Chapter 1 Working scientifically

1.1 Questioning and predicting

1.1 Review

1 An inquiry question guides the investigation and ends in a question mark. A hypothesis is a cause-and-effect statement that can be tested through an investigation. The purpose is a statement outlining the aim of the investigation.

2 A. This is a question. Options B, C and D are statements.

3 **a** independent variable: the surface area of the containers; dependent variable: time taken for the water to reach room temperature

b independent variable: the launch angle; dependent variable: the range of the projectile

c independent variable: the thickness of the foam bumper; dependent variable: force applied to come to a stop

d independent variable: the emf applied to the circuit; dependent variable: the total current

4 qualitative

5 B. This option provides the most detailed inquiry through the questioning of specific properties of the collision.

6 A. B doesn't give a specific test and C is not an inquiry question.

7 A. Hypothesis 1 is written as a specific cause-and-effect statement that could be tested through an investigation.

1.2 Planning investigations

1.2 Review

1 **a** valid

b reliable

c accurate

2 B and D. Sunhat, sunscreen and eye goggles are appropriate personal and protective equipment (PPE) when working in the sun investigating projectiles. A fume hood and gloves should not be required for the tasks mentioned.

3 B. Conducting an experiment multiple times improves the reliability of the results.

4 **a** bumper foam density

b impact force

c cart mass, method of density measurement, equipment used to measure force, velocity of collision.

5 **i** Create a ramp that can be adjusted from 10° to 30° and place it on a table. Place two marks on the table, 20 cm apart. Start with the launcher at 10°.

ii Measure and record the height of the table.

iii Place some carbon paper face down on a piece of paper on the floor. Measure the distance between the edge of the table and the results paper.

iv Roll a ball down the ramp. Measure the time it takes for the ball to pass between the markers. Use this to calculate launch velocity. Ensure the ball lands on the carbon paper.

v Repeat step iv to get an accurate measurement of the launch velocity.

vi Increase the ramp angle by 5° and repeat steps iii to v.

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1.3 Conducting investigations

1.3 Review

1 in a logbook

2 **a** Systematic error—due to improper calibration every recorded weight will be incorrect. Time should be taken to learn how to calibrate all equipment properly before starting an experiment.

b Mistake—the different masses were not correctly labelled or were incorrectly selected. All materials should be clearly labelled and double-checked before they are used in experiments.

c Random error—this result is an outlier. Repeating an experiment will make outliers obvious.

d Systematic error—due to poor definitions of the variables the results will be inconsistent. Variables should be clearly defined before starting an experiment.

3 0.02 s and 0.72 s are mistakes as they are significantly different from the other readings. The average time is 0.24 s. The mistakes are not included in the average.

4 **a** systematic error

b mistake

c random error

1.4 Processing data and information

1.4 Review

1 **a** 23

b 21

c 22

2 **a** 4

b Report the answer to the least number of significant figures.

c Report the answer to the least number of decimal places.

3 **a** *y*-axis

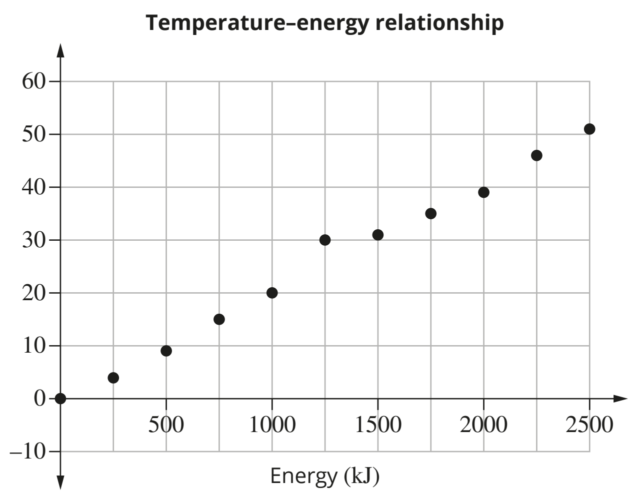
b *x*-axis

4 source A: 8, source B: 60

5 **a** The level of carbon dioxide in the atmosphere slowly rose from 1805. After 1905 it increases at a higher rate. Between 1805 and 1910, the average temperature decreased slightly. From 1910 onwards, the temperature has increased steadily, apart from in 1905, which had a slightly smaller increase.

b In general, both carbon dioxide and average global temperature have been rising from 1805. Note that without further evidence external to this graph, no conclusions may be drawn about the connection between both trends. The consensus among climate scientists, however, is that the average temperature increase can be attributed to the increased carbon dioxide in the atmosphere.

6 **a**



b Data that does not fit an observed pattern or trend.

c data point at 1250 kJ

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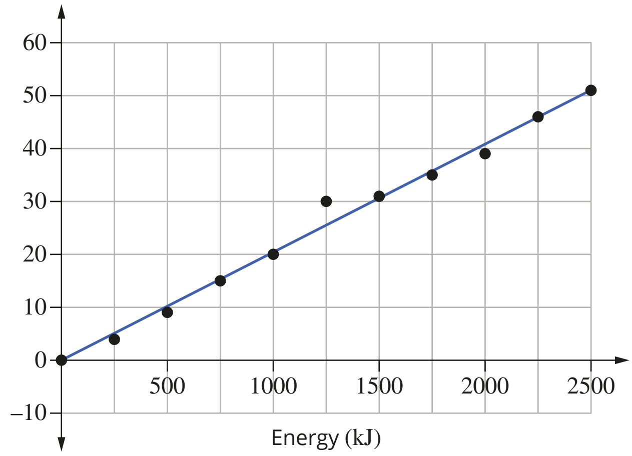
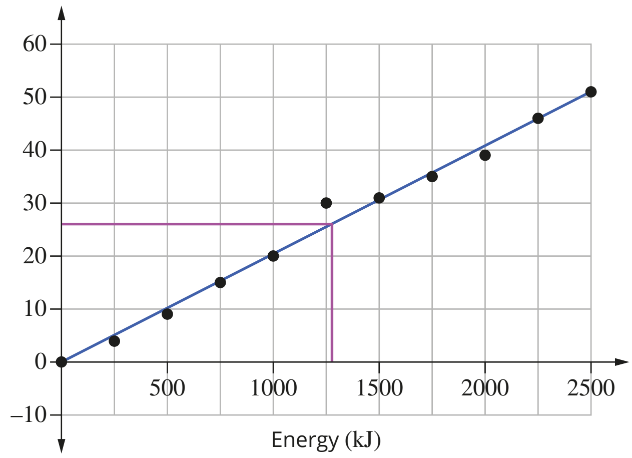
d **Temperature–energy relationship**



e **Temperature–energy relationship**



If you are using a spreadsheet program, the energy found will be 1280 kJ. If you are doing this by hand, there may be some variation in this value.

7 Include units on the *x*-axis (°C), include units on the *y*-axis (A), have a consistent scale on the *x*-axis, include a title, include scale on the *y*-axis.

8 **a** The length is least likely to be accurate or precise because it is difficult to take consistent strides and travel in a straight line.

b The measured length is likely to be more accurate than striding because a trundle wheel has a graduated scale that enables the length to be estimated to the nearest cm, but will not be precise as it is difficult to ensure the shortest distance is being measured.

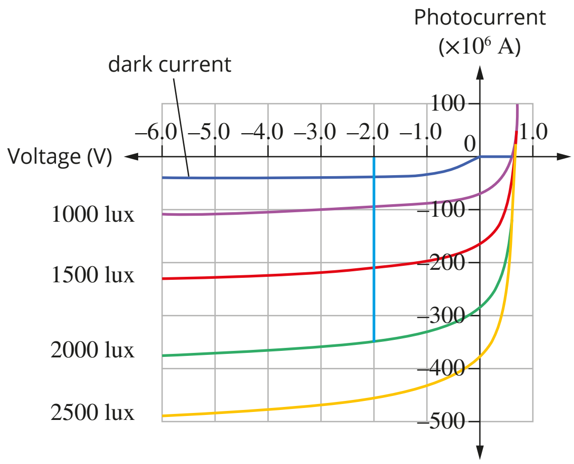
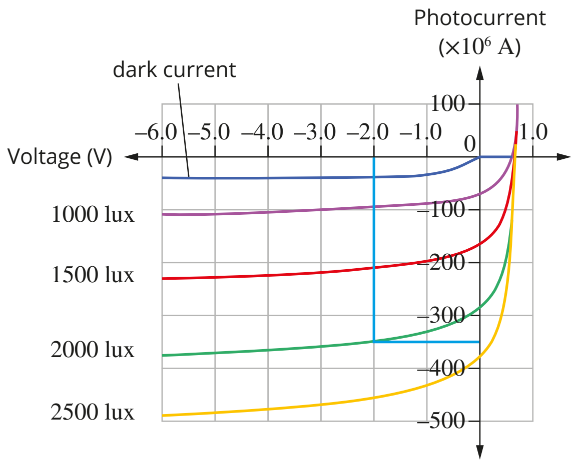
c The length is most likely to be accurate because the tape measure has a graduated scale that measures to the cm, and it is more precise because it will measure in a straight line.

1.5 Analysing data and information

Worked example: Try yourself 1.5.1

READING INFORMATION FROM A GRAPH

Using the graph in Figure 1.5.4, determine the current at 2000 lux in a circuit with a potential difference of –2.0 V.

Thinking	Working
Locate the appropriate lux line. Draw a vertical line from the voltage on the *x*-axis to intersect with the curve of the appropriate lux.	
Draw a horizontal line from the intersection point of the curve to the *y*-axis. The point on the *y*-axis represents the current after the photodiode.	
Read the current, taking note of the units on the axis.	The horizontal line intersects the *y*-axis at -350×10^{-6} A. Therefore -3.5×10^{-4} A will flow through the wire after the photodiode.

1.5 Review

1 Bar graphs, histograms and pie charts are suitable for displaying discrete data, because the data in those graphs does not have to be continuous.

2 Five of: title, *y*-axis label, *y*-axis units, *y*-axis scale, *x*-axis label, *x*-axis scale, *x*-axis units, trend line.

3 The temperature increased at a steady rate for the first 4.5 min, reaching 42°C. From 4.5 min until 10 min, the temperature oscillates from 42°C to 38°C with a period of approximately 1.2 min.

4 E. Graphs A and D have zero velocity above $t = 10$ s. Graph B initially travels at a constant velocity, so has zero acceleration and it also falls in the same category as A and D with zero velocity after $t = 10$ s. The curved gradient of Graph C means it is always accelerating.

5 qualified author, published in a reputable peer-reviewed journal, method detailed enough that the reader could replicate experiment, valid, reliable, accurate, precise, limitations outlined, assumptions outlined, includes suggested future improvements or research directions

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1.6 Problem solving

1.6 Review

1 D. A is incorrect, it's a generalisation to all springs. B is incorrect, it's not a conclusion. C is incorrect, it is an extrapolation of a linear region.

2 **a** Yes it is appropriate; all other variables are kept constant and this velocity is within the range tested. $R = 1.2\,\text{m}$

b No it is not, because that is not within the range of experimental data. It is an extrapolation.

c No it is not. That would be a generalisation that is beyond the scope of the experiment.

3 **a** $\vec{a} = \frac{\vec{F}_{net}}{m}$

b $m = \frac{\vec{F}_{net}}{\vec{a}}$

4 Five repeats of the procedure were conducted.

1.7 Communicating

1.7 Review

1 B. A, C and D use subjective language.

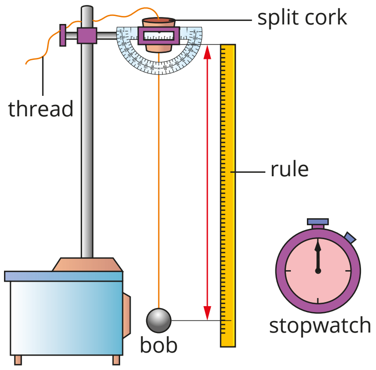
2 A, B and C. D uses first-person narrative.

3 **a** **(i)** independent variable—length of string

(ii) dependent variable—period of motion

(iii) constants—mass of pendulum, release angle, pendulum equipment

b



Method:

1. Set up equipment as shown. Record the mass of the bob, and make a mark on the protractor at 10°.
2. Pull thread through cork until the length is 10 cm.
3. Keeping the thread taut, pull the bob back until the thread is in line with the angle marking on the protractor.
4. Release bob and start stopwatch. Measure the time for 10 periods, and record it.
5. Repeat steps 3 and 4 for thread lengths 15 cm, 20 cm, 25 cm, 30 cm, 35 cm.

4 **a** 2.55×10^5

b 4.32×10^{-7}

c It allows very large and very small numbers to be handled easily.

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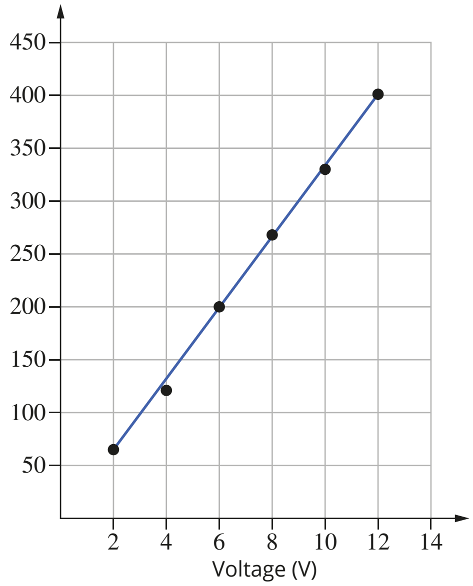
CHAPTER 1 REVIEW

1 **The linear relationship between voltage and current**



2 B. A strong conclusion should be evidence-based and avoid the use of emotive language.

3 C. Secondary sources refer only to outside sources of data or evidence such as from other people's scientific reports, textbooks or magazines.

4 A. An in-text citation must include both the authors and the year of publication. Further details such as the full article name are not needed in text.

5 An aim is a statement outlining the purpose of an investigation. A hypothesis is a cause-and-effect statement that can be tested through an investigation. The variables are the factors that can change in an investigation (the independent variable is the variable changed on purpose by the researcher, the dependent variable is the variable observed or measured in an investigation, controlled variables are all other variables that are maintained in the investigation).

6 **a** dependent variable
b controlled variable
c independent variable

7 independent variable: density of foam in shin pads; dependent variable: force on the ball; controlled variables: method of measurement, ball velocity, ball mass

8 **a** independent variable: material thickness of spring; dependent variable: spring coefficient
b independent variable: metal type; dependent variable: thermal expansion with a temperature increase of 50°C
c independent variable: mass; dependent variable: velocity and acceleration

9 $2 \times 10^6\ \mathrm{W\,m^{-2}\,K^{-1}}$

10 0.03000 L

11 **a** the average of a set of data
b the most frequent value in a set of data
c the middle value in a set of data

12 The average is $\frac{7.02+6.47+6.92+7.21+6.53+6.53}{6} = 6.78$. The range of values is $7.21 - 6.47 = 0.74$. Therefore the uncertainty of the average of the values is $6.78 \pm \frac{0.74}{2} = 6.78 \pm 0.37$.

13 The mean; therefore outliers are excluded before calculating the mean of a set of data.

14 **a** mistake
b random error
c systematic error

15 **a** For example, two significant figures: 0.032; three significant figures: 0.0302; four significant figures: 0.03020: five significant figures: 0.030200.
b Report the final calculation to the least number of significant figures of the measurement used in the calculation.
c Report the final calculation to the least number of decimal places of the measurement used in the calculation.

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16 a accuracy refers to how close a measurement is to the true value, precision refers to how close measurements are to each other

b if more than one independent variable was changed at a time, if an inappropriate method was used, if outliers were included in data analysis, if an insufficient sample size was used

c many possible answers; for example, the experiment was only repeated three times

17 a reliability

b accuracy

c validity

d precision

18 to give credit to others, to avoid plagiarism, to enable the reader to obtain further information

19 a To investigate the effect of temperature on the resistance of wire.

b independent variable: temperature of wire; dependent variable: resistance; controlled variables: wire diameter, wire material, wire length, measurement technique

c quantitative

d glass thermometer: low precision; non-contact thermometer: high precision; analog voltmeter and ammeter: low precision, depending on scale; voltage/current probe (data logger): high precision

e a graph that decreased with increasing temperature

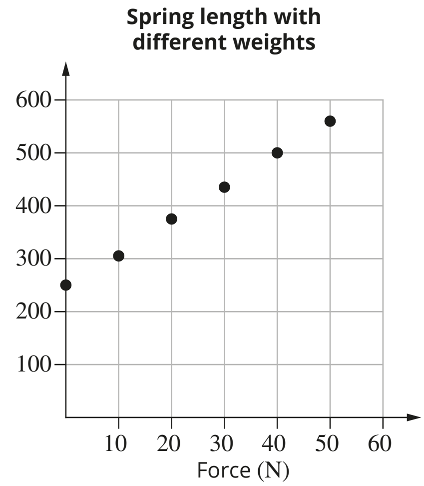
20 extension = 14 × mass

21 a $v = 3t - 8$

b The object started moving at 8 m s^{-1} in the negative direction. It then accelerated at a constant rate of 3 m s^{-2} in a positive direction.

22 The chart needs a title, and the *x*-axis needs a label. The trend line does not fit the data. The data is not linear.

23 a



b length = 6.2 × force + 247.4

The spring starts with an unstretched length of 247.4 mm, and then stretches 6.2 mm for every N of force that is applied.

24 a independent variable: starting temperature, dependent variable: time to cool to room temperature, controlled variables: cup properties (surface area, insulation), coffee properties, room temperature, method of data collection

b Inquiry question: How does the starting temperature of a cup of black coffee, in a disposable paper cup 200 mL in volume, affect the time it takes to cool to room temperature?

Hypothesis: An increase in starting temperature will increase the time taken to reach room temperature exponentially.

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c 1. Make 1 L of black coffee in a jug. Divide it into six different cups with 200 mL in each. Record the mass of each cup.
2. Heat each cup to a different starting temperature using a stove, water bath, or other heating element. Suggested temperatures are 50°C, 55°C, 60°C, 65°C, 70°C, 75°C.
3. Place each cup on the bench away from draughts, and at a distance from each other. Place a thermometer or temperature probe in each cup.
4. Record the starting temperature of each cup. Start a stopwatch. Record room temperature.
5. Record the temperature every minute. Record the time taken to reach room temperature.

d

	Cup 1	**Cup 2**	**Cup 3**	**Cup 4**	**Cup 5**	**Cup 6**
Mass (g)						
Time (s)						
Starting temperature (°C)						
1 min						
2 mins						

25 Student answers will vary depending on the cars chosen.

Example results:

Car	**Frontal offset score**	**Side impact score**	**Total score**
Ford Falcon	14.61	16.00	34.61
Honda Accord	14.79	16.00	35.79
Kia Rio	14.52	16.00	35.52

26 Student responses will vary.

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Chapter 2 Projectile motion

2.1 Projectiles launched horizontally

Worked example: Try yourself 2.1.1

PROJECTILE UNDER FREE FALL

A ball is allowed to free fall from a height of 30 m on Mars. Take the acceleration due to gravity to be $g = 3.8\,\text{m s}^{-2}$ and ignore air resistance.

a Calculate the time that the ball takes to land.	
Thinking	**Working**
Let the downward direction be positive. Write out the information relevant to the vertical component of the motion. Note that at the instant the ball is released it is not moving so its initial vertical velocity is zero.	Down is positive. Vertically: $s = +30\,\text{m}$ $u = 0\,\text{m s}^{-1}$ $a = +3.8\,\text{m s}^{-2}$ $t = ?$
In the vertical direction, the ball has constant acceleration, so use equations for uniform acceleration. Select the equation that best fits the information you have.	$s = ut + \frac{1}{2}at^2$
Substitute values, rearrange and solve for *t*.	$30 = 0 + \frac{1}{2} \times 3.8 \times t^2$ $t^2 = 15.79$ $t = 3.97$ $= 4.0\,\text{s}$ (to two significant figures)

b Calculate the velocity of the ball at the time of impact.	
Thinking	**Working**
Let the downward direction be positive. Write out the information relevant to the vertical component of the motion.	Vertically, with down as positive: $s = +30\,\text{m}$ $u = 0\,\text{m s}^{-1}$ $v = ?$ $a = +3.8\,\text{m s}^{-2}$ $t = 4.0\,\text{s}$
To find the final vertical speed, use the equation for uniform acceleration that best fits the information you have.	$v = u + at$
Substitute values, rearrange and solve for *v*.	Vertically: $v = 0 + 3.8 \times 4.0$ $= 15\,\text{m s}^{-1}$
Indicate the velocity with a magnitude and a direction.	The final velocity of the ball is $15\,\text{m s}^{-1}$ down.

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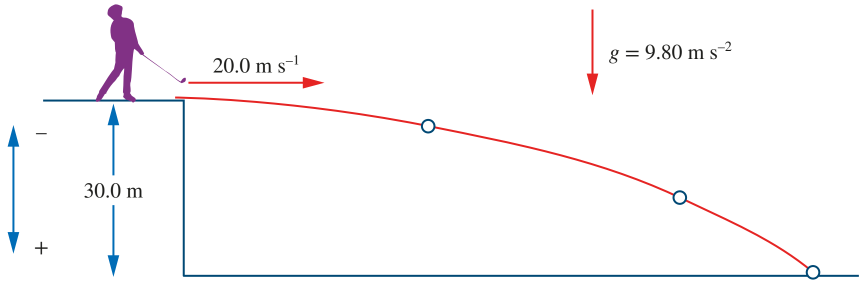
Worked example: Try yourself 2.1.2

PROJECTILE LAUNCHED HORIZONTALLY

A ball of mass 100 g is hit horizontally from the top of a 30.0 m high cliff with a speed of 20.0 m s^{-1}. Using g = 9.80 m s^{-2} and ignoring air resistance, calculate the following values:



a the time that the ball takes to land	
Thinking	**Working**
Let the downward direction be positive. Write out the information relevant to the vertical component of the motion. Note that the instant the ball is hit, it is travelling only horizontally, so its initial vertical velocity is zero.	Down is positive. Vertically: u = 0 m s^{-1} s = +30.0 m a = +9.80 m s^{-2} t = ?
In the vertical direction, the ball has constant acceleration, so use equations for uniform acceleration. Select the equation that best fits the information you have.	$s = ut + \frac{1}{2}at^2$
Substitute values, rearrange and solve for t.	$30.0 = 0 + 4.90t^2$ $t = \sqrt{\frac{30.0}{4.90}}$ = 2.47 s (to three significant figures)

b the distance that the ball travels from the base of the cliff, i.e. the range of the ball	
Thinking	**Working**
Write out the information relevant to the horizontal component of the motion. As the ball is hit horizontally, the initial speed gives the horizontal component of the velocity throughout the flight. Never round off a value until the final calculation, otherwise you will introduce a rounding error. Use the calculated value of t.	Horizontally: u = 20.0 m s^{-1} t = 2.47 s from part (a) s = ?
Select the equation that best fits the information you have.	As horizontal speed is constant, you can use $v_{av} = \frac{s}{t}$.
The distance travelled will be equal to the magnitude of the displacement. Substitute values, rearrange and solve for s. The distance travelled is a scalar quantity so no direction is required.	$20.0 = \frac{s}{2.47}$ $s = 20.0 \times 2.47$ = 49.5 m

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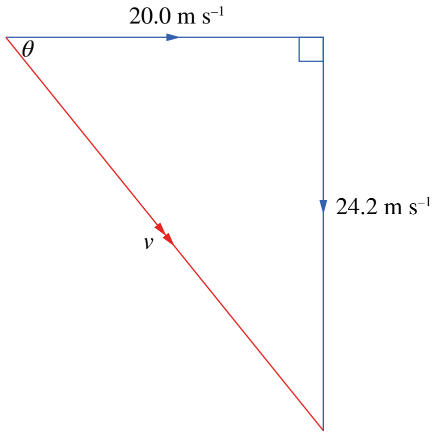
c the velocity of the ball as it lands.

Thinking	Working
Find the horizontal and vertical components of the ball's speed as it lands. Write out the information relevant to both the vertical and horizontal components.	Horizontally: $u = v_H = 20.0\,m\,s^{-1}$ Vertically, with down as positive: $u = 0$ $a = 9.80\,m\,s^{-2}$ $s = +30.0\,m$ $t = 2.47\,s$ $v_V = ?$
To find the final vertical speed, v_V, use the equation for uniform acceleration that best fits the information you have.	$v = u + at$
Substitute values, rearrange and solve for the variable you are looking for, in this case v.	Vertically: $v = u + at$ $= 0 + 9.80 \times 2.47$ $= 24.2\,m\,s^{-1}$ down
Add the components as vectors.	
Use Pythagoras theorem to work out the actual speed, v, of the ball.	$v = \sqrt{v_H{}^2 + v_V{}^2}$ $= \sqrt{20.0^2 + 24\ 2^2}$ $= \sqrt{988}$ $= 31.4\,m\,s^{-1}$
Use trigonometry to solve for the angle, θ. Never round off a value until the final calculation, otherwise you will introduce a rounding error. Use the calculated value of v_V.	$\theta = \tan^{-1}\left(\frac{24.2}{20.0}\right)$ $= 50.5°$
Indicate the velocity with magnitude and direction relative to the horizontal.	The final velocity of ball is $31.4\,m\,s^{-1}$ at 50.5° below the horizontal.

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2.1 Review

1 Let the downward direction be positive.

$u = 0$, $a = +9.80\,\text{m s}^{-2}$, $s = +200\,\text{m}$, $t = ?$

Use $s = ut + \frac{1}{2}at^2$ and rearrange for t.

$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{400}{9.8}}$

$= 6.39\,\text{s}$

Note that square root also gives a legitimate mathematical answer of $t = -6.39\,\text{s}$; however, that makes no physical sense and that answer can be rejected.

2 Let the downward direction be positive.

$u = -5.0\,\text{m s}^{-1}$, $a = +9.80\,\text{m s}^{-2}$, $s = +50\,\text{m}$, $v = ?$

Use $v^2 = u^2 + 2as$.

$v^2 = (-5)^2 + 2 \times 9.80 \times 50$

$v = \sqrt{1005}$

$= 32\,\text{m s}^{-1}$ down

3 Let the upward direction be positive.

$u = +20\,\text{m s}^{-1}$, $a = -9.80\,\text{m s}^{-2}$, $v = 0$, $s = ?$

Use $v^2 = u^2 + 2as$ and rearrange for s.

$s = \frac{v^2 - u^2}{2a}$

$= \frac{0 - 400}{2 \times -9.80}$

$= \frac{-400}{-19.6}$

$s = 20.4\,\text{m}$ (a scalar quantity does not require a direction)

4 B and C. The Moon possesses gravity and no air resistance. The ball travels in a parabolic arc much further than if moving on Earth. The balls did not travel in a straight line and were not hit hard enough to enter orbit (they would need to travel at over $2300\,\text{m s}^{-1}$ to enter the Moon's orbit).

5 **a** Vertically, $s = ut + \frac{1}{2}at^2$, rearrange to give $t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{9.8}{9.80}} = 1.0\,\text{s}$.

b Horizontally, $v_{av} = \frac{s}{t}$. The distance travelled is a scalar quantity.

$s = v \times t$

$= 20 \times 1$

$= 20\,\text{m}$

c The acceleration is constant: $9.80\,\text{m s}^{-2}$ downwards.

d Vertically,

$v = u + at = 0 + 9.80 \times 0.80$

$= 7.8\,\text{m s}^{-1}$ in the positive direction

As we can ignore air resistance, the horizontal velocity is constant: $20\,\text{m s}^{-1}$ in the positive direction.

$v = \sqrt{v_H{}^2 + v_V{}^2}$

$= \sqrt{461.5}$

$= 21\,\text{m s}^{-1}$ (speed is a scalar quantity)

e Vertically,

$v = u + at = 0 + 9.80 \times 1.0$

$= 9.8\,\text{m s}^{-1}$ in the positive direction

As we can ignore air resistance, the horizontal velocity is constant: $20\,\text{m s}^{-1}$ in the positive direction.

$v = \sqrt{v_H{}^2 + v_V{}^2}$

$= \sqrt{496}$

$= 22\,\text{m s}^{-1}$ (speed is a scalar quantity)

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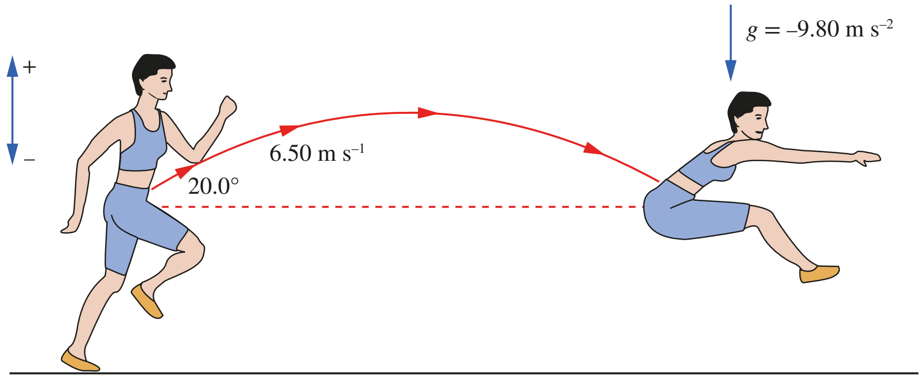
2.2 Projectiles launched obliquely

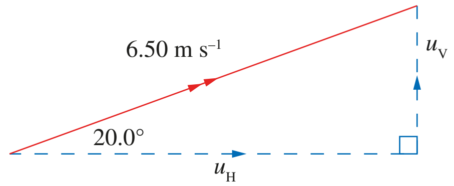
Worked example: Try yourself 2.2.1

LAUNCH AT AN ANGLE

An athlete in a long–jump event leaps with a velocity of 6.50 m s^{-1} at 20° to the horizontal.



For the following questions, treat the athlete as a point mass, ignore air resistance and use $g = 9.80$ m s^{-2}.

a What is the athlete's velocity at the highest point?	
Thinking	**Working**
First find the horizontal and vertical components of the initial speed. Remember, speed is a scalar quantity equal to the magnitude of the velocity.	 Using trigonometry: $u_H = 6.50 \cos 20.0$ $= 6.11$ m s^{-1} $u_V = 6.50 \sin 20.0$ $= 2.22$ m s^{-1}
Projectiles that are launched obliquely move only horizontally at the highest point of their motion. The vertical component of the velocity at this point is therefore zero. The actual velocity is given by the horizontal component of the velocity throughout the motion.	At maximum height: $v = 6.11$ m s^{-1} horizontally to the right.

b What is the maximum height gained by the athlete during the jump?	
Thinking	**Working**
To find the maximum height that is gained, you must use the vertical component. Recall that at the maximum height, the vertical component of velocity is zero. Never round off a value until the final calculation, otherwise you will introduce a rounding error. Use the calculated value of u.	Vertically, taking up as positive: $u = +2.22$ m s^{-1} $a = -9.80$ m s^{-2} $v = 0$ $s = ?$
Substitute these values into an appropriate equation for uniform acceleration.	$v^2 = u^2 + 2as$ $0 = (2.22)^2 + 2 \times (-9.80) \times s$
Rearrange and solve for s. Distance travelled is a scalar quantity.	$s = \frac{2.22^2}{19.6}$ $= 0.252$ m

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c Assuming a return to the original height, what is the total time the athlete is in the air?	
Thinking	**Working**
As the motion is symmetrical, the time required to complete the motion will be double that taken to reach the maximum height. First, the time it takes to reach the highest point must be found.	Vertically, taking up as positive: $u = +2.22\,\text{m s}^{-1}$ $a = -9.80\,\text{m s}^{-2}$ $v = 0$ $t = ?$
Substitute these values into an appropriate equation for uniform acceleration.	$v = u + at$ $0 = 2.22 - 9.80t$
Rearrange the formula and solve for t.	$t = \frac{2.22}{9.80}$ $= 0.227\,\text{s}$
The time to complete the motion is double the time it takes to reach the maximum height.	Total time $= 2 \times 0.227$ $= 0.454\,\text{s}$

2.2 Review

1 B. The horizontal velocity is constant throughout the flight and there is no vertical velocity while the javelin is at its highest point. Therefore, at this point the velocity is lowest. The javelin is always acting under gravitational forces, which causes a constant acceleration downwards.

2 **a** $v_H = 15\cos 25 = 13.6 = 14\,\text{m s}^{-1}$ (speed is a scalar quantity)
b $v_V = 15\sin 25 = 6.3\,\text{m s}^{-1}$ (speed is a scalar quantity)
c At maximum height acceleration is $9.80\,\text{m s}^{-2}$ downwards due to gravity.
d At maximum height the velocity of the ball is:
$v_H = 13.6\,\text{m s}^{-1}$ horizontally

3 **a** $u_H = 8.0\cos 60 = 4.0\,\text{m s}^{-1}$ (speed is a scalar quantity)
b $u_V = 8.0\sin 60 = 6.9\,\text{m s}^{-1}$ (speed is a scalar quantity)
c Take upwards to be positive.
Never round off a value until the final calculation, otherwise you will introduce a rounding error. Use the non-rounded value of u_V.
Vertically,
$u_V = +6.9\,\text{m s}^{-1}$, $v_V = 0$, $a = -9.80\,\text{m s}^{-2}$, $t = ?$
$v = u + at$
$0 = 6.9 + -9.80t$
$t = \frac{6.9}{9.80}$
$= 0.71\,\text{s}$
d Take upwards to be positive.
Vertically,
$u_V = +6.9\,\text{m s}^{-1}$, $v_V = 0$, $a = -9.80\,\text{m s}^{-2}$, $t = 0.71\,\text{s}$, $s = ?$
$s = ut + \frac{1}{2}at^2$
$= 6.9 \times 0.70 + \frac{1}{2} \times -9.80 \times 0.70^2$
$= 2.4\,\text{m}$ (a scalar quantity)
total height from ground $= 2.4 + 1.5 = 3.9\,\text{m}$
e At maximum height, the vertical velocity is zero, and the horizontal velocity is constant for the duration of the flight. i.e. $v_H = 4.0\,\text{m s}^{-1}$, $v_V = 0.0\,\text{m s}^{-1}$. So, total velocity $= 4.0\,\text{m s}^{-1}$ to the right.

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4 First, calculate the horizontal component of the initial speed:

$u_H = u\cos 60 = 0.5u$

The horizontal speed (a scalar) is constant throughout the flight:

$v_{av} = \frac{s}{t}$

$\therefore t = \frac{s}{v_{av}} = \frac{50}{0.5u} = \frac{100}{u}$

In the vertical direction, the projectile reaches the top of the flight in half of the total flight time.

$v_V = 0.0$, $u_V = u\sin 60$, and $a = -9.80\,\text{m s}^{-2}$, $t = \frac{50}{u}$

$v = u + at$

$0 = u\sin 60 - 9.80 \times \frac{50}{u}$

Solve this for u,

$u\sin 60 = 9.80 \times \frac{50}{u}$

$u^2 = \frac{9.80 \times 50}{\sin 60}$

$= 565.8$

$u = 23.79$

$= 24\,\text{m s}^{-1}$ (speed is a scalar quantity)

CHAPTER 2 **REVIEW**

1 Determine up as positive.

Vertically,

$a = -9.80\,\text{m s}^{-2}$, $v = 0$, $s = +15\,\text{m}$, $u = ?$

$v^2 = u^2 + 2as$

$0 = u^2 + 2 \times -9.80 \times 15$

$u = \sqrt{294}$

$= 17\,\text{m s}^{-1}$ upwards

2 Determine up as positive. Vertically,

$a = -3.80\,\text{m s}^{-2}$, $v = 0$, $s = +20\,\text{m}$, $u = ?$

$v^2 = u^2 + 2as$

$0 = u^2 + 2 \times -3.80 \times 20$

$u = \sqrt{152}$

$= 12.3\,\text{m s}^{-1}$ upwards

3 Determine down as positive.

$u = -1.5\,\text{m s}^{-1}$, $a = +9.80\,\text{m s}^{-2}$, $s = +9.0\,\text{m}$, $v = ?$, $t = ?$

There are a couple of ways to solve this problem. One would be to use the formula $s = ut + \frac{1}{2}at^2$ and then by using the quadratic formula you can solve for t.

Alternatively,

$v^2 = u^2 + 2as$

$= (-1.5)^2 + 2 \times 9.80 \times 9.0$

$v = 13.37\,\text{m s}^{-1}$

$s = \frac{1}{2}(u + v)t$

$9.0 = \frac{1}{2}(-1.5 + 13.37) \times t$

$t = 1.5\,\text{s}$

No. The squirrel reaches the ground after the nut.

4 B and D. The only force acting on the stone is gravity. Gravity accelerates the stone and so its speed increases. The stone travels in a parabolic, not circular, path.

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5 **a** Determine down as positive.

$u = 0\,\text{m s}^{-1}$, $a = 9.80\,\text{m s}^{-2}$, $s = 1.2\,\text{m}$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$1.2 = 0 + \frac{1}{2} \times 9.80 \times t^2$

$t = \sqrt{\frac{2 \times 1.2}{9.80}}$

$= \sqrt{0.24}$

$= 0.49\,\text{s}$

b Horizontal velocity is constant for the duration at $4.0\,\text{m s}^{-1}$ for 0.49 s.

So the distance travelled = 4.0 × 0.49 = 1.98 = 2.0 m (to two significant figures).

c The skateboard is accelerating at $9.80\,\text{m s}^{-2}$ downwards for the entire travel, not just before it lands.

$a = 9.80\,\text{m s}^{-2}$ down

6 **a** The distance travelled is a scalar quantity.

$s_H = v_H \times t$

$= 2.0 \times 0.75$

$= 1.5\,\text{m}$

b Speed is a scalar quantity.

$v_V = u_V + at$

$= 0 + 9.80 \times 0.75$

$= 7.4\,\text{m s}^{-1}$

c Speed is a scalar quantity.

$v = \sqrt{{v_H}^2 + {v_V}^2}$

$= \sqrt{4 + 54}$

$= 7.6\,\text{m s}^{-1}$

7 **a** Determine down as positive.

Vertically,

$u = 0.0\,\text{m s}^{-1}$, $a = +9.80\,\text{m s}^{-2}$, $s = +100\,\text{m}$, $v = ?$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2 \times 9.80 \times 100$

$v = \sqrt{1960}$

$v_V = 44.3\,\text{m s}^{-1}$ (speed is a scalar quantity)

b The horizontal speed is constant for the duration of the flight, $v_H = 25.0\,\text{m s}^{-1}$.

The angle of the final velocity can be found with trigonometry:

$\tan\theta = \frac{v_V}{v_H} = \frac{44.3}{25.0} = 1.77$

$\theta = \tan^{-1} 1.77 = 60.6°$

c Never round off a value until the final calculation, otherwise you will introduce a rounding error. Use the non-rounded values for the speeds.

Use Pythagoras theorem to find the final speed:

$v^2 = {v_V}^2 + {v_H}^2$

$v = \sqrt{44.3^2 + 25\,0^2}$

$= 50.8\,\text{m s}^{-1}$ (speed is a scalar quantity)

$\therefore v = 50.8\,\text{m s}^{-1}$ at 60.6° to the horizontal

d Find the flight time:

Vertically,

$s = ut + \frac{1}{2}at^2$

$100 = 0 + \frac{1}{2} \times 9.80 \times t^2$

$t = \sqrt{\frac{2 \times 100}{9.80}}$

$= 4.52\,\text{s}$

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Horizontally,

$s = v \times t$

$120 = v \times 4.52$

$v = 26.6\,\text{m s}^{-1}$

The tourist would need to increase the initial speed by 1.6 m s^{-1}.

8 **a** Determine down as positive.

$u = 0\,\text{m s}^{-1}$, $a = 9.80\,\text{m s}^{-2}$, $s = 2.0\,\text{m}$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$2.0 = 0 + \frac{1}{2} \times 9.80 \times t^2$

$t = \sqrt{\frac{2 \times 2}{9.80}}$

$= 0.64\,\text{s}$

b The acceleration on both balls is the same, so the time it takes ball B to strike the ground is the same as ball A in part a.

$t = 0.64\,\text{s}$

c Ball B is travelling $5\,\text{m s}^{-1}$ faster than ball A.

Therefore it travels $5 \times 0.64 = 3.2\,\text{m}$ further than ball A before hitting the ground. (Alternatively, ball B travels 6.4 m and ball A travels 3.2 m. The difference is 3.2 m.)

9 As the polystyrene has less mass than the hockey ball, any drag force will have a greater acceleration on the polystyrene. The effect on the vertical acceleration, gravity versus drag, is relatively small. The horizontal velocity of the polystyrene ball will reduce much more rapidly than the hockey ball, and thus the polystyrene ball will not travel as far.

10 **a** Vertically,

$u = 0\,\text{m s}^{-1}$, $a = 9.80\,\text{m s}^{-2}$, $s = 20\,\text{m}$, $v_V = ?$

$v_V^2 = u^2 + 2as$

$= 0 + 2 \times 9.80 \times 20$

$= 392$

$v_V = 19.8\,\text{m s}^{-1}$

Horizontally,

$v_H = 50\,\text{m s}^{-1}$

Using Pythagoras theorem:

$v^2 = v_V^2 + v_H^2$

$= 392 + 2500$

$= 2892$

$v = 54\,\text{m s}^{-1}$ (no direction required)

b $\tan\theta = \frac{v_V}{v_H}$

$\theta = \tan^{-1}\frac{19.8}{20}$

$= 45°$

11 **a** Horizontally,

$v_H = 10\,\text{m s}^{-1}$ to the right

Vertically,

$u_V = 0$, $a = 9.80\,\text{m s}^{-1}$, $s = 1.0\,\text{m}$, $v_V = ?$

$v_V^2 = u^2 + 2as$

$= 0 + 2 \times 9.80 \times 1.0$

$v_V = 4.4\,\text{m s}^{-1}$ down

b Using Pythagoras theorem, first calculate the speed.

$v^2 = v_V^2 + v_H^2$

$v = \sqrt{4.4^2 + 10^2}$

$= 10.9$

$= 11\,\text{m s}^{-1}$

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Use trigonometry to find the angle.

$\tan\theta = \frac{v_V}{v_H}$

$\theta = \tan^{-1}\frac{4.4}{10}$

$= 24°$

$v = 11\,m\,s^{-1}$, 24° to the horizontal

c Vertically,

$u = 0$, $a = 9.80\,m\,s^{-2}$, $s = 1.0\,m$, $v = 4.4\,m\,s^{-1}$, $t = ?$

$v = u + at$

$4.4 = 0 + 9.80t$

$t = 0.45\,s$

d Horizontally,

$s = v \times t$

$= 10 \times 0.45$

$= 4.5\,m$ (no direction required)

12 a Horizontally,

$s = v \times t = 2.5 \times 1.0 = 2.5\,m$ (no direction required)

b The acceleration is $9.80\,m\,s^{-2}$ vertically downwards.

c Vertically:

$u = 0$, $a = 9.80\,m\,s^{-2}$, $t = 1.0\,s$, $s = ?$

$s = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times 9.80 \times 1.0^2$

$= 4.9\,m$ down

Therefore, half of the fall height is 2.45 m.

Next calculate the time taken to reach half the fall (using the non-rounded value for the displacement s),

$u = 0$, $a = 9.80\,m\,s^{-2}$, $s = 2.45\,m$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$2.45 = 0 + \frac{1}{2} \times 9.80 \times t^2$

$t = 0.71\,s$

d Calculate the speed at the halfway point.

Vertically,

$u = 0$, $a = 9.80\,m\,s^{-2}$, $s = 2.45\,m$, $t = 0.71\,s$, $v = ?$

$v = u + at$

$= 0 + 9.80 \times 0.71$

$v_V = 6.9\,m\,s^{-1}$ (speed is a scalar)

Horizontally,

$v_H = 2.5\,m\,s^{-1}$ (speed is a scalar)

Using Pythagoras theorem:

$v^2 = v_V^2 + v_H^2$

$v = \sqrt{6.9^2 + 2.5^2}$

$= 7.4\,m\,s^{-1}$ (no direction required)

13 The target has a 1 m radius, so the distance travelled needs to be between 24 m and 26 m.

Take up as positive.

Vertically:

$u = 18\sin 30 = 9.0\,m\,s^{-1}$, $a = -9.80\,m\,s^{-2}$, $s = 0\,m$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 9.0t + \frac{1}{2} \times -9.80 \times t^2$

$t = 1.8\,s$

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Horizontally,

$u = 18\cos 30 = 15.6\,\text{m s}^{-1}$, $t = 1.8\,\text{s}$, $s = ?$

$s = vt$

$= 15.6 \times 1.8$

$= 28.6\,\text{m}$

No, the rocket does not hit the target.

14 **a** $u_H = 16\cos 50$

$= 10.28$

$= 10\,\text{m s}^{-1}$

b $u_V = 16\sin 50$

$= 12.26$

$= 12\,\text{m s}^{-1}$

c Determine up as positive.

Vertically,

$u = 12.26\,\text{m s}^{-1}$, $v = 0$, $a = -9.80\,\text{m s}^{-2}$, $s = ?$

$v^2 = u^2 + 2as$

$0 = 12.26^2 + 2 \times -9.80 \times s$

$s = 7.7\,\text{m}$ up

So total height from ground = 7.7 + 1.2 = 8.9 m

15 At the highest point the ball has zero vertical velocity. The horizontal velocity is constant throughout the flight when air resistance is ignored. So the overall velocity at the highest point is equal to the horizontal speed:

$v_H = v\cos\theta = 20\cos 30 = 17.3\,\text{m s}^{-1}$

16 C. Drag will add a horizontal acceleration against the direction of travel, so the horizontal velocity will continually decrease. A reduced horizontal velocity will reduce the ball's range. Drag will inhibit the vertical acceleration and the ball will not reach a greater height.

17 **a** $u = 15\,\text{m s}^{-1}$

i Take up as positive.

Vertically,

$u_V = 15\sin 45 = 10.6\,\text{m s}^{-1}$, $a = -9.80\,\text{m s}^{-2}$, $s = 0$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 10.6t + \frac{1}{2} \times -9.80 \times t^2$

$t = \frac{10.6}{4.9}$

$= 2.2\,\text{s}$

Horizontally,

$u_H = 15\cos 45 = 10.6\,\text{m s}^{-1}$, $t = 2.2\,\text{s}$, $s = ?$

$s = vt$

$= 10.6 \times 2.2$

$= 22.96$

$= 23\,\text{m}$

ii Take up as positive.

Vertically,

$u_V = 15\sin 55 = 12.29\,\text{m s}^{-1}$, $a = -9.80\,\text{m s}^{-2}$, $s = 0$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 12.29t + \frac{1}{2} \times -9.80 \times t^2$

$t = \frac{12.29}{4.9}$

$= 2.5\,\text{s}$

Horizontally,

$u_H = 15\cos 55 = 8.6\,\text{m s}^{-1}$, $t = 2.5\,\text{s}$, $s = ?$

$s = vt$

$= 8.6 \times 2.5$

$= 21.57$

$= 22\,\text{m}$

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iii Take up as positive.

Vertically,

$u_V = 15\sin 35 = 8.6\,m\,s^{-1}$, $a = -9.80\,m\,s^{-2}$, $s = 0$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 8.6t + \frac{1}{2} \times -9.80 \times t^2$

$t = \frac{8.6}{4.9}$

$= 1.8\,s$

Horizontally,

$u_H = 15\cos 35 = 12.29\,m\,s^{-1}$, $t = 1.8\,s$, $s = ?$

$s = vt$

$= 12.29 \times 1.8$

$= 21.57$

$= 22\,m$

b **i** Yes. By increasing or decreasing the angle away from 45° by the same amount, the displacement is equally reduced.

ii Students' answers may vary.

An investigation will include changing the launch angle and measuring the distance travelled. The initial speed and the type of projectile need to be kept constant.

18 Both cannon balls travel the same distance.

The question gives a launch speed, not a launch force, so the mass of each projectile plays no part in the initial acceleration. When drag is not a consideration, the greatest range is achieved at a launch angle of 45°. Both of the cannon balls are launched at the same difference in angle from the maximum (i.e. 45 = 60 – 15, and 45 = 30 + 15). The maximum range for both launches will be the same.

19 **a** **i** $v_H = 28.0\cos 30 = 24.2\,m\,s^{-1}$

ii $24.2\,m\,s^{-1}$

iii $24.2\,m\,s^{-1}$

b **i** $v_V = 28.0\sin 30 = 14.0\,m\,s^{-1}$ upwards

ii Let upwards be positive. $u = 14.0$, $a = -9.80$, $t = 1$, $v = ?$

Using $v = u + at$,

$v = 14.0 + (-9.80) \times 1 = 4.2\,m\,s^{-1}$ upwards

iii $u = 14.0$, $a = -9.8$, $t = 2$, $v = ?$

Using $v = u + at$,

$v = 14.0 + (-9.80 \times 2) = -5.6\,m\,s^{-1} = 5.6\,m\,s^{-1}$ downwards

c $v_H = 24.2\,m\,s^{-1}$, $v_V = -5.6\,m\,s^{-1}$; therefore using Pythagoras theorem, $v = 24.8\,m\,s^{-1}$

d Calculate flight time from vertical motion. $s = 0$, $u = 14.0$, $a = -9.80$, $t = ?$

Use $s = ut + \frac{1}{2}at^2$ and as $s = 0$ factorise for t.

$0 = t(14 - 4.9t)$, so $t = 2.86\,s$ at landing ($t = 0$ is the launch time solution). At $t = 2.86\,s$, $v_V = -14\,m\,s^{-1}$, $v_H = 24.2\,m\,s^{-1}$, $v = 28\,m\,s^{-1}$

This question could also be solved by knowing that the landing speed is the same as the take-off speed.

e $s = v_H \times t = 14 \times 2.86\,s = 40.0\,m$

20 **a** $u_H = 18\cos\theta$

Horizontally, the speed is constant.

$s = vt$

$t = \frac{s}{v}$

$= \frac{20}{18\cos\theta}$

b $u_V = 18\sin\theta$

Vertically,

$u = 18\sin\theta$, $a = -9.80\,m\,s^{-2}$, $s = 0\,m$, $t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = 18\sin\theta \times t + \frac{1}{2} \times -9.80 \times t^2$

$t = \frac{18\sin\theta}{4.9}$

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c Equate the two equations found in parts a and b.

$\frac{18\sin\theta}{4.9} = \frac{20}{18\cos\theta}$

$\sin\theta \times \cos\theta = \frac{20 \times 4.9}{18 \times 18} = 0.3025$

$2\sin\theta\cos\theta = 0.605$

Therefore if $2\sin\theta\cos\theta = \sin 2\theta$, then $\sin 2\theta = 0.605$

$2\theta = \sin^{-1} = 0.605$

$= 37.2$

$\theta = 18.61$

$= 19°$

21 Student answers may vary.

Projectile motion depends on the initial conditions, projectile properties and the environment.

- Initial conditions include initial velocity, launch angle and the height of the launch compared to landing position.
- Projectile properties include properties that change air resistance such as the shape, surface area and surface smoothness.
- Environment conditions include the force of gravity and any wind.

By using the equations of motion, it is possible to model the expected behaviour of projectiles by substituting in the correct initial conditions.

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Chapter 3 Circular motion

3.1 Circular motion

Worked example: Try yourself 3.1.1

CALCULATING SPEED

A water wheel has blades 2.0 m in length that rotate at a frequency of 10 revolutions per minute. At what speed do the tips of the blades travel? Express your answer in $km\,h^{-1}$.

Thinking	Working
Calculate the period, T. Remember to express frequency in the correct units. Alternatively, recognise that 10 revolutions in 60 s means that each revolution takes 6 s.	10 revolutions per minute $= \frac{10}{60} = 0.167$ Hz $T = \frac{1}{f}$ $= \frac{1}{0.167}$ $= 6.0$ s
Substitute r and T into the formula for speed and solve for v.	$v = \frac{2\pi r}{T}$ $= \frac{2 \times \pi \times 2.0}{6.0}$ $= 2.09\,m\,s^{-1}$
Convert $m\,s^{-1}$ into $km\,h^{-1}$ by multiplying by 3.6.	$2.09 \times 3.6 = 7.5\,km\,h^{-1}$

Worked example: Try yourself 3.1.2

CALCULATING ANGULAR VELOCITY

A truck wheel of diameter 1 m travels over 8 m of ground in 3 s. What is the angular velocity of the wheels? Express your answer in $°\,s^{-1}$.

Thinking	Working
Calculate the angle $\Delta\theta$ in radians.	$\Delta\theta = \frac{l}{r}$ $= \frac{8}{0.5}$ $= 16$ rad
Convert the angle to degrees.	$\Delta\theta = 16 \times \frac{180}{\pi}$ $= 916°$
Substitute $\Delta\theta$ and t into the formula for angular velocity and solve for ω.	$\omega = \frac{\Delta\theta}{}$ $= \frac{916}{3}$ $= 305.6$ $= 300°\,s^{-1}$ (to one significant figure)

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Worked example: Try yourself 3.1.3

CENTRIPETAL FORCES

An athlete in a hammer-throw event is swinging the ball of mass 7.0 kg in a horizontal circular path. The ball is moving at 25 m s^{-1} in a circle of radius 1.2 m.

a Calculate the magnitude of the acceleration of the ball.	
Thinking	**Working**
As the object is moving in a circular path, the centripetal acceleration is towards the centre of the circle. Write down the other variables that are given.	$v = 25\text{ m s}^{-1}$ $r = 1.2\text{ m}$ $a_c = ?$
Find the equation for centripetal acceleration that fits the information you have, and substitute the values.	$a_c = \frac{v^2}{r}$ $= \frac{25^2}{1.2}$ $= 520.8$ $= 520\text{ m s}^{-2}$ (to two significant figures)
Only the magnitude is required, so no direction is needed in the answer.	The acceleration of the ball is 520 m s^{-2}.

b Calculate the magnitude of the tensile force acting in the wire.	
Thinking	**Working**
Identify the unbalanced force that is causing the object to move in a circular path. Write down the information that you are given.	$m = 7.0\text{ kg}$ $a = 520.8\text{ m s}^{-2}$ $F_{net} = ?$
Select the equation for centripetal force, and substitute the variables you have.	$F_c = F_{net} = ma$ $= 7.0 \times 520.8$ $= 3.6 \times 10^3\text{ N}$
Only the magnitude is required, so no direction is needed in the answer.	The force of tension in the wire is the unbalanced force that is causing the ball to accelerate. Tensile force $F_T = 3.6 \times 10^3\text{ N}$

3.1 Review

1 No. Phil's inertia made him stay where he was (stationary) as the tram moved forwards. This made it look like Phil was thrown backwards relative to the tram. This is an example of Newton's first law. Objects will remain at rest unless a net unbalanced force acts to change the motion.

2 Newton's third law $\left(F_{AB} = -F_{BA}\right)$ says that every action has an equal and opposite reaction, and that action–reaction pairs must act on different bodies. The weight force and the normal force of an object both act on the same body, so they cannot be an action–reaction pair.
For a mug sitting on a table, the third law pairs for each will be:
Force on mug by table (F_N) with force on table by mug.
Gravitational force Earth exerts on mug (F_g) with gravitational force mug exerts on Earth.

3 B. A sideways force of friction between the road and the tyres is enabling the car to travel in a circle.

4 $T = \frac{1}{f}$
$= \frac{1}{5}$
$= 0.2\text{ s}$

5 **a** A and D. The speed is constant, but the velocity is changing as the direction is constantly changing. The acceleration is directed towards the centre of the circle.

b **i** 8.0 m s^{-1}

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ii 8.0 m s^{-1} south

iii $a_c = \frac{v^2}{r}$

$= \frac{8.0^2}{9.2}$

$= 6.96$

$= 7.0$ m s^{-2} west

c $\vec{F}_{net} = m\vec{a} = 1200 \times 7.0 = 8.4 \times 10^3$ N west

d **i** 8.0 m s^{-1} north

ii 7.0 m s^{-2} east

e The force needed to give the car a larger centripetal acceleration will eventually exceed the maximum frictional force that could act between the tyres and the road surface. At this time, the car would skid out of its circular path.

6 **a** The magnitude of the acceleration is a scalar quantity.

$a_c = \frac{v^2}{r}$

$= \frac{(2.0)^2}{1.5}$

$= 2.67$

$= 2.7$ m s^{-2} (to two significant figures)

b The skater has an acceleration so the forces are unbalanced. This can be explained using Newton's second law of motion.

c Magnitude of the force is a scalar quantity.

$F_{net} = ma = 50 \times 2.7$

$= 133.3$

$= 130$ N (to two significant figures)

7 **a** $\Delta\theta = \frac{l}{r}$

$= \frac{5}{1.2} = 4.17^c$ (multiply by $\frac{180}{\pi}$ to find the angle in degrees)

$= 238.7$

$= 240°$

b $\omega = \frac{\Delta\theta}{t}$

$= \frac{238.7}{0.5}$

$= 477.5$

$= 480°$ s^{-1}

c 5 revolutions $= 5 \times 360 = 1800°$

$\omega = \frac{\Delta\theta}{t}$

$50 = \frac{1800}{t}$

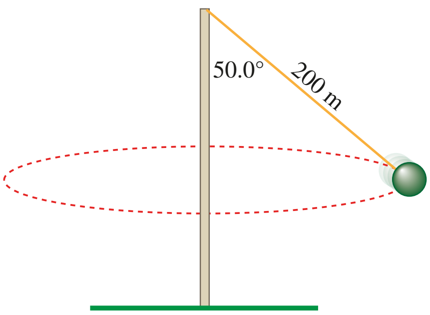
$t = 36$ s

3.2 Circular motion on banked tracks

Worked example: Try yourself 3.2.1

OBJECT ON THE END OF A STRING

During a game of Totem Tennis, the ball of mass 200 g is swinging freely in a horizontal circular path. The cord is 2.00 m long and is at an angle of 50.0° to the vertical shown in the diagram.



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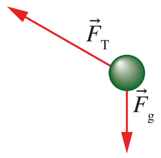
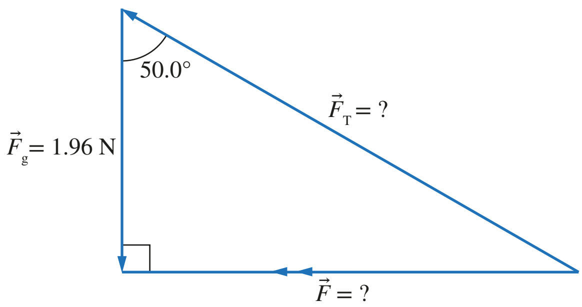
a Calculate the radius of the ball's circular path.	
Thinking	**Working**
The centre of the circular path is not the top end of the cord, but is where the pole is level with the ball. Use trigonometry to find the radius.	$r = 2.00 \sin 50.0$ $= 1.53\,\text{m}$

b Draw and identify the forces that are acting on the ball at the instant shown in the diagram.	
Thinking	**Working**
There are two forces acting—the tension in the cord, $\vec{F}_T$, and gravity, $\vec{F}_g$. These forces are unbalanced.	

c Determine the net force that is acting on the ball at this time.	
Thinking	**Working**
First calculate the weight force, $\vec{F}_g$.	$\vec{F}_g = m\vec{g}$ $= 0.200 \times 9.80$ $= 1.96\,\text{N}$ downwards
The ball has an acceleration that is towards the centre of its circular path. This is horizontal and towards the left at this instant. The net force will also lie in this direction at this instant. A force triangle and trigonometry can be used here.	 $\vec{F}_{net} = 1.96 \tan 50$ $= 2.34\,\text{N}$ towards the left

d Calculate the size of the tensile force in the cord.	
Thinking	**Working**
Use trigonometry to find F_T. The size of the force is a scalar and doesn't require a direction.	$F_T = \frac{1.96}{\cos 50.0}$ $= 3.05\,\text{N}$

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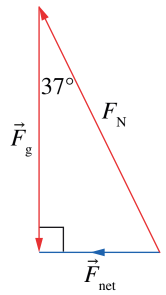
Worked example: Try yourself 3.2.2

BANKED CORNERS

A curved section of track on an Olympic velodrome has a radius of 40 m and is banked at an angle of 37° the horizontal. A cyclist of mass 80 kg is riding on this section of track at the design speed.

a Calculate the net force acting on a cyclist at this instant as they are riding at the design speed.	
Thinking	**Working**
Draw a force diagram and include all forces acting on the cyclist.	
Calculate the weight force, $\vec{F}_g$.	$\vec{F}_g = m\vec{g}$ $= 80 \times 9.80$ $= 784$ N downwards
Use the force triangle and trigonometry to work out the net force, $\vec{F}_{net}$.	$\tan\theta = \frac{\vec{F}_{net}}{\vec{F}_g}$ $\tan 37 = \frac{\vec{F}_{net}}{784}$ $\vec{F}_{net} = 0.75 \times 784$ $= 590$ N towards the centre of the circle

b Calculate the design speed for this section of the track.	
Thinking	**Working**
Write down all the known values.	m = 80 kg r = 40 m $\theta = 37°$ $\vec{F}_g = 784$ N down $\vec{F}_{net} = 590$ N towards the centre $v = ?$
Use the design speed formula.	$v = \sqrt{rg\tan\theta}$ $= \sqrt{40 \times 9.80 \times \tan 37}$ $= 17\ \text{m s}^{-1}$

3.2 Review

1 **a** $r = 2.4\cos 60 = 1.2$ m

b The forces are her weight acting vertically downwards and the tension in the rope acting along the rope towards the top of the maypole.

c She has an acceleration directed right towards point B, the centre of her circular path.

d Use a force triangle for the girl, showing the net force towards B.

$\vec{F}_{net} = \frac{m\vec{g}}{\tan 60} = \frac{294}{1.73} = 170$ N towards B

e $F_{net} = F_c = \frac{mv^2}{r}$

$170 = \frac{30 \times v^2}{1.2}$

$v = 2.6\ \text{m s}^{-1}$

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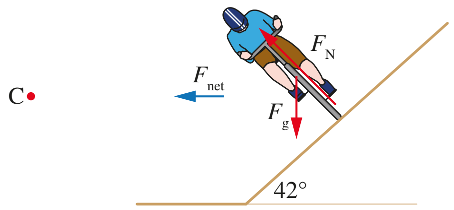
2 In all circular motion, the acceleration is directed towards the centre of the circle.

3 The design speed depends on the bank angle $\tan\theta$ and the radius of the curve. The architect could make the bank angle larger or increase the radius of the track.

4 The car will travel higher up the banked track as the greater speed means that a greater radius is required in the circular path. When travelling faster than the design speed the normal force is not sufficient to keep the car moving in a circle and causes the car to move outwards from the centre.

5 On the horizontal track, the car is depending on the force of *friction* to turn the corner. The size of the *normal* force is equal to the *weight* of the car, so these vertical forces are *balanced*. When driving on the banked track, the *normal* force is not vertical and so is not balanced by the *weight* force. In both cases, the forces acting on the car are unbalanced.

6



7 $v = \sqrt{rg\tan\theta}$

$40 = \sqrt{150 \times 9.80 \times \tan\theta}$

$\tan\theta = \frac{40^2}{150 \times 9.80}$

$\theta = 47°$

8 **a** **i** $F_{net} = F_c = \frac{mv^2}{r}$

$= \frac{1200 \times 18^2}{80}$

$= 4860$

$= 4.9$ kN (only the magnitude is required, so a direction is not needed)

ii $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

$= \tan^{-1}\left(\frac{18^2}{80 \times 9.80}\right)$

$= 22°$

b Since the angle of bank (θ) is fixed, an increasing v increases r for constant θ as $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$.

A greater radius will make the car travel higher up the banked track. The driver would have to turn the front wheels slightly towards the bottom of the bank.

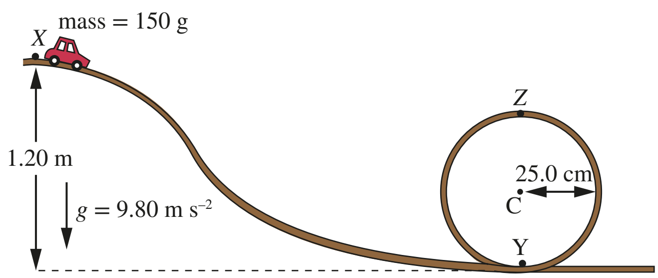
3.3 Work and energy

Worked example: Try yourself 3.3.1

VERTICAL CIRCULAR MOTION

A student arranges a toy car track with a vertical loop of radius 25.0 cm, as shown.

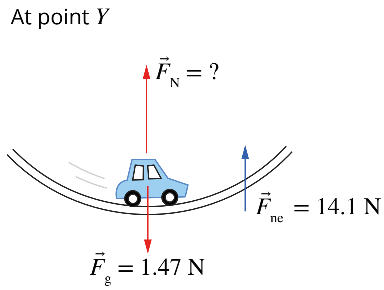
A toy car of mass 150 g is released from rest at a height of 1.20 m at point X. The car rolls down the track and travels around the loop. Assume g is 9.80 m s^{-2}, and ignore friction for the following questions.



a Calculate the speed of the car as it reaches the bottom of the loop, point Y.

Thinking	Working
Note all the variables given to you in the question.	At X: $m = 150\,\text{g} = 0.150\,\text{kg}$ $h = 1.20\,\text{m}$ $v = 0$ $g = 9.80\,\text{m s}^{-2}$
Use an energy approach to calculate the speed. Calculate the total mechanical energy first.	The initial speed is zero, so K at X is zero. Mechanical energy, E_m, at X is: $E_m = K + U$ $= \frac{1}{2}mv^2 + mg\Delta h$ $= 0 + (0.150 \times 9.80 \times 1.20)$ $= 1.764$ $= 1.76\,\text{J}$ (to two significant figures)
Use conservation of energy ($E_m = K + U$) to determine the velocity at point Y. As the car rolls down the track, it loses its gravitational potential energy and gains kinetic energy. At the bottom of the loop (Y), the car has zero potential energy.	At Y: $E_m = 1.76\,\text{J}$ $h = 0$ $U_g = 0$ $E_m = K + U$ $= \frac{1}{2}mv^2 + mg\Delta h$ $1.76 = 0.5 \times 0.150v^2 + 0$ $v^2 = 23.5$ $v = \sqrt{23.5}$ $= 4.85\,\text{m s}^{-1}$

b Calculate the normal reaction force from the track at point Y.

Thinking	Working
To solve for $\vec{F}_N$, start by working out the centripetal force. At Y, the car has a centripetal acceleration towards C (i.e. upwards), so the net (centripetal) force must also be vertically up at this point.	$F_{net} = F_c = \frac{mv^2}{r}$ $= \frac{0.150 \times 4.85^2}{0.250}$ $= 14.1\,\text{N up}$
Calculate the weight force, $\vec{F}_g$, and add it to a force diagram.	 $\vec{F}_g = m\vec{g}$ $= 0.150 \times 9.80$ $= 1.47\,\text{N down}$
Work out the normal force using vectors. Note up as positive and down as negative for your calculations. These forces are unbalanced, as the car has a centripetal acceleration upwards (towards C). The upward (normal) force must be larger than the downward force.	$\vec{F}_N = \vec{F}_{net} - \vec{F}_g = 14.1 - (-1.47)$ $= 15.6\,\text{N up}$

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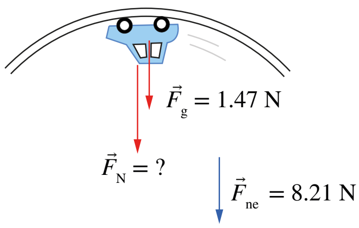
c What is the speed of the car as it reaches point Z?	
Thinking	**Working**
Calculate the speed from the values you have, using $E_m = K + U$.	At Z: $m = 0.150\,\text{kg}$ $h = 2 \times 0.250 = 0.500\,\text{m}$ Mechanical energy is conserved, so use the value from part (a). Be careful not to introduce any rounding errors. At Z: $E_m = K + U$ $= \frac{1}{2}mv^2 + mg\Delta h$ $1.764 = \frac{1}{2} \times 0.15 \times v^2 + 0.150 \times 9.80 \times 0.500$ $1.764 = 0.075 \times v^2 + 0.735$ $0.075v^2 = 1.764 - 0.735$ $v^2 = 13.72$ $v = \sqrt{13.72}$ $= 3.70\,\text{m s}^{-1}$

d What is the normal force acting on the car at point Z?	
Thinking	**Working**
To find $\vec{F}_N$, start by working out the net, or centripetal, force. At Z, the car has a centripetal acceleration towards C (i.e. down), so the net (centripetal) force must also be vertically down at this point. Never round off a value until the final calculation, otherwise you will introduce a rounding error. Use the non-rounded value of v.	$F_{net} = F_c = \frac{mv^2}{r}$ $= \frac{0.150 \times 3.70^2}{0.250}$ $= 8.23\,\text{N down}$
Work out the normal force using vectors. Note up as positive and down as negative for your calculations.	At point Z  $\vec{F}_{net} = \vec{F}_g + \vec{F}_N$ $-8.23 = -1.47 + F_N$ $\vec{F}_N = -8.23 + 1.47$ $= -6.76$ $= 6.76\,\text{N down}$

3.3 Review

1 0 J. There is no change in energy so there is no mechanical work.

2 Take down as positive.

$F_{net} = F_c = \frac{mv^2}{r}$

$= \frac{0.20 \times 1.5^2}{2}$

$= 0.225\,\text{N down}$

$\vec{F}_{net} = \vec{F}_N + \vec{F}_g$

$0.225 = \vec{F}_N + mg$

$\vec{F}_N = 0.22 - 0.20 \times 9.80$

$= -1.735$

$= 1.7\,\text{N up}$

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3 **a** Determine down as positive.

$F_{net} = F_c = \frac{mv^2}{r}$

$= \frac{0.08 \times 4.0^2}{0.5}$

$= 2.56\,\text{N}$ down

$\vec{F}_{net} = \vec{F}_T + \vec{F}_g$

$2.56 = \vec{F}_T + mg$

$\vec{F}_T = 2.56 - 0.08 \times 9.80$

$= 1.776$

$= 1.8\,\text{N}$ down

b According to the conservation of mechanical energy, the change in potential energy will be equal to the change in kinetic energy as the yo-yo goes around the circle. The change in height will be equal to twice the radius: $\Delta h = 2r = 1.0\,\text{m}$.

$K_{top} + U_{top} = K_{bottom} + U_{bottom}$

$U_{top} - U_{bottom} = K_{bottom} - K_{top}$

$mg\Delta h = \frac{1}{2}m({v_{bottom}}^2 - {v_{top}}^2)$

$0.08 \times 9.80 \times 1.0 = \frac{1}{2} \times 0.08\,({v_{bottom}}^2 - 4.0^2)$

${v_{bottom}}^2 = \frac{0.08 \times 9.80 \times 1.0 \times 2}{0.08} + 4.0^2 = 35.6$

$v_{bottom} = 5.97$

$= 6.0\,\text{m s}^{-1}$

c Determine down as positive.

$F_{net} = F_c = \frac{mv^2}{r}$

$= \frac{0.08 \times 35.6}{0.5}$

$= 5.7\,\text{N}$ up

$\vec{F}_{net} = \vec{F}_T + \vec{F}_g$

$-5.7 = \vec{F}_T + mg$

$\vec{F}_T = 0.08 \times 9.80 - 5.7$

$= -4.912$

$= 4.9\,\text{N}$ up

4 **a** At X, mechanical energy is:

$E_m = K + U$

$= \frac{1}{2}\text{mv}^2 + mg\Delta h$

$= 0.5 \times 500 \times 2.00^2 + 500 \times 9.80 \times 50.0$

$= 1000 + 245\,000$

$= 246\,000\,\text{J}$

At Y: U_g is zero so its kinetic energy is 246 000 J

$\frac{1}{2}mv^2 = 246\,000$

$0.5 \times 500 \times v^2 = 246\,000$

$v = \sqrt{984}$

$= 31.4\,\text{m s}^{-1}$

b At Z, mechanical energy = 246 000 J

$E_m = K + U$

$246\,000 = K + 500 \times 9.80 \times 30.0$

$246\,000 = K + 147\,000$

$K = 99\,000\,\text{J}$

$0.5 \times 500\,v^2 = 99\,000$

$v = 19.9\,\text{m s}^{-1}$

c At Z: $F_g = mg = 500 \times 9.80 = 4900\,\text{N}$ down

$F_{net} = F_c = \frac{mv^2}{r} = \frac{500 \times 19.9^2}{15} = 13\,200\,\text{N}$ down

$\vec{F}_{net} = \vec{F}_N + \vec{F}_g$

$13\,200 = \vec{F}_N + 4900$

$\vec{F}_N = 8300\,\text{N}$ down

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d For the cart to just lose contact at Z, $\vec{F}_N = 0$.

$a_c = \frac{v^2}{r} = g$, so:

$v = \sqrt{rg}$

$= \sqrt{15.0 \times 9.80}$

$= 12.1\,\text{m s}^{-1}$

5 **a** $a_c = \frac{v^2}{r}$

$= \frac{6.0^2}{2.0}$

$= 18\,\text{m s}^{-2}$ up

b $F_{net} = F_c = \frac{mv^2}{r}$

$= \frac{55 \times 6.0^2}{2.0}$

$= 990\,\text{N}$ up

$F_g = mg$

$= 55 \times 9.80$

$= 540\,\text{N}$ down

$\vec{F}_{net} = \vec{F}_N + \vec{F}_g$ (and take down as negative)

$990 = \vec{F}_N - 540$

$\vec{F}_N = 990 + 540$

$= 1530\,\text{N}$ up

6 **a** If the ball is just losing contact with track, $\vec{F}_N = 0$ so $\vec{F}_{net} = \vec{F}_g$ and therefore $\vec{a} = 9.8\,\text{m s}^{-2}$ down.

b $v = \sqrt{rg}$

$= \sqrt{0.5 \times 9.80}$

$= 2.2\,\text{m s}^{-1}$

7 Minimum speed would occur at the top of the circle, when the tension in the rope just reaches 0 N, so the net force will come solely from the force of gravity.

$F_{net} = F_c = \frac{mv^2}{r}$

$= \frac{1 \times v^2}{0.8}$

$F_{net} = mg = 1 \times 9.80 = 9.8$

$\frac{v^2}{0.8} = 9.8$

$v^2 = 7.84$

$v = 2.8\,\text{m s}^{-1}$

8 **a** Due to the conservation of mechanical energy, the change in gravitational potential energy will equal the change in kinetic energy.

$\Delta U = \Delta K$

$mg\Delta h = \frac{1}{2}mv^2$

$0.15 \times 9.80 \times 0.4 = \frac{1}{2} \times 0.15 \times v^2$

$v^2 = 7.84$

$v = 2.8\,\text{ms}^{-1}$

b $F_{net} = F_c = \frac{mv^2}{r}$

$= \frac{0.15 \times 2.8^2}{0.4}$

$= 2.94\,\text{N}$ towards the centre

$\vec{F}_{net} = \vec{F}_T + \vec{F}_g = \vec{F}_T - mg$

$2.94 = \vec{F}_T - 0.15 \times 9.80$

$\vec{F}_T = 2.94 + 1.47 = 4.41\,\text{N}$ towards the centre

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3.4 Torque

Worked example: Try yourself 3.4.1

CALCULATING TORQUE

A force of 255 N is required to apply a torque on a sports car steering wheel as it turns left. The force is applied at 90° to the 15.5 cm radius of the steering wheel. Calculate the torque on the steering wheel.

Thinking	Working
Identify the variables involved and state them in their standard form.	$\tau = ?$ $r_{\perp} = 15.5\text{ cm} = 0.155\text{ m}$ $F = 255\text{ N}$
Apply the equation for torque. State the answer with the appropriate direction.	$\tau = r_{\perp}F$ $= 0.155 \times 255$ $= 39.5\text{ N m}$ anticlockwise

Worked example: Try yourself 3.4.2

CALCULATING TORQUE FROM THE PERPENDICULAR COMPONENT OF FORCE

A mechanic uses a 17.0 cm long spanner to tighten a nut on a winch. He applies a force of 104 N at an angle of 75.0° to the spanner.

Calculate the magnitude of the torque that the mechanic applies to the nut.

Thinking	Working
Use the trigonometric relationship $F_{\perp} = F\sin\theta$ to determine the force perpendicular to the spanner.	$F_{\perp} = F\sin\theta$ $= 104 \sin 75.0$ $= 100.5\text{ N}$ perpendicular to the force arm
Convert variables to their standard units.	$r = 17.0\text{ cm}$ $= 0.170\text{ m}$
Apply the equation for torque: $\tau = r_{\perp}F = rF_{\perp}$ State the answer with the appropriate units and direction.	$\tau = rF_{\perp}$ $= 0.170 \times 100.5$ $= 17.1\text{ N m}$ (from the image, the torque is applied in a clockwise direction)

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3.4 Review

1 a The magnitude of the torque produced by a given force is proportional to the length of the force arm. By pushing the door at the handle, rather than the middle, the length to the force arm is increased.

b A crowbar can be used to generate a large torque because the force can be applied at a large distance from the pivot.

2 a $\tau = r_{\perp}F$

$= 2 \times 100$

$= 200\,\text{N m}$ anticlockwise

b The torque is zero because the line of action passes through the pivot point.

3 $r = \frac{\tau}{F_{\perp}}$

$= \frac{15}{30}$

$= 0.5\,\text{m}$ (perpendicular to the force)

4 $F_{\perp} = \frac{\tau}{r}$

$= \frac{9}{0.50}$

$= 18\,\text{N}$ (perpendicular to the force arm)

5 $\tau = rF_{\perp}$

$= 0.40 \times 225$

$= 90\,\text{N m}$ (direction not specified)

6 a The applied force is equal to the weight force, F_g. The size of the torque is a scalar, so no direction is required.

$\tau = rF_g = 0.5 \times 1.0 \times 9.80$

$= 4.9\,\text{N m}$

b $\tau = rF_g = 1.0 \times 1.0 \times 9.80$

$= 9.8\,\text{N m}$

c $\tau = rF_{\perp} = rF_g \sin\theta$

$= 1.0 \times 1.0 \times 9.80 \times \sin 30$

$= 4.9\,\text{N m}$

7 The magnitude of the torque is a scalar, so no direction is required.

$\tau = rF \sin\theta$

$= 0.30 \times 300 \times \sin 30$

$= 0.30 \times 300 \times 0.5$

$= 45\,\text{N m}$

8 a Weight of skip: $F_g = mg = 3500 \times 9.80 = 3.4 \times 10^4\,\text{N}$ down.

b The effective force arm remains at 15 m throughout, so the torque does not change.

c $\tau = rF \sin\theta = 25 \times 3.43 \times 10^4 \times \sin 37 = 5.2 \times 10^5\,\text{N m}$ clockwise about the pivot.

CHAPTER 3 REVIEW

1 a $v = \frac{2\pi r}{T}$

$= \frac{2\pi \times 0.800}{1.36}$

$= 3.70\,\text{m s}^{-1}$

b $a_c = \frac{v^2}{r}$

$= \frac{3.70^2}{0.800}$

$= 17.1\,\text{m s}^{-2}$ towards the centre of the circle

c $F_c = F_{net} = ma$

$= 0.0250 \times 17.1 = 0.430\,\text{N}$ (size only needed)

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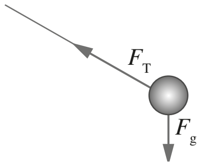
2 **a** 

b Use a force triangle for the ball. The magnitude of the tension is a scalar quantity.

$F_T = \frac{mg}{\sin 30.0}$

$= \frac{0.0250 \times 9.80}{0.50}$

$= 0.49\,N$

3 **a** $a_c = \frac{v^2}{r}$

$= \frac{5^2}{10}$

$= 2.5\,m\,s^{-2}$ towards the centre of the circle

b The centripetal force is created by the friction between the tyres and the ground.

4 $a_c = \frac{v^2}{r}$

$8.88 \times 10^4 = \frac{v^2}{0.9}$

$v = 282.7\,m\,s^{-2}$

$f = \frac{v}{2\pi r}$

$= \frac{282.7}{2\pi \times 0.9}$

$= 50.0\,Hz$

5 Use a force triangle with weight, normal and net force (acting horizontally).

$v = \sqrt{rg \tan\theta}$

$= \sqrt{30 \times 9.80 \times \tan 40}$

$= 15.7\,m\,s^{-1}$

6 **a** $v = \sqrt{rg \tan\theta}$

$= \sqrt{28 \times 9.80 \times \tan 33}$

$= 13.3\,m\,s^{-1}$

$= 13.3 \times 3.6$

$= 48\,km\,h^{-1}$

b $F_N = \frac{mg}{\cos 33}$

$= \frac{539}{\cos 33}$

$= 640\,N$

c On a horizontal track, F_N is equal and opposite to the weight force, so $F_N = mg = 539$ N. This is less than the normal force on the banked track (643 N).

7 $\Delta\theta = 360°$

Calculate the time for one revolution:

$t = \frac{\text{circumference}}{\text{velocity}}$

$= \frac{2\pi r}{v}$

$= \frac{2\pi \times 0.80}{5.0}$

$= 1.01\,s$

$\omega = \frac{\Delta\theta}{t} = \frac{360}{1.01}$

$= 358$

$= 360°\,s^{-1}$

8 $a_c = \frac{v^2}{r}$

$13 = \frac{v^2}{0.020}$

$v = 0.51\,m\,s^{-2}$

$f = \frac{v}{2\pi r}$

$= \frac{0.51}{2\pi \times 0.02}$

$= 4.1\,Hz$

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9 **a** $v = 50\,\text{km h}^{-1}$

$= \frac{50}{3.6} = 13.89\,\text{m s}^{-1}$

$= \frac{2\pi r}{T}$

$T = \frac{2\pi r}{v} = \frac{2\times\pi\times62}{13.89} = 28\,\text{s}$

b $F_c = \frac{mv^2}{r}$

$= \frac{1.6\times13.89^2}{62}$

$= 5.0\ \text{N}$

10 **a** $T = \frac{1}{f}$

$= \frac{1}{2.0}$

$= 0.5\,\text{s}$

b $v = \frac{2\pi r}{T}$

$= \frac{2\times\pi\times0.80}{0.5}$

$= 10\,\text{m s}^{-1}$

c $a_c = \frac{v^2}{r}$

$= \frac{(10)^2}{0.8}$

$= 126\,\text{m s}^{-2}$

$= 130\,\text{m s}^{-2}$ (to two significant figures)

d $F_{net} = ma$

$= 2.5 \times 126$

$= 315.83 = 320\,\text{N}$ (to two significant figures)

11 Since $a_c = \frac{v^2}{r}$

a Since $a_c = \frac{v^2}{r}$, doubling the speed increases the centripetal acceleration by a factor of four.

b Since $a_c = \frac{v^2}{r}$, tripling the radius reduces the centripetal acceleration to be one-third of the original.

c Since $a_c = \frac{v^2}{r}$, mass has no effect on the centripetal acceleration.

12 Orbital radius $= 6.37 \times 10^6\,\text{m} + 3.60 \times 10^4\,\text{m}$

$= 6.406 \times 10^6\,\text{m}$

Orbital distance $= 2\pi r$

$= 2\pi \times 6.406 \times 10^6\,\text{m}$

$= 4.025 \times 10^7\,\text{m}$

$T = (23 \times 60 \times 60) + (56 \times 60) + 5$

$= 86\,165\,\text{s}$

$v = \frac{2\pi r}{T}$

$= \frac{4.025\times10^7}{86165}$

$= 467\,\text{m s}^{-1}$

$a_c = \frac{v^2}{r}$

$= \frac{(4.67\times10^2)^2}{6.406\times10^6}$

$= 3.40 \times 10^{-2}\,\text{m s}^{-2}$

13 **a** $10\,\text{m s}^{-1}$ south

b $10\,\text{m s}^{-1}$. A direction is not required as the question asked for speed.

c $v = \frac{2\pi r}{T}$

$T = \frac{2\pi r}{v}$

$= \frac{2\times\pi\times20}{10}$

$= 13\,\text{s}$

d $a_c = \frac{v^2}{r}$

$= \frac{10^2}{20}$

$= 5.0\,\text{m s}^{-2}$ west

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e The centripetal force is supplied by friction.

$F_c = F_{net} = ma$

$= 1500 \times 5$

$= 7.5 \times 10^3$ N west

14 Recall the equation for force on a moving charge. This provides the centripetal force, so is equal to $\frac{mv^2}{r}$.

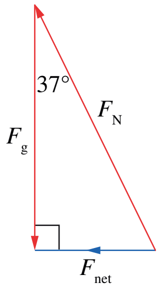
$qvB = \frac{mv^2}{r}$

$r = \frac{mv}{qB}$

$= \frac{1.67 \times 10^{-27} \times 3.50 \times 10^6}{1.60 \times 10^{-19} \times 0.25}$

$= 0.146$ m

15 A. From the triangle, $F_N > F_g$.



16 **a** $v = \frac{2\pi r}{T}$

$= \frac{2 \times \pi \times 3.84 \times 10^8}{27.3 \times 24 \times 60 \times 60}$

$= 1.02 \times 10^3$ m s^{-1}

b $\omega = \frac{360}{27.3 \times 24 \times 60 \times 60}$

$= 1.53 \times 10^{-4}$ °s^{-1}

c $F_c = \frac{mv^2}{r}$

$= \frac{7.36 \times 10^{22} \times (1.02 \times 10^3)^2}{3.84 \times 10^8}$

$= 1.99 \times 10^{20}$ N

17 $\tau_{muscle} = rF_{\perp}$

$= 0.04 \times 500$

$= 20$ N m

$\tau_{object} = rF_{\perp}$

$20 = 0.35 \times mg$

$m = \frac{20}{0.35 \times 9.80}$

$= 5.8$ kg

18 $\tau_{student} = r_{\perp}F = 1.25 \times F_g = 1.25 \times 50 \times 9.8 = 612.5$ N

$\tau_{box} = \tau_{student} = r_{\perp}F = 1.5 \times F_g = 612.5$

$F_g = \frac{612.5}{1.5} = 408.3 = 410$ N (to two significant figures)

$m = \frac{408.3}{9.8} = 41.67 = 42$ kg (to two significant figures)

19 Length is a scalar quantity so no direction is required.

$\tau = rF_{\perp}$

$15 = r \times 40$

$r = 0.38$ m

20 Objects undergo circular motion when the net force acting on the object is radially inward with a magnitude of $F_{net} = F_c = \frac{mv^2}{r}$. In simple situations, this is created with either a normal reaction force (part 1 of the activity), or the tension force in a string (part 3 of the activity). In more complex situations this force is created by adding all the forces acting on the object.

The velocity of objects travelling in a circle is tangential to the circle. This is because all objects have inertia, and when there is no unbalanced force being applied it will continue to travel in a straight line with a constant velocity.

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Chapter 4 Motion in gravitational fields

4.1 Gravity

Worked example: Try yourself 4.1.1

GRAVITATIONAL ATTRACTION BETWEEN SMALL OBJECTS

Two bowling balls are sitting next to each other on a shelf so that the centres of the balls are 60 cm apart. Ball 1 has a mass of 7.0 kg and ball 2 has a mass of 5.5 kg. Calculate the force of gravitational attraction between them.

Thinking	Working
Recall the formula for Newton's law of universal gravitation.	$F = \frac{GMm}{r^2}$
Identify the information required, and convert values into appropriate units when necessary.	$M = 7.0\ \text{kg}$ $m = 5.5\ \text{kg}$ $r = 60\ \text{cm} = 0.60\ \text{m}$ between the two balls $G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$
Substitute the values into the equation.	$F = 6.67 \times 10^{-11} \times \frac{7.0 \times 5.5}{0.60^2}$
Solve the equation.	$F = 7.1 \times 10^{-9}$ N towards one another.

Worked example: Try Yourself 4.1.2

GRAVITATIONAL ATTRACTION BETWEEN MASSIVE OBJECTS

Calculate the force of gravitational attraction between the Earth and the Moon, given the following data:

$m_{Earth} = 6.0 \times 10^{24}\ \text{kg}$

$m_{Moon} = 7.3 \times 10^{22}\ \text{kg}$

$r_{Moon–Earth} = 3.8 \times 10^{8}\ \text{m}$

Thinking	Working
Recall the formula for Newton's law of universal gravitation.	$F = \frac{GMm}{r^2}$
Identify the information required.	$M = 6.0 \times 10^{24}\ \text{kg}$ $m = 7.3 \times 10^{22}\ \text{kg}$ $r = 3.8 \times 10^{8}\ \text{m}$ $G = 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$
Substitute the values into the equation.	$F = 6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24} \times 7.3 \times 10^{22}}{(3.8 \times 10^{8})^2}$
Solve the equation.	$F = 2.0 \times 10^{20}$ N between the Earth and the Moon

Worked example: Try yourself 4.1.3

ACCELERATION CAUSED BY A GRAVITATIONAL FORCE

The force of gravitational attraction between the Sun and the Earth is approximately 3.6×10^{22} N. Calculate the accelerations of the Earth and the Sun caused by this attraction. Compare these accelerations by calculating the ratio $\frac{a_{Earth}}{a_{Sun}}$.

Use the following data:

$m_{Earth} = 6.0 \times 10^{24}\ \text{kg}$

$m_{Sun} = 2.0 \times 10^{30}\ \text{kg}$

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Thinking	Working
Recall the formula for Newton's second law of motion.	$\vec{F}_{net} = m\vec{a}$
Transpose the equation to make $\vec{a}$ the subject.	$\vec{a} = \frac{\vec{F}_{net}}{m}$
Substitute values into this equation to find the accelerations of the Earth and the Sun.	$a_{Earth} = \frac{3.6\times10^{22}}{6.0\times10^{24}} = 6.0 \times 10^{-3}\,N\,kg^{-1}$ $a_{Sun} = \frac{3.6\times10^{22}}{2.0\times10^{30}} = 1.8 \times 10^{-8}\,N\,kg^{-1}$
Compare the two accelerations.	$\frac{a_{Earth}}{a_{Sun}} = \frac{6.0\times10^{-3}}{1.8\times10^{-8}} = 3.3 \times 10^{5}$ The acceleration of the Earth is 3.3×10^5 times greater than the acceleration of the Sun.

Worked example: Try yourself 4.1.4

CALCULATING GRAVITATIONAL FIELD STRENGTH AT DIFFERENT ALTITUDES

Commercial airlines typically fly at an altitude of 11000 m. Calculate the gravitational field strength of the Earth at this height using the following data:

$r_{Earth} = 6.38 \times 10^6\,m$

$m_{Earth} = 5.97 \times 10^{24}\,kg$

Thinking	Working
Recall the formula for gravitational field strength.	$g = \frac{GM}{r^2}$
Add the altitude of the plane to the radius of the Earth.	$r = 6.38 \times 10^6 + 11000\,m$ $= 6.391 \times 10^6\,m$
Substitute the values into the formula.	$g = \frac{GM}{r^2}$ $= 6.67 \times 10^{-11} \times \frac{5.97\times10^{24}}{(6.391\times10^6)^2}$ $= 9.75\,N\,kg^{-1}$ towards the centre of the Earth

Worked example: Try yourself 4.1.5

GRAVITATIONAL FIELD STRENGTH ON ANOTHER PLANET OR MOON

Calculate the strength of the gravitational field on the surface of Mars.

$m_{Mars} = 6.42 \times 10^{23}\,kg$

$r_{Mars} = 3390\,km$

Give your answer correct to three significant figures.

Thinking	Working
Recall the formula for gravitational field strength.	$g = \frac{GM}{r^2}$
Convert Mars's radius to m.	$r = 3390\,km$ $= 3.39 \times 10^6\,m$
Substitute values into the formula.	$g = \frac{GM}{r^2}$ $= 6.67 \times 10^{-11} \times \frac{6.42\times10^{23}}{(3.39\times10^6)^2}$ $= 3.73\,N\,kg^{-1}$ towards the centre of Mars

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4.1 Review

1 The force of attraction between any two bodies in the universe is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

2 $F_g = \frac{GMm}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{2.0\times10^{30}\times6.4\times10^{23}}{(2.2\times10^{11})^2} = 1.8 \times 10^{21}$ N (attractive)

3 $F_g = m_{Mars}a_{Mars}$

$1.8 \times 10^{21} = 6.4 \times 10^{23} \times a_{Mars}$

$a_{Mars} = \frac{1.8\times10^{21}}{6.4\times10^{23}}$

$= 2.8 \times 10^{-3}$ m s^{-2} (towards the Sun)

4 **a** Note: 1 million km = 1×10^6 km = 1×10^9 m

$F_g = \frac{GMm}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{6.0\times10^{24}\times6.4\times10^{23}}{(9.3\times10^{10})^2}$

$= 3.0 \times 10^{16}$ N (attractive)

b $F_g = \frac{GMm}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{2.0\times10^{30}\times6.0\times10^{24}}{(15.3\times10^{10})^2}$

$= 3.4 \times 10^{22}$ N (attractive)

c % comparison $= \frac{3.0\times10^{16}}{3.4\times10^{22}} \times 100 = 0.000088\%$. The Mars–Earth force was 0.000088% of the Sun–Earth force.

5 You can use the scalar values for the force and the distance as no direction is required in the answer.

The distance has been increased three times from 400 km to 1200 km so, in terms of the inverse square law and the original distance, r:

$F \propto \frac{1}{r^2}$

$\propto \frac{1}{(3r)^2}$

$\propto \frac{1}{9(r)^2}$

$\therefore \frac{1}{9}$ of the original

6 $g = \frac{GM}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{1\times10^{13}}{900^2}$

$= 0.0008$ N kg^{-1} or 8×10^{-4} N kg^{-1} (towards the centre of 67P/Churyumov–Gerasimenko)

7 $g = G\frac{M}{r^2}$

Mercury: $g = 6.67 \times 10^{-11} \times \frac{3.30\times10^{23}}{(2.44\times10^6)^2} = 3.7$ N kg^{-1}

Saturn: $g = 6.67 \times 10^{-11} \times \frac{5.69\times10^{26}}{(6.03\times10^7)^2} = 10.4$ N kg^{-1}

Jupiter: $g = 6.67 \times 10^{-11} \times \frac{1.90\times10^{27}}{(7.15\times10^7)^2} = 24.8$ N kg^{-1}

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4.2 Satellite motion

Worked example: Try yourself 4.2.1

WORKING WITH KEPLER'S LAWS

Determine the orbital speed of a satellite, assuming it is in a circular orbit of radius of 42 100 km around the Earth. Take the mass of the Earth to be 5.97×10^{24} kg and use $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

Thinking	Working
Ensure that the variables are in their standard units.	$r = 42\,100\text{ km} = 4.21 \times 10^7\text{ m}$
Choose the appropriate relationship between the orbital speed, v, and the data that has been provided.	$a_c = g = \frac{GM}{r^2} = \frac{v^2}{r}$ $\therefore \frac{GM}{r^2} = \frac{v^2}{r}$ $\therefore \frac{GM}{r} = v^2$
Make v, the orbital speed, the subject of the equation.	$v = \sqrt{\frac{GM}{r}}$
Substitute in values and solve for the orbital speed, v.	$v = \sqrt{\frac{GM}{r}}$ $= \sqrt{\frac{(6.67 \times 10^{-11}) \times (5.97 \times 10^{24})}{4.21 \times 10^7}}$ $= 3.08 \times 10^3\text{ m s}^{-1}$

Worked example: Try yourself 4.2.2

SATELLITES IN ORBIT

Callisto is the second largest of Jupiter's moons. It is about the same size as the planet Mercury. Callisto has a mass of 1.08×10^{23} kg, an orbital radius of 1.88×10^6 km and an orbital period of 1.44×10^6 s (16.7 days).

a Use Kepler's third law to calculate the orbital radius (in km) of Europa, another moon of Jupiter, which has an orbital period of 3.55 days.	
Thinking	**Working**
Note down the values for the known satellite. You can work in days and km.	Callisto: $r = 1.88 \times 10^6$ km $T = 16.7$ days
$\frac{r^3}{T^2}$ = constant for all satellites of a central mass. Work out this ratio for the known satellite.	$\frac{r^3}{T^2}$ = constant $= \frac{(1.88 \times 10^6)^3}{16.7^2}$ $= 2.38 \times 10^{16}$
Use this constant value with the ratio for the satellite in question.	Europa: $\frac{r^3}{T^2}$ = constant $\frac{r^3}{3.55^2} = 2.38 \times 10^{16}$
Make r^3 the subject of the equation.	$r^3 = 3.55^2 \times 2.38 \times 10^{16}$ $= 3.00 \times 10^{17}$
Solve for r.	$r = \sqrt[3]{3.00 \times 10^{17}}$ $= 6.70 \times 10^5$ km Europa has a shorter period than Callisto so you should expect Europa to have a smaller orbit than Callisto.

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b Use the orbital data for Callisto to calculate the mass of Jupiter.	
Thinking	**Working**
Note down the values for the known satellite. You must work in SI units.	Callisto/Jupiter: $r = 1.88 \times 10^9$ m $T = 1.44 \times 10^6$ s $m = 1.66 \times 10^{23}$ kg $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} $M = ?$
Select the expressions from the equation for centripetal acceleration that best suit your data. $a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{GM}{r^2} = g$	$\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2}$ These two expressions use the given variables r and T, and the constant G, so that a solution may be found for M.
Transpose the equation to make M the subject.	$M = \frac{4\pi^2 r^3}{GT^2}$
Substitute values and solve.	$M = \frac{4\pi^2(1.88 \times 10^9)^3}{6.67 \times 10^{-11} \times (1.44 \times 10^6)^2}$ $= 1.90 \times 10^{27}$ kg

c Calculate the orbital speed of Callisto in km s^{-1}.	
Thinking	**Working**
Note values you will need to use in the equation $v = \frac{2\pi r}{T}$.	Callisto: $r = 1.88 \times 10^6$ km $T = 1.44 \times 10^6$ s $v = ?$
Substitute values and solve. The answer will be in km s^{-1} if r is expressed in km.	$v = \frac{2\pi r}{T}$ $= \frac{2\pi \times 1.88 \times 10^6}{1.44 \times 10^6}$ $= 8.20$ km s^{-1}

4.2 Review

1 C. Satellites orbit around a central mass. The Earth does not orbit Mars. The Moon does not orbit the Sun and the Sun does not orbit the Earth.

2 B. In order to be geostationary, the satellite must be in a high orbit.

3 **a** $a = g = 0.22$ m s^{-2}

b $\vec{F}_{net} = \vec{F}_g = m\vec{g}$

$= 2.3 \times 10^3 \times 0.22$

$= 506$

$= 510$ N, towards the Earth (to two significant figures)

4 $\frac{r^3}{T^2}$ = constant for satellites of Saturn, therefore the orbital period for each moon can be calculated.

For Atlas:

$\frac{r^3}{T^2} = \frac{(1.37 \times 10^5)^3}{(0.60)^2}$

$= 7.14 \times 10^{15}$

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For Titan:

$\frac{r^3}{T^2} = 7.14 \times 10^{15}$

$T^2 = \frac{r^3}{7.14 \times 10^{15}}$

$= \frac{(1.20 \times 10^6)^3}{7.14 \times 10^{15}}$

$= 242$

$T = \sqrt{242}$

$= 15.6$ days

5 All of these quantities are given as scalars; the direction for g will always be towards the centre of mass.

a $g = G\frac{M}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{((6380 + 2000) \times 10^3)^2}$

$= 5.67\ \text{N kg}^{-1}$

b $g = G\frac{M}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{((6380 + 10000) \times 10^3)^2}$

$= 1.48\ \text{N kg}^{-1}$

c $g = G\frac{M}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{((6380 + 20200) \times 10^3)^2}$

$= 0.564\ \text{N kg}^{-1}$

d $g = G\frac{M}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{5.97 \times 10^{24}}{((6380 + 35786) \times 10^3)^2}$

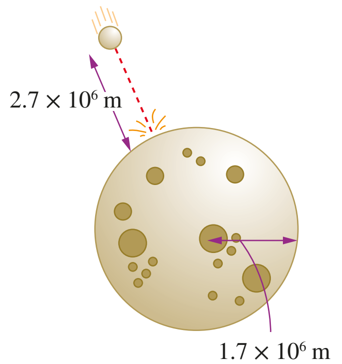
$= 0.224\ \text{N kg}^{-1}$

4.3 Gravitational potential energy

Worked example: Try yourself 4.3.1

GRAVITATIONAL POTENTIAL ENERGY IN A NON-CONSTANT FIELD

A 500 kg lump of space junk is plummeting towards the Moon (see the figure below). The Moon has a radius of 1.7×10^6 m and a mass of 7.3×10^{22} kg. Calculate the gravitational potential energy of the space junk when it is 2.7×10^6 m away from the Moon.



Thinking	Working
Determine the radius of the satellite's orbit.	$r = 1.7 \times 10^6 + 2.7 \times 10^6$ $= 4.4 \times 10^6$ m
Recall the formula for the gravitational potential energy of a satellite.	$U = -\frac{GMm}{r}$
Substitute the values into the formula.	$U = -\frac{6.67 \times 10^{-11} \times 7.3 \times 10^{22} \times 500}{4.4 \times 10^6}$ $= -5.5 \times 10^8$ J

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Worked example: Try yourself 4.3.2

TOTAL ENERGY OF A SATELLITE

Sputnik 1 was the first artificial satellite to be put into orbit. It had a mass of 84.0 kg and orbited the Earth at an altitude of 577 km. (Use $r_{Earth} = 6.38 \times 10^6$ m and $m_{Earth} = 5.97 \times 10^{24}$ kg.)

a Calculate the total mechanical energy of this satellite.

Thinking	Working
Determine the radius of the satellite's orbit.	$r = 577 \times 10^3 + 6.38 \times 10^6$ $= 6957000$ $= 6.96 \times 10^6$ m
Use the definition for total energy: $E = -\frac{GMm}{2r}$	$E = -\frac{GMm}{2r}$ $= -\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 84.0}{2 \times 6.96 \times 10^6}$ $= -2.40 \times 10^9$ J

b Calculate the speed of the satellite.

Thinking	Working
Recall the equation for the kinetic energy of a satellite.	$K = \frac{GMm}{2r}$
Substitute the known values and solve for *K*.	$K = \frac{GMm}{2r}$ $= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 84.0}{2 \times 6.96 \times 10^6}$ $= 2.40 \times 10^9$ J
Remember that the kinetic energy of an object can also be calculated with the equation: $K = \frac{1}{2}mv^2$ Use this to solve for the speed *v*.	$K = \frac{1}{2}mv^2$ $2.40 \times 10^9 = \frac{1}{2} \times 84.0 \times v^2$ $v^2 = 5.72 \times 10^7$ $v = 7570$ m s^{-1}

Worked example: Try yourself 4.3.3

CHANGES IN GRAVITATIONAL POTENTIAL ENERGY

A satellite with a mass of 500 kg is orbiting the Earth with an orbital radius of 7100 km. It moves into a lower orbit at an altitude of 6800 km. Calculate the change in gravitational potential energy of the satellite. (Use $m_{Earth} = 5.97 \times 10^{24}$ kg.)

Thinking	Working
Use the formula for gravitational potential energy to calculate the initial gravitational potential energy, U_i.	$U_i = -\frac{GMm}{r}$ $= -\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 500}{7.1 \times 10^6}$ $= -2.8 \times 10^{10}$ J
Use the formula for gravitational potential energy to calculate the final gravitational potential energy, U_f.	$U_f = -\frac{GMm}{r}$ $= -\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 500}{6.8 \times 10^6}$ $= -2.9 \times 10^{10}$ J
Calculate the change in gravitational potential energy.	$\Delta U = U_f - U_i$ $= -2.9 \times 10^{10} - -2.8 \times 10^{10}$ $= -1 \times 10^9$ J

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Worked example: Try yourself 4.3.4

ESCAPE VELOCITY

Calculate the escape velocity for an 11 900 kg spacecraft being launched from the surface of the Moon. (Use $r_{Moon} = 1.74 \times 10^6$ m and $m_{Moon} = 7.32 \times 10^{22}$ kg.)	
Thinking	**Working**
Recall the definition of the escape velocity.	$v_{esc} = \sqrt{\frac{2GM}{r}}$
Identify the information required, and convert values into appropriate units where necessary.	$G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} $M = 7.32 \times 10^{22}$ kg $r = 1.74 \times 10^6$ m
Substitute the values into the equation and solve for v_{esc}.	$v_{esc} = \sqrt{\frac{2\times 6.67\times10^{-11}\times 7.32\times10^{22}}{1.74\times10^6}}$ $= 2.37 \times 10^3$ m s^{-1}

4.3 Review

1 C. A circular orbit means that its altitude will not change, hence its gravitational potential energy does not change. Its speed will also remain the same in a circular orbit.

2 **a** g increases from point A to point D.

b The meteor is under the influence of the Earth's gravitational field which will cause it to accelerate at an increasing rate as it approaches the Earth.

c A, B and C are all correct. The total energy of the system does not change.

3 $r = 6.7 \times 10^4 + 6.38 \times 10^6 = 6.45 \times 10^6$ m

$$U = -\frac{6.67\times10^{-11}\times5.97\times10^{24}\times3\times10^6}{6.45\times10^6}$$
$$= -1.85\times10^{14}\text{ J}$$

4 $$U_i = -\frac{6.67\times10^{-11}\times5.97\times10^{24}\times240}{(600\times10^3)+(6.38\times10^6)}$$
$$= -1.37\times10^{10}\text{ J}$$

$$U_f = -\frac{6.67\times10^{-11}\times5.97\times10^{24}\times240}{8000\times10^3}$$
$$= -1.19\times10^{10}\text{ J}$$

$$\Delta U = U_f - U_i = (-1.19\ -\ -1.37)\times10^{10}$$
$$= 1\ 8\times10^9\text{ J}$$

5 $$v_{esc} = \sqrt{\frac{2\times6.67\times10^{-11}\times6.42\times10^{23}}{3.39\times10^6}}$$
$$= 5.03\times10^3\text{ m s}^{-1}$$
$$= 5.03\text{ km s}^{-1}$$

CHAPTER 4 REVIEW

1 $$F = \frac{GMm}{r^2}$$
$$= 6.67\times10^{-11}\times\frac{6.0\times10^{24}\times75}{(6.4\times10^6)^2}$$
$$= 730\text{ N (attractive)}$$

2 $$F = \frac{GMm}{r^2}$$
$$2.79\times10^{20} = 6.67\times10^{-11}\times\frac{1.05\times10^{21}\times5.69\times10^{26}}{r^2}$$
$$r^2 = \frac{6.67\times10^{-11}\times1.05\times10^{21}\times5.69\times10^{26}}{2.79\times10^{20}}$$
$$r = 378\,000\,000\text{ m}$$
$$= 3.78\times10^8\text{ m}$$

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3 $F = ma_{Sun}$

$a_{Sun} = \frac{F}{m}$

$= \frac{4.2 \times 10^{23}}{2.0 \times 10^{30}}$

$a = 2.1 \times 10^{-7}\,\text{m s}^{-2}$

4 The Moon has a smaller mass than the Earth and therefore experiences a larger acceleration from the same gravitational force.

5 $g = \frac{GM}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{6.4 \times 10^{23}}{(3400000)^2}$

$= 3.7\,\text{m s}^{-2}$ towards the centre of Mars.

6 **a** $F = \frac{GMm}{r^2}$

$= \frac{(6.67 \times 10^{-11} \times 1.9 \times 10^{27} \times 1000)}{(7.15 \times 10^7)^2}$

$= 2.48 \times 10^4\,\text{N}$

b The magnitude of the gravitational force that the comet exerts on Jupiter is equal to the magnitude of the gravitational force that Jupiter exerts on the comet $= 2.48 \times 10^4\,\text{N}$.

c $a_{comet} = \frac{F_g}{m}$

$= \frac{2.48 \times 10^4}{1000}$

$= 24.8\,\text{m s}^{-2}$

d $a_{Jupiter} = \frac{F_g}{m}$

$= \frac{2.48 \times 10^4}{1.90 \times 10^{27}}$

$= 1.31 \times 10^{-23}\,\text{m s}^{-2}$

7 C. At a height of two Earth radii above the Earth's surface, a person is a distance of three Earth radii from the centre of the Earth.

Then $F = \frac{900}{3^2} = \frac{900}{9} = 100\,\text{N}$

8 $F_g = G\frac{m_1 m_2}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{6.4 \times 10^{23} \times 65}{(3.4 \times 10^6)^2}$

$= 240\,\text{N}$

9 **a** D. $F_{net} = F_N + F_g$

$F_N = 80 \times 30 + 80 \times 9.8 = 3184\,\text{N}$ or $3200\,\text{N}$

b B. From part (a), the force acting on the astronaut is greater than the weight of the astronaut.

c B. Although the astronaut is in free fall during orbit, gravity still exerts a force: $F_g = mg = 80 \times 8.2 = 656\,\text{N}$ or $660\,\text{N}$.

10 When representing a gravitational field with a field diagram, the direction of the arrowhead indicates the *direction* of the gravitational force and the space between the arrows indicates the *magnitude* of the field. In gravitational fields, the field lines always point towards the sources of the field.

11 $g = \frac{F_g}{m} = \frac{1.4}{0.15} = 9.3\,\text{N kg}^{-1}$

12 **a** $g = \frac{Gm_{Earth}}{(r_{Earth})^2}$

$= \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6378 \times 1000)^2}$

$= 9.79\,\text{N kg}^{-1}$

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b $g = \frac{Gm_{Earth}}{(r_{Earth})^2}$

$= \frac{6.67\times10^{-11}\times5.97\times10^{24}}{(6357\times1000)^2}$

$= 9.85\,N\,kg^{-1}$

$\% = \frac{9.85}{9.79} \times 100 = 100.7\%$

Therefore, the gravitational field is 0.7% stronger at the North Pole than the equator.

13 $G\frac{M}{(0.8R)^2} = G\frac{m}{(0.2R)^2}$

$\frac{M}{0.64} = \frac{m}{0.04}$

$\frac{M}{m} = \frac{0.64}{0.04} = 16$

14 $N\,kg^{-1}$

15 $g = \frac{GM}{r^2} = 6.67 \times 10^{-11} \times \frac{3.0\times10^{30}}{(10\times10^3)^2} = 2 \times 10^{12}\,N\,kg^{-1}$

16 $g_{poles} = G\frac{M}{r^2}$

$8 = 6.67 \times 10^{-11} \times \frac{M}{5\,000\,000^2}$

$M = 3 \times 10^{24}\,kg$

$g_{equator} = G\frac{M}{r^2} = 6.67 \times 10^{-11} \times \frac{3\times10^{24}}{6000000^2} = 5.6\,N\,kg^{-1}$

$\frac{8}{5.6} = 1.4$. The gravitational field strength at the poles is 1.4 times that at the equator. (Alternatively, the inverse square law could also be used to find this relationship.)

17 g is proportional to $\frac{1}{r^2}$, so if g becomes $\frac{1}{100}$ of its value, r must become 10 times its value so that $\frac{1}{r^2}$ becomes $\frac{1}{100}$. 10 times r means a distance of 10 Earth radii.

18 B. In order to stay above the same point on the Earth's surface at all times, the satellite must orbit the Earth once every 24 hours.

19 Io

$k = \frac{r^3}{T^2} = \frac{422^3}{42.5^2} = 41600$

Europa

$T = \sqrt{\frac{r^3}{k}} = \sqrt{\frac{671^3}{41600}} = 85.2\,h$

Ganymede

$T = \sqrt{\frac{r^3}{k}} = \sqrt{\frac{1070^3}{41600}} = 172\,h$

Callisto

$T = \sqrt{\frac{r^3}{k}} = \sqrt{\frac{1883^3}{41600}} = 401\,h$

20 $g = \frac{GM}{r^2}$

$= 6.67 \times 10^{-11} \times \frac{5.97\times10^{24}}{((6380+19000)\times10^3)^2}$

$= 0.62\,N\,kg^{-1}$

21 **a** $v = \frac{2\pi r}{T} = \frac{2\pi \times 1.22\times10^9}{15.9\times24\times60\times60} = \frac{7.67\times10^9}{1.37\times10^6} = 5580\,m\,s^{-1}$

b $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$

$\therefore\ M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 \times\ 1.22\times10^9)^3}{6.67\times10^{-11}\times(1.37\times10^6)^2} = 5.69 \times 10^{26}\,kg$

22 **a** $U = -\frac{GMm}{r}$

$= -\frac{6.67\times10^{-11}\times6.0\times10^{24}\times1}{6.5\times10^6}$

$= -6.2\times10^7\,J$

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b $E = K + U = -\frac{GMm}{2r}$

$= -\frac{6.67\times10^{-11}\times6.0\times10^{24}\times1}{2\times6.5\times10^{6}}$

$= -3.1\times10^{7}$ J

c $E = K + U = -\frac{GMm}{2r}$

$\therefore\ r = -\frac{GMm}{2E}$

$= -\frac{6.67\times10^{-11}\times6.0\times10^{24}\times1}{2\times-1\times10^{7}}$

$= 2.0\times10^{7}$ m

23 $U_i = -\frac{6.67\times10^{-11}\times6.0\times10^{24}\times20000}{7.0\times10^{6}}$

$= -1.1\times10^{12}$ J

$U_f = -\frac{6.67\times10^{-11}\times6.0\times10^{24}\times20000}{9.0\times10^{6}}$

$= -8.9\times10^{11}$ J

$\Delta U = U_f - U_i$

$= -8.9\times10^{11} - (-1\ 1\times10^{12})$

$= 2.1\times10^{11}$ J

24 **a** $g = \frac{GM}{r^2} = 6.67\times10^{-11}\times\frac{3.3\times10^{23}}{(3.0\times10^{6}\ ^{2}} = 2.45\ \text{N kg}^{-1}$

b $U = -\frac{GMm}{r}$

$= -\frac{6.67\times10^{-11}\times3.3\times10^{23}\times20}{3.0\times10^{6}}$

$= -1.5\times10^{8}$ J

c $U_f = -\frac{GMm}{r}$

$= -\frac{6.67\times10^{-11}\times3.3\times10^{23}\times20}{2.5\times10^{6}}$

$= -1.8\times10^{8}$ J

$\Delta U = U_f - U_i$

$= -1.8\times10^{8} - (-1\ 5\times10^{8})$

$= -3\times10^{7}$ J

25 The kinetic energy gained is equal to the gravitational potential energy lost.

$U_i = -\frac{6.67\times10^{-11}\times6.0\times10^{24}\times1000}{7.0\times10^{6}}$

$= -5.7\times10^{10}$ J

$U_f = -\frac{6.67\times10^{-11}\times6.0\times10^{24}\times1000}{6.6\times10^{6}}$

$= -6.1\times10^{10}$ J

$\Delta U = U_f - U_i$

$= -6.1\times10^{10} - (-5.7\times10^{10})$

$= -4\times10^{9}$ J

Therefore, the satellite will gain 4×10^{9} J of kinetic energy.

26 $v_{esc} = \sqrt{\frac{2\times6.67\times10^{-11}\times3.30\times10^{23}}{2.44\times10^{6}}}$

$= 4250\ \text{m s}^{-1}$

$= 4.25\ \text{km s}^{-1}$

27 Objects undergo circular motion when the net force acting on the object is radially inward with a magnitude of $F_c = \frac{mv^2}{r}$. For objects in orbit (such as a satellite around Earth or a planet around the Sun), the main force acting is the force of gravity.

In the inquiry activity, the launch direction is investigated to find how satellites enter orbit. Initially the launch direction is directly up to get out of the atmosphere and the associated air resistance, but then the rocket turns to establish an orbital path. Newton's cannonball thought experiment can be used to describe how objects enter an orbital path.

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Module 5 Review

Advanced mechanics

MULTIPLE CHOICE

1 C. Action–reaction pairs always act on different objects. One force acts on the floor and the other force acts on the ball. These forces are equal in magnitude as described in Newton's third law.

2 C. The only force acting is the gravitational force.

3 C. During launch the normal force acting on the astronaut will be greater than usual and so the apparent weight will be greater.

4 D. The gravitational force will be constant during the launch.

5 A. In a stable orbit, there is no normal force acting ($F_N = 0$) on the astronaut so they will experience apparent weightlessness.

6 B. In deep space, there are no planets or large masses to exert a force of gravity on the astronaut so they will experience weightlessness since $g = 0$, $F_g = 0$.

7 B. The ball will increase in speed at a constant rate; that is, with constant acceleration.

8 D. At the top of the ride, $F_N < F_g$ so he would feel lighter than usual.

9 C. The maximum effect is achieved if the force applied is perpendicular (at 90°) to the surface of the object.

10 D. The combination of force and the force arm length provides a greater torque than the other options.

11 A. Either of the perpendicular components can be used and not only one or the other. Both aren't necessary.

12 C. There is no net torque about the reference point and therefore rotation does not occur when the object is in rotational equilibrium.

13 B. For an object to experience static equilibrium it must experience both rotational equilibrium and translational equilibrium.

14 D. The period and the mass of the satellite have an inverse square relationship.

$\frac{1}{T^2} \propto M$

$16\frac{1}{T^2} \propto 16M$

$\frac{1}{\left(\frac{T}{4}\right)^2} \propto 16M$

15 C. The maximum distance will be achieved with a 45° launch angle.

16 B.

Horizontally, $u_H = \frac{s_H}{t} = \frac{100}{4.0} = 25\,\text{m s}^{-1}$

To reach maximum height, the time taken is halved.

Vertically,

$v_V = u_V + at$

$0 = u_V - 9.80 \times 2.0$

$u_V = 19.6\,\text{m s}^{-1}$

Using trigonometry,

$\tan\theta = \frac{u_V}{u_H} = \frac{19.6}{25}$

$\theta = 38.1 = 40°$

17 A. $\Delta\theta = \frac{l}{r}$

$= \frac{130}{60}$

$= 2.17$ rad

$\Delta\theta = 2.17 \times \frac{180}{\pi}$

$= 124°$

$\omega = \frac{\Delta\theta}{t} = \frac{124}{2}$

$= 62.1°\,\text{s}^{-1}$

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18 C. $g = \frac{F_g}{m} = \frac{1.4}{0.15} = 9.3\,\text{N kg}^{-1}$

19 B. $F_g = mg$

$= 0.100 \times 9.80$

$= 0.98\,\text{N downwards}$

$\vec{F}_{net} = 0.980 \tan 55$

$= 1.4\,\text{N towards the left}$

20 C. $v = \sqrt{rg \tan\theta}$

$20^2 = 40 \times 9.80 \times \tan\theta$

$\tan\theta = 1.02$

$\theta = 45.58 \approx 45°$

SHORT ANSWER

21 $F = \frac{GMm}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 24 \times 81}{0.72^2}$

$= 2.5 \times 10^{-7}\,\text{N}$

22 Sum of clockwise torques = sum of anticlockwise torques

$9.80\,\text{N kg}^{-1}\,(35\,\text{kg} \times 3.0\,\text{m} + 25\,\text{kg} \times 3.5\,\text{m}) = 9.80\,\text{N kg}^{-1}\,(d \times 42\,\text{kg})$

Thomas should sit 4.6 m from the pivot opposite his siblings.

23 g is proportional to $\frac{1}{r^2}$, so if g becomes $\frac{1}{100}$ of its value, r must become 10 times its value so that $\frac{1}{r^2}$ becomes $\frac{1}{100}$. 10 times r means a distance of 10 Earth radii.

24 $a = g = G\frac{M}{r^2}$

$g = 6.67 \times 10^{-11} \times \frac{3.3 \times 10^{23}}{(2500000)^2}$

$= 3.5\,\text{m s}^{-2}$

25 $g_{poles} = G\frac{M}{r^2}$

$8 = 6.67 \times 10^{-11} \times \frac{M}{5000000^2}$

$M = 3 \times 10^{24}\,\text{kg}$

$g_{equator} = G\frac{M}{r^2} = 6.67 \times 10^{-11} \times \frac{3 \times 10^{24}}{6000000^2} = 5.6\,\text{N kg}^{-1}$

$\frac{8}{5.6} = 1.4$. The gravitational field strength at the poles is 1.4 times that at the equator. (Alternatively, the inverse square law could also be used to find this relationship.)

26 Horizontal velocity:

$v_H = \frac{s}{t} = \frac{200}{15} = 13.3\,\text{m s}^{-1}$

Initial vertical velocity can be found using the time taken to reach maximum height (i.e. half the total time):

$v_V = u_V + at$

$0 = u_V - 1.6 \times 7.5$

$u_V = 12\,\text{m s}^{-1}$

Find the launch angle:

$\tan\theta = \frac{12}{13.3} = 0.9$

$\theta = 41.99 \approx 42°$

Find the magnitude of the initial velocity:

$v^2 = u_V{}^2 + u_H{}^2$

$= 12^2 + 13.3^2 = 321.78$

$v = 17.9$

$= 18\,\text{m s}^{-1}$ at 42° from the horizontal

27 **a** $F = \frac{GMm}{r^2}$

$= \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 1.2 \times 10^4}{(6.73 \times 10^6)^2}$

$= 1.06 \times 10^5\,\text{N}$

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b $T = \sqrt{\frac{4\pi^2 r^3}{GM}}$

$= \sqrt{\frac{4 \times \pi^2 \times (6.73 \times 10^6)^3}{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}}$

$= 5.5 \times 10^3\,\text{s}$

c The mass of the satellite has no effect on its orbital period.

28 **a** Take up as positive.

vertically: $v = 0\,\text{m s}^{-1}$ (at the top), $a = -9.80\,\text{m s}^{-2}$, $t = 1.0\,\text{s}$, $s = ?$

$s = vt - \frac{1}{2}at^2$

$= 0 - 0.5 \times -9.80 \times 1.0^2$

$= 4.9\,\text{m}$

b $9.80\,\text{m s}^{-2}$ down

c horizontally: $u = ?$, $a = 0\,\text{m s}^{-2}$, $t = 2\text{s}$, $s = 8\,\text{m}$

$s = ut + \frac{1}{2}at^2$

$8 = u \times 2$

$u = 4\,\text{m s}^{-1}$

taking up as positive

vertically: $v = 0\,\text{m s}^{-1}$ (at the top), $a = -9.80\,\text{m s}^{-2}$, $t = 1\text{s}$, $u = ?$

$v = u + at$

$0 = u - 9.80 \times 1$

$u = 9.80\,\text{m s}^{-1}$

Use Pythagoras to find the actual speed at launch:

$u = \sqrt{4^2 + 9.80^2}$

$= 10.6\,\text{m s}^{-1}$

29 **a** $a_c = \frac{v^2}{r} = \frac{6^2}{2} = 18\,\text{m s}^{-2}$ up

b $F_N = F_g + F_{net}$

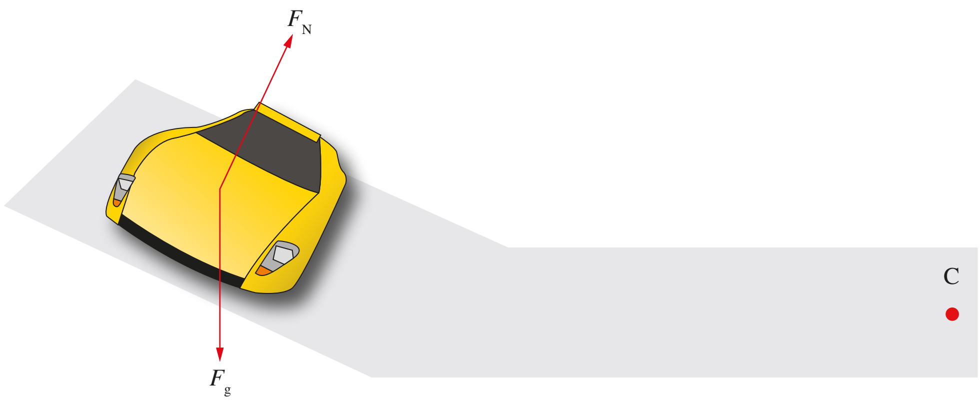
$= mg + ma$

$= 55 \times 9.80 + 55 \times 18$

$= 1.5 \times 10^3\,\text{N}$

c The apparent weight is given by the normal force of 1.5×10^3 N. This is almost three times larger than the weight force and so the skater would feel much heavier than usual.

30 **a**



b $\tan\theta = \frac{v^2}{rg}$

$\theta = \tan^{-1}\frac{v^2}{rg}$

$= \tan^{-1}\left(\frac{40^2}{150 \times 9.80}\right)$

$= 47°$

31 a As the Gravitron spins at a uniform rate and Jodie is pinned to the wall, the horizontal forces acting on her are *unbalanced* and the vertical forces are *balanced*.

b $v = \frac{2\pi r}{T} = \frac{2\pi \times 5}{2.5}$

$= 12.6\,\text{m s}^{-1}$

c $a_c = \frac{v^2}{r} = \frac{12.6^2}{5}$

$= 31.8\,\text{m s}^{-2}$

d $F_N = F_{net} = ma$

$= 60 \times 31.8$

$= 1.9 \times 10^3\,\text{N}$

e $T = \frac{10}{6} = 1.67\,\text{s}$

$f = \frac{1}{T} = \frac{1}{1.67} = 0.6\,\text{Hz}$

32 a The ball bearing just maintains contact with the track so $F_N = 0$, and $F_{net} = F_g$ so $a = 9.80\,\text{m s}^{-2}$ down.

b at point C, $F_N = 0$, so $F_{net} = F_g$

$ma = mg$

$a_c = g$

$\frac{v^2}{r} = g$

$v = \sqrt{rg} = \sqrt{0.5 \times 9.80} = 2.2\,\text{m s}^{-1}$

c apparent weight = F_N and at point C, $F_N = 0$ $\therefore$ apparent weight = 0

d Total energy at point C, $E = K + U_g$

$= \frac{1}{2}mv^2 + mgh$

$= \frac{1}{2} \times 0.025 \times 2.2^2 + 0.025 \times 9.80 \times 1.0$

$= 0.3063\,\text{J}$

Total energy at point B = $K = \frac{1}{2}mv^2 = 0.3063\,\text{J}$

$0.3063 = \frac{1}{2} \times 0.025 v^2$

$v = \sqrt{\frac{2 \times 0.3063}{0.025}}$

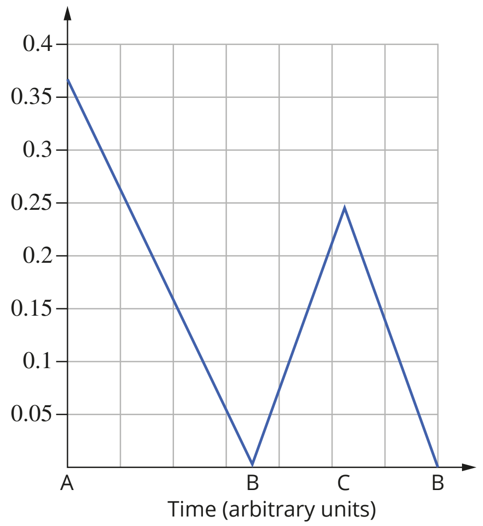
$= 4.9\,\text{m s}^{-1}$

e At the top of the ramp at point A, the ball has some kinetic energy due to its motion and gravitational potential energy due to its height. As it rolls down the ramp, the potential energy is transformed into kinetic energy causing the speed to increase as it hits point B. This added kinetic energy allows the ball to travel up around the loop, again transforming kinetic into potential so that it slows down as it passes point C. The ball then travels out of the system back past point B where the total mechanical energy is now the kinetic energy of the ball.

f At point A: $U = mgh = 0.025 \times 9.80 \times 1.5 = 0.3675\,\text{J}$

At point B: $U = 0\,\text{J}$

At point C: $U = mgh = 0.025 \times 9.80 \times 1.0 = 0.245\,\text{J}$



33 **a** U = area under graph between 7.0×10^6 m and 6.5×10^6 m

Counting squares gives 8.5 squares

area of each square = $1.0 \times 10^4 \times 0.5 \times 10^6 = 5 \times 10^9$ J

$U = 8.5 \times 5 \times 10^9$

$= 4.25 \times 10^{10}$ J

b At 600 km altitude (height of 7000 km): $K = \frac{1}{2}mv^2 = 0.5 \times 10\,000 \times 1500^2 = 1.125 \times 10^{10}$ J

So K at 100 km altitude (height of 6500 km) $= 1.125 \times 10^{10} + 4.25 \times 10^{10} = 5.375 \times 10^{10}$ J

$\frac{1}{2}mv^2 = 5.375 \times 10^{10}$

$0.5 \times 10\,000\,v^2 = 5.375 \times 10^{10}$

$v = 3.3 \times 10^3$ m s^{-1}

c **i** $r = 6400 + 3600 = 10\,000$ km $= 10 \times 10^6$ m

weight = 4.0×10^4 N (from graph)

ii $r = 6.0 \times 10^5 + 6.4 \times 10^6 = 7.0 \times 10^6$ m

weight = 8.1×10^4 N (from graph)

d At 600 km, $F_g = 8.1 \times 10^4$ N, so $a = \frac{F_g}{m} = 8.1$ m s^{-2}

At 100 km, $F_g = 9.2 \times 10^4$ N, so $a = \frac{F_g}{m} = 9.2$ m s^{-2}

The acceleration increases from 8.1 m s^{-2} to 9.2 m s^{-2}.

e Let x be the distance from the centre of the Earth where the Earth's gravity equals the Moon's gravity. Then:

$g_{\text{Earth}} = \frac{6.67\times10^{-11}\times6\times10^{24}}{x^2}$

$g_{\text{Moon}} = \frac{6.67\times10^{-11}\times7.3\times10^{22}}{(3.8\times10^8 - x)^2}$

Equating these two expressions gives:

$\frac{6\times10^{24}}{x^2} = \frac{7.3\times10^{22}}{(3.8\times10^8 - x)^2}$

$\frac{82.2}{x^2} = \frac{1}{(3.8\times10^8 - x)^2}$

Taking square roots of both sides gives:

$\frac{9.07}{x} = \frac{1}{(3.8\times10^8 - x)}$

Inverting both sides gives:

$\frac{x}{9.07} = 3.8 \times 10^8 - x$

$x = 3.45 \times 10^9 - 9.07x$

$10.07x = 3.45 \times 10^9$

$x = 3.4 \times 10^8$ m

34 **a** At 300 km, $g \approx 3.0$ N kg^{-1}

$F_g = mg = 20 \times 3.0 = 60$ N

b Area ≈ 9 squares $= 9 \times 1.0 \times 2.0 \times 10^5 = 1.8 \times 10^6$ J kg^{-1}

ΔK = area × mass $= 1.8 \times 10^6 \times 20 = 3.6 \times 10^7$ J

c Determine the energy associated with each grid square by multiplying each area by the mass of 20 kg. Calculate the altitude at which the total area starting from zero height is equal to 40 MJ.

d On Earth, weight is the gravitational force acting on an object near the Earth's surface, whereas apparent weight is the contact force between the object and the Earth's surface. In many situations, these two forces are equal in magnitude but are in opposite directions. This is because apparent weight is a reaction force to the weight of an object resting on the ground. However, in an elevator accelerating upwards, the apparent weight of an object would be greater than its weight since an additional force would be required to cause the object to accelerate upwards.

35 **a** The maximum height occurs when the vertical velocity is equal to zero: 9 m.

b $v_H = \frac{\text{distance}}{\text{time}} = \frac{18}{3.0} = 6.0$ m s^{-1}

c $\tan\theta = \frac{v_V}{v_H} = \frac{15}{6.0}$

$\theta = 68°$

d $v_V = u_V + at$

$0 = 15 - 9.80t$

$t = 1.53$ s

$s_V = u_V t + \frac{1}{2}at^2$

$= 15 \times 1.53 + \frac{1}{2} \times -9.80 \times 1.53^2$

$= 11$ m

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e First, calculate the magnitude of the original initial velocity:

$v^2 = v_V{}^2 + v_H{}^2$

$= 15^2 + 6^2$

$= 225 + 36$

$= 261$

$v = 16.16\,m\,s^{-1}$

Now, calculate the new initial vertical velocity:

$v_V = v \times \sin\theta$

$= 16.16 \times \sin 50$

$= 12.38$

$= 12\,m\,s^{-1}$

Now, calculate the new initial horizontal velocity:

$v_H = v \times \cos\theta$

$= 16.16 \times \cos 50$

$= 10.38$

$= 10\,m\,s^{-1}$

Calculate the time to the maximum height:

$v_V = u_V + at$

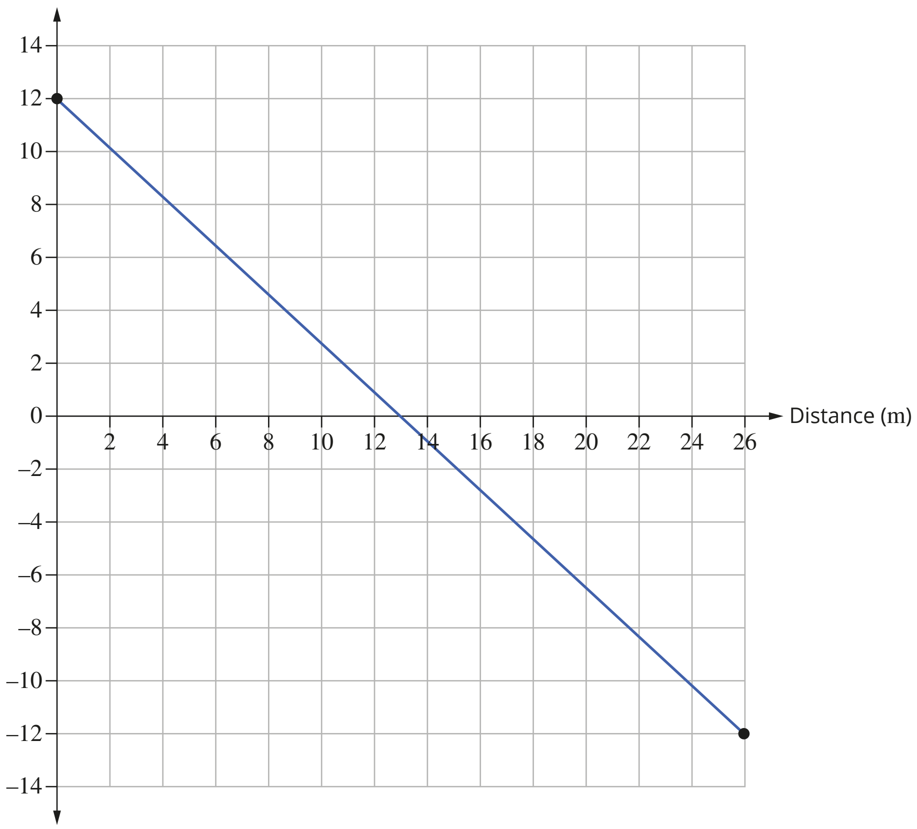
$0 = 12 - 9.80t$

$t = 1.26\,s$

Double this time for the total height, then the total distance travelled can be found using the horizontal velocity.

$s = vt = 10.38 \times 2.53 = 26.23$

$= 26\,m$ (the ball will reach the goal posts)



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Chapter 5 Charged particles, conductors, and electric and magnetic fields

5.1 Particles in electric fields

Worked example: Try yourself 5.1.1

THE ACCELERATION OF A CHARGED PARTICLE

Calculate the magnitude of the acceleration experienced by an electron travelling in a uniform electric field of strength $5 \times 10^{-6}\,\text{N}\,\text{C}^{-1}$.
($q_e = -1.602 \times 10^{-19}\,\text{C}$, $m_e = 9.109 \times 10^{-31}\,\text{kg}$)

Thinking	Working
Recall the formula for the electrostatic force.	$\vec{F} = q\vec{E}$
Substitute the values for electric field strength and the charge of the electron q_e.	$\vec{F} = q\vec{E}$ $= (-1.602 \times 10^{-19}) \times (5 \times 10^{-6})$ $= 8.01 \times 10^{-25}\,\text{N}$ (in the opposite direction to the field)
Recall Newton's second law and solve for a. You are looking for the magnitude of the acceleration so a direction is not needed.	$\vec{F}_{net} = m\vec{a}$ $\vec{a} = \frac{\vec{F}_{net}}{m}$ $= \frac{8.01 \times 10^{-25}}{9.109 \times 10^{-31}}$ $a = 9 \times 10^{5}\,\text{m}\,\text{s}^{-2}$ (to one significant figure)

Worked example: Try yourself 5.1.2

WORK DONE ON A CHARGE IN A UNIFORM ELECTRIC FIELD

A student sets up a parallel plate arrangement so that one plate is at a potential of 36.0 V and the other earthed plate is positioned 2.00 m away. Calculate the work done to move an electron a distance of 75.0 cm towards the negative plate. (Use $q_e = -1.602 \times 10^{-19}\,\text{C}$.)
In your answer identify what does the work and what the work is done on.

Thinking	Working
Identify the variables presented in the problem to calculate the electric field strength E.	$V_2 = 36.0\,\text{V}$ $V_1 = 0\,\text{V}$ $V = 36.0 - 0 = 36.0\,\text{V}$ $d = 2.00\,\text{m}$
Use the equation $E = \frac{V}{d}$ to determine the electric field strength.	$E = \frac{V}{d}$ $= \frac{36.0}{2.00}$ $= 18.0\,\text{V}\,\text{m}^{-1}$
Use the equation $W = qEd$ to determine the work done. Note that d here is the distance that the electron moves.	$W = qEd$ $= 1.602 \times 10^{-19} \times 18.0 \times 0.750$ $= 2.16 \times 10^{-18}\,\text{J}$
Determine if work is done on the charge by the field or if work is done on the field by the charge.	Since the negatively charged electron would normally move away from the negative plate, work is done on the field.

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Worked example: Try yourself 5.1.3

PROJECTILE MOTION OF A CHARGE IN AN ELECTRIC POTENTIAL

Two parallel plates are separated by 2.0 cm. One plate is earthed and the other has a potential of 3.0 kV.

a Determine the final speed of a single electron when accelerating from rest across this potential difference. (Use $q_e = -1.602 \times 10^{-19}$ C and $m_e = 9.1 \times 10^{-31}$ kg.)	
Thinking	**Working**
Write down the variables that are given using appropriate units.	$u = 0\ \text{m s}^{-1}$ $q = 1.602 \times 10^{-19}$ C $m = 9.1 \times 10^{-31}$ kg $V = 5.0 \times 10^3$ V $v = ?$
Recall the electron-gun equation	$\frac{1}{2}mv^2 = qV$
Solve for v.	$v^2 = \frac{2qV}{m}$ $= \frac{2 \times 1.602 \times 10^{-19} \times 3.0 \times 10^3}{9.1 \times 10^{-31}}$ $= 1.1 \times 10^{15}$ $v = 3.3 \times 10^7\ \text{m s}^{-1}$

b Calculate the acceleration of the electron.	
Thinking	**Working**
Find the electric field strength.	$E = \frac{V}{d}$ $= \frac{3.0 \times 10^3}{2.0 \times 10^{-2}}$ $= +1.5 \times 10^5\ \text{V m}^{-1}$
Combine Newton's second law with the electrostatic force.	$\vec{F}_{net} = m\vec{a}$ $a = \frac{F}{m}$ $= \frac{qE}{m}$
Solve for a.	$a = \frac{-1.602 \times 10^{-19} \times 1.5 \times 10^5}{9.1 \times 10^{-31}}$ $= 2.6 \times 10^{16}\ \text{m s}^{-2}$ (towards the charged plate)

c Now assume the initial velocity of the electron was equal to $2.0 \times 10^7\ \text{m s}^{-1}$ travelling horizontally. How long does it take for the electron to hit the positive plate? Use the same direction conventions given in Worked example 5.1.3c.	
Thinking	**Working**
Write down the known quantities. While the horizontal velocity is equal to $5 \times 10^7\ \text{m s}^{-1}$, the initial vertical velocity is equal to zero. Take up and right to be positive.	Vertically, $s = 1.0$ cm $= +0.01$ m (half the distance between plates) $u = 0\ \text{m s}^{-1}$ $a = 2.6 \times 10^{16}\ \text{m s}^{-2}$ $t = ?$
Identify the correction equation to use.	$s = ut + \frac{1}{2}at^2$
Substitute the known values and rearrange the expression so it equals zero.	$0.01 = 0 \times t + \frac{1}{2} \times 2.6 \times 10^{16} \times t^2$ $0 = 1.3 \times 10^{16} \times t^2 + 0t - 0.01$
Use the quadratic equation to solve for t.	$t = \frac{-0 \pm \sqrt{0^2 - 4 \times 1.3 \times 10^1 \times -0.01}}{2 \times 1.3 \times 10^{16}}$ $= 8.8 \times 10^{-10}$ s

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d Assuming the same conditions as for part c, calculate the final horizontal displacement of the electron.	
Thinking	**Working**
Write down the known quantities.	Horizontally, $u = +2.0 \times 10^7\,\text{m s}^{-1}$ $a = 0\,\text{m s}^{-2}$ $t = 8.8 \times 10^{-10}\,\text{s}$ $s = ?$
Identify the correct equation to use.	$s = ut + \frac{1}{2}at^2$
Solve for *s*.	$s = 2.0 \times 10^7 \times 8.8 \times 10^{-10} + \frac{1}{2} \times 0 \times (8.8 \times 10^{-10})^2$ $= +0.176$ $= 1.8\,\text{cm}$ to the right

5.1 Review

1 C. In an electric field, a force is exerted between two charged objects.

2 B. The electric field direction is defined as being the direction that a positively charged test charge moves when placed in the electric field.

3 **a** True. Electric field lines start and end at 90° to the surface, with no gap between the lines and the surface.

b False. Field lines can never cross. If they did it would indicate that the field is in two directions at that point, which can never happen.

c False. Electric fields go from positively charged objects to negatively charged objects.

d True. Around small charged spheres called point charges you should draw at least eight field lines: top, bottom, left, right and in between each of these.

e True. Around point charges the field lines radiate like spokes on a wheel.

f False. Between two point charges the direction of the field at any point is the resultant field vector determined by adding the field vectors due to each of the two point charges.

g False. Between two oppositely charged parallel plates the field between the plates is evenly spaced and is drawn straight from the positive plate to the negative plate.

4 The charge is positive so it travels in the same direction as the field.

$\vec{F} = q\vec{E}$

$= 5.00 \times 10^{-3} \times 2.5$

$= 0.005 \times 2.5$

$= 0.0125$

$= 1.25 \times 10^{-2}\,\text{N}$

5 The charge is negative so it travels against the direction of the field.

$\vec{F} = q\vec{E}$

$= -1.602 \times 10^{-19} \times 3.25$

$= -5.207 \times 10^{-19}\,\text{N}$

and

$\vec{F} = m\vec{a}$

$\vec{a} = \frac{\vec{F}}{m}$

$= \frac{-5.207 \times 10^{-19}}{9.109 \times 10^{-31}}$

$= 5.72 \times 10^{11}\,\text{m s}^{-2}$ (in the opposite direction to the field)

6 $E = \frac{V}{d}$

$4000 = \frac{V}{0.3}$

$V = 4000 \times 0.3 = 1200\,\text{V}$

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7 The field around a monopole is *radial*, *static* and *non-uniform*.

A monopole is a single point source associated with electrical and gravitational fields. The inverse square law applies to the radial fields around monopoles.

8 **a** $E = \frac{V}{d}$

$= \frac{5.0 \times 10^3}{0.012}$

$= 4.2 \times 10^5 \text{ V m}^{-1}$

Using $\vec{F} = q\vec{E}$ and $\vec{F}_{net} = m\vec{a}$,

$q\vec{E} = m\vec{a}$

$\vec{a} = \frac{q\vec{E}}{m}$

$= \frac{-1.6 \times 10^{-19} \times 4.2 \times 10^5}{9.1 \times 10^{-31}}$

$= 7.3 \times 10^{16} \text{ m s}^{-2}$

An electron will move towards the positive plate, so the acceleration is in the opposite direction to the field.

b Take the positive direction to be in the initial horizontal direction and vertically towards the positive plate.

Vertically,

$s = +0.6 \text{ cm} = 0.006 \text{ m}$ (half the vertical distance)

$u = 0 \text{ m s}^{-1}$

$a = 7.3 \times 10^{16} \text{ m s}^{-2}$

$t = ?$

$s = ut + \frac{1}{2}at^2$

$0 = \frac{1}{2}at^2 + ut - s$

$= (3.7 \times 10^{16})t^2 - 0.006$

Using the quadratic formula:

$t = \frac{0 \pm \sqrt{0 - 4 \times 3.7 \times 10^{16} \times -0.006}}{2 \times 3.7 \times 10^{16}}$

$= 4.0 \times 10^{-10} \text{ s}$

5.2 Particles in magnetic fields

Worked example: Try yourself 5.2.1

MAGNITUDE OF FORCE ON A POSITIVELY CHARGED PARTICLE

A single positively charged particle with a charge of $+1.6 \times 10^{-19}$ C travels at a velocity of 50 m s^{-1} perpendicular to a magnetic field of strength 6.0×10^{-5} T.

What is the magnitude of the force the particle will experience from the magnetic field?

Thinking	Working
Establish which quantities are known and which ones are required. All variables are given as scalars as you are looking for the magnitude of the force.	$F = ?$ $q = +1.6 \times 10^{-19}$ C $v = 50 \text{ m s}^{-1}$ $B = 6.0 \times 10^{-5}$ T $\theta = 90°$ as the particle is travelling perpendicular to the magnetic field
Substitute values into the force equation.	$F = qv_{\perp}B = qvB\sin\theta$ $= 1.6 \times 10^{-19} \times 50 \times 6.0 \times 10^{-5} \times \sin 90$
Express the final answer in an appropriate form. Note that only magnitude has been requested so do not include direction.	$F = 4.8 \times 10^{-22}$ N

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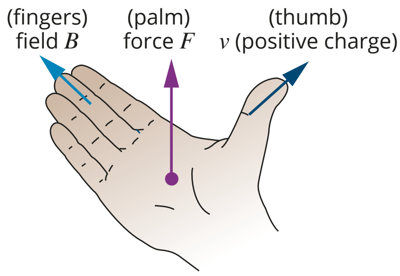
Worked example: Try yourself 5.2.2

DIRECTION OF FORCE ON A CHARGED PARTICLE

A single positively charged particle with a charge of $+1.6 \times 10^{-19}$ C is travelling horizontally from left to right across a computer screen and perpendicular to a magnetic field that runs vertically down the screen. In what direction will the force experienced by the charge act?

Thinking	Working
The right-hand rule is used to determine the direction of the force on a positively charged particle.	Align your hand so that your fingers are pointing downwards in the direction of the magnetic field. The positively charged particle is travelling from left to right across the computer screen. Align your thumb so it is pointing right, in the direction that a positive charge would travel. Your palm is facing into the screen, which is the direction of the force applied by the magnetic field on the positive charge.



Worked example: Try yourself 5.2.3

CALCULATING SPEED AND PATH RADIUS OF ACCELERATED CHARGED PARTICLES

An electron gun releases electrons from its cathode which are accelerated across a potential difference of 25 kV, over a distance of 20 cm between a pair of charged parallel plates. Assume that the mass of an electron is 9.109×10^{-31} kg and the magnitude of the charge on an electron is 1.602×10^{-19} C.

a Calculate the strength of the electric field acting on the electron beam.

Thinking	Working
Ensure that the variables are in their standard units.	$25\text{ kV} = 25 \times 10^3 = 2.5 \times 10^4\text{ V}$ $20\text{ cm} = 0.20\text{ m}$
Apply the correct equation.	$E = \frac{V}{d}$
Solve for E. A direction convention is not specified so assume the electric field is in the positive direction.	$E = \frac{2.5 \times 10^4}{0.20}$ $= 1.3 \times 10^5\text{ V m}^{-1}$

b Calculate the speed of the electrons as they exit the electron-gun assembly.

Thinking	Working
Apply the correct equation.	$\frac{1}{2}mv^2 = qV$
Rearrange the equation to make v the subject.	$v = \sqrt{\frac{2qV}{m}}$
Solve for v.	$v = \sqrt{\frac{2 \times 1.602 \times 10^{-19} \times 2.5 \times 10^4}{9.109 \times 10^{-31}}}$ $= 9.4 \times 10^7\text{ m s}^{-1}$

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c The electrons then travel through a uniform magnetic field perpendicular to their motion. Given that this field is of strength 0.3 T, calculate the expected radius of the path of the electron beam.	
Thinking	**Working**
Apply the correct equation.	$r = \frac{mv}{qB}$
Solve for *r*.	$r = \frac{9.109 \times 10^{-} \times 9.4 \times 10^{7}}{1.602 \times 10^{-} \times 0.3}$ $= 1.8 \times 10^{-3}\,\text{m}$

5.2 Review

1 B. A charged particle moving in a magnetic field will experience a force.

2 Remember, the direction of the charge for the right-hand rule is for a positive charge. When figuring out the direction of the force, your thumb should point west, i.e. in the opposite direction to the velocity.

$F = qvB\sin\theta$

$= 1.6 \times 10^{-19} \times 1.0 \times 1.5 \times 10^{-5} \times \sin 90$

$= 2.4 \times 10^{-24}$ south

3 **a** $F = qvB\sin\theta$

$= 1.6 \times 10^{-19} \times 7.0 \times 10^{6} \times 8.6 \times 10^{-3} \times \sin 90$

$F = 9.6 \times 10^{-15}\,\text{N}$

b $r = \frac{mv}{qB}$

$= \frac{9.1 \times 10^{-} \times 7.0 \times 10^{6}}{1.6 \times 10^{-} \times 8.6 \times 10^{-3}}$

$= 4.6 \times 10^{-3}\,\text{m}$

4 A charged particle in a magnetic field will experience a force ($F = qv_{\perp}B$).

As force ∝ velocity, the force will increase as the velocity increases. This will continue while the charge remains in the magnetic field, continuously accelerating the charge.

5 D. Orientating the right hand with the fingers pointing right and the thumb pointing inwards in the direction of the motion of the charge, the palm is pointing vertically down.

6 **a** South (S). The palm of the hand will be pointing downwards, indicating that the force will be south based on the compass directions provided.

b The path followed is therefore C.

c While the velocity changes as the particle undergoes circular motion, its speed stays constant. Since *v* is constant and energy is a scalar quantity, the kinetic energy remains constant.

d Path A. The palm of the hand will be pointing upwards, indicating that the force will be north based on the compass directions provided. The particle will curve upwards, as the force changes direction with the changing direction of the negative particle.

e Particles with no charge, e.g. neutrons, could follow path B.

7 D. $F = qvB\sin\theta$

$= 1.6 \times 10^{-19} \times 0.5 \times 2 \times 10^{-5} \times \sin 90$

$= 1.6 \times 10^{-24}\,\text{N}$

Using the directions given and the right-hand rule, the force will be towards you.

8 0 N. The particle will experience a force of zero newton because the particle is moving parallel to the magnetic field. That is, there is no component of the motion perpendicular to the field.

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CHAPTER 5 REVIEW

1 As it is a positive charge, the force will be in the direction of the electric field.

$\vec{F} = q\vec{E} = 3.00 \times 10^{-3} \times 7.5$

$= 0.003 \times 7.5$

$= 0.0225\,\text{N}$

2 C. Electric field lines start and end at 90° to the surface, with no gap between the lines and the surface. Electric field lines go from positively charged objects to negatively charged objects, and field lines can never cross.

3 The electrical potential is defined as the work done per unit charge to move a charge from infinity to a point in the electric field. The electrical potential at infinity is defined as zero. When you have two points in an electric field (*E*) separated by a distance (*d*) that is parallel to the field, the potential difference *V* is then defined as the change in the electrical potential between these two points.

4 $E = \frac{V}{d}$

$1000 = \frac{V}{0.025}$

$V = 1000 \times 0.025$

$= 25\,\text{V}$

5 C. For a uniform electric field, the electric field strength is the same at all points between the plates.

6 When a positively charged particle moves across a potential difference from a positive plate towards an earthed plate, work is done by the *field* on the *charged particle*.

7 Use the work done in a uniform electric field, $W = qEd$, equation to determine the work done on the field.

$W = qEd$

$= 2.5 \times 10^{-18} \times 556 \times 3.0 \times 10^{-3}$

$= 4.2 \times 10^{-18}\,\text{J}$

8 The direction cannot be specified from the information given, but it will be perpendicular to the magnetic field.

The magnetic force exerted on the electron is:

$F = qvB\sin\theta$

$= 1.6 \times 10^{-19} \times 7.0 \times 10^{6} \times 8.6 \times 10^{-3} \times \sin 90$

$= 9.6 \times 10^{-15}\,\text{N}$

9 The magnitude of the force is a scalar variable, so no direction is required.

$E = \frac{V}{d} = \frac{15 \times 10^3}{0.12} = 125\,000\,\text{V}\,\text{m}^{-1}$

$F = qE$

$= 1.6 \times 10^{-19} \times 125\,000$

$= 2.0 \times 10^{-14}\,\text{N}$

10 **a** work done by the field

b no work is done

c work done on the field

d no work is done

e work done on the field

f work done by the field

11 **a** $W = qEd$

$= 3.204 \times 10^{-19} \times 34 \times 0.01$

$= 1.09 \times 10^{-19}\,\text{J}$

b Work is done on the field if the charge is forced to go in a direction it would not naturally go. Alpha particles carry a positive charge. So work is done on the field since a positive charged particle is being moved towards a positive potential.

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12 Recall that kinetic energy gained by the ion (ΔK) is equal to work done (W). Therefore, the velocity can be calculated using the equation $K = \frac{1}{2}mv^2$ when the kinetic energy is known.

K can be calculated in two steps by using the work done on a charge in a uniform electric field equation, $W = qEd$ (or $W = qV$), and the equation to determine the electric field, $E = \frac{V}{d}$.

$E = \frac{V}{d} = \frac{1000}{0.020} = 50\,000\,\text{V m}^{-1}$

$W = qEd = 3 \times 1.602 \times 10^{-19} \times 50\,000 \times 0.020 = 4.806 \times 10^{-16}\,\text{J}$

$K = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 4.806 \times 10^{-16}}{3.27 \times 10^{-25}}}$

$= 5.42 \times 10^4\,\text{m s}^{-1}$

13 Find the weight force of the ball using $\vec{F} = m\vec{g}$. Then substitute this value into the equation $\vec{F} = q\vec{E}$ to calculate the charge.

Take down to be positive.

$\vec{F} = m\vec{g}$

$= 5.00 \times 10^{-3} \times 9.80$

$= 4.9 \times 10^{-2}\,\text{N}$

$= q\vec{E}$

The upward force must be in the opposite direction to the weight force.

$q = \frac{\vec{F}}{\vec{E}} = \frac{-4.9 \times 10^{-2}}{-300.0}$

$= +1.63 \times 10^{-4}\,\text{C}$

The charge must be positive to provide an upward force in the vertically upward field.

14 C. The motion of electrons once discharged from an electron gun can be further controlled by additional electric and magnetic fields. Focusing magnets are also used to control the width of the beam.

15 The magnitude of the force is a scalar quantity so it doesn't require a direction.

$F = qvB\sin\theta$

$= 1.6 \times 10^{-19} \times 30 \times 6.0 \times 10^{-5} \times \sin 30$

$F = 1.4 \times 10^{-22}\,\text{N}$

16 The magnitude of the magnetic field is a scalar quantity so it doesn't require a direction.

$F = qvB\sin\theta$

$1.5 \times 10^{-24} = 1.6 \times 10^{-19} \times 60 \times B \times \sin 50$

$B = 2.0 \times 10^{-7}\,\text{T}$

17 **a** The electron will experience a force at right angles to its motion. This acts upwards in the initial moment and causes the electron to curve in an upward arc from its starting position.

b The radius of the electron path is dependent upon its velocity and the magnitude of the magnetic field that is acting.

18 **a** The strength of the electric field between the charged plates is given by:

$E = \frac{V}{d}$

$= \frac{500}{3.5 \times 10^{-2}}$

$= 1.4 \times 10^4\,\text{V m}^{-1}$

b Because the electrons travel in a straight line, the strength of the electric and magnetic fields must be balanced. You can say that:

$\vec{F}_B = \vec{F}_E$

$qvB = qE$

$v = \frac{E}{B}$ (direction is not required)

$= \frac{1.4 \times 10^4}{1.5 \times 10^{-3}}$

$= 9.3 \times 10^6\,\text{m s}^{-1}$

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19 $F = qvB\sin\theta$

$= 1.6 \times 10^{-19} \times 10 \times 3.0 \times 10^{-5} \times \sin 90$

$= 4.8 \times 10^{-23}\,\text{N}$

20 The inquiry activity looks into the effects on charged particles travelling in electric fields, but students may also describe processes of particles in magnetic fields.

In the activity, when the wool is rubbed on the Styrofoam it transfers electrons onto the Styrofoam, creating a negative charge. The Styrofoam is such a good insulator that the electrons cannot move freely and the top surface remains charged when the pie dish is placed on it. This net negative charge on the Styrofoam repels some of the electrons on the pie dish, which are conducted off when touched with a conductor (in this case, your body). The pie dish is left with a net positive charge. The tinsel is initially neutral in charge, but when it falls and touches the pie dish it becomes positively charged.

The tinsel experiences an electrostatic force (repulsion) when it passes through the electric field of the pie dish. The tinsel can be held stationary in the air if the electrostatic repulsion force is equal to the force of gravity.

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Chapter 6 The motor effect

6.1 Force on a conductor

Worked example: Try yourself 6.1.1

MAGNITUDE OF THE FORCE ON A CURRENT-CARRYING WIRE

Determine the magnitude of the force due to the Earth's magnetic field that acts on a suspended power line running east–west near the equator at the moment it carries a current of 50 A from west to east. Assume that the strength of the Earth's magnetic field at this point is 5.0×10^{-5} T.

Thinking	Working
Check the direction of the conductor and determine whether a force will apply. Forces only apply to the component of the wire perpendicular to the magnetic field.	As the current is running east–west and the Earth's magnetic field runs south–north, the current and the field are at right angles and a force will exist.
Establish what quantities are known and what are required. Since the length of the power line hasn't been supplied, consider the force per unit length (i.e. 1 m).	$I = 50$ A $l = 1.0$ m $B = 5.0 \times 10^{-5}$ T $\theta = 90°$ $F = ?$
Substitute values into the force equation and simplify.	$F = lIB\sin\theta$ $= 1.0 \times 50 \times 5.0 \times 10^{-5} \times 1$ $= 2.5 \times 10^{-3}$ N
Express final answer in an appropriate form with a suitable number of significant figures. Note that only magnitude has been requested; so do not include direction.	$F = 2.5 \times 10^{-3}$ N per metre of power line

Worked example: Try yourself 6.1.2

FORCE AND DIRECTION ON A CURRENT-CARRYING WIRE

Santa's house sits at a point that can be considered the Earth's magnetic North Pole (which behaves like the south pole of a magnet).

Assuming the strength of the Earth's magnetic field at this point is 5.0×10^{-5} T, determine:

a the magnetic force on a 1.0 m length of wire carrying a conventional current of 1.0 A vertically up the outside wall of Santa's house.	
Thinking	**Working**
Identify the known quantities. The direction of the magnetic field at the southern magnetic pole will be almost vertically upwards.	$I = 1.0$ A $l = 1.0$ m $B = 5.0 \times 10^{-5}$ T $\theta = 0°$ (The section of the wire running up the wall of the building will be parallel to the magnetic field, $\vec{B}$) $F = ?$
Substitute into the appropriate equation and simplify.	$F = lIB\sin\theta = 0$ N Since there is no force, it is not necessary to state a direction.

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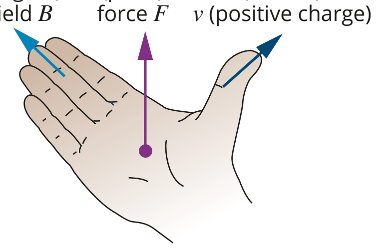
b the magnetic force on a 3.00 m length of wire carrying a conventional current of 15.0 A running horizontally right to left across the outside of Santa's house.	
Identify the known quantities. The direction of the magnetic field at the southern magnetic pole will be almost vertically upwards.	The section of the wire running horizontally through the building will be perpendicular to the magnetic field, *B*. A force *F* with a strength equivalent to *IlB* will apply.
Identify the known quantities.	$I = 15.0\,A$ $l = 3.00\,m$ $B = 5.0 \times 10^{-5}\,T$ $\theta = 90°$ $F = ?$
Substitute into the appropriate equation and simplify.	$F = lIB\sin\theta$ $= 3.00 \times 15.0 \times 5.0 \times 10^{-5} \times \sin 90$ $= 2.25 \times 10^{-3}\,N$
Determine the direction of the magnetic force using the right-hand rule. 	Align your hand so that your fingers are pointing in the direction of the magnetic field, i.e. vertically down. Align your thumb so it is pointing left in the direction of the conventional current i.e. the movement of a positive charge. Your palm should be facing outwards (out from the house). This is the direction of the force applied by the magnetic field on the wire.
State the magnetic force in an appropriate form with a suitable number of significant figures. Include the direction to fully specify the vector quantity.	$F = 2.25 \times 10^{-3}\,N$ outwards

c the magnitude of the magnetic force on a 1.5 m length of wire carrying a conventional current of 2.5 A running at a 30° angle across the exterior of Santa's house.	
Identify the known quantities.	$I = 2.5\,A$ $l = 1.5\,m$ $B = 5.0 \times 10^{-5}\,T$ $\theta = 30°$ $F = ?$
Substitute into the appropriate equation and simplify.	$F = lIB\sin\theta$ $= 1.5 \times 2.5 \times 5.0 \times 10^{-5} \times \sin 30$ $= 9.375 \times 10^{-5}\,N$
State your answer using an appropriate number of significant figures. The magnitude of the force is a scalar quantity which doesn't require a direction.	$F = 9.4 \times 10^{-5}\,N$

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6.1 Review

1 D. Orientating the right hand with the fingers pointing right and the thumb pointing inwards in the direction of the motion of the charge, the palm is pointing vertically down.

2 D. When the conductor is at an angle of 90° (perpendicular) to the magnetic field, the full magnitude of the magnetic force is experienced.

3 Direction: thumb points right (west to east), fingers point into the page (north), palm will face up.

$F = IlB\sin\theta$

$= 100 \times 80 \times 5.0 \times 10^{-5} \times \sin 90$

$= 0.40\,\text{N}$ up

4 $F = IlB\sin\theta$

$= 1.5 \times 5 \times 5.0 \times 10^{-5} \times \sin 30$

$= 1.88 \times 10^{-4}\,\text{N}$. The direction of the force is away from the house.

5 Direction: thumb points into the page, fingers point to the left (west), palm will face up (north).

$F = IlB\sin\theta$

$= 0.05 \times 2.0 \times 2.0 \times 10^{-3} \times \sin 90$

$= 2.0 \times 10^{-4}\,\text{N}$ north

6 **a** Direction: thumb points west, fingers point to the north, palm will face down.

$F = IlB\sin\theta$

$= 80 \times 50 \times 4.5 \times 10^{-5} \sin 90$

$= 0.18$ N downwards

b The same as in (a). The change in height has no effect on the perpendicular components of the magnetic field (south–north) and the wire's direction.

6.2 Forces between conductors

Worked example: Try yourself 6.2.1

FORCES BETWEEN PARALLEL CURRENT-CARRYING CONDUCTORS

Determine the force per unit length acting between two current-carrying conductors, both carrying 5.0 A of current in the same direction, spaced 20 cm apart. Use $\mu_0 = 4\pi \times 10^{-7}\,\text{N A}^{-2}$.

Thinking	Working
Identify the known quantities.	$I_1 = 5.0\,\text{A}$ $I_2 = 5.0\,\text{A}$ $r = 0.20\,\text{m}$ $\mu_0 = 4\pi \times 10^{-7}\,\text{N A}^{-2}$ $F = ?$
Determine if the force is attractive or repulsive.	As both currents are flowing in the same direction, the force between the two conductors would be attractive.
Substitute into the appropriate equation and simplify.	$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$ $= \frac{4\pi \times 10^{-7} \times 5 \times 5}{2\pi \times 0.2}$ $= 2.5 \times 10^{-5}\,\text{N m}^{-1}$ attractive

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Worked example: Try yourself 6.2.2

FORCES BETWEEN PARALLEL CURRENT-CARRYING CONDUCTORS WITH UNEQUAL CURRENTS

Determine the force per unit length acting between two current-carrying conductors, one carrying 20 A of current, the other carrying 15 A of current in the opposite direction, both spaced 5.0 cm apart. Use $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$.

Thinking	Working
Identify the known quantities.	$I_1 = 20\,\mathrm{A}$ $I_2 = 15\,\mathrm{A}$ $r = 0.05\,\mathrm{m}$ $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$ $F = ?$
Determine if the force is attractive or repulsive.	As the currents are flowing in different directions, the force between the two conductors would be repulsive.
Substitute into the appropriate equation and simplify.	$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$ $= \frac{4\pi \times 10^{-7} \times 20 \times 15}{2\pi \times 0.05}$ $= 1.2 \times 10^{-3}\,\mathrm{N\,m^{-1}}$ repulsive

6.2 Review

1 Attractive. If the two parallel conductors carry current in the same direction, the forces attract.

2 C. Recall the force per unit length between the two current-carrying conductors is defined by $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$.

If one current increases by a factor of two and the other decreases by a factor of two, there will be no net change in the force.

3 Direction: because the currents are running in different directions the force will be repulsive.

$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$

$= \frac{4\pi \times 10^{-7} \times 10 \times 5}{2\pi \times 0.02}$

$= 5.0 \times 10^{-4}\,\mathrm{N\,m^{-1}}$ repulsive

4 Direction: because the currents are running in the same direction the force will be attractive.

$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$

$= \frac{4\pi \times 10^{-7} \times 10 \times 10}{2\pi \times 0.1}$

$= 2.0 \times 10^{-4}\,\mathrm{N\,m^{-1}}$ attractive

5 $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$

$1.0 \times 10^{-4} = \frac{4\pi \times 10^{-7} \times I_1 I_2}{2\pi \times 0.2}$

Because $I_1 = I_{2,}$

$I^2 = \frac{1.0 \times 10^{-4} \times 2\pi \times 0.2}{4\pi \times 10^{-7}}$

$= 100$

$I = 10\,\mathrm{A}$

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CHAPTER 6 REVIEW

1 D. The magnitude of the magnetic force on a conductor aligned so that the current is running parallel to a magnetic field is zero.

A component of the conductor's length must be perpendicular to a magnetic field for a force to be created.

2 **a** Attractive. If the two parallel conductors carry current in the same direction, the forces attract.

b Repulsive. If the two parallel conductors carry current in the opposite direction, the forces will repel.

3 The original force was given according to the equation

$$F = k\frac{I_1 I_2}{d} = k\frac{2I \times 3I}{d} = 6k\frac{I^2}{d}$$

The new force will be given by

$$F = k\frac{0.5I_1 \times 0.5I_2}{2d} = k\frac{0.5 \times 2I \times 0.5 \times 3I}{2d} = \frac{3}{4} \times k\frac{I^2}{d} = \frac{3}{4} \times \frac{F}{6} = \frac{1}{8}F$$

4 **a** palm

b fingers

c thumb

5 $F = lIB\sin\theta$

$0.800 = 3.20 \times I \times 0.0900 \times \sin 90$

$$I = \frac{0.800}{0.0900 \times 3.20} = 2.78\,\text{A}$$

6 In each case the force is found from $F = lI_{\perp}B$ as the field is perpendicular to the current.

a $F = 1 \times 10^{-3} \times 1 \times 10^{-3} \times 5 \times 10^{-3}$

$= 5.0 \times 10^{-9}$ N out of the page

b $F = 0.10 \times 2 \times 0.01 = 2.0 \times 10^{-3}$ N out of the page

7 The east–west line would experience the greater magnetic force as it runs perpendicular to the Earth's magnetic field.

8 C. Since the force is $F = lIB\sin\theta$, if the current doubles, the force would double.

9 The current-carrying conductor must be perpendicular to the magnetic field, to utilise the full effect of the magnetic force.

10 Direction: thumb points right (west to east), fingers point into the page (north), palm will face up.

$F = lIB\sin\theta$

$= 200 \times 100 \times 8.0 \times 10^{-5}$

$= 1.6$ N upwards

11 $F = lIB\sin\theta$

$= 3.0 \times 10 \times 5.0 \times 10^{-5} \times \sin 30$

$= 7.5 \times 10^{-4}$ N

12 $F = lIB\sin\theta$

$10 \times 10^{-5} = 10 \times I \times 5.0 \times 10^{-5} \times \sin 90$

$I = 0.2$ A

13 **a** $F = lIB\sin\theta$

$= 0.1 \times 2.0 \times 2.0 \times 10^{-3}$

$= 4.0 \times 10^{-4}$ N north

b $F = lIB\sin\theta$

$= 10 \times 12.0 \times 10.0 \times 10^{-3}$

$= 1.2$ N north

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c $F = lI_{\perp}B$

$5 \times 10^{-2} = 10 \times I \times 5.0 \times 10^{-3}$

$I = 1.0$ A

14 $F = lI_{\perp}B$

$15 \times 10^{-2} = 20 \times 10 \times B$

$B = 7.5 \times 10^{-4}$ T (the magnitude of the field is a scalar variable so no direction is required)

15 $F = lIB\sin\theta$

$= 200 \times 40 \times 7.5 \times 10^{-5}$

$= 0.6$ N downwards

16 B.

Using $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$ if one current decreases by a factor of four and the other decreases by a factor of two, the net force will decrease by a factor of eight.

17 $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$

$= \frac{4\pi \times 10^{-7} \times 10 \times 12}{2\pi \times 0.2}$

$= 1.2 \times 10^{-4}$ N m^{-1} repulsive

18 $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$

$= \frac{4\pi \times 10^{-7} \times 15 \times 15}{2\pi \times 0.5}$

$= 9.0 \times 10^{-5}$ N m^{-1} attractive

19 $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$

$7.5 \times 10^{-4} = \frac{4\pi \times 10^{-7} \times {}^2}{2\pi \times 0.06}$

$I^2 = 225$

$I = 15$ A

20 The force per unit length (1 m) between two parallel current-carrying wires is given by:

$\frac{F}{l} = \frac{\mu_0 I_A I_B}{2\pi r}$

If the conductors are 1 m apart, and each conductor carries 1 A of current, this equation can be simplified to:

$\frac{F}{l} = \frac{\mu_0 \times (1\text{ A}) \times (1\text{ A})}{2\pi \times (1\text{ m})}$

or:

$\frac{F}{l} = \frac{\mu_0}{2\pi} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7}$ N m^{-1}

21 A current carrying wire produces a magnetic field. This magnetic field interacts with an external magnetic field, producing a repulsion force. The force is dependent on the strength of the magnetic field (B), the current (I) the length of the wire in the field (l) and the angle between the magnetic field and the current (θ).

$F = lIB\sin\theta$

The direction of the force produced can be predicted using the right-hand rule.

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Chapter 7 Electromagnetic induction

7.1 Magnetic flux

Worked example: Try yourself 7.1.1

MAGNETIC FLUX

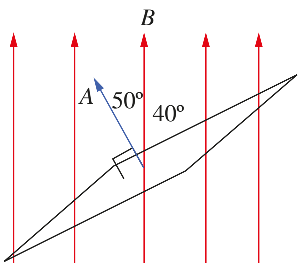
A student places a horizontal square coil of wire of side length 4.0 cm into a uniform vertical magnetic field of 0.050 T. How much magnetic flux ‘threads’ the coil?

Thinking	Working
Calculate the area of the coil perpendicular to the magnetic field.	side length = 4.0 cm = 0.04 m area of the square = $(0.04\,\text{m})^2$ $= 0.0016\,\text{m}^2$
Calculate the magnetic flux.	$\Phi = B_{\parallel}A$ $= 0.050 \times 0.0016$ $= 0.00008\,\text{Wb}$
State the answer in an appropriate form.	$\Phi = 8.0 \times 10^{-5}\,\text{Wb}$

Worked example: Try yourself 7.1.2

MAGNETIC FLUX AT AN ANGLE

A student places a square coil of wire of side length 5.0 cm into a uniform vertical magnetic field of 0.10 T. The plane of the square coil is at an angle of 40° to the magnetic field. How much magnetic flux ‘threads’ the coil?

Thinking	Working
Calculate the area of the coil.	side length = 5.0 cm = 0.05 m area of the square = $(0.0025\,\text{m})^2$ $= 0.0025\,\text{m}^2$
Draw a diagram to calculate the angle θ.	 The plane of the area is 40° to the magnetic field. So the area vector, which is directed normal to the plane, will be at an angle: $\theta = 90 - 40$ $= 50°$
Calculate the magnetic flux.	$\Phi = BA\cos\theta$ $= 0.1 \times 0.0025 \times \cos 50$ $= 0.000161\,\text{Wb}$
State the answer in an appropriate form.	$\Phi = 1.6 \times 10^{-4}$ Wb or 0.16 mWb

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7.1 Review

1 A. There is no change in magnetic flux in this scenario and so there cannot be an induced emf.

2 0 Wb. Since the plane of the coil is parallel to the magnetic field there is no flux passing through the coil.

3 $\Phi = B_{\parallel}A = 2.0 \times 10^{-3} \times 0.04^2 = 3.2 \times 10^{-6}$ Wb

4 The magnetic flux decreases from 3.2×10^{-6} Wb to 0 after one-quarter of a turn. Then it increases again to 3.2×10^{-6} Wb through the opposite side of the loop after half a turn. Then it decreases to 0 again after three-quarters of a turn. After a full turn it is back to 3.2×10^{-6} Wb again.

5 $\Phi = B_{\parallel}A = 1.6 \times 10^{-3} \times \pi \times 0.05^2 = 1.3 \times 10^{-5}$ Wb

6 $\Phi = B_{\parallel}A$

$= 0.10 \times 0.0025$

$= 0.25$ mWb

7 $\theta = 90 - 50$

$= 40°$

$\Phi = B_{\parallel}A = BA\cos\theta$

$= 2.5 \times 10^{-3} \times \pi \times 0.03^2 \times \cos 40$

$= 5.4 \times 10^{-6}$ Wb

7.2 Faraday's and Lenz's laws

Worked example: Try yourself 7.2.1

INDUCED EMF IN A COIL

A student winds a coil of area 50 cm^2 with 10 turns. She places it horizontally in a vertical uniform magnetic field of 0.10 T.

a Calculate the magnetic flux perpendicular to the coil.	
Thinking	**Working**
Identify the quantities to calculate the magnetic flux through the coil and convert to SI units where required.	$\Phi = B_{\parallel}A$ $B = 0.10$ T $A = 50\text{ cm}^2 = 50 \times 10^{-4}\text{ m}^2$
Calculate the magnetic flux and state with appropriate units.	$\Phi = B_{\parallel}A = 0.10 \times 50 \times 10^{-4}$ $= 5.0 \times 10^{-4}$ Wb

b Calculate the magnitude of the average induced emf in the coil when the coil is removed from the magnetic field in a time of 1.0 s.	
Identify the quantities for determining the induced emf.	$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$ $N = 10$ turns $\Delta\Phi = \Phi_2 - \Phi_1$ $= 0 - 5.0 \times 10^{-4}$ = a change of 5.0×10^{-4} Wb $t = 1.0$ s
Calculate the magnitude of the average induced emf, ignoring the negative sign that indicates the direction. Use appropriate units.	$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$ $= 10 \times \frac{5.0 \times 10^{-4}}{1.0}$ $= 5.0 \times 10^{-3}$ V

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Worked example: Try yourself 7.2.2

NUMBER OF TURNS IN A COIL

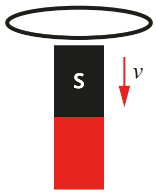
A coil of cross-sectional area $2.0 \times 10^{-3}\,m^2$ experiences a change in the strength of a magnetic field from 0 to 0.20 T over 1.00 s. If the magnitude of the average induced emf is measured as 0.40 V, how many turns must be on the coil?

Thinking	Working
Identify the quantities to calculate the magnetic flux through the coil when in the presence of the magnetic field and convert to SI units where required.	$\Phi = B_{\parallel}A$ $B = 0.20\,T$ $A = 2.0 \times 10^{-3}\,m^2$
Calculate the magnetic flux when in the presence of the magnetic field.	$\Phi = B_{\parallel}A$ $= 0.20 \times 2.0 \times 10^{-3}$ $= 4.0 \times 10^{-4}$ Wb
Identify the quantities from the question required to complete Faraday's law.	$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$ $N = ?$ $\Delta\Phi = \Phi_2 - \Phi_1$ $= 4.0 \times 10^{-4} - 0$ = a change of 4.0×10^{-4} Wb $t = 1.0\,s$ $\varepsilon = 0.40\,V$
Rearrange Faraday's law and solve for the number of turns on the coil. Ignore the negative sign.	$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$ $N = \frac{\varepsilon\Delta t}{\Delta\Phi}$ $= \frac{0.40 \times 1.0}{4.0 \times 10^{-4}}$ = 1000 turns

Worked example: Try yourself 7.2.3

INDUCED CURRENT IN A COIL FROM A PERMANENT MAGNET

The south pole of a magnet is moved downwards away from a horizontal coil held above it. In which direction will the induced current flow in the coil?



Thinking	Working
Consider the direction of the change in magnetic flux.	The magnetic field direction will be downwards towards the south pole. The downward flux from the magnet will decrease as the magnet is moved away from the coil. So the change in flux is decreasing downwards.
What will oppose the change in flux?	The magnetic field that opposes the change would act downwards.
Determine the direction of the induced current required to oppose the change.	In order to oppose the change, the current direction would be clockwise when viewed from above (using the right-hand grip rule).

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Worked example: Try yourself 7.2.4

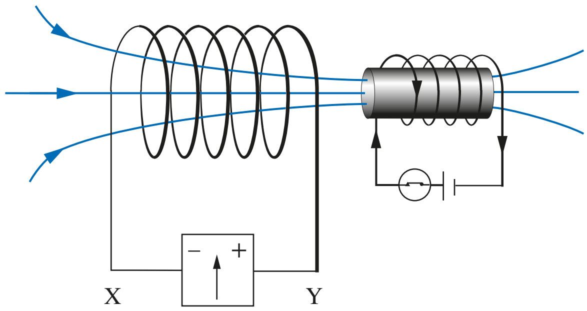
INDUCED CURRENT IN A COIL FROM AN ELECTROMAGNET

What is the direction of the current induced in the solenoid when the electromagnet is:

(i) switched on

(ii) left on

(iii) switched off?



Thinking	Working
Consider the direction of the change in magnetic flux for each case.	**(i)** Initially there is no magnetic flux through the solenoid. When the electromagnet is switched on, the electromagnet creates a magnetic field directed to the right. So the change in flux through the solenoid is increasing to the right. **(ii)** While the current in the electromagnet is steady, the magnetic field is constant and the flux through the solenoid is constant. **(iii)** In this case, initially there is a magnetic field from the electromagnet directed to the right. When the electromagnet is switched off, there is no longer a magnetic field so the change in flux through the solenoid is decreasing to the right.
What will oppose the change in flux for each case?	**(i)** The magnetic field that opposes the change in flux through the solenoid is directed to the left. **(ii)** There is no change in flux and so no opposition is needed and there will be no magnetic field created by the solenoid. **(iii)** The magnetic field that opposes the change in flux through the solenoid is directed to the right.
Determine the direction of the induced current required to oppose the change for each case.	**(i)** In order to oppose the change, the current will flow through the solenoid in the direction from Y to X (through the meter from X to Y), using the right-hand grip rule. **(ii)** There will be no induced emf or current in the solenoid. **(iii)** In order to oppose the change, the current will flow through the solenoid in the direction from X to Y (through the meter from Y to X), using the right-hand grip rule.

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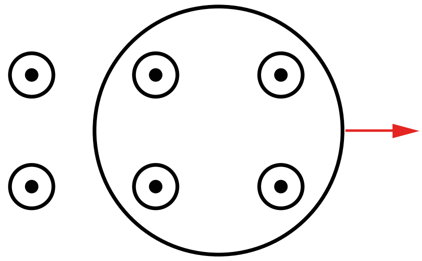
Worked example: Try yourself 7.2.5

FURTHER PRACTICE WITH LENZ'S LAW

A coil is moved to the right and out of a magnetic field that is directed out of the page. In what direction will the induced current flow in the coil while the magnet is moving?



Thinking	Working
Consider the direction of the change in magnetic flux.	Initially, the magnetic flux passes through the full area of the coil and out of the page. Moving the coil out of the field decreases the magnetic flux. So the change in flux is decreasing out of the page.
What will oppose the change in flux?	The magnetic field that opposes the change would act out of the page.
Determine the direction of the induced current required to oppose the change.	In order to oppose the change, the current direction would be anticlockwise (using the right-hand grip rule).

7.2 Review

1 $\Phi = B_{\parallel}A = 2.0 \times 10^{-3} \times 0.02 \times 0.03 = 1.2 \times 10^{-6}$ Wb

2 Zero flux threads the loop when the plane of the loop is parallel to the magnetic field.

3 $\Delta\Phi = 1.2 \times 10^{-6}$ Wb

$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = \frac{1.2 \times 10^{-6}}{0.040} = 3.0 \times 10^{-5}$ V

4 C. The speed of the magnet reduces the time over which the change occurs but there is no change in the strength of the magnetic field or the area of the coil, hence the total flux (area under the curve) is the same.

5 $\Phi = 80 \times 10^{-3} \times 10 \times 10^{-4}$

$= 8 \times 10^{-5}$ Wb

$\varepsilon = -\frac{\Delta\Phi}{\Delta t} = \frac{8 \times 10^{-5}}{0.020} = 4 \times 10^{-3}$ V

6 The effect of using multiple coils is similar to placing cells in series—the emf of each of the coils adds together to produce the total emf.

$\Delta\Phi = 500 \times 4 \times 10^{-3}$ V $= 2$ V

7 C. The magnetic field of the induced current will always oppose the original change.

8 **a** A. When the external magnetic field is switched off this represents a change in flux through the coil that is decreasing out of the page. In order to oppose this change, the induced current will create a magnetic field out of the page.

b A. When the external magnetic field is reversed this represents a change in flux through the coil that is decreasing out of the page, followed by increasing into the page. In order to oppose this change, the induced current will create a magnetic field out of the page.

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7.3 Transformers

Worked example: Try yourself 7.3.1

TRANSFORMER EQUATION—VOLTAGE

A transformer is built into a phone charger to reduce the 240 V supply voltage to the required 6 V for the charger. If the number of turns in the secondary coil is 100, what is the number of turns required in the primary coil?

Thinking	Working
State the relevant quantities given in the question. Choose a form of the transformer equation with the unknown quantity in the top left position.	$V_S = 6\text{ V}$ $V_P = 240\text{ V}$ $N_S = 100$ turns $N_P = ?$ $\frac{N_P}{N_S} = \frac{V_P}{V_S}$
Substitute the quantities into the equation, rearrange and solve for N_P.	$\frac{N_P}{100} = \frac{240}{6}$ $N_P = \frac{100 \times 240}{6}$ $= 4000$ turns

Worked example: Try yourself 7.3.2

TRANSFORMER EQUATION—CURRENT

A phone charger with 4000 turns in the primary coil and 100 turns in its secondary coil draws a current of 0.50 A. What is the current in the primary coil?

Thinking	Working
State the relevant quantities given in the question. Choose a form of the transformer equation with the unknown quantity in the top left position.	$I_S = 0.50\text{ A}$ $N_S = 100$ turns $N_P = 4000$ turns $I_P = ?$ $\frac{I_P}{I_S} = \frac{N_S}{N_P}$
Substitute the quantities into the equation, rearrange and solve for I_P.	$\frac{I_P}{0.50} = \frac{100}{4000}$ $I_P = \frac{0.50 \times 100}{4000}$ $= 0.0125\text{ A} = 0.013\text{ A}$ (to two significant figures)

Worked example: Try yourself 7.3.3

TRANSFORMERS—POWER

The power drawn from the secondary coil of the transformer by a phone charger is 3 W. What power is drawn from the mains supply if the transformer is an ideal transformer?

Thinking	Working
The energy efficiency of a transformer can be assumed to be 100%. The power in the secondary coil will be the same as that in the primary coil.	The power drawn from the mains supply is the power in the primary coil, which will be the same as the power in the secondary coil: $P = 3\text{ W}$

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Worked example: Try yourself 7.3.4

TRANSMISSION-LINE POWER LOSS

300 MW is to be transmitted from the Murray 1 power station in the Snowy Mountains Scheme to Sydney, along a transmission line with a total resistance of 1.0 Ω. What would be the total transmission power loss if the voltage along the line was 500 kV?	
Thinking	**Working**
Convert the values to SI units.	$P = 300\text{ MW} = 300 \times 10^6\text{ W}$ $V = 500\text{ kV} = 500 \times 10^3\text{ V}$
Determine the current in the line based on the required voltage.	$P = VI$ $\therefore I = \frac{P}{V}$ $= \frac{300 \times 10^6}{500 \times 10^3}$ $= 600\text{ A}$
Determine the corresponding power loss.	$P = I^2R$ $= 600^2 \times 1$ $= 3.6 \times 10^5\text{ W or } 0.36\text{ MW}$

Worked example: Try yourself 7.3.5

VOLTAGE DROP ALONG A TRANSMISSION LINE

Power is to be transmitted along a transmission line with a total resistance of 1.0 Ω. The current is 600 A. What voltage would be needed at the power generation end of the transmission line to achieve a supply voltage of 500 kV?	
Thinking	**Working**
Determine the voltage drop along the transmission line.	$\Delta V = IR$ $= 600 \times 1.0$ $= 600\text{ V}$
Determine the initial supply voltage.	$V_{\text{initial}} = V_{\text{supplied}} + \Delta V$ $= 500 \times 10^3 + 600$ $= 500.6\text{ kV}$

7.3 Review

1 B. The power equation is $P = VI$ and the '2' indicates the secondary coil.

2 D. A change in flux through the secondary coil is required for an emf to be induced in the second coil, but a DC input to the primary coil will create a constant flux. Therefore the voltage output is zero.

3 $\frac{N_S}{N_P} = \frac{V_S}{V_P}$

$\frac{N_S}{800} = \frac{12}{240}$

$N_S = \frac{12 \times 800}{240}$

$= 40$ turns

4 **a** In an ideal transformer there should be no power loss, so $P_1 = P_2$.

b $\frac{I_1}{I_2} = \frac{N_2}{N_1}$

5 $I = \frac{P}{V} = \frac{5.0 \times 10^3}{500} = 10\text{ A}$

$P_{\text{loss}} = I^2R = 10^2 \times 4.0 = 400\text{ W}$

6 $I = \frac{P}{V} = \frac{500 \times 10^6}{250 \times 10^3} = 2000\text{ A}$

$P_{\text{loss}} = I^2R = 2000^2 \times 10 = 4 \times 10^7\text{ W or } 40\text{ MW}$

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7 a $I = \frac{P}{V} = \frac{500 \times 10^6}{100 \times 10^3} = 5000\,A$

b $V_{drop} = I \times R$

$V_{drop} = 5000 \times 2 = 10\,000\,V$ or $10\,kV$

$V_{supplied} = 100 - 10 = 90\,kV$

8 B is correct. A is incorrect because the ΔV in the formula indicates the voltage drop in the transmission lines; it does not refer to the voltage being transmitted.

CHAPTER 7 **REVIEW**

1 a B changes from 8.0×10^{-4} T to 16×10^{-4} T which is a change of 8.0×10^{-4} T.

$\Phi = \Delta B \times A = 8.0 \times 10^{-4} \times 40 \times 10^{-4}$

$= 3.2 \times 10^{-6}$ Wb

$\varepsilon = -N\frac{\Delta \Phi}{\Delta t} = \frac{3.2 \times 10^{-6}}{1.0 \times 10^{-3}} = 3.2 \times 10^{-3}\,V$ or $3.2\,mV$

b Clockwise. Doubling the magnetic field strength increases the flux through the coil out of the page. The induced magnetic field will act into the page to oppose the increasing magnetic flux out of the page. Using the right-hand grip rule, the induced current direction is clockwise around the coil.

2 a $\Phi = 20 \times 10^{-3} \times \pi \times 0.04^2$

$= 1 \times 10^{-4}$ Wb

$\varepsilon = -N\frac{\Delta \Phi}{\Delta t} = 40 \times \frac{1 \times 10^{-4}}{0.10} = 0.04\,V$

b From Y to X. As the coil is removed, the magnetic flux through the coil changes from being directed downwards to no magnetic flux. To oppose this change the coil must create a magnetic field that is directed downwards again. Using the right-hand grip rule, this means the current must flow clockwise around the coil when viewed from above.

3 From X to Y. As the rod moves to the right, the area of the loop decreases so the magnetic flux through the loop, which is directed out of the page, decreases. In order to oppose this change the loop will create a magnetic field directed out of the page again. Using the right-hand grip rule, the current will flow through the rod from X to Y.

4 A change in the emf in S_1 produces a current in S_2. So no current flows in S_2 between $t = 1$ s and $t = 4$ s. An increase in emf at a constant rate ($t = 0$ to $t = 1$ s) would produce a constant current, and a decrease in emf at a lower rate ($t = 4$ to $t = 7$ s) would produce a lower current in the opposite direction.

5 $V_p I_p = V_s I_s$

$\frac{I_S}{3.0} = \frac{14}{42}$

$I_S = 1.0\,A$

6 $\frac{N_P}{N_S} = \frac{V_P}{V_S}$

$\frac{N_P}{30} = \frac{14}{42}$

$N_P = 10$

There are 10 turns in the primary coil.

7 A. The spikes in the voltage output occur when the input voltage rises and falls, i.e. when it *changes*.

8 In a quarter of a turn $\Delta\Phi = 80 \times 10^{-3} \times 10 \times 10^{-4} = 8 \times 10^{-5}$ Wb

Frequency is 50 Hz, so a quarter of a turn takes $\frac{1}{4} \times 0.02 = 0.005$ s.

$\varepsilon = -N\frac{\Delta \Phi}{\Delta t} = 500 \times \frac{8 \times 10^{-5}}{0.005}$

$= 8\,V$

9 Doubling the frequency halves the Δt in Faraday's law so it doubles the average emf to 16 V.

10 Any two of:

1. Using a DC power supply means that the voltage cannot be stepped up or down with transformers.
2. There will be significant power loss along the 8 Ω power lines.
3. Damage to any appliances operated in the shed that are designed to operate on 240 V AC and not on 240 V DC.

11 As the coil area is reduced, the flux into the page will decrease. To oppose this the induced current will try to increase the flux again in the same direction. Using the right-hand grip rule the direction of the induced current will be clockwise.

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12 AB and CD. Both the sides AB and CD cut across lines of flux as the coil rotates.

13 $P = VI$

$150 \times 10^3 = 10000 \times I$

$I = \frac{150 \times 10^3}{10000}$

$= 15\,\text{A}$

14 Calculate the voltage drop:

$V = IR$

$= 15 \times 2.0$

$= 30\,\text{V}$

Calculate the final voltage: Initial voltage – voltage drop

$V = 10000 - 30 = 9970\,\text{V}$

15 $P = I^2R$

Using current calculated from Question 13, $I = 15\,\text{A}$

$P = 15^2 \times 2.0$

$= 450\,\text{W}$

16 Without the first transformer, voltage in the transmission lines, $V = 1000\,\text{V}$

Calculate I:

$P = VI$

$150\,\text{kW} = 1000\,I$

$I = \frac{150 \times 10^3}{1000} = 150\,\text{A}$

Power loss in the lines:

$P = I^2R$

$= 150^2 \times 2.0$

$= 45\,\text{kW}$

Power supplied = 150 kW – 45 kW = 105 kW

This represents a 30% power loss—bad idea!

17 Anticlockwise. Initially there is no flux through the coil. As the coil begins to rotate, the amount of flux increases and to the left. To oppose this change, an induced magnetic field will be directed to the right. Using the right-hand grip rule this creates an anticlockwise current in the coil for the orientation shown in the diagram.

18 The student must induce an emf of 1.0 V in the wire by somehow changing the magnetic flux through the coil at an appropriate rate. A change in flux can be achieved by changing the strength of the magnetic field or by changing the area of the coil. The magnetic field can be changed by changing the position of the magnet relative to the coil. The area can be changed by changing the shape of the coil or by rotating the coil relative to the magnetic field.

To calculate the required rate of change of flux to produce 1.0 V:

$\frac{\Delta\Phi}{\Delta t} = \frac{\varepsilon}{N}$

$\frac{\Delta\Phi}{\Delta t} = \frac{1.0}{100} = 0.01\,\text{Wb}\,\text{s}^{-1}$

For example, if the shape was changed from 0.01 m^2 to 0.02 m^2 in a time of 0.1 s, then:

$\frac{\Delta\Phi}{\Delta t} = \frac{(100 \times 10^{-3} \times 0.02 - 00 \times 0^{-3} \times 0.01)}{0.1} = \frac{0.001}{0.1} = 0.01\,\text{Wb}\,\text{s}^{-1}$

19 $\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$

$= -N\frac{\Delta B\,A}{\Delta t}$

$A = \frac{-\varepsilon\Delta t}{N\Delta B}$

$= \frac{0.020 \times 0.050}{1 \times 0.10}$

$= 0.010\,\text{m}^2$

20 $\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$

$\Delta t = -N\frac{\Delta\Phi}{\varepsilon}$

$= 100 \times \frac{0.40 \times 50 \times 10^{-4}}{1600 \times 10^{-3}}$

$= 0.125\,\text{s}$

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21 A change in magnetic flux induces an emf in a conductor. If the conductor is a complete loop, as this one is, a current will result. As the falling magnet enters the wire section, the magnetic flux for the coil is increasing in one direction (depending on the orientation of the magnet). The current induced in the coil will be in the direction to counter the change in flux.

As the magnet falls, the gravitational potential energy is transformed into kinetic energy. When the magnet enters the wire section, some of the kinetic energy is transferred to the wire and transformed into electric potential energy. As the current travels through the wire, the electric potential energy is transformed into light through the LED, and heat.

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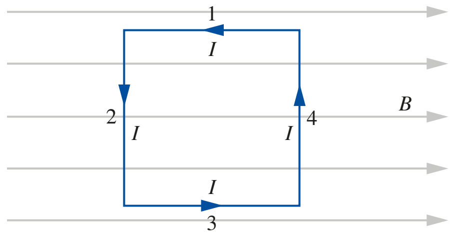
Chapter 8 Applications of the motor effect

8.1 Motors

Worked example: Try yourself 8.1.1

TORQUE ON A COIL

A single square wire coil, with a side length of 4.0 cm, is free to rotate within a magnetic field, *B*, of strength 1.2×10^{-4} T. A current of 1.5 A is flowing through the coil. What is the torque on the coil when it is parallel to the magnetic field? What direction is the coil turning?



Thinking	Working
Confirm that the coil will experience a force based on the magnetic field and current directions supplied.	Using the right-hand rule, confirm that a force applies on side 2 out of the page. A force applies to side 4 into the page. The coil will turn clockwise as viewed from in front of side 3. Sides 1 and 3 lie parallel to the magnetic field and no force will apply.
Identify the variables involved and state them in their standard form. Remember that the angle is between the magnetic field and the area vector which is directed at 90° to the plane of the coil.	$n = 1$ (as there is only one loop in the coil) $B = 1.2 \times 10^{-4}$ T $I = 1.5$ A $A = 0.04 \times 0.04 = 0.0016\ \text{m}^2$ $\theta = 90°$ (the plane of the coil is parallel to the field) $\tau = ?$
Calculate the torque on the coil.	$\tau = nIA_{\perp}B = nIAB\sin\theta$ $= 1.0 \times 1.5 \times 0.0016 \times 1.2 \times 10^{-4} \times \sin 90$ $= 2.9 \times 10^{-7}\ \text{N m}$ The direction is clockwise as viewed from side 3.

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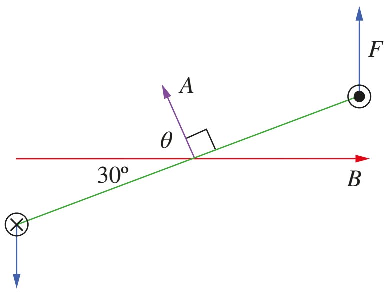
Worked example: Try yourself 8.1.2

TORQUE ON A COIL AT AN ANGLE

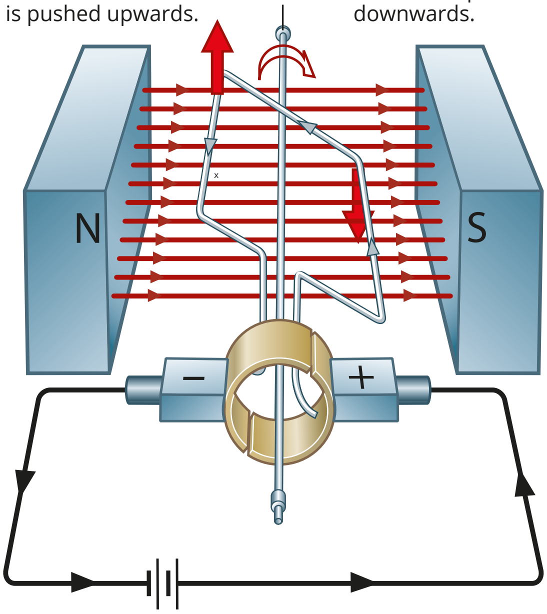
A single square wire coil of side length 5.00 cm is free to rotate within a magnetic field, B, of strength 1.00×10^{-4} T. A current of 2.50 A is flowing through the coil. What is the magnitude of the torque on the coil when it is at an angle of 30.0° to the magnetic field?

Thinking	Working
Determine the angle between the area vector of the coil and the magnetic field. Remember that the angle is between the magnetic field and the area vector which is directed at 90° to the plane of the coil.	 $\theta = 90 - 30$ $= 60°$
Identify the variables involved and state them in their standard form.	$n = 1$ (as there is only one loop in the coil) $B = 1.00 \times 10^{-4}$ T $I = 2.50$ A $A = 0.05 \times 0.05 = 0.0025\ \text{m}^2$ $\theta = 60°$ $\tau = ?$
Calculate the torque on the coil. The magnitude is a scalar quantity so no direction is required.	$\tau = nIA_{\perp}B = nIAB\sin\theta$ $= 1 \times 1.00 \times 10^{-4} \times 2.50 \times 0.0025 \times \sin 60$ $= 5.41 \times 10^{-7}$ N m

8.1 Review

1 A. The maximum torque exists when the force is applied perpendicular to the axis of rotation.

2 The torque is acting clockwise.



3 **a** $F = lI_{\perp}B$

$= 0.05 \times 2.0 \times 0.10$

$= 1.0 \times 10^{-2}$ N, into the page

b $F = lI_{\perp}B$

$= 0.05 \times 2.0 \times 0.10$

$= 1.0 \times 10^{-2}$ N, out of the page

c The force will be 0 N.

Side PQ is parallel to the magnetic field.

d Considering the direction of the forces acting on sides PS and QR, the coil would rotate in an anticlockwise direction.

e D. The direction of the current does not affect the magnitude of the torque. This is the only option that doesn't affect either the distance to the axis of rotation or the magnetic force from the options available.

f $n = 1$ (as there is only one loop in the coil)

$B = 0.1$ T

$A = 0.02\,\text{m} \times 0.05\,\text{m} = 0.001\,\text{m}^2$

$I = 2.0$ A

$\theta = 90°$ (as the plane of the coil is parallel to the magnetic field)

$\tau = nIA_{\perp}B = nIAB\sin\theta$

$= 1 \times 0.1 \times 2.0 \times 0.001 \times \sin 90$

$= 2.0 \times 10^{-4}$ N m

4 In an AC motor, the stator creates a rotating magnetic field that induces a current in the conductors of the rotor. Through application of Faraday's and Lenz's laws, the rotating magnetic field of the stator effectively pulls the rotor.

8.2 Generators

Worked example: Try yourself 8.2.1

PEAK AND RMS AC CURRENT VALUES

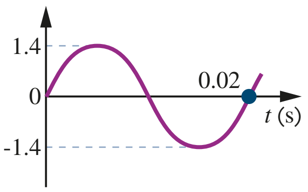
A 1000 W kettle is connected to a 240 V AC power outlet. What is the peak power use of the kettle?

Thinking	Working
Note that the values given in the question represent rms values. Power is $P = VI$ so both V and I must be known to calculate the power use. The voltage V is given, and the current I can be calculated from the rms power supplied.	$P_{rms} = V_{rms} I_{rms}$ $I_{rms} = \frac{P_{rms}}{V_{rms}}$ $= \frac{1000}{240}$ $= 4.17$ A
Substitute in known quantities and solve for peak power.	$P_p = \sqrt{2}\,V_{rms} \times \sqrt{2}\,I_{rms} = 2V_{rms}I_{rms}$ $= 2 \times V_{rms} \times I_{rms}$ $= 2 \times 240 \times 4.17$ $= 2000$ W

8.2 Review

1 B. Applying Lenz's law, the back emf opposes the change in magnetic flux that created it, so the induced back emf will be in the opposite direction to the emf creating it. The net emf used by the motor is then less than the supplied voltage.

2



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3 $V_{rms} = \frac{10}{\sqrt{2}} = 7.07\,V$

$I_{rms} = \frac{6}{\sqrt{2}} = 4.24\,V$

$P_{rms} = V_{rms} \times I_{rms}$

$= 7.07 \times 4.24 = 30\,W$

4 $I_{rms} = \frac{P_{rms}}{V_{rms}} = \frac{600}{240} = 2.5\,A$

$I_p = \sqrt{2} \times I_{rms} = \sqrt{2} \times 2.50 = 3.54\,A$

5 The resulting output of all three phases maintains an emf near the maximum voltage more continuously.

6 $P_{rms} = V_{rms} I_{rms}$

$rms = \frac{P_{rms}}{V_{rms}}$

$= \frac{45}{240} = 0.1875\,A$

$P_p = \sqrt{2} V_{rms} \times \sqrt{2} I_{rms} = 2V_{rms} I_{rms}$

$= 2 \times V_{rms} \times I_{rms}$

$= 2 \times 240 \times 0.1875$

$= 90\,W$

CHAPTER 8 REVIEW

1 **a** down the page
b up the page

2 anticlockwise

3 **a** down the page
b up the page
c Zero torque acts as the forces are trying to pull the coil apart rather than turn it. The force is parallel to the coil, rather than perpendicular to it.

4 C. Providing the coil still has a little momentum, reversing the direction of the current in the loop will ensure that the loop keeps travelling in the same direction. Use the right-hand rule to verify this.

5 The commutator's function is to reverse the current direction in the coil every half turn to keep the coil rotating in the same direction.

6 The magnitude of the force is a scalar quantity.

$F = lI_{\perp}B$

$= 1 \times 1.0 \times 0.50 \times 0.20$

$= 0.1\,N$

7 Current flows into brush P and around the coil from V to X to Y to W. The force on side VX is down and the force on side YW is up, so rotation is anticlockwise.

8 D. As $F = lI_{\perp}B$, the coil will experience more force, and rotate faster, if the current and field strength are increased. Therefore, A and B are correct. Whether C is correct will depend on how the area is increased. But if the length in the field is increased, you would expect it to turn faster. If widened, it will experience more torque but that may not make it turn faster.

9 **a** $V_{rms} = \frac{V_p}{\sqrt{2}}$

$= \frac{25}{\sqrt{2}}$

$= 18\,V$

b $P_p = I_p V_p = 15 \times 25 = 375\,W$

10 D.

$P_{rms} = V_{rms} I_{rms}$

$= \frac{I_p}{\sqrt{2}} \times \frac{V_p}{\sqrt{2}}$

$= \frac{I_p \times V_p}{2}$

$I_p \times V_p = 2 \times P_{rms} = 2 \times 60 = 120\,W$

Option D is the only option that meets this requirement.

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11 $n = 1$ (as there is only one loop in the coil)

$B = 1.0 \times 10^{-4}$ T

$I = 2.0$ A

$A = 0.1\,\text{m} \times 0.1\,\text{m} = 0.01\,\text{m}^2$

$\theta = 90°$ (the plane of the current-carrying coil is parallel to the magnetic field)

$\tau = nIA_{\perp}B = nIAB\sin\theta$

$= 1 \times 1.0 \times 10^{-4} \times 2.0 \times 0.01 \times \sin 90$

$= 2.0 \times 10^{-6}$ N m

12 $n = 1$ (as there is only one loop in the coil)

$B = 5.00 \times 10^{-4}$ T

$I = 1.0$ A

$A = 0.2\,\text{m} \times 0.2\,\text{m} = 0.04\,\text{m}^2$

$\theta = 30°$ (the plane of the current-carrying coil is at 60° to the magnetic field, so the angle is at 90 – 60 = 30°)

$\tau = nIA_{\perp}B = nIAB\sin\theta$

$= 1 \times 5.00 \times 10^{-4} \times 1.0 \times 0.04 \times \sin 30$

$= 1.0 \times 10^{-5}$ N m

13 $n = 5$

$B = 2.5 \times 10^{-4}$ T

$I = 3.00$ A

$A = 0.15\,\text{m} \times 0.15\,\text{m} = 0.0225$

$\theta = 45°$ (the plane of the current-carrying coil is at 60° to the magnetic field, so the angle is at 90 – 45 = 45°)

$\tau = nIA_{\perp}B = nIAB\sin\theta$

$= 1 \times 2.50 \times 10^{-4} \times 1.00 \times 0.0225 \times \sin 45$

$= 1.2 \times 10^{-5}$ N m

14 A commutator is used to prevent the coils in the rotor from becoming tangled as the rotor rotates, and to reverse the current at the point where the coil is perpendicular to the magnetic field.

15 Three-phase generators provide a more constant maximum voltage than a single-phase generator. In addition, each of the three phases can be distributed to different loads, allowing for a more balanced distribution. For a relatively small additional cost (two additional conductors), three times the power can be delivered for a three-phase system compared with a single-phase system.

16 $I_{rms} = \frac{P_{rms}}{V_{rms}} = \frac{2400}{240} = 10$ A

$I_p = \sqrt{2} \times I_{rms} = \sqrt{2} \times 10 = 14$ A

17 Using multiple armature windings can produce a steadier DC output voltage (see Figure 8.2.5b of the Student Book).

18 $P_{rms} = V_{rms}\, I_{rms}$

$\text{rms} = \frac{P_{rms}}{V_{rms}}$

$= \frac{2000}{240} = 8.33$ A

$P_p = \sqrt{2}V_{rms} \times \sqrt{2}I_{rms} = 2V_{rms}\, I_{rms}$

$= 2 \times V_{rms} \times I_{rms}$

$= 2 \times 240 \times 8.33$

$= 4$ kW

19 A DC motor uses a split-ring commutator to prevent the coil in the rotor from reversing every time the coil moves through the plane perpendicular to the magnetic field. The brushes are the electrical contacts to the split-ring commutator.

20 The back emf generated in a DC motor is the result of current produced in response to the rotation of the rotor inside the motor in the presence of an external magnetic field. The back emf, following Lenz's law, opposes the change in magnetic flux that created it, so this induced emf will be in the opposite direction to the emf creating it. The net emf used by the motor is thus always less than the supplied voltage.

21 The motor effect has been used in many different technological applications. The activity on page 222 shows how energy can be converted from an electrical signal into kinetic energy, or vice versa.

A DC motor is a transducer that converts an electrical signal into movement. A DC generator is a transducer that converts movement into an electrical signal. In this activity the first speaker is acting in the normal way for a speaker, converting the electrical signal from the frequency generator into movement. The second speaker is acting like a microphone, converting movement into an electrical signal.

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Module 6 Review

Electromagnetism

MULTIPLE CHOICE

1 C. The alternating current in the primary produces a changing magnetic flux, which induces an emf in the secondary coils (as well as the primary coils).

2 C. The self-induced emf is known as a back emf and opposes the mains emf.

3 B. This is twice the frequency and so the amplitude will be double and the period will halve.

4 A. This has twice the amplitude but the same period and so could be obtained by doubling *N*.

5 C. This time the period has halved (the frequency has doubled) but the amplitude remains the same. Thus a combination of the other quantities must have halved (*B* has doubled, but *N* has reduced to one-quarter).

6 A. The source of the electrons is the heated filament at A.

7 C. For a uniform electric field, the electric field strength is the same at all points between the plates.

8 B.

$E = \frac{V}{d}$

$= \frac{10}{0.40}$

$= 25\,\text{V m}^{-1}$

$W = qEd$

$= 1.602 \times 10^{-19} \times 24 \times 0.05$

$= 2.0 \times 10^{-19}\,\text{J}$

The electron is moving towards the earthed plate against the direction it would naturally go. So work is done on the field.

9 C. Magnetic fields are associated only with dipoles. Only monopoles generate radial fields.

10 A.

$r = \frac{mv}{qB}$

$= \frac{9.11 \times 10^{-31} \times 4.2 \times 10^{6}}{1.6 \times 10^{-19} \times 1.2}$

$= 2 \times 10^{-5}\,\text{m}$

11 D.

$F = \frac{qV}{d}$ (magnitude of the force is a scalar quantity)

$= \frac{1.6 \times 10^{-19} \times 15 \times 10^{3}}{12 \times 10^{-2}}$

$= 2 \times 10^{-14}\,\text{N}$

12 C. The emf, and hence the current, depends on the rate of change. If the rate is increased by a factor of 4, then the current will also increase by a factor of 4: $I = 200\,\mu\text{A}$.

13 B. As the magnet enters the coil, the magnetic field direction from the magnet will be away from the north pole, downwards. Therefore the change in flux is increasing downwards. In order to oppose the change, the current direction would be anticlockwise when viewed from above.

As the magnet exits the coil, the magnetic field direction from the magnet will be away from the north pole, downwards. Therefore the change in flux is decreasing downwards. In order to oppose the change, the current direction would be clockwise when viewed from above.

14 A. As a motor increases speed, the current also increases so that the back emf will increase.

15 B. Using the right-hand rule: the fingers point into the page, the thumb points to the left (direction of conventional current) so the force points downwards.

16 D.

$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$

$= \frac{4\pi \times 10^{-7} \times 5 \times 10^{-3} \times 5 \times 10^{-3}}{2\pi \times 0.05}$

$= 1.0 \times 10^{-10}\,\text{N m}^{-1}$ attractive

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17 A. The ampere is defined (SI definition) as being equal to the amount of current needed through two identical parallel conductors of infinite length when they are 1 metre apart, in order to produce a force per unit length of $2 \times 10^{-7}\,\text{N m}^{-1}$. This means that the equation needed must look at the force produced between two current-carrying conductors.

18 C. The changing current will cause the coil to spin continuously. Use the right-hand rule to show how the current will turn in a clockwise direction.

19 C. In this image, initially, the magnetic flux passes through the full area of the coil and into the page. Moving the coil out of the field decreases the magnetic flux. So the change in flux is decreasing into the page. The magnetic field that opposes the change would act into the page again. In order to oppose the change, the current direction would be clockwise (using the right-hand grip rule).

20 D.

$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$

$$\frac{N_p}{80} = \frac{240}{10}$$

$$N_p = \frac{80 \times 240}{10}$$

$$= 1920 \text{ turns}$$

SHORT ANSWER

21 $E = \frac{kq}{r^2}$

$$= \frac{9 \times 10^9 \times 9.4 \times 10^{-6}}{(3.5 \times 10^{-3})^2}$$

$= 6.9 \times 10^9\,\text{N C}^{-1}$ to the left (away from the charge)

22 **a** $F = BIl = 1.0 \times 0.05 \times 1.0 = 0.05\,\text{N}$

b The right-hand rule tells us that it is to the right.

c $F = BIl = 1.0 \times 0.01 \times 1.0 = 0.01\,\text{N}$

d The direction of the force on side PQ is to the left.

23 **a** $v^2 = \frac{2qV}{m}$

$$= \frac{2 \times 1.6 \times 10^{-19} \times 28 \times 10^3}{9.11 \times 10^{-31}}$$

$v = 9.9 \times 10^7\,\text{m s}^{-1}$

b $E = \frac{V}{d}$

$$= \frac{28 \times 10^3}{0.20}$$

$= 1.4 \times 10^5\,\text{V m}^{-1}$

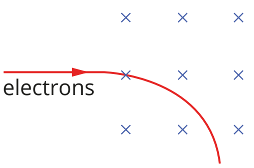
24 **a** Side AB is parallel to the field. Hence there will be no force on it.

b Side DC is parallel to the field. Hence there will be no force on it.

c $F = nBIl = 100 \times 0.2 \times 0.1 \times 0.25 = 0.5\,\text{N}$ out of the page

d $F = nBIl = 100 \times 0.2 \times 0.1 \times 0.25 = 0.5\,\text{N}$ into the page

25



26 Because the forces acting on the electrons due to the electric and magnetic fields are balanced (i.e. there is no deflection), we know that the electric force is equivalent to the magnetic force:

$$F_B = F_E$$

$$qvB = qE$$

$$qvB = q\frac{V}{d}$$

$$d = \frac{V}{vB}$$

$$= \frac{3000}{3.25 \times 10^7 \times 1.6 \times 10^{-3}}$$

$$= 5.8 \times 10^{-2}\,\text{m}$$

27 **a** $\Phi = B\,A$

$$= 1.0 \times 10^{-3} \times 100 \times 10^{-3} \times 50 \times 10^{-3}$$

$$= 5 \times 10^{-6}\,\text{Wb}$$

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b No (zero) flux threads the loop in the new position, as the plane of the loop is now parallel to the magnetic field. Alternatively, $\theta = 90°$ and $\cos\theta = 0$ so no (zero) flux threads the loop in the new position.

c $\varepsilon = N\frac{\Delta\Phi}{\Delta t}$

$= \frac{1\times5\times10^{-6}}{2\times10^{-3}}$

$= 2.5\times10^{-3}$ V

d $I = \frac{V}{R}$

$= \frac{2.5\times10^{-3}}{2.0}$

$= 1.25\times10^{-3}$ A

e No. Once the loop is stationary, there is no change in flux and therefore no emf generated and no current flows in the loop.

28 **a** $F = BIl = 100\times1\times1\times10^{-5} = 1\times10^{-3}$ N

b The right-hand rule tells us that a current from west to east will experience an upward force.

c The weight of 1 m of cable is $mg = 0.05\times9.8 = 0.49$ N. For the magnetic force to equal this: $I = \frac{F}{B} = \frac{0.49}{(1\times10^{-5})} = 4.9\times10^{4}$ A. (Not much chance of magnetic levitation for power cables!)

d The change of force is from 1×10^{-3} N up to 1×10^{-3} N down—a change of 2×10^{-3} N down.

e The horizontal component of the current is now less and so there will be a smaller force per metre of cable.

29 **a** $I_s = \frac{I_p\times V_p}{V_s} = \frac{2.0\times600}{3000} = 0.4$ A

b $V_{p-p} = 2\times3000$

$= 6000$ V

c $N_p = \frac{N_S\times V_P}{V_S} = \frac{1000\times600}{3000} = 200$ turns

d $P_{s\ rms} = V_{s\ rms}\times I_{s\ rms} = \frac{3000}{\sqrt{2}}\times0.4 = 850$ W

e $P_{s\ peak} = V_{s\ peak}\times I_{s\ peak} = 3000\times0.4\times\sqrt{2} = 1700$ W

30 **a** Since the plane of the loop is parallel to the magnetic field direction, no (zero) flux threads the loop.

b Rotate the loop or the magnetic field so they are no longer parallel.

c The maximum flux threads the loop when the plane of the loop and the magnetic field direction are perpendicular (at right angles) to each other.

d $\Phi = B\,A$

$= 0.50\times0.2\times0.1$

$= 0.01$ Wb or 10^{-2} Wb

31 **a** The field is from N to S, so the right-hand rule shows that the force on side AB is upwards and that on side CD is downwards.

b In the position shown (with the coil horizontal), the direction of the forces on the sides AB and CD are at right angles to the radius and the turning effect is maximum.

c The torque becomes zero when the coil is in the vertical position. It continues to rotate for two reasons: (i) its momentum will carry it past the true vertical position; (ii) at the vertical position the commutator reverses the direction of the current through the coil and so the forces reverse; thus it continues to rotate for another half turn, at which point the current reverses again and the rotation continues.

d $I = \frac{F}{n\,B} = \frac{40}{(100\times0.2\times0.5)} = 4.0$ A

e The shorter side will halve the force, twice the current will double the force and half the turns halves the force. The net effect is to halve the force, so $F = 20$ N.

f The force increases with the field, so the new force is $F = 40\times\frac{8}{5} = 64$ N.

32 **a** $f = \frac{1}{T} = \frac{1}{2\times10^{-3}} = 500$ Hz

b $V_{p-p} = 20$ V

c $V_{rms} = \frac{V_{peak}}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7.1$ V

d $I_{rms} = \frac{I_{peak}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.71$ A

e $P_{rms} = V_{rms}\times I_{rms}$

$= 7.1\times0.71$

$= 5$ W

f An alternator has a pair of sliprings instead of a split ring commutator.

g AC is generated in the coils of an alternator. Each slip ring connects to each end of the coil. The slip rings maintain the AC generated in the coil at the output.

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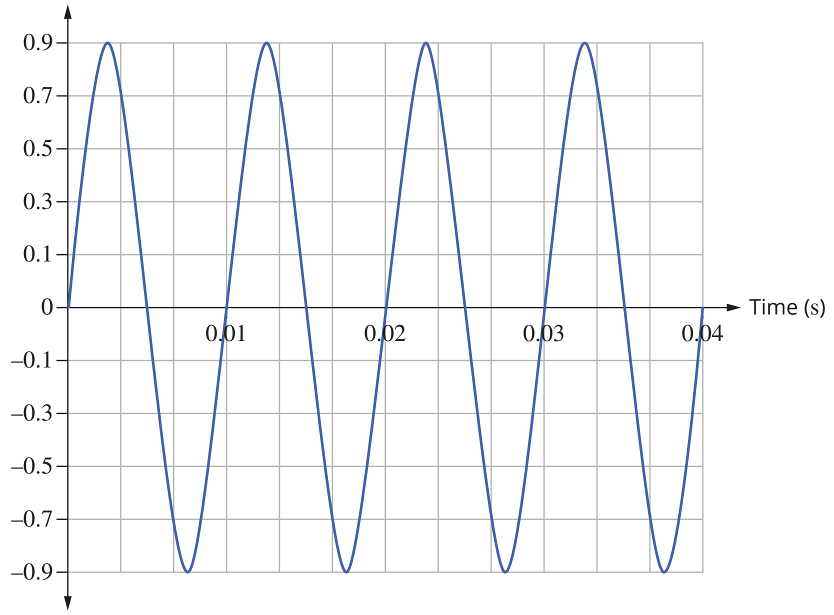
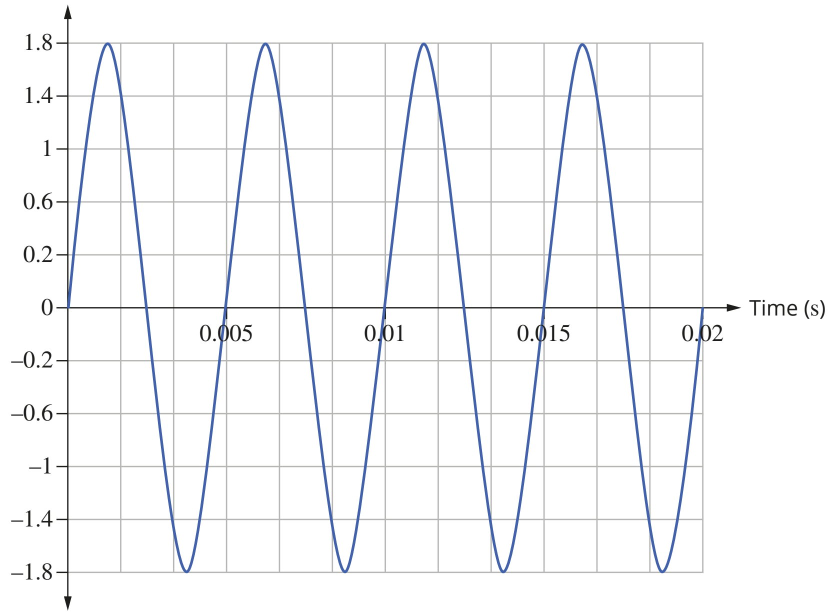
33 **a** $T = \frac{1}{f} = \frac{1}{100} = 0.01\,\text{s}$

The graph is a sine wave with peak amplitude of 0.9 V and a period of 0.01 s (10 ms).



b $V_{rms} = \frac{V_{peak}}{\sqrt{2}} = \frac{0.9}{\sqrt{2}} = 0.64\,\text{V}$

c The output graph would have half the period and twice the amplitude. The rms voltage would be 1.3 V.



d $f = \frac{3000}{60}$

$= 50\,\text{Hz}$

e $T = \frac{1}{f} = \frac{1}{50} = 0.02\,\text{s}$

A quarter turn will take $\frac{0.02}{4} = 0.005\,\text{s}$

$\varepsilon = N\frac{\Delta\Phi}{\Delta t}$

$= \frac{200 \times 0.5 \times 100 \times 10^{-4}}{0.005}$

$= 200\,\text{V}$

34 **a** As the loop enters the magnetic field there is a flux increasing down through the loop. Lenz's law states the induced current in the loop will oppose the change in flux that causes it. Therefore there will be an induced field (or flux) up through the loop. Using the right-hand grip rule with the thumb pointing up, the fingers curl in the direction of the induced current from Y to X.

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b The loop moves at a speed of 5 cm s^{-1}, and with side length 20 cm, it is halfway into the field when it has travelled 10 cm, which takes 2 s.

$\varepsilon = N\frac{\Delta\Phi}{\Delta t}$

$= \frac{1\times 0.40\times 0.2\times 0.1}{2}$

$= 4 \times 10^{-3}$ V

c $I = \frac{V}{R}$

$= \frac{4\times 10^{-3}}{0.5}$

$= 8 \times 10^{-3}$ A

d $P = VI$

$= 4 \times 10^{-3} \times 8 \times 10^{-3}$

$= 3.2 \times 10^{-5}$ W

e The source of this power is the external force that is moving the loop into the magnetic field.

f The loop is moving at a speed of 5 cm s^{-1}, so after 5 s it has moved 25 cm and has been totally within the magnetic field for 1 s. Since there is now no flux change there will be no emf induced in the loop at this moment.

g As the loop emerges from the magnetic field there is a flux decreasing down through the loop. Lenz's law states the induced current in the loop will oppose the change in flux that causes it. Therefore there will be an induced field (or flux) down through the loop. Using the right-hand grip rule with the thumb pointing down, the fingers curl in the direction of the induced current from X to Y.

35 **a** With little or no current in the power line there was almost no voltage drop. When the house appliances were turned on, there was a higher current in the power line and hence a voltage drop along the line, leaving a low voltage at the house.

b As the generator was supplying 4000 W at 250 V, the current in the line was $I = \frac{4000}{250} = 16$ A. The voltage drop along the line was therefore $\Delta V = R = 16 \times 2 = 32$ V and so the voltage at the house was $250 - 32 = 218$ V. The power lost is $P_{loss} = I^2R = 16^2 \times 2 = 512$ W and so the power at the house was $4000 - 512 = 3488$ W.

Alternatively, $P_{house} = VI = 218 \times 16 = 3488$ W.

c At the generator end, a 1:20 step-up transformer is required ($\frac{5000}{250} = 20$). There will be 20 times as many turns in the secondary as in the primary. At the house end, a 20:1 step-down transformer is required.

d $I = \frac{P}{V} = \frac{4000}{5000} = 0.8$ A

e The voltage drop is $V = IR = 0.8 \times 2 = 1.6$ V.

f The power loss is $P = I^2R = 0.8^2 \times 2 = 1.28$ W.

g The voltage at the house will be $\frac{5000-1.6}{20} = 249.92$ V.

h The power at the house will be $4000 - 1.28 = 3998.72$ W.

i The power loss before the transformers were added was 512 W (part (b)) which was 12.8% of the power generated (4000 W), and the power loss with the transformers was 1.28 W (part (f)), which is about 0.03% of the power generated.

j The reason is that the power loss in the power line depends on the square of the current ($P = I^2R$). Since the current was reduced by a factor of 20 and the resistance remains constant, the power loss decreased by a factor of 20^2 or 400.

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Chapter 9 Electromagnetic spectrum

9.1 Electromagnetism

Worked example: Try yourself 9.1.1

USING THE WAVE EQUATION FOR LIGHT

A particular colour of red light has a wavelength of 600 nm. Calculate the frequency of this colour.

Thinking	Working
Recall the wave equation for light.	$c = f\lambda$
Transpose the equation to make frequency the subject.	$f = \frac{c}{\lambda}$
Substitute in values to determine the frequency of this wavelength of light.	$f = \frac{3.00 \times 10^{8}}{600 \times 10^{-9}}$ $= 5.00 \times 10^{14}$ Hz

9.1 Review

1 B. Light waves can travel through a vacuum, while mechanical waves cannot. Both light and mechanical waves have a measurable wavelength, their speeds can be measured and both types of waves undergo diffraction.

2 D. The changing electric and magnetic fields are perpendicular to each other.

3 D. 200 nm is a wavelength just shorter (or more energetic) than the range of visible light.

4 FM radio waves/infrared radiation/visible light/X-rays

5 Using $c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$, and $f = \frac{c}{\lambda}$

a $f = \frac{3.00 \times 10^{8}}{656 \times 10^{-9}}$

$= 4.57 \times 10^{14}$ Hz

b $f = \frac{3.00 \times 10^{8}}{589 \times 10^{-9}}$

$= 5.09 \times 10^{14}$ Hz

9.2 Spectroscopy

9.2 Review

1 If the element is given sufficient energy to excite its atoms, the energy is released as light, which forms the emission spectrum.

2 The lines have been blueshifted and they have been spread out. This means the object is moving towards Earth and it is rotating. The rotation of the object means that each side will be slightly redshifted or blueshifted, causing each of the spectral lines to spread out.

3 3, then 2, then 1. The stars are receding so each spectrum is redshifted and the degree of shift reflects the speed. The two lines near 400 nm in 1 are at around 410 nm in 2 and over 490 nm in 3. There are no shorter-wavelength lines shifted into the ultraviolet in 3, and the lines around 650–690 nm in 1 and 2 are shifted off the scale into the infrared in 3, which indicates 3 must be the fastest moving source.

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CHAPTER 9 REVIEW

1 Percentage difference $= \frac{\text{approximation} - \text{theoretical}}{\text{theoretical}} \times 100\% = \frac{3.00\times10^8 - 299792458}{299792458} \times 100\% = 0.069\%$

2 B. A changing current creates a changing magnetic field which induces an electric field. A changing current does not create a constant magnetic field, a changing magnetic field does not create a constant current, and a constant current does not create a changing magnetic field.

3 **a** Using the wave equation, $c = f\lambda$

$\lambda = \frac{c}{f} = \frac{3.0\times10^8}{5.0\times10^{14}} = 6.0 \times 10^{-7} = 600\,\text{nm}$

b Reading from the image: yellow.

4 $\lambda = \frac{c}{f} = \frac{3.00\times10^8}{7.0\times10^7} = 4.3\,\text{m}$

5 $f = \frac{c}{\lambda} = \frac{3.00\times10^8}{200\times10^{-12}} = 1.5 \times 10^{18}\,\text{Hz}$

6 $\lambda = \frac{c}{f} = \frac{3.0\times10^8}{915\times10^6} = 0.33\,\text{m}$

7 The missing bands in the absorption band match the bands present in the emission spectrum. This rule breaks down at high temperatures when the element emits a continuous spectrum.

8 Using $c = 3.00 \times 10^8\,\text{m s}^{-1}$, and $f = \frac{c}{\lambda}$

a $f = \frac{3.00\times10^8}{486\times10^{-9}}$

$= 6.17 \times 10^{14}\,\text{Hz}$

b $f = \frac{3.00\times10^8}{397\times10^{-9}}$

$= 7.56 \times 10^{14}\,\text{Hz}$

9 To generate electromagnetic radiation, a charge needs to oscillate. Wood is not conductive, therefore there are no charges in the wood that can move, or be induced to move.

10 Using Wien's law,

$\lambda_{\text{max}} \propto \frac{1}{T}$

$\frac{\lambda_{\text{max,A}}}{\lambda_{\text{max,B}}} = \frac{T_B}{T_A}$

$\frac{450}{400} = 1.125$

Therefore, star B is 1.13 times hotter than star A.

11 The two dangerous radiations have shorter wavelengths (i.e. higher energy photons) and can penetrate skin. More importantly, these photons have sufficient energy to disrupt biological molecules. Radio waves do not have sufficient energy to affect atoms, let alone biological molecules.

12 Infrared radiation is absorbed as it travels through the Earth's atmosphere. More infrared radiation from the sources being studied can therefore be detected by a telescope operated outside the atmosphere, providing better measurements. As measurements are also being made in a cooler region than on Earth's surface, the measurements are less subject to interference from ground-based heat sources, again making the measurements more accurate.

13 Average distance between spots: $\frac{5.9 + 5.8 + 6.0 + 5.9}{4} = 5.9$ cm.

The distance between melted spots corresponds to the antinodes of the radiation, so that the wavelength is calculated by:

$\frac{\lambda}{2} = 5.9$ cm

$\lambda = 11.8\,\text{cm}$

$c = f\lambda$

$= 2.45 \times 10^9 \times 0.118$

$= 2.891 \times 10^8$

$= 2.9 \times 10^8\,\text{m s}^{-1}$

The theoretical value for the speed of light is $c = 3.00 \times 10^8\,\text{m s}^{-1}$. The value calculated in this experiment is slightly less than, but within 3% of, the theoretical value. Uncertainty or error may have been introduced within the measurement, so a further understanding of how the experiment was conducted is needed to analyse this result.

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14 Emission spectra are produced when atoms are excited and release photons of particular wavelengths as they return to the ground state. In space, nebulae are heated by nearby hot stars, and can produce emission spectra.

Absorption spectra are produced when atoms absorb particular wavelengths as light passes through them. The dense cores of stars tend to produce continuous spectra, and as this light passes through the outer layers of the stars, particular wavelengths are absorbed, depending on the temperature and atoms present. Most stars therefore produce absorption spectra.

15 red star, orange star, yellow star, blue star

16 It is a young star as it has not had time to form heavier elements. Stars fuse elements such as hydrogen together which create heavier elements such as carbon during a process known as nucleosynthesis. If heavier elements aren't present, the star must be young.

17 To determine the characteristics of a star the visible light can be broken up with a spectrograph and then examined. The spectrum is compared to known spectra of other stars. Stars of the A class will have absorption spectra showing strong hydrogen lines. The temperature of the star can also be determined using Wien's law as stars will produce a specific radiation curve at a specific temperature—in this case in the range 7 500–10 000 K. The colour of the star is related to its temperature.

18 According to Wien's law, the temperature is inversely proportional to the peak wavelength of the spectrum. Therefore, as star A has a lower peak wavelength (approximately 250 nm) than star B (at approximately 350 nm), star A must be hotter.

19 Light can be modelled as mutually perpendicular, oscillating electric and magnetic fields. Light with a wavelength between 380 nm and 750 nm is visible to our eyes. The longer wavelengths are seen as red, and the shorter wavelengths appear as blue and violet, with all the colours of the rainbow in between. White light is made from a combination of different wavelengths.

Our eyes have three different cones that are sensitive to light of different wavelengths. When all three are stimulated, our brains interpret the signal as white light. This can occur when all wavelengths are present (for example, the light coming from the sun), or when red, green and blue light are combined (for example, from an RGB LED light globe).

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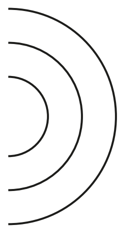
Chapter 10 Light: wave model

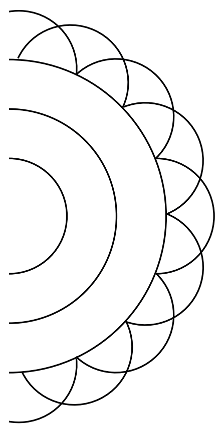
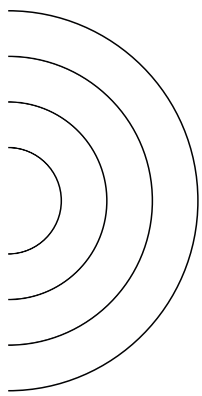
10.1 Diffraction and interference

Worked example: Try yourself 10.1.1

APPLYING HUYGENS' PRINCIPLE

On the circular waves shown below, sketch some of the secondary wavelets on the outer wavefront and draw the appearance of the new wave formed after one period.



Thinking	Working
Sketch a number of secondary wavelets on the advancing wavefront.	
Sketch the new wavefront.	

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Worked example: Try yourself 10.1.2

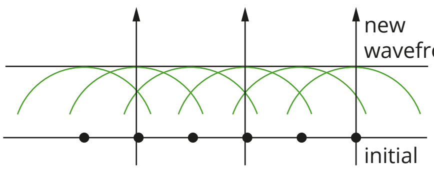
CALCULATING WAVELENGTH FROM FRINGE SEPARATION

Green laser light is directed through a pair of thin slits that are 25 µm apart. The slits are 1.5 m from a screen on which bright fringes are 3.3 cm apart. Calculate the wavelength of the green laser light in nm.

Thinking	Working
Determine the angle θ.	Since the screen is 1.5 m away and the distance between the first two bright fringes is 3.3 cm = 0.033 m: $\tan\theta = \frac{0.033}{1.5}$ $\theta = \tan^{-1}\left(\frac{0.033}{1.5}\right) = 1.3°$
Determine m.	The path difference between the rays creating the first fringe from the centre is 1λ. Therefore, $m = 1$.
Recall the equation for fringe separation.	$d\sin\theta = m\lambda$
Transpose the equation to make λ the subject.	$\lambda = \frac{d\sin\theta}{m}$
Substitute values into the equation and solve.	$\lambda = \frac{25\times10^{-6}\times\sin 1.3}{1} = 5.5\times10^{-7}$ m
Express your answer using the units specified.	The wavelength of the green laser light is 550 nm.

10.1 Review

1 The new wavefront should be a straight line across the front of the secondary wavelets.



2 D. Significant diffraction occurs when $\frac{\lambda}{w}$ is approximately 1 or greater. 700 nm ≈ 10^{-6} m and 0.001 mm = 0.001 × 10^{-3} or 10^{-6} m.

3 A and D. When crests meet troughs, or troughs meet crests, the addition of these out-of-phase waves means that they cancel to form a node.

4 Since the screen is 3.25 m away and the distance between the first two bright fringes is 0.037 m:

$$\tan\theta = \frac{0.037}{3.25}$$

$$\therefore \theta = \tan^{-1}\left(\frac{0.037}{3.25}\right) = 0.65°$$

$$d\sin\theta = m\lambda$$

$$\therefore \lambda = \frac{d\sin\theta}{m}$$

$$= \frac{40\times10^{-6}\times\sin 0.65}{1}$$

$$= 4.6\times10^{-7}\text{ nm}$$

$$= 460\text{ nm}$$

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10.2 Polarisation

Worked example: Try yourself 10.2.1

APPLYING MALUS' LAW TO CALCULATE RELATIVE REDUCTION IN INTENSITY

How much (as a percentage) is the intensity of a ray of light reduced if it passes through a polarising filter that is aligned at 30° to the plane of polarisation of the light?

Thinking	Thinking
Recall Malus' law.	$I = I_{max} \cos^2 \theta$
Substitute the angle between the planes of polarisation of the filter and the light.	$I = I_{max} \cos^2 30$ $= 0.75 I_{max}$
Express the answer as a percentage.	The intensity of the light leaving the filter is only 75% of the intensity of the light entering the filter, i.e. intensity has been reduced by 25%.

Worked example: Try yourself 10.2.2

APPLYING MALUS' LAW TO CALCULATE CHANGE IN INTENSITY

Horizontally polarised laser light with an intensity of 90 cd passes through a polarising filter that is orientated at 60° to the horizontal plane. Calculate the intensity of the light as it leaves the filter.

Thinking	Thinking
Recall Malus' law.	$I = I_{max} \cos^2 \theta$
Substitute the values into the equation.	$I = 90 \cos^2 60$ $= 23$ cd

10.2 Review

1 Polarisation occurs when transverse waves are allowed to vibrate in only one direction. Polarisation can only occur in transverse waves and cannot occur in longitudinal waves. Since light can be polarised, it must be a transverse wave.

2 $I = I_{max} \cos^2 \theta$
$= I_{max} \cos^2 20$
$= 0.88 I_{max}$
The intensity of the light after the filter is 88% of the intensity before the filter. The intensity has been reduced by 12%.

3 $I = I_{max} \cos^2 \theta$
$= 30 \cos^2 35$
$= 20$ cd

4 $I = I_{max} \cos^2 \theta$
$0.9 I_{max} = I_{max} \cos^2 \theta$
$0.9 = \cos^2 \theta$
$\cos \theta = \sqrt{0.9} = 0.95$
$\theta = \cos^{-1} 0.95$
$= 18°$

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CHAPTER 10 REVIEW

1 **a** wave model

b wave model

c particle model

2 C. Newton's esteemed reputation meant that his theory was regarded as correct.

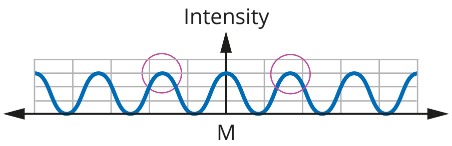
3 A. This shows the bending of the edges of the waves as they pass through a gap.

4 Green light ($\lambda = 525$ nm) has a longer wavelength than blue light ($\lambda = 460$ nm). The longer wavelength of green light results in more-widely spaced fringes and a wider overall pattern.

5 D. Polarisation is a phenomenon in which transverse waves are restricted in their direction of vibration. Polarisation can only occur in transverse waves and cannot occur in longitudinal waves. Since light can be polarised, it must be a transverse wave.

6 D. Young's double-slit experiment produced an interference pattern of alternating bright and dark lines on the screen.

7 The central antinode occurs where both waves have travelled the same distance, i.e. the path difference is 0. The next antinode on either side occurs when the path difference is 1λ.



8 Up until Young's experiment, most scientists supported a particle or 'corpuscular' model of light. Young's experiment demonstrated interference patterns, which are characteristic of waves. This led to scientists abandoning the particle theory and supporting a wave model of light.

9 **a** $\theta = \tan^{-1}\left(\frac{0.031}{4.0}\right) = 0.44°$

b $\lambda = \frac{d \sin\theta}{m} = \frac{75 \times 10^{-6} \times \sin 0.44}{1}$

$= 5.8 \times 10^{-7}$ m

$= 580$ nm

10 $d \sin\theta = m\lambda$

$\therefore \sin\theta = \frac{m\lambda}{d} = \frac{3 \times 6.50 \times 10^{-7}}{80 \times 10^{-5}} = 0.0244$

$\therefore \theta = \sin^{-1} 0.0244 = 1.4°$

11 $d = \frac{m\lambda}{\sin\theta} = \frac{5 \times 4.25 \times 10^{-7}}{\sin 2.0}$

$= 6.1 \times 10^{-5}$ m

$= 61\ \mu$m

12 Young performed his famous experiment in 1803, in which he observed an interference pattern in light. Young shone monochromatic light on a pair of narrow slits. Light passed through the slits and formed a pattern of bright and dark bands (or fringes) on a screen. Young compared this to interference patterns he had observed in water waves, and he identified that these lines corresponded to regions of constructive and destructive interference. This could only be explained by considering light to be a wave.

13 **a** increase

b increase

14 Both snow and water reflect light. This reflected light is known as glare. The light reflected from water and snow is partially polarised. Snowboarders and sailors are likely to wear polarising sunglasses as these will absorb the polarised glare from the snow or water.

15 $I = I_{max} \cos^2\theta$

$I_2 = I_1 \cos^2 50 = 0.41\, I_1$

$I_2 = 41\%$ of I_1

16 $I = I_{max} \cos^2\theta = 15 \cos^2 75$

$= 1.0$ cd

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17 $I = I_{max} \cos^2 \theta$

$0.2 I_{max} = I_{max} \cos^2 \theta$

$0.2 = \cos^2 \theta$

$\cos \theta = \sqrt{0.2} = 0.44$

$\theta = \cos^{-1} 0.44 = 63°$

18 The wavelength of light waves is very small; there are not many natural structures that are small enough to cause diffraction of light waves.

19 The distance to the screen is much greater than the slit separation.

$d \sin\theta = m\lambda$ is for constructive interference which will result in bright bands. Therefore use $d \sin \theta = (m - \frac{1}{2})\lambda$ for dark bands.

$0.0001 \sin \theta = (3 - \frac{1}{2})\ 450 \times 10^{-9}$

$\sin \theta = 0.01125$

$\theta = 0.64°$

20 Some behaviour of light can be explained and predicted using a wave model. Polarisation is one of these behaviours, along with refraction and diffraction.

In the activity, after the light passes through the first polarising film, all wavelengths are oscillating in the same plane. Light that does not pass through the tape will remain in that plane. When the second polarising film is rotated, the amount of light that will pass through will diminish until no light passes through.

Light that passes through the tape will rotate so that when the surrounds are blocked, light of different colours will be seen in these areas. Different thicknesses of tape in different orientations will show up as different colours.

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Chapter 11 Light: quantum model

11.1 Black-body radiation

Worked example: Try yourself 11.1.1

THE TEMPERATURE AT A STAR'S SURFACE

A newly discovered star is observed to emit radiation with a peak wavelength of approximately 90 nm. Based on this wavelength, estimate the surface temperature of this star.

Thinking	Working
Express the peak wavelength in metres.	$\lambda_{max} = 90\,\text{nm} = 90 \times 10^{-9}\,\text{m}$
Rearrange Wien's law to solve for *T*.	$\lambda_{max} = \frac{b}{T}$ $T = \frac{2.898 \times 10^{-3}}{\lambda_{m\ x}}$
Substitute the value for λ_{max} and solve for *T*.	$T = \frac{2.898 \times 10^{-3}}{90 \times 10^{-9}}$ $= 32\,200 = 32\,000\,\text{K}$ (to two significant figures)

Worked example: Try yourself 11.1.2

RE-RADIATED ENERGY FROM THE EARTH

The Earth's average surface temperature at the equator is 300 K. What is the peak wavelength of the re-radiated electromagnetic radiation from this portion of the Earth?

Thinking	Working
State Wien's law.	$\lambda_{max} = \frac{b}{T}$
Substitute the values for *b* and *T* and solve for λ_{max}.	$\lambda_{max} = \frac{2.898 \times 10^{-3}}{300}$ $= 9.66 \times 10^{-6}\,\text{m}$ $= 9.66\,\mu\text{m}$

Worked example: Try yourself 11.1.3

USING PLANCK'S EQUATION

Calculate the energy in joules of a quantum of infrared radiation that has a frequency of 3.6×10^{14} Hz.

Thinking	Working
Recall Planck's equation.	$E = hf$
Substitute in the appropriate values to solve.	$E = 6.626 \times 10^{-34} \times 3.6 \times 10^{14}$ $= 2.4 \times 10^{-19}\,\text{J}$

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Worked example: Try yourself 11.1.4

CONVERTING TO ELECTRON-VOLTS

A quantum of light has 2.4×10^{-19} J. Convert this energy to electron-volts.

Thinking	Working
Recall the conversion rate for joules to electron-volts.	$1\ \text{eV} = 1.602 \times 10^{-19}\ \text{J}$
Divide the value expressed in joules by $1.602 \times 10^{-19}\ \text{J eV}^{-1}$ to convert to electron-volts.	$\frac{2.4 \times 10^{-19}}{1.602 \times 10^{-19}}$ $= 1.5\ \text{eV}$

Worked example: Try yourself 11.1.5

CALCULATING QUANTUM ENERGIES IN ELECTRON-VOLTS

Calculate the energy (in eV) of a quantum of infrared radiation that has a frequency of 3.6×10^{14} Hz. Use $h = 4.14 \times 10^{-15}$ eV s.

Thinking	Working
Recall Planck's equation.	$E = hf$
Substitute in the appropriate values and solve for E.	$E = 4.14 \times 10^{-15} \times 3.6 \times 10^{14}$ $= 1.5\ \text{eV}$

11.1 Review

1 Maxwell's theory permits a continuous range of frequencies that can be emitted by a hot object. Planck proposed that light is radiated in discrete, quantised amounts or energy, rather than in a continuous, unbroken wave.

2 $\lambda_{max}T = 2.898 \times 10^{-3}\ \text{m K}$

$$T = \frac{2.898 \times 10^{-3}}{\lambda_{m\ x}}$$
$$= \frac{2.898 \times 10^{-3}}{800 \times 10^{-9}}$$
$$= 3620\ \text{K}$$

3 $\lambda_{max}T = 2.898 \times 10^{-3}\ \text{m K}$

$$T = \frac{2.898 \times 10^{-3}}{\lambda_{m\ x}}$$
$$= \frac{2.898 \times 10^{-3}}{700 \times 10^{-9}}$$
$$= 4140\ \text{K}$$

4 $\lambda_{max}T = 2.898 \times 10^{-3}\ \text{m K}$

$$\lambda_{max} = \frac{2.898 \times 10^{-3}}{T}$$
$$= \frac{2.898 \times 10^{-3}}{9000}$$
$$= 322\ \text{nm}$$

5 **a** $E = \frac{hc}{\lambda}$

$$= \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{656 \times 10^{-9}}$$
$$= 3.03 \times 10^{-19}\ \text{J}$$
$$= \frac{3.03 \times 10^{-19}}{1.60 \times 10^{-19}}$$
$$= 1.89\ \text{eV}$$

b $E = \frac{hc}{\lambda}$

$$= \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{589 \times 10^{-9}}$$
$$= 3.38 \times 10^{-19}\ \text{J}$$
$$= \frac{3.38 \times 10^{-19}}{1.60 \times 10^{-19}}$$
$$= 2.11\ \text{eV}$$

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c $E = \frac{hc}{\lambda}$

$= \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{486 \times 10^{-9}}$

$= 4.09 \times 10^{-19}\,\text{J}$

$= \frac{4.09 \times 10^{-19}}{1.60 \times 10^{-19}}$

$= 2.56\,\text{eV}$

d $E = \frac{hc}{\lambda}$

$= \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{397 \times 10^{-9}}$

$= 5.01 \times 10^{-19}\,\text{J}$

$= \frac{5.01 \times 10^{-19}}{1.60 \times 10^{-19}}$

$= 3.13\,\text{eV}$

11.2 The photoelectric effect

Worked example: Try yourself 11.2.1

CALCULATING THE WORK FUNCTION OF A METAL

Calculate the work function (in J and eV) for gold, which has a threshold frequency of 1.2×10^{15} Hz.

Thinking	Working
Recall the formula for the work function.	$\phi = hf_0$
Substitute the threshold frequency of the metal into this equation.	$\phi = 6.626 \times 10^{-34} \times 1.2 \times 10^{15}$ $= 8.0 \times 10^{-19}\,\text{J}$
Convert this energy from J to eV.	$\phi = \frac{8.0 \times 10^{-19}}{1.602 \times 10^{-19}}$ $= 5.0\,\text{eV}$

Worked example: Try yourself 11.2.2

CALCULATING THE KINETIC ENERGY OF PHOTOELECTRONS

Calculate the kinetic energy (in eV) of the photoelectrons emitted from lead by ultraviolet light which has a frequency of 1.5×10^{15} Hz. The work function of lead is 4.14 eV. Use $h = 4.14 \times 10^{-15}$ eV s.

Thinking	Working
Recall Einstein's photoelectric equation.	$K_{max} = hf - \phi$
Substitute values into this equation.	$K_{max} = 4.14 \times 10^{-15} \times 1.5 \times 10^{15} - 4.25$ $= 6.21 - 4.14$ $= 2.1\,\text{eV}$

11.2 Review

1 In the photoelectric effect, a metal surface may become positively charged if light shining on it causes electrons to be released.

2 **a** True.

b False: When light sources of the same intensity but different frequencies are used, the higher frequency light has a higher stopping voltage, but it produces the same maximum current as the lower frequency.

c True.

3 **a** $\phi = hf_0 = 4.14 \times 10^{-15} \times 1.0 \times 10^{15} = 4.1\,\text{eV}$

b $\phi = hf_0 = 4.14 \times 10^{-15} \times 1.1 \times 10^{15} = 4.6\,\text{eV}$

c $\phi = hf_0 = 4.14 \times 10^{-15} \times 1.5 \times 10^{15} = 6.2\,\text{eV}$

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4 D.

The threshold frequency is:

$$f_0 = \frac{\phi}{h}$$
$$= \frac{3.66}{4.14 \times 10^{-15}}$$
$$= 8.84 \times 10^{14}\,\text{Hz}$$

In order to release photoelectrons, the light must have a frequency higher than the threshold frequency. Therefore, 9.0×10^{14} Hz is the only frequency that will release photoelectrons.

5 $K_{max} = 4.14 \times 10^{-15} \times 9.0 \times 10^{14} - 3.66$
$$= 0.066\,\text{eV}$$

CHAPTER 11 **REVIEW**

1 Particle model: black-body radiation and photoelectric effect.

Wave model: interference patterns and polarisation.

2 $\lambda_{max}T = 2.898 \times 10^{-3}\,\text{m K}$
$$T = \frac{2.898 \times 10^{-3}}{\lambda_{m\ x}}$$
$$= \frac{2.898 \times 10^{-3}}{502 \times 10^{-9}}$$
$$= 5770\,\text{K}$$

3 $\lambda_{max}T = 2.898 \times 10^{-3}\,\text{m K}$
$$T = \frac{2.898 \times 10^{-3}}{\lambda_{m\ x}}$$
$$= \frac{2.898 \times 10^{-3}}{455 \times 10^{-9}}$$
$$= 6370\,\text{K}$$

4 Rigel has the higher surface temperature.

Since Betelgeuse is red, its peak wavelength must have a longer wavelength than Rigel which is blue. Since it has a longer wavelength, Betelgeuse must have a lower surface temperature than Rigel.

5 $\lambda_{max}T = 2.898 \times 10^{-3}\,\text{m K}$
$$\lambda_{max} = \frac{2.898 \times 10^{-3}}{T}$$
$$= \frac{2.898 \times 10^{-3}}{11000}$$
$$= 263\,\text{nm}$$

6 $E = hf = 6.63 \times 10^{-34} \times 8.0 \times 10^{14} = 5.3 \times 10^{-19}\,\text{J}$

7 $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{(500 \times 10^{-9})} = 3.98 \times 10^{-19}\,\text{J}$
$$= \frac{3.98 \times 10^{-19}}{1.60 \times 10^{-19}} = 2.49\,\text{eV}$$

8 $E = hf = 4.14 \times 10^{-15} \times 6.0 \times 10^{14} = 2.5\,\text{eV}$

9 $E = 5.0 \times 1.602 \times 10^{-19}\,\text{J} = 8.0 \times 10^{-19}\,\text{J}$

10 photoelectrons

11 $\phi = hf_0$
$$f_0 = \frac{\phi}{h}$$
$$= \frac{5.0}{4.14 \times 10^{-15}}$$
$$= 1.2 \times 10^{15}\,\text{Hz}$$

12 $\phi = hf_0$
$$= 4.14 \times 10^{-15} \times 1.5 \times 10^{15}$$
$$= 6.2\,\text{eV}$$

$K_{max} = 4.14 \times 10^{-15} \times 2.2 \times 10^{15} - 6.2$
$$= 2.9\,\text{eV}$$

13 The stopping voltage is equivalent to the maximum kinetic energy of the photoelectrons, so $K_{max} = 1.95$ eV.

14 The work function is given by the *y*-intercept of the K_{max} versus frequency graph. Approximate values are:

Rb = 2.1 eV, Sr = 2.5 eV, Mg = 3.4 eV, W = 4.5 eV.

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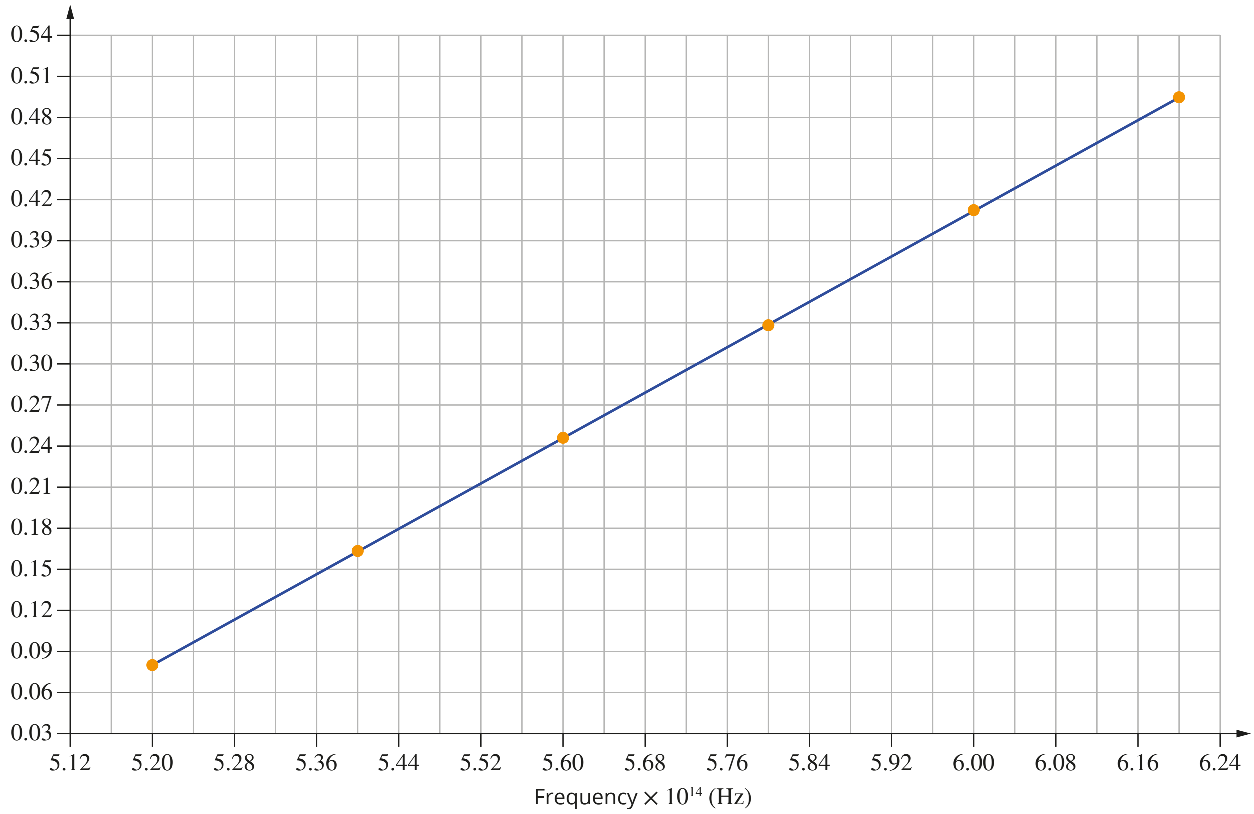
15 a



b gradient $= h = \frac{\text{rise}}{\text{run}}$

$= \frac{0.494 - 0.080}{6.20 \times 10^{4} - 5.20 \times 10^{14}}$

$= \frac{0.414}{1.00 \times 10^{14}}$

$= 4.1 \times 10^{-15}$ eV s

c The *x*-intercept on the graph will give an approximate value of 5.0×10^{14} Hz.

d No. The frequency of red light is below the threshold frequency for rubidium.

Frequency of the red light:

$f = \frac{c}{\lambda}$

$= \frac{3.00 \times 10^{8}}{680 \times 10^{-9}}$

$= 4.41 \times 10^{14}$ Hz

This is less than the threshold frequency of 5.0×10^{14} Hz so no photoelectrons will be emitted.

16 a True.

b False: The stopping voltage is reached when the photocurrent is reduced completely to zero.

c True.

d True.

17 C and D.

$\phi = hf_0 = \frac{hc}{\lambda_0}$

$\lambda_0 = \frac{hc}{\phi} = \frac{4.14 \times 10^{-5} \times 3.00 \times 10^{8}}{1.81}$

$= 6.86 \times 10^{-7}$ m

$= 686$ nm

Photons with wavelengths shorter than the threshold wavelength—i.e. violet light and ultraviolet radiation—will cause photoelectrons to be emitted.

18 $E = \frac{hc}{\lambda}$

$= \frac{4.14 \times 10^{-5} \times 3 \times 10^{8}}{475 \times 10^{-9}} = 2.61$ eV

$K_{max} = 2.61 - 2.36$

$= 0.25$ eV

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19 $K_{max} = hf - \phi$

$= \frac{hc}{\lambda} - \phi$

$0.80 = \frac{4.14 \times 10^{-5} \times 3.0 \times 10^{8}}{500 \times 10^{-9}} - \phi$

$\phi = 2.48 - 0.80$

$= 1.7\,\text{eV}$

20 The development of the quantum model of light is an excellent example of how the scientific community debates and evolves different models as different evidence is observed. The two main competing models of light were the wave model and the particle model. Neither can simply explain all the behaviour of light. While the wave model is needed to explain phenomena like diffraction patterns and polarisation, a particle model is needed when looking at the photoelectric effect.

Einstein proposed the quantum model of light, describing light as a photon. His theory was extended further when the wave properties of matter were observed, and the famous $E = mc^2$ equation mathematically described the conversion from matter to energy and vice versa.

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Chapter 12 Light and special relativity

12.1 Einstein's postulates

12.1 Review

1 D. They believed that all waves needed to travel in some sort of medium, so just as air is the medium for sound they invented another to be the medium for light.

2 A and D. An aircraft taking off is accelerating, as is a car going around a curve. These would be non-inertial frames of reference as they are accelerating.

3 The example should describe the movement of an object which highlights the non-inertial reference frame, i.e. a hanging pendulum in the spaceship will move from its normal vertical position when the spaceship accelerates.

4 The speed of the ball is greater for Jana than it is for Tom. The speed of the sound is greater forwards than it is backwards for Jana, while for Tom it is the same forwards and backwards. The speed of light is the same for Jana and Tom.

5 **a** $340 + 30 = 370\,\text{m s}^{-1}$
b $340 - 40 = 300\,\text{m s}^{-1}$
c $340 + 20 = 360\,\text{m s}^{-1}$
d $340\,\text{m s}^{-1}$

6 A. In order for the same events to be simultaneous in one inertial frame and not simultaneous in another inertial frame, time must act differently in each inertial frame of reference.

7 **a** In Anna and Ben's frame: $v = 25\,\text{m s}^{-1}$, so in Chloe's frame $v = 10 - 25 = 15\,\text{m s}^{-1}$ backwards.
b $d = vt = 15 \times 0.2 = 3\,\text{m}$ backwards
c 0.2 s

8 **a** $t = \frac{d}{v} = \frac{5}{50} = 0.1\text{ s}$
b $50\,\text{m s}^{-1}$ in all frames
c $d = vt = 10 \times 0.1 = 1\,\text{m}$
d $50\,\text{m s}^{-1}$ as always
e The light had to travel ≈4 m, so $t = \frac{4}{50} = 0.08\,\text{s}$ (approx.)

12.2 Evidence for special relativity

Worked example: Try yourself 12.2.1

TIME DILATION

A stationary observer on Earth measures a very fast scooter passing by, travelling at $2.98 \times 10^8\,\text{m s}^{-1}$. On the wrist of the rider is a watch on which the stationary observer sees 60.0 s pass. Calculate how many seconds pass by on the stationary observer's clock during this observation. Use $c = 3.00 \times 10^8\,\text{m s}^{-1}$.

Thinking	Working
Identify the variables: the time for the stationary observer is t, the proper time for the moving clock is t_0, and the velocities are v and the constant c.	$t = ?$ $t_0 = 60.0\,\text{s}$ $v = 2.98 \times 10^8\,\text{m s}^{-1}$ $c = 3.00 \times 10^8\,\text{m s}^{-1}$
Use Einstein's time dilation formula and the Lorentz factor.	$t = t_0\gamma$ $= \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$
Substitute the values for t_0, v and c into the equation and calculate the answer, t.	$t = \frac{60.0}{\sqrt{1-\frac{(2.98\times10^8)^2}{(3.00\times10^8)^2}}}$ $= \frac{60.0}{0.11528}$ $= 520\,\text{s}$

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Worked example: Try yourself 12.2.2

LENGTH CONTRACTION

A stationary observer on Earth measures a very fast scooter travelling by at $2.98 \times 10^8\,\mathrm{m\,s^{-1}}$. The stationary observer measures the scooter's length as 45.0 cm. Calculate the proper length of the scooter, measured when the scooter is at rest. Use $c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$.

Thinking	Working
Identify the variables: the length measured by the stationary observer is l, the proper length of the scooter is l_0, and the velocities are v and the constant c.	$l = ?$ $l_0 = 0.450\,\mathrm{m}$ $v = 2.50 \times 10^8\,\mathrm{m\,s^{-1}}$ $c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$
Use Einstein's length contraction formula and the Lorentz factor.	$l = \frac{l_0}{\gamma}$ $= {}_0\sqrt{1-\frac{v^2}{c^2}}$
Substitute the values for l_0, v and c into the equation and calculate the answer, l.	$_0 = \frac{0.450}{\sqrt{1-\frac{(2.98\times10^8)^2}{(3.00\times10^8)^2}}}$ $= \frac{0.450}{0.11528}$ $= 3.90\,\mathrm{m}$

Worked example: Try yourself 12.2.3

LENGTH CONTRACTION OF DISTANCE TRAVELLED

A stationary observer on Earth measures a very fast train approaching a tunnel at a speed of 0.986c. The stationary observer measures the tunnel's length as 123 m long. Calculate the length of the tunnel as seen by the train's driver.

Thinking	Working
Identify the variables: the length seen by the pilot is l, the proper length of the distance is l_0 and the velocity is v.	$l = ?$ $l_0 = 123\,\mathrm{m}$ $v = 0.986c\,\mathrm{m\,s^{-1}}$
Use Einstein's length contraction formula and the Lorentz factor.	$l = \frac{l_0}{\gamma}$ $= {}_0\sqrt{1-\frac{v^2}{c^2}}$
Substitute the values for l_0 and v into the equation. Cancel c and calculate the answer, l.	$l = 123 \times \sqrt{1-\frac{(0.986c)^2}{c^2}}$ $= 123 \times \sqrt{1-(0.986)^2}$ $= 123 \times 0.16675$ $= 20.5\,\mathrm{m}$

12.2 Review

1 In a device called a *light* clock, the *oscillation* of light is used as a means of measuring *time*, as the speed of light is *constant* no matter from which inertial frame of reference it is viewed.

2 'Proper time' is the time measured at rest with respect to the event. Proper times are always less than any other times.

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3 $t = t_0\gamma$

$$= \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{1.05}{\sqrt{1-\frac{(1.75\times10^8)^2}{(3\times10^{8}\ {}^{2}}}}$$

$$= \frac{1.05}{0.81223}$$

$= 1.29\,\text{s}$

4 $t = t_0\gamma$

$$= \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$75.0 = \frac{t_0}{\sqrt{1-\frac{(2.30\times10^8)^2}{(3\times10^{8}\ {}^{2}}}}$$

$t_0 = 75.0 \times 0.642$

$= 48.2\,\text{s}$

5 $t = \frac{t_0}{\gamma} = \frac{1}{\sqrt{1-0.5^2}} = 1.15\,\text{s}$

6 The equator clock is moving faster relative to the poles. It is also accelerating and hence will run slower. The effect is well below what we can detect, as the speed of the equator is 'only' about 460 m s^{-1}, which is about 1.5 millionths of *c*.

7 The length that a stationary observer measures in their own frame of reference. That is, the object (or distance) that is being measured is at rest with the observer.

8 Correct to three significant figures:

$l = \frac{l_0}{\gamma}$

$= l_0\sqrt{1-\frac{v^2}{c^2}}$

$= 1.00 \times \sqrt{1-\frac{(1.75\times10^8)^2}{(3.00\times10^8)^2}}$

$= 1.00 \times 0.81223$

$= 0.812\,\text{m}$

12.3 Momentum and energy

Worked example: Try yourself 12.3.1

RELATIVISTIC MOMENTUM

a Calculate the momentum, as seen by a stationary observer, provided to an electron with a rest mass of 9.11×10^{-31} kg, as it goes from rest to a speed of 0.985c. Use $c = 3.00 \times 10^8\,\text{m s}^{-1}$.

Thinking	Working
Identify the variables: the rest mass is m_0, and the velocity of the rocket ship is *v*.	$p_v = ?$ $m_0 = 9.11 \times 10^{-31}$ kg $v = 0.985c$
Use the relativistic momentum formula.	$p_v = \gamma m_0 v$
Substitute the values for m_0 and *v* into the equation and calculate the answer p_v.	$p_v = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} m_0 v$ $= \frac{1}{\sqrt{1-\frac{0.985^2c^2}{c^2}}} \times 9.11\times10^{-31}\times0.985\times3.00\times10^2$ $= 1.56 \times 10^{-21}\,\text{kg m s}^{-1}$

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b If three times the relativistic momentum from part (a) is applied to the electron, calculate the new final speed of the electron in terms of c.

Identify the variables: the rest mass is m_0, and the relativistic momentum of the rocket ship is p_v.	$p_v = 3 \times (1.56 \times 10^{-21)}$ $= 4.68 \times 10^{-21}\,\text{kg m s}^{-1}$ $m_0 = 9.11 \times 10^{-31}\,\text{kg}$ $v = ?$
Use the relativistic momentum formula, rearranged.	$p_v = \gamma m_0 v$ $= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} m_0 v$ $v = \frac{p_v}{m_0\sqrt{\left(1+\frac{p_v^2}{m_0^2c^2}\right)}}$
Substitute the values for m_0 and p_v into the rearranged equation and calculate the answer v.	$v = \frac{p_v}{m_0\sqrt{\left(1+\frac{p_v{}^2}{m_0^2c^2}\right)}}$ $= \frac{4.68\times10^{-21}}{9.11\times10^{-3}\sqrt{1+\frac{(4.68\times10^{-21})^2}{(9.11\times10^{-3}\ )\,3.00\times10^8)^2}}}$ $= 2.995 \times 10^8\,\text{m s}^{-1}$ $= 0.998c$

Worked example: Try yourself 12.3.2

FUSION

A further fusion reaction in the Sun fuses two helium nuclides. A helium nucleus and two protons are formed and 30 MeV of energy is released.

${}^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + 2{}^1_1\text{H}$

a How much energy is released in joules?

Thinking	Working
$1\text{ eV} = 1.602 \times 10^{-19}\,\text{J}$	$30\text{ MeV} = 30 \times 10^6 \times 1.602 \times 10^{-19}$ $= 4.8 \times 10^{-12}\,\text{J}$

b Calculate the mass defect for this reaction.

Thinking	Working
Use $\Delta E = \Delta mc^2$.	$\Delta m = \frac{\Delta E}{c^2}$ $= \frac{4.8\times10^{-12}}{(3.0\times10^{8\ 2}}$ $= 5.3 \times 10^{-29}\,\text{kg}$

12.3 Review

1 $p_v = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} m_0 v$

$= \frac{1}{\sqrt{1-\frac{(775)^2}{(3.00\times10^8)^2}}} \times 1230 \times 775$

$= 9.53 \times 10^5\,\text{kg m s}^{-1}$

2 $p_v = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} m_0 v$

$= \frac{1}{\sqrt{1-\frac{(0.850)^2c^2}{c^2}}} \times 1.99264824 \times 10^{-26} \times 0.850 \times 3.00 \times 10^8$

$= 9.65 \times 10^{-18}\,\text{kg m s}^{-1}$

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3 Since $v << c$; $p_0 = m_0 v$

$p = 1.99264824 \times 10^{-26} \times 800$

$= 1.59 \times 10^{-23}\,\text{kg m s}^{-1}$

4 $K = (\gamma - 1)mc^2$

$= \left(\dfrac{1}{\sqrt{1-\frac{(0.750c)^2}{c^2}}} - 1\right) \times 0.0123 \times (3.00 \times 10^8)^2$

$= 5.67 \times 10^{14}\,\text{J}$

5 $K = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.0123 \times (0.750 \times 3.00 \times 10^8)^2$

$= 3.11 \times 10^{14}\,\text{J}$

6 B. Relativistic kinetic energy depends on the momentum of the arrow. For the very fast arrow, the relativistic momentum is larger than the classical momentum.

7 $E_{\text{total}} = \gamma m_0 c^2$

$= \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}} m_0 c^2$

$= \dfrac{1}{\sqrt{1-\frac{(2.55\times10^8)^2}{(3.00\times10^8)^2}}} \times 210 \times (3.00 \times 10^8)^2$

$= 3.59 \times 10^{19}\,\text{J}$

8 $\Delta E = \Delta mc^2$

$= (4.00 \times 10^6 \times 10^3) \times (3.00 \times 10^8)^2$

$= 3.60 \times 10^{26}\,\text{J}$ (per second)

For a full day:

$= (3.60 \times 10^{26}) \times (24 \times 60 \times 60)$

$= 3.11 \times 10^{31}\,\text{J}$

CHAPTER 12 **REVIEW**

1 No object can travel at or beyond the speed of light, so the value of $\frac{v^2}{c^2}$ will always be less than 1. The number under the square root sign will alo, therefre, be a positive number less thanone. The square root of a positive number less than one will always be less than one as well. Note, however, when v is very small that $\frac{v^2}{c^2}$ is also very small and so the number under the square root sign will be very close to one. The result is a number very close to one. Some calculators may not be able to distinguish a number so close to one, but this is just due to the limitations of the calculator.

2 The speed is 0.000167 of c and so $\gamma \approx 1 + \frac{v^2}{2c^2} = 1 + \frac{(0.000167c)^2}{2c^2} = 1.000000014$.

3 A (postulate 2) and C (postulate 1)

4 At the poles. The Earth has a very small circular acceleration which is negligible for most purposes; however, at the poles it is even less.

5 C. There is no ‘fixed space’ in which to measure absolute velocities; we can only measure them relative to some other frame of reference.

6 Space and time are interdependent—motion in space reduces motion in time.

7 Both observers will see the light travel at $3.00 \times 10^8\,\text{m s}^{-1}$. According to Einstein’s second postulate, the speed of light will always be the same no matter what the motion of the light source or observer.

8 A and B. We are in the same frame in either case. C and D may be true, but they are not sufficient conditions as we must be in the same frame. (C did not specify with respect to what we were stationary.)

9 B. Crews A and B have no relative velocity between them so no time dilation occurs. They will both measure the time for C and the Earthlings moving in slow motion as the Earth has a high relative velocity.

10 You could not tell the difference between (i) and (iii), but in (ii) you could see whether an object like a pendulum hangs straight down.

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11 In your frame of reference time proceeds normally. Your heart rate would appear normal. As Mars is moving at a high speed relative to you, the clocks on Mars appear to be moving slowly as time for them, as measured by you, will be dilated.

12 $t = t_0\gamma$

$= \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$= \frac{20.0}{\sqrt{1-\frac{(2.00\times10^8)^2}{(3\times10^{8}\ {}^{2}}}}$

$= 26.8\,\text{s}$

13 a $t = t_0\gamma$

$= \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$1.5 = \frac{t_0}{\sqrt{1-\frac{(2.25\times10^8)^2}{(3\times10^{8}\ {}^{2}}}}$

$t_0 = 1.5 \times 0.6614$

$= 0.992\,\text{s}$

b 0.992 s (the swimmer sees the pool clock as t_0)

14 a $= \frac{l_0}{\gamma}$ and $\frac{l}{l_0} = \frac{1}{2}$

Thus $\sqrt{1-\frac{v^2}{c^2}} = 0.5$

$\frac{v^2}{c^2} = 1 - 0.25$

$v^2 = c^2 \times 0.75$

$v = 0.866c$ or $2.598 \times 10^8\,\text{m s}^{-1}$.

b No, it can't have doubled to over c! The contraction has doubled so this time $\gamma = 4$.

Then $\sqrt{1-\frac{v^2}{c^2}} = 0.25$

$\frac{v^2}{c^2} = 1 - 0.0625$

$v^2 = c^2 \times 0.9375$

$v = 0.968c$ or $2.90 \times 10^8\,\text{m s}^{-1}$.

15 a $t = t_0\gamma$

$= \frac{1.00}{\sqrt{1-\frac{v^2}{c^2}}}$

$= \frac{1.00}{\sqrt{1-\frac{(2.4\times10^{8}\ {}^{2}}{(3\times10^{8}\ {}^{2}}}}$

$= 1.67\,\text{s}$

b Length:

$= \frac{l_0}{\gamma}$

from part a, $\gamma = 1.67$

$= \frac{3.00}{1.67}$

$= 1.80\,\text{m}$

The height is unchanged at 1.0 m

16 a $t = \frac{d}{v} = \frac{5}{0.9} = 5.6$ years

b $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.9c)^2}{c^2}}} = 2.29$

$t_0 = \frac{t}{\gamma}$

$= \frac{5.6}{2.29}$

$= 2.45$ years

c Raqu measures the distance as only

$= \frac{l_0}{\gamma} = \frac{5}{2.29} = 2.183\,\text{ly}$

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17 **a** At 8000 m s^{-1}, $\frac{v}{c} = 2.7 \times 10^{-5}$ and γ will have a value of

$\gamma \approx 1 + \frac{v^2}{2c^2} = 1 + \frac{(2.7 \times 10^{-5})^2}{2} = 1 + 3.6 \times 10^{-10}$.

The difference (in mm) will therefore be $4 \times 10^9 \times 3.6 \times 10^{-10} = 1.4$ mm—hardly a problem!

b No, as the motion is perpendicular to the north–south direction this dimension is not affected.

18 **a** $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.995c)^2}{c^2}}} = \frac{1}{\sqrt{1-(0.995)^2}} = 10.01$

b No, they don't experience any difference in their own time frame.

c $t = \frac{25}{0.995} = 25.1$ years

About 25.1 years from our frame of reference.

d 2.51 years as $\gamma = 10$

e No! They see the distance between Earth and Vega foreshortened because of the high relative speed, so to them the distance is only about 2.5 ly.

19 Earth observer: the observer will not measure the proper time of the muon's life span. Instead they will see that the muon's time is slow according to the equation $t = t_0\gamma$, where t_0 is the rest life span of the muon. The result is that the observer sees the muon live a much longer time, t, and therefore makes it to the surface. Muon: the muon will see the Earth approach at a very high speed (approx. 0.992c) and will see the distance contracted. It will not be 15 km, but instead be much shorter according to the equation $l = \frac{l_0}{\gamma}$. The distance the muon travels is l.

20 B. As objects approach the speed of light, c, their inertia gets larger and larger and they become more and more difficult to accelerate.

21 $E_{total} = K + E_0$

$K = E_0 = mc^2$

$K = (\gamma - 1)\, mc^2 = mc^2$

$\therefore \gamma = 2$

$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 2$

$1 - \frac{v^2}{c^2} = \frac{1}{4}$

$\frac{v^2}{c^2} = \frac{3}{4}$

$v = \sqrt{0.75c^2} = 0.87c$

$= \sqrt{0.75} \times 3.00 \times 10^8$

$= 2.60 \times 10^8$ m s^{-1}

22 From the previous question $\gamma = 2$

relativistic mass $= \gamma m_0$

$= 2m_0$

$m = 2 \times 1.67 \times 10^{-27} = 3.34 \times 10^{-27}$ kg

23 $K = (\gamma - 1)mc^2$

$= \frac{1}{\sqrt{1-\frac{(0.960)^2c^2}{c^2}}} - 1 \times (5.30 \times 10^3) \times (3.00 \times 10^8)^2$

$= 1.23 \times 10^{21}$ J

24 $t = t_0\gamma$

$= \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$5.50 = \frac{t_0}{\sqrt{1-\frac{(2.75 \times 10^8)^2}{(3 \times 10^8)^2}}}$

$t_0 = 5.50 \times 0.3996$

$= 2.20$ s

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25 **a** Simply the height of the clock, 1 m.

b $t_A = \frac{d}{v} = \frac{1}{3.0 \times 10^8} = 3.33 \times 10^{-9}$ s

c $d = vt = ct_c$

d As the distance the ship moves in Chloe's frame is $0.9ct_c$ and the height of the clock is 1 m, the distance d which the light travels is given by $d^2 = (0.9ct_c)^2 + 1^2 = 0.81c^2t_c^2 + 1$.

As this also equals $c^2t_c^2$ (from part c), we find that:

$0.81c^2t_c^2 + 1 = c^2t_c^2$

$0.19c^2t_c^2 = 1$ and so

$t_c^2 = \frac{1}{0.19c^2}$

$t_c = \sqrt{\frac{1}{0.19c^2}}$

$= 7.6 \times 10^{-9}$ s

e $\frac{t_c}{_A} = \frac{7.6}{3.3} = 2.3$ which is the same as γ for $v = 90\%$ of c.

26 **a** $t = t_0\gamma$

$= \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$

$= \frac{2.2 \times 10^{-6}}{\sqrt{1-\frac{(0.992c)^2}{c^2}}}$

$= \frac{2.2 \times 10^{-6}}{\sqrt{1-0.992^2}}$

$= \frac{2.2 \times 10^{-6}}{\sqrt{1-0.992^2}}$

$= 1.74 \times 10^{-5}$ s or 17.4 μs

b Non-relativistic:

$d = vt = 0.992 \times 3 \times 10^8 \times 2.2 \times 10^{-6} = 655$ m

Relativistic:

$d = vt = 0.992 \times 3 \times 10^8 \times 1.74 \times 10^{-5} = 5178$ m

27 Proper time, t_0, because the observer can hold a stopwatch in one location and start it when the front of the carriage is in line with the watch and stop it when the back of the carriage is in line with it.

28 $= l_0\sqrt{1-\frac{v^2}{c^2}}$

$= 23.5 \times \sqrt{1-\frac{(660)^2}{(3.00 \times 10^8)^2}}$

$= 23.5 \times 1.0000$

$= 23.5$ m

At this speed, there is no difference in length.

29 $E = 50 \times 10^6$ J for 1 kg of gas.

So for 5 kg,

$E = 2.5 \times 10^7$ J

$\Delta E = \Delta mc^2$

$2.5 \times 10^7 = \Delta m \times (3.0 \times 10^8)^2$

$\Delta m = 2.78 \times 10^{-9}$ kg

30 In Newtonian physics, time and space is thought of as absolute; that is, that 1 second and 1 m are the same for all observers. Scientists discovered that the speed of light was in fact always 3×10^8 ms^{-1}, in all non-accelerating reference frames. These two situations are in conflict. Einstein developed the theory of special relativity to explain how the speed of light could be absolute in all reference frames. He applied the transformations that Lorentz developed to spacetime in order to have the speed of light absolute, dilating the time axis, and contracting the space axis. This activity looks specifically at the time dilation, rather than the space contraction.

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Module 7 Review

The nature of light

MULTIPLE CHOICE

1 B and E. The spectra in A and C are continuous, whereas the spectra in D is an absorption spectrum.

2 A. The electric and magnetic fields in electromagnetic radiation are perpendicular to each other and are both perpendicular to the direction of propagation of the radiation.

3 **a** B. In vapour lamps the excited electrons drop one or more energy levels and radiate photons as they do so.
b A. As the filament of an incandescent light bulb heats up, the free electrons in the tungsten atoms collide, accelerate and emit photons.
c D. The material that light-emitting diodes (LEDs) are made from contains a conduction band and a valence band.
d C. The photons in laser light all have the same wavelength and frequency and are in phase with each other, i.e. they are coherent.

4 B.

electron: $\lambda = \frac{h}{mv}$

$= \frac{6.63\times10^{-34}}{9.1\times10^{-31}\times7.5\times10^{6}}$

$= 9.7 \times 10^{-11}$ m

blue light: $\lambda = 470 \times 10^{-9} = 4.7 \times 10^{-7}$ m

X-ray: $c = f\lambda$, so $\lambda = \frac{c}{f} = \frac{3.0\times10^{8}}{5\times10^{17}} = 6 \times 10^{-10}$ m

proton: $\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{1.7\times10^{-21}} = 3.9 \times 10^{-13}$ m

Blue light has the longest wavelength.

5 D.

$c = f\lambda$ so $\lambda = \frac{c}{f} = \frac{3.0\times10^{8}}{3.0\times10^{-5}} = 10^{13}$ m

This is in the infrared region.

6 D.

$I_1 = \frac{1}{2}I_0$

$I_2 = I_1\cos^2 30 = 0.75I_1 = \frac{3}{8}I_0$

$I_3 = I_2\cos^2 30 = 0.75I_2 = \frac{9}{32}I_0$

7 B.

$3 \times 555 \times 10^{-9} = 5.00 \times 10^{-6} \times \sin\theta$

$\sin\theta = 0.333$

$\theta = 19.5°$

8 B. As the change in angle between each polarisation axis and the previous polarisation axis is the same, you can pick any two intensities to calculate the angle.

$\frac{I_1}{I_0} = 0.97 = \cos^2\theta$

$0.98 = \cos\theta$

$\theta = \cos^{-1} 0.98$

$= 9.97$

$= 10°$

9 B. V_0 is proportional to the energy of the incident photons. Since blue light has a higher frequency than yellow light, its photons have more energy.

10 C. The applied voltage works against the electrons as they try to reach the collector.

A and B are incorrect because the potential difference between the emitter and the collector does not affect these quantities.

11 A. The colour of the incident light is indicated by the value of V_0, while the intensity of the incident light is indicated by the size of the current.

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12 B, C, D, E. The photoelectric effect treats light as having particle-like properties as well as wave properties.

13 A.

$E = mc^2$

$= 4 \times 10^9 \times (3.00 \times 10^8)^2$

$= 3.6 \times 10^{26}\,\text{J}$

14 C. In this example, γ must be > 1, so A and B are not correct. The speed is much less than c, so D is not correct. C is the only feasible answer.

or

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{50\,000^2}{300\,000\,000^2}}} = 1.000\,000\,014 > 1$$

15 A (postulate 2) and C (postulate 1)

16 A or C. If the other craft is further away, its velocity away from Earth is $4 \times 10^6 - 0.4 \times 10^6 = 3.6 \times 10^6\,\text{m s}^{-1}$. If it is between us and Earth, it is $4 \times 10^6 + 0.4 \times 10^6 = 4.4 \times 10^6\,\text{m s}^{-1}$.

17 C. It was the elegance of Maxwell's equations that convinced Einstein that they, and their implications about light, were correct.

18 C. When the speed increases towards the speed of light, the distance travelled decreases.

19 A. Width and height are not affected as they are at right angles to the direction of motion, so a stationary observer will measure a moving object with a contracted length.

20 C. The greater the impulse, the greater the increase in the momentum. At speeds near that of light, this can be interpreted as an increase in the mass of the object, and so the velocity only increases a very small amount.

SHORT ANSWER

21 **a** $3.00 \times 10^8\,\text{m s}^{-1}$

b $c = f\lambda$

$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8}{10^{16}}$

$= 3 \times 10^{-8}\,\text{m}$

c From the diagram, they are ultraviolet waves.

d Any one of:

- UV lamps are used to sterilise surgical equipment in hospitals
- UV lamps are used to sterilise food and drugs
- UV rays help the body to produce vitamin D
- any other suitable use of UV.

22 A microwave oven is tuned to produce electromagnetic waves with a frequency of 2.45 GHz. This is the resonant frequency of water molecules. When food is bombarded with radiation at this frequency, the water molecules within the food start to vibrate. The energy of the water molecules is then transferred to the rest of the food, heating it up.

23 **a** Since $\Delta x = \frac{\lambda L}{d}$, if d is halved, Δx will be doubled.

b Since $\Delta x = \frac{\lambda L}{d}$, if L is doubled, Δx will be doubled.

c Since $\Delta x = \frac{\lambda L}{d}$ and $\lambda = \frac{v}{f}$, then $\Delta x = \frac{vL}{df}$ and if f is halved, Δx will be doubled.

d There will be a wider central band.

24 **a** The maximum light intensity occurs when the polarising axes are parallel.

b During a full 360° rotation, the polarising axes go from parallel, to perpendicular at 90°, to parallel at 180°, to perpendicular at 270° and back to parallel at 360°. After the start there are two instances of perpendicular polarising axes where the crossed polarising sheets transmit zero light and two instances of parallel polarising axes where the transmitted light intensity will be a maximum.

25 **a** The wave model and the particle (or corpuscular) model.

b Young's experiment resulted in bright and dark bands or fringes being seen on a screen. These can only be due to interference effects. The ability to interfere with one another constructively and destructively is a property of waves. Young's work therefore supported the wave model of light. The particle model could not explain the interference effects observed; it would predict just two bright bands.

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26 For the third dark band, pd = 2.5λ. For the fourth dark band, pd = 3.5λ. That is, the path difference is always one whole wavelength greater for each consecutive dark band. This value has been stated as equal to 500 nm in this example, therefore λ = 500 nm.

27 **a** K_{max} represents the maximum kinetic energy with which the electrons are emitted.

f is the frequency of the light incident on the metal plate (usually after passing through a filter, so it is not sufficient to call this the frequency of light from the source).

ϕ is the work function, which is the minimum energy required to eject an electron. It is a property of the metal.

b K_{max} is not altered.

c More photoelectrons are ejected each second, therefore more current is flowing.

28 In your frame of reference time proceeds normally. As Mars is moving at a high speed relative to you, people on Mars appear to be in slow motion as time for them, as seen by you, will be dilated.

29 $t = \frac{d}{v}$

$= \frac{2.50 \times 10^{-2}}{2.83 \times 10^{8}}$

$= 8.83 \times 10^{-11}$ s

So the moving particle lasts for 8.83×10^{-11} s.

$t = t_0\gamma$

$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

$8.83 \times 10^{-11} = \frac{t_0}{\sqrt{1 - \frac{(2.83 \times 10^8)^2}{c^2}}}$

$t_0 = 2.93 \times 10^{-11}$ s. So the particle lives for 2.93×10^{-11} s in the rest frame. This is reasonable, as the 'normal' lifetime should be shorter than when observed to be travelling at high speeds.

30 Muons have *very short* lives. On average, muons live for approximately 2.2 μs. Their speeds are measured as they travel through the atmosphere. A muon's speed is *very similar* to the speed of light. According to Newtonian laws, muons *should not* reach the Earth's surface. However, many *do*.

31 **a** Redshift is the stretching out of waves due to the Doppler effect. This can be heard when a car is approaching and driving away—the soundwaves are compressed as the car travels towards you and then stretched as the car travels away from you, which changes the pitch. Redshift is when light waves are stretched to the red end of the spectrum as a light source moves away from the observer.

Stellar spectra can be compared to the Sun's spectrum to see how much certain emission/absorption lines have been shifted. This will tell you whether the star is moving away from you and at what speed.

b Stellar spectra can give clues to a star's:

- surface temperature (the peak wavelength of the spectrum is related to the surface temperature by Wien's law)
- density (knowing the surface temperature of the star, it is possible to determine the energy given off each second by a unit of area of the surface, so that the surface area can be determined; if certain assumptions are made (i.e. the star is spherical), then the volume and hence the density can also be approximated)
- chemical composition (absorption/emission lines will be able to tell you what elements the star is made of)
- rotational velocity (similar to translational velocity, rotational velocity will affect the position of the emission spectrum; a broadening of the lines will occur as an object is turning).

c The absorption lines in this spectra are similar to those found in a type M star. This is a red star with a temperature range of 3500–2000 K.

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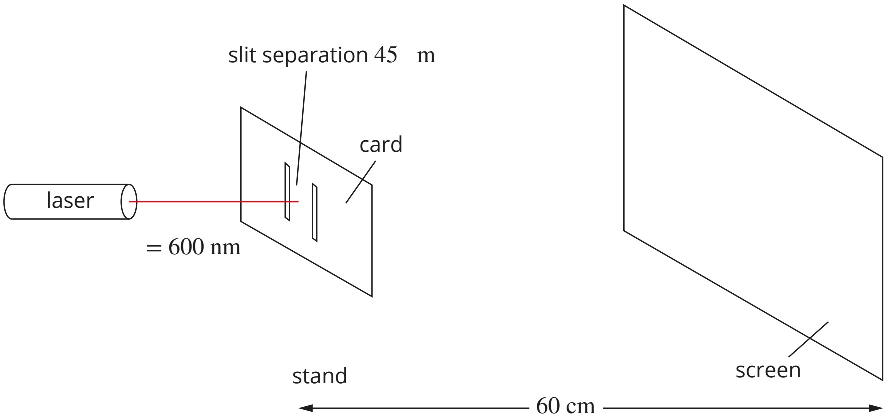
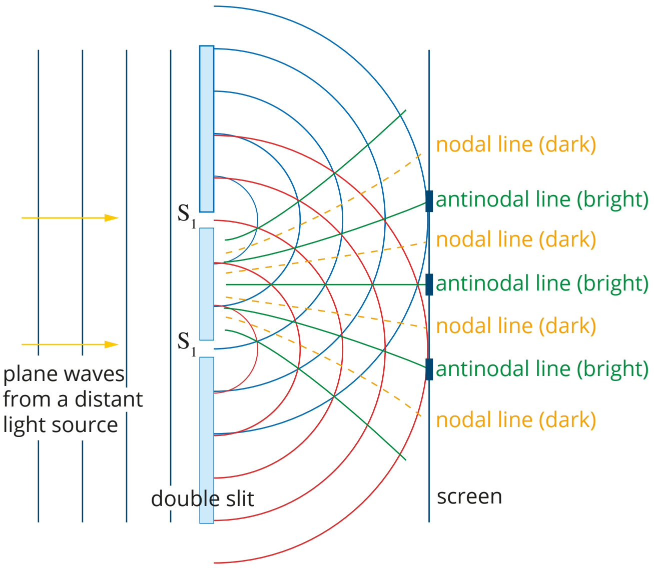
32 a



b



Areas of constructive interference create bands of high-intensity (bright) light on the screen, and areas of destructive interference create dark bands where light is 'cancelled out'.

c $d \sin\theta = m\lambda$

$\sin\theta = \frac{600\times10^{-9}}{45\times10^{-6}}$

$\theta = \sin^{-1} 0.013$

$= 0.76°$

$\sin\theta = \frac{\Delta x}{L}$

$\Delta x = 0.60 \times \sin 0.76$

$= 0.008$

$= 8.0\,\text{mm}$

d $d \sin\theta = m\lambda$

$\sin\theta = \frac{500\times10^{-9}}{45\times10^{-6}}$

$\theta = \sin^{-1} 0.011$

$= 0.64°$

$\sin\theta = \frac{\Delta x}{L}$

$\Delta x = 0.60 \times \sin 0.64$

$= 0.067$

$= 6.7\,\text{mm}$

$\frac{x_2}{x_1}$ 100% $\frac{6.7}{8.0}$ 100% 84%

So the fringe spacing decreased by 16% when the wavelength was changed from 600 nm to 500 nm.

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33 a In the particle model, the energy of the incident photons is set by their frequency according to $E = hf$. Each incident photon interacts with only one electron; therefore, the energy of the emitted electrons will depend only on the frequency of the incident light. Electron energy is not altered by altering the intensity because this only varies the number of photons, not their energy. Therefore, the energy of the emitted electrons is not affected, only the number emitted.

b The wave model predicts that altering the intensity of the light corresponds to waves of greater amplitude. Hence, the wavefronts should deliver more energy to the electrons and, therefore, the emerging electrons should have higher energy. This is not observed.

c The photoelectric effect supports the particle (photon) model of light because:

1 It predicts a minimum frequency (threshold frequency) and energy before electrons are emitted. The wave model predicts that any frequency should work.

2 The energy of the emitted electrons depends only on the frequency of the incident light. The wave model predicts that increasing the intensity of light would increase the energy of the emitted electrons.

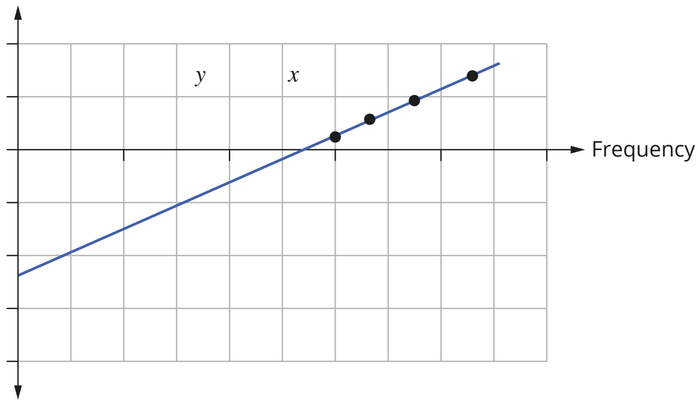
3 It explains an absence of any time delay before electrons are emitted when weak light sources are used. This time delay is suggested by the wave model.

d Use the wave equation, $c = f\lambda$, to find the frequency for each wavelength of light.

Frequency ($\times 10^{14}$ Hz)	Maximum kinetic energy (eV)
8.57	1.4
7.5	0.91
6.67	0.60
6.0	0.25

e

Photoelectric effect



f From the trend line in the graph from part (e),

$h = 4.4 \times 10^{-15}$ eV s

$\phi = 2.4$

g $\phi = 2.4 \pm 0.1$

Potassium fits inside this range and was used for the experiment.

34 a time $= \frac{\text{distance}}{\text{speed}}$

$= \frac{5}{0.9}$

$= 5.6$ years

b At 0.9c Raqu's time will seem to be shortened by a factor $\gamma = 2.3$, thus it will seem to take her only 2.4 years.

c Relative to her, the distance appeared to be foreshortened by the factor γ, thus the distance she travelled was much less than 5 light years.

d Atomic clocks enabled extremely short events to be timed to many decimal places. Differences in time for the same event to occur, when measured by observers in different inertial frames of reference, indicate that time is not uniform between the two inertial frames. These measurements support Einstein's special theory of relativity.

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35 **a** The mass difference is $4 \times 1.673 \times 10^{-27} - 6.645 \times 10^{-27} = 4.7 \times 10^{-29}$ kg

$E = mc^2 = 4.7 \times 10^{-29} \times (3 \times 10^8)^2$

$= 4.23 \times 10^{-12}$ J

b As the total energy produced by the Sun each second is 3.9×10^{26} J and the last answer gives us the energy produced for each helium atom, the number of helium atoms must be given by $\frac{3.9\times10^{26}}{4.2\times10^{-12}} = 9.3 \times 10^{37}$ every second.

c The mass lost by the Sun each second is given by $m = 9.3 \times 10^{37} \times 4.7 \times 10^{-29} = 4.37 \times 10^9$ kg.

In one day this will be $4.37 \times 10^9 \times 24 \times 60 \times 60 = 3.8 \times 10^{14}$ kg.

d $p = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} m_0 v$

$= \frac{1}{\sqrt{1-\frac{750^2}{(3\times10^8)^2}}} \times 1\ 99 \times 10^{-26} \times 750$

$= 1.49 \times 10^{-23}$ kg m s^{-1}

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Chapter 13 Origins of the elements

13.1 The big bang

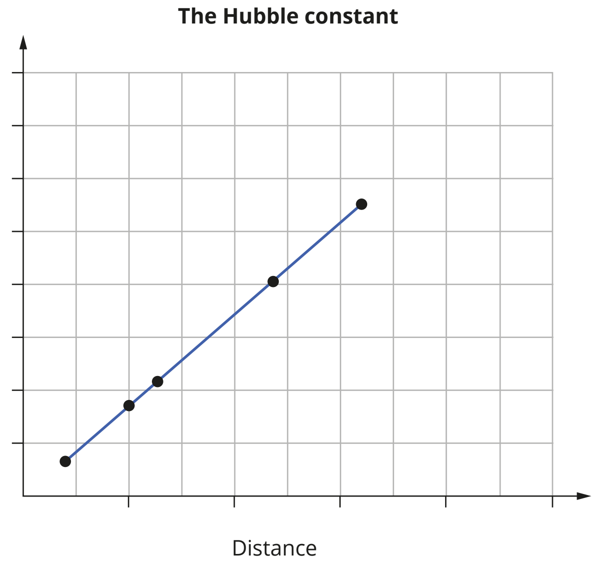
Worked example: Try yourself 13.1.1

CALCULATING THE HUBBLE CONSTANT

Astronomers have found the following data for the speed of distant galaxies:

Distance (Mpc)	Velocity ($km\,s^{-1}$)
20	1300
50	3400
65	4300
120	8100
160	11000

a Analyse the data to find a value for the Hubble constant.

Thinking	Working
Construct a graph of the distance versus the velocity and draw a line of best fit.	
Interpret the graph.	The gradient of the graph is equal to 69.0. This means that for this data the Hubble constant is equal to 69 (to the correct number of significant figures). $H_0 = 69\,km\,s^{-1}\,Mpc^{-1}$

b Using the Hubble constant found in part a, calculate the recessional speed of a galaxy that is 700 Mpc from the Earth.

Thinking	Working
State Hubble's law.	$v = H_0 d$
Substitute the given values into Hubble's law.	$v = 69 \times 700$
Calculate the speed.	$v = 48\,000\,km\,s^{-1}$ (to two significant figures)

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13.1 Review

1 The steady state theory suggests that the universe is infinite and that matter is being created all the time at just the right rate to keep the density constant as it expands.

2 The cosmic microwave background radiation was generated in the initial very hot state of the universe, which the steady state theory did not support. The steady state theory had no explanation for the CMB radiation.

3 During inflation, pair production resulted in matter being created as pairs could not annihilate because of the rapid expansion. As the universe cooled, the energy of photons decreased which consequently decreased pair production and most particles annihilated with their antiparticles. However, there was a slight imbalance of matter and antimatter which left the universe mostly with matter particles.

4 The matter created during inflation would never have condensed to form atoms, and therefore, ultimately, galaxies or stars.

5 Annihilation, which involves the conversion of an electron and positron into two photons.

6 There was slightly more matter than antimatter in the early universe. Therefore, once all matter–antimatter pairs had annihilated only matter remained.

7 **a** At that time hydrogen, helium and lithium were the only elements to exist. They had been formed in the first few minutes while the universe was hot enough for fusion to occur. Other elements didn't form as the universe was still hot enough that photons had enough energy to ionise any atoms that did form.

b Other elements formed from the supernovae of the early, large stars. These supernovae not only formed other stars, but eventually whole solar systems. Elements heavier than iron are actually thought to be produced in neutron star mergers. The recent detection of the kilonova associated with the neutron star merger GW170817 lends support to this theory.

8 Hubble's observations of galaxies showed that the further away a galaxy was, the faster it appeared to be moving away from us. He concluded that this can be explained if the universe between the galaxies is expanding as this would cause them to appear to be moving away from each other. Therefore this supported earlier predictions of an expanding universe.

13.2 The life cycle of a star

Worked example: Try yourself 13.2.1

DETERMINING THE LUMINOSITY OF A STAR USING THE H–R DIAGRAM

A main sequence star is observed to have a surface temperature of 20 000 K. What is its approximate luminosity?

Thinking	Working
Determine where on the H–R diagram this star would sit.	The main sequence is a band that runs from bottom right to top left on the H–R diagram, and a temperature of 20 000 K should be at the top left of this band.
Draw a vertical line from the required temperature on the *x*-axis to intersect with the luminosity curve. Draw a horizontal line from the intersection of the luminosity curve to the *y*-axis.	The star's luminosity is approximately 100 to 1000 times that of the Sun.

13.2 Review

1 A Hertzsprung–Russell diagram plots the luminosity of a star (which is derived from the absolute magnitude) against the spectral type of stars (from which the temperature of the star is derived).

2 Along the main sequence, luminosity increases with the surface temperature.

3 The continuous spectrum provides information about the surface temperature of the star. The absorption spectrum gives information about the elements present.

4 A continuous black-body spectrum and an absorption spectrum both contain a distribution intensity across a range of wavelengths that is determined by the temperature of the object that emitted the radiation. The difference between these two is that the absorption spectrum has had radiation absorbed at various frequencies; these are called absorption lines. An emission spectrum, unlike a continuous black-body spectrum, only contains intensities at certain narrow bands of wavelength; these are called emission lines.

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5 Less-broad spectral patterns indicate a large star. A peak in the red section of the visible spectrum coincides with a cooler star. A red giant would best fit these observations.

6 A. Luminosity (vertical axis) does increase with temperature (horizontal axis) but not in a straight line, so option A is the best answer.

7 An A-type star is to the left of the Sun along the main sequence on the H–R diagram. This means it has a higher surface temperature and a greater luminosity than the Sun.

8 D. The H–R diagram plots the luminosity of a star (which is derived from the absolute magnitude) against the spectral type of stars (from which the temperature of the star is derived).

13.3 The life and death of stars

Worked example: Try yourself 13.3.1

MASS–ENERGY EQUIVALENCE

The visible portion of the energy the Sun is producing each second is approximately equal to $5.0 \times 10^{25}\,\mathrm{J\,s^{-1}}$. At what rate is the Sun losing mass due to this energy loss?
(Use $c = 3.0 \times 10^{8}\,\mathrm{m\,s^{-1}}$.)

Thinking	Working
The energy comes from the fusion of hydrogen into helium with a corresponding loss in the potential energy of the nuclei. This loss of energy will correspond to a mass loss given by Einstein's equation $E = mc^2$.	$E = mc^2$ $E = 5.0 \times 10^{25}\,\mathrm{J\,s^{-1}}$ $c = 3.0 \times 10^{8}\,\mathrm{m\,s^{-1}}$ $m = ?$
Rearrange $E = mc^2$ in terms of mass and solve.	$E = mc^2$ $m = \frac{E}{c^2}$ $= \frac{5.0\times10^{25}}{(3.0\times10^{8})^{2}}$ $= 5.6 \times 10^{8}\,\mathrm{kg\,s^{-1}}$ So the Sun is losing mass due to visible radiation at a rate of $5.6 \times 10^{8}\,\mathrm{kg\,s^{-1}}$.

13.3 Review

1 Nuclear fusion reactions in the Sun involve fusing hydrogen nuclei to produce helium nuclei. In contrast, hydrogen burning in oxygen involves only the electrons in the outer shell of the atoms. Fusion reactions are much more energetic than chemical reactions. The energy involved in nuclear reactions is about 100 million times greater than chemical reactions.

2 $E = mc^2$

$= 6 \times 10^{9} \times (3.0 \times 10^{8})^{2}$

$= 5.4 \times 10^{26}\,\mathrm{J} = 5 \times 10^{26}\,\mathrm{J}$ (to one significant figure)

3 protostar → main-sequence star → red giant → white dwarf

4 The total mass of the products is less than the total mass of the reactants. This mass defect implies there was a release of energy from the system which can be quantified using Einstein's mass–energy equation.

5 planetary nebulae

6 The Sun is most likely to initially expand into a red giant before collapsing and becoming a white dwarf.

7 The force of pressure (mostly thermal pressure with some radiation pressure) pushing outwards is balanced by the inward pull of gravity at hydrostatic equilibrium. The thermal pressure is a result of the energy the particles in the gas or plasma have due to their motion and is exerted through collisions. The radiation pressure is a result of the momentum that photons carry and impart during collisions. The force of gravity is a result of the gravitational field acting on objects that have mass.

8 Nitrogen has an atomic number of 7 which means it has seven protons in the nucleus. Carbon only has six protons in the nucleus so one more proton must have been produced within the nucleus. From the CNO cycle in Figure 13.3.11, this process occurs through the addition of a proton (hydrogen nucleus) to a carbon atom.

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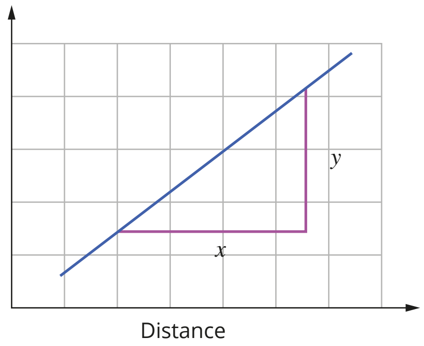
CHAPTER 13 REVIEW

1 Hubble measured the distance and redshift of many galaxies. He then calculated the velocity of recession for these galaxies and noticed that the further away a galaxy was, the faster it was receding.

2 pair production and inflation

3



slope = $\frac{y}{x}\ \ \frac{v}{d} = H_0$

$\therefore v = H_0 d$, which is Hubble's law.

4 Predictions made by physicists as part of the work on the big bang theory say that the early universe should have been composed of mainly hydrogen and some helium. Observations of the cosmic microwave background radiation and the spectra of old stars yield results that agree closely with these predictions.

5 If the universe is expanding and the galaxies are getting further apart as the universe between them expands, this implies that in the past they were closer together. If we extrapolate back in time, the finite amount of mass and energy in the universe would have been contained in a smaller and smaller volume. This would mean the average energy of the particles (i.e. the temperature) would increase. At the very beginning of the universe, this would mean an infinite density and temperature—a hot big bang beginning for the universe.

6 A larger Hubble constant would suggest a smaller age of the universe as the age of the universe can be estimated by calculating the reciprocal of the Hubble constant.

7 the cosmic microwave background (CMB) radiation

8 spectral class, surface temperature and chemical composition

9 Their spectra show the same lines as our Sun, and these lines correspond to the 98 known elements in our periodic table.

10 Betelgeuse, Rigel, Polaris (pole star), Arcturus

11 Spectral class M stars have low surface temperatures of less than 3500 K. These are the only stars with temperatures in their atmospheres low enough for molecules to exist and therefore to produce absorption spectra with strong lines for those molecules.

12 **a** white dwarfs
b main sequence
c supergiant stars
d red giant stars

13 Like all stars Rigel would have started from a dust and gas cloud collapsing to form a protostar.

Rigel would have spent most of its lifetime on the main sequence fusing hydrogen into helium.

As Rigel starts to convert silicon to iron as the main fusion process, less energy will be produced than needed for the fusion process. It will begin to collapse.

As a giant star, the core of Rigel can then be expected to heat to billions of degrees in a fraction of a second. An explosive supernova results.

The final stage will be either a neutron star or a black hole.

14 Stars are born into the middle (approximately) of the main sequence, after rapidly igniting once the protostar collapses.

15 The Sun is close to the centre of the H–R diagram; that is, in terms of the overall range, it is of average temperature and average brightness. However, most stars are actually cooler and fainter than the Sun.

16 $E = mc^2$

$= 4 \times 10^9 \times (3 \times 10^8)^2$

$= 3.6 \times 10^{26}$ J $= 4 \times 10^{26}$ J (to one significant figure)

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17 Hydrogen fusing to helium is the main reaction in main-sequence stars, while helium fusing to carbon via the triple alpha process is the main reaction in red giants.

18 New elements are formed through nuclear fusion reactions in a process called nucleosynthesis. During their lives on the main sequence, stars like the Sun form helium. When they become a red giant they form elements like carbon, nitrogen and oxygen. Larger stars form elements up to iron while they are giants and other heavier elements when they explode as a supernova.

19 The predictions of the big bang theory are very accurate. Theoretical predictions indicate that the elements formed in the early universe should have been approximately 25% helium and the remainder hydrogen. Observations determining the abundance of these elements in stars and galaxies agree closely with the predicted values. The predicted abundance of other elements and isotopes predicted by the big bang theory were tested by the WMAP mission, and again observations closely matched the abundances from theoretical predictions.

20 Experimental evidence and observation are the most fundamental parts of the scientific method. Predictions are made by models and theories. These are then tested against experimental evidence or observation. If the predictions are supported by the evidence then scientists can be confident about their theory or model and can then test them in other ways. If predictions are not supported then the theory or model must be altered or new ones developed and tested against evidence. It is through this process that scientists can develop theories or models that are accepted, continue to be refined and become better at explaining what is being observed.

The steady state theory was another model that was proposed to describe the evolution of the universe. By using observational evidence for the big bang, such as the cosmic microwave background (CMB) radiation and the increasing acceleration of galaxies further away from the Milky Way (seen through cosmological redshift), astrophysicists were able to rule out the steady state theory and instead now use the big bang theory to model the expansion and evolution of the universe. Theories such as this always rely on the most current evidence and are able to change and adapt depending on what scientists have discovered.

21 Non-fusing helium builds up in the core of the star and a shell of hydrogen begins fusing around a non-fusing helium rich core. This causes the outer layers of the star to expand and cool, forming a red giant.

22 Neutron stars are extremely dense objects composed mainly of neutrons held up against gravity. In contrast, the gravity within a black hole has exceeded any force than can resist collapse and have obtained some much higher, possibly infinite, density. The escape velocity of neutron stars, unlike black holes, is not greater than the speed of light so they are visible through radiation that escapes their surface.

23 The expanding balloon is an analogy used to explain the expansion of the universe. The surface of the balloon represents a two-dimensional space. The universe is a three-dimensional space. The space within the balloon and outside the balloon does not represent anything in this analogy; only the two-dimensional surface is significant.

As the balloon is inflated, the distance between the galaxies increases. From each galaxy, all the other galaxies are seen to move away. The further the galaxies are apart, the faster they seem to move away from each other. However, as the galaxies have not moved in the reference frame that was drawn on the balloon, they do not have relative velocities.

Unlike the steady state theory, where matter is being created all the time at just the right rate to keep the density of the universe constant, the big bang theory shows that the origin of matter occurred in the very early universe. Evidence of cosmological redshift (the expansion of space) supports the big bang theory, which helps us to understand the processes by which galaxies (and then stars and planets) were all formed. The heavier elements were then created due to the life cycles of stars. Fusion processes within stars create elements such as carbon, which, due to supernova events, then go on to create planets and even life.

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2 $v = \sqrt{\frac{2qV}{m}}$

$= \sqrt{\frac{2 \times 1.602 \times 10^{-19} \times 10 \times 10^3}{1.88 \times 10^{-28}}}$

$= 4.13 \times 10^6 \, \text{m s}^{-1}$

$r = \frac{mv}{qB}$

$= \frac{1.88 \times 10^{-28} \times 4.13 \times 10^6}{1.602 \times 10^{-19} \times 1.5 \times 10^{-3}}$

$= 3.23 \, \text{m}$

The muon is slower as there is a greater mass to accelerate. And with more mass for the same charge as the electron, the turning radius is greater.

3 Water is more volatile (evaporates easily) than oil, so the mass and size of the droplets change more rapidly than oil drops. (In Millikan's original experiment, the water evaporated in around two seconds!)

4

Atomic model	Description
solid-ball model	An indivisible ball. Where the name atom first appeared as it was referred to as *atomos,* meaning indivisible.
plum pudding model	From Thomson's experiments with electrons, he proposed the plum pudding model in 1904. The atom in this model is a ball of positive charge with negative charges embedded within it.
nuclear model	The majority of the mass of an atom in this model is in a small positive nucleus which is surrounded by negative electrons.
planetary model	The electrons in this model orbit the positive nucleus in specific pathways, like planets orbiting the Sun.

14.2 Nuclear model of the atom

14.2 Review

1 The model predicts particles are affected only by the diffuse electrical charge. Electrical repulsion between the positively charged alpha particles and the diffuse positive atomic charge leads to a small amount of repulsion, observed as scattering of less than 2° from their initial path.

2 Higher-energy (higher velocity) alpha particles will encounter the atoms with a greater momentum, and the effect of the electrical repulsion would be reduced with consequent reduced scattering.

3 The alpha particles and nucleus are both positively charged and experience electrical repulsion. The particles cannot touch as the repulsion becomes infinite as they approach.

4 As neutrons decay (decompose) to a proton and electron, the mass of the neutron is greater, with the difference being approximately the mass of an electron. (Specifically, a proton is 99.86% the mass of a neutron, while an electron is 0.054%.)

CHAPTER 14 REVIEW

1 The number of protons is equal to the number of electrons. The number of neutrons does not affect the electrical charge.

2 There is a gap or hole in the anode through which some of the electrons pass.

3 Charged particles experience a force from the magnetic field that is proportional to the particle's velocity, constantly accelerating the charged particle.

4 $v = \sqrt{\frac{2qV}{m}}$

$= \sqrt{\frac{2 \times 1.602 \times 10^{-19} \times 2.5 \times 10^3}{9.109 \times 10^{-31}}}$

$= 2.97 \times 10^7 \, \text{m s}^{-1} = 3.0 \times 10^7 \, \text{m s}^{-1}$

5 $v = \sqrt{\frac{2qV}{m}}$

$= \sqrt{\frac{2 \times 1.602 \times 10^{-19} \times 4.5 \times 10^3}{9.109 \times 10^{-31}}}$

$= 3.98 \times 10^7 \, \text{m s}^{-1} = 4.0 \times 10^7 \, \text{m s}^{-1}$

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6 **a** $F = qvB\sin\theta$

$= 1.602 \times 10^{-19} \times 6.4 \times 10^{6} \times 9.1 \times 10^{-3} \times \sin 90$

$= 9.3 \times 10^{-15}\,\text{N}$

This net force causes centripetal acceleration, so the direction is towards the centre of the circular motion.

b $r = \frac{mv}{qB}$

$= \frac{9.109 \times 10^{-31} \times 6.4 \times 10^{6}}{1.602 \times 10^{-19} \times 9.1 \times 10^{-3}}$

$= 4.0 \times 10^{-3}\,\text{m}$

7 $B = \frac{mv}{qr}$

$= \frac{9.109 \times 10^{-31} \times 4.3 \times 10^{6}}{1.602 \times 10^{-19} \times 4.2 \times 10^{-2}}$

$= 5.8 \times 10^{-4}\,\text{T}$

8 $v = \frac{rBq}{m} = rB\frac{e}{m}$

$= 0.06 \times 1.50 \times 10^{-4} \times 1.76 \times 10^{11}$

$= 1.58 \times 10^{6}\,\text{m s}^{-1}$

9 **a** $qE = mg$

$q = \frac{mg}{E}$

Substitute $E = \frac{V}{d}$

$q = \frac{mgd}{V}$

b density $= \rho = \frac{\text{mass}}{\text{volume}}$

volume of a sphere $= \frac{4}{3}\pi r^3$

$\rho = \frac{3m}{4\pi r^3}$

$m = \frac{4}{3}\rho \pi r^3$

Substitute this value into the equation for q in part **a**

$q = \frac{4 \quad r^3 gd}{3V}$

c $q = \frac{4 \quad r^3 gd}{3V}$

$V = \frac{4 \quad r^3 gd}{3q}$

$= \frac{4 \times 900 \times \pi \times (0.5 \times 10^{-6})^3 \times 9.8 \times 0.05}{3 \times 1.602 \times 10^{-19}}$

$= 1.4 \times 10^{3}\,\text{V}$

d For a droplet that has a charge of some multiple n of the charge of an electron q_e, the total charge is equal to $q = nq_e$.

$V = \frac{4 \quad r^3 gd}{3nq_e}$

$= \frac{1.4 \times 10^{3}}{n}$

$= \frac{1.4 \times 10^{3}}{3}$

$= 480\,\text{V}$

10 C, A, B

11 The mass of an atom is concentrated in the nucleus, and the nucleus occupies a tiny fraction of the volume of the atom.

12 Electrons. Rutherford's model explained the nucleus but provided no explanation for the arrangement of the electrons. Atomic absorption spectra showed light could be absorbed and emitted by atoms, and in discrete bands. Rutherford's model did not address how this could occur, whether by the electrons or in the atomic nucleus.

13 Rutherford could not have concluded anything from this. The null result would have been consistent with the plum pudding model. However, if the nucleus was very small, such that the backscattering might have occurred with one in one million collisions, it would have still occurred but have been harder to detect. Absence of evidence is not evidence of absence.

14 The mass of the electron is $\frac{1}{1836}$ of the mass of a proton. For each group of 1837 fans, one will be for Team Proton and 1836 for Team Electron. For 51 436 fans, there will be 28 for Team Proton and 51 408 for Team Electron.

15 Divided atoms would have different numbers of protons in their nuclei, and are therefore different elements. This gives the daughter elements different chemical properties to the mother atom.

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16 Protons have a single positive charge, whereas neutrons are electrically neutral. They are approximately the same mass (neutrons are slightly heavier).

17 $v = \frac{qrB}{m}$

$= \frac{1.602 \times 10^{-19} \times 4.6 \times 10^{-2} \times 9.4 \times 10^{-4}}{9.109 \times 10^{-31}}$

$= 7.6 \times 10^{6}\,\text{m s}^{-1}$

18 $v = \sqrt{\frac{2qV}{m}}$

$= \sqrt{\frac{2 \times 1.602 \times 10^{-19} \times 5 \times 10^{3}}{1.88 \times 10^{-28}}}$

$= 2.92 \times 10^{6}\,\text{m s}^{-1}$

$r = \frac{mv}{qB}$

$= \frac{1.88 \times 10^{-28} \times 2.92 \times 10^{6}}{1.602 \times 10^{-19} \times 1.55 \times 10^{-3}}$

$= 2.21\,\text{m}$

19 **a** $qV = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2qV}{m}}$

$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 3.5 \times 10^{3}}{1.67 \times 10^{-27}}}$

$= 8.2 \times 10^{5}\,\text{m s}^{-1}$

b $\frac{mv^2}{r} = qvB$

$B = \frac{mv}{qr}$

$= \frac{1.67 \times 10^{-27} \times 8.2 \times 10^{5}}{1.6 \times 10^{-19} \times 5.0 \times 10^{-2}}$

$= 0.17\,\text{T}$

20 The atom is too small to be visible to the eye, so the evolution of the model relied on other evidence. Through a series of experiments scientists were able to deduce information about electrical charge, size and internal structure.

The plum pudding model of the atom was proposed after the discovery of the charge and mass of electrons, and the knowledge that atoms were electrically neutral and so must also contain positive charge.

The Geiger–Marsden experiment involved firing alpha particles (positive in charge) at a thin piece of metal. The expected result was that all the alpha particles would go straight through the foil, as the plum pudding model had no significant concentration of positive charge that would repel or deviate the alpha particles. However, the alpha particles were seen to deviate or even reflect. From this, Rutherford changed the model of the atom to include the positive charge in the centre, with the electrons on the edge.

The discovery of the neutron was deduced from the ejection of heavy electrically neutral particles, and the quantised energy levels of the electrons have further evolved the model of the atom (discussed in Chapter 15).

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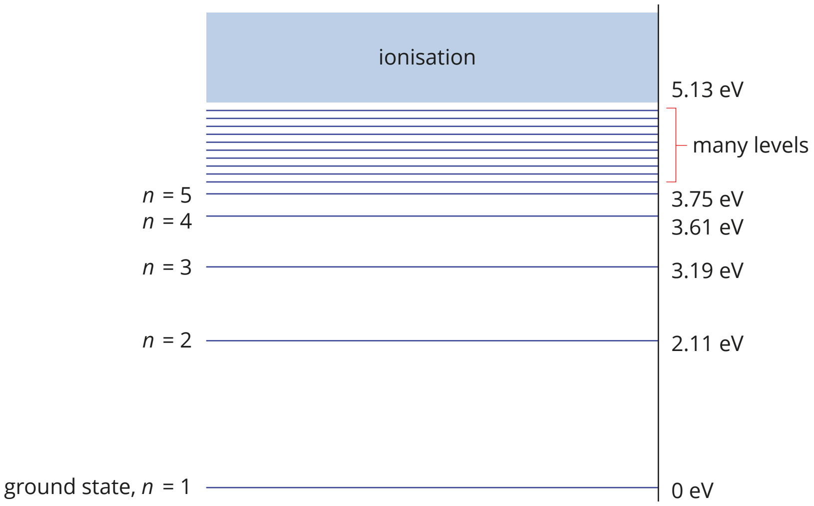
Chapter 15 Quantum mechanical nature of the atom

15.1 Bohr model

Worked example: Try yourself 15.1.1

ENERGY LEVELS

The energy levels for sodium gas are shown. Work out the energy of the light that is produced as an electron drops from the $n = 4$ to the $n = 3$ state.



Thinking	Working
Using the figure, find the energy (in eV) of each level involved.	$n = 4$, $E_4 = 3.61$ eV $n = 3$, $E_3 = 3.19$ eV
Calculate the difference between these levels.	$\Delta E = E_4 - E_3$ $= 3.61 - 3.19$ $= 0.42$ eV

Worked example: Try yourself 15.1.2

SPECTRAL ANALYSIS

In the Sun's absorption spectrum, one of the dark Fraunhofer lines corresponds to a frequency of 6.9×10^{14} Hz. Calculate the energy (in joules) of the photon that corresponds to this line. Use $h = 6.626 \times 10^{-34}$ J s.

Thinking	Working
Recall Planck's equation.	$\Delta E = hf$
Substitute in the appropriate values and solve for ΔE.	$\Delta E = 6.626 \times 10^{-34} \times 6.9 \times 10^{14}$ $= 4.6 \times 10^{-19}$ J

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Worked example: Try yourself 15.1.3

USING THE BOHR MODEL OF THE HYDROGEN ATOM

Calculate the wavelength (in nm) of the photon produced when an electron drops from the $n = 3$ energy level of the hydrogen atom to the $n = 1$ energy level. Identify the spectral series to which this line belongs.

Use Figure 15.1.7 to calculate your answer.

Thinking	**Working**
Identify the energy of the relevant energy levels of the hydrogen atom.	$n = 3$, $E_3 = -1.5\,\text{eV}$ $n = 1$, $E_1 = -13.6\,\text{eV}$
Calculate the change in energy.	$\Delta E = E_3 - E_1$ $= -1.5 - (-13.6)$ $= 12.1\,\text{eV}$
Calculate the wavelength of the photon with this amount of energy.	$\lambda = \frac{hc}{E}$ $= \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{12.1}$ $= 1.03 \times 10^{-7}\,\text{m}$ $= 103\,\text{nm}$
Identify the spectral series.	The electron drops down to the $n = 1$ energy level. Therefore, the photon must be in the Lyman series.

Worked example: Try yourself 15.1.4

USING THE RYDBERG FORMULA

Using the Rydberg formula, calculate the wavelength (in nm) of the photon produced when an electron drops from the $n = 4$ energy level of the hydrogen atom to the $n = 1$ energy level. Identify the spectral series to which this line belongs.

Thinking	**Working**
Identify the known variables.	$n_f = 1$ $n_i = 4$ $R = 1.097 \times 10^{7}\,\text{m}^{-1}$
Recall the Rydberg formula.	$\frac{1}{\lambda} \quad R\left[\frac{1}{n_f^2} \quad \frac{1}{n_i^2}\right]$
Solve for the wavelength, λ.	$\frac{1}{\lambda} \quad R\left[\frac{1}{n_f^2} \quad \frac{1}{n_i^2}\right]$ $= 1.097 \times 10^{7} \times \left[\frac{1}{1^2} - \frac{1}{4^2}\right]$ $= 1.097 \times 10^{7} \times 0.94$ $\lambda = 9.72 \times 10^{-8}$ $= 97.2\ \text{nm}$
Identify the spectral series.	The electron drops down to the $n = 1$ energy level. Therefore, the photon must be in the Lyman series.

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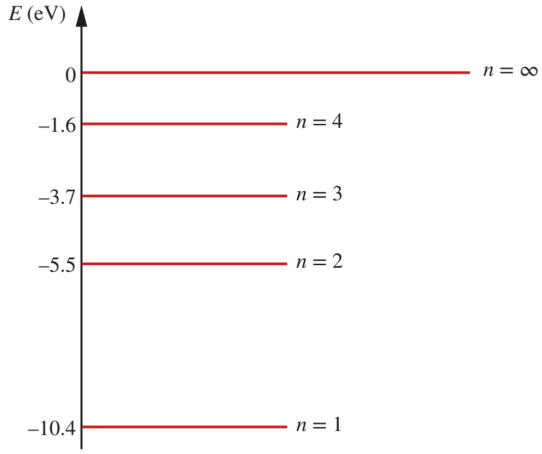
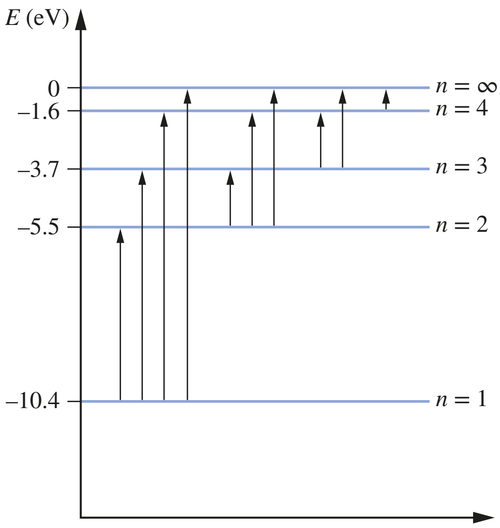
Worked example: Try yourself 15.1.5

ABSORPTION OF PHOTONS

Some of the energy levels for atomic mercury are shown in the diagram below.



Light with photon energies 6.7 eV, 9.0 eV and 11.0 eV is incident on some mercury gas. What could happen as a result of the incident light?

<table>
<tr><th>Thinking</th><th>Working</th></tr>
<tr><td>Check whether the energy of each photon corresponds to any differences between energy levels by determining the difference in energy between each level.</td><td>
</td></tr>
<tr><td>Compare the energy of the photons with the energies determined in the previous step. Comment on the possible outcomes.</td><td>A photon of 6.7 eV corresponds to the energy required to promote an electron from the ground state to the second excited state (n = 1 to n = 3). The photon may be absorbed.
A photon of 9.0 eV cannot be absorbed.
A photon of 11.0 eV may ionise the mercury atom. The ejected electron will leave the atom with 0.6 eV of kinetic energy.</td></tr>
</table>

15.1 Review

1 An emission spectrum for an element typically consists of a series of spaced coloured lines on a black background.

2 The different coloured lines in the emission spectrum of an atom correspond to the possible electron transitions between energy levels within the atom.

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3 The energy levels within an atom are commonly represented as horizontal lines on a graph.

4 $\Delta E = hf$

$= 6.63 \times 10^{-34} \times 6.0 \times 10^{14}$

$= 4.0 \times 10^{-19}\,\text{J}$

5 $\Delta E = \frac{hc}{\lambda}$

$\lambda = \frac{hc}{\Delta E}$

$= \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{0.42}$

$= 3.0 \times 10^{-6}\,\text{m}$

6 $\Delta E = E_4 - E_1$

$= -0.85 - (-13.6)$

$= 12.75\,\text{eV}$

7 $\frac{1}{\lambda} \quad R\left[\frac{1}{n_f^2} \quad \frac{1}{n_i^2}\right]$

$= 1.097 \times 10^{7} \times \left[\frac{1}{1^2} - \frac{1}{5^2}\right]$

$= 1.097 \times 10^{7} \times 0.96$

$\lambda = 9.5 \times 10^{-8}$

$= 95\,\text{nm}$

15.2 Quantum model of the atom

Worked example: Try yourself 15.2.1

CALCULATING THE DE BROGLIE WAVELENGTH

Calculate the de Broglie wavelength of a proton travelling at $7.0 \times 10^{5}\,\text{m s}^{-1}$. The mass of a proton is $1.67 \times 10^{-27}\,\text{kg}$.

Thinking	Working
Recall de Broglie's equation.	$\lambda = \frac{h}{mv}$
Substitute the appropriate values into the equation and solve it.	$\lambda = \frac{h}{mv}$ $= \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 7.0 \times 10^{5}}$ $= 5.7 \times 10^{-13}\,\text{m}$

Worked example: Try yourself 15.2.2

CALCULATING THE DE BROGLIE WAVELENGTH OF A MACROSCOPIC OBJECT

Calculate the de Broglie wavelength of a person with $m = 66\,\text{kg}$ running at $36\,\text{km h}^{-1}$.

Thinking	Working
Convert velocity to SI units.	$v = \frac{36}{3.6}$ $= 10\,\text{m s}^{-1}$
Recall de Broglie's equation.	$\lambda = \frac{h}{mv}$
Substitute the appropriate values into the equation and solve it.	$\lambda = \frac{h}{mv}$ $= \frac{6.63 \times 10^{-34}}{66 \times 10}$ $= 1.0 \times 10^{-36}\,\text{m}$

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Worked example: Try yourself 15.2.3

WAVELENGTH OF ELECTRONS FROM AN ELECTRON GUN

Find the de Broglie wavelength of an electron that has been accelerated from rest through a potential difference of 50 V. The mass of an electron is 9.11×10^{-31} kg and the magnitude of the charge on an electron is 1.6×10^{-19} C.

Thinking	Working
Calculate the kinetic energy of the electron from the work done on it by the electric potential. Recall from earlier chapters that $W = qV$.	$W = qV$ $= 1.6 \times 10^{-19} \times 50$ $= 8.0 \times 10^{-18}$ J
Calculate the velocity of the electron.	$K = \frac{1}{2}mv^2$ $v = \sqrt{\frac{2K}{m}}$ $= \sqrt{\frac{2 \times 8.0 \times 10^{-18}}{9.11 \times 10^{-31}}}$ $= 4.2 \times 10^6$ m s^{-1}
Use de Broglie's equation to calculate the wavelength of the electron.	$\lambda = \frac{h}{mv}$ $= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 4.2 \times 10^6}$ $= 1.7 \times 10^{-10}$ m $= 0.17$ nm

15.2 Review

1 $\lambda = \frac{h}{mv}$

$= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 1.0 \times 10^6}$

$= 7.3 \times 10^{-10}$ m

2 B. Wave behaviour of matter is linked to the mass and the velocity (that is, momentum) of the matter. So only moving particles exhibit wave behaviour.

3 **a** $\lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{8.6 \times 10^{18}} = 3.5 \times 10^{-11}$ m

b $\lambda = \frac{h}{mv}$

$3.5 \times 10^{-11} = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-3} \times v}$

$v = \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3.5 \times 10^{-11}}$

$= 2.1 \times 10^7$ m s^{-1}

4 According to Heisenberg's uncertainty principle, if the uncertainty about the position of a particle were decreased, then the uncertainty about the speed of the particle would increase.

5 Newtonian physics describes the position and velocity of an object as 'known', so its future position can be predicted. Quantum mechanics proposes that you cannot know the position and velocity of a particle at the same time. So the assumptions that Newtonian physics makes do not fit with what happens at the subatomic level, making classical physics inappropriate at this scale.

6 D. According to Heisenberg's uncertainty principle, it is impossible to precisely measure the momentum and position of a particle simultaneously.

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CHAPTER 15 REVIEW

1 2.11 eV

2 3.61 eV

3 No. The photon energy will be exactly equal to the energy difference between the electron's initial and final levels.

4 The light globe produces a continuous spectrum showing all the colours of the rainbow. The vapour lamp produces a discrete spectrum showing just coloured lines.

5 The temperature of the light globe filament increases when it is switched on. As the filament heats up, the free electrons in the tungsten atoms collide, accelerate and emit photons. A wide range of photon wavelengths are emitted due to a wide range of different collisions (some weak, some strong), and some visible light and a lot of infrared radiation are produced.

6

$$\lambda = \frac{h}{mv}$$

$$4.0 \times 10^{-9} = \frac{6.63\times10^{-34}}{9.11\times10^{-3} \times v}$$

$$v = \frac{6.63\times10^{-34}}{9.11\times10^{-31}\times4.0\times10^{-9}}$$

$$= 1.8 \times 10^{5}\,\text{m s}^{-1}$$

7 The location of electrons can't be restricted to specific orbital paths, because their precise location and velocity cannot be known simultaneously.

8 Sodium vapour is heated so that electrons are excited to higher energy levels, emitting light when they transition back to lower energy levels. The most common transitions in sodium produce orange light.

9

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.63\times10^{-34}}{0.040\times1.0\times10^{3}}$$

$$= 1.7 \times 10^{-35}\,\text{m}$$

10 No—the wavelength is much smaller than the size of everyday objects.

The wavelength of the bullet travelling at $1.0 \times 10^{3}\,\text{m s}^{-1}$ is many times smaller than the radius of an atom. Significant diffraction only occurs when the wavelength and gap (or object) sizes are approximately equal, i.e. when $\lambda \geq w$.

11 Electrons in the atom cannot assume a continuous range of energy values but are restricted to certain discrete values, i.e. the levels are quantised.

12

$$\Delta E = E_3 - E_1$$

$$= -1.5 - (-13.6)$$

$$= 12.1\,\text{eV}$$

$$\Delta E = hf$$

$$f = \frac{\Delta E}{h}$$

$$= \frac{12.1}{(4.14\times10^{-5})}$$

$$= 2.9 \times 10^{15}\,\text{Hz}$$

13 Bohr's work on the hydrogen atom convinced many scientists that a particle model was needed to explain the way light behaves in certain situations. It built significantly on the work done by Planck and Einstein.

14 The emission spectrum of hydrogen appears as a series of coloured lines. The absorption spectrum of hydrogen appears as a full visible spectrum with a number of dark lines. The lines in the emission spectrum have the same wavelengths as the missing lines in the absorption spectrum.

15 For the product of the uncertainty in position and the uncertainty in momentum to remain constant, then as the uncertainty in position is decreased, the uncertainty in momentum will increase.

16 It is likely that the photon would knock the electron off course and hence the electron's position would be subject to greater uncertainty.

17 high-energy orbits of multi-electron atoms, the continuous emission spectrum of solids, Zeeman splitting and the two close spectral lines in hydrogen that are revealed at high resolution

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18 $\Delta E = \frac{hc}{\lambda} = E_5 - E_2$

$= \frac{4.14 \times 10^{-15} \times 3.0 \times 10^{8}}{434 \times 10^{-9}}$

$= E_5 - (-3.4)$

$E_5 = 2.86 - 3.4$

$= -0.54\,\text{eV}$

19 The wavelength of a cricket ball is so small that its wave-like behaviour could not be seen by a cricket player.

20 $\lambda = \frac{hc}{E}$

$= \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{6.63 \times 10^{-14}}$

$= 3.0 \times 10^{-12}\,\text{m}$

Speed of the proton to exhibit this wavelength:

$v = \frac{h}{m\lambda}$

$= \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 3.0 \times 10^{-12}}$

$= 1.32 \times 10^{5}\,\text{m s}^{-1}$

21 $W = qV = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2qV}{m}}$

$\lambda = \frac{h}{mv}$

$= \frac{h}{m\frac{\sqrt{2qV}}{\sqrt{m}}}$

$= \frac{h}{\sqrt{2qVm}}$

22 $\lambda = \frac{h}{mv}$

$\lambda mv = h$

$mv = \frac{h}{\lambda}$

$p = \frac{h}{\lambda}$

23 An electron microscope can resolve images in finer detail than an optical microscope because a high-speed electron has a shorter wavelength than a light wave.

24 Rutherford's atomic model, with the electrons travelling in circles around the nucleus, did not agree with the understanding of charged particles at the time. It was known that charged particles accelerating give off light energy, which would reduce their overall energy. The electrons were being modelled travelling in a circular path, which would be constantly accelerating. This would cause the electrons to slow down and spiral into the nucleus.

Bohr refined the model, placing the electrons at distinct orbits and assigned energy levels to them. This was the start of applying quantum mechanics to the atom, and was able to describe the emission and absorption spectra that were seen.

Schrödinger further modified the atomic model, treating the electrons as waves, which is the foundation of modern quantum theory. At the same time, Heisenberg determined that it was not possible to describe the position and momentum of a particle at the same time, which changed the atomic model to include a probability of the electron being in that position.

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Chapter 16 Properties of the nucleus

16.1 Radioactive decay

Worked example: Try yourself 16.1.1

WORKING WITH ISOTOPES

Consider the isotope of thorium, $^{230}_{90}Th$. Work out the number of protons, nucleons and neutrons in this isotope.

Thinking	Working
The lower number is the atomic number.	atomic number = 90 This nuclide has 90 protons.
The upper number is the mass number. This indicates the number of particles in the nucleus, i.e. the number of nucleons.	mass number = 230 This nuclide has 230 nucleons.
Subtract the atomic number from the mass number to find the number of neutrons.	This isotope has 230 – 90 = 140 neutrons.

Worked example: Try yourself 16.1.2

RADIOACTIVE DECAY

Polonium-218 decays by emitting an alpha particle and a gamma ray. The nuclear equation is:

$$^{218}_{84}Po \rightarrow X + ^{4}_{2}\alpha + \gamma$$

Determine the atomic number and mass number for X, then use the periodic table on page 419 of the Student Book to identify the element.

Thinking	Working
Balance the mass numbers.	$218 = b + 4$ mass number = 214
Balance the atomic numbers.	$84 = a + 2$ atomic number = 82
Use the periodic table to look up element 82.	Element 82 is lead.

16.1 Review

1 nucleons

2 79 protons and 118 neutrons (197 – 79)

3 235

4 The number of electrons in a neutral atom is the same as the number of protons, which is given by the atomic number.

5 mass number of X is 218 – 214 = 4
atomic number of X is 86 – 84 = 2
X is an alpha particle.

6 mass number of Y is 214 – 214 = 0
atomic number of Y is 82 – 83 = –1
Y is a beta-minus particle.

7 beta-plus

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8 **a** gamma
b beta-minus
c alpha
d beta
e gamma

16.2 Half-life

Worked example: Try yourself 16.2.1

HALF-LIFE

A sample of the radioisotope sodium-24 contains 4.0×10^{10} nuclei. The half-life of sodium-24 is 15 hours.

a Calculate the decay constant for sodium-24.	
Thinking	**Working**
Recall the formula for the decay constant.	$\lambda = \frac{\ln(2)}{t_{1/2}}$
Substitute $t_{1/2}$ into the equation and solve for λ. As the half-life given for sodium is in hours, your answer will be in hour^{-1}.	$\lambda = \frac{\ln(2)}{t_{1/2}}$ $= \frac{\ln(2)}{15}$ $= 0.046$ hour^{-1}

b How many sodium-24 atoms will remain in the sample after 150 hours?	
Thinking	**Working**
Recall the formula for the number of nuclei remaining after time *t*.	$N = N_0 e^{-\lambda t}$
Substitute $N_0 = 4.0 \times 10^{10}$ and the decay constant from part (a) into the equation. Calculate the number of nuclei remaining.	$N = N_0 e^{-\lambda t}$ $= 4.0 \times 10^{10} \times e^{-0.046 \times 150}$ $= 3.9 \times 10^7$ nuclei

16.2 Review

1 The activity is the count rate or the number of decays each second.

2 One half-life has elapsed, so half of the original sample remains, i.e. 4.0×10^{10} atoms.

3 $\lambda = \frac{\ln(2)}{t_{1/2}}$

$= \frac{\ln(2)}{8}$

$= 0.087$ day^{-1}

$N = N_0 e^{-\lambda t}$

$= 2.4 \times 10^{12} \times e^{-0.087 \times 24}$

$= 3.0 \times 10^{11}$ nuclei

Alternatively, you could use the formula $N_t = N_0\left(\frac{1}{2}\right)^n$.

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4 **a** Halve successively from a starting number, e.g. 800 until 0.1% of 800 (0.8) is reached:

$$800 \rightarrow 400 \rightarrow 200 \rightarrow 100 \rightarrow 50 \rightarrow 25 \rightarrow 12.5 \rightarrow 6.25 \rightarrow 3.125 \rightarrow 1.56 \rightarrow 0.78$$

This takes 10 halvings, $n = 10$.

Alternatively,

$0.1\% = 0.001$

$\left(\frac{1}{2}\right)^n = 0.001$

Take logs of both sides:

$n \log\left(\frac{1}{2}\right) = \log 0.001$

$-0.3n = -3$

$n = 10$

It will take 10 half-lives to drop below 0.1%.

b 10 half-lives must pass = $10 \times 24\,000 = 240\,000$ years

5 The percentage chance any atom has of decaying in a period of time equal to its half-life is always 50%.

6 number of half-lives = 4

$12 = N_0 \times \left(\frac{1}{2}\right)^4$

$N_0 = \frac{12}{0.0625} = 192$

So 192 µg must be produced.

7 $6000 \rightarrow 3000 \rightarrow 1500 \rightarrow 750 \rightarrow 375$

so 4 half-lives have passed:

$\frac{60}{4} = 15$

The half-life of the radioisotope is 15 minutes.

8 **a** time to drop from $800 \rightarrow 400 = 10$ minutes or from $400 \rightarrow 200 = 10$ minutes

b 40 minutes = 4 half-lives; $A = 800 \times \left(\frac{1}{2}\right)^4 = 50$ Bq

9 Lead-210 undergoes beta decay. Its half-life is 20 years.

10 Starting from U-234, seven alpha and four beta-minus decays have occurred.

16.3 Nuclear fission and fusion

16.3 Review

1 The strong nuclear force is a force of attraction that acts between every nucleon but only over relatively short distances. This force acts like a nuclear cement.

2 The decay products of the nuclear fission process comprise many different, often highly radioactive isotopes. This is what makes up the waste. These waste products remain radioactive for centuries or even millennia after they are produced.

3 Since the neutron is neutral, it will only experience attractive forces from other nucleons due to the strong nuclear force.

4 fissile—uranium-235 and plutonium-239

non-fissile—uranium-238 and cobalt-60

5 $1 + 235 = 148 + 85 + a$

$\therefore a = 3$

6 Fusion is the joining together of two small nuclei to form a larger nucleus. Fission is the splitting apart of one large nucleus into smaller fragments.

7 Electrostatic forces of repulsion act on the protons. If the protons are moving slowly they will not have enough energy to overcome the repulsive forces and they will not fuse together.

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8 Initially, electrostatic forces of repulsion act on the protons, but they are travelling fast enough to overcome these forces. The protons will get close enough for the strong nuclear force to take effect and they will fuse together. These protons have overcome the energy barrier.

9 The number of nucleons is conserved as there are five nucleons on each side of the reaction.

16.4 Energy from nuclear reactions

Worked example: Try yourself 16.4.1

FISSION

Plutonium-239 is a fissile material. When a plutonium-239 nucleus is struck by and absorbs a neutron, it can split in many different ways. Consider the example of a nucleus that splits into lanthanum-143 and rubidium-94 and releases some neutrons.

The nuclear equation for this is:

$$^{1}_{0}n + ^{239}_{94}Pu \rightarrow ^{143}_{57}La + ^{94}_{37}Rb + a\,^{1}_{0}n$$

a How many neutrons are released during this fission process, i.e. what is the value of *a*?	
Thinking	**Working**
Analyse the mass numbers (*Z*).	$1 + 239 = 143 + 94 + (a \times 1)$ $a = (1 + 239) - (143 + 94)$ $= 3$ Three neutrons are released during fission.

b During this single fission reaction, there was a loss of mass (a mass defect) of 4.58×10^{-28} kg. Calculate the amount of energy that was released during fission of a single plutonium-239 nucleus. Give your answer in both MeV and joules, to three significant figures.	
Thinking	**Working**
The energy released during the fission of this plutonium nucleus can be found by using $E = mc^2$.	$E = mc^2$ $= (4.58 \times 10^{-28}) \times (3.00 \times 10^8)^2$ $= 4.12 \times 10^{-11}$ J
To convert J into eV, divide by 1.602×10^{-19}. Remember that 1 MeV = 10^6 eV.	$E = \frac{4.12 \times 10^{-11}}{1.602 \times 10^{-19}}$ $= 2.58 \times 10^8$ eV $= 258$ MeV

c The combined mass of the plutonium nucleus and bombarding neutron was 2.86×10^{-25} kg. What percentage of this initial mass was converted into the energy produced during the fission process?	
Thinking	**Working**
Use the relationship: $\%\text{ mass decrease} = \frac{\text{mass defect}}{\text{initial mass}} \times \frac{100}{1}$	$\%\text{ mass decrease} = \frac{\text{mass defect}}{\text{initial mass}} \times \frac{100}{1}$ $= \frac{4.58 \times 10^{-28}}{2.86 \times 10^{-25}} \times \frac{100}{1}$ $= 0.16\%$

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16.4 Review

1 **a** $E = mc^2$

$= (2.12 \times 10^{-28}) \times (3.00 \times 10^8)^2$

$= 1.91 \times 10^{-11}$ J

b $E = \frac{1.91 \times 10^{-11}}{1.602 \times 10^{-19}}$

$= 1.19 \times 10^8$ eV

2 The mass of the products is less than the mass of the reactants. The mass difference is related to the energy released via $E = mc^2$.

3 The amount of energy released per nucleon during a single nuclear fission reaction is less than the amount for a single fusion reaction.

4 less than 1%

5 **a** $2 + 3 = a + 1$

$\therefore a = 4$

$1 + 1 = b + 0$

$\therefore b = 2$

X is helium, ${}^4_2$He

b $E = mc^2$

$m = \frac{E}{c^2}$

$= \frac{33 \times 10^6 \times 1.6 \times 10^{-19}}{(3.0 \times 10^8)^2}$

$= 5.9 \times 10^{-29}$ kg

6 **a** atomic number = 2 + 1 + 1 – 2 = 2, mass number = 4 + 1 + 1 – 3 = 3, particle X is ${}^3_2$He

b $E = 23 \times 10^6 \times 1.602 \times 10^{-19} = 3.7 \times 10^{-12}$ J

c $E = mc^2$

$m = \frac{E}{c^2}$

$= \frac{3.7 \times 10^{-12}}{(3 \times 10^8)^2}$

$= 4.1 \times 10^{-29}$ kg

7 When two hydrogen-2 nuclei are fused together to form a helium-4 nucleus, the binding energy per nucleon increases and the nucleus becomes more stable.

CHAPTER 16 REVIEW

1 20 protons and 25 neutrons (45 – 20)

2 A nuclide that is able to split in two when hit by a neutron is fissile.

3 Cobalt-60 has 27 protons, 33 neutrons (60 – 27) and 60 nucleons.

4 The atomic and mass numbers of X are both 0, so X is a gamma ray.

5 Potassium is element 19. It has 48 – 19 = 29 neutrons. Figure 16.1.13 shows a minus sign so it emits a beta-minus particle.

6 No, only a few nuclides (e.g. uranium-235 and plutonium-239) are fissile.

7 The strong nuclear force causes the proton to be attracted to all other nucleons. It will also experience a smaller electrostatic force of repulsion between itself and other protons.

8 **a** beta-minus

b proton

c alpha

d neutron

e gamma

f beta-positive (positron)

9 atomic number = 5 – 2 = 3, mass number = 11 – 4 = 7, so X is lithium, ${}^7_3$Li

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10 **a** atomic number = 9 – 8 = 1, mass number = 18 – 17 = 1, so X is a proton

b atomic number = 13 – 13 = 0, mass number = 28 – 27 = 1, so Y is a neutron

11 Neutrons are uncharged and are not repelled by the nucleus as alpha particles are.

12 The nuclei are all positively charged and so repel each other. Particles need to be moving at very high speeds to be able to collide without being deflected by electrostatic repulsion. High temperatures correspond to high particle velocity. 100 million degrees provides the required energy for this to occur.

13 **a** $208 = x + 0 \rightarrow x = 208$

$81 = y - 1 \rightarrow y = 82$

b $180 = x + 4 \rightarrow x = 176$

$80 = y + 2 \rightarrow y = 78$

14 $18 = a + 0 \rightarrow a = 18$

$10 = \mathrm{b} + 1 \rightarrow b = 9$

$18 = c + 0 \rightarrow c = 18$

$9 = d + 1 \rightarrow d = 8$

X has atomic number 9, so is fluorine, F.

Y has atomic number 8, so is oxygen, O.

15 atomic number = 12, mass number = 7 – 1 = 6, X is carbon-12

16 Electromagnetic forces are balanced by the strong nuclear force acting between all nucleons in close proximity.

17 **a** gamma

b gamma

18 gamma radiation

19 The bombarding electrons will be strongly repelled by the electron clouds of the atoms as they are all negatively charged. The small mass of the bombarding electrons also makes them relatively easy to repel compared to, for example, a proton.

20 one half-life has passed, so

$N = 6.0 \times 10^{14} \times \left(\frac{1}{2}\right)^1 = 3.0 \times 10^{14}$ atoms

21 two half-lives have passed, so

$N = 6.0 \times 10^{10} \times \left(\frac{1}{2}\right)^2 = 1.5 \times 10^{10}$ atoms

22 The long half-life means that the source will not need to be replaced for many years. The gamma rays have a strong penetrating power so they are able to penetrate the skull and reach the tumour site.

23 $\Delta E = \Delta mc^2$

$= 4.99 \times 10^{-28} \times (3.00 \times 10^8)^2$

$= 4.49 \times 10^{-11}\,\mathrm{J}$

24 **a** The combined mass of the hydrogen and helium-3 nuclei is greater than the combined mass of the helium-4 nucleus, positron and neutrino.

b The energy has come from the lost mass (or mass defect) via $E = mc^2$.

c $21\,\mathrm{MeV} = 21 \times 10^6 \times 1.6 \times 10^{-19} = 3.4 \times 10^{-12}\,\mathrm{J}$

d $E = mc^2 \therefore m = \frac{E}{c^2} = \frac{3.4 \times 10^{-12}}{(3.00 \times 10^8)^2}$

$m = 3.8 \times 10^{-29}\,\mathrm{kg}$

25 Fission produces radioactive fission fragments, whereas fusion produces no radioactive waste products. Fusion creates more energy per nucleon than fission.

26 The binding energy per nucleon increases and the nucleus becomes more stable.

27 The higher the binding energy, the more stable the nucleus. This is because higher binding energy means that it takes more energy to completely separate particles in the nucleus. Iron therefore has the most stable nuclei of all the elements.

28 $E = 5.0 \times 10^4 \times 1.6 \times 10^{-19}$

$= 8.0 \times 10^{-15}\,\mathrm{J}$

29 Radioactive decay of a nucleus is a random process that can't be predicted for an individual atom. Scientists have discovered that the time it takes for half the nuclei to decay is constant for a particular atom; this is called a half-life. The half-life can be used to predict the radiation emitted so that it is usable in medical research, energy production and other industries.

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Chapter 17 Deep inside the atom

17.1 The Standard Model

17.1 Review

1 The strong nuclear force acts between nucleons, i.e. protons and neutrons.
2 weak nuclear: W^+, W^- and Z bosons; strong nuclear: gluons; electromagnetic: photons
3 The gauge bosons are the force-carrier particles. The leptons are fundamental particles that can be found individually and do not experience the strong force. Quarks are the third kind of particles in the standard model. They experience the strong nuclear force and form composite particles called hadrons.
4 Quarks must exist in groups of two or three; leptons can exist individually. All quarks experience the strong force; leptons do not. Quarks have non-integer (fractional) charges; leptons have charges of –1 or 0.
5 The ball represents the force-carrier particle being exchanged, namely bosons.
6 The two groups of hadrons are called mesons and baryons. The mesons contain two quarks. One of the quarks in this group is normal matter, while the other is antimatter. The baryons have three quarks. This group contains the familiar particles called protons and neutrons.
7

Gauge boson	Lepton	Hadron
gluon, photon	electron, neutrino, muon	neutron, proton

17.2 Evidence for the Standard Model

17.2 Review

1 They all use charged particles.
2 They are passed through an electric field.
3 linac, booster ring, storage ring, beamlines
4 very close to the speed of light
5 from the outward spiralling circular path of the particles
6 Cyclotrons use a static magnetic field and an alternating electric field to accelerate particles, so that the particles spiral outwards as they gain energy. In a synchrotron, the magnetic field strength is increased as the particle's energy increases, allowing for a fixed radius even as the energy increases.
7 The LHCb has been designed to look for the bottom or beauty quark. Every collision event forms a broad range of quarks for detection. Since quarks decay quickly, the LHCb has moveable tracking detectors that allow for studying a collision event at multiple points after the collision.

CHAPTER 17 REVIEW

1 A particle collides with its antiparticle and mass is converted into energy.
2 Protons are made of up, up, down quarks and neutrons are made of up, down, down quarks.
3 electromagnetism and the strong and weak nuclear forces
4 A proton is made up of two up quarks ($2 \times +\frac{2}{3}$) and one down quark ($-\frac{1}{3}$) so $\frac{4}{3} - \frac{1}{3} = +1$.

A neutron is made up of two down quarks ($2 \times -\frac{1}{3}$) and one up quark ($+\frac{2}{3}$) so $-\frac{2}{3} + \frac{2}{3} = 0$.
5 The correct order is: weak nuclear, electromagnetic, gravity, (strong nuclear).
6 The Standard Model is based on the assumption that forces arise through the exchange of particles called gauge bosons (or just bosons). Each of the three forces is mediated by a different particle: strong—gluon, electromagnetic—photon, weak—W^+, W^- and Z.
7 An electron is a fundamental particle and it is a lepton.
8 Electrons are 'boiled' off a heated wire element acting as a cathode.
9 linear accelerator

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10 A beamline is typically a stainless steel tube of 15–35 m in length along which synchrotron light travels from the storage ring, where it is produced, to its target for experimental work.

11 X-ray diffraction

12 in the electron gun

13 Synchrotrons produce synchrotron light with frequencies ranging from the infrared region through to the highest-frequency X-rays.

14 An accelerator capable of generating the 2 TeV accelerated particles required wasn't available before the early 1990s.

15 Any two from the following:

- It is unable to explain dark matter.
- It is unable to explain the absence of antimatter in the universe today.
- It is unable to provide a completely unified model for all fundamental forces; interactions with gravity are unexplained.
- It is unable to explain why neutrinos have mass.

16 High-intensity, high-energy sources allow X-ray diffraction techniques to be completed over considerably shorter times than with traditional X-ray sources.

17 Synchrotrons produce a continuous spectrum of radiation, and particular wavelengths within that spectrum are highly selectable or tunable.

18

X-ray tube	Synchrotron
single burst relatively divergent standard intensity	produced over hours highly collimated many times brighter

19 In the storage ring, electrons orbit for hours at a time at speeds near that of light, before being channelled along the beamlines for experimentation.

20 in the booster ring

21 in the linac

22 hydrogen atom source, electrons are stripped away, linac, Proton Synchrotron Booster, Super Proton Synchrotron, storage ring, detectors

23 Reponses will vary.

The fundamental particle model is constantly being updated. It was only recently (2013) that the Higgs boson was discovered even though it had been predicted years before. This is because as our sensing and measurement technology improves, the limits to our understanding grow. In the activity you had to determine what property to test for, and what test is most appropriate.

While the Standard Model is able to describe the fundamental building blocks of matter and the forces that govern them, there are still certain phenomena which it cannot explain (see Question 15 for some examples).

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Module 8 Review questions

From the universe to the atom

MULTIPLE CHOICE

1 C. Rutherford's research led him to believe that there was a positive centre surrounded by negative charges.

2 A. Luminosity (vertical axis) does increase with temperature (horizontal axis) but not in a straight line, so option A is the best answer.

3 C. Luminosity is approximately proportional to the cube of the mass. $2^3 = 8$.

4 D. The H–R diagram plots the luminosity of a star (which is derived from the absolute magnitude) against the spectral type of stars (from which the temperature of the star is derived).

5 B. The Sun is a 'main-sequence' star.

6 C. Blue supergiants have a large luminosity and a high surface temperature.

7 A. A 'Sun-like' star will progress to a white dwarf as part of its life cycle. White dwarf stars are not very luminous and have a moderate surface temperature.

8 D. All stars begin life as a planetary nebula before moving into the main sequence. A massive star will then expand to become a red supergiant before it becomes a supernova to end as a black hole.

9 C. $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$

$24.7\,\text{MeV} = 24.7 \times 10^{6}\,\text{eV}$

$(24.7 \times 10^{6}\,\text{eV}) \times 1.6 \times 10^{-19} = 3.95 \times 10^{-12}\,\text{J}$

10 C. All nuclear equations must balance.

The mass numbers must balance on both sides of the reaction, as should the atomic numbers.

Mass numbers:

$235 + 1 = 92 + 141 + x$

$\therefore x = 3$

11 A. Energy = 30.4 eV + 10.4 eV = 40.8 eV

$\lambda = \frac{hc}{E} = \frac{4.14 \times 10^{-5} \times 3 \times 10^{8}}{40.8}$

$= 3.04 \times 10^{-8}\,\text{m}$

12 D.

$E_{3-2} = -3.7 - (-5.5) = 1.8\,\text{eV};\ \lambda = \frac{hc}{E} = \frac{4.14 \times 10^{-5} \times 3 \times 10^{8}}{1.8} = 689\,\text{nm}$ (red visible)

$E_{3-1} = -3.7 - (-10.4) = 6.7\,\text{eV};\ \lambda = \frac{hc}{E} = \frac{4.14 \times 10^{-5} \times 3 \times 10^{8}}{6.7} = 185\,\text{nm}$ (ultraviolet)

$E_{2-1} = -5.5 - (-10.4) = 4.9\,\text{eV};\ \lambda = \frac{hc}{E} = \frac{4.14 \times 10^{-5} \times 3 \times 10^{8}}{4.9} = 253\,\text{nm}$ (ultraviolet)

The spectrum needs to have one red line corresponding to the visible wavelength, 689 nm.

13 C. The de Broglie wavelength of a particle is given by $\lambda = \frac{h}{p}$ and therefore depends only on the momentum of the particle.

14 B. The probability of a nuclide decaying during the next half-life is always 50%.

15 B. A neutron in the hydrogen atom has transformed into a proton and an electron.

16 B. The radioisotope with the shorter half-life (Bi-211) is less stable by a factor of four and so will initially have a higher activity by a factor of four.

17 A. Hadrons are subatomic particles that are composed of quarks and interact by the strong interaction, e.g. protons and neutrons.

Electrons and neutrinos are leptons.

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18 B. An electron that accelerates across a potential difference of 5 kV gains 5 keV of energy.

$5\,\text{keV} = (5 \times 10^3) \times (1.6 \times 10^{-19}) = 8.0 \times 10^{-16}\,\text{J}$

$\therefore$ Kinetic energy of electron $= 8.0 \times 10^{-16}\,\text{J} = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2 \times 8.0 \times 10^{-16}}{9.1 \times 10^{-31}}}$

$= 4.2 \times 10^7\,\text{m s}^{-1}$

SHORT ANSWER

19 Stellar spectra include the absorption lines for elements for which we know the wavelengths. When these spectra are emitted from objects receding from us, the lines are shifted to longer wavelengths—towards the red end of the spectrum. This is the case for the majority of stellar objects which are redshifted, the more distant objects being shifted the most and hence receding the fastest. This is consistent with a model in which space–time and the whole universe is expanding.

20 **a** The energy of photons results from the mass of the leptons being converted to energy.

b $E = mc^2$. The mass of the two particles forms the mass defect.

$E = 2 \times 9.1 \times 10^{-31} \times (3.0 \times 10^8)^2 = 1.6 \times 10^{-13}\,\text{J}$

21 **a** Bohr's work on the hydrogen atom convinced many scientists that a particle model was needed to explain the way light behaves in certain situations. It built significantly on the work done by Planck and Einstein.

b Neils Bohr would state that if incident light had an energy value less than the minimum energy difference between the lowest and next orbital levels within the hydrogen atom, the light would not result in any orbital changes.

22 **a** Photon energy > ionisation energy, i.e. the photon has enough energy to free the electron.

b $14.0 - 13.6 = 0.4\,\text{eV}$

$0.4\,\text{eV} = 0.4 \times 1.6 \times 10^{-19}\,\text{J}$

$= 6.4 \times 10^{-20}\,\text{J}$

c $\Delta K = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 6.4 \times 10^{-20}}{9.11 \times 10^{-31}}}$

$= 3.74 \times 10^5\,\text{ms}^{-1}$

$p = mv$

$= 9.11 \times 10^{-31} \times 3.74 \times 10^5$

$= 3.41 \times 10^{-25}\,\text{kg m s}^{-1}$

d $\lambda = \frac{h}{p}$

$= \frac{6.63 \times 10^{-34}}{3.41 \times 10^{-25}}$

$= 1.94 \times 10^{-9}\,\text{m}$

e Since there is no energy level 10.0 eV above the ground state, the photon cannot be absorbed.

23 **a** $E = \frac{V}{d}$

$= \frac{240}{(1.6 \times 10^{-3})}$

$= 1.5 \times 10^5\,\text{N C}^{-1}$ (or V m^{-1}) downwards

b $q = \frac{mgd}{V}$

$= \frac{1.96 \times 10^{-4} \times 9.8 \times 1.6 \times 10^{-3}}{240}$

$= 1.28 \times 10^{-18}\,\text{C}$

c $q = nq_e$

$n = \frac{1.28 \times 10^{-18}}{(1.6 \times 10^{-19})}$

$= 8$ electrons

24 **a** $\lambda = \frac{h}{p} = \frac{h}{mv}$

$= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times \left(\frac{0.01}{100}\right) \times 3 \times 10^8}$

$= 2.42 \times 10^{-8}\,\text{m}$

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b A series of bright and dark fringes.

c The high-speed electrons are exhibiting wave-like behaviour.

d $\lambda = \frac{h}{mv} \rightarrow v = \frac{h}{m\lambda}$

$v = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-2} \times 2.42 \times 10^{-9}}$

$= 164\,\text{m s}^{-1}$

25 a $\lambda = \frac{h}{p} = \frac{h}{mv}$

$= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-3} \times 1.75 \times 10^{7}}$

$= 4.16 \times 10^{-11}\,\text{m}$

$= 0.0416\,\text{nm}$

b There would be circular bands or fringes of specific spacing around a common central point.

c As the accelerating voltage is increasd, the electron speed would incrase. Therefore the electron has more momentum. As $\lambda = \frac{h}{p}$, the electron's wavelength is reduced. The amount of diffraction depends on $\frac{\lambda}{w}$ and so less diffraction occurs. Less diffraction means the overall pattern is smaller; that is, the circular bands are more closely spaced.

26 a From the graph the activity halves in approximately 1 minute, hence half-life is 1 minute.

b 5 minutes is 5 half-lives. The radioactive sample will reduce by a factor of $2^5 = 32$. Remaining mass $= \frac{150}{32} = 4.7\,\text{g}$.

c ${}^{26}_{11}\text{Na} \rightarrow {}^{0}_{-1}\beta + {}^{26}_{12}\text{Mg} + \text{energy}$

27 Cs-137: 55 protons, 82 neutrons, 137 nucleons

I-131: 53 protons, 78 neutrons, 131 nucleons

28

Particle	Property
gluon	• mediator of the strong nuclear force • interacts with quarks
photon	• mediator of the electromagnetic force • interacts with charged particles
W^+, W^- and Z	• mediator of the weak nuclear force • causes nuclear decay
(graviton)	• mediator of the gravitational force

29 In the single-slit diffraction experiment, as the slit is made narrower the position of the particle becomes more precisely known. As a consequence, the direction, and therefore the momentum of the particle, becomes less precisely known, because with a narrower slit the diffraction pattern becomes wider.

30 Δp is the uncertainty of a particle's momentum. If this value gets smaller it means the momentum (or velocity) of a particle is known more precisely. As a consequence, as the right-hand side of the relation remains constant, the uncertainty in a particle's position, Δx, becomes greater.

31 a $v = H_0 d = 70\,\text{km s}^{-1}\,\text{Mpc}^{-1} \times d$

The speed is directly proportional to the distance.

Proxima Centauri is 1.3 pc away and so its speed is:

$70 \times 10^{-6}\,\text{km s}^{-1}\,\text{pc}^{-1} \times 1.3\,\text{pc} = 91\,\text{mm s}^{-1}$

The edge of the universe is $4.4 \times 10^{26}\,\text{m} = \frac{4.4 \times 10^{26}\,\text{m}}{3 \times 10^{5}} = 1.4 \times 10^{10}\,\text{pc}$

Speed $= 70 \times 10^{-6}\,\text{km s}^{-1}\,\text{pc}^{-1} \times 1.4 \times 10^{10}\,\text{pc} = 9.8 \times 10^{8}\,\text{m s}^{-1}$

b The edge of the visible universe is receding from us at a speed in excess of the speed of light. That is not a violation of the principles of special relativity, as no object is moving through space at a speed in excess of the speed of light, it is purely a relative velocity.

If there is concern that technically this is not the edge of the visible universe, since light would never reach us, we are only concerned here with a factor of 3 or so and in astronomical terms, this is good enough!

c $H_0 = \frac{v}{d}$, i.e. a velocity divided by a distance. As velocity in SI units is m s^{-1} and distance is simply m, the units of H_0 are $\text{m s}^{-1}\,\text{m}^{-1} = \text{s}^{-1}$. As $70\,\text{km s}^{-1} = 7 \times 10^{4}\,\text{m s}^{-1}$ and $1\,\text{Mpc} = 3.1 \times 10^{22}\,\text{m}$, H_0 has the value $\frac{v}{d} = \frac{7 \times 10^{4}}{3.1 \times 10^{22}} = 2 \times 10^{-18}\,\text{s}^{-1}$.

The relative proximity of a star like Proxima Centauri would mean redshift would not be measurable.

d The extremely small value for H_0 reflects the fact that the recession velocity is only significant for huge distances!

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32 a To produce diffraction patterns with the same fringe separation, they must have equivalent wavelengths.

b $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{8.3 \times 10^{18}}$

$= 3.6 \times 10^{-11}$ m

c $\lambda = 3.6 \times 10^{-11}$ m, since they must have an equivalent wavelength to the X-ray photons.

d $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{3.6 \times 10^{-11}}$

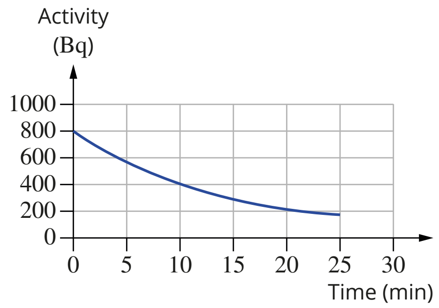
$= 1.8 \times 10^{-23}$ kg m s^{-1}

e No. The energy of the X-rays is given by $E = \frac{hc}{\lambda}$ and the energy of the electrons is given by $\Delta K = \frac{1}{2}mv^2$.

f de Broglie would say that the electrons (with their associated wavelengths) were diffracted as they passed through the gaps between the atoms in the crystal, creating a diffraction pattern.

g In addition to their particle properties, electrons have a de Broglie wavelength. The orbit must fit an integral number of wavelengths so that a standing wave is formed ($2\pi r = n\lambda$). Only energy levels corresponding to these wavelengths exist.

33 a



b From graph, after 13 minutes, activity is about 320 Bq.

c Find the time at which activity has been reduced from 800 Bq to 400 Bq: $t_{1/2} \approx 10$ min.

d $\lambda = \frac{\ln(2)}{t_{1/2}} = \frac{\ln(2)}{10 \times 60} = 0.0012 = 1.2 \times 10^{-3}$ s^{-1}

e From graph, extrapolate to find activity when $t = 30$ min. Activity ≈ 100 Bq.

f i ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$

${}^{218}_{84}Po \rightarrow {}^{218}_{85}At + {}^{0}_{-1}\beta$

ii Po and At have different numbers of protons and this is what makes them distinct elements.

iii ${}^{210}Bi$ can undergo beta decay to form ${}^{210}Po$ and then this undergoes alpha decay to form ${}^{206}Pb$. Alternatively, it can undergo an alpha decay first to form ${}^{206}Tl$ and then the subsequent beta decay results in ${}^{206}Pb$.

iv They all have 84 protons, but differ in their number of neutrons: 214, 210 and 206 neutrons respectively.

34 a

Category	Particle type	Description	Particle name
gauge bosons		mediators of the fundamental forces	photons, gluons, gravitons, W^+, W^-, and Z
fermions (make up all matter)	leptons	• experience the weak nuclear force, exchanging W and Z bosons • charged leptons experience the electromagnetic force, exchanging photons • do not experience the strong force	positrons, electrons, neutrinos, muons
	hadrons	• experience the strong force, exchanging gluons • made up of quarks	
	- baryons	made of three quarks	protons, neutrons, antiprotons
	- mesons	made of two quarks	pions

b Hadrons. The other particles listed are fundamental particles. Fundamental particles do not have an internal structure. Hadrons are less numerous and hence are likely to be made up of other more fundamental particles.

c The Higgs boson essentially gives mass to all elementary particles.

d Dark matter and antimatter are two predicted phenomenon not able to be explained by the Standard Model.

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35 **a** The big bang is an expansion of space–time. Before the big bang there was no space, time or matter, so it is not a case of matter exploding out into space in a time continuum, but space and time itself being created at the big bang event as energy converted to matter, after which space expanded. The energy present allowed the creation of matter–antimatter pairs and the rapid inflation of the universe prevented annihilation taking place immediately, taking the created matter with it. While it is true that the early universe was extremely dense, the big bang theory would suggest that mass/energy, space and time all emerged at once from nothing.

b The radiation which when created would have had a very short wavelength would be expected to 'stretch out' with space itself, and so would have a much longer wavelength as space expanded. Calculations show that this would be in the microwave range today.

c The variations indicate a slightly uneven distribution of light and therefore matter. This allowed gravitational attraction to collect clumps of matter together, ultimately forming stars and galaxies. If there had been completely uniform radiation, there would have been no universe as we know it.

d Pair production is the creation of a matter and antimatter pair of particles, such as a positron and an electron from a photon. This is a mechanism for the creation of particles from photons.

e Normally pairs annihilate rapidly with the release of photons, but the rapid inflation moved the pairs apart so that the particles were able to persist.

f As the universe cooled, the average photon energy dropped to a level at which a photon no longer had the energy required to create a matter–antimatter pair.

g Any atoms formed would immediately be ionised as the photons, although not having enough energy for pair production, certainly possessed the ionisation energy for a hydrogen atom.

h Fusion requires very high densities, temperatures and pressures for charged particles to overcome their mutual repulsion and come close enough for the strong nuclear force to exceed the electrostatic repulsion. This happened in the first few seconds after the big bang, and then particle distances increased and energies dropped below the values required for fusion to be possible. Fusion reignited in stars much later when gravitational forces once again brought particles together at high densities.

i Photon energies had to be below the ionisation energy of the atoms.

j Gravity caused particles to aggregate. As the dust clouds collapsed under their mutual attraction, vast amounts of energy were released and this created the temperatures and pressures for fusion to reignite in the first stars.

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