

## FURTHER MATHEMATICS EXAM 1: SOLUTIONS

**Core:**

**Question 1 Answer D**

The data is discrete numerical data so a dot plot is the most suitable graph of the ones listed. A scatterplot is used for bivariate numerical data and a line graph is usually used for time-series data.

**Question 2 Answer A**

The mean is

$$\frac{0 \times 13 + 1 \times 18 + 2 \times 21 + 3 \times 11 + \dots + 6 \times 2}{74} = 1.9054$$

**Question 3 Answer B**

There are four plants that have a height of at least 110mm.

$$\frac{4}{22} \times 100 = 18.1818... \approx 18\%$$

**Question 4 Answer D**

The distribution of the data would be considered to be symmetric.

The 5-number summary for the data is:

Min. = 84;  $Q_1 = 95$ ; Median = 101.5

$Q_3 = 105$ ; Max. = 121

The range is  $121 - 84 = 37$

IQR is 10 which is the spread of the middle 50%

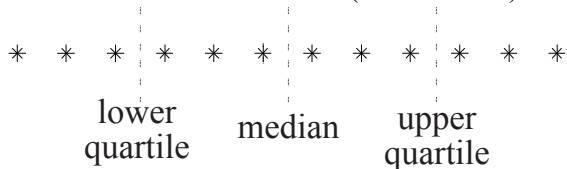
Test for outliers  $95 - 15 = 80$ ;  $105 + 15 = 120$ .

121 is an outlier

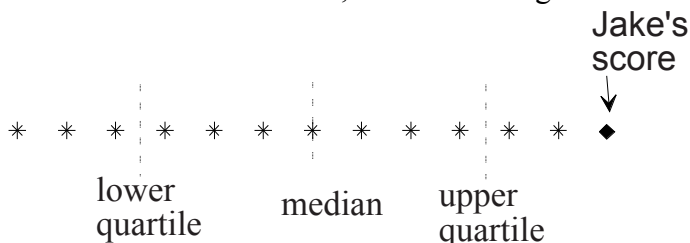
**Question 5 Answer A**

The mean and the standard deviation will change because Jake's marks are included in the calculation of the statistics. The median, and upper and lower quartiles, however, depend on the position of Jake's score.

If there are 12 data values (all different):-



If there are 13 data values; Jake is the highest:-



The lower quartile does not change.

**Question 6 Answer D**

$$z\text{-score} = \frac{\text{raw score} - \text{mean}}{\text{standard deviation}}$$

To obtain a z-score of 2 solve

$$2 = \frac{\text{raw score} - 73}{6.5}$$

$$13 = \text{raw score} - 73$$

$$\text{Raw score} = 86\%$$

**Question 7 Answer B**

The slope,  $m$ , of the regression line  $y = mx + c$  where  $y = \text{Result on Test 2}$  and  $x = \text{Result on Test 1}$  is given by

$$m = r \frac{s_y}{s_x}$$

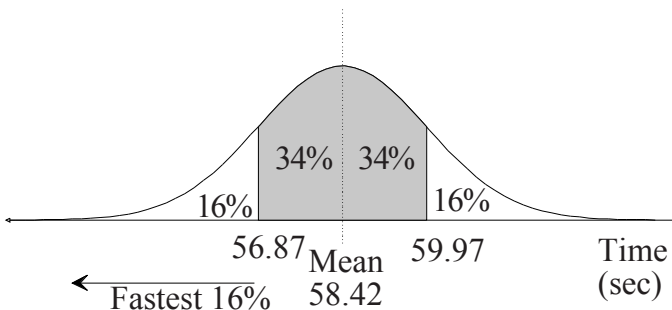
$$\text{Slope} = 0.88 \times \frac{6.5}{9} = 0.63555... \approx 0.64$$

**Question 8 Answer B**

68% of the swimming times would be within one standard deviation of the mean leaving 16% more than one st. dev. and 16% less than one st. dev.

The fastest 16% would have a time less than one standard deviation from the mean.

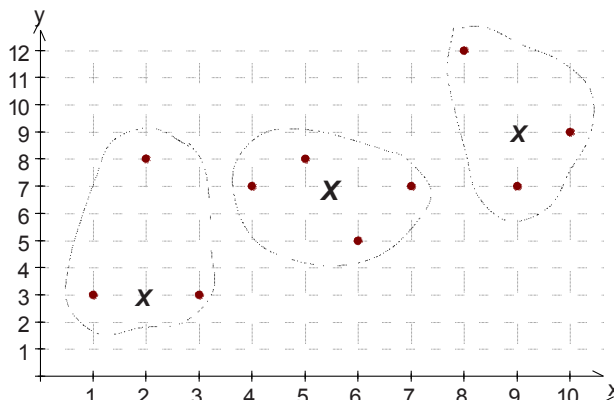
i.e. less than  $58.42 - 1.55 = 56.87$  seconds.



**Question 9 Answer D**

The 10 points on the graph are divided into groups of 3-4-3 points, using the  $x$ -values, and the median of each group is found.

The median of the left group is (2, 3) and the median of the right group is (9, 9)



**Question 10 Answer A**

Deseasonalisation is a means of smoothing a time series graph that shows seasonal fluctuations; the process would bring the highs (Quarter 2) and lows (Quarter 4) back to a value closer to the seasonal average. Quarters 1 and 3 values would be much closer to the seasonal average and so their seasonal indices would be closer to one. The seasonal indices for the four quarters would sum to 4; the seasonal index for Quarter 2 would be well above 1 and the seasonal index for Quarter 4 would be below 1.

**Question 11 Answer C**

Using the given least-squares regression equation the predicted  $y$ -value for an  $x$ -value of 5 is

$$3.2 - 0.45 \times 5 = 0.95$$

The actual  $y$ -value for the  $x$ -value of 5 is 1

$$\begin{aligned} \text{Residual} &= \text{Actual } y\text{-value} - \text{predicted } y\text{-value} \\ &= 1 - 0.95 \\ &= 0.05 \end{aligned}$$

**Question 12 Answer A**

To linearise the given data set either the  $x$ -scale or the  $y$ -scale needs to be compressed. This can be done by either a  $\log x$ ,  $\frac{1}{x}$ ,  $\log y$  or a  $\frac{1}{y}$  transformation.

An  $x^2$  transformation would stretch the  $x$ -scale

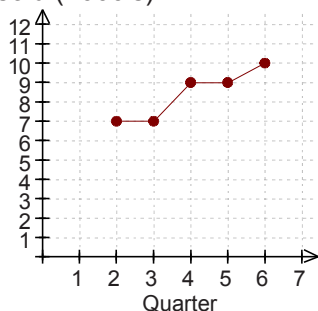
**Question 13 Answer A**

Moving from left to right, the median point of the first three points is found; (2, 7)

Then moving along one point only the median of points 2, 3 and 4 is found; (3, 7)

Continuing in this way the other median points are (4, 9), (5, 9), and (6, 10)

Cars sold (1000's)



**Section B:**

**Module 1: Number patterns**

**Question 1 Answer D**

The sequence is arithmetic with the first term  $a = -8$ . Now, common difference is

$$\begin{aligned} d &= t_2 - t_1 \\ &= -11.5 - -8 = -3.5 \\ t_3 - t_2 &= -15 - -11.5 = -3.5 \end{aligned}$$

Therefore  $a = -8$  and  $d = -3.5$

**Question 2 Answer B**

From sequence  $a = 3$ .

To find the common ratio,  $r$

$$\begin{aligned} r &= \frac{t_2}{t_1} = \frac{t_3}{t_2} \text{ and so on} \\ &= \frac{-9}{3} \\ &= -3 \end{aligned}$$

To find 8<sup>th</sup> term,  $t_8$

$$\begin{aligned} t_n &= ar^{n-1} \\ t_8 &= 3 \times (-3)^{(8-1)} \\ &= 3 \times (-3)^7 \\ &= 3 \times -2187 \\ &= -6561 \end{aligned}$$

**Question 3 Answer A**

A straight line graph represents an arithmetic sequence. Because the graph is decreasing, the common difference is negative.

From the graph,  $t_1 = 4$  and the common difference is  $-3$ . For arithmetic sequences, the difference equation general form is

$$\begin{aligned} t_{n+1} &= t_n + b, \quad t_1 = 4 \\ \text{Therefore} \\ t_{n+1} &= t_n - 3, \quad t_1 = 4 \end{aligned}$$

**Question 4 Answer E**

$$\begin{aligned} f_1 &= 2 \\ f_2 &= 3 \\ f_3 &= 2f_1 + f_2 \\ &= 2 \times 2 + 3 = 7 \\ f_4 &= 2f_2 + f_3 \\ &= 2 \times 3 + 7 = 13 \\ f_5 &= 2f_3 + f_4 \\ &= 2 \times 7 + 13 = 27 \end{aligned}$$

Therefore the sequence is 2, 3, 7, 13, 27.

**Question 5 Answer C**

$$\text{From } t_n = a + (n-1)d$$

$$t_{11} = 43 = a + 10d \quad [1]$$

$$\text{From } S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = 820 = \frac{20}{2}[2a + (19)d]$$

$$820 = 20a + 190d \quad [2]$$

Using elimination technique equation [1]  $\times$  19

$$817 = 19a + 190d \quad [3]$$

$$820 = 20a + 190d \quad [2]$$

Then [2] - [3]

$$3 = 1a$$

$$a = 3$$

**Question 6 Answer B**

Let  $B_n$  = bread sold in a month.  $B_1 = 1000$  sold in the first month

$$B_2 = B_1 + 5\% \text{ of } B_1$$

$$= (1 + 5\%)B_1$$

$$= (1 + 0.05)B_1$$

$$= 1.05B_1$$

$$B_3 = 1.05B_2$$

This is a geometric sequence where  $a = 1000$

and  $r = 1.05$ .

**Method 1 Iteration**

$$B_2 = 1.05B_1 = 1.05 \times 1000 = 1050$$

$$B_3 = 1.05B_2 = 1.05 \times 1050 = 1102.5$$

$$B_4 = 1.05B_3 = 1.05 \times 1102.5 = 1157.625$$

and so on

$$B_{10} = 1.05B_9 = 1.05 \times 1477.455.. = 1551.32...$$

$$B_{11} = 1.05B_{10} = 1.05 \times 1551.32.. \\ = 1628.89 \text{ (11}^{\text{th}} \text{ week)}$$

**Method 2**

$$t_n = ar^{n-1}$$

$$1600 = 1000 \times 1.05^{n-1}$$

$$1.6 = 1.05^{n-1}$$

Using logs

$$\log 1.6 = \log 1.05^{n-1}$$

$$\log 1.6 = (n-1) \log 1.05$$

$$n-1 = \frac{\log 1.6}{\log 1.05}$$

$$n-1 = \frac{0.2041...}{0.021189..}$$

$$n-1 = 9.633$$

$$n = 10.633$$

Using logs

$n = 10$  weeks is not enough

$n = 11^{\text{th}}$  week

**Question 7 Answer D**

From

$$S_{\infty} = \frac{a}{1-r}$$

$$21\frac{1}{3} = \frac{16}{1-r}$$

$$\frac{64}{3} = \frac{16}{1-r}$$

$$64(1-r) = 16 \times 3$$

$$64 - 64r = 48$$

$$-64r = 48 - 64$$

$$r = \frac{-16}{-64}$$

$$= \frac{1}{4}$$

Now for  $a = 16$  and  $r = \frac{1}{4}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_5 = \frac{16(1 - \frac{1}{4}^5)}{1 - \frac{1}{4}}$$

$$S_5 = \frac{16(1 - \frac{1}{4}^5)}{[1 - \frac{1}{4}]}$$

Using a calculator

$$\frac{16(1 - (1/4)^5)}{1 - 1/4}$$

$$\text{Ans} \rightarrow \text{Frac} \quad 21.3125$$

$$.3125 \rightarrow \text{Frac} \quad 341/16$$

$$\blacksquare \quad 5/16$$

$$S_5 = 21\frac{5}{16}$$

**Question 8 Answer C**

The sequence is a Fibonacci-type sequence where the next term is the sum of the two previous terms. Therefore, working backwards to find the first two terms, we have

31 is the sum of 19 and the previous unknown term, that is

$$31 = 19 + f_{n-1}$$

$$f_{n-1} = 31 - 19 = 12$$

19 is the sum of 12 and the previous unknown term, that is

$$19 = 12 + f_{n-1}$$

$$f_{n-1} = 19 - 12 = 7$$

$$12 = 7 + f_{n-1}$$

$$f_{n-1} = 12 - 7 = 5$$

$$7 = 5 + f_{n-1}$$

$$f_{n-1} = 7 - 5 = 2$$

$$5 = 2 + f_{n-1}$$

$$f_{n-1} = 5 - 2 = 3$$

Therefore the sequence is 3, 2, 5, 7, 12, 19, 31, 50, 81, 131 and so

$$f_1 = 3 \text{ and } f_2 = 2$$

**Question 9 Answer E**

Let  $M_n$  = Account balance at the end of the  $n$ th month.

$$M_0 = 100$$

4.8% per annum = 0.4% per month.

$$M_1 = M_0 + 80 + 0.4\% \text{ of } (M_0 + 80)$$

$$= (1 + 0.4\%)(M_0 + 80)$$

$$= (1 + 0.004)(M_0 + 80)$$

$$= 1.004(M_0 + 80)$$

$$M_2 = 1.004(M_1 + 80)$$

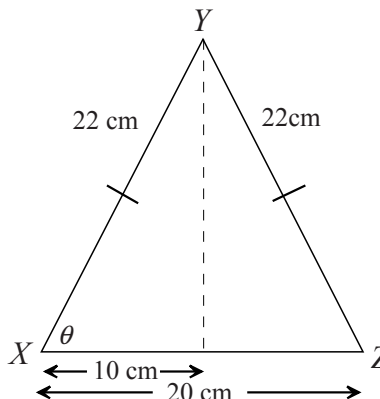
$$\text{In general, } M_{n+1} = 1.004(M_n + 80),$$

$$M_{n+1} = 1.004M_n + 80.32, M_0 = 100$$

**Module 2: Geometry and Trigonometry**

**Question 1 Answer E**

The altitude (perpendicular) from the point  $Y$  to the side  $XZ$  cuts the side  $XZ$  in half because  $XYZ$  is isosceles.



$$\cos(\angle YXZ) = \frac{10}{22} = 0.4545\dots$$

$$\angle YXZ = 62.96^\circ$$

Alternatively the Cosine rule could be used:

$$\cos \theta = \frac{22^2 + 20^2 - 22^2}{2 \times 22 \times 20} = \frac{10}{22}$$

**Question 2 Answer D**

Heron's formula is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \text{ where}$$

$$s = \frac{1}{2}(a+b+c); a, b \text{ and } c \text{ are the lengths of the sides.}$$

$$s = \frac{1}{2}(10 + 13 + 17) = 20$$

$$\text{So Area} = \sqrt{20(20-10)(20-13)(20-17)}$$

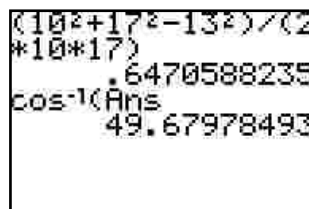
**Question 3 Answer E**

Using the cosine rule:

$$\cos(\angle QPR) = \frac{10^2 + 17^2 - 13^2}{2 \times 10 \times 17} = 0.6471$$

$$\angle QPR = 49.68^\circ$$

Calculator:

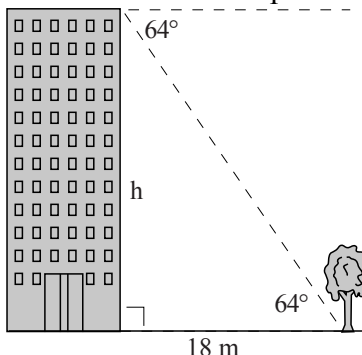


**Question 4 Answer E**

$$\tan 64^\circ = \frac{h}{18}$$

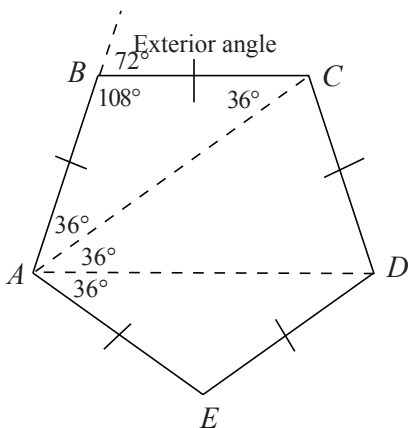
$$h = 18 \tan 64^\circ = 36.91 \text{ m}$$

= 36.9m correct to one dec. place.



**Question 5 Answer B**

The exterior angle of a regular pentagon =  $\frac{360^\circ}{5} = 72^\circ$   
 so the interior angle =  $180^\circ - 72^\circ = 108^\circ$

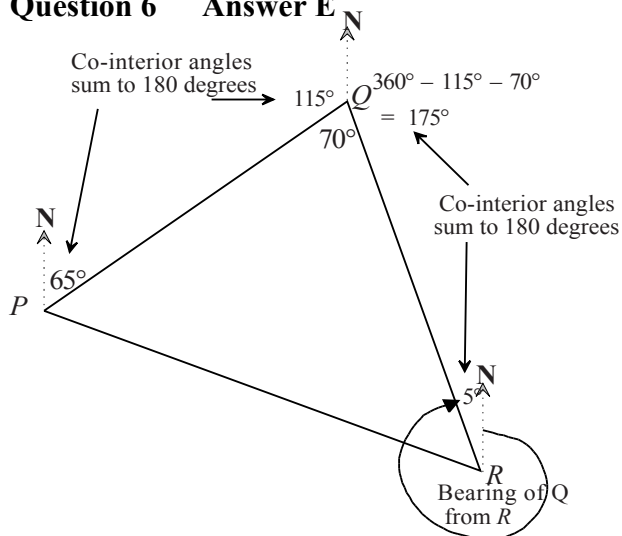


The triangle *ABC* (and *ADE*) is isosceles so the other

angles are  $\frac{72^\circ}{2} = 36^\circ$

This means  $\angle CAD = 36^\circ$

**Question 6 Answer E**



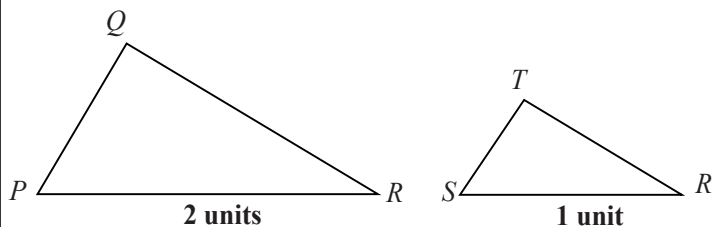
The bearing of point *Q* from point *R* =  $360^\circ - 5^\circ = 355^\circ$

**Question 7 Answer B**

If the contour interval is 50 metres for the graph then point *B* is 200 metres higher than point *A* and the horizontal distance between *B* and *A* is 1.35 km = 1350 m.

$$\text{Average slope} = \frac{\text{rise}}{\text{run}} = \frac{200}{1350} = 0.1481$$

**Question 8 Answer D**

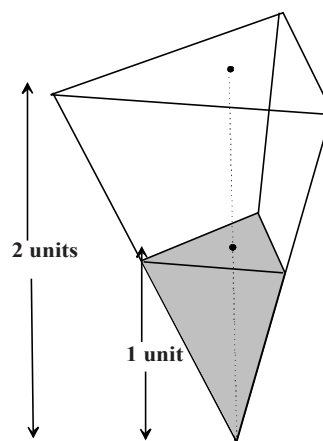


Triangles *PQR* and *TSR* are similar triangles so their corresponding side lengths are in the same ratio.

If their corresponding lengths are in the ratio 2 : 1 then their areas are in the ratio  $2^2 : 1^2 = 4 : 1$

Triangle *STR* (shaded) has area 1 unit and triangle *PQR* has area 4 units but if the shaded area (1 unit) is removed from triangle *PQR* then the unshaded area left is 3 units.

**Question 9 Answer E**



The whole container and the part of the container containing liquid can be considered similar solids.

If their corresponding lengths are in the ratio 2 : 1 then their volumes are in the ratio  $2^3 : 1^3 = 8 : 1$

If the 1 unit of volume is 1250 mL then 8 units are  $1250 \times 8 = 10\,000\text{mL} = 10 \text{ litres}$

### Module 3: Graphs and relations

#### Question 1 Answer A

Speed is the gradient of Distance versus Time graph. The constant speed explains the straightline and the constant gradient.

$$\begin{aligned} \text{speed} &= \frac{\text{distance}}{\text{time}} \\ \text{gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{30 \text{ metres}}{10 \text{ seconds}} \\ &= 3 \text{ metres per second} \end{aligned}$$

#### Question 2 Answer C

The solution is the intersection of the equations. Using the substitution method

$$\begin{aligned} y &= 8 - 2x & [1] \\ y &= 3x + 3 & [2] \\ \text{Substitute [2] into [1] and solve} \\ 3x + 3 &= 8 - 2x \\ 5x &= 5 \\ x &= 1 \\ \text{Substitute } x = 1 &\text{ into [2] and solve.} \\ y &= 3 \times 1 + 3 \\ y &= 6 \end{aligned}$$

The co-ordinates of the point of intersection is (1, 6)

#### Question 3 Answer B

Using the  $x$  and  $y$  intercept method to find the **equation of the line**.

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{7} + \frac{y}{4} = 1$$

Multiply by LCD of 4 and 7.

$$28 \times \frac{x}{7} + 28 \times \frac{y}{4} = 28 \times 1$$

$$4x + 7y = 28$$

The required region is above and thus greater than and therefore  $4x + 7y \geq 28$

Use a test point such as (0,10) which is above the line on the  $y$ -axis.

$$4 \times 0 + 7 \times 10 \geq 28$$

$70 \geq 28$  is true and thus the region is correctly defined.

#### Question 4 Answer D

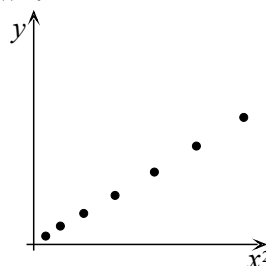
The 1<sup>st</sup> quadrant is the region where the values of  $x$  and  $y$  are positive, that is when  $x \geq 0$  and  $y \geq 0$ .

#### Question 5 Answer A

Setting up the table with the  $x^2$  values

$x$	1	1.5	2	2.5	3	3.5	4
$y$	0.5	1.125	2	3.125	4.5	6.125	8
$x^2$	1	2.25	4	6.25	9	12.25	16

And using the stemplot function of a graphics calculator as shown.



#### Question 6 Answer D

Profit = Revenue – Cost

The revenue function is of the form  $y = mx$  where the gradient is

$$m = \frac{\text{rise}}{\text{run}} = \frac{60}{30} = 2$$

Therefore  $\$R = 2n$ .

$$\$P = \$R - \$C$$

$$= 2n - \left( \frac{2}{3}n + 20 \right)$$

$$= 2n - \frac{2}{3}n - 20$$

$$\$P = \frac{4}{3}n - 20$$

**Question 7 Answer C**

From the shape, the function shown is either

$$y = \frac{k}{x} \text{ or } y = \frac{k}{x^2}$$

Testing the two points given in each of the relationships

$$y = \frac{3200}{x^2} \text{ and } y = \frac{400}{x}$$

$$(8,50)$$

$$y = \frac{3200}{x^2}$$

$$50 = \frac{3200}{8^2}$$

$$50 \times 64 = 3200$$

$$3200 = 3200$$

$$(16,25)$$

$$y = \frac{3200}{x^2}$$

$$50 = \frac{3200}{16^2}$$

$$50 \times 256 = 3200$$

$$12800 = 3200 \text{ False}$$

does not hold true for both points.

$$(8,50)$$

$$y = \frac{400}{x}$$

$$50 = \frac{400}{8}$$

$$50 \times 8 = 400$$

$$400 = 400$$

$$(16,25)$$

$$y = \frac{400}{x}$$

$$25 = \frac{400}{16}$$

$$25 \times 16 = 400$$

$$400 = 400$$

Holds true for both points given.

**Question 8 Answer D**

The feasible region is bounded by 4 points and the value of the objective function is summarized in the table below

Point	Objective function $Z=3x+7y$	Value
(0,0)	$3 \times 0 + 7 \times 0 =$	0
(20,0)	$3 \times 20 + 7 \times 0 =$	60
(10,10)	$3 \times 10 + 7 \times 10 =$	100
(0,15)	$3 \times 0 + 7 \times 15 =$	105

**Question 9 Answer B**

In the description given the key phrases describing the constraints are highlighted as follows

*Creamy Krisps is a donut shop that makes 2 kinds of donuts.*

*Donut X requires 20 grams of icing for each donut, while Donut Y requires 45 grams of icing. There is a total of 20 kilograms (1kg = 1000g) of icing delivered each day.*

$$20x + 45y \leq 20000$$

*The daily expected sales for the two kinds of donuts is at least 3000 in total*

$$x + y \geq 3000$$

*and no more than 4000 of Donut Y.*

Already given as  $0 \leq y \leq 4000$ .

**Module 4: Business-related mathematics**

**Question 1 Answer A**

$$\text{January price} = \frac{(100 - 20)}{100} \times P = 0.8P$$

where P is the original price.

$$\begin{aligned} \text{February price} &= 0.7 \times \text{January price} \\ &= 0.7 \times 0.8P \\ &= 0.56P = 56\% \text{ of } P \end{aligned}$$

The discount is  $100\% - 56\% = 44\%$

**Question 2 Answer B**

Using the perpetuity formula  $Q = \frac{Pr}{100}$

where Q is the perpetuity received each time period (month),

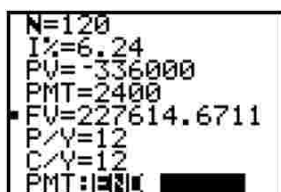
P is the amount invested and r is the interest rate per time period ( $r = 6.24\%/12 = 0.52\%$ )

$$Q = \frac{\$336000 \times 0.52}{100} = \$1747.64$$

**Question 3 Answer E**

The perpetuity amount is the interest earned each month so if Carlo withdraws more than this amount then his invested amount will decrease. (Eliminates answers A and C)

Using the TVM calculator to find the amount left after 10 years and the length of time the investment will last :-



**Question 4 Answer C**

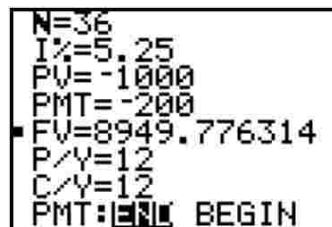
If inflation has been 1.6% compounding over three years then the cost of a ticket will be

$$2.80 \times \left(1 + \frac{1.6}{100}\right)^3 = 2.8 \times 1.016^3 = 2.936 \approx \$2.94$$

**Question 5 Answer D**

Using the TVM facility on the calculator:-

Note: both the PV (present value) and PMT (payment) are investments so are considered *outgoings* and negative.

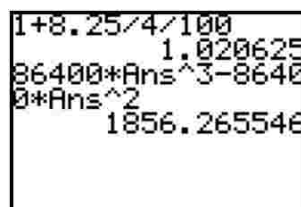


**Question 6 Answer C**

The interest paid in the third month = the value after the third month – the value after the second month =  $PR^3 - PR^2$

where  $P = 86400$  and  $R = 1 + \frac{8.25}{100} = 1.0206\dots$

Using the calculator:



**Question 7 Answer C**

Reducing balance depreciation uses the formula:

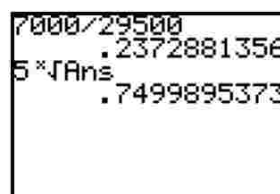
$$\text{Book value} = PR^n$$

where P is the original price and  $R = 1 - \frac{r}{100}$ ; r being the rate of depreciation per year.

Substituting gives

$$7000 = 29500 \times R^5$$

Using the calculator to solve for R:



$$R = 0.74998\dots \approx 0.75$$

Hence  $0.75 = 1 - \frac{r}{100}$  giving  $r = 25\%$

**Question 8 Answer E**

Amount owing after the deposit is paid =  $\$1450 - \$290 = \$1160$

Total repayments are

$$\$115 \times 12 = \$1380$$

$$\text{Interest paid} = \$1380 - \$1160 = \$220$$

As a percentage of the amount owing this is

$$\frac{220}{1160} \times 100 = 18.97\% \approx 19\%$$



**Question 9 Answer A**

This question requires some work with the TVM solver. Generally if repayments are made more frequently then less interest will be paid (and therefore the total amount paid will be less) over the term of the loan; eliminates answer C

Some calculator screens:-

You will need to calculate the fortnightly/monthly payments first.

```

N=26
I%=5.95
PV=156000
PMT=-796.764084
FV=-144232.9917
P/Y=26
C/Y=26
PMT:BEGIN
  
```

```

N=12
I%=5.95
PV=156000
PMT=-1728.0054...
FV=-144228.3521
P/Y=12
C/Y=12
PMT:BEGIN
  
```

The screens above show that using the fortnightly payments they will owe about \$5 more after one year so answer A is the answer that is not true.

The screen below shows that they will pay slightly less in the first year with fortnightly payments.

```

796.764*26
20715.864
1728.005*12
20736.06
  
```

**Module 5: Networks and decision mathematics:****Question 1 Answer C**

A subgraph is part (or all) of the original graph without the addition of any new edges or vertices (This eliminates answers A, B and E). If an edge is included then its connecting vertices must also be included (eliminates answer D).

**Question 2 Answer E**

A planar graph can be drawn with no edges intersecting. The given graph is not

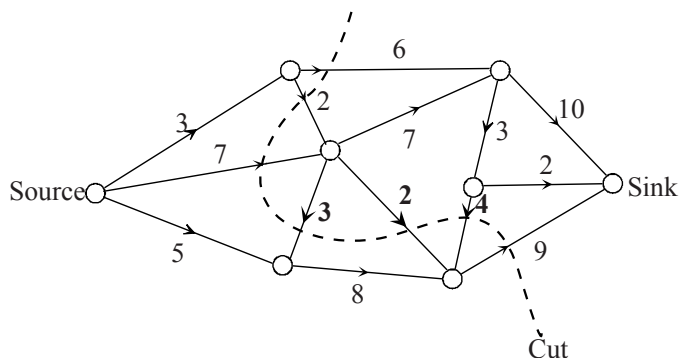
- a directed graph because there are no directional arrows on the edges.
- a complete graph because this would require an edge from every vertex to every other vertex; a total of 21 edges.
- a simple graph because it has multiple edges from C to D.
- a spanning tree because it has circuits.

**Question 3 Answer D**

An additional edge from vertex A to vertex B would mean all vertices, except vertices E and F, are of even degree. The Euler path would start at vertex E and finish at vertex F, or vice-versa.

**Question 4 Answer C**

Only the flow that is in the direction from source to sink is counted in the capacity of a cut.



Capacity of the given cut is

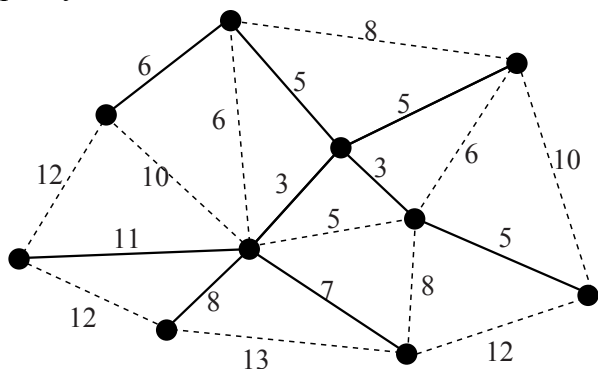
$$6 + 2 + 7 + 9 = 24$$

**Question 5 Answer B**

There is only one edge going to table tennis so Helen must referee table tennis. Answers C, D and E are all possible allocations of the other sports so answer B is the answer that does not follow from the graph.

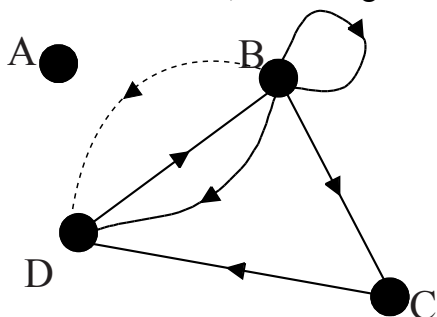
**Question 6 Answer D**

We are looking for a minimum length spanning tree in this case. Start at an edge with minimum weight (3) and connect this to another vertex with an edge of least weight(3). Continue connecting vertices by selecting edges of least weight until all vertices are connected. On a graph with 10 vertices this will require 9 edges. Capacity = 3 + 3 + 5 + 5 + 5 + 6 + 7 + 8 + 11 = 53



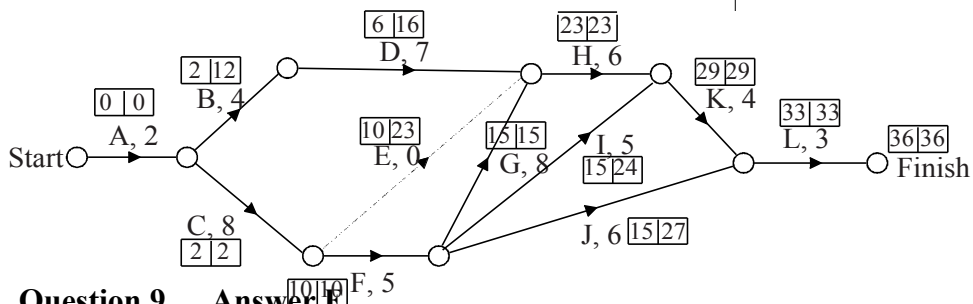
**Question 7 Answer E**

A graph can be easily drawn from the matrix:- Although there is not an edge from vertex C to vertex B there is an edge from C to D then an edge from D to B so B is reachable from C; a two-stage connection.



**Question 8 Answer E**

A forward and backward scan for the project shows the *earliest start times* and the *latest start times* for each of the activities:- The minimum time to complete the whole project is the length of the longest path through the activities; hence 36 days.



**Question 9 Answer E**

The slack time for activity J is the *latest start time* - *earliest start time*.  
 = 27 - 15  
 = 12 days

**Module 6: Matrices**

**Question 1 Answer C**

Element  $a_{2,3}$  is the value found in row 2 and column 3.

$$\begin{matrix} & \text{Column} \\ & \mathbf{3} \\ \text{Row 2} & \begin{bmatrix} 4 & 1 & 1 & 0 \\ -3 & 3 & \mathbf{0} & -1 \\ -2 & 2 & 5 & -4 \end{bmatrix} \end{matrix}$$

$a_{2,3} = 0$

**Question 2 Answer D**

For a product of the matrices to exist the number of columns in the first matrix must be the same as the number of rows in the second matrix.

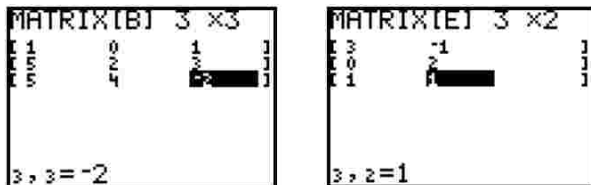
- A.B  $2 \times 2 \times 3 \times 3$  does not exist
- B.C  $3 \times 3 \times 2 \times 4$  does not exist
- D.A  $2 \times 3 \times 2 \times 2$  does not exist
- E.D  $3 \times 2 \times 2 \times 3$  does exist It is a  $3 \times 3$  matrix
- C.E  $2 \times 4 \times 3 \times 2$  does not exist

**Question 3 Answer B**

$BE$  is  $3 \times 2 \times 2 \times 2$  and does exist as  $3 \times 2$

$$B.E = \begin{bmatrix} 1 & 0 & 1 \\ 5 & 2 & 3 \\ 5 & 4 & -2 \end{bmatrix} \times \begin{bmatrix} 3 & -1 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$$

Using a graphics calculator, set up the two matrices as shown



and compute to determine the product

$$B.E = \begin{bmatrix} 4 & 0 \\ 18 & 2 \\ 13 & 1 \end{bmatrix}$$

**Question 4 Answer B**

Firstly, adding the matrices on the right gives:

$$\begin{bmatrix} -3 & -9+d \\ c+2 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix}$$

Equating the matrices:

For  $c$

$$c + 2 = 0$$

$$c = 0 - 2$$

$$c = -2$$

For  $d$

$$-9 + d = 2$$

$$d = 2 + 9$$

$$d = 11$$

**Question 5 Answer C**

For a singular matrix the determinant ( $ad - bc$ ) is equal to 0.

A  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$   $1 \times 1 - 0 \times 0 = 1 - 0 = 1$

B  $\begin{bmatrix} 3 & 6 \\ 4 & 3 \end{bmatrix}$   $3 \times 3 - 6 \times 4 = 9 - 24 = -15$

C  $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$   $1 \times 1 - (-1 \times -1) = 1 - 1 = 0$

D  $\begin{bmatrix} 5 & 8 \\ 6 & 2 \end{bmatrix}$   $5 \times 2 - 8 \times 6 = 10 - 48 = -38$

E  $\begin{bmatrix} 6 & -8 \\ 3 & 4 \end{bmatrix}$   $6 \times 4 - (-8 \times 3) = 24 - (-24) = 48$

**Question 6 Answer A**

Transpose the equations so they are in the form of  $ax + by = c$

$$2x + 1y = 1$$

$$-3x + 1y = 2$$

Now set-up the co-efficients of the  $x$  and  $y$  terms in a matrix and complete the equation as shown.

$$\begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

**Question 7 Answer E**

Using a graphics calculator to setup the matrix  $S$  (use matrix  $A$ ) as shown



$3, 3 = 6$

Use the MATRIX MATH function



to calculate the determinant of a  $3 \times 3$ .

$$\begin{bmatrix} 2 & 4 & 5 \\ 1 & 7 & 3 \\ 4 & 8 & 6 \end{bmatrix}$$

$$\det(A) = -40$$

**Question 8 Answer A**

Identify there are two states the airport in Melbourne and Gold Coast. The transition matrix convention to be adopted is the FROM state will be in the columns. Enter given percentage probabilities in decimal form. Check that the columns add up to 1.

FROM	
Melb	Gold Coast
TO Melb	$\begin{bmatrix} 0.7 & 0.4 \end{bmatrix}$
Gold C	$\begin{bmatrix} 0.3 & 0.6 \end{bmatrix}$

**Question 9 Answer D**

Identify for a long-term steady state  $n$  needs to be large say  $n = 60$ .

Evaluate on a graphic calculator as follows

```

[A]      [[.45 .4]      [A]^60*[B]
          [.55 .6]]    [[46315.78947]
[B]      [[58000]      [63684.21053]]
          [52000]]
    
```

Round-off the value to the nearest thousand.

```

[46000]
[64000]
    
```