

VCE Algorithmics (HESS)

2015–2016

Written examination – End of year

Examination specifications

Overall conditions

The examination will be sat at a time and date to be set annually by the Victorian Curriculum and Assessment Authority (VCAA). VCAA examination rules will apply. Details of these rules are published annually in the *VCE and VCAL Administrative Handbook*.

There will be 15 minutes reading time and 2 hours writing time.

The examination will be marked by a panel appointed by the VCAA.

The examination will contribute 60 per cent to the study score.

Content

The *VCE Algorithmics (HESS) Study Design 2015–2016* is the document for the development of the examination. All outcomes in Units 3 and 4 will be examined.

All of the key knowledge and skills that underpin the outcomes in Units 3 and 4 are examinable.

Students will **not** be required to use information and communications technology (ICT) in the examination.

Format

The examination will be in the form of a question and answer book.

The examination will consist of two sections.

Section A will consist of 20 multiple-choice questions worth 1 mark each and will be worth a total of 20 marks.

Section B will consist of a number of short- and extended-answer questions worth a total of 80 marks. Questions may include short scenarios, case studies, multiple parts and the use of stimulus material.

All questions will be compulsory. The total marks for the examination will be 100.

Answers to Section A are to be recorded on the answer sheet provided for multiple-choice questions.

Answers to Section B are to be recorded in the spaces provided in the question and answer book.

Approved materials and equipment

- Normal stationery requirements (pens, pencils, highlighters, erasers, sharpeners and rulers)
- One scientific calculator

Relevant references

The following publications should be referred to in relation to the VCE Algorithmics (HESS) examination:

- *VCE Algorithmics (HESS) Study Design 2015–2016*
- *VCAA Bulletin*

Advice

During the 2015–2016 accreditation period for VCE Algorithmics (HESS), examinations will be prepared according to the examination specifications above. Each examination will conform to these specifications and will test a representative sample of the key knowledge and skills from all outcomes in Units 3 and 4.

The following sample examination provides an indication of the types of questions teachers and students can expect until the current accreditation period is over.

Answers to multiple-choice questions are provided on page 25.

Answers to other questions are not provided.

Victorian Certificate of Education
Year

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER Letter

ALGORITHMICS (HESS)

Written examination

Day Date

Reading time: *.*.* to *.*.* (15 minutes)

Writing time: *.*.* to *.*.* (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
A	20	20	20
B	14	14	80
			Total 100

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers and one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 24 pages.
- Answer sheet for multiple-choice questions.

Instructions

- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A – Multiple-choice questions**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** or that **best answers** the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

A heuristic is a technique that will attempt to find

- A. a precise solution to a problem.
- B. an optimal solution to a problem.
- C. all of the possible solutions to a problem.
- D. a solution to a problem in a reasonable time frame.

Question 2

Which one of the following types of algorithm is commonly used to determine the minimum spanning tree of a graph?

- A. Prim
- B. PageRank
- C. Bellman-Ford
- D. Floyd-Warshall

Question 3

$$T(n) = T(n/2) + O(1)$$

Which one of the following is the solution to the above recurrence relation using the Master Theorem?

- A. $\Theta(n)$
- B. $\Theta(n^2)$
- C. $\Theta(\log n)$
- D. $\Theta(n \log n)$

Question 4

Depth-first search is an algorithm that can be used to search graph data structures.

The time complexity of searching a graph with V vertices and E edges in the worst case is

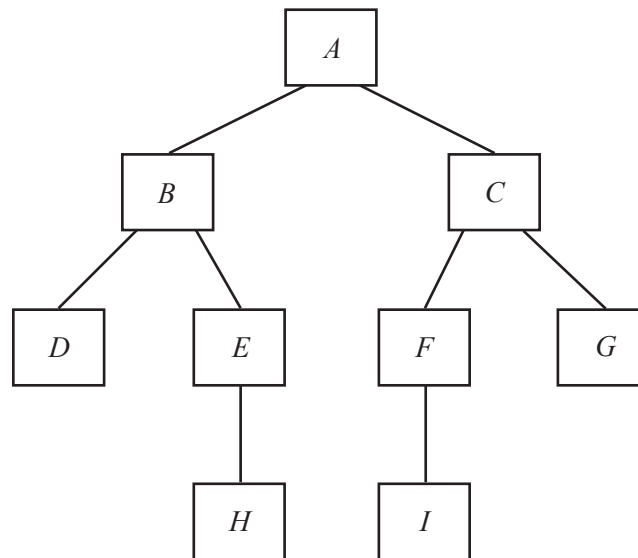
- A. $O(\log |E| + \log |V|)$
- B. $O(|E| + |V|)$
- C. $O(\log |E|)$
- D. $O(\log |V|)$

Question 5

Which one of the following is a benefit of DNA computing, when implementing algorithms, in comparison with digital methods of computation?

- A. DNA computing solves all algorithms much faster than digital methods of computation.
- B. DNA computing provides new capabilities of computability theory that digital methods of computation do not have.
- C. Efficiency is greater and errors are far fewer with DNA computing than with digital methods of computation.
- D. DNA computing can provide massive parallelism that can be used to speed up a search compared to digital methods of computation.

Use the following information to answer Questions 6 and 7.

**Question 6**

Starting at node (vertex) *A*, the first four nodes visited in a depth-first search are

- A. *ABDE*
- B. *ABDC*
- C. *ABCD*
- D. *ACFG*

Question 7

Starting at node (vertex) *A*, the first four nodes visited in a breadth-first search are

- A. *ABDE*
- B. *ABDC*
- C. *ABCD*
- D. *ACFG*

Question 8

Dynamic programming involves using a recurrence relation to identify and solve sub-problems and then

- A. removing duplicate computation of sub-problems.
- B. minimising the difference between sub-problem results.
- C. backtracking if the sub-problem has already been solved.
- D. using an iterative computation that avoids the repeated computation of shared sub-problems.

Question 9

The manager of a very large restaurant has detected a problem with the time taken to wash all of the dirty dishes that are in a single pile near the sinks in the kitchen.

The best abstract data type (ADT) to represent the processing of the dishes is a

- A. tree.
- B. stack.
- C. graph.
- D. queue.

Question 10

An algorithm contains a loop invariant.

If the algorithm is correct, the loop invariant must be

- A. true at all times.
- B. true only at the end of the body of the loop.
- C. true only at the start of the body of the loop.
- D. true before the loop is executed and after the loop has finished.

Question 11

What is a Turing machine useful for?

- A. analysing the logic of computer algorithms as a theoretical tool
- B. the processing of algorithms using practical computing technology
- C. the modelling of the strengths of particular algorithms, including optimisation
- D. effectively modelling concurrency, in particular with always-halting concurrent systems

Question 12

... is an algorithm design pattern wherein a problem is solved by splitting it into smaller, non-overlapping sub-problems, then individually solving the sub-problems and combining their solutions to form a solution to the original problem.

Which one of the following design patterns is defined above?

- A. minimax
- B. backtracking
- C. divide and conquer
- D. dynamic programming

Question 13

Which one of the following operations is typically associated with the graph ADT?

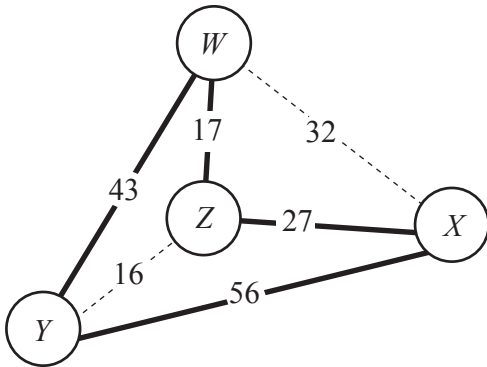
- A. pop
- B. push
- C. index
- D. traverse

Question 14

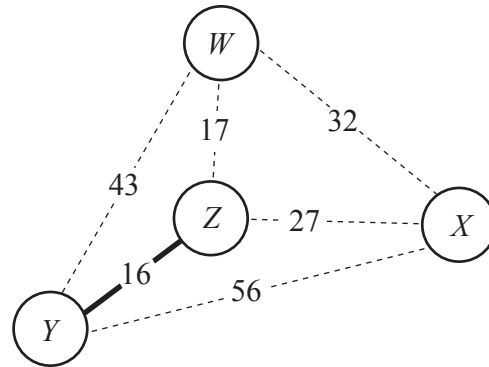
Each of the diagrams below depicts the same weighted graph.

Assuming bolded edges show a solution, which diagram also shows a solution to the travelling salesman problem?

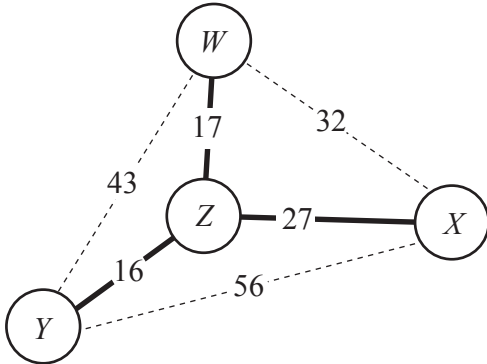
A.



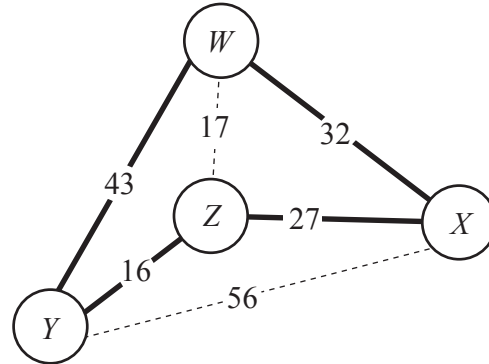
B.



C.



D.

**Question 15**

An 'undecidable' decision problem is one where

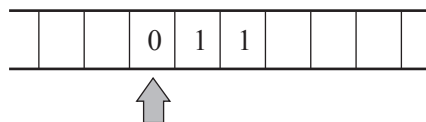
- A. it cannot be decided how a Turing machine can solve the problem.
- B. it cannot be decided by a Turing machine if the problem has a solution.
- C. an algorithm for a Turing machine to solve the problem cannot ever be found.
- D. an algorithm for a Turing machine to solve the problem cannot presently be found.

Use the following information to answer Questions 16 and 17.

A Turing machine is set up with the instructions shown in the following table, where each row represents one step of the machine and H is the halting state. It begins in state 1.

Current state	Tape symbol	Print operation	Head motion	Next state
1	blank	0	left	H
1	0	none	right	1
1	1	none	none	2
2	blank	1	none	H
2	0	1	left	H
2	1	none	right	2
H				

The machine is given the following tape. For this machine, the tape remains stationary while the head moves. The arrow shows the starting position of the head.



Question 16

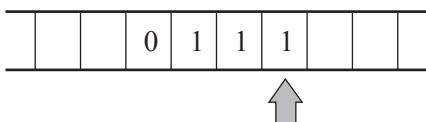
After the Turing machine’s first step, it will be in

- A. state 1 and the head will be one position to the right.
- B. state 2 and the head will still be at the starting position.
- C. state 1 and the head will still be at the starting position.
- D. the halting state and the head will be one position to the left.

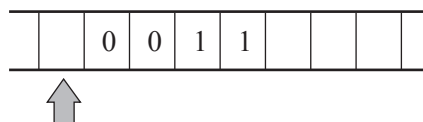
Question 17

Which one of the following best represents the tape’s appearance and the position of the head when the Turing machine halts?

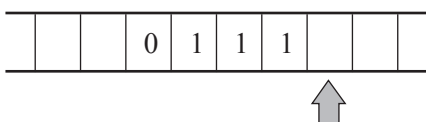
A.



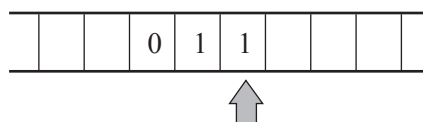
B.



C.



D.



Question 18

A table comparing two advanced algorithm design patterns has been prepared. The left-hand column's heading is 'characteristic'. The remaining two columns are currently labelled ① and ②.

Characteristic	①	②
can make use of recursion	yes	yes
divides problems into sub-problems	no	yes
equivalent to depth-first search	yes	no

Which one of the following statements is correct?

- A. Label ① should read 'divide and conquer' and label ② should read 'backtracking'.
- B. Label ① should read 'backtracking' and label ② should read 'divide and conquer'.
- C. Label ① should read 'dynamic programming' and label ② should read 'divide and conquer'.
- D. Label ① should read 'divide and conquer' and label ② should read 'dynamic programming'.

Question 19

A limitation of the Church-Turing thesis is that it does **not** consider

- A. Turing machines.
- B. analog computing.
- C. the lambda calculus.
- D. algorithms for computing mathematical functions.

Question 20

In the early 1900s, David Hilbert proposed a program to formalise the foundation of mathematics by establishing a formal system for arithmetic that was consistent, complete and decidable.

Church and Turing independently showed that there did not exist a procedure to compute the answer to an arbitrary statement in first-order logic that was

- A. complete.
- B. decidable.
- C. complete or decidable.
- D. complete and decidable at the same time.

SECTION B**Instructions for Section B**

Answer **all** questions in the spaces provided.

Question 1 (5 marks)

Consider an algorithm that has three input variables, A , B and C , and a selection of test data where two possible values for each of the input variables is such that

$$A = \{3, 5\}$$

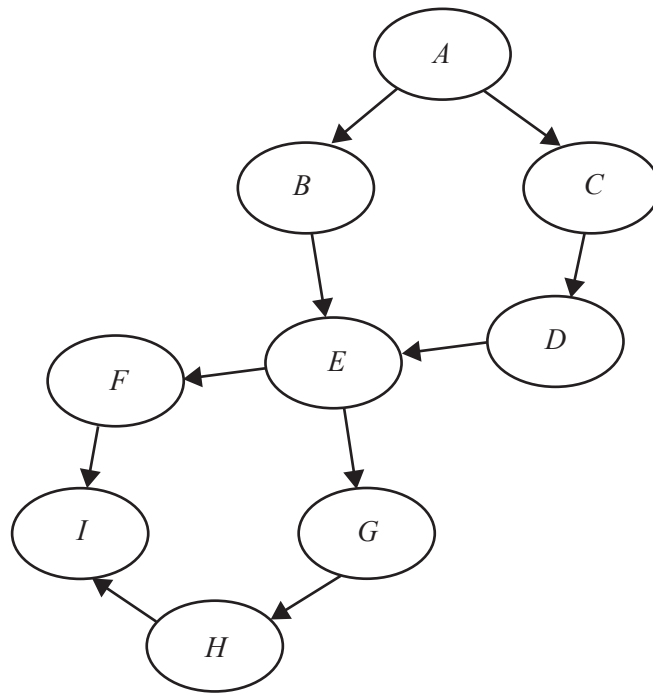
$$B = \{X, Y\}$$

$$C = \{7, 9\}$$

- a. What is the total number of possible test cases required to test the given data? 1 mark

- b. Describe the process that would be undertaken to pair-wise test the algorithm with the given data. In your answer, give all of the possible pair-wise test cases. 4 marks

Question 3 (3 marks)



a. What is the path coverage of the above graph? State all paths as part of your solution. 2 marks

b. How many paths need to be followed to ensure branch coverage of the above graph? 1 mark

Question 5 (4 marks)

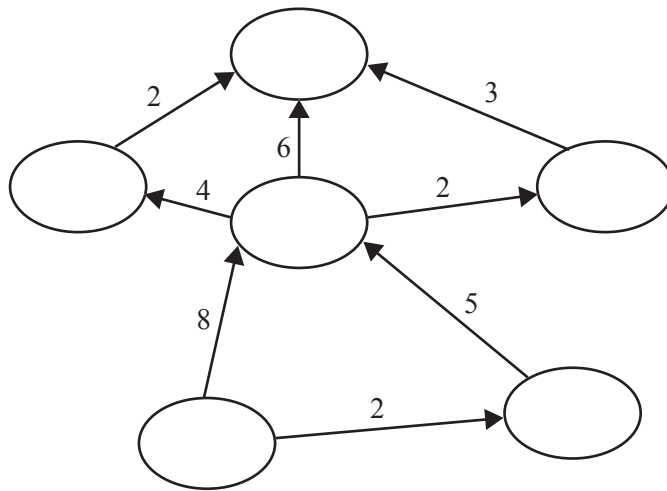
Consider the following pseudocode.

```
Algorithm searchGraph(start, target)
  variables: Q, node           // Q is a queue
  set Q to start
  repeat until Q is empty
    set node to first node in queue
    set Q to every node except first node in queue
    set node.visited to true
    for each successor of node.neighbours
      if successor.visited = false then
        set successor.predecessor to node
        Q.enqueue(successor)
      endif
    endfor
  endrepeat
  if node = target then
    variables: next
    set next to target
    repeat until next = start
      set next.traced to true
      set next to next.predecessor
    endrepeat
  endif
  set start.traced to true
```

The pseudocode above represents a searching algorithm that takes two inputs: a set of nodes (start) and the target that is being searched for (target).

Using Big-O notation, state the time complexity of the algorithm in the worst case, where the number of nodes in the set of nodes is n . Show all working and calculations in your answer.

Question 7 (6 marks)




State and explain two properties of the graph above. For each property, annotate the graph to show where an example of that property is located.

Property 1 _____

Property 2 _____

Question 8 (8 marks)

- a. Sketch a graph that could be used to model the friendship network of a group of six people. 4 marks



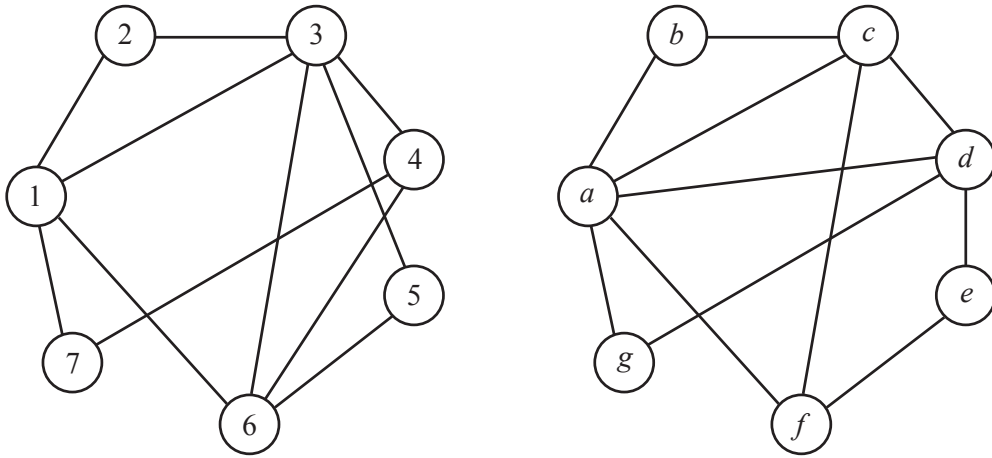
- b. Select two of the properties you included in the graph above. Justify the use of both of these properties in the graph. 4 marks

Property 1 _____

Property 2 _____

Question 9 (2 marks)

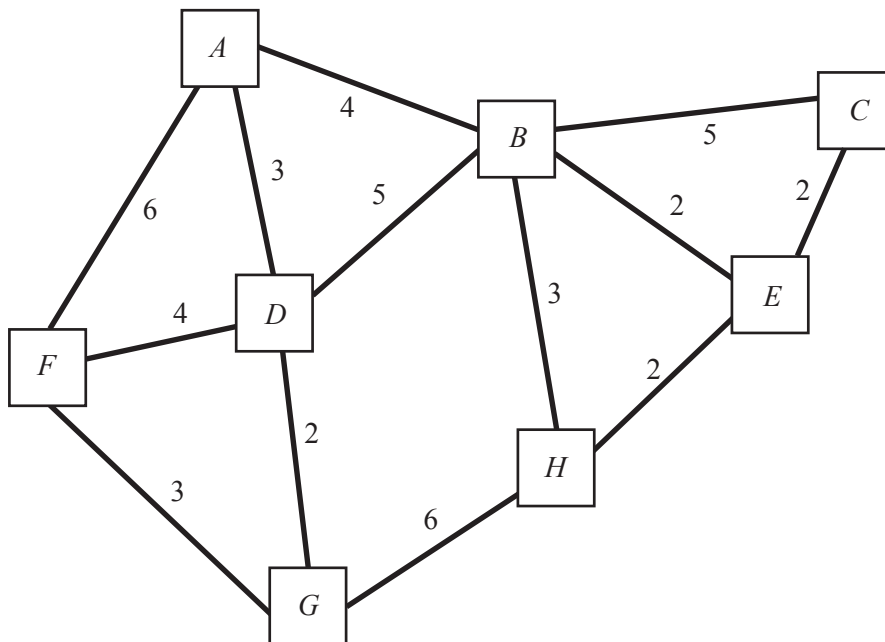
Two researchers use equivalent graphs to represent the social structure of people who belong to a small group. One researcher used letters to label her graph, the other used numbers.



In the space below, draw a new graph that combines the two researchers' graphs. Label each node with **two** labels (letter, number) showing the connections between the letters and numbers in the two graphs.

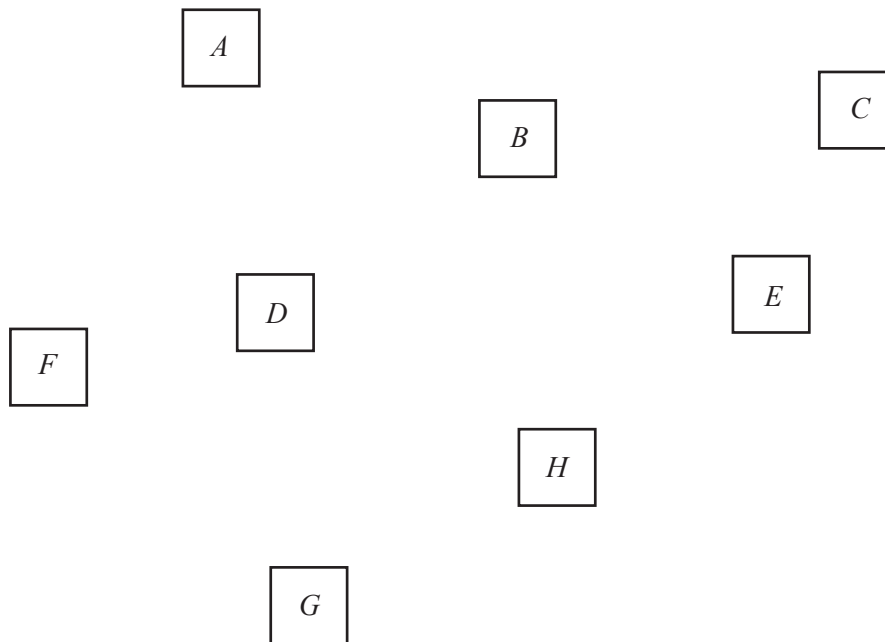
Question 11 (5 marks)

Consider the graph below representing the computers (A – H) in a small local area network. The numbers on the connections represent the lengths (in metres) of cable required to connect each computer to the ones next to it.



- a. Using the template below, draw the connections between computers to show the minimum spanning tree. Label the cable lengths between each computer.

4 marks



- b. State, in metres, the minimum amount of cable required to connect all of the computers as a minimum spanning tree.

1 mark

Question 12 (8 marks)

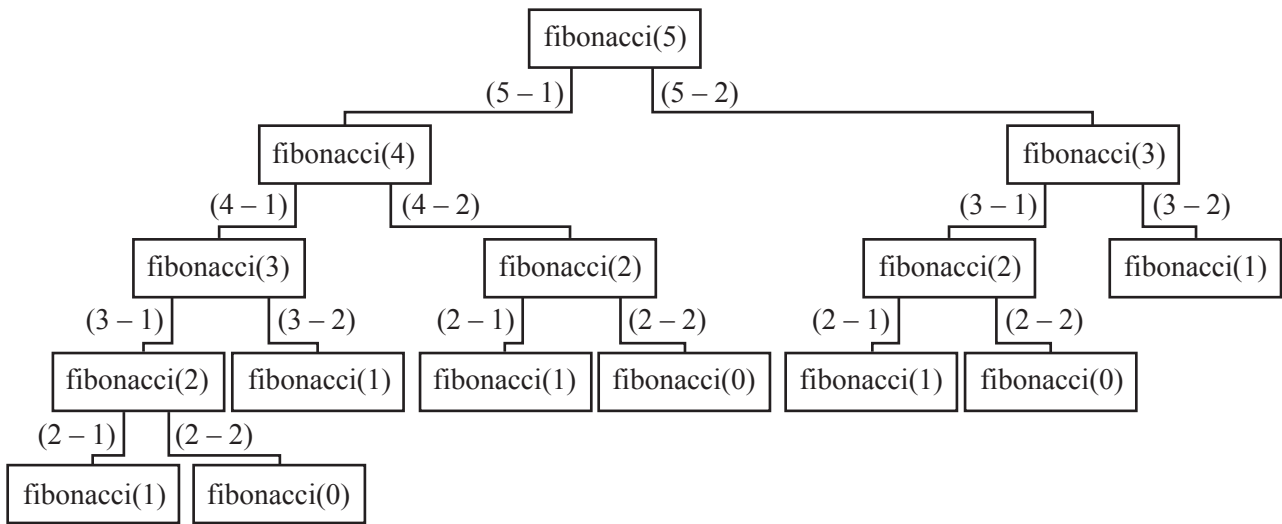
Susan has written a simple recursive function for generating the n th Fibonacci number. For example, if $n = 20$, the 20th Fibonacci number, 6765, is returned.

```

function fibonacci( $n$ )
  if  $n \leq 1$  return  $n$ 
  return fibonacci ( $n - 1$ ) + fibonacci ( $n - 2$ )

```

As part of her testing of the algorithm, Susan measured the time it took to return a value. For $n = 10$, the function returned the correct result almost instantaneously, but for $n = 50$ it took more than an hour. In order to try to work out why it was taking so long, Susan constructed a tree showing each calculation made by the function for $n = 5$.



- a. i.** When looking at her diagram, what should Susan notice as the mostly likely reason for her function's poor performance? 1 mark

- ii.** Explain how this reason accounts for the function's increasingly poor performance for increasing values of n . 2 marks

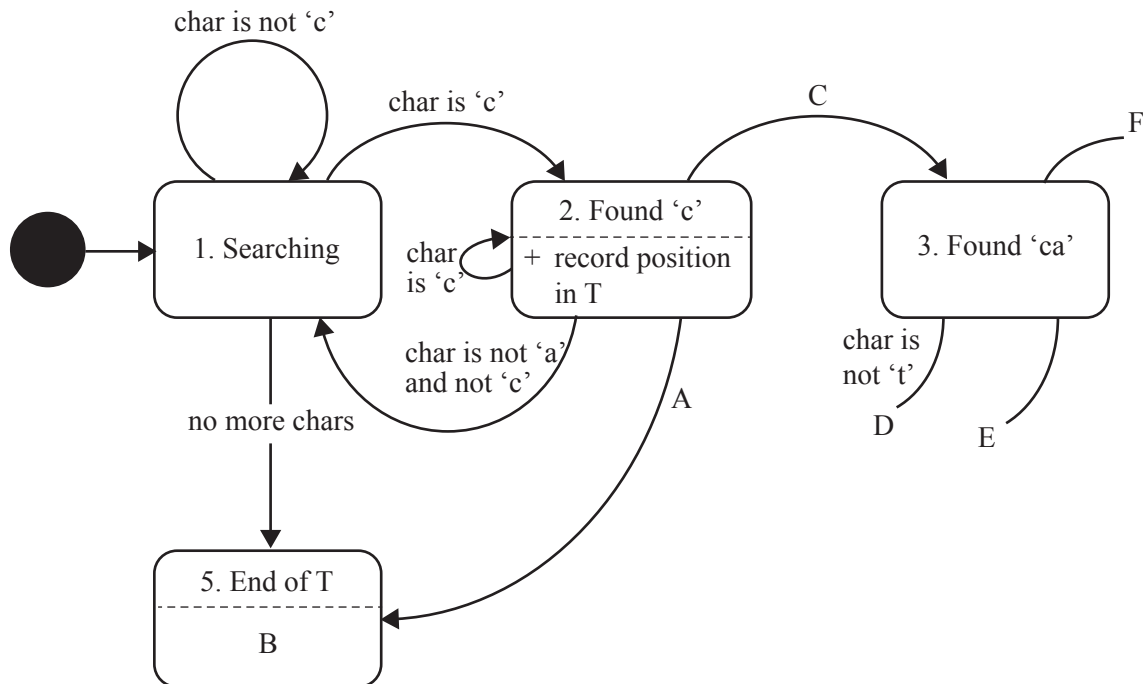
- b. i.** Outline how you would apply dynamic programming to this problem to improve the function's performance. 3 marks

- ii.** Explain how your solution is more efficient than the original solution. 2 marks

Question 13 (10 marks)

Rodney is developing a pattern-matching algorithm. It must search a text string, T, for a given string, G, and stop if it finds it. If G is found in T, the algorithm must output the position of G in T, otherwise it must output zero (0). To help him understand how the algorithm should work, Rodney decides to solve a smaller problem first by drawing a state graph for finding the word 'cat' in T.

The diagram below shows what Rodney has drawn so far. The upper-case letters A to F are labels for the parts of the diagram that Rodney has not yet completed. Each transition in his state graph is triggered by the reading of the next character in T.



- a.** For the transition from state 2 to state 5, what label should Rodney write at A? 1 mark

- b.** If state 5 is reached, Rodney expects some action to take place.

What action should be written at B?

1 mark

- c.** For the transition from state 2 to state 3, what label should be written at C?

1 mark

- d.** Rodney has started to draw a transition from state 3 labelled ‘char is not “t” (D)’.

To which state (1, 2, 3 or 5) should this transition go?

1 mark

- e.** Rodney has also started another transition (E).

What should this transition be labelled, and to which state (1, 2, 3 or 5) should it go?

2 marks

- f.** State 4 has not yet been drawn on the diagram. It is arrived at via a transition from state 3 (F).

i. What should the transition from state 3 to state 4 be labelled?

1 mark

ii. What should state 4 be labelled?

1 mark

iii. When state 4 has been reached, what action(s), if any, should be carried out? If you expect no action to be undertaken, write ‘none’ and explain why not.

2 marks

Answers to multiple-choice questions

Question	Answer
1	D
2	A
3	C
4	B
5	D
6	A
7	C
8	D
9	B
10	D
11	A
12	C
13	D
14	D
15	C
16	A
17	A
18	B
19	B
20	B