Neap

Trial Examination 2021

Suggested solutions

QCE General Mathematics Units 3&4

Paper 1

SECTION 1 – MULTIPLE-CHOICE QUESTIONS

	A	B	C	D
1.	●	○	○	○
2.	○	○	○	●
3.	○	○	●	○
4.	●	○	○	○
5.	○	●	○	○
6.	○	●	○	○
7.	○	○	●	○
8.	○	●	○	○
9.	○	●	○	○
10.	○	○	○	●
11.	○	○	○	●
12.	○	○	○	●
13.	○	○	○	●
14.	○	○	●	○
15.	○	○	○	●
16.	○	○	○	●
17.	○	○	●	○
18.	○	○	○	●
19.	○	●	○	○
20.	○	●	○	○

QUESTION 1 A

A is correct. Using a calculator gives the value –0.5425695681, which rounds to –0.543. **B** is incorrect. This value has been truncated rather than rounded. **C** is incorrect. This option gives the value for the gradient (b). **D** is incorrect. This option gives the value for the y-intercept (a).

QUESTION 2 D

D is correct. The scatterplot has a distinctive negative direction and a non-linear form. **A** is incorrect. The scatterplot is neither positive nor linear. **B** is incorrect. The scatterplot is not positive. **C** is incorrect. The scatterplot is neither weak nor linear.

QUESTION 3 C

C is correct.

$$(-0.7683)^2 = 0.5903$$

$$0.5903 \times 100 = 59.03\%$$

The explanatory variable is the number of days students are absent and the response variable is the school results. So, 59.03% of the variation in the response variable can be explained by the variation in the explanatory variable and 40.97% can be explained by other factors (100% – 59.03%). **A** is incorrect. The correlation coefficient –0.7863 has not been squared. **B** is incorrect. The correlation coefficient –0.7863 has not been squared and the order of the explanatory and response variables has been swapped. **D** is incorrect. The order of the explanatory and response variables has been swapped.

QUESTION 4 A

A is correct. A value of 5 for years of age is predicting outside the range of 7 to 12 years of age. **B**, **C** and **D** are incorrect. The values of 8, 9.5 and 11 years of age are predicting inside the range of 7 to 12 years of age.

QUESTION 5 B

B is correct.

$$\frac{16\,271 + 14\,627 + 15\,763 + 15\,632 + 13\,621}{5} = 15182.80$$

A is incorrect. All values in the table have been added and divided by 7. **C** is incorrect. A three-point moving average has been performed by adding the three values \$14 627, \$15 763 and \$15 632 and dividing by 3. **D** is incorrect. The five values have been added and not divided by 5.

QUESTION 6 B

B is correct. The yearly average is found by adding the values and dividing by 4.

$$\frac{3523 + 2837 + 2648 + 3873}{4} = 3220.25$$

$$\text{seasonal index} = \frac{\text{quarterly value}}{\text{yearly average}}$$

$$\text{seasonal index for autumn} = \frac{2837}{3220.25}$$

$$= 0.881 \text{ (to 3 decimal places)}$$

A is incorrect. This is the seasonal index for winter. **C** is incorrect. This is the seasonal index for summer. **D** is incorrect. This is the yearly average.

QUESTION 7 C

C is correct. The common ratio for geometric sequences between successive terms is found using:

$$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} \ldots$$

$$r = \frac{-18}{5} = \frac{64.8}{-18} = \frac{-233.28}{64.8} = -3.6$$

A is incorrect. This option gives the common difference.

$$t_2 - t_1 = -18 - 5$$
$$= -23$$

B is incorrect. This option gives the incorrect solution.

$$t_2 + t_1 = -18 + 5$$
$$= -13$$

D is incorrect. This option gives the incorrect value of *r*.

$$r = \frac{t_1}{t_2}$$
$$= -0.27$$

QUESTION 8 B

B is correct. City X and city Y are on the same parallel of latitude so the small circle distance formula must be used. The angular distance between the meridians of longitude is found by adding the east and west values:

$110°18' + 93°28' = 203°46'.$

However, as the shortest path between the two cities passes on the opposite side of the world, the angular distance must be subtracted from 360º. So, the shortest angular distance is:

$360° - 203°46' = 156°14'.$

$$D = 111.2 \cos\theta \times \text{angular distance}$$
$$= 111.2 \cos(28°34') \times 156°14'$$
$$= 15258.16 \text{ km}$$

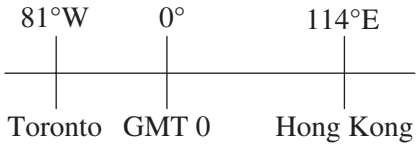
A is incorrect. This option swaps the latitude and angular distance using $D = 111.2 \cos(156°14') \times 28°34'.$

C is incorrect. This option gives the longest distance as the angular distance was not subtracted from 360°.

D is incorrect. This option uses the formula for distance on a great circle, $D = 111.2 \times \text{angular distance}.$

QUESTION 9 B

B is correct. The time zone difference between Hong Kong and Toronto is $81° + 114° = 195°$ and as 1 hour is equal to 15°, the time difference must be $\frac{195°}{15°} = 13 \text{ hours}.$ Hong Kong is ahead of Toronto by 13 hours, so if it is 3:30 pm Friday in Hong Kong then it must be 3:30 pm – 13 hours = 2:30 am Friday in Toronto.



A is incorrect. This option adds 4 hours to Hong Kong's local time. **C** is incorrect. This option adds 13 hours to Hong Kong's local time. **D** is incorrect. This option subtracts 4 hours from Hong Kong's local time.

QUESTION 10 D

D is correct. The lowest interest rate is best when taking out a loan. When compared to the other three home loans, 2.58% compounding daily is the lowest interest rate. Using the effective annual rate of interest formula:

$$\begin{aligned} i_{\text{effective}} &= \left(1+\frac{i}{n}\right)^{n}-1 \\ &= \left(1+\frac{\frac{2.58}{100}}{365}\right)^{365}-1 \\ &= 0.0261\times 100 \\ &= 2.61\%\text{ p.a.} \end{aligned}$$

A is incorrect. For 2.59% p.a. compounding monthly:

$$\begin{aligned} i_{\text{effective}} &= \left(1+\frac{\frac{2.59}{100}}{12}\right)^{12}-1 \\ &= 0.0262\times 100 \\ &= 2.62\%\text{ p.a. (higher than } \mathbf{D}\text{)} \end{aligned}$$

B is incorrect. For 2.63% p.a. compounding quarterly:

$$\begin{aligned} i_{\text{effective}} &= \left(1+\frac{\frac{2.63}{100}}{4}\right)^{4}-1 \\ &= 0.0266\times 100 \\ &= 2.66\%\text{ p.a. (higher than } \mathbf{D}\text{)} \end{aligned}$$

C is incorrect. For 2.62% p.a. compounding annually:

$$\begin{aligned} i_{\text{effective}} &= \left(1+\frac{\frac{2.62}{100}}{1}\right)^{1}-1 \\ &= 0.0262\times 100 \\ &= 2.62\%\text{ p.a. (higher than } \mathbf{D}\text{)} \end{aligned}$$

QUESTION 11 D

D is correct. Using the compound interest formula and solving for the interest rate gives:

$$A = P(1+i)^n$$

$$150\,000 = 120\,000\left(1+\frac{i}{4}\right)^{3\times 4}$$

$$\frac{150\,000}{120\,000} = \left(1+\frac{i}{4}\right)^{12}$$

$$1.25 = \left(1+\frac{i}{4}\right)^{12}$$

$$\sqrt[12]{1.25} = \sqrt[12]{\left(1+\frac{i}{4}\right)^{12}}$$

$$1.01876\ldots = 1+\frac{i}{4}$$

$$1.01876\ldots - 1 = \frac{i}{4}$$

$$\frac{i}{4} = 0.01876\ldots$$

$$i = 0.01876\ldots \times 4$$

$$= 0.075077\ldots \times 100$$

$$= 7.51\%\text{ p.a.}$$

A is incorrect. The quarterly interest rate is not multiplied by 4 to find the annual interest rate and is incorrectly rounded. **B** is incorrect. The quarterly interest rate is not multiplied by 4 to find the annual interest rate. **C** is incorrect. The interest rate has been incorrectly rounded.

QUESTION 12 D

D is correct. The degree of vertex C includes all the edges coming out of it including the loop, which equals 5. **A** is incorrect. This option is the degree of vertex D. **B** is incorrect. This option may be reached if the loop has been ignored. **C** is incorrect. This option may be reached if one edge of the loop has been ignored.

QUESTION 13 D

The adjusted final repayment value is found by using the recurrence relation $A_{n+1} = rA_n - R$ to model the balance of the loan, where:

$$r = 1 + \frac{\frac{5.4}{12}}{100}$$
$$= 1.0045$$
$$R = 1700$$

Using the calculator to show 24 iterations of $A_{n+1} = 1.0045A_n - 1700$ by pressing $38\,500 \times 1.0045 - 1700$ and then pressing '=' 24 times:

$$A_0 = 38\,500$$
$$A_1 = 36\,973.25$$
$$A_2 = 35\,439.63$$
$$A_3 = 33\,899.11$$
$$\vdots$$
$$A_{22} = 3275.80$$
$$A_{23} = 1590.54$$
$$A_{24} = -\$102.30$$

The loan has been overpaid by \$102.30 in the 24th payment, so the repayment of \$1700 is reduced to \$1597.70.

QUESTION 14 C

To find the payment from a reducing balance loan, the annuities present value formula is used and rearranged.

$$A_{PV} = 422\,000$$
$$M = ?$$
$$i = \frac{\frac{3.95}{12}}{100}$$
$$= 0.003291667\ldots$$
$$A_{PV} = \left(\frac{1 - (1+i)^{-n}}{i} \right)$$
$$422\,000 = M \left(\frac{1 - (1 + 0.003291667)^{-7 \times 12}}{0.003291667} \right)$$
$$422\,000 = M \times 73.28260727\ldots$$
$$M = \frac{422\,000}{73.28260727\ldots}$$
$$= \$5758.53$$

QUESTION 15 D

To find the future value of an annuity, the annuities future value formula is used.

$A_{FV} = ?$

$M = 2300$

$$i = \frac{\frac{2.86}{4}}{100}$$

$$= 0.00715\ldots$$

$$A_{FV} = M\left(\frac{(1+i)^n - 1}{i}\right)$$

$$A_{FV} = 2300\left(\frac{(1+0.00715)^{4\times 5} - 1}{0.00715}\right)$$

$$= 2300 \times 21.41859188$$

$$= \$49262.76$$

QUESTION 16 D

Using the perpetuity formula:

$$A = \frac{M}{i}$$

$$= \frac{2500}{\frac{3.25}{100}}$$

$$= \$76923.08$$

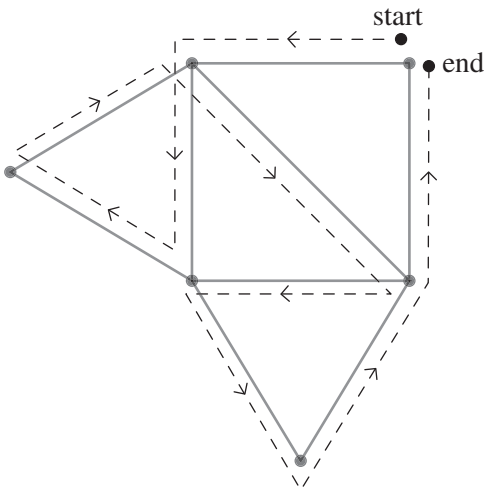
QUESTION 17 C

C is correct. Amelie must organise the cake as there are no other arrows pointing to the cake task. Therefore, Amelie cannot organise the venue, so Danielle must organise the venue. Danielle cannot organise the music, so Carolina must organise the music. Carolina cannot organise the flowers, so Belinda must organise the flowers. **A** and **D** are incorrect. There is no arrow from Belinda to the cake task. **B** is incorrect. There is no arrow from Carolina to the venue task.

QUESTION 18 D

D is correct. An Eulerian graph covers every edge once only and starts and ends at the same vertex. For this to occur, every vertex must have an even degree.

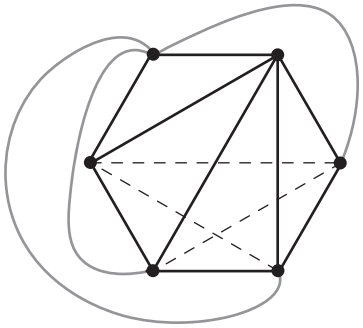
For example:



A is incorrect. A semi-Hamiltonian graph covers every vertex once only and can traverse multiple edges. **B** is incorrect. A semi-Eulerian graph has a pair of vertices with odd degrees and will start and end at different vertices. **C** is incorrect. A Hamiltonian graph covers every vertex once only and can traverse multiple edges.

QUESTION 19 B

B is correct. The complete graph has six vertices, and needs three interior edges to be completely removed so that it can be adjusted to a planar graph with no edges crossing over and eight faces. For example, three edges can be removed completely and three edges can be redrawn outside the hexagon, as shown in the graph below.



A is incorrect. Removing one edge will not allow for the hexagon to be redrawn with no edges crossing over. **C** is incorrect. Removing six edges will create a planar graph; however, only three interior edges need to be removed to create a planar graph. **D** is incorrect. Removing nine edges will remove all interior edges and create a planar graph; however, a planar graph can be created with fewer edges removed.

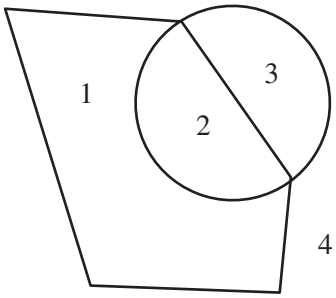
QUESTION 20 B

B is correct. The minimum spanning tree is A–B, A–C, C–D, D–F, F–E which adds up to $4 + 3 + 5 + 2 + 5 = 19$ m. **A** is incorrect. The value of 15 m may be reached by missing one of the edges. **C** is incorrect. This option may be reached if some larger values have been added together. **D** is incorrect. This option may be reached if all the values have been added together.

SECTION 2

QUESTION 21 (3 marks)

a) number of faces = 4



[1 mark]
1 mark for identifying the correct number of faces.
Note: Responses do not require a diagram.

b) $v = 5$

$f = 4$

$e = 7$

$v + f - e = 2$

$5 + 4 - 7 = 2$

$2 = 2$

Euler's formula only works for planar graphs, so the values prove that the planar graph from 21a) has 4 faces.

[2 marks]
1 mark for identifying the correct values for v and e.
1 mark for using Euler's formula.
Note: Identifying the correct values for v and e may be implied by subsequent working.

QUESTION 22 (7 marks)

a) $A = ?$

$P = 57000$

$i = 3.62\%$

$$= \frac{\frac{3.62}{12}}{100}$$

$$= 0.0030166\ldots$$

$n = 6 \text{ years} \times 12$

$= 72$

$$A = P(1+i)^n$$

$$= 57000 \times (1 + 0.0030166\ldots)^{72}$$

$$= \$70804.62442$$

[3 marks]

1 mark for determining the values of i and n.

1 mark for substituting into the compound interest rule OR a correct alternative procedure.

1 mark for determining the correct future value of the lump sum after 6 years.

Note: The values for i and n may be implied by subsequent working. Final answer given to two decimal places or more is acceptable. Accept follow-through errors.

b) $A_{FV} = ?$

$M = 2400$

$i = 3.62\%$

$$= \frac{\frac{3.62}{12}}{100}$$

$$= 0.0030166$$

$n = 6 \text{ years} \times 12$

$= 72$

$$A_{FV} = M\left(\frac{(1+i)^n - 1}{i}\right)$$

$$= 2400\left(\frac{(1+0.0030166\ldots)^{72} - 1}{0.0030166\ldots}\right)$$

$$= 2400 \times 80.28278231\ldots$$

$$= \$192678.6775\ldots$$

[2 marks]

1 mark for substituting into the future value annuity formula OR a correct alternative procedure.

1 mark for determining the correct value of monthly deposits after 6 years.

Note: Accept follow-through errors.

c) üü=üüüüüüüüüüüüüüüü … + 75…

$= 263\,483.30$

Therefore, the doctor will have accumulated $263 483.30 in their account after 6 years.

[2 marks]
1 mark for determining the total amount after 6 years.
1 mark for stating the solution with correct units and rounding to two decimal places.
Note: Final answer given to two decimal places or more is acceptable.

QUESTION 23 (5 marks)

a)

	Barks	Does not bark	Total
Small dogs	5	15	20
Large dogs	5	20	25
Total	10	35	45

[4 marks]
2 marks for correctly completing the Barks and Does not bark columns.
2 marks for correctly completing the Total column.

b) % of large dogs that bark $= \frac{5}{25} \times 100$

$= 20\%$

[1 mark]
1 mark for identifying the percentage of large dogs that bark.

QUESTION 24 (5 marks)

$$b = r\frac{s_y}{s_x}$$
$$= 0.79 \times \frac{1893}{2.37}$$
$$= 631$$
$$a = \overline{y} - b\overline{x}$$
$$= 12\,300 - 631 \times 8.5$$
$$= 6936.5$$
$$y = a + bx$$
$$y = 6936.5 + 631x$$

[5 marks]
1 mark for using the appropriate formula to find the slope.
1 mark for identifying the value for b.
1 mark for using the appropriate formula to find the intercept.
1 mark for identifying the value for a.
1 mark for correctly identifying the equation of the least-squares line.

QUESTION 25 (5 marks)

a) $V_0 = a,\ V_{n+1} = V_n - d$

$V_0 = 66\,000,\ V_{n+1} = V_n - 1200$

[1 mark]

1 mark for identifying a recursive rule for the straight-line depreciation of the arithmetic sequence.

Note: V_0 can be written before or after V_{n+1}.

b) $V_0 = \$66\,000$

$V_1 = \$64\,800$

$V_2 = \$63\,600$

$V_3 = \$62\,400$

$V_4 = \$61\,200$

$V_5 = \$60\,000$

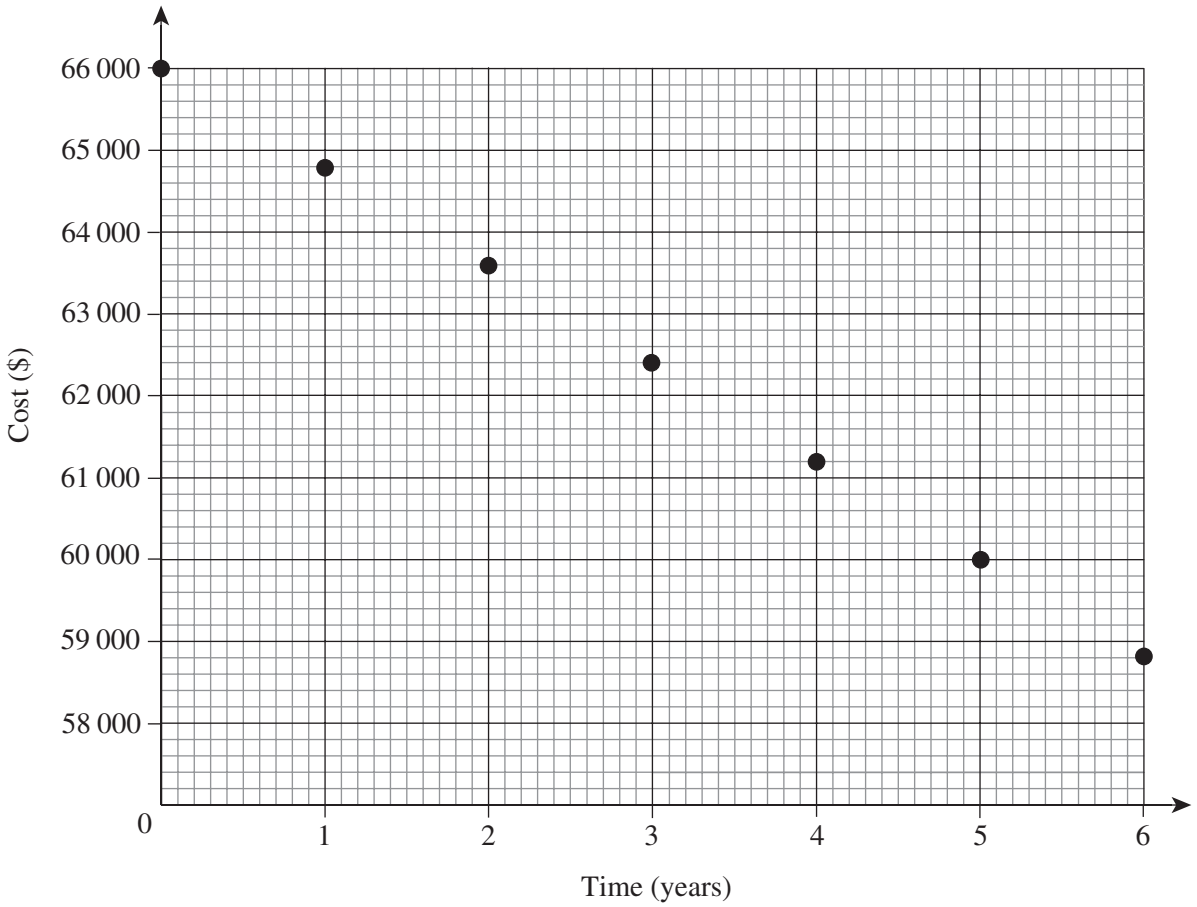
$V_6 = \$58\,800$

[2 marks]

Award 2 marks for determining all six correct values.

Note: Deduct 1 mark for each incorrect answer.

c)



[2 marks]

1 mark for giving time in years on the horizontal axis, value in dollars on the vertical axis and an evenly spaced accurate scale.

1 mark for plotting points correctly in a negative linear direction according to the values found in 25b).

Note: Axes do not need to be labelled and a line does not need to be drawn connecting the points to obtain full marks.

QUESTION 26 (6 marks)

a) $\text{angular distance} = 115°52' - 22°56'$

$= 92°56'$

$D = 111.2\cos\theta \times \text{angular distance}$

$= 111.2\cos(31°57') \times 92°56'$

$= 8768.66295\ldots$

$= 8769 \text{ km}$

[3 marks]
1 mark for identifying the angular distance.
1 mark for identifying the distance formula for a small circle.
1 mark for correctly calculating the distance to the nearest km.

b) $\text{angular distance} = 31°57' + 39°4'$

$= 71°1'$

$D = 111.2 \times \text{angular distance}$

$= 111.2 \times 71°1'$

$= 7897.053333\ldots$

$= 7897 \text{ km}$

[3 marks]
1 mark for calculating the angular distance.
1 mark for identifying the distance formula for a great circle.
1 mark for correctly calculating the distance to the nearest km.

QUESTION 27 (6 marks)

a) $\text{weekly average} = \dfrac{121 + 110 + 153 + 179 + 280 + 410 + 350}{7}$

$= \dfrac{1603}{7}$

$= 229$

[1 mark]
1 mark for giving the correct weekly average.

b) $\text{seasonal index} = \dfrac{\text{weekly value}}{\text{weekly average}}$

Monday:

$\dfrac{121}{229} = 0.528$

Tuesday:

$\dfrac{110}{229} = 0.480$

Wednesday:

$\dfrac{153}{229} = 0.668$

Thursday:

$\dfrac{179}{229} = 0.782$

Friday:

$\dfrac{280}{229} = 1.223$

Saturday:

$\dfrac{410}{229} = 1.790$

Sunday:

$\dfrac{350}{229} = 1.528$

[3 marks]

Award 3 marks for determining the seasonal indices for all 7 days.
Note: Deduct 1 mark for each incorrect answer.

c) There are 7 days in the week, so the seasonal indices should add up to 7.

$0.528 + 0.480 + 0.668 + 0.782 + 1.223 + 1.790 + 1.528 = 6.999$

The seasonal indices add up to 6.999, which is reasonable as rounding each of the indices to three decimal places in 27b) has caused an inaccuracy when adding them.

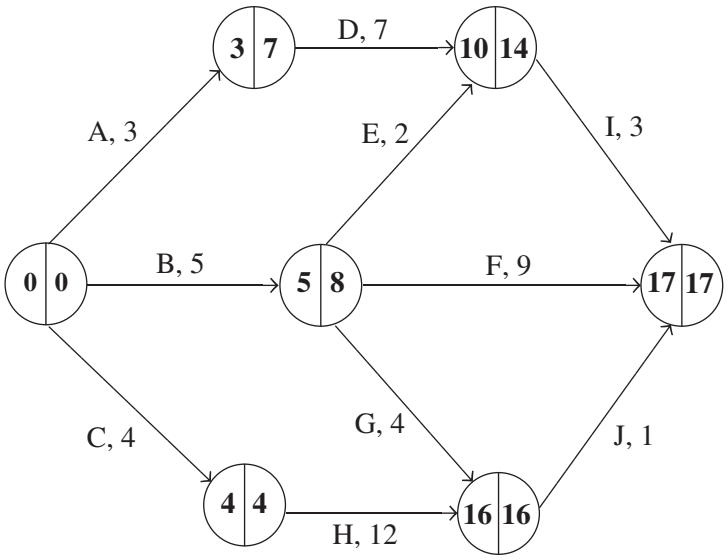
Friday, Saturday and Sunday are all above 1 as they are the busiest days of the week. Monday, Tuesday, Wednesday and Thursday are all below 1 as they are the quietest days of the week.

[2 marks]

1 mark for identifying that seasonal indices should add up to 7, and that rounding causes an inaccuracy in adding up to 7.
1 mark for identifying that Friday, Saturday and Sunday are above average, and that Monday, Tuesday, Wednesday and Thursday are below average.
Note: Accept follow-through errors.

QUESTION 28 (3 marks)

a)



[1 mark]
1 mark for showing the correct values in the empty spaces using forward and backward scanning.

b) $C \rightarrow H \rightarrow J$

[1 mark]
1 mark for correctly identifying the critical path.
Note: Accept responses that indicate the correct critical path on the network diagram.

c) 17 minutes

[1 mark]
1 mark for correctly determining the minimum completion time.