Neap

Trial Examination 2022

Question and Response Booklet

QCE Specialist Mathematics Units 3&4

Paper 1 – Technology-free

Student's Name: ______________________________

Teacher's Name: ______________________________

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- Calculators are **not** permitted.
- Formula booklet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (55 marks)

- 9 short response questions

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

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SECTION 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 55 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (5 marks)

A plane contains three points: A(–2, 5, 8), B(3, 5, –2) and C(4, –1, 1).

a) Determine the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. *[2 marks]*

b) Determine the equation of a line passing through points A and C. *[1 mark]*

c) The centre of a sphere is located at point B. The sphere passes through point A.
Determine the Cartesian equation of the sphere. *[2 marks]*

QUESTION 12 (4 marks)

Evaluate the integral $\int_0^{\pi} \cos\left(\frac{3x}{4}\right)\cos\left(\frac{x}{4}\right)dx$.

QUESTION 13 (5 marks)

A spacecraft orbiting the planet Venus releases a probe to collect data. The gravity on the surface of Venus is $g = 9$ m s^{-2}. As the probe is released from the spacecraft, the atmosphere of Venus provides drag equal to a deceleration of $0.2v$ m s^{-2}, where v is the vertical velocity of the probe t seconds after it is released from the orbiting spacecraft. At its release, the probe's vertical velocity is zero.

a) Terminal velocity is reached when there is no net vertical acceleration acting on the falling object.

Calculate the terminal velocity of the probe. *[2 marks]*

b) Show that the relationship between v and t is $v = 45\left(1 - e^{-0.2t}\right)$. *[3 marks]*

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QUESTION 14 (7 marks)

Use integration by parts to evaluate $\int_{-2}^{2} 5x^2 e^{0.5x}\, dx$.

QUESTION 15 (7 marks)

A balloon initially has a radius of 3 cm. It is inflated at a rate of 20 $cm^3\ s^{-1}$.

Determine the rate at which the surface area of the balloon is increasing when the radius of the balloon is 5 cm.

QUESTION 16 (8 marks)

The polynomial $P(z) = z^4 + bz^3 + cz^2 - 96z + 72,\ z \in \mathbb{C},$ has three unique roots. One of the roots is $z = 6$ and two of the roots are complex.

Determine the coefficients b and c for the polynomial. Assume b and c are real.

QUESTION 17 (8 marks)

Use mathematical induction to prove that, for all $n \geq 1$, $3^n + 6^n + 9^n$ is divisible by 9.

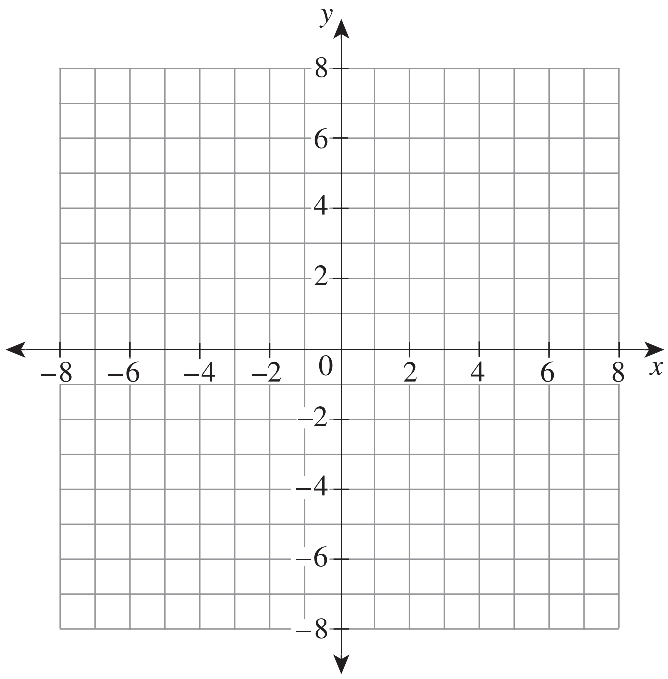
QUESTION 18 (8 marks)

Two inequalities are shown.

$$|z-2| \geq |z-3+i|$$
$$|z-1-2i| \leq 4$$

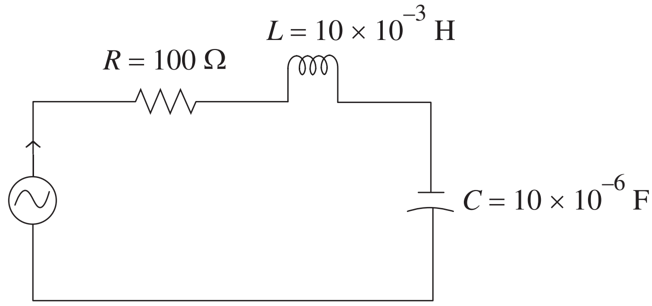
For the inequalities shown:

- describe the key characteristics of each inequality
- sketch the area of intersection between the inequalities on the Argand plane
- calculate the physical area of the intersection of the inequalities.



QUESTION 19 (3 marks)

A series RLC circuit consists of a resistor (R), inductor (L) and capacitor (C) that are connected in series, as shown in the diagram. The resistance of a resistor is measured in Ohms (Ω), the inductance of an inductor is measured in Henries (H), and the capacitance of a capacitor is measured in Farads (F).



A property of the circuit, impedance (Z) can be calculated as

$$Z = R + i(X_L + X_C),$$

where $X_L = 2\pi f L$ and $X_C = \dfrac{-1}{2\pi f C}$.

Determine the frequency, f, at which the magnitude of the impedance, Z, will be at a minimum.

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Formula Booklet

QCE Specialist Mathematics Units 3&4

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Calculus	
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}\sin(x) = \cos(x)$	$\int \sin(x)dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$	$\int \cos(x)dx = \sin(x) + c$
$\frac{d}{dx}\tan(x) = \sec^2(x)$	$\int \sec^2(x)dx = \tan(x) + c$
$\frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\cos^{-1}\left(\frac{x}{a}\right) = \frac{-1}{\sqrt{a^2 - x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\tan^{-1}\left(\frac{x}{a}\right) = \frac{a}{a^2 + x^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

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Calculus		
chain rule	If $h(x) = f\left(g(x)\right)$ then $h'(x) = f'\left(g(x)\right)g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
integration by parts	$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$	$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$
volume of a solid of revolution	about the x-axis	$V = \pi\int_a^b [f(x)]^2\,dx$
	about the y-axis	$V = \pi\int_a^b [f(y)]^2\,dy$
Simpson's rule	$\int_a^b f(x)dx \approx \frac{w}{3}\left[f(x_0) + 4\left[f(x_1) + f(x_3) + \ldots\right] + 2\left[f(x_2) + f(x_4) + \ldots\right] + f(x_n)\right]$	
simple harmonic motion	If $\frac{d^2x}{dt^2} = -\omega^2 x$ then $x = A\sin(\omega t + \alpha)$ or $x = A\cos(\omega t + \beta)$	
	$v^2 = \omega^2\left(A^2 - x^2\right)$	$T = \frac{2\pi}{\omega}$; $f = \frac{1}{T}$
acceleration	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	

Real and complex numbers	
complex number forms	$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$
modulus	$\lvert z\rvert = r = \sqrt{x^2 + y^2}$
argument	$\arg(z) = \theta,\ \tan(\theta) = \frac{y}{x}, -\pi < \theta \le \pi$
product	$z_1 z_2 = r_1 r_2\,\text{cis}\left(\theta_1 + \theta_2\right)$
quotient	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\text{cis}\left(\theta_1 - \theta_2\right)$
De Moivre's theorem	$z^n = r^n\text{cis}(n\theta)$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
permutation	${}^nP_r = \frac{n!}{(n-r)!} = n\times(n-1)\times(n-2)\times\ldots\times(n-r+1)$	
combination	${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$	
sample means	mean	μ
	standard deviation	$\frac{\sigma}{\sqrt{n}}$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$	

Trigonometry	
Pythagorean identities	$\sin^2(A) + \cos^2(A) = 1$ $\tan^2(A) + 1 = \sec^2(A)$ $\cot^2(A) + 1 = \text{cosec}^2(A)$
angle sum and difference identities	$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ $\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
double-angle identities	$\sin(2A) = 2\sin(A)\cos(A)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$ $= 1 - 2\sin^2(A)$ $= 2\cos^2(A) - 1$
product identities	$\sin(A)\sin(B) = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$ $\cos(A)\cos(B) = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$ $\sin(A)\cos(B) = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$ $\cos(A)\sin(B) = \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$

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Vectors and matrices		
magnitude	$\lvert \boldsymbol{a} \rvert = \left\lvert \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \right\rvert = \sqrt{a_1^2 + a_2^2 + a_3^2}$	
scalar (dot) product	$\boldsymbol{a} \cdot \boldsymbol{b} = \lvert \boldsymbol{a} \rvert \lvert \boldsymbol{b} \rvert \cos(\theta)$	
	$\boldsymbol{a} \cdot \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} . \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$	
vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + k\boldsymbol{d}$	
Cartesian equation of a line	$\frac{x - a_1}{d_1} = \frac{y - a_2}{d_2} = \frac{z - a_3}{d_3}$	
vector (cross) product	$\boldsymbol{a} \times \boldsymbol{b} = \lvert \boldsymbol{a} \rvert \lvert \boldsymbol{b} \rvert \sin(\theta) \hat{\boldsymbol{n}}$	
	$\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$	
vector projection	$\boldsymbol{a}$ on $\boldsymbol{b} = \lvert \boldsymbol{a} \rvert \cos(\theta) \hat{\boldsymbol{b}} = \left(\boldsymbol{a} \cdot \hat{\boldsymbol{b}} \right) \hat{\boldsymbol{b}}$	
vector equation of a plane	$\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$	
Cartesian equation of a plane	$ax + by + cz + d = 0$	
determinant	If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(\mathbf{A}) = ad - bc$	
multiplicative inverse matrix	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \ \det(\mathbf{A}) \neq 0$	
linear transformations	dilation	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
	rotation	$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
	reflection (in the line $y = x\tan(\theta)$)	$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$

Physical constant	
magnitude of mean acceleration due to gravity on Earth	$g = 9.8 \text{ m s}^{-2}$