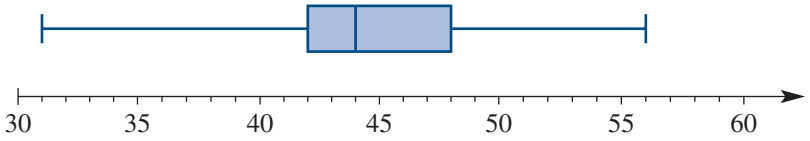
Exercise 10C

LEVEL 1

1 List any outliers for each data set by observation.

a 6, 5, 8, 8, 11, 16, 34, 6, 6
b 33, 33, 34, 33, 33, 32, 43, 48, 35
c 7, 5, 0, 7, 16, 0, 8, 5, 6
d 28, 22, 133, 22, 31, 22
e 11, 3, 7, 6, 53, 6, 1, 6, 3, 1, 3
f 13, 10, 13, 0, 8, 8, 8

2 The results of an assignment are represented in the following box-and-whisker plot.



a What is the lower extreme?
b What is the upper extreme?
c What is the lower quartile?
d What is the upper quartile?
e Find the interquartile range.
f Give the outlier if it exists.

Example 7

3 Give the outlier in the following data sets, if it exists. Justify using the criteria $Q_1 - 1.5 \times \text{IQR}$.

a 1, 6, 7, 7, 8, 9, 9
b 69, 82, 82, 85, 86, 90, 91
c 20, 31, 32, 35, 35, 35, 35, 36, 36
d 10, 18, 19, 20, 21
e 56, 66, 68, 70, 70
f 0, 8, 8, 8, 10, 13, 13, 13

4 Give the outlier in the following data sets, if it exists. Justify using the criteria $Q_3 + 1.5 \times \text{IQR}$.

a 0, 0, 1, 3, 5, 5, 10
b 22, 23, 23, 24, 25, 25, 25, 27, 36
c 70, 72, 73, 74, 82
d 32, 33, 33, 33, 33, 34, 35, 43, 48
e 45, 45, 45, 47, 50, 50, 59
f 101, 102, 102, 102, 103, 115

5 The ages of a group people attending a musical was as follows:
29, 37, 27, 28, 40, 76, 26, 31, 42, 34, 25, 26, 30, 40, 27, 52, 36, 23, 31, 14, 27, 32, 28, 29, 40, 20

a Find the lower quartile.
b Find the upper quartile.
c What is the interquartile range?
d Give the outlier if it exists.

6 Seven students scored the following marks in an assessment task: 35, 49, 56, 59, 65, 69, 96.
Is the mark of 96 an outlier for this set of data? Justify your answer with calculations.

7 The rainfall for the past nine months was as follows: 15, 20, 22, 14, 31, 19, 20, 16, 17.
Is the 31 an outlier for this set of data? Justify your answer with calculations.

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LEVEL 2

8 Jennifer scored the following points in her basketball games this season: 4, 5, 5, 6, 8, 8, 9, 10, 12 and 20.

- **a** What is the range?
- **b** What is the interquartile range?
- **c** What is the value of the outlier?
- **d** How does the outlier affect the range and the interquartile range?

9 Consider the following set of scores: 3, 5, 6, 13, 13, 19, 20, 20, 81.

- **a** Calculate the mean and the median of this set of scores.
- **b** What is the effect on the mean and the median of removing the outlier?

10 **a** Calculate the mean, median and mode for each data set. Answer correct to one decimal place when necessary.

- **i** 26, 27, 21, 26, 57, 22, 23
- **ii** 38, 41, 26, 29, 34, 27, 27, 34, 116
- **iii** 95, 92, 91, 88, 88, 87, 42
- **iv** 21, 61, 62, 62, 65, 68, 69, 71, 73

b Calculate the mean, median and mode for each data set after removing the outlier. Answer correct to one decimal place.

11 Consider the following set of scores: 12, 15, 16, 16, 18, 18, 19, 20, 20, 60.

- **a** Calculate the mean and the median of the set of scores.
- **b** What is the effect on the mean and the median of removing the outlier?

LEVEL 3

12 Nine employees earned the following wages per week: $680, $590, $490, $990, $660, $520, $300, $510 and $540.

- **a** Is the wage of $990 an outlier for this set of data? Justify your answer with calculations.
- **b** Each employee receives a $50 pay increase. What effect will this have on the standard deviation?

13 The players in Noah's football team completed a survey on the number of text messages sent using their mobile phone within the last day. The results are summarised in the table opposite.

Number of messages	Frequency
0	10
1	8
2	5
3	4
6	1

- **a** What is the mean number of messages sent? Answer correct to one decimal place.
- **b** Calculate the median number of messages sent.
- **c** What is the mode?
- **d** Calculate the mean, the median and the mode when the outlier is removed.

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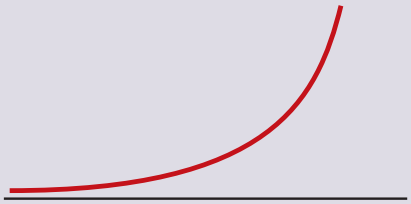
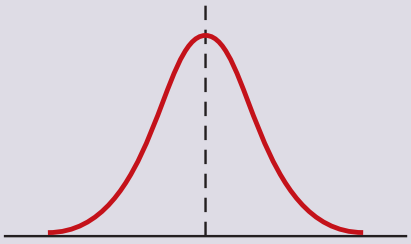
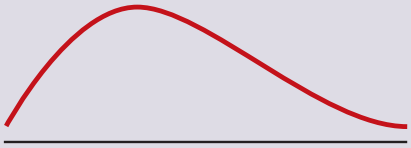
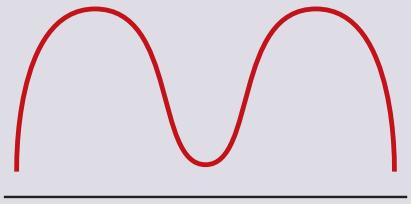
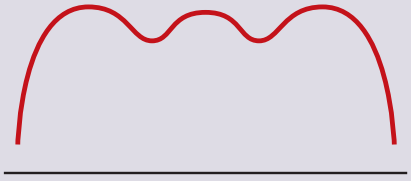
10D Describing distributions

The general shape of a graph can be described in terms of smoothness, symmetry and the number of modes. **Smooth** curves have no breaks or jagged sections, and data gradually increases and/or decreases. **Symmetrical** graphs are balanced about a vertical line in the centre, and the mode is the score that occurs the most (highest frequency). **Unimodal** graphs have one mode. However, it is common for data to have several modes. **Bimodal** graphs have two modes and multimodal graphs have many modes.

SHAPE	DESCRIPTION	GRAPH
Smoothness	Data has no breaks or jagged sections.	
Symmetry	Data is symmetrical and balanced about a centre.	
Unimodal	Data has only one mode or peak.	
Bimodal	Data has two modes or peaks.	
Multimodal	Data has many modes or peaks.	

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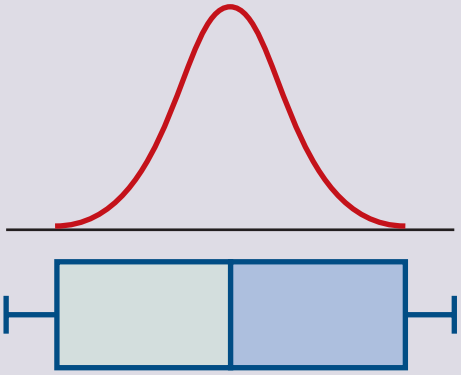
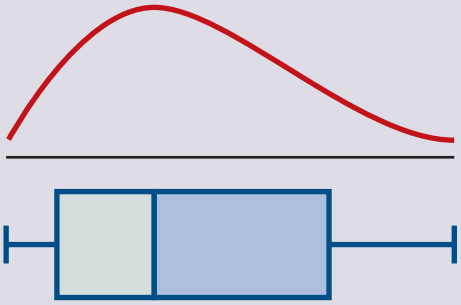
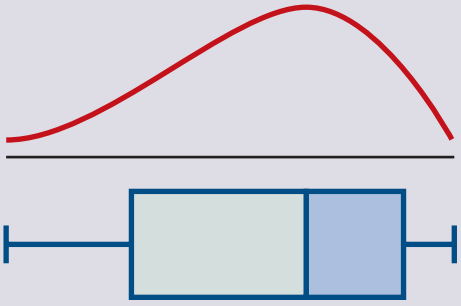
Symmetry or skewness

Data is symmetric if it forms a mirror image of itself when folded in the 'middle' along a vertical axis. Otherwise the data is skewed. Skewed data has more data on one side. There are two types:

- Positively skewed data has a long tail on the right-hand side.
- Negatively skewed data has a long tail on the left-hand side.

SHAPE	DESCRIPTION	GRAPH
No skew (Symmetric)	Data is symmetrical and balanced about a vertical line.	
Positively skewed	There is more data on the left side. The long tail is on the right side (positive side).	
Negatively skewed	There is more data on the right side. The long tail is on the left side (negative side).	

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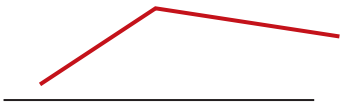
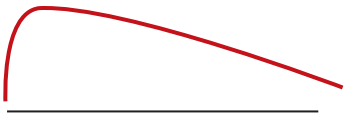
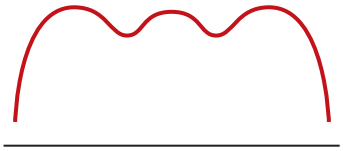
Exercise 10D

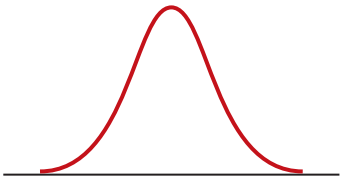
LEVEL 1

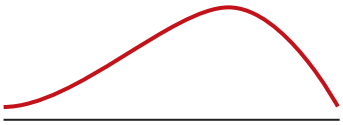
1 Describe the smoothness, symmetry and modality of the following graphs.

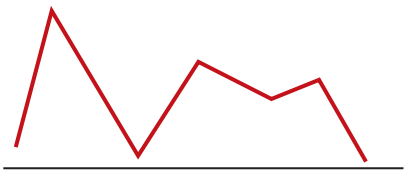
a 

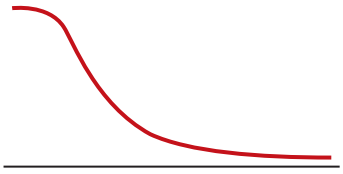
b 

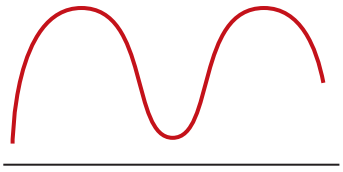
c 

d 

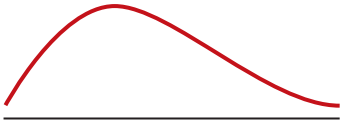
e 

f 

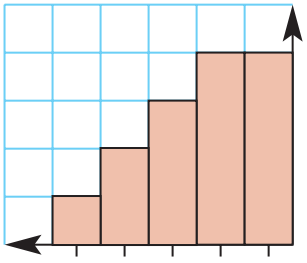
g 

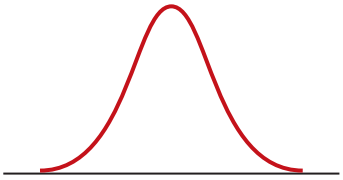
h 

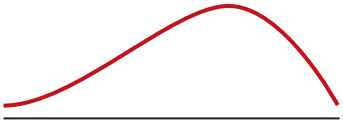
2 Describe the skewness of the following graphs.

a 

b 

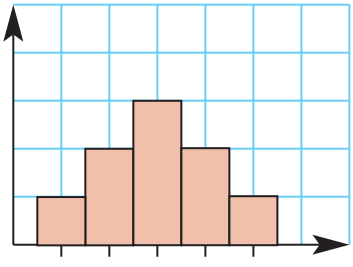
c 

d 

e 

f 

g 

h 

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LEVEL 2

3 The grouped frequency table shows the age of people entering a competition.

Class	Class centre (x)	Frequency (f)	$f \times x$
5–9		1	
10–14		0	
15–19		2	
20–24		3	
25–29		7	
30–34		18	
35–39		16	

a Calculate the mean, median and modal class from the frequency table. Answer correct to one decimal place when necessary.
b What is the effect on the mean and the median of removing the outlier?
c Display the data in a frequency histogram.
d Describe the symmetry and modality of the data.
e Describe the skewness of the distribution.

LEVEL 3

4 There are eight students in a hospitality class. The students sat for a test and the results were 98, 95, 91, 94, 95, 11, 95 and 94.
a Find the mean, the median and the mode. Answer correct to nearest whole number.
b Lily scored 91. She told her father that her result was above the average. Do you agree with Lily's statement? Give a reason for your answer.

5 High school students were surveyed and the results of two samples are shown below.

Sample A

Score	1	2	3	4	5	6	7
Frequency	3	5	8	9	8	5	3

Sample B

Score	1	2	3	4	5	6	7
Frequency	3	9	9	7	6	5	1

a For sample A (correct to one decimal place where necessary), find the:
i median **ii** mean
iii mode **iv** standard deviation
b For sample B (correct to one decimal place where necessary), find the:
i median **ii** mean
iii mode **iv** standard deviation
c Construct a frequency histogram for sample A.
d Construct a frequency histogram for sample B.
e Describe the symmetry and modality of sample A and sample B.
f Describe the skewness of sample A and sample B.

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10E Comparing distributions

The selection and the use of the appropriate measure of location (mean or median) and measure of spread (range, interquartile range or standard deviation) depends on the nature of the data and the relative merits of each measure. For example, if the dataset contains one or two extreme scores then the value of the mean will greatly increase or decrease.

MEASURES OF LOCATION		
Mean	Advantages	Easy to understand and calculate. Depends on every score. Varies least from sample to sample.
	Disadvantages	Distorted by outliers. Not suitable for categorical data.
Median	Advantages	Easy to understand. Not affected by outliers.
	Disadvantages	May not be central. Data needs to be sorted. Varies more than the mean in a sample.
Mode	Advantages	Easy to determine. Not affected by outliers. Suitable for categorical data.
	Disadvantages	There may be no mode or more than one mode. May not be central.
Measures of spread		
Range	Advantages	Easy to understand. Easy to calculate.
	Disadvantages	Dependent on the smallest and largest values. May be distorted by outliers.
Interquartile range	Advantages	Easy to determine for small data sets. Easy to understand. Not affected by outliers.
	Disadvantages	Difficult to calculate for large data sets. Dependent on lower and upper quartiles. Data needs to be sorted.
Standard deviation	Advantages	Depends on every score. Not significantly affected by outliers.
	Disadvantages	Difficult to determine without a calculator. Difficult to understand.

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Example 8: Comparing statistics for two sets of data 10E

The table below shows the number of participants in a fitness class for two instructors.

	M	T	W	T	F	S	S
Bec	8	5	4	8	8	4	5
Rita	10	9	12	14	8	10	1

a Find the mean and median for each set of data. Answer correct to one decimal place where necessary.

b Find the range and interquartile range for each set of data.

c Examine the summary statistics and outline any concerns.

SOLUTION:

1 Determine the mean and median for each instructor.

2 Mean is the sum of all the scores divided by the number of scores; evaluate.

3 Write the scores in increasing order.

4 Median is the middle score. There is an odd number of scores so the median is the 4th (or score 5).

5 Determine the range and IQR for each instructor.

6 Range is the highest score minus the lowest score; evaluate.

7 Interquartile range is the upper quartile minus the lower quartile score; evaluate.

8 Look at the data for any outliers or a score that is separated from the majority of the data.

9 Compare all the statistics and the effect of the outlier.

a

Bec	Rita
$\text{Mean } \bar{x} = \frac{42}{7} = 6$	$\text{Mean } \bar{x} = \frac{64}{7} = 9.1$
$4, 4, 5, 5, 8, 8, 8$	$2, 8, 9, 10, 10, 12, 14$
Median is 5.	Median is 10.

b

Bec	Rita
Range $= 8 - 4 = 4$	Range $= 14 - 2 = 12$
IQR $= 8 - 4 = 4$	IQR $= 12 - 8 = 4$

c There is an outlier (Rita's class on Sunday: 1).

The outlier has affected the calculation for the mean and the range.

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Exercise 10E

LEVEL 1

Example 8

1 Two classes in Year 11 were given a short quiz. The test was marked out of 10 and the results are listed in the table.

Score	Class A	Class B
7	4	1
8	3	3
9	5	6
10	6	8

Answer the following questions, correct to one decimal place when necessary.

a How many students are there in each class?
b What is the mean and median for class A?
c What is the mean and median for class B?
d What is the range and interquartile range for class A?
e What is the range and interquartile range for class B?
f What is the standard deviation for class A?
g What is the standard deviation for class B?
h Which class performed better in these short quizzes? Give reasons for your answer.
i Compare and contrast the two sets of data.

2 The number of males and females retiring in the past 15 years is displayed in the double stem-and-leaf plot.

Males		Females
6	0	9
8532	1	1369
9876442	2	155789
410	3	0125

Answer the following questions, correct to one decimal place when necessary.

a Find the mean and median number of male retirees.
b Find the mean and median number of female retirees.
c Find the range and interquartile range for the number of male retirees.
d Find the range and interquartile range for the number of female retirees.
e Find the standard deviation for the number of male retirees.
f Find the standard deviation for the number of female retirees.
g Compare and contrast the two sets of data.

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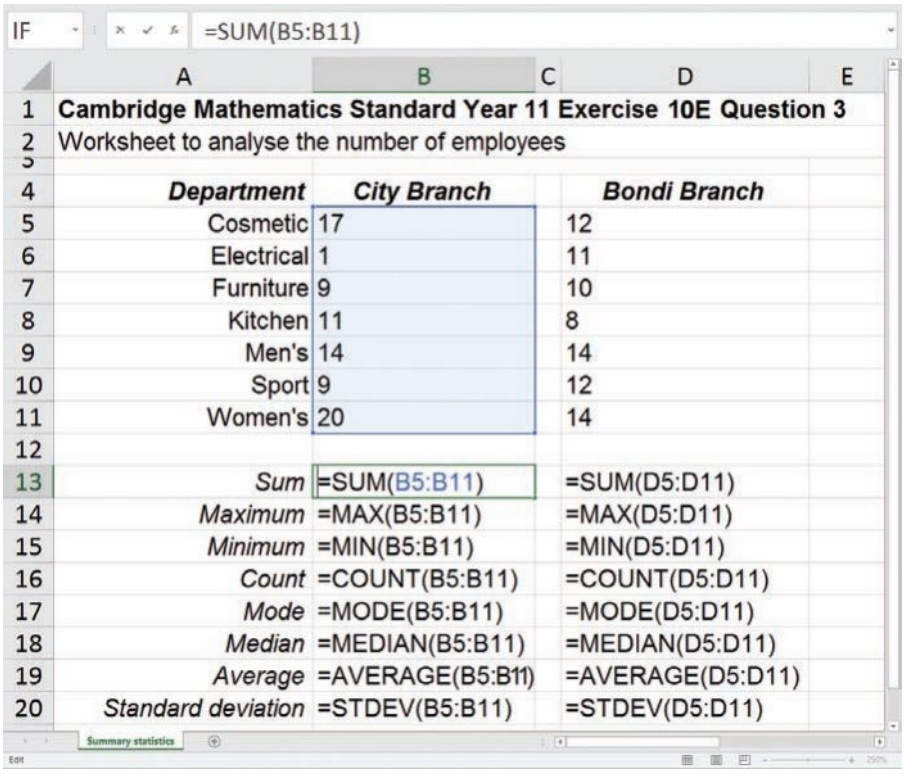
LEVEL 2

XLS 10EQ3

3 Create the spreadsheet below for the number of employees at two branches of a department store.



	A	B	C	D	E
1	**Cambridge Mathematics Standard Year 11 Exercise 10E Question 3**				
2	Worksheet to analyse the number of employees				
4	***Department***	***City Branch***		***Bondi Branch***	
5	Cosmetic	17		12	
6	Electrical	1		11	
7	Furniture	9		10	
8	Kitchen	11		8	
9	Men's	14		14	
10	Sport	9		12	
11	Women's	20		14	
12					
13	*Sum*	=SUM(B5:B11)		=SUM(D5:D11)	
14	*Maximum*	=MAX(B5:B11)		=MAX(D5:D11)	
15	*Minimum*	=MIN(B5:B11)		=MIN(D5:D11)	
16	*Count*	=COUNT(B5:B11)		=COUNT(D5:D11)	
17	*Mode*	=MODE(B5:B11)		=MODE(D5:D11)	
18	*Median*	=MEDIAN(B5:B11)		=MEDIAN(D5:D11)	
19	*Average*	=AVERAGE(B5:B11)		=AVERAGE(D5:D11)	
20	*Standard deviation*	=STDEV(B5:B11)		=STDEV(D5:D11)	

a The formulae in cells B13–B20 and D13–D20 are shown above. Enter these formulae.
b Comment on the measures of location (mean, median and mode) for the city branch.
c Comment on the measures of location (mean, median and mode) for the Bondi branch.
d Comment on the measures of spread (range and standard deviation) for the city branch.
e Comment on the measures of spread (range and standard deviation) for the Bondi branch.
f Compare and contrast the two sets of data.

LEVEL 3

4 The areas of eight suburbs of Sydney and Melbourne are listed below.

Sydney	9.0	7.4	8.2	11.5	6.4	6.3	5.6	7.3
Melbourne	3.0	7.5	9.5	25.2	4.5	3.6	2.0	14.5

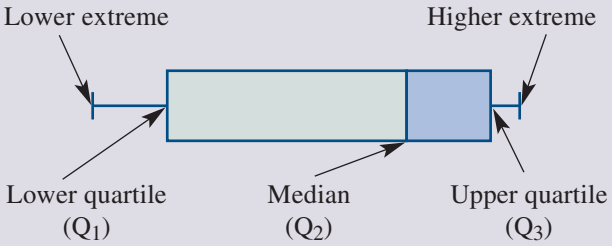
Answer the following questions, correct to one decimal place when necessary.

a Find the mean and median for each set of data.
b Which is the better measure for the centre of this data? Explain your answer.
c Find the range and interquartile range for each set of data.
d Which is the better measure for the spread of this data? Explain your answer.
e Compare and contrast the two sets of data.

10F Parallel box-and-whisker plots

A box-and-whisker plot or box plot is a graph that uses five important statistics: lower extreme (or lowest value); lower quartile (or first quartile, Q_1); median; upper quartile (or third quartile, Q_3), and the higher extreme (or highest value). These statistics are referred to as a five-number summary. A box-and-whisker plot is constructed from a scale of data values. The box is between the two quartiles with a dividing line for the median (Q_2) and the whiskers are drawn to the two extremes.

BOX-AND-WHISKER PLOT



Five-number summary

PARALLEL BOX-AND-WHISKER PLOT

Parallel box-and-whisker plots have two box plots on the same scale.
They are used to compare two sets of data.

Example 9: Constructing a parallel box-and-whisker plot 10F

Construct a parallel box-and whisker plot from this data.

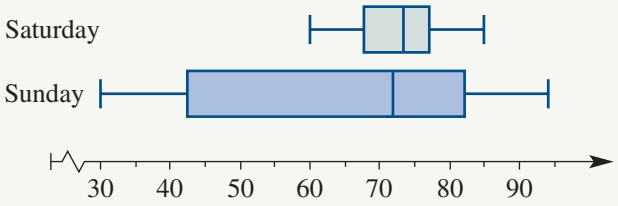
Saturday	60	62	65	69	70	72	75	75	77	78	80	85
Sunday	30	35	40	45	54	66	75	77	80	85	94	94

SOLUTION:

1. Calculate the five-number summary for each day.
2. Draw a labelled and scaled number line that covers the full range of values for each day.
3. For Saturday, draw a box starting at $Q_1 = 67$ and ending at $Q_3 = 77.5$.
4. Mark the median value with a vertical line at 73.5 (Q_2).
5. Draw in the whiskers, lines joining the midpoint of the ends of the box to the minimum and maximum values.
6. Repeat steps 3–5 to construct the box-and-whisker plot for Sunday.
7. Label each box plot.

Saturday: Min = 60
$Q_1 = 67$
Median = 73.5
$Q_3 = 77.5$
Max = 85

Sunday: Min = 30
$Q_1 = 42.5$
Median = 70.5
$Q_3 = 82.5$
Max = 94



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Using a parallel box-and-whisker plot to compare data

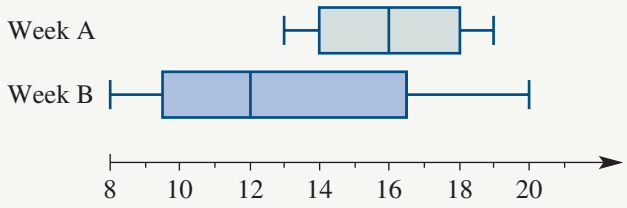
Parallel box-and-whisker plots are useful for comparing two or more sets of data collected on the same variable, such as the assessment results for two different groups of students. A double box-and-whisker plot allows the median and the spread to be easily identified and compared.

Example 10: Comparing data with a parallel box-and-whisker plot 10F

The number of births in a city hospital each day during the past two weeks is shown in the table below and summarised in the parallel box-and whisker plots.

a What is the lower extreme for week A?
b What is the upper quartile for week B?
c What is the median for week A?
d What was the lower quartile for week B?
e Compare and contrast the two sets of data.

Week A	13	13	15	15	17	18	18	19
Week B	8	9	10	12	12	15	18	20



SOLUTION:

1 Read the value from the graph.
2 Read the value from the graph.
3 Read the value from the graph.
4 Read the value from the graph.
5 Spread of the data for Week B is larger than the spread for Week A (box widths and extreme values are larger).

a Lower extreme for week A is 13.
b Upper quartile for week B is 16.5.
c Median for week A is 16.
d Lower quartile for week B is 9.5.
e The number of births each day was more consistent and the median higher in Week A. However, the largest and smallest number of births occurred in Week B.

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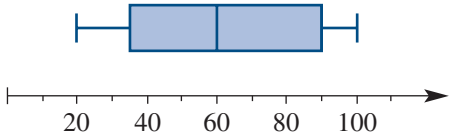
Exercise 10F

LEVEL 1

Example 10

1 From the box-and-whisker plot, find the:



- **a** lower extreme
- **b** higher extreme
- **c** median
- **d** lower quartile
- **e** upper quartile

2 Complete a five-number summary for each set of data.

- **a** 44, 46, 51, 52, 55, 57, 59, 61
- **b** 8, 10, 12, 15, 17, 24, 25, 30, 32, 42

3 Construct a box-and-whisker plot using the following five-number summaries.

- **a** Min = 3, Q_1 = 4, median = 5, Q_3 = 6 and max = 7
- **b** Min = 60, Q_1 = 64, median = 70, Q_3 = 73 and max = 80

4 **a** Given the data set 2, 5, 9, 12, 14, 17 and 19, find the:

i lower extreme **ii** upper extreme **iii** range
iv lower quartile **v** upper quartile **vi** median

b Construct a box-and-whisker plot for the above data set.

5 **a** Given the data set 42, 34, 49, 32, 50, 43, 47, 32, 38, 51, 37 and 35, find the:

i lower extreme **ii** upper extreme **iii** range
iv lower quartile **v** upper quartile **vi** median

b Construct a box plot for the above data set.

6 The heights (in centimetres) of people living in Olivia Parade are listed below.

157	168	133	135	154	167	167	123	140	157	163	176	160	165
145	149	157	154	159	180	143	154	156	123	150	143	132	140

- **a** What is the lower extreme?
- **b** What is the upper extreme?
- **c** What is the median?
- **d** What is the lower quartile?
- **e** What is the upper quartile?
- **f** Construct a box-and-whisker plot for this data set.
- **g** Describe the data in terms of shape, centre and spread.

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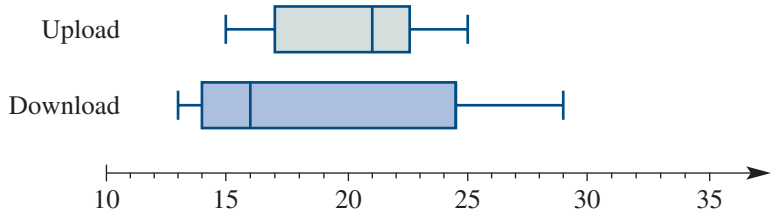
LEVEL 2

7 The parallel box-and-whisker plot shows the amount of data uploaded and downloaded by Anthony each day for the past month.



a What is the highest amount of data uploaded?
b What is the lowest amount of data downloaded?
c What is the median amount of data uploaded?
d Calculate the range for downloading data.
e What is the lower quartile for data downloaded?
f What is the upper quartile for data uploaded?
g What is the interquartile range for data uploaded?
h What is the interquartile range for data downloaded?

Example 9

8 Construct a parallel box-and-whisker plot, using the following five-number summaries.
Part 1: Min = 12, Q_1 = 14, median = 19, Q_3 = 22 and max = 26
Part 2: Min = 14, Q_1 = 19, median = 23, Q_3 = 26 and max = 29

9 Construct a parallel box-and-whisker plot using the following five-number summaries.
Aaron: Min = 20, Q_1 = 21.5, median = 26, Q_3 = 30 and max = 35
Eliza: Min = 18.5, Q_1 = 26, median = 27.5, Q_3 = 34 and max = 35

10 The ages of residents who attended two community meetings are displayed in the double stem-and-leaf plot below.

a What is the five-number summary for meeting A?
b What is the five-number summary for meeting B?
c Construct a parallel box-and-whisker plot using the data in the double stem-and-leaf plot.
d What is the interquartile range for meeting A?
e What is the interquartile range for meeting B?

Meeting A		Meeting B
6	0	9
6521	1	337
987521	2	0266899
2100	3	2335

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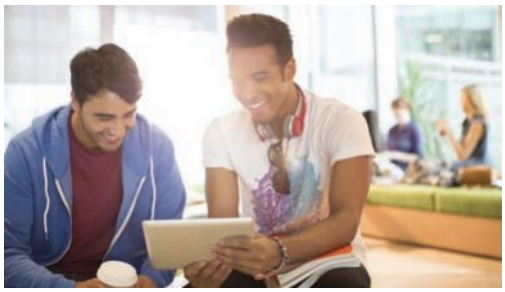
LEVEL 3

11 The TAFE results of two students are listed below.

Ryan	69	76	75	75	70	78	79	72	63	72	73	81
Tane	76	70	76	74	72	74	70	73	77	68	75	75



a What is Ryan's lower score?
b What is Tane's median?
c What is Tane's upper score?
d What is Ryan's median?
e What is Tane's lower quartile?
f What is Ryan's upper quartile?
g Construct a parallel box-and-whisker plot for the two sets of data.
h Determine the range and interquartile range for each data set.
i Who is the better student? Give a reason for your answer.

12 The approval ratings for two local politicians are listed below.

Jade	80	65	69	62	72	70	75	77	78	85	75	60
Ruby	75	85	66	35	92	94	40	30	45	54	77	80

a What is Ruby's lower extreme?
b What is Ruby's upper extreme?
c What is Jade's median?
d What is Jade's lower quartile?
e What is Jade's upper quartile?
f Construct a double box plot.
g Compare and contrast the shapes of the box-and-whisker plots.
h Who is the more popular politician? Justify your answer.

13 The ages of the people attending an environmental conference are recorded below.

Female	35	63	28	30	27	42	53	50	52	63	61	62	45	18
Male	22	63	13	20	12	22	30	25	30	52	37	46	22	12

a What is the five-number summary for females attending the conference?
b What is the five-number summary for males attending the conference?
c Construct a double box-and-whisker plot for the above data.
d Determine the range and interquartile range for each data set.
e Compare and contrast the shapes of the box-and-whisker plots.
f What conclusions can you make about the ages of females and males attending the conference?

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PPT STUDY GUIDE

Key ideas and chapter summary

Measures of central tendency

Mean

$$\text{Mean} = \frac{\text{Sum of scores}}{\text{Number of scores}} \quad \text{or} \quad \bar{x} = \frac{\sum fx}{\sum f}$$

μ – Population mean $\bar{x}$ – Sample mean

Median

The median is the middle score. To find the median, list all the scores in increasing order and select the middle one.

Mode

The mode is the score that occurs the most.

It is the score with the highest frequency.

Measures of spread

Range

Range = Highest score – Lowest score

Interquartile range

IQR = Third quartile – First quartile = $Q_3 - Q_1$

Standard deviation

Standard deviation is the spread of data about the mean.

Population standard deviation – σ_n or σ_x.

Sample standard deviation – σ_{n-1} or S_x.

Outlier

An outlier is a score that is separated from the majority of the data.

Use $Q_1 - 1.5 \times \text{IQR}$ and $Q_3 + 1.5 \times \text{IQR}$ as criteria to determine an outlier.

Describing datasets

- Shape of the graph is described in terms of smoothness, symmetry and the number of modes.
- Positively skewed data has a long tail on the right-hand side.
- Negatively skewed data has a long tail on the left-hand side.

Comparing datasets

The selection and the use of the appropriate measure of location (mean or median) and measure of spread (range, interquartile range or standard deviation) depends on the nature of the data and the relative merits of each measure.

Parallel box-and-whisker plots

Two box-and-whisker plots on the same scale. They are used to compare two sets of data.

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Review

Multiple-choice questions

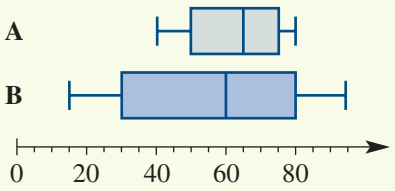
1 Sarah scored 48, 74, 89, 55, 64 and 89 in the Trial HSC. Which measure of location would Sarah prefer to tell her parents?

A Mean **B** Median **C** Mode **D** Range

2 Find the median of this data: 8, 9, 10, 6, 7, 7, 3, 4, 5, 5, 4, 4, 10

A 3 **B** 5.5 **C** 7 **D** 6

3 Two examinations results are displayed in the box plot. What is the interquartile range for exam B?



A 25 **B** 40
C 50 **D** 60

4 The table shows statistics of the results of five tests. Which student has the more consistent results?

A Alice
B Molly
C Hamza
D Ryder

Student	Mean	Standard deviation	Range
Alice	13.6	3.8	8
Hamza	13.9	4.2	8
Molly	13.8	2.7	8
Ryder	14.0	3.4	8

5 What is the difference between the range and interquartile range for the following data: 11, 14, 12, 10, 15, 16, 10, 15, 16, 20, 10

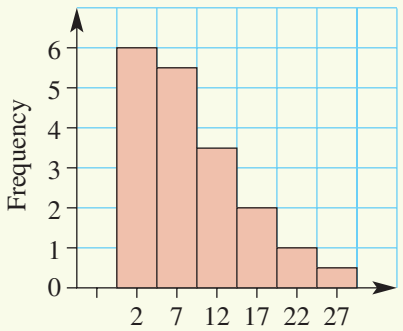
A 4 **B** 5 **C** 6 **D** 10

6 There are 7 scores in a data set. One of the scores is an outlier. Which of the mean, mode, median or interquartile range will be most influenced by the outlier?

A Mode **B** Mean **C** Median **D** Interquartile range

7 How would you describe the symmetry and skewness of the histogram opposite?

A Positively skewed
B Negatively skewed
C Symmetric
D Symmetric with a clear outlier



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Short-answer questions

1 Find the missing number in these data sets.

a The scores 10, 11, 13, 15, ☐, 16, 18, 20 and 20 have a median of 15.

b The scores 4, 4, ☐, 6, 8 and 11 have a median of 5.

c The scores 5, 6, 6, 6, ☐, 9, 10 and 11 have a median of 7.

2 Members of a swimming club recorded the following times (in minutes) at a carnival.

46	48	48	47	44	46	44	45	42	45	45	45	44	46	48	42	42
47	43	45	45	47	47	48	45	44	43	46	41	43	47	43	41	44
43	45	44	41	46	43	44	42	46	42	44	46	43	46	43	43	44

a Construct a frequency table.

b How many times were recorded?

c Add an fx column.

d What is the mean for this data? Answer correct to one decimal place.

3 Philip has completed three mathematics tests. His mean mark is 64%. What mark (out of 100) does he have to get in his next test to increase the mean to 65%?

4 Find the interquartile range for each data set.

a 18, 16, 19, 18, 16, 13, 12, 15

b 19, 22, 17, 18, 23, 15, 15, 13

c 32, 29, 24, 26, 25, 28, 29, 24, 30, 22

d 29, 37, 39, 57, 58, 34, 58, 59, 29, 31

e 43, 57, 39, 61, 31, 34, 57, 38, 41

f 40, 50, 46, 41, 46, 53, 59, 44, 46

5 The number of service calls per day made by an air-conditioning technician is recorded below:
8, 1, 6, 8, 11, 4, 9, 11, 10, 8, 7, 10, 8, 4, 5, 6, 9

a What is the range?

b What is the first quartile?

c What is the third quartile?

d What is the interquartile range?

6 The lifetimes in hours of six batteries of Brand A and Brand B are recorded below.

Brand A	74	57	61	90	89	79
Brand B	67	74	39	109	68	99

a What is the mean and sample standard deviation of each brand of battery?

b Which brand of battery has the more consistent lifetimes? Explain your answer.

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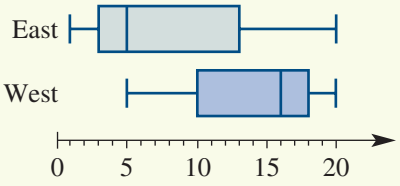
Review

7 The time taken (in minutes) for Liam to travel to work each day for 2 weeks is shown below.

Week 1	32	31	35	33	35	59	40
Week 2	38	30	32	37	41	39	42

a Draw a parallel box plot.
b What is the median of the scores in week 1?
c What is the median of the scores in week 2?
d What is the interquartile range for Week 1?
e What is the interquartile range for Week 2?
f Comment on the skewness of the scores in Week 1

8 The results of a survey are shown in the parallel box-and-whisker plot.



a What is the range of the results in the east?
b What is the range of the results in the west?
c What is the median of the results in the east?
d Describe the skewness of the results in the east.
e Describe the skewness of the results in the west.
f Is 20 an outlier for the East data? Show using calculations.

Extended-response question

9 A survey was conducted with 20 students to see if they took longer to perform a study task when listening to music than when not listening to music. The data is presented below.

Five-number summary	Listening to music (min)	Not listening to music (min)
Lower extreme	17	13
Lower quartile	19	18
Median	24	21
Upper quartile	32	26
Upper extreme	33	30

a Draw a parallel box-and-whisker plot.
b Consider the following comment made by one of the students.
'It does not take any longer to complete a study task if you are listening to music.'
Is this a reasonable comment based on the data? Use the box plots to compare and contrast the data. Fully justify your response.

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11 Working with time

Syllabus topic — MS-M2 Working with time

This topic will develop your understanding of the concepts related to locations on the Earth's surface and calculation of distances and time differences using latitude, longitude and time zones.

Outcomes

- Convert units of time including 12-hour and 24-hour clocks.
- Interpret timetables and use them to solve problems.
- Locate positions on the Earth's surface, using latitude and longitude.
- Find time differences between two places on Earth using time zones.
- Solve problems involving Coordinated Universal Time and the International Date Line.
- Solve practical problems using time zones in Australia and in neighbouring nations.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Solutions (enabled by teacher)
- Literacy worksheet
- Widgets
- Quick Quiz
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

In the Interactive Textbook you can take a test of prior knowledge required for this chapter, and depending on your score you may be directed to revision from the previous years' work.

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11A Units of time

Time is the ongoing sequence of events taking place: the past, present and future. We measure time using seconds, minutes, hours, days, weeks, months and years. One year is the time it takes Earth to travel around the Sun. This is approximately $365\frac{1}{4}$ days. There are 30 days in September, April, June and November; all the rest have 31 except for February, which has 28 days or 29 days in a leap year. Clocks measure time.

Time units

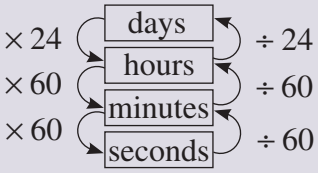
The table below outlines the time conversions for various time units.

Quantity	Name of unit	Symbol	Value
Time	Second	s	Base unit
	Minute	min	1 min = 60 s
	Hour	h	1 h = 60 min
	Day	d	1 d = 24 h

Converting between units

The diagram below outlines the time conversions for various units. Follow the directions along the arrows until you reach the desired unit. Most scientific calculators have a degrees-minutes-seconds key ,,,, or DMS that can be used to convert between hours, minutes and seconds.

TIME CONVERSIONS



Example 1: Converting units of time 11A

Complete **a** 3 h and 15 min = ☐ min **b** 10 080 min = ☐ d

SOLUTION:

1 To change hours to minutes multiply by 60.
2 To change minutes to hours divide by 60.
3 To change hours to days divide by 24.

a 3 h 15 min = $3 \times 60 + 15$ min = 195 min

b 10 080 min = $10\,080 \div 60$
= 168 h
= $168 \div 24 = 7$ d

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24-hour time

The time of day is written in the 24-hour notation in the form hh:mm (hours:minutes). The day begins at midnight (00:00) and the last minute of the day begins at 23:59. The advantage of the 24-hour clock is that it clearly distinguishes the time in the morning and the afternoon without the a.m. and p.m. notation.

The 24-hour notation is used in many applications such as timetables, oven clocks and airport arrivals and departures.

12-hour clock	24-hour clock	12-hour clock	24-hour clock
12:00 (midnight)	00:00	12:00 (noon)	12:00
1:00 a.m.	01:00	1:00 p.m.	13:00
2:00 a.m.	02:00	2:00 p.m.	14:00
3:00 a.m.	03:00	3:00 p.m.	15:00
4:00 a.m.	04:00	4:00 p.m.	16:00
5:00 a.m.	05:00	5:00 p.m.	17:00
6:00 a.m.	05:00	6:00 p.m.	18:00
7:00 a.m.	07:00	7:00 p.m.	19:00
8:00 a.m.	08:00	8:00 p.m.	20:00
9:00 a.m.	09:00	9:00 p.m.	21:00
10:00 a.m.	10:00	10:00 p.m.	22:00
11:00 a.m.	11:00	11:00 p.m.	23:00
12:00 a.m.	12:00	12:00 p.m.	24:00

24-HOUR TIME

24-hour notation is in the form hh:mm (hours:minutes).

Example 2: Using 24-hour time 11A

a Convert 8:15 p.m. to 24-hour time.
b Write 21:45 in 12-hour time.
c How long is it between 18:00 and 23:15?

SOLUTION:

1 8:15 p.m. is in the afternoon. Add 12 to the hour (8).
2 21:45 is a time in the afternoon. Subtract 12 from the hour (21).
3 Subtract the two times to calculate the time difference.
4 Write the answer in words.

a 20:15
b 9:45 p.m.
c 23:15 – 18:00 = 5:15
It is 5 hours and 15 minutes between the two times.

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Exercise 11A

LEVEL 1

Example 1

1 Complete the following

a 2.5 h = ☐ min
b 2 min = ☐ s
c 20 d = ☐ h
d 40 min = ☐ s
e 4.5 d = ☐ h
f 10 h = ☐ min
g 720 min = ☐ h
h 48 000 s = ☐ min
i 96 h = ☐ d
j 1080 h = ☐ d
k 390 min = ☐ h
l 780 s = ☐ min

2 What unit of time is most appropriate to measure each of the following?

a Lesson at school
b Reheating a meal in a microwave
c Age of a person
d School holidays
e Accessing the internet
f Movie

3 Find the number of seconds in:

a 1 hour
b 1 day
c 1 year

4 What time will it be:

a 4 hours after 5:30 p.m.?
b 7 hours before 1:00 p.m.?
c 17 hours before midday?
d 15 hours after 2:00 a.m.?
e 3.5 hours after 1:00 p.m.?
f 2.25 hours before 4:00 a.m.?

5 The time required to bake a cake in an oven is 1 h 9 min. When will the cake be finished if the starting time are as follows?

a 5:00 a.m.
b 9:00 a.m.
c 8:20 p.m.
d 10:34 a.m.
e 5:44 p.m.
f Midnight
g 3:56 p.m.
h 10:51 a.m.

6 Add these times.

a 2 h 24 min + 2 h 14 min
b 4 h 04 min + 1 h 46 min
c 5 h 30 min + 3 h 30 min
d 7 h 41 min + 5 h 50 min
e 11 h 29 min + 6 h 55 min
f 9 h 09 min + 3 h 59 min

7 Subtract these times.

a 10 h 47 min – 2 h 22 min
b 4 h 52 min – 1 h 40 min
c 6 h 20 min – 5 h 08 min
d 8 h 10 min – 4 h 20 min
e 5 h 34 min – 2 h 18 min
f 10 h 46 min – 7 h 56 min

8 Find the difference between these times.

a 8:00 p.m. and 11:00 p.m.
b 2:15 a.m. and 9:25 a.m.
c 1:43 p.m. and 12:40 p.m.
d 11:05 a.m. and 9:35 a.m.
e 1:30 p.m. and midnight
f 6:23 a.m. and 5:25 p.m.

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9 What is the time:

a 11 hours before 2 a.m.?

b 5 hours after 7 p.m.?

c $1\frac{1}{2}$ hours after 6:30 a.m.?

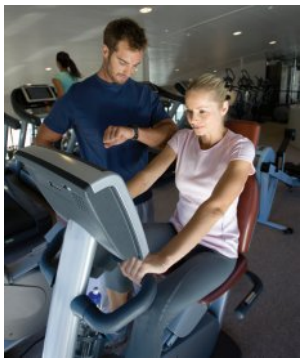
d $2\frac{1}{3}$ hours before 3:30 p.m.?

e 4 h and 11 min after 1:45 p.m.?

f 12 h and 50 min after 6:23 a.m.?

10 Lauren is a cyclist who recorded the following training times at the gym.

1 h 21 min	59 min
2 h 05 min	1 h 47 min
45 min	1 h 35 min



a What is the total training time?

b What is the mean training time?

c What is the median training time?

Example 2

11 Write in 24-hour time.

a 2:00 a.m.

b 10:30 p.m.

c 8:41 p.m.

d 11:12 p.m.

e 1:59 a.m.

f Midnight

g 9:09 p.m.

h 4:39 p.m.

i 5:15 a.m.

12 Write in 12-hour time.

a 03:18

b 10:40

c 12:00

d 17:56

e 09:11

f 23:00

g 18:45

h 03:20

i 20:50

13 What is the time 5 hours and 29 minutes after the following times?

a 07:00

b 13:19

c 22:41

14 What is the time 6 hours and 20 minutes before the following times?

a 15:45

b 14:56

c 02:30

15 Ivy leaves home at 09:28 to go shopping. She returns at 13:48. How long, in hours and minutes, had she been away?

16 A game of football starts at 14:12 and finishes at 15:59. What is the length of the game? Answer in hours and minutes.

17 Alexis's dad is going to pick her up from the cinema once the film has finished. If the film starts at 17:35 and is 99 minutes long, at what time will he need to pick her up?

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LEVEL 2

18 Copy and complete the following table for a Blu-ray player that uses 24-hour time.

	Program (12-hour time)	Time settings (24-hour time)
a	6:00 a.m. to 7:38 a.m.	
b	10:25 a.m. to 12:35 p.m.	
c	3:20 p.m. to 4:54 p.m.	
d	6:30 p.m. to 8:05 p.m.	
e		07:50 to 10:00
f		12:45 to 14:10
g		00:20 to 15:49
h		23:10 to 02:07

19 A movie runs for 212 minutes

- **a** Convert this time to hours and minutes.
- **b** It takes Sophie 34 minutes to drive to the cinema from home. What is the minimum time Sophie needs to leave home if the movie starts at 16:08?
- **c** What is the finishing time of the movie if the movie starts at 14:35?
- **d** What is the starting time if the movie finishes at 11:10?

20 Sydney airport shows the arrival of a number of international flights.

- **a** What is the arrival time of the London flight in 12-hour time?
- **b** How long is there between the flight from Toronto and the flight from New York?
- **c** How long ago did the Rome flight arrive if the current time is 8:35 p.m.?
- **d** How long ago did the Dubai flight arrive if the current time is 9:45 p.m.?

Arrival

FLIGHT	FROM	LANDED	GATE
DB692	LONDON	1632	A2
AK721	NEW YORK	1735	B6
CS169	DUBAI	1805	A5
AL379	ROME	1855	B2
AF4485	PARIS	1900	A1
QA034	MADRID	1910	B4
CA456	TORONTO	2030	B3

LEVEL 3

21 Eliza worked from 10:30 a.m. until 4:00 p.m. on Friday, from 7:30 a.m. till 2:00 p.m. on Saturday, and from 12 noon till 5:00 p.m. on Sunday.

- **a** How many hours did Eliza work during the week?
- **b** Express the time worked on Friday as a percentage of the total time worked during the week. Answer correct to the nearest whole number.

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11B Timetables

A timetable is a list of times at which possible events or actions are intended to take place. There are many different types of timetables, such as your school timetable, train timetables or airline departures. Timetables are written using 12-hour or 24-hour format.

TIMETABLES

A timetable is a list of times at which possible events or actions are intended to take place.

Example 3: Reading and interpreting a train timetable 11B

This timetable is part of a rail timetable from Parramatta to Central.

T1 North Shore & Northern Line: City to Berowra via Gordon, City to Hornsby via Macquarie University

Services departing Parramatta from	03:00	06:00	09:00	12:00	15:00	18:00	21:00	00:00
Stations								
Parramatta	36	---	15:46	15:44	15:52	---	15:59	
Harris Park	-	---	---	15:45	---	---	16:00	
Granville	-	---	---	15:48	---	---	16:03	
Clyde	-	---	---	15:50	---	---	16:05	
Auburn	-	---	---	15:52	---	---	16:07	
Lidcombe	43	---	---	15:55	15:59	---	16:10	
Strathfield	50	15:53	15:58	16:02	16:06	16:08	16:17	
Burwood	15:52	15:55	---	16:04	---	16:10	16:19	
Redfern	16:01	16:04	16:10	16:13	16:16	16:19	16:28	
Central	16:05	16:08	16:14	16:17	16:20	16:23	16:32	

EARLIER TRAINS

a How long does the train take to travel from Granville to Burwood?

b At what time does the 15:59 train from Parramatta depart Strathfield?

c How long does the 15:46 train take to go from Parramatta to Redfern?

d Alan wants to travel from Parramatta and get to Central before 16:30. Describe the different options shown on this timetable.

SOLUTION:

1 Departs Granville at 15:48 and arrives at Burwood at 16:04. Subtract the times.

2 Read the right-hand side of the timetable.

3 Departs Parramatta at 15:46 and arrives at Redfern at 16:10 . Subtract the times.

4 Read the bottom of the timetable to determine which trains arrive before 16:30. Check that the trains depart Parramatta.

a Time taken = 16:04 – 15:48 = 16 minutes

b The 15:59 train departs Strathfield at 16:17.

c Time taken = 16:10 – 15:46 = 24 minutes

d Alan could catch the all stations train departing at 15:44 and arriving at 16:17. Alternatively fast trains depart at 15:46 arriving at 16:14 and at 15:52 arriving at 16:20.

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Example 4: Reading and interpreting a train timetable 11B

The timetable opposite is part of a rail timetable from Condobolin to Stockinbingal.

a How long does the trip take from Condobolin to Stockinbingal?

b Nina joins the train at Lake Cargelligo and gets off at Temora. How long is her trip?

c Another train leaves Condobolin at 12:45 p.m. and travels at the same speed and stops at the same stations. When will this train arrive at Stockinbingal?

Destination	Coota-mundra
Days of Operation	Daily
Service No.	718
Condobolin	07:25
Euabalong West	08:30
Euabalong	08:35
Lake Cargelligo	09:00
Tullibigeal	09:30
Ungarie	10:00
West Wyalong	10:35
Wyalong	10:40
Barmedman	11:00
Temora	11:30
Stockinbingal	12:00

SOLUTION:

1 Departs Condobolin at 07:25 and arrives at Stockinbingal at 12:00.

2 Departs Lake Cargelligo at 09:00 and arrives at Temora at 11:30.

3 Add the time taken for the trip in part **a** to the departure time of 12:45 p.m.

a Time taken = 12:00 – 07:25
= 4:35 or 4 h and 35 min

b Time taken = 11:30 – 09:00
= 2:30 or 2 h and 30 min

c Trips takes 4 h and 35 min
Arrival time = 12:45 + 04:35 = 05:20
∴ Train arrives at 5:20 p.m.

Example 5: Reading and interpreting a bus timetable 11B

Part of a bus timetable is shown opposite.

a Where does the 373 bus start and finish?

b Where is stop 200 081?

c When does the bus arrive at St James?

d How long does it take to travel from Circular Quay to Museum station at Liverpool St?

10:59 **Circular Quay, Alfred St, Stand A** (Stop 2000147)

B **373**

City Circular Quay to Coogee via Belmore Rd service

Hide stop sequence

Time	Stop name
10:59	Sydney, Circular Quay, Alfred St, Stand A (Stop 2000147)
11:01	Sydney, Phillip St opp Museum of Sydney (Stop 200068)
11:03	Sydney, Martin Place Station, Elizabeth St, Stand F (Stop 200081
11:06	Sydney, St James Station, Elizabeth St (Stop 200045)
11:09	Sydney, Museum Station, Elizabeth St, Stand C (Stop 200073)
11:11	Sydney, Museum Station, Liverpool St, Stand A (Stop 200071)

SOLUTION:

1 Read timetable under bus number 373.

2 Read right-hand side of the timetable.

3 Read left-hand side of the timetable.

4 Departs Circular Quay at 10:59 and arrives at Museum station at 11:11.

5 Subtract the times.

a Circular Quay to Coogee

b Martin Place Station

c St James station at 11:06

d Time taken = 11:11 – 10:59
= 12 minutes

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Exercise 11B

LEVEL 1

Example 3 **1** Use the train timetable below to answer the following questions.

T4 Eastern Suburbs & Illawarra Line: Bondi Junction to Waterfall or Cronulla

Services departing Bondi Junction from	03:00	06:00	09:00	12:00	15:00	1
Stations						
Bondi Junction	22:35	22:45	22:55	23:05	---	
Edgecliff	22:38	22:48	22:58	23:08	---	
Kings Cross	22:41	22:51	23:01	23:11	---	
Martin Place	22:43	22:53	23:03	23:13	---	
Town Hall	22:46	22:56	23:06	23:16	---	
Central	22:48	22:59	23:09	23:18	23:23	
Redfern	---	23:01	23:11	---	23:26	
Sydenham	---	23:06	23:16	---	---	
Tempe	---	---	23:19	---	---	
Wolli Creek	---	23:09	23:20	---	23:35	
Arncliffe	---	---	23:22	---	---	

EARLIER TRAINS

a Find the time the 23:06 train from Town Hall arrives at:
i Redfern **ii** Tempe **iii** Arncliffe

b How long does it take for the 23:05 to travel from Bondi Junction to:
i Kings Cross? **ii** Martin Place? **iii** Central?

c How long does it take for the 22:58 train from Edgecliff to reach Arncliffe?

d How long does the train leaving Kings Cross at 22:51 take to get to Sydenham?

e At how many stations will the 22:45 train from Bondi Junction stop at after leaving Bondi Junction?

f Maya needs to be in Central by 23:00. What is the latest train from Bondi Junction she can take to get there on time?

g Dylan gets to Wolli Creek on time at 23:20. At what time did that train leave Martin Place?

Example 5 **2** Use the ferry timetable on the right to answer the following questions.

a Where does the F2 ferry start?

b Where is stop 20883?

c When does the ferry arrive at Mosman?

d How long does it take to travel from Circular Quay to Mosman at 11:00?

11:00 **Circular Quay, Wharf 4** (Stop 20005)

F **F2 Taronga Zoo**

Circular Quay to Taronga Zoo service

Hide stop sequence

Time	Stop name
11:00	Sydney, Circular Quay, Wharf 4 (Stop 20005)
11:12	Mosman, Taronga Zoo Wharf (Stop 20883)

3 Use the bus timetable below to answer the following questions.

	Monday to Friday	♿	♿	♿	♿	♿	♿	♿	♿	♿	♿	♿
map ref	Route Number	426	423	426	423	426	423	426	423	426	423	426
A	**Kingsgrove Bus Depot** Omnibus Road	...	04:10	...	04:56	...	05:20	...	05:40	...	06:00	...
B	**Earlwood** Homer Street & Earlwood Avenue	...	04:18	...	05:04	...	05:28	...	05:48	...	06:09	...
C	**Undercliffe** Cooks River - Illawarra Road	...	04:23	...	05:09	...	05:33	...	05:53	...	06:15	...
D	**Dulwich Hill** Marrickville & New Canterbury Rds	03:51	...	05:02	...	05:20	...	05:40	...	05:58	...	06:20
E	**Marrickville** Marrickville Road & Illawarra Road	03:54	04:27	05:05	05:13	05:23	05:37	05:43	05:57	06:01	06:20	06:24
F	**Newtown Station** Enmore Road	04:02	04:35	05:13	05:21	05:31	05:45	05:51	06:06	06:10	06:29	06:33
G	**The University of Sydney** City Rd & Butlin Ave	04:05	04:38	05:16	05:24	05:34	05:48	05:54	06:10	06:14	06:33	06:37
H	**Central Station** Railway Square	04:09	04:42	05:20	05:28	05:38	05:52	05:58	06:15	06:19	06:38	06:42
I	**City** Sheraton on the Park, Elizabeth Street	04:18	04:51	05:29	05:37	05:47	06:01	06:07	06:24	06:28	06:47	06:51
J	**City - Martin Place** Elizabeth Street	04:21	04:54	05:32	05:40	05:50	06:04	06:11	06:28	06:32	06:51	06:55

a At what time does the earliest 423 bus leave Earlwood?

b How many 423 and 426 buses take passengers to the city from Newtown departing between 5:00 a.m. and 6:00 a.m.?

c What is the latest time to catch a bus from Dulwich Hill and get to Martin Place by 6:30 a.m.?

d Edward is waiting at the Marrickville stop for a bus to the city on Tuesday at 6:00 a.m. What is the earliest time he can expect to be in Martin Place?

e How long does it take to travel on the 06:00 bus from Kingsgrove to Martin Place?

4 Use the Manly to Darling Harbour fast ferry timetable to answer the following questions.

Darling Harbour	Pyrmont Bay	North Sydney	Circular Quay	Manly Wharf 3	Manly Wharf 2	North Sydney	Pyrmont Bay	Darling Harbour
-	-	-	-	6:55	-	7:15	7:28	7:35
-	-	-	-	7:25	-	7:45	7:59	8:02
-	-	-	-	7:55	-	8:17	8:32	8:39
7:35	-	-	-	8:15	-	8:33	8:45	8:53
8:05	-	-	-	8:45	-	9:05	9:19	9:25
16:40	16:47	17:00	17:10	-	17:29	-	-	-
17:15	17:22	17:35	-	17:55	-	-	-	18:25
17:45	17:52	18:05	-	18:25	-	-	-	19:00
18:15	18:22	18:35	-	18:55	-	-	-	19:30
18:35	18:42	18:55	-	19:15	-	-	-	19:45
19:05	19:11	19:24	19:29	-	19:49	-	-	-

a How long does it take to travel on the 7:35 ferry from Darling Harbour to Manly?

b How long does it take to travel on the 19:05 ferry from Darling Harbour to Manly?

c What is the latest time to catch a ferry from Manly and get to Darling Harbour by 9:00?

d What is the latest time to catch a ferry from Pyrmont Bay and get to Manly by 18:45?

e Eve is waiting at North Sydney for a ferry to Darling Harbour at 8:00 a.m. How long does she need to wait?

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5 Use the airline timetable shown (ignore differences in time zones) to answer the following questions.

a What is the scheduled departure time in 12-hour format?

b From where is airline flight JQ27 departing and where is it arriving?

c How long is flight JQ27?

JQ27* Sydney (Terminal 1) to Phuket

Scheduled:	15:10	Depart Sydney **15:10** Local Time	Arrive Phuket **20:20** Local Time
Check in:	K		
Gate:	58		

6 Use the airline timetable shown (ignore differences in time zones) to answer the following questions.

a What is the scheduled arrival time in 12-hour format?

b From where is airline flight SQ232 departing and where is it arriving?

c How long is flight SQ232?

SQ232* Sydney (Terminal 1) to Singapore

Scheduled:	12:15	Depart Sydney **12:15** Local Time	Arrive Singap... **17:35** Local Time
Check in:	J		
Gate:	61		

7 Use the airline timetable shown (ignore differences in time zones) to answer the following questions.

a From where is airline flight JL4026 departing and where is it arriving?

b How long is flight JL4026 on the 25 Mar?

c How long is flight JL4018 on the 25 Mar?

d How long is flight JL4018 on the 26 Mar?

e From where is airline flight QF0026 departing and where is it arriving?

f How long is flight QF0026 on the 25 Mar?

Tokyo to

Singapore					
JL4026*	0005	0645	Daily	-	until 25 Mar
"	0005	0615	Daily	-	from 26 Mar
JL4018*	1050	1730	Daily	-	until 25 Mar
"	1130	1735	Daily	-	from 26 Mar
Sydney					
QF0026	2200	0935+1	Daily	-	until 25 Mar
"	2200	0925+1	12345·7	-	from 26 Mar
"	2200	0825+1	123··67	-	from 01 Apr

8 Use the airline timetable below (ignore differences in time zones) to answer the following questions.

QF1* Sydney (Terminal 1) to London via Dubai

QANTAS

Scheduled:	16:45	Depart Sydney **16:45** Local Time	Via Dubai **00:40** Local Time / **01:40** Local Time	Arrive London **06:50** Local Time
Check in:	C			
Gate:	10			

a From where is airline flight QF1 departing and where is it arriving?

b How long is flight QF1 from Sydney to Dubai?

c How long is flight QF1 from Dubai to London?

d For how long does flight QF1 stopover in Dubai?

e How long is flight QF1?

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LEVEL 2

Example 4 **9** The timetable opposite is part of a rail timetable from Moree Town to Grafton.

a How many stops are there after leaving Moree Town up to Grafton?

b How long does the trip take from Moree Town to Grafton?

c Another train leaves Moree Town at 17:53 and travels at the same speed and stops at the same stations. When will this train arrive at Grafton?

d How long does the trip take from Warialda to Jackadgery?

e Layla joins the train at Inverell and gets off at Grafton. How long is her trip?

f Another train leaves Inverell at 06:43 and travels at the same speed and stops at the same stations. When will this train arrive at Grafton?

Destination	Grafton
Days of Operation	Mon/Wed/Fri
Service No.	142
Moree Town	11:05
Moree	11:10
Biniguy	11:40
Gravesend	11:50
Warialda	12:10
Delungra	12:30
Mount Russell turn off	12:40
Inverell *arr*	12:55
Inverell *dep*	13:40
Glen Innes	14:36
Gibraltar Range	15:26
Jackadgery	16:05
Grafton	16:40

LEVEL 3

10 Research the public transport system in NSW to determine the best type of transport, the travelling time and the cost of public transport for the following trips.

a Sydney at Central to the Blue Mountains

b Sydney at Central to the Bradman museum in Bowral

c Sydney at Central to Hawkesbury River

d Dubbo zoo to Sydney airport

e Byron bay to Newcastle

f Parliament house in Canberra to Sydney Olympic park

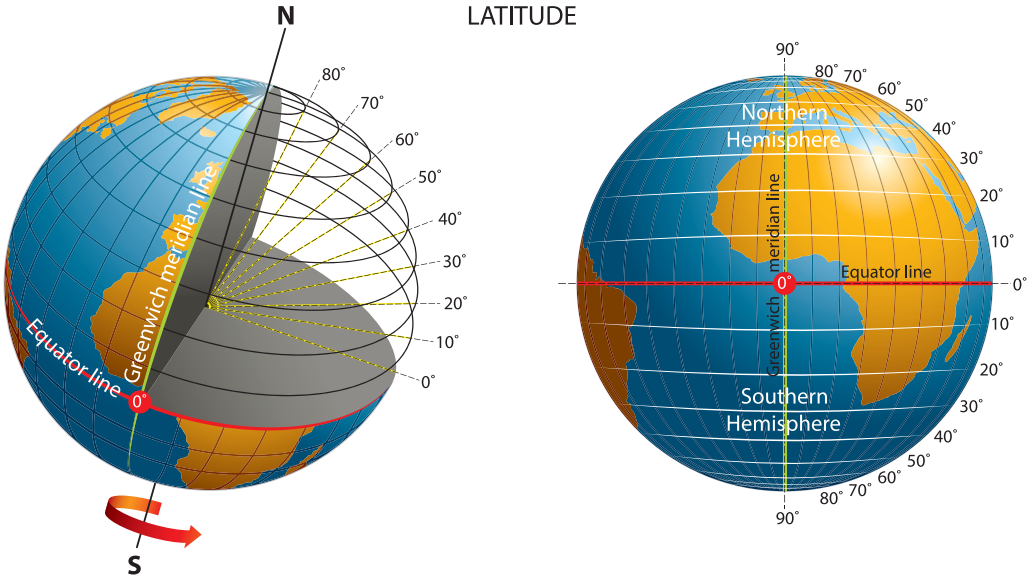
g Armidale to Parkes Observatory

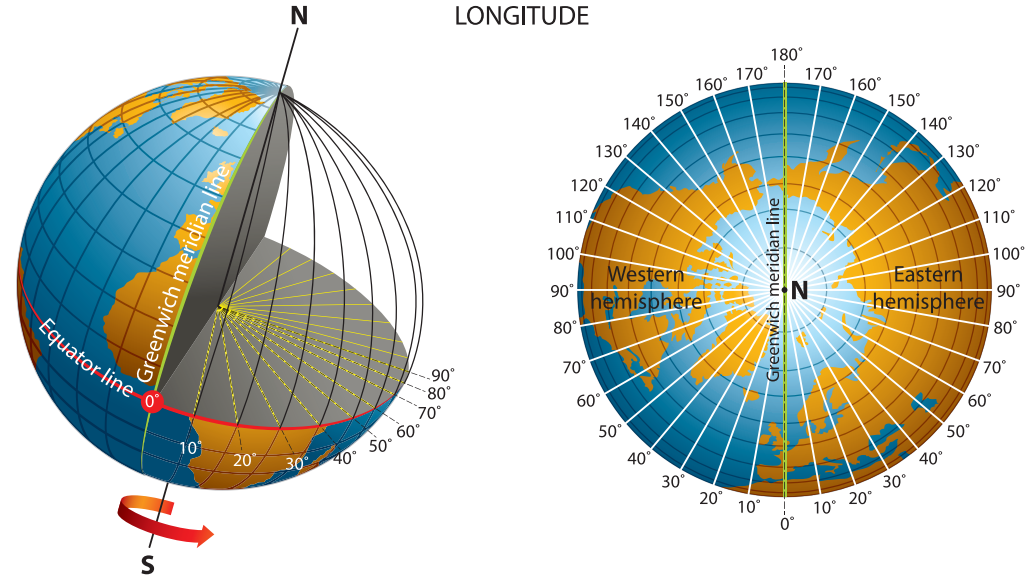
h Wollongong to Tamworth

11C Latitude and longitude

Earth's surface is divided into parallels of latitude and meridians of longitude, which are imaginary lines on the Earth's surface, forming circles and a grid. This imaginary grid is used to specify the position of a point on the Earth's surface. Latitude is an angle north or south of the equator, and longitude is an angle east or west of the Greenwich meridian. The angle of latitude is the angle at the Earth's centre between the plane of the equator and a line from the centre to the parallel of latitude. The angle of longitude is likewise formed at the Earth's centre between the plane of the Greenwich median and a line from the centre of the Earth to the meridian of longitude. The Greenwich or prime meridian is an imaginary north–south line that passes through the town of Greenwich (London).

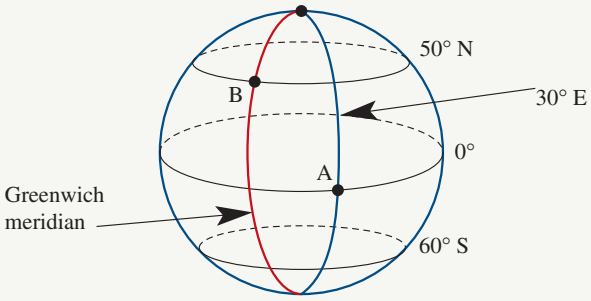
Latitude is given first (N or S) and then the longitude (E or W). For example, the position of Sydney is (33°S, 151°E). It is 33° south of the equator and 151° east of the Greenwich meridian.





Example 6: Finding a position on Earth's surface 11C

A diagram of Earth's surface is shown.



a What is the latitude and longitude of point A?
b What is the latitude and longitude of point B?
c What are the coordinates of a point 60° due south of point A?
d What are the coordinates of a point 10° due east of point B?

SOLUTION:

1. Latitude is the angle north or south of the equator (0°). Point A is on the equator.
2. Longitude is the angle east or west of the Greenwich meridian (30°E).
3. Latitude is the angle north or south of the equator (50°N).
4. Longitude is the angle east or west of the Greenwich meridian (0°). Point B is on the Greenwich meridian.
5. Point A is on the equator (0°) hence the required point is 60°S. It has the same longitude as point *A* (30°E).
6. Point B is on the Greenwich meridian (0°) hence the required point is 10°E. It has the same latitude as point B (50°N).

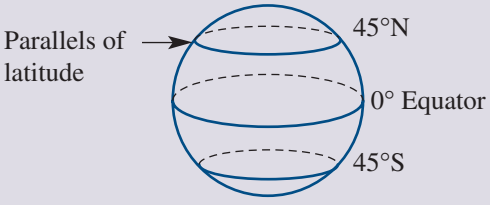
a Point A is at (0°, 30°E).

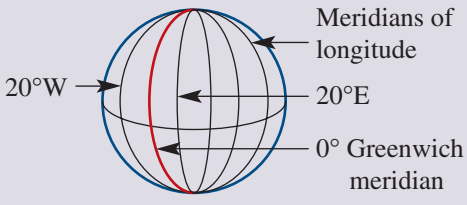
b Point B is at (50°N, 0°).

c Point A is at (60°S, 30°E).

d Point B is at (50°N, 10°E).

Latitude	Longitude
Measures the angular distance north and south of the equator (0°)	Measures the angular distance east and west of the Greenwich meridian (0°)





Example 7: Applying latitude and longitude 11C

Brisbane is located at (27°S, 153°E). Perth is located at (31°S, 115°E).

a What is the difference in longitude between Brisbane and Perth?

b Which city is closer to the equator?

SOLUTION:

1 Longitude of Brisbane is 153°E and Perth is 115°E. Both cities are east of the Greenwich meridian.

2 Subtract the angles to find the difference in longitude.

3 Latitude of Brisbane is 27°S and Perth is 31°S. Both cities are south of the equator.

4 Smaller angle is closer to the equator.

a $\text{Longitude difference} = 153 - 115$
$= 38 \text{ degrees}$

b Brisbane is closer to the equator (smaller angle).

Exercise 11C

LEVEL 1

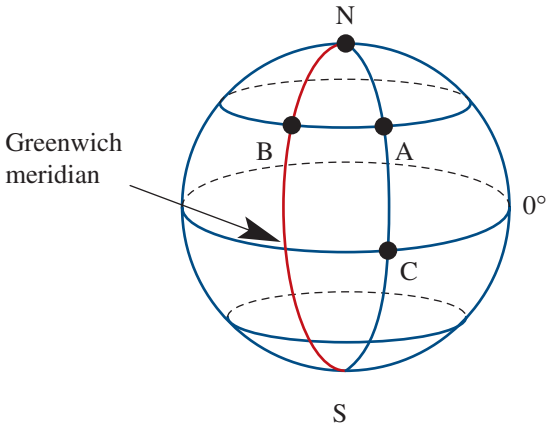
Example 6

1 A diagram of Earth's surface is shown below. The coordinates of point A are (60°N, 70°E).

a What are the coordinates of point B?

b What are the coordinates of point C?

c What is the maximum value possible for latitude? Where does it occur?



Example 7

2 Brussels is located at (50°N, 4°E). Lima is located at (12°S, 77°W).

a Which city is closer to the equator?

b What is the difference in latitude between Brussels and Lima?

c What is the difference in longitude between Brussels and Lima?

3 Tehran is located at (35°N, 51°E). Wellington is located at (41°S, 174°E).

a Which city is closer to the Greenwich meridian?

b What is the difference in latitude between Teheran and Wellington?

c What is the difference in longitude between Teheran and Wellington?

4 Nagoya is located at (35°N, 136°E). Determine the coordinates of a point that is:

a 20° due south of Nagoya

b 20° due north of Nagoya

c 20° due west of Nagoya

d 20° due east of Nagoya.

5 Belgrade is located at (44°N, 20°E). Determine the coordinates of a point that is:

a 40° due south of Belgrade

b 40° due north of Belgrade

c 40° due west of Belgrade

d 40° due east of Belgrade.

6 Munich has the coordinates (48°N, 11°E) and Paris has the coordinates (48°N, 2°E)

a What do these cities have in common in terms of their position on Earth's surface?

b What is the longitude of a point 20° due south of Munich?

c Which city is further from the Greenwich meridian?

d What are the coordinates of a point 80° due east of Paris?

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LEVEL 2

7 The following table shows the latitude and longitude of cities around the world.

City	Latitude	Longitude
Adelaide	34°S	139°E
Athens	37°N	23°E
Birmingham	52°N	1°W
Cairo	30°N	31°E
Montevideo	34°S	56°W
Rio de Janeiro	22°S	43°W
Sofia	42°N	23°E
Warsaw	52°N	21°E
Kuala Lumpur	3°N	101°E

a Which city is closest to the following latitudes?
- **i** 10°N
- **ii** 27°S
- **iii** Equator
- **iv** Latitude of Athens

b Which city is closest to the following longitudes?
- **i** 121°E
- **ii** 50°W
- **iii** Greenwich meridian
- **iv** Longitude of Sofia

c List the cities in the Northern Hemisphere.

d List the cities in the Western Hemisphere.

e Which cities have the same latitude?

f Which cities have the same longitude?

g Which city is closest to the North Pole?

h Which city is closest to the South Pole?

LEVEL 3

8 Hobart is located at (42°S, 147°E) and Port Moresby is located at (9°S, 147°E).

a Describe the relative positions of these cities on Earth's surface.

b What are the coordinates of a point half-way between these cities?

9 Kingston is located at (17°N, 76°W). Jakarta is located at (6°S, 106°E).

a What is the difference in latitude between Kingston and Jakarta?

b What is the difference in longitude between Kingston and Jakarta?

c What are the coordinates of a point 20° due south of Kingston?

d What are the coordinates of a point 80° due east of Jakarta?

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11D Time zones

A time zone is a region of the Earth that has a uniform standard time or local time. There are 24 time zones (from −12 to +12) roughly divided by longitude (see the table of international time zones on page 394). The time zone number indicates how many hours to add or subtract from the Coordinated Universal Time (UTC). The UTC reflects the mean solar time along the Earth's prime meridian and is still widely known as the Greenwich Mean Time (GMT). The International Date Line is an imaginary line through the Pacific Ocean that corresponds to 180° longitude. It runs from the North Pole to the South Pole and defines the change of one calendar day to the next.

TIME ZONE

Everywhere within a time zone has the same time.

- Positive time zone number – ahead of UTC or GMT. Add the time.
- Negative time zone number – behind of UTC or GMT. Subtract the time.

Australian time zones

Australia has three time zones called the Eastern Standard Time (+10), Central Standard Time (+9.5) and Western Standard Time (+8). On the map below, DST stands for Daylight Saving Time.

Example 8: Solving problems using a time zone 11D

Hobart is located in the Australian Eastern Standard Time zone (+10) and Greenwich is at the Coordinated Universal Time (UTC) in time zone (0). What is the time in:

a Hobart if it is 8:00 p.m. in Greenwich?

b Greenwich if it is 5:30 a.m. in Hobart?

SOLUTION:

1 Hobart is in the EST (+10) zone and is 10 hours ahead of UTC at Greenwich. Add the time.

2 Greenwich is at UTC and is 10 hours behind Hobart, which is in EST (+10). Subtract the time.

a Hobart = 8:00 p.m. + 10 hours
= 6:00 a.m.

b Greenwich = 5:30 a.m. – 10 hours
= 7:30 p.m.

Daylight saving time

Daylight Saving Time (DST) is the practice of advancing clocks one hour during the warmer months of the year. There are many countries that adopt daylight savings time to take advantage of the increased hours of sunlight. In Australia, daylight saving is observed in New South Wales, Victoria, South Australia, Tasmania and the Australian Capital Territory. Daylight saving is not observed in Queensland, the Northern Territory or Western Australia. Daylight saving time in Australia begins on the first Sunday in October, when clocks are put forward one hour. It ends on the first Sunday in April, when clocks are put back one hour.

DAYLIGHT SAVING TIME

For daylight saving time, add one hour

Example 9: Solving problems using daylight saving time 11D

Jack, in Sydney, wants to phone his mother in Perth. Sydney is located in UTC (+10) and Perth is in UTC (+8). Daylight saving time is operating in Sydney. When should Jack ring to reach his mother at 6:00 p.m. in Perth?

SOLUTION:

1 Sydney is in UTC (+10) and 2 hours ahead of Perth (+8) Add the time.

Sydney = 6:00 a.m. + 2 hours
= 8:00 p.m.

2 Sydney is in daylight saving, so add one hour.

Sydney = 8:00 p.m. + 1 h
= 9:00 p.m.

3 Write answer in words.

Jack should ring when it is 9:00 p.m. in Sydney.

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Table of international time zones

The standard time zones and the major places in each zone are shown below. This table does not take into account time differences as a result of daylight saving time.

Time zone	Major places
–12	Eniwetok
–11	Midway Island, Samoa
–10	Cook Is, Honolulu, Tahiti
–9	Alaska
–8	Anchorage, Los Angeles, San Francisco, Seattle, Vancouver
–7	Alberta, Denver, Edmonton, Phoenix, Salt Lake City, Santa Fe
–6	Chicago, Dallas, El Paso, Guatemala, New Orleans, Mexico City
–5	Boston, Kingston, Lima, Miami, Montreal, New York, Quebec
–4	Caracas, Labrador, La Paz, Santiago
–3	Brasilia, Buenos Aires, Georgetown, Montevideo, Rio de Janeiro
–2	Mid-Atlantic
–1	Azores, Cape Verde Is
0	Belfast, Dublin, Edinburgh, Iceland, Lisbon, London, Morocco
+1	Amsterdam, Berlin, Madrid, Prague, Paris, Rome, Vienna
+2	Athens, Cairo, Cape Town, Greece, Harare, Helsinki, Istanbul
+3	Baghdad, Beirut, Kuwait, Moscow, Nairobi, Tehran
+4	Abu Dhabi, Baku, Kabul, Kazan, Muscat, Tehran, Volgograd
+5	Colombo, Islamabad, Kolkata, Madras, New Delhi
+6	Dhaka, Kathmandu, Colombo, Sri Lanka
+7	Bangkok, Hanoi, Jakarta, Phnom Penh
+8	Beijing, Hong Kong, Kuala Lumpur, Manila, Perth, Singapore
+9	Osaka, Seoul, Sapporo, Seoul, Tokyo
+9.5	Adelaide, Darwin, Alice Springs, Broken Hill
+10	Brisbane, Canberra, Hobart, Melbourne, Port Moresby, Sydney
+10.5	Lord Howe Is
+11	Magadan, New Caledonia, Solomon Is, Vanuatu
+12	Auckland, Christchurch, Fiji, Marshall Is, Wellington, Suva

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Exercise 11D

LEVEL 1

1 Copy and complete this table to show the equivalent times in three Australian time zones.

WST (+8)	CST (+9.5)	EST (+10)
1:00 p.m.		
	6:30 a.m.	
		8:20 p.m.
4:19 a.m.		
	Midnight	
		9:12 a.m.

Example 8

2 What is the time in the following places if it is 6:30 p.m. in Sydney EST (+10)?

a Perth (WST + 8)
b Darwin (CST + 9.5)
c Adelaide (CST + 9.5)
d Melbourne (EST + 10)

3 What is the time in the following places if it is 9:40 a.m. in Alice Springs (CST + 9.5)?

a Fremantle (WST + 8)
b Wollongong (EST + 10)
c Port Adelaide (CST + 9.5)
d Hobart (EST + 10)

4 What is the time in the following places if it is 2:00 p.m. in Greenwich?

a Athens (UTC + 2)
b Mexico city (UTC − 6)
c Singapore (UTC + 8)
d Santiago (UTC − 4)

5 What is the time in the following places if it is 1:00 a.m. Sunday in Greenwich?

a Kuwait (UTC + 3)
b Alaska (UTC − 9)
c Brisbane (UTC + 10)
d Cape Verde Is (UTC − 1)

6 Find the time difference between the following cities.

a Vanuatu (UTC + 11) and Kolkata (UTC + 5)
b Denver (UTC − 7) and Montevideo (UTC − 3)
c Bangkok (UTC + 7) and Boston (UTC − 5)
d Phoenix (UTC − 7) and Madrid (UTC + 1)
e Baghdad (UTC + 3) and Christchurch (UTC + 12)
f Chicago (UTC − 6) and Cape Verde Is (UTC − 1)
g Abu Dhabi (UTC + 4) and Edmonton (UTC − 7)
h Samoa (UTC − 11) and Rome (UTC + 1)

This line and sculpture mark the Greenwich Meridian.

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LEVEL 2

7 What is the time in the following cities if it is 8:00 a.m. in Cairo (UTC + 2)?

a Abu Dhabi (UTC + 4)
b Brasilia (UTC − 3)
c Berlin (UTC + 1)
d Edmonton (UTC − 7)

8 What is the time in the following cities if it is 11:00 p.m. Monday in Dallas (UTC − 7)?

a Honolulu (UTC − 10)
b Harare (UTC + 2)
c New York (UTC − 7)
d Broken Hill (UTC + 9.5)

9 Find the time in the following Australian cities if it is 4:00 p.m. in Sydney (UTC + 10).

a Brisbane (UTC + 10)
b Perth (UTC + 8)
c Darwin (UTC + 9.5)
d Lord Howe Is (UTC + 10.5)

10 The time in Greenwich on the prime meridian is midnight on Monday 1 November. What is the time and day in Seattle (UTC − 8)?

11 Newcastle is part of the Eastern Standard Time zone and is 10 hours ahead of UTC. What is the standard time in Greenwich when it is 5:00 p.m. Friday in Newcastle?

LEVEL 3

12 Sydney is 10 hours ahead of UTC and Seattle is 8 hours behind UTC.

a What is the time difference between the two cities?
b What is the time difference if Sydney is on daylight saving time and Seattle is on standard time?

Example 9 **13** What is the time difference between Tasmania and Sri Lanka if Tasmania is on daylight saving time and Sri Lanka is on standard time? Tasmania is 10 hours ahead of UTC and Sri Lanka is 5 hours ahead of UTC.

14 People in New South Wales (UTC + 10) turn their clocks forwards one hour for daylight saving. Western Australia (UTC + 8) does not adjust its clocks for daylight saving.

a In summer, what is the time in New South Wales if it is 3:30 p.m. in Western Australia?
b In summer, what is the time in Western Australia if it is 10:30 a.m. in New South Wales?

15 Daylight saving operates in Victoria but not in the Northern Territory. Melbourne is part of the Australian Eastern Standard Time (UTC + 10) zone and Darwin is part of the Australian Central Standard Time (UTC + 9.5) zone.

a In summer, what is the time in the Northern Territory if it is 1:15 a.m. in Victoria?
b In summer, what is the time in Victoria if it is 10:30 p.m. in the Northern Territory?

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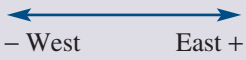
11E Time difference and local time

The international time zones can be used to calculate the local time in two locations. First, find the time difference between the two locations by subtracting the time zones. Then draw an east–west time line and place the locations on the time line. To find the local time you need to add or subtract the time difference. If the required time is east of the given time then add the time difference. Alternatively, if the required time is west of the given time then subtract the time difference.

TIME DIFFERENCE AND LOCAL TIME

To calculate the local time:

1. Find the time difference between the two locations.
2. Place the locations on an east–west time line.
3. Find the local time by adding or subtracting the time difference.
   - East of location with the given time (moving west to east). Add the time difference.
   - West of location with the given time (moving east to west). Subtract the time difference.



Example 10: Finding the time difference and local time 11E

Patrick is working in Seoul (UTC + 9) and needs to travel to Chicago (UTC – 6).

a What is the time difference between these places?

b What is the local time in Chicago if it is 5:00 p.m. in Seoul?

c A flight leaves Seoul at 5:00 p.m. and flies non-stop to Chicago. The flight takes 14 hours. What is the time in Chicago when the flight arrives?

SOLUTION:

1. Time difference is determined by the difference in longitude. Add the longitude of Chicago to that of Seoul.
2. Draw an east–west time line.
3. Mark Chicago (UTC – 6) on the left-hand side and Seoul (UTC + 9) on the right-hand side.
4. Require the time in Chicago. It is west of the location with the time information. Subtract the time difference.
5. Write the answer in words.
6. Flight takes 14 hours. Add the time for the flight to the local time of the destination.
7. Write the answer in words.

a Time difference $= +9 - (-6)$
$= 15$ h

b

Time in Chicago = 5:00 p.m. – 15 h
= 2:00 a.m.

Time in Chicago is 2:00 a.m.

c Time in Chicago = 2:00 a.m. + 14 h
= 4:00 p.m.

Time in Chicago is 4:00 p.m.

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Example 11: Finding the time difference and local time 11E

Moana is planning a trip from Suva in Fiji (UTC + 12) to Miami in Florida (UTC – 5).

a What is the time difference between these cities?

b What is the time and day in Miami if it is 10:10 a.m. on Wednesday 6 January in Suva?

c Moana boards a plane at 2:30 p.m. on 7 April Suva time and the flight lasts 16 hours. What is the local time and date in Miami when she arrives?

SOLUTION:

1 Draw an east–west time line.

2 Mark Miami (–5) on the left-hand side and Suva (+12) on the right-hand side.

3 Time difference is determined by adding the time ahead of UTC (+12) and the time behind UTC (–5).

4 Require the time in Miami. It is west of the location with the time information, so subtract the time.

5 Subtracting 17 hours from 10:10 p.m. results in it being the previous day (5 January).

6 Write the answer in words.

7 Subtract the time difference (17 h) to find the local time in Miami.

8 Flight takes 16 hours. Add the time for the flight to the local time at the destination.

9 Evaluate.

10 Write the answer in words.

a Miami Suva

– West East +

Time difference $= 12 + 5$

$= 17$ h

b Time in Miami $= 10{:}10$ a.m. $- 17$ h

$= 5{:}10$ p.m.

Miami's date is Tuesday 5 January and the time is 5:10 p.m.

c Time in Miami $= 2{:}30$ p.m. $- 17$ h $+ 16$ h

$= 1{:}30$ p.m.

Local time in Miami is 1:30 p.m. on 7 April.

Exercise 11E

LEVEL 1

Example 10

1 Lucas lives in Strasbourg and his sister Kayla lives in Vancouver.

a Calculate the time difference between Strasbourg (UTC + 1) and Vancouver (UTC − 8).

b What is the time in Strasbourg if it is 3:00 p.m. on a Monday afternoon in Vancouver?

2 It is 9:00 p.m. on Friday 30 March in Greenwich.

a What is the date and local time in New Orleans (UTC − 8)?

b What is the date and local time in Canberra (UTC + 10)?

3 Julia lives in Sydney (UTC + 10) and her son Tyler lives in Los Angeles (UTC − 8).

a What is the time difference between Sydney and Los Angeles?

b Julia would like to telephone Tyler at 00:00 Monday for New Year's Eve. At what time and on what day in Sydney should Julia phone?

4 Patrick lives in Melbourne (UTC + 10). He wants to watch a football game being played in Brazil (UTC − 2) starting at 7:00 p.m. on Saturday. What is the time and day in Melbourne when the game starts?

5 A One Day International cricket game between Australia and the West Indies starts at 11:00 a.m. on 26 November, in Brisbane. Blake lives in Kingston, Jamaica, and wants to watch the game on television. Brisbane is 10 hours ahead of UTC and Kingston is 5 hours behind UTC. At what time and date should Blake switch on his television to watch the start of the game? Queensland does not have daylight saving.

6 Jasmine lives in Bathurst (UTC + 11, as Bathurst is on daylight saving time), and wants to telephone her mother in London (UTC 0) so that she receives the call at 8:00 a.m. on Christmas Day. On what day and at what time in Bathurst should Jasmine make the call to London?

7 Andrew is in Moscow (UTC + 3) and wants to record the USA Open tennis final. The tennis match is due to begin at 2:00 p.m. on Sunday in New York (UTC − 5). On what day and at what time will Andrew need to start recording?

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LEVEL 2

Example 11

8 A plane leaves Melbourne (UTC+10) and flies to Paris (UTC+1). The flight takes 23 hours. The plane leaves Melbourne at 4:00 p.m. Thursday. On what day and at what time does the plane arrive in Paris?

9 Na calls a friend in Beijing from Los Angeles. Beijing is 8 hours ahead of UTC and Los Angeles is 8 hours behind UTC. It is 8:15 p.m. in Los Angeles and they talk for 30 minutes. What is the time in Beijing at the end of the phone call?

10 Adelaide (UTC+9.5) is located in the Central Standard Time zone. Melbourne (UTC+10) is located in the Australian Eastern Standard Time zone.

a What is the time in Adelaide if it is 5:45 a.m. in Melbourne?
b What is the time in Melbourne if it is 2:10 p.m. in Adelaide?
c At what time will a train arrive in Adelaide if it departs Melbourne at 7:00 a.m. and travel time is 10 hours?

LEVEL 3

11 Andrew is planning a trip from Townsville (UTC+9.5) to Hong Kong (UTC+8).
a What is the time and day in Hong Kong if it is 7:30 a.m. on 5 June in Townsville?
b A flight leaves Townsville at 7:30 a.m. and flies non-stop to Hong Kong. The flight is in the air for 7 hours. What time and day is it in Hong Kong when the flight arrives?

12 A boat departs Brisbane on Tuesday 12 October at 12:00 p.m. and takes $15\frac{1}{2}$ days to travel west to Honolulu. Brisbane is 10 hours ahead of UTC and Honolulu is 10 hours behind UTC.
a What is the local time and day in Honolulu when the boat departs Brisbane?
b What is the local time and day when the boat arrives in Honolulu?

13 A plane leaves Sydney at 2:00 a.m. Tuesday 17 May for an 8 hour flight to Kabul. Sydney is 10 hours ahead of UTC and Kabul is 4 hours ahead of UTC.
a What is the local time and day when the plane arrives in Kabul?
b The return flight leaves Kabul at 2:00 p.m. on the next Friday. What is the time and day when the plane arrives in Sydney?

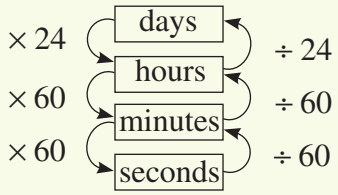
Refer to the Interactive Textbook for Section 11F Longitude and time difference.

PPT STUDY GUIDE

Key ideas and chapter summary

Units of time

1 min = 60 s
1 h = 60 min
1 day = 24 h

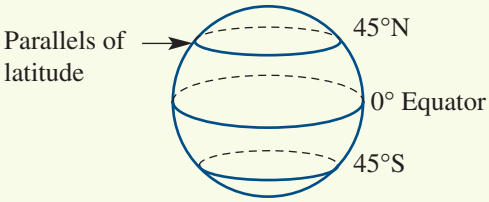


Timetables

A timetable is a list of times at which possible events or actions are intended to take place.

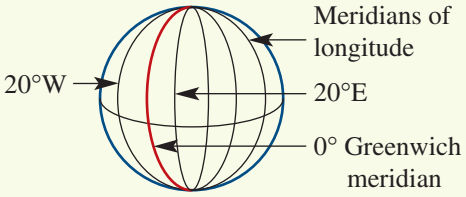
Latitude

Measures the angular distance north and south of the equator (0°)



Longitude

Measures the angular distance east and west of the Greenwich meridian (0°)



Time zones

Time zones have the same time irrespective of time differences.

- Positive time zone number – ahead of UTC or GMT
- Negative time zone number – behind of UTC or GMT

Time difference

To calculate the local time:

1. Find the time difference between the two locations.
2. Place the locations on an east–west time line.
3. Find the local time by adding or subtracting the time difference.
   - East of location with the given time (moving west to east): add the time difference.
   - West of location with the given time (moving east to west): subtract the time difference.

Daylight Saving Time (DST)

The practice of advancing clocks by 1 hour in the warmer months to take advantage of the longer daylight.

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Review

Multiple-choice questions

1 Violet works on her homework from 3:45 p.m. until 5:08 p.m. each weekday. How much time does Violet spend doing her homework over the week?

A 1 h 23 min **B** 5 h 15 min **C** 6 h 55 min **D** 9 h 01 min

2 The school bus takes 49 minutes to get from Joseph's house to the school. If school starts at 8:30 a.m., what is the latest time Joseph can catch the bus to get to school on time?

A 7:19 a.m. **B** 7:41 a.m. **C** 7:51 a.m. **D** 9:19 a.m.

3 A train leaves the station at 7:28 a.m. The journey to its final destination takes 6 hours and 54 minutes. What is the train's arrival time?

A 1:22 a.m. **B** 2:22 a.m. **C** 1:22 p.m. **D** 2:22 p.m.

4 Use the train timetable below to answer the following question.

T2 Inner West & South Line: City to Campbelltown or Leppington

Previous day — Services departing Museum from 03:00 06:

Stations	EARLIER TRAINS			
Museum	1:03	11:12	11:18	11:27
St James	1:05	11:14	11:20	11:29
Circular Quay	1:09	11:18	11:24	11:33
Wynyard	1:11	11:20	11:26	11:35
Town Hall	1:14	11:23	11:29	11:38
Central	1:18	11:27	11:33	11:42
Redfern	1:20	11:29	11:35	11:44

How long does the train leaving Museum at 11:12 take to get to Redfern?

A 17 s **B** 29 s **C** 11 min 29 s **D** 17 min

5 The coordinates of Dublin are (53°N, 6°W). What are the coordinates of Hamburg if it is 16° east of Dublin?

A (53°N, 10°E) **B** (53°N, 22°W) **C** (69°N, 6°W) **D** (37°N, 6°W)

6 What is the time difference between Singapore (UTC + 8) and Canberra (UTC + 10)?

A 2 h **B** 3 h **C** 18 h **D** 80 h

7 What is the time in St John's (UTC − 3.5) if it is 4.00 a.m. in Dallas (UTC − 6)?

A 1:30 a.m. **B** 2:00 a.m. **C** 6:30 a.m. **D** 7:00 a.m.

8 The time in Greenwich on the prime meridian is 3:00 p.m. What is the standard time in Kuala Lumpur (UTC + 8) ?

A 11:00 p.m. **B** 6:00 a.m. **C** 11:00 a.m. **D** 6:00 p.m.

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Review

Short-answer questions

1 Complete the following.

a 120 min = ☐ s
b 3.5 days = ☐ h
c 7 h = ☐ min
d 630 min = ☐ h
e 24 000 s = ☐ min
f 120 h = ☐ days
g 1032 h = ☐ days
h 750 min = ☐ h
i 960 s = ☐ min

2 What is the time 7 hours and 10 minutes after the following times?

a 08:00
b 13:25
c 22:50

3 Elizabeth and Hoshi get 1 hour and 24 minutes of homework each weekday. Elizabeth takes 5 minutes less to do her homework on a Tuesday, and 5 minutes less each subsequent day up to and including Friday.

a How long does Hoshi spend on homework each week?
b How much time does Elizabeth spend doing her homework each week?
c How much more spare time does Elizabeth get than Hoshi each week?

4 The school assembly starts at 9:50 a.m. It lasts for 28 minutes. Then there are sports announcements for another 8 minutes before the students are dismissed. At what time are the students dismissed?

5 Luke's football team starts its game at 10:15 a.m. The team plays two halves that are 25 minutes each and there is a 6-minute break at half time.

a Luke scores a goal 5 minutes after half time. At what time does he score his goal?
b At what time does the game finish?

6 Use the ferry timetable to answer the following questions.

a How long does it take to travel on the 6:22 ferry from Circular Quay to Rose Bay?
b How long does it take to travel on the 6:59 ferry from Circular Quay to Double Bay?
c Stella is waiting at Circular Quay at 6:30 a.m. for a ferry to Double Bay. How long does she need to wait?

Monday to Friday

Service	F7	F7	F7
Circular Quay	06:22	06:46	06:59
Garden Island	...	...	...
Darling Point	...	...	...
Double Bay	...	...	07:14
Rose Bay	06:35	06:57	...
Watsons Bay *arr*	...	...	...
Watsons Bay *dep*	...	...	...
Rose Bay	06:38	07:00	...

7 Karachi is located at (25°N, 67°E) and Salvador is located at (13°S, 39°W).

a Which city is closer to the Greenwich meridian?
b What is the difference in latitude between Karachi and Salvador?
c What is the difference in longitude between Karachi and Salvador?

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Review

8 Monterrey is located at (26°N, 100°W) and Auckland is located at (37°S, 174°E).
- **a** What is the difference in latitude between Monterrey and Auckland?
- **b** What is the difference in longitude between Monterrey and Auckland?
- **c** What are the coordinates of a point 50° due south of Monterrey?
- **d** What are the coordinates of a point 50° due west of Auckland?

9 What is the time in Boston (UTC − 5) when it is 1:00 p.m. in Hanoi (UTC + 7)?

10 A flight from Sydney (UTC + 10) to Adelaide (UTC + 9.5) takes two hours, flying time.
- **a** What is the time difference in minutes between Sydney and Adelaide?
- **b** A plane leaves Sydney at 7:00 a.m. At what time will it arrive in Adelaide, taking into account the flying time?

11 Perth (UTC + 8) is located in the Western Standard Time zone. Brisbane (UTC + 10) is located in the Eastern Standard Time zone.
- **a** What is the time difference in hours between Perth and Brisbane?
- **b** What is the time in Perth if it is 5:00 p.m. in Brisbane?

12 The Gold Coast's time zone is UTC + 10 and Bali's time zone is UTC + 8.
- **a** What is the time in Bali if the time in the Gold Coast is 9:00 a.m.?
- **b** A flight leaves the Gold Coast at 11:00 a.m. and flies non-stop to Bali. The flight is in the air for 3 hours. At what time does the flight arrive in Bali?

Extended-response questions

13 Fremantle (UTC + 8) is located in the Western Standard Time zone. Dubbo (UTC + 10) is located in the Eastern Standard Time zone.
- **a** Isabella, who is in Dubbo, rang a friend in Fremantle at 8:00 p.m. on a Saturday. What day and time was it in Fremantle?
- **b** Isabella caught a plane, at 8:00 p.m. on a Saturday, from Dubbo to Fremantle. The travel time for the trip was 5 hours. When will Isabella arrive in Fremantle?

14 Molly flew from Sydney (UTC + 10) to Chittagong (UTC + 6). Her plane left Sydney at 8:30 a.m. Thursday (Sydney time), stopped for 4 hours in Singapore and arrived in Chittagong at 1:00 p.m. Thursday (Chittagong time). What was the total flying time?

15 Lucy is sailing across the Pacific Ocean. At 8:30 a.m. on Saturday 9 July her position is (UTC − 8) and she is heading for Tahiti at (UTC − 10). At 8:30 a.m. on Saturday 9 July, she rings her sister in Tahiti. What is the local time and date when her sister takes the call?

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12 Budgeting and household expenses

Syllabus topic — F1.3 Budgeting and household expenses

This topic will allow you to develop an appropriate budget for a given situation and justify various types of financial decisions.

Outcomes

- Interpret information about the costs from house bills.
- Use a budget to purchase a motor vehicle.
- Plan the purchase of a car.
- Determine the cost of repayments and total amount repaid on a loan.
- Describe the different types of motor vehicle insurance.
- Calculate the cost of stamp duty on a vehicle.
- Calculate the fuel consumption and running costs of a vehicle.
- Prepare a personal budget for a given income, taking into account expenses.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Widgets
- Literacy worksheet
- Spreadsheets
- Quick Quiz
- Study guide
- Solutions (enabled by teacher)

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

In the Interactive Textbook you can take a test of prior knowledge required for this chapter, and depending on your score you may be directed to revision from the previous years' work.

12A Household bills

Household bills are the costs associated with living in a home. They are a big percentage of the expenses in a budget. Household bills include council rates, electricity and gas bills, water and sewerage rates, telephone bills and internet connection.

Example 1: Reading household bills 12A

Phone company

Tax Invoice
Phone Billing Services Pty. Ltd.
ABN 12 123 479 686
Invoice Number 234688731

Need Help?
Visit www. phone.com.au
Call 1300 123 569 for help
during Mon-Fri (8am-7pm)
Sat (9am-5pm) local time

Customer Account Number 2164 9142 0001 46

Previous Account	Payments and Adjustments	GST Adjustments	Balance Forward
63.42	63.42CR	0.00	0.00

New charges $67.40

New charges due 11 Mar

Total amount due $67.40

SUMMARY OF CHARGES

Recurring charges	52.66
Other charges and credits	0.58
Usage charges	15.44
Discounts	7.41CR
GST	6.13
New charges (including GST)	67.40
Balance forward	0.00
Total Amount Due (including GST)	**$67.40**

Issue date 25 Feb

Account period
25 Jan to 24 Feb

Answer the following questions using the above telephone account.

a What was the amount of the previous account?
b What is the due date?
c How much GST is charged?

SOLUTION:

1 Read 'Previous Account'.
2 Read the box 'New charges due'.
3 Read 'GST'.

a Previous account is $63.42.
b Due date is 11 March.
c GST charged is $6.13.

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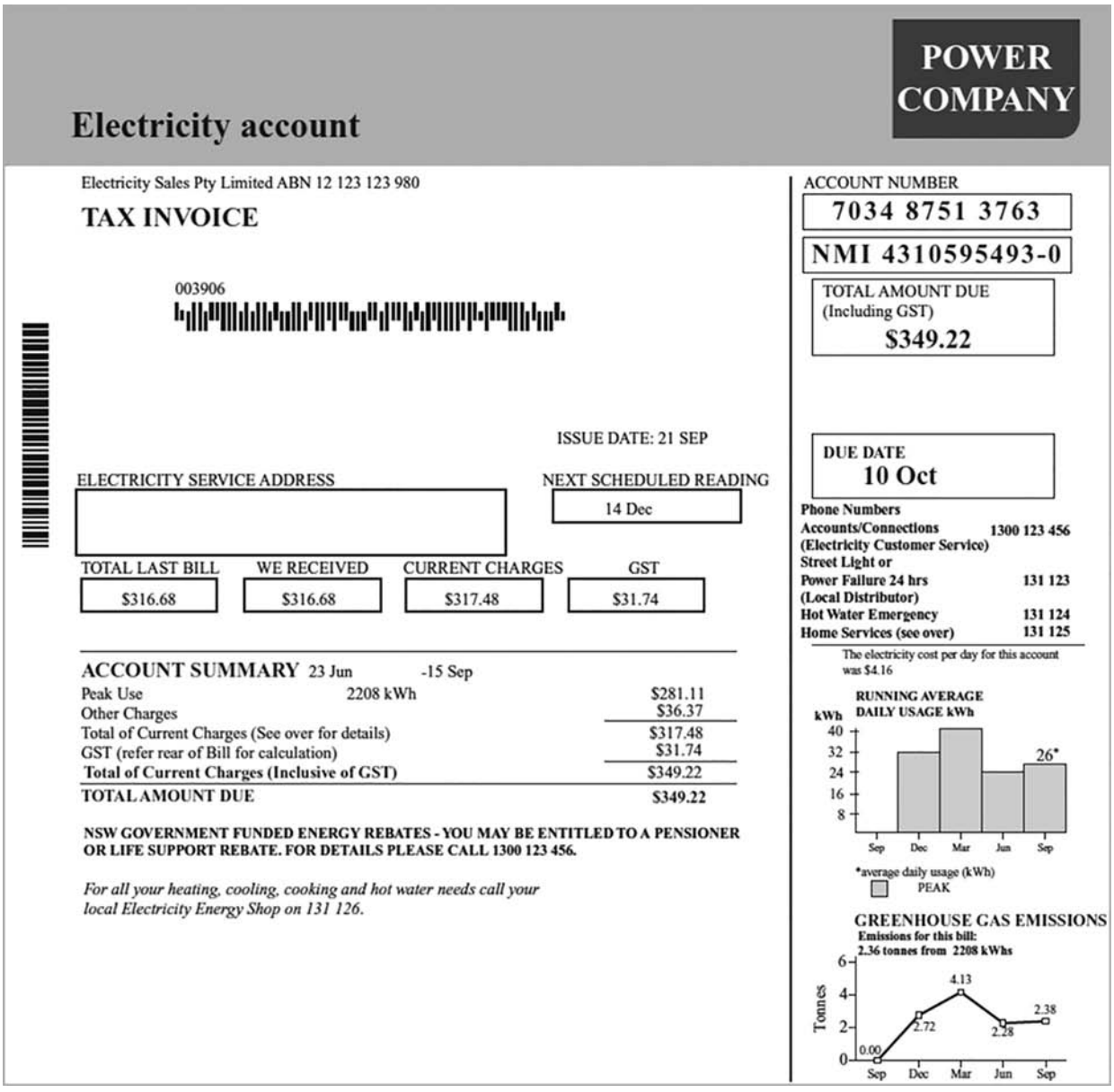
Exercise 12A

LEVEL 1

Example 1

1 Answer the following questions using this electricity account.



POWER COMPANY

Electricity account

Electricity Sales Pty Limited ABN 12 123 123 980

TAX INVOICE

003906

ACCOUNT NUMBER
7034 8751 3763

NMI 4310595493-0

TOTAL AMOUNT DUE
(Including GST)
$349.22

ISSUE DATE: 21 SEP

DUE DATE
10 Oct

ELECTRICITY SERVICE ADDRESS

NEXT SCHEDULED READING
14 Dec

TOTAL LAST BILL	WE RECEIVED	CURRENT CHARGES	GST
$316.68	$316.68	$317.48	$31.74

ACCOUNT SUMMARY 23 Jun -15 Sep

Peak Use	2208 kWh	$281.11
Other Charges		$36.37
Total of Current Charges (See over for details)		$317.48
GST (refer rear of Bill for calculation)		$31.74
Total of Current Charges (Inclusive of GST)		$349.22
TOTAL AMOUNT DUE		$349.22

NSW GOVERNMENT FUNDED ENERGY REBATES - YOU MAY BE ENTITLED TO A PENSIONER OR LIFE SUPPORT REBATE. FOR DETAILS PLEASE CALL 1300 123 456.

For all your heating, cooling, cooking and hot water needs call your local Electricity Energy Shop on 131 126.

Phone Numbers

Accounts/Connections (Electricity Customer Service)	1300 123 456
Street Light or Power Failure 24 hrs (Local Distributor)	131 123
Hot Water Emergency	131 124
Home Services (see over)	131 125

The electricity cost per day for this account was $4.16

RUNNING AVERAGE DAILY USAGE kWh

*average daily usage (kWh)
PEAK

GREENHOUSE GAS EMISSIONS

Emissions for this bill:
2.36 tonnes from 2208 kWhs

a When was this electricity account issued?
b What is the account number?
c What is the total amount due?
d What is the due date?
e How much GST is charged?
f What is the charge for the peak use of electricity?
g How much was the last account?
h By how much has the bill increased from the last bill?
i Express the increase as a percentage of the last bill. Answer correct to two decimal places.

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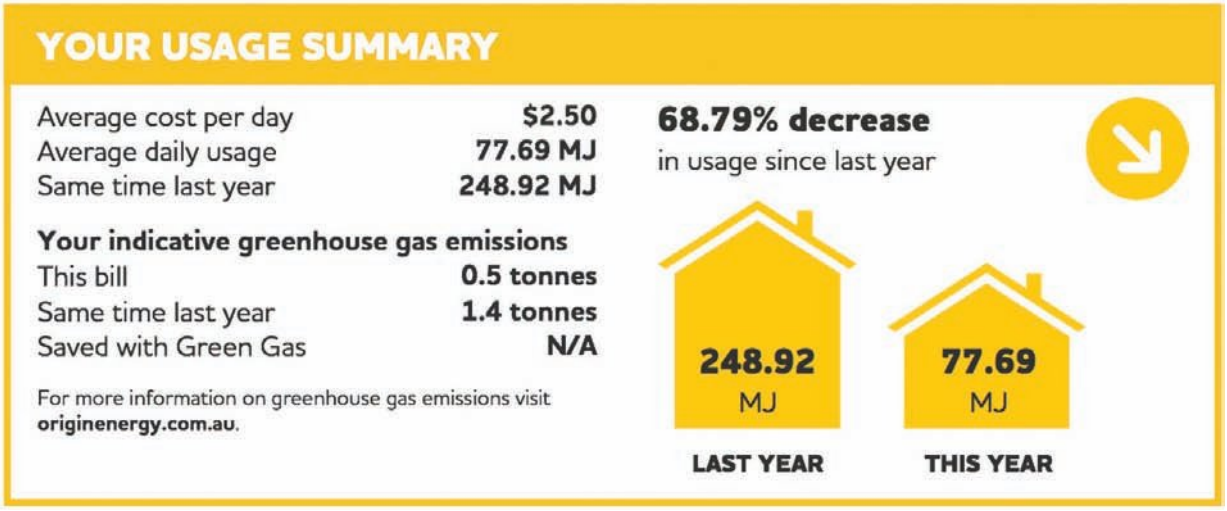
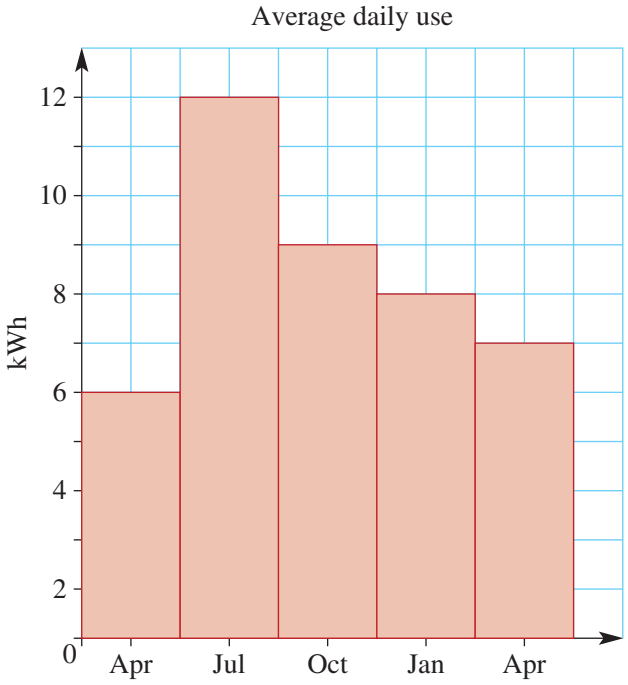
2 The usage summary of a quarterly gas account is shown below.



a What was the average daily gas usage for the quarter?
b What was the average daily gas usage at the same time last year?
c What is the average cost per day for gas?
d What was the percentage decrease in gas usage?
e Describe the greenhouse gas emissions for this quarter.

3 The graph below is part of an electricity account issued to a customer.



a How many times per year is the electricity meter read?
b What unit is used to measure electricity?
c What was the average daily use in the October quarter?
d Which quarter had the greatest usage of electricity?

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LEVEL 2

4 Lucy received her natural gas account.

Amount used	Charges	Cost
First 5500 MJ	1.457 cents per MJ	
Next 13900 MJ	1.360 cents per MJ	
Supply fee	$40.00	

a Calculate the cost of the first 5500 MJ. Answer to the nearest cent.
b Calculate the cost of the next 13 900 MJ. Answer to the nearest cent.
c What is the total charge? Answer to the nearest cent.

5 Local council rates increased by 4%. The old rates for a property were $2200. What is the new rate for this property, correct to the nearest dollar?

6 Amy uses a 1.5 kilowatt hair dryer for a total of 8 hours. She is charged at a rate of 14.81 cents per kilowatt-hour.
a How many kilowatt-hours were used by the hair dryer?
b What is the cost of using the hair dryer? Answer to the nearest cent.

7 Council rates for a property valued at $374 000 are calculated using the table below.

Council rates	Charge
Residential rate	0.004155 cents in the dollar
Garbage rate	$202.80
Water and sewerage service	$114.99
Water usage charge	94 kL at $1.3390 per kL

Find the amount payable to the nearest cent for:
a residential rate
b water usage
c garbage
d total rates payable on this property.

LEVEL 3

8 An energy company's charges for gas over a 3-month period are shown in the table.

Usage charge	First 750 MJ	1.3920 cents per MJ
	Additional MJ over 750	1.3330 cents per MJ

a Lucy used 2250 MJ of gas in this period. What is the cost of this gas? Answer to the nearest cent.
b What percentage of Lucy's gas usage was charged at the lower price? Answer correct to two decimal places.

12B Purchasing a car

The cost of a purchasing a motor vehicle depends on many factors. A motor vehicle is not an investment. It decreases in value immediately. In the first year of ownership, a new car can lose up to 20% of its value, and by the fifth year, your car will have decreased in price by over 65%.
The percentage decrease is determined by dividing the price decrease by the purchase price and multiplying the result by 100.

Example 2: Calculating the percentage decrease 12B

A new vehicle is bought for $25 000 and sold one year later for $19 000. Calculate the percentage decrease in the value of the new vehicle.

SOLUTION:

1 Subtract $19 000 from $25 000 ($6000).
2 Divide the price decrease ($6000) by the purchase price ($25 000).
3 Express as a percentage (multiply by 100).

$$\text{Percentage decrease} = \frac{\$6000}{\$25\,000} \times 100$$
$$= 24\%$$

Percentage decrease is 24%.

Finance

Using car dealers' finance, a purchaser pays a deposit and then makes a large number of repayments. The total cost using finance is greater than the price for cash.

BUYING ON FINANCE

Total cost = Deposit + Total repayments
Total repayments = Repayment × Number of repayments
Interest paid = Total cost − Sale price

Example 3: Calculating the cost of repayments 12B

A four-wheel-drive vehicle is for sale at $45 000. Finance is available at $5000 deposit and monthly repayments of $1470 for 5 years.

a What is the total of the repayments?
b What is the total cost of the finance?
c What is the interest paid?

SOLUTION:

1 Multiply the monthly repayment by the number of repayments.
2 Add the deposit to the total cost of the repayments.
3 Subtract the sale price from the total cost.

a Total repayment $= 1470 \times 12 \times 5$
$= \$88\,200$
b Total cost $= 5000 + 88\,200$
$= \$93\,200$
c Interest paid $= 93\,200 - 45\,000$
$= \$48\,200$

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Exercise 12B

LEVEL 1

Example 2

1 Calculate the percentage decrease in the price of a new vehicle after one year.

- **a** Purchase price is $25 500. Market value after one year is $21 420.
- **b** Purchase price is $36 800. Market value after one year is $27 600.
- **c** Purchase price is $54 250. Market value after one year is $48 825.
- **d** Purchase price is $23 826. Market value after one year is $20 900.

2 Calculate the price of the following cars after the trade-in.

- **a** Sale price is $35 500. Trade-in is worth $6000.
- **b** Sale price is $16 850. Trade-in is worth $2980.
- **c** Sale price is $24 120. Trade-in is worth $9460.
- **d** Sale price is $64 870. Trade-in is worth $11 820.

3 Calculate the amount of the deposit needed to purchase the following cars.

- **a** Sale price is $21 400. Deposit is 25%.
- **b** Sale price is $19 240. Deposit is 15%.
- **c** Sale price is $45 100. Deposit is 35%.
- **d** Sale price is $65 200. Deposit is 40%.

Example 3

4 Calculate the total repayments to purchase the following cars.

- **a** Sale price is $14 800. Monthly repayments of $410 for 5 years.
- **b** Sale price is $19 240. Monthly repayments of $1120 for 2 years.
- **c** Sale price is $45 100. Weekly repayments of $360 for 3 years.
- **d** Sale price is $85 200. Weekly repayments of $610 for 4 years.

5 Charlotte has been offered terms to purchase a car. The price of the car is $24 560 or 50% deposit and repayments of $90 per week for 200 weeks.

- **a** What is the amount of the deposit?
- **b** Find the total cost of the repayments.
- **c** What is the cost of purchasing the car on terms?

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LEVEL 2

6 A utility vehicle is for sale at $42 000. Finance is available at $7500 deposit and monthly repayments of $1280 for 5 years.

a What is the total cost of the repayments?

b How much will the car cost if you use the finance package?

c What is the interest paid?

7 Musa has seen a used car he would like to buy, priced at $13 400. He has saved $7000 towards the cost of the car. His parents have offered to lend him the balance to pay for it. Musa agrees to pay $40 each week to repay his parents.

a How much will Musa need to borrow from his parents?

b How long will it take Musa to repay the loan from his parents?

8 A used car is for sale at $27 000. Finance is available at 10% deposit and monthly repayments of $630 for 4 years.

a How much deposit is to be paid?

b What is the total cost of the repayments?

c How much will the car cost if you use the finance package?

d What is the interest paid?

9 Emily has two choices of finance packages for a new car.
Package A: Deposit of $3000, $1400 per month over 5 years.
Package B: No deposit, $1540 per month over 6 years.

a Determine the total cost of package A.

b Determine the total cost of package B.

c How much will be saved by selecting the cheaper package?

10 A prestige car is for sale at $65 000. Finance from the car dealer is available at a deposit of 40% and weekly repayments of $530 for 4 years. A personal loan of $39 000 is available from the bank at 15% p.a. simple interest for 4 years.

a How much deposit is required?

b What is the interest paid using the finance from the car dealer?

c What is the interest paid using the finance from the bank?

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11 Registration of a motor vehicle involves the payment of a fee. Note: fees may change each year.

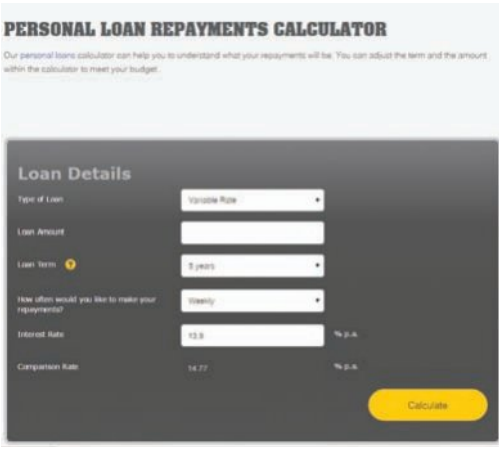
Size of vehicle	Tare weight	Registration fees	
		Private use	Business use
Cars, station wagons and trucks	up to 975 kg	$218	$321
	976 kg to 1154 kg	$239	$354
	1155 kg to 1504 kg	$269	$404
	1505 kg to 2504 kg	$383	$583
Trailers (including caravans)	up to 254 kg	$52	$133
	255 kg to 764 kg	$143	$200
	765 kg to 975 kg	$218	$321
	976 kg to 1154 kg	$239	$354
	1155 kg to 1504 kg	$269	$404
	1505 kg to 2499 kg	$383	$583
Motor cycle		$101	$101

a What is the cost of registering a car for private use, if the weight of the car is 1000 kg?
b What is the cost of registering a truck for business use, if the weight of the truck is 1500 kg?
c What is the cost of registering a car for business use, if the weight of the car is 925 kg?
d What is the cost of registering a motor cycle for private use?

LEVEL 3

12 Ali is buying a used car for $12 000. He is required to pay a transfer fee of $26 and stamp duty of $360. Finance from the car dealer is available at a deposit of 20% and monthly repayments of $380 for 4 years. How much above the price is Ali paying the car dealer?

13 Personal loan calculators on lenders' websites can be used to determine the monthly repayments.



Use a personal loans calculator with a monthly gross salary of $5000 and monthly expense details of $2000 to determine the maximum amount to be borrowed.

a Current variable rate of interest and a loan term of 4 years.
b Current variable rate of interest and a loan term of 2 years.
c Current fixed rate of interest and a loan term of 4 years.
d Current fixed rate of interest and a loan term of 2 years.

12C Car insurance

Insurance is a major cost of keeping a motor vehicle on the road. There are three main types of insurance:

- Green slip or Compulsory Third Party insurance protects vehicle owners and drivers who are legally liable for personal injury to any other party in the event of a personal injury claim made against them by other road users.
- Third Party Property insurance covers you for damage caused by your car to property owned by a third party in the event of an accident.
- Comprehensive insurance covers you for damage to your own vehicle as well as damage your car may cause to another person's vehicle or property.

An insurance premium is the cost of taking out insurance cover. Many insurance companies offer an online calculator for your vehicle insurance premium. It requires information on the make and model of car, your age, driving history, finance, modifications or accessories and location. The cost of insurance is affected by other factors:

- No-claim bonus is a discount on an insurance premium. This discount increases if no-claim is made on the policy until it reaches the maximum discount level.
- Excess is paid when a claim is made on the policy. The standard excess can be varied; there are also excesses for younger drivers.

CAR INSURANCE

The cost of insurance is affected by make and model of car, your age and driving history, finance, modifications or accessories and location. No-claim bonus and excess amount are major factors.

Example 4: Calculating the insurance premium 12C

Elle has been quoted $960 for comprehensive car insurance. She has a no-claim bonus of 40%. How much is Elle required to pay?

SOLUTION:

1 No-claim discount of 40% requires payment of 60%.
2 Calculate 60% of $960.
3 Evaluate.
4 Write the answer in words.

Premium = 60% of $960

$= 0.60 \times 960$

Elle is required to pay $576.

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Exercise 12C

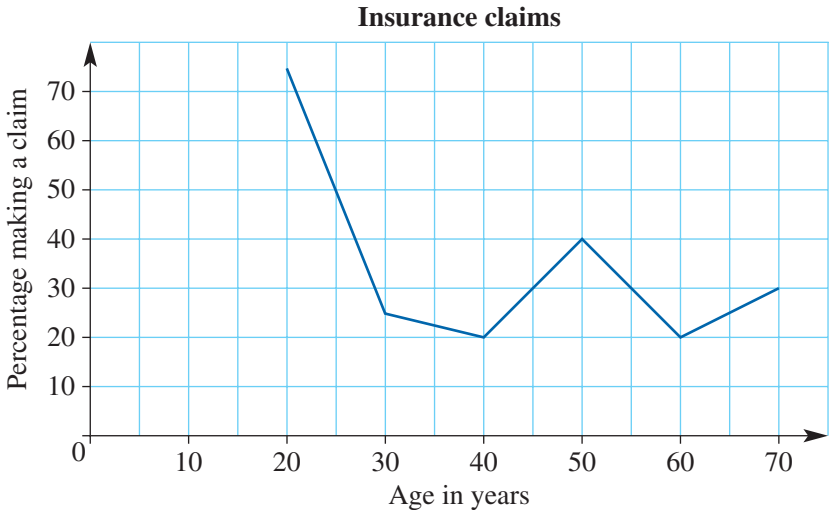
LEVEL 1

Example 4

1 What is the cost of comprehensive car insurance for the following premiums?

- **a** Premium of $1080 with a no-claim bonus of 60%
- **b** Premium of $1690 with a no-claim bonus of 30%
- **c** Premium of $880 with a no-claim bonus of 40%
- **d** Premium of $1320 with a no-claim bonus of 70%
- **e** Premium of $2350 with a no-claim bonus of 50%

2 The graph below shows the percentage of claims for each age group.



- **a** What is the percentage of claims for people 50 years old?
- **b** Which age group made the least number of claims?
- **c** Calculate the gross percentage change in claims between the ages of 60 and 70.
- **d** How do insurance companies cater for the large number of claims made by people 20 years old?

3 Dan is 20 years old and has received this quote for comprehensive insurance.

Premium details	Excesses	Cost
12 month policy $678.00 30% No-claim bonus	Standard	$500
	Male under 21	$1200
	Female under 21	$900

- **a** Calculate the cost of the insurance.
- **b** What is Dan's excess if he makes a claim?

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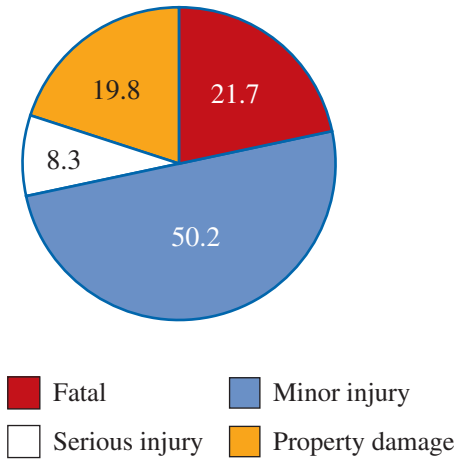
LEVEL 2

4 Connor has been quoted an insurance premium of $980.60 by his insurance company. The company has given him a 20% no-claim bonus, as he had not made a claim in the previous year. What would the insurance premium have been without his no-claim bonus?

5 The sector graph shows the road crash costs according to categories. The total insurance cost was $1.2 billion.

a What is the insurance cost for minor injury?
b What is the insurance cost for serious injury?
c What is the insurance cost for a fatal accident?
d What is the insurance cost for property damage?



LEVEL 3

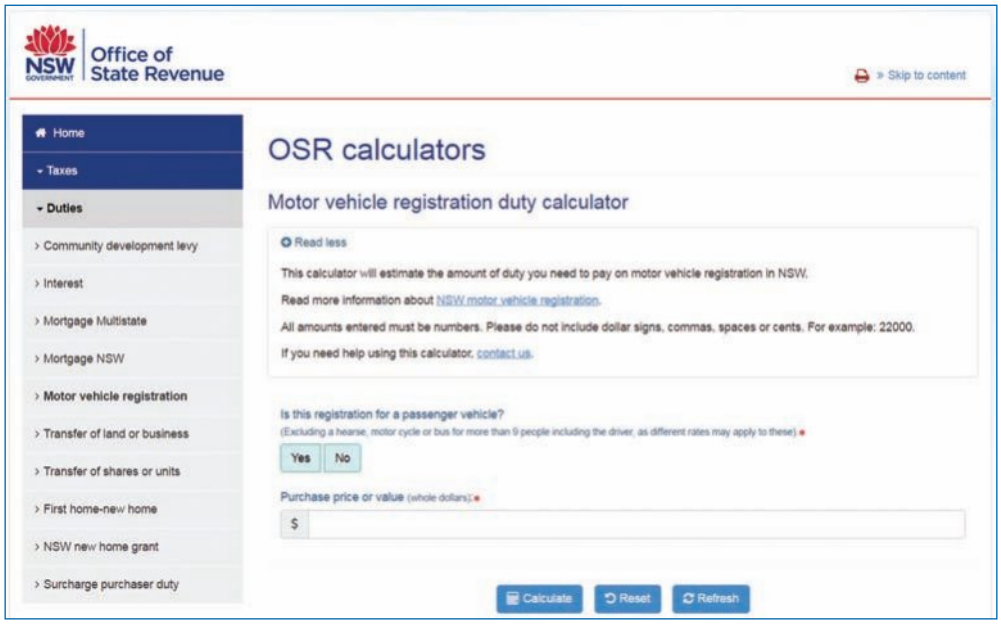
6 The premiums quoted below are for clients with a maximum no-claim bonus. The car is owned outright by a mature-age driver and driven for private use.

Model of car	Agreed value	Premium A		Premium B	
		Mosman	Penrith	Mosman	Penrith
Brand A	$20 100	$540	$605	$600	$760
Brand B	$38 890	$810	$899	$770	$1500
Brand C	$24 400	$615	$650	$615	$860

a Which suburb has the highest premium? Suggest a reason.
b Do expensive cars have higher premiums?
c What is the best quote for the Brand B?
d What is the best quote for the Brand A?
e Which model has the lowest premium?
f What is the average premium for the Brand B?
g What is the average premium for the Brand C?
h What is the average premium for Mosman?
i What is the average premium for Penrith?
j Premium A is being increased by 3%. What would be the new premium for a Brand A car at Mosman?

12D Stamp duty

Stamp duty is the tax you pay to the government when registering or transferring a motor vehicle. The amount of stamp duty payable is based on the price of the motor vehicle. For example, a new passenger car purchased for $40 000 would require a duty of $3 per $100 or $1200 ($0.03 \times 40\,000$). That is, for every $100 you paid for the vehicle, the stamp duty is $3 or a tax of 3%.



STAMP DUTY ON VEHICLES

1. Round up the cost of the vehicle to the nearest $100 (per $100), $200 (per $200) etc.
2. Express the stamp duty as a fraction or decimal. ($3 per $100 is 3/100 or 0.03.)
3. Multiply the answer obtained in step 1 by the fraction or decimal obtained in step 2.

Example 5: Calculating stamp duty on vehicles 12D

A used car is bought for $17 730. Calculate the stamp duty payable if the charge is $3 per $100 or part $100.

SOLUTION:

1 Round $17 730 up to the nearest $100.	Value of vehicle = $17 800
2 Express the stamp duty as a fraction.	$3 per $100 is the fraction $\frac{3}{100}$.
3 Multiply $17 800 by $\frac{3}{100}$.	Stamp duty $= 17\,800 \times \frac{3}{100}$
4 Evaluate.	$= \$534$
5 Write the answer in words.	Stamp duty payable is $534.

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Exercise 12D

LEVEL 1

Example 5

1 The table below is used to calculate the stamp duty payable on a vehicle.

Value of vehicle	Stamp duty payable
$0–$45 000	$3 per $100 or part $100
More than $45 000	$1350 plus $5 per $100 (or part $100) over $45 000

Calculate the stamp duty payable on the following vehicles.

a $32 600 **b** $26 500 **c** $45 000
d $13 790 **e** $35 521 **f** $23 802
g $52 700 **h** $98 435 **i** $120 080

2 The table below is used to calculate the stamp duty payable on a vehicle.

Value of vehicle	Stamp duty payable
$0–$60 000	$5 per $200 or part $200
More than $60 000	$1500 plus $7 per $200 (or part $200) over $60 000

Calculate the stamp duty payable on the following vehicles.

a $13 200 **b** $29 790 **c** $45 410
d $73 800 **e** $61 670 **f** $88 605
g $57 326 **h** $79 190 **i** $91 456

3 The table below is used to calculate the stamp duty payable on a used vehicle.

Value of vehicle	Passenger	Non-passenger
All prices	$5 per $300 (or part $300)	$7 per $300 (or part $300)

Calculate the stamp duty payable on the following vehicles.

a Passenger car $21 300
b Passenger car $69 500
c Non-passenger car $45 880
d Non-passenger car $36 614

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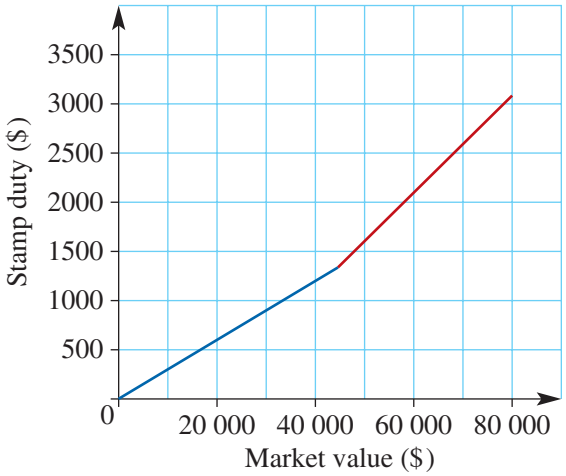
LEVEL 2

4 Stamp duty is calculated at 3% of the market value of a vehicle up to $45 000, plus 5% of the value of the vehicle over $45 000. Use the following graph to answer the questions below.



a How much stamp duty is payable on a car whose market value is $20 000?
b How much stamp duty is payable on a car whose market value is $60 000?
c How much stamp duty is payable on a car whose market value is $45 000?
d How much stamp duty is payable on a car whose market value is $70 000?
e What is the market value of a car if the stamp duty paid was $300?
f What is the market value of a car if the stamp duty paid was $2300?

LEVEL 3

5 Construct a line graph to represent the following stamp duty charge. Stamp duty is calculated at 2.5% of the market value of a vehicle up to $60 000, plus 4% of the value of the vehicle over $60 000. Use your graph to answer the questions below.

a How much stamp duty is payable on a car whose market value is $30 000?
b How much stamp duty is payable on a car whose market value is $60 000?
c How much stamp duty is payable on a car whose market value is $80 000?
d What is the market value of a car if the stamp duty paid was $500?
e What is the market value of a car if the stamp duty paid was $1000?
f What is the market value of a car if the stamp duty paid was $3000?

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12E Running and maintenance costs

Car running costs

Car running costs are a hidden, irregular cost and depend on the make, model and age of the car.

- Maintenance – includes oil changes, tune-ups, new tyres and brakes. Car parts need to be replaced after a certain period of time or after the car has travelled a specific number of kilometres.
- Repairs – damage to the car as a result of a car accident, scratch or dent. These costs are very unpredictable.
- Fuel – cost depends on the distance travelled by the car, the price paid for the fuel, the efficiency of the car and the type of driving.
- Improvements – additional items purchased to improve the car, such as seat covers.
- Parking – money required to park the car. This can expensive in the major cities.
- Tolls – fee charges for travelling a tolled motorway.
- Car washes – cost of maintaining a clean car.
- Fines – traffic ticket issued by the police for breaking the law by speeding.

CAR RUNNING COSTS

Car running costs involve maintenance, repairs, fuel, improvements, parking, tolls, car washes and fines.

Example 6: Calculating the maintenance cost of a car tyre 12E

Julia travels 20 000 km each year in her car. A set of tyres costs $640 and lasts about 40 000 km. Calculate the cost of tyre wear:

a per kilometre **b** per year

SOLUTION:

1 Write the cost of the tyres over the number of kilometres in fraction form.
2 Evaluate.
3 Write answer in words.
4 Divide the length of time the tyres will last by the distance travelled each year.
5 Divide the tyre cost by the number of years.
6 Evaluate.
7 Write answer in words.

a $\text{Cost} = \dfrac{\$640}{40\,000}$
$= \$0.016$

Tyre wear costs $0.016 per kilometre.

b $\text{Years} = \dfrac{40\,000}{20\,000} = 2$

$\text{Cost} = \$640 \div 2$
$= \$320$

Tyre wear costs $320 per year.

Fuel consumption

A motor vehicle's fuel consumption rate (often referred to just as 'fuel consumption') is the number of litres of fuel it uses to travel 100 kilometres. The fuel consumption is calculated by filling the motor vehicle with fuel and recording the kilometres travelled from the odometer. When the motor vehicle is again filled with fuel, record the reading from the odometer and how many litres of fuel it takes to refill the tank. The distance travelled is the difference between the odometer readings.

FUEL CONSUMPTION

$$\text{Fuel consumption} = \frac{\text{Amount of fuel (L)} \times 100}{\text{Distance travelled (km)}}$$

Example 7: Calculating fuel consumption 12E

A medium-size car travelled 750 km using 60 L of petrol.
What was the fuel consumption?

SOLUTION:

1. Write the fuel consumption formula.
2. Substitute 60 for the amount of fuel and 750 for the distance travelled.
3. Evaluate.
4. Write answer in words.

$$\text{Fuel consumption} = \frac{\text{Amount of fuel} \times 100}{\text{Distance travelled}}$$
$$= \frac{60 \times 100}{750}$$
$$= 8.0 \text{ L/100 km}$$

Fuel consumption is 8 L per 100 km.

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Exercise 12E

LEVEL 1

1 Engine oil costs $60 for 5 litres. A car needs 5 litres of oil changed every 5000 km. Shane travels 15 000 km each year.

- **a** How many oil changes are required each year?
- **b** How many litres of engine oil are required each year?
- **c** What is the total cost of engine oil per year?
- **d** What is the cost of engine oil per kilometre?

Example 6

2 The table below shows the running costs of a car in cents per kilometre.

Running costs	50 000 km	75 000 km	100 000 km
Fuel	10.4	10.4	10.4
Tyres	2.3	2.8	2.7
Services	7.8	7.5	8.6

- **a** What are the total running costs if the car travels 50 000 km?
- **b** What are the total running costs if the car travels 75 000 km?
- **c** What are the total running costs if the car travels 100 000 km?

Example 7

3 Calculate the fuel consumption (litres per 100 km) for each of the following vehicles.

- **a** Ahmed's car uses 38.2 litres of petrol to travel 400 km.
- **b** A sports car travelled 900 km using 79.38 litres of petrol.
- **c** Joel's sedan uses 30.36 litres of LPG to travel 600 km.
- **d** A small car uses 41.05 litres of petrol to travel 500 km.
- **e** Zahra's car uses 139.8 litres of petrol to travel 1500 km.
- **f** Max's motorbike uses 70 litres of LPG to travel 2000 km.

4 Chelsea has bought a used car whose fuel consumption is 10 litres of petrol per 100 kilometres. She is planning to travel around Australia. Calculate the number of litres of petrol Chelsea's car will use on the following distances.

- **a** A trip of 2716 km from Perth to Adelaide
- **b** A trip of 732 km from Adelaide to Melbourne
- **c** A trip of 658 km from Melbourne to Canberra
- **d** A trip of 309 km from Canberra to Sydney
- **e** A trip of 982 km from Sydney to Brisbane
- **f** A trip of 3429 km from Brisbane to Darwin
- **g** A trip of 4049 km from Darwin to Perth

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LEVEL 2

5 Stephanie travels 37 km to work and 37 km from work each day.

a How many kilometres does she travel to and from work in a 6-day working week?

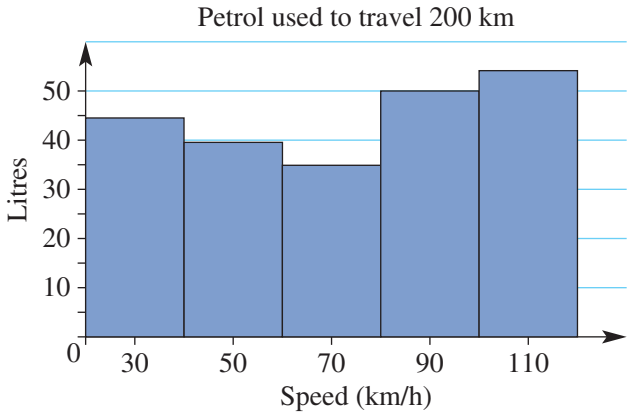
b Stephanie drives an SUV with a fuel consumption of 8.38 L/100 km. How many litres of petrol does Stephanie use travelling to and from work in a week? Answer correct to one decimal place.

c What is Stephanie's petrol bill for travelling to work if petrol costs $1.35 per litre?

6 A family car uses LPG at a rate of 15 L/100 km and the gas tank holds 72 litres. How far can the car travel on a tank of LPG?

7 Grace drives a four-wheel-drive vehicle with a petrol consumption of 15.2 L/100 km and a petrol tank capacity of 95 litres. She is planning a trip from Sydney to Bourke via Dubbo. The distance from Sydney to Dubbo is 412 km and from Dubbo to Bourke is 360 km. Grace filled her petrol tank at Sydney. How many times will she need to fill her tank before arriving at Bourke? Give reasons for your answer.

8 The graph below shows a motor vehicle's fuel consumption at various speeds.



a How many litres of fuel were used at 70 km/h?

b How many litres of fuel were used at 110 km/h?

c What is the fuel consumption rate at 30 km/h?

d What is the fuel consumption rate at 90 km/h?

e What speed used fuel the most efficiently?

f How many litres of fuel were saved by travelling at 90 km/h instead of 110 km/h?

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9 Dylan owns a V8 car with a fuel consumption of 11 L/100 km in the city and 8 L/100 km in the country. Dylan travels 8000 km per year in the city and 10 000 km per year in the country. The average cost of petrol is $1.50 per litre in the city and 10 cents higher in the country.

- **a** Determine the cost of petrol to drive in the city for the year.
- **b** Determine the cost of petrol to drive in the country for the year.
- **c** What is the total cost of petrol for Dylan for one year?
- **d** What is the total cost of petrol for Dylan in one year if the average cost of petrol increased to $1.80 per litre in the city and country prices stayed 10 cents higher?

10 Holly is planning a trip from Sydney to Brisbane using a car with a fuel consumption of 13 litres/100 km. The distance from Sydney to Brisbane via the Pacific Highway is 998 km and via the New England Highway it is 1027 km. The cost of LPG is 79.2 cents per litre.

- **a** How much will the trip cost via the Pacific Highway?
- **b** How much will the trip cost via the New England Highway?
- **c** How much money is saved by travelling via the Pacific Highway?

LEVEL 3

11 Tyler buys a new car with a fuel consumption of 11.2 litres/100 km. Oscar buys the LPG version of Tyler's new car, with a fuel consumption of 15.4 litres/100 km. Both Tyler and Oscar average 300 km in a week in the same conditions. The average price of ULP is $1.40 cents per litre and LPG is $0.79 cents per litre.

- **a** How many litres of fuel are used by Tyler in a week?
- **b** How many litres of fuel are used by Oscar in a week?
- **c** Calculate each car's yearly consumption of fuel.
- **d** What is Tyler's yearly fuel bill?
- **e** What is Oscar's yearly fuel bill?
- **f** Oscar paid an additional $1500 for the LPG version of the car. How many years will it take for the fuel savings to reach $1500 or the break-even point? Answer correct to the nearest whole number.
- **g** Research the current fuel prices of ULP and LPG. How long will it take for the fuel saving to exceed the initial costs?

12 Investigate the costs for two common cars on a family trip in your local area. Calculate the cost for the return trip in each case. You will need to determine the distance of the trip, fuel consumption for each car and the average price of fuel in the local area.

12F Personal budget

Budgeting involves balancing income and expenses. It is planning how to manage your income. Budgets are created for a specified time, such as weekly, monthly or yearly.

CREATING A BUDGET

1. List all the income categories.
2. List all the expense categories.
3. Calculate the total of the income and expenses categories.
4. Balance the budget by modifying the categories or by entering a balance category.

Example 8: Balancing a budget 12F

Balance the following weekly budget.

Income		Expenses	
Salary	$1726.15	Clothing	$73.08
Bonus	$20.00	Gifts and Christmas	$114.80
Investment	$156.78	Groceries	$467.31
Part-time work	$393.72	Insurance	$171.34
		Loan repayments	$847.55
		Motor vehicle costs	$105.96
		Phone	$38.26
		Power and heating	$51.82
		Rates	$54.82
		Recreation	$216.79
		Work-related costs	$68.76
		Balance	
Total		Total	

SOLUTION:

1. Add all the income.

$$\begin{aligned}\text{Income} &= 1726.15 + \ldots + 393.72 \\ &= \$2296.65\end{aligned}$$

2. Add the all the expenses excluding the 'balance'.
3. Subtract the total expenses from the total income.
4. Write the result of step 3 as the balance.

$$\begin{aligned}\text{Expenses} &= 73.08 + \ldots + 68.76 \\ &= \$2210.49 \\ \text{Balance} &= \text{Income} - \text{Expenses} \\ &= 2296.65 - 2210.49 \\ &= \$86.16\end{aligned}$$

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Exercise 12F

LEVEL 1

1 Oscar and Jill are living in a unit. Part of their budget is shown below. Calculate the total amount paid over one year for:

a electricity
b insurance
c food
d rent

Item	When	Cost
Electricity	Quarterly	$384
Food	Weekly	$360
Insurance	Biannually	$1275
Rent	Monthly	$1950

2 Sarah earns $67 365 annually. She has budgeted 20% of her salary for rent. How much does she expect to pay to rent an apartment for one year?

Example 8

3 Adam has constructed a yearly budget as shown below.

Income		Expenses	
Wage	$60 786.22	Clothing	$4634.42
Interest	$674.15	Council rates	$1543.56
		Electricity	$1956.87
		Entertainment	$4987.80
		Food	$17 543.90
		Gifts and Christmas	$5861.20
		Insurance	$2348.12
		Loan repayments	$16 789.34
		Motor vehicle costs	$2458.91
		Telephone	$832.98
		Work-related costs	$812.67
		Balance	
Total		Total	

a Calculate the total income.
b Calculate the total expenses.
c Balance the budget.

4 Nathan has a net weekly income of $1700. This table shows his weekly expenses.

Item	Cost
Rent	$450
Car expenses	$300
Food	$250
Other expenses	$200

a What are Nathan's total weekly expenses?
b How much can Nathan save each week?
c What percentage of his income is paid in car expenses?
d Nathan is planning an overseas holiday for $10 000. For how many weeks does he need to save?

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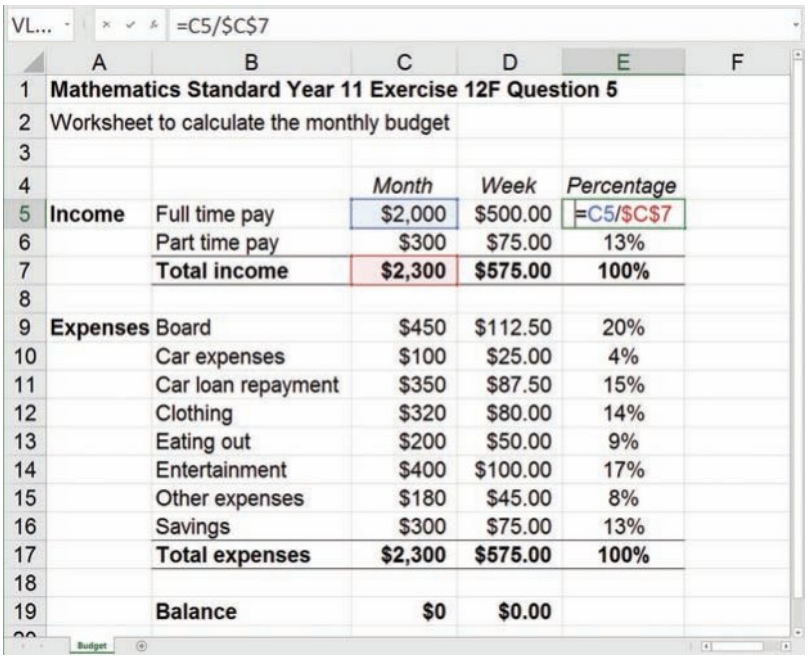
LEVEL 2

5 Create the spreadsheet below.

XLS
12FQ5



=C5/$C$7

	A	B	C	D	E	F
1	**Mathematics Standard Year 11 Exercise 12F Question 5**					
2	Worksheet to calculate the monthly budget					
3						
4			*Month*	*Week*	*Percentage*	
5	**Income**	Full time pay	$2,000	$500.00	=C5/$C$7	
6		Part time pay	$300	$75.00	13%	
7		**Total income**	**$2,300**	**$575.00**	**100%**	
8						
9	**Expenses**	Board	$450	$112.50	20%	
10		Car expenses	$100	$25.00	4%	
11		Car loan repayment	$350	$87.50	15%	
12		Clothing	$320	$80.00	14%	
13		Eating out	$200	$50.00	9%	
14		Entertainment	$400	$100.00	17%	
15		Other expenses	$180	$45.00	8%	
16		Savings	$300	$75.00	13%	
17		**Total expenses**	**$2,300**	**$575.00**	**100%**	
18						
19		**Balance**	**$0**	**$0.00**		

a The formula for cell E5 is '= C5 / $C$7'. It is the formula for relative percentage. Fill down the contents of E5 to E7 using this formula.
b Enter formulae in E9–E17 to calculate the relative percentages for expenses.
c Edit the amount spent per month on eating out from $200 to $240. Observe the changes.
d Edit the amount of savings per month from $300 to $360. Observe the changes.
e Edit the amount of car expenses per month from $100 to $150. Observe the changes.

6 Bella earns $4000 each fortnight and pays 37% of this in tax and other deductions. This table shows her major fortnightly expenses.

a What are Bella's total fortnightly expenses?
b What amount is Bella left with each fortnight after tax and other deductions?
c What is the maximum she can save each fortnight?
d Bella's utilities bill increased by 8%. How much will she be left with at the end of the fortnight?

Item	Cost
Mortgage	$750
Utilities	$250
Groceries	$400
Entertainment	$100

7 Aiden lives with two friends and they agree to split the household costs evenly. For each week, rent is $600; utilities are $850 for each quarter; insurance is $350 for each quarter; food is $500 each fortnight; and cleaning and other expenses are $200 each month.

a Find the household costs for the year.
b How much must Aiden contribute each week? Answer to the nearest dollar.

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LEVEL 3

8 Patrick has a net annual salary of $95 000. He pays $1350 each month in rent and has regular bills that amount to $4450 each year. Patrick estimates that, each month, food costs him $625, entertainment is $375 and other expenses are $3250.

- **a** How much will Patrick save each year?
- **b** Patrick is considering taking out a loan to buy a house. Repayments will be $2750 each month. Will he be able to afford the repayments? Justify your answer.

9 Hayley prepared a monthly budget as shown in the table.

- **a** Calculate her total income.
- **b** What is the difference between the fixed expenses and the variable expenses?
- **c** How much can Hayley save each month?
- **d** Hayley has no savings and is planning a trip to Europe in eight months time. She estimates the trip will cost $30 000. Will Hayley be able to save enough to pay for the trip?

Income	**Amount**
Net wage	$8750
Interest	$300
Fixed expenses	**Amount**
Housing	$2225
Transport	$550
Utilities	$420
Variable expenses	**Amount**
Clothing	$670
Entertainment	$320
Food	$1125
Personal care	$60

10 Dimitri had a total weekly income of $104 made up of a part-time job earning $74 and an allowance of $30. He decided to budget his expenses in the following way: sport $24, movies $22, school $16 and food $20.

- **a** Prepare a weekly budget showing income and expenses.
- **b** What is the balance?

11 Ava has a net fortnightly pay of $1896.

- **a** Ava has a mortgage with an annual repayment of $13 676. Calculate the amount that Ava must budget each fortnight for her mortgage.
- **b** Ava has budgeted $180 per week for groceries, $60 per week for entertainment, $468 per year for medical expenses and $80 per week to run a car. Express these as fortnightly amounts and calculate their total.
- **c** Ava has an electricity bill of $130 per quarter, telephone bill of $91 per quarter and council rates of $1118 per annum. Express these amounts annually and convert to fortnightly amounts. What is the total of these fortnightly amounts?
- **d** Prepare a fortnightly budget showing income and expenses.

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PPT STUDY GUIDE

Key ideas and chapter summary

Household bills

Household bills are the costs associated with living in a home. They are a big percentage of the expenses in a budget. Household bills include council rates, electricity and gas bills, water and sewerage rates, telephone bills and internet connections.

Purchasing a car

Total cost = Deposit + Total repayments

Total repayments = Repayment × Number of repayments

Interest paid = Total cost − Sale price

Car insurance

The cost of insurance is affected by make and model of car, your age and driving history, finance, modifications or accessories and location.

No-claim bonus and excess amount are major factors.

Stamp duty

1. Round the cost of the vehicle up to the nearest $100 (or as required).
2. Express the stamp duty as a fraction or decimal.
3. Multiply the answer in step 1 by the answer in step 2.

Car running costs

Car running costs involve maintenance, repairs, fuel, improvements, parking, tolls, car washes and fines.

Fuel consumption

$$\text{Fuel consumption} = \frac{\text{Amount of fuel (L)} \times 100}{\text{Distance travelled (km)}}$$

Budgeting

Budgeting involves balancing of income and expenses. Budgets are created for a specified time such as weekly.

Creating a budget

1. List all the income categories.
2. List all the expense categories.
3. Calculate the total of the income and expenses categories.
4. Balance the budget by modifying the categories or by entering a balance category.

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Review

Multiple-choice questions

1 The usage summary of a quarterly electricity account is shown opposite. What was the percentage increase in electricity usage?

A 1.11% **B** 5.15%
C 5.43% **D** 21.55%

Average cost per day	**$5.39**
Average daily usage	**21.55 kWh**
Same time last year	**20.44 kWh**
Your indicative greenhouse gas emissions	
Total for this bill	**1.8 tonnes**
Same time last year	**1.8 tonnes**
Saved with Green Power	**N/A**

2 An electricity bill increased by 6%. The previous bill was $860. What is the new electricity bill, correct to the nearest dollar?

A $52 **B** $866 **C** $911 **D** $912

3 Adam has the following bills: electricity $250 per quarter, phone $70 per month, rates $1200 per year and rent $300 per week. What is the total amount Adam should budget for the year?

A $358 **B** $1553 **C** $1820 **D** $18 640

4 A motorbike is for sale at $13 000. Finance is available at $3000 deposit and monthly repayments of $520 for 4 years. What is the interest paid?

A $14 960 **B** $17 960 **C** $24 960 **D** $27 960

5 Jake has been quoted $1280 for comprehensive car insurance. He has a no-claim bonus of 60%. How much is Jake required to pay?

A $512 **B** $768 **C** $1220 **D** $1280

6 A new car is bought for $28 810. Calculate the stamp duty payable if the charge is $3 per $100 or part of $100.

A $840 **B** $864 **C** $867 **D** $870

7 Mia's car uses 8.25 litres per 100 km. How many litres of petrol will her car use on a trip of 1150 km from Broken Hill to Sydney?

A 94.875 L **B** 139.73 L **C** 1397.3 L **D** 9487.5 L

8 Hunter travels 36 000 km each year in his four-wheel drive. A set of tyres costs $880 and lasts about 32 000 km. What is the cost of tyre wear per kilometre? Answer to the nearest cent.

A $0.02 **B** $0.03 **C** $0.36 **D** $0.41

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Short-answer questions

1 Amelia earns $90 345 annually. She has budgeted 30% of her salary for a loan repayment. How much should she expect to pay for the loan repayment for one year?

2 Michael buys a car for $18 000. After one year the market value of the car is $15 000. What is the percentage decrease in the price? Answer correct to one decimal place.

3 A new car is for sale at $39 000. Finance is available at 20% deposit and monthly repayments of $900 for 5 years.

a How much will the car cost if you use the finance package?

b What is the interest paid?

4 Lucy is 18 years old and has received this quote for comprehensive insurance.

Premium details	Excesses	Cost
12-month policy $850.00 10% No-claim bonus	Standard	$600
	Male under 21	$1400
	Female under 21	$1000

a Calculate the cost of the insurance.

b What is Lucy's excess if she makes a claim?

5 Engine oil costs $52 for 5 litres. A car needs 5 litres of oil changed every 6000 km. Victoria travels 24 000 km each year. What is the cost of oil for four years?

6 Rata's car uses 11.26 litres per 100 km.

a How many litres of petrol will his car use on a trip of 155 km from Sydney to Newcastle?

b The cost of petrol is $1.60 per litre. How much will the petrol cost for the 155 km trip?

c How many litres of petrol will his car use on a trip of 294 km from Armidale to Taree?

d The cost of petrol is $1.35 per litre. How much will the petrol cost for the 294 km trip?

7 Sienna filled her car with petrol. The odometer reading was 64 080 km at that time. When she next filled the petrol tank, the odometer reading was 64 605 km. The car required 42 L of petrol.

a How far has the car travelled between fill ups?

b What was the average fuel consumption in litres per 100 km?

c Sienna next filled the petrol tank at 65 200 with 45 L of petrol. What was the average fuel consumption in kilometres per litre from the last fill up? Answer correct to one decimal place.

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Review

8 Jett has constructed a yearly budget shown below.

Income		Expenses	
Salary	$62 609.80	Christmas	$6 037.03
Interest	$705.45	Clothing	$4 773.45
		Council rates	$1589.86
		Electricity	$2 015.57
		Entertainment	$5137.43
		Food	$18 070.22
		Insurance	$2 418.56
		Loan repayments	$17 293.02
		Motor vehicle costs	$2 532.67
		Telephone	$857.97
		Work related costs	$837.05
		Balance	
Total		Total	

a Find the total income. **b** Find the total expenses. **c** Balance the budget.

9 Hannah has budgeted $210 per week for groceries, $70 per week for leisure, $23 per fortnight for medical expenses and $90 per week to run a car. Calculate the monthly expenses. Assume 4 weeks in a month.

Extended-response question

10 Blake took out a loan to purchase a car and will make regular monthly payments over 5 years. However, owning and running a car is not cheap:

- Loan repayments are $680 per month.
- Car registration costs $900 per year.
- Servicing costs $725 twice a year.
- A green slip costs $410 per year.
- Comprehensive insurance costs $190 per month.
- Petrol costs $80 per week.

a How much will Blake eventually pay on the loan?

b What is the cost of the car, given that 40% of the loan repayments cover the interest?

c What is the total annual cost during the purchase period?

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Practice Paper 2

Section I

Attempt Questions 1–15 (15 marks)
Allow about 20 minutes for this section

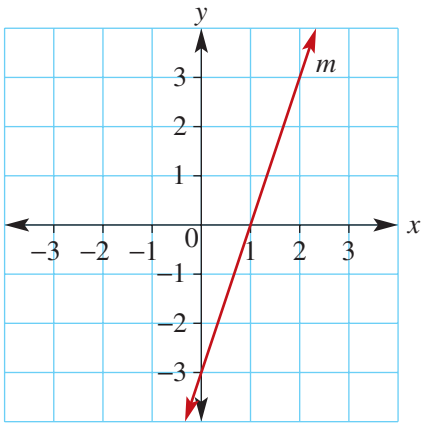
1 A survey required respondents to write down their age last birthday.
Which of the following terms best describes this data?

A Categorical
B Continuous
C Discrete
D Stratified

2 What is the new price when \$60 is increased by 20% then decreased by 20%?

A \$38.40
B \$57.60
C \$60.00
D \$86.40

3



What is the equation of line m?

A $y = 3x - 3$
B $y = \frac{x}{3} - 3$
C $y = -3x - 3$
D $y = -\frac{x}{3} - 3$

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4 For the scores 20, 22, 22, 14, 19, 22, 20, 21, 24 and 12 consider the following statements.

i The median is greater than the mean.

ii The mode is less than the median.

A Both statements I and II are true.

B Both statements I and II are false.

C Statement I is false and statement II is true.

D Statement I is true and statement II is false.

5 A car uses, on average, 7 L per 100 km in fuel. How much fuel would be used on a trip of 382 km?

A 26.74 L

B 34.72 L

C 38. 20 L

D 54.57 L

6 What is the gradient of a line given by the equation $y = 2x - 4$?

A -4 **B** -2

C 2 **D** 4

7 What type of data is information collected on the type of motor vehicle?

A Discrete

B Continuous

C Categorical

D Stratified

8 A car mechanic charges a total of \$165 to repair a motor vehicle. The Goods and Services Tax (GST) of 10% was included in this total. Which of the following statements is correct?

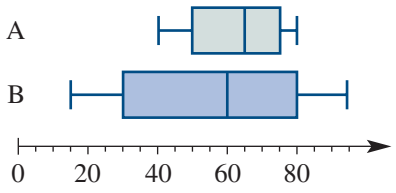
A 90% of \$165 was the price of the repair before the GST was added.

B The total repair price included \$16.50 GST.

C The price before adding the GST is \$165 ÷ 1.10.

D The GST cannot be determined without knowing the original repair cost.

9 Two examinations results are displayed in the box plot. What is the interquartile range for exam A?



A 25

B 40

C 50

D 80

10 A country has 30% of the population between the ages of 20 and 30. How many people aged between 20 and 30 should be included in a stratified sample of 250 people?

A 30

B 45

C 60

D 75

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11 A car is travelling at a constant speed. It travels 60 km in 3 hours. This situation is described by the linear equation $d = mt$. What is the value of m?

A 0.05
B 3
C 20
D 60

12 A cruise ship leaves the wharf at 6:48 a.m. The journey to its next port takes 13 hours and 16 minutes. When does the cruise ship arrive at this port?

A 7:32 a.m.
B 8:04 a.m.
C 7:32 p.m.
D 8:04 p.m.

13 Police checked the blood alcohol content of every fifth driver passing an intersection. What is this method of sampling?

A Census
B Random
C Stratified
D Systematic

14 What is the time difference between Ho Chi Minh (UTC + 7) and Melbourne (UTC + 10)?

A 3 h
B 7 h
C 17 h
D 70 h

15 Which equation correctly gives the relationship between x and y, from the table?

x	0	2	5	9	10
y	–2	10	28	52	58

A $y = 5x - 2$
B $y = 4x + 2$
C $y = 6x - 2$
D $y = 10x + 2$

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Section II

Attempt Questions 16–18 (45 marks)
Allow about 70 minutes for this section
All necessary working should be shown in every question.

Question 16 (15 marks) **Marks**

a Nathan is an auto-electrician who is entitled to a 15% trade discount. In addition, he is given a 10% reduction on the discounted price if he pays cash. Nathan bought electrical equipment with a full retail price of $480 and was given both discounts.

i How much does Nathan pay for the electrical equipment? **1**

ii How much money is saved using the discounts? **1**

iii Express the overall savings as a percentage of the retail price. **1**

b Jacksonville is located at (30°N, 82°W) and Suva is located at (18°S, 178°E).

i What is the difference in latitude between Jacksonville and Suva? **1**

ii What is the difference in longitude between Jacksonville and Suva? **1**

iii What are the coordinates of a point 45° due south of Jacksonville? **1**

c Raymond has been quoted $1540 for comprehensive car insurance. He has a no-claim bonus of 40%. How much is Raymond required to pay? **2**

d Nina's car uses 9.25 litres per 100 km.

i How many litres of petrol will this car use on a trip of 185 km from Parramatta to Nowra? **1**

ii The cost of petrol is $1.37 per litre. How much will the petrol cost for the 185 km trip? **1**

e Alexis's parents want to give her $10 000 for her wedding in five years time. They have found an account that will earn 8% p.a. simple interest. What is the amount of money they need to invest in this account to total $10 000 in 5 years time? **2**

f A new car is purchased for $35 800. After 6 years its salvage value is $24 160. What is the annual amount of depreciation, if the amount of depreciation is constant? **2**

g A truck rental company charges $100 for the hire of a truck plus 40 cents for each kilometre travelled. Write an expression for total cost (c) in dollars of hiring the truck if the truck travelled (x) kilometres. **1**

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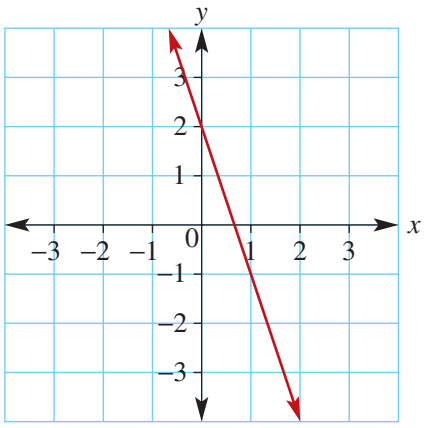
Question 17 (15 marks) **Marks**

a



i What is the gradient of this straight line? **1**

ii What is the y-intercept of this straight line? **1**

iii What is the equation of this straight line? **1**

iv What is the x-intercept of this straight line? **1**

b Max has completed three tests. His mean mark is 64%. What percentage mark does he have to get in his next test to increase the mean to 65%? **2**

c Sarah conducted a survey of students' opinions about the school uniform. She selected the first five people who were not in school uniform for the survey. Why might the results of this survey be biased? **2**

d A Year 11 class is given a short quiz out of 10. Calculate the following measures of location and spread. (Answer correct to one decimal place.)

Score	Number of students
7	4
8	3
9	5
10	6

i Mean **1**

ii Median **1**

iii Mode **1**

iv Range **1**

v Interquartile range **1**

e David earned \$787.50 in interest when he invested \$5000 over a period of 3 years and 6 months. What is the flat annual rate of interest paid on this investment? **2**

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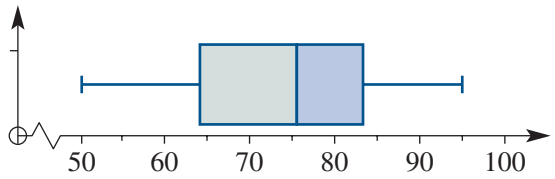
Question 18 (15 marks) **Marks**

a The following box-and-whisker plot shows the weights (kg) of 400 people.



i What is the range of weights? **1**

ii Determine the median weight. **1**

iii Determine the interquartile range of weights. **1**

iv How many people weighed less than 83 kg? **2**

b Logan records the details of each CD in his music collection. State the type of data that would be recorded for each of the following.

i Title of the CD **1**

ii Number of tracks on the CD **1**

iii Playing time for each track **1**

c Philip's water usage in one day was 50 L shower, 44 L toilet, 35 L tap and 18 L washing machine. What percentage of water usage is for the tap? **2**

d Oscar invests $100 000 for 8 months in a term deposit. This investment offers a flat rate of 6% per annum interest. What is the interest earned from this investment? **3**

e The time zone of Dili is UTC + 9 and Napier is UTC + 12.

i What is the time in Dili if the time in Napier is 10:30 a.m.? **1**

ii A flight leaves Napier at 12:30 p.m. and flies non-stop to Dili. The flight is in the air for 6 hours. At what time does the flight arrive in Dili? **1**

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2019 Higher School Certificate Examination Mathematics Standard 1/2 Reference sheet

Measurement

Precision

Absolute error $= \frac{1}{2} \times$ precision

Upper bound = measurement + absolute error

Lower bound = measurement − absolute error

Length, area, surface area and volume

$$l = \frac{\theta}{360} \times 2\pi r$$

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(x + y)$$

$$A \approx \frac{h}{2}(d_f + d_l)$$

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Trigonometry

$$A = \frac{1}{2}ab \sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Financial Mathematics

$$FV = PV(1 + r)^n$$

Straight-line method of depreciation

$$S = V_0 - Dn$$

Declining-balance method of depreciation

$$S = V_0(1 - r)^n$$

Statistical Analysis

$$z = \frac{x - \bar{x}}{s}$$

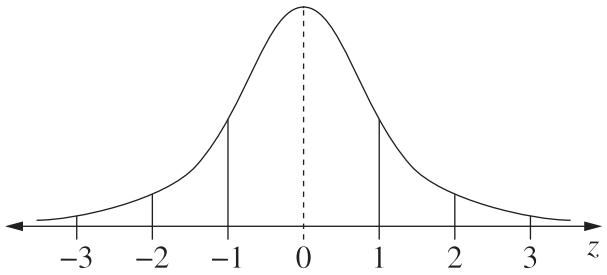
An outlier is a score

less than $Q_1 - 1.5 \times IQR$

or

more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

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Glossary

Note: The interactive textbook-includes more pop-up definitions and a dictionary.

A

Absolute error The difference between the actual value and the measured value indicated by an instrument.

Algebraic expression Mathematical statement built up from numbers and pronumerals (variables)and connected by mathematical operations, but containing no equals sign.

Allowable deduction *See* tax deduction

Amount owed The result when the interest of a loan or investment is added to the principal amount (also known as future value).

Angular distance The distance between two points on a circle or sphere measured as the angle formed at the centre between radii at the points.

Annual leave loading A payment calculated as a fixed percentage of the normal pay over a fixed number of weeks. Annual leave loading is usually at the rate of $17\frac{1}{2}\%$.

Annulus Area between a large and a small circle with a common centre.

Appreciation An increase in value of an item over time. It is often expressed as the rate of appreciation.

Area The amount of surface enclosed by the boundaries of the shape.

B

BAC *See* blood alcohol content.

Balance The amount of money in an account; the total amount an investment is worth at a point in time including amount invested and interest; or the amount still owing on a loan.

Bias When events are not equally likely.

Bimodal Data with two modes or peaks.

Blood alcohol content (BAC) A measure of the amount of alcohol in your blood.

Bonus An extra payment or gift earned as reward for achieving a goal.

Bound A limit to a measurement. *See also* **upper bound** and **lower bound**.

Box-and-whisker plot A graph that uses a five-number summary of a numerical data set.

Box plot *See* Box-and-whisker plot.

Budget A plan used to manage money by listing a person's income and expenditure.

C

Calorie A non-SI unit of heat and energy, once commonly used to measure food energy.

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Capacity The maximum volume of liquid or gas which a container could hold.

Car running costs Car costs, such as maintenance, repairs, fuel, improvements, parking, tolls, car washes and fines.

Casual rate An amount paid for each hour of casual work.

Categorical data Data that is divided into categories such as hair colour. It uses words not numbers.

Census Collecting data from the whole population.

Central tendency *See* measure of central tendency.

Circumference The distance around the outside of a circle; the curved boundary.

Class A group of data.

Class centre Median or middle score of a class in a grouped frequency distribution.

Closed cylinder A cylinder with both circular bases. *See* Open cylinder.

Coefficient A numeral placed before a variable to indicate that the variable is multiplied by that number.

Commission A payment for services, mostly as a percentage of the value of the goods sold.

Common denominator A common denominator is always found by multiplying the denominators of two given fractions together. This is required before adding or subtracting fractions.

Common factors A number or term that divides exactly (no remainder) into each of the given numbers or terms.

Complementary event The outcomes that are not members of the event.

Composite shape A 2D shape made up of two or more 2D shapes.

Composite solid Two or more common solids.

Compound interest A type of interest that is paid on a loan or earned on an investment, which is calculated not only on the initial principal, but also on the interest accumulated during the loan/investment period.

Compounding annually A form of compound interest where the interest is calculated per year.

Compounding time period The compounding period is the time period for the calculation of interest for an investment or loan. Typical compounding periods are yearly, quarterly, monthly or daily.

Concentration The amount of an ingredient in a mixture or solution expressed as a ratio to the amount of the mixture or solution.

Cone A solid figure, with a circular base, that tapers to a point.

Constant of variation He constant coefficient related to pairs of variables that are in direct proportion.

Constant term The part of an equation or expression without any pronumerals.

Consumption Of fuel or energy, the rate expressed as amount used over time or distance or other variable.

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Continuous data Numerical data obtained when quantities are measured rather than counted.

Conversion graph A graph used to change a quantity from one unit to another unit.

Coordinated Universal Time (UTC) *See* Greenwich Mean Time.

Corresponding Matching, for example in two or more similar figures, the sides or angles which match.

Cross-section The intersection of a solid with a plane.

Cumulative frequency The frequency of the score plus the frequency of all the scores less than that score. It is the progressive total of the frequencies.

Cumulative frequency histogram A histogram with equal intervals of the scores on the horizontal axis and the cumulative frequencies associated with these intervals shown by vertical rectangles.

Cumulative frequency polygon A line graph constructed by joining the top right-hand corner of the rectangles in a cumulative frequency histogram. Also called an ogive.

Cylinder A prism right with a circular cross-section. *See* Open cylinder and Closed cylinder.

D

Data Raw scores. Information before it is organised.

Daylight saving time Advancing clocks one hour during the warmer months of the year so that the increased hours of daylight fall in the evening more than in the early morning.

Decile Divides an ordered dataset into 10 equal groups.

Deduction A regular amount of money subtracted from a person's wage or salary. *See also* Tax deduction

Dependent variable If two variables y and x are related in an equation, when a number is substituted for one variable, such as $x = 2$, then this variable is called the independent variable and the other is called the dependent variable.

Deposit An amount put into an account; an initial amount paid to reserve the goods or services; the first payment of a series of installments.

Depreciation *See* straight-line depreciation.

Direct variation The relationship between two quantities that increase or decrease at the same rate. Equation for direct variation: $y = mx$ where y and x are two variables in direct proportion to each other and m is the gradient of the graph of y against x.

Discrete data Data obtained when a quantity is counted. It can only take exact numerical values.

Distributive law A rule for expanding grouping symbols by multiplying each term inside the grouping symbol by the number or term outside the grouping symbol.

Dose The amount of a medication or drug given to a patient; it may refer to the amount of the active ingredient rather than total amount of medication itself.

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Dot plot A graph that consists of a number line with each data point marked by a dot. When several data points have the same value, the points are stacked on top of each other.

Double stem-and-leaf plot A stem-and-leaf plot that uses two sets of similar data together.

Double-time A penalty rate that pays the employee twice the normal hourly rate.

E

Element of the sample space An outcome or data value in probability.

Energy The capacity to do work.

Energy consumption The amount of energy consumed per unit of time.

Equally likely outcomes Outcomes of an event that have the same chance of occurring.

Equation A mathematical statement that says that two things are equal.

Equator Imaginary horizontal line that divides the Earth into two hemispheres. Latitude of the equator is 0°.

Equivalent fractions Fractions that can be reduced to the same basic fraction; fractions that have the same value but are written differently.

Evaluate Calculate the exact value of an expression.

Event In a probability experiment, the outcome or result. *See also* multistage event.

Expand Remove the grouping symbols.

Expanded form An expression with no grouping symbols; a term shown without indices.

Expected frequency The number of times that a particular event should occur.

Expression A mathematical statement written in numbers and symbols, containing two or more numbers or variables connected by operations.

F

Factorise To break up an expression into a product of its factors.

Five-number summary A summary of a data set consisting of the lower extreme, lower quartile, median, upper quartile and upper extreme.

Flat interest *See* Simple interest.

Formula A mathematical relationship between two or more variables.

Fortnight Two weeks or 14 days.

Frequency The number of times a certain event occurs; also the number of times a data value occurs in a set.

Frequency distribution The division of a set of observations into a number of classes, together with a listing of the number of observations (the frequency) in that class. Frequency distributions can be displayed in the form of a frequency table, a two-way-table or in graphical form.

Frequency histogram A histogram with equal intervals of the scores on the horizontal axis, and the frequencies associated with these intervals shown by vertical rectangles.

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Frequency polygon A line graph constructed by joining the midpoints at the tops of the rectangles of a frequency histogram.

Frequency table A table that lists the outcomes and how often (frequency) each outcome occurs. If the data is grouped it is a grouped frequency table.

Fuel consumption rate The number of litres of fuel a vehicle uses to travel 100 kilometres.

Function (mathematical) A mathematical relation where each element of a given set is associated with at most one element of another set.

G

General form A linear equation written in the form $ax + by + c = 0$.

Goods and Services Tax (GST) A tax added to the purchase price of each item. The GST rate in Australia is 10% of the purchase price of the item except for basic food items and some medical expenses.

Gradient The steepness or slope of the line. It is calculated by dividing the vertical rise by the horizontal run.

Gradient–intercept formula A linear equation written in the form $y = mx + b$.

Greenwich Mean Time Time at the Greenwich meridian.

Greenwich meridian Imaginary vertical line that passes through the town of Greenwich (London). Longitude of the Greenwich meridian is 0°.

Gross income The total amount of money earned from all sources. It includes interest, profits from shares and all payments received throughout the year.

Gross pay The total of an employee's pay including allowances, overtime pay, commissions and bonuses.

Grouped data Data organised into small groups or class intervals rather than as individual scores.

Grouped frequency table A table with classes listed in the first column in ascending order, a class centre column, an (optional) tally column and a frequency column with total count of scores in each class.

Grouping symbol Symbols used to indicate the order of operations such as parentheses ( ) and brackets [ ].

H

Histogram A graph using columns to represent frequency or cumulative frequency. *See* Frequency histogram and Cumulative frequency histogram.

Hypotenuse The side in a right-angled triangle opposite the right-angle. It is the longest side.

I

Income tax Tax paid on income received.

Independent variable *See* dependent variable.

Index form *See* Index notation.

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Index notation A method to write expressions in a shorter way such as $a \times a = a^2$.

Inflation A rise in the price of goods and services or Consumer Price Index (CPI). It is often expressed as annual percentage.

Intercept The position where the line cuts the axis.

Interest The amount paid for borrowing money or the amount earned for lending money.

Interest rate The rate at which interest is charged or paid. It is usually expressed as a percentage.

International Date Line An imaginary line through the Pacific Ocean that corresponds to 180° longitude.

Interquartile range The difference between the first quartile and third quartile.

Interval (data) Describes the boundaries of a group of data.

Investment Money deposited into a bank or financial institution that grows at a particular interest rate for the time it is kept there.

Isosceles triangle A triangle where two sides have equal lengths and two angles are equal. The equal angles are the ones joining each same-length side to the third side.

J

Joule SI unit of energy.

K

Kilojoules SI unit of energy, commonly used to measure food and heat energy, equal to 1000 joules accepted measurement for food energy.

Kilowatt-hour Non-SI unit energy commonly used to measure electrical energy.

L

Latitude The angle or angular distance north or south of the equator.

Like term Terms with exactly the same pronumerals, such as $3a$ and $6a$ and the same powers.

Limit of reading *See* **precision.**

Linear equation An equation whose variables are raised to the power of 1.

Linear expression A mathematical statement written in numbers and symbols *See also* Non-linear expression.

Linear function A function that when graphed on a number plane is a straight line. *See also* Non-linear function and Function (mathematical).

Linear modelling A mathematical description of a practical situation using a linear function.

Loan repayment An amount paid to a bank or financial institution at regular intervals to repay a loan with interest included.

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Longitude The angle or angular distance east or west of the Greenwich meridian.

Lower bound Measurement minus absolute error.

Lower extreme Lowest score in the data set.

Lower quartile The lowest 25% of the scores in the data set.

Lowest common denominator For two or more fractions, the lowest common multiple of their denominators.

M

Mass The amount of matter within an object.

Mean A measure of the centre. It is calculated by summing all the scores and dividing by the number of scores.

Measurement Determining the size of a quantity.

Measures of central tendency Also known as measures of location. The most common measures are mean, median and mode.

Measures of spread Measures of spread include range, interquartile range and standard deviation.

Median The middle score or value. To find the median, list all the scores in increasing order and select the middle one.

Medicare levy An additional charge to support Australia's universal healthcare system.

Meridians of longitude Great imaginary circles east and west of the Greenwich meridian.

Modality The number of modes occurring in a set of data.

Mode The score that occurs the most. It is the score with the highest frequency.

Multimodal Data with many modes or peaks.

Multistage event Two or more events such as tossing a coin and rolling a die.

N

Negatively skewed Data more on the right side. The long tail is on the left side (negative side).

Net of a solid A drawing consisting of plane shapes that can be folded to form the solid.

Net pay The amount remaining after deductions have been subtracted from the gross pay.

Nominal data Categorical data whose name does not indicate order.

Non-linear An equation or expression which when graphed does not produce a straight line.

Number pattern A sequence of numbers formed using a rule. Each number in the pattern is called a term.

Number plane A graph with a vertical y-axis and a horizontal x-axis intersecting at right angles at the origin O (0, 0).

Numerator The top number in a fraction

Numerical data Data that is measured using numbers.

O

Ogive *See* Cumulative frequency polygon.

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Open cylinder A cylinder without a circular base. It is the curved part of the cylinder.

Order of operations The sequence in which computations are done.

Ordinal data Categorical data whose name does indicate order.

Outcome A possible result in a probability experiment.

Outlier Data values that appear to stand out from the main body of a data set.

Overtime Extra payment when a person works beyond the normal working day.

P

Parallel box-and-whisker plot A box-and-whisker plot that uses two sets of similar data together.

Parallelogram A quadrilateral with both pairs of opposite sides parallel.

Parallels of latitude Small imaginary circles north and south of the equator.

Pareto chart A graph that combines a frequency histogram and a cumulative frequency line graph. The histogram is plotted in decreasing order of relative frequency.

Pareto principle 20% of the invested input is responsible for 80% of the results obtained.

Pay As You Go (PAYG) Tax deducted from a person's wage or salary throughout the year.

Per annum Calculated or applied each year.

Percentage change The increase or decrease in the quantity as a percentage of the original amount of the quantity.

Percentage error The maximum error in a measurement as a percentage of the measurement given.

Percentile Divides an ordered data set into 100 equal groups.

Perpendicular height The height of a 2D shape or 3D object when measured at right angles to the base.

Piecework A fixed payment for work completed.

Population The entire data set.

Population mean μ The mean of all values of a measure in the entire population.

Population standard deviation A calculation for the standard deviation that uses all the data or the entire population. (σ_n)

Positively skewed Data more on the left side. The long tail is on the right side (positive side).

Power (Physics) The rate at which energy is generated or consumed. (Mathematics) Another name for index or exponent; a way to express a number multiplied by itself one or more times.

Precision The smallest unit on the measuring instrument, also known as the limit of reading.

Prefix The first part of a word. In measurement, it is used to indicate the size of a quantity.

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Principal The initial amount of money borrowed, lent or invested.

Prism A solid shape that has the same cross-section for its entire length or height.

Probability The chance of something happening. The probability of the event is calculated by dividing the number of favourable outcomes by the total number of outcomes.

Pronumeral A letter or symbol used to represent a number.

Pyramid A solid shape with a plane shape as its base and triangular sides meeting at an apex.

Pythagoras' theorem The square of the hypotenuse is equal to the sum of the squares of the other two sides. $c^2 = a^2 + b^2$

Q

Quadrant Quarter of a circle. The arc of a quadrant measures 90°.

Quantile A set of values that divide an ordered dataset into equal groups.

Quantitative data Numerical data. It is data that has been measured.

Quartile Divides an ordered data set into four equal groups. *See* Upper quartile and Lower quartile.

R

Random sample A sample that occurs when members of the population have an equal chance of being selected.

Range The difference between the highest and lowest scores. It is a simple way of measuring the spread of the data.

Rate A comparison of different quantities in definite order.

Rate of interest *See* Interest rate.

Relative error A measurement calculated by dividing the limit of reading (absolute error) by the actual measurement.

Relative frequency The frequency of the event divided by the total number of frequencies. It estimates the chances of something happening or the probability of an event.

Replacement In the selection of cards (or other objects) from a set in a probability experiment, 'with replacement' means the selected card is returned to the set before the next card is selected, so the set always contains the original cards. Selection without replacement means the set reduces with each selection.

Retainer A fixed payment usually paid to a person receiving a commission.

Rhombus A quadrilateral with both pairs of opposite sides parallel and all sides equal.

Royalty A payment for the use of intellectual property such as book or song. It is calculated as a percentage of the revenue or profit received from its use.

S

Salary A payment for a year's work, which is divided into equal monthly, fortnightly or weekly payments.

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Sample A part of the population.

Sample mean $\bar{x}$ The mean of all values of a measure in a particular sample.

Sample space The set of all possible outcomes.

Sample standard deviation A calculation for the standard deviation when the data set is a sample (σ_{n-1}).

Scale factor The amount by which a shape is enlarged or reduced to get a similar figure of different size.

Scientific notation *See* Standard form.

Score A single value from a list of data.

Sector Part of a circle between two radii and an arc.

Self-selected sample Members of the population volunteer themselves.

Semicircle Half a circle. The arc of a semicircle measures 180°.

Significant figures A statement to specify the accuracy of a number. It is often used to round a number.

Similar figures Figures which are exactly the same shape but different sizes.

Simple interest A fixed percentage of the amount invested or borrowed and is calculated on the original amount.

Simulation A mathematical model that represents a real experiment or situation.

Skewed data Data that is not symmetrical. *See* Symmetrical, Positively skewed and Negatively skewed.

Slant height The shortest distance from the apex to an edge of the base in a pyramid or cone.

Smoothness Data whose graph has no breaks or jagged sections.

Sphere A perfectly round object such as a ball.

Stamp duty Tax paid to the government when registering or transferring a motor vehicle.

Standard deviation A measure of the spread of data about the mean. It is an average of the squared deviations of each score from the mean.

Standard drink Any drink containing 10 grams of alcohol.

Standard form (also called scientific notation) A number between 1 and 10 multiplied by a power of 10. It is used to write very large or very small numbers more conveniently.

Stem-and-leaf plot A method of displaying data in which the first part of a number is written in the stem and the second part of the number is written in the leaves.

Stopping distance The distance a vehicle travels from the time a driver sees an event occurring to the time the vehicle is brought to a stop.

Straight-line depreciation The value of an item decreases by the same amount each period.

Strata A group within a population that reflects the characteristics of the entire population.

Stratified sample A sample using categories or strata of a population. Members from each category are randomly selected. For example, one student is selected from each year 7, 8, 9, 10, 11 and 12.

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Subject of a formula or equation When a formula or equation has a pronumeral with no numbers on the left-hand side of the equal sign, such as $C = 40n + 75$ then C is the subject of the formula.

Substitution It involves replacing the pronumeral in an algebraic expression with one or more numbers.

Summary statistic A number such as the mode, mean or median that describes the data.

Superannuation Type of annuity in which money is invested for a person's retirement.

Surface area The sum of the area of each surface of the solid.

Symmetrical Data that forms a mirror image of itself when folded in the 'middle' along a vertical axis.

Symmetry Data evenly balanced about the centre.

Systematic sample A sample that divides the population into a structured sample size. For example, sorting the names of people in alphabetical order and selecting every 5th person.

T

Table of values A set of ordered pairs (arranged in table format) that can be used to plot the points to draw a graph.

Tally To use simple marks or strokes to track the number of times a value occurs; a cross stroke is often used to show groups of 5.

Tax deduction Expenses related to earning an income allowed by the Australian Taxation Office to be deducted from their income for the purposes of calculating taxable income, such as self-education, travel, car or clothing expenses.

Tax payable Tax owed to the taxation office, such as the amount of tax which an employee has to pay according to their taxable income.

Tax refund Money returned to an individual by the Australian Taxation Office when their tax return revels that their PAYG tax paid exceeds their tax payable in a year.

Tax return A form filled out on paper or online and submitted to the Australian Taxation Office which presents details of an individual's tax claims, income and tax deductions.

Taxable income The gross income minus any allowable deductions.

Time 24-hour Time of day written in the form hh : mm (hours : minutes).

Time-and-a-half A penalty rate that pays the employee one and half times the normal hourly rate.

Time zone A region of the Earth that has a uniform standard time or local time.

Timetable A list of times at which possible events or actions are intended to take place.

Trapezium A quadrilateral with at least one pair parallel sides.

Trapezoidal rule A formula to estimate the area of a shape with an irregular boundary.

Tree diagram A technique used to list the outcomes in a probability experiment. It shows each event as a branch of the tree.

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U

Unimodal Data with only one mode or peak.

Upper bound Measurement plus absolute error.

Upper extreme Highest score in the data set.

Upper quartile The highest 25% of the scores in the data set.

V

Value Added Tax (VAT) A tax added to the purchase price of each item. VAT is used in many countries with the rate ranging from 2% to 25%.

Variable A symbol used to represent a number or group of numbers.

Variation A mathematical relationship between two variables.

Volume The amount of space occupied by a three-dimensional object.

W

Wage A payment for work that is calculated on an hourly basis.

Watt SI unit for power.

X

***x*-intercept** The point at which the graph cuts the x-axis.

Y

***y*-intercept** The point at which the graph cuts the y-axis.

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Answers

Chapter 1

Exercise 1A

1a $8g$ **b** $17x$ **c** d
d $4x^2$ **e** $9c^2$ **f** $-3r^3$
g $-3m$ **h** $8t$ **i** $12v^2$

2a $7w+2$ **b** $5q-8$
c $15a+1$ **d** $3x^2+4y$
e $4b^2+a$ **f** $-3n^3+2m$
g $-12x^2-y$ **h** $8a^2-b$
i $4z^2+9y$

3a $-4u^4+6u^2+9$ **b** $7q^3-3q^2$
c b^2 **d** $3x^3+x^2$
e $2p^2+2p$ **f** $2w^2-2w$
g c^2+4c+2 **h** k^3-k^2+3
i $-2y^2+y-5$

4a $\frac{5+y}{7}$ **b** $\frac{3-x}{4}$
c $\frac{2w+4}{5}$ **d** $\frac{8-5t}{9}$

5a $\frac{2m}{5}$ **b** $\frac{y}{7}$ **c** $\frac{5x}{3}$
d $\frac{2b}{11}$ **e** $\frac{3d}{3}=d$ **f** $\frac{5n}{13}$
g $\frac{11b}{2}$ **h** $\frac{w}{6}$

6a $\frac{3x}{4}$ **b** $\frac{5y}{8}$
c $\frac{21m}{10}$ **d** $\frac{3r}{8}$
e $\frac{12x}{14}=\frac{6x}{7}$ **f** $\frac{7b}{6}$
g $\frac{3h}{20}$ **h** $\frac{5p}{12}$

7a $\frac{10k}{12}=\frac{5k}{6}$ **b** $\frac{22x}{24}=\frac{11x}{12}$
c $\frac{4y}{10}=\frac{2y}{5}$ **d** $\frac{10c}{6}=\frac{5c}{3}$

8

+	x^2	$3x^2-2x$	x^2+x
$3x^2$	$4x^2$	$6x^2-2x$	$4x^2+x$
$6x$	x^2+6x	$3x^2+4x$	x^2+7x
x^2-2x	$2x^2-2x$	$4x^2-4x$	$2x^2-x$
$-2x$	x^2-2x	$3x^2-4x$	x^2-x

9 $2x^2+4x$
10 $500-6y^2-3y$
11 $4x^2+x+6y$

12a $\frac{7k}{12}$ **b** $\frac{17b}{24}$ **c** $\frac{17r}{6}$
d $\frac{39r}{70}$ **e** $\frac{23n}{30}$ **f** $\frac{7d}{24}$
g $\frac{11a}{30}$ **h** $\frac{29y}{44}$

13a $\frac{11m}{12}$ **b** $\frac{21x}{20}$
c $\frac{42m}{30}=\frac{7m}{5}$ **d** $\frac{37h}{84}$

14a $\frac{21+y}{7}$ **b** $\frac{10-x}{2}$
c $\frac{57m}{7}$ **d** $\frac{7y}{9}$
e $\frac{14y-1}{3}$ **f** $\frac{11x-2}{4}$
g $\frac{12a+7}{6}$ **h** $\frac{w-3}{20}$

Exercise 1B

1a $15w$ **b** $24p$
c $120f$ **d** $8xy$
e $-9mn$ **f** $-105kr$
g $36gh$ **h** $21n^2$
i $150a^2$

2a m^7 **b** $3y^8$
c $8z^5$ **d** $48y^4$
e $6a^7$ **f** $8w^{15}$
g $14s^6t^7$ **h** $5p^3q^5$
i $-12c^8d^{10}$

3a $10a^9$ **b** $-12x^{12}$
c $36y^9$ **d** m^4n^7
e $14a^4b^5$ **f** $8c^3d^8e^2$

4a $4w$ **b** $9c^2$ **c** $-8s^2$
d 4 **e** $-5h$ **f** $3y$
g a **h** $2m$ **i** $-4z^2$

5a y^5 **b** $12b^6$ **c** $-3x$
d $2m^4$ **e** $-3t^9$ **f** $-b^4$
g $\frac{2}{3}d^{13}$ **h** $\frac{4}{3}q^2$ **i** $\frac{3}{2}e^2$

6a $6w^4$ **b** $5y^{10}$ **c** $3x^3$
d $-14a^4$ **e** $-8c^{14}$ **f** $-28e^6$

7a $2a$ **b** $4t$ **c** 8
d $-\frac{ab}{6}$ **e** n^4 **f** $\frac{x^7y}{6}$
g $-\frac{y^4}{8}$ **h** $\frac{7d^5}{e}$ **i** $\frac{x^2}{2y^2}$

8a x^{12} **b** s^6
c $9y^2$ **d** $64n^9$
e $-a^{20}b^{10}$ **f** $625r^8t^4$
g $a^{12}b^{11}$ **h** $1296e^{10}f^{31}$
i $864a^{11}b^{18}$

9a 1 **b** 1 **c** 5
d 7 **e** a **f** 1
g -10 **h** 1 **i** 7

10a $16q^5$ **b** $-8u^7$
c $24a^8$ **d** $7x^4$
e $2m^7$ **f** $-d^8$
g $125g^{12}$ **h** $c^{20}d^8$
i $64x^{18}y^6$ **j** 20
k 1 **l** 3

11

×	x^2	$(-3x^4)$	$4xy^3$
$2x^2$	$2x^4$	$-6x^6$	$8x^3y^3$
$(-5x^3y^5)$	$-5x^5y^5$	$15x^7y^5$	$-20x^4y^8$
$7x^5y^8$	$7x^7y^8$	$-21x^9y^8$	$28x^6y^{11}$

12

÷	$(-2a^2)$	ab^2	$3a^2b^2$
$6a^4$	$-3a^2$	$\frac{6a^3}{b^2}$	$\frac{2a^2}{b^2}$
$(-3a^5b^2)$	$\frac{3}{2}a^3b^2$	$-3a^4$	$-a^3$
$12a^6b^4$	$-6a^4b^4$	$12a^5b^2$	$4a^4b^2$

13a $\frac{32y^2}{3}$ **b** $\frac{12d^2}{5}$ **c** $\frac{10w^2}{9}$
d 15 **e** 1 **f** $\frac{10w^2}{63}$
g $\frac{15a}{2}$ **h** $\frac{2m^2}{45}$ **i** $\frac{c}{3e}$

14a $4m^{13}$ **b** $-14x^{11}$ **c** $45y^{10}$

15 $\frac{x}{2y}$

16 $\frac{15x^3}{2}$

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17a $8x^4$ **b** $\frac{5a^6}{2}$ **c** $\frac{7v^8}{4}$

d $20x^4y$ **e** m^3n^2 **f** $\frac{4e^7}{3f^4}$

g $25a^2b$ **h** $\frac{m^9}{45}$ **i** $\frac{c^2d^3}{7e^3}$

18a $\frac{15h^2k^2}{4}$ **b** u^2v

c $\frac{a^2b^2}{2}$ **d** $18m^2n$

e $\frac{3m^2}{4}$ **f** $\frac{1}{6y^2}$

19a $\frac{3x^4y}{z^2}$ **b** m^2np^2

c $2a^3c^2$ **d** $\frac{c^2d^2}{2}$

e $\frac{-24w}{u^4}$ **f** $\frac{16c^2d^5}{e^2}$

Exercise 1C

1 Did not multiply –2 by 3. The correct answer is $9x - 6$

2a $3a + 6$ **b** $2d + 2$

c $7b - 14$ **d** $6x + 8$

e $10x - 14$ **f** $36b + 4$

g $20 + 8t$ **h** $6 - 12w$

i $15 + 45d$ **j** $40e - 16d$

k $20a + 45b$ **l** $14h + 56g$

3a $-4x - 12$ **b** $-3y - 15$

c $-b - 8$ **d** $-7k + 14$

e $-6w + 6$ **f** $-2x + 26$

g $-8 - 4q$ **h** $-15 + 20r$

i $14s - 56$

4a $y^2 + y$ **b** $v^2 + 4v$

c $n^2 + 10n$ **d** $2x^2 - 3x$

e $3e^2 + 5e$ **f** $6d^2 - 2d$

g $7ez + 3fz$ **h** $2ab - 3ac$

i $cd + 4ce$

5a $6g + 2$ **b** $8s + 14$

c $y - 27$ **d** $x + 8$

e $8z - 2$ **f** $-4q + 35$

6a $6x + 1$ **b** $25y - 16$

c $9b - 4$ **d** $9r + 2$

e $5n - 2$ **f** $4q - 7$

7a $4y + 12$ **b** $2c + 2$

c $5a - 15$ **d** $-9e^2 - 12$

e $-8b^2 + 4$ **f** $-24g^2 - 18$

8a $k^2 + 3k$ **b** $2b^2 - 6b$

c $-3y^2 + 5y$ **d** $4x^2 - x$

e $-ab - 4ac$ **f** $5pq - 3pr$

9a $7x - 3$ **b** $5y + 8$

c $8a + 22$ **d** $13c - 9$

e $10s + 34$ **f** $9h + 21$

g $10x$ **h** $2z + 59$

i $7c - 41$ **j** $21g + 3$

k $13u - 18$ **l** $-7d + 10$

10 Did not apply the minus sign when multiplying –2 by 3. The correct answer is $18x - 12 - 2x - 6 = 16x - 18$

11a $2x^2 - 3x$ **b** $2b^2 + 4b$

c $2y^2 + 5y$ **d** g^2

e $7v^2 - 3v$ **f** $b^2 - 3b$

g $3u^2 + 5u$ **h** $3n^2 - 25n$

i $5d^2 + 26d$ **j** $-6e^2 + 65e$

k $7k^2 - 15k$ **l** $-10t^2 + 19t$

12 $a^2b^2 - 5ab$

13a $2x^3 + 3x^2 - 2x - 2$

b $a^3 + 2a^2 - 4a - 12$

c $5y^3 + 2y^2 - 3y - 21$

d $-3b^3 - b^2 + 7b$

e $z^3 - 2z^2 - z$

f $-2e^3 - 7e^2 + 7e$

g $5x$

h $3a^3 + 3a$

i v

j $a^3 + a^2b - ab - b^2$

k $x^4 + x^2y - x^2 - 3xy$

l $y^3 + 4y^2z - yz^2 - y$

14a $5m^6 + m^2$

b $2x^6 - 6x^5 + 8x^4$

c $a^{12} - 3a^8 - 2a^4$

15a $21a^6 - 10a^4 + 7a^3$

b $9x^5 - 35x^2$

c $-6n^3 + 3n^2r$

16a $2b^2$ **b** $\frac{3r^4}{4}$ **c** $\frac{1}{8b}$

Exercise 1D

1a 10.0 cm **b** 13.0 cm

c 26.0 mm **d** 22.4 mm

e 4.9 cm **f** 83.1 cm

2a 9.00 cm **b** 14.70 cm

c 11.53 mm **d** 21.17 mm

e 4.21 cm **f** 10.42 cm

3a $y = 55$ mm **b** $a = 55$ mm

c $x = 23$ mm **d** $d = 25$ mm

e $b = 62$ mm **f** $m = 14$ mm

4 9.0 m

5a $x = 126.00$

b $y = 5.74$, $x = 8.31$

6a $x = 3.6$ cm, $y = 1.7$ cm

b $x = 21.1$ cm

Exercise 1E

1a 21.6 cm **b** 41.2 m

c 59.0 m

2 49.2 m

3a 40.0 m **b** 12.4 cm

c 26.5 mm

4 43.69 cm

5a 18.8 m **b** 12.6 cm

c 88.0 mm

6a 25.1 cm **b** 119.4 m

c 213.6 mm **d** 157.1 mm

e 69.1 m **f** 18.8 cm

7a 25.7 m **b** 10.8 m **c** 41.1 m

8a 25.00 m **b** 10.71 mm

c 33.56 cm

9a 36 m **b** 16 m **c** 47 cm

d 22 m **e** 24 m **f** 23 m

10 28 cm

11a 8.49 cm **b** 28.97 cm

Exercise 1F

1a 149.5 m^2 **b** 4 cm^2

c 19.5 mm^2 **d** 72.2 m^2

e 100.8 mm^2 **f** 40.4 m^2

2a 37.2 cm^2 **b** 71.7 m^2

c 198 m^2 **d** 70 mm^2

e 14 m^2 **f** 21 cm^2

3 42 cm^2

4 11 046 m^2

5 154 cm^2

6a 5.8 km^2 **b** 82.8 cm^2

c 15.0 mm^2 **d** 31.8 cm^2

e 75 mm^2 **f** 120 m^2

7 7 m^2

8 49 m^2

9 30 tiles

10a 1380 cm^2 **b** 36 cm^2

c 134 cm^2

11 50 cm^2

12 42 cm^2

13a 253 m^2 **b** \$5060

14a 8 cm **b** 6 cm

c 148.5 cm^2

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15 50.27 mm^2

16a 5.66 m **b** 80 m^2

17a 32 cm^2 **b** 8

c 6.87 cm^2

Exercise 1G

1a \$5850 **b** \$910

c \$37 000 **d** \$1134

e \$15 225

2a \$600 **b** \$1089

c \$5020 **d** \$10 005

e \$11 518.75

3a 1.2% **b** 0.4%

c 2.4% **d** 3.6%

4a \$8 **b** \$144

c \$1440 **d** \$900

5 \$55 200

6 \$69.75

7a \$22 000 **b** \$122 000

8 \$24

9 \$5040

10a \$7680 **b** \$7560 **c** \$7200

11 Computer application

12 \$71.75

13 \$40 000

14 7.5%

15 $3\frac{1}{3}$ years

16a \$51 000 **b** \$2000

Exercise 1H

1a \$899.89 **b** \$11 578.20

c \$15 577.17 **d** \$27 621.40

2a \$10 063.79 **b** \$102 028.69

c \$1307.84 **d** \$23 509.61

3 \$27 209.78

4 \$12 107.45

5 \$2536.50

6 \$6433.75

7a \$20 766.90 **b** \$7266.90

8 \$39 604.68

9 \$53 608.98

10 \$49 662.32

11 Computer application

12 \$2340.76

13 Investment 2 by \$80

Exercise 1I

1a

Score	Tally	Freq
20	𝍸	3
21	𝍸 𝍸	7
22	𝍸 𝍸	6
23	𝍸 𝍸	4
24	𝍸	2

b 21 **c** 22

2a

Number of calls	Tally	Freq
0	𝍸 𝍸	7
1	𝍸 𝍸 𝍸	12
2	𝍸 𝍸	8
3	𝍸	4
4	𝍸	2
5	𝍸	1

b 1 **c** 2 **d** 27

3

Score	Tally	Freq
0	𝍸 𝍸	6
1	𝍸 𝍸	7
2	𝍸 𝍸	6
3	𝍸	5
4	𝍸	3
5	𝍸	2
6	𝍸	1

4

Score	Tally	Freq
7	𝍸	3
8	𝍸 𝍸	7
9	𝍸 𝍸	4
10	𝍸	3
11	𝍸	2
12	𝍸 𝍸	1

5

Score	Tally	Freq
91	𝍸 𝍸	4
92	𝍸	5
93	𝍸	3
94	𝍸 𝍸	4
95	𝍸	3
96	𝍸	3
97	𝍸	5
98	𝍸	3

6a

Score	Tally	Freq
1	𝍸 𝍸 𝍸	11
2	𝍸 𝍸	9
3	𝍸	5
4	𝍸 𝍸 𝍸	11
5	𝍸 𝍸	6
6	𝍸 𝍸	9

b 51

c 31

d 1 and 4

e No: Small sample accounts for the difference in frequencies OR Yes: There is a notable difference between the frequency of the outcomes.

7a

Score	Tally	Freq
32	𝍸 𝍸	6
33	𝍸	5
34	𝍸 𝍸	9
35	𝍸 𝍸	8
36	𝍸 𝍸	8
37	𝍸 𝍸	6
38	𝍸 𝍸	9

b 51 **c** 20

d 34 and 38 s **e** 29.41%

Answers

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8a

Score	Tally	Freq
1	IIII	4
2	IIII	4
3	卌 卌 I	11
4	II	2
5	卌 III	8
6	卌 I	6
7	IIII	4
8	III	3
9	卌	5
10		0
11		0
12	I	1

b 3 letters

c Small amount of data indicates that the conclusions are not reliable. However, a word length of 3 occurs significantly more often than any other word length.

Exercise 1J

1a 9 **b** 28 **c** 8
d 5 **e** 7 **f** 42
g 11 **h** 14
2a 14.0 **b** 7.0 **c** 9.5
d 9.5 **e** 11.0 **f** 2.2
g 10.2 **h** 8.8
3a 1406 **b** 70.30 **c** 69.95
4a 9 **b** 7 **c** 34
d 1100 **e** 2 **f** 100
g 1 **h** 10
5a 7 **b** 16 **c** 20.5
d 12 **e** 120 **f** 3
g 50 **h** 10.5
6 16
7a 13 **b** 5 **c** 9 **d** 39
8a 17 **b** 7 **c** 26 **d** 24
9a 25 **b** 19 **c** 22
d 4 **e** 22
10a 16 **b** 13 **c** 3 **d** 14
11a 7.89 **b** 7 **c** 4
12 1.96
13 15

Review 1

Multiple-choice

1 D **2** D **3** A **4** B **5** D
6 B **7** B **8** D **9** C **10** C

Short-answer

1a $6ab^3$ **b** $8x^2 - 2x$
c $2m^2 + 12m$
2a $\frac{7x}{3}$ **b** $\frac{37b}{15}$ **c** $\frac{3y}{4}$
3a $15m^5$ **b** $12y^6$
c $\frac{3y}{2}$ **d** 7
e x^{12} **f** $16x^4y^6$
4a $2x - 2y^2$
b $6w + 36m$
c $-24a^2 + 8b^2$
d $-8a + 3b$
e $9y + 21$
f $c^5 + 4c^3d$
5a 35.00 **b** 15.65
c 38.47
6a 26.2 cm **b** 69.1 cm
c 40.0 m
7a 27.0 m^2 **b** 17.5 m^2
c 125.4 cm^2 **d** 47.0 cm^2
e 28.0 mm^2 **f** 3.0 m^2
8 $300
9a $135 061 **b** $151 497
10a 11 **b** 40
11a 12 **b** 16

Extended-response

12 $64 286
13a

Class	Freq.
4–8	5
9–13	6
14–18	8
19–23	4
	23

b 12 **c** 21.7%

Chapter 2

Exercise 2A

1a $1782 **b** $3563 **c** $7721
2a $1108, $2217
b $1509, $3019
c $2073, $4146
d $851, $1702
3 $1656
4a $30 160 **b** $39 520
c $39 216 **d** $125 736
5a $3000 **b** $78 000
6 $41 860
7 $81 120
8 Stephanie $49 348.00, Tahlia $45 852.00; Stephanie by $3496.00
9 Laura $32 110.00, Ebony $29 508.00; Laura by $2602.00
10 Tran $98 696.00, Jake $99 960.00; Difference $1264.00
11a $892.50 **b** $943.50
c $1020.00 **d** $1071.00
12 $1130.50
13a $444.00 **b** $351.50
c $858.40 **d** $511.71
14a $15 787.20 **b** $31 720.00
c $39 062.40 **d** $24 731.20
15a 40.50 hours **b** $911.25
16 42 hours
17 9 hours
18a $193.60 **b** $968.00
c $1936.00 **d** $50 336.00
19 Alyssa $332.10, Connor $320.00; Alyssa by $12.10
20 $635 481.60
21 $83 790.00
22 Computer application
23 $1777.58. Calculations for weekly pay are incorrect as not every month has 4 weeks.
$1640.85
24 18.75 hours

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Exercise 2B

1a $1092.00 **b** $897.75
c $1862.00 **d** $966.00
e $1176.00
2a $1421.00 **b** $791.00
c $784.00 **d** $1074.85
3 $3564.95
4 $2326.50
5a $2901.60 **b** $2896.20
c $2065.80 **d** $2943.90
e $2214.50 **f** $2055.00
6 $222
7 $5810.95
8 $542
9 Computer application
10a $1424.00 **b** $35.60
11a $1400.00 **b** $980.00
c $73 528.00
d Pay increases by $728; however, the holiday loading was $980.

Exercise 2C

1a $108.00 **b** $237.00
c $379.20 **d** $262.98
2a $227.94 **b** $114.30
c $292.68 **d** $370.74
3 $1560.00
4 $135.30
5 $1023.40
6a 45.5 hours **b** $1041.60
7 $9.80
8 $16.00
9 $1000.68
10 $1237.50
11a $257.50 **b** $1287.50
12 $820.88
13 $1761
14 $3589.60
15 $2212
16 $3720
17 $451.05
18 $680.00
19a $448.00 **b** $268.80
c 37.5%
20 5.89 hours
21a $453.60 **b** 4 hours
c $22.68 **d** $567.00

Exercise 2D

1a $352.80 **b** $669.60
c $1368.80
2a $9120.00 **b** $8400.00
c $5710.00 **d** $11 814.00
3 $1620.00
4a $431.00 **b** $642.00
c $1640.00
5 $960.00
6a $512.00 **b** $500.00
7a $1000, $3250
b $1000, $4500
c $1000, $5750
d $1000, $7000
8 $50 600.00
9a $3400.00 **b** $4700.00
c $5200.00
10 2%
11 $460.00
12a $11 600.00 **b** $41 600.00
c $2260.00 **d** $1220.00

Exercise 2E

1a $2250 **b** $3870 **c** $2880
2 $5140
3 $500
4 $4473.60
5a $13 096.32 **b** $36 864.00
c $15 712.35
6a $12 105.60 **b** $9240.40
7a $8762.00 **b** $18 312.84
c $28 580.76
8a $58.20 **b** $97.00
c $107.60 **d** $165.80
9a $142.80 **b** $156.40
c $154.20 **d** $103.20
10a $1005.00 **b** $595.00
11 $358.20
12a 11 240 copies
b $42 065.70
c $37 859.13
13 $832.00

Exercise 2F

1a $165 **b** $330
c $45 **d** $68
e $128
Allowable deduction $736
2 $1591
3 $4614
4a i $2349.60 **ii** $844.80
iii $3022.80 **iv** $1544.40
v $2049.30 **vi** $3960.00
b i $2610.00 **ii** $773.40
iii $3300.00 **iv** $2400.00
v $48.00 **vi** $416.40
5 $5107.20
6a $24 000 **b** $48 000
c $72 000 **d** $96 000
7 $1890
8a $775.50 **b** $519.59
c $348.12
9 $47 160
10a $115.20 **b** $67.20
c $54.72 **d** $915.20

Exercise 2G

1a $78 880 **b** $74 429
c $79 280 **d** $59 896
e $75 795 **f** $66 406
g $71 480 **h** $60 110
i $49 440
2 $64 378
3a $3200 **b** $72 280
4a $89 164 **b** $1140
c $88 024
5a $2210 **b** $61 410
6a $58 240 **b** $56 780
7a $64 642 **b** $60 322
8a $72 280 **b** $301
c $824 **d** $71 456
9a $99 580 **b** $224.40
c $1964.40 **d** $97 615.60

Exercise 2H

1a $511.00 **b** $720.00
c $493.00
2a $407.40 **b** $1419.60
c $962.73
3a $15 892.00 **b** $32 858.00
4a $1689.95 **b** $3850.70
c $4003.61
5 $79 485.00

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6a $464.75 **b** $343.75
7a $9.60 **b** $310.40
c $16 140.80
8a $1434.00 **b** $258.12
c $129.06 **d** $542.18
e $891.82
9a $1120.00 **b** $168.00
c $100.80 **d** $363.80
e $756.20
10a $1976.00 **b** $177.84
c $1769.71
11a $989.40 **b** 33.0%
12a $2420.10 **b** $1210.05
c 24.7% **d** $2400.00
13a $2403.00 **b** $1259.40
c 17%

Exercise 2I

1a B **b** A **c** C
d D **e** E **f** C
g C **h** D **i** A
2a $532 **b** $2717
c $15 922 **d** $32 994
e $76 732 **f** $17 547
3 $1862
4 $28 702
5a 19 cents **b** $3572.33
c $19 822.37
6a $460.00 **b** $1770.82
c $812.00 **d** $938.12
e $1356.00 **f** $4011.84
g $3406.00 **h** $315.80
i $1812.80
7a $511.10 **b** $15 585.95
c $3836.55 **d** $30 515
e $61 625.50 **f** $8905.25
8a $1951.50 refund
b $96.16 owing
9a $42 540 **b** $5372.50
c $432.50
10a $147 500
b $42 207
c $44 720
d Refund of $2513
11a $41 535.08 **b** 28.5%
12a $1128 **b** $11 005
13a $2496 **b** $36 304
14 Computer application
15a $21 660 **b** $18 297.75
c Refund of $3362.25
d 22.2%
16a $1275 **b** $5611.38
17 $59 670

Review 2

Multiple-choice

1 B **2** B **3** C **4** A **5** D
6 D **7** A **8** B **9** B

Short-answer

1a $1855.20 **b** $38.65
2 $5371.25
3a $839.16 **b** $5634.36
4 $641.25
5a $4725.00 **b** $235 275.00
6 $4440
7a $31 461.46 **b** $26 742.24
8 $1056.00 **9** $5.35
10 $5773 **11** $77 520
12a $1270.50 **b** $2579.50
13a $96 720 **b** $493
c $3230 **d** $93 490
14 $37 519.10
15 $747.50

Extended-response

16a $513 **b** $102.60
17a $25 480
b $27 136.90
c He pays another $1656.90 in tax.
d 25.4%

Chapter 3

Exercise 3A

1a 19 **b** 19 **c** 7
d 25 **e** −96 **f** $\sqrt{24}$
g 48 **h** 2 **i** 3
2a 25 **b** 10 **c** 0
d −25 **e** 4 **f** 7
3a −2 **b** 6 **c** 97
d 1 **e** −2 **f** 1
g $-2\frac{3}{4}$ **h** 6.61 **i** $-2\frac{24}{25}$
4a 11 **b** 56 **c** 92
d 0.88 **e** $2\frac{5}{9}$ **f** $1\frac{3}{8}$
5a 16 **b** 98 **c** −300
d 2 **e** −45 **f** 14.0625
g $\frac{7}{250}$ **h** $-\frac{27}{128}$ **i** 0.07
6a 2 **b** 5 **c** 2
d 3 **e** 1 **f** 8
g 0.25 **h** $1\frac{3}{4}$ **i** $\frac{1}{25}$
7a 2 **b** 4 **c** 3 **d** 1
8a 485 **b** −153 **c** $\sqrt{48}$
d 11 **e** 25 **f** $\frac{10}{49}$
9 150.8 cm^2 **10** 64
11 3 **12** 2.35 **13** 3.24
14 8 **15** 0.008
16 9.20 **17** 6

Exercise 3B

1a 90 km/h **b** 97 km/h
c 80 km/h **d** 84 km/h
e 96 km/h **f** 48 km/h
2a 168 km **b** 392 km
c 146 km **d** 70 km
e 196 km **f** 154 km
3a 2 h **b** 2.5 h **c** 2.5 h
d 5 h **e** 6 h **f** 8 h
4 227.27 km/h
5a 70 km **b** 0.9 h
c 77.78 km/h
6a 45 minutes **b** 64 km/h
7a 25 m **b** 20 m **c** 45 m
8a 47 m **b** 49 m
9a 12 m **b** 25 m
c 43 m **d** 66 m
e 94 m **f** 126 m
10a No. Stopping distance is 15 m.
b Sarah would hit the child at a greater speed.
11a 89.510 m **b** 90 m
12a 81 km/h **b** 63 km/h
c 48 km/h **d** 115 km/h
13a 34 km **b** 2 km
c 91 km **d** 272 km
14a 5 h 29 min **b** 50 min
c 5 h 6 min **d** 9 h 21 min
e 20 min **f** 1 h 40 min

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15a 1224 km/h **b** 102 km
c 0.34 km **d** 48 min
16a 169 km/h **b** 5 h 55 min
17a

Speed (km/h)	Reaction distance (m)
10	2.1
20	4.2
40	8.3
80	16.7

b Yes. Table shows the reaction distance doubles when the speed doubles.

18a

Speed (km/h)	Braking distance (m)
10	0.60
20	2.35
40	9.41
80	37.65

b Yes. Table shows the braking distance quadruples when the speed doubles.

19a $t = \frac{18}{5V}\left(d - \frac{V^2}{170}\right)$
b 0.6 s **c** 1.8 s **d** 3.0 s

20a

Reaction time (s)	Reaction distance (m)
0.50	8
1.00	17
1.50	25

b Reaction distance increases when the reaction time increases.

Exercise 3C

1a $a = 8$ **b** $y = 11$
c $x = 10$ **d** $g = 7$
e $w = -4$ **f** $z = -13$
g $b = 9$ **h** $m = 18$
i $c = 21$ **j** $q = 7$
k $h = 6$ **l** $n = -5$

2a $n = 3$ **b** $x = 5$
c $c = -4$ **d** $b = 7\frac{1}{2}$
e $m = -5\frac{1}{4}$ **f** $b = -4\frac{1}{6}$
g $s = 16$ **h** $y = 30$
i $a = -77$ **j** $g = 24$
k $c = 45$ **l** $k = -9$

3a Error is in the second line.
b $6y = 2$ or $y = \frac{1}{3}$

4a $w = 5$ **b** $t = 3$ **c** $x = 4$
d $y = -1$ **e** $m = -1$ **f** $a = 7$
g $e = 4$ **h** $b = 1$ **i** $k = -5$
j $w = 3$ **k** $d = -1$ **l** $z = -6$

5a $q = \frac{13}{3} = 4\frac{1}{3}$
b $r = \frac{25}{4} = 6\frac{1}{4}$
c $x = \frac{3}{2} = 1\frac{1}{2}$
d $a = \frac{27}{4} = 6\frac{3}{4}$
e $m = \frac{1}{5}$
f $k = -\frac{17}{2} = -8\frac{1}{2}$
g $c = \frac{13}{2} = 6\frac{1}{2}$
h $e = \frac{16}{5} = 3\frac{1}{5}$
i $p = \frac{15}{6} = 2\frac{1}{2}$
j $r = \frac{9}{2} = 4\frac{1}{2}$
k $w = \frac{44}{3} = 14\frac{2}{3}$
l $c = \frac{8}{9}$

6a $d = \frac{5}{12}$ **b** $y = 1\frac{2}{5}$
c $a = \frac{2}{3}$ **d** $x = 9\frac{2}{3}$
e $b = 7\frac{1}{5}$ **f** $n = 36\frac{1}{4}$

7a $z = 10$ **b** $c = 4$
c $m = 36$ **d** $x = 3\frac{8}{9}$
e $y = 16\frac{4}{5}$ **f** $a = 20$

8a $\frac{a}{4} + 20 = 27$ **b** $a = 28$

9a $g = 16$ **b** $y = 36$
c $c = 50$ **d** $x = -49$
e $b = 4$ **f** $w = 37\frac{1}{2}$

10a $m = 2$ **b** $t = 4$ **c** $e = -4$
d $x = -1\frac{1}{3}$ **e** $d = 6$ **f** $w = -1$
g $a = 3\frac{1}{2}$ **h** $g = 6\frac{1}{5}$ **i** $r = 5$

11a $a = 16$ **b** $z = 12\frac{1}{3}$
c $r = \frac{1}{2}$ **d** $x = -2\frac{5}{6}$
e $b = 2$ **f** $w = -\frac{1}{6}$

12a $d = 5$ **b** $x = -8$
c $a = -4$ **d** $y = 1$
e $m = -3\frac{4}{5}$ **f** $c = 1\frac{5}{6}$
g $s = -\frac{2}{7}$ **h** $z = 2\frac{1}{2}$
i $b = 1$ **j** $x = -\frac{1}{3}$
k $a = 1$ **l** $w = 2\frac{1}{2}$

13a $x = -4\frac{1}{5}$ **b** $w = 7\frac{1}{5}$
c $y = -\frac{2}{11}$ **d** $e = -3\frac{1}{3}$
e $m = -\frac{2}{19}$ **f** $b = -1\frac{1}{14}$
g $r = -1\frac{2}{3}$ **h** $t = 1\frac{2}{13}$
i $n = -1\frac{8}{13}$

14a $3c + 15 = 261$ **b** Cost is \$82
15 Number is -5
16a $v = 12$ **b** $a = 14\frac{2}{7}$
c $x = 13\frac{1}{3}$ **d** $m = 52\frac{1}{2}$
e $b = 84$ **f** $g = 96$
g $y = 3\frac{3}{5}$ **h** $c = 2\frac{10}{13}$
i $h = 40$ **j** $n = 56$
k $p = -20$ **l** $z = \frac{10}{21}$

17a $x = \frac{1}{3}$ **b** $a = -\frac{5}{6}$
c $y = \frac{2}{5}$ **d** $d = -1$
e $c = 1\frac{1}{7}$ **f** $m = \frac{1}{2}$

18a $b = -\frac{2}{5}$ **b** $y = \frac{1}{7}$
c $y = 3$ **d** $x = -3$
e $c = -\frac{1}{2}$ **f** $m = -5\frac{1}{2}$

19a $2x + 6 = x + 2$ **b** $x = -4$
20a $x + 8 = 2x - 6$ **b** $x = 14$

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21a $d = -\frac{3}{5}$ **b** $r = -18$

c $x = -1\frac{7}{9}$ **d** $a = 5\frac{3}{5}$

e $m = -\frac{2}{5}$ **f** $y = 2$

22a $a = 1\frac{3}{4}$ **b** $m = 3\frac{2}{3}$

c $y = -1$ **d** $b = -14$

e $x = -4$ **f** $c = 5\frac{1}{2}$

g $n = 9\frac{1}{2}$ **h** $k = 8\frac{3}{4}$

i $p = 36\frac{1}{2}$

Exercise 3D

1a $A = 113.1$ **b** $A = 804.2$

c $A = 7238.2$

2a $A = 157.08$ **b** $A = 1036.73$

c $A = 75.40$

3a $I = 6.0$ **b** $I = 18.0$

c $I = 50.4$

4a $A = 11.49$ **b** $A = 6.75$

c $A = 10.13$

5a $S = 20.0$ **b** $S = 2.5$

6a $V = 905$ **b** $V = 589$

c $V = 1499$

7a $z = 2$ **b** $z = 4.875$

8a \$1050 **b** \$1112.50

c \$1500

9a 16°C **b** 27°C **c** 38°C

10 $r = 4.14$ cm

11a 540 000 km **b** 3 780 000 km

c 196 560 000 km

12 $V = 9$ volts

13 $n = 60$

14 $h = 4.5$ cm

15 $t = 7.75$

16 $b = 28.72$ mm

17 $s = -13.5$

18 $n = 2$

19 $R_1 = 1.40$

20 $FV = \$27\,160.56$

21 $PV = \$5006$

22 $a = 3\frac{1}{2}$

23 $h = 20$

24 $c = 3.6$

25 $s = 592.5$

26 $A = 0.7$

27 $V = 268\text{ cm}^3$

Exercise 3E

1a $x = y - 3$ **b** $x = \frac{y}{5}$

c $x = y + 7$ **d** $x = \frac{y - 9}{2}$

e $x = 4 - y$ **f** $x = \frac{y + 7}{3}$

2a $x = 8y$ **b** $x = 2(y - 1)$

c $x = 3(y + 1)$ **d** $x = 6(7 - y)$

e $x = 5(y - 5)$ **f** $x = -3(y + 2)$

3a $t = \frac{d}{s}$ **b** $s = \frac{d}{t}$

c 64 km/h

4a $l = \frac{A}{b}$ **b** $b = \frac{A}{l}$

c $b = 6.4$ cm **d** $l = 2.625$ cm

5a $m = \frac{F}{a}$ **b** $a = \frac{F}{m}$

c $a = 4$ **d** $m = 4.5$

6a $b = a - c$ **b** $b = \frac{a + c}{6}$

c $b = a - c$ **d** $b = \frac{a - 2c}{6}$

e $b = \frac{4a + c}{7}$ **f** $b = 8a - 3c$

g $b = \frac{a + c}{-2}$ or $b = \frac{-a - c}{2}$

h $b = \frac{a + c}{5}$ **i** $b = \frac{a + 4c}{9}$

7a $b = y - mx$ **b** $m = \frac{y - b}{x}$

c $x = \frac{y - b}{m}$ **d** $x = 3$

e $m = 4$

8a $l = \frac{P}{2} - b$ **b** $l = 25$ m

c $b = \frac{P}{2} - l$

9a $a = \frac{w}{35} - 1.5b - 2c$

b $b = \left(\frac{w}{35} - a - 2c\right) \div 1.5$

c $c = \left(\frac{w}{35} - a - 1.5b\right) \div 2$

d $c = 5.25$

10a $b = \frac{A}{\pi a}$ **b** $b = 2.07$ cm

11a $x = \pm\sqrt{y}$ **b** $x = \pm\sqrt{\frac{y - 4}{3}}$

c $x = \pm\sqrt{\frac{5 - y}{4}}$ **d** $x = y^2$

e $x = \frac{y^2}{49}$ **f** $x = \frac{(y - 7)^2}{4}$

12 $p = \pm\sqrt{\frac{4m - 5n}{2}}$

13a $h = \frac{V}{\pi r^2}$ **b** $r = \sqrt{\frac{V}{\pi h}}$

c $r = 1.69$ cm

14 $r = \sqrt{\frac{A}{4\pi}}$

15a $m = Bh^2$ **b** $h = \sqrt{\frac{m}{B}}$

c i $h = 1.82$ m

ii $h = 1.84$ m

16 $M = FV\left[\frac{r}{(1 + r)^n - 1}\right]$

17a $L = 9.8T^2$

b $L = 156.8$ metres

18 $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

19a $r = \sqrt[3]{\frac{3V}{4\pi}}$ **b** $r = 1.24$ m

20 $L = \pm\frac{\sqrt{Z^2 - R_2}}{2\pi f}$

21 $A = P(r + 1)^n$

Exercise 3F

1a 0.09 **b** 0.07 **c** 0.10

d 0.10 **e** 0.28 **f** 0.17

2a 0.02 **b** 0.10 **c** 0.01

d 0.20 **e** 0.16 **f** 0.04

3a 0.09 **b** 0.11

c about 6 hours

4a 3 h 44 min **b** 8 h 12 min

c 5 h 48 min **d** 10 h 12 min

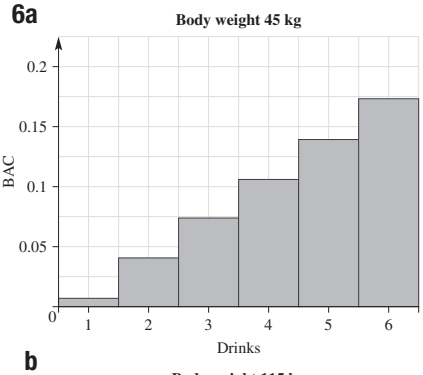
e 6 h 8 min **f** 11 h 28 min

5a 4 h 56 min **b** 1 h 56 min

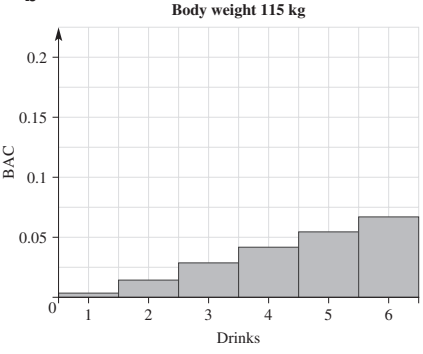
c 11 h 28 min **d** 1 h 52 min

e 3 h 20 min **f** 20 min

6a



b



7a 1.4 **b** 8.0 **c** 1.5

d 0.9 **e** 0.6

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8a 0.075 h **b** 0.045 h
9a 3 h 5 min **b** 5 h 45 min
10a 3.2 **b** 6.8
11a Male – 4.7, Female – 7.6
b Males are larger in size.

Exercise 3G

1 120 mg
2a 50 **b** 150
c 12.5 days
3a 5 mg **b** 40 mg **c** 1800 mg
4a 0.6 mL **b** 0.8 mL
c 2 mL **d** 1.6 mL
5a 3 mL **b** 6 mL
c 2 mL **d** 10 mL
6a 11 mL **b** 4 mL
c 2 mL **d** 7 mL
7a 187.5 mL/h **b** 400 mL/h
8a 1.5 tablets **b** 2.25 tablets
9a 12.5 mL **b** 12 mL
10a 25 mL **b** 8 doses
c 2 days
11a 150 mL/h
b 2.5 mL/min
c 3000 drops/h
d 50 drops/min

Review 3

Multiple-choice

1 C **2** C **3** A **4** D
5 C **6** B **7** D **8** D

Short-answer

1a 44 **b** 102
2 87.96 cm
3a 510 km **b** 38.25 L **c** $58.14
4a 39 m **b** 29 m
5a $e = 8$ **b** $b = 4\frac{1}{2}$
c $w = 9$ **d** $n = 6$
e $a = 5$ **f** $q = 4$
g $h = 98$ **h** $d = \frac{7}{10}$
i $y = \frac{1}{2}$
6a $p = -8$ **b** $s = -1\frac{4}{5}$
c $p = 2$ **d** $d = \frac{6}{7}$
e $y = -1$ **f** $a = -\frac{4}{9}$
7a $c = 6\frac{1}{2}$ **b** $x = 1\frac{1}{2}$ **c** $u = 1$
8a $A = 1.47$ **b** $A = 2.34$
c $A = 4.57$
9a $m = 2.6$ kg **b** $v = 3.138$ m/s
10 $\sin C = \frac{2A}{ab}$
11a $s = \frac{v^2 - u^2}{2a}$
b $u = \pm\sqrt{v^2 - 2as}$
12a 0.084 **b** 5.6 hours

Extended-response

13a 1.5 h
b Anna – 15 km, Bridget – 18 km
14a $C = 120 + 65h$ **b** $380

Chapter 4

Exercise 4A

1a 50 **b** 7800
c 2000 **d** 89 000
e 570 **f** 600 000
g 9.4 **h** 60
i 81 **j** 0.49
k 22 **l** 5.1
2a 3000 **b** 45 000
c 76 000 **d** 8 100 000
e 4 000 000 **f** 520
g 6.8 **h** 9.3
i 45 **j** 0.3
k 2.3 **l** 60
3a 2000 **b** 12 000
c 9 000 000 **d** 7 800 000
e 50 000 **f** 300 000 000
g 6.1 **h** 0.4
i 0.21 **j** 0.08
k 0.079 **l** 8
4a Centimetre **b** Metre
c Millimetre **d** Kilometre
e Centimetre **f** Metre
5a Tonne **b** Gram
c Kilogram **d** Kilogram
e Tonne **f** Gram
6a 19.25 L **b** 50
7 2 495 000 m
8 2.2 kg
9 12 t
10 5.625 kg
11 23.4 g
12 10 750 g
13 3068 kg
14 7.25 kg
15a 159 km **b** 19.875 L
16 12 km
17a 4 **b** 40
18a 0.005 km, 5000 cm, 500 m, 5 000 000 mm
b 5 000 000 mm, 500 m, 5000 cm, 0.005 km
19a 1 000 000 **b** 1 000 000
c 100 **d** 0.1
e 20 **f** 0.005
g 39 000 **h** 310 000 000
i 4 700 000 **j** 0.0743
k 65 **l** 0.4
20a 0.08 **b** 8
21a 1.5 km **b** 50

Exercise 4B

1a A = 10 mm, B = 44 mm, C = 72 mm, D = 89 mm
b 1 mm
c 0.5 mm
d A Lower bound = 9.5
A Upper bound = 10.5
B Lower bound = 43.5
B Upper bound = 44.5
C Lower bound = 71.5
C Upper bound = 72.5
D Lower bound = 88.5
D Upper bound = 89.5
e A 0.050
B 0.011
C 0.007
D 0.006
f A – 5.0%
B – 1.1%
C – 0.7%
D – 0.6%
2a A – 1.82 kg, B – 4.24 kg

Answers

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b 0.02 kg
c 0.01 kg
d A Lower bound = 1.81
A Upper bound = 1.83
B Lower bound = 4.23
B Upper bound = 4.25
e A 0.005, B 0.002
f A 0.5%, B 0.2%

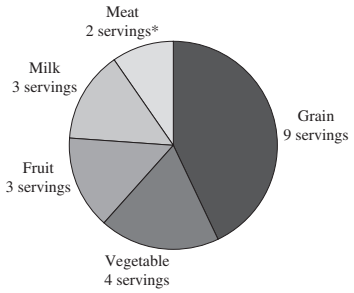
3a 0.40 kg **b** 0.008 **c** 0.80%
4a 16 g **b** 0.068 **c** 6.81%
5a 0.29 kg **b** 0.097 **c** 9.667%
6a 0.15 m **b** 0.006 **c** 0.6%
7a 0.3 m
b 0.2 m
c Cooper – 0.0053
Filip – 0.0036
d Cooper – 0.534%
Filip – 0.358%

Exercise 4C

1a 7.6×10^3 **b** 1.7×10^9
c 5.9×10^5 **d** 6.8×10^6
e 3.5×10^4 **f** 3.1×10^8
g 7.71×10^7 **h** 5.23×10^{11}
i 9.54×10^{10} **j** 5.4×10^2
2a 5.6×10^{-4} **b** 6.87×10^{-5}
c 8.12×10^{-7} **d** 4.3×10^{-3}
e 5.8×10^{-5} **f** 3.12×10^{-6}
g 2.6×10^{-1} **h** 9.2×10^{-2}
i 1.67×10^{-10} **j** 6.0×10^{-5}
3 5.0×10^{-6} s
4a 4.10×10^8 **b** 4×10^8
5a 112 000
b 534 000 000
c 5200
d 86 780 000
e 240
f 7 800 000 000
g 3 900 000
h 28
i 64 000
j 35 000
6a 0.00035
b 0.0000079
c 0.000000163
d 0.00581
e 0.049
f 0.98
g 0.0000000412
h 0.0000633
i 0.000000003
j 0.000000007134
7 5.81×10^{-6} kg
8a 1.475×10^{10} **b** 2.961×10^8
c 2.982×10^{-6} **d** 1.86×10^{-8}
9a 3.25×10^7 **b** 1.5×10^{10}
c 1.5×10^1
10a 1 600 000 **b** 3 678 000
c 789 000 **d** 3 000 000
e 778 000 **f** 3 194 700
g 821 100 **h** 7000
i 49 000
11a 0.004 **b** 0.1918
c 0.00159 **d** 0.111222
e 0.00003 **f** 0.019833
g 0.0081 **h** 0.0927
i 0.00042
12 0.000016 m
13 270 000 000 cg
14 4240 g
15a 2 880 000 **b** 0.00004802
16 4×10^8
17a 1.5×10^8 **b** 1.2×10^{-5}
18a $\pm 262\,000$ **b** ± 2.43
19 13 920
20 3×10^8 m/s
21 2×10^6
22 2.76×10^{13}
23 9.4×10^8 km

Exercise 4D

1a 1884 kJ **b** 7536 kJ
c 63 639 kJ **d** 1815 Cal
e 88 Cal **f** 299 Cal
2a 2793 kJ **b** 3251 kJ
c 1743 kJ **d** 2035 kJ
e 2549 kJ
3a 4 **b** 3
c Fruit and milk **d** Fruit
e Milk **f** 2
4a 52.8 g
b i 95 kJ
ii 475 kJ
iii 4180 kJ
c i 2090 kJ
ii 418 kJ
iii 20.9 kJ
d 16 720 kJ **e** 74
f 1% **g** 1.5625%
h 180 **i** 16.7%
j 26.4%
5a 21
b



c 43%
d 33%
e 3729 kJ
f 3500 kJ

Exercise 4E

1a 21 Sep **b** $349.22
c 10 Oct **d** $31.74
e $281.11 **f** $32.54
g 10.28%
2a $45.01
b $49.83
c $54.65
3a 16 kWh **b** $3.79
4a 12.6 kWh **b** $2.85
5a 12 kWh **b** $2.74
6 $1.03
7a $137.01 **b** $208.36
c $385.37
8a 1.5 kWh **b** 500 Wh
c 30 h **d** 12.5 h

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9a $92.72 **b** $107.53
c 5.9% **d** 3600 MJ
e 1930 MJ **f** $122.32

Review 4

Multiple-choice

1	D	**2**	A	**3**	B	**4**	C	**5**	B
6	D	**7**	D	**8**	D	**9**	C	**10**	B

Short-answer

1 6.246 tonnes
2a 0.05
b 400 000
c 3 000 000
3a 0.1 m
b 1.2 m
c 0.299%
4a 4 800 000
b 0.000 625
c 725 760
5a 5.08×10^4
b 3.6×10^{-3}
c 3.81×10^8
6a 1.512×10^{10}
b 2.0×10^6
c 7×10^{-3}
7 3.6×10^{12}
8 2.3×10^6
9a 2947 kJ
b 3869 kJ
c 2562 kJ
10a 4
b kWh
c 8 kWh
d 12 kWh
e April
f October
g July

Extended-response

11a 0.6 kWh
b 600 Wh
c 40 h

Chapter 5

Exercise 5A

1a Certain **b** Impossible
c Impossible **d** Certain
e Impossible **f** Certain
g Certain **h** Impossible
i Certain **j** Certain
2a Certain **b** Even
c Unlikely **d** Unlikely
e Even **f** Likely
g Even **h** Unlikely
i Even **j** Certain
3a Likely **b** Unlikely
4a Unlikely **b** Even
c Unlikely
5a Even **b** Certain
c Certain **d** Unlikely
6a {H, T}
b {R, R, W, W, W}
c {A, U, S, S, I, E}
d {1, 2, 3, 4, 5}
e {B, G}
f {Win, Draw, Loss}
7a {B, B, R, B, R} **b** 5
c Unlikely **d** Likely
8a $\{1, 2, 3, 4, 5, 6\}$ **b** 6
9a {A, B, C, D, E, F, G, H, I, J}
b Unlikely
10a {Win, Draw, Loss}
b 3
c No. Less chance of a draw
11a {Win, Draw, Loss}
b No. Jacob does not have the same chance of winning the match as the Australian champion.
12a Even **b** Unlikely
c Unlikely **d** Likely
e Likely
13a 0.50 **b** 0.75 **c** 1
d 0.90 **e** 0 **f** 0.25
14a $\{\$10, \$10, \$10, \$10, \$20, \$20, \$50, \$50, \$50\}$
b 3
c $\{\$10, \$10, \$10, \$20, \$20, \$50, \$50, \$50\}$
d $\{\$10, \$10, \$10, \$10, \$20, \$50, \$50, \$50\}$
e $\{\$10, \$10, \$10, \$10, \$20, \$20, \$50, \$50\}$
15a Even **b** Impossible
c Unlikely **d** Even
16 False. Not all the outcomes are equally likely.
17a {Laura, Mia, Emma}
b No. Laura has an 80% chance of winning.
c 10%

Exercise 5B

1a $\frac{1}{4}$ **b** $\frac{2}{7}$ **c** $\frac{1}{2}$
d $\frac{5}{26}$ **e** $\frac{1}{6}$ **f** $\frac{1}{5}$
2a $\frac{5}{8}$ **b** $\frac{3}{8}$ **c** $\frac{5}{8}$
3a $\frac{1}{5}$ **b** $\frac{13}{20}$ **c** $\frac{3}{20}$ **d** $\frac{4}{5}$
4 $\frac{1}{2}$
5 $\frac{1}{3}$
6a $\frac{1}{3}$ **b** $\frac{4}{9}$ **c** $\frac{2}{9}$ **d** $\frac{2}{3}$
7a $\frac{1}{52}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$
d $\frac{3}{26}$ **e** $\frac{1}{13}$ **f** $\frac{1}{52}$
g $\frac{5}{13}$ **h** $\frac{3}{13}$ **i** $\frac{1}{26}$
8 Disagree. Depends on the location and the season.
9a $\frac{1}{2}$ **b** $\frac{7}{8}$ **c** $\frac{5}{16}$
d $\frac{1}{4}$ **e** $\frac{15}{16}$ **f** $\frac{1}{4}$
10a $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{2}$
d $\frac{1}{4}$ **e** 1 **f** $\frac{1}{4}$
11a $\frac{1}{24}$ **b** $\frac{1}{24}$
c $\frac{23}{24}$ **d** $\frac{23}{24}$
12a $\frac{1}{17}$ **b** $\frac{1}{17}$ **c** $\frac{1}{51}$
d $\frac{4}{51}$ **e** $\frac{4}{17}$ **f** $\frac{11}{51}$

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13a $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $\frac{1}{2}$ **d** $\frac{1}{24}$

14 False. Not all outcomes are equally likely.

Exercise 5C

1a

	Boy	Girl
Boy	BB	BG
Girl	GB	GG

b 4

2a

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b 36

3a

+	M1	M2	M3	M4
E1	E1M1	E1M2	E1M3	E1M4
E2	E2M1	E2M2	E2M3	E2M4
E3	E3M1	E3M2	E3M3	E3M4

b 12

4a

	D	E
A	AD	AE
B	BD	BE
C	CD	CE

b 6

5 8

	D	E	F	G
X	XD	XE	XF	XG
Y	YD	YE	YF	YG

6a

	Y1	Y2	Y3	G1
Y1	Y1Y1	Y1Y2	Y1Y3	Y1G1
Y2	Y2Y1	Y2Y2	Y2Y3	Y2G1
Y3	Y3Y1	Y3Y2	Y3Y3	Y3G1
G1	G1Y1	G1Y2	G1Y3	G1G1

b i $\frac{1}{16}$ **ii** $\frac{1}{16}$ **iii** $\frac{9}{16}$

7a

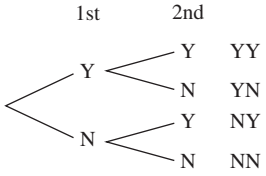
	K	Q	J
K	KK	KQ	KJ
Q	QK	QQ	QJ
J	JK	JQ	JJ

b i $\frac{1}{9}$ **ii** $\frac{1}{9}$ **iii** $\frac{4}{9}$

8a {HH, HT, TH, TT}

b i $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$

9



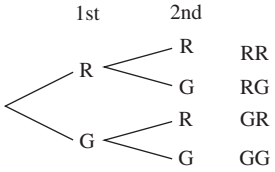
10a {TTT, TTF, TFT, TFF, FTT, FTF, FFT, FFF}

b 8

c i $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{7}{8}$

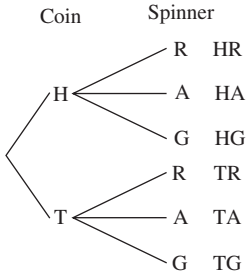
11a {12, 13, 21, 23, 31, 32} **b** $\frac{1}{6}$

12a



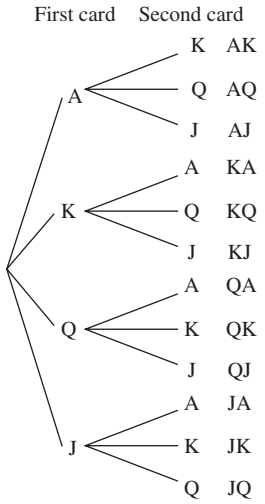
b $\frac{1}{2}$

13a

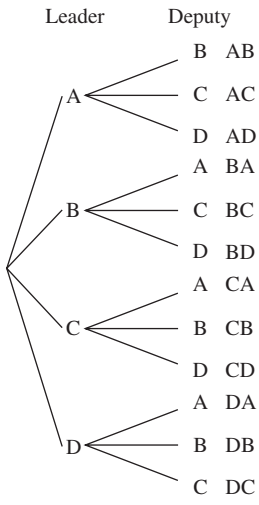


b $\frac{1}{3}$

14

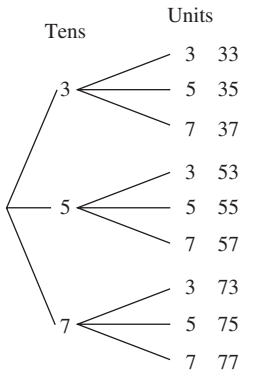


15a



b $\frac{1}{2}$

16a



b $\frac{2}{3}$

17a $\frac{5}{13}$ **b** 75% **c** $\frac{1}{4}$

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Exercise 5D

1a $\frac{3}{10}$ **b** $\frac{7}{15}$ **c** 0
d 1 **e** $\frac{7}{30}$

2a $\frac{57}{100}$ **b** $\frac{2}{25}$ **c** $\frac{7}{20}$

3a $\frac{5}{8}$ **b** $\frac{3}{8}$

4a 0.25 **b** 0.15
c 0.40 **d** 1

5 15%

6a 1 **b** 0.7 **c** 0.6

7a $\frac{23}{65}$ **b** $\frac{7}{18}$
c $\frac{11}{30}$ **d** $\frac{337}{585}$
e $\frac{203}{390}$ **f** $\frac{869}{1170}$

8a 0.51 **b** 0.49 **c** 0.50
d 0.47 **e** 0.26 **f** 0.53

9 $\frac{1}{3}$

10 $\frac{33}{56}$

11 $\frac{13}{20}$

12a i $\frac{3}{5}$ **ii** $\frac{3}{10}$ **iii** $\frac{1}{5}$
iv $\frac{3}{5}$ **v** $\frac{4}{5}$ **vi** $\frac{1}{5}$
b i 2 **ii** 4 **iii** 1
iv 1 **v** 2
c Parramatta

Exercise 5E

1a Selecting a red card from a normal pack of cards
b Not winning first prize in Lotto
c Throwing an odd number when a die is rolled
d Obtaining a head when a coin is tossed
e Drawing club, diamond or heart from a normal pack of playing cards
f Choosing a blue or red ball from a bag containing a blue, red and green ball

2a $\frac{4}{5}$ **b** 0.1
c 38% **d** 3 : 4
e $\frac{8}{11}$ **f** 0.55
g 62.5% **h** 4 : 7

3a 0.71 **b** $\frac{71}{100}$

4 0.4 **5** $\frac{4}{15}$

6a $\frac{3}{8}$ **b** 0.375 **c** 37.5%

7a $\frac{3}{4}$ **b** $\frac{9}{11}$
c $\frac{7}{9}$ **d** $\frac{40}{99}$
e $\frac{17}{36}$ **f** $\frac{259}{396}$
g $\frac{137}{396}$ **h** $\frac{25}{44}$

8a $\frac{12}{13}$ **b** $\frac{25}{26}$

9a $\frac{6}{7}$ **b** $\frac{5}{7}$

10a 0.9 **b** 0.85
c 0.09 **d** 1
e 1 **f** 1
g 0.25 **h** 0.75

11a 92% **b** 93%
c 90% **d** 17%
e 18% **f** 75%

12a $\frac{13}{16}$ **b** 6

Exercise 5F

1a $\frac{2}{7}$ **b** $\frac{3}{14}$
c $\frac{5}{14}$ **d** $\frac{1}{7}$

2a 0.118 **b** 0.294
c 0.412 **d** 0.176

3a 10.5% **b** 21.0%
c 14.5% **d** 17.0%
e 21.5% **f** 15.5%

4a 20% **b** 30%
c 10% **d** 15%
e 25%

5a $\frac{1}{3}$ **b** $\frac{5}{6}$
c $\frac{1}{2}$ **d** $\frac{1}{8}$

6a 10% **b** 75%
c 37.5% **d** 80%

7 Investigation
8 Investigation
9 Investigation
10 6.25%
11 0.96
12 $\frac{71}{134}$

13a 20 times
b Die has nearly twice the outcomes for data values 1 and 6. It appears to be biased.

14 Computer application

15a $\frac{28}{73}$
b Collecting and using data from more than one year.

16a 4 **b** 0.25 **c** 20

17 Investigation
18 Investigation
19 Investigation

Exercise 5G

1 18
2 624
3 2

4a 198 **b** 27

5 117
6 21

7a $\frac{1}{6}$ **b** 80
c $\frac{1}{2}$ **d** 240
e $\frac{2}{3}$ **f** 320
g $\frac{1}{3}$ **h** 160

8a $\frac{1}{25}$ **b** 8
c $\frac{1}{25}$ **d** 8
e $\frac{8}{25}$ **f** 64

Answers

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9a $\frac{1}{3}$ **b** 10
c $\frac{1}{6}$ **d** 5
e $\frac{2}{3}$ **f** 20

10a 192 **b** 160
c 96 **d** 64
e 96 **f** 96

11 Computer application

12a 30 **b** 90
c 90 **d** 30

13a 169 **b** 16
c 144 **d** 1024

14a 4 **b** 4 **c** 36

Exercise 5H

1a i O–, A– **ii** O–, B–
iii O–, B–, A–, AB–
iv O–
v O–, O+, A–, A+
vi O–, O+, B–, B+
vii O–, O+, B–, B+,A–, A+, AB–, AB+
viii O–, O+
b O– **c** O+, B–, A–
d AB+ **e** O–
f AB+, B+

2a 45–64
b 18–24
c 8%
d People over 65 are more likely to be ill, and in low-income countries many people die before 65.

3a 3.33% **b** 820 000
c 256 667 **d** 175 000
e 1 417 000

4a 9265 **b** 4905
c 1090 **d** 545

5a 2.4 **b** 561 700
c 1 101 372
d 3.9 million. Not believable, because one patient often needs many donations' worth of blood products.

Review 5

Multiple-choice

1 B **2** C **3** C
4 C **5** A **6** D
7 C **8** B **9** B

Short-answer

1a Impossible
b Unlikely
c Unlikely

2a {2, 4, 6, 8, 10}
b {a, e, i, o, u}
c {W, W, W, B, B, B, B}

3 132

4a $\frac{1}{50}$ **b** $\frac{1}{2}$ **c** $\frac{19}{50}$
d $\frac{2}{5}$ **e** $\frac{1}{5}$ **f** $\frac{7}{50}$

5 $\frac{1}{2}$

6a $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** 1

7a $\frac{4}{9}$ **b** $\frac{1}{3}$ **c** $\frac{7}{9}$

8a $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** 0
d $\frac{1}{4}$ **e** $\frac{1}{2}$ **f** $\frac{1}{2}$

9a $\frac{2}{5}$ **b** 1 **c** $\frac{1}{5}$

10a 0.125 **b** 0.5
c 0.875 **d** 0.125
e 0.625 **f** 0.375

11a $\frac{32}{35}$ **b** $\frac{3}{35}$
c $\frac{2}{7}$ **d** $\frac{4}{5}$

12a $\frac{12}{13}$ **b** $\frac{3}{4}$ **c** $\frac{25}{26}$

13a 0.20 **b** 0.24
c 0.32 **d** 0.24

14 0.58
15 2
16 336

Extended-response

17a $\frac{19}{33}$ **b** $\frac{14}{19}$

Chapter 6

Exercise 6A

1a 50.3 m^2 **b** 63.6 cm^2
c 16.6 mm^2 **d** 65.0 mm^2
e 113.1 cm^2 **f** 95.0 mm^2

2a 1963.50 mm^2 **b** 28.27 cm^2
c 314.16 m^2 **d** 254.47 m^2
e 7542.96 cm^2 **f** 6.16 mm^2

3a 172.8 mm^2 **b** 40.8 m^2
c 31.0 m^2 **d** 75.4 cm^2
e 173.6 m^2 **f** 31.2 mm^2

4a 103.67 m^2 **b** 84.67 cm^2
c 62.83 cm^2 **d** 386.42 mm^2

5 680.9 cm^2

6 50 cm^2

7a 14.1 cm^2 **b** 34.0 cm^2
c 47.1 mm^2 **d** 12.6 cm^2
e 94.4 mm^2 **f** 24.0 m^2

8a 32.95 cm^2 **b** 13.62 cm^2
c 88.51 mm^2 **d** 49.31 cm^2
e 4.13 m^2 **f** 164.70 cm^2

9 151 cm^2

10a 864 mm^2 **b** 491 mm^2

11 32.3 m^2

12 1.3 m

13 415 m^2

14 10 m

15a 120° **b** 10°

16 $6761

Exercise 6B

1a 52 cm^2 **b** 864 m^2 **c** 29 m^2
d 138 m^2 **e** 14 mm^2 **f** 117 cm^2

2 38 m^2

3 124 cm^2

4 88 cm^2

5 7.7 cm^2

6a 1950 m^2 **b** 1200 m^2
c 3800 m^2 **d** 7350 m^2
e 7992 m^2 **f** 1767 m^2

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7a 1767 m^2 **b** 37 m
8a 345 m^2 **b** 2046 m^2
c 297 m^2 **d** 3350 m^2
e 6038 m^2
9a 5 mm **b** 39.3 mm^2
c 99.3 mm^2
10a 180 cm^2 **b** 78.54 cm^2
c 101.46 cm^2
11a 113.10 cm^2 **b** 30.90 cm^2
12a 35 m **b** 8748.5 m^2
13 53.54 cm^2
14a 128 cm^2 **b** 2 **c** 27.47 cm^2
15a $A = lb + \frac{1}{2}\pi r^2 = 2xy + \frac{1}{2}\pi x^2$
b 65 m^2
16 30 pavers
17a 402.1 mm^2 **b** 30 mm
c 882.1 mm^2
18 2293 m^2
19a $A = \frac{1}{8}\pi x^2$
b $A = \frac{1}{8}\pi x^2 - \pi y^2$
c 13 m^2

Exercise 6C

1a 408 m^2 **b** 672 m^2
c 207 m^2 **d** 200 m^2
e 147 m^2 **f** 234 m^2
2a 264 m^3 **b** 1776 m^3
3 288 cm^2
4a 18.6 m^2 **b** 10.2 m^2
5 126 m^2 **6** 125 m^2
7 20.5 m^2 **8** 3400 m^2

Exercise 6D

1a 1200 mm^2 **b** 32 cm^2
c 150 m^2 **d** 150 mm^2
e 66 cm^2 **f** 158 m^2
2a 100 m^2 **b** 408 cm^2
3a 1200 cm^2 **b** 288 mm^2
c 2141.6 m^2 **d** 408 m^2
e 120 m^2 **f** 360 m^2
4a 1226 cm^2 **b** 240 m^2
c 912 cm^2 **d** 218 cm^2
e 530 m^2 **f** 246 cm^2
5a 15 m^2 **b** $4050
6a 720 cm^2 **b** 475.2 cm^2
c 1002.7 cm^2
7 1248 m^2
8a 6 faces **b** 27.04 cm^2
9 8 cm

Exercise 6E

1a 1659 mm^2 **b** 94 cm^2
c 151 m^2
2a 1156.1 mm^2 **b** 458.2 cm^2
c 2596.5 m^2
3a 452.39 m^2 **b** 78.54 mm^2
c 615.75 cm^2
4a 30.41 m^2 **b** 100.53 cm^2
c 226.19 mm^2
5a 461.81 cm^2 **b** 1140.40 m^2
c 28.86 m^2
6a 4.5 cm **b** 395.8 cm^2
c 523.1 cm^2
7a 0.79 m^2 **b** $18.85 **c** $75.40
8 Computer application

Exercise 6F

1a 102.7 mm^2 **b** 84 cm^2
c 256 cm^2 **d** 114.1 cm^2
e 275 m^2 **f** 160 cm^2
2a 5760 cm^2 **b** 11.2 m^2
c 166.4 m^2
3a 118.44 cm^2 **b** 81.27 cm^2
c 63.92 cm^2 **d** 408.82 cm^2
4a 7.4 m^2 **b** 242.5 cm^2
c 3939.6 cm^2
5a 31 m^2 **b** 404 cm^2
c 1009 cm^2
6a 4.2 cm **b** 55.4 cm^2
c 92.4 cm^2 **d** 147.8 cm^2
7a 396 cm^2 **b** 267 cm^2
8a 20 cm^2 **b** 261 cm^2
9 62 cm^2 **10** 160 cm^2
11a 9.5 cm **b** 8.1 cm
c 228.5 cm^2
12 34.6 cm
13a 6.91 cm **b** 85.22 cm
c 85 cm
14a 910 cm^2 (or 911 cm^2 if unrounded values are used in intermediate calculations)
b 728 cm^2
c 589 cm^2 (or 592 cm^2 if unrounded values are used in intermediate calculations)

Exercise 6G

1a 96 m^3 **b** 4400 m^3 **c** 112 m^3
2 165 mm^3 **3** 750 m^3
4a 108 m^3 **b** 5832 cm^3
c 137 344 cm^3
5 307.1 cm^3
6a 450 m^3 **b** 240 mm^3
c 410 cm^3
7 1200 cm^3
8a 1526 cm^3 **b** 446 cm^3
c 13 357 mm^3
9a 600 m^3 **b** 48 m^2
c 480 m^3
10a 15 m^2 **b** 180 m^3
11 38.971 cm^3
12a 280 cm^3 **b** 376 cm^2

Exercise 6H

1a 31 m^3 **b** 1583 mm^3
c 3079 mm^3 **d** 9640 m^3
e 2262 m^3 **f** 226 m^3
2 1570.80 cm^3
3a 524 cm^3 **b** 2145 mm^3
c 1437 cm^3 **d** 22 m^3
e 262 m^3 **f** 1 526 814 mm^3
4 905 mm^3
5a 4.7 m^3 **b** 1352.2 cm^3
c 904.8 cm^3
6a 100π m^2 **b** 800π m^3
7a 502.7 m^3 **b** 10 472 s
8a 6378 km **b** 1.09×10^{12} km^3
9 Four times
10a 3.5 cm **b** 179.6 cm^3
c 808.2 cm^3 **d** 269.4 cm^3
e 33.3%

Exercise 6I

1a 327 m^3 **b** 14 m^3
c 12 m^3 **d** 270 m^3
e 560 m^3 **f** 96 m^3
2a 160 m^3 **b** 2000 mm^3
3 12 cm^3

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4a 236 m^3 **b** 1139 m^3
c 235 m^3
5 296.88 cm^3
6a 48 400 m^2 **b** 1 677 866.67 m^3
7 49.86 cm^3
8a 37.5 cm^2 **b** 150 cm^3
9 800 m^3
10a 785 m^3 **b** 6283 m^3
c 5498 m^3
11 44 cm^3

Exercise 6J

1a $SA = 18\ cm^2, V = 4\ cm^3$
b $SA = 22\ cm^2, V = 5\ cm^3$
c $SA = 24\ cm^2, V = 6\ cm^3$
2a $SA = 1240\ m^2, V = 2394\ m^3$
b $SA = 160\ m^2, V = 112\ m^3$
c $SA = 5212\ cm^2, V = 20\,520\ cm^3$
3a $SA = 201\ m^2, V = 235\ m^3$
b $SA = 138\ mm^2, V = 117\ mm^3$
c $SA = 686\ cm^2, V = 1593\ cm^3$
4a 137.4 cm^2 **b** 77.0 cm^2
c 214.4 cm^2 **d** 153.9 cm^3
e 89.8 cm^3
5a $SA = 861\ cm^2, V = 1960\ cm^3$
b $SA = 97\ m^2, V = 79\ m^3$
c $SA = 156\ m^2, V = 131\ m^3$
6a 4072 cm^3 **b** 2714 cm^3
7 42 667 cm^3
8a 2.36 m^3 **b** 7.5 m^2
c 27.64 m^3
9a First tank is 1.216 m^3
Second tank is 0.955 m^3
b First tank is 7 m^2
Second tank is 6 m^2
10 2 100 000 cm^3

Exercise 6K

1 Four cans with 300 mL remaining.
2a 300 mL **b** 20
c 60 **d** 30
3a 4 **b** 2
c 70 **d** 34
e 900 **f** 0.5
g 43 **h** 30 000
i 103 **j** 7000
k 5 **l** 8 000 000
4 200 mL
5 720 000 L
6 60 mL
7 3 L
8 421.88 mL
9a 96 000 000 mL
b 0.04 mL
c 5.65 mL
d 270 000 000 mL
e 147 000 000 mL
f 471.24 mL
10a 12.6 m^2
b 31.4 m^3
c 31 kL
11a 217.5 m^3 **b** 218 kL
12a 608.5 m^3
b 608.5 kL
c 305 550 L

Review 6

Multiple-choice

1 C **2** B **3** D **4** A **5** D
6 B **7** B **8** C **9** C **10** B

Short-answer

1a 6.2 mm^2 **b** 22.0 mm^2
c 60.4 m^2
2a 628 cm^2 **b** 9425 cm^3
c 9.4 L
3a 48 cm^2 **b** 12 cm^2
c 36 cm^2
4 266 cm^2 **5** 180 m^2
6 603.2 cm^2 **7** 263.89 cm^2
8a 1257 cm^2 **b** 4189 cm^3
9a 144 m^3 **b** 24 mm^3 **c** 1139 m^3
10a 240 m^3 **b** 96 m^3 **c** 144 m^3
11 6 L

Extended-response

12a 140 m^2 **b** 120 m^3 **c** 128 m^2
13a 88 m^3
b 17.6 h (17.7 h if unrounded value in part a is used)

Practice Paper 1

1 B **2** B **3** C **4** C
5 D **6** A **7** A **8** C
9 D **10** B **11** B **12** D
13 B **14** D **15** B
16a i $1306.60
ii **1** $681.10 **2** $4573.10
b i $59 000 **ii** $10 722
iii $1180 **iv** $11 902
v $353 refund
c i $-10a - 5$ **ii** $5x^5$
17a i $\frac{7}{13}$ **ii** $\frac{12}{13}$
b 47.71 cm^3 **c** $x = 4$
d i 1658.8 cm^2 **ii** 7
e i $x = 15$ **ii** $x = -0.5$
f 1.08×10^9 km
g 0.010%
18a i $r = \sqrt[3]{\frac{3V}{4\pi}}$ **ii** 0.78 m
b i $3764.48
ii Heater C, because the room's volume is 81.081 m^3, too big for heaters A or B.
c i 25.45 m^3 **ii** 25 447 L
d 1.23 **e** $x = \frac{y - b}{m}$
f 0.065

Chapter 7

Exercise 7A

1a Sample **b** Census
c Census **d** Sample
e Sample **f** Sample
g Census **h** Census
i Sample **j** Census
2a Systematic **b** Random
c Self-selected **d** Stratified
e Systematic **f** Random
g Systematic **h** Random
i Stratified **j** Random
3 173 **4** 6%
5a 4 **b** 58 **c** 62
6a 200 **b** 22 **c** 28
7a 440 **b** 15 **c** 18
8 36
9a 440
b i 11 **ii** 15 **iii** 13 **iv** 16

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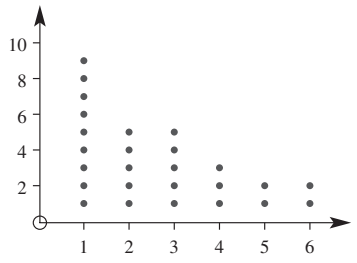
10a 23 **b** 65%
c School population does not represent the views of the entire population.
11 56
12a 13 **b** 12
13a No opportunity to give a reason or state that part of the policy is good.
b Bias, as it states that the boss is lazy. This may not be the interviewee's opinion.
14a Inaccurate. Survey is biased as people at the festival have a liking for country music.
b Systematic survey of people of across NSW.

Exercise 7B

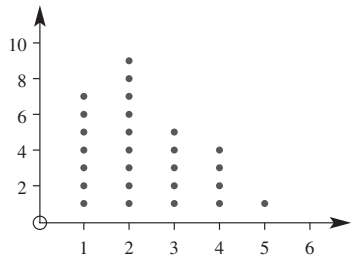
1a Categorical **b** Numerical
c Numerical **d** Categorical
e Numerical **f** Numerical
g Categorical **h** Numerical
i Categorical **j** Numerical
k Numerical **l** Categorical
m Numerical
2a Discrete **b** Discrete
c Continuous **d** Discrete
e Continuous **f** Discrete
g Discrete **h** Continuous
i Continuous **j** Discrete
k Continuous
3a Continuous **b** Continuous
c Categorical **d** Categorical
e Categorical **f** Discrete
g Continuous **h** Discrete
i Categorical **j** Discrete
k Categorical **l** Continuous
m Continuous **n** Discrete
o Continuous **p** Categorical
4a Numerical **b** Continuous
5a Numerical **b** Discrete
6 Categorical and nominal
7 Categorical and ordinal
8 Numerical and discrete
9 Categorical and nominal
10 Numerical and discrete
11 Numerical and continuous
12a Numerical and discrete
b Categorical and ordinal

Exercise 7C

1a 16 **b** 13
c 9 and 16 **d** 332
2a



b 3 **c** 5
3a



b 5 **c** 5
4a 34 **b** 24 **c** 28
d 15 **e** 24
5a

0	4
1	79
2	0124566778
3	00113347
4	016

b 16 **c** 27.5
6a

876554433	0	56
75411	1	24479
1	2	2233689
	3	2

b Australian tourists spent fewer nights away from home in the past year than New Zealand tourists. The median for the Australian tourist is 7 and the New Zealand tourist is 22. New Zealand tourist has a greater spread than the Australian tourist.
7a

9777664	5	58
733	6	1456889
75221	7	123889

b Ages for admission is younger for males than females. The median is 63 for males and 68 for females. The admission age for males has a greater spread than for females.

Exercise 7D

1a

Class	Class centre	Freq.
5–19	12	10
20–34	27	8
35–49	42	6
50–64	57	4
	Total	28

b 10 **c** 5–19 **d** 28
2a

Class	Class centre	Tally	Freq.
3–7	5	IIII	4
8–12	10	𝍸 I	6
13–17	15	II	2
18–22	20	𝍸 II	7
23–27	25	I	1
28–32	30	III	3
33–37	35	𝍸 III	8
38–42	40	𝍸	5

b 8 **c** 23–27 **d** 36
3

Class	Class centre	Tally	Freq.
170–174	172	II	2
175–179	177	II	2
180–184	182	𝍸 𝍸 II	12
185–189	187	𝍸 III	8
190–194	192	𝍸 I	6

4a 0–9, 10–19, 20–29, 30–39 and 40–49
b 4.5, 14.5, 24.5, 34.5 and 44.5

Answers

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c

Class	Class centre	Freq.
0–9	4.5	3
10–19	14.5	11
20–29	24.5	29
30–39	34.5	46
40–49	44.5	1

5a 130–139, 140–149, 150–159, 160–169, 170–179 and 180–189

b 134.5, 144.5, 154.5, 164.5, 174.5 and 184.5

c

Class	Class centre	Freq.
130–139	134.5	2
140–149	144.5	3
150–159	154.5	9
160–169	164.5	16
170–179	174.5	8
180–189	184.5	2

6a 0–9, 10–19, 20–29, 30–39, 40–49, 50–59 and 60–69

b 4.5, 14.5, 24.5, 34.5, 44.5, 54.5 and 64.5

c

Class	Class centre	Freq.
0–9	4.5	7
10–19	14.5	11
20–29	24.5	6
30–39	34.5	3
40–49	44.5	1
50–59	54.5	1
60–69	64.5	1

d 60%

Exercise 7E

1a

Score	Freq	Cum. freq.
4	4	4
5	6	10
6	7	17
7	10	27
8	5	32

b 5 **c** 22 **d** 10 **e** 32

2a

Number of calls	Freq.	Cum. freq.
20	4	4
21	3	7
22	10	17
23	12	29
24	6	35
25	5	40

b 3 **c** 35 **d** 11

3a

Score	Tally	Freq.	Cum. freq.
8	||	2	2
9	卌	5	7
10	卌 卌 |	11	18
11	卌 |	6	24
12	||	2	26
13	||||	4	30

b 30 **c** 11 **d** 5 **e** 10

4a

Score	Tally	Freq	Cum. freq.
4	||||	4	4
5	||||	4	8
6	卌	5	13
7	||	2	15
8	卌	5	20
9	卌	5	25
10	卌	5	30

b See above

c 30 **d** 5 **e** 5 **f** 22

g 20 **h** 6, 8, 9 and 10

i 7 **j** $\frac{1}{15}$ **k** $\frac{2}{15}$

l 73.33% **m** 26.67%

5a

Score	Tally	Freq.	Cum. freq.
61	卌 |	6	6
62	卌 卌 |||	13	19
63	卌 卌 ||||	14	33
64	卌 ||||	9	42
65	||	2	44
66	|	1	45

b See above **c** 45

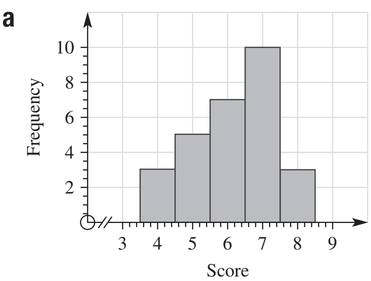
d 6 **e** 33

f $\frac{13}{45}$ **g** 4.44%

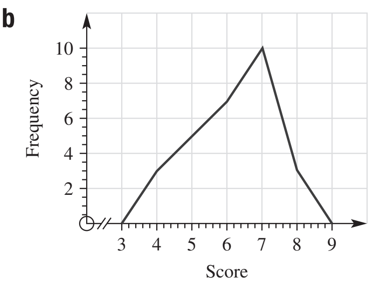
h 93.33%

Exercise 7F

1a



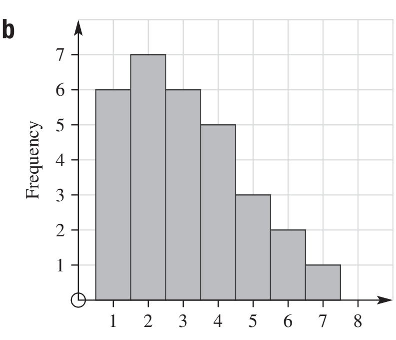
b



2a

Score	Tally	Freq.
1	卌 |	6
2	卌 ||	7
3	卌 |	6
4	卌	5
5	|||	3
6	||	2
7	|	1

b



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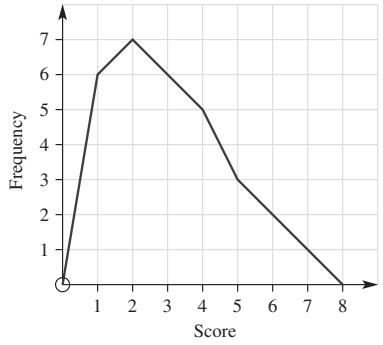
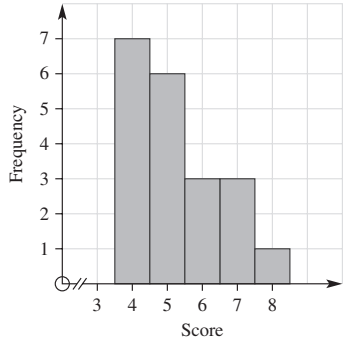
c



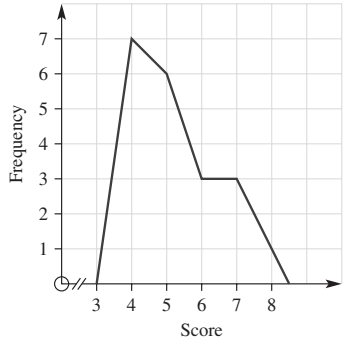
3a

Score	Tally	Freq.
4	IIIIII	7
5	IIIII	6
6	III	3
7	III	3
8	I	1

b



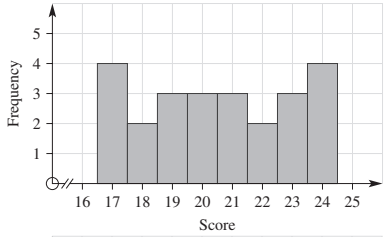
c



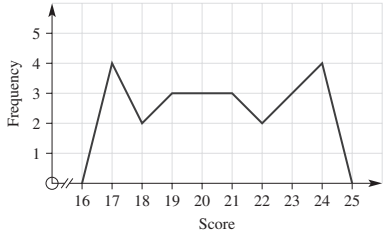
4a

Score	Tally	Freq.
17	IIII	4
18	II	2
19	III	3
20	III	3
21	III	3
22	II	2
23	III	3
24	IIII	4

b



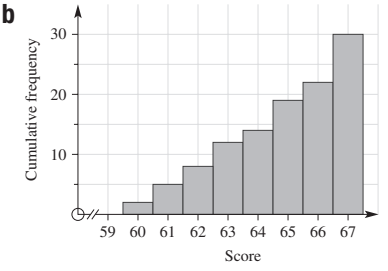
c



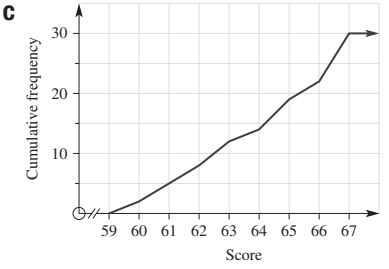
5a

Score	Freq	Cum. freq.
60	2	2
61	3	5
62	3	8
63	4	12
64	2	14
65	5	19
66	3	22
67	8	30

b



c



d 12 days **e** 22 days

6a 10 **b** 90 **c** 50 **d** 50
e 0 **f** 50 **g** 0 **h** 150
i 50 **j** 50 **k** 500

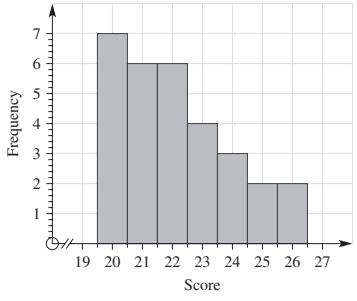
l

Score	Freq	Cum. freq.
10	10	10
20	90	100
30	50	150
40	50	200
50	0	200
60	50	250
70	0	250
80	150	400
90	50	450
100	50	500

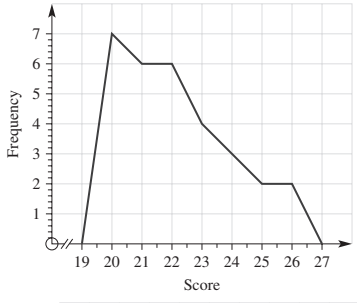
7a

Score	Freq	Cum. freq.
20	7	7
21	6	13
22	6	19
23	4	23
24	3	26
25	2	28
26	2	30

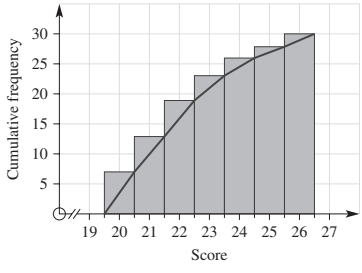
b



c



d



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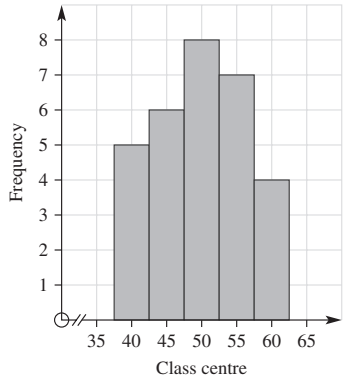
e 22
f $Q_1 = 21$, $Q_3 = 23$
g IQR = 2
8a Suggested classes:
38–42, 43–47, 48–52, 53–57 and 58–62

b

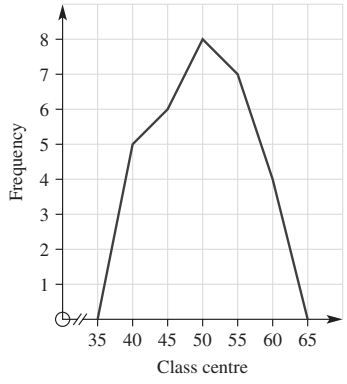
Class	Class centre	Freq.	Cum. freq.
38–42	40	5	5
43–47	45	6	11
48–52	50	8	19
53–57	55	7	26
58–62	60	4	30

c See above

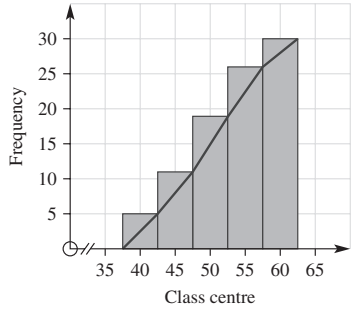
d



e



f



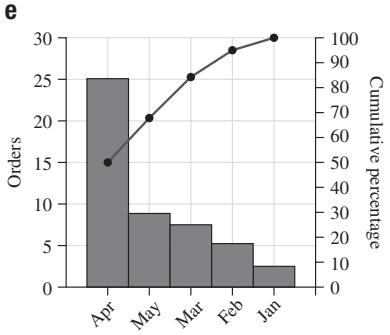
g 50
h $Q_1 = 45$, $Q_3 = 55$
i IQR = 10

Exercise 7G

1a

Month	Freq.	Cum. freq.	Cum. per.
Apr	25	25	50
May	9	34	68
Mar	8	42	84
Feb	5	47	94
Jan	3	50	100

b 5
c Scale 1 to 30 (or 25) with a major guideline of 5.
d Scale 1 to 100 with a major guideline of 10.
e

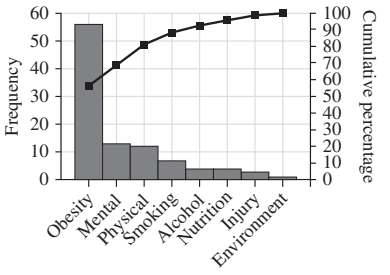


f April and May

2a

Health issue	Freq.	Cum. freq.	Cum. per.
Obesity	56	56	56
Mental health	13	69	69
Physical activity	12	81	81
Smoking	7	88	88
Alcohol	4	92	92
Nutrition	4	96	96
Injury	3	99	99
Environment	1	100	100

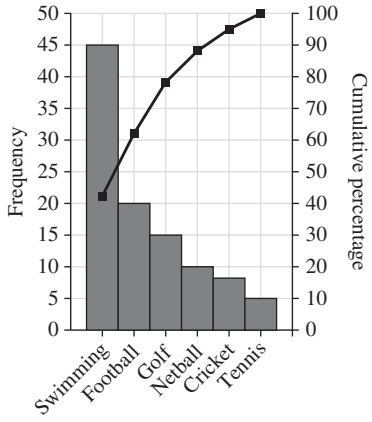
b 8
c Scale 1 to 60 with a major guideline of 10.
d Scale 1 to 100 with a major guideline of 10.
e



f Obesity and mental health
3 Computer investigation
4a

Sport	Freq.	Cum. freq.	Cum. per.
Swimming	45	45	44
Football	20	65	63
Golf	15	80	78
Netball	10	90	87
Cricket	8	98	95
Tennis	5	103	100

b



c Swimming, football and golf

Exercise 7H

1a $4000 **b** $1000
c 2.5 years **d** 3.5 years
e About $1250 **f** About $2600
2a 3.88 cents per kilometre
b Brand A
c $734.50
d $183.00
e $250.00
3a 4L **b** 11L **c** $13.05
4a 10 023 **b** 28.1% **c** $\frac{1786}{10\,023}$
d 2005
e January had the most number of accidents (2814). People often go on holidays in January and drive their car.
5a 44 km/h **b** 30 km/h
c 37 km/h **d** 36 km/h
e 36 km/h **f** 14 km/h
6a 6 hours **b** 14
c 276 **d** 12.5
e 12.5 **f** 12

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7a

Class	Class centre (x)	Freq. (f)	$f \times x$
20–29	24.5	85	2082.5
30–39	34.5	72	2484
40–49	44.5	71	3159.5
50–59	54.5	55	2997.5
60–69	64.5	36	2322

b 319 **c** 41
d 26.65% **e** 28.53%

8a Mean = 1.2, median = 0, mode = 0

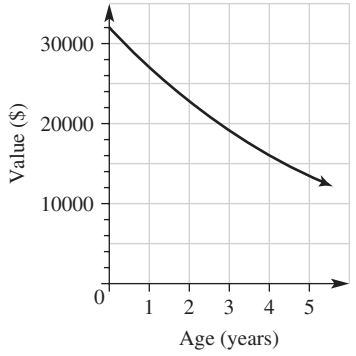
b It depends. Mean has been affected by outlier of 6, but even without this outlier the average would still be 1.1 accidents/day. Median gives a good picture of a typical day, but conceals the fact that many days have multiple accidents.

9a $5120

b

Year	Current value	Depreciation	Depreciated value
1	$32 000	$5120	$26 880
2	$26 880	$4301	$22 579
3	$22 579	$3613	$18 966
4	$18 966	$3035	$15 931
5	$15 931	$2549	$13 382

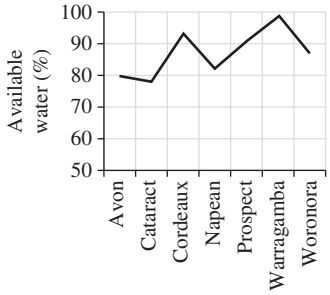
c



Exercise 7I

1a 375 mm **b** June
c Year 2 **d** February
2a Warragamba **b** Warragamba
c 117 360 ML **d** 62 457.3 ML
e 2 537 380 ML **f** 362 483 ML

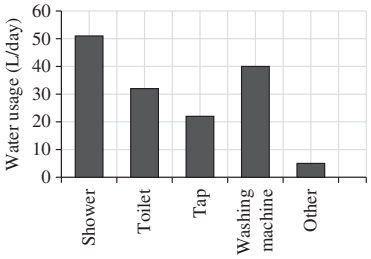
g



3a QLD **b** WA
c QLD **d** WA

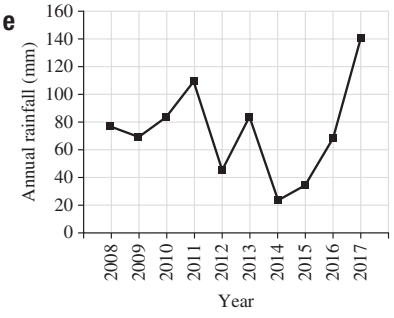
4a 150 L **b** 54 750 L
c 34%

d



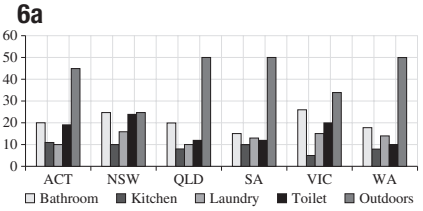
5a 2017 **b** 2014
c Mean = 73.31 mm, median = 73.2
d Range = 118.2

e



f Student opinion. Lowest rainfall occurred in 2014 and 2015. This could be part of the global climate change. However, 2017 had the highest rainfall in January for 10 years.

6a



b

c NSW. Possibly lower use of dual-flush toilets; less use of water outdoors increases the percentage for the other categories.

d QLD, SA and WA. Less rainfall than the other states.

e Kitchen. Water only used in cooking and dishwashing.

f Outdoors. Many gardens require a large amount of water.

g NSW uses a higher percentage of household water in the bathroom and toilet and a lower percentage of household water outdoors.

Review 7

Multiple-choice

1 D **2** C **3** C **4** B
5 B **6** D **7** A **8** A

Short-answer

1a Categorical **b** Categorical
c Numerical **d** Numerical
e Numerical **f** Categorical
g Numerical **h** Numerical
i Numerical

2a 35 **b** 26 **c** 25

3a 18 **b** 8 **c** 22 **d** 22

4a

Class	Class centre	Freq.	Cum. freq.
4–8	6	5	5
9–13	11	6	11
14–18	16	8	19
19–23	21	4	23
	Total	23	

b 23 **c** 12 **d** 48%

5a 60% **b** 2011

c Single-flush toilets are decreasing in popularity and being replaced with dual-flush toilets.

d 12.8 million

Answers

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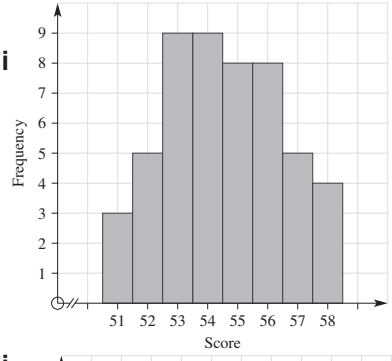
Extended-response

6a

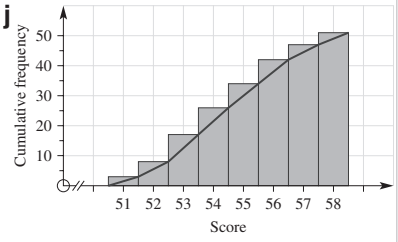
Score	Tally	Freq	Cum. freq.
51	𝍸	3	3
52	𝍸	5	8
53	𝍸 𝍸	9	17
54	𝍸 𝍸	9	26
55	𝍸 𝍸	8	34
56	𝍸 𝍸	8	42
57	𝍸	5	47
58	𝍸	4	51

b 51 **c** 26
d See above **e** 51 min
f 53 and 54 **g** 7
h 54

i



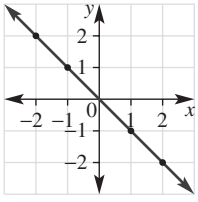
j



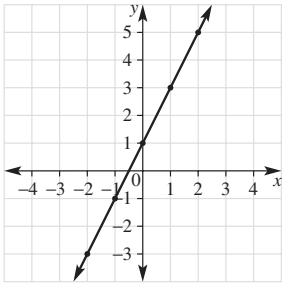
Chapter 8

Exercise 8A

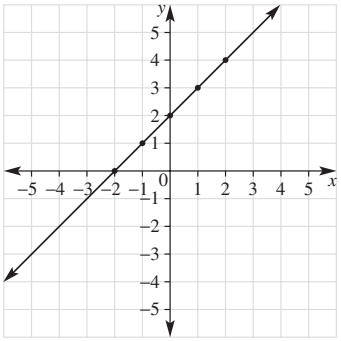
1a



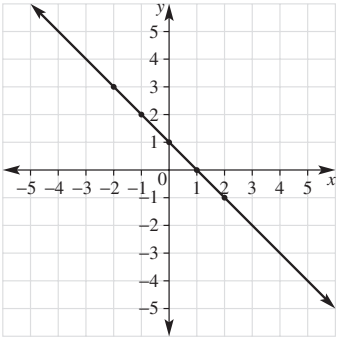
b



c



d



2a

x	−2	−1	0	1	2
y	−4	−2	0	2	4

b

x	−2	−1	0	1	2
y	4	2	0	−2	−4

c

x	−2	−1	0	1	2
y	−1	0	1	2	3

d

x	−2	−1	0	1	2
y	−3	−2	−1	0	1

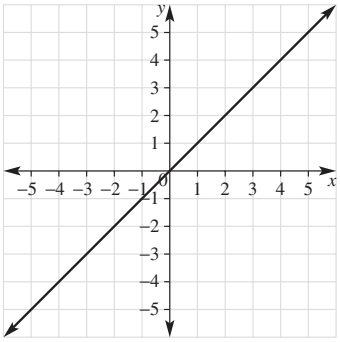
e

x	−2	−1	0	1	2
y	−1	1	3	5	7

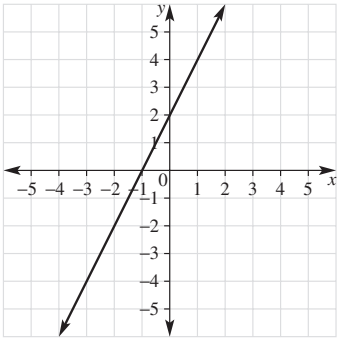
f

x	−2	−1	0	1	2
y	4	3	2	1	0

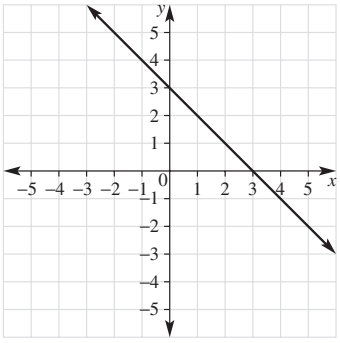
3a



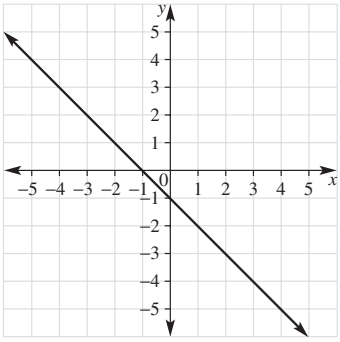
b



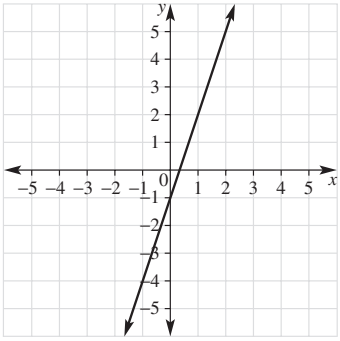
c



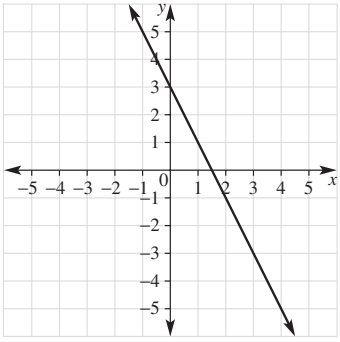
d



e



f



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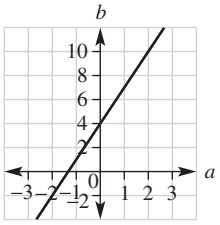
4a i

a	–2	–1	0	1	2
b	–2	1	4	7	10

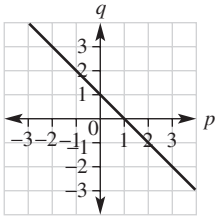
ii

p	–2	–1	0	1	2
q	3	2	1	0	–1

b

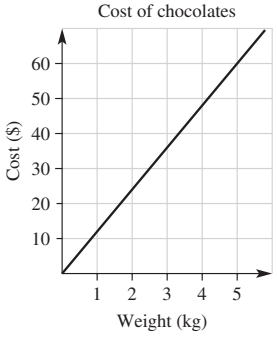


c



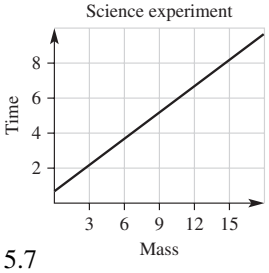
5a Cost (c) **b** Weight (w)

c



d $18

6a

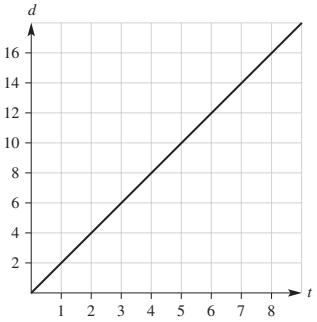


b 5.7

7a $d = 2t$

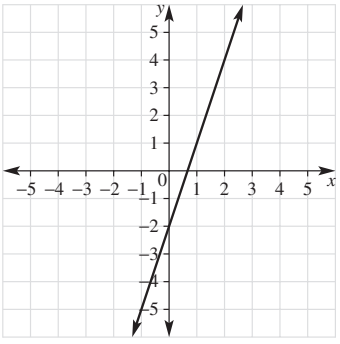
b

t	0	2	4	6	8
d	0	4	8	12	16

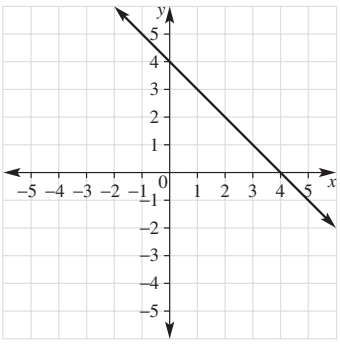


8a False **b** True **c** True
d True **e** False **f** False

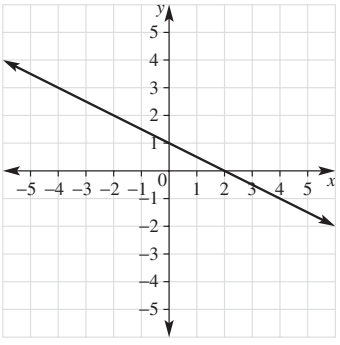
9a



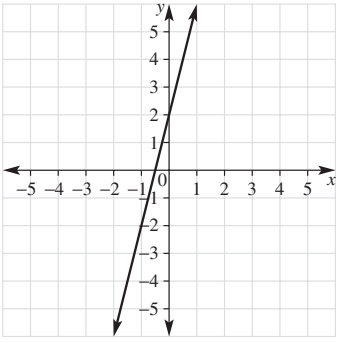
b



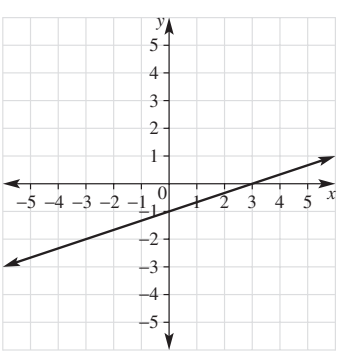
c



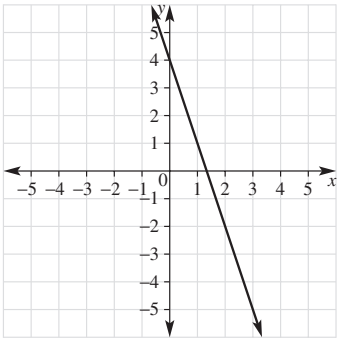
d



e



f



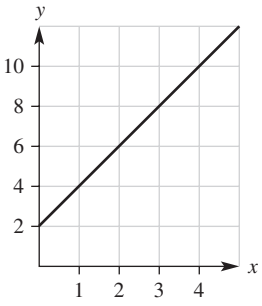
Exercise 8B

1a 4 **b** –1 **c** –12 **d** 4

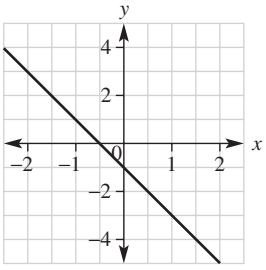
2a 2 **b** –5 **c** $-\frac{1}{2}$

3a 2 **b** –1

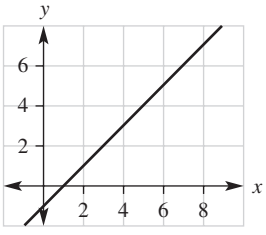
4a Gradient of 2, y-intercept of 2



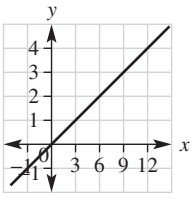
b Gradient of –2, y-intercept of –1



c Gradient of 1, y-intercept of –1



d Gradient of $\frac{1}{3}$, y-intercept of 0



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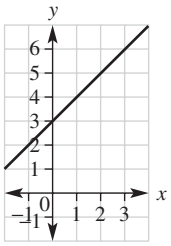
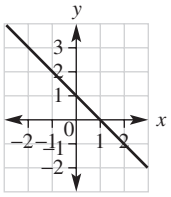
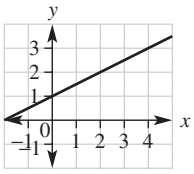
5a Gradient of 1, y-intercept of 3



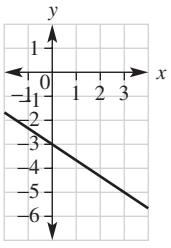
b Gradient of -1, y-intercept of 1



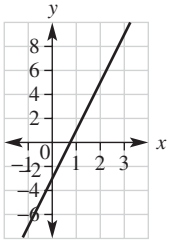
c Gradient of $\frac{1}{2}$, y-intercept of 1



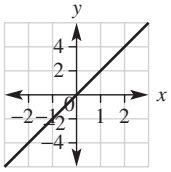
d Gradient of $-\frac{2}{3}$, y-intercept of -3



e Gradient of 4, y-intercept of -3



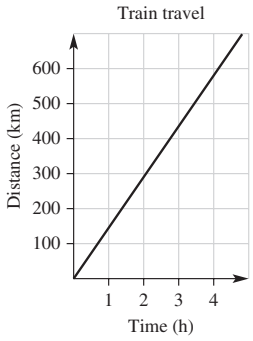
f Gradient of 2, y-intercept of 0



6a

t	0	1	2	3	4
d	0	150	300	450	600

b

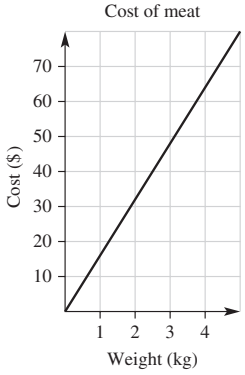


c 150 **d** 0

7a

n	0	1	2	3	4
c	0	16	32	48	64

b



c 16 **d** 0

Exercise 8C

1a Gradient is 4, y-intercept is 2

b Gradient is 3, y-intercept is -7

c Gradient is 5, y-intercept is 0.4

d Gradient is 1.5, y-intercept is -2

e Gradient is $\frac{1}{2}$, y-intercept is 3

f Gradient is -3, y-intercept is 5

g Gradient is 1, y-intercept is 0

h Gradient is 5, y-intercept is 2

2a $y = 3x + 2$ **b** $y = -2x + 10$

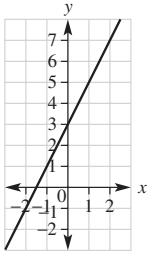
c $y = -4x - 1$ **d** $y = 0.5x + 1$

3a $y = x + 1$ **b** $y = -2x - 1$

c $y = \frac{1}{2}x - 1$ **d** $y = -3x + 12$

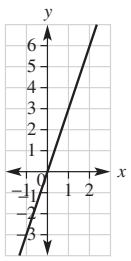
4a Gradient is 2, y-intercept is 3

b



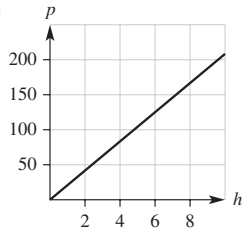
5a $y = 3x$

b



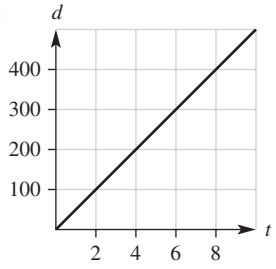
6a $p = 21h$

b

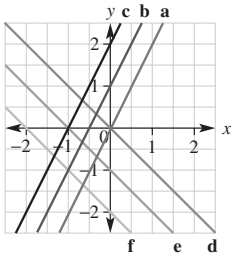


7a $d = 50t$

b

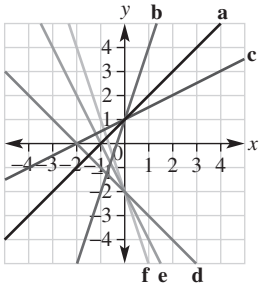


8a–f



g The graphs for equations a, b and c are parallel with a positive gradient. The graphs for equations d, e and f are parallel with a negative gradient.

9a–f



g The graphs for equations a, b and c intersect the y-axis at 1 (y-intercept of 1). The graphs for equations d, e and f intersect the y-axis at -2 (y-intercept of -2).

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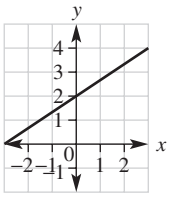
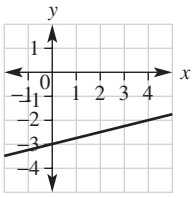
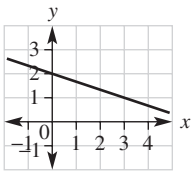
10a



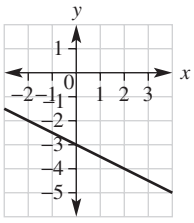
b



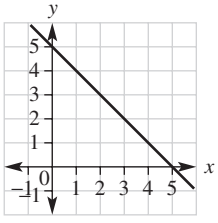
c



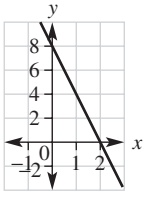
d



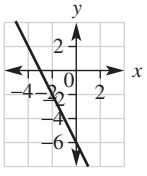
e



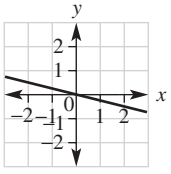
f



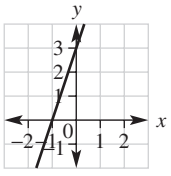
g



h



i



Exercise 8D

1a 500 L **b** 1500 L
c 2100 L **d** 10 minutes
e 1900 L

2a 400 CNY **b** 250 CNY
c 20 AUD **d** 70 AUD
e Gradient is 5

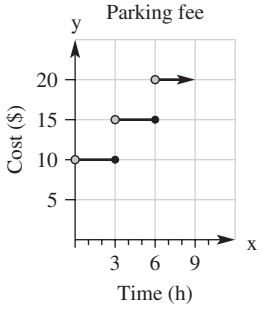
3a \$1.50 **b** \$3.50
c \$1.50 **d** \$3.00

4a \$40 000 **b** \$90 000
c 48 months **d** $v = -2.5t + 120$
e \$115 000 **f** \$2500

5a \$125 billion **b** 300 000
c $A = \frac{1}{4}N$ **d** \$87.5 billion
e \$250 billion

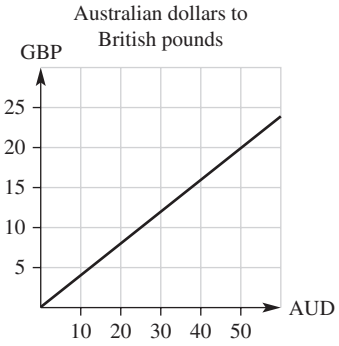
6a \$15 **b** \$25
c 30 calls **d** $C = 0.5n + 15$

7a



b \$15 **c** \$10 **d** \$15

8a



b 16 GBP **c** 25 AUD
d Gradient is 0.4, the vertical intercept is 0
e $GBP = 0.4 \times AUD$ or $y = 0.4x$

Exercise 8E

1a $y = 3x$ **b** $k = 3$
c 12 **d** 5

2a $y = 2x$ **b** $k = 2$
c 16 **d** 6

3a $y = \frac{1}{4}x$ **b** $k = 4$
c 6 **d** 16

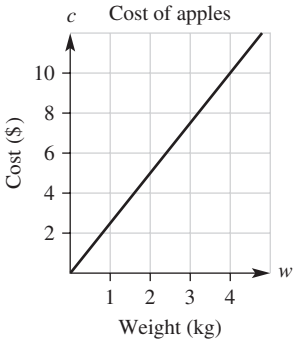
4a $a = 1.5b$ **b** $k = 1.5$
c 33 **d** 24

5a $p = 21.5h$ **b** $k = 21.5$
c \$236.50 **d** = \$139.75
e 17 hours **f** 35 hours

6a

w	0	1	2	3	4
c	0	2.5	5	7.5	10

b

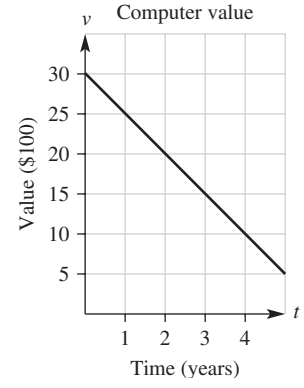


c 6

7a

t	0	1	2	3	4
v	30	25	20	15	10

b

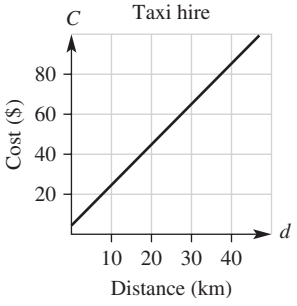


c \$3000 **d** \$2000 **e** 3 years

8a

d	0	10	20	30	40
C	3	23	43	63	83

b



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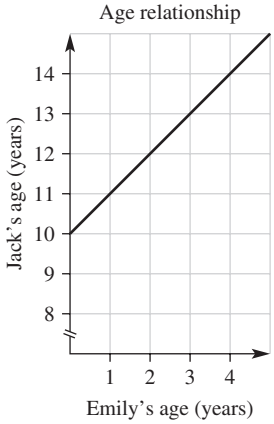
9a

E	0	1	2	3	4
J	10	11	12	13	14

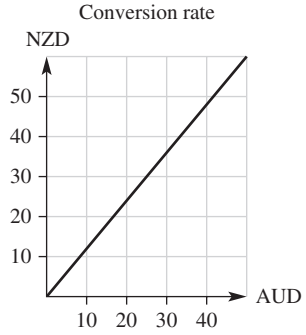
b



10a

AUD	0	10	20	30	40
NZD	0	12	24	36	48

b



Review 8

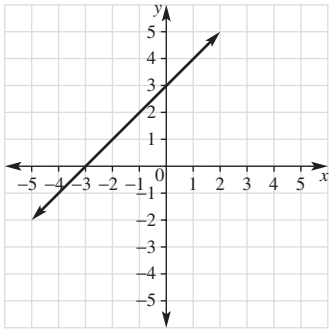
Multiple-choice

1 C **2** C **3** C **4** D
5 A **6** C **7** D **8** C

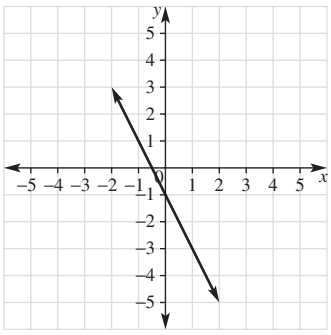
Short-answer

1a y values are 1, 2, 3, 4, 5
b y values are 3, 1, –1, –3, –5

2a



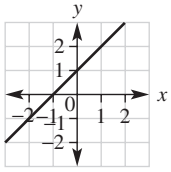
b



3a 1 **b** $\frac{3}{2}$ (or 1.5) **c** –2

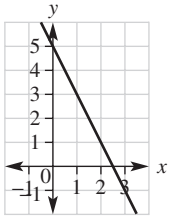
4a $y = 2x - 1$ **b** $y = -\frac{1}{2}x$

5a



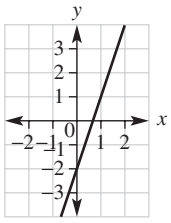
Gradient is 1 and y-intercept is 1.

b



Gradient is –2 and y-intercept is 5.

c

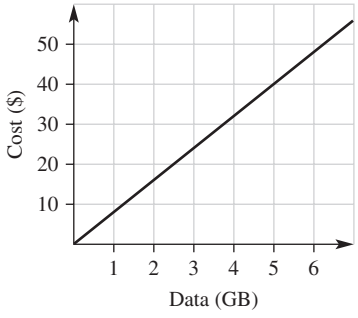


Gradient is 3 and y-intercept is –2.

6a π **b** 2 cm

7a Cost (c) **b** Data (d)

c

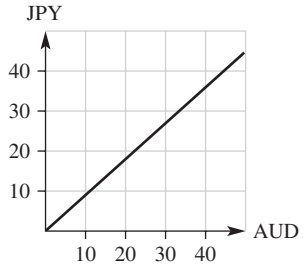


d 2.5

8a

AUD	0	10	20	30	40
JPY	0	9	18	27	36

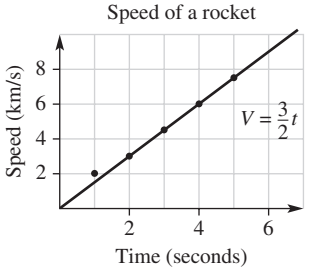
b



Extended-response

9a $18 000
b $24 000
c $v = -0.5t + 30$
d $27 000
e 60 months
f $500

10a



b 9 km/s
c 8 seconds

Chapter 9

Exercise 9A

1a $11 000 **b** $165 000
c $567 000 **d** $777
e $18 500
2a $944 **b** $11 440
c $18 912.50 **d** $6135
e $50 850
3a $33 150 **b** $203 150
4 $97.88
5 $1.67
6 Computer application
7a 24 **b** $450 **c** $93.75
8a $22 000 **b** 10.5% p.a.
c $9240 **d** $650.83

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9a \$14 000 **b** \$3640 **c** \$735
10a \$68 000 **b** \$2000
11 8.98% p.a.
12a \$7200
b \$50 256
c \$14 256
d 16.5% p.a.
13 24% p.a.
14a \$40 650
b 7.1% p.a.
c \$903.33

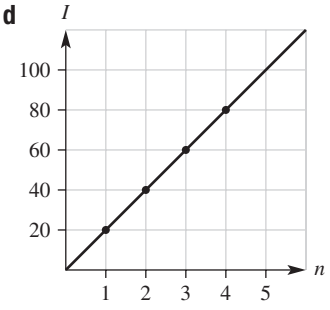
Exercise 9B

1a $I = 20n$

b

n	0	1	2	3	4
I	0	20	40	60	80

c, d

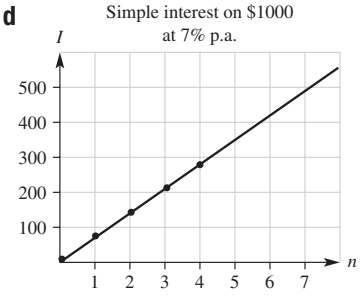


2a $I = 70n$

b

n	0	1	2	3	4
I	0	70	140	210	280

c, d



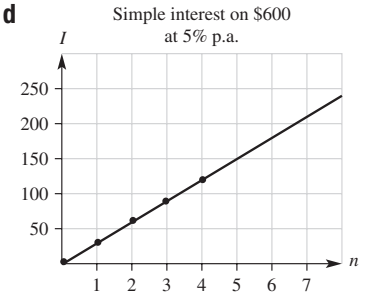
e \$175 **f** \$420 **g** 3 years

3a $I = 30n$

b

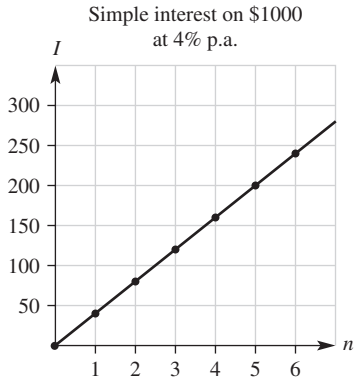
n	0	1	2	3	4	5
I	0	30	60	90	120	150

c, d



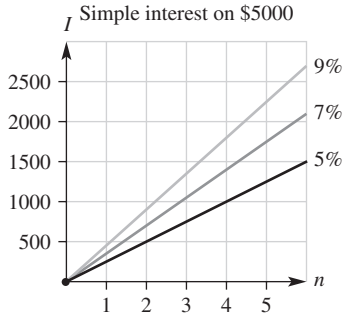
e \$105 **f** \$180
g 12 years

4



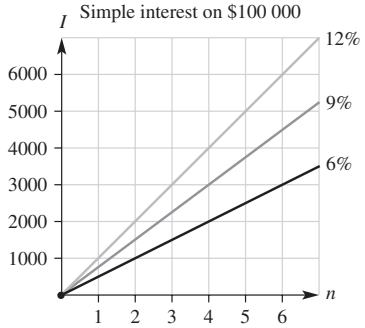
\$240

5a



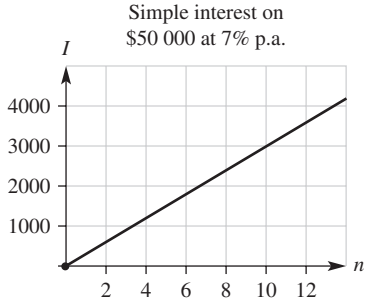
b i \$625 **ii** \$875
iii \$1125
c i \$1250 **ii** \$1750
iii \$2250
d i 4 years
ii About 2.8 years
iii About 2.2 years

6a

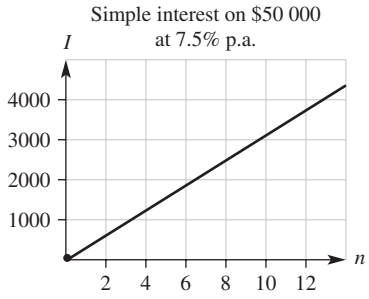


b i \$500 **ii** \$750
iii \$1000
c i \$3000 **ii** \$4500
iii \$6000
d i 4 months
ii about 2.7 months
iii 2 months

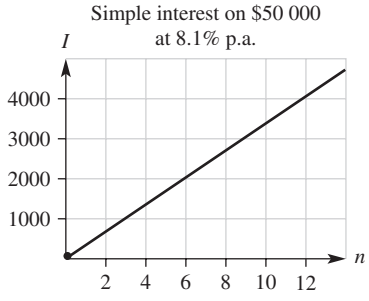
7a



b



c



Exercise 9C

1a \$4.80 **b** \$36 **c** \$9.90
d \$22 **e** \$536 **f** \$70.56
g \$6.75 **h** \$0.12
2a \$22 **b** \$156 **c** \$112
d \$822 **e** \$3 **f** \$11.52
g \$80 **h** \$0.70
3a \$24 **b** \$19.80 **c** \$135
d \$6.60 **e** \$37.2 **f** \$15
4a \$49 014.40 **b** \$100 551.60
c \$131 736.80 **d** \$68 878.80
5a \$185.25 **b** \$234
c \$546 **d** \$68.25

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6a $15 810 **b** $13 438.50
7a $96 **b** $323.40
c $20 **d** $1263.50
e $993.60 **f** $84
8 25%
9a $501.60 **b** $336
c $510 **d** $249.60
10a 10% **b** $198

Exercise 9D

1a $3.60 **b** $14.00 **c** $17.00
d $0.32 **e** $49.00 **f** $4.20
g $4.29 **h** $260 **i** $37.00
2a $62 **b** $682
3a $78 **b** $858
4a £26.25 **b** £14
c £3.50 **d** £1.40
5a 8820 **b** 2940 **c** 7140
d 5250 **e** 7560 **f** 2100
g 5880 **h** 2730
6a $16 **b** $22 **c** $90
d $35 **e** $14 **f** $180
7a $540 **b** $347
8 Computer application
9 $329.09 **10** $1681.82

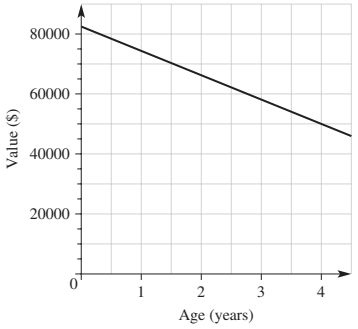
Exercise 9E

1a $3040 **b** $12 160
c $3040 **d** $9120
e $3040 **f** $6080
2a $6950 **b** $5250
c $3550 **d** $1850
3 $33 300
4a $16 000 **b** $4000
c $4000 **d** After 2 years
e $2000 **f** $14 000
5 $49 950
6 $21 400
7a 9 years **b** 14 years
8a After 8 years **b** After 9 years
9a $6400 **b** $19 200
10 $2400
11 $3000
12a $8000
b $32 000

c

Year	Current value	Depre-ciation	Depre-ciated value
1	$82 000	$8000	$74 000
2	$74 000	$8000	$66 000
3	$66 000	$8000	$58 000
4	$58 000	$8000	$50 000

d



Review 9

Multiple-choice

1 B **2** B **3** C **4** B **5** D
6 D **7** D **8** D **9** C **10** A

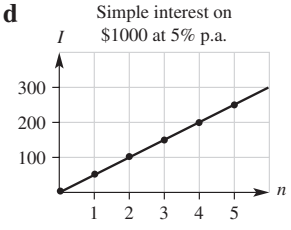
Short-answer

1 $96 000 **2** $2600 **3** $18
4 7% p.a.
5 2 years 3 months
6 16.7% **7** $4.68
8a $I = 50n$

b

n	0	1	2	3	4	5
I	0	50	100	150	200	250

c, d



e $275
9a $7.22 **b** $4.66
c $13.49 **d** 7.98
10 $110.16
11a $28.60 **b** $314.60
12a $2322 **b** $1376

Extended-response

13 $139.22
14a $280 **b** 31.4%
15a $12 000 **b** $48 191.55
c $36 191.55

Chapter 10

Exercise 10A

1a

Class	Class centre	f	$f \times x$
25–29	27	25	675
30–34	32	21	672
35–39	37	35	1295
40–44	42	21	882
45–49	47	26	1222
50–54	52	23	1196
55–59	57	32	1824
		183	7766

b 183 people **c** 42
2 64.9
3a

Class	Class centre	f	$f \times x$
2000–2004	2002	9	18 018
2005–2009	2007	15	30 105
2010–2014	2012	18	36 216
2015–2019	2017	10	20 170
2020–2024	2022	8	16 176
2025–2029	2027	6	12 162
2030–2034	2032	3	6 096
2035–2039	2037	1	2 037
		70	140 980

b 70 vehicles
c 2014

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4a 41 **b** 48
c Median
5a Mean = 17, median = 16, mode = 16
b All measures increased by 5.
c Mean = 22, median = 21, mode = 21
6 14
7a 7.6
b 12
8a 20 **b** 19.5
c

Age	Frequency	Cum. freq.
18	2	2
19	5	7
20	6	13
21	2	15

d 19
9a 10 **b** 7
c

Drinks	Frequency
7	5
8	7
9	4
10	2

d 8 **e** No
10 64.4 km
11a 0 **b** 50 **c** 250 **d** 25
12a–c

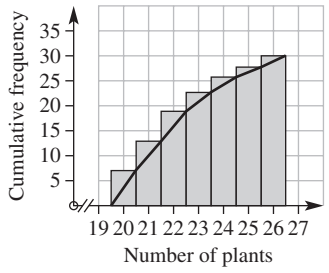
Class	x	f	$f \times x$
65–69	67	4	268
70–74	72	6	432
75–79	77	7	539
80–84	82	11	902
85–89	87	6	522
90–94	92	6	552
		40	3215

d 40 pieces of data
e 80.4
f 79.9

13a

Plants	Frequency	Cum. freq.
20	7	7
21	6	13
22	6	19
23	4	23
24	3	26
25	2	28
26	2	30

b



c Median is about 22

Exercise 10B

1a 28 **b** 11 **c** 16
d 21 **e** 32 **f** 10
2a 5 **b** 8 **c** 5 **d** 8
3a 10 **b** 6.5 **c** 10.5 **d** 4
4a 11 **b** 10 **c** 5
d 27 **e** 21 **f** 9
5a 110 **b** 122
c 185 **d** 63
6a 2.4 **b** 49.9 **c** 0 **d** 10.2
7a 3.6 **b** 3.8 **c** 3.4 **d** 44.9
8a 3.92 **b** 7.10
9a 2.3 **b** 1.7 **c** 2.0
10a 18.4 **b** 18.7
11a 120 **b** 41.48 **c** 70
12 Mean will be increased and the sample standard deviation will decrease.
13a Molly: $\overline{x} = 12.1$ and $\sigma_x = 4.7$
b Harry: $\overline{x} = 14.0$ and $\sigma_x = 3.0$
c Harry's results were more consistent as he has a lower standard deviation.
14a Charlie is more consistent as he has a lower interquartile range.
b Thomas is most likely to make the higher score as his median is 2 less than Charlie's but he has an interquartile range that is 5 higher.
15 Data test is the better result as Amber's result is more than one standard deviation above the class mean. Algebra test is less than one standard deviation above the mean.
16a 16.08 **b** 1.4
c Standard deviation measures the spread of data about the mean.

Exercise 10C

1a 34 **b** 48 **c** 16
d 133 **e** 53 **f** 0
2a 31 **b** 56
c 42 **d** 48
e 6 **f** Outlier 31
3a Yes, 1 **b** Yes, 69 **c** Yes, 20
d No **e** No **f** Yes, 0
4a No **b** Yes, 36 **c** No
d No **e** No **f** Yes, 59
5a 27 **b** 37 **c** 10 **d** 76
6 96 is not an outlier.
Upper limit $= Q_3 + 1.5 \times \text{IQR}$
$= 69 + 1.5 \times 20$
$= 99$
7 31 is an outlier
Upper limit $= Q_3 + 1.5 \times \text{IQR}$
$= 21 + 1.5 \times 5.5$
$= 29.25$
8a 16 **b** 5 **c** 20
d The outlier is the highest score and directly affects the range. It has a smaller effect on the interquartile range.
9a Mean = 20, median 13
b Mean is drastically reduced from 20 to 12.375.
Median is unaltered at 13.

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10a i Mean = 28.9, median = 26, mode = 26

ii Mean = 41.3, median = 34, mode = 27 & 34

iii Mean = 83.3, median = 88, mode = 88

iv Mean = 61.3, median = 65, mode = 62

b i Mean = 24.2, median = 24.5, mode = 26

ii Mean = 32, median = 31.5, mode = 27 & 34

iii Mean = 90.2, median = 89.5, mode = 88

iv Mean = 66.4, median = 66.5, mode = 62

11a Mean = 21.4, median = 18

b Mean = 17.1, median = 18
Removing the outlier has affected the mean.

12a Yes. Outlier is \$990
Upper limit = $Q_3 + 1.5 \times IQR$
$= 670 + 1.5 \times 170$
= \$925

b The dataset is translated by 50 in the positive direction but the standard deviation remains the same.

13a 1.3 **b** 1 **c** 0

d Mean = 1.1, median = 1, mode = 0

Exercise 10D

1a Not smooth, not symmetrical, unimodal

b Smooth, not symmetrical, unimodal

c Smooth, symmetrical, multimodal

d Smooth, symmetrical, unimodal

e Smooth, not symmetrical, unimodal

f Not smooth, not symmetrical, unimodal

g Smooth, not symmetrical, unimodal

h Smooth, not symmetrical, bimodal

2a Positively skewed

b No skew

c Negatively skewed

d No skew

e Negatively skewed

f Positively skewed

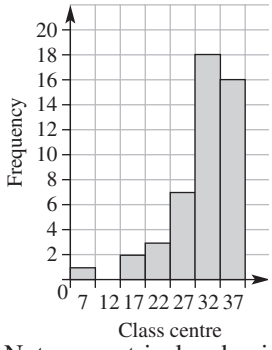
g Negatively skewed

h No skew

3a Mean = 31.1, median = 32, modal class = 30–34

b Mean has increased to 31.7 and the median has stayed at 32.

c



d Not symmetrical and unimodal

e Negatively skewed

4a Mean = 84.1, median = 94.5, mode = 95

b True but misleading. Lily's result is above the mean but below the median and mode. The mean has been affected by the outlier (11) as measure of the centre.

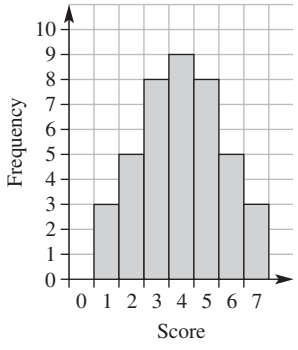
5a i 4 **ii** 4

iii 4 **iv** 1.7

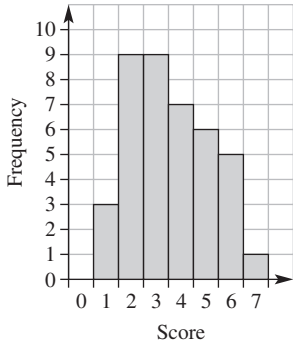
b i 3 **ii** 3.6

iii 2 and 3 **iv** 1.6

c



d



e Sample A is symmetrical and unimodal. Sample B is not symmetrical and bimodal

f Sample A is not skewed. Sample B is positively skewed.

Exercise 10E

1a 18

b Mean = 8.7, median = 9

c Mean = 9.2, median = 9

d Range = 2, IQR = 2

e Range = 3, IQR = 1

f 1.1

g 0.9

h Class B. Measures of the location are higher and measures of spread are smaller (ignore outlier).

i Both classes have performed well, with the measures of location close to 9 out of a possible 10. Class B is more consistent, with only one student scoring less than 8.

2a Mean = 22.6, median = 24

b Mean = 23.4, median = 25

c Range = 28.0, IQR = 14.0

d Range = 26.0, IQR = 14.0

e 7.8

f 7.9

g The two sets of data have similar measures for location and measures for spread. Number of females retiring is slightly higher on average than the number of males (compare mean and median).

3 Computer application

4a Sydney:
Mean = 7.7, median = 7.4
Melbourne:
Mean = 8.7, median = 6

b Median is the better measure. Mean in Melbourne has been distorted by an outlier (25.2).

c Sydney:
Range = 5.9, IQR = 2.25
Melbourne:
Range = 23.2, IQR = 8.7

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d Interquartile range is the better measure. Range in Melbourne has been distorted by an outlier (25.2).

e Area of the Sydney suburbs is more than the area of the Melbourne suburbs. There is a greater spread in the area of the suburbs in Melbourne compared to the area of the suburbs in Sydney.

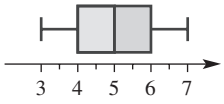
Exercise 10F

1a 20 **b** 100 **c** 60
d 35 **e** 90

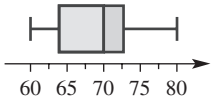
2a Min = 44, Q_1 = 48.5, med = 53.5, Q_3 = 58, max = 61

b Min = 8, Q_1 = 12, med = 20.5, Q_3 = 30, max = 42

3a

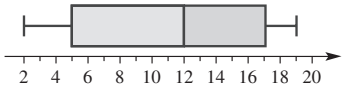


b



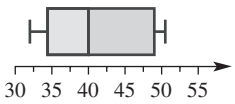
4a i 2 **ii** 19 **iii** 17
iv 5 **v** 17 **vi** 12

b



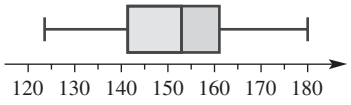
5a i 32 **ii** 51 **iii** 19
iv 34.5 **v** 48 **vi** 40

b



6a 123 **b** 180 **c** 154
d 141.5 **e** 161.5

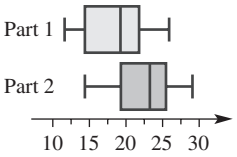
f



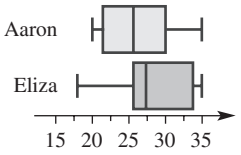
g Data is not skewed, has a median of 154 and an interquartile range of 20.

7a 25 **b** 13 **c** 21 **d** 16
e 14 **f** 22.5 **g** 5.5 **h** 10.5

8



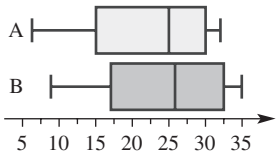
9



10a Min = 6, Q_1 = 15, med = 25, Q_3 = 30, max = 32

b Min = 9, Q_1 = 17, med = 26, Q_3 = 32, max = 35

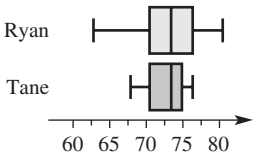
c



d 15 **e** 15

11a 63 **b** 74 **c** 77
d 74 **e** 71 **f** 77

g

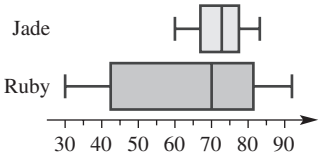


h Ryan: Range = 18, IQR = 6
Tane: Range = 9, IQR = 4.5

i Tane is the better student even though they have the same median. Tane's results are more consistent (less spread).

12a 30 **b** 94 **c** 73.5
d 67 **e** 77.5

f



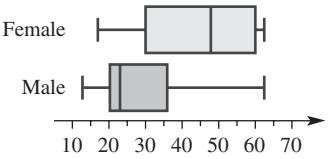
g Ruby's data has a greater upper extreme and a smaller lower extreme than Jade's data. This results in a larger range and interquartile range. Jade's data has a higher median than Ruby's.

h Jade is the more popular politician. Her data has a larger measure of centre (median) and the spread is smaller (interquartile range).

13a Min = 18, Q_1 = 30, med = 47.5, Q_3 = 61, max = 63

b Min = 12, Q_1 = 20, med = 23.5, Q_3 = 37, max = 63

c



d Female: Range = 45, IQR = 31
Male: Range = 51, IQR = 17

e Female data has a greater lower extreme but has the same upper extreme as the male data. In addition, the female data has a larger interquartile range but a smaller range. Female data has a higher median than the male data.

f Age of the females is higher than the age of the males attending the conference. There is a larger spread in the ages of the females attending the conference.

Review 10

Multiple-choice

1 C **2** D **3** C **4** B
5 A **6** B **7** A

Short-answer

1a 15 **b** 4 **c** 8

2a

Score (x)	f	$f \times x$
41	3	123
42	5	210
43	9	387
44	9	396
45	8	360
46	8	368
47	5	235
48	4	192
Total	51	2271

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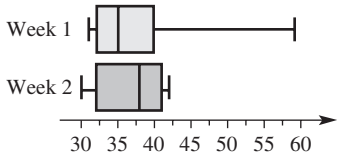
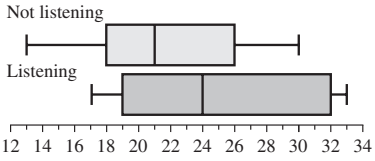
b 51
c See part a
d 44.5
3 68%
4a 4 **b** 5.5 **c** 5
d 27 **e** 21 **f** 9
5a 10 **b** 5.5 **c** 9.5 **d** 4
6a Brand A: Mean = 75 hours, $S_x = 13.8$ hours
Brand B: Mean = 76 hours, $S_x = 25.0$ hours
b Brand A has a more consistent lifetime as the standard deviation is smaller than for Brand B.
7a


b 35 **c** 38 **d** 8 **e** 9
f Week 1 is positively skewed.
8a 19 **b** 15 **c** 5
d Positively skewed
e Negatively skewed
f Upper limit = $Q_3 + 1.5 \times \text{IQR}$
$= 13 + 1.5 \times 10 = 28$
As 20 < 28, 20 is not an outlier.

Extended-response

9a


b The statement is not reasonable. The 'not listening' group not only has a lower minimum and maximum than the 'listening' group, every other summary statistic is lower as well. Although the range of the 'not listening' group is slightly higher, its smaller interquartile range indicates that the scores are more consistent. Significantly, the maximum score of the 'not listening' group is significantly lower that the upper quartile of the 'listening' group.

Chapter 11

Exercise 11A

1a 150 min **b** 120 s **c** 480 h
d 2400 s **e** 108 h **f** 600 min
g 12 h **h** 800 min **i** 4 d
j 45 d **k** 6.5 h **l** 13 min
2a Minute **b** Minute
c Year **d** Week
e Second **f** Hour
3a 3600 s **b** 86400 s
c 31536000 s
4a 9:30 p.m. **b** 6:00 a.m.
c 7:00 p.m. **d** 5:00 p.m.
e 4:30 p.m. **f** 1:45 a.m.
5a 6:09 a.m. **b** 10:09 a.m.
c 9:29 p.m. **d** 11:43 a.m.
e 6:53 p.m. **f** 1:09 a.m.
g 5:05 p.m. **h** 12:00 noon
6a 4 h 38 min **b** 5 h 50 min
c 9 h **d** 13 h 31 min
e 18 h 24 min **f** 13 h 8 min
7a 8 h 25 min **b** 3 h 12 min
c 1 h 12 min **d** 3 h 50 min
e 3 h 16 min **f** 2 h 50 min
8a 3 h **b** 7 h 10 min
c 1 h 3 min **d** 1 h 30 min
e 10 h 30 min **f** 11 h 2 min
9a 3:00 p.m.
b 12:00 a.m. (midnight)
c 8:00 a.m. **d** 1:10 p.m.
e 5:56 p.m. **f** 7:13 p.m.
10a 8 h 32 min **b** 1 h 25 min 20 s
c 1h 28 min
11a 02:00 **b** 22:30 **c** 20:41
d 23:12 **e** 01:59 **f** 00:00
g 21:09 **h** 16:39 **i** 05:15
12a 3:18 a.m. **b** 10:40 a.m.
c Midday or 12:00 noon
d 5:56 p.m. **e** 9:11 a.m.
f 11:00 p.m. **g** 6:45 p.m.
h 3:20 a.m. **i** 8:50 p.m.
13a 12:29 **b** 18:48 **c** 04:10
14a 09:25 **b** 08:36 **c** 20:10
15 4 h 20 min **16** 1 h 47 min
17 19:14
18

	Program (12-h)	Time settings (24-h)
a	6:00 a.m. to 7:38 a.m.	06:00 to 07:38
b	10:25 a.m. to 12:35 p.m.	10:25 to 12:35
c	3:20 p.m. to 4:54 p.m.	15:20 to 16:54
d	6:30 p.m. to 8:05 p.m.	18:30 to 20:05
e	7:50 a.m. to 10:00 a.m.	07:50 to 10:00
f	12:45 p.m. to 2:10 p.m.	12:45 to 14:10
g	0:20 a.m. to 3:49 p.m.	00:20 to 15:49
h	11:10 p.m. to 2:07 a.m.	23:10 to 02:07

19a 3 h 32 min **b** 15:34
c 18:07 **d** 07:38
20a 4:32 p.m. **b** 2 h 55 min
c 1 h 40 min **d** 3 h 40 min
21a 17 h **b** 32%

Exercise 11B

1a i 23:11 **ii** 23:19
iii 23:22
b i 6 min **ii** 8 min
iii 13 min
c 24 min **d** 15 min **e** 8
f 22:45 **g** 23:03
2a Sydney, Circular Quay, Wharf 4
b Mosman, Taronga Zoo Wharf
c 11:12 **d** 12 min
3a 04:18 **b** 5 **c** 05:40
d 06:32 **e** 51 min
4a 40 min **b** 44 min **c** 8:15
d 17:52 **e** 17 min
5a 3:10 p.m.
b Departs from Sydney and arrives at Phuket.
c 5 h 10 min

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6a 5:35 p.m.
b Departs from Sydney and arrives at Singapore.
c 5 h 20 min
7a Departs from Tokyo and arrives at Singapore.
b 6 h 40 min **c** 6 h 40 min
d 6 h 5 min
e Departs Tokyo and arrives at Sydney.
f 11 h 35 min
8a Departs from Sydney and arrives at London.
b 7 h 55 min **c** 5 h 10 min
d 1 h **e** 13 h 5 min
9a 11 **b** 5 h 35 min
c 23:28 **d** 3 h 55 min
e 3 h **f** 09:43
10 Investigation

Exercise 11C

1a (60°N, 0°) **b** (0°, 70°E)
c North Pole 90°N and South Pole 90°S
2a Lima **b** 62° **c** 81°
3a Teheran **b** 76° **c** 123°
4a (15°N, 136°E) **b** (55°N, 136°E)
c (35°N, 116°E) **d** (35°N, 156°E)
5a (4°N, 20°E) **b** (84°N, 20°E)
c (44°N, 20°W) **d** (44°N, 60°E)
6a Latitude of 48°N **b** 11°E
a Munich **b** (48°N, 82°E)
7a **i** Kuala Lumpur
ii Rio de Janeiro
iii Kuala Lumpur
iv Sofia
b **i** Adelaide
ii Montevideo
iii Birmingham
iv Athens
c Athens, Birmingham, Cairo, Sofia, Warsaw and Kuala Lumpur
d Birmingham, Montevideo and Rio de Janeiro
e Adelaide and Montevideo; Birmingham and Warsaw
f Athens and Sofia
g Birmingham and Warsaw
h Adelaide and Montevideo
8a Hobart and Port Moresby are located on the 147°E meridian of longitude. Hobart (42°S) is further south than Port Moresby (9°S).
b (25.5°S, 147°E)
9a 23° **b** 182° or 178°
c (3°S, 76°W) **d** (6°S, 174°W)

Exercise 11D

1

WST (+8)	CST (+9.5)	EST (+10)
1:00 p.m.	2:30 p.m.	3:00 p.m.
5:00 a.m.	6:30 a.m.	7:00 a.m.
6:20 p.m.	7:50 p.m.	8:20 p.m.
4:19 a.m.	5:49 a.m.	6:19 a.m.
10:30 p.m.	Midnight	12:30 a.m.
7:12 a.m.	8:42 a.m.	9:12 a.m.

2a 4:30 p.m. **b** 6:00 p.m.
c 6:00 p.m. **d** 6:30 p.m.
3a 8:10 a.m. **b** 10:10 a.m.
c 9:40 a.m. **d** 10:10 a.m.
4a 4:00 p.m. **b** 8:00 a.m.
c 10:00 p.m. **d** 10:00 a.m.
5a 4:00 a.m. Sunday
b 4:00 p.m. Saturday
c 11:00 a.m. Sunday
d 12:00 Midnight
6a 6 h **b** 4 h **c** 12 h
d 8 h **e** 9 h **f** 5 h
g 11 h **h** 12 h
7a 10:00 a.m. **b** 3:00 a.m.
c 7:00 a.m. **d** 11:00 p.m.
8a 8:00 p.m. Monday
b 8:00 a.m. Tuesday
c 11:00 p.m. Monday
d 3:30 p.m. Tuesday
9a 4:00 p.m. **b** 2:00 p.m.
c 3:30 p.m. **d** 4:30 p.m.
10 4:00 p.m. Sunday 31 October
11 7:00 a.m. Friday
12a 18 h **b** 19 h
13 6 h
14a 6:30 p.m. **b** 7:30 a.m.
15a 11:45 p.m.
b 12:00 (midnight)

Exercise 11E

1a 9 h
b 12:00 a.m. (midnight)
2a 1:00 p.m. on Friday 30 March
b 7:00 a.m. on Saturday 31 March
3a 18 h
b 6:00 p.m. Sunday
4 7:00 a.m. Sunday
5 8:00 p.m. 25 November
6 7:00 p.m. 25 December
7 10:00 p.m. Sunday
8 6:00 a.m. Friday
9 12:45 p.m.
10a 5:15 a.m. **b** 2:40 p.m.
c 4:30 p.m.
11a 6:00 a.m. on 5 June
b 1:00 p.m. on 5 June
12a 4 p.m. Monday 11 October
b 4 a.m. 27 October
13a 4:00 a.m. Tuesday 17 May
b 4:00 a.m. Saturday 21 May

Review 11

Multiple-choice

1 C **2** B **3** D **4** D
5 A **6** A **7** C **8** A

Short-answer

1a 7200 s **b** 84 h **c** 420 min
d 10.5 h **e** 400 min **f** 5 days
g 43 days **h** 12.5 h **i** 16 min
2a 15:10 **b** 20:35 **c** 06:00
3a 7 h **b** 6 h 10 min **c** 50 min
4 10:26 am
5a 10:51 am **b** 11:11 am
6a 0 h 13 min **b** 0 h 15 min
c 29 min
7a Salvador **b** 38°
c 106°
8a 63° **b** 274° or 86°
c 24°S, 100°W **d** 37°S, 124°E
9 1:00 a.m.

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10a 30 min **b** 8:30 a.m.
11a 2 h **b** 3:00 p.m.
12a 7:00 a.m. **b** 12:00 noon

Extended-response

13a 6:00 p.m. Saturday
b 11:00 p.m. Saturday
14 4.5 h
15 6:30 a.m. Saturday 9 July

Chapter 12

Exercise 12A

1a 21 Sep **b** 7034 8751 3763
c $349.22 **d** 10 Oct
e $31.74 **f** $281.11
g $316.68 **h** $32.54
i 10.28%
2a 77.69 MJ **b** 248.92 MJ
c $2.50 **d** 68.79%
e Greenhouse gas emissions have decreased from 1.4 tonnes to 0.5 tonnes.
3a 4 **b** kWh
c 9 kWh **d** July
4a $80.14 **b** $189.04
c $309.18
5 $2288
6a 12 kW **b** $1.78
7a $15.54 **b** $125.87
c $202.80 **d** $459.20
8a $30.44 **b** 66.67%

Exercise 12B

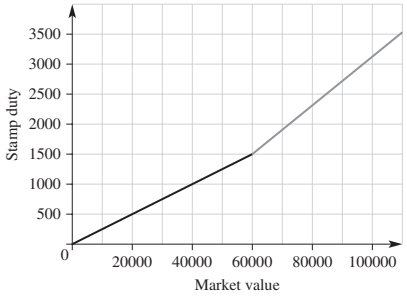
1a 16% **b** 25%
c 10% **d** 12.3%
2a $29 500 **b** $13 870
c $14 660 **d** $53 050
3a $5350 **b** $2886
c $15 785 **d** $26 080
4a $24 600 **b** $26 880
c $56 160 **d** $126 880
5a $12 280 **b** $18 000
c $30 280
6a $76 800 **b** $84 300
c $42 300
7a $6400 **b** 160 weeks
8a $2700 **b** $30 240
c $32 940 **d** $5940
9a $87 000 **b** $110 880
c $23 880
10a $26 000 **b** $71 240
c $23 400
11a $239 **b** $404
c $321 **d** $101
12 $9026
13 Investigation

Exercise 12C

1a $432 **b** $1183 **c** $528
d $396 **e** $1175
2a 40%
b 40 and 60 age groups
c 10%
d Charge a higher excess.
3a $474.60 **b** $1200
4 $1225.75
5a $602 400 000 **b** $99 600 000
c $260 400 000 **d** $237 600 000
6a Penrith **b** Yes
c $770 **d** $540
e Brand A **f** $994.75
g $685.00 **h** $658.33
i $879 **j** $556.20

Exercise 12D

1a $978 **b** $795 **c** $1350
d $414 **e** $1068 **f** $717
g $1735 **h** $4025 **i** $5105
2a $330 **b** $745 **c** $1140
d $1983 **e** $1563 **f** $2508
g $1435 **h** $2172 **i** $2606
3a $355 **b** $1160
c $1071 **d** $861
4a $600 **b** $2100
c $1350 **d** $2600
e $10 000 **f** $64 000
5



a $750 **b** $1500
c $2300 **d** $20 000
e $40 000 **f** $97 500

Exercise 12E

1a 3 **b** 15 L
c $180 **d** $0.012
2a $10 250 **b** $15 525 **c** $21 700
3a 9.55 **b** 8.82 **c** 5.06
d 8.21 **e** 9.32 **f** 3.5
4a 271.6 L **b** 73.2 L **c** 65.8 L
d 30.9 L **e** 98.2 L **f** 342.9 L
g 404.9 L
5a 444 km **b** 37.2 L **c** $50.23
6 480 km
7 Once. Distance required is 772 km. Distance travelled on one tank of petrol is 625 km.
8a 35 L **b** 55 L **c** 45 L
d 50 L **e** 70 km/h **f** 5 L
9a $1320 **b** $1280
c $2600 **d** $3104
10a $102.75 **b** $105.74 **c** $2.99
11a 33.6 L **b** 46.2 L
c Tyler 1747.2 L, Oscar 2402.4 L
d $2446.08 **e** $1897.90
f 3 **g** Investigation
12 Investigation

Exercise 12F

1a $1536 **b** $2550
c $18 720 **d** $23 400
2 $13 473.00
3a $61 460.37 **b** $59 769.77
c $1690.60
4a $1200 **b** $500
c 17.6% **d** 20 weeks
5 Computer application
6a $1500 **b** $2520
c $1020 **d** $1000
7a $51400 **b** $329
8a $23 350
b Yes. Extra amount is $16 800 after subtracting rent.
9a $9050 **b** $1020
c $3680
d Hayley requires another $560 for her trip.

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10a

Income		Expenses	
Job	$74	Sport	$24
Allow	$30	Movies	$22
		School	$16
		Food	$20
		Balance	$22
	$104		$104

b $22

11a $526 **b** $658 total

c $77 total

d

Income		Expenses	
Job	$1896	Mortgage	$526
		Groceries	$360
		Entertain	$120
		Medical	$18
		Car	$160
		Electricity	$20
		Telephone	$14
		Rates	$43
		Balance	$635
	$1896		$1896

Review 12

Multiple-choice

1 C **2** D **3** D **4** A
5 A **6** C **7** A **8** B

Short-answer

1 $27 103.50

2 16.7%

3a $61 800 **b** $22 800

4a $765 **b** $1000

5 $832

6a 17.453 L **b** $27.92

c 33.104 L **d** $44.69

7a 525 km **b** 8 L/100 km

c 7.6 L/100 km

8a $63 315.25 **b** $61 562.83

c $1752.42

9 $1526

Extended-response

10a $40 800 **b** $24 480 **c** $17 360

Practice Paper 2

1 C **2** B **3** A **4** D **5** A
6 C **7** C **8** C **9** A **10** D
11 C **12** D **13** D **14** A **15** C

16a i $367.20 **ii** $112.80

iii 23.5%

b i 48°

ii 260° or 100°

iii (15°S, 82°W)

c $924

d i 17.1125 L **ii** $23.44

e $7142.86

f $1940

g $c = 0.40x + 100$

17a i -3 **ii** 2

iii $y = -3x + 2$ **iv** $\frac{2}{3}$

b 68%

c The sample has not been picked at random. The five students selected are not in school uniform and may not be representative of the school. It is possible that these students do not like the school uniform.

d i 8.7 **ii** 9 **iii** 10

iv 3 **v** 2

e 4.5%

18a i 45 kg **ii** 76 kg

iii 19 kg **iv** 300

b i Qualitative categorical data

ii Quantitative discrete data

iii Quantitative continuous data

c 23.8% **d** $4000

e i 7:30 a.m.

ii 3:30 p.m.

Exercise: Appendix

1a 2 **b** $\frac{1}{4}$ **c** 4 **d** $\frac{1}{3}$ **e** 1.5 **f** $\frac{1}{3}$

2a 2 **b** 3 **c** 15 **d** 30

3a $x = 6$ **b** $x = 16$

c $x = 4.2$ **d** $x = 29.4$

4a $a = 4, b = 6$ **b** $x = 10, y = 8$

c $p = 7.2, q = 15$ **d** $c = 16, d = 5$

5 Height of the tree is 7 m

6 Height of the building is 12 m

7 The tower is 40 m in height.

8 Block of units is 14 m in height.

9 Flagpole has a height of 5 m.

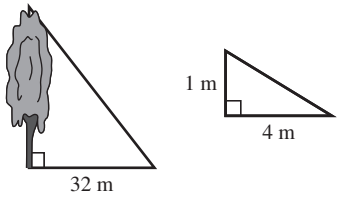
10 Lucas is 2 m tall.

11a Height of the lighthouse is 12 m.

b Height of the wall is 4 m

12a $x = 1.5, y = 2$ **b** $z = 21$

13a



b Three angles of one triangle equal three angles of the second triangle.

c Height of the tree is 8 m.

14 Feet of the ladder are 187 cm apart.

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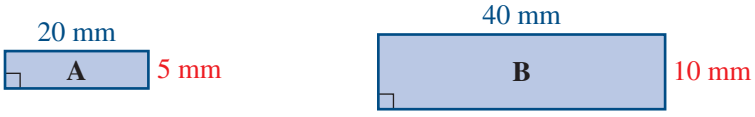
Appendix: Scale factors in similar figures

Similar figures

The pictures of the three pieces of cake are similar. Similar figures are exactly the same shape but they are different sizes.

When we enlarge or reduce a shape by a scale factor, the original and the image are similar. Similar shapes have:

- corresponding angles of equal size
- corresponding sides of different size, but in the same ratio or proportion.



For example, the above rectangles are similar. All the angles are 90°. The corresponding sides are in the same ratio $\left(\frac{10}{5} = \frac{40}{20} = 2\right)$. The measurements in rectangle B are twice the measurements in rectangle A. Rectangle B has been enlarged by a scale factor of 2.

SIMILAR FIGURES

- Similar figures are exactly the same shape but are a different size.
- Corresponding (or matching) angles of similar figures are equal.
- Corresponding (or matching) sides of similar figures are in the same ratio.
- Scale factor is the amount the first shape is enlarged or reduced to get the second shape.

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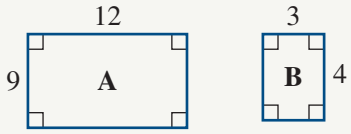
Example 1: Calculating the scale factor

What is the scale factor for these two similar rectangles?



SOLUTION

1. Look carefully at the similar figures.
2. Match the corresponding sides (9 matches with 3 and 12 matches with 4).
3. Write the matching sides as a fraction (measurement in rectangle B divided by the matching measurement in rectangle A).
4. Simplify the fraction by dividing both terms by the same number. This fraction is the scale factor.

Rectangle B is smaller than rectangle A and is rotated.

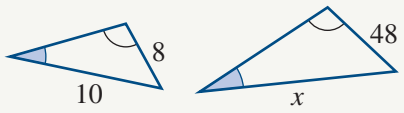
$$\text{Scale factor} = \frac{3}{9} \text{ or } \frac{4}{12}$$

$$= \frac{1}{3} \text{ (or } 1:3)$$

$\therefore$ Rectangle B is $\frac{1}{3}$ the size of rectangle A.

Example 2: Using a scale factor

What is the length of the unknown side in the following pair of similar triangles?



SOLUTION

1. Match the corresponding sides (8 matches with 48).
2. Write the matching sides as a fraction (second shape to the first shape). This fraction is the scale factor.
3. Match the corresponding side for x (side marked with a 10).
4. Calculate x by multiplying 10 by 6 (scale factor is 6).

$$\text{Scale factor} = \frac{48}{6}$$

$$= 6 \text{ (or } 1:3)$$

$$x = 10 \times 6$$

$$= 60$$

Finding an unknown side in similar figures

The lengths of the corresponding (or matching) sides in similar figures are in the same ratio or proportion. This property is used to calculate the length of an unknown side

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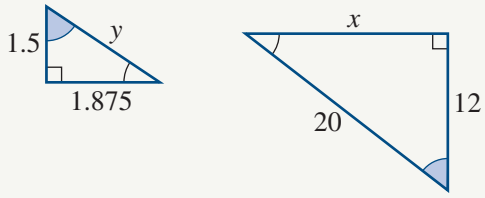
FINDING AN UNKNOWN SIDE IN SIMILAR FIGURES

1. Determine the corresponding or matching sides in the similar figures
2. Write an equation using two fractions formed from matching sides. A measurement from the second shape divided by a matching measurement from the first shape. This is the scale factor.
3. Solve the equation.
4. Check that the answer is reasonable and units are correct.

Example 3: Finding an unknown side in similar figures

What are the lengths of the unknown sides in the following pair of similar triangles?



SOLUTION

1. Determine the matching sides in the similar triangles (x and 1.875, 12 and 1.5, 20 and y).
2. Write an equation using matching sides involving x. Use x (second shape) and 1.875 (first shape) equal to 12 (second shape) and 1.5 (first shape).
3. Solve the equation.
4. Write an equation using matching sides involving y. Use 20 (second shape) and y (first shape) equal to 12 (second shape) and 1.5 (first shape).
5. Solve the equation.
6. Check that the answers are reasonable.

$$\frac{x}{1.875} = \frac{12}{1.5} \left[\frac{\text{Second shape}}{\text{First shape}}\right]$$

$$x = \frac{12}{1.5} \times 1.875$$

$$= 15$$

$$\frac{20}{y} = \frac{12}{1.5} \left[\frac{\text{Second shape}}{\text{First shape}}\right]$$

$$12y = 20 \times 1.5$$

$$y = \frac{20 \times 1.5}{12}$$

$$= 2.5$$

Solving a worded problem using similar figures

Similar figures are used to solve problems that require the length of an object. For example, we can calculate the height of a tree without physically measuring the height. A similar figure is drawn using a metre rule and the length of its shadow is measured (see example below). This is a very useful concept.

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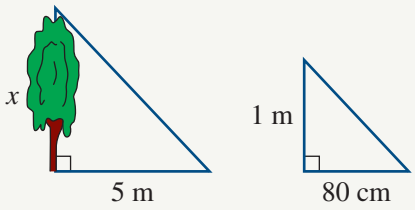
SOLVING A WORDED PROBLEM USING SIMILAR FIGURES

1. Read the question and underline the key terms.
2. Draw similar figures and label the information from the question.
3. Use a pronumeral (x) to represent an unknown side.
4. Write an equation using two fractions formed from matching sides.
5. Solve the equation.
6. Check that the answer is reasonable and units are correct.

Example 4: Solving a problem involving similar figures

A tree casts a shadow on the ground that is 5 m in length. At the same time a one metre ruler casts a shadow that is 80 cm in length. What is the height of the tree? Answer in metres correct to two decimal place.



SOLUTION

1. Divide 80 cm by 100 to convert it to metres.
2. Determine the matching sides in the similar triangles (1 and x, 0.8 and 5).
3. Write an equation using matching sides involving x. Use 1 (second shape) and x (first shape) equal to 0.82 (second shape) and 5 (first shape).
4. Solve the equation.
5. Write the answer correct to two decimal places.
6. Write answer in words.

Let x be the height of the tree.

$80 \text{ cm} = 0.8 \text{ m}$

$$\frac{1}{x} = \frac{0.82}{5} \quad \left[\frac{\text{Second shape}}{\text{First shape}}\right]$$

$$0.8x = 5$$

$$x = \frac{5}{0.8}$$

$$= 6.25 \text{ m}$$

$\therefore$ The height of the tree is 6.25 m

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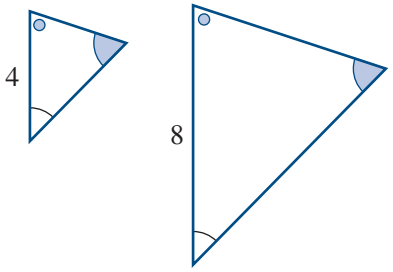
Exercise: Appendix

LEVEL 1

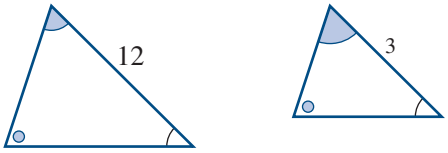
Example 1

1 What is the scale factor for the following pairs of similar figures?

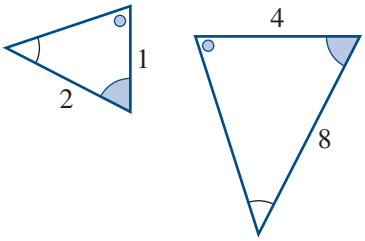
a



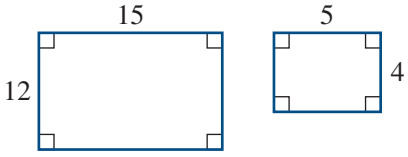
b



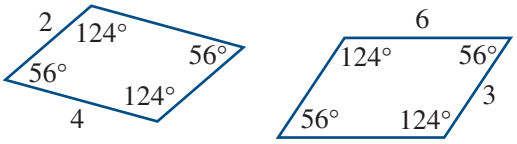
c



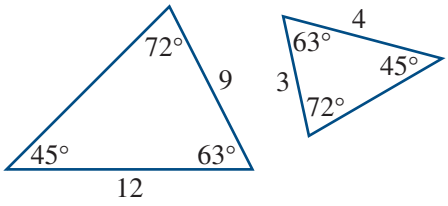
d



e



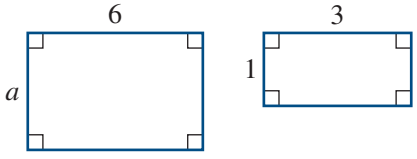
f



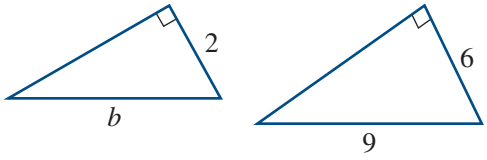
Example 2

2 Use the scale factor to find the length of the unknown side in the following pairs of similar figures.

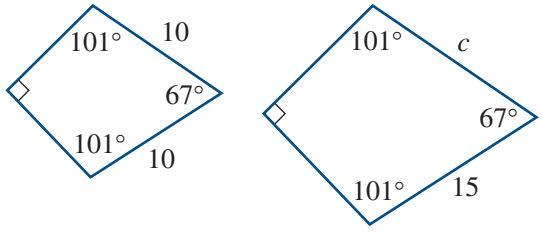
a



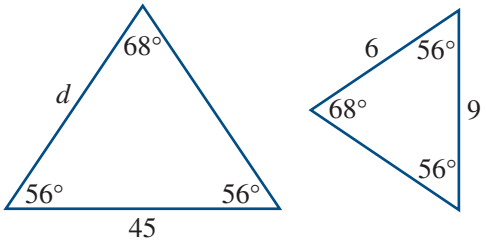
b



c



d



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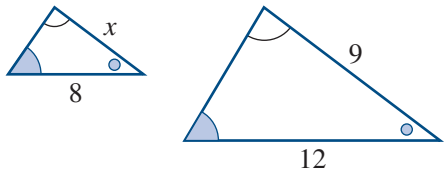
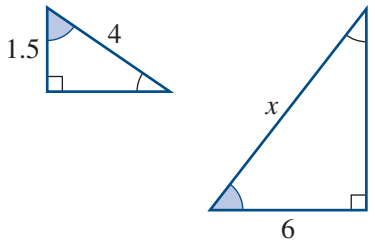
3 Find the length of the pronumeral for the following pairs of similar triangles. All measurements are in centimetres.

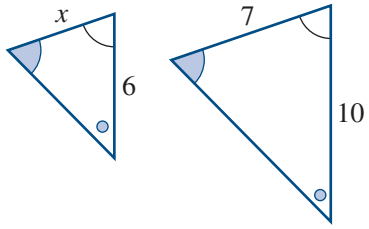
a



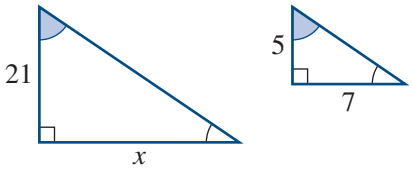
b



c

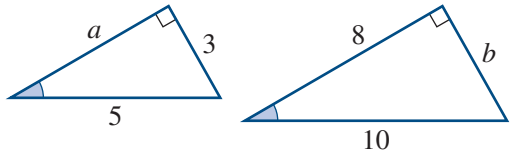


d

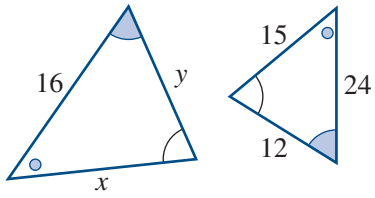


Example 3 **4** Find the length of the unknown sides for the following pairs of similar triangles. All measurements are in centimetres.

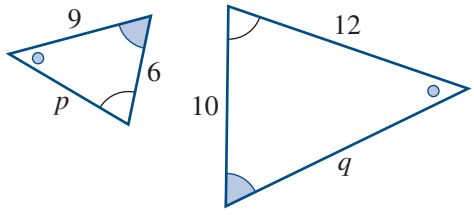
a



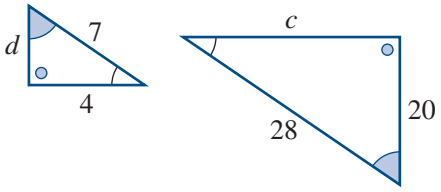
b



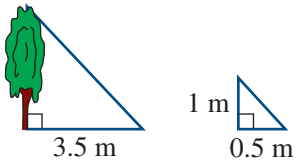
c



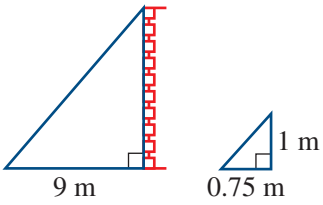
d



Example 4 **5** A tree casts a shadow 3.5 m long. At the same time a one metre ruler casts a shadow 0.5 m long. What is the height of the tree? Answer correct to one decimal place.



6 A building casts a shadow 9 m long. At the same time a one metre ruler casts a shadow 0.75 m long. What is the height of the building? Answer correct to the nearest metre.



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LEVEL 2

7 A stick 2 m high throws a shadow 1.5 m long. At the same time a tower throws a shadow 30 m long. How high is the tower? Answer correct to one decimal place.

8 David is 1.8 m in height. When he is standing out in the sun his shadow is 2.4 m long. At the same time a block of units casts a shadow of 18 m. How tall is the block of units? Answer correct to the nearest metre.

9 Jessica found that her shadow was 3 m long when the shadow of a flagpole was 9 m long. If Jessica's height is 1.5 m, what is the height of the flagpole? Answer to the nearest metre.

10 Lucas and his younger brother Nathan are standing side by side. Nathan is 1.4 m tall and casts a shadow 3.5 m long. How tall is Lucas if his shadow is 5 m long? Answer correct to one decimal place.

11 Wollongong's lighthouse casts a shadow of length 15 m. At the same time a one metre beach umbrella casts a shadow whose length is 1.25 m.

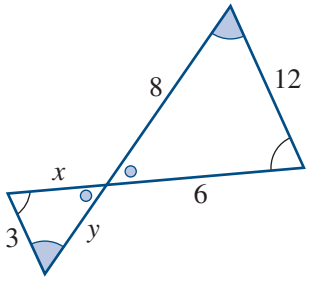
a What is the height of the lighthouse? Answer correct to nearest metre.

b A nearby wall casts a shadow 5 m long. Calculate the height of this wall to the nearest metre.

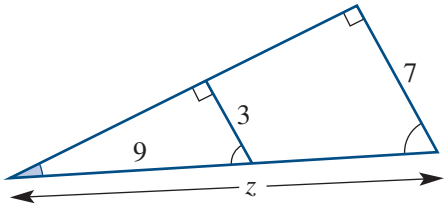
LEVEL 3

12 Find the value of the pronumeral in the following diagrams.

a



b



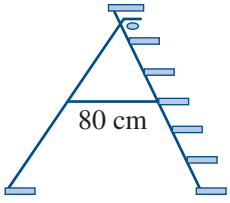
13 A tree and a 1 m vertical stick cast their shadows at a particular time of the day. The shadow of the tree is 32 m and the shadow of the vertical stick is 4 m.

a Draw two triangles to represent the above information.

b Give a reason why the two triangles are similar.

c Find the height of the tree correct to the nearest metre.

14 A 3.5 m ladder has a support 80 cm long placed 1.5 m from the top of the ladder. How far apart are the feet of the ladder? Answer in centimetres correct to the nearest whole number.



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Appendix B: The 'capture–recapture' technique

The 'capture–recapture' technique is used to estimate population size. It has applications in ecology, for example, to estimate animal populations. This method can also be used, for example, to estimate the number of people needing particular services. The technique works by capturing a random sample of the population. This first sample is tagged and then released. At a later time a second sample is captured, and the recaptured or tagged members recorded. The first sample captured divided by the population is approximately proportional to the number recaptured divided by the number in the second sample captured.

THE CAPTURE–RECAPTURE TECHNIQUE

1. Use p to represent the population size.
2. First sample: Write the number captured as the numerator and p as the denominator of a fraction. This represents the sample as a proportion of the population.
3. Second sample: Write the number recaptured or tagged as the numerator and the number captured in the second sample as the denominator.
4. Equate the two fractions in steps 2 and 3.
5. Solve the equation for p.

Example 1: Using the capture–recapture technique

Adam is a biologist who is estimating the population of fish in a lake. He randomly captures and tags 120 fish. Two months later he samples 80 fish and finds he has recaptured 5 fish he had tagged. Estimate the number of fish in the lake.

SOLUTION

1. Use p to represent the population size.
2. Write the number captured (120) as the numerator and p as the denominator of a fraction.
3. Write the number recaptured or tagged (5) as the numerator and the number captured in the second sample (80) as the denominator.
4. Equate the two fractions in steps 2 and 3 $\left(\dfrac{120}{p} = \dfrac{5}{80}\right)$.
5. Solve the equation for p.
6. Write the answer in words.

$$\frac{120}{p} = \frac{5}{80}$$

$$80p \times \frac{120}{p} = \frac{5}{80} \times 80p$$

$$9600 = 5p$$

$$p = \frac{9600}{5}$$

$$= 1920$$

There are approximately 1920 fish in the lake.

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Exercise: Appendix B

LEVEL 1

1 Emily is a biologist who is estimating the population of crabs in a lake. She randomly captures and tags 64 crabs. Three months later she samples 50 crabs and finds she has recaptured 8 crabs, as they are tagged. What is Emily's estimate for the number of crabs in the lake?

2 Patrick is a wildlife officer concerned about the number of rabbits in the park. He catches and tags 80 rabbits and then releases them. A week later he catches 70 rabbits in the park and finds that 10 of them are tagged. Estimate for Patrick the number of rabbits in the park.

3 Lucas has a large pond with goldfish. He would like to estimate the number of goldfish in the pond. He randomly captures and tags 24 goldfish. Five days later he samples 20 goldfish and finds he has recaptured 4 goldfish, as they are tagged. Estimate for Lucas the number of goldfish in the pond.

4 The capture–recapture technique was used to estimate the number of people with a physical disability in the local community. A doctor saw 12 patients with a physical disability on a given day. Two months later he saw 42 patients with 3 of the same patients who had a physical disability. Estimate the number of people with a physical disability in the local community.

5 Marjana would like to estimate the number of parrots in the national park. She randomly captures and marks 36 parrots. After three weeks, a second sample of 35 parrots is caught. Marjana finds she has recaptured 6 parrots. Estimate the number of parrots in the national park.

6 The number of grasshoppers appears to be increasing in a particular town. James needed an estimate of the grasshopper population. He spent the first day collecting 28 grasshoppers and tagging each insect. On the next day James collected another 28 grasshoppers and found that he had recaptured 4 grasshoppers. What is James's estimate for the number of grasshoppers in the town?

7 To determine the number of frogs in a pond, 20 frogs are caught and tagged and then released into the pond. Two days later 10 frogs are caught of which 4 have tags. Estimate the number of frogs in the pond.

8 The capture–recapture technique was used to estimate the number of people with Alzheimer's disease in the local community. A doctor saw 54 patients with Alzheimer's disease in one week. One month later she saw 51 patients including 27 of the same patients who had Alzheimer's disease. Estimate the number of people in the local community with Alzheimer's disease.

9 David is a fisherman who is estimating the number of lobsters in a lake. He captures and tags 28 lobsters. Two months later he captures 15 lobsters and finds he has recaptured 5 lobsters that are tagged. What is David's estimate for the number of lobsters in the lake?

10 The local community is concerned about the number of cane toads. Samantha needed an estimate of the cane toad population. She spent the first day collecting 48 cane toads and tagging each animal. On the next day Samantha collected 44 cane toads and found that she

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had recaptured 6 cane toads. What is Samantha's estimate for the number of cane toads in the community?

11 Katerina would like to estimate the number of dingoes in the national park. She randomly captures and marks 22 dingoes. After four weeks, a second sample of 20 dingoes is caught. Katerina finds she has recaptured 4 dingoes. Estimate the number of dingoes in the national park.

12 Tim is a scientist who is concerned about the number of bats in the park. He catches and tags 60 bats and then releases them. Two weeks later he catches 85 bats in the park and finds that 25 of them are tagged. Estimate the number of bats in the park for Tim.

LEVEL 2

13 Simulate the capture–recapture technique using a normal pack of playing cards.

- **a** Shuffle the deck of cards.
- **b** First sample: Select 10 cards at random. Record the cards selected.
- **c** Replace the 10 cards into the deck. Shuffle the deck of cards.
- **d** Second sample: Select 10 cards at random. Determine the number of cards that have been selected again or 'recaptured'.
- **e** Use the capture–recapture technique to estimate the number of playing cards.
- **f** Repeat the simulation by selecting 20 cards for each sample.
- **g** Repeat the simulation by selecting 30 cards for each sample.

14 Simulate the capture–recapture technique using coloured discs.

- **a** Mix the coloured discs in a container.
- **b** First sample: Select 10 discs at random. Tag or mark each disc.
- **c** Replace these 10 discs into the container. Mix the discs.
- **d** Second sample: Select 10 discs at random. Determine the discs that have been selected again or 'recaptured'.
- **e** Use the capture–recapture technique to estimate the number of coloured discs.
- **f** Repeat the simulation by selecting 20 discs for each sample.
- **g** Repeat the simulation by selecting 30 discs for each sample.

LEVEL 3

15 A ranger is concerned by the number of foxes in a wildlife park. She needs to estimate the population of the foxes, but she knows that the foxes that have been trapped and tagged learn to avoid the trap in the future. If the capture–recapture technique was used, would this lead to population estimate that is higher or lower than the actual population? Explain your answer.

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Answers

Exercise: Appendix B

1 400
2 560
3 120
4 168
5 210
6 196
7 50
8 102
9 84
10 352
11 110
12 204
15 Population estimate is higher than the actual population. The number recaptured that have a tag is smaller and has been affected by the learning of the foxes.

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