CambridgeMATHS NSW

STAGE 4

SECOND EDITION

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INCLUDES INTERACTIVE TEXTBOOK POWERED BY CAMBRIDGE HOTMATHS

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About the Authors

Stuart Palmer was born and educated in NSW. He is a high school mathematics teacher with more than 25 years' experience teaching students from all walks of life in a variety of schools. He has been a head of department in two schools and is now an educational consultant who conducts professional development workshops for teachers all over NSW and beyond. He also works with pre-service teachers at The University of Sydney.

Karen McDaid has more that 17 years' experience teaching mathematics in several primary and secondary schools, and as a lecturer teaching mathematics to primary preservice teachers at Western Sydney University. Karen was the Professional Teachers Association representative on the NSW Board Curriculum Committee during the development and consultation phases of both the Australian Curriculum and the NSW Mathematics K-10 Syllabus for the Australian Curriculum. She has been active on the executive committee of the Mathematical Association of NSW since 2003 and was the NSW Councillor on the board of the Australian Association of Mathematics Teachers from 2014 to 2018. Karen co-authored the CambridgeMATHS NSW GOLD books Years 7 to 10. Karen is currently Head of Mathematics K to 12 at Cluey Learning.

David Greenwood is the Head of Mathematics at Trinity Grammar School in Melbourne and has over 20 years' experience teaching mathematics from Years 7 to 12. He has run workshops within Australia and overseas regarding the implementation of the Australian Curriculum and the use of technology for the teaching of mathematics. He has written more than 30 mathematics titles and has a particular interest in the sequencing of curriculum content and working with the Australian Curriculum proficiency strands.

Bryn Humberstone graduated from University of Melbourne with an Honours degree in Pure Mathematics, and is currently teaching both junior and senior mathematics in Victoria. Bryn is particularly passionate about writing engaging mathematical investigations and effective assessment tasks for students with a variety of backgrounds and ability levels.

Justin Robinson is Head of Positive Education and a mathematics teacher at Geelong Grammar School. Prior to this, he spent 20 years teaching mathematics and was a key writer of in-house maths material. He has a keen interest in engaging all students through a wide variety of effective teaching methods and materials.

Jenny Goodman has worked for over 20 years in comprehensive State and selective high schools in NSW and has a keen interest in teaching students of differing ability levels. She was awarded the Jones medal for education at Sydney University and the Bourke prize for Mathematics. She has written for Cambridge NSW and was involved in the Spectrum and Spectrum Gold series.

Jennifer Vaughan has taught secondary mathematics for over 30 years in NSW, WA, Queensland and New Zealand and has tutored and lectured in mathematics at Queensland University of Technology. She is passionate about providing students of all ability levels with opportunities to understand and to have success in using mathematics. She has taught special needs students and has had extensive experience in developing resources that make mathematical concepts more accessible; hence, facilitating student confidence, achievement and an enjoyment of maths.

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Introduction and guide to this book

The **second edition** of this popular resource features a new interactive digital platform powered by Cambridge HOTmaths, together with improvements and updates to the textbook, and additional online resources such as video demonstrations of all the worked examples, Desmos-based interactives, carefully chosen HOTmaths resources including widgets and walkthroughs, and worked solutions for all exercises, with access controlled by the teacher. The Interactive Textbook also includes the ability for students to complete textbook work, including full working-out online, where they can self-assess their own work and alert teachers to particularly difficult questions. Teachers can see all student work, the questions that students have 'red-flagged', as well as a range of reports. As with the first edition, the complete resource is structured on detailed teaching programs for teaching the NSW Syllabus, now found in the Online Teaching Suite.

The chapter and section structure has been retained, and remains based on a logical teaching and learning sequence for the syllabus topic concerned, so that chapter sections can be used as ready-prepared lessons. Exercises have questions graded by level of difficulty and are grouped according to the **Working Mathematically components** of the NSW Syllabus, with enrichment questions at the end. Working programs for three ability levels (Building Progressing and Mastering) have been subtly embedded inside the exercises to facilitate the management of differentiated learning and reporting on students' achievement (see page X for more information on the Working Programs). In the second edition, the *Understanding* and *Fluency* components have been combined, as have Problem-Solving and Reasoning. This has allowed us to better order questions according to difficulty and better reflect the interrelated nature of the Working Mathematically components, as described in the NSW Syllabus.

Topics are aligned exactly to the NSW Syllabus, as indicated at the start of each chapter and in the teaching program, except for topics marked as:

- REVISION — prerequisite knowledge
- EXTENSION — goes beyond the Syllabus
- FRINGE — topics treated in a way that lies at the edge of the Syllabus requirements, but which provide variety and stimulus.

See the Stage 5 books for their additional curriculum linkage.

The parallel ***CambridgeMATHS Gold*** series for Years 7–10 provides resources for students working at Stages 3, 4, and 5.1. The two series have a content structure designed to make the teaching of mixed ability classes smoother.

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Guide to the working programs

It is not expected that any student would do every question in an exercise. The print and online versions contain working programs that are subtly embedded in every exercise. The suggested working programs provide three pathways through each book to allow differentiation for Building, Progressing and Mastering students.

Each exercise is structured in subsections that match the Working Mathematically strands, as well as Enrichment (Challenge).

The questions suggested for each pathway are listed in three columns at the top of each subsection:

- The left column (lightest-shaded colour) is the Building pathway
- The middle column (medium-shaded colour) is the Progressing pathway
- The right column (darkest-shaded colour) is the Mastering pathway.

	Building	Progressing	Mastering
UNDERSTANDING AND FLUENCY	1–3, 4, 5	3, 4–6	4–6
PROBLEM-SOLVING AND REASONING	7, 8, 11	8–12	8–13
ENRICHMENT	—	—	14

Gradients within exercises and question subgroups

The working programs make use of the gradients that have been seamlessly integrated into the exercises. A gradient runs through the overall structure of each exercise, where there is an increasing level of mathematical sophistication required in the Problem-solving and Reasoning group of questions than in the Understanding and Fluency group, and within each group the first few questions are easier than the last.

The right mix of questions

Questions in the working programs are selected to give the most appropriate mix of *types* of questions for each learning pathway. Students going through the Building pathway will likely need more practice at Understanding and Fluency but should also attempt the easier Problem-Solving and Reasoning questions.

Choosing a pathway

There are a variety of ways of determining the appropriate pathway for students through the course. Schools and individual teachers should follow the method that works for them if the chapter pre-tests can be used as a diagnostic tool.

For classes grouped according to ability, teachers may wish to set one of the Building, Progressing or Mastering pathways as the default setting for their entire class and then make individual alterations, depending on student need. For mixed-ability classes, teachers may wish to set a number of pathways within the one class, depending on previous performance and other factors.

The nomenclature used to list questions is as follows:

- 3, 4: complete all parts of questions 3 and 4
- 1–4: complete all parts of questions 1, 2, 3 and 4
- 10(½): complete half of the parts from question 10 (a, c, e … or b, d, f, …)
- 2–4(½): complete half of the parts of questions 2, 3 and 4
- 4(½), 5: complete half of the parts of question 4 and all parts of question 5
- —: do not complete any of the questions in this section.

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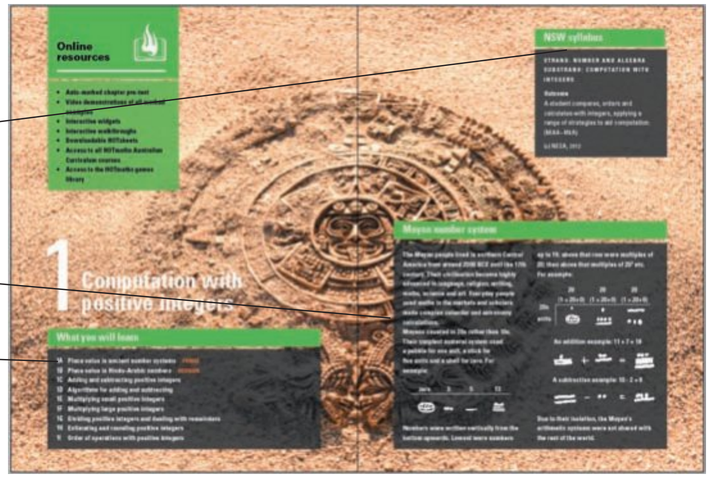
Guide to this book

Features:

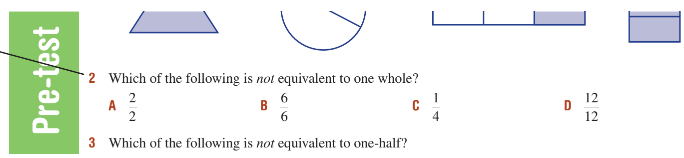
NSW Syllabus: strands, substrands and content outcomes for chapter (see teaching program for more detail)

Chapter introduction: use to set a context for students

What you will learn: an overview of chapter contents



Pre-test: establishes prior knowledge (also available as an auto-marked quiz in the Interactive Textbook as well as a printable worksheet)


Pre-test

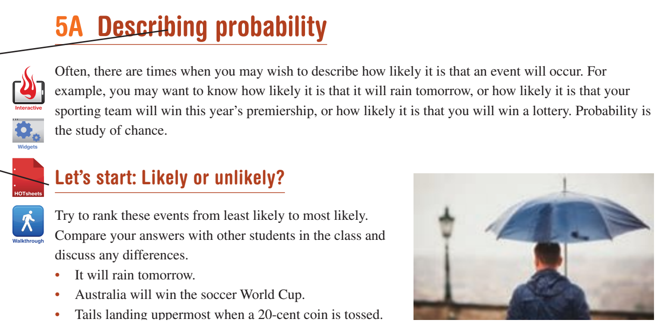
2 Which of the following is *not* equivalent to one whole?

A $\frac{2}{2}$ B $\frac{6}{6}$ C $\frac{1}{4}$ D $\frac{12}{12}$

3 Which of the following is *not* equivalent to one-half?

Topic introduction: use to relate the topic to mathematics in the wider world

Let's start: an activity (which can often be done in groups) to start the lesson


5A Describing probability

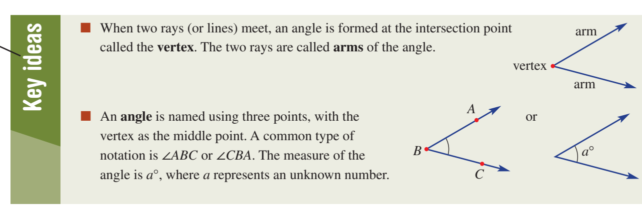
Often, there are times when you may wish to describe how likely it is that an event will occur. For example, you may want to know how likely it is that it will rain tomorrow, or how likely it is that your sporting team will win this year's premiership, or how likely it is that you will win a lottery. Probability is the study of chance.

Let's start: Likely or unlikely?

Try to rank these events from least likely to most likely. Compare your answers with other students in the class and discuss any differences.

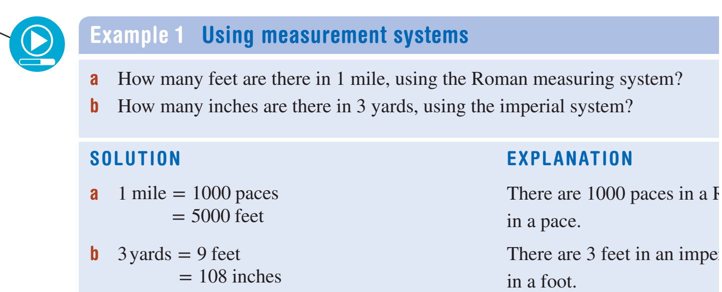
- It will rain tomorrow.
- Australia will win the soccer World Cup.
- Tails landing uppermost when a 20-cent coin is tossed.

Key ideas: summarises the knowledge and skills for the lesson


Key ideas

- When two rays (or lines) meet, an angle is formed at the intersection point called the **vertex**. The two rays are called **arms** of the angle.
- An **angle** is named using three points, with the vertex as the middle point. A common type of notation is $\angle ABC$ or $\angle CBA$. The measure of the angle is $a°$, where a represents an unknown number.

Examples: solutions with explanations and descriptive titles to aid searches. Video demonstrations of every example are included in the Interactive Textbook.


Example 1 Using measurement systems

a How many feet are there in 1 mile, using the Roman measuring system?
b How many inches are there in 3 yards, using the imperial system?

SOLUTION	EXPLANATION
a 1 mile = 1000 paces = 5000 feet	There are 1000 paces in a R in a pace.
b 3 yards = 9 feet = 108 inches	There are 3 feet in an impe in a foot.

Exercise questions categorised by the **working mathematically components** and **enrichment**

Example references link exercise questions to worked examples.

Exercise 10A FRINGE

UNDERSTANDING AND FLUENCY	1–8	4–9	5–9(½)

1 Complete these number sentences.
a Roman system

Example 16 4 Convert to the units shown in brackets.
a 2 t (kg) b 70 kg (g)
c 2.4 g (mg) d 2300 mg (g)
e 4620 mg (g) f 21 600 kg (t)

PROBLEM-SOLVING AND REASONING	10–12, 18	12–14, 18	15–19

10 Arrange these measurements from smallest to largest.

ENRICHMENT	—	—	20

Very long and short lengths

20 When 1 metre is divided into 1 million parts, each part is called a **micrometre** (μm). At the other end of the spectrum, a **light year** is used to describe large distances in space.
a State how many micrometres there are in:
i 1 m ii 1 cm
iii 1 mm iv 1 km

Investigations: inquiry-based activities

Puzzles and **challenges**

Investigation

The perfect billiard

When a billiard ball bounces c (with no side spin), we can ass at which it hits the wall (incon same as the angle at which it l (outgoing angle). This is simil reflects off a mirror.

Single bounce

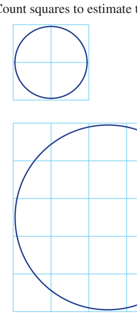
Use a ruler and protractor to d and then answer the questions.
a Find the outgoing angle if:
i the incoming angle is 3
ii the centre angle is 104°
b What geometrical reason d

Puzzles and challenges

1 Without measuring, state w
A

2 You have two sticks of len

3 Count squares to estimate t
a



b

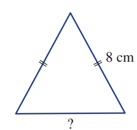
Chapter reviews with **multiple-choice, short-answer** and **extended-response questions**

Multiple-choice questions

1 Which of the following is a metric unit of capacity?
A cm B pace C digit D yard E litre

2 Shonali buys 300 cm of wire that costs \$2 per metre. How much does she pay for the wire?
A \$150 B \$600 C \$1.50 D \$3 E \$6

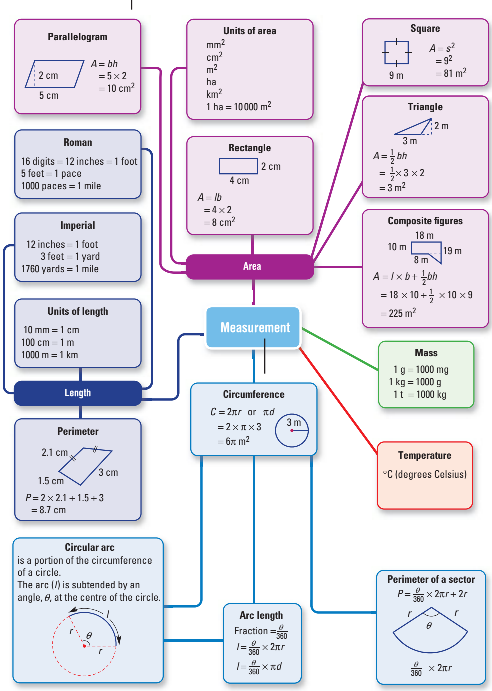
3 The triangle given has a perimeter of 20 cm. What is the missing base length?
A 6 cm B 8 cm C 4 cm
D 16 cm E 12 cm



4 The area of a rectangle with length 2 m and width 5 m is:
A 10 m^2 B 5 m^2 C 5 m D 5 m^3 E 10 m

5 A triangle has base length 3.2 cm and height 4 cm. What is its area?
A 25.6 cm^2 B 12.8 cm C 12.8 cm^2 D 6 cm E 6.4 cm^2

Chapter summary: mind map of key concepts & interconnections



Two **Semester reviews** per book

Semester review 1

Multiple-choice questions
1 Using numerals, thirty-five thousand, two hu
A 350 260 B 35 260 C 3
2 The place value of 8 in 2 581 093 is:
A 8 thousand B 80 thousand C 8
3 The remainder when 23 650 is divided by 4
A 0 B 4 C 1
4 18 − 3 × 4 + 5 simplifies to:
A 65 B 135 C 1
5 800 ÷ 5 × 4 is the same as:
A 160 × 4 B 800 ÷ 20 C 8

Short-answer questions
1 Write the following numbers using words.
a 1030 b 13 000
d 10 030 e 100 300

Textbooks also include:
- Complete **answers**
- **Index**

Overview of the digital resources

Interactive Textbook: general features

The **Interactive Textbook (ITB)** is an online HTML version of the print textbook powered by the HOTmaths platform, included with the print book or available separately. (A **downloadable PDF textbook** is also included for offline use). These are its features, including those enabled when the students' ITB accounts are linked to the teacher's **Online Teaching Suite (OTS)** account.

The features described below are illustrated in the screenshot below.

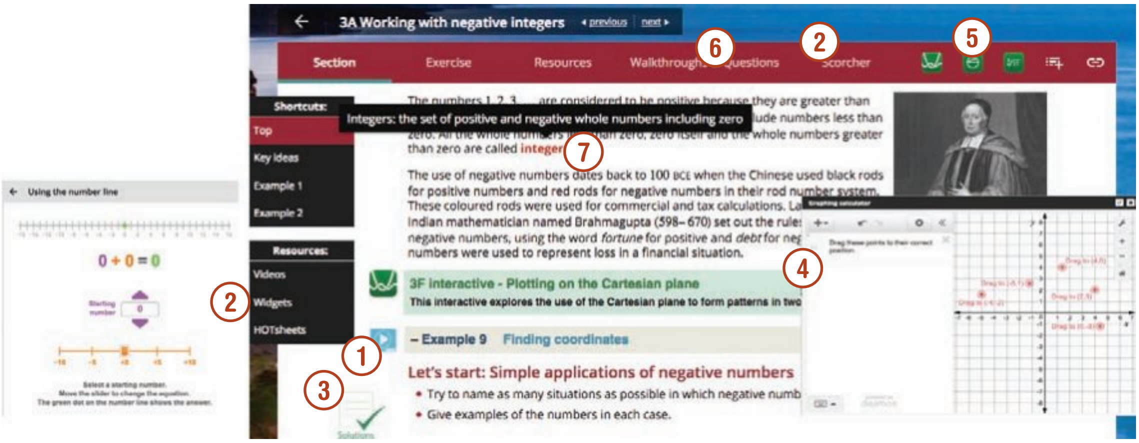
1. Every worked example is linked to a high-quality video demonstration, supporting both in-class learning and the 'flipped classroom'
2. Seamlessly blend with Cambridge HOTmaths, including hundreds of interactive widgets, walkthroughs and games and access to Scorcher
3. **Worked solutions** are included and can be enabled or disabled in the student accounts by the teacher
4. **Desmos interactives** based on embedded graphics calculator and geometry tool windows demonstrate key concepts and enable students to visualise the mathematics
5. The **Desmos scientific calculator** is also available for students to use (as well as the graphics calculator and geometry tools)
6. Auto-marked practice quizzes in each section with saved scores
7. **Definitions** pop up for key terms in the text, and access to the Hotmaths **dictionary**

Not shown but also included:

- Access to alternative HOTmaths lessons is included, including content from previous year levels.
- Auto-marked pre-tests and multiple-choice review questions in each chapter.

INTERACTIVE TEXTBOOK POWERED BY THE HOTmaths PLATFORM

Note: *HOTmaths platform features are updated regularly.*



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Interactive Textbook: Workspaces and self-assessment tools

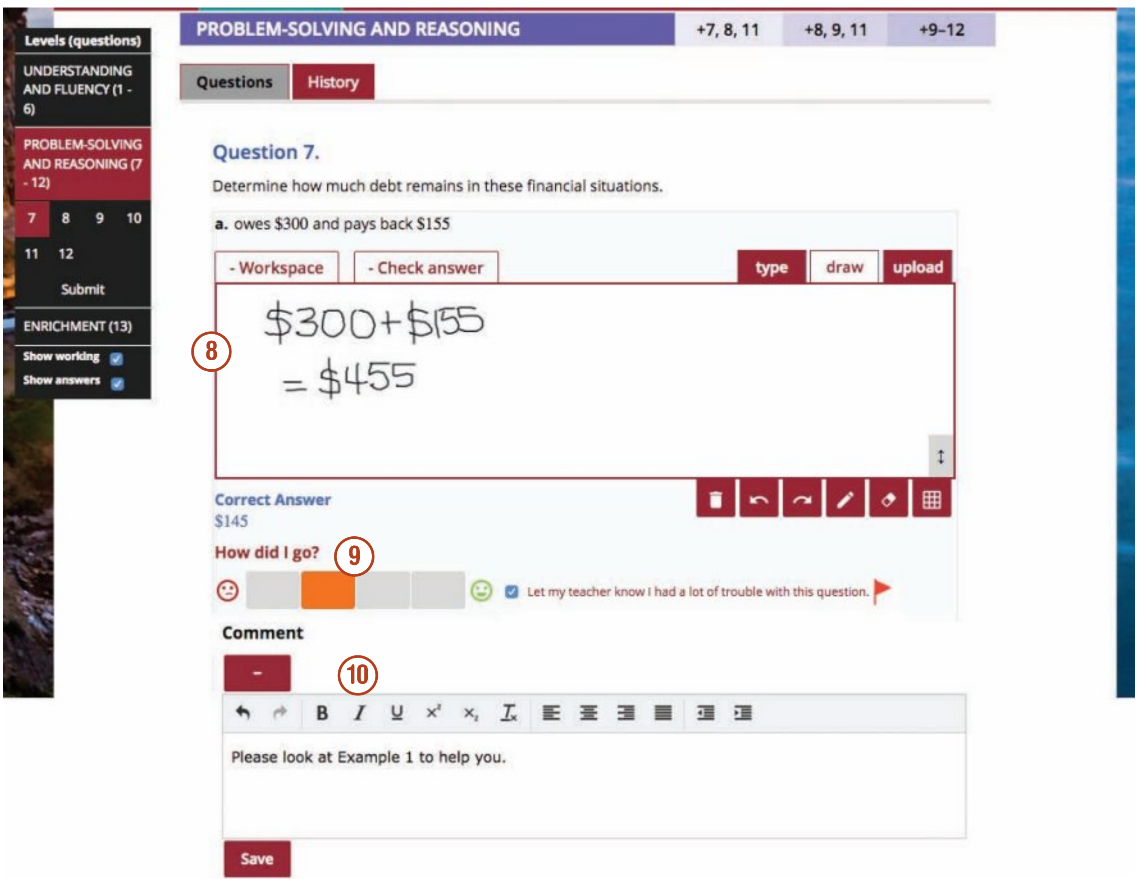
Almost every question in ***CambridgeMATHS NSW Second Edition*** can be completed and saved by students, including showing full working-out and students critically assessing their own work. This is done via the workspaces and self-assessment tools that are found below every question in the Interactive Textbook.

8 The new **Workspaces** enable students to enter working and answers online and to save them. Input is by typing, with the help of a symbol palette, handwriting and drawing on tablets, or by uploding images of writing or drawing.

9 The new **self-assessment tools** enable students to check answers including questions that have been red-flagged, and can rate their confidence level in their work, and alert teachers to questions the student has had particular trouble with. This self-assessment helps develop responsibility for learning and communicates progress and performance to the teacher.

10 Teachers can view the students' self-assessment individually or provide feedback. They can also view results by class.

WORKSPACES AND SELF-ASSESSMENT



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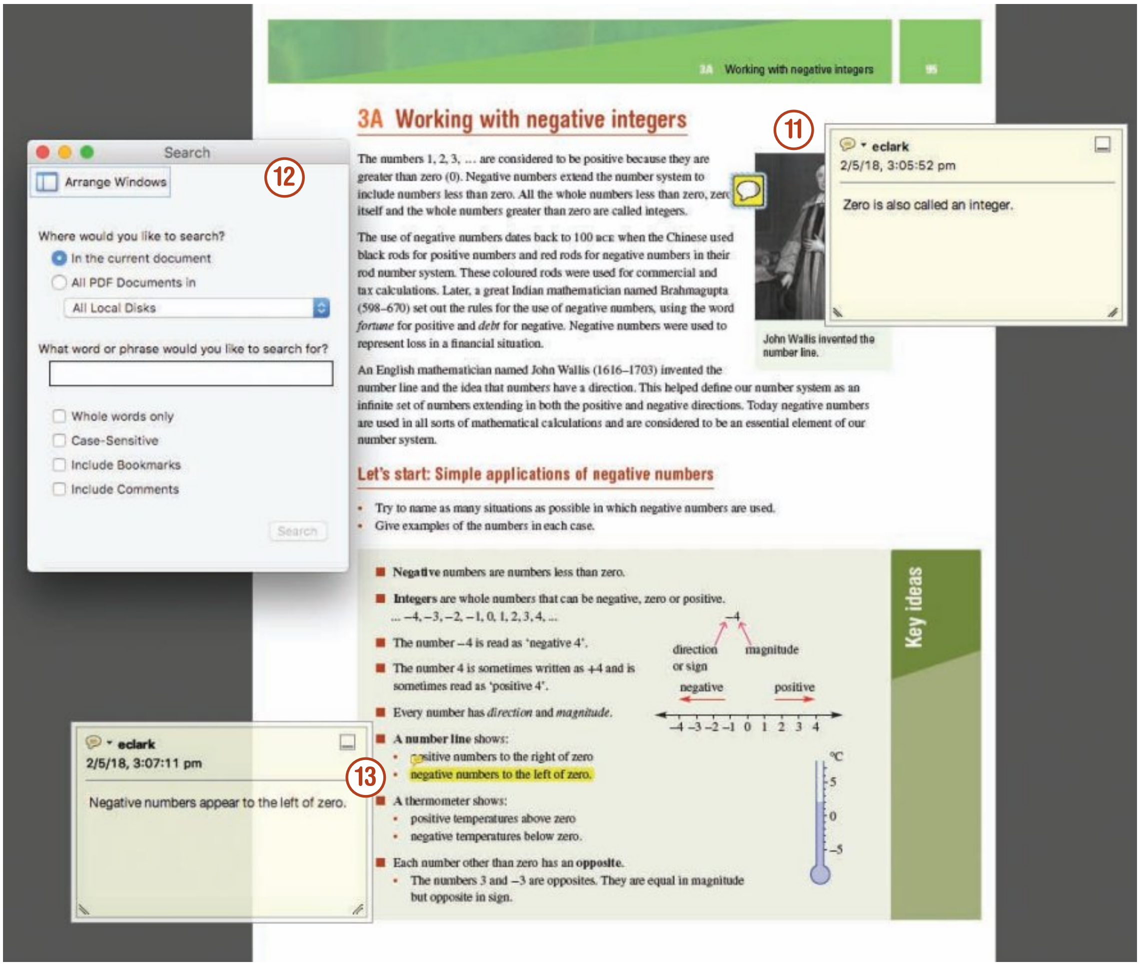
Downloadable PDF Textbook

The convenience of a downloadable PDF textbook has been retained for times when users cannot go online.

The features include:

11 PDF note-taking
12 PDF search features are enabled
13 highlighting functionality.

PDF TEXTBOOK



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Online Teaching Suite

The Online Teaching Suite is automatically enabled with a teacher account and is integrated with the teacher's copy of the Interactive Textbook. All the assets and resources are in one place for easy access. The features include:

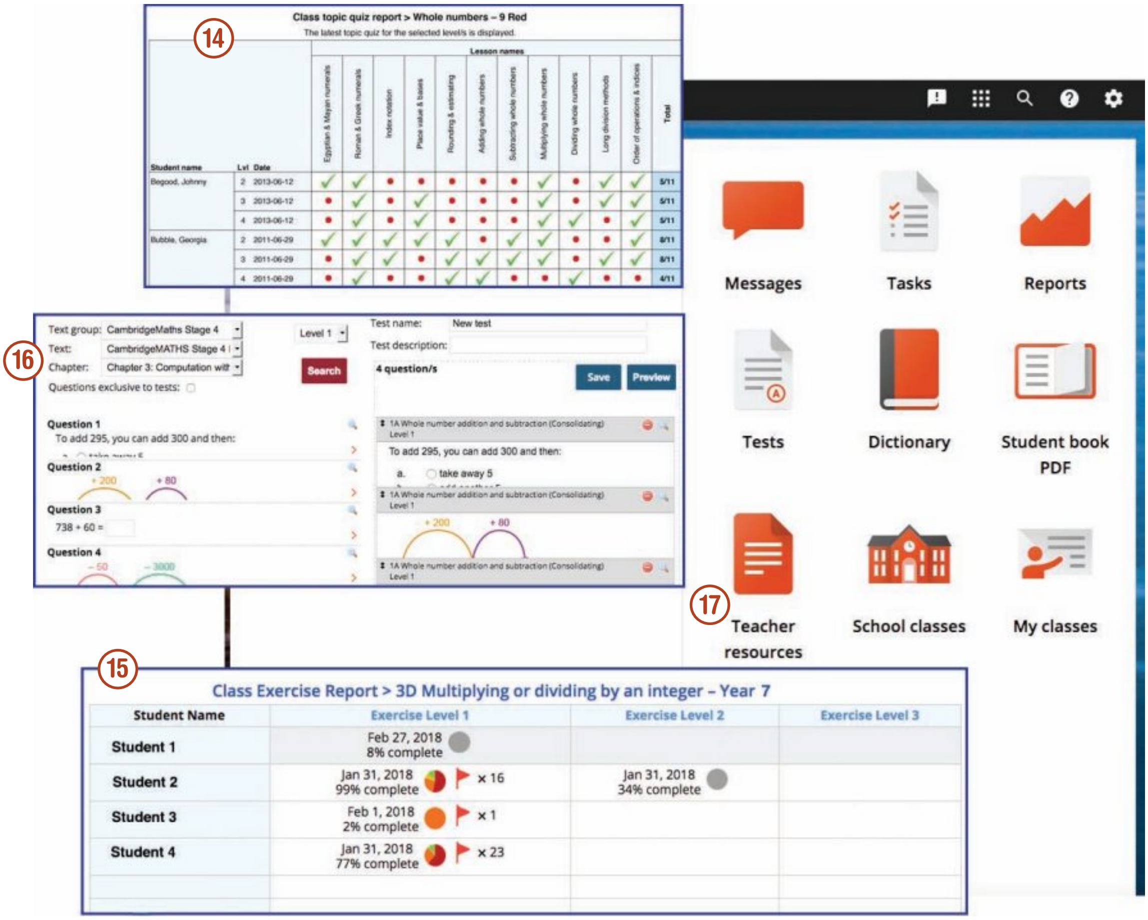
14 The HOTmaths learning management system with class and student analytics and reports, and communication tools
15 Teacher's view of a student's working and self-assessment, including multiple progress and completion reports viewable at both student and class level, as well as seeing the questions that a class has flagged as being difficult
16 A HOTmaths-style test generator
17 Chapter tests and worksheets

Not shown but also available:

- Editable teaching programs and curriculum grids.

ONLINE TEACHING SUITE POWERED BY THE HOTmaths PLATFORM

Note: *HOTmaths platform features are updated regularly.*



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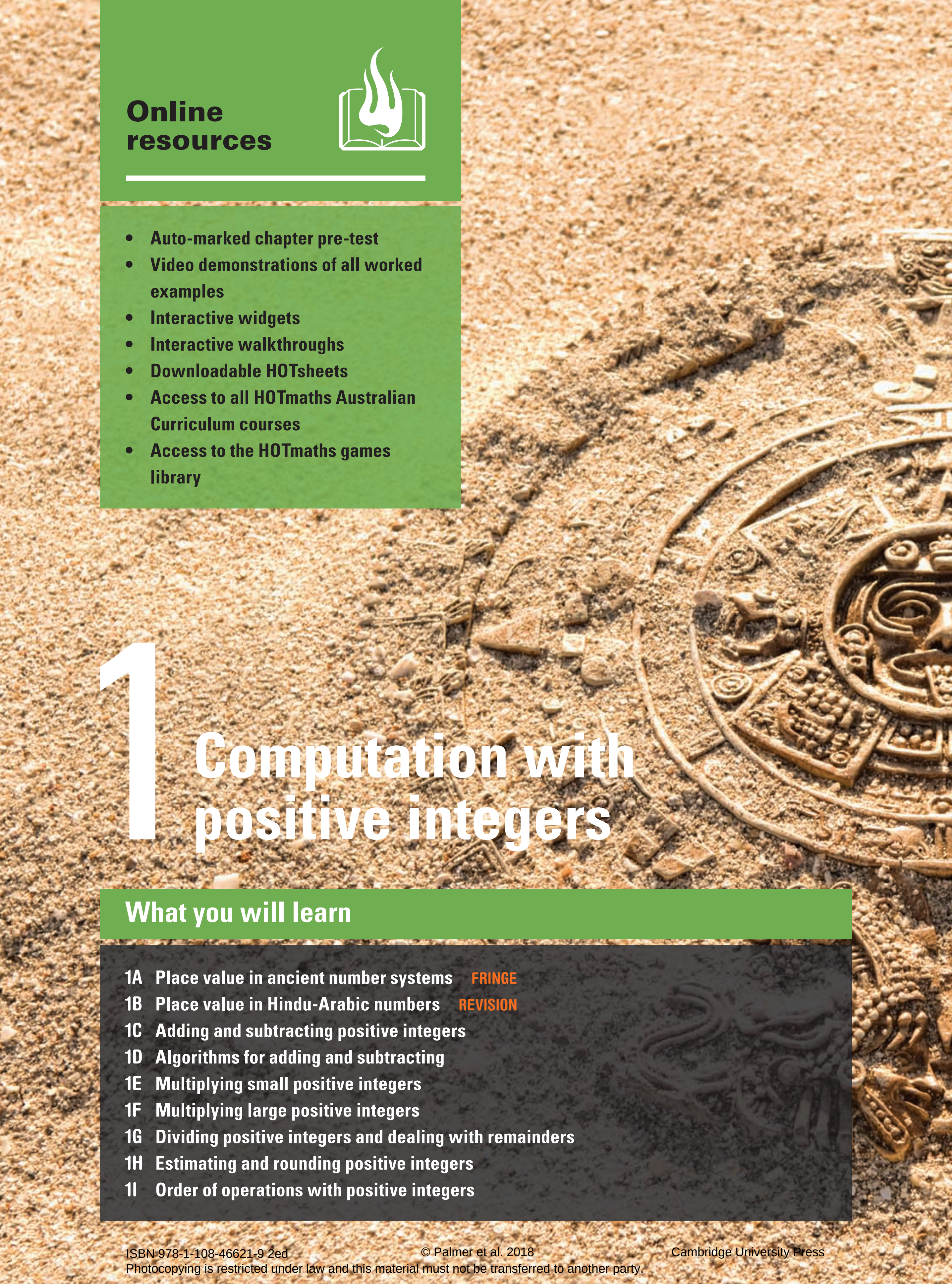


Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

1 Computation with positive integers

What you will learn

1A Place value in ancient number systems FRINGE
1B Place value in Hindu-Arabic numbers REVISION
1C Adding and subtracting positive integers
1D Algorithms for adding and subtracting
1E Multiplying small positive integers
1F Multiplying large positive integers
1G Dividing positive integers and dealing with remainders
1H Estimating and rounding positive integers
1I Order of operations with positive integers

NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRAND: COMPUTATION WITH INTEGERS

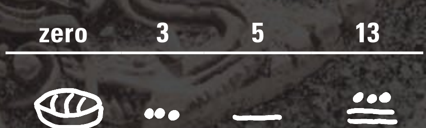
Outcome

A student compares, orders and calculates with integers, applying a range of strategies to aid computation. (MA4–4NA)

Mayan number system

The Mayan people lived in northern Central America from around 2000 BCE until the 17th century. Their civilisation became highly advanced in language, religion, writing, maths, science and art. Everyday people used maths in the markets and scholars made complex calendar and astronomy calculations.

Mayans counted in 20s rather than 10s. Their simplest numeral system used a pebble for one unit, a stick for five units and a shell for zero. For example:



Numbers were written vertically from the bottom upwards. Lowest were numbers up to 19; above that row were multiples of 20; then above that multiples of 20^2 etc. For example:

An addition example: 11 + 7 = 18

A subtraction example: 10 - 2 = 8

Due to their isolation, the Mayan's arithmetic systems were not shared with the rest of the world.

Pre-test

1 For each of the following, match the word with the symbol.

a	add	**A**	−
b	subtract	**B**	÷
c	multiply	**C**	+
d	divide	**D**	×

2 Write each of the following as a number.
- **a** fifty-seven
- **b** one hundred and sixteen
- **c** two thousand and forty-four
- **d** eleven thousand and two

3 Answer which number is:
- **a** 2 more than 11
- **b** 5 less than 42
- **c** 1 less than 1000
- **d** 3 more than 7997
- **e** double 13
- **f** half of 56

4 Complete these patterns, showing the next seven numbers.
- **a** 7, 14, 21, 28, 35, __, __, __, __, __, __, __.
- **b** 9, 18, 27, 36, 45, __, __, __, __, __, __, __.
- **c** 11, 22, 33, 44, 55, __, __, __, __, __, __, __.

5 Find the value of:

a $48 \div 24$ **b** $48 \div 48$ **c** $48 \div 16$ **d** $48 \div 4$

6 Give the result for each of these computations.

a $3 + 11$	**b** $14 + 9$	**c** $99 + 20$	**d** $138 + 12$
e $20 - 11$	**f** $41 - 9$	**g** $96 - 17$	**h** $101 - 22$
i 5×6	**j** 9×7	**k** 12×12	**l** 8×11
m $10 \div 2$	**n** $30 \div 15$	**o** $66 \div 6$	**p** $132 \div 12$

7 Arrange these numbers from smallest to largest.
- **a** 37, 73, 58, 59, 62, 159
- **b** 301, 103, 31, 310, 130
- **c** 29 143, 24 913, 13 429, 24 319, 24 931

8 What is the remainder when each number is divided by 3?

a 12	**b** 10	**c** 37	**d** 62
e 130	**f** 263	**g** 1001	**h** 2704

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1A Place value in ancient number systems FRINGE

Interactive Widgets HOTsheets Walkthrough

Throughout the ages and in different countries, number systems were developed and used to help people count and communicate with numbers. From the ancient Egyptians to the modern day, different systems have used pictures and symbols to represent whole numbers. Some of the well-known number systems are the Egyptian, Babylonian, Roman, modern Chinese and the Hindu-Arabic or decimal system.

Let's start: Count like a Roman

Here are the letters used in the Roman number system for some numbers that you know.

Number	1	2	3	4	5	6	7	8	9	10	50	100
Roman numerals	I	II	III	IV	V	VI	VII	VIII	IX	X	L	C

Note: On some watches and clocks IIII is used to represent the number 4.

- What numbers do you think XVII and XIX represent?
- Can you write the numbers 261 and 139 using Roman numerals?

Key ideas

- **Egyptian number system**
  - Records show that this number system was used from about 3000 BCE.
  - **Hieroglyphics** were used to represent numbers.
  - From about 1600 BCE, hieroglyphics were used to represent groups of 10, 100, 1000 etc.
  - Symbols of the same type were grouped in twos or threes and arranged vertically.

Number	1	10	100	1000	10 000	100 000	1 000 000
Hieroglyphic	𓏺	𓎆	𓍢	𓆼	𓂭	𓆐	𓁨
Description	stick or staff	arch or heel bone	coil of rope	lotus flower	bent finger or reed	tadpole or frog	genie

For example:

3	5	21	342
𓏺𓏺𓏺	𓏺𓏺𓏺 𓏺𓏺	𓎆𓎆𓏺	𓍢𓍢𓍢𓎆𓎆𓏺𓏺 𓎆𓎆

  - Note that the hieroglyphic with the larger value is written in front (i.e. on the left).
  - There was no symbol for the number zero.

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Key ideas

Babylonian number system

- From about 1750 BCE, the ancient Babylonians used a very sophisticated number system and its origins have been traced to about 3000 BCE.
- Symbols called **cuneiform** (wedge shapes) were used to represent numbers.
- The symbols were marked into clay tablets, which were then allowed to dry in the sun.
- The number system is based on the number 60, but a different wedge shape was used to represent groups of 10.
- The system is positional in that the position of each wedge shape helps determine its value. So ▽▽ means 2 but ▽ ▽▽ means 62.
- To represent zero, they used a blank space or sometimes a small slanted wedge shape for zeros inside a number.

Number	1	10	60
Symbol	▽	◁	▽
Description	upright wedge shape	sideways wedge	upright wedge shape

For example:

5	11	72	121
▽▽▽ ▽▽	◁▽	▽ ◁▽▽	▽▽ ▽

Roman number system

- Some capital letters are used and are called Roman numerals.
- The Roman number system was developed in about the third century BCE and remained the dominant system in many parts of the world until about the Middle Ages. It is still used today in many situations.
- A smaller letter value to the left of a larger letter value indicates subtraction. For example, IV means $5 - 1 = 4$ and XC means $100 - 10 = 90$. Only one letter can be placed to the left for subtraction. Only the letter I can be placed before V or X.
  For example, IV $= 4$ and IX $= 9$ but $99 \neq$ IC.
  The number $99 = 90 + 9 =$ XCIX.

Number	1	5	10	50	100	500	1000
Symbol	I	V	X	L	C	D	M

For example:

2	4	21	59	90
II	IV or IIII	XXI	LIX	XC

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Example 1 Using ancient number systems

Write each of the numbers 3, 15 and 144 using the given number systems.

a Egyptian **b** Babylonian **c** Roman

SOLUTION			EXPLANATION	
a	3	|||			means 1
	15	∩||| ||	∩	means 10
	144	𓍢∩∩|| ∩∩||	𓍢	means 100
b	3	𒁹𒁹𒁹	𒁹	means 1
	15	𒌋𒁹𒁹𒁹 𒁹𒁹	𒌋	means 10
	144	𒁹𒁹 𒌋𒌋𒁹𒁹𒁹 𒁹	𒁹	means 60
c	3	III	I	means 1
	15	XV	V X	means 5 means 10
	144	CXLIV	C XL IV	means 100 means 40 means 4

Exercise 1A FRINGE

UNDERSTANDING AND FLUENCY	1–5	3–6	4–6

1 Which number system (Egyptian, Babylonian or Roman) uses these symbols?

a cuneiform (wedge shapes); e.g. 𒁹

b capital letters; e.g. V and L

c hieroglyphics (pictures); e.g. 𓍢 and ∩

2 Draw the symbols used in these number systems for the given numbers.

a Egyptian

i 1 **ii** 10 **iii** 100 **iv** 1000

b Babylonian

i 1 **ii** 10 **iii** 60

c Roman

i 1 **ii** 5 **iii** 10 **iv** 50 **v** 100

3 In the Roman system, IV does not mean 1 + 5 to give 6. What does IV mean?

Example 1

4 Write these numbers using the given number systems.

a Egyptian

i 3 **ii** 21 **iii** 114 **iv** 352

b Babylonian

i 4 **ii** 32 **iii** 61 **iv** 132

c Roman

i 2 **ii** 9 **iii** 24 **iv** 156

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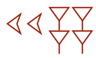
5 What number do these groups of symbols represent?

a Egyptian

i  **ii**  **iii**  **iv** 

b Babylonian

i  **ii**  **iii**  **iv** 

c Roman

i IV **ii** VIII **iii** XVI **iv** XL

6 Work out the answer to each of these problems. Write your answer using the same number system that is given in the question.

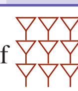
a XIV + XXII

b 

c 

d DCLXIX + IX

PROBLEM-SOLVING AND REASONING	7, 8, 11	8–12	10–13

7 In ancient Babylon, a person adds  goats to another group of  goats.

How many goats are there in total? Write your answer using the Babylonian number system.

8 An ancient Roman counts the number of people in three queues. The first queue has XI, the second has LXII and the third has CXV. How many people are there in total? Write your answer using the Roman number system.

9 One Egyptian house is made from  stones and a second house is made from  stones.

How many more stones does the first house have? Write your answer using the Egyptian number system.

10 Which number system (Egyptian, Babylonian or Roman) uses the least number of symbols to represent these numbers?

a 55 **b** 60 **c** 3104

11 In the Roman system, the letters I, X and C are used to reduce either of the next two larger numerals. So 9 is IX, not VIIII; and 49 is XLIX, not IL. Also, only one numeral can be used to reduce another number. So 8 is VIII, not IIX.

Write these numbers using Roman numerals.

a 4 **b** 9 **c** 14
d 19 **e** 29 **f** 41
g 49 **h** 89 **i** 99
j 449 **k** 922 **l** 3401

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12 The Egyptian system generally uses more symbols than the other systems described here. Can you explain why? How many symbols are used for the number 999?

13 In the Babylonian system 𒁹 stands for 1, but because they did not use a symbol for zero at the end of a number, it also represents 60. People would know what the symbol meant, depending on the situation in which it was used. Here is how it worked for large numbers. The dots represent empty spaces.

1	60	3600
𒁹	𒁹 ……	𒁹 …… ……

a Write these numbers using the Babylonian system.

i 12　　ii 72　　iii 120

iv 191　　v 3661　　vi 7224

b Can you explain why 𒁹 …… …… represents 3600?

c What would 𒁹 …… …… …… represent?

ENRICHMENT	—	—	14

Other number systems

14 Other well-known number systems include:

i Mayan

ii modern Chinese

iii ancient Greek.

Look up these number systems on the internet or elsewhere. Write a brief sentence covering the points below.

a When and where the number systems were used.

b What symbols were used.

c Examples of numbers using these symbols.

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1B Place value in Hindu-Arabic numbers REVISION

The commonly used number system today, called the decimal system or base 10, is also called the Hindu-Arabic number system. Like the Babylonian system, the value of the digit depends on its place in the number, but only one digit is used in each position. A digit for zero is also used. The decimal system originated in ancient India about 3000 BCE and spread throughout Europe through trade and Arabic texts over the next 4000 years.

Let's start: Largest and smallest

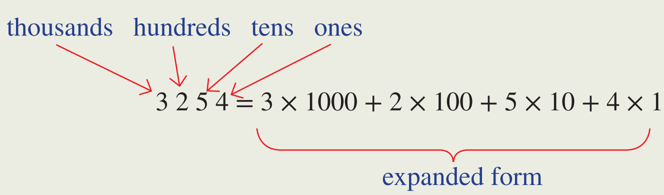
Without using decimal points, repeated digits or a zero (0) at the start of a number, see if you can use all the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 to write down:

- the largest possible number
- the smallest possible number.

Can you explain why your numbers are, in fact, the largest or smallest possible?

Key ideas

- The Hindu-Arabic or **decimal system** uses base 10. This means powers of 10 (1, 10 or 10^1, 100 or 10^2, 1000 or 10^3, …) are used to determine the place value of a digit in a number.
- The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are called **digits**.
- The value of each digit depends on its place in the number. The **place value** of the digit 2 in the number 126, for example, is 20.
- $3 \times 1000 + 2 \times 100 + 5 \times 10 + 4 \times 1$ (or $3 \times 10^3 + 2 \times 10^2 + 5 \times 10^1 + 4 \times 1$) is said to be the **expanded form** of the **basic numeral** 3254.



- Symbols used to compare numbers include the following.

• $=$ (is equal to)	$1 + 3 = 4$	or	$10 - 7 = 3$
• $\neq$ (is not equal to)	$1 + 3 \neq 5$	or	$11 + 38 \neq 50$
• $>$ (is greater than)	$5 > 4$	or	$100 > 37$
• $\geq$ (is greater than or equal to)	$5 \geq 4$	or	$4 \geq 4$
• $<$ (is less than)	$4 < 5$	or	$13 < 26$
• $\leq$ (is less than or equal to)	$4 \leq 5$	or	$4 \leq 4$
• $\approx$ or $\doteqdot$ (is approximately equal to)	$4.02 \approx 4$	or	$8997 \doteqdot 9000$

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Example 2 Finding place value

Write down the place value of the digit 4 in each of these numbers.

a 437 **b** 543 910

SOLUTION	EXPLANATION
a $4 \times 100 = 400$	4 is worth 4×100 3 is worth 3×10 7 is worth 7×1
b $4 \times 10000 = 40000$	5 is worth 5×100000 4 is worth 4×10000 3 is worth 3×1000 9 is worth 9×100 1 is worth 1×10

Example 3 Writing in expanded form

Write 517 in expanded form.

SOLUTION	EXPLANATION
$517 = 5 \times 100 + 1 \times 10 + 7 \times 1$	Write each digit separately and multiply by the appropriate power of 10.

Exercise 1B REVISION

UNDERSTANDING AND FLUENCY	1–5, 6–7(½)	3–7(½), 8	4–8(½)

1 Complete the following:

a $10^1 = \_\_$ **b** $10^2 = \_\_\_$ **c** $10^3 = \_\_\_\_$

2 Write down these numbers using digits.

a two hundred and sixty-three **b** seven thousand four hundred and twenty-one
c thirty-six thousand and fifteen **d** one hundred thousand and one

3 Which symbol (next to the capital letters) matches the given words?

A $=$ **B** $\neq$ **C** $>$ **D** $\geq$
E $<$ **F** $\leq$ **G** $\approx$ **H** $\doteqdot$

a is not equal to **b** is less than **c** is greater than or equal to
d is equal to **e** is greater than **f** is less than or equal to
g is approximately equal to

Example 2

4 Write down the value of the digit 7 in each of these numbers.

a 37 **b** 71 **c** 379 **d** 704
e 1712 **f** 7001 **g** 45 720 **h** 170 966

5 Write down the value of the digit 2 in each of these numbers.

a 126 **b** 2143 **c** 91 214 **d** 1 268 804

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6 State whether each of these statements is true or false.

a $5 > 4$ b $6 = 10$ c $9 \neq 99$ d $1 < 12$
e $22 \leq 11$ f $126 \leq 126$ g $19 \geq 20$ h $138 > 137$
i $13 = 1 + 3$ j $15 + 7 = 22 + 5$ k $16 - 8 = 8 - 16$ l $10 = 1 + 2 + 3 + 4$

Example 3

7 Write these numbers in expanded form.

a 17 b 281 c 935 d 20
e 4491 f 2003 g 10001 h 55555

8 Write these numbers, given in expanded form, as a basic numeral.

a $3 \times 100 + 4 \times 10 + 7 \times 1$ b $9 \times 1000 + 4 \times 100 + 1 \times 10 + 6 \times 1$
c $7 \times 1000 + 2 \times 10$ d $6 \times 100000 + 3 \times 1$
e $4 \times 1000000 + 3 \times 10000 + 7 \times 100$ f $9 \times 10000000 + 3 \times 1000 + 2 \times 10$

PROBLEM-SOLVING AND REASONING	9, 12	9, 10, 12	10–13

9 Arrange these numbers from smallest to largest.

a 55, 45, 54, 44 b 729, 29, 92, 927, 279
c 23, 951, 136, 4 d 435, 453, 534, 345, 543, 354
e 12345, 54321, 34512, 31254 f 1010, 1001, 10001, 1100, 10100

10 How many numbers can be made using the given digits? Digits are not allowed to be used more than once and all digits must be used.

a 2, 8 and 9 b 1, 6 and 7 c 2, 5, 6 and 7

11 Three different digits, not including zero, are chosen. How many numbers can be formed from these three digits if the digits are allowed to be used more than once?

12 The letters used here represent the digits of a number. Write each one in expanded form.

a ab b $abcd$ c $a000a$

13 By considering some of the other number systems (Egyptian, Babylonian or Roman) explained in the previous section, describe the main advantages of the Hindu-Arabic system.

ENRICHMENT	—	—	14

Large numbers and expanded form

14 It is convenient to write very large numbers in expanded form with index notation. Here is an example.

$50000000 = 5 \times 10000000 = 5 \times 10^7$

a Explain why it is convenient to write large numbers in this type of expanded form.
b 3200 can also be written in the form 32×10^2. All the non-zero digits are written down and then multiplied by a power of 10. Similarly, write each of these numbers in the same way.
 i 4100 ii 370000 iii 21770000
c Write each of these numbers as basic numerals.
 i 381×10^2 ii 7204×10^3 iii 1028×10^6
d Write each of these numbers in expanded form, just as you did in the examples above. Research them if you do not know what they are.
 i 1 million ii 1 billion iii 1 trillion
 iv 1 googol v 1 googolplex

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1C Adding and subtracting positive integers

The process of finding the total value of two or more numbers is called addition. The words ‘plus’, ‘add’ and ‘sum’ are also used to describe addition.

The process for finding the difference between two numbers is called subtraction. The words ‘minus’, ‘subtract’ and ‘take away’ are also used to describe subtraction.

Let’s start: Your mental strategy

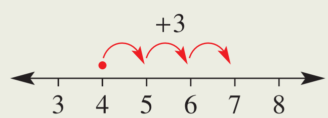
Many problems that involve addition and subtraction can be solved mentally without the use of a calculator or complicated written working.

Consider $98 + 22 - 31 + 29$

How would you work this out? What are the different ways it could be done mentally? Explain your method.

Key ideas

- The symbol $+$ is used to show addition or find a sum.
  For example: $4 + 3 = 7$



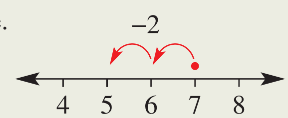
- $a + b = b + a$
  - This is the **commutative law** for addition, meaning that the order does not matter.

  For example: $4 + 3 = 3 + 4$

- $a + (b + c) = (a + b) + c$
  - This is called the **associative law** for addition, meaning it does not matter which pair is added first.

  For example: $4 + (11 + 3) = (4 + 11) + 3$

- The symbol $-$ is used to show subtraction or find a difference.
  For example: $7 - 2 = 5$



- $a - b \neq b - a$ (in general)
  For example: $4 - 3 \neq 3 - 4$

- $a - (b - c) \neq (a - b) - c$ (in general)
  For example: $8 - (4 - 2) \neq (8 - 4) - 2$

- Mental addition and subtraction can be done using different strategies.
  - **Partitioning** (Grouping digits in the same position)
    For example: $171 + 23 = 100 + (70 + 20) + (1 + 3)$
    $= 194$
  - **Compensating** (Making a 10, 100 etc. and then adjusting or compensating by adding or subtracting)
    For example: $46 + 9 = 46 + 10 - 1$
    $= 55$
  - **Doubling or halving** (Making a double or half and then adjusting with addition or subtraction)
    For example:
    $75 + 78 = 75 + 75 + 3$
    $= 150 + 3$
    $= 153$

    $124 - 61 = 124 - 62 + 1$
    $= 62 + 1$
    $= 63$

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Example 4 Mental addition and subtraction

Use the suggested strategy to mentally work out the answer.

a 132 + 156 (partitioning)
b 25 + 19 (compensating)
c 56 − 18 (compensating)
d 35 + 36 (doubling or halving)

SOLUTION	EXPLANATION
a 132 + 156 = 288	100 + 30 + 2 100 + 50 + 6 200 + 80 + 8
b 25 + 19 = 44	25 + 19 = 25 + 20 − 1 = 45 − 1 = 44
c 56 − 18 = 38	56 − 18 = 56 − 20 + 2 = 36 + 2 = 38
d 35 + 36 = 71	35 + 36 = 35 + 35 + 1 = 70 + 1 = 71

Exercise 1C

UNDERSTANDING AND FLUENCY	1–5, 6–10(½)	5–11(½)	6–11(½)

1 Write the word *addition* or *subtraction* for each of the following.
- **a** What is the sum of 6 and 8?
- **b** What number is 6 less than 8?
- **c** What number is six more than eight?
- **d** What is the difference between 6 and 8?

2 Write the number which is:
- **a** 3 more than 7
- **b** 58 more than 11
- **c** 7 less than 19
- **d** 137 less than 157

3 **a** Find the sum of each pair of numbers.
- **i** 2 and 6
- **ii** 19 and 8
- **iii** 62 and 70

b Find the difference between each pair of numbers.
- **i** 11 and 5
- **ii** 29 and 13
- **iii** 101 and 93

4 State whether each of these statements is true or false.
- **a** $4 + 3 > 6$
- **b** $11 + 19 \geqslant 30$
- **c** $13 - 9 < 8$
- **d** $26 - 15 \leqslant 10$
- **e** $1 + 7 - 4 \geqslant 4$
- **f** $50 - 21 + 6 < 35$

5 Give the result for each of these computations.
- **a** 7 plus 11
- **b** 22 minus 3
- **c** the sum of 11 and 21
- **d** 128 add 12
- **e** 36 take away 15
- **f** the difference between 13 and 4

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Example 4a

6 Mentally find the answers to these sums. Hint: Use the partitioning strategy.

a 23 + 41	**b** 71 + 26	**c** 138 + 441
d 246 + 502	**e** 937 + 11	**f** 1304 + 4293
g 140273 + 238410	**h** 390447 + 201132	**i** 100001 + 101010

7 Mentally find the answers to these differences. Hint: Use the partitioning strategy.

a 29 − 18	**b** 57 − 21	**c** 249 − 137
d 1045 − 1041	**e** 4396 − 1285	**f** 10101 − 100

Example 4b

8 Mentally find the answers to these sums. Hint: Use the compensating strategy.

a 15 + 9	**b** 64 + 11	**c** 19 + 76
d 18 + 115	**e** 31 + 136	**f** 245 + 52

Example 4c

9 Mentally find the answers to these differences. Hint: Use the compensating strategy.

a 35 − 11	**b** 45 − 19	**c** 156 − 48
d 244 − 22	**e** 376 − 59	**f** 5216 − 199

Example 4d

10 Mentally find the answers to these sums and differences. Hint: Use the doubling or halving strategy.

a 25 + 26	**b** 65 + 63	**c** 121 + 123
d 240 − 121	**e** 482 − 240	**f** 1006 − 504

11 Mentally find the answers to these computations.

a 11 + 18 − 17	**b** 37 − 19 + 9	**c** 101 − 15 + 21
d 136 + 12 − 15	**e** 28 − 10 − 9 + 5	**f** 39 + 71 − 10 − 10
g 1010 − 11 + 21 − 1	**h** 5 − 7 + 2	**i** 10 − 25 + 18

PROBLEM-SOLVING AND REASONING	12, 13, 18	13, 14, 18, 19	15–17, 19–21

12 Gary worked 7 hours on Monday, 5 hours on Tuesday, 13 hours on Wednesday, 11 hours on Thursday and 2 hours on Friday. What is the total number of hours that Gary worked during the week?

13 In a batting innings, Phil hit 126 runs and Mario hit 19 runs. How many more runs did Phil hit compared to Mario?

14 A farmer reduced his cattle numbers from 86 to 54. How many cows were taken away?

15 Bag A has 18 marbles and bag B has 7 fewer marbles than bag A. What is the total number of marbles?

16 Matt has 36 cards and Andy has 35 more cards than Matt. If they combine their cards, how many do they have in total?

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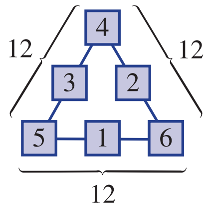
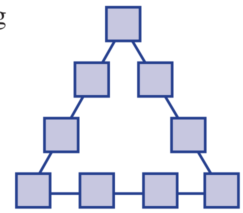
17 Each side on a magic triangle adds up to the same number, as shown in this example with a sum of 12 on each side.



a Place each of the digits from 1 to 6 in a magic triangle with three digits along each side so that each side adds up to the given number.
  i 9 **ii** 10

b Place each of the digits from 1 to 9 in a magic triangle with four digits along each side so that each side adds up to the given number.
  i 20 **ii** 23



18 **a** The mental strategy of partitioning is easy to apply for 23 + 54 but harder for 23 + 59. Explain why.

b The mental strategy of partitioning is easy to apply for 158 − 46 but harder for 151 − 46. Explain why.

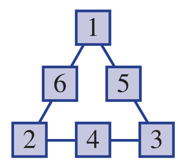
19 Complete these number sentences if the letters a, b and c represent numbers.

a $a + b = c$ so $c - \_\_ = a$ **b** $a + c = b$ so $b - a = \_\_$

20 This magic triangle uses the digits 1 to 6 and has each side adding to the same total.
This example shows a side total of 9.

a How many different side totals are possible using the same digits?

b Explain your method.



21 The sum of two numbers is 87 and their difference is 29. What are the two numbers?

ENRICHMENT	—	—	22

Magic squares

22 A magic square has every row, column and main diagonal adding to the same number, called the magic sum. For example, this magic square has a magic sum of 15.

4	9	2	→15
3	5	7	→15
8	1	6	→15
↓ 15	↓ 15	↓ 15	

Diagonals: ↙ 15, ↘ 15

Find the magic sums for these squares, then fill in the missing numbers.

a

6		
7	5	
2		

b

10		
	11	13
		12

c

15	20	
14		
19		

d

1	15		4
	6		9
		11	
13		2	16

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1D Algorithms for adding and subtracting

Interactive
Widgets
HOTsheets
Walkthrough

To add or subtract larger numbers we can use a process called an algorithm.

Adding can involve carrying a number *to* the next column, whereas subtracting can involve trading *from* the next column.

Let's start: The missing digits

Discuss what digits should go in the empty boxes. Give reasons for your answers.

$$\begin{array}{r} 1\ \square\ 4 \\ +\ 9\ 5\ \square \\ \hline 1\ \square\ 2\ 5 \end{array} \qquad \begin{array}{r} \square\ 5\ \square \\ -\ 1\ \square\ 4 \\ \hline 9\ 4 \end{array}$$

Key ideas

- An **algorithm** is a procedure involving a number of steps.
- **Addition algorithm**
  - Arrange the numbers vertically so that the digits with similar place value are in the same column.
  - Add digits in the same column, starting on the right.
  - If the digits add to more than 9, carry the 10 to the next column.

For example:

$$\begin{array}{r} {}^{1}234 \\ +\ 192 \\ \hline 426 \end{array} \qquad \begin{array}{r} 4+2=6 \\ 3+9=12 \\ 1+2+1=4 \end{array}$$

- **Subtraction algorithm**
  - Arrange the numbers vertically so that the digits with similar place value are in the same column.
  - Subtract digits in the same column top-down and starting on the right.
  - If the digits subtract to less than 0, borrow a 1 from the next column to form an extra 10.

For example:

$$\begin{array}{r} {}^{1}\not{2}\ {}^{1}5\ 9 \\ -\ 1\ 8\ 2 \\ \hline 7\ 7 \end{array} \qquad \begin{array}{r} 9-2=7 \\ 15-8=7 \\ 1-1=0 \end{array}$$

- Calculators may be used to check your answers.

Example 5 Using the addition algorithm

Give the result for each of these sums.

a $\begin{array}{r} 26 \\ +\ 66 \\ \hline \end{array}$

b $\begin{array}{r} 439 \\ +\ 172 \\ \hline \end{array}$

SOLUTION

a $\begin{array}{r} {}^{1}26 \\ +\ 66 \\ \hline 92 \end{array}$

EXPLANATION

Add the digits vertically.
6 + 6 = 12, so carry the 1 to the tens column.

b $\begin{array}{r} {}^{1}4\ {}^{1}3\ 9 \\ +\ 1\ 7\ 2 \\ \hline 6\ 1\ 1 \end{array}$

9 + 2 = 11, so carry a 1 to the tens column.
1 + 3 + 7 = 11, so carry a 1 to the hundreds column.

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Example 6 Using the subtraction algorithm

Give the result for each of these differences.

a $\begin{array}{r} 74 \\ -15 \\ \hline \end{array}$ **b** $\begin{array}{r} 3240 \\ -\ 2721 \\ \hline \end{array}$

SOLUTION

a $\begin{array}{r} {}^{6}\cancel{7}\ {}^{1}4 \\ -1\ \ 5 \\ \hline 5\ \ 9 \end{array}$

EXPLANATION

Trade 1 from 7 to make 14 − 5 = 9
Then subtract 1 from 6 (not 7).

b $\begin{array}{rrrr} {}^{2}\cancel{3} & {}^{1}2 & {}^{3}\cancel{4} & {}^{1}0 \\ -2 & 7 & 2 & 1 \\ \hline & 5 & 1 & 9 \end{array}$

Trade 1 from 4 to make 10 − 1 = 9.
Subtract 2 from 3 (not 4).
Trade 1 from 3 to make 12 − 7 = 5.
Note that 2 − 2 = 0 and you do not need to show a 0 before the 5.

Exercise 1D

UNDERSTANDING AND FLUENCY	1–4(½), 5, 6–7(½)	3–7(½), 8	4–7(½), 8

1 Mentally find the value of each of these sums.

a 8 + 9 **b** 87 + 14 **c** 138 + 6 **d** 99 + 11
e 998 + 7 **f** 19 + 124 **g** 102 + 99 **h** 52 + 1053

2 Mentally find the value of each of these differences.

a 13 − 5 **b** 36 − 9 **c** 75 − 8 **d** 100 − 16
e 37 − 22 **f** 104 − 12 **g** 46 − 17 **h** 1001 − 22

3 What is the missing digit in each of these computations?

a $\begin{array}{r} 2\ 7 \\ +3\ 1 \\ \hline 5\square \end{array}$ **b** $\begin{array}{r} 3\ 6 \\ +1\ 5 \\ \hline 5\square \end{array}$ **c** $\begin{array}{r} 1\ 2\ 3 \\ +\ \ \ 9\ 1 \\ \hline 2\square 4 \end{array}$ **d** $\begin{array}{r} 4\ 6 \\ +\ \square\ 5 \\ \hline 1\ 1\ 1 \end{array}$

e $\begin{array}{r} 2\ 4 \\ -1\ \square \\ \hline 1\ 2 \end{array}$ **f** $\begin{array}{r} 6\ 7 \\ -\ 4\ 8 \\ \hline \square 9 \end{array}$ **g** $\begin{array}{r} 1\ 6\ 2 \\ -\ \ \ \square\ 1 \\ \hline 8\ 1 \end{array}$ **h** $\begin{array}{r} 1\ 4\square 2 \\ -\ \ \ 6\ 2\ 3 \\ \hline 8\ 0\ 9 \end{array}$

Example 5

4 Give the answer to each of these additions. Check your answers with a calculator.

a $\begin{array}{r} 36 \\ +51 \\ \hline \end{array}$ **b** $\begin{array}{r} 74 \\ +25 \\ \hline \end{array}$ **c** $\begin{array}{r} 17 \\ +24 \\ \hline \end{array}$ **d** $\begin{array}{r} 47 \\ +39 \\ \hline \end{array}$

e $\begin{array}{r} 129 \\ +\ 97 \\ \hline \end{array}$ **f** $\begin{array}{r} 458 \\ +287 \\ \hline \end{array}$ **g** $\begin{array}{r} 1041 \\ +\ 882 \\ \hline \end{array}$ **h** $\begin{array}{r} 3092 \\ +1988 \\ \hline \end{array}$

5 Show your working to find the value of each of these sums.

a 85 + 76 **b** 131 + 94 **c** 1732 + 497 **d** 988 + 987

6 Give the result for each of these sums.

a $\begin{array}{r} 17 \\ 26 \\ +34 \\ \hline \end{array}$ **b** $\begin{array}{r} 126 \\ 47 \\ +\ 19 \\ \hline \end{array}$ **c** $\begin{array}{r} 152 \\ 247 \\ +\ 19 \\ \hline \end{array}$ **d** $\begin{array}{r} 2197 \\ 1204 \\ +\ 807 \\ \hline \end{array}$

e 946 + 241 + 27 + 9 **f** 1052 + 839 + 7 + 84

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Example 6

7 Find the answers to these subtractions. Check your answers with a calculator.

a $\begin{array}{r} 54 \\ -23 \\ \hline \end{array}$ **b** $\begin{array}{r} 85 \\ -65 \\ \hline \end{array}$ **c** $\begin{array}{r} 46 \\ -27 \\ \hline \end{array}$ **d** $\begin{array}{r} 94 \\ -36 \\ \hline \end{array}$

e $\begin{array}{r} 125 \\ -\ 89 \\ \hline \end{array}$ **f** $\begin{array}{r} 241 \\ -129 \\ \hline \end{array}$ **g** $\begin{array}{r} 358 \\ -279 \\ \hline \end{array}$ **h** $\begin{array}{r} 491 \\ -419 \\ \hline \end{array}$

8 Show your working to find the answer to each of these differences.

a 32 − 16 **b** 124 − 77 **c** 613 − 128 **d** 1004 − 838

PROBLEM-SOLVING AND REASONING	9, 10, 14	10–12, 14,15	12–17

9 Farmer Green owns 287 sheep, farmer Brown owns 526 sheep and farmer Grey owns 1041 sheep. How many sheep are there in total?

10 A car's odometer shows 12 138 kilometres at the start of a journey and 12 714 kilometres at the end of the journey. What was the journey's distance?

11 Two different schools have 871 and 950 students enrolled.

a How many students are there in total?

b Find the difference between the number of students in the schools.

12 Find the missing digits in these sums.

a $\begin{array}{rrr} & 3 & \square \\ + & 5 & 3 \\ \hline & \square & 1 \end{array}$ **b** $\begin{array}{rrrr} & 1 & \square & 4 \\ + & & 7 & \square \\ \hline & \square & 9 & 1 \end{array}$ **c** $\begin{array}{rrrr} & & \square & \square \\ + & \square & 4 & 7 \\ \hline & 9 & 1 & 4 \end{array}$

13 Find the missing numbers in these differences.

a $\begin{array}{rrr} & 6 & \square \\ - & 2 & 8 \\ \hline & \square & 4 \end{array}$ **b** $\begin{array}{rrrr} & 2 & \square & 5 \\ - & \square & 8 & \square \\ \hline & & 8 & 1 \end{array}$ **c** $\begin{array}{rrrrr} & 3 & \square & \square & 2 \\ - & & 9 & 2 & \square \\ \hline & \square & 1 & 6 & 5 \end{array}$

14 **a** First work out the answer to these simple computations before doing part **b**.

i 28 + 18 − 17 **ii** 36 − 19 + 20

b For part **i** above, is it possible to work out 18 − 17 and then add this total to 28?

c For part **ii** above, is it possible to work out 19 + 20 and then subtract this total from 36?

d Can you suggest a good mental strategy for part **ii** above that gives the correct answer?

15 **a** What are the missing digits in this sum?

b Explain why there is more than one possible set of missing numbers in the sum given opposite. Give some examples.

$\begin{array}{rrrr} & 2 & \square & 3 \\ + & \square & \square & \square \\ \hline & 4 & 2 & 1 \end{array}$

16 The sum of two numbers is 978 and their difference is 74. What are the two numbers?

17 Make up some of your own problems like Question **16** and test them on a friend.

ENRICHMENT	—	—	18

More magic squares

18 Complete these magic squares.

a

62	67	60
		65

b

101		114	
	106		109
	110		
113	103	102	116

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1E Multiplying small positive integers

The multiplication of two numbers represents a repeated addition.

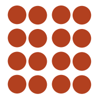
For example, 4×2 could be thought of as 4 groups of 2 or $2 + 2 + 2 + 2$.

4×2

Similarly, 4×2 could be thought of as 2 groups of 4 or 2×4 or $4 + 4$.

2×4

The product 4×4 involves 4 groups of 4, which means the dots form a square. 4×4 can be read as '4 squared' and written as 4^2. The number 16 is a square number.



4×4

Let's start: What is wrong with these strategies?

Three students explain their method for finding the answer to 124×8.

- Billy says that you can do 124×10 to get 1240, then subtract 2 to get 1238.
- Lea says that you halve 124 and 8 twice each to give $31 \times 2 = 62$.
- Surai says that you multiply 8 by 4 to give 32, 8 by 2 to give 16 and 8 by 1 to give 8. She says the total is $32 + 16 + 8 = 56$.

Are any of the students correct and can you explain any errors in their thinking?

Key ideas

- Finding the **product** of two numbers involves multiplication. We say 'the product of 2 and 3 is 6'.
- $a \times b = b \times a$ For example: $2 \times 3 = 3 \times 2$
  - This is the **commutative law** for multiplication, meaning that the order does not matter.
- $(a \times b) \times c = a \times (b \times c)$ For example: $(3 \times 5) \times 4 = 3 \times (5 \times 4)$
  - This is the **associative law** for multiplication, meaning it does not matter which pair is multiplied first.
- The multiplication algorithm for multiplying by a single digit involves:
  - multiplying the single digit by each digit in the other number, starting from the right
  - carry and add any digits with a higher place value to the total in the next column.

  For example:

  $$\begin{array}{r} {}^{1}23 \\ \underline{4} \\ 92 \end{array} \qquad \begin{array}{l} 4 \times 3 = 12 \\ 4 \times 2 + 1 = 9 \end{array}$$

- Mental multiplication can be done using different strategies:
  - memorising the multiplication tables. For example: $9 \times 7 = 63$ $12 \times 3 = 36$
  - using the commutative law by changing the order. For example, 43×2 might be thought of more easily as 2 groups of 43 or 2×43
  - using the commutative and associative law by altering the order if more than one number is being multiplied. For example:

  $$\begin{aligned} 5 \times 11 \times 2 &= 5 \times 2 \times 11 \\ &= 10 \times 11 \\ &= 110 \end{aligned}$$

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Key ideas

- When a number is multiplied by itself, it is said to be 'squared'.
  $a \times a = a^2$
  For example: $5 \times 5 = 5^2 = 25$
  When the number 5 is squared, the result is 25. So the number 25 is a square number.
  - Using the **distributive law** by making a 10, 100 etc. and then adjusting by adding or subtracting. The distributive law is:
    $a \times (b + c) = (a \times b) + (a \times c)$ or $a \times (b - c) = (a \times b) - (a \times c)$.
    This will be used more extensively in the algebra chapters.
    For example:
    $$\begin{aligned} 6 \times 21 &= (6 \times 20) + (6 \times 1) \\ &= 120 + 6 \\ &= 126 \end{aligned} \qquad \begin{aligned} 7 \times 18 &= (7 \times 20) - (7 \times 2) \\ &= 140 - 14 \\ &= 126 \end{aligned}$$
  - Using the doubling and halving strategy by doubling one number and halving the other.
    For example:
    $$\begin{aligned} 15 \times 18 &= 30 \times 9 \\ &= 270 \end{aligned}$$
  - Using factors to split a number.
    For example:
    $$\begin{aligned} 11 \times 16 &= 11 \times 8 \times 2 \\ &= 88 \times 2 \\ &= 176 \end{aligned}$$

Example 7 Using mental strategies for multiplication

Use a mental strategy to find the answer to each of these products.

a 7×6 **b** 3×13 **c** 4×29 **d** 5×24 **e** 7×14 **f** 12^2

SOLUTION	EXPLANATION
a $7 \times 6 = 42$	7×6 or 6×7 should be memorised (from multiplication tables).
b $3 \times 13 = 39$	$3 \times 13 = (3 \times 10) + (3 \times 3) = 30 + 9 = 39$ (The distributive law is being used.)
c $4 \times 29 = 116$	$4 \times 29 = (4 \times 30) - (4 \times 1) = 120 - 4 = 116$ (The distributive law is being used.)
d $5 \times 24 = 120$	$5 \times 24 = 10 \times 12 = 120$ (The doubling and halving strategy is being used.)
e $7 \times 14 = 98$	$7 \times 14 = 7 \times 7 \times 2 = 49 \times 2 = 98$ (Factors of 14 are used.)
f $12^2 = 12 \times 12 = 144$	This is read as '12 squared', which means that 12 is multiplied by itself. The number 144 is a square number.

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Example 8 Using the multiplication algorithm

Give the result for each of these products.

a 31 × 4 **b** 197 × 7

SOLUTION

a
$$\begin{array}{r} 31 \\ \times \quad 4 \\ \hline 124 \end{array}$$

b
$$\begin{array}{r} {}^{6}1{}^{4}97 \\ \times \quad\; 7 \\ \hline 1379 \end{array}$$

EXPLANATION

a $4 \times 1 = 4$
$4 \times 3 = 12$

b $7 \times 7 = 49$ (carry the 4)
$7 \times 9 + 4 = 67$ (carry the 6)
$7 \times 1 + 6 = 13$

Exercise 1E

UNDERSTANDING AND FLUENCY	1–3, 4–5(½), 6, 7(½)	3, 4–5(½), 6, 7(½), 8	4–8(½)

1 Write the next three numbers in these patterns.

a 4, 8, 12, 16, __, __, __ **b** 11, 22, 33, __, __, __ **c** 17, 34, 51, __, __, __

2 Are these statements true or false?

a $4 \times 3 = 3 \times 4$ **b** $2 \times 5 \times 6 = 6 \times 5 \times 2$ **c** $11 \times 5 = 10 \times 5$
d $3 \times 32 = 3 \times 30 + 3 \times 2$ **e** $5 \times 18 = 10 \times 9$ **f** $21 \times 4 = 2 \times 42$
g $19 \times 7 = 20 \times 7 - 19$ **h** $39 \times 4 = 40 \times 4 - 1 \times 4$ **i** $64 \times 4 = 128 \times 8$

3 What is the missing digit in each of these products?

a
$$\begin{array}{r} 21 \\ \times \; 3 \\ \hline 6\square \end{array}$$
b
$$\begin{array}{r} 36 \\ \times \; 5 \\ \hline 18\square \end{array}$$
c
$$\begin{array}{r} 76 \\ \times \; 2 \\ \hline 1\square \end{array}$$
d
$$\begin{array}{r} 402 \\ \times \quad 3 \\ \hline 1\square06 \end{array}$$

Example 7a,f

4 Using your knowledge of multiplication tables, give the answer to each of these products.

a 8×7 **b** 6×9 **c** 12×4 **d** 11^2
e 6×12 **f** 7×5 **g** 12×9 **h** 13×3

Example 7b,c

5 Find the results to these products mentally. Hint: Use the distributive law strategy – subtraction for **a** to **d** and addition for **e** to **h**.

a 3×19 **b** 6×29 **c** 4×28 **d** 38×7
e 5×21 **f** 4×31 **g** 6×42 **h** 53×3

Example 7d,e

6 Find the answers to these products mentally. Hint: Use the double and halve strategy or split a number using its factors.

a 4×24 **b** 3×18 **c** 6×16 **d** 24×3

Example 8

7 Give the result of each of these products, using the multiplication algorithm. Check your results with a calculator.

a
$$\begin{array}{r} 33 \\ \times \; 2 \\ \hline \end{array}$$
b
$$\begin{array}{r} 43 \\ \times \; 3 \\ \hline \end{array}$$
c
$$\begin{array}{r} 72 \\ \times \; 6 \\ \hline \end{array}$$
d
$$\begin{array}{r} 55 \\ \times \; 3 \\ \hline \end{array}$$
e
$$\begin{array}{r} 129 \\ \times \quad 2 \\ \hline \end{array}$$
f
$$\begin{array}{r} 407 \\ \times \quad 7 \\ \hline \end{array}$$
g
$$\begin{array}{r} 526 \\ \times \quad 5 \\ \hline \end{array}$$
h
$$\begin{array}{r} 3509 \\ \times \quad\; 9 \\ \hline \end{array}$$

8 Find the answers to these products, showing your working.

a 47×5 **b** 1391×3 **c** 9×425 **d** 7×4170

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PROBLEM-SOLVING AND REASONING	9, 10, 14	10–12, 14, 15(½)	11–13, 15, 16

9 A circular race track is 240 metres long and Rory runs seven laps. How far does Rory run in total?

10 Eight tickets costing $33 each are purchased for a concert. What is the total cost of the tickets?

11 Reggie and Angelo combine their packs of cards. Reggie has five sets of 13 cards and Angelo has three sets of 17 cards. How many cards are there in total?

12 Sala purchases some goods for a party and has $100 to spend. She selects eight bottles of drink for $2 each, 13 food packs at $6 each and 18 party hats at 50 cents each. Does she have enough money to pay for all the items?

13 Find the missing digits in these products.

a $\begin{array}{r} 39 \\ \times\ 7 \\ \hline 2\square 3 \end{array}$ **b** $\begin{array}{r} 25 \\ \times\ \square \\ \hline 125 \end{array}$ **c** $\begin{array}{r} 79 \\ \times\ \square \\ \hline \square 37 \end{array}$ **d** $\begin{array}{r} 132 \\ \times\ \square \\ \hline 10\square 6 \end{array}$

e $\begin{array}{r} 2\square \\ \times\ 7 \\ \hline \square 89 \end{array}$ **f** $\begin{array}{r} \square\square \\ \times\ 9 \\ \hline 351 \end{array}$ **g** $\begin{array}{r} 23\square \\ \times\ 5 \\ \hline 1\square 60 \end{array}$ **h** $\begin{array}{r} \square\square 4 \\ \times\ \square \\ \hline \square 198 \end{array}$

14 The commutative and associative laws for multiplication mean that numbers can be multiplied in any order. So $(a \times b) \times c = (b \times a) \times c = b \times (a \times c) =$ ___, where the brackets show which numbers are multiplied first. In how many ways can $2 \times 3 \times 5$ be calculated?

15 Write each of the following as single products. For example: $5 \times 8 + 5 \times 2 = 5 \times 10$

a $3 \times 20 + 3 \times 1$ **b** $9 \times 50 + 9 \times 2$ **c** $7 \times 30 + 7 \times 2$

d $5 \times 100 - 5 \times 3$ **e** $a \times 40 - a \times 2$ **f** $a \times 200 + a \times 3$

16 How many different ways can the two spaces be filled in this product? Explain why.

$\begin{array}{r} 2\square 3 \\ \times\ 4 \\ \hline 8\square 2 \end{array}$

ENRICHMENT	—	—	17, 18

Missing digits

17 Find all the missing digits in these products.

a $\begin{array}{r} \square 1\square \\ \times\ 7 \\ \hline \square 51\square \end{array}$ **b** $\begin{array}{r} 29\square \\ \times\ 3 \\ \hline 8\square\square \end{array}$

18 The product of two numbers is 132 and their sum is 28. What are the two numbers?

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1F Multiplying large positive integers

Interactive Widgets HOTsheets Walkthrough

There are many situations that require the multiplication of large numbers; for example, the total revenue from selling 40 000 tickets at \$23 each, or the area of a rectangular park with length and width dimensions of 65 metres by 122 metres. To complete such calculations by hand requires the use of a suitable algorithm.

Let's start: Spot the errors

There are three types of errors in the working shown for this problem. Find the errors and describe them.

$$\begin{array}{r} 271 \\ \times\ \ 13 \\ \hline 613 \\ +\ 271 \\ \hline 1273 \end{array}$$

Key ideas

- When multiplying by 10, 100, 1000, 10 000 etc. each digit moves to the left by the number of zeros. For example: $45 \times 1000 = 45\,000$
- A strategy for multiplying by multiples of 10, 100 etc. is to first multiply by the number without the zeros then add the zeros to the answer later. For example: $21 \times 3000 = 21 \times 3 \times 1000 = 63 \times 1000 = 63\,000$
- The algorithm shown here begins by writing the larger number above the smaller.

$$\begin{array}{rl} 143 & \\ \times\ \ 14 & \\ \hline {}^{1}572 & \leftarrow 143 \times 4 \\ +1430 & \leftarrow 143 \times 10 \\ \hline 2002 & \leftarrow 1430 + 572 \end{array}$$

Example 9 Multiplying large numbers

Give the result for each of these products.

a 37×100 **b** 45×70 **c** 614×14 **d** 25^2

SOLUTION

a $37 \times 100 = 3700$

b $45 \times 70 = 45 \times 7 \times 10$
$= 315 \times 10$
$= 3150$

c
$$\begin{array}{r} 614 \\ \times\ \ \ 14 \\ \hline 2456 \\ +6140 \\ \hline 8596 \end{array}$$

d
$$\begin{array}{r} 25 \\ \times\ \ 25 \\ \hline 125 \\ +\ 500 \\ \hline 625 \end{array}$$

EXPLANATION

a Move the 3 and the 7 two places to the left and add two zeros.

b First multiply by 7, then multiply by 10 later.

$$\begin{array}{r} 45 \\ \times\ \ 7 \\ \hline 315 \end{array}$$

c First multiply 614×4.
Then multiply 614×10.
Add the totals to give the answer.

d First multiply 25×5.
Then multiply 25×20.
Add the totals to give the answer.
This can be written as $25^2 = 625$.

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Exercise 1F

UNDERSTANDING AND FLUENCY	1–3, 4–6(½)	3, 4–6(½)	4–6(½)

1 What is the missing digit in each of these products?

a $72 \times 10 = 7\square0$ **b** $13 \times 100 = 130\square$ **c** $49 \times 100 = 49\square0$ **d** $924 \times 10 = 92\square0$

2 What is the missing number in each of these products?

a $15 \times \_\_ = 1500$ **b** $329 \times \_\_ = 3290$ **c** $92 \times \_\_ = 920000$

3 State if the following calculations are correct. If they are incorrect, find the correct answer.

a
$$\begin{array}{r} 26 \\ \times\ \ 4 \\ \hline 84 \end{array}$$

b
$$\begin{array}{r} 39 \\ \times\ 14 \\ \hline 156 \\ +\ \ 39 \\ \hline 195 \end{array}$$

c
$$\begin{array}{r} 92 \\ \times\ \ 24 \\ \hline 368 \\ +1840 \\ \hline 2208 \end{array}$$

d
$$\begin{array}{r} 102 \\ \times\ \ 24 \\ \hline 408 \\ +240 \\ \hline 648 \end{array}$$

Example 9a

4 Give the result of each of these products.

a 4×100 **b** 29×10 **c** 183×10 **d** 46×100

e 50×1000 **f** 630×100 **g** 1441×10 **h** 2910×10000

Example 9b

5 Use the suggested strategy in Example 9b to find these products.

a 17×20 **b** 36×40 **c** 92×70 **d** 45×500

e 138×300 **f** 92×5000 **g** 317×200 **h** 1043×9000

Example 9c,d

6 Use the multiplication algorithm to find these products.

a
$$\begin{array}{r} 37 \\ \times 11 \\ \hline \end{array}$$

b
$$\begin{array}{r} 72 \\ \times 19 \\ \hline \end{array}$$

c
$$\begin{array}{r} 126 \\ \times\ 15 \\ \hline \end{array}$$

d
$$\begin{array}{r} 428 \\ \times\ 22 \\ \hline \end{array}$$

e
$$\begin{array}{r} 46 \\ \times 46 \\ \hline \end{array}$$

f
$$\begin{array}{r} 416 \\ \times\ 98 \\ \hline \end{array}$$

g
$$\begin{array}{r} 380 \\ \times\ 49 \\ \hline \end{array}$$

h
$$\begin{array}{r} 1026 \\ \times\ \ 33 \\ \hline \end{array}$$

PROBLEM-SOLVING AND REASONING	7, 8, 13	8–10, 13, 14	9–12, 14, 15

7 Estimate the answers to these products, then use a calculator to check.

a 19×11 **b** 26×21 **c** 37×15 **d** 121×18

8 A pool area includes 68 square metres of paving at \$32 per square metre. What is the total cost of paving?

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9 Waldo buys 215 metres of pipe at \$28 per metre. What is the total cost of piping?

10 How many seconds are there in one day?

11 Find the missing digits in these products.

a
$$\begin{array}{r} 2\,\square \\ \times\ 1\,7 \\ \hline 1\,\square\,1 \\ +\ 2\,\square\,0 \\ \hline \square\,\square\,1 \end{array}$$

b
$$\begin{array}{r} 1\,\square\,3 \\ \times\ 1\,\square \\ \hline \square\,2\,9 \\ +\ 1\,\square\,3\,\square \\ \hline \square\,\square\,5\,\square \end{array}$$

c
$$\begin{array}{r} \square\,\square \\ \times\ 3\,7 \\ \hline 3\,4\,3 \\ +\ \square\,4\,\square\,\square \\ \hline \square\,\square\,\square\,\square \end{array}$$

d
$$\begin{array}{r} \square\,2\,\square \\ \times\ 2\,\square \\ \hline 1\,2\,6 \\ +\ \square\,5\,2\,\square \\ \hline \square\,6\,\square\,\square \end{array}$$

12 There are 360 degrees in a full turn. How many degrees does the minute hand on a clock turn in one week?

13 The product of two whole numbers is less than their sum. Neither number is zero or less. What must be true about one of the numbers?

14 If both numbers in a multiplication computation have at least three digits, then the algorithm needs to be expanded. Use the algorithm to find these products.

a
$$\begin{array}{r} 294 \\ \times\ 136 \\ \hline \end{array}$$

b
$$\begin{array}{r} 1013 \\ \times\ 916 \\ \hline \end{array}$$

c
$$\begin{array}{r} 3947 \\ \times\ 1204 \\ \hline \end{array}$$

d
$$\begin{array}{r} 47126 \\ \times\ 3107 \\ \hline \end{array}$$

15 Can you work out these problems using an effective mental strategy? Look to see if you can first simplify each question.

- **a** $98 \times 16 + 2 \times 16$
- **b** $33 \times 26 - 3 \times 26$
- **c** $19 \times 15 + 34 \times 17 - 4 \times 17 + 1 \times 15$
- **d** $22 \times 19 - 3 \times 17 + 51 \times 9 - 1 \times 9 + 13 \times 17 - 2 \times 19$

ENRICHMENT	—	—	16–18

Multiplication puzzle

16 **a** What is the largest number you can make by choosing five digits from the list 1, 2, 3, 4, 5, 6, 7, 8, 9 and placing them into the product shown at right?

b What is the smallest number you can make by choosing five digits from the list 1, 2, 3, 4, 5, 6, 7, 8, 9 and placing them into the product shown at right?

$$\begin{array}{r} \square\,\square\,\square \\ \times\ \square\,\square \\ \hline \end{array}$$

17 The product of two whole numbers is 14 391 and their difference is 6. What are the two numbers?

18 **a** $8^2 = 8 \times 8 = 64$

64 is a square number.

Find all the three-digit square numbers in which the first digit is 1 or 2.

b What is the largest three-digit square number?

c What is the largest four-digit square number?

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1G Dividing positive integers and dealing with remainders

Interactive
Widgets
HOTsheets
Walkthrough

Division is used to split a quantity into equal groups. Examples include:

- 20 apples shared equally between 5 people
- \$10 000 shared equally between 4 people

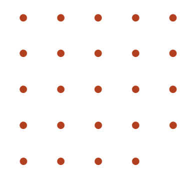
Multiplication and division are reverse operations, and this is shown in this simple example:

$7 \times 3 = 21$ So, $21 \div 3 = 7$ or $21 \div 7 = 3$

Let's start: Arranging counters

A total of 24 counters sit on a table. Using whole numbers, in how many ways can the counters be divided into equal-sized groups with no counters remaining?

- Is it also possible to divide the counters into equal-sized groups but with two counters remaining?
- If five counters are to remain, how many equal-sized groups can be formed and why?



Key ideas

- The number of equal-sized groups formed from the division operation is called the **quotient**.
- The total being divided is called the **dividend** and the size of the equal groups is called the **divisor**.
- Any amount remaining after division into equal-sized groups is called the **remainder**.
  For example:
  $7 \div 3 = 2$ and 1 remainder means
  $7 = 2 \times 3 + 1$
  $37 \div 5 = 7$ and 2 remainder means
  $37 = 7 \times 5 + 2$

  $7 \div 3 = 2$ and 1 remainder $= 2\frac{1}{3}$

  total being divided (dividend) → 7; size of equal groups (divisor) → 3; quotient → 2
- $a \div b \neq b \div a$ (generally)
  - The commutative law does not hold for division.
    For example: $8 \div 2 \neq 2 \div 8$
- $(a \div b) \div c \neq a \div (b \div c)$ (generally)
  - The associative law does not hold for division.
    For example: $(8 \div 4) \div 2 \neq 8 \div (4 \div 2)$
- The short division algorithm involves first dividing into the digit with the highest place value and then carrying any remainder to the next digit, working from left to right.
  For example:

  $413 \div 3 = 137$ and 2 remainder

  $= 137\frac{2}{3}$

  $$3\overline{)4^{1}1^{2}3}$$ with quotient 1 3 7

  $4 \div 3 = 1$ and 1 rem; $11 \div 3 = 3$ and 2 rem; $23 \div 3 = 7$ and 2 rem

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Key ideas

- Mental division can be done using different strategies.
  - Knowing your multiplication tables off by heart.
  - $63 \div 9 = \square$ is the same as asking $9 \times \square = 63$.
  - Making a convenient multiple of the divisor and then adjusting by adding or subtracting. Below is an application of the distributive law.

    For example:

    $$\begin{aligned} 84 \div 3 &= (60 + 24) \div 3 \\ &= (60 \div 3) + (24 \div 3) \\ &= 20 + 8 \\ &= 28 \end{aligned} \qquad \begin{aligned} 84 \div 3 &= (90 - 6) \div 3 \\ &= (90 \div 3) - (6 \div 3) \\ &= 30 - 2 \\ &= 28 \end{aligned}$$

  - Halving both numbers. If both numbers in the division are even, then halve both numbers.

    For example:

    $$\begin{aligned} 70 \div 14 &= 35 \div 7 \\ &= 5 \end{aligned}$$

Example 10 Using mental strategies for division

Use a mental strategy to find each quotient.

a $84 \div 7$ **b** $93 \div 3$ **c** $128 \div 8$

SOLUTION	EXPLANATION
a $84 \div 7 = 12$	$7 \times \square = 84$ (Use your knowledge from multiplication tables.)
b $93 \div 3 = 31$	$93 \div 3 = (90 \div 3) + (3 \div 3) = 30 + 1$ (This uses the distributive law.)
c $128 \div 8 = 16$	$128 \div 8 = 64 \div 4 = 32 \div 2 = 16$ (Halve both numbers repeatedly.)

Example 11 Using the short division algorithm

Use the short division algorithm to find each quotient and remainder.

a $3\overline{)37}$ **b** $7\overline{)195}$

SOLUTION

a

$$\begin{array}{r} 12 \\ 3\overline{)37} \end{array}$$

$37 \div 3 = 12$ and 1 remainder.

$= 12\frac{1}{3}$

b

$$\begin{array}{r} 2\ 7 \\ 7\overline{)19^{5}5} \end{array}$$

$195 \div 7 = 27$ and 6 remainder.

$= 27\frac{6}{7}$

EXPLANATION

a $3 \div 3 = 1$ with no remainder.
$7 \div 3 = 2$ with 1 remainder.

b 7 does not divide into 1.
$19 \div 7 = 2$ with 5 remainder.
$55 \div 7 = 7$ with 6 remainder.

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Exercise 1G

UNDERSTANDING AND FLUENCY	1–3, 4–5(½), 6, 7, 8(½)	3, 4–5(½), 6, 7, 8(½), 9	4–9(½)

1 Complete these statements.
- **a** $6 \times 7 = 42$, so $42 \div 7 =$ __ and $42 \div 6 =$ __
- **b** $3 \times 12 = 36$, so $36 \div 3 =$ __ and $36 \div 12 =$ __

2 What is the remainder when:
- **a** 2 is divided into 7?
- **b** 5 is divided into 37?
- **c** 42 is divided by 8?
- **d** 50 is divided by 9?

3 Write the missing digit in each of these divisions.
- **a** $\begin{array}{r} \square\,7 \\ 3\overline{)5\;1} \end{array}$
- **b** $\begin{array}{r} \square\,2 \\ 7\overline{)8\;4} \end{array}$
- **c** $\begin{array}{r} 2\,\square \\ 5\overline{)1\;2\;5} \end{array}$
- **d** $\begin{array}{r} 4\,\square \\ 3\overline{)1\;3\;5} \end{array}$

Example 10a

4 Use your knowledge of multiplication tables to find each quotient.
- **a** $28 \div 7$
- **b** $36 \div 12$
- **c** $48 \div 8$
- **d** $45 \div 9$
- **e** $56 \div 8$
- **f** $63 \div 7$
- **g** $96 \div 12$
- **h** $121 \div 11$

Example 10b

5 Find the answers to these using a mental strategy. Hint: Use the distributive law strategy.
- **a** $63 \div 3$
- **b** $76 \div 4$
- **c** $57 \div 3$
- **d** $205 \div 5$
- **e** $203 \div 7$
- **f** $189 \div 9$
- **g** $906 \div 3$
- **h** $490 \div 5$

Example 10c

6 Find the answers to these using a mental strategy. Hint: Use the halving strategy by halving both numbers.
- **a** $88 \div 4$
- **b** $124 \div 4$
- **c** $136 \div 8$
- **d** $112 \div 16$

7 Write the answers to these divisions, which involve 0s and 1s.
- **a** $26 \div 1$
- **b** $1094 \div 1$
- **c** $0 \div 7$
- **d** $0 \div 458$

Example 11

8 Use the short division algorithm to find each quotient and remainder. Check your answers using a calculator.
- **a** $3\overline{)71}$
- **b** $7\overline{)92}$
- **c** $2\overline{)139}$
- **d** $6\overline{)247}$
- **e** $4\overline{)2173}$
- **f** $3\overline{)61\,001}$
- **g** $5\overline{)4093}$
- **h** $9\overline{)90\,009}$

9 Use the short division algorithm and express the remainder as a fraction.
- **a** $526 \div 4$
- **b** $1691 \div 7$
- **c** $2345 \div 6$
- **d** $92\,337 \div 8$

PROBLEM-SOLVING AND REASONING	10, 11, 17	11–13, 17, 18	14–16, 19–22

10 If 117 food packs are divided equally among nine families, how many packs does each family receive?

11 Spring Fresh Company sells mineral water in packs of six bottles. How many packs are there in a truck containing 744 bottles?

12 A bricklayer earns $1215 in a week.
- **a** How much does he earn per day if he works Monday to Friday?
- **b** How much does he earn per hour if he works 9 hours per day Monday to Friday?

13 A straight fence has two end posts as well as as other posts that are divided evenly along the fence 4 metres apart. If the fence is to be 264 metres long, how many posts are needed, including the end posts?

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14 Friendly Taxis can take up to four passengers each. How many taxis are required to transport 59 people?

15 A truck can carry up to 7 tonnes of rock. What is the minimum number of trips needed to transport 130 tonnes of rock?

16 All the rows, columns and main diagonals in the magic square multiply to give 216. Can you find the missing numbers?

	9	12
		1

17 Write down the missing numbers.

a $37 \div 3 = 12$ and ☐ remainder means $37 =$ ☐ $\times 3 + 1$.

b $96 \div 7 =$ ☐ and 5 remainder means $93 = 13 \times$ ☐ $+ 5$.

c $104 \div 20 = 5$ and ☐ remainder means $104 =$ ☐ $\times 20 + 4$.

18 Pies are purchased wholesale at nine for \$4. How much will it cost to purchase 153 pies?

19 Give the results to these problems, if a represents any number other than 0.

a $a \div 1$ **b** $0 \div a$ **c** $a \div a$

20 A number less than 30 leaves a remainder of 3 when divided by 5 and a remainder of 2 when divided by 3. What two numbers meet the given conditions?

21 As you know $a \div b$ is not generally equal to $b \div a$. However, can you find a situation where $a \div b = b \div a$?

22 The short division algorithm can also be used to divide by numbers with more than one digit.

For example: $215 \div 12 = 17$ and 11 remainder.
$21 \div 12 = 1$ and 9 remainder.
$95 \div 12 = 7$ and 11 remainder.

$$\begin{array}{r} 1\ 7 \\ 12\overline{)21^{9}5} \end{array}$$

Use the short division algorithm to find each quotient and remainder and express each remainder as a fraction.

a $371 \div 11$ **b** $926 \div 17$ **c** $404 \div 13$
d $1621 \div 15$ **e** $2109 \div 23$ **f** $6914 \div 56$

ENRICHMENT	—	—	23, 24

Long, short division

23 Use the short division algorithm to find each quotient and remainder.

a $1247 \div 326$ **b** $1094 \div 99$ **c** $26401 \div 1432$

24 The magic product for this square is 6720. Find the missing numbers.

1	6		56
40		2	3
14			
			10

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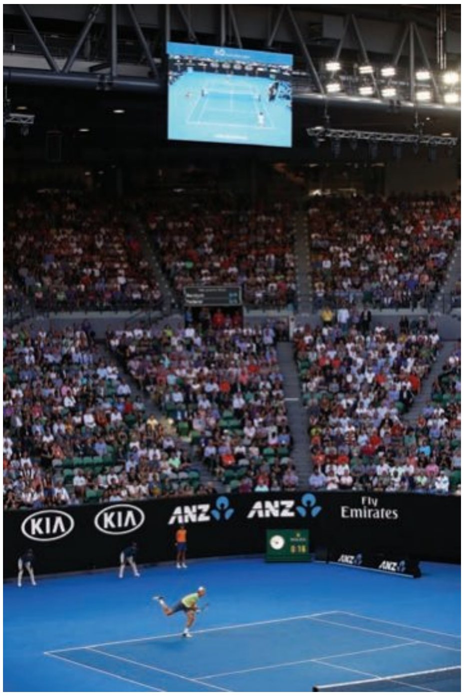
1H Estimating and rounding positive integers

Many theoretical and practical problems do not need precise or exact answers. In such situations reasonable estimations can provide enough informaton to solve the problem.

The total revenue from the Australian Open tennis tournament depends on crowd numbers. Estimates would be used before the tournament begins to predict these numbers. An estimate for the total revenue might be $8 million.



Let's start: The tennis crowd

Here is a photo of a crowd at a tennis match. Describe how you might estimate the number of people in the photo. What is your answer? How different is your answer from those of others in your class?

Key ideas

- Estimates or approximations to the answers of problems can be found by **rounding** numbers to the nearest 10, 100, 1000 etc.
  - If the next digit is 0, 1, 2, 3 or 4, then round down.
  - If the next digit is 5, 6, 7, 8 or 9, then round up.
- **Leading digit approximation** rounds the first digit to the nearest 10 or 100 or 1000 etc.
  For example: For 932 use 900 For 968 use 1000
- The symbol $\approx$ means **'is approximately equal to'**. The symbol $\doteqdot$ can also be used.

Example 12 Rounding

Round these numbers as indicated.

a 86 (to the nearest 10) **b** 4142 (to the nearest 100)

SOLUTION	EXPLANATION
a $86 \approx 90$	The digit after the 8 is greater than or equal to 5, so round up.
b $4142 \approx 4100$	The digit after the 1 is less than or equal to 4, so round down.

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Example 13 Using leading digit approximation

Estimate the answers to these problems by rounding each number to the leading digit.

a 42×7 **b** 95×326

SOLUTION

a $42 \times 7 \approx 40 \times 7$
$= 280$

b $95 \times 326 \approx 100 \times 300$
$= 30\,000$

EXPLANATION

The leading digit in 42 is the 4 in the 'tens' column.

The nearest 'ten' to 95 is 100, and the leading digit in 326 is in the 'hundreds' column.

Example 14 Estimating with operations

Estimate the answers to these problems by rounding both numbers as indicated.

a 115×92 (to the nearest 100) **b** $2266 \div 9$ (to the nearest 10)

SOLUTION

a $115 \times 92 \approx 100 \times 100$
$\therefore 115 \times 92 \approx 10\,000$

b $2266 \div 9 \approx 2270 \div 10$
$\therefore 2266 \div 9 \approx 227$

EXPLANATION

115 rounds to 100 and 92 rounds to 100.

2266 rounds to 2270 and 9 rounds to 10.

Exercise 1H

UNDERSTANDING AND FLUENCY	1, 2, 3–7(½)	2, 3–7(½)	3–7(½)

1 State whether these numbers have been rounded up or down.

a $59 \approx 60$ **b** $14 \approx 10$ **c** $137 \approx 140$
d $255 \doteqdot 260$ **e** $924 \doteqdot 900$ **f** $1413 \doteqdot 1000$

2 Which is larger?

a $58 + 97$ or $60 + 100$ **b** 24×31 or 20×30
c $130 - 79$ or $130 - 80$ **d** $267 - 110$ or $270 - 110$

Example 12

3 Round these numbers as indicated.

a 59 (nearest 10) **b** 32 (nearest 10) **c** 124 (nearest 10)
d 185 (nearest 10) **e** 231 (nearest 100) **f** 894 (nearest 100)
g 96 (nearest 10) **h** 584 (nearest 100) **i** 1512 (nearest 1000)

4 Round these numbers using leading digit approximation; i.e. round to the first digit.

a 21 **b** 29 **c** 136 **d** 857
e 5600 **f** 92 104 **g** 9999 **h** 14

5 Estimate the answers to these problems by first rounding both numbers as indicated.

a $72 + 59$ (nearest 10) **b** $138 - 61$ (nearest 10) **c** $275 - 134$ (nearest 10)
d $841 + 99$ (nearest 10) **e** $203 - 104$ (nearest 100) **f** $815 + 183$ (nearest 100)
g $990 + 125$ (nearest 100) **h** $96 + 2473$ (nearest 100) **i** $1555 - 555$ (nearest 1000)

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Example 13

6 Use leading digit approximation to estimate each answer.

a 29×4 **b** $124 + 58$ **c** $232 - 106$ **d** $61 \div 5$

e $394 \div 10$ **f** 97×21 **g** $1390 + 3244$ **h** $999 - 888$

Example 14

7 Estimate the answers to these problems by rounding both numbers as indicated.

a 29×41 (nearest 10) **b** 92×67 (nearest 10)

c 124×173 (nearest 100) **d** 2402×3817 (nearest 1000)

e $48 \div 11$ (nearest 10) **f** $159 \div 12$ (nearest 10)

g $104 \div 11$ (nearest 10) **h** $2493 \div 103$ (nearest 100)

PROBLEM-SOLVING AND REASONING	8, 9, 13	9–11, 13	10–12, 13c–d

8 Many examples of Aboriginal art include dot paintings. Estimate the number of dots in the painting shown below.

9 A digger can dig 29 scoops per hour and work 7 hours per day. Approximately how many scoops can be dug over 10 days?

10 Most of the pens at a stockyard are full of sheep. There are 55 pens and one of the pens has 22 sheep. Give an estimate for the total number of sheep at the stockyard.

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11 A whole year group of 159 students is roughly divided into 19 groups. Estimate the number in each group.

12 It is sensible sometimes to round one number up if the other number is going to be rounded down. Use leading digit approximation to estimate the answers to these problems.

a 11×19 **b** 129×954 **c** 25×36 **d** 1500×2500

13 The letters a and b represent numbers. Which of the words 'smaller' or 'larger' completes these sentences?

a If a and b are both rounded up, then compared to the true answer the approximate answer to:

i $a + b$ will be ________________. **ii** $a \times b$ will be ________________.

b If only a is rounded up, but b is left as it is, then compared to the true answer the approximate answer to:

i $a - b$ will be ________________. **ii** $a \div b$ will be ________________.

c If only b is rounded up, but a is left as it is, then compared to the true answer the approximate answer to:

i $a - b$ will be ________________. **ii** $a \div b$ will be ________________.

d If only b is rounded down, but a is left as it is, then compared to the true answer the approximate answer to:

i $a - b$ will be ________________. **ii** $a \div b$ will be ________________.

ENRICHMENT	—	—	14

Maximum error

14 When rounding numbers before a calculation is completed, it is most likely that there will be an error. This error can be large or small, depending on the type of rounding involved.

For example: when rounding to the nearest 10, $71 \times 11 \approx 70 \times 10 = 700$.

But $71 \times 11 = 781$, so the error is 81.

a Calculate the errors if these numbers are rounded to the nearest 10 before the multiplication is calculated.

i 23×17 **ii** 23×24 **iii** 65×54 **iv** 67×56

b Explain why the errors in parts **i** and **iii** are much less than the errors in parts **ii** and **iv**.

c Calculate the errors if these numbers are rounded to the nearest 10 before the division is calculated.

i $261 \div 9$ **ii** $323 \div 17$ **iii** $99 \div 11$ **iv** $396 \div 22$

d Explain why the approximate answers in parts **i** and **ii** are less than the correct answer, and why the approximate answers in parts **iii** and **iv** are more than the correct answer.

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1I Order of operations with positive integers

When combining the operations of addition, subtraction, multiplication and division, a particular order needs to be followed. Multiplication and division sit higher in the order than addition and subtraction, and this relates to how we might logically interpret simple mathematical problems put into words.

Consider these two statements.

- 2 groups of 3 chairs plus 5 chairs.
- 5 chairs plus 2 groups of 3 chairs.

In both cases, there are $2 \times 3 + 5 = 11$ chairs. This means that $2 \times 3 + 5 = 5 + 2 \times 3$.

This also suggests that for $5 + 2 \times 3$, the multiplication should be done first.

Let's start: Minimum brackets

- How might you use brackets to make this statement true?

  $2 + 3 \times 5 - 3 \div 6 + 1 = 2$
- What is the minimum number of pairs of brackets needed to make it true?

Key ideas

- When working with more than one operation complete the operations in the following order:
  - Always deal with **brackets** first.
    For example: $8 - (5 - 2)$ becomes $8 - 3$, not $3 - 2$
  - Do **multiplication** and **division** next, working from left to right.
    For example, $18 \div 3 \times 3$ becomes 6×3 not $18 \div 9$.
  - Do **addition** and **subtraction** last, working from left to right.
    For example: $8 - 2 + 3$ becomes $6 + 3$ not $8 - 5$

  For example: $4 \times (2 + 3) - 12 \div 6$ (1st: brackets)

  $= 4 \times 5 - 12 \div 6$ (2nd: 4×5)

  $= 20 - 12 \div 6$ (3rd: $12 \div 6$)

  $= 20 - 2$

  $= 18$

- Recall $(a + b) + c = a + (b + c)$ but $(a - b) - c \neq a - (b - c)$

  $(a \times b) \times c = a \times (b \times c)$ but $(a \div b) \div c \neq a \div (b \div c)$

- Brackets can sit inside other brackets.
  - Square brackets can also be used.
    For example: $[2 \times (3 + 4) - 1] \times 3$
  - Always deal with the inner brackets first.

- Note that some calculators apply the order of operations and some do not.
  For example, $5 + 2 \times 3 = 11$, not 21
  Try this on a variety of calculators and mobile phones.

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Example 15 Using order of operations

Use order of operations to evaluate these expressions.

a $5 + 10 \div 2$ **b** $18 - 2 \times (4 + 6) \div 5$

SOLUTION

a $5 + 10 \div 2 = 5 + 5$
$= 10$

b $18 - 2 \times (4 + 6) \div 5 = 18 - 2 \times 10 \div 5$
$= 18 - 20 \div 5$
$= 18 - 4$
$= 14$

EXPLANATION

Do the division before the addition.

Deal with brackets first. Do the multiplication and division next, working from left to right. Do the subtraction last.

Example 16 Using order of operations in worded problems

Find the difference between 76 and 43, triple this result and, finally, subtract the quotient of 35 and 7.

SOLUTION

$3 \times (76 - 43) - 35 \div 7 = 3 \times 33 - 5$
$= 99 - 5$
$= 94$

EXPLANATION

First, write the problem using symbols and numbers. Use brackets for the difference since this operation is to be completed first.

Exercise 1I

UNDERSTANDING AND FLUENCY	1–4, 5–6(½), 7, 8	2(½), 3, 4, 5–6(½), 7, 8	3, 4, 5–6(½), 7, 8

1 Write the following sentences into your book and place each of these words into one of the empty spaces.

parentheses	subtraction	multiplication
addition	brackets	division

a Brackets are also known as ________________________.

b In an expression containing mixed operations, always do the operations in the ___________________first.

c Next, do __________________ and _________________, working from left to right.

d Then, do __________________ and _________________, working from left to right.

2 In each statement, which operation (+, −, ÷ or ×) must be done first?

a $2 + 5 - 3$ **b** $5 \div 5 \times 2$ **c** $2 \times 3 \div 6$

d $5 \times 2 + 3$ **e** $7 \div 7 - 1$ **f** $(6 + 2) \times 3$

g $(8 \div 4) - 1$ **h** $4 + 7 \times 2$ **i** $8 - 10 \div 5$

j $10 - 2 + 3$ **k** $6 + 2 \times 3 - 1$ **l** $5 \times (2 + 3 \div 3) - 1$

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3 Label these statements as true or false.

a $5 \times 2 + 1 = (5 \times 2) + 1$ **b** $10 \times (3 + 4) = 10 \times 3 + 4$

c $21 - 7 \div 7 = (21 - 7) \div 7$ **d** $9 - 3 \times 2 = 9 - (3 \times 2)$

4 Find the value of:

a $3 + 4 \times 5 + 6$ **b** $(3 + 4) \times 5 + 6$

c $3 + 4 \times (5 + 6)$ **d** $(3 + 4) \times (5 + 6)$

Example 15a

5 Use order of operations to perform these computations. Use a calculator to check your answers.

a $2 + 3 \times 7$ **b** $5 + 8 \times 2$ **c** $10 - 20 \div 2$

d $22 - 16 \div 4$ **e** $6 \times 3 + 2 \times 7$ **f** $1 \times 8 - 2 \times 3$

g $18 \div 9 + 60 \div 3$ **h** $2 + 3 \times 7 - 1$ **i** $40 - 25 \div 5 + 3$

j $63 \div 3 \times 7 + 2 \times 3$ **k** $78 - 14 \times 4 + 6$ **l** $300 - 100 \times 4 \div 4$

Example 15b

6 Use order of operations to perform these computations. Use a calculator to check your answers.

a $2 \times (3 + 2)$ **b** $18 \div (10 - 4)$ **c** $(19 - 9) \div 5$

d $(100 + 5) \div 5 + 1$ **e** $2 \times (9 - 4) \div 5$ **f** $50 \div (13 - 3) + 4$

g $16 - 2 \times (7 - 5) + 6$ **h** $(7 + 2) \div (53 - 50)$ **i** $14 - (7 \div 7 + 1) \times 2$

j $(20 - 10) \times (5 + 7) + 1$ **k** $3 \times (72 \div 12 + 1) - 1$ **l** $48 \div (4 + 4) \times (3 \times 2)$

7 These computations involve brackets within brackets. Work with the inner brackets first.

a $2 \times [(2 + 3) \times 5 - 1]$

b $[10 \div (2 + 3) + 1] \times 6$

c $26 \div [10 - (17 - 9)]$

d $[6 - (5 - 3)] \times 7$

e $2 + [103 - (21 + 52)] - (9 + 11) \times 6 \div 12$

Example 16

8 Write these sentences using numbers and symbols and then complete the computation.

a Triple the sum of 3 and 6.

b Double the quotient of 20 and 4.

c The quotient of 44 and 11 plus 4.

d 5 more than the product of 6 and 12.

e The quotient of 60 and 12 is subtracted from the product of 5 and 7.

f 15 less than the difference of 48 and 12.

g The product of 9 and 12 is subtracted from double the product of 10 and 15.

PROBLEM-SOLVING AND REASONING	9, 10, 13	10–14	11–15

9 A delivery of 15 boxes of books arrives, each box containing eight books. The bookstore owner removes three books from each box. How many books still remain in total?

10 In a class, eight students have three TV sets at home, four have two TV sets, 13 have one TV set and two students have no TV sets. How many TV sets are there in total?

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11 Insert brackets into these statements to make them true.

a $4 + 2 \times 3 = 18$ **b** $9 \div 12 - 9 = 3$ **c** $2 \times 3 + 4 - 5 = 9$
d $3 + 2 \times 7 - 3 = 20$ **e** $10 - 7 \div 21 - 18 = 1$ **f** $4 + 10 \div 21 \div 3 = 2$
g $20 - 31 - 19 \times 2 = 16$ **h** $50 \div 2 \times 5 - 4 = 1$ **i** $25 - 19 \times 3 + 7 \div 12 + 1 = 6$

12 The amount of \$100 is divided into two first prizes of equal value and three second prizes of equal value. Each prize is a whole number of dollars and first prize is at least 4 times the value of the second prize. If second prize is more than \$6, find the amount of each prize.

13 Decide if the brackets given in each statement are actually necessary.

a $2 + (3 \times 6) = 20$ **b** $(2 + 3) \times 6 = 30$ **c** $(20 \times 2) \times 3 = 120$
d $10 - (5 + 2) = 3$ **e** $22 - (11 - 7) = 18$ **f** $19 - (10 \div 2) = 14$
g $(40 \div 10) \div 4 = 1$ **h** $100 \div (20 \div 5) = 25$ **i** $2 \times (3 + 2) \div 5 = 2$

14 The letters a, b and c represent numbers. Decide if the brackets are necessary in these expressions.

a $a + (b + c)$ **b** $a - (b - c)$ **c** $a \times (b \times c)$ **d** $a \div (b \div c)$

15 Write a simpler statement for these. Assume $a \neq 0$ and $b \neq 0$.

a $a + b - a$ **b** $(a - a) \times b$ **c** $a + b \div b$ **d** $a \times b \div a$

ENRICHMENT	—	—	16

Operation in rules

16 Using whole numbers and any of the four operations (+, −, ×, ÷), describe how you would obtain the 'Finish' number from the 'Start' number in each of these tables. Your rule must work for every pair of numbers in its table.

a

Start	Finish
1 →	3
2 →	5
3 →	7
4 →	9

b

Start	Finish
1 →	0
2 →	3
3 →	6
4 →	9

c

Start	Finish
3 →	10
4 →	17
5 →	26
6 →	37

Make up your own table with a 'secret' rule and test it on a friend.

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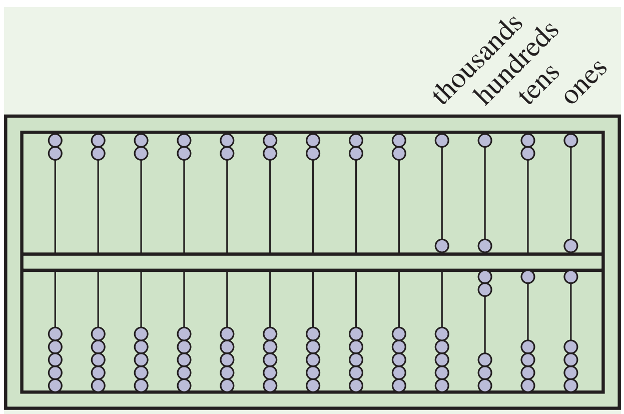
The abacus

The abacus is a counting device that has been used for thousands of years. They were used extensively by merchants, traders, tax collectors and clerks before modern-day number systems were developed. Counting boards called Abax date back to 500 BCE. These were wood or stone tablets with grooves, which would hold beans or pebbles.

The modern abacus is said to have originated in China in about the thirteenth century and includes beads on wires held in a wooden frame.

There are 5 beads on one side of a modern abacus worth 1 each and 2 beads on the opposite side worth 5 each.

- Each wire represents a different unit: ones, tens, hundreds etc.
- Beads are counted only when they are pushed toward the centre.



Here is a diagram showing the number 5716.

A modern abacus with thirteen wires

A German woodcut from 1508 showing an abacus in use by the man on the right, while a mathematician (at left) writes algorithms.

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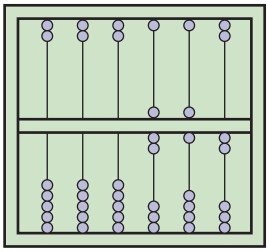
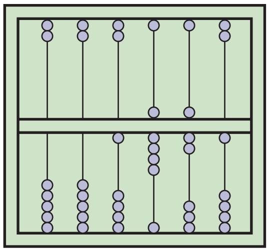
a What numbers are showing on the abacus diagrams below? Only the first six wires are showing.

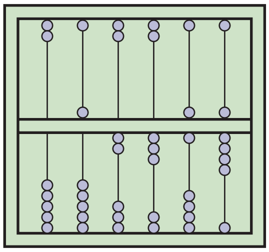
i



ii



iii



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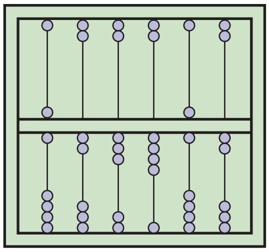

b Draw abacus diagrams showing these numbers.

i 57

ii 392

iii 6804

iv 290 316

c Imagine adding two numbers using an abacus by sliding beads along their wires. Clearly explain the steps taken to add these numbers.

i $11 + 7$

ii $2394 + 536$

d Imagine subtracting two numbers using an abacus by sliding beads along their wires. Clearly explain the steps taken to subtract these numbers.

i $23 - 14$

ii $329 - 243$

e Multiplication is calculated as a repeated addition.
For example: $3 \times 21 = 21 + 21 + 21$
Clearly explain the steps involved when using an abacus to multiply these numbers.

i 3×42

ii 5×156

f Division is calculated as a repeated subtraction.
For example: $63 \div 21 = 3$, since $63 - 21 - 21 - 21 = 0$
Clearly explain the steps involved when using an abacus to divide these numbers.

i $28 \div 7$

ii $405 \div 135$

g See if you can find a real abacus or computer abacus with which to work. Use the abacus to show how you can do the problems in Questions **c** to **f** above.

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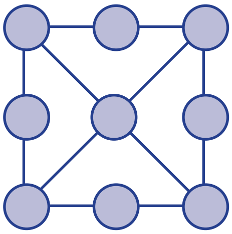
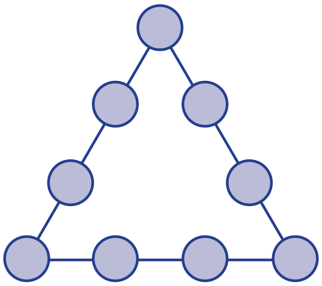
1 The extra dollar. The cost of dinner for two people is \$45 and they both give the waiter \$25 each. Of the extra \$5 the waiter is allowed to keep \$3 as a tip and returns \$1 to each person. So the two people paid \$24 each, making a total of \$48, and the waiter has \$3. The total is therefore $\$48 + \$3 = \$51$. Where did the extra \$1 come from?

2 The sum along each line is 15. Place each of the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 to make this true.



3 The sum along each side of this triangle is 17. Place each of the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 to make this true.



4 Make the total of 100 out of all the numbers 2, 3, 4, 7 and 11, using each number only once. You can use any of the operations ($+$, $-$, $\times$, $\div$), as well as brackets.

5 Sudoku is a popular logic number puzzle made up of a 9 by 9 square, where each column and row can use the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9 only once. Also, each digit is to be used only once in each 3 by 3 square. Solve these puzzles.

	4				2		8	7
2	8		7		9		1	
			6			3		5
	3	7		2				8
	6	5	4	7	8			2
			2	6				
					7	5		
		8	3	9		2	7	

	7			6	9	3		
		4	1		8			7
8					2	9	1	
3		1						
	2	8	5		3			
	5	6		9		2		
	3	9			5			
6				8	4			
5			9		7			

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Chapter summary

Whole numbers

Place value

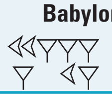
The place value of 3 in 1327 is 300.

$2 \times 100 + 7 \times 10 + 3 \times 1$ is the expanded form of 273.

Ancient number systems

Roman
LXXVI is 76
XCIV is 94

Babylonian



is 23
is 71

Egyptian

is 21
is 143

Addition and subtraction

Algorithms

$$\begin{array}{r} {}^{1}371 \\ +\,843 \\ \hline 1214 \end{array} \qquad \begin{array}{r} {}^{8}\not{9}{}^{1}37 \\ -\,643 \\ \hline 294 \end{array}$$

Mental strategies

$172 + 216 = 300 + 80 + 8$
$= 388$
$98 - 19 = 98 - 20 + 1$
$= 79$

Order of operations

Brackets first,
then
$\times$ and $\div$
(from left to right)
then
$+$ and $-$
(from left to right)

$2 \times (7 + 1) = 2 \times 8$
$= 16$
$8 - 10 \div 2 = 8 - 5$
$= 3$
$2 + 3 \times 4 \div (9 \div 3)$
$= 2 + 12 \div 3$
$= 2 + 4$
$= 6$

Multiplication and division

Algorithms

$$\begin{array}{r} {}^{2}29 \\ \times\,13 \\ \hline 87 \\ 290 \\ \hline 377 \end{array} \qquad \begin{array}{r} 6\;\;8 \\ 3\overline{)2\;0^{2}5} \end{array}$$

with 1 remainder

Mental strategies

$7 \times 31 = 7 \times 30 + 7 \times 1 = 217$
$5 \times 14 = 10 \times 7 = 70$
$64 \div 8 = 32 \div 4 = 16 \div 2 = 8$
$156 \div 4 = 160 \div 4 - 4 \div 4$
$= 40 - 1$
$= 39$

Multiplying by 10, 100, . . .

$38 \times 100 = 3800$
$38 \times 700 = 38 \times 7 \times 100$
$= 26\,600$

Estimation

955 to the nearest 10 is 960
950 to the nearest 100 is 1000

Leading digit approximation
$39 \times 326 \approx 40 \times 300 = 12\,000$

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Multiple-choice questions

1 Which of the following is equal to 24?

A $5 + 3 \times 3$ **B** $(5 + 3) \times 3$ **C** $5 \times 3 + 3$
D $5 \times (3 + 3)$ **E** $(5 - 3) \times 3$

2 $3 \times 1000 + 9 \times 10 + 2 \times 1$ is the expanded form of:

A 3920 **B** 392 **C** 3092
D 3902 **E** 329

3 Which of the following is not true?

A $2 + 3 = 3 + 2$ **B** $2 \times 3 = 3 \times 2$ **C** $(2 \times 3) \times 4 = 2 \times (3 \times 4)$
D $5 \div 2 \neq 2 \div 5$ **E** $7 - 2 = 2 - 7$

4 The sum of 198 and 103 is:

A 301 **B** 304 **C** 299
D 199 **E** 95

5 The difference between 378 and 81 is:

A 459 **B** 297 **C** 303
D 317 **E** 299

6 The product of 7 and 21 is:

A 147 **B** 141 **C** 21
D 140 **E** 207

7 The missing digit in this division is: $\begin{array}{r} 1\ \ 1\ \ 8 \\ 7\overline{)\square\ {}^{1}2\ {}^{5}6} \end{array}$

A 6 **B** 1 **C** 9
D 8 **E** 7

8 The remainder when 317 is divided by 9 is:

A 7 **B** 5 **C** 2
D 1 **E** 0

9 458 rounded to the nearest 100 is:

A 400 **B** 500 **C** 460
D 450 **E** 1000

10 The answer to $[2 + 3 \times (7 - 4)] \div 11$ is:

A 1 **B** 5 **C** 11
D 121 **E** 0

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Short-answer questions

1 The value of the digit 3 in the number 1325 is 300.
What is the value of the digit 3 in these numbers?

a 1235 **b** 3500 **c** 1375
d 235 000 **e** 3 500 000 **f** 1 350 000

2 Write down the place value of the digit 5 in each of these numbers.

a 357 **b** 5249 **c** 356 612

3 Use a mental strategy to find these sums and differences.

a $124 + 335$ **b** $687 - 324$ **c** $59 + 36$ **d** $256 - 39$

4 Use an algorithm and show your working for these sums and differences.

a $\begin{array}{r} 76 \\ +52 \\ \hline \end{array}$ **b** $\begin{array}{r} 1528 \\ +\ 796 \\ \hline \end{array}$ **c** $\begin{array}{r} 329 \\ -138 \\ \hline \end{array}$ **d** $\begin{array}{r} 2109 \\ -1814 \\ \hline \end{array}$

5 Use a mental strategy to perform these computations.

a 5×19 **b** 22×6 **c** 5×44
d $123 \div 3$ **e** $264 \div 8$ **f** $96 \div 4$
g 29×1000 **h** 36×300 **i** $14678 \div 1$

6 Use an algorithm and show your working for these computations.

a $\begin{array}{r} 157 \\ \times\ \ 9 \\ \hline \end{array}$ **b** $\begin{array}{r} 27 \\ \times 13 \\ \hline \end{array}$ **c** $7\overline{)327}$ **d** $4\overline{)30162}$

7 Find the missing digits in these computations.

a $\begin{array}{r} 2\,\square\,3 \\ +\ 7\,3\,\square \\ \hline 9\,6\,1 \end{array}$ **b** $\begin{array}{r} \square\,2\,\square \\ -\ 4\,\square\,3 \\ \hline 2\,5\,3 \end{array}$

c $\begin{array}{r} \square\ 3 \\ \times\ \ 2\ \square \\ \hline \square\ 7\ 1 \\ \square\ 0\ 6\ 0 \\ \hline \square\,\square\ 3\ 1 \end{array}$ **d** $\begin{array}{r} 1\ \square\ 3 \\ 5\overline{)\square\ {}^{4}1\ {}^{1}\square} \end{array}$

8 Round these numbers as indicated.

a 72 (nearest 10) **b** 3268 (nearest 100) **c** 951 (nearest 100)

9 Use leading digit approximation to estimate the answers to these computations.

a $289 + 532$ **b** 22×19 **c** 452×11 **d** $99 \div 11$

10 Use order of operations to find the answers to these computations.

a $3 \times (2 + 6)$ **b** $6 - 8 \div 4$ **c** $2 \times 8 - 12 \div 6$
d $(5 + 2) \times 3 - (8 - 7)$ **e** $0 \times (988234 \div 3)$ **f** $1 \times (3 + 2 \times 5)$

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Extended-response questions

1 A city tower construction uses 4520 tonnes of concrete trucked from a factory that is 7 kilometres from the construction site. Each truck can carry 7 tonnes of concrete. The concrete costs $85 per truck load for the first 30 loads and $55 per load after that.

a How many loads of concrete are needed? Add a full load for any remainder.

b Find the total distance travelled by the trucks to deliver all loads, assuming they need to return to the factory after each load.

c Find the total cost of the concrete needed to construct the tower.

d A different concrete supplier offers a price of $65 per 8-tonne truck, no matter how many loads are needed. Find the difference in the cost of concrete for the tower by this supplier compared to the original supplier.

2 One night Ricky and her brother Micky decided to have some fun at their father's sweet shop. In the shop they found 7 tins of 135 jelly beans each, 9 packets of 121 choc buds, 12 jars of 70 smarties and 32 packets of 5 liquorice sticks.

a Find the total number of sweets that Ricky and Micky found that night.

b Find the difference between the number of choc buds and the number of smarties.

c Ricky and Micky decided to divide each type of sweet into groups of 7 and then eat any remainder. Which type of sweet did they eat the most of and how many?

d After eating the remainders, they round the total of each sweet using leading digit approximation. If they rounded down they put the spare sweets in their pockets. If they rounded up they borrowed any spare sweets from their pockets. Any leftover in their pockets they ate. Did Ricky and Micky get to eat any more sweets?

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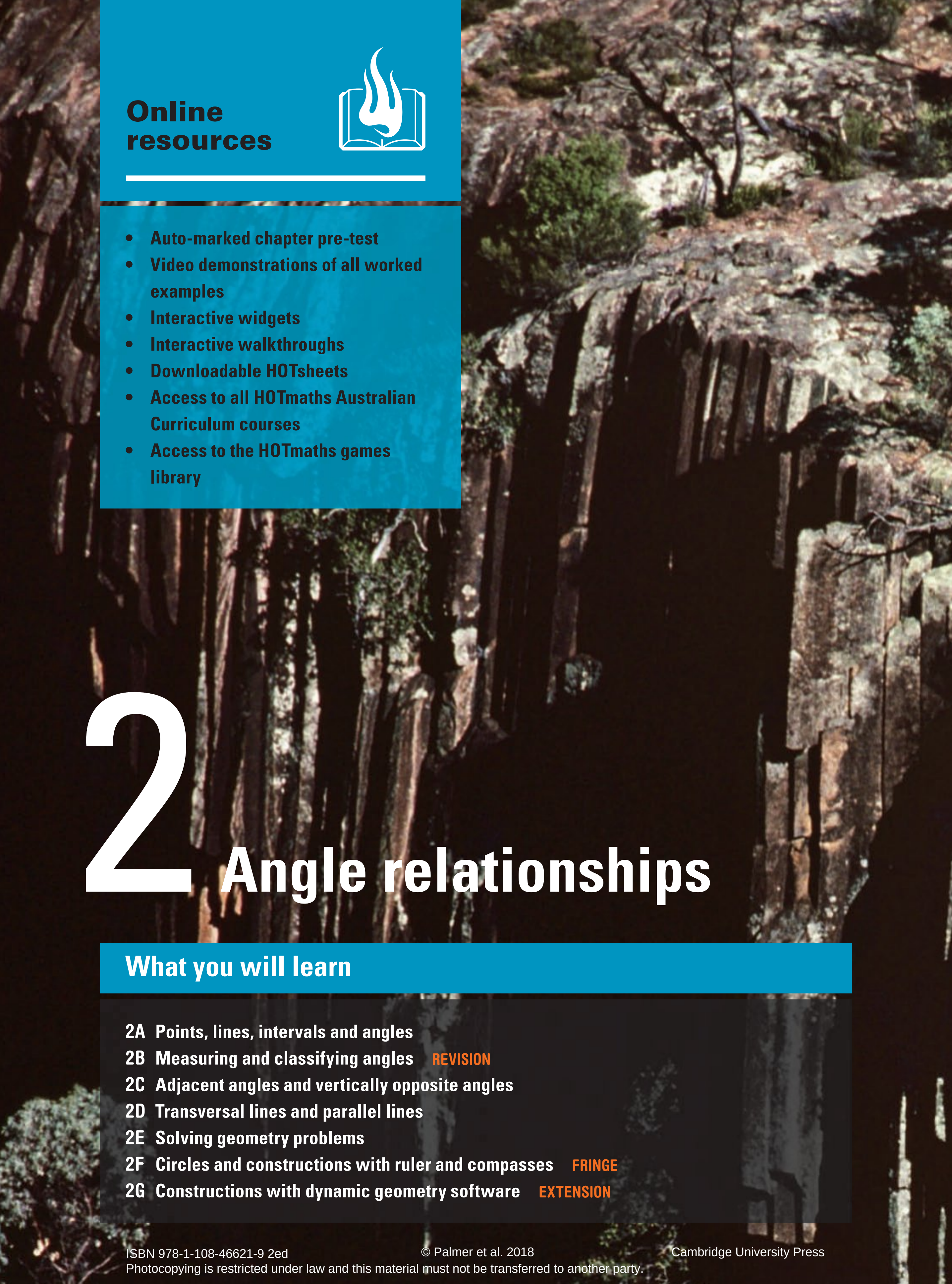

Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

2 Angle relationships

What you will learn

2A Points, lines, intervals and angles
2B Measuring and classifying angles REVISION
2C Adjacent angles and vertically opposite angles
2D Transversal lines and parallel lines
2E Solving geometry problems
2F Circles and constructions with ruler and compasses FRINGE
2G Constructions with dynamic geometry software EXTENSION

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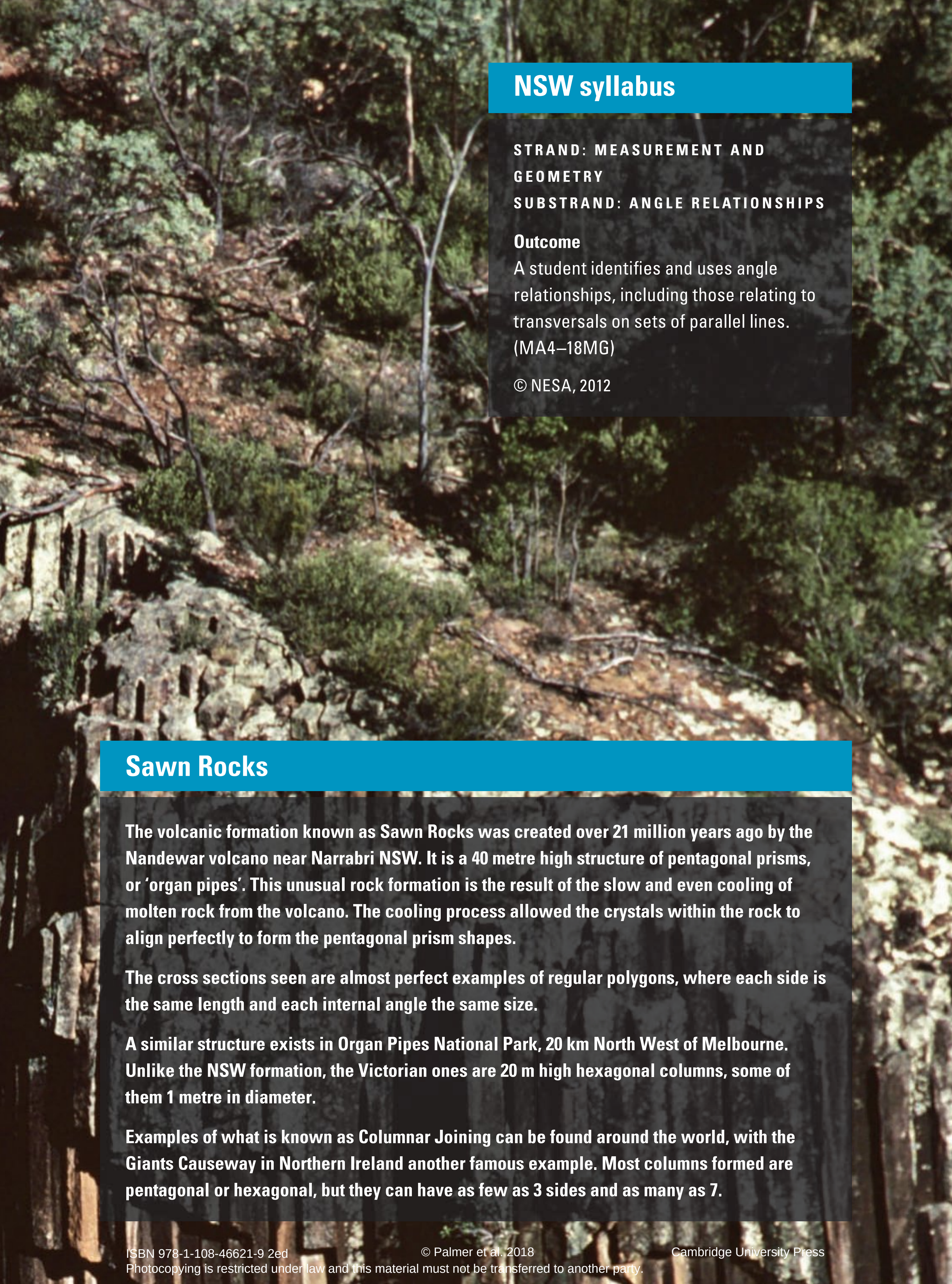


NSW syllabus

STRAND: MEASUREMENT AND GEOMETRY

SUBSTRAND: ANGLE RELATIONSHIPS

Outcome

A student identifies and uses angle relationships, including those relating to transversals on sets of parallel lines. (MA4–18MG)

Sawn Rocks

The volcanic formation known as Sawn Rocks was created over 21 million years ago by the Nandewar volcano near Narrabri NSW. It is a 40 metre high structure of pentagonal prisms, or 'organ pipes'. This unusual rock formation is the result of the slow and even cooling of molten rock from the volcano. The cooling process allowed the crystals within the rock to align perfectly to form the pentagonal prism shapes.

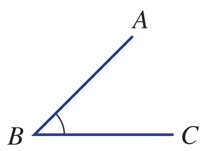
The cross sections seen are almost perfect examples of regular polygons, where each side is the same length and each internal angle the same size.

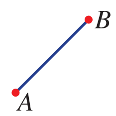
A similar structure exists in Organ Pipes National Park, 20 km North West of Melbourne. Unlike the NSW formation, the Victorian ones are 20 m high hexagonal columns, some of them 1 metre in diameter.

Examples of what is known as Columnar Joining can be found around the world, with the Giants Causeway in Northern Ireland another famous example. Most columns formed are pentagonal or hexagonal, but they can have as few as 3 sides and as many as 7.

Pre-test

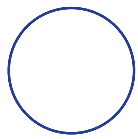
1 Here are some objects labelled I, II, III, IV, V and VI.

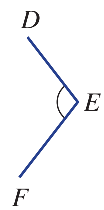
I


II


III


IV

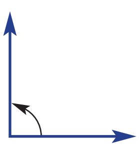

V


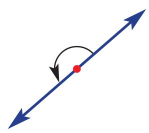
VI


Which object is:

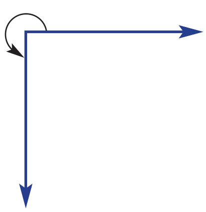
a a point?
b a line?
c a circle?
d a segment *AB* (or interval *AB*)?
e an angle *ABC*?
f an angle *DEF*?

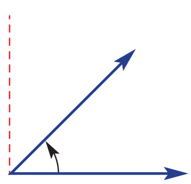
2 Estimate or guess the size of these angles. Remember there are 360° in a full circle.

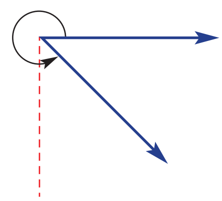
a


b


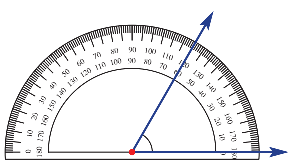
c

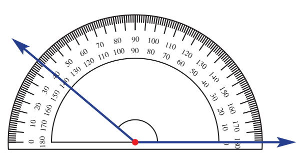

d


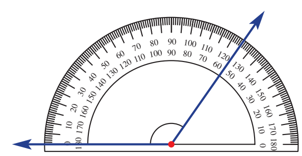
e


f


3 What angle measurements are shown on these protractors?

a


b


c


4 Do the following pairs of numbers add to 180°?

a 75°, 125°
b 132°, 48°
c 19°, 151°

5 Find the missing value in each of these simple statements.

a □ + 10° = 90°
b □ + 30° = 180°
c □ + 210° = 360°
d □ + 20° + 85° = 360°

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2A Points, lines, intervals and angles

The fundamental building blocks of geometry are the point, line and plane. They are the basic objects used to construct angles, triangles and other more complex shapes and objects. Points and lines do not actually occupy any area but can be represented on a page using drawing equipment.

Interactive

Widgets

HOTsheets

Walkthrough

Let's start: Geometry around you

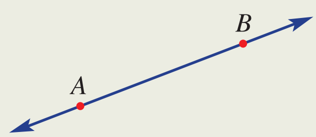
Take a look around the room you are in or consider any solid object near where you are seated (e.g. a book). Discuss what parts of the room or object could be described using:

- single points
- straight lines
- flat planes.

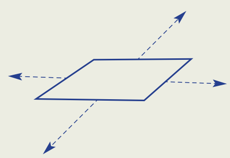
Key ideas

- A **point** is usually labelled with a capital letter.

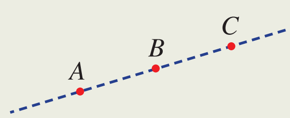
- A **line** passing through two points, *A* and *B*, can be called line *AB* or line *BA* and extends indefinitely in both directions.



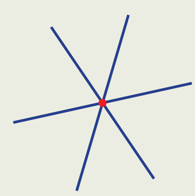
- A **plane** is a flat surface and extends indefinitely.



- Points that all lie on a single line are **collinear**.

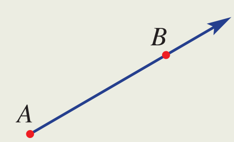


- If two lines meet, an **intersection point** is formed.
- Three or more lines that meet at the same point are **concurrent**.



- A **line segment** (or **interval**) is part of a line with a fixed length and end points. If the end points are *A* and *B* then it would be named line segment *AB* or line segment *BA* (or interval *AB* or interval *BA*).

- A **ray** *AB* is a part of a line with one end point *A* and passing through point *B*. It extends indefinitely in one direction.



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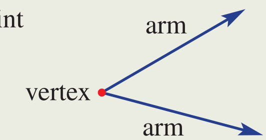
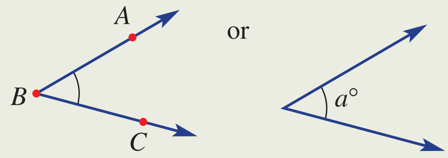
Key ideas

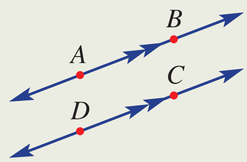
- When two rays (or lines) meet, an angle is formed at the intersection point called the **vertex**. The two rays are called **arms** of the angle.



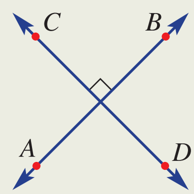
- An **angle** is named using three points, with the vertex as the middle point. A common type of notation is $\angle ABC$ or $\angle CBA$. The measure of the angle is $a°$, where a represents an unknown number.



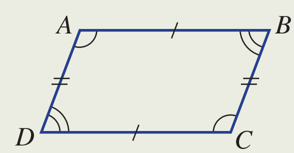
- Lower-case letters are often used to represent the number of degrees in an unknown angle.
- These two lines are parallel. This is written $AB \parallel DC$.



- These two lines are perpendicular. This is written $AB \perp CD$.



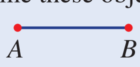
- The markings on this diagram show that $AB = CD$, $AD = BC$, $\angle BAD = \angle BCD$ and $\angle ABC = \angle ADC$.



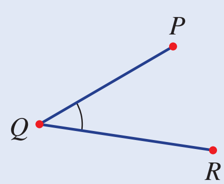
Example 1 Naming objects

Name these objects.

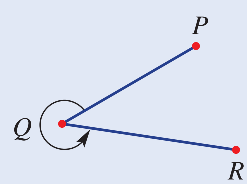
a



b



c



SOLUTION	EXPLANATION
a line segment AB	Line segment BA, interval AB or interval BA are also acceptable.
b $\angle PQR$	Point Q is the vertex and sits in between P and R. $\angle RQP$ is also correct.
c reflex $\angle PQR$	In diagrams **b** and **c**, there are two different angles called PQR: one is acute and the other is reflex. The word 'reflex' is used in situations such as this.

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Exercise 2A

UNDERSTANDING AND FLUENCY	1–8	4–9	5–10

1 Draw a diagram for each of the following objects.

a a point P **b** a line AN **c** an angle $\angle ABC$
d a ray ST **e** a plane **f** three collinear points A, B and C

2 Draw diagrams to show:

a three collinear points **b** three concurrent lines

3 Match the words *line, segment* or *ray* to the correct description.

a Starts from a point and extends indefinitely in one direction.
b Extends indefinitely in both directions, passing through two points.
c Starts and ends at two points.

4 Match the words *point, line* or *plane* with the following descriptions.

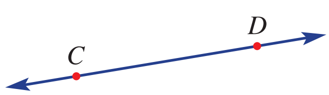
a the edge of a sheet of paper **b** a flat wall
c the surface of a pool of water on a calm day **d** where two walls and a floor meet in a room
e where two walls meet in a room **f** one side of a cereal packet
g where two sides meet on a box **h** where three sides meet on a box

Example 1

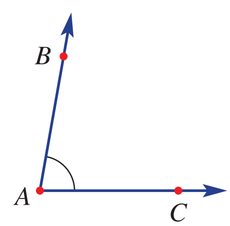
5 Name the following objects.

a

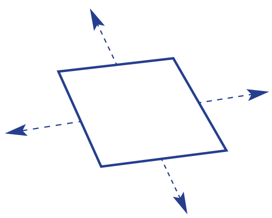
b



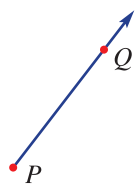
c



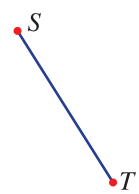
d



e

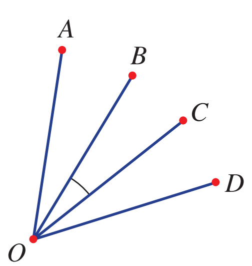


f

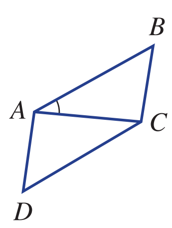


6 Use three letters to name the angle marked in these diagrams.

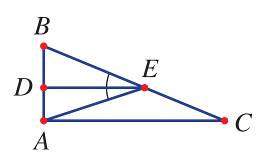
a



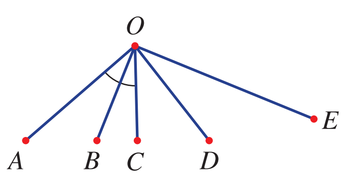
b



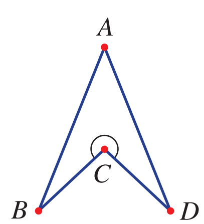
c



d



e



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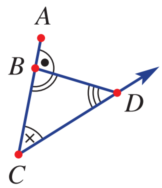
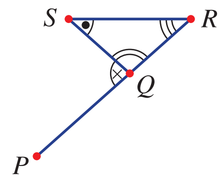
7 For each diagram, name the five line segments and the four marked angles using the given labels.

a

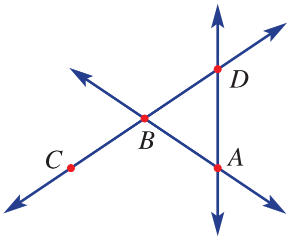


b

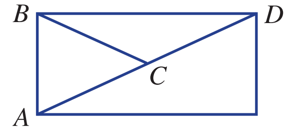


8 Name the set of three labelled points that are collinear in each of these diagrams.

a

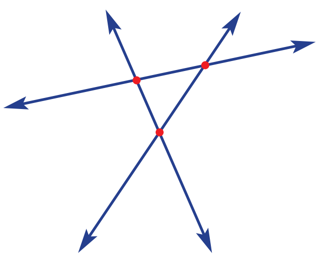


b

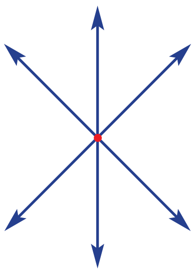


9 State whether the following sets of lines are concurrent.

a

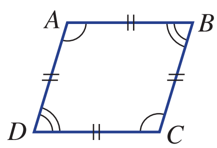


b

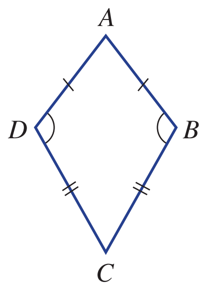


10 In the following diagrams, name the equal sides and equal angles.

a



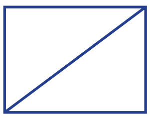
b



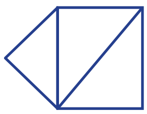
PROBLEM-SOLVING AND REASONING	11, 14	11, 12, 14	12–15

11 Count the number of angles formed inside these shapes. Count all angles, including ones that may be the same size and those angles that are divided by another segment.

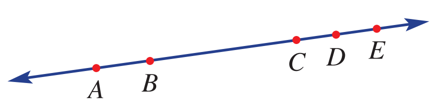
a



b



12 How many line segments are there on this line? Do not count *AB* and *BA* as they represent the same segment.



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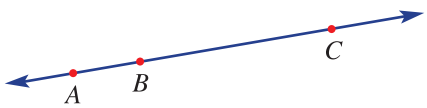
13 A line contains a certain number of labelled points.
For example, this line has three points.

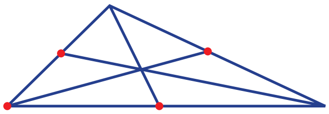


a Complete this table by counting the total number of segments for the given number of labelled points.

Number of points	1	2	3	4	5	6
Number of segments						

b Explain any patterns you see in the table. Is there a quick way of finding the next number in the table?

14 The lines joining each vertex (corner) of a triangle with the midpoint (middle point) of the opposite side are drawn here.



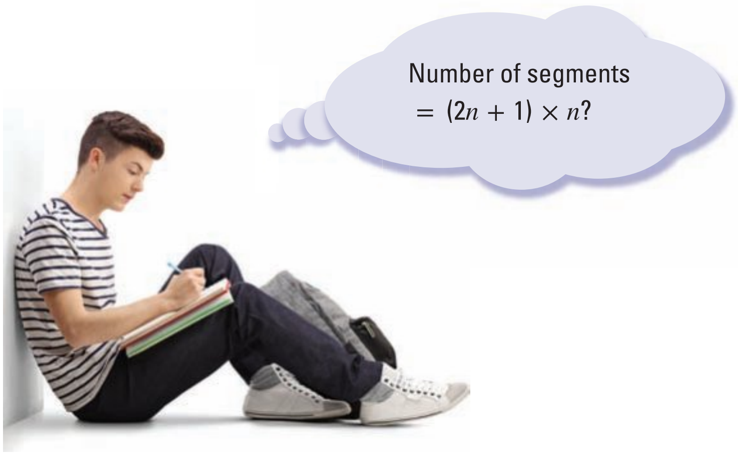
a Draw any triangle and use a ruler to measure and mark the midpoints of each side.
b Join each vertex with the midpoint of the opposite side.
c Are your segments from part **b** concurrent?
d Do you think your answer to part **c** will always be true for any triangle? Try one other triangle of a different size to check.

15 **a** If points A, B and C are collinear and points A, B and D are collinear, does this mean that points B, C and D are also collinear?
b If points A, B and C are collinear and points C, D and E are collinear, does this mean that points B, C and D are also collinear?

ENRICHMENT	—	—	16

The general rule

16 In Question **13** you may have determined a quick method of finding the number of segments for the given number of points. If n is the number of points on the line, can you find a rule (in terms of n) for the number of segments? Test your rule to see if it works for at least three cases.



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2B Measuring and classifying angles REVISION

Angles are usually described using the unit of measurement called the degree, where 360 degrees (360°) describes one full turn. The idea to divide a circle into 360° dates back to the Babylonians, who used a sexagesimal number system based on the number 60. Because both 60 and 360 are numbers that have a large number of factors, many fractions of these numbers are very easy to calculate.

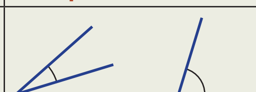
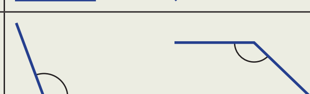
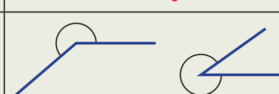
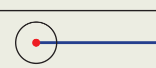
Let's start: Estimating angles

How good are you at estimating the size of angles? Estimate the size of these angles and then check with a protractor.

Alternatively, construct an angle using computer geometry. Estimate and then check your angle using the angle-measuring tool.

Key ideas

- Angles are classified according to their size.

Angle type	Size	Examples
acute	between 0° and 90°	
right	90°	
obtuse	between 90° and 180°	
straight	180°	
reflex	between 180° and 360°	
revolution	360°	

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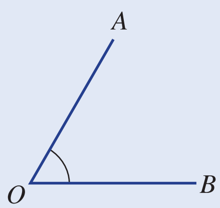
Key ideas

- A **protractor** can be used to measure angles to within an accuracy of about half a degree. Some protractors have increasing scales marked both clockwise and anticlockwise from zero. To use a protractor follow these steps:
  1. Place the centre of the protractor on the vertex of the angle.
  2. Align the base line of the protractor along one arm of the angle.
  3. Measure the angle using the other arm and the scale on the protractor.
  4. A reflex angle can be measured by subtracting a measured angle from 360°.

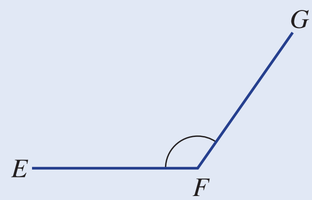
Example 2 Measuring with a protractor

For the angles shown, state the type of angle and measure its size.

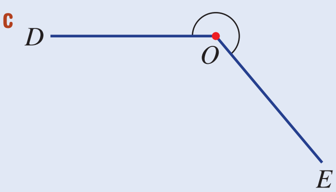
a



b



c



SOLUTION

a acute $\angle AOB = 60°$

b obtuse $\angle EFG = 125°$

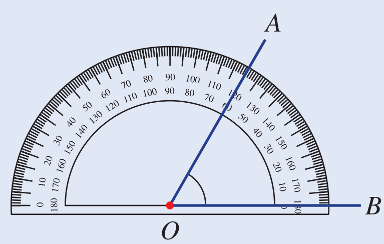
c reflex

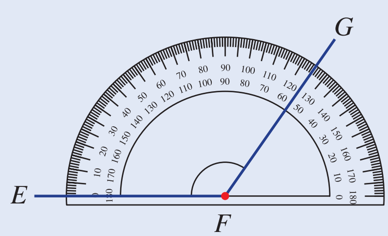
obtuse $\angle DOE = 130°$

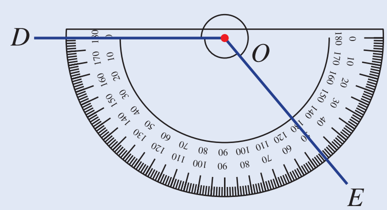
reflex $\angle DOE = 360° - 130°$

$= 230°$

EXPLANATION







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Example 3 Drawing angles

Use a protractor to draw each of the following angles.

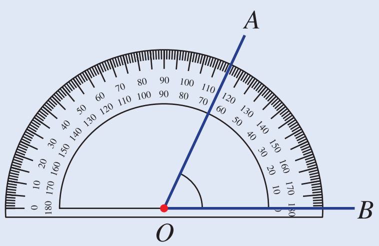
a $\angle AOB = 65^\circ$

b $\angle WXY = 130^\circ$

c $\angle MNO = 260^\circ$

SOLUTION

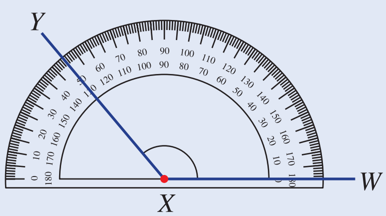
a



EXPLANATION

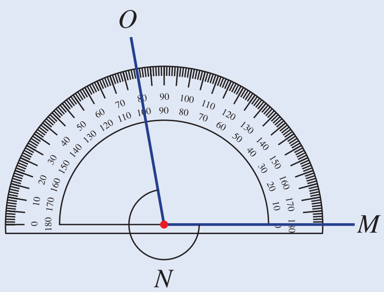
Step 1: Draw a base line *OB*.
Step 2: Align the protractor along the base line with the centre at point *O*.
Step 3: Measure 65° and mark a point, *A*.
Step 4: Draw the arm *OA*.

b



Step 1: Draw a base line *XW*.
Step 2: Align the protractor along the base line with the centre at point *X*.
Step 3: Measure 130° and mark a point, *Y*.
Step 4: Draw the arm *XY*.

c



Step 1: Draw an angle of $360^\circ - 260^\circ = 100^\circ$.
Step 2: Mark the reflex angle on the opposite side to the obtuse angle of 100°.
Alternatively, draw a 180° angle and measure an 80° angle to add to the 180° angle.

Exercise 2B REVISION

UNDERSTANDING AND FLUENCY	1–5, 6(½)	3–5, 6(½), 7	4(½), 5, 6–7(½), 8

1 Without using a protractor, draw an example of the following types of angles.

- **a** acute
- **b** right
- **c** obtuse
- **d** straight
- **e** reflex
- **f** revolution

2 How many right angles (i.e. angles of 90°) make up:

- **a** a straight angle?
- **b** 270°?
- **c** a revolution?

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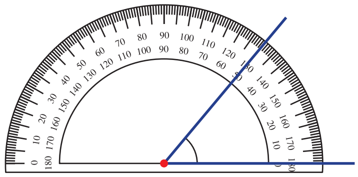
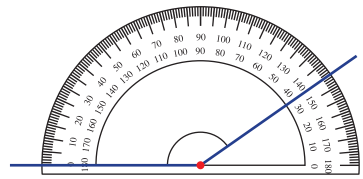
3 What is the size of the angle measured with these protractors?

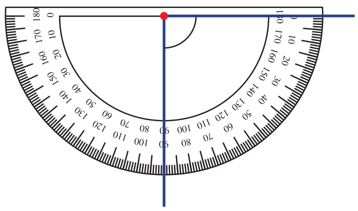
a



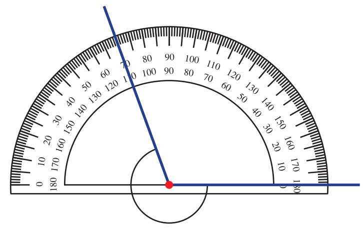
b



c



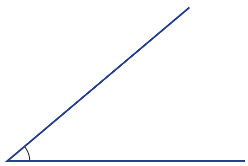
d



Example 2

4 For the angles shown, state the type of angle and measure its size.

a



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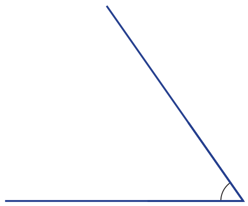

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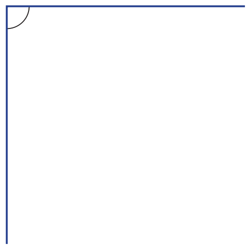

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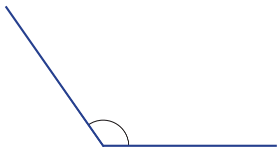

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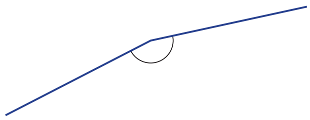

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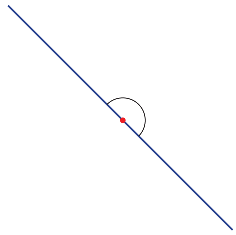
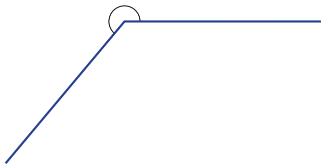


g



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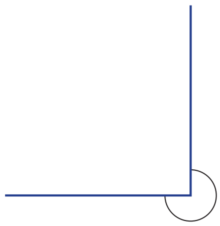
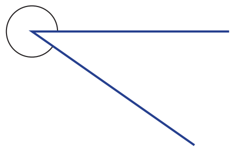


i



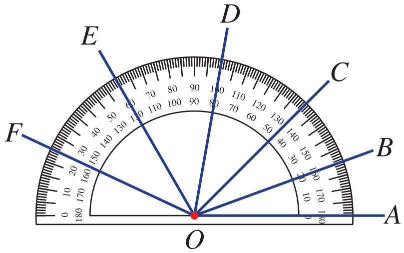
5 **a** Write down the size of the angles shown on this protractor.

i $\angle AOB$ **ii** $\angle BOC$

iii $\angle COD$ **iv** $\angle DOE$

v $\angle EOF$

b Find the sum of all the angles from part **a**. Name a single angle in the diagram that equals this sum.



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Example 3

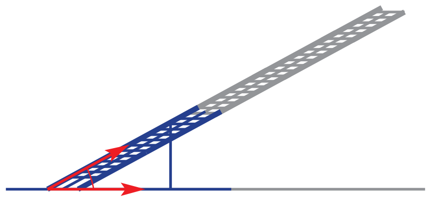
6 Use a protractor to draw each of the following angles.

a 40° **b** 75° **c** 90° **d** 135° **e** 175°

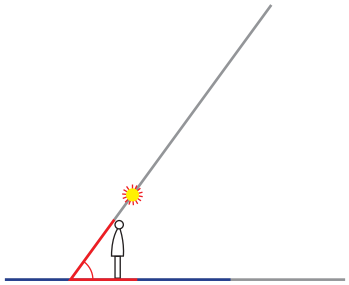
f 205° **g** 260° **h** 270° **i** 295° **j** 352°

7 For each of the angles marked in the situations shown, measure:

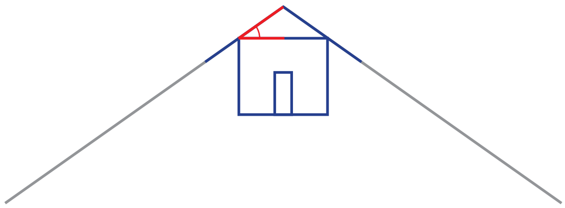
a the angle that this ramp makes with the ground



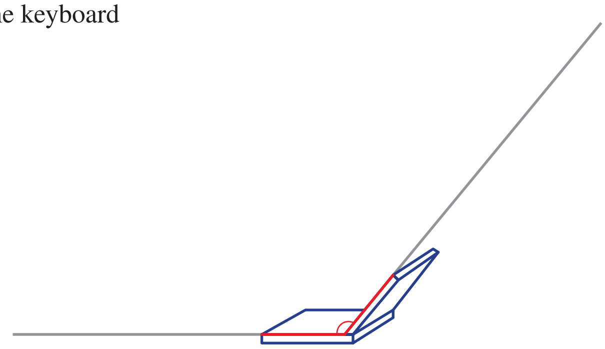
b the angle the sun's rays make with the ground



c the angle or pitch of this roof



d the angle between this laptop screen and the keyboard



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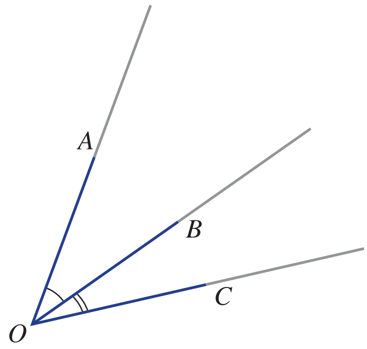
8 In the diagram shown at right, there are two marked angles, $\angle AOB$ and $\angle BOC$. Measure $\angle AOB$, $\angle BOC$ and $\angle AOC$. Does $\angle AOB + \angle BOC = \angle AOC$? Why or why not?



PROBLEM-SOLVING AND REASONING	9, 11	9–11	9–12

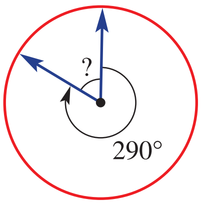
9 A clock face is numbered 1 to 12. Find the angle the minute hand turns in:

a 30 minutes **b** 1 hour **c** 15 minutes **d** 45 minutes
e 5 minutes **f** 20 minutes **g** 55 minutes **h** 1 minute
i 9 minutes **j** 10.5 minutes **k** 42.5 minutes **l** 21.5 minutes

10 A clock face is numbered 1 to 12. Find the angle between the hour hand and the minute hand at:

a 6 p.m. **b** 3 p.m. **c** 4 p.m. **d** 11 a.m.

11 The arrow on this dial starts in an upright position. It then turns by a given number of degrees clockwise or anticlockwise.



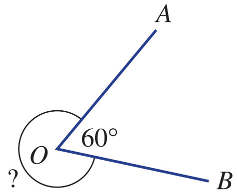
a Find the angle between the arrow in its final position with the arrow in its original position, as shown in the diagram opposite, which illustrates part **i**. Answer with an acute or obtuse angle.

i 290° clockwise **ii** 290° anticlockwise
iii 450° clockwise **iv** 450° anticlockwise
v 1000° clockwise **vi** 1000° anticlockwise

b Did it matter to the answer if the dial was turning clockwise or anticlockwise?

c Explain how you calculated your answer for turns larger than 360°.

12 An acute angle $\angle AOB$ is equal to 60°. Why is it unnecessary to use a protractor to work out the size of the reflex angle $\angle AOB$?



ENRICHMENT	—	—	13

Time challenge

13 Find the angle between the hour hand and the minute hand of a clock at these times.

a 10:10 a.m. **b** 4:45 a.m. **c** 11:10 p.m.
d 2:25 a.m. **e** 7:16 p.m. **f** 9:17 p.m.

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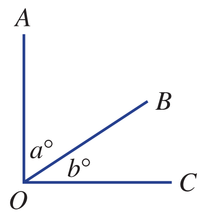
2C Adjacent angles and vertically opposite angles

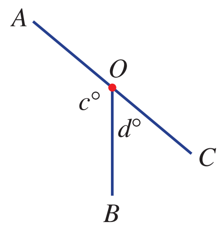
Not all angles in a diagram or construction need to be measured directly. Special relationships exist between pairs of angles at a point and this allows some angles to be calculated exactly without measurement, even if diagrams are not drawn to scale.

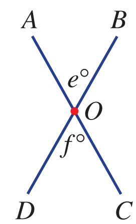
Interactive
Widgets
HOTsheets
Walkthrough

Let's start: Special pairs of angles

By making a drawing or using computer geometry, construct the diagrams below. Measure the two marked angles. What do you notice about the two marked angles?

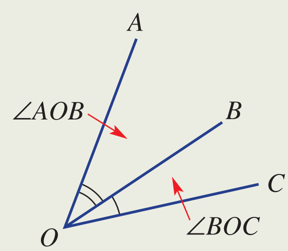




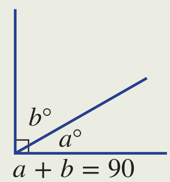


Key ideas

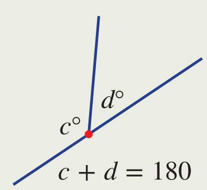
- **Adjacent** angles are side by side and share a vertex and an arm.



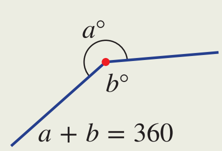
- Two adjacent angles in a right angle are **complementary**.
  They add to 90°.
  If the value of a is 30, then the value of b is 60 because $30° + 60° = 90°$.
  30° is the complement of 60°.



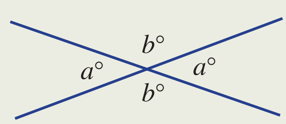
- Two adjacent angles on a straight line are **supplementary**.
  They add to 180°.
  If the value of $c = 130$, then the value of d is 50 because $130° + 50° = 180°$.
  130° is the supplement of 50°.



- Angles in a **revolution** have a sum of 360°.



- **Vertically opposite** angles are formed when two lines intersect.
  The opposite angles are equal.



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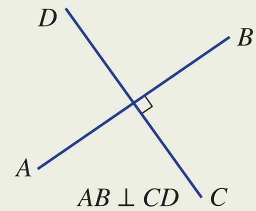
- **Perpendicular** lines meet at right angles (90°).

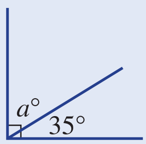


Key ideas

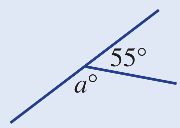
Example 4 Finding angles at a point

Without using a protractor, find the size of each angle marked with the letter a.

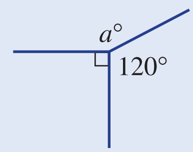
a



b



c

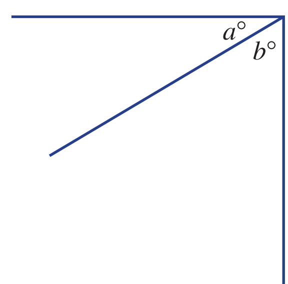


SOLUTION	EXPLANATION
a $a + 35 = 90$ $a = 55$	Angles in a right angle add to 90. $90 - 35 = 55$
b $a + 55 = 180$ $a = 125$	Angles on a straight line add to 180. $180 - 55 = 125$
c $a + 90 + 120 = 360$ $a + 210 = 360$ $a = 150$	The sum of angles in a revolution is 360. Simplify by adding 90 and 120. a is the difference between 210 and 360.

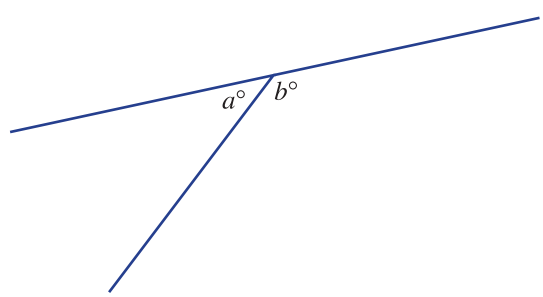
Exercise 2C

UNDERSTANDING AND FLUENCY	1–7	4, 5–6(½), 7	5–6(½), 7, 8(½)

1 **a** Measure the two marked angles in this diagram.
b Calculate $a + b$. Is your answer 90? If not, check your measurements.
c Write the missing word: $a°$ and $b°$ are ____________ angles.



2 **a** Measure the two marked angles in this diagram.
b Calculate $a + b$. Is your answer 180? If not, check your measurements.
c Write the missing word: $a°$ and $b°$ are ____________ angles.



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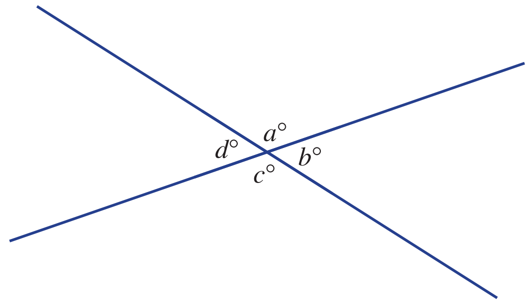
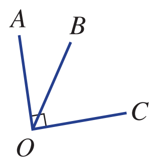
3 **a** Measure the four marked angles in this diagram.

b What do you notice about the sum of the four angles?

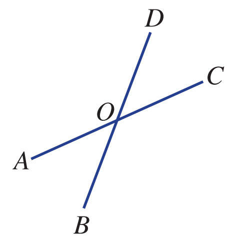
c Write the missing words: b and d are _______ _______ angles.



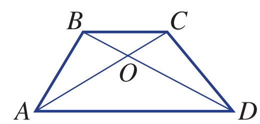
4 **a** Name the angle that is complementary to $\angle AOB$ in this diagram.



b Name the two angles that are supplementary to $\angle AOB$ in this diagram.



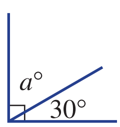
c Name the angle that is vertically opposite to $\angle AOB$ in this diagram.



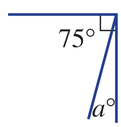
Example 4

5 Without using a protractor, find the value of the letter a in each angle. (The diagrams shown may not be drawn to scale.)

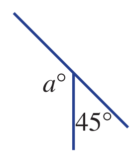
a



b



c



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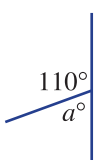
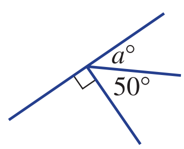


e



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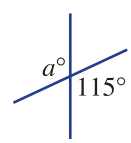

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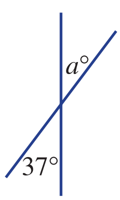

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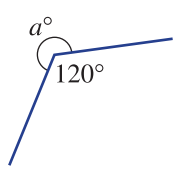
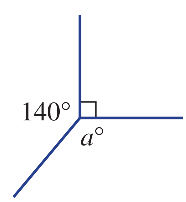


i



6 For each of the given pairs of angles, write C if they are complementary, S if they are supplementary or N if they are neither.

a 21°, 79° **b** 130°, 60° **c** 98°, 82° **d** 180°, 90°

e 17°, 73° **f** 31°, 59° **g** 68°, 22° **h** 93°, 87°

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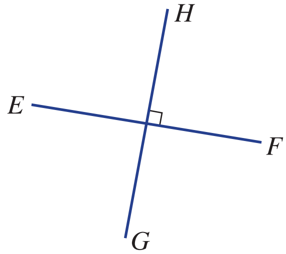
7 Write a statement like $AB \perp CD$ for these pairs of perpendicular line segments.

a


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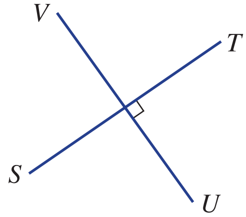
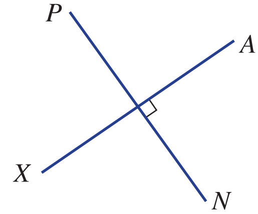
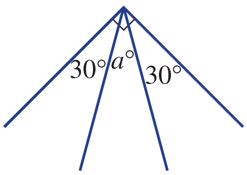


c


8 Without using a protractor, find the value of a in these diagrams.

a


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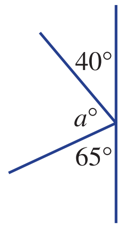

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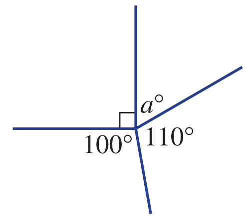

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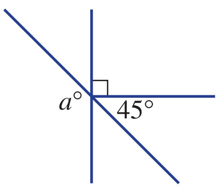

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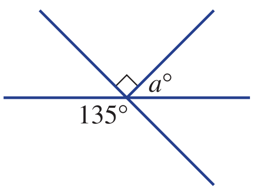

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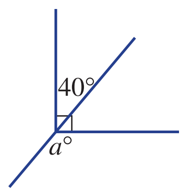
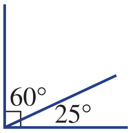


PROBLEM-SOLVING AND REASONING	9, 12	9, 10, 12	10–13

9 Do these diagrams have the correct information? Give reasons.

a


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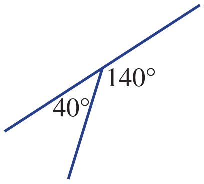

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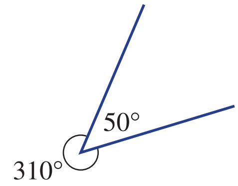

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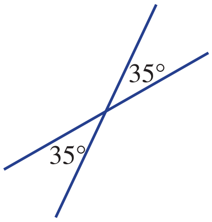

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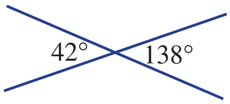

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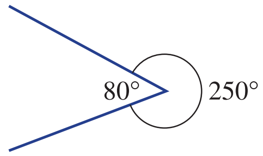

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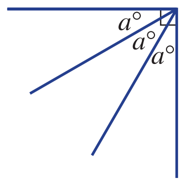
10 Find the value of a in these diagrams.

a


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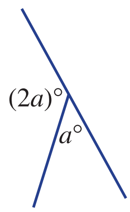

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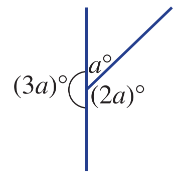

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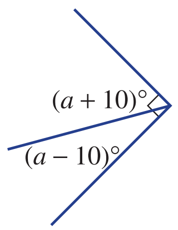
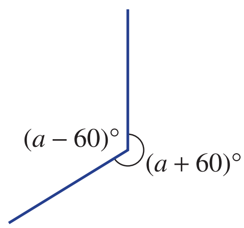
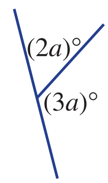
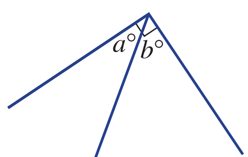


e


f


11 A pizza is divided between four people. Bella is to get twice as much as Bobo, who gets twice as much as Rick, who gets twice as much as Marie. Assuming the pizza is cut into triangular pieces, find the angle at the centre of the pizza for Marie's piece.

12 Write down a rule connecting the letters in these diagrams. For example: $a + b = 180$.

a


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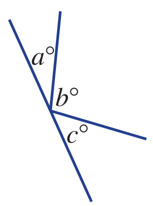
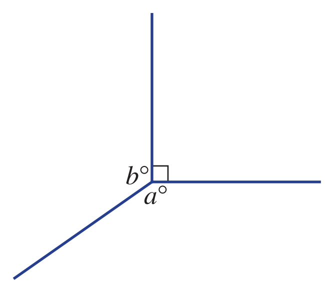
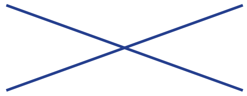


c


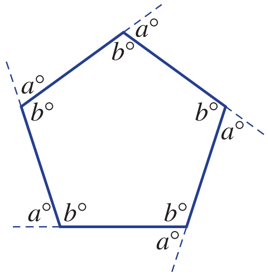
13 What is the minimum number of angles needed in this diagram to determine all other angles? Explain your answer.



ENRICHMENT	—	—	14

Pentagon turns

14 Consider walking around a path represented by this regular pentagon. All sides have the same length and all internal angles are equal. At each corner (vertex) you turn an angle of a, as marked.



a How many degrees would you turn in total after walking around the entire shape? Assume that you face the same direction at the end as you did at the start.

b Find the value of a.

c Find the value of b.

d Explore the outside and inside angles of other regular polygons using the same idea. Complete this table to summarise your results.

Regular shape	a	b
triangle		
square		
pentagon		
hexagon		
heptagon		
octagon		

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2D Transversal lines and parallel lines

Interactive
Widgets
HOTsheets
Walkthrough

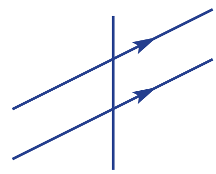
When a line, called a transversal, cuts two or more other lines, a number of angles are formed. Pairs of these angles are either corresponding, alternate or cointerior angles, depending on their relative position. If the transversal cuts two parallel lines then there is a relationship between the sizes of these special pairs of angles.

Let's start: What's formed by a transversal?

Draw a pair of parallel lines using either:

- two sides of a ruler or
- computer geometry (parallel line tool).

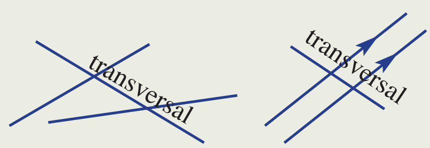
Then cross the two lines with a third line (transversal) at any angle.



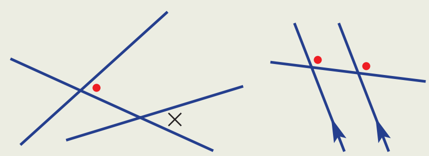
Measure each of the eight angles formed and discuss what you find. If computer geometry is used, drag the transversal and see if your observations apply to all the cases that you observe.

Key ideas

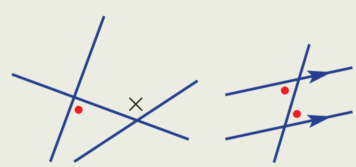
- A **transversal** is a line intersecting two or more other lines that are usually, but not necessarily, parallel.



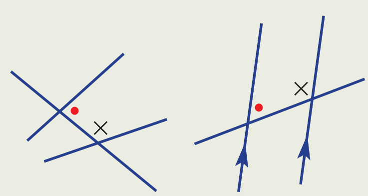
- A transversal crossing two lines will form special pairs of angles. These are:
  - **corresponding** (in corresponding positions)



  - **alternate** (on opposite sides of the transversal and inside the other two lines)



  - **cointerior** (on the same side of the transversal and inside the other two lines).



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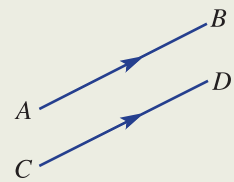
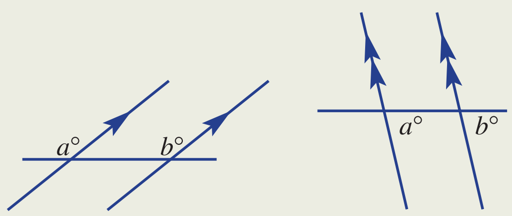
Key ideas

- **Parallel lines** are marked with the same arrow set.
  - If AB is parallel to CD then we write $AB \| CD$.



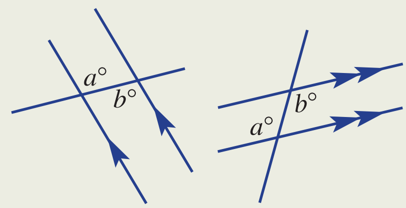
- If a transversal crosses two **parallel** lines, then:
  - corresponding angles are equal
  - alternate angles are equal
  - cointerior angles are supplementary (i.e. sum to 180°).

corresponding



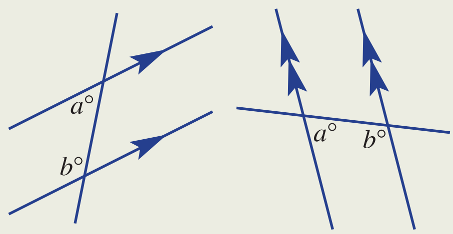
$a = b$ $a = b$

alternate



$a = b$ $a = b$

cointerior



$a + b = 180$ $a + b = 180$

Example 5 Naming pairs of angles

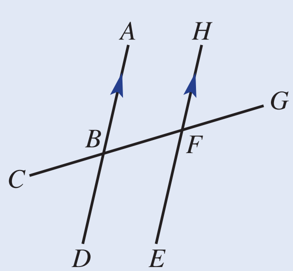
In the diagram below, name the angle that is:

a corresponding to $\angle ABF$

b alternate to $\angle ABF$

c cointerior to $\angle ABF$

d vertically opposite to $\angle ABF$



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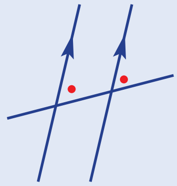
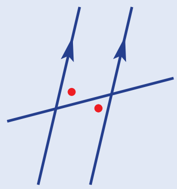
SOLUTION | **EXPLANATION**

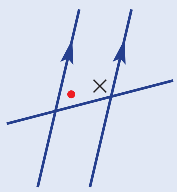
a ∠*HFG*



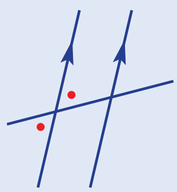
b ∠*EFB*



c ∠*HFB*



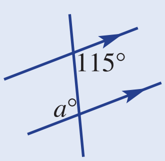
d ∠*CBD*



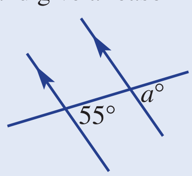
Example 6 Finding angles in parallel lines

Find the value of a in these diagrams and give a reason for each answer.

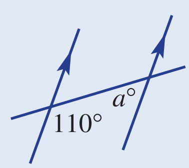
a



b



c



SOLUTION | **EXPLANATION**

a $a = 115$
alternate angles in parallel lines

Alternate angles in parallel lines are equal.

b $a = 55$
corresponding angles in parallel lines

Corresponding angles in parallel lines are equal.

c $a + 110 = 180$
$a = 70$
cointerior angles add to 180 degrees

Cointerior angles in parallel lines sum to 180°.

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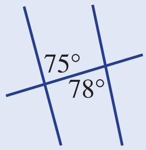
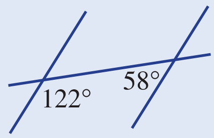
Example 7 Proving lines are parallel

Giving reasons, state whether the two lines cut by the transversal are parallel.

a



b



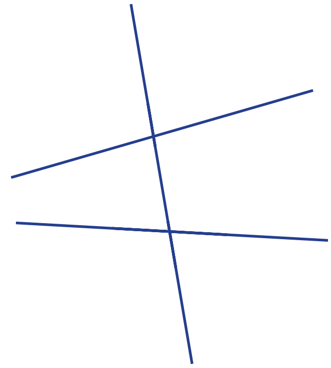
SOLUTION	EXPLANATION
a not parallel The alternate angles are not equal.	Parallel lines have equal alternate angles.
b parallel The cointerior angles sum to 180°.	122° + 58° = 180° Cointerior angles inside parallel lines are supplementary (i.e. sum to 180°).

Exercise 2D

UNDERSTANDING AND FLUENCY	1–6, 7–9(½)	4–6, 7–9(½)	7–9(½)

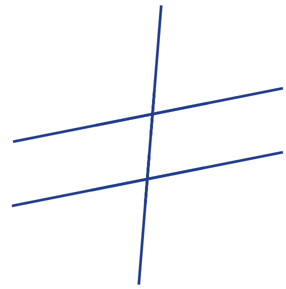
1 Use a protractor to measure each of the eight angles in this diagram.

- **a** How many *different* angle measurements did you find?
- **b** Do you think that the two lines cut by the transversal are parallel?



2 Use a protractor to measure each of the eight angles in this diagram.

- **a** How many *different* angle measurements did you find?
- **b** Do you think that the two lines cut by the transversal are parallel?



3 Choose the word *equal* or *supplementary* to complete these sentences.

If a transversal cuts two parallel lines, then:

- **a** alternate angles are ______________.
- **b** cointerior angles are ______________.
- **c** corresponding angles are _________.
- **d** vertically opposite angles are ______.

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Example 5

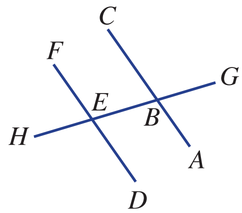
4 Name the angle that is:

a corresponding to $\angle ABE$

b alternate to $\angle ABE$

c cointerior to $\angle ABE$

d vertically opposite to $\angle ABE$



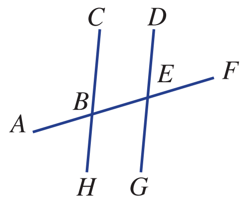
5 Name the angle that is:

a corresponding to $\angle EBH$

b alternate to $\angle EBH$

c cointerior to $\angle EBH$

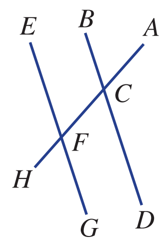
d vertically opposite to $\angle EBH$



6 Name an angle that is:

a corresponding to $\angle ACD$

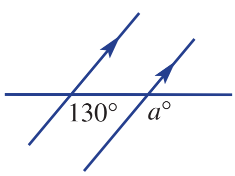
b vertically opposite to $\angle ACD$



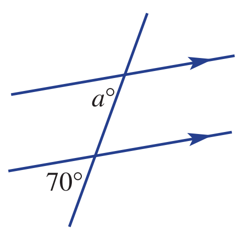
Example 6

7 Find the value of a in these diagrams, giving a reason.

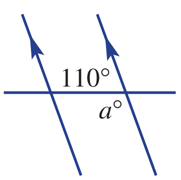
a



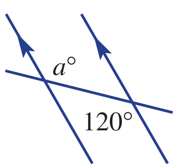
b



c



d



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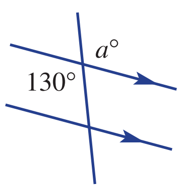

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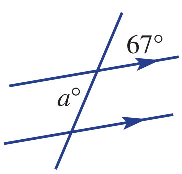
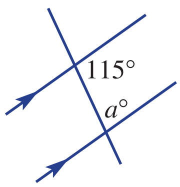
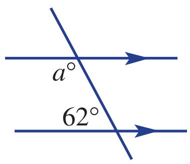


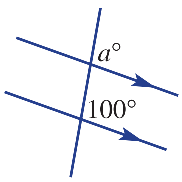
g



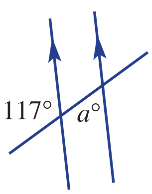
h



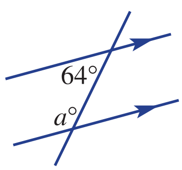
i



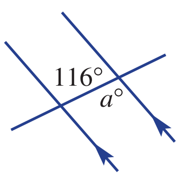
j



k



l



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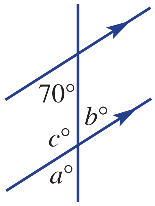
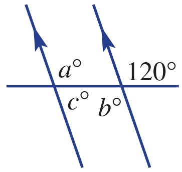
8 Find the value of each unknown pronumeral in the following diagrams.

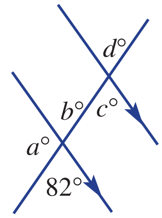
a



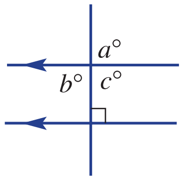
b



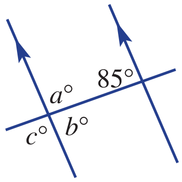
c



d



e



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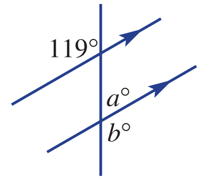
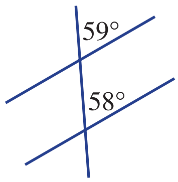


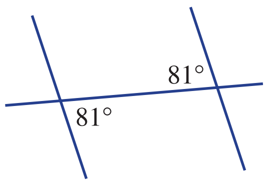
Example 7

9 Giving reasons, state whether the two lines cut by the transversal are parallel.

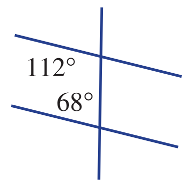
a



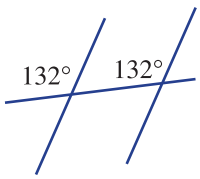
b



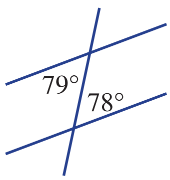
c



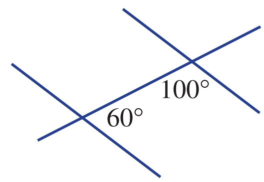
d



e



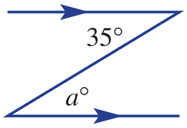
f



PROBLEM-SOLVING AND REASONING	10, 11, 14	11(½), 12, 14, 15	11(½), 12, 14–16

10 Find the value of a in these diagrams.

a



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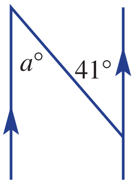
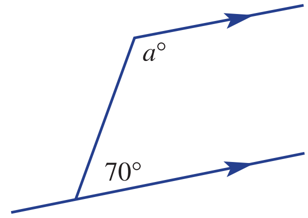


c



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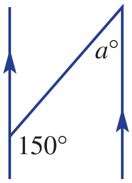

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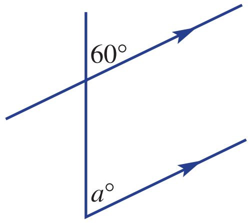

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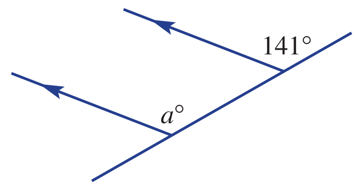

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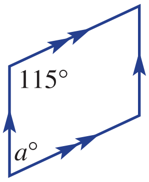
11 Find the value of a in these diagrams.

a


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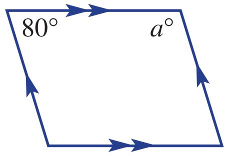

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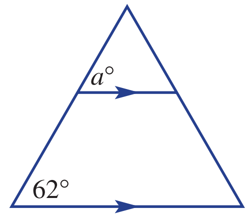

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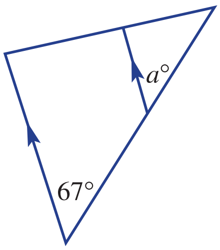

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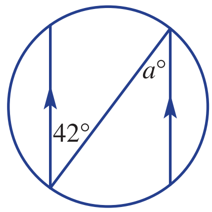

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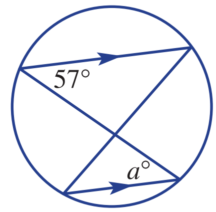

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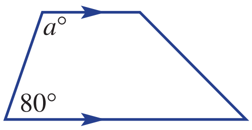

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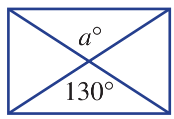

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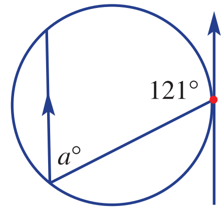
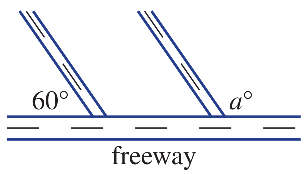


12 A transversal cuts a set of three parallel lines.

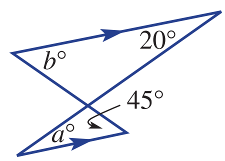
a How many angles are formed?

b How many angles of different sizes are formed if the transversal is *not* perpendicular to the three lines?

13 Two roads merge into a freeway at the same angle, as shown. Find the value of a between the parallel roads and the freeway.



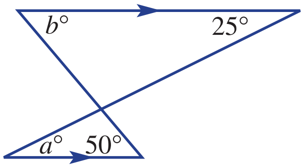
14 This diagram includes two triangles with two sides that are parallel.

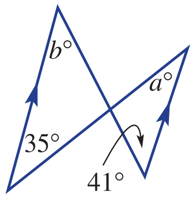


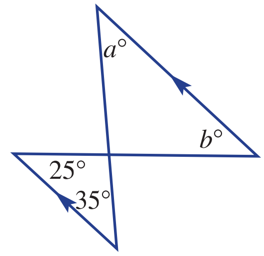
a Give a reason why:

i $a = 20$ **ii** $b = 45$

b Now find the values of a and b in the diagrams below.

i


ii


iii


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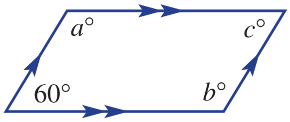
15 This shape is a parallelogram with two pairs of parallel sides.

a Use the 60° angle to find the value of a and b.

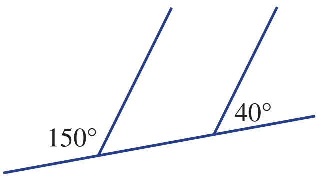
b Find the value of c.

c What do you notice about the angles inside a parallelogram?

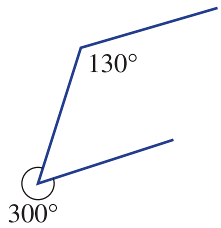


16 Explain why these diagrams do not contain a pair of parallel lines.

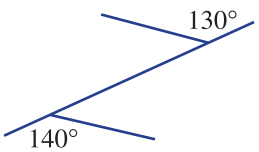
a



b



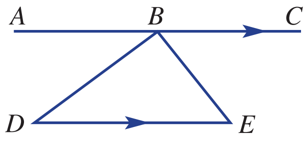
c



ENRICHMENT	—	—	17, 18

Adding parallel lines

17 Consider this triangle and parallel lines.



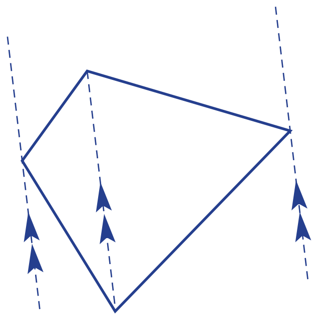
a Giving a reason for your answer, name an angle equal to:

i $\angle ABD$ **ii** $\angle CBE$

b What do you know about the three angles $\angle ABD$, $\angle DBE$ and $\angle CBE$?

c What do these results tell you about the three inside angles of the triangle BDE? Is this true for any triangle? Try a new diagram to check.

18 Use the ideas explored in Question **17** to show that the angles inside a quadrilateral (i.e. a four-sided shape) must sum to 360°. Use this diagram to help.



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2E Solving geometry problems

Interactive

Widgets

HOTsheets

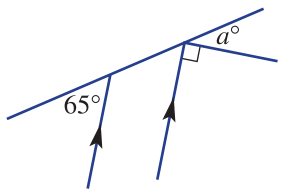
Walkthrough

Parallel lines are at the foundation of construction in all its forms. Imagine the sorts of problems engineers and builders would face if drawings and constructions could not accurately use and apply parallel lines. Angles formed by intersecting beams would be difficult to calculate and could not be transferred to other parts of the building.

Let's start: Not so obvious

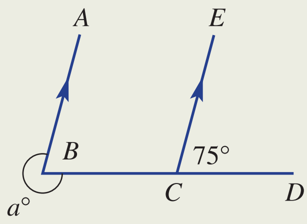
Some geometrical problems require a combination of two or more ideas before a solution can be found. This diagram includes the unknown angle $a°$.

- Discuss if it is possible to find the value of a.
- Describe the steps you would take to find the value of a. Discuss your reasons for each step.



Key ideas

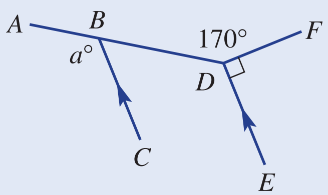
- Some geometrical problems involve more than one step.
  Step 1: $\angle ABC = 75°$ (corresponding angles on parallel lines)
  Step 2: $a + 75 = 360$ (angles in a revolution add to 360°)
  $a = 285$



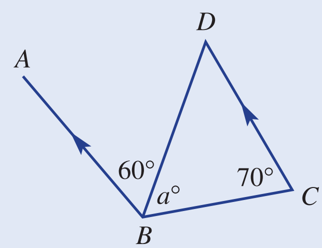
Example 8 Finding angles with two steps

Find the value of a in these diagrams.

a



b



SOLUTION	EXPLANATION
a $\angle BDE + 90° + 170° = 360°$ $\angle BDE = 100°$ $\therefore a = 100$	Angles in a revolution add to 360°. $\angle ABC$ corresponds with $\angle BDE$, and BC and DE are parallel.
b $\angle ABC + 70° = 180°$ $\angle ABC = 110°$ $a + 60 = 110$ $a = 50$	$\angle ABC$ and $\angle BCD$ are cointerior angles, with AB and DC parallel. $\angle ABC = 110°$ and $a° + 60° = 110°$

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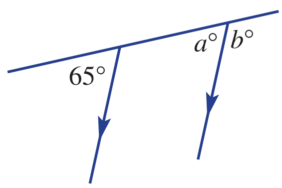
Exercise 2E

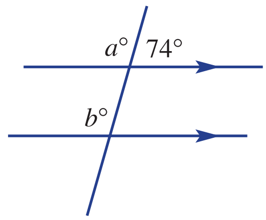
UNDERSTANDING AND FLUENCY	1–3	2, 3(½)	3(½)

1 In these diagrams, first find the value of a then find the value of b.

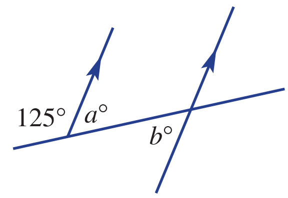
a



b

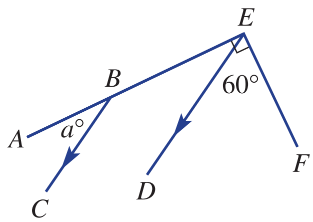


c



2 Name the angle in these diagrams (e.g. ∠ABC) that you would need to find first before finding the value of a. Then find the value of a.

a



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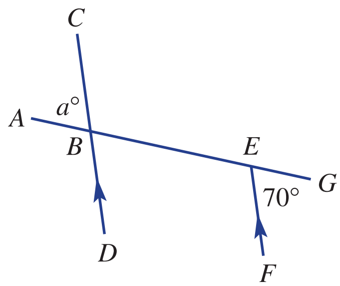

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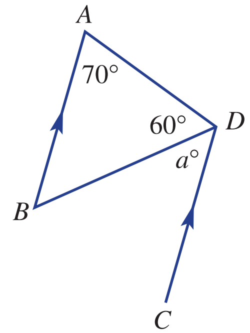
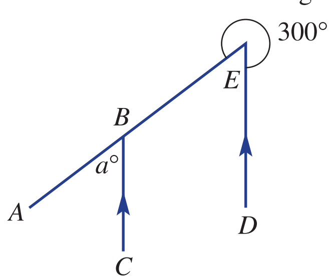


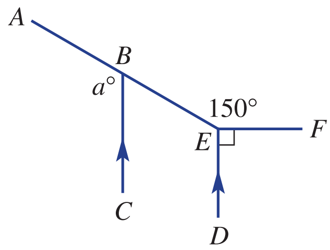
Example 8

3 Find the value of a in these diagrams.

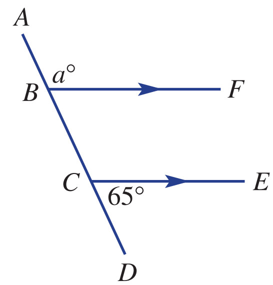
a



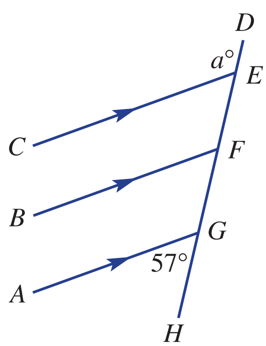
b



c



d



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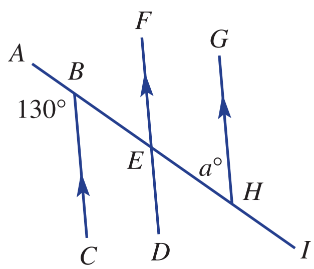

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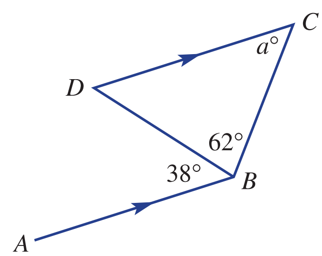

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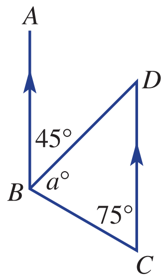
g



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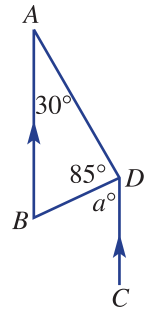

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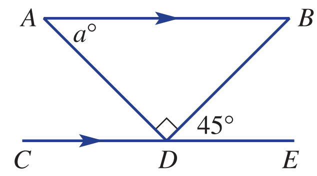

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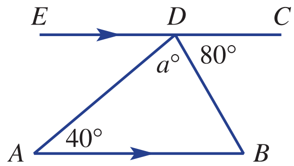

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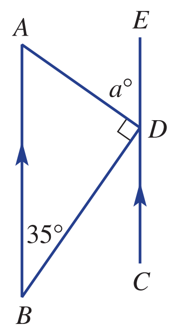
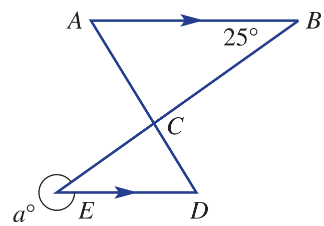


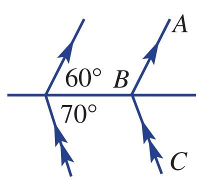
l



PROBLEM-SOLVING AND REASONING	4, 6	4–7	4, 5, 7, 8

4 Find the size of $\angle ABC$ in these diagrams.

a



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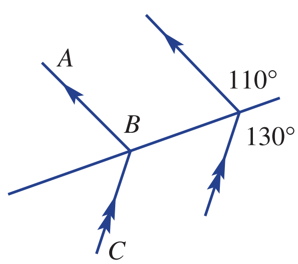

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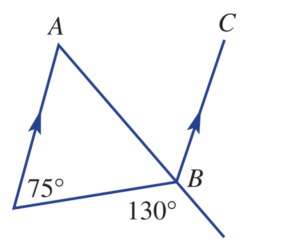

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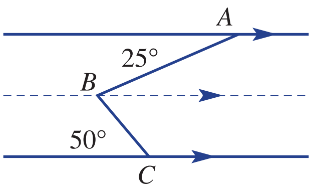

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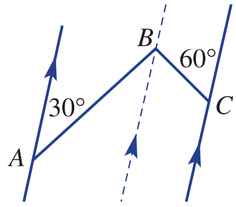

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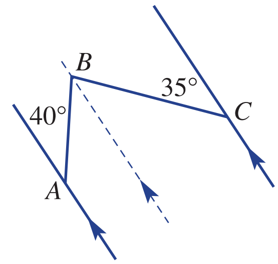

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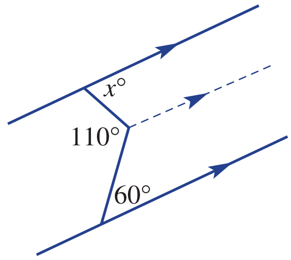
5 Find the value of x in each of these diagrams.

a



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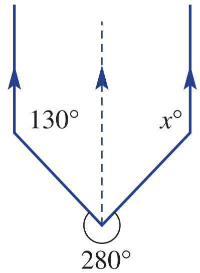

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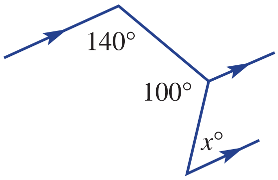
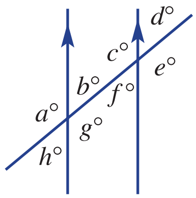


6 What is the minimum number of angles you need to know to find all the angles marked in these diagrams?

a



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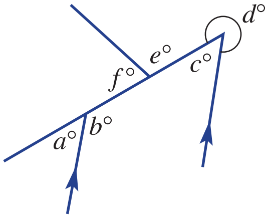

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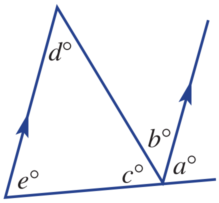
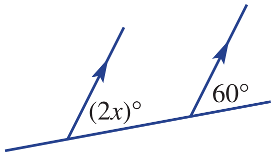


7 In these diagrams, the letter x represents a number and $2x$ means $2 \times x$. Find the value of x.

a



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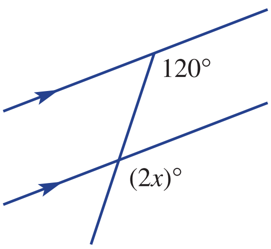

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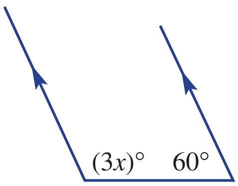

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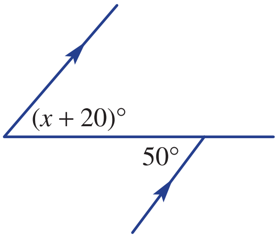

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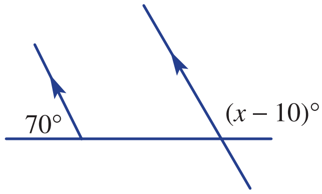

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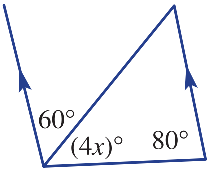

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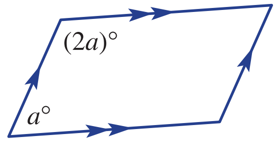
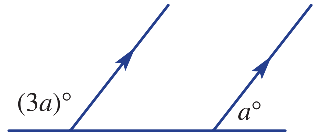
8 Find the value of a in these diagrams.

a



b



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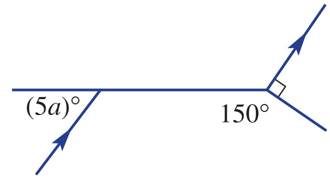
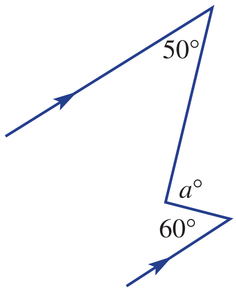


ENRICHMENT	—	—	9

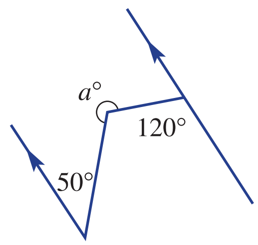
Adding parallel lines

9 Find the value of a in these diagrams. You may wish to add one or more parallel lines to each diagram.

a



b



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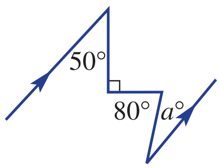
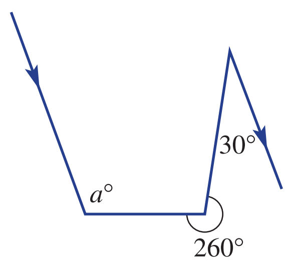
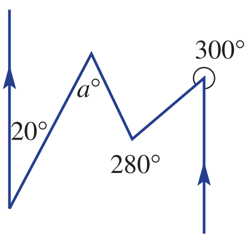


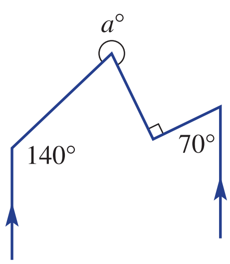
d



e



f



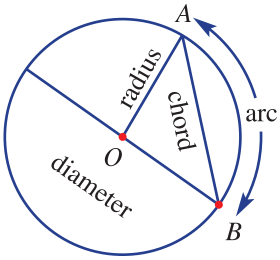
2F Circles and constructions with ruler and compasses FRINGE

Interactive
Widgets
HOTsheets
Walkthrough

One of the most important characteristics of a circle is that the distance from the centre to the circle, called the radius, is always the same. This fact is critical in the construction of geometrical diagrams and other objects that contain circular parts like gears and wheels.

Let's start: Features of a circle

Here is a circle with some common features.



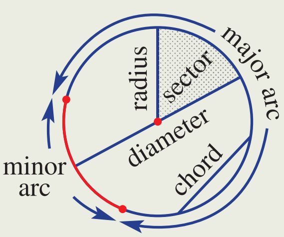
Which of the features (radius, diameter, chord or arc) would change in length if:

- point A is moved around the circle?
- point B is moved away from O so that the size of the circle changes?

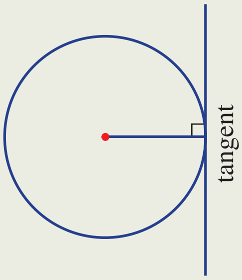
If possible, try constructing this diagram using computer software. Measure lengths and drag the points to explore other possibilities.

Key ideas

- Common circle features include:
  - **centre:** A point at an equal distance from all points on the circle
  - **radius:** A line interval joining the centre to a point on the circle (plural: radii)
  - **chord:** A line interval joining two points on the circle
  - **diameter:** The longest chord passing through the centre
  - **arc:** A part of a circle
  - **sector:** A region bounded by two radii and an arc



- A pair of **compasses** (sometimes called a compass) and a **ruler** can be used to construct geometrical figures precisely.
- The word **bisect** means to cut in half.
- A **tangent** to a circle is a line that touches the circle at a point and is at 90° (perpendicular) to the radius.



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Example 9 Constructing an angle bisector

Use a pair of compasses and a ruler to bisect an angle ∠*AOB* by following steps **a** to **e**.

a Draw and label an angle ∠*AOB*.

b Construct an arc with centre *O* so that it cuts *OA* at point *C* and *OB* at point *D*.

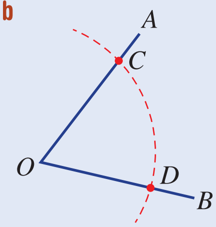
c With the same radius construct an arc with centre *C* and another with centre *D*. Ensure these arcs intersect at a point *E*.

d Mark in the ray *OE*.

e Measure ∠*AOE* and ∠*DOE*. What do you notice?

SOLUTION

a, b

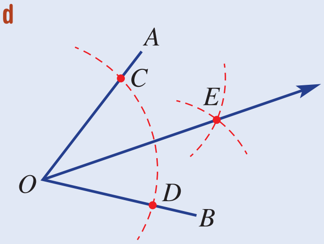


EXPLANATION

First, draw an angle ∠*AOB*. The size of the angle is not important.

Construct an arc using *O* as the centre to produce points *C* and *D*.

c, d



Construct *E* so that the intersecting arcs have the same radius.

Ray *OE* completes the construction.

e ∠*AOE* = ∠*BOE*

The angles are equal, so ray *OE* bisects ∠*AOB*.

Exercise 2F FRINGE

UNDERSTANDING AND FLUENCY	1–3, 5, 6	3–7	5–7

1 Use a pair of compasses and a ruler to draw a circle with a radius of about 3 cm. Then mark and label these features.

a centre *O*

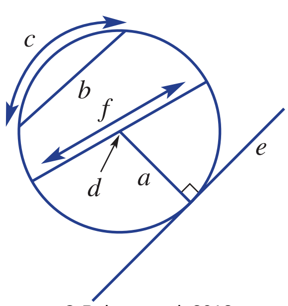
b two points, *A* and *B*, at any place on the circle

c radius *OA*

d chord *AB*

e minor arc *AB*

2 Name the features marked on the circle shown below.



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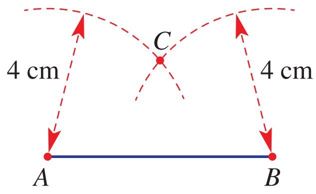
3 Use a ruler to draw a segment AB of length 6 cm and then complete the following.

a Construct a circle with radius 3 cm with centre A. (Use a ruler to help set the pair of compasses.)

b Construct a circle with radius 3 cm with centre B.

c Do your two circles miss, touch or overlap? Is this what you expected?

4 Use a ruler to draw a line segment, AB, of about 5 cm in length.



a Using a pair of compasses, construct arcs with radius 4 cm, as shown, using:

i centre A **ii** centre B

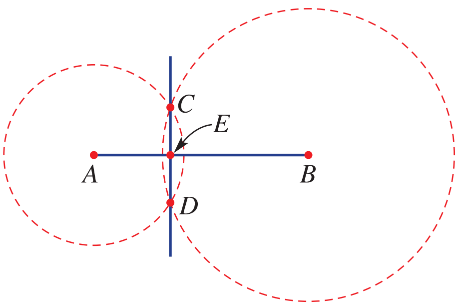
b Mark point C as shown and use a ruler to draw the segments:

i AC **ii** BC

c Measure the angles $\angle BAC$ and $\angle ABC$. What do you notice?

Example 9

5 Follow steps **a** to **e** to construct a perpendicular line.



a Draw a line segment, AB, of about 5 cm in length.

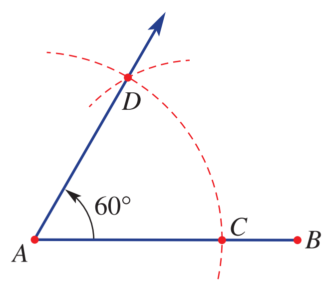
b Construct overlapping circles of different sizes using the two centres A and B.

c Mark the intersecting points of the circles and label these points C and D.

d Draw the line CD and mark the intersection of line CD and segment AB with the point E.

e Measure $\angle AEC$ with a protractor. What do you notice?

6 Follow steps **a** to **d** to construct a 60° angle.



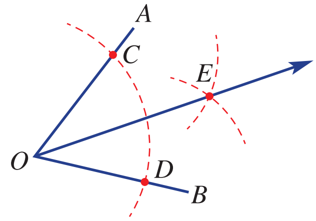
a Draw a line segment, AB, of about 5 cm in length.

b Construct an arc with centre A and intersecting the segment AB at C.

c With the same radius construct an arc with centre C and intersecting the first arc at D.

d Draw the ray AD and measure $\angle BAD$. What do you notice?

7 Follow steps **a** to **e** to construct an angle bisector.



a Draw any angle and label $\angle AOB$.

b Construct an arc with centre O so that it cuts OA and OB at points C and D.

c With the same radius, construct an arc with centre C and another with centre D. Ensure these arcs intersect at a point, E.

d Mark in the ray OE.

e Measure $\angle AOE$ and $\angle BOE$. What do you notice?

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PROBLEM-SOLVING AND REASONING	8, 11	8–9, 11	9–12

8 Consider the construction of the perpendicular line. (See diagram in Question **5**.)

a Explain how to alter the construction so that the point E is the exact midpoint of the segment AB.

b If point E is at the centre of segment AB, then the line CD will be called the perpendicular bisector of segment AB. Complete the full construction to produce a perpendicular bisector.

9 Using the results from Questions **6** and **7**, explain how you could construct the angles below. Try each construction and then check each angle with a protractor.

a 30° **b** 15°

10 Show how you could construct these angles. After each construction, measure the angle using a protractor. (You may wish to use the results from Questions **5** and **7** for help.)

a 45° **b** 22.5°

11 Consider the construction of a perpendicular line. (See the diagram in Question **5**.) Do you think it is possible to construct a perpendicular line using circles with radii of any size? Explain.

12 The diagram in Question **7** shows an acute angle, $\angle AOB$.

a Do you think it is possible to bisect an obtuse angle? If so, show how.

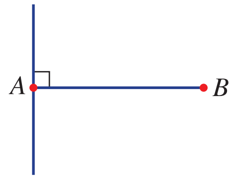
b Do you think it is possible to bisect a reflex angle? If so, show how.

ENRICHMENT	—	—	13

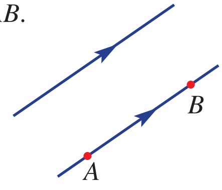
No measurement allowed

13 Using only a pair of compasses and a ruler's edge, see if you can construct these objects. No measurement is allowed.

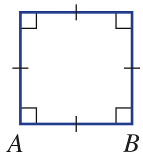
a Perpendicular line at the end point of a segment. Start with segment AB.



b Two parallel lines. Start with line AB.



c A square. Start with segment AB.



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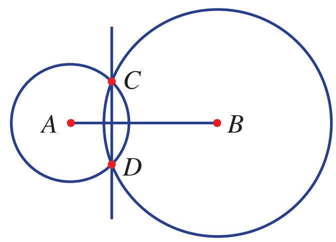
2G Constructions with dynamic geometry software EXTENSION

Dynamic geometry software is an ideal tool for constructing geometrical figures. Constructing with dynamic geometry is like constructing with a ruler and a pair of compasses, but there is the added freedom to drag objects and explore different variations of the same construction. With dynamic geometry, the focus is on 'construction' as opposed to 'drawing'. Although this is more of a challenge initially, the results are more precise and allow for greater exploration.

Let's start: The disappearing line



Use computer geometry to construct this figure starting with segment *AB*.

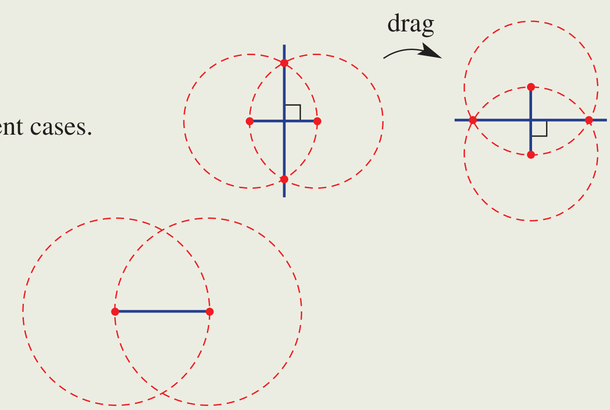
Add the line *CD* and check that it makes a right angle.

Drag the points *A* and *B* or increase the size of the circles.

Can you drag point *A* or *B* to make the line *CD* disappear? Why would this happen?

Key ideas

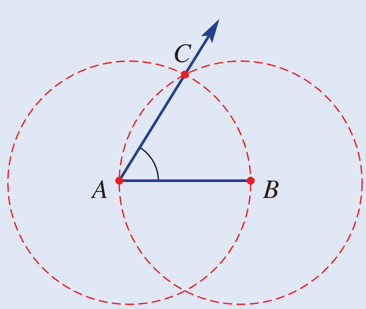
- Using dynamic geometry is like using a pair of compasses and a ruler.
- Objects can be dragged to explore different cases.
- Upon dragging, the geometrical construction should retain the desired properties.
- The same segment can be used to ensure two circles have exactly the same radius.



Example 10 Constructing a 60° angle

Construct an angle of 60° using dynamic geometry software. Then drag one of the starting points to check the construction.

SOLUTION



EXPLANATION

Step 1: Construct and label a segment *AB*.
Step 2: Construct two circles with radius *AB* and centres *A* and *B*.
Step 3: Mark the intersection *C* and draw the ray *AC*.
Step 4: Measure $\angle BAC$ to check.

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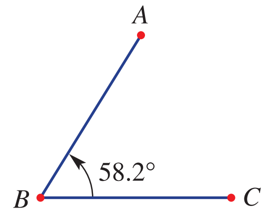
Exercise 2G EXTENSION

UNDERSTANDING AND FLUENCY	1–2	2–4	3, 4

1 **a** Use dynamic geometry software to construct an angle $\angle ABC$. Any size will do.

b Mark and measure the angle using geometry software. Drag the point A around B to enlarge the angle. See whether you can form all these types of angles.

i acute **ii** right **iii** straight
iv reflex **v** revolution



2 Look at the 60° angle construction in **Example 10**.

a Why do the two circles have exactly the same radius?

b What other common geometrical object could be easily constructed simply by adding one more segment?

Example 10

3 Construct each of the following using dynamic geometry software. If necessary, refer back to Exercise 2F to assist you. Check each construction by dragging one of the starting points. All desired properties should be retained.

a perpendicular line **b** perpendicular bisector **c** 60° angle **d** angle bisector

4 **a** Use the 'parallel line' tool to construct a pair of parallel lines and a transversal.

b Measure the eight angles formed.

c Drag the transversal to change the size of the angles. Check that:

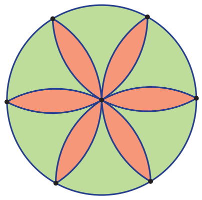
i alternate angles are equal **ii** corresponding angles are equal
iii cointerior angles are always supplementary

PROBLEM-SOLVING AND REASONING	5, 7	5–8	5–8

5 Use dynamic geometry software to construct these angles. You may wish to use the 'angle bisector' shortcut tool.

a 30° **b** 15° **c** 45°

6 Use dynamic geometry software to construct a six-pointed flower. Then drag one of the starting points to increase or decrease its size.



7 **a** When using geometry software it may be necessary to use a full circle instead of an arc. Explain why.

b When constructing a perpendicular bisector, the starting segment AB is used as the radius of the circles. This is instead of using two circles with different radii. Explain why.

8 Explain why geometrical construction is a precise process, whereas drawing using measurement is not.

ENRICHMENT	—	—	9

Intricate designs

9 Sketch your own intricate design or use the internet to find a design that uses circles and lines. Use dynamic geometry to see if it is possible to precisely construct the design. Use colour to enhance your design.

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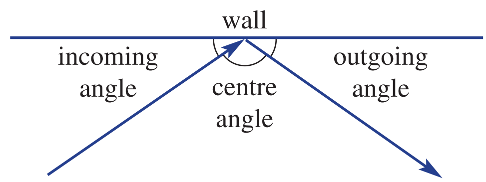
Investigation

The perfect billiard ball path

When a billiard ball bounces off a straight wall (with no side spin), we can assume that the angle at which it hits the wall (incoming angle) is the same as the angle at which it leaves the wall (outgoing angle). This is similar to how light reflects off a mirror.

Single bounce

Use a ruler and protractor to draw a diagram for each part and then answer the questions.

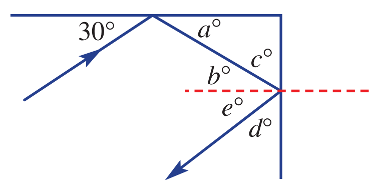


a Find the outgoing angle if:
- **i** the incoming angle is 30°
- **ii** the centre angle is 104°

b What geometrical reason did you use to calculate the answer to part **a ii** above?

Two bounces

Two bounces of a billiard ball on a rectangular table are shown here.



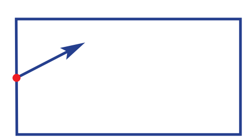
a Find the values of angles a, b, c, d and e, in that order. Give a reason for each.

b What can be said about the incoming angle on the first bounce and the outgoing angle on the second bounce? Give reasons for your answer.

c Accurately draw the path of two bounces using:
- **i** an initial incoming bounce of 20
- **ii** an initial incoming bounce of 55°

More than two bounces

a Draw paths of billiard balls for more than two bounces starting at the midpoint of one side of a rectangular shape, using the starting incoming angles below.
- **i** 45°
- **ii** 30°



b Repeat part **a** but use different starting positions. Show accurate diagrams, using the same starting incoming angle but different starting positions.

c Summarise your findings of this investigation in a report that clearly explains what you have found. Show clear diagrams for each part of your report.

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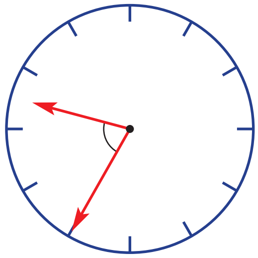
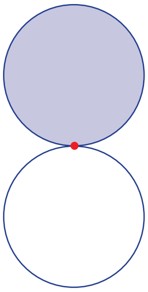
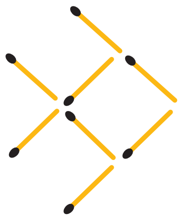
1 What is the angle between the hour hand and minute hand of a clock at 9:35 a.m.?



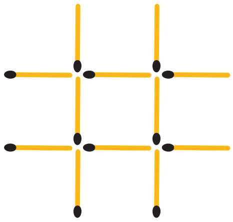
2 Two circles are the same size. The shaded circle rolls around the other circle. How many degrees will it turn before returning to its starting position?



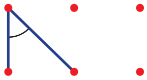
3 Move three matchsticks to turn the fish to face the opposite direction.



4 Move three matchsticks to get three squares of the same size.



5 How many angles of different sizes can you form from joining dots in this 2 by 3 grid? One possible angle (45°) is shown for you. Do not count the 180° angle or reflex angles outside the grid.



6 What is the angle between the hour hand and minute hand of a clock at 2:37 p.m.?

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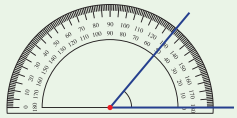
Chapter summary

Introduction to Geometry

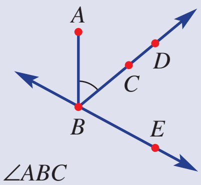
Measuring angles



Angles

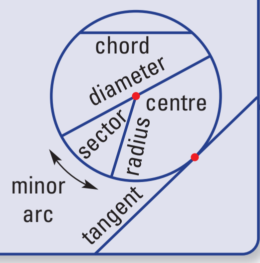
acute 0° to 90°
right 90°
obtuse 90° to 180°
straight 180°
reflex 180° to 360°
revolution 360°

Geometrical objects

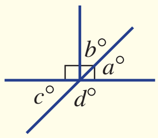


$\angle ABC$
vertex B
ray BD
segment AB
collinear points B, C, D
line BE

Circle features

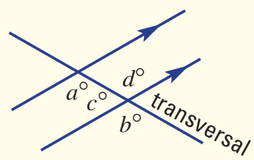


Angles at a point



Complementary
$a + b = 90$
Supplementary
$c + d = 180$
Vertically opposite
$a = c$
Revolution
$a + b + 90 + c + d = 360$

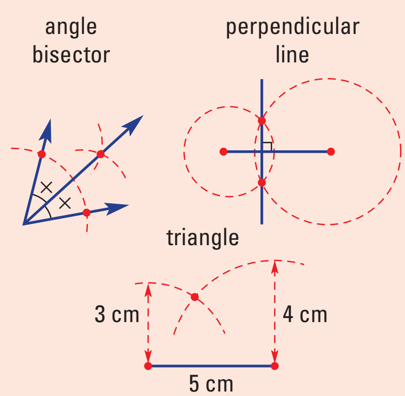
Parallel lines



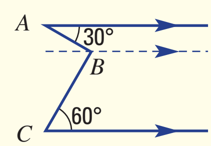
$a = b$ (corresponding)
$a = d$ (alternate)
$a + c = 180$ (cointerior)

If $a = 120$, $b = 120$,
$d = 120$ and $c = 60$.

Constructions



Problem with parallel lines (Ext)



$\angle ABC = 30° + 60° = 90°$

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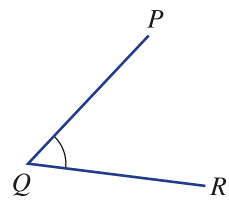
Multiple-choice questions

1 Three points are collinear if:
- **A** they are at right angles
- **B** they form a 60° angle
- **C** they all lie in a straight line
- **D** they are all at the same point
- **E** they form an arc on a circle

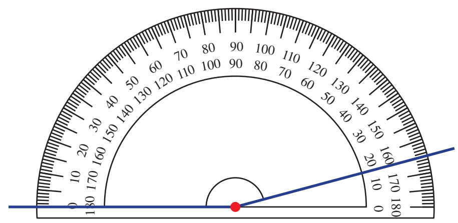
2 The angle shown here can be named:
- **A** $\angle QRP$
- **B** $\angle PQR$
- **C** $\angle QPR$
- **D** $\angle QRR$
- **E** $\angle PQP$



3 Complementary angles:
- **A** sum to 180°
- **B** sum to 270°
- **C** sum to 360°
- **D** sum to 90°
- **E** sum to 45°

4 A reflex angle is:
- **A** 90°
- **B** 180°
- **C** between 180° and 360°
- **D** between 0° and 90°
- **E** between 90° and 180°

5 What is the reading on this protractor?
- **A** 15°
- **B** 30°
- **C** 105°
- **D** 165°
- **E** 195°

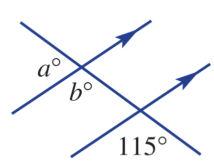


6 The angle a minute hand on a clock turns in 20 minutes is:
- **A** 72°
- **B** 36°
- **C** 18°
- **D** 144°
- **E** 120°

7 If a transversal cuts two parallel lines, then:
- **A** cointerior angles are equal
- **B** alternate angles are supplementary
- **C** corresponding angles are equal
- **D** vertically opposite angles are supplementary
- **E** supplementary angles add to 90°

8 An angle bisector:
- **A** cuts an angle in half
- **B** cuts a segment in half
- **C** cuts a line in half
- **D** makes a 90° angle
- **E** makes a 180° angle

9 The value of a in this diagram is:
- **A** 115
- **B** 75
- **C** 60
- **D** 55
- **E** 65



10 In Question **9**, the value of b in the diagram is
- **A** 65
- **B** 115
- **C** 75
- **D** 55
- **E** 90

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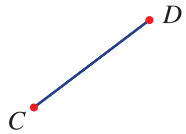
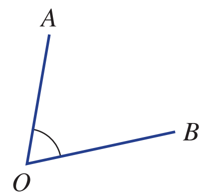
Short-answer questions

1 Name each of these objects.

a

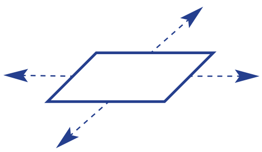


b

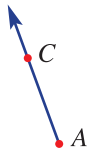


c

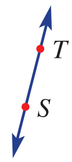
d



e

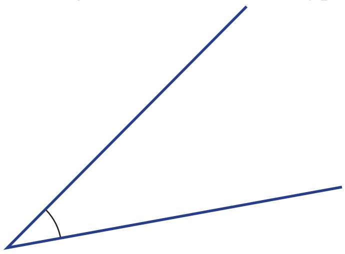


f

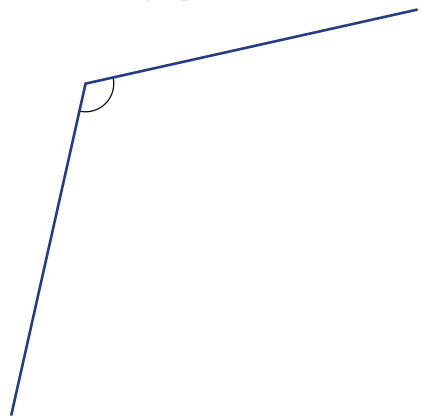


2 For the angles shown, state the type of angle and measure its size using a protractor.

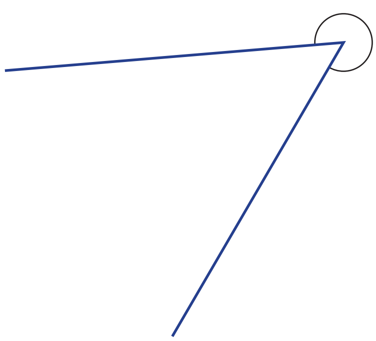
a



b



c



3 Find the angle between the hour and minute hands on a clock at the following times.

a 6 a.m. **b** 9 p.m.

c 3 p.m. **d** 5 a.m.

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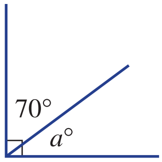
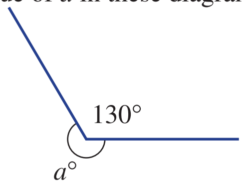
4 Without using a protractor, find the value of a in these diagrams.

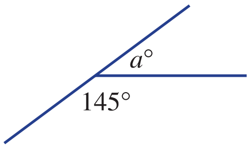
a



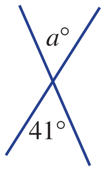
b



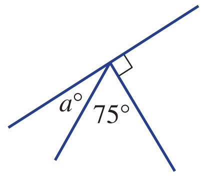
c



d



e



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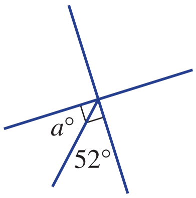
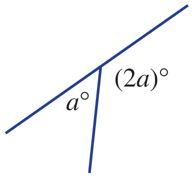
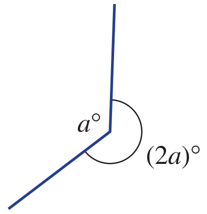


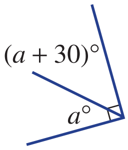
g



h

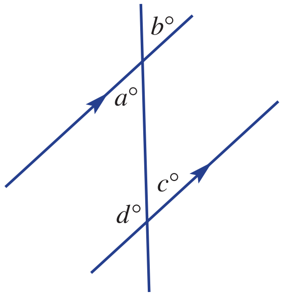


i



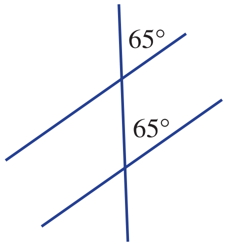
5 Using the letters a, b, c or d given in the diagram, write down a pair of angles that are:

a vertically opposite
b cointerior
c alternate
d corresponding
e supplementary but not cointerior

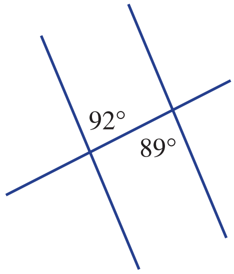


6 For each of the following, state whether the two lines cut by the transversal are parallel. Give reasons for each answer.

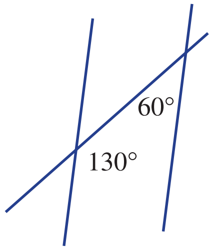
a



b



c



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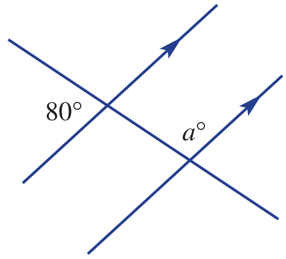
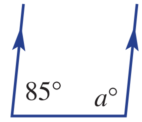
7 Find the value of a in these diagrams.

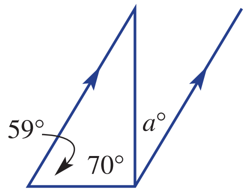
a



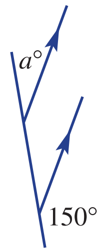
b



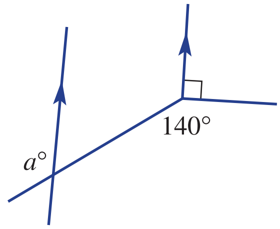
c



d



e



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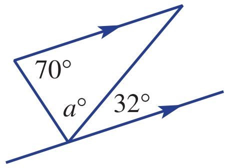
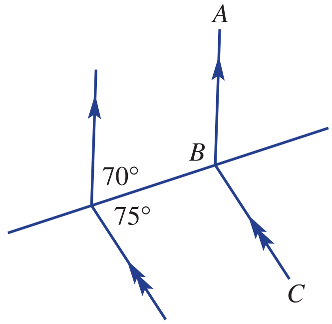
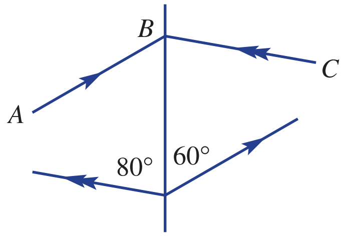


8 Find the size of $\angle ABC$ in these diagrams.

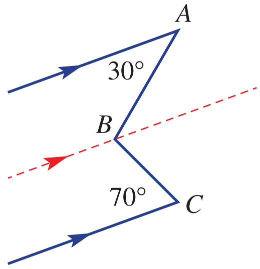
a



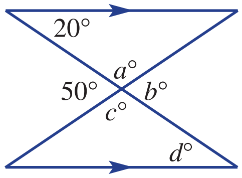
b



c



9 Find the value of a, b, c, and d in this diagram.



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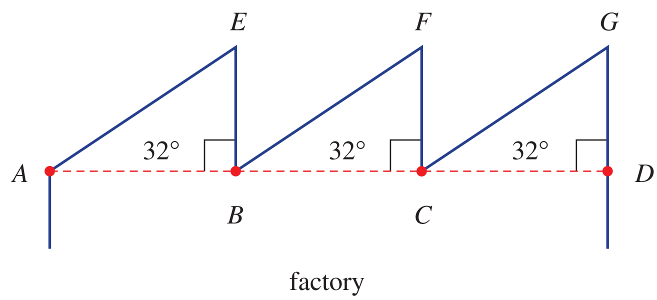
Extended-response questions

1 A factory roof is made up of three sloping sections. The sloping sections are all parallel and the upright supports are at 90° to the horizontal, as shown. Each roof section makes a 32° angle (or pitch) with the horizontal.



a State the size of each of these angles.

i $\angle EAB$ ii $\angle GCD$ iii $\angle ABF$ iv $\angle EBF$

b Complete these sentences.

i $\angle BAE$ is ____________ to $\angle CBF$.

ii $\angle FBC$ is ____________ to $\angle GCB$.

iii $\angle BCG$ is ____________ to $\angle GCD$.

c Solar panels are to be placed on the sloping roofs and it is decided that the angle to the horizontal is to be reduced by 11°. Find the size of these new angles.

i $\angle FBC$ ii $\angle FBA$ iii $\angle FCG$

2 A circular birthday cake is cut into pieces of equal size, cutting from the centre outwards. Each cut has an angle of $a°$ at the centre.

- Tanya's family takes four pieces.
- George's family takes three pieces.
- Sienna's family takes two pieces.
- Anita's family takes two pieces.
- Marcus takes one piece.

a How many pieces were taken all together?

b If there is no cake left after all the pieces are taken, find the value of a.

c Find the value of a if:

i half of the cake still remains

ii one-quarter of the cake still remains

iii one-third of the cake still remains

iv one-fifth of the cake still remains

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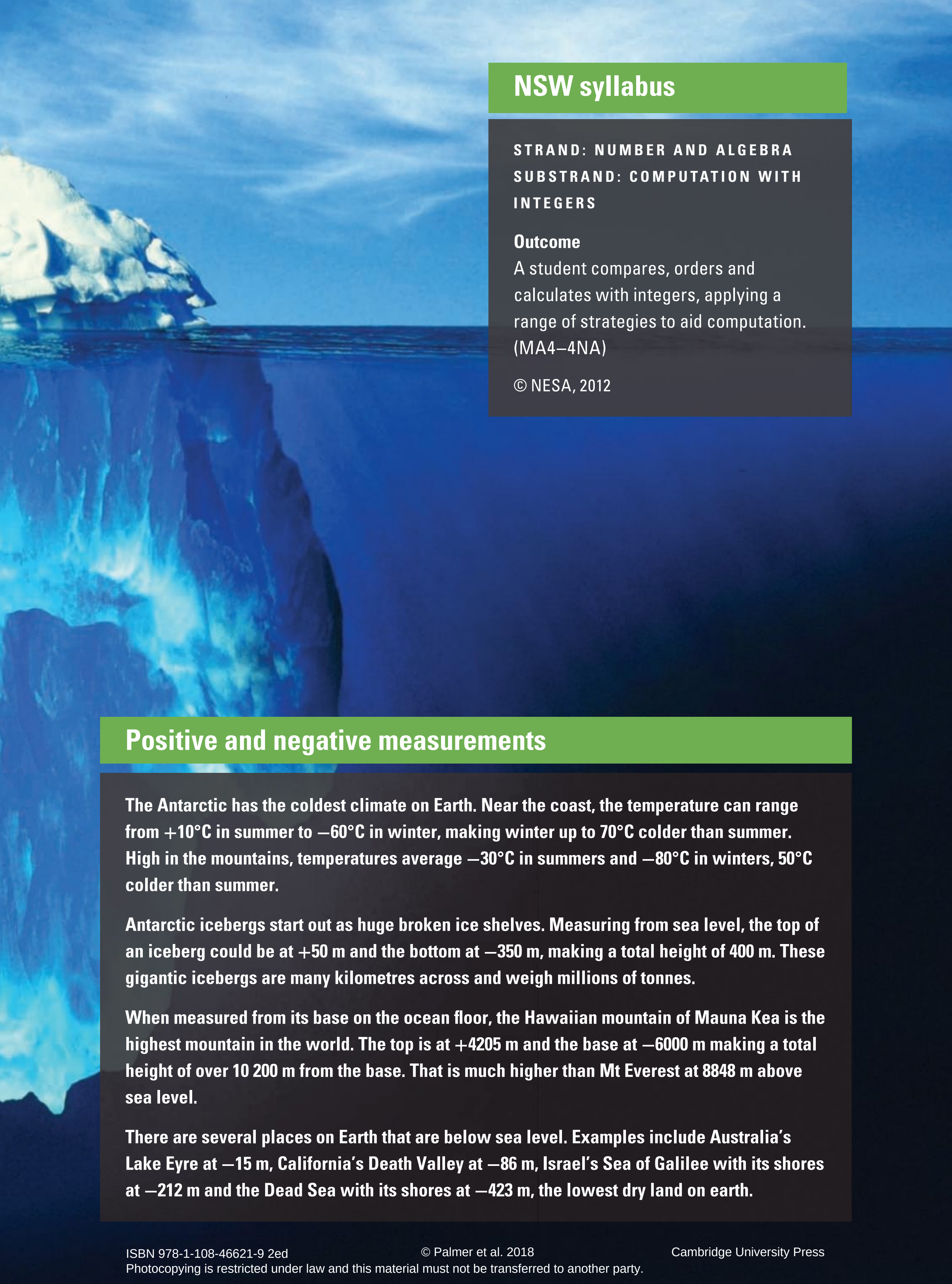
Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

3 Computation with positive and negative integers

What you will learn



NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRAND: COMPUTATION WITH INTEGERS

Outcome

A student compares, orders and calculates with integers, applying a range of strategies to aid computation. (MA4–4NA)

Positive and negative measurements

The Antarctic has the coldest climate on Earth. Near the coast, the temperature can range from +10°C in summer to −60°C in winter, making winter up to 70°C colder than summer. High in the mountains, temperatures average −30°C in summers and −80°C in winters, 50°C colder than summer.

Antarctic icebergs start out as huge broken ice shelves. Measuring from sea level, the top of an iceberg could be at +50 m and the bottom at −350 m, making a total height of 400 m. These gigantic icebergs are many kilometres across and weigh millions of tonnes.

When measured from its base on the ocean floor, the Hawaiian mountain of Mauna Kea is the highest mountain in the world. The top is at +4205 m and the base at −6000 m making a total height of over 10 200 m from the base. That is much higher than Mt Everest at 8848 m above sea level.

There are several places on Earth that are below sea level. Examples include Australia's Lake Eyre at −15 m, California's Death Valley at −86 m, Israel's Sea of Galilee with its shores at −212 m and the Dead Sea with its shores at −423 m, the lowest dry land on earth.

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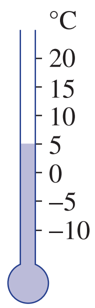
Pre-test

1 Insert the symbols < (is less than) or > (is greater than) to make each statement true.

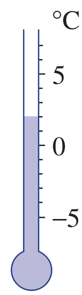
a 5 ☐ 7 **b** 0 ☐ 10 **c** 9 ☐ 11 **d** 3 ☐ 0

2 Read the temperature on these thermometers.

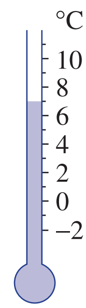
a



b



c



3 Evaluate these products.

a 2×15 **b** 11×7 **c** 3×13 **d** 28×4

4 Evaluate these quotients.

a $35 \div 7$ **b** $121 \div 11$ **c** $84 \div 12$ **d** $340 \div 20$

5 Use order of operations to evaluate the following.

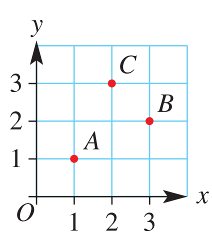
a $2 + 5 \times 4$ **b** $10 \div 2 - 3$ **c** $(11 + 15) \times 2$

d $24 \div (8 - 2)$ **e** $(6 - 3) \times (1 + 9)$ **f** $8 \times (4 - 2) + 10 \div 5$

6 Decide if the answers to these expressions are positive (i.e. greater than zero) or negative (i.e. less than zero).

a $5 - 4$ **b** $4 - 5$ **c** $10 \times 2 - 21$ **d** $30 - 5 \times 4$

7 Write down the coordinates (x, y) of A, B and C for this Cartesian plane.



8 Plot these points on the given Cartesian plane.

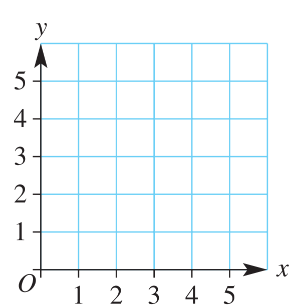
a $A(2, 3)$

b $B(4, 1)$

c $C(5, 4)$

d $D(0, 2)$

e $E(3, 0)$



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3A Working with negative integers

The numbers 1, 2, 3, … are considered to be positive because they are greater than zero (0). Negative numbers extend the number system to include numbers less than zero. All the whole numbers less than zero, zero itself and the whole numbers greater than zero are called integers.

The use of negative numbers dates back to 100 BCE when the Chinese used black rods for positive numbers and red rods for negative numbers in their rod number system. These coloured rods were used for commercial and tax calculations. Later, a great Indian mathematician named Brahmagupta (598–670) set out the rules for the use of negative numbers, using the word *fortune* for positive and *debt* for negative. Negative numbers were used to represent loss in a financial situation.

John Wallis invented the number line.

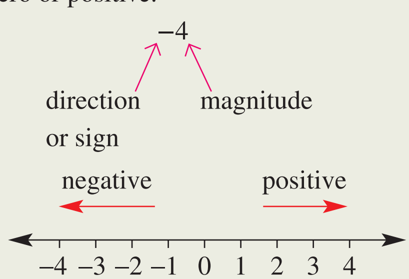
An English mathematician named John Wallis (1616–1703) invented the number line and the idea that numbers have a direction. This helped define our number system as an infinite set of numbers extending in both the positive and negative directions. Today negative numbers are used in all sorts of mathematical calculations and are considered to be an essential element of our number system.

Let's start: Simple applications of negative numbers

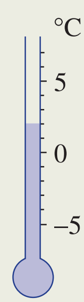
- Try to name as many situations as possible in which negative numbers are used.
- Give examples of the numbers in each case.

Key ideas

- **Negative** numbers are numbers less than zero.
- **Integers** are whole numbers that can be negative, zero or positive.
  ... −4, −3, −2, −1, 0, 1, 2, 3, 4, ...
- The number −4 is read as 'negative 4'.
- The number 4 is sometimes written as +4 and is sometimes read as 'positive 4'.
- Every number has *direction* and *magnitude*.



- A **number line** shows:
  - positive numbers to the right of zero
  - negative numbers to the left of zero.
- A thermometer shows:
  - positive temperatures above zero
  - negative temperatures below zero.



- Each number other than zero has an **opposite**.
  - The numbers 3 and −3 are opposites. They are equal in magnitude but opposite in sign.

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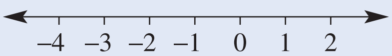
Example 1 Drawing a number line

Draw a number line, showing all integers from −4 to 2.

SOLUTION



EXPLANATION

Use equally spaced markings and put −4 on the left and 2 on the right.

Example 2 Less than or greater than

Insert the symbol < (is less than) or > (is greater than) into these statements to make them true.

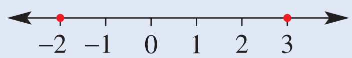
a −2 ☐ 3 **b** −1 ☐ −6

SOLUTION

a −2 < 3

EXPLANATION

−2 is to the left of 3 on a number line.

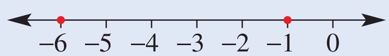


SOLUTION

b −1 > −6

EXPLANATION

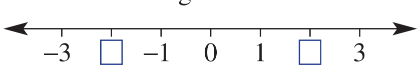
−1 is to the right of −6 on a number line.

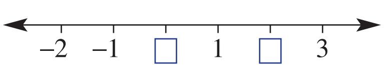


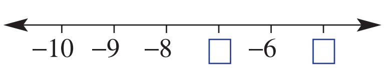
Exercise 3A

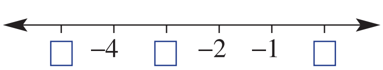
UNDERSTANDING AND FLUENCY	1–5, 6(½)	3, 4, 5–6(½), 7	4–7(½)

1 What are the missing numbers on these number lines?

a


b


c


d


2 −5 is the opposite number of 5, and 5 is the opposite number of −5. Write down the opposite to these numbers.

a 2 **b** 6 **c** −3 **d** −7
e −15 **f** 21 **g** 132 **h** −1071

3 Fill in the blanks using the words *greater* and *less*.

a 5 is ___________ than 0 **b** −3 is ___________ than 0
c 0 is ___________ than −6 **d** 0 is ___________ than 1

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Example 1

4 Draw a number line for each description, showing all the given integers.

a from −2 to 2
b from −5 to 1
c from −10 to −6
d from −32 to −25

5 List all the integers that fit the given description.

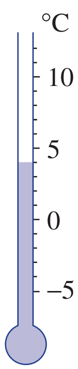
a from −2 up to 4
b from −7 up to 0
c greater than −3 and less than 2
d greater than −5 and less than 1
e less than 4 and greater than −4
f less than −3 and greater than −10

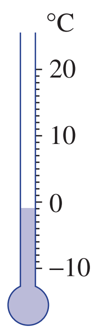
Example 2

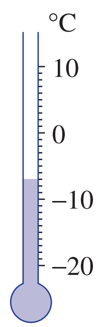
6 Insert the symbol < (is less than) or > (is greater than) into these statements to make them true.

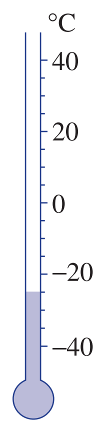
a 7 ☐ 9
b 3 ☐ 2
c 0 ☐ −2
d −4 ☐ 0
e −1 ☐ −5
f −7 ☐ −6
g −11 ☐ −2
h −9 ☐ −13
i −3 ☐ 3
j 3 ☐ −3
k −130 ☐ 1
l −2 ☐ −147

7 Give the temperature for these thermometers.

a


b


c


d


PROBLEM-SOLVING AND REASONING 8, 9, 11 | 9–11 | 10–12

8 Arrange these numbers in *ascending* order.

a −3, −6, 0, 2, −10, 4, −1
b −304, 126, −142, −2, 1, 71, 0

9 Write the next three numbers in these simple patterns.

a 3, 2, 1, ___, ___, ___
b −8, −6, −4, ___, ___, ___
c 10, 5, 0, ___, ___, ___
d −38, −40, −42, ___, ___, ___
e −91, −87, −83, ___, ___, ___
f 199, 99, −1, ___, ___, ___

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10 These lists of numbers show deposits (positive numbers) and withdrawals (negative numbers) for a month of bank transactions. Find the balance at the end of the month.

a

Starting balance	\$200
	\$10
	−\$130
	−\$25
	\$100
	−\$20
Final balance	______

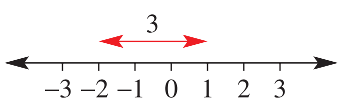
b

Starting balance	\$0
	\$50
	−\$60
	−\$100
	\$200
	−\$100
Final balance	______

11 If the height above sea level for a plane is a positive number, then the height for a submarine could be written as a negative number. What is the height relative to sea level for a submarine at these depths?

a 50 metres
b 212.5 metres
c 0 metres

12 The difference between two numbers could be thought of as the distance between the numbers on a number line. For example, the difference between −2 and 1 is 3.



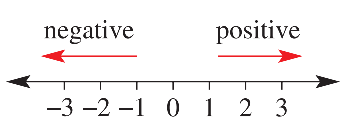
Find the difference between these pairs of numbers.

a −1 and 1 **b** −2 and 2 **c** −3 and 1
d −4 and 3 **e** −3 and 0 **f** −4 and −1
g −10 and −4 **h** −30 and 14

ENRICHMENT	—	—	13

The final position

13 For these sets of numbers, a positive number means to move right and a negative number means to move left. Start at zero each time and find the final position.



a −1, 4, −5
b 3, −5, −1, 4
c −5, −1, 3, 1, −2, −1, 4
d −10, 20, −7, −14, 8, −4
e −250, 300, −49, −7, 36, −81
f −7001, 6214, −132, 1493, −217

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3B Adding or subtracting a positive integer

Interactive
Widgets
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Walkthrough

Adding and subtracting a positive integer can give both positive and negative answers. For example, when a newly installed fridge at 20°C is switched on, the temperature inside the freezer might fall by 25°C. The final temperature is −5°C; i.e. $20 - 25 = -5$. If a temperature of −10°C rises by 5°C, the final temperature is −5°C; i.e. $-10 + 5 = -5$.

Let's start: Positive and negative possibilities

Decide if it is possible to find an example of the following. If so, give a specific example.

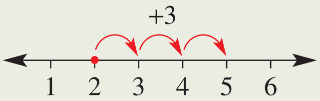
- A positive number added to a positive number gives a positive number.
- A positive number added to a positive number gives a negative number.
- A positive number added to a negative number gives a positive number.
- A positive number added to a negative number gives a negative number.
- A positive number subtracted from a positive number gives a positive number.
- A positive number subtracted from a positive number gives a negative number.
- A positive number subtracted from a negative number gives a positive number.
- A positive number subtracted from a negative number gives a negative number.

Key ideas

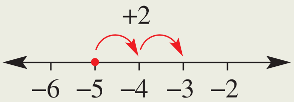
- If a positive number is added to a number, you move right on a number line.

For example:

$2 + 3 = 5$ Start at 2 and move right by 3.



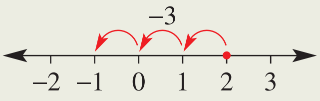
$-5 + 2 = -3$ Start at −5 and move right by 2.



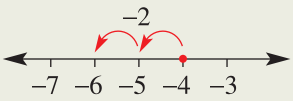
- If a positive number is subtracted from a number, you move left on a number line.

For example:

$2 - 3 = -1$ Start at 2 and move left by 3.



$-4 - 2 = -6$ Start at −4 and move left by 2.



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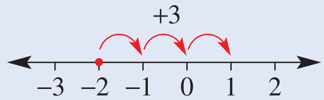
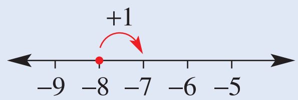
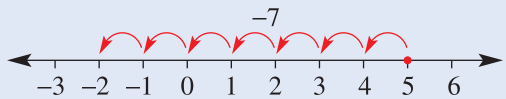
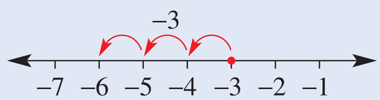
Example 3 Adding and subtracting positive integers

Calculate the answer to these additions and subtractions.

a −2 + 3 **b** −8 + 1 **c** 5 − 7 **d** −3 − 3

SOLUTION	EXPLANATION
a −2 + 3 = 1	
b −8 + 1 = −7	
c 5 − 7 = −2	
d −3 − 3 = −6	

Exercise 3B

UNDERSTANDING AND FLUENCY	1, 2, 3–5(½)	2, 3–6(½)	3–6(½)

1 In which direction (i.e. right or left) on a number line do you move for the following calculations?

a 2 is added to −5

b 6 is added to −4

c 4 is subtracted from 2

d 3 is subtracted from −4

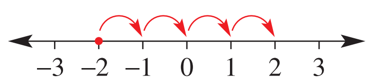
2 Match up the problems **a** to **d** with the number lines **A** to **D**.

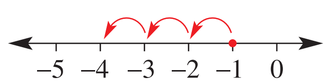
a 5 − 6 = −1

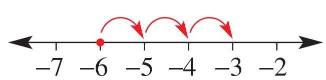
b −2 + 4 = 2

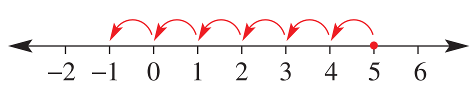
c −1 − 3 = −4

d −6 + 3 = −3

A 

B 

C 

D 

Example 3a, b

3 Calculate the answer to these additions. Check your answers using a calculator.

a −1 + 2	**b** −1 + 4	**c** −3 + 5	**d** −10 + 11
e −4 + 3	**f** −5 + 2	**g** −11 + 9	**h** −20 + 18
i −4 + 0	**j** −8 + 0	**k** −30 + 29	**l** −39 + 41
m −130 + 132	**n** −181 + 172	**o** −57 + 63	**p** −99 + 68

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Example 3c, d

4 Calculate the answer to these subtractions. Check your answers using a calculator.

a $4 - 6$	**b** $7 - 8$	**c** $3 - 11$	**d** $1 - 20$
e $-3 - 1$	**f** $-5 - 5$	**g** $-2 - 13$	**h** $-7 - 0$
i $-37 - 4$	**j** $39 - 51$	**k** $62 - 84$	**l** $-21 - 26$
m $-100 - 200$	**n** $100 - 200$	**o** $328 - 421$	**p** $-496 - 138$

5 Find the missing number.

a $2 + \square = 7$	**b** $-2 + \square = 7$
c $-2 + \square = 3$	**d** $-4 + \square = -2$
e $5 - \square = 0$	**f** $3 - \square = -4$
g $-9 - \square = -12$	**h** $-20 - \square = -30$
i $-6 + \square = -1$	**j** $-8 - \square = -24$
k $\square + 1 = -3$	**l** $\square + 7 = 2$
m $\square - 4 = -10$	**n** $\square - 7 = -20$
o $\square + 6 = -24$	**p** $\square - 100 = -213$

6 Evaluate the following. Remember to work from left to right.

a $3 - 4 + 6$	**b** $2 - 7 - 4$	**c** $-1 - 4 + 6$	**d** $-5 - 7 - 1$
e $-3 + 2 - 7 + 9$	**f** $-6 + 1 - 20 + 3$	**g** $0 - 9 + 7 - 30$	**h** $-15 - 20 + 32 - 1$

PROBLEM-SOLVING AND REASONING	7, 8, 11	8, 9, 11	9–12

7 Determine how much debt remains in these financial situations.

a owes \$300 and pays back \$155

b owes \$20 and borrows another \$35

c owes \$21 500 and pays back \$16 250

8 **a** The reading on a thermometer measuring temperature rises 18°C from −15°C. What is the final temperature?

b The reading on a thermometer measuring temperature falls 7°C from 4°C. What is the final temperature?

c The reading on a thermometer measuring temperature falls 32°C from −14°C. What is the final temperature?

9 For an experiment, a chemical solution starts at a temperature of 25°C, falls to −3°C, rises to 15°C and then falls again to −8°C. What is the total change in temperature? Add all the changes together for each rise and fall.

10 An ocean sensor is raised and lowered to different depths in the sea. Note that −100 metres means 100 metres below sea level.

a If the sensor is initially at −100 metres and then raised to −41 metres, how far does the sensor rise?

b If the sensor is initially at −37 metres and then lowered to −93 metres, how far is the sensor lowered?

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11 Give an example that suits the description.

a A positive number subtract a positive number equals a negative number.

b A negative number subtract a positive number equals a negative number.

c A negative number add a positive number equals a positive number.

d A negative number add a positive number equals a negative number.

12 **a** a is a positive integer, b is a positive integer and $a > b$. For each of the following, decide if the result will be positive, negative or zero.

i $a + b$

ii $a - b$

iii $b - a$

iv $a - a$

b a is a negative integer and b is a positive integer. Decide if each of the following is *always* true.

i $a + b$ is positive

ii $a - b$ is negative

ENRICHMENT	—	—	13

+ or − combinations

13 Insert + or − signs into these statements to make them true.

a $3 \square 4 \square 5 = 4$

b $1 \square 7 \square 9 \square 4 = -5$

c $-4 \square 2 \square 1 \square 3 \square 4 = 0$

d $-20 \square 10 \square 7 \square 36 \square 1 \square 18 = -4$

e $-a \square b \square a \square b = 0$

f $-a \square a \square 3a \square b \square b = a - 2b$

Month	Mean Temperature °C	
	Daily Minimum	Daily Maximum
Jan	–12.9	–5.8
Feb	–11.6	–3.3
Mar	–7.5	0.7
Apr	–1.8	5.9
May	3.8	12.3
Jun	8.4	16.2
Jul	10.7	18.6
Aug	9.7	17.1
Sep	5.2	12.9
Oct	–1.8	4.7
Nov	–9.4	–2.4
Dec	–12.3	–5.1

Positive and negative numbers are used in everyday life.

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3C Adding or subtracting a negative integer

Interactive
Widgets
HOTsheets
Walkthrough

By observing patterns in number calculations, we can see the effect of adding and subtracting negative integers.

Addition		Subtraction	
$2 + 3 = 5$	-1	$2 - 3 = -1$	$+1$
$2 + 2 = 4$	-1	$2 - 2 = 0$	$+1$
$2 + 1 = 3$	-1	$2 - 1 = 1$	$+1$
$2 + 0 = 2$	-1	$2 - 0 = 2$	$+1$
$2 + (-1) = 1$	-1	$2 - (-1) = 3$	$+1$
$2 + (-2) = 0$	-1	$2 - (-2) = 4$	$+1$
$2 + (-3) = -1$		$2 - (-3) = 5$	

So adding -3 is equivalent to subtracting 3, and subtracting -3 is equivalent to adding 3.

Let's start: Dealing with debt

Let $-\$10$ represent \$10 of debt. Write a statement (e.g. $5 + (-10) = -5$) to represent the following financial situations:

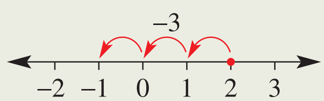
- \$10 of debt is added to a balance of \$5
- \$10 of debt is added to a balance of $-\$5$
- \$10 of debt is removed from a balance of $-\$15$.

Key ideas

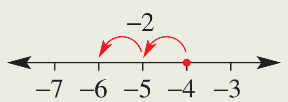
- Adding a negative number is equivalent to subtracting its opposite.

$a + (-b) = a - b$

$2 + (-3) = 2 - 3 = -1$



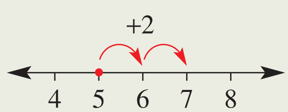
$-4 + (-2) = -4 - 2 = -6$



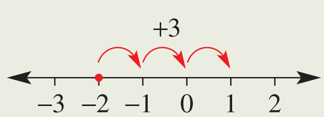
- Subtracting a negative number is equivalent to adding its opposite.

$a - (-b) = a + b$

$5 - (-2) = 5 + 2 = 7$



$-2 - (-3) = -2 + 3 = 1$



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Example 4 Adding and subtracting negative integers

Calculate the answer to these additions and subtractions.

a $7 + (-2)$ **b** $-2 + (-3)$ **c** $1 - (-3)$ **d** $-6 - (-2)$

SOLUTION	EXPLANATION
a $7 + (-2) = 7 - 2$ $= 5$	Adding -2 is equivalent to subtracting 2.
b $-2 + (-3) = -2 - 3$ $= -5$	Adding -3 is equivalent to subtracting 3.
c $1 - (-3) = 1 + 3$ $= 4$	Subtracting -3 is equivalent to adding 3.
d $-6 - (-2) = -6 + 2$ $= -4$	Subtracting -2 is equivalent to adding 2.

Exercise 3C

UNDERSTANDING AND FLUENCY	1, 2, 3–6(½)	3–7(½)	4–7(½)

1 Rewrite each of the following questions as either an addition or a subtraction. The first two have been done for you.

a $6 - (-3) = 6 + 3$ **b** $7 + (-2) = 7 - 2$ **c** $12 - (-8)$ **d** $-9 - (-2)$
e $9 + (-2)$ **f** $-12 - (-4)$ **g** $-12 + (-9)$ **h** $20 - (-9)$

2 Complete these sentences.

a Adding -4 is equivalent to subtracting ☐.
b Adding -6 is equivalent to ____________ 6.
c Adding 5 is equivalent to subtracting ☐.
d Adding -11 is equivalent to ____________ 11.
e Subtracting -2 is equivalent to adding ☐.
f Subtracting -7 is equivalent to ____________ 7.

3 State whether each of the following is true or false.

a $2 + (-3) = 5$ **b** $10 + (-1) = 9$ **c** $-5 + (-3) = -8$ **d** $-6 + (-2) = -4$
e $5 - (-1) = 4$ **f** $3 - (-9) = 12$ **g** $2 - (-3) = 1$ **h** $-11 - (-12) = -1$

Example 4a, b

4 Calculate the answer to each of these additions. Check your answer using a calculator.

a $3 + (-2)$ **b** $8 + (-3)$ **c** $12 + (-6)$ **d** $9 + (-7)$
e $1 + (-4)$ **f** $6 + (-11)$ **g** $20 + (-22)$ **h** $0 + (-4)$
i $-2 + (-1)$ **j** $-7 + (-15)$ **k** $-5 + (-30)$ **l** $-28 + (-52)$
m $-7 + (-3)$ **n** $-20 + (-9)$ **o** $-31 + (-19)$ **p** $-103 + (-9)$

Example 4c, d

5 Calculate the answer to each of these subtractions. Check your answer using a calculator.

a $2 - (-3)$ **b** $5 - (-6)$ **c** $20 - (-30)$ **d** $29 - (-61)$
e $-5 - (-1)$ **f** $-7 - (-4)$ **g** $-11 - (-6)$ **h** $-41 - (-7)$
i $-4 - (-6)$ **j** $-9 - (-10)$ **k** $-20 - (-20)$ **l** $-96 - (-104)$
m $5 - (-23)$ **n** $28 - (-6)$ **o** $-31 - (-19)$ **p** $-104 - (-28)$

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6 Find the missing number.

a $2 + \square = -1$ **b** $3 + \square = -7$ **c** $-2 + \square = -6$

d $\square + (-3) = 1$ **e** $\square + (-10) = -11$ **f** $\square + (-4) = 0$

g $5 - \square = 6$ **h** $2 - \square = 7$ **i** $-1 - \square = 3$

j $\square - (-3) = 7$ **k** $\square - (-10) = 12$ **l** $\square - (-4) = -20$

m $5 - \square = 11$ **n** $\square - (-2) = -3$ **o** $-2 - \square = -4$

p $\square + (-5) = -1$

7 Calculate the answer, working from left to right.

a $3 + (-2) + (-1)$ **b** $2 + (-1) + (-6)$ **c** $3 - (-1) - (-4)$

d $10 - (-6) + (-4)$ **e** $-7 - (-1) + (-3)$ **f** $-20 - (-10) - (-15)$

g $-9 - (-19) + (-16)$ **h** $-15 - (-20) + (-96)$ **i** $-13 - (-19) + (-21)$

j $-2 - (-3) - (-5)$ **k** $-18 - (-16) - (-19)$ **l** $5 + (-20) - (-26)$

PROBLEM-SOLVING AND REASONING	8, 9, 13	9–11, 13, 14	10–12, 14, 15

8 A diver is at a height of -90 metres from the surface of the sea. During a diving exercise, the diver rises 50 metres, falls 138 metres and then rises once again by 35 metres. What is the diver's final height from sea level?

9 A small business has a bank balance of $-\$50000$. An amount of \$20000 of extra debt is added to the balance and, later, \$35000 is paid back. What is the final balance?

10 \$100 of debt is added to an existing balance of \$50 of debt. Later, \$120 of debt is removed from the balance. What is the final balance?

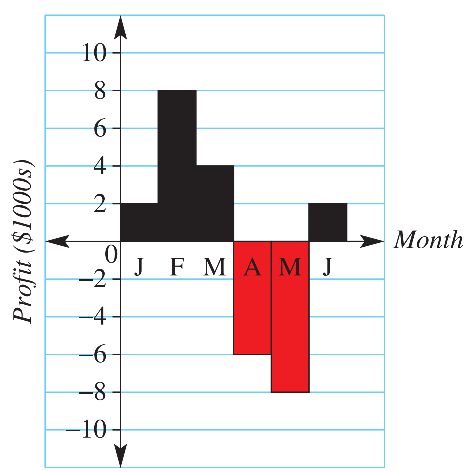
11 Here is a profit graph showing the profit for each month of the first half of the year for a bakery shop.

a What is the profit for:

i February?

ii April?

b What is the overall profit for the 6 months?



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12 Complete these magic squares, using addition. The sum of each row, column and diagonal should be the same.

a

−2		5
	1	
		4

b

		−6
−3		−17
		−7

13 Write these sentences as mathematical statements, e.g. $2 + (-3)$.

a The sum of 3 and 4.

b The sum of −2 and −9.

c The difference between 5 and −2.

d The difference between −2 and 1.

e The sum of a and the opposite of b.

f The difference between a and the opposite of b.

14 Simplify these numbers. Hint: In part **a**, $-(-4)$ is the same as $0 - (-4)$.

a $-(-4)$

b $-(-(-1))$

c $-(-(-(-(-3))))$

15 a If a is a positive number and b is a negative number, decide if each of the following statements is *always* true or false.

i $a + b$ is negative

ii $a - b$ is positive

b If a is a negative number and b is a negative number, decide if each of the following statements is *always* true or false.

i $a + b$ is negative

ii $a - b$ is positive

c If a and b are both negative numbers and $b < a$, is $a - b$ always positive? Give reasons.

ENRICHMENT	—	—	16, 17

Have some fun!

16 Write down the value of these expressions.

a $1 + 2 + 3 + 4 + 5 + \dots + 99 + 100$

b $1 - 2 + 3 - 4 + 5 - \dots + 99 - 100$

c $1 + 2 - 3 - 4 + 5 + 6 - 7 - 8 + 9 + 10 - 11 - 12 \dots - 99 - 100$

17 Use a spreadsheet to check your answers to Question **16**.

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3D Multiplying or dividing by an integer

Interactive Widgets HOTsheets Walkthrough

The rules for multiplication and division of integers can be developed by considering repeated addition.

For example: 4 groups of -3 is $-3 + (-3) + (-3) + (-3) = -12$.

So, $4 \times (-3) = -12$.

Also, $-3 \times 4 = -12$ since $a \times b = b \times a$.

We also know that if $5 \times 7 = 35$, then $35 \div 7 = 5$.

Similarly, if $4 \times (-3) = -12$ then $-12 \div (-3) = 4$. This is saying there are 4 groups of -3 in -12, which we know from the repeated addition above.

Also, $-12 \div 4 = -3$.

These examples give rise to the rules governing the multiplication and division of negative numbers.

Let's start: Patterns in tables

Complete this table of values for multiplication by noticing the patterns. What does the table of values tell you about the rules for multiplying negative integers?

×	–3	–2	–1	0	1	2	3
–3				0			
–2				0			
–1				0			
0	0	0	0	0	0	0	0
1				0	1		
2				0	2	4	
3				0			

Key ideas

- The product or quotient of two numbers of the same sign (i.e. positive or negative) is a positive number.

So	$a \times b = ab$	and	$-a \times (-b) = ab$
For example:	$3 \times 4 = 12$		$-3 \times (-4) = 12$
So	$a \div b = \frac{a}{b}$	and	$-a \div (-b) = \frac{a}{b}$
For example:	$12 \div 4 = 3$		$-12 \div (-4) = 3$

- The product or quotient of two numbers of the opposite sign (i.e. positive and negative) is a negative number.

So	$-a \times b = -ab$	and	$a \times (-b) = -ab$
For example:	$-3 \times 4 = -12$		$3 \times (-4) = -12$
So	$-a \div b = -\frac{a}{b}$	and	$a \div (-b) = -\frac{a}{b}$
For example:	$-12 \div 3 = -4$		$12 \div (-3) = -4$

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Example 5 Multiplying and dividing integers

Calculate these products and quotients.

a $5 \times (-6)$ **b** $-3 \times (-7)$ **c** $-36 \div (-4)$ **d** $-18 \div 9$

SOLUTION	EXPLANATION
a $5 \times (-6) = -30$	The two numbers are of opposite sign, so the answer is negative.
b $-3 \times (-7) = 21$	The two numbers are of the same sign, so the answer is positive.
c $-36 \div (-4) = 9$	The two numbers are of the same sign, so the answer is positive.
d $-18 \div 9 = -2$	The two numbers are of opposite sign, so the answer is negative.

Example 6 Working with mixed operations

Work from left to right to find the answer to $-7 \times 4 \div (-2)$.

SOLUTION	EXPLANATION
$-7 \times 4 \div (-2) = -28 \div (-2)$ $= 14$	First, calculate -7×4. Then calculate $-28 \div (-2)$.

Exercise 3D

UNDERSTANDING AND FLUENCY	1–4, 5–8(½)	4, 5–10(½)	5–10(½)

1 Write down the missing number.

a $2 \times (-3) = -6$, so $-6 \div (-3) = \square$

b $2 \times (-3) = -6$, so $-6 \div 2 = \square$

c $-16 \div 4 = -4$, so $\square \times 4 = -16$

d $16 \div (-4) = -4$, so $\square \times (-4) = 16$

2 Without finding the answer, decide if each of the following is either a positive or negative value.

a -6×7 **b** $-8 \times (-3)$ **c** $-246 \div (-2)$ **d** $7 \div (-1)$

e $-36 \div (-9)$ **f** $48 \times (-1)$ **g** $-9 \times (-3)$ **h** $-36 \div 9$

3 Complete these product tables.

a

×	–2	–1	0	1	2
–2			0		
–1			0		
0	0	0	0	0	0
1			0	1	2
2			0		

b

×	–4	–2	0	2	4
–4	16				
–2					
0					0
2					
4				8	

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4 Complete each sentence by inserting the missing word *positive* or *negative*.

a The product (×) of two positive numbers is ________________.
b The product (×) of two negative numbers is ________________.
c The product (×) of two numbers with opposite signs is________________.
d The quotient (÷) of two positive numbers is ________________.
e The quotient (÷) of two negative numbers is ________________.
f The quotient (÷) of two numbers with opposite signs is ________________.

Example 5a, b

5 Calculate the answer to these products.

a $3 \times (-5)$ **b** $1 \times (-10)$ **c** -3×2 **d** -9×6
e $-8 \times (-4)$ **f** $-2 \times (-14)$ **g** $-12 \times (-12)$ **h** -11×9
i -13×3 **j** $7 \times (-12)$ **k** $-19 \times (-2)$ **l** -36×3
m $-6 \times (-11)$ **n** $5 \times (-9)$ **o** $-21 \times (-3)$ **p** $-36 \times (-2)$

Example 5c, d

6 Calculate the answer to these quotients.

a $14 \div (-7)$ **b** $36 \div (-3)$ **c** $-40 \div 20$ **d** $-100 \div 25$
e $-9 \div (-3)$ **f** $-19 \div (-19)$ **g** $-25 \div 5$ **h** $38 \div (-2)$
i $84 \div (-12)$ **j** $-108 \div 9$ **k** $-136 \div 2$ **l** $-1000 \div (-125)$
m $-132 \div (-11)$ **n** $-39 \div (-3)$ **o** $78 \div (-6)$ **p** $-156 \div (-12)$

Example 6

7 Work from left to right to find the answer. Check your answer using a calculator.

a $2 \times (-3) \times (-4)$ **b** $-1 \times 5 \times (-3)$ **c** $-10 \div 5 \times 2$
d $-15 \div (-3) \times 1$ **e** $-2 \times 7 \div (-14)$ **f** $100 \div (-20) \times 2$
g $48 \div (-2) \times (-3)$ **h** $-36 \times 2 \div (-4)$ **i** $-125 \div 25 \div (-5)$
j $-8 \div (-8) \div (-1)$ **k** $46 \div (-2) \times (-3) \times (-1)$ **l** $-108 \div (-12) \div (-3)$

8 Write down the missing number in these calculations.

a $5 \times \square = -35$ **b** $\square \times (-2) = -8$ **c** $16 \div \square = -4$
d $-32 \div \square = -4$ **e** $\square \div (-3) = -9$ **f** $\square \div 7 = -20$
g $-5000 \times \square = -10000$ **h** $-87 \times \square = 261$ **i** $243 \div \square = -81$
j $50 \div \square = -50$ **k** $-92 \times \square = 184$ **l** $-800 \div \square = -20$

9 Remember that $\frac{9}{3}$ means $9 \div 3$. Use this knowledge to simplify each of the following.

a $\frac{-12}{4}$ **b** $\frac{27}{-9}$ **c** $\frac{-40}{-5}$ **d** $\frac{-124}{-4}$
e $\frac{-15}{-5}$ **f** $\frac{-100}{-20}$ **g** $\frac{-900}{30}$ **h** $\frac{20000}{-200}$

10 Given that $3^2 = 3 \times 3 = 9$ and $(-3)^2 = -3 \times (-3) = 9$, simplify each of the following.

a $(-2)^2$ **b** $(-1)^2$ **c** $(-9)^2$ **d** $(-10)^2$
e $(-6)^2$ **f** $(-8)^2$ **g** $(-3)^2$ **h** $(-1.5)^2$

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PROBLEM-SOLVING AND REASONING	11–14	11, 12, 14	13–16

11 List the different pairs of integers that multiply to give these numbers.

a 6 **b** 16 **c** -5 **d** -24

12 Insert a multiplication or division sign between the numbers to make a true statement.

a $2 \square -3 \square -6 = 1$

b $-25 \square -5 \square 3 = 15$

c $-36 \square 2 \square -3 = 216$

d $-19 \square -19 \square 15 = 15$

13 **a** There are two distinct pairs of numbers whose product is -8 and difference is 6. What are the two numbers?

b The quotient of two numbers is -11 and their difference is 36. What are the two numbers? There are two distinct pairs to find.

14 Given that 2^4 means $2 \times 2 \times 2 \times 2$ and $(-2)^4 = (-2) \times (-2) \times (-2) \times (-2)$

a Calculate:

i $(-2)^3$

ii $(-2)^6$

iii $(-3)^3$

iv $(-3)^4$

b Which questions from part **a** give positive answers and why?

c Which questions from part **a** give negative answers and why?

15 Without evaluating, decide if each of the following would result in a positive or negative answer.

a $(-3)^2 \div 6 \times -8$

b $-15 \times -4 \times (-1)^3$

c $(5 - 9) \times 7 \div (-2)$

16 $a \times b$ is equivalent to ab, and $2 \times (-3)$ is equivalent to $-(2 \times 3)$. Use this information to simplify these expressions.

a $a \times (-b)$ **b** $-a \times b$ **c** $-a \times (-b)$

ENRICHMENT	—	—	17

Multiplication and division with negative integers

17 $(-1) + (-2) + (-3) + (-4) = -10$ and $(-1) \times (-2) \times (-3) \times (-4) = 24$. Therefore, it is possible to use the numbers -1, -2, -3 and -4 to achieve a 'result' of -10 and 24. What other 'results' can you find using those four numbers and any mathematical operations?

For example: What is $(-1) \times (-2) + (-3) \times (-4)$? Can you find expressions for every integer from -20 to 20?

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3E Order of operations with positive and negative integers

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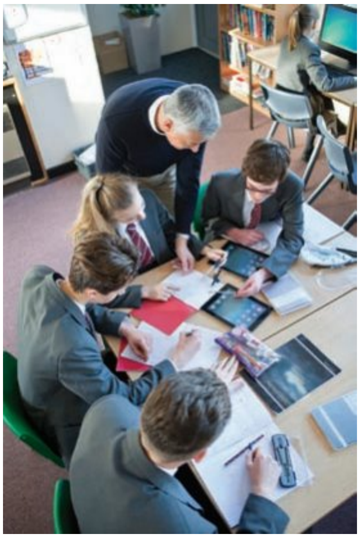
We have learned from our study of positive whole numbers that there is a particular order to follow when dealing with mixed operations and brackets. This order also applies when dealing with negative numbers. For example: $-2 + 3 \times (-4)$ is different from $(-2 + 3) \times (-4)$.

Let's start: Brackets or not?



During a classroom debate about the statement
$3 \times (-4) - 8 \div (-2) = -8$:

- Lil says that the statement needs to have brackets to make it true.
- Max says that even with brackets it is impossible to make it true.
- Riley says that it is correct as it is and there is no need for brackets.

Who is correct and why?

Key ideas

- When working with more than one operation and with positive and/or negative numbers:
  - Deal with brackets first.
  - Do multiplication and division next, working from left to right.
  - Do addition and subtraction last, working from left to right.

$$-2 \times 3 - (10 + (-2)) \div 4$$
(1st: brackets)
$$= -2 \times 3 - 8 \div 4$$
(2nd: -2×3, 3rd: $8 \div 4$)
$$= -6 - 2$$
(last)
$$= -8$$

Example 7 Using order of operations

Use order of operations to evaluate the following.

a $5 + 2 \times (-3)$ **b** $-6 \times 2 - 10 \div (-5)$

SOLUTION	EXPLANATION
a $5 + 2 \times (-3) = 5 + (-6)$ $= -1$	Do the multiplication before the addition.
b $-6 \times 2 - 10 \div (-5) = -12 - (-2)$ $= -12 + 2$ $= -10$	Do the multiplication and division first. When subtracting -2, add its opposite.

Example 8 Order of operations with brackets

Use order of operations to evaluate the following.

a $(-2-1)\times 8$ **b** $5\div(-10+5)+5$ **c** $-6-\frac{10+2}{3}$

SOLUTION

a $(-2-1)\times 8 = -3\times 8$
$= -24$

Deal with brackets first.

b $5\div(-10+5)+5 = 5\div(-5)+5$
$= -1+5$
$= 4$

Deal with brackets first. Then do the division before the subtraction.

c $-6-\frac{10+2}{3} = -6-\frac{12}{3}$
$= -6-4$
$= -10$

Simplify the fraction first.

Exercise 3E

UNDERSTANDING AND FLUENCY	1, 2, 3–4(½)	2–4(½)	3–4(½)

1 Which operation (i.e. addition, subtraction, multiplication or division) is done first in each of the following computations?

a $-2\div 2+1$ **b** $8\times(-6)-4$ **c** $-3+2\times(-6)$
d $7-(-8)\div 4$ **e** $(-2+3)\div 5$ **f** $-6\div(4-(-2))$
g $-4\times 3\div(-6)$ **h** $(2+3\times(-2))+1$ **i** $-11\div(7-2\times(-2))$

2 Classify each of the following statements as true or false.

a $-4+2\times 3=-4+(2\times 3)$ **b** $-4+2\times 3=(-4+2)\times 3$
c $8\times(2-(-2))=8\times 4$ **d** $8\times(2-(-2))=8\times 0$
e $-40-20\div(-5)=(-40-20)\div(-5)$ **f** $-40-20\div(-5)=-40-(20\div(-5))$

Example 7

3 Use order of operations to evaluate the following. Check your answer using a calculator.

a $2+3\times(-3)$ **b** $9+10\div(-5)$ **c** $20+(-4)\div 4$
d $18+(-9)\times 1$ **e** $10-2\times(-3)$ **f** $10-1\times(-4)$
g $-8-(-7)\times 2$ **h** $-2\times 4+8\times(-3)$ **i** $-3(-1)+4\times(-2)$
j $12\div(-6)+4\div(-2)$ **k** $-30\div 5-6\times 2$ **l** $-2\times 3-4\div(-2)$
m $8\times(-2)-(-3)\times 2$ **n** $-1\times 0-(-4)\times 1$ **o** $0\times(-3)-(-4)\times 0+0$

Example 8

4 Use order of operations to evaluate the following. Check your answer using a calculator.

a $(3+2)\times(-2)$ **b** $\frac{8-4}{-2}$ **c** $-3\times(-2+4)$
d $-1\times(7-8)$ **e** $\frac{10}{4-(-1)}$ **f** $(2+(-3))\times(-9)$
g $\frac{24-12}{16+(-4)}$ **h** $(3-7)\div(-1+0)$ **i** $-2\times(8-4)+(-6)$
j $-2-3\times(-1+7)$ **k** $0+(-2)\div(1-2)$ **l** $1-\frac{2\times(-3)}{-3-(-2)}$
m $(-3+(-5))\times(-2-(-1))$ **n** $\frac{3}{1+4}\times 6$ **o** $-5-(8+(-2))+9\div(-9)$

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PROBLEM-SOLVING AND REASONING	5, 6, 9	6, 7, 9, 10	6–8, 10, 11

5 A shop owner had bought socks at \$5 a pair but, during an economic downturn, sold them for \$3 a pair. In a particular week, 124 pairs are sold and there are other costs of \$280. What is the shop owner's overall loss for the week?

6 A debt of \$550 is doubled and then \$350 of debt is removed each month for 3 months. What is the final balance?

7 Insert brackets to make each statement true.

a $-2 + 3 \times 8 = 8$ **b** $-10 \div 4 + 1 = -2$ **c** $-1 + 7 \times 2 - 15 = -3$
d $-5 - 1 \div (-6) = 1$ **e** $3 - 8 \div 5 + 1 = 0$ **f** $50 \times 7 - 8 \times (-1) = 50$
g $-2 \times 3 - (-7) - 1 = -21$ **h** $-3 + 9 \div (-7) + 5 = -3$ **i** $32 - (-8) \div (-3) + 7 = 10$

8 By inserting only *one* pair of brackets, how many different answers are possible for this calculation? Also include the answers for which brackets are not used.
$-2 + 8 \times (-4) - (-3)$

9 If brackets are removed from these problems, does the answer change?

a $(2 \times 3) - (-4)$ **b** $(8 \div (-2)) - 1$ **c** $(-2 + 3) \times 4$
d $9 \div (-4 + 1)$ **e** $(9 - (-3) \times 2) + 1$ **f** $(-1 + 8 \div (-2)) \times 2$

10 State if each of the following is generally true or false.

a $(-3 + 1) + (-7) = -3 + (1 + (-7))$ **b** $(-3 + 1) - (-7) = -3 + (1 - (-7))$
c $(a + b) + c = a + (b + c)$ **d** $(a + b) - c = a + (b - c)$
e $(a - b) + c = a - (b + c)$ **f** $(a - b) - c = a - (b - c)$

11 **a** Given that $5^3 = 5 \times 5 \times 5$, is the answer to each of the following positive or negative?

i $-6 \times (-4) \times (-8) \times (-108) \times (-96)$ **ii** $-100 \div (-2) \div 2 \div (-5)$
iii $(-3)^3$ **iv** $-1 \times (-2)^3$
v $\dfrac{-6 \times (-3) \times 4 \times 7 \times (-3)}{(-2)^2}$ **vi** $\dfrac{(-1)^2 \times (-1)}{(-1)^3 \times (-1)}$

b Explain the strategy you used to answer the questions in part **a**.

ENRICHMENT	—	—	12, 13

Powers and negative numbers

12 First, note that:

- $2^4 = 2 \times 2 \times 2 \times 2 = 16$
- $(-2)^4 = -2 \times (-2) \times (-2) \times (-2) = 16$
- $-2^4 = -(2 \times 2 \times 2 \times 2) = -16$

When evaluating expressions with powers, the power is dealt with first in the order of operations.
For example: $((-2)^3 - 1) \div (-3) = (-8 - 1) \div (-3) = -9 \div (-3) = 3$
Evaluate each of the following.

a 2^2 **b** $(-2)^2$ **c** -2^2
d $(-2)^5$ **e** -2^5 **f** $(32 - 1) \times 4$
g $((-3)^3 - 1) \div (-14)$ **h** $30 \div (1 - 4^2)$ **i** $-10000 \div (-10)^4$

13 Kevin wants to raise -3 to the power of 4. He types -3^4 into a calculator and gets -81. Explain what Kevin has done wrong.

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3F The Cartesian plane

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During the 17th century, two well-known mathematicians, René Descartes and Pierre de Fermat, independently developed the idea of a number plane. The precise positions of points are illustrated using coordinates, and these points can be plotted using the axes as measuring guides. This invention revolutionised the study of mathematics and provided a vital link between geometry and algebra. The number plane, or coordinate plane, is also called the Cartesian plane (named after Descartes). It uses two axes at right angles that extend in both the positive and negative directions.

Mathematician and philosopher René Descartes

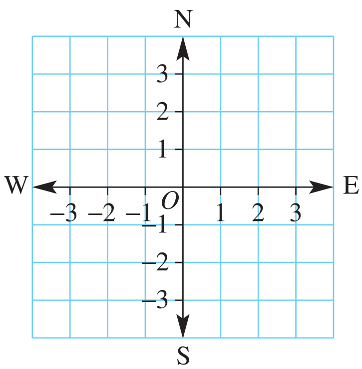
Let's start: North, south, east and west

The units for this grid are in metres.
René starts at position O and moves:

- 3 m east
- 2 m south
- 4 m west
- 5 m north.

Pierre starts at position O and moves:

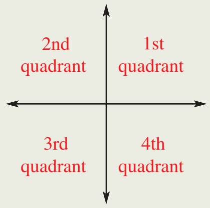
- 1 m west
- 3 m south
- 4 m east
- 5 m north.

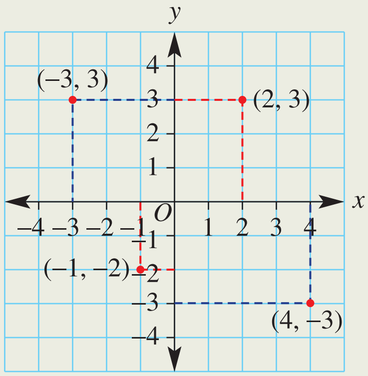


Using the number plane, how would you describe René and Pierre's final positions?

Key ideas

- The **number plane** is also called the Cartesian plane.
- The diagrams show the two axes (the x-axis and the y-axis) and the four quadrants.





- A point plotted on the plane has an x coordinate and y coordinate, which are written as (x, y).
- The point (0, 0) is called the origin and labelled O.
- To plot points, always start at the origin.
  - For (2, 3) move 2 right and 3 up.
  - For (4, –3) move 4 right and 3 down.
  - For (–3, 3) move 3 left and 3 up.
  - For (–1, –2) move 1 left and 2 down.

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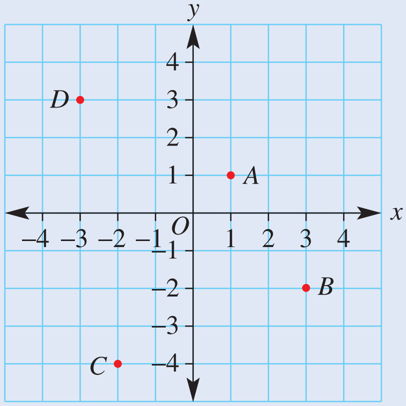
Example 9 Finding coordinates

For the Cartesian plane shown, write down the coordinates of the points labelled A, B, C and D.



SOLUTION

$A = (1, 1)$
$B = (3, -2)$
$C = (-2, -4)$
$D = (-3, 3)$

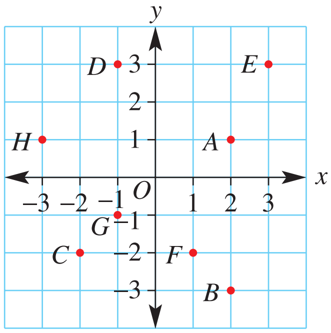
EXPLANATION

For each point, write the x coordinate first (from the horizontal axis) followed by the y coordinate (from the vertical axis).

Exercise 3F

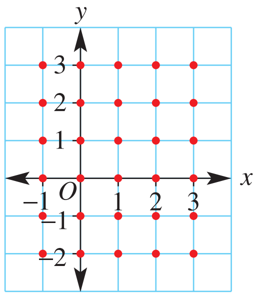
UNDERSTANDING AND FLUENCY	1–6	2–7	3–7

1 Match the points A, B, C, D, E, F, G and H with the given coordinates.



a $(-1, 3)$ **b** $(2, -3)$
c $(2, 1)$ **d** $(-2, -2)$
e $(3, 3)$ **f** $(-3, 1)$
g $(1, -2)$ **h** $(-1, -1)$

2 Count the number of points, shown as dots, on this plane that have:



a both x and y coordinates as positive numbers
b an x coordinate as a positive number
c a y coordinate as a positive number
d an x coordinate as a negative number
e a y coordinate as a negative number
f both x and y coordinates as negative numbers
g neither x nor y as positive or negative numbers

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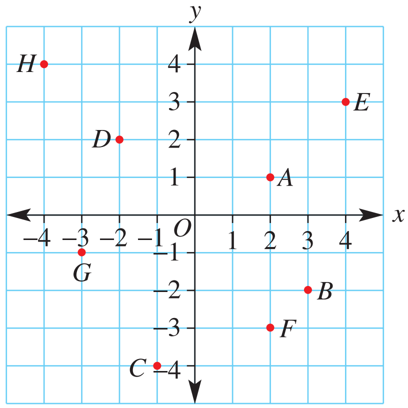
Example 9

3 For the Cartesian plane given, write down the coordinates of the points labelled A, B, C, D, E, F, G and H.



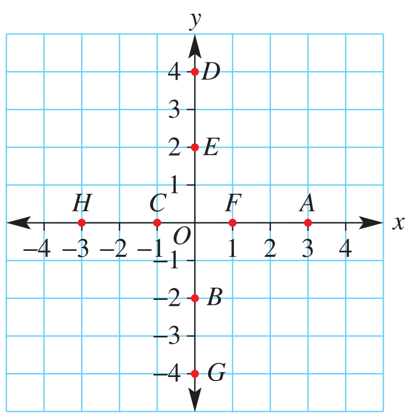
4 **a** Draw a set of axes, using 1 cm spacings. Use −4 to 4 on both axes.

b Now plot these points.

i (−3, 2) **ii** (1, 4) **iii** (2, −1) **iv** (−2, −4)

v (2, 2) **vi** (−1, 4) **vii** (−3, −1) **viii** (1, −2)

5 For the number plane given, write down the coordinates of the points labelled A, B, C, D, E, F, G and H.



6 Seven points have the following x and y coordinates.

x	−3	−2	−1	0	1	2	3
y	−2	−1	0	1	2	3	4

a Plot the seven points on a Cartesian plane. Use −3 to 3 on the x-axis and −2 to 4 on the y-axis.

b What do you notice about these seven points on the Cartesian plane?

7 Seven points have the following x and y coordinates.

x	−3	−2	−1	0	1	2	3
y	5	3	1	−1	−3	−5	−7

a Plot the seven points on a number plane. Use −3 to 3 on the x-axis and −7 to 5 on the y-axis.

b What do you notice about these seven points on the number plane?

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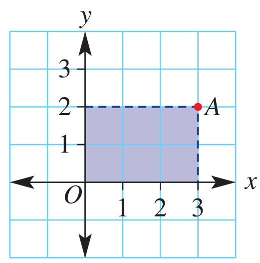
PROBLEM-SOLVING AND REASONING	8, 9, 13	9–11, 13, 14	10–14

8 When plotted on the Cartesian plane, what shape does each set of points form?

a $A(-2, 0)$, $B(0, 3)$, $C(2, 0)$
b $A(-3, -1)$, $B(-3, 2)$, $C(1, 2)$, $D(1, -1)$
c $A(-4, -2)$, $B(3, -2)$, $C(1, 2)$, $D(-1, 2)$
d $A(-3, 1)$, $B(-1, 3)$, $C(4, 1)$, $D(-1, -1)$

9 Using the origin as one corner, the point $A(3, 2)$ as the opposite corner and the axes as two of the sides, a rectangle can be positioned on a set of axes, as shown opposite. Its area is 6 square units. Find the area of the rectangle if the point A is:

a $(2, 2)$
b $(-3, 2)$
c $(-1, -4)$
d $(3, -5)$



10 Karen's bushwalk starts at a point $(2, 2)$ on a grid map. Each square on the map represents 1 kilometre. If Karen walks to the point $(2, -7)$, then $(-4, -7)$, then $(-4, 0)$ and then $(2, 0)$, how far has she walked in total?

11 The points $A(-2, 0)$, $B(-1, ?)$ and $C(0, 4)$ all lie on a straight line. Find the y coordinate of point B.

12 The points $A(-4, 8)$, $B(-1, ?)$ and $C(2, -2)$ all lie on a straight line. Find the y coordinate of point B.

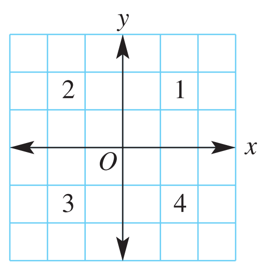
13 Consider the points $A(-2, 2)$, $B(2, -2)$ and $C(3, -2)$.

a Which point is closest to $(0, 0)$?
b Which point is farthest from $(0, 0)$?
c List the given points in order from closest to farthest from the origin, O.

14 A point (a, b) sits on the number plane in one of the four regions 1, 2, 3 or 4, as shown. These regions are called **quadrants**.

a Name the quadrant or quadrants that include the points where:
i $a > 0$
ii $a > 0$ and $b < 0$
iii $b < 0$
iv $a < 0$ and $b < 0$

b Shade the region that includes all points for which $b > a$.



ENRICHMENT	—	—	15

Complete the shape

15 Consider the points $A(0, 0)$ and $B(3, 1)$.

a $ABCD$ is a square. Write down the coordinates of C and D if C is in the first quadrant.
b ABE is an isosceles right-angled triangle. There are four possible locations for point E if AB is not the hypotenuse. List them all.
c G is the point $(1, 3)$ and $ABGH$ is a parallelogram. Write down the coordinates of H.

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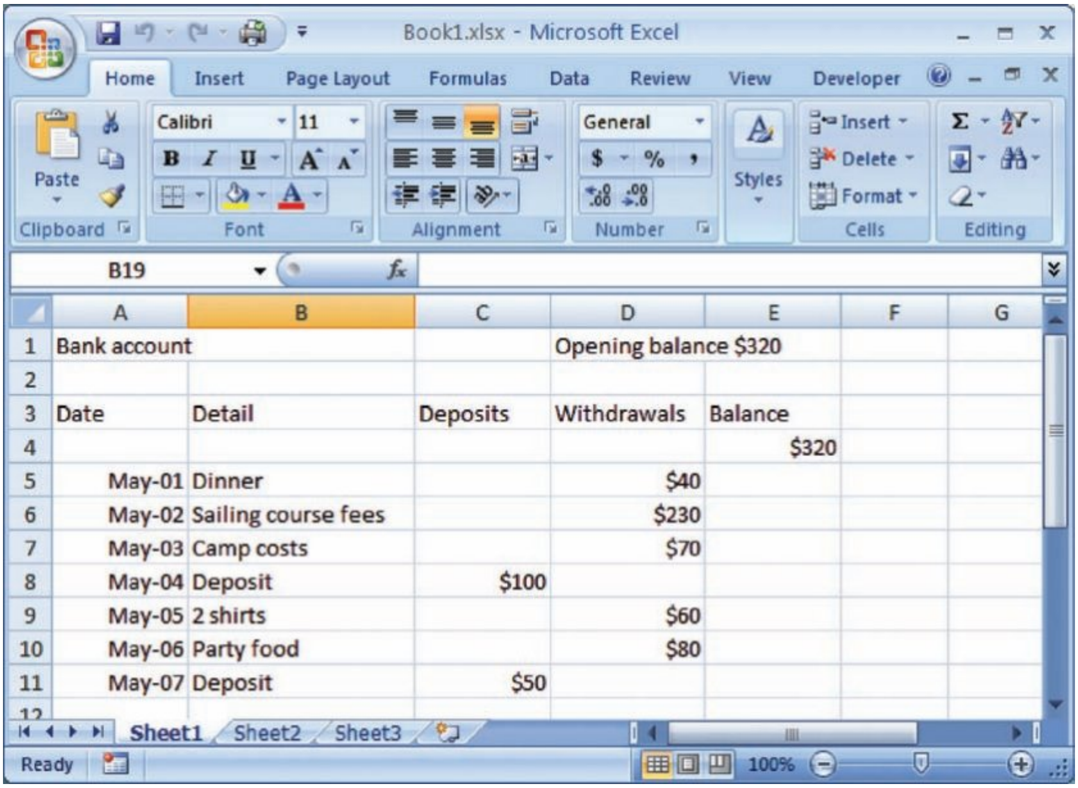
Investigation

Account balance with spreadsheets

If you have money saved in a bank account, your account balance should be positive. If you take out or spend too much money, your account balance may become negative.

a Set up a spreadsheet to record and calculate a bank balance. Enter the given information describing one week of deposits and withdrawals, as shown.



	A	B	C	D	E
1	Bank account			Opening balance $320	
2					
3	Date	Detail	Deposits	Withdrawals	Balance
4					$320
5	May-01	Dinner		$40	
6	May-02	Sailing course fees		$230	
7	May-03	Camp costs		$70	
8	May-04	Deposit	$100		
9	May-05	2 shirts		$60	
10	May-06	Party food		$80	
11	May-07	Deposit	$50		

b **i** For the given spreadsheet, what is the balance at the end of May 1st?

ii On which day does the balance become negative?

c Enter this formula into cell E5: = E4 + C5 − D5
Fill down to reveal the balance after each day.

d Enter another week of deposits and withdrawals so that the balance shows both positive and negative amounts.

e Now alter your opening balance. What opening balance is needed so that the balance never becomes negative? Is there more than one value? What is the least amount?

f Investigate how positive and negative numbers are used on credit card accounts. Give a brief explanation.

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1 Complete these magic squares. All rows, columns and diagonals sum to the same number.

a

		−5
		0
	−6	−1

b

−9	5		−6
	−4		−1
		1	
3		−8	6

c

		−1	
−3			
−7	−5	−4	−10
−2		−13	1

2 Find the next three numbers in these patterns.

a 3, −9, 27, ___, ___, ___

b −32, 16, −8, ___, ___, ___

c 0, −1, −3, −6, ___, ___, ___

d −1, −1, −2, −3, −5, ___, ___, ___

3 Evaluate the following.

a $-100 + (-98) + (-96) + \ldots + 98 + 100$

b $(50 - 53) + (49 - 52) + (48 - 51) + \ldots + (0 - 3)$

4 Insert brackets and symbols (+, −, ×, ÷) into these number sentences to make them true.

a −3 □ 4 □ −2 = −6

b −2 □ 5 □ −1 □ 11 = 21

c 1 □ 30 □ −6 □ −2 = −3

5 **a** The difference between two numbers is 14 and their sum is 8. What are the two numbers?

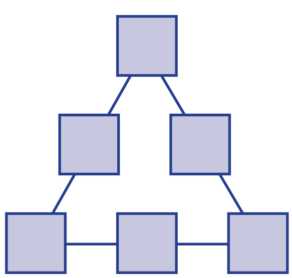
b The difference between two numbers is 31 and their sum is 11. What are the two numbers?

6 Place the integers −3, −2, −1, 0, 1 and 2 into the triangle so that the sum of every side is:

a −3

b 0

c −2



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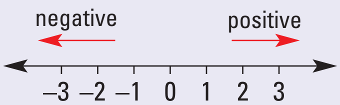
Chapter summary

Integers

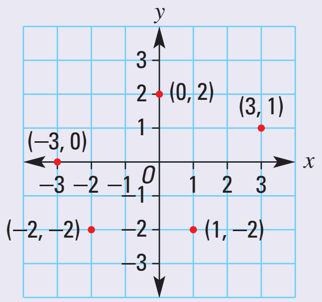
…, −3, −2, −1, 0, 1, 2, 3, …

Number line



$-2 < 3$ $\quad$ $1 > -1$

Cartesian plane



Adding and subtracting positive integers

$-3 + 5 = 2$
$-4 + 3 = -1$
$5 - 7 = -2$
$-1 - 10 = -11$

Adding and subtracting negative integers

$2 + (-3) = 2 - 3 = -1$
$-5 + (-4) = -5 - 4 = -9$
$4 - (-3) = 4 + 3 = 7$
$-10 - (-6) = -10 + 6 = -4$

Multiplication

$2 \times 3 = 6$
$2 \times (-3) = -6$
$-2 \times 3 = -6$
$-2 \times (-3) = 6$

Division

$10 \div 5 = 2$
$10 \div (-5) = -2$
$-10 \div 5 = -2$
$-10 \div (-5) = 2$

Order of operations

First brackets, then × or ÷ then + or −, from left to right.

$$\begin{aligned} & 3 \times (5 - (-2)) + 8 \div (-4) \\ &= 3 \times 7 + (-2) \\ &= 21 + (-2) \\ &= 19 \end{aligned}$$

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Multiple-choice questions

1 When the numbers −4, 0, −1, 7 and −6 are arranged from lowest to highest, the correct sequence is:
A 0, −1, −4, −6, 7 **B** 0, −4, −6, −1, 7 **C** −6, −4, −1, 0, 7
D −1, −4, 6, 0, 7 **E** −6, −1, 0, −4, 7

2 The difference between −19 and 8 is:
A 152 **B** −11 **C** −27 **D** 11 **E** 27

3 The missing number in 2 − □ = 3 is:
A 1 **B** −1 **C** 5 **D** −5 **E** 2

4 5 − (−2) + (−7) is equal to:
A −4 **B** 10 **C** 7 **D** 0 **E** 14

5 The temperature inside a mountain cabin is initially −5°C. After burning a fire for 2 hours the temperature rises to 17°C. What is the rise in temperature?
A −12°C **B** 12°C **C** 22°C **D** −85°C **E** −22°C

6 The product and quotient of two negative numbers is:
A positive **B** negative **C** zero **D** added **E** different

7 −2 × (−5) ÷ (−10) is equal to:
A −5 **B** 10 **C** −20 **D** 1 **E** −1

8 Which operation (i.e. addition, subtraction, multiplication or division) is completed second in the calculation of (−2 + 5) × (−2) + 1?
A addition **B** subtraction **C** multiplication
D division **E** brackets

9 (−2) × 5 − (−2) is equal to:
A −12 **B** −8 **C** 8 **D** 12 **E** 9

10 The points $A(-2, 3)$, $B(-3, -1)$, $C(1, -1)$ and $D(0, 3)$ are joined on a number plane. What shape do they make?
A triangle **B** square **C** trapezium
D kite **E** parallelogram

Chapter review

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Short-answer questions

1 Insert the symbol $<$ (is less than) or $>$ (is greater than) into each statement to make it true.

a $0 \square 7$ **b** $-1 \square 4$ **c** $3 \square -7$ **d** $-11 \square -6$

2 Evaluate:

a $2 - 7$ **b** $-4 + 2$
c $0 - 15$ **d** $-36 + 37$
e $5 + (-7)$ **f** $-1 + (-4)$
g $10 - (-2)$ **h** $-21 - (-3)$
i $1 - 5 + (-2)$ **j** $-3 + 7 - (-1)$
k $0 + (-1) - 10$ **l** $-2 - (-3) - (-4)$

3 Find the missing number for each of the following.

a $-2 + \square = -3$ **b** $-1 + \square = -10$
c $5 - \square = 6$ **d** $-2 - \square = -4$
e $-1 - \square = 20$ **f** $-15 - \square = -13$
g $7 + \square = -80$ **h** $-15 + \square = 15$

4 Evaluate:

a $5 \times (-2)$ **b** -3×7
c $-2 \times (-15)$ **d** $10 \div (-2)$
e $-36 \div 12$ **f** $-100 \div (-25)$
g $-3 \times 2 \div (-6)$ **h** $-38 \div (-19) \times (-4)$

5 Find the missing number.

a $4 \times \square = -8$ **b** $\square \div -5 = 10$
c $\square \div 9 = -4$ **d** $-1 \times \square = 1$

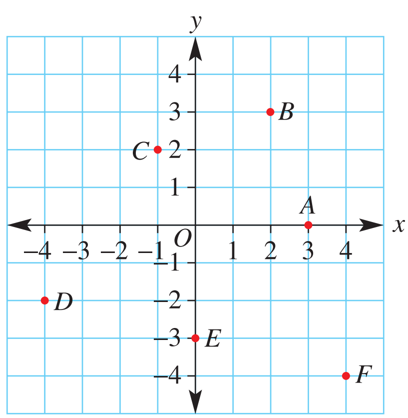
6 Use order of operations to find the answers to these expressions.

a $-2 + 5 \times (-7)$ **b** $-1 - 18 \div (-2)$
c $-15 \div (1 + 4)$ **d** $5 - 4 \times (-3) \div (-3)$
e $(-2 - 5) \times (8 \div (-1))$ **f** $-7 \times ((-4) - 7) + 3$

7 Evaluate:

a $-3 - \dfrac{3 - 3}{3 + 3}$ **b** $(1 - 2) \times 3 - 4$

8 For the Cartesian plane shown, write down the coordinates of the points labelled A, B, C, D, E and F.



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Extended-response questions

1 A scientist, who is camped on the ice in Greenland, records the following details in her notepad regarding the temperature over five days. Note that 'min' stands for minimum and 'max' stands for maximum.

- Monday: min = -18°C, max = -2°C.
- Decreased 29°C from Monday's max to give Tuesday's min.
- Wednesday's min was -23°C.
- Max was only -8°C on Thursday.
- Friday's min is 19°C colder than Thursday's max.

a What is the overall temperature increase on Monday?
b What is Tuesday's minimum temperature?
c What is the difference between the minimum temperatures for Tuesday and Wednesday?
d What is the overall temperature drop from Thursday's maximum to Friday's minimum?
e By how much will the temperature need to rise on Friday if its maximum is 0°C?

2 When joined, these points form a picture on the number plane. What is the picture?
$A(0, 5)$, $B(1, 3)$, $C(1, 1)$, $D(2, 0)$, $E(1, 0)$, $F(1, -2)$, $G(3, -5)$
$H(-3, -5)$, $I(-1, -2)$, $J(-1, 0)$, $K(-2, 0)$, $L(-1, 1)$, $M(-1, 3)$, $N(0, 5)$

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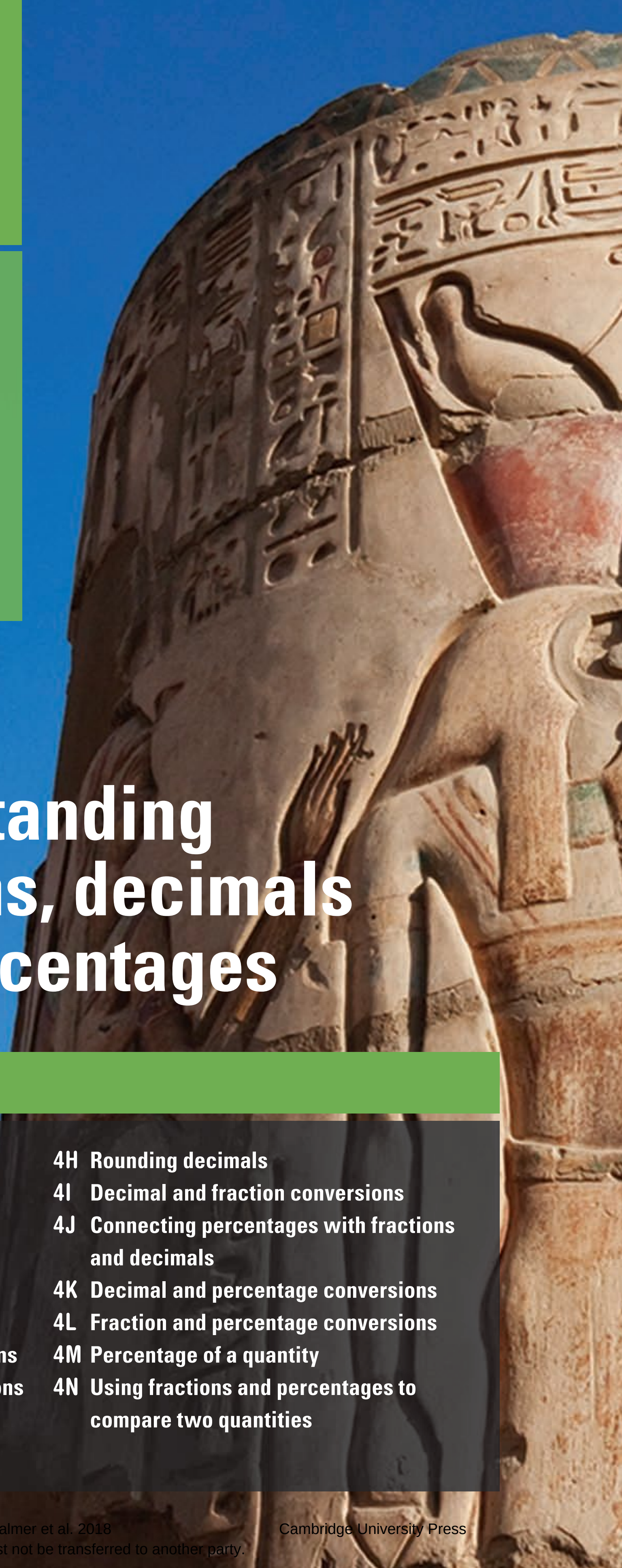
Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library



4 Understanding fractions, decimals and percentages

What you will learn

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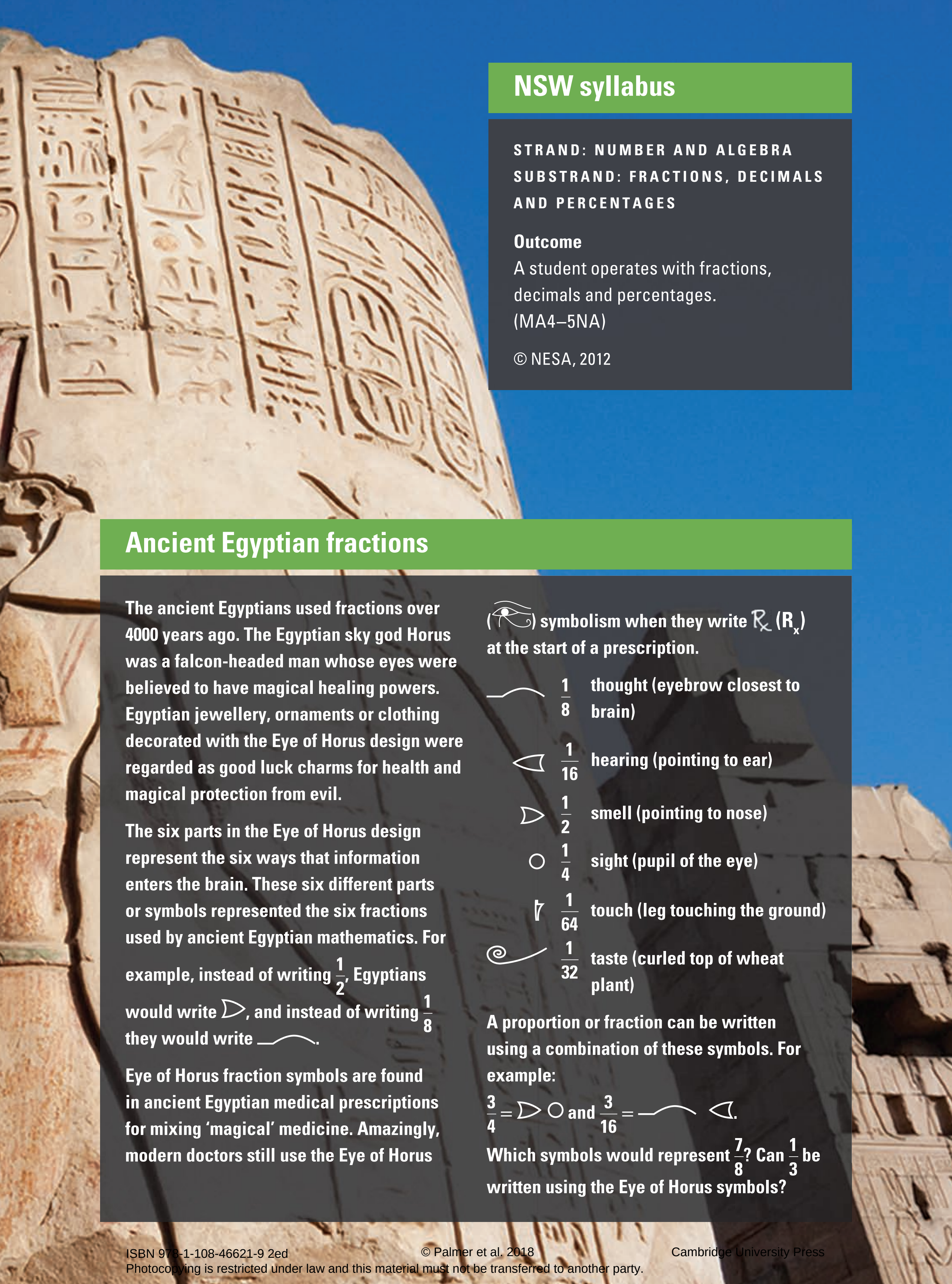

NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRAND: FRACTIONS, DECIMALS AND PERCENTAGES

Outcome

A student operates with fractions, decimals and percentages. (MA4–5NA)

Ancient Egyptian fractions

The ancient Egyptians used fractions over 4000 years ago. The Egyptian sky god Horus was a falcon-headed man whose eyes were believed to have magical healing powers. Egyptian jewellery, ornaments or clothing decorated with the Eye of Horus design were regarded as good luck charms for health and magical protection from evil.

The six parts in the Eye of Horus design represent the six ways that information enters the brain. These six different parts or symbols represented the six fractions used by ancient Egyptian mathematics. For example, instead of writing $\frac{1}{2}$, Egyptians would write ᐅ, and instead of writing $\frac{1}{8}$ they would write ⁀.

Eye of Horus fraction symbols are found in ancient Egyptian medical prescriptions for mixing 'magical' medicine. Amazingly, modern doctors still use the Eye of Horus (eye symbol) symbolism when they write ℞ (R_x) at the start of a prescription.

- $\frac{1}{8}$ thought (eyebrow closest to brain)
- $\frac{1}{16}$ hearing (pointing to ear)
- $\frac{1}{2}$ smell (pointing to nose)
- $\frac{1}{4}$ sight (pupil of the eye)
- $\frac{1}{64}$ touch (leg touching the ground)
- $\frac{1}{32}$ taste (curled top of wheat plant)

A proportion or fraction can be written using a combination of these symbols. For example:

$\frac{3}{4}$ = ᐅ ○ and $\frac{3}{16}$ = ⁀ ᐊ.

Which symbols would represent $\frac{7}{8}$? Can $\frac{1}{3}$ be written using the Eye of Horus symbols?

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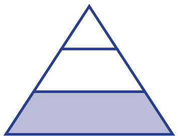
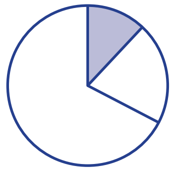
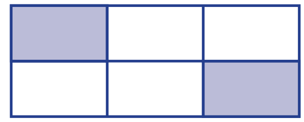
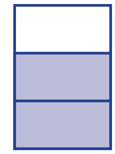
Pre-test

1 In which diagram is one-third shaded?

A 

B 

C 

D 

2 Which of the following is *not* equivalent to one whole?

A $\frac{2}{2}$ **B** $\frac{6}{6}$ **C** $\frac{1}{4}$ **D** $\frac{12}{12}$

3 Which of the following is *not* equivalent to one-half?

A $\frac{2}{4}$ **B** $\frac{3}{9}$ **C** $\frac{5}{10}$ **D** $\frac{10}{20}$

4 Find:

a $1 - \frac{1}{4}$ **b** $1 - \frac{1}{2}$ **c** $1 - \frac{1}{3}$ **d** $1 - \frac{1}{5}$

5 Find:

a $3 - \frac{1}{4}$ **b** $2 - \frac{1}{2}$ **c** $10 - \frac{1}{2}$ **d** $6 - \frac{3}{4}$

6 Tom eats half a block of chocolate on Monday and half of the remaining block on Tuesday. How much chocolate is left for Wednesday?

7 Find the next three terms in these number sequences.

a $0, \frac{1}{2}, 1, 1\frac{1}{2}$, __, __, __

b $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}$, __, __, __

c $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$, __, __, __

d $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$, __, __, __

8 Copy and complete.

a $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \_\_ \times \frac{1}{2}$

b $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \_\_ \times \frac{3}{4}$

c $\frac{5}{6} \times \_\_ = \frac{5}{6}$

d $\frac{6}{8} \div \_\_ = \frac{6}{8}$

9 Find:

a $\frac{1}{2}$ of \$15 **b** $\frac{1}{4}$ of \$160 **c** $\frac{3}{4}$ of \$1 **d** $\frac{1}{3}$ of \$6

10 State whether each of the following is true or false.

a $\frac{1}{2}$ of $16 = 16 \div 2$

b $\frac{16}{4} = \frac{1}{4}$ of 16

c $\frac{3}{4}$ of $100 = 75$

d one-tenth $= \frac{1}{100}$

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4A Factors and multiples

Number patterns are fascinating. Factors and multiples are key building blocks for a broad field known as number theory. Many famous mathematicians have studied number patterns in an attempt to better understand our world and to assist with new scientific discoveries. Around 600 BCE, the Greeks built on the early work of the Egyptians and Babylonians. Thales of Miletus, the 'father of Greek mathematics', is credited for significant advances in number theory. One of his students, Pythagoras of Samos, went on to become one of the most well-known mathematicians to have lived. Pythagoras was primarily a religious leader, but he believed that the understanding of the world could be enhanced through the understanding of number. We start this chapter by explaining the concepts of factors and multiples.


How many factors are there in a set of 12?

One dozen doughnuts are generally packed into bags with 3 rows of 4 doughnuts each. Since $3 \times 4 = 12$, we can say that 3 and 4 are **factors** of 12.

Purchasing 'multiple' packs of one dozen doughnuts could result in buying 24, 36, 48 or 60 doughnuts, depending on the number of packs. These numbers are known as **multiples** of 12.

Let's start: The most factors, the most multiples

Which number that is less than 100 has the most factors?

Which number that is less than 100 has the most multiples less than 100?

Key ideas

- **Factors** of a particular number are numbers that divide exactly into that number.
  - For example: The factors of 20 can be found by considering pairs of numbers that multiply to give 20, which are 1×20, 2×10 and 4×5.
    Therefore, written in **ascending** order, the factors of 20 are 1, 2, 4, 5, 10, 20.
  - Every whole number is a factor of itself and also 1 is a factor of every whole number.
- **Multiples** of a particular number are numbers created by multiplying the particular number by any whole number.
  - For example: The multiples of 20 are 20, 40, 60, 80, 100, 120, …
    Multiples of 20 are also 480, 2000, 68 600. There is an infinite number of multiples!
- Given the statements above, it follows that factors are less than or equal to the particular number being considered and multiples are greater than or equal to the number being considered.

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Example 1 Finding factors

Find the complete set of factors for each of these numbers.

a 15 **b** 40

SOLUTION	EXPLANATION
a Factors of 15 are 1, 3, 5, 15.	$1 \times 15 = 15$, $3 \times 5 = 15$
b Factors of 40 are: 1, 2, 4, 5, 8, 10, 20, 40.	$1 \times 40 = 40$, $2 \times 20 = 40$ $4 \times 10 = 40$, $5 \times 8 = 40$ The last number you need to check is 7.

Example 2 Listing multiples

Write down the first six multiples for each of these numbers.

a 11 **b** 35

SOLUTION	EXPLANATION
a 11, 22, 33, 44, 55, 66	The first multiple is always the given number. Add on the given number to find the next multiple. Repeat this process to get more multiples.
b 35, 70, 105, 140, 175, 210	Start at 35, the given number, and repeatedly add 35 to continue producing multiples.

Example 3 Finding factor pairs

Express 195 as a product of two factors, both of which are greater than 10.

SOLUTION	EXPLANATION
$195 = 13 \times 15$	Systematically divide 195 by numbers greater than 10 in an attempt to find a large factor.

Exercise 4A

UNDERSTANDING AND FLUENCY	1–4(½), 5	2–4(½), 5, 6	3–6(½)

1 For each of the following numbers, state whether they are factors (F), multiples (M) or neither (N) of the number 60.

a 120 **b** 14 **c** 15 **d** 40
e 6 **f** 5 **g** 240 **h** 2
i 22 **j** 600 **k** 70 **l** 1

2 For each of the following numbers, state whether they are factors (F), multiples (M) or neither (N) of the number 26.

a 2 **b** 54 **c** 52 **d** 4
e 210 **f** 27 **g** 3 **h** 182
i 1 **j** 26 000 **k** 13 **l** 39

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Example 1

3 List the complete set of factors for each of the following numbers.

a 10 **b** 24 **c** 17 **d** 36 **e** 60
f 42 **g** 80 **h** 12 **i** 28

Example 2

4 Write down the first six multiples for each of the following numbers.

a 5 **b** 8 **c** 12 **d** 7 **e** 20
f 75 **g** 15 **h** 100 **i** 37

5 Fill in the gaps to complete the set of factors for each of the following numbers.

a 18 1, 2, __, 6, 9, __
b 25 1, __, 25
c 72 __, 2, 3, __, __, 8, __, __, 18, __, 36, 72
d 120 1, 2, __, __, __, 6, __, 10, __, __, 20, __, 30, __, 60, __

6 Which number is the incorrect multiple for each of the following sequences?

a 3, 6, 9, 12, 15, 18, 22, 24, 27, 30
b 43, 86, 129, 162, 215, 258, 301, 344
c 11, 21, 33, 44, 55, 66, 77, 88, 99, 110
d 17, 34, 51, 68, 85, 102, 117, 136, 153, 170

PROBLEM-SOLVING AND REASONING	7, 8, 11	8–10, 11–13	8–10, 13–15

7 Consider the set of whole numbers from 1 to 25 inclusive.

a Which number has the most factors?
b Which number has the fewest factors?
c Which numbers have an odd number of factors?

Example 3

8 Express each of these numbers as a product of two factors, both of which are greater than 10.

a 192 **b** 315 **c** 180
d 121 **e** 336 **f** 494

9 Zane and Matt are both keen runners. Zane takes 4 minutes to jog around a running track and Matt takes 5 minutes. They start at the same time and keep running until they both cross the finish line at the same time.

a How long do they run for? **b** How many laps did Zane run?
c How many laps did Matt run?

10 Anson is preparing for his 12th birthday party. He has invited 12 friends and is making each of them a 'lolly bag' to take home after the party. To be fair, he wants to make sure that each friend has the same number of lollies. Anson has a total of 300 lollies to share among the lolly bags.

a How many lollies does Anson put in each of his friends' lolly bags?
b How many lollies does Anson have left over to eat himself?

Anson then decides that he wants a lolly bag for himself also.

c How many lollies will now go into each of the 13 lolly bags?

After much pleading from his siblings, Anson prepares lolly bags for them also. His sister Monique notices that the total number of lolly bags is now a factor of the total number of lollies.

d What are the different possible number of sibling(s) that Anson could have?
e How many siblings do you expect Anson has?

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11 Are the following statements true or false?

- **a** A multiple of a particular number is always smaller than that number.
- **b** 2 is a factor of every even number.
- **c** 3 is a factor of every odd number.
- **d** A factor is always greater than or equal to the given number.
- **e** When considering a particular number, that number is both a factor and a multiple of itself.

12 60 is a number with many factors. It has a total of 12 factors and, interestingly, it has each of the numbers 1, 2, 3, 4, 5, 6 as a factor.

- **a** What would be the smallest number that could boast having 1, 2, 3, 4, 5, 6, 7 and 8 as factors?
- **b** What would be the smallest number that could boast having 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 as factors?
- **c** Express, in written form, how you can determine the smallest number that contains a particular set of factors.

13 **a** What numbers can claim the number 100 to be a multiple?

- **b** What are the factors of 100?

14 All AM radio stations have frequencies that are multiples of 9. For example, a particular radio station has a frequency of 774 kilohertz (or kHz). Find three other AM radio stations and show their frequencies are, indeed, multiples of 9.

15 Two numbers are chatting with one another when one number asks the other, “Are you a multiple of mine?” The reply comes back, “Well, I have always considered you to be one of my factors.” Explain why this response is enough to help the first number answer her question.

ENRICHMENT	—	—	16

Designing some help from the computer

16 **a** Design a spreadsheet that will enable a user to enter any number between 1 and 100 and it will automatically list the first 30 multiples of that number.

- **b** Design a spreadsheet that will enable a user to enter any particular number between 1 and 100 and it will automatically list the number’s factors.
- **c** Improve your factor program so that it finds the sum of the factors and also states the total number of factors for the particular number.
- **d** Use your spreadsheet program to help you find a pair of **amicable numbers**. A pair of numbers is said to be amicable if the sum of the factors for each number, excluding the number itself, is equal to the other number. Each number that makes up the first such pair of amicable numbers falls between 200 and 300.

  An example of a non-amicable pair of numbers:

  12 – factor sum $= 1 + 2 + 3 + 4 + 6 = 16$

  16 – factor sum $= 1 + 2 + 4 + 8 = 15$

  The factor sum for 16 would need to be 12 for the pair to be amicable numbers.

  Helpful Excel formulas

  INT (number) – Rounds a number down to the nearest integer (whole number).

  MOD (number, divisor) – Returns the remainder after a number is divided by its divisor.

  IF (logical test, value if true, value if false) – Checks whether a condition is met and returns one value if true and another value if false.

  COUNTIF (range, criteria) – Counts the number of cells within a range that meet the given condition.

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4B Highest common factor and lowest common multiple

Interactive
Widgets
HOTsheets
Walkthrough

In the previous exercise, factors and multiples of a number were explained. Remember that factors are less than or equal to a given number and that multiples are greater than or equal to a given number.

There are many applications in mathematics for which the highest common factor (HCF) of two or more numbers must be determined. In particular, the skill of finding the HCF is required for the future topic of factorisation, which is an important aspect of algebra.

given number
e.g. 12

factors
$\leqslant 12$
e.g. 1, 2, 3, 4, 6, 12

multiples
$\geqslant 12$
e.g. 12, 24, 36, 48, …

Similarly, there are many occasions for which the lowest common multiple (LCM) of two or more numbers must be determined. Adding and subtracting fractions with different denominators requires the skill of finding the LCM.

Let's start: You provide the starting numbers!

For each of the following answers, you must determine possible starting numbers. On all occasions, the numbers involved are less than 100.

1. The HCF of two numbers is 12. Suggest two possible starting numbers.
2. The HCF of three numbers is 11. Suggest three possible starting numbers.
3. The LCM of two numbers is 30. Suggest two possible starting numbers.
4. The LCM of three numbers is 75. Suggest three possible starting numbers.
5. The HCF of four numbers is 1. Suggest four possible numbers.
6. The LCM of four numbers is 24. Suggest four possible numbers.

Key ideas

- **HCF** stands for **highest common factor**.
- As the name suggests, it refers to the highest (i.e. largest) factor that is common to the numbers provided in the question.
  For example: Find the HCF of 24 and 40.

  Factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.

  Factors of 40 are 1, 2, 4, 5, 8, 10, 20 and 40.

  Therefore, common factors of 24 and 40 are 1, 2, 4 and 8.

  Therefore, the highest common factor of 24 and 40 is 8.

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Key ideas

- **LCM** stands for **lowest common multiple**.
- As the name suggests, it refers to the lowest (i.e. smallest) multiple that is common to the numbers provided in the question.
  For example: Find the LCM of 20 and 12.

  Multiples of 20 are 20, 40, 60, 80, 100, 120, 140, …

  Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, …

  Therefore, common multiples of 20 and 12 are 60, 120, 180, …

  Therefore, the lowest common multiple of 20 and 12 is 60.
- The LCM of two numbers can always be found by multiplying the two numbers together and dividing by their HCF.
  For example: Find the LCM of 20 and 12.

  The HCF of 20 and 12 is 4.

  Therefore, the LCM of 20 and 12 is $20 \times 12 \div 4 = 60$.

Example 4 Finding the highest common factor (HCF)

Find the highest common factor (HCF) of 36 and 48.

SOLUTION	EXPLANATION
Factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18 and 36.	$1 \times 36 = 36$, $2 \times 18 = 36$, $3 \times 12 = 36$, $4 \times 9 = 36$, $6 \times 6 = 36$
Factors of 48 are: 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48.	$1 \times 48 = 48$, $2 \times 24 = 48$, $3 \times 16 = 48$, $4 \times 12 = 48$, $6 \times 8 = 48$
The HCF of 36 and 48 is 12.	Common factors are 1, 2, 3, 4, 6 and 12, of which 12 is the highest.

Example 5 Finding the lowest common multiple (LCM)

Find the lowest common multiple (LCM) of the following pairs of numbers.

a 5 and 11 **b** 6 and 10

SOLUTION	EXPLANATION
a The LCM of 5 and 11 is 55.	Note that the HCF of 5 and 11 is 1. $5 \times 11 = 55$
b The LCM of 6 and 10 is 30.	Note that the HCF of 6 and 10 is 2. The LCM of 6 and 10 is $6 \times 10 \div 2 = 30$. Multiples of 6 are 6, 12, 18, 24, 30, 36, … Multiples of 10 are 10, 20, 30, 40, …

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Exercise 4B

UNDERSTANDING AND FLUENCY	1–4, 5–7(½)	4, 5–8(½)	5–8(½), 9

1 The factors of 12 are 1, 2, 3, 4, 6 and 12, and the factors of 16 are 1, 2, 4, 8 and 16.
- **a** What are the common factors of 12 and 16?
- **b** What is the HCF of 12 and 16?

2 Fill in the missing numbers to find out the HCF of 18 and 30.

Factors of 18 are 1, __, 3, __, __ and 18.

Factors of ___ are 1, __, __, 5, __, 10, __ and 30.

Therefore, the HCF of 18 and 30 is ___.

3 The first 10 multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72 and 80.

The first 10 multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54 and 60.
- **a** What are two common multiples of 8 and 6?
- **b** What is the LCM of 8 and 6?

4 Fill in the missing numbers to find out the LCM of 9 and 15.

Multiples of 9 are 9, 18, __, 36, __, __, __, __, 81 and __.

Multiples of 15 are __, 30, __, 60, 75, __, __ and 120.

Therefore, the LCM of 9 and 15 is __.

Example 4

5 Find the HCF of the following pairs of numbers.
- **a** 4 and 5
- **b** 8 and 13
- **c** 2 and 12
- **d** 3 and 15
- **e** 16 and 20
- **f** 15 and 60
- **g** 50 and 150
- **h** 48 and 72
- **i** 80 and 120
- **j** 75 and 125
- **k** 42 and 63
- **l** 28 and 42

6 Find the HCF of the following groups of numbers.
- **a** 20, 40, 50
- **b** 6, 15, 42
- **c** 50, 100, 81
- **d** 18, 13, 21
- **e** 24, 72, 16
- **f** 120, 84, 144

Example 5

7 Find the LCM of the following pairs of numbers.
- **a** 4 and 9
- **b** 3 and 7
- **c** 12 and 5
- **d** 10 and 11
- **e** 4 and 6
- **f** 5 and 10
- **g** 12 and 18
- **h** 6 and 9
- **i** 20 and 30
- **j** 12 and 16
- **k** 44 and 12
- **l** 21 and 35

8 Find the LCM of the following groups of numbers.
- **a** 2, 3, 5
- **b** 3, 4, 7
- **c** 2, 3, 4
- **d** 3, 5, 9
- **e** 4, 5, 8, 10
- **f** 6, 12, 18, 3

9 Find the HCF of the following pairs of numbers and then use this information to help calculate the LCM of the same pair of numbers.
- **a** 15 and 20
- **b** 12 and 24
- **c** 14 and 21
- **d** 45 and 27

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PROBLEM-SOLVING AND REASONING	10, 11, 14	10, 11, 13, 14	12–15

10 Find the LCM of 13 and 24.

11 Find the HCF of 45 and 72.

12 Find the LCM and HCF of 260 and 390.

13 Andrew runs laps of ‘the circuit’ in 4 minutes. Bryan runs laps of the same circuit in 3 minutes. Chris can run laps of the same circuit in 6 minutes. They all start together on the starting line and run a ‘race’ that goes for 36 minutes.

- **a** What is the first time, after the start, that they will all cross over the starting line together?
- **b** How many laps will each boy complete in the race?
- **c** How many times does Bryan overtake Andrew during this race?
- **d** Prepare a 1-minute class presentation involving three PowerPoint slides that describes how you solved this problem and explains your answer.

14 Given that the HCF of a pair of different numbers is 8, find the two numbers:

- **a** if both numbers are less than 20
- **b** when one number is in the 20s and the other is in the 30s

15 Given that the LCM of a pair of numbers is 20, find the seven possible pairs of numbers.

ENRICHMENT	—	—	16

LCM of large groups of numbers

16
- **a** Find the LCM of these single-digit numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9.
- **b** Find the LCM of these first 10 natural numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
- **c** Compare your answers to parts **a** and **b**. What do you notice? Explain.
- **d** Find the LCM of the first 11 natural numbers.

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4C What are fractions?

Interactive
Widgets
HOTsheets
Walkthrough

The word fraction comes from the Latin word ‘frangere’, which means ‘to break into pieces’.

Although the following sentences are not directly related to the mathematical use of fractions, they all contain words that are related to the original Latin term ‘frangere’ and they help us gain an understanding of exactly what a fraction is.

- *The fragile vase smashed into a hundred pieces when it landed on the ground.*
- *After the window was broken, several fragments were found on the floor.*
- *She fractured her leg in two places.*
- *The computer was running slowly and needed to be defragmented.*
- *The elderly gentleman was becoming very frail in his old age.*

Can you think of any other related sentences?

Brainstorm specific common uses of fractions in everyday life. The list could include cooking, shopping, sporting, building examples and more.

Let’s start: What strength do you like your cordial?

Imagine preparing several jugs of different strength cordial. Samples could include $\frac{1}{4}$ strength cordial, $\frac{1}{5}$ strength cordial, $\frac{1}{6}$ strength cordial or $\frac{1}{8}$ strength cordial.

- In each case, describe how much water and how much cordial is needed to make a 1-litre mixture. Note: 1 litre (L) = 1000 millilitres (mL).
- On the label of a cordial container, it suggests ‘To make up by glass or jug: add five parts water to one part cordial, according to taste.’
- What fraction of the beverage is cordial?

Key ideas

- A fraction is made up of a **nu**merator (**up**) and a **d**enominator (**d**own).

  For example: $\frac{3}{5}$ ← numerator (3), ← denominator (5)

  - The **denominator** tells you how many parts the whole is divided up into.
  - The **numerator** tells you how many of the divided parts you have selected.
  - The horizontal line separating the numerator and the denominator is called the **vinculum**.

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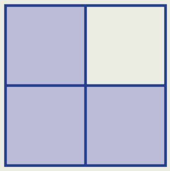
Key ideas

- We can represent fractions using area. If a shape is divided into regions of equal areas, then shading a certain number of these regions will create a fraction of the whole shape.

  For example:  Fraction shaded = $\frac{3}{4}$

- A **proper fraction** or **common fraction** is less than a whole, and therefore the numerator must be smaller than the denominator.

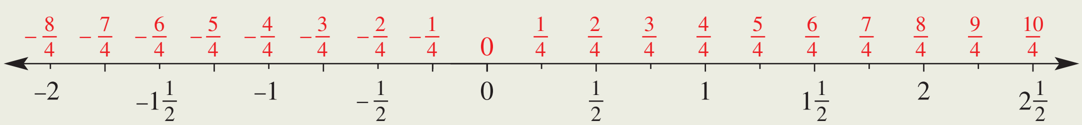
  For example: $\frac{2}{7}$ is a proper fraction.

- An **improper fraction** is greater than a whole, and therefore the numerator must be larger than the denominator.

  For example: $\frac{5}{3}$ is an improper fraction.

- We can represent positive and negative fractions on a number line.

  This number line shows the whole numbers from −2 to 2. Each unit has then been divided equally into four segments, therefore creating 'quarters'.

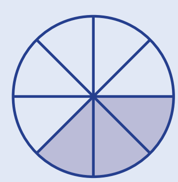


- Whole numbers can be represented as fractions.

  On the number line above we see that 1 is the same as $\frac{4}{4}$ and 2 is the same as $\frac{8}{4}$.

Example 6 Understanding the numerator and the denominator

a Into how many pieces has the whole pizza been divided?
b How many pieces have been selected (i.e. shaded)?
c In simplest form, when representing the shaded fraction of the pizza:
  i what must the denominator equal?
  ii what must the numerator equal?
  iii write the amount of pizza selected (shaded) as a fraction.



SOLUTION	EXPLANATION
a 8	Pizza cut into 8 equal pieces.
b 3	3 of the 8 pieces are shaded in blue.
c **i** 8	Denominator shows the number of parts the whole has been divided into.
ii 3	Numerator tells how many of the divided parts you have selected.
iii $\frac{3}{8}$	Shaded fraction is the numerator over the denominator; i.e. 3 out of 8 divided pieces.

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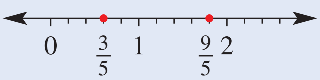
Example 7 Representing fractions on a number line

Represent the fractions $\frac{3}{5}$ and $\frac{9}{5}$ on a number line.

SOLUTION



EXPLANATION

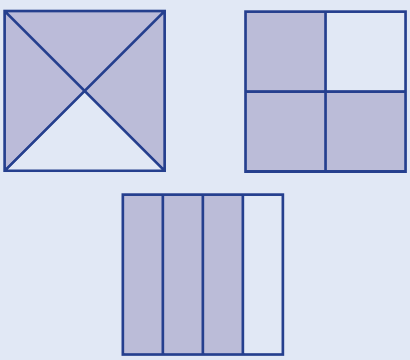
Draw a number line starting at 0 and mark on it the whole numbers 0, 1 and 2.

Divide each whole unit into five segments of equal length. Each of these segments has a length of one-fifth.

Example 8 Shading areas

Represent the fraction $\frac{3}{4}$ in three different ways, using a square divided into four equal regions.

SOLUTION



EXPLANATION

Ensure division of square creates four equal areas.

Shade in three of the four regions.

Exercise 4C

UNDERSTANDING AND FLUENCY	1–4, 5	4, 5(½), 6, 7	5(½), 6, 7(½)

1 **a** State the denominator of this proper fraction: $\frac{2}{9}$.

b State the numerator of this improper fraction: $\frac{7}{5}$.

2 Group the following list of fractions into proper fractions, improper fractions and whole numbers.

A $\frac{7}{6}$ **B** $\frac{2}{7}$ **C** $\frac{50}{7}$ **D** $\frac{3}{3}$

E $\frac{3}{4}$ **F** $\frac{5}{11}$ **G** $\frac{1}{99}$ **H** $\frac{9}{4}$

I $\frac{11}{8}$ **J** $\frac{10}{10}$ **K** $\frac{5}{1}$ **L** $\frac{121}{5}$

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Example 6

3 Answer the following questions for each of the pizzas (**A** to **D**) drawn below.

a Into how many pieces has the whole pizza been divided?

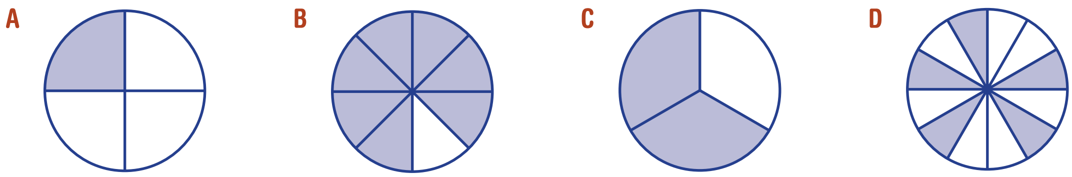
b How many pieces have been selected (shaded)?

c In representing the shaded fraction of the pizza:

i what must the denominator equal?

ii what must the numerator equal?

iii write the amount of pizza selected (shaded) as a fraction.



4 Find the whole numbers among the following list of fractions. Hint: There are five whole numbers to find.

A $\frac{15}{4}$ **B** $\frac{14}{8}$ **C** $\frac{12}{5}$ **D** $\frac{30}{15}$

E $\frac{17}{3}$ **F** $\frac{30}{12}$ **G** $\frac{12}{12}$ **H** $\frac{33}{10}$

I $\frac{53}{3}$ **J** $\frac{9}{3}$ **K** $\frac{50}{20}$ **L** $\frac{28}{7}$

M $\frac{96}{8}$ **N** $\frac{24}{5}$ **O** $\frac{62}{4}$ **P** $\frac{1031}{2}$

Example 7

5 Represent the following fractions on a number line.

a $\frac{3}{7}$ and $\frac{6}{7}$

b $\frac{2}{3}$ and $\frac{5}{3}$

c $\frac{1}{6}$ and $\frac{5}{6}$

d $\frac{2}{4}$ and $\frac{11}{4}$

e $\frac{11}{5}$ and $-\frac{8}{5}$

f $-\frac{5}{4}$, $-\frac{9}{4}$ and $-\frac{3}{2}$

Example 8

6 Represent each of these fractions in three different ways, using a rectangle divided into equal regions.

a $\frac{1}{4}$ **b** $\frac{3}{8}$ **c** $\frac{2}{6}$

7 Write the next three fractions for each of the following fraction sequences.

a $\frac{3}{5}, \frac{4}{5}, \frac{5}{5}, \frac{6}{5}$, __, __, __

b $\frac{5}{8}, \frac{6}{8}, \frac{7}{8}, \frac{8}{8}$, __, __, __

c $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}$, __, __, __

d $\frac{11}{7}, \frac{10}{7}, \frac{9}{7}, \frac{8}{7}$, __, __, __

e $\frac{9}{2}, \frac{7}{2}, \frac{5}{2}, \frac{3}{2}$, __, __, __

f $\frac{18}{4}, \frac{13}{4}, \frac{8}{4}, \frac{3}{4}$, __, __, __

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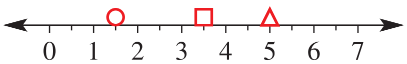
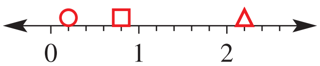
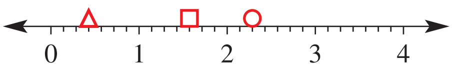
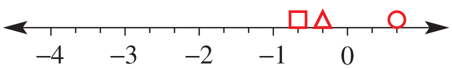
PROBLEM-SOLVING AND REASONING | 8, 11(½) | 8, 9, 11(½), 12 | 9, 10, 12, 13

8 What fractions correspond to each of the different shapes positioned on these number lines?

a 

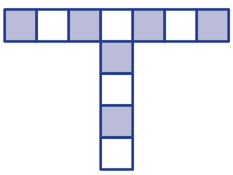
b 

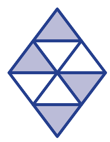
c 

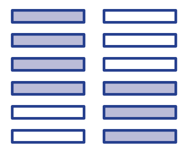
d 

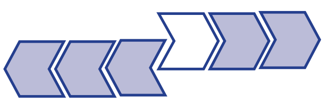
9 What operation (i.e. +, –, × or ÷) does the vinculum relate to?

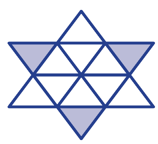
10 For each of the following, state what fraction of the diagram is shaded.

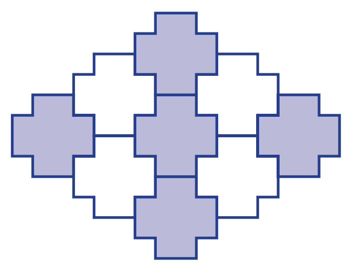
a 

b 

c 

d 

e 

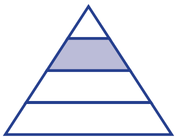
f 

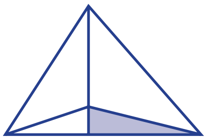
11 For each of the following, write the fraction that is describing part of the total.

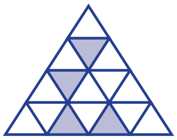
a After one day of a 43-kilometre hike, they had completed 12 kilometres.
b From 15 starters, 13 went on to finish the race.
c Rainfall for 11 months of the year was below average.
d One egg is broken in a carton that contains a dozen eggs.
e Two players in the soccer team scored a goal.
f The lunch stop was 144 kilometres into the 475-kilometre trip.
g Seven members in the class of 20 have visited Australia Zoo.
h One of the car tyres is worn and needs replacing.
i It rained three days this week.

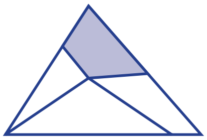
12 Explain the logic behind the terms ‘proper fraction’ and ‘improper fraction’.

13 Which diagram has one-quarter shaded?

A 

B 

C 

D 

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ENRICHMENT	—	—	14

Adjusting concentration

14 a A 250-millilitre glass of cordial is made by mixing four parts water to one part cordial.
- **i** What fraction of the glass is cordial?
- **ii** What amount of cordial is required?

b Fairuz drinks 50 millilitres of the glass of cordial and thinks it is too strong. So he fills the glass back up with 50 millilitres of pure water.
- **i** How much cordial is in the glass now?
- **ii** What fraction of the glass is cordial?

c Fairuz drinks 50 millilitres of the glass of cordial but he still thinks it is too strong. So, once again, he fills the glass back up with 50 millilitres of pure water.
- **i** How much cordial is in the glass now?
- **ii** What fraction of the glass is cordial?

d Lynn prefers her cordial much stronger compared with Fairuz. When she is given a glass of the cordial that is mixed at four parts to one, she drinks 50 millilitres and decides it is too weak. So she fills the glass back up with 50 millilitres of straight cordial.
- **i** How much cordial is in Lynn's glass after doing this once?
- **ii** What fraction of the glass is cordial?

e Like Fairuz, Lynn needs to repeat the process to make her cordial even stronger. So, once again, she drinks 50 millilitres and then tops the glass back up with 50 millilitres of straight cordial.
- **i** How much cordial is in Lynn's glass now?
- **ii** What fraction of the glass is cordial?

f If Fairuz continues diluting his cordial concentration in this manner and Lynn continues strengthening her cordial concentration in this manner, will either of them ever reach pure water or pure cordial? Discuss.

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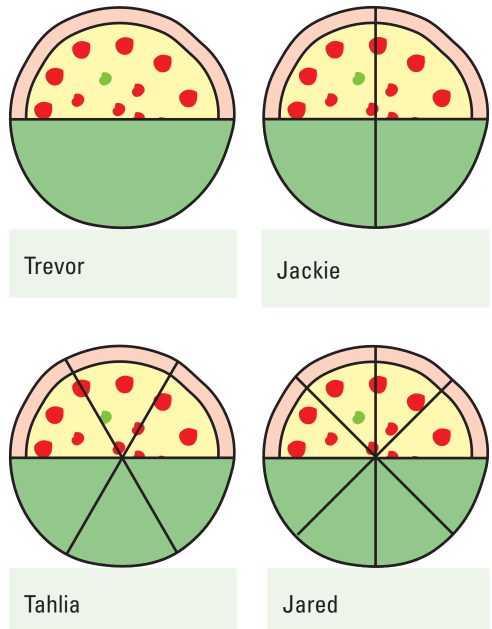
4D Equivalent fractions and simplified fractions

Often fractions may look very different when in fact they have the equivalent value.

For example, in an AFL football match, 'half-time' is the same as 'the end of the second quarter'. We can say that $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions. In both situations, the equivalent fraction of the game has been completed.

Consider a group of friends eating pizzas during a sleepover. The pizzas are homemade and each person cuts up their pizza as they like.

Trevor cuts his pizza into only two pieces, Jackie cuts hers into four pieces, Tahlia cuts hers into six pieces and Jared cuts his into eight pieces. The shaded pieces are the amount that they have eaten before it is time to start the second movie.



By looking at the pizzas, it is clear to see that Trevor, Jackie, Tahlia and Jared have all eaten the same amount of pizza. We can therefore conclude that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$ and $\frac{4}{8}$ are equivalent fractions.

This means that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$.

Let's start: Fraction clumps

Prepare a class set of fraction cards. (Two example sets are provided below.)

- Hand out one fraction card to each student.
- Students then arrange themselves into groups of equivalent fractions.
- Set an appropriate time goal by which this task must be completed.

Repeat the process with a second set of equivalent fraction cards.

Sample sets of fraction cards

Class set 1

$$\frac{1}{2}, \frac{3}{12}, \frac{3}{24}, \frac{10}{80}, \frac{1}{3}, \frac{8}{40}, \frac{1}{5}, \frac{3}{6}, \frac{1}{8}, \frac{5}{40}, \frac{3}{9}, \frac{1}{4}, \frac{1000}{4000}, \frac{100}{200}, \frac{10}{50}, \frac{2}{16}, \frac{10}{30}, \frac{13}{39}, \frac{5}{10}, \frac{7}{14}, \frac{2}{6}, \frac{7}{28}, \frac{2}{10}, \frac{4}{8}, \frac{2}{8}$$

Class set 2

$$\frac{2}{3}, \frac{6}{14}, \frac{3}{18}, \frac{4}{10}, \frac{2}{12}, \frac{24}{64}, \frac{11}{66}, \frac{4}{6}, \frac{3}{7}, \frac{30}{70}, \frac{12}{32}, \frac{3}{8}, \frac{10}{15}, \frac{5}{30}, \frac{1}{6}, \frac{2000}{5000}, \frac{21}{49}, \frac{300}{800}, \frac{6}{9}, \frac{9}{21}, \frac{2}{5}, \frac{14}{35}, \frac{20}{30}, \frac{6}{16}, \frac{22}{55}$$

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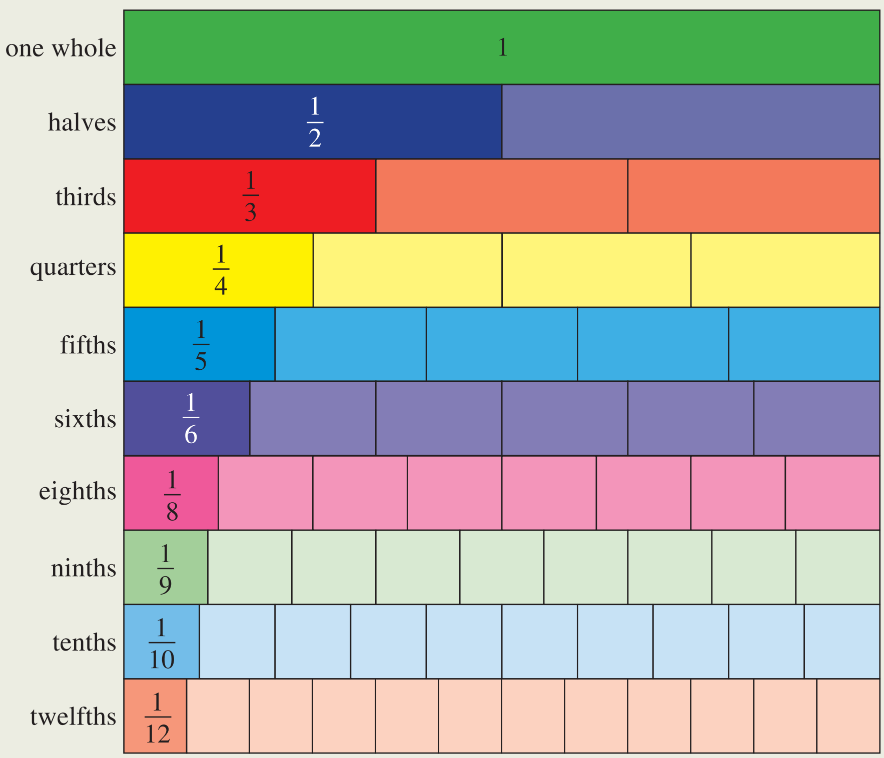
Key ideas

- **Equivalent fractions** are fractions that mark the same place on a number line. For example: $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions.
- Equivalent fractions are produced by multiplying the numerator and denominator by the same number. This number can be any whole number greater than 1.
- Equivalent fractions can also be produced by dividing the numerator and denominator by the same number.
- **Simplifying fractions** involves writing a fraction in its 'simplest form' or 'easiest form' or 'most convenient form'. To do this, the numerator and the denominator must be divided by their **highest common factor (HCF)**.
- It is a mathematical convention to write all answers involving fractions in their simplest form.
- A fraction wall can be helpful when comparing fractions.



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Example 9 Producing equivalent fractions

Write four equivalent fractions for $\frac{2}{3}$.

SOLUTION

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$

Many other fractions are also possible.

Other common choices include:

$\frac{20}{30}, \frac{200}{300}, \frac{2000}{3000}, \frac{40}{60}$

EXPLANATION

×2 ×3 ×4 ×5

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15}$

×2 ×3 ×4 ×5

Example 10 Checking for equivalence

By writing either $=$ or $\neq$ between the fractions, state whether the following pairs of fractions are equivalent or not equivalent.

a $\frac{1}{3}$ □ $\frac{3}{7}$

b $\frac{4}{5}$ □ $\frac{20}{25}$

SOLUTION

a $\frac{1}{3} \neq \frac{3}{7}$

EXPLANATION

Convert to a common denominator.

$\frac{1}{3} = \frac{7}{21}$ (×7) and $\frac{3}{7} = \frac{9}{21}$ (×3)

$\frac{7}{21} \neq \frac{9}{21}$

b $\frac{4}{5} = \frac{20}{25}$

$\frac{4}{5} = \frac{20}{25}$ (×5)

Example 11 Converting to simplest form

Write these fractions in simplest form.

a $\frac{12}{20}$

b $\frac{7}{42}$

SOLUTION

a $\frac{12}{20} = \frac{3 \times \cancel{4}}{5 \times \cancel{4}} = \frac{3}{5}$

b $\frac{7}{42} = \frac{\cancel{7} \times 1}{\cancel{7} \times 6} = \frac{1}{6}$

EXPLANATION

The HCF of 12 and 20 is 4.

Both the numerator and the denominator are divided by the HCF of 4.

The HCF of 7 and 42 is 7.

The 7 is 'cancelled' from the numerator and the denominator.

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Exercise 4D

UNDERSTANDING AND FLUENCY	1–4, 5–9(½)	5–9(½), 10	6–10(½)

1 Which of the following fractions are equivalent to $\frac{1}{2}$?

$\frac{3}{5}, \frac{3}{6}, \frac{3}{10}, \frac{2}{4}, \frac{11}{22}, \frac{7}{15}, \frac{8}{12}, \frac{2}{1}, \frac{5}{10}, \frac{6}{10}$

2 Which of the following fractions are equivalent to $\frac{8}{20}$?

$\frac{4}{10}, \frac{1}{5}, \frac{6}{20}, \frac{8}{10}, \frac{16}{40}, \frac{2}{5}, \frac{4}{12}, \frac{12}{40}, \frac{80}{200}, \frac{1}{4}$

3 Fill in the missing numbers to complete the following strings of equivalent fractions.

a $\frac{1}{3} = \frac{\square}{6} = \frac{4}{\square} = \frac{\square}{30} = \frac{\square}{60} = \frac{100}{\square}$

b $\frac{2}{8} = \frac{\square}{4} = \frac{\square}{12} = \frac{6}{\square} = \frac{\square}{80} = \frac{10}{\square}$

4 In each of these lists of equivalent fractions, circle the fraction that is in its simplest form.

a $\frac{3}{15}, \frac{10}{50}, \frac{2}{10}, \frac{1}{5}$

b $\frac{100}{600}, \frac{3}{18}, \frac{1}{6}, \frac{7}{42}$

c $\frac{4}{6}, \frac{2}{3}, \frac{16}{24}, \frac{20}{30}$

d $\frac{9}{12}, \frac{15}{20}, \frac{6}{8}, \frac{3}{4}$

5 Fill in the gaps to reduce these fractions to their simplest form.

a $\frac{10}{30}$ **i** HCF = $\square$ **ii** $\frac{10}{30} = \frac{1 \times \square}{3 \times \square}$. Therefore, simplest form is $\frac{\square}{3}$.

b $\frac{4}{18}$ **i** HCF = $\square$ **ii** $\frac{4}{18} = \frac{2 \times \square}{9 \times \square}$. Therefore, simplest form is $\frac{\square}{9}$.

c $\frac{4}{28}$ **i** HCF = $\square$ **ii** $\frac{4}{28} = \frac{1 \times \square}{7 \times \square}$. Therefore, simplest form is $\frac{1}{\square}$.

d $\frac{9}{15}$ **i** HCF = $\square$ **ii** $\frac{9}{15} = \frac{3 \times \square}{5 \times \square}$. Therefore, simplest form is $\frac{\square}{\square}$.

Example 9

6 Write four equivalent fractions for each of the fractions listed.

a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{2}{5}$ **d** $\frac{3}{5}$

e $\frac{2}{9}$ **f** $\frac{3}{7}$ **g** $\frac{5}{12}$ **h** $\frac{3}{11}$

7 Find the unknown value to make the equation true.

a $\frac{3}{4} = \frac{\square}{12}$ **b** $\frac{5}{8} = \frac{\square}{80}$ **c** $\frac{6}{11} = \frac{18}{\square}$ **d** $\frac{2}{7} = \frac{16}{\square}$

e $\frac{3}{\square} = \frac{15}{40}$ **f** $\frac{\square}{1} = \frac{14}{7}$ **g** $\frac{\square}{10} = \frac{24}{20}$ **h** $\frac{13}{14} = \frac{\square}{42}$

i $\frac{2}{7} = \frac{10}{\square}$ **j** $\frac{19}{20} = \frac{190}{\square}$ **k** $\frac{11}{21} = \frac{55}{\square}$ **l** $\frac{11}{\square} = \frac{44}{8}$

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Example 10

8 By writing either = or ≠ between the fractions, state whether the following pairs of fractions are equivalent or not equivalent.

a $\frac{1}{2} \square \frac{5}{8}$ **b** $\frac{4}{8} \square \frac{2}{4}$ **c** $\frac{3}{7} \square \frac{30}{60}$

d $\frac{5}{9} \square \frac{15}{18}$ **e** $\frac{11}{15} \square \frac{33}{45}$ **f** $\frac{1}{2} \square \frac{402}{804}$

g $\frac{12}{36} \square \frac{1}{3}$ **h** $\frac{18}{24} \square \frac{21}{28}$ **i** $\frac{6}{18} \square \frac{11}{33}$

Example 11

9 Write the following fractions in simplest form.

a $\frac{15}{20}$ **b** $\frac{12}{18}$ **c** $\frac{10}{30}$ **d** $\frac{8}{22}$

e $\frac{14}{35}$ **f** $\frac{2}{22}$ **g** $\frac{8}{56}$ **h** $\frac{9}{27}$

i $\frac{35}{45}$ **j** $\frac{36}{96}$ **k** $\frac{120}{144}$ **l** $\frac{700}{140}$

10 These lists of fractions are meant to contain only fractions in their simplest form; however, there is one mistake in each list. Find the fraction that is not in simplest form and rewrite it in its simplest form.

a $\frac{1}{3}, \frac{3}{8}, \frac{5}{9}, \frac{7}{14}$ **b** $\frac{2}{5}, \frac{12}{16}, \frac{15}{19}, \frac{13}{37}$

c $\frac{12}{19}, \frac{4}{42}, \frac{5}{24}, \frac{6}{61}$ **d** $\frac{7}{63}, \frac{9}{62}, \frac{11}{81}, \frac{13}{72}$

PROBLEM-SOLVING AND REASONING	11, 13	12–14	13–15

11 A family block of chocolate consists of 12 rows of 6 individual squares. Tania eats 16 individual squares. What fraction of the block, in simplest terms, has Tania eaten?

12 Four people win a competition that allows them to receive $\frac{1}{2}$ a tank of free petrol. Find how many litres of petrol the drivers of these cars receive.

a Ford Territory with a 70-litre tank
b Nissan Patrol with a 90-litre tank
c Holden Commodore with a 60-litre tank
d Mazda 323 with a 48-litre tank

13 Justin, Joanna and Jack are sharing a large pizza for dinner. The pizza has been cut into 12 equal pieces. Justin would like $\frac{1}{3}$ of the pizza, Joanna would like $\frac{1}{4}$ of the pizza and Jack will eat whatever is remaining. By considering equivalent fractions, determine how many slices each person gets served.

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14 J. K. Rowling's first book, *Harry Potter and the Philosopher's Stone*, is 225 pages long. Sam plans to read the book in three days, reading the same number of pages each day.

a How many pages should Sam read each day?

b The fraction $\frac{75}{225}$ of the book is equivalent to what fraction in simplest form?

By the end of the second day, Sam is on track and has read $\frac{2}{3}$ of the book.

c How many pages of the book is $\frac{2}{3}$ equivalent to?

15 A fraction when simplified is written as $\frac{3}{5}$. What could the fraction have been before it was simplified? Explain why the number of answers is infinite.

ENRICHMENT	—	—	16

Equivalent bars of music

16 Each piece of music has a time signature. A common time signature is called $\frac{4}{4}$ time, and is actually referred to as Common time.

Common time, or $\frac{4}{4}$ time, means that there are four 'quarter notes' (or crotchets) in each bar. Listed below are the five most commonly used musical notes.

○ – whole note (fills the whole bar) – semibreve

𝅗𝅥 – half note (fills half the bar) – minim

♩ – quarter note (four of these to a bar) – crotchet

♪ – eighth note (eight to a bar) – quaver

𝅘𝅥𝅯 – sixteenth note (sixteen to a bar) – semi-quaver

a Write six different 'bars' of music in $\frac{4}{4}$ time.

Carry out some research on other types of musical time signatures.

b Do you know what the time signature $\frac{12}{8}$ means?

c Write three different bars of music for a $\frac{12}{8}$ time signature.

d What are the musical symbols for different length rests?

e How does a dot (or dots) written after a note affect the length of the note?

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4E Mixed numerals and improper fractions

Interactive
Widgets
HOTsheets
Walkthrough

As we have seen in this chapter, a fraction is a common way of representing part of a whole number. For example, a particular car trip may require $\frac{2}{3}$ of a tank of petrol.

On many occasions, you may need whole numbers plus a part of a whole number. For example, a long interstate car trip may require $2\frac{1}{4}$ tanks of petrol. When you have a combination of a whole number and a fraction, this number is known as a **mixed numeral**.

Let's start: Pizza frenzy

With a partner, attempt to solve the following pizza problem. There is more than one answer.

At Pete's pizza shop, small pizzas are cut into four equal slices, medium pizzas are cut into six equal slices and large pizzas are cut into eight equal slices.

For a class party, the teacher orders 13 pizzas, which the students ate with ease. After the last slice is eaten, a total of 82 slices of pizza had been eaten by the students. How many pizzas of each size did the teacher order?

Key ideas

- A number is said to be a **mixed numeral** when it is a mix of a whole number plus a proper fraction.

$2\frac{3}{5}$ is a mixed numeral

whole number (2) — proper fraction ($\frac{3}{5}$)

- **Improper fractions** (fractions greater than a whole, where the numerator is greater than the denominator) can be converted to mixed numerals or whole numbers.

$\frac{15}{4} = 3\frac{3}{4}$ (improper fraction = mixed numeral)

$15 \div 4 = 3 \text{ rem } 3$

$\frac{16}{4} = 4$ (improper fraction = whole number)

$16 \div 4 = 4$

- Mixed numerals can be converted to improper fractions.
- In general, improper fractions should be written as mixed numerals, with the fraction part written in simplest form.

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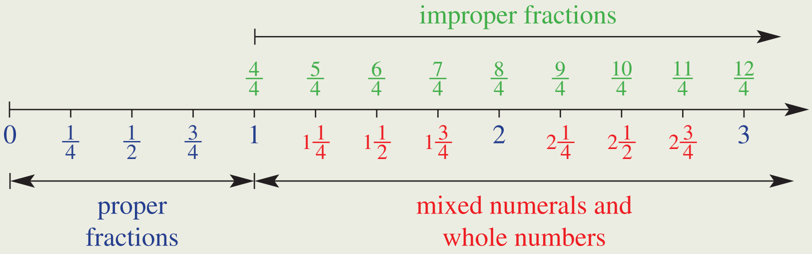
Key ideas

- A number line helps show the different types of fractions.



Example 12 Converting mixed numerals to improper fractions

Convert $3\frac{1}{5}$ to an improper fraction.

SOLUTION

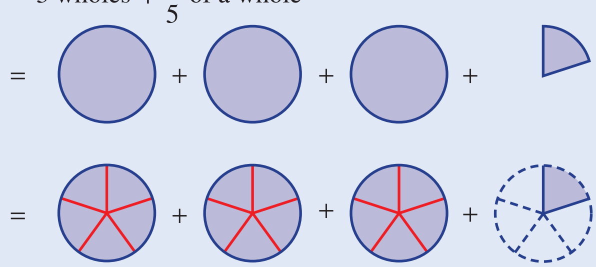
$$3\frac{1}{5} = 1 + 1 + 1 + \frac{1}{5}$$
$$= \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{1}{5}$$
$$= \frac{16}{5}$$

or

$$3\frac{1}{5} = \frac{15}{5} + \frac{1}{5}$$
$$= \frac{16}{5}$$

EXPLANATION

$3\frac{1}{5} = 3$ wholes $+ \frac{1}{5}$ of a whole



Short-cut method:
Multiply the whole number part by the denominator and then add the numerator.
$3 \times 5 + 1 = 16$

Example 13 Converting improper fractions to mixed numerals

Convert $\frac{11}{4}$ to a mixed numeral.

SOLUTION

Method 1

$$\frac{11}{4} = \frac{8+3}{4} = \frac{8}{4} + \frac{3}{4} = 2 + \frac{3}{4} = 2\frac{3}{4}$$

Method 2

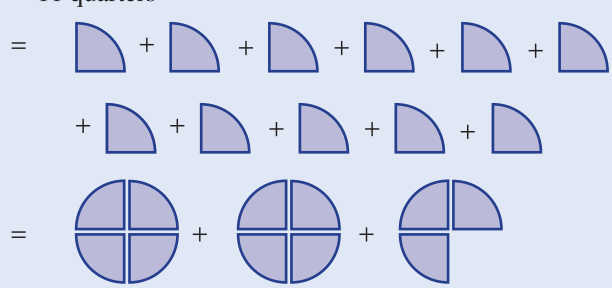
Divide 11 by 4.

$$4\overline{)11}\ \ 2\text{ rem. }3$$

So $\frac{11}{4} = 2\frac{3}{4}$

EXPLANATION

$\frac{11}{4} = 11$ quarters



$= 2\frac{3}{4}$

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Example 14 Writing mixed numerals in simplest form

Convert $\frac{20}{6}$ to a mixed numeral in simplest form.

SOLUTION

$\frac{20}{6} = 3\frac{2}{6} = 3\frac{1 \times \cancel{2}}{3 \times \cancel{2}} = 3\frac{1}{3}$

or

$\frac{20}{6} = \frac{10 \times \cancel{2}}{3 \times \cancel{2}} = \frac{10}{3} = 3\frac{1}{3}$

EXPLANATION

Method 1: Convert to a mixed numeral and then simplify the fraction part.

Method 2: Simplify the improper fraction first and then convert to a mixed numeral.

Each pane of glass is $\frac{1}{12}$ of the whole window.

Exercise 4E

UNDERSTANDING AND FLUENCY	1–6, 7–9(½)	5, 6, 7–9(½)	6, 7–9(½)

1 Between which two whole numbers do the following mixed numerals lie?

a $2\frac{1}{2}$ **b** $11\frac{1}{7}$ **c** $36\frac{8}{9}$

2 Work out the total number of pieces in each of these situations.

a four pizzas cut into six pieces each
b 10 Lego trucks, where each truck is made from 36 Lego pieces
c five jigsaw puzzles with 12 pieces in each puzzle
d three cakes cut into eight pieces each

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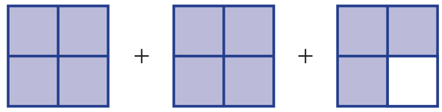
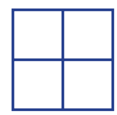
3 The mixed numeral $2\frac{3}{4}$ can be represented in 'window shapes' as

$2\frac{3}{4}$ = 

Represent the following mixed numerals using 'window shapes'.

a $1\frac{1}{4}$ **b** $1\frac{3}{4}$ **c** $3\frac{2}{4}$ **d** $5\frac{2}{4}$

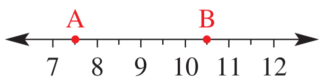
4 A 'window shape' consists of four panes of glass. How many panes of glass are there in the following number of 'window shapes'?

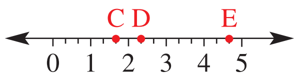


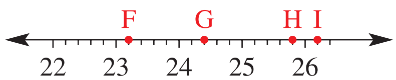
a 2 **b** 3 **c** 7 **d** 11

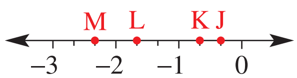
e $4\frac{1}{4}$ **f** $1\frac{3}{4}$ **g** $2\frac{2}{4}$ **h** $5\frac{4}{4}$

5 What mixed numerals correspond to the letters written on each number line?

a


b


c


d


6 On your calculator, what button(s) need to be pressed to:

a convert $\frac{13}{2}$ to a mixed numeral? **b** convert $7\frac{3}{4}$ to an improper fraction?

Example 12

7 Convert these mixed numerals to improper fractions.

a $2\frac{1}{5}$ **b** $1\frac{3}{5}$ **c** $3\frac{1}{3}$ **d** $5\frac{2}{3}$

e $4\frac{1}{7}$ **f** $3\frac{3}{7}$ **g** $2\frac{1}{2}$ **h** $6\frac{1}{2}$

i $4\frac{2}{5}$ **j** $11\frac{1}{2}$ **k** $8\frac{2}{5}$ **l** $10\frac{3}{10}$

m $6\frac{1}{9}$ **n** $2\frac{7}{9}$ **o** $5\frac{2}{8}$ **p** $2\frac{5}{8}$

q $1\frac{11}{12}$ **r** $3\frac{5}{11}$ **s** $4\frac{5}{12}$ **t** $9\frac{7}{12}$

u $5\frac{15}{20}$ **v** $8\frac{3}{100}$ **w** $64\frac{3}{10}$ **x** $20\frac{4}{5}$

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Example 13

8 Convert these improper fractions to mixed numerals.

a $\frac{7}{5}$ b $\frac{4}{3}$ c $\frac{5}{3}$ d $\frac{7}{4}$

e $\frac{11}{3}$ f $\frac{21}{5}$ g $\frac{16}{7}$ h $\frac{10}{4}$

i $\frac{12}{7}$ j $\frac{19}{6}$ k $\frac{20}{3}$ l $\frac{41}{4}$

m $\frac{35}{8}$ n $\frac{26}{5}$ o $\frac{48}{7}$ p $\frac{41}{3}$

q $\frac{37}{12}$ r $\frac{81}{11}$ s $\frac{93}{10}$ t $\frac{78}{7}$

u $\frac{231}{100}$ v $\frac{333}{10}$ w $\frac{135}{11}$ x $\frac{149}{12}$

Example 14

9 Convert these improper fractions to mixed numerals in their simplest form.

a $\frac{10}{4}$ b $\frac{28}{10}$ c $\frac{16}{12}$ d $\frac{8}{6}$

e $\frac{18}{16}$ f $\frac{30}{9}$ g $\frac{40}{15}$ h $\frac{60}{25}$

PROBLEM-SOLVING AND REASONING	10, 12	10, 11, 13	11–13

10 Draw a number line from 0 to 5 and mark on it the following fractions.

a $\frac{2}{3}, 2, \frac{5}{3}, 3\frac{1}{3}$ b $\frac{3}{4}, \frac{12}{4}, 2\frac{1}{4}, 3\frac{1}{2}$ c $\frac{4}{5}, \frac{14}{5}, 3\frac{1}{5}, \frac{10}{5}, \frac{19}{5}$

11 Fill in the gaps for the following number patterns.

a $1\frac{1}{3}, 1\frac{2}{3}, 2,$ ___, $2\frac{2}{3}, 3, 3\frac{1}{3},$ ___, ___, $4\frac{1}{3}, 4\frac{2}{3}, 5$

b $\frac{3}{7}, \frac{5}{7}, 1, 1\frac{2}{7},$ ___, $1\frac{6}{7},$ ___, $2\frac{3}{7}, 2\frac{5}{7},$ ___, $3\frac{2}{7},$ ___, ___

c $\frac{3}{5}, 1\frac{1}{5}, 1\frac{4}{5},$ ___, $3, 3\frac{3}{5},$ ___, ___, $5\frac{2}{5},$ ___, $6\frac{3}{5},$ ___

12 Four friends order three large pizzas for their dinner. Each pizza is cut into eight equal slices. Simone has three slices, Izabella has four slices, Mark has five slices and Alex has three slices.

a How many pizza slices do they eat in total?

b How much pizza do they eat in total? Give your answer as a mixed numeral.

c How many pizza slices are left uneaten?

d How much pizza is left uneaten? Give your answer as a mixed numeral.

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13 a Patricia has three sandwiches that are cut into quarters and she eats all but one-quarter. How many quarters does she eat?
b Phillip has five sandwiches that are cut into halves and he eats all but one-half. How many halves does he eat?
c Crystal has x sandwiches that are cut into quarters and she eats them all but one-quarter. How many quarters does she eat?
d Byron has y sandwiches that are cut into thirds and he eats all but one-third. How many thirds does he eat?
e Felicity has m sandwiches that are cut into n pieces and she eats them all. How many pieces does she eat?

ENRICHMENT	—	—	14

Mixed numeral swap meet

14 a Using the digits 1, 2 and 3 only once, three different mixed numerals can be written.
i Write down the three possible mixed numerals.
ii Find the difference between the smallest and highest mixed numerals.
b Repeat part **a** using the digits 2, 3 and 4.
c Repeat part **a** using the digits 3, 4 and 5.
d Predict the difference between the largest and smallest mixed numeral when using only the digits 4, 5 and 6. Use subtraction to see if your prediction is correct.
e Write down a rule for the difference between the largest and smallest mixed numerals when using any three consecutive integers.
f Extend your investigation to allow mixed numerals where the fraction part is an improper fraction.
g Extend your investigation to produce mixed numerals from four consecutive digits.

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4F Ordering positive and negative fractions

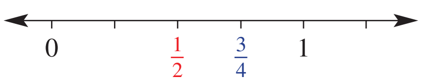
Interactive
Widgets
HOTsheets
Walkthrough

You already know how to order a set of whole numbers.
For example: 3, 7, 15, 6, 2, 10 are a set of six different whole numbers that you could place in ascending or descending order.
In ascending order, the correct order is 2, 3, 6, 7, 10, 15.
In descending order, the correct order is 15, 10, 7, 6, 3, 2.

In this section you will learn how to write different fractions in ascending and descending order. To be able to do this we need to compare different fractions and we do this through our knowledge of equivalent fractions (see Section 4D).

Remember a fraction is greater than another fraction if it lies to the right of that fraction on a number line.

$$\frac{3}{4} > \frac{1}{2}$$



Let's start: The order of five

- As a warm-up activity, ask five volunteer students to arrange themselves in alphabetical order, then in height order and, finally, in birthday order.
- Each of the five students receives a large fraction card and displays it to the class.
- The rest of the class must then attempt to order the students in ascending order, according to their fraction card. It is a group decision and none of the five students should move until the class agrees on a decision.
- Repeat the activity with a set of more challenging fraction cards.

Key ideas

- To **order** (or arrange) positive fractions we must know how to compare different fractions. This is often done by considering equivalent fractions.
- If the numerators are the same, the smallest fraction is the one with the biggest denominator, as it has been divided up into the most pieces.

  For example: $\frac{1}{7} < \frac{1}{2}$
- If the denominators are the same, the smallest fraction is the one with the smallest numerator.

  For example: $\frac{3}{10} < \frac{7}{10}$
- To order two fractions with different numerators and denominators, we can use our knowledge of equivalent fractions to produce fractions with a common denominator and then compare the numerators.
- The **lowest common denominator** (**LCD**) is the lowest common multiple of the different denominators.
- **Ascending** order is when numbers are ordered going *up*, from smallest to largest.
- **Descending** order is when numbers are ordered going *down*, from largest to smallest.

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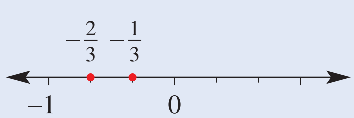
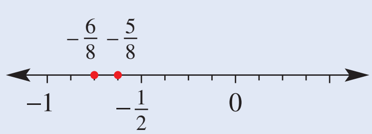
Example 15 Comparing fractions

Place the correct mathematical symbol (i.e. <, = or >) in between the following pairs of fractions to make true mathematical statements.

a $\frac{2}{5} \square \frac{4}{5}$ **b** $\frac{1}{3} \square \frac{1}{5}$ **c** $\frac{2}{3} \square \frac{3}{5}$

d $2\frac{3}{7} \square \frac{16}{7}$ **e** $-\frac{1}{3} \square -\frac{2}{3}$ **f** $-\frac{3}{4} \square -\frac{5}{8}$

SOLUTION	EXPLANATION
a $\frac{2}{5} \boxed{<} \frac{4}{5}$	Denominators are the same, therefore compare numerators.
b $\frac{1}{3} \boxed{>} \frac{1}{5}$	Numerators are the same. Smallest fraction has the biggest denominator.
c $\frac{2}{3} \square \frac{3}{5}$	LCD of 3 and 5 is 15. Produce equivalent fractions.
$\frac{10}{15} \boxed{>} \frac{9}{15}$	Denominators now the same, therefore compare numerators.
Hence, $\frac{2}{3} \boxed{>} \frac{3}{5}$.	
d $2\frac{3}{7} \square \frac{16}{7}$	Convert mixed numeral to an improper fraction. Denominators are the same, therefore compare numerators.
$\frac{17}{7} \boxed{>} \frac{16}{7}$	
Hence, $2\frac{3}{7} \boxed{>} \frac{16}{7}$.	
e $-\frac{1}{3} \boxed{>} -\frac{2}{3}$	 $-\frac{1}{3}$ is further to the right than $-\frac{2}{3}$.
f $-\frac{3}{4} \square -\frac{5}{8}$	
$-\frac{6}{8} \boxed{<} -\frac{5}{8}$	$-\frac{5}{8}$ is further to the right than $-\frac{6}{8}$.

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Example 16 Ordering fractions

Place the following fractions in ascending order.

a $\frac{3}{4}, \frac{4}{5}, \frac{2}{3}$

b $1\frac{3}{5}, \frac{7}{4}, \frac{3}{2}, 2\frac{1}{4}, \frac{11}{5}$

SOLUTION	EXPLANATION
a $\frac{45}{60}, \frac{48}{60}, \frac{40}{60}$	LCD of 3, 4 and 5 is 60. Produce equivalent fractions with denominator of 60.
$\frac{40}{60}, \frac{45}{60}, \frac{48}{60}$	Order fractions in ascending order.
$\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$	Rewrite fractions back in original form.
b $\frac{8}{5}, \frac{7}{4}, \frac{3}{2}, \frac{9}{4}, \frac{11}{5}$	Express all fractions as improper fractions.
$\frac{32}{20}, \frac{35}{20}, \frac{30}{20}, \frac{45}{20}, \frac{44}{20}$	LCD of 2, 4 and 5 is 20. Produce equivalent fractions with a denominator of 20.
$\frac{30}{20}, \frac{32}{20}, \frac{35}{20}, \frac{44}{20}, \frac{45}{20}$	Order fractions in ascending order.
$\frac{3}{2}, 1\frac{3}{5}, \frac{7}{4}, \frac{11}{5}, 2\frac{1}{4}$	Rewrite fractions back in original form.

Exercise 4F

UNDERSTANDING AND FLUENCY	1–4, 5–6(½)	4–6(½), 7, 8(½)	5–6(½), 7, 8(½)

1 Circle the largest fraction in each of the following lists.

a $\frac{3}{7}, \frac{2}{7}, \frac{5}{7}, \frac{1}{7}$ **b** $\frac{4}{3}, \frac{2}{3}, \frac{7}{3}, \frac{5}{3}$

c $\frac{5}{11}, \frac{9}{11}, \frac{3}{11}, \frac{4}{11}$ **d** $\frac{8}{5}, \frac{4}{5}, \frac{6}{5}, \frac{7}{5}$

2 State the lowest common multiple of the following sets of numbers.

a 2, 5 **b** 3, 7 **c** 5, 4 **d** 6, 5

e 3, 6 **f** 2, 10 **g** 4, 6 **h** 8, 6

i 2, 3, 5 **j** 3, 4, 6 **k** 3, 8, 4 **l** 2, 6, 5

3 State the lowest common denominator of the following sets of fractions.

a $\frac{1}{3}, \frac{3}{5}$ **b** $\frac{2}{4}, \frac{3}{5}$ **c** $\frac{4}{7}, \frac{2}{3}$ **d** $\frac{2}{10}, \frac{1}{5}$

e $\frac{4}{6}, \frac{3}{8}$ **f** $\frac{5}{12}, \frac{2}{5}$ **g** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ **h** $\frac{4}{3}, \frac{3}{4}$

4 Fill in the gaps to produce equivalent fractions.

a $\frac{2}{5} = \frac{\square}{15}$ **b** $\frac{2}{3} = \frac{\square}{12}$ **c** $\frac{1}{4} = \frac{\square}{16}$

d $\frac{3}{7} = \frac{\square}{14}$ **e** $\frac{3}{8} = \frac{\square}{40}$ **f** $\frac{5}{6} = \frac{\square}{18}$

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Example 15

5 Place the correct mathematical symbol (i.e. $<$, $=$ or $>$) in between the following pairs of fractions to make true mathematical statements.

a $\frac{3}{5} \square \frac{1}{5}$ **b** $\frac{7}{9} \square \frac{2}{9}$ **c** $\frac{2}{2} \square \frac{3}{3}$ **d** $\frac{13}{18} \square \frac{17}{18}$

e $\frac{1}{4} \square \frac{1}{3}$ **f** $\frac{1}{10} \square \frac{1}{20}$ **g** $\frac{1}{7} \square \frac{1}{5}$ **h** $\frac{3}{5} \square \frac{18}{30}$

i $\frac{2}{3} \square \frac{1}{3}$ **j** $\frac{4}{5} \square \frac{3}{4}$ **k** $\frac{5}{6} \square \frac{9}{10}$ **l** $\frac{5}{7} \square \frac{15}{21}$

m $\frac{7}{11} \square \frac{3}{5}$ **n** $1\frac{2}{3} \square 1\frac{1}{2}$ **o** $3\frac{3}{7} \square \frac{15}{4}$ **p** $\frac{12}{5} \square \frac{19}{8}$

q $-\frac{1}{4} \square -\frac{1}{2}$ **r** $-\frac{2}{3} \square -\frac{3}{4}$ **s** $-\frac{2}{5} \square -\frac{5}{8}$ **t** $-\frac{3}{4} \square -\frac{3}{5}$

Example 16

6 Place the following fractions in ascending order.

a $\frac{3}{5}, \frac{8}{5}, 1\frac{2}{5}$ **b** $\frac{5}{9}, \frac{1}{3}, \frac{2}{9}$

c $\frac{2}{5}, \frac{3}{4}, \frac{4}{5}$ **d** $\frac{5}{6}, \frac{3}{5}, \frac{2}{3}$

e $2\frac{1}{4}, \frac{11}{4}, \frac{5}{2}, 3\frac{1}{3}$ **f** $\frac{15}{8}, \frac{11}{6}, \frac{7}{4}, \frac{5}{3}$

g $2\frac{7}{10}, \frac{9}{4}, \frac{11}{5}, 2\frac{1}{2}, 2\frac{3}{5}$ **h** $4\frac{4}{9}, \frac{15}{3}, 4\frac{10}{27}, 4\frac{2}{3}, 4\frac{1}{6}$

7 Use a number line to place the following fractions in ascending order.

a $-\frac{5}{4}, -2\frac{1}{2}, -\frac{3}{2}$ **b** $-\frac{1}{4}, -\frac{1}{6}, -\frac{1}{3}$

8 Place the following fractions in descending order, without finding common denominators.

a $\frac{1}{3}, \frac{1}{5}, \frac{1}{4}, \frac{1}{2}$ **b** $\frac{3}{5}, \frac{3}{7}, \frac{3}{6}, \frac{3}{8}$

c $\frac{7}{2}, \frac{7}{5}, \frac{7}{8}, \frac{7}{7}$ **d** $\frac{1}{15}, \frac{1}{10}, \frac{1}{50}, \frac{1}{100}$

e $7\frac{1}{11}, 8\frac{3}{5}, 5\frac{4}{9}, 10\frac{2}{3}$ **f** $2\frac{1}{3}, 2\frac{1}{9}, 2\frac{1}{6}, 2\frac{1}{5}$

PROBLEM-SOLVING AND REASONING	9, 12	9, 10, 12–14	10, 11, 13–15

9 Place the following cake fractions in decreasing order of size.

A sponge cake shared equally by four people = $\frac{1}{4}$ cake

B chocolate cake shared equally by eleven people = $\frac{1}{11}$ cake

C carrot and walnut cake shared equally by eight people = $\frac{1}{8}$ cake

10 Four friends, Dean, David, Andrea and Rob, all competed in a marathon. Their respective finishing times were $3\frac{1}{3}$ hours, $3\frac{5}{12}$ hours, $3\frac{1}{4}$ hours and $3\frac{4}{15}$ hours. Write down the correct finishing order of the four friends.

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11 Rewrite the fractions in each set with their lowest common denominator and then write the next two fractions that would continue the pattern.

a $\frac{2}{9}, \frac{1}{3}, \frac{4}{9}$, ___, ___

b $\frac{1}{2}, \frac{5}{4}, 2$, ___, ___

c $\frac{11}{6}, \frac{3}{2}, \frac{7}{6}$, ___, ___

d $\frac{1}{2}, \frac{4}{7}, \frac{9}{14}$, ___, ___

12 Write a fraction that lies between the following pairs of fractions.

a $\frac{3}{5}, \frac{3}{4}$

b $\frac{1}{4}, \frac{1}{2}$

c $\frac{2}{7}, \frac{1}{6}$

d $\frac{17}{20}, \frac{7}{10}$

e $2\frac{1}{3}, 2\frac{1}{5}$

f $8\frac{7}{10}, 8\frac{3}{4}$

13 Explain how to find a fraction that lies between two fractions with different denominators.

14 Write the whole number values that $\square$ can take so that $\frac{\square}{3}$ lies between:

a 2 and 3

b 5 and $5\frac{1}{2}$

15 Thomas and Nathan had a doughnut-eating race to see who could eat the most doughnuts in 1 minute. Before the race started, Thomas cut each of his doughnuts into fifths to make them just the right bite-size. Nathan decided to cut each of his doughnuts into quarters before the race. After 1 minute of frenzied eating, the stop whistle blew. Thomas had devoured 28 fifths of doughnut and Nathan had munched his way through 22 quarters of doughnut.

a Who won the doughnut-eating race?

b What was the winning doughnut margin? Express your answer in simplest form.

ENRICHMENT	—	—	16

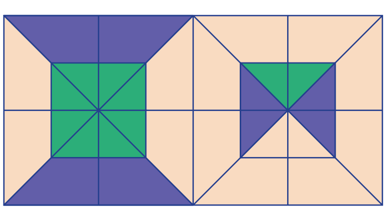
Shady designs

16 **a** For each of the diagrams shown, work out what fraction of the rectangle is coloured blue. Explain how you arrived at each of your answers.

b Redraw the shapes in order from the most blue to least blue.

c Design and shade two more rectangle designs.

i



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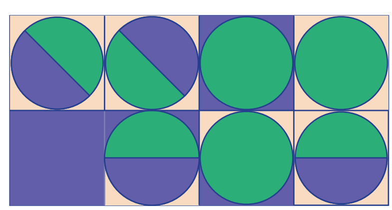

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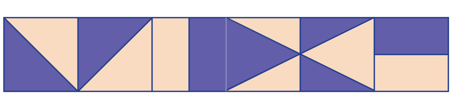

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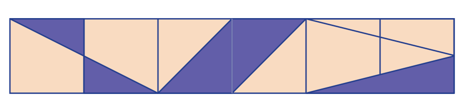

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4G Place value in decimals and ordering decimals

Some quantities change by whole number amounts, such as the number of people in a room, but there are many quantities that increase or decrease continuously, such as your height, weight and age. Often we talk about age as a whole number (e.g. Mike is 12 years old) but, in reality, our age is an ever-increasing (continuous) quantity. For example, if Mike is 12 years, 4 months, 2 weeks, 3 days, 5 hours, 6 minutes and 33 seconds old, then Mike is actually 12.38062147 years old!

There are many numbers in today's society that are not whole numbers. For example, it is unusual to buy an item in a supermarket that is simply a whole number of dollars. The price of almost all shopping items involves both dollars and cents. A chocolate bar may cost $1.95, which is an example of a decimal number.

Let's start: Split-second timing

Organise students into pairs and use a digital stopwatch. Many students will have a watch with a suitable stopwatch function.

- Try to stop the stopwatch on exactly 10 seconds. Attempt this two times.
  Were you able to stop it exactly on 10.00 seconds? What was the closest time?
- Try these additional challenges with your partner.
  - **a** Stop the watch exactly on:
    - **i** 12.5 seconds
    - **ii** 8.37 seconds
    - **iii** 9.7 seconds
    - **iv** 14.25 seconds
  - **b** How quickly can you start and stop the stopwatch?
  - **c** How accurately can you time 1 minute without looking at the stopwatch?

Key ideas

- A **decimal point** is used to separate the whole number from the decimal or fraction part.
- When dealing with decimal numbers, the place value table must be extended to involve tenths, hundredths, thousandths etc.
  The number 428.357 means:

Hundreds	Tens	Units	•	Tenths	Hundredths	Thousandths
4	2	8	•	3	5	7
4×100	2×10	8×1	•	$3 \times \frac{1}{10}$	$5 \times \frac{1}{100}$	$7 \times \frac{1}{1000}$
400	20	8	•	$\frac{3}{10}$	$\frac{5}{100}$	$\frac{7}{1000}$

← whole numbers — decimal point — fractions →

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Example 17 Understanding decimal place value

What is the value of the digit 8 in the following numbers?

a 12.85 **b** 6.1287

SOLUTION

a The value of 8 is $\frac{8}{10}$.

b The value of 8 is $\frac{8}{1000}$.

EXPLANATION

The 8 is in the first column after the decimal point, which is the tenths column.

The 8 is in the third column after the decimal point, which is the thousandths column.

Example 18 Changing to decimals

Express each of the following proper fractions and mixed numerals as decimals.

a $\frac{7}{10}$ **b** $\frac{5}{100}$ **c** $3\frac{17}{100}$

SOLUTION

a $\frac{7}{10} = 0.7$

b $\frac{5}{100} = 0.05$

c $3\frac{17}{100} = 3.17$

EXPLANATION

$\frac{7}{10}$ means seven-tenths, so put the 7 in the tenths column.

$\frac{5}{100}$ means five-hundredths, so put the 5 in the hundredths column.

$3\frac{17}{100}$ means 3 units and 17 one-hundredths.
17 hundredths is one-tenth and seven-hundredths.

Example 19 Arranging decimal numbers in order

Arrange the following decimal numbers in ascending order (i.e. smallest to largest).
3.72, 7.23, 2.73, 2.37, 7.32, 3.27

SOLUTION

2.37, 2.73, 3.27, 3.72, 7.23, 7.32

EXPLANATION

The units column has a higher value than the tenths column, and the tenths column has a higher value than the hundredths column.
2.73 is bigger than 2.37 because it has seven-tenths, which is bigger than three-tenths.

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Exercise 4G

UNDERSTANDING AND FLUENCY	1, 2, 3–6(½)	3–7(½)	4–7(½)

1 For the number 58.237, give the value of the digit (as a fraction):

a 2 **b** 3 **c** 7

2 A stopwatch is stopped at 36.57 seconds.

a What is the digit displayed in the tenths column?
b What is the digit displayed in the units column?
c What is the digit displayed in the hundredths column?
d Is this number closer to 36 or 37 seconds?

Example 17

3 What is the value of the digit 6 in the following numbers?

a 23.612 **b** 17.46 **c** 80.016 **d** 0.693
e 16.4 **f** 8.56813 **g** 2.3641 **h** 11.926

4 State whether each of the following is true or false.

a $7.24 < 7.18$ **b** $21.32 < 20.89$ **c** $4.61 > 4.57$ **d** $8.09 > 8.41$
e $25.8 \leqslant 28.5$ **f** $2.1118 \leqslant 2.8001$ **g** $7.93 \geqslant 8.42$ **h** $11.11 \geqslant 11.109$
i $\frac{3}{10} = \frac{30}{100}$ **j** $\frac{7}{10} = \frac{70}{100}$ **k** $\frac{5}{10} \neq 5$ **l** $\frac{2}{10} \neq \frac{20}{1000}$

Example 18a,b

5 Express each of the following proper fractions as a decimal.

a $\frac{3}{10}$ **b** $\frac{8}{10}$ **c** $\frac{5}{100}$ **d** $\frac{23}{100}$
e $\frac{9}{10}$ **f** $\frac{2}{100}$ **g** $\frac{121}{1000}$ **h** $\frac{74}{1000}$

Example 18c

6 Express each of the following mixed numerals as a decimal.

a $6\frac{4}{10}$ **b** $5\frac{7}{10}$ **c** $212\frac{3}{10}$ **d** $1\frac{16}{100}$
e $14\frac{83}{100}$ **f** $7\frac{51}{100}$ **g** $5\frac{7}{100}$ **h** $18\frac{612}{1000}$

7 Write the following number phrases as decimals.

a seven and six-tenths **b** twelve and nine-tenths
c thirty-three and four-hundredths **d** twenty-six and fifteen-hundredths
e eight and forty-two hundredths **f** ninety-nine and twelve-thousandths

PROBLEM-SOLVING AND REASONING	8–9, 12	9, 10, 12	9–13

8 How close are the following decimal numbers to their nearest whole number?

a 6.9 **b** 7.03 **c** 18.98 **d** 16.5
e 17.999 **f** 4.99 **g** 0.85 **h** 99.11

Example 19

9 Arrange these groups of numbers in ascending order (i.e. smallest to largest).

a 3.52, 3.05, 3.25, 3.55 **b** 30.6, 3.06, 3.6, 30.3
c 17.81, 1.718, 1.871, 11.87 **d** 26.92, 29.26, 29.62, 22.96, 22.69

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10 The batting averages for five retired Australian Cricket test captains are Adam Gilchrist 47.60, Steve Waugh 51.06, Mark Taylor 43.49, Allan Border 50.56 and Kim Hughes 37.41.
   a List the five players in descending order of batting averages (i.e. largest to smallest).
   b Ricky Ponting's test batting average is 51.85. Where does this rank him in terms of the retired Australian test captains listed above?

11 The depth of a river at 9 a.m. on six consecutive days was:

Day 1: 1.53 m	Day 2: 1.58 m	Day 3: 1.49 m
Day 4: 1.47 m	Day 5: 1.52 m	Day 6: 1.61 m

   a On which day was the river level highest?
   b On which day was the river level lowest?
   c On which days was the river level higher than the previous day?

12 a, b and c are digits and $a > b > c$. Write these numbers from smallest to largest. Note that the dot represents the decimal point.
   a $a.b$, $b.c$, $a.c$, $c.c$, $c.a$, $b.a$
   b $a.bc$, $b.ca$, $b.bb$, $c.ab$, $c.bc$, $ba.ca$, $ab.ab$, $a.aa$, $a.ca$

13 Write as decimals, if a is a digit.
   a $\frac{a}{10}$ b $\frac{a}{100}$ c $\frac{a}{10} + \frac{a}{100}$ d $a + \frac{a}{10} + \frac{a}{1000}$

ENRICHMENT	—	—	14

Different decimal combinations

14 a Write as many different decimal numbers as you can and place them in ascending order using:
      i the digits 0, 1 and a decimal point. Each digit must be used only once.
      ii the digits 0, 1, 2 and a decimal point. Each digit must be used only once.
      iii the digits 0, 1, 2, 3 and a decimal point. Each digit must be used only once.
   b Calculate the number of different decimal numbers that could be produced using the digits 0, 1, 2, 3, 4 and a decimal point.

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4H Rounding decimals

Interactive
Widgets
HOTsheets
Walkthrough

Decimal numbers sometimes contain more decimal places than we need. It is important that we are able to round decimal numbers when working with money, measuring quantities (including time and distance), or writing answers to some division calculations.

For example, the distance around the school oval might be 0.39647 km, which rounded to 1 decimal place is 0.4 km or 400 m. The rounded figure, although not precise, is accurate enough for most applications.

Running events are electronically measured and rounded to 2 decimal places. Usain Bolt has repeatedly broken his own world records. In August 2009, he set a new world record of 9.58 seconds over 100 m at the World Championships in Germany, which was 5-hundredths (0.05) of a second faster than his London Olympic Games (August 2012) record of 9.63 seconds.

Let's start: Rounding brainstorm

1 In a group of four, brainstorm occasions when it may be useful to round or estimate decimal numbers. Aim to get more than 10 common applications.

2 In pairs one person states a decimal number and the partner needs to state another decimal number that would allow the two numbers to add up to a whole number. Use mental arithmetic only. Start with 1 decimal place and try to build up to 3 or 4 decimal places.

Key ideas

- **Rounding** involves approximating a decimal number to fewer decimal places.
- To round a decimal:
  - Cut the number after the required decimal place; for example, round to 2 decimal places.
  - To determine whether you should round your answer up or down, consider only the digit *immediately* to the right of the specified place. For rounding purposes this can be referred to as the **critical digit**.

    'cut'

    15.63 | 27 — 2 is the critical digit in this example

  - If the critical digit is *less than* 5 (i.e. 0, 1, 2, 3 or 4), then you *round down*. This means write the original number to the place required, leaving off all other digits. This can be referred to as simply leaving the number as it is.
  - If the critical digit is 5 *or more* (i.e. 5, 6, 7, 8 or 9), then you *round up*. This means write the original number to the place required, but increase last digit by 1. Leave off all other digits.

Example 20 Determining the critical digit

The following decimal numbers need to be rounded to 2 decimal places. Draw a line where the number must be cut and then circle the critical digit.

a 23.5398 **b** 1.75137

SOLUTION	EXPLANATION
a 2 3 . 5 3 | (9) 8	A line is drawn directly after the specified number of decimal places, in this case, 2.
b 1 . 7 5 | (1) 3 7	The critical digit is always the number straight after the specified number of decimal places.

Example 21 Rounding decimals to 1 decimal place

Round each of the following to 1 decimal place.

a 25.682 **b** 13.5458

SOLUTION	EXPLANATION
a 25.7	The critical digit is 8 and therefore the tenths column must be rounded up from a 6 to a 7.
b 13.5	The critical digit is 4 and therefore the tenths column remains the same, in effect rounding the original number down to 13.5.

Example 22 Rounding decimals to different decimal places

Round each of the following to the specified number of decimal places.

a Round 18.34728 to 3 decimal places.
b Round 0.43917 to 2 decimal places.
c Round 7.59967 to 3 decimal places.

SOLUTION	EXPLANATION
a 18.347	The critical digit is 2, therefore round down.
b 0.44	The critical digit is 9, therefore round up.
c 7.600	The critical digit is 6, therefore round up. Rounding up has resulted in digits being carried over. Remember to show the stated number of decimal places; hence, the zeros must be displayed.

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Exercise 4H

UNDERSTANDING AND FLUENCY	1–4, 5–6(½), 7, 8(½)	3, 4–6(½), 7, 8–9(½)	4–6(½), 7, 8–9(½)

1 For each of the following, select the closer alternative.

- **a** Is 5.79 closer to 5.7 or 5.8?
- **b** Is 2.4 closer to 2 or 3?
- **c** Is 83 closer to 80 or 90?
- **d** Is 6.777 closer to 6.77 or 6.78?

Example 20

2 The following decimals need to be rounded, correct to 2 decimal places. Draw a line where the number must be cut and then circle the critical digit that must be checked as to whether to round up or down.

a 12.6453 **b** 4.81932 **c** 157.281 **d** 4 001 565.38471
e 0.06031 **f** 203.5791 **g** 66.6666 **h** 7.995123

3 To round correctly to a specified number of places, you must know which digit is the critical digit. Remember: The critical digit is always the digit immediately to the right of the specified number of places.

a State the critical digit in each of the following numbers.

- **i** 25.8174 rounded to 1 decimal place. Critical digit = ____
- **ii** 25.8174 rounded to 2 decimal places. Critical digit = ____
- **iii** 25.8174 rounded to 3 decimal places. Critical digit = ____
- **iv** 25.8174 rounded to the nearest whole number. Critical digit = ____

b State the correct rounded numbers for the numbers in parts **i** to **iv** above.

Example 21

4 Round each of the following to 1 decimal place.

a 14.82 **b** 7.38 **c** 15.62 **d** 0.87
e 6.85 **f** 9.94 **g** 55.55 **h** 7.98

5 Write each of the following, correct to 2 decimal places.

a 3.7823 **b** 11.8627 **c** 5.9156 **d** 0.93225
e 123.456 **f** 300.0549 **g** 3.1250 **h** 9.849
i 56.2893 **j** 7.121999 **k** 29.9913 **l** 0.8971

Example 22a,b

6 Round each of the following to the specified number of decimal places, given as the number in the brackets.

a 15.913 (1) **b** 7.8923 (2) **c** 235.62 (0) **d** 0.5111 (0)
e 231.86 (1) **f** 9.3951 (1) **g** 9.3951 (2) **h** 34.71289 (3)

Example 22c

7 Round each of the following to the specified number of decimal places.

a 23.983 (1) **b** 14.8992 (2) **c** 6.95432 (0) **d** 29.999731 (3)

8 Round each of the following to the nearest whole number.

a 27.612 **b** 9.458 **c** 12.299 **d** 123.72
e 22.26 **f** 117.555 **g** 2.6132 **h** 10.7532

9 Round each of the following amounts to the nearest dollar.

a \$12.85 **b** \$30.50 **c** \$7.10 **d** \$1566.80
e \$120.45 **f** \$9.55 **g** \$1.39 **h** \$36.19

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PROBLEM-SOLVING AND REASONING	10, 12	10–12	11–13

10 Some wise shoppers have the habit of rounding all items to the nearest dollar as they place them in their shopping basket. They can then keep a running total and have a close approximation as to how much their final bill will cost. Use this technique to estimate the cost of the following.

a Jeanette purchases 10 items:
$3.25, $0.85, $4.65, $8.99, $12.30, $7.10, $2.90, $1.95, $4.85, $3.99

b Adam purchases 12 items:
$0.55, $3.00, $5.40, $8.90, $6.90, $2.19, $3.20, $5.10, $3.15, $0.30, $4.95, $1.11

c Jeanette's actual shopping total is $50.83 and Adam's is $44.75. How accurate were Jeanette's and Adam's estimations?

11 Electronic timing pads are standard in National Swimming competitions. In a recent National Under 15 100-metre freestyle race, Edwina receives a rounded time of 52.83 seconds and Jasmine a time of 53.17 seconds.

a If the timing pads can calculate times only to the nearest second, what will be the time difference between the two swimmers?

b If the timing pads can calculate times only to the nearest tenth of a second, what will be the time difference between the two swimmers?

c What is the time difference between the two swimmers, correct to 2 decimal places?

d If the timing pads can measure to 3 decimal places, what would be the quickest time in which Edwina could have swum the race?

12 Using a calculator, evaluate $15.735629 \div 7$, correct to 2 decimal places. What is the least number of decimal places you need to find in the quotient to ensure that you have rounded correctly to 2 decimal places?

13 Samara believes 0.449999 should be rounded up to 0.5, but Cassandra believes it should be rounded down to 0.4. Make an argument to support each of their statements, but then show the flaw in one girl's logic and clearly indicate which girl you think is correct.

ENRICHMENT	—	—	14, 15

Rounding with technology

14 Most calculators are able to round numbers correct to a specified number of places. Find out how to do this on your calculator and check your answers to Questions **5** and **6**.

15 Spreadsheet software packages can also round numbers correct to a specified number of places. Find out the correct syntax for rounding cells in a spreadsheet program, such as Microsoft Excel, and then check your answers to Questions **7** and **8**.

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4I Decimal and fraction conversions

Interactive
Widgets
HOTsheets
Walkthrough

Decimals and fractions are both commonly used to represent numbers that are not simply whole numbers. It is important that we know how to convert a decimal number to a fraction, and how to convert a fraction to a decimal number.

In the photo opposite, we can see that:

- $\frac{1}{4}$ or 0.25 of the cake has been cut and $\frac{3}{4}$ or 0.75 of the cake remains.

Let's start: Match my call

- In pairs, nominate one student to be 'Fraction kid' and the other to be 'Decimal expert'. 'Fraction kid' starts naming some common fractions and 'Decimal expert' tries to give the equivalent decimal value. Start with easy questions and build up to harder ones.
- After 10 turns, swap around. This time 'Decimal expert' will name some decimal numbers and 'Fraction kid' will attempt to call out the equivalent fraction.
- Discuss the following question in pairs:
  Which is easier, converting fractions to decimals or decimals to fractions?

Key ideas

■ Converting fractions to decimals:

- When the denominator is 2, think halves. Example 1: $\frac{1}{2} = 0.5$
  Example 2: $3\frac{1}{2} = 3.5$
- When the denominator is 4, think quarters. Example 1: $\frac{1}{4} = 0.25$
  Example 2: $\frac{3}{4} = 0.75$
- When the denominator is 10, think tenths; e.g. $\frac{7}{10} = 0.7$
  seven-tenths; 7 in the tenths column
- When the denominator is 100, think hundredths.
  Example 1: $\frac{3}{100} = 0.03$
  Example 2: $\frac{37}{100} = 0.37$
  (the hundredths column)
- When the denominator is 5, think tenths; e.g. $\frac{3}{5} = \frac{6}{10} = 0.6$
- When the denominator is a factor of 100, think hundredths.
  e.g. $\frac{17}{50} = \frac{34}{100} = 0.34$
- When all else fails, do a division by hand (or use a calculator).
  e.g. $\frac{5}{8} = 8\overline{)5.{}^{5}0{}^{2}0{}^{4}0}$ giving 0.625 or $\frac{5}{8} = 5 \div 8 = 0.625$

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Key ideas

- Sometimes, the decimal has a pattern that repeats forever. These are called **recurring decimals** or repeating decimals.

  Example 1: $\frac{2}{3} = 0.666\ldots = 0.\dot{6}$

  Example 2: $\frac{13}{11} = 1.1818\ldots = 1.\dot{1}\dot{8}$ or $1.\overline{18}$

  Example 3: $\frac{5}{7} = 0.\dot{7}1428\dot{5}$ or $0.\overline{714285}$

Example 23 Converting decimals to fractions

Convert the following decimals to fractions in their simplest form.

a 0.239 **b** 10.35

SOLUTION

a $\frac{239}{1000}$

b $10\frac{35}{100} = 10\frac{7}{20}$

EXPLANATION

$0.239 = 239$ thousandths

$0.35 = 35$ hundredths, which can be simplified further by dividing the numerator and denominator by the highest common factor of 5.

Example 24 Converting fractions to decimals

Convert the following fractions to decimals.

a $\frac{17}{100}$ **b** $5\frac{3}{5}$ **c** $\frac{7}{12}$

SOLUTION

a $\frac{17}{100} = 0.17$

b $5\frac{3}{5} = 5\frac{6}{10} = 5.6$

c $\frac{7}{12} = 0.58333\ldots$ or $0.58\dot{3}$

EXPLANATION

17 hundredths

$\frac{6}{10}$ is an equivalent fraction of $\frac{3}{5}$, whose denominator is a power of 10.

$$\begin{array}{r} 0.58333\ldots \\ 12\,\overline{)\,7.{}^{7}0{}^{10}0{}^{4}0{}^{4}0{}^{4}0} \end{array}$$

Exercise 4I

UNDERSTANDING AND FLUENCY	1–6, 7(½)	2, 3–7(½), 8	3–7(½), 8

1 Complete each of these statements, which convert common fractions to decimals.

a $\frac{1}{2} = \frac{\square}{10} = 0.5$ **b** $\frac{1}{4} = \frac{25}{\square} = 0.25$

c $\frac{3}{4} = \frac{\square}{100} = 0.\square 5$ **d** $\frac{2}{\square} = \frac{4}{10} = 0.\square$

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2 Complete each of these statements, which convert decimals to fractions, in simplest form.

a $0.2 = \frac{\square}{10} = \frac{1}{5}$ **b** $0.15 = \frac{\square}{100} = \frac{3}{\square}$ **c** $0.8 = \frac{8}{\square} = \frac{\square}{5}$ **d** $0.64 = \frac{64}{100} = \frac{\square}{25}$

3 Are the following true or false?

a $0.333\ldots = 0.3$ **b** $0.1111\ldots = 0.\dot{1}$ **c** $3.2222\ldots = 3.\dot{2}$
d $1.7272\ldots = 1.7\dot{2}$ **e** $3.161616\ldots = 3.\dot{1}\dot{6}$ **f** $4.216216\ldots = 4.\overline{216}$

Example 23

4 Convert the following decimals to fractions in their simplest form.

a 0.5 **b** 6.4 **c** 10.15 **d** 18.12
e 3.25 **f** 0.05 **g** 9.075 **h** 5.192

Example 24a

5 Convert each of these fractions to decimals.

a $\frac{7}{10}$ **b** $\frac{9}{10}$ **c** $\frac{31}{100}$ **d** $\frac{79}{100}$
e $\frac{121}{100}$ **f** $3\frac{29}{100}$ **g** $\frac{123}{1000}$ **h** $\frac{3}{100}$

Example 24b

6 Convert the following fractions to decimals, by first changing the fraction to an equivalent fraction whose denominator is 10, 100 or 1000.

a $\frac{4}{5}$ **b** $\frac{1}{2}$ **c** $\frac{7}{20}$ **d** $\frac{23}{50}$
e $5\frac{19}{20}$ **f** $3\frac{1}{4}$ **g** $\frac{5}{2}$ **h** $\frac{3}{8}$

Example 24c

7 Convert the following fractions to decimals, by dividing the numerator by the denominator. Use a calculator to check your answers.

a $\frac{1}{2}$ **b** $\frac{3}{6}$ **c** $\frac{3}{4}$ **d** $\frac{2}{5}$
e $\frac{1}{3}$ **f** $\frac{3}{8}$ **g** $\frac{5}{12}$ **h** $\frac{3}{7}$
i $\frac{1}{6}$ **j** $\frac{2}{3}$ **k** $\frac{1}{7}$ **l** $\frac{5}{9}$

8 Copy and complete the following fraction ↔ decimal tables. The quarters table (part **c**) has already been done for you. It's well worth trying to memorise these fractions and their equivalent decimal values.

a halves

Fraction	$\frac{0}{2}$	$\frac{1}{2}$	$\frac{2}{2}$
Decimal			

b thirds

Fraction	$\frac{0}{3}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3}$
Decimal				

c quarters

Fraction	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
Decimal	0	0.25	0.5	0.75	1

d fifths

Fraction	$\frac{0}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$
Decimal						

PROBLEM-SOLVING AND REASONING

9, 10, 12 | 10, 12, 14 | 13–16

9 Arrange the following from smallest to largest.

a $\frac{1}{2}, 0.75, \frac{5}{8}, 0.4, 0.99, \frac{1}{4}$ **b** $\frac{3}{7}, 0.13, \frac{1}{9}, 0.58, 0.84, \frac{4}{5}$

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10 Tan and Lillian are trying to work out who is the better chess player. They have both been playing chess games against their computers. Tan has played 37 games and beaten the computer 11 times. Lillian has played only 21 games and has beaten the computer 6 times.

a Using a calculator and converting the appropriate fractions to decimals, determine who is the better chess player.

b Lillian has time to play another four games of chess against her computer. To be classified as a better player than Tan, how many of these four games must she win?

11 To estimate the thickness of one sheet of A4 paper, Christopher measures the thickness of a ream of paper, which consists of 500 sheets of A4 paper. He determines that the pile is 55 mm thick. How thick is one sheet of A4 paper? Express your answer as a decimal number and also as a fraction.

12 **a** Copy and complete the following fraction ↔ decimal table.

Fraction	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$
Decimal									

b Comment on the trend in the decimal values as the *denominator* increases.

c Try to explain why this makes sense.

13 **a** Copy and complete the following decimal ↔ fraction table.

Decimal	0.1	0.2	0.25	0.4	0.5	0.6	0.75	0.8	0.9
Fraction									

b Comment on the trend in the fractions as the *decimal value* increases.

c Try to explain why this makes sense.

14 **a** Write $\frac{1}{3}$ as a recurring decimal. **b** Write $\frac{2}{3}$ as a recurring decimal.

c Using your calculator, find $2 \div 3$.

d Is the calculator correct or incorrect to display the answer as 0.666666667. Explain.

15 Write three different fractions with different denominators that are between the decimal value of 2.4 and 2.5.

16 When $\frac{4}{7}$ is expressed in decimal form, find the digit in the 23rd decimal place. Give a reason for your answer.

ENRICHMENT	—	—	17

Design a decimal game for the class

17 Using the skill of converting decimals to fractions and vice versa, design an appropriate game that students in your class could play. Ideas may include variations of Bingo, Memory or Dominoes. Try creating a challenging set of question cards.

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4J Connecting percentages with fractions and decimals

Interactive Widgets HOTsheets Walkthrough

Percentages are related closely to fractions. A percentage is a fraction in which the denominator is 100. *Per cent* is Latin for 'out of 100'. One dollar is equivalent to 100 cents and a century is 100 years.

Percentages are used in many everyday situations. Interest rates, discounts, test results and statistics are usually described using percentages rather than fractions or decimals because it is easier to compare two different results.

Let's start: Comparing performance

Consider these netball scores achieved by four students.
Annie scores 30 goals from 40 shots (i.e. 30 out of 40).
Bella scores 19 goals from 25 shots.
Cara scores 4 goals from 5 shots.
Dianne scores 16 goals from 20 shots.

- Discuss ways to compare the accuracy of their goal shooting.
- How might percentages be used?

The chart below might be useful.
Annie divided her 40 shots in the 10 boxes and then shaded the ones she scored.

	Annie (40 shots)	Bella	Cara	Dianne
100%				
	4			
90%				
	4			
80%				
	4			
70%				
	4			
60%				
	4			
50%				
	4			
40%				
	4			
30%				
	4			
20%				
	4			
10%				
	4			
0%				

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Key ideas

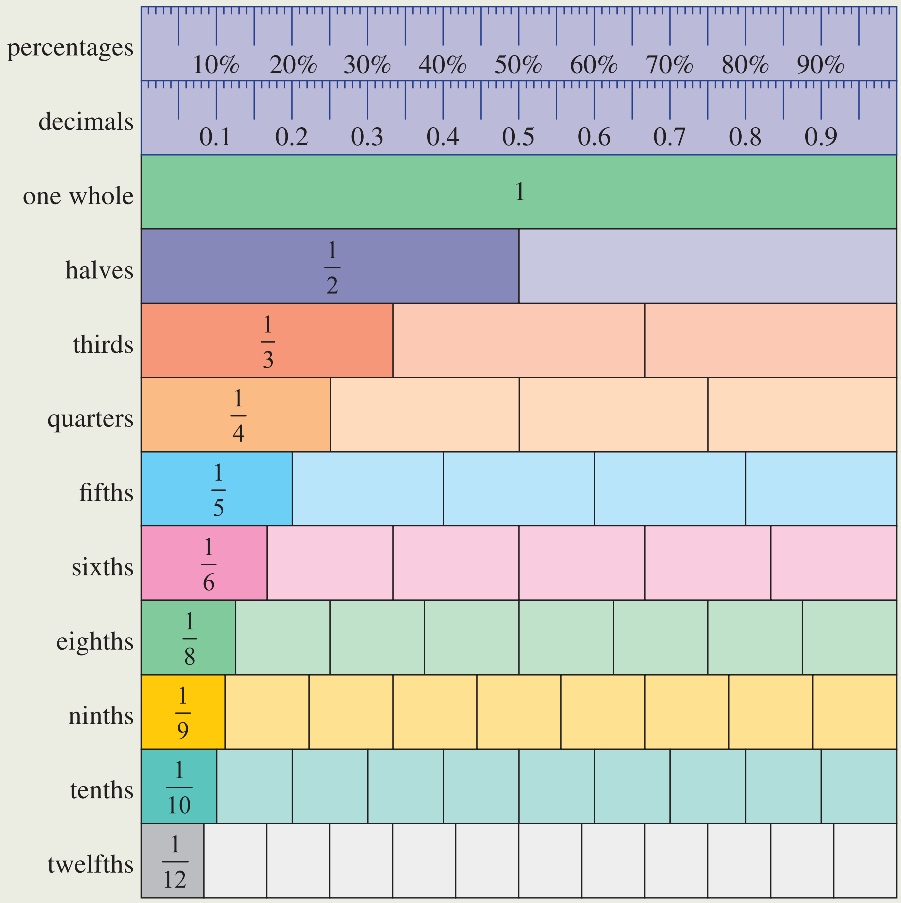
- Percentages have been used for hundreds of years but the symbol we use today is fairly recent. The symbol % means **per cent**. It comes from the Latin words *per centum*, which mean 'out of 100'.

  For example: 35% means '35 out of 100' or $\frac{35}{100}$ or 35 ÷ 100 or 0.35.

- Percentages are a useful way to compare fractions.

  For example: $\frac{3}{4} = \frac{75}{100} = 75\%$ and $\frac{18}{25} = \frac{72}{100} = 72\%$, therefore $\frac{3}{4} > \frac{18}{25}$.

- It is important to understand the relationships and connections between fractions, decimals and percentages. The 'fraction wall' diagram below shows these very clearly.



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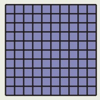
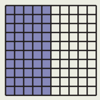
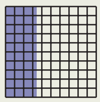
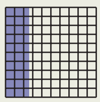
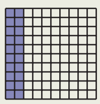
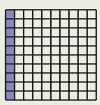
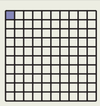
Key ideas

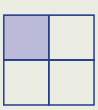
- It is also important to memorise the most commonly used conversions. These are listed in the table below.

Words	Diagram	Fraction	Decimal	Percentage
one whole		1	1	100%
one-half		$\frac{1}{2}$	0.5	50%
one-third		$\frac{1}{3}$	0.333 … or $0.\dot{3}$	$33\frac{1}{3}\%$
one-quarter		$\frac{1}{4}$	0.25	25%
one-fifth		$\frac{1}{5}$	0.2	20%
one-tenth		$\frac{1}{10}$	0.1	10%
one-hundredth		$\frac{1}{100}$	0.01	1%

- The number facts in the table can be used to do other conversions.

For example:

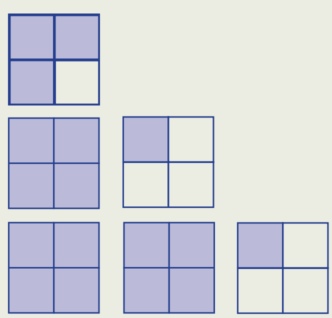
$\frac{1}{4} = 0.25 = 25\%$



$\times 3$: $\frac{3}{4} = 0.75 = 75\%$

$\times 5$: $\frac{5}{4} = 1.25 = 125\%$

$\times 9$: $\frac{9}{4} = 2.25 = 225\%$



Example 25 Using a known number fact to make conversions

Given that $\frac{1}{5} = 20\%$, complete the following.

a $\frac{1}{5} = 20\%$, so $\frac{3}{5} = \_\_\_\_\%$ **b** $\frac{1}{5} = 20\%$, so $\frac{6}{5} = \_\_\_\_\%$ **c** $\frac{1}{5} = 20\%$, so $\frac{11}{5} = \_\_\_\_\%$

SOLUTION	EXPLANATION
$\frac{1}{5} = 20\%$	This should be a memorised number fact.
a $\frac{3}{5} = 20\% \times 3 = 60\%$	Multiply the number fact by 3.
b $\frac{6}{5} = 20\% \times 6 = 120\%$	Multiply the number fact by 6.
c $\frac{11}{5} = 20\% \times 11 = 220\%$	Multiply the number fact by 11.

Example 26 Using memorised number facts

Convert the following fractions to decimals and percentages.

a seventeen-tenths **b** nine-quarters **c** two-thirds

SOLUTION	EXPLANATION
a $\frac{1}{10} = \frac{10}{100} = 0.10 = 10\%$	This should be a memorised number fact.
$\therefore \frac{17}{10} = \frac{170}{100} = 1.70 = 170\%$	Multiply the number fact by 17.
b $\frac{1}{4} = \frac{25}{100} = 0.25 = 25\%$	This should be a memorised number fact.
$\therefore \frac{9}{4} = 2.25 = 225\%$	Multiply the number fact by 9.
c $\frac{1}{3} = 0.333... = 33\frac{1}{3}\%$	This should be a memorised number fact.
$\therefore \frac{2}{3} = 0.666... = 66\frac{2}{3}\%$	Multiply the number fact by 2.

Exercise 4J

UNDERSTANDING AND FLUENCY	1–7, 8–9(½)	4–7, 8–10(½)	5–7, 8–10(½)

1 The percentage equivalent to one-quarter is:
A 4% **B** 2.5% **C** 25% **D** 40%

2 The percentage equivalent to three-quarters is:
A 7.5% **B** 34% **C** 75% **D** 80%

3 The percentage equivalent to 0.1 is:
A 0.1% **B** 1% **C** 10% **D** 100%

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4 Use the fraction wall (see page 173) to complete the following.

a three-quarters = $\frac{\square}{\square}$ = 0.___ = ___%

b nine-tenths = $\frac{\square}{\square}$ = 0.___ = ___%

c two-fifths = $\frac{\square}{\square}$ = 0.___ = ___%

d four-fifths = $\frac{\square}{\square}$ = 0.___ = ___%

Example 25

5 Complete the following.

a $\frac{1}{2} = 50\%$, so $\frac{3}{2} =$ ___%

b $\frac{1}{2} = 50\%$, so $\frac{7}{2} =$ ___%

c $\frac{1}{4} = 25\%$, so $\frac{3}{4} =$ ___%

d $\frac{1}{4} = 25\%$, so $\frac{7}{4} =$ ___%

e $\frac{1}{5} = 20\%$, so $\frac{9}{5} =$ ___%

f $\frac{1}{5} = 20\%$, so $\frac{11}{5} =$ ___%

Example 26

6 Convert the following fractions to decimals and percentages.

a three-tenths b three-fifths c five-quarters d four-thirds

7 a Use the fraction wall (see page 173) to write down:

i six fractions that are equivalent to 50%

ii ten fractions that are greater than 25% but less than 50%

b Comparing your answers to **i** and **ii**, which fraction is closest to 50%?

8 Use the fraction wall (see page 173) to complete these computations. Give your answer as a fraction in simplest form.

a $\frac{1}{2} + \frac{1}{4}$

b $\frac{1}{4} + \frac{1}{4}$

c $\frac{1}{8} + \frac{1}{8}$

d $\frac{1}{2} + \frac{1}{4} + \frac{1}{4}$

e $\frac{1}{6} + \frac{1}{6}$

f $\frac{2}{3} + \frac{2}{3}$

9 Use the fraction wall (see page 173) to do these computations.

a $\frac{1}{2} - \frac{1}{4}$

b $\frac{1}{2} - \frac{1}{6}$

c $\frac{1}{4} - \frac{1}{8}$

d $\frac{1}{2} - \frac{1}{4} - \frac{1}{4}$

e $\frac{2}{3} - \frac{1}{6}$

f $\frac{2}{3} - \frac{1}{12}$

g $1 - \frac{1}{4}$

h $1 - \frac{1}{5}$

i $1 - \frac{3}{8}$

10 Use the fraction wall (see page 173) to complete these computations.

a $1 - 0.1$

b $1 - 0.7$

c $1 - 0.25$

d $1 - 0.65$

e $0.25 + 0.25$

f $0.25 + 0.65$

g 3×0.2

h 6×0.2

i 0.3×7

PROBLEM-SOLVING AND REASONING	11, 12, 15	12, 13, 15, 16	13–16

11 Rachel's birthday cake is cut into two equal pieces, then four equal pieces, then eight equal pieces. Rachel eats three pieces.

a What percentage of the cake did Rachel eat?

b What percentage of the cake remains?

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12 Use the fraction wall (see page 173) to answer the following questions.

a What is bigger: three-quarters or two-thirds?
b Which is bigger: two-thirds or three-fifths?
c What is half of one-half?
d What is half of one-quarter?
e What fraction is exactly halfway between one-half and one-quarter?

13 Sophie's netball team wins six of their first seven games. They have three more games to play.

a What is the highest percentage the team can achieve?
b What is the lowest percentage the team can achieve?

14 Use the fractions in the fraction wall (see page 173) to solve these problems.

a *Two* fractions with the *same* denominator add up to one-half. What could they be? What else could they be? Write down all the possibilities from the fractions in the fraction wall.
b *Two* fractions with *different* denominators add up to one-half. What could they be? What else could they be? Write down all the possibilities from the fractions in the fraction wall.
c *Three* fractions with the *same* denominator add up to one-half. What could they be? What else could they be? Write down all the possibilities from the fractions in the fraction wall.
d *Three* fractions with *different* denominators add up to one-half. What could they be? What else could they be? Write down all the possibilities from the fractions in the fraction wall.
e *Two* fractions with *different* denominators add up to one-half. One of them is one-tenth. What is the other one?

15 Are the following statements true or false? Explain your answers, using the fraction wall.

a $0.5 = 50\%$
b $\frac{1}{3} = 33\%$
c $\frac{1}{5} = 15\%$
d $\frac{9}{10} = 90\%$
e $\frac{1}{8} \approx 12\%$
f $\frac{2}{3} > 66\%$

16 Which one of the following price reductions represents the greatest percentage discount? Explain your answer.

	Before discount	After discount
A	\$40	\$28
B	\$90	\$60
C	\$100	\$69
D	\$80	\$60

ENRICHMENT	—	—	17

A frog named Willy Makeit

17 Willy Makeit is a very small frog. He is on flat ground, 1 metre from his pond. He needs to get back to the pond but he gets very tired when he jumps. His first jump is half a metre. Every jump he makes after that is half the distance of the previous jump.

Use the fraction wall (see page 182) to answer these questions.

a After Willy has made his first jump, how far is he from the pond?
b In metres, how long is his second jump?
c After Willy has made his second jump, how far is he from the pond?
d In metres, how long is his third jump?
e Will Willy make it to the pond?

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4K Decimal and percentage conversions

Interactive
Widgets
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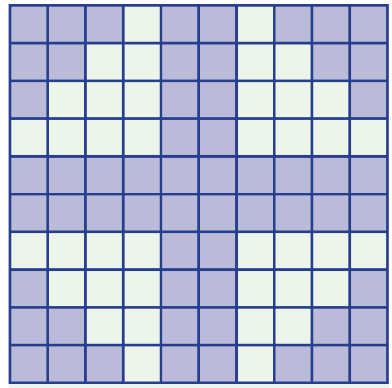
Walkthrough

Percentages give an idea of proportion. For example, if a newspaper states that 2000 people want a council swimming pool constructed, then we know how many want a pool but we don't know what proportion of the community that is. However, if there are 2500 people in this community, the newspaper can state that 80% want a swimming pool. This informs us that a majority of the community (i.e. 80 out of every 100 people) want a swimming pool constructed.

Let's start: Creative shading

- Draw a square of side length 10 cm and shade exactly 20% or 0.2 of this figure.
- Draw a square of side length 5 cm and shade exactly 60% or 0.6 of this figure.
- Draw another square of side length 10 cm and creatively shade an exact percentage of the figure. Ask your partner to work out the percentage you shaded.



What percentage is shaded?

Key ideas

- The symbol **%** means **per cent**. It comes from the Latin words *per centum*, which mean 'out of 100'.

  For example: 23% means 23 out of 100 or $\frac{23}{100}$ or 0.23.

- To convert a percentage to a decimal, divide by 100. This is done by moving the decimal point 2 places to the left.

  For example: $42\% = 42 \div 100 = 0.42$

- To convert a decimal to a percentage, multiply by 100. This is done by moving the decimal point 2 places to the right.

  For example: $0.654 = 0.654 \times 100\% = 65.4\%$

Example 27 Converting percentages to decimals

Express the following percentages as decimals.

a 30% **b** 240% **c** 12.5% **d** 0.4%

SOLUTION	EXPLANATION
a $30\% = 0.3$	$30 \div 100$
b $240\% = 2.4$	$240 \div 100 = 2.4$
c $12.5\% = 0.125$	Decimal point appears to move 2 places to the left.
d $0.4\% = 0.004$	Decimal point appears to move 2 places to the left.

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Example 28 Converting decimals to percentages

Express the following decimals as percentages.

a 0.045 **b** 7.2

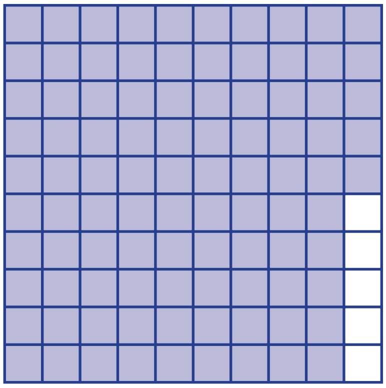
SOLUTION	EXPLANATION
a $0.045 \times 100\% = 4.5\%$	Multiplying by 100 appears to move the decimal point 2 places to the right.
b $7.2 \times 100\% = 720\%$	Multiply 7.2 by 100.

Exercise 4K

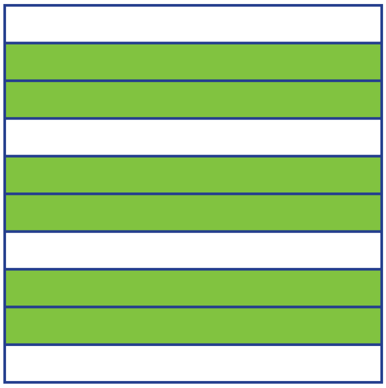
UNDERSTANDING AND FLUENCY	1–6, 7–9(½)	6, 7–9(½)	7–9(½)

1 What percentage of each square has been shaded?

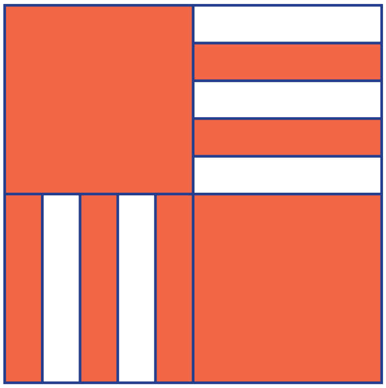
a



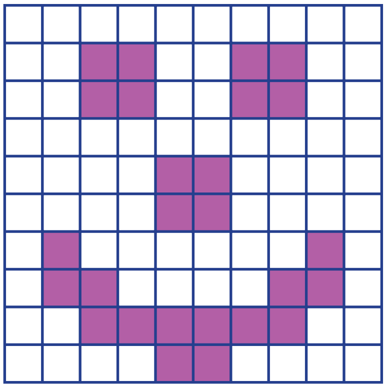
b



c



d



2 72.5% is equivalent to which of the following decimals?

A 72.5 **B** 7.25 **C** 0.725 **D** 725.0

3 1452% is equivalent to which of the following decimals?

A 0.1452 **B** 14.52 **C** 145 200 **D** 145.20

4 0.39 is equivalent to which of the following percentages?

A 39% **B** 3.9% **C** 0.39% **D** 0.0039%

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5 Prue answered half the questions correctly for a test marked out of 100.

a What score did Prue get on the test?

b What percentage did Prue get on the test?

c Find the score you would expect Prue to get if the test was out of:

i 10 **ii** 200 **iii** 40 **iv** 2

d Find the percentage you would expect Prue to get if the test was out of:

i 10 **ii** 200 **iii** 40 **iv** 2

6 Fill in the empty boxes.

a $58\% = 58 \text{ out of } \square = 58 \square 100 = \dfrac{58}{\square} = 0 \square 58$

b $35\% = \square \text{ out of } 100 = 35 \div \square = \dfrac{\square}{100} = \square.35$

c $126\% = 126 \square \square 100 = \square \div 100 = \dfrac{126}{\square} = 1.\square\square$

Example 27a,b

7 Express the following percentages as decimals.

a 32% **b** 27% **c** 68% **d** 54%

e 6% **f** 9% **g** 100% **h** 1%

i 218% **j** 142% **k** 75% **l** 199%

Example 27c,d

8 Express the following percentages as decimals.

a 22.5% **b** 17.5% **c** 33.33% **d** 8.25%

e 112.35% **f** 188.8% **g** 150% **h** 520%

i 0.79% **j** 0.025% **k** 1.04% **l** 0.95%

Example 28

9 Express the following decimals as percentages.

a 0.8 **b** 0.3 **c** 0.45 **d** 0.71

e 0.416 **f** 0.375 **g** 2.5 **h** 2.314

i 0.025 **j** 0.0014 **k** 12.7 **l** 1.004

PROBLEM-SOLVING AND REASONING	10, 11, 15	11, 12, 15, 16	13, 14, 16, 17

10 Place the following values in order from highest to lowest.

a 86%, 0.5%, 0.6, 0.125, 22%, 75%, 2%, 0.78

b 124%, 2.45, 1.99%, 0.02%, 1.8, 55%, 7.2, 50

11 An ice-cream store is offering a discount of 15% on orders over $25. A single-scoop ice-cream in a cone is $5.50 and a double-scoop ice-cream in a cone is $7.25. A family of six purchases three single-scoop ice-creams and three double-scoop ice-creams.

a Will the family receive the discount?

b What percentage will they pay?

12 Last Saturday, Phil spent 24 hours of the day in the following way: 0.42 of the time was spent sleeping, 0.22 was spent playing sport and 0.11 was spent eating. The only other activity Phil did for the day was watch TV.

a What percentage of the day did Phil spend watching TV?

b What percentage of the day did Phil spend either sitting down or lying down?

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13 Sugarloaf Reservoir has a capacity of 96 gigalitres. However, as a result of the drought it is only 25% full. How many gigalitres of water are in the reservoir?

14 The average daily energy intake for adolescent boys is 11 500 kJ. The average serving size of a bowl of Rice Bubbles with $\frac{1}{2}$ cup of reduced-fat milk provides 770 kJ. What percentage of a boy's daily intake is a bowl of Rice Bubbles with milk? Round your answer to 1 decimal place.

15 a, b, c and d are digits. Write the following decimal numbers as percentages.

a $0.abcd$ **b** $a.ac$ **c** $ab.dc$
d $0.0dd$ **e** $c.dba$ **f** $0.cccddd$

16 a, b, c and d are digits. Write the following percentages as decimal numbers.

a $a.b\%$ **b** $bcd\%$ **c** $ac\%$
d $0.da\%$ **e** $abbb\%$ **f** $dd.d\%$

17 Trudy says that it is impossible to have more than 100%. She supports her statement by saying that if you get every question correct in a test, then you get 100% and you cannot get any more.

a Do you agree with Trudy's statement?
b Provide four examples of when it makes sense that you cannot get more than 100%.
c Provide four examples of when it is perfectly logical to have more than 100%.

ENRICHMENT	—	—	18, 19

AFL ladder

18 The Australian Rules football ladder has the following column headings.

Pos	Team	P	W	L	D	F	A	%	Pts
6	Brisbane Lions	22	13	8	1	2017	1890	106.72	54
7	Carlton	22	13	9	0	2270	2055	110.46	52
8	Essendon	22	10	11	1	2080	2127	97.79	42
9	Hawthorn	22	9	13	0	1962	2120	92.55	36
10	Port Adelaide	22	9	13	0	1990	2244	88.68	36

a Using a calculator, can you determine how the percentage column is calculated?
b What do you think the 'F' and the 'A' column stand for?
c In their next match, Essendon scores 123 points for their team and has 76 points scored against them. What will be their new percentage?
d By how much do Hawthorn need to win their next game to have a percentage of 100?
e If Port Adelaide plays Hawthorn in the next round and the final score is Port Adelaide 124 beats Hawthorn 71, will Port Adelaide's percentage become higher than Hawthorn's?

19 Create your own AFL-style ladder using a spreadsheet program. After entering the results, the program should automatically update the points column and the percentage column.
When carrying out a sort on the data, ensure that your program will automatically change any team's position on the ladder, if necessary.

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4L Fraction and percentage conversions

Interactive
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We come across percentages in many everyday situations. Interest rates, discounts, test results and statistics are just some of the common ways in which we deal with percentages. Percentages are closely related to fractions. A percentage is another way of writing a fraction with a denominator of 100. Therefore, 87% means that if something is divided into 100 pieces you would have 87 of them.

Let's start: Student ranking

Five students completed five different mathematics tests. Each of the tests was out of a different number of marks. The results are shown below. Your task is to rank the five students in descending order, according to their test result.

- Matthew scored 15 out of a possible 20 marks.
- Mengna scored 36 out of a possible 50 marks.
- Maria scored 33 out of a possible 40 marks.
- Marcus scored 7 out of a possible 10 marks.
- Melissa scored 64 out of a possible 80 marks.

Change these test results to equivalent scores out of 100, and state the percentage test score for each student.

Key ideas

- We can write percentages as fractions by changing the % sign to a denominator of 100 (meaning out of 100).

  For example: $37\% = \frac{37}{100}$

- We can convert fractions to percentages through our knowledge of equivalent fractions.

  For example: $\frac{1}{4} = \frac{25}{100} = 25\%$

- To convert any fraction to a percentage, multiply by 100.

  For example: $\frac{3}{8} \times 100 = \frac{3}{8} \times \frac{100}{1} = \frac{75}{2} = 37\frac{1}{2}$. Therefore $\frac{3}{8} = 37\frac{1}{2}\% = 37.5\%$

- Common percentages and their equivalent fractions are shown in the table below. It is useful to know these.

Fraction	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{1}{10}$
Percentage	50%	$33\frac{1}{3}\%$	25%	20%	$12\frac{1}{2}\%$	$66\frac{2}{3}\%$	75%	10%

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Example 29 Converting percentages to fractions

Express these percentages as fractions or mixed numerals in their simplest form.

a 17% **b** 36% **c** 140%

SOLUTION

a $17\% = \frac{17}{100}$

Change % sign to a denominator of 100.

b $36\% = \frac{36}{100}$

Change % sign to a denominator of 100.

$= \frac{9 \times 4}{25 \times 4}$

Cancel HCF.

$= \frac{9}{25}$

Answer is now in simplest form.

c $140\% = \frac{140}{100}$

Change % sign to a denominator of 100.

$= \frac{7 \times 20}{5 \times 20}$

Cancel HCF.

$= \frac{7}{5} = 1\frac{2}{5}$

Convert answer to a mixed numeral.

Example 30 Converting to percentages through equivalent fractions

Convert the following fractions to percentages.

a $\frac{5}{100}$ **b** $\frac{11}{25}$ **c** $\frac{5}{8}$ **d** $3\frac{3}{5}$

SOLUTION

a $\frac{5}{100} = 5\%$

Denominator is already 100, therefore simply write number as a percentage.

b $\frac{11}{25} \underset{\times 4}{\overset{\times 4}{=}} \frac{44}{100}$

$= 44\%$

Require denominator to be 100.
Therefore, multiply numerator and denominator by 4 to get an equivalent fraction.

c $\frac{5}{8} = \frac{5 \times 125}{8 \times 125}$

Convert the fraction to thousandths by multiplying the denominator and numerator by 125.

$= \frac{625}{1000}$

$= \frac{62.5}{100}$

Convert the fraction to hundredths by dividing the denominator and numerator by 10.

$= 62.5\%$

Write the fraction as a percentage.

d $3\frac{3}{5} = 3\frac{6}{10}$

Convert the fraction to tenths by multiplying the denominator and numerator by 2.

$= 3\frac{60}{100}$

Convert the fraction to hundredths by multiplying the denominator and numerator by 10.

$= \frac{360}{100}$

Write the mixed numeral as an improper fraction.

$= 360\%$

Write the improper fraction as a percentage.

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Example 31 Converting to percentages by multiplying by 100

Convert the following fractions to percentages.

a $\frac{3}{8}$ **b** $3\frac{3}{5}$

SOLUTION

a $\frac{3}{8} \times 100 = \frac{3}{{}_{2}\cancel{8}} \times \frac{\cancel{100}^{25}}{1}$

$= \frac{75}{2} = 37\frac{1}{2}$

$\therefore \frac{3}{8} = 37\frac{1}{2}\%$

b $3\frac{3}{5} \times 100 = \frac{18}{{}_{1}\cancel{5}} \times \frac{\cancel{100}^{20}}{1}$

$= 360$

$\therefore 3\frac{3}{5} = 360\%$

EXPLANATION

Multiply by 100.
Simplify by cancelling HCF.
Write your answer as a mixed number.

Convert mixed number to improper fraction.
Cancel and simplify.

Exercise 4L

UNDERSTANDING AND FLUENCY	1–3, 4–7(½)	3, 4–7(½)	4–7(½)

1 Change these test results to equivalent scores out of 100, and therefore state the percentage.

a 7 out of 10 = ____ out of 100 = ____%
b 24 out of 50 = ____ out of 100 = ____%
c 12 out of 20 = ____ out of 100 = ____%
d 1 out of 5 = ____ out of 100 = ____%
e 80 out of 200 = ____ out of 100 = ____%
f 630 out of 1000 = ____ out of 100 = ____%

2 Write these fraction sequences into your workbook and write beside each fraction the equivalent percentage value.

a $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$ **b** $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}$ **c** $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}$

3 **a** If 14% of students in Year 7 are absent due to illness, what percentage of Year 7 students are at school?

b If 80% of the geography project has been completed, what percentage still needs to be finished?

Example 29a,b

4 Express these percentages as fractions in their simplest form.

a 11% **b** 71% **c** 43% **d** 49%
e 25% **f** 30% **g** 15% **h** 88%

Example 29c

5 Express these percentages as mixed numerals in their simplest form.

a 120% **b** 180% **c** 237% **d** 401%
e 175% **f** 110% **g** 316% **h** 840%

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Example 30

6 Convert these fractions to percentages, using equivalent fractions.

a $\frac{8}{100}$	**b** $\frac{15}{100}$	**c** $\frac{97}{100}$	**d** $\frac{50}{100}$
e $\frac{7}{20}$	**f** $\frac{8}{25}$	**g** $\frac{43}{50}$	**h** $\frac{18}{20}$
i $\frac{56}{50}$	**j** $\frac{27}{20}$	**k** $\frac{20}{5}$	**l** $\frac{16}{10}$

Example 31

7 Convert these fractions to percentages. Check your answers using a calculator.

a $\frac{1}{8}$	**b** $\frac{1}{3}$	**c** $\frac{4}{15}$	**d** $\frac{10}{12}$
e $1\frac{3}{20}$	**f** $4\frac{1}{5}$	**g** $2\frac{36}{40}$	**h** $\frac{13}{40}$

PROBLEM-SOLVING AND REASONING	8, 9, 13	9–11, 13, 14	10–12, 14, 15

8 A bottle of lemonade is only 25% full.

a What fraction of the bottle has been consumed?
b What percentage of the bottle has been consumed?
c What fraction of the bottle is left?
d What percentage of the bottle is left?

9 A lemon tart is cut into eight equal pieces. What percentage of the tart does each piece represent?

10 Petrina scores 28 out of 40 on her fractions test. What is her percentage score?

11 The nutrition label on a particular brand of sliced bread states that the average serving size of two slices is equal to 55 grams. It also states that there are 2.2 grams of sugar per serve. What percentage of the bread is sugar?

12 The Sydney Kings basketball team have won 14 out of 18 games. They still have two games to play. What is the smallest and the largest percentage of games the Kings could win for the season?

13 Lee won his tennis match with the score 6–4, 6–2, 6–1.

a What fraction of games did he win?
b What percentage of games did he win?

14 Scott and Penny have just taken out a home loan, with an interest rate of $5\frac{1}{2}\%$. Write this interest rate as a fraction.

15 Write each of the following percentages as fractions.

a $2\frac{1}{2}\%$	**b** $8\frac{1}{4}\%$	**c** $12\frac{1}{2}\%$	**d** $33\frac{1}{3}\%$

ENRICHMENT	—	—	16

Lottery research

16 Conduct research on a major lottery competition. If possible:

a Find out, on average, how many tickets are sold each week.
b Find out, on average, how many tickets win a prize each week.
c Determine the percentage chance of winning a prize.
d Determine the percentage chance of winning the various divisions.
e Work out the average profit the lottery competition makes each week.

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4M Percentage of a quantity

Interactive
Widgets
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Walkthrough

A common application of percentages is to find a certain percentage of a given number. Throughout life you will come across many examples where you need to calculate percentages of a quantity. Examples include retail discounts, interest rates, personal improvements, salary increases, commission rates and more.

In this exercise we will focus on the mental calculation of percentages.

Let's start: Percentages in your head

It is a useful skill to be able to quickly calculate percentages mentally.

Calculating 10% or 1% is often a good starting point. You can then multiply or divide these values to arrive at other percentage values.

- In pairs, using mental arithmetic only, calculate these 12 percentages.

a 10% of \$120	**b** 10% of \$35	**c** 20% of \$160	**d** 20% of \$90
e 30% of \$300	**f** 30% of \$40	**g** 5% of \$80	**h** 5% of \$420
i 2% of \$1400	**j** 2% of \$550	**k** 12% of \$200	**l** 15% of \$60

- Check your answers with a classmate or your teacher.
- Design a quick set of 12 questions for a classmate.
- Discuss helpful mental arithmetic skills to increase your speed at calculating percentages.

Key ideas

- To find the percentage of a number, without using a calculator:
  1. Express the required percentage as a fraction.
  2. Change the 'of' to a multiplication sign.
  3. Look for a written or mental strategy to complete the multiplication.

$$\begin{aligned} 25\% \text{ of } 60 &= \frac{1}{4} \text{ of } 60 \\ &= 60 \div 4 \\ &= 60 \div 2 \div 2 \\ &= 30 \div 2 \\ &= 15 \end{aligned}$$

- Useful mental strategies
  - To find 50%, divide by 2
  - To find 10%, divide by 10
  - To find 25%, divide by 4
  - To find 1%, divide by 100
- Using a calculator, 25% of 60 = 25 ÷ 100 × 60.

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Example 32 Finding the percentage of a number

Find:

a 30% of 50

b 15% of 400

SOLUTION

a $30\% \text{ of } 50 = \frac{30}{100} \times \frac{50}{1}$

$= \frac{30}{2} = 15$

Mental arithmetic:

10% of 50 = 5

Hence, 30% of 50 = 15.

b $15\% \text{ of } 400 = \frac{15}{100} \times \frac{400}{1}$

$= \frac{15 \times 4}{1} = 60$

Mental arithmetic:

10% of 400 = 40, $\therefore$ 5% of 400 = 20

Hence, 15% of 400 = 60.

EXPLANATION

Write % as a fraction. Cancel and simplify.

Multiply by 3 to get 30%.

Write % as a fraction.
Cancel and simplify.

5% is half of 10%.
Multiply by 3 to get 15%.

Example 33 Solving a worded percentage problem

Jacqueline has saved up $50 to purchase a new pair of jeans. She tries on many different pairs but only likes two styles, Evie and Next. The Evie jeans are normally $70 and are on sale with a 25% discount. The Next jeans retail for $80 and have a 40% discount for the next 24 hours. Can Jacqueline afford either pair of jeans?

SOLUTION

Evie jeans

Discount = 25% of $70

$= \frac{25}{100} \times \frac{70}{1} = \$17.50$

Sale price = $70 − $17.50

= $52.50

Next jeans

Discount = 40% of $80

$= \frac{40}{100} \times \frac{80}{1} = \$32$

Sale price = $80 − $32

= $48

Jacqueline can afford the Next jeans.

EXPLANATION

Calculate the discount on the Evie jeans.
Find 25% of $70.

Find the sale price by subtracting the discount.

Calculate the discount on the Next jeans.
Find 40% of $80.

Find the sale price by subtracting the discount.

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Exercise 4M

UNDERSTANDING AND FLUENCY	1, 2, 3–4(½), 5	2, 3–4(½), 5, 6(½)	3–4(½), 6(½)

1 Copy and complete the following sentences.

- **a** Finding 10% of a quantity is the same as dividing the quantity by ______.
- **b** Finding 1% of a quantity is the same as dividing the quantity by ______.
- **c** Finding 50% of a quantity is the same as dividing the quantity by ______.
- **d** Finding 100% of a quantity is the same as dividing the quantity by ______.
- **e** Finding 20% of a quantity is the same as dividing the quantity by ______.
- **f** Finding 25% of a quantity is the same as dividing the quantity by ______.

2 Without calculating the exact values, determine which alternative (**i** or **ii**) has the highest value.

- **a** **i** 20% of $400 **ii** 25% of $500
- **b** **i** 15% of $3335 **ii** 20% of $4345
- **c** **i** 3% of $10 000 **ii** 2% of $900
- **d** **i** 88% of $45 **ii** 87% of $35

Example 32

3 Find the following percentages, using a mental strategy. Check your answers with a calculator.

- **a** 50% of 140 **b** 10% of 360 **c** 20% of 50 **d** 30% of 90
- **e** 25% of 40 **f** 25% of 28 **g** 75% of 200 **h** 80% of 250
- **i** 5% of 80 **j** 4% of 1200 **k** 5% of 880 **l** 2% of 9500
- **m** 11% of 200 **n** 21% of 400 **o** 12% of 300 **p** 9% of 700

4 Find:

- **a** 120% of 80 **b** 150% of 400 **c** 110% of 60 **d** 400% of 25
- **e** 125% of 12 **f** 225% of 32 **g** 146% of 50 **h** 3000% of 20

5 Without using a calculator, match the questions with their correct answer.

Question	*Answer*
10% of $200	$8
20% of $120	$16
10% of $80	$20
50% of $60	$24
20% of $200	$25
5% of $500	$30
30% of $310	$40
10% of $160	$44
1% of $6000	$60
50% of $88	$93

6 Without using a calculator, find:

- **a** 30% of $140 **b** 10% of 240 millimetres **c** 15% of 60 kilograms
- **d** 2% of 4500 tonnes **e** 20% of 40 minutes **f** 80% of 500 centimetres
- **g** 5% of 30 grams **h** 25% of 12 hectares **i** 120% of 120 seconds

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PROBLEM-SOLVING AND REASONING	7, 8, 14	8–11, 14, 15	10–13, 16–18

7 Harry scored 70% on his percentages test. If the test is out of 50 marks, how many marks did Harry score?

Example 33

8 Grace wants to purchase a new top and has $40 to spend. She really likes a red top that was originally priced at $75 and has a 40% discount ticket on it. At another shop, she also likes a striped hoody, which costs $55. There is 20% off all items in the store on this day. Can Grace afford either of the tops?

9 In a student survey, 80% of students said they received too much homework. If 300 students were surveyed, how many students claimed that they received too much homework?

10 25% of teenagers say their favourite fruit is watermelon. In a survey of 48 teenagers, how many students would you expect to write watermelon as their favourite fruit?

11 At Gladesbrook College, 10% of students walk to school, 35% of students catch public transport and the remainder of students are driven to school. If there are 1200 students at the school, find how many students:

a walk to school
b catch public transport
c are driven to school

12 Anthea has just received a 4% salary increase. Her wage before the increase was $2000 per week.

a How much extra money does Anthea receive due to her salary rise?
b What is Anthea's new salary per week?
c How much extra money does Anthea receive per year?

13 Sam has 2 hours of 'free time' before dinner is ready. He spends 25% of that time playing computer games, 20% playing his drums, 40% playing outside and 10% reading a book.

a How long does Sam spend doing each of the four different activities?
b What percentage of time does Sam have remaining at the end of his four activities?
c Sam must set the table for dinner, which takes 5 minutes. Does he still have time to get this done?

14 Gavin mows 60% of the lawn in 48 minutes. How long will it take him to mow the entire lawn if he mows at a constant rate?

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15 Find:

a 20% of (50% of 200)

b 10% of (30% of 3000)

c 5% of (5% of 8000)

d 80% of (20% of 400)

16 Write a survey question for the students in your class, such as 'What is your favourite colour?' or 'How many days last week did you catch a bus to school?'. Provide five different answer options. Survey your class and calculate the percentage of students who chose each option. Use a sector graph (i.e. a pie chart) to display your findings.

17 Which is larger: 60% of 80 or 80% of 60?

18 Tom does the following calculation: 120 ÷ 4 ÷ 2 × 3. What percentage of 120 does he find?

ENRICHMENT	—	—	19

Waning interest

19 When someone loses interest or motivation in a task, they can be described as having a 'waning interest'. Jill and Louise are enthusiastic puzzle makers, but they gradually lose interest when tackling very large puzzles.

a Jill is attempting to complete a 5000-piece jigsaw puzzle in 5 weeks. Her interest drops off, completing 100 fewer pieces each week.

i How many pieces must Jill complete in the first week to ensure that she finishes the puzzle in the 5-week period?

ii What percentage of the puzzle does Jill complete during each of the 5 weeks?

iii What is the percentage that Jill's interest wanes each week?

b Louise is attempting to complete an 8000-piece jigsaw puzzle in 5 weeks. Her interest wanes at a constant rate of 5% per week.

i What percentage of the puzzle must Louise complete in the first week to ensure she finishes the puzzle in the 5-week period?

ii Record how many pieces of the puzzle Louise completes each week and the corresponding percentage of the puzzle.

iii Produce a table showing the cumulative number of pieces completed and the cumulative percentage of the puzzle completed over the 5-week period.

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4N Using fractions and percentages to compare two quantities

Interactive
Widgets
HOTsheets
Walkthrough

Sometimes we want to know the proportion of a certain quantity compared to a given total or another quantity. This may be done using a fraction, percentage or ratio. The Earth's surface, for example, is about 70% ocean. So, the proportion of land could be written as 30% (as a percentage) or $\frac{3}{10}$ (as a fraction). The ratio of land to ocean could be described as 30 parts of land to 70 parts of ocean. Alternatively, the ratio could be expressed as 3 parts of land to 7 parts of ocean.

Let's start: Tadpole proportion

Scientists Hugh and Jack take separate samples of tadpoles, which include green and brown tadpoles, from their local water channels. Hugh's sample contains 3 green tadpoles and 15 brown tadpoles, whereas Jack's sample contains 27 green tadpoles and 108 brown tadpoles.

- Find the proportion of green tadpoles in each of Hugh and Jack's samples.
- Use both fractions and percentages to compare the proportions.
- Which sample might be used to convince the local council that there are too many brown tadpoles in the water channels?

Key ideas

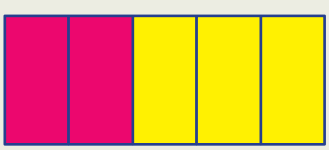
- To express one quantity as a fraction of another:

  $$\text{Fraction} = \frac{\text{amount}}{\text{total}}$$

- To express one quantity as a percentage of another, find an equivalent fraction in which the denominator is 100.

  This idea works well when the denominator is 2, 4, 5, 10, 20, 25 or 50.

- A ratio compares parts of a total.



$$\text{Red fraction} = \frac{2}{5} = \frac{4}{10} = \frac{40}{100}$$

$$\text{Red percentage} = \frac{40}{100} = 40\%$$

Ratio of red parts to all parts = 2 : 5

Ratio of red to yellow = 2 : 3

- This can also be done using a calculator.

  $2 \div 5 \times 100 = 40$

  So $\frac{2}{5} = 40\%$

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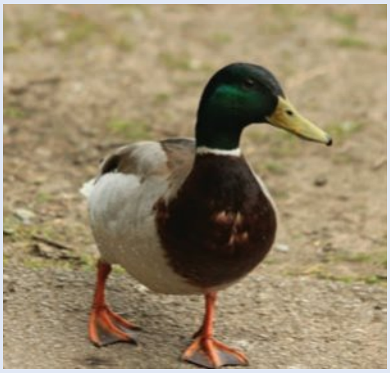
Example 34 Expressing as a proportion

Express the following as both a fraction and a percentage of the total.

a \$40 out of a total of \$200

b 24 green ducks out of a total of 30 ducks



SOLUTION

a Fraction $= \frac{40}{200} = \frac{4}{20} = \frac{1}{5}$

Write the given amount over the total. Then simplify the fraction.

Percentage $= \frac{40}{200} = \frac{20}{100} = 20\%$

Convert to hundredths.

b Fraction $= \frac{24}{30} = \frac{4}{5}$

There is a total of 24 green ducks out of a total of 30. Simplify the fraction.

Percentage $= \frac{4}{5} = \frac{8}{10} = \frac{80}{100}$

$= 80\%$

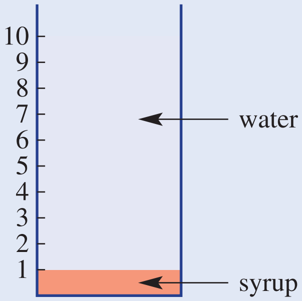
Convert to hundredths.

Example 35 Using ratios

A glass of cordial is 1 part syrup to 9 parts water.

a Express the amount of syrup as a fraction of the total.

b Express the amount of water as a percentage of the total.



SOLUTION

a Fraction $= \frac{1}{10}$

There is a total of 10 parts, including 1 part syrup.

b Percentage $= \frac{9}{10} = \frac{90}{100} = 90\%$

There is a total 9 parts water in a total of 10 parts.

$\frac{9}{10} = \frac{90}{100}$

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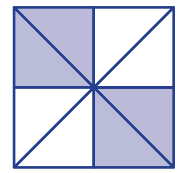
Exercise 4N

UNDERSTANDING AND FLUENCY	1–5	2, 3–4(½), 5, 6	3–4(½), 5–7

Note: The numbers in this exercise have been chosen carefully so that mental strategies may be used, rather than a calculator.

1 This square shows some coloured triangles and some white triangles.



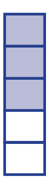
- **a** How many triangles are coloured?
- **b** How many triangles are white?
- **c** What fraction of the total is coloured?
- **d** What percentage of the total is coloured?
- **e** What fraction of the total is white?
- **f** What percentage of the total is white?

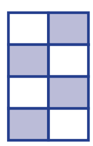
2 A farmer's pen has two black sheep and eight white sheep.
- **a** How many sheep are there in total?
- **b** What fraction of the sheep are black?
- **c** What fraction of the sheep are white?
- **d** What percentage of the sheep are black?
- **e** What percentage of the sheep are white?

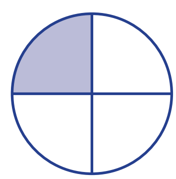
Example 34

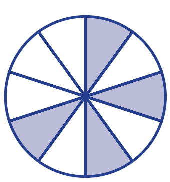
3 Express the following as both a fraction and a percentage of the total. Check your answers using a calculator.
- **a** 30 out of a total of 100
- **b** 3 out of a total of 5
- **c** $10 out of a total of $50
- **d** $60 out of a total of $80
- **e** 2 kg out of a total of 40 kg
- **f** 14 g out of a total of 28 g
- **g** 3 L out of a total of 12 L
- **h** 30 mL out of a total of 200 mL

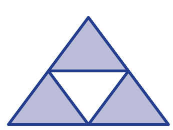
4 Write each coloured area as both a fraction and percentage of the total area.

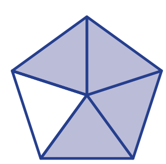
a


b


c


d


e


f


Example 35

5 A jug of lemonade is made up of 2 parts of lemon juice to 18 parts of water.
- **a** Express the amount of lemon juice as a fraction of the total.
- **b** Express the amount of lemon juice as a percentage of the total.

6 A mix of concrete is made up of 1 part of cement to 4 parts of sand.
- **a** Express the amount of cement as a fraction of the total.
- **b** Express the amount of cement as a percentage of the total.
- **c** Express the amount of sand as a fraction of the total.
- **d** Express the amount of sand as a percentage of the total.

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7 A pair of socks is made up of 3 parts of wool to 1 part of nylon.
  a Express the amount of wool as a fraction of the total.
  b Express the amount of wool as a percentage of the total.
  c Express the amount of nylon as a fraction of the total.
  d Express the amount of nylon as a percentage of the total.

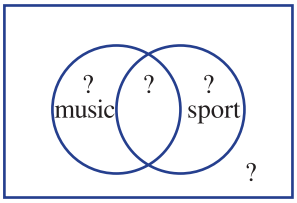
PROBLEM-SOLVING AND REASONING	8, 9, 13	9–11, 13, 14	10–12, 15–17

8 Gillian pays $80 tax out of her income of $1600. What percentage of her income does she keep?

9 Over summer, a dam's water volume reduces from 20 megalitres to 4 megalitres. What fraction of the water in the dam has been lost?

10 Express the following as a fraction and percentage of the total.
  a 20 cents of $5
  b 14 days out of 5 weeks
  c 15 centimetres removed from a total length of 3 metres
  d 3 seconds taken from a world record time of 5 minutes
  e 180 grams of a total of 9 kilograms
  f 1500 centimetres from a total of 0.6 kilometres

11 Of 20 students, 10 play sport and 12 play a musical instrument, with some of these students playing both sport and a musical instrument. Two students do not play any sport or a musical instrument.



  a What fraction of the students play both sport and a musical instrument?
  b What percentage of the students play a musical instrument but not sport?

12 An orchard of 80 apple trees is tested for diseases. 20 of the trees have blight disease, 16 have brown rot disease and some trees have both. A total of 48 trees have neither blight nor brown rot.
  a What percentage of the trees has both diseases?
  b What fraction of the trees has blight but does not have brown rot?

13 For a recent class test, Ross scored 45 out of 60 and Maleisha scored 72 out of 100. Use percentages to show that Ross obtained the higher mark.

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14 The prices of two cars are reduced for sale. A hatch priced at \$20 000 is now reduced by \$3000 and a 4WD priced at \$80 000 is now reduced by \$12 800. Determine which car has the largest percentage reduction, giving reasons.

15 A yellow sports drink has 50 grams of sugar dissolved in fluid and weighs a total of 250 grams. A blue sports drink has 57 grams of sugar dissolved in fluid and weighs a total of 300 grams. Which sports drink has the least percentage of sugar? Give reasons.

16 A room contains a girls and b boys.

a Write an expression using the pronumerals a and b for the fraction of:

i boys in the room **ii** girls in the room

b Write an expression using the pronumerals a and b for the percentage of:

i boys in the room **ii** girls in the room

17 A mixture of dough has a parts of flour to b parts of water.

a Write an expression for the fraction of flour.

b Write an expression for the percentage of water.

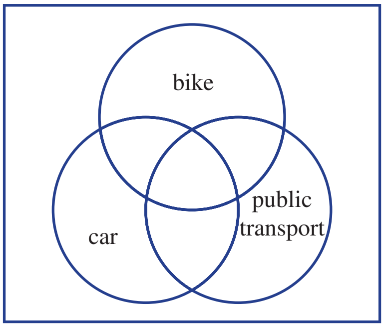
ENRICHMENT	—	—	18

Transport turmoil

18 A class survey of 30 students reveals that the students use three modes of transport to get to school: bike, public transport and car. All of the students used at least one of these three modes of transport in the past week.

Twelve students used a car to get to school and did not use any of the other modes of transport. One student used all three modes of transport and one student used only a bike for the week. There were no students who used both a bike and a car but no public transport. Five students used both a car and public transport but not a bike. Eight students used only public transport.

Use this diagram to help answer the following.



a How many students used both a bike and public transport but not a car?

b What fraction of the students used all three modes of transport?

c What fraction of the students used at least one mode of transport, including a bike?

d What fraction of the students used at least one mode of transport, including public transport?

e What percentage of students used public transport and a car during the week?

f What percentage of students used public transport or a car or both during the week?

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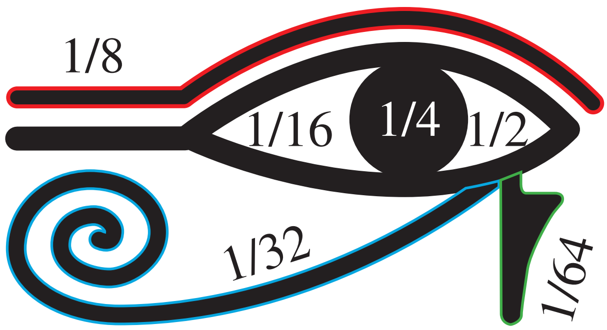
Investigation

Egyptian fractions



The fractions in the ancient Egyptian Eye of Horus were used for dividing up food and land, as well as portions of medicine. They are called **unitary** fractions because all the numerators are 1.

Clearly, the ancient Egyptians had no calculators or precise measuring instruments; nevertheless, by repeatedly dividing a quantity in half, the fractions $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ or $\frac{1}{32}$ were combined to estimate any other fraction.

Imagine that you are an ancient Egyptian baker and wish to share your last three loaves of bread equally between four people.

First, you cut two loaves in half and give half a loaf to each of your four customers.

You have one loaf remaining and you can cut that into quarters (i.e. half and then half again).

So each of your four customers now receives half a loaf and one-quarter of a loaf, which is $\frac{3}{4}$ ( ) of a loaf.

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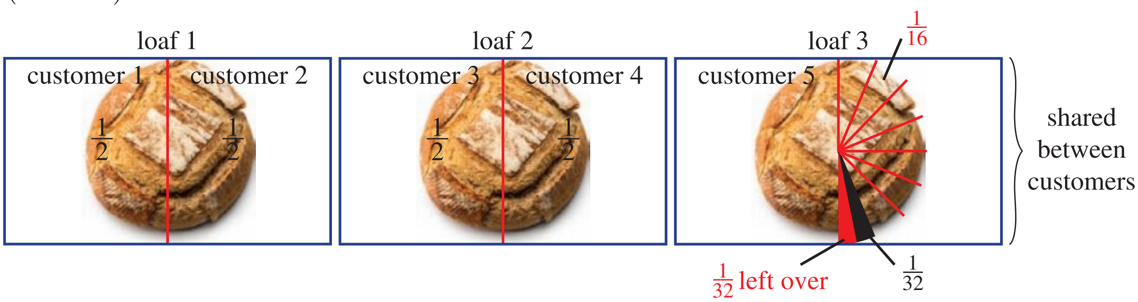
Using ancient Egyptian fractions, how could three loaves be divided equally between five people?

First, cut the loaves in half and give each customer $\frac{1}{2}$ () a loaf. The remaining half loaf can be cut into eight parts and each person is given $\frac{1}{8}$ of $\frac{1}{2} = \frac{1}{16}$th () of a loaf. There is a small portion left $\left(3 \text{ portions of } \frac{1}{16}\right)$, so these portions can be divided in half and each customer given $\frac{1}{2}$ of $\frac{1}{16} = \frac{1}{32}$ () of a loaf.



Each customer has an equal share $\frac{1}{2} + \frac{1}{16} + \frac{1}{32}$ () of the loaf and the baker will have the small $\frac{1}{32}$ () of a loaf left over.

If each loaf is divided *exactly* into five parts, the three loaves would have 15 equal parts altogether and each customer could have three parts of the 15; $\frac{3}{15} = \frac{1}{5}$ of the total or $\frac{3}{5}$ of one loaf. $\frac{3}{5} = 0.6$ and $\frac{1}{2} + \frac{1}{16} + \frac{1}{32} = 0.59375 \approx 0.6$ ($\approx$ means approximately equal).

So even without calculators or sophisticated measuring instruments, the ancient Egyptian method of repeated halving gives quite close approximations to the exact answers.

Task

Using diagrams, explain how the following portions can be divided equally using only the ancient Egyptian unitary fractions of $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ and $\frac{1}{32}$.

- **a** three loaves of bread shared between eight people
- **b** one loaf of bread shared between five people
- **c** two loaves of bread shared between three people

Include the Egyptian Eye of Horus symbols for each answer, and determine the difference between the exact answer and the approximate answer found using the ancient Egyptian method.

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Puzzles and challenges

1 Three cities are known as India's Golden Triangle. Match each of the fractions in the middle row with the equivalent fraction in the bottom row. Place the letter in the code below to find the names of the cities.

1	2	3	4	5	6	7	8	9	10	11	12
$\frac{4}{24}$	$\frac{28}{35}$	$\frac{100}{120}$	$\frac{5}{7}$	$\frac{21}{36}$	$\frac{1}{2}$	$\frac{22}{77}$	$\frac{2}{3}$	$4\frac{2}{5}$	$\frac{81}{90}$	$\frac{25}{3}$	$\frac{43}{9}$
$U = 8\frac{1}{3}$	$A = \frac{5}{6}$	$H = \frac{15}{21}$	$D = \frac{4}{5}$	$G = \frac{1}{6}$	$N = 4\frac{7}{9}$	$I = \frac{7}{12}$	$E = \frac{2}{7}$	$P = \frac{18}{27}$	$J = \frac{9}{10}$	$R = \frac{48}{96}$	$L = \frac{22}{5}$

2	7	9	4	5	3	1	6	3	3	12	2	10	3	5	8	11	6

2 At the end of each practice session, Coach Andy rewards his swim team by distributing 30 pieces of chocolate according to effort. Each swimmer receives a *different* number of whole pieces of chocolate. Suggest possible numbers (all different) of chocolate pieces for each swimmer attending practice when the chocolate is shared between:

a four swimmers **b** five swimmers **c** six swimmers **d** seven swimmers

3 *Forming fractions*

Make a set of cards that look like these shown below.

1	2	3	4	5	6	7	8	9

In the following questions, any two of these cards can be used to make a fraction. One card is used as the numerator and the other is used as the denominator.

a What fraction with the least value can you make?
b What fraction with the greatest value can you make?
c How many fractions can be simplified to give whole numbers?
d What is the fraction you can make with these cards that is closest to 1 but less than 1?
e What fractions can you make that are equal to 0.5?
f What fractions can you make that are equal to 75%?
g How many fractions can you make that are greater than 0.5 but less than 1?
h Use four different cards to make two fractions that add together to give 1. In how many ways can this be done?
i Use six different cards to make three fractions that add together to give 1. In how many ways can this be done?
j How many fractions can you make that are greater than 1 but less than 1.5?

4 When a \$50 item is increased by 20%, the final price is \$60. Yet, when a \$60 item is reduced by 20% the final price is not \$50. Explain.

5 The length and width of a rectangular projector screen in a small theatre is 200% more than the length and width of a television screen in the same room. How much bigger is the area of the screen than that of the television? Give your answer as a percentage.

6 Find the missing number.

$$\frac{7}{4} = 1 + \cfrac{1}{1 + \cfrac{1}{\square}}$$

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Chapter summary

Decimals

Changing decimals to fractions

$0.16 = \frac{16}{100} = \frac{4}{25}$

$2.008 = 2\frac{8}{1000} = 2\frac{1}{125}$

Changing fractions to decimals

$\frac{2}{5} = \frac{4}{10} = 0.4$

$\frac{7}{20} = \frac{35}{100} = 0.35$

Comparing decimals

$12.3 > 12.1$

$6.72 < 6.78$

$0.15 \neq 0.105$

$284.7 \leq 284.7$

Decimals as fractions

$0.184 = \frac{1}{10} + \frac{8}{100} + \frac{4}{1000}$

$= \frac{184}{1000}$

Rounding

≈ means is approximately equal to

The critical digit is circled.

2.34 | ② ⟶ 2.34

2.34 | ⑤ ⟶ 2.35

2.34 | ⑥ ⟶ 2.35

2.89 | ⑨ ⟶ 2.90

2.99 | ⑦ ⟶ 3.00

Place value of digits

0.184

1 tenth

8 hundredths

4 thousandths

Decimals and percentages

$0.63 = 63 \div 100 = 63\%$

$8\% = 8 \div 100 = 0.08$

$240\% = 240 \div 100 = 2.4$

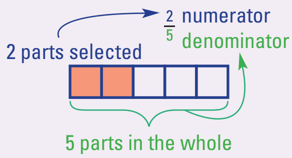
Fractions to decimals

$\frac{13}{100} = 0.13$

$\frac{3}{8} = 0.375$ $\qquad 8\overline{)3.000}$ gives 0.375

$\frac{2}{9} = 0.2222... = 0.\dot{2}$ $\qquad 9\overline{)2.000...}$ gives $0.222...$

Fractions



proper fraction $\frac{3}{4}$

improper fraction $\frac{10}{7}$

mixed numeral $5\frac{3}{4}$

mixed ⟶ improper

$2\frac{3}{5} = \frac{10}{5} + \frac{3}{5} = \frac{13}{5}$

improper ⟶ mixed

$\frac{40}{15} = 2\frac{10}{15} = 2\frac{\cancel{5} \times 2}{\cancel{5} \times 3} = 2\frac{2}{3}$

or

$\frac{40}{15} = \frac{\cancel{5} \times 8}{\cancel{5} \times 3} = \frac{8}{3} = 2\frac{2}{3}$

Equivalent fractions

$\frac{50}{100} = \frac{30}{60} = \frac{21}{42} = \frac{8}{16} = \frac{1}{2}$ (circled)

simplest form

$1\% = \frac{1}{100}$ $\qquad 50\% = \frac{50}{100} = \frac{1}{2}$

$10\% = \frac{10}{100} = \frac{1}{10}$ $\qquad 75\% = \frac{75}{100} = \frac{3}{4}$

$20\% = \frac{20}{100} = \frac{1}{5}$ $\qquad 80\% = \frac{80}{100} = \frac{4}{5}$

$25\% = \frac{25}{100} = \frac{1}{4}$ $\qquad 100\% = \frac{100}{100} = 1$

Recall

$\frac{7}{7} = 7 \div 7 = 1$

Simplify

$\frac{42}{63} = \frac{\cancel{7} \times 6}{\cancel{7} \times 9} = \frac{6}{9} = \frac{\cancel{3} \times 2}{\cancel{3} \times 3} = \frac{2}{3}$

or

$\frac{42}{63} = \frac{\cancel{21} \times 2}{\cancel{21} \times 3} = \frac{2}{3}$

HCF of 42 and 63 is 21.

Facts about fractions, decimals and percentages

$\frac{1}{2} = 0.5 = 50\%$ $\qquad \therefore \frac{3}{2} = 1.5 = 150\%$

$\frac{1}{3} = 0.\dot{3} = 33\frac{1}{3}\%$ $\qquad \therefore \frac{2}{3} = 0.\dot{6} = 66\frac{2}{3}\%$

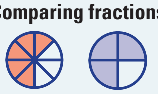
$\frac{1}{4} = 0.25 = 25\%$ $\qquad \therefore \frac{3}{4} = 0.75 = 75\%$

$\frac{1}{5} = 0.2 = 20\%$ $\qquad \therefore \frac{2}{5} = 0.4 = 40\%$

$\frac{1}{10} = 0.1 = 10\%$ $\qquad \therefore \frac{3}{10} = 0.3 = 30\%$

$\frac{1}{100} = 0.01 = 1\%$ $\qquad \therefore \frac{12}{100} = 0.12 = 12\%$

Comparing fractions



$\frac{5}{8}$? $\frac{3}{4}$

$\frac{5}{8} < \frac{6}{8}$

8 is the lowest common denominator (LCD), which is the lowest common multiple (LCM) of 4 and 8.

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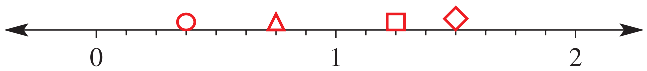
Multiple-choice questions

1 Which set of fractions corresponds to each of the different shapes positioned on the number line?



A $\frac{3}{8}, \frac{6}{8}, 1\frac{3}{8}, \frac{12}{8}$ **B** $\frac{3}{8}, \frac{3}{4}, 1\frac{1}{4}, \frac{12}{8}$ **C** $\frac{1}{2}, \frac{3}{4}, \frac{9}{8}, 1\frac{5}{8}$

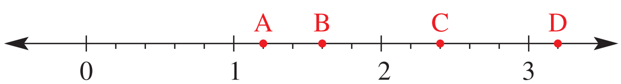
D $\frac{2}{8}, \frac{3}{4}, 1\frac{3}{8}, 1\frac{1}{2}$ **E** $\frac{3}{8}, \frac{3}{4}, 1\frac{1}{2}, \frac{14}{8}$

2 Which of the following statements is not true?

A $\frac{3}{4} = \frac{9}{12}$ **B** $\frac{6}{11} = \frac{18}{33}$ **C** $\frac{3}{10} = \frac{15}{40}$

D $\frac{13}{14} = \frac{39}{42}$ **E** $\frac{2}{7} = \frac{16}{56}$

3 Which set of mixed numerals corresponds to the letters written on the number line?



A $1\frac{1}{5}, 1\frac{3}{5}, 2\frac{2}{5}, 3\frac{1}{5}$ **B** $1\frac{2}{5}, 1\frac{3}{5}, 2\frac{3}{5}, 3\frac{1}{5}$ **C** $1\frac{1}{5}, 1\frac{2}{5}, 2\frac{2}{5}, 3\frac{2}{5}$

D $1\frac{2}{5}, 1\frac{4}{5}, 2\frac{2}{5}, 3\frac{2}{5}$ **E** $1\frac{1}{5}, 1\frac{3}{5}, 2\frac{3}{5}, 3\frac{1}{5}$

4 Which is the lowest common denominator for this set of fractions: $\frac{7}{12}, \frac{11}{15}, \frac{13}{18}$?

A 60 **B** 120 **C** 180 **D** 3240 **E** 90

5 Which of the following fraction groups is in correct descending order?

A $\frac{1}{5}, \frac{1}{3}, \frac{2}{2}$ **B** $\frac{3}{4}, \frac{3}{5}, \frac{3}{8}, \frac{3}{7}$ **C** $\frac{5}{8}, \frac{4}{5}, \frac{3}{8}, \frac{2}{3}$

D $\frac{1}{10}, \frac{1}{20}, \frac{1}{50}, \frac{1}{100}$ **E** $2\frac{1}{5}, 2\frac{8}{15}, 2\frac{2}{3}, 2\frac{3}{4}$

6 Which statement is incorrect?

A $\frac{1}{2} + \frac{1}{2} = 1$ **B** $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$ **C** $\frac{1}{2} + \frac{1}{2} = \frac{2}{4}$

D $\frac{1}{2} + \frac{1}{2} = 1.0$ **E** $\frac{1}{2} + \frac{1}{2} = \frac{4}{4}$

7 Three friends share a pizza. Kate eats $\frac{1}{4}$ of the pizza, Archie eats $\frac{1}{3}$ of the pizza and Luke eats the rest. What fraction of the pizza does Luke eat?

A $\frac{4}{12}$ **B** $\frac{2}{3}$ **C** $\frac{14}{15}$ **D** $\frac{7}{15}$ **E** $\frac{5}{12}$

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8 Which list is in correct ascending order?

A $0.68, \frac{3}{4}, 0.76, 77\%, \frac{13}{40}$

B $\frac{7}{8}, 82\%, 0.87, \frac{12}{15}, 88\%$

C $21\%, 0.02, 0.2, 0.22, \frac{22}{10}$

D $\frac{14}{40}, 0.3666, 0.3\dot{6}, 37\%, \frac{93}{250}$

E $0.76, 72\%, \frac{3}{4}, 0.68, \frac{13}{40}$

9 $\frac{60}{14}$ can be written as:

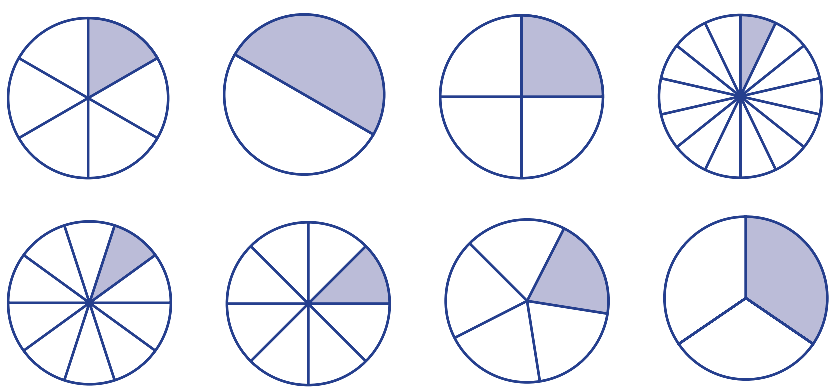
A $4\frac{2}{7}$ **B** $2\frac{4}{7}$ **C** $4\frac{2}{14}$ **D** $7\frac{4}{7}$ **E** $5\frac{1}{7}$

10 $\frac{17}{25}$ of a metre of material is needed for a school project. How many centimetres is this?

A 65 cm **B** 70 cm **C** 68 cm **D** 60 cm **E** 75 cm

Short-answer questions

1 List the shaded fractions in correct ascending order.



2 Write four fractions equivalent to $\frac{3}{5}$ and write a sentence to explain why they are equal in value.

3 Write the following fractions in simplest form.

a $\frac{18}{30}$ **b** $\frac{8}{28}$ **c** $\frac{35}{49}$

4 Convert each of the following to a mixed numeral in simplest form.

a $\frac{15}{10}$ **b** $\frac{63}{36}$ **c** $\frac{45}{27}$ **d** $\frac{56}{16}$

5 Place the correct mathematical symbol (i.e. <, = or >) in between the following pairs of fractions to make true mathematical statements.

a $\frac{2}{7} \square \frac{4}{7}$ **b** $\frac{3}{8} \square \frac{1}{8}$ **c** $1\frac{2}{3} \square 1\frac{3}{5}$ **d** $3\frac{1}{9} \square \frac{29}{9}$

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6 State the largest fraction in each list.

a $\frac{3}{7}, \frac{2}{7}, \frac{5}{7}, \frac{1}{7}$ **b** $\frac{3}{8}, \frac{2}{8}, \frac{5}{8}, \frac{1}{8}$

7 State the lowest common multiple for each pair of numbers.

a 2, 5 **b** 3, 7 **c** 8, 12

8 State the lowest common denominator for each set of fractions.

a $\frac{1}{2}, \frac{3}{5}$ **b** $\frac{2}{3}, \frac{3}{7}$ **c** $\frac{3}{8}, \frac{5}{12}$

9 Rearrange each set of fractions in descending order.

a $1\frac{3}{5}, \frac{9}{5}, 2\frac{1}{5}$ **b** $\frac{14}{8}, \frac{11}{6}, \frac{9}{4}, \frac{5}{3}$ **c** $5\frac{2}{3}, \frac{48}{9}, 5\frac{7}{18}, 5\frac{2}{9}, 5\frac{1}{3}$

10 Determine the simplest answer for each of the following.

a $\frac{3}{8} + \frac{1}{8}$ **b** $\frac{1}{3} + \frac{1}{2}$ **c** $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

d $\frac{1}{3} + \frac{1}{6}$ **e** $\frac{7}{8} - \frac{3}{8}$ **f** $\frac{1}{10} + \frac{3}{10}$

11 Copy the table into your workbook and complete.

Percentage form	36%			140%		18%
Fraction		$2\frac{1}{5}$	$\frac{5}{100}$		$\frac{11}{25}$	

12 Determine which alternative (**i** or **ii**) is the larger discount.

a **i** 25% of \$200 **ii** 20% of \$260

b **i** 5% of \$1200 **ii** 3% of \$1900

13 Express the following as both a fraction and percentage of the total.

a 6 out of 10 **b** \$4 out of 20 **c** 50 cents out of \$8 **d** 600 mL out of 2 L

14 What is the place value of the digit 3 in the following numbers?

a 12.835 **b** 6.1237 **c** 13.5104

15 Write each fraction as a decimal and percentage.

a $\frac{81}{10}$ **b** $\frac{81}{100}$ **c** $\frac{801}{100}$ **d** $\frac{801}{1000}$

16 List all the possible numbers with 3 decimal places that, when rounded to 2 decimal places, result in 45.27.

17 Write down the factors of these numbers.

a 24 **b** 32 **c** 36 **d** 64

e 100 **f** 144 **g** 72 **h** 75

18 Use your answers to Question **17** to find the highest common factor of:

a 24 and 32 **b** 32 and 36 **c** 144 and 72 **d** 75 and 100

19 Find the lowest common multiple of these number pairs.

a 4 and 6 **b** 2 and 5 **c** 3 and 6 **d** 3 and 2

e 5 and 12 **f** 4 and 2 **g** 3 and 5 **h** 2 and 6

i 4 and 8 **j** 4 and 10 **k** 5 and 10 **l** 8 and 10

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Extended-response questions

1 Copy and complete the table below.

Simplified fraction	Decimal	Percentage
		35%
$\frac{3}{5}$		
$1\frac{1}{10}$		
	2.75	
		750%
$\frac{5}{3}$		
$\frac{5}{6}$		
	0.375	

2 Of the following, for which subject did Keira obtain the best test result?

English: 17 out of 25
Mathematics: 20 out of 30
Science: 15 out of 20
History: 7 out of 10

3 Five students, Penny, Philip, Jay, Tiger and Marie, share the winnings of a $100 fête prize. They share the prize in the following way.

Penny 25%
Philip $\frac{1}{5}$
Jay 10%
Tiger 0.15
Marie ?

a Write Penny's share as a:
  i fraction
  ii decimal

b Write Tiger's share as a:
  i percentage
  ii fraction

c How much money do these students receive?
  i Philip
  ii Tiger

d Write Marie's share as:
  i a percentage
  ii a fraction
  iii a decimal
  iv an amount of money

e Philip and Marie combine their share.
  i How much is this, in dollars?
  ii Write this value as a fraction, a decimal and as a percentage.

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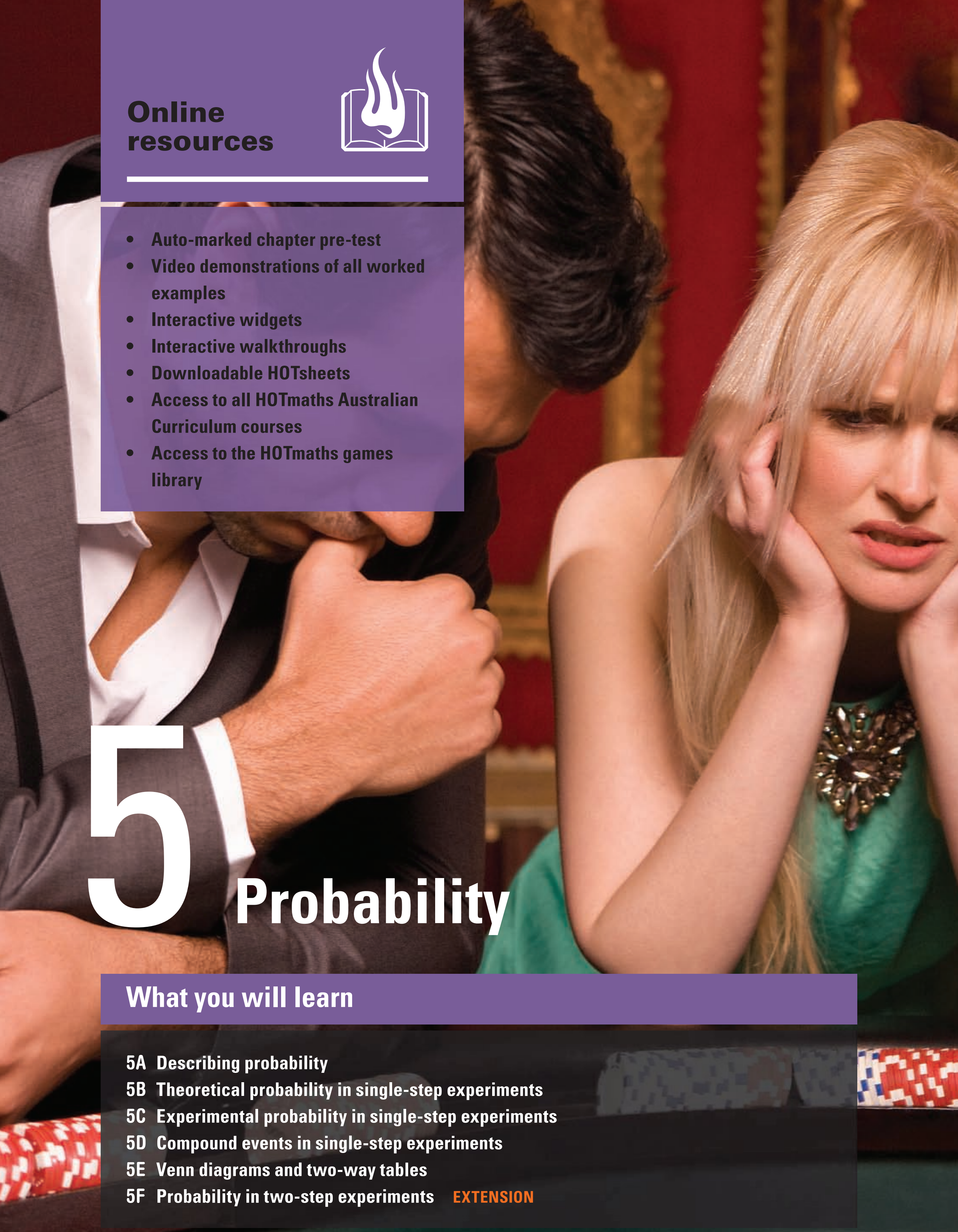

Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

5 Probability

What you will learn

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NSW syllabus

STRAND: STATISTICS AND PROBABILITY

SUBSTRAND: PROBABILITY

Outcome

A student represents probabilities of simple and compound events. (MA4–21SP)

Gambling problem or problem gambling?

Would you like to give away $4000 a year for no gain? That is what the average gaming machine player loses in NSW every year. This contributes to a total gambling loss to Australians of about $20 billion each year. The social cost of this is an extra $5 billion per year as a result of people who become addicted to gambling and become a financial burden on their families and the community.

Gambling activities include lotteries, online gaming, gaming machines, sports betting and table games. The people who invent and run these activities calculate the mathematical probabilities so that, in the long run, the players lose their money.

It is worth thinking about Probability before becoming involved in gambling activities.

Pre-test

1 Write these fractions in simplest form.

a $\frac{10}{20}$ **b** $\frac{20}{30}$

c $\frac{21}{28}$ **d** $\frac{12}{48}$

2 Consider the set of numbers 4, 2, 6, 5, 9.

a How many numbers are in the set?

b How many of the numbers are even?

c What fraction of the numbers are odd?

3 Write the following values as decimals.

a $2 \div 4$ **b** $20 \div 50$

c $12 \div 60$ **d** $11 \div 55$

4 Order these events from least likely to most likely.

A Rolling a die and it landing on the number 3.

B Flipping a coin and it landing with ‘tails’ showing.

C The Prime Minister of Australia being struck by lightning tomorrow.

D The internet being used by somebody in the next 20 minutes.

5 For each of the following events, choose A for low chance, B for even chance or C for high chance.

a Rolling a number greater than one with a six-sided die.

b Rolling a one with a six-sided die.

c Rolling an even number with a six-sided die.

d Being struck by lightning.

e Tossing a head with a coin.

6 Copy this table into your workbook and complete.

Fraction	Decimal	Percentage
$\frac{1}{2}$		
$\frac{1}{3}$		
$\frac{1}{4}$		
$\frac{1}{5}$		
$\frac{1}{10}$		
$\frac{1}{100}$		

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5A Describing probability

Often, there are times when you may wish to describe how likely it is that an event will occur. For example, you may want to know how likely it is that it will rain tomorrow, or how likely it is that your sporting team will win this year's premiership, or how likely it is that you will win a lottery. Probability is the study of chance.

Let's start: Likely or unlikely?

Try to rank these events from least likely to most likely. Compare your answers with other students in the class and discuss any differences.

- It will rain tomorrow.
- Australia will win the soccer World Cup.
- Tails landing uppermost when a 20-cent coin is tossed.
- The sun will rise tomorrow.
- The king of spades is at the top of a shuffled deck of 52 playing cards.
- A diamond card is at the bottom of a shuffled deck of 52 playing cards.

Key ideas

This topic involves the use of sophisticated terminology.

Terminology	Example	Definition
chance experiment	rolling a fair 6-sided die	A chance experiment is an activity that may produce a variety of different results which occur randomly. The example given is a single-step experiment.
trials	rolling a die 50 times	When an experiment is performed one or more times, each occurrence is called a trial. The example given indicates 50 trials of a single-step experiment.
outcome	rolling a 5	An outcome is one of the possible results of a chance experiment.
equally likely outcomes	rolling a 5 rolling a 6	Equally likely outcomes are two or more results that have the same chance of occurring.
sample space	{1, 2, 3, 4, 5, 6}	The sample space is the set of all possible outcomes of an experiment. It is usually written inside braces, as shown in the example.
event	e.g. 1: rolling a 2 e.g. 2: rolling an even number	An event is either one outcome or a collection of outcomes. It is a subset of the sample space.

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Key ideas

Terminology	Example	Definition
compound event	rolling an even number	A compound event is a collection of two or more outcomes from the sample space of a chance experiment.
mutually exclusive events	rolling a 5 rolling an even number	Two or more events are mutually exclusive if they share no outcomes.
non-mutually exclusive events	rolling a 5 rolling an odd number	Events are non-mutually exclusive if they share one or more outcomes. In the given example, the outcome 5 is shared.
comple-mentary events	rolling a 2 or 3 rolling a 1, 4, 5 or 6	If *all* the outcomes in the sample space are divided into two events, they are complementary events.
complement	Rolling 2, 3, 4 or 5 is an event. Rolling a 1 or 6 is the complement.	If an experiment was performed and an event did not occur, then the complement definitely occurred.
favourable outcome(s)	In some games, you must roll a 6 before you can start moving your pieces.	Outcomes are favourable if they are part of some desired event.
theoretical probability or likelihood or chance	The probability of rolling an even number is written as: $P(\text{even}) = \frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$ Probabilities can be expressed as fractions, decimals and percentages.	Theoretical probability is the actual chance or likelihood that an event will occur when an experiment takes place. $P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$ Probabilities range from 0 to 1 or 0% to 100%.
experimental probability	A die is rolled 600 times and shows a 5 on 99 occasions. The experimental probability of rolling a 5 on this die is: $P(5) \approx \frac{99}{600} = 0.165 = 16.5\%$	Sometimes it is difficult or impossible to calculate a theoretical probability, so an estimate can be found using a large number of trials. This is called the experimental probability. If the number of trials is large, the experimental probability should be very close to that of the theoretical.
certain likely even chance unlikely impossible	rolling a number less than 7 rolling a number less than 6 rolling a 1, 2 or 3 rolling a 2 rolling a 7	The probability is 100% or 1. The probability is 50% or 0.5 or $\frac{1}{2}$. The probability is 0% or 0.

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Key ideas

Terminology	Example	Definition
the sum of all probabilities in an experiment	$P(1)=\frac{1}{6}$ $P(2)=\frac{1}{6}$ $P(3)=\frac{1}{6}$ $P(4)=\frac{1}{6}$ $P(5)=\frac{1}{6}$ $P(6)=\frac{1}{6}$ $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{6}{6}=1=100\%$	The sum of the probabilities of all the outcomes of a chance experiment is 1 (or 100%).
the sum of the probabilities of an event and its complement	$P(\text{rolling 1 or 6})=\frac{2}{6}$ $P(\text{rolling 2, 3, 4 or 5})=\frac{4}{6}$ $\frac{2}{6}+\frac{4}{6}=\frac{6}{6}=1=100\%$	The sum of the probabilities of an event and its complement is 1 (or 100%). $P(\text{event})+P(\text{complementary event})=1$

Example 1 Describing chance

Classify each of the following statements as either true or false.

a It is likely that children will go to school next year.
b It is an even chance for a fair coin to display tails.
c Rolling a 3 on a 6-sided die and getting heads on a coin are equally likely.
d It is certain that two randomly chosen odd numbers will add to an even number.

SOLUTION	EXPLANATION
a true	Although there is perhaps a small chance that the laws might change, it is (very) likely that children will go to school next year.
b true	There is a 50 – 50, or an even chance, of a fair coin displaying tails. It will happen, on average, half of the time.
c false	These events are not equally likely. It is more likely to flip heads on a coin than to roll a 3 on a 6-sided die.
d true	No matter what odd numbers are chosen, they will always add to an even number.

Exercise 5A

UNDERSTANDING AND FLUENCY	1–4	2–5	3–5

1 Match each of the events **a** to **d** with a description of how likely they are to occur (**A** to **D**).

a A tossed coin landing heads up.
b Selecting an ace first try from a fair deck of 52 playing cards.
c Obtaining a number other than 6 if a fair 6-sided die is rolled.
d Obtaining a number greater than 8 if a fair 6-sided die is rolled.

A unlikely
B likely
C impossible
D even chance

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2 Fill in the blanks, using the words *unlikely*, *even*, *impossible*, *certain* and *chance*.

a If an event is guaranteed to occur, we say it is __________.

b An event that is equally likely to occur or not occur has an __________ __________.

c A rare event is considered __________.

d An event that will never occur is called __________.

Example 1

3 Consider a fair 6-sided die with the numbers 1 to 6 on it. Answer true or false to each of the following.

a Rolling a 3 is unlikely.

b Rolling a 5 is likely.

c Rolling a 4 and rolling a 5 are equally likely events.

d Rolling an even number is likely.

e There is an even chance of rolling an odd number.

f There is an even chance of rolling a multiple of 3.

4 Match up each of the events **a** to **d** with an equally likely event **A** to **D**.

a rolling a 2 on a 6-sided die

b selecting a heart card from a fair deck of 52 playing cards

c flipping a coin and tails landing face up

d rolling a 1 or a 5 on a 6-sided die

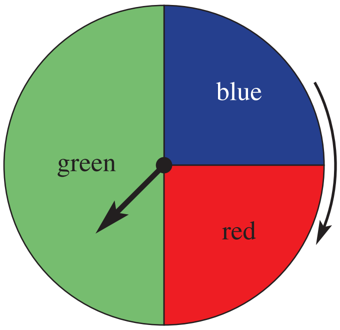
A selecting a black card from a fair deck of 52 playing cards

B rolling a number bigger than 4 on a 6-sided die

C selecting a diamond card from a fair deck of 52 playing cards

D rolling a 6 on a 6-sided die

5 Consider the spinner shown, which is spun and could land with the arrow pointing to any of the three colours. (If it lands on a boundary, it is re-spun until it lands on a colour.)



a State whether each of the following is true or false.

i There is an even chance that the spinner will point to green.

ii It is likely that the spinner will point to red.

iii It is certain that the spinner will point to purple.

iv It is equally likely that the spinner will point to red or blue.

v Green is twice as likely as blue.

b Use the spinner to give an example of:

i an impossible event

ii a likely event

iii a certain event

iv two events that are equally likely

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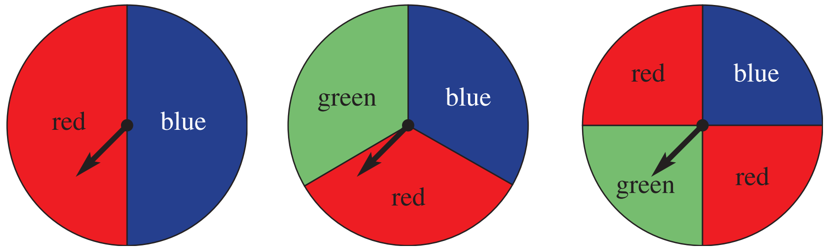
PROBLEM-SOLVING AND REASONING	6, 10	7, 8, 10	8–10

6 Three spinners are shown below. Match each spinner with the description.



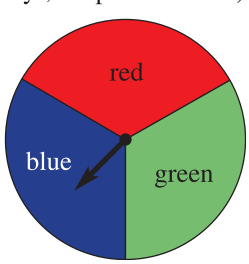
spinner 1 spinner 2 spinner 3

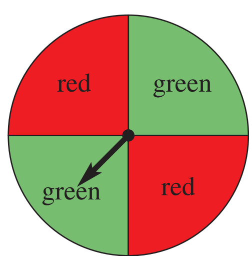
a Has an even chance of red, but blue is unlikely.
b Blue and green are equally likely, but red is unlikely.
c Has an even chance of blue, and green is impossible.

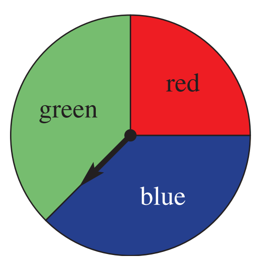
7 Explain why in Question **6** red is twice as likely to occur as blue in spinner 3 but equally likely to occur in spinner 2 even though both spinners have equally-sized sectors.

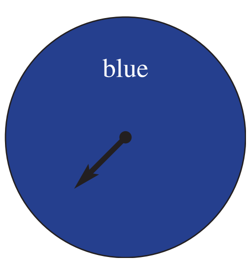
8 Draw spinners to match each of the following descriptions, using blue, red and green as the possible colours.
a Blue is likely, red is unlikely and green is impossible.
b Red is certain.
c Blue has an even chance, red and green are equally likely.
d Blue, red and green are all equally likely.
e Blue is twice as likely as red, but red and green are equally likely.
f Red and green are equally likely and blue is impossible.
g Blue, red and green are all unlikely, but no two colours are equally likely.
h Blue is three times as likely as green, but red is impossible.

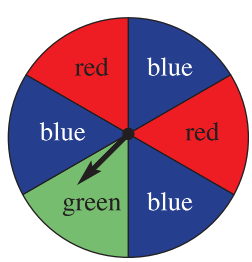
9 For each of the following spinners, give a description of the chances involved so that someone could determine which spinner is being described. Use the colour names and the language of chance (i.e. 'likely', 'impossible' etc.) in your descriptions.

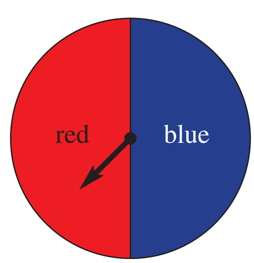
a


b


c


d


e


f


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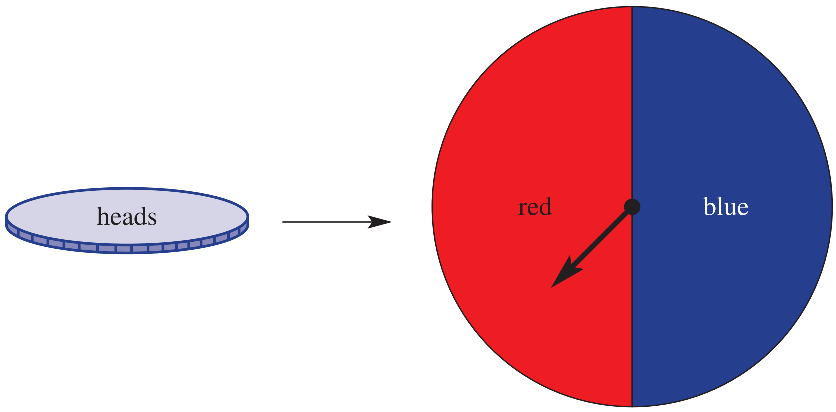
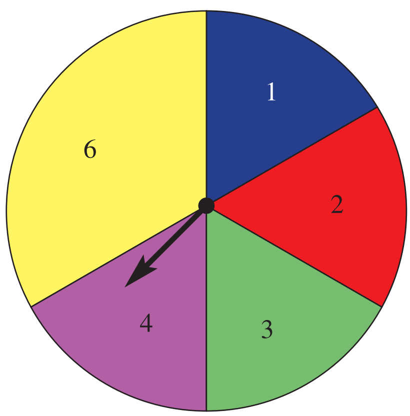
10 A coin consists of two sides that are equally likely to occur when tossed. It is matched up with a spinner that has exactly the same chances, as shown below.



Tossing the coin with heads landing uppermost is equally likely to spinning red on the spinner. Tossing the coin with tails landing uppermost is equally likely to spinning blue on the spinner. Hence, we say that the coin and the spinner are **equivalent**.



a Draw a spinner that is equivalent to a fair 6-sided die. (Hint: The spinner should have six sections of different colours.)

b How can you tell from the spinner you have drawn that it is equivalent to a fair die?

c A die is 'weighted' so that there is an even chance of rolling a 6, but rolling the numbers 1 to 5 are still equally likely. Draw a spinner that is equivalent to such a die.

d How could you make a die equivalent to the spinner shown in the diagram?

e Describe a spinner that is equivalent to selecting a card from a fair deck of 52 playing cards.

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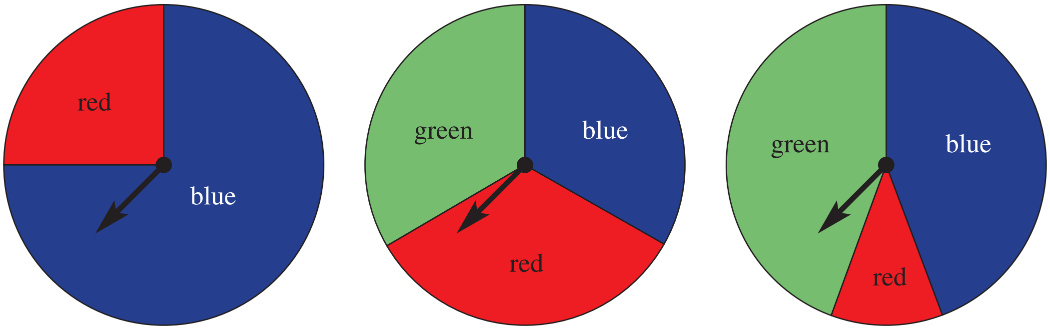
ENRICHMENT	—	—	11

Spinner proportions

11 The language of chance is a bit vague. For example, for each of the following spinners it is 'unlikely' that you will spin red, but in each case the chance of spinning red is different.



spinner 1 spinner 2 spinner 3

Rather than describing this in words, we could give the fraction (or decimal or percentage) of the spinner occupied by a colour.

a For each of the spinners above, give the fraction of the spinner occupied by red.

b What fraction of the spinner would be red if it has an even chance?

c Draw spinners for which the red portion occupies:
- **i** 100% of the spinner
- **ii** 0% of the spinner

d For the sentences below, fill in the gaps with appropriate fraction or percentage values.
- **i** An event has an even chance of occurring if that portion of the spinner occupies __________ of the total area.
- **ii** An event that is impossible occupies __________ of the total area.
- **iii** An event is unlikely to occur if it occupies more than __________ but less than __________ of the total area.
- **iv** An event is likely if it occupies more than __________ of the total area.

e How can the fractions help determine if two events are equally likely?

f Explain why all the fractions occupied by a colour must be between 0 and 1.

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5B Theoretical probability in single-step experiments

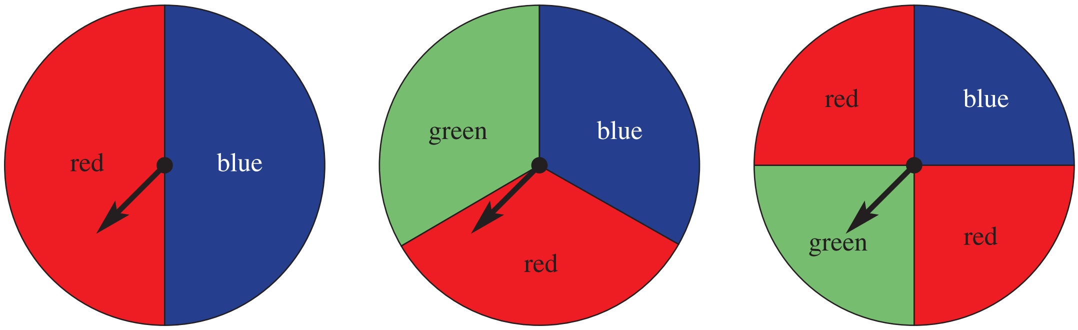
Interactive
Widgets
HOTsheets
Walkthrough

The **probability** of an event occurring is a number between 0 and 1. This number states precisely how likely it is for an event to occur. It is often written as a fraction and can indicate how frequently the event would occur over a large number of trials. For example, if you toss a fair coin many times, you would expect heads to come up half the time, so the probability is $\frac{1}{2}$. If you roll a fair 6-sided die many times, you should roll a 4 about one-sixth of the time, so the probability is $\frac{1}{6}$.

To be more precise, we should list the possible outcomes of rolling the die: 1, 2, 3, 4, 5, 6. Doing this shows us that there is a 1 out of 6 chance that you will roll a 4 and there is a 0 out of 6 $(=0)$ chance of rolling a 9.

Let's start: Spinner probabilities

Consider the three spinners shown below.



- What is the probability of spinning blue for each of these spinners?
- What is the probability of spinning red for each of these spinners?
- Try to design a spinner for which the probability of spinning green is $\frac{4}{7}$ and the probability of spinning blue is 0.

Key ideas

- Many key ideas relevant to this section can be found in the list of terminology in section 5A.
- Some examples of single-step experiments are:
  - tossing a coin once
  - spinning a spinner once
  - rolling a die once
  - choosing one prize in a raffle
  - choosing one card from a deck of playing cards.

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Key ideas

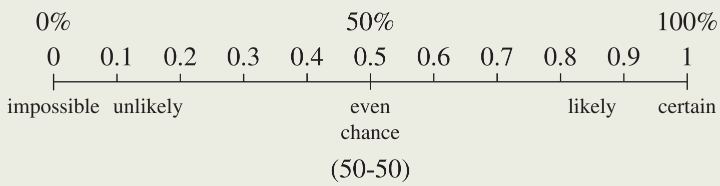
- Theoretical probability is the actual chance or likelihood that an event will occur when an experiment takes place.

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

For example: The chance of rolling a fair die once and getting a 2.

$$P(\text{rolling a 2}) = \frac{1}{6}$$

- Probabilities can be expressed as:
  - fractions or decimals between 0 and 1
  - percentages between 0% and 100%



Example 2 Calculating probability

A fair 6-sided die is rolled.

a List the sample space.

b Find the probability of rolling a 3, giving your answer as a fraction.

c Find the probability of rolling an even number, giving your answer as a decimal.

d Find the probability of rolling a number less than 3, giving your answer as a percentage.

SOLUTION	EXPLANATION
a Sample space = {1, 2, 3, 4, 5, 6}	For the sample space, we list all the possible outcomes. Technically, the sample space is {roll a 1, roll a 2, roll a 3, roll a 4, roll a 5, roll a 6}, but we do not usually include the additional words.
b $P(3) = \frac{1}{6}$	The event can occur in one way (rolling a 3) out of six possible outcomes.
c $P(\text{even}) = \frac{1}{2} = 0.5$	The event can occur in three ways (i.e. 2, 4 or 6). So the probability is $\frac{3}{6} = \frac{1}{2}$ or 0.5
d $P(\text{less than } 3) = \frac{1}{3} = 0.\dot{3}$ $= 33\frac{1}{3}\%$	The event can occur in two ways (1 or 2). So the probability is $\frac{2}{6} = \frac{1}{3}$ or $0.\dot{3}$ or $33\frac{1}{3}\%$. place.

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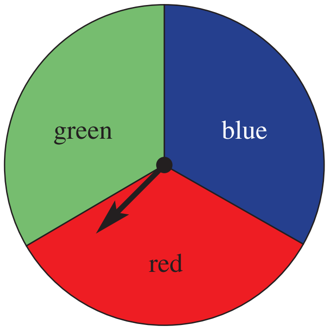
Exercise 5B

UNDERSTANDING AND FLUENCY	1–6	3–7	4–7

1 Match up each event **a** to **d** with the set of possible outcomes **A** to **D**.



- **a** tossing a coin
- **b** rolling a die
- **c** selecting a suit from a fair deck of 52 playing cards
- **d** spinning the spinner shown at right

- **A** {1, 2, 3, 4, 5, 6}
- **B** {red, green, blue}
- **C** {heads, tails}
- **D** {hearts, diamonds, clubs, spades}

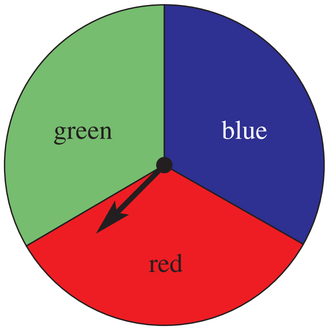
2 Complete the following sentences.
- **a** The _________ _________ is the set of possible outcomes.
- **b** An impossible event has a probability of _________.
- **c** If an event has a probability of 1, then it is _________.
- **d** The higher its probability, the _________ likely the event will occur.
- **e** An event with a probability of $\frac{1}{2}$ has an _________ _________ of occurring.

Example 2a

3 Consider a fair 6-sided die.
- **a** List the sample space.
- **b** List the odd numbers on the die.
- **c** State the probability of throwing an even number.

Example 2b–d

4 Consider the spinner shown.



- **a** How many outcomes are there? List them.
- **b** Find P(red); i.e. find the probability of the spinner pointing to red.
- **c** Find P(red or green).
- **d** Find P(not red).
- **e** Find P(yellow).

5 A spinner with the numbers 1 to 7 is spun. The numbers are evenly spaced.

- **a** List the sample space.
- **b** Find $P(6)$.
- **c** Find $P(8)$.
- **d** Find P(2 or 4).
- **e** Find P(even).
- **f** Find P(odd).
- **g** Give an example of an event having the probability of 1.

6 The letters in the word MATHS are written on five cards and then one is drawn from a hat.
- **a** List the sample space.
- **b** Find P(T), giving your answer as a decimal.
- **c** Find P(consonant is chosen), giving your answer as a decimal.
- **d** Find the probability that the letter drawn is also in the word TAME, giving your answer as a percentage.

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7 The letters in the word PROBABILITY are written on 11 cards and then one is drawn from a hat.

a Find $P(\text{P})$.

b Find $P(\text{P or L})$.

c Find $P(\text{letter chosen is in the word BIT})$.

d Find $P(\text{not a B})$.

e Find $P(\text{a vowel is chosen})$.

f Give an example of an event with the probability of $\frac{3}{11}$.

PROBLEM-SOLVING AND REASONING	8, 11	8, 9, 11	9–12

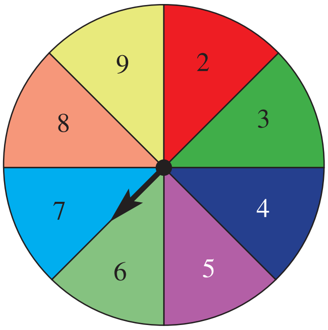
8 A bag of marbles contains 3 red marbles, 2 green marbles and 5 blue marbles. They are all equal in size and weight. A marble is chosen at random.

a What is the probability that a red marble is chosen? (Hint: It is not $\frac{1}{3}$ because the colours are not all equally likely.) Give your answer as a percentage.

b What is the probability that a blue marble is chosen? Give your answer as a percentage.

c What is the probability that a green marble is *not* chosen? Give your answer as a percentage.

9 Consider the spinner opposite, numbered 2 to 9.



a List the sample space.

b A number is prime if it has exactly two factors. Therefore, 5 is a prime number but 6 is not. Find the probability that a prime number will be spun, giving your answer as a decimal. (Remember that 2 is a prime number.)

c Giving your answers as decimals, state the probability of getting a prime number if each number in the spinner opposite is:

i increased by 1

ii increased by 2

iii doubled

(Hint: It will help if you draw the new spinner.)

d Design a new spinner for which the $P(\text{prime}) = 1$.

10 A bag contains various coloured marbles – some are red, some are blue, some are yellow and some are green. You are told that $P(\text{red}) = \frac{1}{2}$, $P(\text{blue}) = \frac{1}{4}$ and $P(\text{yellow}) = \frac{1}{6}$. You are not told the probability of selecting a green marble.

a If there are 24 marbles:

i Find how many marbles there are of each colour.

ii What is the probability of getting a green marble?

b If there are 36 marbles:

i Find how many marbles there are of each colour.

ii What is the probability of getting a green marble?

c What is the minimum number of marbles in the bag?

d Does the probability of getting a green marble depend on the actual number of marbles in the bag? Justify your answer.

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11 **a** State the values of the letters in the following table.

Event	P (event occurs)	P (event does not occur)	Sum of two numbers
rolling a die, get a 3	$\frac{1}{6}$	$\frac{5}{6}$	a
tossing a coin, get H	$\frac{1}{2}$	b	c
rolling a die, get 2 or 5	d	$\frac{2}{3}$	e
selecting letter from 'HEART', get a vowel	f	g	h

b If the probability of selecting a vowel in a particular word is $\frac{3}{13}$, what is the probability of selecting a consonant?

c If the probability of spinning blue with a particular spinner is $\frac{4}{7}$, what is the probability of spinning a colour other than blue?

12 A box contains different coloured counters, with $P(\text{purple}) = 10\%$, $P(\text{yellow}) = 0.\dot{6}$ and $P(\text{orange}) = \frac{1}{7}$.

a Is it possible to obtain a colour other than purple, yellow or orange? If so, state the probability.

b What is the minimum number of counters in the box?

c If the box cannot fit more than 1000 counters, what is the maximum number of counters in the box?

ENRICHMENT	—	—	13

Designing spinners

13 For each of the following, design a spinner using only red, green and blue sectors to obtain the desired probabilities. If it cannot be done, then explain why.

a $P(\text{red}) = \frac{1}{2}, P(\text{green}) = \frac{1}{4}, P(\text{blue}) = \frac{1}{4}$

b $P(\text{red}) = \frac{1}{2}, P(\text{green}) = \frac{1}{2}, P(\text{blue}) = \frac{1}{2}$

c $P(\text{red}) = \frac{1}{4}, P(\text{green}) = \frac{1}{4}, P(\text{blue}) = \frac{1}{4}$

d $P(\text{red}) = 0.1, P(\text{green}) = 0.6, P(\text{blue}) = 0.3$

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5C Experimental probability in single-step experiments

Interactive
Widgets
HOTsheets
Walkthrough

Although the probability of an event tells us how often an event should happen in theory, we will rarely find this being exactly right in practice. For instance, if you toss a coin 100 times, it might come up heads 53 times out of 100, which is not exactly $\frac{1}{2}$ of the times you tossed it. Sometimes we will not be able to find the exact probability of an event, but we can carry out an experiment to estimate it.

Let's start: Tossing coins

For this experiment, each class member needs a fair coin that they can toss.

- Each student should toss the coin 20 times and count how many times heads occurs.
- Tally the total number of heads obtained by the class.
- How close is this total number to the number you would expect that is based on the probability of $\frac{1}{2}$? Discuss what this means.

Key ideas

- The **experimental probability** of an event occurring based on a particular experiment is defined as:

$$\frac{\text{number of times the event occurs}}{\text{total number of trials in the experiment}}$$

- The **expected number** of occurrences = probability × number of trials.
- If the number of trials is large, then the experimental probability should be close to the theoretical probability of an event.

Example 3 Working with experimental probability

When playing with a spinner with the numbers 1 to 4 on it, the following numbers come up:
1, 4, 1, 3, 3, 1, 4, 3, 2, 3.

a What is the experimental probability of getting a 3?
b What is the experimental probability of getting an even number?
c Based on this experiment, how many times would you expect to get a 3 if you spin 1000 times?

SOLUTION	EXPLANATION
a $\frac{2}{5}$ or 0.4 or 40%	$\frac{\text{number of 3s}}{\text{number of trials}} = \frac{4}{10} = \frac{2}{5}$
b $\frac{3}{10}$	$\frac{\text{number of times with even result}}{\text{number of trials}} = \frac{3}{10}$
c 400 times	$\text{probability} \times \text{number trials} = \frac{2}{5} \times 1000 = 400$

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Exercise 5C

UNDERSTANDING AND FLUENCY	1–5, 7	3–8	4–8

Example 3a,b

1 A 6-sided die is rolled 10 times and the following numbers come up: 2, 4, 6, 4, 5, 1, 6, 4, 4, 3.

a What is the experimental probability of getting a 3?

b What is the experimental probability of getting a 4?

c What is the experimental probability of getting an odd number?

d Is the statement 'rolling an even number and rolling a 5 are complementary events' true or false?

2 When a coin is tossed 100 times, the results are 53 heads and 47 tails.

a What is the experimental probability of getting a head?

b What is the experimental probability of getting a tail?

c What is the theoretical probability of getting a tail if the coin is fair?

d If 'tossing a head' is an event, what is the complementary event?

3 A survey is conducted on people's television viewing habits.

Number of hours per week	0–5	5–10	10–20	20–30	30+
Number of people	20	10	15	5	0

a How many people participated in the survey?

b What is the probability that a randomly selected participant watches less than 5 hours of television?

c What is the probability that a randomly selected participant watches 20–30 hours of television?

d What is the probability that a randomly selected participant watches between 5 and 20 hours of television?

e Based on this survey, the experimental probability of watching 30+ hours of television is 0. Does this mean that watching 30+ hours is impossible?

Example 3c

4 A fair coin is tossed.

a How many times would you expect it to show tails in 1000 trials?

b How many times would you expect it to show heads in 3500 trials?

c Initially, you toss the coin 10 times to find the probability of the coin showing tails.

i Explain how you could get an experimental probability of 0.7.

ii If you toss the coin 100 times, are you more or less likely to get an experimental probability close to 0.5?

5 A fair 6-sided die is rolled.

a How many times would you expect to get a 3 in 600 trials?

b How many times would you expect to get an even number in 600 trials?

c If you roll the die 600 times, is it possible that you will get an even number 400 times?

d Are you more likely to obtain an experimental probability of 100% from two throws or to obtain an experimental probability of 100% from 10 throws?

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6 Each time a basketball player takes a free throw there is a 4 in 6 chance that the shot will go in. This can be simulated by rolling a 6-sided die and using numbers 1 to 4 to represent 'shot goes in' and numbers 5 and 6 to represent 'shot misses'.

a Use a 6-sided die over 10 trials to find the experimental probability that the shot goes in.

b Use a 6-sided die over 50 trials to find the experimental probability that the shot goes in.

c Working with a group, use a 6 -sided die over 100 trials to find the experimental probability that the shot goes in.

d Use a 6-sided die over just one trial to find the experimental probability that the shot goes in. (Your answer should be either 0 or 1.)

e Which of the answers to parts **a** to **d** above is closest to the theoretical probability of 66.67%? Justify your answer.

f Is this statement true or false? 'Shot goes in' and 'shot misses' are complementary events.

7 The colour of the cars in a school car park is recorded.

Colour	red	black	white	blue	purple	green
Number of cars	21	24	25	20	3	7

Based on this sample:

a What is the probability that a randomly chosen car is white?

b What is the probability that a randomly chosen car is purple?

c What is the probability that a randomly chosen car is green or black?

d How many purple cars would you expect to see in a shopping centre car park with 2000 cars?

e If 'red or black' is an event, what is the complementary event?

8 The number of children in some families is recorded in the table shown.

Number of children	0	1	2	3	4
Number of families	5	20	32	10	3

a How many families have no children?

b How many families have an even number of children?

c How many families participated in the survey?

d Based on this experiment, what is the probability that a randomly selected family has 1 or 2 children?

e Based on this experiment, what is the probability that a randomly selected family has an even number of children?

f What is the total number of *children* considered in this survey?

g If 'no children' is an event, what is the complementary event?

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PROBLEM-SOLVING AND REASONING	9, 11	9–11	10–12

9 A handful of 10 marbles of different colours is placed into a bag. A marble is selected at random, its colour recorded and then returned to the bag. The results are presented in the table.

Red marble chosen	Green marble chosen	Blue marble chosen
21	32	47

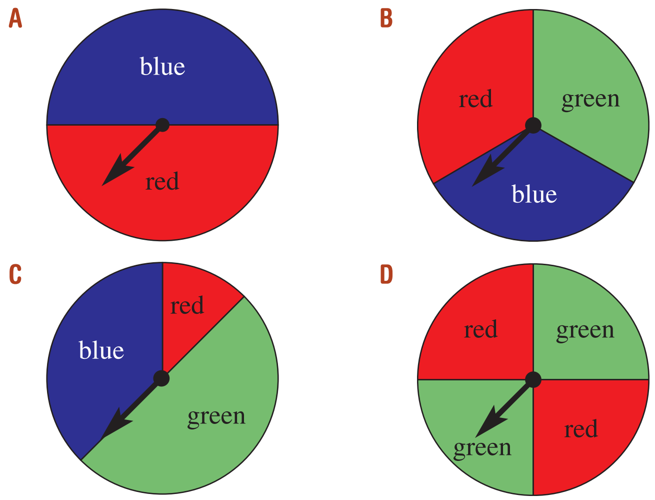
a Based on this experiment, how many marbles of each colour do you think there are? Justify your answer in a sentence.

b For each of the following, state whether or not they are possible outcomes for the 10 marbles.

i 3 red, 3 green, 4 blue
ii 2 red, 4 green, 4 blue
iii 1 red, 3 green, 6 blue
iv 2 red, 3 green, 4 blue, 1 purple
v 2 red, 0 green, 8 blue

10 Match each of the experiment results **a** to **d** with the most likely spinner that was used (**A** to **D**).

	red	green	blue
a	18	52	30
b	27	23	0
c	20	23	27
d	47	0	53



11 Assume that any baby has a 50% chance of being a boy or a girl, and use a coin to simulate a family with four children. Toss the coin four times, using heads to represent boys and tails to represent girls. Count the number of girls in the family. Repeat this experiment 20 times and present your results in a table like the one below.

Number of girls	0	1	2	3	4	Total
Number of families						20

a Based on your simulation, what is the experimental probability that a family will have just one girl?

b Based on your simulation, what is the experimental probability that a family will have four girls?

c Explain why you might need to use simulations and experimental probabilities to find the answers to parts **a** and **b** above.

d If you had repeated the experiment only 5 times instead of 20 times, how might the accuracy of your probabilities be affected?

e If you had repeated the experiment 500 times instead of 20 times, how might the accuracy of your probabilities be affected?

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12 Classify the following statements as true or false. Justify each answer in a sentence.

a If the probability of an event is $\frac{1}{2}$, then it must have an experimental probability of $\frac{1}{2}$.

b If the experimental probability of an event is $\frac{1}{2}$, then it must have a theoretical probability of $\frac{1}{2}$.

c If the experimental probability of an event is 0, then the theoretical probability is 0.

d If the probability of an event is 0, then the experimental probability is also 0.

e If the experimental probability is 1, then the theoretical probability is 1.

f If the probability of an event is 1, then the experimental probability is 1.

ENRICHMENT	—	—	13

Improving estimates

13 A spinner is spun 500 times. The table below shows the tally for every 100 trials.

	red	green	blue
First set of 100 trials	22	41	37
Second set of 100 trials	21	41	38
Third set of 100 trials	27	39	34
Fourth set of 100 trials	25	46	29
Fifth set of 100 trials	30	44	26

a Give the best possible estimate for $P(\text{red})$, $P(\text{green})$ and $P(\text{blue})$ based on these trials.

b If your estimate is based on just one set of trials, which one would cause you to have the most inaccurate results?

c Design a spinner that could give results similar to those in the table. Assume you can use up to 10 sectors of equal size.

d Design a spinner that could give results similar to those in the table if you are allowed to use sectors of different sizes.

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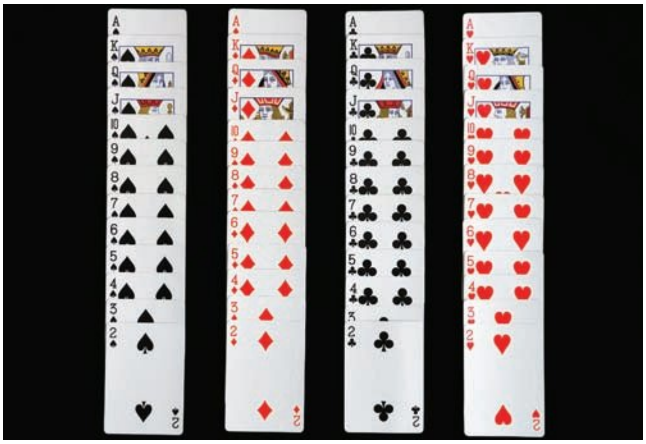
5D Compound events in single-step experiments

When solving probability problems, it is important to read the question very carefully, especially when dealing with compound events. Terminology such as *at least* and *more than* may seem the same but they are not. Even simple words like *and*, *or* and *not* require your careful attention.

Let's start: What is in a standard deck of 52 playing cards?

Do you know what is in a deck of cards?

- When and where was this standard deck of cards first used?
- How many cards are there in a standard deck? Why that number?
- How many cards are red? How many cards are black? How many cards are aces?
- What are 'suits'? How many are there and what are they called?
- How many cards are there in each suit?
- What are 'court cards'? How many are there?
- What are 'jokers'? What are some card games that involve the use of the jokers?
- Why is the first card in every suit called an ace, not a 1?
- Are the decks of cards used in other countries different from this one?
- In how many ways can you choose a card that is red *or* a 7? Is it 26, 28 or 30?



Key ideas

Some of the following key ideas are repeated from earlier pages and some are new.
In the following table, an ace = 1, jack = 11, queen = 12 and a king = 13, but this is not the case in every card game.

Terminology	Example	Definition/Explanation
chance experiment	randomly choosing one card from a standard deck	A chance experiment is an activity which may produce a variety of different results that occur randomly. The example given is a single-step experiment.
event	e.g. 1: choosing the 5 of clubs e.g. 2: choosing a 5	An event is either one outcome or a collection of outcomes. It is a subset of the sample space.

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Key ideas

Terminology	Example	Definition/Explanation
compound event	choosing a court card	A compound event is a collection of two or more outcomes from the sample space of a chance experiment.
mutually exclusive events	choosing a 5 choosing a 6	Two or more events are mutually exclusive if they share no outcomes.
non-mutually exclusive events	choosing a 5 choosing a red card	Events are non-mutually exclusive if they share one or more outcomes. In the example, there are four cards numbered 5, of which two are also red.
'more than' or 'greater than'	choosing a card greater than 10	In this example, the cards numbered 10 are *not* included. There are 12 cards in this compound event.
'at least' or 'greater than or equal to'	choosing a 10 at least	In this example, the cards numbered 10 *are* included. There are 16 cards in this compound event.
'less than'	choosing a card less than 10	In this example, the cards numbered 10 are *not* included. There are 36 cards in this compound event.
'at most' or 'less than or equal to'	choosing a 10 at most	In this example, the cards numbered 10 *are* included. There are 40 cards in this compound event.
'not'	choosing a 10 that is not red	There are four cards numbered 10. Only two of them are not red. There are two cards in this compound event.
exclusive 'or'	choosing a card that is either red or a 10, but not both	There are 26 red cards. There are four cards numbered 10 but two of them are also red. There are 26 cards in this compound event.
inclusive 'or'	choosing a card that is red or a 10 or both	There are 26 cards that are red. There are two black cards that show 10. There are 28 cards in this compound event.
'and'	choosing a card that is red and a 10	There are 26 red cards but only two of them are numbered 10. There are two cards in this compound event.

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Example 4 Choosing one card from a standard deck

One card is chosen randomly from a standard deck of cards. What is the probability that it is:

a red? **b** not red? **c** a club?
d not a club? **e** a 7? **f** neither a 7 nor 8?
g a red ace? **h** a red card or an ace? **i** a red card that is not an ace?

SOLUTION	EXPLANATION
a $P(\text{red}) = \frac{26}{52} = \frac{1}{2}$	There are 52 cards in the deck of which 26 are red.
b $P(\text{not red}) = 1 - \frac{1}{2} = \frac{1}{2}$	Red and not red are complementary events.
c $P(\text{club}) = \frac{13}{52} = \frac{1}{4}$	There are 52 cards in the deck of which 13 are clubs.
d $P(\text{not a club}) = 1 - \frac{1}{4} = \frac{3}{4}$	Club and not club are complementary events.
e $P(7) = \frac{4}{52} = \frac{1}{13}$	There are 52 cards in the deck of which four show a 7.
f $P(\text{neither a 7 nor 8}) = 1 - \frac{8}{52} = \frac{44}{52} = \frac{11}{13}$	There are 52 cards in the deck of which eight show a 7 or 8. That leaves 44 cards that do not show a 7 or 8.
g $P(\text{red ace}) = \frac{2}{52} = \frac{1}{26}$	There are four aces but only two of them are red.
h $P(\text{red or ace}) = \frac{28}{52} = \frac{7}{13}$	There are 26 red cards, including two red aces. There are also two black aces.
i $P(\text{red but not ace}) = \frac{24}{52} = \frac{6}{13}$	There are 26 red cards, including two red aces. So there are only 24 red cards that are not aces.

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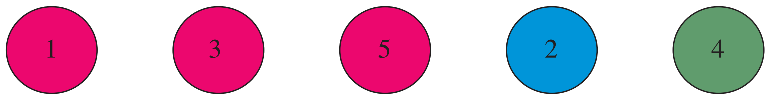
Exercise 5D

UNDERSTANDING AND FLUENCY	1–3	1–3	2, 3

1 Consider the following chance experiment.
These discs are identical except for their colour and their number.



They are placed in a bag and shaken. One disc is chosen randomly from the bag.

Use the terminology given in the first column of the table on page 222–223 in this chapter to fill in the blanks. You may use some of the terminology more than once.

- **a** ‘Choosing a blue disc’ is an example of an e _ _ _ t or o _ _ _ _ _ e.
- **b** ‘Choosing a blue disc’ and ‘choosing a green disc’ are e _ _ _ _ _ y l _ _ _ _ y o _ _ _ _ _ _ s. They are also m _ _ _ _ _ _ _ e _ _ _ _ _ _ _ _ _ e _ _ _ _ _.
- **c** The p _ _ _ _ _ _ _ _ _ _ y of ‘choosing a red disc’ is 60%.
- **d** The chosen number will be a _ _ _ _ _ t one.
- **e** It is c_ _ _ _ _ _ that the chosen number will be less than 6.
- **f** It is c_ _ _ _ _ _ that the chosen disc will be red or even.
- **g** The probability of ‘choosing a number 1 _ _ _ t _ _ _ 5’ is 80%.

2 Complete the following, using the experiment in Question **1**. Give your answers as percentages.
What is the probability that the disc:

- **a** is red or blue?
- **b** is red and blue?
- **c** is red or shows the number 4?
- **d** is red and shows the number 3?
- **e** shows a number of 2 or more?
- **f** shows a number greater than 3?

3 A standard die is rolled once. What is the probability (as a simple fraction) that the number rolled is:

- **a** even or a 5?
- **b** even and a 5?
- **c** at least 5?
- **d** greater than 5?
- **e** less than 5?
- **f** at most 5?
- **g** not 5?
- **h** odd but not 5?
- **i** less than 4 and even?
- **j** less than 4 or even?
- **k** less than 4 or even but not both?

PROBLEM-SOLVING AND REASONING	4, 5	4–6	5, 6

Example 4

4 Sophie has randomly chosen a card from a standard deck and placed it in her pocket. She is going to randomly choose a second card from the deck.

- **a** What is the probability that she chooses the same card as the one in her pocket?
- **b** What is the probability that the second card has the same suit as the first card?
- **c** What is the probability that the second card’s suit is different from that of the first card?

5 Rachel has eight socks in her sock drawer. They are not joined together. Two are red, two are green, two are yellow and two are blue. She has randomly chosen one sock and can see its colour. She is now going to randomly choose another sock.

- **a** What is the probability that it is the same colour as the first sock?
- **b** What is the probability that it is not the same colour as the first sock?

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6 In this exercise you get a chance to be the teacher and make up the questions. You are required to use the terminology in the first column of the table below to write questions for another student in your class. The answers to your questions must not be 0 or 1. The other student fills in the answers in the probability column.

A chance experiment

Every domino tile in the picture shows two numbers. The first tile shows a 5 and a 6.

The six tiles are placed face down and shuffled. One of them is chosen at random.

Terminology	Your question	Probability answer
greater than		
at least		
less than		
at most		
not		
exclusive or		
inclusive or		
and		

ENRICHMENT	—	—	7

Combinations on your calculator

7 Scientific calculators have a button called ${}^{n}C_{r}$ that is useful for combinations. Examples of a combination are given below.

a There are five people in a room (A, B, C, D, E). You must choose two. Write down all the possibilities. How many possibilities are there?

b Enter ${}^{5}C_{2}$. This should confirm your answer to part **a**.

c Now there are 10 people in the room and you must choose two. How many combinations are there?

d **i** There are 40 balls in a barrel and you must draw 6. How many combinations are there?

ii Four extra balls are placed in the barrel. How many combinations are there now?

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5E Venn diagrams and two-way tables

Interactive
Widgets
HOTsheets
Walkthrough

When two events are being considered, Venn diagrams and two-way tables give another way to view the probabilities. They are especially useful when survey results are being considered and converted to probabilities.

Let's start: Are English and mathematics enemies?

Conduct a poll among students in the class, asking whether they like English and whether they like maths. Use a tally like the one shown.

	Like maths	Do not like maths
Like English	𝍸 I	𝍸 IIII
Do not like English	𝍸 𝍸 I	𝍸 II

Use your survey results to debate these questions.
- Are the students who like English more or less likely to enjoy maths?
- If you like maths, does that increase the probability that you will like English?
- Which is the more popular subject within your class?

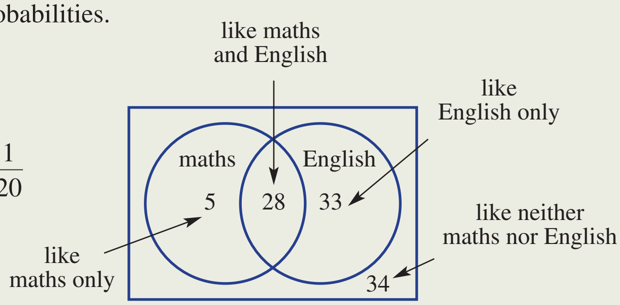
Key ideas

- A **two-way table** lists the number of outcomes or people in different categories, with the final row and column being the total of the other entries in that row or column. For example:

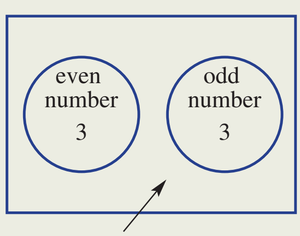
	Like maths	Do not like maths	Total
Like English	28	33	61
Do not like English	5	34	39
Total	33	67	100

- A two-way table can be used to find probabilities.
  For example:
  $P(\text{like maths}) = \frac{33}{100}$
  $P(\text{like maths and not English}) = \frac{5}{100} = \frac{1}{20}$

- A **Venn diagram** is a pictorial representation of a two-way table without the total row and column. The two-way table above can be written as shown.



- **Mutually exclusive** events cannot both occur at the same time. Rolling an even number and rolling an odd number.
  For example:
  rolling an even number on a fair die is 3
  rolling an odd number in a fair die is 3



There is nothing in both circles, so the events are mutually exclusive.

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Example 5 Constructing Venn diagrams and two-way tables

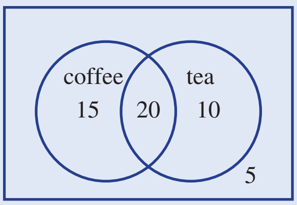
A survey is conducted of 50 people, asking who likes coffee and who likes tea. It was found that 20 people liked both, 15 people liked coffee but not tea, and 10 people liked tea but not coffee.

a How many people liked neither tea nor coffee?
b Represent the survey findings in a Venn diagram.
c How many people surveyed like tea?
d How many people like both coffee and tea?
e How many people like coffee or tea (or both)?
f Represent the survey findings in a two-way table.

SOLUTION

a 5

b



c $20 + 10 = 30$

d 20

e 45

f

	Like coffee	Dislike coffee	Total
Like tea	20	10	30
Dislike tea	15	5	20
Total	35	15	50

EXPLANATION

$50 - 20 - 15 - 10 = 5$ people who do not like either.

The Venn diagram includes four numbers, corresponding to the four possibilities.

For example, the number 15 means that 15 people like coffee but not tea.

10 people like tea but not coffee, but 20 people like both. In total, 30 people like tea.

20 out of 50 people like both coffee and tea.

$15 + 20 + 10 = 45$ people like either coffee or tea or both.

The two-way table has the four numbers from the Venn diagram and also a 'total' column (e.g. $20 + 10 = 30$, $15 + 5 = 20$) and a 'total' row. Note that 50 in the bottom corner is both $30 + 20$ and $35 + 15$.

Example 6 Using two-way tables to calculate probabilities

Consider the two-way table below showing the eating and sleeping preferences of different animals at the zoo.

	Eats meat	No meat	Total
Sleeps during day	20	12	32
Only sleeps at night	40	28	68
Total	60	40	100

a For a randomly selected animal, find:
i P(sleeps only at night) **ii** P(eats meat or sleeps during day)
b If an animal is selected at random and it eats meat, what is the probability that it sleeps during the day?
c What is the probability that an animal that sleeps during the day does not eat meat?

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SOLUTION

a **i** $P(\text{sleeps only at night})$

$$= \frac{68}{100}$$

$$= \frac{17}{25}$$

EXPLANATION

The total number of animals that sleep at night is 68.

So $\frac{68}{100} = \frac{17}{25}$.

ii $P(\text{eats meat or sleeps during day})$

$$= \frac{72}{100}$$

$$= \frac{18}{25}$$

$20 + 12 + 40 = 72$ animals eat meat or sleep during the day (or both).

$\frac{72}{100} = \frac{18}{25}$

b $P(\text{sleeps during day, given that the animal eats meat})$

$$= \frac{20}{60}$$

$$= \frac{1}{3}$$

Of the 60 animals that eat meat, 20 sleep during the day, so the probability is $\frac{20}{60} = \frac{1}{3}$.

c $P(\text{does not eat meat, given that animal sleeps during the day})$

$$= \frac{12}{32}$$

$$= \frac{3}{8}$$

Of the 32 animals that sleep during the day, 12 do not eat meat. The probability is $\frac{12}{32} = \frac{3}{8}$.

Exercise 5E

UNDERSTANDING AND FLUENCY 1–5 3–7 4–7

1 **a** Copy and complete the two-way table by writing in the missing totals.

	Like bananas	Dislike bananas	Total
Like apples	30	15	45
Dislike apples	10	20	
Total		35	75

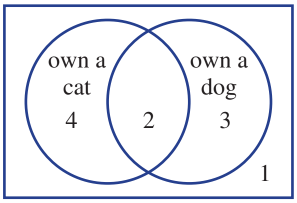
b How many people like both apples and bananas?

c How many people dislike both apples and bananas?

d How many people participated in the survey?

e It is not possible to like apples and dislike apples. These two events are _______ _______.

2 Consider the Venn diagram representing cat and dog ownership.



a State the missing number (1, 2, 3 or 4) to make the following statements true.

i The number of people surveyed who own a cat and a dog is _____.

ii The number of people surveyed who own a cat but do not own a dog is _____.

iii The number of people surveyed who own neither a cat nor a dog is _____.

iv The number of people surveyed who own a dog but do not own a cat is _____.

b Is owning a cat and owning a dog a mutually exclusive event? Why/why not?

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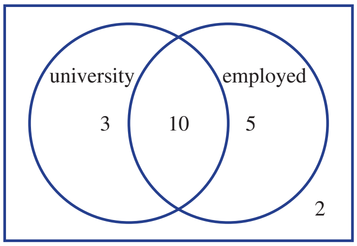
Example 5a–e

3 In a group of 30 students it is found that 10 play both cricket and soccer, 5 play only cricket and 7 play only soccer.

a How many students do not play cricket or soccer?
b Represent the survey findings in a Venn diagram.
c How many of the students surveyed play cricket?
d How many of the students surveyed play cricket or soccer or both?
e How many of the students surveyed play either cricket or soccer but not both?

Example 5f

4 Consider this Venn diagram, showing the number of people surveyed who have a university degree and the number of those surveyed who are employed.



a What is the total number of people surveyed who are employed?
b Copy and complete the two-way table shown below.

	Employed	Unemployed	Total
University degree			
No university degree			
Total			

c If the 10 in the centre of the Venn diagram is changed to 11, which cells in the two-way table would change?

Example 6a

5 The two-way table below shows the results of a poll conducted of a group of students who own mobile phones to find out who pays their own bills.

	Boys	Girls	Total
Pay own bill	4	7	11
Do not pay own bill	8	7	15
Total	12	14	26

a How many students participated in this poll?
b How many participants were boys?
c How many of the students surveyed pay their own bill?
d Find the probability that a randomly selected participant:
 i is a boy who pays his own bill
 ii is a girl who pays her own bill
 iii is a girl
 iv does not pay their own bill
e There are four events shown in the table above (i.e. being a boy, being a girl, paying own bill, not paying own bill). Which pair(s) of events are mutually exclusive?

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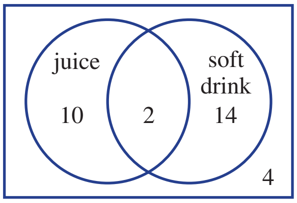
Example 6b, c

6 Forty men completed a survey about home ownership and car ownership. The results are shown in the two-way table below.

	Own car	Do not own car	Total
Own home	8	2	10
Do not own home	17	13	30
Total	25	15	40

a Represent the two-way table above as a Venn diagram.
b If a survey participant is chosen at random, give the probability that:
  i he owns a car and a home
  ii he owns a car but not a home
  iii he owns a home
c If a survey participant is selected at random and he owns a car, what is the probability that he also owns a home?
d If a survey participant is selected at random and he owns a home, what is the probability that he also owns a car?

7 The Venn diagram shows the number of people surveyed who like juice and/or soft drinks.



a What is the total number of people surveyed who like juice?
b What is the probability that a randomly selected survey participant likes neither juice nor soft drink?
c What is the probability that a randomly selected survey participant likes juice or soft drink or both?
d What is the probability that a randomly selected survey participant likes juice or soft drink but not both?
e Explain the difference between *inclusive or* used in part **c** and *exclusive or* used in part **d**. Make two copies of the Venn diagram and use shading to illustrate the difference.

PROBLEM-SOLVING AND REASONING	8, 10	8–10	9–11

8 A car salesperson notes that among 40 cars for sale, there are 15 automatic cars and 10 sports cars. Only two of the sports cars are automatic.
a Create a two-way table of this situation.
b What is the probability that a randomly selected car will be a sports car that is not automatic?
c What is the probability that a randomly selected car will be an automatic car that is not a sports car?
d If an automatic car is chosen at random, what is the probability that it is a sports car?

9 A page of text is analysed and of the 150 words on it, 30 are nouns, 10 of which start with a vowel. Of the words that are not nouns, 85 of them do not start with vowels.
a If a word on the page is chosen at random, what is the probability that it is a noun?
b How many of the words on the page start with vowels?
c If a word on the page starts with a vowel, what is the probability that it is a noun?
d If a noun is chosen at random, what is the probability that it starts with a vowel?

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10 In a two-way table, there are nine spaces to be filled with numbers.

a What is the minimum number of spaces that must be filled before the rest of the table can be determined? Explain your answer.

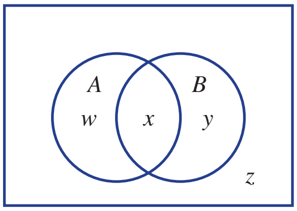
b If you are given a two-way table with five spaces filled, can you always determine the remaining spaces? Justify your answer.

c Explain why the following two-way table must contain an error.

	B	**Not *B***	**Total**
A	20		
Not *A*		29	
Total	62		81

11 In this Venn diagram, w, x, y and z are all unknown positive integers. Copy and complete this two-way table.

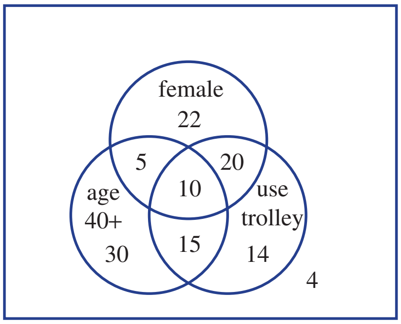
	B	**Not *B***	**Total**
A	x		
Not *A*		z	
Total	$x + y$		



ENRICHMENT — — 12

Triple Venn diagrams

12 A group of supermarket shoppers is surveyed on their age, gender and whether they shop using a trolley or a basket. This Venn diagram summarises the results.



a How many shoppers participated in the survey?

b How many of the participants are aged 40 or over?

c Give the probability that a randomly selected survey participant:

i uses a trolley **ii** is female

iii is aged 40 or over

iv is male and uses a trolley

v is female and younger than 40

vi is younger than 40 and uses a trolley

d If a female survey participant is chosen at random, what is the probability that she:

i uses a trolley? **ii** is aged 40 or over?

e If a survey participant that uses a trolley is chosen at random, what is the probability that they:

i are male? **ii** are under 40?

f Describe what you know about the four participants outside of the three circles in the diagram.

g If all you know about a survey participant is that they use a trolley, are they more likely to be male or female? Justify your answer.

h If a female survey participant is shopping, are they more likely to use a trolley or a basket?

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5F Probability in two-step experiments EXTENSION

Interactive
Widgets
HOTsheets
Walkthrough

Sometimes an experiment consists of two independent components, such as when a coin is tossed and then a die is rolled. Or perhaps a card is pulled from a hat and then a spinner is spun. We can use tables to list the sample space.

Consider the following example in which a coin is flipped and then a die is rolled.

		Die					
		1	2	3	4	5	6
Coin	Heads	H1	H2	H3	H4	H5	H6
	Tails	T1	T2	T3	T4	T5	T6

There are 12 outcomes listed in the table. So the probability of getting a 'tail' combined with the number 5 is $\frac{1}{12}$.

Let's start: Dice dilemma

In a board game, two dice are rolled and the player moves forward according to their sum.

- What are the possible values that the sum could have?
- Are some values more likely than others? Discuss.
- How likely is it that the numbers showing on the two dice will add to 5?

Key ideas

- If two independent events occur, the outcomes can be listed as a table.
- The probability is still given by

$$P(\text{event}) = \frac{\text{number of outcomes in which the event occurs}}{\text{total number of possible outcomes}}$$

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Example 7 Using a table for multiple events

A spinner with the numbers 1, 2 and 3 is spun, and then a card is chosen at random from the letters ATHS.

a Draw a table to list the sample space of this experiment.
b How many outcomes does the experiment have?
c Find the probability of the combination 2S.
d Find the probability of an odd number being spun and the letter H being chosen.

SOLUTION

a

	A	T	H	S
1	1A	1T	1H	1S
2	2A	2T	2H	2S
3	3A	3T	3H	3S

b There are 12 outcomes.

c $P(2S) = \frac{1}{12}$

d $P(\text{odd}, H) = \frac{2}{12} = \frac{1}{6}$

EXPLANATION

a The sample space of the spinner $\{1, 2, 3\}$ is put into the left column.
The sample space of the cards $\{A, T, H, S\}$ is put into the top row.

b The table has $4 \times 3 = 12$ items in it.

c All 12 outcomes are equally likely. Spinning 2 and choosing an S is one of the 12 outcomes.

d Possible outcomes are 1H and 3H, so probability $= 2 \div 12$.

Exercise 5F EXTENSION

UNDERSTANDING AND FLUENCY	1–5	2–6	3–6

1 A coin is flipped and then a spinner is spun. The possible outcomes are listed in the table below.

	1	2	3	4	5
H	H1	H2	H3	H4	H5
T	T1	T2	T3	T4	T5

a How many outcomes are possible?
b List the four outcomes in which an even number is displayed on the spinner.
c Hence, state the probability that an even number is displayed.
d List the outcomes for which tails is flipped and an odd number is on the spinner.
e What is $P(\text{T, odd number})$?

2 Two coins are tossed and the four possible outcomes are shown below.

		20-cent coin	
		H	T
20-cent coin	H	HH	HT
	T	TH	TT

a What is the probability that the 50-cent coin will be heads and the 20-cent coin will be tails?
b For which outcomes are the two coins displaying the same face?
c What is the probability of the two coins displaying the same face?

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Example 7

3 A coin is flipped and then a die is rolled.

a Draw a table to list the sample space of this experiment.

b How many possible outcomes are there?

c Find the probability of the pair H3.

d Find the probability of flipping ‘heads’ and rolling an odd number.

4 A letter is chosen from the word LINE and another is chosen from the word RIDE.

a Draw a table to list the sample space.

b How many possible outcomes are there?

c Find P(NR); i.e. the probability that N is chosen from LINE and R is chosen from RIDE.

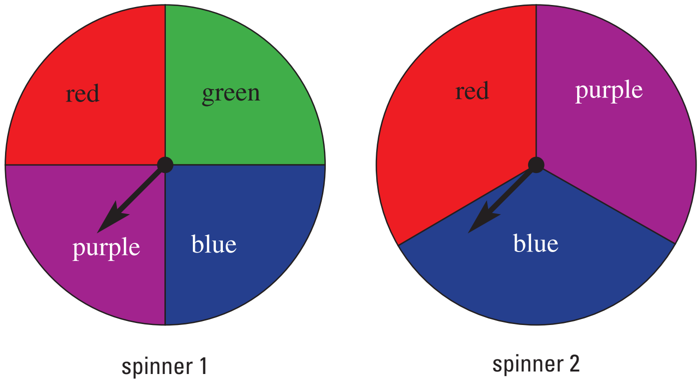
d Find P(LD).

e Find the probability that two vowels are chosen.

f Find the probability that two consonants are chosen.

g Find the probability that the two letters chosen are the same.

5 The spinners shown below are each spun.



a Draw a table to list the sample space. Use R for red, P for purple and so on.

b Find the probability that spinner 1 will display red and spinner 2 will display blue.

c Find the probability that both spinners will display red.

d What is the probability that spinner 1 displays red and spinner 2 displays purple?

e What is the probability that one of the spinners displays red and the other displays blue?

f What is the probability that both spinners display the same colour?

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6 A letter from the word EGG is chosen at random and then a letter from ROLL is chosen at random. The sample space is shown below.

	R	O	L	L
E	ER	EO	EL	EL
G	GR	GO	GL	GL
G	GR	GO	GL	GL

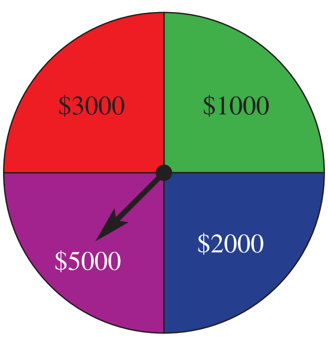
a Find P(ER).
b Find P(GO).
c Find P(both letters are vowels).
d Find P(both letters are consonants).

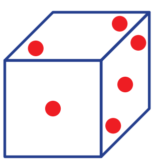
PROBLEM-SOLVING AND REASONING	7, 8, 10	7–10	8–11

7 Two dice are rolled for a board game. The numbers showing are then added together to get a number between 2 and 12.
a Draw a table to describe the sample space.
b Find the probability that the two dice add to 5.
c Find the probability that the two dice add to an even number.
d What is the most likely sum to occur?
e What are the two least likely sums to occur between 2 and 12?

8 In Rosemary's left pocket she has two orange marbles and one white marble. In her right pocket she has a yellow marble, a white marble and three blue marbles. She chooses a marble at random from each pocket.
a Draw a table to describe the sample space. (Hint: The left pocket outcomes are W, O, O.)
b Find the probability that she will choose an orange marble and a yellow marble.
c What is the probability that she chooses a white marble and a yellow marble?
d What is the probability that she chooses a white marble and an orange marble?
e Find the probability that a white and a blue marble are selected.
f What is the probability that the two marbles selected are the same colour?

9 In a game show, a wheel is spun to determine the prize money and then a die is rolled. The prize money shown is multiplied by the number on the die to give the total winnings.
a What is the probability that a contestant will win \$6000?
b What is the probability that they will win more than \$11 000?





10 Two different experiments are conducted simultaneously. The first has seven possible outcomes and the second has nine outcomes. How many outcomes are there in the combined experiment?

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11 In a standard deck of 52 playing cards there are four suits (diamonds, hearts, clubs and spades) and 13 cards in each suit (A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K).

a What is the probability that a randomly chosen card is of the diamond suit?

b If a card is chosen at random, what is the probability that it will be 3♦?

c What is the probability of selecting a card that is red and a king?

d If two cards are chosen at random from separate decks, what is the probability that:

i they are both diamonds? (Hint: Do not draw a 52×52 table.)

ii they are both red cards?

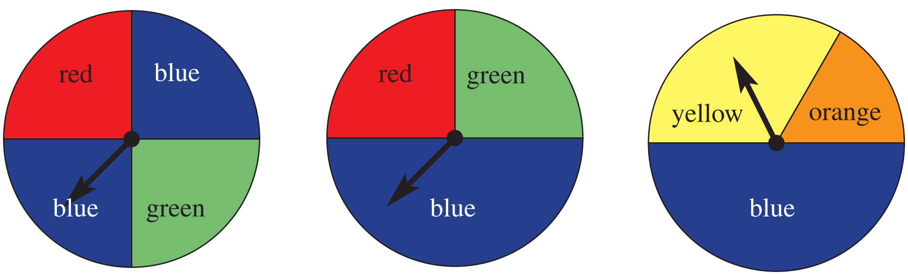
iii 3♦ is chosen from both decks?

e How would your answers to part **d** change if the two cards were drawn from the same deck?

ENRICHMENT	—	—	12

Spinners with unequal areas

12 Consider the spinners below.



spinner 1
outcomes: R, G, B, B

spinner 2
outcomes: R, G, B

spinner 3

a Find the following probabilities for spinner 2.

i $P(\text{red})$

ii $P(\text{blue})$

b Find the probability of the following occurring when spinner 2 is spun twice.

i two reds

ii two blues

iii a red, then a green

iv a red and a green (in either order)

c Spinner 3 has $P(\text{orange}) = \frac{1}{6}$, $P(\text{yellow}) = \frac{1}{3}$ and $P(\text{blue}) = \frac{1}{2}$.

What six letters could be used to describe the six equally likely outcomes when spinner 3 is spun?

d If spinner 3 is spun twice, find the probability of obtaining:

i yellow twice

ii the same colour twice

iii orange and then blue

iv orange and blue (either order)

v at least one orange

vi at least one blue

e Spinners 2 and 3 are both spun. Find the probability of obtaining:

i red then orange

ii green then blue

iii orange and not blue

iv both blue

v neither blue

vi neither red

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Investigation

Develop a spreadsheet simulation of two dice rolling

1 Set up a table in a spreadsheet to randomly generate 500 outcomes for tossing two dice.
- Include a column for each die.
- Include a column to show the sum of each pair of outcomes.

	A	B	C	D	E
1	Rolling two dice				
2					
3	Die 1	Die 2	Sum	Possible Sum	Count
4	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A4+B4	2	=COUNTIF(C$4:C$503,D4)
5	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A5+B5	3	=COUNTIF(C$4:C$503,D5)
6	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A6+B6	4	=COUNTIF(C$4:C$503,D6)
7	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A7+B7	5	=COUNTIF(C$4:C$503,D7)
8	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A8+B8	6	=COUNTIF(C$4:C$503,D8)
9	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A9+B9	7	=COUNTIF(C$4:C$503,D9)
10	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A10+B10	8	=COUNTIF(C$4:C$503,D10)
11	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A11+B11	9	=COUNTIF(C$4:C$503,D11)
12	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A12+B12	10	=COUNTIF(C$4:C$503,D12)
13	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A13+B13	11	=COUNTIF(C$4:C$503,D13)
14	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A14+B14	12	=COUNTIF(C$4:C$503,D14)
15	=RANDBETWEEN(1,6)	=RANDBETWEEN(1,6)	=A15+B15		

2 Using the spreadsheet software, count how many times each sum from 2 to 12 is achieved. Plot the data as a histogram (or a 2D column graph if your spreadsheet software does not include histograms). Comment on whether your distribution is symmetrical or skewed.

3 Use a table to calculate the theoretical probability of each of the dice sums, and compare the results with the experimental probability.

Considering other dice sums

Using the spreadsheet software, conduct a large simulation (1000 or more rows) of rolling three dice and noting the sums. Use the spreadsheet software to generate a frequency column graph of your results and comment on how this graph looks compared to the simulation of two dice being rolled.

Monopoly risk

In the game of Monopoly, two fair 6-sided dice are rolled to work out how far a player should go forward. For this investigation, you will need two 6-sided dice or a random number simulator that simulates numbers between 1 and 6.

a Roll the two dice and note what they add up to. Repeat this 100 times and complete this table.

Dice sum	2	3	4	5	6	7	8	9	10	11	12	Total
Tally												100

b Represent the results in a column graph. Describe the shape of the graph. Do you notice any patterns?

c Use the results of your experiment to give the experimental probability of two dice adding to:

i 3 **ii** 6 **iii** 8 **iv** 12 **v** 15

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d What is the most likely sum for the dice to add to, based on your experiment?

e If the average Monopoly game involves 180 rolls, find the expected number of times, based on your experiment, that the dice will add to:

i 3 **ii** 6 **iii** 8 **iv** 12 **v** 15

f Why do you think that certain sums happen more often than others? Explain why this might happen by comparing the number of times the dice add to 2 and the number of times they add to 8.

g What is the mean dice sum of the 100 trials you conducted above?

To conduct many experiments, a spreadsheet can be used. For example, the spreadsheet below can be used to simulate rolling three 6-sided dice. Drag down the cells from the second row to row 1000 to run the experiment 1000 times.

	A	B	C	D
1	Die 1	Die 2	Die 3	=MODE(D2:D1001)
2	=INT(6*RAND())+1	=INT(6*RAND())+1	=INT(6*RAND())+1	=A2+B2+C2

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Puzzles and challenges

1 At the local sports academy, everybody plays netball or tennis. Given that half the tennis players also play netball and one-third of the netballers also play tennis, what is the probability that a randomly chosen person at the academy plays both?

2 For each of the following, find an English word that matches the description.

a $P(\text{vowel}) = \frac{1}{2}$

b $P(\text{F}) = \frac{2}{3}$

c $P(\text{vowel}) = \frac{1}{4}$ and $P(\text{D}) = \frac{1}{4}$

d $P(\text{I}) = \frac{2}{11}$ and $P(\text{consonant}) = \frac{7}{11}$

e $P(\text{M}) = \frac{1}{7}$ and $P(\text{T}) = \frac{1}{7}$ and $P(\text{S}) = \frac{1}{7}$

f $P(\text{vowel}) = 0$ and $P(\text{T}) = \frac{1}{3}$

3 In a particular town, there are 22 women who can cook and 18 men who cannot cook. Given that half the town is male and 54% of the town can cook, how many men in the town can cook?

4 In the following game, the player flips a fair coin each turn to move a piece. If the coin shows 'heads' the piece goes right, and if it is 'tails' the coin goes left. What is the approximate probability that the player will win this game?

WIN				**START**					**LOSE**

5 If a person guesses all the answers on a 10-question true or false test, what is the probability that they will get them all right?

6 A bag contains eight counters that are red, blue or yellow. A counter is selected from the bag, its colour noted and the counter replaced. If 100 counters were selected and 14 were red, 37 were blue and 49 were yellow, how many counters of each colour are likely to be in the bag?

7 Each of the eight letters of a word is written on a separate card. Given the following probabilities, what is the word?

$P(\text{letter P}) = P(\text{letter R}) = 12.5\%$, $P(\text{letter B}) = \frac{1}{4}$, $P(\text{vowel}) = 0.375$

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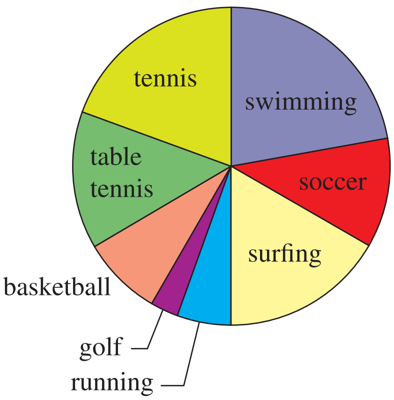
8 What is the capital city of Iceland?
Find the answer to this question by looking at the pie chart and finding the answers below. You'll need a protractor to measure each angle in the graph. Match up the letter with the correct numerical answer given below.

A school of 1080 students asks its students to nominate their favourite sport offered by the school program.



A the probability that a randomly chosen student prefers golf

E the number of students who prefer swimming

I the probability of a student choosing basketball

J the number of students who nominate table tennis

K the probability that a randomly chosen student nominates soccer

R If golf and table tennis are cut from the school program, how many students must choose a different sport?

V the probability that a student does not choose swimming or surfing

Y the probability of a student being a keen surfer

Letter									
probability	180	240	$\frac{1}{6}$	$\frac{1}{9}$	150	$\frac{1}{36}$	$\frac{11}{18}$	$\frac{1}{12}$	$\frac{1}{9}$

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Chapter summary

Probability

Theoretical probability

Playing cards

Diamonds

Hearts

Spades

Clubs

Trial: Roll a fair die
Sample space (possible outcomes):
{1, 2, 3, 4, 5, 6}

$P(\text{odd number}) = \frac{3}{6} = \frac{1}{2}$

Trial: Select a playing card and note its suit.
Sample space: {spade, diamond club, heart}

Theoretical probabilities

$P(\text{black}) = \frac{26}{52} = \frac{1}{2}$

$P(\text{heart}) = \frac{13}{52} = \frac{1}{4}$

$P(\text{not spade}) = \frac{39}{52} = \frac{3}{4}$

$P(\text{either red or a spade}) = \frac{39}{52} = \frac{3}{4}$

$P(\text{red ace}) = \frac{2}{52} = \frac{1}{26}$

Expected number of outcomes

Outcome	Frequency	Experimental probability
Heart	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$
Diamond	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$
Club	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$
Spade	$\frac{13}{52} = \frac{1}{4}$	$\frac{1}{4} \times 20 = 5$

Experimental probability

Use an experiment or survey or simulation to estimate probability.
e.g. Spinner lands on blue 47 times out of 120 ⟶
Experimental probability = $\frac{47}{120}$

An experiment can be used if the exact probability cannot be calculated.

Experimental probability

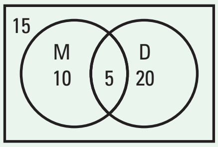
Playing card selected and replaced 20 times, and its suit noted.

Outcome	Frequency	Experimental probability
Heart	4	$\frac{4}{20}$
Diamond	5	$\frac{5}{20}$
Club	4	$\frac{4}{20}$
Spade	7	$\frac{7}{20}$
	$n = 20$	

Presenting data from a survey

50 people were asked:
- Are you male? (Yes/No)
- Do you drive a car? (Yes/No)

Venn diagram



Two-way table

	Drive	Don't drive
Male	5	10
Not male	20	15

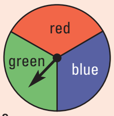
Probability: How likely an event will occur

$$P(\text{event}) = \frac{\text{number of favourable}}{\text{total number of outcomes}}$$

unlikely $\frac{1}{2}$ likely
0 — 1
impossible even chance certain
more likely ⟶

Probabilities can be given as fractions, decimals or percentages.
e.g. 25%, $\frac{1}{4}$, 0.25
e.g. 70%, $\frac{7}{10}$, 0.7

e.g. Spin spinner
Sample space: {red, green, blue}
$P(\text{spin red}) = \frac{1}{3}$
$P(\text{don't spin blue}) = \frac{2}{3}$



Chance experiment
e.g. Select a playing card and note its suit.
Sample space: {spades, diamonds, hearts, clubs}
$P(\text{diamonds}) = \frac{1}{4}$
$P(\text{hearts or clubs}) = \frac{2}{4} = \frac{1}{2}$

Expected number is $P(\text{event}) \times$ number of trials
e.g. Flip coin 100 times, expected number of heads
$= \frac{1}{2} \times 100 = 50$
e.g. Roll die 36 times, expected number of 5s
$= \frac{1}{6} \times 36 = 6$

e.g. Roll a fair die
Sample space: {1, 2, 3, 4, 5, 6}
$P(\text{roll a 5}) = \frac{1}{6}$
$P(\text{roll odd number}) = \frac{3}{6} = \frac{1}{2}$

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Multiple-choice questions

1 The results of a survey are shown below. Note that each student learns only one instrument.

Instrument learned	piano	violin	drums	guitar
Number of students	10	2	5	3

Based on the survey, the experimental probability that a randomly selected survey participant is learning the guitar is:

A $\frac{1}{4}$ **B** $\frac{1}{2}$ **C** 3 **D** $\frac{3}{5}$ **E** $\frac{3}{20}$

2 Which of the following events has the same probability as rolling an odd number on a fair 6-sided die?

A rolling a number greater than 4 on a fair 6-sided die
B choosing a vowel from the word CAT
C tossing a fair coin and getting heads
D choosing the letter T from the word TOE
E spinning an odd number on a spinner numbered 1 to 7

3 Each letter of the word APPLE is written separately on five cards. One card is then chosen at random. P(letter P) is:

A 0 **B** 0.2 **C** 0.4 **D** 0.5 **E** 1

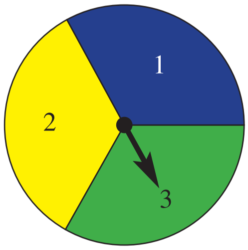
4 A fair 6-sided die is rolled 600 times. The expected number of times that the number rolled is either a 1 or a 2 is:

A 100 **B** 200 **C** 300 **D** 400 **E** 600

5 The letters of the word STATISTICS are placed on 10 different cards and placed into a hat. If a card is drawn at random, the probability that it will show a vowel is:

A 0.2 **B** 0.3 **C** 0.4 **D** 0.5 **E** 0.7

6 A fair die is rolled and then the spinner shown at right is spun. The probability that the die will display the same number as the spinner is:



A $\frac{1}{36}$ **B** $\frac{1}{18}$ **C** $\frac{1}{6}$

D $\frac{1}{2}$ **E** 1

7 A coin is tossed three times. The probability of obtaining at least two tails is:

A $\frac{2}{3}$ **B** 4 **C** $\frac{1}{2}$ **D** $\frac{3}{8}$ **E** $\frac{1}{8}$

8 An experiment is conducted in which three dice are rolled and the sum of the faces is added. In 12 of the 100 trials, the sum of the faces is 11. Based on this, the experimental probability of having three faces add to 11 is:

A $\frac{11}{100}$ **B** $\frac{12}{111}$ **C** $\frac{3}{25}$ **D** 12 **E** $\frac{1}{2}$

9 Rachel has a fair coin. She has tossed 'heads' five times in a row. Rachel tosses the coin one more time. What is the probability of tossing 'tails'?

A 0 **B** 1 **C** $\frac{1}{2}$ **D** less than $\frac{1}{2}$ **E** more than $\frac{1}{2}$

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10 When a fair die is rolled, what is the probability that the number is even but not less than 3?

A 0 **B** $\frac{1}{6}$ **C** $\frac{1}{3}$ **D** $\frac{1}{2}$ **E** $\frac{2}{3}$

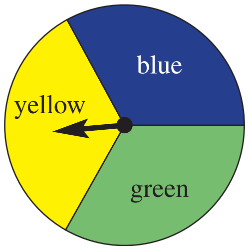
Short-answer questions

1 For each of the following descriptions, choose the probability from the set $0, \frac{1}{8}, \frac{3}{4}, 1, \frac{19}{20}$, that matches best.

a certain **b** highly unlikely **c** highly likely
d likely **e** impossible

2 List the sample space for each of the following experiments.

a A fair 6-sided die is rolled.
b A fair coin is tossed.
c A letter is chosen from the word DESIGN.
d Spinning the spinner shown opposite.



3 Vin spins a spinner with nine equal sectors, which are numbered 1 to 9.

a How many outcomes are there?
b Find the probability of spinning:
i an odd number **ii** a multiple of 3 **iii** a number greater than 10
iv a prime number less than 6 **v** a factor of 8

4 One card is chosen at random from a standard deck of 52 playing cards. Find the probability of drawing:

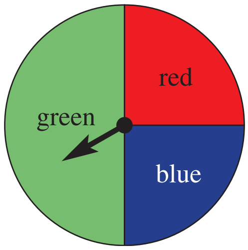
a a red king
b a king or queen
c a jack of diamonds
d a picture card (i.e. king, queen or jack)

5 A coin is tossed 100 times, with the outcome 42 heads and 58 tails.

a What is the experimental probability of getting heads? Give your answer as a percentage.
b What is the actual probability of getting heads if the coin is fair? Give your answer as a percentage.

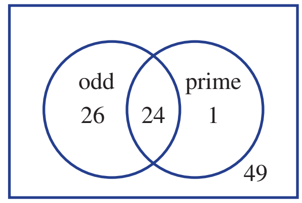
6 Consider the spinner shown.

a State the probability that the spinner lands in the green section.
b State the probability that the spinner lands in the blue section.
c Grace spins the spinner 100 times. What is the expected number of times it would land in the red section?
d She spins the spinner 500 times. What is the expected number of times it would land in the green section?



Extended-response questions

1 The Venn diagram shows how many numbers between 1 and 100 are odd and how many are prime.



Consider the numbers 1 to 100.

a How many are odd?
b How many prime numbers are there?
c What is the probability that a randomly selected number will be odd and prime? Give your answer as a percentage.
d What is the probability that a randomly selected number will be prime but not odd? Give your answer as a percentage.
e If an odd number is chosen, what is the probability that it is prime?
f If a prime number is chosen, what is the probability that it is odd?

2 The two-way table below shows the results of a survey on car ownership and public transport usage. You can assume the sample is representative of the population.

	Uses public transport	Does not use public transport	Total
Owns a car	20	80	
Does not own a car	65	35	
Total			

a Copy and complete the table.
b How many people participated in the survey?
c What is the probability that a randomly selected person will have a car?
d What is the probability that a randomly selected person will use public transport even though they own a car?
e What is the probability that someone owns a car given that they use public transport?
f If a car owner is selected, what is the probability that they will catch public transport?
g In what ways could the survey produce biased results if it had been conducted:
  i outside a train station?
  ii in regional New South Wales?

3 A spinner is made using the numbers 1, 3, 5 and 10 in four sectors. The spinner is spun 80 times, and the results obtained are shown in the table.

Number on spinner	Frequency
1	30
3	18
5	11
10	21
	80

a Which sector on the spinner occupies the largest area? Explain.
b Two sectors of the spinner have the same area. Which two numbers do you think have equal areas, and why?
c What is the experimental probability of obtaining a 1 on the next spin?
d Draw an example of what you think the spinner might look like, in terms of the area covered by each of the four numbers.

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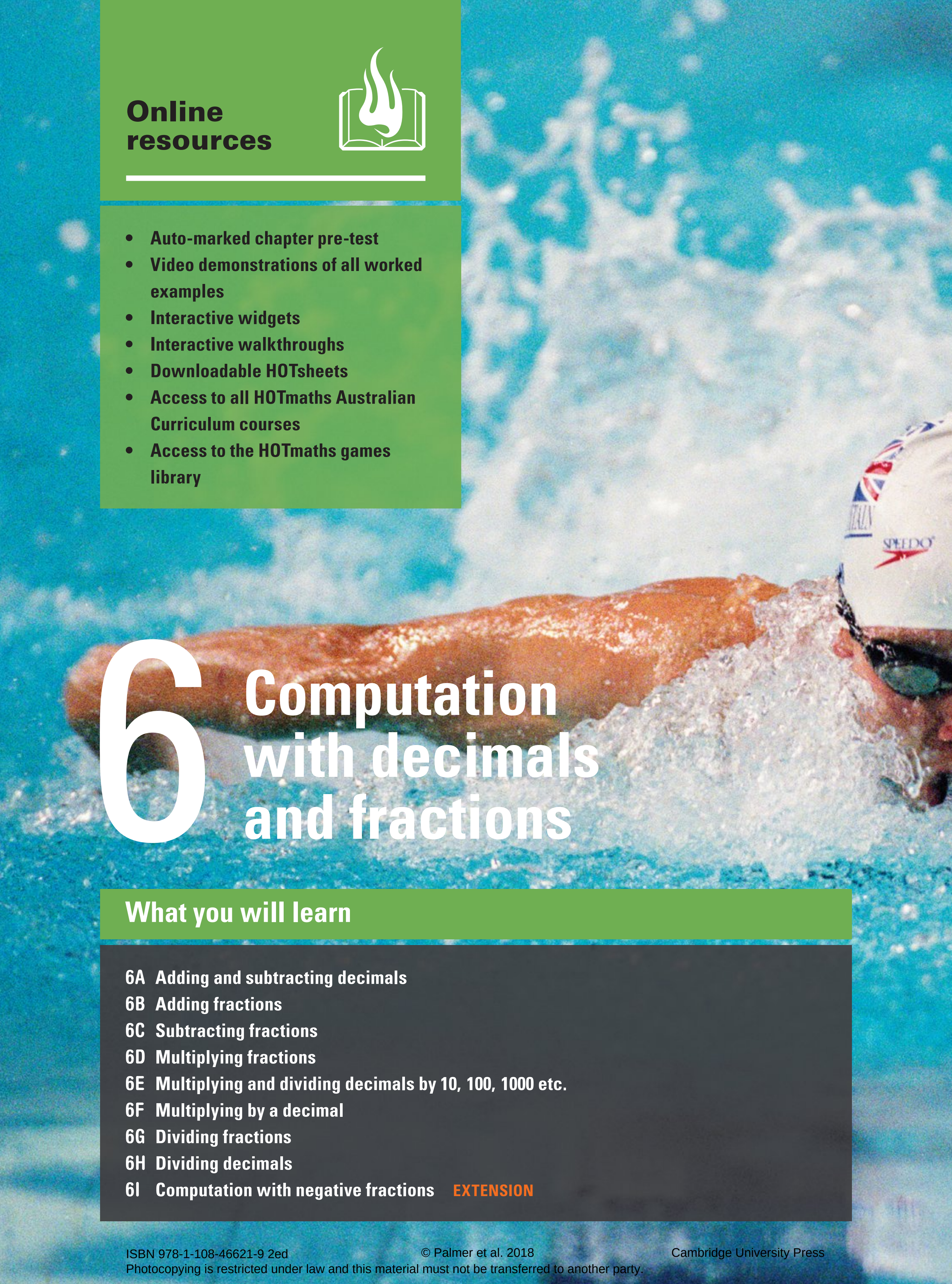

Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

6 Computation with decimals and fractions

What you will learn

NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRAND: FRACTIONS, DECIMALS AND PERCENTAGES

Outcome

A student operates with fractions, decimals and percentages.
(MA4–5NA)

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Swimming records and decimal places

Swimming times are electronically measured and recorded, with the seconds given to 2 decimal places. Accuracy is very important so that the right decision is made for placegetters and records. Sometimes winners are separated by as little as one-hundredth of a second. Ian Thorpe, who set 13 world records, had his 400 m freestyle record (3:40.06) broken in 2009 by German swimmer Paul Biedermann, who swam only 0.01 of a second faster than Thorpe.

Australian swimmers Melanie Schlanger, Brittany Elmslie, Alicia Coutts and Cate Campbell won the Women's 4 × 100 m Freestyle Relay at the London 2012 Olympic Games in an Olympic record time of 3 minutes and 33.15 seconds. Second and third placegetters were The Netherlands (3:33.79) and USA (3:34.24).

The decimal system allows us to express quantities with great accuracy. There are many instances in our everyday lives in which accuracy of measurement is highly important. For example, bicycle, car and aeroplane parts must be manufactured to precise measurements; medicine production requires measurement of chemicals in precise quantities; and investment rates, stock market prices and values are measured to many decimal places (8% per year equals 0.153846% per week).

Can you think of any other quantities that require decimal measurement?

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Pre-test

1 Complete the following.

a $\frac{1}{10} = 0.\_$ **b** $\frac{3}{10} = 0.\_$ **c** $\frac{17}{10} = 1.\_$

d $\frac{1}{100} = 0.0\_$ **e** $\frac{1}{1000} = 0.\_\ \_\ \_$ **f** $\frac{47}{10} = \_.\_$

2 Write the decimal for:

a one-half **b** one-quarter **c** three-quarters

3 Write the following cents as dollars.

a 70 cents **b** 85 cents **c** 100 cents

d 5 cents **e** 105 cents **f** 3 cents

4 Find how many cents are in:

a half a dollar **b** one-quarter of $1

c three-quarters of $1 **d** half of $5

5 Find the cost of:

a six labels at 45 cents each

b 12 pears at $1.05 each

c $1\frac{1}{2}$ boxes of mangoes at $15 a box

d seven pens at 27 cents a pen

6 Tom paid $50 for 200 photos to be printed. What was the cost of each print?

7 $124 is shared between eight people. If each share is the same amount, how much does each person receive?

8 Complete:

a $\$8.50 \times 10 = \_\_\_\_$

b $\$6 - \$5.90 = \_\_\_\_$

c $\$10 - \$7.30 = \_\_\_\_$

d $\$70 \div 100 = \_\_\_\_$

e $\$6.90 + \$4.30 = \_\_\_\_$

f $\$20 - \$19.76 = \_\_\_\_$

9 Petrol is 124 cents a litre. Calculate how much change from $100 Calvin receives when he buys:

a 10 litres **b** 50 litres **c** 70 litres

10 Find the total of these amounts: $7, $5.50, $4.90, $12, $56, $10.10 and $9.15.

11 Complete these computations.

a $\begin{array}{r} 329 \\ +194 \\ \hline \\ \hline \end{array}$ **b** $\begin{array}{r} 1024 \\ -\ 185 \\ \hline \\ \hline \end{array}$ **c** $\begin{array}{r} 104 \\ \times\ 13 \\ \hline \\ \hline \end{array}$ **d** $5\overline{)6185}$

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6A Adding and subtracting decimals

Interactive Widgets HOTsheets Walkthrough

Addition and subtraction of decimals follows the same procedures as those for whole numbers. To add or subtract whole numbers you must line up the units, tens, hundreds and so on, and then you add or subtract each column. When dealing with the addition or subtraction of decimals the routine is the same.

Consider how similar the following two sums are:

$$\begin{array}{r} 5^{1}14 \\ 272 \\ \underline{206} \\ \underline{892} \end{array} \qquad \begin{array}{r} 5^{1}1.4 \\ 27.2 \\ \underline{10.6} \\ \underline{89.2} \end{array}$$

Let's start: What's the total?

Each student thinks of three coins (gold or silver) and writes their total value on a sheet of paper. Each student in the class then estimates the total value of the amounts written down in the classroom. Record each student's estimated total.

- Each student then writes the value of the three coins they thought of on the board (e.g. $2.70, $0.80).
- Students copy down the values into their workbooks and add the decimal numbers to determine the total value of the coins in the classroom.
- Which student has the closest estimation?

Key ideas

- When adding or subtracting decimals, the decimal points and each of the decimal places must be lined up.

$$\begin{array}{r} 1.56 \\ +\,2.70 \\ \hline 4.26 \\ \hline \end{array}$$

Writing an extra zero will help.

Line up the decimal points and the digits.

Example 1 Adding decimals

Find:

a 8.31 + 5.93 **b** 64.8 + 3.012 + 5.94

SOLUTION

a

$$\begin{array}{r} {}^{1}8.31 \\ +\ 5.93 \\ \hline 14.24 \\ \hline \end{array}$$

EXPLANATION

Make sure all decimal points and decimal places are correctly aligned directly under one another.

b

$$\begin{array}{r} {}^{1}6^{1}4.800 \\ 3.012 \\ +\ \ 5.940 \\ \hline 73.752 \\ \hline \end{array}$$

Align decimal points directly under one another. Fill in missing decimal places with additional zeros.

Carry out addition, following the same procedure as that for addition of whole numbers.

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Example 2 Subtracting decimals

Find:

a 5.83 − 3.12 **b** 146.35 − 79.5

SOLUTION	EXPLANATION
a $\begin{array}{r} 5.83 \\ -\ 3.12 \\ \hline 2.71 \end{array}$	Make sure all decimal points and decimal places are correctly aligned directly under one another.
b $\begin{array}{r} {}^{13}\,{}^{15}\quad{}^{1} \\ \not{1}\,\not{4}\,\not{6}\,.\,3\,5 \\ -\quad 7\,9\,.\,5\,0 \\ \hline 6\,6\,.\,8\,5 \end{array}$	Align decimal points directly under one another and fill in missing decimal places with additional zeros. Carry out subtraction, following the same procedure as that for subtraction of whole numbers.

Exercise 6A

UNDERSTANDING AND FLUENCY	1–3, 4(½), 5, 6(½)	2, 3–7(½)	4–7(½)

1 The values 7.12, 8.5 and 13.032 must be added together. Which of the following is the best way to prepare these numbers ready for addition?

A $\begin{array}{l} 7.12 \\ 8.5 \\ +13.032 \\ \hline \\ \hline \end{array}$ **B** $\begin{array}{r} 7.12 \\ 8.5 \\ +13.032 \\ \hline \\ \hline \end{array}$ **C** $\begin{array}{r} 7.120 \\ 8.500 \\ +13.032 \\ \hline \\ \hline \end{array}$ **D** $\begin{array}{l} 7.12 \\ 8.5 \\ +13.032 \\ \hline \\ \hline \end{array}$

2 Which of the following is the correct way to perform the computation 77.81 − 6.3?

A $\begin{array}{r} 77.81 \\ -\ 6.3\ \\ \hline 84.11 \end{array}$ **B** $\begin{array}{r} 77.81 \\ -\ 6.30 \\ \hline 71.51 \end{array}$ **C** $\begin{array}{r} 77.81 \\ -\ 6.3\ \ \ \\ \hline 14.81 \end{array}$ **D** $\begin{array}{r} 77.8\ 1 \\ -\ \ 6.3 \\ \hline 77.18 \end{array}$

3 Find each of the following.

a $\begin{array}{r} 13.25 \\ +14.72 \\ \hline \\ \hline \end{array}$ **b** $\begin{array}{r} 7.23 \\ 16.31 \\ +\ 2.40 \\ \hline \\ \hline \end{array}$ **c** $\begin{array}{r} 210.0 \\ 22.3 \\ +\ 15.1 \\ \hline \\ \hline \end{array}$ **d** $\begin{array}{r} 47.81 \\ 6.98 \\ +\ 3.52 \\ \hline \\ \hline \end{array}$

Example 1

4 Find each of the following.

a 7.6 + 3.1 **b** 19.4 + 3.7 **c** 43.5 + 7.6
d 9.4 + 3.8 **e** 12.6 + 7.4 **f** 8.0 + 0.8
g 12.45 + 3.61 **h** 5.37 + 13.81 + 2.15 **i** 312.5 + 31.25
j 1.567 + 3.4 + 32.6 **k** 5.882 + 3.01 + 12.7 **l** 323.71 + 3.4506 + 12.9

5 Find:

a $\begin{array}{r} 17.2 \\ -\ 5.1 \\ \hline \\ \hline \end{array}$ **b** $\begin{array}{r} 128.63 \\ -\ 14.50 \\ \hline \\ \hline \end{array}$ **c** $\begin{array}{r} 23.94 \\ -\ 17.61 \\ \hline \\ \hline \end{array}$ **d** $\begin{array}{r} 158.32 \\ -87.53 \\ \hline \\ \hline \end{array}$

Example 2

6 Find:

a 12.8 − 5.4 **b** 42.6 − 2.5 **c** 12.9 − 0.9 **d** 12 − 9.4
e 14.8 − 2.5 **f** 234.6 − 103.2 **g** 25.9 − 3.67 **h** 31.657 − 18.2
i 412.1 − 368.83 **j** 5312.271 − 364.93 **k** 120 − 39.7 **l** 500 − 257.3

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7 Find, using a calculator.

a 46.189 + 23.85 − 7.816 **b** 282.375 − 159.483 − 72.689

PROBLEM-SOLVING AND REASONING	8, 9, 14	9–11, 14	11–15

8 Find the missing numbers in the following sums. Check your final answers using a calculator.

a
$$\begin{array}{r} 3.\square \\ +4.6 \\ \hline \square.3 \end{array}$$

b
$$\begin{array}{r} 8.\square 9 \\ +\ \square.75 \\ \hline \square 4.4\square \end{array}$$

c
$$\begin{array}{r} 1.\square 1 \\ +\ \square\square.11 \\ \hline 11.1\square \end{array}$$

d
$$\begin{array}{r} \square.3\square 6 \\ 2.\square 43 \\ +\ 1.89\square \\ \hline \square 1.395 \end{array}$$

9 How much greater is 262.5 than 76.31?

10 Stuart wants to raise $100 for the Rainbow Club charity. He already has three donations of $30.20, $10.50 and $5.00. How much does Stuart still need to raise?

11 Daily rainfalls for four days over Easter were 12.5 mm, 3.25 mm, 0.6 mm and 32.76 mm. What was the total rainfall over the four-day Easter holiday?

12 Complete the addition table below.

+		0.05		
0.3				1.72
0.75			1.13	
	1.21		1.58	
				3.03

13 Michelle earned $3758.65 working part-time over a 1-year period. However, she was required to pay her parents $20 per week for board for 52 weeks. Michelle also spent $425.65 on clothing and $256.90 on presents for her family and friends during the year. She placed the rest of her money in the bank. How much did Michelle bank for the year?

14 If $a = 2.8$, $b = 1.31$ and $c = 3.928$, find:

a $a + b + c$ **b** $a + b - c$ **c** $c + b - a$ **d** $c - (b + b)$

15 **a** Write down three numbers between 1 and 10, each with 2 decimal places, that would add to 11.16.

b Can you find a solution to part **a** that uses each digit from 1 to 9 exactly once?

ENRICHMENT	—	—	16

Money, money, money…

16 Investigate the following procedures and share your findings with a friend.

a Choose an amount of money that is less than $10.00 (e.g. $3.25).

b Reverse the order of the digits and subtract the smaller number from the larger number (e.g. $5.23 − $3.25 = $1.98).

c Reverse the order of the digits in your new answer and now add this number to your most recent total (e.g. $1.98 + $8.91 = $10.89).

Did you also get $10.89? Repeat the procedure using different starting values.

Try to discover a pattern or a rule. Justify your findings.

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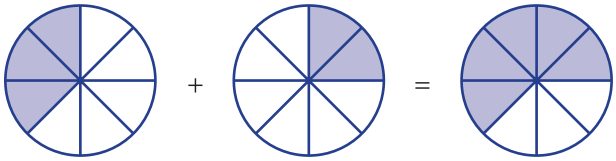
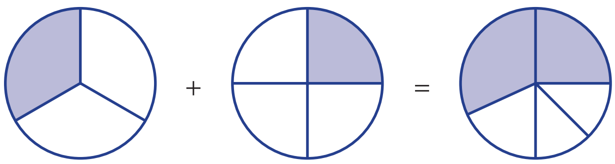
6B Adding fractions

Fractions with the same denominator can be easily added together.



$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

Fractions with different denominators cannot be added together so easily.



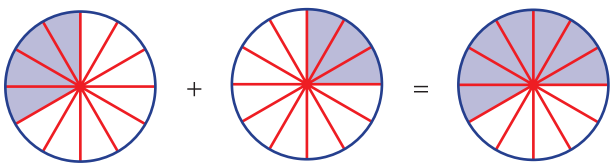
$\frac{1}{3} + \frac{1}{4} = ?$

Note:

$\frac{1}{3} + \frac{1}{4} \neq \frac{1}{7}$

$\frac{1}{3} + \frac{1}{4} \neq \frac{2}{7}$

But with a common denominator it is possible.



$\frac{1}{3} + \frac{1}{4} = ?$

$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

Let's start: 'Like' addition

Pair up with a classmate and discuss the following.

1 Which of the following pairs of numbers can be simply added together without having to carry out any form of conversion?

- **A** 6 goals, 2 goals
- **B** 11 goals, 5 behinds
- **C** 56 runs, 3 wickets
- **D** 6 hours, 5 minutes
- **E** 21 seconds, 15 seconds
- **F** 47 minutes, 13 seconds
- **G** 15 cm, 3 m
- **H** 2.2 km, 4.1 km
- **I** 5 kg, 1680 g
- **J** $\frac{2}{7}, \frac{3}{7}$
- **K** $\frac{1}{4}, \frac{1}{2}$
- **L** $2\frac{5}{12}, 1\frac{1}{3}$

Does it become clear that we can only add pairs of numbers that have the *same* unit? In terms of fractions, we need to have the same ______________?

2 By choosing your preferred unit (when necessary), work out the answer to each of the problems above.

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Key ideas

- Fractions can be simplified easily using addition if they are 'like' fractions; that is, if they have the **same denominator**. This means they have been divided up into the same number of pieces.

Same denominators

- If two or more fractions have the same denominator, to add them together simply add the numerators and keep the denominator. This allows you to find the total number of divided pieces.

Different denominators

- If the denominators are different, we must use our knowledge of equivalent fractions to convert them to fractions with the same **lowest common denominator (LCD)**.
  To do this, carry out these steps.
  1. Find the LCD (often, but not always, found by multiplying denominators).
  2. Convert fractions to their equivalent fractions with the LCD.
  3. Add the numerators and write this total above the LCD.
- After adding fractions, always look to see if your answer needs to be simplified.

Example 3 Adding fractions with the same denominators

Add the following fractions together.

a $\frac{1}{5}+\frac{3}{5}$

b $\frac{3}{11}+\frac{5}{11}+\frac{6}{11}$

SOLUTION

a $\frac{1}{5}+\frac{3}{5}=\frac{4}{5}$

The denominators are the same (like) therefore simply add the numerators.

b $\frac{3}{11}+\frac{5}{11}+\frac{6}{11}=\frac{14}{11}$

Denominators are the same, so add numerators.

$=1\frac{3}{11}$

Simplify answer by converting to a mixed numeral.

Example 4 Adding fractions with different denominators

Add the following fractions together.

a $\frac{1}{5}+\frac{1}{2}$

b $\frac{3}{4}+\frac{5}{6}$

SOLUTION

a $\frac{1}{5}+\frac{1}{2}=\frac{2}{10}+\frac{5}{10}$

$=\frac{7}{10}$

LCD is 10.
Write equivalent fractions with the LCD.
Denominators are the same, so add numerators.

b $\frac{3}{4}+\frac{5}{6}=\frac{9}{12}+\frac{10}{12}$

$=\frac{19}{12}$

$=1\frac{7}{12}$

LCD is 12.
Write equivalent fractions with the LCD.
Denominators are the same, so add numerators.
Simplify answer to a mixed numeral.

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Example 5 Adding mixed numerals

Simplify:

a $3\frac{2}{3} + 4\frac{2}{3}$

b $2\frac{5}{6} + 3\frac{3}{4}$

SOLUTION	EXPLANATION
a Method 1	
$3 + 4 + \frac{2}{3} + \frac{2}{3} = 7 + \frac{4}{3}$	Add the whole number parts together. Add the fraction parts together.
$= 8\frac{1}{3}$	Noting that $\frac{4}{3} = 1\frac{1}{3}$, simplify the answer.
Method 2	
$\frac{11}{3} + \frac{14}{3} = \frac{25}{3}$	Convert mixed numerals to improper fractions. Have the same denominators, so add numerators.
$= 8\frac{1}{3}$	Convert improper fraction back to a mixed numeral.
b Method 1	
$2 + 3 + \frac{5}{6} + \frac{3}{4}$	Add the whole number parts together. LCD of 6 and 4 is 12.
$= 5 + \frac{10}{12} + \frac{9}{12}$	Write equivalent fractions with LCD.
$= 5 + \frac{19}{12}$	Add the fraction parts together.
$= 6\frac{7}{12}$	Noting that $\frac{19}{12} = 1\frac{7}{12}$, simplify the answer.
Method 2	
$\frac{17}{6} + \frac{15}{4} = \frac{34}{12} + \frac{45}{12}$	Convert mixed numerals to improper fractions. Write equivalent fractions with LCD.
$= \frac{79}{12}$	Add the numerators together.
$= 6\frac{7}{12}$	Simplify answer back to a mixed numeral.

Exercise 6B

UNDERSTANDING AND FLUENCY	1, 2, 3–7(½)	4–9(½)	5–9(½)

1 Copy the following sentences into your workbook and fill in the gaps using these words:

lowest	simplified	common	numerators	denominator
check	denominator	denominator	denominator	

a To add two fractions together, they must have the same ______________.

b When adding fractions together, if they have the same ______________, you simply add the ______________.

c When adding two or more fractions where the ______________ are different, you must find the ___________ ___________ ___________.

d After carrying out the addition of fractions, you should always ______________ your answer to see if it can be ______________.

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2 Copy the following sums into your workbook and fill in the empty boxes.

a $\frac{3}{8}+\frac{2}{8}=\frac{\square}{8}$

b $\frac{4}{7}+\frac{1}{7}=\frac{\square}{7}$

c $\frac{1}{3}+\frac{1}{4}$
$=\frac{\square}{12}+\frac{\square}{12}$
$=\frac{\square}{12}$

d $\frac{2}{5}+\frac{3}{4}$
$=\frac{\square}{20}+\frac{\square}{20}$
$=\frac{\square}{20}=1\frac{\square}{20}$

3 State the LCD for the following pairs of 'incomplete' fractions.

a $\frac{}{5}+\frac{}{3}$ **b** $\frac{}{4}+\frac{}{5}$ **c** $\frac{}{2}+\frac{}{3}$ **d** $\frac{}{6}+\frac{}{3}$

e $\frac{}{2}+\frac{}{8}$ **f** $\frac{}{5}+\frac{}{10}$ **g** $\frac{}{7}+\frac{}{11}$ **h** $\frac{}{3}+\frac{}{9}$

i $\frac{}{12}+\frac{}{8}$ **j** $\frac{}{2}+\frac{}{18}$ **k** $\frac{}{15}+\frac{}{10}$ **l** $\frac{}{12}+\frac{}{16}$

4 The following sums have been completed, but only six of them are correct. Copy them into your workbook, then place a tick beside the six correct answers and a cross beside the six incorrect answers.

a $\frac{1}{6}+\frac{3}{6}=\frac{4}{6}$ **b** $\frac{1}{3}+\frac{1}{4}=\frac{2}{7}$ **c** $\frac{2}{5}+\frac{4}{5}=\frac{6}{10}$ **d** $\frac{1}{11}+\frac{3}{11}=\frac{4}{11}$

e $\frac{3}{5}+\frac{4}{5}=1\frac{2}{5}$ **f** $\frac{2}{7}+\frac{2}{7}=\frac{2}{7}$ **g** $\frac{7}{12}+\frac{4}{12}=\frac{11}{12}$ **h** $\frac{4}{9}+\frac{4}{5}=\frac{4}{14}$

i $\frac{3}{10}+\frac{4}{10}=\frac{7}{10}$ **j** $\frac{1}{2}+\frac{2}{5}=\frac{3}{7}$ **k** $2\frac{2}{7}+3\frac{1}{7}=5\frac{3}{7}$ **l** $1\frac{2}{3}+2\frac{1}{5}=3\frac{3}{8}$

Example 3a

5 Add the following fractions.

a $\frac{1}{8}+\frac{4}{8}$ **b** $\frac{2}{7}+\frac{3}{7}$ **c** $\frac{1}{5}+\frac{3}{5}$ **d** $\frac{3}{11}+\frac{6}{11}$

e $\frac{5}{8}+\frac{2}{8}$ **f** $\frac{1}{12}+\frac{6}{12}$ **g** $\frac{3}{15}+\frac{4}{15}$ **h** $\frac{3}{9}+\frac{2}{9}$

Example 3b

i $\frac{6}{7}+\frac{3}{7}$ **j** $\frac{7}{10}+\frac{6}{10}$ **k** $\frac{2}{5}+\frac{3}{5}+\frac{4}{5}$ **l** $\frac{12}{19}+\frac{3}{19}+\frac{8}{19}$

Example 4a

6 Add the following fractions.

a $\frac{1}{2}+\frac{1}{4}$ **b** $\frac{1}{3}+\frac{3}{5}$ **c** $\frac{1}{2}+\frac{1}{6}$ **d** $\frac{1}{4}+\frac{1}{3}$

e $\frac{2}{5}+\frac{1}{4}$ **f** $\frac{1}{5}+\frac{3}{4}$ **g** $\frac{2}{7}+\frac{1}{3}$ **h** $\frac{3}{8}+\frac{1}{5}$

Example 4b

i $\frac{3}{5}+\frac{5}{6}$ **j** $\frac{4}{7}+\frac{3}{4}$ **k** $\frac{8}{11}+\frac{2}{3}$ **l** $\frac{2}{3}+\frac{3}{4}$

Example 5a

7 Simplify:

a $1\frac{1}{5}+2\frac{3}{5}$ **b** $3\frac{2}{7}+4\frac{1}{7}$ **c** $11\frac{1}{4}+1\frac{2}{4}$ **d** $1\frac{3}{9}+4\frac{2}{9}$

e $5\frac{2}{3}+4\frac{2}{3}$ **f** $8\frac{3}{6}+12\frac{4}{6}$ **g** $9\frac{7}{11}+9\frac{7}{11}$ **h** $4\frac{3}{5}+7\frac{4}{5}$

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Example 5b

8 Simplify:

a $2\frac{2}{3}+1\frac{3}{4}$ **b** $5\frac{2}{5}+1\frac{5}{6}$ **c** $3\frac{1}{2}+8\frac{2}{3}$ **d** $5\frac{4}{7}+7\frac{3}{4}$

e $8\frac{1}{2}+6\frac{3}{5}$ **f** $12\frac{2}{3}+6\frac{4}{9}$ **g** $17\frac{8}{11}+7\frac{3}{4}$ **h** $9\frac{7}{12}+5\frac{5}{8}$

9 Simplify, using a calculator.

a $15\frac{3}{8}+8\frac{2}{7}$ **b** $62\frac{1}{9}+17\frac{3}{4}$ **c** $143\frac{1}{3}+56\frac{7}{8}$ **d** $125\frac{3}{10}+134\frac{17}{100}$

PROBLEM-SOLVING AND REASONING	10, 11, 14	11, 12, 14, 15	12–15

10 Myles, Liza and Camillus work at a busy cinema complex. For a particular movie, Myles sells $\frac{3}{5}$ of all the tickets and Liza sells $\frac{1}{3}$.

a What fraction of movie tickets are sold by Myles and Liza, together?

b If all of the movie's tickets are sold, what is the fraction sold by Camillus?

11 Martine loves to run and play. Yesterday, she ran for $2\frac{1}{4}$ kilometres, walked for $5\frac{2}{5}$ kilometres and skipped for $\frac{1}{2}$ a kilometre. What was the total distance that Martine ran, walked and skipped?

12 Jackson is working on a 1000-piece jigsaw puzzle. All the pieces are the same size. After 1 week, he has completed $\frac{1}{10}$ of the puzzle. After 2 weeks he has completed another $\frac{2}{5}$ of the puzzle. In the third week, Jackson completed another $\frac{1}{4}$ of the puzzle.

a By the end of the third week, what fraction of the puzzle has Jackson completed?

b How many pieces of the puzzle does Jackson place in the second week?

c What fraction of the puzzle is still unfinished by the end of the third week? How many pieces is this?

13 A survey of Year 7 students' favourite sport is carried out. A total of 180 students participate in the survey. One-fifth of students reply that netball is their favourite, one-quarter reply rugby and one-third reply soccer. The remainder of students leave the question unanswered.

a What fraction of the Year 7 students answered the survey question?

b What fraction of the Year 7 students left the question unanswered?

c How many students did not answer the survey question?

14 Fill in the empty boxes to make the following fraction computations correct.

a $\frac{1}{\square}+\frac{1}{\square}=\frac{7}{10}$ **b** $\frac{1}{\square}+\frac{1}{\square}+\frac{1}{\square}=\frac{7}{8}$

c $\frac{3}{\square}+\frac{\square}{4}=\frac{17}{20}$ **d** $\frac{2}{\square}+\frac{\square}{3}+\frac{4}{\square}=1$

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15 Four students each read a portion of the same English novel over two nights, for homework. The table shows what fraction of the book was read on each of the two nights.

Student	First night	Second night
Mikhail	$\frac{2}{5}$	$\frac{1}{4}$
Jim	$\frac{1}{2}$	$\frac{1}{10}$
Vesna*	$\frac{1}{4}$	$\frac{1}{5}$
Juliet	$\frac{7}{12}$	$\frac{1}{20}$

*Vesna woke up early on the third morning and read another $\frac{1}{6}$ of the novel before leaving for school.

Place the students in order, from least to most, according to what fraction of the book they had read by their next English lesson.

ENRICHMENT	—	—	16

Raise it to the max, lower it to the min

16 **a** Using the numbers 1, 2, 3, 4, 5 and 6 only once, arrange them in the boxes below to, first, produce the maximum possible answer, and then the minimum possible answer. Work out the maximum and minimum possible answers.

$$\frac{\square}{\square} + \frac{\square}{\square} + \frac{\square}{\square}$$

b Repeat the process for four fractions using the digits 1 to 8 only once each. Again, state the maximum and minimum possible answers.

c Investigate maximum and minimum fraction statements for other sets of numbers and explain your findings.

d Explain how you would arrange the numbers 1 to 100 into 50 different fractions if you were trying to achieve the maximum or minimum sum.

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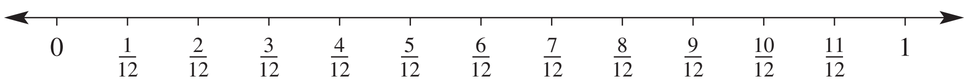
6C Subtracting fractions

Subtracting fractions is very similar to adding fractions. You must establish the **lowest common denominator (LCD)** if one does not exist and this is done through producing equivalent fractions. Then, instead of adding numerators at the final step, you simply carry out the correct subtraction.

Complications can arise when subtracting mixed numerals and **Example 7b** shows the available methods that can be used to overcome such problems.

Let's start: Alphabet subtraction



- Copy into your workbook the number line above.
- Place the following letters in the correct position on the number line.

  $A = \frac{2}{3}$ $B = \frac{5}{12}$ $C = \frac{1}{2}$ $D = \frac{11}{12}$ $E = \frac{1}{12}$ $F = \frac{1}{4}$ $G = \frac{0}{12}$

  $H = \frac{1}{3}$ $I = \frac{7}{12}$ $J = \frac{5}{6}$ $K = \frac{12}{12}$ $L = \frac{3}{4}$ $M = \frac{1}{6}$
- Complete the following alphabet subtractions, giving your answer as a fraction and also the corresponding alphabet letter.

  a J − F **b** A − G **c** D − F − M **d** C − B
  e K − C **f** L − H − E **g** K − J − E **h** L − I − M
- What does A + B + C + D + E + F + G + H + I − J − K − L − M equal?

Key ideas

- Fractions with the same denominator can be easily subtracted.
- When subtracting mixed numerals, you may need to trade a whole.
  For example:

  $7\frac{1}{8} - 2\frac{3}{8}$ $\frac{1}{8}$ is not big enough to have $\frac{3}{8}$ subtracted from it.

  $6\frac{9}{8} - 2\frac{3}{8}$ Therefore, we choose to trade a whole from the 7. (Note: $7\frac{1}{8}$ is equivalent to $6\frac{9}{8}$).
- An alternative method involves first converting both mixed numerals to improper fractions.

  For example: $7\frac{1}{8} - 2\frac{3}{8} = \frac{57}{8} - \frac{19}{8}$

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Example 6 Subtracting fractions

Simplify:

a $\frac{7}{9} - \frac{2}{9}$ **b** $\frac{5}{6} - \frac{1}{4}$

SOLUTION	EXPLANATION
a $\frac{7}{9} - \frac{2}{9} = \frac{5}{9}$	Denominators are the same, therefore we are ready to subtract the second numerator from the first.
b $\frac{5}{6} - \frac{1}{4} = \frac{10}{12} - \frac{3}{12}$ $= \frac{7}{12}$	Need to find the LCD, which is 12. Write equivalent fractions with the LCD. We have the same denominators now, so subtract the second numerator from the first.

Example 7 Subtracting mixed numerals

Simplify:

a $5\frac{2}{3} - 3\frac{1}{4}$ **b** $8\frac{1}{5} - 4\frac{3}{4}$

SOLUTION	EXPLANATION
Method 1: Converting to an improper fraction	
a $5\frac{2}{3} - 3\frac{1}{4} = \frac{17}{3} - \frac{13}{4}$ $= \frac{68}{12} - \frac{39}{12}$	Convert mixed numerals to improper fractions. Need to find the LCD, which is 12. Write equivalent fractions with the LCD.
$= \frac{29}{12}$ $= 2\frac{5}{12}$	We have the same denominators now, so subtract second numerator from the first and convert back to mixed numeral.
b $8\frac{1}{5} - 4\frac{3}{4} = \frac{41}{5} - \frac{19}{4}$ $= \frac{164}{20} - \frac{95}{20}$	Convert mixed numerals to improper fractions. Need to find the LCD, which is 20. Write equivalent fractions with the LCD.
$= \frac{69}{20}$ $= 3\frac{9}{20}$	We have the same denominators now, so subtract second numerator from the first and convert back to mixed numeral.
Method 2: Borrowing a whole number	
a $5\frac{2}{3} - 3\frac{1}{4} = \left(5 + \frac{2}{3}\right) - \left(3 + \frac{1}{4}\right)$	Understand that a mixed numeral is the addition of a whole number and a proper fraction.
$= (5 - 3) + \left(\frac{2}{3} - \frac{1}{4}\right)$	Group whole numbers awnd group proper fractions.
$= 2 + \left(\frac{8}{12} - \frac{3}{12}\right)$	Simplify whole numbers; simplify proper fractions.
$= 2\frac{5}{12}$	Borrowing a whole was not required.

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SOLUTION	EXPLANATION
b $8\frac{1}{5} - 4\frac{3}{4} = \left(8 + \frac{1}{5}\right) - \left(4 + \frac{3}{4}\right)$	$\frac{3}{4}$ cannot be taken away from $\frac{1}{5}$ easily.
$= \left(7 + \frac{6}{5}\right) - \left(4 + \frac{3}{4}\right)$	Therefore, we must borrow a whole.
$= (7 - 4) + \left(\frac{6}{5} - \frac{3}{4}\right)$	Group whole numbers and group proper fractions.
$= 3 + \left(\frac{24}{20} - \frac{15}{20}\right)$	Simplify whole numbers; simplify proper fractions.
$= 3\frac{9}{20}$	Borrowing a whole was required.

Exercise 6C

UNDERSTANDING AND FLUENCY	1, 2–7(½)	4–9(½)	5–9(½)

1 Copy the following sentences into your workbook and fill in the blanks using these words:

simplify	denominator	multiply	multiply

a To subtract one fraction from another, you must have a common _______________.
b One fail-safe method of producing a common denominator is to simply _______________ the two denominators.
c The problem with finding a common denominator that is not the lowest common denominator is that you have to deal with larger numbers and you also need to ____________ your answer at the final step.
d To find the LCD you can _______________ the denominators and then divide by the HCF of the denominators.

2 State the LCD for the following pairs of 'incomplete' fractions.

a $\frac{-}{4} - \frac{-}{6}$ **b** $\frac{-}{2} - \frac{-}{10}$ **c** $\frac{-}{15} - \frac{-}{5}$ **d** $\frac{-}{6} - \frac{-}{9}$

e $\frac{-}{8} - \frac{-}{12}$ **f** $\frac{-}{12} - \frac{-}{20}$ **g** $\frac{-}{14} - \frac{-}{8}$ **h** $\frac{-}{9} - \frac{-}{21}$

3 Copy these equations into your workbook, and fill in the empty boxes.

a $\frac{3}{7} - \frac{2}{7} = \frac{\square}{7}$

b $\frac{8}{13} - \frac{5}{13} = \frac{\square}{13}$

c $\frac{1}{3} - \frac{1}{4}$
$= \frac{\square}{12} - \frac{\square}{12}$
$= \frac{\square}{12}$

d $\frac{4}{5} - \frac{2}{3}$
$= \frac{\square}{15} - \frac{\square}{15}$
$= \frac{\square}{15}$

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4 The following equations have been completed, but only six of them are correct. Copy them into your workbook, then place a tick beside the six correct answers and a cross beside the six incorrect answers.

a $\frac{8}{10} - \frac{5}{10} = \frac{3}{10}$ **b** $\frac{3}{5} - \frac{2}{3} = \frac{1}{2}$ **c** $\frac{5}{12} - \frac{5}{10} = \frac{5}{2}$ **d** $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$

e $\frac{8}{11} - \frac{8}{10} = \frac{0}{1} = 0$ **f** $\frac{12}{15} - \frac{3}{15} = \frac{9}{15}$ **g** $\frac{2}{3} - \frac{2}{3} = 0$ **h** $\frac{5}{7} - \frac{2}{7} = \frac{2}{7}$

i $\frac{3}{20} - \frac{2}{20} = \frac{1}{20}$ **j** $2\frac{5}{9} - 1\frac{4}{9} = 1\frac{1}{9}$ **k** $2\frac{8}{14} - \frac{5}{14} = 2\frac{3}{0}$ **l** $\frac{12}{21} - \frac{7}{11} = \frac{5}{10} = \frac{1}{2}$

Example 6a

5 Simplify:

a $\frac{5}{7} - \frac{3}{7}$ **b** $\frac{4}{11} - \frac{1}{11}$ **c** $\frac{12}{18} - \frac{5}{18}$ **d** $\frac{2}{3} - \frac{1}{3}$

e $\frac{3}{5} - \frac{3}{5}$ **f** $\frac{6}{9} - \frac{2}{9}$ **g** $\frac{5}{19} - \frac{2}{19}$ **h** $\frac{17}{23} - \frac{9}{23}$

i $\frac{84}{100} - \frac{53}{100}$ **j** $\frac{41}{50} - \frac{17}{50}$ **k** $\frac{23}{25} - \frac{7}{25}$ **l** $\frac{7}{10} - \frac{3}{10}$

Example 6b

6 Simplify:

a $\frac{2}{3} - \frac{1}{4}$ **b** $\frac{3}{5} - \frac{1}{2}$ **c** $\frac{3}{5} - \frac{3}{6}$ **d** $\frac{4}{7} - \frac{1}{4}$

e $\frac{1}{2} - \frac{1}{3}$ **f** $\frac{3}{4} - \frac{1}{9}$ **g** $\frac{8}{11} - \frac{1}{3}$ **h** $\frac{4}{5} - \frac{2}{3}$

i $\frac{3}{4} - \frac{5}{8}$ **j** $\frac{11}{20} - \frac{2}{5}$ **k** $\frac{5}{12} - \frac{7}{18}$ **l** $\frac{7}{9} - \frac{2}{3}$

Example 7a

7 Simplify:

a $3\frac{4}{5} - 2\frac{1}{5}$ **b** $23\frac{5}{7} - 15\frac{2}{7}$ **c** $8\frac{11}{14} - 7\frac{9}{14}$ **d** $3\frac{5}{9} - \frac{3}{9}$

e $6\frac{2}{3} - 4\frac{1}{4}$ **f** $5\frac{3}{7} - 2\frac{1}{4}$ **g** $9\frac{5}{6} - 5\frac{4}{9}$ **h** $14\frac{3}{4} - 7\frac{7}{10}$

Example 7b

8 Simplify:

a $5\frac{1}{3} - 2\frac{2}{3}$ **b** $8\frac{2}{5} - 3\frac{4}{5}$ **c** $13\frac{1}{2} - 8\frac{5}{6}$ **d** $12\frac{2}{9} - 7\frac{1}{3}$

e $8\frac{5}{12} - 3\frac{3}{4}$ **f** $1\frac{3}{5} - \frac{7}{9}$ **g** $11\frac{1}{11} - 1\frac{1}{4}$ **h** $6\frac{3}{20} - 3\frac{2}{3}$

9 Simplify, using a calculator.

a $15\frac{1}{5} - 8\frac{4}{5}$ **b** $27\frac{1}{3} - 9\frac{3}{4}$ **c** $\frac{85}{7} - 10\frac{2}{9}$ **d** $421\frac{1}{5} - 213\frac{3}{11}$

PROBLEM-SOLVING AND REASONING	10, 11, 15	11, 12, 15, 16	13, 14, 16, 17

10 Tiffany poured herself a large glass of cordial. She noticed that the cordial jug has $\frac{3}{4}$ of a litre in it before she poured her glass and only $\frac{1}{5}$ of a litre in it after she filled her glass. How much cordial did Tiffany pour into her glass?

11 A family block of chocolate is made up of 60 small squares of chocolate. Marcia eats 10 squares, Jon eats 9 squares and Holly eats 5 squares. What fraction of the block of chocolate is left?

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12 Three friends split a restaurant bill. One pays $\frac{1}{2}$ of the bill and one pays $\frac{1}{3}$ of the bill. What fraction of the bill must the third friend pay?

13 Patty has $23\frac{1}{4}$ dollars, but owes her parents $15\frac{1}{2}$ dollars. How much money does Patty have left after she pays back her parents? Repeat this question using decimals and dollars and cents. Do you get the same answer?

14 Three cakes were served at a birthday party: an ice-cream cake, a chocolate cake and a sponge cake. $\frac{3}{4}$ of the ice-cream cake was eaten. The chocolate cake was cut into 12 equal pieces, of which 9 were eaten. The sponge cake was divided into 8 equal pieces, with only 1 piece remaining.

- **a** What fraction of each cake was eaten?
- **b** What fraction of each cake was left over?
- **c** What was the total amount of cake eaten during the party?
- **d** What was the total amount of cake left over after the party?

15 Fill in the empty boxes to make the following fraction computations correct.

- **a** $\frac{1}{\square} - \frac{1}{\square} = \frac{1}{12}$
- **b** $\frac{\square}{5} - \frac{\square}{2} = \frac{1}{10}$
- **c** $2\frac{\square}{3} - 1\frac{\square}{3} = \frac{2}{3}$
- **d** $8\frac{1}{\square} - 6\frac{\square}{4} = 1\frac{1}{2}$

16 Today David's age is one-seventh of Felicity's age. Felicity is a teenager.

- **a** In 1 year's time David will be one-fifth of Felicity's age. What fraction of her age will he be in 2 years' time?
- **b** How many years must pass until David is one-third of Felicity's age?
- **c** How many years must pass until David is half Felicity's age?

17 Simplify:

- **a** **Example 7** shows two possible methods for subtracting mixed numerals: 'Borrowing a whole number' and 'Converting to an improper fraction'. Simplify the following two expressions and discuss which method is the most suitable for each question.
  - **i** $2\frac{1}{5} - 1\frac{2}{3}$
  - **ii** $27\frac{5}{11} - 23\frac{4}{5}$
- **b** If you have an appropriate calculator, work out how to enter fractions and check your answers to parts **i** and **ii** above.

ENRICHMENT	—	—	18

Letter to an absent friend

18 Imagine that a friend in your class is absent for this lesson on the subtraction of fractions. They were present yesterday and understood the process involved when adding fractions. Your task is to write a letter to your friend, explaining how to subtract mixed numerals. Include some examples, discuss both possible methods but also justify your favourite method. Finish off with three questions for your friend to attempt and include the answers to these questions on the back of the letter.

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6D Multiplying fractions

What does it mean to multiply two fractions together?

Do you end up with a smaller amount or a larger amount when you multiply two proper fractions together?

What does $\frac{1}{3} \times \frac{2}{3}$ equal?

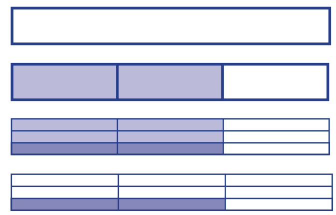
- ***'Strip' method***

  Imagine you have a strip of paper.

  You are told to shade $\frac{2}{3}$ of the strip.

  You are now told to shade in a darker colour $\frac{1}{3}$ of your $\frac{2}{3}$ strip.

  The final amount shaded is your answer.



$\frac{2}{9}$

- ***'Number line' method***

  Consider the number line from 0 to 1 (shown opposite).

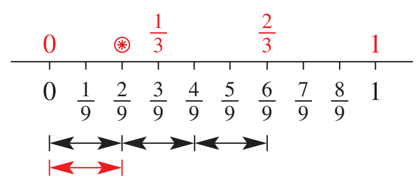
  It is divided into ninths.

  Locate $\frac{2}{3}$.

  Divide this position into three equal pieces (shown as |⟵⟶|).

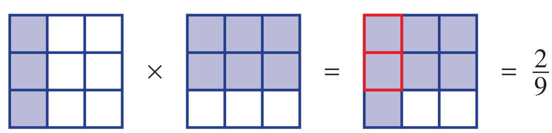
  To locate $\frac{1}{3} \times \frac{2}{3}$ you have only one of the three pieces.

  The final location is your answer (shown as |⟵⟶|); i.e. $\frac{2}{9}$.



- ***'Shading' method***

  Consider $\frac{1}{3}$ of a square multiplied by $\frac{2}{3}$ of a square.



- ***'The rule' method***

  When multiplying fractions, multiply the numerators together and multiply the denominators together.

$$\frac{1}{3} \times \frac{2}{3} = \frac{1 \times 2}{3 \times 3} = \frac{2}{9}$$

Let's start: 'Clock face' multiplication

Explain and discuss the concept of fractions of an hour on the clock face.

In pairs, students match up the following 10 'clock face' multiplication questions with their correct answer. You may like to place a time limit of 5 minutes on the activity.

Discuss answers at the end of the activity.

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Questions	Answers
1. $\frac{1}{2}$ of 4 hours	**A.** 25 minutes
2. $\frac{1}{3}$ of 2 hours	**B.** $1\frac{1}{2}$ hours
3. $\frac{1}{4}$ of 6 hours	**C.** 5 minutes
4. $\frac{1}{3}$ of $\frac{1}{4}$ hour	**D.** $\frac{1}{4}$ hour
5. $\frac{1}{4}$ of $\frac{1}{3}$ hour	**E.** 2 hours
6. $\frac{1}{3}$ of $\frac{3}{4}$ hour	**F.** 2 hours 40 minutes
7. $\frac{1}{10}$ of $\frac{1}{2}$ hour	**G.** $\frac{1}{12}$ hour
8. $\frac{1}{5}$ of $\frac{1}{2}$ hour	**H.** 40 minutes
9. $\frac{2}{3}$ of 4 hours	**I.** $\frac{1}{10}$ hour
10. $\frac{5}{6}$ of $\frac{1}{2}$ hour	**J.** 3 minutes

Key ideas

- Fractions do *not* need to have the same denominator to be multiplied together.
- To multiply fractions, multiply the numerators together and multiply the denominators together.
  - In symbols: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

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Key ideas

- If possible, 'simplify', 'divide' or 'cancel' fractions before multiplying.
  - Cancelling can be done *vertically* or *diagonally*.
  - Cancelling can never be done *horizontally*.

$\frac{3}{5} \times \frac{\cancel{4}^{1}}{\cancel{8}_{2}}$	cancelling vertically	✓
$\frac{^{1}\cancel{3}}{5} \times \frac{4}{\cancel{6}_{2}}$	cancelling diagonally	✓
Never do this! → $\frac{^{1}\cancel{3}}{5} \times \frac{\cancel{6}^{2}}{7}$	cancelling horizontally	✗

- A whole number can be written as a fraction with a denominator of 1.
- 'of', ' × ', 'times', 'lots of' and 'product' all refer to the same mathematical operation of multiplying.
- Mixed numerals must be changed to improper fractions before multiplying.
- Final answers should be written in simplest form.

Example 8 Finding a simple fraction of a quantity

Find:

a $\frac{2}{3}$ of 15 bananas **b** $\frac{3}{10}$ of 50 lollies

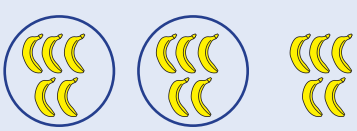
SOLUTION

a $\frac{2}{3}$ of 15 bananas

$\left(\frac{1}{3} \text{ of } 15\right) \times 2 = 10$

EXPLANATION

Divide 15 bananas into 3 equal groups.
Therefore, 5 in each group.
Take 2 of the groups.



Answer is 10 bananas.

SOLUTION

b $\frac{3}{10}$ of 50 lollies

$\left(\frac{1}{10} \text{ of } 50\right) \times 3 = 15$

EXPLANATION

Divide 50 into 10 equal groups.
Therefore, 5 in each group.
Take 3 of the groups.
Therefore, answer is 15 lollies.

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Example 9 Multiplying proper fractions

Find:

a $\frac{2}{3} \times \frac{1}{5}$ **b** $\frac{3}{4} \times \frac{8}{9}$ **c** $\frac{4}{8}$ of $\frac{3}{6}$

SOLUTION	EXPLANATION
a $\frac{2}{3} \times \frac{1}{5} = \frac{2 \times 1}{3 \times 5} = \frac{2}{15}$	Multiply the numerators together. Multiply the denominators together. The answer is in simplest form.
b $\frac{3}{4} \times \frac{8}{9} = \frac{{}^{1}\cancel{3} \times \cancel{8}^{2}}{{}^{1}\cancel{4} \times \cancel{9}^{3}} = \frac{2}{3}$	Cancel first. Then multiply numerators together and denominators together.
c $\frac{4}{8}$ of $\frac{3}{6} = \frac{4}{8} \times \frac{3}{6} = \frac{{}^{1}\cancel{4} \times \cancel{3}^{1}}{{}^{2}\cancel{8} \times \cancel{6}^{2}} = \frac{1}{4}$	Change 'of' to multiplication sign. Cancel and then multiply the numerators and the denominators. The answer is in simplest form.

Example 10 Multiplying proper fractions by whole numbers

Find:

a $\frac{1}{3} \times 21$ **b** $\frac{2}{5}$ of 32

SOLUTION	EXPLANATION
a $\frac{1}{3} \times 21 = \frac{1}{{}^{1}\cancel{3}} \times \frac{\cancel{21}^{7}}{1} = \frac{7}{1} = 7$	Rewrite 21 as a fraction with a denominator equal to 1. Cancel and then multiply numerators and denominators. $7 \div 1 = 7$
b $\frac{2}{5}$ of $32 = \frac{2}{5} \times \frac{32}{1} = \frac{64}{5} = 12\frac{4}{5}$	Rewrite 'of' as a multiplication sign. Write 32 as a fraction. Multiply numerators and denominators. Convert answer to a mixed numeral.

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Example 11 Multiplying improper fractions

Find:

a $\frac{5}{3} \times \frac{7}{2}$

b $\frac{8}{5} \times \frac{15}{4}$

SOLUTION

a $\frac{5}{3} \times \frac{7}{2} = \frac{5 \times 7}{3 \times 2}$

$= \frac{35}{6} = 5\frac{5}{6}$

b $\frac{8}{5} \times \frac{15}{4} = \frac{^{2}\cancel{8} \times \cancel{15}^{3}}{_{1}\cancel{5} \times \cancel{4}_{1}}$

$= \frac{6}{1} = 6$

EXPLANATION

Multiply the numerators together.
Multiply the denominators together.
Convert the answer to a mixed numeral.

Cancel first.
Multiply 'cancelled' numerators together and 'cancelled' denominators together.
Write the answer in simplest form.

Example 12 Multiplying mixed numerals

Find:

a $2\frac{1}{3} \times 1\frac{2}{5}$

b $6\frac{1}{4} \times 2\frac{2}{5}$

SOLUTION

a $2\frac{1}{3} \times 1\frac{2}{5} = \frac{7}{3} \times \frac{7}{5}$

$= \frac{49}{15}$

$= 3\frac{4}{15}$

b $6\frac{1}{4} \times 2\frac{2}{5} = \frac{^{5}\cancel{25}}{_{1}\cancel{4}} \times \frac{\cancel{12}^{3}}{\cancel{5}_{1}}$

$= \frac{15}{1}$

$= 15$

EXPLANATION

Convert mixed numerals to improper fractions.
Multiply numerators together.
Multiply denominators together.

Write the answer in simplest form.

Convert to improper fractions.
Simplify fractions by cancelling.
Multiply numerators and denominators together.

Write the answer in simplest form.

Exercise 6D

UNDERSTANDING AND FLUENCY	1–5, 6–8(½), 9, 10(½)	5, 6–8(½), 9, 10(½)	6–8(½), 9, 10(½), 11

1 a Label each of the following as proper, improper or mixed.

i $\frac{2}{5}$ ii $1\frac{1}{3}$ iii $\frac{7}{5}$ iv $\frac{1}{3}$ v $\frac{5}{3}$ vi $1\frac{9}{10}$

b Copy these sentences into your workbook and fill in the blanks.

i A proper fraction has a value that is between ________ and ________.

ii An improper fraction is always greater than __________.

iii A mixed numeral consists of two parts, a __________ __________ part and a __________ ______________ part.

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2 Use a calculator to investigate the following question. When multiplying a whole number by a proper fraction, do you get a smaller or larger answer when compared with the whole number? Explain your answer.

3 Copy into your workbook the grid shown opposite.

- **a** On your diagram, shade in blue $\frac{1}{3}$ of the grid.
- **b** Now shade in red $\frac{1}{4}$ of the shaded blue.
- **c** You have now shaded $\frac{1}{4}$ of $\frac{1}{3}$. What fraction is this of the original grid?

Example 8

4 Calculate the following, using a diagram or a drawing if necessary.

- **a** $\frac{1}{3}$ of 12 lollies
- **b** $\frac{1}{5}$ of 10 pencils
- **c** $\frac{2}{3}$ of 18 donuts
- **d** $\frac{3}{4}$ of 16 boxes
- **e** $\frac{3}{8}$ of 32 dots
- **f** $\frac{3}{7}$ of 21 triangles

5 There are four methods shown below for finding the product of one-half and one-fifth. Only one of them is correct. Copy the correct method into your workbook.

A $\frac{1}{2} \times \frac{1}{5}$
$= \frac{1+1}{2+5}$
$= \frac{2}{7}$

B $\frac{1}{2} \times \frac{1}{5}$
$= \frac{1 \times 1}{2 \times 5}$
$= \frac{2}{10}$

C $\frac{1}{2} \times \frac{1}{5}$
$= \frac{5}{10} \times \frac{2}{10}$
$= \frac{7}{20}$

D $\frac{1}{2} \times \frac{1}{5}$
$= \frac{1 \times 1}{2 \times 5}$
$= \frac{1}{10}$

Example 10

6 Find:

- **a** $\frac{1}{3}$ of 18
- **b** $\frac{1}{5}$ of 45
- **c** $\frac{2}{3}$ of 24
- **d** $\frac{3}{5}$ of 25
- **e** $\frac{2}{7}$ of 42
- **f** $\frac{1}{4}$ of 16
- **g** $\frac{4}{5}$ of 100
- **h** $\frac{3}{7}$ of 77

Example 9

7 Evaluate:

- **a** $\frac{3}{4} \times \frac{1}{5}$
- **b** $\frac{2}{7} \times \frac{1}{3}$
- **c** $\frac{2}{3} \times \frac{5}{7}$
- **d** $\frac{4}{9} \times \frac{2}{5}$
- **e** $\frac{2}{3} \times \frac{3}{5}$
- **f** $\frac{4}{7} \times \frac{1}{4}$
- **g** $\frac{3}{4} \times \frac{1}{3}$
- **h** $\frac{5}{9} \times \frac{9}{11}$
- **i** $\frac{3}{6} \times \frac{5}{11}$
- **j** $\frac{2}{3} \times \frac{4}{8}$
- **k** $\frac{8}{11} \times \frac{3}{4}$
- **l** $\frac{2}{5} \times \frac{10}{11}$
- **m** $\frac{2}{7}$ of $\frac{3}{5}$
- **n** $\frac{3}{4}$ of $\frac{2}{5}$
- **o** $\frac{5}{10}$ of $\frac{4}{7}$
- **p** $\frac{6}{9}$ of $\frac{3}{12}$

Example 11

8 Find:

- **a** $\frac{5}{2} \times \frac{7}{3}$
- **b** $\frac{6}{5} \times \frac{11}{7}$
- **c** $\frac{6}{4} \times \frac{11}{5}$
- **d** $\frac{9}{6} \times \frac{13}{4}$
- **e** $\frac{8}{5} \times \frac{10}{3}$
- **f** $\frac{21}{4} \times \frac{8}{6}$
- **g** $\frac{10}{7} \times \frac{21}{5}$
- **h** $\frac{14}{9} \times \frac{15}{7}$

Example 12

9 Find:

- **a** $1\frac{3}{5} \times 2\frac{1}{3}$
- **b** $1\frac{1}{7} \times 1\frac{2}{9}$
- **c** $3\frac{1}{4} \times 2\frac{2}{5}$
- **d** $4\frac{2}{3} \times 5\frac{1}{7}$

10 Find:

- **a** $\frac{6}{5} \times \frac{8}{3}$
- **b** $\frac{1}{2} \times \frac{3}{8}$
- **c** $\frac{3}{4}$ of $5\frac{1}{3}$
- **d** $7\frac{1}{2} \times 4\frac{2}{5}$
- **e** $\frac{3}{7}$ of $\frac{2}{3}$
- **f** $1\frac{1}{2} \times 2\frac{1}{4}$
- **g** $\frac{8}{9} \times \frac{6}{20}$
- **h** $\frac{15}{4} \times \frac{8}{5}$

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11 Repeat Question **10**, using a calculator. Don't forget to estimate your answer first, so that you can pick up any calculator keystroke errors.

PROBLEM-SOLVING AND REASONING	12, 13, 16	13, 14, 16, 17	13–15, 17, 18

12 At a particular secondary college, $\frac{2}{5}$ of the Year 7 students are boys.

a What fraction of the Year 7 students are girls?

b If there are 120 Year 7 students, how many boys and girls are there?

13 To paint one classroom, $2\frac{1}{3}$ litres of paint are required.

How many litres of paint are required to paint five identical classrooms?

14 A scone recipe requires $1\frac{3}{4}$ cups of self-raising flour and $\frac{3}{4}$ of a cup of cream. James is catering for a large group and needs to quadruple the recipe. How much self-raising flour and how much cream will he need?

15 Julie has finished an injury-plagued netball season during which she was able to play only $\frac{2}{3}$ of the matches. The season consisted of 21 matches. How many games did Julie miss as a result of injury?

16 Not all of the following fraction equations are correct. Copy them into your workbook, then place a tick beside those that are correct and a cross beside those that are wrong. Provide the correct solution for those you marked as incorrect.

a $\frac{1}{3}+\frac{1}{4}=\frac{1}{7}$

b $\frac{1}{3}+\frac{1}{4}=\frac{1}{12}$

c $\frac{1}{3}\times\frac{1}{4}=\frac{2}{7}$

d $\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$

e $\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$

f $\frac{1}{3}-\frac{1}{4}=\frac{0}{-1}$

17 Circle the correct alternative for the following statement and justify your answer. Using an example, explain why the other alternatives are incorrect.

When multiplying a proper fraction by another proper fraction the answer is…

A a whole number **B** a mixed numeral **C** an improper fraction **D** a proper fraction

18 Write two fractions that:

a multiply to $\frac{3}{5}$

b multiply to $\frac{3}{4}$

c multiply to $\frac{1}{7}$

ENRICHMENT	—	—	19

Who are we?

19 **a** Using the clues provided, work out which two fractions are being discussed.

- We are two proper fractions.
- Altogether we consist of four different digits.
- When added together our answer will still be a proper fraction.
- When multiplied together you could carry out some cancelling.
- The result of our product, when simplified, contains no new digits from our original four.
- Three of our digits are prime numbers and the fourth digit is a cube number.

b Design your own similar question and develop a set of appropriate clues. Ask a classmate to answer your question.

c Design the ultimate challenging 'Who are we?' question. Make sure that there is only one possible answer.

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6E Multiplying and dividing decimals by 10, 100, 1000 etc.

In this section, we will be multiplying decimals by numbers such as 10, 100, 1000 etc. We will be using the phrase ‘powers of 10’ to represent numbers, such as those shown opposite.

> 10 to the power of 1 $= 10^1 = 10$
> 10 to the power of 2 $= 10^2 = 10 \times 10 = 100$
> 10 to the power of 3 $= 10^3 = 10 \times 10 \times 10 = 1000$

Note that 10 to the power of 3 is 1000, which contains three zeros.

This pattern continues, so 10 to the power of 6 is written as 1 followed by 6 zeros.

Let’s start: Does the decimal point really ‘move’?

Consider the number 2.58.

Working from left to right:

- The digit 2 is in the units column.
- The decimal point sits between the units and the tenths, as it always does.
- The digit 5 is in the tenths column.
- The digit 8 is in the hundredths column.

Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths
		2	.	5	8	

Now, imagine that you buy 10 items for $2.58 each. The cost is $25.80.

- Did the digits change?
- Did the decimal point move? If so, which way and how many places?
- Or was it that the digits moved and the decimal point stayed still?
- If so, which way did the digits move? By how many places?

Key ideas

- Every number contains a decimal point but it is usually not shown in integers.
  For example: 345 is 345.0 and 2500 is 2500.0.
- Extra zeros can be added in the columns to the right of the decimal point without changing the value of the decimal.
  For example: $12.5 = 12.50 = 12.500 = 12.5000$ etc.
- When a decimal is **multiplied** by 10 (which is 10 to the power of 1), **the decimal point stays still** and the digits all move 1 place to the left.
  However, it is easier to ‘visualise the decimal point moving’ 1 place to the **right**.
  For example: $23.758 \times 10 = 237.58$

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Key ideas

Operation	Visualisation	Example
Multiplying a decimal by 1000 (10 to the power of 3)	Decimal point moves 3 places to the right.	$23.758 \times 1000 = 23758$
Multiplying a decimal by 100 (10 to the power of 2)	Decimal point moves 2 places to the right.	$23.758 \times 100 = 2375.8$
Multiplying a decimal by 10 (10 to the power of 1)	Decimal point moves 1 place to the right.	$23.758 \times 10 = 237.58$
Dividing a decimal by 10 (10 to the power of 1)	Decimal point moves 1 place to the left.	$23.758 \div 10 = 2.3758$
Dividing a decimal by 100 (10 to the power of 2)	Decimal point moves 2 places to the left.	$23.758 \div 100 = 0.23758$
Dividing a decimal by 1000 (10 to the power of 3)	Decimal point moves 3 places to the left.	$23.758 \div 1000 = 0.023758$

Example 13 Multiplying by 10, 100, 1000 etc

Evaluate:

a 36.532×100

b 4.31×10000

SOLUTION

a $36.532 \times 100 = 3653.2$

b $4.31 \times 10000 = 43100$

EXPLANATION

100 has two zeros, therefore the decimal point appears to move 2 places to the right. 36.532

Decimal point appears to move 4 places to the right and additional zeros are inserted as necessary. 4.3100

Example 14 Dividing by 10, 100, 1000 etc

Evaluate:

a $268.15 \div 10$

b $7.82 \div 1000$

SOLUTION

a $268.15 \div 10 = 26.815$

b $7.82 \div 1000 = 0.00782$

EXPLANATION

10 has one zero, therefore the decimal point appears to move 1 place to the left. 268.15

Decimal point appears to move 3 places to the left and additional zeros are inserted as necessary. 007.82

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Example 15 Working with the 'missing' decimal point

Evaluate:

a 567×10000 **b** $23 \div 1000$

SOLUTION

a $567 \times 10000 = 5670000$

EXPLANATION

If no decimal point is shown in the question, it must be at the very end of the number. Four additional zeros must be inserted to move the invisible decimal point 4 places to the right. 5670000.

b $23 \div 1000 = 0.023$

Decimal point appears to move 3 places to the left. 0.023

Example 16 Evaluating using order of operations

Calculate this expression, using the order of operations:

$426 \div 100 + 10(0.43 \times 10 - 1.6)$

SOLUTION

$$\begin{aligned} 426 \div 100 &+ 10(0.43 \times 10 - 1.6) \\ &= 4.26 + 10(4.3 - 1.6) \\ &= 4.26 + 10 \times 2.7 \\ &= 4.26 + 27 \\ &= 31.26 \end{aligned}$$

EXPLANATION

First, we must calculate the brackets.

The division by 100 can also be done in the first step.

$10(4.3 - 2.6)$ means $10 \times (4.3 - 2.6)$.

Exercise 6E

UNDERSTANDING AND FLUENCY	1–3, 4–7(½)	3, 4–7(½)	4–7(½)

1 Fill in the correct number of zeros in the multiplier to make the following product statements correct. The first one has been done for you.

a $56.321 \times 1\boxed{0\,0} = 5632.1$

b $27.9234 \times 1\boxed{} = 27923.4$

c $0.03572 \times 1\boxed{} = 3.572$

d $3200 \times 1\boxed{} = 320000000$

2 Fill in the correct number of zeros in the divisor to make the following division statements correct. The first one has been done for you.

a $2345.1 \div 1\boxed{0\,0\,0} = 2.3451$

b $7238.4 \div 1\boxed{} = 72.384$

c $0.00367 \div 1\boxed{} = 0.000367$

d $890 \div 1\boxed{} = 0.0089$

3 a How many places and in what direction must the decimal point in the number move if the following operations occur?

i $\times 100$ **ii** $\div 10$ **iii** $\times 1000000$ **iv** $\div 1$

v $\div 1000$ **vi** $\times 1000$ **vii** $\times 10$ **viii** $\div 10000000$

b If all of the operations above had taken place on a number, one after the other, what would be the final position of the decimal place relative to its starting position?

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Example 13

4 Calculate:

a 4.87×10 **b** 35.283×10 **c** 422.27×10
d 14.304×100 **e** 5.69923×1000 **f** 1.25963×100
g 12.7×1000 **h** 154.23×1000 **i** 0.34×10000
j 213.2×10 **k** 867.1×100000 **l** 0.00516×100000000

Example 14

5 Calculate:

a $42.7 \div 10$ **b** $353.1 \div 10$ **c** $24.422 \div 10$
d $5689.3 \div 100$ **e** $12135.18 \div 1000$ **f** $93261.1 \div 10000$
g $2.9 \div 100$ **h** $13.62 \div 10000$ **i** $0.54 \div 1000$
j $36.7 \div 100$ **k** $0.02 \div 10000$ **l** $1000.04 \div 100000$

6 Calculate:

a 22.913×100 **b** 0.03167×1000 **c** $4.9 \div 10$
d $22.2 \div 100$ **e** 6348.9×10000 **f** $1.0032 \div 1000$

Example 15

7 Calculate:

a 156×100 **b** 43×1000 **c** $2251 \div 10$
d $16 \div 1000$ **e** 2134×100 **f** $2134 \div 100$
g $7 \div 1000$ **h** 99×100000 **i** $34 \div 10000$

PROBLEM-SOLVING AND REASONING	8(½), 9, 13	9–11, 13, 14	10–12, 14, 15

Example 16

8 Calculate the following, using the order of operations.

a $1.56 \times 100 + 24 \div 10$ **b** $16 \div 100 + 32 \div 10$
c $3 + 10(24 \div 100 + 8)$ **d** $10(6.734 \times 100 + 32)$
e $35.4 + 4.2 \times 10 - 63.4 \div 10$ **f** $4.7 - 24 \div 10 + 0.52 \times 10$
g $14 \div 100 + 1897 \div 1000$ **h** $78.1 - 10(64 \div 100 + 5)$

9 A service station charges \$1.47 per litre of petrol. How much will it cost Tanisha to fill her car with 100 litres of petrol?

10 A large bee farm produces 1200 litres of honey per day.

a If there are 1000 millilitres in 1 litre, how many millilitres of honey can the farm's bees produce in one day?

b The farm's honey is sold in 100-millilitre jars. How many jars of honey can the farm's bees fill in one day?

11 Wendy is on a mobile phone plan that charges her 3 cents per text message. On average, Wendy sends 10 text messages per day. What will it cost Wendy for 100 days of sending text messages at this rate? Give your answer in cents and then convert your answer to dollars.

12 Darren wishes to purchase 10000 shares at \$2.12 per share. Given that there is also an additional \$200 brokerage fee, how much will it cost Darren to purchase the shares?

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13 The weight of a matchstick is 0.00015 kg. Find the weight of 10 000 boxes of matches, with each box containing 100 matches. The weight of one empty match box is 0.0075 kg.

14 Complete the table below, listing at least one possible combination of operations that would produce the stated answer from the given starting number. Justify your answers to a friend.

Starting number	Answer	Possible two-step operations
12.357	1235.7	$\times$ 1000, $\div$ 10
34.0045	0.0340045	
0.003601	360.1	
bac.dfg	*ba.cdfg*	$\div$ 100, $\times$ 10
d.swkk	*dswkk*	
fwy	*f.wy*	

15 The number 12 345.6789 undergoes a series of multiplication and division operations by different powers of 10. The first four operations are: $\div$ 1000, $\times$ 100, $\times$ 10 000 and $\div$ 10. What is the fifth and final operation if the final number is 1.23456789?

ENRICHMENT	—	—	16

Standard form

16 Extremely large numbers and extremely small numbers are often written in a more practical way, known as standard form or scientific notation.

For example, the distance from the Earth to the Sun is 150 000 000 kilometres! The distance of 150 million kilometres can be written in standard form as 1.5×10^8 kilometres.

1.5×10^8 indicates that the decimal place needs to be moved 8 places to the right.

$$\begin{aligned} 1.5 \times 10^8 &= 1.5 \times 100\,000\,000 \\ &= 150\,000\,000. \end{aligned}$$

a Represent these numbers in standard form.
  i 50 000 000 000 000 **ii** 42 000 000 **iii** 12 300 000 000 000 000
b Use a calculator to evaluate the following.
  i 40 000 000 000 $\times$ 500 000 000 **ii** 9 000 000 $\times$ 120 000 000 000 000
c The distance from the Earth to the Sun is stated above as 150 million kilometres. The more precise figure is 149 597 892 kilometres. Research how astronomers can calculate the distance so accurately. Hint: It is linked to the speed of light.
d Carry out further research on very large numbers. Create a list of 10 very large numbers (e.g. distance from Earth to Pluto, the number of grains in 1 kg of sand, the number of stars in the galaxy, the number of memory bytes in a terabyte …). Rank your 10 large numbers in ascending order.
e How are very small numbers, such as 0.000000000035, represented in standard form?
f Represent the following numbers in standard form.
  i 0.000001 **ii** 0.0000000009 **iii** 0.000000000007653

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6F Multiplying by a decimal

There are countless real-life applications that involve the multiplication of decimal numbers. For example, finding the area of a block of land that is 34.5 m long and 5.2 m wide, or pricing a 4.5-hour job at a rate of \$21.75 per hour. In general, the procedure for multiplying decimal numbers is the same as multiplying whole numbers. There is, however, one extra final step, which involves placing the decimal point in the correct position in the answer.

Let's start: Multiplication musings

Consider the following questions within your group.

- What happens when you multiply by a number that is less than 1?
- Consider the product 15×0.75. Will the answer be more or less than 15? Why?
- Estimate an answer to 15×0.75.
- What is the total number of decimal places in the numbers 15 and 0.75?
- Calculate 15×0.75. How many decimal places are there in the answer?

Key ideas

- When multiplying decimals, start by ignoring any decimal points and perform the multiplication as you would normally. On arriving at your answer, you must now place the decimal point in the correct position.
- The product of three-tenths and seven-hundredths is twenty-one thousandths. In figures:
  $\frac{3}{10} \times \frac{7}{100} = \frac{21}{1000}$
  Here is the same computation, using decimals: $0.3 \times 0.07 = 0.021$
  The first decimal, 0.3, has one decimal place.
  The other decimal, 0.07, has two decimal places.
  The result, 0.021, has three decimal places.
- The correct position of the decimal point in the answer is found by following the rule that the total number of decimal places in the question must equal the number of decimal places in the answer.

  For example: 5.34×1.2 ← 3 decimal places in the question

  $$\begin{array}{r} 534 \\ \times \quad 12 \\ \hline 1068 \\ 5340 \\ \hline 6408 \end{array}$$

  decimal points ignored here

  $5.34 \times 1.2 = 6.408$ 3 decimal places in the answer
- It is always worthwhile estimating your answer. This allows you to check that your decimal point is in the correct place and that your answer makes sense.
- When multiplying by multiples of 10, initially ignore the zeros in the multiplier and any decimal points and perform routine multiplication. On arriving at your answer, position your decimal point, remembering to move your decimal point according to the rules of multiplying by powers of 10.

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Example 17 Multiplying decimals

Calculate:

a 12.31 × 7 **b** 3.63 × 6.9

SOLUTION	EXPLANATION
a $\begin{array}{r} 1231 \\ \times \quad 7 \\ \hline 8617 \end{array}$	Perform multiplication, ignoring decimal point. There are 2 decimal places in the question, so there will be 2 decimal places in the answer.
12.31 × 7 = 86.17	Estimation is less than 100 (≈ 12 × 7 = 84).
b $\begin{array}{r} 363 \\ \times \ 69 \\ \hline 3267 \\ 21780 \\ \hline 25047 \end{array}$	Ignore both decimal points. Perform routine multiplication. Total of 3 decimal places in the question, so there must be 3 decimal places in the answer.
3.63 × 6.9 = 25.047	Estimation is less than 28 (≈ 4 × 7 = 28).

Example 18 Multiplying decimals by multiples of 10

Calculate:

a 2.65 × 40 000 **b** 0.032 × 600

SOLUTION	EXPLANATION
a 2.65 × 40 000 = 106 000	Ignore the decimal point and zeros. Multiply 265 × 4.
$\begin{array}{r} 265 \\ \times \quad 4 \\ \hline 1060 \end{array}$	Position the decimal point in your answer. There are 2 decimal places in the question, so must have 2 decimal places in the answer.
∴ 10.60 × 10 000 = 106 000.	Move the decimal point 4 places to the right because there are four zeros in 10 000.
b 0.032 × 600 = 19.2	Ignore the decimal point and zeros. Multiply 32 × 6.
$\begin{array}{r} 32 \\ \times 6 \\ \hline 192 \end{array}$	Position decimal point in the answer. Shift decimal place 2 places to the right because there are two zeros in 600.
∴ 0.192 × 100 = 19.2	

Exercise 6F

UNDERSTANDING AND FLUENCY	1(½), 2, 3, 4–6(½)	3, 4–6(½), 7	5–6(½), 7, 8

1 Work out the total number of decimal places in each of the following product statements.

a 4 × 6.3 **b** 3.52 × 76 **c** 42 × 5.123
d 8.71 × 11.2 **e** 5.283 × 6.02 **f** 2.7 × 10.3
g 4.87 × 3241.21 **h** 0.003 × 3 **i** 0.00103 × 0.0045

2 Insert the decimal point into each of the following answers so that the multiplication is true.

a 6.4 × 3 = 192 **b** 6.4 × 0.3 = 192 **c** 0.64 × 0.3 = 192

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3 Why is it worthwhile to estimate an answer to a multiplication question involving decimals?

Example 17

4 Calculate:

a 5.21×4	b 3.8×7	c 22.93×8
d 14×7.2	e 3×72.82	f 1.293×12
g 3.4×6.8	h 5.4×2.3	i 0.34×16
j 43.21×7.2	k 0.023×0.042	l 18.61×0.071

Example 18

5 Calculate:

a 2.52×40	b 6.9×70	c 31.75×800
d 1.4×7000	e 3000×4.8	f 7.291×50000
g 0.0034×200	h 0.0053×70000	i 3.004×30

6 Calculate and then round your answer to the nearest dollar. Check your answers using a calculator.

a $5 \times \$6.30$	b $3 \times \$7.55$	c $4 \times \$18.70$
d $\$1.45 \times 12$	e $\$30.25 \times 4.8$	f $7.2 \times \$5200$
g $34.2 \times \$2.60$	h $0.063 \times \$70.00$	i $0.085 \times \$212.50$

7 a What is the difference between a decimal point and a decimal place?
b How many decimal points and how many decimal places are in the number 423.1567?

8 Copy and complete the rule for multiplying decimal numbers (see the Key Ideas in the section).
The total number of decimal places ________________________ must equal the number of ________________________ in the answer.

PROBLEM-SOLVING AND REASONING	9, 10, 14	10–12, 14, 15	11–13, 15–17

9 Anita requires 4.21 m of material for each dress she is making. She is planning to make a total of seven dresses. How much material does she need?

10 The net weight of a can of spaghetti is 0.445 kg. Find the net weight of eight cans of spaghetti.

11 Jimbo ran 5.35 km each day for the month of March. How many kilometres did he run for the month?

12 Bernard is making a cubby house for his children. He needs 32 lengths of timber, each 2.1 metres long.
a What is the total length of timber needed to build the cubby house?
b What is the cost of the timber if the price is \$2.95 per metre?

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13 A lawyer charges $125.00 per hour to assist her client. How much does the lawyer charge the client if she works on the job for 12.25 hours?

14 According to its manufacturer, a particular car can travel 14.2 km on 1 litre of petrol.

- **a** How far could the car travel on 52 litres of petrol?
- **b** The car has 23.4 litres of fuel in the tank and must complete a journey of 310 km. Will it make the journey without refuelling?
- **c** If the car does make the journey, how much petrol is left in the tank at the end of the trip? If the car doesn't make the journey, how many extra litres of fuel are needed?

15 Write down two numbers, each with 2 decimal places, that when multiplied by 1.83 will give an answer between 0.4 and 0.5.

16 Write down one number with 4 decimal places that when multiplied by 345.62 will give an answer between 1 and 2.

17
- **a** If $68 \times 57 = 3876$, what is the answer to 6.8×5.7? Why?
- **b** If $23 \times 32 = 736$, what is the answer to 2.3×32? Why?
- **c** If $250 \times 300 = 75\,000$, what is the answer to 2.5×0.3? Why?
- **d** What is 7×6? What is the answer to 0.7×0.6? Why?

ENRICHMENT	—	—	18

Creating a simple cash register

18 Using a spreadsheet program, such as Excel, design a user-friendly cash register interface. You must be able to enter up to 10 different items into your spreadsheet. You will need a quantity column and a cost per item column.

Using appropriate formulae, the total cost of the bill should be displayed, and there should then be room to enter the amount of money paid and, if necessary, what change should be given.

When your spreadsheet is set up, enter the following items.

4 chocolate bars @$1.85 each
3 loaves of bread @$3.19 each
newspaper @$1.40
2×2 litres of milk @$3.70 each
washing powder @$8.95
toothpaste @$4.95
2 kg sausages @$5.99 per kg
tomato sauce @$3.20
2 packets of Tim Tams @$3.55 each
5×1.25 litres of soft drink @$0.99 each
Money paid $= \$80.00$

If your program is working correctly, the amount of change given should be $13.10.

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6G Dividing fractions

Interactive Widgets HOTsheets Walkthrough

Remember that division used to be referred to as 'how many'.

Thinking of division as 'how many' helps us to understand dividing fractions.

For example, to find $\frac{1}{2} \div \frac{1}{4}$, think of $\frac{1}{2}$ how many $\frac{1}{4}$ s, or how many $\frac{1}{4}$ s are in a $\frac{1}{2}$?

Consider this strip of paper that is divided into four equal sections.

In our example of $\frac{1}{2} \div \frac{1}{4}$, we have only $\frac{1}{2}$ a strip, so we will shade in half the strip.

By thinking of the ÷ sign as 'how many', the question is asking how many quarters are in half the strip. From our diagram, we can see that the answer is 2. Therefore, $\frac{1}{2} \div \frac{1}{4} = 2$

In a game of football, when it is half-time, you have played two quarters. This is another way of confirming that $\frac{1}{2} \div \frac{1}{4} = 2$.

Let's start: 'Divvy up' the lolly bag

To 'divvy up' means to divide up or divide out or share equally.

Consider a lolly bag containing 24 lollies. In pairs, students answer the following questions.

- How many lollies would each person get if you 'divvy up' the lollies between three people?
- If you got $\frac{1}{3}$ of the lollies in the bag, how many did you get?

Can you see that 'divvying up' by 3 is the same as getting $\frac{1}{3}$? Therefore, $\div 3$ is the same as $\times \frac{1}{3}$.

- How many lollies would each person get if you 'divvy up' the lollies between eight people?
- If you got $\frac{1}{8}$ of the lollies in the bag, how many did you get?

Can you see that 'divvying up' by 8 is the same as getting $\frac{1}{8}$? Therefore, $\div 8$ is the same as $\times \frac{1}{8}$.

- What do you think is the same as dividing by n?
- What do you think is the same as dividing by $\frac{a}{b}$?

Key ideas

- To find the **reciprocal** of a fraction, you must **invert** the fraction. This is done by swapping the numerator and the denominator. 'Inverting' is sometimes known as turning the fraction upside down or flipping the fraction.
  - The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

  For example: The reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.

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Key ideas

- Dividing by a number is the same as multiplying by its reciprocal.

  For example: $15 \div 3 = 5$ and $15 \times \frac{1}{3} = 5$.
  - Dividing by 2 is the same as multiplying by $\frac{1}{2}$.
- When asked to divide by a fraction, instead choose to multiply by the fraction's reciprocal.

  Therefore, to divide by $\frac{a}{b}$ we multiply by $\frac{b}{a}$.
- When dividing, mixed numerals must be changed to improper fractions.

Example 19 Finding reciprocals

State the reciprocal of the following.

a $\frac{2}{3}$ **b** 5 **c** $1\frac{3}{7}$

SOLUTION	EXPLANATION
a Reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.	The numerator and denominator are swapped.
b Reciprocal of 5 is $\frac{1}{5}$.	Think of 5 as $\frac{5}{1}$ and then invert.
c Reciprocal of $1\frac{3}{7}$ is $\frac{7}{10}$.	Convert $1\frac{3}{7}$ to an improper fraction; i.e. $\frac{10}{7}$, and then invert.

Example 20 Dividing a fraction by a whole number

Find:

a $\frac{5}{8} \div 3$ **b** $2\frac{3}{11} \div 5$

SOLUTION

a
$$\frac{5}{8} \div 3 = \frac{5}{8} \times \frac{1}{3} = \frac{5}{24}$$

b
$$2\frac{3}{11} \div 5 = \frac{25}{11} \div \frac{5}{1} = \frac{\overset{5}{\cancel{25}}}{11} \times \frac{1}{\underset{1}{\cancel{5}}} = \frac{5}{11}$$

EXPLANATION

a Change the ÷ sign to a × sign and invert the 3.
Multiply the numerators and denominators.

b Convert the mixed numeral to an improper fraction.
Write 5 as an improper fraction.
Change the ÷ sign to a × sign and invert the divisor.
Simplify by cancelling.
Multiply numerators and denominators.

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Example 21 Dividing a whole number by a fraction

Find:

a $6 \div \frac{1}{3}$

b $24 \div \frac{3}{4}$

SOLUTION

a $6 \div \frac{1}{3} = \frac{6}{1} \times \frac{3}{1}$
$= \frac{18}{1} = 18$

b $24 \div \frac{3}{4} = \frac{{}^{8}\cancel{24}}{1} \times \frac{4}{\cancel{3}^{1}}$
$= 32$

EXPLANATION

Instead of $\div \frac{1}{3}$, change to $\times \frac{3}{1}$.

Simplify.

Instead of $\div \frac{3}{4}$, change to $\times \frac{4}{3}$.

Cancel and simplify.

Example 22 Dividing fractions by fractions

Find:

a $\frac{3}{5} \div \frac{3}{8}$

b $2\frac{2}{5} \div 1\frac{3}{5}$

SOLUTION

a $\frac{3}{5} \div \frac{3}{8} = \frac{3}{5} \times \frac{8}{3}$
$= \frac{8}{5} = 1\frac{3}{5}$

b $2\frac{2}{5} \div 1\frac{3}{5} = \frac{12}{5} \div \frac{8}{5}$
$= \frac{{}^{3}\cancel{12}}{{}_{1}\cancel{5}} \times \frac{\cancel{5}^{1}}{\cancel{8}^{2}}$
$= \frac{3}{2} = 1\frac{1}{2}$

EXPLANATION

Change the ÷ sign to a × sign and invert the divisor. (Note: The divisor is the second fraction.)

Cancel and simplify.

Convert mixed numerals to improper fractions.

Change the ÷ sign to a × sign and invert the divisor.

Cancel, multiply and simplify.

Exercise 6G

UNDERSTANDING AND FLUENCY	1–5, 6–9(½)	4(½), 5, 6–11(½)	5, 6–11(½)

1 Write down the reciprocal of each of the following.

a $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{2}{7}$ **d** $\frac{1}{3}$

e 6 **f** $\frac{3}{2}$ **g** $\frac{7}{4}$ **h** $2\frac{1}{2}$

2 Complete the following pattern:

$24 \div 4 =$ ____

$24 \div 2 =$ ____

$24 \div 1 =$ ____

$24 \div \frac{1}{2} =$ ____ $(24 \div \frac{1}{2} = 24 \times$ ____ $)$

$24 \div \frac{1}{4} =$ ____ $(24 \div \frac{1}{4} = 24 \times$ ____ $)$

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3 Which of the following is the correct first step for finding $\frac{3}{5} \div \frac{4}{7}$?

A $\frac{3}{5} \times \frac{7}{4}$ **B** $\frac{5}{3} \times \frac{4}{7}$ **C** $\frac{5}{3} \times \frac{7}{4}$

4 Write the correct first step for each of these division questions. (Do not go on and find the final answer.)

a $\frac{5}{11} \div \frac{3}{5}$ **b** $\frac{1}{3} \div \frac{1}{5}$ **c** $\frac{7}{10} \div \frac{12}{17}$ **d** $\frac{8}{3} \div 3$

e $6 \div \frac{3}{4}$ **f** $7 \div \frac{1}{2}$ **g** $\frac{5}{4} \div \frac{1}{2}$ **h** $\frac{3}{5} \div \frac{3}{7}$

5 When dividing mixed numerals, the first step is to convert to improper fractions and the second step is to multiply by the reciprocal of the divisor. Write the correct first and second steps for each of the following mixed numeral division questions. (Do not go on and find the final answer.)

a $2\frac{1}{2} \div 1\frac{1}{3}$ **b** $24 \div 3\frac{1}{5}$ **c** $4\frac{3}{11} \div 5\frac{1}{4}$ **d** $\frac{8}{3} \div 11\frac{3}{7}$

Example 19 **6** State the reciprocal of each of the following.

a $\frac{5}{7}$ **b** $\frac{3}{5}$ **c** $\frac{2}{9}$ **d** $\frac{1}{8}$

e $2\frac{1}{3}$ **f** $4\frac{3}{5}$ **g** $1\frac{5}{6}$ **h** $8\frac{2}{3}$

i 12 **j** 101 **k** $\frac{1}{9}$ **l** 1

Example 20 **7** Find:

a $\frac{3}{4} \div 2$ **b** $\frac{5}{11} \div 3$ **c** $\frac{8}{5} \div 4$ **d** $\frac{15}{7} \div 3$

e $2\frac{1}{4} \div 3$ **f** $5\frac{1}{3} \div 4$ **g** $12\frac{4}{5} \div 8$ **h** $1\frac{13}{14} \div 9$

Example 21 **8** Find:

a $5 \div \frac{1}{4}$ **b** $7 \div \frac{1}{3}$ **c** $10 \div \frac{1}{10}$ **d** $24 \div \frac{1}{5}$

e $12 \div \frac{2}{5}$ **f** $15 \div \frac{3}{8}$ **g** $14 \div \frac{7}{2}$ **h** $10 \div \frac{3}{2}$

Example 22 **9** Find:

a $\frac{2}{7} \div \frac{2}{5}$ **b** $\frac{1}{5} \div \frac{1}{4}$ **c** $\frac{3}{7} \div \frac{6}{11}$ **d** $\frac{2}{3} \div \frac{8}{9}$

e $2\frac{1}{4} \div 1\frac{1}{3}$ **f** $4\frac{1}{5} \div 3\frac{3}{10}$ **g** $12\frac{1}{2} \div 3\frac{3}{4}$ **h** $9\frac{3}{7} \div 12\frac{4}{7}$

10 Find:

a $\frac{3}{8} \div 5$ **b** $22 \div \frac{11}{15}$ **c** $2\frac{2}{5} \div 1\frac{3}{4}$ **d** $\frac{3}{4} \div \frac{9}{4}$

e $7 \div \frac{1}{4}$ **f** $2\frac{6}{15} \div 9$ **g** $7\frac{2}{3} \div 1\frac{1}{6}$ **h** $\frac{3}{5} \div \frac{2}{7}$

11 Repeat Question **10**, using a calculator. Don't forget to estimate your answer first, so that you pick up any calculator keystroke errors.

PROBLEM-SOLVING AND REASONING	12, 13, 17	13–15, 17, 18	14–16, 18–20

12 Make each sentence correct, by inserting the word *more* or *less* in the gap.

a $10 \div 2$ gives an answer that is _________ than 10.

b $10 \div \frac{1}{2}$ gives an answer that is _________ than 10.

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c $\frac{3}{4} \div \frac{2}{3}$ gives an answer that is ________ than $\frac{3}{4}$.

d $\frac{3}{4} \times \frac{3}{2}$ gives an answer that is ________ than $\frac{3}{4}$.

e $\frac{5}{7} \div \frac{8}{5}$ gives an answer that is ________ than $\frac{5}{7}$.

f $\frac{5}{7} \times \frac{5}{8}$ gives an answer that is ________ than $\frac{5}{7}$.

13 If $2\frac{1}{4}$ leftover pizzas are to be shared between three friends, what fraction of pizza will each friend receive?

14 A property developer plans to subdivide $7\frac{1}{2}$ acres of land into blocks of at least $\frac{3}{5}$ of an acre. Through some of the land runs a creek, where a protected species of frog lives. How many of the blocks can the developer sell if two blocks must be reserved for the creek and its surroundings?

15 Miriam cuts a 10-millimetre sisal rope into four equal pieces. If the rope is $3\frac{3}{5}$ metres long before it is cut, how long is each piece?

16 A carpenter takes $\frac{3}{4}$ of an hour to make a chair. How many chairs can he make in 6 hours?

17 Justin is a keen runner and regularly runs at a pace of $3\frac{1}{2}$ minutes per kilometre. Justin finished a Sunday morning run in 77 minutes. How far did he run?

18 Pair up the equivalent expressions and state the simplified answer.

$\frac{1}{2}$ of 8	$12 \div 4$	$10 \times \frac{1}{2}$	$10 \div 2$
$3 \div \frac{1}{2}$	$12 \times \frac{1}{4}$	$\frac{1}{2} \div \frac{1}{8}$	3×2

19 Find:

a $\frac{3}{8} \times \frac{4}{5} \div \frac{2}{3}$ **b** $\frac{3}{8} \div \frac{4}{5} \div \frac{2}{3}$ **c** $\frac{3}{8} \div \frac{4}{5} \times \frac{2}{3}$ **d** $\frac{3}{8} \times \frac{4}{5} \times \frac{2}{3}$

20 a A car travels 180 kilometres in $1\frac{1}{2}$ hours. How far will it travel in 2 hours if it travels at a constant speed?

b A different car took $2\frac{1}{4}$ hours to travel 180 kilometres. How far did it travel in 2 hours, if it maintained a constant speed?

ENRICHMENT	—	—	21

You provide the question

21 Listed below are six different answers.

You are required to make up six questions that will result in the following six answers.

All questions must involve a division sign. Your questions should increase in order of difficulty by adding extra operation signs and extra fractions.

a Answer 1: $\frac{3}{5}$ **b** Answer 2: $2\frac{1}{3}$ **c** Answer 3: $\frac{7}{1}$

d Answer 4: 0 **e** Answer 5: $\frac{1}{100}$ **f** Answer 6: $4\frac{4}{5}$

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6H Dividing decimals

Similar to multiplication of decimal numbers, there are countless real-life applications that involve the division of decimal numbers. However, unlike multiplying decimal numbers, where we basically ignore the decimal points until the very end of the question, with division we try to manipulate the question in such a way as to prevent dividing by a decimal number.

Terminology reminders:

Example: $24 \div 4 = 6$ or $\frac{24}{4} = 6$ or $\begin{array}{r} 6 \\ 4\overline{)24} \end{array}$

24 is known as the **dividend** (the amount you have or the number being divided), 4 is known as the **divisor** (the number doing the dividing) and 6 is known as the **quotient** (or the answer).

Let's start: Division decisions

Consider the following questions within your group.

- What happens when you divide by a number that is less than 1?
- Consider the answer of $10 \div 0.2$. Will the answer be more or less than 10? Why?
- Estimate an answer to $10 \div 0.2$.
- Calculate the answer of $100 \div 2$. How does this compare to the answer of $10 \div 0.2$?
- Can you think of an easier way to calculate $21.464 \div 0.02$?

Key ideas

- **Division of decimal numbers by whole numbers**
  - Complete as you would normally with any other division question.
  - The decimal point in the quotient (answer) goes directly above the decimal point in the dividend.

  For example: $60.524 \div 4$

  $$\begin{array}{r} 15.131 \\ 4\overline{)6{}^{2}0.5{}^{1}24} \end{array}$$

- **Division of decimal numbers by other decimals**
  - Change the divisor into a whole number.
  - Whatever change is made to the divisor must also be made to the dividend.

    For example: $24.562 \div 0.02$

    $24.562 \div 0.02 = 2456.2 \div 2$
  - When dividing by multiples of 10, initially ignore the zeros in the divisor and perform routine division. On arriving at your answer, you must then re-position your decimal point according to the rules of dividing by powers of 10. For each zero in the question that you ignored initially, the decimal point must move 1 place to the left.

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Example 23 Dividing decimals by whole numbers

Calculate:

a $42.837 \div 3$ **b** $0.0234 \div 4$

SOLUTION

a 14.279

$$3\overline{)4^{1}2.8^{2}3^{2}7}\quad = 14.279$$

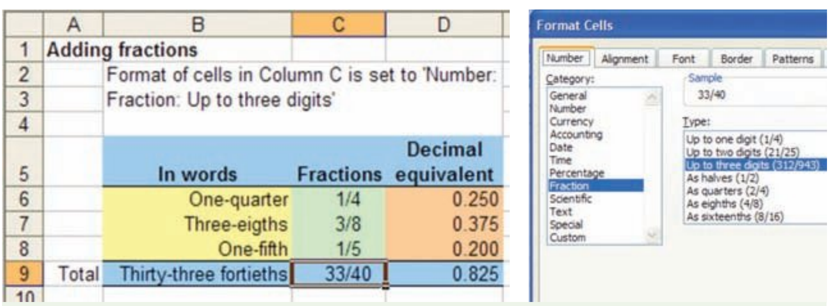
b 0.00585

$$4\overline{)0.02^{2}3^{3}4^{2}0}\quad = 0.00585$$

EXPLANATION

a Carry out division, remembering that the decimal point in the answer is placed directly above the decimal point in the dividend.

b Remember to place zeros in the answer every time the divisor 'doesn't go'. Again, align the decimal place in the answer directly above the decimal place in the question.
An additional zero is required at the end of the dividend to terminate the decimal answer.



Basic arithmetic calculators automatically treat fractions as division operations and convert them to decimals, but mathematical calculators and spreadsheets can be set to work with fractions.

Example 24 Dividing decimals by decimals

Calculate:

a $62.316 \div 0.03$ **b** $0.03152 \div 0.002$

SOLUTION

a $62.316 \div 0.03$
$= 6231.6 \div 3 = 2077.2$

$$3\overline{)62^{2}3^{2}1.6}\quad = 2077.2$$

b $0.03152 \div 0.002$
$= 31.52 \div 2 = 15.76$

$$2\overline{)3^{1}1.^{1}5^{1}2}\quad = 15.76$$

EXPLANATION

a Need to divide by a whole number.
$62.316 \div 0.03$
Move each decimal point 2 places to the right. Carry out the division question $6231.6 \div 3$.

b Multiply divisor and dividend by 1000.
$0.03152 \div 0.002$
Move each decimal point 3 places to the right. Carry out the division question $31.52 \div 2$.

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Example 25 Dividing decimals by multiples of 10

Calculate 67.04 ÷ 8000.

SOLUTION

$$8\overline{)67.04}^{\;08.38}$$

8.38 ÷ 1000 = 0.00838
67.04 ÷ 8000 = 0.00838

EXPLANATION

Ignore the three zeros in the 8000.
Divide 67.04 by 8.
Now divide by 1000, resulting in moving the decimal point 3 places to the left.

Example 26 Evaluating using order of operations

Calculate using the order of operations:
3.8 − 1.6 × 0.45 + 5 ÷ 0.4

SOLUTION

3.8 − 1.6 × 0.45 + 5 ÷ 0.4
= 3.8 − 0.72 + 12.5
= 3.08 + 12.5
= 15.58

EXPLANATION

First carry out × and ÷, working from left to right.
Then carry out + and −, working from left to right.

Exercise 6H

UNDERSTANDING AND FLUENCY	1–3, 4(½), 5, 6(½), 7–9	4(½), 5, 6(½), 7, 8, 9(½)	5, 6(½), 7, 9(½), 10(½)

1 For the question 36.52 ÷ 0.4 = 91.3, which of the following options uses the correct terminology?
- **A** 36.52 is the divisor, 0.4 is the dividend and 91.3 is the quotient.
- **B** 36.52 is the dividend, 0.4 is the divisor and 91.3 is the quotient.
- **C** 36.52 is the quotient, 0.4 is the dividend and 91.3 is the divisor.
- **D** 36.52 is the divisor, 0.4 is the quotient and 91.3 is the dividend.

2 Copy and complete so that each statement is correct
- **a** 9.6 ÷ 0.2 = ______ ÷ 2
- **b** 10.64 ÷ 0.5 = ______ ÷ 5
- **c** 0.064 ÷ 0.2 = ______ ÷ 2
- **d** 15.639 ÷ 0.3 = ______ ÷ 3
- **e** 15.639 ÷ 0.03 = ______ ÷ 3

3 For each of the following pairs of numbers, move the decimal points the same number of places so that the second number becomes a whole number.
- **a** 3.2456, 0.3
- **b** 120.432, 0.12
- **c** 0.00345, 0.0001
- **d** 1234.12, 0.004

Example 23

4 Calculate:
- **a** 8.4 ÷ 2
- **b** 30.5 ÷ 5
- **c** 64.02 ÷ 3
- **d** 2.822 ÷ 4
- **e** 4.713 ÷ 3
- **f** 2.156 ÷ 7
- **g** 38.786 ÷ 11
- **h** 1491.6 ÷ 12
- **i** 0.0144 ÷ 6
- **j** 234.21 ÷ 2
- **k** 3.417 ÷ 5
- **l** 0.01025 ÷ 4

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5 Explain where you place the decimal point in the quotient (i.e. answer), when dividing a decimal by a whole number.

Example 24

6 Calculate:

a $6.14 \div 0.2$ **b** $23.25 \div 0.3$ **c** $2.144 \div 0.08$
d $5.1 \div 0.6$ **e** $0.3996 \div 0.009$ **f** $45.171 \div 0.07$
g $0.0032 \div 0.04$ **h** $0.04034 \div 0.8$ **i** $10.78 \div 0.011$
j $4.003 \div 0.005$ **k** $0.948 \div 1.2$ **l** $432.2 \div 0.0002$

7 Calculate:

a $1200 \div 20$ **b** $120 \div 2$ **c** $12 \div 0.2$ **d** $1.2 \div 0.02$
e Explain why these questions all give the same answer.

8 Design three decimal division questions for your partner. Make sure you calculate the answer to each question. Swap questions with your partner. Go to work solving your partner's questions. Pass your answers back for your partner to correct. Discuss any mistakes made by either person.

Example 25

9 Calculate:

a $236.14 \div 200$ **b** $413.35 \div 50$ **c** $3.71244 \div 300$
d $0.846 \div 200$ **e** $482.435 \div 5000$ **f** $0.0313 \div 40$

10 Calculate the following, rounding your answers to 2 decimal places. Check your answers using a calculator.

a 35.5 kg ÷ 3 **b** \$213.25 ÷ 7 **c** 182.6 m ÷ 0.6 m
d 287 g ÷ 1.2 **e** 482.523 L ÷ 0.5 **f** \$5235.50 ÷ 9

PROBLEM-SOLVING AND REASONING	11–12(½), 17	12–14, 17, 18	14–16, 18, 19

Example 26

11 Calculate the following, using the order of operations. Check your answers using a calculator.

a $3.68 \div 2 + 5.7 \div 0.3$
b $6(3.7 \times 2.8 + 5.2)$
c $17.83 - 1.2(8.1 - 2.35)$
d $9.81 \div 0.9 + 75.9 \div 10$
e $(56.7 - 2.4) \div (0.85 \div 2 + 0.375)$
f $34.5 \times 2.3 + 15.8 \div (0.96 - 0.76)$

12 Find the missing digits in these division questions.

a 0.□ □ over 3)2. 6 7
b 0. 6 4 over 3)1.□2
c 2.□ 5 over □)10. 7 □
d 2. 1 4 □ over □)15. □ 2 9

13 Charlie paid \$12.72 to fill her ride-on lawnmower with 8 L of fuel. What was the price per litre of the fuel that she purchased?

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14 Dibden is a picture framer and has recently purchased 214.6 m of timber. The average-sized picture frame requires 90 cm (0.9 m) of timber. How many average picture frames could Dibden make with his new timber?

15 A water bottle can hold 600 mL of water. How many water bottles can be filled from a large drink container that can hold 16 L?

16 Six friends go out for dinner. At the end of the evening, the restaurant's bill is $398.10.

- **a** As the bill is split equally among the six friends, how much does each person pay?
- **b** Given that the friends are happy with the food and service, they decide to round the amount they each pay to $70. What is the waiter's tip?

17 Clara purchases 1.2 kg of apples for $3.90. Her friend Sophia buys 900 g of bananas for $2.79 at the same shop. Find the cost per kilogram of each fruit. Which type of fruit is the best value in terms of price per kilogram?

18 A police radar gun measures a car to be 231.5 m away. At 0.6 seconds later, the radar gun measures the same car to be 216.8 m away.

- **a** Determine the speed of the car in metres per second (m/s).
- **b** Multiply your answer to part **a** by 3.6 to convert your answer to km/h.
- **c** The car is travelling along an 80 km/h stretch of road. Is the car speeding?

19 Given that $24.53 \times 1.97 = 48.3241$, write down the value of each of the following questions, without using a calculator.

- **a** $48.3241 \div 1.97$
- **b** $48.3241 \div 2.453$
- **c** $4832.41 \div 1.97$
- **d** $483.241 \div 245.3$
- **e** $0.483241 \div 0.197$
- **f** $483\,241 \div 2453$

ENRICHMENT	—	—	20

What number am I?

20 I am thinking of a number. Given the following clues for each, find the number.

- **a** When I add 4.5 and then multiply by 6, the answer is 30.
- **b** When I divide it by 3 and then add 2.9, the answer is 3.
- **c** When I multiply it by 100 and then add 9, the answer is 10.
- **d** When I multiply it by 5 and then add a half, the answer is 6.
- **e** When I subtract 0.8, then divide by 0.2 and then divide by 0.1, the answer is 200.
- **f** Make up three of your own number puzzles to share with the class.

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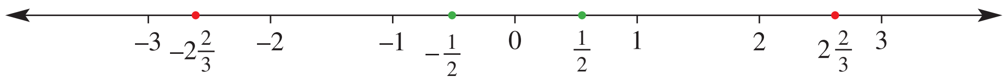
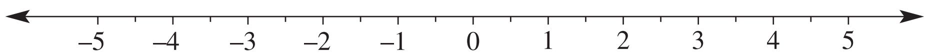
6I Computation with negative fractions EXTENSION

The English mathematician John Wallis (1616–1703) invented a number line that displayed numbers extending in both the positive and negative directions.

So, just as we can have negative integers, we can also have negative fractions. In fact, each positive fraction has an opposite (negative) fraction. Two examples are highlighted on the number line below.



Let's start: Where do you end up?



You are given a starting point and a set of instructions to follow. You must determine where the finishing point is. The first set of instructions reviews the addition and subtraction of integers. The other two sets involve the addition and subtraction of positive and negative fractions.

- Starting point is +1. Add 3, subtract 5, add −2, subtract −4, subtract 3.

  Finishing point = ☐

- Starting point is 0. Subtract $\frac{3}{5}$, add $\frac{1}{5}$, add $-\frac{4}{5}$, subtract $\frac{2}{5}$, subtract $-\frac{3}{5}$.

  Finishing point = ☐

- Starting point is $\frac{1}{2}$. Subtract $\frac{3}{4}$, add $-\frac{1}{3}$, subtract $-\frac{1}{2}$, subtract $\frac{1}{12}$, add $\frac{1}{6}$.

  Finishing point = ☐

Key ideas

- The techniques for adding, subtracting, multiplying or dividing positive fractions also apply to negative fractions.
- The arithmetic rules we observed for integers (Chapter 1) also apply to fractions.
- Subtracting a larger positive fraction from a smaller positive fraction will result in a negative fraction.

  For example: $\frac{1}{5} - \frac{2}{3} = \frac{3}{15} - \frac{10}{15} = -\frac{7}{15}$

- Adding a negative fraction is equivalent to subtracting its opposite.

  For example: $\frac{1}{2} + \left(-\frac{1}{3}\right) = \frac{1}{2} - \left(+\frac{1}{3}\right) = \frac{1}{2} - \frac{1}{3}$

- Subtracting a negative fraction is equivalent to adding its opposite.

  For example: $\frac{1}{2} - \left(-\frac{1}{3}\right) = \frac{1}{2} + \left(+\frac{1}{3}\right) = \frac{1}{2} + \frac{1}{3}$

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Key ideas

- The product or quotient of two fractions of the same sign (positive or negative) is a positive fraction.
  - Product: $\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$ or $-\frac{1}{3} \times \left(-\frac{2}{5}\right) = \frac{2}{15}$
  - Quotient: $\frac{2}{15} \div \frac{1}{3} = \frac{2}{5}$ or $-\frac{2}{15} \div \left(-\frac{1}{3}\right) = \frac{2}{5}$
- The product or quotient of two fractions of the opposite sign (positive and negative) is a negative fraction.
  - Product: $\frac{1}{2} \times \left(-\frac{1}{4}\right) = -\frac{1}{8}$ or $-\frac{1}{2} \times \frac{1}{4} = -\frac{1}{8}$
  - Quotient: $\frac{1}{8} \div \left(-\frac{1}{2}\right) = -\frac{1}{4}$ or $-\frac{1}{8} \div \frac{1}{2} = -\frac{1}{4}$

Example 27 Adding and subtracting negative fractions

Evaluate:

a $\frac{2}{7} + \left(-\frac{5}{7}\right)$ **b** $\frac{2}{3} - \left(-\frac{4}{3}\right)$ **c** $\frac{1}{5} + \left(-\frac{1}{4}\right)$ **d** $-\frac{7}{3} - \left(-3\frac{2}{3}\right)$

SOLUTION

a $\frac{2}{7} + \left(-\frac{5}{7}\right) = \frac{2}{7} - \frac{5}{7}$
$= -\frac{3}{7}$

EXPLANATION

Adding $-\frac{5}{7}$ is equivalent to subtracting $\frac{5}{7}$.

b $\frac{2}{3} - \left(-\frac{4}{3}\right) = \frac{2}{3} + \frac{4}{3}$
$= \frac{6}{3} = 2$

Subtracting $-\frac{4}{3}$ is equivalent to adding $\frac{4}{3}$.

c $\frac{1}{5} + \left(-\frac{1}{4}\right) = \frac{1}{5} - \frac{1}{4}$
$= \frac{4}{20} - \frac{5}{20}$
$= -\frac{1}{20}$

Adding $-\frac{1}{4}$ is equivalent to subtracting $\frac{1}{4}$.
The LCM of 5 and 4 is 20.
Write equivalent fractions with LCD of 20.
Subtract the numerators.

d $-\frac{7}{3} - \left(-3\frac{2}{3}\right) = -\frac{7}{3} + 3\frac{2}{3}$
$= -\frac{7}{3} + \frac{11}{3}$
$= \frac{4}{3} = 1\frac{1}{3}$

Subtracting $-3\frac{2}{3}$ is equivalent to adding $3\frac{2}{3}$.
Convert mixed numeral to improper fraction.
Denominators are the same, therefore add numerators $-7 + 11 = 4$.

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Example 28 Multiplying with negative fractions

Evaluate:

a $\frac{2}{3} \times \left(-\frac{4}{5}\right)$

b $-\frac{6}{5} \times \left(-\frac{3}{4}\right)$

SOLUTION

a $\frac{2}{3} \times \left(-\frac{4}{5}\right) = -\frac{8}{15}$

b $-\frac{6}{5} \times \left(-\frac{3}{4}\right) = \frac{6}{5} \times \frac{3}{4}$

$= \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$

EXPLANATION

The two fractions are of opposite sign, so the answer is a negative.

The two fractions are of the same sign, so the answer is a positive.

Cancel where possible, then multiply numerators and multiply denominators.

Example 29 Dividing with negative fractions

Evaluate:

a $-\frac{2}{5} \div \left(-\frac{3}{4}\right)$

b $-1\frac{1}{3} \div 3$

SOLUTION

a $-\frac{2}{5} \div \left(-\frac{3}{4}\right) = -\frac{2}{5} \times \left(-\frac{4}{3}\right)$

$= \frac{2}{5} \times \frac{4}{3}$

$= \frac{8}{15}$

b $-1\frac{1}{3} \div 3 = -\frac{4}{3} \times \frac{1}{3}$

$= -\frac{4}{9}$

EXPLANATION

The reciprocal of $\left(-\frac{3}{4}\right)$ is $\left(-\frac{4}{3}\right)$.

The two fractions are of the same sign, so the answer is a positive.

The answer is in simplest form.

The reciprocal of 3 is $\frac{1}{3}$.

The two numbers are of opposite sign, so the answer is a negative.

Exercise 6I EXTENSION

UNDERSTANDING AND FLUENCY	1–3, 4–7(½)	3, 4–7(½), 8	4–7(½), 8

1 Draw a number line from −4 to 4 and on it mark the following negative and positive fractions.

a $-\frac{1}{4}$ **b** $\frac{1}{4}$

c $1\frac{1}{2}$ **d** $-2\frac{1}{2}$

e $-3\frac{4}{5}$ **f** $-\frac{7}{3}$

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2 Complete these sentences.

a Adding $\left(-\frac{1}{4}\right)$ is equivalent to subtracting ______.

b Adding $\frac{1}{3}$ is equivalent to subtracting ______.

c Subtracting $\left(-\frac{3}{5}\right)$ is equivalent to adding ______.

d Subtracting $\frac{2}{7}$ is equivalent to adding ______.

3 Do not evaluate the following expressions. Simply state whether the answer will be positive or negative.

a $-\frac{3}{5} \times \left(-\frac{1}{3}\right)$ **b** $-5\frac{1}{5} \times \frac{9}{11}$ **c** $\frac{5}{3} \div \left(-\frac{3}{5}\right)$ **d** $-2\frac{1}{7} \div \left(-8\frac{1}{3}\right)$

Example 27a,b

4 Evaluate:

a $-\frac{6}{7} + \frac{2}{7}$ **b** $-\frac{3}{5} + \frac{4}{5}$ **c** $-\frac{5}{9} - \frac{2}{9}$ **d** $-\frac{11}{3} - \frac{5}{3}$

e $\frac{1}{3} + \left(-\frac{2}{3}\right)$ **f** $\frac{1}{5} + \left(-\frac{3}{5}\right)$ **g** $\frac{1}{4} - \left(-\frac{5}{4}\right)$ **h** $\frac{3}{11} - \left(-\frac{4}{11}\right)$

Example 27c,d

5 Evaluate:

a $\frac{1}{4} + \left(-\frac{1}{3}\right)$ **b** $\frac{3}{7} + \left(-\frac{4}{5}\right)$ **c** $\frac{1}{2} - \left(-\frac{3}{5}\right)$ **d** $\frac{2}{9} - \left(-\frac{2}{3}\right)$

e $-\frac{3}{2} - \left(-\frac{5}{4}\right)$ **f** $-\frac{5}{8} - \left(-\frac{3}{4}\right)$ **g** $-\frac{7}{5} - \left(-1\frac{1}{4}\right)$ **h** $-\frac{8}{3} - \left(-2\frac{2}{5}\right)$

Example 28

6 Evaluate:

a $\frac{3}{5} \times \left(-\frac{4}{7}\right)$ **b** $-\frac{2}{5} \times \frac{8}{11}$ **c** $-\frac{1}{3} \times \left(-\frac{4}{5}\right)$ **d** $-\frac{5}{9} \times \left(-\frac{3}{2}\right)$

e $-\frac{3}{9} \times \frac{4}{7}$ **f** $\frac{2}{6} \times \left(-\frac{3}{8}\right)$ **g** $-1\frac{1}{2} \times \left(-\frac{2}{7}\right)$ **h** $-\frac{3}{8} \times 3\frac{1}{5}$

Example 29

7 Evaluate:

a $-\frac{5}{7} \div \frac{3}{4}$ **b** $-\frac{1}{4} \div \frac{5}{9}$ **c** $-\frac{2}{3} \div \left(-\frac{5}{4}\right)$ **d** $-\frac{4}{9} \div \left(-\frac{1}{3}\right)$

e $-\frac{4}{7} \div 2$ **f** $-\frac{3}{5} \div 4$ **g** $-1\frac{1}{2} \div (-2)$ **h** $-5\frac{1}{3} \div \left(-2\frac{2}{9}\right)$

8 Evaluate, using a calculator. Estimate your answer first.

a $\frac{13}{5} - \left(-\frac{17}{5}\right)$ **b** $-\frac{7}{13} + \left(-4\frac{2}{3}\right)$ **c** $-\frac{24}{33} \times \left(-\frac{15}{40}\right)$ **d** $-2\frac{3}{7} \div 5\frac{4}{5}$

PROBLEM-SOLVING AND REASONING	9, 10, 13	10–14	10–12, 14–16

9 Arrange these fractions from smallest to largest.

$\frac{3}{4}, -\frac{1}{2}, -\frac{5}{3}, -\frac{3}{4}, -1\frac{1}{2}, \frac{1}{16}, -\frac{1}{5}, 3\frac{1}{10}$

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10 Toolapool has an average maximum temperature of $13\frac{1}{2}$°C and an average minimum temperature of $-3\frac{1}{4}$°C. The average temperature range is calculated by subtracting the average minimum temperature from the average maximum temperature. What is the average temperature range for Toolapool?

11 Xaio aims to get 8 hours sleep per week night. On Monday night he slept for $6\frac{1}{3}$ hours, on Tuesday night $7\frac{1}{2}$ hours, on Wednesday night $5\frac{3}{4}$ hours and on Thursday night $8\frac{1}{4}$ hours.

a State the difference between the amount of sleep Xaio achieved each night and his goal of 8 hours. Give a negative answer if the amount of sleep is less than 8 hours.

b After four nights, how much is Xaio ahead or behind in terms of his sleep goal?

c If Xaio is to meet his weekly goal exactly, how much sleep must he get on Friday night?

12 Maria's mother wants to make eight curtains that each require $2\frac{1}{5}$ metres of material in a standard width, but has only $16\frac{1}{4}$ metres. She asks Maria to buy more material. How much more material must Maria buy?

13 Place an inequality sign (< or >) between the following fraction pairs to make a true statement.

a $-\frac{1}{3}\ \square\ -\frac{1}{2}$ **b** $-3\frac{1}{5}\ \square\ -2\frac{3}{7}$ **c** $\frac{1}{4}\ \square\ -\frac{1}{2}$ **d** $-\frac{3}{5}\ \square\ \frac{1}{11}$

e $2\frac{1}{5}\ \square\ -4\frac{3}{5}$ **f** $0\ \square\ -\frac{1}{100}$ **g** $\frac{4}{9}\ \square\ \frac{5}{9}$ **h** $-\frac{4}{9}\ \square\ -\frac{5}{9}$

14 Do not evaluate the following expressions. Simply state whether the answer will be positive or negative.

a $-\frac{2}{7}\times\left(-\frac{1}{7}\right)\times\left(-\frac{3}{11}\right)$ **b** $-4\frac{1}{5}\times\left(-\frac{9}{11}\right)^2$

c $-\frac{5}{6}\div\left(-\frac{2}{7}\right)\times\frac{1}{3}\times\left(-\frac{4}{9}\right)$ **d** $\left(-\frac{3}{5}\right)^3\div\left(-4\frac{1}{5}\right)^3$

15 Using a calculator, evaluate the expressions given in Question **14**.

16 If $a > 0$, $b > 0$ and $a < b$, place an inequality sign between the following fraction pairs to make a true statement.

a $\frac{a}{b}\ \square\ \frac{b}{a}$ **b** $\frac{a}{b}\ \square\ -\frac{a}{b}$ **c** $-\frac{a}{b}\ \square\ -\frac{b}{a}$ **d** $-\frac{b}{a}\ \square\ -\frac{a}{b}$

ENRICHMENT	—	—	17

Positive and negative averages

17 a Calculate the average (also known as the *mean*) of the following sets of numbers, by adding and then dividing by the number of numbers.

i $1\frac{1}{2}, \frac{3}{4}, \frac{5}{4}, 2\frac{1}{2}$ **ii** $-\frac{2}{3}, \frac{5}{6}, \frac{7}{6}, \frac{1}{3}, -1\frac{1}{3}$

iii $-2\frac{1}{5}, -\frac{3}{5}, 0, \frac{1}{5}, -1\frac{3}{5}$ **iv** $-7\frac{1}{3}, -2\frac{1}{2}, -5\frac{1}{6}, -3\frac{3}{10}$

b List a set of five different fractions that have an average of 0.

c List a set of five different fractions that have an average of $-\frac{3}{4}$.

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Investigation

Best buy

The concept of a 'best buy' relates to purchasing a product that is the best value for money. To determine the 'best buy' you need to compare the prices of similar products for the same weight.

STRAWBERRY JAM jar 375 g $3.95 $10.53 per kg	STRAWBERRY JAM jar 250 g $2.95 $11.80 per kg

Converting units

a Convert the following to a price per kg.
- **i** 2 kg of apples for $3.40
- **ii** 5 kg of sugar for $6.00
- **iii** 1.5 kg of cereal for $4.50
- **iv** 500 g of butter for $3.25

b Convert the following to a price per 100 g.
- **i** 300 g of grapes for $2.10
- **ii** 1 kg of cheese for $9.60
- **iii** 700 g of yoghurt for $7.49
- **iv** 160 g of dip for $3.20

Finding 'best buys'

a By converting to a price per kg, determine which is the best buy.
- **i** 2 kg of sauce A for $5.20 or 1 kg of sauce B for $2.90
- **ii** 4 kg of pumpkin A for $3.20 or 3 kg of pumpkin B for $2.70
- **iii** 500 g of honey A for $5.15 or 2 kg of honey B for $19.90
- **iv** 300 g of milk A for $0.88 or 1.5 kg of milk B for $4.00

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b By converting to a price per 100 g, determine which is the best buy.

i 500 g of paper A for \$3.26 or 200 g of paper B for \$1.25

ii 250 g of salami A for \$4.50 or 150 g of salami B for \$3.10

iii 720 g of powder A for \$3.29 or 350 g of powder B for \$1.90

iv 1.1 kg of shampoo A for \$12.36 or 570 g of shampoo B for \$6.85

Problem-solving

a Star Washing Liquid is priced at \$3.85 for 600 g, whereas Best Wash Liquid is priced at \$5.20 for 1 kg. Find the difference in the price per 100 g, correct to the nearest cent.

b Budget apples cost \$6.20 per 5 kg bag. How much would a 500 g bag of Sunny apples have to be if it was the same price per 100 g?

c 1.5 kg of cheddar cheese costs \$11.55, and 800 g of feta cheese costs \$7.25. Sally works out the best value cheese, then buys \$5 worth of it. How much and what type of cheese did Sally buy?

Investigate

Go to a local supermarket and choose a type of product of which there are many brands to choose from.

a Record the following information for each brand of the same type of product.

i price **ii** weight **iii** brand name

b Calculate the price of each brand per:

i kg **ii** 100 g

c Compare the prices of each brand. Comment on the following.

i Which brand is the 'best buy'?

ii The differences between the cheapest and most expensive brands.

iii The reasons why some brands might be more expensive.

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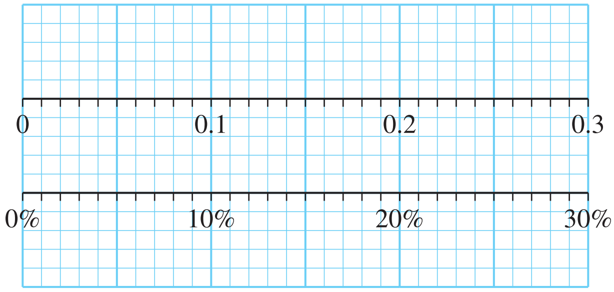
Investigation

Decimal vs percentage

Draw two horizontal lines on a long sheet of graph paper that is 100 small squares wide. On line 1, make a number line from 0 to 1, labelling every tenth and marking hundredths. On line 2, mark percentages from 0% to 100%, labelling every 10% and marking every percentage.

Example of the number lines



Answer the following questions, using your number lines to help you.

a Which is larger, 73% or 0.74?

b Which is smaller, 26% or 0.3?

c List in ascending order: 45%, 0.72, 49%, 37%, 0.58, 0.7, 51%, 0.64, 60%, 0.5.

d List in descending order: 37%, 0.03, 82%, 0.37, 0.8, 77%, 0.23, 38%, 2%, 0.4.

e List in ascending order: 78%, 0.683, 77.5%, 79.9%, 0.78452, 0.76, 0.784, 69.9%, 0.6885.

f Write three decimal numbers between 0.47 and 0.57.

g In decimal form, list all the whole tenths that are greater than 0.34 and less than 0.78.

h In decimal form, list all the decimal hundredths that are greater than 0.32 and less than 0.41.

i Write down three pairs of decimals so that each pair has a difference of 0.02. Now write each of your number pairs as percentages. What is the difference between these percentage values?

j Play some decimal/percentage games in pairs.
- Person A gives a decimal (between 0 and 1) and person B states the percentage that is equivalent to two-tenths less than person A's decimal.
- Person A states a percentage and person B gives the decimal that is equivalent to 5% more than person A's percentage.
- Use your decimal and percentage number lines to make up your own games.

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1 According to legend, this sank to the bottom of the Atlantic Ocean.
To find the answer:

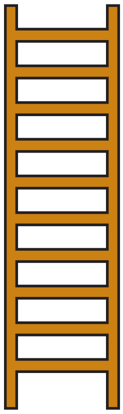
- Work out the problems.
- Locate each answer in the table.
- Place the letter by the answer in the blank next to the questions.
- Match the numbers and letters in the puzzle below.

1. ______	The sum of 21.36 and 14.4 minus 8.2
2. ______	Digit in the hundredths place in 347.879
3. ______	The square of 0.9
4. ______	5.1 divided by 0.3
5. ______	The decimal equivalent of $\frac{7}{8}$
6. ______	The sum of 0.0415, 0.415 and 0.0041, less 0.062, to 1 decimal place
7. ______	3.15×0.05, to 1 decimal place
8. ______	The area of a rectangle with length 6.2 cm and width 2.3 cm
9. ______	The difference between 9 and 8.0091
10. ______	The number of decimal places when 0.6235 is multiplied by 6.23

A = 0.2
C = 0.875
E = 0.4
F = 0.81
I = 14.26
L = 17
N = 0.9909
O = 27.56
S = 6
T = 7

5	1	9	2	8	9	6	9	2	1	3	7	2	4	7	9	2	8	10

2 Consider the ladder in the diagram. The heights from the ground of each rung on the ladder are separated by an equal amount. Determine the heights from the ground for each rung of the ladder.



3 Find the digits represented by the letters in these decimal problems.

a
$$\begin{array}{r} A.2B \\ +9.C5 \\ \hline 11.12 \end{array}$$

b
$$\begin{array}{r} 2A.43 \\ -9.B4 \\ \hline C7.8D \end{array}$$

c $3.A \times B.4 = 8.16$

d
$$A\overline{)2.2B1} = 0.757$$

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Chapter summary

Fractions

Adding fractions

$\frac{4}{5} + \frac{2}{3}$
$= \frac{12}{15} + \frac{10}{15}$
$= \frac{22}{15} = 1\frac{7}{15}$

Subtracting fractions

$3\frac{1}{4} - 1\frac{2}{3}$
$= 2\frac{5}{4} - 1\frac{2}{3}$
$= (2-1) + \left(\frac{5}{4} - \frac{2}{3}\right)$
$= 1 + \left(\frac{15}{12} - \frac{8}{12}\right)$
$= 1\frac{7}{12}$

Or

$3\frac{1}{4} - 1\frac{2}{3}$
$= \frac{13}{4} - \frac{5}{3}$
$= \frac{39}{12} - \frac{20}{12}$
$= \frac{39-20}{12}$
$= \frac{19}{12} = 1\frac{7}{12}$

Multiplying fractions

$\frac{5}{10}$ of $\frac{20}{50}$
$= \frac{\cancel{5}^{1}}{\cancel{10}_{2}} \times \frac{\cancel{20}^{2}}{\cancel{50}_{5}}$
$= \frac{1}{5}$

$3\frac{1}{5} \times 2\frac{1}{4}$
$= \frac{\cancel{16}^{4}}{5} \times \frac{9}{\cancel{4}_{1}}$
$= \frac{36}{5} = 7\frac{1}{5}$

Dividing fractions

$4\frac{1}{6} \div 1\frac{1}{9}$
$= \frac{25}{6} \div \frac{10}{9}$
$= \frac{\cancel{25}^{5}}{\cancel{6}_{2}} \times \frac{\cancel{9}^{3}}{\cancel{10}_{2}}$
$= \frac{15}{4}$
$= 3\frac{3}{4}$

Reciprocal of $\frac{10}{9}$ is $\frac{9}{10}$.

Percentage of a quantity

25% of \$40
$= \frac{1}{4} \times 40$
$=$ \$10

120% of 50 minutes
$= \frac{120}{\cancel{100}_{2}} \times \frac{\cancel{50}}{1}$
$=$ 60 minutes
$=$ 1 hour

Decimals

Subtraction

$\overset{1}{2}\overset{1}{1}6.94$
$-\ 31.53$
185.41

Addition

$\overset{1}{}9.807$
$+\ \overset{1}{}26.350$
36.157

Align decimal points.

Multiplication by powers of 10

$2.76 \times 10\,000$
$= 27\,600.0$
Decimal point moves right.

Division by powers of 10

$2.76 \div 10\,000$
$= 0.000276$
Decimal point moves left.

Division

$8.547 \div 0.03$
$= 854.7 \div 3$

$$\begin{array}{r} 2\,8\,4.9 \\ 3\,\overline{)\,8\,{}^{2}5\,{}^{1}4.{}^{2}7} \end{array}$$

Multiplication

$$\begin{array}{r} 278 \\ \times\ 34 \\ \hline 1112 \\ 8340 \\ \hline 9452 \end{array}$$

$2.78 \times 34 = 94.52$
$2.78 \times 3.4 = 9.452$
$0.278 \times 3.4 = 0.9452$
$0.278 \times 0.34 = 0.09452$

Number of decimal places in the question equals number of decimal places in the answer.

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Multiple-choice questions

1 The next number in the pattern 0.023, 0.025, 0.027, 0.029 is:
A 0.0003 **B** 0.030 **C** 0.0031 **D** 0.031 **E** 0.033

2 0.05 is equivalent to:
A $\frac{5}{10}$ **B** $\frac{5}{100}$ **C** $\frac{5}{1000}$ **D** $\frac{5}{500}$ **E** 5

3 The smallest number out of 0.012, 10.2, 0.102, 0.0012 and 1.02 is:
A 0.012 **B** 0.102 **C** 0.0012 **D** 1.02 **E** 10.2

4 $0.36 \div 1000$ is equal to:
A 3.6 **B** 360 **C** 0.036 **D** 0.0036 **E** 0.00036

5 6.2×0.2 is equal to:
A 1.24 **B** 12.4 **C** 0.124 **D** 124 **E** 0.0124

6 What is the answer to 0.08×0.6?
A 0.48 **B** 4.8 **C** 0.0048 **D** 0.048 **E** 48

7 When rounded to 1 decimal place, 84.553 becomes:
A 80 **B** 84 **C** 84.5 **D** 84.6 **E** 84.55

8 As a decimal, $\frac{23}{90}$ is equal to:
A 0.2 **B** $0.2\dot{5}$ **C** 0.26 **D** $0.\dot{2}\dot{8}$ **E** $0.25\dot{6}$

9 $7 + 0.7 + 0.07 + 0.007$, to 2 decimal places, is:
A 7.78 **B** 7.77 **C** 7 **D** 7.7 **E** 7.777

10 $5.\overline{624}$ means:
A 5.62444… **B** 6.6242424… **C** 5.624624624…
D 5.6246464… **E** 5.62456245624…

Short-answer questions

1 Arrange each group in descending order, from largest to smallest.
a 0.4, 0.04, 0.44 **b** 2.16, 2.016, 2.026 **c** 0.932, 0.98, 0.895

2 Write each fraction as a decimal.
a $\frac{81}{10}$ **b** $\frac{81}{100}$ **c** $\frac{801}{100}$ **d** $\frac{801}{1000}$

3 What is the place value of the digit 3 in the following numbers?
a 12.835 **b** 6.1237 **c** 13.5104

4 State whether each of the following is true or false.
a $8.34 < 8.28$ **b** $4.668 > 4.67$ **c** $8.2 > 8.182$
d $3.08 \leqslant \frac{308}{100}$ **e** $\frac{62}{100} \geqslant 6.20$ **f** $\frac{7}{10} = \frac{70}{100}$

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5 Evaluate:

a $2841 \div 3$ **b** $2841 \div 0.3$ **c** $2841 \div 0.03$

6 Round each of the following to the specified number of decimal places (which is given in brackets).

a 423.46 (1)
b 15.8892 (2)
c 7.25432 (1)
d 69.999531 (3)
e $2\frac{3}{4}$ (1)
f $\frac{2}{3}$ (2)
g $\frac{5}{11}$ (3)
h $\frac{1}{81}$ (44) (Hint: Look for the short method!)

7 Evaluate:

a $13.85 - 4.32$
b $19.12 - 14.983$
c $27.6 + 15.75$
d $204.70837 + 35.7902$
e $472.427 - 388.93$
f $210.8 - (26.3 - 20.72)$

8 State whether each of the following is true or false.

a $10.34 \div 100 = 0.1034$
b $3.125 \times 0.1 = 31.25$
c $115.23 \div 10 = 1.1523 \times 1000$
d 115.23 has 3 decimal places
e $24.673 = 24.7$ when rounded to 1 decimal place

9 State the number of decimal places in each answer to these calculations.

a A number with 2 decimal places × a number with 3 decimal places
b A number with 1 decimal place × a number with 4 decimal places
c A number with 2 decimal places × a number with 2 decimal places
d A number with 2 decimal places × a number with 0 decimal places

10 Solve each of the following, using the order of operations.

a 1.37×100
b 0.79×1000
c 225.1×10
d $96.208 \div 1000$
e $75.68 + 6.276 \times 100 - 63.24 \div 10$
f $3.56 \times 100 + 45 \div 10$
g $100 \times (56.34 \times 100 + 0.893)$

11 Calculate the following.

a 2.4×8
b 9×7.11
c 2.3×8.4
d $3.8 \div 4$
e $12.16 \div 8$
f $3 \div 0.5$
g $4 \div 0.25$
h $1.2 \div 0.4$
i $3.42 \div 1.1$

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12 Copy and complete this table, stating fractions both with the denominator 100 and in their simplest form.

Decimal	Fraction	Percentage
0.45		
	$\frac{?}{100} = \frac{7}{10}$	
		32%
0.06		
	$\frac{79}{100}$	
1.05		
	$\frac{?}{100} = \frac{7}{20}$	
		65%
	$\frac{?}{1000} = \frac{1}{8}$	

Extended-response questions

1 Find the answer in these practical situations.

a Jessica is paid \$125.70 for 10 hours of work and Jaczinda is paid \$79.86 for 6 hours of work. Who receives the higher rate of pay per hour, and by how much?

b Petrol is sold for 144.9 cents per litre. Jacob buys 30 L of petrol for his car. Find the total price he pays, to the nearest 5 cents.

c The Green family are preparing to go to the Great Barrier Reef for a holiday. For each of the four family members, they purchase a goggles and snorkel set at \$37.39 each, fins at \$18.99 each and rash tops at \$58.48 each. How much change is there from \$500?

d For her school, a physical education teacher buys 5 basketballs, 5 rugby union balls and 5 soccer balls. The total bill is \$711.65. If the rugby balls cost \$38.50 each and the basketballs cost \$55.49 each, what is the price of a soccer ball?

2 A car can use 25% less fuel per kilometre when travelling at 90 km/h than it would when travelling at 110 km/h. Janelle's car uses 7.8 litres of fuel per 100 km when travelling at 110 km/h, and fuel costs 145.6 cents per litre.

a How much money could Janelle save on a 1000 km trip from Sydney to Brisbane if she travels at a constant speed of 90 km/h instead of 110 km/h?

b During a 24-hour period, 2000 cars travel the 1000 km trip between Sydney and Brisbane. How much money could be saved if 30% of these cars travel at 90 km/h instead of 110 km/h?

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Chapter 1: Computation with positive integers

Multiple-choice questions

1 Using numerals, thirty-five thousand, two hundred and six is:
A 350 260 B 35 260 C 35 000 206 D 3526 E 35 206

2 The place value of 8 in 2 581 093 is:
A 8 thousand B 80 thousand C 8 hundred D 8 tens E 8 ones

3 The remainder when 23 650 is divided by 4 is:
A 0 B 4 C 1 D 2 E 3

4 $18 - 3 \times 4 + 5$ simplifies to:
A 65 B 135 C 11 D 1 E 20

5 $800 \div 5 \times 4$ is the same as:
A 160×4 B $800 \div 20$ C $800 \div 4 \times 5$ D 40 E $4 \times 5 \div 800$

Short-answer questions

1 Write the following numbers using words.
a 1030 b 13 000 c 10 300
d 10 030 e 100 300 f 1 300 000

2 Write the numeral for:
a $6 \times 10000 + 7 \times 1000 + 8 \times 100 + 4 \times 10 + 9 \times 1$
b $7 \times 100000 + 8 \times 100 + 5 \times 10$

3 Calculate:
a $96481 + 2760 + 82$ b $10963 - 4096$ c 147×3
d 980×200 e $4932 \div 3$ f $9177 \div 12$

4 State whether each of the following is true or false.
a $18 < 20 - 2 \times 3$ b $9 \times 6 > 45$ c $23 = 40 \div 2 + 3$

5 How much more than 17×18 is 18×19?

6 Calculate:
a $7 \times 6 - 4 \times 3$ b $8 \times 8 - 16 \div 2$ c $12 \times (6 - 2)$
d $16 \times [14 - (6 - 2)]$ e $24 \div 6 \times 4$ f $56 - (7 - 5) \times 7$

7 State whether each of the following is true or false.
a $4 \times 25 \times 0 = 1000$ b $0 \div 10 = 0$ c $8 \div 0 = 0$
d $8 \times 7 = 7 \times 8$ e $20 \div 4 = 20 \div 2 \div 2$ f $8 + 5 + 4 = 8 + 9$

8 Insert brackets to make $18 \times 7 + 3 = 18 \times 7 + 18 \times 3$ true.

9 How many times can 15 be subtracted from 135 before an answer of zero occurs?

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10 Write 3 859 643 correct to the nearest:
a 10 **b** thousand **c** million

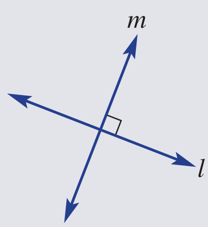
Extended-response question

1 Tom works as a labourer, earning \$25 an hour on weekdays and \$60 an hour on weekends.
  a During a particular week, Tom works from 7 a.m. to 2 p.m. Monday to Thursday. How many hours does he work that week?
  b How much does Tom earn for this work?
  c If Tom works 5 hours on Saturday in the same week, what is his total income for the week?
  d How many more hours on a Friday must Tom work to earn the same amount as working 5 hours on a Saturday?

Chapter 2: Angle relationships

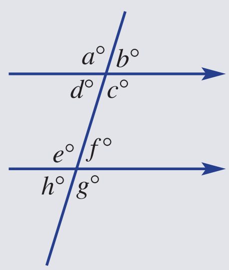
Multiple-choice questions

1 Which statement is correct?
  A Line m is perpendicular to line l.
  B Line m bisects line l.
  C Line m is parallel to line l.
  D Line m is shorter than line l.
  E Line m is longer than line l.

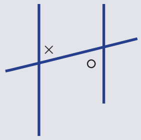


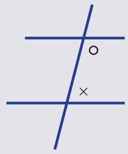
2 An angle of 181° is classified as:
A acute **B** reflex **C** straight **D** obtuse **E** sharp

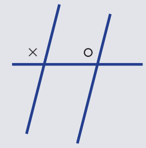
3 Which two angles represent alternate angles?
  A $a°$ and $e°$
  B $d°$ and $f°$
  C $a°$ and $f°$
  D $g°$ and $b°$
  E $c°$ and $f°$

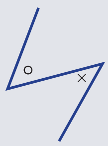


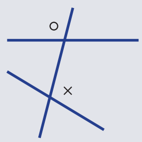
4 Which of the following shows a pair of supplementary angles?

A


B


C


D


E


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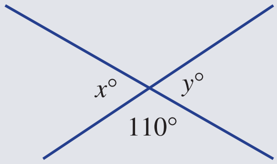
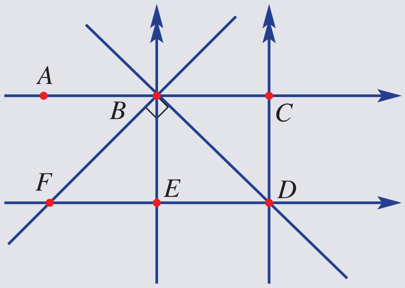
5 The value of $x + y$ is:



A 70
B 220
C 35
D 140
E 110

Short-answer questions

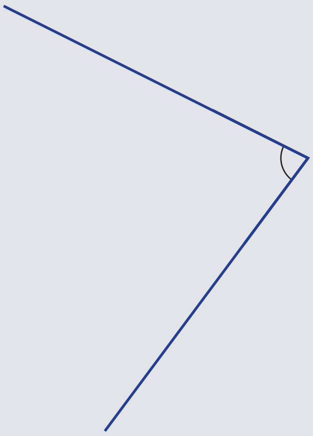


1 **a** Name two pairs of parallel lines.
b Name a pair of perpendicular lines.
c List any three lines that are concurrent. At what point do they cross?
d Name two points that are collinear with point C.
e Name the point at which line BE and line FD intersect.

2 Measure these angles.

a

b



c

3 What is the complement of 65°?

4 What is the supplement of 102°?

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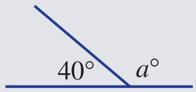
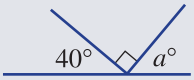
5 Find the value of a in each of the following angles.

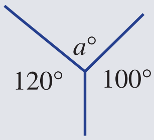
a



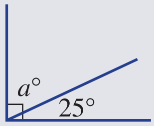
b



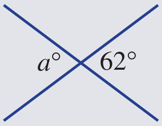
c



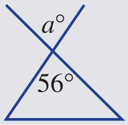
d



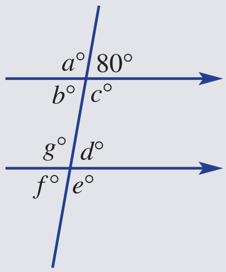
e



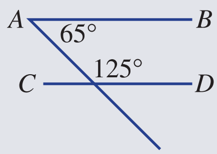
f



6 Find the value of a, b, c, d, e, f and g.



7 Explain why AB is *not* parallel to CD.



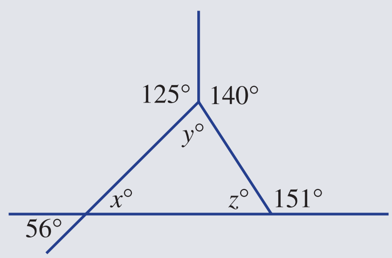
Extended-response question

1 Consider the diagram shown.

a Find the value of:

i x **ii** y **iii** z

b What is the value of $x + y + z$?



Chapter 3: Computation with positive and negative integers

Multiple-choice questions

1 Which of the following statements is incorrect?

A $-2 > -4$ **B** $0 < 5$ **C** $0 < -10$ **D** $-9 < -8$ **E** $-5 < 3$

2 $12 + (-9) - (-3)$is the same as:

A $12 + 9 + 3$ **B** $12 - 9 + 3$ **C** $12 - 9 - 3$ **D** $12 - 12$ **E** 12

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3 The value of $2 \times (-3)^2$ is:
A 36 **B** −36 **C** 18 **D** −18 **E** 12

4 The coordinates of the point that is 3 units below (3, 1) is:
A (0, 1) **B** (0, −2) **C** (0, −1) **D** (3, 4) **E** (3, −2)

5 $12 \times (-4 + (-8) \div 2)$ equals:
A −96 **B** 72 **C** −72 **D** 60 **E** 96

Short-answer questions

1 For each of the following, insert <, = or >.
a $-3 \ \square \ 3$ **b** $-10 \div 2 \ \square \ 5$ **c** $-20 \times (-1) \ \square \ \frac{-40}{-2}$

2 Calculate:
a $-5 + (-8)$ **b** $12 - 96$ **c** $-12 - 96$
d $-4 - 8 - 9$ **e** $-12 + 96$ **f** $-7 - (-7)$

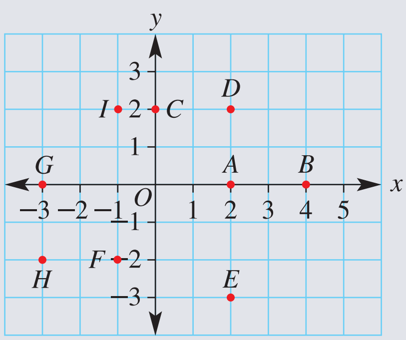
3 Find:
a -6×4 **b** $-9 \times 8 \times (-1)$ **c** $(-12)^2$
d $\frac{-9 \times (-7)}{3}$ **e** $-150 \div (-2 - 3)$ **f** $-10 + 7 \times (-3)$

4 State whether the answer to each of the following is positive or negative.
a $-3 \times (-3) \times (-3)$ **b** $-109 \times 142 \times (-83)$ **c** $-2 \times (-1 - (-3))$

5 Copy and complete.
a $\square + 9 = -6$ **b** $\square \times (-3) = -6 \times (-4)$ **c** $16 \times \square = -64$

Extended-response question

1 Refer to the given Cartesian plane when answering these questions.



a Name any point that lies in the first quadrant.
b Name any point(s) with a y value of zero. Where does each point lie?
c Which point has coordinates (−1, −2)?
d Find the distance between points:
  i A and B **ii** D and E
e What shape is formed by joining the points $IDAG$?
f What is the area of $IDAG$?
g $ABXD$ are the vertices of a square. What are the coordinates of X?
h Decode: (2, 2), (2, −3), (0, 2), (−1, 2), (2, 2), (2, −3)

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Chapter 4: Understanding fractions, decimals and percentages

Multiple-choice questions

1 Which of the following is equivalent to $\frac{12}{7}$?

A $\frac{24}{7}$ **B** $1\frac{5}{7}$ **C** $1\frac{5}{12}$ **D** $\frac{112}{17}$ **E** $\frac{7}{12}$

2 $\frac{3}{8}$ is the same as:

A 0.375 **B** 3.8 **C** 0.38 **D** $2.\dot{6}$ **E** 38%

3 $\frac{350}{450}$ in simplest form is:

A $\frac{35}{45}$ **B** $\frac{4}{5}$ **C** $\frac{3}{4}$ **D** $\frac{3.5}{4.5}$ **E** $\frac{7}{9}$

4 What fraction of \$2 is 40 cents?

A $\frac{1}{20}$ **B** $\frac{20}{1}$ **C** $\frac{5}{1}$ **D** $\frac{1}{5}$ **E** $\frac{1}{40}$

5 Which fraction is largest?

A $\frac{1}{2}$ **B** $\frac{2}{3}$ **C** $\frac{3}{4}$ **D** $\frac{4}{5}$ **E** $\frac{5}{6}$

Short-answer questions

1 Arrange $\frac{1}{2}, \frac{1}{3}, \frac{2}{5}$ and $\frac{3}{10}$ in *ascending* order.

2 Express $5\frac{2}{3}$ as an improper fraction.

3 State if each of the following is true or false.

a $0.5 = 50\%$ **b** $0.15 = \frac{5}{20}$ **c** $38\% = 0.19$

d $126\% = 1.26$ **e** $\frac{4}{5} = 0.08$ **f** $1\frac{3}{4} = 1.75$

4 Write 15% as a simple fraction.

5 Find 25% of \$480.

6 Find $12\frac{1}{2}\%$ of \$480.

7 State whether each of the following is true or false.

a 25% of $x = x \div 4$ **b** 10% of $w = \frac{w}{10}$

c 20% of $50 = 50\%$ of 20 **d** 1% of $x = 100x$

8 Which is larger, $\frac{2}{3}$ or 67%?

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Extended-response question

1 Caleb's cold and flu prescription states: 'Take two pills three times a day with food.' The bottle contains 54 pills.

a How many pills does Caleb take each day?
b What fraction of the bottle remains after Day 1?
c How many days will it take for the pills to run out?
d If Caleb takes his first dose Friday night before going to bed, on what day will he take his last dose?

Chapter 5: Probability

Multiple-choice questions

1 What is the probability of rolling a 6 with a fair 6-sided die?

A 6% B 16% C $16\frac{2}{3}\%$ D 17% E 60%

2 Sophie has some jelly beans. Six are red and four are green. She eats a red one, then randomly chooses another jelly bean. The probability that it is red is:

A $\frac{3}{5}$ B $\frac{2}{5}$ C $\frac{4}{9}$ D $\frac{5}{9}$ E $\frac{1}{2}$

3 The letters of the alphabet are written on cards and placed in a hat. One letter is chosen randomly. The probability that it is a vowel is closest to:

A 0.05 B 0.1 C 0.2 D 0.3 E 0.26

4 There are 30 students in a class, of which 17 are boys. A student is chosen at random. The probability of choosing a girl is:

A $\frac{17}{30}$ B $\frac{13}{17}$ C $\frac{13}{30}$ D $\frac{17}{13}$ E $\frac{1}{13}$

5

	Right handed	Left handed
Boys	25	7
Girls	20	8

A survey of some Year 7 students is conducted, asking them whether they are left handed or right handed. If a student is chosen at random, the chance of choosing a left-handed girl is:

A $\frac{1}{3}$ B $\frac{2}{15}$ C $\frac{2}{7}$ D $\frac{5}{7}$ E $\frac{8}{15}$

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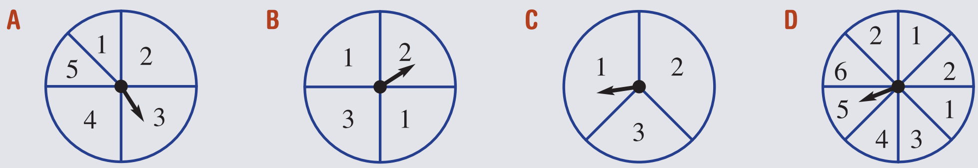
Short-answer questions

1 A spinner is designed with different numbers in each sector. From the spinners **A** to **D** shown below:

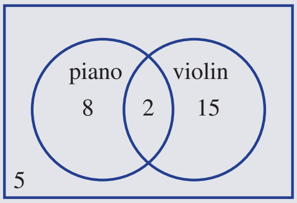


a Which spinner has the lowest probability of landing on the number 1 in a single spin?
b Which spinner has a 50% probability of landing on the number 1 in a single spin?
c List the spinners in order, from the most likely to land on the number 1 to the least likely.

2 One card is randomly selected from a standard deck of 52 playing cards. Find the probability that the selected card is:

a red
b black
c a heart
d an ace
e a king
f a red 7

3 The students attending an after-school music programme are asked, ‘Do you play the piano?’ and ‘Do you play the violin?’. Their responses are shown in the Venn diagram. One of the music students is chosen at random. Find the probability that:



a the student plays neither the piano nor the violin.
b the student plays the piano and the violin.
c the student plays the piano but not the violin.
d the student plays the violin.
e the student plays the piano or the violin.

4 Arrange these events from least likely to most likely.
A: tossing ‘heads’ with a fair coin
B: randomly choosing a king from a standard deck of playing cards
C: rolling a 6 with a fair 6-sided die
D: randomly choosing a red card or a spade from a standard deck of playing cards
E: rolling a number greater than 1 on a fair 6-sided die

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Extended-response question

1 A standard deck of playing cards includes 13 cards for each suit: hearts, diamonds, clubs and spades. Each suit has an ace, king, queen, jack, 2, 3, 4, 5, 6, 7, 8, 9 and 10. One card is drawn at random from the deck.

Find the following probabilities.

a P(heart) b P(club) c P(diamond or spade)
d P(ace of hearts) e P(number less than 4 and not an ace)
f P(king) g P(ace or heart) h P(queen or club)

Chapter 6: Computation with decimals and fractions

Multiple-choice questions

1 $80 + \frac{6}{100} + \frac{7}{1000}$ is the same as:
A 8067 B 867 C 80.67 D 80.067 E 80.607

2 Select the incorrect statement.
A $0.707 > 0.7$
B $0.770 = \frac{77}{100}$
C $0.07 \times 0.7 = 0.49$
D $0.7 \times \frac{1}{10} = 0.07$
E $0.7 \times 10 = 7$

3 The best estimate for 23.4×0.96 is:
A 234 B 230 C 0.234 D 23 E 20

4 $\frac{1}{2} + \frac{1}{3}$ is equal to:
A $\frac{2}{5}$ B $\frac{2}{6}$ C $\frac{5}{6}$ D $\frac{1}{5}$ E $\frac{7}{6}$

5 $6.8 \div 0.04$ is the same as:
A $68 \div 4$ B $680 \div 4$ C 17 D $\frac{4}{68}$ E $7 \div 0.05$

Short-answer questions

1 Write each of the following as a decimal.
a two-tenths
b $\frac{13}{100}$
c $\frac{17}{10}$

2 In the decimal 136.094:
a What is the value of the digit 6?
b What is the value of the digit 4?
c What is the decimal, correct to the nearest tenth?

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3 Round 18.398741 correct to:
- **a** the nearest whole
- **b** 1 decimal place
- **c** 2 decimal places

4 Evaluate:
- **a** $15 - 10.93$
- **b** $19.7 + 240.6 + 9.03$
- **c** $20 - 0.99$
- **d** 0.6×0.4
- **e** $(0.3)^2$
- **f** $\frac{12}{0.2}$

5 Find:
- **a** $1.24 - 0.407$
- **b** $1.2 + 0.6 \times 3$
- **c** $1.8 \times 0.2 \div 0.01$

6 If $369 \times 123 = 45\,387$, write down the value of:
- **a** 3.69×1.23
- **b** 0.369×0.123
- **c** $45.387 \div 36.9$

7 Find:
- **a** 36.49×1000
- **b** $1.8 \div 100$
- **c** 19.43×200

8 For each of the following, circle the larger of each pair.
- **a** $\frac{4}{5}$, 0.79
- **b** 1.1, 11%
- **c** $\frac{2}{3}$, 0.6

9 Find each of the following.
- **a** $\frac{2}{3} + \frac{1}{4}$
- **b** $4 - 1\frac{1}{3}$
- **c** $2\frac{1}{2} + 3\frac{3}{4}$
- **d** $\frac{2}{5} \times \frac{1}{2}$
- **e** $\frac{2}{3} \div \frac{1}{6}$
- **f** $1\frac{1}{5} \times \frac{5}{12}$

Extended-response question

1 The cost of petrol is 146.5 cents per litre.
- **a** Find the cost of 55 L of petrol, correct to the nearest cent.
- **b** Mahir pays cash for his 55 L of petrol. What is the amount that he pays, correct to the nearest 5 cents?
- **c** If the price of petrol is rounded to the nearest cent before the cost is calculated, how much would 55 L of petrol cost now?
- **d** By how much is Mahir better off if the rounding occurs at the end rather than the beginning?
- **e** Is the result the same if the price drops to 146.2 cents per litre?

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Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

7 Time

What you will learn

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NSW syllabus

STRAND: MEASUREMENT AND GEOMETRY

SUBSTRAND: TIME

Outcome

A student performs calculations of time that involve mixed units, and interprets time zones.
(MA4–15MG)

Luxor temple obelisk

Thousands of years before the use of clocks and even the introduction of the Gregorian calendar, sundials were used to tell the time during the day. Egyptian obelisks dating back to 3500 BCE were some of the earliest sundials used.

The two 3300-year-old twin obelisks, once marking the entrance of the Luxor temple in Egypt, are still standing today. One of them, however, was gifted to France and in 1836 was placed at the centre of Place de la Concorde in Paris, where it still stands.

Pre-test

1 How many:
a hours in one day?
b seconds in one minute?
c minutes in one hour?
d days in one week?
e months in one year?
f days in one year (not a leap year)?

2 What day is it:
a 3 days after Tuesday?
b 6 days before Sunday?
c 3 weeks after Wednesday?
d 10 minutes after 11:55 p.m. Saturday?

3 Give the time, using a.m. or p.m., that matches these descriptions.
a 2 hours after 3 p.m.
b 1 hour before 2:45 a.m.
c 6 hours before 10:37 a.m.
d 4 hours after 4:49 p.m.
e $1\frac{1}{2}$ hours after 2:30 p.m.
f $3\frac{1}{3}$ hours before 7:15 p.m.
g 2 hours before 12:36 p.m.
h 5 hours after 9:14 a.m.

4 Convert the following to the units shown in brackets.
a 60 seconds (minutes)
b 120 minutes (hours)
c 49 days (weeks)
d 6 hours (minutes)

5 Melissa watched two movies on the weekend. One lasted 1 hour 36 minutes and the other lasted 2 hours 19 minutes.
a What was the total time Melissa spent watching movies, in hours and minutes?
b What was the total time in minutes?

6 Write the following times as they would be displayed on a digital clock; e.g. 8:15.
a 3 o'clock
b half past 2
c a quarter to 6

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7A Units of time

Time in minutes and seconds is based on the number 60. Other units of time, including the day and year, are defined by the rate at which the Earth spins on its axis and the time that the Earth takes to orbit the Sun.

The origin of the units seconds and minutes dates back to the ancient Babylonians, who used a base 60 number system. The 24-hour day dates back to the ancient Egyptians, who described the day as 12 hours of day and 12 hours of night. Today, we use a.m. (*ante meridiem*, which is Latin for 'before noon') and p.m. (*post meridiem*, which is Latin for 'after noon') to represent the hours before and after noon (midday). During the rule of Julius Caesar, the ancient Romans introduced the solar calendar, which recognised that the Earth takes about $365\frac{1}{4}$ days to orbit the Sun. This gave rise to the leap year, which includes one extra day (in February) every 4 years.

The calendar we use today is called the Gregorian calendar. It was formally introduced by Pope Gregory XIII in 1582 and includes the 12 months, each with 30 or 31 days, except for February, which has 28 days or 29 days in a leap year.

Number	1	2	3	4	5	6	7	8	9	10	11	12
Month	Jan	Feb	Mar	April	May	June	July	Aug	Sep	Oct	Nov	Dec
Days	31	28/29	31	30	31	30	31	31	30	31	30	31

Time after Christ (AD) is now often referred to as the Common Era (CE) and the time before Christ (BC) is also referred to as Before the Common Era (BCE).

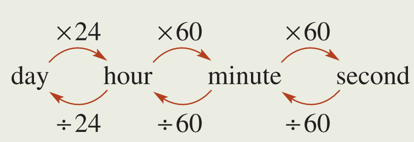
Let's start: Knowledge of time

Do you know the answers to these questions about time and the calendar?

- When is the next leap year?
- Why do we have a leap year?
- Which months have 31 days?
- Why are there different times in different countries or parts of a country?
- What do BCE (or BC) and CE (or AD) mean on time scales?

Key ideas

- The standard unit of time is the **second** (s).
- Units of time include:
  - 1 **minute** (min) = 60 seconds (s)
  - 1 **hour** (h) = 60 minutes (min)
  - 1 **day** = 24 hours (h)
  - 1 **week** = 7 days
  - 1 **year** = 12 **months**



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Key ideas

- Units of time smaller than a second.
  - millisecond = 0.001 second (1000 milliseconds = 1 second)
  - microsecond = 0.000001 second (1 000 000 microseconds = 1 second)
  - nanosecond = 0.000000001 second (1 000 000 000 nanoseconds = 1 second)
- a.m. or p.m. is used to describe the 12 hours before and after noon (midday).
- **24-hour time** shows the number of hours and minutes after midnight.
  - 0330 is 3:30 a.m.
  - 1530 is 3:30 p.m.
- DMS conversion: Most scientific and graphics calculators have a DMS (Degrees, Minutes and Seconds) button or function that converts time in fraction or decimal form to hours, minutes and seconds.
  For example: 2.26 hours → $2^\circ 15' 36''$, meaning 2 hours, 15 minutes and 36 seconds.

Example 1 Converting units of time

Convert these times to the units shown in brackets.

a 3 days (minutes) **b** 30 months (years)

SOLUTION

a $3\text{ days} = 3 \times 24\text{ h}$
$= 3 \times 24 \times 60\text{ min}$
$= 4320\text{ min}$

EXPLANATION

1 day = 24 hours
1 hour = 60 minutes

b $30\text{ months} = 30 \div 12\text{ years}$
$= 2\frac{1}{2}\text{ years}$

There are 12 months in 1 year.

Example 2 Using 24-hour time

Write these times using the system given in brackets.

a 4:30 p.m. (24-hour time) **b** 1945 hours (a.m./p.m.)

SOLUTION

a 4:30 p.m. = 1200 + 0430
= 1630 hours

EXPLANATION

Since the time is p.m., add 12 hours to 0430 hours.

b 1945 hours = 7:45 p.m.

Since the time is after 1200 hours, subtract 12 hours.

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Example 3 Converting to hours, minutes and seconds

Convert the following to hours, minutes and seconds.

a Convert mentally $4\frac{1}{3}$ hours.

b Use a calculator to convert 4.42 hours.

SOLUTION	EXPLANATION
a $4\frac{1}{3} = 4$ hours, 20 minutes	$\frac{1}{3}$ of an hour is 20 minutes because $\frac{1}{3}$ of $60 = 20$.
b $4.42 \rightarrow 4°25'12'' = 4$ hours, 25 minutes and 12 seconds	Use the DMS bottom on your calculator. Ensure your calculator is in Degree mode.

Exercise 7A

UNDERSTANDING AND FLUENCY	1–4, 5–6(½), 7, 8	3, 4–6(½), 7–9	5–6(½), 7, 8–10

1 Which months of the year contain:

- **a** 28 or 29 days?
- **b** 30 days?
- **c** 31 days?

2 From options **A** to **F**, match up the time units with the most appropriate description.

- **a** single heartbeat
- **b** 40 hours of work
- **c** duration of a university lecture
- **d** bank term deposit
- **e** 200-metre run
- **f** flight from Australia to the UK

- **A** 1 hour
- **B** 1 minute
- **C** 1 day
- **D** 1 week
- **E** 1 year
- **F** 1 second

3 State whether you would multiply by 60 (M) or divide by 60 (D) when converting:

- **a** hours to minutes
- **b** seconds to minutes
- **c** minutes to hours
- **d** minutes to seconds

4 Find the number of:

- **a** seconds in 2 minutes
- **b** minutes in 180 seconds
- **c** hours in 120 minutes
- **d** minutes in 4 hours
- **e** hours in 3 days
- **f** days in 48 hours
- **g** weeks in 35 days
- **h** days in 40 weeks

Example 1

5 Convert these times to the units shown in brackets.

- **a** 3 h (min)
- **b** 10.5 min (s)
- **c** 240 s (min)
- **d** 90 min (h)
- **e** 6 days (h)
- **f** 72 h (days)
- **g** 1 week (h)
- **h** 1 day (min)
- **i** 14 400 s (h)
- **j** 20 160 min (weeks)
- **k** 2 weeks (min)
- **l** 24 h (s)
- **m** 3.5 h (min)
- **n** 0.25 min (s)
- **o** 36 h (days)
- **p** 270 min (h)
- **q** 75 s (min)
- **r** 7200 s (h)

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Example 2

6 Write these times, using the system shown in brackets.

a 1:30 p.m. (24-hour) **b** 8:15 p.m. (24-hour) **c** 10:23 a.m. (24-hour)
d 11:59 p.m. (24-hour) **e** 0630 hours (a.m./p.m.) **f** 1300 hours (a.m./p.m.)
g 1429 hours (a.m./p.m.) **h** 1938 hours (a.m./p.m.) **i** 2351 hours (a.m./p.m.)

7 Write each of these digital clock displays as a number of hours expressed as a decimal; e.g. 4:30 is 4.5 hours.

a 1:30 **b** 4:45 **c** 7:15 **d** 3:20

Example 3a

8 Write these times in hours and minutes.

a $2\frac{1}{2}$ hours **b** $4\frac{1}{4}$ hours **c** $1\frac{1}{3}$ hours
d 6.5 hours **e** 3.75 hours **f** 9.25 hours

9 Round these times to the nearest hour.

a 1:32 p.m. **b** 5:28 a.m. **c** 1219 hours **d** 1749 hours

Example 3b

10 Use the DMS button/function on your calculator to convert the following to hours, minutes and seconds.

a 7.12 hours **b** 2.28 hours **c** 3.05 hours **d** 8.93 hours

PROBLEM-SOLVING AND REASONING	11, 12, 14	11, 12, 14, 15	12, 13, 15, 16

11 Marion reads the following times on an airport display panel. Re-write the times using a.m. or p.m.

a 0630 **b** 1425 **c** 1927

12 When there are 365 days in a year, how many weeks are there in a year? Round your answer to 2 decimal places.

13 Assuming there are 365 days in a year and my birthday falls on a Wednesday this year, on what day will my birthday fall in 2 years' time?

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14 Explain why:
- **a** 4.2 hours is 4 hours and 12 minutes
- **b** 2 hours and 10 minutes is 2.16 hours

15
- **a** To convert from hours to seconds, what single number do you multiply by?
- **b** To convert from days to minutes, what single number do you multiply by?
- **c** To convert from seconds to hours, what single number do you divide by?
- **d** To convert from minutes to days, what single number do you divide by?

16 Without the use of the DMS function on your calculator, but allowing the use of your calculator's basic functions, convert the following to hours, minutes and seconds.
- **a** 2.4 hours
- **b** 7.18 hours
- **c** 9.92 hours

ENRICHMENT	—	—	17

Historical devices

17 There is a rich history associated with the way civilisations have recorded time and with the development of today's calendar. Use the internet as a research tool and write 5–10 points summarising this topic.

You may want to include topics relating to:
- the introduction of the Gregorian calendar
- the number of days in the year and in a leap year
- the lunar calendar
- some ancient methods of recording time
- the Roman influence on today's calendar
- the use of sundials.

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7B Working with time

Interactive
Widgets
HOTsheets
Walkthrough

It is a common activity to make a calculation involving time. For example, working out the duration of a sporting event or show, finding a train's time of arrival, using a timetable, or estimating time durations for trade quotes or cooking. These calculations may involve operations such as addition or subtraction and the use of the different ways that time can be displayed.

Let's start: Your mental strategy

Mental strategies are helpful in working out the sums and differences associated with time. Try to work out the answer to these simple problems and then describe the mental strategy you used to your group or class.

- A football match begins at 1:45 p.m. and finishes at 4:10 p.m. What is the duration of the match?
- A train leaves the city station at 8:40 a.m. and arrives in town $2\frac{1}{2}$ hours later. At what time does the train reach town?
- The construction of the Great Pyramid of Giza began in 2560 BCE. How old does that make the pyramid now?

Key ideas

- Mental strategies should be used to solve simple problems involving time.
  - The total time to build two models, which took 45 minutes and 55 minutes each, is $55 + 5 + 40 = 1$ hour 40 minutes.
  - The time duration of a taxi ride beginning at 2:50 p.m. and ending at 3:35 p.m. is $35 + 10 = 45$ minutes or $60 - 15 = 45$ minutes.
- When solving problems to do with time, it may be helpful to use the same type of units.

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Example 4 Calculating time intervals

Mentally calculate the time interval between these pairs of times.

a 4:35 p.m. to 9:10 p.m.

b 5 h 20 min 20 s to 7 h 40 min 10 s

SOLUTION

a Time interval = 4 h + 25 min + 10 min
= 4 h 35 min

b Time interval = 2 h + 20 min − 10 s
= 2 h 19 min 50 s

EXPLANATION

a There are 4 hours from 5 p.m. to 9 p.m., another 25 minutes before 5 p.m., and then 10 minutes after 9 p.m.

b 2 hours and 20 minutes after 5 h 20 min and 20 s is 10 seconds too many, so subtract 10 seconds. A calculator method might include entering 7°40′10″ − 5°20′20″, using the DMS button or °, ′ and ″ symbols.

Example 5 Using timetables

Use this train timetable for Bathurst to Penrith to answer these questions.

Station	a.m.	p.m.
Bathurst	7:11	2:41
Lithgow	8:15	3:45
Bell	8:32	4:02
Mount Victoria	8:42	4:13
Katoomba	8:57	4:29
Springwood	9:29	5:01
Penrith	9:54	5:30

a How long does it take to travel from:
- **i** Bathurst to Lithgow in the morning?
- **ii** Lithgow to Penrith in the morning?
- **iii** Bathurst to Penrith in the afternoon?

b Luke travels from Lithgow to Bell in the morning and then from Bell to Katoomba in the afternoon. What is Luke's total travel time?

SOLUTION

a **i** 1 h 4 min
ii 1 h + 45 min – 6 min = 1 h 39 min
iii 3 h – 11 min = 2h 49 min

b 17 min + 27 min = 44 min

EXPLANATION

a **i** 8:15 is 1 hour plus 4 minutes after 7:11.
ii 1 hour and 45 minutes takes 8:15 to 10:00, so subtract 6 minutes to get 9:54.
iii 3 hours after 2:41 is 5:41, so subtract 11 minutes.

b 8:15 to 8:32 is 17 minutes, and 4:02 to 4:29 is 27 minutes. This gives a total of 44 minutes.

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Exercise 7B

UNDERSTANDING AND FLUENCY	1–3, 4(½), 5, 6	2, 3, 4–6(½)	4–7(½)

1 State whether each of the following is true or false.
- **a** There are 60 seconds in 1 hour.
- **b** 12 noon is between morning and afternoon.
- **c** There are 35 minutes between 9:35 a.m. and 10:10 a.m.
- **d** There are 17 minutes between 2:43 p.m. and 3:10 p.m.
- **e** The total of 39 minutes and 21 minutes is 1 hour.

2 What is the time difference between these times?
- **a** 12 noon and 6:30 p.m.
- **b** 12 midnight and 10:45 a.m.
- **c** 12 midnight and 4:20 p.m.
- **d** 11 a.m. and 3:30 p.m.

3 Add these time durations to give a total time.
- **a** 1 h 30 min and 2 h 30 min
- **b** 4 h 30 min and 1 h 30 min
- **c** 2 h 15 min and 1 h 15 min
- **d** 6 h 15 min and 2 h 30 min
- **e** 3 h 45 min and 1 h 30 min
- **f** 4 h 45 min and 2 h 45 min

Example 4a

4 Mentally calculate the time interval between these pairs of times.
- **a** 2:40 a.m. to 4:45 a.m.
- **b** 4:20 p.m. to 6:30 p.m.
- **c** 1:50 p.m. to 5:55 p.m.
- **d** 12:07 p.m. to 2:18 p.m.
- **e** 6:40 a.m. to 8:30 a.m.
- **f** 1:30 a.m. to 5:10 a.m.
- **g** 10:35 p.m. to 11:22 p.m.
- **h** 3:25 a.m. to 6:19 a.m.
- **i** 6:18 a.m. to 9:04 a.m.
- **j** 7:51 p.m. to 11:37 p.m.

5 Write the time for these descriptions.
- **a** 4 hours after 2:30 p.m.
- **b** 10 hours before 7 p.m.
- **c** $3\frac{1}{2}$ hours before 10 p.m.
- **d** $7\frac{1}{2}$ hours after 9 a.m.
- **e** $6\frac{1}{4}$ hours after 11:15 a.m.
- **f** $1\frac{3}{4}$ hours before 1:25 p.m.

Example 4b

6 Calculate the time interval between these pairs of times. You may wish to use the DMS or °, ′, ″ buttons on your calculator.
- **a** 2 h 10 min 20 s to 4 h 20 min 30 s
- **b** 5 h 30 min 15 s to 8 h 45 min 21 s
- **c** 9 h 46 min 13 s to 10 h 50 min 27 s
- **d** 1 h 30 min 10 s to 2 h 25 min 5 s
- **e** 6 h 43 min 28 s to 8 h 37 min 21 s
- **f** 4 h 51 min 42 s to 10 h 36 min 10 s

7 For each of the following, add the time durations to find the total time. Give your answers in hours, minutes and seconds.
- **a** $2\frac{2}{4}$ hours and $3\frac{1}{2}$ hours
- **b** $5\frac{1}{3}$ hours and $2\frac{2}{3}$ hours
- **c** 6.2 hours and 2.9 hours
- **d** 0.3 hours and 4.2 hours
- **e** 2 h 40 min 10 s and 1 h 10 min 18 s
- **f** 10 h 50 min 18 s and 2 h 30 min 12 s

PROBLEM-SOLVING AND REASONING	8, 9, 14	9–11, 14, 15	11–13, 15, 16

8 A ferry takes Selina from Cabarita to Circular Quay in 23 minutes and 28 seconds. The return trip takes 19 minutes and 13 seconds. What is Selina's total travel time?

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Example 5

9 Use this train timetable for Fairfield to Redfern to answer these questions.

Station	a.m.	p.m.
Fairfield	7:32	2:43
Granville	7:44	2:56
Auburn	7:48	2:59
Ashfield	8:01	3:12
Redfern	8:11	3:23

a How long does it take to travel from:
- **i** Fairfield to Auburn in the morning?
- **ii** Granville to Redfern in the morning?
- **iii** Auburn to Redfern in the afternoon?
- **iv** Fairfield to Redfern in the afternoon?

b Does it take longer to travel from Fairfield to Redfern in the morning or afternoon?

c Jeremiah travels from Fairfield to Auburn in the morning and then from Auburn to Redfern in the afternoon. What is Jeremiah's total travel time?

10 A scientist argues that dinosaurs died out 52 million years ago, whereas another says they died out 108 million years ago. What is the difference in their time estimates?

11 Three essays are marked by a teacher. The first takes 4 minutes and 32 seconds to mark, the second takes 7 minutes and 19 seconds, and the third takes 5 minutes and 37 seconds. What is the total time taken to complete marking the essays?

12 Adrian arrives at school at 8:09 a.m. and leaves at 3:37 p.m. How many hours and minutes is Adrian at school?

13 On a flight to Europe, Janelle spends 8 hours and 36 minutes on a flight from Melbourne to Kuala Lumpur, Malaysia, 2 hours and 20 minutes at the airport at Kuala Lumpur, and then 12 hours and 19 minutes on a flight to Geneva, Switzerland. What is Janelle's total travel time?

14 To convert the speed 10 metres per second (m/s) to kilometres per hour (km/h), you must multiply by 3600 and divide by 1000 to give a factor of 3.6. Explain why.

15 Give a reason why airports and other workplaces might use a 24-hour display rather than an a.m. or p.m. display.

16 Calculate a rent of $400 per week as a yearly amount. Assume 365 days in a year and show your working.

ENRICHMENT	—	—	17–19

Time challenges

17 A doctor earns $180 000 working 40 weeks per year, 5 days per week, 10 hours per day. What does the doctor earn in each of these time periods?

a per day **b** per hour **c** per minute **d** per second (in cents)

18 What is the angle between the hour and minute hands on an analogue clock at these times?

a 6:15 a.m. **b** 4:55 p.m. **c** 5:47 a.m.

19 Rex takes 3 hours to paint a standard-sized bedroom, whereas his mate Wilbur takes 4 hours to paint a room of the same size. How long will it take to paint a standard-sized room if they work together?

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7C Using time zones

Interactive
Widgets
HOTsheets
Walkthrough

During the 19th century, as railways and telecommunications developed, it became increasingly important to deal with the difference in local times. Standard time zones were introduced around the world, most of which are one-hourly deviations from standard time, which is taken to be the time in Greenwich, England (United Kingdom). Standard time or Greenwich Mean Time (GMT) is now called Coordinated Universal Time (UTC).

In Australia, we use three main time zones: the Western, Central and Eastern Standard Time zones, which alter for daylight saving in some states and territories.

Let's start: Time zone discussion

In groups or as a class, discuss what you know about Australian and international time zones. You may wish to include:

- time zones and time differences within Australia
- the current time in other cities
- daylight saving
- the timing of telecasts of sporting events around the world
- jet lag.

Key ideas

- The map on the following pages shows the time zones all around the world.
- Time is based on the time in a place called Greenwich, United Kingdom, and this is called Coordinated Universal Time (UTC) or Greenwich Mean Time (GMT).
- Australia has three time zones:
  - Eastern Standard Time (EST), which is UTC plus 10 hours.
  - Central Standard Time (CST), which is UTC plus 9.5 hours.
  - Western Standard Time (WST), which is UTC plus 8 hours.

Perth (WA) (4 p.m.) — add 2 hours → Sydney (NSW) (6 p.m.)
Sydney (NSW) (6 p.m.) — subtract 2 hours → Perth (WA) (4 p.m.)

- Daylight saving is the practice of moving clocks forward, usually by one hour, to create more daylight in the evening and less daylight in the morning. In some parts of Australia, daylight saving runs from the first Sunday in October to the first Sunday in April.
- The International Date Line separates one calendar day from the next. So crossing the date line from west to east subtracts one day.

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Example 6 Working with Australian time zones

Use the Australian standard time zones map (on pages 326–327) to help with these questions. When it is 8:30 a.m. in New South Wales, what time is it in each of the following?

a Queensland **b** Northern Territory **c** Western Australia

SOLUTION	EXPLANATION
a 8:30 a.m.	Using standard time, NSW and Qld are in the same time zone.
b 8:00 a.m.	NT is UTC + $9\frac{1}{2}$ hours, whereas NSW is UTC + 10 hours. So NT is $\frac{1}{2}$ hour behind.
c 6:30 a.m.	WA is UTC + 8 hours and so is 2 hours behind NSW.

Example 7 Using time zones

Coordinated Universal Time (UTC) is based on the time in Greenwich, United Kingdom. Use the world time zone map to answer the following.

a When it is 2 p.m. UTC, find the time in these places.
- **i** France
- **ii** China
- **iii** Queensland
- **iv** Alaska

b When it is 9:35 a.m. in New South Wales, Australia, find the time in these places.
- **i** Alice Springs
- **ii** Perth
- **iii** London
- **iv** central Greenland

SOLUTION	EXPLANATION
a i 2 p.m. + 1 hour = 3 p.m.	Use the time zone map to see that France is to the east of Greenwich and is in a zone that is 1 hour ahead.
ii 2 p.m. + 8 hours = 10 p.m.	From the time zone map, China is 8 hours ahead of Greenwich.
iii 2 p.m. + 10 hours = 12 midnight	Queensland uses Eastern Standard Time, which is 10 hours ahead of Greenwich.
iv 2 p.m. − 9 hours = 5 a.m.	Alaska is to the west of Greenwich, in a time zone that is 9 hours behind.
b i 9:35 a.m. $-\frac{1}{2}$ hour = 9:05 a.m.	Alice Springs uses Central Standard Time, which is $\frac{1}{2}$ hour behind Eastern Standard Time.
ii 9:35 a.m. − 2 hours = 7:35 a.m.	Perth uses Western Standard Time, which is 2 hours behind Eastern Standard Time.
iii 9:35 a.m. − 10 hours = 11:35 p.m. (the day before)	UTC (time in Greenwich, United Kingdom) is 10 hours behind EST.
iv 9:35 a.m. − 13 hours = 8:35 p.m. (the day before)	Central Greenland is 3 hours behind UTC in Greenwich, so is 13 hours behind EST.

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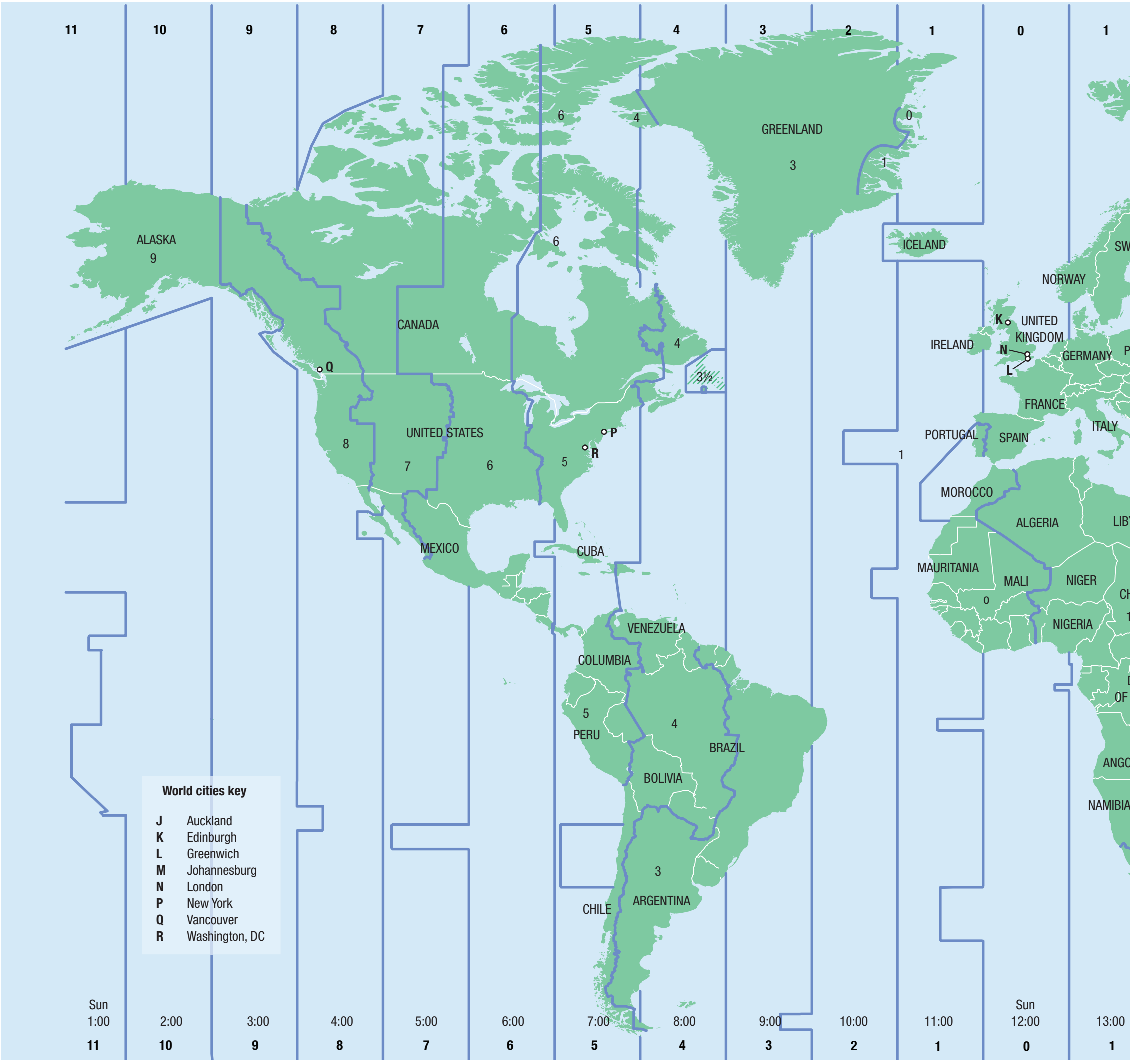

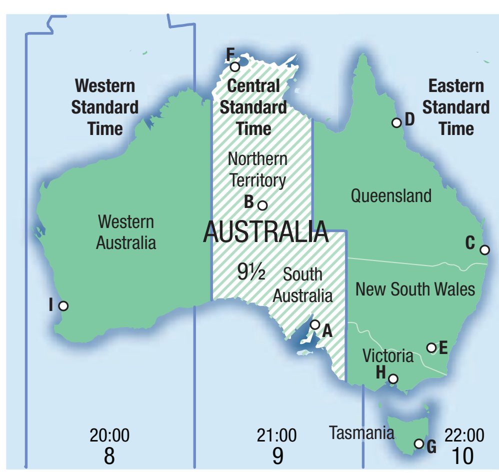

Australian cities key

A Adelaide
B Alice Springs
C Brisbane
D Cairns
E Canberra, ACT
F Darwin
G Hobart
H Melbourne
I Perth

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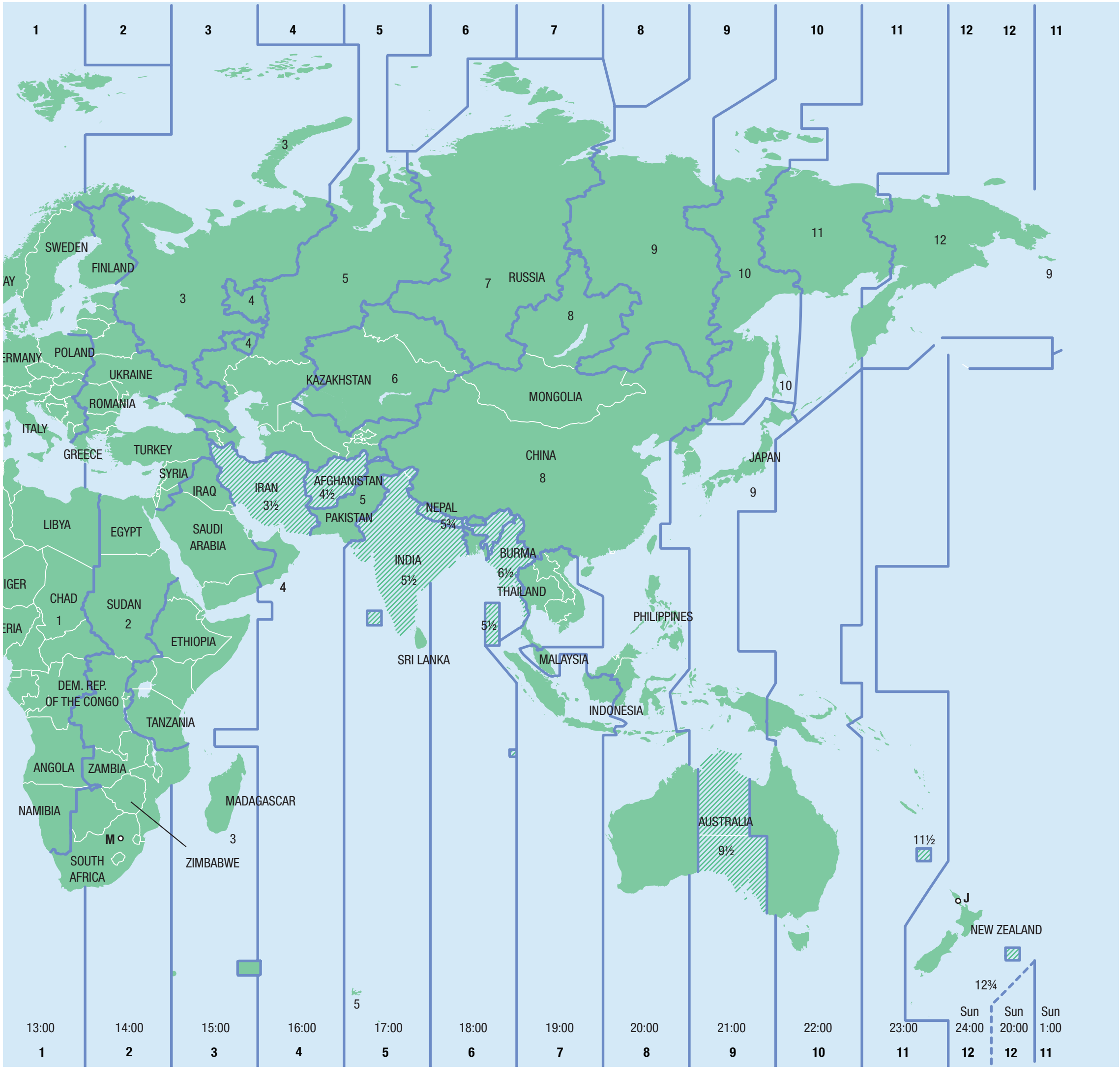

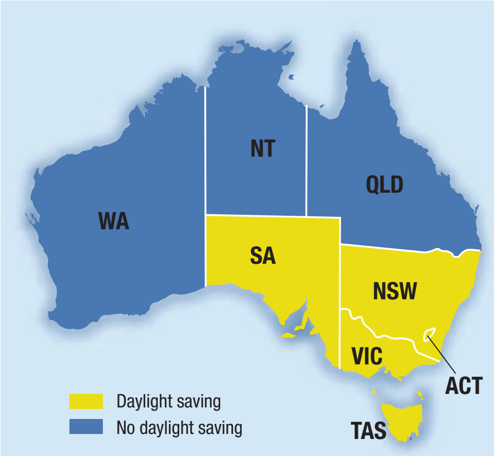

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Exercise 7C

UNDERSTANDING AND FLUENCY	1–6	3–5, 6–7(½)	4(½), 5, 6–7(½)

1 Which Australian states and territories use daylight saving time?

2 **a** How many hours in front of Coordinated Universal Time (UTC) are these countries and Australian states?

i Victoria, Australia **ii** South Australia **iii** Western Australia
iv Thailand **v** China **vi** Egypt

b How many hours behind Coordinated Universal Time (UTC) are the following countries?

i Iceland **ii** eastern Brazil
iii Columbia **iv** Peru

3 When it is 10 a.m. Monday in New Zealand, what day of the week is it in the USA?

Example 6

4 Use the Australian standard time zones map (on pages 326–327) to help answer the following. When it is 10 a.m. in New South Wales, what time is it in these states and territories?

a Victoria **b** South Australia
c Tasmania **d** Northern Territory
e Western Australia **f** Australian Capital Territory

5 Use the Australian standard time zones map to help answer the following. When it is 4:30 p.m. in Western Australia, what time is it in the following states?

a South Australia **b** New South Wales **c** Tasmania

Example 7a

6 Use the time zone map to find the time in the following places, when it is 10 a.m. UTC.

a Spain **b** Turkey **c** Tasmania **d** Darwin
e Argentina **f** Peru **g** Alaska **h** Portugal

Example 7b

7 Use the time zone map to find the time in these places, when it is 3:30 p.m. in New South Wales.

a United Kingdom **b** Libya **c** Sweden **d** Perth
e Japan **f** central Greenland **g** Alice Springs **h** New Zealand

PROBLEM-SOLVING AND REASONING	8, 9, 13, 14	9–11, 14, 15	10–12, 16, 17

8 What is the time difference between these pairs of places?

a United Kingdom and Kazakhstan
b South Australia and New Zealand
c Queensland and Egypt
d Peru and Angola (in Africa)
e Mexico and Germany

9 Rick in Wollongong, NSW wants to watch a soccer match that is being televised at 2 p.m. in England (United Kingdom). What time will he need to switch on his television in Wollongong? (Use Eastern Standard Time.)

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10 At the London Olympics a rowing race is scheduled to begin at 11:35 a.m. What time will this be in Broome, Western Australia?

11 A 2-hour football match starts at 2:30 p.m. Eastern Standard Time (EST) in Newcastle, NSW. What time will it be in the United Kingdom when the match finishes?

12 If the date is 29 March and it is 3 p.m. in Perth, what is the time and date in these places?
- a Italy
- b Alaska
- c Chile

13 Use the daylight saving time zone map (on pages 326–327) to help answer the following. During daylight saving time, when it is 9:30 a.m. in Sydney, what time is it in the following states?
- a Queensland
- b Victoria
- c South Australia
- d Western Australia

14 During daylight saving time, Alice drives from Kingscliff in New South Wales to the Gold Coast in Queensland. How will she need to adjust her wristwatch when she crosses the border?

15 Explain why Eastern Standard Time in Australia is 11 hours ahead of the United Kingdom for a proportion of the year.

16 Monty departs on a 20-hour flight from Brisbane to London, United Kingdom, at 5 p.m. on 20 April. Give the time and date of his arrival in London (ignoring UK daylight saving time).

17 Elsa departs on an 11-hour flight from Johannesburg, South Africa, to Perth at 6:30 a.m. on 25 October. Give the time and date of her arrival in Perth. (Note: South Africa does not use daylight saving time.)

ENRICHMENT	—	—	18

Time anomalies

18 There are a number of interesting anomalies associated with time zones. You may wish to use the internet to help explore these topics.
- a Usually, states and territories to the east are ahead of those in the west. During daylight saving time, however, this is not true for all states in Australia. Can you find these states and explain why?
- b Broken Hill is in New South Wales but does not use the New South Wales time zone. Explore.
- c Does Lord Howe Island (part of New South Wales) use the same time as New South Wales all year round? Discuss.
- d Are there any other time zone anomalies in Australia or overseas that you can describe?

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Investigation

Around-the-world race

Imagine you are planning a race around the world starting in Sydney and stopping for 3 hours in each of Kuala Lumpur, Abu Dhabi, London, New York and Honolulu, in that order, before returning to Sydney. The 3-hour stopover in each city is required to pick up special tokens.

Travel time estimates

a Using a rough guess, estimate how many hours it might take to complete the Around-the-world race including the 3-hour stopovers.

b What do you think the approximate cruising speed of a passenger aircraft is in km/h? Use the internet to check your estimate. Also, find out the approximate circumference of the earth.

c Hence, estimate the total non-stop travel time around the world. You may wish to use the rule Time = Distance ÷ Speed.

d Add the 3-hour stopover times to calculate the total time required for the Around-the-world race.

Time zones

a Using the map on pages 326–327, state the time zones for the following cities. An example answer might look like UTC + 8 hours.

i Sydney
ii Kuala Lumpur
iii Abu Dhabi
iv London
v New York
vi Honolulu

b Find the UTC time difference between these cities.

i Sydney and Kuala Lumpur
ii Kuala Lumpur and Abu Dhabi
iii Abu Dhabi and London
iv London and New York
v New York and Honolulu
vi Honolulu and Sydney

c Between which two cities would you cross the International Date Line in this Around-the-world race?

Distance and travel time

a Complete this table to analyse the time required to travel between each pair of cities. Use the internet to find the approximate distances between cities and assume that the average speed of the aircraft is 800 km/h. Use the rule Time = Distance ÷ Speed to calculate the travel time.

Route	Distance	Travel time
Sydney to Kuala Lumpur		
Kuala Lumpur to Abu Dhabi		
Abu Dhabi to London		
London to New York		
New York to Honolulu		
Honolulu to Sydney		

b By adding in the 3-hour stopover time in each city except Sydney, recalculate the approximate time required to complete the entire race.

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Arrival and departure times

a Complete this table to estimate the landing and take-off times for each city. Use the following information.

- Starting time in Sydney is 6:00 a.m.
- Estimated travel times from the previous page.
- 3-hour stopover times in each city except Sydney
- The UTC time difference between each city calculated in the Time Zones section on the previous page.

City	Arrival time	Departure time
Sydney		6:00 a.m.
Kuala Lumpur		
Abu Dhabi		
London		
New York		
Honolulu		
Sydney		

b Some cities have curfews where no arrivals or departures are allowed in a given timeframe. For this activity we will assume that all cities have a 12:00 midnight to 6:00 a.m. curfew. Departure and arrival times will need to be delayed in some cases to account for these curfews. Complete this table to recalculate the arrival and departure times for each city using this curfew.

City	Arrival time	Departure time
Sydney		6:00 a.m.
Kuala Lumpur		
Abu Dhabi		
London		
New York		
Honolulu		
Sydney		

c Compare the total time taken to complete the Around-the-world race including curfews with the total time taken without curfews. Did the curfews make much difference?

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Puzzles and challenges

1 Teaghan takes 7 hours to fly from Dubbo, New South Wales to Esperance, Western Australia. She departs at 7 a.m. What is the time in Esperance when she arrives? (Use Australian standard time.)

2 Albert can dig a post hole in 6 minutes and Sue can dig a post hole in 8 minutes. How long will it take for them to dig one hole if they work together?

3 When it is a Tuesday on 25 October in a particular year, what day will it be on 25 October in the following year, if it is not a leap year?

4 The average time for five snails to complete a race is 2 min 30 s. Four of the snails' race times are 2 min 20 s, 3 min, 2 min 10 s and 1 min 50 s. What is the fifth snail's race time?

5 At a speedway, the yellow car completes a lap every 30 seconds and the blue car completes a lap every 50 seconds. If the cars both start at the same place, how long will it take for the blue car to 'lap' the yellow car?

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Time

Conversion

day	hour	minute	second

day → hour: × 24; hour → minute: × 60; minute → second: × 60

second → minute: ÷ 60; minute → hour: ÷ 60; hour → day: ÷ 24

24-hour time

9:26 a.m. is 0926
5:03 p.m. is 1703

Time zones

UTC (Coordinated Universal Time) is the time in Greenwich, England (United Kingdom).

- Countries to the east are ahead
- Countries to the west are behind

DMS conversion

1.47 hours → 1 h 28′ 12″

Australian time zones

State/ Territory	Standard	Daylight saving
WA	+ 8	+ 8
NT	$+9\frac{1}{2}$	$+9\frac{1}{2}$
SA	$+9\frac{1}{2}$	$+10\frac{1}{2}$
Tas.	+ 10	+ 11
Vic.	+ 10	+ 11
NSW	+ 10	+ 11
ACT	+ 10	+ 11
Qld	+ 10	+ 10

Mental strategies

Finding duration
4:35 to 6:44 = 2 h + 9 min
6:19 to 8:07 = 2 h − 12 min
= 1 h 48 min

Addition

2 h 26 min 12s + 3 h 24 min + 56 s
= 5 h + 50 min + 68 s
= 5 h 51 min 8 s

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Multiple-choice questions

1 The number of minutes in 3 hours is:

A 180 **B** 60 **C** 90
D 300 **E** 200

2 How many years are there in 42 months?

A 2 **B** 2.5 **C** 2
D 3.5 **E** 4

3 When written using 24-hour time, 4:26 p.m. is:

A 0626 **B** 1226 **C** 0426
D 1426 **E** 1626

4 Converting $2\frac{2}{3}$ hours to hours and minutes gives:

A 2 h 67 min **B** 2 h 35 min **C** 2 h 23 min
D 2 h 40 min **E** 2 h 30 min

5 Converting 2.64 hours to hours, minutes and seconds gives:

A 2 h 40 min 12 s **B** 2 h 38 min 24 s **C** 3 h 4 min 0 s
D 2 h 30 min 10 s **E** 2 h 60 min 4 s

6 The time taken to make and assemble two chairs is 3 hours 40 minutes and 15 seconds and 2 hours 38 minutes and 51 seconds. Hence, the total build time is:

A 5 h 58 min 6 s **B** 6 h 20 min 6 s **C** 6 h 19 min 6 s
D 6 h 19 min 66 s **E** 6 h 18 min 6 s

7 The time interval from 3:36 a.m. to 4:27 a.m. is:

A 51 min **B** 49 min **C** 41 min
D 39 min **E** 61 min

8 How many hours is Western Australia behind New South Wales during Australian standard time?

A 5 **B** 4 **C** 3
D 2 **E** 1.5

9 If it is 12 noon during daylight saving time in South Australia, what time is it in Queensland?

A 2 p.m. **B** 2:30 p.m. **C** 1 p.m.
D 12:30 p.m. **E** 11:30 a.m.

10 When it is 4 a.m. UTC, the time in Sydney is:

A 1:30 p.m. **B** 1 p.m. **C** 2 p.m.
D 3 p.m. **E** 3 a.m.

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Short-answer questions

1 Convert these times to the units shown in brackets.

- **a** $1\frac{1}{2}$ h (min)
- **b** 120 s (min)
- **c** 48 h (days)
- **d** 3 weeks (days)
- **e** 1 day (min)
- **f** 1800 s (h)

2 Re-write these times, using the system shown in brackets.

- **a** 4 a.m. (24-hour time)
- **b** 3:30 p.m. (24-hour time)
- **c** 7:19 p.m. (24-hour time)
- **d** 0635 (a.m./p.m.)
- **e** 1251 (a.m./p.m.)
- **f** 2328 (a.m./p.m.)

3 Re-write these times, using hours and minutes.

a $3\frac{1}{2}$ hours **b** $4\frac{1}{3}$ hours **c** 6.25 hours **d** 1.75 hours

4 Use the DMS button/function on your calculator to convert the following to hours, minutes and seconds.

a 3.6 hours **b** 6.92 hours **c** 11.44 hours

5 Margaret is catching a train leaving at 1330 in London and arriving at 1503 in York. What will be Margaret's travel time?

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6 Calculate the time interval between these pairs of times. Give your answer in hours, minutes and seconds.

a 7:43 a.m. to 1:36 p.m.

b 2 h 30 min 10 s to 6 h 36 min 5 s

c 5 h 52 min 6 s to 7 h 51 min 7 s

d 0931 to 1309

7 Use this train timetable for Telarah to Newcastle to answer the following questions.

Station	a.m.	p.m.
Telarah	7:30	2:52
Metford	7:42	3:04
Sandgate	7:55	3:16
Hamilton	8:10	3:30
Newcastle	8:16	3:36

a How long does it take to travel from:

i Telarah to Sandgate in the morning?

ii Metford to Newcastle in the morning?

iii Sandgate to Newcastle in the afternoon?

iv Telarah to Newcastle in the afternoon?

b Does it take longer to travel from Telarah to Newcastle in the morning or afternoon?

c Ashdi travels from Telarah to Sandgate in the morning, then from Sandgate to Newcastle in the afternoon. What is Ashdi's total travel time?

8 Use the Australian time zone maps (on pages 326–327) to help answer these questions.

a During Australian standard time it is 7:45 a.m. in South Australia. What time is it in:

i New South Wales?

ii Western Australia?

b During Australian daylight saving time it is 4:36 p.m. in New South Wales. What time is it in:

i Western Australia?

ii Queensland?

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9 An AFL match telecast begins at 2:10 p.m. Eastern Standard Time. At what time will someone in the Northern Territory need to switch on the television if they want to watch the game?

Extended-response questions

1 Use the International time zone maps (on pages 326–327) to answer these questions.

a When it is 11 a.m. UTC, state the time in:
- i Sydney
- ii Ethiopia
- iii Pakistan

b When it is 3:30 p.m. in New South Wales, state the time in:
- i Zimbabwe
- ii China
- iii Bolivia

c When it is 6 a.m. Tuesday in New South Wales, state the day of the week in:
- i India
- ii Canada

d Chris flies from Sydney, leaving at 8 a.m., and travels for 7 hours, arriving in Kuala Lumpur, Malaysia. What is the time in Kuala Lumpur when he arrives?

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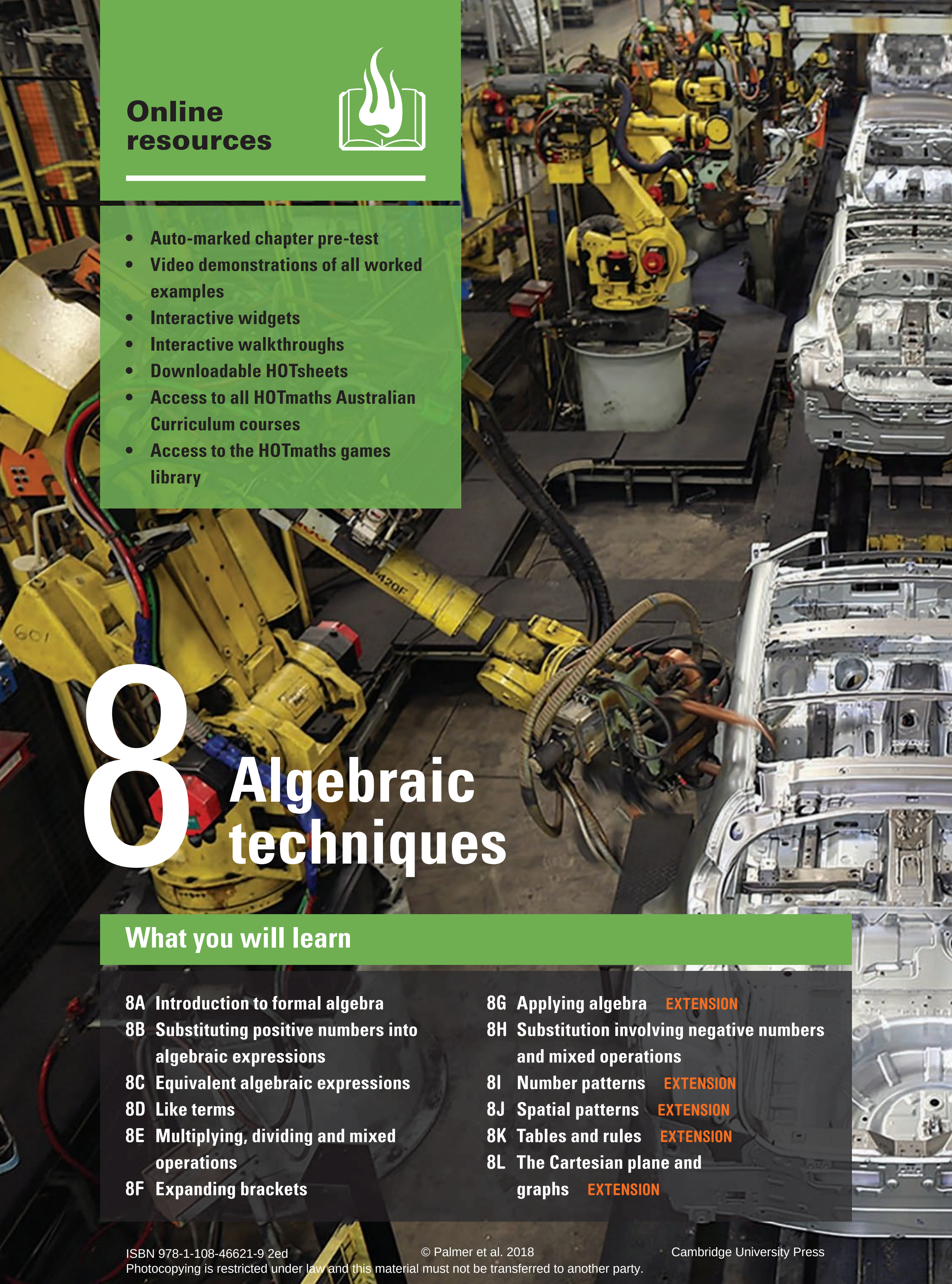

Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

8 Algebraic techniques

What you will learn

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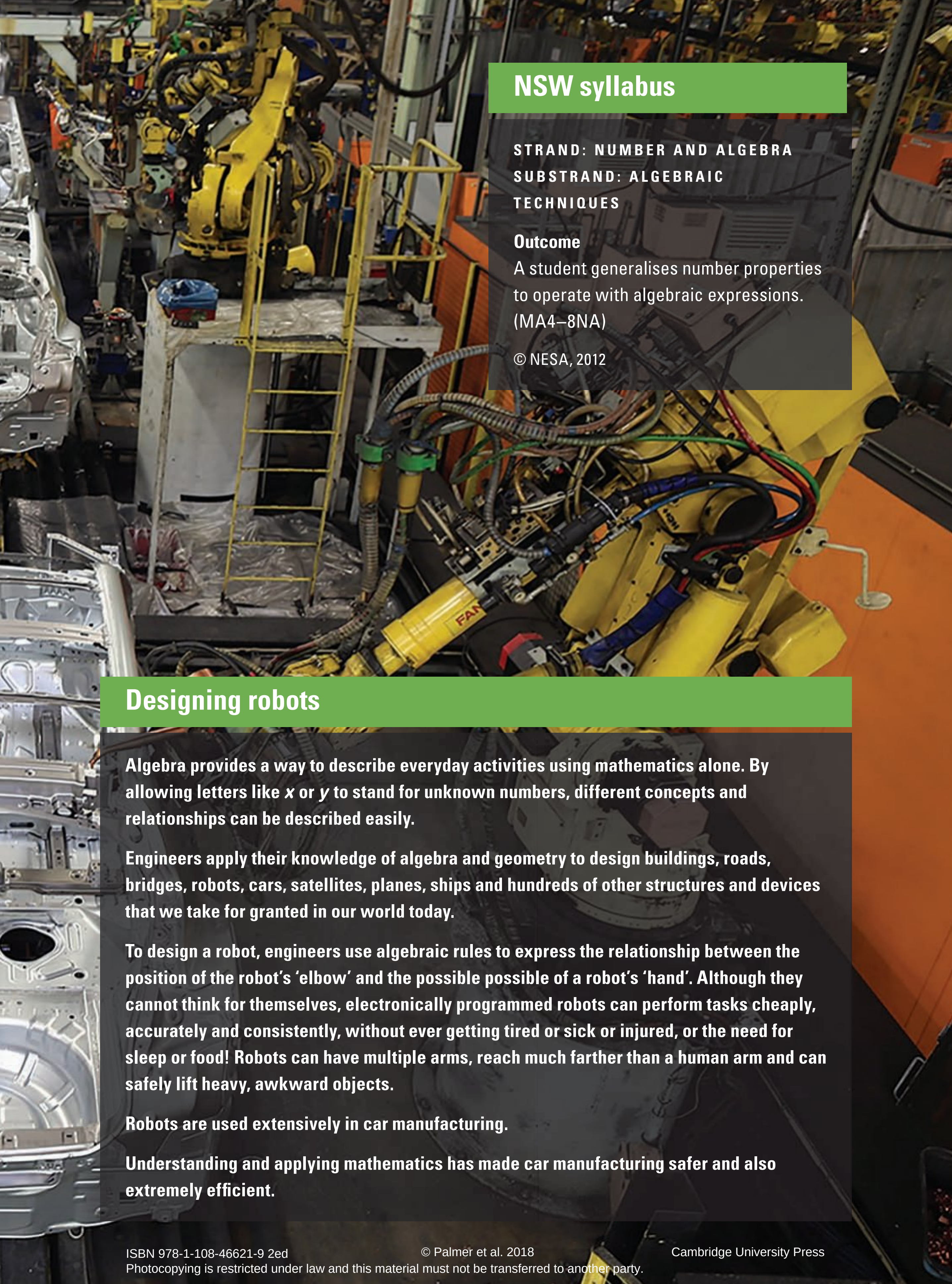

NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRAND: ALGEBRAIC TECHNIQUES

Outcome

A student generalises number properties to operate with algebraic expressions. (MA4–8NA)

Designing robots

Algebra provides a way to describe everyday activities using mathematics alone. By allowing letters like *x* or *y* to stand for unknown numbers, different concepts and relationships can be described easily.

Engineers apply their knowledge of algebra and geometry to design buildings, roads, bridges, robots, cars, satellites, planes, ships and hundreds of other structures and devices that we take for granted in our world today.

To design a robot, engineers use algebraic rules to express the relationship between the position of the robot's 'elbow' and the possible possible of a robot's 'hand'. Although they cannot think for themselves, electronically programmed robots can perform tasks cheaply, accurately and consistently, without ever getting tired or sick or injured, or the need for sleep or food! Robots can have multiple arms, reach much farther than a human arm and can safely lift heavy, awkward objects.

Robots are used extensively in car manufacturing.

Understanding and applying mathematics has made car manufacturing safer and also extremely efficient.

Pre-test

1 If □ = 7, write the value of each of the following.
a □ + 4   b □ − 2   c 12 − □   d 3 × □

2 Write the value of □ × 4 if:
a □ = 2   b □ = 9   c □ = 10   d □ = 2.5

3 Write the answer to each of the following computations.
a 4 and 9 are added
b 3 is multiplied by 7
c 12 is divided by 3
d 10 is halved

4 Write down the following, using numbers and the symbols +, ÷, × and −.
a 6 is tripled
b 10 is halved
c 12 is added to 3
d 9 is subtracted from 10

5 For each of the tables, describe the rule relating the *input* and *output* numbers. For example: *output* = 2 × *input*.

a

input	1	2	3	5	9
output	3	6	9	15	27

b

input	1	2	3	4	5
output	6	7	8	9	10

c

input	1	5	7	10	21
output	7	11	13	16	27

d

input	3	4	5	6	7
output	5	7	9	11	13

6 If the value of x is 8, find the value of:
a $x + 3$ b $x - 2$ c $x \times 5$ d $x \div 4$

7 Find the value of each of the following.
a $4 \times 3 + 8$ b $4 \times (3 + 8)$ c $4 \times 3 + 2 \times 5$ d $4 \times (3 + 2) \times 5$

8 Find the value of each of the following.
a $50 - (3 \times 7 + 9)$ b $24 \div 2 - 6$ c $24 \div 6 - 2$ d $24 \div (6 - 2)$

9 If □ = 5, write the value of each of the following.
a □ − 4   b □ × 2 − 1   c □ ÷ 5 + 2   d □ × 7 + 10
e □ × □   f □ × □ ÷ □   g 3 × □ − 15   h □²

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8A Introduction to formal algebra

Interactive
Widgets
HOTsheets
Walkthrough

A pronumeral is a letter that can represent any number. The choice of letter used is not significant mathematically, but can be used as an aide to memory. For instance, h might stand for someone's height and w might stand for someone's weight.

The table shows the salary Petra earns for various hours of work if she is paid \$12 an hour.

Number of hours	Salary earned (\$)
1	$12 \times 1 = 12$
2	$12 \times 2 = 24$
3	$12 \times 3 = 36$
n	$12 \times n = 12n$

Rather than writing $12 \times n$, we write $12n$ because multiplying a pronumeral by a number is common and this notation saves space. We can also write $18 \div n$ as $\frac{18}{n}$.

Using pronumerals, we can work out a total salary for any number of hours of work.

Let's start: Pronumeral stories

Ahmed has a jar with b biscuits. He eats 3 biscuits and then shares the rest equally among 8 friends. Each friend receives $\frac{b-3}{8}$ biscuits. This is a short story for the expression $\frac{b-3}{8}$.

- Try to create another story for $\frac{b-3}{8}$, and share it with others in the class.
- Can you construct a story for $2t + 12$? What about $4(k + 6)$?

Key ideas

- A list of all the terminology used in this chapter is provided on the following pages.
- Algebra is used to describe the rules and conventions of numbers and arithmetic.
- When doing algebra, we use the equals symbol (=) to indicate that two or more things have exactly the same value. This is called an **identity** because they are identical.

  For example: $3 + 5$ gives 8, which is the same as $5 + 3$.

  So we can write $3 + 5 = 5 + 3$.

  'is identical to' or 'has the same value as' (pointing to =)

Key ideas

- In the following table, the letters a, b and c could represent any numbers.

	Number fact	Algebra fact	Notes
1	$5 + 3 = 3 + 5$	$a + 3 = 3 + a$	Numbers can be added in any order.
2	$5 + 3 = 3 + 5$	$a + b = b + a$	
3	$4 + 5 + 1 = 1 + 4 + 5$	$b + c + a = a + b + c$	
4	$-3 + 5 = 5 - 3$	$-3 + a = a - 3$	The negative sign 'belongs' to the number 3.
5	$5 \times 3 = 3 \times 5$ Numbers can be multiplied in any order.	$a \times 3 = 3 \times a$	The multiplication symbol is usually 'invisible'. $a \times 3 = 3 \times a = 3a$
6	$5 \times 3 = 3 \times 5$	$a \times b = b \times a$	This can be written as: $a \times b = ab$
7	$5 + 5 + 5 = 3 \times 5$ meaning '3 lots of 5'.	$a + a + a = 3 \times a$	This can be written as: $a + a + a = 3a$
8	$2 \div 8 = \frac{2}{8}$	$a \div 8 = \frac{a}{8}$	Division and fractions are related to each other. The first number in the division is the numerator in the fraction.
9	$2 \div 8 = \frac{2}{8}$	$a \div b = \frac{a}{b}$	

Terminology	Example	Definition
pronumeral	a or b or c	A letter of an alphabet or a symbol used to represent one or more numerical values
variable	a or b or c	A pronumeral that represents more than one numerical value
expression or algebraic expression	$3a + 5$	A statement containing numbers and pronumerals that are connected by mathematical operations but containing no equals sign
term	The expression $3a + 5$ contains two terms.	One of the components of an expression
like terms	$3a$ and $5a$ are like terms. $3a$ and $5a^2$ are *not* like terms.	Two or more terms that contain the same pronumerals
constant or constant term	In the expression $3a + 5$, the number 5 is called the constant or the constant term.	The part of an expression without any pronumerals
coefficient	In the expression $3a + b + 5$: • The coefficient of a is 3. • The coefficient of b is 1.	A numeral placed before a pronumeral to indicate that the pronumeral is multiplied by that factor
equivalent expressions	$3a + 5$ and $5 + 3a$	Expressions that will always have the same numerical value as each other when the pronumerals are replaced with any number

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Key ideas

Terminology	Example	Definition
simplify	$3a + 5a$ simplifies to $8a$. $3a + 5$ can't be simplified.	To find the simplest possible equivalent expression
identity	$3a + 5 = 5 + 3a$ or $3a + 5 \equiv 5 + 3a$	A statement that indicates that two expressions will have the same numerical value when the pronumerals are replaced with numbers. The symbol $\equiv$ is sometimes used in identities.
substitute	When $a = 3$, $a + 5$ becomes $3 + 5$.	To replace pronumerals with numerical values
substitution	When $a = 3$, the value of $a + 5$ is 8.	A process in which pronumerals are replaced with numbers
evaluate	Evaluate $a + 5$ when $a = 3$.	To calculate the numerical value of an expression in which all the pronumerals have been given a value

Example 1 The terminology of algebra

a List the individual terms in the expression $3a + b + 13c$.
b State the coefficient of each pronumeral in the expression $3a + b + 13c$.
c Give an example of an expression with exactly two terms, one of which is a constant term.

SOLUTION	EXPLANATION
a There are three terms: $3a$, b and $13c$.	Each part of an expression is a term. Terms get added (or subtracted) to make an expression.
b The coefficient of a is 3, the coefficient of b is 1 and the coefficient of c is 13.	The coefficient is the number in front of a pronumeral. For b the coefficient is 1 because b is the same as $1 \times b$.
c $27a + 19$ (There are many other expressions.)	This expression has two terms, $27a$ and 19, and 19 is a constant term because it is a number without any pronumerals.

Example 2 Writing expressions from word descriptions

Write an expression for each of the following.

a 5 more than k
b 3 less than m
c the sum of a and b
d double the value of x
e the product of c and d

SOLUTION	EXPLANATION
a $k + 5$	5 must be added to k to get 5 more than k.
b $m - 3$	3 is subtracted from m.
c $a + b$	a and b are added to obtain their sum.
d $2 \times x$ or just $2x$	x is multiplied by 2. The multiplication sign is optional.
e $c \times d$ or just cd	c and d are multiplied to obtain their product.

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Example 3 Expressions involving more than one operation

Write an expression for each of the following without using the × or ÷ symbols.

a p is halved, then 4 is added

b the sum of x and y is taken and then divided by 7

c the sum of x and one-seventh of y

d 5 is subtracted from k and the result is tripled

SOLUTION	EXPLANATION
a $\frac{p}{2} + 4$	p is divided by 2, then 4 is added.
b $(x + y) \div 7 = \frac{x + y}{7}$	x and y are added. This whole expression is divided by 7. By writing the result as a fraction, the brackets are no longer needed.
c $x + \frac{y}{7}$ or $x + \frac{1}{7}y$	x is added to one-seventh of y, which is $\frac{y}{7}$.
d $(k - 5) \times 3 = 3(k - 5)$	5 subtracted from k gives the expression $k - 5$. Brackets must be used to multiply the whole expression by 3.

Exercise 8A

UNDERSTANDING AND FLUENCY	1–5	2, 3, 4–5(½), 6	3–5(½), 6

Example 1

1 The expression $4x + 3y + 24z + 7$ has four terms.

a List the terms.

b What is the constant term?

c What is the coefficient of x?

d Which letter has a coefficient of 24?

Example 2

2 Match each of the word descriptions on the left with the correct mathematical expression on the right.

a the sum of x and 4 — **A** $x - 4$

b 4 less than x — **B** $\frac{x}{4}$

c the product of 4 and x — **C** $4 - x$

d one-quarter of x — **D** $4x$

e the result from subtracting x from 4 — **E** $\frac{4}{x}$

f 4 divided by x — **F** $x + 4$

3 For each of the following expressions, state:

i the number of terms

ii the coefficient of n.

a $17n + 24$

b $31 - 27a + 15n$

c $15nw + 21n + 15$

d $15a - 32b + \frac{4}{3}xy + 2n$

e $n + 51$

f $5bn - 12 + \frac{d}{2} + 12n$

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4 Write an expression for each of the following without using the × or ÷ symbols.
a 1 more than x
b the sum of k and 5
c double the value of u
d 4 lots of y
e half of p
f one-third of q
g 12 less than r
h the product of n and 9
i t is subtracted from 10
j y is divided by 8

Example 3

5 Write an expression for each of the following without using the × or ÷ symbols.
a 5 is added to x, then the result is doubled.
b a is tripled, then 4 is added.
c k is multiplied by 8, then 3 is subtracted.
d 3 is subtracted from k, then the result is multiplied by 8.
e The sum of x and y is multiplied by 6.
f x is multiplied by 7 and the result is halved.
g p is halved and then 2 is added.
h The product of x and y is subtracted from 12.

6 Describe each of these expressions in words.
a $7x$
b $a + b$
c $(x + 4) \times 2$
d $5 - 3a$

PROBLEM-SOLVING AND REASONING	7, 8, 12	8–10, 12, 13	9–11, 13, 14

7 Nicholas buys 10 lolly bags from a supermarket.
a If there are 7 lollies in each bag, how many lollies does he buy in total?
b If there are n lollies in each bag, how many lollies does he buy in total? Hint: Write an expression involving n.

8 Mikayla is paid $$x$ per hour at her job. Write an expression for each of the following.
a How much does Mikayla earn if she works 8 hours?
b If Mikayla gets a pay rise of $3 per hour, what is her new hourly wage?
c If Mikayla works for 8 hours at the increased hourly rate, how much does she earn?

9 Recall that there are 100 centimetres in 1 metre and 1000 metres in 1 kilometre. Write expressions for each of the following.
a How many metres are there in x km?
b How many centimetres are there in x metres?
c How many centimetres are there in x km?

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10 A group of people go out to a restaurant, and the total amount they must pay is \$$A$. They decide to split the bill equally. Write expressions to answer the following questions.

a If there are 4 people in the group, how much do they each pay?

b If there are n people in the group, how much do they each pay?

c One of the n people has a voucher that reduces the total bill by \$20. How much does each person pay now?

11 There are many different ways of describing the expression $\frac{a+b}{4}$ in words. One way is 'The sum of a and b is divided by 4.' What is another way?

12 If x is a whole number between 10 and 99, classify each of these statements as true or false.

a x must be smaller than $2 \times x$.

b x must be smaller than $x + 2$.

c $x - 3$ must be greater than 10.

d $4 \times x$ must be an even number.

e $3 \times x$ must be an odd number.

13 If b is an even number greater than 3, classify each of these statements as true or false.

a $b + 1$ must be even.

b $b + 2$ could be odd.

c $5 + b$ could be greater than 10.

d $5b$ must be greater than b.

14 If c is a number between 10 and 99, sort the following in ascending order (i.e. smallest to largest).
$3c$, $2c$, $c - 4$, $c \div 2$, $3c + 5$, $4c - 2$, $c + 1$, $c \times c$.

ENRICHMENT	—	—	15

Many words compressed

15 One advantage of writing expressions in symbols rather than words is that it takes up less space. For instance, 'twice the value of the sum of x and 5' uses eight words and can be written as $2(x + 5)$.

Give an example of a worded expression that uses more than 10 words and then write it as a mathematical expression.

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8B Substituting positive numbers into algebraic expressions

Interactive
Widgets
HOTsheets
Walkthrough

Substitution involves replacing pronumerals (like x and y) with numbers and obtaining a single number as a result. For example, we can evaluate $4 + x$ when x is 11, to get 15.

Let's start: Sum to 10

The pronumerals x and y could stand for any number.

- What numbers could x and y stand for if you know that $x + y$ must equal 10? Try to list as many pairs as possible.
- If $x + y$ must equal 10, what values could $3x + y$ equal? Find the largest and smallest values.

Key ideas

- To **evaluate** an expression or to **substitute** values means to replace each pronumeral in an expression with a number to obtain a final value. For example, if $x = 3$ and $y = 8$, then $x + 2y$ evaluated gives $3 + 2 \times 8 = 19$.
- A term like $4a$ means $4 \times a$. When substituting a number we must include the multiplication sign, since two numbers written as 42 is very different from the product 4×2.
- Once an expression contains no pronumerals, evaluate using the normal order of operations seen in Chapter 1:
  - operations inside brackets, followed by
  - multiplication and division from left to right, followed by
  - addition and subtraction from left to right.

  For example:
  $$\begin{aligned}(4 + 3) \times 2 - 20 \div 4 + 2 &= 7 \times 2 - 20 \div 4 + 2 \\ &= 14 - 5 + 2 \\ &= 9 + 2 \\ &= 11\end{aligned}$$

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Example 4 Substituting a pronumeral

Given that $t = 5$, evaluate:

a $t + 7$ **b** $8t$ **c** $\frac{10}{t} + 4 - t$

SOLUTION

a $t + 7 = 5 + 7$
$= 12$

b $8t = 8 \times t$
$= 8 \times 5$
$= 40$

c $\frac{10}{t} + 4 - t = \frac{10}{5} + 4 - 5$
$= 2 + 4 - 5$
$= 1$

EXPLANATION

Replace t with 5 and then evaluate the expression, which now contains no pronumerals.

Insert × between 8 and t, then substitute in 5. If the multiplication sign is not included, we might get a completely incorrect answer of 85.

Replace all occurrences of t with 5 before evaluating. Note that the division $(10 \div 5)$ is calculated before the addition and subtraction.

Example 5 Substituting multiple pronumerals

Substitute $x = 4$ and $y = 7$ to evaluate these expressions.

a $5x + y + 8$ **b** $80 - (2xy + y)$

SOLUTION

a $5x + y + 8 = 5 \times x + y + 8$
$= 5 \times 4 + 7 + 8$
$= 20 + 7 + 8$
$= 35$

b $80 - (2xy + y) = 80 - (2 \times x \times y + y)$
$= 80 - (2 \times 4 \times 7 + 7)$
$= 80 - (56 + 7)$
$= 80 - 63$
$= 17$

EXPLANATION

Insert the implied multiplication sign between 5 and x before substituting the values for x and y.

Insert the multiplication signs, and remember the order in which to evaluate.
Note that both occurrences of y are replaced with 7.

Example 6 Substituting with powers and roots

If $p = 4$ and $t = 5$, find the value of:

a $3p^2$ **b** $t^2 + p^3$ **c** $\sqrt{p^2 + 3^2}$

SOLUTION

a $3p^2 = 3 \times p \times p$
$= 3 \times 4 \times 4$
$= 48$

EXPLANATION

Note that $3p^2$ means $3 \times p \times p$, not $(3 \times p)^2$.

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SOLUTION	EXPLANATION
b $t^2 + p^3 = 5^2 + 4^3$ $= 5 \times 5 + 4 \times 4 \times 4$ $= 25 + 64$ $= 89$	t is replaced with 5, and p is replaced with 4. Remember that 4^3 means $4 \times 4 \times 4$.
c $\sqrt{p^2 + 3^2} = \sqrt{4^2 + 3^2}$ $= \sqrt{25}$ $= 5$	Recall that the square root of 25 must be 5 because $5 \times 5 = 25$.

Exercise 8B

UNDERSTANDING AND FLUENCY	1–5, 6–8(½), 9	5, 6–8(½), 9, 10(½)	6–8(½), 9, 10(½)

1 Use the correct order of operations to evaluate the following.

a $4 + 2 \times 5$ **b** $7 - 3 \times 2$ **c** $3 \times 6 - 2 \times 4$ **d** $(7 - 3) \times 2$

Example 4a

2 What number would you get if you replaced b with 5 in the expression $12 + b$?

3 What number is obtained when $x = 3$ is substituted into the expression $5 \times x$?

4 What is the result of evaluating $10 - u$ if u is 7?

5 Calculate the value of $12 + b$ if:

a $b = 5$ **b** $b = 8$ **c** $b = 60$ **d** $b = 0$

Example 4b,c

6 If $x = 5$, evaluate each of the following. Set out your solution in a manner similar to that shown in **Example 4**.

a $x + 3$ **b** $x \times 2$ **c** $14 - x$
d $2x + 4$ **e** $3x + 2 - x$ **f** $13 - 2x$
g $2(x + 2) + x$ **h** $30 - (4x + 1)$ **i** $\frac{20}{x} + 3$
j $(x + 5) \times \frac{10}{x}$ **k** $\frac{x + 7}{4}$ **l** $\frac{10 - x}{x}$
m $7x + 3(x - 1)$ **n** $40 - 3x - x$ **o** $x + x(x + 1)$
p $\frac{30}{x} + 2x(x + 3)$ **q** $100 - 4(3 + 4x)$ **r** $\frac{6(3x - 8)}{x + 2}$

Example 5

7 Substitute $a = 2$ and $b = 3$ into each of these expressions and evaluate.

a $2a + 4$ **b** $3a - 2$ **c** $a + b$
d $3a + b$ **e** $5a - 2b$ **f** $7ab + b$
g $ab - 4 + b$ **h** $2 \times (3a + 2b)$ **i** $100 - (10a + 10b)$
j $\frac{12}{a} + \frac{6}{b}$ **k** $\frac{ab}{3} + b$ **l** $\frac{100}{a + b}$

8 Evaluate the expression $5x + 2y$ when:

a $x = 3$ and $y = 6$ **b** $x = 4$ and $y = 1$ **c** $x = 7$ and $y = 3$
d $x = 0$ and $y = 4$ **e** $x = 2$ and $y = 0$ **f** $x = 10$ and $y = 10$

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9 Copy and complete each of these tables.

a

n	1	2	3	4	5	6
$n + 4$	5			8		

b

x	1	2	3	4	5	6
$12 - x$			9			

c

b	1	2	3	4	5	6
$2(b - 1)$						

d

q	1	2	3	4	5	6
$10q - q$						

Example 6

10 Evaluate each of the following, given that $a = 9$, $b = 3$ and $c = 5$.

a $3c^2$ **b** $5b^2$ **c** $a^2 - 3^3$ **d** $2b^2 + \frac{a}{3} - 2c$

e $\sqrt{a} + \sqrt{3ab}$ **f** $\sqrt{b^2 + 4^2}$ **g** $24 + \frac{2b^3}{6}$ **h** $(2c)^2 - a^2$

PROBLEM-SOLVING AND REASONING 11, 14 | 11, 12, 14 | 12–15

11 A number is substituted for b in the expression $7 + b$ and gives the result 12. What is the value of b?

12 A number is substituted for x in the expression $3x - 1$. If the result is a two-digit number, what value might x have? Try to describe all the possible answers.

13 Copy and complete the table.

x	5	9	12			
$x + 6$	11			7		
$4x$	20				24	28

14 Assume x and y are two numbers, where $xy = 24$.

a What values could x and y equal if they are whole numbers? Try to list as many as possible.

b What values could x and y equal if they can be decimals, fractions or whole numbers?

15 Dugald substitutes different whole numbers into the expression $5 \times (a + a)$. He notices that the result always ends in the digit 0. Try a few values and explain why this pattern occurs.

ENRICHMENT — | — | 16

Missing numbers

16 **a** Copy and complete the following table, in which x and y are whole numbers.

x	5	10	7			
y	3	4		5		
$x + y$			9	14	7	
$x - y$	2				3	8
xy		40			10	0

b If x and y are two numbers where $x + y$ and $x \times y$ are equal, what values might x and y have? Try to find at least three (they do not have to be whole numbers).

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8C Equivalent algebraic expressions

Interactive
Widgets
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Walkthrough

In algebra, as when using words, there are often many ways to express the same thing. For example, we can write 'the sum of x and 4' as $x + 4$ or $4 + x$, or even $x + 1 + 1 + 1 + 1$.

No matter what number x is, $x + 4$ and $4 + x$ will always be equal. We say that the expressions $x + 4$ and $4 + x$ are equivalent.

By substituting different numbers for the pronumerals it is possible to see whether two expressions are equivalent. Consider the four expressions in this table.

	$3a + 5$	$2a + 6$	$7a + 5 - 4a$	$a + a + 6$
$a = 0$	5	6	5	6
$a = 1$	8	8	8	8
$a = 2$	11	10	11	10
$a = 3$	14	12	14	12
$a = 4$	17	14	17	14

From this table it becomes apparent that $3a + 5$ and $7a + 5 - 4a$ are equivalent, and that $2a + 6$ and $a + a + 6$ are equivalent.

Let's start: Equivalent expressions

Consider the expression $2a + 4$.

- Write as many different expressions as possible that are equivalent to $2a + 4$.
- How many equivalent expressions are there?
- Try to give a logical explanation for why $2a + 4$ is equivalent to $4 + a \times 2$.

Key ideas

- Two expressions are **equivalent** if they are always equal, regardless of which numbers are substituted for the pronumerals.

  For example:
  - $x + 12$ is equivalent to $12 + x$, because the order in which numbers are added is not important.
  - $3k$ is equivalent to $k + k + k$, because multiplying by a whole number is the same as adding repeatedly.
- The rules of algebra are used to prove that two expressions are equivalent, but a table of values can be helpful to test whether expressions are likely to be equivalent.

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Example 7 Equivalent expressions

Which two of these expressions are equivalent: $3x + 4$, $8 - x$, $2x + 4 + x$?

SOLUTION

$3x + 4$ and $2x + 4 + x$ are equivalent.

EXPLANATION

By drawing a table of values, we can see straight away that $3x + 4$ and $8 - x$ are not equivalent, since they differ for $x = 2$.

	$x = 1$	$x = 2$	$x = 3$
$3x + 4$	7	10	13
$8 - x$	7	6	5
$2x + 4 + x$	7	10	13

$3x + 4$ and $2x + 4 + x$ are equal for all values, so they are equivalent.

Exercise 8C

UNDERSTANDING AND FLUENCY	1, 2, 4	2–5	3–5

Example 7

1 **a** Copy the following table into your workbook and complete.

	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$2x + 2$				
$(x + 1) \times 2$				

b Fill in the gap: $2x + 2$ and $(x + 1) \times 2$ are __________ expressions.

2 **a** Copy the following table into your workbook and complete.

	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$5x + 3$				
$6x + 3$				

b Are $5x + 3$ and $6x + 3$ equivalent expressions?

3 Show that $6x + 5$ and $4x + 5 + 2x$ are equivalent by completing the table.

	$6x + 5$	$4x + 5 + 2x$
$x = 1$		
$x = 2$		
$x = 3$		
$x = 4$		

4 For each of the following, choose a pair of equivalent expressions.

a $4x$, $2x + 4$, $x + 4 + x$

b $5a$, $4a + a$, $3 + a$

c $2k + 2$, $3 + 2k$, $2(k + 1)$

d $b + b$, $3b$, $4b - 2b$

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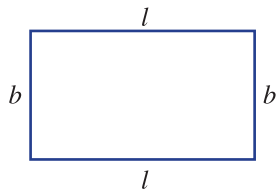
5 Match up the equivalent expressions below.

a	$3x + 2x$	**A**	$6 - 3x$
b	$4 - 3x + 2$	**B**	$2x + 4x + x$
c	$2x + 5 + x$	**C**	$5x$
d	$x + x - 5 + x$	**D**	$4 - x$
e	$7x$	**E**	$3x + 5$
f	$4 - 3x + 2x$	**F**	$3x - 5$

PROBLEM-SOLVING AND REASONING	6, 9	6, 7, 9, 10	7, 8, 10–12

6 Write two different expressions that are equivalent to $4x + 2$.

7 The rectangle shown below has a perimeter given by $b + l + b + l$. Write an equivalent expression for the perimeter.



8 There are many expressions that are equivalent to $3a + 5b + 2a - b + 4a$. Write an equivalent expression with as few terms as possible.

9 The expressions $a + b$ and $b + a$ are equivalent and only contain two terms. How many expressions are equivalent to $a + b + c$ and contain only three terms? Hint: Rearrange the pronumerals.

10 Prove that no two of these four expressions are equivalent: $4 + x$, $4x$, $x - 4$, $x \div 4$.

11 Generalise each of the following patterns in numbers to give two equivalent expressions. The first one has been done for you.

- **a** Observation: $3 + 5 = 5 + 3$ and $2 + 7 = 7 + 2$ and $4 + 11 = 11 + 4$.
  Generalised: The two expressions $x + y$ and $y + x$ are equivalent.
- **b** Observation: $2 \times 5 = 5 \times 2$ and $11 \times 5 = 5 \times 11$ and $3 \times 12 = 12 \times 3$.
- **c** Observation: $4 \times (10 + 3) = 4 \times 10 + 4 \times 3$ and $8 \times (100 + 5) = 8 \times 100 + 8 \times 5$.
- **d** Observation: $100 - (4 + 6) = 100 - 4 - 6$ and $70 - (10 + 5) = 70 - 10 - 5$.
- **e** Observation: $20 - (4 - 2) = 20 - 4 + 2$ and $15 - (10 - 3) = 15 - 10 + 3$.
- **f** Observation: $100 \div 5 \div 10 = 100 \div (5 \times 10)$ and $30 \div 2 \div 3 = 30 \div (2 \times 3)$.

12
- **a** Show that the expression $4 \times (a + 2)$ is equivalent to $8 + 4a$ using a table of values for a between 1 and 4.
- **b** Write an expression using brackets that is equivalent to $10 + 5a$.
- **c** Write an expression without brackets that is equivalent to $6 \times (4 + a)$.

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ENRICHMENT	—	—	13, 14

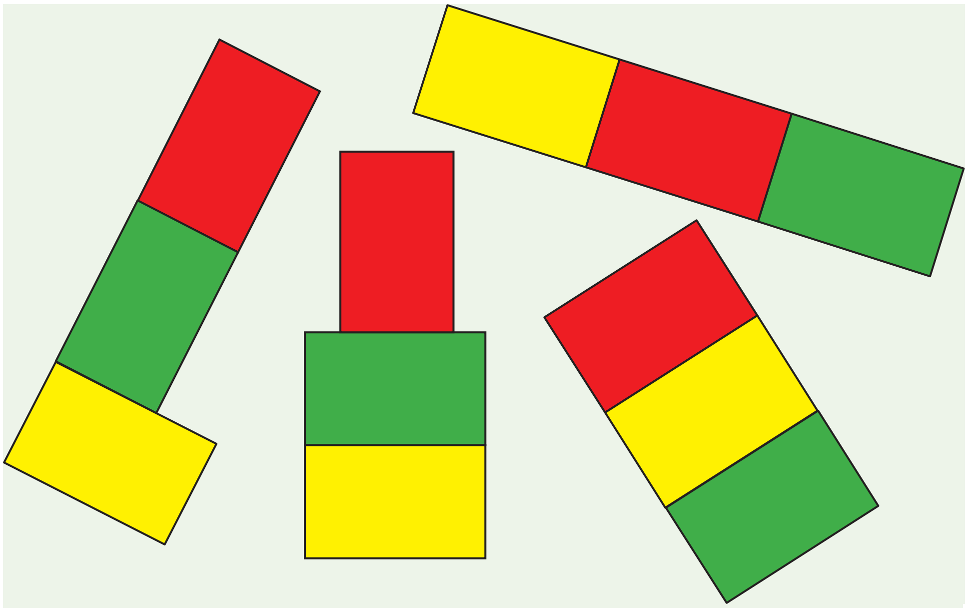
Thinking about equivalence

13 $3a + 5b$ is an expression containing two terms. List two expressions containing three terms that are equivalent to $3a + 5b$.

14 Three expressions are given: expression A, expression B and expression C.

a If expressions A and B are equivalent, and expressions B and C are equivalent, does this mean that expressions A and C are equivalent? Try to prove your answer.

b If expressions A and B are not equivalent, and expressions B and C are not equivalent, does this mean that expressions A and C are not equivalent? Try to prove your answer.



Each shape above is made from three identically sized tiles of length l and breadth b. Which of the shapes have the same perimeter?

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8D Like terms

Interactive
Widgets
HOTsheets
Walkthrough

Whenever we have terms with exactly the same pronumerals, they are called '*like terms*' and can be collected and combined. For example, $3x + 5x$ can be simplified to $8x$. If the two terms do not have exactly the same pronumerals, they must be kept separate; for example, $3x + 5y$ cannot be simplified – it must be left as it is.

Recall from arithmetic that numbers can be multiplied in any order (e.g. $5 \times 3 = 3 \times 5$). This means pronumerals can appear in a different order within a term and give equivalent expressions (e.g. *ab* and *ba* are equivalent).

Let's start: Simplifying expressions

- Try to find a simpler expression that is equivalent to
$$1a + 2b + 3a + 4b + 5a + 6b + \ldots + 19a + 20b$$
- What is the longest possible expression that is equivalent to $10a + 20b + 30c$? Assume that all coefficients must be whole numbers greater than zero.
- Compare your expressions to see who has the longest one.

Key ideas

- **Like terms** are terms containing exactly the same pronumerals, although not necessarily in the same order.

Like	Not like
$3x$ and $5x$	$3x$ and $5y$
$-12a$ and $7a$	$11d$ and $4c$
$5ab$ and $6ba$	$-8ab$ and $5a$
$4x^2$ and $3x^2$	x^2 and x

- Like terms can be combined within an expression to create a simpler expression that is equivalent. For example, $5ab + 6ab$ can be simplified to $11ab$.
- If two terms are not like terms (such as $4x$ and $5y$), they can still be added to get an expression like $4x + 5y$, but this expression cannot be simplified further.

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Example 8 Identifying like terms

Which of the following pairs are like terms?

a $3x$ and $2x$ **b** $3a$ and $3b$ **c** $2ab$ and $5ba$
d $4k$ and k **e** $2a$ and $4ab$ **f** $7ab$ and $9aba$

SOLUTION	EXPLANATION
a $3x$ and $2x$ are like terms.	The pronumerals are the same.
b $3a$ and $3b$ are not like terms.	The pronumerals are different.
c $2ab$ and $5ba$ are like terms.	The pronumerals are the same, even though they are written in a different order (one a and one b).
d $4k$ and k are like terms.	The pronumerals are the same.
e $2a$ and $4ab$ are not like terms.	The pronumerals are not exactly the same (the first term contains only a and the second term has a and b).
f $7ab$ and $9aba$ are not like terms.	The pronumerals are not exactly the same (the first term contains one a and one b, but the second term contains two copies of a and one b).

Example 9 Simplifying using like terms

Simplify the following by collecting like terms.

a $7b + 2 + 3b$ **b** $12d - 4d + d$ **c** $5 + 12a + 4b - 2 - 3a$
d $13a + 8b + 2a - 5b - 4a$ **e** $12uv + 7v - 3vu + 3v$

SOLUTION	EXPLANATION
a $7b + 2 + 3b = 10b + 2$	$7b$ and $3b$ are like terms, so they are combined. They cannot be combined with 2 because it is not 'like' $7b$ or $3b$.
b $12d - 4d + d = 9d$	All the terms here are like terms. Remember that d means $1d$ when combining them.
c $5 + 12a + 4b - 2 - 3a$ $= 12a - 3a + 4b + 5 - 2$ $= 9a + 4b + 3$	$12a$ and $3a$ are like terms. We subtract $3a$ because it has a minus sign in front of it. We can also combine the 5 and the 2 because they are like terms.
d $13a + 8b + 2a - 5b - 4a$ $= 13a + 2a - 4a + 8b - 5b$ $= 11a + 3b$	Combine like terms, remembering to subtract any term that has a minus sign in front of it.
e $12uv + 7v - 3vu + 3v$ $= 12uv - 3vu + 7v + 3v$ $= 9uv + 10v$	Combine like terms. Remember that $12uv$ and $3vu$ are like terms (i.e. they have the same pronumerals), so $12uv - 3uv = 9uv$.

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Exercise 8D

UNDERSTANDING AND FLUENCY	1, 2, 3–5(½)	2, 3–5(½)	3–6(½)

1 Write down the terms from the list below that are like $4x$

a $3x$ **b** $3xy$ **c** x **d** $4x^2$ **e** $4xy$
f $-2x$ **g** yx **h** $4x$ **i** $4ab$ **j** x^2

2 Copy the following sentences into your workbook and fill in the gaps to make the sentences true. Use these words:

terms	equivalent	like

a $3x$ and $5x$ are ____________ terms.
b $4x$ and $3y$ are not ____________ ____________.
c $4xy$ and $4yx$ are like ____________.
d $x + x + 7$ and $2x + 7$ are ____________ expressions.

Example 8

3 Classify the following pairs as like terms (L) or not like terms (N).

a $7a$ and $4b$ **b** $3a$ and $10a$
c $18x$ and $32x$ **d** $4a$ and $4b$
e 7 and $10b$ **f** x and $4x$
g $5x$ and 5 **h** $12ab$ and $4ab$
i $7cd$ and $12cd$ **j** $3abc$ and $12abc$
k $3ab$ and $2ba$ **l** $4cd$ and $3dce$

4 Decide which of the following can be simplified by collecting like terms. Write 'Y' if it can be simplified, or 'N' if it cannot be simplified.

a $7x + 3y + 2x$ **b** $10m - 9m$
c $5m + n$ **d** $9pq - 10p^2q$
e $-8w - 10w + w$ **f** $7 + 5 + m - m$
g $ab + ba$ **h** $5x^2 - 5x$

5 Simplify the following by collecting like terms.

a $a + a$ **b** $3x + 2x$
c $4b + 3b$ **d** $12d - 4d$
e $15u - 3u$ **f** $14ab - 2ab$
g $8ab + 3ab$ **h** $4xy - 3xy$

Example 9

6 Simplify the following by collecting like terms.

a $2a + a + 4b + b$ **b** $5a + 2a + b + 8b$ **c** $3x - 2x + 2y + 4y$
d $4a + 2 + 3a$ **e** $7 + 2b + 5b$ **f** $3k - 2 + 3k$
g $7f + 4 - 2f + 8$ **h** $4a - 4 + 5b + b$ **i** $3x + 7x + 3y - 4x + y$
j $10a + 3 + 4b - 2a$ **k** $4 + 10h - 3h$ **l** $10x + 4x + 31y - y$
m $10 + 7y - 3x + 5x + 2y$ **n** $11a + 4 - 3a + 9$ **o** $3b + 4b + c + 5b - c$
p $7ab + 4 + 2ab$ **q** $9xy + 2x - 3xy + 3x$ **r** $2cd + 5dc - 3d + 2c$
s $5uv + 12v + 4uv - 5v$ **t** $7pq + 2p + 4qp - q$ **u** $7ab + 32 - ab + 4$

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PROBLEM-SOLVING AND REASONING	7, 8, 10(½)	8, 9, 10(½), 11	8, 9, 11, 12

7 Ravi and Marissa each work for n hours per week. Ravi earns \$27 per hour and Marissa earns \$31 per hour.

a Write an expression for the amount Ravi earns in one week.

b Write an expression for the amount Marissa earns in one week.

c Write a simplified expression for the total amount Ravi and Marissa earn in one week.

8 The length of the line segment shown could be expressed as $a + a + 3 + a + 1$.

a Write the length in the simplest form.

b What is the length of the segment if a is equal to 5?

9 Let x represent the number of marbles in a standard-sized bag. Xavier bought 4 bags and Cameron bought 7 bags. Write simplified expressions for:

a the number of marbles Xavier has

b the number of marbles Cameron has

c the total number of marbles that Xavier and Cameron have

d the number of *extra* marbles that Cameron has compared to Xavier

10 Simplify the following by collecting like terms.

a $3xy + 4xy + 5xy$

b $4ab + 5 + 2ab$

c $5ab + 3ba + 2ab$

d $10xy - 4yx + 3$

e $10 - 3xy + 8xy + 4$

f $3cde + 5ecd + 2ced$

g $4 + x + 4xy + 2xy + 5x$

h $12ab + 7 - 3ab + 2$

i $3xy - 2y + 4yx$

11 a Test, using a table of values, that $3x + 2x$ is equivalent to $5x$.

b Prove that $3x + 2y$ is not equivalent to $5xy$.

12 a Test that $5x + 4 - 2x$ is equivalent to $3x + 4$.

b Prove that $5x + 4 - 2x$ is not equivalent to $7x + 4$.

c Prove that $5x + 4 - 2x$ is not equivalent to $7x - 4$.

ENRICHMENT	—	—	13

How many rearrangements?

13 The expression $a + 3b + 2a$ is equivalent to $3a + 3b$.

a List two other expressions with three terms that are equivalent to $3a + 3b$.

b How many expressions, consisting of exactly three terms added together, are equivalent to $3a + 3b$? All coefficients must be whole numbers greater than 0.

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8E Multiplying, dividing and mixed operations

Interactive Widgets HOTsheets Walkthrough

To multiply a number by a pronumeral, we have already seen that we can write them next to each other. For example, $7a$ means $7 \times a$, and $5abc$ means $5 \times a \times b \times c$. The order in which numbers or pronumerals are multiplied is unimportant, so $5 \times a \times b \times c = a \times 5 \times c \times b = c \times a \times 5 \times b$. When writing a product without × signs, the numbers are written first.

We write $\frac{7xy}{3xz}$ as shorthand for $(7xy) \div (3xz)$.

We can simplify fractions like $\frac{10}{15}$ by dividing by common factors, such as $\frac{10}{15} = \frac{\cancel{5} \times 2}{\cancel{5} \times 2} = \frac{2}{3}$

Similarly, common variables can be cancelled in a division like $\frac{7xy}{3xz}$, giving $\frac{7\cancel{x}y}{3\cancel{x}z} = \frac{7y}{3z}$.

Let's start: Rearranging terms

$5abc$ is equivalent to $5cab$ because the order of multiplication does not matter. In what other ways could $5abc$ be written? $5 \times a \times b \times c = ?$

Key ideas

- $a \times b$ is written as ab and $a \times a$ is written as a^2

  For example: $a \times 2 = 2a$
  $a \times 1 = 1a = a$
  $a \times 0 = 0a = 0$

- $a \div b$ is written as $\frac{a}{b}$.

  For example: $a \div 2 = \frac{a}{2}$
  $a \div 1 = \frac{a}{1} = a$
  $1 \div a = \frac{1}{a}$

- Because of the commutative property of multiplication (e.g. $2 \times 7 = 7 \times 2$), the order in which values are multiplied is not important. So $3 \times a$ and $a \times 3$ are equivalent.

- Because of the associative property of multiplication (e.g. $3 \times (5 \times 2)$ and $3 \times (5 \times 2)$ are equal), brackets are not required when only multiplication is used. So $3 \times (a \times b)$ and $(3 \times a) \times b$ are both written as $3ab$.

- In a product, numbers should be written first in a term and pronumerals are usually written in alphabetical order. For example, $b \times 2 \times a$ is written as $2ab$.

- When dividing, any common factor in the numerator and denominator can be cancelled.

  For example: $\frac{{}^{2}\cancel{4}a\,{}^{1}\cancel{b}}{{}^{1}\cancel{2}\,{}^{1}\cancel{b}c} = \frac{2a}{c}$

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Example 10 Simplifying expressions with multiplication

a Write $4 \times a \times b \times c$ without multiplication signs.
b Simplify $4a \times 2b \times 3c$, giving your final answer without multiplication signs.
c Simplify $3w \times 4w$.

SOLUTION

a $4 \times a \times b \times c = 4abc$

When pronumerals are written next to each other they are being multiplied.

b
$$\begin{aligned} 4a \times 2b \times 3c &= 4 \times a \times 2 \times b \times 3 \times c \\ &= 4 \times 2 \times 3 \times a \times b \times c \\ &= 24abc \end{aligned}$$

First, insert the missing multiplication signs.
Rearrange to bring the numbers to the front.
$4 \times 2 \times 3 = 24$ and $a \times b \times c = abc$, giving the final answer.

c
$$\begin{aligned} 3w \times 4w &= 3 \times w \times 4 \times w \\ &= 3 \times 4 \times w \times w \\ &= 12w^2 \end{aligned}$$

First, insert the missing multiplication signs.
Rearrange to bring numbers to the front.
$3 \times 4 = 12$ and $w \times w$ is written as w^2.

Example 11 Simplifying expressions with division

a Write $(3x + 1) \div 5$ without a division sign.
b Simplify the expression $\frac{8ab}{12b}$.

SOLUTION

a $(3x + 1) \div 5 = \frac{3x + 1}{5}$

The brackets are no longer required as it becomes clear that all of $3x + 1$ is being divided by 5.

b
$$\begin{aligned} \frac{8ab}{12b} &= \frac{8 \times a \times b}{12 \times b} \\ &= \frac{2 \times \cancel{4} \times a \times \cancel{b}}{3 \times \cancel{4} \times \cancel{b}} \\ &= \frac{2a}{3} \end{aligned}$$

Insert multiplication signs to help spot common factors.
8 and 12 have a common factor of 4.

Cancel out the common factors of 4 and b.

Exercise 8E

UNDERSTANDING AND FLUENCY	1–4, 5–8(½)	4, 5–8(½)	5–9(½)

1 Chen claims that $7 \times d$ is equivalent to $d \times 7$.
- **a** If $d = 3$, find the values of $7 \times d$ and $d \times 7$.
- **b** If $d = 5$, find the values of $7 \times d$ and $d \times 7$.
- **c** If $d = 8$, find the values of $7 \times d$ and $d \times 7$.
- **d** Is Chen correct in his claim?

2 Classify each of the following statements as true or false.
- **a** $4 \times n$ can be written as $4n$.
- **b** $n \times 3$ can be written as $3n$.
- **c** $4 \times b$ can be written as $b + 4$.
- **d** $a \times b$ can be written as ab.

3 a Simplify the fraction $\frac{12}{18}$. $\left(\text{Note: This is the same as } \frac{2 \times 6}{3 \times 6}.\right)$

b Simplify the fraction $\frac{2000}{3000}$. $\left(\text{Note: This is the same as } \frac{2 \times 1000}{3 \times 1000}.\right)$

c Simplify $\frac{2a}{3a}$. $\left(\text{Note: This is the same as } \frac{2 \times a}{3 \times a}.\right)$

4 Match up these expressions with the correct way to write them.

a $2 \times u$ — A $3u$

b $7 \times u$ — B $\frac{5}{u}$

c $5 \div u$ — C $2u$

d $u \times 3$ — D $\frac{u}{5}$

e $u \div 5$ — E $7u$

Example 10a

5 Write each of these expressions without any multiplication signs.

a $2 \times x$ b $5 \times p$ c $8 \times a \times b$
d $3 \times 2 \times a$ e $5 \times 2 \times a \times b$ f $2 \times b \times 5$
g $x \times 7 \times z \times 4$ h $2 \times a \times 3 \times b \times 6 \times c$ i $7 \times 3 \times a \times 2 \times b$
j $a \times 2 \times b \times 7 \times 3 \times c$ k $9 \times a \times 3 \times b \times d \times 2$ l $7 \times a \times 12 \times b \times c$

Example 10b

6 Simplify these expressions.

a $3a \times 12$ b $7d \times 9$ c $2 \times 4e$
d $3 \times 5a$ e $4a \times 3b$ f $7e \times 9g$
g $8a \times bc$ h $4d \times 7af$ i $a \times 3b \times 4c$
j $2a \times 4b \times c$ k $4d \times 3e \times 5fg$ l $2cb \times 3a \times 4d$

Example 10c

7 Simplify these expressions.

a $w \times w$ b $a \times a$ c $3d \times d$
d $2k \times k$ e $p \times 7p$ f $q \times 3q$
g $6x \times 2x$ h $3z \times 5z$ i $9r \times 4r$

Example 11a

8 Simplify these expressions.

a $x \div 5$ b $z \div 2$ c $a \div 12$
d $b \div 5$ e $2 \div x$ f $5 \div d$
g $x \div y$ h $a \div b$ i $(4x + 1) \div 5$
j $(2x + y) \div 5$ k $(2 + x) \div (1 + y)$ l $(x - 5) \div (3 + b)$
m $2x + y \div 5$ n $2 + x \div 1 + y$ o $x - 5 \div 3 + b$
p $3 \times 2b - 2b$ q $3 \times (2b - 2b)$ r $6b + 15b \div 3$
s $(6b + 15b) \div 3$ t $(c - 2c) \times 4$ u $c - 2c \times 4$

Example 11b

9 Simplify the following expressions by dividing by any common factors. Remember that $\frac{a}{1} = a$.

a $\frac{2x}{5x}$ b $\frac{5a}{9a}$ c $\frac{9ab}{4b}$ d $\frac{2ab}{5a}$
e $\frac{2x}{4}$ f $\frac{9x}{12}$ g $\frac{10a}{15a}$ h $\frac{30y}{40y}$
i $\frac{4a}{2}$ j $\frac{21x}{7x}$ k $\frac{4xy}{2x}$ l $\frac{9x}{3xy}$

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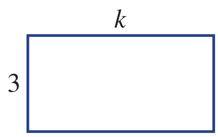
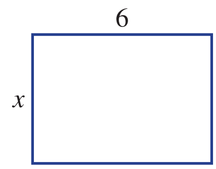
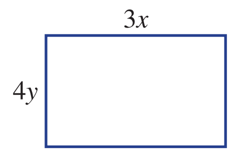
PROBLEM-SOLVING AND REASONING	10, 11, 14	11, 12, 14, 15	12–15

10 Write a simplified expression for the area of the following rectangles. Recall that for rectangles, $Area = length \times breadth$.

a


b


c


11 The weight of a single muesli bar is x grams.

a What is the weight of 4 bars? Write an expression.

b If Jamila buys n bars, what is the total weight of her purchase?

c Jamila's cousin Roland buys twice as many bars as Jamila. What is the total weight of Roland's purchase?

12 We can factorise a term like $15ab$ by writing it as $3 \times 5 \times a \times b$. Numbers are written in prime factor form and pronumerals are given with multiplication signs. Factorise the following.

a $6ab$ **b** $21xy$ **c** $4efg$ **d** $33q2r$

13 Five friends go to a restaurant. They split the bill evenly, so each spends the same amount.

a If the total cost is \$100, how much do they each spend?

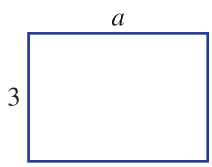
b If the total cost is \$$C$, how much do they each spend? Write an expression.

14 The expression $3 \times 2p$ is the same as the expression $\underset{(1)}{2p} + \underset{(2)}{2p} + \underset{(3)}{2p}$.

a What is a simpler expression for $2p + 2p + 2p$? (Hint: Combine like terms.)

b $3 \times 2p$ is shorthand for $3 \times 2 \times p$. How does this relate to your answer in part **a**?

15 The area of the rectangle shown is $3a$. The length and breadth of this rectangle are now doubled.



a Draw the new rectangle, showing its dimensions.

b Write a simplified expression for the area of the new rectangle.

c Divide the area of the new rectangle by the area of the old rectangle. What do you notice?

d What happens to the area of the original rectangle if you triple both the length and the breadth?

ENRICHMENT	—	—	16

Managing powers

16 The expression $a \times a$ can be written as a^2 and the expression $a \times a \times a$ can be written as a^3.

a What is $3a^2b^2$ when written in full with multiplication signs?

b Write $7 \times x \times x \times y \times y \times y$ without any multiplication signs.

c Simplify $2a \times 3b \times 4c \times 5a \times b \times 10c \times a$.

d Simplify $4a^2 \times 3ab^2 \times 2c^2$.

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8F Expanding brackets

Interactive
Widgets
HOTsheets
Walkthrough

We have already seen that there are different ways of writing two equivalent expressions. For example, $4a + 2a$ is equivalent to $2 \times 3a$, even though they look different.

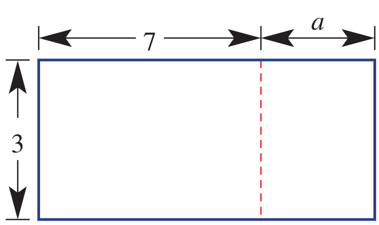
Note that $3(7 + a) = 3 \times (7 + a)$, which is equivalent to 3 lots of $(7 + a)$.

So, $3(7 + a) = 7 + a + 7 + a + 7 + a$
$= 21 + 3a$

It is sometimes useful to have an expression that is written with brackets, like $3 \times (7 + a)$, and sometimes it is useful to have an expression that is written without brackets, like $21 + 3a$.

Let's start: Total area

What is the total area of the rectangle shown? Try to write two expressions, only one of which includes brackets.



Key ideas

- **Expanding** (or **eliminating**) brackets involves writing an equivalent expression without brackets. This can be done by writing the bracketed portion a number of times or by multiplying each term.

  $2(a + b) = a + b + a + b$ or $2(a + b) = 2 \times a + 2 \times b$
  $= 2a + 2b$ $= 2a + 2b$

- To eliminate brackets, you can use the **distributive law**, which states that:

  $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$

- The distributive law is used in arithmetic.

  For example: $5 \times 27 = 5(20 + 7)$
  $= 5 \times 20 + 5 \times 7$
  $= 100 + 35$
  $= 135$

- The process of removing brackets using the distributive law is called **expansion**.
- When expanding, every term inside the brackets must be multiplied by the term outside the brackets.

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Example 12 Expanding brackets by simplifying repeated terms

Repeat the expression that is inside the brackets and then collect like terms. The number outside the brackets is the number of repeats.

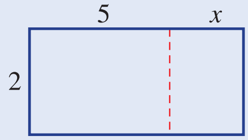
a $2(a + k)$ **b** $3(2m + 5)$

SOLUTION	EXPLANATION
a $2(a + k) = a + k + a + k$ $= 2a + 2k$	Two repeats of the expression $a + k$. Simplify by collecting the like terms.
b $3(2m + 5) = 2m + 5 + 2m + 5 + 2m + 5$ $= 6m + 15$	Three repeats of the expression $2m + 5$. Simplify by collecting the like terms.

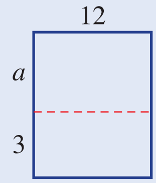
Example 13 Expanding brackets using rectangle areas

Write two equivalent expressions for the area of each rectangle shown, only one of which includes brackets.

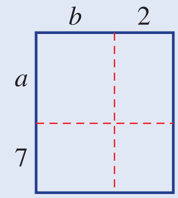
a



b



c



SOLUTION	EXPLANATION
a Using brackets: $2(5 + x)$ Without brackets: $10 + 2x$	The whole rectangle has height 2 and breadth $5 + x$. The smaller rectangles have area $2 \times 5 = 10$ and $2 \times x = 2x$, so they are added.
b Using brackets: $12(a + 3)$	The dimensions of the whole rectangle are 12 and $a + 3$. Note that, by convention, we do not write $(a + 3)12$.
Without brackets: $12a + 36$	The smaller rectangles have area $12 \times a = 12a$ and $12 \times 3 = 36$.
c Using brackets: $(a + 7)(b + 2)$	The whole rectangle has height $a + 7$ and breadth $b + 2$. Note that brackets are used to ensure we are multiplying the entire height by the entire breadth.
Without brackets: $ab + 2a + 7b + 14$	The diagram can be split into four rectangles, with areas ab, $2a$, $7b$ and 14.

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Example 14 Expanding using the distributive law

Expand the following expressions.

a $5(x + 3)$ **b** $8(a - 4)$ **c** $3(a + 2b)$ **d** $5a(3p - 7q)$

SOLUTION

a $5(x + 3) = 5 \times x + 5 \times 3$
$= 5x + 15$

EXPLANATION

Use the distributive law.
$5(x + 3) = 5 \times x + 5 \times 3$
Simplify the result.

SOLUTION

b $8(a - 4) = 8 \times a - 8 \times 4$
$= 8a - 32$

EXPLANATION

Use the distributive law with subtraction.
$8(a - 4) = 8 \times a - 8 \times 4$
Simplify the result.

SOLUTION

c $3(a + 2b) = 3 \times a + 3 \times 2b$
$= 3a + 6b$

EXPLANATION

Use the distributive law.
$3(a + 2b) = 3 \times a + 3 \times 2b$
Simplify the result, remembering that $3 \times 2b = 6b$.

SOLUTION

d $5a(3p - 7q) = 5a \times 3p - 5a \times 7q$
$= 15ap - 35aq$

EXPLANATION

Use the distributive law.
$5a(3p - 7q) = 5a \times 3p - 5a \times 7q$
Simplify the result, remembering that $5a \times 3p = 15ap$ and $5a \times 7q = 35aq$.

Exercise 8F

UNDERSTANDING AND FLUENCY	1–5, 6–7(½)	4, 5, 6–8(½)	5, 6–8(½)

Example 12

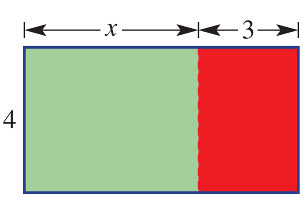
1 Fill in the boxes.

The expression $3(a + 2)$ can be expanded in two different ways.

a $3(a + 2) = (a + 2) + (a + 2) + (a + 2)$
$= 3a + \square$

b $3(a + 2) = 3 \times \square + 3 \times 2$
$= 3a + 6$

2 The area of the rectangle shown can be written as $4(x + 3)$.



a What is the area of the green rectangle?
b What is the area of the red rectangle?
c Write the total area as an expression, without using brackets.

3 Copy and complete the following computations, using the distributive law.

a $3 \times 21 = 3 \times (20 + 1)$
$= 3 \times 20 + 3 \times 1$
$= \_\_\_ + \_\_\_$
$= \_\_\_$

b $7 \times 34 = 7 \times (30 + 4)$
$= 7 \times \_\_\_ + 7 \times \_\_\_$
$= \_\_\_ + \_\_\_$
$= \_\_\_$

c $5 \times 19 = 5 \times (20 - 1)$
$= 5 \times \_\_\_ - 5 \times \_\_\_$
$= \_\_\_ - \_\_\_$
$= \_\_\_$

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4 **a** Copy and complete the following table. Remember to follow the rules for correct order of operations.

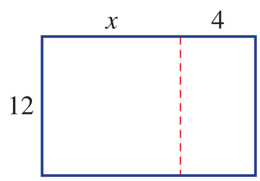
	$4(x+3)$	$4x+12$
$x=1$	$=4(1+3)$ $=4(4)$ $=16$	$=4(1)+12$ $=4+12$ $=16$
$x=2$		
$x=3$		
$x=4$		

b Fill in the gap: The expressions $4(x+3)$ and $4x+12$ are _____________.

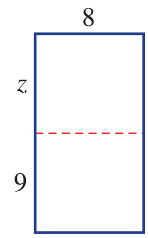
Example 13

5 For the following rectangles, write two equivalent expressions for the area.

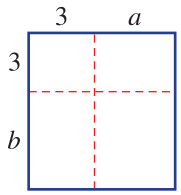
a



b



c



Example 14a,b

6 Use the distributive law to expand the following.

a $6(y+8)$ **b** $7(l+4)$ **c** $8(s+7)$ **d** $4(2+a)$
e $7(x+5)$ **f** $3(6+a)$ **g** $9(9-x)$ **h** $5(j-4)$
i $8(y-8)$ **j** $8(e-7)$ **k** $6(e-3)$ **l** $10(8-y)$

Example 14c

7 Use the distributive law to expand the following.

a $10(6g-7)$ **b** $5(3e+8)$ **c** $5(7w+10)$ **d** $5(2u+5)$
e $7(8x-2)$ **f** $3(9v-4)$ **g** $7(q-7)$ **h** $4(5c-v)$
i $2(2u+6)$ **j** $6(8l+8)$ **k** $5(k-10)$ **l** $9(o+7)$

Example 14d

8 Use the distributive law to expand the following.

a $6i(t-v)$ **b** $2d(v+m)$ **c** $5c(2w-t)$ **d** $6e(s+p)$
e $d(x+9s)$ **f** $5a(2x+3v)$ **g** $5j(r+7p)$ **h** $i(n+4w)$
i $8d(s-3t)$ **j** $f(2u+v)$ **k** $7k(2v+5y)$ **l** $4e(m+10y)$

PROBLEM-SOLVING AND REASONING	9, 10, 13	10, 11, 13, 14	10–12, 14, 15

9 Write an expression for each of the following and then expand it.

a A number, x, has 3 added to it and the result is multiplied by 5.
b A number, b, has 6 added to it and the result is doubled.
c A number, z, has 4 subtracted from it and the result is multiplied by 3.
d A number, y, is subtracted from 10 and the result is multiplied by 7.

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10 In a school classroom there is one teacher as well as an unknown number of boys and girls.

- **a** If the number of boys is b and the number of girls is g, write an expression for the total number of people in the classroom, including the teacher.
- **b** The teacher and all the students are each wearing two socks. Write two different expressions for the total number of socks being worn, one with brackets and one without.

11 When expanded, $4(3x + 6y)$ gives $12x + 24y$. Find two other expressions that expand to $12x + 24y$.

12 The distance around a rectangle is given by the expression $2(l + b)$, where l is the length and b is the breadth. What is an equivalent expression for this distance?

13 Use a diagram of a rectangle like that in Question **2** to prove that $5(x + 3) = 5x + 15$.

14 Use a diagram of a rectangle to prove that $(a + 2)(b + 3) = ab + 2b + 3a + 6$.

15 When expanded, $5(2x + 4y)$ gives $10x + 20y$.

- **a** How many different ways can the missing numbers be filled with whole numbers for the equivalence $\square(\square x + \square y) = 10x + 20y$?
- **b** How many different expressions expand to give $10x + 20y$ if fractions or decimals are included?

ENRICHMENT	—	—	16

Expanding sentences

16 Using words, people do a form of expansion. Consider these two statements.

Statement A: 'John likes tennis and football.'

Statement B: 'John likes tennis and John likes football.'

Statement B is an 'expanded form' of statement A, which is equivalent in its meaning but shows more clearly that two facts are being communicated. Write an 'expanded form' of the following sentences.

- **a** Rosemary likes maths and English.
- **b** Priscilla eats fruit and vegetables.
- **c** Bailey and Lucia like the opera.
- **d** Frank and Igor play video games.
- **e** Pyodir and Astrid like chocolate and tennis. (Note: There are four facts being communicated here.)

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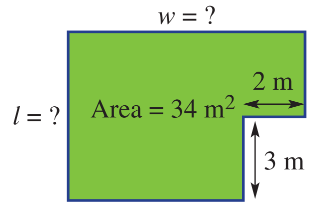
8G Applying algebra EXTENSION

An algebraic expression can be used to describe problems relating to many different areas, including costs, speeds and sporting results. Much of modern science relies on the application of algebraic rules and formulas. It is important to be able to convert word descriptions of problems to mathematical expressions in order to solve these problems mathematically.

Let's start: Garden bed area

The garden shown has an area of 34 m^2, but the width and length are unknown.

- What are some possible values that w and l could equal?
- Try to find the dimensions of the garden that make the fencing around the outside as small as possible.



Key ideas

- Many different situations can be modelled with algebraic expressions.
  For example, an algebraic expression for perimeter is $2l + 2b$
- To apply an expression, the pronumerals should be defined clearly. Then known values should be substituted for the pronumerals.
  For example: If $l = 5$ and $b = 10$, then

$$\begin{aligned} \text{perimeter} &= 2 \times 5 + 2 \times 10 \\ &= 30 \end{aligned}$$

Example 15 Applying an expression

The perimeter of a rectangle is given by the expression $2l + 2b$, where l is the length and b is the breadth.

a Find the perimeter of a rectangle if $l = 5$ and $b = 7$.

b Find the perimeter of a rectangle with length 8 cm and breadth 3 cm.

SOLUTION	EXPLANATION
a Perimeter is given by $2l + 2b = 2(5) + 2(7)$ $= 10 + 14$ $= 24$	To apply the rule, we substitute $l = 5$ and $b = 7$ into the expression. Evaluate using the normal rules of arithmetic (i.e. multiplication before addition).
b Perimeter is given by $2l + 2b = 2(8) + 2(3)$ $= 16 + 6$ $= 22$ cm	Substitute $l = 8$ and $b = 3$ into the expression. Evaluate using the normal rules of arithmetic, remembering to include appropriate units (cm) in the answer.

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Example 16 Constructing expressions from problem descriptions

Write expressions for each of the following.

a The total cost, in dollars, of 10 bottles, if each bottle costs $$x$.

b The total cost, in dollars, of hiring a plumber for n hours. The plumber charges a $30 call-out fee plus $60 per hour.

c A plumber charges a $60 call-out fee plus $50 per hour. Use an expression to find how much an 8-hour job would cost.

SOLUTION	EXPLANATION
a $10x$	Each of the 10 bottles costs $$x$, so the total cost is $10 \times x = 10x$.
b $30 + 60n$	For each hour, the plumber charges $60, so must pay $60 \times n = 60n$. The $30 call-out fee is added to the total bill.
c Expression for cost: $60 + 50n$ If $n = 8$, then cost is $60 + 50 \times 8 = \$460$	Substitute $n = 8$ to find the cost for an 8-hour job. Cost will be $460.

Exercise 8G EXTENSION

UNDERSTANDING AND FLUENCY	1–7	3–8	4–8

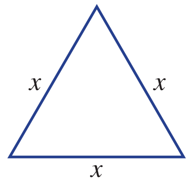
Example 15a **1** The area of a rectangle is given by the expression $l \times b$, where l is its length and b is its breadth.

- **a** Find the area if $b = 5$ and $l = 7$.
- **b** Find the area if $b = 2$ and $l = 10$.

Example 15b **2** The perimeter of a square with breadth b is given by the expression $4b$.

- **a** Find the perimeter of a square with breadth 6 cm (i.e. $b = 6$).
- **b** Find the perimeter of a square with breadth 10 m (i.e. $b = 10$).

3 Consider the equilateral triangle shown.



- **a** Write an expression that gives the perimeter of this triangle.
- **b** Use your expression to find the perimeter if $x = 12$.

Example 16a **4** If pens cost $2 each, write an expression for the cost of n pens.

5 If pencils cost $$x$ each, write an expression for the cost of:

- **a** 10 pencils
- **b** 3 packets of pencils, if each packet contains 5 pencils
- **c** k pencils

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6 A car travels at 60 km/h, so in n hours it has travelled $60n$ kilometres.

a How far does the car travel in 3 hours (i.e. $n = 3$)?

b How far does the car travel in 30 minutes?

Example 16b

c Write an expression for the total distance travelled in n hours for a motorbike with speed 70 km/h.

Example 16c

7 A carpenter charges a \$40 call-out fee and then \$80 per hour. This means the total cost for x hours of work is $\$(40 + 80x)$.

a How much would it cost for a 2-hour job (i.e. $x = 2$)?

b How much would it cost for a job that takes 8 hours?

c The call-out fee is increased to \$50. What is the new expression for the total cost of x hours?

8 Match up the word problems with the expressions (**A** to **E**) below.

a The area of a rectangle with height 5 and breadth x. — **A** $10 + 2x$

b The perimeter of a rectangle with height 5 and breadth x. — **B** $5x$

c The total cost, in dollars, of hiring a DVD for x days if the price is \$1 per day. — **C** $5 + x$

d The total cost, in dollars, of hiring a builder for 5 hours if the builder charges a \$10 call-out fee and then $\$x$ per hour. — **D** x

e The total cost, in dollars, of buying a \$5 magazine and a book that costs $\$x$. — **E** $10 + 5x$

PROBLEM-SOLVING AND REASONING	9, 10, 14	10–12, 14	11–15

9 A plumber charges a \$50 call-out fee and \$100 per hour.

a Copy and complete the table below.

No. of hours (t)	1	2	3	4	5
Total cost (\$)					

b Find the total cost, in dollars, if the plumber works for t hours. Give an expression.

c Substitute $t = 30$ into your expression to find how much it will cost for the plumber to work 30 hours.

10 To hire a tennis court, you must pay a \$5 booking fee plus \$10 per hour.

a What is the cost of booking a court for 2 hours?

b What is the cost, in dollars, of booking a court for x hours? Write an expression.

c A tennis coach hires a court for 7 hours. Substitute $x = 7$ into your expression to find the total cost.

11 In Australian Rules football a goal is worth 6 points and a 'behind' is worth 1 point. This means the total score for a team is $6g + b$, if g goals and b behinds are scored.

a What is the score for a team that has scored 5 goals and 3 behinds?

b What are the values of g and b for a team that has scored 8 goals and 5 behinds?

c If a team has a score of 20, this could be because $g = 2$ and $b = 8$. What are the other possible values of g and b?

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12 Adrian's mobile phone costs 30 cents to make a connection, plus 60 cents per minute of talking. This means that a t-minute call costs $30 + 60t$ cents.

a What is the cost of a 1-minute call?

b What is the cost of a 10-minute call? Give your answer in dollars.

c Write an expression for the cost of a t-minute call in dollars.

13 During a sale, a shop sells all CDs for $\$c$ each, books cost $\$b$ each and DVDs cost $\$d$ each. Claudia buys 5 books, 2 CDs and 6 DVDs.

a What is the cost, in dollars, of Claudia's order? Give your answer as an expression involving b, c and d.

b Write an expression for the cost of Claudia's order if CDs doubled in price and DVDs halved in price.

c As it happens, the total price Claudia ends up paying is the same in both situations. Given that CDs cost \$12 and books cost \$20 (so $c = 12$ and $b = 20$), how much do DVDs cost?

14 A shop charges $\$c$ for a box of tissues.

a Write an expression for the total cost, in dollars, of buying n boxes of tissues.

b If the original price is tripled, write an expression for the total cost of buying n boxes of tissues.

c If the original price is tripled and twice as many boxes are bought, write an expression for the total cost.

15 To hire a basketball court costs \$10 for a booking fee, plus \$30 per hour.

a Write an expression for the total cost, in dollars, to hire the court for x hours.

b For the cost of \$40, you could hire the court for 1 hour. How long could you hire the court for the cost of \$80?

c Explain why it is *not* the case that hiring the court for twice as long costs twice as much.

d Find the average cost per hour if the court is hired for a 5-hour basketball tournament.

e Describe what would happen to the *average* cost per hour if the court is hired for many hours (e.g. more than 50 hours).

ENRICHMENT	—	—	16

Mobile phone mayhem

16 Rochelle and Emma are on different mobile phone plans, as shown below.

	Connection	Cost per minute
Rochelle	20 cents	60 cents
Emma	80 cents	40 cents

a Write an expression for the cost, in dollars, of making a t-minute call using Rochelle's phone.

b Write an expression for the cost of making a t-minute call using Emma's phone.

c Whose phone plan would be cheaper for a 7-minute call?

d What is the length of call for which it would cost exactly the same for both phones?

e Investigate current mobile phone plans and describe how they compare to those of Rochelle's and Emma's plans.

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8H Substitution involving negative numbers and mixed operations

Interactive
Widgets
HOTsheets
Walkthrough

The process known as substitution involves replacing a pronumeral or letter with a number. As a car accelerates, its speed can be modelled by the rule $10 + 4t$. So, after 8 seconds we can calculate the car's speed by substituting $t = 8$ into $10 + 4t$

So $10 + 4t = 10 + 4 \times 8 = 42$ metres per second.

We can also look at the car's speed before time $t = 0$. So at 2 seconds before $t = 0$ (i.e. $t = -2$), the speed would be $10 + 4t = 10 + 4 \times (-2) = 2$ metres per second.

Let's start: Order matters

Two students substitute the values $a = -2$, $b = 5$ and $c = -7$ into the expression $ac - bc$. Some of the different answers received are 21, −49, −21 and 49.

- Which answer is correct and what errors were made in the computation of the three incorrect answers?

Key ideas

- Substitute into an expression by replacing pronumerals (or letters) with numbers.
- Use brackets around negative numbers to avoid confusion with other symbols.

For example:
If $a = -3$ then

$$\begin{aligned} 3 - 7a &= 3 - 7 \times (-3) \\ &= 3 - (-21) \\ &= 3 + 21 \\ &= 24 \end{aligned}$$

Example 17 Substituting integers

Evaluate the following expressions using $a = 3$ and $b = -5$.

a $2 + 4a$ **b** $7 - 4b$ **c** $b \div 5 - a$

SOLUTION	EXPLANATION
a $2 + 4a = 2 + 4 \times 3$ $= 2 + 12$ $= 14$	Replace a with 3 and evaluate the multiplication first.
b $7 - 4b = 7 - 4 \times (-5)$ $= 7 - (-20)$ $= 7 + 20$ $= 27$	Replace the b with −5 and evaluate the multiplication before the subtraction.
c $b \div 5 - a = -5 \div 5 - 3$ $= -1 - 3$ $= -4$	Replace b with −5 and a with 3, and then evaluate.

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Exercise 8H

UNDERSTANDING AND FLUENCY	1–3, 4–6(½)	3, 4–6(½), 7	4–6(½), 7

1 Which of the following shows the correct substitution of $a = -2$ into the expression $a - 5$?
A $2 - 5$ **B** $-2 + 5$ **C** $-2 - 5$ **D** $2 + 5$

2 Which of the following shows the correct substitution of $x = -3$ into the expression $2 - x$?
A $-2 - (-3)$ **B** $2 - (-3)$ **C** $-2 + 3$ **D** $-3 + 2$

3 Rafe substitutes $c = -10$ into $10 - c$ and gets 0. Is he correct? If not, what is the correct answer?

Example 17a, b

4 Evaluate the following expressions using $a = 6$ and $b = -2$.
a $5 + 2a$ **b** $-7 + 5a$ **c** $b - 6$ **d** $b + 10$
e $4 - b$ **f** $7 - 2b$ **g** $3b - 1$ **h** $-2b + 2$
i $5 - 12 \div a$ **j** $1 - 60 \div a$ **k** $10 \div b - 4$ **l** $3 - 6 \div b$

Example 17c

5 Evaluate the following expressions using $a = -5$ and $b = -3$.
a $a + b$ **b** $a - b$ **c** $b - a$ **d** $2a + b$
e $5b + 2a$ **f** $6b - 7a$ **g** $-7a + b + 4$ **h** $-3b - 2a - 1$

6 Evaluate these expressions for the values given.
a $26 - 4x$ $(x = -3)$ **b** $-2 - 7k$ $(k = -1)$
c $10 \div n + 6$ $(n = -5)$ **d** $-3x + 2y$ $(x = 3, y = -2)$
e $18 \div y - x$ $(x = -2, y = -3)$ **f** $-36 \div a - ab$ $(a = -18, b = -1)$

7 These expressions contain brackets. Evaluate them for the values given. (Remember that ab means $a \times b$.)
a $2 \times (a + b)$ $(a = -1, b = 6)$ **b** $10 \div (a - b) + 1$ $(a = -6, b = -1)$
c $ab \times (b - 1)$ $(a = -4, b = 3)$ **d** $(a - b) \times bc$ $(a = 1, b = -1, c = 3)$

PROBLEM-SOLVING AND REASONING	8, 9, 11	8, 9, 11, 12	9, 10, 12, 13

8 The area of a triangle, in m^2, for a fixed base of 4 metres is given by the rule $2h$, where h metres is the height of the triangle. Find the area of such a triangle with these heights.
a 3 m **b** 8 m

9 A motorcycle's speed, in metres per second, after a particular point on a racing track is given by the expression $20 + 3t$, where t is in seconds.
a Find the motorcycle's speed after 4 seconds.
b Find the motorcycle's speed at $t = -2$ seconds (i.e. 2 seconds before passing the $t = 0$ point).
c Find the motorcycle's speed at $t = -6$ seconds.

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10 The formula for the perimeter, P, of a rectangle is $P = 2l + 2b$, where l and b are the length and the breadth, respectively.

a Use the given formula to find the perimeter of a rectangle with:

i $l = 3$ and $b = 5$

ii $l = 7$ and $b = -8$

b What problems are there with part **a ii** above?

11 Write two different expressions involving x that give an answer of -10 if $x = -5$.

12 Write an expression involving the pronumeral a combined with other integers, so if $a = -4$ the expression would equal these answers.

a -3

b 0

c 10

13 If a and b are any non–zero integer, explain why these expressions will always give the result of zero.

a $a - b + b - a$

b $\frac{a}{a} - 1$

c $\frac{(a - a)}{b}$

d $\frac{ab}{b} - a$

ENRICHMENT	—	—	14

Celsius/Fahrenheit

14 The Fahrenheit temperature scale (°F) is still used today in some countries, but most countries use the Celsius scale (°C). 32°F is the freezing point for water (0°C). 212°F is the boiling point for water (100°C).

The formula for converting °F to °C is $C = \frac{5}{9} \times (F - 32)$.

a Convert these temperatures from °F to °C.

i 41°F

ii 5°F

iii −13°F

b Can you work out the formula that converts from °C to °F?

c Use your rule from part **b** to check your answers to part **a**.

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8I Number patterns EXTENSION

Interactive Widgets HOTsheets Walkthrough

Mathematicians commonly look at lists of numbers in an attempt to discover a pattern. They also aim to find a rule that describes the number pattern to allow them to predict future numbers in the sequence.

Here is a list of professional careers that all involve a high degree of mathematics and, in particular, involve looking at data so that comments can be made about past, current or future trends.

Statistician, economist, accountant, market researcher, financial analyst, cost estimator, actuary, stock broker, data analyst, research scientist, financial advisor, medical scientist, budget analyst, insurance underwriter and mathematics teacher!

Let's start: What's next?

A number sequence consisting of five terms is placed on the board. Four gaps are placed after the last number.

20, 12, 16, 8, 12, ___, ___, ___, ___

- Can you work out and describe the number pattern?
  This number pattern involves a repeated process of subtracting 8 and then adding 4.
- Make up your own number pattern and test it on a class member.

Key ideas

- Number patterns are also known as **sequences**, and each number in a sequence is called a **term**.
  - Each number pattern has a particular starting number and terms are generated by following a particular rule.
- Strategies to determine the pattern involved in a number sequence include:
  - Looking for a common difference
    Are terms increasing or decreasing by a constant amount?
    For example: 2, 6, 10, 14, 18, … Each term is increasing by 4.
  - Looking for a common ratio
    Is each term being multiplied or divided by a constant amount?
    For example: 2, 4, 8, 16, 32, … Each term is being multiplied by 2.
  - Looking for an increasing/decreasing difference
    Is there a pattern in the difference between pairs of terms?
    For example: 1, 3, 6, 10, 15, … The difference increases by 1 each term.
  - Looking for two interlinked patterns
    Is there a pattern in the odd-numbered terms, and another pattern in the even-numbered terms?
    For example: 2, 8, 4, 7, 6, 6, … The odd-numbered terms increase by 2, the even-numbered terms decrease by 1.
  - Looking for a special type of pattern
    Could it be a list of square numbers, prime numbers, Fibonacci numbers etc.?
    For example: 1, 8, 27, 64, 125, … This is the pattern of cube numbers: 1^3, 2^3, 3^3, …

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Example 18 Identifying patterns with a common difference

Find the next three terms for these number patterns, which have a common difference.

a 6, 18, 30, 42, ___, ___, ___ **b** 99, 92, 85, 78, ___, ___, ___

SOLUTION

a 54, 66, 78

The common difference is 12. Continue adding 12 to generate the next three terms.

b 71, 64, 57

The pattern indicates the common difference is 7. Continue subtracting 7 to generate the next three terms.

Example 19 Identifying patterns with a common ratio

Find the next three terms for the following number patterns that have a common ratio.

a 2, 6, 18, 54, ___, ___, ___ **b** 256, 128, 64, 32, ___, ___, ___

SOLUTION

a 162, 486, 1458

The common ratio is 3. Continue multiplying by 3 to generate the next three terms.

b 16, 8, 4

The common ratio is $\frac{1}{2}$. Continue dividing by 2 to generate the next three terms.

Exercise 8I EXTENSION

UNDERSTANDING AND FLUENCY	1, 2, 3–7(½)	3–8(½)	4–8(½)

1 Generate the first five terms of the following number patterns.

a start with 8 and keep adding 3
b start with 32 and keep subtracting 1
c start with 2 and keep subtracting 4
d start with 123 and keep adding 7

2 Generate the first five terms of the following number patterns.

a start with 3 and keep multiplying by 2
b start with 5 and keep multiplying by 4
c start with 240 and keep dividing by 2
d start with 625 and keep dividing by 5

3 State whether the following number patterns have a common difference (+ or −), a common ratio (× or ÷) or neither.

a 4, 12, 36, 108, 324, …
b 19, 17, 15, 13, 11, …
c 212, 223, 234, 245, 256, …
d 8, 10, 13, 17, 22, …
e 64, 32, 16, 8, 4, …
f 5, 15, 5, 15, 5, …
g 2, 3, 5, 7, 11, …
h 75, 72, 69, 66, 63, …

Example 18

4 Find the next three terms for the following number patterns, which have a common difference.

a 3, 8, 13, 18, ___, ___, ___
b 4, 14, 24, 34, ___, ___, ___
c 26, 23, 20, 17, ___, ___, ___
d 106, 108, 110, 112, ___, ___, ___
e 63, 54, 45, 36, ___, ___, ___
f 4, 3, 2, 1, ___, ___, ___
g 101, 202, 303, 404, ___, ___, ___
h 17, 11, 5, −1, ___, ___, ___

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Example 19

5 Find the next three terms for the following number patterns, which have a common ratio.

a 2, 4, 8, 16, ___, ___, ___
b 5, 10, 20, 40, ___, ___, ___
c 96, 48, 24, ___, ___, ___
d 1215, 405, 135, ___, ___, ___
e 11, 22, 44, 88, ___, ___, ___
f 7, 70, 700, 7000, ___, ___, ___
g 256, 128, 64, 32, ___, ___, ___
h 1216, 608, 304, 152, ___, ___, ___

6 Find the missing numbers in each of the following number patterns.

a 62, 56, ___, 44, 38, ___, ___
b 15, ___, 35, ___, ___, 65, 75
c 4, 8, 16, ___, ___, 128, ___
d 3, 6, ___, 12, ___, 18, ___
e 88, 77, 66, ___, ___, ___, 22
f 2997, 999, ___, ___, 37
g 14, 42, ___, ___, 126, ___, 182
h 14, 42, ___, ___, 1134, ___, 10206

7 Write the next three terms in each of the following sequences.

a 3, 5, 8, 12, ___, ___, ___
b 1, 2, 4, 7, 11, ___, ___, ___
c 1, 4, 9, 16, 25, ___, ___, ___
d 27, 27, 26, 24, 21, ___, ___, ___
e 2, 3, 5, 7, 11, 13, ___, ___, ___
f 2, 5, 11, 23, ___, ___, ___
g 2, 10, 3, 9, 4, 8, ___, ___, ___
h 14, 100, 20, 80, 26, 60, ___, ___, ___

8 Generate the next three terms for the following number sequences and give an appropriate name to the sequence.

a 1, 4, 9, 16, 25, 36, ___, ___, ___
b 1, 1, 2, 3, 5, 8, 13, ___, ___, ___
c 1, 8, 27, 64, 125, ___, ___, ___
d 2, 3, 5, 7, 11, 13, 17, ___, ___, ___
e 4, 6, 8, 9, 10, 12, 14, 15, ___, ___, ___
f 121, 131, 141, 151, ___, ___, ___

PROBLEM-SOLVING AND REASONING	9, 10, 13	10, 11, 13, 14	10–12, 14, 15

9 Complete the next three terms for the following challenging number patterns.

a 101, 103, 106, 110, ___, ___, ___
b 162, 54, 108, 36, 72, ___, ___, ___
c 3, 2, 6, 5, 15, 14, ___, ___, ___
d 0, 3, 0, 4, 1, 6, 3, ___, ___, ___

10 When making human pyramids, there is one less person on each row above, and it is complete when there is a row of only one person on the top.

Write down a number pattern for a human pyramid with 10 students on the bottom row. How many people are needed to make this pyramid?

11 The table below represents a seating plan with specific seat numbering for a section of a grandstand at a soccer ground. It continues upwards for another 20 rows.

Row 4	25	26	27	28	29	30	31	32
Row 3	17	18	19	20	21	22	23	24
Row 2	9	10	11	12	13	14	15	16
Row 1	1	2	3	4	5	6	7	8

a What is the number of the seat directly above seat number 31?
b What is the number of the seat on the left-hand edge of row 8?
c What is the third seat from the right in row 14?
d How many seats are in the grandstand?

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12 Find the next five numbers in the following number pattern.

1, 4, 9, 1, 6, 2, 5, 3, 6, 4, 9, 6, 4, 8, 1, ___, ___, ___, ___, ___

13 Jemima writes down the following number sequence: 7, 7, 7, 7, 7, 7, 7, …
Her friend Peta declares that this is not really a number pattern. Jemima defends her number pattern, stating that it is most definitely a number pattern as it has a common difference and also has a common ratio. What are the common difference and the common ratio for the number sequence above? Do you agree with Jemima or Peta?

14 Find the sum of the following number sequences.
- **a** $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$
- **b** $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$
- **c** $1 + 2 + 3 + 4 + 5 + \ldots + 67 + 68 + 69 + 70$
- **d** $5 + 8 + 11 + 14 + 17 + 20 + 23 + 26 + 29 + 32 + 35 + 38$

15 The great handshake problem. There are a certain number of people in a room and they must all shake one another's hand. How many handshakes will there be if there are:
- **a** 3 people in the room?
- **b** 5 people in the room?
- **c** 10 people in the room?
- **d** 24 people in a classroom?
- **e** n people in the room?

ENRICHMENT	—	—	16

What number am I?

16 Read the following clues to work out the mystery number.
- **a** I have three digits.
  I am divisible by 5.
  I am odd.
  The product of my digits is 15.
  The sum of my digits is less than 10.
  I am less than 12×12.
- **b** I have three digits.
  The sum of my digits is 12.
  My digits are all even.
  My digits are all different.
  I am divisible by 4.
  The sum of my units and tens digits equals my hundreds digit.
- **c** I have three digits.
  I am odd and divisible by 5 and 9.
  The product of my digits is 180.
  The sum of my digits is less than 20.
  I am greater than 30^2.
- **d** Make up two of your own mystery number puzzles and submit your clues to your teacher.

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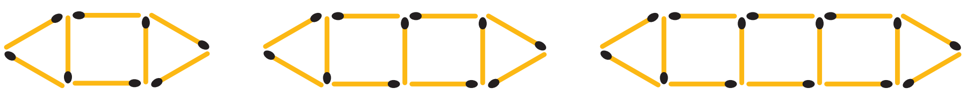
8J Spatial patterns EXTENSION

Patterns can also be found in geometric shapes. Mathematicians examine patterns carefully to determine how the next term in the sequence is created. Ideally, a rule is formed that shows the relationship between the geometric shape and the number of objects (e.g. tiles, sticks or counters) required to make such a shape. Once a rule is established, it can be used to make predictions about future terms in the sequence.

Let's start: Stick patterns

Materials required: One box of toothpicks or matchsticks per student.

- Generate a spatial pattern using your matchsticks.
- You must be able to make at least three terms in your pattern. For example:



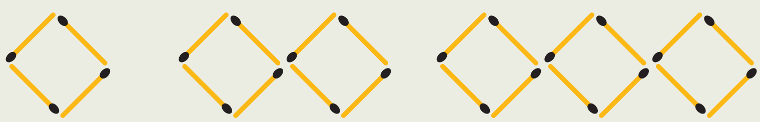
- Ask your partner how many matchsticks would be required to make the next term in the pattern.
- Repeat the process with a different spatial design.

Key ideas

- A **spatial pattern** is a sequence of geometrical shapes that can be described by a **number pattern**.

  For example:

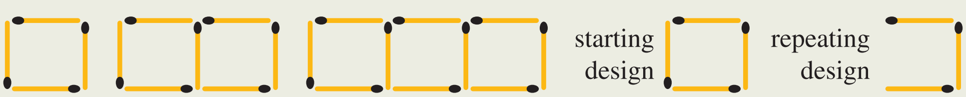
  spatial pattern:



  number pattern: 4 8 12

- A spatial pattern starts with a simple geometric design. Future terms are created by adding on repeated shapes of the same design. If designs connect with an edge, the repetitive shape added on will be a subset of the original design, as the connecting edge does not need to be repeated.

  For example:



- To help describe a spatial pattern, it is generally converted to a number pattern and a common difference is observed.
- The common difference is the number of objects (e.g. sticks) that need to be added on to create the next term.
- A table of values shows the number of shapes and the number of sticks.

Number of squares	1	2	3	4	5
Number of sticks	4	8	12	16	20

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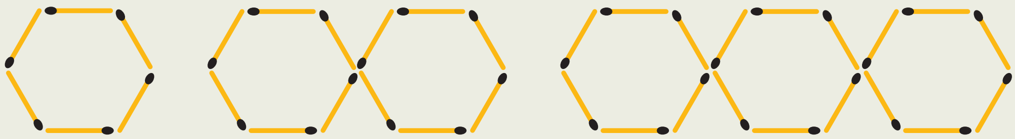
Key ideas

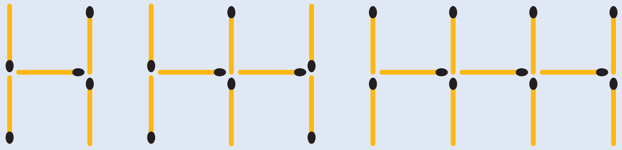
- A pattern rule tells how many sticks are needed for a certain number of shapes.
  For example: Number of sticks $= 4 \times$ number of shapes
- Rules can be found that connect the number of objects (e.g. sticks) required to produce the number of designs.
  For example: hexagon design



Rule is: Number of sticks used $= 6 \times$ number of hexagons formed

Example 20 Drawing and describing spatial patterns

a Draw the next two shapes in the spatial pattern shown.

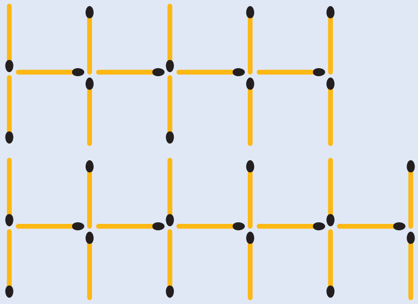


b Write the spatial pattern above as a number pattern in regard to the number of matchsticks required to make each shape.

c Describe the pattern by stating how many matchsticks are required to make the first term, and how many matchsticks are required to make the next term in the pattern.

SOLUTION

a



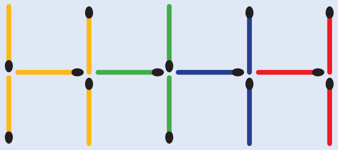
b 5, 8, 11, 14, 17

c 5 matchsticks are required to start the pattern, and an additional 3 matchsticks are required to make the next term in the pattern.

EXPLANATION

Follow the pattern.

Count the number of matchsticks in each term. Look for a pattern.



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Example 21 Finding a general rule for a spatial pattern

a Draw the next two shapes in this spatial pattern.

b Complete the table.

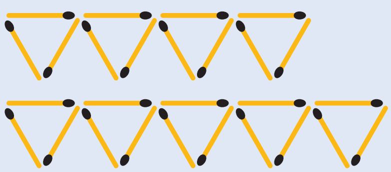
Number of triangles	1	2	3	4	5
Number of matchsticks required	3				

c Describe a rule connecting the number of matchsticks required to the number of triangles.

d Use your rule to predict how many matchsticks would be required to make 20 triangles.

SOLUTION

a



EXPLANATION: Follow the pattern by adding one triangle each time.

b

No. of triangles	1	2	3	4	5
No. of matchsticks required	3	6	9	12	15

EXPLANATION: An extra 3 matchsticks are required to make each new triangle.

c Number of matchsticks $= 3 \times$ number of triangles

EXPLANATION: 3 matchsticks are required per triangle.

d Number of matchsticks $= 3 \times 20$ triangles $= 60$ sticks

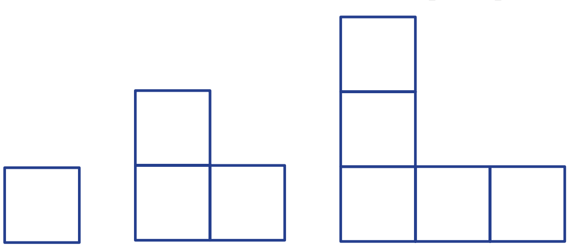
EXPLANATION: 20 triangles $\times$ 3 matchsticks each

Exercise 8J EXTENSION

UNDERSTANDING AND FLUENCY	1–4	3–5	4–6

1 Draw the next two terms for each of these spatial patterns.

a



b

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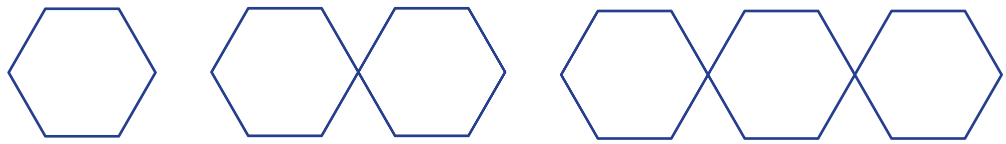
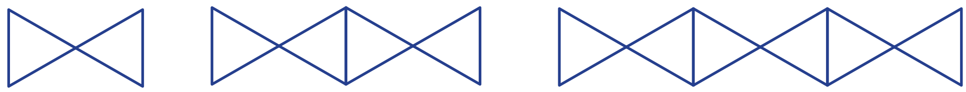
c

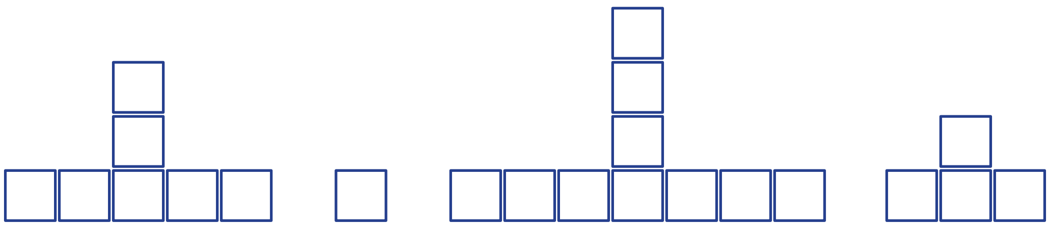


d



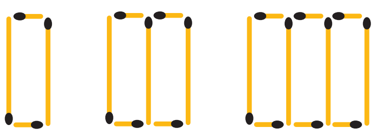
e

2 Draw the following geometrical designs in sequential ascending (i.e. increasing) order and draw the next term in the sequence.

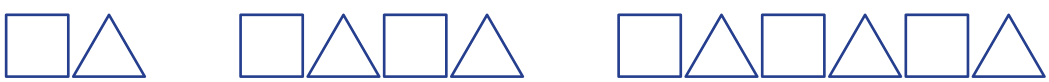


3 For each of the following spatial patterns, draw the starting geometrical design and also the geometrical design that is added on repetitively to create new terms. (For some patterns the repetitive design is the same as the starting design.)

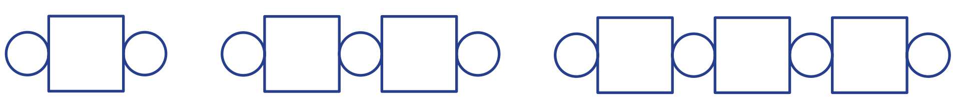
a



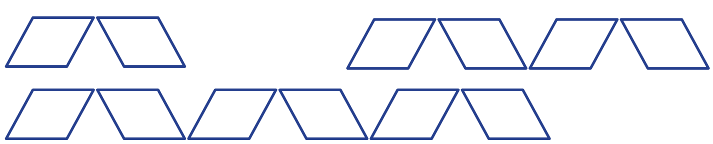
b



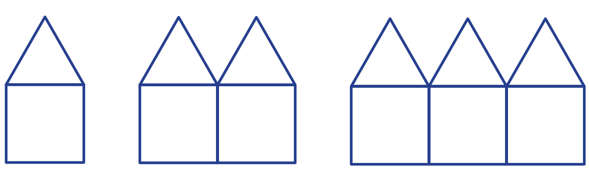
c



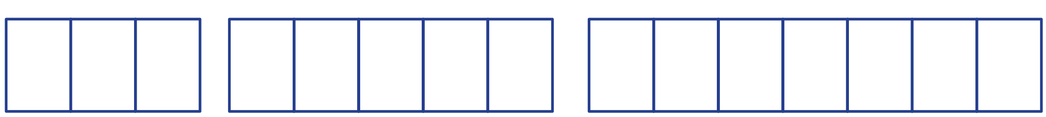
d



e



f



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Example 20

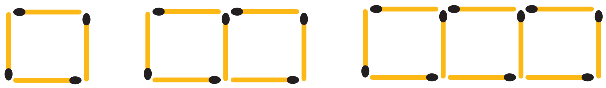
4 For each of the spatial patterns below:

i Draw the next two shapes.

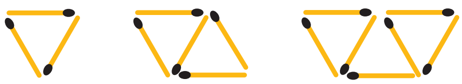
ii Write the spatial pattern as a number pattern.

iii Describe the pattern by stating how many matchsticks are required to make the first term and how many more matchsticks are required to make the next term in the pattern.

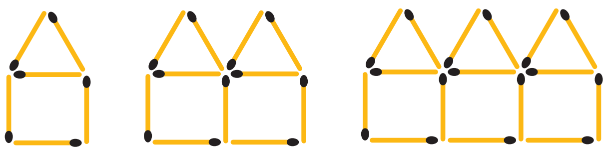
a



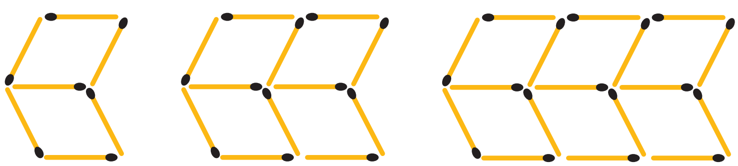
b



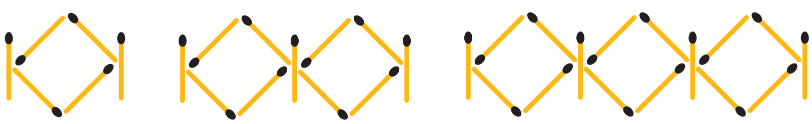
c



d



e



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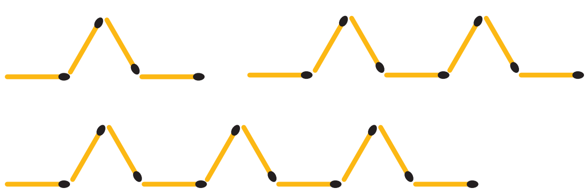
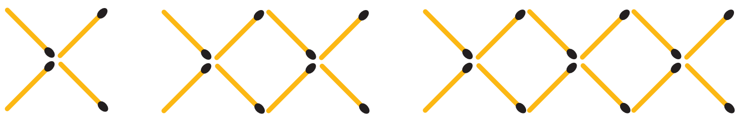


Example 21

5 **a** Draw the next two shapes in this spatial pattern.



b Copy and complete the table.

Number of crosses	1	2	3	4	5
Number of matchsticks required					

c Describe a rule connecting the number of matchsticks required to the number of crosses produced.

d Use your rule to predict how many matchsticks would be required to make 20 crosses.

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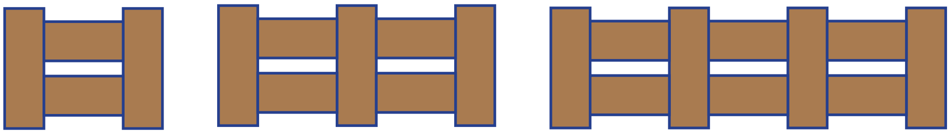
6 **a** Draw the next two shapes in this spatial pattern.



b Copy and complete the table. Planks are vertical and horizontal.

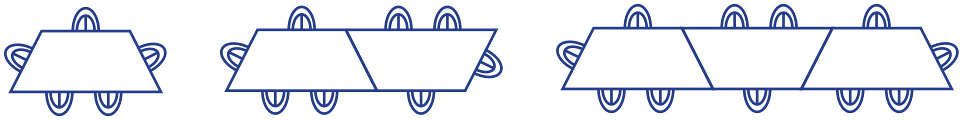
Number of fence sections	1	2	3	4	5
Number of planks required					

c Describe a rule connecting the number of planks required to the number of fence sections produced.

d Use your rule to predict how many planks would be required to make 20 fence sections.

PROBLEM-SOLVING AND REASONING	7, 8, 12	8, 9, 12, 13	10, 11, 13, 14

7 At North Park Primary School, the classrooms have trapezium-shaped tables. Mrs Greene arranges her classroom's tables in straight lines, as shown.



a Draw a table of results showing the relationship between the number of tables in a row and the number of students that can sit at the tables. Include results for up to five tables in a row.

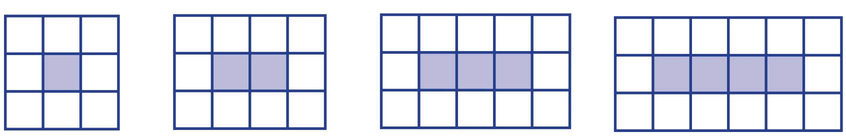
b Describe a rule that connects the number of tables placed in a straight row to the number of students that can sit around the tables.

c The room allows seven tables to be arranged in a straight line. How many students can sit around the tables?

d There are 65 students in Grade 6 at North Park Primary School. Mrs Greene would like to arrange the tables in one straight line for an outside picnic lunch. How many tables will she need?

8 The number of tiles required to pave around a spa is related to the size of the spa.

The approach is to use large tiles that are the same size as that of a small spa.



A spa of length 1 unit requires 8 tiles to pave around its perimeter, whereas a spa of length 4 units requires 14 tiles to pave around its perimeter.

a Complete a table of values relating length of spa and number of tiles required, for values up to and including a spa of length 6 units.

b Describe a rule that connects the number of tiles required for the length of the spa.

c The largest size spa manufactured is 15 units long. How many tiles would be required to pave around its perimeter?

d A paving company has only 30 tiles left. What is the largest spa they would be able to tile around?

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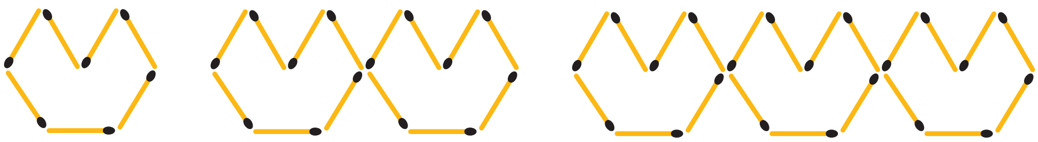
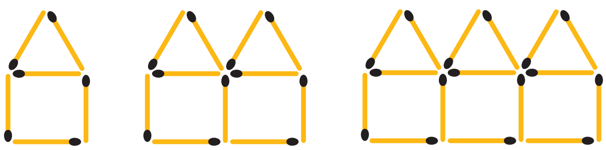
9 Present your answers to either Question 7 or 8 in an A4 or A3 poster form. Express your findings and justifications clearly.

10 Which rule correctly describes this spatial pattern?



A Number of matchsticks $= 7 \times$ number of 'hats'
B Number of matchsticks $= 7 \times$ number of 'hats' $+ 1$
C Number of matchsticks $= 6 \times$ number of 'hats' $+ 2$
D Number of matchsticks $= 6 \times$ number of 'hats'

11 Which rule correctly describes this spatial pattern?



A Number of matchsticks $= 5 \times$ number of houses $+ 1$
B Number of matchsticks $= 6 \times$ number of houses $+ 1$
C Number of matchsticks $= 6 \times$ number of houses
D Number of matchsticks $= 5 \times$ number of houses

12 Design a spatial pattern to fit the following number patterns.
a 4, 7, 10, 13, …
b 4, 8, 12, 16, …
c 3, 5, 7, 9, …
d 3, 6, 9, 12, …
e 5, 8, 11, 14, …
f 6, 11, 16, 21, …

13 A rule to describe a special window spatial pattern is written as $y = 4 \times x + 1$, where y represents the number of 'sticks' required and x is the number of windows created.
a How many sticks are required to make one window?
b How many sticks are required to make 10 windows?
c How many sticks are required to make g windows?
d How many windows can be made from 65 sticks?

14 A rule to describe a special fence spatial pattern is written as $y = m \times x + n$, where y represents the number of pieces of timber required and x represents the number of fencing panels created.
a How many pieces of timber are required to make one panel?
b What does m represent?
c Draw the first three terms of the fence spatial pattern for $m = 4$ and $n = 1$.

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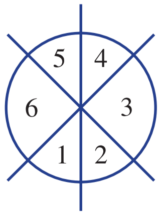
ENRICHMENT	—	—	15

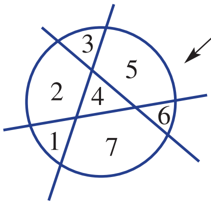
Cutting up a circle

15 What is the *greatest* number of sections into which you can divide a circle using only a particular number of straight line cuts?

a Explore the problem above.
Note: The greatest number of sections is required and, hence, only one of the two diagrams below is correct for three straight line cuts.



Incorrect.
Not the maximum number of sections.



Correct.
The maximum number of sections.

b Copy and complete this table of values.

Number of straight cuts	1	2	3	4	5	6	7
Number of sections created			7				

c Can you discover a pattern for the maximum number of sections created? What is the maximum number of sections that could be created with 10 straight line cuts?

d The formula for determining the maximum number of cuts is quite complex.

$$\text{Sections} = \frac{1}{2}\text{cuts}^2 + \frac{1}{2}\text{cuts} + 1$$

Verify that this formula works for the values you listed in the table above.
Using the formula, how many sections could be created with 20 straight cuts?

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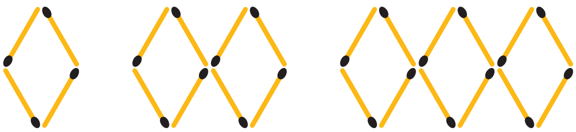
8K Tables and rules EXTENSION

In the previous section on spatial patterns, it was observed that rules can be used to connect the number of objects (e.g. matchsticks) required to make particular designs.

A table of values can be created for any spatial pattern. Consider this spatial pattern and the corresponding table of values.



Number of diamonds (*input*)	Number of matchsticks (*output*)
1	4
2	8
3	12

What values would go in the next row of the table?
A rule that produces this table of values is:

number of matchsticks = 4 × number of diamonds

Alternatively, if we consider the number of diamonds as the *input* variable and the number of matchsticks as the *output* variable, then the rule could be written as:

output = 4 × *input*

If a rule is provided, a table of values can be created.
If a table of values is provided, often a rule can be found.

Let's start: Guess the output

- A table of values is drawn on the board with three completed rows of data.
- Additional values are placed in the *input* column. What *output* values should be in the *output* column?
- After adding *output* values, decide which rule fits (models) the values in the table and check that it works for each *input* and *output* pair.

Four sample tables are listed below.

input	*output*
2	6
5	9
6	10
1	?
8	?

input	*output*
12	36
5	15
8	24
0	?
23	?

input	*output*
2	3
3	5
9	17
7	?
12	?

input	*output*
6	1
20	8
12	4
42	?
4	?

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Key ideas

- A **rule** shows the relation between two varying quantities.
  For example: *output* = *input* + 3 is a rule connecting the two quantities *input* and *output*.
  The values of the *input* and the *output* can vary, but we know from the rule that the value of the *output* will always be 3 more than the value of the *input*.
- A **table of values** can be created from any given rule. To complete a table of values, the *input* (one of the quantities) is replaced by a number. This is known as **substitution**. After substitution, the value of the other quantity, the *output*, is calculated.
  For example: If *input* = 4, then

$$\begin{aligned} output &= input + 3 \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

- Often, a rule can be determined from a table of values. On close inspection of the values, a relationship may be observed. Each of the four operations should be considered when looking for a connection.

input	1	2	3	4	5	6
output	6	7	8	9	10	11

  By inspection, it can be observed that every *output* value is 5 more than the corresponding *input* value. The rule can be written as: *output* = *input* + 5.

Example 22 Completing a table of values

Complete each table for the given rule.

a *output* = *input* − 2

input	3	5	7	12	20
output					

b *output* = (3 × *input*) + 1

input	4	2	9	12	0
output					

SOLUTION

a *output* = *input* − 2

input	3	5	7	12	20
output	1	3	5	10	18

EXPLANATION

Replace each *input* value in turn into the rule.
e.g. When *input is* 3:
output = 3 − 2 = 1

b *output* = (3 × *input*) + 1

input	4	2	9	12	0
output	13	7	28	37	1

Replace each *input* value in turn into the rule.
e.g. When *input* is 4:
output = (3 × 4) + 1 = 13

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Example 23 Finding a rule from a table of values

Find the rule for each of these tables of values.

a

input	3	4	5	6	7
output	12	13	14	15	16

b

input	1	2	3	4	5
output	7	14	21	28	35

SOLUTION

a *output* = *input* + 9

Each *output* value is 9 more than the *input* value.

b *output* = *input* × 7 or *output* = 7 × *input*

By inspection, it can be observed that each *output* value is 7 times bigger than the *input* value.

Exercise 8K EXTENSION

UNDERSTANDING AND FLUENCY	1–7	4–7	5–7

1 State whether each of the following statements is true or false.

a If *output* = *input* × 2, then when *input* = 7, *output* = 14.

b If *output* = *input* − 2, then when *input* = 5, *output* = 7.

c If *output* = *input* + 2, then when *input* = 0, *output* = 2.

d If *output* = *input* ÷ 2, then when *input* = 20, *output* = 10.

2 Which table of values matches the rule *output* = *input* − 3?

A

input	10	11	12
output	13	14	15

B

input	5	6	7
output	15	18	21

C

input	8	9	10
output	5	6	7

D

input	4	3	2
output	1	1	1

3 Which table of values matches the rule *output* = *input* ÷ 2?

A

input	20	14	6
output	18	12	4

B

input	8	10	12
output	4	5	6

C

input	4	5	6
output	8	10	12

D

input	4	3	2
output	6	5	4

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4 Match each rule (A to D) with the correct table of values (**a** to **d**).

Rule A: *output* = *input* − 5

Rule B: *output* = *input* + 1

Rule C: *output* = 4 × *input*

Rule D: *output* = 5 + *input*

a

input	20	14	6
output	15	9	1

b

input	8	10	12
output	13	15	17

c

input	4	5	6
output	5	6	7

d

input	4	3	2
output	16	12	8

Example 22a

5 Copy and complete each table for the given rule.

a *output* = *input* + 3

input	4	5	6	7	10
output					

b *output* = *input* × 2

input	5	1	3	21	0
output					

c *output* = *input* − 8

input	11	18	9	44	100
output					

d *output* = *input* ÷ 5

input	5	15	55	0	100
output					

Example 22b

6 Copy and complete each table for the given rule.

a *output* = (10 × *input*) − 3

input	1	2	3	4	5
output					

b *output* = (*input* ÷ 2) + 4

input	6	8	10	12	14
output					

c *output* = (3 × *input*) + 1

input	5	12	2	9	0
output					

d *output* = (2 × *input*) − 4

input	3	10	11	7	50
output					

Example 23

7 State the rule for each of these tables of values.

a

input	4	5	6	7	8
output	5	6	7	8	9

b

input	1	2	3	4	5
output	4	8	12	16	20

c

input	10	8	3	1	14
output	21	19	14	12	25

d

input	6	18	30	24	66
output	1	3	5	4	11

PROBLEM-SOLVING AND REASONING

8, 9, 11 | 8, 9, 11, 12 | 9, 10, 12, 13

8 Copy and complete the missing values in the table and state the rule.

input	4	10	13	24			5	11	2
output			39		42	9	15		6

9 Copy and complete the missing values in the table and state the rule.

input	12	93	14	17		10			
output	3			8	12	1	34	0	200

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10 Copy and complete each table for the given rule.

a $output = (input \times input) - 2$

input	3	6	8	12	2
output					

b $output = (24 \div input) + 1$

input	6	12	1	3	8
output					

c $output = input^2 + input$

input	5	12	2	9	0
output					

d $output = 2 \times input \times input - input$

input	3	10	11	7	50
output					

11 Copy and complete each table for the given rule.

a $output = input + 6$

input	c	d	$2p$	b^2	www
output					

b $output = 3 \times input - 2$

input	t	k	p^2	$2f$	ab
output					

12 Copy and complete the missing values in the table and state the rule.

input	b		e	g^2		x	c		1
output		cd			cmn	xc		0	c

13 It is known that for an *input* value of 3, the *output* value is 7.

a State two different rules that work for these values.

b How many different rules are possible? Explain.

ENRICHMENT	—	—	14

Finding harder rules

14 a The following rules all involve two operations. Find the rule for each of these tables of values.

i

input	4	5	6	7	8
output	5	7	9	11	13

ii

input	1	2	3	4	5
output	5	9	13	17	21

iii

input	10	8	3	1	14
output	49	39	14	4	69

iv

input	6	18	30	24	66
output	3	5	7	6	13

v

input	4	5	6	7	8
output	43	53	63	73	83

vi

input	1	2	3	4	5
output	0	4	8	12	16

b Write three of your own two-operation rules and produce a table of values for each rule.

c Swap your tables of values with those of a classmate and attempt to find one another's rules.

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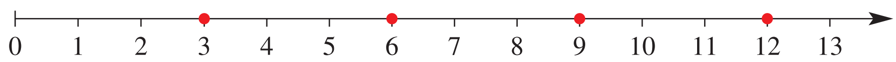
8L The Cartesian plane and graphs EXTENSION

In previous sections we looked at number sequences and spatial patterns. We used rules and tables of values to describe them. Another way of showing a pattern is by plotting points. For example, we could use a number line to show the simple pattern 3, 6, 9, 12, …

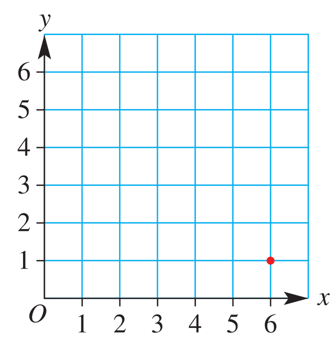


However, when we work with two sets of values (i.e. inputs and outputs) we need two dimensions. Instead of a number line, we use a Cartesian plane.

Let's start: I'm thinking of two numbers

I am thinking of two positive numbers.
They add together to give 7.
They could be $x = 6, y = 1$, which is graphed as (6, 1) on the Cartesian plane opposite.

- What else could they be?
  Find another 5 points and plot them.
- What do you notice about the position of those points?

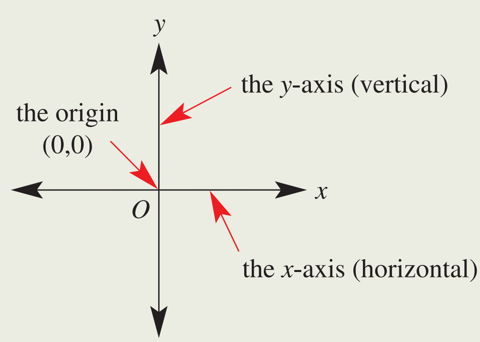


Key ideas

- A **number plane** is a grid for plotting points.

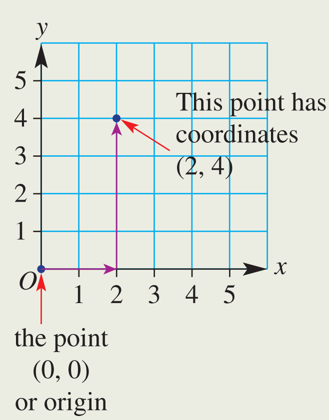
  It is also called the **Cartesian plane**.

  Important features of a number plane are:
  - the ***x*-axis** and ***y*-axis**: these are horizontal and vertical number lines
  - the **origin**: the point where the *x*-axis and *y*-axis meet



- Points are located by a grid reference system of **coordinates**.
  - The point (x, y) means:
    - Start from the origin.
    - Go x units across to the left or right.
    - Go y units up or down.
  - For (2, 4) the x coordinate is 2 and the y coordinate is 4. To plot this point, start at the origin and go 2 units right, then 4 units up.

- For a rule describing a pattern with *input* and *output*:
  - The *input* is the x value.
  - The *output* is the y value.



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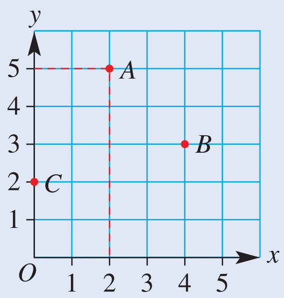
Example 24 Plotting points on a Cartesian plane

Plot these points on a Cartesian plane.

$A(2, 5)$ $B(4, 3)$ $C(0, 2)$

SOLUTION



EXPLANATION

Draw a Cartesian plane, with both axes labelled from 0 to 5.
The first coordinate is the x coordinate.
The second coordinate is the y coordinate.
To plot point A, go along the horizontal axis to the number 2, then move vertically up 5 units. Place a dot at this point, which is the intersection of the line passing through the point 2 on the horizontal axis and the line passing through the point 5 on the vertical axis.

Example 25 Drawing a graph

For the given rule *ouput* = *input* + 1:

a Complete the given table of values.

b Plot each pair of points in the table to form a graph.

input (x)	*output* (y)
0	1
1	
2	
3	

SOLUTION

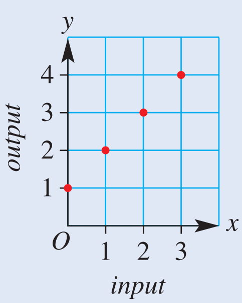
a

input (x)	*output* (y)
0	1
1	2
2	3
3	4

EXPLANATION

Use the given rule to find each *output* value for each *input* value.
The rule is:
ouput = *input* + 1, so add 1 to each *input* value.

b



Plot each (x, y) pair.
The pairs are $(0, 1)$, $(1, 2)$, $(2, 3)$ and $(3, 4)$.

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Exercise 8L EXTENSION

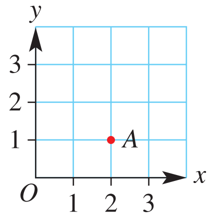
UNDERSTANDING AND FLUENCY	1–8	5–11	6, 8–12

1 Which point lies on the x-axis $(0, 3)$ or $(3, 0)$?

2 Write down the coordinates of the point which is 5 units above the origin.

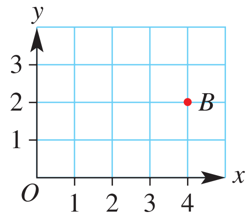
3 Which of the following is the correct way to describe point A?

A 2 1
B 2, 1
C (2, 1)
D $(x2, y1)$
E $(2_x, 1_y)$



4 Which of the following is the correct set of coordinates for point B?

A (2, 4)
B 4, 2
C (4, 2)
D (2 4)
E $x = 4, y = 2$



5 Copy and complete the following sentences, using the terminology in the table.

first	y-axis	x
y	second	y
x-axis	origin	x

a The horizontal axis is known as the _______________.
b The _______________ is the vertical axis.
c The point at which the axes intersect is called the _______________.
d The x coordinate is always written _______________.
e The second coordinate is always the _______________.
f _______________ comes before _______________ in the dictionary, and the _______________ coordinate comes before the _______________ coordinate on the Cartesian plane.

Example 24

6 Plot the following points on a Cartesian plane.

a $A(4, 2)$
b $B(1, 1)$
c $C(5, 3)$
d $D(0, 2)$
e $E(3, 1)$
f $F(5, 4)$
g $G(5, 0)$
h $H(0, 0)$

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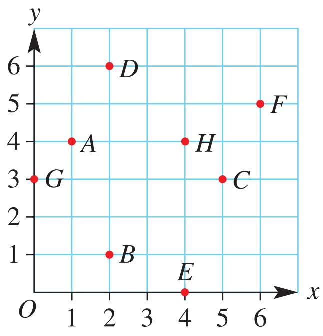
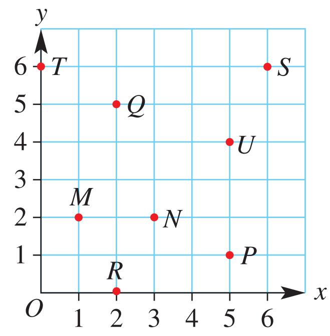
7 Write down the coordinates of each of these labelled points.

a



b



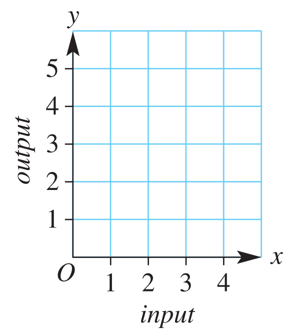
Example 25

8 For the given rule *ouput* = *input* + 2:

a Copy and complete the given table of values.

b Plot each pair of points in the table to form a graph.

input (x)	*output* (y)
0	2
1	
2	
3	

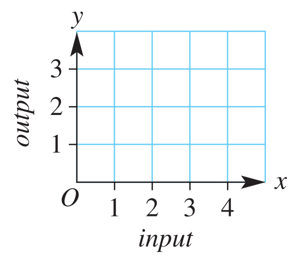


9 For the given rule *ouput* = *input* − 1:

a Copy and complete the given table of values.

b Plot each pair of points in the table to form a graph.

input (x)	*output* (y)
1	
2	
3	
4	

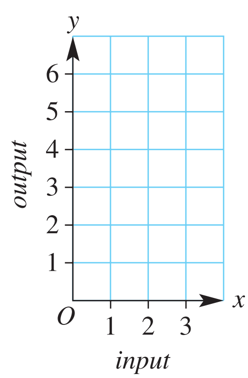


10 For the given rule *ouput* = *input* × 2:

a Copy and complete the given table of values.

b Plot each pair of points in the table to form a graph.

input (x)	*output* (y)
0	
1	
2	
3	



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11 Draw a Cartesian plane from 0 to 5 on both axes. Place a cross on each pair of coordinates that have the same x and y value.

12 Draw a Cartesian plane from 0 to 8 on both axes. Plot the following points on the grid and join them in the order they are given.
(2, 7), (6, 7), (5, 5), (7, 5), (6, 2), (5, 2), (4, 1), (3, 2), (2, 2), (1, 5), (3, 5), (2, 7)

PROBLEM-SOLVING AND REASONING	13, 14, 17	14, 15, 17	14–16, 17, 18

13 **a** Plot the following points on a Cartesian plane and join the points in the order given, to draw the basic shape of a house.
(1, 5), (0, 5), (5, 10), (10, 5), (1, 5), (1, 0), (9, 0), (9, 5)

b Describe a set of four points to draw a door.

c Describe two sets of four points to draw two windows.

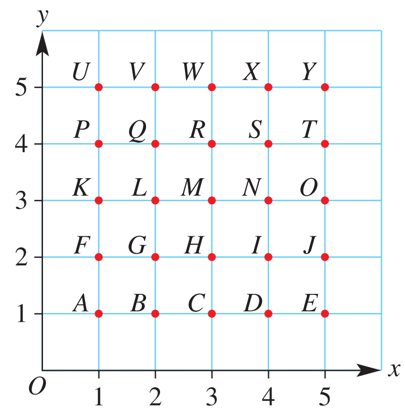
d Describe a set of four points to draw a chimney.

14 Point $A(1, 1)$ is the bottom left-hand corner of a square of side length 3.

a State the other three coordinates of the square.

b Draw the square on a Cartesian plane and shade in half of the square where the x coordinates are greater than the y coordinates.

15 A grid system can be used to make secret messages. Jake decides to arrange the letters of the alphabet on a Cartesian plane in the following manner.



a Decode Jake's message:
(3, 2), (5, 1), (2, 3), (1, 4)

b Code the word 'secret'.

c To increase the difficulty of the code, Jake does not include brackets or commas and he uses the origin to indicate the end of a word.
What do the following numbers mean?
135155001543415134001453540014231114354.

d Code the phrase: 'Be here at seven'.

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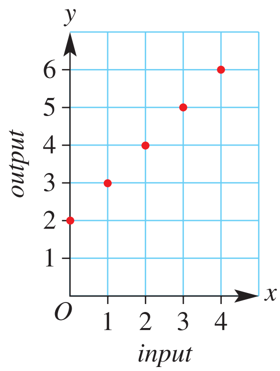
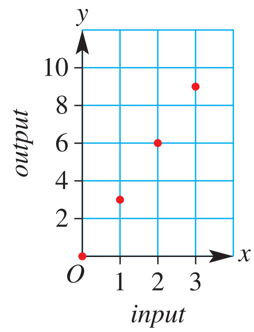
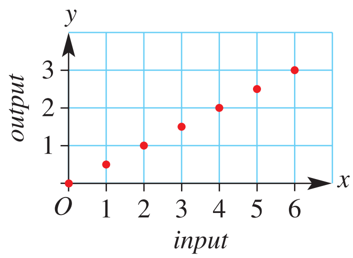
16 *ABCD* is a rectangle. The coordinates of *A*, *B* and *C* are given below. Draw each rectangle on a Cartesian plane and state the coordinates of the missing corner, *D*.

a $A(0, 5)$ $B(0, 3)$ $C(4, 3)$ $D(?, ?)$
b $A(4, 4)$ $B(1, 4)$ $C(1, 1)$ $D(?, ?)$
c $A(0, 2)$ $B(3, 2)$ $C(3, 0)$ $D(?, ?)$
d $A(4, 1)$ $B(8, 4)$ $C(5, 8)$ $D(?, ?)$

17 Write a rule (e.g. $output = input \times 2$) that would give these graphs.

a


b


c


18 $A(1, 0)$ and $B(5, 0)$ are the base points of an isosceles triangle.

a Find the coordinates of a possible third vertex.
b Show on a Cartesian plane that the possible number of answers for this third vertex is infinite.
c Write a sentence to explain why the possible number of answers for this third vertex is infinite.
d The area of the isosceles triangle is 10 square units. State the coordinates of the third vertex.

ENRICHMENT	—	—	19

Locating midpoints

19 a Plot the points $A(1, 4)$ and $B(5, 0)$ on a Cartesian plane. Draw the line segment *AB*. Find the coordinates of *M*, the midpoint of *AB*, and mark it on the grid.
b Find the midpoint, *M*, of the line segment *AB*, which has coordinates $A(2, 4)$ and $B(0, 0)$.
c Determine a method for locating the midpoint of a line segment without having to draw the points on a Cartesian plane.
d Find the midpoint, *M*, of the line segment *AB*, which has coordinates $A(6, 3)$ and $B(2, 1)$.
e Find the midpoint, *M*, of the line segment *AB*, which has coordinates $B(2, 1)$ and $B(4, 3)$.
f Find the midpoint, *M*, of the line segment *AB*, which has coordinates $A(-3, 2)$ and $B(2, -3)$.
g $M(3, 4)$ is the midpoint of *AB* and the coordinates of *A* are $(1, 5)$. What are the coordinates of *B*?

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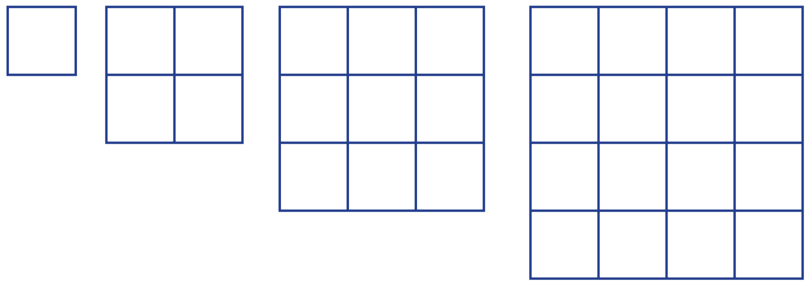
Investigation

Fencing paddocks

A farmer is interested in fencing off a large number of 1 m × 1 m foraging regions for the chickens. Consider the pattern below.



$n = 1$ $n = 2$ $n = 3$ $n = 4$

a For $n = 2$, the outside perimeter is 8 m, the area is 4 m^2 and the total length of fencing required is 12 m. Copy and complete the table.

n	1	2	3	4	5	6
Outside perimeter (m)		8				
Area (m^2)		4				
Fencing required		12				

b Write an expression for:

  i the total outside perimeter of the fenced section

  ii the total area of the fenced section

c The farmer knows that the expression for the total amount of fencing is one of the following. Which one is correct? Prove to the farmer that the others are incorrect.

  i $6n$ **ii** $(n + 1)^2$ **iii** $n \times 2 \times (n + 1)$

d Use the correct formula to work out the total amount of fencing required if the farmer wishes to have a total area of 100 m^2 fenced off.

In a spreadsheet application these calculations can be made automatically. Set up a spreadsheet as shown.

	A	B	C	D
1	n	Perimeter	Area	Total fencing
2	0	0	0	0
3	=A2+1	=A3*4	=A3*A3	=A3*2*(A3+1)
4	=A3+1	=A4*4	=A4*A4	=A4*2*(A4+1)
5	=A4+1	=A5*4	=A5*A5	=A5*2*(A5+1)
6	=A5+1	=A6*4	=A6*A6	=A6*2*(A6+1)
7	=A6+1	=A7*4	=A7*A7	=A7*2*(A7+1)
8	=A7+1	=A8*4	=A8*A8	=A8*2*(A8+1)
9	=A8+1	=A9*4	=A9*A9	=A9*2*(A9+1)
10	=A9+1	=A10*4	=A10*A10	=A10*2*(A10+1)
11	=A10+1	=A11*4	=A11*A11	=A11*2*(A11+1)
12	=A11+1	=A12*4	=A12*A12	=A12*2*(A12+1)
13	=A12+1	=A13*4	=A13*A13	=A13*2*(A13+1)
14	=A13+1	=A14*4	=A14*A14	=A14*2*(A14+1)
15	=A14+1	=A15*4	=A15*A15	=A15*2*(A15+1)
16	=A15+1	=A16*4	=A16*A16	=A16*2*(A16+1)
17	=A16+1	=A17*4	=A17*A17	=A17*2*(A17+1)
18	=A17+1	=A18*4	=A18*A18	=A18*2*(A18+1)
19	=A18+1	=A19*4	=A19*A19	=A19*2*(A19+1)
20	=A19+1	=A20*4	=A20*A20	=A20*2*(A20+1)
21	=A20+1	=A21*4	=A21*A21	=A21*2*(A21+1)
22	=A21+1	=A22*4	=A22*A22	=A22*2*(A22+1)
23	=A22+1	=A23*4	=A23*A23	=A23*2*(A23+1)
24	=A23+1	=A24*4	=A24*A24	=A24*2*(A24+1)
25	=A24+1	=A25*4	=A25*A25	=A25*2*(A25+1)
26	=A25+1	=A26*4	=A26*A26	=A26*2*(A26+1)
27	=A26+1	=A27*4	=A27*A27	=A27*2*(A27+1)
28	=A27+1	=A28*4	=A28*A28	=A28*2*(A28+1)
29	=A28+1	=A29*4	=A29*A29	=A29*2*(A29+1)
30	=A29+1	=A30*4	=A30*A30	=A30*2*(A30+1)
31	=A30+1	=A31*4	=A31*A31	=A31*2*(A31+1)
32				

Drag down the cells until you have all the rows from $n = 0$ to $n = 30$.

e Find the amount of fencing needed if the farmer wants the total area to be at least:

  i 25 m^2 **ii** 121 m^2

  iii 400 m^2 **iv** 500 m^2

f If the farmer has 144 m of fencing, what is the maximum area his grid could have?

g For each of the following lengths of fencing, give the maximum area, in m^2, that the farmer could contain in the grid.

  i 50 m **ii** 200 m

  iii 1 km **iv** 40 km

h In the end, the farmer decides that the overall grid does not need to be a square, but could be any rectangular shape. Design rectangular paddocks with the following properties.

  i perimeter = 20 m and area = 21 m^2

  ii perimeter = 16 m and fencing required = 38 m

  iii area = 1200 m^2 and fencing required = 148 m

  iv perimeter = 1 km and fencing required is less than 1.5 km

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1 Find the values of the pronumerals in the following sum/product tables.

a

			Sum
	a	b	c
	d	24	32
Sum	12	e	48

b

			Product
	a	b	18
	2	c	d
Product	12	e	180

2 Copy and complete the following table, in which x and y are always whole numbers.

x	2				
y	7	6		12	
$3x$		6	9		
$x + 2y$			9		7
xy				0	5

3 What is the coefficient of x once the expression $x + 2(x + 1) + 3(x + 2) + 4(x + 3) + \ldots + 100(x + 99)$ is simplified completely?

4 In a mini-Sudoku, the digits 1 to 4 occupy each square such that no row, column or 2×2 block has the same digit twice. Find the value of each of the pronumerals in the following mini-Sudoku.

a	3	2	c
c	d	e	f
2	g	$d + 1$	h
i	1	j	k

5 In a magic square the sum of each row, column and diagonal is the same. Find the value of the pronumerals to make the following into magic squares. Confirm your answer by writing out the magic square as a grid of numbers.

a

A	B	C
$A - 1$	$A + 1$	$B - C$
$B - 1$	$C - 1$	$A + C$

b

$2D$	$\frac{E}{2}$	F	$3G - 2$
G	$2G$	$D + 3$	D
$4F + 1$	E	$F + G$	EF
$G - 1$	$\frac{EG}{2}$	$2(F + G)$	$F - 1$

6 Think of any number and then perform the following operations: Add 5, then double the result, then subtract 12, then subtract the original number, then add 2. Use algebra to explain why you now have the original number again. Then design a puzzle like this yourself and try it on friends.

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Chapter summary

Algebra

Creating expressions

6 more than k: $k + 6$
Product of 4 and x: $4x$
10 less than b: $b - 10$
Half of q: $\frac{q}{2}$
The sum of a and b is tripled: $3(a + b)$

Mathematical convention

$3a$ means $3 \times a$
$\frac{b}{10}$ means $b \div 10$

Algebraic expressions

Combination of numbers, pronumerals and operations
e.g. $2xy + 3yz$, $\frac{12}{x} - 3$

Pronumerals

Letters used to represent numbers
e.g. g: number of grapes in a bunch
d: distance travelled by a hockey ball

Terms are pronumerals and numbers combined with $\times$ or $\div$.
e.g. $4x$, $10y$, $\frac{a}{3}$, 12

Equivalent expressions

Always equal when pronumerals are substituted
e.g. $2x + 3$ and $3 + 2x$ are equivalent.
$4(3x)$ and $12x$ are equivalent.

Substitution

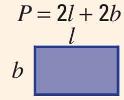
Replacing pronumerals with values
e.g. $5x + 2y$ when $x = 10$ & $y = 3$ becomes $5(10) + 2(3) = 50 + 6 = 56$
e.g. q^2 when $q = 7$ becomes $7^2 = 49$

To **simplify** an expression, find a simpler expression that is equivalent.

Combining like terms gives a way to simplify.
e.g. $4a + 2 + 3a = 7a + 2$
$3b + 5c + 2b - c = 5b + 4c$
$12xy + 3x - 5yx = 7xy + 3x$

Applications

Expressions are used widely
$A = l \times b$
$P = 2l + 2b$



Cost is $50 + 90x$
call-out fee ↗ ↖ hourly rate

Expanding brackets

$3(a + 4) = 3a + 12$
$5k(10 - 2j) = 50k - 10kj$
Using the distributive law gives an equivalent expression.

Like terms have exactly the same pronumerals.
$5a$ and $3a$ ✓
$2ab$ and $12ba$ ✓
$7ab$ and $2a$ ✗

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Chapter review

Multiple-choice questions

1 In the expression $3x + 2y + 4xy + 7yz$ the coefficient of y is:
A 3 **B** 2 **C** 4 **D** 7 **E** 16

2 If $t = 5$ and $u = 7$, then $2t + u$ is equal to:
A 17 **B** 32 **C** 24 **D** 257 **E** 70

3 If $x = 2$, then $3x^2$ is equal to:
A 32 **B** 34 **C** 12 **D** 25 **E** 36

4 Which of the following pairs does *not* consist of two like terms?
A $3x$ and $5x$ **B** $3y$ and $12y$ **C** $3ab$ and $2ab$
D $3cd$ and $5c$ **E** $3xy$ and yx

5 A fully simplified expression equivalent to $2a + 4 + 3b + 5a$ is:
A 4 **B** $5a + 5b + 4$ **C** $10ab + 4$
D $7a + 3b + 4$ **E** $11ab$

6 The simplified form of $4x \times 3yz$ is:
A $43xyz$ **B** $12xy$ **C** $12xyz$ **D** $12yz$ **E** $4x3yz$

7 The simplified form of $\frac{21ab}{3ac}$ is:
A $\frac{7b}{c}$ **B** $\frac{7ab}{ac}$ **C** $\frac{21b}{c}$ **D** 7 **E** $\frac{b}{7c}$

8 When brackets are expanded, $4(2x + 3y)$ becomes:
A $8x + 3y$ **B** $2x + 12y$ **C** $8x + 8y$
D $24x$ **E** $8x + 12y$

9 The fully simplified form of $2(a + 7b) - 4b$ is:
A $2a + 10b$ **B** $2a + 3b$ **C** $a + 3b$
D $2a + 14b - 4b$ **E** $2a + 18b$

10 A number is doubled and then 5 is added. The result is then tripled. If the number is represented by k, then an expression for this description is:
A $3(2k + 5)$ **B** $6(k + 5)$ **C** $2k + 5$
D $2k + 15$ **E** $30k$

Short-answer questions

1 **a** List the four individual terms in the expression $5a + 3b + 7c + 12$.
b What is the constant term in the expression above?

2 Write an expression for each of the following.
a 7 is added to u **b** k is tripled **c** 7 is added to half of r
d 10 is subtracted from h **e** the product of x and y **f** x is subtracted from 12

3 If $u = 12$, find the value of:
a $u + 3$ **b** $2u$ **c** $\frac{24}{u}$ **d** $3u - 4$

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Chapter review

4 If $p = 3$ and $q = -5$, find the value of:
a pq **b** $p + q$ **c** $2(q - p)$ **d** $4p + 3q$

5 If $t = 4$ and $u = 10$, find the value of:
a t^2 **b** $2u^2$ **c** $3 + \sqrt{t}$ **d** $\sqrt{10tu}$

6 For each of the following pairs of expressions, state whether they are equivalent (E) or not equivalent (N).
a $5x$ and $2x + 3x$ **b** $7a + 2b$ and $9ab$
c $3c - c$ and $2c$ **d** $3(x + 2y)$ and $3x + 2y$

7 Classify the following pairs as like terms (L) or not like terms (N).
a $2x$ and $5x$ **b** $7ab$ and $2a$ **c** $3p$ and p
d $9xy$ and $2yx$ **e** $4ab$ and $4aba$ **f** $8t$ and $2t$
g $3p$ and 3 **h** $12k$ and $120k$

8 Simplify the following by collecting like terms.
a $2x + 3 + 5x$ **b** $12p - 3p + 2p$ **c** $12b + 4a + 2b + 3a + 4$
d $12mn + 3m + 2n + 5nm$ **e** $1 + 2c + 4h - 3o + 5c$ **f** $7u + 3v + 2uv - 3u$

9 Simplify the following expressions involving products.
a $3a \times 4b$ **b** $2xy \times 3z$ **c** $12f \times g \times 3h$ **d** $8k \times 2 \times 4lm$

10 Simplify the following expressions involving quotients.
a $\frac{3u}{2u}$ **b** $\frac{12y}{20y}$ **c** $\frac{2ab}{6b}$ **d** $\frac{12xy}{9yz}$

11 Expand the following expressions using the distributive law.
a $3(x + 2)$ **b** $4(p - 3)$ **c** $7(2a + 3)$ **d** $12(2k + 3l)$

12 Give two examples of expressions that expand to give $12b + 18c$.

13 If a tin of paint weighs 9 kg, write an expression for the weight of t tins of paint.

14 If there are g girls and b boys in a room, write an expression for the total number of children in the room.

15 Write an expression for the total number of books that Analena owns if she has x fiction books and twice as many non-fiction books.

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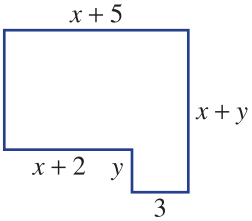
Extended-response questions

1 A taxi driver charges \$3.50 to pick up passengers and then \$2.10 per kilometre travelled.

- **a** State the total cost if the trip length is:
  - **i** 10 km **ii** 20 km **iii** 100 km
- **b** Write an expression for the total cost, in dollars, of travelling a distance of k kilometres.
- **c** Use your expression to find the total cost of travelling 40 km.
- **d** Prove that your expression is not equivalent to $2.1 + 3.5k$ by substituting a value for k.
- **e** Another taxi driver charges \$6 to pick up passengers and then \$1.20 per kilometre. Write an expression for the total cost of travelling k kilometres in this taxi.

2 An architect has designed a room, shown below, for which x and y are unknown. (All measurements are in metres.)



- **a** Find the perimeter of this room if $x = 3$ and $y = 2$.
- **b** It costs \$3 per metre to install skirting boards around the perimeter of the room. Find the total cost of installing skirting boards if the room's perimeter is $x = 3$ and $y = 2$.
- **c** Write an expression for the perimeter of the room and simplify it completely.
- **d** Write an expanded expression for the total cost, in dollars, of installing skirting boards along the room's perimeter.
- **e** Write an expression for the total area of the floor in this room.

Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

9 Equations

What you will learn

9A Introduction to equations
9B Solving equations by inspection
9C Equivalent equations
9D Solving equations systematically
9E Equations with fractions
9F Equations with brackets
9G Formulas and relationships EXTENSION
9H Using equations to solve problems EXTENSION

NSW syllabus

STRAND: NUMBER AND ALGEBRA

SUBSTRAND: EQUATIONS

Outcome

A student uses algebraic techniques to solve simple linear and quadratic equations.
(MA4–10NA)

Theme park equations

Equations are used widely in mathematics and in many other fields. Whenever two things are equal, or should be equal, there is the potential to use the study of equations to help deal with such a situation.

Knowledge of mathematics and physics is vitally important when designing theme park rides. Engineers use equations to 'build' model rides on a computer so that safety limits can be determined in a virtual reality in which no one gets injured.

Algebraic equations are solved to determine the dimensions and strengths of structures required to deal safely with the combined forces of weight, speed and varying movement. Passengers might scream with a mixture of terror and excitement but they must return unharmed to earth!

At Dreamworld on the Gold Coast, Queensland, 'The Claw' swings 32 people upwards at 75 km/h to a maximum height of 27.1 m (9 storeys), simultaneously spinning 360° at 5 r.p.m. (revolutions per minute). 'The Claw' is the most powerful pendulum ride on the planet. It is built to scare!

Pre-test

1 Fill in the missing number.

a $\square + 3 = 10$ **b** $41 - \square = 21$ **c** $\square \times 3 = 48$ **d** $100 \div \square = 20$

2 If $\square = 5$, state whether each of these equations is true or false.

a $\square - 2 = 5$ **b** $\square \times 3 = 15$ **c** $20 \div 4 = \square$ **d** $\square \times \square = 36$

3 If $x = 3$, find the value of:

a $x + 4$ **b** $8 - x$ **c** $5x$ **d** $3 + 7x$

4 If $n = 6$, state the value of:

a $n \div 2$ **b** $4n + 3$ **c** $8 - n$ **d** $12 \div n + 4$

5 The expression $n + 3$ can be described as 'the sum of n and 3'. Write expressions for:

a the sum of k and 5
b double p
c the product of 7 and y
d one-half of q

6 Simplify each of the following algebraic expressions.

a $3x + 2x$
b $7 \times 4b$
c $2a + 7b + 3a$
d $4 + 12a - 2a + 3$

7 State the missing values in the tables below.

a

n	1	3	c	d	12
$5 \times n$	a	b	20	35	e

b

n	2	4	c	12	e
$n - 2$	a	b	10	d	39

c

n	a	b	3	0	e
$2n + 1$	5	11	c	d	27

d

n	a	4	c	d	2
$6 - n$	3	b	5	6	e

8 For each of the following, state the opposite operation.

a $\times$ **b** $+$ **c** $\div$ **d** $-$

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9A Introduction to equations

An equation is a mathematical statement stating that two expressions have the same value. It consists of two expressions that are separated by an equals sign (=).

Sample equations include:

$$\begin{aligned} 3 + 3 &= 6 \\ 30 &= 2 \times 15 \\ 100 - 30 &= 60 + 10 \end{aligned}$$

which are all true equations.
An equation does not have to be true.
For instance:

$$\begin{aligned} 2 + 2 &= 17 \\ 5 &= 3 - 1 \text{ and} \\ 10 + 15 &= 12 + 3 \text{ are all false equations.} \end{aligned}$$

Let's start: Equations – True or false?

Rearrange the following five symbols to make as many different equations as possible.

$$5,\ 2,\ 3,\ +,\ =$$

- Which of them are true? Which are false?
- Is it always possible to rearrange a set of numbers and operations to make true equations?

Key ideas

- An **expression** is a collection of pronumerals, numbers and operators without an equals sign (e.g. $2x + 3$).
- An **equation** is a mathematical statement stating that two things are equal (e.g. $2x + 3 = 4y - 2$).
- Equations have a left-hand side (LHS), a right-hand side (RHS) and an equals sign in between.

$$\underbrace{2 + 3}_{\text{LHS}} \;=\; \underbrace{4y - 2}_{\text{RHS}}$$

equals sign

- The equals sign indicates that the LHS and RHS have the same numerical value.
- Equations are mathematical statements that can be true (e.g. $2 + 3 = 5$) or false (e.g. $5 + 7 = 21$).
- If a pronumeral is included in an equation, you need to know the value to substitute before deciding whether the equation is true.

  For example, $3x = 12$ would be true if 4 is substituted for x, but it would be false if 10 is substituted.

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Example 1 Identifying equations

Which of the following are equations?

a $3 + 5 = 8$ **b** $7 + 7 = 18$ **c** $2 + 12$ **d** $4 = 12 - x$ **e** $3 + u$

SOLUTION	EXPLANATION
a $3 + 5 = 8$ is an equation.	There are two expressions (i.e. $3 + 5$ and 8) separated by an equals sign.
b $7 + 7 = 18$ is an equation.	There are two expressions separated by an equals sign. Although this equation is false, it is still an equation.
c $2 + 12$ is not an equation.	This is just a single expression. There is no equals sign.
d $4 = 12 - x$ is an equation.	There are two expressions separated by an equals sign.
e $3 + u$ is not an equation.	There is no equals sign, so this is not an equation.

Example 2 Classifying equations

For each of the following equations, state whether it is true or false.

a $7 + 5 = 12$

b $5 + 3 = 2 \times 4$

c $12 \times (2 - 1) = 14 + 5$

d $3 + 9x = 60 + 6$, if $x = 7$

e $10 + b = 3b + 1$, if $b = 4$

f $3 + 2x = 21 - y$, if $x = 5$ and $y = 8$

SOLUTION	EXPLANATION
a true	The left-hand side (LHS) and right-hand side (RHS) are both equal to 12, so the equation is true.
b true	LHS $= 5 + 3 = 8$ and RHS $= 2 \times 4 = 8$, so both sides are equal.
c false	LHS $= 12$ and RHS $= 19$, so the equation is false.
d true	If x is 7, then: LHS $= 3 + 9 \times 7 = 66$ RHS $= 60 + 6 = 66$
e false	If b is 4, then: LHS $= 10 + 4 = 14$ RHS $= 3(4) + 1 = 13$
f true	If $x = 5$ and $y = 8$, then: LHS $= 3 + 2(5) = 13$ RHS $= 21 - 8 = 13$

Example 3 Writing equations from a description

Write equations for each of the following scenarios.

a The sum of x and 5 is 22.

b The number of cards in a deck is x. In 7 decks there are 91 cards.

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c Priya's age is currently j. In 5 years' time her age will equal 17.

d Corey earns \$$w$ per year. He spends $\frac{1}{12}$ on sport and $\frac{2}{13}$ on food. The total amount Corey spends on sport and food is \$15 000.

SOLUTION	EXPLANATION
a $x + 5 = 22$	The sum of x and 5 is written $x + 5$.
b $7x = 91$	$7x$ means $7 \times x$ and this number must equal the 91 cards.
c $j + 5 = 17$	In 5 years' time Priya's age will be 5 more than her current age, so $j + 5$ must be 17.
d $\frac{1}{12} \times w + \frac{2}{13} \times w = 15000$	$\frac{1}{12}$ of Corey's wage is $\frac{1}{12} \times w$ and $\frac{2}{13}$ of his wage is $\frac{2}{13} \times w$.

Exercise 9A

UNDERSTANDING AND FLUENCY	1–4, 5(½), 6–8	4, 5(½), 6, 7, 8–9(½)	5(½), 6, 7, 8–9(½)

Example 1

1 Classify each of the following as an equation (E) or not an equation (N).

a $7 + x = 9$ **b** $2 + 2$ **c** $2 \times 5 = t$
d $10 = 5 + x$ **e** $2 = 2$ **f** $7 \times u$
g $10 \div 4 = 3p$ **h** $3 = e + 2$ **i** $x + 5$

Example 2a–c

2 Classify each of these equations as true or false.

a $2 + 3 = 5$ **b** $3 + 2 = 6$ **c** $5 - 1 = 6$

3 Consider the equation $4 + 3x = 2x + 9$.

a If $x = 5$, state the value of the left-hand side (LHS).
b If $x = 5$, state the value of the right-hand side (RHS).
c Is the equation $4 + 3x = 2x + 9$ true or false when $x = 5$?

Example 2d,e

4 If $x = 2$, is $10 + x = 12$ true or false?

5 For each of the following equations, state whether it is true or false.

a $10 \times 2 = 20$ **b** $12 \times 11 = 144$ **c** $3 \times 2 = 5 + 1$
d $100 - 90 = 2 \times 5$ **e** $30 \times 2 = 32$ **f** $12 - 4 = 4$
g $2(3 - 1) = 4$ **h** $5 - (2 + 1) = 7 - 4$ **i** $3 = 3$
j $2 = 17 - 14 - 1$ **k** $10 + 2 = 12 - 4$ **l** $1 \times 2 \times 3 = 1 + 2 + 3$
m $2 \times 3 \times 4 = 2 + 3 + 4$ **n** $100 - 5 \times 5 = 20 \times 5$ **o** $3 - 1 = 2 + 5 - 5$

6 If $x = 3$, state whether each of these equations is true or false.

a $5 + x = 7$ **b** $x + 1 = 4$ **c** $13 - x = 10 + x$ **d** $6 = 2x$

7 If $b = 4$, state whether each of the following equations is true or false.

a $5b + 2 = 22$ **b** $10 \times (b - 3) = b + b + 2$ **c** $12 - 3b = 5 - b$ **d** $b \times (b + 1) = 20$

Example 2f

8 If $a = 10$ and $b = 7$, state whether each of these equations is true or false.

a $a + b = 17$ **b** $a \times b = 3$ **c** $a \times (a - b) = 30$
d $b \times b = 59 - a$ **e** $3a = 5b - 5$ **f** $b \times (a - b) = 20$
g $21 - a = b$ **h** $10 - a = 7 - b$ **i** $1 + a - b = 2b - a$

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Example 3a

9 Write equations for each of the following.

a The sum of 3 and x is equal to 10.
b When k is multiplied by 5, the result is 1005.
c The sum of a and b is 22.
d When d is doubled, the result is 78.
e The product of 8 and x is 56.
f When p is tripled, the result is 21.
g One-quarter of t is 12.
h The sum of q and p is equal to the product of q and p.

PROBLEM-SOLVING AND REASONING	10, 11, 13	11, 12, 14	12–14

Example 3b–d

10 Write true equations for each of these problems. You do not need to solve them.

a Chairs cost \$$c$ at a store. The cost of 6 chairs is \$546.
b Patrick works for x hours each day. In a 5-day working week, he works $37\frac{1}{2}$ hours in total.
c Pens cost \$$a$ each and pencils cost \$$b$. Twelve pens and three pencils cost \$28 in total.
d Amy is f years old. In 10 years' time her age will be 27.
e Andrew's age is j and Hailey's age is m. In 10 years' time their combined age will be 80.

11 Find a value of m that would make this equation true: $10 = m + 7$.

12 Find two possible values of k that would make this equation true: $k \times (8 - k) = 12$.

13 If the equation $x + y = 6$ is true, and x and y are both whole numbers between 1 and 5, what values could they have?

14 Equations involving pronumerals can be split into three groups:
A: Always true, no matter what values are substituted.
N: Never true, no matter what values are substituted.
S: Sometimes true but sometimes false, depending on the values substituted.
Categorise each of these equations as either A, N or S.

a $x + 5 = 11$
b $12 - x = x$
c $a = a$
d $5 + b = b + 5$
e $a = a + 7$
f $5 + b = b - 5$
g $0 \times b = 0$
h $a \times a = 100$
i $2x + x = 3x$
j $2x + x = 4x$
k $2x + x = 3x + 1$
l $a \times a + 100 = 0$

ENRICHMENT	—	—	15, 16

Equation permutations

15 For each of the following, rearrange the symbols to make a true equation.

a 6, 2, 3, ×, =
b 1, 4, 5, −, =
c 2, 2, 7, 10, −, ÷, =
d 2, 4, 5, 10, −, ÷, =

16 a How many different equations can be produced using the symbols 2, 3, 5, +, = ?
b How many of these equations are true?
c Is it possible to change just one of the numbers above and still produce true equations by rearranging the symbols?
d Is it possible to change just the operation above (i.e. +) and still produce true equations?

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9B Solving equations by inspection

Interactive Widgets HOTsheets Walkthrough

Solving an equation is the process of finding the values that pronumerals must take in order to make the equation true. Pronumerals are also called 'unknowns' when solving equations. For simple equations, it is possible to find a solution by using known number facts or by trying a few values for the pronumeral. This method does not guarantee that we have found all the solutions (if there is more than one) and it will not help if there are no solutions, but it can be a useful and quick method for simple equations.

$x+5=12$ → $x=7$	$\frac{2x}{3}-7=11$, $x=???$
Easily solved!	You need a plan B!
Section 9B	Section 9C to 9G

Let's start: Finding the missing value

- Find the missing values to make the following equations true.
  $10 \times \square - 17 = 13$
  $27 = 15 + 3 \times \square$
  $2 \times \square + 4 = 17$
- Can you always find a value to put in the place of $\square$ in any equation?

Key ideas

- In the equation $x + 5 = 12$, the pronumeral x is called an **unknown**.
- In the equation $x + 5 = 12$, there is only one value of x that makes the equation true.
- Some equations can be solved without using a formal method.
- It is quite easy to solve the equation $x + 5 = 12$ if you know that $7 + 5 = 12$. This is called solving by inspection.
- The **solution** is $x = 7$ because $7 + 5 = 12$ is a true statement.

Example 4 Finding the missing number

For each of these equations, find the value of the missing number that would make it true.

a $\square \times 7 = 35$ **b** $20 - \square = 14$

SOLUTION	EXPLANATION
a 5	$5 \times 7 = 35$ is a true equation.
b 6	$20 - 6 = 14$ is a true equation.

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Example 5 Solving equations

Solve each of the following equations by inspection.

a $c + 12 = 30$ **b** $5b = 20$ **c** $2x + 13 = 21$ **d** $x^2 = 9$

SOLUTION	EXPLANATION
a $c + 12 = 30$ $c = 18$	The unknown variable here is c. $18 + 12 = 30$ is a true equation.
b $5b = 20$ $b = 4$	The unknown variable here is b. Recall that $5b$ means $5 \times b$, so if $b = 4$, $5b = 5 \times 4 = 20$.
c $2x + 13 = 21$ $x = 4$	The unknown variable here is x. Trying a few values: $x = 10$ makes LHS $= 20 + 13 = 33$, which is too large. $x = 3$ makes LHS $= 6 + 13 = 19$, which is too small. $x = 4$ makes LHS $= 21$.
d $x^2 = 9$ $x = -3, x = 3$ $x = \pm 3$	$(-3)^2 = 9$ is a true equation and $(3)^2 = 9$ is also a true equation. This equation has two solutions.

Exercise 9B

UNDERSTANDING AND FLUENCY	1–4, 5–6(½)	4, 5–7(½)	5–7(½)

1 If the missing number is 5, classify each of the following equations as true or false.

a $\square + 3 = 8$
b $10 \times \square + 2 = 46$
c $10 - \square = 5$
d $12 = 6 + \square \times 2$

2 For the equation $\square + 7 = 13$:

a Find the value of the LHS (left-hand side) if $\square = 5$.
b Find the value of the LHS if $\square = 10$.
c Find the value of the LHS if $\square = 6$.
d What value of $\square$ would make the LHS equal to 13?

Example 4

3 Find the value of the missing numbers.

a $4 + \square = 7$
b $2 \times \square = 12$
c $13 = \square + 3$
d $10 = 6 + \square$
e $42 = \square \times 7$
f $100 - \square = 30$
g $15 + 6 = \square + 1$
h $\square + 11 = 49 - \square$

4 Name the unknown pronumeral in each of the following equations.

a $4 + x = 12$
b $50 - c = 3$
c $4b + 2 = 35$
d $5 - 10d = 2$

Example 5a, b

5 Solve the following equations by inspection.

a $8 \times y = 64$
b $6 \div l = 3$
c $l \times 3 = 18$
d $4 - d = 2$
e $l + 2 = 14$
f $a - 2 = 4$
g $s + 7 = 19$
h $x \div 8 = 1$
i $12 = e + 4$
j $r \div 10 = 1$
k $13 = 5 + s$
l $0 = 3 - z$

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Example 5c

6 Solve the following equations by inspection.

a $2p - 1 = 5$ **b** $3p + 2 = 14$ **c** $4q - 4 = 8$ **d** $4v + 4 = 24$

e $2b - 1 = 1$ **f** $5u + 1 = 21$ **g** $5g + 5 = 20$ **h** $4(e - 2) = 4$

i $45 = 5(d + 5)$ **j** $3d - 5 = 13$ **k** $8 = 3m - 4$ **l** $8 = 3o - 1$

Example 5d

7 Solve the following equations by inspection. (All solutions are whole numbers between 1 and 10.)

a $4 \times (x + 1) - 5 = 11$ **b** $7 + x = 2 \times x$ **c** $(3x + 1) \div 2 = 8$

d $10 - x = x + 2$ **e** $2 \times (x + 3) + 4 = 12$ **f** $15 - 2x = x$

g $x^2 = 4$ **h** $x^2 = 100$ **i** $36 = x^2$

PROBLEM-SOLVING AND REASONING	8, 9, 13	9–11, 13	10–14

8 Find the value of the number in each of these examples.

a A number is doubled and the result is 22. **b** 3 less than a number is 9.

c Half of a number is 8. **d** 7 more than a number is 40.

e A number is divided by 10, giving a result of 3

f 10 is divided by a number, giving a result of 5.

9 Justine is paid \$10 an hour for x hours. During a particular week, she earns \$180.

a Write an equation involving x to describe this situation.

b Solve the equation by inspection to find the value of x.

10 Karim's weight is w kg and his brother is twice as heavy, weighing 70 kg.

a Write an equation involving w to describe this situation.

b Solve the equation by inspection to find the value of w.

11 Taylah buys x kg of apples at \$4.50 per kg. She spends a total of \$13.50.

a Write an equation involving x to describe this situation.

b Solve the equation by inspection to find x.

12 Yanni's current age is y years old. In 12 years' time he will be three times as old.

a Write an equation involving y to describe this situation.

b Solve the equation by inspection to find y.

13 **a** Solve the equation $x + (x + 1) = 19$ by inspection.

b The expression $x + (x + 1)$ can be simplified to $2x + 1$. Use this observation to solve $x + (x + 1) = 181$ by inspection.

14 There are three consecutive whole numbers that add to 45.

a Solve the equation $x + (x + 1) + (x + 2) = 45$ by inspection to find the three numbers.

b An equation of the form $x + (x + 1) + (x + 2) = ?$ has a whole number solution only if the right-hand side is a multiple of 3. Explain why this is the case. (Hint: Simplify the LHS.)

ENRICHMENT	—	—	15

Multiple variables

15 When multiple variables are involved, inspection can still be used to find a solution. For each of the following equations find, by inspection, one pair of values for x and y that make them true.

a $x + y = 8$ **b** $x - y = 2$ **c** $3 = 2x + y$

d $x \times y = 6$ **e** $12 = 2 + x + y$ **f** $x + y = x \times y$

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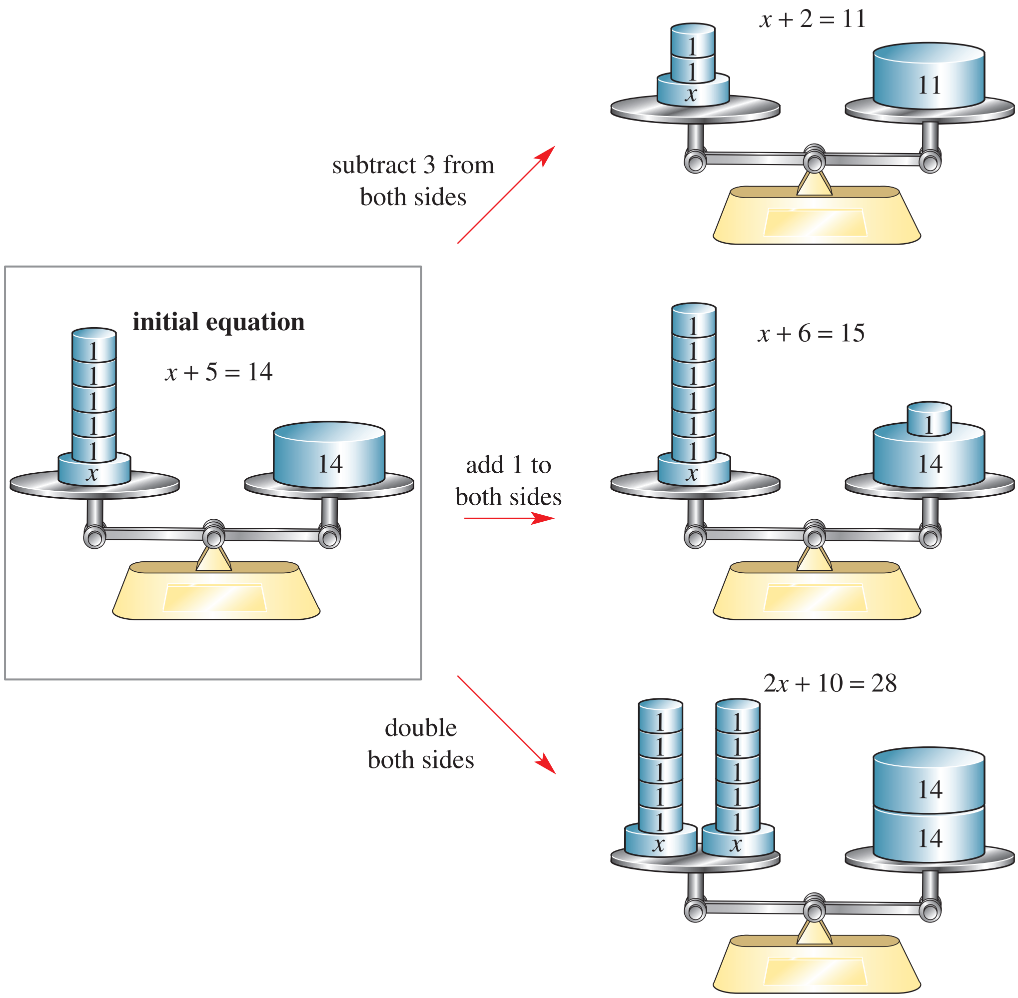
9C Equivalent equations

Interactive Widgets HOTsheets Walkthrough

Sometimes, two equations essentially express the same thing. For example, the equations $x + 5 = 14$, $x + 6 = 15$ and $x + 7 = 16$ are all made true by the same value of x. Each time, we have added one to both sides of the equation.

We can pretend that true equations are about different objects that have the same weight. For instance, to say that $3 + 5 = 8$ means that a 3 kg block added to a 5 kg block weighs the same as an 8 kg block.



A true equation stays true if we 'do the same thing to both sides', such as adding a number or multiplying by a number. The exception to this rule is that multiplying both sides of any equation by zero will always make the equation true, and dividing both sides of any equation by zero is not permitted because nothing can be divided by zero. If we do the same thing to both sides, we will have an equivalent equation.

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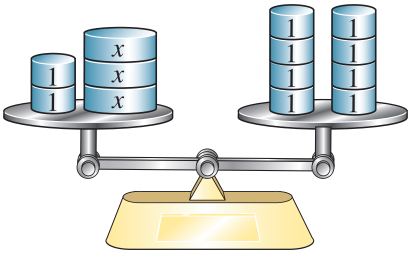
Let's start: Equations as scales

The scales in the diagram show $2 + 3x = 8$.

- What would the scales look like if two '1 kg' blocks were removed from both sides?
- What would the scales look like if the two '1 kg' blocks were removed only from the left-hand side? (Try to show whether they would be level.)
- Use scales to illustrate why $4x + 3 = 4$ and $4x = 1$ are equivalent equations.



Key ideas

- Two equations are **equivalent** if you can get from one to the other by repeatedly:
  - adding the same number to both sides
  - subtracting the same number from both sides
  - multiplying both sides by the same number (other than zero)
  - dividing both sides by the same number (other than zero)
  - swapping the left-hand side with the right-hand side of the equation

Example 6 Applying an operation

For each equation, find the result of applying the given operation to both sides and then simplify.

a $2 + x = 5$ [add 4 to both sides]
b $7x = 10$ [multiply both sides by 2]
c $30 = 20b$ [divide both sides by 10]
d $7q - 4 = 10$ [add 4 to both sides]

SOLUTION	EXPLANATION
a $2 + x = 5$	The equation is written out, and 4 is added to both sides.
$2 + x + 4 = 5 + 4$	
$x + 6 = 9$	Simplify the expressions on each side.
b $7x = 10$	The equation is written out, and both sides are multiplied by 2.
$7x \times 2 = 10 \times 2$	
$14x = 20$	Simplify the expressions on each side.
c $30 = 20b$	The equation is written out, and both sides are divided by 10.
$\frac{30}{10} = \frac{20b}{10}$	
$3 = 2b$	Simplify the expressions on each side.
d $7q - 4 = 10$	The equation is written out, and 4 is added to both sides.
$7q - 4 + 4 = 10 + 4$	
$7q = 14$	Simplify the expressions on each side.

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Example 7 Showing that equations are equivalent

Show that these pairs of equations are equivalent by stating the operation used.

a $2x + 10 = 15$ and $2x = 5$

b $5 = 7 - x$ and $10 = 2(7 - x)$

c $10(b + 3) = 20$ and $b + 3 = 2$

SOLUTION	EXPLANATION
a Both sides have had 10 subtracted. $2x + 10 = 15$ -10 ↓ ↓ -10 $2x = 5$	$2x + 10 - 10$ simplifies to $2x$, so we get the second equation by subtracting 10.
b Both sides have been multiplied by 2 $5 = 7 - x$ $\times 2$ ↓ ↓ $\times 2$ $10 = 2(7 - x)$	$2(7 - x)$ represents the RHS; i.e. $7 - x$, being multiplied by 2.
c Both sides have been divided by 10. $10(b + 3) = 20$ $\div 10$ ↓ ↓ $\div 10$ $b + 3 = 2$	Remember $10(b + 3)$ means $10 \times (b + 3)$. If we have $10(b + 3)$, we get $b + 3$ when dividing by 10.

Exercise 9C

UNDERSTANDING AND FLUENCY	1–4	2, 3–5(½)	3–5(½)

1 Match up each of these equations (**a** to **e**) with its equivalent equation (i.e. **A** to **E**), where 3 has been added.

a $10 + x = 14$
b $x + 1 = 13$
c $12 = x + 5$
d $x + 10 = 11x$
e $12x = 120$

A $12x + 3 = 123$
B $x + 13 = 11x + 3$
C $13 + x = 17$
D $x + 4 = 16$
E $15 = x + 8$

Example 6a

2 Write an equation that results from adding 10 to both sides of each of these equations.

a $10d + 5 = 20$ **b** $7e = 31$ **c** $2a = 12$ **d** $x = 12$

Example 6b–d

3 For each equation, show the result of applying the listed operations to both sides. (Note: [+ 1] means 'add 1 to both sides'.)

a $5 + x = 10$ [+ 1]
b $3x = 7$ [× 2]
c $12 = 8q$ [÷ 4]
d $9 + a = 13$ [− 3]
e $7 + b = 10$ [+ 5]
f $5 = 3b + 7$ [− 5]
g $2 = 5 + a$ [+ 2]
h $12x - 3 = 3$ [+ 5]
i $7p - 2 = 10$ [+ 2]

Example 7

4 Show that these pairs of equations are equivalent by stating the operation used.

a $4x + 2 = 10$ and $4x = 8$
b $7 + 3b = 12$ and $9 + 3b = 14$
c $20a = 10$ and $2a = 1$
d $4 = 12 - x$ and $8 = 2(12 - x)$
e $18 = 3x$ and $6 = x$
f $12 + x = 3$ and $15 + x = 6$
g $4(10 + b) = 80$ and $10 + b = 20$
h $12x = 5$ and $12x + 4 = 9$

5 For each of the following equations, show the equivalent equation that is the result of adding 4 to both sides and then multiplying both sides by 3.

a $x = 5$
b $2 = a + 1$
c $d - 4 = 2$
d $7 + a = 8$
e $3y - 2 = 7$
f $2x = 6$

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PROBLEM-SOLVING AND REASONING	6, 9	6, 7, 9, 10	7, 8, 10, 11

6 Match up each of these equations (**a** to **e**) with its equivalent equation (i.e. **A** to **E**), stating the operation used.

a $m + 10 = 12$ **A** $7 - m = 6$
b $3 - m = 2$ **B** $5m = 18$
c $12m = 30$ **C** $6m = 10$
d $5m + 2 = 20$ **D** $6m = 15$
e $3m = 5$ **E** $m + 12 = 14$

7 For each of the following pairs of equations, show that they are equivalent by listing the two steps required to transform the first equation to the second.

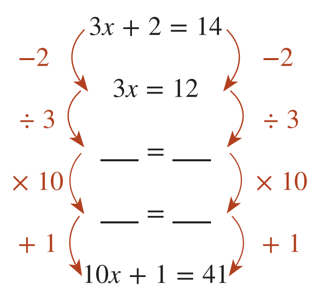
a $x = 5$ and $3x + 2 = 17$ **b** $m = 2$ and $10m - 3 = 17$
c $5(2 + x) = 15$ and $x = 1$ **d** $10 = 3x + 10$ and $0 = x$

8 For each of the following equations, write an equivalent equation that you can get in one operation. Your equation should be simpler (i.e. smaller) than the original.

a $2q + 7 = 9$ **b** $10x + 3 = 10$ **c** $2(3 + x) = 40$ **d** $x \div 12 = 5$

9 Sometimes two equations that look quite different can be equivalent.

a Show that $3x + 2 = 14$ and $10x + 1 = 41$ are equivalent by copying and completing the equation shown.
b Show that $5x - 3 = 32$ and $x + 2 = 9$ are equivalent. (Hint: Try to go via the equation $x = 7$.)
c Show that $(x \div 2) + 4 = 9$ and $(x + 8) \div 2 = 9$ are equivalent.



10 As stated in the rules for equivalence, which are listed in Key ideas, multiplying both sides by zero is not permitted.

a Write the result of multiplying both sides of the following equations by zero.
i $3 + x = 5$ **ii** $2 + 2 = 4$ **iii** $2 + 2 = 5$
b Explain in a sentence why multiplying by zero does not give a useful equivalent equation.

11 Substituting pronumerals can be thought of as finding equivalent equations. Show how you can start with the equation $x = 3$ and find an equivalent equation with:

a $7x + 2$ on the LHS **b** $8 + 2x$ on the LHS

ENRICHMENT	—	—	12

Equivalence relations

12 Classify each of the following statements as true or false, justifying your answer.

a Every equation is equivalent to itself.
b If equation 1 and equation 2 are equivalent, then equation 2 and equation 1 are equivalent.
c If equation 1 and equation 2 are equivalent, and equation 2 and equation 3 are equivalent, then equation 1 and equation 3 are equivalent.
d If equation 1 and equation 2 are *not* equivalent, and equation 2 and equation 3 are *not* equivalent, then equation 1 is *not* equivalent to equation 3.

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9D Solving equations systematically

Interactive Widgets HOTsheets Walkthrough

A soccer player preparing for a game will put on shin pads, then socks and, finally, soccer boots. When the game is over, these items are removed in reverse order: first the boots, then the socks and, finally, the shin pads. Nobody takes their socks off before their shoes. A similar reversal of procedures occurs with equivalent equations.

Here are three equivalent equations.

$$\begin{aligned} x &= 3 \\ 2x &= 6 \quad &&(\times 2 \text{ both sides}) \\ 2x + 4 &= 10 \quad &&(+4 \text{ both sides}) \end{aligned}$$

We can undo the operations around x by doing the opposite operation in the reverse order.

$$\begin{aligned} 2x + 4 &= 10 \\ 2x &= 6 \quad &&(-4 \text{ both sides}) \\ x &= 3 \quad &&(\div 2 \text{ both sides}) \end{aligned}$$

Because these equations are equivalent, this means that the solution to $2x + 4 = 10$ is $x = 3$. An advantage with this method is that solving equations by inspection can be very difficult if the solution is not just a small whole number.

Let's start: Attempting solutions

Georgia, Khartik and Lucas try to solve the equation $4x + 8 = 40$. They present their attempted solutions below.

Georgia

$$\begin{aligned} 4x + 8 &= 40 \\ x + 8 &= 10 \quad &&(\div 4 \text{ both sides}) \\ x &= 2 \quad &&(-8 \text{ both sides}) \end{aligned}$$

Khartik

$$\begin{aligned} 4x + 8 &= 40 \\ 4x &= 48 \quad &&(-8 \text{ left}, +8 \text{ right}) \\ x &= 12 \quad &&(\div 4 \text{ both sides}) \end{aligned}$$

Lucas

$$\begin{aligned} 4x + 8 &= 40 \\ 4x &= 32 \quad &&(-8 \text{ both sides}) \\ x &= 8 \quad &&(\div 4 \text{ both sides}) \end{aligned}$$

- Which of the students has the correct solution to the equation? Justify your answer by substituting each student's final answer.
- For each of the two students with the incorrect answer, explain the mistake they have made in their attempt to have equivalent equations.
- What operations would you do to both sides if the original equation was $7x - 10 = 11$?

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Key ideas

- Sometimes it is very difficult to solve an equation by inspection, so a systematic approach is required.
- To solve an equation, find a simpler equation that is equivalent. Repeat this until the solution is found.
- A simpler equation can be found by applying the opposite operations in reverse order.
  e.g. For $5x + 2 = 17$, we have:

$$5x + 2 = 17$$
-2 (both sides)
$$5x = 15$$
$\div 5$ (both sides)
$$x = 3$$

- A solution can be checked by substituting the value to see if the equation is true.

$$\begin{aligned} \text{LHS} &= 5x + 2 = 17 \\ &= 5 \times 3 + 2 \\ &= 17 \end{aligned} \qquad \text{RHS} = 17$$

Example 8 Solving one-step equations

Solve each of the following equations algebraically.

a $5x = 30$ **b** $17 = y - 21$ **c** $10 = \frac{q}{3}$

SOLUTION / EXPLANATION

a

$$5x = 30$$
$\div 5$ (both sides)
$$x = 6$$

So the solution is $x = 6$.

The opposite of $\times 5$ is $\div 5$.
By dividing both sides by 5, we get an equivalent equation. Recall that $5x \div 5$ simplifies to x.

b

$$17 = y - 21$$
$+21$ (both sides)
$$38 = y$$

So the solution is $y = 38$.

The opposite of -21 is $+21$.

Write the pronumeral on the LHS.

c

$$10 = \frac{q}{3}$$
$\times 3$ (both sides)
$$30 = q$$

So the solution is $q = 30$.

Multiplying both sides by 3 gives an equivalent equation that is simpler. Note that $\frac{q}{3} \times 3 = q$.

Write the pronumeral on the LHS.

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Example 9 Solving two-step equations

Solve each of the following equations algebraically and check the solution.

a $7 + 4a = 23$ **b** $\frac{d}{3} - 2 = 4$ **c** $12 = 2(e + 1)$

SOLUTION

a

$$7 + 4a = 23$$
-7 (both sides)
$$4a = 16$$
$\div 4$ (both sides)
$$a = 4$$

Check:

$$\begin{aligned} \text{LHS} &= 7 + 4a \\ &= 7 + 4 \times 4 \\ &= 7 + 16 \\ &= 23 \end{aligned} \qquad \text{RHS} = 23$$

b

$$\frac{d}{3} - 2 = 4$$
$+2$ (both sides)
$$\frac{d}{3} = 6$$
$\times 3$ (both sides)
$$d = 18$$

Check:

$$\begin{aligned} \text{LHS} &= \frac{d}{3} - 2 \\ &= \frac{18}{3} - 2 \\ &= 6 - 2 \\ &= 4 \end{aligned} \qquad \text{RHS} = 4$$

c

$$12 = 2(e + 1)$$
$\div 2$ (both sides)
$$6 = e + 1$$
-1 (both sides)
$$5 = e$$

So the solution is $e = 5$.

Check:

$$\text{LHS} = 12 \qquad \begin{aligned} \text{RHS} &= 2(e + 1) \\ &= 2(5 + 1) \\ &= 2 \times 6 \\ &= 12 \end{aligned}$$

EXPLANATION

a At each step, try to make the equation simpler by applying an operation to both sides.

Choose the opposite operations based on $7 + 4a$:

$a \xrightarrow{\times 4} 4a \xrightarrow{+7} 7 + 4a$

Opposite operations: -7, then $\div 4$.

Check that our equation is true by substituting $a = 4$ back into the LHS and RHS. Both sides are equal, so $a = 4$ is a solution.

b At each step, try to make the equation simpler by applying an operation to both sides.

The opposite of -2 is $+2$ and the opposite of $\div 3$ is $\times 3$.

Check that our equation is true by substituting $d = 18$ back into the LHS and RHS. Both sides are equal, so $d = 18$ is a solution.

c At each step, try to make the equation simpler by applying an operation to both sides.
The opposite of $\times 2$ is $\div 2$ and the opposite of $+1$ is -1.
Write the solution on the LHS.

Check that our equation is true by substituting $e = 5$ back into the equation.

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Exercise 9D

UNDERSTANDING AND FLUENCY	1–4, 5(½), 6, 7, 8(½)	4, 5(½), 6, 7, 8–9(½)	5(½), 6, 7, 8–10(½)

1 State whether each of the following equations is true or false.

a $x + 4 = 7$, if $x = 3$
b $b - 2 = 7$, if $b = 5$
c $7(d - 6) = d$, if $d = 7$
d $g + 5 = 3g$, if $g = 2$

2 Consider the equation $7x = 42$.

a Copy and complete the following.
b What is the solution to the equation $7x = 42$?

$7x = 42$
$\div 7$ $\quad$ $\div 7$
$x =$ ___

3 The equations $g = 2$ and $12(g + 3) = 60$ are equivalent. What is the solution to the equation $12(g + 3) = 60$?

4 Copy and complete the following, showing which operation was used. Remember that the same operation must be used for both sides.

a $5 + a = 30$ → □ □ → $a = 25$

b $10b = 72$ → □ □ → $b = 7.2$

c $12 = \frac{c}{4}$ → □ □ → $48 = c$

d $8 = c - 12$ → □ □ → $20 - c$

Example 8

5 Solve the following equations algebraically.

a $6m = 54$
b $g - 9 = 2$
c $s \times 9 = 81$
d $i - 9 = 1$
e $7 + t = 9$
f $8 + q = 11$
g $4y = 48$
h $7 + s = 19$
i $24 = j \times 6$
j $12 = l + 8$
k $1 = v \div 2$
l $19 = 7 + y$
m $k \div 5 = 1$
n $2 = y - 7$
o $8z = 56$
p $13 = 3 + t$
q $b \times 10 = 120$
r $p - 2 = 9$
s $5 + a = 13$
t $n - 2 = 1$

6 Copy and complete the following to solve the given equations algebraically.

a $7a + 3 = 38$ → □ □ → $7a = 35$ → □ □ → ___ = ___

b $4b - 10 = 14$ → $+10$ $+10$ → ___ = ___ → □ □ → ___ = ___

c $2(q + 6) = 20$ → $\div 2$ $\div 2$ → $q + 6 =$ ___ → □ □ → ___ = ___

d $5 = \frac{x}{10} + 3$ → -3 -3 → ___ $= \frac{x}{10}$ → □ □ → ___ = ___

7 For each of these equations, state the first operation you would apply to both sides to solve it.

a $2x + 3 = 9$
b $4x - 7 = 33$
c $5(a + 3) = 50$
d $22 = 2(b - 17)$

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Example 9

8 For each of the following equations:

- **i** Solve the equation algebraically, showing your steps.
- **ii** Check your solution by substituting the value into the LHS and RHS.

a $6f - 2 = 64$ **b** $\frac{k}{4} + 9 = 10$ **c** $5x - 4 = 41$

d $3(a - 8) = 3$ **e** $5k - 9 = 31$ **f** $\frac{a}{3} + 6 = 8$

g $2n - 8 = 14$ **h** $\frac{n}{4} + 6 = 8$ **i** $1 = 2g - 7$

j $30 = 3q - 3$ **k** $3z - 4 = 26$ **l** $17 = 9 + 8p$

m $10d + 7 = 47$ **n** $38 = 6t - 10$ **o** $9u + 2 = 47$

p $7 = 10c - 3$ **q** $10 + 8q = 98$ **r** $80 = 4(y + 8)$

s $4(q + 8) = 40$ **t** $7 + 6u = 67$

9 Solve the following equations, giving your solutions as fractions.

a $4x + 5 = 8$ **b** $3 + 5k = 27$ **c** $22 = (3w + 7) \times 2$

d $10 = 3 \times (2 + x)$ **e** $3 = (8x + 1) \div 2$ **f** $3(x + 2) = 7$

10 Solve the following equations algebraically. (Note: The solutions for these equations are negative numbers.)

a $4r + 30 = 2$ **b** $2x + 12 = 6$ **c** $10 + \frac{t}{2} = 2$

d $\frac{y}{4} + 10 = 4$ **e** $-3x = 15$ **f** $4 = 2k + 22$

g $2x = -12$ **h** $5x + 20 = 0$ **i** $0 = 2x + 3$

PROBLEM-SOLVING AND REASONING	11, 12, 16	12–14, 16, 17	13–15, 17, 18

11 For each of the following, write an equation and solve it algebraically.

- **a** The sum of x and 5 is 12.
- **b** The product of 2 and y is 10.
- **c** When b is doubled and then 6 is added, the result is 44.
- **d** 7 is subtracted from k. This result is tripled, giving 18.
- **e** 3 is added to one-quarter of b, giving a result of 6.
- **f** 10 is subtracted from half of k, giving a result of 1.

12 Danny gets paid \$12 per hour, plus a bonus of \$50 for each week. In one week he earned \$410.

- **a** Write an equation to describe this, using n for the number of hours worked.
- **b** Solve the equation algebraically and state the number of hours worked.

13 Jenny buys 12 pencils and 5 pens for the new school year. The pencils cost \$1.00 each.

- **a** If pens cost \$$x$ each, write an expression for the total cost, in dollars.
- **b** The total cost was \$14.50. Write an equation to describe this.
- **c** Solve the equation algebraically, to find the total cost of each pen.
- **d** Check your solution by substituting your value of x into $12 + 5x$.

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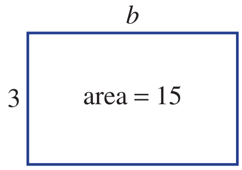
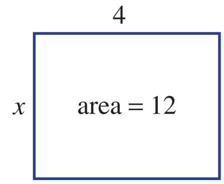
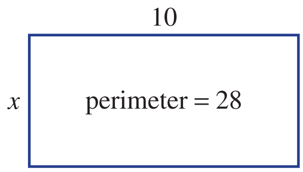
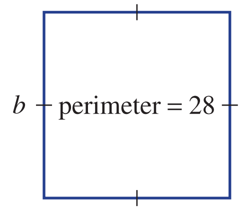
14 Write equations and solve them algebraically to find the unknown value in each of the following diagrams.

a


b


c


d


15 Solve the following equations algebraically.

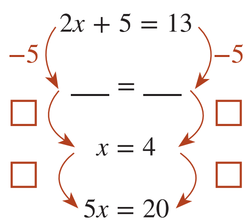
a $7(3 + 5x) - 21 = 210$

b $(100x + 13) \div 3 = 271$

c $3(12 + 2x) - 4 = 62$

16 Write five different equations that give a solution of $x = 2$.

17 a Show that $2x + 5 = 13$ and $5x = 20$ are equivalent by filling in the missing steps.

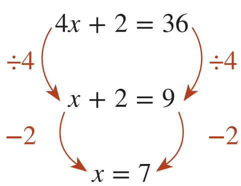


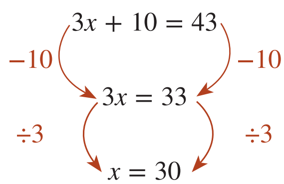
b Show that $10 + 2x = 20$ and $2(x - 3) = 4$ are equivalent.

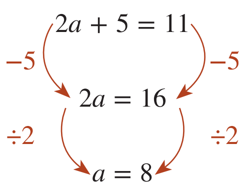
c If two equations have exactly the same solutions, does this guarantee they are equivalent? Justify your answer.

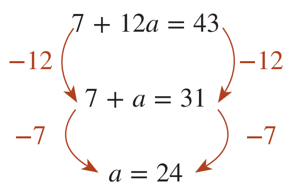
d If two equations have different solutions, does this guarantee they are not equivalent? Justify your answer.

18 Nicola has attempted to solve four equations. Describe the error she has made in each case.

a


b


c


d


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ENRICHMENT	—	—	19

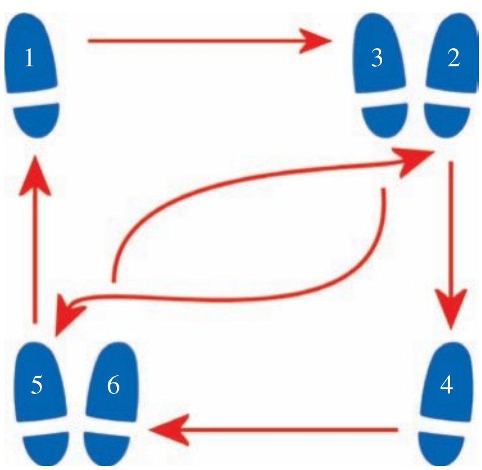
Equations with pronumerals on both sides

19 If an equation has a pronumeral on both sides, you can subtract it from one side and then use the same method as before. For example:

$$5x + 4 = 3x + 10$$
$$-3x \qquad -3x$$
$$2x + 4 = 10$$
$$-4 \qquad -4$$
$$2x = 6$$
$$\div 2 \qquad \div 2$$
$$x = 3$$

Solve the following equations using this method.

a $5x + 2 = 3x + 10$
b $8x - 1 = 4x + 3$
c $5 + 12l = 20 + 7l$
d $2 + 5t = 4t + 3$
e $12s + 4 = 9 + 11s$
f $9b - 10 = 8b + 9$
g $5j + 4 = 10 + 2j$
h $3 + 5d = 6 + 2d$



Start

Just like dance steps, a strict order must be followed when solving equations algebraically.

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9E Equations with fractions

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Solving equations that involve fractions is straightforward once we recall that, in algebra, $\frac{a}{b}$ means $a \div b$. This means that if we have a fraction with b on the denominator, we can multiply both sides by b to get a simpler, equivalent equation.

Let's start: Fractional differences

Consider these three equations.

a $\frac{2x + 3}{5} = 7$ **b** $\frac{2x}{5} + 3 = 7$ **c** $2\left(\frac{x}{5}\right) + 3 = 7$

- Solve each of them (by inspection or algebraically).
- Compare your solutions with those of your classmates.
- Why do two of the equations have the same solution?

Key ideas

- Recall that $\frac{a}{b}$ means $a \div b$, so $\frac{x}{5}$ means $x \div 5$ and $\frac{x}{2}$ is 'half of x'.
- To solve an equation that has a fraction on one side, multiply *both* sides by the denominator.

$$\frac{x}{5} = 4$$
×5 ×5
$$x = 20$$

- If neither side of an equation is a fraction, do not multiply by the denominator.

$$\frac{x}{3} + 5 = 8$$
×3 ×3
. . .
✘ Do not do this

$$\frac{x}{3} + 5 = 8$$
−5 −5
$$\frac{x}{3} = 3$$
×3 ×3
$$x = 9$$
✓ Do this

- Sometimes it is wise to swap the LHS and RHS

For example, $12 = \frac{x}{3} + 1$ becomes $\frac{x}{3} + 1 = 12$, which is easier to solve

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Example 10 Solving equations with fractions

Solve each of the following equations.

a $\frac{a}{7} = 3$ **b** $\frac{5y}{3} = 10$ **c** $\frac{3x}{4} + 7 = 13$ **d** $\frac{2x - 3}{5} = 3$

SOLUTION

a

$\frac{a}{7} = 3$

(×7 both sides)

$a = 21$

EXPLANATION

Multiplying both sides by 7 removes the denominator of 7.

b

$\frac{5y}{3} = 10$

(×3 both sides)

$5y = 30$

(÷5 both sides)

$y = 6$

Multiplying both sides by 3 removes the denominator of 3.
Then the equation $5y = 30$ can be solved normally.

c

$\frac{3x}{4} + 7 = 13$

(−7 both sides)

$\frac{3x}{4} = 6$

(×4 both sides)

$3x = 24$

(÷3 both sides)

$x = 8$

In the LHS, $\frac{3x}{4} + 7$ means $3x \div 4 + 7$. The simplest first step is to subtract 7 from both sides.
Once there is a fraction by itself, multiply by its denominator (in this case, 4) and solve the equation $3x = 24$ as you would normally.

d

$\frac{2x - 3}{5} = 3$

(×5 both sides)

$2x - 3 = 15$

(+3 both sides)

$2x = 18$

(÷2 both sides)

$x = 9$

In the LHS, $\frac{2x - 3}{5}$ means $2x - 3$ is divided by 5.
The simplest first step is to multiply both sides by 5.

Then solve the equation $2x - 3 = 15$ as you would normally.

Exercise 9E

UNDERSTANDING AND FLUENCY	1–4, 5–6 (½)	4, 5–7(½)	5–7(½)

1 Classify each of the following as true or false.

a $\frac{a}{5}$ means $a \div 5$

b $\frac{q}{12}$ means $12 \div q$.

c $\frac{4 + a}{3}$ means $(4 + a) \div 3$.

d $\frac{4 + a}{3}$ means $4 + (a \div 3)$.

e $\frac{12 + 3q}{4}$ means $(12 + 3q) \div 4$.

f $2 + \frac{x}{5}$ means $(2 + x) \div 5$.

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2 Rewrite each of the following equations, using fraction notation.

a $x \div 3 + 1 = 5$
b $(x + 1) \div 3 = 5$
c $x - 1 \div 5 = 6$
d $2 \times x \div 3 = 7$
e $2 \times x \div 3 - 1 = 7$
f $(2x - 5) \div 4 = 2$

Example 10a

3 Fill in the missing steps to solve each of these equations.

a $\frac{b}{4} = 11$ → ×4 (both sides) → $b = \_\_$

b $\frac{d}{5} = 3$ → ×5 (both sides) → $\_\_ = \_\_$

c $\frac{h}{4} = 7$ → □ (both sides) → $\_\_ = \_\_$

d $\frac{p}{13} = 2$ → □ (both sides) → $\_\_ = \_\_$

4 For each of the following equations (**a** to **d**), choose the appropriate first step (i.e. **A** to **D**) needed to solve it.

a $\frac{x}{3} = 10$
b $\frac{x}{3} + 2 = 5$
c $\frac{x - 3}{2} = 1$
d $\frac{x}{2} - 3 = 5$

A Multiply both sides by 2.
B Multiply both sides by 3.
C Subtract 2 from both sides.
D Add 3 to both sides.

Example 10b

5 Solve the following equations algebraically.

a $\frac{m}{6} = 2$
b $\frac{c}{9} = 2$
c $\frac{s}{8} = 2$
d $\frac{r}{5} = 2$
e $\frac{3u}{5} = 12$
f $\frac{2y}{9} = 4$
g $\frac{5x}{2} = 10$
h $\frac{3a}{8} = 6$
i $\frac{4h}{5} = 8$
j $\frac{3j}{5} = 9$
k $\frac{5v}{9} = 5$
l $\frac{3q}{4} = 6$

Example 10c,d

6 Solve the following equations algebraically. Check your solutions using substitution.

a $\frac{h + 15}{12} = 2$
b $\frac{y + 5}{11} = 1$
c $\frac{j + 8}{11} = 1$
d $\frac{b - 2}{2} = 1$
e $\frac{7u - 12}{9} = 1$
f $14 + \frac{4t}{9} = 18$
g $1 = \frac{w + 5}{11}$
h $1 = \frac{4r - 13}{3}$
i $\frac{2q}{2} + 2 = 4$
j $\frac{s + 2}{5} = 1$
k $\frac{3l}{2} + 9 = 21$
l $12 = \frac{2z}{7} + 10$
m $1 = \frac{v - 4}{7}$
n $\frac{f - 2}{7} = 1$
o $9 = 4 + \frac{5x}{2}$
p $3 = \frac{7 + 4d}{9}$
q $\frac{7n}{9} + 14 = 21$
r $\frac{7m + 7}{4} = 21$
s $3 = \frac{7p}{4} - 11$
t $\frac{4a - 6}{5} = 6$

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7 Solve the following equations algebraically. (Note: The solutions to these equations are negative numbers.)

a $\frac{y+4}{3} = 1$ **b** $\frac{a}{10} + 2 = 1$ **c** $\frac{2x}{5} + 10 = 6$ **d** $\frac{x}{4} + 12 = 0$

e $0 = 12 + \frac{2u}{5}$ **f** $\frac{3y}{5} + 8 = 2$ **g** $1 = \frac{-2u - 3}{5}$ **h** $-2 = \frac{4d}{5} + 2$

PROBLEM-SOLVING AND REASONING	8, 11	9, 10, 12	10–13

8 In each of the following cases, write an equation and solve it to find the number.

a A number, t, is halved and the result is 9.
b One-third of q is 14.
c A number, r, is doubled and then divided by 5. The result is 6.
d 4 is subtracted from q and this is halved, giving a result of 3.
e 3 is added to x and the result is divided by 4, giving a result of 2.
f A number, y, is divided by 4 and then 3 is added, giving a result of 5.

9 A group of five people go out for dinner and then split the bill evenly. They each pay \$31.50.

a If b represents the total cost of the bill, in dollars, write an equation to describe this situation.
b Solve this equation algebraically.
c What is the total cost of the bill?

10 Lee and Theo hired a tennis court for a cost of \$$x$, which they split evenly. Out of his own pocket, Lee also bought some tennis balls for \$5.

a Write an *expression* for the total amount of money that Lee paid.
b Given that Lee paid \$11 in total, write an equation and solve it to find the total cost of hiring the court.
c State how much money Theo paid for his share of hiring the tennis court.

11 Sometimes the solution for an equation will be a fraction. For example, $2x = 1$ has the solution $x = \frac{1}{2}$.

a Give another equation that has $x = \frac{1}{2}$ as its solution.
b Find an equation that has the solution $x = \frac{5}{7}$.
c Could an equation have the solution $x = -\frac{1}{2}$? Justify your answer.

12 a Explain, in one sentence, the difference between the expressions $\frac{2x+3}{5}$ and $\frac{2x}{5} + 3$.

d What is the first operation you would apply to both sides to solve the equation $\frac{2x+3}{5} = 7$?

c What is the first operation you would apply to both sides to solve the equation $\frac{2x}{5} + 3 = 7$?

d Are there any values of x for which the expressions $\frac{2x+3}{5}$ and $\frac{2x}{5} + 3$ are equal to each other?

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13 Dividing by 2 and multiplying by $\frac{1}{2}$ have the same effect.

For example, $6 \div 2 = 3$ and $6 \times \frac{1}{2} = 3$.

a Show how each of these equations can be solved algebraically.

i $\frac{x}{2} = 5$

ii $\frac{1}{2} \times x = 5$

b Solve the two equations $\frac{x+4}{3} = 10$ and $\frac{1}{3}(x + 4) = 10$ algebraically, showing clearly the steps you would use at each stage.

c How does rewriting divisions as multiplications change the first step when solving equations?

ENRICHMENT	—	—	14, 15

Fractional solutions

14 Solve each of the following equations, giving your solutions as a fraction.

a $\frac{2x + 5}{4} = 3$

b $\frac{3x - 4}{6} = \frac{3}{4}$

c $\left(\frac{7 + 2x}{4}\right) \times 3 = 10$

d $\frac{1}{2} = \frac{3x - 1}{5}$

15 Consider the equation $\frac{5x - 3}{7} = 6$. The solution is $x = 9$. Change one number or one operator (i.e. $\times$, $-$ or $\div$) in the equation so that the solution will be $x = 12$.

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9F Equations with brackets

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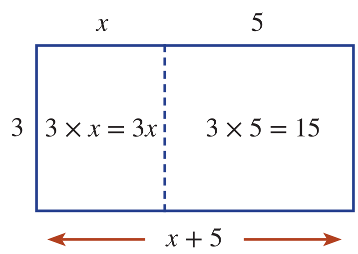
Recall from Chapter 8 that expressions with brackets can be expanded using the diagram shown below about rectangles' areas.

So $3(x + 5)$ is equivalent to $3x + 15$.

When solving $3(x + 5) = 21$, we could first divide both sides by 3 *or* we could first expand the brackets, giving $3x + 15 = 21$, and then subtract 15. For some equations, the brackets must be expanded first.



Let's start: Removing brackets

a Draw two rectangles, with areas $4(x + 3)$ and $5(x + 2)$.
b Use these to show that $4(x + 3) + 5(x + 2)$ is equivalent to $9x + 22$.
c Can you solve the equation $4(x + 3) + 5(x + 2) = 130$?

Key ideas

- To expand brackets, use the **distributive law**, which states that:

  $a(b + c) = ab + ac$ For example: $3(x + 4) = 3x + 12$

  $a(b - c) = ab - ac$ For example: $4(b - 2) = 4b - 8$

- **Like terms** are terms that contain exactly the same pronumerals and can be collected to simplify expressions. For example, $3x + 4 + 2x$ can be simplified to $5x + 4$.
- Equations involving brackets can be solved by first expanding brackets and collecting like terms.

Example 11 Expanding brackets

Expand each of the following.

a $4(x + 3)$ **b** $6(q - 4)$ **c** $5(3a + 4)$

SOLUTION	EXPLANATION
a $4(x + 3) = 4x + 12$	Using the distributive law: $4(x + 3) = 4x + 12$
b $6(q - 4) = 6q - 24$	Using the distributive law: $6(q - 4) = 6q - 24$
c $5(3a + 4) = 15a + 20$	Using the distributive law: $5(3a + 4) = 5 \times 3a + 20$

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Example 12 Simplifying expressions with like terms

Simplify each of these expressions.

a $2x + 5 + x$ **b** $3a + 8a + 2 - 2a + 5$

SOLUTION

a $2x + 5 + x = 3x + 5$

b $3a + 8a + 2 - 2a + 5 = 9a + 7$

EXPLANATION

Like terms are $2x$ and x.
These are combined to get $3x$.

Like terms are combined:
$3a + 8a - 2a = 9a$
$2 + 5 = 7$

Example 13 Solving equations by expanding brackets

Solve each of these equations by expanding brackets first.

a $3(x + 2) = 18$ **b** $7 = 7(4q - 3)$ **c** $3(b + 5) + 4b = 29$

SOLUTION

a $3(x + 2) = 18$

$3x + 6 = 18$

-6 -6

$3x = 12$

$\div 3$ $\div 3$

$x = 4$

b $7 = 7(4q - 3)$

$7 = 28q - 21$

$+21$ $+21$

$28 = 28q$

$\div 28$ $\div 28$

$1 = q$

So $q = 1$ is the solution.

c $3(b + 5) + 4b = 29$

$3b + 15 + 4b = 29$

$7b + 15 = 29$

-15 -15

$7b = 14$

$\div 7$ $\div 7$

$b = 2$

EXPLANATION

a Use the distributive law to expand the brackets.

Solve the equation by performing the same operations to both sides.

b Use the distributive law to expand brackets.

Solve the equation by performing the same operations to both sides.

c Use the distributive law to expand brackets.
Collect like terms to simplify the expression.
Solve the equation by performing the same operations to both sides.

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Exercise 9F

UNDERSTANDING AND FLUENCY	1–5, 6–8(½)	3, 4–9(½)	6–9(½)

1 Choose the equation which is equivalent to: $2(x - 3) = 9$

A $2x - 3 = 9$ **B** $2x - 6 = 9$ **C** $2x - 9 = 9$ **D** $x - 3 = 18$

2 Choose the equation which is equivalent to: $5(3x + 1) = 7$

A $15x + 5 = 7$ **B** $3x + 5 = 7$ **C** $15x = 6$ **D** $3x + 1 = 35$

3 Choose the equation that is equivalent to: $2(1 - 3x) = 8$

A $2 + 6x = 8$ **B** $6x + 2 = 8$ **C** $2 - 6x = 8$ **D** $1 - 3x = 16$

Example 11

4 Expand each of the following.

a $2(x + 1)$ **b** $5(2b + 3)$ **c** $2(3a - 4)$ **d** $5(7a + 1)$
e $4(3x + 4)$ **f** $3(8 - 3y)$ **g** $12(4a + 3)$ **h** $2(u - 4)$

Example 12

5 Simplify these expressions by collecting like terms.

a $3a + a + 2$ **b** $5 + 2x + x$
c $2b - 4 + b$ **d** $5a + 12 - 2a$
e $5x + 3 + x$ **f** $3k + 6 - 2k$
g $7 + 2b - 1$ **h** $6k - k + 1$

Example 13a

6 Solve the following equations by expanding the brackets first. Check your solutions by substituting them in.

a $2(10 + s) = 32$ **b** $2(5 + l) = 12$
c $3(p - 7) = 6$ **d** $8(y + 9) = 72$
e $8(4 + q) = 40$ **f** $7(p + 7) = 133$
g $8(m + 7) = 96$ **h** $22 = 2(b + 5)$
i $25 = 5(2 + p)$ **j** $63 = 7(p + 2)$
k $9(y - 6) = 27$ **l** $2(r + 8) = 32$

Example 13b

7 Solve these equations by expanding the brackets first.

a $6(3 + 2d) = 54$ **b** $8(7x - 7) = 56$
c $3(2x - 4) = 18$ **d** $27 = 3(3 + 6e)$
e $44 = 4(3a + 8)$ **f** $30 = 6(5r - 10)$
g $10 = 5(9u - 7)$ **h** $3(2q - 9) = 39$

Example 13c

8 Solve the following equations by first expanding the brackets. You will need to simplify the expanded expressions by collecting like terms.

a $5(4s + 4) + 4s = 44$ **b** $5i + 5(2 + 2i) = 25$ **c** $3(4c - 5) + c = 50$
d $3(4 + 3v) - 4v = 52$ **e** $5(4k + 2) + k = 31$ **f** $4q + 6(4q - 4) = 60$
g $40 = 4y + 6(2y - 4)$ **h** $44 = 4f + 4(2f + 2)$ **i** $40 = 5t + 6(4t - 3)$

9 Solve the following equations. (Note: The solutions to these equations are negative numbers.)

a $3(u + 7) = 6$ **b** $2(k + 3) = 0$
c $6(p - 2) = -18$ **d** $16 = 8(q + 4)$
e $5(2u + 3) = 5$ **f** $3 = 2(x + 4) + 1$
g $4(p - 3) + p = -32$ **h** $3(r + 4) + 2r + 40 = 2$

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PROBLEM-SOLVING AND REASONING	10, 11	10–12	10, 12, 13

10 For each of the following problems:

- **i** Write an equation.
- **ii** Solve your equation by first expanding any brackets.

a 5 is added to x and then this is doubled, giving a result of 14.
b 3 is subtracted from q and the result is tripled, giving a final result of 30.
c A number, x, is doubled and then 3 is added. This number is doubled again to get a result of 46.
d 4 is added to y and this is doubled. Then the original number, y, is subtracted, giving a result of 17.

11 For each of the following equations, prove that there are no solutions.

a $2(x + 5) - 2x = 7$
b $3(2x + 1) + 6(2 - x) = 4$
c $4(2x + 1) - 10x + 2(x + 1) = 12$

12 Consider the equation $2(3x + 4) - 6x + 1 = 9$.

a Show that this equation is true if $x = 0$.
b Show that this equation is true if $x = 3$.
c Explain why this equation is always true.
d Give an example of another equation involving brackets that is always true, where one side contains a pronumeral but the other side is just a number.

13 For equations like $4(3x + 2) = 44$, you have been expanding the brackets first.
Since $4(3x + 2) = 44$ is the same as $4 \times (3x + 2) = 44$, you can just start by dividing both sides by 4.
Without expanding brackets, solve the equations in Question **6** by dividing first.

ENRICHMENT	—	—	14

Equations with multiple brackets

14 Solve each of the following equations.

a $6(2j - 4) + 4(4j - 3) = 20$
b $3(4a + 5) + 5(1 + 3a) = 47$
c $2(5a + 3) + 3(2a + 3) = 63$
d $222 = 3(4a - 3) + 5(3a + 3)$
e $77 = 2(3c - 5) + 3(4c + 5)$
f $240 = 4(3d + 3) + 6(3d - 2)$
g $2(x + 3) + 4(x + 5) = 32$
h $4(x + 5) + 4(x - 5) = 24$
i $2(3x + 4) + 5(6x + 7) + 8(9x + 10) = 123$

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9G Formulas and relationships EXTENSION

Often, two or more pronumerals (or variables) represent related quantities. For example, the speeds at which a car travels and the time (t) it takes to arrive at its destination are related variable quantities. A formula is an equation that contains two or more pronumerals and shows how they are related.

Let's start: Fahrenheit and Celsius

In Australia, we measure temperature in degrees Celsius, whereas in the USA it is measured in degrees Fahrenheit. A formula to convert between them is $F = \frac{9C}{5} + 32$.

- At what temperature in degrees Fahrenheit does water freeze?
- At what temperature in degrees Fahrenheit does water boil?
- What temperature is 100° Fahrenheit in Celsius? Do you know what is significant about this temperature?

Key ideas

- A variable is a pronumeral that represents more than one value.
- A **formula** is a **rule** or **equation** that shows the relationship between two or more **variables**. For example:

  $A = l\,b$

  A: area of rectangle, l: length, b: breadth
- In the example above there are three variables, whereby $A = 6$, $l = 2$ and $b = 3$ is one solution for this formula because $6 = 2 \times 3$.
- $A = lb$ has three variables: A, l and b. If two variables are known, then the formula can be used to find the unknown variable.
- The **subject** of an equation is a pronumeral that occurs by itself on the left-hand side. For example: T is the subject of $T = 4x + 1$.
- To use a formula, first substitute all the known values into the formula, then solve the resulting equation, if possible.

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Example 14 Applying a formula involving two pronumerals

Consider the rule $k = 3b + 2$. Find the value of:

a k if $b = 5$ **b** k if $b = 10$ **c** b if $k = 23$

SOLUTION

a $k = 3 \times 5 + 2$
$= 17$

Substitute $b = 5$ into the equation.

b $k = 3 \times 10 + 2$
$= 32$

Substitute $b = 10$ into the equation.

c
$23 = 3b + 2$ (−2 both sides)
$21 = 3b$ (÷3 both sides)
$7 = b$

Therefore, $b = 7$.

Substitute $k = 23$ into the equation. Now solve the equation to find the value of b.

Example 15 Applying a formula involving three pronumerals

Consider the rule $Q = w(4 + t)$. Find the value of:

a Q if $w = 10$ and $t = 3$ **b** t if $Q = 42$ and $w = 6$

SOLUTION

a $Q = 10(4 + 3)$
$= 10 \times 7$
$= 70$

Substitute $w = 10$ and $t = 3$ to evaluate.

b $42 = 6(4 + t)$

Substitute $Q = 42$ and $w = 6$.

$42 = 24 + 6t$ (−24 both sides)
$18 = 6t$ (÷6 both sides)
$3 = t$

Therefore, $t = 3$.

Expand the brackets and then solve the equation.

Exercise 9G EXTENSION

UNDERSTANDING AND FLUENCY	1–5	3–6	5, 6

1 True or false?
The values $F = 10, m = 5, a = 2$, satisfy the formula $F = ma$?

2 Substitute:

a $x = 3$ into the expression $5x$

b $x = 7$ into the expression $4(x + 2)$

c $y = 3$ into the expression $20 - 4y$

d $y = 10$ into the expression $\frac{y + 4}{7}$

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Example 14

3 Consider the rule $h = 2m + 1$. Find:

a h if $m = 3$
b h if $m = 4$
c m if $h = 17$. Set up an equation and solve it algebraically.
d Find m if $h = 21$. Set up an equation and solve it algebraically.

4 Consider the formula $y = 5 + 3x$. Find:

a y if $x = 6$
b x if $y = 17$
c x if $y = 26$

5 Consider the rule $A = q + t$. Find:

a A if $q = 3$ and $t = 4$
b q if $A = 5$ and $t = 1$
c t if $A = 3$ and $q = 3$

Example 15

6 Consider the formula $G = 7x + 2y$. Find:

a G if $x = 3$ and $y = 3$
b x if $y = 2$ and $G = 11$
c y if $G = 31$ and $x = 3$

PROBLEM-SOLVING AND REASONING	7, 10	7, 8, 10	8–11

7 A formula for the area of a rectangle is $A = b \times h$, where b is the rectangle's base and h is the rectangle's height.

a Set up and solve an equation to find the base of a rectangle with $A = 20$ and $h = 4$.
b A rectangle is drawn for which $A = 25$ and $b = 5$.
  i Set up and solve an equation to find h.
  ii What type of rectangle is this?

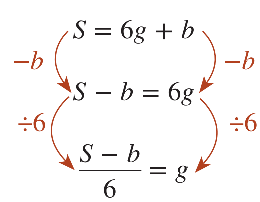
8 The perimeter for a rectangle is given by $P = 2(b + h)$. Find the:

a perimeter when $b = 3$ and $h = 5$
b value of h when $P = 10$ and $b = 2$
c area of a rectangle if its perimeter is 20 and base is 4

9 To convert between temperatures in Celsius and Fahrenheit the rule is $F = \frac{9C}{5} + 32$.

a Find F if $C = 20$.
b Find the value of C if $F = 50$.
c Find the temperature in Celsius if it is 53.6° Fahrenheit.
d Marieko claims the temperature in her city varies between 68° Fahrenheit and 95° Fahrenheit. What is the difference, in Celsius, between these two temperatures?

10 Rearranging a formula involves finding an equivalent equation that has a different variable on one side by itself. For example, as shown at right, the formula $S = 6g + b$ can be rearranged to make g by itself.



Now we have a formula that can be used to find g once S and b are known.

a Rearrange $S = 5d + 3b$ to make a rule where d is by itself.
b Rearrange the formula $F = \frac{9C}{5} + 32$ to make C by itself.
c Rearrange the formula $Q = 3(x + 12) + x$ to make x by itself. (Hint: You will need to expand the brackets first.)

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11 A taxi company charges different amounts of money based on how far the taxi travels and how long the passenger is in the car. Although the company has not revealed the formula it uses, some sample costs are shown below.

Distance (D) in km	Time (t) in minutes	Cost (C) in dollars
10	20	30
20	30	50

a Show that the rule $C = D + t$ is consistent with the values above.

b Show that the rule $C = 3D$ is not consistent with the values above.

c Show that the rule $C = 2D + 10$ is consistent with the values above.

d Try to find at least two other formulas that the taxi company could be using, based on the values shown.

ENRICHMENT	—	—	12

AFL equations

12 In Australian Rules Football (AFL), the score, S, is given by $S = 6g + b$, where g is the number of goals scored and b is the number of 'behinds' (i.e. near misses).

a Which team is winning if the Abbotsford Apes have scored 11 goals ($g = 11$) and 9 behinds ($b = 9$), and the Box Hill Baboons have scored 12 goals and 2 behinds?

b The Camperdown Chimpanzees have scored 7 behinds and their current score is $S = 55$. Solve an equation algebraically to find how many goals the team has scored.

c In some AFL competitions, a team can score a 'supergoal', which is worth 9 points. If q is the number of supergoals that a team kicks, write a new formula for the team's score.

d For some rare combinations of goals and behinds, the score equals the product of g and b. For example, 4 goals and 8 behinds gives a score of $6 \times 4 + 8 = 32$, and $4 \times 8 = 32$. Find all the other values of g and b that make the equation $6g + b = gb$ true.

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9H Using equations to solve problems EXTENSION

Interactive Widgets HOTsheets Walkthrough

Equations can be used to solve problems which may arise in the real world.

Let's start: Stationery shopping

Sylvia bought 10 pencils and 2 erasers for \$20.40.
Edward bought 5 pencils and 3 erasers for \$12.60.

- Use the information above to work out how much Karl will pay for 6 pencils and 5 erasers.
- Describe how you got your answer.
- Is there more than one possible solution?

Key ideas

- To solve a problem, follow these steps.

Use a pronumeral to stand in for the unknown.	Let p = the cost of a pencil.
↓ Write an equation to describe the problem.	$10p + 2 \times 3.5 = 25$.
↓ Solve the equation.	This can be done by inspection or algebraically.
↓ Make sure that you answer the original question and the solution seems reasonable and realistic.	Don't forget to include the correct units (e.g. dollars, years, cm).

Example 16 Solving a problem using equations

The sum of Kate's current age and her age next year is 19. How old is Kate?

SOLUTION	EXPLANATION
Let k = Kate's current age.	Define a pronumeral to stand for the unknown number.
$k + (k + 1) = 19$	Write an equation to describe the situation. Note that $k + 1$ is Kate's age next year.
$2k + 1 = 19$ (−1 both sides) $2k = 18$ (÷2 both sides) $k = 9$	Simplify the LHS and then solve the equation algebraically.
Kate is currently 9 years old.	Answer the original question.

Exercise 9H EXTENSION

UNDERSTANDING AND FLUENCY	1–5	3–6	4–7

1 The product of k and 7 is 42.
a Write an equation to describe this fact.
b Solve the equation to find the value of k.

2 The sum of x and 19 is 103.
a Write an equation to describe this fact.
b Solve the equation to find the value of x.

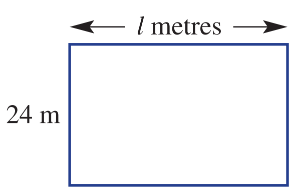
Example 16

3 Launz buys a car and a trailer for a combined cost of \$40 000. The trailer costs \$2000.
a Define a pronumeral for the car's cost.
b Write an equation to describe the problem.
c Solve the equation algebraically.
d Hence, state the cost of the car.

4 Meghan buys 12 pens for a total cost of \$15.60.
a Define a pronumeral for the cost of one pen.
b Write an equation to describe the problem.
c Solve the equation algebraically.
d Hence, state the cost of one pen.

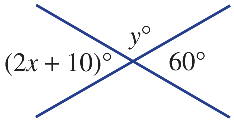
5 Jonas is paid \$17 per hour and gets paid a bonus of \$65 each week. One particular week he earned \$643.
a Define a pronumeral for the number of hours Jonas worked.
b Write an equation to describe the problem.
c Solve the equation algebraically.
d How many hours did Jonas work in that week?

6 This rectangular paddock has an area of 720 m^2.



a Write an equation to describe the problem, using l for the paddock's length.
b Solve the equation algebraically.
c How long is the paddock?
d What is the paddock's perimeter?

7 Write an equation in terms of x and an equation in terms of y for each of these diagrams.

a


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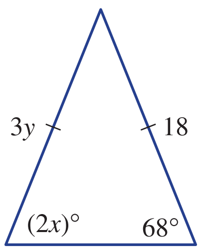
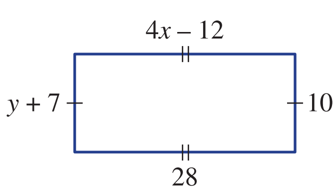


c


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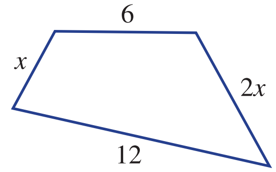
PROBLEM-SOLVING AND REASONING	8, 9, 15	9–12, 15	13–17

8 A number is doubled, then 3 is added and the result is doubled again. This gives a final result of 34. Set up and solve an equation to find the original number, showing all the steps clearly.

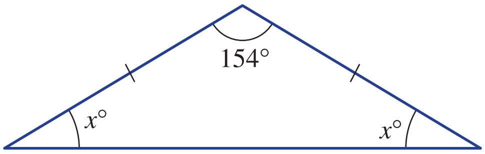
9 The perimeter of the shape shown is 30. Find the value of x.



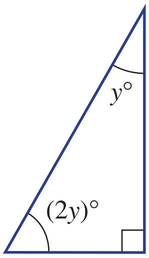
10 Alexa watches some television on Monday, then twice as many hours on Tuesday, then twice as many hours again on Wednesday. If she watches a total of $10\frac{1}{12}$ hours from Monday to Wednesday, how much television did Alexa watch on Monday?

11 Marcus and Sara's combined age is 30. Given that Sara is 2 years older than Marcus, write an equation and find Marcus' age.

12 An isosceles triangle is shown below. Write an equation and solve it to find $x°$, the unknown angle. (Remember: The sum of angles in a triangle is 180°.)



13 Find the value of y in the triangle shown here, by first writing an equation.



14 A rectangle has base b and height h. The perimeter and area of the rectangle are equal. Write an equation and solve it by inspection to find some possible values for b and h. (Note: There are many solutions to this equation. Try to find a few.)

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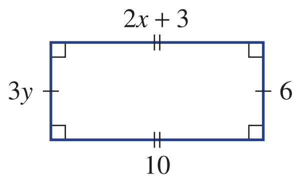
15 Find the values of x and y in the rectangle shown.



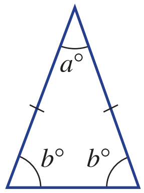
16 If photocopying costs 35 cents a page and p is the number of pages photocopied, which of the following equations have possible solutions? Justify your answers. (Note: Fraction answers are not possible because you must still pay 35 cents even if you photocopy only part of a page.)

a $0.35p = 4.20$

b $0.35p = 2.90$

c $0.35p = 2.80$

17 Assume that an isosceles triangle is drawn so that each of its three angles is a whole number of degrees. Prove that the angle a must be an even number of degrees.

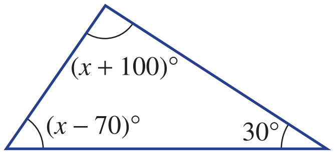


ENRICHMENT	—	—	18

Strange triangles

18 Recall that the sum of angles in a triangle is 180°.

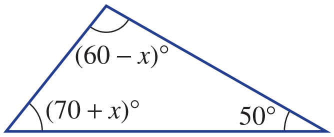
a David proposes the following triangle, which is not drawn to scale.



i Find the value of x.

ii Explain what makes this triangle impossible.

b Helena proposes the following triangle, which is also not drawn to scale.



i Explain why the information in the diagram is not enough to find x.

ii What are the possible values that x could take?

c Design a geometric puzzle, like the one in part **a**, for which the solution is impossible.

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Investigation

Theme parks

There are thousands of theme parks all over the world that offer a vast array of rides that are built to thrill. By surfing the internet, you can discover the longest, tallest, fastest and scariest rides. Although prices are kept competitive, theme parks need to make a profit so that they can maintain safety standards and continue to build new and more exciting rides.

Thrill World and Extreme Park are two theme parks. Both charge different prices for entry and for each ride. Their prices are:

- Thrill World: $20 entry and $5 per ride
- Extreme Park: $60 entry and $3 per ride

a Copy and complete the table below for each theme park. The total cost for the day includes the entry cost and cost of the rides.

Number of rides (n)	1	2	3	4	5	6	7	8	…	20	21	22	23	24	25
Thrill World total cost $$T$	$25								…						
Extreme Park total cost $$E$	$63								…						

b Write an equation for:

i T, the total cost, in dollars, for n rides at Thrill World

ii E, the total cost, in dollars, for n rides at Extreme Park

c For each of these thrill seekers, use an equation to calculate how many rides they went on.

i Amanda, who spent $105 at Thrill World

ii George, who spent $117 at Extreme Park

d Refer to your completed table to determine the number of rides that will make the total cost for the day the same at each theme park.

e A third theme park, Fun World, decides to charge no entry fee but charges $10 per ride. Find the minimum number of rides that you could go on at Fun World before it becomes cheaper at:

i Thrill World

ii Extreme Park

f Fun World changes its pricing policy after it decides that it will be unable to compete by pricing its rides so high, so it decides to charge an entry fee and then make all rides free.
Investigate how much Fun World should charge to attract customers while still making profits that are similar to those of Thrill World and Extreme Park. Provide some mathematical calculations to support your conclusions.

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1 Find the unknown number in the following puzzles.

a A number is added to half of itself and the result is 39.

b A number is doubled, then tripled, then quadrupled. The result is 696.

c One-quarter of a number is subtracted from 100 and the result is 8.

d Half of a number is added to 47, and the result is the same as the original number doubled.

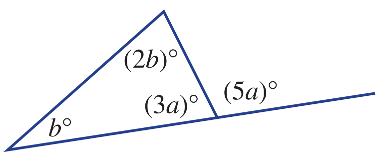
e A number is increased by 4, the result is doubled and then 4 is added again to give an answer of 84.

2 Find the same values of x and y that will make both of these equations true.
$x + y = 20$ and $x \times y = 91$

3 Find the same values of a, b and c, given the clues:
$5(a + 2) + 3 = 38$ and $2(b + 6) - 2 = 14$ and $3a + 2b + c = 31$

4 Find the values of a and b for each of these geometric figures.

a



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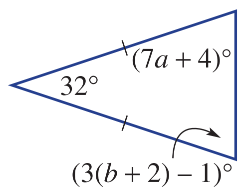

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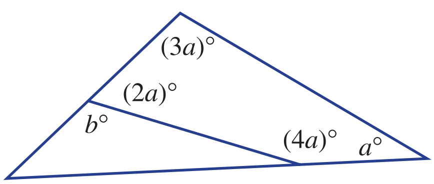
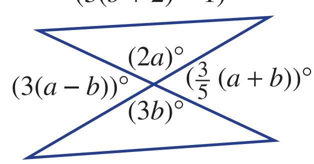


d



5 By solving equations, find the answer to the question: *What did the student expect when she solved the puzzle?*
If the solution is $x = 1$, then the letter is A. If the solution is $x = 2$, then the letter is B and so on.

$3x + 2 = 5$	$16 = 2(x + 5)$	$5(x - 2) = 65$	$\frac{x}{3} + 1 = 7$	$2 = \frac{x + 2}{8}$

$2x + 10 = 60$	$\frac{x}{2} + 3 = 8$	$2(x + 1) + x = 47$	$20 - x = 9$	$4x - 3 = 17$

6 In a farmer's paddock there are sheep and ducks. Farmer John says to his grandson, "There are 41 animals in this paddock." Grandson James says to his grandpa, "There are 134 animal legs in this paddock."
How many sheep and how many ducks are in the paddock?

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Chapter summary

Equations

Solving an equation

Finding pronumeral values to make equation true

e.g. $15 + x = 20$

solution: $x = 5$

$2 + 3 = 5$, $4 + x = 12$, $19 = 12 - 3q$ } equations

$7 + a$, $2 + 3 =$ } not equations

Formulas or rules are types of equations.

e.g. $F = ma$, $A = b \times h$, $P = 4x$

Equivalent equations

are obtained by performing the same operation to both sides

e.g. $12 + 3x = 5$ $(+8)$ → $20 + 3x = 13$

To solve algebraically, use equivalent equations and opposite operations.

Main expression	First step
$3x + 5$	-5
$12x$	$\div 12$
$5x - 2$	$+2$
$\frac{x}{4}$	$\times 4$

e.g. $3x + 5 = 38$ (-5) → $3x = 33$ $(\div 3)$ → $x = 11$

e.g. $\frac{x}{4} = 7$ $(\times 4)$ → $x = 28$

e.g. $12x = 48$ $(\div 12)$ → $x = 4$

e.g. $5x - 2 = 33$ $(+2)$ → $5x = 35$ $(\div 5)$ → $x = 7$

Equations with fractions are solved by multiplying by the denominator when the fraction is by itself.

e.g. $2 + \frac{x}{7} = 4$ (-2) → $\frac{x}{7} = 2$ $(\times 7)$ → $x = 14$

e.g. $\frac{3x + 9}{4} = 6$ $(\times 4)$ → $3x + 9 = 24$ (-9) → $3x = 15$ $(\div 3)$ → $x = 5$

Brackets should be expanded and like items collected. Use the distributive law to expand.

e.g. $4(x + 2) = 4x + 8$

$3x(2 + y) = 6x + 3xy$

e.g. $5(x + 3) + 2x = 29$

$5x + 15 + 2x = 29$

$7x + 15 = 29$ (-15) → $7x = 14$ $(\div 7)$ → $x = 2$

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Multiple-choice questions

1 If $x = 3$, which one of the following equations is true?
A $4x = 21$ **B** $2x + 4 = 12$ **C** $9 - x = 6$ **D** $2 = x + 1$ **E** $x - 3 = 4$

2 When 11 is added to the product of 3 and x, the result is 53. This can be written as:
A $3x + 11 = 53$ **B** $3(x + 11) = 53$ **C** $\frac{x}{3} + 11 = 53$ **D** $\frac{x + 11}{3} = 53$ **E** $3x - 11 = 53$

3 Which of the following values of x make the equation $2(x + 4) = 3x$ true?
A 2 **B** 4 **C** 6 **D** 8 **E** 10

4 The equivalent equation that results from subtracting 3 from both sides of $12x - 3 = 27$ is:
A $12x = 24$ **B** $12x - 6 = 24$ **C** $12x - 6 = 30$ **D** $9x - 3 = 24$ **E** $12x = 30$

5 To solve $3a + 5 = 17$, the first step to apply to both sides is to:
A add 5 **B** divide by 3 **C** subtract 17 **D** divide by 5 **E** subtract 5

6 The solution to $2t - 4 = 6$ is:
A $t = 1$ **B** $t = 3$ **C** $t = 5$ **D** $t = 7$ **E** $t = 9$

7 The solution of $\frac{2x}{7} = 10$ is:
A $x = 35$ **B** $x = 70$ **C** $x = 20$ **D** $x = 30$ **E** $x = 5$

8 The solution to the equation $10 = \frac{3p + 5}{2}$ is:
A $p = 5$ **B** $p = 20$ **C** $p = 15$ **D** $p = 7$ **E** $p = 1$

9 The solution of $3(u + 1) = 15$ is:
A $u = 5$ **B** $u = 4$ **C** $u = 11$ **D** $u = 6$ **E** $u = 3$

10 Which equation has solution $x = 5$?
A $x - 5 = 5$ **B** $x + 5 = 5$ **C** $5x = 0$ **D** $5x = 5$ **E** $5x = 25$

Short-answer questions

1 Classify each of the following equations as true or false.
a $4 + 2 = 10 - 2$ **b** $2(3 + 5) = 4(1 + 3)$ **c** $5w + 1 = 11$, if $w = 2$
d $2x + 5 = 12$, if $x = 4$ **e** $y = 3y - 2$, if $y = 1$ **f** $4 = z + 2$, if $z = 3$

2 Write an equation for each of the following situations. You do not need to solve the equations.
a The sum of 2 and u is 22. **b** The product of k and 5 is 41.
c When z is tripled the result is 36. **d** The sum of a and b is 15.

3 Solve the following equations by inspection.
a $x + 1 = 4$ **b** $x + 8 = 14$ **c** $9 + y = 10$
d $y - 7 = 2$ **e** $5a = 10$ **f** $\frac{a}{5} = 2$

4 For each equation, find the result of applying the given operation to both sides and then simplify.
a $2x + 5 = 13$ $[-5]$ **b** $7a + 4 = 32$ $[-4]$
c $12 = 3r - 3$ $[+3]$ **d** $15 = 8p - 1$ $[-1]$

Chapter review

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5 Solve each of the following equations algebraically and check your solutions by substituting.

a $5x = 15$ **b** $r + 25 = 70$ **c** $12p + 17 = 125$ **d** $12 = 4b - 12$

e $5 = \frac{x}{3} + 2$ **f** $13 = 2r + 5$ **g** $10 = 4q + 2$ **h** $8u + 2 = 66$

6 Solve the following equations algebraically.

a $\frac{3u}{4} = 6$ **b** $\frac{8p}{3} = 8$ **c** $3 = \frac{2x + 1}{3}$

d $\frac{5y}{2} + 10 = 30$ **e** $4 = \frac{2y + 20}{7}$ **f** $\frac{4x}{3} + 4 = 24$

7 Expand the brackets in each of the following expressions.

a $2(3 + 2p)$ **b** $4(3x + 12)$ **c** $7(a + 5)$ **d** $9(2x + 1)$

8 Solve each of these equations by expanding the brackets first. Check your solutions by substituting.

a $2(x - 3) = 10$ **b** $27 = 3(x + 1)$ **c** $48 = 8(x - 1)$

d $60 = 3y + 2(y + 5)$ **e** $7(2z + 1) + 3 = 80$ **f** $2(5 + 3q) + 4q = 40$

9 Consider the equation $4(x + 3) + 7x - 9 = 10$.

a Is $x = 2$ a solution?

b Show that the solution to this equation is *not* a whole number.

10 **a** Does $3(2x + 2) - 6x + 4 = 15$ have a solution? Justify your answer.

b State whether the following are solutions to $5(x + 3) - 3(x + 2) = 2x + 9$.

i $x = 2$ **ii** $x = 3$

11 The following equations contain two variables. If $y = 5$, what is the value of x?

a $x + y = 8$ **b** $x - y = 8$

c $xy = 40$ **d** $xy^2 = 100$

12 The solution of $3(2x - 5) = 15$ is $x = 5$. Change one number in the equation so that the solution will be $x = 6$.

13 For each of the following problems, write an equation and solve it to find the unknown value.

a A number is added to three times itself and the result is 20. What is the number?

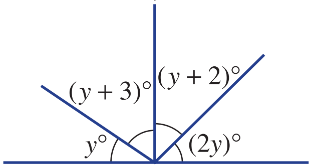
b The product of 5 and a number is 30. What is the number?

c Juanita's mother is twice as old as Juanita. The sum of their ages is 60. How old is Juanita?

d A rectangle has a length of 21 cm and a perimeter of 54 cm. What is its breadth?

14 Find the value of y for each of these figures.

a



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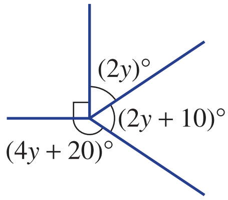

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Extended-response questions

1 Udhav's mobile phone plan charges a 15-cents connection fee and then 2 cents per second for every call.

a How much does a 30-second call cost?

b Write a rule for the total cost, C, in cents, for a call that lasts t seconds.

c Use your rule to find the cost of a call that lasts 80 seconds.

d If a call cost 39 cents, how long did it last? Solve an equation to find t.

e If a call cost $1.77, how long did it last?

f On a particular day, Udhav makes two calls — the second one lasting twice as long as the first, with a total cost of $3.30. What was the total amount of time he spent on the phone?

2 Gemma is paid $$x$ per hour from Monday to Friday, but earns an extra $2 per hour during weekends. During a particular week, she worked 30 hours during the week and then 10 hours on the weekend.

a If $x = 12$, calculate the total wages Gemma was paid that week.

b Explain why her weekly wage is given by the rule $W = 30x + 10(x + 2)$.

c Use the rule to find Gemma's weekly wage if $x = 16$.

d If Gemma earns $620 in one week, find the value of x.

e If Gemma earns $860 in one week, how much did she earn from Monday to Friday?

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Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

10 Measurement and computation of length, perimeter and area

What you will learn

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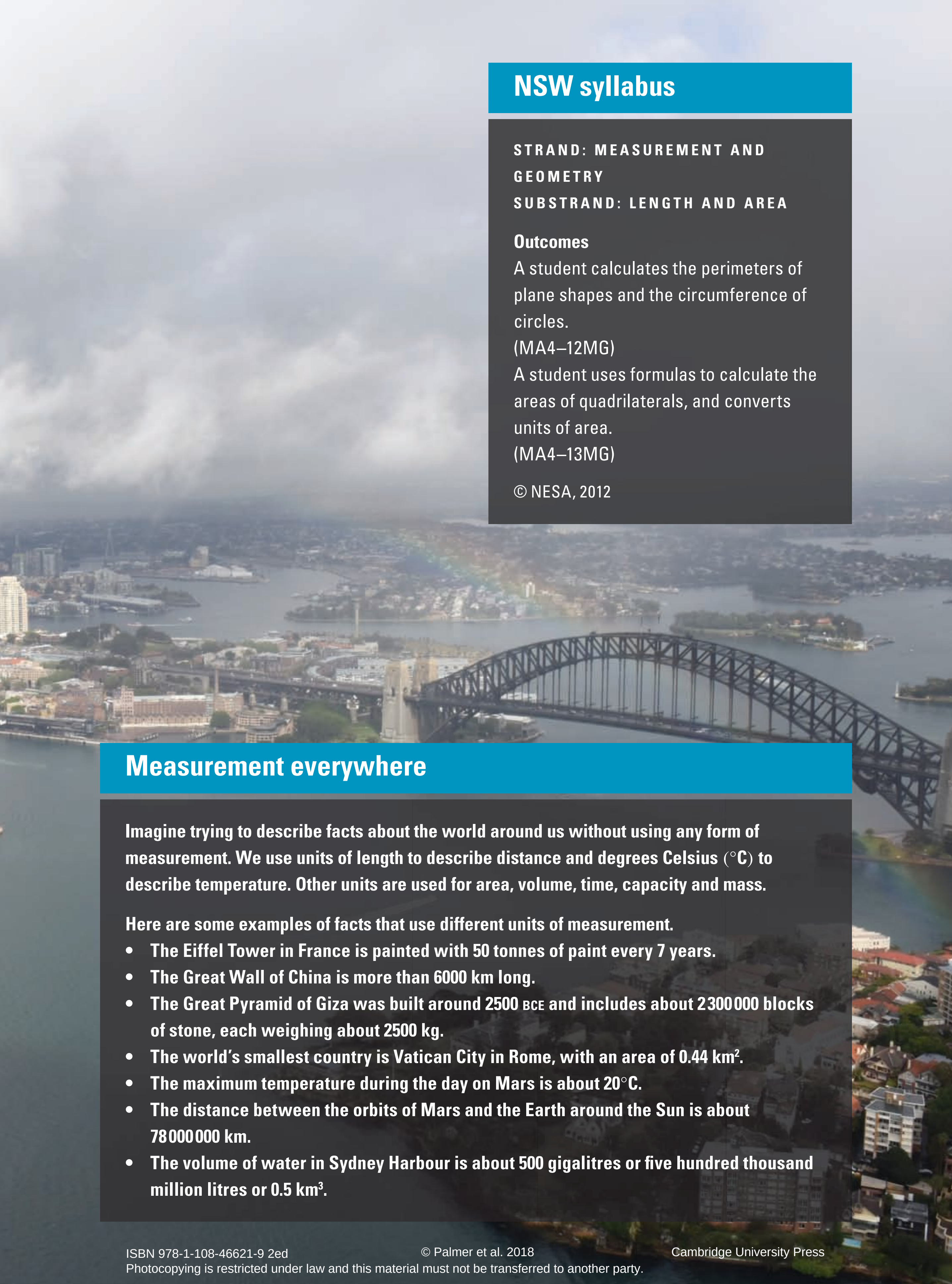

NSW syllabus

STRAND: MEASUREMENT AND GEOMETRY

SUBSTRAND: LENGTH AND AREA

Outcomes

A student calculates the perimeters of plane shapes and the circumference of circles.
(MA4–12MG)

A student uses formulas to calculate the areas of quadrilaterals, and converts units of area.
(MA4–13MG)

Measurement everywhere

Imagine trying to describe facts about the world around us without using any form of measurement. We use units of length to describe distance and degrees Celsius (°C) to describe temperature. Other units are used for area, volume, time, capacity and mass.

Here are some examples of facts that use different units of measurement.

- **The Eiffel Tower in France is painted with 50 tonnes of paint every 7 years.**
- **The Great Wall of China is more than 6000 km long.**
- **The Great Pyramid of Giza was built around 2500 BCE and includes about 2 300 000 blocks of stone, each weighing about 2500 kg.**
- **The world's smallest country is Vatican City in Rome, with an area of 0.44 km^2.**
- **The maximum temperature during the day on Mars is about 20°C.**
- **The distance between the orbits of Mars and the Earth around the Sun is about 78 000 000 km.**
- **The volume of water in Sydney Harbour is about 500 gigalitres or five hundred thousand million litres or 0.5 km^3.**

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Pre-test

1 Measure the length of these lines in millimetres.

a ______________________________

b ____

c __________

2 Arrange these units from smallest to largest.

a centimetre (cm), kilometre (km), metre (m), millimetre (mm)

b gram (g), kilogram (kg), milligram (mg), tonne (t)

c hour (h), year, second (s), day, minute (min)

3 For each of the following, find how many:

a millimetres are in a centimetre

b centimetres are in a metre

c grams are in a kilogram

d seconds are in a minute

e minutes are in an hour

f millilitres are in a litre

4 Calculate the answer to each of the following.

a 2×1000

b $200 \div 100$

c $56\,000 \div 1000$

d 2.5×1000

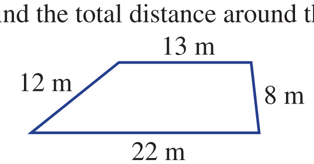
e 3×60

f $2 \times 60 \times 60$

g $1400 \div 1000$

h $27 \div 1000$

5 Find the total distance around these shapes.

a


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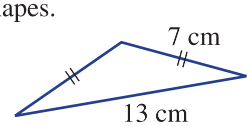
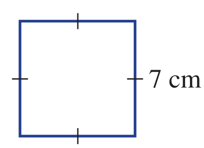
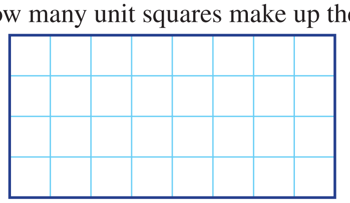


c


6 How many unit squares make up the area of these shapes?

a


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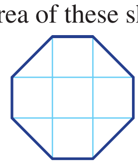

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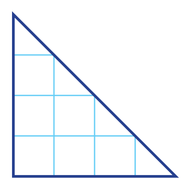

7 Give the most appropriate unit (e.g. metres) for measuring each of the following.

a distance between two towns

b your weight

c length of a school lesson

d width of a large plate

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10A Measurement systems of the past and present FRINGE

From about 3000 BCE a unit of measure called the *cubit* was used in ancient Egypt to measure lengths. It is known that the cubit, which is the length of an arm from the elbow to the end of the middle finger, was used to measure the depth of the Nile River in flood. Other unit measures based on the human body were also used. Some include the *digit* (width of a finger), *palm* (width of four fingers) and *span* (distance from the tip of the thumb to the tip of the little finger in an outstretched hand).

Ancient Egyptian monuments like this giant statue of the pharaoh Ramses II would have been constructed according to specifications given in cubits.

Because each individual's arm length and finger width is different, there was a need to develop a standard unit of length. The Egyptians defined a *standard royal cubit* (about 524 mm), and this was represented as a stone rod. From this cubit the following divisions were made: 28 digits in a cubit, 4 digits in a palm and 14 digits in a span.

Many of these units of measurement were adapted and developed by the Babylonians, Greeks, Romans, English and French over many centuries. The English imperial system, which was adapted from the Roman and Greek systems, is commonly used in the United Kingdom and the United States today, and was used in Australia until the 1970s. Many people today still prefer to describe lengths and other measures using imperial units, such as the *inch* and *mile*.

The metric system was developed in France in the 1790s and is the universally accepted system today. The word *metric* comes from the Greek word *metron*, meaning 'measure'. It is a decimal system where length measures are based on the unit called the *metre*. The definition of the metre has changed over time. Originally it was proposed to be the length of a pendulum that beats at a rate of one per second. It was later defined as 1/10 000 000 of the distance from the North Pole to the equator on a line on the Earth's surface passing through Paris. In 1960, a metre became 1 650 763.73 wave lengths of the spectrum of the krypton-86 atom in a vacuum. In 1983, the metre was defined as the distance that light travels in 1/299 792 458 seconds inside a vacuum.

Let's start: Egyptian trader

Imagine you are in ancient Egypt and you are trading goods at a market. You use the Egyptian units: *digit* (width of a finger), *palm* (width of four fingers) and *span* (distance from the top of the thumb to the tip of the little finger in an outstretched hand).

- Use a ruler to find the metric equivalent of your *digit*, *palm* and *span*.
- You purchase a wad of papyrus paper that is 1 digit thick. Which students in the class would get the least paper if they used their own index finger width?
- You purchase a bowl of grain 1 span deep. Which student in the class gets the most grain?
- You purchase 5 cubits of cloth. Which student gets the most cloth?

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Key ideas

- Ancient measurement systems that developed from about 3000 BCE include the Egyptian, Babylonian, Greek and Roman systems. The **metric system** is the commonly used system today in many countries, including Australia.
- **Roman system**
  - 1 foot = 12 inches = 16 digits = 4 palms
  - 1 cubit = 6 palms
  - 1 pace (double step) = 5 feet
  - 1 mile = 1000 paces
- **imperial system**
  - 1 foot = 12 inches (1 inch is about 2.5 cm)
  - 1 yard = 3 feet (1 yard is about 91.5 cm)
  - 1 rod = 16.5 feet
  - 1 chain = 22 yards
  - 1 furlong = 40 rods
  - 1 mile = 8 furlongs = 1760 yards (1 mile is about 1.6 km)
- **metric system**
  - 1 centimetre (cm) = 10 millimetres (mm)
  - 1 metre (m) = 100 centimetres (cm)
  - 1 kilometre (km) = 1000 metres (m)

Example 1 Using measurement systems

a How many feet are there in 1 mile, using the Roman measuring system?
b How many inches are there in 3 yards, using the imperial system?

SOLUTION	EXPLANATION
a 1 mile = 1000 paces = 5000 feet	There are 1000 paces in a Roman mile and 5 feet in a pace.
b 3 yards = 9 feet = 108 inches	There are 3 feet in an imperial yard and 12 inches in a foot.

Example 2 Choosing metric lengths

Which metric unit would be the most appropriate for measuring these lengths?
a width of a large room
b thickness of glass in a window

SOLUTION	EXPLANATION
a metres (m)	Using mm or cm would give a very large number, and using km would give a number that is very small.
b millimetres (mm)	The thickness of glass is likely to be around 5 mm.

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Exercise 10A FRINGE

UNDERSTANDING AND FLUENCY	1–8	4–9	5–9

1 Complete these number sentences.

a Roman system

i 1 ______________ = 12 inches = 16 ______________ = ______________ palms

ii 1 ________________________ = 1000 paces

b imperial system

i 1 foot = 12______________

ii 3 ______________ = 1 yard

iii ______________ = 1760 yards

c metric system

i 1 m = ______________ cm

ii 1 cm = ______________ mm

iii ______________ km = 1000 m

2 List the units of length (e.g. cubit), from smallest to largest, commonly used in the Roman system.

3 List the units of length (e.g. inch), from smallest to largest, commonly used in the imperial system.

4 List the units of length (e.g. centimetre), from smallest to largest, commonly used in the metric system.

Example 1

5 Use the Roman system to state how many:

a feet are in 1 pace
b feet are in 1 mile
c palms are in 1 foot
d palms are in 1 pace
e digits are in 1 foot
f digits are in 1 pace

6 Use the imperial system to state how many:

a inches are in 1 foot
b feet are in 1 yard
c inches are in 1 yard
d yards are in 1 mile
e yards are in 1 chain
f rods are in 1 furlong

7 Use the metric system to state how many:

a millimetres are in 1 centimetre
b centimetres are in 1 metre
c metres are in 1 kilometre
d millimetres are in 1 metre
e centimetres are in 1 kilometre
f millimetres are in 1 kilometre

Example 2

8 Which metric unit would be the most appropriate for measuring the following?

a the distance between two towns
b diameter of a small drill bit
c height of a flag pole
d length of a garden hose
e width of a small desk
f distance across a city

A drill bit

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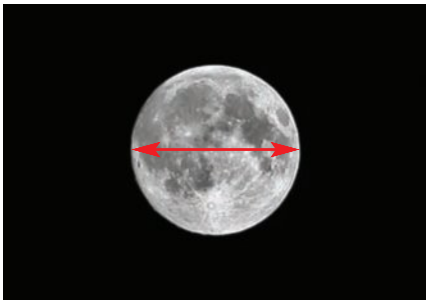
9 Choose which metric unit would be the most suitable for measuring the real-life length indicated in these photos.

a

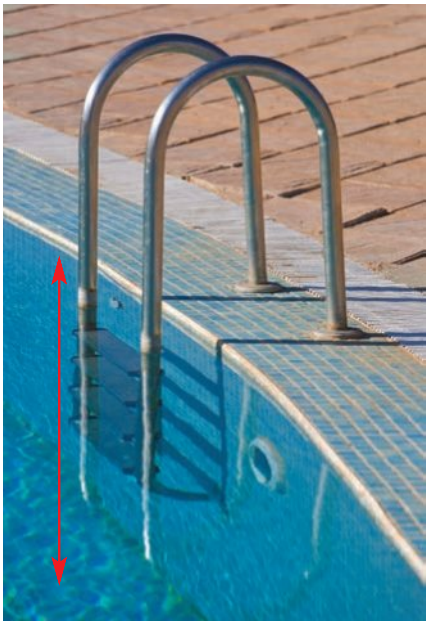
b

c

d



e



f

PROBLEM-SOLVING AND REASONING	10, 11, 14	11–14, 16	14–17

10 A Roman offers you either 12 palms or 1 pace of cloth. Which option gives you the most cloth?

11 The Roman army marches 5 Roman miles to the next post. How many paces is the journey?

12 An English cricketer runs 1 chain for each run made. How many yards will he run if he makes 20 runs?

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13 Here is the length of 1 mm and 1 cm. Use these diagrams as a guide to estimate the length of these lines.

1 mm 1 cm

a

b

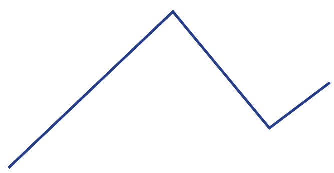
c

d

e

14 Estimate the length of each line or curve, in centimetres.

a



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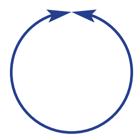
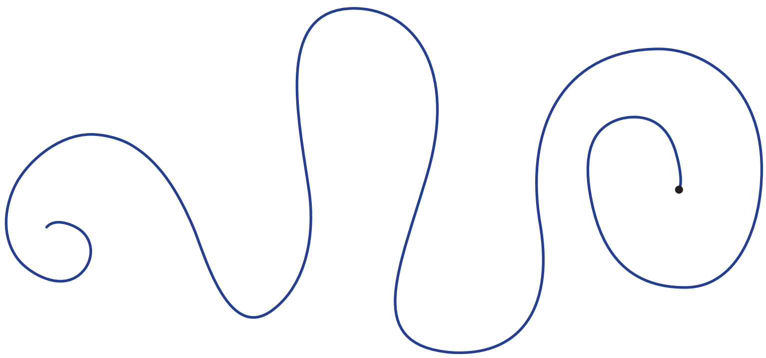


c



15 Complete these tables.

a metric

	mm	cm	m	km
mm	1	$\frac{1}{10}$		
cm	10	1		
m	1000		1	
km				1

b imperial

	inch	feet	yard	mile
inch	1		$\frac{1}{36}$	
feet	12	1		
yard			1	
mile			1760	1

c Roman

	digit	palm	feet	pace	mile
digit	1		$\frac{1}{16}$		
palm	4	1			
feet			1		
pace				1	
mile					1

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16 Why would it be more difficult to include the imperial units of chains and rods in the table in Question **15b**?

17 Generally speaking, why is the metric system easier to use than either the imperial or Roman systems?

ENRICHMENT	—	—	18

Walking paces

18 The Roman pace involves 2 steps, and 1000 of these paces make up a Roman mile. These units would have been used to estimate distances for the Roman armies that spread throughout much of the world during that time.

- **a** Estimate how many paces (i.e. double steps) you would take in 1 kilometre (1000).
- **b** Calculate how many paces you would take to cover 1 kilometre.
- **c** If each pace takes 1 second, find how long it would take to walk from Sydney to Melbourne (about 900 km) non-stop. Convert your answer to number of hours.

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10B Using and converting units of length REVISION

Interactive
Widgets
HOTsheets
Walkthrough

To avoid the use of very large and very small numbers, an appropriate unit is often chosen to measure a length or distance. It may also be necessary to convert units of length. For example, 150 pieces of timber, each measured in centimetres, may need to be communicated as a total length using metres. Another example might be that 5 millimetres is to be cut from a length of timber 1.4 metres long because it is is too wide to fit a door opening that is 139.5 centimetres wide.

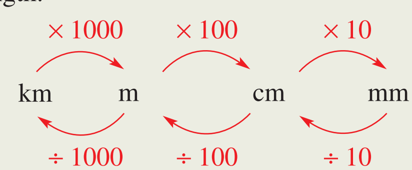
Let's start: How good is your estimate?

In less than 3 seconds, guess the length of your desk, in centimetres.

- Now use a ruler to find the actual length in centimetres.
- Convert your answer to millimetres and metres.
- If you lined up all the class desks end to end, how many desks would be needed to reach 1 kilometre? Explain how you got your answer.

Key ideas

- The **metre** (m) is the basic metric unit of length.
  - 1 km = 1000 m
  - 1 m = 100 cm
  - 1 m = 10 cm



- Conversion
  - When converting to a smaller unit, multiply by 10 or 100 or 1000. The decimal point appears to move to the right. For example:

    2.3 m = (2.3 × 100) cm
    = 230 cm

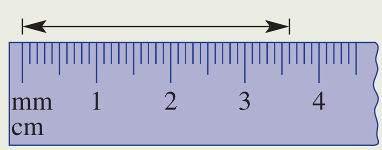
    28 cm = (28 × 10) mm
    = 280 mm

  - When converting to a larger unit, divide by a power of 10 (i.e. 10, 100, 1000). The decimal point appears to move to the left. For example:

    47 m = (47 ÷ 10) cm
    = 4.7 cm

    4600 m = (4600 ÷ 1000) km
    = 4.6 km

- When reading scales, be sure about what units are showing on the scale. This scale shows 36 mm.



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Example 3 Converting metric units of length

Convert to the units given in brackets.

a 3 m (cm) **b** 25 600 cm (km)

SOLUTION

a 3 m = 3 × 100 cm
= 300 cm

b 25 600 cm = 25 600 ÷ 100 000
= 0.256 km

EXPLANATION

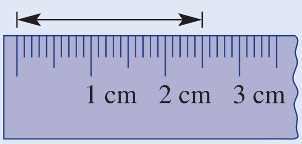
1 m = 100 cm
Multiply since you are converting to a smaller unit.

There are 100 cm in 1 m and 1000 m in 1 km and 100 × 1000 = 100 000.

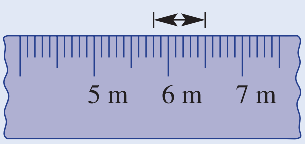
Example 4 Reading length scales

Read the scales on these rulers to measure the marked length.

a



b



SOLUTION

a 25 mm

b 70 cm

EXPLANATION

2.5 cm is also accurate.

Each division is $\frac{1}{10}$ of a metre, which is 10 cm.

Exercise 10B REVISION

UNDERSTANDING AND FLUENCY	1–6, 7(½)	3, 4(½), 5, 6, 7(½), 8	4–7(½), 8, 9

1 Write down the missing number or word in these sentences.
- **a** When converting from metres to centimetres, you multiply by ____________.
- **b** When converting from metres to kilometres, you divide by _______________________.
- **c** When converting from centimetres to metres, you _________________ by 100.
- **d** When converting from kilometres to metres, you _________________ by 1000.
- **e** When converting to a smaller unit, you _________________.
- **f** When converting to a larger unit, you _________________.

2 Calculate each of the following.
a 100 × 10 **b** 10 × 100 **c** 100 × 1000 **d** 10 × 100 × 1000

3 a When multiplying by a positive power of 10, in which direction does the decimal point appear to move – left or right?
b When dividing by a positive power of 10, in which direction does the decimal point appear to move – left or right?

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Example 3a

4 Convert these measurements to the units shown in brackets.

a 5 cm (mm)	**b** 2 m (cm)	**c** 3.5 km (m)	**d** 26.1 m (cm)
e 40 mm (cm)	**f** 500 cm (m)	**g** 4200 m (km)	**h** 472 mm (cm)
i 6.84 m (cm)	**j** 0.02 km (m)	**k** 9261 mm (cm)	**l** 4230 m (km)

5 Add these lengths together and give the result in the units shown in brackets.

a 2 cm and 5 mm (cm)	**b** 8 cm and 2 mm (mm)	**c** 2 m and 50 cm (m)
d 7 m and 30 cm (cm)	**e** 6 km and 200 m (m)	**f** 25 km and 732 m (km)

Example 3b

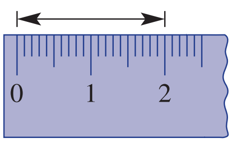
6 Convert to the units shown in the brackets.

a 3 m (mm)	**b** 6 km (cm)
c 2.4 m (mm)	**d** 0.04 km (cm)
e 47 000 cm (km)	**f** 913 000 mm (m)
g 216 000 mm (km)	**h** 0.5 mm (m)

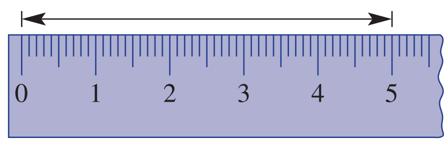
Example 4

7 These rulers show centimetres with millimetre divisions. Read the scale to measure the marked length.

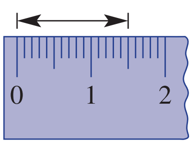
a



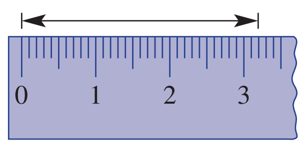
b



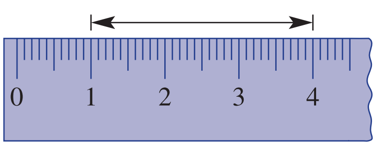
c



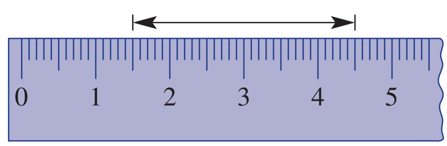
d



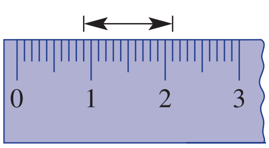
e



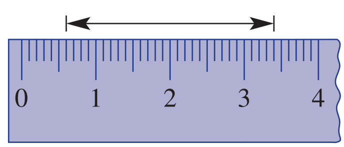
f



g

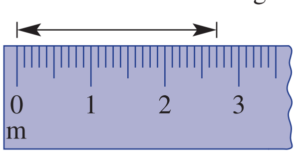


h

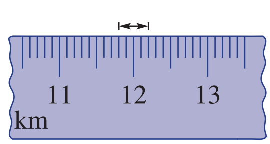


8 Read the scale on these diagrams. Be careful with the units shown!

a



b



9 Use subtraction to find the difference between the measurements, and give your answer with the units shown in brackets.

a 9 km, 500 m (km)

b 3.5 m, 40 cm (cm)

c 0.2 m, 10 mm (cm)

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PROBLEM-SOLVING AND REASONING	10–12, 18	12–14, 18	15–19

10 Arrange these measurements from smallest to largest.

a 38 cm, 540 mm, 0.5 m
b 0.02 km, 25 m, 160 cm, 2100 mm
c 0.003 km, 20 m, 3.1 m, 142 mm
d 0.001 km, 0.1 m, 1000 cm, 10 mm

11 Joe widens a 1.2 m doorway by 50 mm. What is the new width of the doorway, in centimetres?

12 Three construction engineers individually have plans to build the world's next tallest tower. The Titan tower is to be 1.12 km tall, the Gigan tower is to be 109 500 cm tall and the Bigan tower is to be 1210 m tall. Which tower will be the tallest?

13 Steel chain costs $8.20 per metre. How much does is cost to buy chain of the following lengths?

a 1 km
b 80 cm
c 50 mm

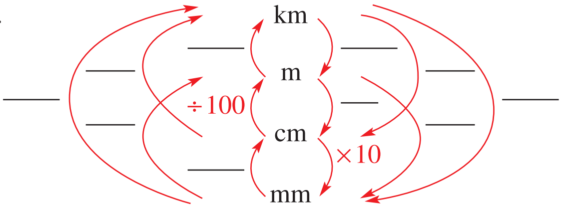
14 A house is 25 metres from a cliff above the sea. The cliff is eroding at a rate of 40 mm per year. How many years will pass before the house starts to fall into the sea?

15 Mount Everest is moving with the Indo-Australian plate at a rate of about 10 cm per year. How many years will it take to move 5 km?

16 A ream of 500 sheets of paper is 4 cm thick. How thick is 1 sheet of paper, in millimetres?

17 A snail slithers 2 mm every 5 seconds. How long will it take to slither 1 m?

18 Copy this chart and fill in the missing information.



19 Many tradespeople measure and communicate with millimetres, even for long measurements like timber beams or pipes. Can you explain why this might be the case?

ENRICHMENT	—	—	20

Very long and short lengths

20 When 1 metre is divided into 1 million parts, each part is called a **micrometre** (μm). At the other end of the spectrum, a **light year** is used to describe large distances in space.

a State how many micrometres there are in:

i 1 m
ii 1 cm
iii 1 mm
iv 1 km

b A virus is 0.000312 mm wide. How many micrometres is this?

c Research the length called the light year. Explain what it is and give examples of distances using light years, such as to the nearest star other than the Sun.

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10C Perimeter of rectilinear figures

Interactive

Widgets

HOTsheets

Walkthrough

The distance around the outside of a two-dimensional shape is called the perimeter. The word *perimeter* comes from the Greek words *peri*, meaning 'around', and *metron*, meaning 'measure'. We associate perimeter with the outside of all sorts of regions and objects, like the length of fencing surrounding a block of land or the length of timber required to frame a picture.

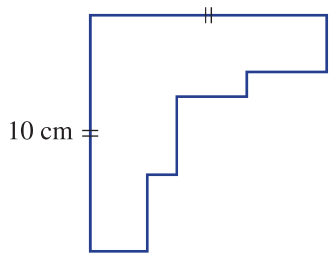
Below is a list of words that are used to describe measurements of shapes.

- dimensions
- length (l)
- width (w)
- breadth (b)
- base (b)
- height (h)
- side (s)
- perpendicular height (h)

The 'length' could be the longer side or the shorter side of a rectangle.

Let's start: Is there enough information?

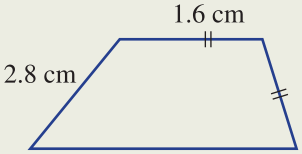
This diagram, which is not drawn to scale, includes only 90° angles and only one side length is given. Discuss if there is enough information given in the diagram to find the perimeter of the shape. What additional information, if any, is required?



Key ideas

- **Perimeter**, sometimes denoted as P, is the distance around the outside of a two-dimensional shape.
- Sides with the same markings are of equal length.
- The unknown lengths of some sides can sometimes be determined by considering the given lengths of other sides.

  For example:



$$P = 1.6 + 1.6 + 2.8 + 4.1$$
$$= 10.1 \text{ cm}$$

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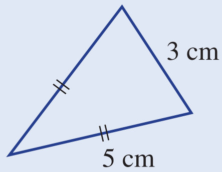
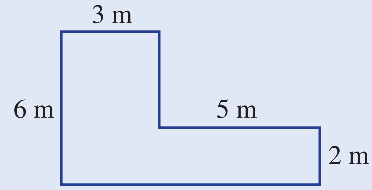
Example 5 Finding the perimeter

Find the perimeter of each of these shapes.

a



b



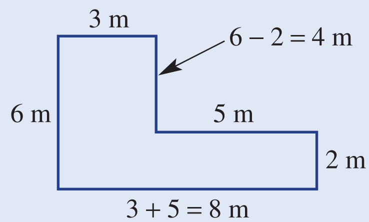
SOLUTION

a Perimeter $= 2 \times 5 + 3$
$= 13$ cm

b Perimeter $= 2 \times 6 + 2 \times 8$
$= 28$ m

EXPLANATION

There are two equal lengths of 5 cm and one length of 3 cm.

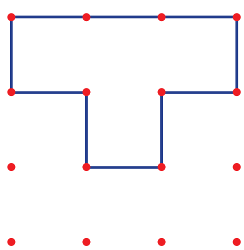


Exercise 10C

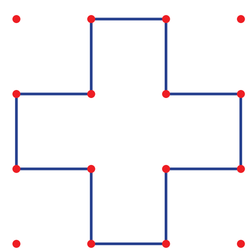
UNDERSTANDING AND FLUENCY	1–4	2–5	3(½), 4–5

1 These shapes are drawn on 1 cm grids. Give the perimeter of each.

a

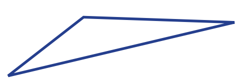


b

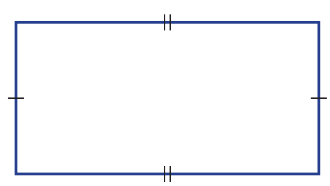


2 Use a ruler to measure the lengths of the sides of these shapes, and then find the perimeter.

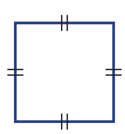
a



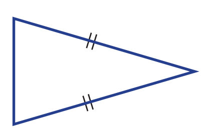
b



c



d



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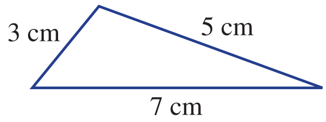
Example 5a

3 Find the perimeter of each of these shapes. (Diagrams are not drawn to scale.)

a



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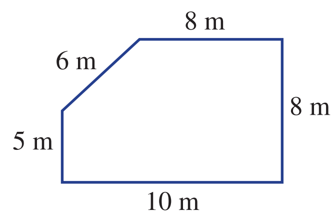
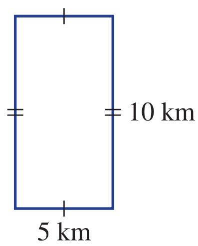
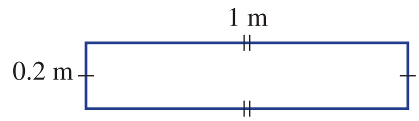


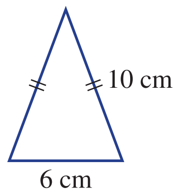
c



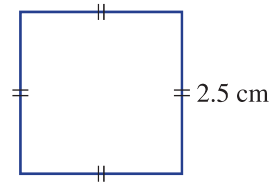
d



e



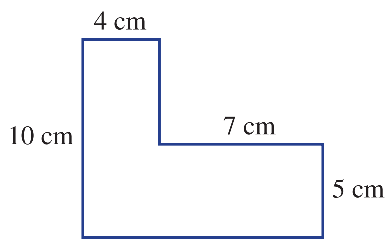
f



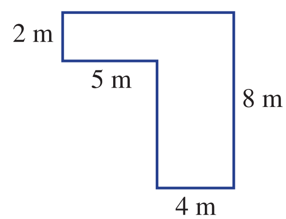
Example 5b

4 Find the perimeter of each of these shapes. All corner angles are 90°.

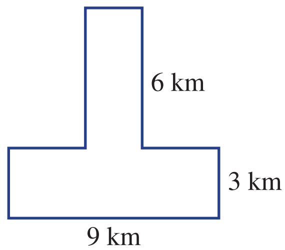
a



b



c



5 **a** A square has a side length of 2.1 cm. Find its perimeter.

b A rectangle has a length of 4.8 m and a width of 2.2 m. Find its perimeter.

c An equilateral triangle has all sides the same length. If each side is 15.5 mm, find its perimeter.

PROBLEM-SOLVING AND REASONING	6, 7, 13	7–9, 13, 14	10–12, 14, 15

6 A grazing paddock is to be fenced on all sides. It is rectangular in shape, with a length of 242 m and a breadth of 186 m. If fencing costs $25 per metre, find the cost of fencing required.

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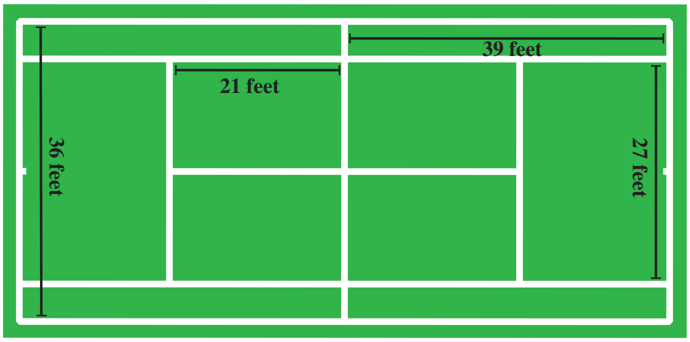
7 A grass tennis court is lined with chalk. All the measurements are shown in the diagram and given in feet.

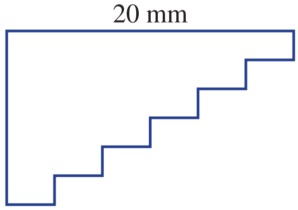
a Find the total number of feet of chalk required to do all the lines of the given tennis court.

b There are 0.305 metres in 1 foot. Convert your answer to part **a** to metres.

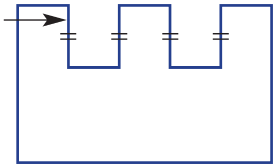


8 Only some side lengths are shown for these shapes. Find the perimeter of each of these shapes.

a

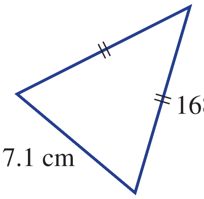


b



9 Find the perimeter of each of these shapes. Give your answers in centimetres.

a



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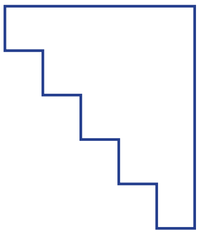

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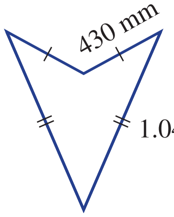
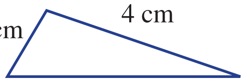


10 A square paddock has 100 equally-spaced posts that are 4 metres apart, including one in each corner. What is the perimeter of the paddock?

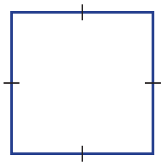
11 The perimeter of each shape is given. Find the missing length of each.

a



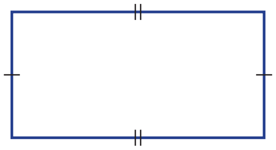
$P = 11$ cm

b



$P = 20$ m

c



$P = 38$ km

12 A rectangle has a perimeter of 16 cm. Using only whole numbers for the length and width, how many different rectangles can be drawn? Do not count rotations of the same rectangle.

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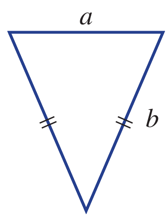
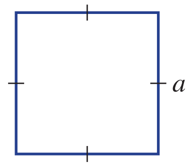
13 Write an equation (e.g. $P = 2a + b$) to describe the perimeter of each shape.

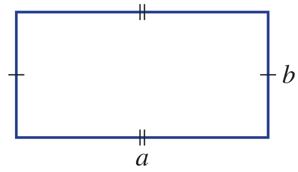
a



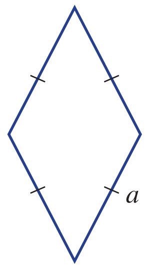
b



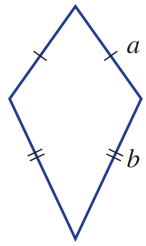
c



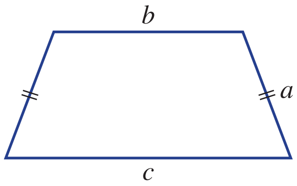
d



e

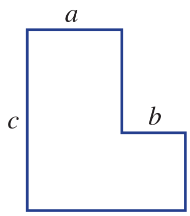


f

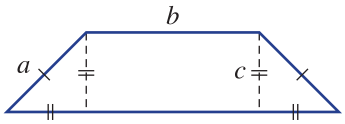


14 Write an algebraic rule for the perimeter of each given shape.

a



b



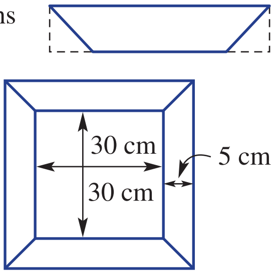
15 **a** A square has perimeter P. Write an expression for its side length.

b A rectangle has perimeter P and width a. Write an expression for its length.

ENRICHMENT	—	—	16

Picture frames

16 The amount of timber used to frame a picture depends on the outside lengths of the overall frame. These are then cut at 45° to make the frame.



a A square painting of side length 30 cm is to be framed with timber of width 5 cm. Find the total length of timber required for the job.

b A rectangular photo with dimensions 50 cm by 30 cm is framed with timber of width 7 cm. Find the total length of timber required to complete the job.

c Kimberley uses 2 m of timber of width 5 cm to complete a square picture frame. What is the side length of the picture?

d A square piece of embroidery has side length a cm and is framed by timber of width 4 cm. Write an expression for the total amount of timber used.

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10D Pi and circumference of circles

Interactive
Widgets
HOTsheets
Walkthrough

Since the ancient times, people have known about a special number that links a circle's diameter to its circumference. We know this number as pi (π). Pi is a mathematical constant that appears in formulas relating to circles, but it is also important in many other areas of mathematics. The actual value of pi has been studied and approximated by ancient and more modern civilisations over thousands of years.

The Egyptians knew pi was slightly more than 3 and approximated it to be $\frac{356}{81} \approx 3.16$. The Babylonians used $\frac{25}{8} \approx 3.125$ and the ancient Indians used $\frac{339}{108} \approx 3.139$.

It is believed that Archimedes of Syracus (287–212 BCE) was the first person to use a mathematical technique to evaluate pi. He was able to prove that pi was greater than $\frac{223}{71}$ and less than $\frac{22}{7}$. In 480 CE, the Chinese mathematician Zu Chongzhi showed that pi was close to $\frac{335}{113} \approx 3.1415929$, which is accurate to seven decimal places.

Before the use of calculators, the fraction $\frac{22}{7}$ was commonly used as a good and simple approximation to pi. Interestingly, mathematicians have been able to prove that pi is an irrational number, which means that there is no fraction that can be found that is exactly equal to pi. If the exact value of pi was written down as a decimal, the decimal places would continue forever with no repeated pattern.

Archimedes was able to show that the value of pi was somewhere between $\frac{223}{71}$ and $\frac{22}{7}$.

Let's start: Discovering pi

In the table below, the diameters and circumferences for three circles are given, correct to 2 decimal places. Use a calculator to work out the value of Circumference ÷ Diameter and put your results in the third column. Add your own circle measurements by measuring the diameter and circumference of circular objects, such as a can.

Diameter d (mm)	Circumference C (mm)	$C \div d$
4.46	14.01	
11.88	37.32	
40.99	128.76	
Add your own	Add your own	

- What do you notice about the numbers $C \div d$ in the third column?
- Why might the numbers in the third column vary slightly from one set of measurements to another?
- What rule can you write down which links C with d?

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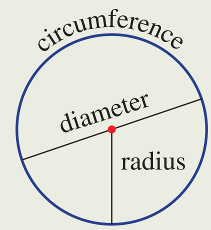
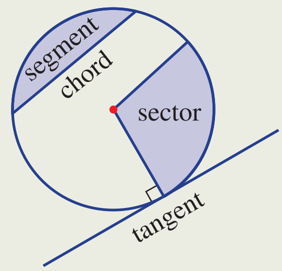
Key ideas

- Features of a circle
  - **Diameter** (d) is the distance across the centre of a circle.
  - **Radius** (r) is the distance from the centre to the circle. Note: $d = 2r$.
  - **Chord**: A line interval connecting two points on a circle.
  - **Tangent**: A line that touches the circle at a point.
    - A tangent to a circle is at right angles to the radius.
  - **Sector**: A portion of a circle enclosed by two radii and a portion of a circle (arc).
  - **Segment**: An area of a circle 'cut off' by a chord.



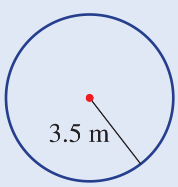
- **Circumference** (C) is the distance around a circle.
  - $C = 2\pi r$ or $C = \pi d$
  - **Pi (π)** is a constant numerical value and is an irrational number, meaning that it cannot be expressed as a fraction.
    - As a decimal, the digits have no pattern and continue forever.
  - The ratio of the circumference to the diameter of any circle is equal to pi (π); i.e. $\pi = \frac{C}{d}$.
  - $\pi = 3.14159$ (correct to 5 decimal places)
    - Common approximations include 3, 3.14 and $\frac{22}{7}$.
    - A more precise estimate for pi can be found on most calculators or on the internet.



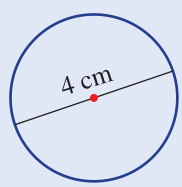
Example 6 Finding the circumference with a calculator

Find the circumference of each of these circles, correct to 2 decimal places. Use a calculator for the value of pi.

a



b



SOLUTION	EXPLANATION
a $C = 2\pi r$ $= 2 \times \pi \times 3.5$ $= 7\pi$ $= 21.99$ m	Since r is given, you can use $C = 2\pi r$. Alternatively, use $C = \pi d$ with $d = 7$. Round off as instructed.
b $C = \pi d$ $= \pi \times 4$ $= 4\pi$ $= 12.57$ cm	Substitute $d = 4$ into the rule $C = \pi d$ or use $C = 2\pi r$ with $r = 2$. Round off as instructed.

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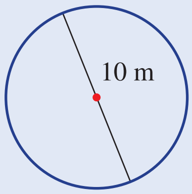
Example 7 Finding the circumference without a calculator

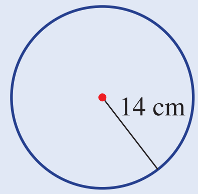
Calculate the circumference of each of these circles using the given approximation of π.

a



$\pi \approx 3.14$

b



$\pi \approx \frac{22}{7}$

SOLUTION

a $C = \pi d$
$= 3.14 \times 10$
$= 31.4$ m

b $C = 2\pi r$
$= 2 \times \frac{22}{7} \times 14$
$= 88$ cm

EXPLANATION

Use $\pi \approx 3.14$ and multiply mentally. Move the decimal point one place to the right. Alternatively, use $C = 2\pi r$ with $r = 5$.

Use $\pi \approx \frac{22}{7}$ and cancel the 14 with the 7 before calculating the final answer.

$2 \times \frac{22}{7} \times 14 = 2 \times 22 \times 2$

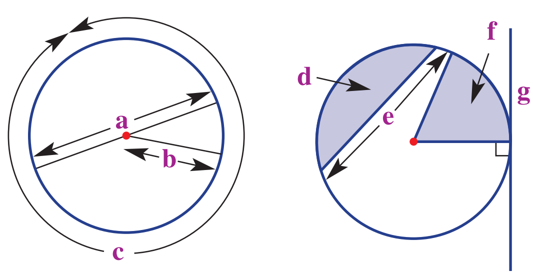
Exercise 10D

UNDERSTANDING AND FLUENCY	1–7	4, 5, 6(½), 7, 8	6(½), 7, 8

1 Evaluate the following using a calculator and round to 2 decimal places.
a $\pi \times 5$ **b** $\pi \times 13$ **c** $2 \times \pi \times 3$ **d** $2 \times \pi \times 37$

2 Write down the value of π, correct to:
a 1 decimal place **b** 2 decimal places **c** 3 decimal places

3 Name the features of the circles shown.



4 A circle has circumference (C) 81.7 m and diameter (d) 26.0 m, correct to 1 decimal place. Calculate $C \div d$. What do you notice?

5 Answer true or false to the following questions.
a The distance from the centre of a circle to its circumference is called the diameter.
b $\pi = \frac{C}{d}$ **c** $C = \pi r$

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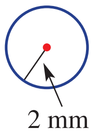
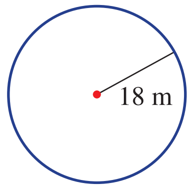
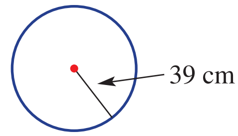
d Some reasonable approximations for pi include 3, 3.14 and $\frac{22}{7}$.
e A tangent to a circle will touch the circle only once.
f A tangent is at 100° to the radius.
g Pi can be expressed exactly as a fraction.

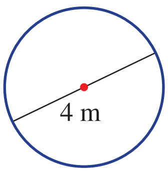
Example 6

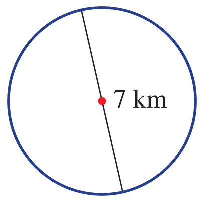
6 Find the circumference of each of these circles, correct to 2 decimal places. Use a calculator for the value of pi.

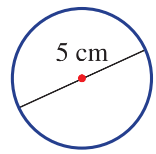
a


b


c


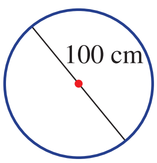
d


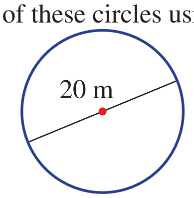
e


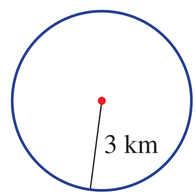
f


Example 7a

7 Calculate the circumference of each of these circles using $\pi \approx 3$.

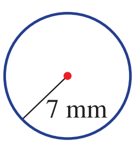
a


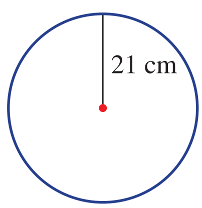
b


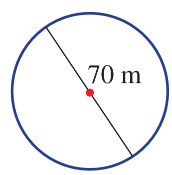
c


Example 7b

8 Calculate the circumference of each of these circles using $\pi \approx \frac{22}{7}$.

a


b


c


PROBLEM-SOLVING AND REASONING

9, 10, 15	10–12, 15, 16	13, 14, 16, 17

9 A water tank has a diameter of 3.5 m. Find its circumference, correct to 1 decimal place.

10 An athlete trains on a circular track of radius 40 m and jogs 10 laps each day, 5 days a week. How far does he jog each week? Round the answer to the nearest whole number of metres.

11 Here are some students' approximate circle measurements. Which students have incorrect measurements?

	r	C
Mick	4 cm	25.1 cm
Svenya	3.5 m	44 m
Andre	1.1 m	1.38 m

12 Which of the following circles has the largest circumference?
A a circle with radius 5 cm
B a circle with diameter 11 cm
C a circle with diameter 9.69 cm
D a circle with radius 5.49 cm

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13 What is the difference in the circumference of a circle with radius 2 cm and a circle with diameter 3 cm? Use 3 as an approximation to pi.

14 Use a mental strategy to calculate the following.

a Using $\pi = 3$, find the diameter of a circle if its circumference is:

i 6 m **ii** 15 cm **iii** 45 km

b Using $\pi = 3$, find the radius of a circle if its circumference is:

i 12 mm **ii** 18 m **iii** 3 cm

15 Explain why the rule $C = 2\pi r$ is equivalent to (i.e. the same as) $C = \pi d$.

16 It is more precise in mathematics to give 'exact' values for circle calculations in terms of π. For example, $C = 2 \times \pi \times 3$ gives $C = 6\pi$. This gives the final exact answer and is not written as a rounded decimal. Find the exact answers for Question **6** in terms of π.

17 We know that $C = 2\pi r$ or $C = \pi d$.

a Rearrange these rules to write a rule for:

i r in terms of C **ii** d in terms of C

b Use the rules you found in part **a** to find the following, correct to 2 decimal places.

i the radius of a circle with circumference 14 m

ii the diameter of a circle with circumference 20 cm

ENRICHMENT	—	—	18

Memorising pi

18 The box below shows π correct to 100 decimal places. The Guinness World record for the most number of digits of π recited from memory is held by Lu Chao, a Chinese student. He recited 67 890 digits non-stop over a 24-hour period.

3.1415926535 8979323846 2643383279 5028841971 6939937510 5820974944 5923078164 0628620899 8628034825 3421170679

Challenge your friends to see who can remember the most number of digits in the decimal representation of π.

Number of digits memorised	Report
10+	A good show
20+	Great effort
35+	Superb
50+	Amazing memory

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10E Arc length and perimeter of sectors and composite figures

Whenever a portion of a circle's circumference is used in a diagram or construction, an arc is formed. To determine the arc's length, the particular fraction of the circle is calculated by considering the angle at the centre of the circle that defines the arc.

Interactive

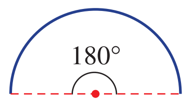
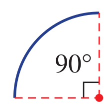
Widgets

HOTsheets

Walkthrough

Let's start

Complete this table to develop the rule for finding an arc length (l).

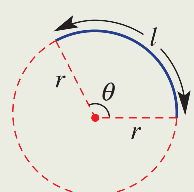
Angle	Fraction of circle	Arc length	Diagram
180°	$\frac{180}{360} = \frac{1}{2}$	$l = \frac{1}{2} \times \pi d$	
90°	$\frac{90}{360} =$ ______	$l =$ ______ $\times$ ______	
45°			
30°			
150°			
θ			

Key ideas

- A circular **arc** is a portion of the circumference of a circle.
  In the diagram:
  - r = radius of circle
  - θ = number of degrees in the angle at the centre of a circle
  - l = **arc length**

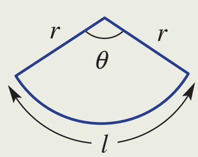
  Formula for arc length:

  $$l = \frac{\theta}{360} \times 2\pi r \text{ or } l = \frac{\theta}{360} \times \pi d$$



- The sector also has two straight edges.
  Formula for perimeter of **sector**:

  $$P = \frac{\theta}{360} \times 2\pi r + 2r \text{ or } P = \frac{\theta}{360} \times \pi d + d$$



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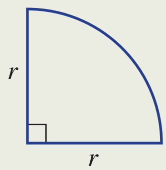
Key ideas

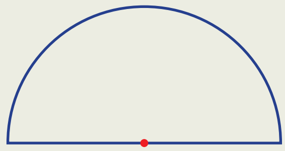
- Common circle portions

quadrant



$P = \frac{1}{4} \times 2\pi r + 2r$

semicircle



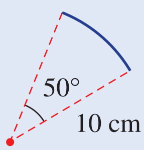
$P = \frac{1}{2} \times 2\pi r + 2r$

- A **composite figure** is made up of more than one basic shape.

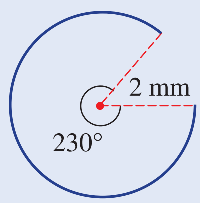
Example 8 Finding an arc length

Find the length of each of these arcs for the given angles, correct to 2 decimal places.

a



b



SOLUTION

a $l = \frac{50}{360} \times 2\pi \times 10$
$= 8.73$ cm

b $l = \frac{230}{360} \times 2\pi \times 2$
$= 8.03$ mm

EXPLANATION

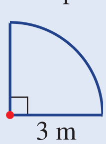
The fraction of the full circumference is $\frac{50}{360}$ and the full circumference is $2\pi r$, where $r = 10$.

The fraction of the full circumference is $\frac{230}{360}$ and the full circumference is $2\pi r$.

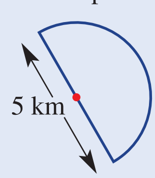
Example 9 Finding the perimeter of a sector

Find the perimeter of each of these sectors, correct to 1 decimal place.

a



b



SOLUTION

a $P = \frac{1}{4} \times 2\pi \times 3 + 2 \times 3$
$= 10.7$ m

b $P = \frac{1}{2} \times \pi \times 5 + 5$
$= 12.9$ km

EXPLANATION

The arc length is one-quarter of the circumference and included are two radii, each of 3 m.

A semicircle's perimeter consists of half the circumference of a circle plus a full diameter.

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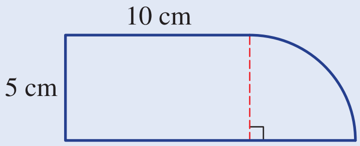
Example 10 Finding the perimeter of a composite shape

Find the perimeter of each of the following composite shape, correct to 1 decimal place.



SOLUTION

$$P = 10 + 5 + 10 + 5 + \frac{1}{4} \times 2\pi \times 5$$
$$= 37.9 \text{ cm}$$

EXPLANATION

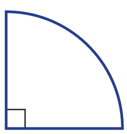
There are two straight sides of 10 cm and 5 cm shown in the diagram. The radius of the circle is 5 cm, so the straight edge at the base of the diagram is 15 cm long. The arc is a quarter circle.

Exercise 10E

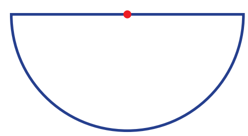
UNDERSTANDING AND FLUENCY	1–5	2, 3, 4–5(½)	4(½), 5(½)

1 What fraction of a circle is shown in these diagrams? Name each shape.

a

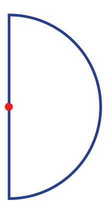


b

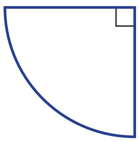


2 What fraction of a circle is shown in these sectors? Simplify your fraction.

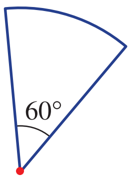
a



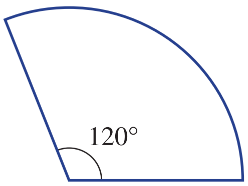
b



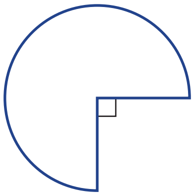
c



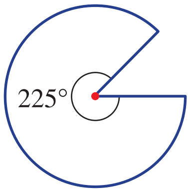
d



e

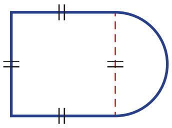


f



3 Name the two basic shapes that make up these composite figures.

a



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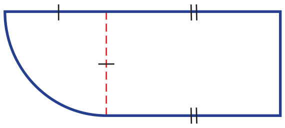
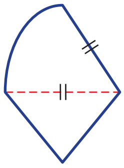


c



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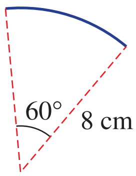
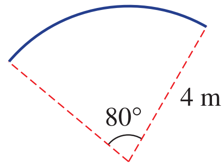
Example 8

4 Find the length of each of the following arcs for the given angles, correct to 2 decimal places.

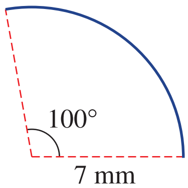
a



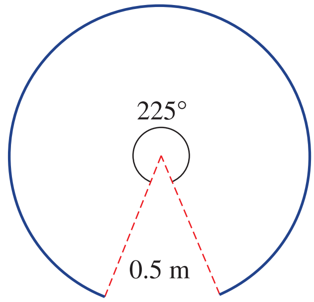
b



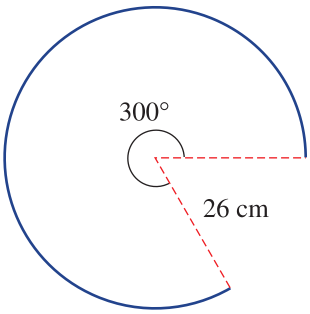
c



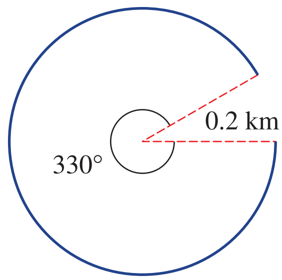
d



e



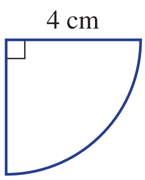
f



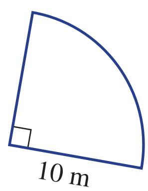
Example 9

5 Find the perimeter of each of these sectors, correct to 1 decimal place.

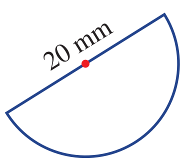
a



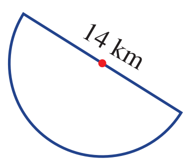
b



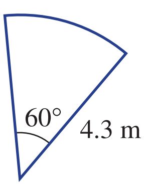
c



d



e



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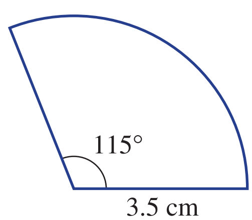

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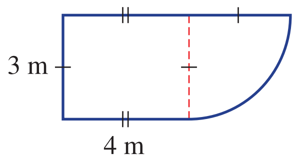
PROBLEM-SOLVING AND REASONING | 6–8 | 6–11 | 6(½), 9–12

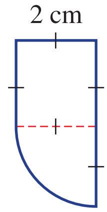
Example 10

6 Find the perimeter of each of these composite shapes, correct to 1 decimal place.

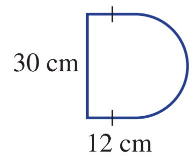
a



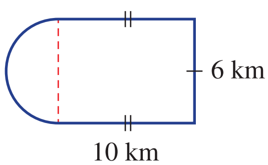
b



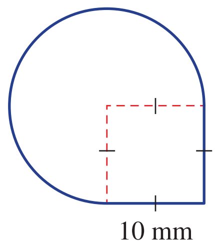
c



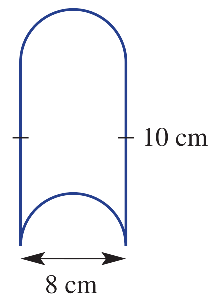
d



e



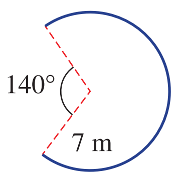
f



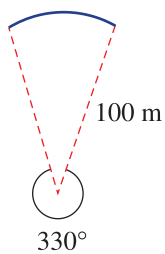
7 A window consists of a rectangular part of length 2 m and breadth 1 m, with a semicircular top having a diameter of 1 m. Find its perimeter, correct to the nearest cm.

8 For these sectors, find only the length of the arc, correct to 2 decimal places.

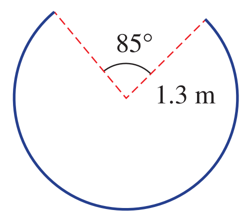
a



b

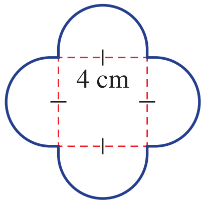


c

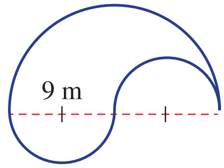


9 Calculate the perimeter of each of these shapes, correct to 2 decimal places.

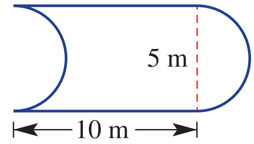
a



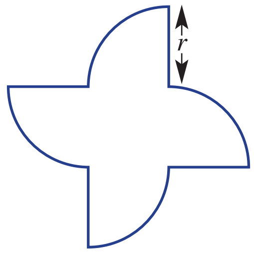
b



c



10 Give reasons why the circumference of this composite shape can be found by simply using the rule $P = 2\pi r + 4r$.



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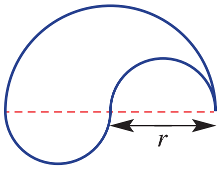
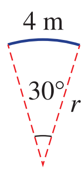
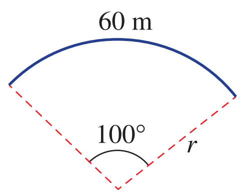
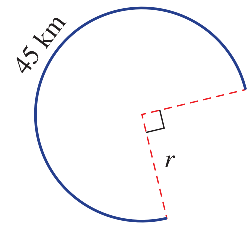
11 Explain why the perimeter of this shape is given by $P = 2\pi r$.



12 Find the radius of each of these sectors for the given arc lengths, correct to 1 decimal place.

a


b


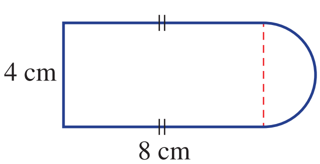
c


ENRICHMENT	—	—	13, 14

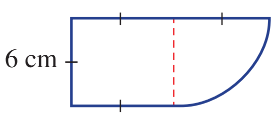
Exact values and perimeters

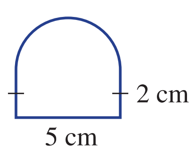
13 The working to find the exact perimeter of this composite shape is given by:

$$P = 2 \times 8 + 4 + \frac{1}{2}\pi \times 4$$
$$= 20 + 8\pi \text{ cm}$$



Find the exact perimeter of each of the following composite shapes.

a


b


14 Find the exact answers for Question **9** in terms of π.

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10F Units of area and area of rectangles

Area is measured in square units. It is often referred to as the amount of space contained inside a flat (i.e. plane) shape; however, curved three-dimensional (3D) solids also have surface areas. The amount of paint needed to paint a house and the amount of chemical needed to spray a paddock are examples of when area would be considered.

Interactive
Widgets
HOTsheets
Walkthrough

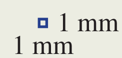
Let's start: The 12 cm^2 rectangle

A rectangle has an area of 12 square centimetres (12 cm^2).

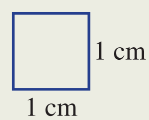
- Draw examples of rectangles that have this area, showing the length and width measurements.
- How many different rectangles with whole number dimensions are possible?
- How many different rectangles are possible if there is no restriction on the type of numbers allowed to be used for length and width?

Key ideas

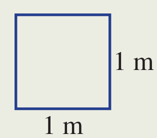
- The metric units of area include:
  - 1 square millimetre (1 mm^2)



  - 1 square centimetre (1 cm^2)
    There are 100 mm^2 in 1 cm^2.

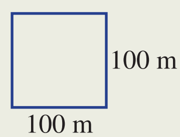


  - 1 square metre 1 m^2
    There are 10 000 cm^2 in 1 m^2.



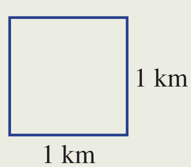
(Not drawn to scale.)

  - 1 hectare (1 ha)
    There are 10 000 m^2 in 1 ha.



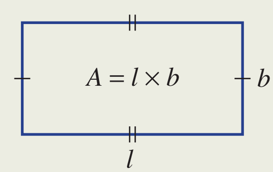
(Not drawn to scale.)

  - 1 square kilometre (1 km^2)
    There are 1 000 000 m^2 in 1 km^2.



(Not drawn to scale.)

- The word dimensions is used for the lenghs of the sides of rectangles. Other words include: length (l), breadth (b), base (b), height (h) and width (w).
- The area of a rectangle is given by the number of rows multiplied by the number of columns. Written as a formula, this looks like $A = l \times b$, where l is the length and b is the breadth (or width). This also works for numbers that are not whole.



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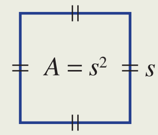
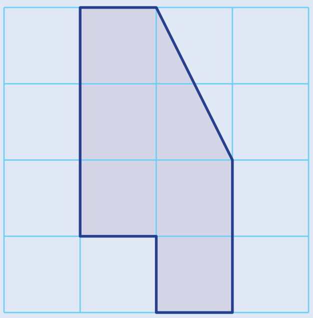
Key ideas

- The area of a square is given by $A = s \times s = s^2$.



Example 11 Counting areas

Count the number of squares to find the area of the shape drawn on this centimetre grid.

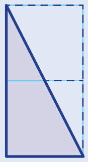


SOLUTION

6 cm^2

EXPLANATION

There are 5 full squares and half of 2 squares in the triangle, giving 1 more.

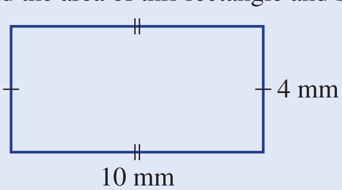


$\frac{1}{2}$ of $2 = 1$

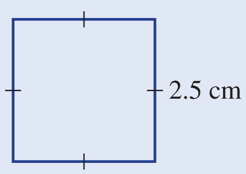
Example 12 Areas of rectangles and squares

Find the area of this rectangle and square.

a



b



SOLUTION

a $\text{Area} = l \times b$
$= 10 \times 4$
$= 40\text{ mm}^2$

EXPLANATION

The area of a rectangle is the product of the length and breath.

SOLUTION

b $\text{Area} = s^2$
$= 2.5^2$
$= 6.25\text{ cm}^2$

EXPLANATION

The width is the same as the length, so $A = s \times s = s^2$.
$(2.5)^2 = 2.5 \times 2.5$

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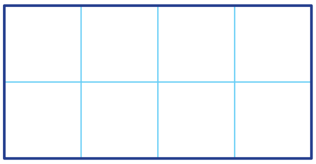
Exercise 10F

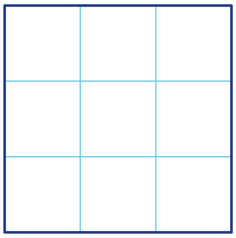
UNDERSTANDING AND FLUENCY	1–5, 6(½), 8, 9	5–6(½), 7–10	5–6(½), 8–11

1 For this rectangle drawn on a 1 cm grid, find each of the following.



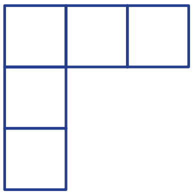
a the number of single 1 cm squares
b the length and the breadth
c length × breadth

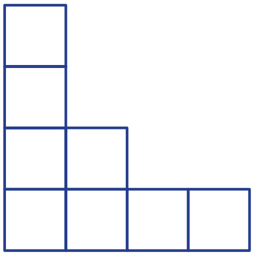
2 For this square drawn on a centimetre grid, find the following.

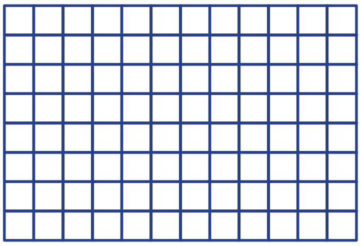


a the number of single 1 cm squares
b the length and the breadth
c length × breadth

3 Count the number of squares to find the area of these shapes.

a


b


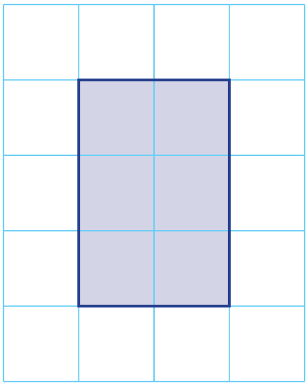
c


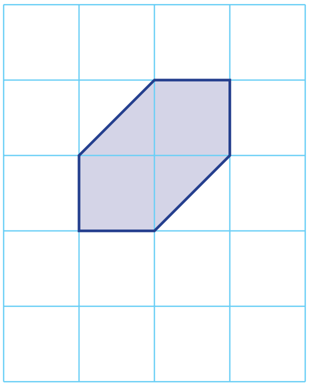
4 Which unit of area (mm^2, cm^2, m^2, ha or km^2) would you choose to measure these areas? Note that 1 km^2 is much larger than 1 ha.

a area of an A4 piece of paper
b area of a wall of a house
c area of a small farm
d area of a large desert
e area of a large football oval
f area of a nail head

Example 11

5 Count the number of squares to find the area of these shapes on centimetre grids.

a


b


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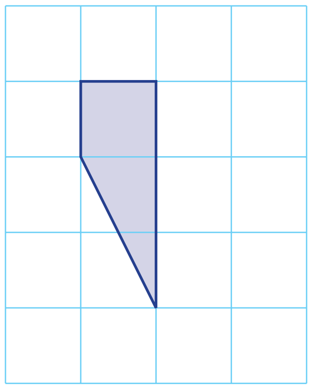
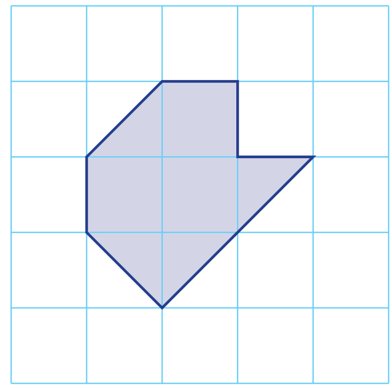
c



d



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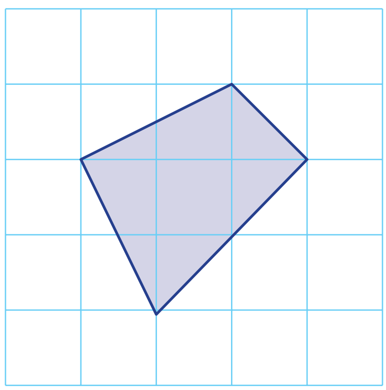

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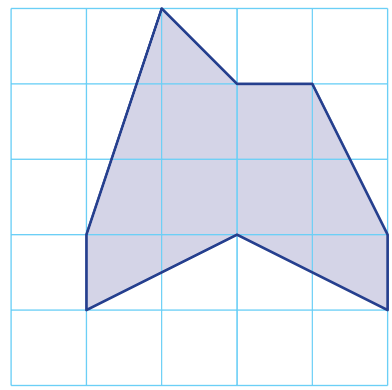
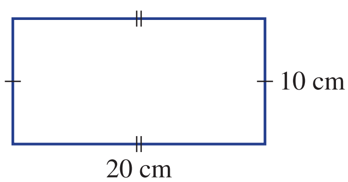


Example 12

6 Find the areas of these rectangles and squares. Diagrams are not drawn to scale.

a



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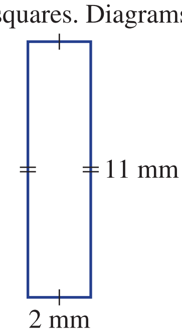

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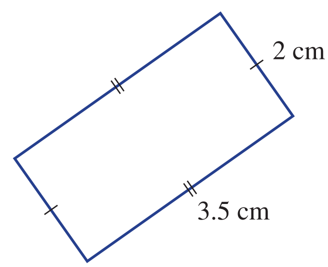

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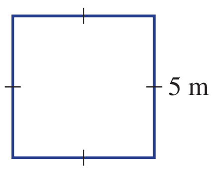

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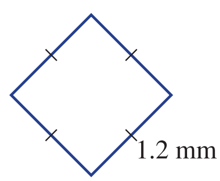

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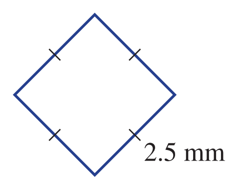

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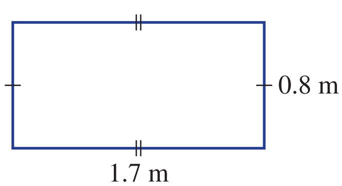

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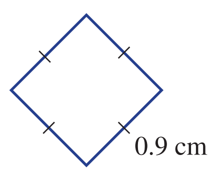
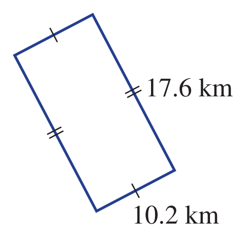


i



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7 Find the side length of a square with each of these areas. Use trial and error if you are unsure.

a $4\ \text{cm}^2$ **b** $25\ \text{m}^2$ **c** $144\ \text{km}^2$

8 There are $10000\ \text{m}^2$ in one hectare (ha). Convert these measurements to hectares.

a $20000\ \text{m}^2$ **b** $100000\ \text{m}^2$ **c** $5000\ \text{m}^2$

9 A rectangular soccer field is to be laid with new grass. The field is 100 m long and 50 m wide. Find the area of grass to be laid.

10 Glass is to be cut for a square window of side length 50 cm. Find the area of glass required for the window.

11 Two hundred square tiles, each measuring 10 cm by 10 cm, are used to tile an open floor area. Find the area of flooring that is tiled.

PROBLEM-SOLVING AND REASONING	12, 13, 16	13, 14, 16, 17	13–15, 17–19

12 **a** A square has a perimeter of 20 cm. Find its area.

b A square has an area of $9\ \text{cm}^2$. Find its perimeter.

c A square's area and perimeter are the same number. How many units is the side length?

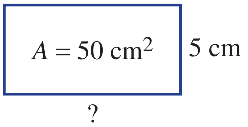
13 The carpet chosen for a room costs \$70 per square metre. The room is rectangular and is 6 m long by 5 m wide. What is the cost of carpeting the room?

14 Troy wishes to paint a garden wall that is 11 m long and 3 m high. Two coats of paint are needed. The paint suitable to do the job can be purchased only in whole numbers of litres and covers an area of $15\ \text{m}^2$ per litre. How many litres of paint will Troy need to purchase?

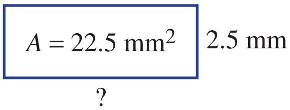
15 A rectangular area of land measures 200 m by 400 m. Find its area in hectares.

16 **a** Find the missing length for each of these rectangles.

i



ii



b Explain the method that you used for finding the missing lengths of the rectangles above.

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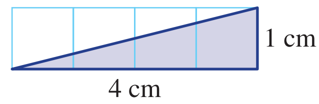
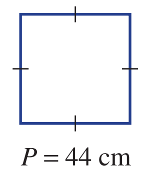
17 Explain why the area shaded here is exactly 2 cm^2.



18 A square has perimeter P cm.

a If $P = 44$ cm, find the area of the square.

b If P is unknown, write an expression for the area of the square, using P.

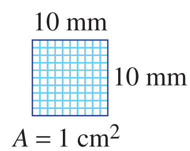


19 A square has all its side lengths doubled. How does this change the area? Investigate and justify your answer.

ENRICHMENT — — 20

Area conversions

20 a Use this diagram or similar to help answer the following.

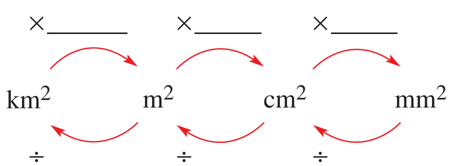


i How many mm^2 in 1 cm^2?

ii How many cm^2 in 1 m^2?

iii How many m^2 in 1 km^2?

b Complete the diagram below.



c Convert these units to the units shown in brackets.

i 2 cm^2 (mm^2)

ii 10 m^2 (cm^2)

iii 3.5 km^2 (m^2)

iv 300 mm^2 (cm^2)

v 21 600 cm^2 (m^2)

vi 4 200 000 m^2 (km^2)

vii 0.005 m^2 (mm^2)

viii 1 km^2 (ha)

ix 40 000 000 cm^2 (ha)

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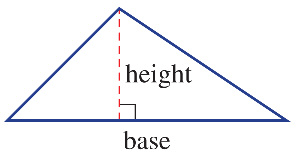
10G Area of triangles

Interactive
Widgets
HOTsheets
Walkthrough

In terms of area, a triangle can be considered to be half a rectangle, which is why the formula for the area of a triangle looks very much like the formula for the area of a rectangle but with the added factor of $\frac{1}{2}$. One of the sides of a triangle is called the base (b), and the height (h) is the distance between the base and the opposite vertex. This is illustrated using a line that is perpendicular (i.e. at 90°) to the base.

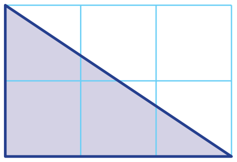


Any shape with all straight sides (i.e. polygons) can be divided up into a combination of rectangles (or squares) and triangles. This can help to find areas of such shapes.

Let's start: Half a rectangle

Look at these triangles. For each one, discuss why the area could be considered as half a rectangle. Give reasons for each case.

1



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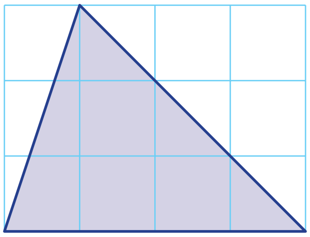

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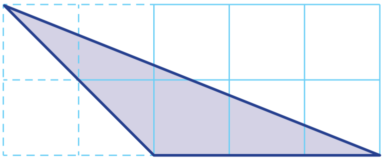
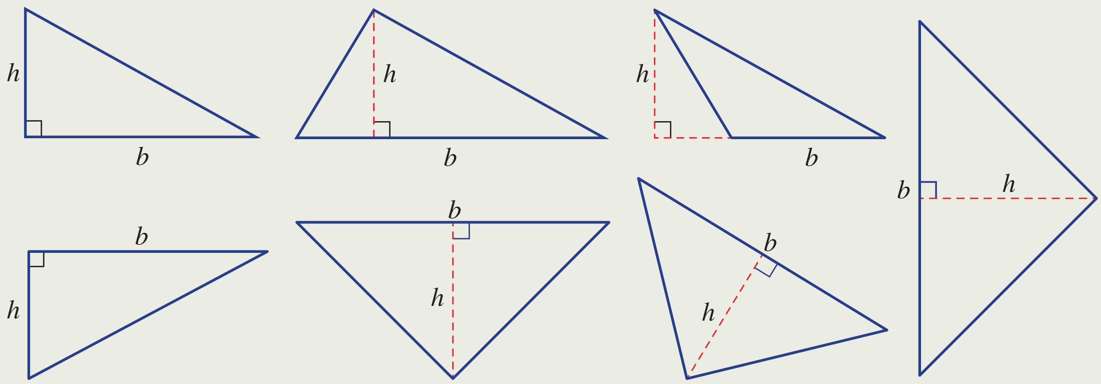


Key ideas

- In rectangles, the dimensions are often called length (l) and width (w).
- In triangles, the dimensions are usually called **base** (b) and **height** (h).
- The base can be *any* side of the triangle.
- The height is **perpendicular** to the base.



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Key ideas

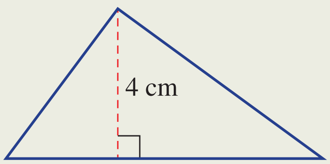
- A triangle is half of a rectangle, so the area is $\frac{1}{2} \times$ base $\times$ height.
- The formula for the area of a triangle is:

$$\boldsymbol{A = \frac{1}{2}bh}$$

A = area
b = length of the base
h = perpendicular height

Note: This can be also written as $A = \frac{bh}{2}$ or $A = b \times h \div 2$.

For example:



$$A = \frac{1}{2} \times 6 \times 4 \quad \text{or} \quad A = \frac{1}{2} \times 6 \times 4 \quad \text{or} \quad A = \frac{1}{2} \times 6 \times 4$$

$$= \frac{1}{2} \times 24 \qquad = 3 \times 4 \qquad = 6 \times 2$$

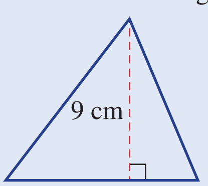
$$= 12\ \text{cm}^2 \qquad = 12\ \text{cm}^2 \qquad = 12\ \text{cm}^2$$

Note: It is alright to 'halve' the product or the base or the height.

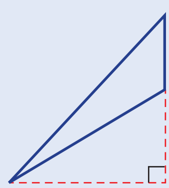
Example 13 Finding areas of triangles

Find the area of each given triangle.

a



b



SOLUTION	EXPLANATION
a Area $= \frac{1}{2}bh$ $= \frac{1}{2} \times 10 \times 9$ $= 45\ \text{cm}^2$	Use the formula and substitute the values for base length and height.
b Area $= \frac{1}{2}bh$ $= \frac{1}{2} \times 6 \times 9$ $= 27\ \text{m}^2$	The length measure of 9 m is marked at 90° to the side marked 6 m. So 6 m is the length of the base and 9 m is the perpendicular height. Any side can be used as the base.

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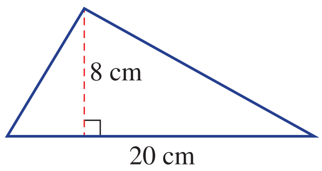
Exercise 10G

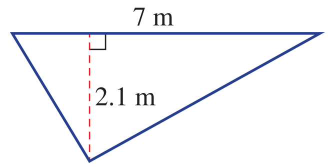
UNDERSTANDING AND FLUENCY	1–5	3, 4(½), 5–7	4(½), 5–7

1 For each of these triangles, what length would be used as the base?

a



b



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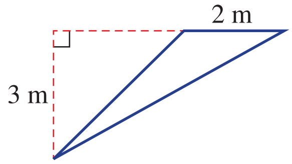

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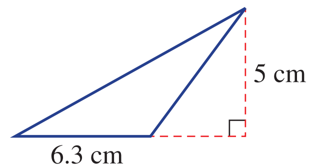
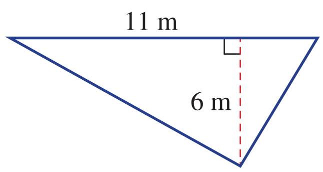


2 For each of these triangles, what length would be used as the height?

a



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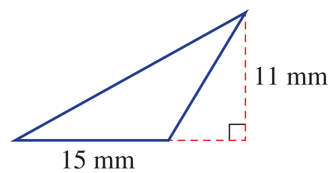

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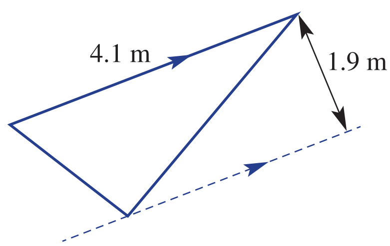

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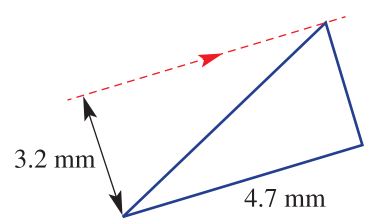


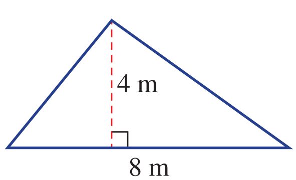
3 Find the value of A in $A = \frac{1}{2}bh$ if:

a $b = 5$ and $h = 4$ **b** $b = 7$ and $h = 16$ **c** $b = 2.5$ and $h = 10$

Example 13

4 Find the area of each triangle given.

a



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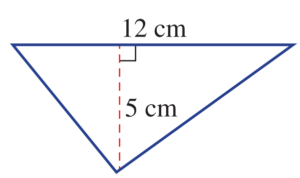

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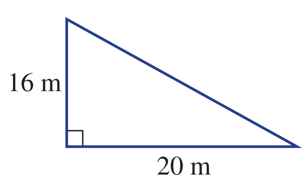

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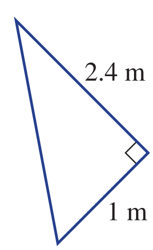

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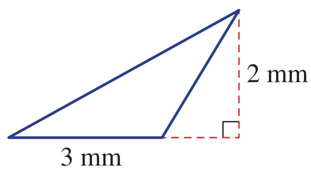

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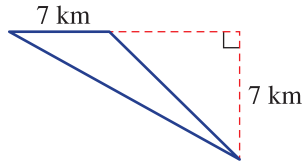

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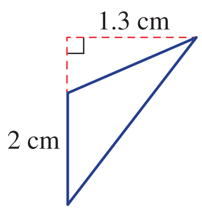

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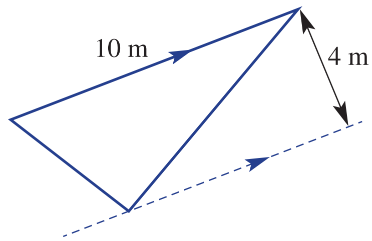

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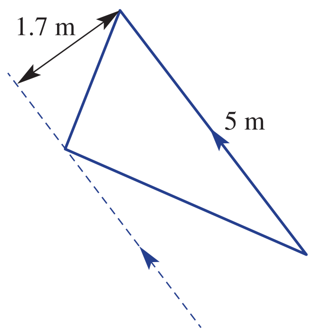
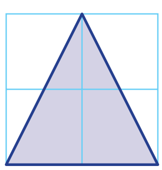


5 Find the area of each of these triangles, which have been drawn on 1 cm grids. Give your answer in cm^2.

a



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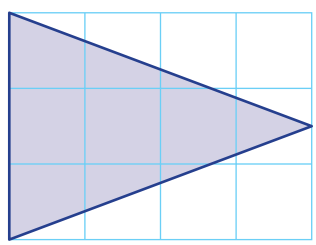

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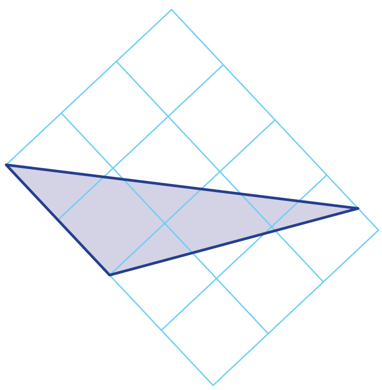

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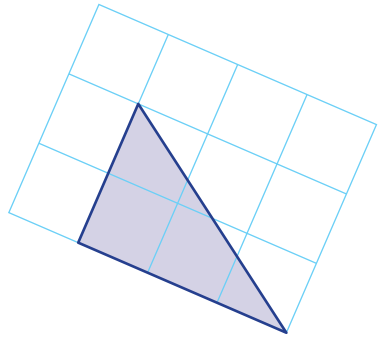

6 A rectangular block of land measuring 40 m long by 24 m wide is cut in half along a diagonal. Find the area of each triangular block of land.

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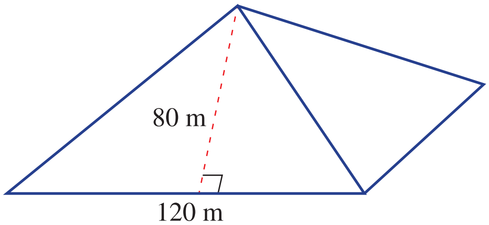
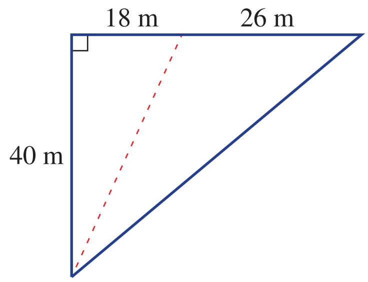
7 A square pyramid has a base length of 120 m and a triangular face of height 80 m. Find the area of one triangular face of the pyramid.



PROBLEM-SOLVING AND REASONING	8, 9, 12	9, 10, 12, 13	9–11, 13, 14

8 Each face of a 4-sided die is triangular, with a base of 2 cm and a height of 1.7 cm. Find the total area of all 4 faces of the die.

9 A farmer uses fencing to divide a triangular piece of land into two smaller triangles, as shown. What is the difference in the two areas?

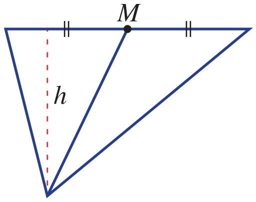


10 A yacht must have two of its sails replaced as they have been damaged by a recent storm. One sail has a base length of 2.5 m and a height of 8 m, and the bigger sail has a base length of 4 m and a height of 16 m. If the cost of sail material is $150 per square metre, find the total cost to replace the yacht's damaged sails.

11 a The area of a triangle is 10 cm^2 and its base length is 4 cm. Find its height.

b The area of a triangle is 44 mm^2 and its height is 20 mm. Find its base length.

12 The midpoint, M, of the base of a triangle joins the opposite vertex. Is the triangle area split in half exactly? Give reasons for your answer.



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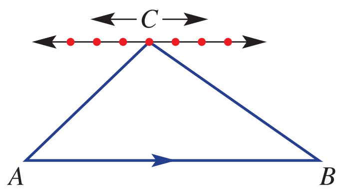
13 If the vertex, C, for this triangle moves parallel to the base AB, will the area of the triangle change? Justify your answer.

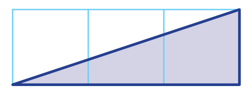


14 The area of a triangle can be found using the formula $A = \frac{1}{2}bh$. Write down the formula to find the base, b, if you are given the area, A, and height, h.

ENRICHMENT	—	—	15

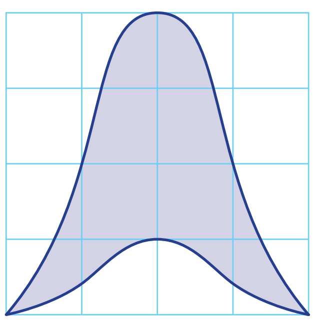
Estimating areas with curves

15 This diagram shows a shaded region that is $\frac{1}{2}$ of $3\text{ cm}^2 = 1.5\text{ cm}^2$.

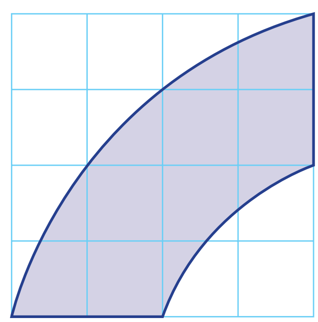


Using triangles like the one shown here, and by counting whole squares also, estimate the areas of these shapes below.

a



b



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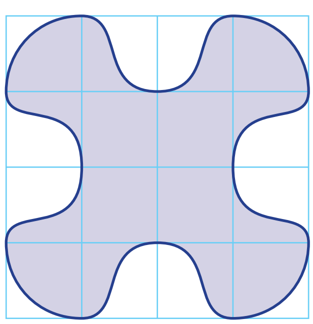

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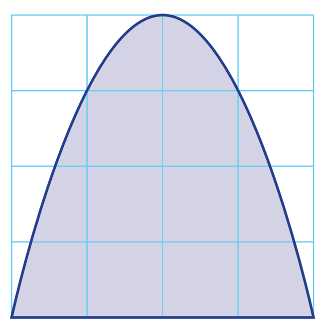

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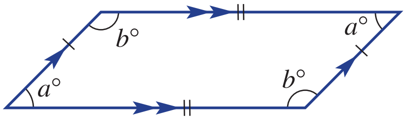
10H Area of parallelograms

Recall that a parallelogram is a quadrilateral with two pairs of parallel sides. Opposite sides are of the same length and opposite angles are equal.



Like a triangle, the area of a parallelogram is found by using the length of one side (called the base) and the height (which is perpendicular to the base.)

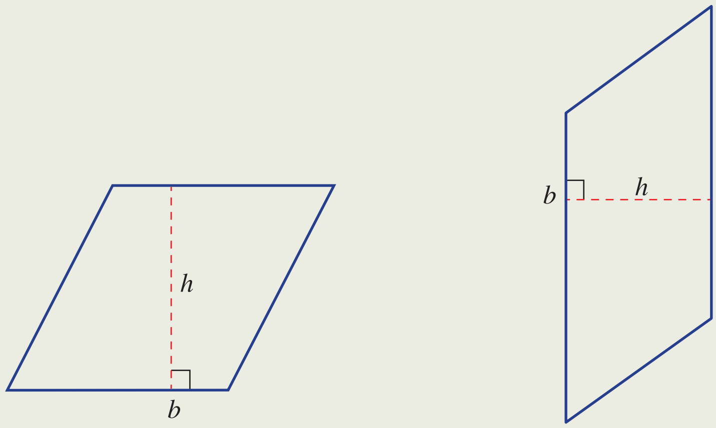
Let's start: Developing the rule

Start this activity by drawing a large parallelogram on a loose piece of paper. Ensure the opposite sides are parallel and then use scissors to cut it out. Label one side as the base and label the height, as shown in the diagram.

- Cut along the dotted line.
- Now shift the triangle to the other end of the parallelogram to make a rectangle.
- Now explain how to find the area of a parallelogram.

Key ideas

- A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel
- For parallelograms, the dimensions are called base (b) and perpendicular height (h).
- The base can be *any* side of the parallelogram.
- The height is perpendicular to the base.



- The area of a parallelogram is base × perpendicular height.
- The formula for the area of a parallelogram is:

$$A = b \times h$$

A = area
b = length of the base
h = perpendicular height

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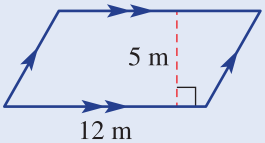
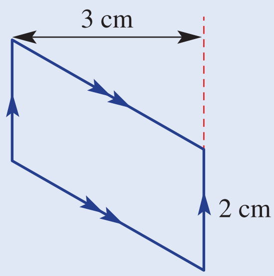
Example 14 Finding the area of a parallelogram

Find the areas of these parallelograms.

a



b



SOLUTION	EXPLANATION
a $A = bh$ $= 12 \times 5$ $= 60\text{ m}^2$	Choose the given side as the base (12 m) and note the perpendicular height is 5 m.
b $A = bh$ $= 2 \times 3$ $= 6\text{ cm}^2$	Use the given side as the base (2 cm), noting that the height is 3 cm.

Exercise 10H

UNDERSTANDING AND FLUENCY	1, 2, 3(½), 4	2–3(½), 4, 5	3(½), 4, 5

1 Copy and complete the following, using the given values of b and h.

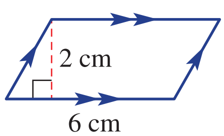
a $b = 5, h = 7$
$A = bh$
$= \_\_ \times \_\_$
$= 35$

b $b = 20, h = 3$
$A = \_\_$
$= 20 \times \_\_$
$= \_\_$

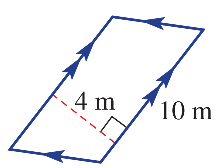
c $b = 8, h = 2.5$
$A = \_\_$
$= 8 \times \_\_$
$= \_\_$

2 For each of these parallelograms, state the side length of the base and the height that might be used to find the area.

a



b



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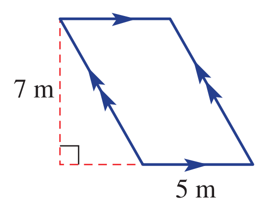

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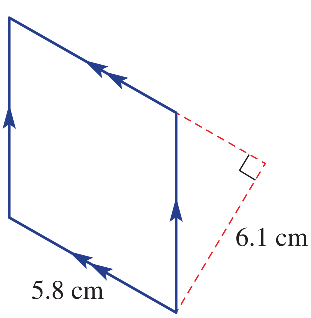

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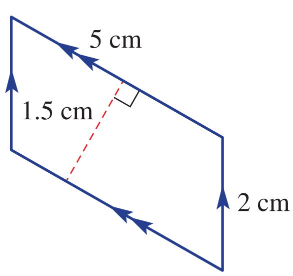

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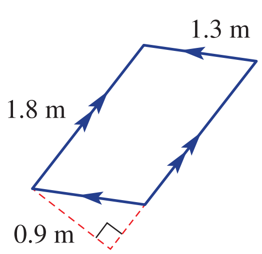

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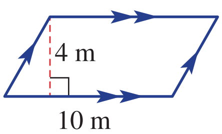
Example 14

3 Find the area of these parallelograms.

a



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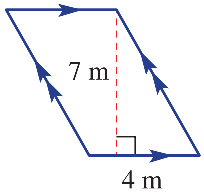

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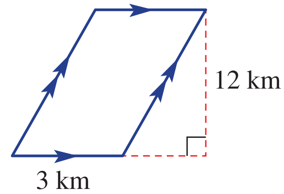

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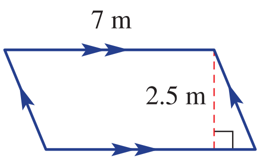

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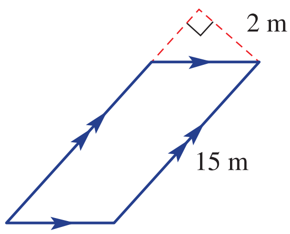

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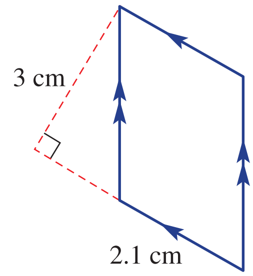
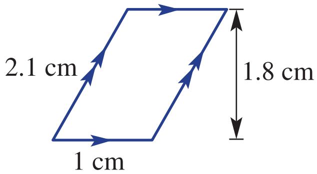
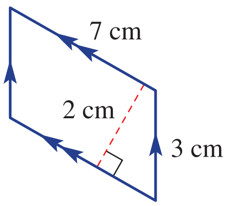


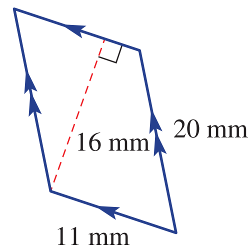
g



h

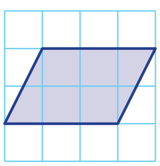


i



4 These parallelograms are on 1 cm grids (not to scale). Find their area.

a



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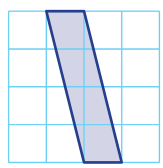

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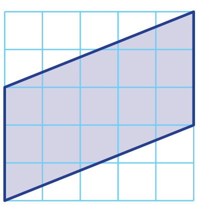

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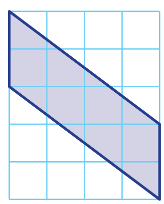

5 The floor of an office space is in the shape of a parallelogram. The longest sides are 9 m and the distance between them is 6 m. Find the area of the office floor.

PROBLEM-SOLVING AND REASONING	6, 7, 10	7, 8, 10, 11	7–9, 11, 12

6 Find the height of a parallelogram when its:

a area $= 10\text{ m}^2$ and base $= 5\text{ m}$

b area $= 28\text{ cm}^2$ and base $= 4\text{ cm}$

c area $= 2.5\text{ mm}^2$ and base $= 5\text{ mm}$

7 Find the base of a parallelogram when its:

a area $= 40\text{ cm}^2$ and height $= 4\text{ cm}$

b area $= 150\text{ m}^2$ and height $= 30\text{ m}$

c area $= 2.4\text{ km}^2$ and height $= 1.2\text{ km}$

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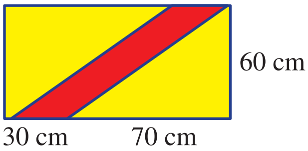
8 A large wall in the shape of a parallelogram is to be painted with a special red paint, which costs $20 per litre. Each litre of paint covers 5 m^2. The wall has a base length of 30 m and a height of 10 m. Find the cost of painting the wall.

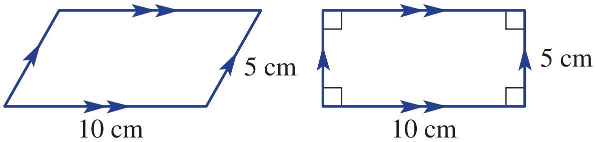
9 A proposed rectangular flag for a new country is yellow with a red stripe in the shape of a parallelogram, as shown. Find:



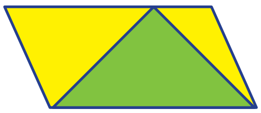
a the area of the red stripe

b the yellow area

10 Explain why this parallelogram's area will be less than the given rectangle's area.

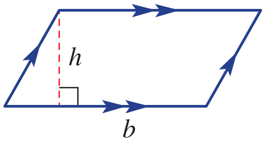


11 A parallelogram includes a green triangular area, as shown. What fraction of the total area is the green area? Give reasons for your answer.



12 The area of a parallelogram can be thought of as twice the area of a triangle. Use this idea to complete this proof of the rule for the area of a parallelogram.

Area = twice triangle area
= 2 × ________
= ________



ENRICHMENT	—	—	13

Glass façade

13 The Puerta de Europa (Gate of Europe) towers are twin office buildings in Madrid, Spain. They look like normal rectangular glass-covered skyscrapers but they lean towards each other at an angle of 15° to the vertical. Two sides are parallelograms and two sides are rectangles. Each tower has a vertical height of 120 m, a slant height of 130 m and a square base of side 50 m.

All four sides are covered with glass. If the glass costs $180 per square metre, find the cost of covering *one* of the towers with glass. (Assume the glass covers the entire surface, ignoring the beams.)

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10I Area of composite figures

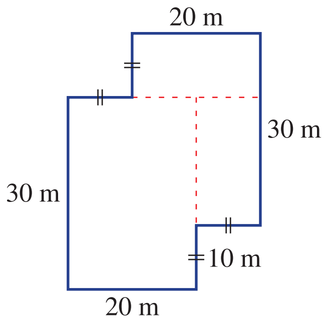
Interactive
Widgets
HOTsheets
Walkthrough

The areas of more complicated shapes can be found by dividing them up into more simple shapes, such as the rectangle and triangle. We can see this in an aerial view of any Australian city. Such a view will show that many city streets are not parallel or at right angles to each other. As a result, this causes city blocks to form interesting shapes, many of which are composite figures made up of rectangles and triangles.

Let's start: Dividing land to find its area

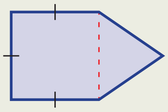
Working out the area of this piece of land could be done by dividing it into three rectangles, as shown.

- Can you work out the area using this method?
- What is another way of dividing the land to find its area? Can you use triangles?
- What is the easiest method to find the area? Is there a way that uses subtraction instead of addition?



Key ideas

- **Composite shapes** are made up of more than one simple shape.
- The area of composite shapes can be found by adding or subtracting the areas of simple shapes.


A square plus a triangle


A rectangle subtract a triangle

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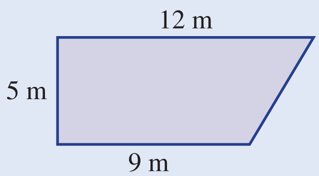
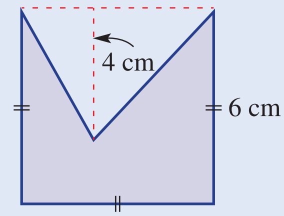
Example 15 Finding the areas of composite shapes

Find the area of each of these composite shapes.

a



b



SOLUTION

a $A = l \times b + \frac{1}{2}bh$

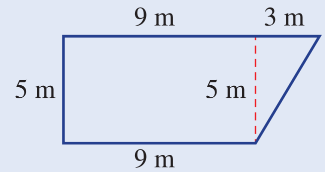
$= 9 \times 5 + \frac{1}{2} \times 3 \times 5$

$= 45 + 7.5$

$= 52.5\,\text{m}^2$

EXPLANATION

Divide the shape into a rectangle and a triangle and find the missing lengths.



SOLUTION

b $A = s^2 - \frac{1}{2}bh$

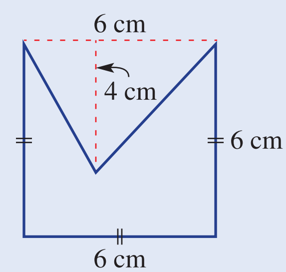
$= 6^2 - \frac{1}{2} \times 6 \times 4$

$= 36 - 12$

$= 24\text{ cm}^2$

EXPLANATION

Subtract the triangle ($\frac{1}{2} \times 6 \times 4$) at the top of the shape from the larger square (6×6).

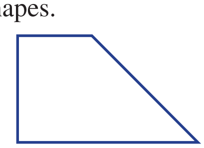


Exercise 10I

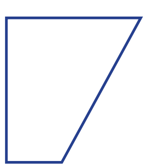
UNDERSTANDING AND FLUENCY	1–5	3, 4–5(½)	4–5(½)

1 Copy these diagrams and draw a dotted line where you might divide these shapes into two simple shapes.

a



b



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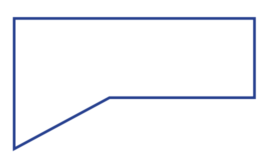
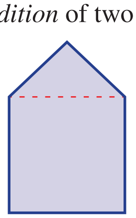
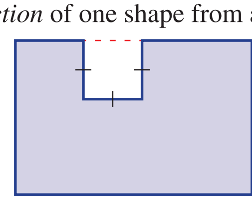


2 To find the area of each of the following shapes, decide if the easiest method would involve the *addition* of two shapes or the *subtraction* of one shape from another.

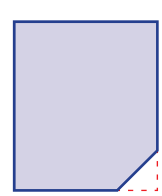
a



b



c



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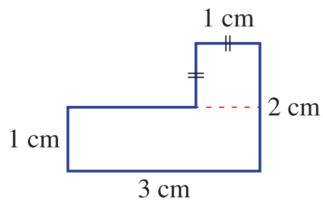
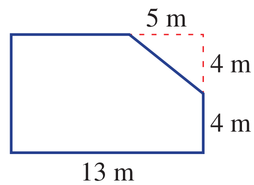
3 Copy and complete the solutions for the areas of these shapes.

a



$A = s^2 + \_\_\_$
$= 1^2 + 3 \times \_\_\_$
$= \_\_\_ + \_\_\_$
$= \_\_\_ \text{cm}^2$

b

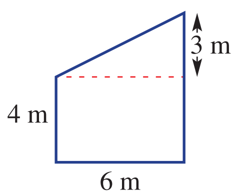


$A = lb - \_\_\_$
$= \_\_\_ \times 8 - \frac{1}{2} \times \_\_\_ \times \_\_\_$
$= \_\_\_ - \_\_\_$
$= \_\_\_ \text{m}^2$

Example 15a

4 Find the areas of these composite figures by adding together the areas of simpler shapes.

a



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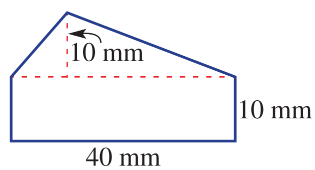

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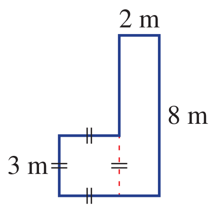

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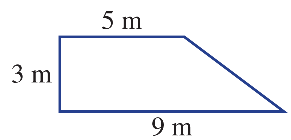

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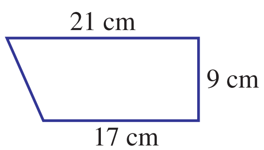

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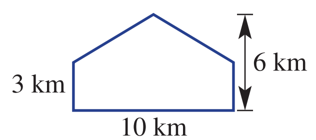
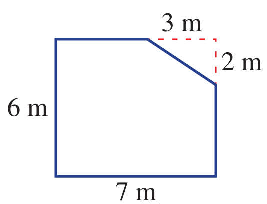


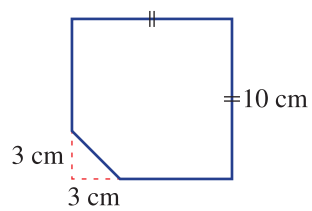
Example 15b

5 Use subtraction to find the areas of these composite shapes.

a



b



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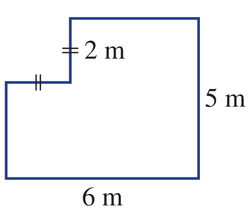

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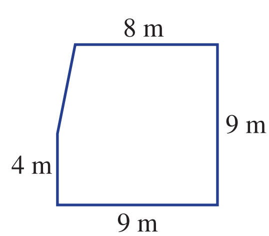

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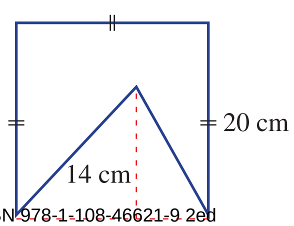

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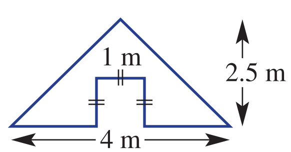

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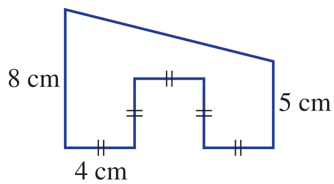
PROBLEM-SOLVING AND REASONING	6, 7, 10	7, 8, 10	8–11

6 Find the areas of these composite figures.

a


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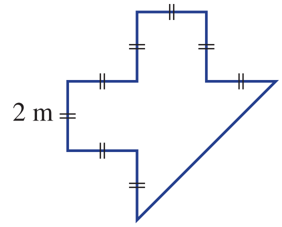

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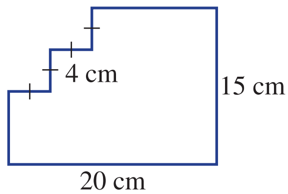
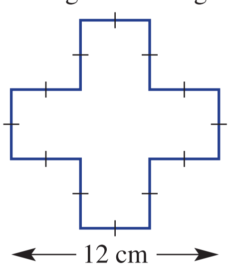


7 By finding the missing lengths first, calculate the areas of these composite shapes.

a


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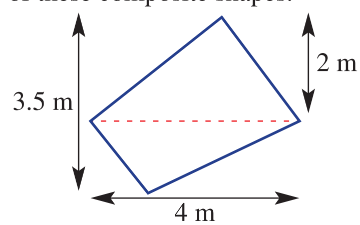

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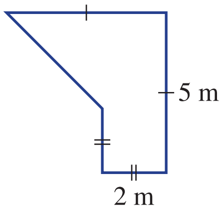

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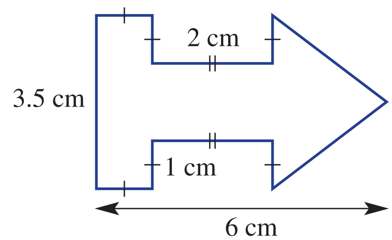
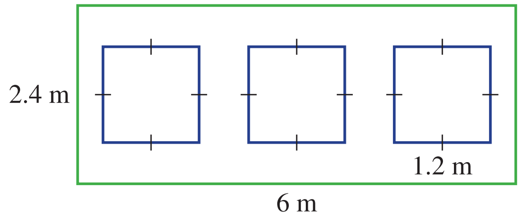
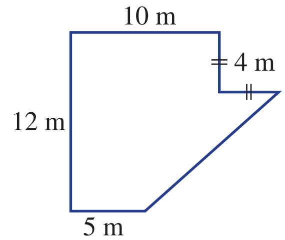


8 A wall has three square holes cut into it to allow for windows, as shown. Find the remaining area of the wall.



9 A factory floor, with dimensions shown opposite, is to be covered with linoleum. Including underlay and installation, the linoleum will cost $25 per square metre. The budget for the job is $3000. Is there enough money in the budget to cover the cost?



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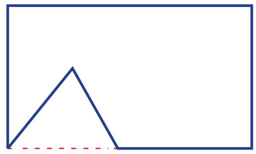
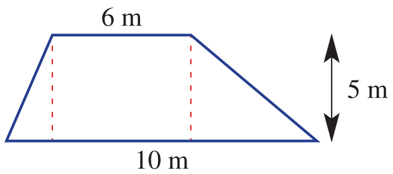
10 Explain why using subtraction is sometimes quicker than using addition to find the area of a composite shape. Refer to the diagram as an example.



11 The 4-sided shape called the trapezium has one pair of parallel sides.

a For the trapezium shown below, is it possible to find the base length of each triangle on the sides? Justify your answer.

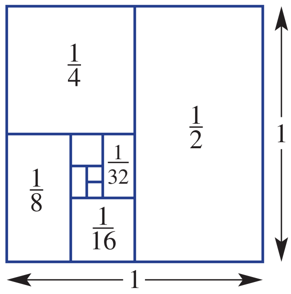


b Can you come up with a method for finding the area of a trapezium using the rectangle and triangles shown in the diagram? Use diagrams to explain your method.

ENRICHMENT	—	—	12

Adding to infinity

12 The square given below, which has an area of 1 unit, is divided to show the areas of $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$



a Similar to the one shown, draw your own square, showing as many fractions as you can. Try to follow the spiral pattern shown. Note: The bigger the square you start with, the more squares you will be able to show.

b **i** Write the next 10 numbers in this number pattern.

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

ii Will the pattern ever stop?

c What is the total area of the starting square?

d What do your answers to parts **b ii** and **c** tell you about the answer to the sum below?

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$ (continues forever)

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10J Mass and temperature REVISION

The scales for both mass and temperature are related to the properties of water. In France in 1795, the gram was defined as being the weight of 1 cm^3 of water at 0°C. Later it was redefined to be the weight at 4°C, as this is considered to be the temperature at which water is the most dense. So, 1 litre of water is very close to 1 kilogram, which is the basic unit for mass. Other units for mass include the tonne, gram and milligram. A small car has a mass of about 1 tonne and a 20-cent coin has a mass of about 11 grams.

Temperature tells us how hot or cold something is. Anders Celsius (1701–1744), a Swedish scientist, worked to define a scale for temperature. After his death, temperature was officially defined by:

- 0°C (0 degrees Celsius) – the freezing point of water.
- 100°C (100 degrees Celsius) – the boiling point of water (at one standard degree of pressure).

This is still the common understanding of degrees Celsius. As mentioned in Chapter 9, Fahrenheit is another scale used for temperature. This is investigated further in Enrichment questions.

Let's start: Choose a unit of mass

Choosing objects for given masses

Name five objects of which their mass would commonly be measured in:

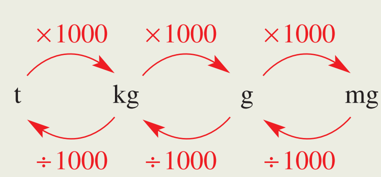
- tonnes
- kilograms
- grams
- milligrams.

Choosing places for given temperatures

Is it possible for the temperature to drop below 0°C? How is this measured and can you give examples of places or situations where this might be the case?

Key ideas

- The basic unit for mass is the **kilogram** (kg). 1 litre of water has a mass of 1 kilogram.
- Metric units for mass include:
  - 1 **gram** (g) = 1000 **milligrams** (mg)
  - 1 kilogram (kg) = 1000 grams (g)
  - 1 **tonne** (t) = 1000 kilograms (kg)



- The common unit for temperature is **degrees Celsius** (°C).
  - 0°C is the freezing point of water.
  - 100°C is the boiling point of water.

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Example 16 Converting units of mass

Convert to the units shown in brackets.

a 2.47 kg (g) **b** 170 000 kg (t)

SOLUTION	EXPLANATION
a $2.47 \text{ kg} = 2.47 \times 1000 \text{ g}$ $= 2470 \text{ g}$	1 kg = 1000 g Multiply because you are changing to a smaller unit.
b $170\,000 \text{ kg} = 170\,000 \div 1000 \text{ t}$ $= 170 \text{ t}$	1 t = 1000 kg Divide because you are changing to a larger unit.

Exercise 10J REVISION

UNDERSTANDING AND FLUENCY	1–3, 4(½), 5–8	3, 4(½), 5–10	4(½), 5, 6(½), 8–11

1 Circle or write down which mass measurements are the same.

a 1 kg, 100 g, 1000 g, 10 t

b 1000 mg, 10 kg, 1 g, 1000 t

2 From options **A** to **F**, choose the mass that best matches the given object.

a human hair — **A** 300 g

b 10-cent coin — **B** 40 kg

c bottle — **C** 100 mg

d large book — **D** 1.5 kg

e large bag of sand — **E** 13 t

f truck — **F** 5 g

3 From options **A** to **D**, choose the temperature that best matches the description.

a temperature of coffee — **A** 15°C

b temperature of tap water — **B** 50°C

c temperature of oven — **C** −20°C

d temperature in Antarctica — **D** 250°C

Example 16

4 Convert to the units shown in brackets.

a 2 t (kg) **b** 70 kg (g)

c 2.4 g (mg) **d** 2300 mg (g)

e 4620 mg (g) **f** 21 600 kg (t)

g 0.47 t (kg) **h** 312 g (kg)

i 27 mg (g) **j** $\frac{3}{4}$ t (kg)

k $\frac{1}{8}$ kg (g) **l** 10.5 g (kg)

m 210 000 kg (t) **n** 0.47 t (kg)

o 592 000 mg (g) **p** 0.08 kg (g)

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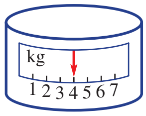
5 Read these mass scales.

a


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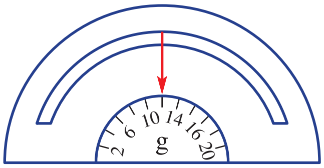

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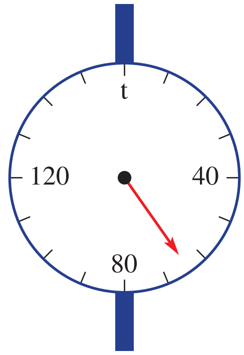
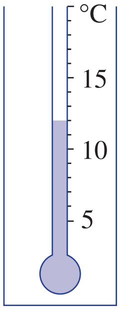


6 Read these temperature scales.

a


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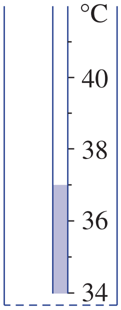

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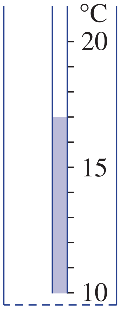

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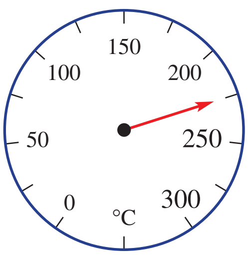

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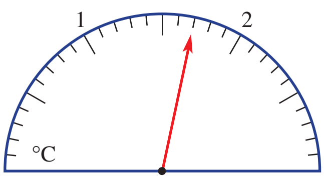

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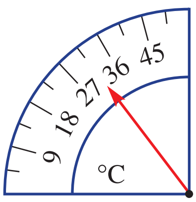

7 A small truck delivers 0.06 t of stone for a garden. Write the mass of stones using these units.

a kg **b** g **c** mg

8 A box contains 20 blocks of cheese, each weighing 150 g. What is the approximate mass of the box in the following units?

a g **b** kg

9 The temperature of water in a cup is initially 95°C. After half an hour the temperature is 62°C. What is the drop in temperature?

10 An oven is initially at a room temperature of 25°C. The oven dial is turned to 172°C. What is the expected increase in temperature?

11 Add all the mass measurements and give the result in kg.

a 3 kg, 4000 g, 0.001 t

b 2.7 kg, 430 g, 930000 mg, 0.0041 t

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PROBLEM-SOLVING AND REASONING	12, 13, 17	13, 14, 16, 17	14–18

12 Arrange these mass measurements from smallest to largest.

a 2.5 kg, 370 g, 0.1 t, 400 mg **b** 0.000 32 t, 0.41 kg, 710 g, 290 000 mg

13 The highest and lowest temperatures recorded over a 7-day period are as follows.

Day	1	2	3	4	5	6	7
Lowest temperature (°C)	8	6	10	9	7	8	10
Highest temperature (°C)	24	27	31	32	21	19	29

a Which day had the largest temperature increase?

b What is the largest temperature drop from the highest temperature on one day to the lowest temperature on the next day?

c What would have been the final temperature on Day 7 if the temperature increase had been 16°C?

14 A 10 kg bag of flour is used at a rate of 200 g per day. How many days will the bag of flour last?

15 A boat has a weight limit of 3.5 t carrying capacity. Loaded onto the boat are 1500 tins of coffee at 500 g each, 36 bags of grain at 20 kg each, 190 boxes of tobacco at 5.5 kg each and 15 people, averaging 80 kg each. Is the load too much for the weight limit of the boat?

16 A truck tare mass (i.e. mass with no load) is 13.2 t. The truck's gross mass is 58.5 t. This is the total maximum mass allowed, including the load.

a What is the maximum load the truck can carry?

b The truck is loaded with 120 timber beams at 400 kg each. Will it exceed its gross weight limit?

17 Water weighs 1 kg per litre. What is the mass of these volumes of water?

a 1 mL

b 1 kL

c 1 ML

18 The kelvin (K) is a temperature unit used by many scientists, where 273 K is approximately 0°C. (The kelvin used to be called the 'degree kelvin' or K.) An increase in 1 K is the same as an increase in 1°C.

a Write the following temperatures in °C.

- **i** 283 K
- **ii** 300 K
- **iii** 1000 K

b Write the following temperatures in kelvins.

- **i** 0°C
- **ii** 40°C
- **iii** −273°C

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ENRICHMENT	—	—	19

Fahrenheit

19 Daniel Fahrenheit (1686–1736) proposed the Fahrenheit temperature scale in 1724. It was commonly used in Australia up until the mid 20th century, and is still used today in the United States.

32°F is the freezing point of water.

212°F is the boiling point of water.

a What is the difference between the temperature, in Fahrenheit, for the boiling point of water and the freezing point of water?

b 1°F is what fraction of 1°C?

c 1°C is what fraction of 1°F?

d To convert from °F to °C, we use the formula $C = (F - 32) \times \frac{5}{9}$. Convert these Fahrenheit temperatures to °C.

i 32°F **ii** 68°F **iii** 140°F **iv** 221°F

e Find the rule to convert from °C to °F. Test your rule to see if it works and write it down.

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Opal mining

Greg, Sally and Alston apply for a mining licence to look for opals at Coober Pedy in South Australia. They are required to choose an area and mark it out with special orange tape so that others will know which areas are already taken. Their length of tape is 200 m.

Square mining areas

They first decide to mark out an area as a square.

a Make a drawing of their square area.

b Calculate the side length and area and show this on your diagram. Also show any working.

Rectangular mining areas

They then change the mining area and experiment with different side lengths.

a Show three possible lengths and areas for rectangular mining sites.

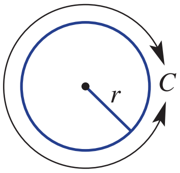
b Complete a table similar to the one below. Fill in the missing numbers for the side lengths given, then add your own rectangle measurements from part **a** above.

Length	Breadth	Perimeter	Area
10		200	
20		200	
35		200	
		200	
		200	
		200	

c Are there any rectangles that give a larger area than the square mining area from above?

Circular mining areas

They now decide to try to arrange the tape to form a circle. For this section you will need the rule to calculate the distance around a circle (circumference). The circumference C is given by $C = 2 \times \pi \times r$, where r is the length of the radius and $\pi \approx 3.14$.



a Calculate the radius of the circle, correct to 1 decimal place. Use a trial and error (guess and check) technique and remember that the circumference will be 200. Explain and show your method using a table of values.

b Calculate the area of the circular mining area, correct to the nearest square metre. Use the special rule for the area of a circle, A, which is given by $A = \pi \times r^2$.

The largest area

Compare the areas marked out with the 200 m tape by Greg, Sally and Alston. Comment on any differences. Which shape gives the largest area for the given perimeter? Would your answer be the same if any shape were allowed to be used? Explain.

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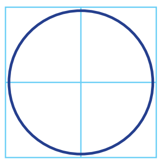
Puzzles and challenges

1 Without measuring, state which line looks longer: A or B? Then measure to check your answer.

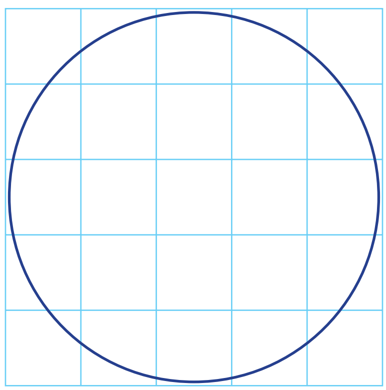
2 You have two sticks of length 3 m and 5 m, both with no scales. How might you mark a length of 1 m?

3 Count squares to estimate the area of these circles, where one grid square = 1 cm.

a

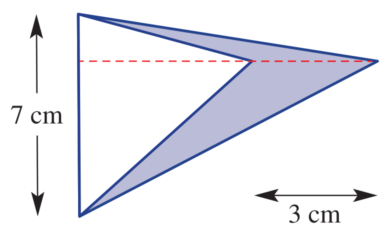


b

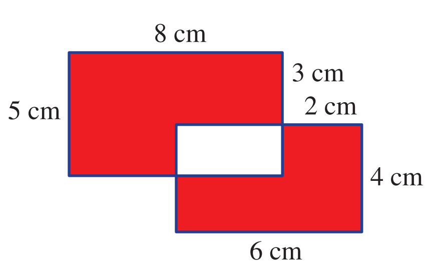


4 A house roof has 500 m^2 of area. If there is 1 mm of rainfall, how much water, in litres, can be collected from the roof?

5 Find the area of the shaded region.



6 These two rectangles overlap, as shown. Find the total area of the shaded region.



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Chapter summary

Measurement

Length

Roman

16 digits = 12 inches = 1 foot
5 feet = 1 pace
1000 paces = 1 mile

Imperial

12 inches = 1 foot
3 feet = 1 yard
1760 yards = 1 mile

Units of length

10 mm = 1 cm
100 cm = 1 m
1000 m = 1 km

Perimeter

2.1 cm, 3 cm, 1.5 cm

$P = 2 \times 2.1 + 1.5 + 3$
$= 8.7$ cm

Area

Parallelogram

2 cm, 5 cm

$A = bh$
$= 5 \times 2$
$= 10\ \text{cm}^2$

Units of area

mm^2
cm^2
m^2
ha
km^2
$1\ \text{ha} = 10\,000\ \text{m}^2$

Rectangle

2 cm, 4 cm

$A = lb$
$= 4 \times 2$
$= 8\ \text{cm}^2$

Square

9 m

$A = s^2$
$= 9^2$
$= 81\ \text{m}^2$

Triangle

2 m, 3 m

$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 3 \times 2$
$= 3\ \text{m}^2$

Composite figures

18 m, 10 m, 19 m, 8 m

$A = l \times b + \frac{1}{2}bh$
$= 18 \times 10 + \frac{1}{2} \times 10 \times 9$
$= 225\ \text{m}^2$

Mass

1 g = 1000 mg
1 kg = 1000 g
1 t = 1000 kg

Temperature

°C (degrees Celsius)

Circumference

$C = 2\pi r$ or πd
$= 2 \times \pi \times 3$
$= 6\pi\ \text{m}^2$

3 m

Circular arc

is a portion of the circumference of a circle.
The arc (l) is subtended by an angle, θ, at the centre of the circle.

l, r, θ, r

Arc length

Fraction $= \frac{\theta}{360}$
$l = \frac{\theta}{360} \times 2\pi r$
$l = \frac{\theta}{360} \times \pi d$

Perimeter of a sector

$P = \frac{\theta}{360} \times 2\pi r + 2r$

r, r, θ

$\frac{\theta}{360} \times 2\pi r$

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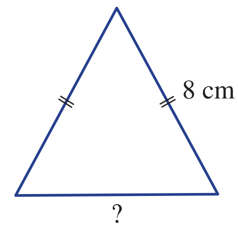
Chapter review

Multiple-choice questions

1 Which of the following is a metric unit of capacity?
A cm **B** pace **C** digit **D** yard **E** litre

2 Shonali buys 300 cm of wire that costs $2 per metre. How much does she pay for the wire?
A $150 **B** $600 **C** $1.50 **D** $3 **E** $6

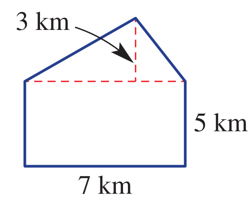
3 The triangle given has a perimeter of 20 cm. What is the missing base length?
A 6 cm **B** 8 cm **C** 4 cm
D 16 cm **E** 12 cm



4 The area of a rectangle with length 2 m and width 5 m is:
A 10 m^2 **B** 5 m^2 **C** 5 m **D** 5 m^3 **E** 10 m

5 A triangle has base length 3.2 cm and height 4 cm. What is its area?
A 25.6 cm^2 **B** 12.8 cm **C** 12.8 cm^2 **D** 6 cm **E** 6.4 cm^2

6 The total area of this composite shape is:
A 56 km^2 **B** 45.5 km^2 **C** 35 km^2
D 10.5 km^2 **E** 24.5 km^2



7 9 tonnes of iron ore is being loaded onto a ship at a rate of 20 kg per second. How many minutes will it take to load all of the 9 tonnes of ore?
A 0.75 min **B** 45 min **C** 7.3 min **D** 450 min **E** 7.5 min

8 The base length of a parallelogram is 10 cm and its area is 30 cm^2. The parallelogram's height is:
A 10 cm **B** 3 cm **C** 30 cm **D** 3 cm^2 **E** 10 m^2

9 What is the exact circumference of a circle with radius 10 cm?
A 30 cm **B** 31.4 cm **C** 10π cm **D** 20π cm **E** π cm

10 What is the exact perimeter of a quadrant cut from a circle of radius 10 cm?
A 50 cm **B** 51.4 cm **C** $(10\pi + 20)$ cm
D $(5\pi + 20)$ cm **E** 100π cm

Short-answer questions

1 Using the metric system, state how many:
a millimetres in one centimetre
b centimetres in one metre
c millimetres in one metre
d square metres in one hectare

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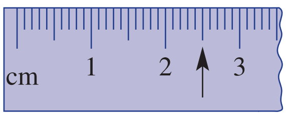
2 Convert to the units shown in brackets.

a 5 cm (mm)
b 200 cm (m)
c 3.7 km (m)
d 421 000 cm (km)
e 7.1 kg (g)
f 24 900 mg (g)
g 28 490 kg (t)
h 0.009 t (g)
i 4000 mL (L)
j 29 903 L (kL)
k 0.4 ML (kL)
l 0.001 kL (mL)
m 1 day (min)
n 3600 s (min)
o 84 h (days)
p 2.5 h (s)

3 Read these scales.

a


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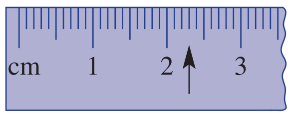

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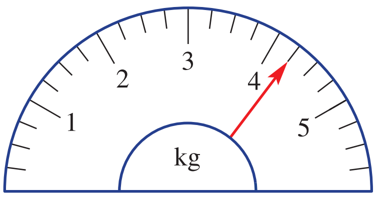

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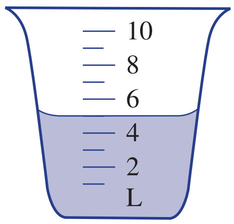
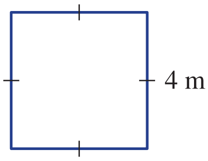


4 Find the perimeter of each of these shapes.

a


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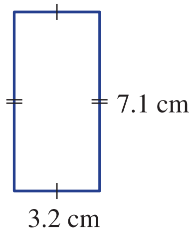

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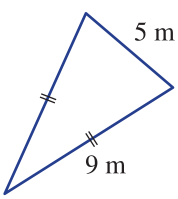

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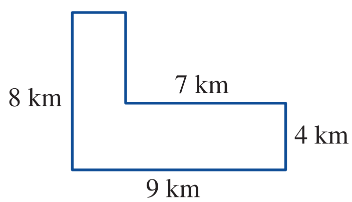
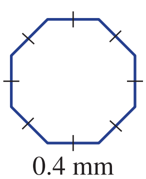
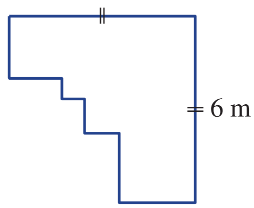


e


f


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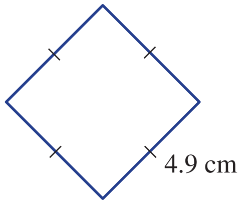
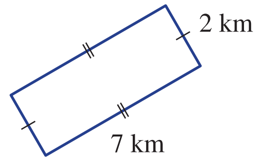
5 Find the area of each of the following shapes.

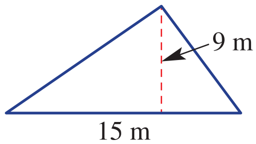
a



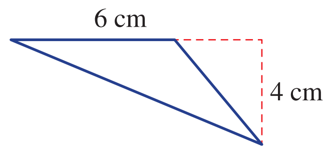
b



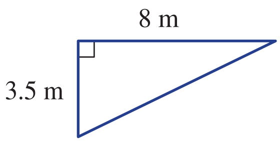
c



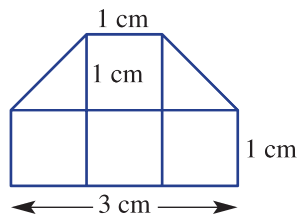
d



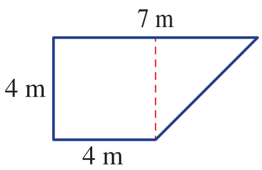
e



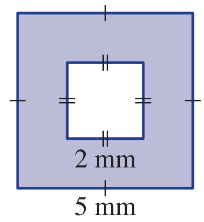
f



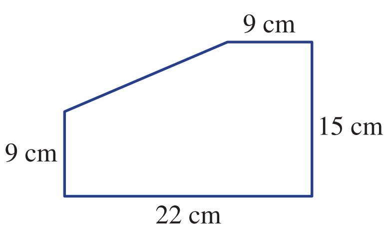
g



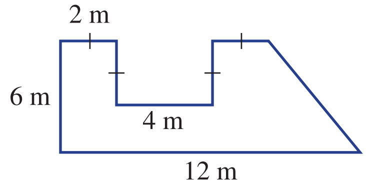
h



i



j



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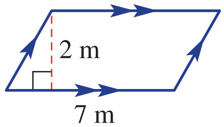
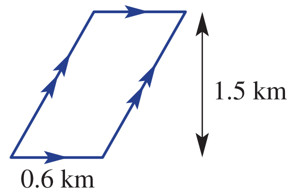
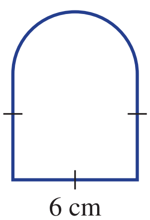


l

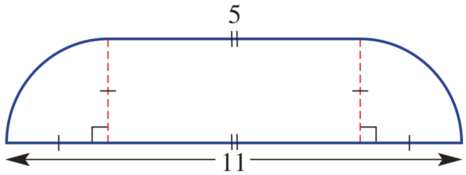


6 Calculate the exact perimeter of each of these composite figures.

a



b



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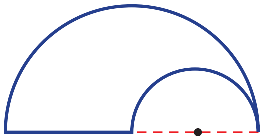
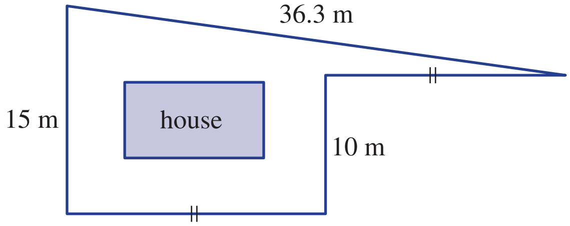
7 Find the exact perimeter of this shape. The curved sections are semicircles.



Extended-response questions

1 A regulation NBA basketball court is a rectangle with dimensions 94 feet by 50 feet. There are 12 inches in 1 foot. One inch is very close to 2.54 cm. Calculate the area of a basketball court in square metres, correct to 2 decimal places.

2 Lachlan builds a race track around the outside of his family house block. The block combines a rectangular and triangular area, as shown in the diagram.



a How far is one complete circuit of the track?

b Lachlan can jog 10 laps at about 33 seconds each. What is the total time, in minutes and seconds, that it takes him to complete the 10 laps?

c What is the total area of the block?

d The house occupies 100 m^2 and the rest of the block is to have instant turf, costing \$12 per square metre. What will be the cost for the instant turf?

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Online resources

- Auto-marked chapter pre-test
- Video demonstrations of all worked examples
- Interactive widgets
- Interactive walkthroughs
- Downloadable HOTsheets
- Access to all HOTmaths Australian Curriculum courses
- Access to the HOTmaths games library

11 Introducing indices

What you will learn

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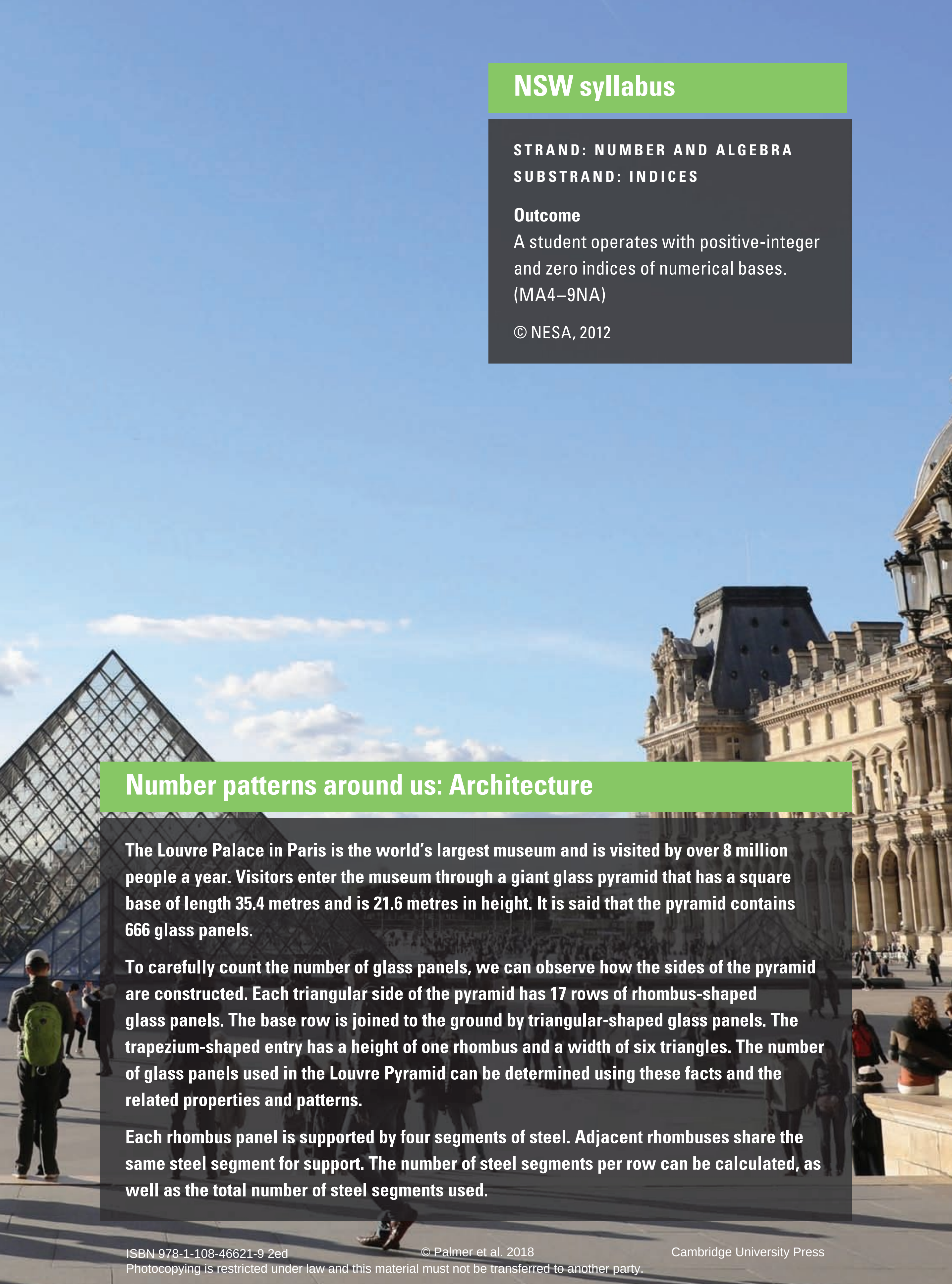

NSW syllabus

STRAND: NUMBER AND ALGEBRA
SUBSTRAND: INDICES

Outcome

A student operates with positive-integer and zero indices of numerical bases. (MA4–9NA)

Number patterns around us: Architecture

The Louvre Palace in Paris is the world's largest museum and is visited by over 8 million people a year. Visitors enter the museum through a giant glass pyramid that has a square base of length 35.4 metres and is 21.6 metres in height. It is said that the pyramid contains 666 glass panels.

To carefully count the number of glass panels, we can observe how the sides of the pyramid are constructed. Each triangular side of the pyramid has 17 rows of rhombus-shaped glass panels. The base row is joined to the ground by triangular-shaped glass panels. The trapezium-shaped entry has a height of one rhombus and a width of six triangles. The number of glass panels used in the Louvre Pyramid can be determined using these facts and the related properties and patterns.

Each rhombus panel is supported by four segments of steel. Adjacent rhombuses share the same steel segment for support. The number of steel segments per row can be calculated, as well as the total number of steel segments used.

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Pre-test

1 Find:

a 6×8 **b** 9×8 **c** 5×9
d 13×7 **e** 12×15 **f** 11×11

2 Which of the solutions to Question **1** is not divisible by 3?

3 Complete these statements.

a $8 = 4 \times$ _____
b $8 = 1 \times$ _____
c $16 = 4 \times$ _____
d $15 =$ _____ $\times 3$
e $12 \times 2 =$ _____ $\times 4$
f $7 \times 3 = 21 \times$ _____
g $15 \times 2 =$ _____ $\times 3$
h $24 \times 3 = 12 \times$ _____

4 State yes or no. Do the solutions to the following divisions contain remainders?

a $15\,465 \div 2$
b $15\,465 \div 3$
c $15\,465 \div 5$
d $15\,465 \div 6$

5 The number 5 is prime because it has only two factors (i.e. 1 and 5). The number 6 is not prime. State whether the following numbers are prime (P) or not prime (N).

a 9 **b** 11 **c** 2 **d** 51

6 State whether each of the following is true or false.

a $6 \times 4 \times 5 = 30 \times 4$
b $20 + 5 = 4 \times 5 + 5$
c $3 \times 2 \times 5 = 6 \times 5$
d $40 \div 2 \div 2 = 40 \div 4$

7 Find:

a 3×3 **b** 5×5 **c** 6×6
d 14×14 **e** 11×11 **f** 10×10

8 Complete:

a $9 \times$ _____ $= 81$
b $13 \times$ _____ $= 169$
c _____ $\times 15 = 225$

9 Find:

a $2 \times 2 \times 2 \times 2 \times 2$ **b** $3 \times 3 \times 3 \times 3$
c $5 \times 5 \times 5$ **d** $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$

10 Complete these statements.

a $4^2 = 4 \times$ _____ $= 16$ **b** $7^2 =$ _____ $\times$ _____ $= 49$
c $6^2 =$ _____ $\times 6 = 36$ **d** $11^2 =$ _____ $\times$ _____ $=$ _____

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11A Divisibility tests

It is useful to know whether a large number is exactly divisible by another number. Although we can always carry out the division algorithm, this can be a difficult and tedious process for large numbers. There are simple divisibility tests for each of the single-digit numbers, with the exception of 7. These divisibility tests determine whether or not the number is divisible by the chosen divisor.

Interactive
Widgets
HOTsheets
Walkthrough

Let's start: Five questions in 5 minutes

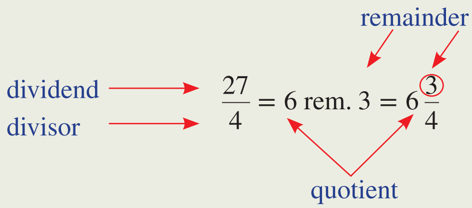
In small groups, attempt to solve the following five questions in 5 minutes.

1. Some numbers are only divisible by 1 and themselves. What are these numbers called?
2. Is 21 541 837 divisible by 3?
3. What two-digit number is the 'most divisible' (i.e. has the most factors)?
4. Find the smallest number that is divisible by 1, 2, 3, 4, 5 and 6.
5. Find a number that is divisible by 1, 2, 3, 4, 5, 6, 7 and 8.

Key ideas

- A number is said to be **divisible** by another number if there is **no remainder** after the division has occurred.
- If the **divisor** divides into the **dividend** exactly, then the divisor is said to be a **factor** of that number.
- **Division notation**

  Example: $27 \div 4 = 6$ remainder 3



  Another way of representing this information is $27 = 4 \times 6 + 3$.
- Key terms:

Dividend	The starting number; the total; the amount you have
Divisor	The number doing the dividing; the number of groups
Quotient	The number of times the divisor went into the dividend, also known as 'the answer'
Remainder	The number left over; the number remaining (sometimes written as 'rem.')

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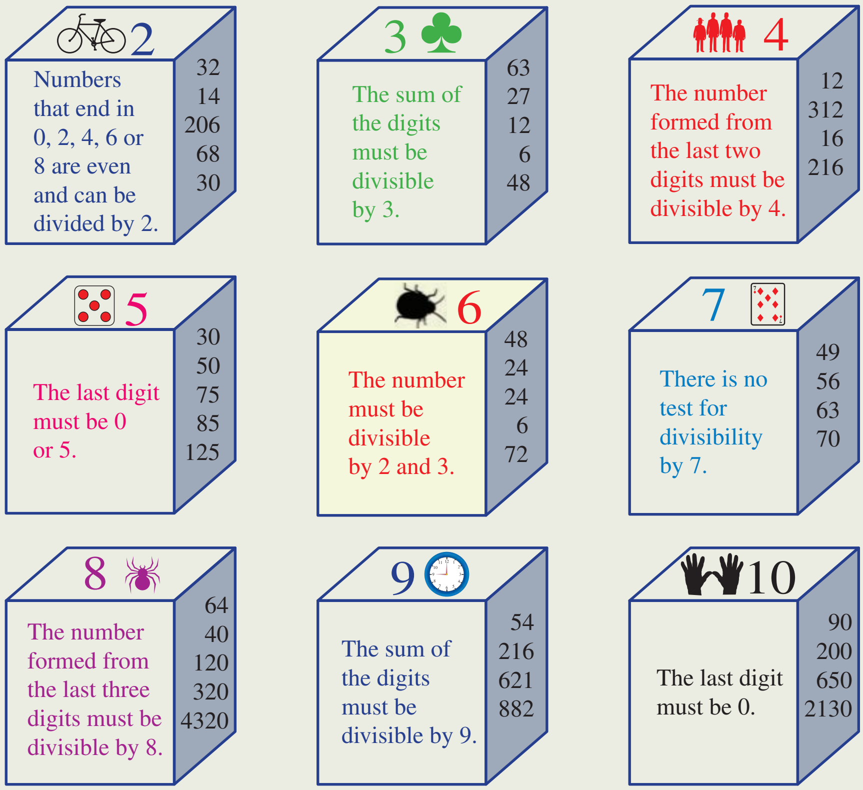
Key ideas

- Divisibility tests
  All positive integers are divisible by 1.



Example 1 Applying divisibility tests

Determine whether or not the following calculations are possible without leaving a remainder.

a 54 327 ÷ 3 **b** 765 146 ÷ 8

SOLUTION

a Digit sum = 21
Yes, 54 327 is divisible by 3.

b $8\overline{)14^{6}6}$ = 18 rem 2.
No, 765 146 is not divisible by 8.

EXPLANATION

$5 + 4 + 3 + 2 + 7 = 21$
21 is divisible by 3.

Check whether the last three digits are divisible by 8.

Example 2 Testing divisibility

Carry out divisibility tests on the given number and fill in the table with ticks or crosses.

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
48 569 412								

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SOLUTION

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
48 569 412	✓	✓	✓	✗	✓	✗	✗	✗

EXPLANATION

48 569 412 is an even number and therefore is divisible by 2.
48 569 412 has a digit sum of 39 and therefore is divisible by 3, but not by 9.
48 569 412 is divisible by 2 and 3, therefore it is divisible by 6.
The last two digits are 12, which is divisible by 4.
The last three digits are 412, which is not divisible by 8.
The last digit is a 2 and therefore is not divisible by 5 or 10.

Exercise 11A

UNDERSTANDING AND FLUENCY	1–7	4, 5(½), 7	6–8

1 Circle the numbers that are factors (divide into without leaving a remainder) of 765.
1 2 3 4 5 6 8 9 10

2 Refer to the divisibility tests in **Example 2** and answer the following as either True (T) or False (F).
- **a** 6790 is divisible by 2
- **b** 6790 is divisible by 5
- **c** 6790 is divisible by 10
- **d** 6790 is divisible by 3
- **e** 9874 is divisible by 2
- **f** 9874 is divisible by 3
- **g** 9874 is divisible by 5
- **h** 1 263 848 is divisible by 8

3 Which three divisibility tests involve calculating the sum of the digits?

4 If you saw only the last digit of a 10-digit number, which three divisibility tests could you still apply?

Example 1

5 **i** Without using a calculator, determine whether the following calculations are possible without leaving a remainder.
- **a** 23 562 ÷ 3
- **b** 39 245 678 ÷ 4
- **c** 1 295 676 ÷ 9
- **d** 213 456 ÷ 8
- **e** 3 193 457 ÷ 6
- **f** 2 000 340 ÷ 10
- **g** 51 345 678 ÷ 5
- **h** 215 364 ÷ 6
- **i** 9543 ÷ 6
- **j** 25 756 ÷ 2
- **k** 56 789 ÷ 9
- **l** 324 534 565 ÷ 5
- **m** 2 345 176 ÷ 8
- **n** 329 541 ÷ 10
- **o** 225 329 ÷ 3
- **p** 356 781 276 ÷ 9
- **q** 164 567 ÷ 8
- **r** 2 002 002 002 ÷ 4

ii Repeat the process, using a calculator. Which way is quicker?

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Example 2

6 Carry out divisibility tests on the given numbers and fill in the table with ticks or crosses.

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
243 567								
28 080								
189 000								
1 308 150								
1 062 347								

7 Write down five two-digit numbers that are divisible by:

a 5 **b** 3 **c** 2 **d** 6
e 8 **f** 9 **g** 10 **h** 4

8 Give a reason why:

a 8631 is not divisible by 2
b 31 313 is not divisible by 3
c 426 is not divisible by 4
d 5044 is not divisible by 5
e 87 548 is not divisible by 6
f 214 125 is not divisible by 8
g 3 333 333 is not divisible by 9
h 56 405 is not divisible by 10

PROBLEM-SOLVING AND REASONING	9, 10, 15	10–12, 15, 16	12–18

9 **a** Can Julie share $41.75 equally among her three children?
b Julie finds one more dollar on the floor and realises that she can now share the money equally among her three children. How much do they each receive?

10 The game of 'clusters' involves a group getting into smaller-sized groups as quickly as possible once a particular cluster size has been called out. If a year level consists of 88 students, which 'cluster' sizes would ensure no students are left out of a group?

11 How many of the whole numbers between 1 and 250 inclusive are not divisible by 5? Explain, in written form, how you arrived at your answer.

12 How many two-digit numbers are divisible by 2 and 3?

13 Find the largest three-digit number that is divisible by both 4 and 5.

14 Find the largest three-digit number that is divisible by both 6 and 7.

15 **a** Is the number 968 362 396 392 139 963 359 divisible by 3?
b Many of the digits in the number above can actually be ignored when calculating the digit sum. Which numbers can be ignored and why?
c To determine if the number above is divisible by 3, only five of the 21 digits actually need to be added together. Find this 'reduced' digit sum.
d Prepare a one-minute talk to explain your answers to Question **15 a** to **c**.
e Give your one-minute talk to your partner and ask for feedback on how clearly you expressed yourself.

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16 The divisibility test for the numeral 4 is to consider whether the number formed by the last two digits is a multiple of 4. Complete the following sentences to make a more detailed divisibility rule.

a If the second-last digit is even, the last digit must be either a __, __ or __.
b If the second-last digit is odd, the last digit must be either a __ or __.

17 Blake's age is a two-digit number. It is divisible by 2, 3, 6 and 9. How old is Blake if you know that he is older than 20 but younger than 50?

18 Find the smallest number that satisfies each of the conditions below.
The number must be larger than the divisor and leave:

a a remainder of 5 when divided by 6
b a remainder of 4 when divided by 5
c a remainder of 3 when divided by 4
d a remainder of 2 when divided by 3
e a remainder of 1 when divided by 2

ENRICHMENT	—	—	19

Divisible by 11?

19 **a** Write down the first nine multiples of the numeral 11.
b What is the difference between the two digits for each of these multiples?
c Write down some three-digit multiples of 11.
d What do you notice about the sum of the first digit and the last digit?

The following four-digit numbers are all divisible by 11:
1606, 2717, 6457, 9251, 9306

e Find the sum of the odd-placed digits and the sum of the even-placed digits. Then subtract the smaller sum from the larger. What do you notice?
f Write down a divisibility rule for the number 11.
g Which of the following numbers are divisible by 11?

i 2 594 669
ii 45 384 559
iii 488 220
iv 14 641
v 1 358 024 679
vi 123 456 789 987 654 321

An alternative method is to alternate adding and subtracting each of the digits.
For example: 4 134 509 742 is divisible by 11.
Alternately adding and subtracting the digits will give the following result:
$4 - 1 + 3 - 4 + 5 - 0 + 9 - 7 + 4 - 2 = 11$

h Try this technique on some of your earlier numbers.

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11B Prime numbers

Interactive
Widgets
HOTsheets
Walkthrough

It is believed that prime numbers (i.e. positive whole numbers with two factors) were first studied by the ancient Greeks. More recently, the introduction of computers has allowed for huge developments in this field. Computers have allowed mathematicians to determine which large numbers are primes. Programs have also been written to automatically generate huge prime numbers that could not be calculated previously by hand.

There continues to be much debate as to whether or not 1 is a prime number. The current thinking is that 1 should not be considered a prime number, the basic reason being that it does not have two distinct factors.

Remarkable fact: There are some interesting prime numbers that have patterns in their digits; for example, 12 345 678 901 234 567 891. This is known as an ascending prime.

You can also get palindromic primes, such as 111 191 111 and 123 494 321.

Below is a palindromic prime number that reads the same upside down or when viewed in a mirror.

1888081808881

Let's start: How many primes?

How many numbers from 1 to 100 are prime?
You and a classmate have 4 minutes to come up with your answer.

Key ideas

- A **prime number** is a positive integer that has only two factors: 1 and itself.
- The prime number less than 20 are 2, 3, 5, 7, 11, 13, 17 and 19.
- A number that has more than two factors is called a **composite number**.
- The number 1 is neither a prime nor a composite number.
- The number 2 is prime. It is the only even prime number.

Example 3 Determining whether a number is a prime or composite

State whether each of these numbers is a prime or composite: 22, 35, 17, 11, 9, 5.

SOLUTION	EXPLANATION
Prime: 5, 11, 17	5, 11, 17 have only two factors (1 and itself).
Composite: 9, 22, 35	$9 = 3 \times 3,\ 22 = 2 \times 11,\ 35 = 5 \times 7$

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Example 4 Finding prime factors

Find the prime numbers that are factors of 30.

SOLUTION	EXPLANATION
Factors of 30 are: 1, 2, 3, 5, 6, 10, 15, 30 Prime numbers from this list of factors are 2, 3 and 5.	Find the entire set of factors first. Determine which factors are prime according to the given definition.

Exercise 11B

UNDERSTANDING AND FLUENCY	1–8	6, 7–8(½), 9	7–8(½), 9

1 The factors of 12 are 1, 2, 3, 4, 6 and 12. Is 12 a prime number?

2 The factors of 13 are 1 and 13. Is 13 a prime number?

3 List the first 10 prime numbers.

4 List the first 10 composite numbers.

5 What is the first prime number greater than 100?

6 What is the first prime number greater than 200?

Example 3

7 State whether each of the following is a prime (P) or composite (C) number.

a 14 b 23 c 70 d 37
e 51 f 27 g 29 h 3
i 8 j 49 k 99 l 59
m 2 n 31 o 39 p 89

Example 4

8 Find the prime numbers that are factors of:

a 42 b 39
c 60 d 25
e 28 f 36

9 List the composite numbers between:

a 30 and 50 b 50 and 70
c 80 and 100

PROBLEM-SOLVING AND REASONING	10, 14	11, 12, 14, 15	11–13, 15, 16

10 The following are not prime numbers, yet they are the product (×) of two primes. Find the two primes for each of the following numbers.

a 55 b 91
c 143 d 187
e 365 f 133

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11 Which one of these numbers has factors that are only itself, 1 and prime numbers?
12, 14, 16, 18, 20

12 Twin primes are pairs of primes that are separated from each other by only one even number; for example, 3 and 5 are twin primes. Find three more pairs of twin primes.

13 13 and 31 are known as a pair of 'reverse numbers'. They are also both prime numbers. Find any other two-digit pairs of prime reverse numbers.

14 Find three different prime numbers that are less than 100 and which sum to a fourth different prime number. Can you find more than five sets of such numbers?

15 Many mathematicians believe that every even number greater than 2 is the sum of two prime numbers. Show this is true for even numbers between 30 and 50.

16 Give two examples of a pair of primes that add to a prime number. Explain why all possible pairs of primes that add to a prime must contain the number 2.

ENRICHMENT	—	—	17

Prime or not prime?

17 Design a spreadsheet that will check whether or not any number entered between 1 and 1000 is a prime number.
If your spreadsheet is successful, someone should be able to enter the number 773 and very quickly be informed whether or not this is a prime number.
You may choose to adapt your factor program (see Enrichment activity Exercise 4A, Question 16).

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11C Using indices

Interactive
Widgets
HOTsheets
Walkthrough

When repeated multiplication of the same factor occurs, the expression can look quite cumbersome. Mathematicians have a method for simplifying such expressions by writing them as **powers**. This involves writing the repeated factor as the base number and then including an index number to indicate how many times this factor must be multiplied by itself. This is also known as writing a number in **index form**.

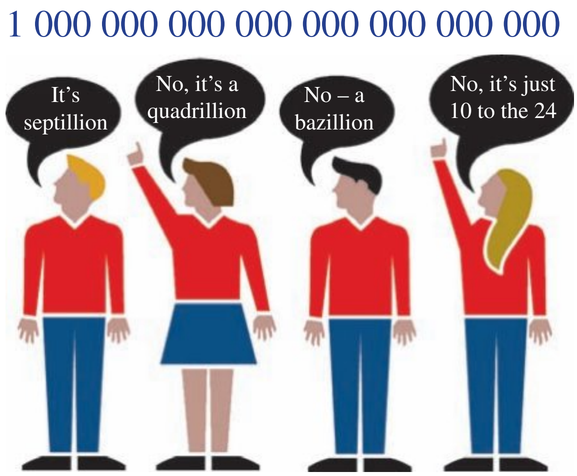
Powers are also used to represent very large and very small numbers. For example, 400 000 000 000 000 would be written as 4×10^{14}. This way of writing a number is called **standard form** or **scientific notation** and you will come across this concept in future years.

Let's start: A better way...

- What is a better way of writing $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$ (that is not the answer, 20)?
- What is a better way of writing $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ (that is not the answer, 1024)?

You may need to access the internet to find out some of the following answers.
Computers have the capacity to store a lot of information. As you most likely know, computer memory is given in bytes.

- How many bytes (B) are in a kilobyte (kB)?
- How many kilobytes are in a megabyte (MB)?
- How many megabytes are in a gigabyte (GB)?
- How many gigabytes are in a terabyte (TB)?
- How many bytes are in a gigabyte?
  Hint: It is over 1 billion and it is far easier to write this number as a power!
- Why do computers frequently use base 2 (binary numbers)?



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Key ideas

- **Powers** are used to help write expressions involving repeated multiplication in a simplified form using indices

  For example: $8 \times 8 \times 8 \times 8 \times 8$ can be written as 8^5

- When writing a **basic numeral** as a power, you need a base number and an index number. This is also known as writing an expression in **index form**.

  index number

  $8^5 = 32\,768$

  base number — basic numeral

- a^b reads as 'a to the power of b'. In expanded form it would look like:

  $$\underbrace{a \times a \times a \times a \times a \ldots \times a}_{\text{The number } a \text{ appears } b \text{ times.}}$$

- Powers take priority in the order of operations.

  For example: $3 + 2 \times 4^2 = 3 + 2 \times 16$
  $= 3 + 32$
  $= 35$

- Note: $2^3 \neq 2 \times 3$, therefore $2^3 \neq 6$. This is a common mistake that must be avoided.

  Instead: $2^3 = 2 \times 2 \times 2 = 8$.

Example 5 Converting to index form

Simplify the following expressions by writing them in index form.

a $5 \times 5 \times 5 \times 5 \times 5 \times 5$ **b** $3 \times 3 \times 2 \times 3 \times 2 \times 3$

SOLUTION	EXPLANATION
a $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$	The number 5 is the repeated factor and it appears six times.
b $3 \times 3 \times 2 \times 3 \times 2 \times 3 = 2^2 \times 3^4$	2 is written two times. 3 is written four times.

Example 6 Expanding a power

Expand and evaluate the following terms.

a 2^4 **b** $2^3 \times 5^2$

SOLUTION	EXPLANATION
a $2^4 = 2 \times 2 \times 2 \times 2$ $= 16$	Write 2 down four times and multiply.
b $2^3 \times 5^2 = 2 \times 2 \times 2 \times 5 \times 5$ $= 8 \times 25$ $= 200$	Repeat the number 2 three times, and the number 5, two times.

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Example 7 Evaluating expressions with powers

Evaluate:

a $7^2 - 6^2$

b $2 \times 3^3 + 10^2 + 1^7$

SOLUTION

a $7^2 - 6^2 = 7 \times 7 - 6 \times 6$
$= 49 - 36$
$= 13$

EXPLANATION

Write in expanded form (optional).
Powers are evaluated before the subtraction occurs.

b $2 \times 3^3 + 10^2 + 1^7 = 2 \times 3 \times 3 \times 3 + 10 \times 10 + 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$
$= 54 + 100 + 1$
$= 155$

Write in expanded form (optional).
Follow order of operation rules.
Carry out the multiplication first, then carry out the addition.

Exercise 11C

UNDERSTANDING AND FLUENCY	1–3, 4–10(½)	3, 4–10(½)	4–11

1 Select the correct answer from the following alternatives.
3^7 means:

A 3×7 **B** $3 \times 3 \times 3$ **C** $7 \times 7 \times 7$
D $3 \times 7 \times 3 \times 7 \times 3 \times 7 \times 3$ **E** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ **F** 37

2 Select the correct answer from the following alternatives.
$9 \times 9 \times 9 \times 9 \times 9$ can be simplified to:

A 9×5 **B** 5×9 **C** 5^9
D 9^5 **E** 99 999 **F** 95

3 Copy and complete the table.

Index form	Base number	Index number	Basic numeral
2^3	2	3	8
5^2			
10^4			
2^7			
1^{12}			
12^1			
0^5			

Example 5a

4 Simplify the following expressions by writing them as powers.

a $3 \times 3 \times 3$ **b** $2 \times 2 \times 2 \times 2 \times 2$ **c** $15 \times 15 \times 15 \times 15$
d $10 \times 10 \times 10 \times 10$ **e** 6×6 **f** $20 \times 20 \times 20$
g $1 \times 1 \times 1 \times 1 \times 1 \times 1$ **h** $4 \times 4 \times 4$ **i** 100×100

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Example 5b

5 Simplify the following expressions by writing them as powers.

a $3 \times 3 \times 5 \times 5$
b $7 \times 7 \times 2 \times 2 \times 7$
c $12 \times 9 \times 9 \times 12$
d $8 \times 8 \times 5 \times 5 \times 5$
e $6 \times 3 \times 6 \times 3 \times 6 \times 3$
f $13 \times 7 \times 13 \times 7 \times 7 \times 7$
g $4 \times 13 \times 4 \times 4 \times 7$
h $10 \times 9 \times 10 \times 9 \times 9$
i $2 \times 3 \times 5 \times 5 \times 3 \times 2 \times 2$

6 Simplify by writing using powers.
$2 \times 3 \times 5 \times 5 \times 3 \times 3 \times 2 \times 2 \times 2 \times 5 \times 3 \times 2 \times 2 \times 5 \times 3$

Example 6a

7 Expand these terms. (Do not evaluate.)

a 2^4 **b** 17^2 **c** 9^3 **d** 3^7
e 14^4 **f** 8^8 **g** 10^5 **h** 54^3

Example 6b

8 Expand these terms. (Do not evaluate.)

a $3^5 \times 2^3$ **b** $4^3 \times 3^4$ **c** $7^2 \times 5^3$ **d** $4^6 \times 9^3$
e 5×7^4 **f** $2^2 \times 3^3 \times 4^1$ **g** $11^5 \times 9^2$ **h** $20^3 \times 30^2$

9 Evaluate:

a 2^5 **b** 8^2 **c** 10^3 **d** $3^2 \times 2^3$
e 10^4 **f** $2^3 \times 5^3$ **g** $1^6 \times 2^6$ **h** $11^2 \times 1^8$

Example 7

10 Evaluate:

a $3^2 + 4^2$ **b** $2 \times 5^2 - 7^2$ **c** $8^2 - 2 \times 3^3$
d $(9 - 5)3$ **e** $2^4 \times 2^3$ **f** $2^7 - 1 \times 2 \times 3 \times 4 \times 5$
g $1^4 + 2^3 + 3^2 + 4^1$ **h** $10^3 - 10^2$ **i** $(1^{27} + 1^{23}) \times 2^2$

11 Use a calculator to evaluate the following.

a $15^2 - 13^2$ **b** $9^3 + 3^4$ **c** $5^4 \times 5^2 + 5^3$
d $22^2 + 19^3$ **e** $(12^3 - 17^2) \times 4^3$ **f** $100^3 - 99^3$

PROBLEM-SOLVING AND REASONING	12, 13, 16	13, 14, 16, 17	14, 15, 17, 18

12 Determine the index number for the following basic numerals.

a $16 = 2^{\square}$ **b** $16 = 4^{\square}$ **c** $64 = 4^{\square}$ **d** $64 = 2^{\square}$
e $27 = 3^{\square}$ **f** $100 = 10^{\square}$ **g** $49 = 7^{\square}$ **h** $625 = 5^{\square}$

13 Write one of the symbols <, = or > in the box to make the following statements true.

a $2^6 \square 2^9$ **b** $8^3 \square 8^2$ **c** $2^4 \square 4^2$ **d** $3^2 \square 4^2$
e $6^4 \square 5^3$ **f** $12^2 \square 3^4$ **g** $11^2 \square 2^7$ **h** $1^8 \square 2^3$

14 A text message is sent to five friends. Each of the five friends then forwards it to five other friends and each of these people also sends it to five other friends. How many people does the text message reach, not including those who forwarded the message?

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15 Jane writes a chain email and sends it to five friends. If each person who received the email reads it within 5 minutes of the email arriving and then sends it to five other people:

- **a** How many people, including Jane, will have read the email 15 minutes after Jane first sent it?
- **b** If the email always goes to a new person, and assuming every person in Australia has an email address and access to email, how long would it take until everyone in Australia has read the message? (Australian population is approx. 25 million people.)
- **c** How many people will read the email within 1 hour?
- **d** Using the same assumptions as above, how long would it take until everyone in the world has read the message? (World population is approx. 7 billion people.)
- **e** How many people will have read the email in 2 hours?

16 Write the correct operation (+, −, ×, ÷) in the box to make the following equations true.

- **a** $3^2 \square 4^2 = 5^2$
- **b** $2^4 \square 4^2 = 4^4$
- **c** $2^7 \square 5^3 = 3^1$
- **d** $9^2 \square 3^4 = 1^{20}$
- **e** $10^2 \square 10^2 = 10^4$
- **f** $10^2 \square 8^2 = 6^2$

17 A chain email is initiated by an individual and sent to x number of recipients. This process is repeated (i.e. is forwarded to x new recipients) y times. How many people receive the email, not including those who forwarded the message?

18 Find a value for a and for b such that $a \neq b$ and $a^b = b^a$.

ENRICHMENT	—	—	19

Investigating factorials

19 In mathematics, the exclamation mark (!) is the symbol for factorials.

$4! = 4 \times 3 \times 2 \times 1 = 24$

$n! = n \times (n-1) \times (n-2) \times (n-3) \times (n-4) \times \ldots \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

- **a** Evaluate 1!, 2!, 3!, 4!, 5! and 6!

  Factorials can be written in prime factor form, which involves powers.

  For example:
  $$\begin{aligned} 6! &= 6 \times 5 \times 4 \times 3 \times 2 \times 1 \\ &= (2 \times 3) \times 5 \times (2 \times 2) \times 3 \times 2 \times 1 \\ &= 2^4 \times 3^2 \times 5 \end{aligned}$$
- **b** Write these numbers in prime factor form.
  - **i** 7!
  - **ii** 8!
  - **iii** 9!
  - **iv** 10!
- **c** Write down the last digit of 12!
- **d** Write down the last digit of 99!
- **e** Find a method of working out how many consecutive zeros would occur on the right-hand end of each of the following factorials if they were evaluated. Hint: Consider prime factor form.
  - **i** 5!
  - **ii** 6!
  - **iii** 15!
  - **iv** 25!
- **f** 10! = 3! × 5! × 7! is an example of one factorial equal to the product of three factorials. Express 24! as the product of two or more factorials.

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11D Prime decomposition

Interactive
Widgets
HOTsheets
Walkthrough

All composite numbers can be broken down (i.e. decomposed) into a unique set of prime factors.
A common way of performing the decomposition into prime factors is using a factor tree. Starting with the given number, 'branches' come down in pairs, representing a pair of factors that multiply to give the number above it. This process continues until prime factors are reached.

Let's start: Composition of numbers from prime factors

'Compose' composite numbers from the following sets of prime factors. The first one has been done for you.

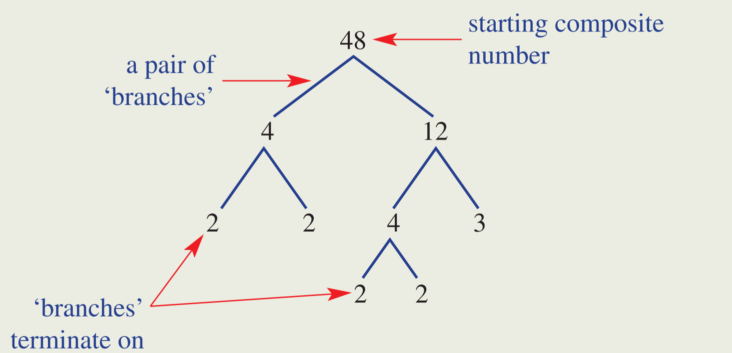
a $2 \times 3 \times 5 = 30$
b $2 \times 3 \times 7 \times 3 \times 2$
c $3^2 \times 2^3$
d $5 \times 11 \times 2^2$
e $13 \times 17 \times 2$
f $2^2 \times 5^2 \times 7^2$
g $2^5 \times 3^4 \times 7$
h $11 \times 13 \times 17$

Note that this process is the reverse of **decomposition**.

Key ideas

- Every **composite number** can be expressed as a product of its **prime factors**.
- A **factor tree** can be used to show the prime factors of a composite number.
- Each 'branch' of a factor tree eventually terminates in a prime factor.
- Powers are often used to efficiently represent composite numbers in prime factor form.

  For example:



$$\therefore 48 = 2 \times 2 \times 2 \times 2 \times 3$$
$$= 2^4 \times 3$$

- It does not matter with which pair of factors you start a factor tree. The final set of prime factors will always be the same.
- It is conventional to write the prime factors in **ascending** (i.e. increasing) order.

  For example: $600 = 2^3 \times 3 \times 5^2$

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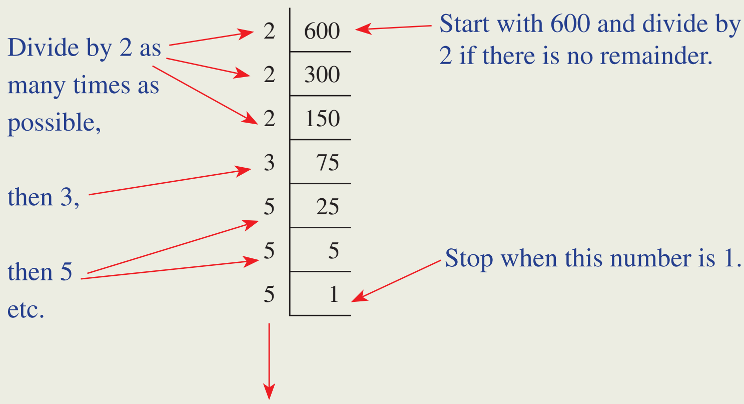
Key ideas

- Here is another way to decompose a number, using division by prime numbers.



$600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5$
$600 = 2^3 \times 3 \times 5^2$

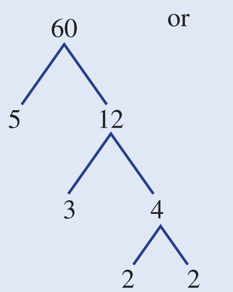
Sometimes it may be necessary to divide by 2, 3, 5, 7, 11, 13 or any prime number.

- Some calculators can decompose a number. The Casio fx-82AU PLUS 11 and fx-100AU PLUS have the word FACT above the ° ’ ” button. Press 600 = SHIFT FACT

Example 8 Expressing composites in prime factor form

Express the number 60 in prime factor form.

SOLUTION



or

2	60
2	30
3	15
5	5
	1

$\therefore 60 = 2 \times 2 \times 3 \times 5$
$60 = 2^2 \times 3 \times 5$

EXPLANATION

A pair of factors for 60 is 5×12.

The 5 branch terminates since 5 is a prime factor.
A pair of factors for 12 is 3×4.

The 3 branch terminates since 3 is a prime factor.
A pair of factors for 4 is 2×2.
Both these branches are now terminated.

Hence, the composite number 60 can be written as a product of each terminating branch.

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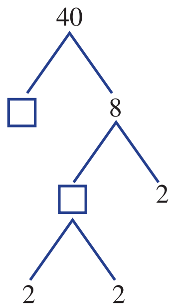
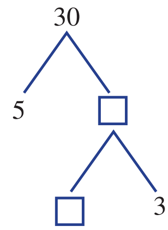
Exercise 11D

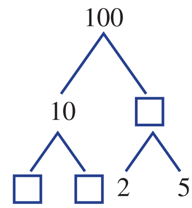
UNDERSTANDING AND FLUENCY	1–5	4, 5–6(½)	5–6(½)

1 Sort the following list of numbers into two groups: composite numbers and prime numbers.
15, 13, 7, 5, 8, 9, 27, 23, 11, 4, 12, 2

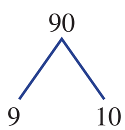
2 Fill in the gaps to complete the following factor trees.

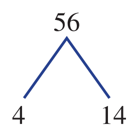
a


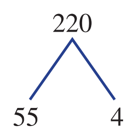
b


c


3 Complete each of the following factor trees.

a


b


c


4 Write the following prime factors, using powers.

a $2 \times 3 \times 3 \times 2 \times 2$
b $5 \times 3 \times 3 \times 3 \times 3 \times 5$
c $7 \times 2 \times 3 \times 7 \times 2$
d $3 \times 3 \times 2 \times 11 \times 11 \times 2$

Example 8

5 Express the following numbers in prime factor form.

a 72
b 24
c 38
d 44
e 124
f 80
g 96
h 16
i 75
j 111
k 64
l 56

6 Express these numbers in prime factor form.

a 600
b 800
c 5000
d 2400
e 1 000 000
f 45 000
g 820
h 690

PROBLEM-SOLVING AND REASONING	7, 8, 11	8, 9, 11, 12	8–10, 12–14

7 Match the correct composite number (**a** to **d**) to its set of prime factors (**A** to **D**).

a 120
b 150
c 144
d 180

A $2 \times 3 \times 5^2$
B $2^2 \times 3^2 \times 5$
C $2^4 \times 3^2$
D $2 \times 3 \times 2 \times 5 \times 2$

8 Find the smallest composite number that has the five smallest prime numbers as factors.

9 **a** Express 144 and 96 in prime factor form.
b By considering the prime factor form, determine the HCF of 144 and 96.

10 **a** Express 25 200 and 77 000 in prime factor form.
b By considering the prime factor form, determine the HCF of 25 200 and 77 000.

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11 Represent the number 24 with four different factor trees, each resulting in the same set of prime factors. Note that simply swapping the order of a pair of factors does not qualify it as a different form of the factor tree.

12 Only one of the following is the correct set of prime factors for 424.

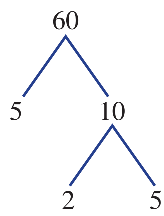
A $2^2 \times 3^2 \times 5$ **B** $2 \times 3^2 \times 5^2$

C 53×8 **D** $2^3 \times 53$

- **a** Justify why you can eliminate alternatives **A** and **B** straight away.
- **b** Why can option **C** be discarded as an option?
- **c** Show that option **D** is the correct answer.

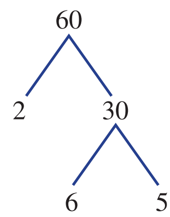
13 **a** State the error in each of the following prime factor trees.

i



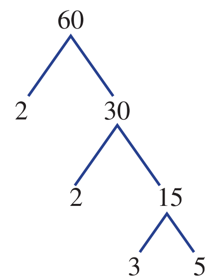
$60 = 2 \times 5^2$

ii



$60 = 2 \times 5 \times 6$

iii



$60 = 2 \times 3 \times 5$

b What is the correct way to express 60 in prime factor form?

14 Write 15 different (i.e. distinct) factor trees for the number 72.

ENRICHMENT	—	—	15–17

Four distinct prime factors

15 There are 16 composite numbers less than 1000 which have four distinct (i.e. different) numbers in their prime decomposition. For example: $546 = 2 \times 3 \times 7 \times 13$.

By considering the prime factor possibilities, find the other 15 composite numbers and express each of them in prime factor form.

16 A conjecture is a statement that may appear to be true but has not been proved conclusively. Goldbach's conjecture states that 'Every even number greater than 2 is the sum of two prime numbers.' For example, $53 = 47 + 5$.

Challenge: Try this for every even number from 4 to 50.

17 Use the internet to find the largest-known prime number.

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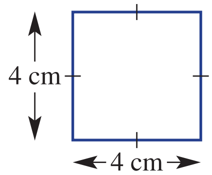
11E Squares, square roots, cubes and cube roots

A square number can be illustrated by considering the area of a square with a whole number as its side length.

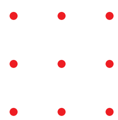
For example:



Area of square $= 4 \text{ cm} \times 4 \text{ cm} = 16 \text{ cm}^2$

Therefore, 16 is a square number.

Another way of representing square numbers is through a square array of dots. For example:



Number of dots $= 3$ rows of 3 dots
$= 3 \times 3$ dots
$= 3^2$ dots
$= 9$ dots

Therefore, 9 is a square number.

To produce a square number you must multiply the number by itself. All square numbers written in index form will have a power of 2.

Finding a square root of a number is the opposite of squaring a number.
For example: $4^2 = 16$ and therefore $\sqrt{16} = 4$.

To find square roots we use our knowledge of square numbers. A calculator is also frequently used to find square roots.

Geometrically, the square root of a number is the side length of a square whose area is that number.

If 4×4 is '4 squared', then $4 \times 4 \times 4$ is '4 cubed'.

Note that '5 cubed' is $5 \times 5 \times 5 = 125$ (not 15).

The opposite of this is called a cube root. For example: $2 \times 2 \times 2 = 8$, so $\sqrt[3]{8} = 2$.

Let's start: Speed squaring tests

In pairs, test one another's knowledge of square numbers.
- Ask 10 quick questions, such as '3 squared', '5 squared'.
- Have two turns each. Time how long it takes each of you to answer the 10 questions.
- Aim to be quicker on your second attempt.

Write down the first 10 square numbers.
- Begin to memorise these important numbers.
- Time how quickly you can recall the first 10 square numbers without looking at a list of numbers.
- Can you go under 5 seconds?

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Key ideas

- Any whole number multiplied by itself produces a **square number**.

  For example: $5^2 = 5 \times 5 = 25$. Therefore, 25 is a square number.

  - Square numbers are also known as **perfect squares**.
  - The first 12 square numbers are:

Index form	1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	11^2	12^2
Basic numeral	1	4	9	16	25	36	49	64	81	100	121	144

  - All square numbers have an odd number of factors.
  - The symbol for squaring is $(\ )^2$. The brackets are optional, but can be very useful when simplifying more difficult expressions.

- The **square root** of a given number is the 'non-negative' number that, when multiplied by itself, produces the given number.

  - The symbol for square rooting is $\sqrt{\ }$.
  - Finding a square root of a number is the opposite of squaring a number.
    For example: $4^2 = 16$; hence, $\sqrt{16} = 4$.
    We read this as: '4 squared equals 16, therefore, the square root of 16 equals 4.'
  - Squaring and square rooting are 'opposite' operations.
    $(\sqrt{x})^2 = x$ also $\sqrt{(x)^2} = x$ (assuming $x \geqslant 0$)
  - A list of common square roots are:

Square root form	$\sqrt{1}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{49}$	$\sqrt{64}$	$\sqrt{81}$	$\sqrt{100}$	$\sqrt{121}$	$\sqrt{144}$
Basic numeral	1	2	3	4	5	6	7	8	9	10	11	12

- Any whole number when multiplied by itself twice produces a cubic number.
  For example: $4^3 = 4 \times 4 \times 4 = 64$

- The cube root of a given number is the number that, when multiplied by itself twice, produces the given number.

  - The symbol used is $\sqrt[3]{\ }$.
    For example: $\sqrt[3]{64} = 4$
    We read this as 'The cube root of 64 is 4.'

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Example 9 Evaluating squares, square roots, cubes and cube roots

Evaluate:

a 6^2 b $\sqrt{64}$ c $\sqrt{1600}$

d 6^3 e $\sqrt[3]{64}$

SOLUTION	EXPLANATION
a $6^2 = 36$	$6^2 = 6 \times 6$
b $\sqrt{64} = 8$	$8 \times 8 = 64$ $\therefore \sqrt{64} = 8$
c $\sqrt{1600} = 40$	$40 \times 40 = 1600$ $\therefore \sqrt{1600} = 40$
d $6^3 = 6 \times 6 \times 6 = 216$	$6 \times 6 = 36$ $36 \times 6 = 216$
e $\sqrt[3]{64} = 4$	$4 \times 4 \times 4 = 64$

Example 10 Evaluating expressions involving squares, square roots, cubes and cube roots

Evaluate:

a $3^3 - \sqrt{9} + 1^2$ b $\sqrt{8^2 + 6^2}$ c $\sqrt[3]{\dfrac{100 - 28}{9}}$

SOLUTION	EXPLANATION
a $3^3 - \sqrt{9} + 1^2 = 27 - 3 + 1$ $= 25$	$3^3 = 3 \times 3 \times 3$, $\sqrt{9} = 3$, $1^2 = 1 \times 1$
b $\sqrt{8^2 + 6^2} = \sqrt{64 + 36}$ $= \sqrt{100}$ $= 10$	$8^2 = 8 \times 8$, $6^2 = 6 \times 6$ $\sqrt{100} = 10$
c $\sqrt[3]{\dfrac{100 - 28}{9}} = \sqrt[3]{\dfrac{72}{9}}$ $= \sqrt[3]{8}$ $= 2$	Simplify the fraction first.

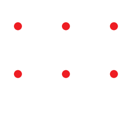
Exercise 11E

UNDERSTANDING AND FLUENCY	1–8, 9(½)	5, 6–7(½), 8, 9–10(½)	6–7(½), 8, 9–10(½)

1 Draw a square of side length 6 cm. What would be the area of this shape? What special type of number is your answer?

2 a Write down the first 15 square numbers in index form and as basic numerals.
  b Repeat part **a** for the first 10 cubic numbers.

3 We can confirm that 9 is a square number by drawing the diagram shown.
  a Show, using dots, why 6 is not a square number.
  b Show, using dots, why 16 is a square number.



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Example 9d

4 Evaluate:

a 6^2 **b** 5 squared **c** $(11)^3$
d 10 to the power of 2 **e** 20^3 **f** 12×12

Example 9b,c,e

5 Evaluate:

a $\sqrt{25}$ **b** square root of 16 **c** $\sqrt[3]{1}$
d the cube root of 27 **e** $\sqrt[3]{1\,000\,000}$

Example 9a

6 Evaluate:

a 8^2 **b** 7^2 **c** 1^2 **d** 12^2
e 3^2 **f** 15^2 **g** 5^2 **h** 0^2
i 11^2 **j** 100^2 **k** 17^2 **l** 33^2

Example 9b,c

7 Evaluate:

a $\sqrt{25}$ **b** $\sqrt{9}$ **c** $\sqrt{1}$ **d** $\sqrt{121}$
e $\sqrt{0}$ **f** $\sqrt{81}$ **g** $\sqrt{49}$ **h** $\sqrt{16}$
i $\sqrt{4}$ **j** $\sqrt{144}$ **k** $\sqrt{400}$ **l** $\sqrt{169}$

Example 9e

8 Using a calculator, evaluate the following.

a $\sqrt[3]{3375}$ **b** $\sqrt[3]{15\,625}$ **c** $\sqrt[3]{9261}$ **d** $\sqrt[3]{6859}$

Example 10

9 Without a calculator, evaluate the following. Then use a calculator to check your answers.

a $3^2 + 5^2 - \sqrt{16}$ **b** 4×4^2
c $8^2 - 0^2 + 1^2$ **d** $1^2 \times 2^2 \times 3^2$
e $\sqrt{5^2 - 3^2}$ **f** $\sqrt{81} - 3^2$
g $6^2 \div 2^2 \times 3^2$ **h** $\sqrt{9} \times \sqrt{64} \div \sqrt{36}$
i $\sqrt{12^2 + 5^2}$ **j** $\sqrt{\dfrac{100 - 64}{9}}$
k $\sqrt{\dfrac{9 - 5}{9}}$ **l** $\sqrt{\dfrac{28 + 4}{28 + 22}}$

10 Using a calculator, evaluate the following.

a $\sqrt{256 + 15^2}$ **b** $\sqrt{333 + 196}$
c $\sqrt{\dfrac{144 - 132}{3}}$ **d** $\sqrt{\dfrac{27 \times 18 \times 2}{3}}$
e $5 \times 4^2 \times \sqrt{81}$ **f** $16^2 \div 4^2 + 5^2$

PROBLEM-SOLVING AND REASONING	11, 12, 15	12, 13, 15, 16	13, 14, 16–18

11 List all the square numbers between 50 and 101.

12 List all the square numbers between 101 and 200. Hint: There are only four.

13 **a** Find two square numbers that add to 85.
b Find two square numbers that have a difference of 85.

14 Find three different square numbers that sum to 59.

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15 **a** Evaluate $3^2 \times 4^2$.

b Evaluate 12^2.

c The rule $a^2 \times b^2 = (a \times b)^2$ can be used to link $3^2 \times 4^2$ and 12^2. What are the values of a and b if $3^2 \times 4^2 = 12^2$?

d Check this formula using other numbers.

16 **a** Show that $3^2 + 4^2 = 5^2$.

b What does $6^2 + 8^2$ equal?

c What does $9^2 + 12^2$ equal?

d What does $30^2 + 40^2$ equal?

17 **a** Evaluate 11^2 and 111^2.

b Predict an answer for 1111^2.

c Evaluate 1111^2 and test your prediction.

18 Stuart decides there are no odd square numbers. His justification is that 'because an even number multiplied by an even number produces an even number, and that an odd number multiplied by an odd number also produces an even number, then there are no odd square numbers'. Do you agree with Stuart's claim? If not, give an example to explain your answer.

ENRICHMENT	—	—	19

Properties of square roots

19 Trial different numbers in the following formulas to determine whether these algebraic statements involving square roots are true or false.

a $\sqrt{a} + \sqrt{b} = \sqrt{a + b}$

b $\sqrt{a} - \sqrt{b} = \sqrt{a - b}$

c $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

d $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

e $\sqrt{a^2} = a$

f $a^3 = a\sqrt{a}$

g $\sqrt{a^2 + b^2} = a + b$

h $\sqrt{a^2 - b^2} = a - b$

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11F The zero index and index laws

Interactive Widgets HOTsheets Walkthrough

Recall that $5^4 = 5 \times 5 \times 5 \times 5$, $5^3 = 5 \times 5 \times 5$ and $5^2 = 5 \times 5$. Therefore, it makes sense that $5^1 = 5$. What about 5 to the power of 0? Does 5^0 have a value?

Also, what happens when we multiply 5^4 and 5^3 together? Is there any easy way to do that?

Let's start: Patterns of powers

- What comes next in these patterns of equations?

$5^4 = 625$		$3^4 = 81$
$\div 5$ ↓		$\div 3$ ↓
$5^3 = 125$		$3^3 = 27$
$\div 5$ ↓		$\div 3$ ↓
$5^2 = 25$		$3^2 = 9$
$\div 5$ ↓		$\div 3$ ↓
$5^1 = 5$		$3^1 = 3$
$\div 5$ ↓		$\div 3$ ↓
$5^? = ?$		$3^? = ?$

We know that $5^4 = 5 \times 5 \times 5 \times 5$. We also know that $5^3 = 5 \times 5 \times 5$.
Therefore, $5^4 \times 5^3 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^7$.

- Can you see a shortcut?
- Can you simplify $8^4 \times 8^5$?
- Does the shortcut work for the expression $6^4 \times 8^5$?

We know that $5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$.
We also know that $5^4 = 5 \times 5 \times 5 \times 5$.

Therefore, $5^6 \div 5^4 = \dfrac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5} = 5^2$.

- Can you see a shortcut?
- Can you simplify $8^9 \div 8^5$?
- Does the shortcut work for this expression $6^9 \times 8^5$?

We know that $5^4 = 5 \times 5 \times 5 \times 5$.
Therefore, $(5^4)^3 = (5 \times 5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5) = 5^{12}$.

- Can you see a shortcut?
- Can you simplify $(8^9)^5$?

$6^0 = 1$	
$8^0 = 1$	
$10^0 = 1$	
$500^0 = 1$	
$a^0 = 1$	(assuming $a \neq 0$)

Any positive number raised to the power of zero is equal to 1.

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Key ideas

- **The zero index**
  - When a number other than 0 is raised to the power 0, the result is 1. For example: $5^0 = 1$ and $10^0 = 1$ but $0^0 \neq 1$.
  - In general, if a is any number other than 0, $a^0 = 1$.
- **The index law for multiplication**
  - When multiplying terms with the same base, add the powers. For example: $8^9 \times 8^5 = 8^{14}$
- **The index law for division**
  - When dividing terms with the same base, subtract the powers. For example: $8^9 \div 8^5 = 8^4$
- **The index law for power of a power**
  - When raising a term in index form to a power, retain the base and multiply the indices. For example: $(8^9)^5 = 8^{45}$

Example 11 Using the zero index

Simplify each of the following terms.

a 25^0 **b** $(-25)^0$ **c** $-(25)^0$ **d** $(6 \times 5)^0$ **e** 6×5^0

SOLUTION	EXPLANATION
a $25^0 = 1$	Any non-zero number raised to the power of 0 is 1.
b $(-25)^0 = 1$	Any non-zero number raised to the power of 0 is 1.
c $-(25)^0 = -1 \times (25)^0$ $= -1 \times 1 = -1$	The negative symbol is not in the brackets, so the zero index does not apply to it.
d $(6 \times 5)^0 = 30^0$ $= 1$	Both the 6 and the 5 are in the brackets. The zero index applies to all the numbers in the brackets.
e $6 \times 5^0 = 6 \times 1 = 6$	The zero index applies to the 5 but not the 6.

Example 12 Using the index laws for multiplication and division

Simplify, giving your answer in index form.

a $7^3 \times 7^5$ **b** $10^{10} \times 10^{10}$ **c** $2^5 \times 2^4 \times 2^3$ **d** $7^8 \div 7^5$ **e** $7^8 \div 7^8$

SOLUTION	EXPLANATION
a $7^3 \times 7^5 = 7^8$	In a multiplication, if the bases are equal then add the powers.
b $10^{10} \times 10^{10} = 10^{20}$	In a multiplication, if the bases are equal then add the powers.
c $2^5 \times 2^4 \times 2^3 = 2^{12}$	In a multiplication, if the bases are equal then add the powers.
d $7^8 \div 7^5 = 7^3$	In a division, if the bases are equal then subtract the powers.
e $7^8 \div 7^8 = 7^0 = 1$	In a division, if the bases are equal then subtract the powers.

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Example 13 Using the index law for the power of a power

Simplify, giving your answer in index form.

a $(7^3)^5$

b $(7^3)^0$

SOLUTION

a $(7^3)^5 = 7^{15}$

b $(7^3)^0 = 7^0 = 1$

EXPLANATION

In this situation, multiply the powers.

In this situation, multiply the powers.

Exercise 11F

UNDERSTANDING AND FLUENCY	1–8	4–9	6–9

1 Copy and complete to give an answer in index form. Use cancelling in parts **c** and **d**.

a $3^2 \times 3^4 = 3 \times \square\ 3 \times \square \times \square \times \square$
$= 3^{\square}$

b $6^4 \times 6^3 = 6 \times \square \times \square \times \square \times 6 \times \square \times \square$
$= 6^{\square}$

c $5^5 \div 5^3 = \dfrac{5 \times \square \times \square \times \square \times \square}{5 \times \square \times \square}$
$= 5^{\square}$

d $9^4 \div 9^2 = \dfrac{9 \times \square \times \square \times \square}{9 \times \square}$
$= 9^{\square}$

2 Decide if these statements are true or false.

a $5 \times 5 \times 5 \times 5 = 5^4$ **b** $2^6 \times 2^2 = 2^{6+2}$ **c** $7^2 \times 7^4 = 7^{4-2}$ **d** $8^4 \div 8^2 = 8^{4+2}$

3 Write the missing words or numbers in these sentences.

a When raising a term or numbers in index form to another power, ________ the indices.

b Any number (except 0) raised to the power 0 is equal to ________.

4 Write the missing numbers in these tables.

a

Index form	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Basic numeral	64	32					

b

Index form	4^5	4^4	4^3	4^2	4^1	4^0
Basic numeral	1024	256				

5 Copy and complete this working.

a $(4^2)^3 = 4 \times \square \times 4 \times \square \times 4 \times \square$
$= 4^{\square}$

b $(12^3)^3 = (12 \times \square \times \square) \times (12 \times \square \times \square) \times (12 \times \square \times \square)$
$= 12^{\square}$

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Example 11

6 Evaluate each of the following.

a 5^0 **b** 9^0 **c** $(-6)^0$ **d** $(-3)^0$

e $-(4^0)$ **f** $\left(\frac{3}{4}\right)^0$ **g** $\left(-\frac{1}{7}\right)^0$

Example 12

7 Simplify, giving your answers in index form.

a $2^4 \times 2^3$ **b** $5^6 \times 5^3$ **c** $7^2 \times 7^4$ **d** $8^9 \times 8$

Example 13

8 Apply the index law for power of a power to simplify each of the following. Leave your answers in index form.

a $(3^2)^3$ **b** $(4^3)^5$ **c** $(3^5)^6$ **d** $(7^5)^2$

9 Simplify each of the following by combining various index laws.

a $4 \times (4^3)^2$

b $(3^4)^2 \times 3$

c $7^8 \div (7^3)^2$

d $(4^2)^3 \div 4^5$

e $(3^6)^3 \div (3^5)^2$

PROBLEM-SOLVING AND REASONING	10, 12	10–13	10–13

10 Evaluate, without using a calculator.

a $7^7 \div 7^5$ **b** $10^6 \div 10^5$ **c** $13^{11} \div 13^9$ **d** $2^{20} \div 2^{17}$

e $101^5 \div 101^4$ **f** $200^{30} \div 200^{28}$ **g** $7 \times 31^{16} \div 31^{15}$ **h** $3 \times 50^{200} \div 50^{198}$

11 Evaluate these without using a calculator.

a $(2^4)^8 \div 2^{30}$ **b** $(10^3)^7 \div 10^{18}$

c $((-1)^{11})^2 \times ((-1)^2)^{11}$ **d** $-2((-2)^3)^3 \div (-2)^8$

12 Explain why the following statements are incorrect and give the correct answer.

a $2 \times 5^0 = 1$ **b** $2^2 \times 2^2 = 4^4$ **c** $2 \times 3^6 = 6^6$ **d** $2^3 + 2^3 = 2^6$

13 Write down the next three equations in this pattern, using fractions on the right-hand side.

$$\begin{aligned}3^4 &= 81\\3^3 &= 27\\3^2 &= 9\\3^1 &= 3\\3^0 &= 1\end{aligned}$$

ENRICHMENT	—	—	14

Changing the base

14 The base of a number in index form can be changed using the index law for power of a power.

For example: $8^2 = (2^3)^2$
$= 2^6$

Change the base numbers and simplify the following using the smallest possible base integer.

a 8^4 **b** 32^3 **c** 9^3

d 81^5 **e** 25^5 **f** 243^{10}

g 256^9 **h** 2401^{20} **i** $100\,000^{10}$

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Fibonacci sequences

Leonardo Fibonacci was a famous 13th century mathematician who discovered some very interesting patterns of numbers that are found in nature.

Fibonacci's rabbits

These rules determine how fast rabbits can breed in ideal circumstances.

- Generation 1: One pair of newborn rabbits is in a paddock. A pair is one female and one male.
- Generation 2: When it is 2 months old, the female produces another pair of rabbits.
- Generation 3: When it is 3 months old, this same female produces another pair of rabbits.
- Every female rabbit always produces one new pair *every month* from age 2 months.

a Using the 'rabbit breeding rules', complete a drawing of the first five generations of rabbit pairs. Use it to complete the table below.

Month	1	2	3	4	5
Number of rabbits	2				
Number of pairs					

b Write down the numbers of pairs of rabbits at the end of each month for 12 months. This is the Fibonacci sequence.

c How many rabbits will there be after 1 year?

d Explain the rule for the Fibonacci sequence.

Fibonacci sequence in plants

a Count the clockwise and anticlockwise spiralling 'lumps' of some pineapples and show how these numbers relate to the Fibonacci sequence.

b Find three examples of flowers that have two terms of the Fibonacci sequence as the ratio of the numbers of clockwise and anticlockwise spirals of petals.

c On many plants, the number of petals is a Fibonacci number. Research the names and images of some of these 'Fibonacci' flowers.

Fibonacci sequence and the golden ratio

a Write down the next 10 terms of the Fibonacci sequence: 1, 1, 2, 3, 5, ...

b Write down a new set of numbers that is one Fibonacci number divided by its previous Fibonacci number. Copy and complete this table.

Fibonacci sequence	1	1	2	3	5			
Ratio	1	1	2	1.5				
		↑ $1 \div 1$	↑ $2 \div 1$	↑ $3 \div 2$				

cont.

c What do you notice about the new sequence (ratio)?

d Research the *golden ratio* and explain how it links to your new sequence.

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Puzzles and challenges

1 A number is said to be a 'perfect number' if the sum of its factors equals the number.
For this exercise, we must exclude the number itself as one of the factors.
The number 6 is the first perfect number.
Factors of 6 (excluding the numeral 6) are 1, 2 and 3.
The sum of these three factors is $1 + 2 + 3 = 6$. Hence, we have a perfect number.

a Find the next perfect number. Hint: It is less than 50.
b The third perfect number is 496. Find all the factors for this number and show that it is a perfect number.

2 When presented with a difficult multiplication question, such as 75×24, you can use your knowledge of factors to help you calculate the answer.
For example: $75 \times 24 = 25 \times 3 \times 6 \times 4$
Choosing appropriate pairs of factors can suddenly make the whole question easier.
Therefore:
$$\begin{aligned} 75 \times 24 &= (25 \times 4)\,(3 \times 6) \\ &= 100 \times 18 \\ &= 1800 \end{aligned}$$
Use factors to help you calculate the following.

a 28×50
b 15×32
c 36×15
d 18×35
e 120×36
f 44×25

3 Instead of carrying out a complex division algorithm, you could convert the divisor into a smaller pair of factors and complete two simpler division questions to arrive at the correct answer.
For example:
$$\begin{aligned} 1458 \div 18 &= (1458 \div 2) \div 9 \\ &= 729 \div 9 \\ &= 81 \end{aligned}$$
Use factors to help you calculate the following.

a $555 \div 15$
b $860 \div 20$
c $3600 \div 48$
d $1456 \div 16$
e $6006 \div 42$
f $2024 \div 22$

4 The easiest way of coming up with a large quantity of prime numbers is to place numbers in a grid and simply cross off multiples of 2, 3, 4, 5 and so on, until you reach the square root of the largest number in the grid.
For example: To find all the prime numbers that are less than 100, we could place the numbers in a 10×10 grid and cross off multiples of 2, 3, 4, 5, 6, 7, 8, 9 and 10. We can stop at 10 because 10 is the square root of 100. Actually, because some of these multiples are simply multiples of other numbers, we can cross out 6, 8, 9 and 10. This means that on our grid of 100 numbers, we only have to cross out the multiples of 2, 3, 5 and 7. Once you have crossed out all these multiples, the numbers that remain and are not crossed out are prime.

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Using the blank sieve (grid) below as an example, work out all the prime numbers less than 100.

01	02	03	04	05	06	07	08	09	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

5 Complete this sequence.

$2^2 = 1^2 + 3$

$3^2 = 2^2 + 5$

$4^2 = 3^2 + 7$

$5^2 =$ ________

$6^2 =$ ________

6 Use the digits 1, 9, 7 and 2, in that order, and any operations and brackets you like, to make as your answers the whole numbers 0 to 10.

Can you do any in more than one way?

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Chapter summary

Number properties

Factors

20: 1, 2, 4, ⑤, 10, 20
15: 1, 3, ⑤, 15
Highest common factor
HCF = ⑤

Multiples

20: 20, 40, ⑥⓪, 80, ...
15: 15, 30, 45, ⑥⓪, 75, ...
Lowest common multiple
LCM = ⑥⓪

Factor trees

90
9 10
3 3 2 5

$90 = 2 \times 3 \times 3 \times 5$
$90 = 2 \times 3^2 \times 5$

Prime numbers

5: factors 1 and 5
17: factors 1 and 17
A prime number only has two factors: 1 and itself.

Composite numbers

10: factors 1, 2, 5, 10
9: factors 1, 3, 9
A composite number has more than two factors.

The number 1 is neither a prime nor a composite number.

Powers

$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$

243: basic numeral; $3 \times 3 \times 3 \times 3 \times 3$: expanded form; 3^5: index form (3: base number; 5: index number)

Square number

$16 = 4 \times 4 = 4^2$
$25 = 5 \times 5 = 5^2$

Square root

$\sqrt{16} = 4$

$\sqrt{25} = 5$

Cube and cube roots

$2 \times 2 \times 2 = 2^3 = 8$
We say, '2 cubed is 8'.
Therefore, the cube root of 8 is 2.
e.g. $\sqrt[3]{8} = 2$

Division

$$\frac{52}{3} = 17\frac{1}{3}$$

52: dividend; 3: divisor; 17: quotient; 1: remainder

$52 = 3 \times 17 + 1$

Divisibility tests

2: last digit even or 0
3: sum digits ÷ 3
4: last two digits ÷ 4
5: last digit 0 or 5
6: ÷ by 2 and 3
8: last 3 digits ÷ 8
9: sum digits ÷ 9
10: last digit 0

The zero index

Any positive number raised to the power of zero gives 1.
e.g. $2^0 = 1$

Index law for multiplication

If the bases are the same, add the powers.
e.g. $2^6 \times 2^4 = 2^{10}$

Index law for division

If the bases are the same, subtract the powers.
e.g. $2^6 \div 2^4 = 2^2$

Index law for power of a power

Multiply the powers
e.g. $(2^6)^4 = 2^{24}$

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Multiple-choice questions

1 Which number is not a power of 2?
3, 6, 9, 12, 15, 18, 22, 24, 27, 30
A 16 **B** 22 **C** 32 **D** 8 **E** 2

2 Which group of numbers contains every factor of 60?
A 2, 3, 4, 5, 10, 12, 15, 60
B 2, 3, 4, 5, 10, 12, 15, 20, 30
C 1, 2, 3, 4, 5, 10, 12, 15, 20, 30
D 2, 3, 4, 5, 10, 15, 20, 30, 60
E 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

3 Which of the following numbers is *not* divisible by only prime numbers, itself and 1?
A 21 **B** 77 **C** 110 **D** 221 **E** 65

4 Which of the following groups of numbers include one prime and two composite numbers?
A 2, 10, 7 **B** 54, 7, 11 **C** 9, 32, 44 **D** 5, 17, 23 **E** 18, 3, 12

5 $7 \times 7 \times 7 \times 7 \times 7$ can be simplified to:
A 5^7 **B** 7^5 **C** 7×5 **D** 75 **E** 77 777

6 Evaluate $\sqrt{3^2 + 4^2}$.
A 7 **B** 5 **C** 14 **D** 25 **E** 6

7 The prime factor form of 48 is:
A $2^4 \times 3$ **B** $2^2 \times 3^2$ **C** 2×3^3 **D** 3×4^2 **E** $2^3 \times 6$

8 Evaluate $4^3 - 3 \times (2^4 - 3^2)$.
A 427 **B** 18 **C** 43 **D** 320 **E** 68

9 Factors of 189 are:
A 3, 7, 9, 18, 21, 27
B 3, 9, 18, 21
C 3, 9, 18
D 3, 7, 9, 17, 21
E 3, 7, 9, 21, 27, 63

10 Which number is *not* divisible by 3?
A 25 697 403 **B** 31 975 **C** 7 297 008 **D** 28 650 180 **E** 38 629 634 073

Short-answer questions

1 **a** Find the complete set of factors of 120 and circle those that are composite numbers.
b Determine three numbers between 1000 and 2000 that each have factors 1, 2, 3, 4, 5 and itself.

2 **a** Write down the first 12 multiples for each of 8 and 7 and circle the odd numbers.
b Which two prime numbers less than 20 have multiples that include both 1365 and 1274?

3 **a** State whether each of these numbers is a prime or composite number.
21, 30, 11, 16, 7, 3, 2
b How many prime multiples are there of 13?

4 **a** State the prime factors of 770.
b Determine three composite numbers less than 100, each with only three factors that are all prime numbers less than 10.

5 Simplify these expressions by writing them in index form.
a $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$
b $5 \times 5 \times 5 \times 5 \times 2 \times 2 \times 2 \times 2 \times 2$

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6 Write these numbers as a product of prime numbers. Use a factor tree and then index form.
a 32 **b** 200 **c** 225

7 Determine which number to the power of 5 equals each of the following.
a 100 000 **b** 243 **c** 1024

8 Evaluate each of the following.
a $5^2 - 3^2$ **b** $2 \times 4^2 - 5^2$ **c** $5 \times 3^4 - 3^2 + 1^6$ **d** $12^2 - (7^2 - 6^2)$

9 State yes or no. Do the answers to the following divisions contain remainders?
a $32\,766 \div 4$ **b** $1136 \div 8$ **c** $2417 \div 3$

10 a Carry out divisibility tests on the given number and fill in the table with ticks or crosses. State the explanation for each result.

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
84 539 424								

b Use divisibility rules to determine a 10-digit number that is divisible by 3, 5, 6 and 9.
c Determine a six-digit number that is divisible by 2, 3, 5, 6, 9 and 10.

11 Evaluate:
a $\sqrt{25}$ **b** $\sqrt{2500}$ **c** $\sqrt{5^2 + 12^2}$
d $4^2 - \sqrt{25} + \sqrt{7^2}$ **e** $\sqrt{16 \times 49} \div \sqrt{4}$ **f** $10^2 \div \sqrt{3^2 + 4^2}$

12 Write down the smallest number greater than 100 that is:
a divisible by 2 **b** divisible by 3 **c** divisible by 4
d divisible by 5 **e** divisible by 6 **f** divisible by 8
g divisible by 9 **h** divisible by 10 **i** prime

13 Write down the value of:
a cube root of 1728 **b** 6 cubed

14 Write down the value of:
a 5^0 **b** $2 + 5^0$ **c** $(2 + 5)^0$
d 2×5^0 **e** $(2 \times 5)^0$ **f** $2^0 + 5^0$

15 Simplify the following expressions using a power of 5.
a $5^{12} \times 5^3$ **b** $5^{12} \div 5^3$ **c** $(5^{12})^3$ **d** 5×5^8

Extended-response question

1 For the following questions, write the answers in index notation (i.e. b^x) and simplify where possible.
a A rectangle has breadth 27 cm and length 125 cm. Determine power expressions for its area and perimeter.
b A square's side length is equal to 4^3. Determine three power expressions for each of the area and perimeter of this square.
c $a \times a \times a \times a \times c \times c$
d $4^3 + 4^3 + 4^3 + 4^3$
e $\dfrac{(3^x + 3^x + 3^x)}{3}$

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Chapter 7: Time

Multiple-choice questions

1 An ancient building is dated back to 2500 BCE. How old was it in 2014? (Note: The year before 1 CE is 1 BCE; there is no year 0.)

A 4514 years B 486 years C 10 000 years D 2500 years E 2014 years

2 Anastasia walks to school in 25 minutes and 54 seconds, then home in 37 minutes and 17 seconds. What is Anastasia's total walking time?

A 62 min 11 s B 1 h 2 min 11 s C 53 min 11 s
D 17 min 37 s E 1 h 3 min 11 s

3 How many hours behind New South Wales is South Australia?

A $\frac{1}{2}$ h B 1 h C $1\frac{1}{2}$ h D 2 h E $2\frac{1}{2}$ h

4 6.1 hours is the same as:

A 6 h 1 min B 6 h 10 min C 6 h 12 min
D 6 h 6 min E 6 h 10 min 10 s

5 When it is 3 a.m. in New York, what time is it in Sydney, using Australian standard time?

A 3 a.m. B 12 noon C 5 p.m. D 3 p.m. E 6 p.m.

Short-answer questions

1 Write the following, using the units shown in brackets.

a $2\frac{1}{2}$ days (h) b 180 min (h) c 3.5 min (min and s)
d $6\frac{1}{2}$ h (h and min) e 9:45 p.m. (24-h time) f 1326 (a.m./p.m.)
g 5.75 h (h and min) h 6.32 h (h, min, s)

2 Calculate these time intervals.

a 2:25 a.m. to 3:10 a.m. b 6:18 p.m. to 8:09 p.m.
c 6 h 40 min 10 s to 7 h 51 min 11 s d 2 h 18 min 50 s to 4 h 10 min 40 s

3 a When it is 9 a.m. UTC, what time is it in the following places?

i New South Wales ii Western Australia
iii Iraq iv Central Greenland
v Alaska vi New Zealand

b When it is 4:20 p.m. in New South Wales during daylight saving time, what time is it in the following places?

i Western Australia ii South Australia
iii Queensland iv Tasmania

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4 Use the given train timetable for Richmond to Chatswood to answer the following questions.

Station	a.m.	p.m.
Richmond	6:37	2:56
Seven Hills	7:21	3:40
Parramatta	7:32	3:52
Central	8:07	4:25
Chatswood	8:35	4:53

a How long does it take to travel from:
  i Richmond to Parramatta in the morning?
  ii Seven Hills to Chatswood in the morning?
  iii Parramatta to Chatswood in the afternoon?
  iv Richmond to Chatswood in the afternoon?

b Does it take longer to travel from Richmond to Chatswood in the morning or afternoon?

c Domenic travels from Richmond to Parramatta in the morning, then from Parramatta to Chatswood in the afternoon. What is Domenic's total travel time?

5 How many hours and minutes are there between 2:30 p.m. Monday and 11:45 a.m. Tuesday?

6 A busy airport operates 18 hours a day. An aircraft lands every 9 minutes. How many aircraft arrive at the airport each day?

7 The local time in Sydney and Melbourne is 2 hours behind Auckland and 10 hours ahead of London. When it is 5 p.m. in Auckland, what time is it in London?

Extended-response question

1 Anchen is researching his next big trip. He plans to travel from Sydney to Cairns, to Alice Springs, to Perth and then return home.

a It is currently 4:30 p.m. at home (Australian standard time). What is the current time in these places?
  i Cairns
  ii Alice Springs
  iii Perth

b Anchen leaves Sydney at 0635 hours and arrives in Cairns at 0914 hours. What is the duration of the flight?

c After staying a week in Cairns, Anchen leaves on a 2 hour and 30 minute flight to Alice Springs. If he leaves at 1:30 p.m., what will be the time in Alice Springs when he arrives?

Chapter 8: Algebraic techniques

Multiple-choice questions

1 $12 - x$ means:

A 12 less than x
B x less than 12
C x has the value of 12
D x is less than 12
E x is more than 12

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2 Double the sum of x and y is:

A $2(x + y)$ B $2x + y$ C $x + 2y$ D $(x + y)^2$ E $x + y + 2$

3 Half the product of a and b is:

A $2ab$ B $\frac{a + b}{2}$ C $\frac{ab}{2}$ D $\frac{1}{2}a + \frac{1}{2}b$ E $\frac{a}{2} + b$

4 $4a + 3b + c + 5b - c$ is the same as:

A $32ab$ B $4a + 8b + 2c$ C $8a + 4b$ D $64abc$ E $4a + 8b$

5 If $a = 3$ and $b = 7$, then $3a^2 + 2b$ is equal to:

A 66 B 95 C 23 D 41 E 20

Short-answer questions

1 Consider the expression $5x + 7y + 3x + 9$.

a How many terms are in this expression?
b Can the expression be simplified?
c What is the value of the constant term?
d What is the coefficient of y?

2 Write an algebraic expression for each of the following.

a the sum of x and 3
b the product of a and 12
c the sum of double x and triple y
d w divided by 6
e double x taken from y

3 Find how many:

a cents are in $\$m$
b hours are in x days
c millimetres are in p kilometres
d days are in y hours

4 If $m = 6$, find the value of each of the following.

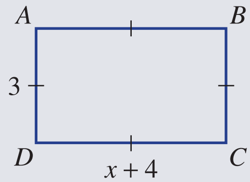
a $m + 7$
b $2m - 1$
c $6m + 3$
d $2(m - 3)$
e $\frac{m + 6}{2}$
f $\frac{m}{2} + 4m - 3$

5 Evaluate the expression $3(2x + y)$ when $x = 5$ and $y = 2$.

6 Simplify each of the following.

a $6a + 4a$
b $7x - 3x$
c $9a + 2a + a$
d $m + m - m$
e $6 + 2a + 3a$
f $x + y + 3x + y$

7 a Write an expression for the perimeter of rectangle $ABCD$.
b Write an expression for the area of rectangle $ABCD$.



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8 Find the missing term.

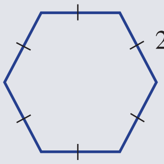
a $3a \times \_\_\_\_ = 18abc$ **b** $10ab \div \_\_\_\_ = 2a$

c $2p + 2p + 2p = 6\_\_\_\_$

9 Expand:

a $2(a + 3)$ **b** $12(a - b)$ **c** $8(3m + 4)$

10 Write the simplest expression for the perimeter of this figure.



Extended-response question

1 A bottle of soft drink costs $3 and a pie costs $2.

a Find the cost of:

i 2 bottles of soft drink and 3 pies

ii x bottles of soft drink and 3 pies

iii x bottles of soft drink and y pies

b If Anh has $50, find his change if he buys x bottles of soft drink and y pies.

Chapter 9: Equations

Multiple-choice questions

1 The solution to the equation $x - 3 = 7$ is:

A 4 **B** 10 **C** 9 **D** 11 **E** 3

2 The solution to the equation $2(x + 3) = 12$ is:

A 4.5 **B** 2 **C** 7 **D** 6 **E** 3

3 $m = 4$ is a solution to:

A $3m + 12 = 0$ **B** $\frac{m}{4} = 16$ **C** $10 - 2m = 2$

D $m + 4 = 0$ **E** $3m - 6 = 2$

4 The solution to $2p - 3 = 7$ is:

A $p = 4$ **B** $p = 5$ **C** $p = 2$ **D** $p = 10$ **E** $p = 3$

5 Ying thinks of a number. If he adds 4 to his number and then multiplies the sum by 5, the result is 35. What equation represents this information?

A $y + 9 = 35$ **B** $5y - 4 = 35$ **C** $5y + 4 = 35$

D $5(y + 4) = 35$ **E** $y + 20 = 35$

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Short-answer questions

1 Solve:

a $x + 9 = 12$ **b** $\frac{x}{9} = 12$ **c** $x - 9 = 12$ **d** $9x = 12$

2 Solve:

a $3x + 3 = 9$ **b** $\frac{x}{2} + 6 = 12$ **c** $3(m - 1) = 18$

3 Solve:

a $2x + 3 = 5$ **b** $\frac{2x}{3} = 5$ **c** $2(x - 3) = 5$

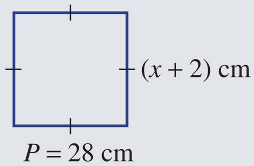
4 If $P = S - C$, find:

a P when $S = 190$ and $C = 87$ **b** S when $P = 47.9$ and $C = 13.1$

c C when $P = 384$ and $S = 709$

5 Use your knowledge of geometry and shapes to find the value of x in each of the following.

a



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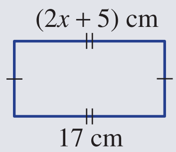
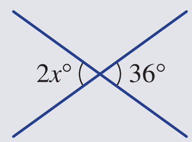
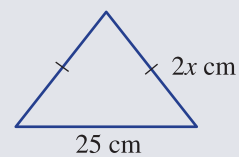


c



6 The perimeter of this triangle is 85 cm. Write an equation and then solve it to find the value of x.



Extended-response question

1 The cost of hiring a hall for an event is \$200 plus \$40 per hour.

a What is the cost of hiring the hall for 3 hours?

b What is the cost of hiring the hall for 5 hours?

c What is the cost of hiring the hall for n hours?

d If the cost of hiring the hall totals \$460, for how many hours was it hired?

Chapter 10: Measurement and computation of length, perimeter and area

Multiple-choice questions

1 17 mm is the same as:

A 0.17 m **B** 0.17 cm **C** 0.017 m **D** 170 cm **E** 1.7 m

2 0.006 L is the same as:

A 6 mL **B** 36 mL **C** 10 mL **D** 60 mL **E** 600 mL

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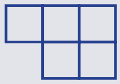
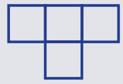
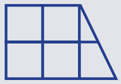
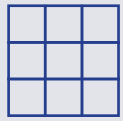
3 Which of the following shapes has the largest perimeter?

A 

B 

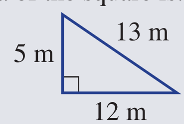
C 

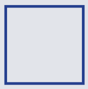
D 

E 

4 The perimeters of the two shapes shown below are equal. The area of the square is:

A 30 m^2 **B** 7.5 m^2
C 56.25 m^2 **D** 120 m^2
E 60 m^2





5 Which is the correct formula for the circumference of a circle?

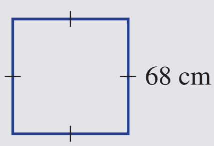
A $C = 2\pi r$ **B** $C = \pi r^2$ **C** $C = \pi r$ **D** $C = 2\pi d$ **E** $C = \pi d^2$

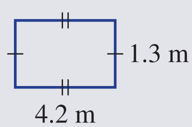
Short-answer questions

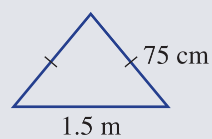
1 Complete these conversions.

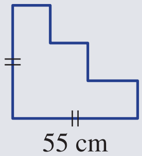
a 5 m = ____ cm **b** 6 km = _____ m **c** 1800 mm = _____ m
d 1.7 cm = _____ m **e** 180 cm = _____ m **f** $5\frac{1}{2}$ km = _____ m

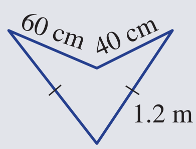
2 Find the perimeter of each of the following.

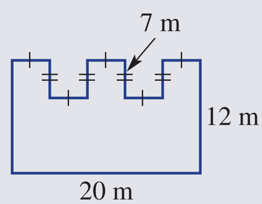
a 

b 

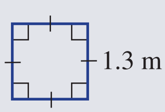
c 

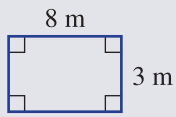
d 

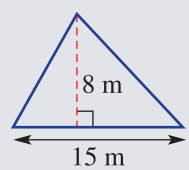
e 

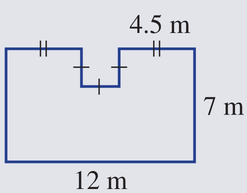
f 

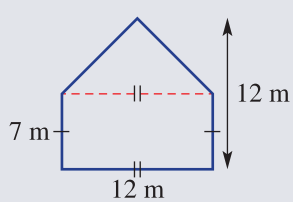
3 Find the area of each of the following.

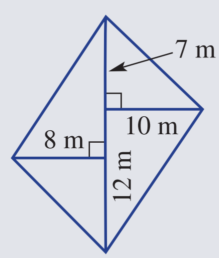
a 

b 

c 

d 

e 

f 

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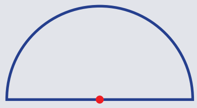
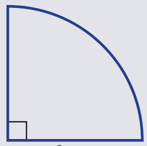
4 Find the exact perimeter of the following sectors.

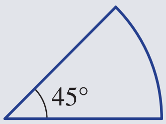
a



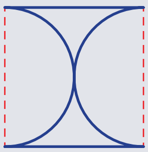
b



c



5 Find the exact perimeter of this composite figure, in which two semicircles have been removed from a square.



Extended-response question

1 Robert has been given 36 m of fencing with which to build the largest rectangular enclosure that he can, using whole number side lengths.

- **a** Draw three possible enclosures and calculate the area of each one.
- **b** What are the dimensions of the rectangle that gives the largest possible area?
- **c** If Robert chooses the dimensions in part **b** and puts a post on each corner, and then posts every metre along the boundary, how many posts will he need?
- **d** If it takes 15 minutes to dig each hole for each post, how many hours will Robert spend digging?

Chapter 11: Introducing indices

Multiple-choice questions

1 The first prime number greater than 90 is:

A 91 **B** 92 **C** 97 **D** 95 **E** 93

2 The highest common factor (HCF) of 12 and 18 is:

A 6 **B** 12 **C** 4 **D** 2 **E** 9

3 $2 \times 2 \times 2 \times 3$ is the same as:

A 6×3 **B** $2^3 \times 3$ **C** 8^3 **D** 6^3 **E** 4^3

4 Evaluating $3^2 - \sqrt{25} + 3$ gives:

A 8 **B** 5 **C** 4 **D** 17 **E** 7

5 The number 48 in prime factor form is:

A $2^4 \times 5$ **B** $2 \times 3 \times 5$ **C** $2^3 \times 3^2$ **D** $2^4 \times 3$ **E** $2^3 \times 3$

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Short-answer questions

1 List the factors of:
a 15 **b** 30 **c** 100

2 List the first five multiples of:
a 3 **b** 7 **c** 11

3 List all factors common to 30 and 36.

4 What is the highest factor common to 36 and 40?

5 Find the value of:
a 11^2 **b** $6^2 \times 2^2$ **c** $33 - 2^3$

6 What is the square root of 14 400?

7 Is the expression $\sqrt{3^2 + 4^2} = 3 + 4$ true or false?

8 Find the smallest number that must be added to 36 791 so that it becomes divisible by:
a 2 **b** 3 **c** 4

9 Simplify the following, and express your answer using indices.
a $(8^5)^2$ **b** $8^5 \times 8^2$ **c** $8^5 \div 8^2$
d $8^5 \times 8$ **e** $8^5 \div 8$ **f** $8^5 \times 8^5$

10 Write down the value of each of the following.
a 8^0 **b** 8×8^0 **c** $(8 \times 8)^0$
d $8 + 8^0$ **e** $(8 + 8)^0$ **f** $8^0 + 8^0$

Extended-response question

1 Copy and complete the following pattern.

a $(-1)^2 = -1 \times -1 = 1$
$(-1)^3 = -1 \times -1 \times -1 = -1$
$(-1)^4 =$
$(-1)^5 =$

b What is the value of -1 to the power of 279? How do you know?

c Will the value of -3 to the power of 79 be positive or negative? Explain.

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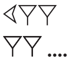
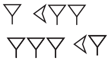
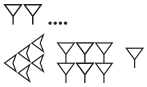
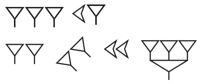
Chapter 1

Pre-test

1 **a** C **b** A **c** D **d** B
2 **a** 57 **b** 116 **c** 2044 **d** 11 002
3 **a** 13 **b** 37 **c** 999 **d** 8000
e 26 **f** 28
4 **a** 42, 49, 56, 63, 70, 77, 84 **b** 54, 63, 72, 81, 90, 99, 108
c 66, 77, 88, 99, 110, 121, 132
5 **a** 2 **b** 1 **c** 3 **d** 12
6 **a** 14 **b** 23 **c** 119 **d** 150
e 9 **f** 32 **g** 79 **h** 79
i 30 **j** 63 **k** 144 **l** 88
m 5 **n** 2 **o** 11 **p** 11
7 **a** 37, 58, 59, 62, 73, 159 **b** 31, 103, 130, 301, 310
c 13 429, 24 319, 24 913, 24 931, 29 143
8 **a** 0 **b** 1 **c** 1 **d** 2
e 1 **f** 2 **g** 2 **h** 1

Exercise 1A

1 **a** Babylonian **b** Roman **c** Egyptian
2 **a** **i** | **ii** ∩ **iii**  **iv** 
b **i**  **ii**  **iii** 
c **i** I **ii** V **iii** X
iv L **v** C
3 5 − 1 = 4
4 **a** **i** III **ii** ∩∩|
iii  **iv** 
b **i**  **ii** 
iii  **iv** 
c **i** II **ii** IX **iii** XXIV **iv** CLVI
5 **a** **i** 33 **ii** 111 **iii** 213 **iv** 241
b **i** 12 **ii** 24 **iii** 71 **iv** 205
c **i** 4 **ii** 8 **iii** 16 **iv** 40
6 **a** XXXVI **b** 
c  **d** DCLXXVIII
7  (21)
8 CLXXXVIII(188)
9  (64)
10 **a** Roman **b** Babylonian **c** Roman
11 **a** IV **b** IX **c** XIV **d** XIX
e XXIX **f** XLI **g** XLIX **h** LXXXIX
i XCIX **j** CDXLIX **k** CMXXII **l** MMMCDI
12 A separate picture is to be used for each 1, 10, 100 etc. The number 999 uses 27 pictures.
13 **a** **i**  **ii** 
iii  **iv** 
v  **vi** 
b third position = 60 × 60 = 3600 **c** 216 000
14 Answers may vary.

Exercise 1B

1 **a** 10 **b** 100 **c** 1000
2 **a** 263 **b** 7421 **c** 36 015 **d** 100 001
3 **a** B **b** E **c** D **d** A
e C **f** F **g** G and H
4 **a** 7 **b** 70 **c** 70 **d** 700
e 700 **f** 7000 **g** 700 **h** 70 000
5 **a** 20 **b** 2000 **c** 200 **d** 200 000
6 **a** true **b** false **c** true **d** true
e false **f** true **g** false **h** true
i false **j** false **k** false **l** true
7 **a** $1 \times 10 + 7 \times 1$
b $2 \times 100 + 8 \times 10 + 1 \times 1$
c $9 \times 100 + 3 \times 10 + 5 \times 1$
d 2×10
e $4 \times 1000 + 4 \times 100 + 9 \times 10 + 1 \times 1$
f $2 \times 1000 + 3 \times 1$
g $1 \times 10000 + 1 \times 1$
h $5 \times 10000 + 5 \times 1000 + 5 \times 100 + 5 \times 10 + 5 \times 1$
8 **a** 347 **b** 9416 **c** 7020
d 600 003 **e** 4 030 700 **f** 90 003 020
9 **a** 44, 45, 54, 55
b 29, 92, 279, 729, 927
c 4, 23, 136, 951
d 345, 354, 435, 453, 534, 543
e 12 345, 31 254, 34 512, 54 321
f 1001, 1010, 1100, 10 001, 10 100
10 **a** 6 **b** 6 **c** 24
11 27
12 **a** $a \times 10 + b \times 1$
b $a \times 1000 + b \times 100 + c \times 10 + d \times 1$
c $a \times 100000 + a \times 1$
13 Position gives the place value and only one digit is needed for each place. Hindu-Arabic system also gives us a way to represent the number zero.
14 **a** You do not need to write the zeros.
b **i** 41×10^2 **ii** 37×10^4 **iii** 2177×10^4
c **i** 38 100 **ii** 7 204 000
iii 1 028 000 000
d **i** 1×10^6 **ii** 1×10^9 **iii** 1×10^{12}
iv 1×10^{100} **v** $1 \times 10^{\text{googol}}$

Exercise 1C

1 **a** addition **b** subtraction
c addition **d** subtraction
2 **a** 10 **b** 69 **c** 12 **d** 20
3 **a** **i** 8 **ii** 27 **iii** 132
b **i** 6 **ii** 16 **iii** 8
4 **a** true **b** true **c** true
d false **e** true **f** false
5 **a** 18 **b** 19 **c** 32
d 140 **e** 21 **f** 9

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6 a 64 b 97 c 579
d 748 e 948 f 5597
g 378 683 h 591 579 i 201 011

7 a 11 b 36 c 112
d 4 e 3111 f 10 001

8 a 24 b 75 c 95
d 133 e 167 f 297

9 a 24 b 26 c 108
d 222 e 317 f 5017

10 a 51 b 128 c 244
d 119 e 242 f 502

11 a 12 b 27 c 107
d 133 e 14 f 90
g 1019 h 0 i 3

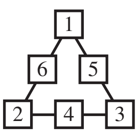
12 38 hours

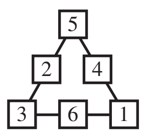
13 107 runs

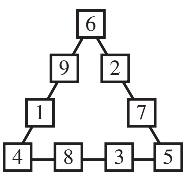
14 32 cows

15 29 marbles

16 107 cards

17 a i


ii


b i


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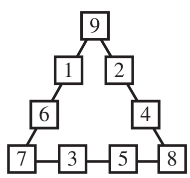

18 a Because 3 + 9 is more than 10, so you have to carry.
b Because 8 − 6 is easy, but 1 − 6 means you have to carry.

19 a $c - b = a$ b $b - a = c$

20 a four ways (totals are 9, 10, 11 and 12)
b Answers may vary.

21 29 and 58

22 a

6	1	8
7	5	3
2	9	4

b

10	15	8
9	11	13
14	7	12

c

15	20	13
14	16	18
19	12	17

d

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

Exercise 1D

1 a 17 b 101 c 144 d 110
e 1005 f 143 g 201 h 1105

2 a 8 b 27 c 67 d 84
e 15 f 92 g 29 h 979

3 a 8 b 1 c 1 d 6
e 2 f 1 g 8 h 3

4 a 87 b 99 c 41 d 86
e 226 f 745 g 1923 h 5080

5 a 161 b 225 c 2229 d 1975

6 a 77 b 192 c 418 d 4208
e 1223 f 1982

7 a 31 b 20 c 19 d 58
e 36 f 112 g 79 h 72

8 a 16 b 47 c 485 d 166

9 1854 sheep

10 576 kilometres

11 a 1821 students
b 79 students

12 a $\begin{array}{r} 3\boxed{8} \\ +\;5\;3 \\ \hline \boxed{9}1 \end{array}$ b $\begin{array}{r} 1\boxed{1}4 \\ +\;\;7\boxed{7} \\ \hline \boxed{1}9\;1 \end{array}$ c $\begin{array}{r} \boxed{6}\boxed{7} \\ +\boxed{8}\;4\;7 \\ \hline 9\;1\;4 \end{array}$

13 a $\begin{array}{r} 6\boxed{2} \\ -\;2\;8 \\ \hline \boxed{3}4 \end{array}$ b $\begin{array}{r} 2\boxed{6}5 \\ -\boxed{1}8\boxed{4} \\ \hline 8\;1 \end{array}$ c $\begin{array}{r} 3\boxed{0}\boxed{9}2 \\ -\;\;9\;2\boxed{7} \\ \hline \boxed{2}\;1\;6\;5 \end{array}$

14 a i 29 ii 37
b yes
c no
d The balance of −19 + 20 is +1, so add 1 to 36.

15 a Answers may vary.
b Different combinations in the middle column can be used to create the sum.

16 452 and 526

17 Students' own answers.

18 a

62	67	60
61	63	65
66	59	64

b

101	115	114	104
112	106	107	109
108	110	111	105
113	103	102	116

Exercise 1E

1 a 20, 24, 28 b 44, 55, 66 c 68, 85, 102

2 a true b true c false
d true e true f true
g false h true i false

3 a 3 b 0 c 5 d 2

4 a 56 b 54 c 48 d 121
e 72 f 35 g 108 h 39

5 a 57 b 174 c 112 d 266
e 105 f 124 g 252 h 159

6 a 96 b 54 c 96 d 72

7 a 66 b 129 c 432 d 165
e 258 f 2849 g 2630 h 31 581

8 a 235 b 4173 c 3825 d 29 190

9 1680 metres

10 $264

11 116 cards

12 no

13 a $\begin{array}{r} 3\;9 \\ \times\;\;7 \\ \hline 2\boxed{7}3 \end{array}$ b $\begin{array}{r} 2\;5 \\ \times\;\boxed{5} \\ \hline 1\;2\;5 \end{array}$
c $\begin{array}{r} 7\;9 \\ \times\;\boxed{3} \\ \hline \boxed{2}3\;7 \end{array}$ d $\begin{array}{r} 1\;3\;2 \\ \times\;\;\boxed{8} \\ \hline 1\;0\boxed{5}6 \end{array}$

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e $\begin{array}{r} 2\boxed{7} \\ \times\quad 7 \\ \hline \boxed{1}\,8\,9 \end{array}$ f $\begin{array}{r} \boxed{3}\boxed{9} \\ \times\quad 9 \\ \hline 3\,5\,1 \end{array}$

g $\begin{array}{r} 2\,3\boxed{2} \\ \times\quad 5 \\ \hline 1\boxed{1}6\,0 \end{array}$ h $\begin{array}{r} \boxed{3}\boxed{1}4 \\ \times\quad \boxed{7} \\ \hline \boxed{2}1\,9\,8 \end{array}$

14 twelve ways

15 a 3×21 b 9×52 c 7×32
d 5×97 e $a \times 38$ f $a \times 203$

16 three ways: (0, 1), (1, 5), (2, 9). You cannot carry a number to the hundreds column.

17 a Answers may vary; e.g. $\begin{array}{r} \boxed{2}1\boxed{7} \\ \times\quad \boxed{7} \\ \hline \boxed{1}5\,1\boxed{9} \end{array}$

b Answers may vary; e.g. $\begin{array}{r} 2\,9\boxed{5} \\ \times\quad 3 \\ \hline 8\boxed{8}\boxed{5} \end{array}$

18 6, 22

Exercise 1F

1 a 2 b 0 c 0 d 4
2 a 100 b 10 c 10000
3 a incorrect, 104 b incorrect, 546
c correct d incorrect, 2448
4 a 400 b 290 c 1830
d 4600 e 50000 f 63000
g 14410 h 29100000
5 a 340 b 1440 c 6440
d 22500 e 41400 f 460000
g 63400 h 9387000
6 a 407 b 1368 c 1890 d 9416
e 2116 f 40768 g 18620 h 33858
7 a 209 b 546 c 555 d 2178
8 $2176
9 $6020
10 86400 seconds

11 a $\begin{array}{r} 2\boxed{3} \\ \times\ 1\,7 \\ \hline 1\boxed{6}1 \\ 2\boxed{3}0 \\ \hline \boxed{3}\boxed{9}1 \end{array}$ b $\begin{array}{r} 1\boxed{4}3 \\ \times\quad 1\boxed{3} \\ \hline \boxed{4}2\,9 \\ 1\boxed{4}3\boxed{0} \\ \hline \boxed{1}\boxed{8}5\boxed{9} \end{array}$

c $\begin{array}{r} \boxed{4}\boxed{9} \\ \times\quad 3\,7 \\ \hline 3\,4\,3 \\ \boxed{1}4\boxed{7}\boxed{0} \\ \hline \boxed{1}\boxed{8}\boxed{1}\boxed{3} \end{array}$ d $\begin{array}{r} \boxed{1}2\boxed{6} \\ \times\quad 2\boxed{1} \\ \hline 1\,2\,6 \\ \boxed{2}5\,2\boxed{0} \\ \hline \boxed{2}6\boxed{4}\boxed{6} \end{array}$

12 60480 degrees
13 one number is a 1
14 a 39984 b 927908
c 4752188 d 146420482
15 a 1600 b 780 c 810 d 1000
16 a 84000 b 3185
17 123, 117
18 a 100, 121, 144, 169, 196, 225, 256, 289
b 961
c 9801

Exercise 1G

1 a 6, 7 b 12, 3
2 a 1 b 2 c 2 d 5
3 a 1 b 1 c 5 d 5
4 a 4 b 3 c 6 d 5
e 7 f 9 g 8 h 11
5 a 21 b 19 c 19 d 41
e 29 f 21 g 302 h 98
6 a 22 b 31 c 17 d 7
7 a 26 b 1094 c 0 d 0
8 a 23 rem. 2 b 13 rem. 1 c 69 rem. 1
d 41 rem. 1 e 543 rem. 1 f 20333 rem. 2
g 818 rem. 3 h 10001 rem. 0
9 a $131\frac{2}{4}$ or $131\frac{1}{2}$ b $241\frac{4}{7}$
c $390\frac{5}{6}$ d $11542\frac{1}{8}$
10 13 packs
11 124 packs
12 a $243 b $27
13 67 posts
14 15 taxis
15 19 trips; any remainder needs 1 more trip
16

2	9	12
36	6	1
3	4	18

17 a 1, 12 b 13, 7 c 4, 5
18 $68
19 a a b 0 c 1
20 8 or 23
21 $a = b$ or $a = -b$
22 a $33\frac{8}{11}$ b $54\frac{8}{17}$ c $31\frac{1}{13}$ d $108\frac{1}{15}$
e $91\frac{16}{23}$ f $123\frac{26}{56}$ or $123\frac{13}{28}$
23 a 3 rem. 269 b 11 rem. 5 c 18 rem. 625
24

1	6	20	56
40	28	2	3
14	5	24	4
12	8	7	10

Exercise 1H

1 a up b down c up
d up e down f down
2 a $60 + 100$ b 24×31
c $130 - 79$ d $270 - 110$
3 a 60 b 30 c 120
d 190 e 200 f 900
g 100 h 600 i 2000
4 a 20 b 30 c 100 d 900
e 6000 f 90000 g 10000 h 10
5 a 130 b 80 c 150
d 940 e 100 f 1000
g 1100 h 2600 i 1000

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6 a 120 **b** 160 **c** 100
d 12 **e** 40 **f** 2000
g 4000 **h** 100
7 a 1200 **b** 6300 **c** 20 000
d 8 000 000 **e** 5 **f** 16
g 10 **h** 25
8 Answers will vary. Compare yours with others.
9 ≈ 2100 scoops
10 ≈ 1200 sheep
11 ≈ 8 people
12 a 200 **b** 100 000 **c** 800 **d** 3 000 000
13 a i larger **ii** larger
b i larger **ii** larger
c i smaller **ii** smaller
d i larger **ii** larger
14 a i 9 **ii** 152 **iii** 10 **iv** 448
b One number is rounded up and the other is rounded down.
c i 3 **ii** 3 **iii** 1 **iv** 2
d If the numerator is decreased, then the approximation will be larger. If the denominator is increased, then the approximation will also be smaller. If the opposite occurs, the approximation will be larger.

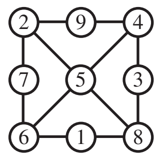
Exercise 1I

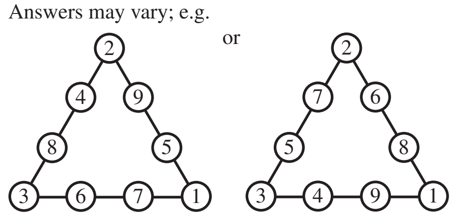
1 a parentheses **b** brackets
c multiplication, division **d** addition, subtraction
2 a + **b** ÷ **c** × **d** ×
e ÷ **f** + **g** ÷ **h** ×
i ÷ **j** − **k** × **l** ÷
3 a true **b** false **c** false **d** true
4 a 29 **b** 41 **c** 47 **d** 77
5 a 23 **b** 21 **c** 0 **d** 18
e 32 **f** 2 **g** 22 **h** 22
i 38 **j** 153 **k** 28 **l** 200
6 a 10 **b** 3 **c** 2 **d** 22
e 2 **f** 9 **g** 18 **h** 3
i 10 **j** 121 **k** 20 **l** 36
7 a 48 **b** 18 **c** 13
d 28 **e** 22
8 a 27 **b** 10 **c** 8 **d** 77
e 30 **f** 21 **g** 192
9 75 books
10 45 TV sets
11 a $(4 + 2) \times 3 = 18$
b $9 \div (12 - 9) = 3$
c $2 \times (3 + 4) - 5 = 9$
d $(3 + 2) \times (7 - 3) = 20$
e $(10 - 7) \div (21 - 18) = 1$
f $(4 + 10) \div (21 \div 3) = 2$
g $[20 - (31 - 19)] \times 2 = 16$
h $50 \div (2 \times 5) - 4 = 1$
i $(25 - 19) \times (3 + 7) \div 12 + 1 = 6$

12 first prize \$38, second prize \$8
13 a no **b** yes **c** no
d yes **e** yes **f** no
g no **h** yes **i** yes
14 a no **b** yes **c** no **d** yes
15 a b **b** 0 **c** $a + 1$ **d** b
16 a multiply by 2 and add 1
b multiply by 3 and subtract 3
c multiply by itself and add 1

Puzzles and challenges

1 The two people pay \$24 each, which is \$48 in total. Of that \$48 the waiter has \$3, leaving a balance of \$45 for the bill.

2


3 Answers may vary; e.g.


4 one way is $(2 + 7) \times 11 + 4 - 3$

5

5	4	6	1	3	2	9	8	7
2	8	3	7	5	9	6	1	4
7	9	1	6	8	4	3	2	5
8	2	4	5	1	3	7	6	9
1	3	7	9	2	6	4	5	8
9	6	5	4	7	8	1	3	2
3	7	9	2	6	5	8	4	1
6	1	2	8	4	7	5	9	3
4	5	8	3	9	1	2	7	6

1	7	5	4	6	9	3	8	2
2	9	4	1	3	8	6	5	7
8	6	3	7	5	2	9	1	4
3	4	1	2	7	6	8	9	5
9	2	8	5	4	3	7	6	1
7	5	6	8	9	1	2	4	3
4	3	9	6	2	5	1	7	8
6	1	7	3	8	4	5	2	9
5	8	2	9	1	7	4	3	6

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Multiple-choice questions

1 B 2 C 3 E 4 A 5 B
6 A 7 D 8 C 9 B 10 A

Short-answer questions

1 a 30 b 3000
c 300 d 30 000
e 3 000 000 f 300 000
2 a 50 b 5000 c 50 000
3 a 459 b 363 c 95 d 217
4 a 128 b 2324 c 191 d 295
5 a 95 b 132 c 220
d 41 e 33 f 24
g 29 000 h 10 800 i 14 678
6 a 1413 b 351 c $46\frac{5}{7}$ d $7540\frac{1}{2}$

7 a
$$\begin{array}{r} 2\ \boxed{2}\ 3 \\ +7\ 3\ \boxed{8} \\ \hline 9\ 6\ 1 \end{array}$$
b
$$\begin{array}{r} \boxed{7}\ 2\ \boxed{6} \\ -4\ \boxed{7}\ 3 \\ \hline 2\ 5\ 3 \end{array}$$
c
$$\begin{array}{r} \boxed{5}\ 3 \\ \times\ 2\ \boxed{7} \\ \hline \boxed{3}\ 7\ 1 \\ \boxed{1}\ 0\ 6\ 0 \\ \hline \boxed{1}\ \boxed{4}\ 3\ 1 \end{array}$$
d
$$\begin{array}{r} 1\ \boxed{8}\ 3 \\ 5\overline{)\boxed{9}\ {}^{4}1^{1}\ \boxed{5}} \end{array}$$
with no remainder

8 a 70 b 3300 c 1000
9 a 800 b 400 c 5000 d 10
10 a 24 b 4 c 14
d 20 e 0 f 13

Extended-response questions

1 a 646 loads b 9044 kilometres
c $36 430 d $295
2 a 3034 sweets b 249
c liquorice sticks, 6 d yes (124)

Chapter 2

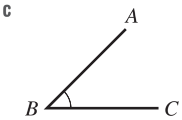
Pre-test

1 a III b IV c V
d II e I f VI
2 a 90° b 180° c 360°
d 270° e 45° f 315°
3 a 60° b 140° c 125°
4 a no b yes c no
5 a 80° b 150° c 150° d 255°

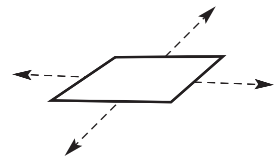
Exercise 2A

1 a • P

b


c


d


e


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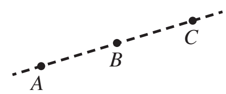
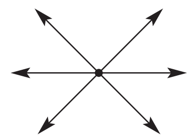


2 a

b


3 a ray b line c segment
4 a line b plane c plane d point
e line f plane g line h point
5 a point T b line CD c angle BAC
d plane e ray PQ f segment ST
6 a $\angle BOC$ b $\angle BAC$ c $\angle BEA$
d $\angle AOC$ e reflex $\angle BCD$
7 a segments AB, BC, AC, BD, CD; angles ABD, CBD, CDB, BCD
b segments PQ, PR, QR, RS, QS; angles PQS, RQS, QRS, RSQ
8 a C, B and D b A, C and D
9 a no b yes
10 a $AB = BC = CD = DA$
$\angle DAB = \angle BCD$
$\angle ABC = \angle ADC$
b $AB = AD$
$BC = CD$
$\angle ADC = \angle ABC$
11 a 8 b 14
12 10
13 a Missing numbers are 0, 1, 3, 6, 10, 15.
b For 5 points, add 4 to the previous total; for 6 points, add 5 to the previous total, and so on.
14 All segments should intersect at the same point; i.e. are concurrent.
15 a yes b no
16 number of segments $= \frac{n}{2}(n-1)$

Exercise 2B

1 Answers may vary.
2 a 2 b 3 c 4
3 a 50° b 145° c 90° d 250°
4 a acute, 40° b acute, 55° c right, 90°
d obtuse, 125° e obtuse, 165° f straight, 180°
g reflex, 230° h reflex, 270° i reflex, 325°
5 a i 20° ii 25° iii 35°
iv 40° v 35°
b $\angle AOF$ (155°)

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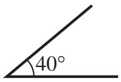
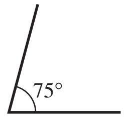
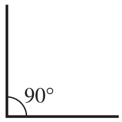
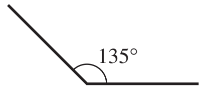
6 a 

b 

c 

d 

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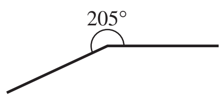
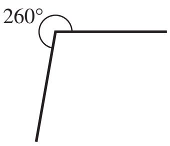
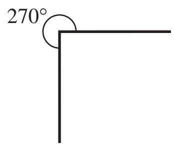
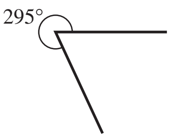
f 

g 

h 

i 

j 

7 a 29° b 55° c 35° d 130°
8 Yes, the two smaller angles make up the larger angle.
9 a 180° b 360° c 90° d 270°
e 30° f 120° g 330° h 6°
i 54° j 63° k 255° l 129°
10 a 180° b 90° c 120° d 30°
11 a i 70° ii 70° iii 90°
iv 90° v 80° vi 80°
b no
c Subtract 360° until you have a number that is less than 180° and then change the sign if it's negative.
12 Use the revolution to get 360° − 60° = 300°.
13 a 115° b 127.5° c 85°
d 77.5° e 122° f 176.5°

Exercise 2C

1 a, b angles should add to 90°
c complementary
2 a, b angles should add to 180°
c supplementary
3 a, b angles should add to 360°
c vertically opposite angles
4 a $\angle BOC$
b $\angle AOD$ and $\angle BOC$
c $\angle COD$
5 a 60 b 15 c 135
d 70 e 40 f 115
g 37 h 240 i 130
6 a N b N c S d N
e C f C g C h S
7 a $EF \perp GH$ b $ST \perp UV$ c $AX \perp NP$
8 a 30 b 75 c 60
d 135 e 45 f 130
9 a No, should add to 90. b Yes, they add to 180.
c Yes, they add to 360. d Yes, they are equal.
e No, they should be equal. f No, should add to 360.
10 a 30 b 60 c 60
d 45 e 180 f 36
11 24°
12 a $a + b = 90$
b $a + b + c = 180$
c $a + b = 270$
13 Only one angle – the others are either supplementary or vertically opposite.
14 a 360° b 72 c 108

b

Regular shape	*a*	*b*
triangle	120	60
square	90	90
pentagon	72	108
hexagon	60	120
heptagon	(360 ÷ 7)	(900 ÷ 7)
octagon	45	135

Exercise 2D

1 a 4 b no
2 a 2 b yes
3 a equal b supplementary
c equal d equal
4 a $\angle DEH$ b $\angle BEF$ c $\angle DEB$ d $\angle CBG$
5 a $\angle FEG$ b $\angle DEB$ c $\angle GEB$ d $\angle ABC$
6 a $\angle CFG$ b $\angle BCF$
7 a 130, corresponding b 70, corresponding
c 110, alternate d 120, alternate
e 130, vertically opposite f 67, vertically opposite
g 65, cointerior h 118, cointerior
i 100, corresponding j 117, vertically opposite
k 116, cointerior l 116, alternate
8 a $a = 70, b = 70, c = 110$
b $a = 120, b = 120, c = 60$
c $a = 98, b = 82, c = 82, d = 82$
d $a = 90, b = 90, c = 90$
e $a = 95, b = 85, c = 95$
f $a = 61, b = 119$
9 a No, corresponding angles should be equal.
b Yes, alternate angles are equal.
c Yes, cointerior angles are supplementary.
d Yes, corresponding angles are equal.
e No, alternate angles should be equal.
f No, cointerior angles should be supplementary.
10 a 35 b 41 c 110
d 30 e 60 f 141

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11 a 65 **b** 100 **c** 62
d 67 **e** 42 **f** 57
g 100 **h** 130 **i** 59

12 a 12 angles **b** two angles

13 120

14 a i The angle marked a is alternate to the 20° angle.
ii The angle marked b is alternate to the 45° angle.
b i $a = 25, b = 50$ **ii** $a = 35, b = 41$
iii $a = 35, b = 25$

15 a $a = 120, b = 120$ **b** 60
c Opposite angles are equal.

16 a Both angles do not add to 180°.
b The cointerior angles do not add to 180°.
c Alternate angles are not equal.

17 a i $\angle BDE$, alternate **ii** $\angle BED$, alternate
b add to 180°
c Three inside angles of a triangle add to 180°, which is always true.

18 Each triangle adds to 180°, so the total is 360°.

Exercise 2E

1 a $a = 65, b = 115$ **b** $a = 106, b = 106$
c $a = 55, b = 55$

2 a $\angle BED$, $a = 30$ **b** $\angle EBD$, $a = 70$ **c** $\angle ADC$, $a = 50$

3 a 60 **b** 120 **c** 115 **d** 123
e 50 **f** 80 **g** 60 **h** 65
i 45 **j** 60 **k** 55 **l** 335

4 a 130° **b** 120° **c** 55°
d 75° **e** 90° **f** 75°

5 a 50 **b** 150 **c** 60

6 a 1 **b** 2 **c** 2

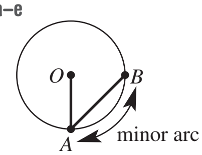
7 a 30 **b** 60 **c** 40
d 30 **e** 120 **f** 10

8 a 60 **b** 45 **c** 12

9 a 110 **b** 250 **c** 40
d 110 **e** 40 **f** 300

Exercise 2F

1 a–e



2 a radius **b** chord **c** minor arc
d centre **e** tangent **f** diameter

3 Your two circles should just touch.

4 $\angle BAC$ and $\angle ABC$ should be equal.

5 $\angle AEC$ should be 90°.

6 $\angle BAD$ should be 60°.

7 $\angle AOE$ and $\angle BOE$ should be equal.

8 a Construct the two circles so that they have the same radius.
b Use two circles of the same size. Point E should be the midpoint of AB.

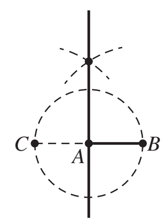
9 a Construct a 60° angle (see Question **5**) and then bisect this angle by constructing the angle bisector to form two 30° angles (see Question **6**).
b Construct an angle bisector of one of the 30° angles from part **a**.

10 a First, construct a 90° angle by constructing a perpendicular line and then construct the angle bisector of the 90° angle.
b Construct the angle bisector of one of the 45° angles from part **a**.

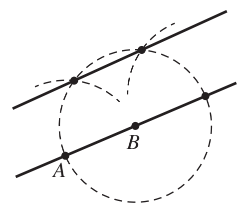
11 No, the circles must overlap.

12 a Yes, follow the procedure as in Question **7**.
b Yes, construct as for an acute or obtuse angle and draw the angle bisector on the reverse side of the vertex.

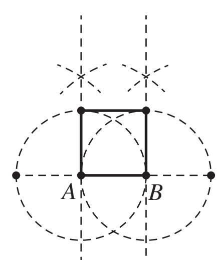
13 a



b



c



Exercise 2G

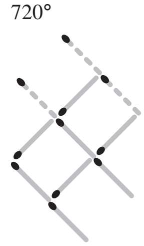
1–9 Answers may vary and can be checked by testing the properties of the constructions.

Puzzles and challenges

1 77.5°

2 720°

3



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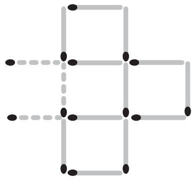
4 

5 5

6 143.5°

Multiple-choice questions

1 C	2 B	3 D	4 C	5 D
6 E	7 C	8 A	9 E	10 B

Short-answer questions

1 a segment *CD* b angle *AOB* c point *P*
d plane e ray *AC* f line *ST*

2 a acute, 35° b obtuse, 115° c reflex, 305°

3 a 180° b 90° c 90° d 150°

4 a 20 b 230 c 35
d 41 e 15 f 38
g 60 h 120 i 30

5 a $a°$ and $b°$ b $a°$ and $d°$
c $a°$ and $c°$ d $b°$ and $c°$
e $c°$ and $d°$ or $b°$ and $d°$

6 a Yes, corresponding angles are equal.
b No, alternate angles should be equal.
c No, cointerior angles should be supplementary.

7 a 100 b 95 c 51
d 30 e 130 f 78

8 a 145° b 140° c 100°

9 $a = 130$, $b = 50$, $c = 130$, $d = 20$

Extended-response questions

1 a i 32° ii 32° iii 148° iv 58°
b i corresponding ii cointerior
iii supplementary
c i 21° ii 159° iii 69°

2 a 12 pieces b 30
c i 15 ii 22.5 iii 20 iv 24

Chapter 3

Pre-test

1 a < b < c < d >

2 a 5°C b 2°C c 7°C

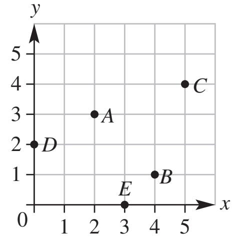
3 a 30 b 77 c 39 d 112

4 a 5 b 11 c 7 d 17

5 a 22 b 2 c 52 d 4
e 30 f 18

6 a positive b negative c negative d positive

7 $A(1, 1)$, $B(3, 2)$, $C(2, 3)$

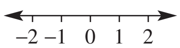
8 

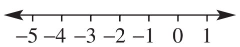
Exercise 3A

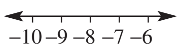
1 a −2, 2 b 0, 2 c −7, −5 d −5, −3, 0

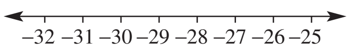
2 a −2 b −6 c 3 d 7
e 15 f −21 g −132 h 1071

3 a greater b less c greater d less

4 a 

b 

c 

d 

5 a −2, −1, 0, 1, 2, 3, 4
b −7, −6, −5, −4, −3, −2, −1, 0
c −2, −1, 0, 1
d −4, −3, −2, −1, 0
e −3, −2, −1, 0, 1, 2, 3
f −9, −8, −7, −6, −5, −4

6 a < b > c > d <
e > f < g < h >
i < j > k < l >

7 a 4°C b −1°C c −7°C d −25°C

8 a −10, −6, −3, −1, 0, 2, 4
b −304, −142, −2, 0, 1, 71, 126

9 a 0, −1, −2 b −2, 0, 2
c −5, −10, −15 d −44, −46, −48
e −79, −75, −71 f −101, −201, −301

10 a $5 b −$10

11 a −50 m b −212.5 m c 0 m

12 a 2 b 4 c 4 d 7
e 3 f 3 g 6 h 44

13 a −2 b 1 c −1
d −7 e −51 f 357

Exercise 3B

1 a right b right c left d left

2 a D b A c B d C

3 a 1 b 3 c 2 d 1
e −1 f −3 g −2 h −2
i −4 j −8 k −1 l 2
m 2 n −9 o 6 p −31

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4 a −2 b −1 c −8 d −19
e −4 f −10 g −15 h −7
i −41 j −12 k −22 l −47
m −300 n −100 o −93 p −634

5 a 5 b 9 c 5 d 2
e 5 f 7 g 3 h 10
i 5 j 16 k −4 l −5
m −6 n −13 o −30 p −113

6 a 5 b −9 c 1 d −13
e 1 f −22 g −32 h −4

7 a \$145 b \$55 c \$5250

8 a 3°C b −3°C c −46°C

9 69°C

10 a 59 m b 56 m

11 Answers may vary.

12 a i positive ii positive
iii negative iv zero
b i no ii yes

13 Other combinations may be possible.
a −, + b +, −, − c +, +, −, +
d −, +, +, +, − e +, +, − f −, +, −, −

Exercise 3C

1 a 6 + 3 b 7 − 2 c 12 + 8 d −9 + 2
e 9 −2 f −12 + 4 g −12 −9 h 20 + 9

2 a 4 b subtracting c −5
d subtracting e 2 f adding

3 a false b true c true d false
e false f true g false h false

4 a 1 b 5 c 6 d 2
e −3 f −5 g −2 h −4
i −3 j −22 k −35 l −80
m −10 n −29 o −50 p −112

5 a 5 b 11 c 50 d 90
e −4 f −3 g −5 h −34
i 2 j 1 k 0 l 8
m 28 n 34 o −12 p −76

6 a −3 b −10 c −4 d 4
e −1 f 4 g −1 h −5
i −4 j 4 k 2 l −24
m −6 n −5 o 2 p 4

7 a 0 b −5 c 8 d 12
e −9 f 5 g −6 h −91
i −15 j 6 k 17 l 11

8 −143 m

9 −\$35 000

10 −\$30

11 a i \$8000 ii −\$6000 b \$2000

12 a

−2	0	5
8	1	−6
−3	2	4

b

−13	−11	−6
−3	−10	−17
−14	−9	−7

13 a 3 + 4 b −2 + (−9) c 5 − (−2)
d 1 − (−2) e $a + (-b)$ f $a - (-b)$

14 a 4 b −1 c −3

15 a i no ii yes
b i yes ii no
c Yes, if $b < a$ then subtracting b takes the result to a number bigger than zero.

16 a 5050 b −50 c −100

Exercise 3D

1 a 2 b −3 c −4 d −4

2 a negative b positive c positive
d negative e positive f negative
g positive h negative

3 a

×	−2	−1	0	1	2
−2	4	2	0	−2	−4
−1	2	1	0	−1	−2
0	0	0	0	0	0
1	−2	−1	0	1	2
2	−4	−2	0	2	4

b

×	−4	−2	0	2	4
−4	16	8	0	−8	−16
−2	8	4	0	−4	−8
0	0	0	0	0	0
2	−8	−4	0	4	8
4	−16	−8	0	8	16

4 a positive b positive c negative
d positive e positive f negative

5 a −15 b −10 c −6 d −54
e 32 f 28 g 144 h −99
i −39 j −84 k 38 l −108
m 66 n −45 o 63 p 72

6 a −2 b −12 c −2 d −4
e 3 f 1 g −5 h −19
i −7 j −12 k −68 l 8
m 12 n 13 o −13 p 13

7 a 24 b 15 c −4 d 5
e 1 f −10 g 72 h 18
i 1 j −1 k −69 l −3

8 a −7 b 4 c −4 d 8
e 27 f −140 g 2 h −3
i −3 j −1 k −2 l 40

9 a −3 b −3 c 8 d 31
e 3 f 5 g −30 h −100

10 a 4 b 1 c 81 d 100
e 36 f 64 g 9 h 2.25

11 a (1, 6), (2, 3), (−1, −6), (−2, −3)
b (1, 16), (2, 8), (4, 4), (−1, −16), (−2, −8), (−4, −4)
c (−1, 5), (−5, 1)
d (−1, 24), (−24, 1), (−2, 12), (−12, 2), (−3, 8), (−8, 3), (−4, 6), (−6, 4)

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12 a ×, ÷ **b** ÷, × **c** ×, × **d** ÷, ×

13 a (4, −2), (−4, 2) **b** (33, −3), (−33, 3)

14 a **i** −8 **ii** 64 **iii** −27 **iv** 81

b parts **ii** and **iv**, even number of negative factors

c parts **i** and **iii**, odd number of negative factors

15 a negative **b** negative **c** positive

16 a $-ab$ **b** $-ab$ **c** ab

17 Answers will vary. You may use a calculator to check that your answers are correct.

Exercise 3E

1 a division **b** multiplication **c** multiplication
d division **e** addition **f** subtraction
g multiplication **h** multiplication **i** multiplication

2 a true **b** false **c** true
d false **e** false **f** true

3 a −7 **b** 7 **c** 19 **d** 9
e 16 **f** 14 **g** 6 **h** −32
i −5 **j** −4 **k** −18 **l** −4
m −10 **n** 4 **o** 0

4 a −10 **b** −2 **c** −6 **d** 1
e 2 **f** 9 **g** 1 **h** 4
i −14 **j** −20 **k** 2 **l** −5
m 8 **n** −6 **o** −12

5 \$528

6 −\$50

7 a $(-2 + 3) \times 8 = 8$ **b** $-10 \div (4 + 1) = -2$
c $(-1 + 7)\ 2 - 15 = -3$ **d** $(-5 - 1) \div (-6) = 1$
e $(3 - 8) \div 5 + 1 = 0$
f $50 \times (7 - 8) \times (-1) = 50$
g $-2 \times (3 - (-7)) - 1 = -21$
h $(-3 + 9) \div (-7 + 5) = -3$
i $(32 - (-8)) \div (-3 + 7) = 10$

8 three answers (−10, −21, −31)

9 a no **b** no **c** yes
d yes **e** no **f** yes

10 a true **b** true **c** true
d true **e** false **f** false

11 a **i** negative **ii** negative **iii** negative
iv positive **v** negative **vi** negative
b If there is an even number of negative factors, the result will be positive; if odd then negative

12 a 4 **b** 4 **c** −4
d −32 **e** −32 **f** 32
g 2 **h** −2 **i** −1

13 Kevin should have typed $(-3)^4$ to raise −3 to the power of 4. -3^4 is -1×3^4.

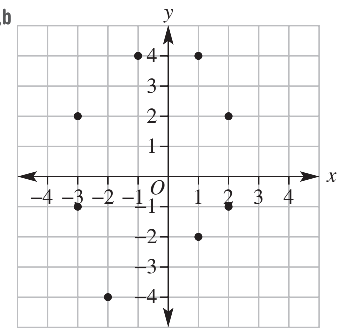
Exercise 3F

1 a D **b** B **c** A **d** C
e E **f** H **g** F **h** G

2 a 9 **b** 18 **c** 15 **d** 6
e 10 **f** 2 **g** 1

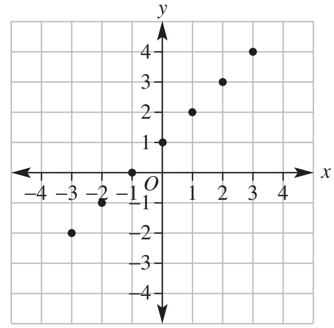
3 $A(2, 1)$, $B(3, -2)$, $C(-1, -4)$, $D(-2, 2)$, $E(4, 3)$, $F(2, -3)$, $G(-3, -1)$, $H(-4, 4)$

4 a,b



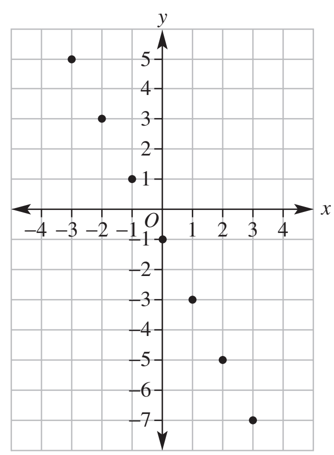
5 $A(3, 0)$, $B(0, -2)$, $C(-1, 0)$, $D(0, 4)$, $E(0, 2)$, $F(1, 0)$, $G(0, -4)$, $H(-3, 0)$

6 a



b They lie in a straight line.

7 a



b They lie in a straight line.

8 a triangle **b** rectangle
c trapezium **d** kite

9 a 4 square units **b** 6 square units
c 4 square units **d** 15 square units

10 28 km

11 $y = 2$

12 $y = 3$

13 a B **b** C **c** B, A, C

14 a **i** 1 and 4 **ii** 4
iii 3 and 4 **iv** 3

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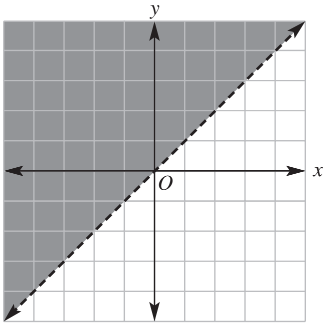
b



15 a $C(2, 4)$, $D(-1, 3)$
b $(2, 4)$, $(-1, 3)$, $(1, -3)$, $(4, -2)$
c $H(-2, 2)$

Puzzles and challenges

1 a

–3	2	–5
–4	–2	0
1	–6	–1

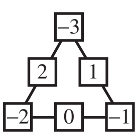
b

–9	5	4	–6
2	–4	–3	–1
–2	0	1	–5
3	–7	–8	6

c

–14	0	–1	–11
–3	–9	–8	–6
–7	–5	–4	–10
–2	–12	–13	1

2 a $-81, 243, -729$ **b** $4, -2, 1$
c $-10, -15, -21$ **d** $-8, -13, -21$
3 a 0 **b** -153
4 a $-3 \times (4 + -2) = -6$
b $-2 \times 5 \times -1 + 11 = 21$ or $-2 \times 5 \div -1 + 11 = 21$
c $(1 - 30 \div -6) \div -2 = -3$ or $1 \times 30 \div -6 - -2 = -3$
5 a 11 and -3 **b** 21 and -10
6 a



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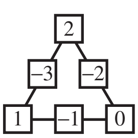

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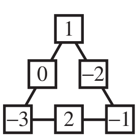

Multiple-choice questions

1 C **2** E **3** B **4** D **5** C
6 A **7** E **8** C **9** B **10** C

Short-answer questions

1 a $<$ **b** $<$ **c** $>$ **d** $<$
2 a -5 **b** -2 **c** -15 **d** 1
e -2 **f** -5 **g** 12 **h** -18
i -6 **j** 5 **k** -11 **l** 5
3 a -1 **b** -9 **c** -1 **d** 2
e -21 **f** -2 **g** -87 **h** 30
4 a -10 **b** -21 **c** 30 **d** -5
e -3 **f** 4 **g** 1 **h** -8
5 a -2 **b** -50 **c** -36 **d** -1
6 a -37 **b** 8 **c** -3
d 1 **e** 56 **f** 80
7 a -3 **b** -7
8 $A(3, 0)$, $B(2, 3)$, $C(-1, 2)$, $D(-4, -2)$, $E(0, -3)$, $F(4, -4)$

Extended-response questions

1 a 16°C **b** -31°C **c** 8°C
d 19°C **e** 27°C
2 rocket

Chapter 4

Pre-test

1 C
2 C
3 B
4 a $\frac{3}{4}$ **b** $\frac{1}{2}$ **c** $\frac{2}{3}$ **d** $\frac{4}{5}$
5 a $2\frac{3}{4}$ **b** $1\frac{1}{2}$ **c** $9\frac{1}{2}$ **d** $5\frac{1}{4}$
6 one-quarter of a block
7 a $2, 2\frac{1}{2}, 3$ **b** $\frac{4}{3}, \frac{5}{3}, \frac{6}{3}$ **c** $\frac{5}{4}, \frac{6}{4}, \frac{7}{4}$ **d** $\frac{2}{3}, \frac{5}{6}, 1$
8 a $3 \times \frac{1}{2}$ **b** $4 \times \frac{3}{4}$
c $\frac{5}{6} \times 1 = \frac{5}{6}$ **d** $\frac{6}{8} \div 1 = \frac{6}{8}$
9 a \$7.50 **b** \$40 **c** 75c **d** \$2
10 a true **b** true **c** true **d** false

Exercise 4A

1 a M **b** N **c** F **d** N
e F **f** F **g** M **h** F
i N **j** M **k** N **l** F
2 a F **b** N **c** M **d** N
e N **f** N **g** N **h** M
i F **j** M **k** F **l** N
3 a 1, 2, 5, 10
b 1, 2, 3, 4, 6, 8, 12, 24

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c 1, 17
d 1, 2, 3, 4, 6, 9, 12, 18, 36
e 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
f 1, 2, 3, 6, 7, 14, 21, 42
g 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
h 1, 2, 3, 4, 6, 12
i 1, 2, 4, 7, 14, 28

4 a 5, 10, 15, 20, 25, 30
b 8, 16, 24, 32, 40, 48
c 12, 24, 36, 48, 60, 72
d 7, 14, 21, 28, 35, 42
e 20, 40, 60, 80, 100, 120
f 75, 150, 225, 300, 375, 450
g 15, 30, 45, 60, 75, 90
h 100, 200, 300, 400, 500, 600
i 37, 74, 111, 148, 185, 222

5 a 3, 18
b 5
c 1, 4, 6, 9, 12, 24
d 3, 4, 5, 8, 12, 15, 24, 40, 120

6 a 22 **b** 162 **c** 21 **d** 117

7 a 24 **b** 1 **c** 1, 4, 9, 16, 25

8 a 12×16 **b** 21×15
c 12×15 **d** 11×11
e 12×28 or 24×14 or 21×16
f 19×26

9 a 20 min **b** 5 laps **c** 4 laps

10 a 25 **b** 0 **c** 23
d 2, 7, 12, 17, 37, 47, 62, 87, 137, 287 **e** 2

11 a false **b** true **c** false
d false **e** true

12 a 840 **b** 2520
c Answers will vary but should include cancelling out common factors.

13 a 1, 2, 4, 5, 10, 20, 25, 50, 100
b 1, 2, 4, 5, 10, 20, 25, 50, 100

14 Answers may vary, but they should be multiples of 9.

15 The larger number gives the reply. Any number is a multiple of its factors. So the answer is 'yes'.

16 Check the *output* each time.

Exercise 4B

1 a 1, 2, 4 **b** 4

2 Factors of 18 are 1, 2, 3, 6, 9 and 18.
Factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.
Therefore, the HCF of 18 and 30 is 6.

3 a 24, 48 **b** 24

4 Multiples of 9 are 9, 18, 27, 36, 45, 54, 63, 72, 81 and 90.
Multiples of 15 are 15, 30, 45, 60, 75, 90, 105 and 120.
Therefore, the LCM of 9 and 15 is 45.

5 a 1 **b** 1 **c** 2 **d** 3
e 4 **f** 15 **g** 50 **h** 24
i 40 **j** 25 **k** 21 **l** 14

6 a 10 **b** 3 **c** 1
d 1 **e** 8 **f** 12

7 a 36 **b** 21 **c** 60
d 110 **e** 12 **f** 10
g 36 **h** 18 **i** 60
j 48 **k** 132 **l** 105

8 a 30 **b** 84 **c** 12
d 45 **e** 40 **f** 36

9 a HCF = 5, LCM = 60 **b** HCF = 12, LCM = 24
c HCF = 7, LCM = 42 **d** HCF = 9, LCM = 135

10 312

11 9

12 LCM = 780, HCF = 130

13 a 12 min
b Andrew 9 laps, Bryan 12 laps, Chris 6 laps
c 3 times (including the finish)
d Answer will vary.

14 a 8, 16 **b** 24, 32

15 1 and 20; 2 and 20; 4 and 20; 5 and 20; 10 and 20; 4 and 5; 4 and 10.

16 a 2520
b 2520
c Identical answers; 2520 is already divisible by 10, so adding 10 to list does not alter LCM.
d 27720 ($2^3 \times 5 \times 7 \times 9 \times 11$)

Exercise 4C

1 a 9 **b** 7

2 proper: B, E, F, G
improper: A, C, H, I, K, L
whole numbers: D, J, K

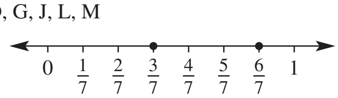
3 A a 4 **b** 1
c i 4 **ii** 1 **iii** $\frac{1}{4}$

B a 8 **b** 7
c i 8 **ii** 7 **iii** $\frac{7}{8}$

C a 3 **b** 2
c i 3 **ii** 2 **iii** $\frac{2}{3}$

D a 12 **b** 5
c i 12 **ii** 5 **iii** $\frac{5}{12}$

4 D, G, J, L, M

5 a


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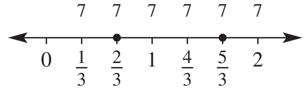

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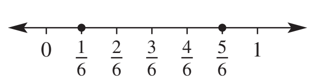

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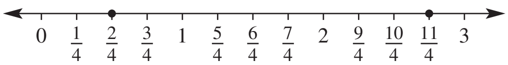
d



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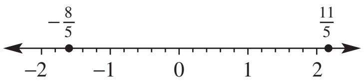

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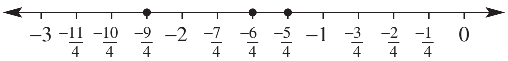
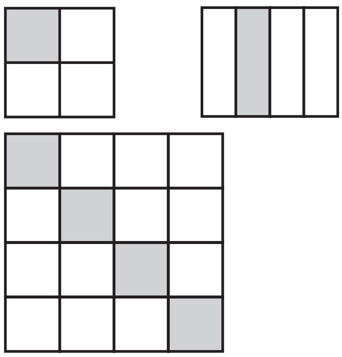


6 Answers may vary.

a



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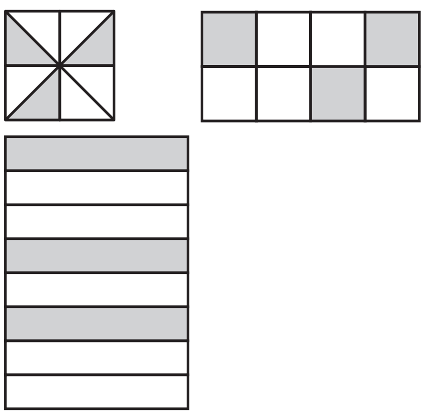
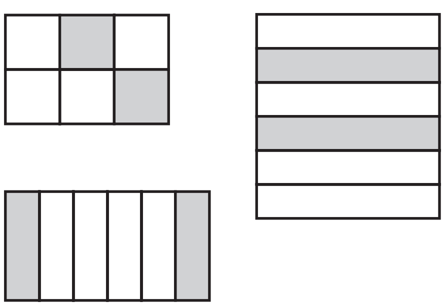


c



7 a $\frac{7}{5}, \frac{8}{5}, \frac{9}{5}$ b $\frac{9}{8}, \frac{10}{8}, \frac{11}{8}$ c $\frac{5}{3}, \frac{6}{3}, \frac{7}{3}$

d $\frac{7}{7}, \frac{6}{7}, \frac{5}{7}$ e $\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$ f $-\frac{2}{4}, -\frac{7}{4}, -\frac{12}{4}$

8 a $\bigcirc = 1\frac{1}{2}, \frac{3}{2}$; $\square = 3\frac{1}{2}, \frac{7}{2}$; $\triangle = 5, \frac{10}{2}$

b $\bigcirc = \frac{1}{5}$; $\square = \frac{4}{5}$; $\triangle = 2\frac{1}{5}, \frac{11}{5}$

c $\triangle = \frac{3}{7}$; $\square = 1\frac{4}{7}, \frac{11}{7}$; $\bigcirc = 2\frac{2}{7}, \frac{16}{7}$

d $\square = -\frac{2}{3}$; $\triangle = -\frac{1}{2}$; $\bigcirc = \frac{2}{3}$

9 division

10 a $\frac{6}{11}$ b $\frac{4}{8} = \frac{1}{2}$ c $\frac{7}{12}$

d $\frac{5}{6}$ e $\frac{3}{12} = \frac{1}{4}$ f $\frac{5}{9}$

11 a $\frac{12}{43}$ b $\frac{13}{15}$ c $\frac{11}{12}$

d $\frac{1}{12}$ e $\frac{2}{11}$ f $\frac{144}{475}$

g $\frac{7}{20}$ h $\frac{1}{4}$ i $\frac{3}{7}$

12 Proper fraction: When you have part of a whole, and therefore you have a numerator that is smaller than the denominator.

Improper fraction: It is called improper because it is impossible to break a whole into parts and end up with more than a whole. An improper fraction is when the numerator is greater than the denominator.

13 C

14 a i $\frac{1}{5}$ ii 50 mL b i 40 mL ii $\frac{4}{25}$

c i 32 mL ii $\frac{16}{125}$ d i 90 mL ii $\frac{9}{25}$

e i 122 mL ii $\frac{61}{125}$

f Approximately, yes they will.

Exercise 4D

1 $\frac{3}{6}, \frac{2}{4}, \frac{11}{22}, \frac{5}{10}$ **2** $\frac{4}{10}, \frac{16}{40}, \frac{2}{5}, \frac{80}{200}$

3 a 2, 12, 10, 20, 300 b 1, 3, 24, 20, 40

4 a $\frac{1}{5}$ b $\frac{1}{6}$ c $\frac{2}{3}$ d $\frac{3}{4}$

5 a 10, 10, 10, 1 b 2, 2, 2, 2

c 4, 4, 4, 7 d 3, 3, 3, 3, 5

6 Answers may vary.

a $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{10}{20}$ b $\frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{10}{40}$

c $\frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \frac{10}{25}$ d $\frac{6}{10}, \frac{12}{20}, \frac{24}{40}, \frac{48}{80}$

e $\frac{4}{18}, \frac{6}{27}, \frac{8}{36}, \frac{20}{90}$ f $\frac{6}{14}, \frac{9}{21}, \frac{12}{28}, \frac{30}{70}$

g $\frac{10}{24}, \frac{50}{120}, \frac{500}{1200}, \frac{5000}{12\,000}$ h $\frac{6}{22}, \frac{9}{33}, \frac{12}{44}, \frac{30}{110}$

7 a 9 b 50 c 33 d 56

e 8 f 2 g 12 h 39

i 35 j 200 k 105 l 2

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8 **a** $\neq$ **b** $=$ **c** $\neq$
d $\neq$ **e** $=$ **f** $=$
g $=$ **h** $=$ **i** $=$

9 **a** $\frac{3}{4}$ **b** $\frac{2}{3}$ **c** $\frac{1}{3}$ **d** $\frac{4}{11}$
e $\frac{2}{5}$ **f** $\frac{1}{11}$ **g** $\frac{1}{7}$ **h** $\frac{1}{3}$
i $\frac{7}{9}$ **j** $\frac{3}{8}$ **k** $\frac{5}{6}$ **l** $\frac{5}{1} = 5$

10 **a** $\frac{7}{14} = \frac{1}{2}$ **b** $\frac{12}{16} = \frac{3}{4}$ **c** $\frac{4}{42} = \frac{2}{21}$ **d** $\frac{7}{63} = \frac{1}{9}$

11 $\frac{2}{9}$

12 **a** 35 **b** 45 **c** 30 **d** 24

13 Justin 4, Joanna 3, Jack 5

14 **a** 75 **b** $\frac{1}{3}$ **c** 150

15 Answers may vary; $\frac{6}{10}$; The number of answers is infinite because there are an infinite number of equivalent fractions that can be created simply by multiplying the numerator and the denominator by a common value.

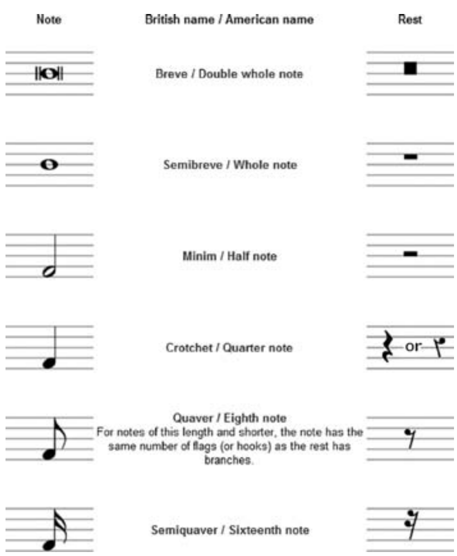
16 Answers may vary.

a

b 12 quavers (eighth notes) to a bar

c

d



e Each dot increases the length of the note by another 50%.

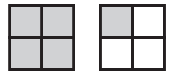
e.g.  $= \frac{3}{4}$ note (value = 3 beats)
= half note + 50%
= half note + quarter note

Exercise 4E

1 **a** 2 and 3 **b** 11 and 12 **c** 36 and 37

2 **a** 24 **b** 360 **c** 60 **d** 24

3 **a**



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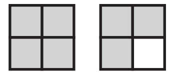
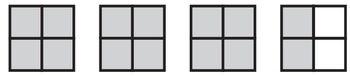
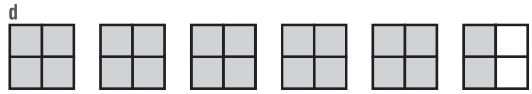


c



d



4 **a** 8 **b** 12 **c** 28 **d** 44
e 17 **f** 7 **g** 10 **h** 24

5 **a** A $7\frac{1}{2}$, B $10\frac{1}{2}$
b C $1\frac{2}{3}$, D $2\frac{1}{3}$, E $4\frac{2}{3}$
c F $23\frac{1}{5}$, G $24\frac{2}{5}$, H $25\frac{4}{5}$, I $26\frac{1}{5}$
d J $-\frac{1}{3}$, K $-\frac{2}{3}$, L $-1\frac{2}{3}$, M $-2\frac{1}{3}$

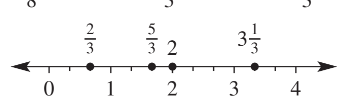
6 **a** Answers may vary.
b Answers may vary.

7 **a** $\frac{11}{5}$ **b** $\frac{8}{5}$ **c** $\frac{10}{3}$ **d** $\frac{17}{3}$
e $\frac{29}{7}$ **f** $\frac{24}{7}$ **g** $\frac{5}{2}$ **h** $\frac{13}{2}$
i $\frac{22}{5}$ **j** $\frac{23}{2}$ **k** $\frac{42}{5}$ **l** $\frac{103}{10}$
m $\frac{55}{9}$ **n** $\frac{25}{9}$ **o** $\frac{42}{8}$ **p** $\frac{21}{8}$
q $\frac{23}{12}$ **r** $\frac{38}{11}$ **s** $\frac{53}{12}$ **t** $\frac{115}{12}$
u $\frac{115}{20}$ **v** $\frac{803}{100}$ **w** $\frac{643}{10}$ **x** $\frac{104}{5}$

8 **a** $1\frac{2}{5}$ **b** $1\frac{1}{3}$ **c** $1\frac{2}{3}$ **d** $1\frac{3}{4}$
e $3\frac{2}{3}$ **f** $4\frac{1}{5}$ **g** $2\frac{2}{7}$ **h** $2\frac{1}{2}$
i $1\frac{5}{7}$ **j** $3\frac{1}{6}$ **k** $6\frac{2}{3}$ **l** $10\frac{1}{4}$
m $4\frac{3}{8}$ **n** $5\frac{1}{5}$ **o** $6\frac{6}{7}$ **p** $13\frac{2}{3}$
q $3\frac{1}{12}$ **r** $7\frac{4}{11}$ **s** $9\frac{3}{10}$ **t** $11\frac{1}{7}$
u $2\frac{31}{100}$ **v** $33\frac{3}{10}$ **w** $12\frac{3}{11}$ **x** $12\frac{5}{12}$

9 **a** $2\frac{1}{2}$ **b** $2\frac{4}{5}$ **c** $1\frac{1}{3}$ **d** $1\frac{1}{3}$
e $1\frac{1}{8}$ **f** $3\frac{1}{3}$ **g** $2\frac{2}{3}$ **h** $2\frac{2}{5}$

10 **a**



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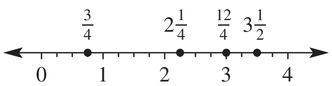
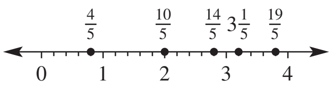
b



c



11 a $2\frac{1}{3}, 3\frac{2}{3}, 4$

b $1\frac{4}{7}, 2\frac{1}{7}, 3, 3\frac{4}{7}, 3\frac{6}{7}$

c $2\frac{2}{5}, 4\frac{1}{5}, 4\frac{4}{5}, 6, 7\frac{1}{5}$

12 a 15 **b** $1\frac{7}{8}$ **c** 9 **d** $1\frac{1}{8}$

13 a 11 **b** 9 **c** $4x - 1$

d $3y - 1$ **e** mn

14 a i $1\frac{2}{3}, 2\frac{1}{3}, 3\frac{1}{2}$ **ii** $1\frac{5}{6}$

b i $2\frac{3}{4}, 3\frac{2}{4}, 4\frac{2}{3}$ **ii** $1\frac{11}{12}$

c i $3\frac{4}{5}, 4\frac{3}{5}, 5\frac{3}{4}$ **ii** $1\frac{19}{20}$

d $1\frac{29}{30}$

e A mixed numeral with a whole number part equal to 1. Denominator equal to product of two largest numbers, numerator is one less than denominator.

Exercise 4F

1 a $\frac{5}{7}$ **b** $\frac{7}{3}$ **c** $\frac{9}{11}$ **d** $\frac{8}{5}$

2 a 10 **b** 21 **c** 20 **d** 30

e 6 **f** 10 **g** 12 **h** 24

i 30 **j** 12 **k** 24 **l** 30

3 a 15 **b** 20 **c** 21 **d** 10

e 24 **f** 60 **g** 12 **h** 12

4 a 6 **b** 8 **c** 4 **d** 6

e 15 **f** 15

5 a $>$ **b** $>$ **c** $=$ **d** $<$

e $<$ **f** $>$ **g** $<$ **h** $=$

i $>$ **j** $>$ **k** $<$ **l** $=$

m $>$ **n** $>$ **o** $<$ **p** $>$

q $>$ **r** $>$ **s** $>$ **t** $<$

6 a $\frac{3}{5}, 1\frac{2}{5}, \frac{8}{5}$ **b** $\frac{2}{9}, \frac{1}{3}, \frac{5}{9}$

c $\frac{2}{5}, \frac{3}{4}, \frac{4}{5}$ **d** $\frac{3}{5}, \frac{2}{3}, \frac{5}{6}$

e $2\frac{1}{4}, \frac{5}{2}, \frac{11}{4}, 3\frac{1}{3}$ **f** $\frac{5}{3}, \frac{7}{4}, \frac{11}{6}, \frac{15}{8}$

g $\frac{11}{5}, \frac{9}{4}, 2\frac{1}{2}, 2\frac{3}{5}, 2\frac{7}{10}$ **h** $4\frac{1}{6}, 4\frac{10}{27}, 4\frac{4}{9}, 4\frac{2}{3}, 5$

7 a $-2\frac{1}{2}, -\frac{3}{2}, -\frac{5}{4}$ **b** $-\frac{1}{3}, -\frac{1}{4}, -\frac{1}{6}$

8 a $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ **b** $\frac{3}{5}, \frac{3}{6}, \frac{3}{7}, \frac{3}{8}$

c $\frac{7}{2}, \frac{7}{5}, \frac{7}{7}, \frac{7}{8}$ **d** $\frac{1}{10}, \frac{1}{15}, \frac{1}{50}, \frac{1}{100}$

e $10\frac{2}{3}, 8\frac{3}{5}, 7\frac{1}{11}, 5\frac{4}{9}$ **f** $2\frac{1}{3}, 2\frac{1}{5}, 2\frac{1}{6}, 2\frac{1}{9}$

9 A, C, B

10 Andrea, Rob, Dean, David

11 a $\frac{5}{9}, \frac{6}{9}$ **b** $\frac{11}{4}, \frac{14}{4}$

c $\frac{5}{6}, \frac{3}{6}$ **d** $\frac{10}{14}, \frac{11}{14}$

12 Answers may vary.

a $\frac{13}{20}$ **b** $\frac{1}{3}$ **c** $\frac{5}{21}$

d $\frac{3}{4}$ **e** $2\frac{1}{4}$ **f** $8\frac{29}{40}$

13 Answers will vary. The first step is to ensure both fractions have a common denominator. This is done through the technique of equivalent fractions. Once you have a common denominator, you simply have to choose a new numerator that fits between the two existing numerators.

14 a 7, 8 **b** 16

15 a Thomas **b** $\frac{1}{10}$ of a donut

16 a i $\frac{1}{4}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{3}$

b Teacher to check

c Answers will vary

Exercise 4G

1 a $\frac{2}{10}$ **b** $\frac{3}{100}$ **c** $\frac{7}{1000}$

2 a 5 **b** 6 **c** 7 **d** 37

3 a $\frac{6}{10}$ **b** $\frac{6}{100}$ **c** $\frac{6}{1000}$ **d** $\frac{6}{10}$

e 6 **f** $\frac{6}{100}$ **g** $\frac{6}{100}$ **h** $\frac{6}{1000}$

4 a false **b** false **c** true **d** false

e true **f** true **g** false **h** true

i true **j** true **k** true **l** true

5 a 0.3 **b** 0.8 **c** 0.15 **d** 0.23

e 0.9 **f** 0.02 **g** 0.121 **h** 0.074

6 a 6.4 **b** 5.7 **c** 212.3 **d** 1.16

e 14.83 **f** 7.51 **g** 5.07 **h** 18.612

7 a 7.6 **b** 12.9 **c** 33.04

d 26.15 **e** 8.42 **f** 99.012

8 a 0.1 **b** 0.03 **c** 0.02 **d** 0.5

e 0.001 **f** 0.01 **g** 0.15 **h** 0.11

9 a 3.05, 3.25, 3.52, 3.55

b 3.06, 3.6, 30.3, 30.6

c 1.718, 1.871, 11.87, 17.81

d 22.69, 22.96, 26.92, 29.26, 29.62

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10 a Waugh, Border, Gilchrist, Taylor, Hughes
b first
11 a Day 6
b Day 4
c Days 2, 5 and 6
12 a *c.c*, *c.a*, *b.c*, *b.a*, *a.c*, *a.b*
b *c.bc*, *c.ab*, *b.ca*, *b.bb*, *a.ca*, *a.bc*, *a.aa*, *ba.ca*, *ab.ab*
13 a 0.*a* **b** 0.0*a* **c** 0.*aa* **d** *a.a*0*a*
14 a i 0.1, 1.0 (2 ways)
ii 0.12, 0.21, 1.02, 1.20, 2.01, 2.10, 10.2, 12.0, 20.1, 21.0 (10 ways)
iii 0.123, 0.132, 0.213, 0.231, 0.312, 0.321, 1.023, 1.032, 1.203, 1.230, 1.302, 1.320, 2.013, 2.031, 2.103, 2.130, 2.301, 2.310, 3.012, 3.021, 3.102, 3.120, 3.201, 3.210, 10.23, 10.32, 12.03, 12.30, 13.02, 13.20, 20.13, 20.31, 21.03, 21.30, 23.01, 23.10, 30.12, 30.21, 31.02, 31.20, 32.01, 32.10, 102.3, 103.2, 120.3, 123.0, 130.2, 132.0, 201.3, 203.1, 210.3, 213.0, 230.1, 231.0, 301.2, 302.1, 310.2, 312.0, 320.1, 321.0 (60 ways)
b 408 ways

Exercise 4H

1 a 5.8 **b** 2 **c** 80 **d** 6.78
2 a 5 **b** 9 **c** 1 **d** 4
e 0 **f** 9 **g** 6 **h** 5
3 a i 1 **ii** 7 **iii** 4 **iv** 8
b i 25.8 **ii** 25.82 **iii** 25.817 **iv** 26
4 a 14.8 **b** 7.4 **c** 15.6 **d** 0.9
e 6.9 **f** 9.9 **g** 55.6 **h** 8.0
5 a 3.78 **b** 11.86 **c** 5.92 **d** 0.93
e 123.46 **f** 300.05 **g** 3.13 **h** 9.85
i 56.29 **j** 7.12 **k** 29.99 **l** 0.90
6 a 15.9 **b** 7.89 **c** 236 **d** 1
e 231.9 **f** 9.4 **g** 9.40 **h** 34.713
7 a 24.0 **b** 14.90 **c** 7 **d** 30.000
8 a 28 **b** 9 **c** 12 **d** 124
e 22 **f** 118 **g** 3 **h** 11
9 a \$13 **b** \$31 **c** \$7 **d** \$1567
e \$120 **f** \$10 **g** \$1 **h** \$36
10 a \$51 **b** \$44 **c** very accurate
11 a 0 s **b** 0.4 s **c** 0.34 s **d** 52.825 s
12 2.25, 3 decimal places
13 Samara: Round to 2 decimal places = 0.45, then round this to 1 decimal = 0.5. Cassandra: Rounding to 1 decimal place, critical digit is the second 4, which is less than 5, therefore rounded to 1 decimal place = 0.4.
Samara has a flaw of rounding an already rounded number. Cassandra is correct.
14 Depends on your calculator
15 Depends on your software package

Exercise 4I

1 a 5 **b** 100 **c** 75, 7 **d** 5, 4
2 a 2 **b** 15, 20 **c** 10, 4 **d** 16
3 a false **b** true **c** true
d false **e** true **f** true
4 a $\frac{1}{2}$ **b** $6\frac{2}{5}$ **c** $10\frac{3}{20}$ **d** $18\frac{3}{25}$
e $3\frac{1}{4}$ **f** $\frac{1}{20}$ **g** $9\frac{3}{40}$ **h** $5\frac{24}{125}$
5 a 0.7 **b** 0.9 **c** 0.31 **d** 0.79
e 1.21 **f** 3.29 **g** 0.123 **h** 0.03
6 a $\frac{8}{10} = 0.8$ **b** $\frac{5}{10} = 0.5$ **c** $\frac{35}{100} = 0.35$
d $\frac{46}{100} = 0.46$ **e** $5\frac{95}{100} = 5.95$ **f** $3\frac{25}{100} = 3.25$
g $\frac{25}{10} = 2.5$ **h** $\frac{375}{1000} = 0.375$
7 a 0.5 **b** 0.5 **c** 0.75 **d** 0.4
e $0.\dot{3}$ **f** 0.375 **g** $0.41\dot{6}$ **h** $0.\overline{428571}$
i $0.1\dot{6}$ **j** $0.\dot{6}$ **k** $0.\dot{1}4285\dot{7}$ **l** $0.\dot{5}$
8 a 0, 0.5, 1
b 0, $0.\dot{3}$, $0.\dot{6}$, $0.\dot{9}$ (0.9999999... = 1)
c 0, 0.25, 0.5, 0.75, 1
d 0, 0.2, 0.4, 0.6, 0.8, 1.0
9 a $\frac{1}{4}$, 0.4, $\frac{1}{2}$, $\frac{5}{8}$, 0.75, 0.99 **b** $\frac{1}{9}$, 0.13, $\frac{3}{7}$, 0.58, $\frac{4}{5}$, 0.84
10 a Tan: $= \frac{11}{37} = 0.\overline{297}$; Lillian: $= \frac{6}{21} = 0.\overline{285714}$; hence, Tan is the better chess player.
b two or more
11 0.11 mm, $\frac{11}{100}$ mm
12 0.5, $0.\dot{3}$, 0.25, 0.2, $0.1\dot{6}$, $0.\overline{142857}$, 0.125, $0.\dot{1}$, 0.1
13 $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{2}{5}$, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{9}{10}$
14 a $0.\dot{3}$ **b** $0.\dot{6}$ **c** 0.666666667
d The calculator is correct. It can show only a certain number of digits. As the answer is an infinitely recurring decimal, the calculator, correctly, needs to round the answer to the maximum number of digits it can display. Hence, it rounds the final digit it can display to a 7. Note that the calculator answer is a rounded answer and not an exact answer in this case.
15 Answers may vary; $2\frac{4}{9}$, $2\frac{45}{100}$, $2\frac{3}{7}$
16 2, $\frac{4}{7} = 0.\overline{571428}$, repeating pattern
17 Answers will vary.

Exercise 4J

1 C
2 C
3 C

4 a $\frac{3}{4} = 0.75 = 75\%$ b $\frac{9}{10} = 0.9 = 90\%$
c $\frac{2}{5} = 0.4 = 40\%$ d $\frac{4}{5} = 0.8 = 80\%$
5 a 150 b 350 c 75 d 175
e 180 f 220
6 a $\frac{3}{10} = 0.3 = 30\%$ b $\frac{3}{5} = 0.6 = 60\%$
c $\frac{5}{4} = 1.25 = 125\%$ d $\frac{4}{3} = 1.\dot{3} = 133\frac{1}{3}\%$
7 a i $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$
ii $\frac{1}{3}, \frac{2}{5}, \frac{2}{6}, \frac{3}{8}, \frac{3}{9}, \frac{4}{9}, \frac{3}{10}, \frac{4}{10}, \frac{4}{12}, \frac{5}{12}$
b $\frac{4}{9}$
8 a $\frac{3}{4}$ b $\frac{1}{2}$ c $\frac{1}{4}$
d 1 e $\frac{1}{3}$ f $\frac{4}{3} = 1\frac{1}{3}$
9 a $\frac{1}{4}$ b $\frac{1}{3}$ c $\frac{1}{8}$
d 0 e $\frac{1}{2}$ f $\frac{7}{12}$
g $\frac{3}{4}$ h $\frac{4}{5}$ i $\frac{5}{8}$
10 a 0.9 b 0.3 c 0.75
d 0.35 e 0.5 f 0.9
g 0.6 h 1.2 i 2.1
11 a 37.5% b 62.5%
12 a three-quarters b two-thirds
c one-quarter d one-eighth
e three-eighths
13 a 90% b 60%
14 a $\frac{1}{4} + \frac{1}{4}, \frac{2}{6} + \frac{1}{6}, \frac{2}{8} + \frac{2}{8}, \frac{1}{8} + \frac{3}{8}, \frac{1}{10} + \frac{4}{10}, \frac{2}{10} + \frac{3}{10}, \frac{1}{12} + \frac{5}{12}, \frac{2}{12} + \frac{4}{12}, \frac{3}{12} + \frac{3}{12}$
b $\frac{1}{3} + \frac{1}{6}, \frac{1}{4} + \frac{2}{8}, \frac{1}{5} + \frac{3}{10}, \frac{1}{3} + \frac{2}{12}, \frac{3}{9} + \frac{1}{6}, \frac{3}{12} + \frac{2}{8}, \frac{2}{6} + \frac{2}{12}, \frac{1}{6} + \frac{4}{12}, \frac{3}{9} + \frac{2}{12}$ etc.
c $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}, \frac{1}{8} + \frac{1}{8} + \frac{2}{8}, \frac{1}{10} + \frac{1}{10} + \frac{3}{10}, \frac{1}{10} + \frac{2}{10} + \frac{2}{10}, \frac{2}{12} + \frac{2}{12} + \frac{2}{12}, \frac{1}{12} + \frac{2}{12} + \frac{3}{12}, \frac{1}{12} + \frac{1}{12} + \frac{4}{12}$
d $\frac{1}{4} + \frac{1}{6} + \frac{1}{12}$
e $\frac{2}{5}$
15 a This is true because 0.5 is one-half and so is 50%.
b This is false because one-third is $\frac{33}{99}$ not $\frac{33}{100}$.
c This is false because $\frac{1}{5} = \frac{2}{10} = \frac{20}{100} = 20\%$, not 15%.
d This is true because $\frac{9}{10} = \frac{90}{100} = 90\%$
e This is true because $\frac{1}{2} = 50\%$ and $\frac{1}{4} = 25\%$ so $\frac{1}{8} = 12.5\%$, which is close to 12%.
f This is true because $\frac{1}{3} = 33\frac{1}{3}\%$ so $\frac{2}{3} = 66\frac{2}{3}\%$, which is greater than 66%.
16 B $\frac{30}{90} = \frac{1}{3} = 33\frac{1}{3}\%$ discount; A 30%, C 31%, D 25%
17 a 0.5 m b 0.25 m c 0.25 m
d 0.125 m e no

Exercise 4K

1 a 95% b 60% c 75% d 26%
2 C
3 B
4 A
5 a 50 b 50%
c i 5 ii 100 iii 20 iv 1
d i 50% ii 50% iii 50% iv 50%
6 a 100, ÷, 100, .
b 35, 100, 35, 0
c out of, 126, 100, 26
7 a 0.32 b 0.27 c 0.68 d 0.54
e 0.06 f 0.09 g 1 h 0.01
i 2.18 j 1.42 k 0.75 l 1.99
8 a 0.225 b 0.175 c 0.3333 d 0.0825
e 1.1235 f 1.888 g 1.50 h 5.20
i 0.0079 j 0.00025 k 0.0104 l 0.0095
9 a 80% b 30% c 45% d 71%
e 41.6% f 37.5% g 250% h 231.4%
i 2.5% j 0.14% k 1270% l 100.4%
10 a 86%, 0.78, 75%, 0.6, 22%, 0.125, 2%, 0.5%
b 50, 7.2, 2.45, 1.8, 124%, 55%, 1.99%, 0.02%
11 a yes b 85%
12 a 25% b 78%
13 24 gigalitres
14 6.7%
15 a *ab.cd%* b *aac%* c *abdc%*
d *d.d%* e *cdb.a%* f *cc.cddd%*
16 a 0.0*ab* b *b.cd* c 0.*ac*
d 0.00*da* e *ab.bb* f 0.*ddd*
17 a no
b Answers may vary. Examples include: percentage score on a Maths test, percentage of damaged fruit in a crate, percentage of spectators wearing a hat, percentage of the day spent sleeping.
c Answers may vary. Examples include: percentage profit, percentage increase in prices, percentage increase in the price of a house, percentage score on a Maths test with a bonus question.
18 a F ÷ A × 100

b F: points scored for the team; A: points scored against the team
c 100%
d 158 points
e yes; Hawthorn 90.60%, Port Adelaide 91.32%

Exercise 4L

1 a 70, 70 **b** 48, 48 **c** 60, 60
d 20, 20 **e** 40, 40 **f** 63, 63

2 a $\frac{1}{4} = 25\%, \frac{2}{4} = 50\%, \frac{3}{4} = 75\%, \frac{4}{4} = 100\%$
b $\frac{1}{5} = 20\%, \frac{2}{5} = 40\%, \frac{3}{5} = 60\%, \frac{4}{5} = 80\%, \frac{5}{5} = 100\%$
c $\frac{1}{3} = 33\frac{1}{3}\%, \frac{2}{3} = 66\frac{2}{3}\%, \frac{3}{3} = 100\%$

3 a 86% **b** 20%

4 a $\frac{11}{100}$ **b** $\frac{71}{100}$ **c** $\frac{43}{100}$ **d** $\frac{49}{100}$
e $\frac{1}{4}$ **f** $\frac{3}{10}$ **g** $\frac{3}{20}$ **h** $\frac{22}{25}$

5 a $1\frac{1}{5}$ **b** $1\frac{4}{5}$ **c** $2\frac{37}{100}$ **d** $4\frac{1}{100}$
e $1\frac{3}{4}$ **f** $1\frac{1}{10}$ **g** $3\frac{4}{25}$ **h** $8\frac{2}{5}$

6 a 8% **b** 15% **c** 97% **d** 50%
e 35% **f** 32% **g** 86% **h** 90%
i 112% **j** 135% **k** 400% **l** 160%

7 a $12\frac{1}{2}\%$ **b** $33\frac{1}{3}\%$ **c** $26\frac{2}{3}\%$ **d** $83\frac{1}{3}\%$
e 115% **f** 420% **g** 290% **h** $32\frac{1}{2}\%$

8 a $\frac{3}{4}$ **b** 75% **c** $\frac{1}{4}$ **d** 25%

9 $12\frac{1}{2}\%$

10 70%

11 4%

12 70%, 80%

13 a $\frac{18}{25}$ **b** 72%

14 $\frac{55}{1000} = \frac{11}{200}$

15 a $\frac{25}{1000}$ **b** $\frac{825}{10000}$ **c** $\frac{125}{1000}$ **d** $\frac{1}{3}$

Exercise 4M

1 a 10 **b** 100 **c** 2
d 1 **e** 5 **f** 4

2 a ii **b** ii **c** i **d** i

3 a 70 **b** 36 **c** 10 **d** 27
e 10 **f** 7 **g** 150 **h** 200
i 4 **j** 48 **k** 44 **l** 190
m 22 **n** 84 **o** 36 **p** 63

4 a 96 **b** 600 **c** 66 **d** 100
e 15 **f** 72 **g** 73 **h** 600

5 10% of \$200 = \$20
20% of \$120 = \$24
10% of \$80 = \$8
50% of \$60 = \$30
20% of \$200 = \$40
5% of \$500 = \$25
30% of \$310 = \$93
10% of \$160 = \$16
1% of \$6000 = \$60
50% of \$88 = \$44

6 a \$42 **b** 24 mm **c** 9 kg
d 90 tonnes **e** 8 min **f** 400 cm
g 1.5 g **h** 3 hectares **i** 144 seconds

7 35

8 no (red \$45, striped \$44)

9 240

10 12

11 a 120 **b** 420 **c** 660

12 a \$80 **b** \$2080 **c** \$4160

13 a computer games 30 min, drums 24 min, outside 48 min, reading 12 min
b 5% time remaining
c yes, with 1 min to spare

14 80 min

15 a 20 **b** 90 **c** 20 **d** 64

16 Answers will vary.

17 They are the same.

18 $37\frac{1}{2}\%$

19 a i 1200
ii 24%, 22%, 20%, 18%, 16%
iii 2%
b i 30%
ii week 1: 30%, 2400 pieces; week 2: 25%, 2000 pieces; week 3: 20%, 1600 pieces; week 4: 15%, 1200 pieces; week 5: 10%, 800 pieces
iii

Week	Cumulative %	Pieces completed
Week 1	30%	2400
Week 2	55%	4400
Week 3	75%	6000
Week 4	90%	7200
Week 5	100%	8000

Exercise 4N

1 a 4 **b** 4 **c** $\frac{1}{2}$
d 50% **e** $\frac{1}{2}$ **f** 50%

2 a 10 **b** $\frac{1}{5}$ **c** $\frac{4}{5}$
d 20% **e** 80%

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3 **a** $\frac{3}{10}$, 30% **b** $\frac{3}{5}$, 60% **c** $\frac{1}{5}$, 20%
d $\frac{3}{4}$, 75% **e** $\frac{1}{20}$, 5% **f** $\frac{1}{2}$, 50%
g $\frac{1}{4}$, 25% **h** $\frac{3}{20}$, 15%

4 **a** $\frac{3}{5}$, 60% **b** $\frac{1}{2}$, 50% **c** $\frac{1}{4}$, 25%
d $\frac{2}{5}$, 40% **e** $\frac{3}{4}$, 75% **f** $\frac{4}{5}$, 80%

5 **a** $\frac{1}{10}$ **b** 10%

6 **a** $\frac{1}{5}$ **b** 20% **c** $\frac{4}{5}$ **d** 80%

7 **a** $\frac{3}{4}$ **b** 75% **c** $\frac{1}{4}$ **d** 25%

8 95%

9 80%

10 **a** $\frac{1}{25}$, 4% **b** $\frac{2}{5}$, 40% **c** $\frac{1}{20}$, 5%
d $\frac{1}{100}$, 1% **e** $\frac{1}{50}$, 2% **f** $\frac{1}{40}$, 2.5%

11 **a** $\frac{1}{5}$ **b** 40%

12 **a** 5% **b** $\frac{1}{5}$

13 Ross 75%, Maleisha 72%

14 hatch 15%, 4WD 16%; hence, 4WD has larger price reduction.

15 yellow 20%, blue 19%; hence, blue has least percentage of sugar.

16 **a** **i** $\frac{b}{a+b}$ **ii** $\frac{a}{a+b}$
b **i** $\frac{100b}{a+b}$ **ii** $\frac{100a}{a+b}$

17 **a** $\frac{a}{a+b}$ **b** $\frac{100b}{a+b}$

18 **a** 3 **b** $\frac{1}{30}$ **c** $\frac{1}{6}$
d $\frac{17}{30}$ **e** 20% **f** 96.67%

Puzzles and challenges

1 Delhi, Agra and Jaipur

2 Answers will vary.

3 **a** $\frac{1}{9}$ **b** $\frac{9}{1}$ **c** 14
d $\frac{8}{9}$ **e** $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$ **f** $\frac{3}{4}, \frac{6}{8}$
g 16 **h** eight ways **i** three ways
j 9

4 20% of 50 is not the same as 20% of 60.

5 nine times bigger; 800%

6 3

Multiple-choice questions

1 B **2** C **3** A **4** C **5** D
6 C **7** E **8** D **9** A **10** C

Short-answer questions

1 $\frac{1}{14}, \frac{1}{10}, \frac{1}{8}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$

2 $\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25}$ etc.
Multiplying by the same number in the numerator and denominator is equivalent to multiplying by 1, and so does not change the value of the fraction.

3 **a** $\frac{3}{5}$ **b** $\frac{2}{7}$ **c** $\frac{5}{7}$

4 **a** $1\frac{1}{2}$ **b** $1\frac{3}{4}$ **c** $1\frac{2}{3}$ **d** $3\frac{1}{2}$

5 **a** $\frac{2}{7} \boxed{<} \frac{4}{7}$ **b** $\frac{3}{8} \boxed{>} \frac{1}{8}$ **c** $1\frac{2}{3} \boxed{>} 1\frac{3}{5}$ **d** $3\frac{1}{9} \boxed{<} \frac{29}{9}$

6 **a** $\frac{5}{7}$ **b** $\frac{5}{8}$

7 **a** 10 **b** 21 **c** 24

8 **a** 10 **b** 21 **c** 24

9 **a** $2\frac{1}{5}, \frac{9}{5}, 1\frac{3}{5}$ **b** $\frac{9}{4}, \frac{11}{6}, \frac{14}{8}, \frac{5}{3}$
c $5\frac{2}{3}, 5\frac{7}{18}, 5\frac{1}{3} = \frac{48}{9}, 5\frac{2}{9}$

10 **a** $\frac{1}{2}$ **b** $\frac{5}{6}$ **c** $1\frac{1}{2}$
d $\frac{1}{2}$ **e** $\frac{1}{2}$ **f** $\frac{2}{5}$

11

Percentage form	Fraction
36%	$\frac{9}{25}$
220%	$2\frac{1}{5}$
5%	$\frac{5}{100}$
140%	$1\frac{2}{5}$
44%	$\frac{11}{25}$
18%	$\frac{9}{50}$

12 **a** ii **b** i

13 **a** $\frac{3}{5}$, 60% **b** $\frac{1}{5}$, 20% **c** $\frac{1}{16}$, 6.25% **d** $\frac{3}{10}$, 30%

14 **a** 3 hundredths $= \frac{3}{100}$
b 3 thousandths $= \frac{3}{1000}$
c 3 units $= 3$

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15 a $8.1 = 810\%$ b $0.81 = 81\%$
c $8.01 = 801\%$ d $0.801 = 80.1\%$

16 45.265
45.266
45.267
45.268
45.270
45.271
45.272
45.273
45.274

17 a 1, 2, 3, 4, 6, 8, 12, 24
b 1, 2, 4, 8, 16, 32
c 1, 2, 3, 4, 6, 9, 12, 18, 36
d 1, 2, 4, 8, 16, 32, 64
e 1, 2, 4, 5, 10, 20, 25, 50, 100
f 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144
g 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
h 1, 3, 5, 15, 25, 75

18 a 8 b 4 c 72 d 25

19 a 12 b 10 c 6 d 6
e 60 f 4 g 15 h 6
i 8 j 20 k 10 l 40

Extended-response questions

1

Simplified fraction	Decimal	Percentage
$\frac{7}{20}$	0.35	35%
$\frac{3}{5}$	0.6	60%
$1\frac{1}{10}$	1.1	110%
$2\frac{3}{4}$	2.75	275%
$7\frac{1}{2}$	7.5	750%
$\frac{5}{3}$	$1.\dot{6}$	$166.\dot{6}$
$\frac{5}{6}$	$0.8\dot{3}$	$83.\dot{3}$
$\frac{3}{8}$	0.375	37.5

2 Science

3 a i $\frac{1}{4}$ ii 0.25
b i 15% ii $\frac{15}{100} = \frac{3}{20}$
c i \$20 ii \$15
d i 30% ii $\frac{3}{10}$ iii 0.3 iv \$30
e i \$50 ii $\frac{1}{2}$, 0.5, 0%

Chapter 5

Pre-test

1 a $\frac{1}{2}$ b $\frac{2}{3}$ c $\frac{3}{4}$ d $\frac{1}{4}$

2 a 5 b 3 c $\frac{2}{5}$

3 a 0.5 b 0.4 c 0.2 d 0.2

4 C, A, B, D

5 a C b A c B d A e B

6 $\frac{1}{2} = 0.5 = 50\%$, $\frac{1}{3} = 0.\dot{3} = 33\frac{1}{3}\%$, $\frac{1}{4} = 0.25 = 25\%$,
$\frac{1}{5} = 0.2 = 20\%$, $\frac{1}{10} = 0.1 = 10\%$, $\frac{1}{100} = 0.01 = 1\%$

Exercise 5A

1 a D b A c B d C

2 a certain b even chance
c unlikely d impossible

3 a true b false c true
d false e true f false

4 a D b C c A d B

5 a i true ii false iii false iv true
v true
b i spinner landing on yellow
ii spinner not landing on red
iii spinner landing on green, blue or red
iv spinner landing on blue or on red

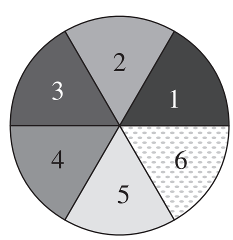
6 a spinner 3 b spinner 2 c spinner 1

7 Spinner 3 contains two red sections. Together, they are twice the size of the blue section. In spinner 2, the red and blue sections are the same size.

8 Answers will vary.

9 a Blue, red and green equally likely.
b Red and green both have an even chance.
c Green and blue equally likely, red and blue are not equally likely.
d Blue is certain.
e Blue, red and green all possible, but no two colours are equally likely.
f Red and blue both have an even chance.

10 a



b All sectors have the same size; i.e. 60°.

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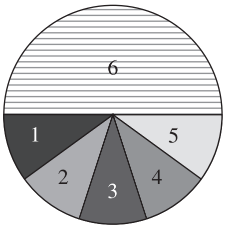
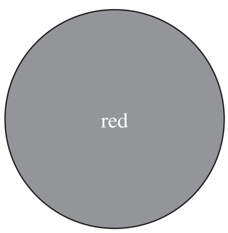
c 

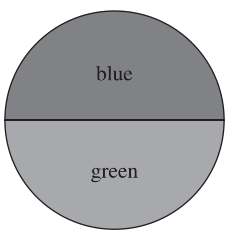
d by replacing the 5 with a 6 (so that there are two faces with 6)

e with 52 equal segments

11 a spinner 1: $\frac{1}{4}$, spinner 2: $\frac{1}{3}$, spinner 3: $\frac{1}{9}$

b $\frac{1}{2}$

c i 

ii 

Other answers possible

d i 50% ii 0% iii 0%, 50% iv 50%

e If the two fractions are equal, the two events are equally likely.

f The proportion of the spinner's area cannot exceed 100% (or 1) and must be greater than 0%.

Exercise 5B

1 a C b A c D d B

2 a sample space b 0 c certain
d more e even chance

3 a {1, 2, 3, 4, 5, 6} b {1, 3, 5} c $\frac{1}{2}$

4 a 3: red, green, blue
b $\frac{1}{3}$ c $\frac{2}{3}$ d $\frac{2}{3}$ e 0

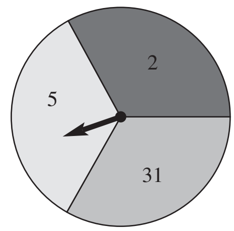
5 a {1, 2, 3, 4, 5, 6, 7}
b $\frac{1}{7}$ c 0 d $\frac{2}{7}$ e $\frac{3}{7}$ f $\frac{4}{7}$
g Number chosen is less than 10; other solutions possible.

6 a {M, A, T, H, S} b 0.2 c 0.8 d 60%

7 a $\frac{1}{11}$ b $\frac{2}{11}$ c $\frac{5}{11}$
d $\frac{9}{11}$ e $\frac{4}{11}$
f Choosing a letter in the word ROPE; other solutions possible.

8 a 30% b 50% c 80%

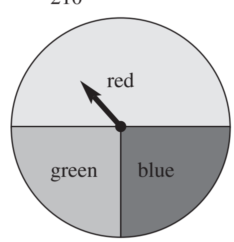
9 a {2, 3, 4, 5, 6, 7, 8, 9} b 0.5
c i 0.375 ii 0.375 iii 0
d Possible spinner shown:

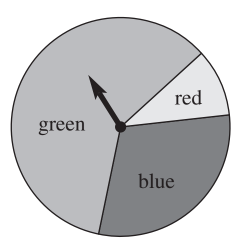


10 a i 12 red, 6 blue, 4 yellow, 2 green ii $\frac{1}{12}$
b i 18 red, 9 blue, 6 yellow, 3 green ii $\frac{1}{12}$
c 12
d No, because it is always $1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{6}$.

11 a $a = 1, b = \frac{1}{2}, c = 1, d = \frac{1}{3}, e = 1, f = \frac{2}{5}, g = \frac{3}{5}, h = 1$
b $\frac{10}{13}$ c $\frac{3}{7}$

12 a yes, $\frac{19}{210}$ b 210 c 840

13 a 

b Cannot be done because adds to more than 1.
c Cannot be done because adds to less than 1.
d 

Exercise 5C

1 a $\frac{1}{10}$ b $\frac{2}{5}$ c $\frac{3}{10}$ d false

2 a $\frac{53}{100}$ b $\frac{47}{100}$ c $\frac{1}{2}$
d tossing a tail

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3 **a** 50 **b** $\frac{2}{5}$ **c** $\frac{1}{10}$ **d** $\frac{1}{2}$

e No, just that nobody did it within the group surveyed.

4 **a** 500 **b** 1750 **c** **i** 7 tails **ii** more

5 **a** 100 **b** 300 **c** yes (but this is very unlikely)

d from 2 rolls

6 **a–e** Answers will vary. **f** true

7 **a** $\frac{1}{4}$ **b** $\frac{3}{100}$ **c** $\frac{31}{100}$ **d** 60

e white, blue, purple or green

8 **a** 5 **b** 40 **c** 70 **d** $\frac{26}{35}$

e $\frac{4}{7}$ **f** 126 **g** 1, 2, 3 or 4 children

9 **a** 2 red, 3 green and 5 blue

b **i** yes **ii** yes **iii** yes **iv** yes **v** no

10 **a** C **b** D **c** B **d** A

11 **a** Answers will vary. **b** Answers will vary.

c No technique for finding theoretical probability has been taught yet.

d less accurate

e more accurate

12 **a** False; there is no guarantee it will occur exactly half of the time.

b False; e.g. in two rolls, a die might land 3 one time. The theoretical probability is not $\frac{1}{2}$, though.

c False; perhaps the event did not happen yet but it could.

d True; if it is theoretically impossible it cannot happen in an experiment.

e False; experiment might have been lucky.

f True; if it is certain, then it must happen in an experiment.

13 **a** red: 25%, green: 42.2%, blue: 32.8%

b Fifth set is furthest from the final estimate.

c have 4 green sectors, 3 blue and 3 red sectors

d red: 90°, green: 150°, blue: 120°

Exercise 5D

1 **a** event; outcome

b equally likely outcomes; mutually exclusive events

c probability

d at least **e** certain

f certain **g** less than

2 **a** 80% **b** 0% **c** 80% **d** 20%

e 80% **f** 40%

3 **a** $\frac{2}{3}$ **b** 0 **c** $\frac{1}{3}$

d $\frac{1}{6}$ **e** $\frac{2}{3}$ **f** $\frac{5}{6}$

g $\frac{5}{6}$ **h** $\frac{1}{3}$ **i** $\frac{1}{6}$

j $\frac{5}{6}$ **k** $\frac{2}{3}$

4 **a** 0 **b** $\frac{12}{51} = \frac{4}{17}$ **c** $\frac{39}{51} = \frac{13}{17}$

5 **a** $\frac{1}{7}$ **b** $\frac{6}{7}$

6 Answers will vary.

7 **a** There are 10: AB, AC, AD, AE, BC, BD, BE, CD, CE, CE

b 10 **c** 45

d **i** 3838380 **ii** 7059052

Exercise 5E

1 **a**

	Like bananas	Dislike bananas	Total
Like apples	30	15	45
Dislike apples	10	20	30
Total	40	35	75

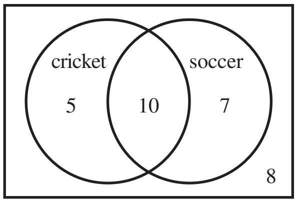
b 30 **c** 20 **d** 75

e mutually exclusive

2 **a** **i** 2 **ii** 4 **iii** 1 **iv** 3

b No, there are two people who own a dog AND a cat.

3 **a** 8 **b**



c 15 **d** 22 **e** 12

4 **a** 15

b

	Employed	Unemployed	Total
University degree	10	3	13
No university degree	5	2	7
Total	15	5	20

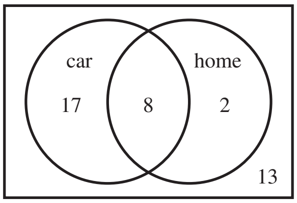
c The 10, 13, 15 and 20 would all increase by 1.

5 **a** 26 **b** 12 **c** 11

d **i** $\frac{2}{13}$ **ii** $\frac{7}{26}$ **iii** $\frac{7}{13}$ **iv** $\frac{15}{26}$

e Being a boy or being a girl; paying own bill or not paying own bill.

6 **a**



b **i** $\frac{1}{5}$ **ii** $\frac{17}{40}$ **iii** $\frac{1}{4}$

c $\frac{8}{25}$ **d** $\frac{4}{5}$

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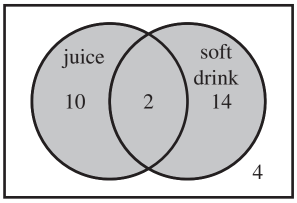
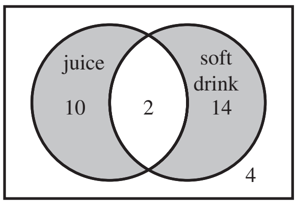
7 **a** 12 **b** $\frac{2}{15}$ **c** $\frac{13}{15}$ **d** $\frac{24}{30}$ or $\frac{4}{5}$

e The 'inclusive or' includes the people who like one or the other or both.



The 'exclusive or' does not include the 2 people who like BOTH juice AND soft drink.



8 **a**

	Sports	Not sports	Total
Automatic	2	13	15
Not automatic	8	17	25
Total	10	30	40

b $\frac{1}{5}$ **c** $\frac{13}{40}$ **d** $\frac{2}{15}$

9 **a** $\frac{1}{5}$ **b** 45 **c** $\frac{2}{9}$ **d** $\frac{1}{3}$

10 **a** 4; if only the totals are missing, they can be calculated from the first four spaces.

b No, not if the 5 spots are in the last row/column.

c Filling it requires a negative number, which is impossible.

11

	B	Not B	Total
A	x	w	$x + w$
Not A	y	z	$y + z$
Total	$x + y$	$w + z$	$w + x + y + z$

12 **a** 120 **b** 60

c **i** $\frac{59}{120}$ **ii** $\frac{19}{40}$ **iii** $\frac{1}{2}$ **iv** $\frac{29}{120}$

v $\frac{21}{60} = \frac{7}{20}$ **vi** $\frac{17}{60}$ **d** **i** $\frac{10}{19}$ **ii** $\frac{5}{19}$

e **i** $\frac{29}{59}$ **ii** $\frac{34}{59}$

f They are male, under 40 and not using a trolley.

g More likely to be female $\left(\frac{30}{59} > \frac{29}{59}\right)$.

h More likely to use trolley $\left(\frac{30}{57} > \frac{27}{57}\right)$.

Exercise 5F

1 **a** 10 **b** H2, H4, T2, T4 **c** $\frac{2}{5}$

d T1, T3, T5 **e** $\frac{3}{10}$

2 **a** $\frac{1}{4}$ **b** HH, TT **c** $\frac{1}{2}$

3 **a**

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

b 12 **c** $\frac{1}{12}$ **d** $\frac{1}{4}$

4 **a**

	R	I	D	E
L	LR	LI	LD	LE
I	IR	II	ID	IE
N	NR	NI	ND	NE
E	ER	EI	ED	EE

b 16 **c** $\frac{1}{16}$ **d** $\frac{1}{16}$ **e** $\frac{1}{4}$

f $\frac{1}{4}$ **g** $\frac{1}{8}$

5 **a**

	R	P	B
R	RR	RP	RB
P	PR	PP	PB
G	GR	GP	GB
B	BR	BP	BB

b $\frac{1}{12}$ **c** $\frac{1}{12}$ **d** $\frac{1}{12}$ **e** $\frac{1}{6}$ **f** $\frac{1}{4}$

6 **a** $\frac{1}{12}$ **b** $\frac{1}{6}$ **c** $\frac{1}{12}$ **d** $\frac{1}{2}$

7 **a**

Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b $\frac{1}{9}$ **c** $\frac{1}{2}$ **d** 7 **e** 2 and 12

8 **a**

	Y	W	B	B	B
W	WY	WW	WB	WB	WB
O	OY	OW	OB	OB	OB
O	OY	OW	OB	OB	OB

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b $\frac{2}{15}$ c $\frac{1}{15}$ d $\frac{2}{15}$

e $\frac{1}{5}$ f $\frac{1}{15}$

9 a $\frac{1}{8}$ b $\frac{1}{3}$

10 63

11 a $\frac{1}{4}$ b $\frac{1}{52}$ c $\frac{1}{26}$

d i $\frac{1}{16}$ ii $\frac{1}{4}$ iii $\frac{1}{2704}$

e The second card would then depend on the first card (e.g. if a red card was selected, it would be a little less likely that the next card would be red).

12 a i $\frac{1}{4}$ ii $\frac{1}{2}$

b i $\frac{1}{16}$ ii $\frac{1}{4}$ iii $\frac{1}{16}$ iv $\frac{1}{8}$

c OYYBBB (or some rearrangement of those letters)

d i $\frac{1}{9}$ ii $\frac{7}{18}$ iii $\frac{1}{12}$ iv $\frac{1}{6}$

v $\frac{11}{36}$ vi $\frac{3}{4}$

e i $\frac{1}{24}$ ii $\frac{1}{8}$ iii $\frac{1}{12}$ iv $\frac{1}{4}$

v $\frac{1}{4}$ vi $\frac{3}{4}$

Puzzles and challenges

1 0.25

2 a MOON b OFF c DING

d PROBABILITY e STUMBLE f TRY

3 32

4 $\frac{5}{9}$

5 0.000977

6 1 red, 3 blue, 4 yellow

7 PROBABLE

8 Reykjavik

Multiple-choice questions

1 E 2 C 3 C 4 B 5 B

6 C 7 C 8 C 9 C 10 C

Short-answer questions

1 a 1 b $\frac{1}{8}$ c $\frac{19}{20}$

d $\frac{3}{4}$ e 0

2 a {1, 2, 3, 4, 5, 6} b {heads, tails}

c {D, E, S, I, G, N} d {blue, yellow, green}

3 a 9

b i $\frac{5}{9}$ ii $\frac{1}{3}$ iii 0 iv $\frac{1}{3}$ v $\frac{4}{9}$

4 a $\frac{1}{26}$ b $\frac{2}{13}$ c $\frac{1}{52}$ d $\frac{3}{13}$

5 a 42% b 50%

6 a $\frac{1}{2}$ b $\frac{1}{4}$ c 25 d 250

Extended-response questions

1 a 50 b 25 c 24% d 1%

e $\frac{12}{25}$ f $\frac{24}{25}$

2 a

	Uses public transport	Does not use public transport	Total
Owns a car	20	80	100
Does not own a car	65	35	100
Total	85	115	200

b 200 c $\frac{1}{2}$ d $\frac{1}{10}$

e $\frac{4}{17}$ f $\frac{1}{5}$

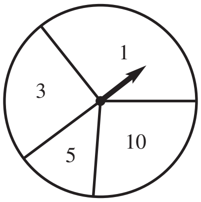
g i More public transport users expected.

ii People less likely to use public transport in regional area.

3 a 1, it has the most occurrences.

b 3 and 10, as they have the closest outcomes.

c $\frac{3}{8}$ d



Chapter 6

Pre-test

1 a 0.1 b 0.3 c 1.7

d 0.01 e 0.001 f 4.7

2 a 0.5 b 0.25 c 0.75

3 a $0.70 b $0.85 c $1.00

d $0.05 e $1.05 f $0.03

4 a 50 b 25 c 75 d 250

5 a $2.70 b $12.60 c $22.50 d $1.89

6 25c

7 $15.50

8 a $85 b $0.10 c $2.70

d $0.70 e $11.20 f $0.24

9 a $87.60 b $38 c $13.20

10 $104.65

11 a 523 b 839 c 1352 d 1237

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Exercise 6A

1 C

2 B

3 a 27.97 b 25.94 c 247.4 d 58.31

4 a 10.7 b 23.1 c 51.1 d 13.2
e 20 f 8.8 g 16.06 h 21.33
i 343.75 j 37.567 k 21.592 l 340.0606

5 a 12.1 b 114.13 c 6.33 d 70.79

6 a 7.4 b 40.1 c 12 d 2.6
e 12.3 f 131.4 g 22.23 h 13.457
i 43.27 j 4947.341 k 80.3 l 242.7

7 a 62.223 b 50.203

8 a 7, 8 b 6, 5, 1, 4 c 0, 1, 0, 2 d 7, 5, 1, 6, 1

9 186.19

10 $54.30

11 49.11 mm

12

+	0.01	0.05	0.38	1.42
0.3	0.31	0.35	0.68	1.72
0.75	0.76	0.80	1.13	2.17
1.20	1.21	1.25	1.58	2.62
1.61	1.62	1.66	1.99	3.03

13 $2036.10

14 a 8.038 b 0.182 c 2.438 d 1.308

15 a Answers will vary; e.g. 3.57 + 4.15 + 3.44
b Answers may vary; e.g. 1.35 + 2.87 + 6.94 = 11.16

16 Always end up with $10.89, unless the starting value has the same first and last digit, in which case we end up with zero. The 8 forms as it comes from adding two 9s. This produces a 1 to be carried over, which results in the answer being a $10 answer, rather than a $9 answer.

Exercise 6B

1 a denominator
b denominator, numerators
c denominators, lowest common denominator
d check, simplified

2 a 5 b 5 c 4, 3, 7 d 8, 15, 23, 3

3 a 15 b 20 c 6 d 6
e 8 f 10 g 77 h 9
i 24 j 18 k 30 l 48

4 a ✓ b ✗ c ✗ d ✓
e ✓ f ✗ g ✓ h ✗
i ✓ j ✗ k ✓ l ✗

5 a $\frac{5}{8}$ b $\frac{5}{7}$ c $\frac{4}{5}$ d $\frac{9}{11}$
e $\frac{7}{8}$ f $\frac{7}{12}$ g $\frac{7}{15}$ h $\frac{5}{9}$
i $1\frac{2}{7}$ j $1\frac{3}{10}$ k $1\frac{4}{5}$ l $1\frac{4}{19}$

6 a $\frac{3}{4}$ b $\frac{14}{15}$ c $\frac{2}{3}$ d $\frac{7}{12}$
e $\frac{13}{20}$ f $\frac{19}{20}$ g $\frac{13}{21}$ h $\frac{23}{40}$
i $1\frac{13}{30}$ j $1\frac{9}{28}$ k $1\frac{13}{33}$ l $1\frac{5}{12}$

7 a $3\frac{4}{5}$ b $7\frac{3}{7}$ c $12\frac{3}{4}$ d $5\frac{5}{9}$
e $10\frac{1}{3}$ f $21\frac{1}{6}$ g $19\frac{3}{11}$ h $12\frac{2}{5}$

8 a $4\frac{5}{12}$ b $7\frac{7}{30}$ c $12\frac{1}{6}$ d $13\frac{9}{28}$
e $15\frac{1}{10}$ f $19\frac{1}{9}$ g $25\frac{21}{44}$ h $15\frac{5}{24}$

9 a $23\frac{31}{35}$ b $79\frac{31}{36}$ c $200\frac{5}{24}$ d $259\frac{47}{100}$

10 a $\frac{14}{15}$ b $\frac{1}{15}$

11 $8\frac{3}{20}$ km

12 a $\frac{3}{4}$ b 400 c $\frac{1}{4}$, 250 pieces

13 a $\frac{47}{60}$ b $\frac{13}{60}$ c 39

14 a 5, 2 b 2, 4, 8 c 5, 1
d 5, 1, 15; other answers possible.

15 Jim $\left(\frac{36}{60}\right)$, Vesna $\left(\frac{37}{60}\right)$, Juliet $\left(\frac{38}{60}\right)$, Mikhail $\left(\frac{39}{60}\right)$

16 a maximum: $\frac{6}{1}+\frac{5}{2}+\frac{4}{3}=9\frac{5}{6}$
minimum: $\frac{1}{4}+\frac{2}{5}+\frac{3}{6}=1\frac{3}{20}$
b maximum: $\frac{8}{1}+\frac{7}{2}+\frac{6}{3}+\frac{5}{4}=14\frac{3}{4}$
minimum: $\frac{1}{5}+\frac{2}{6}+\frac{3}{7}+\frac{4}{8}=1\frac{97}{210}$
c Maximum: Largest numbers as numerators, smallest numbers as denominators; combine largest numerator available with smallest denominator available to produce the fractions.
Minimum: Smallest numbers as numerators, largest numbers as denominators; combine smallest numerator available with smallest denominator available to produce the fractions.

Exercise 6C

1 a denominators b multiply
c simplify d multiply

2 a 12 b 10 c 15 d 18
e 24 f 60 g 56 h 63

3 a 1 b 3 c 4, 3, 1 d 12, 10, 2

4 a ✓ b ✗ c ✗ d ✓
e ✗ f ✓ g ✓ h ✗
i ✓ j ✓ k ✗ l ✗

5 a $\frac{2}{7}$ **b** $\frac{3}{11}$ **c** $\frac{7}{18}$ **d** $\frac{1}{3}$
e 0 **f** $\frac{4}{9}$ **g** $\frac{3}{19}$ **h** $\frac{8}{23}$
i $\frac{31}{100}$ **j** $\frac{24}{50} = \frac{12}{25}$ **k** $\frac{16}{25}$ **l** $\frac{4}{10} = \frac{2}{5}$

6 a $\frac{5}{12}$ **b** $\frac{1}{10}$ **c** $\frac{1}{10}$ **d** $\frac{9}{28}$
e $\frac{1}{6}$ **f** $\frac{23}{36}$ **g** $\frac{13}{33}$ **h** $\frac{2}{15}$
i $\frac{1}{8}$ **j** $\frac{3}{20}$ **k** $\frac{1}{36}$ **l** $\frac{1}{9}$

7 a $1\frac{3}{5}$ **b** $8\frac{3}{7}$ **c** $1\frac{1}{7}$ **d** $3\frac{2}{9}$
e $2\frac{5}{12}$ **f** $3\frac{5}{28}$ **g** $4\frac{7}{18}$ **h** $7\frac{1}{20}$

8 a $2\frac{2}{3}$ **b** $4\frac{3}{5}$ **c** $4\frac{2}{3}$ **d** $4\frac{8}{9}$
e $4\frac{2}{3}$ **f** $\frac{37}{45}$ **g** $9\frac{37}{44}$ **h** $2\frac{29}{60}$

9 a $6\frac{2}{5}$ **b** $17\frac{7}{12}$ **c** $1\frac{58}{63}$ **d** $207\frac{51}{55}$

10 $\frac{11}{20}$ **11** $\frac{3}{5}$ **12** $\frac{1}{6}$ **13** $\$7\frac{3}{4}$, \$7.75

14 a ice-cream $\frac{3}{4}$, chocolate $\frac{3}{4}$, sponge $\frac{7}{8}$
b ice-cream $\frac{1}{4}$, chocolate $\frac{1}{4}$, sponge $\frac{1}{8}$
c $2\frac{3}{8}$ **d** $\frac{5}{8}$

15 a 3, 4 **b** 3, 1 **c** 1, 2 **d** 4, 3

16 a $\frac{1}{4}$ **b** 4 years **c** 10 years

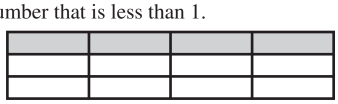
17 a $\frac{8}{15}$: 'Converting to an improper fraction' is quick and efficient for this question.
b $3\frac{36}{55}$: 'Borrowing a whole number' keeps the numbers smaller and easier to deal with for this question.

Exercise 6D

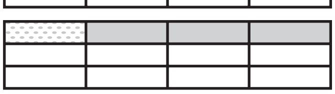
1 a i proper **ii** mixed **iii** improper
iv proper **v** improper **vi** mixed
b i 0, 1 **ii** 1 **iii** whole number, proper fraction

2 You get a smaller answer because you are multiplying by a number that is less than 1.

3 a

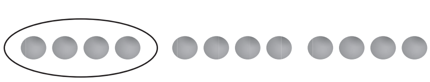


b

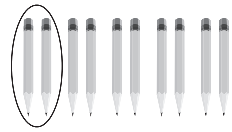


c $\frac{1}{12}$

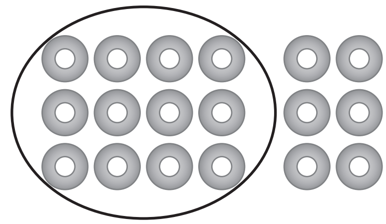
4 a



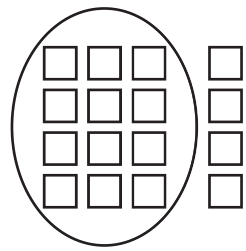
b



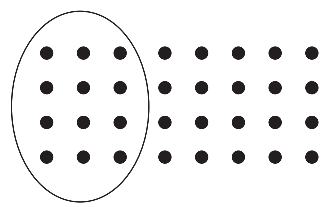
c



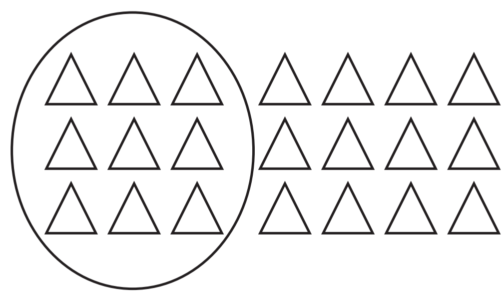
d



e



f



5 D

6 a 6 **b** 9 **c** 16 **d** 15
e 12 **f** 4 **g** 80 **h** 33

7 a $\frac{3}{20}$ **b** $\frac{2}{21}$ **c** $\frac{10}{21}$ **d** $\frac{8}{45}$
e $\frac{2}{5}$ **f** $\frac{1}{7}$ **g** $\frac{1}{4}$ **h** $\frac{5}{11}$
i $\frac{5}{22}$ **j** $\frac{1}{3}$ **k** $\frac{6}{11}$ **l** $\frac{4}{11}$
m $\frac{6}{35}$ **n** $\frac{3}{10}$ **o** $\frac{2}{7}$ **p** $\frac{1}{6}$

8 a $5\frac{5}{6}$ **b** $1\frac{31}{35}$ **c** $3\frac{3}{10}$ **d** $4\frac{7}{8}$
e $5\frac{1}{3}$ **f** 7 **g** 6 **h** $3\frac{1}{3}$

9 a $3\frac{11}{15}$ **b** $1\frac{25}{63}$ **c** $7\frac{4}{5}$ **d** 24

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10 a $3\frac{1}{5}$ b $\frac{3}{16}$ c 4 d 33
e $\frac{2}{7}$ f $3\frac{3}{8}$ g $\frac{4}{15}$ h 6

11 a $3\frac{1}{5}$ b $\frac{3}{16}$ c 4 d 33
e $\frac{2}{7}$ f $3\frac{3}{8}$ g $\frac{4}{15}$ h 6

12 a $\frac{3}{5}$ b 48 boys, 72 girls

13 $11\frac{2}{3}$ L

14 7 cups of self-raising flour, 3 cups of cream

15 7 games

16 a $\times \frac{7}{12}$ b $\times \frac{7}{12}$ c $\times \frac{1}{12}$
d ✓ e ✓ f $\times \frac{1}{12}$

17 D; e.g. $\frac{2}{7} \times \frac{3}{5} = \frac{6}{35}$. The two numerators will always multiply to give a smaller number than the two larger denominators. Hence, the product of two proper fractions will always be a proper fraction.

18 Answers may vary.
a $\frac{2}{5} \times \frac{3}{2}$ b $\frac{5}{4} \times \frac{3}{5}$ c $\frac{3}{7} \times \frac{2}{6}$

19 a $\frac{2}{7}, \frac{3}{8}$
b–c Answers will vary.

Exercise 6E

1 a 00 b 000 c 00 d 00000

2 a 000 b 00 c 0 d 00000

3 a i right 2 places ii left 1 place
iii right 6 places iv no change
v left 3 places vi right 3 places
vii right 1 place viii left 7 places
b right 1 place

4 a 48.7 b 352.83 c 4222.7 d 1430.4
e 5699.23 f 125.963 g 12 700 h 154 230
i 3400 j 2132 k 86 710 000 l 516 000

5 a 4.27 b 35.31 c 2.4422 d 56.893
e 12.13518 f 9.32611 g 0.029 h 0.001362
i 0.00054 j 0.367 k 0.000002 l 0.0100004

6 a 2291.3 b 31.67 c 0.49
d 0.222 e 63 489 000 f 0.0010032

7 a 15 600 b 43 000 c 225.1
d 0.016 e 213 400 f 21.34
g 0.007 h 9 900 000 i 0.0034

8 a 158.4 b 3.36 c 85.4 d 7054
e 71.06 f 7.5 g 2.037 h 21.7

9 $147

10 a 1 200 000 mL b 12 000

11 3000 c, $30

12 $21 400

13 225 kg

14 Answers may vary.

Starting number	Answer	Possible two-step operations
12.357	1235.7	$\times$ 1000, $\div$ 10
34.0045	0.0340045	$\div$ 100, $\div$ 10
0.003601	360.1	$\times$ 100, $\times$ 1000
bac.dfg	*ba.cdfg*	$\div$ 100, $\times$ 10
d.swkk	*dswkk*	$\times$ 100 000, $\div$ 10
fwy	*f.wy*	$\div$ 1000, $\times$ 10

15 $\div$ 1 000 000

16 a i 5×10^{13} ii 4.2×10^{7} iii 1.23×10^{16}
b i 2×10^{19} ii 1.08×10^{21}
c–d Answers will vary.
e 3.5×10^{-11}
f i 1×10^{-6} ii 9×10^{-10} iii 7.653×10^{-12}

Exercise 6F

1 a 1 b 2 c 3 d 3 e 5
f 2 g 4 h 3 i 9

2 a 19.2 b 1.92 c 0.192

3 It helps you check the position of the decimal point in the answer.

4 a 20.84 b 26.6 c 183.44 d 100.8
e 218.46 f 15.516 g 23.12 h 12.42
i 5.44 j 311.112 k 0.000966 l 1.32131

5 a 100.8 b 483 c 25 400
d 9800 e 14 400 f 364 550
g 0.68 h 371 i 90.12

6 a $31.50, $32 b $22.65, $23 c $74.80, $75
d $17.40, $17 e $145.20, $145 f $37 440, $37 440
g $88.92, $89 h $4.41, $4 i $18.0625, $18

7 a The decimal point is the actual 'dot'; decimal places are the numbers after the decimal point.
b 1 decimal point, 4 decimal places

8 in the question; decimal places

9 29.47 m 10 3.56 kg

11 165.85 km

12 a 67.2 m b $198.24

13 $1531.25

14 a 738.4 km b yes c 1.57 L left in the tank

15 Answers may vary; 0.25, 0.26

16 Answers may vary; 0.0043

17 a 38.76 b 73.6 c 0.75 d 42, 0.42

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Exercise 6G

1 a $\frac{2}{1}=2$ b $\frac{4}{3}$ c $\frac{3}{1}=3$ d $\frac{1}{6}$
e $\frac{2}{3}$ f $\frac{4}{7}$ g $\frac{2}{5}$ h $\frac{2}{5}$

2 6, 12, 24, 48, 96

3 A

4 a $\frac{5}{11}\times\frac{5}{3}$ b $\frac{1}{3}\times\frac{5}{1}$ c $\frac{7}{10}\times\frac{17}{12}$ d $\frac{8}{3}\times\frac{1}{3}$
e $\frac{6}{1}\times\frac{4}{3}$ f $\frac{7}{1}\times\frac{2}{1}$ g $\frac{5}{4}\times\frac{2}{1}$ h $\frac{3}{5}\times\frac{7}{3}$

5 a $\frac{5}{2}\div\frac{4}{3}, \frac{5}{2}\times\frac{3}{4}$ b $24\div\frac{16}{5}, \frac{24}{1}\times\frac{5}{16}$
c $\frac{47}{11}\div\frac{21}{4}, \frac{47}{11}\times\frac{4}{21}$ d $\frac{8}{3}\div\frac{80}{7}, \frac{8}{3}\times\frac{7}{80}$

6 a $\frac{7}{5}$ b $\frac{5}{3}$ c $\frac{9}{2}$ d $\frac{8}{1}$
e $\frac{3}{7}$ f $\frac{5}{23}$ g $\frac{6}{11}$ h $\frac{3}{26}$
i $\frac{1}{12}$ j $\frac{1}{101}$ k 9 l 1

7 a $\frac{3}{8}$ b $\frac{5}{33}$ c $\frac{2}{5}$ d $\frac{5}{7}$
e $\frac{3}{4}$ f $1\frac{1}{3}$ g $1\frac{3}{5}$ h $\frac{3}{14}$

8 a 20 b 21 c 100 d 120
e 30 f 40 g 4 h $6\frac{2}{3}$

9 a $\frac{5}{7}$ b $\frac{4}{5}$ c $\frac{11}{14}$ d $\frac{3}{4}$
e $1\frac{11}{16}$ f $1\frac{3}{11}$ g $3\frac{1}{3}$ h $\frac{3}{4}$

10 a $\frac{3}{40}$ b 30 c $1\frac{13}{35}$ d $\frac{1}{3}$
e 28 f $\frac{4}{15}$ g $6\frac{4}{7}$ h $2\frac{1}{10}$

11 a $\frac{3}{40}$ b 30 c $1\frac{13}{35}$ d $\frac{1}{3}$
e 28 f $\frac{4}{15}$ g $6\frac{4}{7}$ h $2\frac{1}{10}$

12 a less b more c more
d more e less f less

13 $\frac{3}{4}$

14 10

15 $\frac{9}{10}$ m

16 8

17 22 km

18 $\frac{1}{2}$ of 8 and $\frac{1}{2}\div\frac{1}{8}=4$, $12\div4$ and $12\times\frac{1}{4}=3$, $10\times\frac{1}{2}$ and $10\div2=5$, $3\div\frac{1}{2}$ and $3\times2=6$

19 a $\frac{9}{20}$ b $\frac{45}{64}$ c $\frac{5}{16}$ d $\frac{1}{5}$

20 a 240 km b 160 km

21 Answers will vary

Exercise 6H

1 B

2 a 96 b 106.4 c 0.64 d 156.39 e 1563.9

3 a 32.456, 3 b 12 043.2, 12
c 34.5, 1 d 1 234 120, 4

4 a 4.2 b 6.1 c 21.34 d 0.7055
e 1.571 f 0.308 g 3.526 h 124.3
i 0.0024 j 117.105 k 0.6834 l 0.0025625

5 directly above the decimal point in the dividend

6 a 30.7 b 77.5 c 26.8 d 8.5
e 44.4 f 645.3 g 0.08 h 0.050425
i 980 j 800.6 k 0.79 l 2 161 000

7 a 60 b 60 c 60 d 60
e An identical change has occurred in both the dividend and the divisor.

9 a 1.1807 b 8.267 c 0.0123748
d 0.00423 e 0.096487 f 0.0007825

10 a 11.83 kg b \$30.46 c 304.33
d 239.17 g e 965.05 L f \$581.72

11 a 20.84 b 93.36 c 10.93
d 18.49 e 67.875 f 158.35

12 a 8, 9 b 9 c 5, 5, 1 d 0, 7, 7

13 \$1.59/L

14 238 frames

15 26.67, 26 can be filled

16 a \$66.35 b \$21.90

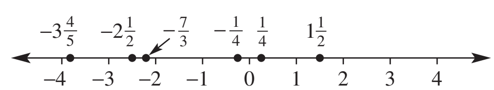
17 apples \$3.25/kg; bananas \$3.10/kg; hence, bananas are better value.

18 a 24.5 m/s b 88.2 km/h c yes

19 a 24.53 b 19.7 c 2453
d 1.97 e 2.453 f 197

20 a 0.5 b 0.3 c 0.01 d 1.1 e 4.8

Exercise 6I

1 

2 a $\frac{1}{4}$ b $-\frac{1}{3}$ c $\frac{3}{5}$ d $-\frac{2}{7}$

3 a positive b negative c negative d positive

4 a $-\frac{4}{7}$ b $\frac{1}{5}$ c $-\frac{7}{9}$ d $-5\frac{1}{3}$
e $-\frac{1}{3}$ f $-\frac{2}{5}$ g $\frac{3}{2}$ h $\frac{7}{11}$

5 a $-\frac{1}{12}$ b $-\frac{13}{35}$ c $1\frac{1}{10}$ d $\frac{8}{9}$
e $-\frac{1}{4}$ f $\frac{1}{8}$ g $-\frac{3}{20}$ h $-\frac{4}{15}$

6 a $-\frac{12}{35}$ b $-\frac{16}{55}$ c $\frac{4}{15}$ d $\frac{5}{6}$
e $-\frac{4}{21}$ f $-\frac{1}{8}$ g $\frac{3}{7}$ h $-1\frac{1}{5}$

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7 a $-\frac{20}{21}$ **b** $-\frac{9}{20}$ **c** $\frac{8}{15}$ **d** $1\frac{1}{3}$
e $-\frac{2}{7}$ **f** $-\frac{3}{20}$ **g** $\frac{3}{4}$ **h** $2\frac{2}{5}$

8 a 6 **b** $-5\frac{8}{39}$ **c** $\frac{3}{11}$ **d** $-\frac{85}{203}$

9 $-\frac{5}{3}, -1\frac{1}{2}, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{5}, \frac{1}{16}, \frac{3}{4}, 3\frac{1}{10}$

10 $16\frac{3}{4}$° C

11 a Mon $= -1\frac{2}{3}$, Tue $= -\frac{1}{2}$, Wed $= -2\frac{1}{4}$, Thur $= \frac{1}{4}$ hours
b $-4\frac{1}{6}$ hours **c** $12\frac{1}{6}$ hours

12 $1\frac{7}{20}$ metres

13 a > **b** < **c** > **d** <
e > **f** > **g** < **h** >

14 a negative **b** negative
c negative **d** positive

15 a $-\frac{6}{539}$ **b** $-2\frac{491}{605}$ **c** $-\frac{35}{81}$ **d** $\frac{1}{343}$

16 a < **b** > **c** > **d** <

17 a i $1\frac{1}{2}$ **ii** $\frac{1}{15}$ **iii** $-\frac{21}{25}$ **iv** $-4\frac{23}{40}$
b Answers may vary: $-\frac{5}{8}, -\frac{3}{8}, -\frac{2}{8}, -\frac{1}{8}, \frac{11}{8}$
c Answers may vary: $-\frac{7}{4}, -\frac{5}{4}, -\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}$

Puzzles and challenges

1 Continent of Atlantis
2 Height of rungs: 0.25 m, 0.40 m, 0.54 m, 0.68 m, 0.83 m, 0.97 m, 1.12 m, 1.26 m, 1.40 m, and 1.55 m
3 a A = 1, B = 7, C = 8 **b** A = 7, B = 5, C = 1, D = 9
c A = 4, B = 2 **d** A = 3, B = 7

Multiple-choice questions

1 D **2** B **3** C **4** E **5** A
6 D **7** D **8** B **9** A **10** C

Short-answer questions

1 a 0.44, 0.4, 0.04 **b** 2.16, 2.026, 2.016
c 0.98, 0.932, 0.895
2 a 8.1 **b** 0.81 **c** 8.01 **d** 0.801
3 a 3 hundredths $= \frac{3}{100}$
b 3 thousandths $= \frac{3}{1000}$
c 3 ones = 3
4 a false **b** false **c** true
d true **e** false **f** true
5 a 947 **b** 9470 **c** 94 700
6 a 423.5 **b** 15.89 **c** 7.3 **d** 70.000
e 2.8 **f** 0.67 **g** 0.455
h 0.01234567901234567901234567901234567901234568
7 a 9.53 **b** 4.137 **c** 43.35
d 240.49857 **e** 83.497 **f** 205.22
8 a true **b** false **c** false **d** false **e** true
9 a 5 **b** 5 **c** 4 **d** 2
10 a 137 **b** 790 **c** 22.51 **d** 0.096208
e 696.956 **f** 360.5 **g** 563 489.3
11 a 19.2 **b** 63.99 **c** 19.32
d 0.95 **e** 1.52 **f** 6
g 16 **h** 3 **i** $3.1\dot{0}\dot{9}$
12

Decimal	Fraction	Percentage
0.45	$\frac{45}{100} = \frac{9}{20}$	45%
0.7	$\frac{70}{100} = \frac{7}{10}$	70%
0.32	$\frac{32}{100} = \frac{8}{25}$	32%
0.06	$\frac{6}{100} = \frac{3}{50}$	6%
0.79	$\frac{79}{100}$	79%
1.05	$\frac{105}{100} = \frac{21}{20}$	105%
0.35	$\frac{35}{100} = \frac{7}{20}$	35%
0.65	$\frac{65}{100} = \frac{13}{20}$	65%
0.125	$\frac{125}{1000} = \frac{1}{8}$	12.5%

Extended-response questions

1 a Jessica \$12.57; Jaczinda \$13.31; hence, Jaczinda earns higher pay rate by 74c per hour.
b \$43.47, \$43.45 to the nearest 5 cents
c \$40.56
d \$48.34
2 a \$28.39 **b** \$17 035.20

Semester review 1

Chapter 1: Computation with positive integers

Multiple-choice questions

1 E **2** B **3** D **4** C **5** A

Short-answer questions

1 a One thousand and thirty
 b Thirteen thousand
 c Ten thousand, three hundred
 d Ten thousand, and thirty
 e One hundred thousand, three hundred
 f One million, three hundred thousand
2 a 67 849 b 700 850
3 a 99 323 b 6867 c 441
 d 196 000 e 1644 f $764\frac{3}{4}$
4 a false b true c true
5 36
6 a 30 b 56 c 48 d 160 e 16 f 42
7 a false b true c false d true
 e true f true
8 $18 \times (7 + 3)$
9 9 times
10 a 3 859 640 b 3 860 000 c 4 000 000

Extended-response question

1 a 28 b $700 c $1000 d 12 h

Chapter 2: Angle relationships

Multiple-choice questions

1 A 2 B 3 B 4 B 5 D

Short-answer questions

1 a $AC \parallel FD$ or $EB \parallel DC$
 b BF and BD
 c AC, BF, BD at point B or CD, ED, BD at point D or BF, BE, BD at B
 d A and B
 e E
2 a 30° b 80° c 150°
3 25°
4 78°
5 a $a = 140$ b $a = 50$ c $a = 140$
 d $a = 65$ e $a = 62$ f $a = 56$
6 $a = 100$, $b = 80$, $c = 100$, $d = 80$, $e = 100$, $f = 80$, $g = 100$
7 Because the cointerior angles add to more than 180°.

Extended-response question

1 a i $x = 56$ ii $y = 95$ iii $z = 29$
 b $x + y + z = 180$

Chapter 3: Computation with positive and negative integers

Multiple-choice questions

1 C 2 B 3 C 4 E 5 A

Short-answer questions

1 a < b < c =
2 a −13 b −84 c −108
 d −21 e 84 f 0
3 a −24 b 72 c 144
 d 21 e 30 f −31
4 a negative b positive c negative
5 a −15 b −8 c −4

Extended-response question

1 a D b A, B, O and G; all lie on the x-axis.
 c F d i 2 units ii 5 units
 e trapezium f 8 square units
 g $X(4, 2)$ h DECIDE

Chapter 4: Understanding fractions, decimals and percentages

Multiple-choice questions

1 B 2 A 3 E 4 D 5 E

Short-answer questions

1 $\frac{3}{10}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}$
2 $\frac{17}{3}$
3 a true b false c false
 d true e false f true
4 $\frac{3}{20}$
5 $120
6 $60
7 a true b true c true d false
8 67%

Extended-response question

1 a 6 b $\frac{8}{9}$ c 9
 d second dose on Sunday week

Chapter 5: Probability

Multiple-choice questions

1 C 2 D 3 C 4 C 5 B

Short-answer questions

1 a A b B c B, C, D, A
2 a $\frac{1}{2}$ b $\frac{1}{2}$ c $\frac{1}{4}$
 d $\frac{1}{13}$ e $\frac{1}{13}$ f $\frac{1}{26}$

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3 a $\frac{1}{6}$ b $\frac{1}{15}$ c $\frac{4}{15}$
d $\frac{17}{30}$ e $\frac{5}{6}$
4 B, C, A, D, E

Extended-response question

1 a $\frac{1}{4}$ b $\frac{1}{4}$ c $\frac{1}{2}$ d $\frac{1}{52}$
e $\frac{2}{13}$ f $\frac{1}{13}$ g $\frac{4}{13}$ h $\frac{4}{13}$

Chapter 6: Computation with decimals and fractions

Multiple-choice questions

1 D 2 C 3 D 4 C 5 B

Short-answer questions

1 a 0.2 b 0.13 c 1.7
2 a 6 units b $\frac{4}{1000}$ c 136.1
3 a 18 b 18.4 c 18.40
4 a 4.07 b 269.33 c 19.01
d 0.24 e 0.09 f 60
5 a 0.833 b 3 c 36
6 a 4.5387 b 0.045387 c 1.23
7 a 36 490 b 0.018 c 3886
8 a $\frac{4}{5}$ b 1.1 c $\frac{2}{3}$
9 a $\frac{11}{12}$ b $2\frac{2}{3}$ c $6\frac{1}{4}$
d $\frac{1}{5}$ e 4 f $\frac{1}{2}$

Extended-response question

1 a \$80.58 b \$80.60 c \$80.85 d 25c
e No, it will be 11c cheaper if rounding occurs at the beginning.

Chapter 7

Pre-test

1 a 24 b 60 c 60
d 7 e 12 f 365
2 a Friday b Monday
c Wednesday d Sunday
3 a 5 p.m. b 1:45 a.m. c 4:37 a.m.
d 8:49 p.m. e 4 p.m. f 3:55 p.m.
g 10:36 a.m. h 2:14 p.m.
4 a 1 min b 2 h c 7 weeks d 360 min
5 a 3 h 55 min b 235 min
6 a 3.00 b 2.30 c 5.45

Exercise 7A

1 a February
b April, June, September, November
c January, March, May, July, August, October, December
2 a F b D c A
d E e B f C
3 a M b D c D d M
4 a 120 s b 3 min c 2 h d 240 min
e 72 h f 2 days g 5 weeks h 280 days
5 a 180 min b 630 s c 4 min
d 1.5 h e 144 h f 3 days
g 168 h h 1440 min i 4 h
j 2 weeks k 20 160 min l 86 400 s
m 210 min n 15 s o 1.5 days
p 4.5 h q 1.25 min r 2 h
6 a 1330 h b 2015 h c 1023 h
d 2359 h e 6:30 a.m. f 1 p.m.
g 2:29 p.m. h 7:38 p.m. i 11:51 p.m.
7 a 1.5 h b 4.75 h c 7.25 h d $3.\dot{3}$ h
8 a 2 h 30 min b 4 h 15 min c 1 h 20 min
d 6 h 30 min e 3 h 45 min f 9 h 15 min
9 a 2 p.m. b 5 a.m.
c 1200 hours d 1800 hours
10 a 7 h 7′12″ b 2 h 16′48″ c 3 h 3′ d 8 h 55'48"
11 a 6:30 a.m. b 2:25 p.m. c 7:27 p.m.
12 52.14 weeks
13 Friday
14 a 0.2 of 1 hour is one-fifth of 1 hour = 12 min.
b 10 min $= \frac{1}{6}$ of 1 hour and $\frac{1}{6} = 0.1\dot{6}$
15 a 3600 b 1440 c 3600 d 1440
16 a 2 h 24 min b 7 h 10 min 48 s c 9 h 55 min 12 s
17 Answers will vary.

Exercise 7B

1 a false b true c true
d false e true
2 a 6 h 30 min b 10 h 45 min
c 16 h 20 min d 4 h 30 min
3 a 4 h b 6 h c 3 h 30 min
d 8 h 45 min e 5 h 15 min f 7 h 30 min
4 a 2 h 5 min b 2 h 10 min c 4 h 5 min
d 2 h 11 min e 1 h 50 min f 3 h 40 min
g 47 min h 2 h 54 min i 2 h 46 min
j 3 h 46 min
5 a 6:30 p.m. b 9 a.m.

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c 6:30 p.m. d 4:30 p.m.
e 5:30 p.m. f 11:40 a.m.

6 a 2 h 10 min 10 s b 3 h 15 min 6 s
c 1 h 4 min 14 s d 54 min 55 s
e 1 h 53 min 53 s f 5 h 44 min 28 s

7 a 5 h 45 min b 8 h
c 9 h 6 min d 4 h 30 min
e 3 h 50 min 28 s f 13 h 20 min 30 s

8 42 min 41 s

9 a i 16 min ii 27 min iii 24 min iv 40 min
b afternoon c 40 min

10 56 million years

11 17 min 28 s

12 7 h 28 min

13 23 h 15 min

14 1 km = 1000 m and 1 h = 3600 s, so multiply to go from hours to seconds and divide to go from metres to kilometres.

15 So that morning and afternoon times are not confused

16 $20 857.14

17 a $900 b $90
c $1.50 d 2.5c

18 a 97.5° b 182.5° or 177.5°
c 108.5°

19 $1\frac{5}{7}$ h or 1 h 42′ 51″

Exercise 7C

1 SA, Vic., Tas., NSW, ACT

2 a i 10 ii 9.5 iii 8
iv 7 v 8 vi 2
b i 0 ii 2 iii 5 iv 5

3 Sunday

4 a 10 a.m. b 9:30 a.m. c 10 a.m.
d 9:30 a.m. e 8 a.m. f 10 a.m.

5 a 6 p.m. b 6:30 p.m. c 6:30 p.m.

6 a 11 a.m. b 12 noon c 8 p.m.
d 7:30 p.m. e 7 a.m. f 5 a.m.
g 1 a.m. h 10 a.m.

7 a 5:30 a.m. b 6:30 a.m. c 6:30 a.m.
d 1:30 p.m. e 2:30 p.m. f 2:30 a.m.
g 3 p.m. h 5:30 p.m.

8 a 6 h b 2.5 h c 8 h
d 6 h e 7 h

9 midnight

10 7:35 p.m.

11 6:30 a.m.

12 a 8 a.m. 29 March
b 10 p.m. 28 March
c 3 a.m. 29 March

13 a 8:30 a.m. b 9:30 a.m.
c 9 a.m. d 6:30 a.m.

14 turn back 1 hour

15 During daylight saving time, it is UTC plus 11 h.

16 3 a.m. 21 April

17 11:30 p.m. 25 October

18 a SA is ahead of Qld in daylight saving time. Qld does not use daylight saving time.
b Broken Hill uses Central Standard Time, as per SA.
c Lord Howe Island is UTC + 10:30 or 11:00 during daylight saving
d Central Western Time and The Indian Pacific Train

Puzzles and challenges

1 12 noon

2 $3\frac{3}{7}$ or 3 min 26 s

3 Wednesday

4 3 min 10 s

5 75 s

Multiple-choice questions

1 A 2 D 3 E 4 D
5 B 6 C 7 A 8 D
9 E 10 C

Short-answer questions

1 a 90 min b 2 min c 2 days
d 21 days e 1440 min f 0.5 h

2 a 0400 b 1530 c 1919
d 6:35 a.m. e 12:51 p.m. f 11:28 p.m.

3 a 3 h 30 min b 4 h 20 min
c 6 h 15 min d 1 h 45 min

4 a 3 h 36′ b 6 h 55′ 12″ c 11 h 26′ 24″

5 1 h 33 min

6 a 5 h 53 min b 4 h 5 min 55 s
c 1 h 59 min 1 s d 3 h 38 min

7 a i 25 min ii 34 min iii 20 min iv 44 min
b morning c 45 min

8 a i 8:15 a.m. ii 6:15 a.m.
b i 1:36 p.m. ii 3:36 p.m.

9 1:40 p.m.

Extended-response question

1 a i 9 p.m. ii 2 p.m. iii 4 p.m.
b i 7:30 a.m. ii 1:30 p.m. iii 1:30 a.m.
c i Tuesday ii Monday
d 1 p.m.

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Chapter 8

Pre-test

1 a 11 b 5 c 5 d 21
2 a 8 b 36 c 40 d 10
3 a 13 b 21 c 4 d 5
4 a 6×3 b $10 \div 2$ c $12 + 3$ d $10 - 9$
5 a $output = 3 \times input$
b $output = input + 5$
c $output = input + 6$
d $output = 2 \times input - 1$
6 a 11 b 6 c 40 d 2
7 a 20 b 44 c 22 d 100
8 a 20 b 6 c 2 d 6
9 a 1 b 9 c 3 d 45
e 25 f 5 g 0 h 25

Exercise 8A

1 a $4x, 3y, 24z, 7$ b 7
c 4 d z
2 a $x + 4$ b $x - 4$ c $4x$ d $\frac{x}{4}$
e $4 - x$ f $\frac{4}{x}$
3 a i 2 ii 17
b i 3 ii 15
c i 3 ii 21
d i 4 ii 2
e i 2 ii 1
f i 4 ii 12
4 a $x + 1$ b $5 + k$ c $2u$ d $4y$
e $\frac{p}{2}$ f $\frac{q}{3}$ g $r - 12$ h $9n$
i $10 - t$ j $\frac{y}{8}$
5 a $2(x + 5)$ b $3a + 4$ c $8k - 3$ d $8(k - 3)$
e $6(x + y)$ f $\frac{7x}{2}$ g $\frac{p}{2} + 2$ h $12 - xy$
6 a The product of 7 and x.
b The sum of a and b.
c The sum of x and 4 is doubled.
d a is tripled and subtracted from 5.
7 a 70 b $10n$
8 a $\$8x$ b $\$(x + 3)$ c $\$8(x + 3)$
9 a $1000x$ b $100x$ c $100\,000x$
10 a $\frac{\$A}{4}$ b $\frac{\$A}{n}$ c $\frac{\$A - 20}{n}$
11 'One-quarter of the sum of a and b.' Other answers possible.
12 a true b true c false
d true e false
13 a false b false
c true d true
14 $c \div 2$, $c - 4$, $c + 1$, $2c$, $3c$, $3c + 5$, $4c - 2$, $c \times c$
15 'The sum of twice the value of x is taken from 3' becomes $3 - 2x$. Other answers possible.

Exercise 8B

1 a 14 b 1 c 10 d 8
2 17
3 15
4 3
5 a 17 b 20 c 72 d 12
6 a 8 b 10 c 9 d 14
e 12 f 3 g 19 h 9
i 7 j 20 k 3 l 1
m 47 n 20 o 35 p 86
q 8 r 6
7 a 8 b 4 c 5 d 9
e 4 f 45 g 5 h 24
i 50 j 8 k 5 l 20
8 a 27 b 22 c 41
d 8 e 10 f 70
9 a

n	1	2	3	4	5	6
$n + 4$	5	6	7	8	9	10

b

x	1	2	3	4	5	6
$12 - x$	11	10	9	8	7	6

c

b	1	2	3	4	5	6
$2(b - 1)$	0	2	4	6	8	10

d

q	1	2	3	4	5	6
$10q - q$	9	18	27	36	45	54

10 a 75 b 45 c 54 d 11
e 12 f 5 g 33 h 19
11 5
12 x is between 4 and 33 inclusive.
13

x	5	9	12	1	6	7
$x + 6$	11	15	18	7	12	13
$4x$	20	36	48	4	24	28

14 a 1 and 24, 2 and 12, 3 and 8, 4 and 6
b Infinitely many answers; e.g. $x = \frac{1}{5}$, $y = 120$.
15 Because $5 \times (a + a)$ is $5 \times 2 \times a$, which is $10a$. Every multiple of 10 ends with 0.
16 a

x	5	10	7	9	5	8
y	3	4	2	5	2	0
$x + y$	8	14	9	14	7	8
$x - y$	2	6	5	4	3	8
xy	15	40	14	45	10	0

b 2 and 2, 3 and $1\frac{1}{2}$, 6 and $1\frac{1}{5}$; other answers possible.

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Exercise 8C

1 a

	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$2x + 2$	2	4	6	8
$(x + 1) \times 2$	2	4	6	8

b equivalent

2 a

	$x = 0$	$x = 1$	$x = 2$	$x = 3$
$5x + 3$	3	8	13	18
$6x + 3$	3	9	15	21

b no

3

	$6x + 5$	$4x + 5 + 2x$
$x = 1$	11	11
$x = 2$	17	17
$x = 3$	23	23
$x = 4$	29	29

They are equivalent because they are always equal.

4 a $2x + 4$ and $x + 4 + x$ b $5a$ and $4a + a$
c $2k + 2$ and $2(k + 1)$ d $b + b$ and $4b - 2b$

5 a C b A c E
d F e B f D

6 $2x + 2 + 2x$, $2(2x + 1)$; other answers possible.

7 $2(l + b)$; other answers possible.

8 $9a + 4b$; other answers possible (must have 2 terms).

9 6

10 If $x = 8$, all four expressions have different values.

11 b $x \times y$ and $y \times x$
c $a \times (b + c)$ and $a \times b + a \times c$
d $a - (b + c)$ and $a - b - c$
e $a - (b - c)$ and $a - b + c$
f $a \div b \div c$ and $a \div (b \times c)$

12 a

	$4 \times (a + 2)$	$8 + 4a$
$x = 1$	12	12
$x = 2$	16	16
$x = 3$	20	20
$x = 4$	24	24

b $5(2 + a)$
c $24 + 6a$

13 $2a + a + 5b$, $3a + 12b - 7b$; others answers possible.

14 a Yes; for any value of x expressions A and B are equal, and expression B and C are equal, so expressions A and C are equal.
b No; e.g. if expression A is $7x$, expression B is $3x$ and expression C is $5x + 2x$.

Exercise 8D

1 a a, c, f, h

2 a like b like terms
c terms d equivalent

3 a N b L c L d N
e N f L g N h L
i L j L k L l N

4 a Y b Y c N d N
e Y f Y g Y h N

5 a $2a$ b $5x$ c $7b$ d $8d$
e $12u$ f $12ab$ g $11ab$ h xy

6 a $3a + 5b$ b $7a + 9b$ c $x + 6y$
d $7a + 2$ e $7 + 7b$ f $6k - 2$
g $5f + 12$ h $4a + 6b - 4$ i $6x + 4y$
j $8a + 4b + 3$ k $7h + 4$ l $14x + 30y$
m $2x + 9y + 10$ n $8a + 13$ o $12b$
p $9ab + 4$ q $6xy + 5x$ r $7cd - 3d + 2c$
s $9uv + 7v$ t $11pq + 2p - q$ u $6ab + 36$

7 a $\$27n$ b $\$31n$ c $\$58n$

8 a $3a + 4$ b 19

9 a $4x$ b $7x$ c $11x$ d $3x$

10 a $12xy$ b $6ab + 5$ c $10ab$
d $6xy + 3$ e $5xy + 14$ f $10cde$
g $6xy + 6x + 4$ h $9ab + 9$ i $7xy - 2y$

11 a

	$3x + 2x$	$5x$
$x = 1$	5	5
$x = 2$	10	10
$x = 3$	15	15

b For example, if $x = 5$ and $y = 10$, then $3x + 2y = 35$ but $5xy = 250$.

12 a

	$4x + 5 - 2x$	$3x + 4$
$x = 1$	7	7
$x = 2$	10	10
$x = 3$	13	13

b For example, if $x = 10$, $5x + 4 - 2x = 34$ but $7x + 4 = 74$.
c For example, if $x = 1$, $5x + 4 - 2x = 7$ but $7x - 4 = 3$.

13 a $2a + a + 3b$, $b + 3a + 2b$; other answers possible.
b 12

Exercise 8E

1 a both are 21 b both are 35
c both are 56 d yes

2 a true b true c false d true

3 a $\frac{2}{3}$ b $\frac{2}{3}$ c $\frac{2}{3}$

4 a C b E c B
d A e D

5 a $2x$ b $5p$ c $8ab$ d $6a$
e $10ab$ f $10b$ g $28xz$ h $36abc$
i $42ab$ j $42abc$ k $54abd$ l $84abc$

6 a $36a$ b $63d$ c $8e$ d $15a$
e $12ab$ f $63eg$ g $8abc$ h $28adf$
i $12abc$ j $8abc$ k $60defg$ l $24abcd$

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7 a w^2 b a^2 c $3d^2$
d $2k^2$ e $7p^2$ f $3q^2$
g $12x^2$ h $15z^2$ i $36r^2$

8 a $\frac{x}{5}$ b $\frac{z}{2}$ c $\frac{a}{12}$ d $\frac{b}{5}$
e $\frac{2}{x}$ f $\frac{5}{d}$ g $\frac{x}{y}$ h $\frac{a}{b}$
i $\frac{4x+1}{5}$ j $\frac{2x+y}{5}$ k $\frac{2+x}{1+y}$ l $\frac{x-5}{3+b}$
m $2x+\frac{y}{5}$ n $2+x+y$ o $x-\frac{5}{3}+b$ p $4b$
q 0 r $11b$ s $7b$ t $-4c$
u $-7c$

9 a $\frac{2}{5}$ b $\frac{5}{9}$ c $\frac{9a}{4}$ d $\frac{2b}{5}$
e $\frac{x}{2}$ f $\frac{3x}{4}$ g $\frac{2}{3}$ h $\frac{3}{4}$
i $2a$ j 3 k $2y$ l $\frac{3}{y}$

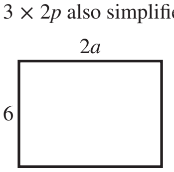
10 a $3k$ b $6x$ c $12xy$

11 a $4x$ grams b nx grams
c $2nx$ grams

12 a $2 \times 3 \times a \times b$ b $3 \times 7 \times x \times y$
c $2^2 \times e \times f \times g$ d $3 \times 11 \times q \times q \times r$

13 a \$20 b $\frac{\$C}{5}$

14 a $6p$
b $3 \times 2p$ also simplifies to $6p$, so they are equivalent.

15 a


b $12a$
c $\frac{12a}{3a}$ simplifies to 4. It has four times the area.
d area is multiplied by 9

16 a $3 \times a \times a \times b \times b$ b $7x^2y^3$
c $1200a^3b^2c^2$ d $24a^3b^2c^2$

Exercise 8F

1 a 6 b a

2 a $4x$ b 12 c $4x + 12$

3 a 60, 3, 63 b 30, 4, 210, 28, 238
c 20, 1, 100, 5, 95

4 a

	$4(x+3)$	$4x+12$
$x = 1$	16	16
$x = 2$	20	20
$x = 3$	24	24
$x = 4$	28	28

b equivalent expressions

5 a $12(x+4)$ and $12x+48$ b $8(z+9)$ and $8z+72$
c $(3+a)(3+b)$ and $9+3a+3b+ab$

6 a $6y+48$ b $7l+28$ c $8s+56$ d $8+4a$
e $7x+35$ f $18+3a$ g $81-9x$ h $5j-20$
i $8y-64$ j $8e-56$ k $6e-18$ l $80-10y$

7 a $60g-70$ b $15e+40$ c $35w+50$ d $10u+25$
e $56x-14$ f $27v-12$ g $7q-49$ h $20c-4v$
i $4u+12$ j $48l+48$ k $5k-50$ l $9o+63$

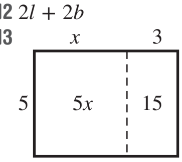
8 a $6it-6iv$ b $2dv+2dm$ c $10cw-5ct$
d $6es+6ep$ e $dx+9ds$ f $10ax+15av$
g $5jr+35jp$ h $in+4iw$ i $8ds-24dt$
j $2fu+fv$ k $14kv+35ky$ l $4em+40ey$

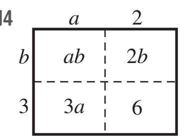
9 a $5(x+3)=5x+15$ b $2(b+6)=2b+12$
c $3(z-4)=3z-12$ d $7(10-y)=70-7y$

10 a $b+g+1$ b $2(b+g+1)=2b+2g+2$

11 $3(4x+8y)$ and $2(6x+12y)$; others answers possible.

12 $2l+2b$

13


14


15 a 4 ways, including $1(10x+20y)$
b infinitely many ways

16 a Rosemary likes maths and Rosemary likes English.
b Priscilla eats fruit and Priscilla eats vegetables.
c Bailey likes the opera and Lucia likes the opera.
d Frank plays video games and Igor plays video games.
e Pyodir likes chocolate and Pyodir likes tennis and Astrid likes chocolate and Astrid likes tennis.

Exercise 8G

1 a 35 b 20

2 a 24 cm b 40 cm

3 a $3x$ b 36

4 $\$2n$

5 a $\$10x$ b $\$15x$ c $\$kx$

6 a 180 km b 30 km c $70n$ km

7 a \$200 b \$680 c $\$(50+80x)$

8 a B b A c D d E e C

9 a

No. of hours (t)	1	2	3	4	5
Total cost (\$)	150	250	350	450	550

b $100t+50$ c \$3050

10 a \$25 b $10x+5$ c \$75

11 a 33 b $g=8, b=5$
c $g=3$ and $b=2$, $g=1$ and $b=14$, $g=0$ and $b=20$

12 a $90c$ b \$6.30 c $0.3+0.6t$

13 a $5b+2c+6d$ b $5b+4c+3d$
c \$8

14 a cn b $3cn$ c $6cn$

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15 a $10 + 30x$ b 2 h 20 min
c Because you don't pay the booking fee twice.
d \$32 e It will get closer to \$30.

16 a $0.2 + 0.6t$ b $0.8 + 0.4t$ c Emma's
d 3 min
e Answers will vary.

Exercise 8H

1 C 2 B 3 no, 20

4 a 17 b 23 c −8 d 8
e 6 f 11 g −7 h 6
i 3 j −9 k −9 l 6

5 a −8 b −2 c 2 d −13
e −25 f 17 g 36 h 18

6 a 38 b 5 c 4 d −13
e −4 f −16

7 a 10 b −1 c −24 d −6

8 a 6 m^2 b 16 m^2

9 a 32 metres per second b 14 metres per second
c 2 metres per second

10 a i 16 ii −2
b A negative width is not possible.

11 Answer may vary.

12 Answers may vary.

13 a $a - a = 0$ and $-b + b = 0$
b $\frac{a}{a} = 1$
c $a - a = 0$
d $\frac{ab}{b}$ cancels to simply give a.

14 a i 5°C ii −15°C iii −25°C
b $F = \frac{9C}{5} + 32$

Exercise 8I

1 a 8, 11, 14, 17, 20 b 32, 31, 30, 29, 28
c 2, −2, −6, −10, −14 d 123, 130, 137, 144, 151

2 a 3, 6, 12, 24, 48 b 5, 20, 80, 320, 1280
c 240, 120, 60, 30, 15 d 625, 125, 25, 5, 1

3 a ratio of 3 b difference subtracting 2
c difference adding 11 d neither
e ratio of $\frac{1}{2}$ f neither
g neither h difference subtracting 3

4 a 23, 28, 33 b 44, 54, 64
c 14, 11, 8 d 114, 116, 118
e 27, 18, 9 f 0, −1, −2
g 505, 606, 707 h −7, −13, −19

5 a 32, 64, 128 b 80, 160, 320
c 12, 6, 3 d 45, 15, 5
e 176, 352, 704 f 70 000, 700 000, 7 000 000
g 16, 8, 4 h 76, 38, 19

6 a 50, 32, 26 b 25, 45, 55
c 32, 64, 256 d 9, 15, 21
e 55, 44, 33 f 333, 111
g 70, 98, 154 h 126, 378, 3402

7 a 17, 23, 30 b 16, 22, 29
c 36, 49, 64 d 17, 12, 6
e 17, 19, 23 f 47, 95, 191
g 5, 7, 6 h 32, 40, 38

8 a 49, 64, 81; square numbers
b 21, 34, 55; Fibonacci
c 216, 343, 512; cubes (i.e. powers of 3)
d 19, 23, 29; primes
e 16, 18, 20; composite
f 161, 171, 181; palindromes

9 a 115, 121, 128 b 24, 48, 16
c 42, 41, 123 d 9, 6, 13

10 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (total = 55)

11 a 39 b 57 c 110 d 192

12 1, 0, 0, 1, 2

13 difference is 0, ratio is 1, Jemima

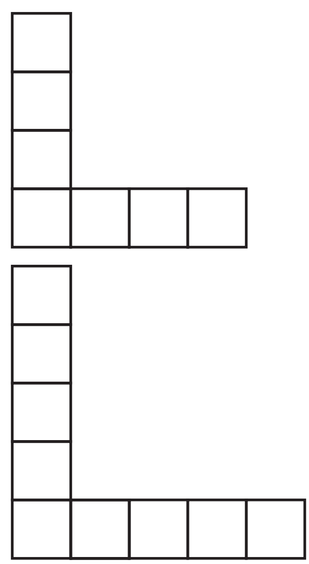
14 a 55 b 100 c 2485 d 258

15 a 3 b 10 c 45 d 276
e $n \times (n - 1) \div 2$

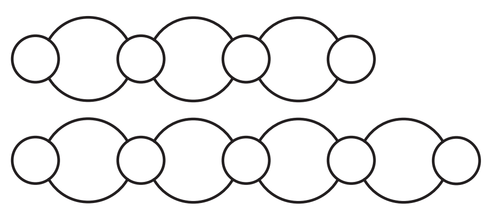
16 a 135 b 624 c 945

Exercise 8J

1 a



b



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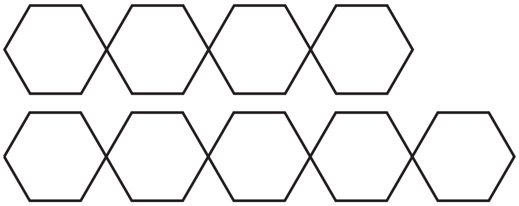
c


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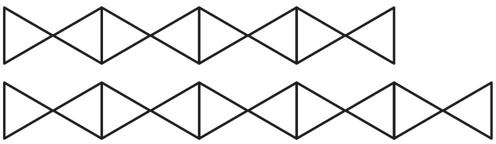
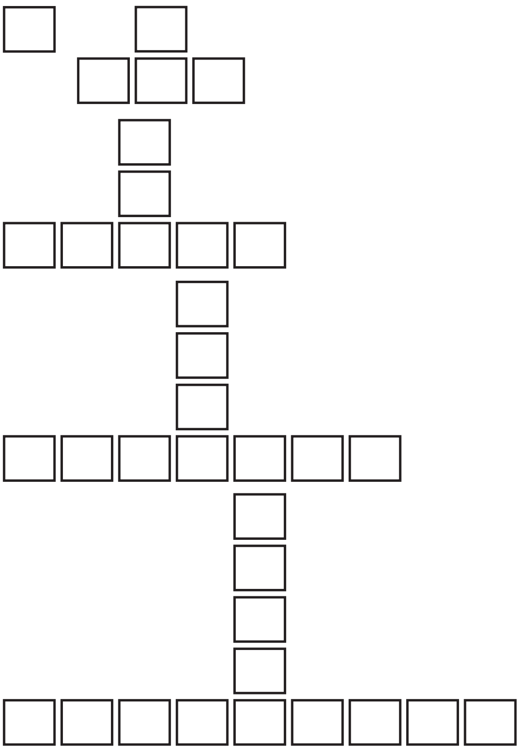
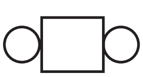
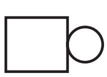


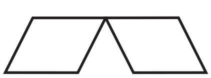
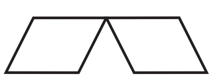
e

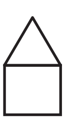

2


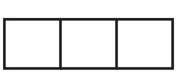
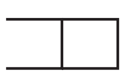
3 a  and 

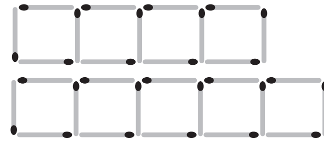
b  and 

c  and 

d  and 

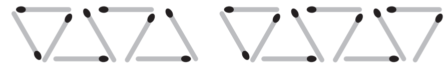
e  and 

f  and 

4 a i


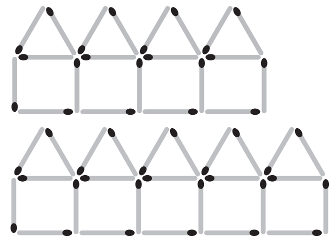
ii 4, 7, 10, 13, 16

iii 4 matchsticks are required to start, then 3 are added

b i


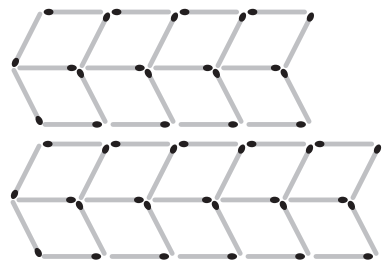
ii 3, 5, 7, 9, 11

iii 3 matchsticks are required at the start, then 2 are added

c i


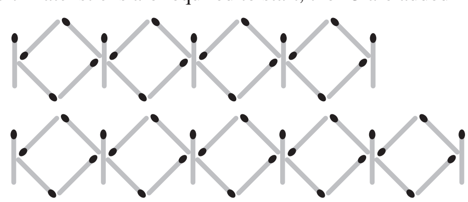
ii 6, 11, 16, 21, 26

iii 6 matchsticks are required to start, then 5 are added

d i


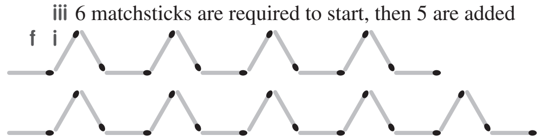
ii 7, 12, 17, 22, 27

iii 7 matchsticks are required to start, then 5 are added

e i


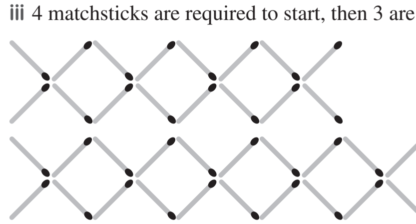
ii 6, 11, 16, 21, 26

iii 6 matchsticks are required to start, then 5 are added

f i


ii 4, 7, 10, 13, 16

iii 4 matchsticks are required to start, then 3 are added

5 a


b

No. of crosses	1	2	3	4	5
No. of matchsticks required	4	8	12	16	20

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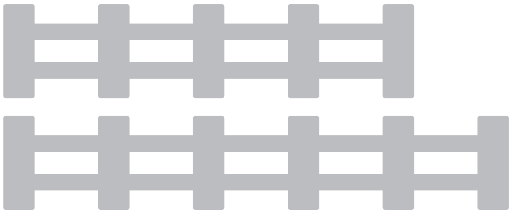
c Number of matchsticks = $4 \times$ number of crosses

d 80 sticks

6 a



b

No. of fence sections	1	2	3	4	5
No. of planks required	4	7	10	13	16

c Number of planks = $3 \times$ number of fence sections + 1

d 61 planks

7 a

No. of tables	1	2	3	4	5
No. of students	5	8	11	14	17

b Number of students = $3 \times$ number of tables + 2

c 23 students **d** 21 tables

8 a

Spa length	1	2	3	4	5	6
No. of tiles	8	10	12	14	16	18

b Number of tiles = $2 \times$ spa length + 6

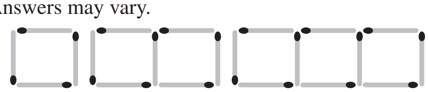
c 36 tiles **d** 12 units

9 Check with your teacher.

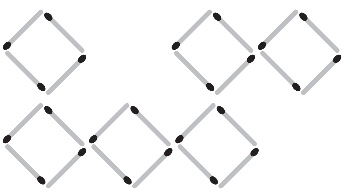
10 A **11** A

12 Answers may vary.

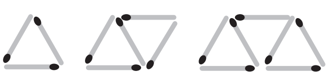
a



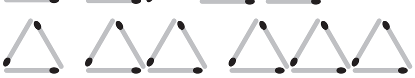
b



c

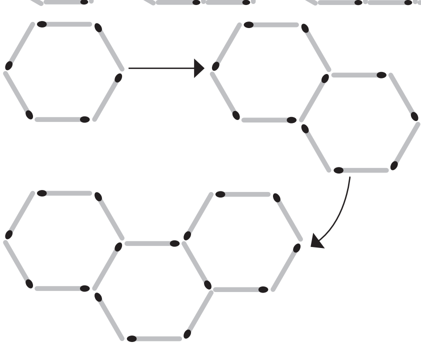


d



e

f



13 a 5 **b** 41 **c** $4g + 1$ **d** 16

14 a $m + n$

b number of pieces for each new panel

c

15 b

No. of straight cuts	1	2	3	4	5	6	7
No. of sections created	2	4	7	11	16	22	29

c 56 **d** 211

Exercise 8K

1 a true **b** false **c** true **d** true

2 C

3 B

4 a A **b** D **c** B **d** C

5 a

input	4	5	6	7	10
output	7	8	9	10	13

b

input	5	1	3	21	0
output	10	2	6	42	0

c

input	11	18	9	44	100
output	3	10	1	36	92

d

input	5	15	55	0	100
output	1	3	11	0	20

6 a

input	1	2	3	4	5
output	7	17	27	37	47

b

input	6	8	10	12	14
output	7	8	9	10	11

c

input	5	12	2	9	0
output	16	37	7	28	1

d

input	3	10	11	7	50
output	2	16	18	10	96

7 a $output = input + 1$ **b** $output = 4 \times input$

c $output = input + 11$ **d** $output = input \div 6$

8 $output = 3 \times input$

9 $output = input - 9$

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10 a

input	3	6	8	12	2
output	7	34	62	142	2

b

input	6	12	1	3	8
output	5	3	25	9	4

c

input	5	12	2	9	0
output	30	156	6	90	0

d

input	3	10	11	7	50
output	15	190	231	91	4950

11 a

input	c	d	$2p$	b^2	www
output	$c+6$	$d+6$	$2p+6$	b^2+6	$www+6$

b

input	t	k	p^2	$2f$	ab
output	$3t-2$	$3k-2$	$3p^2-2$	$6f-2$	$3ab-2$

12

input	b	d	e	g^2	mn	x	c	0	1
output	bc	cd	ec	g^2c	cmn	xc	c^2	0	c

$ouput = c \times input$

13 a $output = 2 \times input + 1; output = 3 \times input - 2$

b infinite

14 a i $output = 2 \times input - 3$

ii $output = 4 \times input + 1$

iii $output = 5 \times input - 1$

iv $output = input \div 6 + 2$

v $output = 10 \times input + 3$

vi $output = 4 \times input - 4$

b–c Answers will vary.

Exercise 8L

1 (3, 0)

2 (0, 5)

3 C

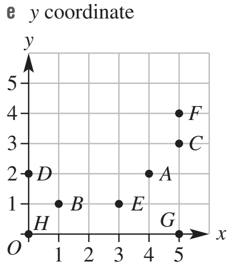
4 C

5 a x-axis **b** y-axis

c origin **d** first

e y coordinate **f** x, y, x, y

6



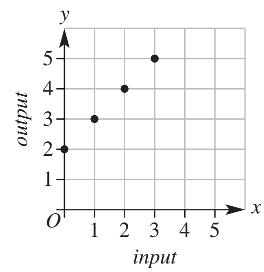
7 a $A(1, 4), B(2, 1), C(5, 3), D(2, 6), E(4, 0), F(6, 5), G(0, 3), H(4, 4)$

b $M(1, 2), N(3, 2), P(5, 1), Q(2, 5), R(2, 0), S(6, 6), T(0, 6), U(5, 4)$

8 a

input (x)	*output* (y)
0	2
1	3
2	4
3	5

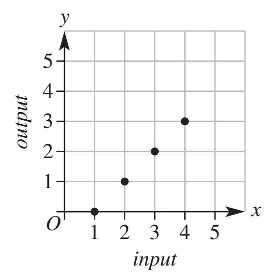
b



9 a

input (x)	*output* (y)
1	0
2	1
3	2
4	3

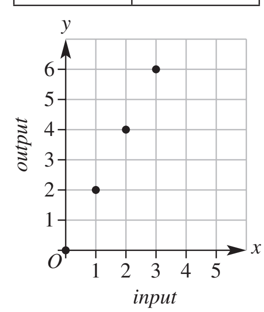
b



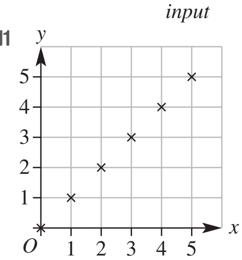
10 a

input (x)	*output* (y)
0	0
1	2
2	4
3	6

b



11



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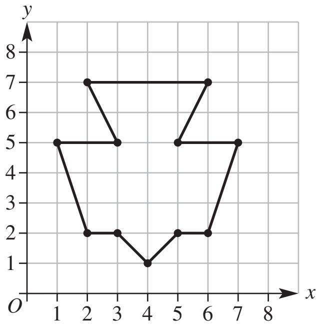
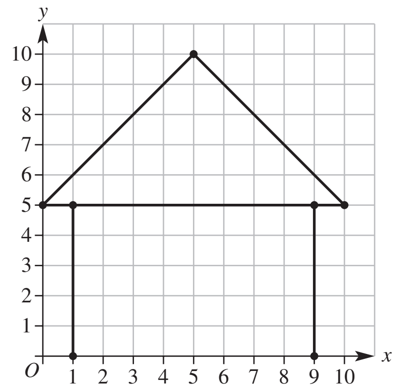
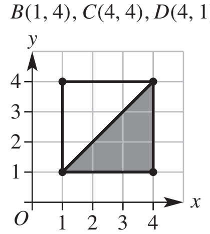
12 

13 a 

b (4, 0), (4, 3), (6, 3), (6, 0), but answers may vary.

c Answers may vary. d Answers may vary.

14 a $B(1, 4)$, $C(4, 4)$, $D(4, 1)$

b 

15 a HELP b (4, 4), (5, 1), (3, 1), (3, 4), (5, 1), (5, 4)

c key under pot plant

d 21510032513451001154004451255143

16 a $D(4, 5)$ b $D(4, 1)$ c $D(0, 0)$ d $D(1, 5)$

17 a $output = input + 2$ b $output = input \times 3$

c $output = input \div 2$

18 a (3, 6) or any with $x = 3$

b any point on a vertical line with $x = 3$

c Answers may vary. If you are given the base of an isosceles triangle, you can have an infinite number of possible triangles simply by varying the height of the triangle.

d (3, 5)

19 a $M(3, 2)$ b (1, 2)

c Find the average of the x values, then the average of the y values.

d (4, 2) e (2.5, 3.5) f (−0.5, −0.5) g (5, 3)

Puzzles and challenges

1 a $a = 4, b = 12, c = 16, d = 8, e = 36$

b $a = 6, b = 3, c = 5, d = 10, e = 15$

2

x	2	2	3	0	5
y	7	6	3	12	1
$3x$	6	6	9	0	15
$x + 2y$	16	14	9	24	7
xy	14	12	9	0	5

3 5050

4 $a = 1, c = 4, d = 2, e = 1, f = 3, g = 4, h = 1, i = 3, j = 4, k = 2$

5 a $A = 4, B = 9, C = 2$

b $D = 8, E = 6, F = 2, G = 5$

6 Because $2(x + 5) - 12 - x + 2$ simplifies to x.

Multiple-choice questions

1 B 2 A 3 C 4 D 5 D
6 C 7 A 8 E 9 A 10 A

Short-answer questions

1 a $5a, 3b, 7c, 12$ b 12

2 a $u + 7$ b $3k$ c $7 + \frac{r}{2}$

d $h - 10$ e xy f $12 - x$

3 a 15 b 24 c 2 d 32

4 a −15 b −2 c −16 d −3

5 a 16 b 200 c 5 d 20

6 a E b N c E d N

7 a L b N c L d L

e N f L g N h L

8 a $7x + 3$ b $11p$ c $7a + 14b + 4$

d $3m + 17mn + 2n$ e $1 + 7c + 4h - 3o$

f $4u + 3v + 2uv$

9 a $12ab$ b $6xyz$ c $36fgh$ d $64klm$

10 a $\frac{3}{2}$ b $\frac{3}{5}$ c $\frac{a}{3}$ d $\frac{4x}{3z}$

11 a $3x + 6$ b $4p - 12$ c $14a + 21$ d $24k + 36l$

12 $2(6b + 9c)$, $6(2b + 3c)$; other answers possible.

13 $9t$ kg

14 $g + b$

15 $3x$

Extended-response questions

1 a i \$24.50 ii \$45.50 iii \$213.50

b $3.5 + 2.1k$ c \$87.50

d If $k = 40$, $2.1 + 3.5k = \$142.10$, not \$87.50

e $6 + 1.2k$

2 a 26 b \$78 c $4x + 2y + 10$

d $12x + 6y + 30$

e $x(x + 5) + 3y$; other answers possible.

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Chapter 9

Pre-test

1 a 7 b 20 c 16 d 5
2 a false b true c true d false
3 a 7 b 5 c 15 d 24
4 a 3 b 27 c 2 d 6
5 a $k + 5$ b $2p$ c $7y$ d $\frac{q}{2}$
6 a $5x$ b $28b$ c $5a + 7b$ d $7 + 10a$
7 a $a = 5, b = 15, c = 4, d = 7, e = 60$
b $a = 0, b = 2, c = 12, d = 10, e = 41$
c $a = 2, b = 5, c = 7, d = 1, e = 13$
d $a = 3, b = 2, c = 1, d = 0, e = 4$
8 a $\div$ b $-$ c $\times$ d $+$

Exercise 9A

1 a E b N c E d E e E
f N g E h E i N
2 a true b false c false
3 a 19 b 19 c true
4 true
5 a true b false c true d true
e false f false g true h false
i true j true k false l true
m false n false o true
6 a false b true c false d true
7 a true b true c false d true
8 a true b false c true d true
e true f false g false h true
i true
9 a $3 + x = 10$ b $5k = 1005$ c $a + b = 22$
d $2d = 78$ e $8x = 56$ f $3p = 21$
g $\frac{t}{4} = 12$ h $q + p = q \times p$
10 a $6c = 546$ b $5x = 37.5$ c $12a + 3b = 28$
d $f + 10 = 27$ e $j + 10 + m + 10 = 80$
11 $m = 3$
12 $k = 2, k = 6$
13 $x = 1$ and $y = 5$, $x = 2$ and $y = 4$, $x = 3$ and $y = 3$, $x = 4$ and $y = 2$, $x = 5$ and $y = 1$
14 a S b S c A d A
e N f N g A h S
i A j S k N l N
15 a $6 = 2 \times 3$; other solutions possible.
b $5 - 4 = 1$; other solutions possible.
c $10 \div 2 = 7 - 2$; other solutions possible.
d $4 - 2 = 10 \div 5$; other solutions possible.
16 a 12 b 4
c Yes; if 5 is changed to 1, then $1 + 2 = 3$ is a true equation.
d Yes; if + is changed to −, then $5 - 3 = 2$ is a true equation.

Exercise 9B

1 a true b false c true d false
2 a 12 b 17 c 13 d 6
3 a 3 b 6 c 10 d 4
e 6 f 70 g 20 h 19
4 a x b c c b d d
5 a $y = 8$ b $l = 2$ c $l = 6$ d $d = 2$
e $l = 12$ f $a = 6$ g $s = 12$ h $x = 8$
i $e = 8$ j $r = 10$ k $s = 8$ l $z = 3$
6 a $p = 3$ b $p = 4$ c $q = 3$ d $v = 5$
e $b = 1$ f $u = 4$ g $g = 3$ h $e = 3$
i $d = 4$ j $d = 6$ k $m = 4$ l $o = 3$
7 a $x = 3$ b $x = 7$ c $x = 5$
d $x = 4$ e $x = 1$ f $x = 5$
g $x = \pm 2$ h $x = \pm 10$ i $x = \pm 6$
8 a 11 b 12 c 16
d 33 e 30 f 2
9 a $10x = 180$ b $x = 18$
10 a $2w = 70$ b $w = 35$
11 a $4.5x = 13.5$ b $x = 3$
12 a $y + 12 = 3y$ b $y = 6$
13 a $x = 9$ b $2x + 1 = 181$ so $x = 90$
14 a $x = 14$, so 14, 15 and 16 are the numbers.
b LHS is $3x + 3$ or $3(x + 1)$, which will always be a multiple of 3.
15 a $x = 2$ and $y = 6$; other solutions possible.
b $x = 12$ and $y = 10$; other solutions possible.
c $x = 1$ and $y = 1$; other solutions possible.
d $x = 12$ and $y = 0.5$; other solutions possible.
e $x = 10$ and $y = 0$; other solutions possible.
f $x = 2$ and $y = 2$; other solutions possible.

Exercise 9C

1 a C b D c E d B e A
2 a $10d + 15 = 30$ b $7e + 10 = 41$
c $2a + 10 = 22$ d $x + 10 = 22$
3 a $6 + x = 11$ b $6x = 14$ c $3 = 2q$
d $6 + a = 10$ e $12 + b = 15$ f $0 = 3b + 2$
g $4 = 7 + a$ h $12x + 2 = 8$ i $7p = 12$
4 a subtracting 2 b adding 2
c dividing by 10 d multiplying by 2
e dividing by 3 f adding 3
g dividing by 4 h adding 4
5 a $3(x + 4) = 27$ b $18 = 3(a + 5)$
c $3d = 18$ d $3(11 + a) = 36$
e $3(3y + 2) = 33$ f $3(2x + 4) = 30$
6 a E (+2) b A (+4) c D (÷2)
d B (−2) e C (×2)
7 a ×3 then +2 b ×10 then −3
c ÷5 then −2 d −10 then ÷3
8 a $2q = 2$ b $10x = 7$ c $3 + x = 20$ d $x = 60$

9 a $3x + 2 = 14$ → (−2) → $3x = 12$ → (÷3) → $x = 4$ → (×10) → $10x = 40$ → (+1) → $10x + 1 = 41$

b $5x - 3 = 32$ → (+3) → $5x = 35$ → (÷5) → $x = 7$ → (+2) → $x + 2 = 9$

c $(x \div 2) + 4 = 9$ → (−4) → $x \div 2 = 5$ → (×2) → $x = 10$ → (+8) → $x + 8 = 18$ → (÷2) → $(x + 8) \div 2 = 9$

10 a **i** $0 = 0$ **ii** $0 = 0$ **iii** $0 = 0$

b Regardless of original equation, will always result in $0 = 0$.

11 a $x = 3$ → (×7) → $7x = 21$ → (+2) → $7x + 2 = 23$

b $x = 3$ → (×2) → $2x = 6$ → (+8) → $8 + 2x = 14$

12 a True; you can +3 to both sides and then −3 to get the original equation again.

b True; simply perform the opposite operations in the reverse order, so +4 becomes −4.

c True; use the operations that take equation 1 to equation 2 and then the operations that take equation 2 to equation 3.

d false; e.g. equation 1: $x = 4$, equation 2: $x = 5$, equation 3: $2x = 8$.

Exercise 9D

1 a true **b** false **c** true **d** false

2 a 6 **b** $x = 6$

3 $g = 2$

4 a −5 **b** ÷10 **c** ×4 **d** +12

5 a $m = 9$ **b** $g = 11$ **c** $s = 9$ **d** $i = 10$
e $t = 2$ **f** $q = 3$ **g** $y = 12$ **h** $s = 12$
i $j = 4$ **j** $l = 4$ **k** $v = 2$ **l** $y = 12$
m $k = 5$ **n** $y = 9$ **o** $z = 7$ **p** $t = 10$
q $b = 12$ **r** $p = 11$ **s** $a = 8$ **t** $n = 3$

6 a $7a + 3 = 38$ → (−3) → $7a = 35$ → (÷7) → $a = 5$

b $4b - 10 = 14$ → (+10) → $4b = 24$ → (÷4) → $b = 6$

c $2(q + 6) = 20$ → (÷2) → $q + 6 = 10$ → (−6) → $q = 4$

d $5 = \frac{x}{10} + 3$ → (−3) → $2 = \frac{x}{10}$ → (×10) → $20 = x$

7 a subtract 3 **b** add 7
c divide by 5 **d** divide by 2

8 a $f = 11$ **b** $k = 4$ **c** $x = 9$ **d** $a = 9$
e $k = 8$ **f** $a = 6$ **g** $n = 11$ **h** $n = 8$
i $g = 4$ **j** $q = 11$ **k** $z = 10$ **l** $p = 1$
m $d = 4$ **n** $t = 8$ **o** $u = 5$ **p** $c = 1$
q $q = 11$ **r** $y = 12$ **s** $q = 2$ **t** $u = 10$

9 a $x = \frac{3}{4}$ **b** $k = \frac{24}{5}$ **c** $w = \frac{4}{3}$
d $x = \frac{4}{3}$ **e** $x = \frac{5}{8}$ **f** $x = \frac{1}{3}$

10 a $r = -7$ **b** $x = -3$ **c** $t = -16$
d $y = -24$ **e** $x = -5$ **f** $k = -9$
g $x = -6$ **h** $x = -4$ **i** $x = -\frac{3}{2}$

11 a $x + 5 = 12 \rightarrow x = 7$ **b** $2y = 10 \rightarrow y = 5$
c $2b + 6 = 44 \rightarrow b = 19$ **d** $3(k - 7) = 18 \rightarrow k = 13$
e $\frac{b}{4} + 3 = 6 \rightarrow b = 12$ **f** $\frac{k}{2} - 10 = 1 \rightarrow k = 22$

12 a $12n + 50 = 410$ **b** $n = 30$ h

13 a $12 + 5x$ **b** $12 + 5x = 14.5$
c $x = 0.5$, so pens cost 50 cents.
d $12 + 5\,(0.5) = 14.5$

14 a $3b = 15 \rightarrow b = 5$ **b** $4x = 12 \rightarrow x = 3$
c $2(10 + x) = 28 \rightarrow x = 4$ **d** $4b = 28 \rightarrow b = 7$

15 a $x = 6$ **b** $x = 8$ **c** $x = 5$

16 Examples include: $x + 1 = 3$, $7x = 14$, $21 - x = 19$, $\frac{4}{x} = x$, $\frac{x}{2} = 1$.

17 a $2x + 5 = 13$ → (−5) → $2x = 8$ → (÷2) → $x = 4$ → (×5) → $5x = 20$

b $10 + 2x = 20$ → (−10) → $2x = 10$ → (÷2) → $x = 5$ → (−3) → $x - 3 = 2$ → (×2) → $2(x - 3) = 4$

c yes **d** yes

18 a First step, $4x + 2$ is not completely divided by 4.
b Second step, LHS divided by 3, RHS has 3 subtracted.
c First step, RHS has 5 added not subtracted.
d First step, LHS has $11a$ subtracted, not 12.

19 a $x = 4$ **b** $x = 1$ **c** $l = 3$ **d** $t = 1$
e $s = 5$ **f** $b = 19$ **g** $j = 2$ **h** $d = 1$

Exercise 9E

1 a true **b** false **c** true
d false **e** true **f** false

2 a $\frac{x}{3} + 1 = 5$ **b** $\frac{x + 1}{3} = 5$ **c** $x - \frac{1}{5} = 6$
d $\frac{2x}{3} = 7$ **e** $\frac{2x}{3} - 1 = 7$ **f** $\frac{2x - 5}{4} = 2$

3 a $b = 44$ **b** $d = 15$ **c** $h = 28$ **d** $p = 26$

4 a B **b** C **c** A **d** D

5 a $m = 12$ **b** $c = 18$ **c** $s = 16$ **d** $r = 10$
e $u = 20$ **f** $y = 18$ **g** $x = 4$ **h** $a = 16$
i $h = 10$ **j** $j = 15$ **k** $v = 9$ **l** $q = 8$

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6 a $h = 9$ b $y = 6$ c $j = 3$
d $b = 4$ e $u = 3$ f $t = 9$
g $w = 6$ h $r = 4$ i $q = 9$
j $s = 3$ k $l = 8$ l $z = 7$
m $v = 11$ n $f = 9$ o $x = 2$
p $d = 5$ q $n = 5$ r $m = 11$
s $p = 8$ t $a = 9$

7 a $y = -1$ b $a = -10$ c $x = -10$
d $x = -48$ e $u = -30$ f $y = -10$
g $u = -4$ h $d = -5$

8 a $\frac{t}{2} = 9 \rightarrow t = 18$ b $\frac{q}{3} = 14 \rightarrow q = 42$
c $\frac{2r}{5} = 6 \rightarrow r = 15$ d $\frac{q-4}{2} = 3 \rightarrow q = 10$
e $\frac{x+3}{4} = 2 \rightarrow x = 5$ f $\frac{y}{4} + 3 = 5 \rightarrow y = 8$

9 a $\frac{b}{5} = 31.50$ b $b = 157.5$
c $157.50

10 a $\frac{x}{2} + 5$ b $\frac{x}{2} + 5 = 11 \rightarrow x = \$12$
c $6

11 a $6x = 3$; other solutions possible.
b $7x = 5$ c yes; e.g. $2x + 1 = 0$

12 a The different order in which 3 is added and the result is multiplied by 5.
b multiply by 5
c subtract 3
d No, the difference between them is always 2.4 for any value of x.

13 a i multiply by 2 ii divide by $\frac{1}{2}$
b $x = 26$ for both of them.
c Makes the first step a division (by a fraction) rather than multiplication.

14 a $x = \frac{7}{2}$ b $x = \frac{17}{6}$
c $x = \frac{19}{6}$ d $x = \frac{7}{6}$

15 Answers will vary. Substitute $x = 12$ into your equation to ensure that it is a valid solution.

Exercise 9F

1 B
2 A
3 C

4 a $2x + 2$ b $10b + 15$ c $6a - 8$
d $35a + 5$ e $12x + 16$ f $24 - 9y$
g $48a + 36$ h $2u - 8$

5 a $4a + 2$ b $5 + 3x$ c $3b - 4$
d $3a + 12$ e $6x + 3$ f $k + 6$
g $2b + 6$ h $5k + 1$

6 a $s = 6$ b $l = 1$ c $p = 9$
d $y = 0$ e $q = 1$ f $p = 12$
g $m = 5$ h $b = 6$ i $p = 3$
j $p = 7$ k $y = 9$ l $r = 8$

7 a $d = 3$ b $x = 2$ c $x = 5$
d $e = 1$ e $a = 1$ f $r = 3$
g $u = 1$ h $q = 11$

8 a $s = 1$ b $i = 1$ c $c = 5$
d $v = 8$ e $k = 1$ f $q = 3$
g $y = 4$ h $f = 3$ i $t = 2$

9 a $u = -5$ b $k = -3$ c $p = -1$
d $q = -2$ e $u = -1$ f $x = -3$
g $p = -4$ h $r = -10$

10 a i $2(5 + x) = 14$ ii $x = 2$
b i $3(q - 3) = 30$ ii $q = 13$
c i $2(2x + 3) = 46$ ii $x = 10$
d i $2(y + 4) - y = 17$ ii $y = 9$

11 a LHS simplifies to 10, but $10 = 7$ is never true.
b LHS simplifies to 15, not 4.
c LHS simplifies to 6, not 12.

12 a LHS = 9, RHS = 9, therefore true.
b LHS = 9, RHS = 9, therefore true.
c LHS simplifies to 9.
d For example, $2(x + 5) - 3 - 2x = 7$. Others possible.

13 a $s = 6$ b $l = 1$ c $p = 9$
d $y = 0$ e $q = 1$ f $p = 12$
g $m = 5$ h $b = 6$ i $p = 3$
j $p = 7$ k $y = 9$ l $r = 8$

14 a $j = 2$ b $a = 1$ c $a = 3$
d $a = 8$ e $c = 4$ f $d = 8$
g $x = 1$ h $x = 3$ i $x = 0$

Exercise 9G

1 true

2 a 15 b 36 c 8 d 2

3 a $h = 7$ b $h = 9$ c $m = 8$ d $m = 10$

4 a $y = 23$ b $x = 4$ c $x = 7$

5 a $A = 7$ b $q = 4$ c $t = 0$

6 a $G = 27$ b $x = 1$ c $y = 5$

7 a $20 = b \times 4 \rightarrow b = 5$
b i $25 = 5h \rightarrow h = 5$ ii square

8 a $P = 16$ b $h = 3$
c 24 units squared

9 a $F = 68$ b $C = 10$
c 12°C d 15°C

10 a $d = \frac{S - 3b}{5}$ b $C = \frac{5(F - 32)}{9}$
c $x = \frac{Q - 36}{4}$

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11 a Check by substituting values back into equation.
b If $D = 20$, C should equal 60 not 50, as in row 2.
c Check by substituting values back into equation.
d For example, $C = 2t - 10$, $C = \frac{Dt}{20} + 20$; other solutions possible.

12 a Abbotsford Apes b 8 goals
c $S = 9q + 6g + b$
d 0 goals and 0 behinds, 2 goals and 12 behinds, 3 goals and 9 behinds, 7 goals and 7 behinds.

Exercise 9H

1 a $7k = 42$ b $k = 6$
2 a $x + 19 = 103$ b $x = 84$
3 a Let c = car's cost b $c + 2000 = 40\,000$
c $c = 38\,000$ d $38000
4 a Let p = cost of one pen b $12p = 15.6$
c $p = 1.3$ d $1.30
5 a Let h = number of hours worked
b $17h + 65 = 643$
c $h = 34$ d 34 hours
6 a $24l = 720$ b $l = 30$
c 30 m d 108 m
7 a $2x + 10 = 60,\ y + 60 = 180$
b $3y = 18,\ 2x = 68$
c $4x - 12 = 28,\ y + 7 = 10$
8 $2(2x + 3) = 34 \rightarrow x = 7$
9 $x = 4$
10 1.5 h
11 14 years old
12 $2x + 154 = 180 \rightarrow x = 13$
13 $3y = 90 \rightarrow y = 30$
14 Examples include: $h = 3$, $b = 6$ or $h = 12$, $b = 2.4$; other solutions possible.
15 $x = 3.5$, $y = 2$
16 a possible ($p = 12$)
b Not possible because solution is not a whole number.
c possible ($p = 8$)
17 $a + 2b = 180$, so $a = 180 - 2b = 2(90 - b)$ is always even.
18 a i $x = 60$
ii One angle is −10°, which is impossible.
b i $60 - x + 70 + x + 50$ is always 180, regardless of the value of x.
ii any value less than 60 and greater than −70
c Answers will vary.

Puzzles and challenges

1 a 26 b 9 c 368
d $31\frac{1}{3}$ e 36
2 7 and 13
3 $a = 5$, $b = 2$, $c = 12$
4 a $a = 22.5$, $b = 37.5$ b $a = 10$, $b = 23$
c $a = 36$, $b = 108$ d $a = 60$, $b = 40$
5 A corny joke.
6 26 sheep, 15 ducks

Multiple-choice questions

1 C 2 A 3 D 4 B
5 E 6 C 7 A 8 A
9 B 10 E

Short-answer questions

1 a false b true c true
d false e true f false
2 a $2 + u = 22$ b $5k = 41$
c $3z = 36$ d $a + b = 15$
3 a $x = 3$ b $x = 6$ c $y = 1$
d $y = 9$ e $a = 2$ f $a = 10$
4 a $2x = 8$ b $7a = 28$ c $15 = 3r$ d $16 = 8p$
5 a $x = 3$ b $r = 45$ c $p = 9$ d $b = 6$
e $x = 9$ f $r = 4$ g $q = 2$ h $u = 8$
6 a $u = 8$ b $p = 3$ c $x = 4$
d $y = 8$ e $y = 4$ f $x = 15$
7 a $6 + 4p$ b $12x + 48$ c $7a + 35$ d $18x + 9$
8 a $x = 8$ b $x = 8$ c $x = 7$
d $y = 10$ e $z = 5$ f $q = 3$
9 a no
b Solution is $x = \frac{7}{11}$, which is not whole.
10 a No; LHS simplifies to 10.
b i Is a solution. ii Is a solution.
11 a $x = 3$ b $x = 13$
c $x = 8$ d $x = 4$
12 $3(2x - 5) = 21$, for example
13 a 5 b 6 c 20 years d 6 cm
14 a $y = 35$ b $y = 30$

Extended-response questions

1 a 75 cents b $C = 15 + 2t$
c $1.75 d 12 seconds
e 81 seconds
f 2.5 min in total (50 seconds for the first call, then 100 seconds)
2 a $500
b 30 hours at \$$x$/hour, and 10 hours at \$$(x + 2)$/hour.
c $660
d $x = 15$
e $x = 21$, so Gemma earned $630 from Monday to Friday.

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Chapter 10

Pre-test

1 a 40 mm b 60 mm c 8 mm
2 a mm, cm, m, km b mg, g, kg, t
c s, min, h, day, year
3 a 10 b 100 c 1000
d 60 e 60 f 1000
4 a 2000 b 2 c 56 d 2500
e 180 f 7200 g 1.4 h 0.027
5 a 55 m b 27 cm c 28 cm
6 a 32 b 7 c 8
7 a kilometres b kilograms
c minutes d centimetres

Exercise 10A

1 a i 1 foot = 12 inches = 16 digits = 4 palms
ii 1 mile = 1000 paces
b i 1 foot = 12 inches
ii 3 feet = 1 yard
iii 1 mile = 1760 yards
c i 1 m = 100 cm ii 1 cm = 10 mm iii 1 km = 1000 m
2 digit, inch, palm, foot, cubit, pace, mile
3 inch, foot, yard, rod, chain, furlong, mile
4 millimetre, centimetre, metre, kilometre
5 a 5 b 5000 c 4
d 20 e 16 f 80
6 a 12 b 3 c 36
d 1760 e 22 f 40
7 a 10 b 100 c 1000
d 1000 e 100 000 f 1 000 000
8 a kilometres b millimetres
c metres d metres
e centimetres f kilometres
9 a metres b millimetres
c kilometres d kilometres
e metres f centimetres
10 1 pace
11 5000
12 440
13 a 2 mm b 5 mm c 2 cm
d 5 cm e 8 cm
14 a 6 cm b 5 cm c 25 cm
15 a

	mm	cm	m	km
mm	1	$\frac{1}{10}$	$\frac{1}{1000}$	$\frac{1}{1000000}$
cm	10	1		$\frac{1}{100000}$
m	1000	100	1	$\frac{1}{1000}$
km	1 000 000	100 000	1000	1

b

	inch	feet	yard	mile
inch	1	$\frac{1}{12}$	$\frac{1}{36}$	$\frac{1}{63360}$
feet	12	1	$\frac{1}{3}$	$\frac{1}{5280}$
yard	36	3	1	$\frac{1}{1760}$
mile	63 360	5280	1760	1

c

	digit	palm	feet	pace	mile
digit	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{80}$	$\frac{1}{80000}$
palm	4	1	$\frac{1}{4}$	$\frac{1}{20}$	$\frac{1}{20000}$
feet	16	4	1	$\frac{1}{5}$	$\frac{1}{5000}$
pace	80	20	5	1	$\frac{1}{1000}$
mile	80 000	20 000	5000	1000	1

16 4 rods = 1 chain, but conversion to other units is less simple.
17 All conversions involve a power of 10.
18 Answers may vary.

Exercise 10B

1 a 100 b 1000 c divide
d multiply e multiply f divide
2 a 1000 b 1000 c 100 000 d 1 000 000
3 a right b left
4 a 50 mm b 200 cm c 3500 m d 2610 cm
e 4 cm f 5 m g 4.2 km h 47.2 cm
i 684 cm j 20 m k 926.1 cm l 4.23 km
5 a 2.5 cm b 82 mm c 2.5 m
d 730 cm e 6200 m f 25.732 km
6 a 3000 mm b 600 000 cm c 2400 mm
d 4000 cm e 0.47 km f 913 m
g 0.216 km h 0.0005 m
7 a 2 cm b 5 cm c 1.5 cm d 3.2 cm
e 3 cm f 3 cm g 1.2 cm h 2.8 cm
8 a 2.7 m b 0.4 km
9 a 8.5 km b 310 cm c 19 cm
10 a 38 cm, 0.5 m, 540 mm
b 160 cm, 2100 mm, 0.02 km, 25 m
c 142 mm, 20 cm, 0.003 km, 3.1 m
d 10 mm, 0.1 m, 0.001 km, 1000 cm
11 125 cm
12 Bigan tower
13 a $8200 b $6.56 c 41c
14 625 years

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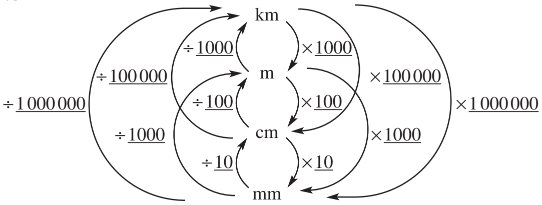
15 50 000 years

16 0.08 mm

17 2500 s

18



19 So that only one unit is used and mm deliver a high degree of accuracy.

20 a i 1 million ii 10 000 iii 1000
iv 1 billion (1 000 000 000)
b 0.312 μm
c The distance you travel in 1 year at light speed

Exercise 10C

1 a 10 cm **b** 12 cm

2 a 6 cm **b** 12 cm **c** 5.2 cm **d** 6.4 cm

3 a 15 cm **b** 37 m **c** 30 km
d 2.4 m **e** 26 cm **f** 10 cm

4 a 42 cm **b** 34 m **c** 36 km

5 a 8.4 cm **b** 14 m **c** 46.5 mm

6 $21 400

7 a 516 ft **b** 157.38 m

8 a 70 mm **b** 72 cm

9 a 40.7 cm **b** 130.2 cm **c** 294 cm

10 400 m

11 a 5 cm **b** 5 m **c** 7 km

12 4, including a square

13 a $P = a + 2b$ **b** $P = 4a$ **c** $P = 2a + 2b$
d $P = 4a$ **e** $P = 2a + 2b$ **f** $P = 2a + b + c$

14 a $P = 2a + 2b + 2c$ or $2(a + b + c)$
b $P = 2a + 2b + 2c$ or $2(a + b + c)$

15 a $\frac{p}{4}$ **b** $\frac{(p - 2a)}{2}$

16 a 160 cm **b** 216 cm **c** 40 cm
d $4a + 32$ or $4(a + 8)$ cm

Exercise 10D

1 a 15.71 **b** 40.84 **c** 18.85 **d** 232.48

2 a 3.1 **b** 3.14 **c** 3.142

3 a diameter **b** radius **c** circumference
d segment **e** chord **f** sector
g tangent

4 Answer is close to pi.

5 a false **b** true **c** false **d** true
e true **f** false **g** false

6 a 12.57 mm **b** 113.10 m **c** 245.04 cm **d** 12.57 m
e 21.99 km **f** 15.71 cm

7 a 300 cm **b** 60 m **c** 18 km

8 a 44 mm **b** 132 cm **c** 220 m

9 11.0 m

10 12 566 m

11 Svenya and Andre

12 B

13 3 cm

14 a i 2 m ii 5 cm iii 15 km
b i 2 mm ii 3 m iii 0.5 cm

15 $d = 2r$, so $2\pi r$ is the same as πd.

16 a 4π mm **b** 36π m **c** 78π cm **d** 4π m
e 7π km **f** 5π cm

17 a i $r = \frac{C}{2\pi}$ ii $D = \frac{C}{\pi}$
b i 2.23 m ii 6.37 cm

Exercise 10E

1 a $\frac{1}{4}$, quadrant **b** $\frac{1}{2}$, semicircle

2 a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{1}{6}$
d $\frac{1}{3}$ **e** $\frac{3}{4}$ **f** $\frac{5}{8}$

3 a square, semicircle **b** quadrant, rectangle
c sector, triangle

4 a 8.38 cm **b** 5.59 m **c** 12.22 mm
d 1.96 m **e** 136.14 cm **f** 1.15 km

5 a 14.3 cm **b** 35.7 m **c** 51.4 mm
d 36.0 km **e** 13.1 m **f** 14.0 cm

6 a 18.7 m **b** 11.1 cm **c** 101.1 cm
d 35.4 km **e** 67.1 mm **f** 45.1 cm

7 657 cm

8 a 26.88 m **b** 52.36 m **c** 6.24 m

9 a 25.13 cm **b** 56.55 m **c** 35.71 m

10 The four arcs make one full circle ($2\pi r$) and the four radii make $4r$.

11 The perimeter includes one large semicircle $\left(\frac{1}{2} \times 2\pi r = \pi r\right)$ and the two smaller semicircles, which make one full smaller circle ($\pi \times r$). So, the total is $2\pi r$.

12 a 7.6 m **b** 34.4 m **c** 9.5 km

13 a $24 + 3\pi$ cm **b** $9 + 2.5\pi$ cm

14 a 8π cm **b** 18π m **c** $5\pi + 20$ cm

Exercise 10F

1 a 8 **b** 4 cm and 2 cm **c** 8 cm^2

2 a 9 **b** 3 cm and 3 cm **c** 9 cm^2

3 a 5 square units **b** 8 square units
c 96 square units

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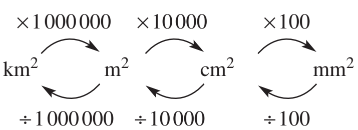
4 a cm^2 b m^2 c ha
d km^2 e ha f mm^2
5 a 6 cm^2 b 3 cm^2 c 2 cm^2
d 5 cm^2 e 4.5 cm^2 f 9 cm^2
6 a 200 cm^2 b 22 mm^2 c 7 cm^2 d 25 m^2
e 1.44 mm^2 f 6.25 mm^2 g 1.36 m^2 h 0.81 cm^2
i 179.52 km^2
7 a 2 cm b 5 m c 12 km
8 a 2 ha b 10 ha c 0.5 ha
9 5000 m^2
10 2500 cm^2
11 20 000 cm^2
12 a 25 cm^2 b 12 cm c 4 units
13 $2100
14 5 L
15 8 ha
16 a i 10 cm ii 9 mm
b Divide the area by the given length.
17 half of a rectangle with area 4 cm^2
18 a 121 cm^2 b $\left(\frac{P}{4}\right)^2$
19 Area is quadrupled (× 4).
20 a i 100 ii 10 000 iii 1 000 000
b


c i 200 mm^2 ii 100 000 cm^2 iii 3 500 000 m^2
iv 3 cm^2 v 2.16 m^2 vi 4.2 km^2
vii 5000 mm^2 viii 100 ha ix 0.4 ha

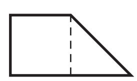
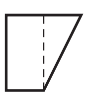
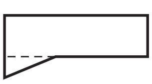
Exercise 10G

1 a 20 cm b 7 m c 2 m d 6.3 cm
2 a 6 m b 11 mm c 1.9 m d 3.2 mm
3 a 10 b 56 c 12.5
4 a 16 m^2 b 30 cm^2 c 160 m^2 d 1.2 m^2
e 3 mm^2 f 24.5 km^2 g 1.3 cm^2 h 20 m^2
i 4.25 m^2
5 a 2 cm^2 b 6 cm^2 c 3 cm^2 d 3 cm^2
6 480 m^2
7 4800 m^2
8 6.8 cm^2
9 160 m^2
10 $6300
11 a 5 cm b 4.4 mm
12 Yes, the base and height for each triangle are equal.
13 No, the base and height are always the same.
14 $b = \frac{2A}{h}$
15 a 7 cm^2 b 8 cm^2 c 11 cm^2 d 11 cm^2

Exercise 10H

1 a $A = bh$
$= 5 \times 7$
$= 35$
b $A = bh$
$= 20 \times 3$
$= 60$
c $A = bh$
$= 8 \times 2.5$
$= 20$
2 a $b = 6$ cm, $h = 2$ cm
b $b = 10$ m, $h = 4$ m
c $b = 5$ m, $h = 7$ m
d $b = 5.8$ cm, $h = 6.1$ cm
e $b = 5$ cm, $h = 1.5$ cm
f $b = 1.8$ m, $h = 0.9$ m
3 a 40 m^2 b 28 m^2 c 36 km^2 d 17.5 m^2
e 30 m^2 f 6.3 cm^2 g 1.8 cm^2 h 14 cm^2
i 176 mm^2
4 a 6 cm^2 b 4 cm^2 c 15 cm^2 d 8 cm^2
5 54 m^2
6 a 2 m b 7 cm c 0.5 mm
7 a 10 cm b 5 m c 2 km
8 $1200
9 a 1800 cm^2 b 4200 cm^2
10 Because height must be less than 5 cm.
11 half; Area (parallelogram) = bh and Area (triangle) = $\frac{1}{2}bh$
12 Area = twice triangle area
$= 2 \times \frac{1}{2}bh$
$= bh$
13 $4 500 000

Exercise 10I

1 a

b

c

2 a addition b subtraction c subtraction
3 a $A = s^2 + lb$
$= 1^2 + 3 \times 1$
$= 1 + 3$
$= 4\ cm^2$
b $A = lb - \frac{1}{2}bh$
$= 13 \times 8 - \frac{1}{2} \times 5 \times 4$
$= 104 - 10$
$= 94\ m^2$
4 a 33 m^2 b 600 mm^2 c 25 m^2
d 21 m^2 e 171 cm^2 f 45 km^2
5 a 39 m^2 b 95.5 cm^2 c 26 m^2
d 78.5 m^2 e 260 cm^2 f 4 m^2
6 a 62 cm^2 b 16 m^2 c 252 cm^2
7 a 80 cm^2 b 7 m^2 c 14.5 m^2 d 11.75 cm^2
8 10.08 m^2
9 yes, with $100 to spare
10 Subtraction may involve only two simple shapes.

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11 a No; bases could vary depending on the position of the top side.
b Yes, 40 m^2; take out the rectangle and join the triangles to give a base of 10 − 6 = 4.

12 a See given diagram in textbook.
b i $\frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}, \frac{1}{512}, \frac{1}{1024}, \frac{1}{2048}, \frac{1}{4096}, \frac{1}{8192}$
ii no
c 1 square unit d Total must equal 1.

Exercise 10J

1 a 1 kg, 1000 g b 1000 mg, 1 g
2 a C b F c A
d D e B f E
3 a B b A c D d C
4 a 2000 kg b 70 000 g
c 2400 mg d 2.3 g
e 4.620 g f 21.6 t
g 470 kg h 0.312 kg
i 0.027 g j 750 kg
k 125 g l 0.0105 kg
m 210 t n 470 kg
o 592 g p 80 g
5 a 4 kg b 12 g c 65 t
6 a 12°C b 37°C c 17°C
d 225°C e 1.7°C f 31.5°C
7 a 60 kg b 60 000 g c 60 000 000 mg
8 a 3000 g b 3 kg
9 33°C
10 147°C
11 a 8 kg b 8.16 kg
12 a 400 mg, 370 g, 2.5 kg, 0.1 t
b 290 000 mg, 0.000 32 t, 0.41 kg, 710 g
13 a 4th day b 25°C c 26°C
14 50 days
15 yes, by 215 kg
16 a 45.3 t b yes, by 2.7 t
17 a 1 g b 1 t c 1000 t
18 a i 10°C ii 27°C iii 727°C
b i 273 K ii 313 K iii 0 K
19 a 180°F b $\frac{5}{9}$ c $\frac{9}{5}$
d i 0°C ii 20°C iii 60°C iv 105°C
e $F = \frac{9C}{5} + 32$

Puzzles and challenges

1 Both lines are the same length.
2 Mark a length of 5 m, then use the 3 m stick to reduce this to 2 m. Place the 3 m stick on the 2 m length to show a remainder of 1 m.
3 a 3 cm^2 b 20 cm^2
4 500 L
5 10.5 cm^2
6 48 cm^2

Multiple-choice questions

1 E 2 E 3 C 4 A 5 E
6 B 7 E 8 B 9 D 10 D

Short-answer questions

1 a 10 b 100 c 1000 d 10 000
2 a 50 mm b 2 m c 3700 m d 4.21 km
e 7100 g f 24.9 g g 28.49 t h 9000 g
i 4 L j 29.903 kL k 400 kL l 1000 mL
m 1440 min n 60 min o 3.5 days p 9000 s
3 a 2.5 cm b 2.3 cm c 4.25 kg d 5 L
4 a 16 m b 20.6 cm c 23 m
d 34 km e 3.2 mm f 24 m
5 a 24.01 cm^2 b 14 km^2 c 67.5 m^2 d 12 cm^2
e 14 m^2 f 5 cm^2 g 22 m^2 h 21 mm^2
i 291 cm^2 j 52 m^2 k 14 m^2 l 0.9 km^2
6 a (3π + 18) cm b (3π + 16) cm
7 (15π + 10) cm

Extended-response questions

1 436.64 m^2
2 a 97.3 m b 5 min 30 s
c 270 m^2 d $2040

Chapter 11

Pre-test

1 a 48 c 45 e 180
b 72 d 91 f 121
2 13 × 7 and 11 × 11
3 a 2 b 8 c 4 d 5
e 6 f 1 g 10 h 6
4 a yes b no c no d yes
5 a N b P c P d N
6 a true b true c true d true
7 a 9 b 25 c 36
d 196 e 121 f 100
8 a 9 b 13 c 15
9 a 32 b 81 c 125 d 1
10 a 4 b 7 × 7
c 6 d 11 × 11 = 121

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Exercise 11A

1 (1) 2 (3) 4 (5) 6 8 (9) 10

2 a T b T c T d F
e T f F g F h T

3 3, 6 and 9

4 2, 5 and 10

5 i a yes b no c yes
d yes e no f yes
g no h yes i no
j yes k no l yes
m yes n no o no
p yes q no r no
ii Using a calculator

6

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
243 567	✗	✓	✗	✗	✗	✗	✓	✗
28 080	✓	✓	✓	✓	✓	✓	✓	✓
189 000	✓	✓	✓	✓	✓	✓	✓	✓
1 308 150	✓	✓	✗	✓	✓	✗	✓	✓
1 062 347	✗	✗	✗	✗	✗	✗	✗	✗

7 a 10, 15, 20, 25, 30 b 12, 15, 18, 21, 24
c 10, 12, 14, 16, 18 d 12, 18, 24, 30, 36
e 16, 24, 32, 40, 48 f 18, 27, 36, 45, 54
g 10, 20, 30, 40, 50 h 12, 16, 20, 24, 28

8 a not even
b digits do not sum to a multiple of 3
c 26 is not divisible by 4
d last digit is not 0 or 5
e not divisible by 3 (sum of digits is not divisible by 3)
f 125 is not divisible by 8 and it is not even
g sum of digits is not divisible by 9
h last digit is not 0

9 a no b $14.25

10 2, 4, 8, 11, 22, 44

11 Start by working out how many numbers are divisible by 5. This answer is found by dividing 250 by 5, which equals 50. As there are 50 multiples of 5 in 250, there are 200 other numbers between 1 and 250 that are not multiples of 5 and, therefore, these 200 numbers are not divisible by 5.

12 15

13 980

14 966

15 a yes
b Multiples of 3; adding a multiple of 3 does not change the result of the divisibility test for 3.
c 18 d, e Answers will vary

16 a 0, 4, 8 b 2, 6

17 36

18 a 11 b 9 c 7 d 5 e 3

19 a 11, 22, 33, 44, 55, 66, 77, 88, 99
b 0
c 110, 121, 132, 143, 154, …, 209, …, 308, 319, …
d equals the centre digit or 11 + the centre digit
e difference is 0 or 11
f Sum the odd- and even-placed digits. If the difference between these two sums is 0 or is divisible by 11, then the number is divisible by 11.
g i yes ii yes iii no iv yes
v yes vi yes

Exercise 11B

1 no 2 yes

3 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

4 4, 6, 8, 9, 10, 12, 14, 15, 16, 18

5 101

6 211

7 a C b P c C d P
e C f C g P h P
i C j C k C l P
m P n P o C p P

8 a 2, 3, 7 b 3, 13 c 2, 3, 5
d 5 e 2, 7 f 2, 3

9 a 32, 33, 34, 35, 36, 38, 39, 40, 42, 44, 45, 46, 48, 49
b 51, 52, 54, 55, 56, 57, 58, 60, 62, 63, 64, 65, 66, 68, 69
c 81, 82, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 98, 99

10 a 5, 11 b 7, 13 c 11, 13
d 11, 17 e 5, 73 f 7, 19

11 14

12 5 and 7, 11 and 13, 17 and 19, as well as other pairs

13 17 and 71, as well as other pairs

14 (5, 7, 11), as well as other groups

15 $32 = 29+3$, $34 = 29+5$, $36 = 29+7$, $38 = 31+7$, $40 = 37 + 3$, $42 = 31+11$, $44 = 41+3$, $46 = 41+5$, $48 = 41+7$

16 $2 + 3 = 5$ and $2 + 5 = 7$. All primes other than 2 are odd and two odds sum to give an even number that is not a prime. So any pair that sums to a prime must contain an even prime, which is 2.

17 Check your spreadsheet using smaller primes.

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Exercise 11C

1 E

2 D

3

Value	Base number	Index number	Basic numeral
2^3	2	3	8
5^2	5	2	25
10^4	10	4	10 000
2^7	2	7	128
1^{12}	1	12	1
12^1	12	1	12
0^5	0	5	0

4 **a** 3^3 **b** 2^5 **c** 15^4 **d** 10^4 **e** 6^2
f 20^3 **g** 1^6 **h** 4^3 **i** 100^2

5 **a** $3^2 \times 5^2$ **b** $2^2 \times 7^3$ **c** $9^2 \times 12^2$
d $5^3 \times 8^2$ **e** $3^3 \times 6^3$ **f** $7^4 \times 13^2$
g $4^3 \times 7^1 \times 13^1$ **h** $9^3 \times 10^2$ **i** $2^3 \times 3^2 \times 5^2$

6 $2^6 \times 3^5 \times 5^4$

7 **a** $2 \times 2 \times 2 \times 2$ **b** 17×17
c $9 \times 9 \times 9$ **d** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
e $14 \times 14 \times 14 \times 14$ **f** $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
g $10 \times 10 \times 10 \times 10 \times 10$ **h** $54 \times 54 \times 54$

8 **a** $3 \times 3 \times 3 \times 3 \times 3 \times 2 \times 2 \times 2$
b $4 \times 4 \times 4 \times 3 \times 3 \times 3 \times 3$
c $7 \times 7 \times 5 \times 5 \times 5$
d $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 9 \times 9 \times 9$
e $5 \times 7 \times 7 \times 7 \times 7$
f $2 \times 2 \times 3 \times 3 \times 3 \times 4$
g $11 \times 11 \times 11 \times 11 \times 11 \times 9 \times 9$
h $20 \times 20 \times 20 \times 30 \times 30$

9 **a** 32 **b** 64 **c** 1000 **d** 72
e 10 000 **f** 1000 **g** 64 **h** 121

10 **a** 25 **b** 1 **c** 10
d 64 **e** 128 **f** 8
g 22 **h** 900 **i** 8

11 **a** 56 **b** 810 **c** 15 750
d 7343 **e** 92 096 **f** 29 701

12 **a** 4 **b** 2 **c** 3 **d** 6
e 3 **f** 2 **g** 2 **h** 4

13 **a** $<$ **b** $>$ **c** $=$ **d** $<$
e $>$ **f** $>$ **g** $<$ **h** $<$

14 125

15 **a** 126 **b** 55 min
c 244 140 626 people **d** 75 min
e 59 604 644 775 390 626

16 **a** $+$ **b** $\times$ **c** $-$
d $\div$ **e** $\times$ **f** $-$

17 x^y

18 $a = 2$, $b = 4$

19 **a** 1, 2, 6, 24, 120, 720
b **i** $2^4 \times 3^2 \times 5 \times 7$ **ii** $2^7 \times 3^2 \times 5 \times 7$
iii $2^7 \times 3^4 \times 5 \times 7$ **iv** $2^8 \times 3^4 \times 5^2 \times 7$
c 0 **d** 0
e It is the index number on the base 5.
f e.g. $23! \times 4!$

Exercise 11D

1 composite: 15, 8, 9, 27, 4, 12; prime: 13, 7, 5, 23, 11, 2

2 **a** 5, 4 **b** 6, 2 **c** 10, 2, 5

3 **a** 3, 3, 2, 5 **b** 7, 2, 2, 2 **c** 5, 11, 2, 2

4 **a** $2^3 \times 3^2$ **b** $3^4 \times 5^2$
c $2^2 \times 3 \times 7^2$ **d** $2^2 \times 3^2 \times 11^2$

5 **a** $2^3 \times 3^2$ **b** $2^3 \times 3$ **c** 2×19 **d** $2^2 \times 11$
e $2^2 \times 31$ **f** $2^4 \times 5$ **g** $2^5 \times 3$ **h** 2^4
i 3×5^2 **j** 3×37 **k** 2^6 **l** $2^3 \times 7$

6 **a** $2^3 \times 3 \times 5^2$ **b** $2^5 \times 5^2$ **c** $2^3 \times 5^4$
d $2^5 \times 3 \times 5^2$ **e** $2^6 \times 5^6$ **f** $2^3 \times 3^2 \times 5^4$
g $2^2 \times 5 \times 41$ **h** $2 \times 3 \times 5 \times 23$

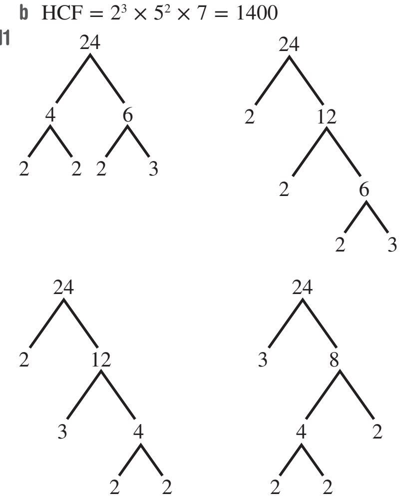
7 **a** D **b** A **c** C **d** B

8 2310

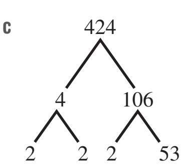
9 **a** $144 = 2^4 \times 3^2$, $96 = 2^5 \times 3$
b HCF $= 2^4 \times 3 = 48$

10 **a** $25\,200 = 2^4 \times 3^2 \times 5^2 \times 7$, $77\,000 = 2^3 \times 5^3 \times 7 \times 11$
b HCF $= 2^3 \times 5^2 \times 7 = 1400$

11



12 **a** 424 cannot have a factor of 5.
b 8 is not a prime number.
c



$\therefore 424 = 2^3 \times 53$

13 **a** **i** $5 \times 10 \neq 60$ **ii** 6 is not a prime number.
iii a 2 has been left off
b $60 = 2^2 \times 3 \times 5$

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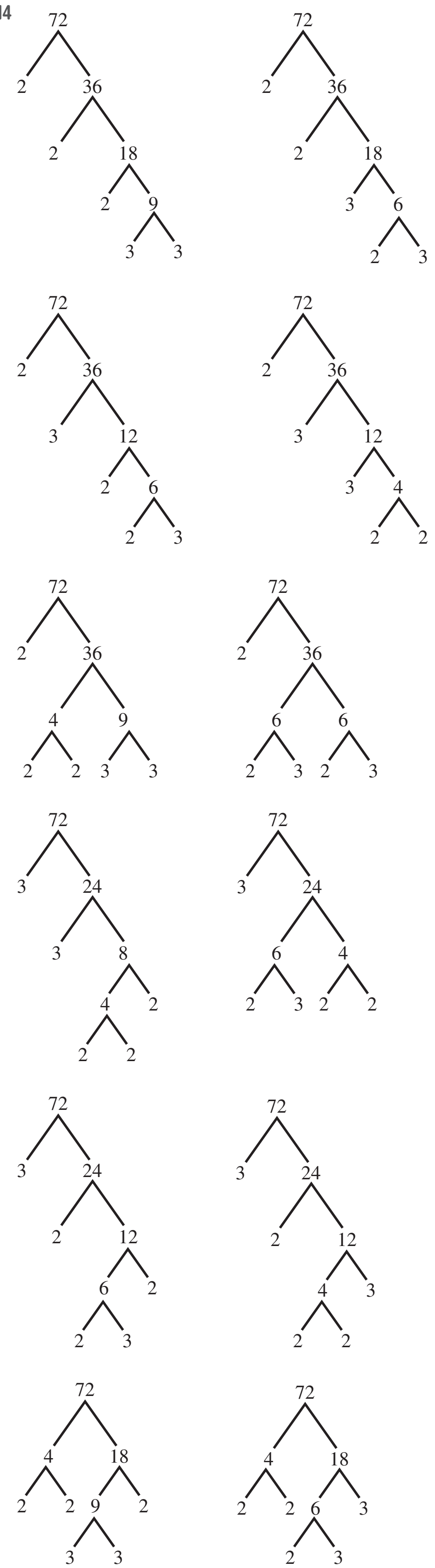
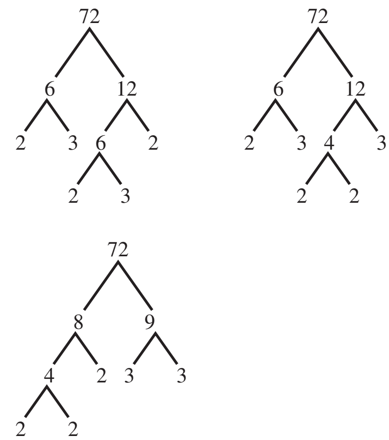
14





15 $2 \times 3 \times 5 \times 7 = 210$
$2 \times 3 \times 5 \times 11 = 330$
$2 \times 3 \times 5 \times 13 = 390$
$2 \times 3 \times 5 \times 17 = 510$
$2 \times 3 \times 5 \times 19 = 570$
$2 \times 3 \times 5 \times 23 = 690$
$2 \times 3 \times 5 \times 29 = 870$
$2 \times 3 \times 5 \times 31 = 930$
$2 \times 3 \times 7 \times 11 = 462$
$2 \times 3 \times 7 \times 17 = 714$
$2 \times 3 \times 7 \times 19 = 798$
$2 \times 3 \times 7 \times 23 = 966$
$2 \times 5 \times 7 \times 11 = 770$
$2 \times 5 \times 7 \times 13 = 910$
$2 \times 3 \times 11 \times 13 = 858$

16 Check with your teacher.

17 Check with your teacher.

Exercise 11E

1 $36\,\text{cm}^2$, a square number

2 a $1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25, 6^2 = 36, 7^2 = 49, 8^2 = 64, 9^2 = 81, 10^2 = 100, 11^2 = 121, 12^2 = 144, 13^2 = 169, 14^2 = 196, 15^2 = 225$

b 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

3 a A square is not possible. b draw a 4 by 4 square

4 a 36 b 25 c 1331
d 100 e 8000 f 144

5 a 5 b 4 c 1
d 3 e 100

6 a 64 b 49 c 1 d 144
e 9 f 225 g 25 h 0
i 121 j 10 000 k 289 l 1089

7 a 5 b 3 c 1 d 11
e 0 f 9 g 7 h 4
i 2 j 12 k 20 l 13

8 a 15 b 25 c 21 d 19

9 a 30 b 64 c 65 d 36
e 4 f 0 g 81 h 4
i 13 j 2 k $\frac{2}{3}$ l $\frac{4}{5}$

10 a 241 b 23 c 2
d 18 e 720 f 41

11 64, 81, 100

12 121, 144, 169, 196

13 a 4 and 81 b 36 and 121

14 1, 9 and 49

15 a 144 **b** 144 **c** $a = 3, b = 4$

d Answers may vary; e.g. $4^2 \times 5^2$ and 20^2

16 a $3^2 + 4^2 = 9 + 16 = 25$ **b** 10^2 **c** 15^2 **d** 50^2

17 a 121 and 12 321

b 1 234 321 **c** 1 234 321

18 no, $9^2 = 81$

19 a false **b** false **c** true

d true **e** true (if $a \geqslant 0$) **f** false

g false **h** false

Exercise 11F

1 a $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

b $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^7$

c $\frac{5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5} = 5^2$ **d** $\frac{9 \times 9 \times 9 \times 9}{9 \times 9} = 9^2$

2 a true **b** true **c** false **d** false

3 a multiply **b** 1

4 a 16, 8, 4, 2, 1 **b** 64, 16, 4, 1

5 a $4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4^6$

b $12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 \times 12 = 12^9$

6 a 1 **b** 1 **c** 1 **d** 1

e −1 **f** 1 **g** 1

7 a 2^7 **b** 5^9 **c** 7^6 **d** 8^{10}

8 a 3^6 **b** 4^{15} **c** 3^{30} **d** 7^{10}

9 a 4^7 **b** 3^9 **c** 7^2 **d** 4 **e** 3^8

10 a $7^2 = 49$ **b** 10 **c** $13^2 = 169$

d $2^3 = 8$ **e** 101 **f** $200^2 = 40\,000$

g $7 \times 31 = 217$ **h** $3 \times 50^2 = 7500$

11 a 4 **b** 1000 **c** 1 **d** 4

12 a The zero index applies only to the 5, not the 2. Correct answer is $2 \times 1 = 2$.

b When doing multiplication, the indices are added, not the bases. Correct answer is 2^4.

c $2 \times 3^6 = 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ but $6^6 = 6 \times 6 \times 6 \times 6 \times 6 \times 6$. These are not equal.

d The rule that says 'add the powers' applies only when multiplying terms, not adding.

13 $3^{-1} = \frac{1}{3}$, $3^{-2} = \frac{1}{9}$, $3^{-3} = 27$

14 a 2^{12} **b** 2^{15} **c** 3^6

d 3^{20} **e** 5^{10} **f** 3^{50}

g 2^{72} **h** 7^{80} **i** 10^{50}

Puzzles and challenges

1 a 28(1, 2, 4, 7, 14) **b** 1, 2, 4, 8, 16, 31, 62, 124, 248

2 a 1400 **b** 480 **c** 540

d 630 **e** 4320 **f** 1100

3 a 37 **b** 43 **c** 75

d 91 **e** 143 **f** 92

4 ~~01~~, 02, 03, ~~04~~, 05, ~~06~~, 07, ~~08~~, ~~09~~, ~~10~~,
11, ~~12~~, 13, ~~14~~, ~~15~~, ~~16~~, 17, ~~18~~, 19, ~~20~~
~~21~~, ~~22~~, 23, ~~24~~, ~~25~~, ~~26~~, ~~27~~, ~~28~~, 29, ~~30~~,
31, ~~32~~, ~~33~~, ~~34~~, ~~35~~, ~~36~~, 37, ~~38~~, ~~39~~, ~~40~~,
41, ~~42~~, 43, ~~44~~, ~~45~~, ~~46~~, 47, ~~48~~, ~~49~~, ~~50~~,
~~51~~, ~~52~~, 53, ~~54~~, ~~55~~, ~~56~~, ~~57~~, ~~58~~, 59, ~~60~~,
61, ~~62~~, ~~63~~, ~~64~~, ~~65~~, ~~66~~, 67, ~~68~~, ~~69~~, ~~70~~,
71, ~~72~~, 73, ~~74~~, ~~75~~, ~~76~~, ~~77~~, ~~78~~, 79, ~~80~~,
~~81~~, ~~82~~, 83, ~~84~~, ~~85~~, ~~86~~, ~~87~~, ~~88~~, 89, ~~90~~,
~~91~~, ~~92~~, ~~93~~, ~~94~~, ~~95~~, ~~96~~, 97, ~~98~~, ~~99~~, ~~100~~

5 $5^2 = 4^2 + 9$, $6^2 = 5^2 + 11$

6 Answers may vary. Check that the answer is the whole number that you are looking for.

Multiple-choice questions

1 B **2** E **3** C **4** E **5** B

6 B **7** A **8** C **9** E **10** B

Short-answer questions

1 a 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120

b 1200, 1440, 1800

2 a 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96
7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84,

b 7, 13

3 a composite 21, 30, 16; prime 11, 7, 3, 2

b only one, 13

4 a 2, 5, 7, 11

b 30, 42, 70

5 a 6^8 **b** $5^4 \times 2^5$

6 a 2^5 **b** $2^3 \times 5^2$ **c** $3^2 \times 5^2$

7 a 10 **b** 3 **c** 4

8 a 16 **b** 7 **c** 397 **d** 131

9 a yes **b** no **c** yes

10 a

Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5
84 539 424	✓	✓	✓	✗

Number	Divisible by 6	Divisible by 8	Divisible by 9	Divisible by 10
84 539 424	✓	✓	✗	✗

Explanation: 84 539 424 is an even number, therefore is divisible by 2. 84 539 424 has a digit sum of 39, therefore is divisible by 3, but not by 9. 84 539 424 is divisible by 2 and 3, therefore is divisible by 6. The last two digits are 24, which is divisible by 4. The last three digits are 424 , which is divisible by 8. The last digit is a 4, therefore not divisible by 5 or 10.

b Many possible answers.

c Any six-digit number ending in 0 with sum of digits equals any multiple of 9.

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11 a 5 b 50 c 13 d 18
e 14 f 20
12 a 102 b 102 c 104
d 105 e 102 f 104
g 108 h 110 i 101
13 a 12 b 216
14 a 1 b 3 c 1
d 2 e 1 f 2
15 a 5^{15} b 5^9 c 5^{36} d 5^9

Extended-response question

1 a area = $3^3 \times 5^3\,\text{cm}^2$ perimeter = $(2 \times 3^3 + 2 \times 5^3)$ cm
b area = 16^3 or 4^6 or 2^{12} perimeter = 16^2 or 4^4 or 2^8
c $a^4 \times c^2$ d 16^2 or 4^4 or 2^8 e 3^x

Semester review 2

Chapter 7: Time

Multiple-choice questions

1 A 2 E 3 A 4 D 5 E

Short-answer questions

1 a 60 h b 3 h c 3 min 30 s
d 6 h 30 min e 2145 f 1:26 p.m.
g 5 h 45 min h 6 h 19 min 12 s
2 a 45 min b 1 h 51 min
c 1 h 11 min 1 s d 1 h 51 min 50 s
3 a i 7 p.m. ii 5 p.m.
iii 12 noon iv 6 a.m.
v 12 midnight vi 9 p.m.
b i 1:20 p.m. ii 3:50 p.m.
iii 3:20 p.m iv 4:20 p.m.
4 a i 55 min ii 1 h 14 min
iii 1 h 1 min iv 1 h 57 min
b morning
c 1 h 56 min
5 21h 15 min
6 120
7 5 a.m.

Extended-response question

1 a i 4:30 p.m. ii 4 p.m. iii 2:30 p.m.
b 2 h 39 min
c 3:30 p.m.

Chapter 8: Algebraic techniques

Multiple-choice questions

1 B 2 A 3 C 4 E 5 D

Short-answer questions

1 a 4 b yes c 9 d 7
2 a $x + 3$ b $12a$ c $2x + 3y$
d $\frac{w}{6}$ e $y - 2x$
3 a $100m$ b $24x$ c $1\,000\,000p$ d $\frac{y}{24}$
4 a 13 b 11 c 39 d 6 e 6 f 24
5 36
6 a $10a$ b $4x$ c $12a$
d m e $6 + 5a$ f $4x + 2y$
7 a $2x + 14$ b $3(x + 4) = 3x + 12$
8 a $6bc$ b $5b$ c p
9 a $2a + 6$ b $12a - 12b$ c $24m + 32$
10 $12xy$

Extended-response question

1 a i \$12 ii \$(3x + 6) iii \$(3x + 2y)
b \$50 − 3x − 2y

Chapter 9: Equations

Multiple-choice questions

1 B 2 E 3 C 4 B 5 D

Short-answer questions

1 a $x = 3$ b $x = 108$ c $x = 21$ d $x = \frac{4}{3}$
2 a $x = 2$ b $x = 12$ c $m = 7$
3 a $x = 1$ b $x = 7.5$ c $x = 5.5$
4 a $P = 103$ b $S = 61$ c $C = 325$
5 a $x = 5$ b $x = 6$ c $x = 18$
6 $4x + 25 = 85; x = 15$

Extended-response question

1 a \$320 b \$400 c \$(200 + 40n) d $6\frac{1}{2}$ h

Chapter 10: Measurement and computation of length, perimeter and area

Multiple-choice questions

1 C 2 A 3 E 4 C 5 A

Short-answer questions

1 a 500 b 6000 c 1.8
d 0.017 e 1.8 f 5500
2 a 272 cm b 11 m c 3 m
d 220 cm e 3.4 m f 92 m
3 a 1.69 m^2 b 24 m^2 c 60 m^2
d 75 m^2 e 114 m^2 f 171 m^2

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4 **a** $(6\pi + 12)$ cm **b** $(3\pi + 12)$ cm **c** $(\pi + 8)$ cm
5 $(8\pi + 16)$ cm

Extended-response question

1 **a** Many answers possible; e.g.
8 m × 10 m, 4 m × 14 m, 15 m × 3 m
b 9 m by 9 m (area = 81 m^2)
c 36 posts
d 9 h

Chapter 11: Introducing indices

Multiple-choice questions

1 C 2 A 3 B 4 E 5 D

Short-answer questions

1 **a** 1, 3, 5, 15 **b** 1, 2, 3, 5, 6, 10, 15, 30
c 1, 2, 4, 5, 10, 20, 25, 50, 100
2 **a** 3, 6, 9, 12, 15 **b** 7, 14, 21, 28, 35 **c** 11, 22, 33, 44, 55
3 1, 2, 3 and 6
4 4
5 **a** 121 **b** 144 **c** 25
6 120
7 false
8 **a** 1 **b** 1 **c** 1
9 **a** 8^{10} **b** 8^7 **c** 8^3
d 8^6 **e** 8^4 **f** 8^{10}
10 **a** 1 **b** 8 **c** 1
d 9 **e** 1 **f** 2

Extended-response question

1 **a** $(-1)^4 = 1; (-1)^5 = -1$
b −1 because 279 is an odd number.
c Will be negative because 79 is an odd number.

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