Neap

Trial Examination 2023

Suggested Solutions

QCE Specialist Mathematics Units 1&2

Paper 2 – Technology-active

SECTION 1 – MULTIPLE CHOICE QUESTIONS

	A	B	C	D
1.	○	○	●	○
2.	●	○	○	○
3.	○	○	○	●
4.	●	○	○	○
5.	●	○	○	○
6.	○	●	○	○
7.	●	○	○	○
8.	○	○	○	●
9.	○	●	○	○
10.	○	○	●	○

QUESTION 1 C

C is correct.

$n! = n(n-1)!$

$\therefore n!(n-1)! = n(n-1)!(n-1)!$

$= n\left((n-1)!\right)^2$

A is incorrect. This option may be reached by distributing the factorial operation over the bracket, resulting in $(n - 1)! = n! - 1!$, and thus simplifying $n!(n! - 1)$ as $(n!)^2 - 1$.

B is incorrect. This option may be reached by distributing the factorial operation over the bracket, resulting in $(n - 1)! = n! - 1!$, then confusing multiplication with addition, resulting in $n! + (n! - 1) = 2(n!) - 1$.

D is incorrect. This option may be reached by incorrectly expressing $(n - 1)!$ in terms of $n!$. Instead of $(n-1)! = \frac{n!}{n}$, the expression reached is $(n-1)! = \frac{n!}{n-1}$, which results in $\frac{(n!)^2}{n-1}$.

QUESTION 2 A

A is correct. The vertex of the transformed absolute value function appears at $(-1, -2)$, indicating that that the function has undergone a horizontal shift of 1 unit to the right and a vertical shift of 2 units down. Substituting $h = -1$ and $k = -2$ into $f(x) = |x - h| + k$ yields $f(x) = |x + 1| - 2$.

B is incorrect. This expression may be reached using the incorrect general form of $f(x) = |x + h| + k$.

C is incorrect. This expression may be reached by incorrectly substituting the values of -1 and -2 into the function.

D is incorrect. This expression may be reached using the incorrect general form of $f(x) = |x - h| - k$.

QUESTION 3 D

D is correct. The dilation described by $(x, y) \to (-x, 2y)$ corresponds to the transformation matrix $\mathbf{T} = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$. Hence, to undo the operation, $\mathbf{T}^{-1}$ is required. Using a graphics calculator,

$$\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

A is incorrect. This option is the transformation matrix $\mathbf{T}$, not $\mathbf{T}^{-1}$.

B is incorrect. This option recognises that $\mathbf{T}^{-1}$ is required, but incorrectly determines the transformation matrix $\mathbf{T}$ to be $\begin{bmatrix} -1 & 0 \\ 2 & 0 \end{bmatrix}$.

C is incorrect. This option may be reached by failing to multiply the rearranged 2×2 matrix by the reciprocal of the determinant, or calculating the determinant to be 1.

QUESTION 4 A

A is correct.

$$\begin{aligned}\text{Arg}(z) &= \tan^{-1}\left(-\frac{2}{1}\right)\\ &= -1.1071\ldots\\ &\approx -1.11\end{aligned}$$

B is incorrect. This option may be reached by incorrectly rounding –1.107... as –1.10. Students may be confused by the negative numbers, or they may not understand the difference between rounding and truncating.

C is incorrect. This option may be reached by evaluating the modulus, not the argument, of the complex number, resulting in $\sqrt{5} = 2.2360\ldots,$ and rounding the solution incorrectly.

D is incorrect. This option may be reached by evaluating the modulus, not the argument, of the complex number, resulting in $\sqrt{5} = 2.2360\ldots.$

QUESTION 5 A

A is correct.

$$\begin{aligned}|\boldsymbol{w}| &= \sqrt{(-3)^2 + (-4)^2}\\ &= 5\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{-4}{-3}\right) + 180°\\ &= 223.13°\end{aligned}$$

B is incorrect. This option may be reached by not adding 180° to the angle as it is in the third quadrant. Using a graphics calculator to perform the inverse tan operation results in 53.13°.

C is incorrect. This option may be reached by mistakenly negating the angle reached using $\tan^{-1}\left(\frac{y}{x}\right)$, which would be appropriate for a second solution for cos, and not adding 180° to the angle.

D is incorrect. This option may be reached by performing the incorrect calculation $\sqrt{(-3)^2 + (-4)^2} = -5$ and not adding 180° to the angle.

QUESTION 6 B

B is correct. Using a graphics calculator gives:

$$\det\begin{bmatrix}1 & 0 & -5\\ 2 & 3 & -6\\ 0 & 11 & 4\end{bmatrix} = -32$$

A is incorrect. This option may be reached by assuming that matrix **A** does not have an inverse and thus has a determinant of 0. However, 'singular' is a description of a matrix, not a scalar value as required by the question.

C is incorrect. This option may be reached if the determinant cannot be found using technology and the matrix is assumed to be singular, which means that the determinant is 0.

D is incorrect. This option may be reached by finding the determinant and removing the negative sign, reaching 32. This may indicate an inability to recognise that the argument to the | | function is a matrix, rather than a real number, as the notation is potentially confusing for students.

QUESTION 7 A

A is correct. As cos(A + B) = cos(A)cos(B) – sin(A)sin(B), using a graphics calculator gives:

$$\begin{aligned}\cos(A+B) &= -0.6\times -0.5 - 0.8\times\frac{\sqrt{3}}{2}\\ &= -0.3928\ldots\\ &\approx -0.393\end{aligned}$$

B is incorrect. This option may be reached by solving cos(A + B) = cos(A)cos(B) + sin(A)sin(B).

C is incorrect. This option may be reached by finding the angle A + B (in radians), rather than cos(A + B). $\because \cos^{-1}(-0.393) = 1.97$.

D is incorrect. This option may be reached by finding the angle A + B (in degrees), rather than cos(A + B). $\because \cos^{-1}(-0.393) = 113.13°$.

QUESTION 8 D

D is correct. The function $f(|x|)$ is produced by reflecting the section of $f(x)$ where $x > 0$ through the y-axis; this describes the graphs of functions 1 and 2.

A is incorrect. As $f(x) = 0$ at some point, the function $\frac{1}{f(x)}$ would be undefined at some point; therefore, an asymptote would appear in the graph of function 2, which is not accurate.

B is incorrect. This option may be reached by not understanding the difference between $|f(x)|$ and $f(|x|)$. As function 1 is always above the x-axis, $|f(x)| = f(x)$. As function 2 is not identical to function 1, this description of function 2 as $|f(x)|$ is incorrect.

C is incorrect. A pair of inverse functions need to be reflections through the line $y = x$, which is not the case in this question.

QUESTION 9 B

B is correct. The contrapositive of the statement $p \to q$ is not $q \to$ not p. In the given statement, the specification p is 'x is a multiple of 6' and the specification q is 'x is a multiple of 3'. Hence, the contrapositive is 'If x is not a multiple of 3, then x is not a multiple of 6.'

A is incorrect. This option is the converse of the original statement, $q \to p$.

C is incorrect. This option incorrectly orders the specifications in the contrapositive; that is, it states not $p \to$ not q, rather than not $q \to$ not p.

D is incorrect. This option is a contradiction of the original statement, $p \to$ not q.

QUESTION 10 C

C is correct. As $|w|>1,$ the solid line must represent z and the dashed line must represent the product wz. This is because wz must have a greater modulus than z as $|w|>1.$ Arg(wz) is observed to be $\frac{7\pi}{12}$, and $|wz|$ is observed to be 2. Using $\frac{wz}{z}$ to determine w gives:

$$\begin{aligned}\frac{wz}{z} &= \frac{2cis\left(\frac{7\pi}{12}\right)}{cis\left(\frac{\pi}{6}\right)} \\ &= \frac{2}{1}cis\left(\frac{7\pi}{12}-\frac{\pi}{6}\right) \\ &= 2cis\left(\frac{5\pi}{12}\right)\end{aligned}$$

A is incorrect. This option may be reached by incorrectly interpreting the inequality $|w|>1$ and thus assuming that the dashed line represents z and the solid line represents wz.

B is incorrect. This option may be reached by finding the complex number represented by the solid line, which is z.

D is incorrect. This option may be reached by finding the complex number represented by the dashed line, which is wz, not w.

SECTION 2

QUESTION 11 (4 marks)

$z = -2 + bi$ and $w = a + (a + b)i$

$\bar{z} = w$

$\bar{z} = -2 - bi$

$\therefore -2 - bi = a + (a + b)i$

Equating the real parts gives:

$-2 = a$

Equating the imaginary parts gives:

$$\begin{aligned} -b &= a + b \\ -b &= -2 + b \\ -b - b &= -2 \\ b &= 1 \end{aligned}$$

[4 marks]
1 mark for stating that $\bar{z} = -2 - bi$.
1 mark for equating the real and imaginary parts of two complex numbers.
1 mark for finding $a = -2$.
1 mark for finding $b = 1$.

QUESTION 12 (5 marks)

a) $$\begin{aligned} \boldsymbol{c} &= \begin{pmatrix} 3 \\ 4 \end{pmatrix} + 2\begin{pmatrix} 11 \\ -1 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 22 \\ -2 \end{pmatrix} \\ &= \begin{pmatrix} 3 + 22 \\ 4 - 2 \end{pmatrix} \\ &= \begin{pmatrix} 25 \\ 2 \end{pmatrix} \end{aligned}$$

[2 marks]
1 mark for showing $2\boldsymbol{b} = \begin{pmatrix} 22 \\ -2 \end{pmatrix}$.
1 mark for providing the correct solution.

b) $$\begin{aligned} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 11 \\ -1 \end{pmatrix} &= 3 \times 11 + 4 \times -1 \\ &= 33 - 4 \\ &= 29 \end{aligned}$$

[1 mark]
1 mark for providing the correct solution.

c) $|\boldsymbol{a}| = \sqrt{3^2 + 4^2}$

$= 5$

$\hat{\boldsymbol{a}} = \frac{1}{5}a$

$= \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$

[2 marks]

1 mark for finding $|a| = 5$.

1 mark for finding $\hat{a} = \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Note: Accept equivalent expressions; for example, $0.6i + 0.8j$.

QUESTION 13 (4 marks)

a) $A = \frac{70}{2}$

$= 35$

[1 mark]

1 mark for providing the correct solution.

Note: The units are specified in the question; do not award marks for 0.35 m as the final answer.

b) $B = \frac{2\pi}{T}$

$= \frac{2\pi}{0.5}$

$= 4\pi$

≈ 12.57

[2 marks]

1 mark for showing valid working. Note: This could be shown by stating the formula or interpreting that the period is 0.5.

1 mark for providing the correct solution. Note: Accept equivalent decimal expressions.

c) $h(0) = 35\sin(4\pi \times 0) + 35$

$= 35$ cm

[1 mark]

1 mark for providing the correct solution.

*Note: Consequential on answers to **Questions 13a)** and **13b)**. Accept follow-through errors from **Question 13a)** that result from the incorrect units being used or similar.*

QUESTION 14 (4 marks)

Method 1:

Students may identify the equations of the two lines that make up $g(x)$ and construct the reciprocal graph using technology. No justification is requested, but students may show that $g(x)=\begin{cases}-2x+1 & -2\le x\le 1\\ 0.5x-1.5 & 1\le x\le 2\end{cases}$.

Students may then use the graphics calculator to graph the reciprocal $\frac{1}{g(x)}$.

Method 2:

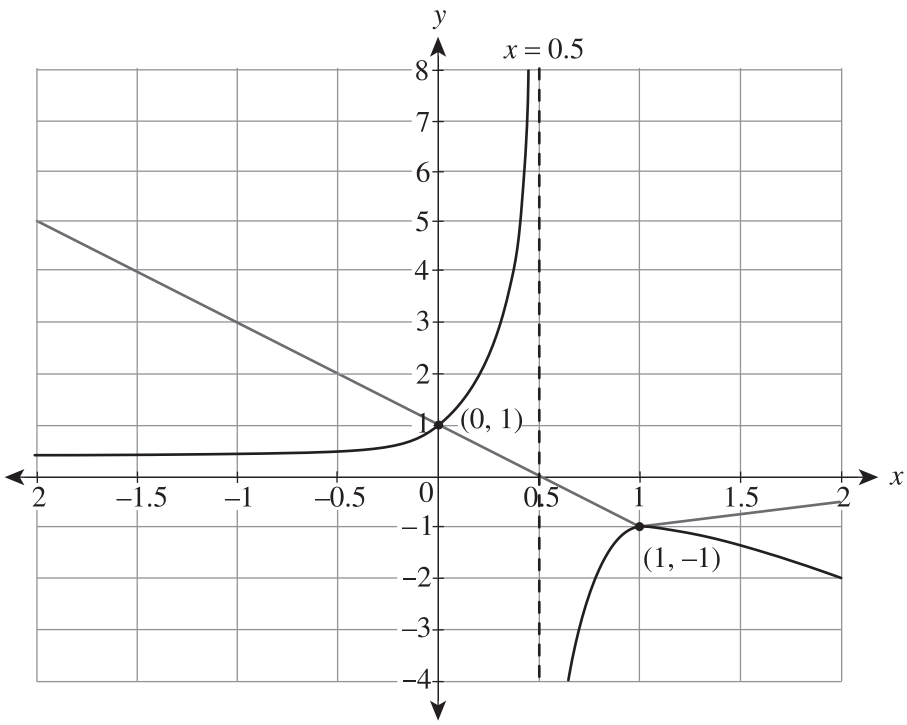
An asymptote will appear on $\frac{1}{g(x)}$ at $x = a$ where $g(a) = 0$. $\therefore$ there is a vertical asymptote at $x = 0.5$.

As $g(x) > 0$ when $x < 0.5$, $\frac{1}{g(x)} > 0$ when $x > 0.5$. As $g(x) < 0$ when $x > 0.5$, $\frac{1}{g(x)} < 0$ when $x > 0.5$.

As $g(0) = 1$, $\frac{1}{g(0)} = 1$. $\therefore$ (0, 1) is a point on reciprocal function.

As $g(1) = –1$, $\frac{1}{g(1)} = -1$. $\therefore$ (1, 1) is a point on reciprocal function.

As an increase in the magnitude of $g(x)$ corresponds to a decrease in the magnitude of $\frac{1}{g(x)}$ and vice versa, asymptotic shapes will occur.



[4 marks]

1 mark for sketching a vertical line on the graph at $x = 0.5$ OR annotating the graph to indicate the asymptote as $x = 0.5$.

1 mark for showing that the graph has two parts, including the part with the domain of $x < 0.5$ is wholly above the x-axis AND the the part with the domain of $x > 0.5$ is wholly below the x-axis.

1 mark for showing that the graph passes through (0, 1) and (1, –1).

1 mark for sketching appropriately shaped curves for both domains. Note: Asymmetry of the reciprocal is required to award this mark.

Note: The grey line in the diagram is the graph of g(x) shown in the question.

QUESTION 15 (5 marks)

a) $n = {}^7C_2$

[1 mark]
1 mark for providing the correct solution.

b) Using technology gives:

$${}^7C_2 = 21$$

[1 mark]
1 mark for providing the correct solution.
*Note: Consequential on answer to **Question 15a)**.*

c) There are six batteries that are not faulty and two batteries will be chosen. Therefore:

$$\begin{aligned} n(\text{selections of good batteries}) &= {}^6C_2 \\ &= 15 \end{aligned}$$

[2 marks]
1 mark for providing reasoning.
1 mark for providing the correct solution.

d) **Method 1:**

$$\begin{aligned} \Pr(\text{operational}) &= \frac{n(\text{selection of good batteries})}{n(\text{unrestricted selections})} \\ &= \frac{15}{21} \\ &= \frac{5}{7} \end{aligned}$$

[1 mark]
1 mark for providing the correct solution.
*Note: Consequential on answers to **Questions 15b)** and **15c)**.*

Method 2:

$$\begin{aligned} \Pr(\text{both good}) &= \Pr(\text{first good}) \times \Pr(\text{second good} \mid \text{first good}) \\ &= \frac{6}{7} \times \frac{5}{6} \\ &= \frac{5}{7} \end{aligned}$$

[1 mark]
1 mark for providing the correct solution.

QUESTION 16 (7 marks)

a) $\det(\mathbf{T}) = 4$

$\therefore\ 2 \times 3 - k \times k = 4$ or $2 \times 3 - k \times k = -4$

$6 - k^2 = 4$

$k = \sqrt{2}$ or $-\sqrt{2}$

$6 - k^2 = -4$

$k^2 = 12$

$|k| = 2\sqrt{3}$

$= 3.4$

Considering the image of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$:

$$\begin{bmatrix} 2 & k \\ k & 3 \end{bmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2+k \\ 3+k \end{pmatrix}$$

From the diagram, it can be seen that vertex B is the image of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Since vertex B is located at approximately $\begin{pmatrix} 3.4 \\ 4.4 \end{pmatrix}$, this is consistent with $k = \sqrt{2}$, and inconsistent with $k = -\sqrt{2}$ and $|k| = 2\sqrt{3}$.

$\therefore k = \sqrt{2}$

[3 marks]

*1 mark for showing that det(**T**) = 4.*

1 mark for reaching $6 - k^2 = 4$

1 mark for providing valid reasoning to exclude $-\sqrt{2}$ as the solution for k.

b) To prove that *OABC* is a parallelogram, it is sufficient to show that $\overrightarrow{OA} = \overrightarrow{CB}$ OR $\overrightarrow{OC} = \overrightarrow{AB}$.

Method 1:

Showing $\overrightarrow{OA} = \overrightarrow{CB}$:

It can be seen that vertex *A* is the image of (0, 1).

$$\therefore \overrightarrow{OA} = \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 3 \end{bmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{2} \\ 3 \end{pmatrix}$$

$$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC}$$

$$= \begin{pmatrix} 2+\sqrt{2} \\ 3+\sqrt{2} \end{pmatrix} - \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 3 \end{bmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{2} \\ 3 \end{pmatrix}$$

$\therefore$ As $\overrightarrow{OA} = \overrightarrow{CB}$, *OABC* is a parallelogram.

[4 marks]

1 mark for identifying the condition required to prove that OABC is a parallelogram.

1 mark for calculating the image of at least one non-origin vertex of the unit square.

*Note: Matrix **T** should be used with $k = \sqrt{2}$ substituted.*

1 mark for calculating a vector that represents a side of the quadrilateral.

1 mark for showing valid proof that OABC is a parallelogram. Note: Award this mark even if students do not present an efficient proof.

Method 2:

Showing $\overrightarrow{OC} = \overrightarrow{AB}$:

It can be seen that vertex *C* is the image of (1, 0).

$$\therefore \overrightarrow{OC} = \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 3 \end{bmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ \sqrt{2} \end{pmatrix}$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

$$= \begin{pmatrix} 2+\sqrt{2} \\ 3+\sqrt{2} \end{pmatrix} - \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 3 \end{bmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ \sqrt{2} \end{pmatrix}$$

As $\overrightarrow{OC} = \overrightarrow{AB}$, *OABC* is a parallelogram.

[4 marks]

1 mark for identifying the condition required to prove that OABC is a parallelogram.

1 mark for calculating the image of at least one non-origin vertex of the unit square.

*Note: Matrix **T** should be used with $k = \sqrt{2}$ substituted.*

1 mark for calculating a vector that represents a side of the quadrilateral.

1 mark for showing valid proof that OABC is a parallelogram. Note: Award this mark even if students do not present an efficient proof.

QUESTION 17 (8 marks)

a)
$$\mathbf{R}_{45^\circ} = \begin{bmatrix} \cos(45^\circ) & -\sin(45^\circ) \\ \sin(45^\circ) & \cos(45^\circ) \end{bmatrix}$$
$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$
$$\mathbf{R}_{-45^\circ} = \begin{bmatrix} \cos(-45^\circ) & -\sin(-45^\circ) \\ \sin(-45^\circ) & \cos(-45^\circ) \end{bmatrix}$$
$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$
$$\mathbf{M}_{x\text{-axis}} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
$$\mathbf{F} = \mathbf{R}_{45^\circ} \times \mathbf{M}_{x\text{-axis}} \times \mathbf{R}_{-45^\circ}$$
$$= \left(\frac{1}{\sqrt{2}}\right)^2 \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

[4 marks]

1 mark for using $\boldsymbol{R}_\theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ *to find the transformation matrices for the rotations.*

1 mark for finding the transformation matrix for the reflection.

1 mark for placing the transformation matrices in the correct order to perform matrix multiplication.

1 mark for providing the correct solution.

b) $y = x = \tan(45°)x$

$\therefore\ \theta = 45°$

$$\mathbf{T} = \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(90°) & \sin(90°) \\ \sin(90°) & -\cos(90°) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

[2 marks]
1 mark for showing understanding that $\theta = 45°$.
1 mark for providing the correct solution.

c) **Method 1:**

From parts a) and b), it is apparent that $\mathbf{F} = \mathbf{T}$.

$\therefore$ The claim is reasonable.

[2 marks]
*1 mark for comparing matrix **F** and **T**.*
1 mark for concluding that the claim is reasonable if $\boldsymbol{F} = \boldsymbol{T}$ or unreasonable if $\boldsymbol{F} \neq \boldsymbol{T}$.
Note: Accept follow-through for previous errors.
*Note: Consequential on answers to **Questions 17a)** and **17b)**.*

Method 2:

Students may present a diagram, or investigate the image of a point that undergoes both transformations, and hence draw a conclusion.

[2 marks]
1 mark for using a valid method of comparison that demonstrates an understanding of transformations.
1 mark for drawing a conclusion about the reasonableness of the claim.

QUESTION 18 (4 marks)

Method 1:

$$\begin{aligned} f(x) &= 2\cos^2(2x) + \sin^2(2x) \\ &= \cos^2(2x) + \cos^2(2x) + \sin^2(2x) \\ &= \cos^2(2x) + 1 \text{ (Pythagorean identity)} \end{aligned}$$

Using $\cos(2A) = 2\cos^2(A) - 1$:

$$\begin{aligned} \cos^2(2x) &= 0.5(\cos(2 \times 2x) + 1) \\ &= 0.5\cos(4x) + 0.5 \end{aligned}$$

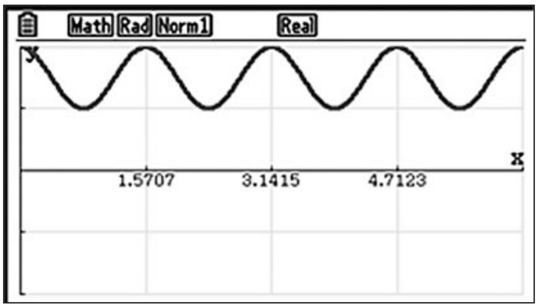
$$\begin{aligned} \therefore f(x) &= 0.5\cos(4x) + 0.5 + 1 \\ &= 0.5\cos(4x) + 1.5 \end{aligned}$$

Hence, $A = 0.5$, $B = 4$, $C = 0$ and $D = 1.5$.

[4 marks]
1 mark for using a Pythagorean identity to convert f(x) into a cosine function.
1 mark for finding A = 0.5.
1 mark for finding B = 4.
1 mark for finding D = 1.5.

Method 2:

Graphing on the graphics calculator, it is observed that $f(x)$ appears over $0 \le x \le 2\pi$.



Therefore:

$$\begin{aligned} A &= \frac{2-1}{2} \\ &= 0.5 \end{aligned}$$

$$\begin{aligned} D &= \frac{2+1}{2} \\ &= 1.5 \end{aligned}$$

$B = 4$ (frequency in 2π)

$C = 0$

[4 marks]
1 mark for finding A = 0.5.
1 mark for finding B = 4.
1 mark for finding C = 0.
1 mark for finding D = 1.5.

QUESTION 19 (5 marks)

Finding the number of triangles that can be created gives:

$$^{15}C_3 = \frac{15\times14\times13}{3\times2\times1}$$
$$= 455$$

Finding the number of comparisons that must be performed gives:

$$^{455}C_2 = 455\times\frac{454}{2}$$
$$= 103\,285$$

$$\text{time required to construct and calculate area of triangles} = 20\times455 \text{ seconds}$$
$$= 20\times\frac{455}{60\times60\times24\times7} \text{ weeks}$$
$$= 0.01505 \text{ weeks}$$

$$\text{time required to compare triangles} = 5\times103\,285 \text{ seconds}$$
$$= 5\times\frac{103\,285}{60\times60\times24\times7} \text{ weeks}$$
$$= 0.854 \text{ weeks}$$

$$\text{fraction of a week} = 0.01505 + 0.854$$
$$\approx 0.87$$

[5 marks]

1 mark for using $^{15}C_3$ to find the number of triangles.

1 mark for using a combination of form $^{x}C_2$ to find the number of comparisons to be made.

Note: Allow follow-through errors for the number of triangles.

1 mark for calculating the total time. Note: Accept follow-through errors for the number of triangles and/or comparisons.

1 mark for converting seconds to weeks.

1 mark for providing the correct solution.

QUESTION 20 (4 marks)

Assuming that both $\mathbf{K}$ and $\mathbf{L}$ can act as inverses for $\mathbf{M}$, and $\mathbf{K} \neq \mathbf{L}$, then $\mathbf{KM} = \mathbf{MK} = \mathbf{I}_3$.

$\therefore\ \mathbf{LM} = \mathbf{ML} = \mathbf{I}_3$ (definition of inverse)

Consider $\mathbf{KML}$:

As matrix multiplication is associative, $\mathbf{KML} = (\mathbf{KM})\mathbf{L} = \mathbf{K}(\mathbf{ML})$.

$$\begin{aligned}\therefore \mathbf{KML} &= (\mathbf{KM})\mathbf{L} \\ &= \mathbf{I}_3\mathbf{L} \\ &= \mathbf{L} \text{ (as } \mathbf{I}_3 \text{ is identity matrix)}\end{aligned}$$

$$\begin{aligned}\therefore \mathbf{KML} &= \mathbf{K}(\mathbf{ML}) \\ &= \mathbf{KI}_3 \\ &= \mathbf{K} \text{ (as } \mathbf{I}_3 \text{ is identity matrix)}\end{aligned}$$

As $\mathbf{KML} = \mathbf{K} = \mathbf{L}$, this is a contradiction of the assumption $\mathbf{K} \neq \mathbf{L}$.

Hence, it cannot be true that $\mathbf{K}$ and $\mathbf{L}$ are both inverses, and $\mathbf{K} \neq \mathbf{L}$.

Hence, it is true that only one possible matrix can serve as $\mathbf{M}^{-1}$ ($\mathbf{M}$ is non-singular).

[4 marks]

*1 mark for assuming that **K** and **L** are both inverses, where **K** ≠ **L** (as directed in question).*
*1 mark for providing evidence that the inverse operates with **M** to produce **I**.*
1 mark for attempting to disprove the assertion using the notion of matrix associativity.
1 mark for identifying a contradiction and providing an appropriate conclusion.

