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Further Mathematics

2006

Trial Examination 2

Core – Data analysis
Module 2 – Geometry and trigonometry
Module 3 – Graphs and relations
Module 4 – Business-related mathematics

Instructions:
Answer all questions in the core and the three modules.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.

Core – Data analysis

Question 1

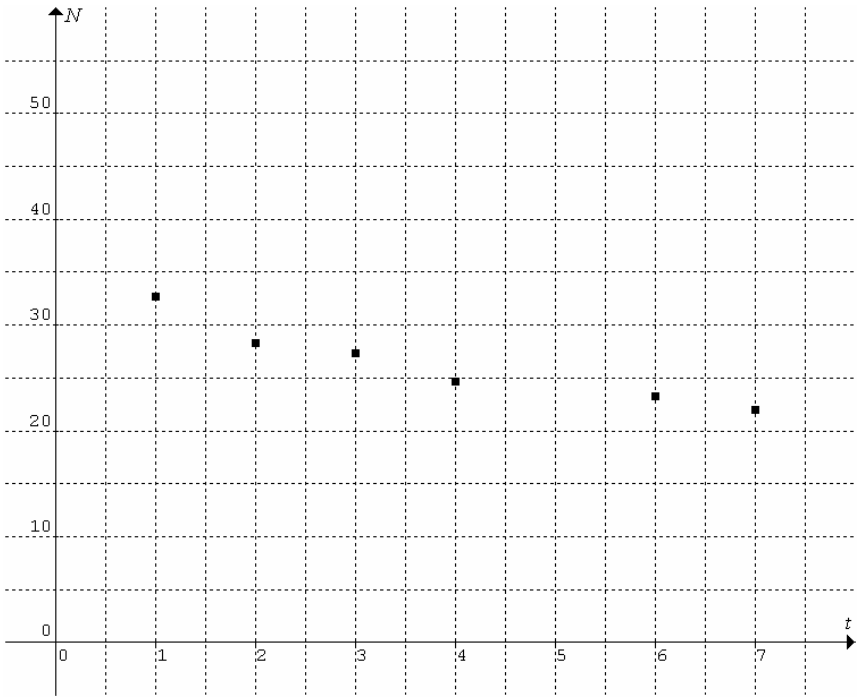
The number of people killed in road accidents in a region in Australia from 1997 to 2005 is shown in the table below. The three-year moving averages (except one) of the number of people killed are also shown.

Year	Number of people killed	Three-year moving average
1997	36	
1998	29	32.7
1999	33	28.3
2000	23	27.3
2001	26	24.7
2002	25	
2003	22	23.3
2004	23	22
2005	21	

a. Calculate the missing three-year moving average (correct to 1 decimal place). 1 mark

b. Take year 1998 as $t = 1$ and year 1999 as $t = 2$ etc, the graph of three-year moving average N against time t is shown below. There is a missing point in the graph, plot it.

1 mark



c. A least squares regression line can be fitted to the graph in part **b**.

i. Find the value of the correlation coefficient correct to three decimal places. 1 mark

ii. Find the percentage (correct to the nearest whole number) of the variation of N that can be accounted for by the variation in t.

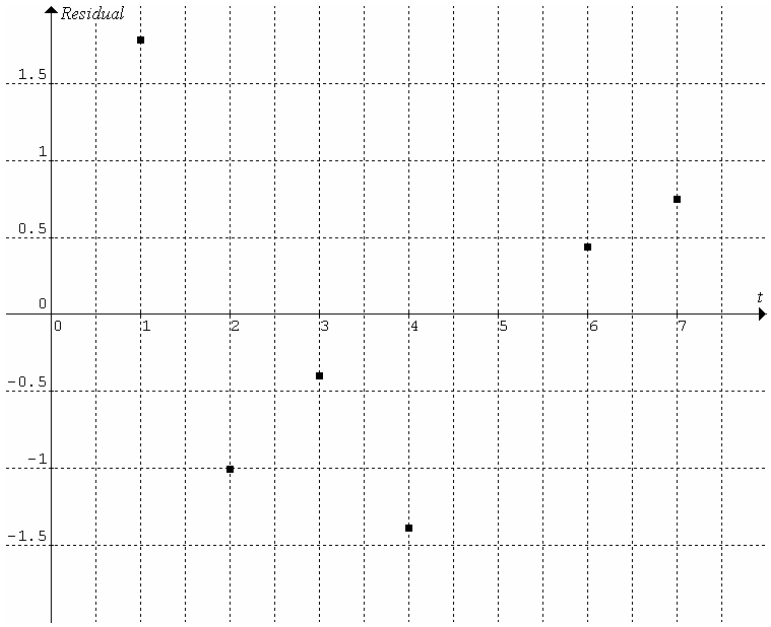
1 mark

iii. Write down the equation of the least squares regression line. 1 mark

iv. Accurately draw the least squares regression line in the graph in part **b**. 2 marks

d. To further investigate the relationship between N and t, a residual plot is constructed as shown below. There is a missing point in the residual plot, calculate this residual (correct to 1 decimal place) and plot it in the graph below.

2 marks



e. What is/are the feature(s) of the residual plot above that suggest(s) a nonlinear relationship will provide a better fit for the data?

1 mark

f. The graph of three-year moving average N against time t indicates that a logarithmic transformation of t may linearise the data. Complete the following table.

1 mark

Year	t	log(t)	Three-year moving average
1998	1	0	32.7
1999	2	0.30	28.3
2000	3	0.48	27.3
2001	4	0.60	24.7
2002	5		
2003	6	0.78	23.3
2004	7	0.85	22

g. Find the equation of the least squares regression line for the relationship between N and $\log(t)$.

1 mark

h. Justify that the least squares regression line for the relationship between N and $\log(t)$ provides a better fit for the data than the least squares regression line for the relationship between N and t.

1 mark

i. Use the equation of the least squares regression line for the relationship between N and $\log(t)$ to predict the number of people killed in road accidents in the region under consideration in 2006.

1 mark

j. Suggest another transformation that may linearise the original data and explain why it will work.

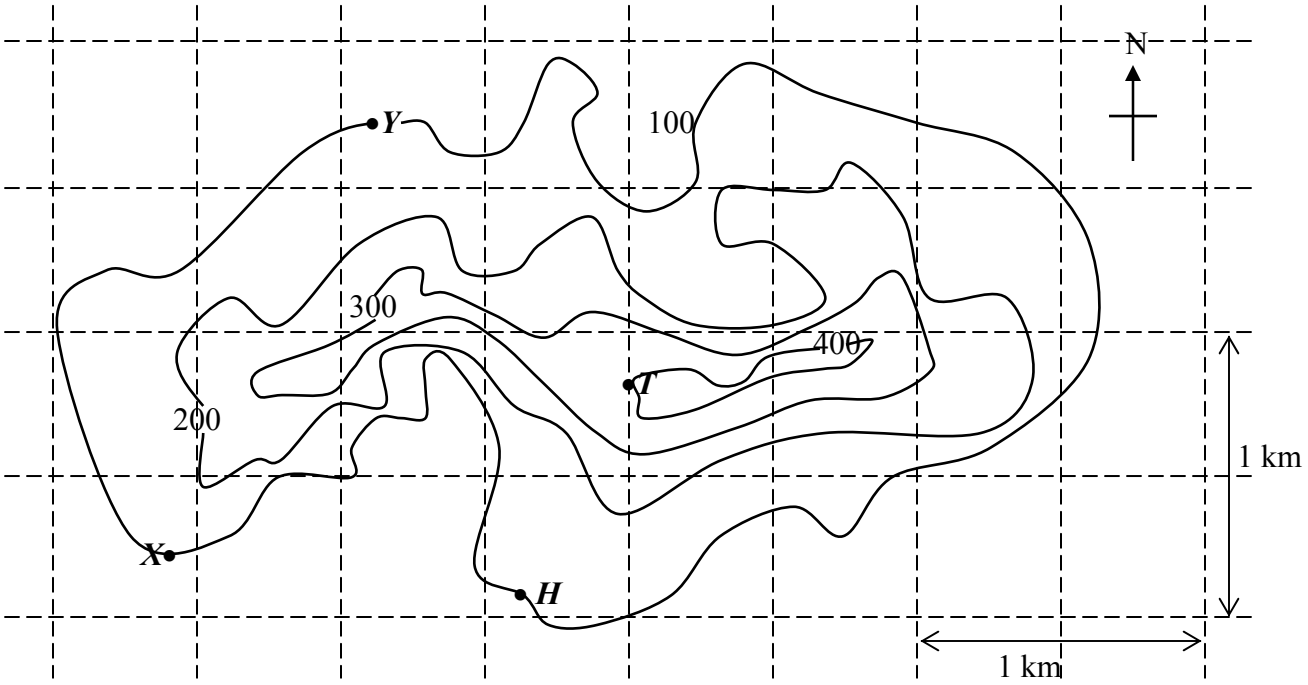
1 mark

Total 15 marks

Module 2: Geometry and trigonometry

Question 1

The contour map below shows a region near the Grampians. It has contours drawn at 100-metre intervals. A bushranger moves from location ***X*** to location ***Y***. Her home is at ***H*** and there is a TV-tower at ***T***.



a. Is it possible for the bushranger to see her home at ***H*** from location ***X*** and/or location ***Y***? Explain.

2 marks

b. Estimate the straight line distance (correct to the nearest 0.1 km) from ***X*** to ***Y***. 2 marks

c. The bushranger observes the TV-tower at ***T*** from both locations ***X*** and ***Y***. At which location will the angle of elevation be greater? Explain.

1 mark

d. Calculate the angle of elevation (correct to the nearest degree) of the TV-tower from location ***Y***.

2 marks

e. Calculate the compass bearing of the TV tower from location ***Y***. 1 mark

f. The bushranger arrives at ***T*** and looks towards location ***X*** and then looks towards ***Y***. Calculate the angle (correct to the nearest degree) between the lines of sight ***TX*** and ***TY***.

2 marks

g. Calculate the ***land area*** (correct to the nearest 10000 m^2) enclosed by the triangle with vertices at ***T***, ***X*** and ***Y***.

2 marks

h. A three dimensional scaled model of the terrain is to be made of clay, starting from the 100-m contour. The scale to be used is 10 cm : 1 km (i.e. 1 : 10000). Estimate the volume of clay (nearest 100 cm^3) required to make the scaled model of the terrain.

3 marks

Total 15 marks

Module 3: Graphs and relations

Question 1

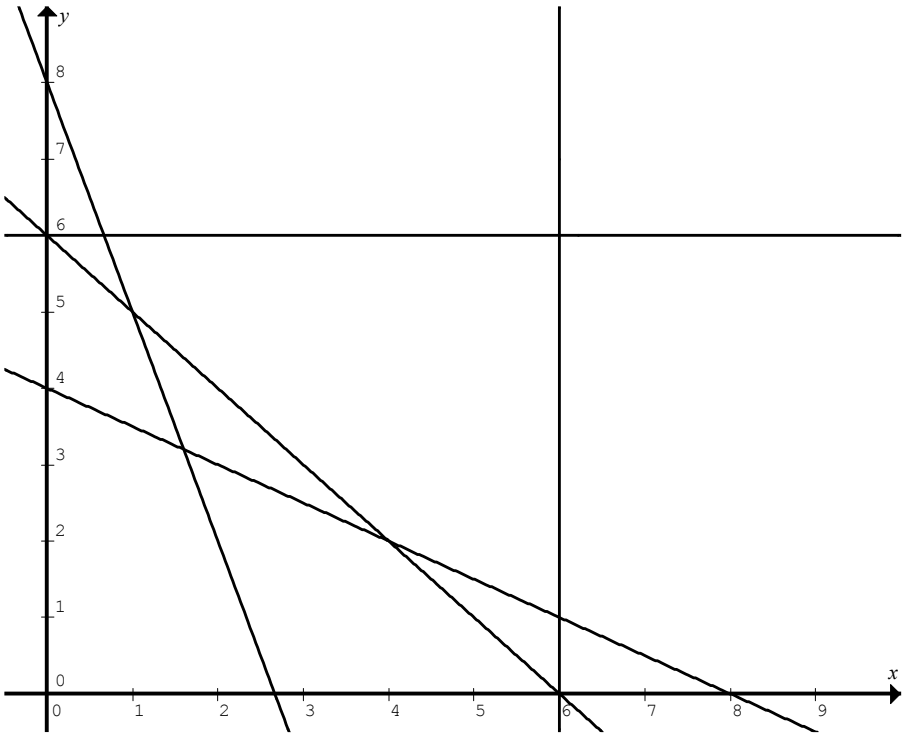
PHB mining owns two mines in Western Australia. Both mines produce high, medium and low grade ores. Mine 1 costs $1200 per day to operate whilst mine 2 costs $960 per day. The manager wants to find out the number of whole days each week that the mines should operate in order to minimise the weekly total operating cost. The following table gives the production in tonnes from each mine per day and the production requirements in tonnes per day for the three grades of ores.

Grade	Mine 1	Mine 2	Production requirements
High	18	36	144
Medium	12	12	72
Low	72	24	192

Let whole number x be the number of days per week that mine 1 operates and whole number y be the number of days per week that mine 2 operates. The mines do not operate on Sundays.

a. Write down five inequalities that constrain the values of x and y. 2 marks

b. Label clearly each of the lines in the following graph with the correct equation. 2 marks



c. Shade the feasible region for operating the two mines for each week. 1 mark

d. Determine the x and y values for each vertex in the feasible region. 2 marks

e. Let C be the weekly cost of operating the two mines. Write an equation for C in terms of x and y.
1 mark

f. Determine the number of whole days each week that the mines should operate in order to minimise the total operating cost. What is the minimum weekly total operating cost of the mines?
2 marks

g. In a particular week new machinery is to be installed in mine 1 and therefore mine 1 is operational for three days only in that week. What is the minimum weekly total operating cost of the mines in ore production for this particular week? Explain your answer.
2 marks

h. After installing the new machinery in mine 1 the daily operating cost in ore production for mine 1 is reduced by \$240. Find all solutions for the number of days per week that each mine can operate for ore production so that the minimum weekly total operating cost is achieved.
2 marks

i. Determine the new minimum weekly total operating cost of the mines in ore production. 1 mark
Total 15 marks

Module 4: Business-related mathematics

Question 1
Sally has a savings account that pays interest at a rate of 4.75% per annum on the minimum monthly balance. The following table shows the entries in Sally's savings account.

Date	Debit	Credit	Balance
30 June		120.00	900.00
15 July	200.00		700.00
25 July		300.00	1000.00
8 August		1500.00	2500.00
31 August	200.00		2300.00
1 September		1700.00	4000.00

Calculate the total interest payable for the two months, July and August. 2 marks

Question 2
Sally enters into a hire-purchase contract to get a new car costing $22000. She agrees to pay a deposit of $2000 and make 30 monthly repayments of $850.

a. Calculate the total amount of interest that she has to pay. 1 mark

b. What is the flat rate of interest charged per annum? 1 mark

c. Determine the effective interest rate per annum (correct to 1 decimal place). 1 mark

Question 3
To buy a home unit Sally also negotiates a 25-year mortgage of $180000 at a fixed rate of 7% per annum compounded monthly for the first five years, then at the market rate (variable) for the remainder of the loan. She agrees to repay $1270 monthly for the first five years. Sally has a net income of $3800 per month.

a. Can Sally afford to keep the car and the home unit? Explain. 1 mark

b. Calculate the amount that she still owes on the home unit after the first five years. 2 marks

c. Calculate the new monthly repayments required to pay off the loan if the market rate has risen to 8% after the initial five years.

2 marks

Question 4
Sally's brother Bobby runs a taxi service. A taxi is considered to have no value when it has travelled 270000 km. The cost of a new taxi is $35100 and it is expected to be driven 1800 km each week.

a. Calculate the rate of depreciation (in $ per km), assuming it is straight-line depreciation. 1 mark

b. Calculate the number of weeks it takes before the taxi has no value. 1 mark

c. Calculate the number of weeks it takes before the book value is $11700. 1 mark

d. If reducing balance depreciation is used to calculate the book value, determine the weekly depreciation rate in % (correct to 4 decimal places) required to reduce the book value to $11700 in 100 weeks.

2 marks
Total 15 marks

End of exam 2