YEAR **12**

MATHEMATICS STANDARD 1

CambridgeMATHS

STAGE 6

GK POWERS

INCLUDES INTERACTIVE TEXTBOOK POWERED BY CAMBRIDGE HOTMATHS

CAMBRIDGE UNIVERSITY PRESS

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Introduction

CambridgeMATHS Mathematics Standard 1 Year 12 provides complete and close coverage of the NSW Stage 6 Mathematics Standard 1 Year 12 syllabus to be implemented from 2018.

Now part of the *CambridgeMATHS* series, this resource is part of a continuum from Year 7 through to 12. The series includes advice on pathways from Stage 5 to Stage 6. The Year 12 resource gives access to selected previous years' lessons for revision of prior knowledge.

The four components of *Mathematics Standard 1 Year 12* – the print book, downloadable PDF textbook, online Interactive Textbook and Online Teaching Resource – contain a huge range of resources available to schools in a single package at a convenient low price. There are no extra subscriptions or per-student charges to pay.

Interactive Textbook powered by the HOTmaths platform – included with the print book or available separately *(shown on the page opposite)*

The Interactive Textbook is an online HTML version of the print textbook powered by the HOTmaths platform, completely designed and reformatted for on-screen use, with easy navigation. Its features include:

1 All examples have video versions to encourage independent learning.
2 All exercises including chapter reviews have the option of being done interactively on line, using **workspaces** and **self-assessment tools**. Students rate their level of confidence in answering the question and can flag the ones that gave them difficulty. Working and answers, whether typed or handwritten, and drawings, can be saved and submitted to the teacher electronically. Answers displayed on screen if selected and worked solutions (if enabled by the teacher) open in pop-up windows.
3 Teachers can give feedback to students on their self-assessment and flagged answers.
4 The full suite of the HOTmaths learning management system and communication tools are included in the platform, with similar interfaces and controls.
5 Worked solutions are included and can be enabled or disabled in the student accounts by the teacher.
6 Interactive widgets and activities based on embedded Desmos windows demonstrate key concepts and enable students to visualise the mathematics.
7 Desmos scientific and graphics calculator windows are also included.
8 Revision of previous years' material is included.
9 Every section in a chapter has a Quick Quiz of automatically marked multiple-choice questions for students to test their progress.
10 Definitions pop up for key terms in the text, and are also provided in a dictionary.
11 Each chapter has a Study Guide – a concise summary in PowerPoint slides that can be used for revision and preparation for assessment.
12 Literacy worksheets can be accessed via the Interactive Textbook, with answers in the Online Teaching Suite, providing activities to help with mathematical terminology.
13 Spreadsheet files are provided for spreadsheet questions.

Downloadable PDF textbook *(shown on the page opposite)*

14 The convenience of a downloadable PDF textbook has been retained for times when users cannot go online.

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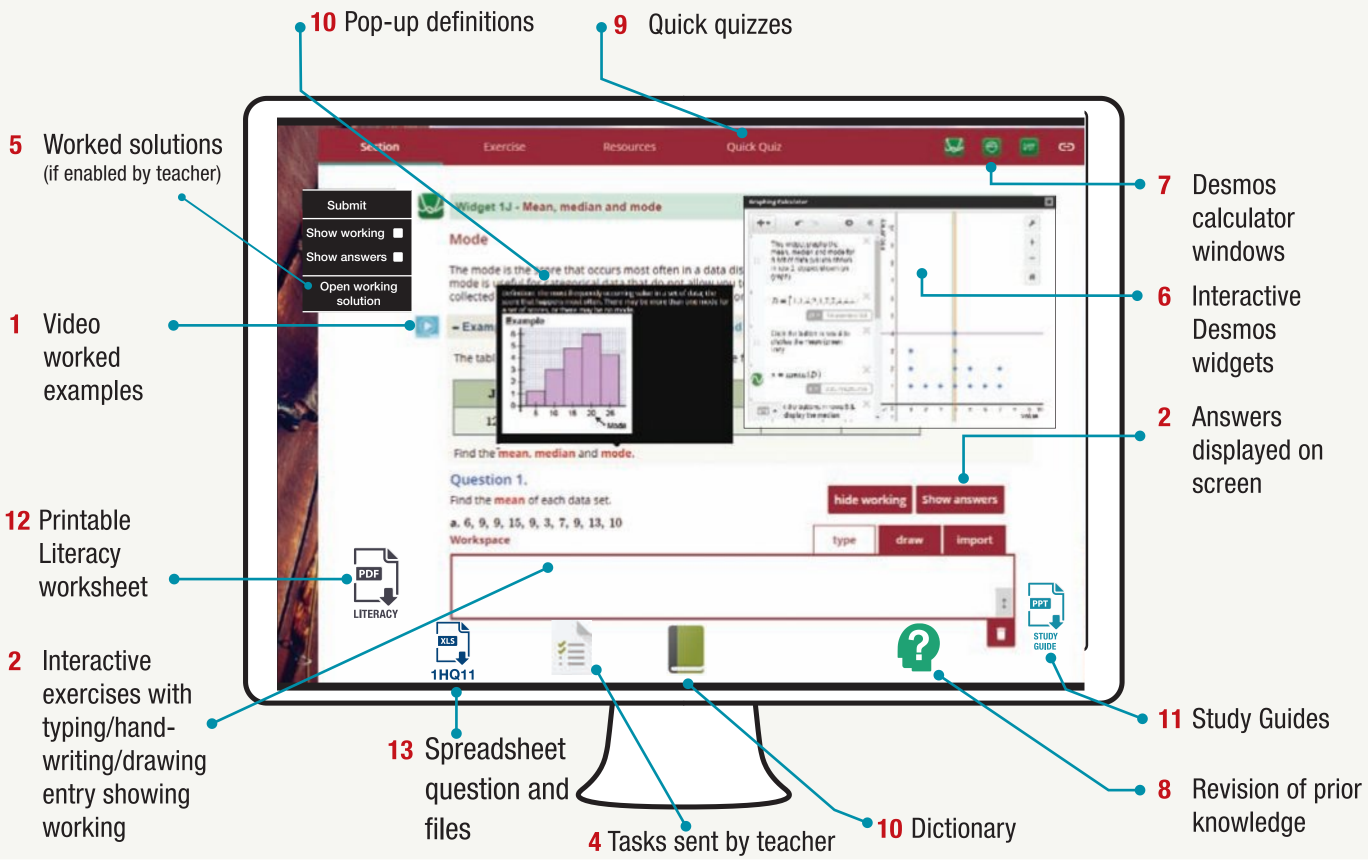
INTERACTIVE TEXTBOOK POWERED BY THE *HOTmaths* PLATFORM

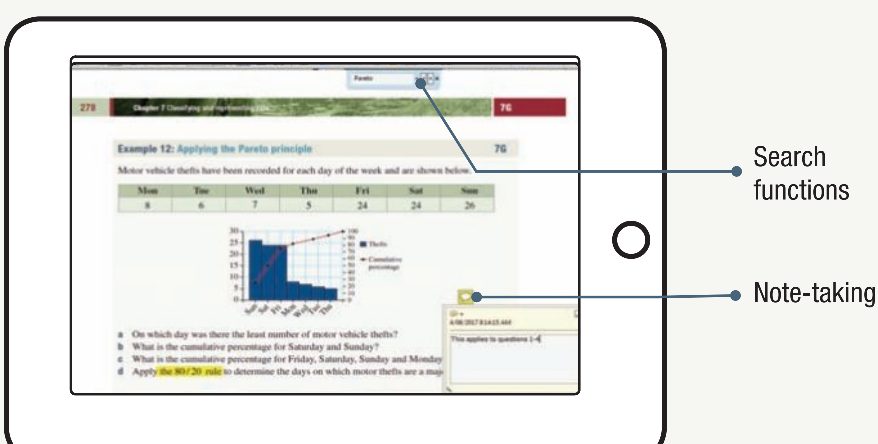
Numbers refer to the descriptions on the opposite page. HOTmaths platform features are updated regularly. Screenshots show Standard 2 content, Standard 1 has the same features.



PDF TEXTBOOK

DOWNLOADABLE

14 Included with Interactive Textbook



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Online Teaching Suite powered by the HOTmaths platform *(shown on the page opposite)*

The Online Teaching Suite is automatically enabled with a teacher account and appears in the teacher's copy of the Interactive Textbook. All the assets and ersources are in one place for easy access. Many of them are opened by clicking on icons in the pages of the Interactive Textbook. The features include:

15 Editable teaching programs with registers, a scope and sequence document and curriculum grid.

16 Topic test worksheets A and B – based on the knowledge, skills and understanding gained in each chapter, and Revision Quiz worksheets provide HSC-standard questions for further revision for each topic, with worked solutions. NESA requirements for problem-solving investigative tasks will also be addressed.

17 A HOTmaths-style test generator provides additional multiple-choice questions, as well as digital versions of the multiple-choice questions in the test worksheets.

18 The HOTmaths learning management system with class and student reports and communication tools is included.

19 Teacher's lesson notes – pop-up text boxes containing lesson notes and additional examples that can be used in class, also available as editable PowerPoint slides which can be given to students as tutorials.

Other features:

20 The textbook is divided into smaller manageable topics to assist teaching.

21 Syllabus topic focus and outcomes are listed at the beginning of each chapter.

22 Each section and exercise begins at the top of the page to make them easy to find and access.

23 Each exercise develops a student's skills to work mathematically at their level.

24 Step-by-step worked examples with precise explanations (and video versions for most of them) encourage independent learning, and are linked to exercises.

25 Important concepts are formatted in boxes for easy reference.

26 Spreadsheet activities are integrated throughout the text, with accompanying Excel files in the Interactive Textbook.

27 Chapter reviews contain a chapter summary and multiple-choice and short-answer questions.

28 A comprehensive glossary and HSC Reference sheet are included.

29 There are two complete HSC practice papers.

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ONLINE TEACHING SUITE POWERED BY THE *HOTmaths* PLATFORM

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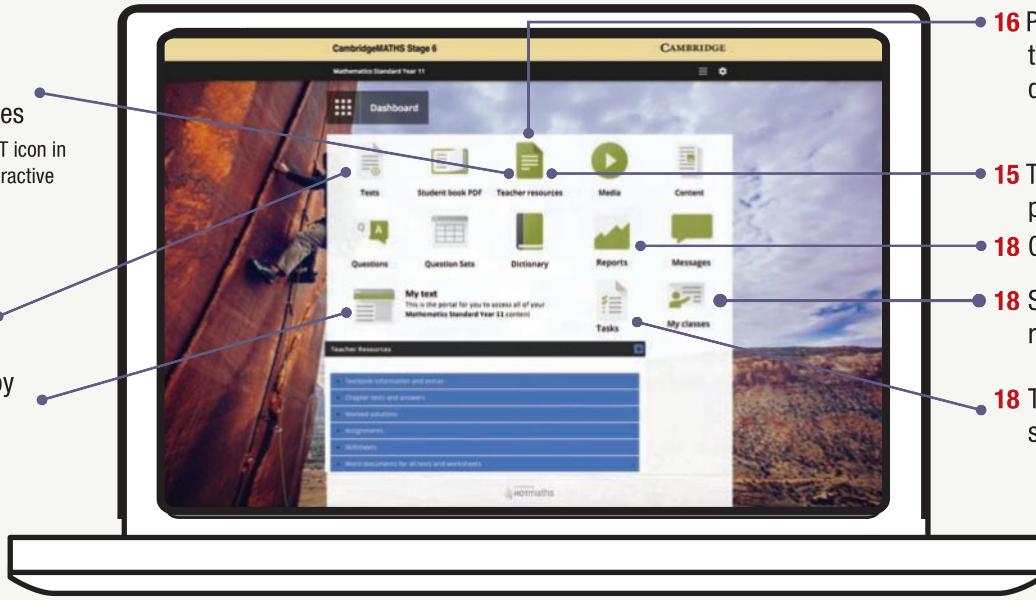


19 Teacher's lesson notes (Indicated by T icon in teacher's interactive textbook)

17 Test generator

Teacher's copy of Interactive Textbook

16 Printable test and quiz sheets

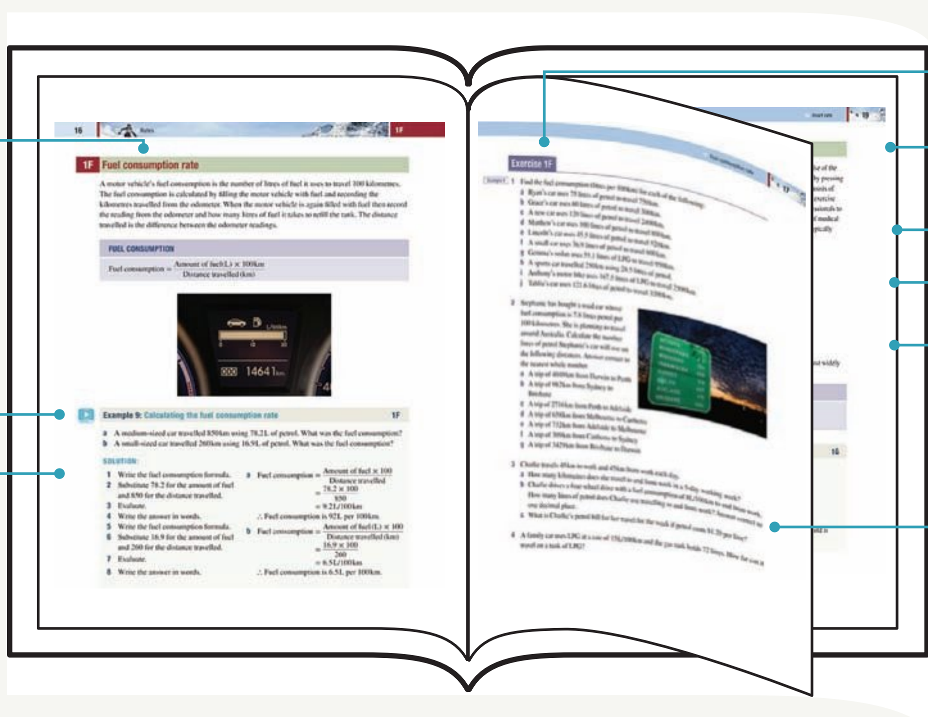
15 Teaching programs

18 Class reports

18 Student results

18 Tasks sent to students

PRINT TEXTBOOK



20 Smaller manageable topics to assist teaching

Icons indicate digital assets

24 Step-by-step worked examples with precise explanations

23 Exercises

27 Chapter summary

28 Glossary

Answers

29 HSC practice papers

24 Questions linked to examples

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1 Rates

Syllabus topic — M4 Rates

This topic focuses on the use of rates to solve problems in practical contexts.

Outcomes

- Use, simplify and convert between units of rates.
- Use rates to make comparisons such as best buys.
- Use rates to determine costs.
- Use rates to solve problems related to speed, distance and time.
- Calculate the fuel consumption rate.
- Solve problems involving heart rates and blood pressure.
- Describe heart rate as a rate expressed in beats per minute.
- Calculate target heart rate ranges.
- Express blood pressure using measures of systolic pressure and diastolic pressure.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Literacy worksheet
- Quick Quiz
- Solutions (enabled by teacher)
- Desmos widgets
- Spreadsheets
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

The Interactive Textbook provides a test of prior knowledge for this chapter, and may direct you to revision from the previous years' work.

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1A Rates

Rates

A rate is a comparison of amounts with different units. For example, we may compare the distance travelled with the time taken. In a rate the units are different and must be specified.

The order of a rate is important. A rate is written as the first amount per one of the second amount. For example, \$2.99/kg represents \$2.99 per one kilogram or 80 km/h represents 80 kilometres per one hour.

We are constantly interested in rates of change and how things change over a period of time. There are many examples of rates such as:

- Growth rate: The average growth rate of a child from 0 to 15 years of age.
- Running rate: Your running pace in metres per second.
- Typing rate: Your typing speed in words per minute.
- Wage rate: The amount of money you are paid per hour.

CONVERTING A RATE

1. Write the rate as a fraction. First quantity is the numerator and 1 is the denominator.
2. Convert the first amount to the required unit.
3. Convert the second amount to the required unit.
4. Simplify the fraction.

Example 1: Converting a rate — 1A

Convert each rate to the units shown.

a 55 200 m/h to m/min **b** \$6.50/kg to c/g

SOLUTION:

1. Write the rate as a fraction.
2. The numerator is 55 200 m and the denominator is 1 h.
3. No conversion required for the numerator.
4. Convert the 1 hour to minutes by multiplying by 60.
5. Simplify the fraction.

a $55\,200 = \dfrac{55\,200 \text{ m}}{1 \text{ h}}$

$= \dfrac{55\,200 \text{ m}}{1 \times 60 \text{ min}}$

$= 920 \text{ m/min}$

6. Write the rate as a fraction.
7. The numerator is \$6.50 and the denominator is 1 kg.
8. Convert the \$6.50 to cents by multiplying by 100.
9. Convert the 1 kg to g by multiplying by 1000.
10. Simplify the fraction.

b $6.50 = \dfrac{\$6.50}{1 \text{ kg}}$

$= \dfrac{6.50 \times 100 \text{ c}}{1 \times 1000 \text{ g}}$

$= 0.65 \text{ c/g}$

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Exercise 1A

1 Which of the following are examples of rates?

a \$7.50
b 150 mL/min
c \$80/h
d 500 cm^2
e 7/22
f \$1.54/L

Example 1

2 Convert to the rate shown.

a \$100 in 4 h is a rate of \$□ /h
b 240 m in 20 s is a rate of □ m/s
c 700 L in 10 h is a rate of L□ /h
d \$39 in 12 h is a rate of \$□ /h
e \$1.20 for 2 kg is a rate of □ c/kg
f 630 km in 60 L is a rate of □ km/L
g 1200 rev in 4 min is a rate of □ rev/min
h A rise of 20° in 4 h is a rate of □ °/h
i 275 m in 25 s is a rate of □ m/s
j 630 L in 9 h is a rate of □ L/h

3 Express each rate in simplest form using the rates shown.

a 300 km on 60 L [km per L]
b 15 m in 10 s [cm per s]
c \$640 for 5 m [\$ per m]
d 56 L in 0.5 min [L per min]
e 78 mg for 13 g [mg per g]
f 196 g for 14 L [g per L]
g 20 g for 8 m^2 [g per m^2]
h 75 mL for 5 min [mL per min]
i \$1.80 for 9 phone calls [c/call]
j \$630 for 36 h work [\$/h]

4 Write each of the following as a simplified rate.

a 18 goals in 3 games
b 12 days in 4 years
c \$1.50 for 5 kilograms
d \$180 in 6 hours
e 49 500 cans in 11 hours
f \$126 000 to purchase 9 hectares
g 80 mm rainfall in 5 days
h 19 000 revolutions in 10 minutes
i 15 kilometres run in 60 minutes
j 60 minutes to run 15 kilometres

5 Convert each rate to the units shown.

a 39 240 m/min [m/s]
b 2 m/s [cm/s]
c 88 cm/h [mm/h]
d 55 200 m/h [m/min]
e 0.4 km/s [m/s]
f 57.5 m/s [km/s]
g 6.09 g/mL [mg/mL]
h 4800 L/kL [mL/kL]
i 300 m/s [cm/s]
j 36 km/h [km/min]

6 A dripping tap filled a 9 litre bucket in 3 hours.

a What was the dripping rate of the tap in litres/hour?
b How long would it take the tap to fill a 21 litre bucket?

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7 If 30 kebabs were bought to feed 20 people at a picnic, and the total cost was $120, find the following rates.

a kebabs/person

b cost/person

8 The number of hours of sunshine was recorded each day for one week in May. The results were:

Monday 6 hours
Tuesday 8 hours
Wednesday 3 hours
Thursday 5 hours
Friday 7 hours
Saturday 6 hours
Sunday 7 hours

Find the average number of hours of sunshine:

a per weekday

b per weekend day

c per week

d per day.

9 Sebastian finished a 10 kilometre race in 37 minutes and 30 seconds. Alexander finished a 15 kilometre race in 53 minutes and 15 seconds. Calculate the running rate of each runner expressed as minutes per kilometre.

10 A football club had 12 000 members. After five successful years and two premierships, they now have 18 000 members. What has been the average rate of membership growth per year for the past 5 years?

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1B The unitary method

The unitary method involves finding one unit of an amount by division. This result is then multiplied to solve the problem. The unitary method is often used to make comparisons.

USING THE UNITARY METHOD

1. Find one unit of an amount by dividing by the amount.
2. Multiply the result in step 1 by a number to solve the problem.

Example 2: Using the unitary method — 1B

A car travels 360 km on 30 L of petrol. How far does it travel on 7 L?

SOLUTION:

1. Write a statement using information from the question. — $30\text{ L} = 360\text{ km}$
2. Find 1 L of petrol by dividing 360 km by the amount or 30. — $1\text{ L} = \frac{360}{30}\text{ km}$
3. Multiply both sides by 7. — $7\text{ L} = \frac{360}{30} \times 7\text{ km}$
4. Evaluate to an appropriate degree of accuracy. — $= 84\text{ km}$
5. Write the answer in words. — The car travels 84 km.

Example 3: Using the unitary method — 1B

a Bella can touch type at 70 words per minute. How many words can she type in 20 minutes?

b A brand of 400 mL soft drink cans sell singly for $2.40, in a six-pack for $11.95, or in a carton of 24 for $39.95. Compare the cost of one can in each option, to the nearest cent.

c What is the cost of 14 cans of the soft drink at the cheapest option?

SOLUTION:

1. Typing rate is 70 words in one minute. — **a** Number of words $= 70 \times 20$
2. Multiply 70 by 20 to determine the number of words typed in 20 minutes. — $= 1400$
3. Write your answer in words. — Bella types 1400 words in 20 minutes.
4. Write down the price of a single can. — **b** $2.40
5. Find the cost of one can in a six-pack by dividing its price by 6, and rounding to the nearest cent. — $\$11.95 \div 6 \approx \$1.99$
6. Find the cost of one can in a carton by dividing its price by 24, and rounding to the nearest cent. — $\$39.95 \div 24 \approx \$1.66$
7. Write the answer in words. — A can costs $2.40 bought singly, $1.99 each in a six-pack and $1.66 each in a carton.
8. Look for the cheapest can and multiply by 14. — **c** $\$1.66 \times 14 = \$23.34$
9. Write the answer in words. — 14 cans at the cheapest price option will cost $23.24

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Exercise 1B

Example 2, 3

1 Use the rate provided to answer the following questions.

a Cost of apples is \$2.50/kg. What is the cost of 5 kg?

b Tax charge is \$28/m^2. What is the tax for 7 m^2?

c Cost savings are \$35/day. How much is saved in 5 days?

d Cost of a chemical is \$65/100 mL. What is the cost of 300 mL?

e Cost of mushrooms is \$5.80/kg. What is the cost of $\frac{1}{2}$ kg?

f Distance travelled is 1.2 km/min. What is the distance travelled in 30 minutes?

g Concentration of a chemical is 3 mL/L. How many mL of the chemical is needed for 4 L?

h Concentration of a drug is 2 mL/g. How many mL is needed for 10 g?

2 A cricket team scores runs at a rate of 5 runs/over in a match. How many runs are scored in 18 overs?

3 If one dozen tennis balls cost \$9.60, how much would 22 tennis balls cost?

4 If Leo can march at 7 km/h, how far can he march in 2.5 hours?

5 If 8 kg of chicken fillets cost \$72, how much would 3 kg of chicken fillets cost?

6 If three pairs of socks cost \$12.99, how much would 10 pairs of socks cost?

7 If 500 g of mincemeat costs \$4.50, how much would 4 kg of mincemeat cost?

8 A courier delivers 1 parcel on average every 20 minutes. How many hours does it take to deliver 18 parcels?

9 Water is dripping from a tap at a rate of 5 L/h. How much water will leak in one day?

10 A bulldozer is moving soil at a rate of 22 t/h. How long will it take at this rate to move 55 tonnes?

11 Edward saves \$40/week, how long should it take to save \$1000?

12 A professional footballer scores an average of 3 goals every 6 games. How many goals is he likely to score in a full season of 22 games?

13 A computer processor can process 500 000 kilobytes of information in 4 seconds. How much information can it process in 15 seconds?

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1C Using rates to make comparisons

Rates are used to solve practical problems such as calculating wages, best buy and costs.

Example 4: Using rates to calculate the best buy 1C

Which is the best buy: Option 1: 12 rose plants for \$195, or Option 2: 10 rose plants for \$162?

SOLUTION:

1 Find the unit cost for option 1.	Option 1: 12 plants = \$195
2 Divide the cost of 12 plants (\$195) by the 12.	1 plant = \$16.25
3 Find the unit cost for option 2.	Option 2: 10 plants = \$162
4 Divide the cost of 120 plants (\$162) by the 10.	1 plant = \$16.20
5 Option 2 has the lowest cost; write the answer.	∴ Best buy is option 2

Example 5: Using rates to determine costs 1C

Alice's mobile phone contract charges a flagfall of \$0.25 and a call rate of \$0.45 per 30 seconds.

a What is the charge if Alice makes a 2 minute call?

b What is the charge if Alice made 200 calls of duration less than 30 seconds?

SOLUTION:

1 Start with the flagfall. 2 minutes is 120 seconds. Divide 120 by 30 to find number of 30 second blocks. Evaluate.	**a** Charge = (0.25 + (120 ÷ 30) × 0.45) = \$2.05
2 Write the answer in words.	∴ Alice is charged \$2.05.
3 Add the flagfall to the rate charge for each call.	**b** Charge = (0.25 + 0.45) × 200
4 Multiply calculation by 200. Evaluate.	= \$140
5 Write the answer in words.	∴ Alice is charged \$140 for the calls.

Example 6: Using rates to calculate wages 1C

Hamish works for a building construction company. Find Hamish's wage for 35 hours at the normal rate of \$22 an hour, 3 hours at time-and-a-half rates and 1 hour at double time rates.

SOLUTION:

1 Write the quantity to be found.	Wage = normal + 1.5 × time + 2 × time
2 Normal wage is 35 multiplied by \$22.	Wage = (35 × 22) normal pay
3 Payment for time-and-a-half is 3 multiplied by \$22 multiplied by 1.5.	+(3 × 22 × 1.5) time-and-a-half pay
4 Payment for double time is 1 multiplied by \$22 multiplied by 2. Evaluate.	+(1 × 22 × 2) double time pay = \$913.00
5 Write your answer in words.	∴ Hamish's wage is \$913.

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Exercise 1C

Example 4 **1** Calculate the best buy between option 1 and 2.

a Option 1: 6 calculators for \$126 Option 2: 24 calculators for \$552
b Option 1: 21 g for \$8.61 Option 2: 27 g for \$15.39
c Option 1: \$16.92 for 36 L Option 2: \$4.68 for 12 L
d Option 1: 5 batteries for \$8.00 Option 2: 12 batteries for \$14.76
e Option 1: 22 pens for \$8.36 Option 2: 30 pens for \$10.80
f Option 1: \$598 for 23 pairs of shoes Option 2: \$891 for 33 pairs of shoes
g Option 1: 36 chocolate bars for \$66.60 Option 2: 20 chocolate bars for \$35
h Option 1: 19 kg for \$37.62 Option 2: 28 kg for \$49.56
i Option 1: 10L of soft drink for \$6.20 Option 2: 3L of soft drink for \$1.89
j Option 1: 9 g for \$4.77 Option 2: 24 g for \$10.08

2 A breakfast cereal is sold in boxes of three different sizes:
small (400 g) for \$5.00, medium (600 g) for \$7.20, large (750 g) for \$8.25
a Find the value of each box in \$/100 g.
b What is the cheapest way to buy a minimum of 3 kg of the cereal?

Example 5 **3** Xavier has a mobile phone contract that charges a flagfall of \$0.30 and a call rate of \$0.43 per 30 seconds.
a What the charge if Xavier makes a 30 second call?
b What the charge if Xavier makes a 2 minute call?
c What the charge if Xavier makes a 5 minute call?
d What the charge if Xavier made 100 calls whose duration was less than 30 seconds?
e What the charge if Xavier made 50 calls whose duration was less than 60 seconds?

4 A mobile phone plan has a monthly charge of \$59 on a 24 month contract. In addition, the calls are charged at a rate of \$0.90 per 60 second block with a \$0.35 connection fee.
a What is the charge for a call lasting 1 minute?
b What is the charge for a call lasting 2 minutes?
c What is the charge for a call lasting 3 minutes and 30 seconds?
d What is the charge for a call lasting 4 minutes and 20 seconds?
e Determine the monthly charge for making 40 calls (60 seconds)
f Determine the monthly charge for making 90 calls (60 seconds)

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5 Nails cost \$4.80/kg. How many kilograms can be bought for \$30?

6 Natural gas is charged at a rate of \$0.014 per MJ.
- **a** Find the charge for 12 500 MJ of natural gas. Answer to the nearest dollar.
- **b** Find the charge for 16 654 MJ of natural gas. Answer to the nearest dollar.

7 Olivia pays council rates of \$2915 p.a. for land valued at \$265 000. Lucy pays council rates of \$3186 on land worth \$295 000 from another council.
- **a** What is Olivia's council charge as a rate \$/\$1000 valuation?
- **b** What is Lucy's council charge as a rate \$/\$1000 valuation?

Example 6 **8** Ryan works as a builder and charges \$45.50 an hour. How much does he earn for working the following hours?
- **a** 35 hours
- **b** 37 hours
- **c** 40 hours
- **d** 42 hours

9 Nathan is a plumber who earned \$477 for a days work. He is paid \$53 per hour. How many hours did Zachary work on this day?

10 Mia is an apprentice electrician who earns \$37.50 per hour.
- **a** How much a does Mia earn for working a 9-hour day?
- **b** How many hours does Mia work to earn \$1200?
- **c** What is Mia's annual income if she works 40 hours a week? Assume she works 52 weeks in the year.

11 Elizabeth is a hairdresser who earns \$24.20 per hour. She works an 8-hour day.
- **a** How much does Elizabeth earn per day?
- **b** How much does Elizabeth earn per week? Assume she works 5 days a week.
- **c** How much does Elizabeth earn per fortnight?
- **d** How much does Elizabeth earn per year? Assume 52 weeks in the year.

12 Logan earns \$32.50 an hour as a driver. He works 38 hours a week at normal time and 5 hours a week at double time. Find his weekly wage. Answer correct to the nearest cent.

13 Grace is a casual who worked 8 hours at normal time and 2 hours at time-and-a-half. Her normal rate of pay is \$12.30 per hour. What is her pay for the above time?

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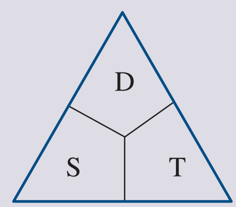
1D Speed as a rate

Speed is a rate that compares the distance travelled to the time taken. The speed of a car is measured in kilometres per hour (km/h). The speedometer in a car measures the instantaneous speed of the car. They are not totally accurate but have a tolerance of about 5%. GPS devices are capable of showing speed readings based on the distance travelled per one-hertz interval. Most cars also have an odometer to indicate the distance travelled by a vehicle.

SPEED

$S = \frac{D}{T}$ or $T = \frac{D}{S}$ or $D = S \times T$

D – Distance
S – Speed
T – Time



Example 7: Solving problems involving speed 1D

a Find the average speed of a car that travels 341 km in 5 hours.
b How long will it take a vehicle to travel 294 km at a speed of 56 km/h?

SOLUTION:

1 Write the formula. **a** $S = \frac{D}{T}$

2 Substitute 341 for D and 5 for T into the formula. $= \frac{341}{5}$

3 Evaluate. $= 68.2$ km/h

4 Write the formula. **b** $T = \frac{D}{S}$

5 Substitute 294 for D and 56 for S into the formula. $= \frac{294}{56}$

6 Evaluate and express the answer correct to the nearest hour. $= 5.25$ h or 5 h 15 min

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Exercise 1D

Example 7 **1** Find the average speed (in km/h) of a vehicle which travels:

a 160 km in 2 hours
b 582 km in 6 hours
c 280 km in 3.5 hours
d 22 km in $\frac{1}{4}$ hour
e 432 km in $4\frac{1}{2}$ hours
f 18 km in 20 minutes

2 Find the distance travelled by a car whose average speed is 62 km/h if the journey lasts for the following time. (Answer correct to the nearest kilometre.)

a 4 hours
b 5 hours
c 2.6 hours
d $1\frac{1}{4}$ hour
e $3\frac{1}{2}$ hours
f $2\frac{3}{4}$ hour

3 How long will it take a vehicle to travel (correct to the nearest hour):

a 240 km at a speed of 80 km/h?
b 175 km at a speed of 70 km/h?
c 160 km at a speed of 48 km/h?
d 225 km at a speed of 45 km/h?
e 240 km at a speed of 40 km/h?
f 556 km at a speed of 69.5 km/h?

4 The Melbourne Formula 1 track is 5.303 km in length. The track record is 1 minute and 24 seconds.

a What is the track record in hours?
b What is the average speed (km/h) for the lap record? Answer correct to two decimal places.

5 Emily lives in Wollongong and travels to Sydney daily. The car trip requires her to travel at different speeds. Most often she travels 30 kilometres at 60 km/h and 40 kilometres at 100 km/h.

a What is the total distance of the trip?
b How long (in hours) does the trip take?
c What is her average speed (in km/h) when travelling to Sydney? (Answer correct to two decimal places.)

6 Thomas drives his car to work 3 days a week. The distance of the trip is 48 km. The trip took 43 minutes on Monday, 50 minutes on Tuesday and 42 minutes on Wednesday.

a Calculate the average time taken to travel to work.
b What is the average speed (in km/h) for the three trips?

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7 Use the information provided on speed to answer the following questions.

- **a** Walking at 5 km/h. How far can I walk in 4 hours?
- **b** Car travelling at 80 km/h. How far will it travel in 2.5 hours?
- **c** Plane is travelling at 600 km/h. How far will it travel in 30 minutes?
- **d** A train took 7 hours to travel 665 km. What was its average speed?
- **e** Ryder runs a 42.4 km marathon in 2 hours 30 minutes. Calculate his average speed.
- **f** A spacecraft travels at 1700 km/h for a distance of 238 000 km. How many hours did it take?

8 Alexandra jogs 100 metres in 20 seconds. How many seconds would it take her to jog one kilometre?

9 A car travels at a rate of 50 metres each second. How many kilometres does it travel in:

- **a** one minute?
- **b** one hour?

10 Convert the following speeds to metres per second. Answer to the nearest whole number.

- **a** 60 km/h
- **b** 260 km/h

11 An athlete runs 100 metres in 10 seconds. If he could continue at this rate, what is his speed in kilometres per hour?

12 A snail travelling at a constant speed travels 400 mm in 8 minutes. How far does it travel in 7 minutes?

13 Find the average speed (in km/h to the nearest whole number) of a vehicle which travels:

- **a** 350 km in 1 hour and 30 minutes
- **b** 600 km in 2 hours and 15 minutes
- **c** 500 km in 6 hours and 10 minutes
- **d** 64 km in 1 hour and 30 seconds
- **e** 36 000 m in 45 minutes
- **f** 320 m in 10 seconds.

14 Find the distances travelled by a car whose average speed is 68 km/h if the journeys last for the following times. (Answer correct to the nearest kilometre.)

- **a** 3 hours 15 minutes
- **b** 5 hours and 30 minutes
- **c** 30 minutes
- **d** 2 minutes
- **e** 1 hour and 20 minutes
- **f** 4 hours and 10 seconds

15 Find how long will it take a vehicle to travel (correct to the nearest minute):

- **a** 450 km at a speed of 82 km/h
- **b** 50 km at a speed of 60 km/h
- **c** 250 km at a speed of 49 km/h
- **d** 580 000 m at a speed of 62 km/h
- **e** 24 000 m at a speed of 72 km/h
- **f** 100 km at a speed of 1 km/min.

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1E Distance–time graphs

A distance–time graph describes a journey involving different events. Each event is a line segment on the distance–time graph and represents travelling at a constant speed. The steeper the line segment the faster the object is travelling. If the distance–time graph has a horizontal line then the object is not moving or is at rest.

DISTANCE–TIME GRAPHS

Line graph with time on the horizontal axis and distance on the vertical axis.

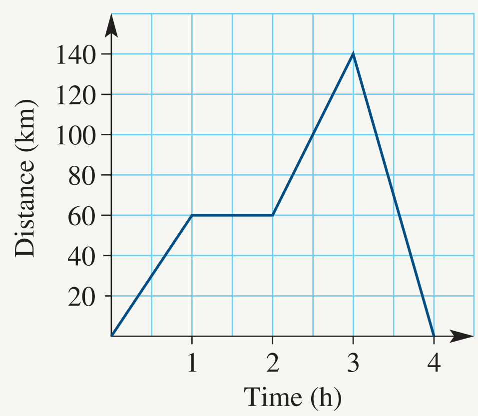
1. Gradient of the line $= \dfrac{\text{Vertical rise}}{\text{Horizontal run}} = \dfrac{\text{Distance}}{\text{Time}} = \text{Speed}$
2. The steepness of a line (or gradient) indicates the speed of the object.
3. A horizontal line indicates that the object is stationary or not moving.

Example 8: Reading a distance–time graph — 1E

The distance–time graph describes a car trip taken last Sunday.

a How long was the rest stop?
b How far did the car travel from its starting point?
c What was the total distance travelled?
d Determine the average speed during the first hour of the trip.

Distance–time graph of a car trip



SOLUTION:

1. Car is at rest when it is not travelling (horizontal line).

a Time for rest stop is 1 hour.

2. Largest value for distance. (140 km)

b Distance is 140 km.

3. The car has travelled on a trip of 140 km and returned.

c Total distance $= 140 \times 82$
$= 280\,\text{km}$

4. Average speed is distanced travelled divided by the time taken.

d $S = \dfrac{D}{T}$
$= \dfrac{60}{1}$
$= 60\,\text{km/h}$
$\therefore$ Average speed is 60 km/h

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Exercise 1E

1 A car journey of 300 km takes 5 hours. The distance–time graph for this journey is shown opposite. For each description below, choose the letter on the graph that matches it.

a A half-hour rest break is taken after travelling 250 km.

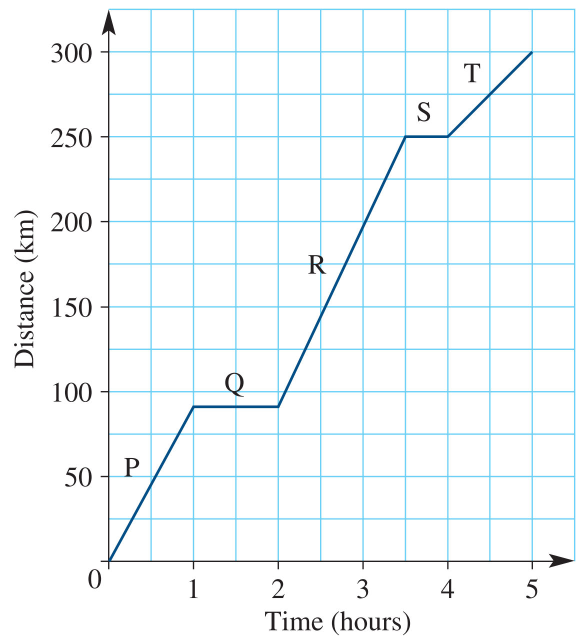
b In the first hour the car travels 90 km.

c The car is at rest for 1 hour, 90 km from the start.

d The car takes 1.5 hours to travel 90 km to 250 km.

e The distance from 250 km to 350 km takes 1 hour.

Distance–time graph of car journey



2 A bicycle journey is shown on distance–time graph opposite.

a What is the total distance travelled?

b What is the time taken for the journey?

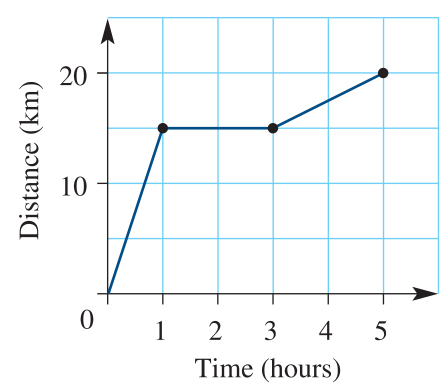
c How long was the cyclist at rest?

d How far had the cyclist travelled after 1 hour?

e How far had the cyclist travelled after 4 hours?

f Determine the average speed during the last two hours of the trip.

Distance–time graph of a bicycle journey



Example 8

3 The distance–time graph describes Alexander's train trip.

a How long was the rest stop?

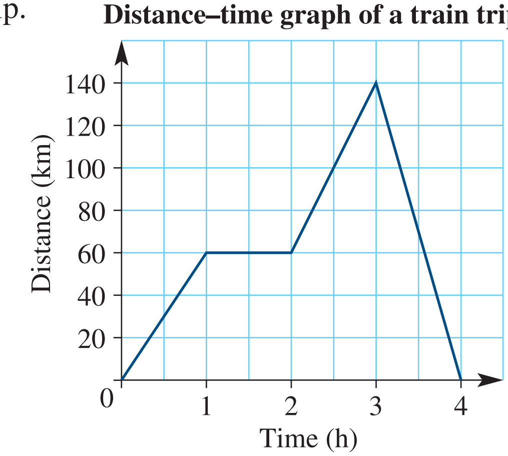
b How far did the train travel from its starting point?

c What was the total distance travelled?

d Determine the average speed during the third hour of the trip.

e Determine the average speed during the last hour of the trip.

Distance–time graph of a train trip



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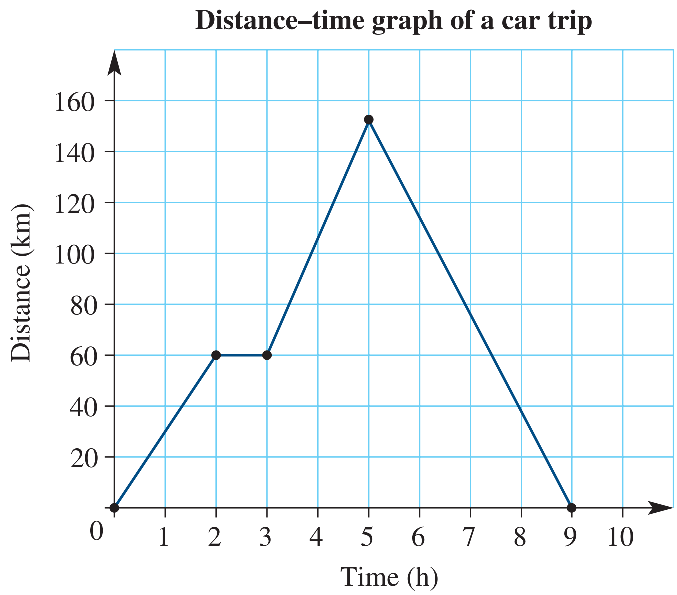
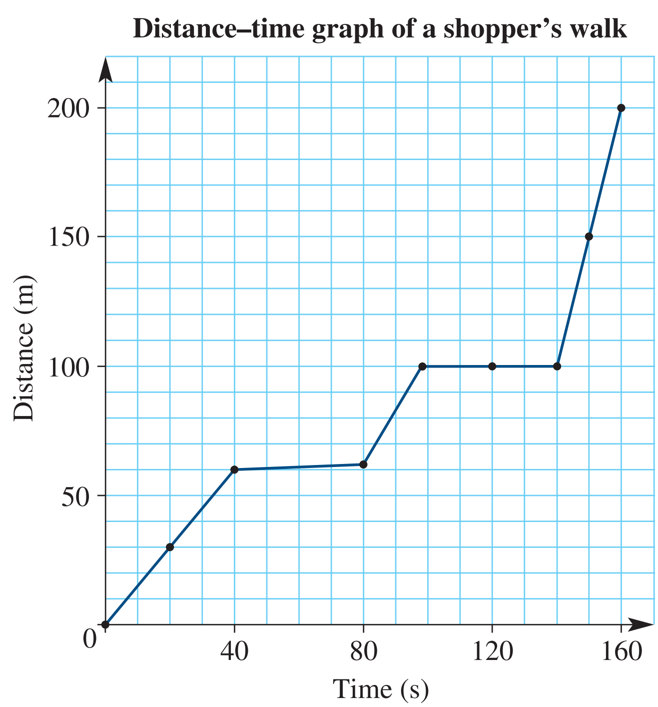
4 The distance–time graph describes Ella's car trip.

- **a** How long was the rest stop?
- **b** How far did the car travel from its starting point?
- **c** How long did the trip take?
- **d** Determine the average speed during the first five hours of the trip.
- **e** Determine the average speed during the last four hours of the trip.
- **f** Determine the average speed for the entire trip, correct to two decimal places.



5 The distance–time graph below shows a shopper's walk in a shopping mall.



- **a** What is the total distance the shopper travelled?
- **b** What was the total time the shopper was not walking?
- **c** What was the total distance the shopper had travelled by the following times?
  - **i** 20 seconds
  - **ii** 80 seconds
  - **iii** 150 seconds

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1F Fuel consumption rate

A motor vehicle's fuel consumption is the number of litres of fuel it uses to travel 100 kilometres. The fuel consumption is calculated by filling the motor vehicle with fuel and recording the kilometres travelled from the odometer. When the motor vehicle is again filled with fuel then record the reading from the odometer and how many litres of fuel it takes to refill the tank. The distance travelled is the difference between the odometer readings.

FUEL CONSUMPTION

$$\text{Fuel consumption} = \frac{\text{Amount of fuel(L)} \times 100\,\text{km}}{\text{Distance travelled (km)}}$$

Example 9: Calculating the fuel consumption rate 1F

a A medium-sized car travelled 850 km using 78.2 L of petrol. What was the fuel consumption?
b A small-sized car travelled 260 km using 16.9 L of petrol. What was the fuel consumption?

SOLUTION:

1. Write the fuel consumption formula.
2. Substitute 78.2 for the amount of fuel and 850 for the distance travelled.
3. Evaluate.
4. Write the answer in words.

a

$$\text{Fuel consumption} = \frac{\text{Amount of fuel} \times 100}{\text{Distance travelled}}$$
$$= \frac{78.2 \times 100}{850}$$
$$= 9.2\,\text{L}/100\,\text{km}$$

∴ Fuel consumption is 9.2 L per 100 km.

5. Write the fuel consumption formula.
6. Substitute 16.9 for the amount of fuel and 260 for the distance travelled.
7. Evaluate.
8. Write the answer in words.

b

$$\text{Fuel consumption} = \frac{\text{Amount of fuel (L)} \times 100}{\text{Distance travelled (km)}}$$
$$= \frac{16.9 \times 100}{260}$$
$$= 6.5\,\text{L}/100\,\text{km}$$

∴ Fuel consumption is 6.5 L per 100 km.

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Exercise 1F

Example 9

1 Find the fuel consumption (litres per 100 km) for each of the following:
- a Ryan's car uses 75 litres of petrol to travel 750 km.
- b Grace's car uses 60 litres of petrol to travel 300 km.
- c A new car uses 120 litres of petrol to travel 2400 km.
- d Matthew's car uses 100 litres of petrol to travel 800 km.
- e Lincoln's car uses 45.5 litres of petrol to travel 520 km.
- f A small car uses 36.9 litres of petrol to travel 600 km.
- g Gemma's sedan uses 55.1 litres of LPG to travel 950 km.
- h A sports car travelled 250 km using 28.5 litres of petrol.
- i Anthony's motor bike uses 167.5 litres of LPG to travel 2500 km.
- j Tahlia's car uses 121.6 litres of petrol to travel 3200 km.

2 Stephanie has bought a used car whose fuel consumption is 7.8 litres petrol per 100 kilometres. She is planning to travel around Australia. Calculate the number litres of petrol Stephanie's car will use on the following distances. Answer correct to the nearest whole number.

- a A trip of 4049 km from Darwin to Perth
- b A trip of 982 km from Sydney to Brisbane
- c A trip of 2716 km from Perth to Adelaide
- d A trip of 658 km from Melbourne to Canberra
- e A trip of 732 km from Adelaide to Melbourne
- f A trip of 309 km from Canberra to Sydney
- g A trip of 3429 km from Brisbane to Darwin

3 Charlie travels 45 km to work and 45 km from work each day.
- a How many kilometres does she travel to and from work in a 5-day working week?
- b Charlie drives a four-wheel drive with a fuel consumption of 8L/100 km to and from work. How many litres of petrol does Charlie use travelling to and from work? Answer correct to one decimal place.
- c What is Charlie's petrol bill for her travel for the week if petrol costs $1.20 per litre?

4 A family car uses LPG at a rate of 15 L/100 km and the gas tank holds 72 litres. How far can it travel on a tank of LPG?

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5 Evie drives a car with a petrol consumption of 9 litres of petrol per 100 km. Petrol costs $1.50 per litre.

- **a** How many litres of fuel does the car use for 300 km?
- **b** How many litres of fuel does the car use for 50 km?
- **c** What is the cost of travelling 100 km?
- **d** What is the cost of travelling 200 km?
- **e** How far can she drive using $10 worth of petrol? Answer to the nearest km.
- **f** How far can she drive using $50 worth of petrol? Answer to the nearest km

6 Max drives a truck whose petrol consumption is 16 L/100 km and the petrol tank holds 90 litres. He is planning a trip from Moorebank to Melbourne. The distance from Moorebank to Melbourne is 840 km. Max filled up the petrol tank at Moorebank.

- **a** What is the distance travelled on one tank of petrol?
- **b** How many litres of petrol are needed on this trip?
- **c** How many times will he need to fill his tank before arriving at Melbourne? Give reasons for your answer.

7 Natalie is planning a trip from Parramatta to Canberra using a car with a fuel consumption of 9.6 L/100 km. The distance from Parramatta to Canberra via the highway is 278 km and avoiding the highway is 363 km. The cost of LPG is 68.5 cents per litre.

- **a** What is the amount of fuel used on the trip via the highway?
- **b** How much will the trip cost via the highway?
- **c** What is the amount of fuel used on the trip avoiding the highway?
- **d** How much will the trip cost avoiding the highway?
- **e** How much money is saved by travelling via the highway?

8 Austin owns an SUV with a fuel consumption of 10.9 L/100 km in the city and 8.4 L/100 km in the country. Austin travels 12 000 km per year in the city and 20 000 km per year in the country. The average cost of petrol is $1.48 per litre in the city and 12 cents higher in the country.

- **a** What is the amount of fuel needed to drive in the city for the year?
- **b** Find the cost of petrol to drive in the city for the year.
- **c** What is the amount of fuel needed to drive in the country for the year?
- **d** Find the cost of petrol to drive in the country for the year.
- **e** What is the total cost of petrol for Austin in one year?
- **f** What is the total cost of petrol for Austin in one year if the average cost of petrol increased to $2.00 in the city and 12 cents higher in the country?

9 Investigate the costs for two common cars on a family trip in your local area. Calculate the cost for the return trip in each case. You will need to determine the distance of the trip, fuel consumption for each car and the average price of fuel in the local area.

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1G Heart rate

Heart rate is the number of heartbeats per minute (bpm). It is measured by finding the pulse of the body. This pulse rate is measured where the pulsation of an artery can be felt on the skin by pressing with the index and middle fingers, such as on the wrist and neck. A heart rate monitor consists of a chest strap with electrodes that transmit to a wrist receiver for display. It is used during exercise when manual measurements are difficult. An electrocardiograph is used by medical professionals to obtain a more accurate measurement of heart rate to assist in the diagnosis and tracking of medical conditions. The resting heart rate is measured while a person is at rest but awake and is typically between 60 and 80 beats per minute.

There are many different formulas used to estimate maximum heart rate (MHR). The most widely used formula is MHR = 220 − Age where age is in years.

HEART RATE

Heart rate is the number of heartbeats per minute (bpm).

MHR = 220 − Age(years)

Example 10: Estimating maximum heart rate 1G

Estimate the maximum heart rate for an 18 year old.

SOLUTION:

1	Write the formula.	MHR = 220 − Age
2	Substitute 18 for age.	= 220 − 18
3	Evaluate.	= 202
4	Write the answer in words.	Maximum heart rate for an 18 year old is estimated to be 202 bpm.

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Example 11: Interpreting trends in heart rate 1G

The table below shows the average resting heart rate for men.

Health	18–25 years	26–35 years	36–45 years	46–55 years	56–65 years	65+ years
Athlete	49–55	49–54	50–56	50–57	51–56	50–55
Excellent	56–61	55–61	57–62	58–63	57–61	56–61
Good	62–65	62–65	63–66	64–67	62–67	62–65
Above average	66–69	66–70	67–70	68–71	68–71	66–69
Average	70–73	71–74	71–75	72–76	72–75	70–73
Below average	74–81	75–81	76–82	77–83	76–81	74–79
Poor	82+	82+	83+	84+	82+	80+

a What is the average resting heart rate for a man aged 47 years in good health?
b What is the average resting heart rate for a man aged 25 years in below-average health?
c What is the health of a man aged 57 years with a resting heart rate of 60?
d What is the health of a man aged 30 years with a resting heart rate of 84?

SOLUTION:

1 Locate the column for the age.
2 Locate row for good health, read value at row/column intersection.

a Age of 47 is in the range 46–55 years. Average rest heart rate is 64–67.

3 Locate the column for the age.
4 Locate row for below-average health, read value at row/column intersection.

b Age of 25 is in the range 18–25 years. Average rest heart rate is 74–81.

5 Locate the column for the age.
6 In that column, locate the heart rate.
7 Read the row header.

c Age of 57 is in the range 56–65 years. Heart rate of 60 is in the range 57–61. Health is excellent.

8 Locate the column for the age.
9 In that column, locate the heart rate.
10 Read the row header.

d Age of 30 is in the range 26–35 years. Heart rate of 84 is in the range 82+. Health is poor.

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Target heart rate

The target heart rate (THR) is the desired range of heart rate during exercise that enables the heart and lungs to receive the most benefit from a workout. This range depends on the person's age, physical condition, gender and previous training. The THR is calculated as a range between 65% and 85% of the MHR. For example, for an 18-year-old with a MHR of 202 the THR is between 131.3 (0.65×202) and 171.7 (0.85×202).

Exercise 1G

Example 10

1 Estimate the maximum heart rate using the formula MHR = 220 − Age for a person who is:

- **a** 20 years old
- **b** 30 years old
- **c** 40 years old
- **d** 50 years old
- **e** 60 years old
- **f** 70 years old
- **g** 80 years old
- **h** 90 years old
- **i** 100 years old.

2 Identify the trends in the maximum heart rate (MHR) with age.

- **a** Draw a number plane with 'Age' as the horizontal axis and 'MHR' as the vertical axis.
- **b** Plot the answers from question **1** on the number plane.
- **c** Join the points to make a straight line.
- **d** Use the graph to estimate the MHR for a person who is 25 years old.
- **e** Use the graph to estimate the MHR for a person who is 38 years old.
- **f** Use the graph to estimate the age of a person with a MHR of 155 bpm.
- **g** Use the graph to estimate the age of a person with a MHR of 175 bpm.

3 Calculate the target heart rate (65% to 85% of the MHR) for questions **1a** to **i**.

Example 11

4 The table below shows the average resting heart rate for women.

Health	18–25 years	26–35 years	36–45 years	46–55 years	56–65 years	65+ years
Athlete	54–60	54–59	54–59	54–60	54–59	54–59
Excellent	61–65	60–64	60–64	61–65	60–64	60–64
Good	66–69	65–68	65–69	66–69	65–68	65–68
Above average	70–73	69–72	70–73	70–73	69–73	69–72
Average	74–78	73–76	74–78	74–77	74–77	73–76
Below average	79–84	77–82	79–84	78–83	78–83	77–84
Poor	85+	83+	85+	84+	84+	85+

- **a** What is the average resting heart rate for a woman aged 35 years in below-average health?
- **b** What is the average resting heart rate for a woman aged 56 years in excellent health?
- **c** What is the health of a woman aged 68 years with a resting heart rate of 78?
- **d** What is the health of a woman aged 37 years with a resting heart rate of 59?

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5 Perform an experiment to measure your heart rate.

Activity	Heart rate
Rest before walk	
End of a 15 minute walk	
3 minutes after the walk	
5 minutes after the walk	

a Copy the table. Measure your resting heart rate and write the result in the table.
b Walk quickly for 15 minutes. Measure your heart rate and write the result in the table.
c Measure your heart rate for 3 and 5 minutes after the walk. Write the results in the table.
d Draw a number plane with ‘Time’ as the horizontal axis and ‘Heart rate’ as the vertical axis.
e Plot the results from the table on the number plane.
f Do you think the first 15 minutes of the graph is a straight line?
g Did your heart rate return to the resting heart rate after 5 minutes?
h Calculate your maximum heart rate using the formula $\text{MHR} = 220 - \text{Age}$.
i Calculate your target heart rate. How does it compare to the results in the table?
j Complete the same experiment by jogging for 15 minutes instead of walking.
k What was the change in your heart rate at the end of the activity?

6 Use your resting heart rate measured in question **5**.
a How many times does your heart beat in 1 hour?
b How many times does your heart beat in 1 day?
c How many times does your heart beat in 1 year?
d How many times has your heart been beating since you were born?
e How many times would your heart beat if you lived to 100 years?

7 Twenty people measured their heart rate using a heart rate monitor. The results were:

64	68	64	72	75	67	91	80	77	73
68	81	73	72	60	62	74	68	55	62

a What is the maximum heart rate?
b What is the minimum heart rate?
c What is the sum of these heart rates?
d Find the mean heart rate. Answer correct to one decimal place.
e Find the interquartile range of these heart rates. Answer correct to one decimal place.
f Find the population standard deviation of these heart rates. Answer correct to one decimal place.

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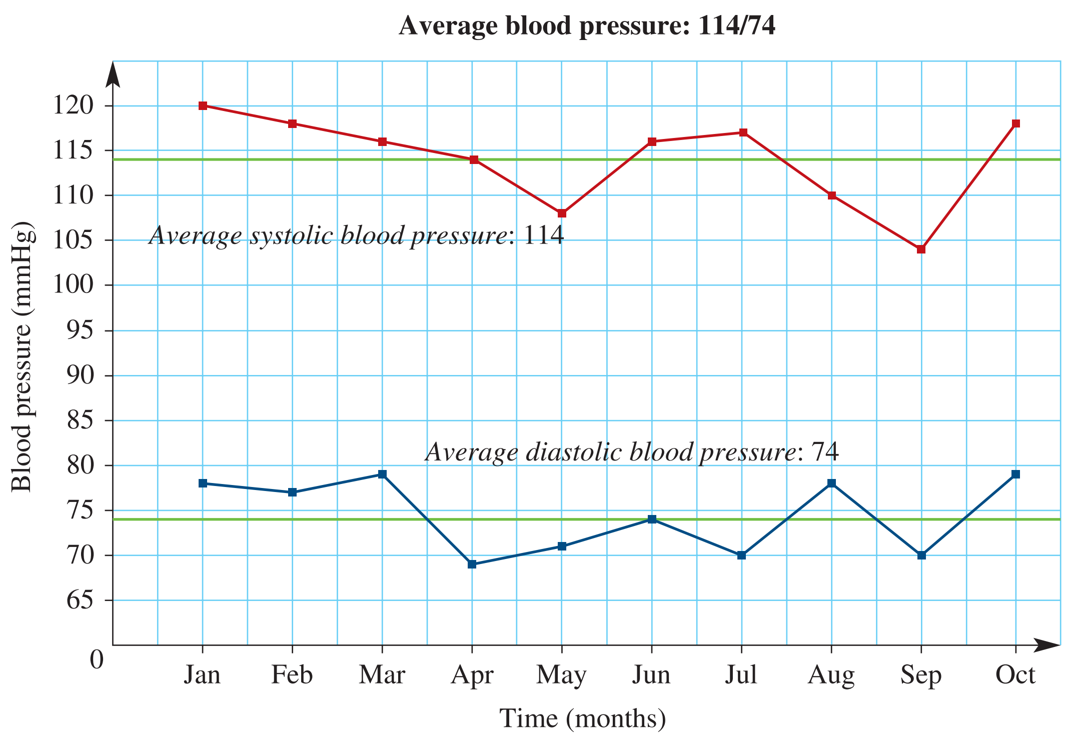
1H Blood pressure

Blood pressure is the pressure of the blood in the arteries as it is pumped around the body by the heart. During each heart beat, blood pressure varies between a maximum (systolic) and a minimum (diastolic) pressure.

Blood pressure is measured by wrapping an inflatable pressure cuff around the upper arm. This cuff is part of a machine called a sphygmomanometer. Blood pressure is expressed in millimetres of mercury (mmHg).

A normal healthy adult has a blood pressure of 120 mmHg systolic and 80 mmHg diastolic, which is expressed as 120/80 mmHg. Many factors affect blood pressure, such as stress, disease, exercise and drugs.

Blood pressure changes to meet your body's needs. A doctor may request a patient's blood pressure be measured on a regular basis. The chart below shows the changes in blood pressure over time for a patient.



BLOOD PRESSURE

- Blood pressure is the pressure of the blood in the arteries as it is pumped around the body.
- Blood pressure varies between a maximum (systolic) and a minimum (diastolic) pressure.
- Blood pressure is measured in mmHg.

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Example 12: Reading a blood pressure table 1H

The table below shows the classification of blood pressure.

Category	Systolic (mmHg)	Diastolic (mmHg)
Normal	<120	<80
Normal to high	120–139	80–89
High	140–179	90–109
Very high	≥ 180	≥ 110

a What is the systolic blood pressure for very high blood pressure?
b What is the diastolic blood pressure for normal to high blood pressure?
c Stephanie has blood pressure of 117/74 mmHg. In what category is she classified?

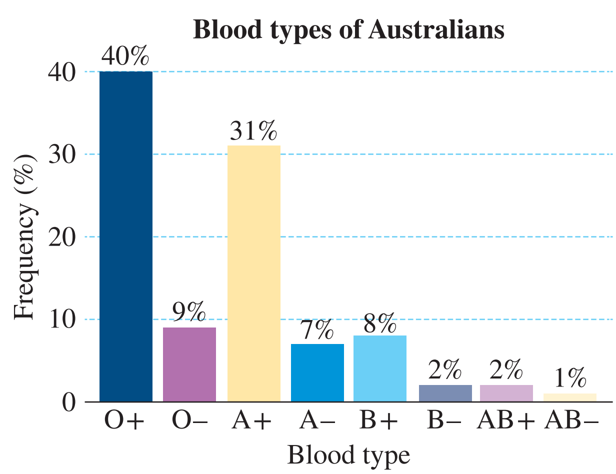
SOLUTION:

1 Read the value in the table.
2 Read the value in the table.
3 Read the value in the table.

a Systolic blood pressure is ≥ 180.
b Diastolic blood pressure is 80–89.
c Blood pressure category is normal.

Blood types

A person's blood type is described by the appropriate letter (A, B, AB or O) and whether or not their blood is Rh positive or Rh negative. The column graph opposite shows the percentage of blood type frequency in Australia (Source: Australian Red Cross Blood Service). Blood is vital to life and for many people blood donors are their lifeline. Most of the blood donated is used to treat people with cancer and other serious illnesses.



Example 13: Calculating the number of people of a particular blood type 1H

The table opposite shows the number of males and females living in NSW. Use the above column graph and this table to answer how many males in NSW have blood type B+.

NSW population	
Males	3.72% million
Females	3.83% million
Total	7.55% million

SOLUTION:

1 Read the percentage of blood type B+ in the column graph (8%).
2 Read the male population of NSW in the table.
3 Multiply the percentage by the population.

a 8% of the Australian population are B+.

$$\begin{aligned}8\% \text{ of } 3.72 \text{ million} &= 0.08 \times 3\,720\,000\\ &= 297\,600\end{aligned}$$

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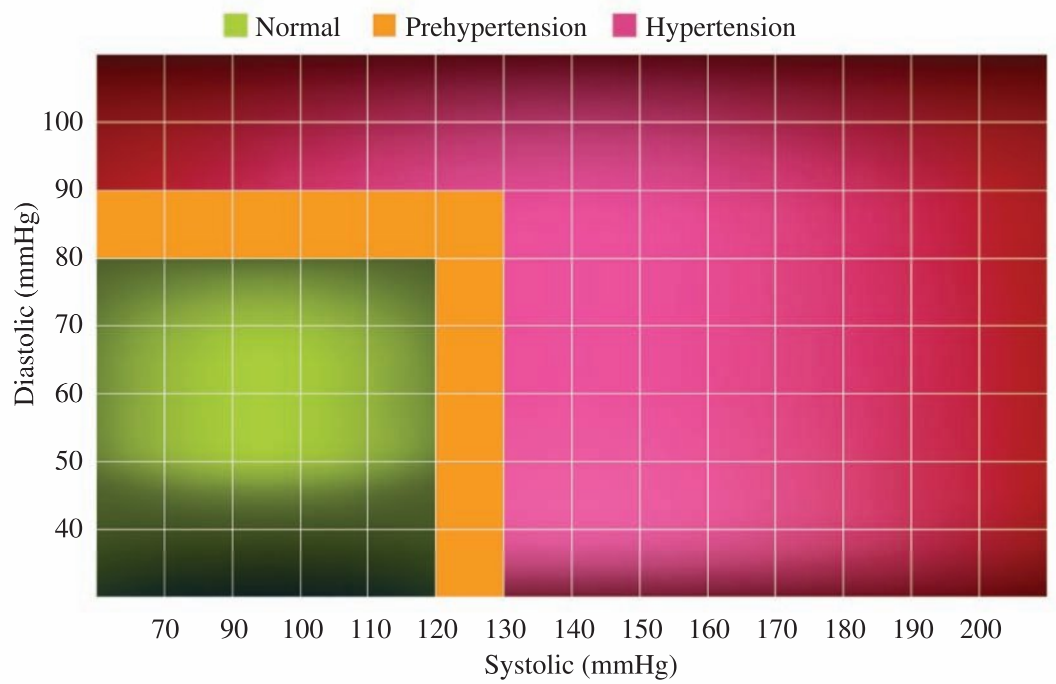
Exercise 1H

Example 12 **1** The blood pressure for normal, prehypertension and hypertension is shown below.

Blood Pressure Management Chart



Category	Systolic	Diastolic
Normal	Less than 120	Less than 80
Pre Hypertension	120 - 139	80 - 89
Hypertension Stage 1	140 - 159	90 - 99
Hypertension Stage 2	160– 179	100 or higher
Hypertensive Crisis	180 or higher	110 or higher

a What is the systolic blood pressure for normal?
b What is the diastolic blood pressure for hypertension stage 1?
c What is the systolic blood pressure for hypertension crisis?
d What is the diastolic blood pressure for prehypertension?
e Caitlin has blood pressure of 125/82 mmHg. In what category is she classified?
f Andrew has blood pressure of 146/96 mmHg. In what category is he classified?
g Heidi has blood pressure of 181/112 mmHg. In what category is she classified?
h Joseph has blood pressure of 165/104 mmHg. In what category is he classified?

2 Owen has a very high blood pressure of 180/110.
a A drug is expected to reduce blood pressure by 20%. What will be his blood pressure?
b A drug is expected to reduce blood pressure by 25%. What will be his blood pressure?

3 Calculate the percentage change in the following blood pressures.
a Gabriel's systolic blood pressure decreases from 144 to 126.
b Lilly's diastolic blood pressure decreases from 96 to 72.

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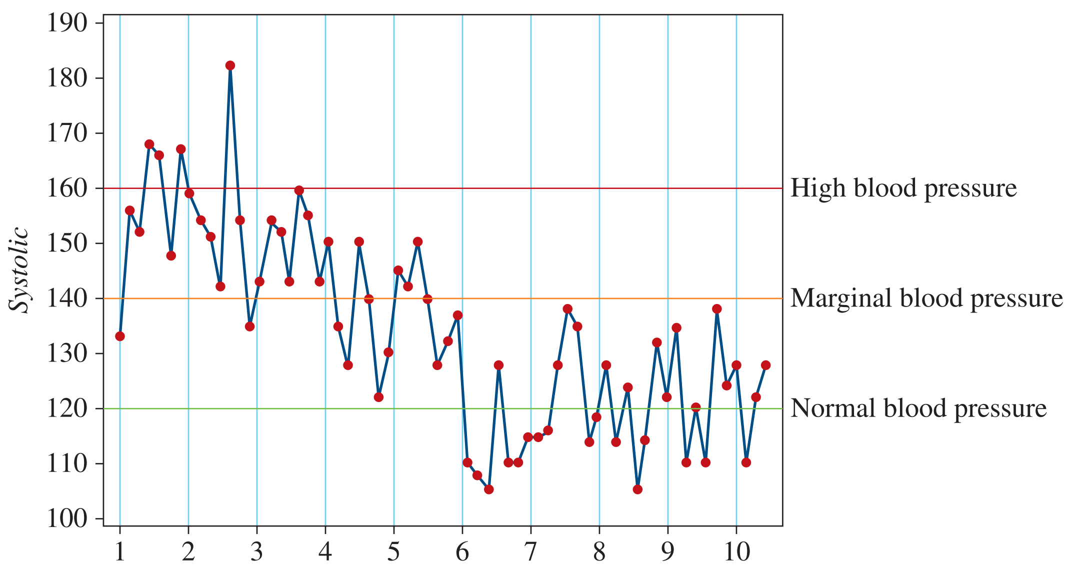
4 Twenty people measured their systolic blood pressure. The results were:

118	123	132	127	140	115	165	133	122	171
128	136	121	117	141	126	139	134	125	130.

a What is the maximum systolic blood pressure?
b What is the minimum systolic blood pressure?
c Find the mean systolic blood pressure. Answer correct to two decimal places.
d Find the population standard deviation systolic blood pressure. Answer correct to two decimal places.

5 Eliza's systolic blood pressure for the past 10 days is shown below.



a What was the highest blood pressure?
b What was the lowest blood pressure?
c What was Eliza's first blood pressure reading on day 2?
d What was Eliza's first blood pressure reading on day 5?
e When did Eliza's blood pressure first reach a normal level?
f How many blood pressure measurements were high (≥160)?
g How many blood pressure measurements were normal (<120)?

Example 13 **6** The tables below show the percentage blood type of Australians and the number of males and females living in NSW.

Blood type percentage			
O+	40%	B+	8%
O−	9%	B−	2%
A+	31%	AB+	2%
A−	7%	AB−	1%

NSW population	
Males	3.72 million
Females	3.83 million
Total	7.55 million

a How many females in NSW have blood type O−?
b How many people in NSW have blood type A+?

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PPT STUDY GUIDE

Summary

Key ideas and chapter summary

Rate

1. Write the rate as a fraction. First quantity is the numerator and 1 is the denominator.
2. Convert the first amount to the required unit.
3. Convert the second amount to the required unit.
4. Simplify the fraction.

Unitary method

1. Find one unit of an amount by dividing by the amount.
2. Multiply the result in step 1 by the number.

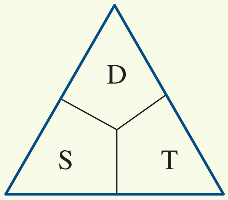
Using rates to make comparisons

Rates are used to solve practical problems such as calculating the best buy, determining costs and calculating wages.

Speed as a rate

Speed is a rate that compares the distance travelled to the time taken.

$S = \frac{D}{T}$ or $T = \frac{D}{S}$ or $D = S \times T$



D – Distance

S – Speed

T – Time

Distance–time graphs

Line graph with time on the horizontal axis and distance on the vertical axis.

- Gradient of the line $= \frac{\text{Vertical rise}}{\text{Horizontal run}} = \frac{\text{Distance}}{\text{Time}} = \text{Speed}$
- The steepness of a line (or gradient) indicates the speed of the object.
- The horizontal line indicates that the object is stationary or not moving.

Fuel consumption rate

The number of litres of fuel used to travel 100 km.

$$\text{Fuel consumption} = \frac{\text{Amount of fuel (L)} \times 100}{\text{Distance travelled (km)}}$$

Heart rate

Heart rate is the number of heartbeats per minute (bpm).

MHR = 220 − Age (years)

Blood pressure

- Blood pressure is the pressure of the blood in the arteries as it is pumped around the body.
- Blood pressure varies between a maximum (systolic) and a minimum (diastolic) pressure.

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Review

Multiple-choice

1 What is $160 in 5 h converted to a rate of $/h?
A $32/h **B** $155/h **C** $165/h **D** $800/h

2 Christian is a delivery driver who delivers one parcel, on average, every 25 minutes. How many hours does it take to deliver 24 parcels?
A 10 **B** 11 **C** 12 **D** 13

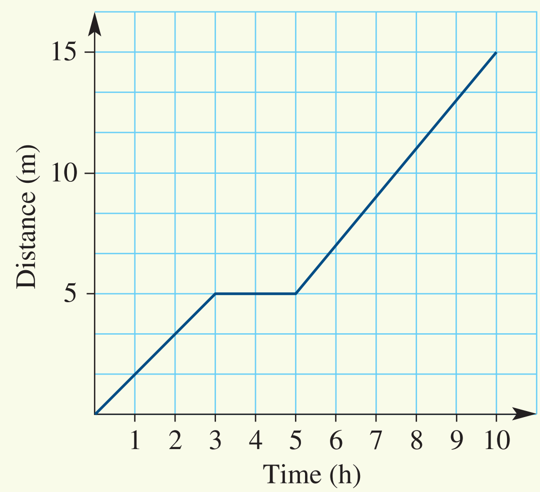
3 A hose fills a 10 L bucket in 20 seconds. What is the rate of flow in litres per hour?
A 0.0001 **B** 30 **C** 1800 **D** 7200

4 Which of the following is the slowest speed?
A 60 km/h **B** 100 m/s **C** 10 000 m/min **D** 6000 m/h

5 How long will it take a vehicle to travel 342 km at a speed of 70 km/h?
A 0.20 h **B** 2.394 h **C** 4.89 h **D** 272 h

Questions **6** and **7** refer to the distance–time graph of the movement of a snail.

Distance–time graph of the movement of a snail



6 The total number of hours the snail was at rest is:
A 2 **B** 3
C 5 **D** 10

7 The speed of the snail in the last 5 hours was:
A 2 m/h **C** 10 m/h
B 5 m/h **D** 15 m/h

8 What is the fuel consumption for a vehicle that travelled 340 km using 51 litres of petrol?
A 7 L/100 km **C** 15 L/100 km
B 9 L/100 km **D** 17 L/100 km

9 Eden's maximum heart rate (MHR) is 175. Which of the following heart rates is within her target heart rate (65–85%)?
A 75 **B** 100 **C** 125 **D** 150

10 A systolic blood pressure decreased from 150 to 120. What is the percentage decrease?
A 10% **B** 20% **C** 30% **D** 40%

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Short-answer

1 Convert each rate to the units shown.

a \$15/kg to \$/g
b 14 400 m/h to m/min
c 120 cm/s to mm/min
d 4800 kg/g to kg/mg
e 14 L/g to mL/kg
f \$3600/g to c/mg

2 If 20 metres of curtain material costs \$580, what would be the cost of 35 metres of the same material?

3 A 5 kg bag of rice costs \$9.20. What is the cost of the following amounts?

a 10 kg
b 40 kg
c 3 kg
d 7 kg
e 500 kg
f 250 kg

4 Calculate the best buy between option 1 and 2.

a Option 1: 5 books for \$110 Option 2: 7 books for \$161
b Option 1: \$175 for 50L Option 2: \$54 for 15L
c Option 1: 35 g for \$357 Option 2: 27 g for \$270
d Option 1: 4 picture frames for \$50 Option 2: 6 picture frames for \$74.70

5 Find the average speed (in km/h) of a vehicle which travels:

a 784 km in 8 hours
b 315 km in 4.5 hours
c 48 km in $\frac{1}{4}$ hour
d 64 km in 40 minutes.

6 Find the distance travelled by a car (correct to the nearest kilometre) whose average speed is 76 km/h if the journey lasts:

a 10 hours
b 4.1 hours
c $3\frac{1}{4}$ hour
d $8\frac{1}{2}$ hours.

7 Maddison runs 200 metres in 45 seconds. How many seconds would it take her to run one kilometre at the same rate?

8 Daniel drives to his mother's house. It takes 45 minutes. Calculate Daniel's average speed if his mother lives 48 km away. Answer correct to the nearest km/h.

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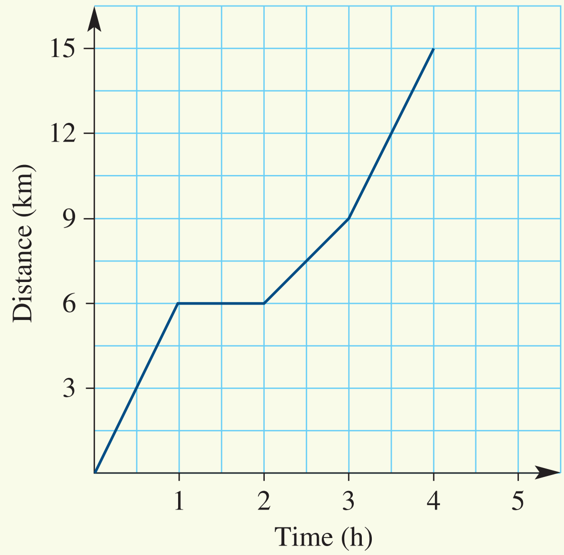
Review

9 How long does it take Stella to drive 180 km along the freeway to work if she manages to average 100 km/h for the trip?

10 The distance–time graph describes the journey of a walker.

a What is the total distance travelled?
b How long was the person actually walking?
c How far had the person walked after:
i 1 hour?
ii 2 hours?
iii 4 hours?
d How long did it take to walk a distance of 12 km?

Distance–time graph of a walker



11 A car travels 960 km on 75 litres of petrol. How far does it travel on 50 litres?

12 Thomas travels 51 km to work and 51 km from work each day.

a How many kilometres does he travel to and from work in a 5-day working week?
b Thomas drives a car with a fuel consumption of 7.5 L/100 km to and from work. How many litres of petrol does Thomas use travelling to and from work per week?
c What is Thomas's petrol bill for work per week if petrol costs are $1.52 per litre?

13 Estimate the maximum heart rate using the formula MHR = 220 − Age for these ages:

a 18 years old
b 28 years old
c 38 years old
d 48 years old

14 The systolic blood pressure for a sample of 20 people is listed below.

203	124	180	210	105	148	161	131	192	125
159	106	170	138	100	120	109	144	190	193

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a What is the minimum?
b What is the maximum?
c What is the range?
d What is the mean?

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2 Networks and paths

Syllabus topic — N1.1 Networks, N1.2 Shortest paths

This topic will develop your skills to be able to identify and use network terminology and to solve problems involving networks.

Outcomes

- Identify and use network terminology.
- Recognise the circumstances when networks can be used to solve a problem.
- Draw a network to represent a map.
- Draw a network to represent information given in a table.
- Define a tree and a minimum spanning tree for a given network.
- Determine and use minimum spanning trees to solve problems.
- Identify the shortest path on a network diagram.
- Recognise when the shortest path is not necessarily the best path.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Literacy worksheet
- Quick Quiz
- Solutions (enabled by teacher)
- Desmos widgets
- Spreadsheets
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

The Interactive Textbook provides a test of prior knowledge for this chapter, and may direct you to revision from the previous years' work.

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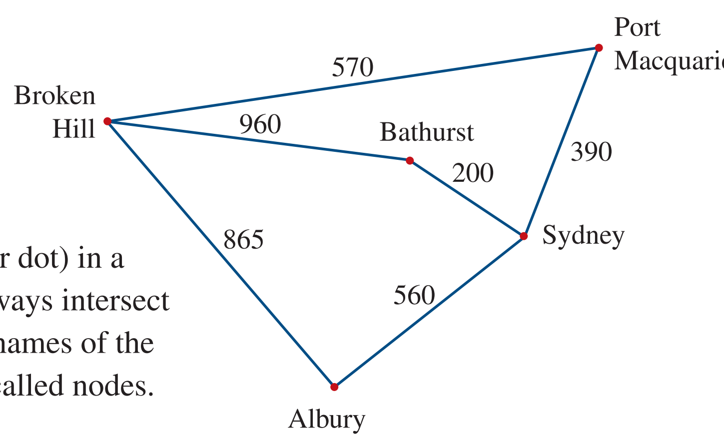
2A Networks

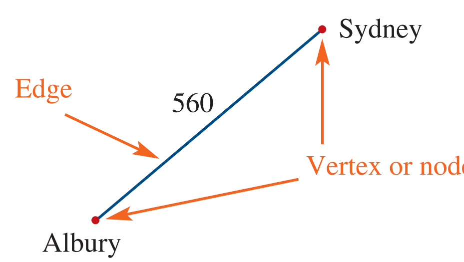
A network is a term to describe a group or system of interconnected objects. There are many situations in everyday life that involve connections between objects. Cities are connected by roads, computers are connected to the internet and people connect to each other through being friends on social media. The diagram below shows the distances in kilometres between some NSW cities. This is referred to as a network or a network diagram. Note that the lengths of the lines in a network diagram are not generally drawn to scale.

Networks: Basic concepts Watch the video in the Interactive Textbook for an illustration of the terms and concepts in action.

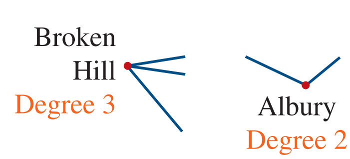
The common terms used in a network are described below.



- A **network diagram** is a representation of a group of objects called vertices that are connected together by lines. Network diagrams are also called **graphs**.
- A **vertex (plural: vertices)** is a point (or dot) in a network diagram at which lines of pathways intersect or branch. In the diagram opposite, the names of the cities are the vertices. Vertices are also called nodes.
- An **edge** is the line that connects the vertices. In the diagram opposite, the line marked with 560 is an edge. Edges can cross each other without intersecting at a node.



- The **degree** of a vertex is the number of edges that are connected to it. The degree of the Broken Hill vertex is 3 because there are three edges attached to the vertex. This is written as deg(Broken Hill) = 3.
  The degree of a vertex is either even or odd.
  - The degree of a vertex is even if it has an even number of edges attached to the vertex. For example, in the above network diagram the vertices of even degree are Port Macquarie, Bathurst and Albury.



  - The degree of a vertex is odd if it has an odd number of edges attached to the vertex. For example, in the above network diagram the vertices of odd degree are Sydney and Broken Hill.
  - There can be multiple edges between vertices, as shown.

- A **loop** starts and ends at the same vertex as shown in the diagram. It counts as one edge, but it contributes two to the degree of the vertex.

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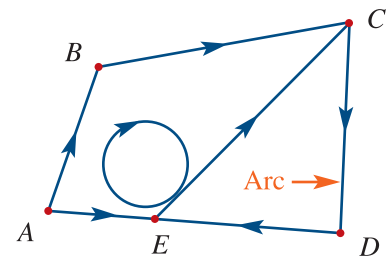
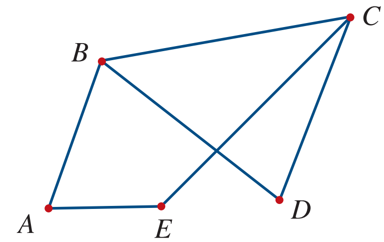
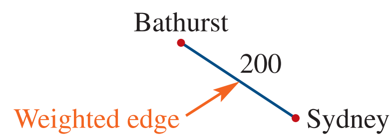
- A **directed** edge, also called an arc, has an arrow and travel is only possible in the direction of the arrow. An **undirected** edge has no arrow and travel is possible in both directions. A network or graph may have both directed and undirected edges.



- In a **directed network** or graph all the edges are directed, as in the diagram opposite, which has five vertices and six edges (arcs). It shows a path can be taken from A to B to C, however there is no path from C to B to A.
- In an **undirected network** or graph all the edges are undirected and travel on an edge is possible in both directions. The diagram opposite is an undirected graph with five vertices and six edges. It shows that there is a path from A to B and from B to A.



- In a **simple network** like the one opposite there are no multiple edges or loops.
- **Labelling of vertices:** in addition to labelling vertices on a diagram, 'labelling of vertices' in a network means listing them all in curly brackets like this, using the network above as an example: $V = \{A, B, C, D, E\}$.
- **Labelling of edges:** An edge between vertex A and B would be labelled (A, B). A loop at B would be (B, B). A complete list of edges for the diagram above would be $E = (A, B), (B, C), (B, D), (C, D), (C, E), (A, E)$.
- A **weighted edge** is an edge of a network diagram that has a number assigned to it that implies some numerical value such as cost, distance or time. The diagram opposite shows a weighted edge that indicates a distance of 200 km between Sydney and Bathurst. See also the first network on the previous page.



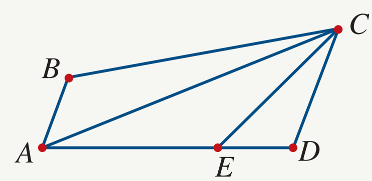
NETWORK

A network is a term to describe a group or system of interconnected objects. It consists of vertices and edges. The edges indicate a path or route between two vertices.

Example 1: Identifying and using network terminology — 2A

For the network shown opposite, find the:

a number of vertices
b number of edges
c degree of vertex C
d number of vertices of odd degree.



SOLUTION:

1	Count the dots in the network diagram.	**a**	Five vertices
2	Count the lines in the network diagram.	**b**	Seven edges
3	Count the number of edges connected to C.	**c**	$\deg(C) = 4$
4	Count the number of edges for each vertex.	**d**	A 3, B 2, C 4, D 2, E 3
5	List the vertices of odd degree.		Two vertices of odd degree (A, E)

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Exercise 2A

1 Copy and complete the following sentences:

a A network is a term to describe a group or system of ____________ objects.

b In a network diagram the vertices are connected together by lines called ____________.

c A ____________ is a point in a network diagram at which lines of pathways intersect.

d The ____________ of a vertex is the number of edges that are connected to it.

e A directed graph is when the edges of a network have ____________.

2 True or false?

a Vertices are represented as a point or dot in a network diagram.

b Directed networks are a connected sequence of the edges showing a route between vertices.

c A loop starts and ends at the same vertex.

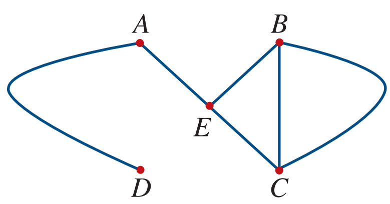
d The degree of a vertex is either even or odd.

e Degree of a vertex is odd if it has an odd number of vertices attached to the edges.

f If an edge has a number assigned to it is called a directed edge.

g The edges in a directed network are usually called arcs.

Example 1 **3** Using the network diagram shown find the:



a number of vertices

b number of edges

c degree of vertex A

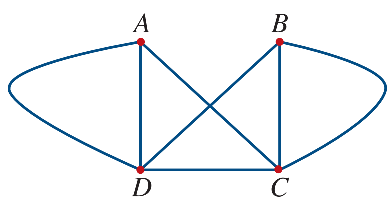
d degree of vertex B

e degree of vertex C

f degree of vertex D

g degree of vertex E.

4 Using the network diagram shown find the:



a number of vertices

b number of edges

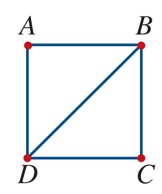
c degree of vertex A

d degree of vertex B

e degree of vertex C

f degree of vertex D.

5 Using the network diagram shown find:



a $\deg(A)$

b $\deg(B)$

c $\deg(C)$

d $\deg(D)$

e the sum of the degrees of all the vertices

f the number of edges.

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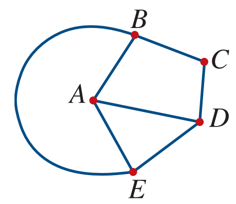
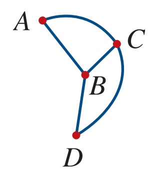
6 Using the network diagram shown find:

a deg(A)
b deg(B)
c deg(C)
d deg(D)
e the sum of the degrees of all the vertices
f the number of edges.



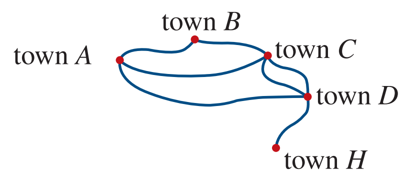
7 Using the network diagram shown find:

a deg(A)
b deg(B)
c deg(C)
d deg(D)
e the sum of the degrees of all the vertices
f the number of edges.



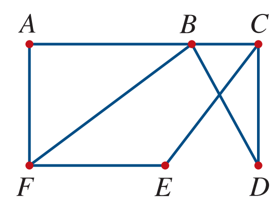
8 Find the degree of the following towns in the network diagram.

a A
b B
c C
d D



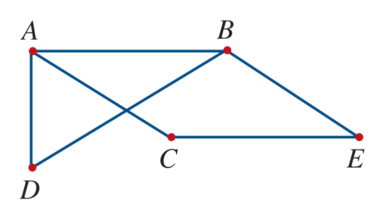
9 Using the network diagram shown find the:

a vertex with the largest degree
b vertex with the smallest degree
c vertices with an even degree
d vertices with an odd degree.



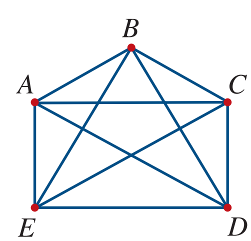
10 Using the network diagram shown find the:

a vertex with the largest degree
b vertex with the smallest degree
c vertices with an even degree
d vertices with an odd degree.



11 Using the network diagram shown find the:

a vertex with the largest degree
b vertex with the smallest degree
c vertices with an even degree
d vertices with an odd degree.



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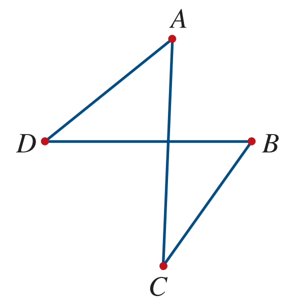
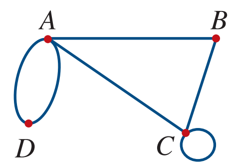
12 Using the network diagram opposite, find the:

a number of vertices
b number of edges
c degree of vertex *A*
d degree of vertex *B*
e degree of vertex *C*
f degree of vertex *D*
g number of vertices of odd degree
h number of vertices of even degree.



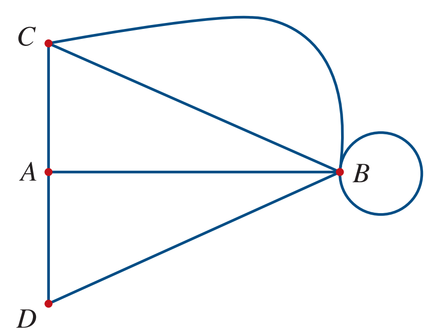
13 Using the network diagram opposite, find the:

a number of vertices
b number of edges
c degree of vertex *A*
d degree of vertex *B*
e number of vertices of odd degree
f number of vertices of even degree.



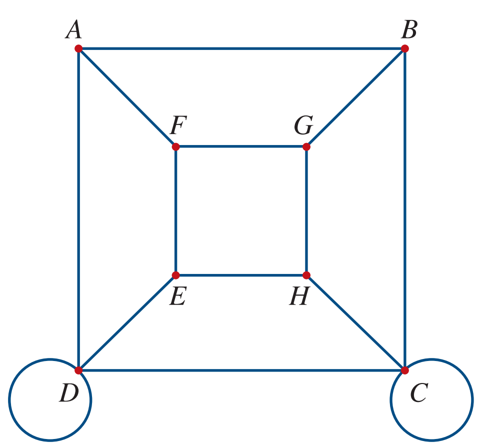
14 Using the network diagram opposite, find the:

a number of vertices
b number of edges
c degree of vertex *B*
d degree of vertex *D*
e number of vertices of odd degree
f number of vertices of even degree.



15 Using the network diagram opposite, find the:

a number of vertices
b number of edges
c degree of vertex *A*
d degree of vertex *C*
e degree of vertex *F*
f number of loops
g number of vertices of odd degree
h number of vertices of even degree.



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2B Travelling a network

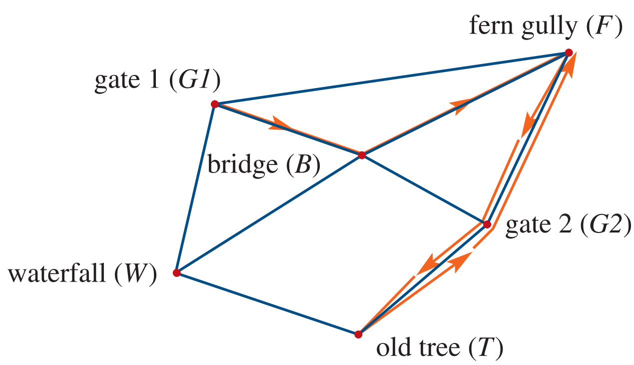
Many practical problems, such as travel routes, that can be modelled by a network involve moving around a graph. To solve such problems you will need to know about a number of concepts to describe the different ways to travel a network.

- A **walk** is a connected sequence of the edges showing a route between vertices where the edges and vertices may be visited multiple times. When there is no ambiguity, a walk in a network diagram can be specified by listing the vertices visited on the walk.

  For example, the network diagram opposite shows a walk in a forest. The forest tracks are the edges (shown in blue) and the places in the forest are the vertices. The red arrows trace out a walk in the forest and is stated as:

  $$G1\text{-}B\text{-}F\text{-}G2\text{-}T\text{-}G2\text{-}F$$

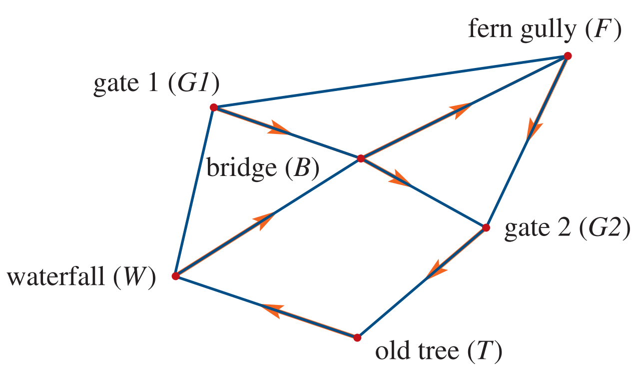
  Note: A walk does not require all of its edges or vertices to be different.



- A **trail** is a walk with no repeated edges. For example, the network diagram opposite shows a trail in a forest. The red arrows trace out a trail in the forest and is stated as:

  $$G1\text{-}B\text{-}F\text{-}G2\text{-}T\text{-}W\text{-}B\text{-}G2$$

  Note: A trail has no repeated edges, however there are two repeated vertices (B and $G2$).

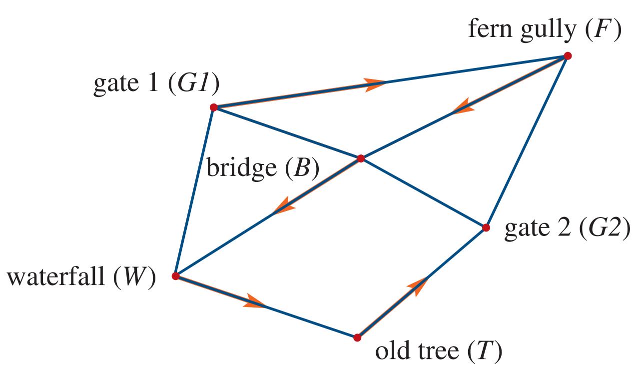


- A **path** is a walk with no repeated vertices. Open paths start and finish at different vertices while closed paths start and finish at the same vertex. Closed paths are also called circuits.

  For example, the network diagram opposite shows a path in a forest. The red arrows trace out a path in the forest and is stated as:

  $$G1\text{-}F\text{-}B\text{-}W\text{-}T\text{-}G2$$

  Note: A path has no repeated edges or vertices.



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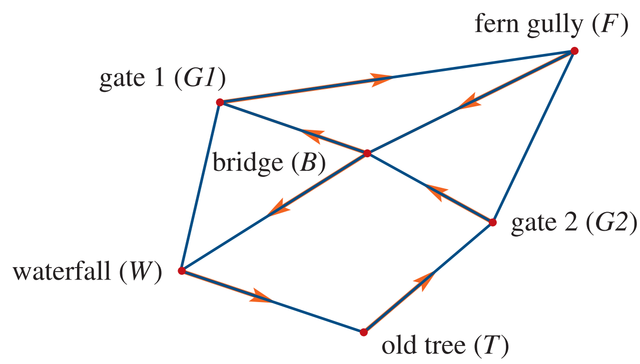
- A **circuit** is a walk with no repeated edges that starts and ends at the same vertex. Circuits are also called closed trails. Alternatively, open trails start and finish at different vertices.

  For example, the network diagram opposite shows a circuit in a forest. The red arrows trace out a circuit in the forest and is stated as:

  $G1\text{-}F\text{-}B\text{-}W\text{-}T\text{-}G2\text{-}B\text{-}G1$

  Note: This circuit starts and ends at the same vertex $(G1)$. There are no repeated edges however the circuit passes through the vertex B twice.

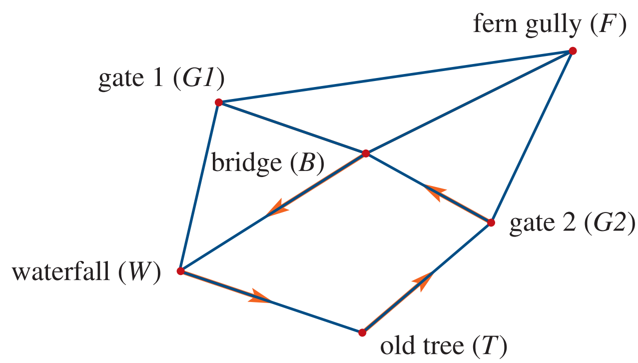


- A **cycle** is a walk with no repeated vertices that starts and ends at the same vertex. There are no repeated edges in a cycle as there are no repeated vertices. Cycles are closed paths.

  For example, the network diagram opposite shows a cycle in a forest. The red arrows trace out a cycle in the forest and is stated as:

  $G2\text{-}B\text{-}W\text{-}T\text{-}G2$

  Note: This cycle starts and ends at the same vertex $(G2)$. There are no repeated vertices or edges.



Travelling through a network Watch the video in the Interactive Textbook to see the five types of routes that can be travelled through networks.

TRAVELLING A NETWORK

Walk is a connected sequence of the edges showing a route between vertices and edges.
Trail is a walk with no repeated edges.
Path is a walk with no repeated vertices.
Circuit is a walk with no repeated edges that starts and ends at the same vertex.
Cycle is a walk with no repeated vertices that starts and ends at the same vertex.

Type of route	Are repeated edges permitted?	Are repeated vertices permitted?
Walk	Yes	Yes
Trail	No	Yes
Path	No	No
Circuit	No	Yes
Cycle	No	No (except first and last)

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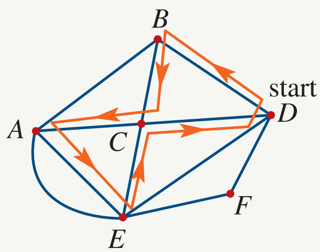
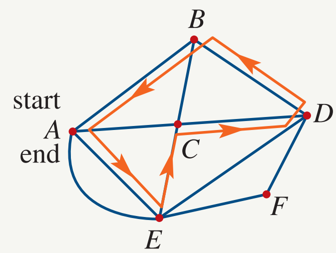
Example 2: Identifying walks, trails, paths, circuits and cycles 2B

Identify the walk in each of the graphs below as a trail, path, circuit, cycle or walk only.

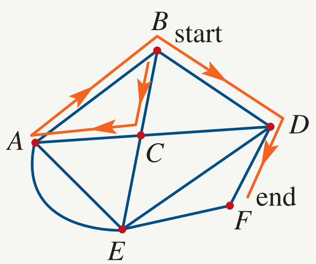
a



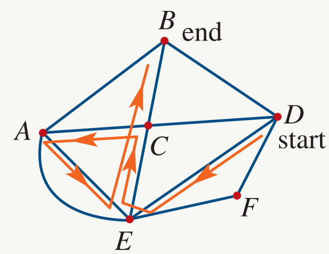
b



c



d

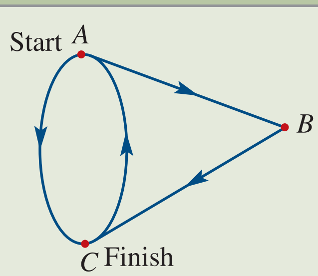
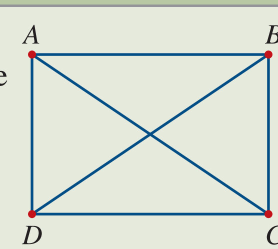


SOLUTION:

1	This walk starts and ends at the same vertex without repeated edges so it is either a circuit or a cycle. The walk passes through vertex *C* twice without repeated edges, so it must be a circuit.	**a** Circuit
2	This walk starts and ends at the same vertex with no repeated edges so it is either a circuit or a cycle. The walk has no repeated vertex so it is a cycle.	**b** Cycle
3	This walk starts at one vertex and ends at a different vertex, so it is not a circuit or a cycle. It has one repeated vertex (*B*) and no repeated edge, so it must be a trail.	**c** Trail
4	This walk starts at one vertex and ends at a different vertex so it is not a circuit or a cycle. It has repeated vertices (*C* and *E*) and repeated edges (the edge between *C* and *E*), so it must be a walk only.	**d** Walk only

Traversable graphs

Many practical problems involve finding a trail in a graph that includes every edge. You can trace out a trail on the graph without repeating an edge or taking the pen off the paper. Graphs that have this property are called traversable graphs. They will be met again in section 2D.

Traversable graph	Non-traversable graph
A traversable graph has a trail that includes every edge. The trail *A*–*B*–*C*–*A*–*C* is one example. 	Not all graphs are traversable. It is impossible to find a trail in a non-traversable graph that includes every edge. 

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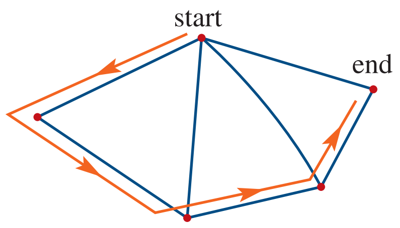
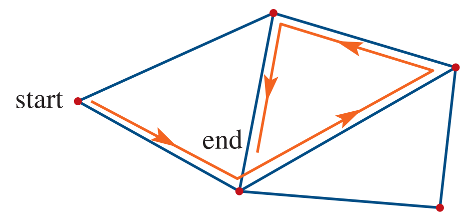
Exercise 2B

Example 2 **1** Identify the walk in each of the graphs below as a trail, path, circuit or walk only.

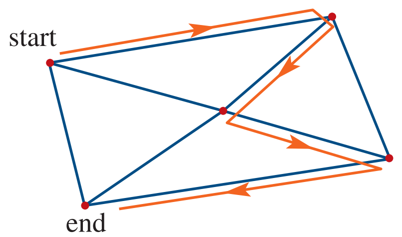
a



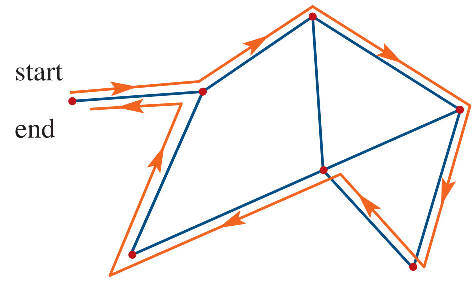
b



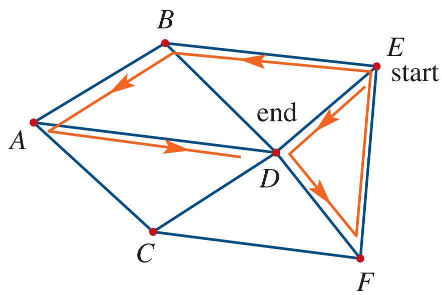
c



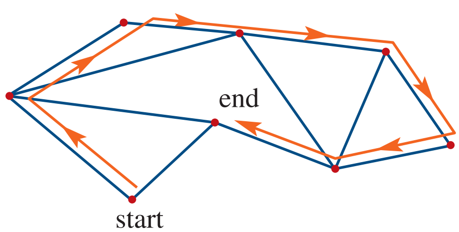
d



e

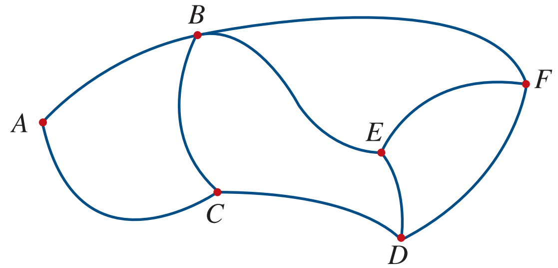


f



2 Using the graph below, identify the walks below as a trail, path, circuit, cycle or walk only.

a *A–B–E–B–F*
b *B–C–D–E–B*
c *C–D–E–F–B–A*
d *A–B–E–F–B–E–D*
e *E–F–D–C–B*
f *C–B–E–F–D–E–B–C–A*



3 Identify the following sequence of vertices as either a trail or a cycle.

a *A–C–B–D–A*
b *P–R–S–Q*
c *M–N–O–P–M*
d *C–B–E–A–F–E–G–D*
e *D–E–A–F–C–B–D*

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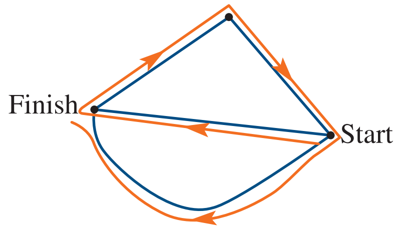
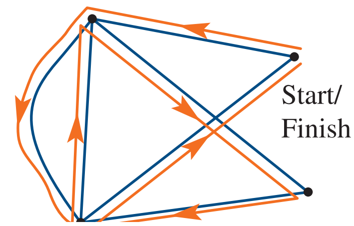
4 Identify the walk in each of the graphs below as a circuit or trail.

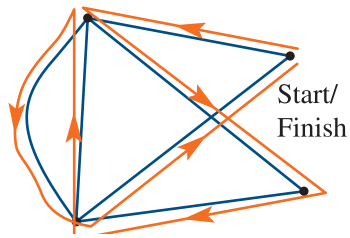
a



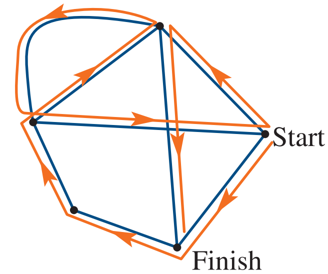
b



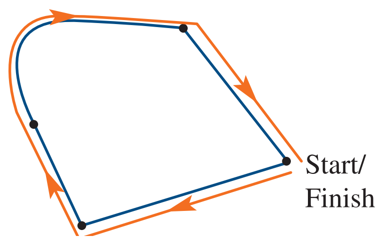
c



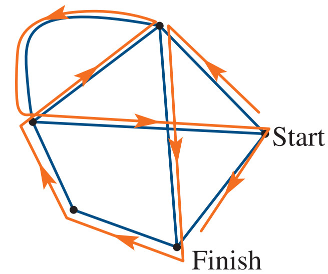
d



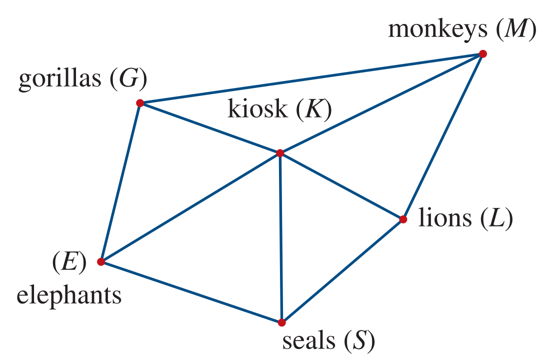
e



f



5 The network diagram below shows the pathway linking five animal enclosures in a zoo to each other and to the kiosk.



a Which of the following represents a trail?

- **i** S-L-K-M-K
- **ii** G-K-L-S-E-K-M
- **iii** E-K-L-K

b Which of the following represents a path?

- **i** K-E-G-M-L
- **ii** E-K-L-M
- **iii** K-H-E-K-G-M

c Which of the following represents a circuit?

- **i** K-E-G-M-K-L-K
- **ii** E-S-K-L-M-K-E
- **iii** K-S-E-K-G-K

d Which of the following represents a cycle?

- **i** K-E-G-K
- **ii** G-K-M-L-K-G
- **iii** L-S-E-K-L

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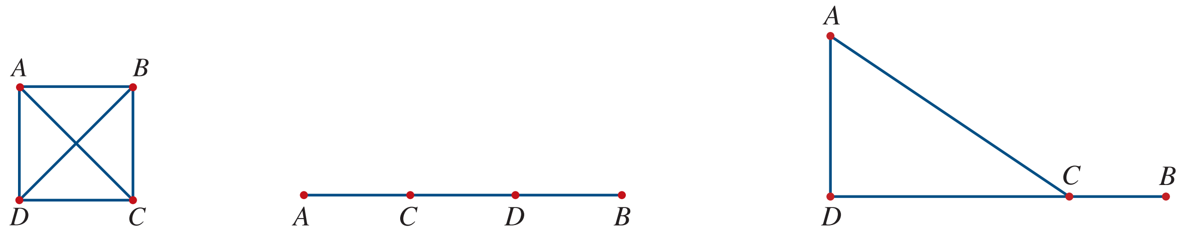
2C Drawing a network diagram

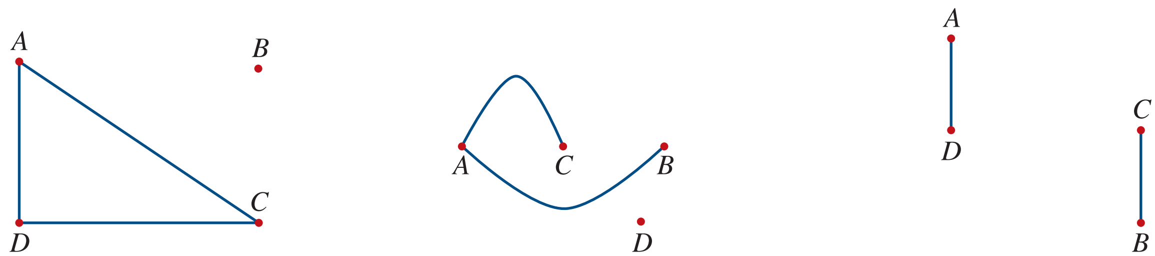
There are many situations in everyday life that involve connections between people or objects. Towns are connected by roads, computers are connected to the internet and families are connected to each other. The network diagram opposite demonstrates some of the connections on social media. When constructing a network, the graphs are either connected or not connected.

Connected graphs

A connected graph has every vertex connected to every other vertex, either directly or indirectly via other vertices. That is, every vertex in the graph can be reached from every other vertex in the graph. The three graphs shown below are all connected.



The graphs are connected because, starting at any vertex, say A, you can always find a path along the edges of the graph to take you to every other vertex. However, the three graphs below are not connected, because there is not a path along the edges that connects vertex A (for example) to every other vertex in the graph.



CONNECTED GRAPH

A graph is connected if every vertex in the graph is accessible from every other vertex in the graph along a path formed by the edges of the graph.

Isomorphic graphs

Different looking graphs can contain the same information. When this happens, we say that these graphs are equivalent or isomorphic. For example, the following three graphs look quite different but, in graphical terms, they are equivalent.

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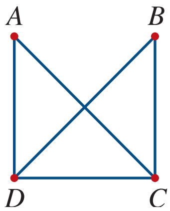
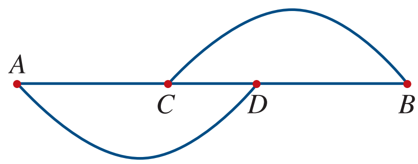
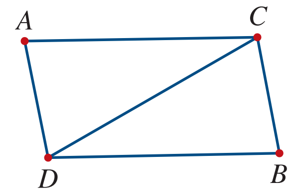
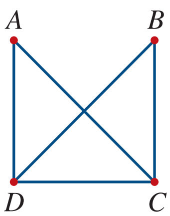
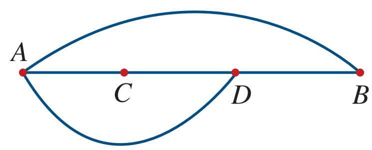


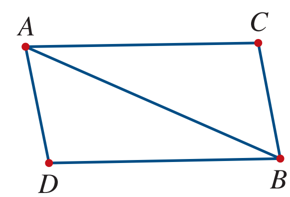




Each of the above graphs has the same number of edges (5), vertices (4), the corresponding vertices have the same degree and the edges join the vertices in the same way (*A* to *C*, *A* to *D*, *B* to *C*, *B* to *D*, and *D* to *C*). However, the three graphs below, although having the same numbers of edges and vertices, are not isomorphic. This is because corresponding vertices do not have the same degree and the edges do not connect the same vertices







ISOMORPHIC GRAPHS

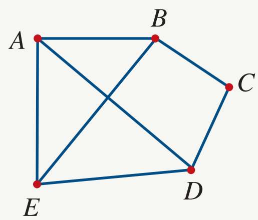
Two graphs are isomorphic (equivalent) if:

- they have the same numbers of edges and vertices
- corresponding vertices have the same degree and the edges connect to the same vertices.

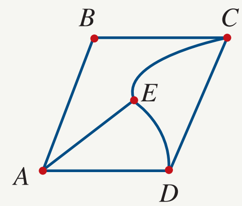
Example 3: Identifying an isomorphic graph 2C

Which of the following graphs is not isomorphic to the other three graphs?

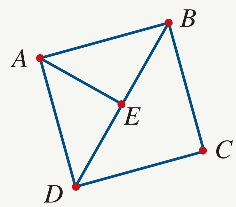
1



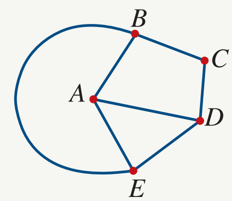
2



3



4



SOLUTION:

1 Check that each graph has the same number of vertices and edges.

Every graph has five vertices and seven edges.

2 Check that corresponding vertices have the same degree.

In Graph 2, vertex *B* has degree 2 and *C* has degree 3; in all others, *B* has degree 3 and *C* has degree 2.

3 Check that edges connect to the same vertices.

In graphs 1, 3 and 4, the edges are *A*–*B*, *A*–*D*, *A*–*E*, *B*–*C*, *B*–*E*, *C*–*D* and *D*–*E*, so these graphs are isomorphic.
Graph 2 does not have edge *B*–*E* and does have edge *C*–*E*, which does not appear in the other graphs, showing again that it is not isomorphic to the others.
Hence, graph 2 cannot be isomorphic to any of the other graphs shown.

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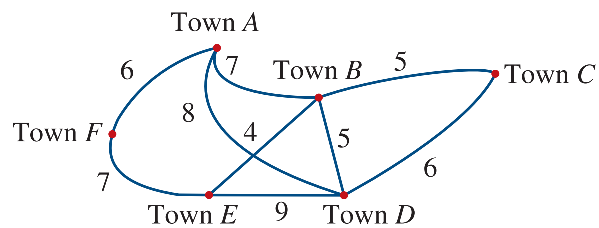
Weighted graphs

The edges of graphs represent connections between the vertices. Sometimes there is more information known about that connection. If the edge of a graph represents a road between two towns, we might also know the length of this road, or the time it takes to travel this road. Extra numerical information about the edge that connects vertices can be added to a graph by writing the number next to the edge. This is called a weighted edge. Graphs that have a number associated with each edge are called weighted graphs.

WEIGHTED GRAPH

A weighted graph is a network diagram that has weighted edges or an edge with number assigned to it that implies some numerical value such as cost, distance or time.

The weighted graph in the diagram on the right shows towns, represented by vertices, and the roads between those towns, represented by edges. The numbers, or weights, on the edges are the distances along the roads. A problem often presented by this road network is, 'What is the shortest distance between certain towns?'



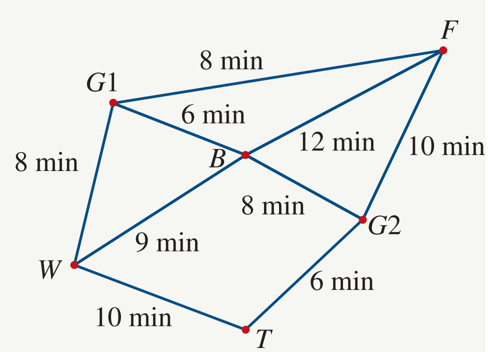
While this question is easily answered if all the towns are directly connected such as Town *A* to Town *B*, the answer is not so obvious if we have to travel through towns to get there such as Town *F* to Town *C*.

Example 4: Solving a practical network problem — 2C

The network diagram opposite is used to model the tracks in a forest connecting a suspension bridge(B), a waterfall (W), a very old tree (T) and a fern gully (F). Walkers can enter or leave the forest through either gate 1 ($G1$) or gate 2 ($G2$). The numbers on the edges represent the times (in minutes) taken to walk directly between these places.

a How long does it take to walk from the bridge directly to the fern gully?

b How long does it take to walk from the old tree to the fern gully via the waterfall and the bridge?



SOLUTION:

1 Identify the edge that directly links the bridge with the fern gully and read off the time.

a The edge is B–F.
The time taken is 12 minutes.

2 Identify the path that links the old tree to the fern gully, visiting the waterfall and the bridge on the way. Add up the times.

b The path is T–W–B–F.
Time = $10 + 9 + 12 = 31$.
The time taken is 31 minutes.

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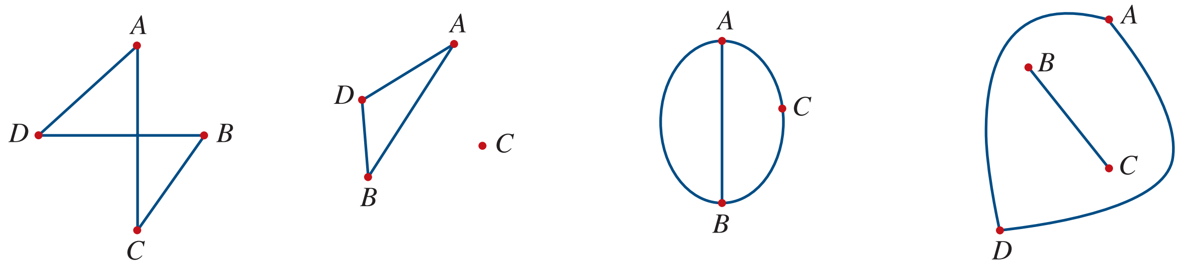
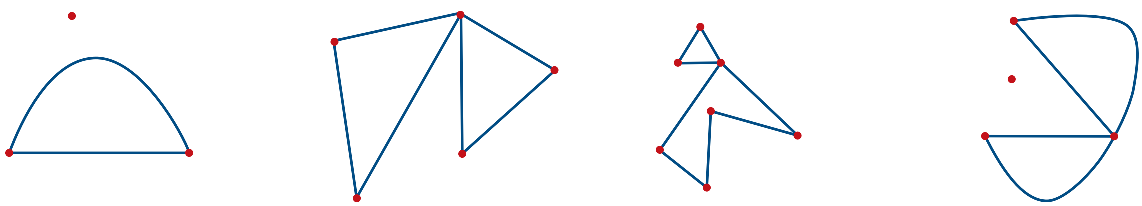
Exercise 2C

1 Which of the following graphs are connected in each question?

a Graph 1 Graph 2 Graph 3 Graph 4



b Graph 1 Graph 2 Graph 3 Graph 4



2 Draw a connected graph with:

a three vertices and three edges

b three vertices and five edges

c four vertices and six edges

d five vertices and five edges.

3 Draw a graph that is not connected with:

a three vertices and two edges

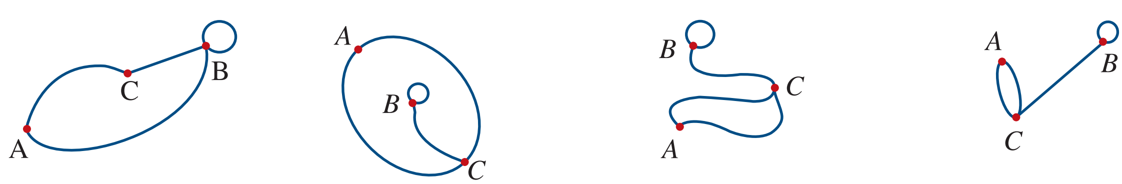
b four vertices and three edges

c four vertices and four edges

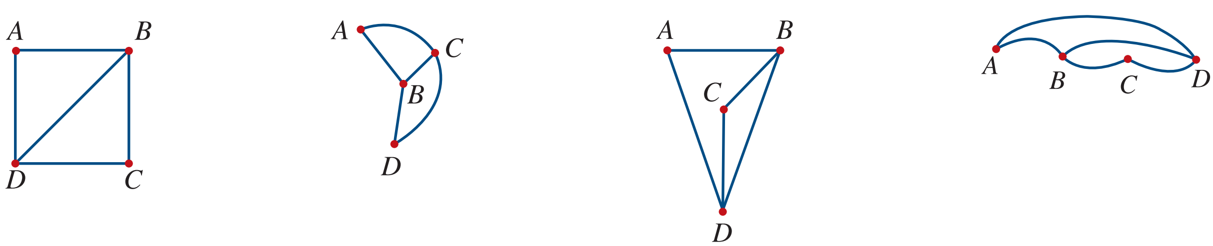
d five vertices and three edges.

Example 3 **4** Which of the following graphs is not isomorphic to the other three graphs in each question?

a Graph 1 Graph 2 Graph 3 Graph 4



b Graph 1 Graph 2 Graph 3 Graph 4



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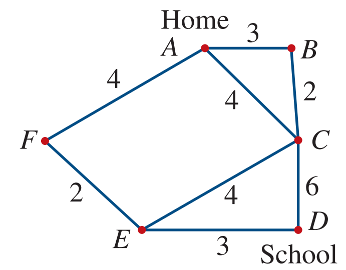
Example 4

5 Evelyn has drawn a network diagram to represent several streets for travelling from her home to school. The numbers indicate the times in minutes.

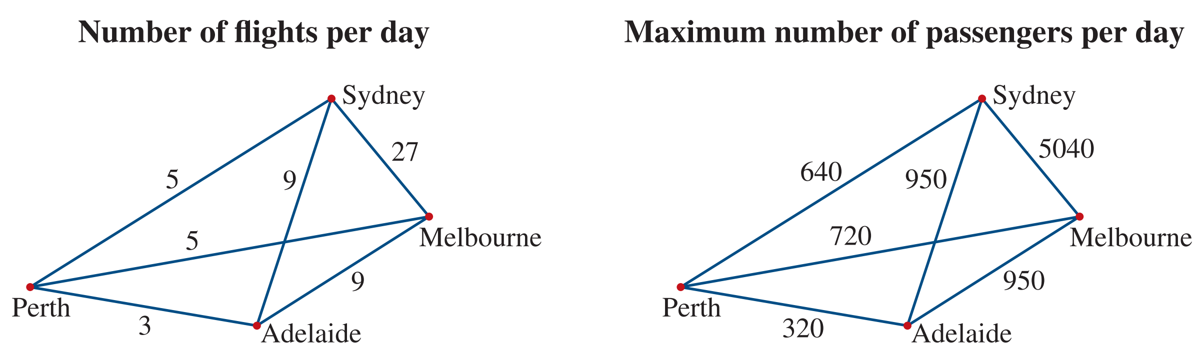


a How long does it take to walk from home to school using the following paths?
  i *A–F–E–D*
  ii *A–F–E–C–D*
  iii *A–C–D*
  iv *A–B–C–D*
  v *A–C–E–D*

b Which of the above walks is the longest journey?

c Which of the above walks is the shortest journey?

6 The network graph below shows details about air travel between Australian cities. The first graph shows the number of flights in each direction between cities, and the second graph shows capacity in each direction.



a How many flights are available per day from Sydney to Adelaide?

b How many flights are available per day from Sydney to Melbourne?

c How many flights are available per day between these Australian cities?

d What is the maximum number of passengers per day from Melbourne to Adelaide?

e What is the maximum number of passengers per day from Sydney to Perth?

f What is the maximum number of passengers per day from Melbourne to Perth?

g What is the maximum number of passengers per day that can fly out of Sydney?

h Seth is wishing to fly from Perth to Melbourne but is told that the direct flights are fully booked. List any other ways of completing this journey.

i What is the greatest number of people per day that can be flown from Perth to Melbourne?

j Amy is wishing to fly from Sydney to Melbourne but is told that the direct flights are fully booked. List any other ways of completing this journey.

k What is the greatest number of people per day that can be flown from Sydney to Melbourne?

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2D Eulerian and Hamiltonian walks

Eulerian trails and circuits

An Eulerian trail is a trail that uses every edge of a graph exactly once. Eulerian trails start and end at different vertices. Similarly, an Eulerian circuit is a circuit that uses every edge of a graph exactly once. Eulerian circuits start and end at the same vertex. Eulerian trails and circuits are important for some real-life applications. For example, if a graph shows towns as vertices and roads as edges, then being able to identify a route through the graph that follows every road is important for mail delivery. A graph with an Eulerian trail is an example of a traversable graph.

An Eulerian trail will exist if the graph is connected and has exactly two vertices with an odd degree. These two vertices of odd degree will form the start and end of the Eulerian trail. Eulerian circuits will exist if every vertex of the graph has an even degree. These results were discovered by the Swiss mathematician called Leonhard Euler.

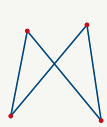
EULERIAN TRAIL	EULERIAN CIRCUIT
A trail that uses every edge of a graph exactly once and starts and ends at different vertices. Eulerian trails exist if the graph has exactly two vertices with an odd degree.	A circuit that uses every edge of a network graph exactly once, and starts and ends at the same vertex. Eulerian circuits exist if every vertex of the graph has an even degree.

Eulerian trails and circuits Watch the video in the Interactive Textbook to see them in action.

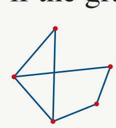
Example 5: Identifying Eulerian trails and circuits 2D

For each of the following graphs, determine whether the graph has an Eulerian trail, an Eulerian circuit or neither. Show one example if the graph has an Eulerian trail or Eulerian circuit.

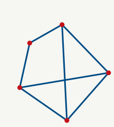
a



b



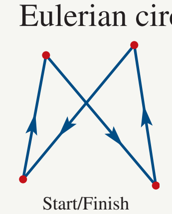
c



SOLUTION:

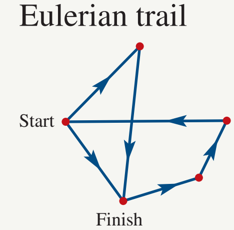
1 All the vertices in the graph have an even degree (degree 2).
2 Even degree indicates there is an Eulerian circuit.
3 Start and finish from the vertex on the bottom left-hand side and travel through each edge once.

a Eulerian circuit



4 Two of the vertices in the graph have an odd degree (degree 3) and the remaining vertices have an even degree (degree 2)
5 Two odd degrees indicates there is an Eulerian trail.
6 Start and finish from the vertices with the odd degrees.

b Eulerian trail



7 Four vertices are odd and one is even. No Eulerian trail or circuit.

c Neither

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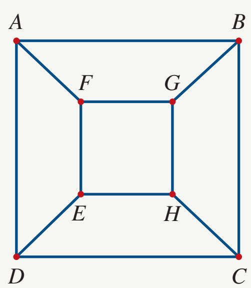
ENRICHMENT: Hamiltonian paths and cycles

Eulerian trails and circuits are focused on the edges (though the degree of the vertices will tell you if walk is Eulerian). Hamiltonian paths and cycles are focused on the vertices. A Hamiltonian path passes through every vertex of a graph once and only once. It may or may not involve all the edges of the graph. A Hamiltonian cycle is a Hamiltonian path that starts and finishes at the same vertex. Hamiltonian paths and cycles have real-life applications where every vertex of a graph needs to be visited, but the route taken is not important. For example, if the vertices of a graph represent people and the edges of the graph represent email connections between those people, a Hamiltonian path would ensure that every person in the graph received the email message. Unlike Eulerian trails and circuits, Hamiltonian paths and cycles do not have a convenient rule or feature that identifies them. Inspection is the only way to identify them. Hamiltonian paths and cycles are named after an Irish mathematician called Sir William Hamilton.

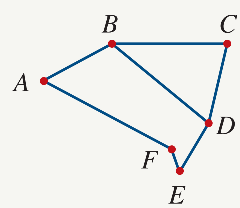
HAMILTONIAN PATH	HAMILTONIAN CYCLE
A path passes through every vertex of a graph once and only once.	A Hamiltonian path that starts and finishes at the same vertex.

Example 6: Identifying a Hamiltonian path and cycle 2D

a List a Hamiltonian path for the network graph below that starts at *A* and finishes at *D*.



b Identify a Hamiltonian cycle for the network graph below starting at *A*.



SOLUTION:

1 A Hamiltonian path involves all the vertices but not necessarily all the edges.

a *A–F–G–B–C–H–E–D*

2 The solution *A–F–G–B–C–H–E–D* is not unique. There are other solutions such as *A–F–E–H–G–B–C–D*.

3 A Hamiltonian circuit is a Hamiltonian path that starts and finishes at the same vertex.

b *A–B–C–D–E–F–A*

4 The solution *A–B–C–D–E–F–A* is not unique. There are other solutions such as *A–F–E–D–C–B–A*.

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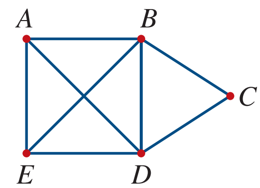
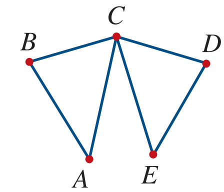
Exercise 2D

1 A network graph is shown opposite.
  a What is the degree of each vertex?
  b Why does this graph have an Eulerian trail?
  c List the Eulerian trail.

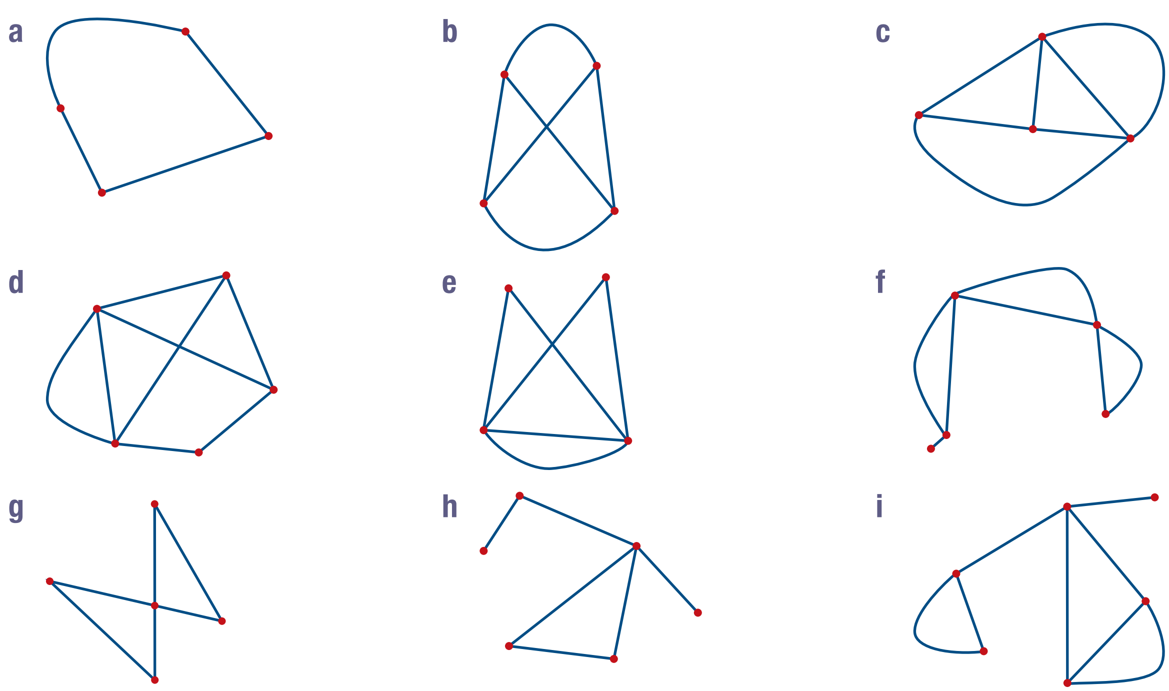


2 A network graph is shown opposite.
  a What is the degree of each vertex?
  b Why does this graph have an Eulerian circuit?
  c List an Eulerian circuit.

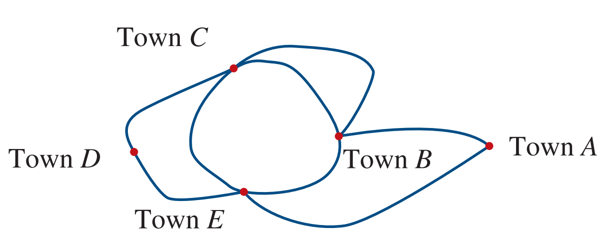


Example 5

3 For each of the following graphs, determine whether the graph has an Eulerian trail, an Eulerian circuit or neither. Show one example if the graph has an Eulerian trail or circuit.



4 A road inspector lives in town A and is required to inspect all roads connecting the neighbouring towns B, C, D and E.
  a Is it possible for the inspector to travel over every road linking the five towns only once and return to town A? Explain.
  b Show one possible route he can follow.



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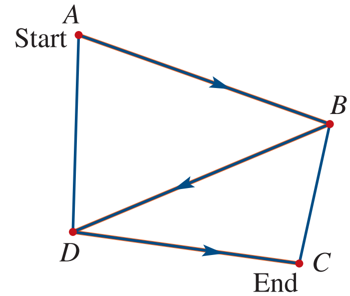
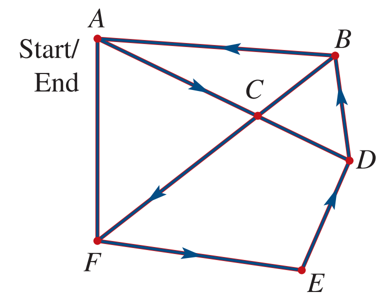
5 A network graph is shown opposite.

a List the path shown in the graph.
b Does the path pass through every vertex?
c Does the path pass through every edge?
d ENRICHMENT: Why is this path a Hamiltonian path?
e ENRICHMENT: List another Hamiltonian path starting at A.



6 A network graph is shown opposite.

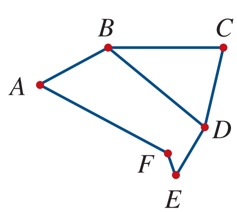
a List the path shown in the graph.
b Does the path pass through every vertex?
c Does the path pass through every edge?
d ENRICHMENT: Why is this path a Hamiltonian cycle?
e ENRICHMENT: List another Hamiltonian cycle starting at A.



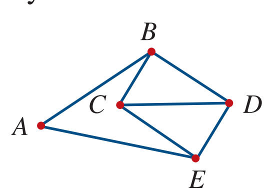
Example 6

7 ENRICHMENT: List a Hamiltonian cycle for each of the following.

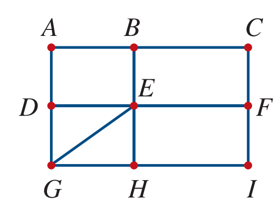
a



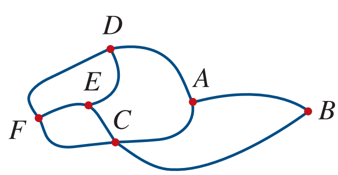
b



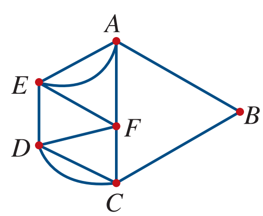
c



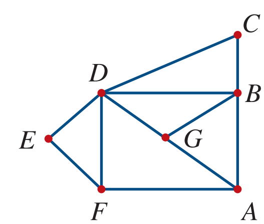
d



e

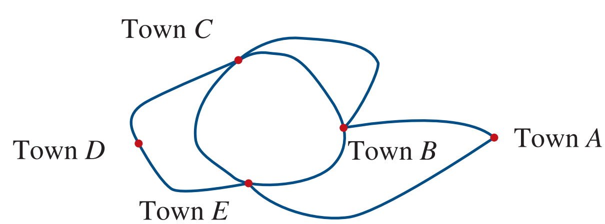


f



8 ENRICHMENT: A tourist wants to visit each of five towns shown in the graph opposite only once. Identify one possible route for the tourist to start the tour at:

a C and finish at A. What is the mathematical name for this route?
b E and finish at E. What is the mathematical name for this route?



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2E Network problems

Example 7: Drawing a network diagram to represent a map 2E

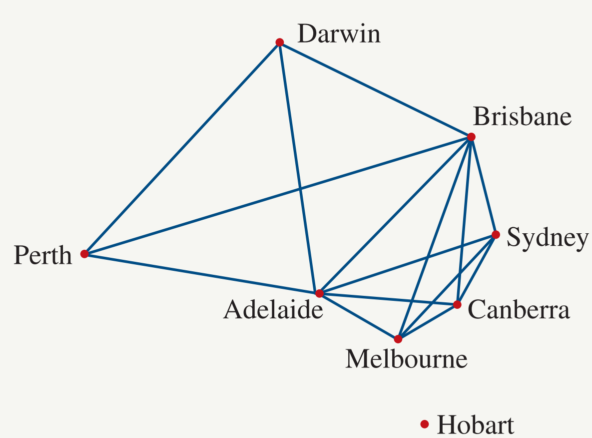
The map shows the main highways between the capital cities of the states and territories of Australia. Construct a network diagram of the main highway connections between the cities. Let each capital city be a vertex and the highway route between the cities an edge.

SOLUTION:

1. Vertices are the dots of the network diagram. In this situation the capital city will be a vertex.
2. Edges are the connections or pathways between the vertices of the network diagram.
3. Start by drawing a dot for each vertex (capital city).
4. Label each vertex with the name of the capital city.
5. Draw a line to represent an edge if the capital cities have a highway route between them
6. The network diagram is not connected, as Hobart does not have a highway linking it to any other city.
7. The proportions of the network diagram do not have to match the real-life situation.

The vertices will be Brisbane, Sydney, Canberra, Melbourne, Hobart, Adelaide, Perth and Darwin.

Highway routes connect the cities with the exception of Hobart.



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Example 8: Drawing a network diagram to represent a table 2E

A group of four students worked in pairs on four different problems. The table below shows the problem number and the two students who found the correct solution to that problem. A network diagram is to be constructed to represent the table.

Problem	Students who solved it	
1	Darcy	Beau
2	Alyssa	Beau
3	Alyssa	Claire
4	Darcy	Claire

a What will be the vertices of the network diagram?
b What will be an edge in the network diagram?
c Draw a network diagram to represent the information in the table.
d Draw an isomorphic graph of the network diagram.
e Which students have not been able to solve a problem together?

SOLUTION:

1 Vertices are the dots of the network diagram. In this situation the student will be a vertex.

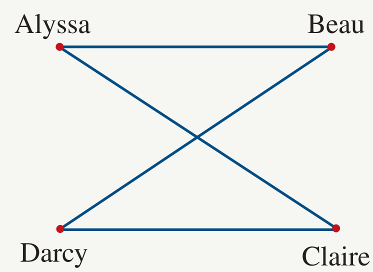
a Alyssa, Beau, Claire and Darcy.

2 Edges are the connections or pathways between the vertices of the network diagram.

b Edges are drawn if the students have worked together to solve the problem.

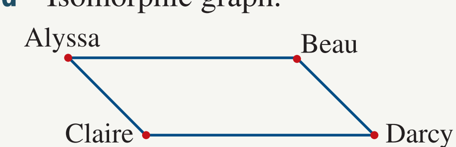
3 Start by drawing a dot for each vertex (student).
4 Label each vertex with the student's name.
5 Draw a line to represent an edge if the students have worked together to solve the problem.
6 The network diagram is connected since the path along the edges can take you to every other vertex.

c Network diagram



7 Isomorphic graphs have the same number of vertices and edges. They must also have to show the same connections.

d Isomorphic graph.



8 Vertices not connected with an edge represent the students who have not been able to solve a problem.

e Alyssa and Darcy, Beau and Claire

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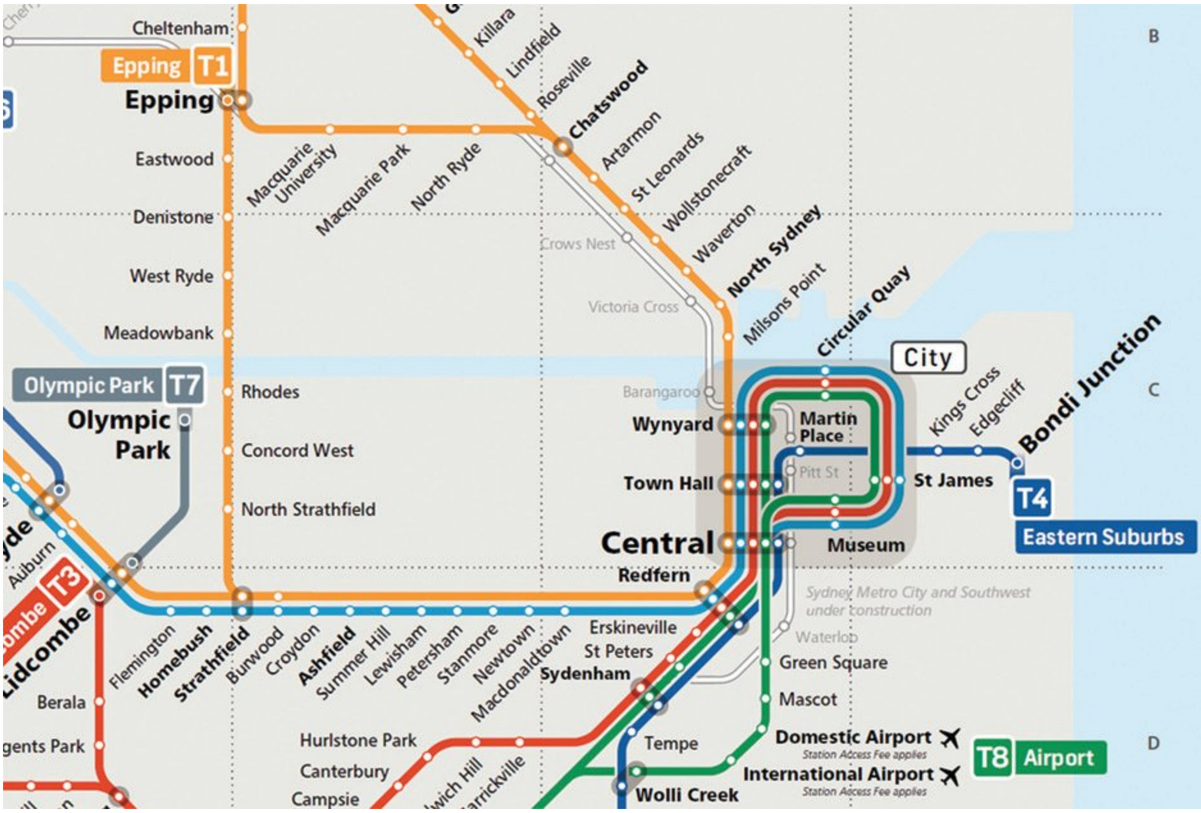
Exercise 2E

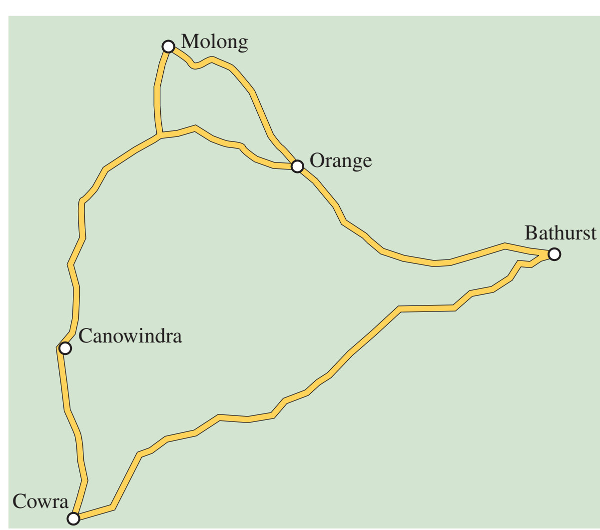
1 This is a network diagram for Sydney trains.



a What are the vertices of the network diagram?
b What are the edges in the network diagram?
c Is this network diagram connected?
d How many vertices are there in the city circle (shaded circle)?
e Draw a network diagram to represent the city circle.

Example 7

2 A map from a NSW region is shown on the right.



a What are the vertices of the network diagram?
b What are the edges in the network diagram?
c Is this network diagram connected?
d Draw an isomorphic graph to this network diagram.

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Example 8

3 Four friends live close to each other. The table opposite shows the friends and the number of minutes to walk between their homes.

a Draw a network diagram to represent the information in the table.

b What are the vertices of the network diagram?

c What does a weighted edge represent in the network diagram?

d Which friends do not have a direct path between their homes?

e What is the shortest total walking time for Alex to leave home and visit Zac, Max and Harvey in that order, and to return home. Ignore any time spent in each house.

Friends		Minutes to walk between homes
Alex	Zac	1
Harvey	Zac	3
Max	Zac	2
Harvey	Max	4
Alex	Harvey	2

4 There are six motorways between six cities labelled A, B, C, D, E and F. The table opposite shows which cities are linked by the motorways and the length of each one in kilometres.

a Draw a network diagram to represent the information in the table.

b What are the vertices of the network diagram?

c What does a weighted edge represent in the network diagram?

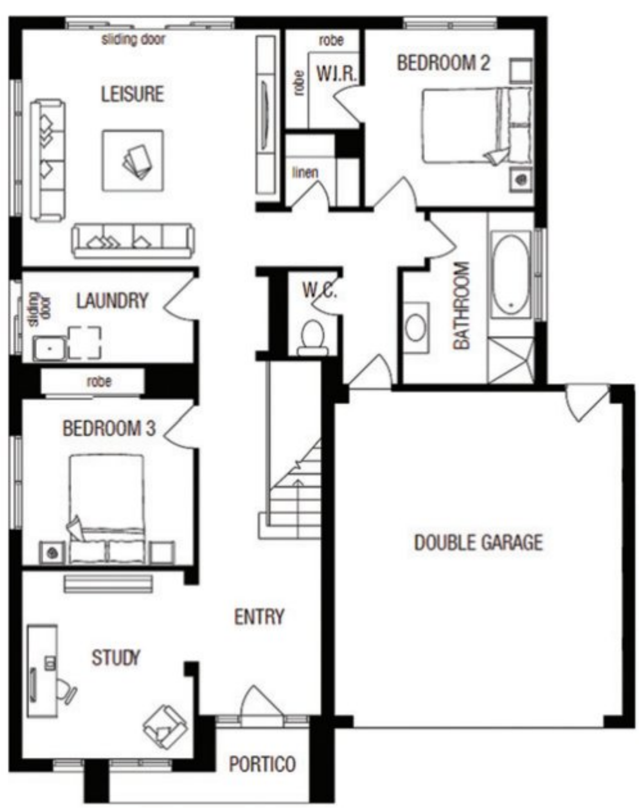
d Which cities are not directly linked to city A?

e How would you travel from city F to city D?

f What is the shortest journey between city F and city D?

	A	B	C	D	E	F
A	–	–	–	27	51	35
B	–	–	48	24	–	–
C	–	48	–	12	–	–
D	27	24	12	–	–	–
E	51	–	–	–	–	–
F	35	–	–	–	–	–

5 The first floor plan of a house is shown opposite. Draw a network diagram by letting the rooms be the vertices and the doorways the edges. Make the hall a vertex.



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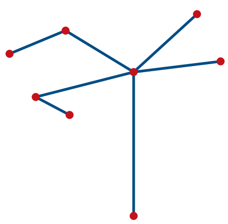
2F Minimal spanning trees

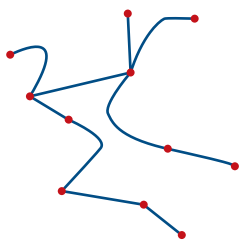
In some network problems it is important to minimise the number and weights of the edges to keep all vertices connected to the graph. For example, a number of towns might need to be connected to a water supply. The cost of connecting the towns can be minimised by connecting each town into a network or water pipes only once, rather than connecting each town to every other town. To solve these problems we need an understanding of trees.

Tree

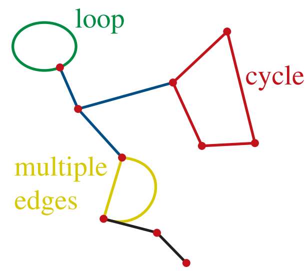
A tree is a connected graph that contains no cycles, multiple edges or loops. The diagram below shows two network graphs that are trees and one network graph that is not a tree.



Graph 1: a tree



Graph 2: a tree



Graph 3: not a tree

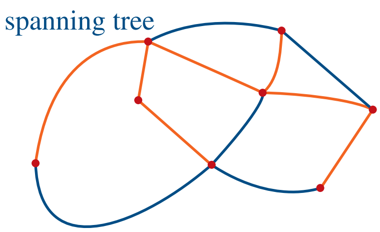
Graphs 1 and 2 are trees: they are connected and have no cycles, multiple edges or loops. Graph 3 is not a tree: it has several cycles (loops and multiple edges count as cycles). For trees, there is a relationship between the number of vertices and the number of edges.

- Graph 1, a tree, has 8 vertices and 7 edges.
- Graph 2, a tree, has 11 vertices and 10 edges.

In general, the number of edges is always one less than the number of vertices. In other words, a tree with n vertices has $n - 1$ edges.

Spanning trees

Every connected graph will have at least one subgraph that is a tree. If a subgraph is a tree, and if that tree connects all of the vertices in the graph, then it is called a spanning tree. An example of a spanning tree is shown opposite. There are several other possibilities. Note: the spanning tree opposite, like the network diagram, has 8 vertices. However it has only 7 edges ($8 - 1 = 7$).



TREE

A tree is a connected graph that contains no cycles, multiple edges or loops.
A tree with n vertices has $n - 1$ edges.
A spanning tree is a tree that connects all of the vertices of a graph.

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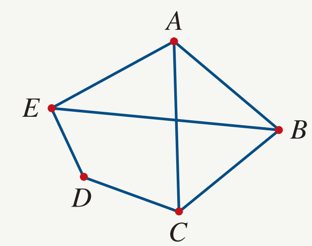
Example 9: Finding a spanning tree in a network 2F

Find two spanning trees for the graph shown opposite.

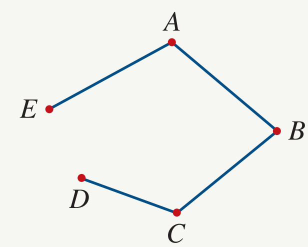


SOLUTION:

1 The graph has five vertices and seven edges. A spanning tree will have five (n) vertices and four ($n - 1$) edges.

2 To form a spanning tree, remove any three edges, provided that all the vertices remain connected, and there are no multiple edges or loops.

Spanning tree 1

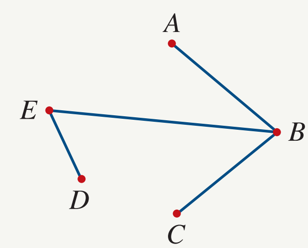


3 Spanning tree 1 is formed by removing edges *EB*, *ED* and *CA*.

4 Spanning tree 2 is formed by removing edges *EA*, *AC* and *CD*.

5 There are several other possible spanning trees.

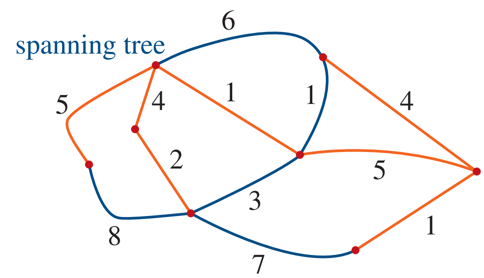
Spanning tree 2



Minimum spanning trees

For a weighted graph, it is possible to determine the 'length' of each spanning tree by adding up the weights of the edges in the tree. For the spanning tree opposite:

Length $= 5 + 4 + 2 + 1 + 5 + 4 + 1$
$= 22$ units



A minimum spanning tree is a spanning tree of minimum length. It connects all the vertices together with the minimum total weighting for the edges.

MINIMUM SPANNING TREE

A minimum spanning tree is a spanning tree of minimum length. It connects all the vertices together with the minimum total weighting for the edges.

A guide to trees Watch the video in the Interactive Textbook to see trees, spanning trees and minimum spanning trees in action.

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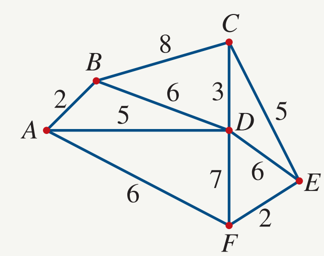
Prim's algorithm

Prim's algorithm is a set of rules to determine a minimum spanning tree for a graph.

1. Choose a starting vertex (any will do).
2. Inspect the edges starting from the starting vertex and choose the one with the lowest weight. (If there are two edges that have the same weight, it does not matter which one you choose). You now have two vertices and one edge.
3. Inspect all of the edges starting from both of the vertices you have in the tree so far. Choose the edge with the lowest weight, ignoring edges that would connect the tree back to itself. You now have three vertices and two edges.
4. Keep repeating step 3 until all of the vertices are connected.

Example 10: Finding the minimum spanning tree 2F

Apply Prim's algorithm to find the minimum spanning tree for the network graph shown on the right. Calculate the length of the minimum spanning tree.



SOLUTION:

1 Start with vertex A. List the weighted edges from vertex A and find the smallest.

$(A, B) = 2$
$(A, F) = 6$
$(A, B) = 2$ is lowest.

2 Look at vertices A and B. List the weighted edges from vertex A and vertex B (apart from (A, B) which you have already found).

$(A, D) = 5$
$(A, F) = 6$
$(B, C) = 8$
$(B, D) = 6$
$(A, D) = 5$ is lowest.

3 Repeat to find the smallest weighted edge from vertex A, B or D.

$(C, D) = 3$ is lowest.

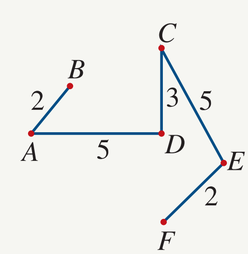
4 Repeat to find the smallest weighted edge from vertex A, B, D or C.

$(C, E) = 5$ is lowest.

5 Repeat to find the smallest weighted edge from vertex A, B, D, C or E.

$(E, F) = 2$ is lowest.

6 All vertices have been included in the graph. Draw the minimum spanning tree.



7 Find the length of the minimum spanning tree by adding the weights of the edges.

$$\text{Length} = 2 + 5 + 3 + 5 + 2 = 17 \text{ units}$$

Kruskal's algorithm

Kruskal's algorithm This alternative method is covered in the Interactive Textbook.

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Exercise 2F

1 Copy and complete the following sentences.

a A tree is a connected graph that contains no cycles, multiple __________ or loops.

b A minimum spanning tree is a spanning tree of minimum _________.

c Prim’s algorithm is a set of rules to determine a minimum ___________ tree for a graph.

d A connected graph has eight vertices. Its spanning tree has ___________ vertices.

e A connected graph has 10 vertices. Its spanning tree has ___________ edges.

2 **a** How many edges are there in a tree with 15 vertices?

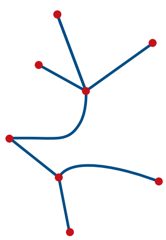
b How many vertices are there in a tree with five edges?

c Draw two different trees with four vertices.

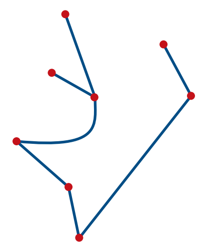
d Draw three different trees with five vertices.

3 Which of the following graphs are trees?

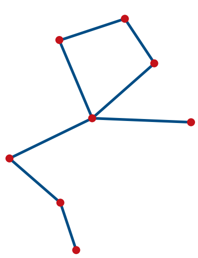
a



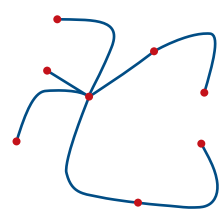
b



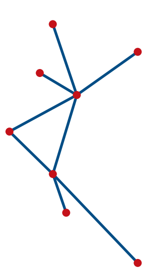
c



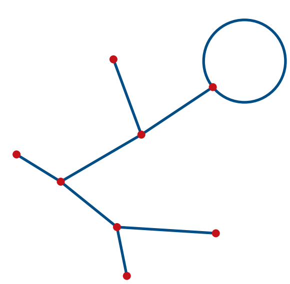
d



e



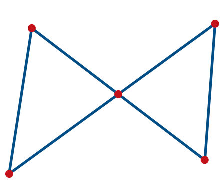
f



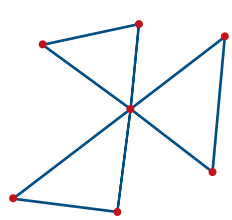
Example 9

4 For each of the following graphs, draw three different spanning trees.

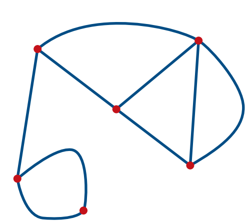
a



b



c



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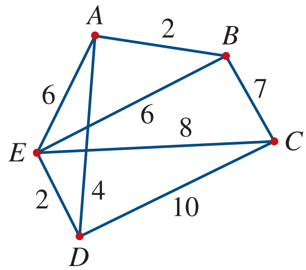
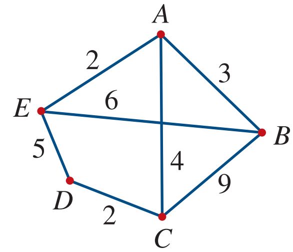
Example 10 **5** Find the minimum spanning tree and its length, for each of the following graphs.

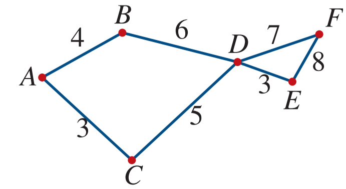
a



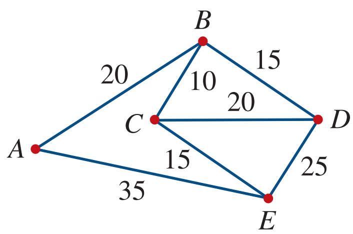
b



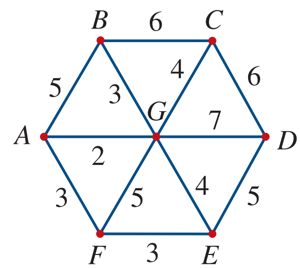
c



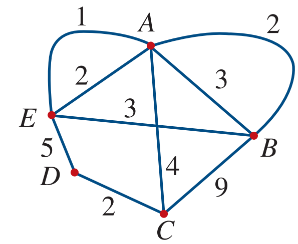
d



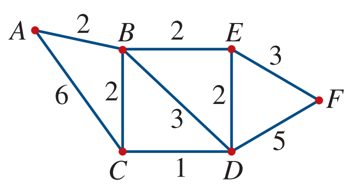
e



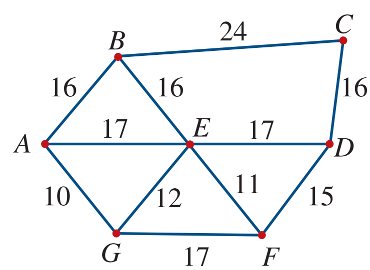
f



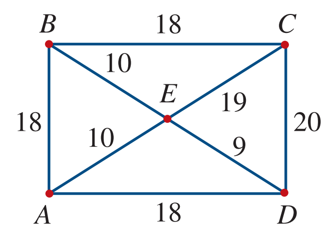
g



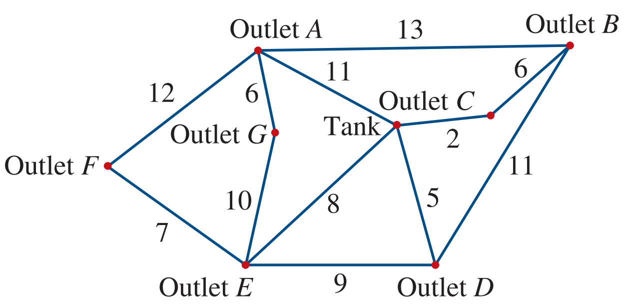
h



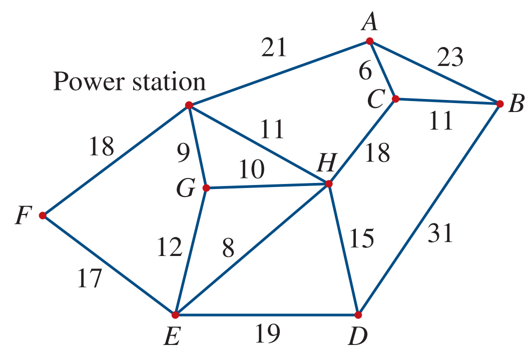
i



6 Water is to be piped from a water tank to seven outlets on a property. The distances (in metres) of the outlets from the tank and from each other are shown in the network opposite. Starting at the tank, determine the minimum length of pipe needed.



7 Power is to be connected by cable from a power station to eight substations (*A* to *H*). The distances (in kilometres) of the substations from the power station and from each other are shown in the network opposite. Determine the minimum length of cable needed.



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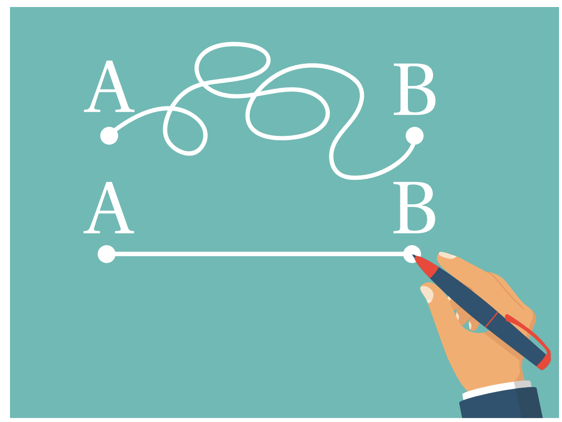
2G Shortest path

The shortest path in a network is the path between two vertices where the sum of the weights of its edges is minimised. Finding the shortest path is often very useful. For example, if the weights of a network represent time, you can choose a path that will allow you to travel in the shortest time. If the weights represent distance, you can determine a path that will allow you to travel the shortest distance. However, be aware that travelling the shortest distance between two places is not necessarily the best path. For example, if shortest path has a speed limit of 60 km/h but another path has a speed limit of 110 km/h then the shortest path may take longer to reach the destination. In such a case, redraw the network diagram with time taken to travel each edge, rather than distance.

While there are sophisticated techniques for solving shortest path problems the method of inspection involves identifying and comparing the lengths of likely candidates for the shortest path. All of the possible paths should be listed and the length of the path calculated. When finding the shortest path it is important to be aware there can be more than one shortest path between two vertices and the shortest path may not pass through all of the vertices.

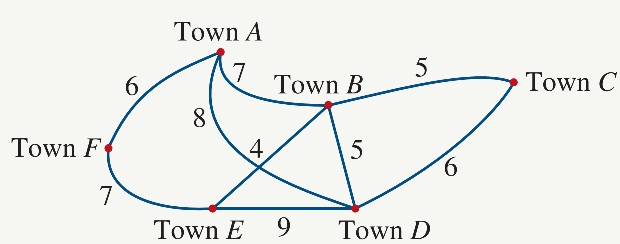


SHORTEST PATH

The shortest path between two vertices in a network is the path where the sum of the weights of its edges is minimised.

Example 11: Finding the shortest path by inspection 2G

Find the shortest path between Town C and Town F.



SOLUTION:

1 Identify all the likely shortest routes between Town C and Town F and calculate their lengths.
Note: Time can be saved by eliminating any route that 'takes the long way around' rather than the direct route. For example, when travelling from Town B to Town D, ignore the route that goes via Town A because it is longer.

C–D–E–F $= 6 + 9 + 7$
$= 22$ km
C–B–E–F $= 5 + 4 + 7$
$= 16$ km
C–B–A–F $= 5 + 7 + 6$
$= 18$ km

2 Compare the different path lengths and identify the shortest path. Write your answer in words.

The shortest path is C–B–E–F.

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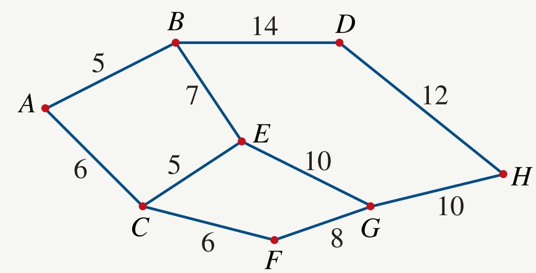
Example 12: Finding the length of the shortest path 2G

Find the length of the shortest path between the following vertices.

a A and E
b A and F
c A and G
d A and H



SOLUTION

1 Identify the shortest paths between vertex A and vertex E.
2 Add the weighted edges to find the length of the path.

a $A–C–E = 6 + 5$
$= 11$

3 Identify the shortest paths between vertex A and vertex F.
4 Add the weighted edges to find the length of the path.

b $A–C–F = 6 + 6$
$= 12$

5 Identify the shortest paths between vertex A and vertex G.
6 Add the weighted edges to find the length of the path.

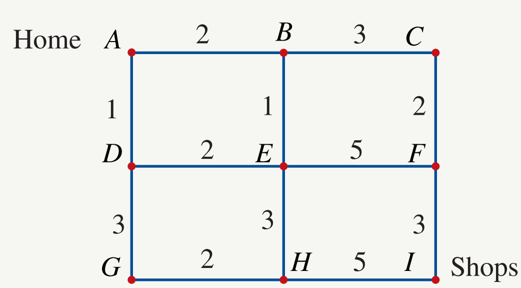
c $A–C–F–G = 6 + 6 + 8$
$= 20$

7 Identify the shortest paths between vertex A and vertex H.
8 Add the weighted edges to find the length of the path.

d $A–C–F–G–H$
$= 6 + 6 + 8 + 10 = 30$

Example 13: Solving a shortest path problem 2G

Darcy has drawn a network diagram to represent several streets for travelling from his home to the shops. The numbers indicate the times in minutes. Describe the shortest path and minimum travelling time.



SOLUTION

1 Identify all the likely shortest routes between home and the shops and calculate their lengths.
2 Compare the different path lengths and identify the shortest path.

$A–D–E–H–I = 1 + 2 + 3 + 5$
$= 11\text{ min}$
$A–D–G–H–I = 1 + 3 + 2 + 5$
$= 11\text{ min}$
$A–B–E–H–I = 2 + 1 + 3 + 5$
$= 11\text{ min}$
$A–B–C–F–I = 2 + 3 + 2 + 3$
$= 10\text{ min}$

3 Write your answer in words.

The shortest path is $A–B–C–F–I$ and the time taken is 10 minutes.

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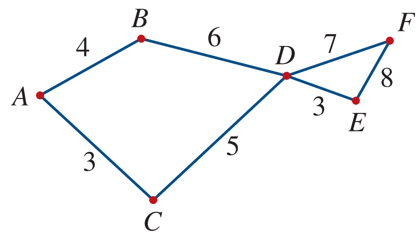
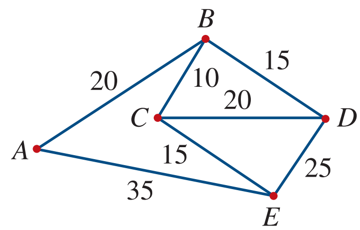
Exercise 2G

Example 11 **1** The numbers in the weighted graph opposite represent time in hours. Find the length of the shortest path between the following vertices.



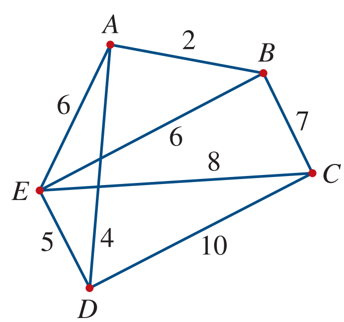
a *A* and *D*
b *A* and *E*
c *A* and *F*
d *C* and *F*

Example 12 **2** The numbers in the weighted graph opposite represent length in metres. Find the length of the shortest path between the following vertices.



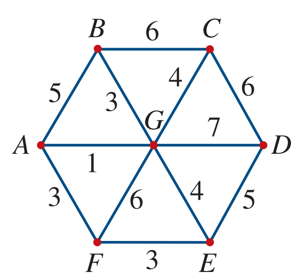
a *C* and *D*
b *A* and *C*
c *A* and *D*
d *B* and *E*

3 The numbers in the weighted graph opposite represent cost in dollars. Find the length of the shortest path between the following vertices.



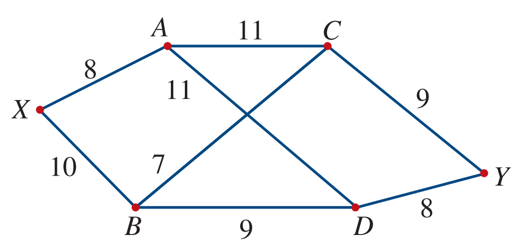
a *E* and *C* **b** *B* and *E*
c *C* and *D* **d** *A* and *E*
e *A* and *C* **f** *A* and *D*
g *B* and *D* **h** *A* to *D* to *E*

4 The numbers in the weighted graph opposite represent time in minutes. Find the length of the shortest path between the following vertices.



a *A* and *C*
b *A* and *E*
c *B* and *D*
d *B* and *F*
e *C* and *F*
f *C* and *E*

5 The numbers in the weighted graph opposite represent distance in metres. Find the length of the shortest path between the following vertices.



a *X* and *C* **b** *X* and *D*
c *A* and *Y* **d** *B* and *Y*
e *A* and *B* **f** *X* and *Y*

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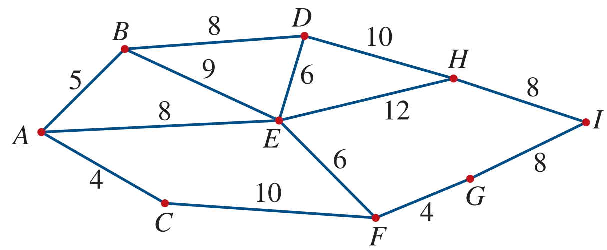
Example 13

6 The network below shows the distance, in kilometres, along walkways that connect landmarks *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H* and *I* in a national park.



a What distance is travelled on the path *A–B–E–H–I*?

b What distance is travelled on the path *I–G–F–E–D–B–A*?

c What distance is travelled on the circuit *F–E–D–H–E–A–C–F*?

d What distance is travelled on the circuit *D–E–A–B–E–F–G–I–H–D*?

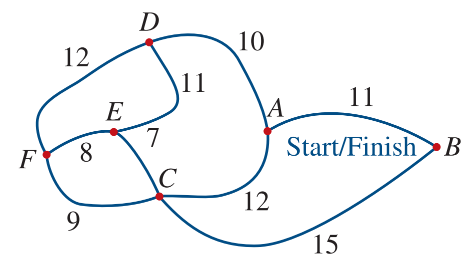
e What is the distance travelled on the shortest cycle starting and finishing at *E*?

f What is the distance travelled on the shortest cycle starting and finishing at *F*?

g Find the shortest path and distance travelled from *A* to *I*.

h Find the shortest path and distance travelled from *C* to *D*.

7 The graph opposite shows a mountain bike rally course. Competitors must pass through each of the checkpoints, *A*, *B*, *C*, *D*, *E* and *F*. The average times (in minutes) taken to ride between the checkpoints are shown on the edges of the graph. Competitors must start and finish at checkpoint A but can pass through the other checkpoints in any order they wish.



a What is the average time travelled on the circuit *A–D–E–F–C–B–A*?

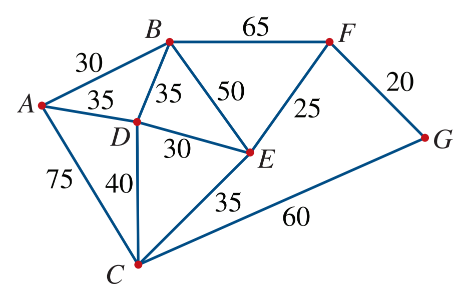
b What is the average time travelled on the circuit *A–B–C–F–E–D–A*?

c What is the average time travelled on the circuit *A–C–E–D–F–C–B–A*?

d Which path would have the shortest average time?

e Will the path with the shortest average time always be the best path? Explain your answer.

8 The network below shows the time (in minutes) of train journeys between seven stations.



a What is the time taken to travel *A–B–E–D–A*?

b What is the time taken to travel *F–G–C–D–B–F*?

c Find the shortest time it would take to travel from *A* to *G*.

d Will the path with the shortest time always be the best path? Explain your answer.

e Find the shortest time it would take to travel from *A* to *G* if in reality each time the train passes through a station, excluding *A* and *G*, an extra 10 minutes is added to the journey.

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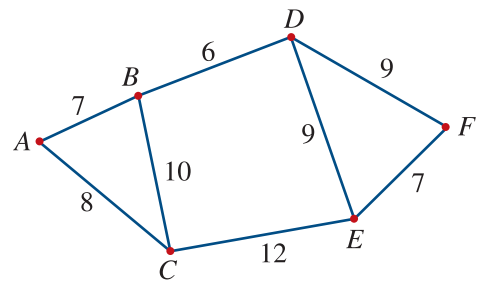
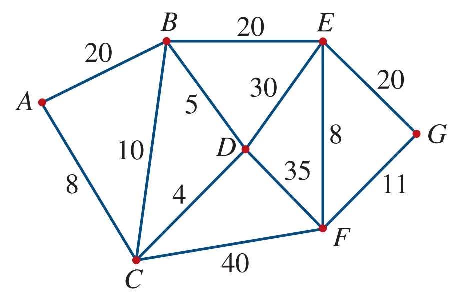
9 The network diagram opposite shows possible water pipes connecting six towns. The numbers on each edge represent the distance (in km) between the towns.



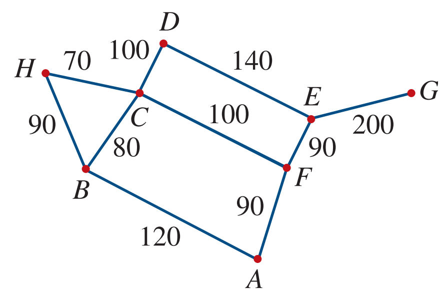
- **a** Which town is closest to *A*? What is the distance?
- **b** Which town is closest to either *A* or your answer in part **a**? What is the distance?
- **c** Which town is closest to either *A* or your answer in part **a** or **b**? What is the distance?
- **d** Which town is closest to either *A* or your answer in part **a** or **b** or **c**? What is the distance?
- **e** Which town is closest to either *A* or your answer in part **a** or **b** or **c** or **d**? What is the distance?
- **f** What is the shortest distance connecting water pipes to the six towns?

10 The network diagram opposite shows possible railway lines connecting seven cities. The numbers on each edge represent the cost (in millions of dollars) in setting up a rail link.



- **a** Which city is closest to *A*? What is the cost?
- **b** Which city is closest to either *A* or your answer in part **a**? What is the cost?
- **c** Which city is closest to either *A* or your answer in part **a** or **b**? What is the cost?
- **d** Which city is closest to either *A* or your answer in part **a** or **b** or **c**? What is the cost?
- **e** Which city is closest to either *A* or your answer in part **a** or **b** or **c** or **d**? What is the cost?
- **f** Which city is closest to either *A* or your answer in part **a** or **b** or **c** or **d** or **e**? What is the cost?
- **g** What is the minimum cost of connecting the seven cities with a railway line?

11 The network diagram opposite shows the major roads connecting eight towns. The numbers on each edge represent the distance in kilometres between the towns.



- **a** Which town is closest to *A*? What is the distance?
- **b** Which town is closest to either *A* or your answer in part **a**? What is the distance?
- **c** Which town is closest to either *A* or your answer in part **a** or **b**? What is the distance?
- **d** Which town is closest to either *A* or your answer in the parts **a** to **c** above? What is the distance?
- **e** Which town is closest to either *A* or your answer in the parts **a** to **d** above? What is the distance?
- **f** Which town is closest to either *A* or your answer in the parts **a** to **e** above? What is the distance?
- **g** Which town is closest to either *A* or your answer in the parts **a** to **f** above? What is the distance?
- **h** What is the minimum distance connecting the eight towns?

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Summary

PPT STUDY GUIDE

Key ideas and chapter summary

Network	A network is a term to describe a group or system of interconnected objects. It consists of vertices and edges that indicate a path or route between two objects.	
Network terminology	Network diagram, vertex, edge, degree, loop, directed network, undirected network, weighted edge, walk, trail, path, circuit, cycle, Eulerian trail, Eulerian circuit.	
Drawing a network diagram	Connected graph	A graph is connected if every vertex in the graph is accessible from every other vertex in the graph along a path formed by the edges of the graph.
	Isomorphic graph	Two graphs are isomorphic (equivalent) if: • they have the same numbers of edges and vertices • corresponding vertices have the same degree and the edges connect to the same vertices.
	Weighted graph	A weighted graph is a network diagram that has weighted edges or an edge with a number assigned to it that implies some numerical value such as cost, distance or time.
Network problems	Network diagrams are used to solving problems involving maps and tables.	
Minimum spanning tree	A tree is a connected graph that contains no cycles, multiple edges or loops. A tree with n vertices has $n - 1$ edges. A spanning tree is a tree that connects all of the vertices of a graph. A minimum spanning tree is a spanning tree of minimum length. It connects all the vertices together with the minimum total weighting for the edges. Prim's and Kruskal's algorithms are a set of rules to determine a minimum spanning tree for a graph.	
Shortest path	The shortest path between two vertices in a network is the path where the sum of the weights of its edges is minimised. Problems using the minimal spanning tree to the find least cost to link locations or objects.	

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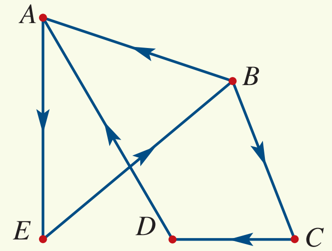
Review

Multiple-choice

Questions **1** to **4** relate to the network diagram opposite.



1 Which of the following vertices has the highest degree?

A Vertex *B*
B Vertex *C*
C Vertex *D*
D Vertex *E*

2 How many edges are in the network diagram?

A 4 **B** 5 **C** 6 **D** 7

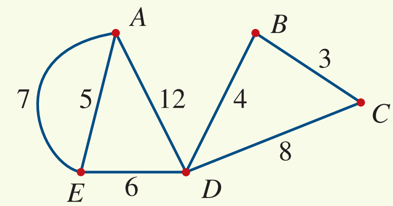
3 Which of the following is a valid path?

A *A–E–B–C* **B** *C–D–A–E–B–A* **C** *C–D–A–E–B–C* **D** *A–E–B*

4 The sequence *C–D–A–E–B* represents:

A a walk only
B a path
C a trail
D a cycle

Questions **5** to **6** relate to the network diagram opposite.



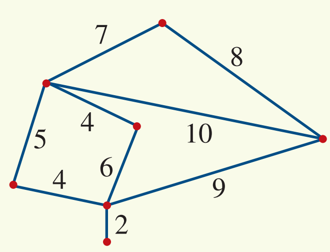
5 Which one of the following is a trail?

A *A–E–D–B–C*
B *E–D–A–B–C–E*
C *A–E–D–C–B–D–A–E*
D *E–D–C–B–D–C–A*

6 What is the length of the shortest path from *A* to *C* in the above network?

A 17 **B** 8 **C** 19 **D** 20

Questions **7** to **8** relate to the network diagram opposite.



7 What is the length of the minimum spanning tree?

A 30
B 31
C 33
D 34

8 What is the length of the shortest path from the bottom vertex to the top vertex?

A 18 **B** 19 **C** 20 **D** 21

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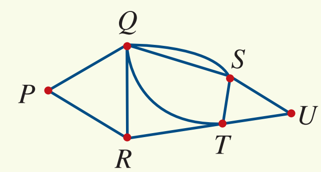
Review

Short-answer

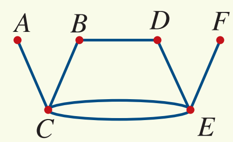
1 Find the degree of the following vertices in the network diagram opposite.

a P **b** Q **c** R
d S **e** T **f** U



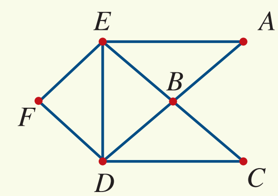
2 A network graph with six vertices is shown opposite.

a What is the degree of each vertex?
b Why does this graph have an Eulerian trail?
c List an Eulerian trail.



3 A network graph with six vertices is shown opposite.

a What is the degree of each vertex?
b Why does this graph have an Eulerian circuit?
c List an Eulerian circuit.

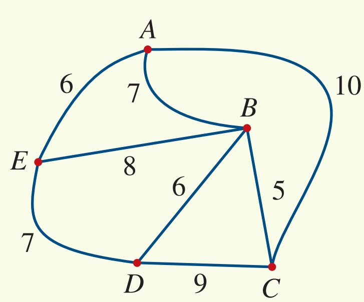


4 A chess tournament is completed between 5 players. Each game has 2 players competing against each other. The table opposite shows the games and the players.

a Draw a network diagram to represent the information in the table.
b What are the vertices of the network diagram?
c Which players have not played a game against each other?

Match	Players	
1	Toby	Jett
2	Amy	Beau
3	Amy	Toby
4	Jett	Ellie
5	Beau	Toby
6	Toby	Ellie

5 For the weighted graph shown, determine the length of the minimum spanning tree.



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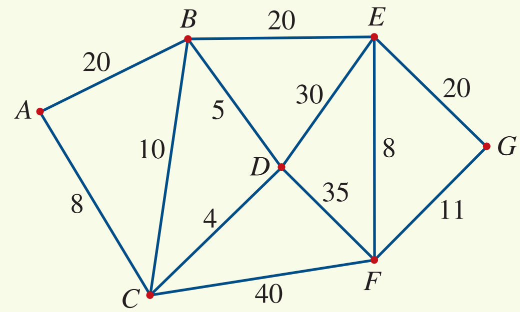
Review

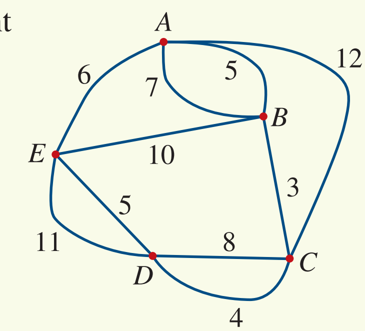
6 What is the length of the minimum spanning tree in the network shown opposite?



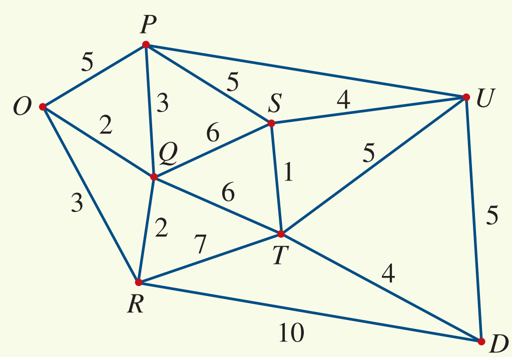
7 In the network shown opposite, the numbers on the edges represent distances in kilometres. Determine the length of:

a the shortest path between vertex *A* and vertex *D*

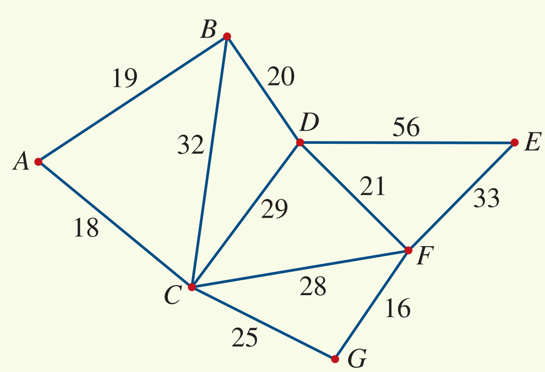
b the length of the minimum spanning tree.



8 What is the length of the shortest path between *O* and *D* in the network shown opposite?



9 Seven towns on an island have been surveyed for transport and communications needs. The towns (labelled *A*, *B*, *C*, *D*, *E*, *F*, *G*) form the network shown here. The road distances between the towns are marked in kilometres. To establish a cable network for communications on the island, it is proposed to put the cable underground beside the existing roads.



a Draw a minimum spanning tree that will ensure that all the towns are connected to the network but that also minimises the amount of cable used.

b What is the minimum length of cable required here if back-up links are not considered necessary; that is, there are no loops in the cable network?

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3 Investments

Syllabus topic — F2 Investments

This topic will develop your skills to calculate and compare the value of different types of investments over a period of time. In addition, you will gain an understanding of the impact of inflation on prices and wages and the appreciation of items.

Outcomes

- Calculate simple interest for different rates and periods.
- Compare simple interest graphs for different rates and periods.
- Use the future value formula to calculate the compound interest.
- Solve practical problems involving compound interest.
- Calculate compound interest for different rates and periods.
- Compare compound interest graphs for different rates and periods.
- Compare the growth of simple interest and compound interest.
- Compare and contrast different investment strategies.
- Determine the impact of inflation on prices and wages.
- Calculate the appreciated value of items.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Literacy worksheet
- Quick Quiz
- Solutions (enabled by teacher)
- Desmos widgets
- Spreadsheets
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

The Interactive Textbook provides a test of prior knowledge for this chapter, and may direct you to revision from the previous years' work.

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3A Simple interest

Interest is the amount paid for borrowing money or the amount earned for lending money. There are different ways of calculating interest. Simple interest (or flat interest) is a fixed percentage of the amount invested or borrowed and is calculated on the original amount. For example, if you invest \$100 in a bank account that pays interest at the rate of 5% per annum (per year), you would receive \$5 each year. That is,

$$\text{Interest} = \$100 \times \frac{5}{100} = \$5$$

This amount of interest would be paid each year. Simple interest is always calculated on the initial amount, or the principal.

SIMPLE INTEREST

$I = Prn$
I – Interest (simple or flat) earned for the use of money
P – Principal is the initial amount of money borrowed
r – Rate of simple interest per period expressed as a decimal
n – Number of time periods

Example 1: Finding simple interest 3A

Calculate the amount of simple interest paid on an investment of \$16 000 at 8% simple interest per annum for 3 years.

SOLUTION:

1. Write the simple interest formula. $I = Prn$
2. Substitute $P = 16000$, $r = 0.08$ and $n = 3$ into the formula. $= 16000 \times 0.08 \times 3$
3. Evaluate. $= \$3840$
4. Write the answer in words. Simple interest is \$3840.

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Amount owed or future value

The interest is added to the principal to determine the amount owed on a loan or the future value of an investment.

FORMULA FOR AMOUNT OWED OR FUTURE VALUE

$A = P + I$

A – Amount or final balance

I – Interest (simple or flat) earned

P – Principal is the initial quantity of money

Example 2: Calculating the amount owed — 3A

Find the amount owed on a loan of \$50 000 at 7% per annum simple interest at the end of two years and six months.

SOLUTION:

1	Write the simple interest formula.	$I = Prn$
2	Substitute $P = 50000$, $r = 0.07$ and $n = 2.5$ into the formula.	$= 50000 \times 0.07 \times 2.5$
3	Evaluate.	$= \$8750$
4	Write the amount owed formula.	$A = P + I$
5	Substitute $P = 50000$ and $I = 8750$ into the formula.	$= 50000 + 8750$
6	Evaluate.	$= \$58750$
7	Write the answer in words.	Amount owed is \$58 750.

Example 3: Calculating value of an investment — 3A

Joel plans to make an investment of \$200 000 at $7\frac{1}{2}\%$ p.a. simple interest for 2 years. What is the total value of his investment at the end of 2 years?

SOLUTION:

1	Write the simple interest formula.	$I = Prn$
2	Substitute $P = 200000$, $r = 0.075$ and $n = 2$ into the formula.	$= \$200000 \times 0.075 \times 2$
3	Evaluate.	$\$ = \$30000$
4	Write the amount owed formula.	$A = P + I$
5	Substitute $P = 50000$ and $I = 8750$ into the formula.	$= \$200000 + \$30000$
6	Evaluate.	$= \$230000$
7	Write the answer in words.	Total value is \$230 000.

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Exercise 3A

Example 1 **1** Calculate the amount of simple interest for each of the following.

a Principal = \$15 000, Interest rate = 13% p.a., Time period = 3 years

b Principal = \$2000, Interest rate = $6\frac{1}{2}$% p.a., Time period = 7 years

c Principal = \$200 000, Interest rate = $9\frac{1}{4}$% p.a., Time period = 2 years

d Principal = \$3600, Interest rate = 9% p.a., Time period = $3\frac{1}{2}$ years

e Principal = \$40 000, Interest rate = 7.25% p.a., Time period = $5\frac{1}{4}$ years

Example 2 **2** Calculate the amount owed for each of the following.

a Principal = \$500, Simple interest rate = 5% p.a., Time period = 4 years

b Principal = \$900, Simple interest rate = 3% p.a., Time period = 7 years

c Principal = \$4000, Simple interest rate = $8\frac{1}{2}$ p.a., Time period = 3 years

d Principal = \$6900, Simple interest rate = 10% p.a., Time period = $4\frac{1}{2}$ years

e Principal = \$10 000, Simple interest rate = 6.75% p.a., Time period = $2\frac{1}{4}$ years

3 The simple interest rate is given as 4.8% per annum.

a What is the interest rate per quarter?

b What is the interest rate per month?

c What is the interest rate per six months?

d What is the interest rate per nine months?

4 Calculate the amount of simple interest for each of the following.

a Principal = \$800, Interest rate = 12% p.a., Time period = 1 month

b Principal = \$1600, Interest rate = 18% p.a., Time period = 6 months

c Principal = \$60 000, Interest rate = 9.6% p.a., Time period = 3 months

d Principal = \$20 000, Interest rate = 6% p.a., Time period = 9 months

5 Andrew takes a loan of \$30 000 for a period of 6 years, at a simple interest rate of 14% per annum. Find the amount owing at the end of 6 years.

6 A loan of \$1800 is taken out at a simple interest rate of 15.5% per annum. How much interest is owing after 3 months?

Example 3 **7** A sum of \$100 000 was invested in a fixed-term account for 4 years. Calculate:

a the simple interest earned if the rate of interest is 5.5% per annum

b the future value of the investment at the end of 4 years.

8 Sophie decides to buy a car for \$28 000. She has saved \$7000 for the deposit and takes out a loan over 2 years for the balance. The flat rate of interest charged is 12% per annum. What is the total amount of interest to be paid?

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9 Domenico has borrowed $24 000 to buy furniture. He wishes to repay the loan over 4 years. Calculate the simple interest on the following rates of interest.

a 8% per annum for the entire period
b 9% per annum after a 6-month interest-free period
c 10% per annum after a 12-month interest-free period

10 Create the spreadsheet below.

3AQ10

SUM =A5*B5*C5

	A	B	C	D	E	F
1	**Mathematics Standard 1**					
2	Worksheet to calculate simple interest					
3						
4	*Principal*	*Rate*	*Time (yr)*	*Interest*	*Amount*	
5	$500	5.00%	4.0	=A5*B5*C5	$600	
6	$15,000	13.00%	3.00	$5,850	$20,850	
7	$2,000	6.25%	7.00	$875	$2,875	
8	$200,000	9.25%	2.00	$37,000	$237,000	
9	$3,600	9.00%	3.50	$1,134	$4,734	
10	$400,000	7.25%	5.25	$152,250	$552,250	
11	$800	10.00%	0.50	$40	$840	
12	$20,000	11.50%	0.75	$1,725	$21,725	
13						

Simple Interest +

a Cell D5 has a formula that calculates the simple interest. Enter this formula.
b The formula for cell E5 is '= A5 + D5'. Fill down the contents of E6 to E12 using this formula.

11 Isabelle buys a TV for $1400. She pays it off monthly over 2 years at an interest rate of 11.5% per annum flat. How much per month will she pay (to the nearest dollar)?

12 Riley wants to earn $4000 a year in interest. How much must he invest if the simple interest rate is 10% p.a.?

13 Samira invests $16 000 for $2\frac{1}{2}$ years. What is the minimum rate of simple interest needed for her to earn $3000?

14 Gurrumul pays back $20 000 on a $15 000 loan at a flat interest rate of 10%. What is the term of the loan?

15 Harry borrowed $300 000 at a flat rate of interest of 8.5% per annum. This rate was fixed for 2 years on the principal. He pays back the interest only over this period.

a How much interest is to be paid over the 2 years?
b After paying the fixed rate of interest for the first year, Harry finds the bank will decrease the flat interest rate to 7.5% if he pays a charge of $1000. How much will he save by changing to the lower interest rate for the last year?

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3B Simple interest graphs

When graphing simple interest make the horizontal axis the time period and the vertical axis the interest earned. Simple interest will increase by a constant amount each time period. This will result in a straight-line graph.

SIMPLE INTEREST GRAPHS

1. Construct a table of values for I and n using the simple interest formula.
2. Draw a number plane with n the horizontal axis and I the vertical axis. Plot the points.
3. Join the points to make a straight line.

Example 4: Constructing a simple interest graph 3B

Draw a graph showing the amount of simple interest earned over a period of 10 years if \$1000 is invested at 8% p.a. Use the graph to estimate the interest earned after 7.5 years.

SOLUTION:

1. Write the simple interest formula.
2. Substitute $P = 1000$, $r = 0.08$ and n into the formula.

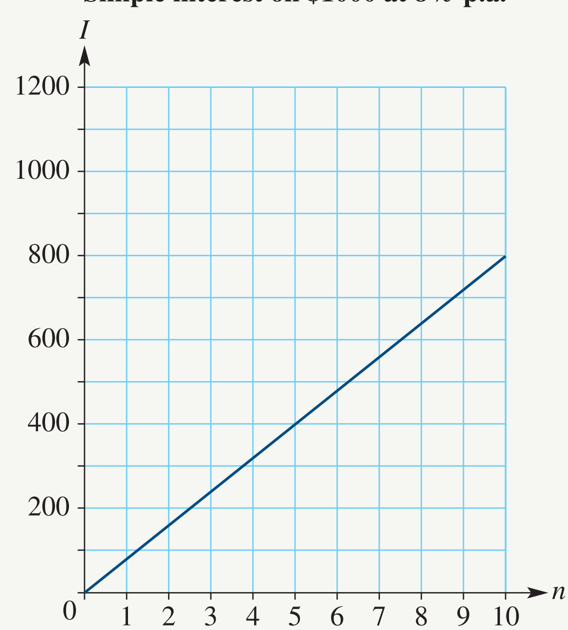
$$I = Prn$$
$$= 1000 \times 0.08 \times n$$
$$= 80n$$

3. Draw a table of values for I and n.
4. Let $n = 0, 2, 4, \ldots$ Find the interest (I) using $I = 80n$.

n	0	2	4	6	8	10
I	0	160	320	480	640	800

5. Draw a number plane with n as the horizontal axis and I as the vertical axis.
6. Plot the points (0, 0), (2, 160), (4, 320), (6, 480), (8, 640) and (10, 800).
7. Draw a straight line between the points. Simple interest graphs are linear.
8. Read the graph to estimate I ($I = 600$ when $n = 7.5$).

Simple interest on \$1000 at 8% p.a.



9. Write the answer in words.

Interest after 7.5 years is approximately \$600.

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Exercise 3B

Example 4

1 Aiden invested \$1000 at 2% per annum simple interest for 3 years.

a Simplify the simple interest formula ($I = Prn$) by substituting values for the principal and the interest rate.

b Use this formula to complete the following table of values.

n	0	1	2	3	4
I					

c Draw a number plane with n as the horizontal axis and I as the vertical axis.

d Plot the points from the table of values. Join the points to make a straight line.

2 Riley invested \$2000 at 6% per annum simple interest for 5 years.

a Simplify the simple interest formula ($I = Prn$) by substituting values for the principal and the interest rate.

b Use this formula to complete the following table of values.

n	0	1	2	3	4	5
I						

c Draw a number plane with n as the horizontal axis and I as the vertical axis.

d Plot the points from the table of values. Join the points to make a straight line.

e Use the graph to find the interest after $2\frac{1}{2}$ years.

f Extend the graph to find the interest after 6 years.

g Estimate the interest earned after 6 years using the graph.

3 Charlotte invested \$800 at 7% per annum simple interest for 6 years.

a Simplify the simple interest formula ($I = Prn$) by substituting values for the principal and the interest rate.

b Use this formula to complete the following table of values.

n	0	1	2	3	4	5	6
I							

c Draw a number plane with n as the horizontal axis and I as the vertical axis.

d Plot the points from the table of values. Join the points to make a straight line.

e Use the graph to find the interest after $2\frac{1}{2}$ years.

f Extend the graph to find the interest after 7 years.

g Estimate the interest earned after 7 years using the graph.

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4 Alice is comparing three different interest rates for a possible investment.

a Draw on the same number plane the graph to represent the interest earned over 5 years on:

i \$1000 invested at 4% per annum simple interest.

ii \$1000 invested at 6% per annum simple interest.

iii \$1000 invested at 8% per annum simple interest.

b How much does each investment earn after 5 years?

c Use the graph to estimate the interest earned after $3\frac{1}{2}$ years.

d Find the time for each investment to earn \$200 in interest.

5 Chloe is comparing three different interest rates for a possible investment.

a Draw a graph to represent the interest earned over 5 years on:

i \$5000 invested at 5% per annum simple interest

ii \$5000 invested at 7% per annum simple interest

iii \$5000 invested at 9% per annum simple interest.

b How much does each investment earn after $2\frac{1}{2}$ years?

c How much does each investment earn after 5 years?

d Find the time for each investment to earn \$1000 in interest.

6 Mick is comparing three different interest rates for a possible investment.

a Draw a graph to represent the interest earned for 6 months on:

i \$100 000 invested at 6% p.a. simple interest

ii \$100 000 invested at 9% p.a. simple interest

iii \$100 000 invested at 12% p.a. simple interest

b How much does each investment earn after 1 month?

c How much does each investment earn after 6 months?

d Find the time for each investment to earn \$2000 in interest.

7 The table below gives details for a fixed-term deposit.

Time period	Simple interest rate per annum
Less than 3 months	6.5%
3 to less than 6 months	7.0%
6 to less than 12 months	7.5%
12 to less than 24 months	8.1%
24 to less than 48 months	8.3%

Chris has \$50 000 to invest in a fixed-term deposit. Draw a separate graph to represent the interest earned on these investments.

a Fixed-term deposit for 3 months

b Fixed-term deposit for 6 months

c Fixed-term deposit for 12 months

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3C Compound interest – Future value

Compound interest is calculated from the initial amount borrowed or principal plus any interest that has been earned. It calculates interest on the interest. For example, if \$100 is invested at a compound interest rate of 10% per annum.

First year –	Interest = \$100 × 0.10 × 1 = \$10 Amount owed = \$100 + \$10 = \$110
Second year –	Interest = \$110 × 0.10 × 1 = \$11 Amount owed = \$110 + \$11 = \$121
Third year –	Interest = \$121 × 0.10 × 1 = \$12.10 Amount owed = \$121 + \$12.10 = \$133.10

These calculations show the interest earned increased each year. In the first year it was \$10, the second year \$11 and the third year \$12.10.

COMPOUND INTEREST FORMULA – FUTURE VALUE

$FV = PV(1 + r)^n$
A – Amount (final balance) or future value of the loan
P – (initial quantity of money) present value of the loan or principal
r – Rate of interest per compounding time period expressed as a decimal
n – Number of compounding time periods

Calculating compound interest

The compound interest is calculated by subtracting the principal from the amount borrowed or invested. Alternatively, finance companies provide an investment calculator as an estimate to the value of an investment.

COMPOUND INTEREST EARNED OR OWED

$I = FV - PV$
FV – future value of the loan or amount (final balance)
PV – Present value of the loan or principal (initial quantity of money)
I – Interest (compound) earned

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Example 5: Finding the compound interest 3C

Paige invests \$5000 over 5 years at a compound interest rate of 6.5% p.a. Calculate:

a the amount of the investment after 5 years, correct to the nearest cent

b the interest earned after 5 years, correct to the nearest cent.

SOLUTION:

1 Write the compound interest formula.
2 Substitute $P = 5000$, $r = 0.065$ and $n = 5$ into the formula.
3 Evaluate.
4 Write the answer in words.

a $A = P(1 + r)^n$
$= 5000(1 + 0.065)^5$
$= 6850.433317$
$= \$6850.43$
Amount is \$6850.43

5 Write the amount borrowed formula.
6 Substitute $P = 5000$ and $I = 6850.43$ into the formula.
7 Evaluate.
8 Write the answer in words.

b $I = A - P$
$= 6850.43 - 5000$
$= \$1850.43$
Interest earned is \$1850.43.

Example 6: Finding compound interest using a graphics calculator 3C

James borrowed \$50000 for 4 years at 11% p.a. interest compounding monthly.

a What is the amount owed after the 4 years?

b the interest paid after 4 years, correct to the nearest cent.

SOLUTION:

1 Write the compound interest formula.
2 Substitute $PV = 50000$, $r = \frac{0.11}{12}$ and $n = 4 \times 12$ into the formula.
3 Evaluate.
4 Write the answer in words.

a $FV = PV(1 + r)^n$
$= 50000 \times \left(1 + \frac{0.11}{12}\right)^{4\times 12}$
$= 77479.902...$
$\approx \$77479.90$
Amount owed is \$77479.90.

5 Write the amount borrowed formula.
6 Substitute $PV = 50000$ and $FV = 77479.90$ into the formula.
7 Evaluate.
8 Write the answer in words.

b $I = FV - PV$
$= 77479.90 - 50000$
$= \$27479.90$
Interest paid is \$27479.90.

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Exercise 3C

Example 5a

1 Calculate the future value, to the nearest cent, for each of the following.

a Present value = \$400, Compound interest rate = 3% p.a., Time period = 2 years

b Presentvalue = \$26 000, Compound interest rate = 8% p.a., Time period = 4 years

c Present value = \$48 000, Compound interest rate = 3.95% p.a., Time period = 10 years

d Present value = \$3000, Compound interest rate = $5\frac{1}{2}$% p.a., Time period = 5 years

e Present value = \$18 000, Compound interest rate = 10% p.a., Time period = $2\frac{1}{2}$ years

f Present value = \$65 000, Compound interest rate = 5.9% p.a., Time period = $3\frac{1}{4}$ years

g Present value = \$240 000, Compound interest rate = 11.3% p.a., Time period = 4.5 years

h Present value = \$14 000, Compound interest rate = $2\frac{1}{4}$% p.a., Time period = $7\frac{3}{4}$ years

2 Use the formula $FV = PV(1 + r)^n$ to calculate the value of an investment of \$16 000, over a period of 2 years with an interest rate of 5% compounding annually.

3 Tyler sold his car for \$35 600. He invested this amount at 7.2% p.a. with interest compounded annually. What is the value of his investment in 15 years?

4 Sarah wishes to invest \$5000 for a period of 8 years. The following investment strategies are suggested to her. What is the interest that will be earned on each investment strategy? Answer to the nearest dollar.

a Simple interest at 7% p.a.

b Compound interest at 7% p.a. compounded annually

c Simple interest at 14% p.a.

d Compound interest at 14% p.a. compounded annually

Example 5b

5 Calculate the amount of compound interest for each of the following.

a Futurevalue = \$25 000, Interest rate = 7% p.a., Time period = 5 years

b Future value = \$300 000, Interest rate = $10\frac{1}{4}$% p.a., Time period = 3 years

c Future value = \$6500, Interest rate = 13% p.a., Time period = $1\frac{1}{2}$ years

d Future value = \$80 000, Interest rate = 8.25% p.a., Time period = $3\frac{1}{4}$ years

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6 Alexis and Oscar invested $27 000 for 6 years at 9% p.a. interest compounding annually. What is the amount of interest earned? Answer to the nearest cent.

7 What is the interest earned if $15 720 is invested for 5 years, compounding yearly at 8% p.a.? Answer to the nearest dollar.

8 Find the amount $7800 will grow to if it is compounded at 6% p.a. for:

a 3 years **b** 10 years **c** 30 years.

9 Create the spreadsheet below.

XLS 3CQ9

SUM fx =A5*(1+B5)^C5

	A	B	C	D	E	F
1	**Mathematics Standard 1**					
2	Worksheet to calculate compound interest					
3						
4	*Principal*	*Rate*	*Time (yr)*	*Amount*	*Interest*	
5	$500	5.00%	4.0	C5	$108	
6	$15,000	13.00%	3.00	$21,643	$6,643	
7	$2,000	6.25%	7.00	$3,057	$1,057	
8	$200,000	9.25%	2.00	$238,711	$38,711	
9	$3,600	9.00%	3.50	$4,867	$1,267	
10	$400,000	7.25%	5.25	$577,625	$177,625	
11	$800	10.00%	0.50	$839	$39	
12	$20,000	11.50%	0.75	$21,701	$1,701	
13						
14						

Compound interest +

a Cell D5 has a formula that calculates the compound interest. Enter this formula.

b The formula for cell E5 is '= A5 + D5'. Fill down the contents of E6 to E12 using this formula.

10 Jack has $50 000 to invest for two years.
Which is the better investment? Why?

Investment 1	**Investment 2**
Simple interest rate 4% p.a.	Compound interest rate 4% p.a.

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Example 6

11 Calculate the future value, to the nearest cent, for each of the following.

a Present value of \$680 invested for 4 years at 5% p.a. compounded biannually
b Present value of \$1250 invested for 8 years at 3% p.a. compounded biannually
c Present value of \$5000 invested for 6 years at 6% p.a. compounded quarterly
d Present value of \$23 000 invested for 5 years at 7% p.a. compounded quarterly
e Present value of \$1400 invested for 3 years at 4.2% p.a. compounded monthly
f Present value of \$4680 invested for 10 years at 8% p.a. compounded monthly
g Present value of \$780 invested for 5 years at 9.8% p.a. compounded weekly
h Present value of \$1340 invested for 6 years at 6% p.a. compounded weekly
i Present value of \$290 invested for 7 years at 10% p.a. compounded fortnightly

12 Which of the following is the best investment over 25 years? Justify your answer.

Investment A: Simple interest at 4% p.a. with \$100 000
Investment B: Compound interest at 4% p.a. compounded annually with \$100 000
Investment C: Compound interest at 4% p.a. compounded biannually with \$100 000
Investment D: Compound interest at 4% p.a. compounded quarterly with \$100 000
Investment E: Compound interest at 4% p.a. compounded monthly with \$100 000

13 Find the future value in a bank account after 3 years if the present value of \$4000 earns 4.6% p.a. compound interest, paid quarterly.

14 Jackson invested \$16 400 over 6 years at 7.4% p.a. interest compounding monthly. Calculate the:

a value of the investment after 4 years
b compound interest earned.

15 Use the formula $A = P(1 + r)^n$ to calculate the value of an investment of \$10 000, over a period of 2 years with a monthly interest rate of 0.8% compounding monthly.

16 Sebastian invested \$20 000 at 12% p.a. interest compounding monthly. What is the amount of interest earned in the first year?

17 Find the amount of money in a bank account after 6 years if an initial amount of \$4000 earns 8% p.a. compound interest, paid quarterly.

18 Isabella invested \$13 500 over 7 years at 6.2% p.a. interest compounding quarterly. Calculate the:

a value of the investment after 7 years (to the nearest cent)
b compound interest earned (to the nearest cent).

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3D Compound interest – Present value

The compound interest formula to find the future value can be rearranged with the present value as the subject of the formula. This process is shown here.

$$FV = PV(1 + r)^n$$

$$\frac{FV}{(1 + r)^n} = \frac{PV(1 + r)^n}{(1 + r)^n}$$

$$\frac{FV}{(1 + r)^n} = PV$$

$$PV = \frac{FV}{(1 + r)^n}$$

COMPOUND INTEREST FORMULA – PRESENT VALUE

$$PV = \frac{FV}{(1 + r)^n}$$

FV – Future value of the loan or amount (final balance)

PV – Present value of the loan or principal (initial quantity of money)

r – Rate of interest per compounding time period expressed as a decimal

n – Number of compounding time periods

Example 7: Calculating the present value 3D

a Calculate the present value of an annuity whose future value is \$8723.27 over 5 years at a compound interest rate of 4.5% p.a. Answer correct to the nearest dollar.

b Calculate the present value of an annuity whose future value is \$500 000 over 8 years with an interest rate of 8.5% per annum compounded monthly. Answer correct to the nearest cent.

SOLUTION:

1 Write the present value formula.

a $PV = \frac{FV}{(1 + r)^n}$

2 Substitute $FV = \$8723.27$, $r = 0.045$ and $n = 5$ into the formula.

$= \frac{8723.27}{(1 + 0.045)^5}$

3 Evaluate to the nearest cent.

$= 6999.997..$

$\approx \$7000$

4 Write the answer in words.

∴ Present value is \$7000

5 Write the present value formula.

b $PV = \frac{FV}{(1 + r)^n}$

6 The investment is compounding per month hence the rate (r) and time period (n) are expressed in months.

7 Substitute $FV = 500\,000$, $r = \frac{0.085}{12}$ and $n = 8 \times 12 = 96$.

$= \frac{500\,000}{\left(1 + \frac{0.085}{12}\right)^{96}}$

8 Evaluate to the nearest cent.

$= \$253\,916.41$

9 Write the answer in words.

Present value is \$253 916.41.

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Exercise 3D

Example 7a **1** Calculate the present value, to the nearest cent, for each of the following.

a Future value = \$34 000, Interest rate = 4% p.a., Time period = 4 years

b Future value = \$87 000, Interest rate = 5% p.a., Time period = 12 years

c Future value = \$190 000, Interest rate = 3% p.a., Time period = 15 years

d Future value = \$200 000, Interest rate = $12\frac{1}{4}$% p.a., Time period = 5 years

e Future value = \$4600, Interest rate = 15% p.a., Time period = $2\frac{1}{2}$ years

f Future value = \$60 000, Interest rate = 6.25% p.a., Time period = $1\frac{1}{4}$ years

g Future value = \$320 000, Interest rate = 5.5% p.a., Time period = $9\frac{3}{4}$ years

h Future value = \$450 000, Interest rate = $9\frac{1}{2}$% p.a., Time period = 25 years

2 What sum of money would Zoe need to invest to accumulate a total of \$50 000 at the end of 4 years at 6% p.a. compound interest? Answer to the nearest cent.

3 Calculate the amount that must be invested at 9.3% p.a. interest compounding annually to have \$70 000 at the end of 3 years. Answer to the nearest cent.

4 What sum of money needs to be invested to accumulate to a total of \$100 000 in 10 years at 7.25% p.a. compound interest? Answer to the nearest cent.

5 Find the present value of money in a bank account if the future value after four years earning 9% p.a. compound interest, paid annually, is \$5000. Answer to the nearest dollar.

Example 7b **6** Calculate the present value, to the nearest dollar, for each of the following.

a Future value of \$1243, interest rate at 6% p.a. compounded biannually for 5 years

b Future value of \$8200, interest rate at 4% p.a. compounded quarterly for 8 years

c Future value of \$1580, interest rate at 4.8% p.a. compounded monthly for 4 years

d Future value of \$19 600, interest rate at 8% p.a. compounded weekly for 3 years

e Future value of \$3800, interest rate at 5% p.a. compounded fortnightly for 7 years

7 What sum of money would Levi need to invest to accumulate a total of \$100 000 at the end of 7 years at 8% p.a. interest compounding biannually? Answer to the nearest cent.

8 What sum of money needs to be invested to accumulate to a total of \$40 000 in 10 years at 9.25% p.a. interest compounding monthly? Answer to the nearest cent.

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3E Compound interest graphs

When graphing compound interest make the horizontal axis the compounding time periods (n) and the vertical axis the interest earned (I). Compound interest will increase by a different amount each time period. This will result in an exponential curve.

COMPOUND INTEREST GRAPHS

1. Construct a table of values for I and n using the compound interest formula.
2. Draw a number plane with n the horizontal axis and I the vertical axis. Plot the points.
3. Join the points to make an exponential curve.

Example 8: Constructing a compound interest graph — 3E

Draw a graph showing the interest earned over a period of 10 years if \$1000 is invested at a compound interest rate of 8% p.a. Use the graph to estimate the interest earned after 7.5 years.

SOLUTION:

1. Write the future value and interest earned formulas.
2. Substitute $PV = 1000$, $r = 0.08$ and n into the formula.

$$FV = PV(1 + r)^n$$
$$= 1000 \times (1.08)^n$$

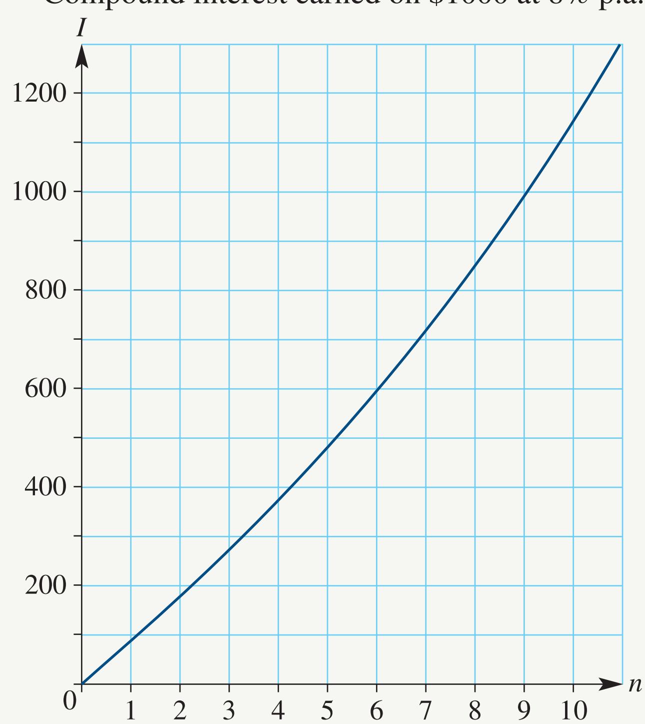
$$I = FV - PV$$
$$= PV(1 + r)^n - PV$$
$$= 1000(1.08)^n - 1000$$

3. Draw a table of values for n, FV and I
4. Let $n = 0, 2, 4, \ldots$ Find the future value and interest earned.

n	0	2	4	6	8	10
FV	1000	1166	1360	1587	1851	2159
I	0	166	360	587	851	1159

5. Draw a number plane with n as the horizontal axis and I as the vertical axis.
6. Plot the points (0, 0), (2, 166), (4, 360), (6, 587), (8, 851) and (10, 1159).
7. Draw an exponential curve (not a straight line) between the points.
8. Read the graph to estimate I when $n = 7.5$ years ($I = \$780$ when $n = 7.5$).

Compound interest earned on \$1000 at 8% p.a.



9. Write the answer in words.

Interest on the loan after 7.5 years is about \$780.

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Exercise 3E

Example 8 **1** Chloe invested \$2000 at 6% per annum interest compounding annually for 5 years.

a Substitute the present value and the interest rate into the formula $FV = PV(1 + r)^n$ to obtain an expression for the future value.

b Substitute the future value expression and the present value into the formula $I = FV - PV$.

c Use these formulas to complete the following table of values. Answer to nearest dollar.

n	0	1	2	3	4	5
FV						
I						

d Draw a number plane with n as the horizontal axis and I as the vertical axis.

e Plot the points from the table of values. Join the points to make a curve.

f Extend the graph to find the interest after 6 years.

2 Samuel invested \$800 at 7% p.a. compound interest, paid annually, for 6 years.

a Substitute the present value and the interest rate into the formula $FV = PV(1 + r)^n$ to obtain an expression for the future value.

b Substitute the future value expression and the present value into the formula $I = FV - PV$.

c Use these formulas to complete the following table of values. Answer to nearest dollar.

n	0	1	2	3	4	5	6
FV							
I							

d Draw a number plane with n as the horizontal axis and I as the vertical axis.

e Plot the points from the table of values. Join the points to make a curve.

f Use the graph to find the interest after $2\frac{1}{2}$ years.

g Extend the graph to find the interest after 7 years.

3 Mitchell is comparing three different interest rates for a possible investment.

a Draw on the same number plane the graph to represent the interest earned over 5 years on:

i \$1000 invested at 4% per annum interest compounding annually

ii \$1000 invested at 6% per annum interest compounding annually

iii \$1000 invested at 8% per annum interest compounding annually.

b What is the approximate value of the interest on each investment after 5 years?

c What is the approximate value of the interest on each investment after $3\frac{1}{2}$ years?

d Find the approximate time for each investment to earn \$200 in interest.

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4 Draw a graph showing the amount of the loan over a period of 6 years if $1000 is borrowed at a compound interest rate of 10% p.a. Use the graph to estimate the interest after $5\frac{1}{2}$ years.

5 Henry is comparing three different interest rates for a possible investment.
  a Draw a graph to represent the interest earned for 1 year on:
    i $5000 invested at 4% p.a. interest compounding quarterly
    ii $5000 invested at 8% p.a. interest compounding quarterly
    iii $5000 invested at 12% p.a. interest compounding quarterly.
  b How much does each investment earn after 1 quarter?
  c How much does each investment earn after 3 quarters?
  d Find the time for each investment to earn $200 in interest.

6 Ruby is comparing three different interest rates for a possible investment.
  a Draw a graph to represent the interest earned over 6 months on:
    i $100000 invested at 6% p.a. interest compounding monthly
    ii $100000 invested at 9% p.a. interest compounding monthly
    iii $100000 invested at 12% p.a. interest compounding monthly.
  b What is the approximate value of interest earned on each investment after 2 months?
  c How much does each investment earn after 6 months?
  d Find the time for each investment to earn $3000 in interest.

7 The table below gives details for an investment product. The compound interest earned is paid quarterly.

Investment	Rate of compound interest
A	4% p.a.
B	6% p.a.
C	8% p.a.
D	10% p.a.

Ethan is prepared to invest $50000 in the above product.
  a Draw a graph to represent the interest earned on these investments after 3 years.
  b What is the interest earned on investment B after 2 years?
  c What is the interest earned on investment C after 18 months?
  d Find the approximate time for investment D to earn $10000 in interest.

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3F Appreciation and inflation

Appreciation

Appreciation is the increase in value of items such as art, gold or land. This increase in value is often expressed as the rate of appreciation. Calculating the appreciation is similar to calculating the compound interest. For example, a painting worth \$100 000 that has an annual rate of appreciation of 10% will be worth \$110 000 after one year (an increase of \$10 000). In the second year its value will increase by \$11 000. The amount of appreciation has increased.

APPRECIATION

$FV = PV(1 + r)^n$ or $A = P(1 + r)^n$

FV – Future value of the item

PV – Present value of the item

r – Rate of appreciation per compounding time period expressed as a decimal

n – Number of compounding time periods

Example 9: Finding the appreciated value — 3F

Joel bought a unit for \$690 000. If the unit appreciates at 9% p.a., what is its value after 7 years? Answer to the nearest dollar.

SOLUTION:

1	Write the formula for appreciation $FV = PV(1 + r)^n$.	$FV = PV(1 + r)^n$
2	Substitute $PV = \$690\,000$, $r = 0.09$ (9% expressed as a decimal) and $n = 7$ into the formula.	$= 690\,000(1 + 0.09)^7$
3	Evaluate.	$= 1\,261\,346.993$
4	Write the answer to the correct degree of accuracy.	$\approx \$1\,261\,347$
5	Answer the question in words.	Unit is valued at \$1 261 347.

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Inflation

Inflation is a rise in the price of goods and services or Consumer Price Index (CPI). It is measured by comparing the prices of a fixed basket of goods and services. If inflation rises then a person's spending power decreases. The inflation rate is the annual percentage change in the CPI. In Australia, the Reserve Bank aims to keep the inflation rate in a 2% to 3% band.

Calculating inflation is similar to calculating appreciation or compound interest.

INFLATION

Inflation rate is the annual percentage change in the CPI.

Use the formula $FV = PV(1 + r)^n$ to calculate the future value of an item following inflation.

Example 10: Finding the price of goods following inflation 3F

a What is the price of a \$650 clothes dryer after one year following inflation? (Inflation rate is 2.6% p.a.)

b What is the price of a \$400 clothes dryer after three years following inflation? (Inflation rate is 3.2% p.a.)

SOLUTION:

1 Write the formula for inflation.	**a** $FV = PV(1 + r)^n$
2 Substitute $PV = 650$, $r = 0.026$ and $n = 1$ into the formula.	$= 650\,(1 + 0.026)^1$
3 Evaluate correct to two decimal places.	$= \$666.90$
4 Write the answer in words.	Clothes dryer will cost \$666.90.
1 Write the formula for inflation.	**b** $FV = PV(1 + r)^n$
2 Substitute $PV = 400$, $r = 0.032$ and $n = 3$ into the formula.	$= 400\,(1 + 0.032)^3$
3 Evaluate correct to two decimal places.	$= \$439.64$
4 Write the answer in words.	Clothes dryer will cost \$439.64.

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Exercise 3F

Example 9 1 A vintage car was bought for $70 000 and appreciated at the rate of 6% p.a. What will be the value of the car after 4 years? Answer correct to the nearest cent.

2 The price of a house has increased by 4.5% for each of the last two years. It was bought for $490 000 two years ago. What is the new current value?

3 William bought the following antiques.

a Tall boy valued at $4450. Each year its value appreciated by 5%. Calculate the value of the tall boy after 3 years. Answer correct to the nearest cent.

b Table valued at $6200. Each year its value appreciated by 4%. Calculate the value of the table after 5 years. Answer correct to the nearest cent.

c Chair valued at $1250. Each year its value appreciated by 9%. Calculate the value of the chair after 4 years. Answer correct to the nearest cent.

4 The collection of dolls was valued at $1500 four years ago. If it appreciated at 12% p.a., find its current value. Answer correct to the nearest cent.

5 The price of a diamond ring has increased from $3400 to $5300 during the past five years due to inflation. What is the rise in the price of the ring?

6 Patrick bought an apartment for $740 000. If the apartment appreciates at 8% p.a., what is its value after 6 years? Answer to the nearest dollar.

7 A retailer advertises an antique bed for $1250. If the bed appreciates at 5% p.a., what is its value after 4 years? Answer to the nearest dollar.

8 Scarlett bought a painting for $240. If the painting appreciates at 11% p.a., what is its value after 7 years? Answer to the nearest dollar.

9 Gold was bought for $4800. It appreciates by 2% p.a. Find the value of the gold after 3.5 years. Answer to the nearest dollar.

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Example 10

10 What is the price of a $1900 television after one year following inflation? (Inflation rate is 2.86% p.a.) Answer to the nearest dollar.

11 What is the price of a $500 lawn mower after 3 years following inflation? (Inflation rate is 3.5% p.a.) Answer to the nearest dollar.

12 What is the price of a $390 printer after 4 years following inflation? (Inflation rate is 5.2% p.a.) Answer to the nearest dollar.

13 The average inflation for the next 5 years is predicted to be 3%. Calculate the price of the following goods in 5 years time. Answer correct to the nearest cent.

- **a** 3 L of milk for $3.57
- **b** Loaf of bread for $3.30
- **c** 250 g honey for $4.50
- **d** 800 g of eggs for $5.20

14 If the inflation rate is 5% p.a., what would you expect to pay, to the nearest dollar, in 4 years time for a house that costs:

- **a** $280 000?
- **b** $760 000?
- **c** $324 000?
- **d** $580 000?
- **e** $1 260 000?
- **f** $956 000?

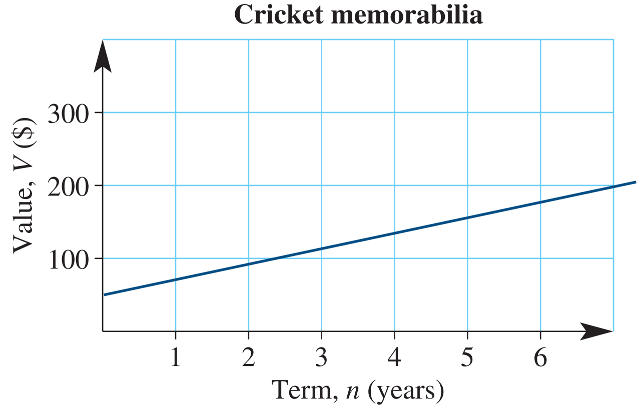
15 If the inflation rate is 5% p.a., what would you expect to pay, to the nearest dollar, in 4 years time for a new motor vehicle that costs:

- **a** $34 000?
- **b** $22 500?
- **c** $65 000?
- **d** $19 990?
- **e** $120 000?
- **f** $57 200?

16 The graph below shows the value of cricket memorabilia for the past 6 years.

- **a** What was the value of the memorabilia after 4 years?
- **b** What was the value of the memorabilia after 6 years?
- **c** What was the initial value?
- **d** How much did the memorabilia appreciate each year?

Cricket memorabilia



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Summary

PPT STUDY GUIDE

Key ideas and chapter summary

Simple interest	$I = Prn \qquad A = P + I$ I – Interest (simple or flat) earned for the use of money P – Principal is the initial amount of money borrowed r – Rate of simple interest per period expressed as a decimal n – Number of time periods A – Amount or final balance
Simple interest graphs	**1** Construct a table of values for I and n using $I = Prn$. **2** Draw a number plane – n is the horizontal axis, I is the vertical axis **3** Plot the points and join them to make a straight line.
Compound interest future value	$FV = PV(1 + r)^n$ or $I = FV - PV$ FV – Future value of the loan or amount (final balance) PV – Present value of the loan or principal (initial quantity of money) r – Rate of interest per compounding time period as a decimal n – Number of compounding time periods I – Interest (compounded) earned
Compound interest present value	$PV = \dfrac{FV}{(1 + r)^n}$ FV – Future value of the loan or amount (final balance) PV – Present value of the loan or principal (initial quantity of money) r – Rate of interest per compounding time period as a decimal n – Number of compounding time periods
Compound interest graphs	**1** Construct a table of values for n and I using $FV = PV(1 + r)^n$ and $I = FV - PV$. **2** Draw a number plane – n is the horizontal axis, I is the vertical axis **3** Plot the points and join them to make a curve.
Appreciation and inflation	Use the formula $FV = PV(1 + r)^n$ for appreciation and inflation. Inflation rate is the annual percentage change in the CPI.

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Review

Multiple-choice

1 What is the flat-rate interest on $1400 at 7% p.a. for 3 years?
A $98 **B** $196 **C** $294 **D** $498

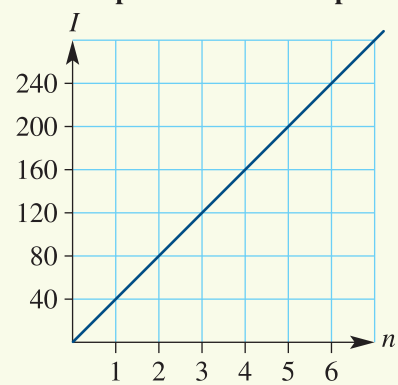
2 Gemma invested $800 for 2 years at a simple interest rate of 4% per annum. What is the total amount of interest earned by the investment?
A $32 **B** $64 **C** $160 **D** $320

3 Lincoln wants to earn $9000 a year in interest. What must he invest if the simple interest rate is 15% p.a.? Answer to the nearest dollar.
A $1350 **B** $10 350 **C** $60 000 **D** $600 000.

4 Using the graph, what is the interest after $3\frac{1}{2}$ years?
A $120 **B** $130
C $140 **D** $240

5 What was the amount of the investment shown in the graph?
A $1000 **B** $2000
C $3000 **D** $4000

Simple interest at 4% p.a.



6 Alana invests $8000 at 10% p.a. interest compounding annually. What is the future value after 3 years? (Answer to the nearest dollar.)
A $242 **B** $2648 **C** $8242 **D** $10 648

7 George borrows $3000 at 10% p.a. interest compounding annually. What is the interest earned after 2 years? (Answer to the nearest dollar.)
A $630 **B** $1500 **C** $6000 **D** $3630

8 The compound interest on $4600 at 12% p.a. for 2 years is:
A $1104 **B** $1170 **C** $4600 **D** $5700

9 A painting was bought for $460 000 and appreciated at the rate of 7% p.a. What will be the value of the painting after 4 years? (Answer to the nearest dollar.)
A $473 016 **B** $492 200 **C** $588 800 **D** $602 966

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Review

Short-answer

1 Charles takes out a flat-rate loan of \$60 000 for a period of 5 years, at a simple interest rate of 12% per annum. Find the amount owing at the end of 5 years.

2 Keira would like to purchase a \$2000 TV from an electronics shop. However, to buy the TV, she has applied for a flat-rate loan over 2 years at 15% p.a. How much does Amelia pay altogether for the TV?

3 Nate borrowed \$1800 at 6% per annum. What is the simple interest accrued between 30 June and 1 September?

4 Kayla borrowed \$36 000 at a flat rate of interest of 7% per annum for $3\frac{1}{2}$ years. How much interest did she pay? Answer to the nearest dollar.

5 Sam invested \$1000 at 7% per annum simple interest for 4 years.

a Simplify the simple interest formula ($I = Prn$) by substituting values for the principal and the interest rate.

b Use this formula to complete the following table of values.

n	0	1	2	3	4
I					

c Draw a number plane with n as the horizontal axis and I as the vertical axis.

d Plot the points from the table of values. Join the points to make a straight line.

e Use the graph to find the interest after $2\frac{1}{2}$ years.

f Extend the graph to find the interest after 6 years.

g Find the time when the interest is \$210.

6 Caitlin invested \$1000 at 5% per annum simple interest for 6 years.

a Simplify the simple interest formula ($I = Prn$) by substituting values for the principal and the interest rate.

b Use this formula to complete the following table of values.

n	0	1	2	3	4	5
I						

c Draw a number plane with n as the horizontal axis and I as the vertical axis.

d Plot the points from the table of values. Join the points to make a straight line.

e What is the interest after $5\frac{1}{2}$ years?

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Review

7 Calculate the future value, to the nearest cent, for each of the following.

a Present value = \$920, Compound interest rate = 5% p.a., Time period = 4 years

b Present value of \$2100 invested for 3 years at 6.1% p.a. compounded monthly.

8 Calculate the present value, to the nearest cent, for each of the following.

a Future value = \$26 000, Interest rate = 4.9% p.a., Time period = 3 years

b Future value of \$10 400, Interest rate at 9% p.a. compounded quarterly for 5 years.

9 What sum of money would Emma need to invest to accumulate a total of \$200 000 at the end of 10 years at 12% p.a. interest compounding biannually? Answer to the nearest cent.

10 Declan invested \$1600 at 10% p.a. compound interest, paid annually, for 6 years.

a Substitute the present value and the interest rate into the formula $FV = PV(1 + r)^n$ to obtain an expression for the future value.

b Substitute the future value expression and the present value into the formula $I = FV - PV$.

c Use these formulas to complete the following table of values. Answer to nearest dollar.

n	0	1	2	3	4	5	6
FV							
I							

d Draw a number plane with n as the horizontal axis and I as the vertical axis.

e Plot the points from the table of values. Join the points to make a curve.

f Use the graph to find the interest after $1\frac{1}{2}$ years.

11 An investment is appreciating at a rate of 4% of its value each year. Ruby decides to invest \$480 000.

a What will be the investment's value after 10 years? Answer to the nearest dollar.

b How much does the investment increase during the first 10 years?

PDF LITERACY

12 The average inflation for the next five years is predicted to be 2.5%. Calculate the price of the following goods in 3 years time. Answer to the nearest cent.

a 2 L of soft drink for \$2.80

b Apple pie for \$4.60

c Hamburger for \$6.00

d Bottle of water for \$1.60

e Punnet of strawberries for \$4.50

f 500 g of chicken breast for \$8.90

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4 Right-angled triangles

Syllabus topic — M3 Right-angled triangles

This topic is focused on solving problems involving right-angled triangles in a variety of contexts.

Outcomes

- Find unknown sides using Pythagoras' theorem.
- Solve problems using Pythagoras' theorem.
- Define the trigonometric ratios – sine, cosine and tangent.
- Use a calculator in trigonometry with angles to the nearest minute.
- Find unknown sides using trigonometry.
- Find unknown angles using trigonometry.
- Using trigonometry to solve practical problems.
- Solve problems involving compass and true bearings.
- Solve problems involving angles of elevation and depression.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Literacy worksheet
- Quick Quiz
- Solutions (enabled by teacher)
- Desmos widgets
- Spreadsheets
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

The Interactive Textbook provides a test of prior knowledge for this chapter, and may direct you to revision from the previous years' work.

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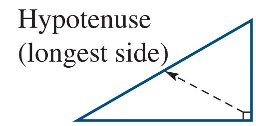
4A Pythagoras' theorem

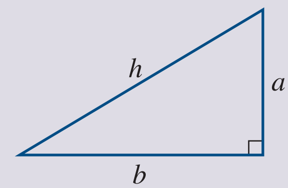
Pythagoras' theorem links the sides of a right-angled triangle. In a right-angled triangle the side opposite the right angle is called the hypotenuse. The hypotenuse is always the longest side.



PYTHAGORAS' THEOREM

Pythagoras' theorem states that the square of the hypotenuse is equal to the sum of the squares of the other two sides.

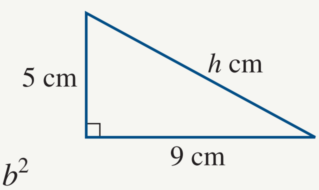
(Hypotenuse)^2 = (side)^2 + (other side)^2 $\quad h^2 = a^2 + b^2$



Pythagoras' theorem is used to find a missing side of a right-angled triangle if two of the sides are given. It can also be used to prove that a triangle is right angled.

Example 1: Finding the length of the hypotenuse 4A

Find the length of the hypotenuse, correct to two decimal places.



SOLUTION:

1 Write Pythagoras' theorem. $\quad h^2 = a^2 + b^2$

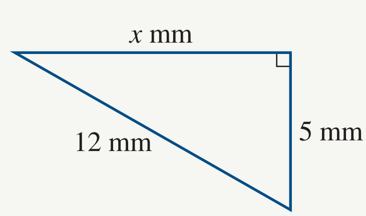
2 Substitute the length of the sides. $\quad = 9^2 + 5^2$

3 Take the square root to find h. $\quad h = \sqrt{9^2 + 5^2}$

4 Express the answer correct to two decimal places. $\quad \approx 10.30\,\text{cm}$

Example 2: Finding the length of a shorter side 4A

What is the value of x, correct to one decimal place?



SOLUTION:

1 Write Pythagoras' theorem. $\quad h^2 = a^2 + b^2$

2 Substitute the length of the sides. $\quad 12^2 = x^2 + 5^2$

3 Make x^2 the subject. $\quad x^2 = 12^2 - 5^2$

4 Take the square root to find x. $\quad x = \sqrt{12^2 - 5^2}$

5 Express the answer to correct one decimal place. $\quad \approx 10.9\,\text{mm}$

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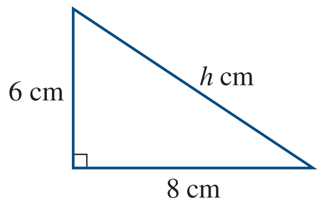
Exercise 4A

Example 1 **1** Find the length of the hypotenuse, correct to one decimal place.

a



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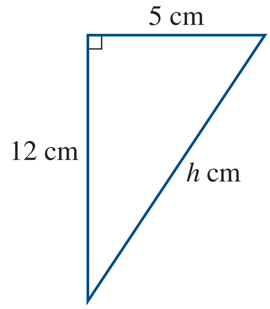

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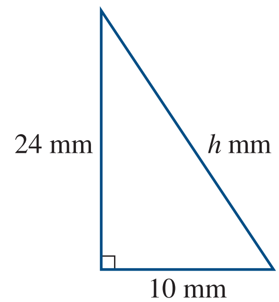

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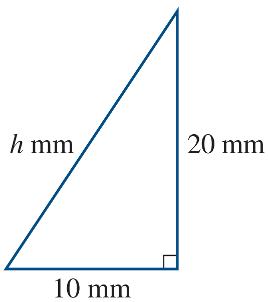

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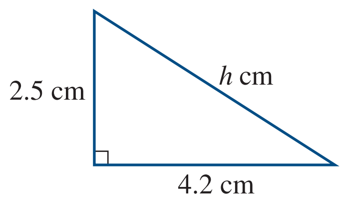

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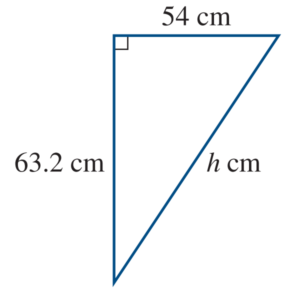
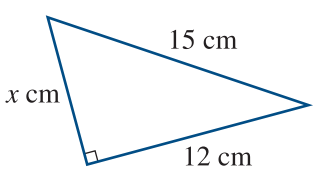


Example 2 **2** Find the value of x, correct to two decimal places.

a



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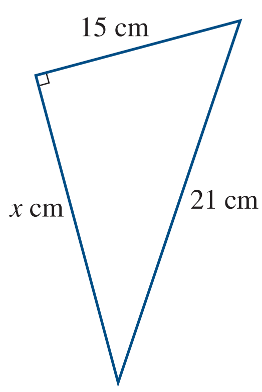

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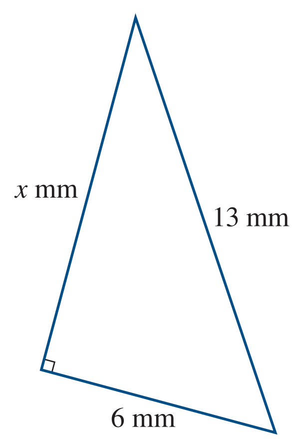

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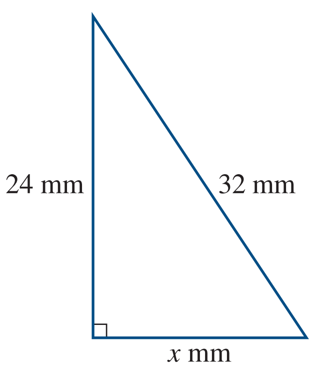

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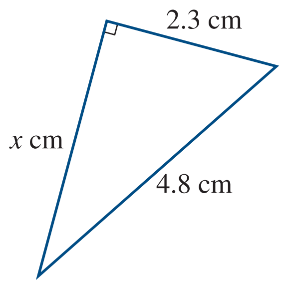

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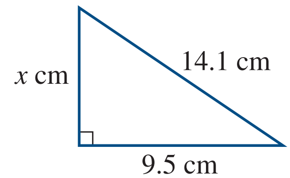

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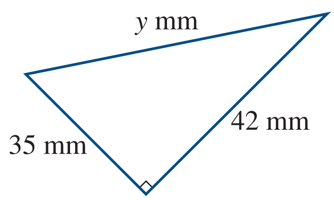
3 Calculate the length of the side marked with the pronumeral. (Answer to the nearest millimetre.)

a



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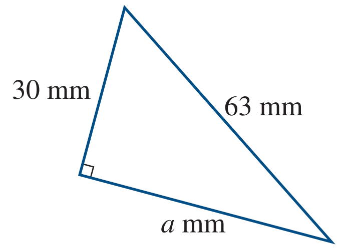

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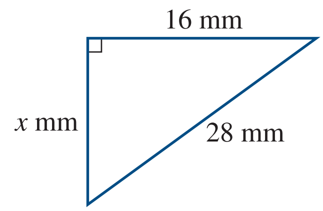

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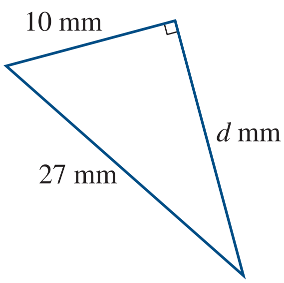

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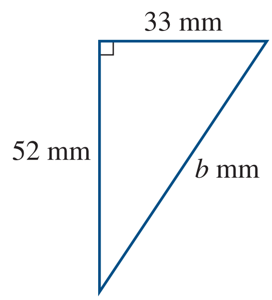

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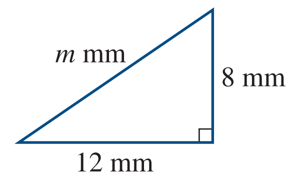
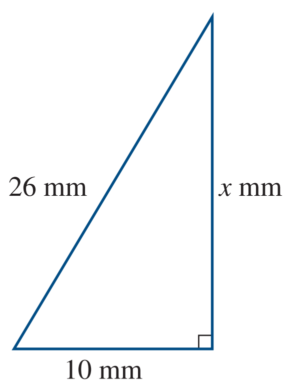


g



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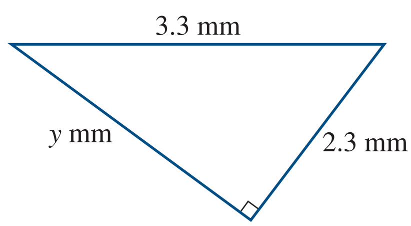
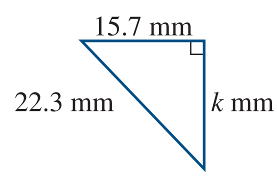
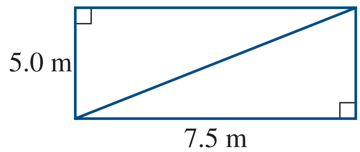


i

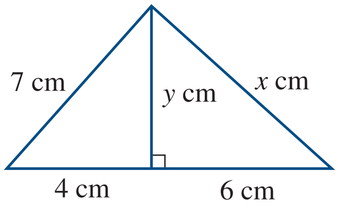


4 Find, correct to one decimal place, the length of the diagonal of a rectangle with dimensions 7.5 metres by 5.0 metres.



5 **a** Find the value of y, correct to two decimal places.

b Find the value of x using the value of y from part **a**, correct to two decimal places.



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4B Applying Pythagoras' theorem

Pythagoras' theorem is used to solve many practical problems. These problems are represented by a right-angle triangle and require the use of Pythagoras' theorem to determine the hypotenuse or the length of a shorter side.

SOLVING A PROBLEM USING PYTHAGORAS' THEOREM

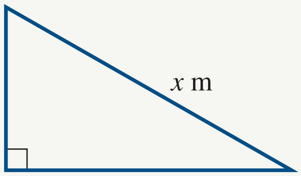
1. Read the question and underline the key terms.
2. Draw a diagram and label the information from the question.
3. Decide whether to determine the hypotenuse or the length of a shorter side.
4. Use Pythagoras' theorem to calculate a solution.
5. Check that the answer is reasonable and units are correct.
6. Explain the answer in words and ensure the question has been answered.

Example 3: Solving problems using Pythagoras' theorem 4B

A helicopter is at a height of 200 m above the ground and is a horizontal distance of 320 m from a landing pad. Find the direct distance of the helicopter from the landing pad, correct to two decimal places.

SOLUTION:

1. Draw a diagram and label the information from the question.
2. Label the hypotenuse with x. This represents the distance of the helicopter from the landing pad.



3. Write Pythagoras' theorem.
4. Substitute the length of the sides.
5. Take the square root to find x.
6. Express the answer correct to two decimal places.
7. Write your answer in words.

$$x^2 = a^2 + b^2$$
$$= 320^2 + 200^2$$
$$x = \sqrt{320^2 + 200^2}$$
$$\approx 377.36\text{ m}$$

∴ The helicopter is 377.36 metres from the landing pad.

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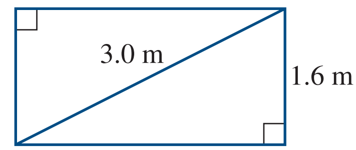
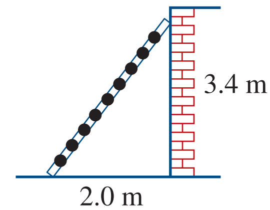
Exercise 4B

Example 3

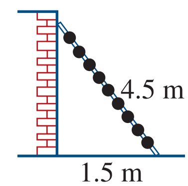
1 A farm gate that is 1.6 m high is supported by a diagonal bar of length 3.0 m. Find the width of the gate, correct to one decimal place.



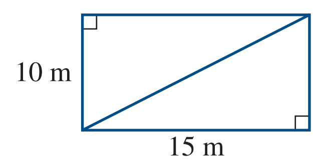
2 A ladder rests against a brick wall as shown in the diagram on the right. The base of the ladder is 2.0 m from the wall, and reaches 3.4 m up the wall. Find the length of the ladder, correct to one decimal place.



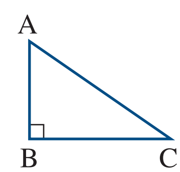
3 The base of a ladder leaning against a wall is 1.5 m from the base of the wall. If the ladder is 4.5 m long, find how high the top of the ladder is from the ground, correct to one decimal place.



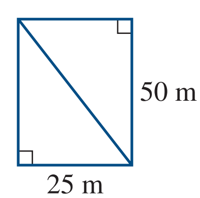
4 Find, correct to one decimal place, the length of the diagonal of a rectangle with dimensions 15 metres by 10 metres.



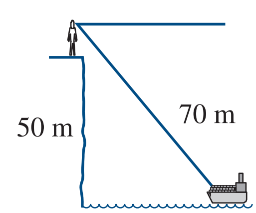
5 In a triangle ABC, there is a right angle at B. AB is 12 cm and BC is 16 cm. Find the length of AC.



6 A rectangular block of land measures 25 m by 50 m. John wants to put a fence along the diagonal. How long will the fence be? (Answer correct to three decimal places.)



7 Use the measurements on the diagram to determine the distance the boat is out to sea. (Answer correct to the nearest metre.)



8 The hypotenuse of a right-angled triangle is 40 cm long and one of the shorter sides measures 20 cm. What is the length of the remaining side in the triangle? (Answer correct to two decimal places.)

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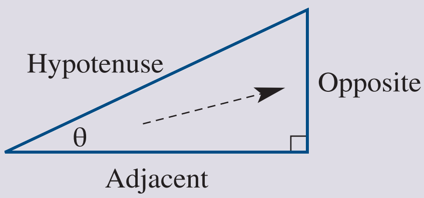
4C Trigonometric ratios

Trigonometric ratios are defined using the sides of a right-angled triangle. The hypotenuse is opposite the right angle, the opposite side is opposite the angle ˜ and the adjacent side is the remaining side.

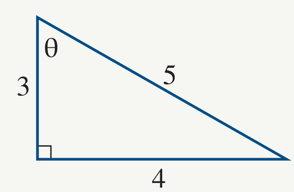
NAMING SIDES OF A RIGHT-ANGLED TRIANGLE



The opposite and adjacent sides are located in relation to the position of angle ˜. If ˜ was in the other angle, the sides would swap their labels. The letter ˜ is the Greek letter theta. It is commonly used to label an angle.

Example 4: Naming the sides of a right-angled triangle 4C

What are the values of the hypotenuse, the opposite side and the adjacent side of the triangle shown?



SOLUTION:

1	Hypotenuse is opposite the right angle.	Hypotenuse is 5 ($h = 5$)
2	Opposite side is opposite the angle ˜.	Opposite side is 4 ($o = 5$)
3	Adjacent side is beside the angle ˜, but not the hypotenuse.	Adjacent side is 3 ($a = 5$)

The trigonometric ratios

The trigonometric ratios sin ˜, cos ˜ and tan ˜ are defined using the sides of a right-angled triangle.

$$\sin \tilde{} = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos \tilde{} = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan \tilde{} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\sin \tilde{} = \frac{o}{h}\ \text{(SOH)} \qquad \cos \tilde{} = \frac{a}{h}\ \text{(CAH)} \qquad \tan \tilde{} = \frac{o}{a}\ \text{(TOA)}$$

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TRIGONOMETRIC RATIOS

The mnemonic 'SOH CAH TOA' is pronounced as a single word.

SOH: **S**ine-**O**pposite-**H**ypotenuse

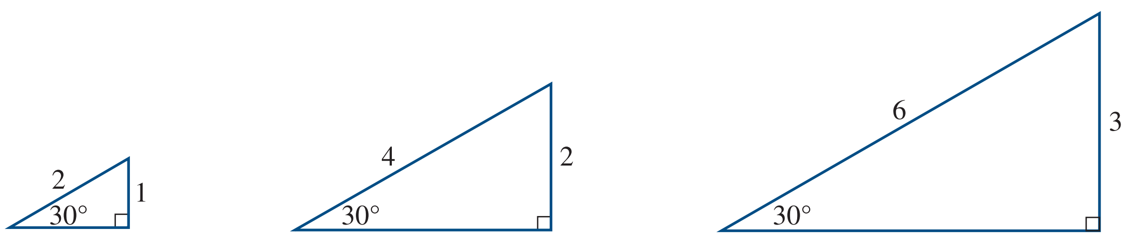
CAH: **C**osine-**A**djacent-**H**ypotenuse

TOA: **T**angent-**O**pposite-**A**djacent

The order of the letters matches the ratio of the sides.

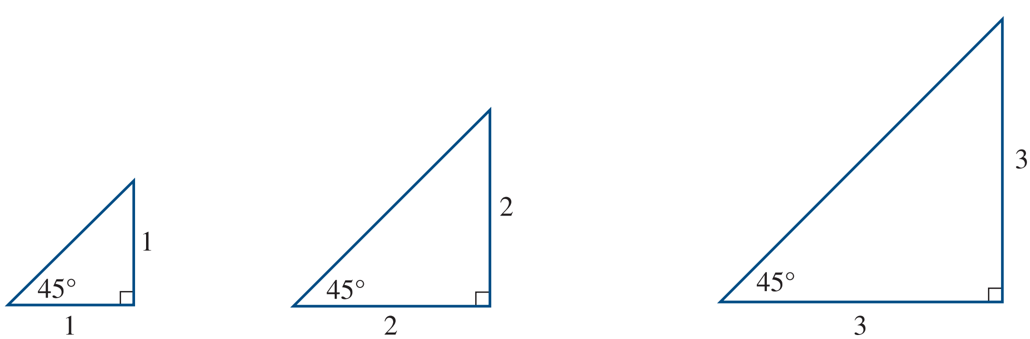
The meaning of the trigonometric ratios

Consider the three triangles drawn below.



The three triangles drawn above show the ratio of the opposite side to the hypotenuse as 0.5. $\left(\frac{1}{2}, \frac{2}{4} \text{ or } \frac{3}{6}\right)$. This is called the sine ratio. All right-angled triangles with an angle of 30° have a sine ratio of 0.5. If the angle is not 30° the ratio will be different, but any two right-angled triangles with the same angle will have the same value for their sine ratio.

Similarly, the three triangles drawn below show the ratio of the opposite side to the adjacent side as 1 $\left(\frac{1}{1}, \frac{2}{2} \text{ or } \frac{3}{3}\right)$. This is called the tangent ratio. All right-angled triangles with an angle of 45° have a tangent ratio of 1.



The ratio of the opposite side to the hypotenuse (sine ratio), the ratio of the adjacent side to the hypotenuse (cosine ratio) and the ratio of the opposite side to the adjacent side (tangent ratio) will always be constant irrespective of the size of the right-angled triangle.

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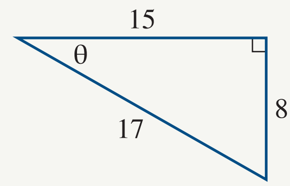
Example 5: Finding the trigonometric ratio 4C

Find the sine, cosine and tangent ratios for angle ˜ in the triangle shown.



SOLUTION:

1 Name the sides of the right-angled triangle. a (15), o (8), h (17)

2 Write the sine ratio (SOH). $\sin ˜ = \frac{o}{h}$

3 Substitute the values for the opposite side and the hypotenuse. $= \frac{8}{17}$

4 Write the cosine ratio (CAH). $\cos ˜ = \frac{a}{h}$

5 Substitute the values for the adjacent side and the hypotenuse. $= \frac{15}{17}$

6 Write the tangent ratio (TOA). $\tan ˜ = \frac{o}{a}$

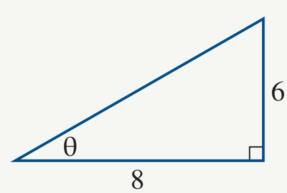
7 Substitute the values for the adjacent side and the opposite side. $= \frac{8}{15}$

Example 6: Finding a trigonometric ratio 4C

Find sin ˜ in simplest form given tan ˜ $= \frac{6}{8}$.

SOLUTION:

1 Draw a triangle and label the opposite and adjacent sides.



2 Find the hypotenuse using Pythagoras' theorem.

3 Substitute the length of the sides into Pythagoras' theorem. $h^2 = 6^2 + 8^2$

4 Take the square root to find the hypotenuse (h). $h = \sqrt{6^2 + 8^2}$

5 Evaluate. $= 10$

6 Write the sine ratio (SOH). $\sin ˜ = \frac{o}{h}$

7 Substitute the values for the opposite side and the hypotenuse. $= \frac{6}{10}$

8 Simplify the ratio. $= \frac{3}{5}$

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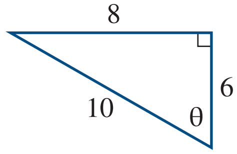
Exercise 4C

Example 4 **1** State the values of the hypotenuse, opposite side and adjacent side in each triangle.

a


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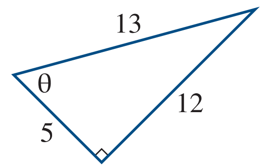

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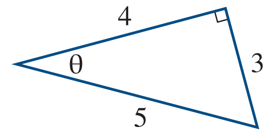

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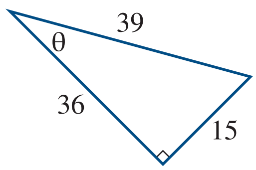

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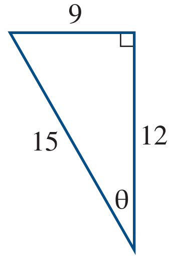

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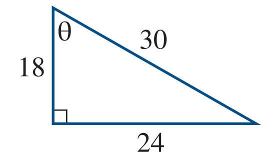
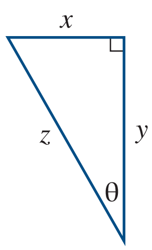


2 State the values of the hypotenuse, opposite side and adjacent side in each triangle.

a


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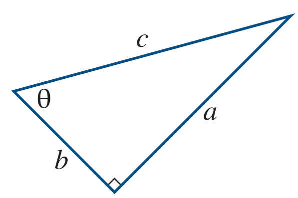

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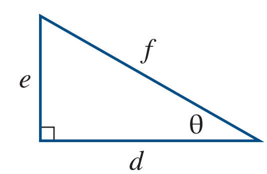
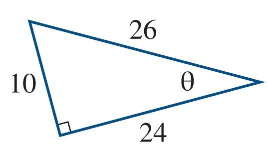


Example 5 **3** Write the ratios for sin ˜, cos ˜ and tan ˜ for each triangle in question **1**.

4 Write the ratios for sin ˜, cos ˜ and tan ˜ for each triangle in question **2**.

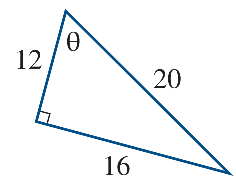
5 Name the trigonometric ratio represented by the following fractions.

a


i $\frac{10}{26}$

ii $\frac{24}{26}$

iii $\frac{10}{24}$

b


i $\frac{12}{20}$

ii $\frac{16}{12}$

iii $\frac{16}{20}$

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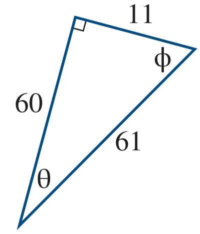
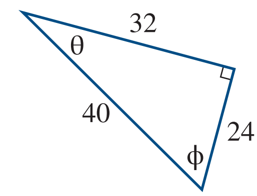
6 Find the sine, cosine and tangent ratios in simplest form for each angle.

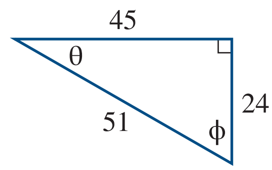
a **i** angle ˜
ii angle °



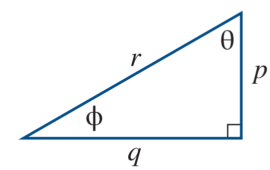
b **i** angle ˜
ii angle °



c **i** angle ˜
ii angle °

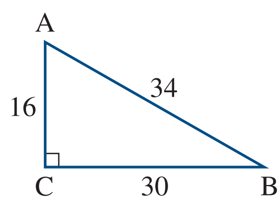


d **i** angle ˜
ii angle °

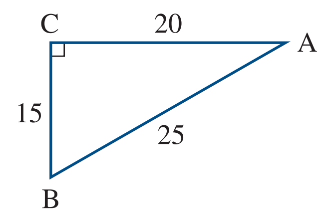


7 Find the sine and cosine ratios in simplest form for angle A and B for each triangle.

a



b



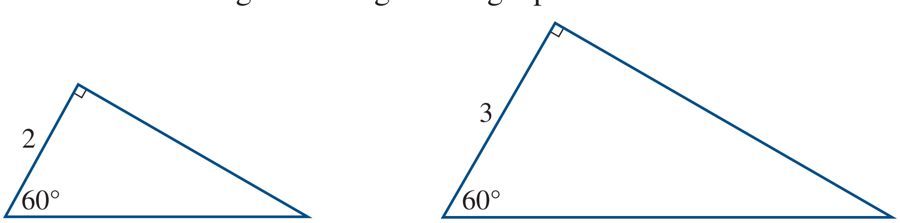
Example 6 **8** Draw a right-angled triangle for each of the following trigonometric ratios and
i find the length of the third side
ii find the other two trigonometric ratios in simplest form.

a $\tan ˜ = \frac{3}{4}$

b $\sin ˜ = \frac{8}{10}$

c $\cos ˜ = \frac{7}{25}$

9 Draw the following two triangles using a protractor and a ruler.



a Measure the length of the hypotenuse, adjacent and opposite sides in each triangle.
b What is the value of the cosine ratio for 60° in both triangles?
c What is the value of the sine ratio for 60° in both triangles?

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4D Using the calculator in trigonometry

In trigonometry, an angle is usually measured in degrees, minutes and seconds. Make sure the calculator is set up to accept angles in degrees. It is essential in this course that the degree mode is selected.

DEGREES	MINUTES
1 degree = 60 minutes	1 minute = 60 seconds
$1° = 60'$	$1' = 60''$

Finding a trigonometric ratio

A calculator is used to find a trigonometric ratio of a given angle. It requires the [sin], [cos] and [tan] keys. The trigonometric ratio key is pressed followed by the angle. The degrees, minutes and seconds [° ′ ″] or [DMS] is then selected to enter minutes and seconds. Some calculators may require you to choose degrees, minutes and seconds from an options menu.

Example 7: Finding a trigonometric ratio 4D

Find the value of the following, correct to two decimal places.

a $\sin 60°$ **b** $2\cos 40.5°$ **c** $\dfrac{3}{\tan 75°}$ **d** $\sin 34°20'$

SOLUTION:

1 Press [sin] 60 [=] or [exe]

a $\sin 60° = 0.866\,025\,403\,8$
≈ 0.87

2 Press 2 [cos] 40.5 [=] or [exe]

b $2\cos 40.5° = 1.520\,811\,31$
≈ 1.52

3 Press 3 ÷ [tan] 75 [=] or [exe]

c $\dfrac{3}{\tan 75°} = 0.803\,847\,577\,3$
≈ 0.80

4 Press [sin] 34 [° ′ ″] 20 [° ′ ″] [=] or [exe]

d $\sin 34°20' = 0.564\,006\,558\,1$
≈ 0.56

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Finding an angle from a trigonometric ratio

A calculator is used to find a given angle from a trigonometric ratio. Check that the degree mode is selected. To find an angle, use the $\boxed{\sin^{-1}}$, $\boxed{\cos^{-1}}$ and $\boxed{\tan^{-1}}$ keys. To select these keys, press the SHIFT or a 2nd function key. The degrees, minutes and seconds $\boxed{{}^{\circ}\,{}'\,{}''}$ or $\boxed{\text{DMS}}$ is then selected to find the angle in minutes and seconds.

Example 8: Finding an angle from a trigonometric ratio — 4D

a Given sin ˜ = 0.6123, find the value of ˜ to the nearest degree.
b Given tan ˜ = 1.45, find the value of ˜ to the nearest minute.

SOLUTION:

1 Press SHIFT $\boxed{\sin^{-1}}$ 0.6123 $\boxed{=}$ or $\boxed{\text{exe}}$.

a sin ˜ = 0.6123
˜ = 37.755 994 38
≈ 38°

2 Press SHIFT $\boxed{\tan^{-1}}$ 1.45 $\boxed{=}$ or $\boxed{\text{exe}}$.
Convert the answer to minutes by using the $\boxed{{}^{\circ}\,{}'\,{}''}$ or $\boxed{\text{DMS}}$.

b tan ˜ = 1.45
˜ = 55°24′27.76″
≈ 55°24′

Example 9: Finding an angle from a trigonometric ratio — 4D

a Given sin ˜ $= \frac{4}{5}$, find the value of ˜ to the nearest degree.

b Given cos ˜ $= \frac{1}{\sqrt{2}}$, find the value of ˜ to the nearest degree.

SOLUTION:

1 Press SHIFT $\boxed{\sin^{-1}}$ $\frac{4}{5}$ $\boxed{=}$ or $\boxed{\text{exe}}$.

a sin ˜ $= \frac{4}{5}$
˜ = 53.130102 … ≈ 53°

2 Press SHIFT $\boxed{\cos^{-1}}$ $\frac{1}{\sqrt{2}}$ $\boxed{=}$ or $\boxed{\text{exe}}$.

b cos ˜ $= \frac{1}{\sqrt{2}}$
˜ = 45°

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Exercise 4D

1 What is the value of the following angles in minutes?

a $1°$ **b** $3°$ **c** $5°$ **d** $7°$
e $10°$ **f** $15°$ **g** $20°$ **h** $60°$
i $0.5°$ **j** $\frac{1}{3}°$ **k** $0.2°$ **l** $0.25°$

2 What is the value of the following angles in degrees?

a 120 minutes **b** 480 minutes
c 60 minutes **d** 600 minutes
e 360 minutes **f** 240 minutes
g 900 minutes **h** 720 minutes
i 30 minutes **j** 15 minutes
k 45 minutes **l** 20 minutes

Example 7a
3 Find the value of the following trigonometric ratios, correct to two decimal places.

a $\sin 20°$ **b** $\cos 43°$
c $\tan 65°$ **d** $\cos 72°$
e $\tan 13°$ **f** $\sin 82°$
g $\cos 15°$ **h** $\tan 48°$

Example 7b
4 Find the value of the following trigonometric ratios, correct to two decimal places.

a $\cos 63°30'$ **b** $\sin 40°10'$ **c** $\cos 52°45'$ **d** $\tan 35°23'$
e $\sin 22°56'$ **f** $\tan 53°42'$ **g** $\tan 68°2'$ **h** $\cos 65°57'$

Example 7c
5 Find the value of the following trigonometric ratios, correct to one decimal place.

a $4\cos 30°$ **b** $3\tan 53°$ **c** $5\sin 74°$ **d** $6\sin 82°$
e $6\tan 77°$ **f** $2\cos 43°$ **g** $8\sin 12°$ **h** $9\tan 54°$

6 Find the value of the following trigonometric ratios, correct to one decimal place.

a $4\sin 65°20'$ **b** $5\tan 23°55'$ **c** $12\cos 10°41'$ **d** $8\sin 21°9'$
e $11\sin 21°30'$ **f** $7\cos 32°40'$ **g** $4\sin 25°12'$ **h** $8\tan 39°24'$

Example 7d
7 Find the value of the following trigonometric ratios, correct to two decimal places.

a $\frac{5}{\tan 40°}$ **b** $\frac{1}{\sin 63°}$ **c** $\frac{12}{\cos 25°}$ **d** $\frac{3}{\sin 42°}$
e $\frac{4}{\cos 38°9'}$ **f** $\frac{5}{\tan 72°36'}$ **g** $\frac{6}{\sin 55°48'}$ **h** $\frac{7}{\cos 71°16'}$

Example 8a
8 Given the following trigonometric ratios, find the value of ˜ to the nearest degree.

a sin ˜ = 0.5673 **b** cos ˜ = 0.1623 **c** tan ˜ = 0.2782
d cos ˜ = 0.7843 **e** tan ˜ = 0.5047 **f** sin ˜ = 0.1298

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Example 8b **9** Given the following trigonometric ratios, find the value of ˜ to the nearest minute.

a tan ˜ = 0.3891
b sin ˜ = 0.6456
c cos ˜ = 0.1432
d sin ˜ = 0.8651
e cos ˜ = 0.3810
f tan ˜ = 0.8922

Example 9a **10** Given the following trigonometric ratios, find the value of ˜ to the nearest degree.

a tan ˜ = $\frac{3}{4}$
b sin ˜ = $\frac{1}{2}$
c cos ˜ = $\frac{5}{8}$
d cos ˜ = $\frac{1}{4}$
e sin ˜ = $\frac{3}{5}$
f tan ˜ = $1\frac{1}{3}$

Example 9b **11** Given the following trigonometric ratios, find the value of ˜ to the nearest degree.

a sin ˜ = $\frac{\sqrt{3}}{2}$
b tan ˜ = $\frac{1}{\sqrt{5}}$
c cos ˜ = $\frac{\sqrt{5}}{6}$
d tan ˜ = $\frac{4}{\sqrt{6}}$
e cos ˜ = $\frac{\sqrt{3}}{2}$
f sin ˜ = $\frac{1}{\sqrt{2}}$

12 Given the following trigonometric ratios, find the value of ˜ to the nearest minute.

a cos ˜ = $\frac{2}{\sqrt{7}}$
b sin ˜ = $\frac{\sqrt{3}}{4}$
c tan ˜ = $\frac{\sqrt{5}}{12}$
d sin ˜ = $\frac{\sqrt{7}}{7}$
e tan ˜ = $\frac{\sqrt{2}}{7}$
f cos ˜ = $\frac{3}{\sqrt{11}}$

13 Given that sin ˜ = 0.4 and angle ˜ is less than 90°, find the value of:

a ˜ to the nearest degree
b cos ˜ correct to one decimal place
c tan ˜ correct to two decimal places.

14 Given that cos ˜ = 0.8 and angle ˜ is less than 90°, find the value of:

a ˜ to the nearest degree
b sin ˜ correct to one decimal place
c tan ˜ correct to two decimal places.

15 Given that tan ˜ = 2.1 and angle ˜ is less than 90°, find the value of:

a ˜ to the nearest minute
b sin ˜ correct to three decimal places
c cos ˜ correct to four decimal places.

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4E Finding an unknown side

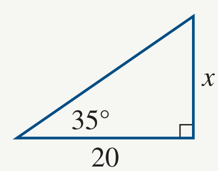
Trigonometric ratios are used to find an unknown side in a right-angled triangle, given at least one angle and one side. The method involves labelling the sides of the triangle and using the mnemonic SOH CAH TOA. The resulting equation is rearranged to make x the subject and the calculator used to find the unknown side.

FINDING AN UNKNOWN SIDE IN A RIGHT-ANGLED TRIANGLE

1. Name the sides of the triangle – h for hypotenuse, o for opposite and a for adjacent.
2. Use the given side and unknown side x to determine the trigonometric ratio. The mnemonic SOH CAH TOA helps with this step.
3. Rearrange the equation to make the unknown side x the subject.
4. Use the calculator to find x. Remember to check the calculator is set up for degrees.
5. Write the answer to the specified level of accuracy.

Example 10: Finding an unknown side 4E

Find the length of the unknown side x in the triangle shown. Answer correct to three decimal places.

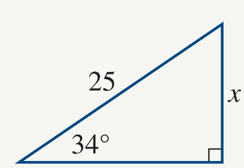


SOLUTION:

1	Name the sides of the right-angled triangle.	a (20), o (x), h
2	Determine the ratio (TOA).	$\tan \tilde{\ } = \frac{o}{a}$
3	Substitute the known values.	$\tan 35° = \frac{x}{20}$
4	Multiply both sides of the equation by 20.	$x = 20 \times \tan 35°$
5	Press 20 [tan] 35 [=]	$= 14.004150...$
6	Write the answer correct to three decimal places.	≈ 14.004

Example 11: Finding an unknown side 4E

Find the length of the unknown side x in the triangle shown. Answer correct to two decimal places.



SOLUTION:

1	Name the sides of the right-angled triangle.	a, o (x), h (25)
2	Determine the ratio (SOH).	$\sin \tilde{\ } = \frac{o}{h}$
3	Substitute the known values.	$\sin 34° = \frac{x}{25}$
4	Multiply both sides of the equation by 25.	$x = 25 \times \sin 34°$
5	Press 25 [sin] 34 [=]	$= 13.979822...$
6	Write the answer correct to two decimal places.	≈ 13.98

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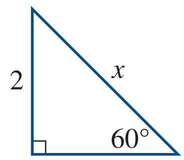
Finding an unknown side in the denominator

It is possible that the unknown side (x) is the denominator of the trigonometric ratio. For example, in the triangle below, the unknown x is the hypotenuse of the triangle. This results in the trigonometric ratio $\frac{2}{x}$.



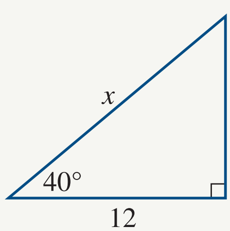
$$\sin \text{~} = \frac{o}{h}$$

$$\sin 60° = \frac{2}{x}$$

To solve these types of equations, multiply both sides by x. Then divide both sides by the trigonometric expression (sin 60°) to make x the subject.

Example 12: Finding an unknown side in the denominator — 4E

Find the length of the unknown side x in the triangle shown. Answer correct to two decimal places.



SOLUTION:

1. Name the sides of the right-angled triangle. — a (12), o, h (x)
2. Determine the ratio (CAH). — $\cos \text{~} = \frac{a}{h}$
3. Substitute the known values. — $\cos 40° = \frac{12}{x}$
4. Multiply both sides of the equation by x. — $x \times \cos 40° = 12$
5. Divide both sides by cos 40°. — $x = \frac{12}{\cos 40°}$
6. Press 12 ÷ [cos] 40 [=] or [exe] — $x = 15.66488747$
7. Write the answer correct to two decimal places. — ≈ 15.66

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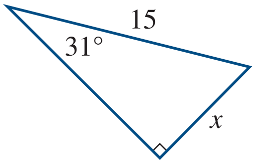
Exercise 4E

Example 10 **1** Find the length of the unknown side x in each triangle, correct to two decimal places.

a


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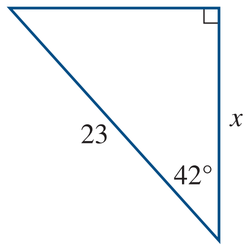

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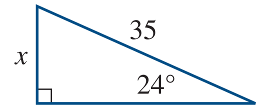

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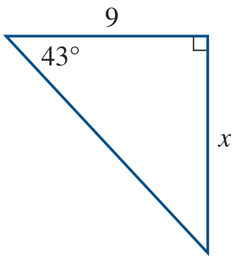

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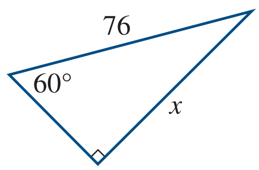

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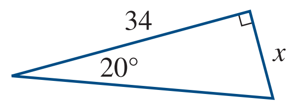

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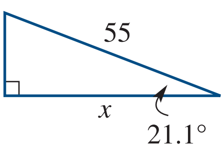
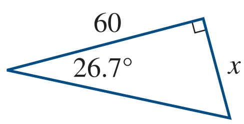


h


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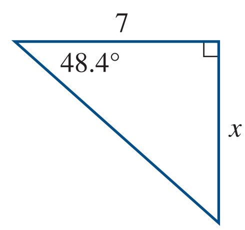
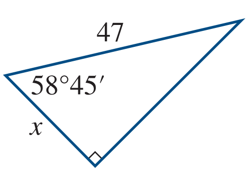


Example 11 **2** Find the length of the unknown side x in each triangle, correct to two decimal places.

a


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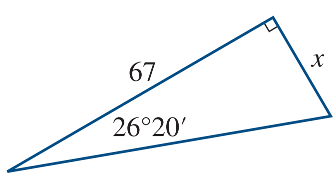

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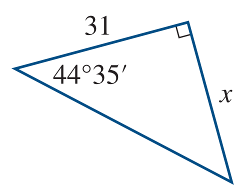

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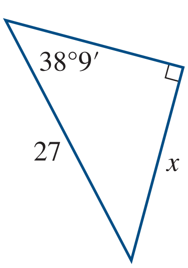

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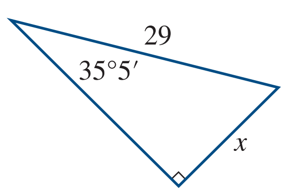

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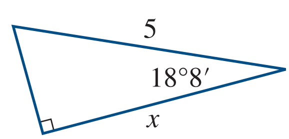

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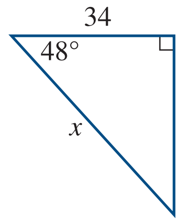
Example 12 **3** Find the length of the unknown side x in each triangle, correct to two decimal places.

a


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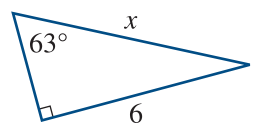

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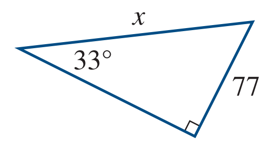

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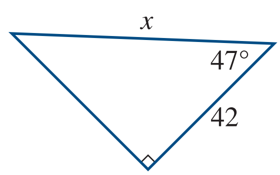

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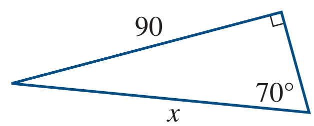

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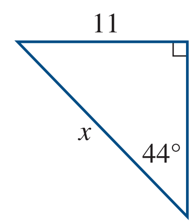
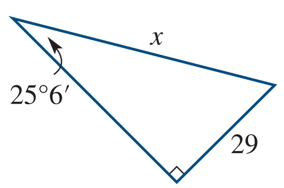


4 Find the length of the unknown side x in each triangle, correct to one decimal place.

a


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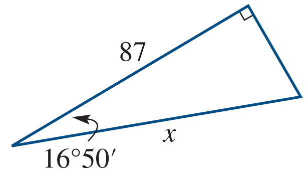

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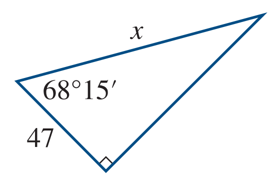

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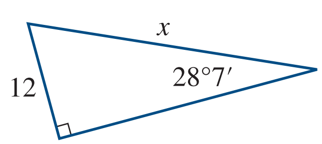

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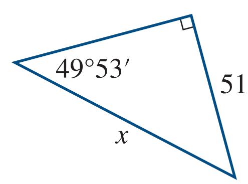

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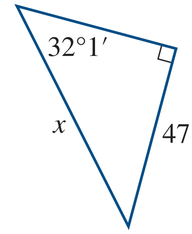

5 Find the length of the unknown side x in each triangle, correct to three decimal places.

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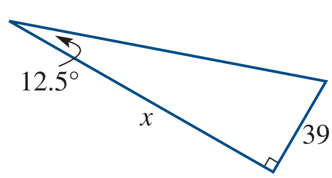

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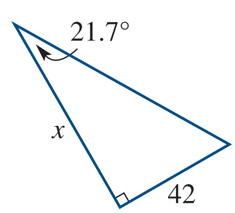

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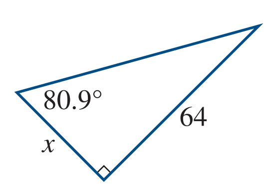

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4F Finding an unknown angle

Trigonometric ratios are used to find an unknown angle in a right-angled triangle, given at least two sides. The method involves labelling the sides of the triangle and using the mnemonic SOH CAH TOA. The resulting equation is rearranged to make ˜ the subject and the calculator is used to find the unknown angle.

FINDING AN UNKNOWN ANGLE IN A RIGHT-ANGLED TRIANGLE

1. Name the sides of the triangle – *h* for hypotenuse, *o* for opposite and *a* for adjacent.
2. Use the given sides and unknown angle ˜ to determine the trigonometric ratio. The mnemonic SOH CAH TOA helps with this step.
3. Rearrange the equation to make the unknown angle ˜ the subject.
4. Use the calculator to find ˜. Remember to check the calculator is set up for degrees.
5. Write the answer to the specified level of accuracy.

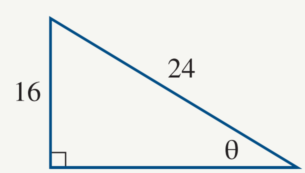
Example 13: Finding an unknown angle 4F

Find the angle ˜ in the triangles shown:

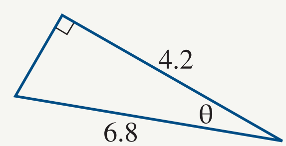
a to the nearest degree.

b to the nearest minute.

a



b



SOLUTION:

1	Name the sides of the right-angled triangle.	**a** *h* (24), *o* (16), *a*
2	Determine the ratio (SOH).	$\sin ˜ = \frac{o}{h}$
3	Substitute the known values.	$\sin ˜ = \frac{16}{24}$
4	Make ˜ the subject of the equation.	$˜ = \sin^{-1}\left(\frac{16}{24}\right)$
5	Press SHIFT [$\sin^{-1}$] (16 ÷ 24) [=] or [exe]	$= 41.8103149$
	or Press SHIFT [$\sin^{-1}$] 16 [$a^b/_c$] 24 [=] or [exe]	
6	Write the answer correct to the nearest degree.	$˜ = 42°$
7	Name the sides of the right-angled triangle.	**b** *h* (6.8), *o*, *a* (4.2)
8	Determine the ratio (CAH).	$\cos ˜ = \frac{a}{h}$
9	Substitute the known values.	$\cos ˜ = \frac{4.2}{6.8}$
10	Make ˜ the subject of the equation.	$˜ = \cos^{-1}\left(\frac{4.2}{6.8}\right)$
11	Press SHIFT [$\cos^{-1}$] (4.2 ÷ 6.2) [=]	$= 51.855486 \ldots$
	or Press SHIFT [$\cos^{-1}$] 4.2 [$a^b/_c$] 6.8 [=]	
12	Write the answer correct to the nearest minute.	$\approx 51°51'$

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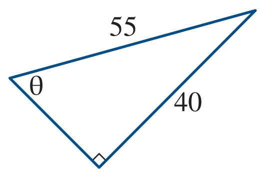
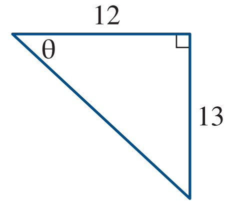
Exercise 4F

Example 13 **1** Find the unknown angle ~ in each triangle. Answer correct to the nearest degree.

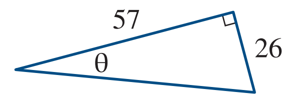
a



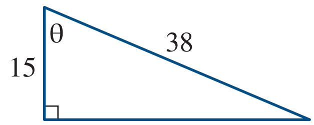
b



c



d



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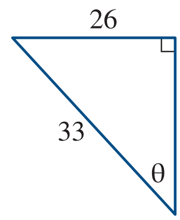

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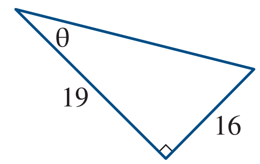

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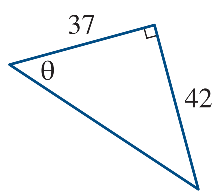

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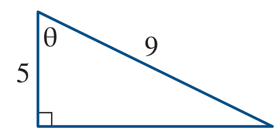
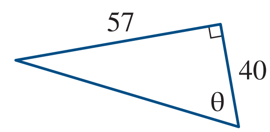
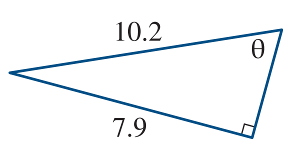


i



2 Find the unknown angle ~ in each triangle. Answer correct to the nearest degree.

a



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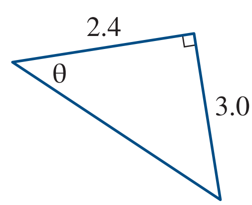

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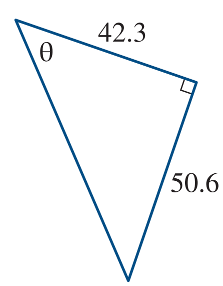
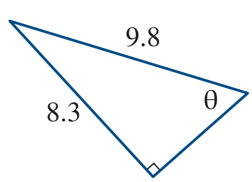


d



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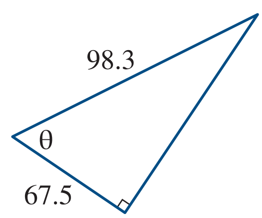

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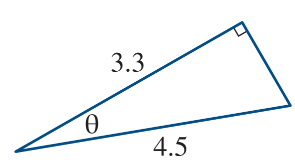

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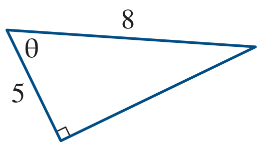
3 Find the unknown angle ˜ in each triangle. Answer correct to the nearest minute.

a


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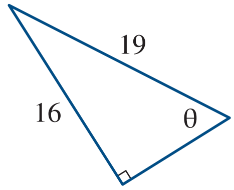

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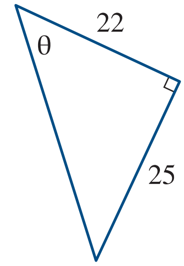

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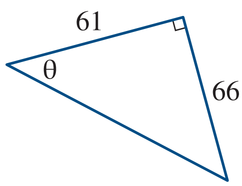

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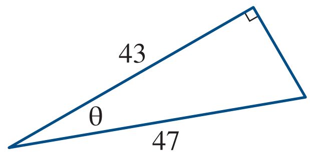
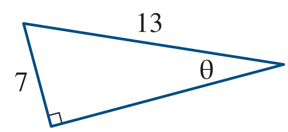
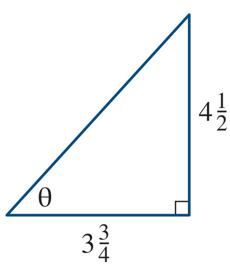


f


4 Find the unknown angle ˜ in each triangle. Answer correct to the nearest degree.

a


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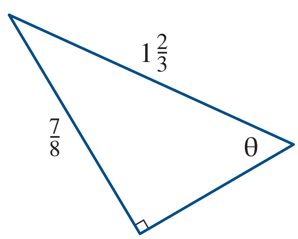

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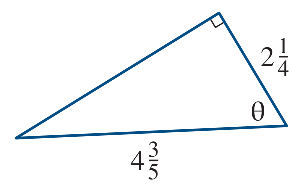
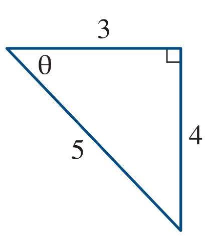


5 Find the unknown angle ˜ in each triangle. Answer correct to the nearest degree.

a


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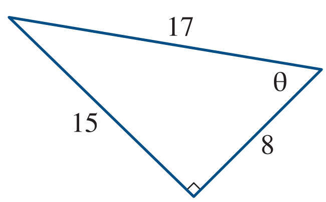

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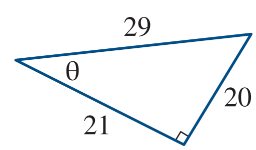
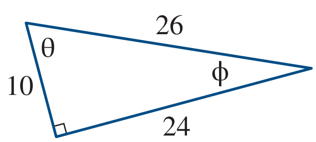


6 Find the angle ˜ and ° in each triangle. Answer correct to the nearest minute.

a


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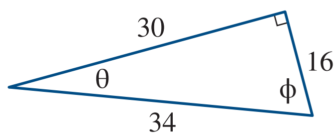

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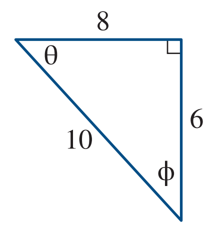

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4G Solving practical problems

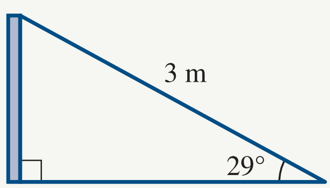
Trigonometry is used to solve many practical problems. How high is that tree? What is the height of the mountain? Calculate the width of the river. When solving a trigonometric problem, make sure you read the question carefully and draw a diagram. Label all the information given in the question on this diagram.

SOLVING A TRIGONOMETRIC WORDED PROBLEM

1. Read the question and underline the key terms.
2. Draw a diagram and label the information from the question.
3. Use trigonometry to calculate a solution.
4. Check that the answer is reasonable and units are correct.
5. Write the answer in words and ensure the question has been answered.

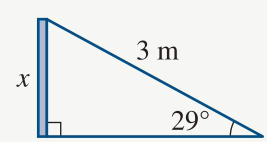
Example 14: Application requiring the length of a side — 4G

A vertical tent pole is supported by a rope tied to the top of the pole and to a peg on the ground. The rope is 3 m in length and makes an angle of 29° to the horizontal. What is the height of the tent pole? Answer correct to two decimal places.



SOLUTION:

1. Draw a diagram and label the required height as x.



2. Name the sides of the right-angled triangle. — a, o (x), h (3 m)
3. Determine the ratio (SOH). — $\sin\ \tilde{} = \frac{o}{h}$
4. Substitute the known values. — $\sin 29° = \frac{x}{3}$
5. Multiply both sides of the equation by 3. — $x = 3 \times \sin 29°$
6. Press 3 [sin] 29 [=] — $= 1.454428\ldots$
7. Write the answer correct to two decimal places. — ≈ 1.45
8. Write the answer in words. — $\therefore$ Height of the tent pole is 1.45 m.

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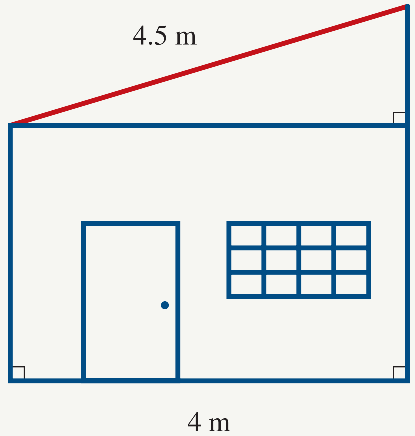
Applications requiring an angle

Trigonometry has many applications, such as in building and construction. Any vertical parts of a structure make a right angle with horizontal parts. Sloping lines in the structure complete a right-angled triangle, and trigonometry can be used to calculate its other angles and side lengths.

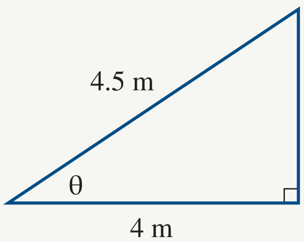
Example 15: Application requiring an angle 4G

The sloping roof of a shed uses sheets of Colorbond steel 4.5 m long on a shed 4 m wide. There is no overlap of the roof past the sides of the walls. Find the angle the roof makes with the horizontal. Answer correct to the nearest degree.



SOLUTION:

1 Draw a diagram and label the required angle as ˜.



2 Name the sides of the right-angled triangle. — *a* (4 m), *o*, *h* (4.5 m)

3 Determine the ratio (CAH). — cos ˜ $= \frac{a}{h}$

4 Substitute the known values. — cos ˜ $= \frac{4}{4.5}$

5 Make ˜ the subject of the equation. — ˜ $= \cos^{-1}\left(\frac{4}{4.5}\right)$

6 Press SHIFT [$\cos^{-1}$] (4 ÷ 4.5) [=] or [exe] — $= 27.26604445$

7 Write the answer correct to the nearest degree. — ˜ $\approx 27°$

8 Write the answer in words. — The roof makes an angle of 27°.

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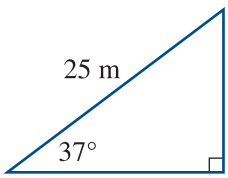
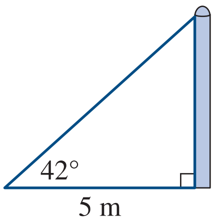
Exercise 4G

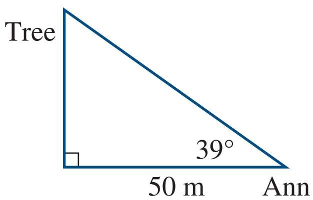
Example 14 **1** A balloon is tied to a string 25 m long. The other end of the string is secured by a peg to the surface of a level sports field. The wind blows so that the string forms a straight line making an angle of 37° with the ground. Find the height of the balloon above the ground. Answer correct to one decimal place.



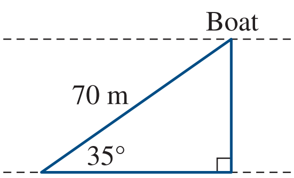
2 A pole is supported by a wire that runs from the top of the pole to a point on the level ground 5 m from the base of the pole. The wire makes an angle of 42° with the ground. Find the height of the pole, correct to two decimal places.



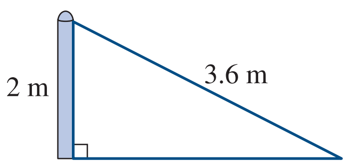
3 Ann noticed a tree was directly opposite her on the far bank of the river. After she walked 50 m along the side of the river, she found her line of sight to the tree made an angle of 39° with the river bank. Find the width of the river, to the nearest metre.



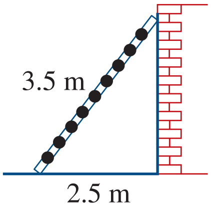
4 A ship at anchor requires 70 m of anchor chain. If the chain is inclined at 35° to the horizontal, find the depth of the water, correct to one decimal place.



Example 15 **5** A vertical tent pole is supported by a rope of length 3.6 m tied to the top of the pole and to a peg on the ground. The pole is 2 m in height. Find the angle the rope makes to the horizontal. Answer correct to the nearest degree.



6 A 3.5 m ladder has its foot 2.5 m out from the base of a wall. What angle does the ladder make with the ground? Answer correct to the nearest degree.



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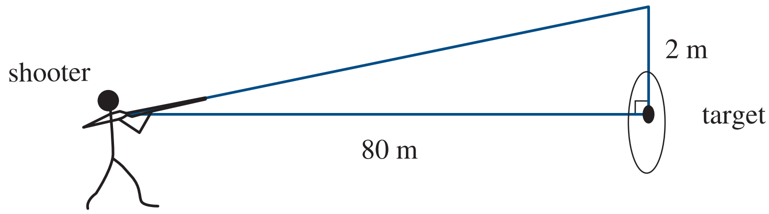
7 A plane maintains a flight path of 19° with the horizontal after it takes off. It travels for 4 km along the flight path. Find, correct to one decimal place:

a the horizontal distance of the plane from its take-off point

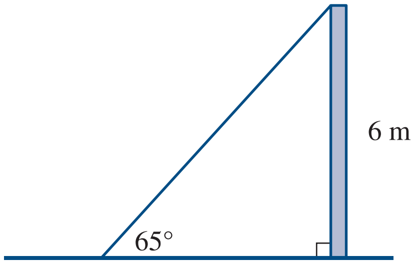
b the height of the plane above ground level.

8 A wheelchair ramp is being provided to allow access to the first floor shops. The first floor is 3 m above the ground floor. The ramp requires an angle of 20° with the horizontal. How long will the ramp be, measured along its slope? Answer correct to two decimal places.

9 A shooter 80 m from a target and level with it, aims 2 m above the bullseye and hits it. What is the angle, to the nearest minute, that his rifle is inclined to the line of sight from his eye to the target?



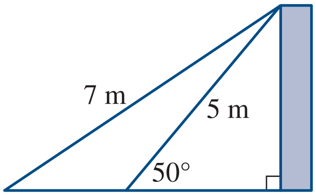
10 A rope needs to be fixed with one end attached to the top of a 6 m vertical pole and the other end pegged at an angle of 65° with the level ground. Find the required length of rope. Answer correct to one decimal place.



11 Two ladders are the same distance up the wall. The shorter ladder is 5 m long and makes an angle of 50° with the ground. The longer ladder is 7 m long. Find:

a the distance the ladders are up the wall, correct to two decimal places

b the angle the longer ladder makes with the ground, correct to the nearest degree.



12 A pole is supported by a wire that runs from the top of the pole to a point on the level ground 7.2 m from the base of the pole. The height of the pole is 5.6 m. Find the angle, to the nearest degree, that the wire makes with the ground.

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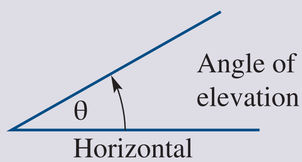
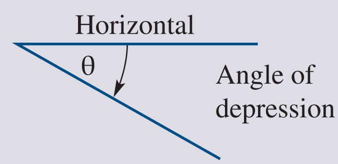
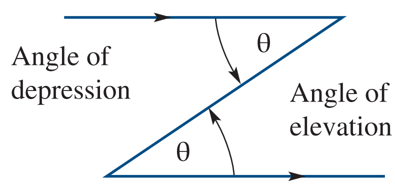
4H Angles of elevation and depression

The angle of elevation is the angle measured upwards from the horizontal. The angle of depression is the angle measured downwards from the horizontal.

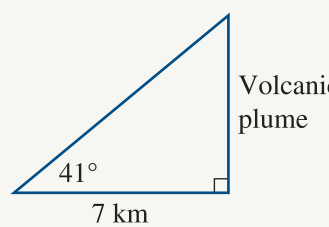
ANGLE OF ELEVATION	ANGLE OF DEPRESSION
	

The angle of elevation is equal to the angle of depression as they form alternate angles between two parallel lines. This information is useful to solve some problems.



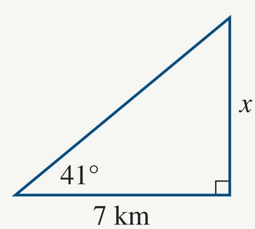
Example 16: Angle of elevation (4H)

A park ranger measured the top of a plume of volcanic ash to be at an angle of elevation of 41°. From her map she noted that the volcano was 7 km away. Calculate the height of the plume of volcanic ash. Answer correct to two decimal places.



SOLUTION:

1 Draw a diagram and label the required height as x.



2 Determine the ratio (TOA).

$$\tan \sim = \frac{o}{a}$$

3 Substitute the known values.

$$\tan 41° = \frac{x}{7}$$

4 Multiply both sides of the equation by 7.

$$7 \times \tan 41° = x$$
$$x = 7 \times \tan 41°$$

5 Write the answer correct to two decimal places.

$$x = 6.085\,007\,165$$
$$\approx 6.09$$

6 Write the answer in words.

The height of the volcanic plume was 6.09 km.

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Example 17: Finding a distance using angle of depression 4H

The top of a cliff is 85 m above sea level. Minh saw a tall ship. He estimated the angle of depression to be 17°.

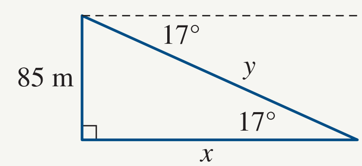
a How far was the ship from the base of the cliff? Answer to the nearest metre.

b How far is the ship in a straight line from the top of the cliff? Answer to the nearest metre.

SOLUTION:

1 Draw a diagram and label the distance to the base of the cliff as x and the distance to the top of the cliff as y.

a



2 Determine the ratio (TOA).

$$\tan \text{˜} = \frac{o}{a}$$

3 Substitute the known values.

$$\tan 17° = \frac{85}{x}$$

4 Multiply both sides of the equation by x.

$$x \times \tan 17° = 85$$

5 Divide both sides by tan 17°.

$$x = \frac{85}{\tan 17°}$$

$$= 278.022...$$

6 Write the answer correct to nearest metre.

$$\approx 278\,\text{m}$$

7 Write the answer in words.

∴ The ship is 278 metres from the base of the cliff.

8 Determine the ratio (SOH).

b $$\sin \text{˜} = \frac{o}{h}$$

9 Substitute the known values.

$$\sin 17° = \frac{85}{y}$$

10 Multiply both sides of the equation by y.

$$y \times \sin 17° = 85$$

11 Divide both sides by sin 17°.

$$y = \frac{85}{\sin 17°}$$

$$= 290.7258...$$

12 Write the answer correct to nearest metre.

$$\approx 291\,\text{m}$$

13 Write the answer in words.

∴ The ship is 291 metres from the top of the cliff.

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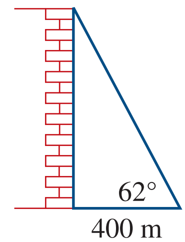
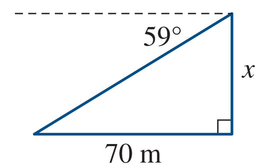
Exercise 4H

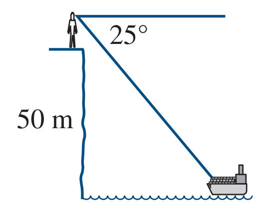
Example 16 **1** Luke walked 400 m away from the base of a tall building, on level ground. He measured the angle of elevation to the top of the building to be 62°. Find the height of the building. Answer correct to the nearest metre.



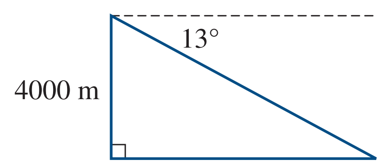
2 The angle of depression from the top of a TV tower to a satellite dish near its base is 59°. The dish is 70 m from the centre of the tower's base on flat land. Find the height of the tower. Answer correct to one decimal place.



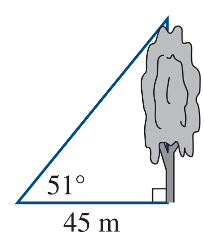
Example 17 **3** When Sarah looked from the top of a cliff 50 m high, she noticed a boat at an angle of depression of 25°. How far was the boat from the base of the cliff? Answer correct to two decimal places.



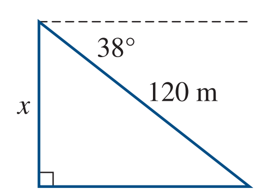
4 The pilot of an aeroplane saw an airport at sea level at an angle of depression of 13°. His altimeter showed that the aeroplane was at a height of 4000 m. Find the horizontal distance of the aeroplane from the airport. Answer correct to the nearest metre.



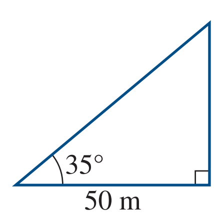
5 The angle of elevation to the top of a tree is 51° at a distance of 45 m from the point on level ground directly below the top of the tree. What is the height of the tree? Answer correct to one decimal place.



6 A iron ore seam of length 120 m slopes down at an angle of depression from the horizontal of 38°. The mine engineer wishes to sink a vertical shaft, x, as shown. What is the depth of the required vertical shaft? Answer correct to the nearest metre.



7 Jack measures the angle of elevation to the top of a tree from a point on level ground as 35°. What is the height of the tree if Jack is 50 m from the base of the tree? Answer to the nearest metre.



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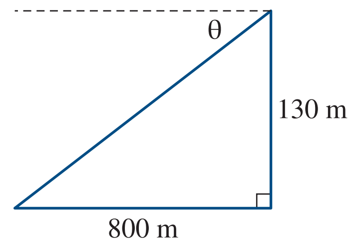
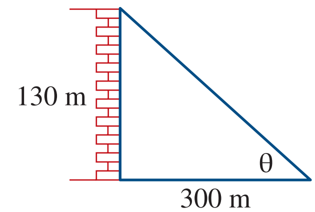
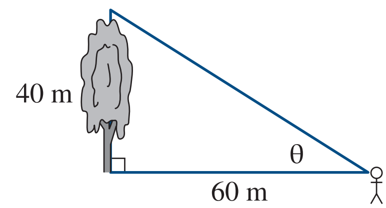
8 A tourist viewing Sydney Harbour from a building 130 m above sea level observes a ferry that is 800 m from the base of the building. Find the angle of depression. Answer correct to the nearest degree.



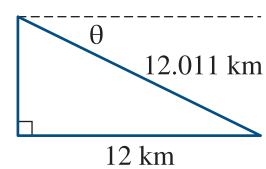
9 What would be the angle of elevation to the top of a radio transmitting tower 130 m tall and 300 m from the observer? Answer correct to the nearest degree.



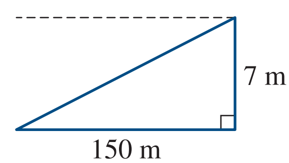
10 Lachlan observes the top of a tree at a distance of 60 m from the base of the tree. The tree is 40 m high. What is the angle of elevation to the top of the tree? Answer correct to the nearest degree.



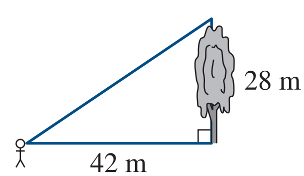
11 A town is 12 km from the base of a mountain. The town is also a distance of 12.011 km in a straight line to the mountain. What is the angle of depression from the top of a mountain to the town? Answer to the nearest degree, correct to one decimal place.



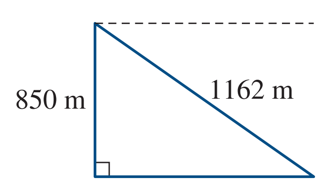
12 Find, to the nearest degree, the angle of elevation of a railway line that rises 7 m for every 150 m along the track.



13 The distance from the base of a tree is 42 m. The tree is 28 m in height. What is the angle of elevation measured from ground level to the top of a tree? Answer correct to the nearest degree.



14 A helicopter is flying 850 m above sea level. It is also 1162 m in a straight line to a ship. What is the angle of depression from the helicopter to the ship? Answer correct to the nearest degree.



15 The angle of elevation to the top of a tree from a point A on the ground is 25°. The point A is 22 m from the base of the tree. Find the height of the tree. Answer correct to nearest metre.

16 A plane is 460 m directly above one end of a 1200 m runway. Find the angle of depression to the far end of the runway. Answer correct to the nearest minute.

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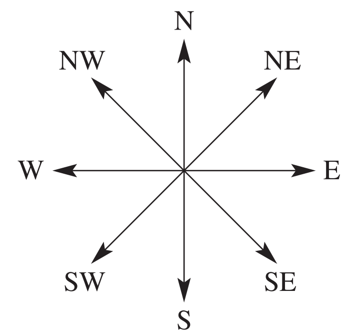
4I Compass and true bearings

A bearing is the direction one object is from another object or an observer or a fixed point. There are two types of bearings: compass bearings and true bearings.

Compass bearings

Compass bearings use the four directions of the compass: north, east, south and west (N, E, S and W). The NS line is vertical and the EW line is horizontal. In-between these directions are another four directions: north-east, south-east, south-west and north-west (NE, SE, SW and NW). Each of these directions makes an angle of 45° with the NS and EW lines.

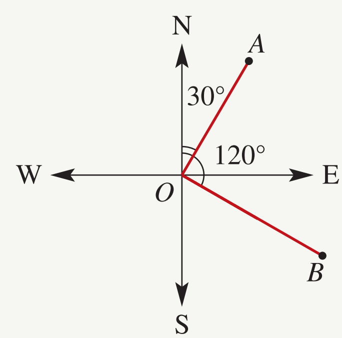


A direction is given using a compass bearing by stating the angle either side of north or south. For example, a compass bearing of S50°W is found by measuring an angle of 50° from the south direction towards the west side.

Example 18: Understanding a compass bearing — 4I

Find the compass bearing of:

a *A* from *O*
b *B* from *O*.



SOLUTION:

1 Determine the quadrant of the compass bearing.	**a** The line *OA* is in the north/east quadrant.
2 Find the angle the direction makes with the vertical (north/south) line.	30°
3 Write the compass bearing with N or S first, then the angle with the vertical line and finally either E or W.	Compass bearing of *A* from *O* is N30°E.
4 Determine the quadrant of the compass bearing.	**b** The line *OB* is in the south/east quadrant.
5 Find the angle the direction makes with the vertical (north/south) line.	180° − 120° = 60°
6 Write the compass bearing with N or S first, then the angle with vertical line and finally either E or W.	Compass bearing of *B* from *O* is S60°E.

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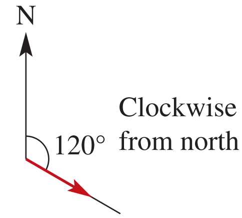
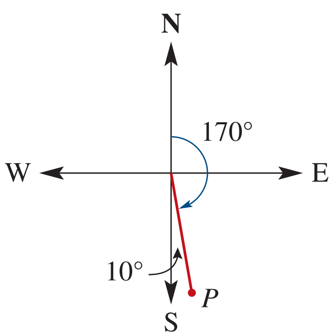
True bearings

A true bearing is the angle measured clockwise from north around to the required direction, and it is written with the letter T after the degree or minutes or seconds symbol. True bearings are sometimes called three-figure bearings because they are written using three numbers or figures. For example, 120°T is the direction measured 120° clockwise from north. It is the same bearing as S60°E.

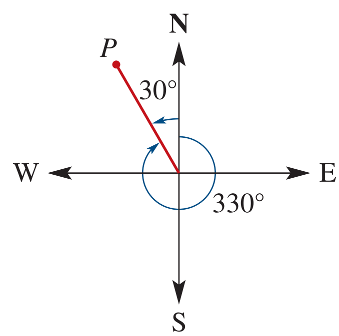


The smallest true bearing is 000°T and the largest true bearing is 360°T. The eight directions of the compass have the following true bearings: north is 000°T, east is 090°T, south is 180°T, west is 270°T, north-east is 045°T, south-east is 135°T, south-west is 225°T and north-west is 315°T.

The bearings in the following diagrams are given using both methods.



Compass bearing S 10°E True bearing 170°T



Compass bearing N 30°W True bearing 330°T

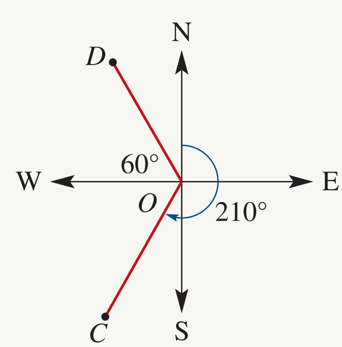
COMPASS BEARING	TRUE BEARING
A direction given by stating the angle either side of north or south, such as S60°E.	A direction given by measuring the angle clockwise from north to the required direction, such as 120°T.

Example 19: Understanding a true bearing 4I

Find the true bearing of:

a *C* from *O*

b *D* from *O*.



SOLUTION:

1 Find the angle the bearing makes in the clockwise direction with the north direction.	**a** 210°
2 Write the true bearing using this angle. Add the letter 'T'.	*C* from *O* is 210°T.
3 Write the true bearing of west.	**b** 270°T
4 Add angle between west and *D* to true bearing for west.	270° + 60° = 330°
5 Write the true bearing using this sum. Add the letter 'T'.	*D* from *O* is 330°T.

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Exercise 4I

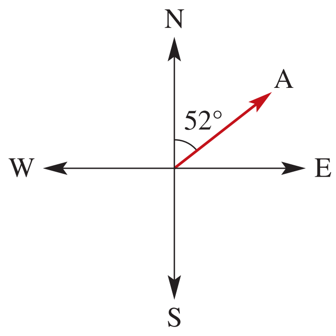
1 State the compass bearing and the true bearing of each of the following directions.

a NE **b** NW **c** SE **d** SW

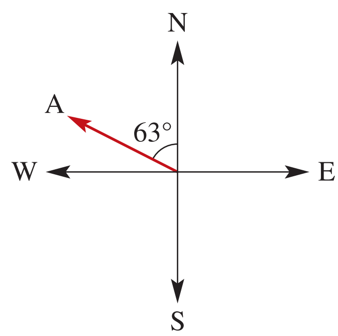
Example 18, 19

2 State the compass bearing and the true bearing of each of the following directions.

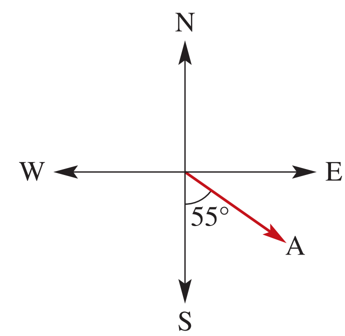
a



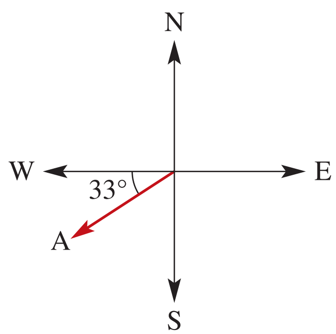
b



c



d



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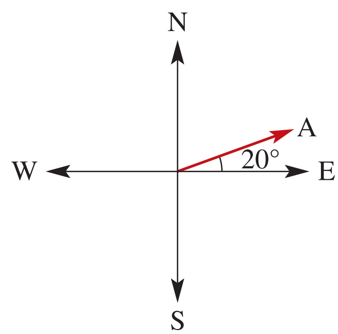

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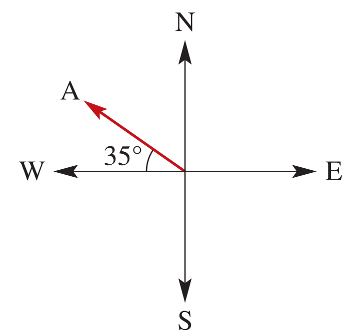

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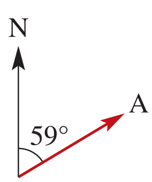

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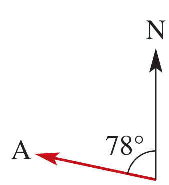
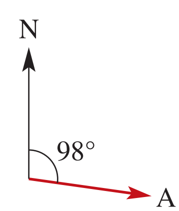


i



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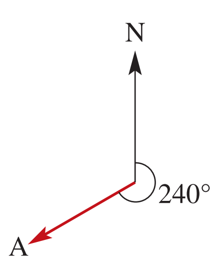

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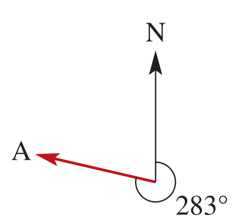

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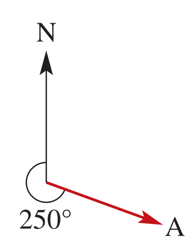
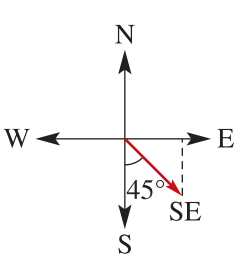


3 Sketch each of these bearings on a separate diagram.

a N10°E **b** S25°W **c** N60°W **d** S42°E
e 300° **f** 105° **g** 219° **h** 050°

4 Aaron runs a distance of 7.2 km in the SE direction. How far east has Aaron run? Answer correct to one decimal place.



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5 A plane is travelling on a true bearing of 030° from A to B.

a What is the compass bearing of A to B?

b What is the true bearing of B to A?

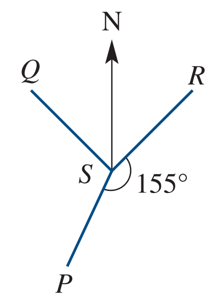
c What is the compass bearing of B to A?

6 The diagram shows the position of P, Q and R relative to S. In the diagram, R is NE of S, Q is NW of S and $\angle PSR$ is 155°.

a What is the true bearing of R from S?

b What is the true bearing of Q from S?

c What is the true bearing of P from S?

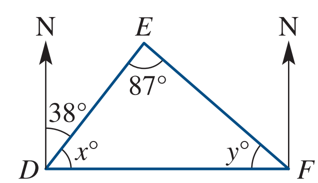


7 The bearing of E from D is $N38°E$, F is east of D and $\angle DEF$ is 87°

a Find the values of x and y.

b What is the compass bearing of E from F?

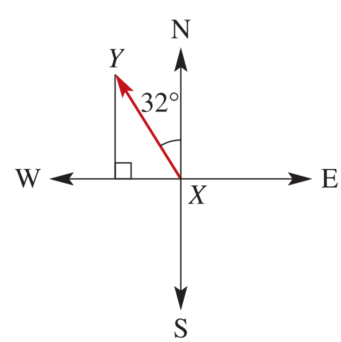
c What is the true bearing of E from F?



8 Riley travels from X to Y for 125 km on a bearing of $N32°W$.

a How far did Riley travel due north, to the nearest kilometre?

b How far did Riley travel due west, to the nearest kilometre?

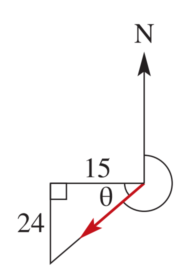


9 Mia cycled for 15 km west and then 24 km south.

a What is the value of ˜ to the nearest degree?

b What is Mia's true bearing from her starting point?

c What is Mia's compass bearing from her starting point?



10 A boat sails 137 km from Port Stephens on a bearing of 065°T.

a How far east has the boat sailed? Answer correct to one decimal place.

b How far north has the boat sailed? Answer correct to one decimal place.

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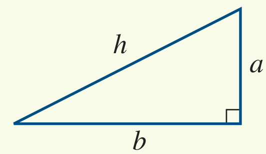
Summary

PPT STUDY GUIDE

Key ideas and chapter summary

Pythagoras' theorem

Pythagoras' theorem states that the square of the hypotenuse is equal to the sum of the squares of the other two sides ($h^2 = a^2 + b^2$).



Applying Pythagoras' theorem

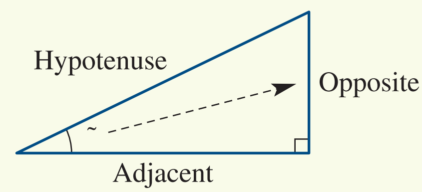
1. Read the question and underline the key terms.
2. Draw a diagram and label the information from the question.
3. Decide whether to determine the hypotenuse or the length of a shorter side.
4. Use Pythagoras' theorem to calculate a solution.
5. Check that the answer is reasonable and units are correct.

Trigonometric ratios

$\sin \theta = \frac{o}{h}$ (SOH)

$\cos \theta = \frac{a}{h}$ (CAH)

$\tan \theta = \frac{o}{a}$ (TOA)



Using the calculator in trigonometry

1 degree = 60 minutes $\quad 1^\circ = 60'$

1 minute = 60 seconds $\quad 1' = 60''$

Finding an unknown side

1. Name the sides of the triangle.
2. Use the given side and unknown side x to determine the trigonometric ratio. The mnemonic SOH CAH TOA helps.
3. Rearrange the equation to make the unknown side x the subject then use the calculator to find x.

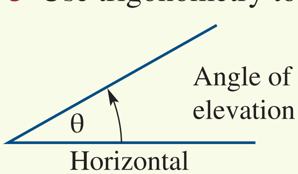
Finding an unknown angle

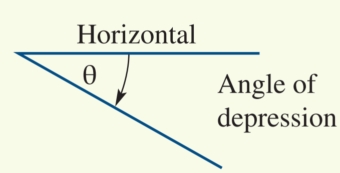
1. Name the sides of the triangle.
2. Use the given sides and unknown angle θ to determine the trigonometric ratio. The mnemonic SOH CAH TOA helps.
3. Rearrange the equation to make the unknown angle θ the subject then use the calculator to find θ.

Solving practical problems

1. Read the question and underline the key terms.
2. Draw a diagram and label the information from the question.
3. Use trigonometry to calculate a solution.

Angles of elevation and depression





Bearings

Compass bearing A direction given by stating the angle either side of north or south such as S60°E.

True bearing A direction given by measuring the angle clockwise from north to the required direction such as 120°.

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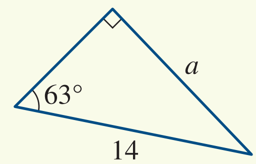
Review

Multiple-choice

1 What is the length of the hypotenuse if the two other sides are 12 cm and 16 cm?

A 400 cm **B** 20 cm **C** 28 cm **D** 7.46 cm

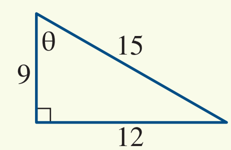
2 What is the length of a?



A $14\cos 63°$ **B** $14\sin 63°$

C $\dfrac{14}{\cos 63°}$ **D** $\dfrac{14}{\sin 63°}$

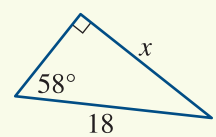
3 What is the value of cos ~ ?



A $\dfrac{9}{15}$ **B** $\dfrac{12}{9}$

C $\dfrac{12}{15}$ **D** $\dfrac{9}{12}$

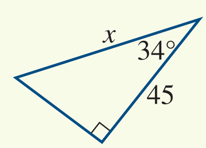
4 What is the length of x?



A $18\cos 58°$ **B** $18\sin 58°$

C $\dfrac{18}{\cos 58°}$ **D** $\dfrac{18}{\sin 58°}$

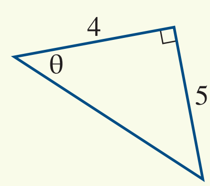
5 What is the length of x?



A $45\cos 34°$ **B** $45\sin 34°$

C $\dfrac{45}{\cos 34°}$ **D** $\dfrac{45}{\sin 34°}$

6 How would angle ~ be calculated?



A $\tan^{-1}\left(\dfrac{5}{4}\right)$ **B** $\tan^{-1}\left(\dfrac{4}{5}\right)$

C $\tan\left(\dfrac{5}{4}\right)$ **D** $\tan\left(\dfrac{4}{5}\right)$

7 What is the size of angle ~ in question **6**? (Answer correct to one decimal place.)

A 38.6° **B** 39.0° **C** 51.0° **D** 51.3°

8 What is the angle of elevation to the top of a tower 80 m tall and 100 m from the observer? Answer in degrees correct to one decimal place.

A 51.3° **B** 51.4° **C** 38.6° **D** 38.7°

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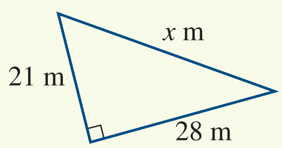
Review

Short-answer

1 Find the value of x, correct to two decimal places.

a



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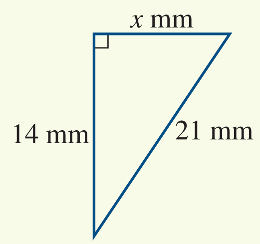

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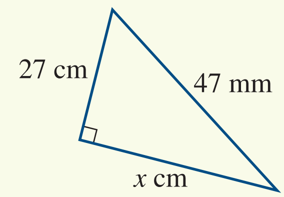
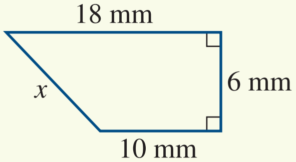


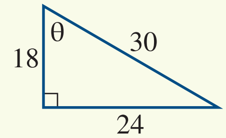
2 A rectangular block of land measures 20 m by 27 m. A fence is required along its diagonal. How long will the fence be? (Answer correct to one decimal place.)

3 Calculate the length of x, correct to the nearest millimetre.



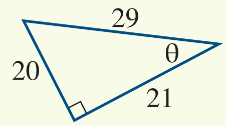
4 In the triangle shown, state the value of the:

a hypotenuse **b** opposite side **c** adjacent side.



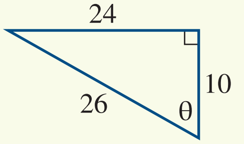
5 What are these ratios?

a sin θ **b** cos θ **c** tan θ



6 What are these ratios in simplest form?

a sin θ **b** cos θ **c** tan θ



7 Find the value of the following trigonometric ratios, correct to two decimal places.

a tan 68° **b** cos 13° **c** sin 23° **d** cos 82°

8 Given the following trigonometric ratios, find the value of θ to the nearest degree.

a $\cos\theta = 0.4829$ **b** $\sin\theta = \frac{1}{3}$ **c** $\tan\theta = 0.2$

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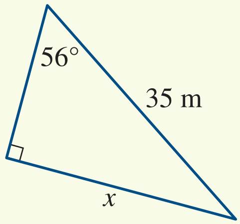
Review

9 Find the value of x, correct to two decimal places.

a


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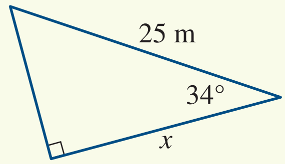

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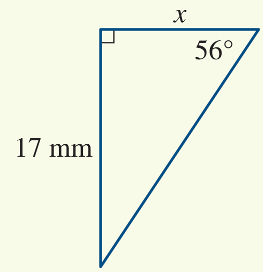

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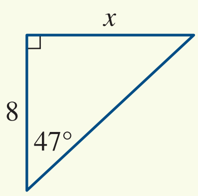

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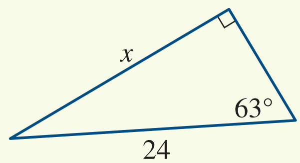

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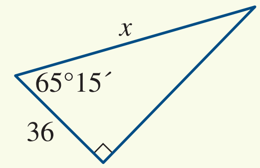
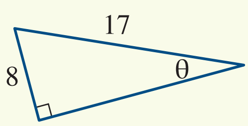


10 Find the unknown angle ~ in each triangle. Answer correct to the nearest minute.

a


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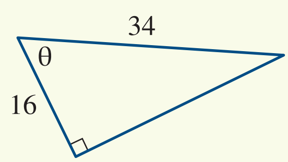

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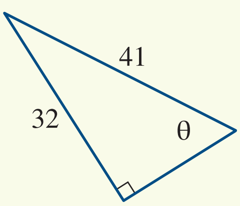

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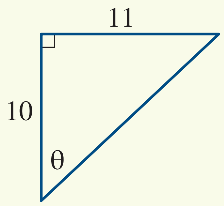

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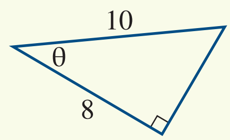

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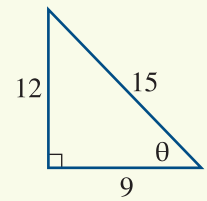
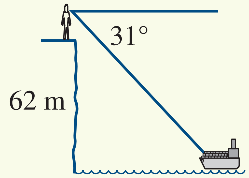


11 Susan looked from the top of a cliff, 62 m high, and noticed a ship at an angle of depression of 31°. How far was the ship from the base of the cliff? Answer correct to one decimal place.



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12 Emma rode for 8.5 km on a bearing of N43°W from her home.

a How far north is Emma from home? Answer correct to one decimal place.

b How far west is Emma from home? Answer correct to one decimal place.

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5 Simultaneous linear equations

Syllabus topic — A3.1 Simultaneous linear equations

This topic will develop your understanding of the use of simultaneous linear equations in solving practical problems.

Outcomes

- Graph linear functions.
- Interpret linear functions as models of physical phenomena.
- Develop linear equations from descriptions of situations.
- Solve a pair of simultaneous linear equations using graphical methods.
- Finding the point of intersection between two straight-line graphs.
- Develop a pair of simultaneous linear equations to model a practical situation.
- Solve practical problems by modelling with a pair of simultaneous linear functions.
- Apply break-even analysis to solve simple problems.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Literacy worksheet
- Quick Quiz
- Solutions (enabled by teacher)
- Desmos widgets
- Spreadsheets
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

The Interactive Textbook provides a test of prior knowledge for this chapter, and may direct you to revision from the previous years' work.

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5A Linear functions

A linear function makes a straight line when graphed on a number plane. The linear function $y = 3x - 2$ has two variables y and x. When a number is substituted for a variable, such as $x = 2$, then this variable is called the independent variable. The dependent variable depends on the number substituted for the independent variable. That is, when $x = 2$ (independent) then $y = 3 \times 2 - 2$ or 4 (dependent).

To graph a linear function, construct a table of values with the independent variable as the first row and the dependent variable as the second row. Plot these points on the number plane with the independent variable on the horizontal axis and the dependent variable as the vertical axis. Join the points to make a straight line.

GRAPHING A LINEAR FUNCTION

1 Construct a table of values with the independent variable as the first row and the dependent variable as the second row.
2 Draw a number plane with the independent variable on the horizontal axis and the dependent variable as the vertical axis. Plot the points.
3 Join the points to make a straight line.

Example 1: Drawing a linear function 5A

Draw the graph of $y = -2x - 1$.

SOLUTION:

1 Draw a table of values for x and y.

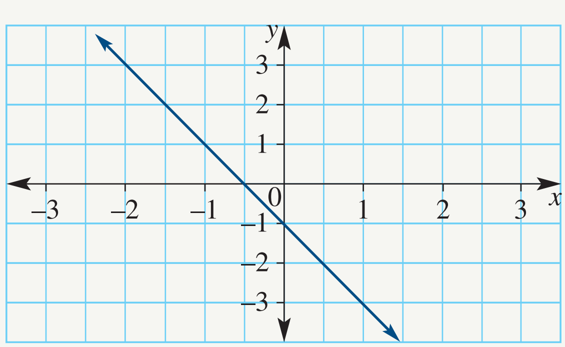
2 Let $x = -2, -1, 0, 1$ and 2. Find y using the linear function $y = -2x - 1$.

x	−2	−1	0	1	2
y	3	1	−1	−3	−5

3 Draw a number plane with x as the horizontal axis and y as the vertical axis.

4 Plot the points (−2, 3), (−1, 1), (0, −1) and (1, −3). The point (2, −5) has not been plotted as it does not fit the scale of the number plane.

5 Join the points to make a straight line.



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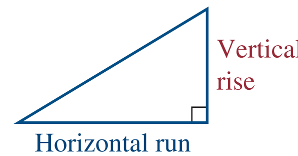
Gradient–intercept formula

When the equation of a straight line is written in the form $y = mx + c$ (or $y = mx + b$) it is called the gradient–intercept formula. The gradient is m or the coefficient of x. It is the slope or steepness of the line. The gradient of a line is calculated by dividing the vertical rise by the horizontal run. Lines that go up to the right (/) have positive gradients and lines that go down to the right (\) have negative gradients.



$$\text{Gradient (or } m) = \frac{\text{Vertical rise}}{\text{Horizontal run}}$$

The intercept of a line is where the line cuts the axis. The intercept on the vertical axis is called the y-intercept and is denoted by the letter c. (Previously in this course, b was used.)

GRADIENT–INTERCEPT FORMULA

Linear equation: $y = mx + c$.
m – Slope or gradient of the line (vertical rise over the horizontal run).
c – y-intercept. Where the line cuts the y-axis or vertical axis.

Example 2: Draw a graph from a table of values, find gradient and *y*-intercept 5A

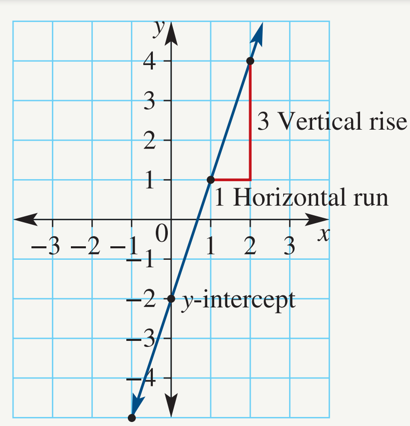
Draw the graph of $y = 3x - 2$ from a table of values. Find the gradient and y-intercept of this line, and check that they form a linear equation that is the same as the original one.

SOLUTION:

1 Construct a table of values for x and y.
2 Let $x = -2, -1, 0, 1$ and 2. Find y using the linear function $y = 3x - 2$.

x	-2	-1	0	1	2
y	-8	-5	-2	1	4

3 Draw a number plane with x as the horizontal axis and y as the vertical axis.
4 Plot the points $(-1, -5)$, $(0, -2)$, $(1, 1)$ and $(2, 4)$. The point $(-2, -8)$ has not been plotted as it does not fit the scale of the number plane.
5 Join the points to make a straight line. Calculate the gradient of the line using the 'rise over run formula' and read the value of the y-intecept where the line crosses the vertical axis.



6 Write the values of the gradient and y-intercept.

Gradient m is 3, y-intercept c is -2.

7 Write equation of the line in the form $y = mx + c$. The gradient is the coefficient of x and the y-intercept is -2.

$y = 3x - 2$

8 Compare this equation with the question.

The equations are the same.

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Sketching a straight line requires at least two points. When an equation is written in gradient–intercept form, one point on the graph is immediately available: the y-intercept. A second point can be quickly calculated using the gradient.

Example 3: Sketching a linear function using the gradient and y-intercept 5A

Sketch the graph of $3y + 6x = 9$.

SOLUTION:

1 Rearrange the equation into gradient form $y = mx + c$. $3y + 6x = 9$

2 Subtract $6x$ from both sides. $3y = 9 - 6x$

3 Divide both sides by 3 and simplify. $y = \frac{9 - 6x}{3}$

$y = 3 - 2x$

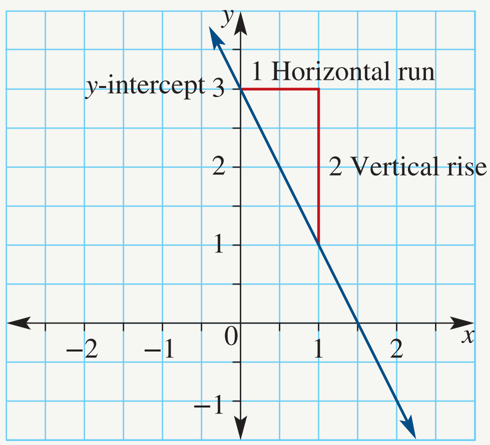
4 The equation is now written in the form $y = mx + c$. $y = -2x + 3$

5 Therefore $m = -2$ and $c = 3$. Gradient is -2 and y-intercept is 3

6 A gradient of -2 means that for every unit across in the positive x-axis direction, you go down 2 in the negative y-axis direction.

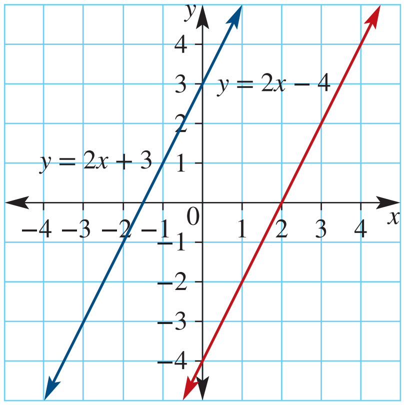
7 Plot the y-intercept (0, 3), then move across 1 (horizontal run) and down 2 (vertical rise) to plot the point (1, 1).

8 Join the points (0, 3) and (1, 1) to make a s traight line.



Parallel lines

Consider the linear function $y = 2x + 3$.
It has a gradient of 2 and y-intercept of 3.
Consider the linear function $y = 2x - 4$.
It has a gradient of 2 and y-intercept of -4.
The graph of these linear functions is shown opposite.
They are parallel because they both have the same gradient of $m = 2$.



PARALLEL LINES

If the value of m is the same for two linear functions, then the lines are parallel.

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Exercise 5A

1 Plot the following points on a number plane and join them to form a straight line.

a

x	-2	-1	0	1	2
y	2	1	0	-1	-2

b

x	-2	-1	0	1	2
y	-3	-1	1	3	5

2 Complete the following table of values and graph each linear function.

a $y = x - 1$

x	0	1	2	3	4
y					

b $y = -2x$

x	0	2	4	6	8
y					

c $y = 2x + 3$

x	-2	-1	0	1	2
y					

d $y = -x + 2$

x	-2	-1	0	1	2
y					

Example 1

3 Draw the graphs of these linear functions by first completing a table of values.

a $y = 2x + 2$
b $y = -x + 3$
c $y = \frac{2}{3}x - 1$
d $y = -\frac{1}{2}x + 1$
e $y = 3x - 1$
f $y = -2x + 3$

Example 2

4 Find the gradient of the following straight lines.

a $y = 5x + 1$
b $y = x - 2$
c $y = -2x$
d $y = \frac{1}{2}x + 4$
e $y = -\frac{2}{3}x + 3$
f $y = \frac{3}{4}x + \frac{1}{2}$

5 Find the y-intercept of the following straight lines.

a $y = 3x - 5$
b $y = -x + 2$
c $y = -4x$
d $y = -\frac{1}{3}x - 1$
e $y = -\frac{2}{5}x + 7$
f $y = \frac{1}{4}x + \frac{3}{5}$

6 Find the equation of the following straight lines defined by:

a gradient 2, passing through (0, 1)
b gradient -3, passing through (0, 4)
c gradient 0.5, passing through (0, -2)
d gradient 0, passing through (0, 6)
e gradient $-\frac{2}{5}$, passing through (0, 4)
f gradient $\frac{1}{3}$, passing through (0, 0).

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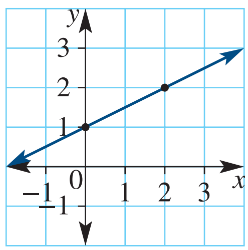
7 What is the equation of each of the following line graphs?

a



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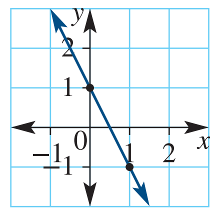

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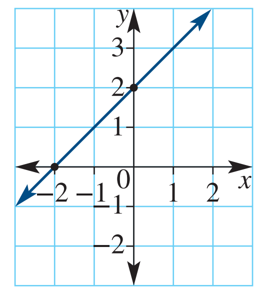

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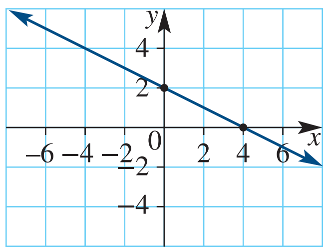

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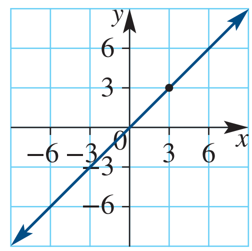

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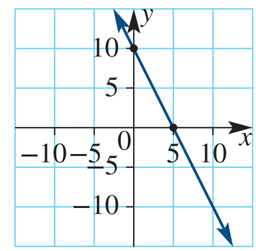

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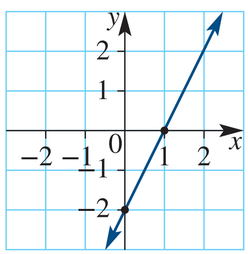

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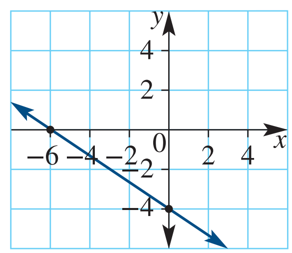

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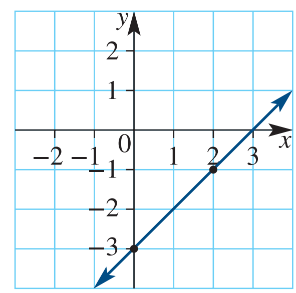

8 Which of the following lines are parallel?

a $y = 2x + 1$ and $y = x + 2$

b $y = -2x + 4$ and $y = 2x + 4$

c $y = 3x + 1$ and $y = 3x + 2$

d $y = -4x + 1$ and $y = -4x$

9 Find the equation of the line passing through each of the following pairs of points.

a (0, 3), (3, 0)

b (−2, 0), (0, 4)

c (2, 0), (0, 2)

d (1, 0), (0, −1)

e (−1, 0), (0, 3)

f (0, 4), (4, 2)

10 Express the following linear equations in gradient–intercept form ($y = mx + c$).

a $y + 2 = 3x$

b $x + y - 4 = 0$

c $y - \frac{1}{2}x = 1$

d $4x - y + 2 = 0$

e $\frac{1}{3}x - y = 1$

f $4 - y = 3x$

11 Draw the graph of the linear functions in question **10** using a table of values.

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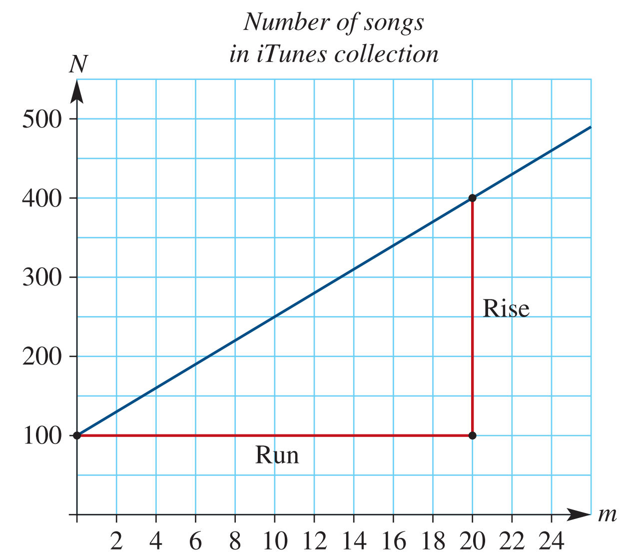
5B Linear models

Linear modelling occurs when a practical situation is described mathematically using a linear function. For example, the gradient–intercept form of a straight-line graph can be used to model an iTunes collection. Logan owns 100 songs in his iTunes collection and adds 15 new songs each month. Using this information, we can write a linear equation to model the number of songs in his collection. Letting N be the number of songs and m be the number of months, we can write $N = 15m + 100$.

Note: The number of months (m) must be greater than zero and a whole number.



The graph of this linear model has been drawn opposite. There are two important features of this linear model:

1. Gradient is the rate per month or $15\left(\frac{300}{20}\right)$ songs.
2. The vertical axis intercept is the initial number of songs or 100.

LINEAR MODELS

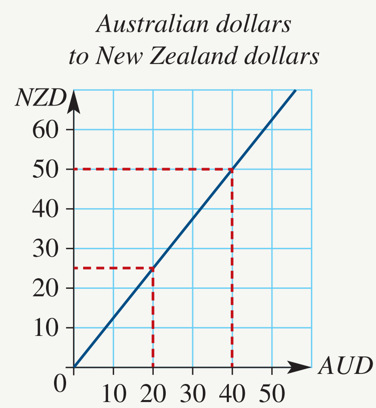
Linear models describe a practical situation mathematically using a linear function.

Example 4: Using linear models 5B

The graph opposite is used to convert Australian dollars (AUD) to New Zealand dollars (NZD). Use the graph to convert:

a 40 AUD to NZD

b 25 NZD to AUD.



SOLUTION:

1 Read from the graph (when AUD = 40, NZD = 50).	**a** 50 NZD
2 Read from the graph (when NZD = 25, AUD = 20).	**b** 20 AUD

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Example 5: Interpreting linear models 5B

Grace sells insurance. She earns a base salary and a commission on each new insurance policy she sells. The graph shows Grace's weekly income (I) plotted against the number of new policies (n) she sells in that week. The relationship between I and n is linear.

a What is Grace's base salary?

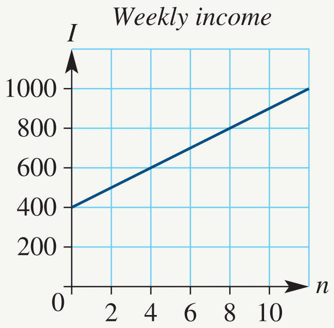
b What is Grace's salary in a week in which she sells 8 new policies?

c How many policies does Grace need to sell to earn \$700 for the week?

d Find the equation of the straight line in terms of I and n.

e Use the equation to calculate the weekly income when Grace sells 3 new policies.

f How much does Grace earn for each new policy?



SOLUTION:

1 Read from the graph (when $n = 0$, $I = 400$).
2 Read from the graph (when $n = 8$, $I = 800$).
3 Read from the graph (when $I = 700$, $n = 6$).
4 Find the gradient by choosing two suitable points. (0, 400) and (8, 800).
5 Calculate the gradient (m) between these points using the gradient formula.
6 Determine the vertical intercept (400).
7 Substitute the gradient and y-intercept into the gradient–intercept form $y = mx + c$.
8 Use the appropriate variables (I for y, n for x).
9 Substitute $n = 3$ into the equation.
10 Evaluate.
11 Check the answer using the graph.
12 The gradient of the graph is the commission for each new policy.

a \$400

b \$800

c 6 new policies

d $m = \dfrac{\text{Rise}}{\text{Run}}, \; c = 400$

$$= \frac{800 - 400}{8 - 0}$$
$$= 50$$
$$y = mx + c$$
$$I = 50n + 400$$

e
$$I = 50n + 400$$
$$= 50 \times 3 + 400$$
$$= \$550$$

f \$50

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Exercise 5B

1 Complete the following tables of values and graph each linear function.

a $c = d + 5$

c	0	1	2	3	4
d					

b $I = -3n$

I	0	2	4	6	8
n					

2 Draw the graph of these linear functions using a table of values from 1 to 10.

a $C = 2n + 10$ **b** $V = -5t + 30$ **c** $M = \frac{1}{2}n - 10$

Example 4

3 **a** The conversion graph opposite is used to convert Australian dollars to British pounds. Use the graph to calculate these exchanges.

i 30 Australian dollars to pounds

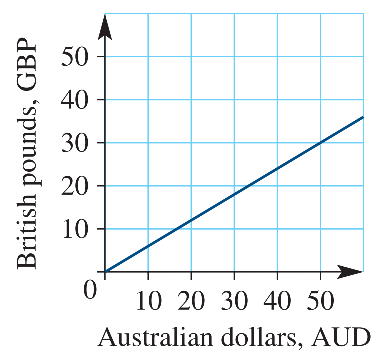
ii 50 Australian dollars to pounds

iii 20 pounds to Australian dollars

iv 10 pounds to Australian dollars

b What is the gradient of the conversion graph?

Australian dollars to British pounds



4 The relationship of the age of machinery (a) in years to its value (v) in \$1000 is $v = -4a + 20$.

a Construct a table of values for age against value. Use values of a from 0 to 4.

b Draw the graph of age (a) against value (v).

c What is the initial cost of the machinery?

d What is the age of the machinery if its current value is \$15 000?

e What is the value of the machinery after $3\frac{1}{2}$ years?

f What will be the value of the machinery after 2 years?

g When will the machinery be worth half its initial cost?

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Example 5

5 An ultrasound machine was purchased by a medical centre for $150 000. Its value is depreciated each month as shown in the graph.

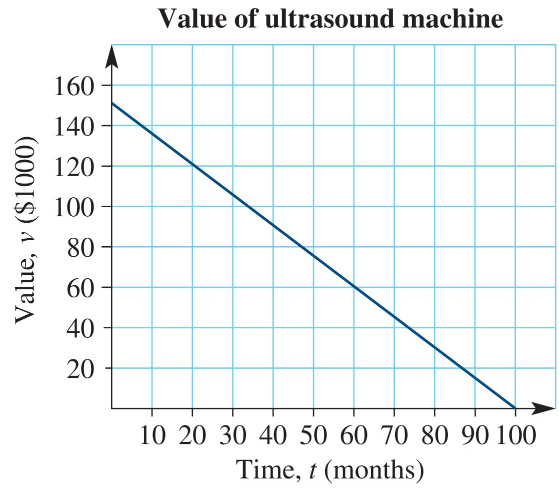
a What was the value of the machine after 40 months?

b What was the value of the machine after five years?

c When does the line predict the machine will have no value?

d Find the equation of the straight line in terms of v and t.

e Use the equation to predict the value of the machine after 6 months.



6 A car is travelling at constant speed. It travels 360 km in 9 hours.

a Write a linear equation in the form $d = mt$ to describe this situation.

b Draw the graph of d against t.

7 The cost ($$C$) of hiring a taxi consists of two elements: a fixed flagfall and a figure that varies with the number (n) of kilometres travelled. If the flagfall is $2.60 and the cost per kilometre is $1.50, determine a rule that gives C in terms of n.

8 The weekly wage, $$w$, of a vacuum cleaner salesperson consists of a fixed sum of $350 plus $20 for each cleaner sold. If n cleaners are sold per week, construct a rule that describes the weekly wage of the salesperson.

9 A telecommunications company's rates for local calls from private telephones consist of a quarterly rental fee of $40 plus 25$c$ for every call. Construct a linear rule that describes the quarterly telephone bill. Let C be the cost (in cents) of the quarterly telephone bill and n the number of calls.

10 Blake converted 100 Australian dollars (AUD) to 60 euros (EUR).

a Draw a conversion graph with Australian dollars on the horizontal axis and euros on the vertical axis.

b How many euros is 25 Australian dollars? Use the conversion graph.

c How many Australian dollars is 45 euros? Use the conversion graph.

d Find the gradient and vertical intercept for the conversion graph.

e Write an equation that relates Australian dollars (AUD) to euros (EUR).

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5C Simultaneous equations – graphically

Two straight lines will always intersect unless they are parallel. The point at which two straight lines intersect can be found by sketching the two graphs on the one set of axes and reading off the coordinates of the point of intersection. Finding the point of intersection is said to be 'solving the equations simultaneously'. In addition to graphing the straight lines, the point of intersection could be determined by looking at the table of values. If the same value for x and y occurs in both tables it is the point of intersection. See example below.

SOLVING A PAIR OF SIMULTANEOUS EQUATIONS GRAPHICALLY

1. Draw a number plane.
2. Graph both linear equations on the number plane.
3. Read the point of intersection of the two straight lines.
4. Interpret the point of intersection for practical applications.

Example 6: Finding the solution of simultaneous linear equations 5C

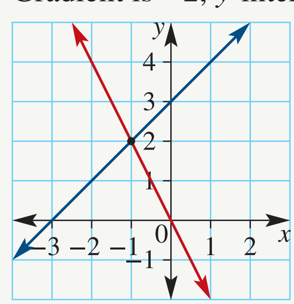
Find the simultaneous solution of $y = x + 3$ and $y = -2x$.

SOLUTION:

1. Use the gradient–intercept form to determine the gradient (coefficient of x) and y-intercept (constant term) for each line.

$y = x + 3$
Gradient is $+1$, y-intercept is 3.
$y = -2x$
Gradient is -2, y-intercept is 0.

2. Draw a number plane.
3. Sketch $y = x + 3$ using the y-intercept of 3 and a gradient of 1.
4. Sketch $y = -2x$ using the y-intercept of 0 and a gradient of -2.



5. Find the point of intersection of the two lines.

$(-1, 2)$

6. The simultaneous solution is the point of intersection.

Simultaneous solution is $x = -1$ and $y = 2$, $(-1, 2)$.

7. Alternatively, construct a table of values for x and y. Let $x = -2, -1, 0, 1$ and 2. Find y using the linear function $y = x + 3$.

x	−2	−1	0	1	2
y	1	2	3	4	5

8. Repeat to find y using the linear function $y = -2x$.

x	−2	−1	0	1	2
y	4	2	0	−2	−4

9. The same value of x and y occurs in both tables when $x = -1$ and $y = 2$.

Simultaneous solution is $x = -1$ and $y = 2$.

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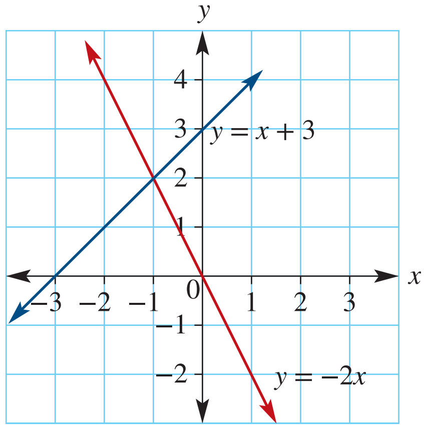
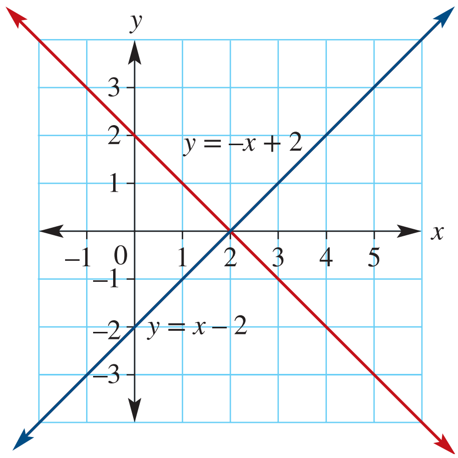
Exercise 5C

1 What is the point of intersection for each of these pairs of straight lines?

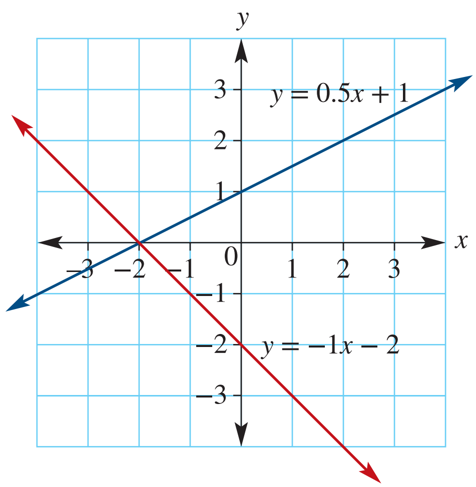
a



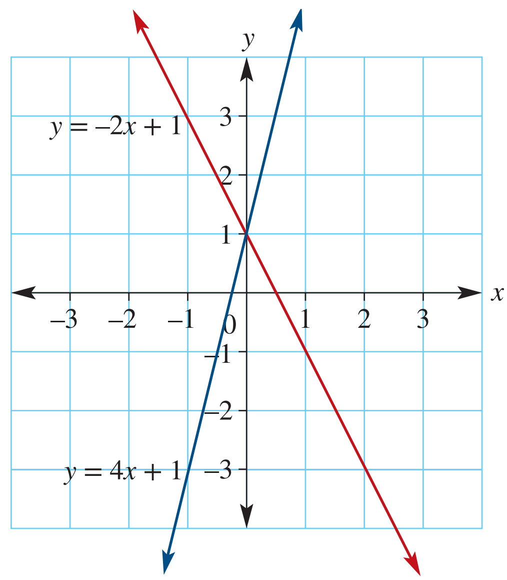
b



c



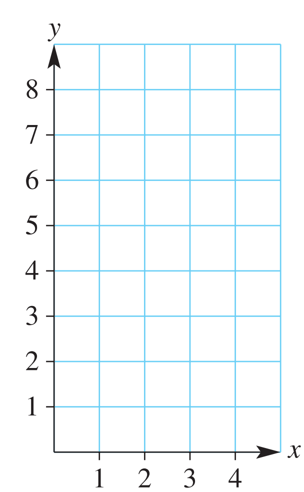
d



Example 6 **2** Plot the following points on a number plane and join them to form two straight lines. What is the point of intersection of these straight lines?

x	0	1	2	3	4
y	0	2	4	6	8

x	0	1	2	3	4
y	6	5	4	3	2



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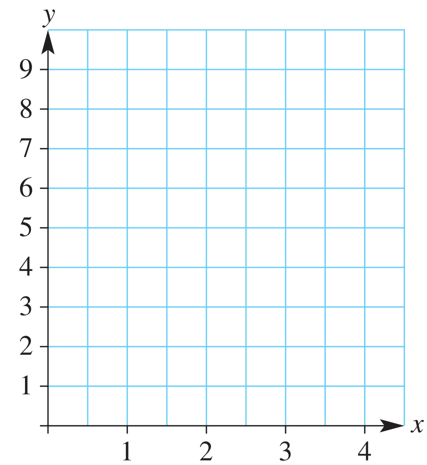
3 Plot the following points on a number plane and join them to form two straight lines. What is the point of intersection of these straight lines?

x	0	1	2	3	4
y	1	3	5	7	9

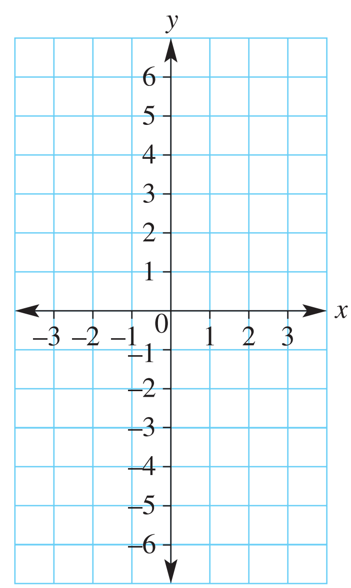
x	0	1	2	3	4
y	4	3	2	1	0



4 Plot the following points on a number plane and join them to form two straight lines. What is the point of intersection of these straight lines?

x	−2	−1	0	1	2
y	−6	−5	−4	−3	−2

x	−2	−1	0	1	2
y	6	3	0	−3	−6



5 Complete the following table of values and plot the points on a number plane. Find the simultaneous solution of these pairs of equations. All solutions are whole numbers.

a $y = 2x + 3$

x	−2	−1	0	1	2
y					

$y = -x$

x	−2	−1	0	1	2
y					

b $y = x + 4$

x	−2	−1	0	1	2
y					

$y = 2x$

x	−2	−1	0	1	2
y					

c $y = 3x + 1$

x	−2	−1	0	1	2
y					

$y = 5x - 3$

x	−2	−1	0	1	2
y					

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6 Complete the following table of values and plot the points on a number plane. What is the solution to each pair of simultaneous equations? Some of the solutions are not whole numbers.

a $y = 3x - 3$

x	−2	−1	0	1	2
y					

$y = x + 1$

x	−2	−1	0	1	2
y					

b $y = 3x - 2$

x	−2	−1	0	1	2
y					

$y = -x + 4$

x	−2	−1	0	1	2
y					

c $y = x$

x	−2	−1	0	1	2
y					

$y = 4x + 3$

x	−2	−1	0	1	2
y					

d $y = -x$

x	−6	−3	0	3	6
y					

$y = 4 - 2x$

x	−6	−3	0	3	6
y					

e $y = 5x + 1$

x	−6	−4	−2	0	2
y					

$y = 3x - 7$

x	−6	−4	−2	0	2
y					

f $y = x + 1$

x	−2	−1	0	1	2
y					

$y = -2x$

x	−2	−1	0	1	2
y					

g $y = 2x - 4$

x	0	1	2	3	4
y					

$y = -x + 5$

x	0	1	2	3	4
y					

h $y = x + 1$

x	−2	−1	0	1	2
y					

$y = -3x + 2$

x	−2	−1	0	1	2
y					

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5D Simultaneous equation models

When two practical situations are described mathematically using a linear function then the point of intersection has an important and often different meaning depending on the situation. For example, when income is graphed against costs the point of intersection represents the point where a business changes from a loss to a profit.

SIMULTANEOUS EQUATIONS AS MODELS

Simultaneous equation models use two linear functions to describe a practical situation and the point of intersection is often the solution to a problem.

Example 7: Using simultaneous equations as models — 5D

Zaina buys and sells books. Income received by selling a book is calculated using the formula $I = 16n$. Costs associated in selling a book are calculated using the formula $C = 8n + 24$.

a What is the income when 6 books are sold?
b What is the costs when 6 books are sold?
c Draw the graph of $I = 16n$ and $C = 8n + 24$ on same number plane.
d Use the graph to determine the number of books needed to be sold for the costs to equal the income.

SOLUTION:

1 Substitute 6 for n into the formula for income $I = 16n$.

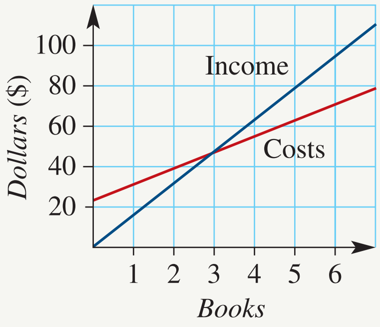
a $I = 16n = 16 \times 6 = 96$
$\therefore$ Income for six books is \$96

2 Substitute 6 for n into the formula for costs $C = 8n + 24$.

b $C = 8n + 24 = 8 \times 6 + 24 = \$72$
$\therefore$ Costs for six books is \$72

3 Draw a number plane.
4 Use the gradient–intercept form to determine the gradient and vertical intercept for each line. Gradient is the coefficient of n. Vertical intercept is the constant term.
5 Sketch $I = 16n$ using the vertical intercept of 0 and gradient of 16.
6 Sketch $C = 8n + 24$ using the vertical intercept of 24 and gradient of 8.

c



7 Find the point of intersection of the two lines (3, 48).

d Income is equal to costs when $n = 3$
$\therefore$ 3 books

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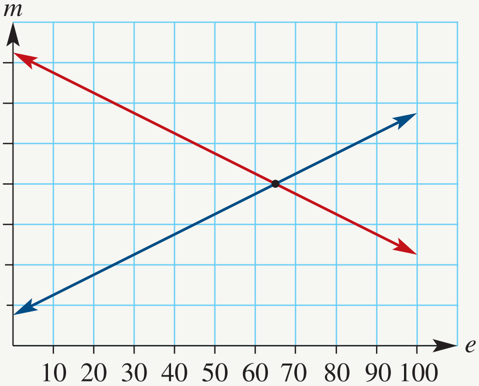
Example 8: Solving problems using intersecting graphs 5D

Isabella's Mathematics mark exceeded her English mark by 15. She scored a total of 145 for both tests. Find Isabella's marks in each subject by plotting intersecting graphs.

SOLUTION:

1 Express the relationship between the Mathematics and the English mark as a linear equation.	Let the Mathematics mark be m. Let the English mark be e. $m = e + 15$
2 Use the gradient–intercept form to determine the gradient and vertical intercept for the line. Gradient is the coefficient of e. Vertical intercept is the constant term.	Gradient is 1, vertical intercept is 15.
3 Express the total of the two marks as a linear equation.	$m + e = 145$
4 Use the gradient–intercept form to determine the gradient and vertical intercept for the line.	$m = -e + 145$ Gradient is -1, vertical intercept is 145.
5 Draw a number plane. **6** Sketch $m = e + 15$ using the vertical intercept of 15 and gradient of 1. **7** Sketch $m = -e + 145$ using the vertical intercept of 145 and a gradient of -1. **8** The simultaneous solution is the point of intersection.	
9 Find the point of intersection of the two lines.	Intersection is (65, 80) so $e = 65$ and $m = 80$
10 Write the solution in words using the context of the question.	Isabella scored 65 in English and 80 in Mathematics.

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Exercise 5D

1 Matilda and Nathan earn wages m and n respectively.

a Matilda earns $100 more than Nathan. Write an equation to describe this information.

b The total of Matilda's wages and Nathan's wages is $1200. Write an equation to describe this information.

c Draw a graph of the two equations on the same number plane. Use n as the horizontal axis and m as the vertical axis.

d Use the intersection of the two graphs to find Matilda's and Nathan's wages.

2 Let one number be represented by a and the other number by b.

a The sum of the two numbers is 42. Write an equation to describe this information.

b The difference of the two numbers is 6. Write an equation to describe this information.

c Draw a graph of the two equations on the same number plane. Use a as the horizontal axis and b as the vertical axis.

d Use the intersection of the two graphs to find the two numbers.

3 Let one number be represented by p and another number by q.

a The sum of the two numbers is 15. Write an equation to describe this information.

b One of the numbers is twice the other number. Write an equation to describe this information.

c Draw a graph of the two equations on the same number plane. Use p as the horizontal axis and q as the vertical axis.

d Use the intersection of the two graphs to find the two numbers.

4 Amy and Nghi work for the same company and their wages are a and b respectively.

a Amy earns $100 more than Nghi. Write an equation to describe this information.

b The total of Amy's and Nghi's wages is $1500. Write an equation to describe this information.

c Draw a graph of the above two equations on the same number plane. Use a as the horizontal axis and b as the vertical axis.

d Use the intersection of the two graphs to find Amy's and Nghi's wages.

Example 7, 8

5 A factory produces items whose costs are $1000 plus $10 for every item. The factory receives $60 for every item sold.

a Write an equation to describe the relationship between the:

i costs (C) and number of items (n)

ii income (I) and number of items (n).

b Draw a graph and find the number of items when income equals costs.

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5E Break-even analysis

The break-even point is reached when costs or expenses and income are equal. There is no profit or loss at the break-even point. For example, if the break-even point for a business is 100 items per month, the business will make a loss if it sells fewer than 100 items each month; if it sells more than 100 items per month, it will make a profit. A profit (or loss) is calculated by subtracting the costs from the income (Profit = Income – Costs). Income is a linear function of the form $I = mx$, where x is the number of items sold and m is the selling price of each item. Cost is a linear function of the form $C = mx + c$, where x is the number of items sold, m is the cost price per item manufactured and c is the fixed costs of production.

BREAK-EVEN ANALYSIS

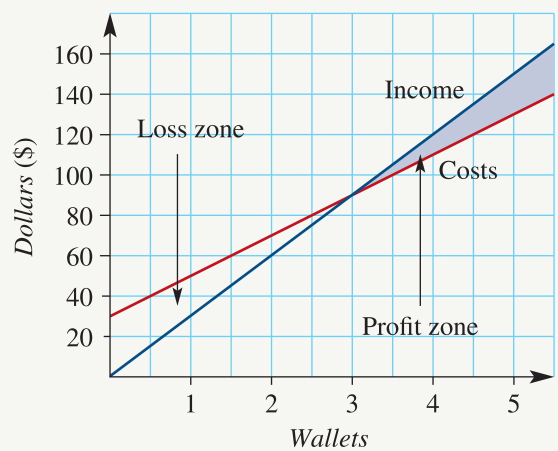
Break-even point occurs when costs equal income.
Profit = Income – Costs
Income: $I = mx$
Costs: $C = mx + c$

Example 9: Interpreting the point of intersection of two graphs 5E

Grace buys and sells wallets. Income received by selling wallets is calculated using the formula $I = 30x$. Costs associated with selling wallets are calculated using the formula $C = 20x + 30$.

a Use the graph to determine the number of wallets that Grace needs to sell to break even.

b How much profit or loss does she make when four wallets are sold?



SOLUTION:

1 Consider when the break-even point occurs.	**a** When the income equals the costs.
2 Read the point of intersection of the two linear graphs.	Intersection is at (3, 90). So $x = 3$. Number of wallets $= 3$
3 Profit is determined by subtracting the costs from the income.	**b** Profit = Income – Cost
4 Read from the graph the values of I and C when $x = 4$.	$I = 120$ and $C = 110$
5 Evaluate.	$= 120 - 110$ $= \$10$
6 Write the answer in words.	Profit for selling 4 wallets is \$10.

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Example 10: Break-even analysis 5E

A firm sells its product at \$20 per unit. The cost of production (\$$C$) is given by the rule $C = 4x + 48$, where x is the number of units produced.

a Find the value of x for which the cost of the production of x units is equal to the income or revenue received by the firm for selling x units.

b Check your answer algebraically.

SOLUTION:

1 Set up the income equation and determine the gradient and vertical intercept.

a Let the income or revenue for producing x units be \$$I$. Formula is:

$$I = 20x$$

Gradient is 20, vertical intercept is 0

2 Set up the cost of production equation and determine the gradient and vertical intercept.

Cost of production (\$$C$) is given by:

$$C = 4x + 48$$

Gradient is 4, vertical intercept is 48

3 Draw a number plane.

4 Use x as the horizontal axis.

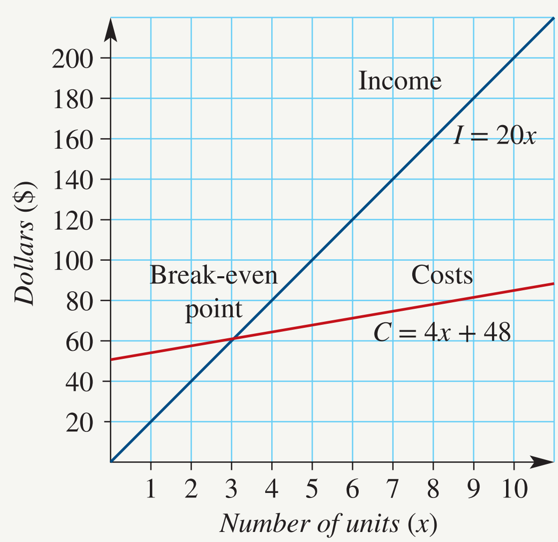
5 Use I and C as the vertical axis.

6 Sketch $I = 20x$ using the vertical intercept of 0 and gradient of 20.

7 Check this line using some valid points such as (1, 20).

8 Sketch $C = 4x + 48$ using the vertical intercept of 48 and gradient of 4.

9 Check this line using some valid points such as (1, 52).



10 Read the value of x at the point of intersection of the two linear graphs.

The point of intersection of the two linear graphs occurs when $x = 3$. This is the break-even point, the value of x for which cost of production is equal to income.

11 Substitute $x = 3$ into the formula $I = 20x$.

b Check algebraically.

12 Substitute $x = 3$ into the formula $C = 4x + 48$.

Income	Costs
$I = 20x$	$C = 4x + 48$
$= 20 \times 3$	$= 4 \times 3 + 48$
$= 60$	$= 60$

13 Check that I is equal to C.

Income equals costs, so answer to **a** is correct.

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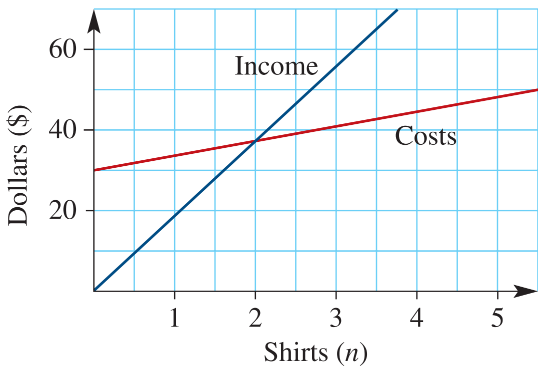
Exercise 5E

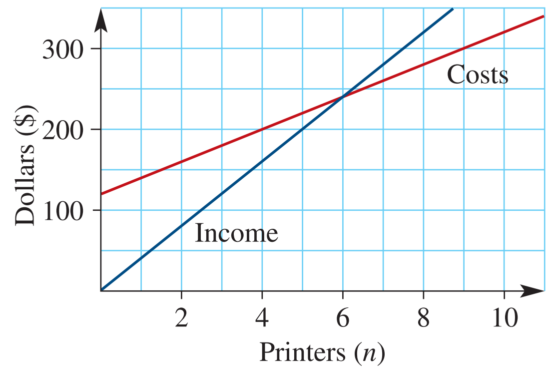
Example 9,10

1 What is the break-even point for the following graphs?

a



b



2 The graph on the right shows the cost of producing boxes of chocolates and the income received from their sale.

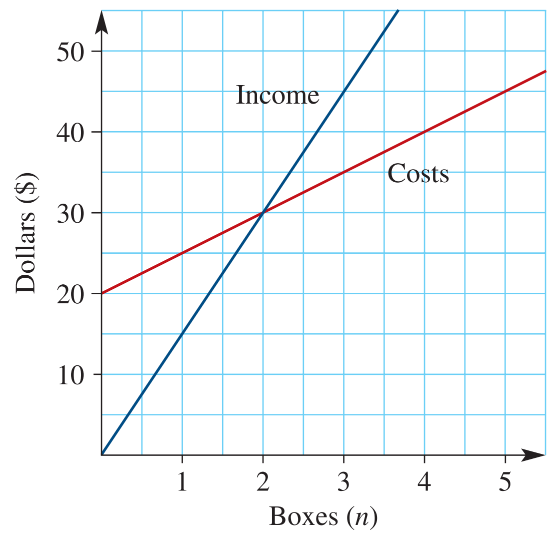
a Use the graph to determine the number of boxes that need to be sold to break even.

b How much profit or loss is made when 3 boxes are sold?

c How much profit or loss is made when one box is sold?

d What are the initial costs?

Cost of chocolate production



3 The graph below shows the cost of producing cartons of soft drinks and the income received from their sale.

a Use the graph to determine the number of cartons that need to be sold to break even.

b How much profit or loss is made when five cartons are sold?

c How much profit or loss is made when two cartons are sold?

d What is the initial cost?

e What is the gradient of the straight line that represents income?

f What is the vertical intercept of the straight line that represents income?

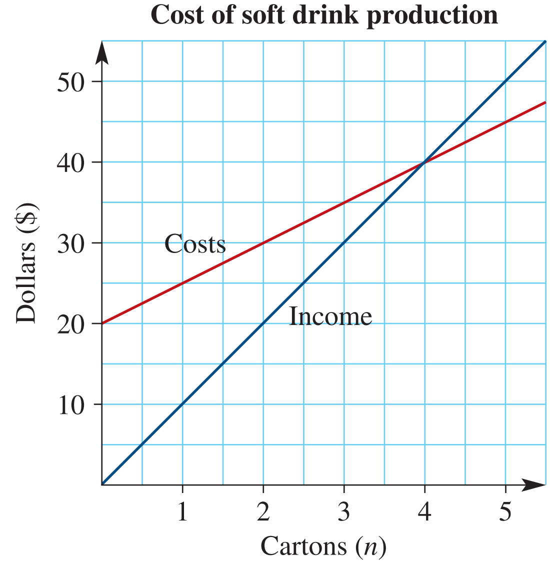
g Write an equation to describe the relationship between income and the number of cartons.

h What is the gradient of the straight line that represents costs?

i What is the vertical intercept of the straight line that represents costs?

j Write an equation to describe the relationship between costs and the number of cartons.

Cost of soft drink production



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PPT STUDY GUIDE

Summary

Key ideas and chapter summary

Linear functions

1 Construct a table of values with the independent variable as the first row and the dependent variable as the second row.
2 Draw a number plane with the independent variable on the horizontal axis and the dependent variable as the vertical axis. Plot the points.
3 Join the points to make a straight line.

Gradient–intercept formula

Linear equation: $y = mx + c$.

m – Slope or gradient of the line
(vertical rise over the horizontal run).

c – y-intercept
Where the line cuts the y-axis or vertical axis.

Linear models

Linear models describe a practical situation mathematically using a linear function.

Simultaneous equations – graphically

1 Draw a number plane.
2 Graph both linear equations on the number plane.
3 Read the point of intersection of the two straight lines.
4 Interpret the point of intersection for practical applications (break-even point).

Simultaneous equation as models

Simultaneous equation models use two linear functions to describe a practical situation and the point of intersection is often the solution to a problem.

Break-even analysis

Break-even point occurs when costs equal income.

Profit = Income − Costs

Income: $I = mx$

Costs: $C = mx + c$

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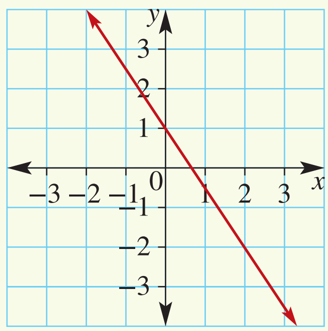
Review

Multiple-choice



1 What is the gradient of this line?

A $-\frac{3}{2}$ **B** $-\frac{2}{3}$ **C** $\frac{2}{3}$ **D** $\frac{3}{2}$

2 What is the y-intercept of this line?

A −2 **B** −1 **C** 1 **D** 2

3 A straight line has the equation of $y = -x - 3$. What is the y-intercept?

A −3 **B** −1 **C** +1 **D** +3

4 The cost of manufacturing bags (C) is given by the formula $c = 40x + 150$, where x is the number of bags sold. What is the cost of manufacturing two bags?

A \$40 **B** \$150 **C** \$190 **D** \$230

5 A car is travelling at a constant speed. It travels 80 km in 4 hours. This situation is described by the linear equation $d = mt$. What is the value of m?

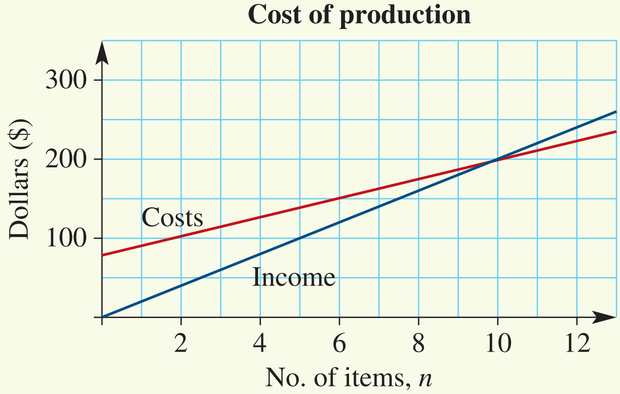
A 0.05 **B** 3 **C** 20 **D** 60

6 What is the point of intersection of the lines $y = x + 2$ and $y = -x + 2$?

A (2, 0) **B** (0, 2) **C** (0, −2) **D** (1, 1)

Use the graph below to answer questions **7–9**.

Cost of production



7 What is the profit for selling 12 items?

A \$10 **B** \$20 **C** \$220 **D** \$240

8 What is the break-even point?

A 10 items **B** 12 items **C** 20 items **D** 80 items

9 What is the loss for selling 5 items?

A \$20 **B** \$30 **C** \$40 **D** \$50

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Review

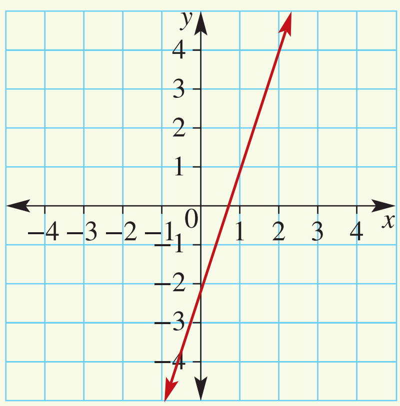
Short-answer

1 Draw the graph of these linear functions.

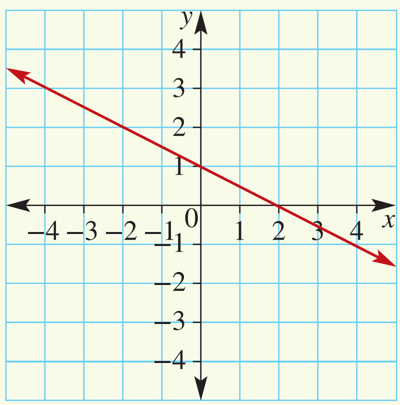
a $y = x + 2$ **b** $y = -3x + 1$ **c** $y = 2x - 2$

2 Find the equation of the following straight-line graphs.

a



b



3 The table below shows the speed v (in m/s) of a plane at time t seconds.

Time (t)	1	2	3	4	5
Speed (v)	2.5	4	5.5	7	8.5

a Draw a number plane with t as the horizontal axis and v as the vertical axis. Plot the points and join them to make a straight line.
b Determine a linear model in the form $y = mx + c$ to describe this situation.
c What does the model predict will be the plane's speed when $t = 2.5$ seconds?
d What does the model predict will be the plane's speed when $t = 6$ seconds?
e What does the model predict will be the plane's speed when $t = 7$ seconds?
f What does the model predict will be the plane's speed when $t = 10$ seconds?

4 An internet access plan charges an excess fee of \$12 per GB.

Data (d)	1	2	3	4	5	6
Cost (c)	12	24	36	48	60	72

a Draw a graph of data against cost.
b Use the graph to find d if c is 30.
c Use the graph to find c if d is 3.5.
d Estimate the cost of 7 GB of data.
e Estimate the cost of 10 GB of data.
f Estimate the cost of 8.5 GB of data.

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Review

5 What is the point of intersection of the lines $y = 3x + 3$ and $y = x - 2$?

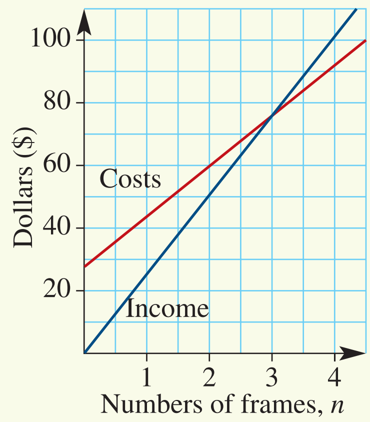
6 The graph opposite shows the cost of making picture frames and the income received from their sale.

a Use the graph to determine the number of picture frames that need to be sold to break even.

b How much profit or loss is made when one picture frame is sold?

c How much profit or loss is made when four picture frames are sold?

d What is the initial cost?



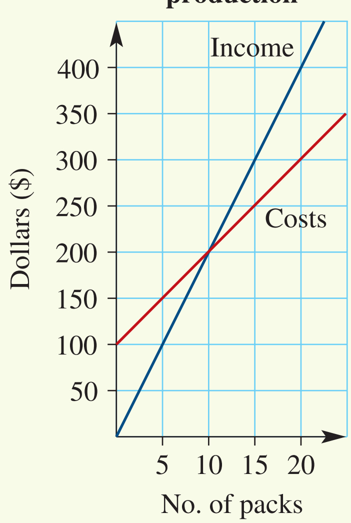
7 The graph opposite shows the cost of producing packs of batteries and the income received from their sale.

a Use the graph to determine the number of packs that need to be sold to break even.

b How much profit or loss is made when 5 packs are sold?

c How much profit or loss is made when 20 packs are sold?

d What are the initial costs?



8 The graph opposite shows the cost of manufacturing tables and the income received from their sale.

a Use the graph to determine the number of tables that need to be sold to break even.

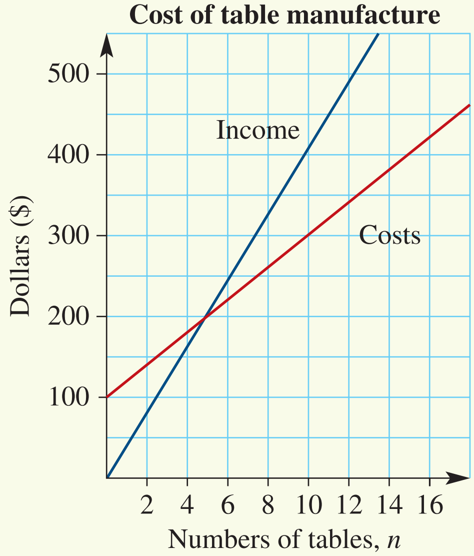
b Write an equation to describe the relationship between income and the number of tables.

c Write an equation to describe the relationship between costs and the number of tables.

d How much profit or loss is made when 10 tables are sold?

e How much profit or loss is made when 4 tables are sold?

f How much profit or loss is made when 16 tables are sold?Simultaneous equations as models



PDF LITERACY

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Practice Paper 1

Section I

Attempt Questions 1–15 (15 marks).
Allow about 20 minutes for this section.

1 A car is travelling at a constant speed. It travels 60 km in 3 hours. This situation is described by the linear equation $d = st$. What is the value of s?

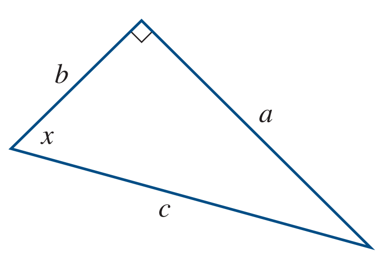
A 0.05
B 3
C 10
D 20

2 What is the amount of interest paid on a \$150 000 loan over 25 years if the interest rate is 7.2% p.a. compounding annually? Answer to the nearest dollar.

A \$10 800
B \$27 000
C \$703 023
D \$853 023

3 What is the value of $\sin x$ in the triangle below?

A $\frac{b}{c}$
B $\frac{a}{b}$
C $\frac{b}{a}$
D $\frac{a}{c}$



4 A car uses on average 7 L per 100 km in fuel. How much fuel would be used on a trip of 382 km?

A 26.74 L
B 34.72 L
C 38.20 L
D 54.57 L

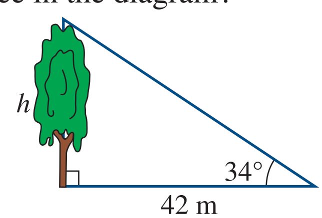
5 Which of the following expressions would give the height, (h), of the tree in the diagram?

A $\dfrac{42}{\tan 34°}$

B $42 \times \tan 34°$

C $\dfrac{42}{\cos 34°}$

D $42 \times \cos 34°$



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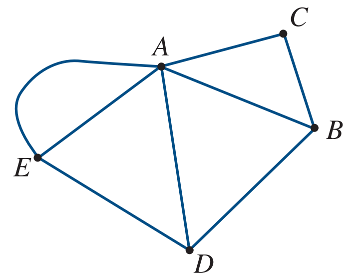
Use the following network graph to answer questions **6** to **9**.



6 What is the degree of vertex B?

A 1
B 2
C 3
D 4

7 How many edges are there in the graph?

A 5
B 6
C 7
D 8

8 How many vertices of even degree are there in the network diagram?

A 1
B 2
C 3
D 4

9 The graph has:

A an Eulerian trail but not an Eulerian circuit
B several Eulerian trails but no Eulerian circuits
C an Eulerian circuit
D neither an Eulerian trail nor an Eulerian circuit

10 Aiden breathes about 15 times each minute. How many times would he breathe in 9 hours?

A 90
B 216
C 8100
D 486 000

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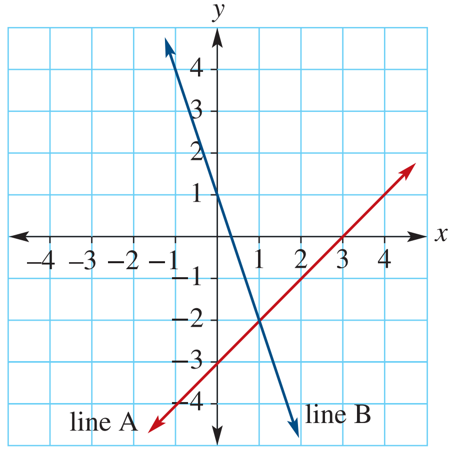
Use the following graph to answer questions **11** to **13**.



11 The slope of line B is:

A –3

B 3

C $-\frac{1}{3}$

D $\frac{1}{3}$

12 The equation of line A is:

A $y - 2x = 3$

B $x - y = 3$

C $y - x = 3$

D $-2y - x = 6$

13 What is the simultaneous solution of line A and line B?

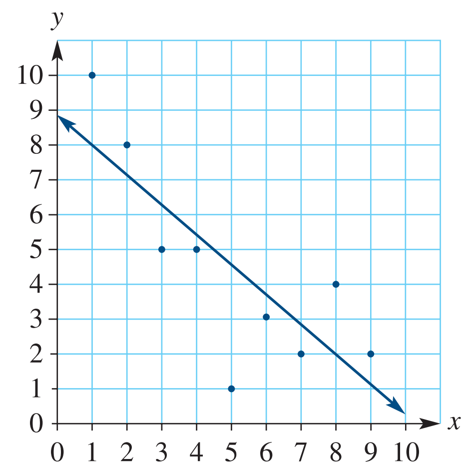
A $x = -1$ and $y = -2$

B $x = -1$ and $y = 2$

C $x = 1$ and $y = -2$

D $x = -3$ and $y = 3$

14 The equation of the line of best fit plotted on the scatterplot opposite is closest to:



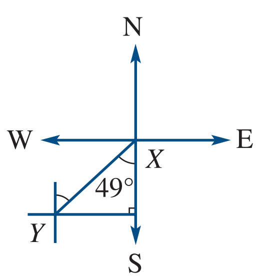
A $y = -0.9x + 9$

B $y = -9x + 0.9$

C $y = 0.9x - 97$

D $y = 9x - 0.9$

15 The compass bearing of Y from X is S49°W. What is the compass bearing of X from Y?



A N41°E

B N49°E

C S41°W

D S49°W

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Section II

Attempt Questions 16–18 (45 marks).
Allow about 70 minutes for this section.
All necessary working should be shown in every question.

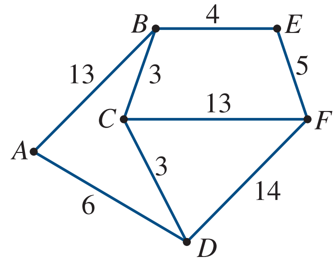
Question 16 (15 marks) **Marks**

a Use the rate provided to answer the following questions.

i Cost of bananas is \$4.76/kg. What is the cost of $\frac{1}{2}$ kg? **1**

ii Cost savings are \$42/day. How much is saved in 7 days? **1**

b A network graph is shown below.



i Does this graph have an Eulerian circuit? Give a reason. **1**

ii Find the minimum spanning tree and its length for the network. **2**

iii What is the length of the shortest path from A to E in the network? **1**

iv What is the length of the shortest path from A to F in the network? **1**

v What is the length of the shortest Hamiltonian cycle for the network? **2**

c A painting was bought for \$695 at the beginning of 2017. It is expected that the painting will appreciate in value by 8% each year. What will be the value of the painting at the beginning of 2028? Answer to the nearest dollar. **2**

d i Copy and complete the table of ordered pairs for $y = -\frac{1}{3}x - \frac{4}{3}$. **1**

x	−4	−1	0	2	6
y					

ii Copy and complete the table of ordered pairs for $y = -x + 2$. **1**

x	−2	−1	0	1	2
y					

iii Graph the lines $y = -\frac{1}{3}x - \frac{4}{3}$ and $y = -x + 2$ on a number plane. **1**

iv Find the point of intersection of $y = -\frac{1}{3}x - \frac{4}{3}$ and $y = -x + 2$. **1**

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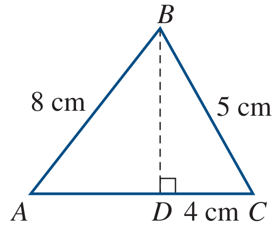
Question 17 (15 marks) **Marks**

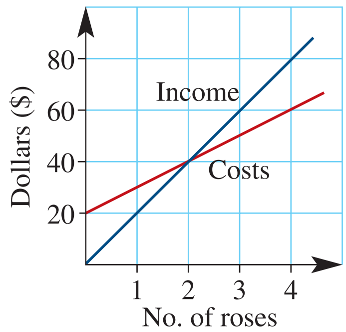
a In the triangle ABC, $AB = 8\,\text{cm}$, $BC = 5\,\text{cm}$ and $CD = 4\,\text{cm}$.



i Find the size of angle BCD, correct to the nearest degree. **1**

ii What is the length of AD? Answer correct to two decimal places. **2**

b The graph shows the cost of growing a rose plant and the income received.



i Use the graph to determine the number of rose plants that need to be sold to break even. **1**

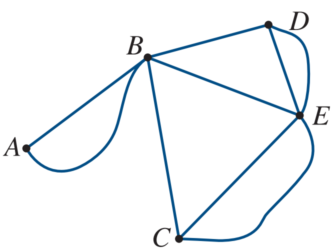
ii How much profit or loss is made when 1 rose is sold? **1**

iii How much profit or loss is made when 4 roses are sold? **1**

iv What are the initial costs? **1**

c Find the distance travelled by a car whose average speed is $54\,\text{km/h}$ if the journey lasts 2 hours and 30 minutes. **2**

d A network diagram is shown opposite.



i What is the degree of vertex E? **1**

ii What is a Hamiltonian path? **1**

iii Give an example of a Hamiltonian path in this graph. **1**

e A straight line has the equation of $2x + y = -2$.

i What is the gradient of this line? **1**

ii What is the y-intercept? **1**

iii What is the point of intersection between $2x + y = -2$ and $y = 2$? **1**

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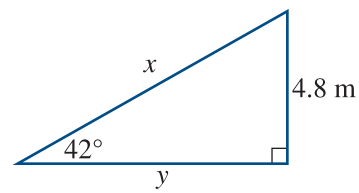
Question 18 (15 marks) **Marks**

a Karen's motor vehicle has a fuel consumption of 9.9 L/100 km in the city and 7.8 L/100 km in the country. Karen travels 8 000 km per year in the city and 15 000 km per year in the country. The average cost of petrol is \$1.53 in the city and 15 cents higher in the country.

- **i** What is the amount of fuel needed to drive in the city for the year? **1**
- **ii** What is the amount of fuel needed to drive in the country for the year? **1**
- **iii** Find the cost of petrol to drive in the city for the year. **1**
- **iv** Find the cost of petrol to drive in the country for the year. **1**

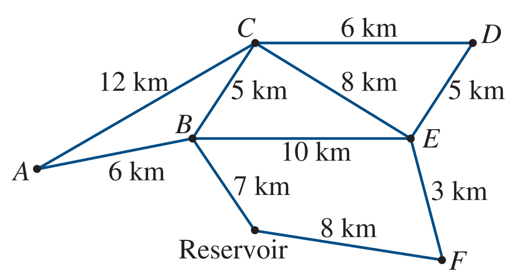
b Use trigonometry to find the values of x and y. **2**
Answer correct to one decimal place.



c A sum of \$290 000 was invested in a bank account for 6 years. Answer the following questions to the nearest dollar.

- **i** Find the simple interest earned if the rate of interest is 3.6% p.a. **1**
- **ii** Find the future value of this investment at the end of 6 years at this simple rate of interest. **1**
- **iii** Find the future value of this investment at the end of 6 years if the interest rate is compounding at 3.6% p.a. **2**

d The diagram at right shows the network of pipes providing water from a reservoir to six small settlements. These pipes connect the settlements to each other and to the reservoir. The lengths of the pipes (in km) are shown opposite.



The pipe inspector plans to start her inspection at the reservoir, travel along each of the pipes and return to the reservoir without having to travel along any pipe section more than once.

- **i** What is the technical name for the route she wants to follow? **1**
- **ii** Explain why such a route is possible for this network of pipes. **1**
- **iii** Name one such route she could follow. **1**
- **iv** The system of pipes is due for replacement. What is the minimum length of pipe that can be used to service all of the settlements? **2**

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6 Further statistical analysis

Syllabus topic — S3 Further statistical analysis

This topic will introduce a variety of methods for identifying, analysing and describing associations between pairs of variables.

Outcomes

- Construct a bivariate scatterplot in identify patterns in data.
- Use bivariate scatterplot to describe the patterns, features and associations of bivariate datasets.
- Identify the dependent and independent variables within bivariate datasets.
- Model a linear association by fitting an appropriate line of best fit to a scatterplot and using it to describe patterns and associations.
- Use an appropriate line of best fit to make predictions by either interpolation and extrapolation.
- Implement the statistical investigation process that involves two numerical variables.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Literacy worksheet
- Quick Quiz
- Solutions (enabled by teacher)
- Desmos widgets
- Spreadsheets
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

The Interactive Textbook provides a test of prior knowledge for this chapter, and may direct you to revision from the previous years' work.

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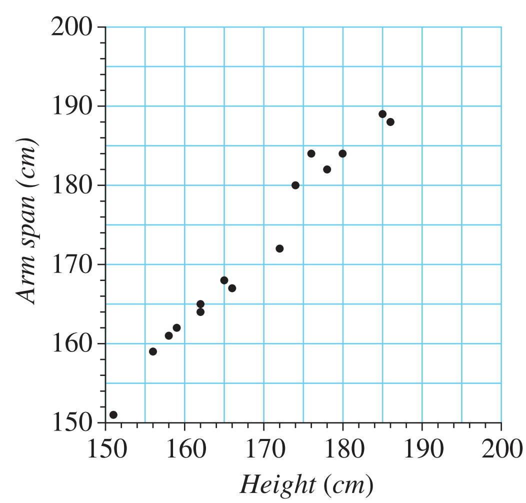
6A Constructing a bivariate scatterplot

Bivariate data is data that has two variables.

A scatterplot is used to determine if there is a relationship between two numerical variables. Data is collected on the two variables and often displayed in a table of ordered pairs. A scatterplot is a graph of the ordered pairs of numbers. Each ordered pair is a dot on the graph. To illustrate this process, a scatterplot has been constructed to determine the relationship between the height and arm span. The data collected on these variables is shown below in the table of ordered pairs.

Height (in cm)	172	159	178	162	156	174	151	162	165	185	186	176	166	180	158
Arm span (in cm)	172	162	182	164	159	180	151	165	168	189	188	184	167	184	161

Each person has two numerical variables, height and arm span. To construct a scatterplot, draw a number plane with the height on the horizontal axis and arm span on the vertical axis. Plot each ordered pair as a dot. The scatterplot shows there is a relationship between these variables.



CONSTRUCTING A SCATTERPLOT

1. Draw a number plane.
2. Determine a scale and a title for the horizontal or x-axis.
3. Determine a scale and a title for the vertical or y-axis.
4. Plot each ordered pair of numbers with a dot.

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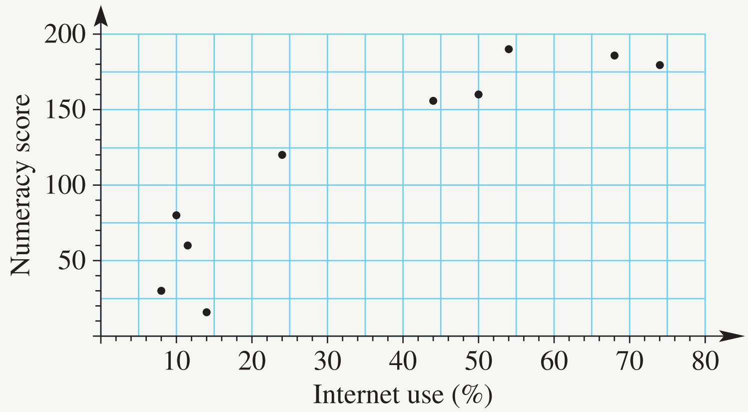
Example 1: Reading a scatterplot 6A

The average numeracy score for year 6 students and their general rate of internet use (%) for 10 countries are displayed in the scatterplot below.



a What is the scale for the vertical axis?
b What is the average numeracy score for the country which has an internet use rate of 24%?
c What is the internet use (%) for the country which has an average numeracy score 160?
d How many countries have internet use of less than 50%?
e How many countries have a numeracy score greater than 100?
f Is there a relationship between these two variables?

SOLUTION:

1 Count the number of divisions between 0 and 50 (5). Therefore 1 unit is 50 divided by 5 (10).

a 1 unit = 10

2 Read from the scatterplot (when internet use is 24% the numeracy score is 120).

b 120

3 Read from the scatterplot (when the numeracy score is 160 the internet use is 50%).

c 50%

4 Count the number of dots less than 50% (left-hand side).

d 6 countries

5 Count the number of dots greater than 100 (top-half).

e 6 countries

6 Look for any pattern in the dots. In this scatterplot when the internet use is greater than 20%, there is a clear increase in the numeracy score. However, this relationship does not exist when the internet use is less than 20%.

f Yes, there is a relationship. When the internet use is greater than 20%, both the variables are increasing.

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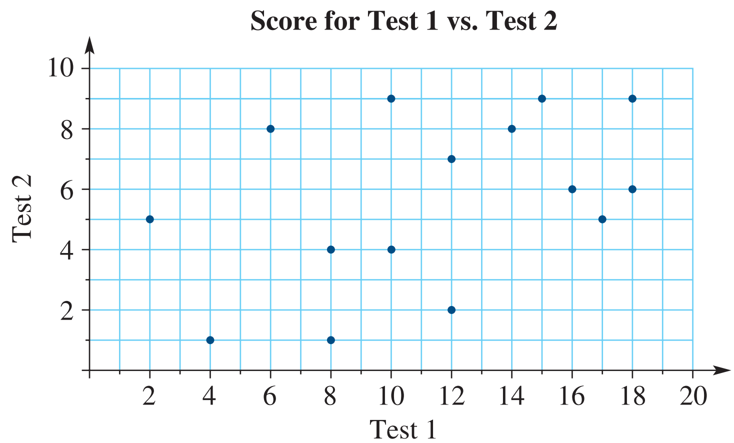
Exercise 6A

Example 1

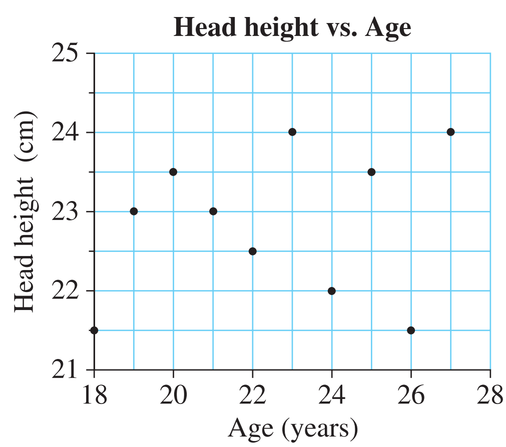
1 The scatterplot shows the results for 15 students in two tests.

a What is the highest mark in test 2?
b What is the lowest mark in test 1?
c What is the range for test 1?
d What is the mode for test 2?
e How many students scored greater than 6 in test 1?



2 The scatterplot shows the head height to age for 10 people.

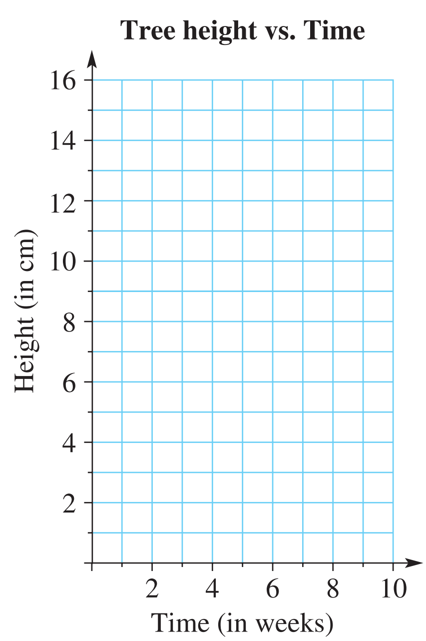
a What is the head height for a person who is 21 years old?
b What is the age of the person who has a head height of 22 cm?
c What is the largest head height?
d What is the age of the youngest person?
e How many people have a head height greater than 23 cm?
f Is there a clear relationship between these two variables?



3 The table below shows the height (in cm) of a eucalyptus tree seedling as it grows.

Time (in weeks)	0	1	2	3	4	5	6	7	8	9	10
Height (in cm)	0	6.6	8.8	9.0	10.5	12.0	13.5	15.2	15.4	15.8	15.9

a Copy the number plane opposite to construct a scatterplot using the above table.
b What is the increase in the height of the seedling during the first week?
c What is the increase in the height of the seedling during the last week?
d How many weeks does it take for the seedling to increase in height from 9 cm to 12 cm?
e Estimate the height of the seedling after 4.5 weeks.
f Estimate the time taken for the seedling to grow to a height of 14 cm.



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4 Adrian is a political commentator who has been studying the effects of television exposure time on the approval ratings of nine politicians. The data is shown below.

Time (in minutes)	5	15	15	75	25	70	40	55	20
Approval rating (%)	60	30	50	90	25	55	55	45	40

a Construct a scatterplot of the data given in the table.
b Are there any conclusions to be drawn from the scatterplot?

5 The table shows the number of runs scored and the number of balls faced by batsmen in a one-day cricket match.

Balls faced	49	29	26	16	19	13	16	10	28	40	6
Runs scored	47	27	10	8	21	3	13	6	15	30	2

a Construct a scatterplot of the data given in the table.
b Are there any conclusions to be drawn from the scatterplot?

6 The maximum wind speed and maximum temperature were recorded for 2 weeks. The data is displayed in the table below.

Wind speed (in km/h)	2	6	12	15	19	20	22	25	17	14	5	11	24	13
Temperature (in °C)	28	26	23	22	21	22	19	16	20	24	25	24	19	26

a Construct a scatterplot of the data given in the table.
b Are there any conclusions to be drawn from the scatterplot?

7 The table below shows the age of the car (in months) and the minimum stopping distance (in metres) when the car is travelling at 60 km/h.

Age of car (in months)	48	12	65	42	98	34
Stopping distance (in metres)	29	28	38	35	36	37

a Prepare a scatterplot using the above data.
b Are there any conclusions to be drawn from the scatterplot?

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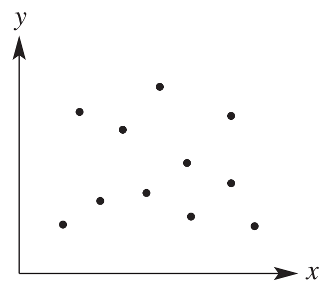
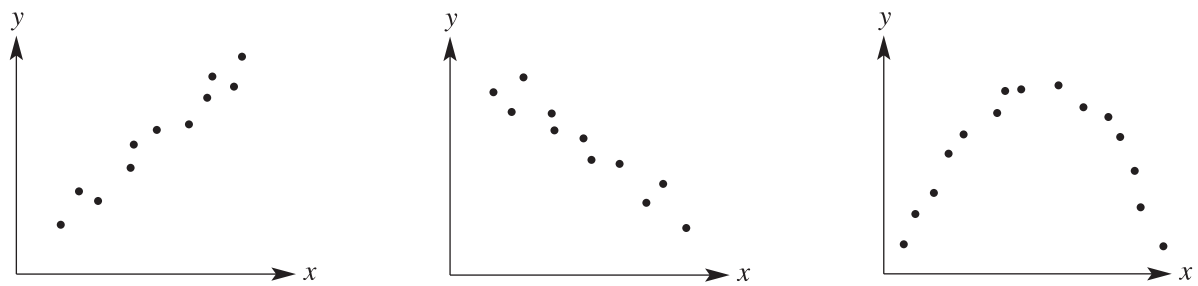
6B Using a bivariate scatterplot

What are the features in a scatterplot that will identify and describe any relationship? First look for a clear pattern. In the scatterplot opposite, there is no clear pattern in the points: they are just randomly spread on the scatterplot. There is no relationship or association between the variables.



For the three examples below, there is a clear (but different) pattern in each set of points, so we conclude that there is an association in each case.



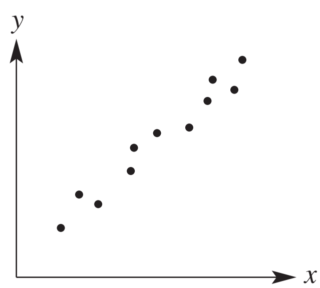
Having found a clear pattern, we need to be able to describe these associations clearly, as they are obviously quite different. There are three things we look for in the pattern of points: form, direction and strength.

Form of an association

If an association exists between the variables then the points in a scatterplot tend to follow a linear pattern or a curved pattern. This is called the form of an association.

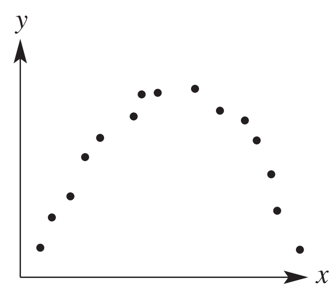
Linear form

If the points seem to approximate a straight line, the association is a linear form.



Non-linear form

If the points seem to approximate a curve, the association is a non-linear form.



FORM OF AN ASSOCIATION

Linear form – when the points tend to follow a straight line.
Non-linear form – when the points tend to follow a curved line.

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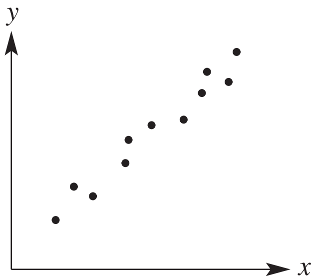
Direction of an association

There are two types of direction if the association is in linear form.

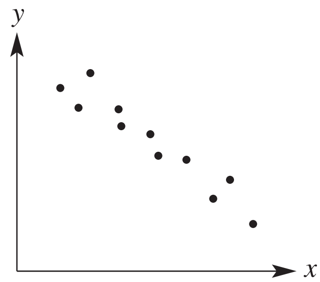
Positive

Positive association exists between the variables if the gradient of the line is positive. That is, the dots on the scatterplot tend to go up as we go from left to right.



Negative

Negative association exists between the variables if the gradient of the line is negative. That is, the dots on the scatterplot tend to go down as we go from left to right.



DIRECTION OF AN ASSOCIATION

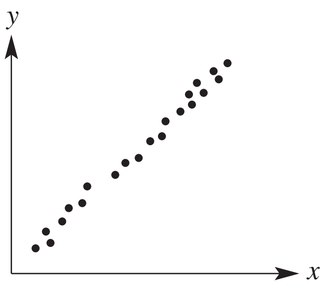
Positive – gradient of the line is positive.
Negative – gradient of the line is negative.

Strength of an association

The strength of an association is a measure of how much scatter there is in the scatterplot.

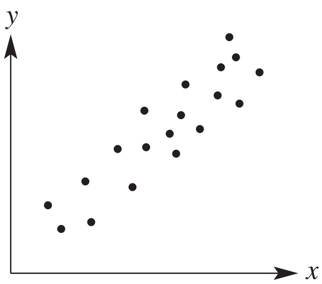
Strong

In strong association the dots will tend to follow a single stream. A pattern is clearly seen. There is only a small amount of scatter in the plot.



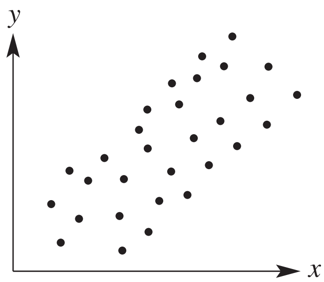
Moderate

In moderate association the amount of scatter in the plot increases and the pattern becomes less clear. This indicates that the association is less strong.



Weak

In weak association the amount of scatter increases further and the pattern becomes even less clear. Linear form is less evident.



STRENGTH OF AN ASSOCIATION

Strong – small amount of scatter in the plot.
Moderate – modest amount of scatter in the plot.
Weak – large amount of scatter in the plot

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Example 2: Describing bivariate datasets 6B

The table below shows the height (cm) and weight (kg) of nine people.

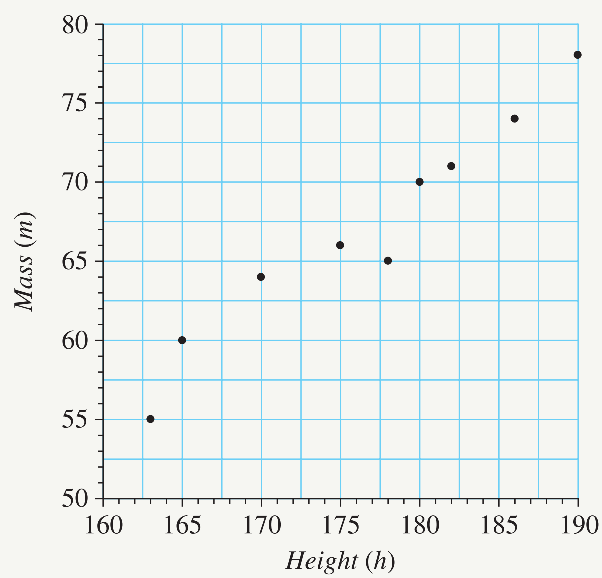
Height (h)	163	165	170	175	178	180	182	186	190
Mass (m)	55	60	64	66	65	70	71	74	78

a Construct a scatterplot using the above table.
b Describe the form of the association.
c Describe the direction of the association.
d Describe the strength of the association.
e Predict the mass of a person who is 173 cm tall using the scatterplot.
f Predict the height of a person who has a mass of 75 kg using the scatterplot.

SOLUTION:

1 Draw a number plane with h as the horizontal axis and m as the vertical axis.
2 Determine a scale for the horizontal axis. Let each unit represent 1 cm.
3 Determine a scale for the vertical axis. Let each unit represent 1 kg.
4 Write titles for the horizontal and vertical axes.
5 Plot the points (163, 55), (165, 60), (170, 64), (175, 66), (178, 65), (180, 70), (182, 71), (186, 74) and (190, 78).

a



6 Look for a pattern. The points approximate a straight line.

b Linear form

7 Gradient of the line is positive. The dots tend to go up as you move from left to right.

c Positive

8 There is a small amount of scatter in the scatterplot.

d Strong

9 Draw an imaginary vertical line from 173 cm.
10 Try to maintain the linear relationship and guess the weight.

e The person weighs approximately 65 kg.

11 Draw an imaginary horizontal line from 75 kg.
12 Try to maintain the linear relationship and guess the height.

f The person's height is approximately 187 cm.

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Independent and dependent variables

Bivariate data has two variables that are often identified as the independent and dependent variables. The independent variable is the input. It is not affected by the other variable and is represented on the horizontal axis of the scatterplot. The dependent variable is the output and is 'dependent' on the independent variable. It is represented on the vertical axis of a scatterplot.

Example 3: Identifying independent and dependent variables 6B

The table below shows the time taken (hours) relative to the distance travelled (km).

Distance (*d*)	0	10	20	30	40	50
Time (*t*)	0	0.25	0.38	0.59	0.82	1.00

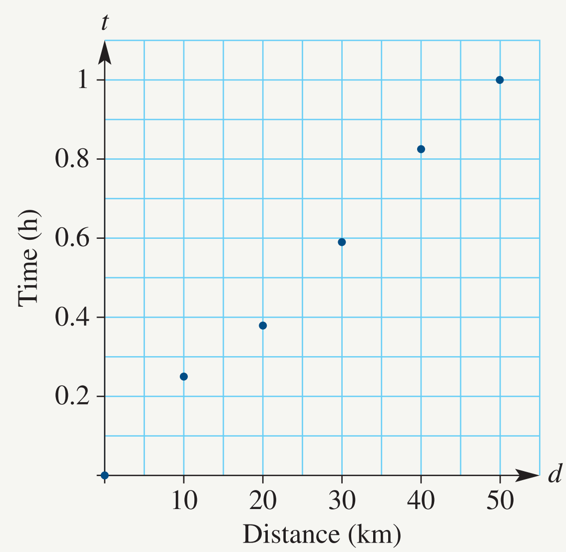
a Draw a scatterplot using the above table.

b Which are the independent and dependent variables?

SOLUTION:

1. Draw a number plane with d as the horizontal axis and t as the vertical axis.
2. Determine a scale for the horizontal axis. Let each unit represent 10 km.
3. Determine a scale for the vertical axis. Let each unit represent 0.2 hours.
4. Write titles for the horizontal and vertical axes.
5. Plot the points (0, 0) (10, 0.25) (20, 0.38) (30, 0.59) (40, 0.82) (50, 1).

a



6. The independent variable is the input and represented on the horizontal axis of the scatterplot.

b Independent variable is distance (d).

7. The dependent variable is the output and represented on the vertical axis of the scatterplot.

Dependent variable is time (t).

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Exercise 6B

Example 2

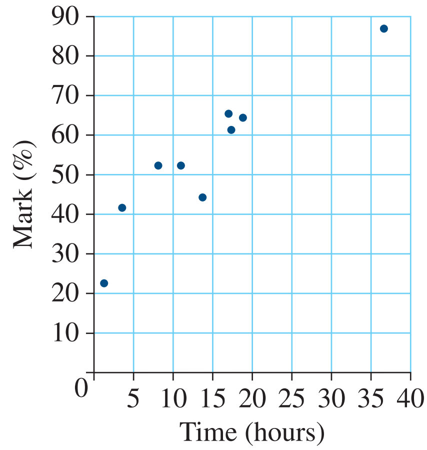
1 Describe the association in the following scatterplots as:

i linear or non-linear

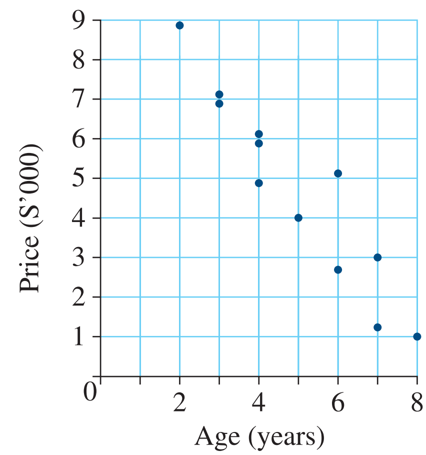
ii positive or negative

iii strong, moderate or weak.

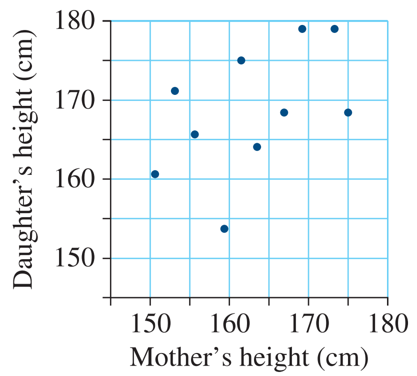
a



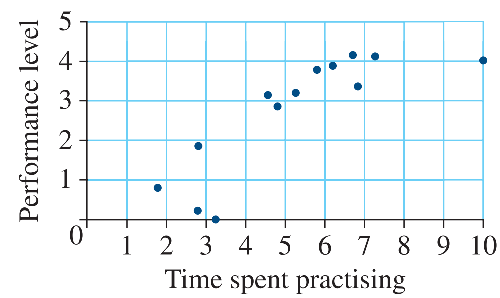
b



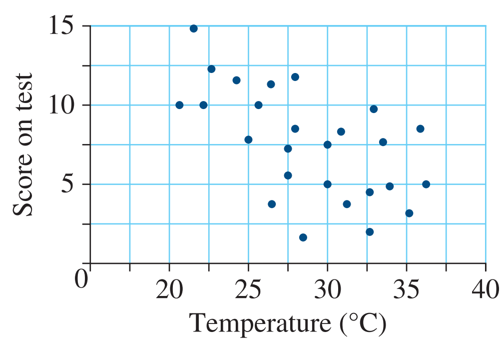
c



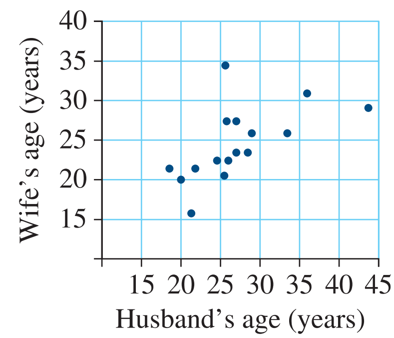
d



e



f



2 For each of the following pairs of variables, indicate whether you expect an association to exist and, if so, whether you would expect the association to be positive or negative.

a Independent variable: Distance travelled Dependent variable: Time taken

b Independent variable: Amount of daily exercise Dependent variable: Fitness level

c Independent variable: Foot length of an adult Dependent variable: Intelligence

d Independent variable: Number of pages Dependent variable: Book price

e Independent variable: Temperature above 30°C Dependent variable: Comfort level

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Example 3 **3** Chocolates are sold for $12 per kg. The table below shows weight against cost.

Weight (w)	1	2	3	4	5
Cost (c)	12	24	36	48	60

a Which is the independent variable?
b Which is the dependent variable?
c Draw a scatterplot of weight against cost.
d Is the form linear or non-linear?
e Is the direction positive or negative?
f Is the strength strong, moderate or weak?

4 The table below shows the drug dosage against reaction time.

Drug dosage (d)	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	6.0
Reaction time (t)	66	48	35	19	18	17	11	15	10	10	11

a Which is the independent variable?
b Which is the dependent variable?
c Draw a scatterplot of time against cost.
d Is the form linear or non-linear?
e Is the direction positive or negative?
f Is the strength strong, moderate or weak?

5 Kayla conducted a science experiment and presented the results in a table.

Mass (m)	3	6	9	12	15
Time (t)	8.2	6.7	5.2	3.7	2.2

a Which is the independent variable?
b Which is the dependent variable?
c Draw a scatterplot of mass against time.
d Is the form linear or non-linear?
e Is the direction positive or negative?
f Is the strength strong, moderate or weak?

6 The table below shows leg length compared with height.

Leg length (in cm)	83	83	85	87	89	89	92	93	94
Height (in cm)	166	167	170	174	179	178	183	185	188

a Which is the independent variable?
b Which is the dependent variable?
c Draw a scatterplot of leg length against height.
d Is the form linear or non-linear?
e Is the direction positive or negative?
f Is the strength strong, moderate or weak?

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6C Line of best fit

If the points on the scatterplot tend to lie on a straight line, then we can fit a line on the scatterplot. The process of fitting a straight line to the data is known as linear regression. Linear regression is completed in many different ways. The simplest method is to draw a line that seems to be a balance of the points above and below the line. The aim of a linear regression is to model the association between two numerical variables by using the equation of a straight line. This equation of the straight line is found using the gradient–intercept formula: $y = mx + c$ where m is the gradient and c is the y-intercept.

LINE OF BEST FIT

A line of best fit is a straight line that approximates a linear association between points.
The equation of the line of best fit is found using the gradient–intercept formula: $y = mx + c$.

The line of best fit is used to make a prediction about one of the variables. When it is used to make a prediction within the data range it is called interpolation. Extrapolation is a prediction outside the data range and must be used carefully, as the line of best fit may not apply, for example, predicting an adult's height based on their increasing height as a child. Interpolation and extrapolation will be examined in detail in section 6D.

Example 4: Drawing a line of best fit by eye 6C

The table below shows the height (cm) and weight (kg) of nine people.

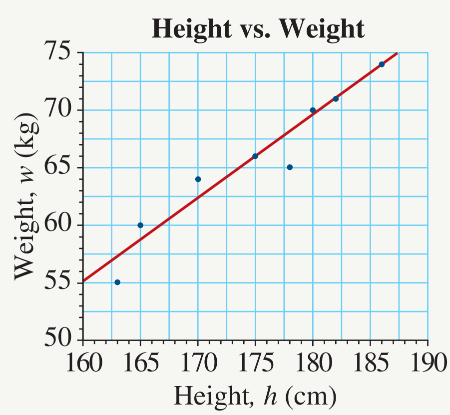
Height (h)	163	165	170	175	178	180	182	186
Weight (w)	55	60	64	66	65	70	71	74

a Construct a scatterplot and draw a line of best fit.
b Describe the association between height and weight.

SOLUTION:

1 Draw a number plane with h as the horizontal axis and w as the vertical axis.
2 Plot the points (163, 55), (165, 60), (170, 64), (175, 66), (178, 65), (180, 70), (182, 71) and (186, 74).
3 Draw a straight line as close as possible to every point. There should be some points above, below and on the line.

a



4 The line of best fit has a positive gradient and is close to the points.

b Strong positive linear association.

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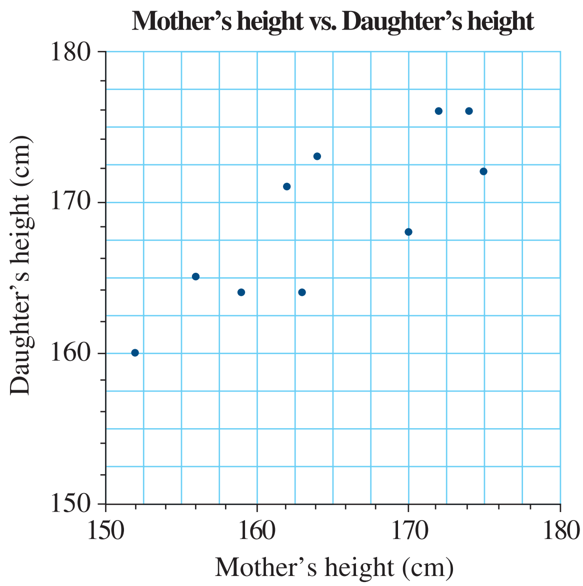
Exercise 6C

Example 4 **1** Draw a scatterplot and a line of best fit by eye for the following points.

- **a** (0, 0) (10, 30) (20, 67) (30, 93) (40, 126) (50, 158) (60, 178)
- **b** (5, 20) (10, 42) (15, 73) (20, 94) (25, 122) (30, 150) (35, 165)
- **c** (0, 6) (2, 24) (3, 39) (4, 44) (5, 59) (6, 64) (7, 79) (8, 84)
- **d** (10, 55) (12, 45) (14, 20) (16, 40) (18, 30) (20, 28) (22, 25)

2 The scatterplot shows the mother's height (in cm) and her daughter's height (in cm).

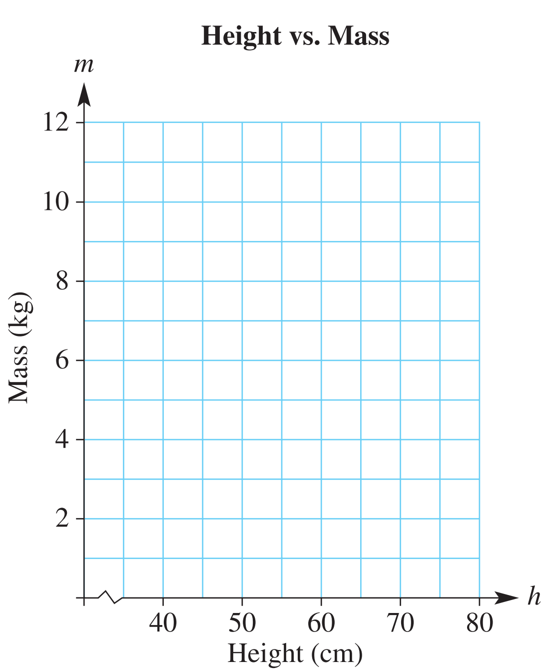
- **a** Copy the scatterplot and draw a line of best fit by eye.
- **b** Describe the strength of the relationship as strong, moderate or weak.
- **c** Estimate the daughter's height if the mother's height is 170 cm.
- **d** Estimate the mother's height if the daughter's height is 162 cm.



3 The height and masses of young children are measured and recorded below.

Height h(cm)	40	45	50	55	60	65	70	75	80	85
Mass m(kg)	1.5	3.1	3.6	5.5	6.0	6.9	7.6	8.6	10.0	11.2

- **a** Complete the scatterplot opposite and draw a line of best fit by eye.
- **b** What is the expected mass of a child given their height is 73 cm?
- **c** What is the expected height of a child given their mass is 4.8 kg?
- **d** What is the expected mass of a child given their height is 48 cm?
- **e** What is the expected height of a child given their mass is 9.0 kg?



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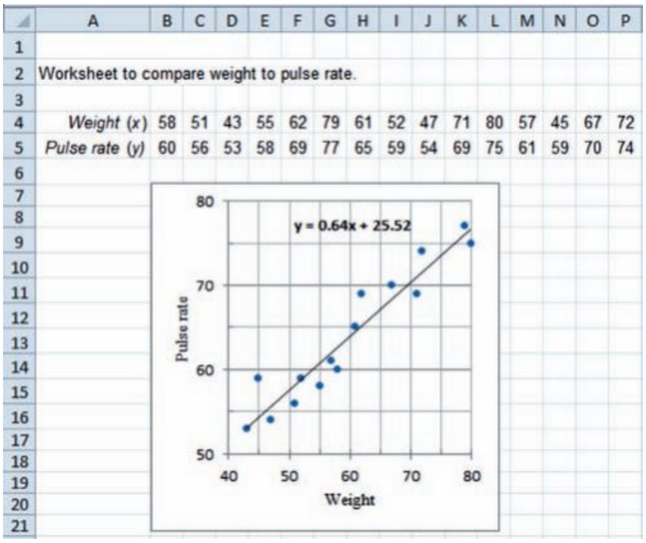
4 Create the spreadsheet and scatterplot below.

6CQ4



Worksheet to compare weight to pulse rate.

Weight (x)	58	51	43	55	62	79	61	52	47	71	80	57	45	67	72
Pulse rate (y)	60	56	53	58	69	77	65	59	54	69	75	61	59	70	74

a Use the trendline tool to insert the least-squares line of best fit onto the scatterplot.
b What is the pulse rate when the weight is 65 kg?
c What is the weight when the pulse rate is 60 beats per minute?

5 The table below shows the amount of energy (in megajoules, MJ) used per day for 12 people of various mass (in kg).

Energy (MJ)	1.5	1.6	1.7	1.8	1.9	2.0	2.0	2.1	2.2	2.3	2.4	2.5
Mass (kg)	50	54	70	71	78	88	98	101	110	115	119	125

a Draw a scatterplot using energy for the horizontal axis and mass for the vertical axis.
b Draw a line of best fit by eye.
c What is the mass when 1.55 MJ of energy is used?
d What is the mass when 2.15 MJ of energy is used?
e What is the energy used when the mass is 100 kg?
f What is the energy used when the mass is 80 kg?

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6D Interpolation and extrapolation

The equation of the line of best fit or regression line can provide important information and be used to make predictions. The gradient (m) indicates the change in dependent variable as the independent variable increases by 1 unit. The vertical intercept (b) indicates the value of the dependent variable when the independent variable is zero. In addition to this information, the equation of best fit is used for interpolation and extrapolation.

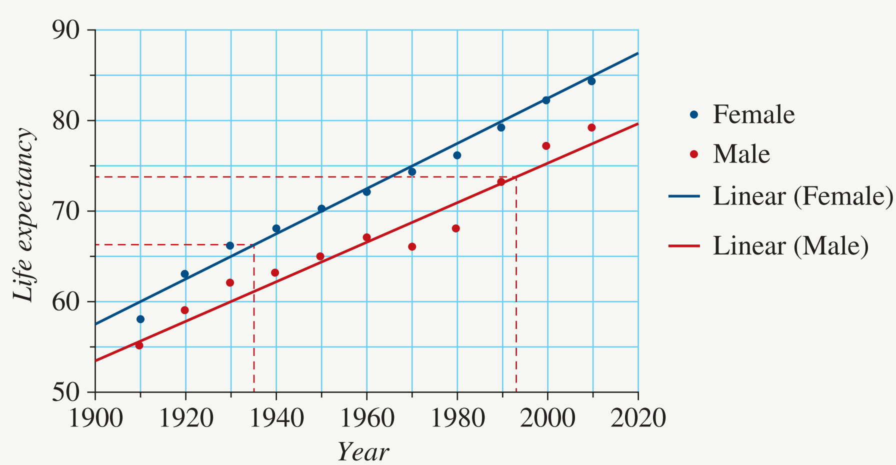
Interpolation

Interpolation is the use of the linear regression line to predict values within the range of the dataset. If the data has a strong linear association then we can be confident our predictions are accurate. However, if the data has a weak linear association, we are less confident with our predictions.

Example 5: Making predictions using interpolation 6D

Life expectancy at birth for females and males is shown below.

Year	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000	2010
Female	58	63	66	68	70	72	74	76	79	82	84
Male	55	59	62	63	65	67	66	68	73	77	79



a What was the life expectancy in 1935 for females?

b What was the life expectancy in 1995 for males?

SOLUTION:

1 Draw a vertical line from 1935 until it intersects the blue line. At this point draw a horizontal line until it reaches the vertical axis. Read the value.

a Life expectancy for females in 1935 is approximately 67 years.

2 Draw a vertical line from 1995 until it intersects the red line. At this point draw a horizontal line until it reaches the vertical axis. Read the value.

b Life expectancy for males in 1995 is approximately 74 years.

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Extrapolation

Extrapolation is the use of the linear regression line to predict values outside the range of the dataset. Predicted values are either smaller or larger than the dataset. The accuracy of predictions using extrapolation depends on the strength of the linear association similar to interpolation. It may not be reasonable to extrapolate too far as this example shows.

Example 6: Making predications using extrapolation 6D

The table below shows the age of a student and their height in centimetres.

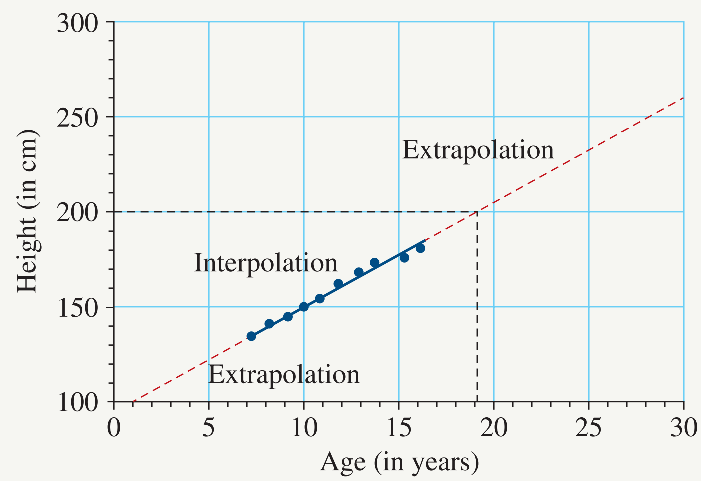
Age (in years)	7	8	9	10	11	12	13	14	15	16
Height (in cm)	133	139	144	149	156	163	170	174	177	181

a Construct a scatterplot from the table using age from 0 to 30 and height from 100 to 300.
b Draw the line of best fit and describe the association between age and height.
c Predict the height of the student when they are aged 19 years.
d What are the limitations of this linear model?

SOLUTION:

1 Draw a number plane with age as the horizontal axis and height as the vertical axis.
2 Determine a scale for the horizontal axis. Let each unit represent 1 year.
3 Determine a scale for the vertical axis. Let each unit represent 10 cm.
4 Write a title for the horizontal and vertical axes.
5 Plot the points (7, 133) (8, 139) (9, 144),…

a



6 There is a small amount of scatter in the scatterplot.

b Strong positive linear association.

7 Read the height from the scatterplot when age is 19.

c Height of the student is 200 cm when they are 19 years old.

8 Extrapolation too far from the dataset needs to be done carefully.

d Adult height does not grow at the same rate as a child. Using the model to extrapolate is flawed, e.g., the prediction is the height will be 260 cm at age 30.

INTERPOLATION	EXTRAPOLATION
Predicting values within the dataset range	Predicting values outside the dataset range

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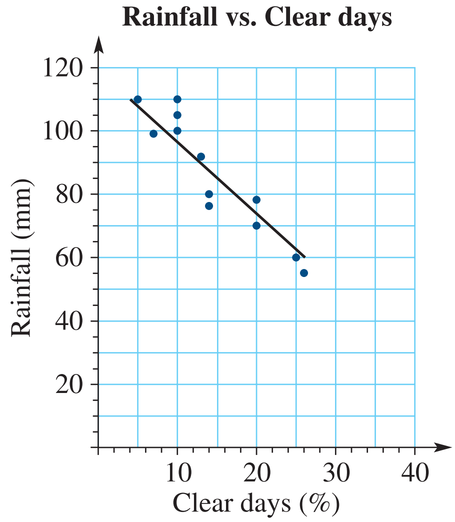
Exercise 6D

Example 5, 6

1 The scatterplot opposite shows the rainfall (in mm) and the percentage of clear days for each month in 2018.



a How many months had 10% of clear days?

b What was the percentage of clear days when the rainfall was 70 mm?

c Predict the rainfall in the month given the following percentage of clear days:

i 4%

ii 22%

iii 26%.

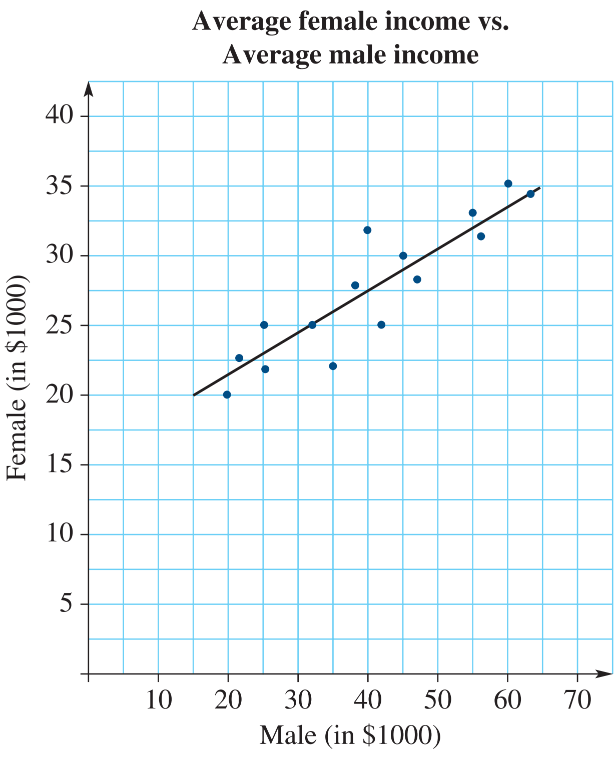
d Predict the percentage of clear days in the month given the following rainfall:

i 80 mm

ii 90 mm

iii 100 mm.

2 The scatterplot shows the average annual female income plotted against average annual male income for 15 countries.



a What was the female income for a country whose average annual male income was $45 000?

b How many countries had an average annual female income of $25 000?

c Predict the female income given the following male income:

i $20 000

ii $40 000

iii $60 000.

d Predict the male income given the following female income:

i $25 000

ii $30 000

iii $35 000.

3 For time ranging from 5 to 25 seconds, the equation relating the number of errors to time is:

$$errors = -0.53 \times time + 15$$

Use this equation to predict the number of errors (to nearest whole number) with the following times. Are you interpolating or extrapolating?

a 10 seconds **b** 20 seconds **c** 30 seconds

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4 For minimum temperatures from 5° C to 20° C, the equation relating the maximum and minimum temperature (in °C) at a weather station is shown below:

$$maximum = 0.67 \times minimum + 13$$

Use this equation to predict the maximum temperature given the following minimum temperatures. Are you interpolating or extrapolating?

a 10°C
b 20°C
c 30°C

5 The equation relating life expectancy at birth from 1900 to the current year for a particular country is given below:

$$life\ expectancy = 0.21 \times year - 353.78$$

Use this equation to predict a life expectancy in the following years. Are you interpolating or extrapolating?

a 1900
b 1950
c 1870
d 2000
e 2030
f 1970

6 When a person's height is between 160 cm and 190 cm, the equation relating weight (in kg) to the height (in cm) is shown below:

$$weight = 0.75 \times height - 65.63$$

Use this equation to predict a person's weight with the following heights. Are you interpolating or extrapolating?

a 150 cm **b** 175 cm **c** 200 cm

7 When a worker's average pay rate is between \$5 and \$25, the equation relating a country's development index (%) to the average pay rate (in dollars per hour) is shown below:

$$development\ index = 0.272 \times pay\ rate + 81.3$$

Use this equation to predict a country's development index with the following average pay rates. Are you interpolating or extrapolating?

a \$40 per hour **b** \$20 per hour **c** \$10 per hour

8 When the area in a large city is between 1 km^2 and 8 km^2, the equation relating a population to area (in square kilometres) is shown below:

$$population = 2680 \times area + 5330$$

Use this equation to predict the population with the following areas. Are you interpolating or extrapolating?

a 2.5 km^2 **b** 5.0 km^2 **c** 7.5 km^2

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6E Statistical investigation

Statistical investigation is the process of gathering statistics. The information gained from a statistical investigation is a vital part of our society. A statistical investigation involves four steps.

Four steps in a statistical investigation

1. **Collect the data**
   Collecting data involves deciding what to collect, locating it and collecting it. The gathering of statistical data may take the form of a:
   - census – data is collected from the whole population
   - survey – data is collected from a smaller group of the population.

   It is important that procedures are in place to ensure the collection of data is accurate, up-to-date, relevant and secure. If the data collected comes from unreliable sources or is inaccurate, the information gained from it will be incorrect. When taking a sample, the data gathered must be representative of the entire population otherwise the information collected may be biased towards a particular outcome.
2. **Organise the data**
   Organising data is the process of arranging, representing and formatting data. It is carried out after the data is collected. The organisation of the data depends on the purpose of the statistical investigation. For example, to store and search a large amount of data, the data needs to be categorised. Organising gives structure to the data.
3. **Summarise and display the data**
   Displaying data is the presentation of the data and information. Information must be well organised, readable, attractively presented and easy to understand. Information is often displayed using graphs such as scatterplots, dot plots, histograms, line graphs, stem-and-leaf plots and box plots. Data is summarised using statistics such as the mean, median, mode and standard deviation.

4. **Analyse the data**
   Analysing data is the process of interpreting data and transforming it into information. It involves examining the data and giving meaning to it. When analysing bivariate data, the form, direction and strength of the association is determined. Scatterplots and lines of best fit are commonly used to analyse the data. They make it easy to interpret data by making instant comparisons and revealing trends. Predictions and conclusions are completed by interpolating and extrapolating the data.

STATISTICAL INVESTIGATION

A statistical investigation involves four steps: collecting data, organising data, summarising and displaying data and analysing data.

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Example 7: Case study of a statistical investigation 6E

James has been asked to complete a statistical investigation on whether the blood glucose level (in mg/100 mL) of an adult can be predicted from their weight (in kg).

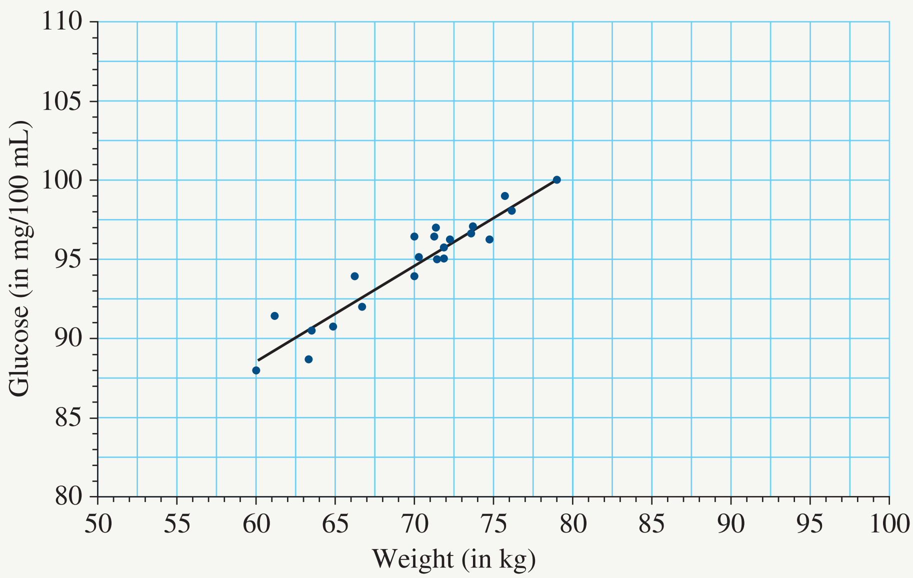
James performed the following steps.

1 Collecting the data – James accessed the medical data on 20 adults.

2 Organising the data – James categorised the data into blood glucose levels and weight.

3 Summarising and displaying the data – James presented the bivariate data into the table shown opposite and the scatterplot shown below.

Weight	Glucose	Weight	Glucose
60.2	88.1	71.6	94.9
61.3	91.5	72.0	95.7
63.5	88.7	72.0	95.1
63.7	90.6	72.5	96.4
65.0	90.9	73.7	96.6
66.4	94.0	73.8	97.0
66.9	92.1	74.8	96.3
70.1	96.5	75.9	99.1
70.2	93.9	76.3	98.2
70.5	95.2	78.9	99.9



4 Analysing the data –

a James calculated Pearson's coefficient to measure the strength of a linear association.

$r = 0.9507479097 \ldots$

This indicates a strong positive linear association between weight and blood glucose levels.

b James calculated the equation and graphed the least-squares regression line on the scatterplot.

$A = 52.78718161 \ldots \qquad B = 0.5966957534 \ldots$

$y = mx + b$

$= Bx + A$

$= 0.60x + 52.79$

$glucose = 0.60 \times weight + 53$

c James applied the results of his statistical investigation to predict the glucose level of a person who weighs 75 kg.

$glucose = 0.60 \times 75 + 53$

$= 98$

$\therefore$ A person weighing 75 kg has a blood glucose level of 98 mg/100 mL.

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Issues in a statistical investigation

A statistical investigation raises a number of ethical issues such as bias, accuracy, copyright and privacy.

- Data needs to be free from bias. Bias means that the data is unfairly skewed or gives too much weight to a particular result. For example, if a survey about favourite music was only completed by teenagers, and the results were generalised to the entire population, it would have a bias. Several checks should be made to limit the impact of bias.
- The accuracy of the collected data is a vital ingredient of a statistical investigation. It depends on the source of the data and whether the data has been recorded correctly. The accuracy of the data is often difficult to check in a reasonable time. It is often necessary to compare data from a number of different sources and determine which data is accurate.
- Copyright is the right to use, copy or control the work of authors and artists. It is against the law to infringe copyright. You are not allowed to use or copy the work of another person without their permission. If data is collected from the internet, it should be assumed to be protected by copyright.
- Privacy is the ability of an individual to control personal data. Data collected on individuals is not always accurate. Inaccuracies can be caused by mistakes in gathering or entering the data, by mismatch of the data and the person or by information being out-of-date. Most people give information about themselves to selected parts of the outside world. Often people are quite willing to tell A something but would be shocked if B knew. But what prevents A telling B?

ISSUES IN A STATISTICAL INVESTIGATION

A statistical investigation raises a number of ethical issues such as bias, accuracy, copyright and privacy.

Causation

Causation indicates that one event is the result of the occurrence of another event (or variable). This is often referred to as the cause and effect. That is, one event is the cause of another event happening. For example, the bell at the end of the period is an event that causes students to leave for the next period. When completing a statistical investigation it is important to be aware that two events (or variables) may have a high correlation but be unrelated. That is, high correlation does not imply causation. For example, the increase in the use of mobile phones has a strong correlation to the increase in life expectancy. However, the use of mobile phones does not cause the increase in life expectancy.

CAUSATION

Causation indicates that one event is the result of the occurrence of another event (or variable).

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Exercise 6E

1 Copy and complete the following sentences.
- **a** A statistical ________________ involves four steps: collecting data, organising data, summarising and displaying data, and analysing data.
- **b** Census data is collected from the whole ________________.
- **c** When taking a ________________ the data gathered must be representative of the entire population.
- **d** Displaying data is the ________________ of the data and information.
- **e** When analysing ________________ data the form, direction and strength of the association is determined.
- **f** Bias means that the data is unfairly ________________ or gives too much weight to a particular result.

2 True or false?
- **a** A survey is when data is collected from a smaller group of the population.
- **b** Data collected from unreliable sources results in incorrect information.
- **c** Data is often displayed using graphs such as scatterplots, dot plots, histograms, line graphs, stem-and-leaf plots and box plots.
- **d** Analysing data is the process that interprets data, transforms it into information.
- **e** You are allowed to use or copy the work of another person without their permission.
- **f** Data collected on individuals is always accurate.

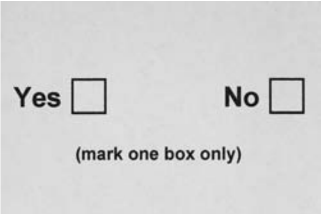
3 The Australian Bureau of Statistics collects data for our society. Collecting data is one step in a statistical investigation. List the four steps involved in a statistical investigation.

4 Explain the difference between a census and a survey.

5 How can you limit the impact of biased data?

6 There is a strong positive correlation between number of car accidents and the number of teachers in cities around the world. Can we conclude from this that teachers are causing car accidents? Give a possible explanation.

7 There is a strong positive correlation between the number of churches in a town and the amount of alcohol consumed by its inhabitants. Does this mean that religion is encouraging people to drink? What common cause might counter this conclusion?



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PPT STUDY GUIDE

Summary

Key ideas and chapter summary

Scatterplot

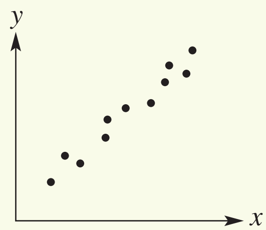
To construct a scatterplot:

1. Draw a number plane.
2. Determine a scale and a title for the horizontal or x-axis.
3. Determine a scale and a title for the vertical or y-axis.
4. Plot each ordered pair of numbers with a dot.

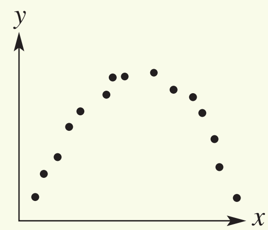
Using a bivariate scatterplot

Form of an association

Linear form – a straight line

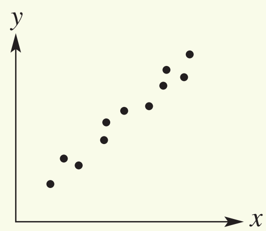


Non-linear form – a curved line

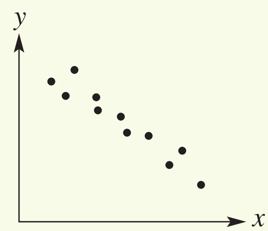


Direction of an association

Positive gradient

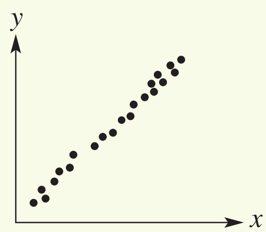


Negative gradient

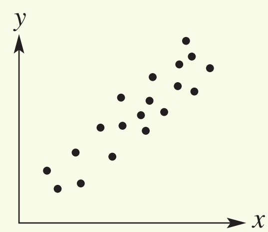


Strength of an association

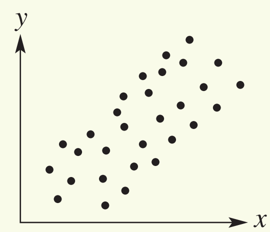
Strong – small amount of scatter



Moderate – modest amount of scatter



Weak – large amount of scatter



Line of best fit by eye

Line of best fit is a straight line that approximates a linear association between points.

Interpolation

Predicting values within the range of the dataset.

Extrapolation

Predicting values outside the range of the dataset.

Statistical investigation

Four steps: collecting data, organising data, summarising and displaying data, and analysing data.

A statistical investigation raises a number of ethical issues such as bias, accuracy, copyright and privacy.

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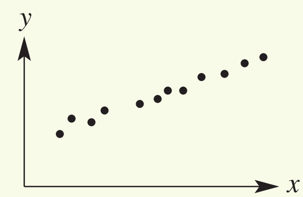
Review

Multiple-choice

1 Blood pressure levels for women increase as they get older. What is the best description for the association between blood pressure levels and a woman's age?

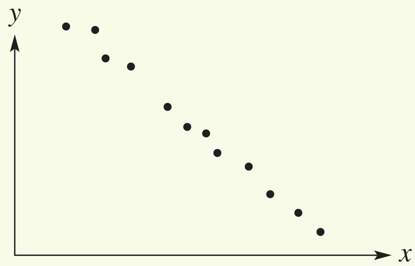
A Positive correlation **B** Zero correlation
C Negative correlation **D** Constant correlation

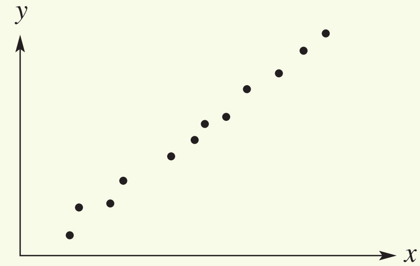
2 What is the correlation between the variables in the scatterplot?

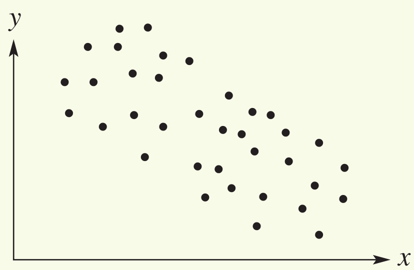


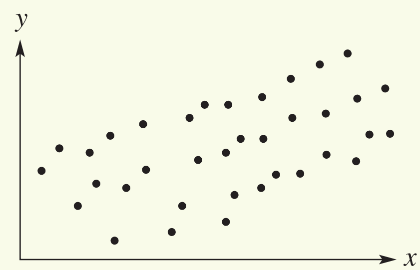
A Strong positive **B** Weak positive
C Strong negative **D** Weak negative

3 Which of the following scatterplots shows weak negative correlation?

A


B


C


D


4 The birth weight and weight at age 21 of eight women are given in the table below.

Birth weight	1.9	2.4	2.6	2.7	2.9	3.2	3.4	3.6
Weight at 21	47.6	53.1	52.2	56.2	57.6	59.9	55.3	56.7

Which of the following conclusions is correct from the above table?

A There is no clear pattern in the bivariate data.
B The form of the association is linear.
C The direction of the association is negative.
D Strength of the association is weak.

5 The linear equation that enables weekly amount spent on entertainment (in dollars) to be predicted from weekly income is given by: $amount = 0.10 \times income + 40$. What is the predicted amount spent on entertainment with a weekly income of \$600?

A \$40 **B** \$46 **C** \$100 **D** \$240

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Review

Short-answer

1 The scatterplot shows the navel height and the body height for 9 students.

a Which has been plotted as the independent variable?

b Which has been plotted as the dependent variable?

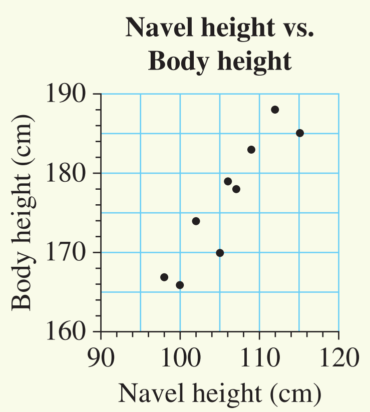
c Is the association between these two variables linear or non-linear?

d Describe the association as strong, moderate or weak.

e What is the body height for the student with a navel height of 112 cm?

f What is the navel height for the student with a body height of 166 cm?

g Use the scatterplot to predict the body height of a student with a navel height of 110 cm.



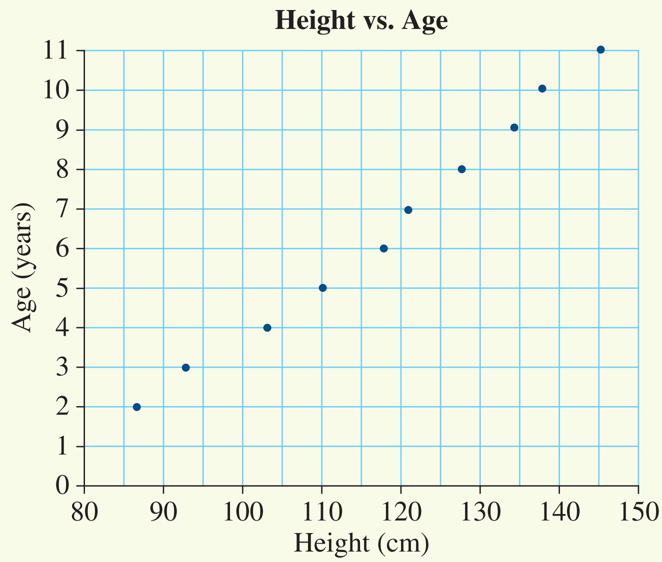
2 The scatterplot shows a student's height (in cm) and their age (in years).

a What is the age for the student when their height is 120 cm?

b What is the height for the student whose age is 11 years?

c State whether the association is positive or negative.

d Describe the strength of the association as strong, moderate or weak.



3 The table below shows the length of the right foot (in cm) and body height (in cm).

Length of right foot	27.5	24	22.6	23.7	26.4	27.1	25.5	26.1
Body height	174.4	156	155.3	160.5	170.7	169.3	163.3	164.9

a Draw a scatterplot using the above table.

b State whether the association is positive or negative.

c Describe the strength of the association as strong, moderate or weak.

4 A strong positive linear association exists between the hours spent studying for an exam and the mark achieved. The equation for this association is *mark* = 4.5 × *study hours* + 2.

a Predict the exam mark if the student studied for 12 hours a week.

b Predict the exam mark if the student studied for 20 hours a week.

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Review

5 The table below shows the age (in months) and the height (in cm) of a young plant.

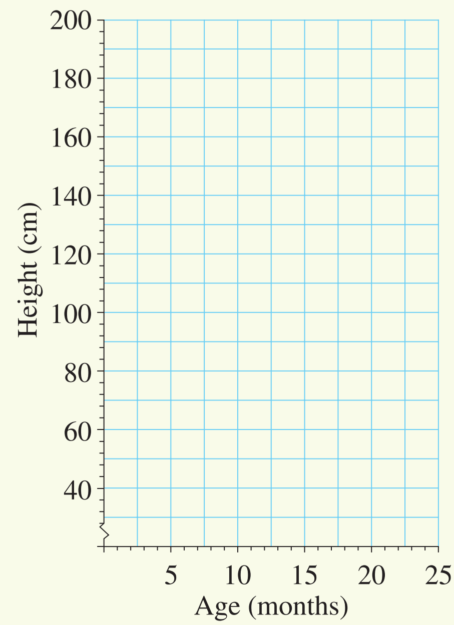
Age (in months)	1	4	5	8	12	14	15	19	22	24
Height (in cm)	48	65	78	87	114	128	131	159	169	188

a Complete the scatterplot and draw a line of best fit by eye.

b What is the predicted height of the plant after 11 months? Answer to the nearest centimetre.

c What is the predicted height of the plant after 6 months? Answer to the nearest centimetre.

d Do these questions involve interpolation or extrapolation?



6 The table shows the birth rate (live births per 1000) and the life expectancy (in years).

Birth rate	30	31	34	38	40	42	43
Life expectancy	66	64	46	54	48	45	42

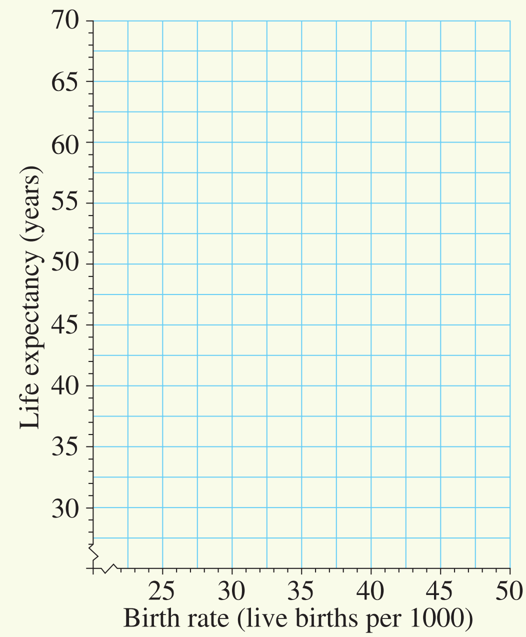
a Complete the scatterplot and draw a line of best fit by eye.

b State whether the association is positive or negative.

c Describe the strength of the association as strong, moderate or weak.

d What is the life expectancy when the birth rate is 35?

e What is the birth rate when the life expectancy is 60?



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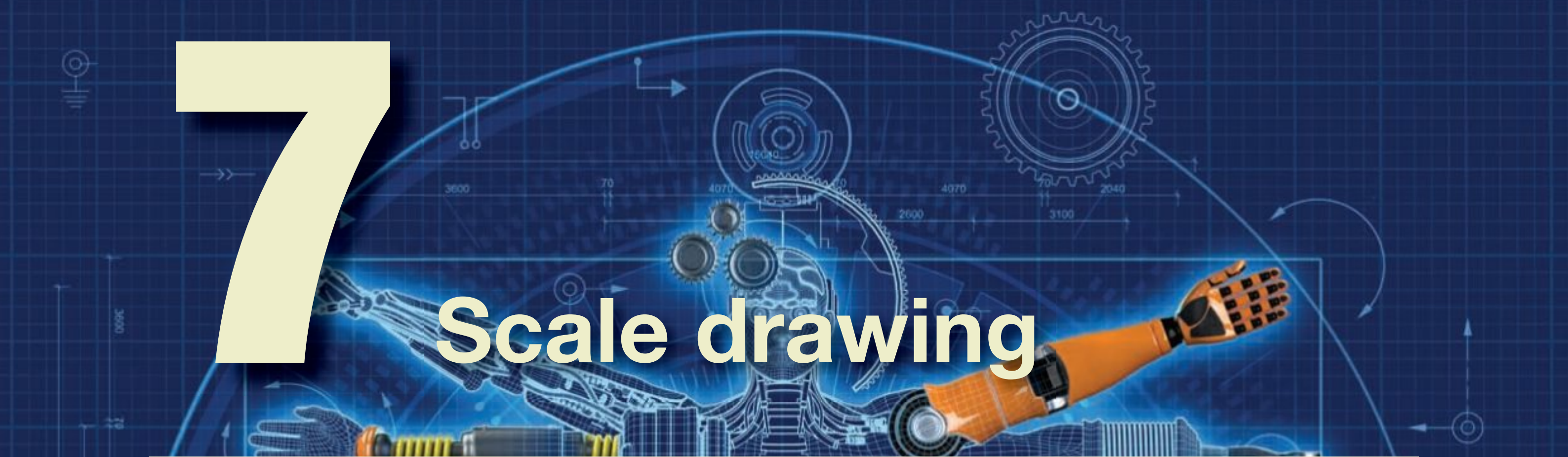

7 Scale drawing

Syllabus topic — M5 Scale drawing

This topic focuses on the use of ratios to solve problems in practical contexts, including the interpretation of scale drawings.

Outcomes

- Express a ratio in simplest terms.
- Find the ratio of two quantities.
- Divide a quantity in a given ratio.
- Solve practical problems involving ratios.
- Use ratio to describe map scales.
- Use the scale factor of two similar figures to solve linear scaling problems.
- Obtain measurements from scale drawings.
- Interpret symbols and abbreviations on building plans and elevation views.
- Estimate and compare quantities using a scale drawing.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Literacy worksheet
- Quick Quiz
- Solutions (enabled by teacher)
- Desmos widgets
- Spreadsheets
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

The Interactive Textbook provides a test of prior knowledge for this chapter, and may direct you to revision from the previous years' work.

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7A Ratios

A ratio is used to compare amounts of the same units in a definite order. For example, the ratio 3 : 4 represents 3 parts to 4 parts or $\frac{3}{4}$ or 0.75 or 75%. A ratio is a fraction and can be simplified in the same way as a fraction. For example, the ratio 15 : 20 can be simplified to 3 : 4 by dividing each number by 5. Equivalent ratios are obtained by multiplying or dividing each amount in the ratio by the same number.

$\overset{\div 3}{15} : \overset{\div 3}{12} = 5 : 4$ $\quad$ $\overset{\times 3}{5} : \overset{\times 3}{4} = 15 : 12$

15 : 12 and 5 : 4 are equivalent ratios.

When simplifying a ratio with fractions, multiply each of the amounts by the lowest common denominator. For example, to simplify $\frac{1}{8} : \frac{3}{4}$ multiply both sides by 8. This results in the equivalent ratio of 1 : 6.

RATIO

A ratio is used to compare amounts of the same units in a definite order.
Equivalent ratios are obtained by multiplying or dividing by the same number.

Example 1: Simplifying a ratio 7A

Write the ratios in simplest form.

a 20 : 4 **b** $3 : \frac{1}{2}$ **c** 1.5 : 3.5

SOLUTION:

1. Divide both sides of the ratio by 4.
2. Evaluate.

a $20 : 4 = 20 \div 4 : 4 \div 4$
$= 5 : 1$

3. Multiply both sides of the ratio by 2.
4. Evaluate.

b $3 : \frac{1}{2} = 3 \times 2 : \frac{1}{2} \times 2$
$= 6 : 1$

5. Multiply both sides of the ratio by10.
6. Divide both sides of the ratio by 5 (highest common factor).
7. Evaluate.

c $1.5 : 3.5 = 1.5 \times 10 : 3.5 \times 10$
$= 15 : 35 = \frac{15}{5} : \frac{35}{5}$
$= 3 : 7$

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Exercise 7A

1 Complete each pair of equivalent ratios.

a $1:3 = 4:\square$ **b** $1:7 = 2:\square$ **c** $2:5 = \square:10$
d $3:7 = \square:21$ **e** $5:10 = 1:\square$ **f** $12:16 = 3:\square$
g $12:18 = \square:3$ **h** $20:50 = \square:25$ **i** $4:7 = 44:\square$

2 Complete each pair of equivalent ratios.

a $1:2:3 = 4:\square:\square$ **b** $4:12:16 = \square:6:\square$ **c** $1:7:9 = \square:\square:63$

Example 1a
3 Write three equivalent ratios for each of the following ratios.

a 1 : 2 **b** 2 : 5 **c** 8 : 6

4 Express each ratio in simplest form.

a 15 : 3 **b** 10 : 40 **c** 24 : 16
d 14 : 30 **e** 8 : 12 **f** 49 : 14
g 81 : 27 **h** 48 : 32 **i** 17 : 51
j 9 : 18 : 9 **k** 5 : 10 : 20 **l** 27 : 9 : 3

5 Express each ratio in simplest form.

a \$24 : \$18 **b** 3 kg to 12 kg **c** 56 t : 16 t
d 40 c to 72 c **e** 3 h to 1 day **f** 2 mm to 1 cm
g 1 km : 250 m **h** 3 m : 50 cm **i** 6 km : 300 m
j 7 cm : 21 mm **k** 8 months : 4 years **l** 2 L : 450 mL

6 There are 14 boys and 10 girls in a class. What is the ratio of:

a boys to girls?
b girls to boys?
c boys to the total number?

7 A clothing store has a discount sale. A dress marked at \$250 is sold for \$200. What is the ratio of the discount to the marked price?

8 Madeleine and Nathan invest \$4500 and \$2500 into a managed fund. What is the ratio of Madeleine's share to Nathan's share?

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9 There were 80 blocks of land available for sale in a new land release. After one month 32 blocks have been sold. What is the ratio of sold blocks to the total number of blocks?

10 A 5 kg bag of potatoes costs $12.80. Find the cost of:

a 1 kg
b 10 kg
c 14 kg
d 6 kg.

11 The cost of 3 pens is $42.60. Find the cost of:

a 1 pen
b 4 pens
c 6 pens
d 10 pens.

12 During a recent dry spell, it rained on only 3 days during the month of November.

a What was the ratio of wet days to total days for the month of November?
b What was the ratio of wet days to dry days for the month of November?

Example 1b

13 Express each ratio in simplest form.

a $\frac{1}{2} : \frac{1}{5}$
b $\frac{2}{3} : \frac{3}{7}$
c $\frac{3}{4} : 1$
d $\frac{1}{2} : \frac{1}{10}$
e $\frac{1}{10} : \frac{1}{5}$
f $\frac{2}{3} : \frac{3}{5}$
g $1 : \frac{3}{4}$
h $\frac{2}{3} : 1$
i $\frac{9}{10} : 1$

14 Express each ratio in simplest form.

a $2x : x$
b $3y : 9y$
c $4a : 2$
d $4a : 8a$
e $3xy : 12x$
f $7m^2 : m$

15 A bag contains 10 green marbles, 14 blue marbles and 6 yellow marbles. What is the ratio of green to yellow to blue marbles in simplest form?

16 In an election, Aiden scored 250 votes, Billie 175 votes and Chelsea 125 votes. Find the ratio of Aiden's votes : Billie's votes : Chelsea's votes.

17 Samantha and Mathilde own a restaurant. Samantha gets $\frac{3}{5}$ of the profits and Mathilde receives the remainder.

a What is the ratio of profits?
b Last week the profit was $2250. How much does Mathilde receive?
c This week the profit is $2900. How much does Samantha receive?

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7B Dividing a quantity in a given ratio

Ratio problems may be solved by dividing a quantity in a given ratio. This method divides each amount in the ratio by the total number of parts.

DIVIDING A QUANTITY IN A GIVEN RATIO

1 Calculate the total number of parts by adding each amount in the ratio.
2 Divide the quantity by the total number of parts to determine the value of one part.
3 Multiply each amount of the ratio by the result in step 2.
4 Check by adding the answers for each part. The result should be the original quantity.

Example 2: Dividing a quantity in a given ratio 7B

Mikhail and Ilya were given \$450 to share in the ratio 4:5. How much did each get?

SOLUTION:

1 Calculate the total number of parts by adding each amount in the ratio (4 parts to 5 parts).

$$\text{Total parts} = 4 + 5 = 9$$
$$9 \text{ parts} = \$450$$

2 Divide the quantity (\$450) by the total number of parts (9 parts) to determine the value of one part.

$$1 \text{ part} = \frac{\$450}{9} = \$50$$

3 Multiply each amount of the ratio by the result in step 2 or \$50.

$$4 \text{ parts} = 4 \times \$50 = \$200$$
$$5 \text{ parts} = 5 \times \$50 = \$250$$

4 Check by adding the answers for each part. The result should be the original quantity or \$450.

$$(\$200 + \$250 = \$450)$$

5 Write the answer in words.

Mikhail got \$200, Ilya got \$250.

Example 3 Dividing a quantity in a given ratio 7B

A man left \$6000 to be divided among his three children, Xia, Yui and Zi, in the ratio 5:8:7, in that order. How much did each get?

SOLUTION:

1 Calculate the total number of parts in the ratio by adding 5 parts to 8 parts to 7 parts.

$$\text{Total parts} = 5 + 8 + 7 = 20$$
$$20 \text{ parts} = \$6000$$

2 Divide the quantity (\$6000) by the total number of parts (20 parts) to determine the value of one part.

$$1 \text{ part} = \frac{\$6000}{20} = \$300$$

3 Multiply each amount of the ratio by the result in step 2 or \$300.

$$5 \text{ parts} = 5 \times \$300 = \$1500$$
$$8 \text{ parts} = 8 \times \$300 = \$2400$$
$$7 \text{ parts} = 7 \times \$300 = \$2100$$

4 Write the answer in words.

Xia got \$1500, Yui got \$2400, Zi got \$2100.

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Exercise 7B

Example 2

1 Find the total number of parts in the following ratios.

a 2 : 9 **b** 1 : 5 **c** 11 : 3 **d** 2 : 3 : 4

2 The ratio of girls to boys in a class is 2 : 7.

a What fraction of the class is girls?

b What fraction of the class is boys?

3 Calculate how much each person receives if $100 is shared in the following ratios.

a 7 : 3 **b** 2 : 3 **c** 11 : 9 **d** 7 : 8 : 10

4 Divide 240 into the following ratios.

a 2 : 1 **b** 3 : 2 **c** 1 : 5 **d** 7 : 5

5 Share each amount in the ratio given.

a $20 in the ratio 4 : 1

b $20 in the ratio 7 : 3

c $15 in the ratio 1 : 2

d 77 drinks in the ratio 3 : 4

e 100 lollies in the ratio 7 : 13

f 45 kg in the ratio 4 : 5

g 160 books in the ratio 5 : 3

h 360 pencils in the ratio 2 : 7

i 50 g in the ratio 1 : 3

j 60 km in the ratio 8 : 7

6 A bag of 500 grams of chocolates is divided into the ratio 7 : 3. What is the mass of the smaller amount?

7 At a concert there were 7 girls for every 5 boys. How many girls were in the audience of 8616?

8 Divide:

a $200 in the ratio 1 : 2 : 2

b $400 in the ratio 1 : 3 : 4

c 12 kg in the ratio 1 : 2 : 3

d 88 kg in the ratio 2 : 1 : 5

e 440 kg in the ratio 12 : 13 : 15

f $63 000 in the ratio 1 : 2 : 4.

9 Share $600 in the ratio:

a 1 : 9

b 2 : 1 : 3

c 2 : 5 : 5

d 13 : 8 : 9.

Example 3

10 Molly, Patrick and Andrew invest in a business in the ratio 6 : 5 : 1. The total amount invested is $240 000. How much was invested by the following people?

a Molly **b** Patrick **c** Andrew

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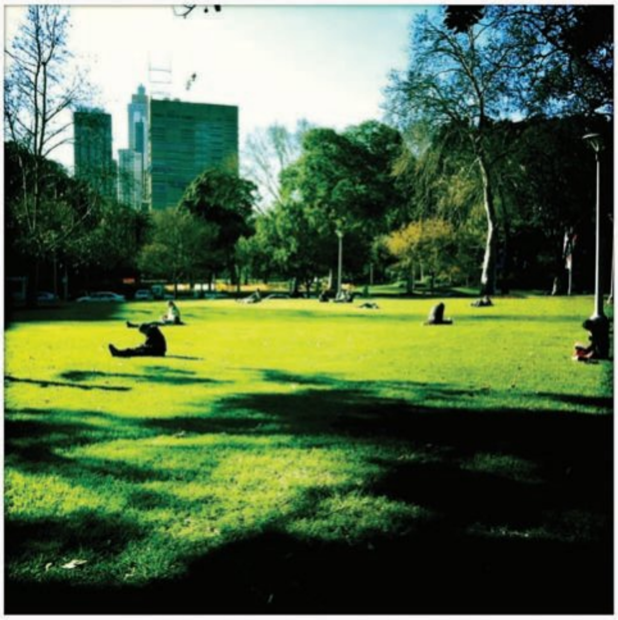
11 The ratio of residential area to parks in a local community is 17 : 3. The total area of the local community is 40 km^2. What is the area of parks?



12 Hayley is 15 years old and her brother is 5 years younger. If \$200 is shared between them in the ratio of their ages, how much will Hayley receive?

13 In a country town, a census showed that there were 5 adults to every 7 children. If the population of the town was 7200, how many children lived there?

14 A punch is made from pineapple juice, lemonade and orange juice in the ratio 5 : 3 : 2.
a How much lemonade is needed if one litre of pineapple juice is used?
b How much pineapple juice is required to make 15 litres of punch?

15 Angus, Ruby and Lily share an inheritance of \$500 000 in the ratio of 7 : 5 : 4. How much will be received by the following people?
a Angus
b Ruby
c Lily

16 In a boiled fruit cake recipe the ratio of mixed fruit to flour to sugar is 5 : 3 : 2. A 250 g packet of mixed fruit is used to make the cake. How much sugar and flour are required?

17 A delivery load of 8.5 tonnes is to be divided between two stores in the ratio 11 : 6. How much will each store receive?

18 The load on a bridge is applied in three positions, *A*, *B* and *C*, in the ratio 5 : 7 : 5. If the total load on the bridge is 782 tonnes, what is the load taken at each point?

19 A jam is made by adding 5 parts fruit to 4 parts of sugar. How much fruit should be added to $2\frac{1}{2}$ kilograms of sugar in making the jam?

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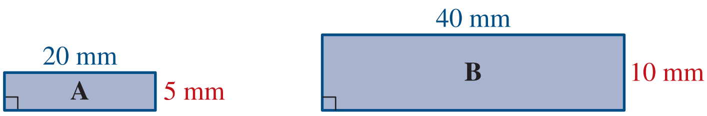
7C Similarity and scale factors

The pictures of the three pieces of cake are similar. Similar figures are exactly the same shape but they are different sizes.



When we enlarge or reduce a shape by a scale factor, the original and the image are similar. Similar shapes have:

- corresponding angles of equal size
- corresponding sides of different size, but in the same ratio or proportion.



For example, the above rectangles are similar. All the angles are 90°. The corresponding sides are in the same ratio $\left(\frac{10}{5} = \frac{40}{20} = 2\right)$. The measurements in rectangle B are twice the measurements in rectangle A. Rectangle B has been enlarged by a scale factor of 2.

SIMILAR FIGURES

- Similar figures are exactly the same shape but are a different size.
- Corresponding (or matching) angles of similar figures are equal.
- Corresponding (or matching) sides of similar figures are in the same ratio.
- Scale factor is the amount the first shape is enlarged or reduced to get the second shape.

We can also compare the ratio of the area of the rectangles.

$$\text{Area rectangle A} = 100\,\text{mm}^2$$
$$\text{Area rectangle B} = 400\,\text{mm}^2$$
$$\text{Ratio of areas} = \frac{400}{100} = 4$$

The area of rectangle A has been enlarged by a scale factor of 4.

We notice that, as the length dimensions are enlarged by a scale factor of 2, the area is enlarged by a scale factor of $2^2 = 4$.

USING SCALE FACTOR FOR AREA

When all the dimensions are multiplied by a scale factor of k, the area is multiplied by a scale factor of k^2.

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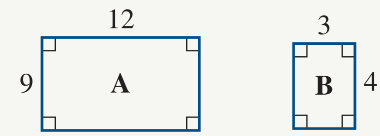
Example 4: Calculating the scale factor 7C

What is the scale factor for these two similar rectangles?

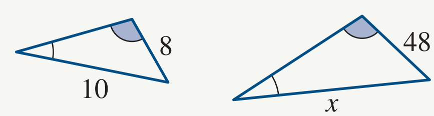


SOLUTION:

1. Look carefully at the similar figures. — Rectangle B is smaller than rectangle A and is rotated.
2. Match the corresponding sides. — 9 matches with 3 and 12 matches with 4.
3. Write the matching sides as a fraction (measurement in rectangle B divided by the matching measurement in rectangle A). — $\text{Scale factor} = \frac{3}{9} \text{ or } \frac{4}{12}$
4. Simplify — $= \frac{1}{3}$ (or 1 : 3)

Example 5: Using a scale factor for length 7C

What is the length of the unknown side in the pair of similar triangles?

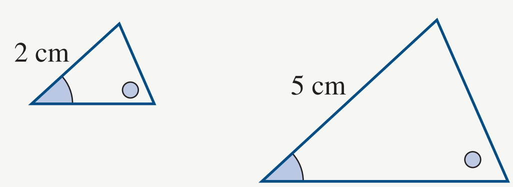


SOLUTION:

1. Match the corresponding sides. — 8 matches with 48
2. Write the matching sides as a fraction (second shape to the first shape). This fraction is the scale factor. — $\text{Scale factor} = \frac{48}{8}$
3. Simplify — $= 6$ (or 6 : 1)
4. Match the corresponding side for x. — $x = 10 \times 6 = 60$ — x matchs side marked with a 10.
5. Calculate x by multiplying 10 by 6 (scale factor is 6).

Example 6: Using a scale factor for area 7C

The two given triangles are similar.
The area of the small triangle is $3\,\text{cm}^2$.
What is the area of the larger triangle?



SOLUTION:

1. Match the corresponding sides. — 2 matches with 5
2. Write the matching sides as a fraction (second shape to the first shape). This fraction is the scale factor. — $\text{Scale factor} = \frac{5}{2}$
3. Calculate the area by multiplying 3 cm^2 by the square of the scale factor. — $\text{Area} = 3 \times \left(\frac{5}{2}\right)^2 = 18.75\,\text{cm}^2$

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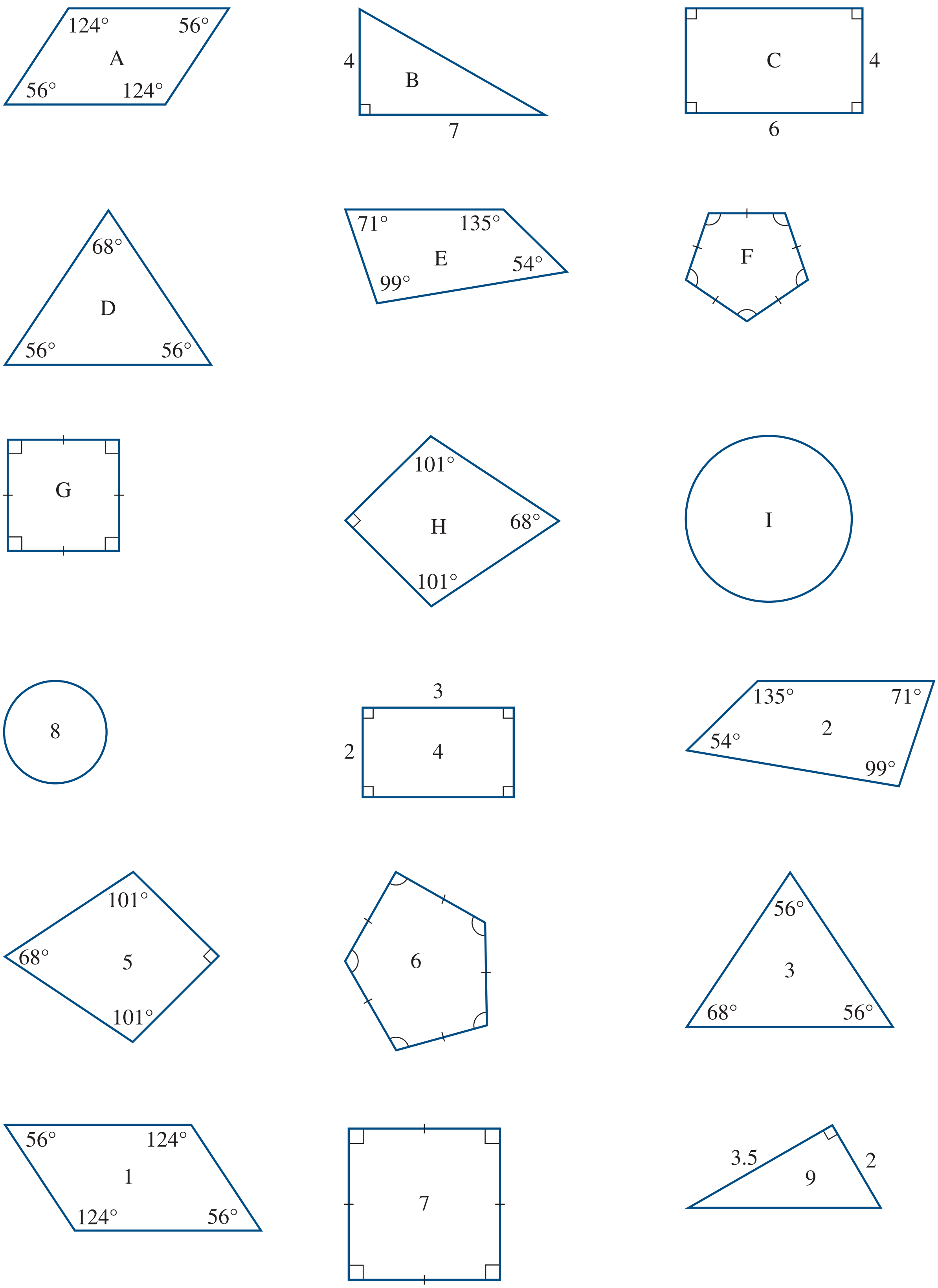
Exercise 7C

1 Match the each shape A to I with its similar shape 1 to 8.



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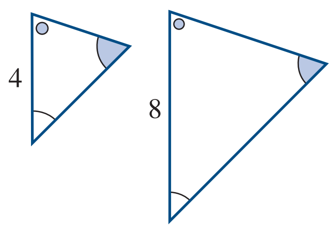
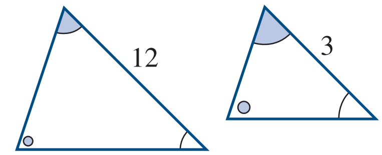
Example 4 **2** What is the scale factor for the following pairs of similar figures?

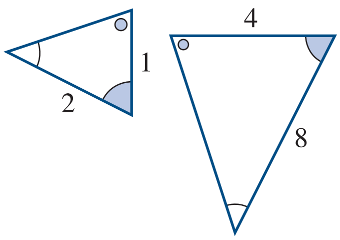
a



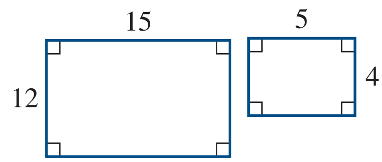
b



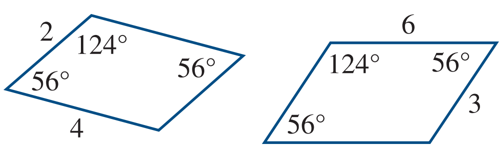
c



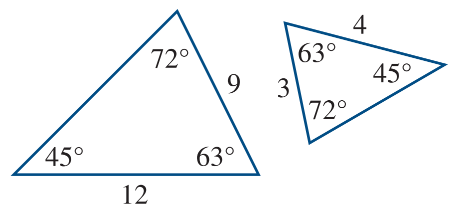
d



e

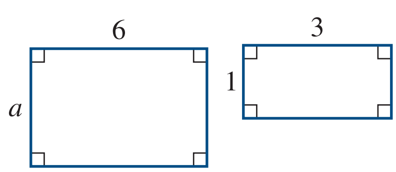


f

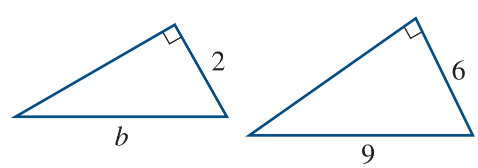


Example 5 **3** Use the scale factor to find the length of the unknown side in the following pairs of similar figures.

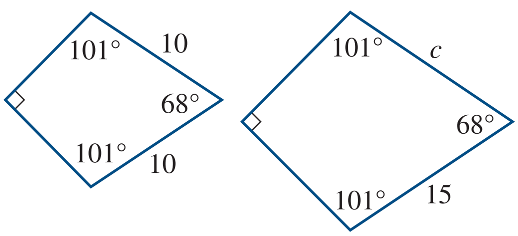
a



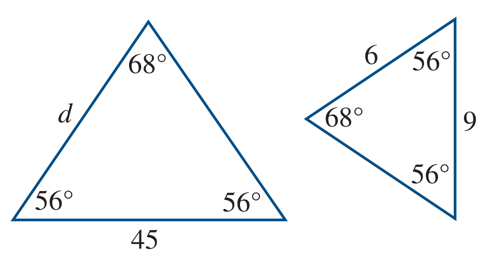
b



c



d



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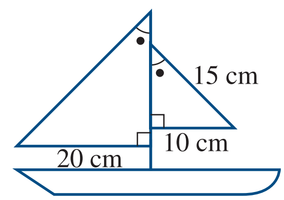
4 A data projector is used to display a computer image measuring 12 cm by 15 cm onto a screen. The scale factor used by the data projector is 1 : 9. What are the dimensions of the screen?

5 A toy yacht consists of two sails with measurements and angles as shown opposite.

a In what way are the two sails similar in shape?

b Find the scale factor for the side lengths of the sails.

c Find the length of the longest side of the large sail.

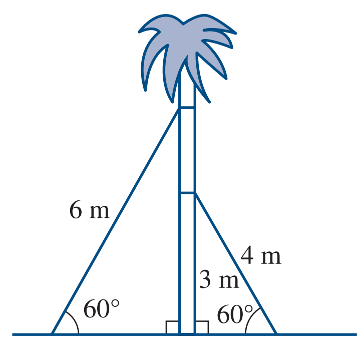


6 A tall palm tree is held in place with two cables of length 6 m and 4 m as shown.

a In what way are the two triangles created by the cables similar in shape?

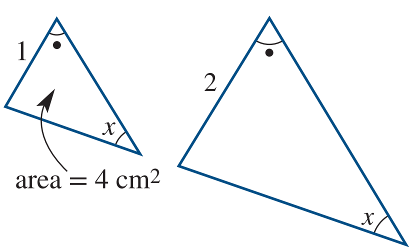
b Find the scale factor for the side lengths of the cables.

c Find the height of the point above the ground where the longer cable is attached to the palm tree.

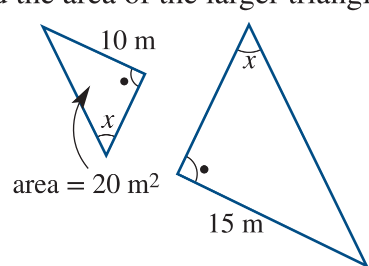


Example 6 7 The two given triangles are known to be similar. Find the area of the larger triangle.

a

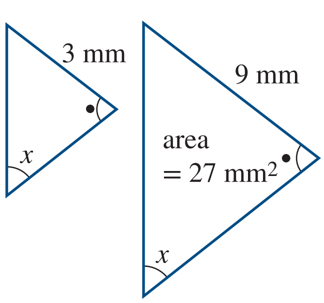


b

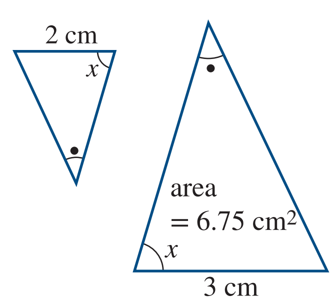


8 The two given triangles are known to be similar. Find the area of the smaller triangle.

a



b



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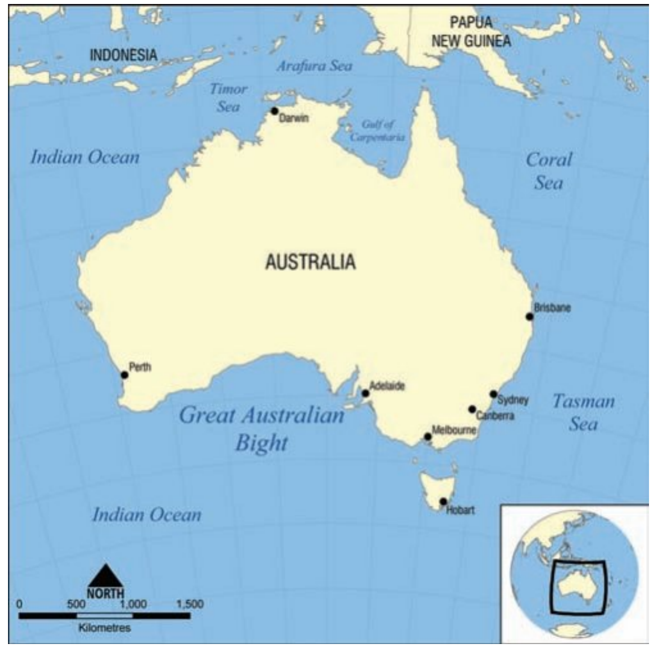
7D Scale drawing

A scale drawing is a drawing that represents the actual object. The scale factor of a scale drawing is the ratio of the size of the drawing to the actual size of the object. For example, a map is a scale drawing. It is not the same size as the area it represents. The measurements have been reduced to make the map a convenient size. The scale of a drawing may be expressed with or without units. For example, a scale of 1 cm to 1 m means 1 cm on the scale drawing represents 1 m on the actual object. Alternatively, a scale of 1:100 means the actual distance is 100 times the length of 1 unit on the scale drawing.



SCALE DRAWING

Scale of a drawing = Drawing length:Actual length

Scale is expressed in two ways:

- Using units such as 1 cm to 1 m (or 1 cm = 1 m).
- No units using a ratio such as 1:100.

Example 7: Using a scale — 7D

A scale drawing has a scale of 1:50.

a Find the actual length if the drawing length is 30 mm. Answer to the nearest centimetre.

b Find the drawing length if the actual length is 4.5 m. Answer to the nearest millimetre.

SOLUTION:

1 Multiply the drawing length by 50 to determine the actual length.

2 Divide by 10 to change millimetres to centimetres.

a Actual length $= 30 \times 50$ mm
$= 1500$ mm
$= 150$ cm

3 Divide the actual length by 50 to determine the drawing length.

4 Multiply by 1000 to change metres to millimetres.

b Drawing length $= 4.5 \div 50$ m
$= 0.09$ m
$= 90$ mm

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Exercise 7D

Example 7

1 A scale drawing has a scale of 1 : 100. What is the actual length of these drawing lengths? Express your answer in metres.

a 2 cm **b** 10 mm **c** 3.4 cm
d 28 mm **e** 8.5 cm **f** 49 mm

2 A scale drawing has a scale of 1 : 25 000. What is the drawing length of these actual lengths? Express your answer in millimetres.

a 2 km **b** 750 m **c** 4000 cm
d 3.5 km **e** 50 000 mm **f** 1375 m

3 Express each of the following scales as a ratio in the form 1 : x.

a 1 cm to 2 cm **b** 1 mm to 5 cm **c** 1 cm to 3 km

4 The scale on a map is 1 : 1000. Calculate the actual distances if these are the distances on the map. Express your answer in metres.

a Road 20 cm **b** Shops 10 cm **c** Pathway 5 cm
d Parking area 10 mm **e** Bridge 34 mm **f** Park 80 mm

5 The scale on a map is given as 1 cm = 15 km. What is the actual distance if the distance on the map is:

a 2.5 cm?
b 45 cm?

6 The scale on a map is 1 : 5000. Calculate the map distances if these are actual distances. Express your answer in millimetres.

a 50 m **b** 80 m
c 100 m **d** 120 m
e 150 m **f** 240 m

7 The scale on a map is given as 1 mm = 50 m. If the distance between two points is 350 m, what is the map distance between these points?

8 A scale drawing has a scale of 1 : 75. Find the:

a actual length if the drawing length is 15 mm
b drawing length if the actual length is 3 m.

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9 The Parkes radio telescope dish has a diameter of 64 metres. The image opposite uses a photograph of the dish.

a Determine a scale for the image.

b Estimate the height of the top of the antenna above the ground.

10 The scale of a model is 2 : 150. Calculate the model lengths if these are actual lengths. Express your answer in millimetres.

a 75 cm
b 180 cm
c 300 cm
d 45 m
e 6 m
f 36 m

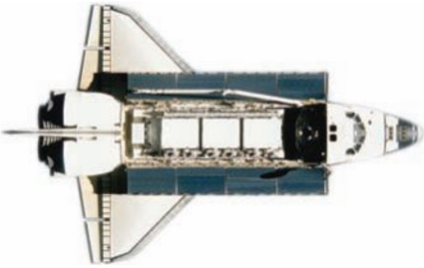
11 A scale drawing of the space shuttle is shown opposite. The actual length of the space shuttle is 56 metres.

a What is the scale factor?

b Calculate the length of the wing span to the nearest metre.

c Calculate the width of the shuttle to the nearest metre.

d What is the length of the nose of the shuttle to the nearest metre?



12 The total length of the Sydney Harbour Bridge is 1150 metres. A scale model is built for a coffee table of length 1.2 metres using the picture below.

a What scale would be suitable?

b What is the maximum height of the bridge if the scale model has a height of 20 cm?

c Estimate the height of the bridge pillars.

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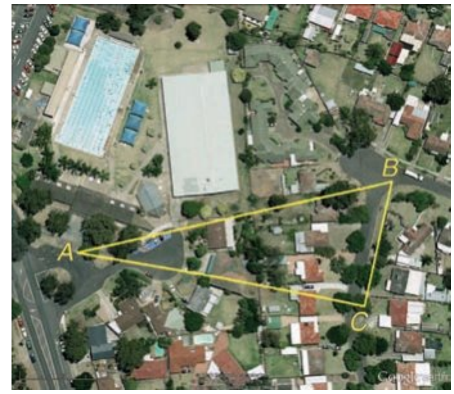
13 The length of the Olympic swimming pool shown opposite is 50 metres.

- **a** Calculate a scale for the aerial photograph.
- **b** What are the dimensions of the aerial photograph? (Answer to the nearest metre.)
- **c** What is the length from A to B?
- **d** What is the length from B to C?
- **e** What is the length from C to A?
- **f** What is the perimeter of ABC?
- **g** Calculate the area of the land marked ABC. Assume a right angle at C.



14 A football field is shown opposite. The scale is marked by the red bar, which represents 50 m on the ground. Answer the following questions, correct to the nearest metre or square metre.

- **a** What is the length of the grandstand highlighted by the yellow bar?
- **b** Calculate the length of the field.
- **c** Calculate the width of the field.
- **d** What is the perimeter of the field?
- **e** What is the area of the field?
- **f** What is the length of the diagonal of the field?

15 Two circular buildings, A and B, at Sydney airport are shown. The red scale represents 25 m on the ground. Answer the following questions, correct to the nearest metre or square metre.

- **a** What is the radius of building A?
- **b** What is the diameter of building B?
- **c** What is the circumference of building A?
- **d** What is the area of building A?
- **e** What is the radius of the building B?
- **f** What is the circumference of building B?

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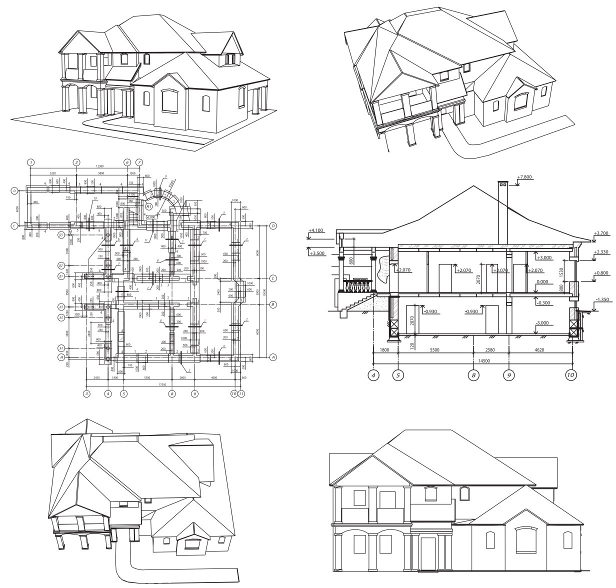
7E Plans and elevations

A plan is a view of an object from the top. It is looking down on the object. A house plan is a horizontal section cut through the building showing the walls, windows, door openings, fittings and appliances. An elevation is a view of an object from one side, such as a front elevation or side elevation. House elevations are rarely a simple rectangular shape but show all the parts of the building that are seen from a particular direction. House elevations are a vertical section parallel to one side of the building.

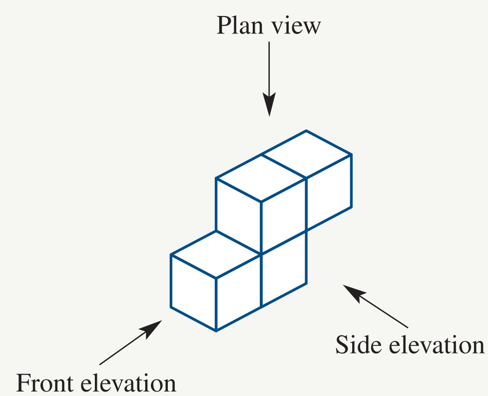


PLANS AND ELEVATIONS

A plan is a view of an object from the top.
An elevation is a view of an object from one side, such as a front elevation or side elevation.

Example 8: Drawing a plan and elevation 7E

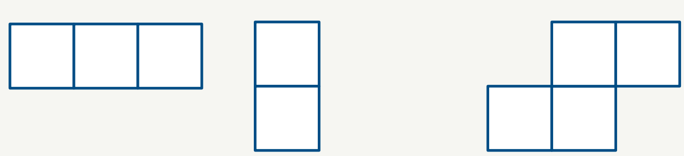
Draw the plan view, front elevation and side elevation of this object.



SOLUTION:

1 Look down on the object for the plan.
2 Look from the front for the front elevation and from the side for the side elevation.

Plan Front elevation Side elevation



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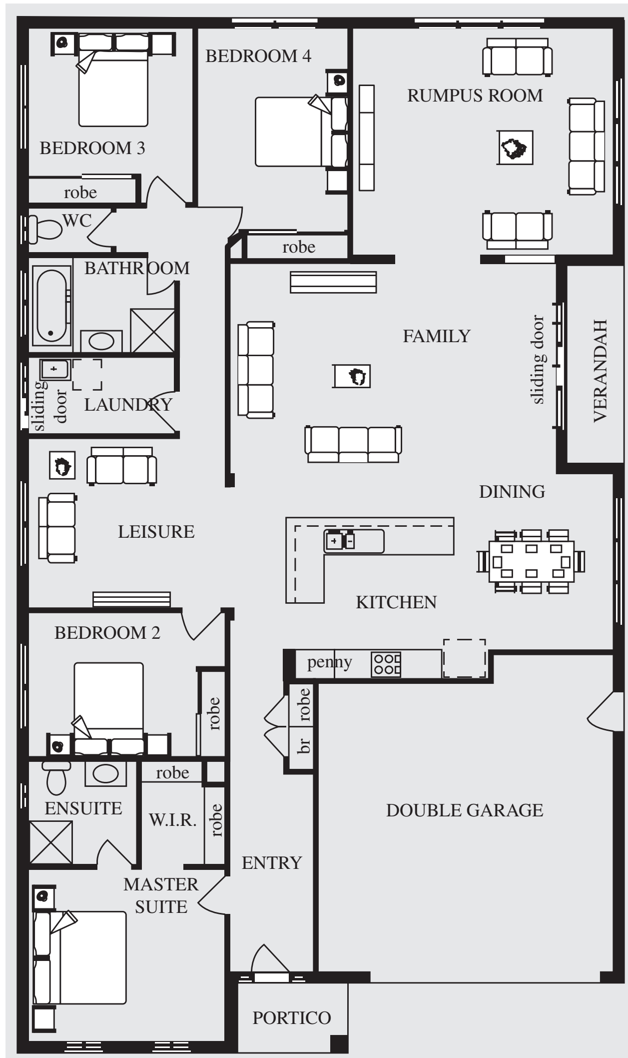
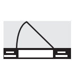
Building plans

A floor plan for a Metricon home is shown below. Building plans are a very common application of similar figures. They are drawn using a scale factor such as 1:150. This allows the dimensions of a house to be determined by measurement and calculation.



Scale 1:150

Common floor plan symbols

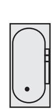

Door swing – indicates direction the door opens

Window – glass window in a solid wall


Kitchen sink – two-compartment kitchen sink


Shower – shower without a bathtub


Toilet – toilet located on wall


Bathtub – bathtub showing location of drain

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Example 9: Finding measurements from a house plan 7E

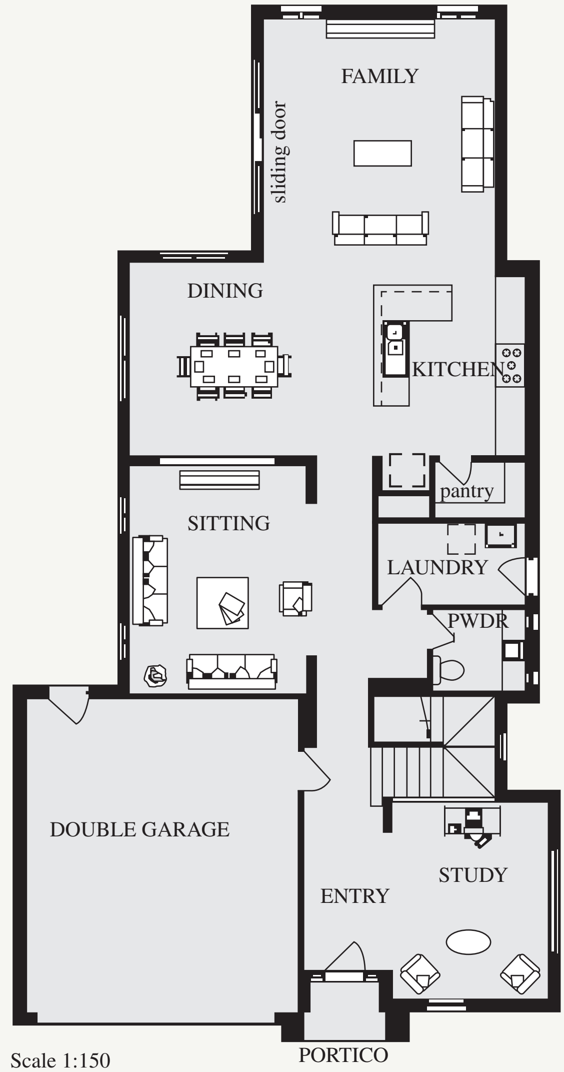
A building plan is shown for the ground floor of a Metricon home.

a How many internal doors are there?

b What is the meaning of PWDR?

c What is the length of the house?

d What are the dimensions of the double garage?



SOLUTION:

1 Count the number of internal doors (find the door symbol).

a 4 internal doors

2 PWDR is an abbreviation for the powder room.

b Powder room

3 Use a ruler to measure the length of the house on the floor plan.

c Drawing length is 12.6 cm

4 Multiply the measurement by 150 (scale 1:150).

$$\text{Actual length} = 12.6 \times 150 \text{ cm}$$
$$= 1890 \text{ cm or } 18.9 \text{ m}$$

5 Use a ruler to measure the dimensions of the double garage on the floor plan.

d Drawing length is 4.1×3.5 cm

6 Multiply the measurements by 150 (scale 1:150).

$$\text{Actual length} = 4.1 \times 150 \text{ cm}$$
$$= 615 \text{ cm or } 6.2 \text{ m}$$

$$\text{Actual breath} = 3.5 \times 150 \text{ cm}$$
$$= 525 \text{ cm or } 5.3 \text{ m}$$

Dimensions are 6.2×5.3 m.

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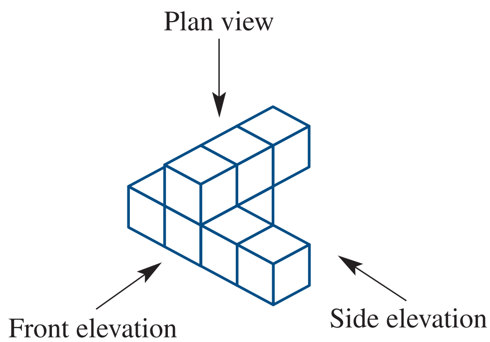
Exercise 7E

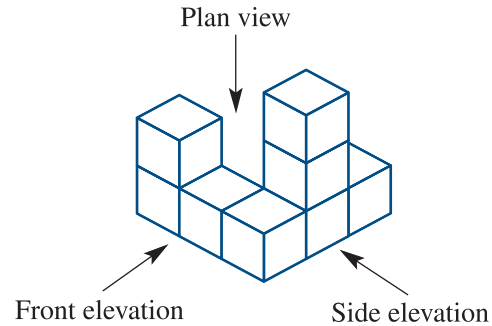
Example 8

1 Draw the plan, front elevation and side elevation for these objects.

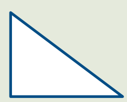
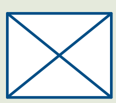
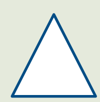
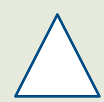
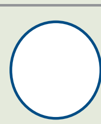
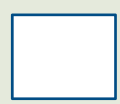
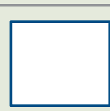
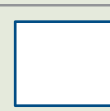
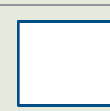
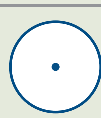
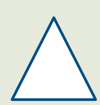
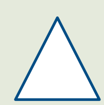
a



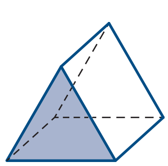
b

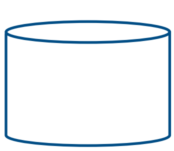


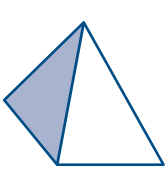
2 What are these shapes?

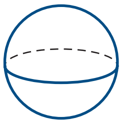
	Plan	Front elevation	Side elevation
a			
b			
c			
d			
e			

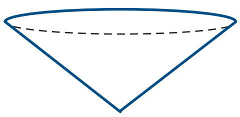
3 Draw the plan, front elevation and side elevation for these objects.

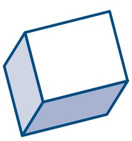
a 

b 

c 

d 

e 

f 

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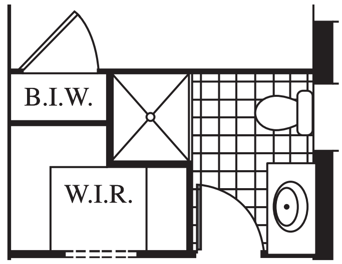
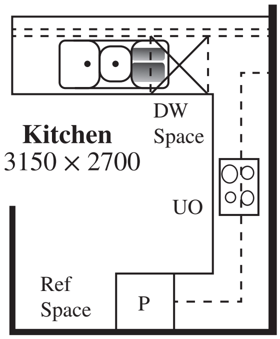
4 A section of a floor plan is shown opposite.
- a What room is shown in the diagram?
- b What symbol is used for a shower?
- c What symbol is used for a door?
- d What does 'W.I.R.' represent on the plan?

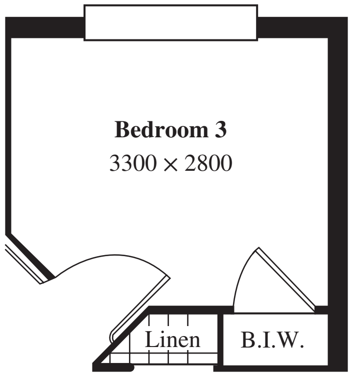


5 A section of a floor plan is shown opposite.
- a What are the dimensions of the kitchen?
- b What symbol is used for the sink?
- c What symbol is used for the cooktop?
- d What does 'Ref' represent on the plan?
- e What does 'P' represent on the plan?

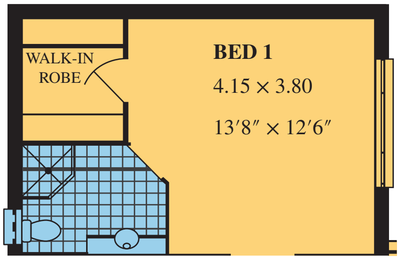


Example 9

6 A section of a floor plan is shown opposite.
- a What are the dimensions of the bedroom?
- b What symbol is used for a window?
- c What does 'B.I.W.' represent on the plan?
- d What is the length of the bedroom on the plan?
- e Calculate a scale for the floor plan.



7 A section of a floor plan is shown opposite.
- a What are the dimensions of the bedroom?
- b What symbol is used for the toilet?
- c What is the length of the bedroom on the plan?
- d Calculate a scale for the floor plan.
- e What are the dimensions of the walk-in robe?
- f What is the area of the bedroom? Answer in square metres, correct to two decimal places.



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Example 9

8 A second-storey building plan is shown for a Masterton home.

a What are the dimensions of the third bedroom? Answer in metres.

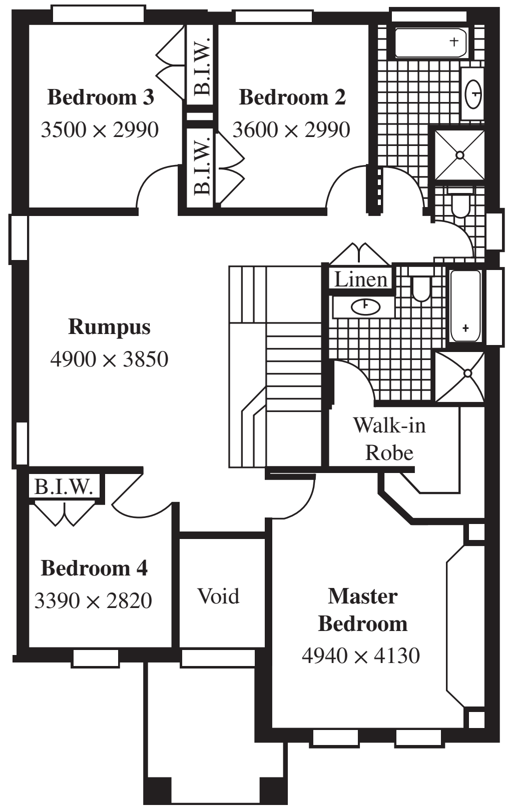
b What are the dimensions of the master bedroom? Answer in metres.

c By measurement, estimate a scale for this plan.

d By measurement, find the width of the house. Answer in metres.

e Calculate the area of the void. Answer to the nearest square metre.

f Calculate the area of the ensuite. Answer to the nearest square metre.



9 A rumpus room is built measuring 5.5 m by 4.7 m. The floor plan uses a scale of 1 : 100. A concrete slab with a depth of 100 mm is used to build the rumpus room.

a What is the area of the rumpus room on the plan? Answer in square millimetres.

b What is the volume of concrete for the rumpus room? Answer in cubic millimetres.

c What is the volume of concrete for the rumpus room if the slab depth is 200 mm? Answer in cubic millimetres.

10 The front elevation of a house is shown opposite (scale 1 : 200).

a What is the width of the house? Answer in metres.

b What is the height of the chimney? Answer in metres.

c What are the dimensions of the front door? Answer in metres.

d What are the dimensions of the window on the right-hand side? Answer to the nearest centimetre.

e What is the area of the window? Answer to the nearest square centimetre.

f What is the area of the large triangular gable? Answer to the nearest square centimetre.

g What is the area of the small triangular gable? Answer to the nearest square centimetre.

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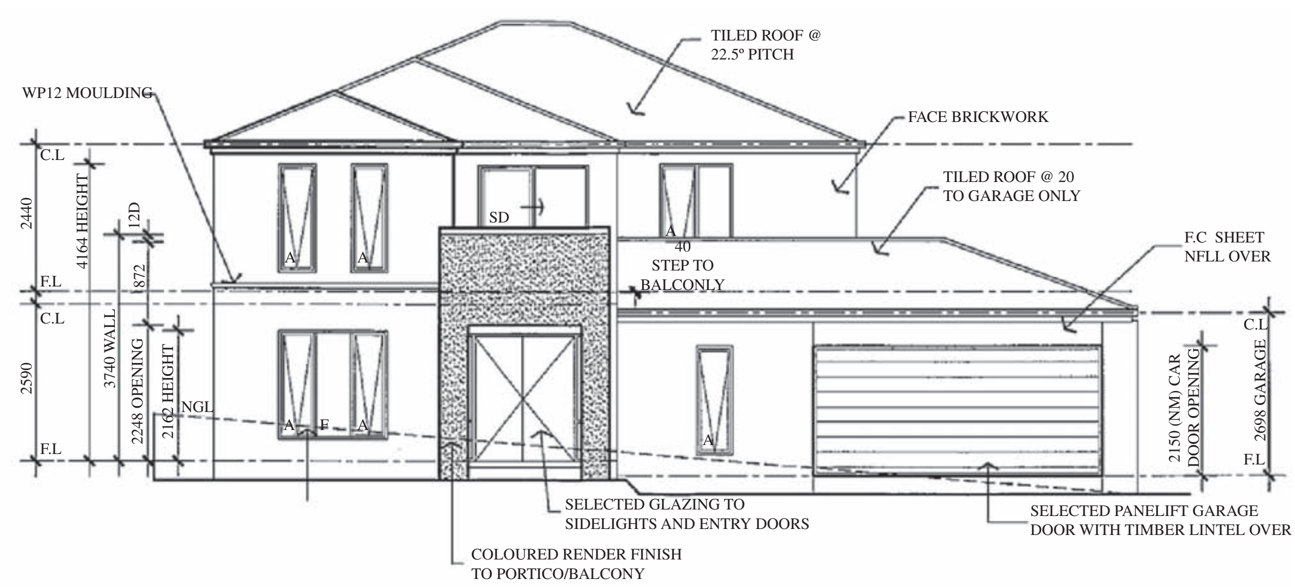
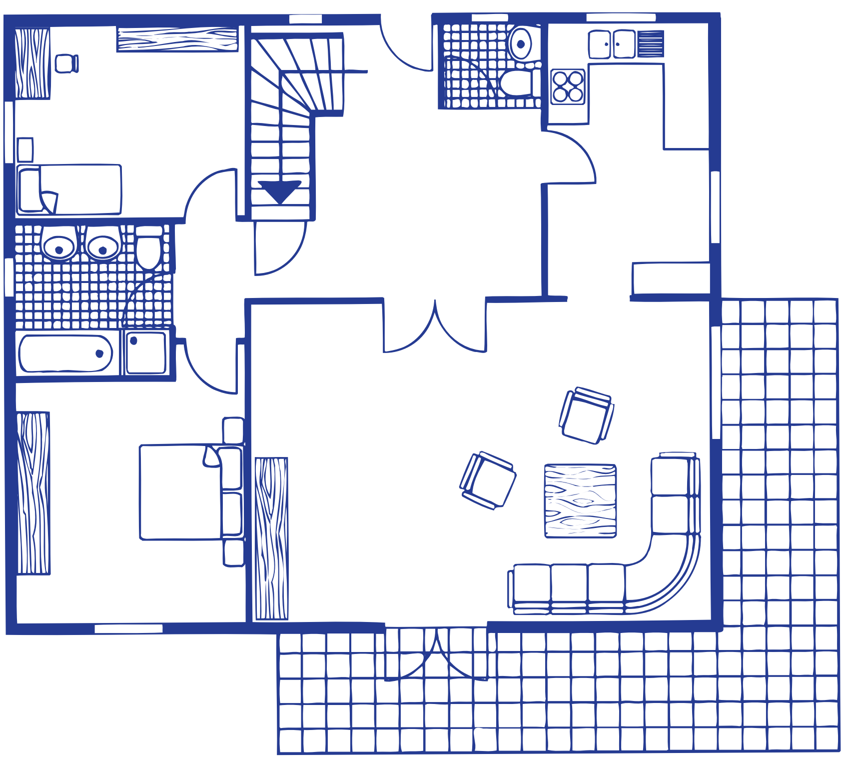
11 The front elevation of a house is shown below.



a What is the height of the first storey? Answer in metres.
b What is the height of the garage? Answer in metres.
c What is the angle of the pitch of the roof?
d How many windows are at the front of the house?

12 A building plan is shown below. The house length is 20 m and the width is 18 m.



a What are the dimensions of the living room? Answer in metres.
b What is the area of the living room? Answer to the nearest square metre.
c Considering only the area of the living room, how much concrete was used in the concrete slab whose thickness is 200 mm? Answer to the nearest cubic metre.

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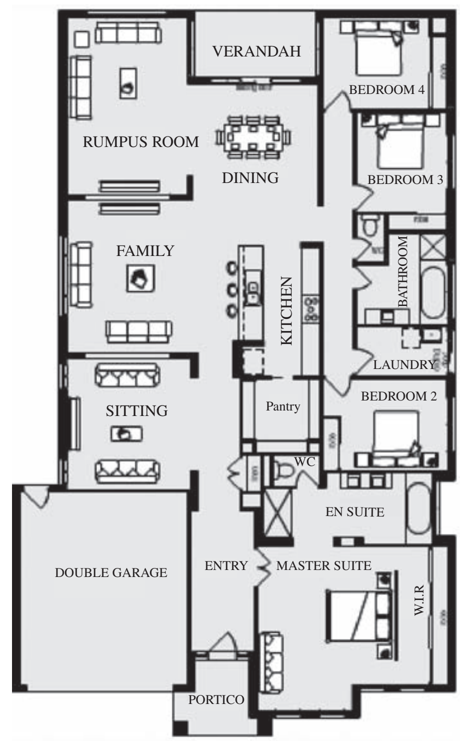
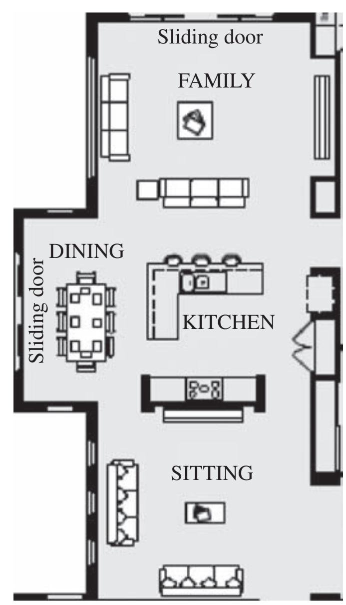
13 A building plan is shown for a Metricon home. The house length is 24 m (includes portico) and the width is 15 m (includes garage).

- **a** What is a suitable scale for this plan?
- **b** What are the dimensions of the verandah? Answer correct to the nearest tenth of a metre.
- **c** What are the dimensions of the double garage? Answer correct to the nearest tenth of a metre.
- **d** What are the dimensions of bedroom 3? Answer correct to the nearest tenth of a metre.
- **e** Calculate the area of the sitting room. Answer correct to the nearest square metre.
- **f** What is the cost of carpeting the sitting room if the cost of the carpet is \$140 per square metre? Answer to the nearest dollar.



14 A section of a building plan is shown opposite. The dimensions of the family room are 5.5 metres by 6.0 metres.

- **a** Estimate a suitable scale for this building plan.
- **b** What is the combined length of the family, kitchen and sitting rooms? Answer correct to the nearest tenth of a metre.
- **c** The family, dining, kitchen and sitting rooms are to be tiled. Calculate the combined area of these rooms. Answer to nearest square metre.
- **d** Ceramic tiles measuring 300×300 mm are to be laid in these rooms. How many tiles are required?
- **e** What assumption has been made to the answer in part **d**?
- **f** The family, dining, kitchen and sitting rooms are built on a concrete slab with a thickness of 0.15 m. What is the volume of concrete used for the slab?



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PPT STUDY GUIDE

Summary

Key ideas and chapter summary

Ratio

A ratio is used to compare amounts of the same units in a definite order.
Equivalent ratios are obtained by multiplying or dividing by the same number.

$$15 : 12 = 5 : 4 \quad (\div 3, \div 3) \qquad 5 : 4 = 15 : 12 \quad (\times 3, \times 3)$$

15 : 12 and 5 : 4 are equivalent ratios.

Dividing a quantity in a given ratio

1. Find the total number of parts by adding each amount in the ratio.
2. Divide the quantity by the total number of parts to find one part.
3. Multiply each amount of the ratio by the result in step **2**.
4. Check by adding the answers for each part. The result should be the original quantity.

Similarity and scale factors

Similar figures are exactly the same shape but are a different size.

- Corresponding (or matching) angles of similar figures are equal.
- Corresponding (or matching) sides of similar figures are in the same ratio.
- Scale factor is the amount the first shape is enlarged or reduced to get the second shape.

Using scale factor for area

When all the dimensions are multiplied by a scale factor of k, the area is multiplied by a scale factor of k^2.

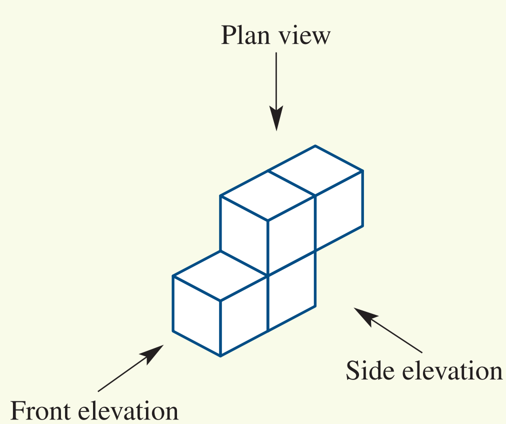
Scale drawing

Scale of a drawing = Drawing length : Actual length
Scale is expressed in two ways:

- Using units such as 1 cm to 1 m (or 1 cm = 1 m).
- No units using a ratio such as 1 : 100.

Plans and elevations

- A plan is a view of an object from the top.
- An elevation is a view of an object from one side, such as a front elevation or side elevation.



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Review

Multiple-choice

1 A school has 315 boys, 378 girls and 63 teachers. The ratio of students to teachers is:
A 5 : 6 **B** 6 : 5 **C** 11 : 1 **D** 1 : 11.

2 The ratio 500 mm to $\frac{1}{5}$ m is the same as:
A 50 : 2 **B** 2500 : 1 **C** 5 : 2 **D** 2 : 5.

3 The ratio of adults to child in a park is 5 : 9. How many adults are in the park if there are 630 children?
A 70 **B** 126 **C** 280 **D** 350

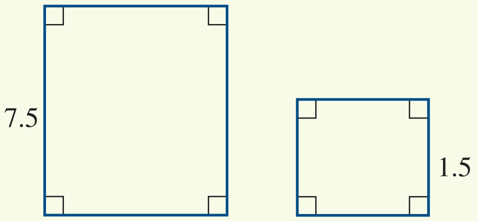
4 $750 is divided in the ratio 1 : 3 : 2. The smallest share is:
A $250 **B** $125 **C** $375 **D** $750.

5 A 360-gram bag of lollies is divided in the ratio 7 : 5. What is the mass of the smaller amount?
A 150 g **B** 168 g **C** 192 g **D** 210 g

6 A scale drawing has a scale of 1 : 20. What is the actual length if the drawing length of an object is 20 mm?
A 1 mm **B** 20 mm **C** 40 mm **D** 400 mm

7 The scale on a map is given as 1 mm = 150 m. If the distance between two points is 600 m, what is the map distance between these points?
A 4 mm **B** 0.25 mm **C** 2.5 cm **D** 40 cm

8 What is the scale factor for these squares?
A $\frac{1}{5}$ **B** 5
C 6 **D** 7.5



9 On a map, Sydney and Melbourne are 143.2 mm apart. If the cities are 716 km apart, what scale has been used?
A 1 : 5 **B** 1 : 5000 **C** 1 : 50 000 **D** 1 : 5 000 000

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Review

Short-answer

1 Complete each pair of equivalent ratios.

a 4 : 30 = 2 : □
b 6 : 10 = □ : 20
c 2 : 11 = 22 : □
d 13 : 3 = □ : 9
e 2 : 5 = □ : 25
f 4 : 9 = □ : 36

2 True or false?

a 1 : 3 = 5 : 9
b The ratio 2 : 3 is the same as 3 : 2.
c The ratio 3 : 5 is written in simplest form.
d 30 cm : 1 m is written as 30 : 1 in simplest form.
e $\frac{2}{3} = 4 : 6$

3 Simplify the following ratios.

a 10 : 40
b 36 : 24
c 75 : 100
d 8 : 64
e 27 : 9
f 5 : 25
g 6 : 4
h 52 : 26
i 12 : 36
j 500 : 100
k 20 : 30
l 28 : 7
m 10 : 15 : 30
n 12 : 9
o 56 : 88
p 4.8 : 1.6
q $\frac{3}{4} : \frac{1}{2}$
r $\frac{2}{7} : \frac{5}{7}$

4 Divide:

a \$80 in the ratio 7 : 9
b 200 kg in the ratio 1 : 4
c 40 m in the ratio 6 : 2
d \$1445 in the ratio 4 : 7 : 6.

5 The ratio of the cost price of a TV to its retail price is 5 : 12. If its cost price is \$480, calculate its retail price.

6 Daniel and Eddie own a business and share the profits in the ratio 3 : 4.

a The profit last week was \$3437. How much does Daniel receive?
b The profit this week is \$2464. How much does Eddie receive?

7 Patrick mixes sand and cement in the ratio 5 : 2 by volume. If he uses 5 buckets of cement, how much sand should he use?

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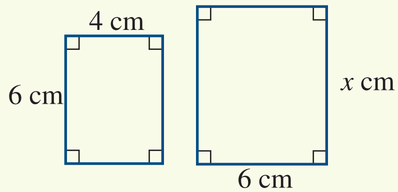
Review

8 Find the scale factor for the following pairs of similar figures, and find the value of x.

a



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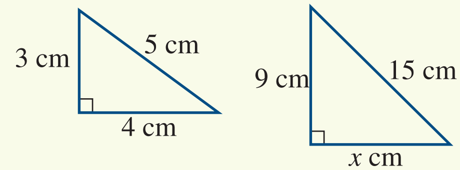


9 The ratio of Victoria's height to Willow's is 8 : 7. If Victoria is 176 cm tall, how tall is Willow?

10 Express each of the following scales as a ratio in the form 1 : a.

a 1 cm to 3 m **b** 1 mm to 6 cm **c** 1 m to 2.5 km

11 Two cities are 50 km apart. How many millimetres apart are they on a map that has a scale of 1 : 100 000?

12 The scale on a map is given as 1 cm = 5 km. If the distance between two points on the map is 46 mm, what is the actual distance between these points? Answer in kilometres.

13 A scale drawing has a scale of 1 : 50 000. What is the drawing length of these actual lengths? Express your answer in millimetres.

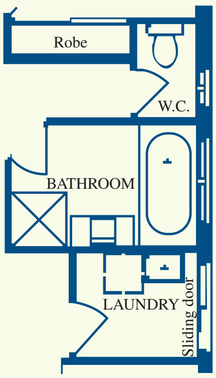
a 4 km **b** 1250 m **c** 5000 cm
d 6.5 km **e** 20 000 mm **f** 2 125 m

14 The scale on a map is 1 : 400. Calculate the actual distances if these are the distances on the map. Express your answer in metres.

a Bike path 180 cm **b** Town centre 20 cm **c** Street 5 cm
d Beach 210 mm **e** River 62 cm **f** Park 60 mm

15 A section of a floor plan is shown opposite. The longer dimension of the laundry is 3 metres.

a Estimate a suitable scale for the floor plan.
b What symbol is used for the bath?
c What does 'W.C.' represent on the plan?
d What are the dimensions of the laundry?



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8 Depreciation and loans

Syllabus topic — F3 Depreciation and loans

This topic will develop your understanding of reducing balance loans and that an asset may depreciate over time rather than appreciate.

Outcomes

- Calculate the depreciation of an asset using declining-balance method.
- Solve practical problems involving reducing-balance loans.
- Solve problems involving credit cards.
- Interpret credit card statements.
- Identify the various fees and charges associated with credit card usage.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Desmos widgets
- Literacy worksheet
- Spreadsheets
- Quick Quiz
- Study guide
- Solutions (enabled by teacher)

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

The Interactive Textbook provides a test of prior knowledge for this chapter, and may direct you to revision from the previous years' work.

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8A Declining-balance depreciation

Declining-balance depreciation occurs when the value of the item decreases by a fixed percentage each time period. For example, if you buy a car for \$20 000 and it depreciates by 10% each year then the value of the car after one year is \$20 000 − \$2000 or \$18 000. After the second year the value of the car is \$20 000 − \$2000 − \$1800 or \$16 200. Notice that the amount of depreciation has decreased in the second year. Depreciation calculations have similarities with compound interest, except that the depreciation is subtracted from the value not added to it.

DECLINING-BALANCE DEPRECIATION

$S = V_0(1 - r)^n$

S – Salvage value or current value of an item. Also referred to as the book value.

V_0 – Purchase price of the item. Value of the item when $n = 0$.

r – Rate of depreciation per time period expressed as a decimal.

n – Number of time periods.

Example 1: Calculating the declining-balance depreciation 8A

Eva purchased a new car two years ago for \$32 000. During the first year it had depreciated by 25% and during the second it had depreciated 20% of its value after the first year. What is the current value of the car?

SOLUTION:

1	Write the declining-balance depreciation formula.	$S = V_0(1 + r)^n$
2	For the first year, substitute $V_0 = 32\,000$, $r = 0.25$ and $n = 1$ into the formula.	$= 32\,000 \times (1 - 0.25)^1$
3	Evaluate the value of the car after the first year.	$= \$24\,000$
4	Write the declining-balance depreciation formula.	$S = V_0(1 - r)^n$
5	For the second year substitute $V_0 = 24\,000$, $r = 0.20$ and $n = 1$ into the formula.	$= 24\,000 \times (1 - 0.20)^1$
6	Evaluate the value of the car after the second year.	$= \$19\,200$
7	Write the answer in words.	Current value is \$19 200.

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Example 2: Calculating the purchase price 8A

Angus buys a car that depreciates at the rate of 26% per annum. After five years the car has a salvage value of \$17 420. How much did Angus pay for the car, to the nearest dollar?

SOLUTION:

1 Write the declining-balance depreciation formula. $S = V_0(1 - r)^n$

2 Substitute $S = 17\,420$, $r = 0.26$ and $n = 5$ into the formula. $17\,420 = V_0 \times (1 - 0.26)^5$

3 Make V_0 the subject of the equation.

4 Evaluate. $V_0 = \dfrac{17\,420}{(1 - 0.26)^5}$

5 Express the answer correct to the nearest whole dollar. $= \$78\,503.596\,21$ $= \$78\,504$

6 Write the answer in words. Angus paid \$78 504 for the car.

Example 3: Calculating the percentage rate of depreciation 8A

Madison bought a delivery van four years ago for \$27 500. Using the declining-balance method for depreciation, she estimates its present value to be \$8107. What annual percentage rate of depreciation did she use? Answer to the nearest whole number.

SOLUTION:

1 Write the declining-balance depreciation formula. $S = V_0(1 - r)^n$

2 Substitute $S = 8107$, $V_0 = 27\,500$ and $n = 4$ into the formula. $8107 = 27\,500 \times (1 - r)^4$

3 Make $(1 - r)^4$ the subject of the equation. $(1 - r)^4 = \dfrac{8107}{27\,500}$

4 Take the fourth root of both sides. $1 - r = \sqrt[4]{\dfrac{8107}{27\,500}}$

5 Rearrange to make r the subject. $r = 1 - \sqrt[4]{\dfrac{8107}{27\,500}}$

6 Evaluate. $= 0.263\,145\,28$

7 Express the answer correct to the nearest whole number. $= 26\%$

8 Write the answer in words. Rate of depreciation is 26%.

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Exercise 8A

Example 1

1 A motor vehicle is bought for $22000. It depreciates at 16% per annum and is expected to be used for 5 years. What is the salvage value of the motor vehicle after the following time periods? Answer to the nearest cent.

a 1 year

b 2 years

c 3 years

2 Emma purchased a used car for $6560 two years ago. Use the declining-balance method to determine the salvage value of the used car if the depreciation rate is 15% per annum. Answer to the nearest dollar.

3 Bailey purchased a motor cycle for $17500. It depreciates at 28% per year. Answer to the nearest dollar.

a What is the book value of the motor cycle after 3 years?

b How much has the motor cycle depreciated over the 3 years?

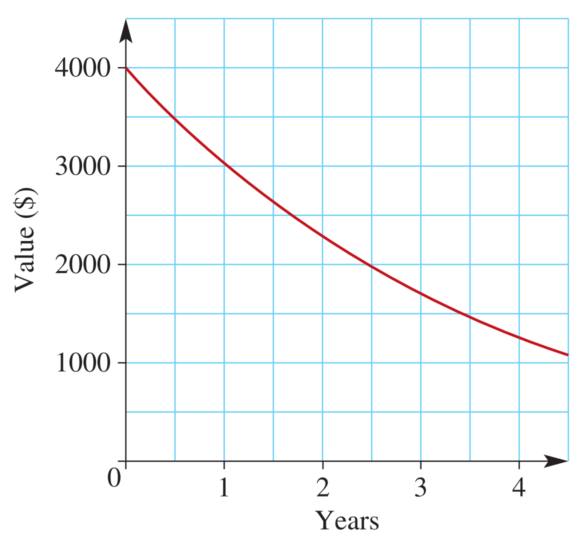
4 A new car is bought for $52000. It depreciates at 22% per annum and is expected to be used for 4 years. How much has the car depreciated over the 4 years? Answer to the nearest dollar.

5 Chloe purchased a car for $19900. It depreciates at 24% per year. Answer to the nearest dollar.

a What is the salvage value of the car after 5 years?

b How much has the car depreciated over the 5 years?

6 The depreciation of a used car over 4 years is shown in the graph below.



a What is the initial value of the used car?

b How much did the used car depreciate during the first year?

c When is the value of the used car $2000?

d When is the value of the used car $1500?

e What is the value of the used car after 4 years?

f What is the value of the used car after $1\frac{1}{2}$ years?

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Example 2 **7** A hatchback was purchased for $16 980 three years ago. By using the declining-balance method of depreciation, find the current value of the hatchback if the annual percentage rate of depreciation is 17.27%. Answer to the nearest dollar.

8 A new car is valued at $35 000. It has a rate of depreciation of 27.14%.

a What is the value of the new car after one years?

b What is the value of the new car after three years?

Example 3 **9** Philip bought a luxury car that depreciates at the rate of 8.9% per annum. After five years the car has a salvage value of $104 350. How much did Philip pay for the car, to the nearest dollar?

10 Mary bought a new car for her business. It depreciates at the rate of 11% per annum. After four years the car has a salvage value of $16 240. How much did Mary pay for the car, to the nearest dollar?

11 A motor vehicle is bought for $32 000. It depreciates at 16% per annum and is expected to be used for 8 years.

a How much does the motor vehicle depreciate in the first year?

b Copy and complete the following depreciation table for the first five years. Answer to the nearest dollar.

Year	Current value	Depreciation	Depreciated value
1			
2			
3			
4			
5			

c Graph the value in dollars against the age in years.

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8B Reducing-balance loans

Reducing-balance loans are calculated on the balance owing and not on the initial amount of money borrowed as with a flat-rate loan ('flat' meaning the interest rate does not change during the life of the loan). As payments are made, the balance owing is reduced and therefore the interest charged is reduced. This can save thousands of dollars on the cost of a loan. The calculations for reducing-balance loans are complicated and financial institutions publish tables related to loans.

LOAN REPAYMENTS

Total to be paid = Loan payment × Number of repayments
Total to be paid = Principal + Interest

Example 4: Using a table for a reducing-balance loan 8B

The table below shows the monthly repayments for a reducing-balance loan. Calculate the amount of interest to be paid on a loan of $200 000 over 13 years.

Term	Amount of the loan			
	$100 000	$150 000	$200 000	$250 000
12 years	$1664	$2096	$2794	$3493
13 years	$1700	$2150	$2856	$3569
14 years	$1726	$2218	$2898	$3622

SOLUTION:

1. Loan is $200 000 and time period is 13 years.
   Total to be paid = Loan payment × Number of repayments
2. Find the intersection value from the table ($2856).
3. Multiply the intersection value by the number of years and 12 (months in a year) to determine the total to be paid.
   $= 2856 \times 13 \times 12$
   $= \$445\,536$
   Total to be paid for the loan is $445 536.
4. Substitute the total to be paid ($445 536) and principal ($200 000) into the formula.
   Total to be paid = Principal + Interest
   $445\,536 = 200\,000 + I$
5. Evaluate.
   $= \$245\,536$
6. Write the answer in words.
   Interest paid is $245 536.

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Example 5: Using a table for a reducing-balance loan 8B

The table shows the monthly payments for each $1000 borrowed. Molly is planning to borrow $280 000 to buy a house at 8% per annum over a period of 20 years.

Interest rate	Period of loan		
	10 years	15 years	20 years
6% p.a.	$11.10	$8.44	$7.10
7% p.a.	$11.61	$9.00	$7.75
8% p.a.	$12.13	$9.56	$8.36

a What is Molly's monthly payment on this loan?
b How much would Molly pay in total to repay this loan?
c How much would Molly save if she repaid the loan over 15 years?

SOLUTION:

1 Find the intersection value from the table for interest rate 8% p.a. and time period 20 years.

a $8.36

2 Multiply the intersection value by the number of thousands borrowed (280).

Monthly repayment = $8.36 × 280
= $2340.80

3 Multiply the monthly repayment by the number of years and 12 (months in a year) to determine the total to be paid.

b Total to be paid
= Loan repayment × Number of repayments
= 2340.80 × 20 × 12

4 Evaluate.

= $561 792

5 Write the answer in words.

Total to be paid for the loan is $561 792.

6 Repeat the above calculations using 15 years instead of 20 years.

c 15 years
Monthly repayment = $9.56 × 280
= $2676.80
Total to be paid
= Loan repayment × Number of repayments
= 2676.80 × 15 × 12
= $481 824

7 Subtract the total to be paid for 15 years from the total to be paid for 20 years.

Amount saved = $561 792 − $481 824

8 Evaluate.

= $79 968

9 Write the answer in words.

The amount saved is $79 968.

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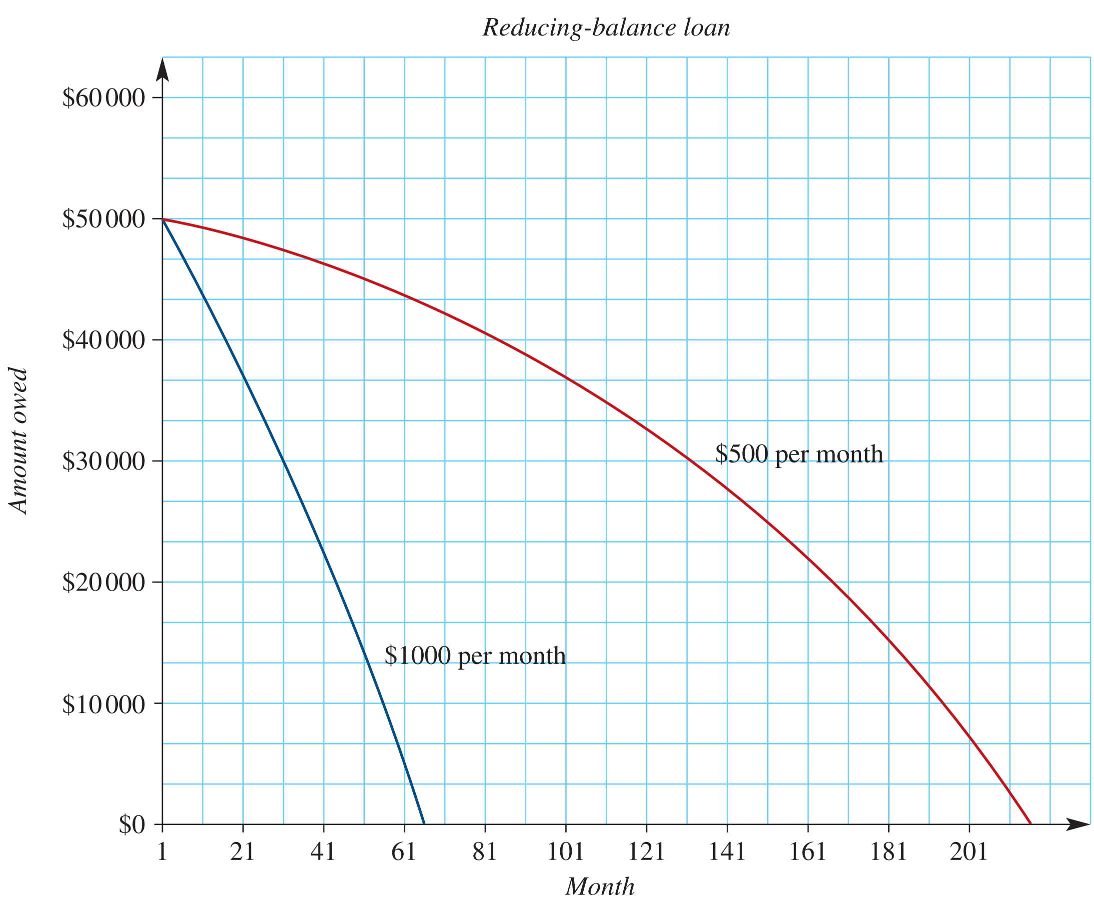
Fees and charges for a loan

Banks and financial institutions charge their customers for borrowing money. A loan account is created and an account service fee is charged per month. In addition to this fee there are a number of other loan fees and charges, depending on the financial institution. Many of these fees are negotiable and customers are advised to compare the fees and charges with the interest rate charged. Fees and charges for a loan may include:

- loan application fee – costs in setting up the loan.
- loan establishment fee – initial costs in processing the loan application.
- account service fee – ongoing account-keeping fee.
- valuation fee – assessment of the market value of a property.
- legal fee – legal processing of a property.

Graph of a reducing-balance loan

The graph below shows the amount owed after each month on a reducing-balance loan. The amount borrowed is \$50 000 at an interest rate of 10% p.a. It illustrates the difference between making repayments of \$500 per month and making repayments of \$1000 per month. When paying \$500 a month, it takes 215 months to pay off the loan, and the interest charged is \$57 500. However, when paying \$1000 a month, it only takes 65 months to pay off the loan, and the interest charged is \$15 000. Each graph is a gradual curve as each payment reduces the amount owed and slowly decreases the interest charged.



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Exercise 8B

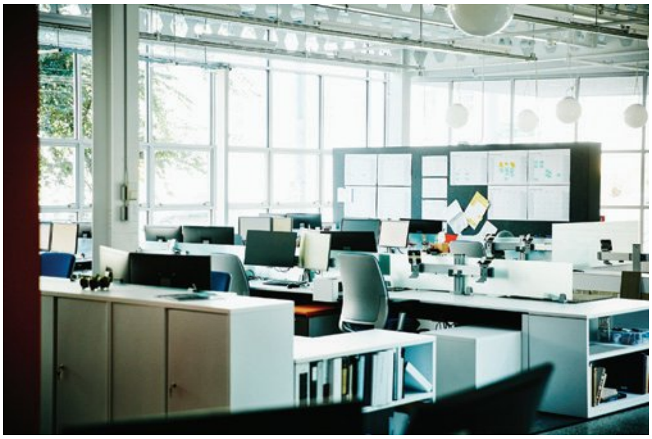
Example 4

1 Tyler is considering an investment loan from the bank at an interest rate of 9.9% p.a. reducible. The table below shows the monthly repayment for an investment loan.

Term	Investment loan				
	\$50 000	**\$75 000**	**\$100 000**	**\$125 000**	**\$150 000**
5 years	\$1060	\$1590	\$2120	\$2650	\$3180
10 years	\$658	\$987	\$1316	\$1645	\$1974
15 years	\$534	\$801	\$1068	\$1336	\$1603

a What is the monthly repayment for a loan of \$75 000 over 15 years?
b What is the monthly repayment for a loan of \$150 000 over 10 years?
c What is the monthly repayment for a loan of \$100 000 over 5 years?
d What is the monthly repayment for a loan of \$50 000 over 15 years?
e What is the monthly repayment for a loan of \$125 000 over 5 years?

2 Kevin is applying for an investment loan from a bank of \$75 000 over 5 years using the table in question **1**.
a What is the monthly repayment?
b What is the total amount paid for this loan?
c What is the interest paid on this loan?



3 The table below shows the monthly repayments per \$1000 on a bank loan.

Term	7.00%	7.25%	7.50%	7.75%
10 years	\$16.39	\$16.78	\$17.18	\$17.58
15 years	\$15.33	\$15.87	\$16.44	\$17.02

Calculate the monthly repayment on the following loans.
a \$310 000 at 7.50% p.a. for 15 years
b \$120 000 at 7.00% p.a. for 10 years
c \$450 000 at 7.75% p.a. for 10 years
d \$180 000 at 7.25% p.a. for 15 years

4 Blake is borrowing \$35 000 at 7% p.a. for 10 years. Use the table in question **3** to answer these questions.
a What is the monthly repayment?
b How much interest will he pay?

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Example 5

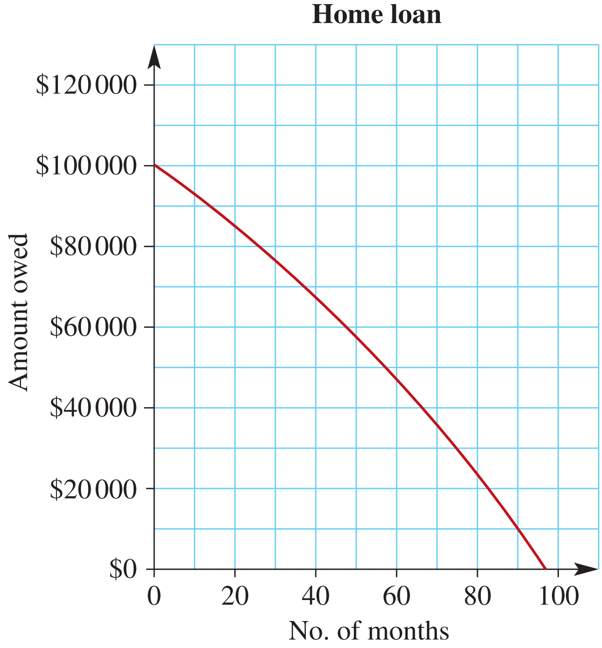
5 The table below shows the monthly payments for a loan of $1000 for varying interest rates. Jack is planning to borrow $340 000 to buy a house at 10% p.a. over a period of 15 years.

Interest rate	Period of loan		
	10 years	15 years	20 years
7% p.a.	$11.61	$9.00	$7.75
8% p.a.	$12.13	$9.56	$8.36
9% p.a.	$12.67	$10.14	$9.00
10% p.a.	$13.22	$10.75	$9.65

a Calculate Jack's monthly payment on this loan.
b How much does Jack pay in total to repay this loan?
c How much interest does Jack pay on this loan?
d How much would Jack save if he repaid the loan over 10 years?

6 Hannah and Mitchell borrow $180 000 over 20 years at a reducible interest rate of 8.5% p.a. They pay $1754 per month.
a Calculate the total amount to be paid on this loan.
b How much interest do they pay on the loan?

7 The graph opposite shows the amount owed each month on a reducing-balance loan. Use the graph to estimate the answer to these questions.
a How much was borrowed?
b How much is owed after 20 months?
c How much is owed after 40 months?
d How much is owed after 60 months?
e When is the amount owing $20 000?
f When is the amount owing $60 000?
g When is the loan paid?



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8C Credit cards

Credit cards are used to buy goods and services and pay for them later. The time when interest is not charged on your purchases is called the interest-free period. If payment is not received when the statement is due then interest is charged from the date of purchase. Interest on credit cards is usually calculated daily on the outstanding balance using compound interest.

The interest rate is usually much higher than for other kinds of loans and credit facilities.

CREDIT CARDS

$$\text{Daily interest rate} = \frac{\text{Annual interest rate}}{365}$$

$FV = PV(1 + r)^n \quad I = FV - PV$

FV – Amount owing on the credit card

PV – Principal is the purchases made on the credit card plus the outstanding balance

r – Rate of interest per compounding time period expressed as a decimal

n – Number of compounding time periods

I – Interest (compound) charged on the outstanding balance

Example 6: Calculating the cost of using a credit card 8C

Samantha has a credit card with a compound interest rate of 18% p.a. and no interest-free period. Samantha used her credit card to pay for clothing costing \$280. She paid the credit card account 14 days later. What is the total amount she paid for the clothing, including the interest charged?

SOLUTION:

1 Write the formula for compound interest.	$FV = PV(1 + r)^n$
2 Substitute $P = 280$, $r = (0.18 \div 365)$ and $n = 14$ into the formula.	$= 280\left(1 + \frac{0.18}{365}\right)^{14}$
3 Evaluate.	$= 281.939\,3596$
4 Express the answer correct to two decimal places.	$= \$281.94$
5 Answer the question in words.	Clothing costs \$281.94

Desmos widget 8C Credit card interest rate compared to a bank loan

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Exercise 8C

1 A credit card has a daily interest rate of 0.05% per day. Find the interest charged on these outstanding balances. Answer correct to the nearest cent.

a $840 for 12 days
b $742.40 for 20 days
c $5680 for 30 days
d $128 for 18 days
e $240 for 6 days
f $1450 for 15 days

Example 6

2 Joel has a credit card with an interest rate of 0.04% compounding per day and no interest-free period. He uses his credit card to pay for a mobile phone costing $980. Calculate the total amount paid for the mobile phone if Joel paid the credit card account in the following time period. Answer correct to the nearest cent.

a 10 days later
b 20 days later
c 30 days later
d 40 days later
e 50 days later
f 60 days later

3 Calculate the compound interest charged on these outstanding balances. Answer correct to the nearest cent.

a Balance = $6820, Daily interest rate of 0.08%, Time period 70 days
b Balance = $23 648, Daily interest rate of 0.06%, Time period 35 days
c Balance = $1550, Daily interest rate of 0.05%, Time period 20 days
d Balance = $35 800, Daily interest rate of 0.09%, Time period 100 days
e Balance = $4500, Daily interest rate of 0.05%, Time period 27 days
f Balance = $7680, Daily interest rate of 0.04%, Time period 180 days

4 Andrew's credit card charges 0.045% compound interest per day on any outstanding balances. How much interest is Andrew charged on an amount of $450, which is outstanding on his credit card for 35 days? Answer correct to the nearest cent.

5 Olivia received a new credit card with no interest-free period and a daily compound interest rate of 0.05%. She used her credit card to purchase food for $320 and petrol for $50 on 18 July. This amount stayed on the credit card for 24 days. What is the total interest charged? Answer correct to the nearest cent.

6 Jett used his credit card to buy a holiday to New Zealand. The cost of the package was $6500. The charge on the credit card is 1% interest per month on the unpaid balance. How much does Jett owe for his holiday after six months? Answer correct to the nearest cent.

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7 Calculate the amount owed, to the nearest cent, for each of the following credit card transactions. The credit card has no interest-free period.

- **a** Transactions = \$540, Compound interest rate = 14% p.a., Time period = 15 days
- **b** Transactions = \$270, Compound interest rate = 11% p.a., Time period = 9 days
- **c** Transactions = \$1400, Compound interest rate = 18% p.a., Time period = 22 days
- **d** Transactions = \$480, Compound interest rate = 16% p.a., Time period = 18 days
- **e** Transactions = \$680, Compound interest rate = 10% p.a., Time period = 9 days

8 Calculate the interest charged for each of the following credit card transactions. The credit card has no interest-free period. Answer correct to the nearest cent.

- **a** Transactions = \$680, Compound interest rate = 15% p.a., Time period = 20 days
- **b** Transactions = \$740, Compound interest rate = 12% p.a., Time period = 13 days
- **c** Transactions = \$1960, Compound interest rate = 17% p.a., Time period = 30 days
- **d** Transactions = \$820, Compound interest rate = 21% p.a., Time period = 35 days
- **e** Transactions = \$1700, Compound interest rate = 19% p.a., Time period = 32 days

9 Luke has a credit card with a compound interest rate of 18.25% per annum.

- **a** What is the daily percentage interest rate, correct to two decimal places?
- **b** Luke has an outstanding balance of \$4890 for a period of 30 days. How much interest, to the nearest cent, will he be charged?

10 Alyssa uses a credit card with a no interest-free period and a compound interest rate of 15.5% p.a. from the purchase date. During April she makes the following transactions.

Transaction details		
04 April	IGA Supermarket	\$85.00
09 April	KMart	\$115.00
12 April	David Jones	\$340.00
27 April	General Pants	\$80.00
28 April	JB HiFi	\$30.00

- **a** What is the daily compound interest rate, correct to three decimal places?
- **b** Alyssa's account is due on 30 April. What is the total amount due if you disregard the amount of interest to be paid?
- **c** How much interest has Alyssa paid on the IGA transaction during the month? Answer correct to the nearest cent.
- **d** How much interest has Alyssa paid on the KMart transaction during the month? Answer correct to the nearest cent.

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8D Credit card statements

Credit card statements are issued each month and contain information such as account number, opening balance, new charges, payments, refunds, reward points, payment due data, minimum payment and closing balance. The credit card statement includes the date and cost of each purchase and could be regarded as a ledger. A ledger documents your spending.

If the minimum payment is not made by the due date, the consequences can be expensive. You may be charged a late payment fee and, of course, you will be charged interest on it.

Example 7: Reading a credit card statement 8D

Your Bank
Your Bank of Australia
ABN 12 345 678 901

Page number	1 of 2
Statement begins	5 Oct
Statement ends	5 Nov

Enquiries
Credit Card **13 2221**
(24 hours a day, 7 days a week)
Your Bank Awards **13 1661**
(8am to 8pm Mon-Fri)

MR JOHN CITIZEN
123 SAMPLE STREET
SUBURBIA NSW 2000

MasterCard 5353 1801 0001 0001

Opening balance	$207.72
New charges	$460.14
Payments/refund	-$207.72

Payment due date
30th November

Minimum payment
$25.00

Closing balance
$460.14

Your Bank Awards 1000123456

Opening points balance	50,500
Total points earned	460
points redeemed	-15,600

Total Points Balance
34,910

Answer the following questions using the above credit card statement.

a What is the credit card account number?
b What is the opening balance?
c What is the payment due date?
d What is the minimum payment?
e What is the closing balance?

SOLUTION:

1 Read the number after ‘MasterCard’.	**a** 5353 1801 0001 0001
2 Read ‘Opening balance’.	**b** Opening balance is $207.72.
3 Read the box ‘Payment due date’.	**c** Payment due date is 30 Nov.
4 Read the box ‘Minimum payment’.	**d** Minimum payment is $25.00.
5 Read the box ‘Closing balance’.	**e** Closing balance is $460.14.

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Exercise 8D

Example 7 **1** Use the credit card statement opposite to answer these questions.
- **a** What is the due date?
- **b** What is the cost of the purchases?
- **c** What is the closing account balance?
- **d** What is the minimum amount due?
- **e** What payment was made last month?
- **f** How much interest was charged?
- **g** What was the opening balance?
- **h** What is the cardholder's credit balance?

Account summary	
Opening balance	**$743.42**
Payments and other credits	$743.42
Purchases	$172.91
Cash advances	$0.00
Interest and other charges	$0.00
Closing account balance	**$172.91**
Cardholder credit balances	4511.88
Payment summary	
Card balances renewal	$4684.79
Monthly payment	$10.00
Due date	**21 Apr**
Minimum amount due	**$10.00**

2 The transactions on a credit card are shown below.
- **a** What is the credit limit?
- **b** What is the account balance?
- **c** How many transactions are shown?
- **d** What is the available credit?

Account summary			
Available credit	**Account balance**		**Credit limit**
$15 549.18	$3950.82		$19 500.00
	Payment due date		**Minimum payment due**
	7 Dec		$57.00
Last 5 transactions View more			
Date	**Transaction description**	**Debit**	**Credit**
30 Nov	WW Petrol	$24.38	
29 Nov	Coles	$55.03	
29 Nov	Woolworths	$34.63	
28 Nov	Myer	$49.13	
28 Nov	David Jones	$23.40	

- **e** How much was spent on 29 November?
- **f** How much was spent on 28 November?
- **g** Where was $49.13 spent on 28 November?
- **h** Where was $24.38 spent on 30 November?
- **i** What is the payment due date?
- **j** What is the minimum amount due?

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3 Create the spreadsheet below.

8DQ3

C10 =SUM(C5:C9)

	A	B	C
1			
2	Worksheet to create a ledger		
3			
4	*Date*	*Details*	*Amount*
5	20-Nov	Manly Vale Pharmacy Manly Vale	-$20.00
6	25-Nov	Manly Vale Pharmacy Manly Vale	-$18.95
7	01-Dec	Virgin Mobile North Sydney	-$25.00
8	05-Dec	Target 78 Brookvale	-$12.99
9	05-Dec	Pulse Warringah Brookvale	-$30.98
10			**-$107.92**

a How many transactions are shown on the ledger?
b How much has been spent at Manly Vale Pharmacy?
c If the account begins on 15 November and ends on 14 December, how many days does it account for?
d If the card has a $5000 credit limit, what is the available credit on 14 December?
e If the minimum payment is $10 and is paid on the due date, what is the balance owing?
f This credit card charges 0.06% per day compound interest on the unpaid balance. What is the interest charged per day on the closing balance? Answer to the nearest cent.

4 Consider the credit card statement shown opposite.
a What is the opening balance?
b What is the credit limit?
c What is the available credit?
d What is the closing balance?
e How much has been spent on purchases, cash advances and special promo debits this month?
f How much interest and other charges were incurred last month?

Visa Account number	**4557 0756 0833 1234**
Credit limit	$12, 000
Available credit	**$6, 361**
Account summary	
– Opening balance	$5, 821.31 DR
+ Payment & other credits received	$781.25 CR
– Purchases, cash advances & special promo debits	$511.93 DR
– Interest & other charges	$86.26 DR
= Closing balance	$5, 638.25 DR

This credit card charges 0.05% per day compound interest on the unpaid balances.
g What is the interest charged per day on the closing balance? Answer to the nearest cent.
h How much interest would be accrued on the closing balance for a year? Answer to the nearest cent.

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8E Fees and charges for credit card usage

Banks and financial institutions charge their customers an annual card fee for maintaining a credit card account. In addition to this fee, customers may be charged fees for late payment, cash advances and balance transfers. The late payment fee applies if the minimum payment has not been received by the due date. Interest is charged for retail purchases and the amount still owing from the previous month.

FEES AND CHARGES FOR CREDIT CARD USAGE

- Annual card fee – maintaining credit card account
- Interest charge – interest charged for retail purchases
- Late payment fee – when minimum payment has not been received by the due date
- Cash advances – withdrawing cash from the credit card account
- Balance transfers – moving balance to another account, often held at another institution

Example 8: Calculating fees and charges 8E

Hilary has a debit of $6000 on a credit card with an interest rate of 14.75% p.a. that compounds daily. She decided to transfer the balance to a new card with a 0% balance transfer for 6 months. However, after 6 months the new card reverted to an interest rate of 19.75% p.a that compounds daily. Is Hilary better off after 12 months?

SOLUTION:

1 Write the formula.
2 Substitute $PV = 6000$, $r = 0.1475$ and $n = 365$ into the formula.
3 Evaluate correct to two decimal places.

Old card $FV = PV(1 + r)^n$

$$= \$6000\left(1 + \frac{0.1475}{365}\right)^{365}$$

$$\approx \$6953.39$$

4 Write the formula.
5 Substitute $PV = 6000$, $r = 0.1975$ and $n = 182.5$ (6 months only) into the formula.
6 Evaluate correct to two decimal places.

New card $FV = PV(1 + r)^n$

$$= \$6000\left(1 + \frac{0.1975}{365}\right)^{182.5}$$

$$\approx \$6622.57$$

7 Calculate the saving by subtracting the future value of the new card from the old card.

Saving = $6953.39 − $6622.57
= $330.82

8 Write the answer in words.

Hilary is better off with the new card by $330.82.

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Exercise 8E

Example 8 **1** Alicia's bank charged an annual credit card fee of $350, a cash advance fee of $2.50 and a late payment fee of $20. Calculate Alicia's banking costs for the year if she made:

- **a** 11 cash advances and 4 late payments
- **b** 20 cash advances and 12 late payments
- **c** 50 cash advances and 6 late payments
- **d** 0 cash advances and 12 late payments
- **e** 100 cash advances and 0 late payments
- **f** 0 cash advances and 0 late payments.

2 The table below shows the credit card usage charges for four banks.

Bank	Annual fee	Cash advance	Late payment
A	$225	$2.00	$15
B	$200	$2.20	$20
C	$250	$1.80	$12
D	$240	$1.90	$16

- **a** What is the cost of the cash advance fee at bank B?
- **b** What is the cost of the late payment fee at bank D?
- **c** Which bank has the lowest annual fee?
- **d** Which bank has the highest cash advance fee?
- **e** Calculate the difference between the late payment fees at bank C and bank D.
- **f** Calculate the difference between the cash advance fees at bank B and bank C.
- **g** What is the average annual fee for these banks?
- **h** What is the average late payment fee for these banks?
- **i** What are the annual banking costs for 30 cash advances and 1 late payment at:
  - **i** Bank A?
  - **ii** Bank B?
  - **iii** Bank C?
  - **iv** Bank D?
- **j** What are the annual banking costs for 100 cash advances and 6 late payments at:
  - **i** Bank A?
  - **ii** Bank B?
  - **iii** Bank C?
  - **iv** Bank D?

3 Elijah's bank charged an annual credit card fee of $320, cash advance fee of $2.30 and late payment fee of $18. What are Elijah's banking costs for the year if he made 80 cash advances and had 1 late payment fee?

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PPT STUDY GUIDE

Key ideas and chapter summary

Summary

Declining-balance depreciation

$S = V_0(1 - r)^n$

S – Salvage value or current value
V_0 – Purchase price of the item
r – Rate of interest per time period (decimal)
n – Number of time periods

Reducing-balance loans

Total to be paid = Loan repayment × Number of repayments
Total to be paid = Principal + Interest

Fees and charges for a loan

- Loan application fee – costs in setting up the loan
- Loan establishment fee – initial costs in process the loan application
- Account service fee – ongoing account-keeping fee.
- Valuation fee – assessment of the market value of a property.
- Legal fee – legal processing of a property.

Credit cards

$$\text{Daily interest rate} = \frac{\text{Annual interest rate}}{365}$$

$FV = PV(1 + r)^n$ and $I = FV - PV$

FV – Future value or the amount owing on the credit card
PV – Present value or the purchases made on the credit card
r – Rate of interest per compounding time period as a decimal
n – Number of compounding time periods
I – Interest (compound) charged for the use of their credit card

Credit card statements

Credit card statements are issued each month and contain information such as account number, opening balance, new charges, payments, refunds, reward points, payment due data, minimum payment and closing balance.

Fees and charges for credit card usage

- Annual card fee – maintaining credit card account
- Interest charge – interest charged for retail purchases
- Late payment fee – when minimum payment has not been received by the due date
- Cash advances – withdrawing cash from the credit card account
- Balance transfers – moving balance to another account, often held at another institution

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Review

Multiple-choice

1 A new car bought for $39000 depreciates at 25% per annum and is expected to be used for 4 years. How much is the car worth after 4 years?

A $9390 **B** $9750 **C** $12340 **D** $29250

2 The table shows the monthly repayment of $1000 on a reducing-balance loan. What is the monthly repayment on $290000 at 8.75% for 20 years?

Term	8.00%	8.25%	8.50%	8.75%
20 years	$6.38	$6.77	$7.17	$7.57

A $1850.20 **B** $1963.30 **C** $2079.30 **D** $2195.30

3 Lachlan borrows $245000 over 20 years at a reducible interest rate of 6.5% p.a. He pays $1856 per month. What is the total paid on this loan?

A $200440 **B** $318500 **C** $445440 **D** $563500

4 A credit card has a compound interest rate of 16% p.a. (no interest free period). Find the interest charged on $4200 for 30 days. Answer correct to the nearest dollar.

A $22 **B** $56 **C** $674 **D** $4256

5 A credit card has a daily interest rate of 0.05% per day (no interest free period). Find the interest charged on $1530 for 14 days. Answer correct to the nearest cent.

A $0.77 **B** $10.74 **C** $76.50 **D** $1540.74

6 Elijah's bank charged an annual credit card fee of $320, cash advance fee of $2.30 and late payment fee of $18. What are Elijah's banking costs for the year if he made 80 cash advances and had 1 late payment fee?

A $202.00 **B** $340.30 **C** $504.00 **D** $522.00

7 Michael has a debt of $16000 on a credit card with a compound interest rate of 14% p.a. He decided to transfer the balance to a new card with a 0% balance transfer for 6 months. How much does he save in the first 12 months if the new card has an interest rate of 16% p.a.?

A 1008 **B** $1232 **C** $2240 **D** $18240

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Review

Short-answer

1 Alexis purchased a car for $19 900. It depreciates at 24% per year.

a What is the salvage value of the car after 5 years? Answer to the nearest dollar.

b How much has the car depreciated over the 5 years?

2 Paige takes out a loan of $21 000 over 36 months. The repayment rate is $753.42 per month.

a How much will Paige pay back altogether? Answer to the nearest dollar.

b What was the interest charged on Paige's loan?

3 James borrows $280 000 and repays the loan in equal fortnightly repayments of $1250 over 20 years. What was the interest charged on James's loan?

4 Madison has a credit card with an interest rate of 17% p.a. compounding daily and no interest-free period. Madison used her credit card to pay for shoes costing $170. She paid the credit card account 26 days later. What is the total amount she paid for the shoes including the interest charged? Answer to the nearest cent.

5 Hayley's bank charged an annual credit card fee of $300, a cash advance fee of $4.00 and a late payment fee of $20. Calculate Hayley's banking costs for the year if she made:

a 9 cash advances and 5 late payments

b 15 cash advances and 7 late payments.

6 Benjamin uses a credit card with a no interest-free period and a compound interest rate of 18.5% p.a. compounding daily from and including the purchase date and due date. Benjamin's account is due on February 28. During February he makes the following transactions.

Transaction Details		
06 February	Coles	$278.00
07 February	Myer	$87.00
18 February	Big W	$259.00
18 February	Jag	$120.00
20 February	Bunnings	$460.00
21 February	Woolworths	$300.00

How much interest will Benjamin pay during the month on the following transactions? Answer correct to the nearest cent.

a Coles transaction

b Big W transaction

c Bunning transaction

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Review

7 Transactions on a credit card with an interest rate of 20% p.a. are shown below.

Post date	Tran date	Description	Amount
Derrick Tan		4512-XXXX-XXXX-6650	
		Previous statement balance	3696.05
31 May	26 May	Payment – Thank you	110.88CR
15 Jun	15 Jun	Best Denki-Plaza	99.56
15 Jun	15 Jun	Harvey Norman	$104.08
15 Jun	15 Jun	Finance charge	81.30
Danielle Tan		4512-XXXX-XXXX-7344	
13 Jun	13 Jun	IKEA	100.00
		Total due	**$3970.11**

a What is the previous statement balance?
b How much was paid on 26 May?
c What is the balance owing on 1 Jun?
d How much did Danielle Tan spend on 13 Jun?
e What is the balance owing on 14 Jun?
f How much was spent at Harvey Norman on 15 Jun?
g How much was the finance charge?
h What is the closing balance?
i How much interest would be paid on the closing balance for a year?
j How much interest would be paid on the closing balance for two years?

8 Marcus has a debit of $12 000 on a credit card with an interest rate of 13% p.a. He decided to transfer the balance to a new card with a 0% balance transfer for 6 months. However, after 6 months the new card reverted to an interest rate of 21.25% p.a. Is Marcus better off after 24 months? Answer to the nearest dollar.

9 A credit card statement shows a closing balance of $5620.60 and a charge of 0.06% per day compound interest on the unpaid balances. What is the interest charged per day on the closing balance? Answer to the nearest cent.

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10 Jenny's bank charged an annual credit card fee of $400, cash advance fee of $4.30 and late payment fee of $14. What are Jenny's banking costs for the year if she made 80 cash advances and had 3 late payment fees?

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9 Graphs of practical situations

Syllabus topic — A3.2 Graphs of practical situations

This topic will develop your skills in graphing non-linear functions and how they can be used to model and solve a range of practical problems.

Outcomes

- Construct a graph of an exponential function using a table of ordered pairs.
- Use an exponential model to solve a practical problem.
- Construct a graph of an quadratic function using a table of ordered pairs.
- Use a quadratic model to solve a practical problem.
- Construct a graph of an reciprocal function using a table of ordered pairs.
- Use a reciprocal model to solve a practical problem.

Digital Resources for this chapter

In the Interactive Textbook:

- Videos
- Literacy worksheet
- Quick Quiz
- Solutions (enabled by teacher)
- Desmos widgets
- Spreadsheets
- Study guide

In the Online Teaching Suite:

- Teaching Program
- Tests
- Review Quiz
- Teaching Notes

Knowledge check

The Interactive Textbook provides a test of prior knowledge for this chapter, and may direct you to revision from the previous years' work.

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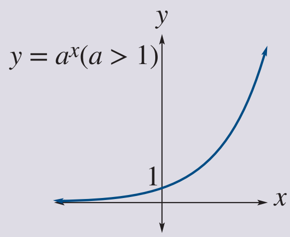
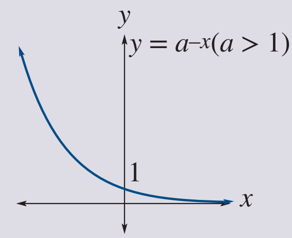
9A Graphs of exponential functions

Exponential functions have x as the power of a constant (e.g. 3^x). They are defined by the general rule $y = a^x$ and $y = a^{-x}$ where the constant $a > 0$. Their graphs are shown below.

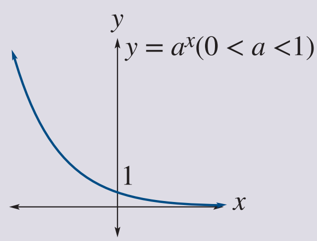
GRAPHS OF EXPONENTIAL FUNCTIONS

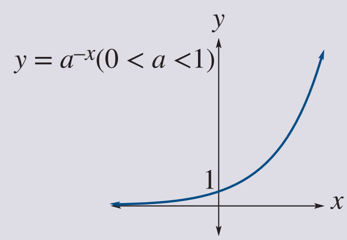
Most practical uses of exponential functions have $a > 1$:





When a is greater than 0 but less than 1, the shape of the curve is reversed horizontally:





When $a = 1$ the graph is flat line, $y = 1$.

Key features of exponential graphs

- The graph lies wholly above the x-axis because a^x is always positive, regardless of the value of x. It is impossible for the y values to be zero or negative.
- The graph always passes through the point (0, 1) because when $x = 0$ then $y = a^0 = 1$, regardless of the value of a.
- The x-axis is an asymptote. That is, it is a line that the curve approaches by getting closer and closer to it but never reaching it.
- The graph $y = a^{-x}$ is the reflection of the graph $y = a^x$ about the y-axis.
- Increasing the value of a such as changing $y = 2^x$ to $y = 3^x$ affects the steepness of the graph. The y values increase at a greater rate when the x values increase.
- The exponential function $y = a^x$ when $a > 1$ is often referred to as a growth function because as the x values increase, the y values increase.
- The exponential function $y = a^{-x}$ when $a > 1$ is often referred to as a decay function because as the x values increase, the y values decrease.

To graph an exponential function:

1. Construct a table of values.
2. Draw a number plane.
3. Plot the points.
4. Join the points to make a curve.

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Example 1: Graphing an exponential function 9A

Draw the graph of $y = 3^x$.

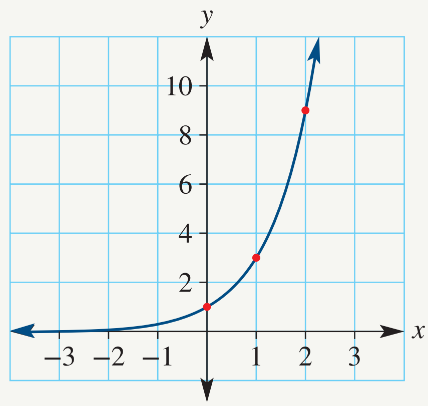
SOLUTION:

1 Construct a table of values for x and y.
2 Let $x = -3, -2, -1, 0, 1, 2$ and 3. Find y using the exponential function $y = 3^x$.

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27

3 Draw a number plane with x as the horizontal axis and y as the vertical axis.
4 Plot the points $(-3, \frac{1}{27})$, $(-2, \frac{1}{9})$, $(-1, \frac{1}{3})$, $(0, 1)$, $(1, 3)$, $(2, 9)$ and $(3, 27)$.
5 Join the points to make a curve.

Note: Sometimes it is necessary to rescale the axes to plot the points as some points are impractical to plot, such as $(-3, \frac{1}{27})$.



Example 2: Graphing an exponential function 9A

Draw the graph of $y = 3^{-x}$.

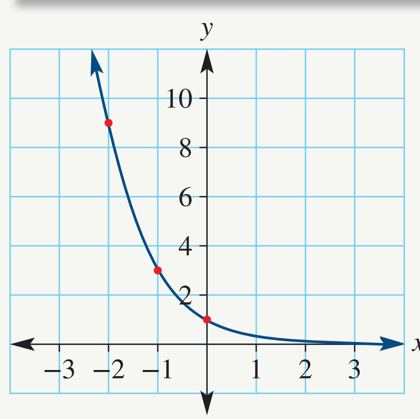
SOLUTION:

1 Construct a table of values for x and y.
2 Let $x = -3, -2, -1, 0, 1, 2$ and 3. Find y using the exponential function $y = 3^{-x}$.

x	-3	-2	-1	0	1	2	3
y	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$

3 Draw a number plane with x as the horizontal axis and y as the vertical axis.
4 Plot the points $(-3, 27)$, $(-2, 9)$, $(-1, 3)$, $(0, 1)$, $(1, \frac{1}{3})$, $(2, \frac{1}{9})$ and $(3, \frac{1}{27})$.
5 Join the points to make a curve.

Note: The exponential curve $y = 3^{-x}$ is the reflection of $y = 3^x$ about the y-axis. Both curves pass through $(0, 1)$ and have the x-axis as an asymptote.



Desmos widget 9A Graphing exponential functions with technology

9A Spreadsheet activity: Graphing exponential functions

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Exercise 9A

Example 1 **1** Complete the following table of values and graph each exponential function.

a $y = 2^x$

x	–2	–1	0	1	2
y		$\frac{1}{2}$			

b $y = 2^{-x}$

x	–2	–1	0	1	2
y					$\frac{1}{4}$

2 Use the graph of $y = 2^x$ in question **1a** to answer these questions.

a Is it possible for y to have negative values?
b Is it possible for x to have negative values?
c Is it possible to calculate y when $x = 0$? If so, what is it?
d Is it possible to calculate x when $y = 0$? If so, what is it?
e What is the approximate value of y when $x = 0.5$?
f What is the approximate value of y when $x = 1.5$?

Example 2 **3** Complete the following table of values by expressing the y values, correct to two decimal places. Graph each exponential function.

a $y = 4^x$

x	–3	–2	–1	0	1	2	3
y	0.02						

b $y = 4^{-x}$

x	–3	–2	–1	0	1	2	3
y						0.06	

4 Use the graph of $y = 4^x$ in question **3a** to answer these questions.

a Is it possible for y to have negative values?
b Is it possible for x to have negative values?
c Is it possible to calculate y when $x = 0$? If so, what is it?
d Is it possible to calculate x when $y = 0$? If so, what is it?
e What is the value of y when $x = 1.5$?
f What is the value of y when $x = 2.5$?

5 Sketch the graph of the following functions on the same set of axes.

a $y = 2^x$ **b** $y = 3^x$ **c** $y = 4^x$

6 Sketch the graph of the following functions on the same set of axes.

a $y = 2^{-x}$ **b** $y = 3^{-x}$ **c** $y = 4^{-x}$

7 What is the effect on the graph of changing the value of a in $y = a^x$?
Hint: Use your graphs in questions **5** and **6**.

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9B Exponential models

Exponential modelling occurs when a practical situation is described mathematically using an exponential function. The quantity usually experiences fast growth or decay.

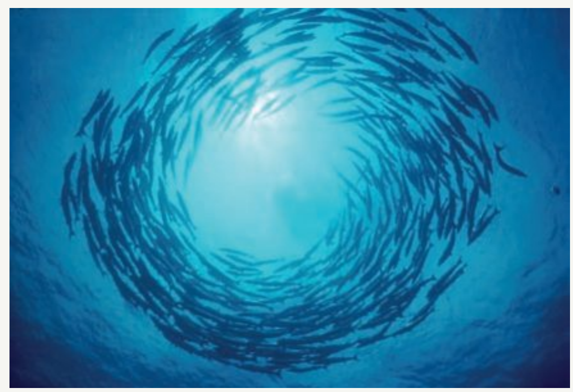
EXPONENTIAL MODEL

Exponential growth – Quantity increases rapidly according to the function $y = a^x$ where $a > 1$.

Exponential decay – Quantity decreases rapidly according to the function $y = a^{-x}$ where $a > 1$.

Example 3: Using an exponential model 9B

The fish population is predicted using the formula, $N = 500 \times 1.5^t$ where N is the number of fish and t is the time in years.



a Construct a table of values for t and N. Use values for t from 0 to 4. Approximate the number of fish to the nearest whole number.
b Draw the graph of $N = 500 \times 1.5^t$.
c How many fish were present after 2 years?
d How many extra fish will be present after 4 years compared to 2 years?
e Estimate the number of fish after 18 months.

SOLUTION:

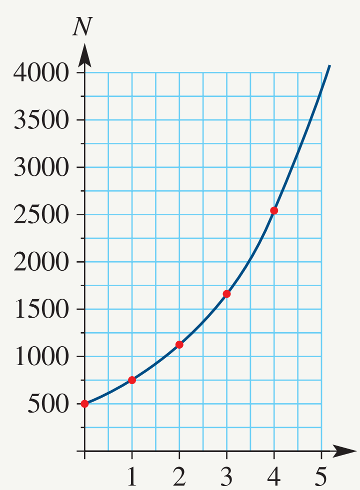
1 Construct a table of values for t and N.
2 Let $t = 0, 1, 2, 3$ and 4. Find N using the exponential function $N = 500 \times 1.5^t$. Express the values for N as a whole number.

a

t	0	1	2	3	4
N	500	750	1125	1688	2531

3 Draw a number plane with t as the horizontal axis and N as the vertical axis.
4 Plot the points (0, 500), (1, 750), (2, 1125), (3, 1688) and (4, 2531).
5 Join the points to make the curve.

b



6 Look up $t = 2$ in the table and find N.
c 1125 fish

7 Subtract the number of fish for 2 years from 4 years.
d Extra = 2531 − 1125 = 1406

8 Read the approximate value of N from the graph when $t = 1.5$.
e Approximately 900 fish

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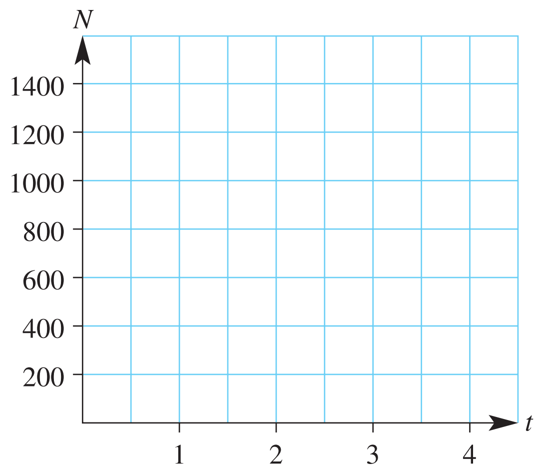
Exercise 9B

Example 3 **1** The exponential function, $N = 6^t$, is used to model the growth in the number of insects (N) after t days.

a Copy and complete the table of values for t and N.

t	0	1	2	3	4
N					

b Copy and draw the graph of $N = 6^t$ on the number plane below.



c What was the initial number of insects?

d How many insects were present after 3 days?

e How many extra insects will be present after 4 days compared with 2 days?

f How many days did it take for the number of insects to exceed 1000?

2 The population of an endangered reptile is decreasing exponentially according to the formula $P = 1.5^{-t} \times 100$, where P is the population of reptiles after t years.

a Copy and complete the table of values for t and P. Express the population of reptiles to the nearest whole number.

t	0	2	4	6	8
N					

b Graph $P = 1.5^{-t} \times 100$ using the table of ordered pairs in part **a**.

c What is the initial population of reptiles?

d Estimate the population of reptiles after 3 years?

e Estimate the population of reptiles after 7 years?

f What is the difference in the population of reptiles after 2 years compared with 6 years?

g Estimate the time taken (to the nearest year) for the population of reptiles to be less than 1.

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3 The size of a flock of birds, F, after t years is decaying exponentially using the function $F = 200 \times 0.5^t$.

- **a** Make a table of values for t and F. Use values for t from 0 to 5. Express F correct to the nearest whole number.
- **b** Draw the graph of $F = 200 \times 0.5^t$.
- **c** What was the initial flock of birds?
- **d** How many birds were present after 6 months (0.5 years)?
- **e** How many birds were present after 3 years?
- **f** How many birds were present after 5 years?
- **g** How many extra birds will be present after 1 year compared with 3 years?
- **h** How many fewer birds will be present after 2 years compared with 4 years?
- **i** How many years will it take for the number of birds to fall to less than one bird?

4 The number of algae grows exponentially according to the function, $b = 30 \times 1.2^t$ where b is the number of algae after t hours.

- **a** Construct a table of ordered pairs using 0, 5, 10, 15 and 20 as values for t. Express the number of algae to the nearest whole number.
- **b** Graph $b = 30 \times 1.2^t$ using the table of ordered pairs in part **a**.
- **c** What is the initial number of algae?
- **d** What is the number of algae after 4 hours?
- **e** What is the number of algae after 8 hours?
- **f** What is the number of algae after 12 hours?
- **g** What is the number of algae after 16 hours?
- **h** Estimate the time taken for the algae to reach 120.

5 Tom invested \$1000 into a managed fund that appreciated in value for 5 years. The amount of money (A) in the fund for each year (t) is shown below.

t	0	1	2	3	4	5
A	\$1000	\$1300	\$1690	\$2197	\$2856	\$3712

- **a** Draw a number plane with t as the horizontal axis and A as the vertical axis.
- **b** Plot the points from the table of values. Join the points to make a curve.
  An exponential growth model in the form $y = 1000 \times 1.3^x$ describes this situation.
- **c** Use the model to find the value (to the nearest dollar) of the fund after $2\frac{1}{2}$ years.
- **d** Use the model to find the value (to the nearest dollar) of the fund after 7 years.
- **e** What is the time when the value of the fund is approximately \$1480?

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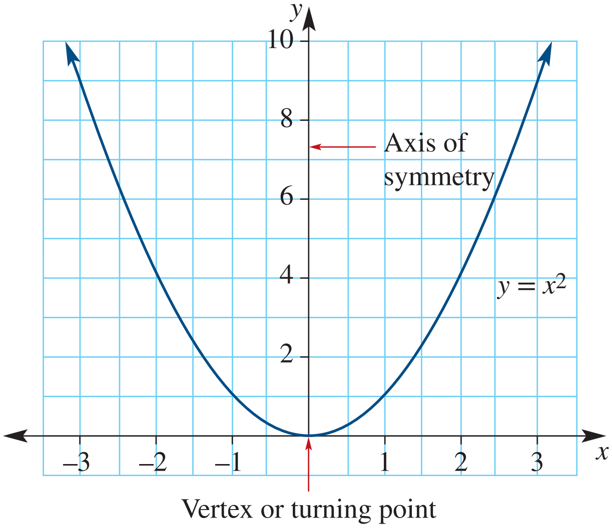
9C Quadratic functions

A quadratic function is a curve whose equation has an x squared (x^2). It is defined by the general rule $y = ax^2 + bx + c$ where a, b and c are numbers. Quadratic functions are graphed in a similar method to exponential functions except the points are joined to make a curve in the shape of a parabola.

Key features of a parabola

The basic parabola has the equation $y = x^2$.

- The vertex (or turning point) is (0, 0).
- It is a minimum turning point.
- Axis of symmetry is $x = 0$ (the y-axis)
- y-intercept is 0 and x-intercept is 0.
- The graph $y = -x^2$ is the reflection of the graph $y = x^2$ about the x-axis.
- Changing the coefficient of the equation such as $y = 2x^2$ or $y = \frac{1}{2}x^2$ affects the height of the parabola.

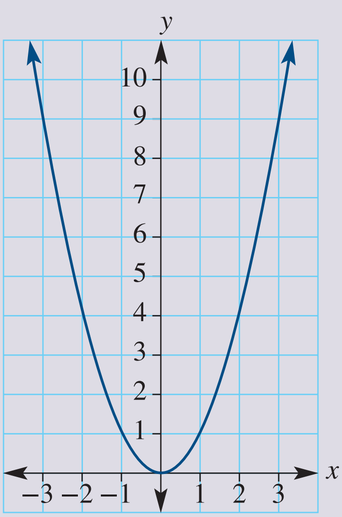


- Adding or subtracting a number to the equation such as $y = x^2 + 1$ or $y = x^2 - 1$ does not change the shape but moves the parabola up or down.

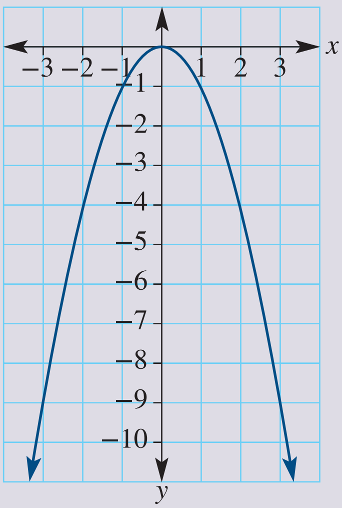
QUADRATIC FUNCTION

A quadratic function has the form $y = ax^2 + bx + c$ where a, b and c are numbers.

Parabola ($y = x^2$) – Minimum turning point



Parabola ($y = -x^2$) – Maximum turning point



To graph a parabola:

1. Construct a table of values.
2. Draw a number plane.
3. Plot the points.
4. Join the points to make a parabola.

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Example 4: Graphing a quadratic function 9C

Draw the graph of $y = x^2 + 1$.

SOLUTION:

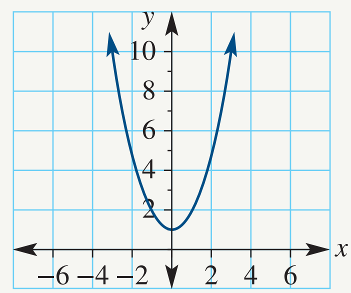
1 Construct a table of values for x and y using $x = -3, -2, -1, 0, 1, 2$ and 3. Find y by substituting into $y = x^2 + 1$.

2 Draw a number plane with x as the horizontal axis and y as the vertical axis.

3 Plot the points $(-3, 10)$, $(-2, 5)$, $(-1, 2)$, $(0, 1)$, $(1, 2)$, $(2, 5)$ and $(3, 10)$.

4 Join the points to make a curve in the shape of a parabola.

x	−3	−2	−1	0	1	2	3
y	10	5	2	1	2	5	10



Example 5: Determining the features of a parabola 9C

Draw the graph of $y = x^2 - 4x + 3$ (use $x = -1, 0, 1, 2, 3, 4, 5$) and find the following features.

a turning point **b** axis of symmetry **c** y-intercept

d x-intercepts **e** minimum value

SOLUTION:

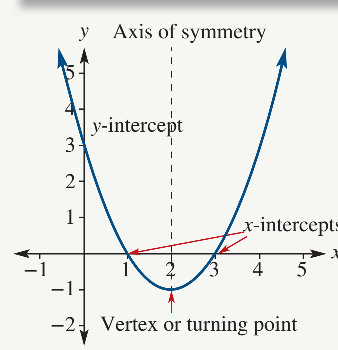
1 Construct a table of values for x and y using $x = -1, 0, 1, 2, 3, 4$ and 5. Find y by substituting into $y = x^2 - 4x + 3$.

2 Draw a number plane with x as the horizontal axis and y as the vertical axis.

3 Plot the points $(-1, 8)$, $(0, 3)$, $(1, 0)$, $(2, -1)$, $(3, 0)$, $(4, 3)$ and $(5, 8)$.

4 Join the points to make a curve in the shape of a parabola.

x	−1	0	1	2	3	4	5
y	8	3	0	−1	0	3	8



5 Find where the graph changes direction.

6 Find the line that splits the graph into two.

7 Find the point where the graph cuts the y-axis.

8 Find the point where the graph cuts the x-axis.

9 Determine the smallest value of y.

a Turning point is $(2, -1)$

b Axis of symmetry is $x = 2$

c y-intercept is 3 $(0, 3)$

d x-intercepts are 1 and 3

e Minimum value is -1

Desmos widget 9C Graphing a quadratic function with technology

9C Spreadsheet activity: Graphing a quadratic function with a spreadsheet

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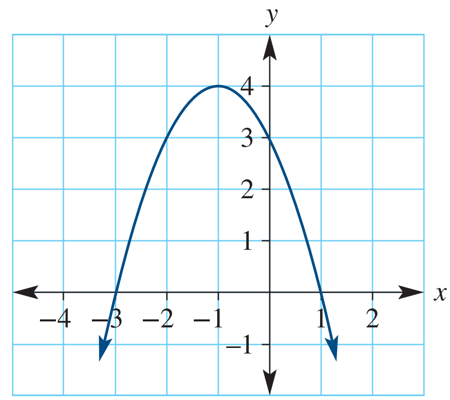
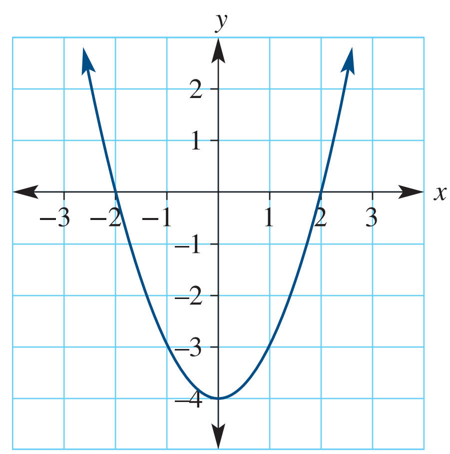
Exercise 9C

1 Write the missing features for this graph.



a The coordinates of the turning point are __________.
b The y-intercept is __________.
c The x-intercepts are __________ and __________.
d The axis of symmetry is __________.
e The maximum value is __________.

2 Write the missing features for this graph.



a The coordinates of the turning point are __________.
b The y-intercept is __________.
c The x-intercepts are __________ and __________.
d The axis of symmetry is __________.
e The minimum value is __________.

3 Complete the following tables of values and graph each quadratic function.

a $y = x^2$

x	–3	–2	–1	0	1	2	3
y							

b $y = 2x^2$

x	–3	–2	–1	0	1	2	3
y							

c $y = 3x^2$

x	–3	–2	–1	0	1	2	3
y							

d $y = \frac{1}{2}x^2$

x	–3	–2	–1	0	1	2	3
y							

e What is the axis of symmetry for each of the above quadratic functions?
f Is the turning point for each of the above quadratic functions a maximum or minimum?
g What is the effect of changing the coefficient of x^2 in the quadratic equation $y = x^2$?

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4 Complete each table of values and graph the quadratic functions on the same number plane.

a $y = -x^2$

x	–3	–2	–1	0	1	2	3
y							

b $y = -2x^2$

x	–3	–2	–1	0	1	2	3
y							

c $y = -3x^2$

x	–3	–2	–1	0	1	2	3
y							

d $y = -\frac{1}{2}x^2$

x	–3	–2	–1	0	1	2	3
y							

e What is the axis of symmetry for each of the above quadratic functions?

f Is the turning point for each of the above quadratic functions a maximum or minimum?

g What is the effect of changing the coefficient of x^2 in the quadratic equation $y = -x^2$?

Example 4

5 Complete the following table of values and graph each quadratic function on the same number plane.

a $y = x^2 + 1$

x	–3	–2	–1	0	1	2	3
y							

b $y = x^2 - 1$

x	–3	–2	–1	0	1	2	3
y							

c $y = x^2 + 2$

x	–3	–2	–1	0	1	2	3
y							

d $y = x^2 - 2$

x	–3	–2	–1	0	1	2	3
y							

e What is the axis of symmetry for each of the above quadratic functions?

f Is the turning point for each of the above quadratic functions a maximum or minimum?

g What is the effect of adding or subtracting a number to the quadratic function $y = x^2$?

Example 5

6 Complete the following table of values and graph each quadratic function on the same number plane.

a $y = x^2 + 2x + 1$

x	–6	–4	–2	0	2	4	6
y							

b $y = x^2 + 4x + 4$

x	–6	–4	–2	0	2	4	6
y							

c $y = x^2 - 2x + 1$

x	–6	–4	–2	0	2	4	6
y							

d $y = x^2 - 4x + 4$

x	–6	–4	–2	0	2	4	6
y							

e What do all of the above quadratic functions have in common?

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9D Quadratic models

Quadratic modelling occurs when a practical situation is described mathematically using a quadratic function.

QUADRATIC MODEL

A quadratic model describes a practical situation using a function in the form $y = ax^2 + bx + c$ where a, b and c are numbers. Quadratic functions are graphed to make a curve in the shape of a parabola.

Example 6: Using a quadratic model 9D

The area (A) of a rectangular garden of length x metres is given by $A = 6x - x^2$.

x	0	1	2	3	4	5	6
A							

a Draw the graph of $A = 6x - x^2$ using the table of ordered pairs.
b Use the graph to estimate the area of the garden when the length of the garden is 4.5 m.
c What is the maximum area of the garden?
d What is the garden length in order to have maximum area?

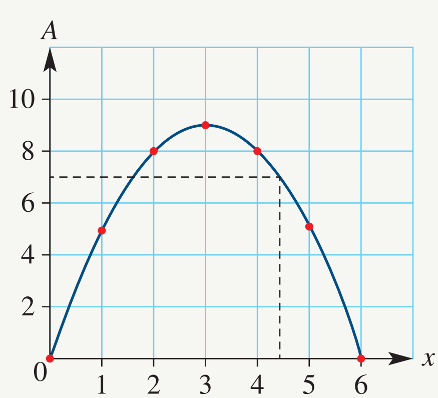
SOLUTION:

1 Let $x = 0, 1, 2, 3, 4, 5$ and 6 and find A by substituting into the quadratic function $A = 6x - x^2$.

a

x	0	1	2	3	4	5	6
A	0	5	8	9	8	5	0

2 Draw a number plane with x as the horizontal axis and A as the vertical axis.
3 Plot the points (0, 0), (1, 5), (2, 8), (3, 9), (4, 8), (5, 5) and (6, 0).
4 Join the points to make a parabolic curve.
5 Draw a vertical line from $x = 4.5$ on the horizontal axis until it intersects the parabola. At this point draw a horizontal line until it connects with the vertical axis.



6 Read this value.

b About 7 m^2. Check your solution algebraically.

$$A = 6x - x^2$$
$$= 6 \times 4.5 - 4.5^2$$
$$= 6.75\,m^2$$

7 Read the largest value for A.
c Maximum area of the garden is 9.

8 Read the value on the x-axis when the A is largest.
d Maximum area occurs when $x = 3$.

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Exercise 9D

Example 6

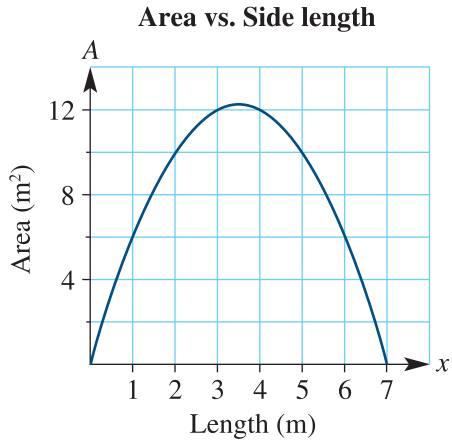
1 The area (A) of a rectangular enclosure of length x metres is given by the formula $A = x(7 - x)$. The graph of this formula is shown opposite.

a What is the area of the enclosure when x is 1 metre?

b What is the area of the enclosure when x is 5 metres?

c What is the enclosure's length in order to have maximum area?

d What is the maximum area of the enclosure?



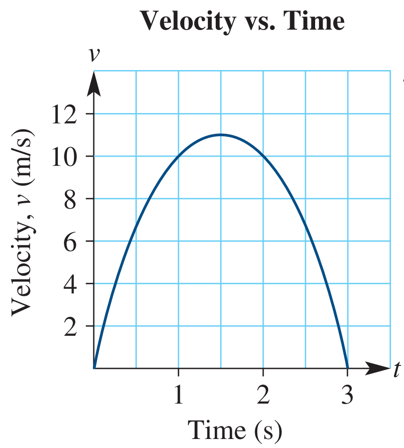
2 The movement of an object with a velocity v (in m/s) at time t (s) is given by the formula $v = 15t - 5t^2$. The graph of this formula is shown opposite.

a What was the initial velocity of the object?

b What was the greatest velocity reached by the object?

c How many seconds did it take for the object to reach maximum velocity?

d Determine the number of seconds when the velocity is greater than 6 m/s. Answer to the nearest second.



3 The price ($$P$) of rope depends on the diameter (d), in metres, of the rope when it is rolled into a circle. The quadratic equation $P = 3d^2$ is used to model this situation.

a Complete the following table of values, correct to the nearest whole number.

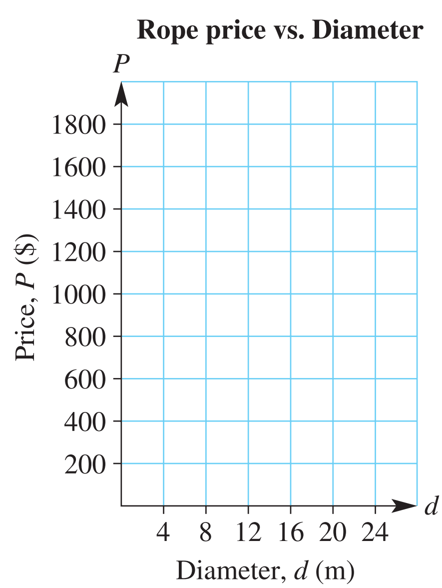
d	0	2	4	6	8	10	12	14	16	18	20	22	24
P													

b Draw the graph of $P = 3d^2$ using the number plane shown opposite.

c What is the price of the rope when the diameter of the rope is 12 metres?

d What is the price of the rope when the diameter of the rope is 23 metres?

e What is the difference between the price of the rope when the diameter of the rope is 5 metres compared with a diameter of 25 metres?



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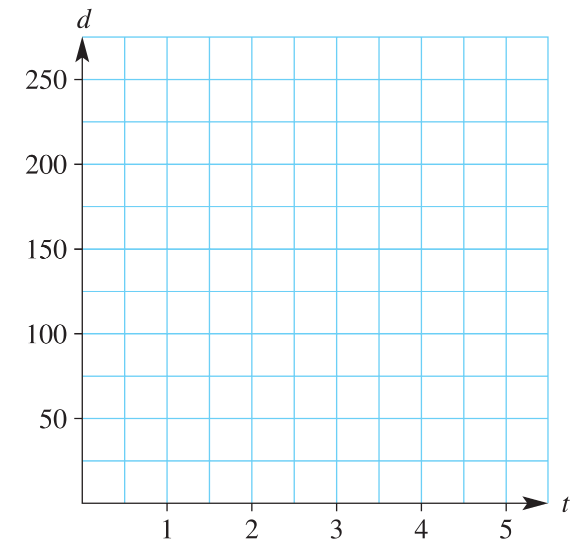
4 A stone falls from rest down a mine shaft. The distance it falls, d metres, at time t seconds is given by the quadratic equation $d = 9.8t^2$.

a Complete the following table of values, correct to the nearest whole number.

t	0	1	2	3	4	5
d						

b Draw the graph of $d = 9.8t^2$ using the number plane shown opposite.



c What is the distance travelled by the stone after 1 second?

d What is the distance travelled by the stone after 5 seconds?

e What is the distance travelled by the stone after 2.5 seconds?

f How long did it take for the stone to travel 100 metres?

g How long did it take for the stone to travel 200 metres?

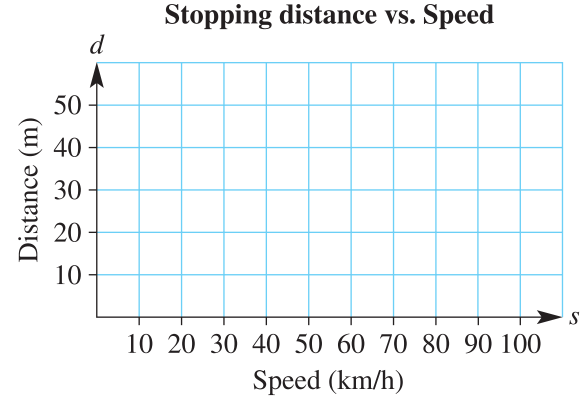
5 The equation $d = 0.005s(s - 1)$ is used to model the stopping distance for a train where d is the stopping distance in metres and s is the train's speed in km/h.

a Complete the following table of values, correct to the nearest whole number.

s	0	20	40	50	60	80	100
d							

b Draw the graph of $d = 0.005s(s - 1)$ using the number plane shown opposite.

Stopping distance vs. Speed



c What is the stopping distance when the train is travelling at 20 km/h?

d What is the stopping distance when the train is travelling at 75 km/h?

e What is the maximum speed (in km/h) a train could be travelling to stop within 15 m?

f What is the maximum speed (in km/h) a train could be travelling to stop within 30 m?

g What is the difference between the stopping distances when a train is travelling at a speed of 40 km/h compared with travelling at a speed of 80 km/h?

h What is the difference between the stopping distances when a train is travelling at a speed of 15 km/h compared with travelling at a speed of 95 km/h?

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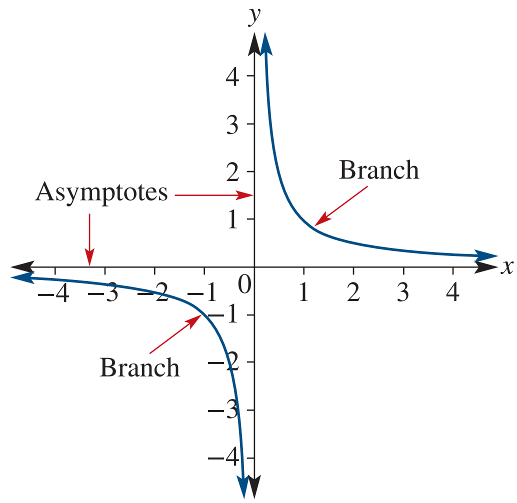
9E Graphs of reciprocal function

A reciprocal function is a curve whose equation has a variable in the denominator such as $\frac{1}{x}$. It is defined by the general rule $y = \frac{k}{x}$ where k is a number. Reciprocal functions are graphed in a similar method to other non-linear functions and make a curve called a hyperbola.

Key features of a hyperbola

The basic hyperbola has the equation $y = \frac{1}{x}$.

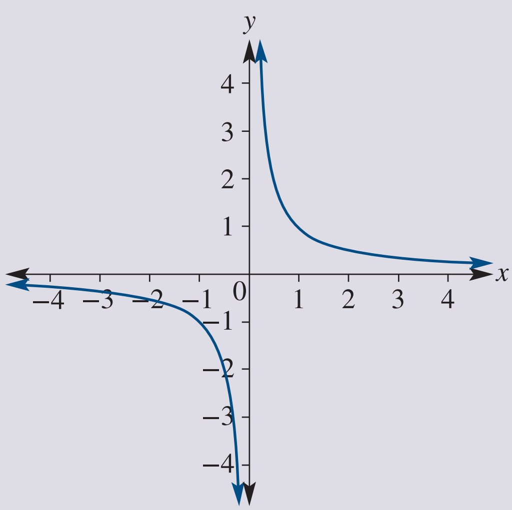
- No value exists for y when $x = 0$.
- The curve has two parts called branches. Each branch is the same shape and size; they are symmetrical and are in opposite quadrants.
- The x-axis and the y-axis are asymptotes of the curve. That is, the curve approaches the x-axis and the y-axis but never touches them.
- The asymptotes are at right angles to each other, so the curve is also called a rectangular hyperbola.



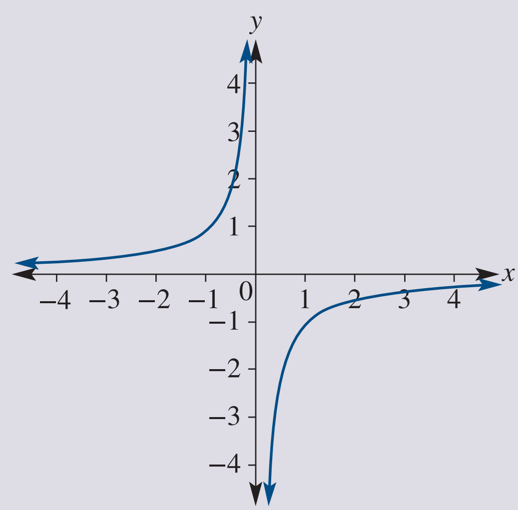
RECIPROCAL FUNCTION

A reciprocal function has the form $y = \frac{k}{x}$ where k is a number.

Hyperbola: $y = \frac{1}{x}$



Hyperbola: $y = -\frac{1}{x}$



To graph a hyperbola:

1. Construct a table of values.
2. Draw a number plane.
3. Plot the points.
4. Join the points to make a hyperbola.

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Example 7: Graphing a reciprocal function 9E

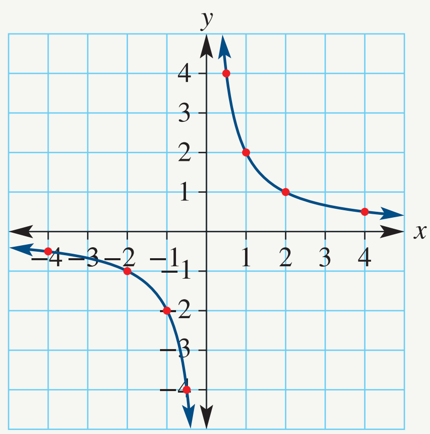
Draw the graph of $y = \frac{2}{x}$.

SOLUTION:

1 Construct a table of values for x and y.
2 Let $x = -4, -2, -1, -0.5, 0.5, 1, 2$ and 4. Find y using the reciprocal function.

x	−4	−2	−1	−0.5	0.5	1	2	4
y	−0.5	−1	−2	−4	4	2	1	0.5

3 Draw a number plane with x as the horizontal axis and y as the vertical axis.
4 Plot the points (−4, −0.5), (−2, −1), (−1, −2), (−0.5, −4), (0.5, 4), (1, 2), (2, 1) and (4, 0.5).
5 No value exists for y when $x = 0$. This results in the curve having two branches.
6 Join the points to make a curve in the shape of a hyperbola.



Example 8: Graphing a reciprocal function 9E

a Draw the graph of $y = -\frac{2}{x}$. **b** What are the asymptotes for this graph?

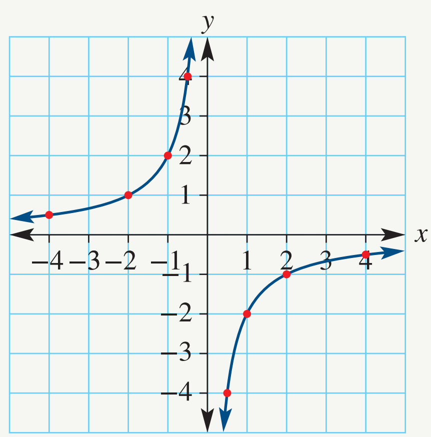
SOLUTION:

1 Construct a table of values for x and y.
2 Let $x = -4, -2, -1, -0.5, 0.5, 1, 2$ and 4. Find y using the reciprocal function.

a

x	−4	−2	−1	−0.5	0.5	1	2	4
y	0.5	1	2	4	−4	−2	−1	−0.5

3 Draw a number plane with x as the horizontal axis and y as the vertical axis.
4 Plot the points (−4, 0.5), (−2, 1), (−1, 2), (−0.5, 4), (0.5, −4), (1, −2), (2, −1) and (4, 0.5).
5 No value exists for y when $x = 0$. This results in the curve having two branches.
6 Join the points to make a curve in shape of a hyperbola.



7 The curve approaches the x-axis and the y-axis but never touches them.

b Asymptotes are $x = 0$ and $y = 0$.

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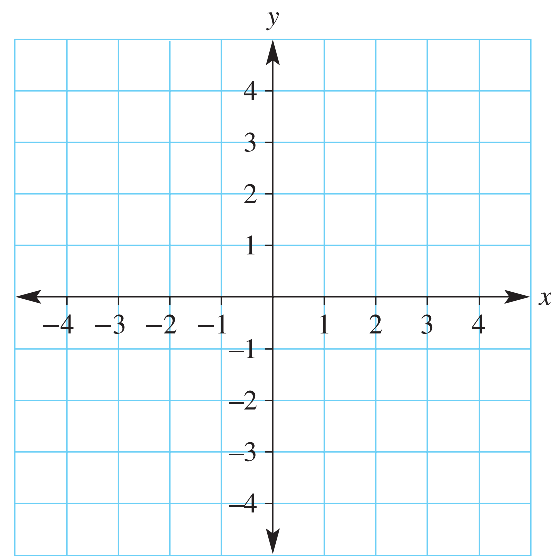
Exercise 9E

Example 7 **1** A reciprocal function is $y = \frac{1}{x}$.

a Complete the following table of values.

x	-4	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	4
y								

b Graph the reciprocal function using the number plane opposite.

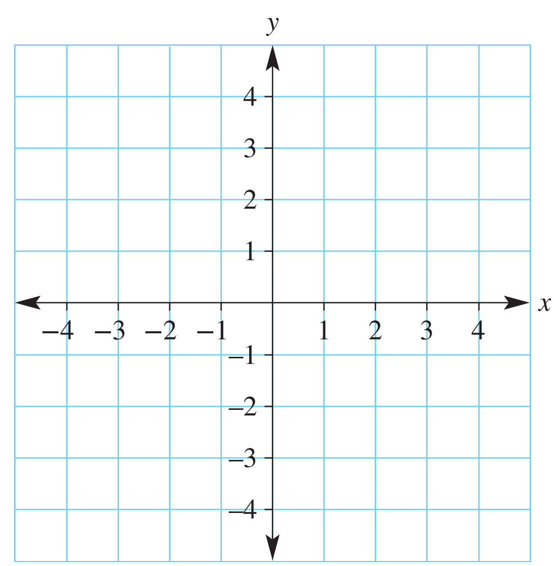


Example 8 **2** A reciprocal function is $y = -\frac{1}{x}$.

a Complete the following table of values.

x	-4	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	4
y								

b Graph the reciprocal function using the number plane opposite.



3 Complete the following table of values and graph each reciprocal function on the same number plane.

a $y = \frac{3}{x}$

x	-9	-3	-1	$-\frac{1}{3}$	$\frac{1}{3}$	1	3	9
y								

b $y = -\frac{3}{x}$

x	-9	-3	-1	$-\frac{1}{3}$	$\frac{1}{3}$	1	3	9
y								

c $y = \frac{4}{x}$

x	-8	-4	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	4	8
y								

d $y = -\frac{4}{x}$

x	-8	-4	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	4	8
y								

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9F Reciprocal models

Reciprocal modelling occurs when a practical situation is described mathematically using a reciprocal function. The quantity usually experiences fast growth or decay.

RECIPROCAL MODELS

A reciprocal model describes a practical situation using a function in the form $y = \frac{k}{x}$ where k is a number. Reciprocal functions are graphed to make a curve in the shape of a hyperbola.

Example 9: Using a reciprocal model 9F

The time taken (t), in hours, for a road trip, at speed (s), in km/h, is given by the reciprocal function $t = \frac{2000}{s}$.

a Construct a table of values for s and t.

b Draw the graph of $t = \frac{2000}{s}$.

c How long did the road trip take at a speed of 70 km/h?

d Why is it impossible to complete the road trip in 10 hours?

SOLUTION:

1 Construct a table of values for s and t.

2 Choose appropriate values for s, the speed of the car. Let $s = 10, 20, 40, 50, 60, 80$ and 100.

3 Find t using the reciprocal function $t = \frac{2000}{s}$. Express the values for t as a whole number.

a

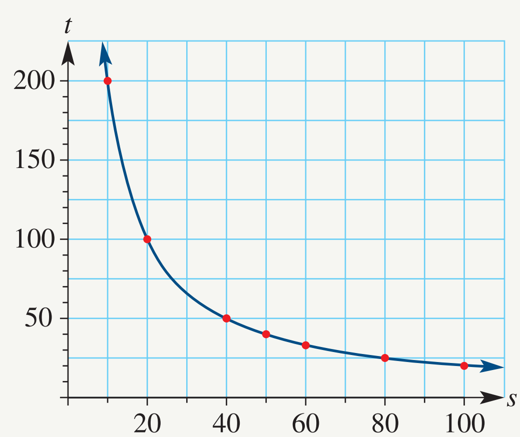
s	10	20	40	50	60	80	100
t	200	100	50	40	33	25	20

4 Draw a number plane with s as the horizontal axis and t as the vertical axis.

5 Plot the points (10, 200), (20, 100), (40, 50), (50, 40), (60, 33), (80, 25) and (100, 20).

6 Join the points to make a branch of a hyperbola.

b



7 Read the approximate value of t from the graph when $s = 70$.

c Approximately 30 hours

8 Read the value of s from the table.

9 Make sense of the result.

d Speed required to complete the trip in 10 h is 200 km/h, which is above the speed limit on Australian roads.

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Example 10: Using a reciprocal model 9F

The cost per person of sharing a pizza ($\$C$) is dependent on the number of people (n) eating the pizza.

The reciprocal equation $C = \frac{24}{n}$ is used to model this situation.

a Describe the possible values for n.

b Construct a table of values for n and C.

c Draw the graph of $C = \frac{24}{n}$.

d What is the cost per person if six people are sharing a pizza?

e How many people shared a pizza if the cost was \$2.40 per person?

SOLUTION:

1 The variable n represents the number of people sharing a pizza.

a n is a positive whole number and likely to be less than 10.

2 Construct a table of values for n and C.

3 Choose appropriate values for n.
Let $n = 1, 2, 3, 4, 5, 6, 7$ and 8.

4 Find C using $C = \frac{24}{n}$.

b

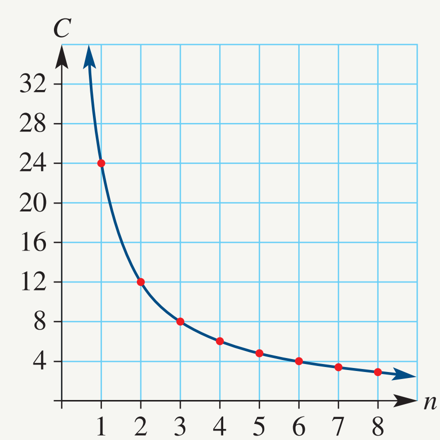
n	1	2	3	4	5	6	7	8
C	24	12	8	6	4.8	4	3.4	3

5 Draw a number plane with n as the horizontal axis and C as the vertical axis.

6 Plot the points $(1, 24)$, $(2, 12)$, $(3, 8)$, $(4, 6)$, $(5, 4.8)$, $(6, 4)$, $(7, 3.4)$ and $(8, 3)$.

7 Join the points to make a branch of a hyperbola.

c



8 Read the value of C from the table or graph when $n = 6$.

d Cost per person is \$4.

9 Substitute 2.4 for C into the reciprocal equation.

e $2.4 = \frac{24}{n}$

10 Solve the equation for n by rearranging the formula and evaluate.

$n = \frac{24}{2.4} = 10$

11 Check that the answer is reasonable.

12 Write the answer in words.

$\therefore$ number of people sharing the pizza was 10

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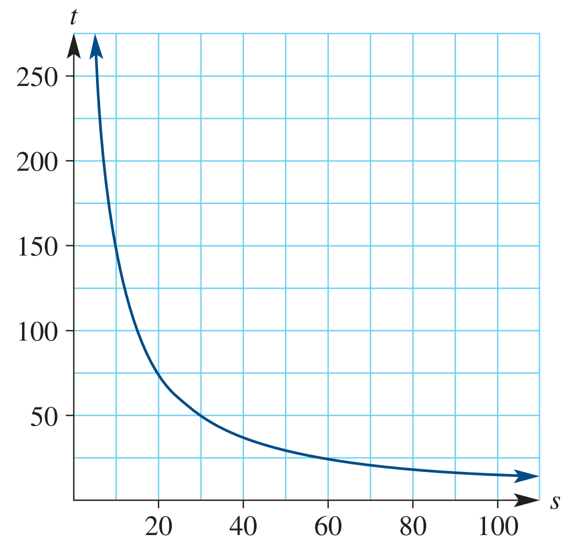
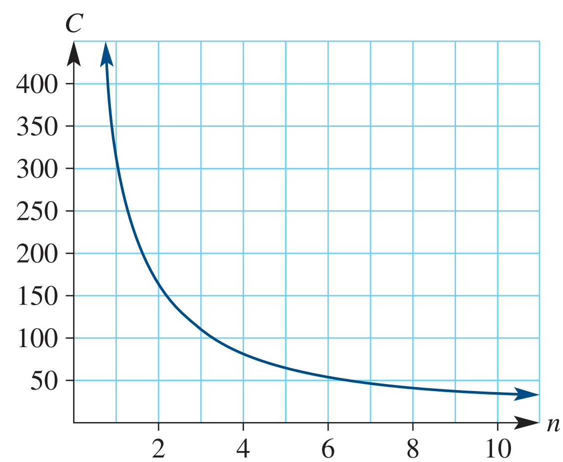
Exercise 9F

Example 9 **1** The time taken (t), in hours, for a road trip, at speed (s), in km/h, is given by the formula $t = \frac{1500}{s}$. The graph of this formula is shown opposite.



- **a** How long did the road trip take at a speed of 50 km/h?
- **b** How long did the road trip take at a speed of 75 km/h?
- **c** What is the speed required to complete the road trip in 25 hours?
- **d** What is the speed required to complete the road trip in 100 hours?
- **e** Why is it impossible to complete the road trip in 5 hours?

Example 10 **2** The cost per person of hiring a yacht (\$$C$) is dependent on the number of people (n) sharing the total cost. The reciprocal equation $C = \frac{320}{n}$ is used to model this situation.



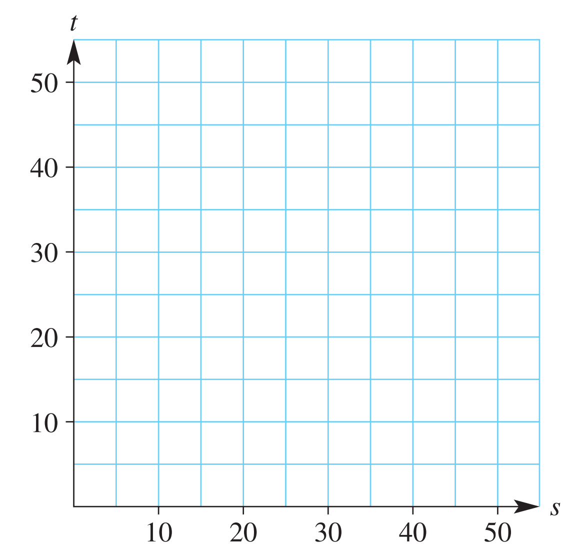
- **a** What is the cost per person of hiring the yacht if 2 people share the total cost?
- **b** What is the cost per person of hiring the yacht if 8 people share the total cost?
- **c** How many people are required to share the cost of hiring a yacht for \$80?
- **d** How many people are required to share the cost of hiring a yacht for \$320?
- **e** Is it possible for the cost per person to be \$1?

3 The time taken (t in minutes) to type an essay depends on the typing speed (s in words per minute). The reciprocal function $t = \frac{150}{s}$ is used to model this situation.

- **a** Complete the following table of values, correct to the nearest whole number.

s	5	10	15	25	30	50
t						

- **b** Draw the graph of $t = \frac{150}{s}$ using the number plane shown opposite.



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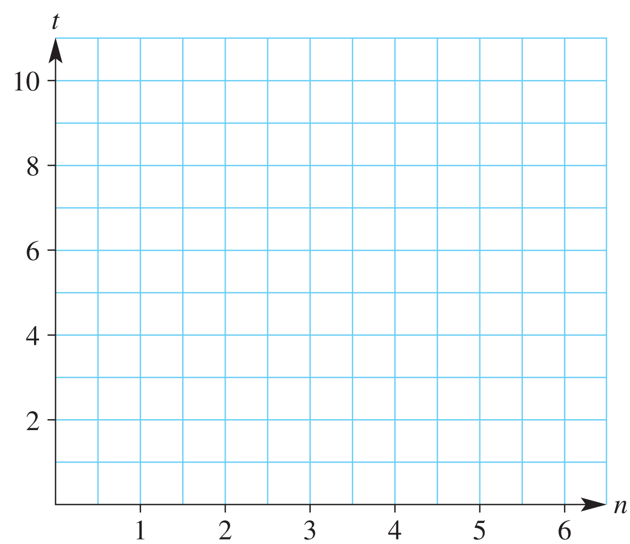
4 The time taken (t in hours) to dig a hole is dependent on the number of people (n) digging the hole. This relationship is modelled using the formula $t = \frac{6}{n}$.

a Complete the following table of values, correct to one decimal place.

n	1	2	3	4	5	6
t						

b Draw the graph of $t = \frac{6}{n}$ using the number plane shown opposite.



c What is the time taken to dig a hole by 1 people?

d What is the time taken to dig a hole by 3 people?

e What is the time taken to dig a hole by 6 people?

f How many people could dig the hole in two hours?

g How many people could dig the hole in 30 minutes?

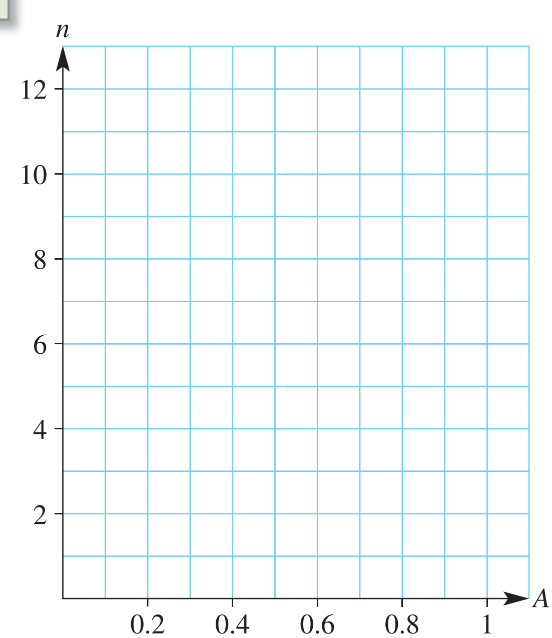
h How long would it take for 360 people to dig the hole? Is this possible?

5 The maximum number of people (n in 1000s) attending an outdoor concert is dependent on the area (A in m^2) allowed per person. The reciprocal equation $n = \frac{1.2}{A}$ models this practical situation.

a Complete the following table of values, correct to the nearest whole number.

A	0.1	0.2	0.3	0.4	0.5	0.6	0.8	0.9	1.0
n									

b Draw the graph of $n = \frac{1.2}{A}$ using the number plane shown opposite.



c How many people can attend this concert if the area allowed is $0.5\,m^2$?

d How many people can attend this concert if the area allowed is $0.25\,m^2$?

e What is the area allowed per person if the maximum number of people attending the concert is 2000?

f What is the area allowed per person if the maximum number of people attending the concert is 5000?

g Is it possible for 12 000 people to attend this concert? Justify your answer.

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9G Miscellaneous problems

Algebraic modelling occurs when a practical situation is described mathematically using an algebraic function. This involves gathering data and analysing the data to determine possible functions. Determining the function is made easier using technology.

ALGEBRAIC MODEL

- Algebraic models are used to describe practical situations.
- Algebraic models may have limitations that restrict their use.

Example 11: Modelling physical phenomena 9G

The mass M kg of a baby orang-utan and its age after x months are given below.

x	0	1	2	3	4	5	6
M	1.5	1.8	2.2	2.6	3.1	3.7	4.5

a Plot the points from the table onto a number plane.

b The formula $M = 1.5(1.2)^x$ models the data in the table. Graph $M = 1.5(1.2)^x$ on the same number plane.

c Use the model to determine the mass of the orang-utan after 2.5 months.

d This model only applies when x is less than or equal to 6. Why?

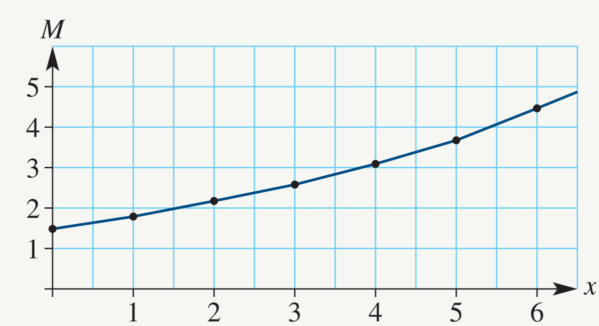
SOLUTION:

1 Draw a number plane with x as the horizontal axis and M as the vertical axis.

2 Plot the points (0, 1.5), (1, 1.8), (2, 2.2), (3, 2.6), (4, 3.1), (5, 3.7) and (6, 4.5).

3 The formula $M = 1.5(1.2)^x$ has the same table of values. Join the points to make a curve.

a, b



4 Substitute 2.5 for x into the formula.

5 Evaluate, correct to one decimal place.

c $M = 1.5(1.2)^{2.5}$

$= 2.4$ kg

6 Use the model for x greater than 6. Let x be 48 months or 4 years. Substitute 48 for x into the formula.

7 Evaluate.

8 Write the answer in words.

d $M = 1.5(1.2)^{48}$

$= 9479.6$ kg

Orang-utans are less than 100 kg in general so the answer of 9479.6 kg is unreasonable.

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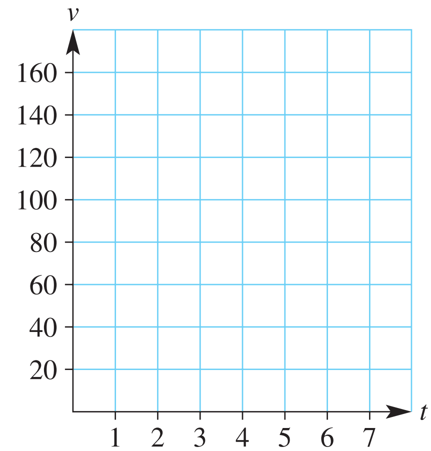
Exercise 9G

1 A new piece of equipment is purchased by a business for \$150 000. The value of the equipment (v in \$1000), to the nearest whole number, is depreciated each year (t) using the table below.

t	0	1	2	3	4	5	6
v	150	75	38	19	9	5	2



a Draw a number plane shown opposite.

b Plot the points from the table of values. Join the points to make a curve.

An exponential model in the form $v = 2^{-t} \times 150$ describes this situation.

c Use the model to predict the value of the equipment after 1.5 years.

d Use the model to predict the value of the equipment after 2.5 years.

e Use the model to predict the value of the equipment after 3.5 years.

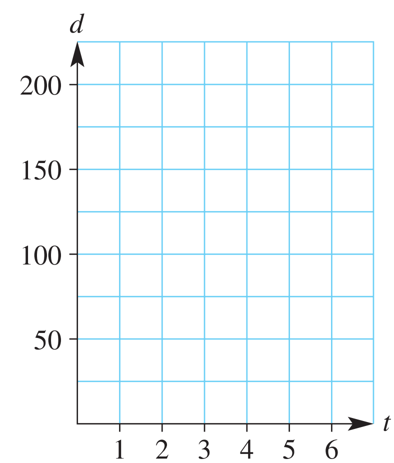
f Use the model to predict the value of the equipment after 6 months.

g When will the value of the equipment be \$75 000?

h Use the model to predict the value of the equipment after 20 years. Explain your answer.

2 The distance (d metres) that an object falls in t seconds is shown in the table below.

t	0	1	2	3	4	5	6
d	0	5	20	45	80	125	180



a Plot the points from the table on the number plane. Join the points to make a curve.

A quadratic model in the form $d = 5t^2$ describes this situation.

b Use the model to find the distance fallen after 1.5 seconds.

c Use the model to find the distance fallen after 2.5 seconds.

d Use the model to find the distance fallen after 3.5 seconds.

e Use the model to find the distance fallen after 10 seconds.

f What is the time taken for an object to fall 320 metres?

g Earth's atmosphere is approximately 100 km. What limitation would you place on this model?

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3 The number of tadpoles (N) in a pond after t months is shown in the table below.

t	0	2	4	6	8	10	12	14
N	0	24	96	216	384	600	864	1176

a Draw a number plane with t as the horizontal axis and N as the vertical axis.

b Plot the points from the table of values. Join the points to make a curve.

A quadratic model in the form $N = 6t^2$ describes this situation.

c Use the model to find the number of tadpoles after 3 months.

d Use the model to find the number of tadpoles after 5 months.

e Use the model to find the number of tadpoles after 7 months.

f Use the model to find the number of tadpoles after 11 months.

g Use the model to find the time taken for the number of tadpoles to reach 2400.

h Use the model to predict the number of tadpoles after 4.5 months. What limitations would you place on this model?

4 The speed of a car (s in km/h) and the time taken (t in hours) is shown below.

t	1	2	3	4	5	6
s	120	60	40	30	24	20

a Draw a number plane with t as the horizontal axis and s as the vertical axis.

b Plot the points from the table of values. Join the points to make a curve.

A hyperbolic model in the form $s = \dfrac{120}{t}$ describes this situation.

c Use the model to find the speed of the car if time taken is 1.5 seconds.

d Use the model to find the speed of the car if time taken is 2.5 seconds.

e Use the model to find the speed of the car if time taken is 3.5 seconds.

f Use the model to find the speed of the car if time taken is 8 seconds.

g What is the time taken if the car is travelling at a speed of 48 km/h?

h Use the model to predict the speed of the car after $\frac{1}{2}$ second. Is this possible? Explain your answer.

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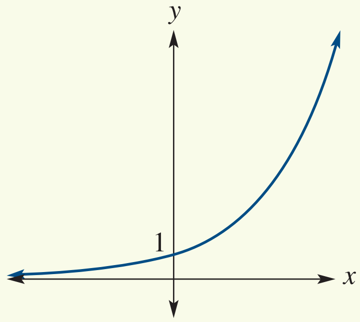
Summary

Key ideas and chapter summary

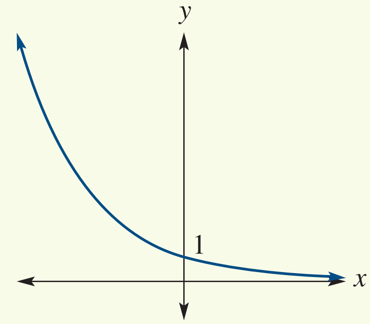
PPT STUDY GUIDE

Exponential function

$y = a^x, a > 1$



$y = a^{-x}, a > 1$



Exponential model

Exponential growth

Quantity increases rapidly using $y = a^x$

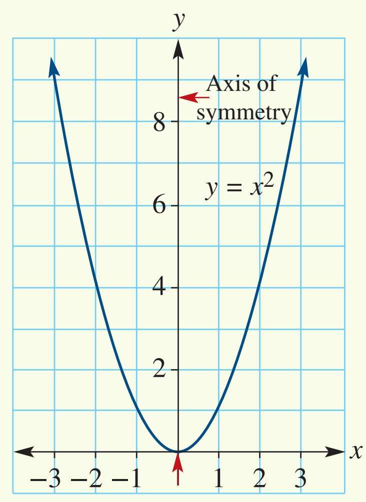
Exponential decay

Quantity decreases rapidly using $y = a^{-x}$

Quadratic function

Quadratic function has the form $y = ax^2 + bx + c$ where a, b and c are numbers.

- Parabola ($y = x^2$)
  Minimum turning point
- Parabola ($y = -x^2$)
  Maximum turning point



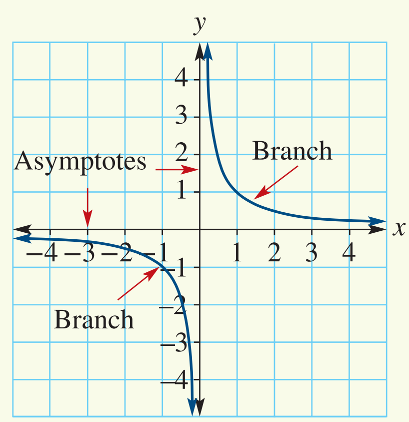
Quadratic model

A quadratic model describes a practical situation using a function in the form $y = ax^2 + bx + c$, where a, b and c are numbers.

Reciprocal function

A reciprocal function has the form $y = \frac{k}{x}$, where k is a number.

- Hyperbola: $y = \frac{1}{x}$
- Hyperbola: $y = -\frac{1}{x}$



Reciprocal model

A reciprocal model describes a practical situation using a function in the form, $y = \frac{k}{x}$ where k is a number.

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Review

Multiple-choice

1 What is the y-intercept of the exponential function $y = 2^{-x}$?

A (0, –1) **B** (0, 0) **C** (0, 1) **D** (0, 2)

2 Which of the following points lies on the quadratic curve $y = 2x^2$?

A (–1, 0) **B** (0, –1) **C** (1, 2) **D** (2, 16)

3 What is the maximum y value of the quadratic function $y = -x^2 + 4x - 3$?

x	–1	0	1	2	3	4	5
y							

A –3 **B** 1 **C** 2 **D** 4

4 The equation $d = 0.4(s^2 + s)$ is used to model the stopping distance for a bicycle where d is the stopping distance in metres and s is the bicycle's speed in m/s. What is the stopping distance given a speed of 5 metres per second?

A 5 m **B** 10 m **C** 12 m **D** 15 m

5 Which of the following points lies on the reciprocal function $y = \frac{8}{x}$?

A (–2, 8) **B** (–1, 8) **C** (0, 8) **D** (2, 4)

6 The speed in km/h (s) of a vehicle is given by the formula $s = \frac{200}{t}$ where t is the time in hours. What is the time taken if the average speed was 100 km/h?

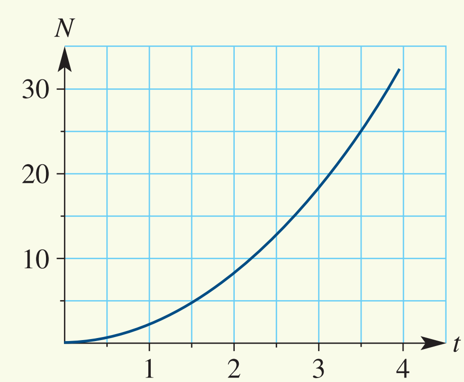
A 0.4 hours **B** 2 hours **C** 100 hours **D** 300 hours

7 A hyperbola has the equation $y = \frac{2}{x}$. Which of the following is an equation of the asymptote?

A $x = 0$ **B** $x = 1$ **C** $x = 2$ **D** $x = \frac{2}{y}$

8 The graph opposite shows the insect population (N) plotted against the time (t) in days.

t	0	1	2	3	4
N	0	2	8	18	32



What type of function would model this data?

A Exponential

B Hyperbolic

C Quadratic

D Reciprocal

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Review

Short-answer

1 Complete the following table of values by expressing the y values, correct to one decimal place. Graph each exponential function.

a $y = 1.5^x$

x	–3	–2	–1	0	1	2	3
y							

b $y = 0.5^x$

x	–3	–2	–1	0	1	2	3
y							

2 The height h cm of a plant and its age after x months is given below.

x	0	1	2	3	4	5	6
h	1.1	2.4	5.3	11.7	25.8	56.7	124.7

a Plot the points from the above table onto a number plane.

b The formula $h = 2.2^x \times 1.1$ models the data in the table. Draw this function.

c Use the model to determine the height of a plant after 1.5 months. Answer correct to one decimal place.

d Use the model to determine the height of a plant after 3.5 months. Answer correct to one decimal place.

3 The population of earthworms grows exponentially according to the formula $w = 1.1^t \times 25$, where w is the number of earthworms after t days.

a Construct a table of ordered pairs using 0, 5, 10, 15 and 20 as values for t. Express the number of earthworms to the nearest whole number.

b Graph $w = 1.1^t \times 25$ using the table of ordered pairs in part **a**.

c What is the initial number of earthworms?

d What is the number of earthworms after 3 days?

e Estimate the time taken for the earthworms to reach a population of 75.

4 Complete the following table of values and graph each quadratic function.

a $y = 3x^2$

x	–3	–2	–1	0	1	2	3
y							

b $y = -\frac{1}{3}x^2$

x	–9	3	–1	0	1	3	9
y							

c $y = x^2 + 3$

x	–3	–2	–1	0	1	2	3
y							

d $y = x^2 - 5x - 4$

x	0	1	2	3	4	5	6
y							

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Review

5 Abbey throws a rock and it takes 6 seconds to reach the ground. The height it reaches is given by the formula $h = -t^2 + 6t$ a where h is the height (in metres) and t is the number of seconds after it has been thrown.

a Complete the following table of values.

t	0	1	2	3	4	5	6
h							

b Draw the graph of $h = -t^2 + 6t$.

c What was the maximum height reached by the rock?

d When was the maximum height reached?

6 Complete the following table of values and graph each reciprocal function on the same number plane.

a $y = \frac{7}{x}$

x	-7	-1	$-\frac{1}{7}$	$\frac{1}{7}$	1	7
y						

b $y = -\frac{7}{x}$

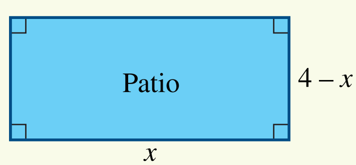
x	-7	-1	$-\frac{1}{7}$	$\frac{1}{7}$	1	7
y						

7 The number of chairs (c) in a row varies inversely with the distance (d in metres) between them. When the chairs are 2 m apart the row can accommodate 60 chairs.

a How many chairs can be placed in a row if the distance between them is 1.5 m?

b What is the distance between the chairs if the number of chairs is 40?

8 A rectangular patio has a length of x metres and a breadth of $(4 - x)$ metres.



a Show that the area of the patio is $A = x \times (4 - x)$.

b Complete the table using the above equation.

x	0	0.5	1	1.5	2	2.5	3	3.5	4
A									

c Draw the graph of this quadratic equation using the table above.

d Use the graph to estimate the area of the patio when the length is 0.75 m.

e Use the graph to estimate the area of the patio when the length is 2.75 m.

f What is the maximum area of the patio?

g What is the patio length in order to have maximum area?

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Practice Paper 2

Section I

Attempt Questions 1–15 (15 marks).
Allow about 20 minutes for this section.

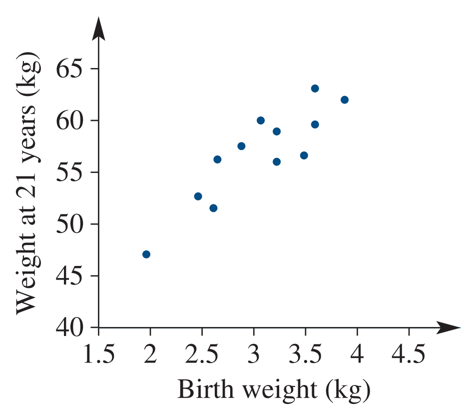
1 A 4 litre tin of paint is made using a mixture of blue, white and green paint in the ratio 3 : 5 : 2. How much blue paint is needed per tin?

A 300 mL
B 1200 mL
C 1800 mL
D 2000 mL

2 A bank charges 0.05753% simple interest per day on the amount owing on a credit card. What is the interest charged in four weeks on a balance of $1200?

A $19.33
B $27.61
C $69.04
D $276.14

3 The scatterplot shows the weights at age 21 and at birth of 12 women. The association is best described as:



A weak positive non-linear
B weak negative non-linear
C strong positive linear
D strong negative linear

4 The graph of $y = 3x^2 - 6x + 7$ meets the y-axis at:

A (0, 1)
B (0, 7)
C (7, 0)
D (1, 0)

5 The ratio of 1.5 m to 10 cm is:

A 1.5 : 10
B 1 : 15
C 15 : 10
D 15 : 1

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6 Which symbol represents a toilet in a floor plan?

A 

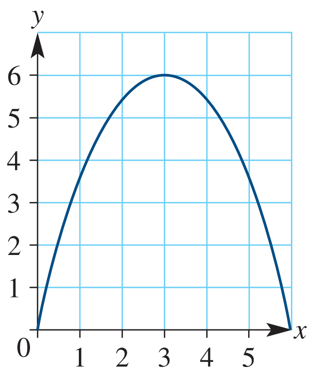
B 

C 

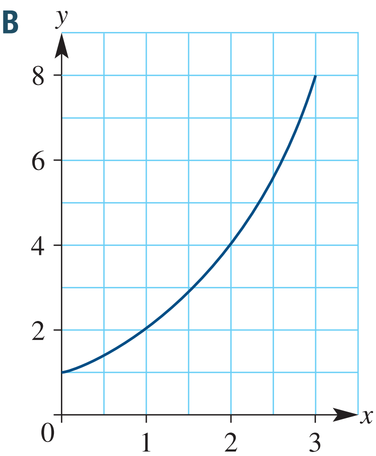
D 

7 Which graph best represents $y = 2^x$?

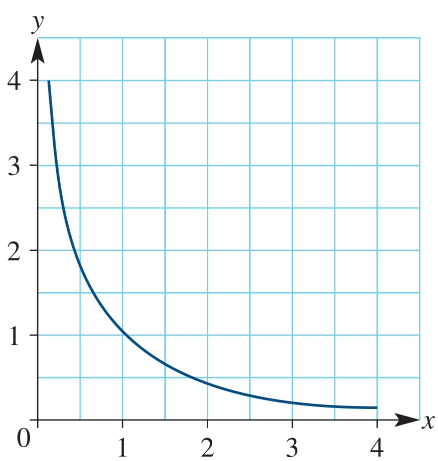
A



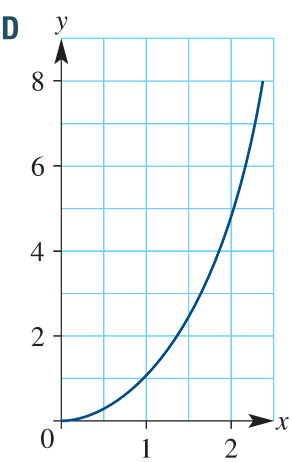
B



C



D



8 The daily interest rate for the outstanding balance on a credit card was 0.037%. The interest charged for 29 days was $9.12. How much was the outstanding balance? Answer to the nearest dollar.

A $246

B $850

C $971

D $24 649

9 Lucy has planted red and white rose bushes in the ratio 2 : 3. How many white rose bushes are there if she planted a total of 30 rose bushes?

A 6

B 12

C 18

D 20

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10 The following linear equation finds the weekly amount spent on food (in dollars) to be predicted from weekly income is given by:

$amount = 0.30 \times income - 20.$

What is the predicted amount spent on food with a weekly income of $900?

A $250 **B** $280

C $290 **D** $320

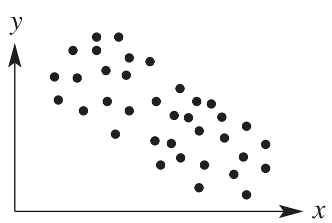
11 A motor vehicle is bought for $16 900. It depreciates at 14% per annum and is expected to be used for 5 years. How much is the motor vehicle worth after 5 years?

A $909
B $2366
C $5070
D $7950

12 What is the y-intercept of the exponential function $y = 3^x$?

A –3
B 0
C 1
D 3

13 What is the correlation between the variables in the scatterplot?



A Strong positive
B Weak positive
C Strong negative
D Weak negative

14 Which of the following points lies on the reciprocal function $y = \frac{12}{x}$?

A (–2, –6)
B (–1, 12)
C (0, 12)
D (2, –6)

15 Square A has a side length of 36 mm and square B has a side length of 18 mm. What is the scale factor?

A –2
B $-\frac{1}{2}$
C 2
D $\frac{1}{2}$

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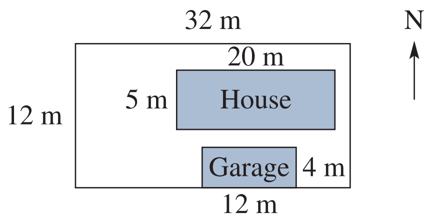
Section II

Attempt Questions 16–18 (45 marks).
Allow about 70 minutes for this section.
All necessary working should be shown in every question.

Question 16 (15 marks) **Marks**

a Three people share a sum of \$1200 in the ratio 5 : 3 : 8. How much does each person receive? **2**

b The diagram shows a site plan.



Not to scale

i What is the total area of land of the house and garage? **1**

ii What percentage of the site is taken up by the house and garage? **1**

iii There is a brick wall, 800 mm high, along the northern boundary of the site. What is the area of the brick wall in square metres? **1**

iv 600 bricks are needed to build 10 square metres of wall. How many bricks were used to build the wall in part **iii**? **1**

v The guttering around the perimeter of the house is to be replaced. Guttering costs \$60 per metre. How much will the new guttering cost? **2**

c i Mia has a debit of \$15 890 on a credit card with an interest rate of 17% p.a. compound. What is the future value of this debit for 2 years? **1**

ii How much interest would Mia pay on this credit card if she made no repayments for 2 years and then paid off the entire debt? **1**

iii Mia transferred the debit to a new card with a compound interest rate of 20% p.a. The new card has a 0% balance transfer for 6 months. How much is saved after 2 years? **2**

d For time ranging from 30 to 60 seconds, the equation relating the number of correct answers to time is:

$$\textit{correct answers} = 0.72 \times \textit{time} + 30.$$

Use this equation to predict the number of correct answers with the following times. Is the method used interpolation or extrapolation?

i 25 seconds

ii 50 seconds

iii 75 seconds **3**

Mathematics Standard 1
Cambridge Maths Stage 6
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Question 17 (15 marks) **Marks**

a **i** Complete the table of values for the equation $y = x^2 - 4$. **1**

x	–3	–2	–1	0	1	2	3
y							

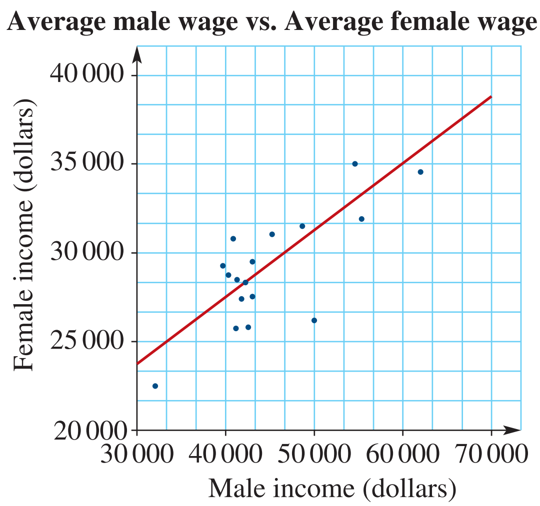
ii Draw the graph of the quadratic function $y = x^2 - 4$. **1**

iii Complete the table of values for the equation $y = -x^2 + 2x + 8$. **1**

x	–2	–1	0	1	2	3	4
y							

iv Draw the graph of the quadratic function $y = -x^2 + 2x + 8$. **1**

b The scatterplot shows the average annual female income plotted against average annual male income for 16 countries. A line of best fit is also shown.



i What was the female income for a country whose average annual male income was \$60 000? **1**

ii How many countries had an average annual female income greater than \$30 000? **1**

iii Find the gradient of the line of best fit. Answer correct to one decimal place. **2**

c A rectangular poster is to be made to advertise energy-efficient housing. It is to be 1.6 m long and 1 m wide. Draw a scale drawing of the rectangle using the scale 1 : 20. **2**

d The table below shows the banking charges for credit card usage.

Fee	Charge
Cash advance	5%
Late payment	\$30
Balance transfer	1.25%

i Find the cost of 40 cash advances for \$100. **1**

ii Find the cost of a balance transfer of \$7000. **1**

iii Find the cost of 6 late payments and 5 cash advances of \$200. **1**

e Graph the exponential function $y = 3^x$ by completing the table of values. **2**
Express the y values correct to one decimal place.

x	–2	–1	0	1	2
y					

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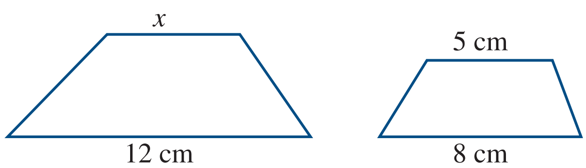
Question 18 (15 marks) **Marks**

a The two figures below are similar.



Not to scale

i What is the scale factor, in simplest form? **1**

ii What is the length of the unknown side, x, in the above trapezium? **2**

b The birth weight and weight at age 18 of eight men are given in the table below.

Birth weight (kg)	1.8	2.3	2.7	2.8	2.9	3.0	3.5	3.6
Weight at 18 (kg)	48.9	54.6	57.2	66.9	69.9	75.8	80.6	86.7

i Draw a scatterplot using the above table. **2**

ii State whether the association is positive or negative. **1**

iii Describe the strength of the association as strong, moderate or weak. **1**

c The table shows the monthly repayment of \$1000 on a reducing-balance loan.

Interest rate per annum	9.00%	9.25%	9.50%	9.75%	10.00%
Monthly repayment	\$7.97	\$8.27	\$8.67	\$9.07	\$9.56

i What is the monthly repayment on \$120 000 at 9.75% for 20 years? **1**

ii Find the total amount to be repaid on this loan. **1**

iii How much is saved if the interest rate is reduced to 9.00%? **2**

d A hyperbola has the equation $y = \frac{2}{x}$.

i What is the equation of the vertical asymptote of the hyperbola? **1**

ii Complete the following table of values. **1**

x	–4	–2	–1	–0.5	0.5	1	2	4
y								

iii Draw the graph of $y = \frac{2}{x}$. **1**

iv What is the value of y when x is $\frac{1}{2}$? **1**

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2019 Higher School Certificate Examination Mathematics Standard 1/2 Reference sheet

Measurement

Precision

Absolute error $= \frac{1}{2} \times$ precision

Upper bound = measurement + absolute error

Lower bound = measurement − absolute error

Length, area, surface area and volume

$$l = \frac{\sim}{360} \times 2^\circ r$$

$$A = \frac{\sim}{360} \times {}^\circ r^2$$

$$A = \frac{h}{2}(x + y)$$

$$A \approx \frac{h}{2}(d_f + d_l)$$

$$A = 2^\circ r^2 + 2^\circ rh$$

$$A = 4^\circ r^2$$

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}{}^\circ r^3$$

Trigonometry

$$A = \frac{1}{2}ab \sin C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Financial Mathematics

$$FV = PV(1 + r)^n$$

Straight-line method of depreciation

$$S = V_0 - Dn$$

Declining-balance method of depreciation

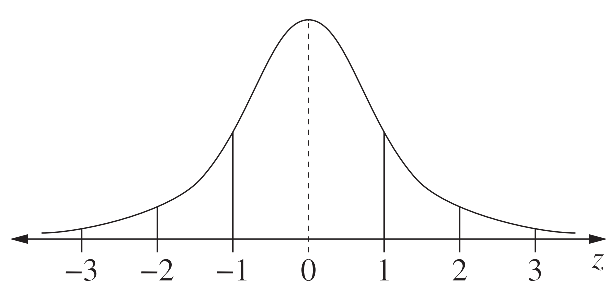
$$S = V_0(1 - r)^n$$

Statistical Analysis

$$z = \frac{x - \bar{x}}{s}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

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Glossary

A

Absolute error – The difference between the actual value and the measured value indicated by an instrument. It is also calculated by finding half the smallest unit on the measuring device.
Account servicing fee – Ongoing account keeping fees.
Adjacent side – A side in a right-angled triangle next to the reference angle but not the hypotenuse.
Algebraic modelling – When a practical situation is described mathematically using an algebraic function.
Allowable deduction – Deductions allowed by the Australian Taxation Office such as work-related, self-education, travel, car or clothing expenses.
Angle of depression – The angle between the horizontal and the direction below the horizontal.
Angle of elevation – The angle between the horizontal and the direction above the horizontal.
Annual leave loading – A payment calculated as a fixed percentage of the normal pay over a fixed number of weeks. Annual leave loading is usually at the rate of 17½%.
Annually – Once a year.
Annulus – Area between a large and a small circle.
Appreciation – An increase in value of an item over time. It is often expressed as the rate of appreciation.
Arc – See *Edge*.
Area – The amount of surface enclosed by the boundaries of the shape.
Association – A connection or relationship between the variables of function.
Asymptote – A line that the curve approaches by getting closer and closer to it but never reaching it.
Axis of symmetry – A vertical line that divides the shape into two congruent or equal halves.

B

Balance transfer – Fee for moving balance to another account, often held at another institution.
BASIX – Building Sustainability Index (BASIX) is a scheme to regulate the energy efficient of residential buildings.
Bearing – The direction one object is from another object. See *Compass bearing* and *True bearing*.
Biannually – Every six months or twice a year.
Bias – When events are not equally likely.
Bimodal – Data with two modes or peaks.
Bivariate data – Data relating to two variables that have both been measured on the same set of items or individuals.
Blood alcohol content (BAC) – A measure of the amount of alcohol in your blood.
Blood pressure – Pressure of the blood in the arteries as it is pumped around the body.
Bonus – An extra payment or gift earned as reward for achieving a goal.
Book value – See *Salvage value.*
Box plot – See *Box-and-whisker plot*.
Box-and-whisker plot – A graph that uses five-number summary of a numerical dataset.
Braking distance – The distance travelled by the vehicle after the driver presses the brake.
Break-even point – The point of intersection when income equals costs in some practical problems.

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Budget – A plan used to manage money by listing a person's income and expenditure.
Building plan – See *House plan*.

C

Calorie – A measurement for food energy.
Capacity – The amount of liquid within a solid figure or the weights on a directed graph of a flow problem.
Car running costs – Car costs such as maintenance, repairs, fuel, improvements, parking, tolls, car washes and fines.
Cash advance – Withdrawing cash from a bank account.
Casual rate – An amount paid for each hour of casual work.
Categorical data – Data that is divided into categories such as hair colour. It uses words not numbers.
Causation – Indicates that one event is the result of the occurrence of another event (or variable).
Census – Collecting data from the whole population.
Circuit – A walk with no repeated edges that starts and ends at the same vertex.
Class centre – Median or middle score of a class in a grouped frequency distribution.
Closed cylinder – A cylinder with both circular bases. See *Open cylinder*.
Coefficient – The number in front of a particular letter in an algebraic expression. For example, the term $3y$ has a coefficient of 3.
Commission – A payment for services, mostly as a percentage of the value of the goods sold.
Compass bearing – A bearing that uses the four directions of the compass (north, south, east and west) such as N37°E.
Complementary event – The outcomes that are not members of the event.
Composite shape – Two or more plane shapes.
Composite solid – Two or more common solids.
Compound interest – Interest calculated from the initial amount borrowed or principal plus any interest that has been earned. It calculates interest on the interest.
Compounding period – The length of time between interest payments in a compound interest investment.
Cone – A solid figure, with a circular base, that tapers to a point.
Connected graph – A graph is connected if every vertex in the graph is accessible from every other vertex in the graph along a path formed by the edges of the graph.
Continuous data – Numerical data obtained when quantities are measured rather than counted.
Conversion graph – A graph used to change one quantity from one unit to another unit.
Coordinated universal time (UTC) – See *Greenwich Mean Time*.
Correlation – Strength of the relationship between two variables.
Cosine ratio – The ratio of the adjacent side to the hypotenuse in a right-angled triangle.
Credit card – A small plastic card used to buy goods and services and pay for them later.
Credit card statement – Information sent to the credit card user each month. It includes an account number, opening balance, new charges, payments, refunds, reward points, payment due data, minimum payment and closing balance.
Cross section – The intersection of a solid with a plane.
Cumulative frequency – The frequency of the score plus the frequency of all the scores less than that score. It is the progressive total of the frequencies.
Cumulative frequency histogram – A histogram with equal intervals of the scores on the horizontal axis and the cumulative frequencies associated with these intervals shown by vertical rectangles.

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Cumulative frequency polygon – A line graph constructed by joining the top right-hand corner of the rectangles in a cumulative frequency histogram.
Cycle – A walk with no repeated vertices that starts and ends at the same vertex.
Cylinder – A prism with a circular base. See *Open cylinder* and *Closed cylinder*.

D

Data – Raw scores. Information before it is organised.
Decile – Divides an ordered dataset into 10 equal groups.
Declining-balance depreciation – A method of depreciation when the value of an item decreases by a fixed percentage each time period.
Deduction – A regular amount of money subtracted from a person's wage or salary.
Degree – A unit for measuring angles or the number of edges that are connected to a vertex in a network diagram.
Dependent variable – A variable that depends on the number substituted for the independent variable.
Diastolic – Minimum blood pressure.
Direct proportion – See *Direct variation*.
Direct variation – Relationship between two variables when one variable depends directly on another variable.
Directed network – A network whose edges have arrows and travel is only possible in the direction of the arrows.
Discrete data – Data obtained when a quantity is counted. It can only take exact numerical values.
Distance-time graphs – Line graph with time on the horizontal axis and distance on the vertical axis.
Distributive law – A rule for expanding grouping symbols by multiplying each term inside the grouping symbol by the number or term outside the grouping symbol.
Dot plot – A graph that consists of a number line with each data point marked by a dot. When several data points have the same value, the points are stacked on top of each other.
Double stem-and-leaf plot – A stem-and-leaf plot that uses two sets of similar data together.
Double time – A penalty rate that pays the employee twice the normal hourly rate.

E

Edge – The line that connects the vertices in a network diagram.
Elevation – A view of an object from one side such as a front elevation or side elevation.
Energy – The capacity or power to do work.
Energy consumption – The amount of energy consumed per unit of time.
Enlargement – A similar figure drawn larger than the original figure.
Equally likely outcomes – Outcomes of an event that have the same chance of occurring.
Equation – A mathematical statement that says that two things are equal.
Equator – Imaginary horizontal line that divides the earth into two hemispheres. Latitude of the equator is 0°.
Eulerian circuit – A circuit that uses every edge of a network graph exactly once.
Eulerian trail – A trail that uses every edge of a graph exactly once.
Evaluate – Work out the exact value of an expression.
Expand – Remove the grouping symbols.
Expected frequency – The number of times that a particular event should occur.
Exponential decay – Quantity decreases rapidly according to the function $y = a^{-x}$.
Exponential function – A curve whose equation has an x as the power (e.g. 3^x). It is defined by the general rule $y = a^x$ and $y = a^{-x}$ where $a > 0$.

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Exponential growth – Quantity increases rapidly according to the function $y = a^x$.
Exponential model – A practical situation described mathematically using an exponential function. The quantity usually experiences fast growth or decay.
Expression – A mathematical statement written in numbers and symbols.
Extrapolation – Predicting values outside the range of the dataset.

F

Factorise – To break up an expression into a product of its factors.
Five-number summary – A summary of a dataset consisting of the lower extreme, lower quartile, median, upper quartile and upper extreme.
Flat interest – See *Simple interest.*
Flat rate loan – Loan that uses simple interest.
Formula – A mathematical relationship between two or more variables.
Fortnight – Two weeks or 14 days.
Frequency – The number of times a certain event occurs.
Frequency distribution – See *Frequency table.*
Frequency histogram – A histogram with equal intervals of the scores on the horizontal axis and the frequencies associated with these intervals shown by vertical rectangles.
Frequency polygon – A line graph constructed by joining the midpoints at the tops of the rectangles of a frequency histogram.
Frequency table – A table that lists the outcomes and how often (frequency) each outcome occurs.
Fuel consumption – The number of litres of fuel a motor vehicle uses to travel 100 kilometres.
Fuel consumption rate – The number of litres of fuel a vehicle uses to travel 100 kilometres.
Future value – The sum of money contributed plus the compound interest earned.

G

General form – A linear equation written in the form $ax + by + c = 0$.
Goods and Services Tax (GST) – A tax added to the purchase price of each item. The GST rate in Australia is 10% of the purchase price of the item except for basic food items and some medical expenses.
Gradient – The steepness or slope of the line. It is calculated by dividing the vertical rise by the horizontal run.
Gradient–intercept formula – A linear equation written in the form $y = mx + c$.
Greenwich Mean Time – Time at the Greenwich meridian.
Greenwich meridian – Imaginary vertical line that passes through the town of Greenwich (London). Longitude of the Greenwich meridian is 0°.
Gross income – The total amount of money earned from all sources. It includes interest, profits from shares or any payment received throughout the year.
Gross pay – The total of an employee’s pay including allowances, overtime pay, commissions and bonuses.
Grouped data – Data organised into small groups rather than as individual scores.
Grouping symbol – Symbols used to indicate the order of operations such as parentheses ( ) and brackets [ ].

H

Hamiltonian cycle – A Hamiltonian path that starts and finishes at the same vertex.
Hamiltonian path – A path passes through every vertex of a graph once and only once.
Heart rate – The number of heartbeats per minute (bpm).

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Histogram – A graph using columns to represent frequency or cumulative frequency. See *Frequency histogram* and *Cumulative frequency histogram*.
House plan – A horizontal section cut through the building showing the walls, windows, door openings, fittings and appliances.
Hyperbola – A curve graphed from a reciprocal function.
Hyperbolic function – See *Reciprocal function*.
Hypotenuse – A side in right-angled triangle opposite the right angle. It is the longest side.

I

Income tax – Tax paid on income received.
Independent variable – A variable that does not depend on another variable for its value.
Index form – See *Index notation*.
Index notation – A method to write expressions in a shorter way such as $a \times a = a^2$.
Inflation – A rise in the price of goods and services or Consumer Price Index (CPI). It is often expressed as annual percentage.
Inflation rate – The annual percentage change in the Consumer Price Index (CPI).
Intercept – The position where the line cuts the axes.
Interest – The amount paid for borrowing money or the amount earned for leading money.
Interest rate – The rate at which interest is charged or paid. It is usually expressed as a percentage.
International date line – An imaginary line through the Pacific ocean that corresponds to 180° longitude.
Interpolation – Predicting values within the range of the dataset.
Interquartile range – The difference between the first quartile and third quartile.
Isomorphic graphs – Two or more different looking graphs that can contain the same information.

K

Kilojoules (kJ) – Internationally accepted measurement for food energy.

L

Late payment fee – A fee charged when the minimum payment has not been received by the due date.
Latitude – The angle or angular distance north or south of the equator.
Like term – Terms with exactly the same pronumerals such as $3a$ and $6a$.
Limit of reading – The smallest unit on measuring instrument.
Line of best fit – A straight line used to approximately model the linear relationship between two variables.
Linear association – A connection between the variables of function that results in the points on a scatterplot following a linear pattern.
Linear equation – An equation whose variables are raised to the power of 1.
Linear function – A function when graphed on a number plane is a straight line.
Linear modelling – A mathematical description of a practical situation using a linear function.
Linear regression – The process of fitting a straight line to the data.
Loan application fee – Initial costs in processing the loan application.
Loan repayment – The amount of money to be paid at regular intervals over the time period.
Longitude – The angle or angular distance east or west of the Greenwich meridian.
Loop – Edge that starts and ends at the same vertex.

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Lower extreme – Lowest score in the dataset.
Lower quartile – The lowest 25% of the scores in the dataset.

M

Mass – The amount of matter within an object.
Mean – A measure of the centre. It is calculated by summing all the scores and dividing by the number of scores.
Measurement – Determining the size of a quantity.
Measures of central tendency – Also known as measures of location. The most common measures are mean, median and mode.
Measures of spread – Measures of spread include range, interquartile range and standard deviation.
Median – The middle score or value. To find the median, list all the scores in increasing order and select the middle one.
Medicare levy – An additional charge to support Australia's universal health care system.
Meridians of longitude – Great imaginary circles east and west of the Greenwich meridian.
Minimum spanning tree – A spanning tree of minimum length. It connects all the vertices together with the minimum total weighting for the edges.
Minute – A measure of time or an angle. There are sixty minutes in one hour and one degree.
Modality – The number of modes occurring in a set of data.
Mode – The score that occurs the most. It is the score with the highest frequency.
Modelling – See *Algebraic modelling*.
Monthly – Every month or twelve times a year.
Mortgage – A loan given to buy a house or unit.
Multi-stage event – Two or more events such as tossing a coin and rolling a die.
Multimodal – Data with many modes or peaks.

N

Negative association – Linear association between the variables with a positive negative.
Negatively skewed – Data more on the right side. The long tail is on the left side (negative side).
Net of a solid – A drawing consisting of plane shapes that can be folded to form the solid.
Net pay – The amount remaining after deductions have been subtracted from the gross pay.
Network – A term to describe a group or system of interconnected objects. It consists of vertices and edges that indicate a path or route between two objects.
Network diagram – A representation of a group of objects called vertices that are connected together by line.
Nominal data – Categorical data whose name does not indicate order.
Non-linear association – A connection between the variables of function that results in the points on a scatterplot following a curved pattern.
Non-traversable graph – A network diagram that is not traversable. See *Traversable graph*.
Number pattern – A sequence of numbers formed using a rule. Each number in the pattern is called a term.

O

Offset survey – A survey involving the measurement of distances along a suitable diagonal or traverse. The perpendicular distances from the traverse to the vertices of the shape are called the offsets.
Open cylinder – A cylinder without a circular base. It is the curved part of the cylinder.
Opposite side – A side in right-angled triangle opposite the reference angle.
Ordinal data – Categorical data whose name does indicate order.

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Outcome – A possible result in a probability experiment.
Outlier – Data values that appear to stand out from the main body of a dataset.
Overtime – Extra payments when a person works beyond the normal working day.

P

Parallel box-and-whisker plot – A box-and-whisker plot that uses two sets of similar data together.
Parallel lines – Two or more straight lines that do not intersect. The gradient of parallel lines are equal.
Parallel of latitude – Small imaginary circles north and south of the equator.
Pareto chart – A graph that combines a frequency histogram and cumulative frequency line graph.
Path – A walk with no repeated vertices.
Pay As You Go (PAYG) – Tax deducted from a person's wage or salary throughout the year.
Per annum – Per year.
Percentage change – The increase or decrease in the quantity as a percentage of the original amount of the quantity.
Percentage error – The maximum error in a measurement as a percentage of the measurement given.
Percentile – Divides an ordered dataset into 100 equal groups.
Piecework – A fixed payment for work completed.
Plan – A view of an object from the top.
Population – The entire dataset.
Population standard deviation – A calculation for the standard deviation that uses all the data or the entire population. (σ_n)
Positive association – Linear association between the variables with a positive gradient.
Positively skewed – Data more on the left side. The long tail is on the right side (positive side).
Prefix – The first part of a word. In measurement it is used to indicate the size of a quantity.
Present value – The amount of money if invested now would equal the future value of the annuity.
Prim's algorithm – A set of rules to determine a minimum spanning tree for a graph.
Principal – The initial amount of money borrowed.
Prism – A solid shape that has the same cross-section for its entire height.
Probability – Probability is the chance of something happening. The probability of the event is calculated by dividing the number of favourable outcomes by the total number of outcomes.
Pronumeral – A letter or symbol used to represent a number.
Pyramid – A solid shape with a plane shape as its base and triangular sides meeting at an apex.
Pythagoras' theorem – The square of the hypotenuse is equal to the sum of the squares of the other two sides. $h^2 = a^2 + b^2$

Q

Quadrant – Quarter of a circle. The arc of a quadrant measures 90°.
Quadratic function – A curve whose equation has an x squared (x^2). It is defined by the general rule $y = ax^2 + bx + c$ where a, b and c are numbers.
Quadratic model – A practical situation using a function in the form $y = ax^2 + bx + c$ where a, b and c are numbers. Quadratic functions are graphed to make a curve in the shape of a parabola.
Quantile – A set of values that divide an ordered dataset into equal groups.

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Quantitative data – Numerical data. It is data that has been measured.
Quarterly – Every three months or four times a year.
Quartile – Divides an ordered dataset into 4 equal groups. See *Upper quartile* and *Lower quartile*.
Random sample – A sample that occurs when members of the population have an equal chance of being selected.
Range – The difference between the highest and lowest scores. It is a simple way of measuring the spread of the data.

R

Rate – A comparison of different quantities in definite order.
Rate of interest – See *Interest rate*.
Ratio – A number used to compare amounts of the same units in a definite order such as 3 : 4.
Reaction distance – The distance travelled by the vehicle when a driver decides to brake to when the driver first commences braking.
Reciprocal function – A curve whose equation has a variable in the denominator such as $\frac{1}{x}$. It is defined by the general rule $y = \frac{k}{x}$ where k is a number.

Reciprocal model – A practical situation using a function in the form $y = \frac{k}{x}$ where k is a number. Reciprocal functions are graphed to make a curve in the shape of a hyperbola.

Reducing-balance loan – Loan calculated on the balance owing not on the initial amount of money borrowed.
Reduction – A similar figure drawn smaller than the original figure.
Relative error – A measurement calculated by dividing the limit of reading (absolute error) by the actual measurement.
Relative frequency – The frequency of the event divided by the total number of frequencies. It estimates the chances of something happening or the probability of an event.
Retainer – A fixed payment usually paid to a person receiving a commission.
Royalty – A payment for the use of intellectual property such as book or song. It is calculated as a percentage of the revenue or profit received from its use.

S

Salary – A payment for a year's work, which is divided into equal monthly, fortnightly or weekly payments.
Salvage value – The depreciated value of an item.
Sample – A part of the population.
Sample space – The set of all possible outcomes.
Sample standard deviation – A calculation for the standard deviation when the dataset is a sample (σ_{n-1}).
Scale drawing – A drawing that represents the actual object.
Scale factor – The ratio of the size of the drawing to the actual size of the object.
Scatterplot – A graph of the ordered pairs of numbers. Each ordered pair is a dot on the graph.
Scientific notation – A way of writing very large or very small numbers. For example, $98\,000\,000 = 9.8 \times 10^7$.
Sector – Part of a circle between two radii and an arc.
Self-selected sample – Members of the population volunteer themselves.
Semicircle – Half a circle. The arc of a semicircle measures 180°.
Shortest path – A path between two vertices in a network where the sum of the weights of its edges is minimised.
Significant figures – A statement to specify the accuracy of a number. It is often used to round a number.

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Similar figure – Figures that have exactly the same shape but they are different sizes.
Simple interest – A fixed percentage of the amount invested or borrowed and is calculated on the original amount.
Simulation – A mathematical model that represents a real experiment or situation.
Simultaneous equations – Two or more equations whose values are common to all the equations. It is the point of intersection of the equations.
Sine ratio – The ratio of the opposite side to the hypotenuse in a right-angled triangle.
Skewed data – Data that is not symmetrical. See *Symmetrical, Positively skewed* and *Negatively skewed*.
Slope – See *Gradient*.
Smoothness – Data whose graph has no breaks or jagged sections.
Spanning tree – A tree that connects all of the vertices in the graph.
Speed – A rate that compares the distance travelled to the time taken.
Sphere – A perfectly round object such as a ball.
Stamp duty – Tax paid to the government when registering or transferring a motor vehicle.
Standard deviation – A measure of the spread of data about the mean. It is an average of the squared deviations of each score from the mean.
Standard drink – Any drink containing 10 grams of alcohol.
Standard form – A number between 1 and 10 multiplied by a power of ten. It is used to write very large or very small numbers more conveniently.
Standard notation – See *Scientific notation*.
Statistical investigation – A process of gathering statistics. It involves four steps: collecting data, organising data, summarising and displaying data, and analysing data.
Stem-and-leaf plot – A method of displaying data where the first part of a number is written in the stem and the second part of the number is written in the leaves.
Stopping distance – The distance a vehicle travels from the time a driver sees an event occurring t the time the vehicle is brought to stop.
Straight line depreciation – The value of an item decreases by the same amount each period.
Strata – A group within a population that reflects the characteristics of the entire population.
Stratified sample – A sample using categories or strata of a population. Members from each category are randomly selected. For example, one student is selected from each year 7, 8, 9, 10, 11 and 12.
Strength of an association – A measure of how much scatter there is in the scatterplot.
Subject of the formula – When a formula or equation has a pronumeral with no numbers on the left-hand side of the equal sign, such as $C = 40n + 75$, then C is the subject of the formula.
Substitution – It involves replacing the pronumeral in an algebraic expression with one or more numbers.
Summary statistic – A number such as the mode, mean or median that describes the data.
Superannuation fund – Type of annuity where money is invested for a person's retirement.
Surface area – The sum of the area of each surface of the solid.
Symmetrical – Data that forms a mirror image of itself when folded in the 'middle' along a vertical axis.
Symmetry – Data evenly balanced about the centre.
Systematic sample – A sample that divides the population into a structured sample size; for example, sorting the names of people in alphabetical order and selecting every 5th person.
Systolic – Maximum blood pressure.

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T

Tangent ratio – The ratio of the opposite side to the adjacent side in a right-angled triangle.
Taxable income – The gross income minus any allowable deductions.
Time 24-hour – Time of day written in form hh : mm (hours : minutes).
Time zone – A region of the earth that has a uniform standard time or local time.
Time-and-a-half – A penalty rate that pays the employee one and half times the normal hourly rate.
Timetable – A list of times at which possible events or actions are intended to take place.
Trail – A walk with no repeated edges.
Trapezoidal rule – A formula to estimate the area of a shape with an irregular boundary.
Traversable graph – A network diagram with a trail that includes every edge.
Traverse survey – See *Offset survey*.
Tree – A connected graph that contains no cycles, multiple edges or loops.
Tree diagram – A technique used to list the outcomes in a probability experiment. It shows each event as a branch of the tree.
Trigonometry – A branch of mathematics involving the measurement of triangles.
True bearing – A bearing using the angle measured clockwise from the north, around to the required direction, such as 120°.
Undirected network – A network whose edges have no arrows and travel is possible in both directions.

U

Unimodal – Data with only one mode or peak.
Unitary method – A technique used to solve a problem that involves finding one unit of an amount by division.
Upper extreme – Highest score in the dataset.
Upper quartile – The highest 25% of the scores in the dataset.
Value Added Tax (VAT) – A tax added to the purchase price of each item. VAT is used in many countries with the rate ranging from 2% to 25%.

V

Variable – A symbol used to represent a number or group of numbers.
Vertex – A point (or dot) in a network diagram at which lines of pathways intersect or the turning point of a parabola.
Volume – The amount of space occupied by a three-dimensional object.

W

Wage – A payment for work that is calculated on an hourly basis.
Walk – A connected sequence of the edges showing a route between vertices where the edges and vertices may be visited multiple times.
Weighted edge – The edge of a network diagram that has a number assigned to it that implies some numerical value such as cost, distance or time.
Weighted graph – A network diagram that has weighted edges.

X

***x*-intercept** – The point at which the graph cuts the x-axis.

Y

***y*-intercept** – The point at which the graph cuts the y-axis.

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Answers

Chapter 1

Exercise 1A

1a Not a rate **b** Rate
c Rate **d** Not a rate
e Not a rate **f** Rate
2a \$25/h **b** 12 m/s
c 70 L/h **d** \$3.25/h
e 60c/kg **f** 10.5 km/L
g 300 rev/min **h** 5°/h
i 11 m/s **j** 70 L/h
3a 5 km/L **b** 150 cm/s
c \$128/m **d** 112 L/min
e 6 mg/g **f** 14 g/L
g 2.5 g/m^2 **h** 15 mL/min
i 20 c/call **j** \$17.50/hour
4a 6 goals/game **b** 3 days/year
c \$0.30/kg **d** \$30/h
e 4500 cans/h **f** \$14 000/hectare
g 16 mm/day **h** 1900 revs/min
i 0.25 km/min **j** 4 min/km
5a 654 m/s **b** 200 cm/s
c 880 mm/h **d** 920 m/min
e 400 m/s **f** 0.0575 km/s
g 6090 mg/mL
h 4 800 000 mL/kL
i 30 000 cm/s **j** 0.6 km/min
6a 3 L/h **b** 7 h
7a 1.5 kebabs/person
b \$6/person
8a 5.8 h/weekday
b 6.5 h/weekend day
c 42 h/week
d 6 h/day
9 Sebastian: 3.75 min/km
Alexander: 3.55 min/km
10 Growth per year is 1200 members.

Exercise 1B

1a \$12.50 **b** \$196 **c** \$175
d \$195 **e** \$2.90 **f** 36 km
g 12 mL **h** 20 mL
2 90 runs
3 \$17.60
4 17.5 km
5 \$27
6 \$43.30
7 \$36.00
8 6 h
9 120 L
10 2.5 h
11 25 weeks
12 11 goals
13 1 875 000 kb

Exercise 1C

1a Option 1 **b** Option 1
c Option 2 **d** Option 2
e Option 2 **f** Option 1
g Option 2 **h** Option 2
i Option 1 **j** Option 2
2a Small \$1.25/100 g
Medium \$1.20/100 g
Large \$1.10/100g
b Large cereal
3a \$0.73 **b** \$2.02 **c** \$4.60
d \$73 **e** \$58
4a \$1.25 **b** \$2.15 **c** \$3.95
d \$4.85 **e** \$109 **f** \$171.50
5 6.25 kg
6a \$175 **b** \$233
7a \$11/\$1000 **b** \$10.80/\$1000
8a \$1592.50 **b** \$1683.50
c \$1820 **d** \$1911
9 9 h
10a \$337.50 **b** 32 h **c** \$78 000
11a \$193.60 **b** \$968
c \$1936 **d** \$50 336
12 \$1560
13 \$135.30

Exercise 1D

1a 80 km/h **b** 97 km/h
c 80 km/h **d** 88 km/h
e 96 km/h **f** 54 km/h
2a 248 km **b** 310 km
c 161.2 km **d** 77.5 km
e 217 km **f** 170.5 km
3a 3 h **b** 3 h **c** 3 h
d 5 h **e** 6 h **f** 8 h
4a 7/300 h **b** 227.27 km/h
5a 70 km **b** 0.9 h
c 77.78 km/h
6a 45 min **b** 64 km/h
7a 20 km **b** 200 km
c 300 km **d** 95 km/h
e 16.96 km/h **f** 140 h
8 200 s
9a 3 km **b** 180 km
10a 17 m/s **b** 72 m/s
11 36 km/h
12 350 mm
13a 233 km/h **b** 267 km/h
c 81 km/h **d** 63 km/h
e 48 km/h **f** 115 km/h
14a 221 km **b** 374 km
c 34 km **d** 2 km
e 91 km **f** 272 km
15a 5 h 29 min **b** 50 min
c 5 h 6 min **d** 9 h 21 min
e 20 min **f** 1 h 40 min

Exercise 1E

1a S **b** P **c** Q
d R **e** T
2a 20 km **b** 5 h **c** 2 h
d 15 km
e Approximately 17
f 2.5 km/h
3a 1 h **b** 140 km
c 280 km **d** 80 km/h
e 140 km/h
4a 1 h **b** 150 km
c 9 h **d** 30 km/h
e 37.5 km/h **f** 33.33 km/h

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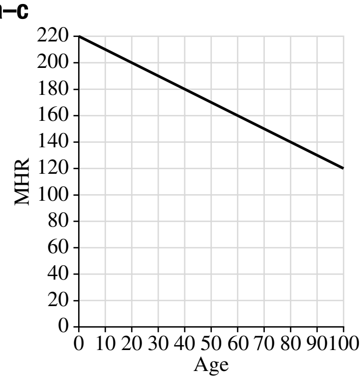
5a 200 m **b** 80 s
c i 30 m
ii 62 m
iii 150 m

Exercise 1F

1a 10 L/100 km **b** 20 L/100 km
c 5 L/100 km **d** 12.5 L/100 km
e 8.75 L/100 km **f** 6.15 L/100 km
g 5.8 L/100 km **h** 11.4 L/100 km
i 6.7 L/100 km **j** 3.8 L/100 km
2a 316 L **b** 77 L **c** 212 L
d 51 L **e** 57 L **f** 24 L
g 267 L
3a 450 km **b** 36 L **c** $43.20
4 480 km
5a 27 L **b** 4.5 L **c** $13.50
d $27.00 **e** 74 km **f** 370 km
6a 562.5 km **b** 134.4 L **c** Once
7a 26.688 L **b** $18.28 **c** 34.848 L
d $23.87 **e** $5.59
8a 1308 L **b** $1935.84
c 1680 L **d** $2688
e $4623.84 **f** $6153.60
9 Student investigation.

Exercise 1G

1a 200 **b** 190 **c** 180
d 170 **e** 160 **f** 150
g 140 **h** 130 **i** 120
2a–c



d 195 **e** 182
f 65 **g** 45
3a 130–170 **b** 123.5–161.5
c 117–153 **d** 110.5–144.5
e 104–136 **f** 97.5–127.5
g 91–119 **h** 84.5–110.5
i 78–102

4a 77–82 **b** 60–64
c Below average
d Athlete
5 Investigation
6 Investigation
7a 91 **b** 55 **c** 1406
d 70.30 **e** 10.5 **f** 8.1

Exercise 1H

1a less than120
b 90–99
c 180 or higher
d 80–89
e Prehypertension
f Hypertension stage 1
g Hypertension crisis
h Hypertension stage 2
2a $\frac{144}{88}$ **b** $\frac{135}{82.5}$
3a 12.5% **b** 25%
4a 171 **b** 115
c 132.15 **d** 14.08
5a 182 **b** 105 **c** 167
d 130 **e** day 6 **f** 4
g 16
6a 344 700 **b** 2 340 500

Review 1

Multiple-choice

1 A **2** A **3** C **4** D **5** C
6 A **7** A **8** C **9** C **10** B

Short-answer

1a $0.015/g **b** 240 m/min
c 20 mm/min **d** 4.8 kg/mg
e 14 000 000 mL/kg
f 360 c/mg
2 $1015
3a $18.40 **b** $73.60
c $5.52 **d** $12.88
e $920 **f** $460
4a Option 1 **b** Option 1
c Option 2 **d** Option 2
5a 98 km/h **b** 70 km/h
c 192 km/h **d** 96 km/h

6a 760 km **b** 312 km
c 247 km **d** 646 km
7 225 s
8 64 km/h
9 1 h and 48 min
10a 15 km **b** 3 hours
c i 6 km **ii** 6 km **iii** 15 km
d 3.5 hours
11 640 km
12a 510 km **b** 38.25 L
c $58.14
13a 202 **b** 192
c 182 **d** 172
14a 100 **b** 210
c 110 **d** 150.40

Chapter 2

Exercise 2A

1a interconnected
b edges or arcs
c vertex
d degree
e arrows
2a True **b** False
c True **d** True
e False **f** False
g True
3a 5 **b** 6 **c** 2 **d** 3
e 3 **f** 1 **g** 3
4a 4 **b** 7 **c** 3
d 3 **e** 4 **f** 4
5a 2 **b** 3 **c** 2
d 3 **e** 10 **f** 5
6a 3 **b** 3 **c** 2
d 3 **e** 14 **f** 7
7a 2 **b** 3 **c** 3
d 2 **e** 10 **f** 5
8a 3 **b** 2 **c** 4 **d** 4
9a B **b** A, D & E
c A, B, D & E **d** C & F
10a A & B **b** C, D & E
c C, D & E **d** A & B
11a All vertices have a degree of 4.
b All vertices have a degree of 4.
c All 5 vertices have even degrees.
d There are no vertices with odd degrees.

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12a 4 **b** 4 **c** 2 **d** 2
e 2 **f** 2 **g** 0 **h** 4
13a 4 **b** 6 **c** 4
d 2 **e** 0 **f** 4
14a 4 **b** 7 **c** 6 **d** 2
e There are two vertices with odd degrees (A & C).
f There are two vertices with even degrees (B & D).
15a 8 **b** 14 **c** 3
d 5 **e** 3 **f** 2
g All 8 vertices have a odd degree.
h There are no vertices with even degrees.

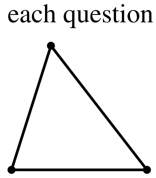
Exercise 2B

1a path **b** trail **c** path
d walk **e** trail **f** path
2a walk **b** cycle **c** path
d walk **e** path **f** walk
3a Cycle **b** Trail **c** Cycle
d Trail **e** Cycle
4a Trail **b** Circuit **c** Circuit
d Trail **e** Circuit **f** Trail
5a ii *G–K–L–S–E–K–M*
b i *K–E–G–M–L*
ii *E–K–L–M*
c ii *E–S–K–L–M–K–E*
d i *K–E–G–K*
iii *L–S–E–K–L*

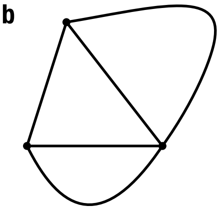
Exercise 2C

1a Graphs 1 and 3 are connected. Graphs 2 and 4 are not connected.
b Graphs 2 and 3 are connected. Graphs 1 and 4 are not connected.
2 There are many possible answers to this question. An example for each question is shown below.

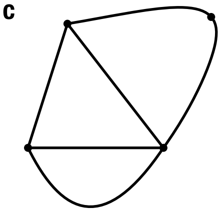
a



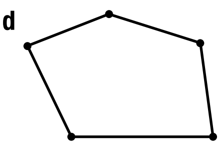
b



c

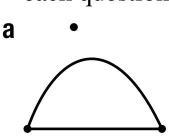


d

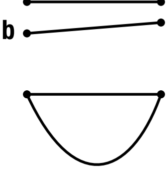


3 There are many possible answers to this question. An example for each question is shown below.

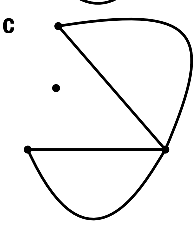
a



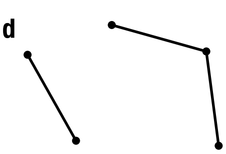
b



c



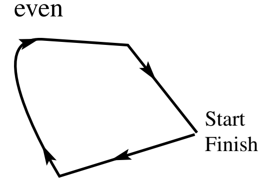
d



4a Graph 1 (Vertex *C* has degree 3 in graphs 2, 3 and 4 but degree 2 in graph 1. Graphs 2, 3, and 4 have the same edges.)
b Graph 2 (Vertex *C* has degree 2 in graphs 1, 3 and 4 but degree 3 in graph 2. Graphs 1, 3, and 4 have the same edges.)
5a i 9 min **ii** 16 min
iii 10 min **iv** 11 min
v 11 min
b *A–F–E–C–D*
c *A–F–E–D*
6a 9 **b** 27 **c** 116 **d** 950
e 640 **f** 720 **g** 8620
h Perth–Sydney–Melbourne
Perth–Sydney–Adelaide–Melbourne
Perth–Adelaide–Melbourne
Perth–Adelaide–Sydney–Melbourne
i 1920
j Sydney–Adelaide–Melbourne
Sydney–Adelaide–Perth–Melbourne
Sydney–Perth–Melbourne
Sydney–Perth–Adelaide–Melbourne
k 7270

Exercise 2D

1a $\deg(A) = 3$, $\deg(B) = 4$, $\deg(C) = 2$, $\deg(D) = 4$, $\deg(E) = 3$
b Eulerian trails exist if the graph is connected and has exactly two vertices with an odd degree. Vertices *A* and *E* are the only vertices with odd degrees.
c Example:
A–B–E–D–B–C–D–A–E
2a $\deg(A) = 2$, $\deg(B) = 2$, $\deg(C) = 4$, $\deg(D) = 2$, $\deg(E) = 2$
b Eulerian circuits exist if the graph is connected and every vertice of the graph has an even degree. All vertices are even.
c Example: *A–B–C–E–D–C–A*
3a Eulerian circuit: all vertices are even



b Neither: more than two odd vertices

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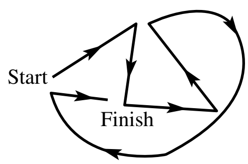
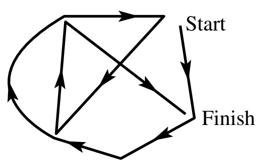
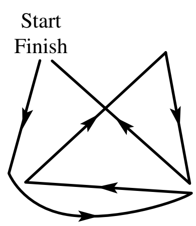
c Eulerian trail: two odd vertices, rest are even



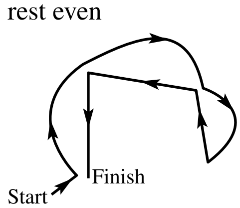
d Eulerian trail: two odd vertices, rest are even



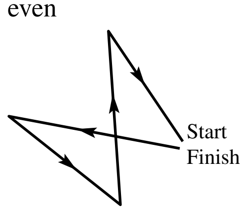
e Eulerian circuit: all vertices are even



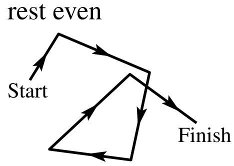
f Eulerian trail: two odd vertices, rest even



g Eulerian circuit: all vertices are even



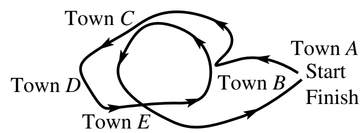
h Eulerian trail: two odd vertices, rest even



i Neither: more than two odd vertices

4a Yes, all vertices are even.

b Other routes are possible.



5a *A–B–D–C*

b Yes.

c No

d A Hamiltonian path passes through every vertex of a graph once and only once.

e *A–D–B–C*

6a *A–C–F–E–D–B–A*

b Yes

c No

d A Hamiltonian cycle is a Hamiltonian path that starts and finishes at the same vertex.

e *A–F–E–D–C–B–A*

7 Other answers are possible.

a *A–B–C–D–E–F–A*

b *A–B–C–D–E–A*

c *A–B–C–F–I–H–E–G–D–A*

d *F–E–D–A–B–C–F*

e *A–E–F–D–C–B–A*

f *A–F–E–D–C–B–G–A*

8a *C–D–E–B–A*. Hamiltonian path

b *E–A–B–C–D–E*. Hamiltonian cycle

Exercise 2E

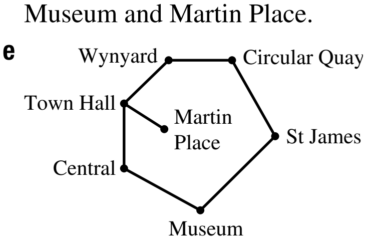
1a Vertices are the train stations.

b Edges are the rail lines that connect the train stations.

c Yes. Every vertex in the graph is accessible from every other vertex.

d City circle has seven stations – Central, Town Hall, Wynyard, Circular Quay, St James, Museum and Martin Place.

e

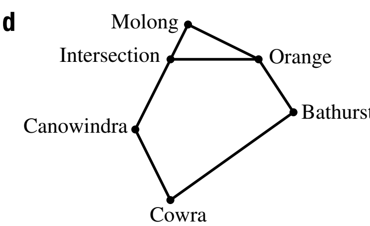


2a Vertices are the regional towns and cities and other places where roads meet.

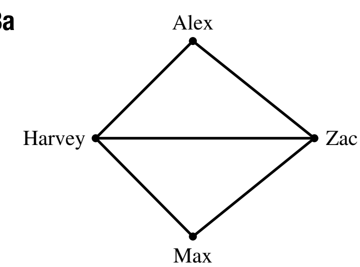
b Edges are the main roads linking the towns.

c Yes. Every vertex in the graph is accessible from every other vertex.

d



3a

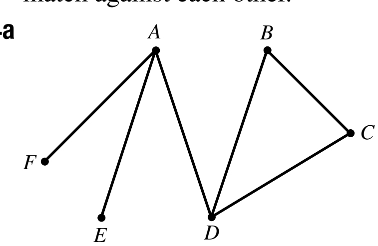


b Vertices are the names of the players.

c Edges indicate that a match has taken place between the two players.

d Alex and Max have not played a match against each other.

4a



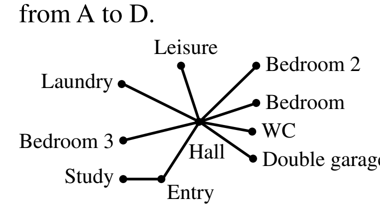
b Vertices are the cities A to F.

c Edges are the motorways M1 to M6.

d B and C are not linked to A.

e Travel from F to A and then from A to D.

5



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Exercise 2F

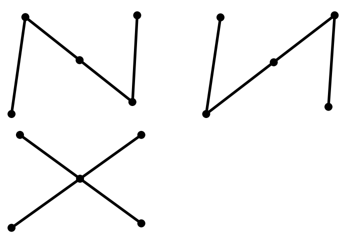
1a Edges **b** Length
c Spanning **d** Eight
e Nine

2a 14 **b** 6
c Various answers are possible.
d Various answers are possible.

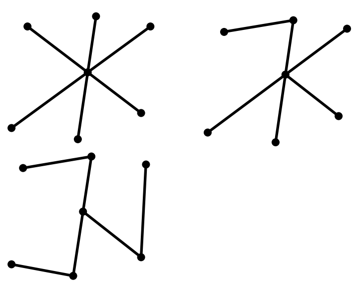
3a Tree **b** Tree
c Not a tree **d** Tree
e Not a tree
f Not a tree as it has a loop

4 Other answers are possible.

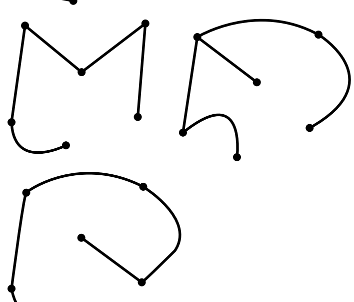
a



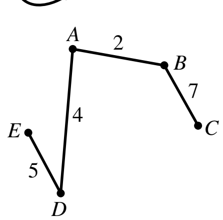
b



c

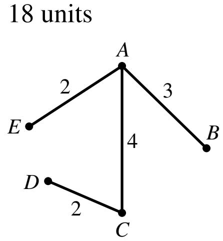


5a



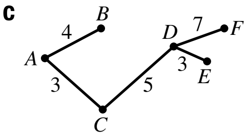
18 units

b



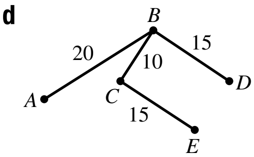
11 units

c



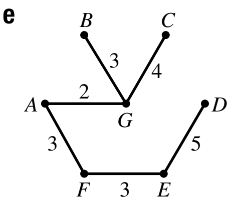
22 units

d



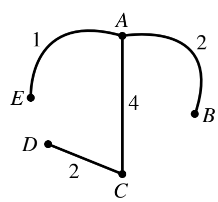
60 units

e



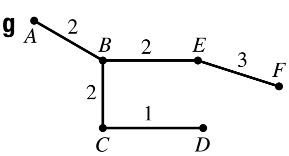
20 units

f



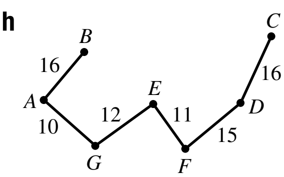
9 units

g



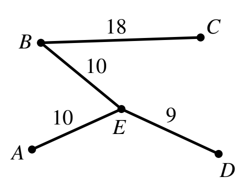
10 units

h



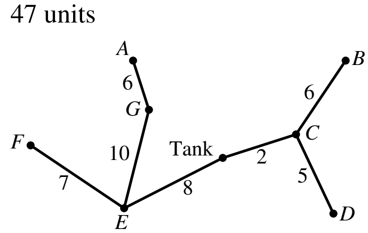
80 units

i



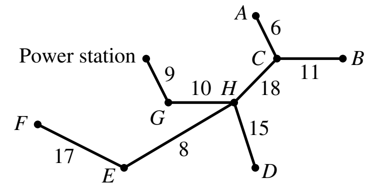
47 units

6



44 m

7



94 km

Exercise 2G

1a 8 h **b** 11 h **c** 15 h **d** 12 h

2a 20 m **b** 30 m **c** 35 m **d** 25 m

3a \$8 **b** \$6 **c** \$10 **d** \$6
e \$9 **f** \$4 **g** \$6 **h** \$9

4a 5 min **b** 5 min
c 10 min **d** 7 min
e 8 min **f** 8 min

5a 17 m **b** 19 m **c** 19 m
d 16 m **e** 18 m **f** 26 m

6a 34 km **b** 37 km **c** 56 km
d 64 km **e** 22 km **f** 28 km
g *A–E–F–G–I* or *A–C–F–G–I*, 26 km
h *C–A–B–D*, 17 km

7a 64 min **b** 64 min
c 77 min
d *A–B–C–E–F–D–A*, 63 min
e The shortest average time may not be the best path for a competitor. For example, if a particular competitor is faster than average going uphill, then they could be quicker going through the checkpoints with hills.

8a 145 min **b** 220 min
c *A–B–E–F–G* 115 min
d The shortest time in a train journey may not be the best path if the train is crowded and you do not get a seat. The best path could be to catch a train that takes longer but you have a seat on the train.
e 135 minutes, using the route *A–B–F–G*.

9a Town *B*, 7 km
b Town *D*, 6 km
c Town *C*, 8 km
d Town *E* or *F*, 9 km
e Town *E* or *F*, 7 km
f 37 km

Answers

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10a Town C, \$8 million
b Town D, \$4 million
c Town B, \$5 million
d Town E, \$20 million
e Town F, \$8 million
f Town G, \$11 million
g \$56 million
11a Town F, 90 km
b Town E, 90 km
c Town C, 100 km
d Town H, 70 km
e Town B, 80 km
f Town D, 100 km
g Town G, 200 km
h 730 km

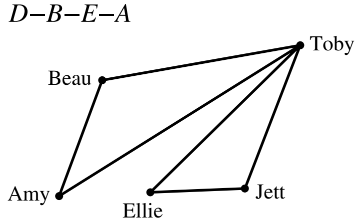
Review 2

Multiple-choice

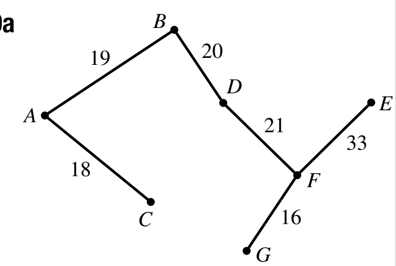
1	A	**2**	C	**3**	D	**4**	B and C
5	C	**6**	B	**7**	A	**8**	A

Short-answer

1a 2 **b** 5 **c** 3
d 4 **e** 4 **f** 2
2a deg(A) = 1, deg(B) = 2, deg(C) = 4, deg(D) = 2, deg(E) = 4, deg(F) = 1
b Eulerian trails exist if the graph is connected and has exactly two vertices with an odd degree. Vertices A and F are the only vertices with odd degrees.
c $A–C–B–D–E–C–E–F$
3a deg(A) = 2, deg(B) = 4, deg(C) = 2, deg(D) = 4, deg(E) = 4, deg(F) = 2
b Eulerian circuits exist if the graph is connected and every vertex has an even degree. All vertices are even.
c $A–B–C–D–E–F$
$D–B–E–A$
4a



b Vertices are the names of the players.
c Amy & Ellie, Amy & Jett, Beau & Ellie, Beau & Jett
5 24
6 56
7a 11 km
b 17 km
8 12
9a



b 127 km

Chapter 3

Exercise 3A

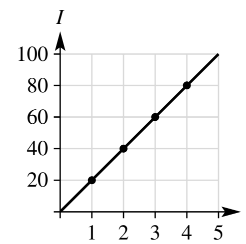
1a \$5850 **b** \$910
c \$37 000 **d** \$1134
e \$15225
2a \$600 **b** \$1089
c \$5020 **d** \$10 005
e \$11 518.75
3a 1.2% **b** 0.4%
c 2.4% **d** 3.6%
4a \$8 **b** \$144
c \$1440 **d** \$900
5 \$55 200
6 \$69.75
7a \$22 000
b \$122 000
8 \$5040
9a \$7680
b \$7560
c \$7200
10 Computer application
11 \$71.75
12 \$40 000
13 7.5%
14 $3\frac{1}{3}$ years
15a \$51000
b \$2000

Exercise 3B

1a $I = 120n$
b

n	0	1	2	3	4
I	0	20	40	60	80

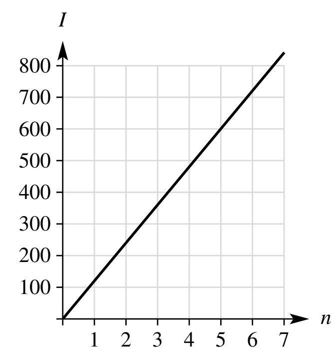
cd



2a $I = 120n$
b

n	0	1	2	3	4	5
I	0	120	240	360	480	600

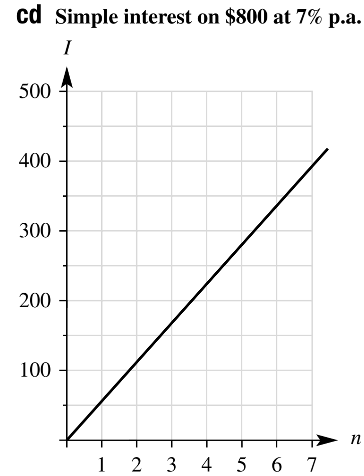
cd Simple interest on \$2000 at 6% p.a.



e \$300 **fg** \$720
3a $I = 56n$
b

n	0	1	2	3	4	5	6
I	0	56	112	168	224	280	336

cd Simple interest on \$800 at 7% p.a.



e \$140 **fg** \$390

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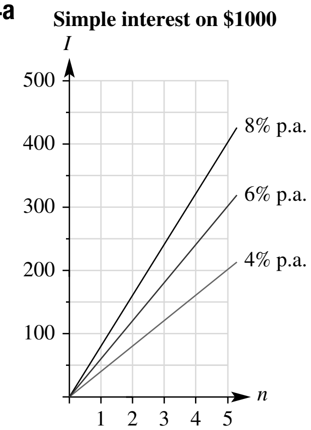
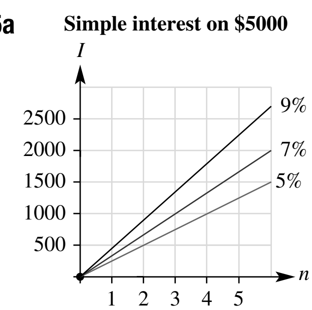
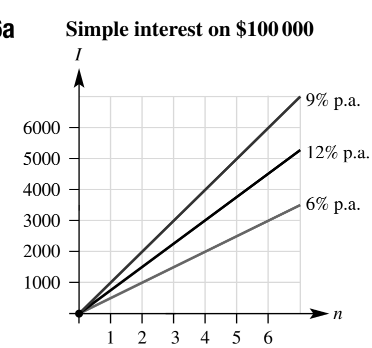
4a


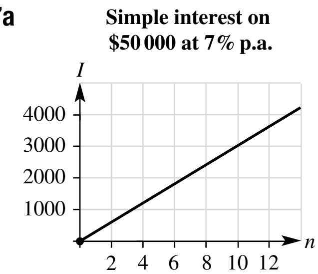
b i $200 **ii** $300 **iii** $400
c i $140 **ii** $210 **iii** $280
d i 5 years **ii** 3.3 years
iii 2.5 years

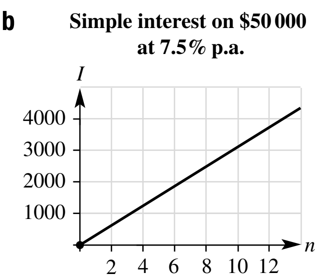
5a


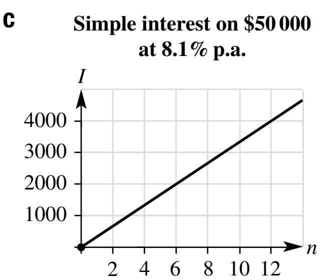
b 5% earns $625, 7% earns $875 and 9% earns $1125
c 5% earns $1250, 7% earns $1750 and 9% earns $2250
d 5% takes 4 years, 7% takes about 2.8 years and 9% takes 2.2 years

6a


b 6% earns $500, 9% earns $750 and 12% earns $1000
c 6% earns $3000, 9% earns $4500 and 12% earns $6000
d 6% takes 4 months, 9% takes about 2.7 months and 12% takes 2 months

7a


b


c


Exercise 3C

1a $424.36 **b** $35 372.71
c $70 710.87 **d** $3920.88
e $22 843.06 **f** $78 311.44
g $388 543.35 **h** $16 634.84
2 $17640
3 $101 011.73
4a $2800 **b** $3591
c $5600 **d** $9263
5a $10 063.69 **b** $102 028.69
c $1307.84 **d** $23 509.61
6 $18 281.70
7 $7378
8a $9289.92 **b** $13 968.61
c $44 799.23
9 Computer application
10 Investment 2 by $80
11a $828.51 **b** $1586.23
c $7147.51 **d** $32 539.90
e $1587.65 **f** $10 387.92
g $1272.62 **h** $1920.26
i $583.20
12 Investment E
13 $4588.29
14a $25 531.63 **b** $9131.63
15 $12 107.45
16 $2536.50
17 $6433.75
18a $20 766.90 **b** $7266.90

Exercise 3D

1a $29 063.34
b $48 444.86
c $121 953.77
d $112 227.23
e $3243.49
f $55 621.16
g $189 862.19
h $46 543.55
2 $39 604.68
3 $53 608.98
4 $49 662.32
5 $3542
6a $925 **b** $5964
c $1304 **d** $15 421
e $2679
7 $57 747.51
8 $15 917.61

Exercise 3E

1a $FV = 2000 \times (1.06)^n$
b $I = 2000(1.06)^n - 2000$
c

n	0	1	2	3	4	5
FV	2000	2120	2247	2382	2525	2676
I	0	120	247	382	525	676

Answers

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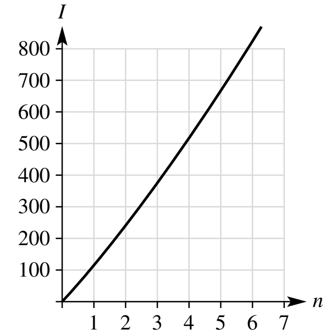
de **Compound interest on $2000 at 6% p.a.**



f $840

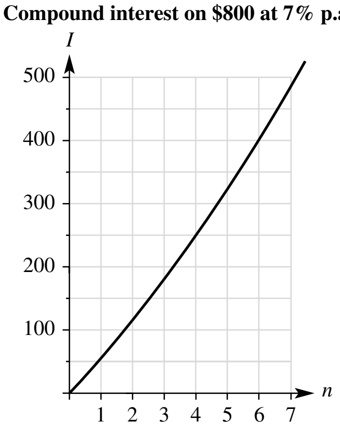
2a $FV = 800 \times (1.07)^n$

b $I = 800(1.07)^n - 800$

c

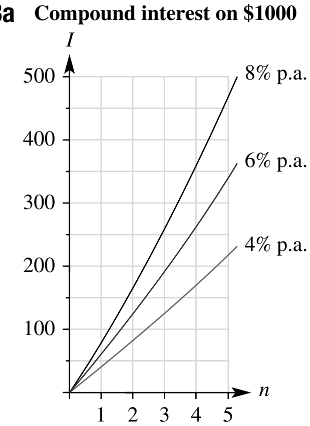
n	0	1	2	3	4	5	6
FV	800	856	916	980	1049	1122	1201
I	0	56	116	180	249	322	401

de **Compound interest on $800 at 7% p.a.**



f $150 **g** $485

3a **Compound interest on $1000**



b i $217

ii $338

iii $469

c i $150

ii $220

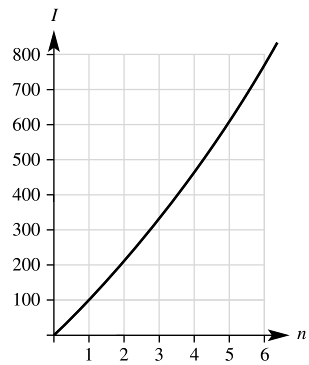
iii $310

d i 4.6 years

ii 3.1 years

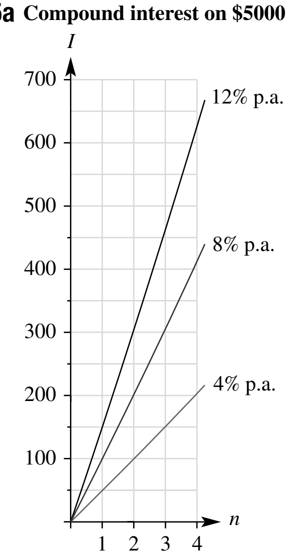
iii 2.3 years

4 **Compound interest on $1000 at 10% p.a.**



Interest is about $690

5a **Compound interest on $5000**



b i $50

ii $100

iii $150

c i $152

ii $306

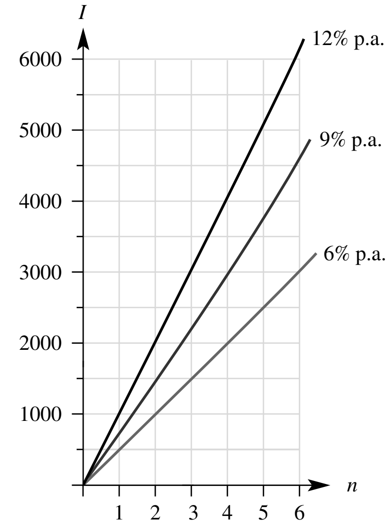
iii $463

d i about 4 quarters

ii about 2 quarters

iii about 1.3 quarters

6a **Compound interest on $100 000**



b i $1003

ii $1506

iii $2010

c i $3038

ii $4585

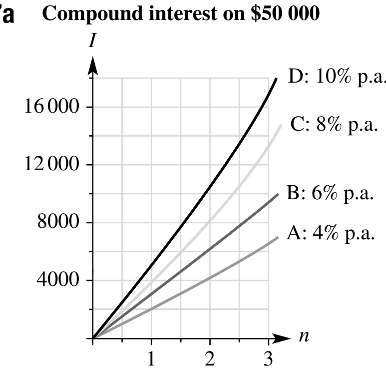
iii $6152

d i 6 months

ii 4 months

iii 3 months

7a **Compound interest on $50 000**



b About $6100 **c** About $6000

d Just under 2 years

Exercise 3F

1 $88 373.39

2 $535 092.25

3a $5151.43 **b** $7543.25

c $1764.48

4 $2360.28

5 $1900

6 $1 174 287

7 $1519

8 $498

9 $5144

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10 $1954
11 $554
12 $478
13a $4.14 **b** $3.83
c $5.22 **d** $6.03
14a $340 342 **b** $923 785
c $393 824 **d** $704 994
e $1 531 538 **f** $1 162 024
15a $41 327 **b** $27 349
c $79 008 **d** $24 298
e $145 861 **f** $69 527
16a About $140 **b** $180
c $50 **d** About $22

Review 3

Multiple-choice

1 C **2** B **3** C **4** C **5** A
6 D **7** A **8** B **9** D

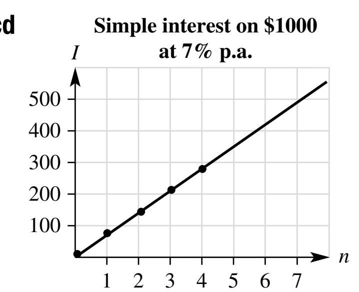
Short-answer

1 $96 000
2 $2600
3 $18
4 $8820
5a $I = 70n$

b

n	0	1	2	3	4
I	0	70	140	210	280

cd

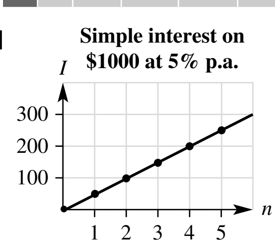


e $175 **f** $420 **g** 3 years
6a $I = 50n$

b

n	0	1	2	3	4	5
I	0	50	100	150	200	250

cd

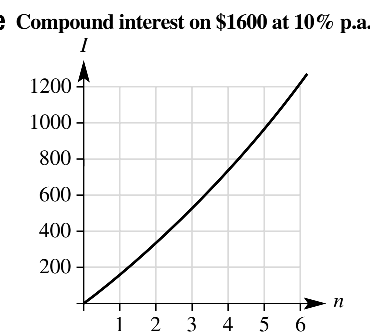


e $275

7a $1118.27 **b** $2520.54
8a $22 524.07 **b** $6664.49
9 $62 360.95
10a $FV = 1600 \times (1.10)^n$
b $I = 1600 \times (1.01)^n - 1600$
c

n	0	1	2	3	4	5	6
FV	1600	1760	1936	2130	2343	2577	2834
I	0	176	336	530	743	977	1234

de



f about $250
11a $710 517 **b** $230 517
12a $3.02 **b** $4.95
c $6.46 **d** $1.72
e $4.85 **f** $9.58

Chapter 4

Exercise 4A

1a 10.0 cm **b** 13.0 cm
c 26.0 mm **d** 22.4 mm
e 4.9 cm **f** 83.1 cm
2a 9.00 cm **b** 14.70 cm
c 11.53 mm **d** 21.17 mm
e 4.21 cm **f** 10.42 cm
3a $y = 55$ mm **b** $b = 55$ mm
c $x = 23$ mm **d** $d = 25$ mm
e $b = 62$ mm **f** $m = 14$ mm
g 24 mm **h** 2 mm
i 16 mm
4a 9.0 m
5a $y = 5.74$ cm **b** $x = 8.31$ cm

Exercise 4B

1 Width of gate is 2.5 m
2 Length of ladder is 3.9 m
3 Height is 4.2 m
4 Length is 11.2 m
5 AC = 20 m
6 Fence is 55.902 m
7 Distance is 49 m
8 Side length is 34.64 cm

Exercise 4C

1a $h = 10, o = 8, a = 6$
b $h = 13, o = 12, a = 5$
c $h = 5, o = 3, a = 4$
d $h = 39, o = 15, a = 36$
e $h = 15, o = 9, a = 12$
f $h = 30, o = 24, a = 18$
2a $h = z, o = x, a = y$
b $h = c, o = a, a = b$
c $h = f, o = e, a = d$
3a $\sin {\sim} = \frac{4}{5}, \cos {\sim} = \frac{3}{5},$ $\tan {\sim} = \frac{4}{3}$
b $\sin {\sim} = \frac{12}{13}, \cos {\sim} = \frac{5}{13},$ $\tan {\sim} = \frac{12}{5}$
c $\sin {\sim} = \frac{3}{5}, \cos {\sim} = \frac{4}{5},$ $\tan {\sim} = \frac{3}{4}$
d $\sin {\sim} = \frac{15}{39}, \cos {\sim} = \frac{36}{39},$ $\tan {\sim} = \frac{15}{36}$
e $\sin {\sim} = \frac{9}{15}, \cos {\sim} = \frac{12}{15},$ $\tan {\sim} = \frac{9}{12}$
f $\sin {\sim} = \frac{12}{15}, \cos {\sim} = \frac{9}{15},$ $\tan {\sim} = \frac{12}{9}$
4a $\sin {\sim} = \frac{x}{z}, \cos {\sim} = \frac{y}{z}, \tan {\sim} = \frac{x}{y}$
b $\sin {\sim} = \frac{a}{c}, \cos {\sim} = \frac{b}{c}, \tan {\sim} = \frac{a}{b}$
c $\sin {\sim} = \frac{e}{f}, \cos {\sim} = \frac{d}{f}, \tan {\sim} = \frac{e}{d}$
5a i sin ~ **ii** cos ~ **iii** tan ~
b i cos ~ **ii** tan ~ **iii** sin ~

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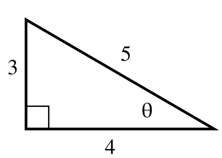
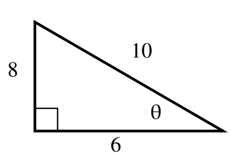
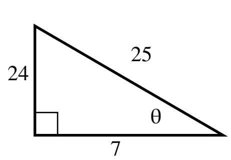
6a i $\sin \tilde{} = \frac{11}{61}, \cos \tilde{} = \frac{60}{61},$
$\tan \tilde{} = \frac{11}{60}$
ii $\sin {}^\circ = \frac{60}{61}, \cos {}^\circ = \frac{11}{61},$
$\tan {}^\circ = \frac{60}{11}$
b i $\sin \tilde{} = \frac{3}{5}, \cos \tilde{} = \frac{4}{5}, \tan \tilde{} = \frac{3}{4}$
ii $\sin {}^\circ = \frac{4}{5}, \cos {}^\circ = \frac{3}{5}, \tan {}^\circ = \frac{4}{3}$
c i $\sin \tilde{} = \frac{8}{17}, \cos \tilde{} = \frac{15}{17},$
$\tan \tilde{} = \frac{8}{15}$
ii $\sin {}^\circ = \frac{15}{17}, \cos {}^\circ = \frac{8}{17},$
$\tan {}^\circ = \frac{15}{8}$
d i $\sin \tilde{} = \frac{q}{r}, \cos \tilde{} = \frac{p}{r}, \tan \tilde{} = \frac{q}{p}$
ii $\sin {}^\circ = \frac{p}{r}, \cos {}^\circ = \frac{q}{r}, \tan {}^\circ = \frac{p}{q}$
7a $\sin A = \frac{15}{17}, \cos A = \frac{8}{17},$
$\sin B = \frac{8}{17}, \cos B = \frac{15}{17}$
b $\sin A = \frac{3}{5}, \cos A = \frac{4}{5}, \sin B = \frac{4}{5},$
$\cos B = \frac{3}{5}$

8a


i 5
ii $\sin \tilde{} = \frac{3}{5}, \cos \tilde{} = \frac{4}{5}$

b


i 6
ii $\cos \tilde{} = \frac{3}{5}, \tan \tilde{} = \frac{4}{3}$

c


i 24
ii $\sin \tilde{} = \frac{24}{25}, \tan \tilde{} = \frac{24}{7}$
9a Hypotenuse is 4 and 6. Adjacent is 2 & 3. Opposite is 3.46 and 5.20.
b $\cos \tilde{} = 0.5$ **c** $\sin \tilde{} = 0.87$

Exercise 4D

1a 60′ **b** 180′ **c** 300′
d 420′ **e** 600′ **f** 900′
g 1200′ **h** 3600′ **i** 30′
j 20′ **k** 12′ **l** 15′
2a 2° **b** 8° **c** 1°
d 10° **e** 6° **f** 4°
g 15° **h** 12° **i** 0.5°
j 0.25° **k** 0.75° **l** 0.33°
3a 0.34 **b** 0.73 **c** 2.14
d 0.31 **e** 0.23 **f** 0.99
g 0.97 **h** 1.11
4a 0.45 **b** 0.65 **c** 0.61
d 0.71 **e** 0.39 **f** 1.36
g 2.48 **h** 0.41
5a 3.5 **b** 4.0 **c** 4.8
d 5.9 **e** 26.0 **f** 1.5
g 1.7 **h** 12.4
6a 3.6 **b** 2.2 **c** 11.8
d 2.9 **e** 4.0 **f** 5.9
g 1.7 **h** 6.6
7a 5.96 **b** 1.12 **c** 13.24
d 4.48 **e** 5.09 **f** 1.57
g 7.25 **h** 21.80
8a 35° **b** 81° **c** 16°
d 38° **e** 27° **f** 7°
9a 21°16′ **b** 40°13′ **c** 81°46′
d 59°54′ **e** 67°36′ **f** 41°44′
10a 37° **b** 30° **c** 51°
d 76° **e** 37° **f** 53°
11a 60° **b** 24° **c** 68°
d 59° **e** 30° **f** 45°
12a 40°54′ **b** 25°40′ **c** 10°33′
d 22°12′ **e** 11°25′ **f** 25°14′
13a 24° **b** 0.9 **c** 0.44
14a 37° **b** 0.6 **c** 0.75
15a 64°32′ **b** 0.903 **c** 0.4299

Exercise 4E

1a 7.73 **b** 17.09 **c** 14.24
d 8.39 **e** 65.82 **f** 12.37
g 51.31 **h** 30.18 **i** 7.88
2a 24.38 **b** 33.16 **c** 30.55
d 16.68 **e** 16.67 **f** 4.75
3a 50.81 **b** 6.73 **c** 141.38
d 61.58 **e** 95.78 **f** 15.84
4a 68.4 **b** 90.9 **c** 126.8
d 25.5 **e** 66.7 **f** 88.7
5a 175.918 **b** 105.541 **c** 10.251

Exercise 4F

1a 47° **b** 47° **c** 25°
d 67° **e** 52° **f** 40°
g 49° **h** 56° **i** 55°
2a 51° **b** 51° **c** 50°
d 58° **e** 47° **f** 43°
3a 51°19′ **b** 57°22′ **c** 48°39′
d 47°15′ **e** 23°49′ **f** 32°35′
4a 50° **b** 32° **c** 61°
5a 53° **b** 62° **c** 44°
6a $\tilde{} = 67°23′, {}^\circ = 22°37′$
b $\tilde{} = 28°4′, {}^\circ = 61°56′$
c $\tilde{} = 36°52′, {}^\circ = 53°8′$

Exercise 4G

1 Height is 15.0 m
2 Pole is 4.50 m high
3 River is 40 m wide
4 Depth is 40.2 m
5 Angle is 34°
6 Angle is 44°
7a Horizontal distance is 3.8 km
b Height is 1.3 km
8 Ramp is 8.77 m
9 Angle is 1°26′
10 Length of the rope is 6.6 m
11a Ladders reach 3.83 m
b Angle is 33°
12 Angle is 38°

Exercise 4H

1 Height is 752 m
2 Height of the tower is 116.5 m
3 Boat is 107.23 m from the base of the cliff
4 Plane was 17 326 m from the airport
5 Height of the tree is 55.6 m
6 Depth of the shaft is 74 m
7 Height of tree is 35 m

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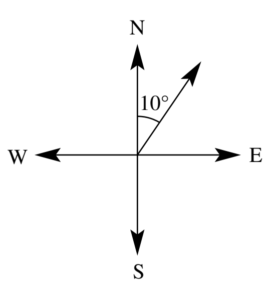
8 Angle of depression is 9°
9 Angle of elevation is 23°
10 Angle of elevation is 34°
11 Angle of depression is 2.5°
12 Angle of elevation is 3°
13 Angle of elevation is 34°
14 Angle of depression is 47°
15 Height of the tree is 10 m
16 Angle of depression is 20°58′

Exercise 4I

1a N45°E, 045°
b N45°W, 315°
c S45°E, 135°
d S45°W, 225°
2a N52°E, 052°
b N63°W, 297°
c S55°E, 125°
d S57°W, 237°
e N70°E, 070°
f N55°W, 305°
g N59°E, 059°
h N78°W, 282°
i S82°E, 098°
j S60°W, 240°
k N77°W, 283°
l S70°E, 110°

3a


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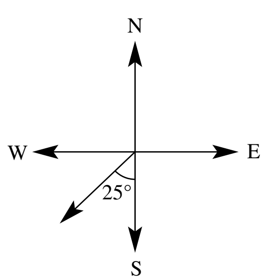
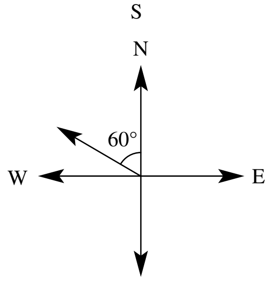


c


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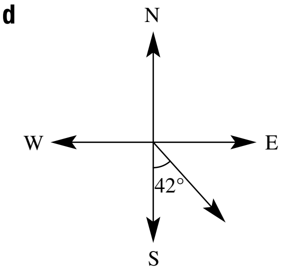
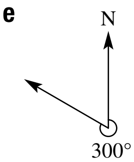
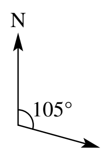
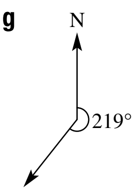
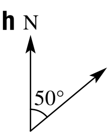


e


f


g


h


4 Aaron has run 5.1 km.
5a N30°E **b** 210° **c** S30°W
6a 045° **b** 315° **c** 200°
7a $x = 52$ and $y = 41$
b N49°W
c 311°
8a 106 km **b** 66 km
9a 58° **b** 212° **c** S32°W
10a 124.2 km **b** 57.9 km

Review 4

Multiple-choice

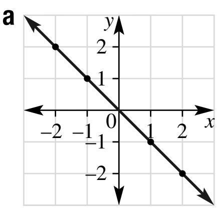
1 B	**2** B	**3** A	**4** B
5 C	**6** A	**7** D	**8** D

Short-answer

1a 35 **b** 15.65 **c** 38.47
2 Fence is 33.6 m
3 10 mm
4a 30 **b** 24 **c** 18
5a $\frac{20}{29}$ **b** $\frac{21}{29}$ **c** $\frac{20}{21}$
6a $\frac{12}{13}$ **b** $\frac{5}{13}$ **c** $\frac{12}{5}$
7a 2.48 **b** 0.97 **c** 0.39 **d** 0.14
8a 61° **b** 19° **c** 11°
9a 29.02 **b** 20.73 **c** 11.47
d 8.58 **e** 21.38 **f** 85.99
10a 28°4′ **b** 61°56′ **c** 51°18′
d 47°44′ **e** 36°52′ **f** 53°8′
11 103.2 m from the base of the cliff
12a 6.2 km **b** 5.8 km

Chapter 5

Exercise 5A

1a


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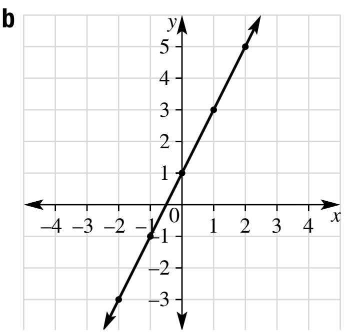
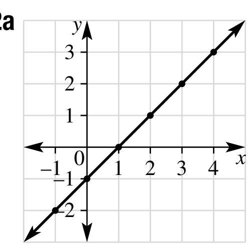
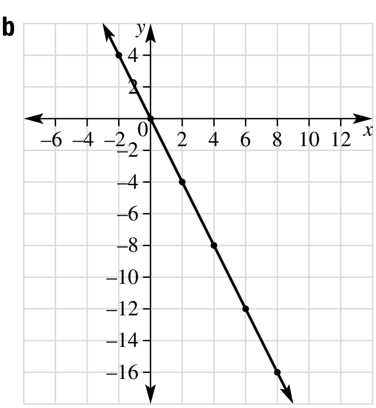


2a


b


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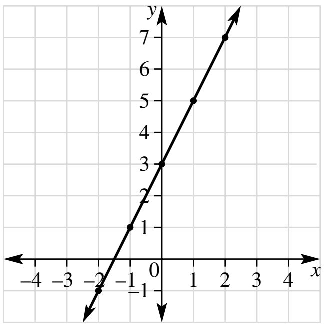
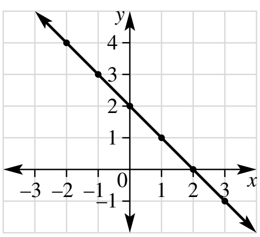
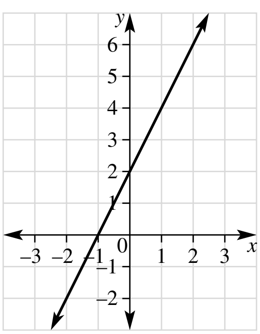
c



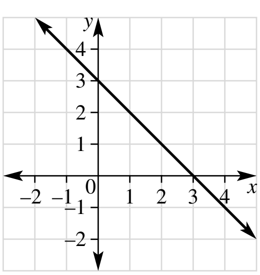
d



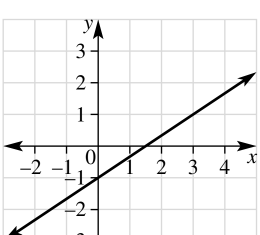
3a



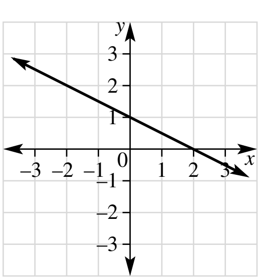
b



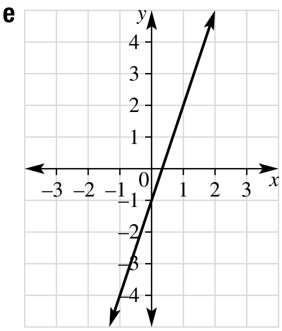
c



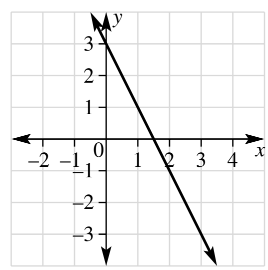
d



e



f



4a 5 **b** 1 **c** −2

d $\frac{1}{2}$ **e** $\frac{-2}{3}$ **f** $\frac{3}{4}$

5a −5 **b** 2 **c** 0

d −1 **e** 7 **f** $\frac{3}{5}$

6a $y = 2x + 1$ **b** $y = -3x + 4$

c $y = 0.5x - 2$ **d** $y = 6$

e $y = -\frac{2}{5}x + 4$ **f** $y = \frac{1}{3}x$

7a $y = \frac{1}{2}x + 1$ **b** $y = -2x + 1$

c $y = x + 2$ **d** $y = -\frac{1}{2}x + 2$

e $y = x$ **f** $y = -2x + 10$

g $y = 2x - 2$ **h** $y = -\frac{2}{3}x - 4$

i $y = x - 3$

8a Not parallel **b** Not parallel

c Parallel **d** Parallel

9a $y = -x + 3$ **b** $y = 2x + 4$

c $y = -x + 2$ **d** $y = x - 1$

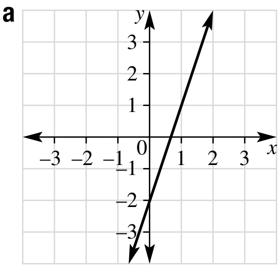
e $y = 3x + 3$ **f** $y = -\frac{1}{2}x + 4$

10a $y = 3x - 2$ **b** $y = -x + 4$

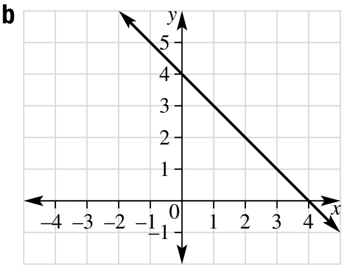
c $y = \frac{1}{2}x + 1$ **d** $y = 4x + 2$

e $y = \frac{1}{3}x - 1$ **f** $y = -3x + 4$

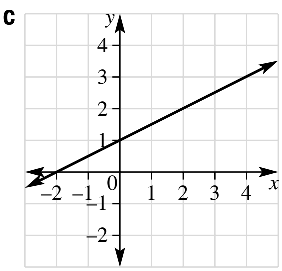
11a



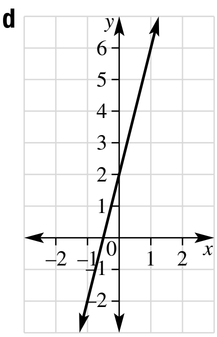
b



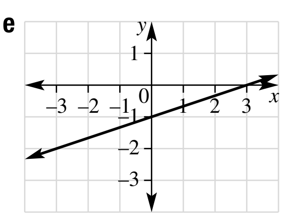
c



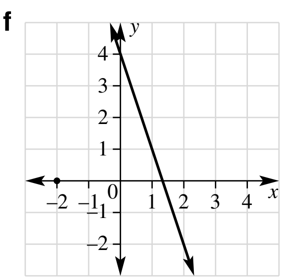
d



e



f



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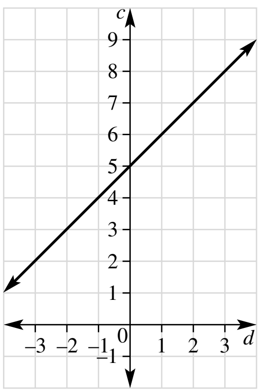
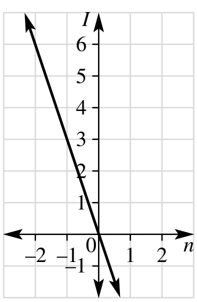
Exercise 5B

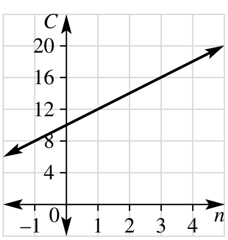
1a



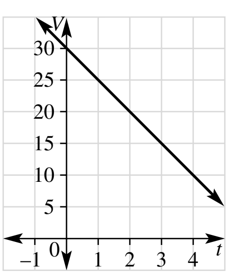
b



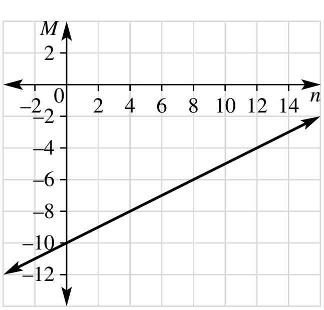
2a



b



c



3a i 18 GBP **ii** 30 GBP

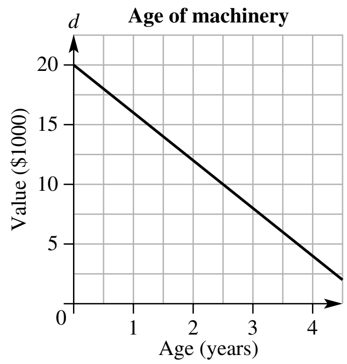
iii 33 AUD **iv** 17 AUD

b $m = \frac{3}{5}$ or 0.6

4a

a	0	1	2	3	4
v	20	16	12	8	4

b



c \$20 000 **d** 1.25 years

e \$6000 **f** \$12 000

g 2.5 years

5a \$90 000

b \$60 000

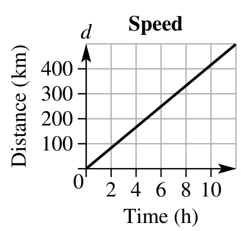
c 100 months

d $v = -1.5t + 150$

e \$141 000

6a $d = 40t$

b

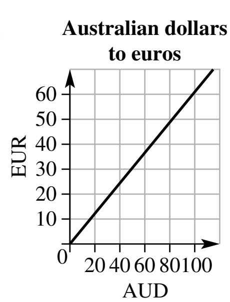


7 $C = 1.5n + 2.6$

8 $w = 20n + 350$

9 $C = 25n + 4000$

10a



b 15 EUR

c 75 AUD

d Gradient is 0.6 and the vertical intercept is 0.

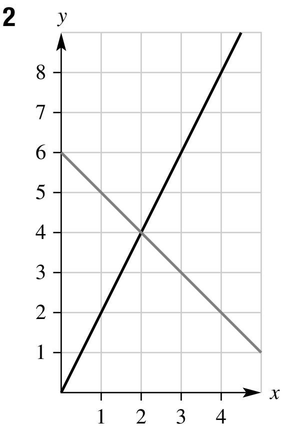
e EUR = 0.6 × AUD

Exercise 5C

1a (−1, 2) **b** (2, 0)

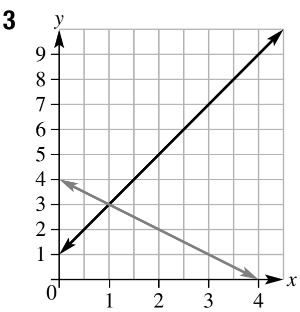
c (−2, 0) **d** (0, 1)

2



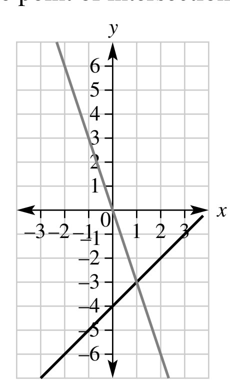
The point of intersection is (2, 4)

3



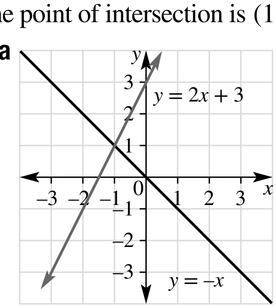
The point of intersection is (1, 3)

4



The point of intersection is (1, –3)

5a



∴ Simultaneous solution is $x = -1$ and $y = 1$

Answers

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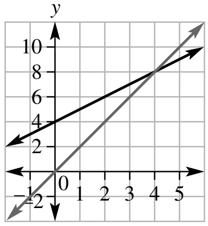
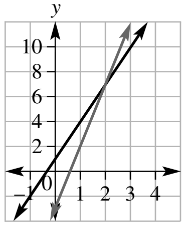
b



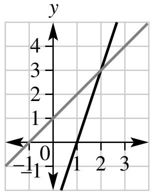
∴ Simultaneous solution is $x = 4$ and $y = 8$

c



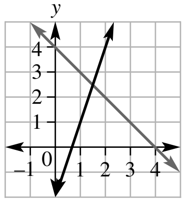
∴ Simultaneous solution is $x = 2$ and $y = 7$

6a



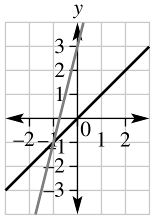
∴ Simultaneous solution is $x = 2$ and $y = 3$

b



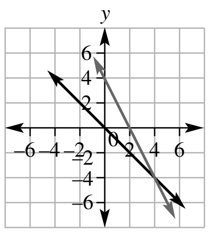
∴ Simultaneous solution is $x = 1\frac{1}{2}$ and $y = 2\frac{1}{2}$

c



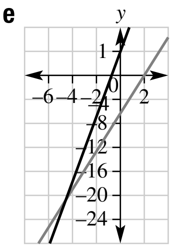
∴ Simultaneous solution is $x = -1$ and $y = -1$

d



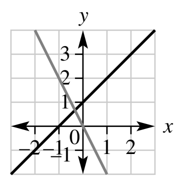
∴ Simultaneous solution is $x = 4$ and $y = -4$

e



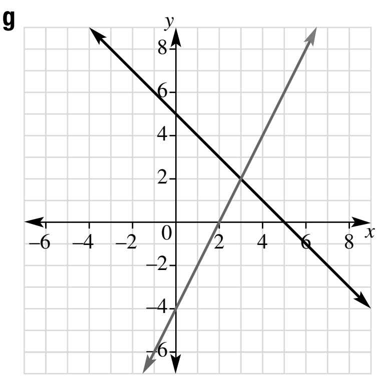
∴ Simultaneous solution is $x = -4$ and $y = -19$

f



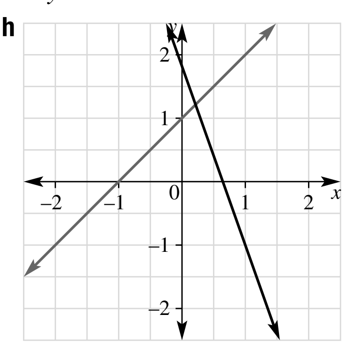
∴ Simultaneous solution is $x = -\frac{1}{3}$ and $y = \frac{2}{3}$

g



∴ Simultaneous solution is $x = 3$ and $y = 2$

h



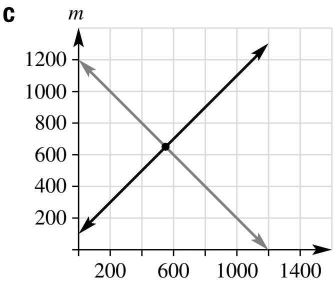
∴ Simultaneous solution is $x = \frac{1}{4}$ and $y = \frac{5}{4}$

Exercise 5D

1a $m = n + 100$

b $m + n = 1200$

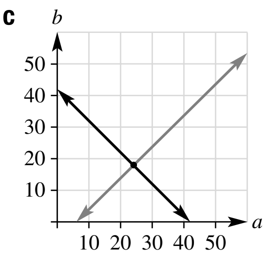
c



d Matilda's wage is \$650 and Nathan's wage is \$550.

2a $a + b = 42$ **b** $a - b = 6$

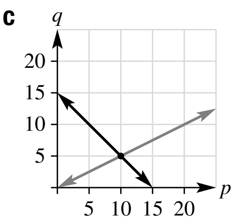
c



d The numbers are 18 and 24.

3a $p + q = 15$ **b** $p = 2q$

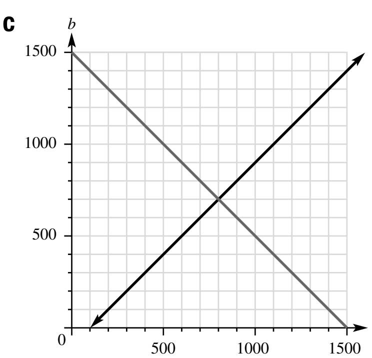
c



d The numbers are 5 and 10.

4a $a = 100 + b$ **b** $a + b = 1500$

c



d Amy's wage is \$800 and Nghi's wage is \$700.

5a i $C = 1000 + 10n$

ii $I = 60n$

b 20 items

Exercise 5E

1a 2 shirts **b** 6 printers

2a 2 boxes **b** \$10 profit

c \$10 loss **d** \$20

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3a 4 cartons to break even.
b Profit of \$5
c Loss of \$10
d Initial costs are \$20
e Gradient is 10
f Vertical intercept is \$0
g $I = 10n$
h Gradient is 5
i Vertical intercept is \$20
j $C = 5n + 20$

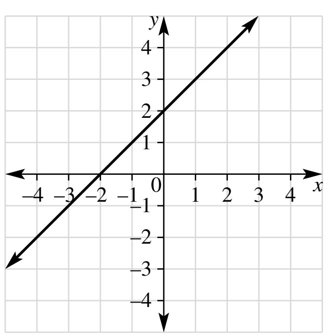
Review 5

Multiple-choice

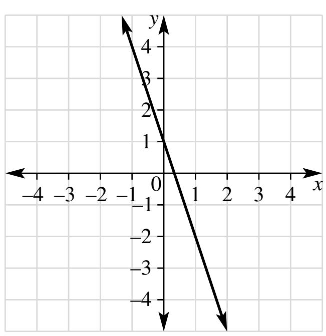
1 A **2** C **3** A **4** D **5** C
6 B **7** B **8** A **9** C

Short answer

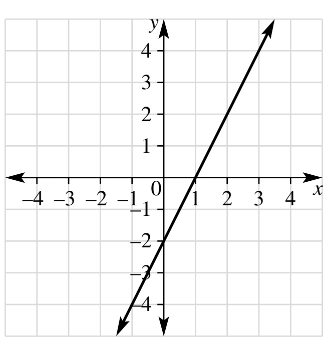
1a



b

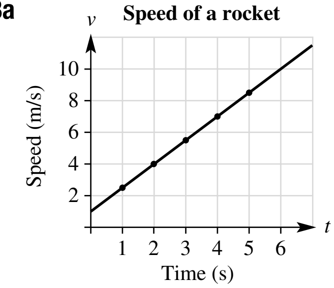


c



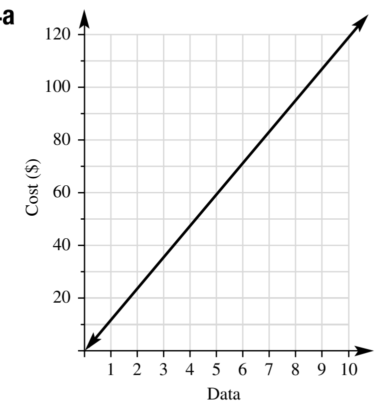
2a $y = 3x - 2$
b $y = -\frac{1}{2}x + 1$

3a



b $v = 1.5t + 1$ **c** 4.75 m/s
d 10 m/s **e** 11.5 m/s
f 16 m/s

4a



b 2.5 GB **c** \$42
d \$84 **e** \$120
f \$102

5 (–2.5, –4.5)

6a 3
b loss of about \$17
c profit of about \$8
d \$28

7a 10
b loss of \$50
c Profit of \$100
d \$100

8a 5 **b** $I = 40n$
c $C = 20n + 100$ **d** \$100 profit
e \$20 loss **f** \$220 profit

Practice paper 1

1 D **2** C **3** D
4 A **5** B **6** C
7 D **8** A **9** D
10 C **11** A **12** B
13 C **14** A **15** B

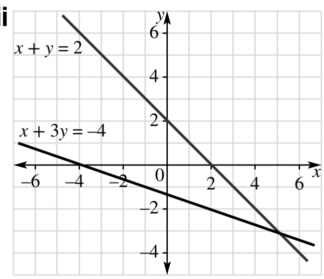
16a i \$2.38 **ii** \$294
b i Eulerian circuit does not exist. Graph has vertices with odd degree.
ii 21 **iii** 16
iv 20
v *E–F–C–D–A–B–E* or *E–B–A–D–C–F–E* length is 44
c \$1620
d i

x	−4	−1	0	2	6
y	0	−1	$-\frac{4}{3}$	−2	$-\frac{10}{3}$

ii

x	−2	−1	0	1	2
y	4	3	2	1	0

iii



iv (5, −3)

17a i 37°
ii $BD = 3$ cm and A$BD = 7.42$ cm
b i 2
ii Loss of \$10
iii Profit of \$20
iv \$20
c 135 km
d i 5
ii A Hamiltonian path passes through every vertex of a graph once and only once.
iii One example is *A–B–D–E–C*
e i −2
ii −2
iii (−2, 2)

18a i 792 L
ii 1170 L
iii \$1211.76
iv \$1965.60
b $x = 7.2$ m and $y = 6.5$ m
c i \$62 640
ii \$352 640
iii \$358 556
d i Eulerian circuit
ii All vertices in the network graph are even.

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iii One possible route: Reservoir–B–A–C–D–E–C–B–E–F–Reservoir

iv 32 km

Chapter 6

Exercise 6A

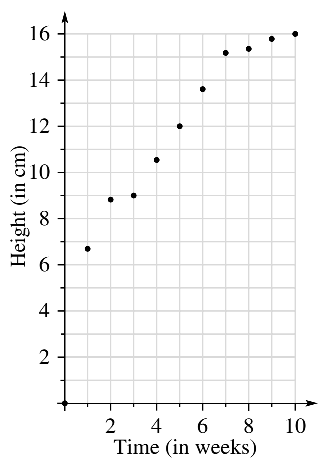
1a 9 **b** 2 **c** 16

d 9 **e** 12

2a 23 cm **b** 24 **c** 24 cm

d 18 **e** 4 **f** No

3a



b 6.6 cm

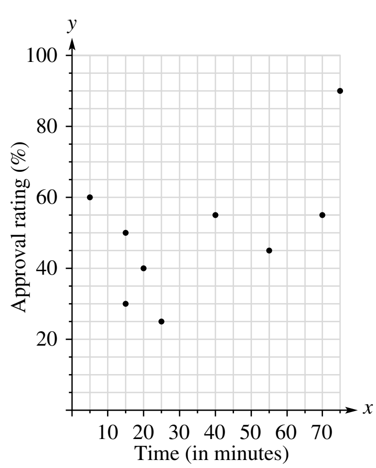
c 0.1 cm

d 2 weeks

e 11 cm

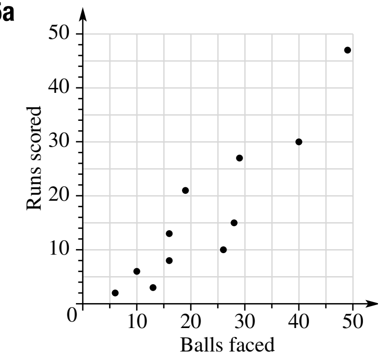
f 6.5 weeks

4a



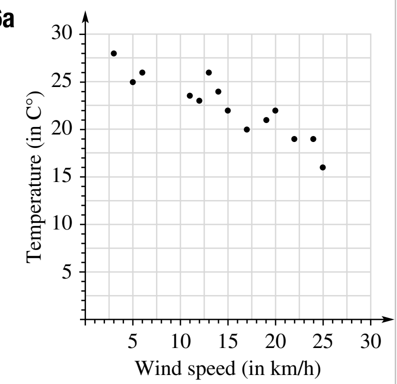
b There is no clear relationship. However, it could be argued that the more a politician appears on television the greater the approval rating.

5a



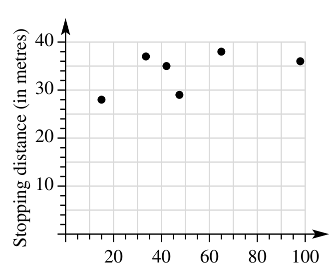
b As the balls faced increased, the number of runs increased.

6a



b As the wind speed increases, the temperature decreases.

7a



b Data is only provided for six cars, which makes predictions unreliable. Furthermore, other factors affect the result such as the type of car, road conditions and car servicing.

Exercise 6B

1a i Linear
ii Positive
iii Strong

b i Linear
ii Negative
iii Strong

c i Linear
ii Positive
iii Moderate or weak

d i Non-linear
ii Positive
iii Moderate

e i Linear
ii Negative
iii Weak

f i Linear
ii Positive
iii Moderate

2a Positive associated

b Positively associated

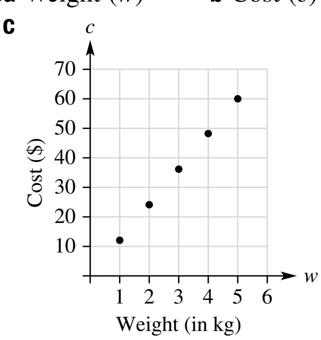
c No association

d Positively associated

e Negatively associated

3a Weight (w) **b** Cost (c)

c



d Linear

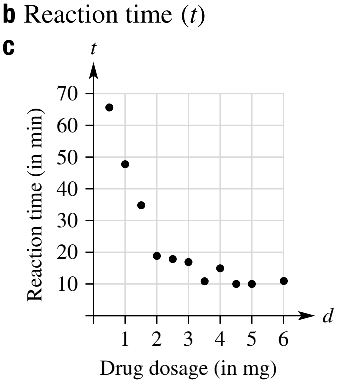
e Positive

f Strong

4a Drug dosage (d)

b Reaction time (t)

c



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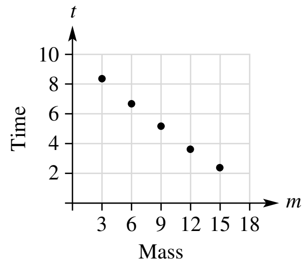
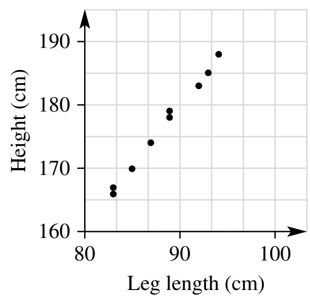
d Non-linear
e Negative
f Moderate
5a Mass (m) **b** Time (t)
c



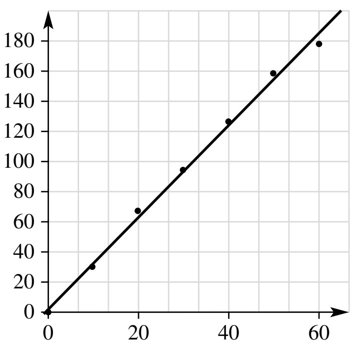
d Linear
e Negative
f Strong
6a Leg length **b** Height
c



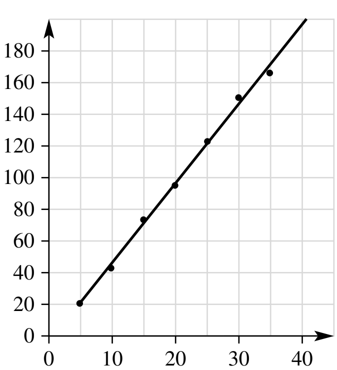
d Linear
e Positive
f Strong

Exercise 6C

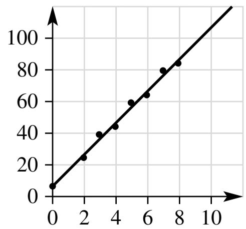
1a



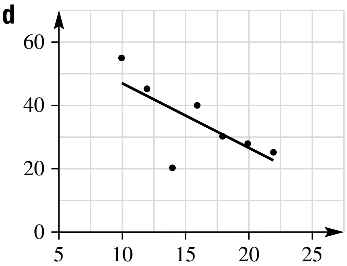
b



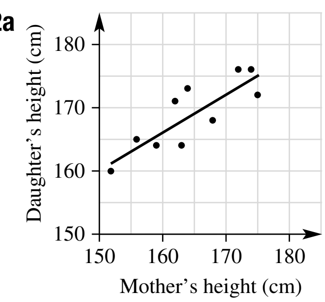
c



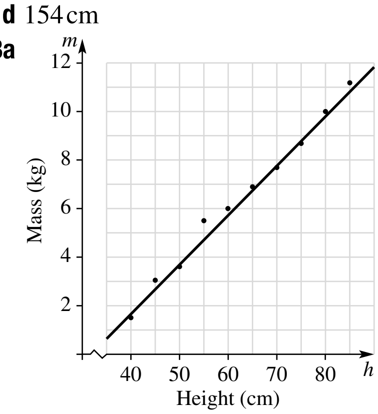
d



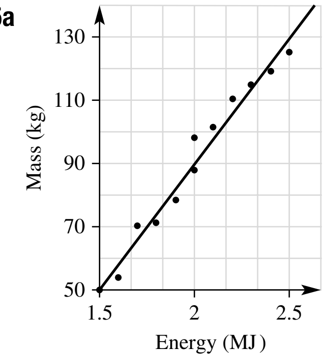
2a



b strong
c 172 cm
d 154 cm
3a



b 8.5 kg
c 54.9 cm
d 3.4 kg
e 75.7 cm
4a Shown on spreadsheet
b 67.12 beats per minute
c 53.875 kg
5a



b Shown on the above scatterplot
c About 54 kg
d About 102 kg
e About 2.1 MJ
f About 1.9 MJ

Exercise 6D

1a 3 days
b 20%
c i 110 mm
ii 70 mm
iii 60 mm
d i 17%
ii 13%
iii 8%
2a \$29 000 **b** 3 countries
c i \$22 000
ii \$27 000
iii \$33 000
d i \$32 000
ii \$49 000
iii \$65 000
3a 10 errors (interpolation)
b 4 errors (extrapolation)
c −1 error (interpolation)
4a 19.7°C (interpolation)
b 26.4°C (interpolation)
c 33.1°C (extrapolation)
5a 45.22 (interpolation)
b 55.72 (interpolation)
c 38.92 (extrapolation)
d 66.22 (interpolation)
e 72.52 (extrapolation)
f 59.92 (interpolation)
6a 46.87 kg (extrapolation)
b 65.62 kg (interpolation)
c 84.37 kg (extrapolation)

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7a 92.18% (extrapolation)
b 86.74% (interpolation)
c 84.02% (interpolation)
8a 12 030 (interpolation)
b 18 730 (interpolation)
c 25 430 (interpolation)

Exercise 6E

1a Investigation **b** Population
c Sample **d** Presentation
e Bivariate **f** Skewed
2a True **b** True
c True **d** True
e False **f** False
3 A statistical investigation involves four steps: collecting data, organising data, summarising and displaying data, and analysing data.
4 Census is data is collected from the whole population. A survey is data is collected from a smaller group of the population.
5 Several checks should be made to limit the impact of bias. This includes how the data is collected and whether this is likely to influence who responds and how they respond. Also to compare the demographics of survey respondents to the general population to check whether the sample is representative.
6 Since the number of car accidents and the number of school teachers will both increase with the size of the city, then the size of the city is likely to explain this correlation.
7 Not necessarily. While one possible explanation is that religion is encouraging people to drink, a better explanation is that towns with a large number of churches also have large populations, thus explaining that larger amount of alcohol consumed. Town size is the probable common cause for this association.

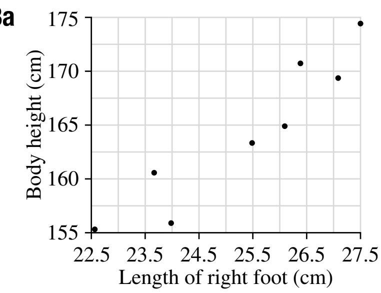
Review 6

Multiple-choice

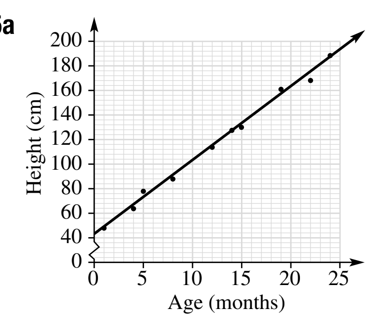
1 A **2** A **3** C **4** B **5** C

Short-answer

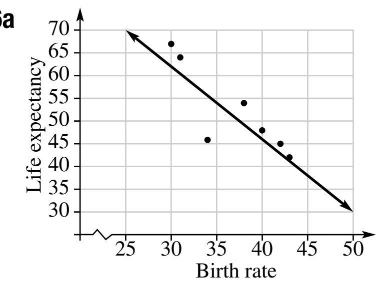
1a Navel height **b** Body height
c Linear **d** Moderate
e 188 cm **f** 100 cm
g 182 cm
2a 7 years **b** 145 cm
c Positive **d** Strong
3a



b Positive
c Strong
4a 56 **b** 92
5a



b About 110 cm
c About 80 cm
d Interpolation
6a



b Negative
c Strong
d About 54
e About 31

Chapter 7

Exercise 7A

1a 12 **b** 14 **c** 4
d 9 **e** 2 **f** 4
g 2 **h** 10 **i** 77
2a 4 : 8 : 12 **b** 2 : 6 : 8
c 7 : 49 : 63
3 Answers may vary.
a 2 : 4, 3 : 6, 5 : 10
b 4 : 10, 20 : 50, 200 : 500
c 4 : 3, 16 : 12, 40 : 30
4a 5 : 1 **b** 1 : 4 **c** 3 : 2
d 7 : 15 **e** 2 : 3 **f** 7 : 2
g 3 : 1 **h** 3 : 2 **i** 1 : 3
j 1 : 2 : 1 **k** 1 : 2 : 4 **l** 9 : 3 : 1
5a 4 : 3 **b** 1 : 4 **c** 7 : 2
d 5 : 9 **e** 1 : 8 **f** 1 : 5
g 4 : 1 **h** 6 : 1 **i** 20 : 1
j 10 : 3 **k** 1 : 6 **l** 40 : 9
6a 7 : 5 **b** 5 : 7 **c** 7 : 12
7 1 : 5
8 9 : 5
9 2 : 5
10a \$2.56 **b** \$25.60
c \$35.84 **d** \$15.36
11a \$14.20 **b** \$56.80
c \$85.20 **d** \$142.00
12a 1 : 10 **b** 1 : 9
13a 5 : 2 **b** 14 : 9 **c** 3 : 4
d 5 : 1 **e** 1 : 2 **f** 10 : 9
g 4 : 3 **h** 2 : 3 **i** 9 : 10
14a 2 : 1 **b** 1 : 3 **c** $2a : 1$
d 1 : 2 **e** $y : 4$ **f** $7m : 1$
15 5 : 7 : 3
16 10 : 7 : 5
17a 3 : 2 **b** \$900 **c** \$1 740

Exercise 7B

1a 11 **b** 6 **c** 14 **d** 9
2a $\frac{2}{9}$ **b** $\frac{7}{9}$

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3a \$70 : \$30 **b** \$40 : \$60
c \$55 : \$45
d \$28 : \$32 : \$40
4a 160 : 80 **b** 144 : 96
c 40 : 200 **d** 140 : 100
5a \$16 : \$4 **b** \$14 : \$6
c \$5 : \$10
d 33 drinks : 44 drinks
e 35 lollies : 65 lollies
f 20 kg : 25 kg
g 100 books : 60 books
h 80 pencils : 280 pencils
i 12.5 g : 37.5 g
j 32 km : 28 km
6 150 g
7 5026
8a \$40 : \$80 : \$80
b \$50 : \$150 : \$200
c 2 kg : 4 kg : 6 kg
d 22 kg : 11 kg : 55 kg
e 132 kg : 143 kg : 165 kg
f \$9000 : \$18 000 : \$36 000
9a \$60 : \$540
b \$200 : \$100 : \$300
c \$100 : \$250 : \$250
d \$260 : \$160 : \$180
10a \$120 000 **b** \$100 000
c \$20 000
11 6 km^2
12 \$120
13 4200
14a 0.6 L **b** 7.5 L
15a \$218 750 **b** \$156 250
c \$125 000
16 150 g of flour and 100 g of sugar
17 3 t and 5.5 t
18 A–230 t, B–322 t, C–230 t
19 $3\frac{1}{8}$ or 3.125 kg

Exercise 7C

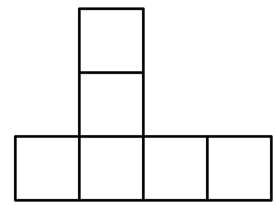
1 A–1, B–9, C–4, D–3, E–2, F–6, G–7, H–5, I–8
2a 2 **b** $\frac{1}{4}$ **c** 4 **d** $\frac{1}{3}$
e 1.5 **f** $\frac{1}{3}$
3a 2 **b** 3 **c** 15 **d** 30
4 Dimensions of the screen are 108 cm by 135 cm
5a Corresponding angles of similar figures are equal.
b $\frac{1}{2}$ **c** 30 cm
6a Corresponding angles of similar figures are equal.
b $\frac{2}{3}$ **c** 4.5 m
7a 16 cm^2 **b** 45 m^2
8a 3 mm^2 **b** 3 cm^2

Exercise 7D

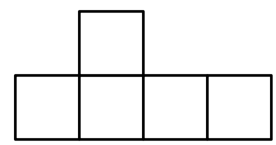
1a 2 m **b** 1 m
c 3.4 m **d** 2.8 m
e 8.5 m **f** 4.9 m
2a 80 mm **b** 30 mm
c 1.6 mm **d** 140 mm
e 2 mm **f** 55 mm
3a 1 : 2 **b** 1 : 50
c 1 : 300 000
4a 200 m **b** 100 m **c** 50 m
d 10 m **e** 34 m **f** 80 m
5a 37.5 km **b** 675 km
6a 10 mm **b** 16 mm
c 20 mm **d** 24 mm
e 30 mm **f** 48 mm
7 Map distance is 7 mm
8a 1125 mm or 1.125 m
b 0.04 m or 40 mm
9a 5 : 6400 or 1 : 1280 (approximation)
b Height of the antenna is approximately 32 m
10a 10 mm **b** 24 mm
c 40 mm **d** 600 mm
e 80 mm **f** 480 mm
11a 1 : 1000 **b** 35 m
c 16 m **d** 13 m
12a 1 to 1000 **b** 200 m
c 15 m
13a 1.5 cm to 5000 cm or 1 : 3333.33…
b 183 m by 208 m
c 153 m **d** 60 m **e** 137 m
f 350 m **g** 4110 m^2
14a 77 m **b** 133 m **c** 83 m
d 432 m **e** 11 039 m^2 **f** 157 m
15a 33 m **b** 10 m **c** 209 m
d 3421 m^2 (3484 m^2)
e 5 m **f** 31 m

Exercise 7E

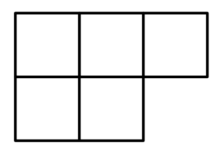
1a Plan



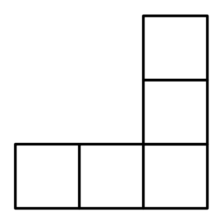
Front elevation



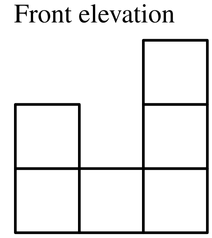
Side elevation



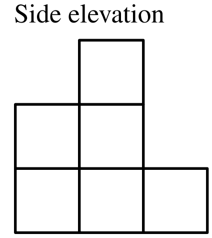
b Plan



Front elevation



Side elevation



2a Triangular prism
b Square pyramid
c Cylinder

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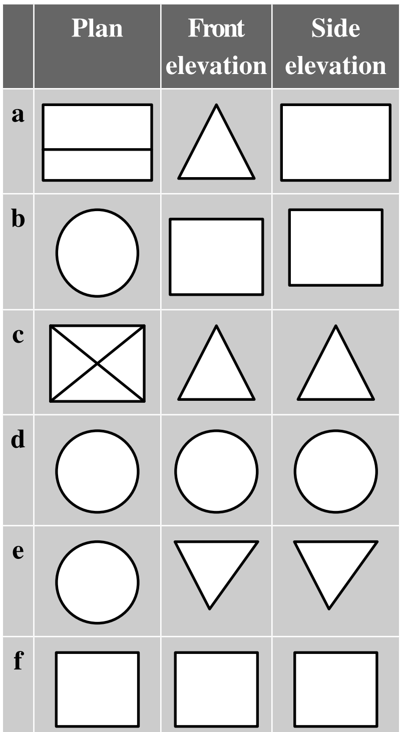
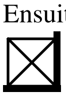
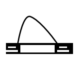
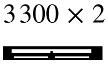
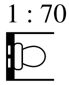
d Cube
e Cone
3



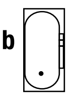
4a Ensuite or bathroom
b  **c** 
d Walk In Robe
5a 3150 × 2700 or 3.15 m by 2.7 m
b  **c** 
d Refrigerator **e** Pantry
6a 3300 × 2800 or 3.3 m by 2.8 m
b 
c Built In Wardrobe
d 3.3 m **e** 1 : 70
7a 4.15 m × 3.80 m **b** 
c 5.2 cm **d** 1 : 80
e 1.12 m × 0.72 m **f** 15.77 m^2
8a 3500 × 2990 or 3.5 m by 2.99 m
b 4940 × 4130 or 4.94 m by 4.13 m
c 1 : 165 **d** 7.92 m
e 3 m^2 **f** 9 m^2
9a 2585 mm^2
b 2 585 000 000 mm^3
c 5 170 000 000 mm^3
10a 14 m **b** 9 m
c 1 m by 2 m
d 200 cm by 160 cm
e 32 000 cm^2
f 207 000 cm^2
g 30 000 cm^2
11a 2.44 m **b** 2.698 m
c 22.5° **d** 6
12a 14 m by 9 m **b** 126 m^2
c 25 m^3
13a 1 : 250
b 2.5 m by 5.0 m
c 6.8 m by 7.5 m
d 3.8 m by 3.8 m
e 25 m^2
f $3500
14a 1 : 175 **b** 13.5 m
c 90 m^2 **d** 1000
e Tiles will fit exactly into the three rooms and no breakage will occur.
f 13.5 m^3

Review 7

Multiple-choice

1 C **2** C **3** D **4** B **5** A
6 D **7** A **8** A **9** D

Short-answer

1a 15 **b** 12 **c** 121
d 39 **e** 10 **f** 16
2a False **b** False
c True **d** False
e True
3a 1 : 4 **b** 3 : 2 **c** 3 : 4 **d** 1 : 8
e 3 : 1 **f** 1 : 5 **g** 3 : 2 **h** 2 : 1
i 1 : 3 **j** 5 : 1 **k** 2 : 3 **l** 4 : 1
m 2 : 3 : 6 **n** 4 : 3 **o** 7 : 11 **p** 3 : 1
q 3 : 2 **r** 2 : 5
4a $35 : $45
b 40 kg : 160 kg
c 30 m : 10 m
d $340 : $595 : $510
5 $1152
6a $1473 **b** $1408
7 12.5 buckets of sand
8a 9 cm **b** 12
9 154 cm
10a 1 : 300 **b** 1 : 60 **c** 1 : 2500
11 500 mm
12 23 km
13a 80 mm **b** 25 mm
c 1 mm **d** 130 mm
e 0.4 mm **f** 42.5 mm
14a 720 m **b** 80 m **c** 20 m
d 84 m **e** 248 m **f** 24 m
15a 1 : 150
b 
c Water closet or toilet
d 2.4 m by 3 m

Chapter 8

Exercise 8A

1a $18 480.00 **b** $15 523.20
c $13 039.49
2 $4740
3a $6532 **b** $10 968
4 $32 752
5a $5046 **b** $14 854
6a $4000 **b** $1000
c 2.5 years **d** 3.5 years
e About $1250 **f** About $2600
7 $9615
8a $25 501 **b** $13 537
9 $166 303
10 $25 884
11a $5120
b

Year	Current value	Depreciation	Depreciated value
1	$32 000	$5120	$26 880
2	$26 880	$4301	$22 579
3	$22 579	$3612	$18 967
4	$18 966	$3034	$15 932
5	$15 931	$2548	$13 383

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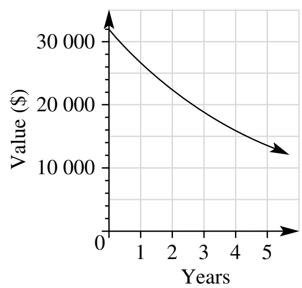
c



Exercise 8B

1a \$801 **b** \$1974
c \$2120 **d** \$534
e \$2650
2a \$1590 **b** \$95 400
c \$20 400
3a \$5 096.40 **b** \$1966.80
c \$7911 **d** \$2 856.60
4a \$573.65 **b** \$33 838
5a \$3655 **b** \$657 900
c \$317 900 **d** \$118 524
6a \$420 960 **b** \$240 960
7a \$100 000 **b** \$84 000
c \$68 000 **d** \$48 000
e about 84 months
f about 48 months
g 96 months

Exercise 8C

1a \$5.05 **b** \$7.46
c \$85.82 **d** \$1.16
e \$0.72 **f** \$10.91
2a \$983.93 **b** \$987.87
c \$991.83 **d** \$995.80
e \$999.79 **f** \$1003.80
3a \$392.65 **b** \$501.71
c \$15.57 **d** \$3369.85
e \$61.15 **f** \$573.23
4 \$7.14
5 \$4.47
6 \$6899.88
7a \$543.12 **b** \$270.73
c \$1415.27 **d** \$483.80
e \$681.68
8a \$5.61 **b** \$3.17 **c** \$27.57
d \$16.67 **e** \$28.55
9a 0.05% **b** \$73.88
10a 0.042% **b** \$650
c \$0.94 **d** \$1.03

Exercise 8D

1a 21 Apr **b** \$172.91
c \$172.91 **d** \$10.00
e \$743.42 **f** \$0.00
g \$743.42 **h** \$4511.88
2a \$19 500.00 **b** \$3950.82
c 5 **d** \$15 549.18
e \$89.66 **f** \$72.53
g Myer **h** WW petrol
i 7 Dec **j** \$57.00
3a 5 **b** \$38.95
c 30 days **d** \$4892.08
e \$97.92 **f** \$0.06
4a \$5821.31 **b** \$12 000
c \$6361 **d** \$5638.25
e \$511.93 **f** \$86.26
g \$2.82 **h** \$1128.55

Exercise 8E

1a \$457.50 **b** \$640
c \$595 **d** \$590
e \$600 **f** \$350
2a \$2.20 **b** \$16
c Bank B **d** Bank B
e \$4 **f** \$0.40
g \$228.75 **h** \$15.75
i **i** \$300 **ii** \$286
iii \$316 **iv** \$313
j **i** \$515 **ii** \$540
iii \$502 **iv** \$526
3 \$522

Review 8

Multiple-choice

1 C **2** D **3** C **4** B
5 B **6** D **7** A

Short-answer

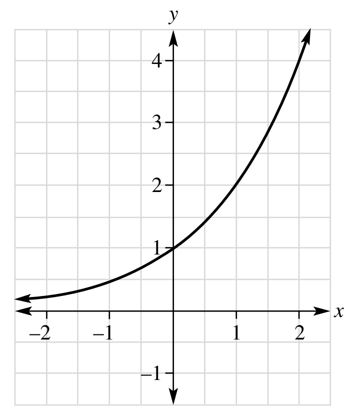
1a \$5046 **b** \$14 854
2a \$27 123 **b** \$6123
3 \$370 000
4 \$172.07
5a \$436 **b** \$500
6a \$3.26 **b** \$1.45
c \$2.10
7a \$3696.05 **b** \$110.88
c \$3585.17 **d** \$100.00
e \$3685.17 **f** \$104.08
g \$81.30 **h** \$3970.11
i \$79.40 **j** \$160.39
8a No. \$699 worse off.
9 \$3.37
10 \$786

Chapter 9

Exercise 9A

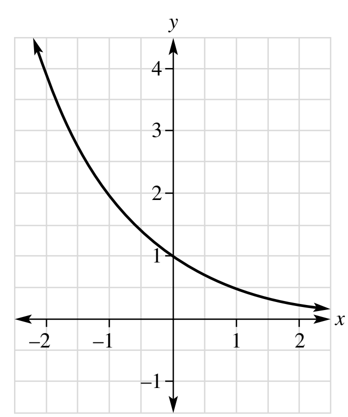
1a

x	−2	−1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



b

x	−2	−1	0	1	2
y	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$



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2a No

b Yes

c Yes. The value of y is 1 when x equals 0.

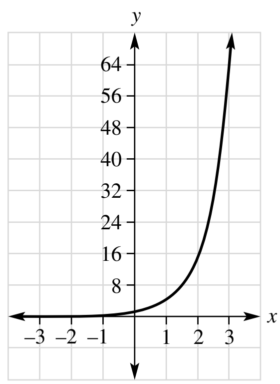
d No. The x-axis ($y = 0$) is an asymptote.

e 1.4

f 2.8

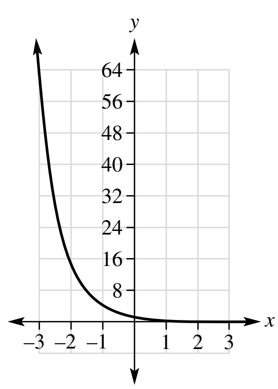
3a

x	−3	−2	−1	0	1	2	3
y	0.02	0.06	0.25	1	4	16	64



b

x	−3	−2	−1	0	1	2	3
y	64	16	4	1	0.25	0.06	0.02



4a No.

b Yes

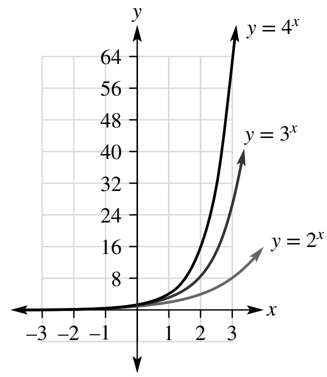
c Yes. The value of y is 1 when x equals 0.

d No. The x-axis ($y = 0$) is an asymptote.

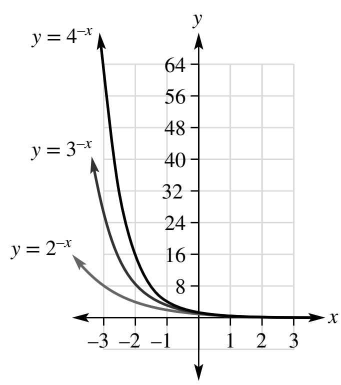
e 8

f 32

5



6



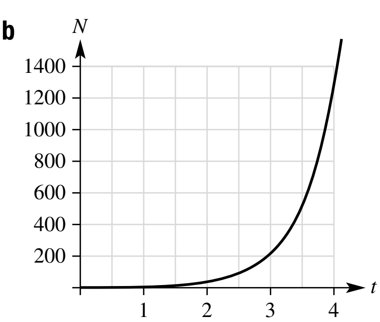
7 When the value of a changes it affects the steepness of the graph. That is, the increasing a means the y values increase or decrease.

Exercise 9B

1a

t	0	1	2	3	4
N	1	6	36	216	1296

b



c 1

d 216

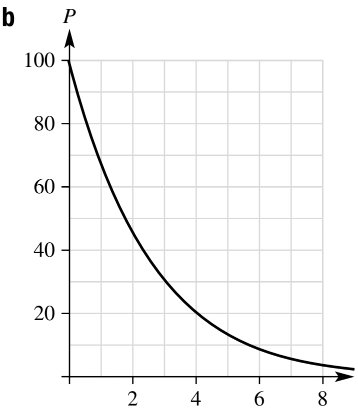
e 1260

f Approximately 3.9 days

2a

t	0	2	4	6	8
N	100	44	20	9	4

b



c 100

d 30

e 6

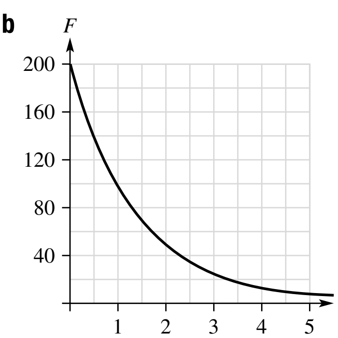
f 35

g 11

3a

t	0	1	2	3	4	5
F	200	100	50	25	13	6

b



c 200

d 140

e 25

f 6

g 75

h 37

i 8

4a

t	0	5	10	15	20
b	30	75	186	462	1150

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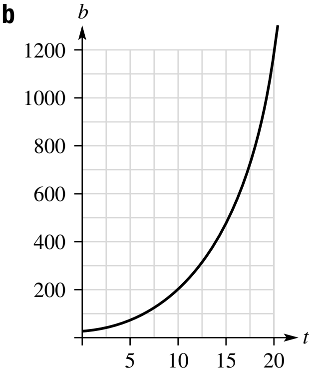
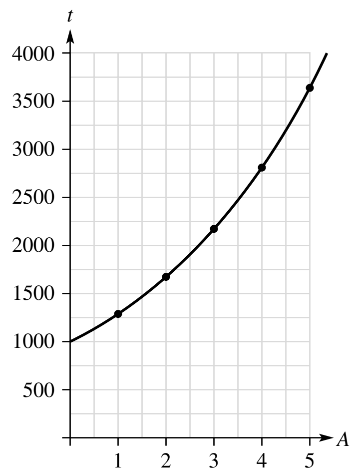
b



c 30 **d** About 60
e 130 **f** 270
g 550 **h** About 7.5 h

5ab



c $1927 **d** $6275
e 1.5 years

Exercise 9C

1a (−1, 4) **b** (0, 3)
c (−3, 0) and (1, 0)
d $x = -1$
e 4

2a (0, −4) **b** (0, −4)
c (−2, 0) and (2, 0)
d $x = 0$
e −4

3a

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

b

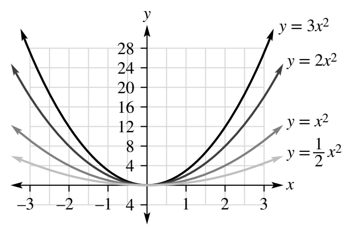
x	−3	−2	−1	0	1	2	3
y	18	8	2	0	2	8	18

c

x	−3	−2	−1	0	1	2	3
y	27	12	3	0	3	12	27

d

x	−3	−2	−1	0	1	2	3
y	4.5	2	0.5	0	0.5	2	4.5



e $x = 0$
f Minimum
g Affects the shape of the parabola. A larger coefficient makes the sides of the parabola steeper.

4a

x	−3	−2	−1	0	1	2	3
y	−9	−4	−1	0	−1	−4	−9

b

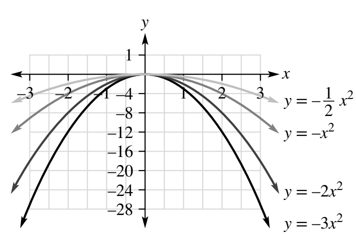
x	−3	−2	−1	0	1	2	3
y	−18	−8	−2	0	−2	−8	−18

c

x	−3	−2	−1	0	1	2	3
y	−27	−12	−3	0	−3	−12	−27

d

x	−3	−2	−1	0	1	2	3
y	−4.5	−2	−0.5	0	−0.5	−2	−4.5



e $x = 0$
f Maximum
g Affects the shape of the parabola. A larger coefficient makes the sides of the parabola steeper.

5a

x	−3	−2	−1	0	1	2	3
y	10	5	2	1	2	5	10

b

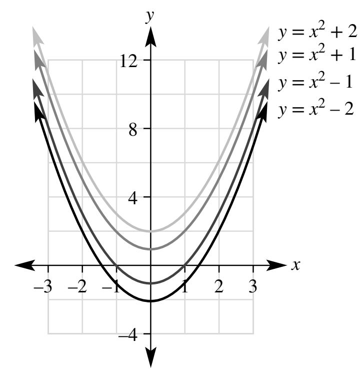
x	−3	−2	−1	0	1	2	3
y	8	3	0	−1	0	3	8

c

x	−3	−2	−1	0	1	2	3
y	11	6	3	2	3	6	11

d

x	−3	−2	−1	0	1	2	3
y	7	2	−1	−2	−1	2	7



e $x = 0$
f Minimum
g Adding or subtracting a number to the quadratic function $y = x^2$ moves the parabola up or down.

6a

x	−6	−4	−2	0	2	4	6
y	25	9	1	1	9	25	49

b

x	−6	−4	−2	0	2	4	6
y	16	4	0	4	16	36	64

c

x	−6	−4	−2	0	2	4	6
y	49	25	9	1	1	9	25

d

x	−6	−4	−2	0	2	4	6
y	64	36	16	4	0	4	16

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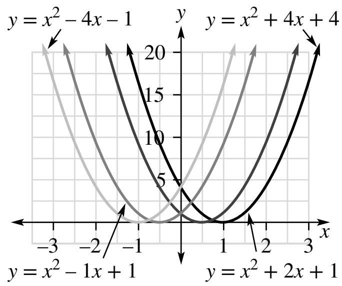


e All these graphs have a minimum turning point that touches the x-axis.

Exercise 9D

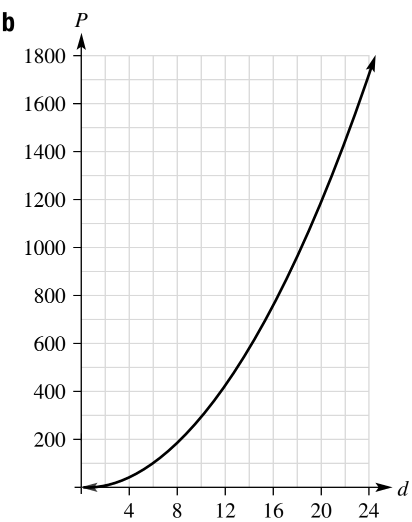
1a $6\,m^2$ **b** $10\,m^2$ **c** 3.5 m **d** $12.25\,m^2$

2a 0 m/s **b** 11.25 m/s **c** 1.5 s **d** 2

3a

d	0	2	4	6	8	10	12	14	16	18	20	22	24
P	0	12	48	108	192	300	432	588	768	972	1 200	1 452	1 728

b

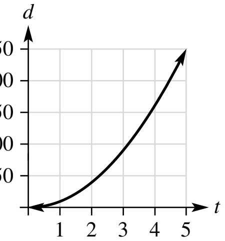


c \$432 **d** \$1 587 **e** \$1 800

4a

t	0	1	2	3	4	5
d	0	10	39	88	157	245

b

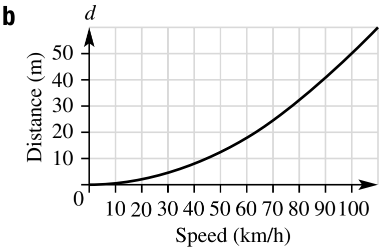


c 10 m **d** 245 m **e** 61 m **f** 3.2 s **g** 4.5 s

5a

s	0	20	40	50	60	80	100
d	0	2	8	12	18	32	50

b



c $d = 2$ m

d $d = 28$ m

e About 55 km/h

f About 78 km/h

g 24 m

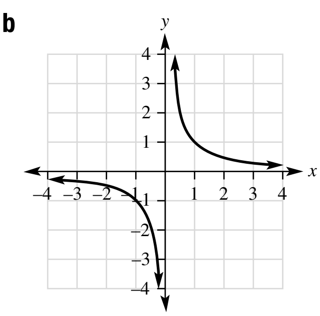
h 44 m

Exercise 9E

1a $y = \frac{1}{x}$

x	-4	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	4
y	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	-2	2	1	$\frac{1}{2}$	$\frac{1}{4}$

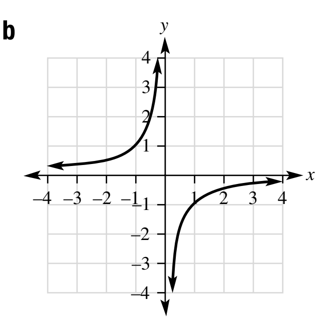
b



2a $y = -\frac{1}{x}$

x	-4	-2	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1	2	4
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	-2	-1	$-\frac{1}{2}$	$-\frac{1}{4}$

b



3a $y = \frac{3}{x}$

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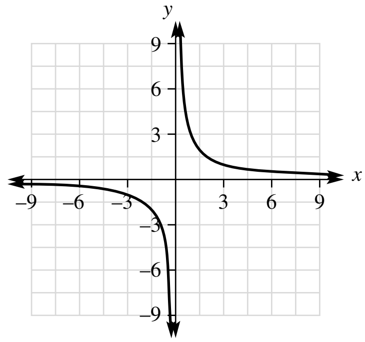
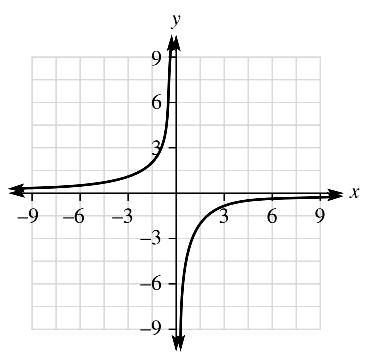
x	−9	−3	−1	$-\frac{1}{3}$	$\frac{1}{3}$	1	3	9
y	$-\frac{1}{3}$	−1	−3	−9	9	3	1	$\frac{1}{3}$



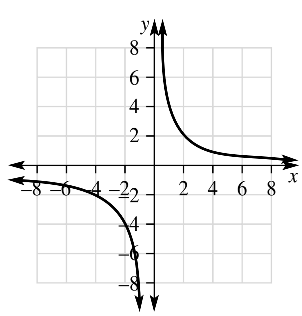
b $y = -\frac{3}{x}$

x	−9	−3	−1	$-\frac{1}{3}$	$\frac{1}{3}$	1	3	9
y	$\frac{1}{3}$	1	3	9	−9	−3	−1	$-\frac{1}{3}$



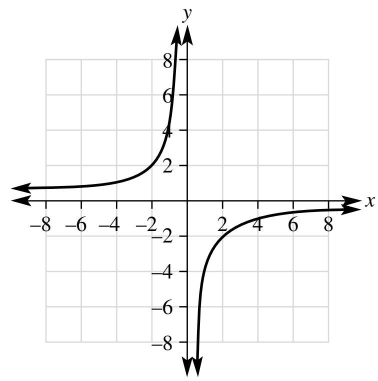
c $y = \frac{4}{x}$

x	−8	−4	−1	$-\frac{1}{2}$	$\frac{1}{2}$	1	4	8
y	$-\frac{1}{2}$	−1	−4	−8	8	4	1	$\frac{1}{2}$



d $y = -\frac{4}{x}$

x	−8	−4	−1	$-\frac{1}{2}$	$\frac{1}{2}$	1	4	8
y	$\frac{1}{2}$	1	4	8	−8	−4	−1	$-\frac{1}{2}$



Exercise 9F

1a 30 h **b** 20 h

c 60 km/h **d** 15 km/h

e Road trip taking 5 hours requires a speed of 300 km/h. This is not possible on Australian roads.

2a \$160 **b** \$40

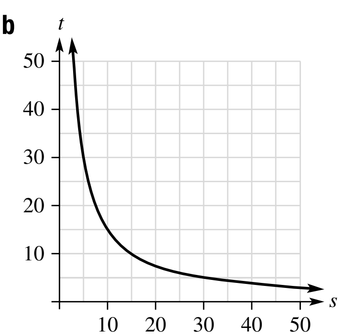
c 4 **d** 1

e Yacht would have to be able to accommodate 320 people if the cost per person is \$1.

3a

s	5	10	15	25	30	50
t	30	15	10	6	5	3

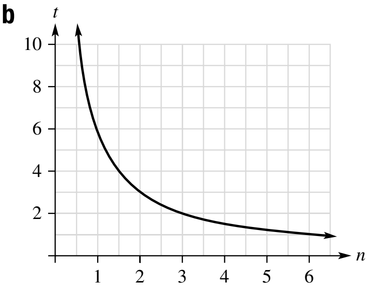
b



4a

n	1	2	3	4	5	6
t	6	3	2	1.5	1.2	1

b



c 6 h **d** 2 h

e 1 h **f** 3

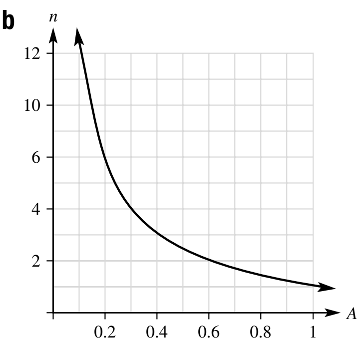
g 12

h It would take 1 minute for 360 people to dig the hole. Coordinating 360 people to dig a hole in 1 minute is impractical.

5a

A	0.1	0.2	0.3	0.4	0.5	0.6	0.8	0.9	1.0
n	12	6	4	3	2.4	2	1.5	1.3	1.2

b



c 2400 **d** 4800

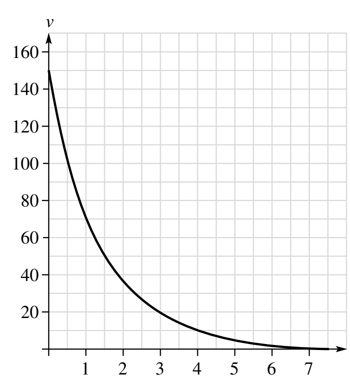
e 0.6 m^2

f About 0.25 m^2

g When 12 000 attend the concert then 0.1 m^2 is allowed per person. This is insufficient room for each person.

Exercise 9G

1ab



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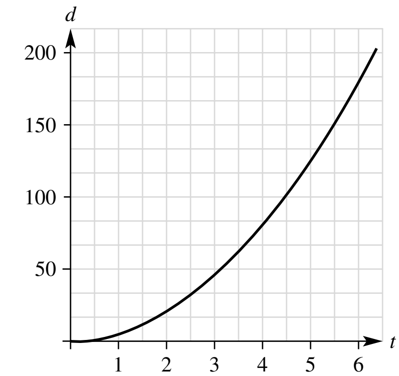
c \$53 000 **d** \$27 000

e \$13 000 **f** \$106 000

g After 1 year

h \$0.14. Equipment is worth no value after 20 years.

2a



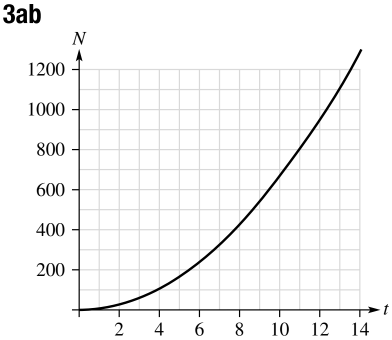
b 11.25 m **c** 31.25 m

d 61.25 m **e** 500 m

f 8 s

g Model is not appropriate for distances greater than 100 km

3ab



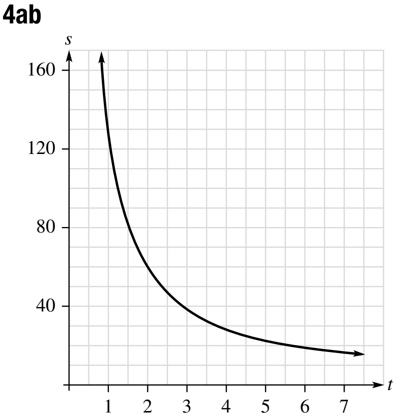
c 54 **d** 150

e 294 **f** 726

g 20 months

h 121.5 tadpoles. Not possible to have half a tadpole. The variable *N* needs to be a whole number.

4ab



c 80 km/h

d 48 km/h

e 34 km/h

f 15 km/h

g 2.5 h

h 240 km/h. This speed exceeds the speed limits on Australian roads.

Review 9

Multiple-choice

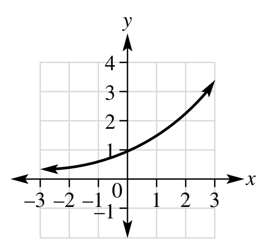
1 C **2** C **3** B **4** C

5 D **6** B **7** A **8** C

Short-answer

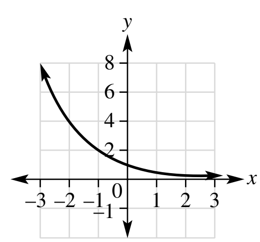
1a

x	−3	−2	−1	0	1	2	3
y	0.3	0.4	0.7	1	1.5	2.3	3.4

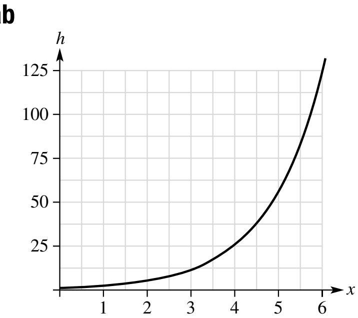


b

x	−3	−2	−1	0	1	2	3
y	8	4	2	1	0.5	0.3	0.1



2ab



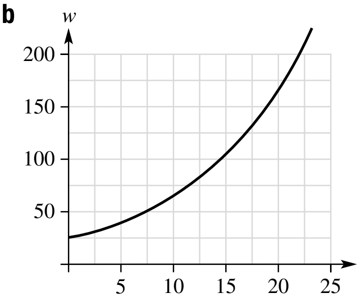
c 3.6 cm

d 17.4 cm

3a

t	0	5	10	15	20
w	25	40	65	104	168

b



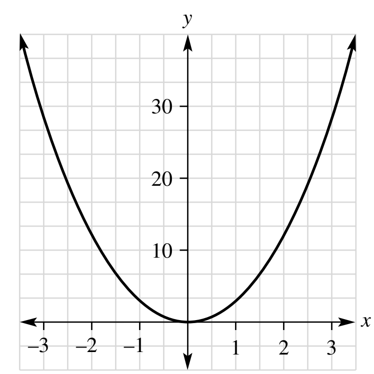
c 25 earthworms

d 33

e About 11.5 days

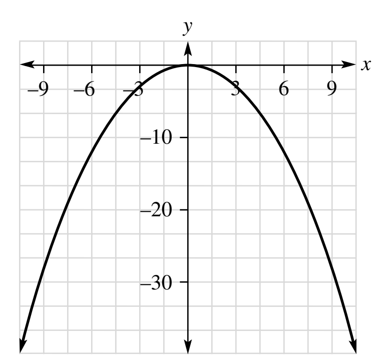
4a

x	−3	−2	−1	0	1	2	3
y	27	12	3	0	3	12	27



b

x	−9	−3	−1	0	1	3	9
y	−27	−3	$\frac{1}{3}$	0	$-\frac{1}{3}$	−3	−27



c

x	−3	−2	−1	0	1	2	3
y	12	7	4	3	4	7	12

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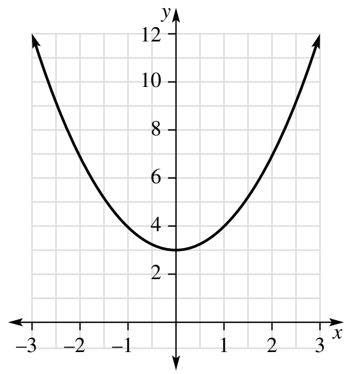
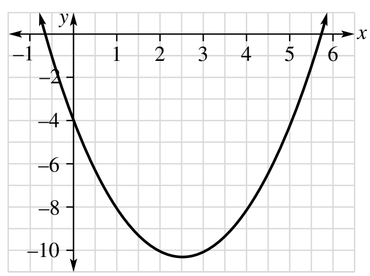


d

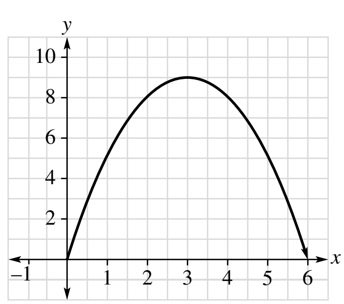
x	0	1	2	3	4	5	6
y	−4	−8	−10	−10	−8	−4	2



5a

t	0	1	2	3	4	5	6
h	0	5	8	9	8	5	0

b

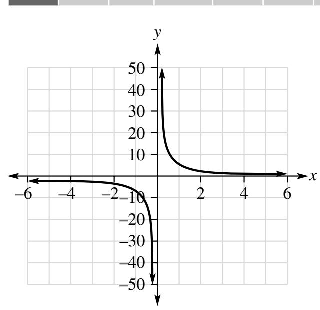


c 9 m

d 3 s

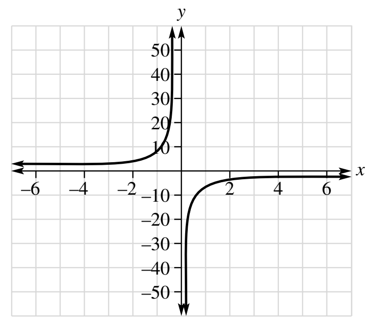
6a

x	−7	−1	$-\frac{1}{7}$	$\frac{1}{7}$	1	7
y	−1	−7	−49	49	7	1



b

x	−7	−1	$-\frac{1}{7}$	$\frac{1}{7}$	1	7
y	1	7	49	−49	−7	−1



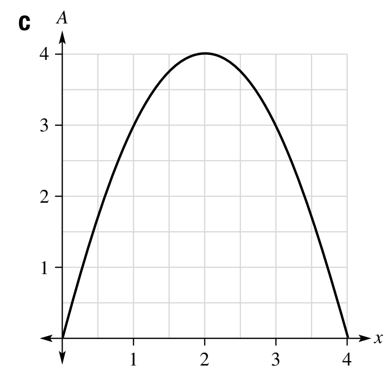
7a 80 **b** 3 m

8a $A = 4x - x^2$

b

x	0	0.5	1	1.5	2	2.5	3	3.5	4
A	0	1.8	3	3.8	4	3.8	3	1.8	0

c



d About 2.5 m^2

e About 3.5 m^2

f 4 m^2

g 2 m

Practice paper 2

1 B **2** A **3** C **4** B **5** D
6 C **7** B **8** B **9** C **10** A
11 D **12** C **13** D **14** A **15** D

16a \$375, \$225 and \$600

b i 148 m^2

ii 39%

iii 25.6 m^2

iv 1 536 bricks

v \$3000

c i \$21 751.82

ii \$5 861.82

iii \$863.87

d i 48, extrapolation

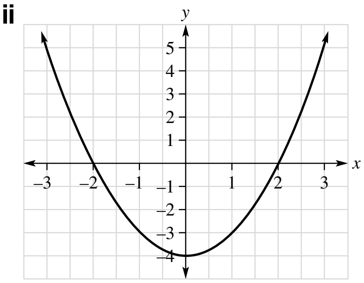
ii 66, interpolation

iii 84, extrapolation

17ai

x	−3	−2	−1	0	1	2	3
y	5	0	−3	−4	−3	0	5

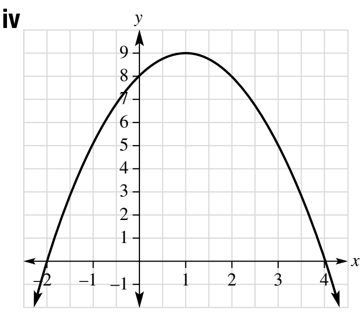
ii



iii

x	−2	−1	0	1	2	3	4
y	0	5	8	9	8	5	0

iv



b i \$35 000

ii 6

iii 0.4

c Rectangle 8 cm by 5 cm drawn accurately

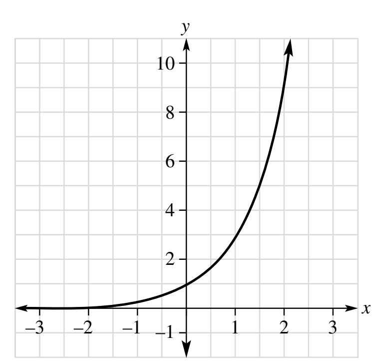
d i \$200

ii \$87.50

iii \$230

e

x	−2	−1	0	1	2
y	0.1	0.3	1	3	9



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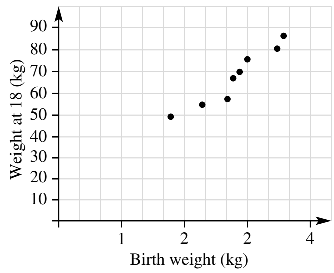
18a i $\frac{2}{3}$

ii 7.5 cm

b i



ii Positive

iii Moderate

c i \$1088.40

ii \$261216

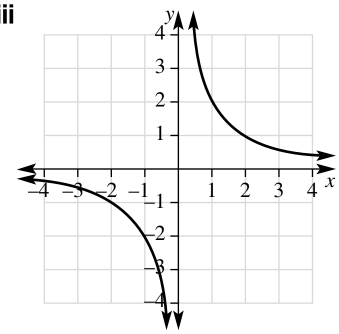
iii \$31680

d i $x = 0$

ii

x	−4	−2	−1	−0.5	0.5	1	2	4
y	−0.5	−1	−2	−4	4	2	1	0.5

iii



iv 4

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