2008 Further Mathematics Trial Examination 2 Suggested Solutions

## 2008

VCE Further Mathematics Trial Examination 2

# **Suggested Solutions**

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#### Core

#### Question 1 a.

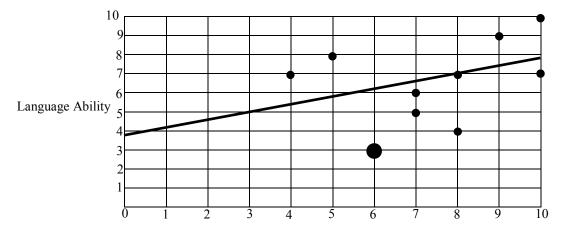
For T183 calculator enter data in stat edit in column  $L_1$ . Go to stat calc, 1-var stats and enter  $L_1$ . This gives mean = 6.1 and standard deviation = 1.6Mean = 6.1, Standard Deviation = 1.6(1 mark) for each correct value **b.(i)** b.(ii) 6 is two standard deviations above the mean.  $Z = \frac{x-4}{1} < -2$ This student is in the top 2.5% of Australian students for artistic ability. x - 4 < -2(1 mark) *x* < 2 Since no student in the sample got a score of less than 2 then no student in the sample was in the lowest 2.5% for artistic ability. (1 mark) c.(i) Dependent Variable: Mathematical Ability Independent Variable: Year level (1 mark)c.(ii) For T183 calculator enter data in stat edit in column  $L_2$ . Go to 2<sup>nd</sup> stat plot and ensure all plots are off. Then,  $2^{nd}$  stat plot enter on, box plot,  $2^{nd}L_2$ . Graph. Use the trace to find Min = 4, LQ = 6, Med = 7.5, UQ = 9, Max = 10Year 9 Year 10 0 2 3 6 8 104 (1 mark) Mathematical Ability

#### Core

**c.(iii)** Year 10 students on the whole have better mathematical ability than year 9 students. The lowest year 10 mark is equal to the year 9 lower quartile and 75% of year 10 students score a mark greater than the year 9 upper quartile.

(1 mark)

#### Question 2 a. & c.



Mathematical Ability (1 mark) for least squares regression line and (1 mark) for required point.

2 b.	2 d.
For T183 calculator enter year 10 language ability	Using the same calculator screen as for 2b.
data in stat edit in column $L_3$ . Go to stat calc	$r^2 = 0.1198 = 12\%$ to the nearest whole number
linear regression (ax+b) $L_2$ , $L_3$	
This gives	
Language Ability = $0.4 \times Mathematical$ Ability+3.8	
	(1 mark)
(1 mark)	
2 e.	2 f.
12% of the variation in English ability can be	<i>residual</i> = <i>data value</i> – <i>predicted</i> value
explained by the variation in mathematical ability.	residual = 3 - 6.2 = -3.2
(1	(1 mark)
mark)	
2 g.	
$x_L = 5, y_L = 7, x_U = 10, y_U = 9$	
$m = \frac{9-7}{10-5} = 0.4$	
$m^{-1}10-5^{-0.4}$	
y = 0.4x + c	
When $x = 5, y = 7$	
$7 = 0.4 \times 5 + c$	
7 = 2 + c	
<i>c</i> = 5	

*Language ability* =  $0.4 \times$  *mathematical ability* + 5

## (1 mark)

Question 1 a.		b.
$7 \times 1 = 7 \text{ cm}^2$		$Length = 4 + 6 \times 3 = 22$
	(1 mark)	Area = $22 \times 1 = 22 \text{ cm}^2$
		(1 mark)
		,
<b>c.</b> Areas = 4, 7, 10		<b>d.</b> Use T1-83 graphics calculator in sequence mode.
This is an arithmetic sequence		y =
$S_n = \frac{n}{2} [2a + (n-1)d]$		enter nMin = 1
$S_{20} = \frac{20}{2} [2 \times 4 + 19 \times 3]$		$\mu_n = 4 + (n-1) \times 3$
2		$\mu(nMin) = 4$
$S_{20} = 650 \text{ cm}^2$		Press 2nd table
	(1 morte)	Scroll down till the value is 79,
	(1 mark)	the question asks how many times would it increase
		This occurs one less than the value of
		<i>n</i> given in the table, so $n = 25$
		(1 mark)
Question 2 a.		<b>b.</b>
$A = 9 \times 1.2 = 10.8 \text{ cm}^2$	(1	Use graphics calculator in sequence mode. $y = -$
	(1 mark)	y = enter
		nMin = 1
		$\mu_n = 9 \times (1.2)^{n-1}$
		$\mu(nMin) = 9$
		Press 2nd table
		The 5th press of the button will correspond with
		<i>n</i> =6
		$\mu_n = 22.4$ to one decimal place
		(1 mark)

#### Module 1 Number patterns and applications

#### Question 2 c.

Use graphics calculator in sequence mode. y =enter nMin = 1  $\mu_n = 9 \times (0.4)^{n-1}$   $\mu(nMin) = 9$ Press 2nd table When n = 7,  $\mu_n = 0.03686$ n = 7 corresponds to 6 presses of the button

(1 mark)

Question 3 a.		<b>b.</b>
$t_1 = t_3 - t_2$		Sequence is 400,700,1100
$t_3 = t_1 + t_2 = 400 + 700 = 1100 \mathrm{m}$		$\frac{700}{\neq} \frac{1100}{\neq}$
	(1 mark)	$\frac{1}{400} \neq \frac{1}{700}$
		not geometric sequence.
		(1 mark)
с.		d.
$t_4 = 1100 + 700 = 1800$		Use graphics calculator in sequence mode.
$t_5 = 1800 + 1100 = 2900$		y = optor
$t_6 = 2900 + 1800 = 4700$ m.		enter nMin = 1
	(1 mark)	$\mu(n) = \mu(n-2) + \mu(n-1)$
	· · · ·	
		$\mu(nMin) = \{700, 400\}$
		Press 2nd table and look for 357100 in the $\mu(n)$
		column
		This corresponds to $n = 15$
		(1 mark)
<b>e.</b> Using the same screen as for the pro-	evious	
question		
$t_{11} = 52100$		
$t_{10} = 32200$		
Distance between these stops = $t_{11}$ -	$-t_{10}$	
= 52100 - 32200 = 19900 m.		
	(1 mark)	

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## Module 1 Number patterns and applications

Question 4 a.	b.
$S_3 = 4^3 - 1 = 63$	$S_1 = 4 - 1 = 3$
(1 mark)	$S_1 = 4 - 1 = 3$ $S_2 = 4^2 - 1 = 15$ $S_3 = 4^3 - 1 = 63$
	$S_3 = 4^3 - 1 = 63$
	$t_1 = 3$
	$t_2 = 15 - 3 = 12$ $t_3 = 63 - 15 = 48$
	$t_3 = 63 - 15 = 48$
	Distance run on each of first 3 days is 3,12,48
	(1 mark)
<b>c.</b> 3, 12, 48 forms a geometric sequence	
because it has a common ratio of 4	
(1 mark)	

## Module 2 Geometry and trigonometry

Question 1 a.	b.
Isosceles triangles have base angles equal.	In triangle $BCX$ , $\angle BCX = 51.5^{\circ}$ , $BC = 230$
Sum of angles of a triangle = $180^{\circ}$	$\cos 51.5^{\circ} = \frac{CX}{250}$
$\angle ABC = 180 - 2 \times 51.5 = 77^{\circ}$	$\cos 51.5^\circ = \frac{1}{230}$
	$CX = 230\cos 51.5^{\circ} = 143.1784$
(1 mark)	$AC = 2 \times CX = 2 \times 143.1784 = 286.36$ cm
	(1 mark)
	d.
In triangle <i>BCX</i> ,	CF is 2.5 times larger than $AC$
$\sin 51.5^{\circ} = \frac{BX}{230}$	$CF = 2.5 \times 286.36 = 716$ cm
$BX = 230 \sin 51.5^{\circ} = 180$ cm.	(1 mark)
(1 mark)	
е.	Question 2 a.
Area of $ACFD = AC \times CF$	AC
$= 286.36 \div 100 \times 715.9 \div 100 = 21 \text{ m}^2$	350 50 Y
(1 mark)	$\sin 50^\circ = \frac{CY}{350}$
	$CY = 350 \sin 50^\circ = 268.12$
	CY + BX = 268 + 180 = 448 cm.
	(1 mark)

## Module 2 Geometry and trigonometry

Question 2 (continued) b.	Question 3 a.
$V = Area of triangular base \times height$	$OC = \text{ radius of sphere} = \frac{1}{2} \times AB$
V = Area of triangular $ABC \times CF$	$OC = \frac{1}{2} \times 130 = 65 \text{ mm}$
$V = \frac{1}{2} \times 286.36 \div 100 \times 180 \div 100 \times 715.9 \div 100$	$2^{\times 100-00}$ mm
$V = 18 m^3$	
(1 mark)	(1 mark)
<b>b.</b> $SA = \frac{1}{2} \times 4\pi r^{2} + \pi r^{2}$ $SA = 3\pi r^{2}$	c. $Q = \frac{65}{60}$ Y
$SA=3\pi \times 65 \div 10 \times 65 \div 10$	Using Pythagoras rule or knowing your triads $OQ = 25$
SA=398 cm <sup>2</sup> (1 mark)	OQ = 25 OQ = 25 OC = 65 QC = 65 - 25 = 40 mm
	(1 mark)
Question 4 a. The triangles are similar because 3 angles of triangle <i>ABD</i> equal three angles of triangle <i>BCD</i> . $\angle CBD = \angle EFD$ , so $\angle BAC = \angle CBD$ $\angle BDC$ is common So the remaining angles of the triangles are equal.	<b>b.</b> Because the triangles are similar, then their corresponding sides are in the same ratio. $\frac{CD}{BD} = \frac{BD}{AD}$ $BD^{2} = CD \times AD$ $BD^{2} = 5 \times 20 = 100$ $BD = 10 \text{ cm}$
(1 mark)	(1  mark)
c. $\frac{AB}{BC} = \frac{AD}{BD}$ $\frac{AB}{7} = \frac{20}{10}$ $AB = 14 \text{ cm}$	(1 mark)

## Module 3 Graphs and relations

Question 1 a.		b.	
6 km.	(1 mark)	Fastest speed is steepest gradient so A	
	(1 mark)		(1 mark)
<b>c.</b>			
speed= $\frac{\text{distance}}{\text{time}} = \frac{6}{\frac{1}{4}} = 24 \text{ km/hr}$			
$\frac{1}{4}$			
		-	(1 mark)
Question 2 a.		<b>b.</b> $I = 5x$	
C = 69 + 2x		1 - 5x	
	(1 mark)	-	(1 mark)
<b>c.</b> $69 + 2x = 5x$		$\begin{array}{l} \mathbf{d.} \\ P = 5x - 69 - 2x \end{array}$	
69 = 3x		P = 3x - 69	
<i>x</i> = 23		If $x = 30$	
	(1 mark)	P = 90 - 69 = \$21	
Question 3 a.	(1 mark)	b.	(1 mark)
$x \ge 10$		$x \ge 2y$	
$y \ge 10$		or $y \le \frac{1}{2}x$	
	(1 mark)	2	(1 mark)
с.		d.	(1 111411)
C = 20x + 10y	(1 1)	$20x + 10y \le 600$	
	(1 mark)	$2x + y \le 60$	(1 1)
e.			(1 mark)
	<i>y</i> <b>≜</b>		
		x = 10	
		$y = \frac{1}{2}x$	
		y = 10	
		y 10	
		x  2x + y = 60	
			(1 mark)

## Module 3 Graphs and relations

#### Question 3 (continued)

<b>f.</b> Number of prizes = $x + y$	$\begin{array}{c} \mathbf{g.} \\ C = 20x + 10y \end{array}$
Points of intersection	$C = 20 \times 24 + 10 \times 12 = $600$
$y = 10 \text{ and } y = \frac{1}{2}x$ (20,10) No. prizes = 30	(1 mark)
y = 10 and $2x + y = 60$ (25,10) No. prizes = 35	
$y = \frac{1}{2}x$ and $2x + y = 60$	
$2\frac{1}{2}x = 60$	
x = 24 (24,12) No. prizes = 36	
Maximum number of prizes = 36 (1 mark) for realizing number of prizes is $x + y$ and (1 mark) for correct answer)	

#### Module 4 Business-related mathematics

Question 1 a.	<b>b.</b> $85\% \equiv 1200$
Profit $= 302 - 234 = 68$	
% Profit $=\frac{68}{234} \times 100 = 29.1\%$	$1\% = \frac{1200}{85}$
(1 mark)	$100\% \equiv \frac{1200}{85} \times 100 = \$1412$
	(1 mark)
c. (i)	c.(ii)
Amount paid $= 300 + 70 \times 24 = $1980$	Interest = $1980 - 1200 = $780$
(1 mark)	(1 mark)
c.(iii)	Question 2 a.(i)
$R = \frac{100I}{PT} = \frac{100 \times 780}{900 \times 2} = 43.33\%$	$I = \frac{PRT}{100} = \frac{28000 \times 8.5 \times 0.5}{100} = \$1190$
(1 mark)	
	(1 mark)

## Module 4 Business-related mathematics

Question 2 (continued)	
a.(ii)	b.(i)
Amount repaid = $620 \times 6 = 3720$ (1 mark)	Use graphics calculator
Amount repaid off principal $= 3720 - 1190$	Press Apps Finance Enter TVM Solver Enter
Amount repaid off principal = 2530	<i>N</i> = 6
Amount owing = $28000 - 2530 = $25,470$	I = 8.5
(1 mark)	PV = 28000
	PMT = -620
	FV =
	P/Y = 12
	C/Y = 12
	End
	Put cursor on FV and press alpha solve.
	FV = \$25,424.77
	$1^{\circ}v = 323,424.77$ (1 mark)
b.(ii)	b.(iii)
Amount paid over 6 months = $620 \times 6 = 3720$	Use graphics calculator
Amount paid off principal = $PV - FV$	Press Apps Finance Enter TVM Solver Enter
Amount paid off principal =28000-25424.77	<i>N</i> = 96
Amount paid off principal =2575.23 (1 mark)	I = 8.5
Amount paid in interest $=3720 - 2575.23$	PV = 28000
Amount paid in interest =\$1,144.77	PMT =
in Franciscus () in the	FV = 0
(1 mark)	P/Y = 12
	C/Y = 12
	End Det surray on <i>DMT</i> and surray alaba salar
	Put cursor on <i>PMT</i> and press alpha solve.
	PMT = \$402.98 (1 mark)
Question 3	a. ii.
a. (i)	$BV = 60000(1 - 0.09)^6 = $34,072.16$
Depreciation per annum = $\frac{11}{100} \times 60000 = 6600$	(1 mark)
Depreciation over 6 years = $6600 \times 6 = 39600$	
BV = 60000 - 39600 = \$20,400	
(1 mark)	
<b>b.</b> Depreciation = $60000 - 24000 = 36000$	
Ĩ	nark)
number of patients = $\frac{36000}{2.4}$ =15,000	······,

## Module 5 Networks and decision mathematics

Question 1 a.	b.(i)
То	2
A  B  C  D	(1 mark)
$A \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$	
<i>From B</i> 0 0 2 0	
From B     0     0     2     0       C     0     0     1       D     0     1     0     1	
$D \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$	
(1 mark)	
<b>b.(ii)</b> The number of roads leaving A	c. The number of roads coming into D
(1 mark)	•
	•
<b>Question 2 a.</b> It is possible to drive along each road once and	<b>b.</b> 6+9+3+4+2+6+8+7=45 km.
visit each site if you start at Botanical Gardens	0+7+5+4+2+0+6+7=45 KIII.
and finish at Daffodil display, or vice versa	(1 mark)
B-A-B-C-D-E-F-G-D is one possible route.	
(1 mark)	Question 3 a.
2,00	
	E, 31
	A, 15
A-B-C-D-E-F-G 6+3+4+2+6+8=29 km.	B, 23 G, 29
	C, 17 D, 19 J, 22
(1 mark)	
	(1 mark)
b.	c.
The critical path is the longest path.	17+19+18+22=76 days
C-D-F-I.	(1 mark)
(1 mark)	e.
17 + 19 - 23 = 13 days	0 days, because F is on the critical path.
(1 mark)	(1 mark)
Question 3 f.	g.
CDFI are still on the critical path. Cost to reduce time for $C = $1,200$	There is now a new critical path. A-E-I. 15+31+22 = 68 days.
Cost to reduce time for $D = \$900$	15+51+22 00 days.
Cost to reduce time for $F = \$800$	(1 mark)
Cost to reduce time for $I = $4,000$	and a)
Total cost of reduction = $6,900$ (1 m Number of days saved = $1+1+2+4=8$	aik)
Cost of 8 days = $$40,000$	
Amount saved = $40,000 - 6,900 = $33,100$ (1 m	ark)

## Module 6 Matrices

Oraction 1 -		
Question 1 a. Cotton Silk Denim		$\begin{bmatrix} 100 + 240 + 80 \end{bmatrix} \begin{bmatrix} 420 \end{bmatrix}$
	-	<b>b.</b> $A = \begin{vmatrix} 150 + 80 + 10 \end{vmatrix} = \begin{vmatrix} 240 \end{vmatrix}$
X 80 10 100		120+360+100 580
$M = Y \mid 40  5  150$		
Z 60 3 200		(1 mark)
	-	
	(1 mark)	
c.		d.
3×1	<i></i>	100+240+80 = 420
	(1 mark)	150+80+10 = 240
		120+360+100 = 580 250+120+40 = 410
		250+120+40 - 410 164+60+5 = 229
е.		80+280+150 = 510
		160+175+60 = 395
Cotton 35		260+70+3 = 333
C = Silk 70		75+200+200=475
Denim 20		
		V V 7
	(1 mark)	X Y Z
		Cotton 420 410 395
		A = Silk   240   229   333
		Denim 580 510 475
		(1 mark)
f.		g.
Cotton Silk Denim		Use graphics calculator to get
$X \begin{bmatrix} 420 & 240 & 580 \end{bmatrix}$		
P = Y 410 229 510		$\begin{bmatrix} 420 & 240 & 580 \end{bmatrix} \begin{bmatrix} 35 \end{bmatrix} X \begin{bmatrix} 43100 \end{bmatrix}$
		$Q = \begin{bmatrix} 410 & 229 & 510 \\ 395 & 333 & 475 \end{bmatrix} \times \begin{bmatrix} 70 \\ 20 \end{bmatrix} = \begin{bmatrix} 7 \\ 40580 \\ 46635 \end{bmatrix}$
$Z \begin{bmatrix} 395 & 333 & 475 \end{bmatrix}$	<i></i>	395 333 475 20 Z 46635
	(1 mark)	(1 mark)
		(T Hark)
h.		
156000 - 40580 = \$115,420		
	<i>.</i>	
	(1 mark)	

## Module 6 Matrices

Question 2 a.		b.
$\begin{array}{c} \text{Today} \\ W & \text{NR} \\ \text{Tomorrow} \begin{array}{c} W \begin{bmatrix} 0.7 & 0.6 \\ 0.3 & 0.4 \end{bmatrix} \end{array}$		$S_0 = \begin{bmatrix} 60\\20 \end{bmatrix}$
	(1 mark)	(1 mark)
<b>c.</b> $S_{1} = \begin{bmatrix} 0.7 & 0.6 \\ 0.3 & 0.4 \end{bmatrix} \begin{bmatrix} 60 \\ 20 \end{bmatrix} = \begin{bmatrix} 54 \\ 26 \end{bmatrix}$ 54 are working and 26 need repair.	(1 mark)	<b>d.</b> $S_{30} = T^{30}S_{0}$ $S_{30} = \begin{bmatrix} 0.7 & 0.6 \\ 0.3 & 0.4 \end{bmatrix}^{30} \begin{bmatrix} 60 \\ 20 \end{bmatrix} = \begin{bmatrix} 53 \\ 27 \end{bmatrix}$ In the long term Rex can expect 53 of the Orange computers to be working. (1 mark)
<b>e.</b> $S_2 = \begin{bmatrix} 0.8 & 0.5 \\ 0.2 & 0.5 \end{bmatrix}^2 \begin{bmatrix} 70 \\ 10 \end{bmatrix} = \begin{bmatrix} 58 \\ 22 \end{bmatrix}$	(1 mark)	<b>f.</b> $S_{30} = T^{30}S_{0}$ $S_{30} = \begin{bmatrix} 0.8 & 0.5 \\ 0.2 & 0.5 \end{bmatrix}^{30} \begin{bmatrix} 70 \\ 10 \end{bmatrix} = \begin{bmatrix} 57 \\ 23 \end{bmatrix}$ He should buy more of the Lemon brand because in the long term 57 out of 80 would be expected to be working whereas only 53 of the Orange computers would be expected to be working.
		(1 mark)

#### End of suggested solutions 2008 Further Mathematics VCE Trial Examination 2

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