Neap

Trial Examination 2023

Suggested Solutions

QCE Specialist Mathematics Units 1&2

Paper 1 – Technology-free

SECTION 1 – MULTIPLE CHOICE QUESTIONS

	A	B	C	D
1.			●	
2.			●	
3.	●			
4.		●		
5.			●	
6.				●
7.	●			
8.				●
9.		●		
10.		●		

QUESTION 1 C

C is correct. Using the formula for the inverse of a 2 × 2 matrix gives:

$$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{2\times3-(1\times-6)}\begin{bmatrix} 3 & -1 \\ -(-6) & 2 \end{bmatrix}$$
$$= \frac{1}{12}\begin{bmatrix} 3 & -1 \\ 6 & 2 \end{bmatrix}$$

A is incorrect. This option may be reached by making a sign error in the evaluation of $ad - bc$, such as $2 \times 3 - (1 \times -6) = 0$. If the determinant is 0, then the inverse will not exist.

B is incorrect. This option may be reached by not recognising that **I** is the identity matrix; thus, while $\mathbf{MM}^{-1} = \mathbf{I}$, $\mathbf{M}^{-1} \neq \mathbf{I}$.

D is incorrect. As the inverse of **M** exists, it follows that $\mathbf{MM}^{-1} = \mathbf{I}$.

QUESTION 2 C

C is correct. Substituting $\theta = \frac{\pi}{3}$ into $2\sin^2(\theta)$ gives:

$$2\sin^2(\theta) = 2\sin^2\left(\frac{\sqrt{3}}{2}\right)$$
$$= 2\times\left(\frac{\sqrt{3}}{2}\right)^2$$
$$= 2\times\left(\frac{3}{4}\right)$$
$$= 1.5$$

Substituting $\theta = \frac{\pi}{3}$ into $\sin\left(\frac{\theta}{2}\right)$ gives:

$$\sin\left(\frac{\theta}{2}\right) = \sin\left(\frac{\pi}{3\times2}\right)$$
$$= \sin\left(\frac{\pi}{6}\right)$$
$$= 0.5$$

Therefore:

$$2\sin^2(\theta) - \sin\left(\frac{\theta}{2}\right) = 2\sin^2\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{6}\right)$$
$$= 1.5 - 0.5$$
$$= 1$$

A is incorrect. This expression may be reached by assuming that $\sin\left(\frac{\pi}{3}\right) = 0.5$ and $\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.

B is incorrect. This expression may be reached by assuming that $\sin\left(\frac{\pi}{3}\right) = 0.5$ and $\sin\left(\frac{\theta}{2}\right) = \frac{1}{2}\sin(\theta)$.

D is incorrect. This expression may be reached by neglecting to square $\sin(\theta)$ when substituting.

QUESTION 3 A

A is correct. Given that $\cot(\theta) = \dfrac{1}{\tan(\theta)}$, the vertical asymptotes will occur when $\tan(x) = 0$.

As $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$, $\sin(x)$ must be 0, which occurs for integer multiples of π.

B is incorrect. This option may be reached by misinterpreting the vertical dilation of 3 as a horizontal dilation of $\dfrac{1}{3}$.

C is incorrect. This option may be reached by finding an expression for the asymptotes of the function $f(x) = \tan(x)$, and assuming that the period of a tan function is $\dfrac{\pi}{2}$.

D is incorrect. This option may be reached by finding an expression for the asymptotes of the function $f(x) = \tan(x)$.

QUESTION 4 B

B is correct.

$$\begin{aligned} z^2 - 4z + 5 &= z^2 - 4z + 4 + 1 \\ &= (z-2)^2 + 1 \end{aligned}$$

Hence, equating to 0 gives:

$$\begin{aligned} (z-2)^2 + 1 &= 0 \\ z &= \pm\sqrt{-1} + 2 \\ &= 2 \pm i \end{aligned}$$

A is incorrect. This option may be reached by factorising $z^2 - 4z + 4$ as $(z + 2)^2$ or implementing the quadratic formula as $z = \dfrac{b - \sqrt{b^2 - 4ac}}{2a}$, which omits the negative coefficient of b.

C is incorrect. This option may be reached by implementing the quadratic formula as $z = b - \dfrac{\sqrt{b^2 - 4ac}}{2a}$, which omits the negative coefficient of b and applies the divisor to only the second term.

D is incorrect. This option may be reached by implementing the quadratic formula as $z = -b - \dfrac{\sqrt{b^2 - 4ac}}{2a}$, which applies the divisor to only the second term.

QUESTION 5 C

C is correct.

$$\binom{6}{2}=\frac{6!}{2!(6-2)!}$$
$$=\frac{6\times5\times4!}{2!4!}$$
$$=\frac{30}{2}$$
$$=15$$

A is incorrect. This option may be reached by interpreting the notation $\binom{6}{2}$ as $\frac{6}{2}$.

B is incorrect. This option may be reached by misinterpreting the notation $\binom{6}{2}$ as 6×2.

D is incorrect. This option may be reached by evaluating $^6P_2$ instead of $^6C_2$.

QUESTION 6 D

D is correct. Using the addition principle $\Pr(A\cup B)=\Pr(A)+\Pr(B)-\Pr(A\cap B)$ gives:

$$\Pr(A\cup B)=0.3+0.3-0.2$$
$$=0.4$$
$$n(A\cup B)=0.4\times100$$
$$=40$$

A is incorrect. This option may be reached by not realising that both $\Pr(A)$ and $\Pr(B)$ are 0.3 and thus finding $\Pr(A\cup B)=0.1$.

B is incorrect. This option may be reached by misinterpreting the symbols $\Pr(A\cap B)$ and $\Pr(A\cup B)$.

C is incorrect. This option may be reached by finding either $n(A)$ or $n(B)$, not $n(A\cup B)$. This may result from the misinterpretation of the statement $\Pr(A)=\Pr(B)=0.3$, or the inability to interpret the word ‘union’.

QUESTION 7 A

A is correct.

$$z_1=2cis\left(-\frac{\pi}{2}\right)$$
$$=-2i$$
$$\therefore\overline{z}_1=-(-2i)$$
$$=2i$$
$$=z_3$$

B is incorrect. This option may be reached by moving the negative sign from the argument to the magnitude.

C is incorrect. This option may be reached by incorrectly converting z_2 to polar form. In polar form, $z_2=2cis\left(-\frac{\pi}{4}\right)$, which may be confused with $2cis\left(-\frac{\pi}{2}\right)$ by students who are inexperienced with radians.

D is incorrect. This option may be reached by assuming that $\overline{2i}=2i$, or misinterpreting the notation

as representing the conjugate operation.

QUESTION 8 D

D is correct. When $a = 4$ and $b = 3$, the values are compliant with the assumption as $a > b$. Substituting into $a^2 - b^2$ gives:

$$\begin{aligned} 4^2 - 3^2 &= 16 - 9 \\ &= 7 \end{aligned}$$

The number 7 is not even, so option **D** is a counter example that disproves the assertion.

A is incorrect. This option may be reached by assuming that an example of the assumption is required. Substituting $a = 5$ and $b = 3$ into $a^2 - b^2$ gives:

$$\begin{aligned} a^2 - b^2 &= 5^2 - 3^2 \\ &= 25 - 9 \\ &= 16 \text{ (even)} \end{aligned}$$

B is incorrect. This option may be reached by misinterpreting the assumption and thus selecting a and b values that are both even.

C is incorrect. This option may be reached by misinterpreting the assumption and thus selecting values that fulfil $a < b$, rather than $a > b$.

QUESTION 9 B

B is correct. The vector $\boldsymbol{v}$ lies in the $\boldsymbol{j}$ direction. Hence, the projection of $\boldsymbol{u}$ on $\boldsymbol{v}$ is the term in the component expression that is in the $\boldsymbol{j}$ direction; that is, $\sqrt{2}\boldsymbol{j}$.

A is incorrect. This option may be reached by interpreting $\boldsymbol{v}$ as being in the $\boldsymbol{i}$ direction, or finding the component of $\boldsymbol{u}$ that is perpendicular to $\boldsymbol{v}$.

C is incorrect. This option may be reached by failing to distinguish between the projection, which applies to vectors, and the magnitude of the projection, which applies to scalars.

D is incorrect. This option may be reached by determining the magnitude of $\boldsymbol{u}$ instead of the projection.

QUESTION 10 B

B is correct. The number of seating arrangements for the bearded men is 2!. The number of seating arrangements for the beardless men is 3!.

$$\begin{aligned} \therefore n(\text{arrangements}) &= 2! \times 3! \\ &= 2 \times 6 \\ &= 12 \end{aligned}$$

A is incorrect. This option may be reached by evaluating 3!, which overlooks the 2! ways to arrange the men with beards.

C is incorrect. This option may be reached by evaluating 4!.

D is incorrect. This option may be reached by evaluating 5! after misinterpreting the restriction.

SECTION 2

QUESTION 11 (4 marks)

Method 1:

$$\begin{aligned} z^2+6z+9+4 &= (z+3)^2+4 \\ &= (z+3)^2-(-4) \\ &= (z+3)^2-(2i)^2 \\ &= (z+3-2i)(z+3+2i) \end{aligned}$$

[4 marks]

1 mark for completing the square using z^2+6z+9.

1 mark for rewriting +4 as –(–4) to show progress towards the difference of two squares. Note: This may be implied by subsequent working.

1 mark for showing evidence that $-4=(2i)^2$.

1 mark for providing the correct solution.

Method 2:

Solving $z^2+6z+13=0$ using the quadratic formula gives $a=1$, $b=6$, $c=13$. Therefore:

$$\begin{aligned} z &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\ &= \frac{-6\pm\sqrt{6^2-52}}{2} \\ &= -3\pm\frac{\sqrt{-16}}{2} \\ &= -3\pm 2i \end{aligned}$$

$$\begin{aligned} \therefore P(z) &= (z-(-3+2i))(z-(-3-2i)) \\ &= (z+3-2i)(z+3+2i) \end{aligned}$$

[4 marks]

1 mark for showing evidence of using the quadratic formula to solve $Q(z)=0$.

1 mark for substituting $a=1$, $b=6$ and $c=13$.

1 mark for obtaining $-3\pm 2i$.

1 mark for factorising $Q(z)$ using brackets of the form (z – (first solution))(z – (second solution)).

Note: Accept follow-through errors.

QUESTION 12 (5 marks)

a) $\angle AOC = 180° - 2\theta$ as ΔAOC isosceles (radii). The internal angles of ΔAOC sum to 180°.

[1 mark]
1 mark for providing the correct solution.
Note: Accept equivalent expressions; for example, 180 – (θ + θ).

b) $\angle BOC = 2\theta°$ as $\angle AOC$ and $\angle BOC$ are supplementary.

[1 mark]
1 mark for providing the correct solution.
Note: Accept equivalent expressions.

c) **Method 1:**

$$\angle OCB = \frac{180 - 2\theta}{2}$$
$$= (90 - \theta)°$$ as ΔOCB isosceles (radii). The internal angles of ΔOCB sum to 180°.

$$\angle ACB = \angle ACO + \angle OCB$$
$$= \theta + 90 - \theta$$
$$= 90°$$ (as required)

[3 marks]
1 mark for calculating $\angle OCB = (90 - \theta)°$.
1 mark for showing that $\angle ACB = 90°$.
1 mark for using appropriate reasoning, units and sequencing.

Method 2:

$$\angle OBC = \frac{180 - 2\theta}{2}$$
$$= (90 - \theta)°$$ as ΔOCB isosceles (radii). The internal angles of ΔOCB sum to 180°.

Given that the sum of internal angles of ΔABC is 180°:

$$\angle ACB = 180 - (\angle BAC + \angle ABC)$$
$$= 180 - (\theta + 90 - \theta)$$
$$= 90°$$ (as required)

[3 marks]
1 mark for calculating $\angle OCB = (90 - \theta)°$.
1 mark for showing that $\angle ACB = 90°$.
1 mark for using appropriate reasoning, units and sequencing.

Method 3 (vectors):

When $\overrightarrow{OC} = c$ and $\overrightarrow{OB} = b$, $\overrightarrow{OA} = -b$.

RTP: $\overrightarrow{AC} \cdot \overrightarrow{BC} = 0$

$$(c - a) \cdot (c - b) = (c + b) \cdot (c - b)$$
$$= c \cdot c - b \cdot b$$
$$= |c|^2 - |b|^2$$
$$= 0 \text{ (radii)}$$

[3 marks]
1 mark for showing that the dot product must equal zero for orthogonal vectors.
1 mark for expressing vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$.
1 mark for evaluating the dot product as 0.

QUESTION 13 (5 marks)

a) **Method 1:**

$$\begin{array}{r} 0.45 \\ 11\overline{)5.0000} \\ \underline{-4.40} \\ 0.60 \\ \underline{-0.55} \\ 0.050 \end{array}$$

$$\therefore \frac{5}{11} = 0.\dot{4}\dot{5}$$

[2 marks]

1 mark for setting out the division using 5 inside the division sign and 11 outside the division sign.

1 mark for providing the correct solution. Note: Accept variations in notation; for example, $0.\overline{45}$.

Method 2:

$$\because \frac{5}{11} = \frac{45}{99}$$

$$\frac{5}{11} = 0.\dot{4}\dot{5}$$

[2 marks]

1 mark for providing the correct solution. Note: Accept variations in notation; for example, $0.\overline{45}$.

Note: Only 1 mark can be awarded for Method 2 as the question requires an appropriate division, which is not present in this method.

b) **Method 1:**

$$0.1232323\dot{2}\dot{3} = 0.1 + 0.0\dot{2}\dot{3}$$
$$= \frac{1}{10} + \frac{23}{99} \times \frac{1}{10}$$
$$= \frac{99}{990} + \frac{23}{990}$$
$$= \frac{122}{990}$$
$$= \frac{61}{495}$$

[3 marks]
1 mark for separating the decimal into the sum of non-recurring and recurring parts.
1 mark for providing the correct solution without simplification. Note: This mark may be implied by the simplified solution.
1 mark for simplifying the solution.

Method 2:

$$x = 0.1\dot{2}\dot{3}$$
$$\therefore 100x = 12.3\dot{2}\dot{3}$$
$$100x - x = 99x$$
$$= 12.3\dot{2}\dot{3} - 0.1\dot{2}\dot{3}$$
$$= 12.2$$
$$x = \frac{12.2}{99}$$
$$= \frac{122}{990}$$
$$= \frac{61}{495}$$

[3 marks]
1 mark for using a strategy where the original number is multiplied by 100.
1 mark for providing the correct solution without simplification. Note: Accept any equivalent expression. This mark may be implied by the simplified solution.
1 mark for simplifying the solution.

QUESTION 14 (5 marks)

a)
$$\mathbf{A}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$
$$= \frac{1}{5\times1-2\times2}\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$$

[1 mark]
1 mark for providing the correct solution.

b)
$$\mathbf{A}^{-1}\mathbf{B} = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 15-2 & -5-4 \\ -6+1 & 2+2 \end{bmatrix}$$
$$= \begin{bmatrix} 13 & -9 \\ -5 & 4 \end{bmatrix}$$

[2 marks]
1 mark for evaluating at least two elements of $\mathbf{A}^{-1}\mathbf{B}$.
1 mark for evaluating all four elements of $\mathbf{A}^{-1}\mathbf{B}$.
Note: Accept follow through errors. Consequential on answer to ***Question 14a)****.*

c)
$$\mathbf{AX} = \mathbf{B}$$
$$\mathbf{A}^{-1}\mathbf{AX} = \mathbf{A}^{-1}\mathbf{B}$$
$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$$

[1 mark]
1 mark for providing the correct solution.
Note: The order of the notation is important. Do not accept $\mathbf{BA}^{-1}$ *for this mark.*

d) Given that $\mathbf{A}^{-1}\mathbf{B}$ was evaluated in Question 14b):
$$\mathbf{X} = \begin{bmatrix} 13 & -9 \\ -5 & 4 \end{bmatrix}$$

[1 mark]
1 mark for providing the correct solution.
Note: Consequential on answer to ***Question 14b)****.*

QUESTION 15 (5 marks)

a) $M = \left(\dfrac{-1+c}{2}, \dfrac{3+c}{2}\right)$

[1 mark]
1 mark for providing the correct solution.
Note: Accept equivalent expressions; for example, (0.5c – 0.5, 0.5c + 0.5).

b) In order to produce a right angle, $\overrightarrow{AB} \cdot \overrightarrow{BC} = 0$.

$$\begin{aligned}\overrightarrow{AB} &= \boldsymbol{b} - \boldsymbol{a}\\ &= \begin{pmatrix} 2+1 \\ 1-3 \end{pmatrix}\\ &= \begin{pmatrix} 3 \\ -2 \end{pmatrix}\end{aligned}$$

$$\begin{aligned}\overrightarrow{BC} &= c - b\\ &= \begin{pmatrix} c-2 \\ c-1 \end{pmatrix}\end{aligned}$$

$$\begin{pmatrix} 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} c-2 \\ c-1 \end{pmatrix} = 3c - 6 - 2c + 2$$

$$c - 4 = 0$$

$$c = 4$$

[4 marks]
1 mark for showing that the dot product of perpendicular vectors must be 0.
1 mark for finding a displacement vector using the definition $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$.
1 mark for expressing the dot product as either 3c – 6 – 2c + 2 or –(3c – 6 – 2c + 2).
1 mark for providing the correct solution.

QUESTION 16 (8 marks)

a) $p = zw$

$= 1cis\left(\frac{\pi}{4}\right) \times 2cis\left(-\frac{\pi}{2}\right)$

$= 1 \times 2cis\left(\frac{\pi}{4} - \frac{\pi}{2}\right)$

$= 2cis\left(-\frac{\pi}{4}\right)$

[2 marks]
1 mark for showing evidence of the method for multiplying magnitudes and adding angles.
1 mark for providing the correct solution in polar form. Note: Accept equivalent expressions; for example, $-\frac{2\pi}{8}$ for $-\frac{\pi}{4}$.

b) $q = \frac{z}{w}$

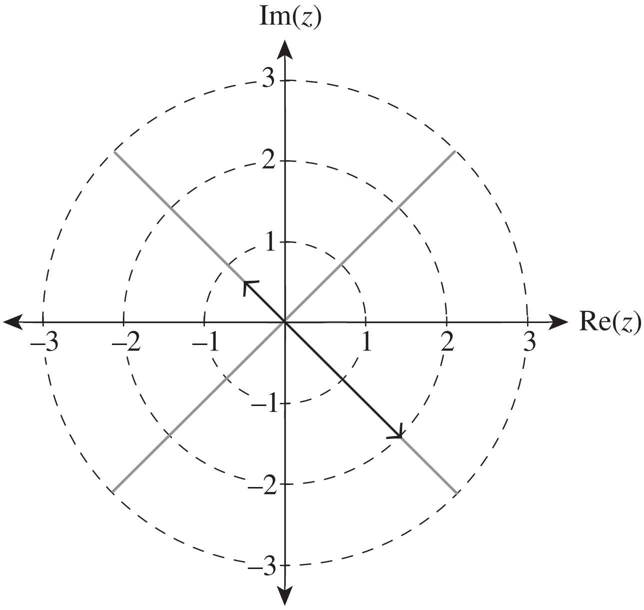
$= \frac{cis\left(\frac{\pi}{4}\right)}{2cis\left(-\frac{\pi}{2}\right)}$

$= \frac{1}{2}cis\left(\frac{\pi}{4} + \frac{\pi}{2}\right)$

$= 0.5cis\left(\frac{3\pi}{4}\right)$

[2 marks]
1 mark for showing evidence of using the method for dividing magnitudes and subtracting angles.
1 mark for providing the correct solution in polar form. Note: Accept equivalent expressions; for example, $\frac{6\pi}{8}$ for $\frac{3\pi}{4}$; $\frac{1}{2}$ for 0.5.

c)



[2 marks]

1 mark for plotting the value of p consistently on the polar grid.

1 mark for plotting the value of q consistently on the polar grid.

*Note: The values of p and q may be flawed. Consequential on answers to **Question 16a)** and **16b)**.*

d) **Method 1:**

From the diagram, it is apparent that the angle between p and q is 180°. Hence, the addition of these two terms is equivalent to the vector addition.

$$s = (2 - 0.5)cis\left(-\frac{\pi}{4}\right)$$

$$= 1.5cis\left(-\frac{\pi}{4}\right)$$

[2 marks]

1 mark for providing an appropriate supporting statement for working. Note: This statement may refer to vectors or show the (2 – 0.5) step.

*1 mark for providing the correct solution. Note: Accept equivalent expressions. Consequential on answers to **Questions 16a), 16b)** and **16c)**.*

Method 2:

$$p + q = 2\cos\left(-\frac{\pi}{4}\right) + 2i\sin\left(\frac{-\pi}{4}\right) + 0.5\cos\left(\frac{3\pi}{4}\right) + 0.5i\sin\left(\frac{3\pi}{4}\right)$$

$$= 2\left(\frac{1}{\sqrt{2}}\right) + 2i\left(-\frac{1}{\sqrt{2}}\right) + 0.5\left(-\frac{1}{\sqrt{2}}\right) + 0.5i\left(\frac{1}{\sqrt{2}}\right)$$

$$= 1.5\left(\frac{1}{\sqrt{2}}\right) + 1.5\left(-\frac{1}{\sqrt{2}}\right)$$

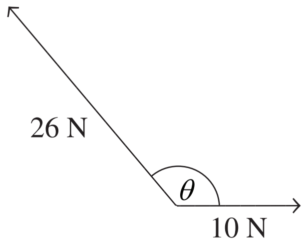
$$= 1.5cis\left(-\frac{\pi}{4}\right)$$

[2 marks]

1 mark for converting to component form by substituting $\cos\theta + i\sin\theta$ for cis.

*1 mark for providing the correct solution. Note: Accept equivalent expressions. Consequential on answers to **Questions 16a)** and **16b)**.*

QUESTION 17 (4 marks)



Considering the sum of these forces, assign *i* to the direction of the 10 N force and *j* to the direction of the 26 N force.

Resolving into components:

$$i:\ 10+26\cos(\theta)=10+26\times-\frac{5}{13}$$
$$=10-10$$
$$=0$$

$$j:\ 26\sin(\theta)=26\times\frac{12}{13}$$
$$=24$$

In order for the object to remain stationary, all forces must be balanced.

Therefore, the magnitude of the third force is 24 N.

[4 marks]

1 mark for drawing a diagram that shows two vectors positioned tail to tail with the angle θ between them.

1 mark for resolving the forces into components to conduct a vector addition.

1 mark for determining the i direction.

1 mark for providing the correct solution.

QUESTION 18 (6 marks)

Given the facts (1) $R\cos\theta = 1$ and (2) $R\sin\theta = \sqrt{3}$:

$$\therefore \frac{R\sin\theta}{R\sin\theta} = \tan\theta$$

$$= \frac{\sqrt{3}}{1}$$

$$\therefore \theta = \frac{\pi}{3} \text{ OR } 60$$

$$\cos\left(\frac{\pi}{3}\right) = 0.5$$

$$\therefore R \times 0.5 = 1$$

$$R = 2$$

$$\therefore \cos(x) - \sqrt{3}\sin(x) = 2\cos\left(x + \frac{\pi}{3}\right)$$

Solving $\cos(x) - \sqrt{3}\sin(x) = \sqrt{2}$ using $2\cos\left(x + \frac{\pi}{3}\right) = \sqrt{2}$ gives:

$$2\cos\left(x + \frac{\pi}{3}\right) = \sqrt{2}$$

$$\cos\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

$$x + \frac{\pi}{3} = \pm\frac{\pi}{4}$$

Using $x + \frac{\pi}{3} = \frac{\pi}{4}$ gives:

$$x = \frac{\pi}{4} - \frac{\pi}{3}$$

$$= \frac{3\pi}{12} - \frac{4\pi}{12}$$

$$= -\frac{\pi}{12}$$

$$= -15°$$

Using $x + \frac{\pi}{3} = -\frac{\pi}{4}$ gives:

$$x = -\frac{\pi}{4} - \frac{\pi}{3}$$

$$= -\frac{3\pi}{12} - \frac{4\pi}{12}$$

$$= -\frac{7\pi}{12}$$

$\therefore$ the solutions for x are $-\frac{\pi}{12}$ and $-\frac{7\pi}{12}$

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(continued)

[6 marks]

1 mark for recognising at least one of the facts (1) $R\cos\theta = 1$ OR (2) $R\sin\theta = \sqrt{3}$.

Note: This mark may be implied by subsequent working.

1 mark for determining $\theta = \frac{\pi}{3}$. Note: Accept 60° for this mark.

1 mark for determining $R = 2$.

1 mark for converting, formulating and solving $R\cos(x \pm \theta) = \sqrt{2}$. Note: This expression may be flawed.

1 mark for providing the first correct solution for x. Note: Accept responses in degrees.

1 mark for providing the second correct solution for x. Note: Do not accept responses in degrees.

Note: Accept follow-through errors for the values of R and θ.

QUESTION 19 (8 MARKS)

a) **Method 1:**

If **A** is in P and $\mathbf{A}^{-1}$ exists, then $\mathbf{A} = \mathbf{I}_2$.

[1 mark]

1 mark for interpreting ∈ as 'in', ∃ as 'exists' and ⇒ as 'if ... then'.

Method 2:

A in P and $\mathbf{A}^{-1}$ exists implies $\mathbf{A} = \mathbf{I}_2$.

[1 mark]

1 mark for interpreting ∈ as 'in', ∃ as 'exists' and ⇒ as 'implies'.

b) **Method 1:**

The logic that precedes statement 6 is all sound.

The only idempotent matrix for which an inverse exists is the identity matrix. Therefore, the statement is reasonable.

[2 marks]

1 mark for providing a valid comment relating to the logic of statements 1–5.
1 mark for drawing a consistent conclusion regarding the reasonableness of statement 6.

Method 2:

It is true that the identity matrix is idempotent.

The statement is reasonable as all powers of the identity matrix will yield the identity matrix.

[2 marks]

1 mark for stating that powers of the identity matrix produce the identity matrix.
1 mark for drawing a consistent conclusion regarding the reasonableness of statement 6.

c) A sufficient condition for inclusion in P is $\mathbf{K}^2 = \mathbf{K}$.

$$\mathbf{K}^2 = \mathbf{K}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$a^2 + bc = a \quad (1)$

$ab + bd = b \quad (2)$

$ac + cd = c \quad (3)$

$bc + d^2 = d \quad (4)$

Rearranging (2) gives:

$ab + bd - b = 0$

$b(a + d - 1) = 0$

As $b \neq 0$:

$a + d - 1 = 0$

$a + d = 1$

$d = 1 - a$

Rearranging (1) gives:

$bc = a - a^2$

$= a(1 - a)$

$c = \dfrac{a(1-a)}{b}$ OR $\dfrac{a - a^2}{b}$ (As $b \neq 0$, this is valid.)

Hence, so that $\mathbf{K} \in$ P, it is required that $d = 1 - a$ and $c = \dfrac{a - a^2}{b}$.

[5 marks]

1 mark for using the statement $\boldsymbol{K}^2 = \boldsymbol{K}$. Note: This mark may be implied by subsequent working.

*1 mark for substituting $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ into **K** and performing a multiplication of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ to determine $\boldsymbol{K}^2$. Note: Multiplication may be flawed.*

*1 mark for attempting to equate the results of the matrix multiplication $\boldsymbol{K}^2$ with **K** itself, by equating corresponding elements. Note: Equations 1–4 do not all need to be present and correct. Look for evidence of at least one equation from 1–4.*

1 mark for determining $d = 1 - a$.

1 mark for determining $c = \dfrac{a - a^2}{b}$.

Note: As this is a proof, responses will vary. Marks should be awarded for demonstrating the concepts identified in the marking guide.