Neap

Trial Examination 2021

Suggested solutions

QCE Mathematical Methods Units 3&4

Paper 1 – Technology-free

SECTION 1 – MULTIPLE-CHOICE QUESTIONS

	A	B	C	D
1.	○	○	●	○
2.	○	●	○	○
3.	○	○	○	●
4.	○	○	●	○
5.	○	●	○	○
6.	●	○	○	○
7.	○	○	●	○
8.	○	●	○	○
9.	○	●	○	○
10.	○	○	●	○

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QUESTION 1 C

C is correct. The original function appears to be a cosine graph shifted up one unit. After deriving, the constant disappears. The derivative of a cosine graph is a negative sine graph. **A** and **B** are incorrect. Both options show a sine graph with a positive coefficient. **D** is incorrect. This option shows a cosine graph with a negative coefficient.

QUESTION 2 B

Let $f(x) = \ln(x)$ and $g(x) = x$.

$$f'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$$

$$= \frac{\frac{1}{x}x - \ln(x) \times 1}{x^2}$$

$$= \frac{1 - \ln(x)}{x^2}$$

QUESTION 3 D

$$\int e^{3x}\left(\cos x - \frac{1}{3}\sin x\right)dx = \frac{1}{3}\int e^{3x}\left(3\cos x - \sin x\right)dx$$

$$= \frac{1}{3}e^{3x}\cos(x) + c$$

QUESTION 4 C

$$\int_0^3 4x + 1\, dx = \left[\frac{4x^2}{2} + x\right]_0^3$$

$$= \frac{4 \times 3^2}{2} + 3$$

$$= 21 \text{ units}^2$$

QUESTION 5 B

$$\frac{0.71 - 0.31}{2} = 0.20$$

QUESTION 6 A

A is correct. The point $x = a$ is a minimum point. The point $x = c$ is a maximum point. A continuous function cannot have two maximum points or two minimum points in a row. Therefore, the point $x = b$ is an inflection point. **B** is incorrect. For the reasons stated above, there cannot be a maximum or minimum point at $x = b$. **C** is incorrect. If $x = b$ were a root of the function, then $f(b) = 0$. **D** is incorrect. There is enough information available to determine the nature of the point at $x = b$.

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QUESTION 7 C

$\frac{2\pi}{3}$ is in the second quadrant.

Thus, $\tan\left(\frac{2\pi}{3}\right) = -\tan\left(\frac{\pi}{3}\right)$
$= -\sqrt{3}$

QUESTION 8 B

$22 + 23 + \frac{15}{2} \approx 53$

Therefore, the probability is approximately 0.53.

QUESTION 9 B

B is correct. The interval is close enough to being one standard deviation from the mean, and therefore should be close to 68%. **A** is incorrect. The probability is too low for an interval roughly one standard deviation from the mean. **C** is incorrect. The probability is too high for an interval roughly one standard deviation from the mean. **D** is incorrect. This option is the probability for two standard deviations from the mean.

QUESTION 10 C

C is correct. This statement is false because the distribution of the sample proportion may not be normal for small sample sizes. **A**, **B** and **C** are incorrect. These statements are true and can be verified from inspecting the formulas for sample proportion.

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SECTION 2

QUESTION 11 (5 marks)

a) $\log_3 5 + \log_3(9x) = \log_3(5 \times 9x)$

$= \log_3(45x)$

$\log_3(45x) = 2$

$\therefore 45x = 3$

$= 9$

Thus, $x = \frac{9}{45}$

$= \frac{1}{5}$

[2 marks]
1 mark for using logarithmic identity (product rule) to combine terms.
1 mark for correctly solving for x.

b) $\log_{11}(2x+1) - \log_{11}(x-2) = \log_{11}(3)$

$\log_{11}\left(\frac{2x+1}{x-2}\right) = \log_{11}(3)$

$\frac{2x+1}{x-2} = 3$

$x = 7$

[3 marks]
1 mark for using logarithmic identity (quotient rule) to combine terms.
1 mark for establishing the equation that is without logarithmic terms.
1 mark for correctly solving for x.

QUESTION 12 (5 marks)

a) $f(x) = 2e^{3x^2+x+1}$

$f'(x) = 2e^{3x^2+x+1} \times (6x+1)$

[1 mark]
1 mark for correctly determining the derivative.

b) $f(x) = \ln(5x-2)$

$f'(x) = \frac{5}{5x-2}$

[1 mark]
1 mark for correctly determining the derivative.

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c) $f(x)=\dfrac{\ln(3x^7)}{\ln(5)}$

$$f'(x)=\frac{1}{\ln(5)}\times\frac{1}{3x^7}\times 3x^6\times 7$$
$$=\frac{7}{x\ln(5)}$$

[3 marks]
1 mark for rewriting the logarithmic expression using change of base method.
1 mark for determining the derivative using the chain rule.
1 mark for expressing the correct answer in simplified form.

QUESTION 13 (5 marks)

x-intercepts:

$$y=\frac{3}{2}x-x^2$$
$$=x\left(\frac{3}{2}-x\right)$$

Using the Null Factor Law, $x\left(\frac{3}{2}-x\right)=0$ implies that $x=0$ or $x=\frac{3}{2}$.

$$\text{area}=\int_0^{\frac{3}{2}}\frac{3}{2}x-x^2dx$$
$$=\left[\frac{3}{2}\times\frac{x^2}{2}-\frac{x^3}{3}\right]_0^{\frac{3}{2}}$$
$$=\left[\frac{3}{4}x^2-\frac{1}{3}x^3\right]_0^{\frac{3}{2}}$$
$$=\frac{3}{4}\left(\frac{3}{2}\right)^2-\frac{1}{3}\left(\frac{3}{2}\right)^3$$
$$=\frac{3^3}{2^4}-\frac{3^2}{2^3}$$
$$=\frac{9}{16}\text{ units}^2$$

[5 marks]
1 mark for identifying where the function intersects the x-axis.
1 mark for setting up the appropriate definite integral.
1 mark for determining the anti-derivative of the x term.
1 mark for determining the anti-derivative of the x^2 term.
1 mark for correctly substituting into the determined anti-derivative.

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QUESTION 14 (5 marks)

$$22\sin(2x) = -11$$
$$\sin(2x) = -\frac{1}{2}$$

$2x$ lies in either the third or fourth quadrant, since $(2x) < 0$.

$$2x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}, \ldots$$

Therefore, the possible values of x are $\frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}$, and $\frac{23\pi}{12}$.

[5 marks]

1 mark for determining that $\sin(2x) = -\frac{1}{2}$.

1 mark for identifying the value $\frac{\pi}{6}$ *as related to the value of 2x (this may be inferred by later working).*

1 mark for identifying a solution for 2x that belongs to a correct quadrant.

1 mark for providing one correct solution for x.

1 mark for providing all correct solutions for x.

QUESTION 15 (5 marks)

a) $$P'(t) = \frac{6}{5}e^{\frac{t}{5}}$$

$$P(t) = \int \frac{6}{5}e^{\frac{t}{5}}dt$$
$$= \frac{\frac{6}{5}e^{\frac{t}{5}}}{\frac{1}{5}} + c$$
$$= 6e^{\frac{t}{5}} + c$$

Substituting $P(0) = 36$ gives:

$$36 = 6e^{\frac{0}{5}} + c$$
$$36 = 6 + c$$
$$c = 30$$

So, $P(t) = 6e^{\frac{t}{5}} + 30.$

[3 marks]
1 mark for determining the integral of the exponential component.
1 mark for determining the value of the constant of integration.
1 mark for establishing the equation for P(t).

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b) At $t = \ln\left(3^5\right)$:

$$P(t) = 6e^{\frac{\ln\left(3^5\right)}{5}} + 30$$

$$= 6e^{\ln\left(3^5\right)^{\frac{1}{5}}} + 30$$

$$= 6e^{\ln 3} + 30$$

$$= 6 \times 3 + 30$$

$$= 48$$

Therefore, the rock-rat population will be approximately 48 at 5.5 months.

[2 marks]
1 mark for using logarithmic laws correctly to simplify after substitution.
1 mark for providing the correct final solution.
Note: Accept follow-through errors from 15a).

QUESTION 16 (5 marks)

a) $x(t) = 9\sin\left(\frac{2t+1}{3}\right)$

$$x'(t) = 9\cos\left(\frac{2t+1}{3}\right) \times \frac{2}{3}$$

$$= 6\cos\left(\frac{2t+1}{3}\right)$$

[2 marks]
1 mark for using the chain rule correctly in the determination of either the first derivative or the second derivative.
1 mark for providing the correct first derivative.

b) $$x''(t) = -6\sin\left(\frac{2t+1}{3}\right) \times \frac{2}{3}$$

$$= -4\sin\left(\frac{2t+1}{3}\right)$$

[1 mark]
1 mark for providing the correct second derivative.
Note: Accept follow-through errors from 16a).

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c) The acceleration function will reach a stationary point when $\frac{2t+1}{3}=\frac{\pi}{2}$. However, this will be a minimum as the coefficient of the sine component is negative. Thus, the acceleration function will reach a maximum when $\frac{2t+1}{3}=\frac{3\pi}{2}$.

$$\frac{2t+1}{3}=\frac{3\pi}{2}$$
$$4t+2=9\pi$$
$$t=\frac{9\pi}{4}-\frac{1}{2}$$

[2 marks]

1 mark for identifying $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ *as the possible value in the equation to solve for t.*

1 mark for correctly solving for t.

Note: $\frac{\pi}{2}$ *will lead to a minimum instead of a maximum.*

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QUESTION 17 (7 marks)

Using the total change theorem gives:

$$\begin{aligned}
\text{total change} &= \int_{10}^{\frac{20}{3}} \frac{\pi}{8}\cos\left(\frac{\pi}{8}\left(\frac{t}{5}-8\right)\right)dt \\
&= \int_{10}^{\frac{20}{3}} \frac{\pi}{8}\cos\left(\frac{\pi t}{40}-\pi\right)dt \\
&= \left[\frac{\pi}{8}\sin\left(\frac{\pi t}{40}-\pi\right)\times\frac{40}{\pi}\right]_{10}^{\frac{20}{3}} \\
&= \left[5\sin\left(\frac{\pi t}{40}-\pi\right)\right]_{10}^{\frac{20}{3}} \\
&= 5\sin\left(\frac{\pi}{40}\times\frac{20}{3}-\pi\right)-5\sin\left(\frac{10\pi}{40}-\pi\right) \\
&= 5\sin\left(-\frac{5\pi}{6}\right)-5\sin\left(-\frac{3\pi}{4}\right) \\
&= 5\times\frac{-1}{2}-5\times\frac{-1}{\sqrt{2}} \\
&= \frac{5\left(\sqrt{2}-1\right)}{2}
\end{aligned}$$

Therefore, the distance is $\frac{5\left(\sqrt{2}-1\right)}{2}$ metres.

[7 marks]

1 mark for calculating the definite integral with the correct bounds placed in order.

1 mark for determining the integral of y'.

1 mark for substituting into the integrated expression.

1 mark for determining the exact value of $\sin\left(-\frac{5\pi}{6}\right)$.

1 mark for determining the exact value of $\sin\left(-\frac{3\pi}{6}\right)$.

1 mark for correctly determining the distance.

1 mark for including the correct units.

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QUESTION 18 (5 marks)

a)
$$z = \frac{x-\mu}{\sigma} = \frac{a^2 - 2a^2}{a} = -a$$

$$P\left(X > a^2\right) = P(Z > -a) = 1 - \frac{1}{2}P(Z < -a \text{ or } Z > a) = 1 - \frac{b}{2}$$

[3 marks]

1 mark for standardising at least one of a^2, $2a^2$ or $3a^2$.

1 mark for determining that $P(Z < -a \text{ or } Z > a)$ needs to be halved.

1 mark for correctly determining the value of $P(X > a^2)$.

b)
$$z = \frac{2a^2 - 2a^2}{a} = 0 \text{ and } z = \frac{3a^2 - 2a^2}{a} = a$$

$$\therefore P\left(2a^2 < X < 3a^2\right) = P(0 < Z < a) = \frac{1-b}{2}$$

[1 mark]

1 mark for correctly determining the value of $P(2a^2 < X < 3a^2)$.

c)
$$P\left(2a^2 < X < 3a^2 \middle| X > a^2\right) = P(0 < Z < a | Z > -a) = \frac{\frac{1-b}{2}}{\left(1 - \frac{b}{2}\right)} = \frac{\frac{1-b}{2}}{\frac{2-b}{2}} = \frac{1-b}{2-b}$$

[1 mark]

1 mark for determining the value of $P(2a^2 < X < 3a^2 \mid X > a^2)$.

Note: Accept follow-through errors from 18a) and 18b).

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QUESTION 19 (8 marks)

The first point of inflection occurs when $f''(x) = 0$.

$$f'(x) = x^3 - \frac{3(a+b)x^2}{2} + 3abx$$

$$\begin{aligned} f''(x) &= 3x^2 - 3(a+b)x + 3ab \\ &= 0 \end{aligned}$$

$$\begin{aligned} 3\left(x^2 - (a+b)x + ab\right) &= 0 \\ x^2 - (a+b)x + ab &= 0 \\ (x-a)(x-b) &= 0 \end{aligned}$$

Therefore, $x - a = 0$ or $x - b = 0$.

Since $x = 1$ is a point of inflection, $1 - a = 0$ or $1 - b = 0$. However, since a and b cannot both equal 0, let $a = 1$.

$x^3 - \frac{3(a+b)x^2}{2} + 3abx = 0$ when $x = 3$.

Letting $x = 3$ and $a = 1$ gives:

$$\begin{aligned} 3^3 - \frac{3(1+b)3^2}{2} + 9b &= 0 \\ 3 - \frac{3(1+b)}{2} + b &= 0 \\ 6 - 3(1+b) + 2b &= 0 \\ 6 - 3 - 3b + 2b &= 0 \\ b &= 3 \end{aligned}$$

The second point of inflection also occurs when $f'' = 0$.

$$f''(x) = 3\left(x^2 - (a+b)x + ab\right) = 0$$

$$\therefore (x-1)(x-3) = 0$$

Therefore, the x-coordinate of the second inflection point is $x = 3$.

[8 marks]
1 mark for recognising the need to solve $f'' = 0$.
1 mark for determining f'.
1 mark for determining f''.
1 mark for solving that a or b is 1.
1 mark for using the fact that $f' = 0$ when $x = 3$ to solve for the other variable.
1 mark for determining that the other variable is 3.
1 mark for concluding that the second inflection point is $x = 3$ with appropriate reasoning.
1 mark for showing logical organisation in the response.
Note: When concluding that the other inflection point is $x = 3$, the response should include the equation $(x - 1)(x - 3) = 0$ and use the Null Factor Law to solve.

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