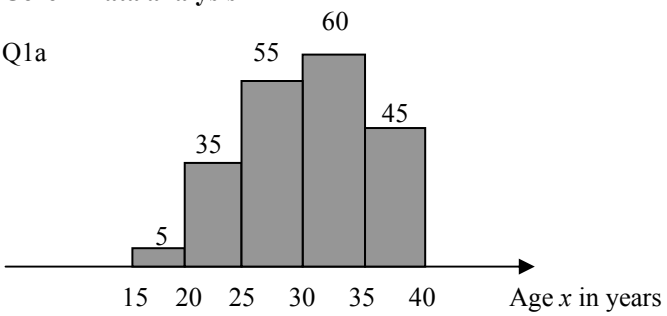
2007 Further Mathematics Trial Exam 2 Solutions

Core – Data analysis

Q1a



Q1b $30 \le x < 35$

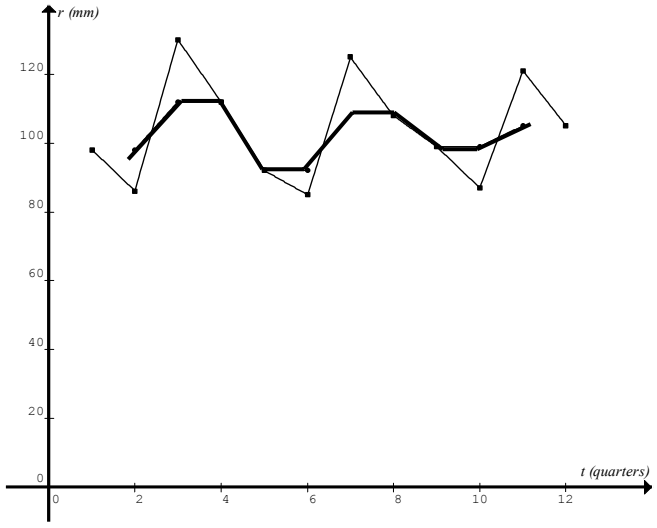
Q1c Average age

$$= \frac{17.5\times 5 + 22.5\times 35 + 27.5\times 55 + 32.5\times 60 + 37.5\times 45}{200} \approx 30$$

Q2a Graphics calculator $R = -0.90 + 130.50D$, $r = 0.94$.

Q2b The residual plot shows a random pattern that indicates the least squares regression line and not a non-linear relationship is a suitable model for the data set.

Q3a



Q3b $Seasonal\ index = \dfrac{actual figure}{deaseasonalised figure}$

First quarter $S.I. = \dfrac{98}{105}$ or $\dfrac{92}{99.3}$ or $\dfrac{99}{106.9} = 0.926$

Second quarter $S.I. = \dfrac{86}{104.0} = 0.827$

Third quarter $S.I. = \dfrac{130}{107.9} = 1.205$

Fourth quarter $S.I. = \dfrac{112}{107.5} = 1.042$

Q3c There was a slight downward trend in quarterly rainfall over the 3-year period.

Q3d For $t = 13$,
deseasonalised rainfall $r = -0.3329\times 13 + 106.2 = 101.9$ mm,
$\therefore$ *seasonal rainfall = deseasonalised rainfall* $\times$ *S.I.*
$= 101.9 \times 0.926 = 94.4$ mm

Q3e The extrapolation is not reliable because |*correlation coefficient*| $= 0.4256$ is too low, which shows that there were large fluctuations in quarterly rainfall.

Module 2: Geometry and trigonometry

Q1a $PQ = QR = RP = \sqrt{1^2 + 1^2} = \sqrt{2}$
$\angle PQR = \angle QRP = \angle RPQ = 60°$

Area of $\Delta PQR = \dfrac{1}{2}\times\sqrt{2}\times\sqrt{2}\times\sin 60° = 0.866 \approx 0.9\ \text{m}^2$

Q1b Volume of each corner $= \dfrac{1}{3}\left(\dfrac{1}{2}\times 1\times 1\right)\times 1 = \dfrac{1}{6}\ \text{m}^3$

Volume of resulting shape $= 2^3 - 8\times\dfrac{1}{6} = 6.7\ \text{m}^3$

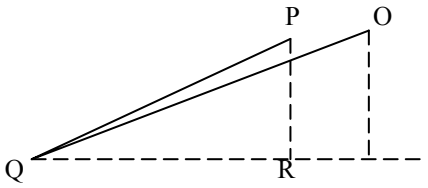
Q1c Total surface area of 2-m cube with one corner removed

$$= 6(2\times 2) - 3\left(\frac{1}{2}\times 1\times 1\right) + 0.866 = 23.366\ \text{m}^2$$

Linear scale factor = 2, $\therefore$ area scale factor $= 2^2 = 4$.
$\therefore$ total surface area $= 23.366\times 4 = 93.5\ \text{m}^2$

Q2a Horizontal distance $= 450 + 150 = 600$ m
Vertical distance $400 - 50 = 350$ m
Average slope of QO $= \dfrac{350}{600} = \dfrac{7}{12}$.

Q2b



Average slope of QP $= \dfrac{300}{450} = \dfrac{8}{12}$, $\therefore$ QP is steeper than QO and point P is above the line of sight OQ. $\therefore$ the view of Q from O is blocked by PR.

Q3a $\angle CAB = 82 - 22 = 60°$, $\angle CBA = 98 - 28 = 70°$

Q3b $\angle ACB = 180 - 60 - 70 = 50°$

The sine rule, $\frac{CA}{\sin 70°} = \frac{120}{\sin 50°}$, $CA = 147.2$ m

Q3c Let h be the height of the flag pole.

$\frac{h}{147.2} = \tan 5°$, $h = 12.8785 \approx 12.9$ m

Q3d Use the sine rule to find CB, $\frac{CB}{\sin 60°} = \frac{120}{\sin 50°}$,

$CB = 135.662$ m.

Angle of elevation $= \tan^{-1}\left(\frac{12.8785}{135.662}\right) = 5.4°$

Module 3: Graphs and relations

Q1a $n = 55$

Q1b When $n = 55$, $R = \frac{3200}{11} \times 55 = 16000$.

Gradient (slope) of cost graph $= \frac{16000 - 5000}{55} = 200$,

$\therefore C = 200n + 5000$

Q1ci $P = R - C = \frac{3200}{11}n - (200n + 5000)$,

$P = \frac{1000}{11}n - 5000$.

Q1cii When $n = 50$, $P = \frac{1000}{11} \times 50 - 5000 = -454.55$.

The loss is \$454.55.

Q2a From graph, distance = 3.5 km

Q2b Average speed $= \frac{3.5}{40/60} = 5.25$ km per hour

Q2c Highest speed (maximum gradient) occurred at $t = 33$ min.

Highest speed = gradient of tangent at $t = 33$ min

$\approx \frac{2}{10/60} = 12$ km per hour.

Q3a $P = 1.80x + 1.20y$

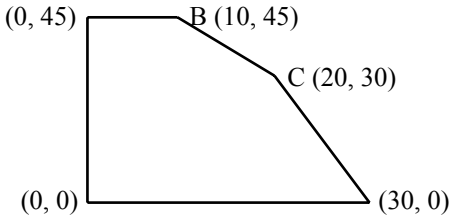
Q3b $x \geq 0$, $y \geq 0$.

Cabernet wine: $\frac{1}{2}x + \frac{1}{6}y \leq 15$, $\therefore 3x + y \leq 90$

Shiraz wine: $\frac{1}{2}x + \frac{1}{3}y \leq 20$, $\therefore 3x + 2y \leq 120$

Mataro wine: $\left(1 - \frac{1}{6} - \frac{1}{3}\right)y \leq 22.5$, $\therefore y \leq 45$

Q3c



Q3d Any point on line segment BC gives maximum profit.
Only point C (20, 30) has the greatest number of litres of X and the least number of litres of Y.
Greatest number of litres of X = 20 000
Least number of litres of Y = 30 000

Q3e Max $P = 1.80 \times 20 + 1.20 \times 30 = 72$
Max possible profit = \$72 000.

Module 4: Business-related mathematics

Q1a Let A_{2005} be the total cost at the start of 2005.

$A_{2006} = \left(1 + \frac{5}{100}\right)A_{2005}$ and $A_{2007} = \left(1 + \frac{10}{100}\right)A_{2006}$.

$\therefore A_{2007} = \left(1 + \frac{10}{100}\right)\left(1 + \frac{5}{100}\right)A_{2005}$

$\therefore 100 = 1.10 \times 1.05 \times A_{2005}$, $\therefore A_{2005} = \frac{100}{1.10 \times 1.05} = \$86.58$

Q1b Inflation $= 100 - 86.58 = 13.42$

Annual inflation rate $= \frac{13.42}{86.58 \times 2} = 0.0775 = 7.75\%$

Q2a

Date	Deposit	Withdrawal	Balance
1/8			\$2325.80
3/8		\$201.50	\$2124.30
12/8	\$570.00		\$2694.30
17/8		\$89.75	\$2604.55
23/8		\$364.20	\$2240.35
29/8	\$230.00		\$2470.35

Minimum balance for August is \$2124.30.

Q2b Interest $= 2124.30 \times \frac{3.5}{100} \times \frac{1}{12} = \$6.20$

Q2c Balance $= 2470.35 + 6.20 = \$2476.55$

Q3a TVM Solver \$1505.59

Q3b Interest $= 1505.5921 \times 240 - 180000 = \$181342.10$

Q3c TVM Solver 217.4838191 months,
i.e. 218 months or 18 years 2 months.

Q3d Amount owing after 217 months, TVM Solver \$722.45

Final repayment $= 722.45 \times \left(1 + \frac{7.35}{100 \times 12}\right) = \$726.88$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors