Neap

Trial Examination 2021

Question and response booklet

QCE Specialist Mathematics Units 3&4

Paper 2 – Technology-active

Student's Name: ______________________

Teacher's Name: ______________________

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- QCAA-approved calculator **permitted**.
- Formula booklet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (55 marks)

- 10 short response questions

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has ten questions and is worth 55 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (5 marks)

a) Use the binomial theory to expand $(\cos\theta + i\sin\theta)^3$. Express your answer in simplest form. *[2 marks]*

b) Use De Moivre's theorem to expand $(\cos\theta + i\sin\theta)^3$. *[1 mark]*

c) Use the results from 11a) and 11b) to show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. *[2 marks]*

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QUESTION 12 (6 marks)

The time taken for customers to be served at a convenience store is an exponential random variable with a mean value of 10 minutes. The exponential random variable follows a probability density function given by $f(x) = \lambda e^{-\lambda x}$, $x \geq 0$. The mean, X, is given by $\frac{1}{\lambda}$, where λ is the parameter of the variable $(\lambda > 0)$.

a) Determine λ and thus write out an expression for $f(x)$. *[2 marks]*

b) Use the result from 12a) to determine the probability that a customer takes between 8 and 12 minutes to be served. *[2 marks]*

c) It has been determined that 50% of customers are served in less than k minutes. Determine the value of k. *[2 marks]*

QUESTION 13 (5 marks)

A particle follows a path determined by the equation $x^2 + x = y^3 + 3y^2 + 2y + 2$. The x-coordinate of the particle changes at a constant rate given by $\frac{dx}{dt} = 4$.

Determine the rate at which the y-coordinates are changing when $y = 0$.

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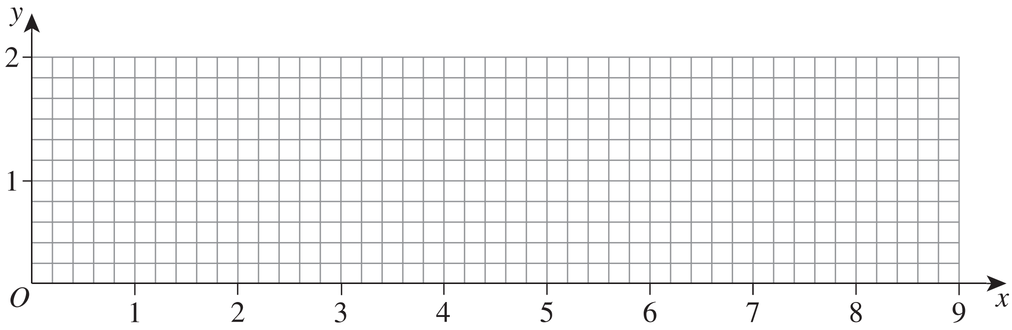
QUESTION 14 (6 marks)

a) Sketch the graph of $y = f(x)$, where $f(x) = \dfrac{e^{0.25x}}{x+1}$. *[2 marks]*



b) Using the graph from 14a), use Simpson's rule with 4 subdivisions to determine $\int_0^8 f(x)\,dx$. *[4 marks]*

QUESTION 15 (5 marks)

The lifetime of a brand of batteries was found to be normally distributed with a known mean of μ hours and a standard deviation of σ hours.

A sample of 50 boxes of the batteries was taken. A box contains 96 batteries. The average lifetime of the batteries in each box was calculated. The mean of the averages was found to be 6.42 hours, and the standard deviation of this sample was found to be 0.14 hours.

a) Determine μ and σ. *[1 mark]*

b) The batteries in one box lasted for a total of 624 hours.

Use this sample to construct a 95% confidence interval for the lifetime of this brand of battery. *[3 marks]*

c) Outline the meaning of the 95% confidence interval calculated in 15b). *[1 mark]*

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QUESTION 16 (6 marks)

A species of frog lives up to three years of age. The survival rate of these frogs in their first and second years is 7% and 19% respectively. Each female frog produces 140 offspring in their third year, and none in their first two years.

a) Represent this information in a 3×3 Leslie matrix. *[1 mark]*

Initially, there are 1500, 150 and 15 females in the first, second, and third age groups respectively.

b) Determine the expected number of females in the first age group after one year. *[2 marks]*

c) Determine the total number of frogs in the population after 4 years if the population is 55% female. *[3 marks]*

QUESTION 17 (6 marks)

A cylindrical water tank is being filled with water at a rate of $0.48\pi\ \text{m}^3\ \text{min}^{-1}$. The diameter of the tank is 6 m. At time t (min), the depth of water in the tank is h (m). There is a tap at the bottom of the tank. When the tap is open, water flows out of the tank at a rate of $0.6h\pi\ \text{m}^3\ \text{min}^{-1}$.

a) Show that $75\dfrac{dh}{dt} = 4 - 5h$ at t minutes after the tap has been opened. *[2 marks]*

b) When $t = 0$, $h = 0.2$.
Determine the value of t when $h = 0.5$. *[4 marks]*

QUESTION 18 (6 marks)

A particle of mass 1 kg is projected vertically upwards with initial velocity U. The numerical value of the air resistance is $\frac{v}{10}$, where the velocity v (m s^{-1}) at time t (s) is such that the acceleration of the particle is given by $a = -\frac{10g + v}{10}$.

a) Determine the maximum height reached by the particle. *[3 marks]*

b) Determine the time taken for the particle to reach the height determined in 18a). *[3 marks]*

QUESTION 19 (5 marks)

Six teams participated in a competition. The results are as follows.

- Team 1 defeated teams 5 and 6.
- Team 2 defeated teams 1 and 3.
- Team 3 defeated team 1.
- Team 4 defeated teams 1, 2 and 3.
- Team 5 defeated teams 2, 3, 4 and 6.
- Team 6 defeated teams 2, 3, 4 and 5.

a) By allocating 1 to represent ‘defeated’ and 0 to represent ‘was defeated by’, complete dominance matrix **D**. *[1 mark]*

Losing teams

$$\mathbf{D} = \text{Winning teams}\ \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} \square & \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square & \square \end{bmatrix} \end{array}$$

It was decided that a dominance matrix would be used to rank the teams into individual places from first to sixth place. This dominance matrix was determined to be **D**.

In order to determine the final rank, a ranking model of $\mathbf{D} + \frac{1}{2}\mathbf{D}^2 + \frac{1}{3}\mathbf{D}^3$ was used.

b) Use this model to rank the teams. *[3 marks]*

c) Identify a possible limitation of the ranking model. *[1 mark]*

QUESTION 20 (5 marks)

Two smooth marbles, X and Y, have masses 400 g and 1.2 kg respectively. The marbles are connected to each other by an infinitely light, inelastic string and lie on a frictionless ramp inclined at 46° to the horizontal. Marble Y is further up the ramp than marble X. The marbles are pulled up the ramp by a force of 15 N, acting in a direction parallel to the ramp, so that the marbles accelerate up the ramp.

a) Draw a diagram outlining the situation above. *[1 mark]*

b) Determine the acceleration of the system. *[2 marks]*

c) The system was initially stationary.

Determine the velocity of the system after it has moved 85 cm. *[2 marks]*

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Formula sheet

QCE Specialist Mathematics Units 3&4

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Calculus	
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}\sin(x) = \cos(x)$	$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$	$\int \cos(x) dx = \sin(x) + c$
$\frac{d}{dx}\tan(x) = \sec^2(x)$	$\int \sec^2(x) dx = \tan(x) + c$
$\frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\cos^{-1}\left(\frac{x}{a}\right) = \frac{-1}{\sqrt{a^2 - x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\tan^{-1}\left(\frac{x}{a}\right) = \frac{a}{a^2 + x^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

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Calculus		
chain rule	If $h(x)=f\left(g(x)\right)$ then $h'(x)=f'\left(g(x)\right)g'(x)$	If $y=f(u)$ and $u=g(x)$ then $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$
product rule	If $h(x)=f(x)g(x)$ then $h'(x)=f(x)g'(x)+f'(x)g(x)$	$\frac{dy}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	If $h(x)=\frac{f(x)}{g(x)}$ then $h'(x)=\frac{f'(x)g(x)-f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
integration by parts	$\int f(x)g'(x)dx=f(x)g(x)-\int f'(x)g(x)dx$	$\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx$
volume of a solid of revolution	about the x-axis	$V=\pi\int_a^b\left[f(x)\right]^2dx$
	about the y-axis	$V=\pi\int_a^b\left[f(y)\right]^2dy$
Simpson's rule	$\int_a^b f(x)dx\approx\frac{w}{3}\left[f\left(x_0\right)+4\left[f\left(x_1\right)+f\left(x_3\right)+\ldots\right]+2\left[f\left(x_2\right)+f\left(x_4\right)+\ldots\right]+f\left(x_n\right)\right]$	
simple harmonic motion	If $\frac{d^2x}{dt^2}=-\omega^2x$ then $x=A\sin\left(\omega t+\alpha\right)$ or $x=A\cos\left(\omega t+\beta\right)$	
	$v^2=\omega^2\left(A^2-x^2\right)$	$T=\frac{2\pi}{\omega}$ $f=\frac{1}{T}$
acceleration	$a=\frac{dv}{dt}=\frac{d^2x}{dt^2}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$	

Real and complex numbers	
complex number forms	$z=x+yi=r\left(\cos(\theta)+i\sin(\theta)\right)=r\,\mathrm{cis}(\theta)$
modulus	$\lvert z\rvert=r=\sqrt{x^2+y^2}$
argument	$\arg(z)=\theta,\ \tan(\theta)=\frac{y}{x},-\pi<\theta\le\pi$
product	$z_1z_2=r_1r_2\,\mathrm{cis}\left(\theta_1+\theta_2\right)$
quotient	$\frac{z_1}{z_2}=\frac{r_1}{r_2}\mathrm{cis}\left(\theta_1-\theta_2\right)$
De Moivre's theorem	$z^n=r^n\mathrm{cis}\left(n\theta\right)$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1} x^{n-1} y + \ldots + \binom{n}{r} x^{n-r} y^r + \ldots + y^n$	
permutation	${}^nP_r = \frac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \ldots \times (n-r+1)$	
combination	${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$	
sample means	mean	μ
	standard deviation	$\frac{\sigma}{\sqrt{n}}$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$	

Trigonometry	
Pythagorean identities	$\sin^2(A) + \cos^2(A) = 1$ $\tan^2(A) + 1 = \sec^2(A)$ $\cot^2(A) + 1 = \operatorname{cosec}^2(A)$
angle sum and difference identities	$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ $\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
double-angle identities	$\sin(2A) = 2\sin(A)\cos(A)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$ $= 1 - 2\sin^2(A)$ $= 2\cos^2(A) - 1$
product identities	$\sin(A)\sin(B) = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$ $\cos(A)\cos(B) = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$ $\sin(A)\cos(B) = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$ $\cos(A)\sin(B) = \frac{1}{2}\left(\sin(A+B) - \sin(A+B)\right)$

Vectors and matrices		
magnitude	$\lvert \boldsymbol{a} \rvert = \left\lvert \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \right\rvert = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}$	
scalar (dot) product	$\boldsymbol{a} \,.\, \boldsymbol{b} = \lvert \boldsymbol{a} \rvert \lvert \boldsymbol{b} \rvert \cos(\theta)$	
	$\boldsymbol{a} \,.\, \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} . \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$	
vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + k\boldsymbol{d}$	
Cartesian equation of a line	$\dfrac{x - a_1}{d_1} = \dfrac{y - a_2}{d_2} = \dfrac{z - a_3}{d_3}$	
vector (cross) product	$\boldsymbol{a} \times \boldsymbol{b} = \lvert \boldsymbol{a} \rvert \lvert \boldsymbol{b} \rvert \sin(\theta)\hat{\boldsymbol{n}}$	
	$\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$	
vector projection	$\boldsymbol{a}$ on $\boldsymbol{b} = \lvert \boldsymbol{a} \rvert \cos(\theta)\hat{\boldsymbol{b}} = \left(\boldsymbol{a} \,.\, \hat{\boldsymbol{b}}\right)\hat{\boldsymbol{b}}$	
vector equation of a plane	$\boldsymbol{r} \,.\, \boldsymbol{n} = \boldsymbol{a} \,.\, \boldsymbol{n}$	
Cartesian equation of a plane	$ax + by + cz + d = 0$	
determinant	If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(\mathbf{A}) = ad - bc$	
multiplicative inverse matrix	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{\det(\mathbf{A})} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, $\det(\mathbf{A}) \neq 0$	
linear transformations	dilation	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
	rotation	$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
	reflection (in the line $y = x\tan(\theta)$)	$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$

Physical constant	
magnitude of mean acceleration due to gravity on Earth	$g = 9.8$ m s^{-2}