The Mathematical Association of Victoria
FURTHER MATHEMATICS

Trial written examination 2
(Extended Answer)

2006

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: ....

QUESTION AND ANSWER BOOK

Structure of book

Core		
Number of questions	*Number of questions to be answered*	*Number of marks*
3	3	15
Module		
Number of modules	*Number of modules to be answered*	*Number of marks*
6	3	45
		Total 60

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2006 Further Mathematics Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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Working Space

Instructions

This examination consists of a core and six modules. Students should answer **all** the questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, e, surds or fractions.

Page

Core

Question 1

The medals tally table for the 2006 Melbourne Commonwealth Games are summarised below.

Country	Gold	Silver	Bronze	Total
Australia	84	69	68	221
England	36	40	34	110
Canada	26	29	31	86
India	22	17	11	50
South Africa	12	13	13	38
Scotland	11	7	11	29
Jamaica	10	4	8	22
Malaysia	7	12	10	29
New Zealand	6	12	13	31
Kenya	6	5	7	18
Others combined	25	36	48	109

a. Place the two variables **country** and **number of medals** in the correct row/column position for the dependent/independent variables of a two-way frequency table for the data presented in the above table.

	VARIABLE ____________________	Totals
VARIABLE _________________________		
Totals		

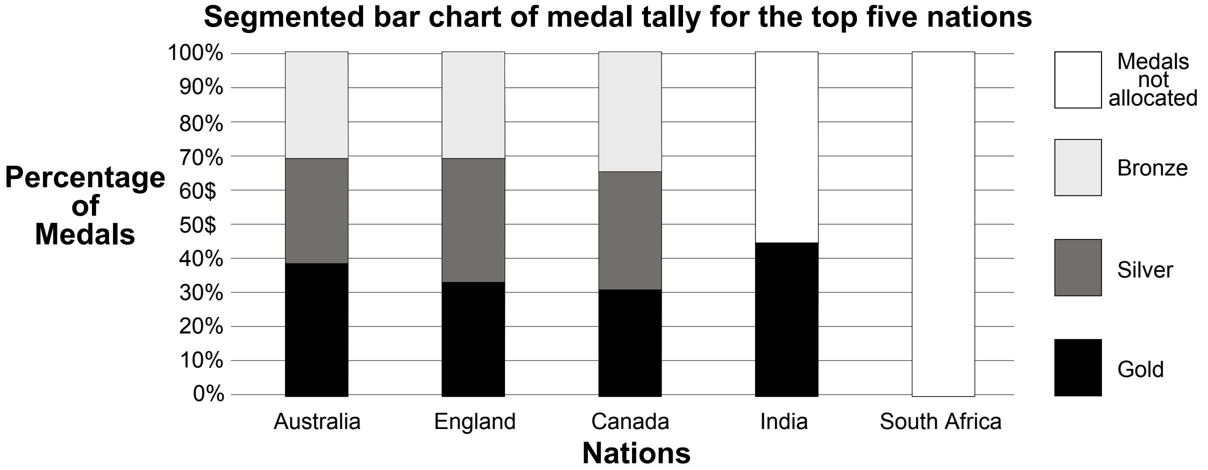
1 mark

b. From the following list, circle **two** suitable statistical tools that could be used to display the number of **gold medals** won by each of the **countries**.

scatterplot	stemplots
histogram	dotplots
barchart	segmented barchart
boxplots	parallel boxplots
back-to-back stemplots	pictograms

1 mark

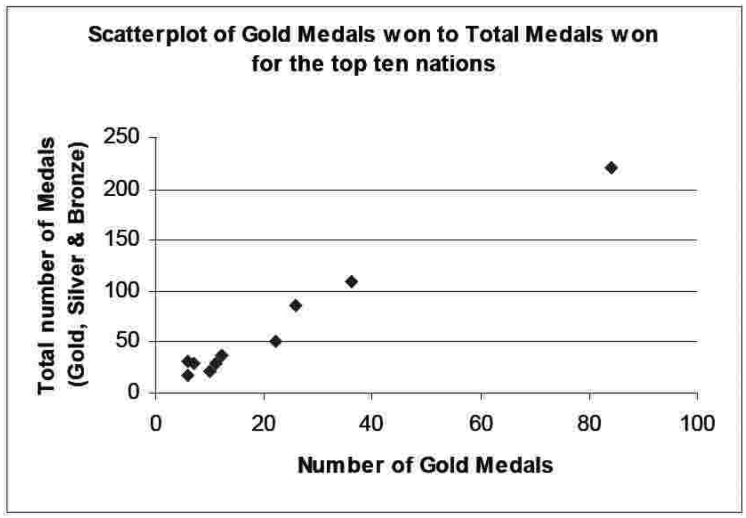
c. The following is a segmented barchart for the top five nations. Complete the segments for India (Silver & Bronze) and South Africa (Gold, Silver & Bronze).



2 marks

Question 2

The top ten nations won 83% of all the medals won at 2006 Melbourne Games. A scatterplot of the top ten nations to investigate the relationship between Gold Medals and Total Medals won is given below.



For the following questions 2a, 2b and 2c, use only the calculated statistical values for the above scatterplot.

Pearson's product-moment correlation coefficient	0.99
mean number of Gold Medals won	22.0
standard deviation of number of Gold Medals won	23.9
mean number of Total Medals won	63.4
standard deviation of number of Total Medals won	62.8

a. Complete the following statement that can be made about the relationship between gold medals and total medals won for the top ten nations.

"We can conclude from this that ☐% of the variation in the total number of medals can be explained by the variation in ☐. The other ☐% variation in total number of medals is due to other factors".

2 marks

b. Using the data provided in the table above, calculate the approximate values of coefficients a and b in the least-squares regression line. Give answers to 2 decimal places.

Number of Total Medals = $a + b$ x number of gold medals .

2 marks

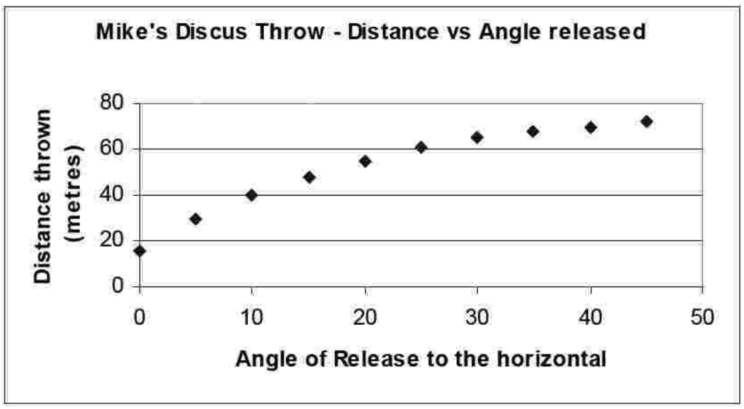
c. i Calculate the z-score for Australia who won 84 Gold Medals.

ii The z-score for England who won 110 Medals in Total was 0.75. Explain what a z -score of +1 represents.

1 + 1 = 2 marks

Question 3

Mike, the discus thrower, has collected data on the distances he has thrown depending on the angle from the horizontal the discus is released. Mike only measured for angles up to 45º, which is the optimum angle for achieving his personal best of 72 metres.



A linear model is applied to the data and a residual analysis is to be performed

a. Sketch the general shape of the residual plot for the above scatterplot

1 mark

The following analysis of three possible transformations and their accompanying coefficient of determinations (r^2 values) are listed below.

Type of Transformation	r^2 value
No transformation	0.958
Reciprocal of Angle of release	0.857
Reciprocal of Distance thrown	0.784
Log of Angle of release	0.996

b. Which transformation is suggested to be the most appropriate and explain your choice.

____

____

1 mark

c. What other transformation not listed above would also be considered most suitable. Give a reason.

____

____

1 mark

The final regression equation decided on was

$$\text{Distance (metres)} = 46.41 \times \log(\text{Angle}°) - 4.55$$

d. Use this equation to complete the following table.

Angle of release (º)	Distance Thrown (metres)
23.0	58.6
37.0	
	71.0

2 marks

Total 15 marks

END OF CORE

Module 1: Number patterns and applications

Question 1

It is expected that the number of people attending the morning athletics program at the MCG during the 2006 Melbourne Commonwealth Games will increase by 2,600 each day.

a. How many people would be expected to watch the athletics on the third morning of the games if 64 400 attended on the first morning?

1 mark

The athletics morning and evening program is to be run for **seven days**.

b. Calculate the expected total attendance for the **morning** athletics program (to the nearest thousand).

2 marks

For the evening sessions of the athletics, the number of people attending followed the following pattern

Evening	1st	2nd	3rd
Attendance	64 400	67 620	71 001

c. i) Show that the pattern shown in the above table is a geometric pattern.

ii) For the above written as a difference equation in the form

$A_{n+1} = aA_n + b, \quad A_o = c$

State the value of a, b and c

a =	b =	c =

1 + 2 = 3 marks

The capacity of the MCG for the games was set at 88 000 people.

d. i) Show that the expected attendance on the **last evening** session does not exceed the 88 000 capacity of the MCG.

ii) Find the total expected attendance for the **seven evenings** (to the nearest thousand).

1 + 2 = 3 marks

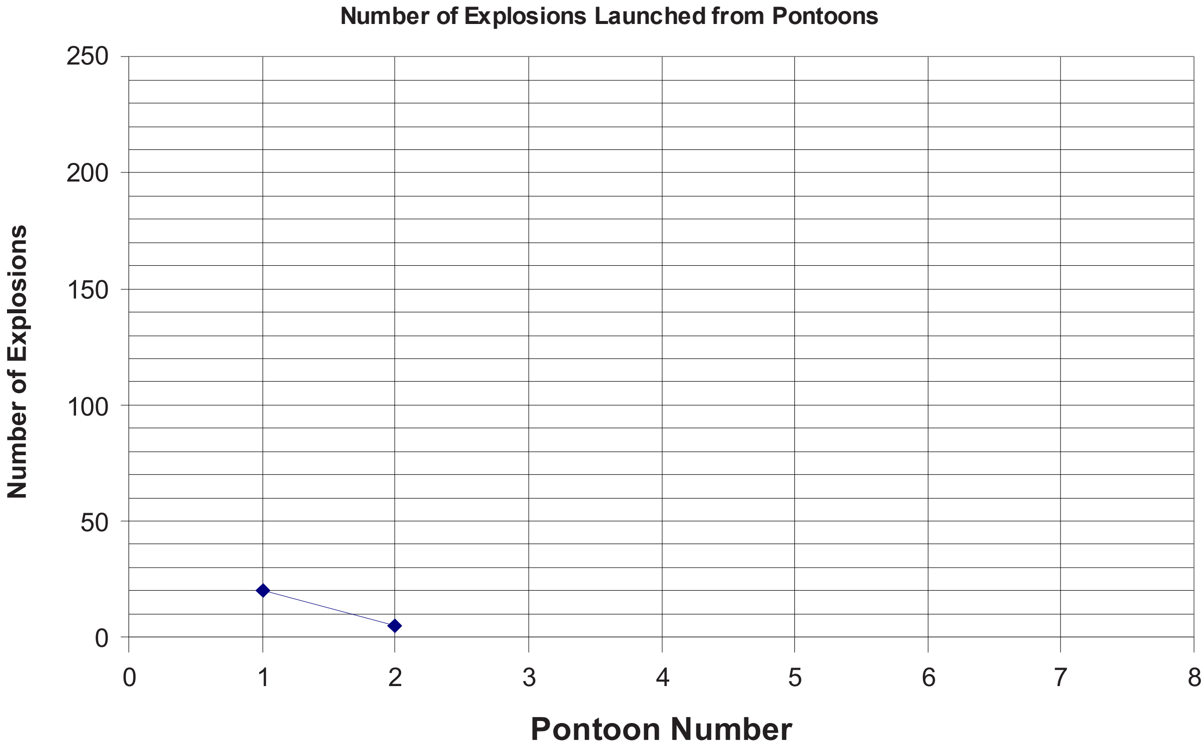
Question 2

The closing ceremony fireworks is choreographed so that the fish pontoons on the river are used to launch the explosions starting with the first pontoon and moving along to the next pontoon along the river.

The number of individual explosions from each pontoon increases and is given by the sequence

$$f_{n+2} = f_n + f_{n+1}, \qquad f_1 = 20, \ f_2 = 5$$

Plot the pattern on the graph below for pontoons 3 to 8



2 marks

Question 3

During the 2006 Melbourne Games, a small pistol shooting sporting club is formed and starts with 10 members and expects to increase its membership by 8 each year. The membership fees started at $88 per year and are expected to increase by $11 each year.

a) How much money is collected over 8 years?

2 marks

b) What percentage of the total is collected in the last 2 years?

2 marks

Total 15 marks

END OF MODULE 1

Module 2 : Geometry and trigonometry

A garden in the shape of a regular octagon, with side length 6 metres, is surrounded by a concrete path, as illustrated in Diagram below:

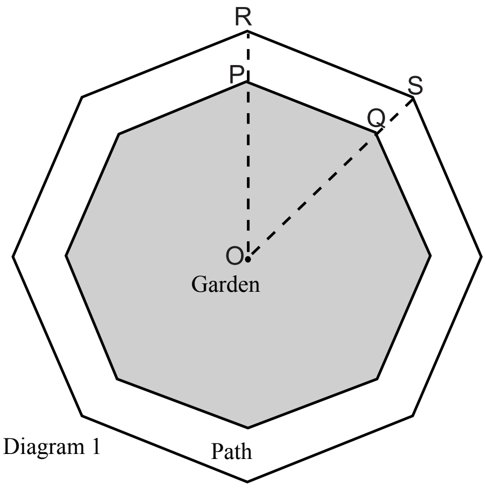


Diagram 1

Question 1

a. Show that the angle *POQ* in triangle *OPQ* has a magnitude of 45°

1 mark

b. Calculate the magnitude of the angle *OPQ* in triangle *OPQ*

1 mark

Part of the garden and path is shown in the diagram below:

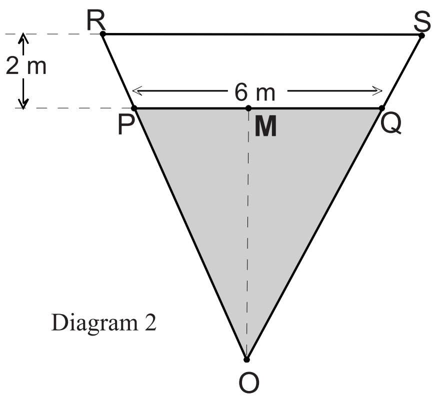


Diagram 2

c. Calculate the length of OM, where M is the midpoint of PQ, giving your answer in metres correct to two decimal places.

2 marks

d. If the width of the path is 2 metres, as shown in **Diagram 2**, use similar triangles, or otherwise, to calculate the length of RS. Give your answer in metres correct to two decimal places.

3 marks

e. Use your previous calculations, or otherwise, to calculate the area of the trapezium $PRSQ$.

Give your answer in square metres correct to two decimal places.

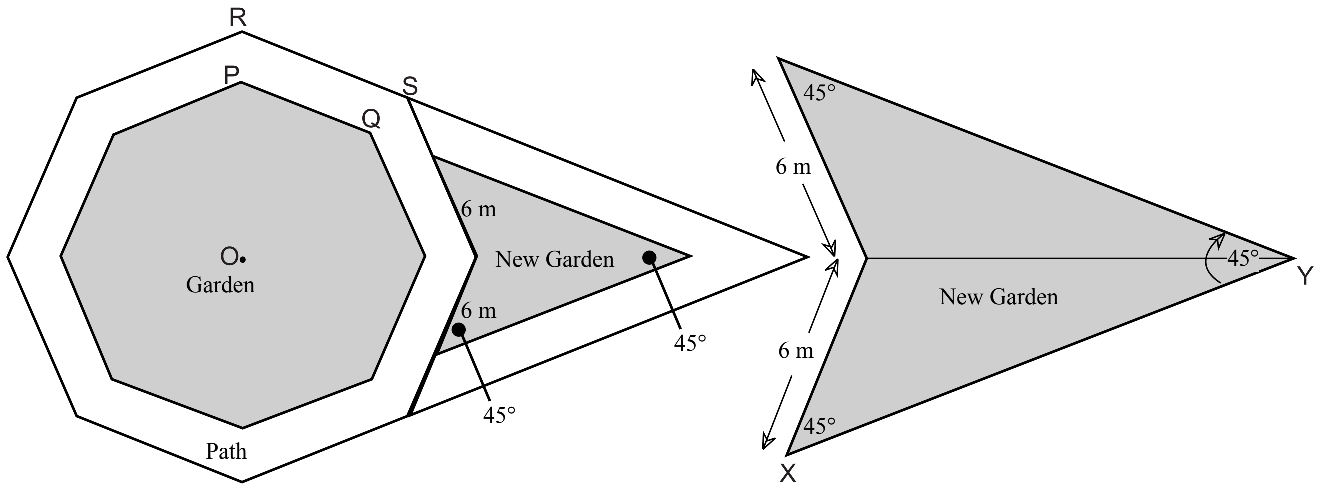
2 marks

f. The path is to be made of concrete to a depth of 15 centimetres. Calculate the volume of concrete needed to lay the path on all eight sides of the garden. Give your answer in cubic metres correct to one decimal place.

2 marks

Question 2

An additional new garden is to be created as shown on the diagram below.



a. Calculate the length of the boundary XY of the new garden. Give your answer in metres correct to two decimal places.

2 marks

b. Calculate the area of the new garden. Give your answer in square metres correct to two decimal places.

2 marks

Total 15 marks

END OF MODULE 2

WORKING SPACE

Module 3: Graphs and relations

John is an entrepreneur who is constantly monitoring and planning for his financial future.

Question 1

John regularly visits a financial adviser who charges the following fee:

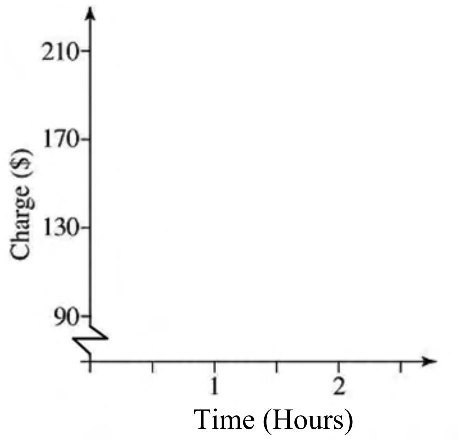
$50 appointment fee

$40 per half hour or part thereof

a. Calculate the total cost for his past two appointments that went for 46 minutes and 1 hour 22 minutes.

2 marks

b. Using the information of the financial adviser's fee schedule, draw the graph of charge (C) against time (t).



2 marks

Question 2

John sells insurance. He is paid \$480 per week plus \$320 for each insurance policy he sells.

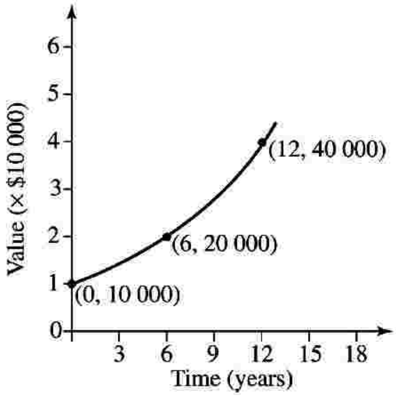
a. How much does John receive if he sells 4 policies in a week?

1 mark

b. Write a rule that relates his weekly pay (P) and the number of policies (n) he sells.

1 mark

John has been investigating the benefits of investing and the importance of time. Whilst reading a book on investing, he read the following extract: *A share portfolio increases in value over time. The graph below shows the growth in value of the shares. Initial amount invested was \$10 000*



Question 3

a. How long does it take the value of the shares to double in value?

1 mark

b. Assuming that the trend in Question 3a continues, predict the value of the shares 24 years after their purchase

1 mark

Question 4

John has a maximum of $1 000 000 to invest. He can purchase either bonds or invest in shares. However, the financial adviser, being conservative, insists that the amount John invests in bonds be **at least** twice as much as that invested in shares.

Based on past experience, the financial adviser knows that bonds will return 6% profit, while shares will return 11%.

a. State the other decision variables where

First decision variable: x = the amount invested in bonds

Second decision variable: y = ☐

1 mark

b. Complete the constraints

$x + y \leq$ ☐

$x \geq$ ☐ y

$x \geq 0$

$y \geq 0$

2 marks

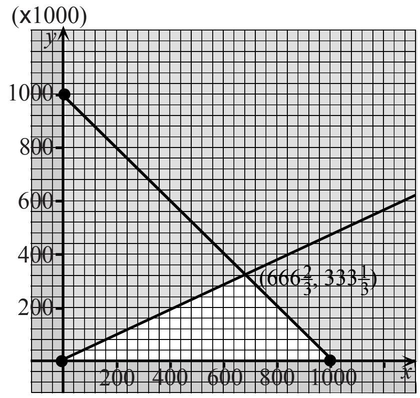
c. John wants to maximise his profit from the investment.

State the objective function that needs to be maximised.

$Z =$ ☐ $x +$ ☐ y

1 mark

The constraints were graphed and are shown below as the unshaded region.



d. List all the co-ordinates that need to be considered and thus determine the optimal "mix" of investments.

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3 marks

Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics

Jesse is starting a three year course at a university and, as he will be living away from home, he has organized accommodation. To help pay for this accommodation he has considered using a perpetuity amount from his inheritance or a student loan from a bank.

Question 1

Jesse has to pay $98 per week for his accommodation for the 44 weeks in first year, paid in 10 monthly payments.

a. How much, to the nearest cent, will Jesse pay per month for his accommodation?

1 mark

b. The cost of Jesse's accommodation increases with inflation and in his second year it increases by 2.3%
Calculate the cost per week, to the nearest cent, of Jesse's accommodation in his second year?

1 mark

c. In his third year Jesse pays $103.06 per week for his accommodation. Calculate, correct to one decimal place, the inflation increase in his third year.

2 marks

Question 2

Jesse has an inheritance that has amounted to $108,000 by the time he starts university. His inheritance is earning 5.8% p.a. interest, compounding monthly.

Jesse considered using this inheritance to provide a monthly perpetuity to help with his accommodation while he was at university.

Calculate the perpetuity amount that Jesse would receive each month.

2 marks

Jesse decides that he does not want to use his inheritance money yet, so he has applied for and gained approval for a student loan to help pay for his accommodation. The loan starts when the first withdrawal is made and amounts borrowed are charged 7.2% p.a. interest, compounding monthly.

He does not have to repay any amount until he has finished his course.

Question 3

Jesse is going to withdraw an initial amount of $1000 and then withdraw $520 at the end of the first and every month thereafter.

a. Determine the amount, to the nearest dollar, that Jesse will owe the lender after one year.

1 mark

b. There is a limit of $20 000 on the amount that Jesse can borrow. After how many months will he exceed this limit if he makes an initial withdrawal of $1000 and continues to withdraw $520 per month ?

1 mark

c. Jesse withdraws the initial $1000 then $520 per month for a total of 32 months until his course is finished. Determine the total amount owing, to the nearest dollar, after this time.

1 mark

d. Determine the amount of interest, to the nearest dollar, that Jesse has accumulated on this loan over the 32 months.

1 mark

Question 4

Jesse plans to start repaying his student loan three months after his course finishes. The borrowing rate continues to be 7.2% p.a., compounding monthly.

a. Assuming that he owes \$19 500 when his course finishes, fill in the figure below (R value) so that the calculation will determine the amount owing when he starts to make repayments three months later.

$$19500 \times \boxed{}^{3} = 19853.11$$

2 marks

b. If Jesse repays \$19 853.11, over three years in monthly repayments, determine, to the nearest cent, the amount he will pay each month.

1 mark

c. If Jesse decides to repay this loan in fortnightly repayments (26 per year and interest is calculated fortnightly) rather than monthly repayments, over three years, how much, to the nearest dollar, in interest will he save?

2 marks

Total 15 marks

END OF MODULE 4

Module 5 : Networks and decision mathematics

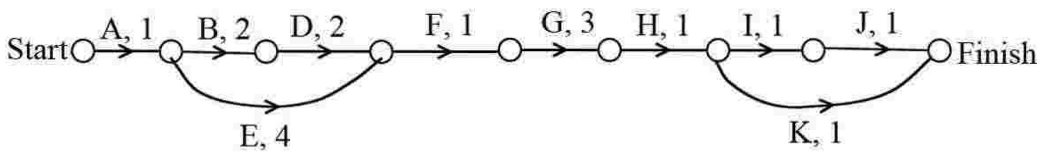
Jim is a volunteer worker for a sporting club and he has noticed that the verandah on the clubrooms is rotting and needs replacing.

Jim proposes to organize the replacement of the verandah and to do most of the work himself.

He has listed the tasks involved and the time, in days, that he estimates each task will take:

Activity	*Description*	*Immediate predecessor(s)*	*Duration of activity (days)*
A	Plan and measure	**-**	**1**
B	Remove old verandah	**A**	**2**
C	Order timber and finishing products	**A**	**8**
D	Check/repair foundations	**B**	**2**
E	Organise tools required	**A**	**4**
F	Cut timber	**C, D, E**	**1**
G	Construct new verandah	**F**	**3**
H	Sand verandah	**G**	**1**
I	Apply first finishing coat	**H**	**1**
J	Apply second finishing coat	**I**	**1**
K	Remove rubbish	**G**	**1**

Jim has also constructed a network diagram for the project:



a. Activity C is missing from the network diagram. Draw in activity C on the diagram above.

1 mark

b. State the tasks that cannot be delayed without delaying the whole project.

2 marks

c. State the earliest completion time of the project.

1 mark

d. State the slack time for activity E

1 mark

e. State the latest finish time for activity B.

1 mark

As part of their pre-season training the club arranges an inter-club round-robin tournament with mixed-ability teams. The club has enough members for five teams.

Question 2

a. If each team is to play every other team once only then how many games will be played in the tournament?

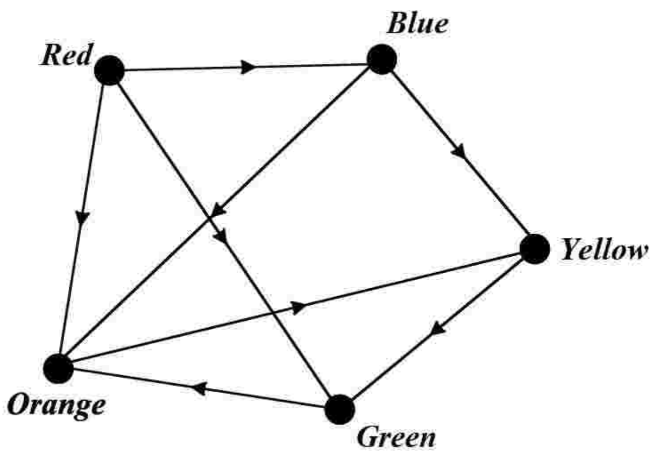
1 mark

b. The teams are named *Red, Blue, Yellow, Green* and *Orange*. The results of the games (there are no draws) are given below:

Red defeated *Green*, *Blue* and *Orange*
Blue defeated *Yellow*, *Green* and *Orange*
Yellow defeated *Green* and *Red*
Green defeated *Orange*
Orange defeated *Yellow*

A network graph has been partially constructed below to represent the results of the tournament.

Complete this graph.



2 marks

c. A dominance matrix, M, for the results of the tournament is partially constructed below; there are two elements missing.

If the element '1' in the matrix means 'defeated', and the element '0' means 'did not defeat', fill-in the two missing elements.

$$\begin{array}{r} \\ \text{Red} \\ \text{Blue} \\ \text{Yellow} \\ \text{Green} \\ \text{Orange} \end{array}
\begin{array}{c} \begin{array}{ccccc} R & B & Y & G & O \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & .... & 1 \\ 1 & .... & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

2 marks

The matrix M^2 is given below:

$$M^2 = \begin{bmatrix} 0 & 0 & 2 & 1 & 2 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

d. Explain the meaning of the element '1' in the second row, column one, giving the teams and results of games that are involved in establishing this element in M^2

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___

___

___

2 marks

e. Determine and write down

i. the matrix $M + M^2$

and hence

ii. the dominance vector associated with the matrix $M + M^2$

1 + 1 = 2 marks

f. Rank the teams in the order first to last for the tournament.

1 mark

Total 15 marks

END OF MODULE 5

TURN OVER

Module 6: Matrices

Question 1

a. List the appropriate details for each of the three matrices given by completing the following table.

Matrix	**Order of the Matrix** $m \times n$	Name the element that has the value of 1 e.g. $a_{2,3}$
$A = \begin{bmatrix} 2 & 0 & 3 \\ 5 & -2 & 1 \end{bmatrix}$		
$B = \begin{bmatrix} -4 & -1 \\ 2 & 1 \\ 3 & 0 \end{bmatrix}$		
$C = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$		

2 marks

b. Complete the following table for the products that exist, stating the order of matrix. If the product does not exist write **does not exist**.

Possible Products	*Order of Matrix*
$A.B$	
$A.C$	
$B.A$	
$B.C$	
$C.A$	
$C.B$	

1 mark

c. Calculate the product $A.B$

$$\begin{bmatrix} 2 & 0 & 3 \\ 5 & -2 & 1 \end{bmatrix} \times \begin{bmatrix} -4 & -1 \\ 2 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

1 mark

Question 2

A student studying arithmetic sequences and series used the following information:

"The sum of first 15 terms is –195 and the sum of the first 20 terms is –410".

To find the first term, *a,* and the common difference, *d,* he used known formulae to set up the following two simultaneous equations.

$$-195 = 15a + 105d$$
$$-410 = 20a + 190d$$

a. The following is the matrix form of the simultaneous equations. Complete the elements of the two blank matrices.

$$\begin{bmatrix} 15 & 105 \\ 20 & 190 \end{bmatrix} \begin{bmatrix} \quad \\ \quad \end{bmatrix} = \begin{bmatrix} \qquad \\ \qquad \end{bmatrix}$$

1 mark

b. Calculate the determinant of the above **square** matrix. Show your workings.

---

1 mark

c. Solve for the value of *a* and *d*.

---

2 marks

Question 3

A large inner Melbourne health gym offers its members 12-month and 3-month membership plans.
A review of their list of memberships for the past five years indicates the following trends:
In general, from one year to the next year

- 60% of 12-month members remain as 12-month members the following year
- 15% of 12-month members change to 3-month memberships the following year
- 25% of 12-month members leave the club (become non-members) the following year
- 50% of 3-month members remain as 3-month members the following year
- 30% of 3-month members change to 12-month memberships the following year
- 20% of 3-month members leave the club (become non-members) the following year

To maintain the level of membership the gym management uses their reception personnel to do telemarketing where from past experience of all the people marketed

- 98% of people do not take up the offer to join.
- 1% of people do take up the offer to join on a 12-month membership plan.
- 1% of people do take up the offer to join on a 3-month membership plan

a. Enter this information (converting percentages to decimals) into the transition matrix, *T,* as indicated below.

$$\begin{array}{cc} & \textit{Current Year} \\ & \begin{array}{ccc} 12 & 3 & Non \end{array} \\ T = & \begin{bmatrix} & & \\ & & \\ & & 0.98 \end{bmatrix} \begin{array}{l} 12 \\ 3 \quad \textit{Next Year} \\ Non \end{array} \end{array}$$

2 marks

In 2006, there were 300 members in total comprising of 200 with 12-month memberships and 100 with 3-month memberships. The number of people telemarketed by the reception personnel has in the past been varied from **none** to 4000 people in a year. For 2006 they're planning to do **no** telemarketing.

b. Write this information in the form of a column matrix as indicated below.

1 mark

$$M_{2006} = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \begin{array}{l} 12-month \\ 3-month \\ People\ Telemarketed \end{array}$$

c. **i.** Use the transition matrix, T, and current membership matrix, M_{2006} to write a matrix product for M_{2007} that can be used to determine the expected membership mix in 2007.

ii. Complete the matrix multiplication to determine M_{2007}.

1 + 1 = 2 marks

To avoid a reduction in membership the marketing manager proposes that 3500 people be telemarketed each year to maintain their membership levels.

d. Determine the long term membership numbers if 3500 people are telemarketed. Give your answer in total number of members.

2 marks

Total 15 marks

END OF QUESTION BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

April 2006

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET