Trial Examination 2006

VCE Further Mathematics Units 3 & 4

Written Examination 1

Suggested Solutions

SECTION A – DATA ANALYSIS – CORE MATERIAL

Question 1

A. Incorrect (lowest value).

B. Incorrect (second lowest value).

C. Correct. Place percentage returns in order from lowest to highest, as shown below.

9.8% 11.2% 12.6% 15.3% 17.5% 20.3% 25.9%

15.3% is the exact middle value.

D. Incorrect (second highest value).

E. Incorrect (highest value).

Answer C

Question 2

A. Incorrect (lowest value).

B. Incorrect because it is the difference between 20.3 (not the highest value) and 9.8 (the lowest value).

C. Incorrect because it is the difference between 25.9 (the highest value) and 11.2 (not the lowest value).

D. Correct.
Range = highest value – lowest value

$$= 25.9 - 9.8$$
$$= 16.1$$

E. Incorrect (highest value).

Answer D

Question 3

A. Correct. For normal distribution approximately 95% of the results are within two standard deviations from the mean.

$$\text{i.e.} \quad 46 \xleftarrow{-2\sigma} 68 \xrightarrow{+2\sigma} 90$$

We have gone 22 either side of the mean (or 2σ).

$$\therefore \quad \sigma = \frac{22}{2}$$
$$= 11$$

B. Incorrect (lower end of 95% range).

C. Incorrect (mean).

D. Incorrect (upper end of 95% range).

E. Incorrect (refers to the percentage of students who score between 46 and 90).

Answer A

Question 4

A. Incorrect (refers to the percentage of individuals who have not owned a car during the 10-year period in question).

B. Incorrect (refers to the percentage of individuals who have owned less than two cars during the 10-year period in question).

C. Incorrect (refers to the percentage of individuals who have owned exactly two cars during the 10-year period in question).

D. Correct. 34 people owned two or more cars (28 + 5 + 1).

$$\therefore \quad \frac{34}{50} \times 100 = 68\%$$

E. Incorrect (refers to the percentage of individuals who have owned less than or equal to two cars during the 10-year period).

Answer D

Question 5

A. Incorrect (assumes 50 cars collectively; there are actually 88 cars involved).

B. Incorrect (this average relies on 57.5 cars collectively which is impossible).

C. Incorrect (assumes 69 cars collectively).

D. Correct.

$$\begin{aligned}\bar{x}(\text{mean}) &= \frac{(3 \times 0) + (13 \times 1) + (28 \times 2) + (5 \times 3) + (1 \times 4)}{50} \\ &= \frac{88}{50} \\ &= 1.76\end{aligned}$$

E. Incorrect (assumes 94 cars collectively).

Answer D

Question 6

A. Incorrect (observed class average for a class size of 21).

B. Correct. By substitution (letting class size = 20)

$$\begin{aligned}\text{class average} &= 109.35 - 1.827 \times 20 \\ &= 72.81\% \\ &= 73\% \text{ (to the nearest whole number)}\end{aligned}$$

C. Incorrect (observed class average for a class size of 19).

D. Incorrect because it is the estimated class average for a class size of 18.

E. Incorrect (observed class average for a class size of 12).

Note: The students should know that they are expected to use the equation supplied to estimate or predict the class average when we have a class size of 20. If this is done correctly the answer is a class average of 73% for a class size of 20.

Answer B

Question 7

A. Correct. From the equation it can be deduced that as class size increases, the class average decreases. A graphics calculator could be used to plot the regression line.

B. Incorrect (the information supplied shows that a smaller class size tends to lead to a higher class average).

C. Incorrect (the information supplied shows that a larger class size tends to lead to a lower class average).

D. Incorrect (the information supplied shows that the class average is affected by class size).

E. Incorrect (once again the information supplied shows that the class average is affected by class size).

Answer A

Question 8

A. Correct.

$$\begin{aligned} r^2 &= 0.7471 \\ \therefore \quad r &\cong -0.8644 \\ &\cong -0.86 \end{aligned}$$

(The solution is negative because of negative correlation (see **Question 7**) and –0.86 is correct to two decimal places).

B. Incorrect (wrong correlation co-efficient).

C. Incorrect (wrong correlation co-efficient).

D. Incorrect (wrong correlation co-efficient).

E. Incorrect (the correlation is -0.86 i.e. a negative correlation. The data shows that as one variable increases the other decreases).

Answer A

Question 9

A. Correct. The range and interquartile range can be seen to be decreasing as the age of drivers increases. i.e. there are less accidents per year as the age of drivers increases, as well as less variation in the number of accidents per year as the age of drivers increases.
$\therefore$ the number of accidents decreases on average and becomes less variable.

B. Incorrect (as the age of a driver increases the number of accidents per year decreases on average but is also less variable).

C. Incorrect (as the age of a driver increases the number of accidents per year does change and is also less variable).

D. Incorrect (as the age of a driver increases the number of accidents per year decreases on average).

E. Incorrect (as the age of a driver increases the number of accidents per year decreases on average and is also less variable).

Answer A

Question 10

A. Incorrect (the relationship between the two variables does not resemble a hyperbolic function).

B. Incorrect (the relationship between the two variables does not resemble a hyperbolic function).

C. Correct. From the scatterplot there seems to be a non-linear or quadratic relationship between x and y.
$\therefore$ an x^2 transformation is most likely to linearise the data.

D. Incorrect (the relationship between the two variables does not resemble a logarithmic function).

E. Incorrect (the relationship between the two variables does not resemble a logarithmic function).

Answer C

Question 11

A. Incorrect (the data shows that sales are increasing over time whereas this equation shows decreasing sales over time).

B. Incorrect (this is the least squares regression equation).

C. Incorrect (once again the data shows that sales are increasing over time whereas this equation shows decreasing sales over time).

D. Correct. Using the graphics calculator, the equation should be

sales = 6.25 × month + 20.625

OR

sales = 20.625 + 6.25 × month

E. Incorrect (this is a possible equation for the line of best fit by eye).

Note: the students should use their graphics calculator to find the 3-Median regression equation. If this is done correctly then the matching answer is simply option **D**. *They also need to understand that the variable x on their graphics calculator refers to the number of months and that the variable y refers to the sales figures (in thousands).*

Answer D

Question 12

A. Incorrect (this answer states that there were only 78 people at the round 1 clash during 1995).

B. Incorrect (this answer states that there were only 79 people at the round 1 clash during 1995).

C. Incorrect (this answer states that there were only 80 people at the round 1 clash during 1995).

D. Correct.

Year	Attendance (x1000)	
1990	78	
1991	72	
1992	66	
1993	75	
1994	83	80 (1994 to 1995)
1995	86	$\longrightarrow$ 79 (1995)
1996	76	78 (1995 to 1996)
1997	67	
1998	74	
1999	90	
2000	79	
2001	84	

The student only needs to calculate the 1994 to 1995 and the 1995 to 1996 four-point moving averages and then centre these two figures (by finding the average of the two results) to give the smoothed value of the attendance in 1995.

$\therefore$ Answer = 79 000

E. Incorrect (figure too high).

Answer D

Question 13

A. Incorrect (represents a way of finding a linear trend line).

B. Incorrect (represents a way of finding a linear trend line).

C. Incorrect (represents a way of finding a linear trend line).

D. Correct. De-seasonalisation is used to attempt to take out the “seasonal” effects rather than to find a linear or straight line trend.

E. Incorrect (represents a way of finding a linear trend line).

Answer D

SECTION B – MODULES

Module 1: Number patterns

Question 1

$t_1 = 8$

$t_3 = -2$

$d = \frac{-2-8}{2} = -5$

$$\begin{aligned} S_4 &= \frac{4}{2}[2(8) + (4-1)(-5)] \\ &= 2(16-15) \\ &= 2 \end{aligned}$$

Answer C

Question 2

$$\begin{aligned} S_\infty &= 10 \\ \frac{a}{1-r} &= 10 \\ a &= 10 - 10r \end{aligned}$$

$$\begin{aligned} t_3 &= \frac{1}{4}a \\ ar^2 &= \frac{1}{4}a \\ r^2 &= \frac{1}{4} \\ r &= \pm\frac{1}{2} \end{aligned}$$

Either $a = 10 - 10\left(\frac{1}{2}\right) = 5$

or $a = 10 - 10\left(-\frac{1}{2}\right) = 15$

Answer D

Question 3

To be arithmetic, the terms must be evenly spaced. The second and fourth terms must be separated by $2d$, where d is the common difference. This would mean that d must be –4. However, the fourth and sixth terms must be separated by $2d$ also, which would require d to be –2.
To be a geometric sequence, there must be a common ratio, r, between terms.

Using the second and fourth terms we find that $r^2 = \frac{t_4}{t_2} = \frac{1}{2}$. Using the fourth and sixth terms to verify this, $r^2 = \frac{t_6}{t_4} = \frac{1}{2}$. Thus, a geometric sequence can be used with $r = \pm\frac{1}{\sqrt{2}}$.

Thus, $t_3 = ar^2 = t_2 r = \frac{\pm 16}{\sqrt{2}} = \pm 8\sqrt{2}$.

The third term could be $8\sqrt{2}$, but could also be $-8\sqrt{2}$ (or even a different type of sequence).

Answer E

Question 4

Consider marks lost as a geometric sequence.

$a = 64$

$r = 0.5$

$$
\begin{aligned}
S_5 &= \frac{a(1 - r^5)}{1 - r} \\
&= \frac{64\left(1 - \frac{1}{32}\right)}{\frac{1}{2}} \\
&= 124
\end{aligned}
$$

Thus a total of 124 marks were lost throughout the five tests. This is a mean of 24.8 per test.
Her mean mark will be 75.2%.

Answer C

Question 5

It would not be possible to have an arithmetic sequence where successive terms alternate in sign. Thus options **A** and **B** are not correct. If the common ratio of a geometric sequence is positive, the signs will not alternate either. If the ratio of any two successive terms is calculated, the result is approximately –1.1. This confirms that a geometric sequence with a negative common ratio will successfully explain the sequence.

Answer D

Question 6

Options **A** and **C** are not difference equations. Option **D** will find the size of the current bet since it will always be \$1 more than the preceding bet. However, this is not what is required. We require the total of all bets. Each bet is \$1 more than the number of preceding bets. Thus $n + 1$ must be added to the total each time.

Answer B

Question 7

Trying out the difference equation for $n = 1$ gives the equation

$$t_2 = bt_1 + 4$$
$$-8 = 6b + 4$$
$$-12 = 6b$$
$$b = -2$$

Clearly, there is more information given than is required, although trying out the equation for $n = 2$ and $n = 3$ could be used to verify the answer.

Answer C

Question 8

A first-order sequence can have terms that are either increasing, decreasing or a mixture of both. Option **A** is thus nonsensical. Likewise, it is not true that having the sequence diverge is evidence that the sequence is either first-order or higher-order. Options **C** and **E** are thus false. The fact that terms 2 and 3 are the same is not proof of anything either. Option **B** is thus likewise untrue.

It is true, however, that for a first-order sequence, a particular term can have only one possible successor. Term 2 is followed by an identical term 3 for the sequence in this question. If the sequence was to be first-order, term 4 must be the same also. It is not. Option **D** is correct.

Answer D

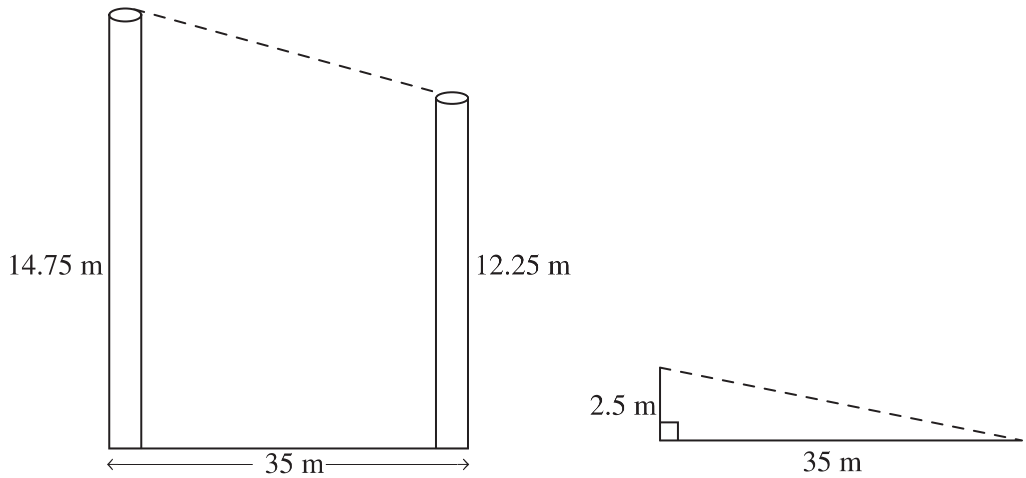
Question 9

This is best solved by comparing the result given by the solution with the result from the difference equation itself for $n = 1, 2, 3...$ It is not necessary to try terms higher than $n = 2$. Option **C** fails for $n = 0$. Options **A** and **E** fail when $n = 1$ is tried. Option **D** fails for $n = 2$.

Answer B

Module 2: Geometry and trigonometry

Question 1



$h^2 = a^2 + b^2$

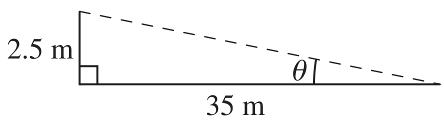
$h^2 = 2.5^2 + 35^2$

$h = 35.08917...$

$h \approx 35.09$ m

Answer D

Question 2



$$\frac{\text{opposite}}{\text{adjacent}} = \tan(\theta)$$

$$\frac{2.5}{35} = \tan(\theta)$$

$$4.08561...° = \theta$$

$$4° = \theta$$

Answer D

Question 3

$$\frac{\text{distance large}}{\text{distance small}} = k$$

$$\Rightarrow \; k = 4$$

$$\frac{\text{volume large}}{\text{volume small}} = k^3$$

$$= 4^3$$

$$= 64$$

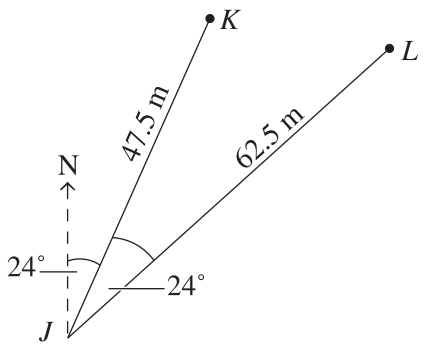
If the radius of a large sphere is four times the radius of small sphere then the scale factor is four. The ratio of the volume of the large object to the volume of the small object is always the cube of the scale factor. Therefore

$$k^3 = 4^3$$

$$= 64$$

Answer D

Question 4



$$\text{Area} = \frac{1}{2} \times b \times c \times \sin(A)$$

$$= \frac{1}{2} \times 47.5 \times 62.5 \times \sin(24°)$$

$$= 603.747\ldots \text{ m}^2$$

$$\approx 604 \text{ m}^2$$

Answer B

Question 5

$$a^2 = b^2 + c^2 - 2bc \times \cos(A)$$

$$a^2 = 47.5^2 + 62.5^2 - 2 \times 47.5 \times 62.5 \times \cos(24)$$

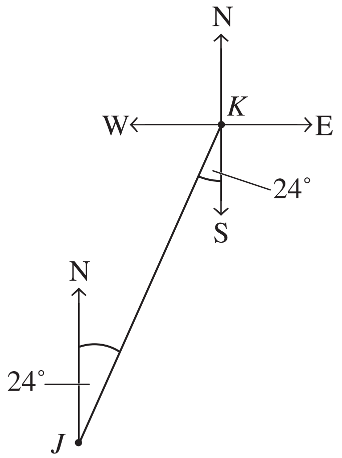
$$a^2 = 738.323$$

$$a = 27.17211 \text{ m}$$

$$a \approx 27.17 \text{ m}$$

Answer B

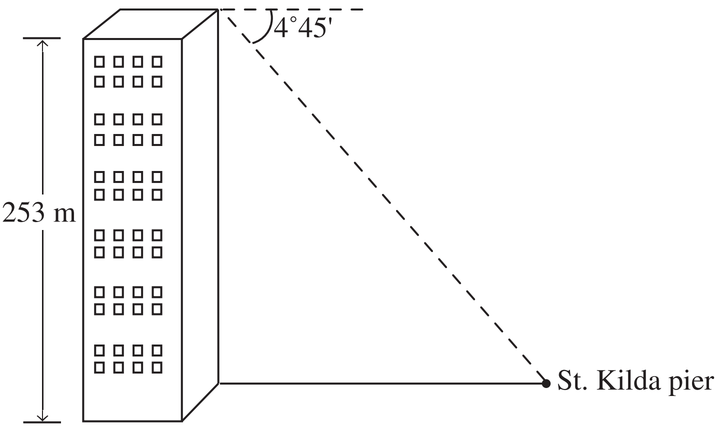
Question 6



Bearing $\overrightarrow{KJ} = \text{S } 24°\text{W}$

Answer C

Question 7



$90° - 85°15' = 4°45'$

$$\frac{\text{opposite}}{\text{adjacent}} = \tan(85°15')$$

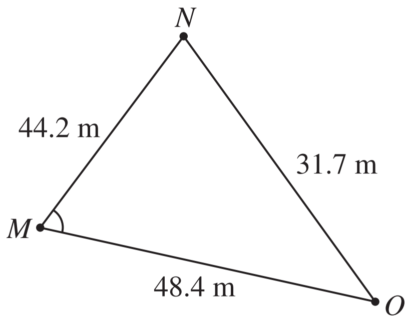
$$\frac{\text{opposite}}{253} = \tan(85°15')$$

$$\text{opposite} = 3044.759 \text{ m}$$

$$\approx 3045 \text{ m}$$

Answer C

Question 8



$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos(A) = \frac{48.4^2 + 44.2^2 - 31.7^2}{2 \times 44.2 \times 48.4}$$

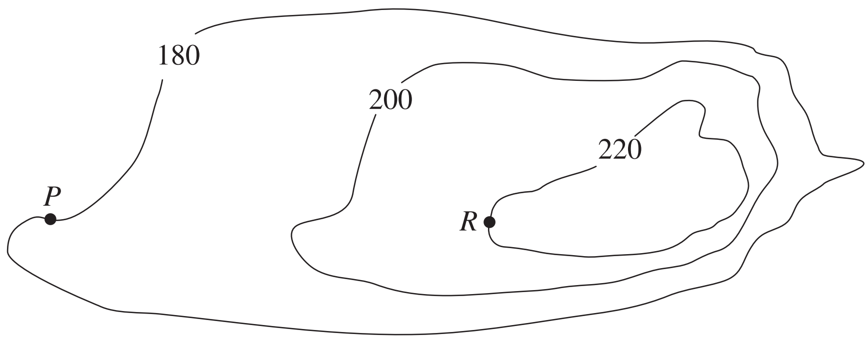
$$\cos(A) = 0.7692$$

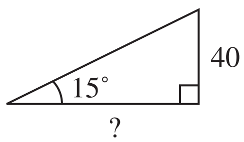
$$A = 39.71...°$$

$$A \approx 40°$$

Answer B

Question 9





$$\frac{\text{opposite}}{\text{adjacent}} = \tan(15°)$$

$$\frac{40}{\text{adjacent}} = \tan(15°)$$

$$\frac{40}{\tan(15°)} = \text{adjacent}$$

$$149.28... = \text{adjacent}$$

$$149.3 \approx \text{adjacent}$$

Answer D

Module 3: Graphs and relations

Question 1

$$m = \frac{0 - -2}{-1 - 3} = \frac{-1}{2}$$

$$y - -2 = \frac{-1}{2}(x - 3)$$

$$y = \frac{-x - 1}{2}$$

$$2y = -x - 1$$

$$x + 2y = -1$$

Answer D

Question 2

All of the possible equations except option **E** have the same basic equations. The difference between them is in the intervals of x-values that are applicable to each. The first section of the graph should include end point $x = 0$ but exclude $x = 3$. Only option **A** and option **D** do this. The distinction is that the third (right-most) section should include neither end point. Option **D** includes both.

Answer A

Question 3

Multiplying the first equation by –2 gives the equation $-6x + 4y = -16$. This is clearly parallel to the second equation, since both x and y coefficients are now the same. Option **A** is true. As parallel lines they have the same gradient. Rearranging the first equation gives $y = -4 + 1.5x$. Thus the gradients of both are positive, making option **B** true. That option **D** is a true statement follows from option **B** being true since at least one has a positive gradient if both do. Likewise option **E** must be true since we already know that these lines are parallel.

To obtain x-intercepts we must find x when $y = 0$ for both equations.

$$3x - 2y = 8$$

$$y = 0$$

$$3x = 8$$

$$x = \frac{8}{3}$$

$$-6x + 4y = 8$$

$$y = 0$$

$$-6x = 8$$

$$x = \frac{-4}{3}$$

These x-intercept values are clearly different and so option **C** is the false option.

Answer C

Question 4

The profit is given by the equation $P = 11x - 12\ 600$. We require $P > 0$.
Thus,

$$\begin{aligned} 11x - 12\ 600 &> 0 \\ 11x &> 12\ 600 \\ x &> 1145.45 \end{aligned}$$

The least value satisfying this condition is 1146.

Answer C

Question 5

This may be done using the graphics calculator or otherwise.

$y = \frac{k}{x} = \frac{110}{x}$

Answer D

Question 6

The greatest increase occurs between April and May. In any case, it is not possible to measure increases or decreases during a month, only between months.

Answer E

Question 7

The border line is not solid, so it is clear that both option **D** and option **E** are incorrect. Both inequations include the line itself, which would require a solid line on the graph. Option **A** is an equation, which is also inappropriate.
The region shaded is below the line so it must require an equation of the form "$y <$", or an equivalent. This confirms that option **B** is correct. This result could be easily verified by substitution of the point (0, 0).

Answer B

Question 8

All of the inequations listed as options **A** to **E** are attempts to describe the sloping boundaries through *a* and *b*. The gradient of the line through *a* is –1 and thus its equation must be $y = -x + a$ or $x + y = a$. Option **A** is a rearrangement of this. The gradient of the line through *b* is 1 and thus its equation is $y = x - b$ or $x - y = b$. Option **B** and option **C** are eliminated on this basis as they have incorrect gradients. Option **D**, however, would also require a solid line on the graph and thus must also be eliminated. Option **E** applies to the region below the boundary (while the unshaded region is above it).

Answer A

Question 9

Let us use *x* to represent the number of kilograms of *Supplement A* and *y* to represent the number of kilograms of *Supplement B*. We need the basic inequations specifying that both *x* and *y* must both be non-negative. All of the suggested answers have these, however, so it can't be used to distinguish between them. We also need to have an equation specifying that the total quantity of riboflavin must be at least 45.
$5x + 3y \leq 45$ is the resulting inequation. Only option **B** and option **E** have this. A similar process yields the inequation $5x + 4y \leq 20$ due to the fat constraint. Notice that this time the quantities must be **less than** 20. It is a maximum, not a minimum. This confirms option **B** as the correct response.

Answer B

Module 4: Business-related mathematics

Question 1

Bank A $I = 500\ 000 \times \frac{4}{12} \times \frac{6}{100}$

$= 10\ 000$

Bank B $I = 500\ 000 \times \frac{3}{12} \times \frac{6.5}{100}$

$= 8125$

The difference is $1875.

Answer B

Question 2

$$I = \frac{Prt}{100}$$

$$r = \frac{100I}{(Pt)}$$

$$= \frac{100 \times 1125}{(5000 \times 5)}$$

$$= 4.5$$

Answer D

Question 3

A credit of $200 makes the balance $456.50.

Minimum balance $= (456.50 - 200)$

$= 256.50$

Answer E

Question 4

$n = 3 \times 12$

$= 36$

$$\text{Effective rate} = \text{Flat rate} \times \frac{2n}{(n+1)}$$

$$= 5.8 \times \frac{2 \times 36}{36 + 1}$$

$$= 5.8 \times \frac{72}{37}$$

$$= 11.3$$

Answer A

Question 5

Use the TVM Solver.

```
N=60
I%=?
PV=-1500
PMT=0
FV=2200
P/Y=12
C/Y=12
PMT:END BEGIN
```

$I = 7.68$

$\therefore\ I = 7.7\%$

Answer B

Question 6

Total depreciation $= 28\ 500 - 3000$

$= 25\ 500$

Annual depreciation $= \dfrac{25\ 000}{6}$

$= 4250$

Answer D

Question 7

Use the TVM Solver.

```
N=24
I%=7.8
PV=35000
PMT=?
FV=0
P/Y=4
C/Y=4
PMT:END BEGIN
```

PMT $= -1840.02$

Total payment $= 1840 \times 24$

$= 44\ 160$

Interest paid $= 44\ 160 - 35\ 000$

$= 9160$

So the interest is $9200.

Answer D

Question 8

Interest is calculated on the amount owing so the interest is reducing as part of the principal is paid. As a result, more of the principal is paid each time an instalment is paid.

Answer C

Question 9

$$\text{Normal price (1)} = 1800 \times \frac{100}{75}$$

$$= 2400$$

$$\text{Normal price (2)} = 1890 \times \frac{100}{70}$$

$$= 2700$$

$$\text{Increase in the price} = 2700 - 2400$$

$$= 300$$

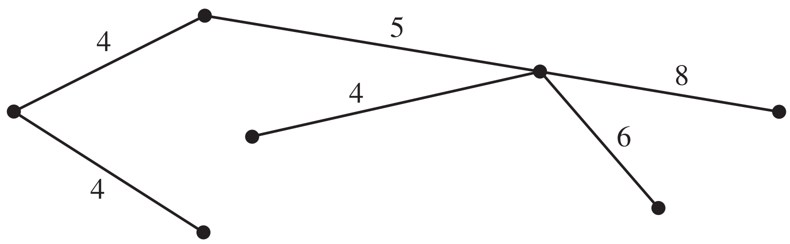
$$\text{Percentage increase} = \frac{300}{2400} \times 100$$

$$= 12.5$$

Answer C

Module 5: Networks and decision mathematics

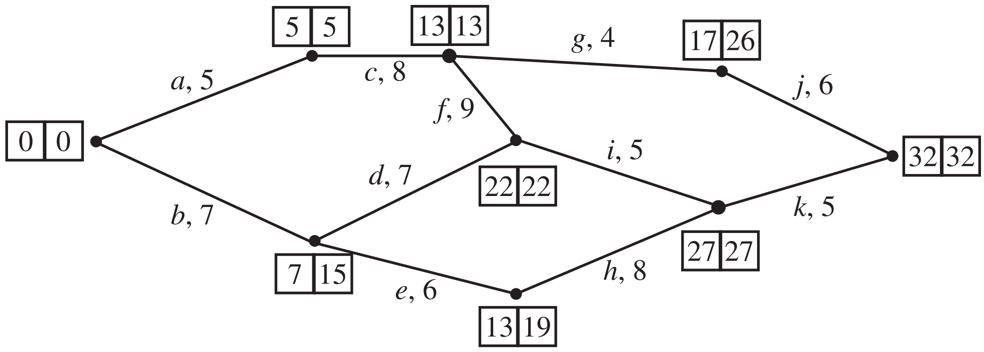
Question 1



$4 + 4 + 5 + 4 + 6 + 8 = 31$

Answer C

Question 2, 3 and 4 are all based on the network below.



Question 2

Critical path = *a, c, f, i, k.*

Options **B, C, D** and **E** contain incorrect edges *e, b* or *j*.

Answer E

Question 3

Activity *d* must be finished by 22 minutes. It cannot begin prior to 7 minutes. $22 - 7 = 15$ minutes.
Of this 15 minutes, activity *d* takes 7 minutes. Therefore slack or float time is equal to $15 - 7$ minutes.

$22 - 7 = 15$

$15 - 7 = 8$

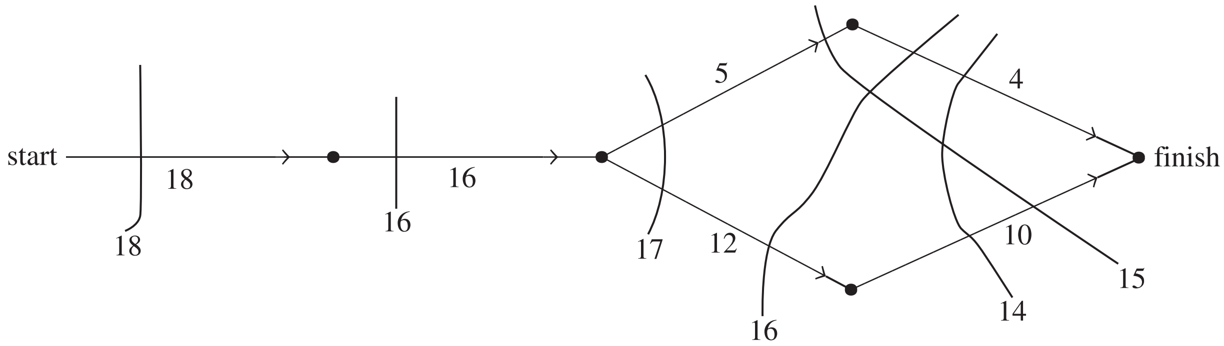
Answer B

Question 4

The duration of the critical path is 32 minutes.

Answer E

Question 5



All possible cuts are shown above. The minimum of these cuts is 14.

Answer B

Question 6

Joe $\longrightarrow$ Further Mathematics

Pat $\longrightarrow$ Accounting

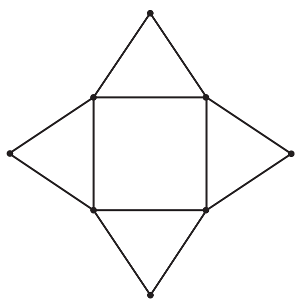
Clare $\longrightarrow$ English Literature

Mary $\longrightarrow$ English

Joe must teach Further Mathematics. English must be taught by Mary. If Mary teaches English this leaves Clare the only teacher able to teach English Literature. The remaining teacher, Pat, must teach the remaining subject – accounting.

Answer D

Question 7



$$vertices\ (v) = 8$$
$$edges\ (e) = 12$$
$$faces\ (f) = 6$$

Answer **A** is correct. Options **B, C, D** and **E** use the values 6, 8 and 12 in an incorrect order. Options **C** and **E** use the incorrect value of faces = 5.

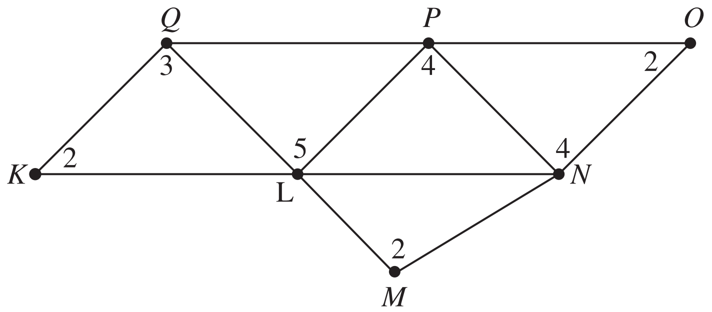
Answer A

Question 8

Two graphs are isomorphic if they have the same number of edges and vertices and the same number of connections between them. Options **A, B, C** and **D** do not necessarily define an isomorphic graph.

Answer E

Question 9



Q and *L* have an odd degree, all other vertices have an even degree. An Euler circuit can only contain vertices of even degree.

Answer B

Module 6: Matrices

Question 1

The element required is in the fourth row and second column of matrix A. This is –4.

Answer A

Question 2

When matrices are multiplied it is necessary that the number of columns on the left matches the number of rows on the right. Option **A**, option **B** and option **C** all pass this test quite simply. However, in option **D** the product, PR, must be calculated. This is undefined. Thus the entire product is undefined.

Answer D

Question 3

This is a simple scalar product. Each element in the matrix is multiplied by the number 3. Thus the resulting matrix is $\begin{bmatrix} 15 & 6 \\ -3 & -9 \end{bmatrix}$.

Answer D

Question 4

It is possible to swap rows in the matrix as long as it occurs for both matrix A and matrix B. In option **A** this has occurred only for matrix B. In option **B**, matrix A has been transposed. This will result in the wrong matrix products completely. Option **C** is similar but matrix B elements have been reversed in an order which does nothing to fix the problem. In option **D**, the columns of matrix A have been swapped. This results in the values of x and y being reversed – another incorrect outcome.

Answer E

Question 5

This is best done by a graphics calculator. However, the calculation on paper would be

$$2[2-1]-1[0-0]+2[0-0]=2$$

Answer D

Question 6

The process of calculating matrix D – matrix C involves subtracting the company's cost (per minute of usage, in each of the three years) from the income for these services in the same years.
This will create a 3×3 matrix containing profits in the same format as matrix C and matrix D.

Answer A

Question 7

It is notable that the matrix concerned has non-zero elements only on the leading diagonal. Thus, it is clear that the price for 2007 will only be influenced by the same item's price from 2006. Option **B** is thus true.
It is clear that all prices are altered. Prices y and z are multiplied by 1.15 and 1.1 respectively. This would result in a 15% increase for y since the new price is 115% of the old one. Similar logic shows that the increase for z would be 10%. The same logic again results in a 10% decrease for x since the new price is 90% of the old price.

Answer E

Question 8

The matrix required is T^3V_0. This can be calculated on a graphics calculator and is given in option **D**. Option **B** gives T^3. This is the transformation matrix, not the new state. Option **E** is the steady state. This is not what we require either.

Answer D

Question 9

From calculating T^{32} and T^{64} it is evident that a steady state is approached. This eliminates option **E**. Trying a variety of initial state vectors results in the same vector being approached every time:

$$\begin{bmatrix} 0.410959 \\ 0.30137 \\ 0.287671 \end{bmatrix}$$

In fact, students should be aware that steady states are not dependent on the initial state unless the initial state is a zero vector. The zero vector is not a possible initial state for this question, however, so it is not relevant.

Answer D