Neap

Trial Examination 2022

Question and Response Booklet

QCE Specialist Mathematics Units 1&2

Paper 2 – Technology-active

Student's Name: ______________________

Teacher's Name: ______________________

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- QCAA-approved calculator **permitted**.
- Formula booklet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (55 marks)

- 9 short response questions

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

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SECTION 2

Instructions

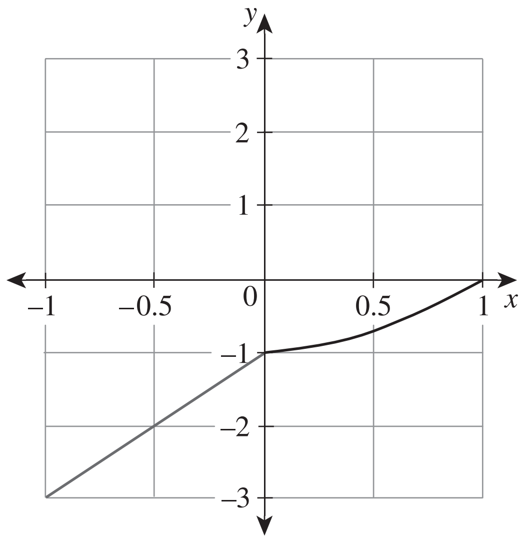
- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 55 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

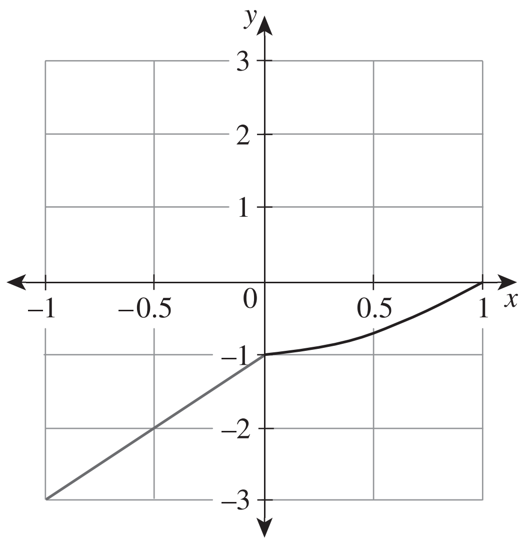
QUESTION 11 (3 marks)

a) Consider the graph of $f(x)$ over the domain $-1 < x < 1$.



On the axes above, sketch the graph of $|f(x)|$ as it would appear over the domain $-1 < x < 1$. *[2 marks]*

b) The graph of $f(x)$ over the domain $-1 < x < 1$ is presented again.



On the axes above, sketch the graph of $f(|x|)$ as it would appear over the domain $-1 < x < 0$. *[1 mark]*

QUESTION 12 (6 marks)

For all 2×2 matrices of real numbers, it is asserted that the determinant of the sum of any two matrices is the same as the sum of the determinants.

That is, $\forall\, \mathbf{M}, \mathbf{N} \in$ set of 2×2 matrices, $\det(\mathbf{M}+\mathbf{N}) = \det(\mathbf{M}) + \det(\mathbf{N})$.

a) Determine whether the assertion is true when $\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\mathbf{N} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. *[3 marks]*

b) Determine whether the assertion is true for all 2×2 matrices. *[3 marks]*

QUESTION 13 (6 marks)

On a school camp, nine students are to be allocated to three cabins. Each cabin can accommodate only three students, and all students must be allocated to a cabin. An example of one allocation is as follows.

- Cabin 1: Ann, Beth and Cathy
- Cabin 2: Dee, Emma and Fiona
- Cabin 3: Gina, Heidi and Isla

a) Calculate the number of possible ways that the nine students might be allocated to the cabins. *[2 marks]*

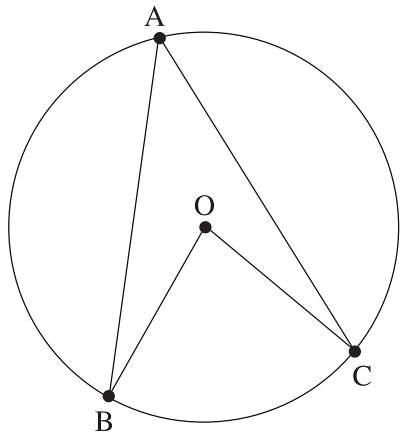
b) If students are randomly allocated to the cabins, calculate the probability that Dee and Heidi will be allocated to the same cabin. *[2 marks]*

c) The decision is made that Dee and Heidi should not be allocated to the same cabin.

Calculate the number of possible ways that the students might now be allocated to the cabins. *[2 marks]*

QUESTION 14 (5 marks)

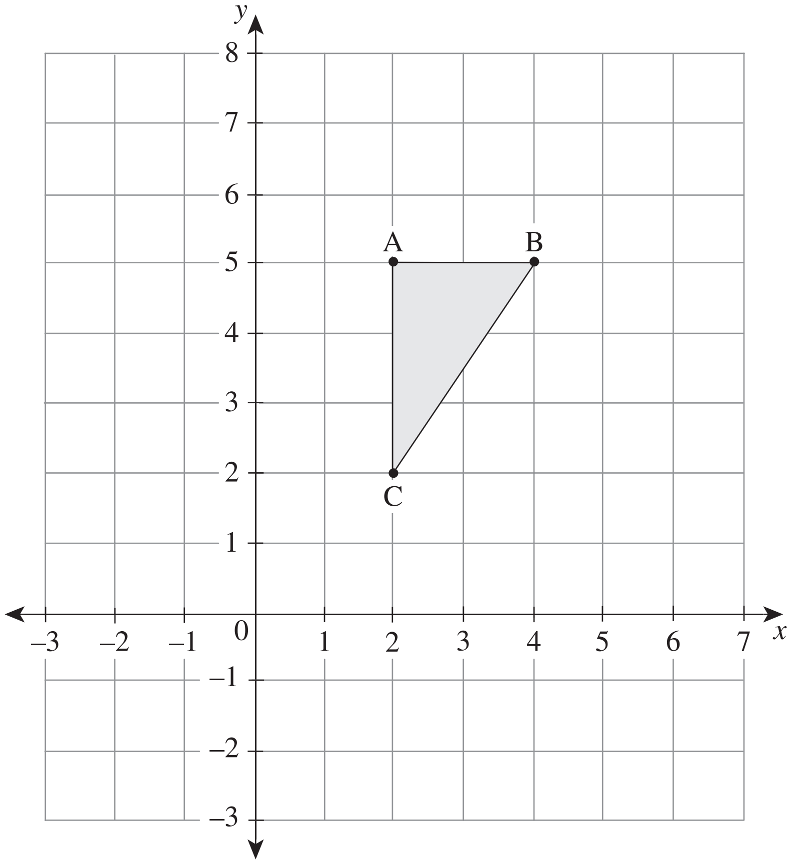
The diagram shows a circle with centre O. The points A, B and C lie on the circumference of the circle.



Prove that $\angle BOC = 2\angle BAC$.

QUESTION 15 (9 marks)

The triangle ABC is shown.



a) Determine $\overrightarrow{BC}$ in polar form. *[3 marks]*

b) If $\boldsymbol{v} = \overrightarrow{BC}$, state the unit vector $\hat{\boldsymbol{v}}$ in component form. *[2 marks]*

c) The triangle ABC is mapped to $A'B'C'$ by applying the transformation represented by the matrix $\mathbf{T} = \begin{bmatrix} 0 & -2 \\ a & 0 \end{bmatrix}$.

Determine the vertices of triangle $A'B'C'$ in terms of a. *[2 marks]*

d) If the area of triangle $A'B'C'$ is 9 units^2, calculate the value of a. *[2 marks]*

QUESTION 16 (6 marks)

Consider the matrices $\mathbf{A}=\begin{bmatrix}1 & 2 & 0\\ -1 & 0.5 & 2\\ 1 & 1 & -2\end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix}18 & 6 & 4\\ 14 & 3.5 & -9\\ 5 & 1 & 3\end{bmatrix}$.

a) State the determinant of **A**. *[1 mark]*

b) State $6\mathbf{A}^{-1}$. *[2 marks]*

c) Matrix X exists such that $\mathbf{AX}+\mathbf{X}=\mathbf{B}$.
Using factorisation to make **X** the subject of the matrix equation, solve for **X**. *[3 marks]*

QUESTION 17 (6 marks)

The parallelogram ABCD is shown.

$\overrightarrow{AB} = \boldsymbol{p}$ and $\overrightarrow{BC} = \boldsymbol{q}$ are defined.

a) Express $\overrightarrow{AC}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$. *[1 mark]*

b) Express $\overrightarrow{BD}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$. *[1 mark]*

c) Using a vector method, prove that the sum of the squares of the lengths of a parallelogram's diagonals is equal to the sum of the squares of the lengths of its sides. *[4 marks]*

QUESTION 18 (7 marks)

The line segment AB forms the diameter of a circle. The coordinates of A are (–2, –3) and the coordinates of B are (1, 1). Point C is located at coordinates (–1, 1.5).

a) Calculate the scalar product $\overrightarrow{AC} \cdot \overrightarrow{BC}$. *[2 marks]*

b) With reference to the scalar product calculated in Question 18a), draw a justified conclusion about whether point C lies inside, outside, or on the circumference of the circle with diameter AB. *[2 marks]*

c) Given that M is the midpoint of AB, determine $\overrightarrow{MC}$ and $|\overrightarrow{MC}|$. *[2 marks]*

d) Evaluate the reasonableness of your responses to Questions 18a) and 18b) with reference to the consistency of the responses. *[1 mark]*

QUESTION 19 (7 marks)

A patrol boat and a fishing boat are initially positioned so that the fishing boat is 10 kilometres due north of the patrol boat. From these initial positions, the patrol boat moves with a velocity of 8 kilometres per hour on a bearing of 040°, while the fishing boat moves with a velocity of 4 kilometres per hour on a bearing of 110°.

a) Calculate the velocity of the fishing boat relative to the patrol boat. Express your answer in kilometres per hour and include a direction. *[3 marks]*

b) The pilot of the patrol boat will see the fishing boat if it is ever less than 2 kilometres away.

Using the relative velocity, or otherwise, determine whether the pilot of the patrol boat will see the fishing boat. *[4 marks]*

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Write the question number you are responding to.

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Formula Booklet

QCE Specialist Mathematics Units 1&2

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Calculus	
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x \rvert + c$
$\frac{d}{dx}\sin(x) = \cos(x)$	$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$	$\int \cos(x) dx = \sin(x) + c$
$\frac{d}{dx}\tan(x) = \sec^2(x)$	$\int \sec^2(x) dx = \tan(x) + c$
$\frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\cos^{-1}\left(\frac{x}{a}\right) = \frac{-1}{\sqrt{a^2 - x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\tan^{-1}\left(\frac{x}{a}\right) = \frac{a}{a^2 + x^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

Calculus		
chain rule	If $h(x)=f\left(g(x)\right)$ then $h'(x)=f'\left(g(x)\right)g'(x)$	If $y=f(u)$ and $u=g(x)$ then $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$
product rule	If $h(x)=f(x)g(x)$ then $h'(x)=f(x)g'(x)+f'(x)g(x)$	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	If $h(x)=\frac{f(x)}{g(x)}$ then $h'(x)=\frac{f'(x)g(x)-f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
integration by parts	$\int f(x)g'(x)dx=f(x)g(x)-\int f'(x)g(x)dx$	$\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx$
volume of a solid of revolution	about the x-axis	$V=\pi\int_a^b[f(x)]^2\,dx$
	about the y-axis	$V=\pi\int_a^b[f(y)]^2\,dy$
Simpson's rule	$\int_a^b f(x)dx\approx\frac{w}{3}\left[f(x_0)+4\left[f(x_1)+f(x_3)+\ldots\right]+2\left[f(x_2)+f(x_4)+\ldots\right]+f(x_n)\right]$	
simple harmonic motion	If $\frac{d^2x}{dt^2}=-\omega^2x$ then $x=A\sin(\omega t+\alpha)$ or $x=A\cos(\omega t+\beta)$	
	$v^2=\omega^2\left(A^2-x^2\right)$	$T=\frac{2\pi}{\omega}$; $f=\frac{1}{T}$
acceleration	$a=\frac{dv}{dt}=\frac{d^2x}{dt^2}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$	

Real and complex numbers	
complex number forms	$z=x+yi=r\left(\cos(\theta)+i\sin(\theta)\right)=r\,\mathrm{cis}(\theta)$
modulus	$\lvert z\rvert=r=\sqrt{x^2+y^2}$
argument	$\arg(z)=\theta,\ \tan(\theta)=\frac{y}{x},-\pi<\theta\le\pi$
product	$z_1z_2=r_1r_2\,\mathrm{cis}\left(\theta_1+\theta_2\right)$
quotient	$\frac{z_1}{z_2}=\frac{r_1}{r_2}\mathrm{cis}\left(\theta_1-\theta_2\right)$
De Moivre's theorem	$z^n=r^n\mathrm{cis}(n\theta)$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
permutation	${}^nP_r = \frac{n!}{(n-r)!} = n\times(n-1)\times(n-2)\times\ldots\times(n-r+1)$	
combination	${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$	
sample means	mean	μ
	standard deviation	$\frac{\sigma}{\sqrt{n}}$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$	

Trigonometry	
Pythagorean identities	$\sin^2(A) + \cos^2(A) = 1$ $\tan^2(A) + 1 = \sec^2(A)$ $\cot^2(A) + 1 = \mathrm{cosec}^2(A)$
angle sum and difference identities	$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ $\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
double-angle identities	$\sin(2A) = 2\sin(A)\cos(A)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$ $= 1 - 2\sin^2(A)$ $= 2\cos^2(A) - 1$
product identities	$\sin(A)\sin(B) = \frac{1}{2}(\cos(A-B) - \cos(A+B))$ $\cos(A)\cos(B) = \frac{1}{2}(\cos(A-B) + \cos(A+B))$ $\sin(A)\cos(B) = \frac{1}{2}(\sin(A+B) + \sin(A-B))$ $\cos(A)\sin(B) = \frac{1}{2}(\sin(A+B) - \sin(A-B))$

Vectors and matrices		
magnitude	$\lvert \boldsymbol{a} \rvert = \left\lvert \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \right\rvert = \sqrt{a_1^{\,2} + a_2^{\,2} + a_3^{\,2}}$	
scalar (dot) product	$\boldsymbol{a} \cdot \boldsymbol{b} = \lvert \boldsymbol{a} \rvert \lvert \boldsymbol{b} \rvert \cos(\theta)$	
	$\boldsymbol{a} \cdot \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} . \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$	
vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + k\boldsymbol{d}$	
Cartesian equation of a line	$\dfrac{x - a_1}{d_1} = \dfrac{y - a_2}{d_2} = \dfrac{z - a_3}{d_3}$	
vector (cross) product	$\boldsymbol{a} \times \boldsymbol{b} = \lvert \boldsymbol{a} \rvert \lvert \boldsymbol{b} \rvert \sin(\theta) \hat{\boldsymbol{n}}$	
	$\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$	
vector projection	$\boldsymbol{a}$ on $\boldsymbol{b} = \lvert \boldsymbol{a} \rvert \cos(\theta) \hat{\boldsymbol{b}} = \left(\boldsymbol{a} \cdot \hat{\boldsymbol{b}} \right) \hat{\boldsymbol{b}}$	
vector equation of a plane	$\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$	
Cartesian equation of a plane	$ax + by + cz + d = 0$	
determinant	If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(\mathbf{A}) = ad - bc$	
multiplicative inverse matrix	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{\det(\mathbf{A})} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, $\det(\mathbf{A}) \neq 0$	
linear transformations	dilation	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
	rotation	$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
	reflection (in the line $y = x\tan(\theta)$)	$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$

Physical constant	
magnitude of mean acceleration due to gravity on Earth	$g = 9.8 \text{ m s}^{-2}$