2004 Further Mathematics
Written Examination 2 (Analysis task)
Suggested answers and solutions

Core: Data analysis

Question 1

12 26 27 28 39 40 41 42 44 50 55 56 56 63 64 70 70 71

Lowest score is 12

Highest score is 71

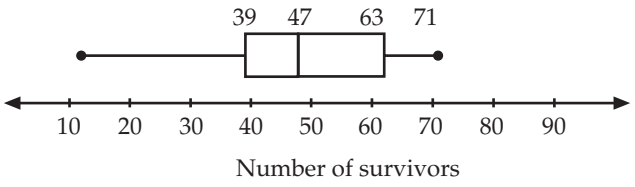
Median is between the 9th and 10th score

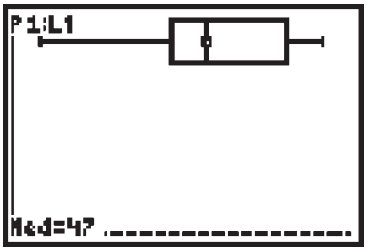
$$\text{Median} = \frac{44 + 50}{2} = 47 \quad \textbf{[A1]}$$

1st Quartile is the 5th score or 39

3rd Quartile is the 14th score or 63 **[A1]**

Number of survivors on 18 lifeboats of the Titanic





b Interquartile Range =

3rd Quartile – 1st Quartile = 63 – 39 = 24 **[consequential A1]**

Lower limit =

1st Quartile – 1.5 x IQR =

39 – 1.5 x 24 = 39 – 36 = 3

Upper Limit =

3rd Quartile + 1.5 x IQR =

63 + 1.5 x 24 = 63 + 36 = 99

All scores are within these limits and so there were no outliers **[A1 with workings]**

c The distribution is negatively skewed. **[A1]**

It has a centre of 47 passengers and a range of 59 passengers. **[A1]**

Question 2

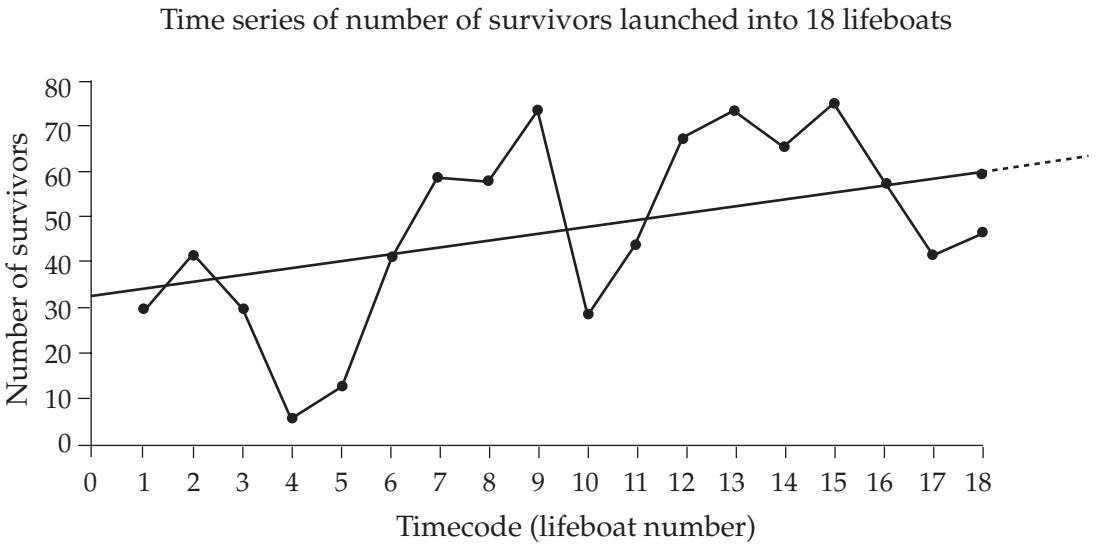
From the table:

a The group that had the best survival rate was **female** (296 survivors and 106 deaths or 296 out of 402 survived $\frac{296}{402} \times \frac{100}{1} \approx 74\%$). **[A1]**

b For the men the class with the worst survival rate was **2nd class** (14 survivors and 154 deaths or 14 out of 168 survived $\frac{14}{168} \times \frac{100}{1} = 8.3\%$). **[A1]**

Question 3

a



The above graph is **random (or cyclical** is acceptable) **[A1]**

with an overall **upward trend** **[A1]**

b **Y intercept** above 30 at approximately 33 **[A1]**

Gradient of 1.5. **[A1]**

c Using

Number of survivors = 33 + 1.5 × Timecode

For the 19th boat

Number of survivors = 33 + 1.5 × 19

= 33 + 28.5

= 61.5 ≈ 62 passengers **[A1]**

For the 20th boat

Number of survivors = 33 + 1.5 × 20

= 33 + 30

= 63 passengers **[A1]**

This gives a total predicted number of 125 extra survivors (62 + 63) **[A1]**

Total Core = 15 marks

Module 1 : Number patterns and applications

Question 1

a 4, 7, 10, 13 **[A1]**

b Show, for example, that $t_2 - t_1 = t_3 - t_2$

i.e. $7 - 4 = 10 - 7 = 3$ **[M1]**

c Use $t_n = a + (n-1)d$ where

$a = 4$ and $d = 3$ **[M1]**

$t_n = 4 + (n-1)3$

$t_n = 4 + 3n - 3$

$t_n = 3n + 1$ **[A1]**

d Let $t_n = 100$ therefore $100 = 3n + 1$ **[M1]**

$3n = 99$

$n = 33 \Rightarrow$ on the 33rd Sunday **[A1]**

e Use $S_n = \frac{n}{2}[2a + (n-1)d]$ where

$a = 4$, $d = 3$, $n = 52$ **[M1]**

$S_{52} = 26[8 + 51 \times 3]$

$S_{52} = 4186$ **[A1]**

Question 2

a $a = 2, b = 1, c = 3$

Any two correct **[A1]**

All three correct **[A1]**

b $t_1 = 3, t_2 = 7, t_3 = 15, t_4 = 31$ **[A1]**

$7 - 3 \neq 15 - 7 \Rightarrow$ not arithmetic

$\frac{7}{3} \neq \frac{15}{7} \Rightarrow$ not geometric **[M1]**

c Use formula $t_n = a^{n-1}t_1 + b\frac{(a^{n-1} - 1)}{a - 1}$

where $a = 2$, $b = 1$ and $t_1 = 3$ **[M1]**

$t_n = 2^{n-1} \times 3 + 1\frac{(2^{n-1} - 1)}{1}$

$t_n = 3 \times 2^{n-1} + 2^{n-1} - 1$

$t_n = 4 \times 2^{n-1} - 1$ **[M1]**

d $t_7 = 3 \times 2^6 + 2^6 - 1$

$t_7 = 255$ **[A1]**

Total Module 1 = 15 marks

Module 2 : Geometry and trigonometry

Question 1

a When 3 sides of a non right-angled triangle are given, the cosine rule may be used to determine any of the angles. **[A1]**

b Using the cosine rule

$\cos N = \frac{750^2 + 600^2 - 1000^2}{2 \times 750 \times 600}$ **[M1]**

$\cos N = -0.0861$

$N = 94.94°$ **[A1]**

c Angle = $180° - (94.94° + 36.72°)$

$= 48.34°$ **[A1]**

d Choose any 2 sides and the included angle e.g.

Area = $\frac{1}{2} \times 750 \times 1000 \times \sin 36°43'$ **[M1]**

$= 224200$ km^2 to the nearest hundred square kilometres **[A1]**

e There are 1000 metres in 1 kilometre therefore there are

1000^2 metres^2 in 1 km^2.

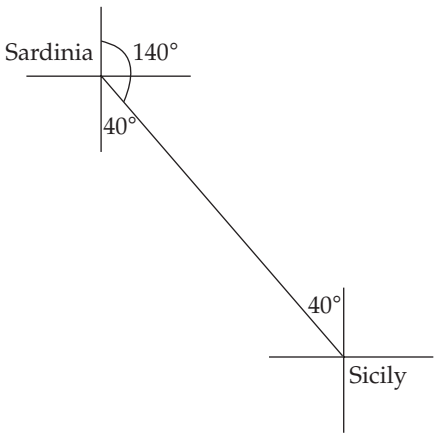
So the answer must be multiplied by

$1000^2 = 1{,}000{,}000$ **[A1]**

f The bearing of Sicily from Sardinia is

$103.28° + 36.72° = 140°T$ **[A1]**

g Calculation of the angles shown in the diagram i.e. $180° - 140° = 40°$ **[M1]**



Bearing of Sardinia from Sicily is

$360° - 40° = 320°T$ **[A1]**

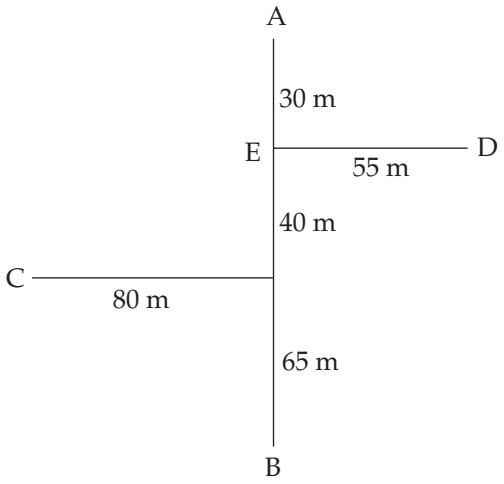
Question 2

a Using Pythagoras' theorem

$CD^2 = 135^2 + 40^2$ **[M1]**

$CD = 140.8$ m , correct to 1 decimal place. **[A1]**

b Angle ADB = angle ADE + angle EDB



angle ADE $= \tan^{-1}\frac{30}{55} = 28.61°$ **[M1]**

angle EDB $= \tan^{-1}\frac{105}{55} = 62.35°$ **[M1]**

Angle ADB = angle ADE + angle EDB

$= 28.61° + 62.35°$

$= 91°$, to the nearest degree **[A1]**

Total Module 2 = 15 marks

Module 3 : Graphs and relations

Question 1

a i 25 dogs require at least 200g of soft food.

25×200 g $= 5000$ g $= 5$ kilograms **[A1]**

ii $x \geq 5$ **[A1]**

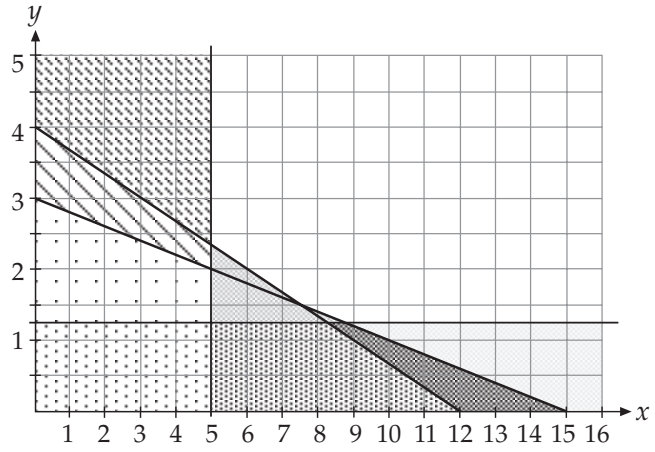
b $50x + 150y \geq 24 \times 25$

"$50x + 150y$" **[A1]**

"24×25" **[A1]**

Question 2

a The feasible region is the unshaded region on the diagram below.



Region $x + 5y \geq 15$ correct **[A1]**

Other regions correct **[H1]**

b Points $(5, 2\frac{1}{3})$ and (7.5, 1.5) **[H1]**

Point (8.75, 1.25) **[A1]**

Three points only **[A1]**

Question 3

a $C = 1.5x + 5y$ **[A1]**

b Substitution of all points **[M1]**

(5, 7/3) gives C = \$19.17

(7.5, 1.5) gives min.cost \$18.75 **[H1]**

(8.75, 1.25) gives C = \$19.38

c $\frac{\$18.75}{25} = \$0.75 = 75$ cents **[A1]**

Question 4

At the break-even point:

Costs = $207 + 15 \times 0.8$ **[M1]**

Revenue = $15 \times d$ where d is the cost of boarding a dog for a day

So $207 + 15 \times 0.8 = 15d$ **[A1]**

$\Rightarrow d = \$14.60$

Total Module 3 = 15 marks

Module 4 : Business-related mathematics

Question 1

a Percentage is $\frac{6149}{286000} \times 100 = 2.15\%$ **[A1]**

b 47% of $40 000

$= \frac{47}{100} \times \$40000 = \$18\,800$ **[A1]**

c 6% of ($564 000 – $115,000) **[M1]**

$= \frac{6}{100} \times \$449\,000$

$= \$26\,940$

plus $2560

Stamp duty = $29,500 **[A1]**

Question 2

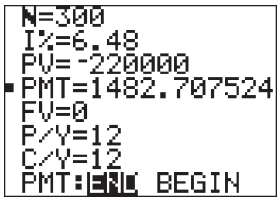
a Monthly payments so interest rate, r, is

$\frac{6.48}{12} = 0.54$

$R = I + \frac{r}{100} = 1.0054$ **[A1]**

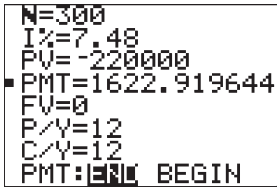
$n = 25 \times 12 = 300$ **[A1]**

b Using the TVM solver on the calculator:



Payments are $1482.71 per month **[A1]**

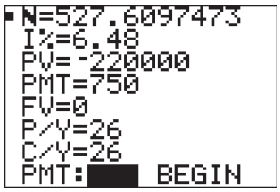
c Using the TVM solver on the calculator:



Payments are $1622.92 per month; **[A1]**

an increase of $140.21 **[H1]**

d Using the TVM solver on the calculator:



Term of the loan is 528 fortnights **[M1]**

$\frac{528}{26} = 20.3$ years **[A1]**

Question 3

a $\$128 \times 52 \times 3 = \$19\,968$

$19968 – $16500 = $3468 **[A1]**

b $\frac{\$3468}{3} = \$1156$ interest per year

$\frac{1156}{16500} \times 100 = 7.006\%$

Flat rate of interest = 7.01% **[H1]**

c Substituting $n = 156$ and

flat rate = 7.01 in the formula for

effective rate of interest

effective rate = $\frac{2n}{n+1} \times$ flat rate **[M1]**

gives

effective rate = $\frac{2 \times 156}{156 + 1} \times 7.01$

$= 13.93$

Effective rate of interest is 13.9% **[A1]**

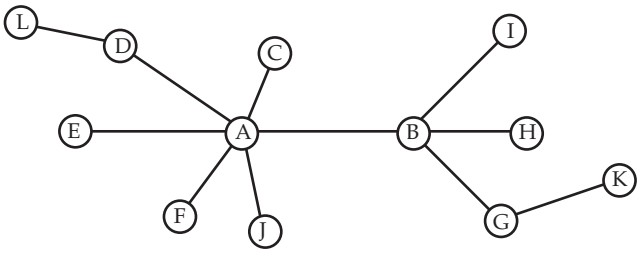
Total Module 4 = 15 marks

Module 5 : Networks and decision mathematics

Question 1

a Set up servers A and B and add computers C-F, G-I.

Connect printers to A and G, scanner to D. **[A2]**



b Output from computer H would travel via a pathway

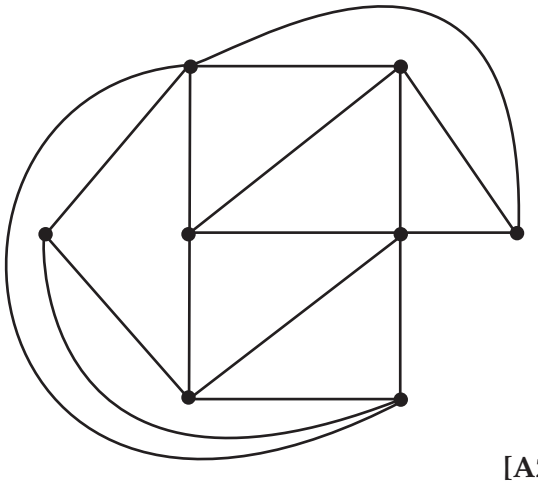
H – B – G – K or H – B – A – J **[A1]**

Question 2

a Move the connection between 1 and 6 outside 2.

Move the connection between 3 and 8 outside 7.

Move the connnection between 1 and 8 to be outside.



[A2]

b This is a planar graph. **[A1]**

Question 3

a Only vertices 3 or 7 are of odd degree. **[A1]**

b If started with 3, end at 7, if started with 7 end at 3. **[A1]**

c Starting at 3:

3 – 4 – 5 – 6 – 7 – 5 – 3 – 2 – 1 – 7

3 – 5 – 7 – 6 – 5 – 4 – 3 – 2 – 1 – 7

3 – 2 – 1 – 7 – 6 – 5 – 4 – 3 – 5 – 7

and other paths too

Starting at 7:

7 – 6 – 5 – 4 – 3 – 5 – 7 – 1 – 2 – 3

7 – 5 – 3 – 4 – 5 – 6 – 7 – 1 – 2 – 3

7 – 1 – 2 – 3 – 5 – 7 – 6 – 5 – 4 – 3 **[A1]**

and other paths too

Question 4

a

Activity	Immediate Predecessor(s)
A	–
B	–
C	A
D	B
E	B
F	C,D
G	E,F
H	C,D
I	C,D
J	G,I

[A2]

b Earliest is 14 weeks.

Enter latest completion time for each pathway.

Activity A: 2 weeks
Activity B: 3 weeks
Activity C: 6 weeks (via A)
Activity D: 7 weeks (via B).
Activity E: 5 weeks (via B)
Activity F: 8 weeks (via D)
Activity G: 11 weeks (via D and F)
Activity H: 11 weeks (via D)
Activity I: 8 weeks (via D)
Activity J: 14 weeks (via D, F and G) **[A1]**

c From above analysis the critical path is B – D – F – G – J **[A1]**

d From earliest completion time subtract the next critical path activity tome and any activities leading from there.

Non Critical activities are A, C, E, H and I

Float(H) = 14 – 7 – 4 = 3 weeks

Float(I) = 14 – 7 – 3 – 1 = 3 weeks

Float(E) = 14 – 8 – 3 = 3 weeks

Float(C) = 14 – 7 – 6 = 1 week

Float(A) = 14 – 7 – 6 = 1 week **[A2]**

Total Module 5 = 15 marks