Further Mathematics Exam 2: Solutions
CORE: DATA ANALYSIS

Question 1

a. Mileage **A1**

b. i $r = -0.902$ **A1**

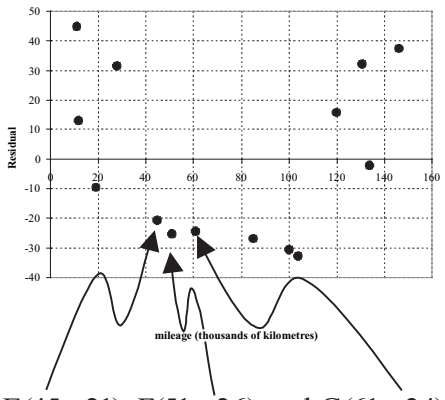
ii. 81.35% **A1**

c. *Price*

$= \boxed{207.82} + \boxed{-1.22} \times Mileage$

A1 **A1**

d.



Residuals E(45, -21), F(51, -26) and G(61, -24)

- for correct grouping of points (ie F below E and G) **A1**
- for correct positions **A1**

e. Yes. The nonlinear relationship will provide a better fit because the residual plot shows a *pattern* (is not random). **A1**

f. 2.16 **A1**

g. Strong **A1**
Negative **A1**
Linear **A1**

h. Price = $395 - 159 \times \log(125)$
= \$61 591
= \$61 600 **A1**

i. 91.5% of variation in price is explained by the variation in log(mileage). **A1**

j. Transformation is successful because the residual plot shows a more ***random*** distribution of residuals. **A1**

Total 15 marks

Further Mathematics Exam 2: Solutions – Multiple choice
MODULE 1: NUMBER PATTERNS

Question 1

a. $t_1 = 80$, $t_2 = 96$, $t_3 = 112$

$t_2 - t_1 = 96 - 80 = 16$
$t_3 - t_2 = 112 - 96 = 16$ **M1**

Therefore there is a common difference so the sequence is arithmetic.

b. $t_{14} = a + 13d$
$= 80 + 13 \times 16$
$= 288$ **A1**

c. $S_n = \frac{n}{2}[2a + (n-1)d]$
$S_{20} = 10[2 \times 80 + 19 \times 16]$ **M1**
$= 4640$ kg **A1**

Question 2

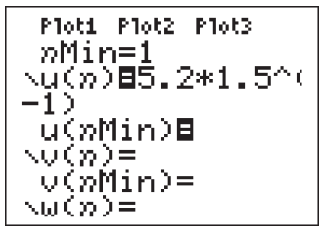
a. $\frac{7.8}{5.2} = 1.5$ **A1**

b. $t_3 = ar^2$
$t_3 = 5.2 \times 1.5^2$
$= 11.7$ m **A1**

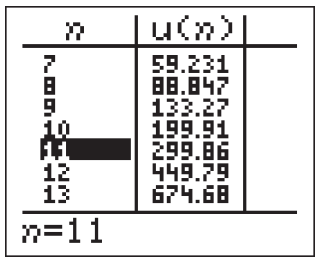
c. $t_n > 200$
$ar^{n-1} > 200$ **M1**

Method 1: Using the Calculator

In the sequence mode
Type $u(n) = 5.2 \times 1.5\text{^}(n - 1)$



Then go to TABLE
And scroll down until $u(n) > 200$



This occurs when $n = 11$

Method 2: Using Logs

$5.2 \times 1.5^{n-1} > 200$

$1.5^{n-1} > \frac{200}{5.2}$

$n - 1 > \frac{\log(38.46)}{\log(1.5)}$

$n. > 1 + 9$

$n > 10$

$\therefore n = 11$

A1
for either method

Question 3

a. The height of beanstalk at the end of these 3 weeks is

$15 + S_3$
$= 15 + 3 + 2.55 + 2.1675$
$= 22.72$ metres **A1**

b. The beanstalk grows

$$S_\infty = \frac{a}{1-r}$$
$$S_\infty = \frac{3}{1-0.85}$$
$$= \frac{3}{0.15}$$
$= 20$ m **A1**

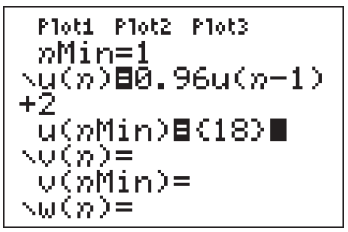
The maximum height of the beanstalk is
$20 + 15 = 35$ m **A1**

Question 4

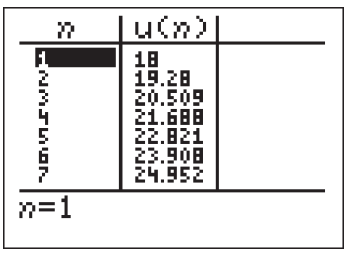
a. $A_2 = 0.96A_1 + 2$
$= 0.96 \times 18 + 2$
$= 19.28$ litres

$A_3 = 0.96A_2 + 2$
$= 0.96 \times 19.28 + 2$
$= 20.51$ litres **A1**

b. Using the sequence mode on the calculator
$u(n) = 0.96\ u(n-1) + 2$
$u(n \text{ min}) = \{18\}$



Go to TABLE to find a list of the first seven terms



$A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7$ **M1**
$= 18 + 19.28 + 20.509 + 21.688 + 22.821$
$+ 23.908 + 24.952 = 151.158$ litres **A1**

c. Let $A_n = 50$

Then $A_{n+1} = 0.96 \times 50 + 2$

$= 48 + 2$

$= 50$

i.e. the sequence is being reduced by the same amount that is being added on. So the sequence becomes stable at 50 **A1**

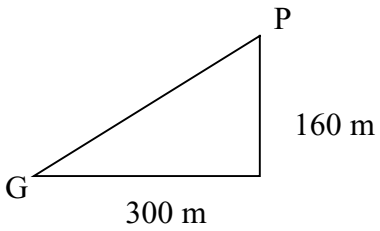
Total 15 marks

Further Mathematics Exam 2: Solutions – Multiple choice
MODULE 2: GEOMETRY & TRIGONOMETRY

Question 1

a. 480 – 240 = 240 metres **A1**

b.



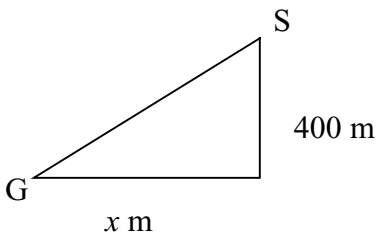
Using Pythagoras

$GP = \sqrt{160^2 + 300^2}$

$\sqrt{115600}$

340 m **A1**

c.



$\text{slope} = \frac{\text{rise}}{\text{run}}$

$1.3 = \frac{400}{x}$

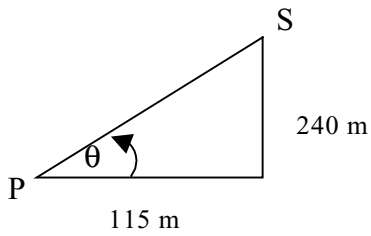
$x = \frac{400}{1.3}$

$x = 307.692...$

$x = 308\,\text{m}$ **A1**

d. i 23 × 5000
= 115000 mm
= 115 m **A1**

d. ii



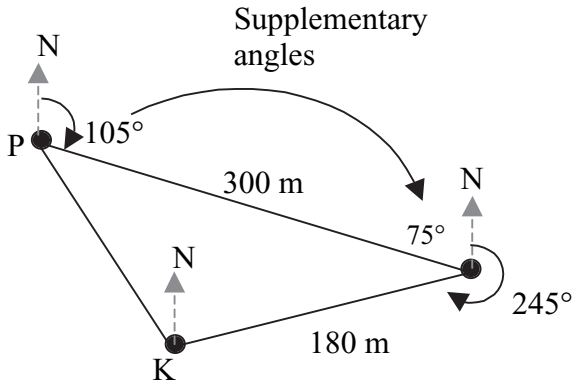
$\tan\theta = \frac{240}{115}$

$\theta = \tan^{-1}(\frac{240}{115})$

$= 64^o$ **A1**

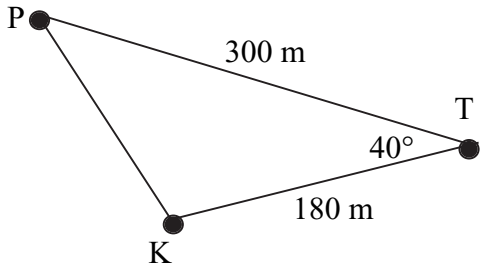
Question 2

a.



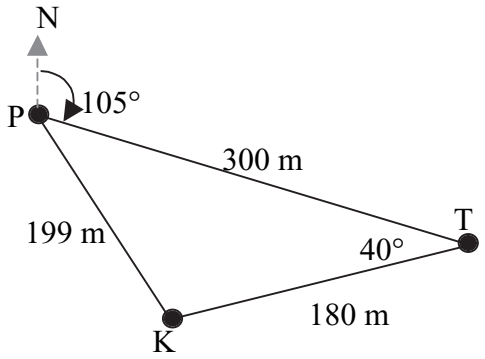
The supplement of 105 is 75
$\angle PTK + 245° + 75° = 360°$
$\angle PTK = 360° - 320°$ **M1**
$\angle PTK = 40°$

b.



Using cosine rule
$PK^2 = 180^2 + 300^2 - 2(180)(300)\cos 40°$
$PK^2 = 39667.2$
$\therefore PK = 199$ m **A1**

c.



To find angle KPT
Use sine rule

$$\frac{\sin P}{180} = \frac{\sin 40°}{199}$$

$$\sin P = \frac{180 \sin 40}{199}$$

$$\sin P = 0.5814...$$

$$P = \sin^{-1}(0.5814...)$$

$$P = 35.55°$$ **M1**

Bearing of Kiosk from Picnic area
$= 105° + 35.55°$
$= 141°T$ **A1**

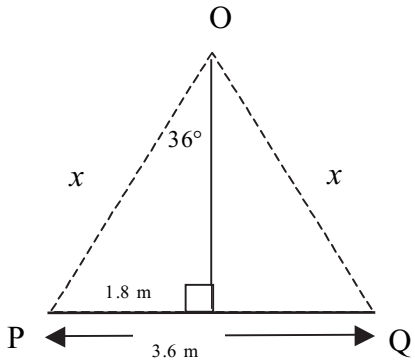
d. $A = \frac{1}{2} \times 180 \times 300 \sin 40°$

$= 17355 \text{ m}^2$ **A1**

Question 3

a. $\frac{360}{5} = 72°$ **A1**

b.



$$\sin 36° = \frac{1.8}{x}$$

$$x = \frac{1.8}{\sin 36°}$$

$x = 3.06 \text{ cm}$ **A1**

c. $A_{POQ} = \frac{1}{2} \times 3.06 \times 3.06 \sin 72°$

$= 4.4526...$

Area of pentagon $= 5 \times 4.4526$
$= 22.26 \text{ cm}^2$

Exact answer
Area of pentagon

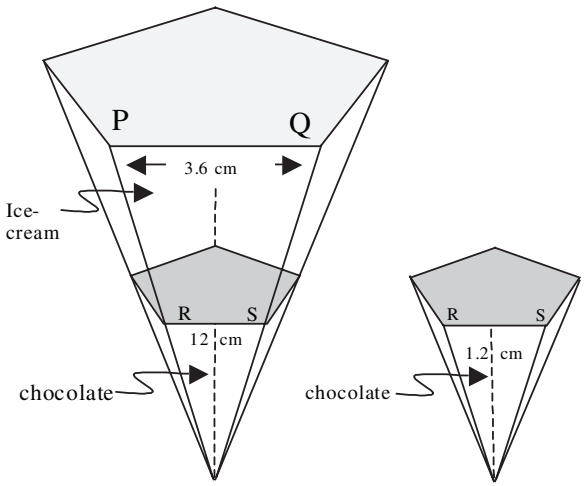
$$= 5 \times \frac{1}{2} \times \frac{1.8}{\sin 36°} \times \frac{1.8}{\sin 36°} \times \sin 72°$$

$$= 22.297...$$

$$= 22.30 \ cm^2$$

[accept answers between 22.26 and 22.30] **A1**

d.



Length ratio = 3.6 : 1.2
= 3 : 1

Volume ratio = cone : chocolate
3^3 : 1
27 : 1 **A1**

So $\frac{26}{27}$ of the cone consists of ice-cream. **A1**

Further Mathematics Exam 2: Solutions – Multiple choice
MODULE 3: LINEAR RELATIONS AND GRAPHS

Question 1

a. 6000 **A1**

b. Select two points from second segment and use $m = \frac{R_2 - R_1}{x_2 - x_1}$ for example

$m = \frac{150\,000 - 60\,000}{6000 - 1500} = \frac{90\,000}{4500} = 20$ **M1**

Substitute a point (x, R) in $R = 20x + k$

eg $60\,000 = 20 \times 1500 + k$ **M1**

$60\,000 = 30\,000 + k$

$\therefore k = 30\,000$

c. i. \$40 000 **A1**

ii. \$94 000 **A1**

d. Straight line with y-intercept (0, 60 000) **A1**

Correct end-point (60 000, 102 000) **A1**

e. Revenue = Cost

$20x + 30\,000 = 60\,000 + 7x$ **M1**

$13x = 30\,000$

$x = 2307.69$

2308 pairs of shoes **A1**

f. Profit = Revenue – Cost

$= (20x + 30000) - (7x + 60000)$

$= 13x - 30000$

$= 13 \times 6000 - 30000$

= \$48 000 **A1**

Alternatively find Revenue and Cost separately

Revenue $= 20x + 30\,000$

$= 20 \times 6000 + 30\,000$

$= 120\,000 + 30\,000$

$= 150\,000$

Cost $= 7x + 60\,000$

$= 7 \times 6000 + 60\,000$

$= 42\,000 + 60\,000$

$= 102\,000$

Profit = Revenue – Cost

$= 150\,000 - 102\,000$

= \$48 000

g. Revenue = Cost

$40x = 30\,000 + 12x$ **M1**

$28x = 30\,000$

$x = 1071.429$

1072 pairs of shoes **A1**

Question 2

a.

Week	1	2	3	4
Deficit	**4000**	**2000**	**1333**	**1000**

A1

b. $k = d \times t$

$= 4000 \times 1$

$= 4000$ **A1**

c. If the production reaches 5000 then the deficit is zero, but

$t = \frac{4000}{d}$ becomes $t = \frac{4000}{0} = \textit{undefined}$ **A1**

Total 15 marks

Further Mathematics Exam 2: Solutions – Multiple choice
MODULE 4: BUSINESS-RELATED MATHEMATICS

Question 1

a. This is a compound interest calculation.
Substitute in the formula

$A = P \times R^n$ where $P = 5000$,

$R = 1 + \frac{\frac{5.4}{12}}{100} = 1.0045$ and **A1**

$n = 5 \times 12 = 60$ **A1**

$5000 \times 1.0045^{60} = 6545.856.... \approx 6546$

b. Using the 'compound interest' formula for the inflated amount

$P = \$240$

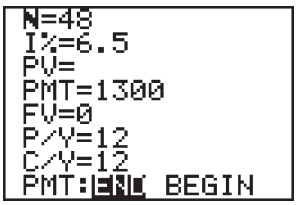
$R = 1 + \frac{2.8}{100} = 1.028$

$n = 8$

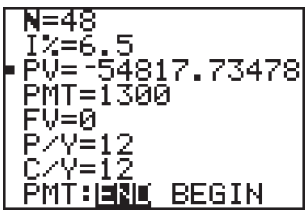
Inflated amount =
$240 \times 1.028^8 = 299.334...$
$\approx \$299.33$

Use of the correct type of formula **M1**
Correct answer **A1**

c. Using the TVM solver on the calculator set up the screen for $4 \times 12 = 48$ months, interest rate 6.5% p.a., present value ?, payment of \$1300 per month and future value of 0.



With the cursor on PV key ⊆:



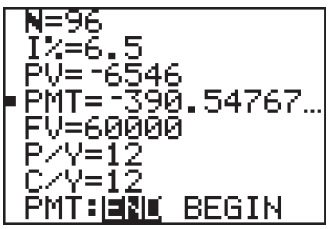
Imogen's parents will need \$54 818 to provide \$1300 per month for 4 years.

Something written down that shows they are on the right track **M1**
Correct answer **A1**

(Note: Both marks awarded if only the correct answer is given)

d. Using the TVM solver on the calculator set up the screen for $8 \times 12 = 96$ months, interest rate 6.5% p.a., present value -6546, payment of ? per month, future value of \$60 000.

With the cursor on PMT key ⊆:



Imogen's parents will need to save \$390.55 each month to have \$60 000 after 8 years if they start with \$6546 in the account.

Something written down that shows they are on the right track **M1**
Correct answer **A1**

Question 2

a. $5500 – $4400 = $1100
As a percentage this is

$$\frac{1100}{5500}\times\frac{100}{1}=20\% \qquad \textbf{A1}$$

b.

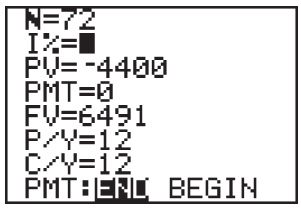
HECS/HELP debt	Years of inflation	Inflated amount ($)
$5500	6	6491.15
$5800	**5**	$5800 \times 1.028^5 = 6658.76$
$6100	**4**	$6100 \times 1.028^4 = 6812.43$

Total = $19 962.34 ≈ $19 962

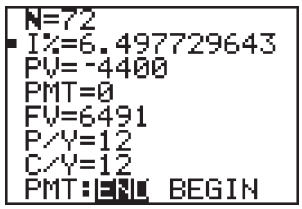
'5' and '4' in table **A1**
One other calculation correct **A1**
Total correct **A1**

c. $\frac{4}{100}\times\$19962.34 \approx \$798.49 \approx \$798$ **A1**

d. Using the TVM solver on the calculator set up the screen for 6 × 12 = 72 months, interest rate ? p.a., present value -4400, payment of $0 per month and future value of $6491.



With the cursor on I%, key ⊆:



The interest rate will need to be 6.498%

Something written down that shows they are on the right track **M1**
Correct answer **A1**

Further Mathematics Exam 2: Solutions – Multiple choice
MODULE 5:NETWORKS AND DECISION MATHEMATICS

Question 1

a. Either CBAFEDC or CDEFABC

Circuit starting and ending at C **M1**
Correct circuit **A1**

b. A Hamiltonian circuit visits each of the vertices once only and starts and ends at the same vertex. **A1**

Question 2

a. Starting with the person who has only one task that he/she can do

Denise can only do *Equipment* which leaves *Catering co-ordinator* for Paolo, *Drive bus* for Rex, *Timekeeper* for Gina and *First Aid* for Stuart.

Volunteer	***Task***
Gina	Timekeeper
Rex	Drive bus
Stuart	First Aid

Two correct **A1**
All three correct **A1**

Question 3

Using the Hungarian algorithm
subtract the minimum from each row.

38	30	0	0	0
3	0	32	20	11
38	32	0	19	28
22	34	0	23	48
2	0	32	25	0

Only four lines needed to cover the zeros (allocation can be made when five lines are needed to cover the zeros) so subtract the minimum from each column.

36	30	0	0	0
1	0	32	20	11
36	32	0	19	28
20	34	0	23	48
0	0	32	25	0

Still only four lines needed.
Subtract the minimum(19) from the uncovered elements and add it to the elements covered by two lines.

36	30	19	0	0
1	0	51	20	11
17	13	0	0	9
1	15	0	4	29
0	0	51	25	0

Five lines needed so start allocation.
Only one zero in rows 2 and 4.

36	30	19	✥	[0]
1	[0]	51	20	11
17	13	✥	[0]	9
1	15	[0]	4	29
[0]	✥	51	25	✥

This eliminates zeros in columns 2 and 3.
Continuing in this manner gives the final allocation (see table on next page).

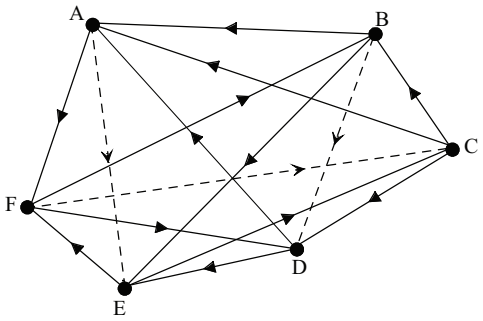
SOLUTIONS – continued

Referee	Town
Greg	**F**
Harry	**C**
Jane	**E**
Ken	**D**
Mario	**B**

Method, something written down. **M2**
Answer correct **A1**

Question 4

a.



All correct **A1**

b. The team from Town A defeated the team from Town F so a '1' is in that position.

The team from Town C did not play the team from Town C so a '0' is in that position.

Defeated town

$$M = \text{Town} \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & \boxed{1} \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & \boxed{0} & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Both correct **A1**

c. A second level win is when, for example, the team from town A defeated the team from town F, who defeated the team from Town D. The team from Town A is said to have a second level win over the team from Town D.

From the graph it can be seen that **the team from Town D** did not have a second level win over the team from town A.

Alternatively the second level wins are given in the matrix M^2. A zero appears in the 1st column only alongside A and D.

```
Ans²
 [[0 1 2 1 0 1]
  [1 0 1 0 2 2]
  [2 0 0 1 3 1]
  [0 0 1 0 1 2]
  [1 2 1 2 0 0]
  [3 1 0 2 2 0]]
```

A1

d. Adding the elements of the rows will give the number of first and second level wins.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{bmatrix} 7 \\ 9 \\ 10 \\ 6 \\ 8 \\ 11 \end{bmatrix}$$

The team from Town F is the overall winner. **A1**

Question 5

a. The minimum completion time is the path that takes the longest time (the critical path) from start to finish.

In this case the critical path is BFIJ which has a total duration of 24 days. **A1**

b. Only reducing the activities on the critical path will reduce the overall completion time.

If activity F is reduced by 3 days then a new critical path, BEHJ (22 days) will become operational. However, activity E can be reduced by one day giving both paths a completion time of 21 days.

So activity E should be reduced by 1 day and activity F by 3 days.

Reducing activity F **M1**
Correct answer **A1**

Total 15 marks

Further Mathematics Exam 2: Solutions – Multiple choice
MODULE 6 : MATRICES

Question 1

a. $\begin{bmatrix} 4 & 4 & 6 \\ 10 & 10 & 20 \\ 15 & 20 & 30 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6.7 \\ 18.0 \\ 29.5 \end{bmatrix}$ **A1**

b. The inverse of $\begin{bmatrix} 4 & 4 & 6 \\ 10 & 10 & 20 \\ 15 & 20 & 30 \end{bmatrix}$ is needed to solve the equations.

Using the calculator:

```
     [[4  4  6 ]
      [10 10 20]
      [15 20 30]]
[A]-1
 [[1    0    -.2]
  [0    -.3  .2 ]
  [-.5  .2   0  ]]
```

A1

c. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.2 \\ 0 & -0.3 & 0.2 \\ -0.5 & 0.2 & 0 \end{bmatrix}\begin{bmatrix} 6.7 \\ 18.0 \\ 29.5 \end{bmatrix}$

Using the calculator:

```
[A]-1*[C]
          [[.8 ]
           [.5 ]
           [.25]]
```

A chop costs \$0.80, a hamburger \$0.50 and a sausage \$0.25 (answers can be in cents) **A1**

Question 2

a. Matrix M must have order 3 × 3
$(3 \times 3) \times (3 \times 1) = (3 \times 1)$ **A1**

b. $M = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.4 & 0 \\ 0 & 0 & 1.3 \end{bmatrix}$

Correct figures on the diagonal **A1**
All correct **A1**

Question 3

a. $P = \begin{bmatrix} 21 & 35 & 9 \\ 12 & 18 & 25 \\ 7 & 4 & 15 \end{bmatrix}\left(\begin{bmatrix} 10.05 \\ 25.20 \\ 38.35 \end{bmatrix} - \begin{bmatrix} 6.7 \\ 18.0 \\ 29.5 \end{bmatrix}\right)$

OR

$P = \begin{bmatrix} 21 & 35 & 9 \\ 12 & 18 & 25 \\ 7 & 4 & 15 \end{bmatrix}\begin{bmatrix} 3.35 \\ 7.20 \\ 8.85 \end{bmatrix}$

3 × 3 matrix correct
(figures transposed from table) **A1**
Profit on each pack matrix correct **A1**

b. Using the calculator:

```
[A]*([B]-[C])
       [[402   ]
        [391.05]
        [185   ]]
```

Budget Meats makes \$402 profit at store R, \$391.05 profit at store S and \$185 profit at store Q. **A1**

Question 4

a.

$$T = \begin{array}{c} \begin{matrix} \geq 3 & 1,2 & 0 \end{matrix} \\ \begin{bmatrix} 0.6 & 0.1 & 0.3 \\ 0.3 & 0.8 & 0.4 \\ 0.1 & 0.1 & 0.3 \end{bmatrix} \end{array} \begin{matrix} \geq 3 \\ 1,2 \\ 0 \end{matrix}$$

This week

next week

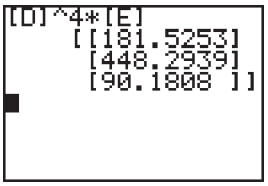
Correct figures in columns **M1**

All correct **A1**

b. $I_0 = \begin{bmatrix} 131 \\ 386 \\ 203 \end{bmatrix}$ **A1**

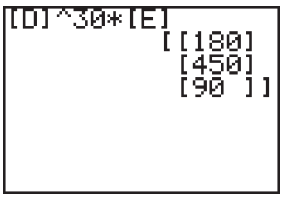
c. The expected numbers are given by $T^4 \times I_0$

Using the calculator:



182 members are expected to attend at least three times, 448 members are expected to attend 1 or 2 times and 90 members are expected to not attend. **M1, A1**

d. Using a high power of T



In the long term 180 members are expected to attend at least three times a week. **A1**

Total 15 marks