FURTHER MATHEMATICS

Units 3 & 4 – Written examination 1

TSSM
Creating VCE Success

2007 Trial Examination

SOLUTIONS

SECTION A: Core – Multiple-choice questions (1 mark each)

Question 1

Answer: C

Explanation:

This distribution is positively skewed with an outlier.

Question 2

Answer: A

Explanation:

$$\bar{x} = \frac{15 \times 0.5 + 5 \times 1.5 + 4 \times 2.5 + 3 \times 3.5 + 1 \times 8.5}{28} = \frac{44}{28} = 1.6$$

Question 3

Answer: B

Explanation:

148.7 is 2 standard deviations below the mean. Therefore 2.5% of women fit into this category.

Question 4

Answer: D

Explanation:

Independent variable is written as the column heading. Therefore the independent variable is Gender and the dependent variable is Type of soft drink.

Question 5

Answer: B

Explanation:

$$\frac{14}{92} \times 100 = 15\%$$

This is the number of females who purchase the sugar drink as a percentage of the total number of females.

Question 6

Answer: E

Explanation:

The greater range and greater IQR mean that there is greater variability in the birth weights of babies whose mothers smoke compared to those whose mothers do not smoke.

Question 7

Answer: D

Explanation:

For the babies of mothers who do not smoke, 0.26 is the interquartile range.
$Q_3 + 1.5 \times 0.26 = 3.86 + 0.39 = 4.25$. Thedot is at 4.3

Question 8

Answer: E

Explanation:

Three point median smoothing was carried out. The results are closest to

Sales ($'000)	35	42	15	19	32	46	20	25
Three point median smoothing		35	19	19	32	32	25	

Question 9

Answer: D

Explanation:

The information in this graph shows a negative cyclic trend. This is not seasonal as the pattern repeats in more than one year.

Question 10

Answer: D

Explanation:

The equation for the three median regression line is $y = 219 + 15x$ Time variables should be 1, 2, 3 etc. Enter these in L1 and the sales figures in L2. STAT, EDIT, 3 will give the 3 median regression line.

Question 11

Answer: C

Explanation:

The value of the seasonal index for the second quarter is closest to $4 - 0.85 - 0.62 - 1.34 = 1.19$.

Question 12

Answer: C

Explanation:

A prediction of the sales in the first quarter of 2007 using this information is closest to
$y = -1.4 \times 5 + 22.8 = 15.8$
$15.8 \times 0.72 = 11.376$
The answer should be in thousands of dollars = $11376.

Question 13

Answer: C

Explanation:

$$m = \frac{rs_y}{s_x} = \frac{0.8054 \times 3.3}{14.3} = 0.186$$

Gives the equation $y = 0.66 + 0.19x$

$$c = \overline{y} - m\overline{x} = 6.5 - 0.186 \times 31.4 = 0.66$$

SECTION B: Module 1: Number Patterns – Multiple-choice questions (1 mark each)

Question 1

Answer: A

Explanation:

The common difference d is $15 - 19 = 11 - 15 = -4$.

Question 2

Answer: A

Explanation:

$a + 4d = 32$

$a + d = 5$ $\quad a + 9 = 5$ The first three terms are -4, 5, 14.

$3d = 27$ $\quad a = -4$

$d = 9$

Question 3

Answer: B

Explanation:

$S_{12} = 6(2a + 11d) = 258$ $\quad 5d = 15$

$2a + 11d = 43$ $\qquad\qquad d = 3$

$a + 3d = 14$ $\qquad\qquad a + 9 = 14$ There are 5 seats in the first row.
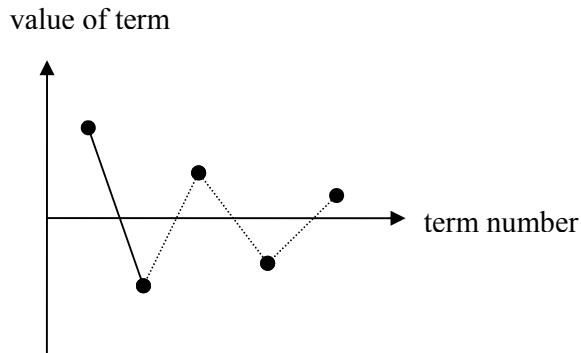
$2a + 6d = 28$ $\qquad\qquad a = 5$

Question 4

Answer: C

Explanation:

$a = 1$

$r = -\dfrac{2}{3}$ $\quad S_\infty = \dfrac{a}{1-r} = \dfrac{1}{1 - \dfrac{-2}{3}} = \dfrac{1}{1 + \dfrac{2}{3}} = \dfrac{1}{\dfrac{5}{3}} = \dfrac{3}{5}$

Question 5

Answer: D

Explanation:

value of term

The first five terms in the sequence are 2, -1, 0.5, -0.25, 0.125. This produces a graph like the one above.

Question 6

Answer: D

Explanation:

$t_2 = 5 \times -2 - 1$ $\quad$ $t_3 = 5 \times -11 - 1$
$\quad = -10 - 1$ $\quad\quad\quad = -55 - 1$
$\quad = -11$ $\quad\quad\quad\quad\; = -56$

Question 7

Answer: B

Explanation:

$D_n = 0.65D_{n-1}$ $\quad\quad$ *where* $\quad\quad$ $D_0 = 25000$; Depreciation is $1 - 0.35 = 0.65$

Question 8

Answer: E

Explanation:

This equation generates a pattern which is neither geometric nor arithmetic. The first few terms are: 120, 477, 1905, 7617, 30465.

Question 9

Answer: C

Explanation:

$t_3 = 3 \times 2 - 2 \times 1 = 4$
$t_4 = 3 \times 4 - 2 \times 2 = 8$

SECTION B: Module 2: Geometry and trigonometry – Multiple-choice questions (1 mark each)

Question 1

Answer: A

Explanation:

For the triangle shown, the size of the angle θ is closest to $\theta = \cos^{-1}\left(\dfrac{18.7}{22.4}\right) \approx 33°$.

Question 2

Answer: D

Explanation:

$$A = \sin^{-1}\left(\frac{2.6 \sin 39}{1.8}\right) \approx 65°$$
$$C = 180 - 39 - 65 \approx 76°$$

Question 3

Answer: D

Explanation:

For the shape shown, the volume is closest to $V = 6 \times 8 \times 4 + 0.5 \times 6 \times 4 \times 7 = 276 cm^2$.

Question 4

Answer: B

Explanation:

$$V_1 = \frac{\pi r^2 h}{3}$$

$$V_2 = \frac{\pi \left(\frac{r}{2}\right)^2 \left(\frac{h}{2}\right)}{3}$$

$V_1 = V_2 \times 8 = 60 \times 8 = 480 cm^3$

Question 5

Answer: C

Explanation:

Using the information given the area of triangle *DEF* is found by using Heron's rule.

$$s = \frac{1}{2}(24 + 20 + 16) = 30$$

$$Area = \sqrt{(30(30-24)(30-20)(30-16))}$$

Question 6

Answer: A

Explanation:
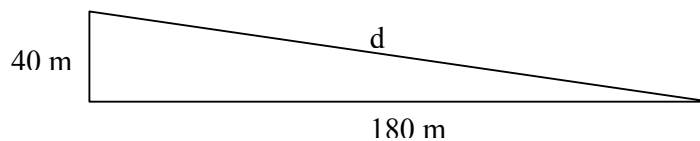
The distance between islands *B* and *C* is closest to

$$a = \sqrt{b^2 + c^2 - 2bc\cos A} = \sqrt{4^2 + 8^2 - 2 \times 4 \times 8 \times \cos 32} \approx 5km$$

Question 7

Answer: E

Explanation:

First $B = \cos^{-1}\left(\dfrac{a^2 + c^2 - b^2}{2ac}\right) = 125.10°$, using $a \approx 5$. The bearing of *C* from *B* is closest to

$360 - (125.10 + 110) \approx 125°$.

Question 8

Answer: C

Explanation:

The length *MN* is closest to $l = \dfrac{15 \times (12 + 24)}{24} = 22.5cm$

Question 9

Answer: D

Explanation:

$Rise = 40m, \quad run = \dfrac{3 \times 6000}{100}m = 180m, \quad d = \sqrt{180^2 + 40^2} \approx 184m$

SECTION B: Module 3: Graphs and relations – Multiple-choice questions (1 mark each)

Question 1

Answer: E

Explanation:

Comparing with $y = mx + c$ .gives a *y*-intercept of -5.

Question 2

Answer: D

Explanation:

The total amount Nettie will have to pay over this time period is $5 \times 60 \times 6 = \$1800$.

Question 3

Answer: D

Explanation:

$(40 \times 2 + 60 + 70 \times 2) \times 6 = \$1680$

Question 4

Answer: E

Explanation:

Brian passed Betty on their way to work. This occurs where the two lines cross on the graph. All other assertions given in the question are false.

Question 5

Answer: B

Explanation:

$2 \times 3 + 3 \times 1 = 9 \geq 8$

Question 6

Answer: D

Explanation:

$3x + 2y \leq 60$ *line*   2
$2x + 5y \geq 50$ *line*   3
$10x + 3y \geq 150$ *line*   1

In each case the gradient and y-intercept was found from the graph and then the appropriate greater than or less than symbol was chosen.

Question 7

Answer: B

Explanation:

Solving each pair of simultaneous equations and rounding to the nearest whole number in each case. Eg for lines 2 and 3.

$$11y = 30$$
$$3x + 2y = 60 \quad 6x + 4y = 120$$
$$y = 2.\overline{72} \approx 3$$
$$2x + 5y = 50 \quad 6x + 15y = 150$$
$$x \approx 18.2$$

Question 8

Answer: C

Explanation:

Using $\$P = x + 2y$ for the point $(11,14)$ the profit is maximised: $\$P = 11 + 28 = \$39$.

Question 9

Answer: B

Explanation:

$$m = \frac{1}{3} \quad c = 2 \qquad 3y = x + 6 \qquad 0 \leq x \leq 6$$
$$\qquad\qquad\qquad\qquad y = x - 2 \qquad 6 \leq x \leq 10$$
$$m = 1 \quad c = -2$$

SECTION B: Module 4: Business-related mathematics – Multiple-choice questions (1 mark each)
Question 1

Answer: B

Explanation:

The amount of interest earned over the six years is: $SI = \dfrac{PRT}{100} = 7500 \times 0.0625 \times 6 = \$2812.50$.

Question 2

Answer: B

Explanation:

The amount of interest earned over the six years is closest to:

$I = A - P = PR^n - P = 7500 \times \left(1 + \dfrac{4.35}{400}\right)^{24} - 7500 = \$2223$

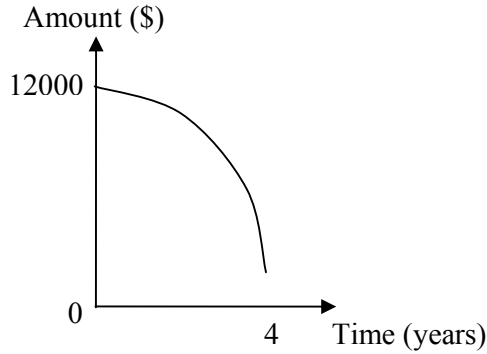**Question 3**

Answer: A

Explanation:

$\dfrac{(6400 - 3800)}{2} = 1300, \qquad \dfrac{1300}{6400} \times 100 \approx 20\%$

Question 4

Answer: D

Explanation:

$(22340 + 28480) \times 0.32 \approx \$16260$

Question 5

Answer: D

Explanation:

$48000 \times \left(1 + \dfrac{7.8}{1200}\right) - 350 = \$47962$

Question 6

Answer: A

Explanation:

Reducing balance loans take the shape of the graph above. The loan would not be paid back in 4 years so the graph should end above the axis.

Question 7

Answer: C

Explanation:

Using the TVM Solver, $N = 52 \times 4, I = 3.5, PV = -150, PMT = -10, \dfrac{P}{Y} = 52, \dfrac{C}{Y} = 52$ therefore $FV = \$2404.37$

Question 8

Answer: C

Explanation:

Find R first, $R = \dfrac{R_e(N+1)}{2N} \approx 13.27\%$.

Then $398 + I = 398 + \dfrac{PRT}{100} = 398 \times 13.27 \times \dfrac{12}{5200} + 398 = \$410.19$. Now divide by 12 weeks, this yields $\$34.18$.

Question 9

Answer: B

Explanation:

$$398 + I = 398 + 398 \times 15 \times \frac{12}{5200} = 411.78. \ \text{Difference} = 411.78 - 410.19 = \$1.59$$

SECTION B: Module 5: Networks and decision mathematics – Multiple-choice questions (1 mark each)

Question 1

Answer: C

Explanation:

An Euler path is possible for this network if you start at K. This vertex has odd degree and there is one other vertex with odd degree (M) so the path will start at one and end at the other.

Question 2

Answer: C

Explanation:

Quentin - banana, Roger – blueberry, Sarah – choc chip, Tina – raspberry, Ursula - cinnamon
This is the only solution which allocates each person to a speciality.

Question 3

Answer: E

Explanation:

The ratings are maximised. To minimise the matrix first subtract each result from 10. Then apply a row reduction followed by a column reduction to get the allocation.

$$\begin{bmatrix} 3 & 1 & 3 & 7 \\ 3 & 6 & 6 & 5 \\ 7 & 2 & 9 & 5 \\ 5 & 4 & 6 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 7 & 9 & 7 & 3 \\ 7 & 4 & 4 & 5 \\ 3 & 8 & 1 & 4 \\ 5 & 6 & 4 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 6 & 4 & 0 \\ 3 & 0 & 0 & 1 \\ 2 & 7 & 0 & 3 \\ 1 & 2 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 6 & 4 & 0 \\ 2 & 0 & 0 & 1 \\ 1 & 7 & 0 & 3 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

Question 4

Answer: B

Explanation:

$v + f - e = 2, v = e + 2 - f = 12 + 2 - 9 = 5$

Question 5

Answer: B

Explanation:

Cut 2: 800 +800 = 1600. This is the minimum cut. (550 is not included in this calculation because it is going towards the source)

Question 6

Answer: C

Explanation:

Activities *E* and *F* must be completed before *H* can begin.

Question 7

Answer: B

Explanation:

36 days along the critical path *ABEHJ*.
Answer A is the total of all paths.
Answer C is found using path *ABDGJ*.
Answers D & E are the maximum and minimum cuts.

Question 8

Answer: C

Explanation:

Using a back scan activity *G* can start on day 20.
Answer A was the reference number on the network.
Answer B was the earliest start time.
Answers D & E were the earliest and latest finishing times.

Question 9

Answer: D

Explanation:

Because D is not on the critical path and has a float time of 5 days a delay of 4 days has no effect on the overall project time.

SECTION B: Module 6: Matrices – Multiple-choice questions (1 mark each)

Question 1

Answer: D

Explanation:

If you multiply the scalar 7 by each of the numbers in the matrix the answer is D.

Question 2

Answer: A

Explanation:

$$\begin{bmatrix} 6 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 4 & -1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 6\times4+2\times3 & 6\times-1+2\times-1 \\ 4\times4+3\times3 & 4\times-1+3\times-1 \end{bmatrix} = \begin{bmatrix} 30 & -8 \\ 25 & -7 \end{bmatrix}$$

Question 3

Answer: D

Explanation:

$4\times\text{-}1-3\times\text{-}1 = \text{-}4+3 = \text{-}1$

Question 4

Answer: C

Explanation:

There are no unique solutions for the second and last equation sets as the two equations in each set are identical except for a scalar.

Question 5

Answer: B

Explanation:

40% of people who purchased The Standard will change to The Argus should read:
40% of people who purchased The Argus will change to The Standard, so this sentence is false.

Question 6

Answer: E

Explanation:

$$\begin{bmatrix} 0.95 & 0.3 \\ 0.05 & 0.7 \end{bmatrix}$$

This matrix explains that 95% of Venus bar purchasers will not change and 30 % of Giggles bar purchasers will change to Venus.

Question 7

Answer: B

Explanation:

$$\begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$$

This matrix explains that 60% of chocolate purchases were for Venus brand and therefore 40% were for the Giggles brand.

Question 8

Answer: D

Explanation:

$$\begin{bmatrix} 0.95 & 0.3 \\ 0.05 & 0.7 \end{bmatrix}^3 \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix} = \begin{bmatrix} 0.787 \\ 0.213 \end{bmatrix}$$

Therefore 79% of chocolate lovers will purchase Venus bars in three weeks time.

Question 9

Answer: C

Explanation:

$$\begin{bmatrix} 0.95 & 0.3 \\ 0.05 & 0.7 \end{bmatrix}^5 \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix} = \begin{bmatrix} 0.827 \\ 0.173 \end{bmatrix}$$

Therefore 82.7% of chocolate lovers will purchase Venus bars in five weeks time.