Specific instructions

This paper consists of a core and five modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

Core : Data analysis

Question 1

The relationship between blood pressure and gender in adults is being studied and data has been collected from a large sample of the population.

The blood pressure of 6504 adults has been measured and the data grouped into three categories: High, Normal and Low.

Of the 3458 males surveyed 930 were found to have high blood pressure and 344 had low blood pressure and of the females surveyed 742 were found to have high blood pressure and 1235 had low blood pressure.

A two-way frequency table is to be used to organize the data:

	Gender		
Blood pressure	Male	Female	Total
Low	344	1235	1579
Normal		1069	3253
High	930	742	1672
Total	3458	3046	6504

a Complete the table by writing in the figure for males with normal blood pressure.

1 mark

b Of the two categorical variables being compared in this table which is the independent variable?

- c i What percentage of the people measured had high blood pressure? Give your answer correct to one decimal place.
 - ii Of the people with high blood pressure what percentage were males? Give your answer correct to one decimal place.

1 + 1 = 2 marks

d The table is to be percentaged by columns so that the relationship between the variables can be observed.

Complete the percentaged table below.

	Gender		
Blood pressure	Male	Female	
Low	9.9	40.5	
Normal		35.1	
High		24.4	
% Total	100	100	

e Use the figures in the table to comment on the statement "Blood pressure is related to gender". Quote percentages from the table to support your statements.

Another study is investigating the relationship between *diastolic blood pressure* and *weight* in males. The diastolic blood pressure and weight from a sample of 50 adult males was recorded and graphed on the scatterplot below.



The equation of the least-squares regression line for this data is found to be:

Diastolic blood pressure = $66.4 + 0.22 \times Weight$

and the product-moment correlation co-efficient is calculated to be 0.5114

a Interpret the association between the variables in terms of direction and strength.

1 mark

b Use the least-squares regression line to predict the diastolic blood pressure for a man weighing 80 kilogram.

с	Find and plot the co-ordinates of two points on the regression line and then draw the regression line on the scatterplot.
	2 marks
d	Interpret the slope of the regression line in terms of the variables.
2	2 marks Complete the following interpretation of the coefficient of determination .
	"% of the variation in <i>diastolic blood pressure</i> can be explained by
	2 marks Total: 15 marks

Module 1 : Number patterns and applications

Tim, a champion athlete, has been competing in triathlons comprising of three disciplines swimming, cycling and running.

Question 1

In a recent triathlon, the ratio of the distance of the **swim**, **cycle** and **run** legs were **1** : **19** : **7** respectively. If the swim leg was 800 metres,

a how far was the run leg (in metres)?

1 mark

b what was the total distance of the race (in kilometres to 1 decimal place)?

1 mark

In his next race of a total distance of 21 kilometres, the cycle leg of 12 kilometres was twice the distance of the run leg.

c Give the ratio of the swim, cycle and run legs in simplest form.

At the conclusion of the race the number of competitors finishing in each minute interval was observed to follow a pattern, as shown in the table.

Time Interval	0 – 1 min	– 2 min	– 3 min	– 4 min	– 5 min
Number of competitors finishing	2	6	18		162

a Using the first three values, does the table above show an arithmetic or geometric progression? Give reasons for your choice.

			2 marks
			\angle mains

b Complete the table above.

c Using $S_n = \frac{a(r^n - 1)}{r - 1}$, if the difference in race time between the first and last place athlete was 8 minutes, how many triathletes were there in total?

3 marks

Tim, competing on a very hot day, sweats at a rate of 120 ml/km. He drinks 100 mL of sports drink at the end of each kilometre to replenish his fluids. Note that his initial total body-fluid content at the **end of the first kilometre** is 7800 ml.

a Write this situation as a difference equation $t_{n+1} = t_n + b$; t_n where t_{n+1} represents body fluid content.

2 marks

b State whether or not Tim should continue to run the event given the race distance is 25 km and his body will not tolerate body fluid content levels below 5000 ml. Show your calculations.

3 marks Total: 15 marks

Module 2 : Geometry and trigonometry

Georgia and Casey are planning to build a new house which has a patio in the shape of a hexagon, as shown by the figure ABCDEF in the diagram below.



They are interested to know the area of their proposed patio.

Question 1

a Use the cosine rule to show that the length of **AC** is 5.543 metres, correct to three decimal places.

1 mark

b Calculate the area of triangle **ABC**. Give your answer in square metres correct to three decimal places.



c Using the fact that the patio area is symmetric about the line **XY**, find the perpendicular distance, **h** metres, from the boundary **AF** to the boundary **CD**, as shown in the diagram above. Give your answer correct to two decimal places.

2 marks

d Calculate

i the area of the trapezium, shaded in the diagram above.

and hence

ii the area of the whole patio. Give your final answer in square metres correct to one decimal place.

2 + 1 = 3 marks

Casey has a **1:50** scale drawing of his house and patio and he is investigating a 'sail' to shade part of the patio. He has drawn one of his prospective sails on his scale drawing and this is depicted as the shaded triangular area, **PQR**, on the diagram below.



Casey has measured the sides of the triangle on his scale drawing and he has calculated the area to be 25.2 square centimetres.

Calculate the actual area of the 'sail'. Give your answer in square metres correct to one decimal place.



Casey's and Georgia's house is to be built on a sloping block. The contour lines are drawn on the plan of the block below. The contour interval is 2 metres.



a On the set of axes below sketch the cross-section of the block along the boundary MN



b If the horizontal distance from **M** to **N** is 42 metres find the length of the **extra fencing** needed along **MN** to account for the slope of the block. Give your answer in metres, correct to the nearest metre.

2 marks Total: 15 marks

Module 3 : Graphs and relations

A UBOOT factory direct outlet for sports runners is reviewing its marketing strategy.

Question 1

The graph below shows the manufacturing cost and retail revenue functions for their ULTRA FAST X-Trainers.



a From the above graph complete the following table.

	Gradient	y-intercept
Manufacturing Cost		
Retail Revenue		

(working space)

4 marks

b Using simultaneous equations or otherwise, show that the break even point is 400 sports runners per week.

2 marks

c Calculate the amount of profit or loss if 450 runners are sold in a week.

b

The UBOOT company is organising a marketing campaign including a weekend of extreme sports involving the nation's finest athletes to be show-cased at a holiday resort. They have been able to acquire special rates for accommodation for 2-person units at \$50 per night and 3-person units at \$100 per night. They have a budget of \$4000 to accommodate up to 150 athletes.

The two decision variables are defined as

Let *x* = the number of 2-person units Let *y* = the number of 3-person units

One of the constraints is on the number of athletes accommodated as given below.

 $2x + 3y \le 150$

a Determine the constraint based upon the total budget.

1 mark

Co	Complete the missing inequations				
1		$_{-}$ (your answer from (a))			
2	$2x + 3y \le 150$				
3		_			
4		_			

c Graph this set of constraints on the axes provided, label the co-ordinates of the point of intersection and clearly indicate the feasible region.



d From the above graph, list the three most likely solutions to this linear programming problem.

2 marks Total: 15 marks

Module 4 : Business related mathematics

Eric is paying off a reducing balance loan for the house in which he lives. He originally borrowed \$102 000 and as at January 1 2003, the balance owing on the loan is \$50 000.

Eric has decided to purchase a block of land worth \$80 000 as a further investment and will need to borrow this money to finance his plan. He is considering several borrowing options.

Question 1

a Determine the total amount that Eric will be owing if he borrows the money for the block of land on January 1 2003.

1 mark

A friend of Eric's, Brenton, offers to lend him the \$80 000 provided he pays a flat rate of interest of 5% per annum and repays the entire loan in 10 years.

b Determine the interest paid by Eric to Brenton each year.

1 mark

c Determine the **total amount of repayments** that Eric will have made to Brenton once the loan has been entirely paid off.

1 mark

A second friend Margaret also offers to lend Eric the \$80 000 with a flat rate of interest. Under Margaret's plan, Eric will repay the entire loan in 5 years and will be charged a total of \$25 000 interest.

d Calculate, to 2 decimal places, the annual flat rate of interest charged by Margaret.

e Compared to Brenton's offer, how much would Eric save by accepting the proposal put forward by Margaret?

Another option for Eric is to borrow from the bank on top of the \$50 000 he currently owes. The bank agrees to create a new mortgage arrangement for him where he will repay the \$130 000 owing over 30 years at 6.5% per annum on the reducing monthly balance. Repayments will be made monthly.

a Given that Eric uses the annuity formula $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$ where $R = 1 + \frac{r}{100}$ determine the values of *n* and *R* (to 4 decimal places) that he will require to pay off the loan.

2 marks

b Using the formula or your calculator, determine the monthly repayments that Eric must make in order to pay off the loan in the specified time. Give your answer to the nearest ten dollars.



2 marks

c Eric decides he could actually repay \$900 per month instead of the figure found in **b**. Calculate the number of monthly payments he would save by paying this extra amount.

Eric decides to purchase some new furniture costing \$3000 for his home under a hire purchase agreement. He agrees to pay \$600 deposit and then monthly instalments of \$125 dollars for 2 years.

a How much interest will Eric pay over the 2 years?

1 mark

b Given that Eric is paying a flat rate of interest of 12.5% per annum, calculate the **effective** interest rate per annum that Eric is paying.

2 marks Total: 15 marks

Module 5: Networks and decision mathematics

Hannah Enterprises is a firm contracted to design a new prototype for a solar powered vehicle. They have offices in 6 locations represented by nodes A-F in the following network. The numbers represent the distances in kilometres between offices along existing roads.



Question 1

a Mike works in the office at D. One morning he is required to deliver a package to each of the other offices then return to his own office. If he only visits each other office once, write down a possible route that Mike may take.

- c Draw in the minimum spanning tree on the network above. 1 mark
- **d** Determine the length of cable needed using this approach.

Hannah Enterprises have identified 7 activities that must be completed in order to construct the prototype. These activities and their durations are given in Table 1.

Table 1

Activity	Duration (weeks)	Predecessors
А	3	-
В	6	А
С	6	А
D	4	А
E	3	В
F	1	C,E
G	2	D,F

a The project network shown below is incomplete. Complete the network by including activities E and F.



2 marks

b Complete the following table.

Activity	Earliest start time	Latest start time
А	0	0
В	3	3
С	3	
D	3	9
Е	9	9
F		12
G	13	13

- c Write down the critical path for this network.
- d Determine the project completion time.

1 mark

Hannah Enterprises finds that it may be possible to speed up some of the activities, however this will result in increased costs. Table 2 shows which activities may be reduced in time, the cost per week of this reduction and the maximum possible time reduction.

Table 2

Activity	Cost per week (\$)	Maximum reduction (weeks)
В	5000	3
С	4000	2
D	3000	2
Е	2000	1

a By drawing the network or otherwise, determine the shortest time to complete the task under this new arrangement.

2 marks

b Complete the following table

Time saved (weeks)	Activities reduced in time	Cost(\$)
1	E	2000
2	E,B	7000
3	E,B	
4		

(working space)