Neap:

Trial Examination 2007

VCE Further Mathematics Units 3 & 4

Written Examination 2

Suggested Solutions

SECTION A – DATA ANALYSIS – CORE MATERIAL

Question 1

a. mean = 21.92 A1

standard deviation = 20.08 A1

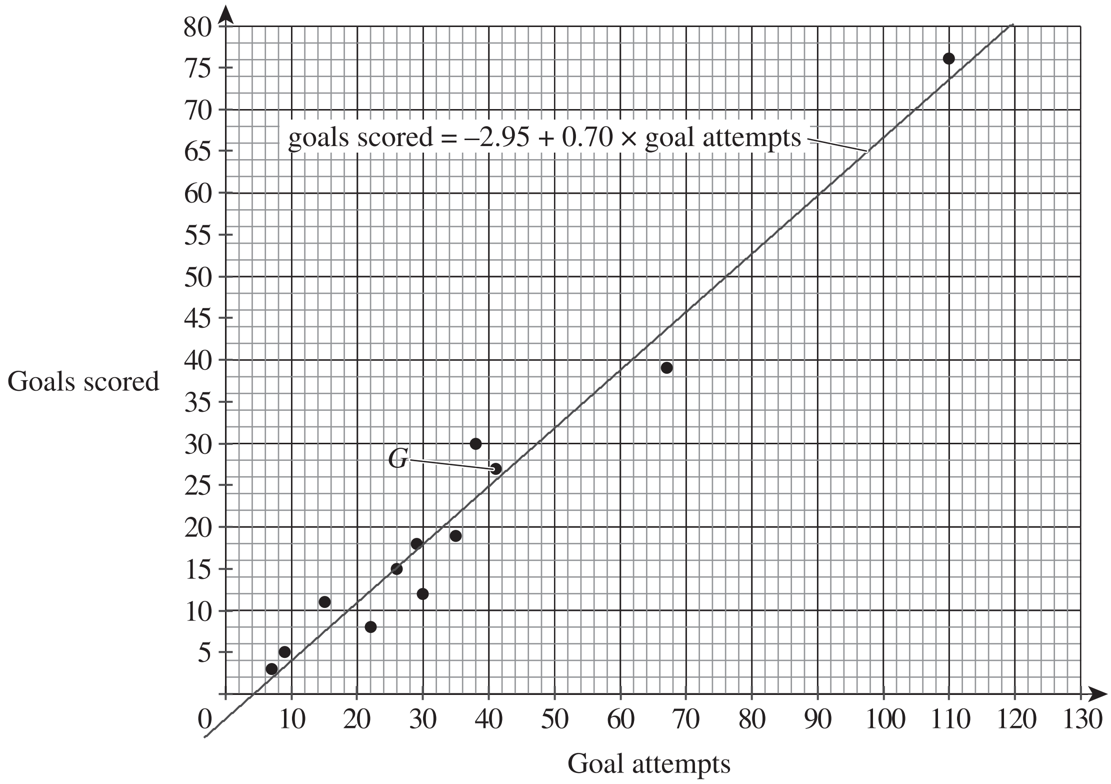
b. percentage = 33.3.% = 33% (to the nearest whole percentage) A1

c. **i.** goals scored = – 2.95 + 0.70 × number of goal attempts A2

ii. $r = 0.9831$ A1

d. linear A1

e.



A3

Two marks for correctly positioning the least squares regression line.

One mark for correctly plotting person G.

f. goals scored A1

g. **i.** 0.7 goals A1

ii. 97% (r^2 value) A1

h. Actual: 41 attempts becomes 27 goals

Estimate (using equation in **c.i.**): 41 attempts become 25.75.

$\therefore$ residual value = 1.25 A1

i. Interpolation (35 goals is within the observed range of recorded results). A1

SECTION B – MODULES

Module 1: Number patterns

Question 1

a. This is a geometric sequence.

$a = 40\,000$

$r = 1.06$

$t_4 = ar^3$

$= 40\,000(1.06)^3$

$= 47\,641$

Her salary would be $47 641. A1

b. $S_6 = \dfrac{a(r^6 - 1)}{r - 1}$ M1

$= \dfrac{40\,000(1.06^6 - 1)}{1.06 - 1}$

$= \dfrac{40\,000(0.4185)}{0.06}$

$= 279\,013$

She earns $279 013 during this time. A1

c. This is an arithmetic sequence.

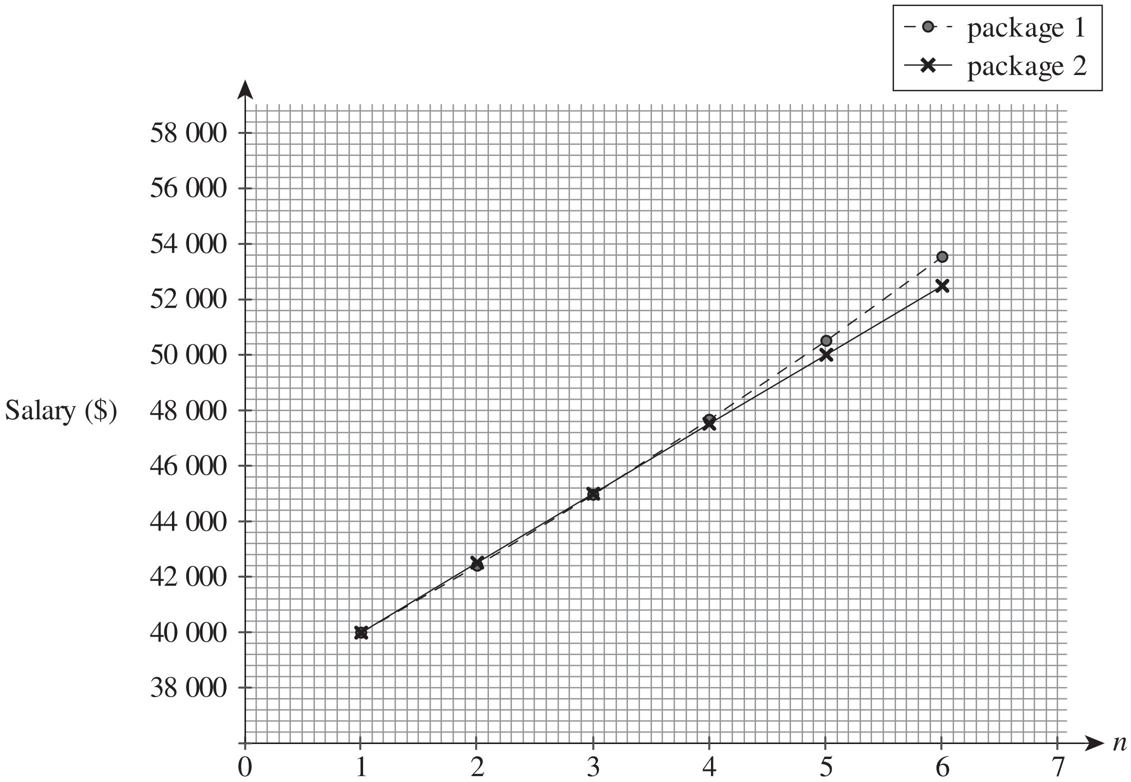
$a = 40\,000$

$d = 2500$

$t_4 = a + 3d$

$= 47\,500$ A1

d.



A2

One mark for plotting packages 1 and 2 correctly.
One mark for correct labelling of graphs.

e. According to the graph, $n = 4$ is the first term where salary package 1 is larger.
Thus the year is 2010. A1

f. Package 1

$$S_4 = \frac{a(r^4 - 1)}{r - 1}$$

$$= \frac{40\,000(1.06^4 - 1)}{1.06 - 1}$$

$$= 174\,984.64$$ M1

Package 2

$$S_4 = \frac{4}{2}[2a + 3d]$$

$$= 2[80\,000 + 7500]$$

$$= 2(87\,500)$$

$$= 175\,000$$ M1

Thus package 2 is a better deal. The margin is:

$175\,000 - 174\,984.64 = \$15.36$ A1

Question 2

a. $t_{n+1} = 1.04t_n + 0.50 \qquad t_1 = 7.0$ A1

b. $t_2 = 1.05(7) + 1 = 8.35$ M1

$t_3 = 1.05(8.35) + 1 = 9.77$

$t_4 = 1.05(9.77) + 1 = 11.26$

Thus it occurs three years later in 2010. A1

c. $T_3 = 1.03T_2 + 0.4(T_2 - T_1) = 9.1405$

$T_4 = 1.03T_3 + 0.4(T_3 - T_2) = 9.7309$

$T_5 = 1.03T_4 + 0.4(T_4 - T_3) = 10.2590$

$T_6 = 1.03T_5 + 0.4(T_5 - T_4) = 10.7780$ M1

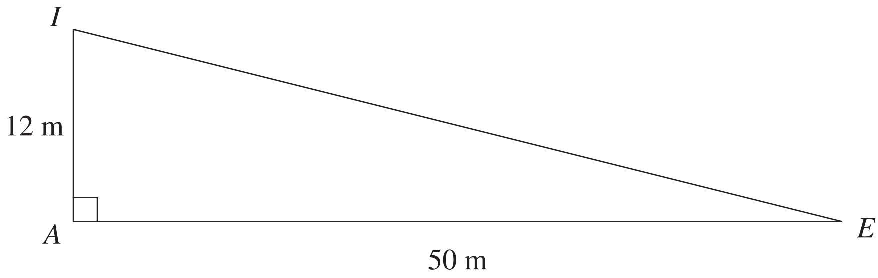
Clearly, term 5 is the first for which the sequence in part **a.** exceeds that of the corresponding term from this section. Term 6 verifies that the corresponding term of part **a.** is further ahead and the difference is increasing. Thus the years 2008 to 2010 are the years when the turnover in part **c.** exceeds the turnover given by the difference equation in part **a.** A1

Module 2: Geometry and trigonometry

Question 1

a. trapezium A1\

b.



$h^2 = a^2 + b^2$

$h^2 = 12^2 + 50^2$

$h^2 = 2644$

$h \approx 51.42$ m A1

c. Area *ABCF*

$\text{area} = \dfrac{\text{top + bottom}}{2} \times \text{height}$

$= \dfrac{0.9 + 2.2}{2} \times 25$

$= 38.75$

Area *CDEF*

$\text{area} = l \times w$

$= 25 \times 0.9$

$= 22.5$ M1

$\text{total area} = 38.75 + 22.5$

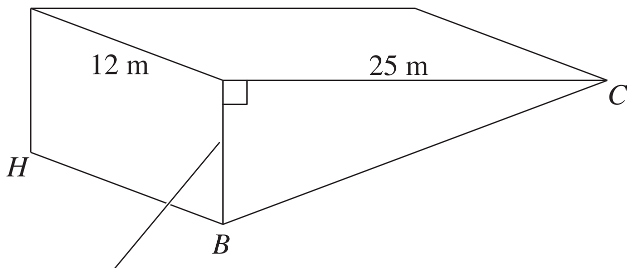
$\approx 61.3 \text{ m}^2$ A1

d. $\text{volume} = \text{end area} \times \text{height}$

$= 61.3 \times 12$

$= 736 \text{ m}^3$ A1

e.



2.2 – 0.9 = 1.3 m

$$\text{area} = \frac{1}{2}b \times h$$
$$= \frac{1}{2} \times 25 \times 1.3$$
$$= 16.25$$ M1

volume = end area × height
= 16.25 × 12
= 195 m^3 A1

f. The body of water above the sloping base volume is a rectangular prism.

The volume of this rectangular prism = 567 – 195
= 372 m^3 M1

Calculate the height of the rectangular prism.

volume = length × width × height
$372 = 50 \times 12 \times h$
$$\frac{372}{50 \times 12} = \text{height}$$
0.62 = height M1

Calculate the distance between the water and the top of the pool

distance = 0.9 m – 0.62 m
= 0.28 metres A1

Question 2

The volume of water in the pool is directly related to the cross-sectional **area** of the hose.
An error of k in distance causes an error of k^2 in area.

Distance
8 : 12 = 2 : 3

Area
4 : 9 A1

Question 3

a. $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \dfrac{55.7^2 + 74.2^2 - 89.3^2}{2 \times 55.7 \times 74.2}$

$A = 85.6035\ldots$ M1

$\text{area} = \dfrac{1}{2} bc \sin A$

$= \dfrac{1}{2} \times 55.7 \times 74.2 \times \sin 85.6035°$

$\approx 2060 \text{ m}^2$

$\text{landscaped area} = 2060 - (50 \times 12)$

$= 1460 \text{ m}^2$ (rounded to the nearest m^2) A1

b. bearing $AB = 21.7°$

bearing $AC = 21.7° + \text{angle } BAC$

bearing $AC = 21.7° + 85.6035°$

bearing $AC = 107.3035°$ M1

bearing $CA = \text{bearing } AC + 180°$

bearing $CA = 107.3035° + 180°$

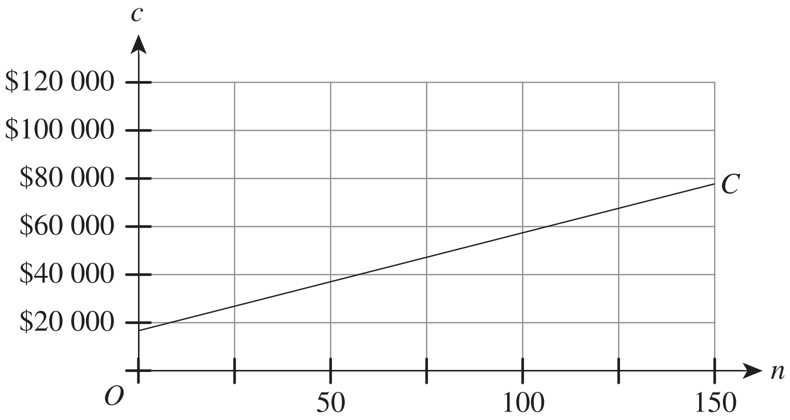
$\approx 287.3°$ A1

Module 3: Graphs and relations

Question 1

a. $C = 15\ 000 + 400n,\ 0 \le n \le 150$ A1

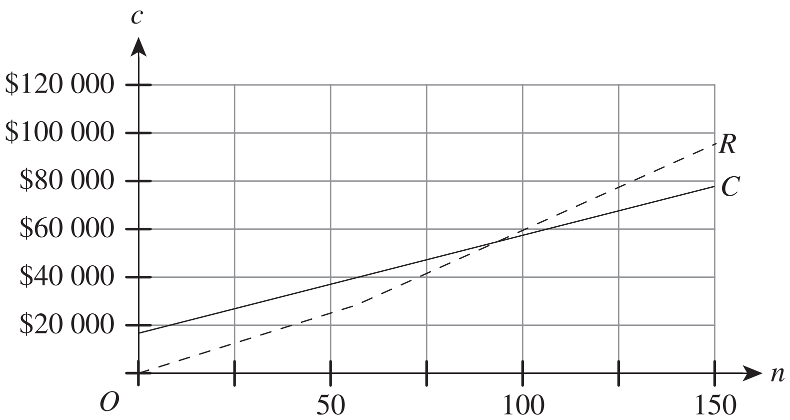
b.



A1

c. $R = \begin{cases} 500n & 0 \le n \le 60 \\ 700n - 12\ 000 & 60 < n \le 150 \end{cases}$ A1

d.



A1

e. From the graph, it can be seen that the break-even point occurs in the $n > 60$ section of the revenue line.

Thus $R = 700n - 12\ 000$ should be used.

$$\text{Find } C = R$$
$$15\ 000 + 400n = 700n - 12\ 000$$
$$\therefore\ 27\ 000 = 300n$$
$$n = 90$$

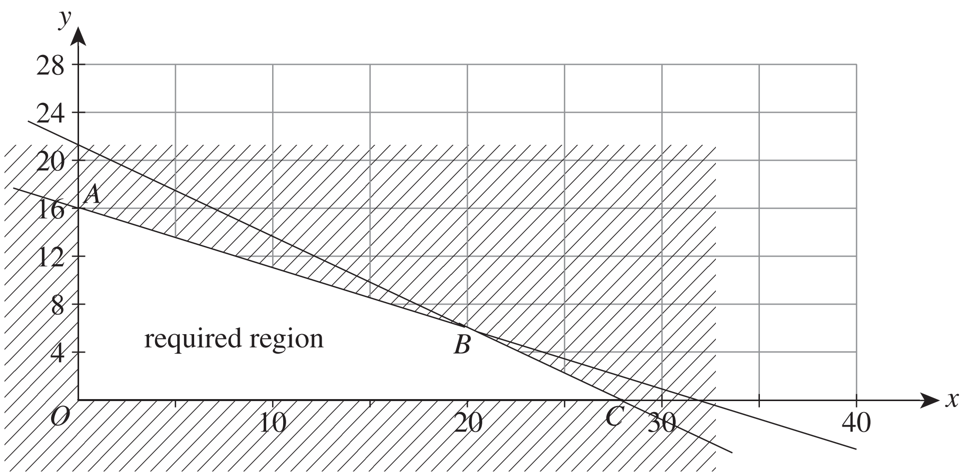
To make a profit, Greenozone must sell at least 91 tanks. A1

Question 2

a. $2x + 4y \leq 64$ welding A1

$3x + 4y \leq 84$ testing A1

b.



A1

One mark for both lines drawn correctly.

$2x + 4y = 64$

x intercept $y = 0$ $\qquad$ y intercept $x = 0$

$2x = 64$ $\qquad$ $4y = 64$

$x = 32$ (32, 0) $\qquad$ $y = 16$ (0, 16)

$3x + 4y = 84$

x intercept $y = 0$ $\qquad$ y intercept $x = 0$

$3x = 84$ $\qquad$ $4y = 84$

$x = 28$ (28, 0) $\qquad$ $y = 21$ (0, 21)

A1

One mark for the full set of intercepts.

Find intersection point B.

$3x + 4y = 84$ ... equation 1

$2x + 4y = 64$ ... equation 2

Subtracting equation 2 from equation 1,

$x = 20$

Substitute into equation 2,

$$2(20) + 4y = 64$$
$$4y = 24$$
$$y = 6$$

$B(20, 6)$ M1

Thus $A(0, 16)$, $B(20, 6)$ and $C(28, 0)$ are the corner points. A1

c. $P = 200x + 310y$ A1

d.

	x	y	P
A	0	16	4960
B	20	6	5860
C	28	0	5600

M1

From this it is clear that option B represents the maximum profit.

Each week, 20 domestic tanks and 6 garden tanks should be made. A1

e. From the table above, it is clear that a profit of \$5860 is the maximum that can be obtained each week. A1

Module 4: Business-related mathematics

Question 1

a. original price = $4295 \div (1 - 0.125)$

$= 4908.57$ M1

The original price was \$4909. A1

b. amount owing = $4295 - 1500$

$= 2795$ A1

interest paid = $145 \times 24 - 2795$

$= 685$ A1

c. interest rate = $\dfrac{1 \times 100}{(P \times t)}$

interest rate = $\dfrac{685 \times 100}{(2795 \times 2)}$ M1

$= 12.25$ A1

d. effective rate = $\dfrac{r_f \times 2n}{(n+1)}$

$= 12.25 \times \dfrac{(2 \times 12)}{(12+1)}$

$= 22.62$ A1

e. *Using a graphics calculator, use the TVM Solver:*

```
N=24
I%=11.45
PV=2795
PMT=
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

M1

The payment would be \$130.85 per month. A1

f. interest paid = $130.85 \times 24 - 2795$

$= 345.40$

Interest is \$345.40. A1

Question 2

a. amount of contribution = $30\ 000 \times 0.09 \times 0.85$

$= 2295$

The net amount is \$2295. A1

b. salary = $30\ 000 \times (1.03)^{40}$ M1

$= 97\ 861$

His salary would be \$97 861 per year. A1

c. *Using a graphics calculator, use the TVM Solver:*

```
N=
I%=6.5
PV=-1035000
PMT=75000
FV=0
P/Y=1
C/Y=1
PMT:END BEGIN
```

M1

He would be paid for 36 years. A1

Module 5: Networks and decision mathematics

Question 1

a. A Hamilton circuit. A1

b. Preston to Reservoir to Thomastown to Epping to Mill Park to Bundoora to Heidelberg to Preston.

OR

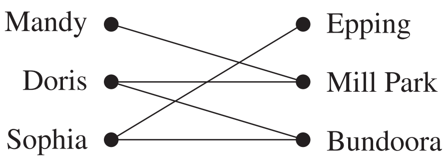
Preston to Heidelberg to Bundoora to Mill Park to Epping to Thomastown to Reservoir to Preston. A1

c. $4 + 5 + 2 + 8 + 6 + 9 + 9 = 43$ km A1

Question 2

a. *Solution: Method 1*

$$\begin{matrix} 42 & 33 & 36 \\ 38 & 34 & 33 \\ 31 & 31 & 27 \end{matrix} \qquad \begin{matrix} \cancel{9} & \cancel{0} & \cancel{3} \\ 5 & 1 & 0 \\ 4 & 4 & 0 \end{matrix} \qquad \begin{matrix} 9 & 0 & 4 \\ 4 & 0 & 0 \\ 3 & 3 & 0 \end{matrix} \qquad \begin{matrix} 6 & 0 & 4 \\ 1 & 0 & 0 \\ 0 & 3 & 0 \end{matrix}$$

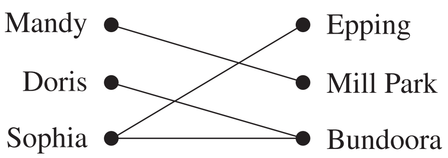


A1

OR

Solution: Method 2

$$\begin{matrix} 42 & 33 & 36 \\ 38 & 34 & 33 \\ 31 & 31 & 27 \end{matrix} \qquad \begin{matrix} 9 & 0 & 3 \\ 5 & 1 & 0 \\ 4 & 4 & 0 \end{matrix} \qquad \begin{matrix} 5 & 0 & 3 \\ 1 & 1 & 0 \\ 0 & 4 & 0 \end{matrix}$$



A1

	Suburb
Mandy	Mill Park
Doris	Bundoora
Sophia	Epping

A1

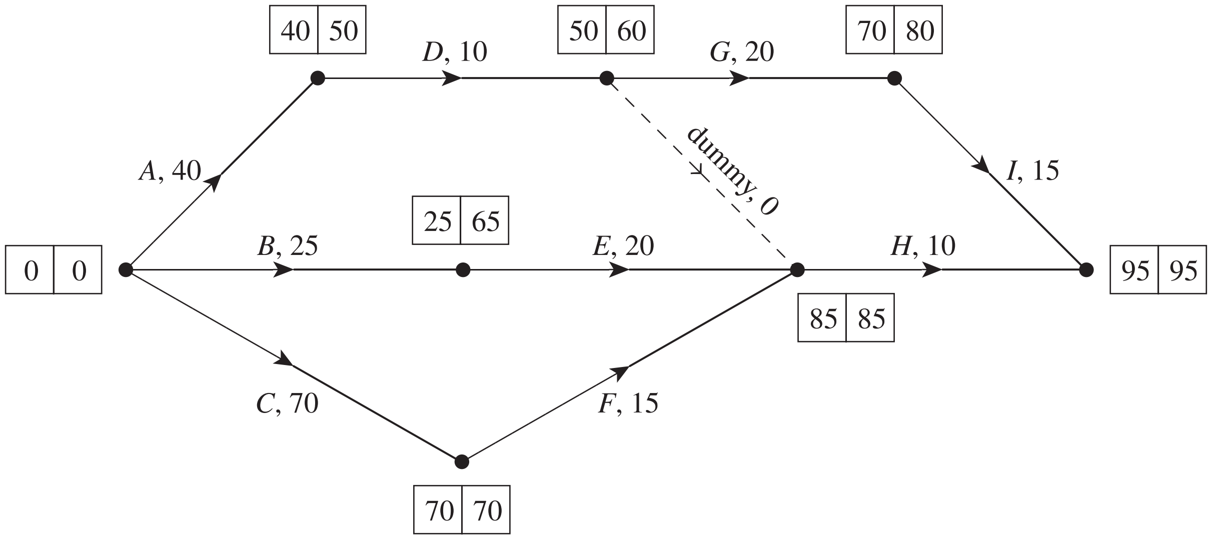
b. Mandy to Mill Park takes 33 minutes.

Doris to Bundoora takes 33 minutes.

Sophia to Epping takes 31 minutes.

Total = 97 minutes A1

Question 3



a. A, B, C, D, E, F. A1

b. The earliest starting time and latest starting time for each activity are shown in the table below.

Activity	Earliest starting time (minutes)	Latest starting time (minutes)
A	0	10
B	0	40
C	0	0
D	40	50
E	25	65
F	70	70
G	50	60
H	85	85
I	70	80

A2

One mark for the correct latest starting time for activity D.
One mark for the correct earliest starting time for activity H.

c. A, B, D, E, G, I. A1

d. There is a gap of 30 minutes between the latest finish time for G and the earliest starting time for G $(80 - 50 = 30)$.

G takes 20 minutes.

Therefore slack time $= 30 - 20$

$= 10$ minutes A1

e. All non-critical activities and their respective slack times are listed in the table below.

Activity	Minutes
A	10
B	40
D	10
E	40
G	10
I	10

A1

Activities *B* and *E* can be delayed the longest without delaying the entire project. A1

This can clearly be seen since activities *B* and *E* have a slack time of 40 minutes, while all other activities have only 10 minutes of slack time.

Question 4

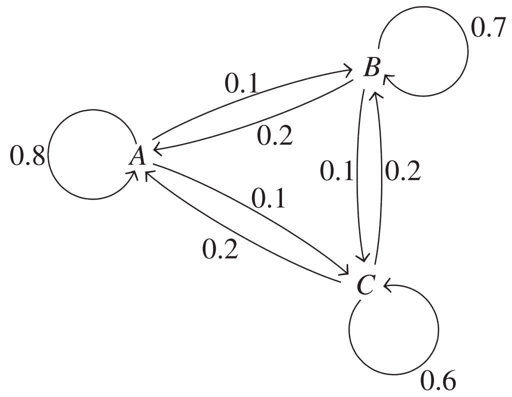
a. Activity *D* should be chosen to allocate the compulsory delay. This is because activity *F* is on the critical path. A1

b. Activity *D* has a slack (float time) of 10 minutes. Therefore, if it is delayed by 20 minutes then the overall effect on the project is that the project is delayed by 10 minutes. A1

Module 6: Matrices

Question 1

a.



The diagram above shows the annual transitions in populations. The matrix below summarises this.

$$T = \begin{bmatrix} 0.8 & 0.2 & 0.2 \\ 0.1 & 0.7 & 0.2 \\ 0.1 & 0.1 & 0.6 \end{bmatrix}$$ A1

b. $S_1 = TS_0$

$$= \begin{bmatrix} 0.8 & 0.2 & 0.2 \\ 0.1 & 0.7 & 0.2 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \begin{bmatrix} 0.4 \\ 0.3 \\ 0.3 \end{bmatrix} = \begin{bmatrix} 0.44 \\ 0.31 \\ 0.25 \end{bmatrix}$$ M1

$S_2 = TS_1$

$$S_2 = \begin{bmatrix} 0.8 & 0.2 & 0.2 \\ 0.1 & 0.7 & 0.2 \\ 0.1 & 0.1 & 0.6 \end{bmatrix} \begin{bmatrix} 0.44 \\ 0.31 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 0.464 \\ 0.311 \\ 0.225 \end{bmatrix}$$ M1

In 2008, 44% nest at area *A*, 31% nest at area *B* and 25% nest at area *C*.

In 2009, 46.4% nest at area *A*, 31.1.% nest at area *B* and 22.5% nest at area *C*.

A1

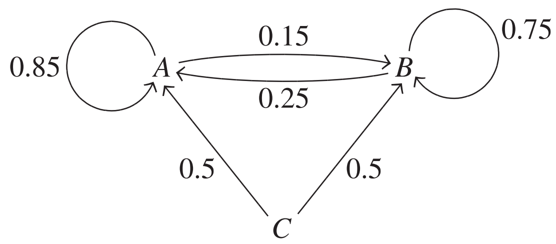
To earn the mark, correct calculations for both years are required.

c. To determine the long-term trend, calculate T^{50} and T^{51} on the calculator.

$$T^{50} = \begin{bmatrix} 0.5000 & 0.5000 & 0.5000 \\ 0.3000 & 0.3000 & 0.3000 \\ 0.2000 & 0.2000 & 0.2000 \end{bmatrix} \qquad S_{50} = \begin{bmatrix} 0.500 \\ 0.300 \\ 0.200 \end{bmatrix}$$ M1

In fact, T^{51} is identical (to an accuracy of four decimal places) to T^{50}. Thus it can be said that the proportions of the populations located across the different areas do stabilise. In all, 50% of the birds will end up at area *A* with 30% and 20% at area *B* and area *C*, respectively. A1

d.



M1

To earn the mark, either a correct diagram or a correct calculation is required.

The birds from area C have been divided equally between area A and area B, as required by the question.

$T = \begin{bmatrix} 0.85 & 0.25 & 0.50 \\ 0.15 & 0.75 & 0.50 \\ 0 & 0 & 0 \end{bmatrix}$ is the new transition matrix. A1

e. $S_{50} = T^{50} S_0$

$$= \begin{bmatrix} 0.625 & 0.625 & 0.625 \\ 0.375 & 0.375 & 0.375 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.40 \\ 0.30 \\ 0.30 \end{bmatrix}$$

$$= \begin{bmatrix} 0.625 \\ 0.375 \\ 0 \end{bmatrix}$$

The long-term proportion of the stork population at area A is 62.5% and at area B is 37.5%. A1

Question 2

a. Let x be the number of adult females.

Let y be the number of immature females.

eggs:

$$1.9x + 0.8y = 289$$
$$x + y = 210$$

M1

Thus $\begin{bmatrix} 1.9 & 0.8 \\ 1.0 & 1.0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 289 \\ 210 \end{bmatrix}$ A1

b. inverse $= \frac{1}{1.1}\begin{bmatrix} 1.0 & -0.8 \\ -1.0 & 1.9 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1.1}\begin{bmatrix} 1.0 & -0.8 \\ -1.0 & 1.9 \end{bmatrix}\begin{bmatrix} 289 \\ 210 \end{bmatrix}$$

$$= \frac{1}{1.1}\begin{bmatrix} 121 \\ 110 \end{bmatrix}$$

$$= \begin{bmatrix} 110 \\ 100 \end{bmatrix}$$

Thus 110 adult females and 100 immature females are in the area. A1

c. $[N] = [2.50 \;\; 1.00]\begin{bmatrix} 110 \\ 100 \end{bmatrix}$

$$= [375]$$

Thus there are 375 eggs. A1

d. Let a = eggs per adult female

Let b = eggs per immature female

$110a + 100b = 500 \qquad \therefore\ 11a + 10b = 50$

$a = b + 1.5$

$\therefore\ a - b = 1.5$

$$\begin{bmatrix} 11 & 10 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 50 \\ 1.5 \end{bmatrix}$$ M1

inverse $= \frac{-1}{21}\begin{bmatrix} -1 & -10 \\ -1 & 11 \end{bmatrix}$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{-1}{21}\begin{bmatrix} -1 & -10 \\ -1 & 11 \end{bmatrix}\begin{bmatrix} 50 \\ 1.5 \end{bmatrix}$$

$$= \frac{-1}{21}\begin{bmatrix} -65 \\ -33.5 \end{bmatrix} = \begin{bmatrix} 3.10 \\ 1.60 \end{bmatrix}$$

Thus each adult female would need to produce an average of 3.10 eggs per year and each immature female would need to produce an average of 1.60 eggs per year. A1