Neap

Trial Examination 2023

Question and Response Booklet

QCE Specialist Mathematics Units 3&4

Paper 2 – Technology-active

Student's Name: _____

Teacher's Name: _____

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- QCAA-approved calculator **permitted**.
- Formula booklet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 9 short response questions

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2023 QCE Specialist Mathematics Units 3&4 Written Examination.

SECTION 1

Instructions

- Choose the best answer for Questions 1–10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

SECTION 2

Instructions

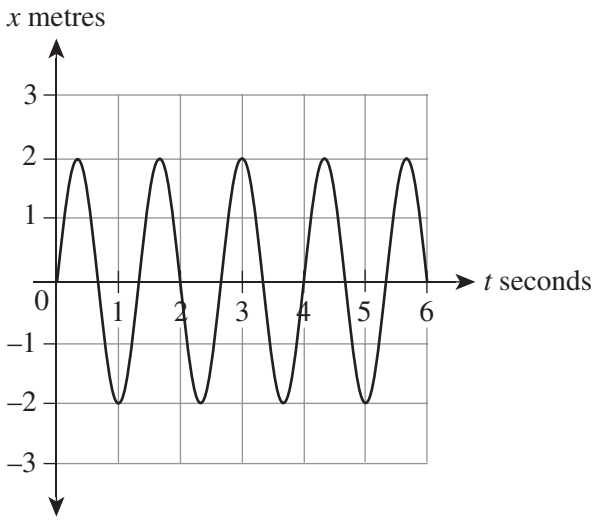
- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 50 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (5 marks)

An oscillating object undergoes simple harmonic motion. The periodic graph represents the object's displacement, x metres, over time, t seconds.



a) Determine the value of A and ω for the object's motion. *[2 marks]*

b) Determine the velocity of the object at $x = 1$ m. *[1 mark]*

c) Determine the maximum acceleration of the object. *[2 marks]*

QUESTION 12 (7 marks)

A school has decided to revise its communication policy. Using a random sample of 60 students, the school records the time, in minutes, that it takes each student to respond to an email from their teacher. The sample has a 95% confidence interval of (84.6, 89.2).

a) Determine the sample mean. *[1 mark]*

b) Calculate the predicted population standard deviation. *[2 marks]*

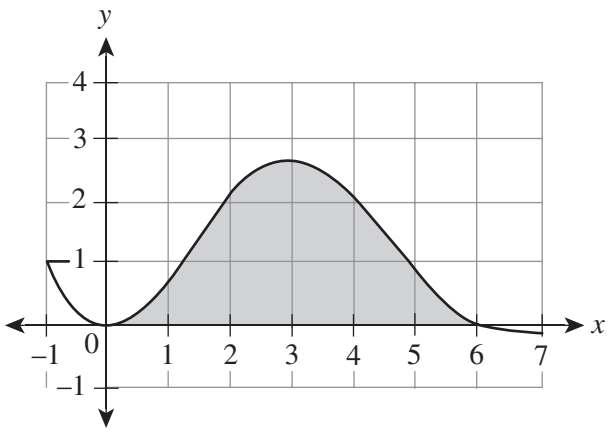
c) State the 99% confidence interval for the sample. *[1 mark]*

d) The school decides to improve the accuracy of the statistical analysis by using a second, larger sample that is increased by x number of students. The margin of error of the second sample's 95% confidence interval has decreased by 30% when compared to the first sample, but the sample standard deviation remains the same for both samples.

Determine the value of x. *[3 marks]*

QUESTION 13 (5 marks)

The graph shows the function $y = (6 - x)\sin^2\left(\frac{2x}{5}\right)$.



a) Using Simpson's rule with six strips, determine an approximation of the shaded area of the graph. *[4 marks]*

b) Evaluate the reasonableness of your approximation from Question 13a). *[1 mark]*

QUESTION 14 (6 marks)

There are five teams playing in a local netball competition. The table summarises the results after each team has played against each other once.

Team	Teams they defeated
A	C, E
B	A, D, E
C	B, D, E
D	A
E	D

a) State the first-order dominance matrix, D_1, that could be used to determine the ranking of the five teams. *[1 mark]*

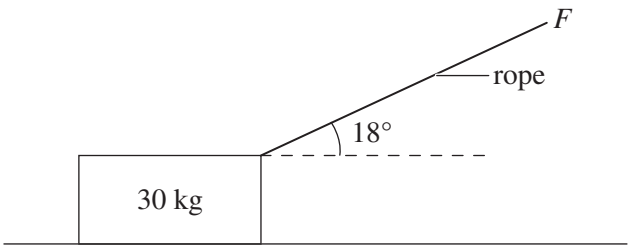
b) Using matrix D_1 from Question 14a), state the ranking of the five teams. *[1 mark]*

c) Using the rule $D_1 + \frac{1}{2}D_2$, determine the ranking of the five teams when first- and second-order wins are considered. *[3 marks]*

d) Evaluate the reasonableness of the ranking found in Question 14c) by comparing it with the ranking found in Question 14b). *[1 mark]*

QUESTION 15 (5 marks)

Charlotte is moving into a new home. She pulls a box along a wooden floor using a rope, as shown in the diagram.



Not to scale

Charlotte is pulling the box with a force, F, of 110 N. The box has an acceleration of 2.2 m s^{-1}.

a) Determine the magnitude of the resistive force being exerted by the floor on the box. *[3 marks]*

b) Determine the magnitude of the normal reactive force. *[2 marks]*

QUESTION 16 (6 marks)

The velocity of a particle is represented by the function

$$v = \frac{\sqrt{1+x^2}}{x},\ x > 0,$$

where v is the velocity, in m s^{-1}, and x is the displacement, in metres, from the origin.

a) Determine the acceleration of the particle when $x = 4.5$ m. *[3 marks]*

b) Given that the particle begins its movement at a point of two units from the origin, determine the displacement of the particle after three seconds. *[3 marks]*

QUESTION 17 (5 marks)

A lake in a national park is home to a population of Golden Perch fish. The park rangers are assessing the weight of mature Golden Perch in the population for conservation purposes. The healthy weight of a mature Golden Perch is 390 g.

A ranger weighs a sample of Golden Perch to approximate the population mean weight. The ranger notes that the sample has a 90% confidence interval of (397 g, 436 g).

Determine the lowest confidence level that contains the healthy weight of a mature Golden Perch. Give your answer correct to the nearest whole number.

QUESTION 18 (5 marks)

A curve is represented by the vector $r(\theta) = (a\cos(\theta))\boldsymbol{i} + (\sin(2\theta))\boldsymbol{j}$, where $a \in R^+$.

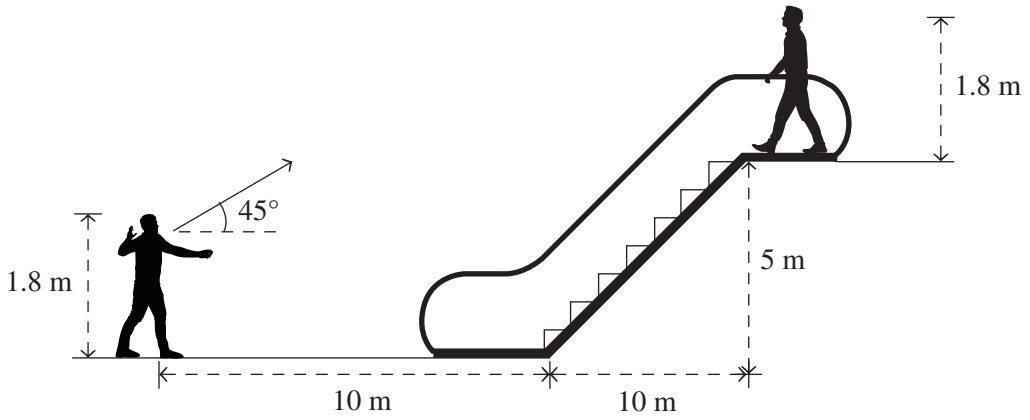
An artist requires a number of teardrop-shaped objects for a sculpture. They plan to create these teardrops by rotating $r(\theta)$ between $x = 0$ and $x = a$ about the x-axis.

If each teardrop has a volume of 8 cm^3, determine the value of a.

QUESTION 19 (6 marks)

Chelsea is standing 10 m from the base of an escalator and plans to throw a ball at an angle of 45° above the horizontal towards Barry, who is about to get on the escalator and travel down. The escalator is moving at a speed of 0.5 m s^{-1}, and has the dimensions shown in the diagram.

Barry is aiming to catch the ball at a point halfway along the escalator. Assume that the ball is thrown and caught 1.8 m above the ground in both cases.



Not to scale

Determine the speed and time at which Chelsea should throw the ball so that Barry can catch it when he is halfway down the escalator.

END OF PAPER

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

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Trial Examination 2023

Formula Booklet

QCE Specialist Mathematics Units 3&4

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$A = \frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Calculus	
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx}\ln(x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + c$
$\frac{d}{dx}\sin(x) = \cos(x)$	$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}\cos(x) = -\sin(x)$	$\int \cos(x) dx = \sin(x) + c$
$\frac{d}{dx}\tan(x) = \sec^2(x)$	$\int \sec^2(x) dx = \tan(x) + c$
$\frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right) = \frac{1}{\sqrt{a^2 - x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\cos^{-1}\left(\frac{x}{a}\right) = \frac{-1}{\sqrt{a^2 - x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\tan^{-1}\left(\frac{x}{a}\right) = \frac{a}{a^2 + x^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

Calculus		
chain rule	If $h(x)=f\left(g(x)\right)$ then $h'(x)=f'\left(g(x)\right)g'(x)$	If $y=f(u)$ and $u=g(x)$ then $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$
product rule	If $h(x)=f(x)g(x)$ then $h'(x)=f(x)g'(x)+f'(x)g(x)$	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	If $h(x)=\frac{f(x)}{g(x)}$ then $h'(x)=\frac{f'(x)g(x)-f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
integration by parts	$\int f(x)g'(x)dx=f(x)g(x)-\int f'(x)g(x)dx$	$\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx$
volume of a solid of revolution	about the x-axis	$V=\pi\int_a^b\left[f(x)\right]^2dx$
	about the y-axis	$V=\pi\int_a^b\left[f(y)\right]^2dy$
Simpson's rule	$\int_a^b f(x)dx\approx\frac{w}{3}\left[f\left(x_0\right)+4\left[f\left(x_1\right)+f\left(x_3\right)+\ldots\right]+2\left[f\left(x_2\right)+f\left(x_4\right)+\ldots\right]+f\left(x_n\right)\right]$	
simple harmonic motion	If $\frac{d^2x}{dt^2}=-\omega^2x$ then $x=A\sin\left(\omega t+\alpha\right)$ or $x=A\cos\left(\omega t+\beta\right)$	
	$v^2=\omega^2\left(A^2-x^2\right)$	$T=\frac{2\pi}{\omega}$ $\quad f=\frac{1}{T}$
acceleration	$a=\frac{dv}{dt}=\frac{d^2x}{dt^2}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$	

Real and complex numbers	
complex number forms	$z=x+yi=r\left(\cos(\theta)+i\sin(\theta)\right)=r\,\mathrm{cis}(\theta)$
modulus	$\left\|z\right\|=r=\sqrt{x^2+y^2}$
argument	$\arg(z)=\theta,\ \tan(\theta)=\frac{y}{x},-\pi<\theta\le\pi$
product	$z_1z_2=r_1r_2\,\mathrm{cis}\left(\theta_1+\theta_2\right)$
quotient	$\frac{z_1}{z_2}=\frac{r_1}{r_2}\mathrm{cis}\left(\theta_1-\theta_2\right)$
De Moivre's theorem	$z^n=r^n\mathrm{cis}\left(n\theta\right)$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \ldots + \binom{n}{r}x^{n-r}y^r + \ldots + y^n$	
permutation	${}^nP_r = \frac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \ldots \times (n-r+1)$	
combination	${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$	
sample means	mean	μ
	standard deviation	$\frac{\sigma}{\sqrt{n}}$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$	

Trigonometry	
Pythagorean identities	$\sin^2(A) + \cos^2(A) = 1$ $\tan^2(A) + 1 = \sec^2(A)$ $\cot^2(A) + 1 = \text{cosec}^2(A)$
angle sum and difference identities	$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
double-angle identities	$\sin(2A) = 2\sin(A)\cos(A)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$ $= 1 - 2\sin^2(A)$ $= 2\cos^2(A) - 1$
product identities	$\sin(A)\sin(B) = \frac{1}{2}\left(\cos(A - B) - \cos(A + B)\right)$ $\cos(A)\cos(B) = \frac{1}{2}\left(\cos(A - B) + \cos(A + B)\right)$ $\sin(A)\cos(B) = \frac{1}{2}\left(\sin(A + B) + \sin(A - B)\right)$ $\cos(A)\sin(B) = \frac{1}{2}\left(\sin(A + B) - \sin(A - B)\right)$

Vectors and matrices		
magnitude	$\lvert\boldsymbol{a}\rvert = \left\lvert\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}\right\rvert = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$	
scalar (dot) product	$\boldsymbol{a} \cdot \boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\cos(\theta)$	
	$\boldsymbol{a} \cdot \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} . \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3$	
vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + k\boldsymbol{d}$	
Cartesian equation of a line	$\dfrac{x - a_1}{d_1} = \dfrac{y - a_2}{d_2} = \dfrac{z - a_3}{d_3}$	
vector (cross) product	$\boldsymbol{a} \times \boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\sin(\theta)\hat{\boldsymbol{n}}$	
	$\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$	
vector projection	$\boldsymbol{a}$ on $\boldsymbol{b} = \lvert\boldsymbol{a}\rvert\cos(\theta)\hat{\boldsymbol{b}} = \left(\boldsymbol{a} \cdot \hat{\boldsymbol{b}}\right)\hat{\boldsymbol{b}}$	
vector equation of a plane	$\boldsymbol{r} \cdot \boldsymbol{n} = \boldsymbol{a} \cdot \boldsymbol{n}$	
Cartesian equation of a plane	$ax + by + cz + d = 0$	
determinant	If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(\mathbf{A}) = ad - bc$	
multiplicative inverse matrix	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{\det(\mathbf{A})}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, $\det(\mathbf{A}) \neq 0$	
linear transformations	dilation	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
	rotation	$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
	reflection (in the line $y = x\tan(\theta)$)	$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$

Physical constant	
magnitude of mean acceleration due to gravity on Earth	$g = 9.8 \text{ m s}^{-2}$