2007

VCE

Further Mathematics

Trial Examination 2

PURPOSE OF THIS TRIAL EXAMINATION

This Further Mathematics Trial Examination is designed to assess

- understanding and communication of mathematical ideas
- interpretation, analysis and solution of routine problems
- interpretation, analysis and solution of non-routine problems

Assessment is by extended answer questions involving multi-stage solutions of increasing complexity.

STUDENT NUMBER

VICTORIAN CERTIFICATE OF EDUCATION
2007

FURTHER MATHEMATICS

Trial Written Examination 2 (Analysis task)

Reading time: 15 minutes
Total writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book
Core

Number of questions	Number of questions to be answered
3	3

Modules

Number of modules	Number of modules to be answered
6	3

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator.
  Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 28 pages.
- Working space is provided throughout the book.
- There is a detachable sheet of miscellaneous formula supplied.

Instructions
- Detach the formula sheet from the book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score:
$$z = \frac{x - \bar{x}}{s_x}$$

least squares line:
$$y = a + bx \text{ where } b = r\frac{s_y}{s_x} \text{ and } a = \bar{y} - b\bar{x}$$

residual value:
residual value = actual value – predicted value

seasonal index:
$$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$$

Module 1: Number patterns

arithmetic series:
$$a + (a+d) + \ldots + (a+(n-1)d) = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$$

geometric series:
$$a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r}, r \neq 1$$

infinite geometric series:
$$a + ar + ar^2 + ar^3 + \ldots = \frac{a}{1-r}, |r| < 1$$

Module 2: Geometry and trigonometry

area of a triangle:
$$\frac{1}{2}bc \sin A$$

Heron's formula:
$$A = \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{1}{2}(a+b+c)$$

circumference of a circle:
$$2\pi r$$

area of a circle:
$$\pi r^2$$

volume of a sphere:
$$\frac{4}{3}\pi r^3$$

surface area of a sphere:
$$4\pi r^2$$

volume of a cone:
$$\frac{1}{3}\pi r^2 h$$

volume of a cylinder:
$$\pi r^2 h$$

volume of a prism:
area of base × height

volume of a pyramid:
$$\frac{1}{3} \text{ area of base} \times \text{height}$$

Pythagoras' theorem: $\qquad c^2 = a^2 + b^2$

sine rule: $\qquad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $\qquad c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $\qquad m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $\qquad y = mx + c$

Module 4: Business-related mathematics

simple interest: $\qquad I = \dfrac{\Pr T}{100}$

compound interest: $\qquad A = PR^n \text{ where } R = 1 + \dfrac{r}{100}$

hire purchase: $\qquad$ effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

annuities: $\qquad A = PR^n - \dfrac{Q(R^n - 1)}{R - 1}, \text{ where } R = 1 + \dfrac{r}{100}$

Module 5: Networks and decision mathematics

Euler's formula: $\qquad v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix: $\qquad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; \ \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix: $\qquad A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ where } \det A \neq 0$

END OF FORMULA SHEET

Specific Instructions

This task paper consists a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

Question 1

Students at a particular school were asked to state, anonymously, if they smoked or not. The results
are listed in **Table 1.** There are 75 students in each year level at this school.

Table 1

	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Total
Smoke	4	16	21	36	39		166
Do not Smoke	71		54	39	36		

a.   Complete table 1 by filling in the four spaces.

2 marks

b.   What percentage of students in year 9 or year10 do not smoke?

_____

_____

_____

1 mark

c.   Classify the variable smoking/ non smoking.

_____

_____

1 mark

Question 2

Twelve students who smoked agreed to be timed when they ran 200m. They were also asked how many cigarettes they had smoked in the previous week. The results are listed in **Table 2.**

Table 2

Student	Number of cigarettes smoked	Time for 200 m run (sec)
Meg	3	32.3
Pat	2	24.8
Jan	1	26.0
Mick	6	35.4
Mary	2	25.4
Sun Yi	4	28.3
Leo	12	39.3
Geoff	14	35.6
Bill	7	31.8
Rob	4	23.4
Tim	8	33.9
Sunyana	5	29.8

For the number of cigarettes smoked by these 12 students, find

a. the median

_____

$\frac{1}{2}$ mark

b. the lower quartile

_____

$\frac{1}{2}$ mark

c. the upper quartile

_____

$\frac{1}{2}$ mark

d. the interquartile range

_____

$\frac{1}{2}$ mark

e.   Show if there are any outliers.

_____

_____

_____

                                                                                   1mark

f.   Use the above information to draw a box plot for the number of cigarettes smoked.

0   1   2   3   4   5   6   7   8   9   10   11   12   13   14   15
                        Number of Cigarettes smoked
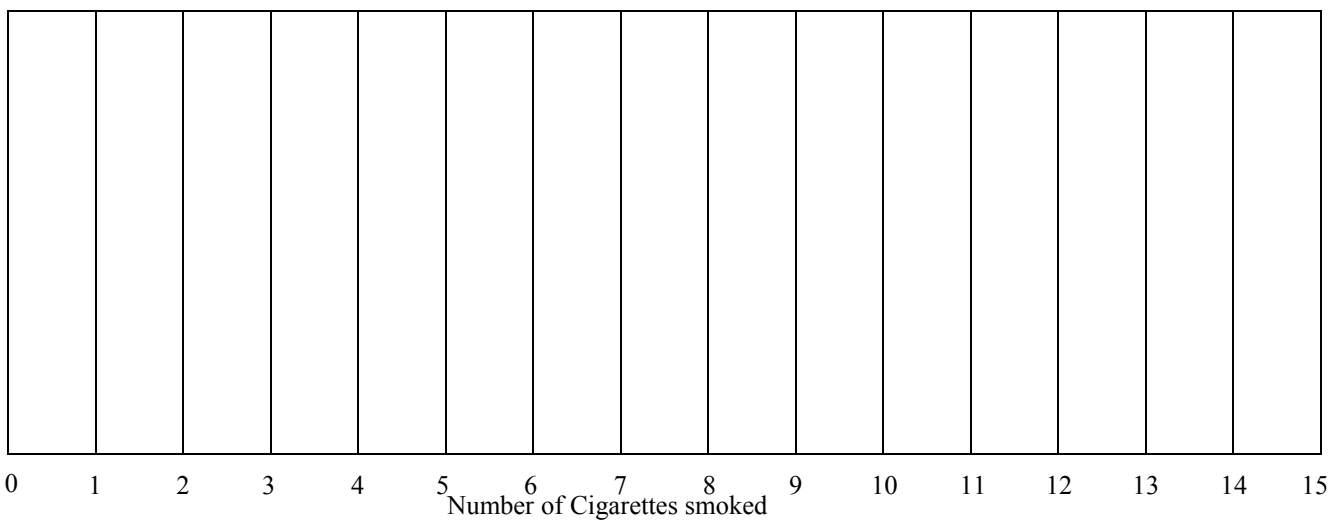
                                                                                   1 mark

g.   Complete the following statement

%

of the students smoked fewer than 7.5 cigarettes in the previous week.

                                                                                   1 mark

Question 3

For the data in **Table 2**

a.   Find the mean and the standard deviation for the times of the runs to one decimal place.

_____

                                                                                   1mark

b.   Approximately what percentage of all secondary students who smoke will run between
            standardised times of -1 and 2.5? Give your answer to one decimal place.

_____

                                                                                   1mark

c.   On the axes below, complete the scatter graph for the data given in **Table 2**, by marking in
the last three points.

1 mark

Time for
200 m run
(sec)

Number of cigarettes smoked
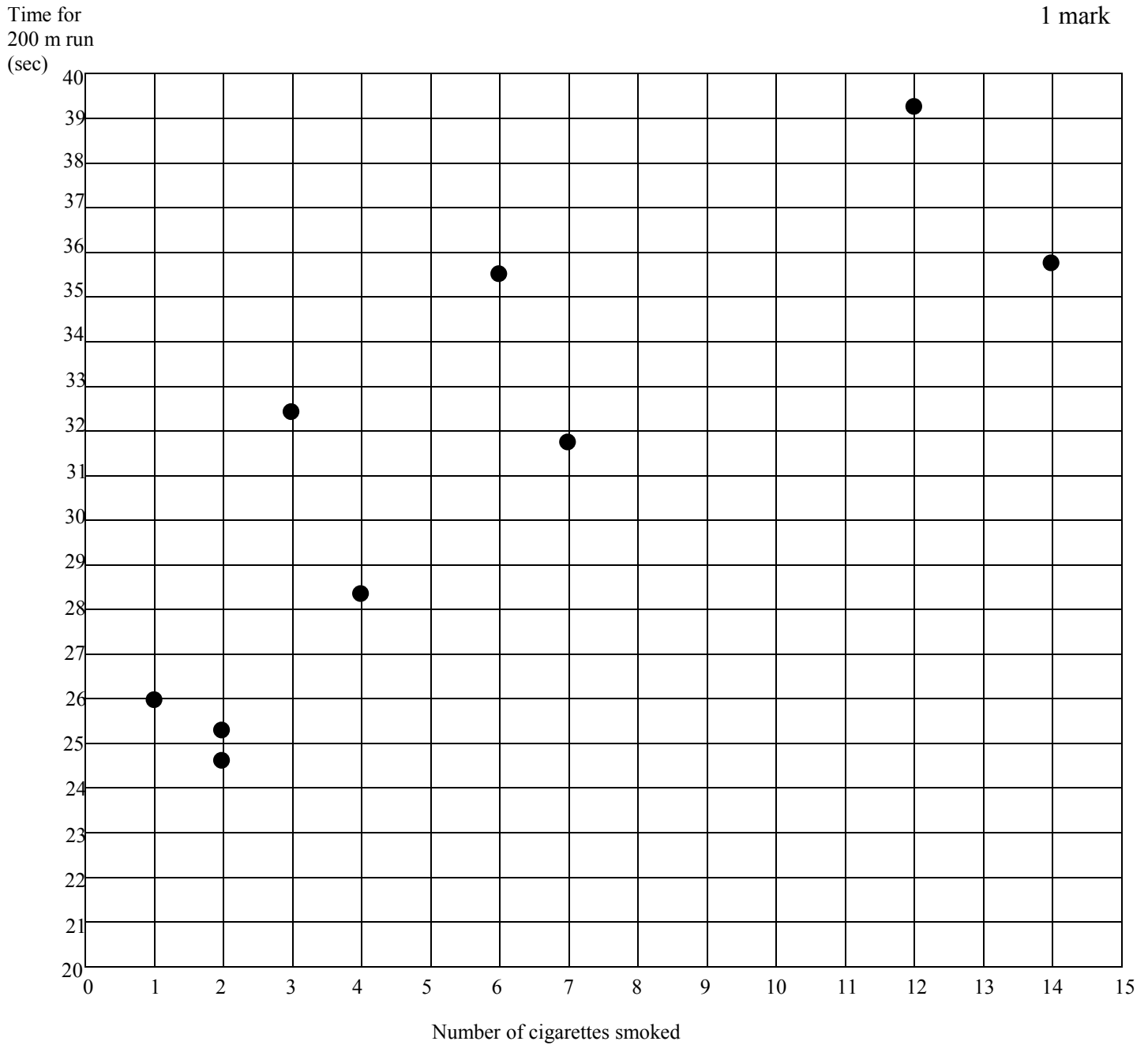
Question 3 (continued)

d.   Using the scatter graph in **c**, find the gradient of the three median regression line. Give your
           answer to two decimal places.

_____

_____

_____

_____

_____

_____

                                                                              2 marks

e.   Draw the three median regression line on the scatter graph.

                                                                              1 mark

                                                                       Total = 15 marks

                            End of Core: Data analysis

If you choose this module, all questions are to be answered.

Question 1
Jane sets up a florist business in 2001 and lists the average number of customers she has each day in the first years of her business in the following table.

Year	2001	2002	2003	2004	2005
Average number of customers per day	80	110	140		

If the average number of customers per day continued to increase in this way,

a.   What would be the average number of customers per day in 2005?

_____

_____

_____

                                                                                          1 mark

b.   What would be the average number of customers per day in 2011?

_____

_____

_____

                                                                                          1 mark

c.   In what year will the average number of customers per day reach 500?

_____

_____

                                                                                          1 mark

d.   How many customers in total will enter the shop between 2009 and 2019 inclusive?

_____

_____

_____

                                                                                          1 mark

Question 1 (continued)

e.   Once the average number of customers reaches 800 per day, it is expected that the number of customers, N, entering the florist each day, d, will decrease by 40 per day. Write an equation that can be used to calculate the average number of customers each day from now on.

_____

                                                                                                                      1 mark

Question 2

In 2001, the cost of a long stem rose was $3.00. This cost increased by 4% each year until now and the trend is expected to continue.

a.   What was the price of a long stem rose in 2002?

_____

                                                                                                                      1 mark

b.   Write an equation for the cost of a rose, C_n, in the n th year of Jane's florist business.

_____

                                                                                                                      1 mark

c.   How much more will a long stem rose cost in the tenth year of the business than it cost in the fourth year of the business? Give your answer to the nearest cent.

_____

_____

                                                                                                                      1 mark

d.   What will be Jane's total income from the sale of roses from 2001 to 2010 inclusive, if she sells 8000 roses in each of these years? Give your answer to the nearest cent.

_____

_____

_____

_____

_____

_____

                                                                                                                    2 marks

Question 3
Jane has overheads to consider in her business and she finds that her profit in the nth year, P_n, can be expressed as a difference equation

$$P_{n+1} = aP_n + b \text{ where } P_0 = \$65000$$

She notes that each year her profit increases by 20% of the previous year's profit, but that the overheads are always $10,000.

a.   Find the value of a.

_____

                                                                   1 mark

b.   Find the value of b.

_____

                                                                   1 mark

c.   How much profit will Jane make in her eighth year of business?

_____

                                                                     1 mark

d.   If Jane's overheads increase to $30,000 at the end of the eighth year of her business and remain at this value, in what year will she first make a loss?

_____

_____

_____

_____

_____

_____

                                                                2 marks

                                                         Total $= 15$ marks

End of Module 1: Number patterns and applications

If you choose this module, all questions are to be answered.

This diagram refers to Questions 1, 2 and 3.
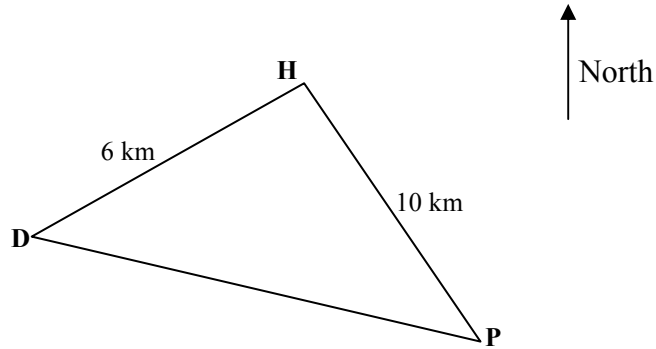
Question 1

Coal is dug from a hole in the ground, H, which is 10 km in a direction 330^0 T from a platform, P. The coal is then transported by truck to a depot, D, where it is stored in a conical pile. D is 6 km from H on a bearing 220^0 T from H.

a. Find angle *DHP*.

_____

_____

_____

1 mark

b. Find the distance, *DP*, to two decimal places.

_____

_____

_____

1 mark

Question 1 (continued)

c. Find the true bearing of D from P. Give your answer to the nearest degree.

_____

_____

_____

_____

_____

2 marks

d. Find the area of triangle *DHP*. Give your answer to two decimal places.

_____

_____

_____

1 mark

Question 2

The conical coal pile has the centre of its base at *D* and has a base radius of 100m and a slant height of 195 m.
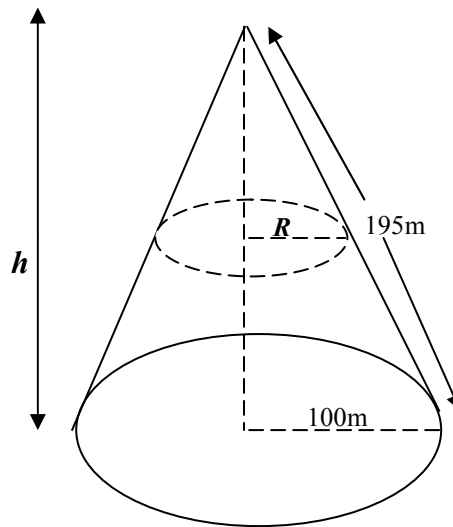
a. Find the height of the coal pile to one decimal place.

_____

_____

_____

1 mark

Question 2 (continued)

b.   Coal is removed from the pile so that the height of the coal that is left is reduced to a quarter
          of the previous height. Find the radius, R, of the top of the pile that is left.

_____

_____

_____

                                                                                                                              1 mark

c.   Find the volume of coal remaining in the pile**.** Give your answer to the nearest tenth of a
          cubic metre.

_____

_____

_____

_____

_____

_____

                                                                                                                              2 marks

Question 3

A platform 210 m high stands at P. A man whose eye is 170 cm above his feet, stands on the
platform and, using a telescope, sees a 16cm high cylindrical can sitting at a point X on the line DP
and 250 m from P.

a.   Find the shortest distance between the feet of the man on the platform and point X.
          Give your answer to one decimal place.

_____

_____

_____

                                                                                                                              1 mark

Question 3 (continued)

b.   The cylindrical can contains a solid ball that fits snugly in the container, touching all sides. Find the volume of the can that is empty space. Give your answer to the nearest cubic centimetre.

_____

_____

_____

_____

_____

_____

                                                                    2 marks

c.   Draw a diagram to show the angle of depression, θ, when the man looks at the top of the can.

                                                                    1 mark

d.   Find θ. Give your answer to the nearest minute.

_____

_____

_____

_____

_____

_____

                                                                    2 marks
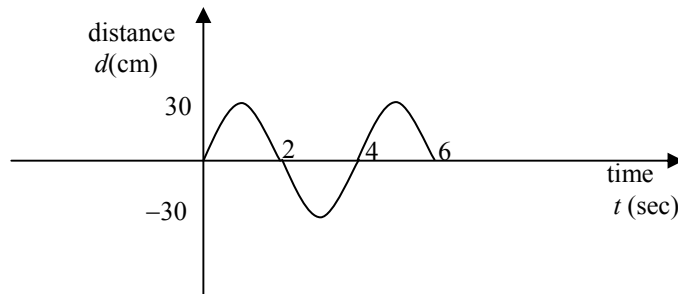
                                                            Total = 15 marks

End of Module 2: Geometry and trigonometry

> If you choose this module, all questions are to be answered.

Question 1

A pendulum swings back and forth and the horizontal distance the bob is from the rest position is shown in the above graph. When the bob swings to the right, the position is taken as positive and when it swings to the left the position is taken as negative.

a. What is the distance that the pendulum swings to the right of the rest position?

_____

                                                                     1 mark

b. How long does it take for the pendulum to make one complete swing? (A complete swing is when the pendulum reaches the rest position after having swung to the right and the left.)

_____

                                                                     1 mark

c. Where would you expect the pendulum bob to be when $t = 33$ seconds?

_____

                                                                     1 mark

Question 2

Martine plants a shrub that is initially 10 cm tall, and it grows to a height of 60 cm after 110 days. The growth of the plant can be modelled with a linear equation.

a.   On the axes below, draw a graph that shows the growth of the plant.

1 mark
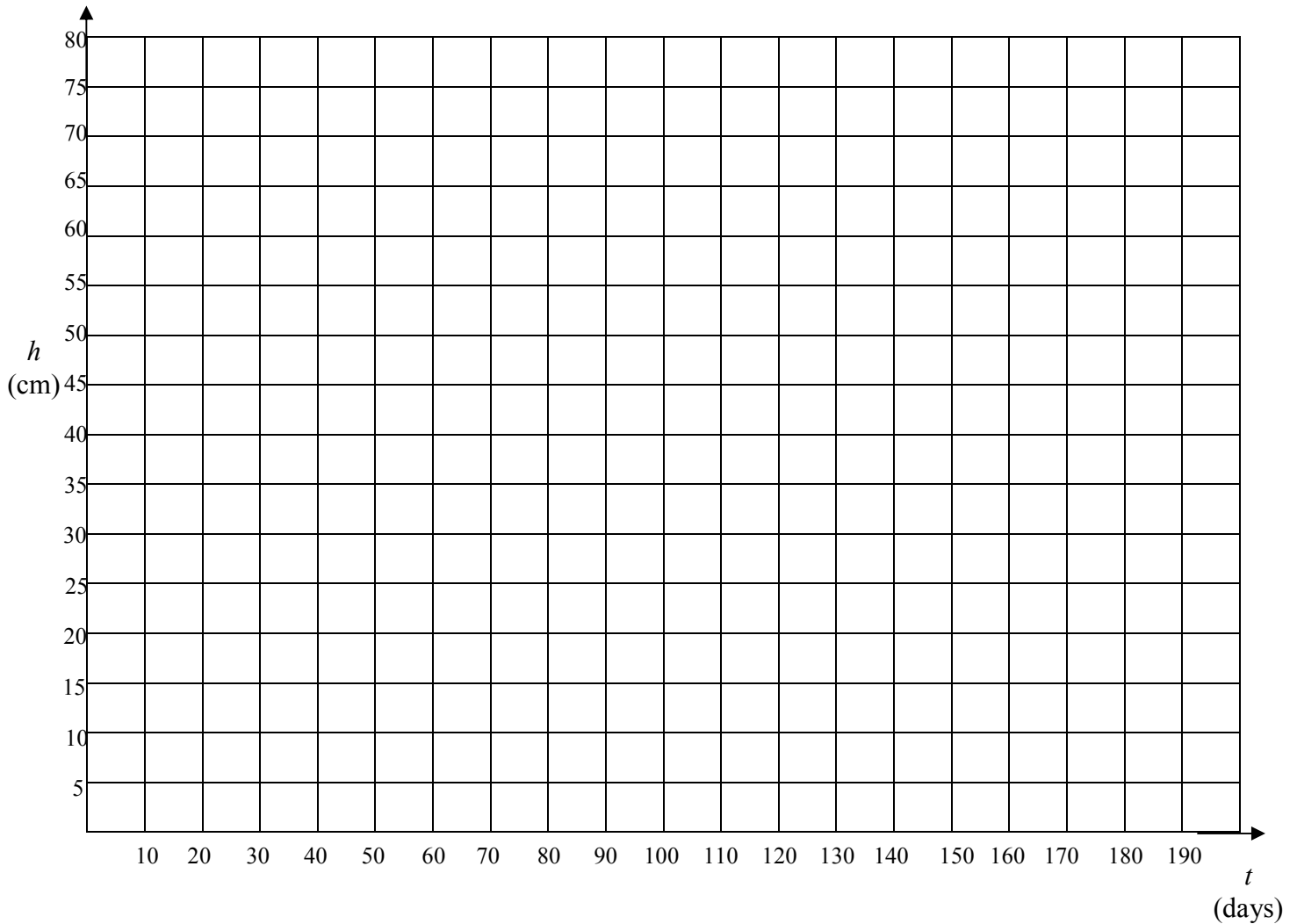
b.   What is the growth rate of the plant per day? Give your answer to two decimal places.

_____

1 mark

c.   Write the linear rule that models the growth of this plant in terms of *h* and *t*.

_____

1 mark

Question 2 (continued)

d.   From day 110 until the plant reaches its full height on day 160, the growth can be modelled
        by a different rule, namely,

$$h = 0.1t + a$$

What is the value of a?

_____

1 mark

e.   On the axes provided in **a.** complete the graph for the growth of the plant until it reaches its
        full height.

_____

_____

1 mark

Question 3

Martine runs a cosmetic business where she makes two types of face cream – one for the day, and
one for the night. She draws up the following table that shows the profit and the number of
kilograms of each item required to service customer demand per month.

Face Cream	**Profit per kilogram**	**Number of kilograms to service customer demand**
Day	$45	260
Night	$60	180

Martine's company cannot make more than 500 kg of face cream per month.

Let x be the number of kg of day cream made each month and y be the number of kg of night cream
made each month.

One of the inequalities is $x \geq 260$

a.   Write an inequality in terms of y.

_____

1 mark

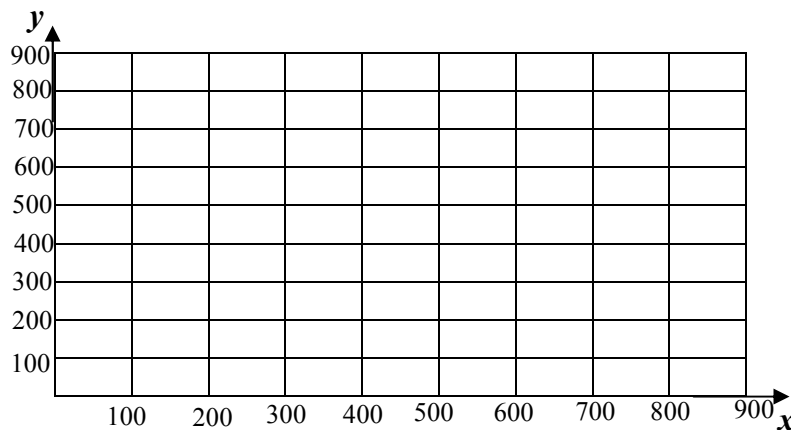**Question 3 continued**

b.   Write another inequality relating to the total amount of face cream the company can make per month.

_____

                                                                                    1 mark

c.   Draw three lines on the axes below and **shade the region that satisfies the three inequalities.**

                                                                                    2 marks

d.   Write down in terms of x and y an equation for the profit, P.

_____

                                                                                    1 mark

e.   How many kilograms of each type of face cream should *Martine* make per month, in order to maximize her profit?

_____

_____

_____

_____

                                                                                    1 mark

f.   What is the maximum profit that *Martine* can make in a month?

_____

                                                                                    1 mark

                                                                         Total $= 15$ marks
                           End of Module 3: Graphs and relations

> If you choose this module, all questions are to be answered.

Question 1

The *Petra Davis Art Gallery* receives the following bank statement from their bank. The bank pays interest at the rate of 8% per annum on the minimum monthly balance, and this interest is credited to the account at the end of each month.

Date	Transaction Details	Withdrawals($)	Deposits($)	Balance($)
2006 1 Dec	Opening Balance			140,247
18 Dec	Cheque deposit		17,335	
27 Dec	Cheque wages	12,036.55		
2007 21 Jan	Cheque deposit		8,642	x
28 Jan	Cheque wages	12,036.55		
12 Feb	Cheque deposit		122,400	
18 Feb	Cheque deposit		12,300	
27 Feb	Cheque wages	12,036.55		
28 Feb	Interest			

a.   What was the minimum monthly balance for December?

_____

                                                                                    1 mark

b.   Find the value of x in the above table.

_____

                                                                                    1 mark

c.   How much interest was credited to the account on 28 February?

_____

_____

_____

_____

                                                                                    2 marks

Module 4: Business-related mathematics

Question 1 (continued)

d.   The money received from the sale of paintings in this three-month period was $160,677. This amount included 12% G.S.T. What amount of G.S.T. does the *Petra Davis Art Gallery* have to pay to the government from this income?

_____

_____

1 mark

Question 2

The *Petra Davis Art Gallery* finds that paintings by *Anton Zinc* appreciate each year, by a percentage of their previous year's value. If a particular painting by this artist was valued at $35,000 on 1st January 2004 and $38,150 on 1st January 2005,

a.   What was the percentage rate of appreciation for the year 2004?

_____

_____

1 mark

b.   If the rate of appreciation remains constant each year, what will be the value of the painting at the end of four years? Give your answer to the nearest dollar.

_____

_____

1 mark

c.   If the rate of appreciation remains constant each year, in what year will the value of this painting reach $200,000?

_____

_____

_____

2 marks

d.   If the appreciation on the painting was a flat rate appreciation of 16% per annum, what would the value of the painting be after 5 years?

_____

1 mark

Question 2 (continued)

d.   In which year will the value of the *Anton Zinc* painting, using the compounding appreciation
model, first exceed the value of the painting using the flat rate appreciation model?

_____

_____

                                                                        1 mark

Question 3
The *Petra Davis Art Gallery* decides to extend their premises for a cost of $250,000. They need to
borrow $200,000 from their bank. The bank's interest rate is 9.8% per annum, compounding
monthly. The *Petra Davis Art Gallery* agrees to repay $2,000 per month.

a.   In the formula $A = PR^n - \dfrac{Q(R^n - 1)}{R - 1}$,

i.   What is the value of *Q*?

_____

                                                                        1 mark

ii.   What is the value of *R*? Give your answer to three decimal places.

_____

                                                                        1 mark

b.   How long will it take the Petra Davis art gallery to repay the loan? Give your answer to the
nearest year.

_____

                                                                        1 mark

c.   How much interest will they have paid over the period of the loan? Give your answer to the
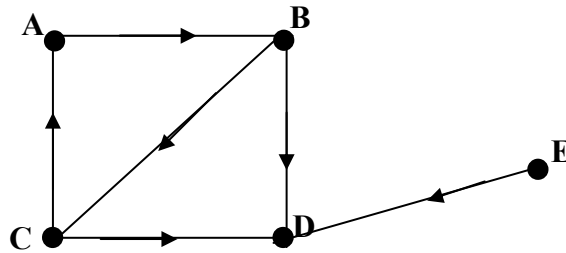nearest cent.

_____

                                                                        1 mark

                                                                Total = 15 marks
                End of Module 4: Business-related mathematics

Module 5: Networks and decision mathematics

Question 1

People live in five country towns, Ambert, A, Bordershire, B, Crossfern, C, Dawson, D and Ericvale, E. A directed graph of the drainage pipes between these towns is shown below.

a.   What is the sum of the degrees of all the vertices?

_____

1 mark

b.   In which town should the central drainage system, into which all the drains from the given towns empty, be built? Give a reason for your answer.

_____

2 marks

c.   Which towns are reachable from Bordershire?

_____

1 mark

d.   Complete the following reachability matrix for the directed graph above.

$$\begin{array}{c} \text{To} \\ \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{array} \\ \text{From} \begin{array}{c} \text{A} \\ \text{B} \\ \text{C} \\ \text{D} \\ \text{E} \end{array} \left[\begin{array}{ccccc} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{array} \right] \end{array}$$

2 marks

Question 2

A business department in Ericville has four different tasks that have to be fulfilled, namely, Research, Web Site Maintenance, Computer Programming and Word Processing. The time in hours for each of the four employees, Joan, Kara, Leo and Ming, for each of these various tasks, is given in the table below.

	Joan	**Kara**	**Leo**	**Ming**
Research	6	7	5	11
Web Site Maintenance	10	5	3	12
Computer Programming	11	3	4	13
Word Processing	8	4	6	10

a. Who should perform each task so that the work is completed in the minimum amount of time?

_____

_____

_____

_____

_____

_____

_____

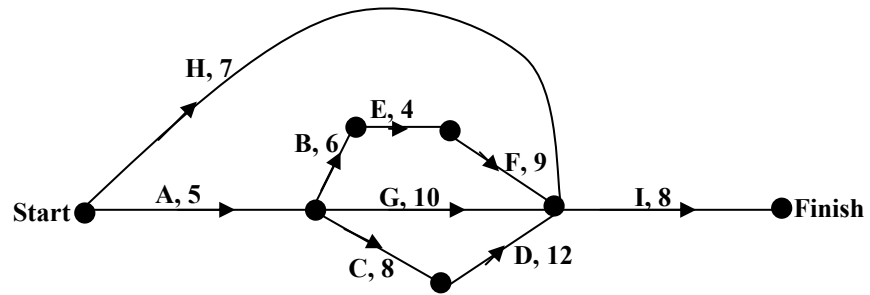_____

_____

_____

_____

3 marks

b. What is the minimum time for all tasks to be completed?

_____

1 mark

Question 3

Activities **A, B, C, D, E, F, G, H** and **I** are necessary to establish a garden. The directed graph of these activities and the time in hours for each activity is shown above.

a.   Using this information, complete the following table.

Activity	Immediate Predecessor
A	-----
B	A
C	A
D	B
E	C
F	E
G	A ,B
H	-----
I	

1 mark

b.   What is the critical path for this project?

_____
1 mark

c.   What is the shortest time possible to complete this project?

_____
1 mark

Question 3 (continued)

d.   If the usual person who completes activity **F** is sick and her replacement takes five hours
          longer to complete this activity, what effect will this have on the completion time of the
          project?

_____

_____

_____
                                                                                    1 mark

e.   If the replacement person for activity **F** is more efficient and completes this activity in six
          hours, what effect will this have on the completion time of the project?

_____

_____
                                                                                    1 mark

                                                              Total = 15 marks

                     End of Module 5: Networks and decision mathematics

If you choose this module, all questions are to be answered.

Question 1
Two companies, **X** and **Y** produce wine. **X** produces *a* bottles of red wine, *b* bottles of white wine and *c* bottles of champagne in one year. **Y** produces *d* bottles of red wine, *e* bottles of white wine and *f* bottles of champagne in the same time.

a.   Complete the following matrix *M*.

$$M = \begin{array}{c} \\ X \\ Y \end{array} \begin{array}{ccc} R & W & C \\ \left[\right. & & \left. \right] \end{array}$$

1 mark

b.   What is the order of matrix *M*?

_____

1 mark

c.   The selling prices of red wine, white wine and champagne are \$P, \$Q and \$R, respectively. Find *N*, the column matrix, which represents the selling price of these three types of wine.

1 mark

d.   Find *MN*.

1 mark

e.   What does the element in the second row, first column of *MN* represent?

_____

1 mark

Question 2

On Monday, Michael bought 3 tins of dog food, 2 cans of cat food and 4 packets of soap powder for $36.10
One week later, he bought 6 tins of dog food, 3 cans of cat food and 2 packets of soap powder for $30.65
The following week, he bought 5 tins of dog food, 1 can of cat food and 3 packets of soap powder for $32.45

If the prices for this three week period for a can of dog food, a can of cat food and a packet of soap powder were x, y and z respectively,

a. Complete the following matrices to find x, y and z. Give the answers to the first matrix in common fractions.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix} \begin{bmatrix} 36.1 \\ 30.65 \\ 32.45 \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

_____

3 marks

b. What is the cost of a tin of dog food?

_____

1 mark

Question 3

A certain town has three pharmacies, Alpha, **A**, Begood, **B** and Charisma, **C**. A recent survey found that the only factor influencing the choice of pharmacy for the next purchase made, was that of the prior purchase. The analysed data is shown in the following table.

Last Purchase (From)	Current Purchase (To)		
	A	**B**	**C**
A	0.9	0.05	0.05
B	0.1	0.8	0.1
C	0.2	0.9	0.9

a.   What percentage of people who bought from Alpha last time will buy from Charisma this time?

_____

<div align="right">1 mark</div>

b.   Write the information from the above table in the transition matrix, **T**, below.

$$\begin{array}{c} \text{Next Purchase} \\ \begin{array}{ccc} A & B & C \end{array} \\ T = \text{Current Purchase} \begin{array}{c} A \\ B \\ C \end{array} \left[\right] \end{array}$$

<div align="right">1 mark</div>

c.   The research showed that the current market share is 30% for Alpha, 60% for Begood and 10% for Charisma. Using decimals, write this information as a 3×1 matrix, S_0.

<div align="right">1 mark</div>

d. i.   Find the matrix, S_1, showing the percentage of customers expected at each pharmacy next week.

<div align="right">1 mark</div>

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Question 3 (continued)

d. ii. Write down the expected percentage of customers to the Charisma pharmacy next week.

_____

1 mark

e. What would be the expected percentage of customers to the Charisma pharmacy in the long term?

_____

1 mark

Total = 15 marks

End of Module 6: Matrices

END OF QUESTION AND ANSWER BOOK
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