Neap

Trial Examination 2008

VCE Further Mathematics Units 3 & 4

Written Examination 2

Suggested Solutions

SECTION A – DATA ANALYSIS – CORE MATERIAL

Question 1

a. **i.** 32 A1

ii. $\frac{(70+85+30+7+17+32)}{6} = \frac{241}{6}$

$= 40.1\dot{6}$

$= 40.2$ A1

iii. Enter war memorial figures into L1, and use Stat → Calc → 1-varStat

$\sigma_x = 94.11$ A1

b. **i.** $\frac{45}{32.25} = 1.395$

$\frac{52}{35} = 1.486$

$\frac{61}{41.75} = 1.461$

$\frac{1.395 + 1.486 + 1.461}{3} = 1.447$

$= 1.45$ A2

ii.

	1st quarter	2nd quarter	3rd quarter	4th quarter
Seasonal index	1.45	0.84	0.73	0.98

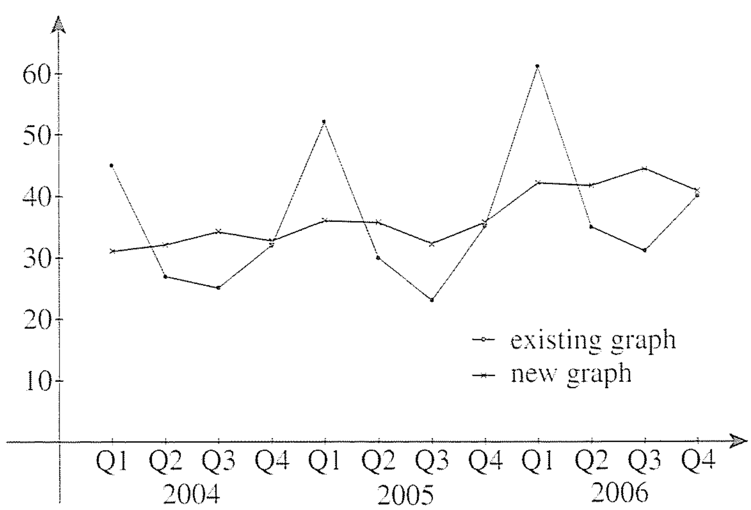
A1

iii.

	1st quarter	2nd quarter	3rd quarter	4th quarter
2004	31.0	32.1	34.2	32.7
2005	35.9	35.7	31.5	35.7
2006	42.1	41.7	42.5	40.8

A1

iv.



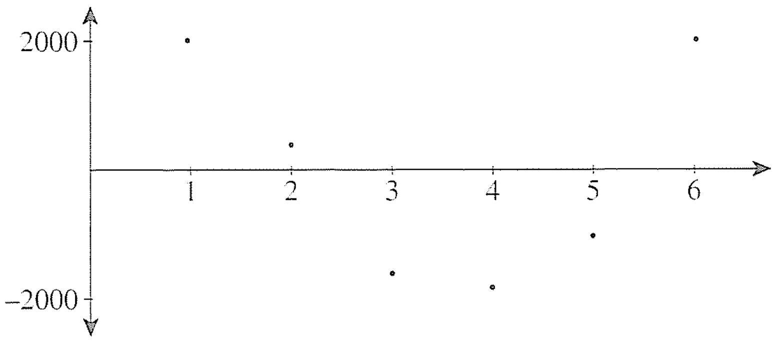
A1

v. Deseasonalising has smoothed the curve and shows a slight positive trend. A1

c. **i.** r = 0.91 using stat → calc → linreg A1

average visitors per day = – 4466 + 2422.86 × year A1

ii.



A1

Plot shows a pattern and does not support the assumption of linearity. A1

iii. x^2 transf. $r = 0.97$

$\log_x$ transf. $r = 0.79$

x^2 gives the best transformation

average visitors per year = –1483.3 + 362.5 × $(\text{year})^2$

SECTION B – MODULES

Module 1: Number patterns

Question 1

a. The lengths and widths of the bricks form a geometric sequence, where the first term is 1.0 and the common ratio is 0.8.

$l_4 = ar^3$

$= 1.0(0.8)^3$

$= 0.512$

Thus 512 mm.

$w_4 = 0.4(0.8)^3$

$= 0.2048$ A1

Thus 205 mm.

Both l_4 and w_4 required for 1 mark

b. $0.01 = 1.0(0.8)^{n-1}$

$0.01 = 0.8^{n-1}$

$\log_{10}(0.01) = (n-1)\log_{10}(0.8)$ M1

$-2 = -0.9691(n-1)$

$20.638 = n - 1$

$n = 21.638$

$l_{21} = 0.0115$

$l_{22} = 0.00922$

Therefore, the 22nd brick is the last, so 19 more bricks are required. A1

Alternatively, trial and error may be used to find the term in the sequence l_n that is closest to 0.01.

For full marks, evidence that various terms have been calculated is required.

c. The distance travelled by the ant is a geometric sequence, where $a = 1.32$. This question requires finding the sum of the first 22 terms. M1

$S_{22} = \frac{1.32(1 - 0.8^{22})}{1 - 0.8}$ M1

$= \frac{1.32(0.9926)}{0.2}$

$= 6.552$ A1

d. **i.** The area of the first brick is 0.4 m^2. The common ratio of areas is 0.64, i.e. the length common ratio squared. A1

ii.
$$S_\infty = \frac{a}{1-r}$$
$$= \frac{0.4}{1-0.64}$$
$$= \frac{10}{9}\ \text{m}^2$$ M1

area = length × width
$$= 1.32 \times 1.056$$
$$= 1.39392\ \text{m}^2$$

The percentage of region enclosed by bricks $= \dfrac{\left(\frac{10}{9}\right)}{1.39392} \times 100 = 79.7\%$ A1

Question 2

a.
$$P_n = a + (n-1)d$$
$$P = 1800 - (n-1)350$$ A1
$$P = 2150 - 350n$$

Either the expanded or unexpanded version of the answer is acceptable.

b. This is arithmetic. The sequence F_n is actually the sum of n years of fees.

$$F_n = \frac{n}{2}[2a + (n-1)d]$$ M1
$$F_n = \frac{n}{2}[3600 - 350(n-1)]$$
$$F_n = n(1975 - 175n)$$ A1
$$= 1975n - 175n^2$$

Either the expanded or unexpanded version of the answer is acceptable.

c. Firstly determine when the fee is due to become negative.

$$0 = 1800 - 350n$$
$$350n = 1800$$
$$n = 5.14$$

So the first five years of fees are to be paid.

$$F_5 = 1975(5) - 175(25)$$
$$= 9875 - 4375$$
$$= \$5500$$ A1

d. $P_{n+2} = 0.5(P_n + P_{n+1}) - 100,\ P_1 = 1500,\ P_2 = 1200$ A1

e. $P_3 = 0.5(1500 + 1200) - 100 = 1250$

$P_4 = 0.5(1200 + 1250) - 100 = 1125$ A1

Module 2: Geometry and trigonometry

Question 1

a. $c = \pi \times d$

$d = \frac{c}{\pi}$

$d = \frac{48.2}{\pi}$

$d = 15.342536\ldots$

$d \cong 15.34$ cm A1

b. $\text{area} = \pi \times r^2$

$= \pi \times \left(\frac{15.3425365}{2}\right)^2$

$= 185.0651577$

$\text{area} \cong 185 \text{ cm}^2$ A1

c. $\angle LKM = 45°$ A1

d. distance KL = radius of circle

$= \frac{\text{diameter}}{2}$

$= \frac{\left(\frac{c}{\pi}\right)}{2}$

$= \frac{\left(\frac{48.2}{\pi}\right)}{2}$

$= 7.67126\ldots \cong 7.67$ cm A1

e. surface area of one pole = circumference × height

$= 48.2 \times 80$

$= 3856 \text{ cm}^2$ M1

surface area of 12 poles $= 12 \times 3856$

$= 46\ 272$

total surface area $\cong 46\ 300 \text{ cm}^2$ A1

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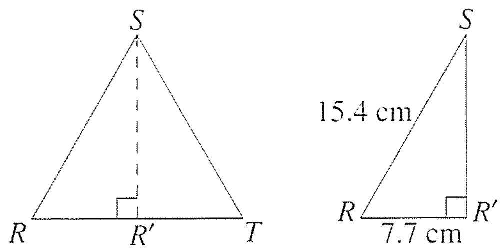
f. **i.** perimeter = $6 \times$ radius

$= 6 \times 7.7$

$= 46.2$ cm A1

ii.



$a^2 = h^2 - b^2$

$= 15.4^2 - 7.7^2$ M1

$= 177.87$

$a = 13.33679\ldots$

distance $UV = 7.7 + 13.33679 + 7.7$

$= 28.73679\ldots$

$\cong 28.7$ cm A1

Question 2

a. $\angle DEF = 18° + 90° + 26°$

$= 134°$ A1

b. $a^2 = b^2 + c^2 - 2bc \times \cos(A)$

$= 145^2 + 55^2 - 2 \times 55 \times 145 \times \cos(134°)$

$= 35129.801\ldots$

$a = 187.42945\ldots$

$a \cong 187$ metres A1

c. $\cos(A) = \dfrac{b^2 + c^2 - a^2}{2bc}$

$= \dfrac{55^2 + 187.429456^2 - 145^2}{2 \times 55 \times 187.429456}$

$= 0.83084839\ldots$

$A = 33.814\ldots$ M1

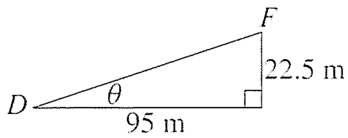
$A \cong 34°$

bearing $\overrightarrow{FD} = 180° + 26° + 34°$

$= 240°$ True A1

Question 3

The scale is 1 : 1000, so the horizontal distance between D and F is $9.5 \times 1000 = 9500$ cm = 95 m. M1
The vertical distance between D and F is $55 - 35 + 2.5 = 22.5$ m.



$$\frac{\text{opposite}}{\text{adjacent}} = \tan(\theta)$$

$$\frac{22.5}{95} = \tan(\theta)$$

$\theta = 13.3245\ldots$

$\theta = 13°$ A1

Module 3: Graphs and relations

Question 1

a. 9 minutes A1

b. The runner is slowest (i.e. on Hill street) between $t = 3$ and $t = 5$. Therefore, the duration is 2 minutes. A1

c. average speed $= \frac{\text{distance}}{\text{time}}$,

where $t = 2$ minutes $= \frac{1}{30}$ hours and distance = 0.2 km (from graph)

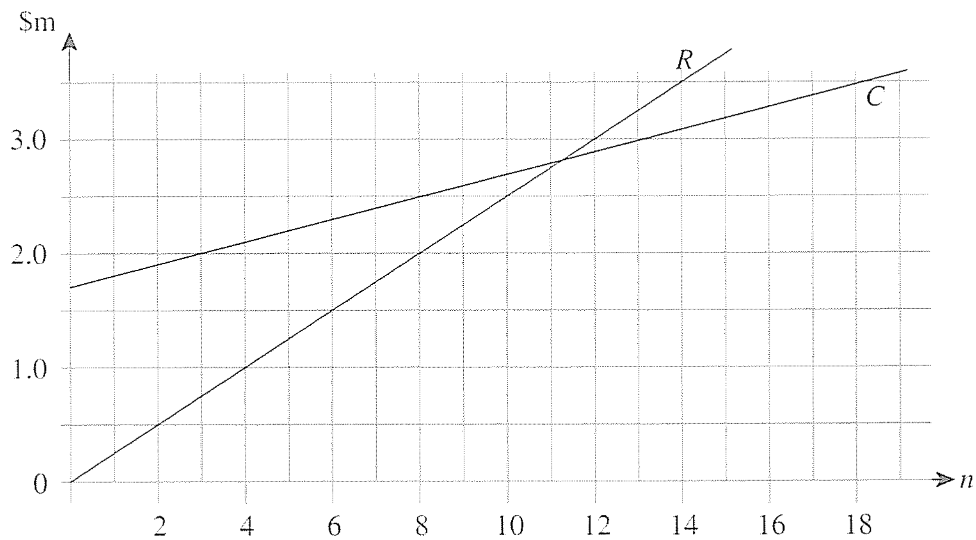
average speed $= \frac{0.2}{\left(\frac{1}{30}\right)} = 6$ km/h A1

Question 2

a. $C = 1.7 + 0.1n$ A1

b. $R = 0.25n$ A1

c.



A1

Requires both lines clearly marked

d. $n = 11$ represents a loss since the line C is higher. $n = 12$ is the least value of n for which R is greater. Therefore, twelve events are required in order to make a profit. A1

Question 3

a. The data in the question is summarised in the table below.

	Maximum event per hour		Events capacity
	Location A	**Location B**	
track	3	2	24
field	2	4	20

M1

$3x + 2y \geq 24$

$2x + 4y \geq 20$

$x \leq 9$

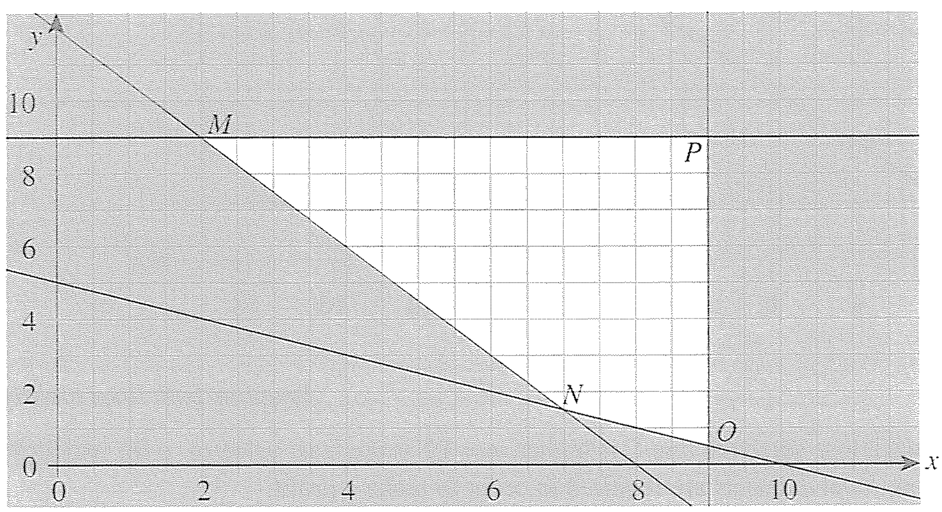
$y \leq 9$

A1

b.

$2x + 4y = 20$	$2x + 3y = 24$
y-intercept, $x = 0$	y-intercept, $x = 0$
$4y = 20$	$3y = 24$
$y = 5$	$y = 8$
$(0,5)$	$(0,8)$
x-intercept, $x = 0$	x-intercept, $x = 0$
$2x = 20$	$2x = 24$
$x = 10$	$x = 12$
$(10,0)$	$(12,0)$

M1



The unshaded region is required.

A2

1 mark for correct boundaries
1 mark for correct shading

c. **i.** $C = 50x + 70y$ A1

ii. We need to check the critical points that form the boundary of the required region.

at M, $y = 9$

$3x + 18 = 24$

$3x = 6$

$x = 2$

$M(2,9)$

at O, $x = 9$

$18 + 4y = 20$

$4y = 2$

$y = 0.5$

$O(9,0.5)$

at N

$3x + 2y = 24$ ...1

$2x + 4y = 20$ ...2

$(1) \times 2 : 6x + 4y = 48$ ...1a

$(1a) - (2) : 4x = 28$

$x = 7$

(sub in 1)$21 + 2y = 24$

$y = 1.5$

$N(7,1.5)$

Location of critical points method. A1

Calculating the cost (C) at each of these points.

at M, $C = 100 + 630 = 730$

at N, $C = 350 + 105 = 455$

at O, $C = 450 + 35 = 485$

at P, $C = 450 + 630 = 1080$

The cheapest is $455, when location A is used for 7 hours and location B for 1.5 hours. A1

Module 4: Business-related mathematics

Question 1

a. $340 - 40 = 300$

$I = \frac{Prt}{400}$

$= \frac{300 \times 8.1 \times 3}{100}$

$= \$72.90$ A1

b. $\frac{372.90}{36} = \$10.36$ per month A1

c. $10.36 \times 36 + 40 = \$412.96$ or $300 + 72.90 + 40 = \$412.90$ A1

d. $r_e = \frac{8.1 \times 2 \times 36}{36 + 1} = 15.76\%$ A1

e. $340 \times 0.65^3 = \$93.37$ A1

Question 2

a. $X = 3800 + 750 = 4550$ A1

$Y = 5400 + 1000 = 6400$ A1

b. **i.** $\frac{5.1}{12} = 0.425$ A1

ii. January, February and March minimum balance = 3800

$\therefore I = \frac{3800 \times 0.425 \times 3}{100}$

$= \$48.45$ A1

iii. April minimum balance $= 3800 \times \frac{0.425}{100} \times 1 = 16.15$

May minimum balance $= 2550 \times \frac{0.425}{100} \times 1 = 10.84$

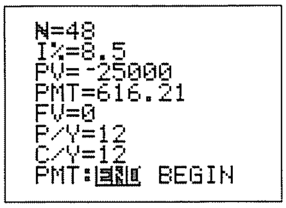
June minimum balance $= 5400 \times \frac{0.425}{100} \times 1 = 22.95$

interest = $98.39 A1

$\therefore$ balance = $5498.39 A1

Question 3

a. **i.** Using a graphics calculator, use the TVM Solver:



The payment would be \$616.21 per month. A1

ii. $616.21 \times 48 = \$29\,578.08$ A1

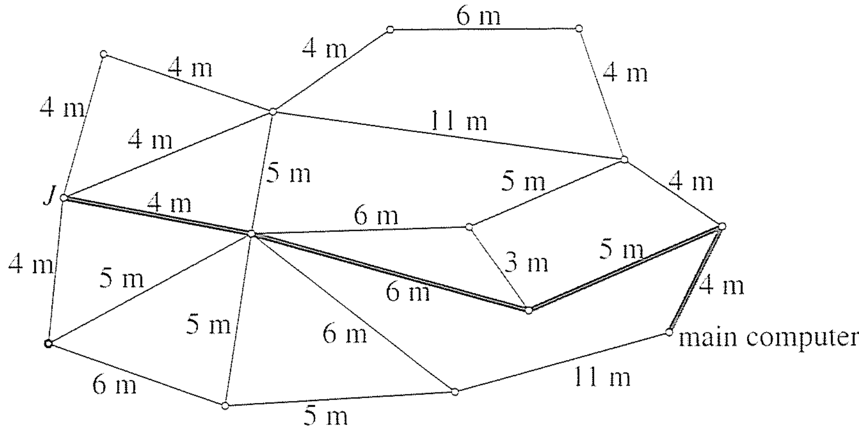
iii. $\$29\,578.08 - 6500 = \$23\,078.08$ A1

b. $(200 \times 12) + (0.25 \times 18\,000) = \$6900$ A1

Module 5: Networks and decision mathematics

Question 1

a. **i.**

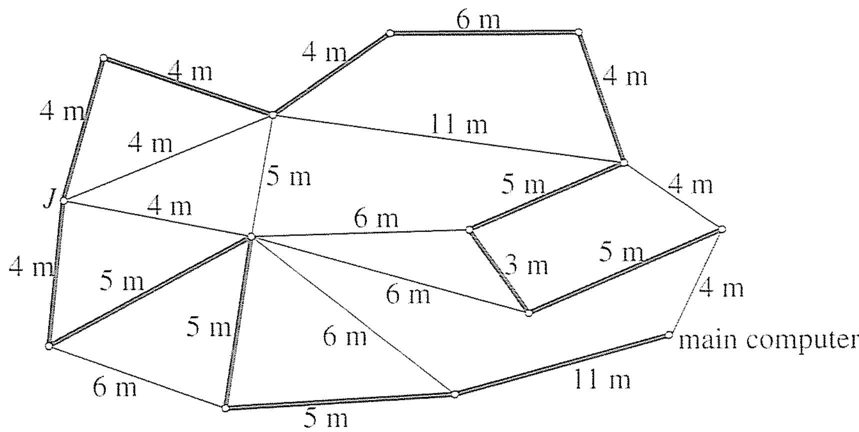


A1

ii. shortest path = 4 + 5 + 6 + 4 = 19 metres A1

b. **i.** Hamilton path A1

ii.



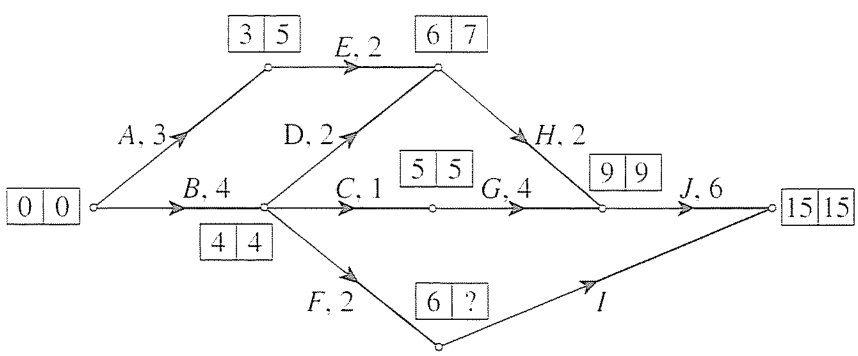
There are other journeys that Kylie could take. A1

Question 2

If Geoff is allocated to the keyboard, Anthony must be allocated to the processor since there is no other option. Sam and Brian remain, and since Brian cannot be allocated to the motherboard, Sam must perform the motherboard task. A1

Question 3

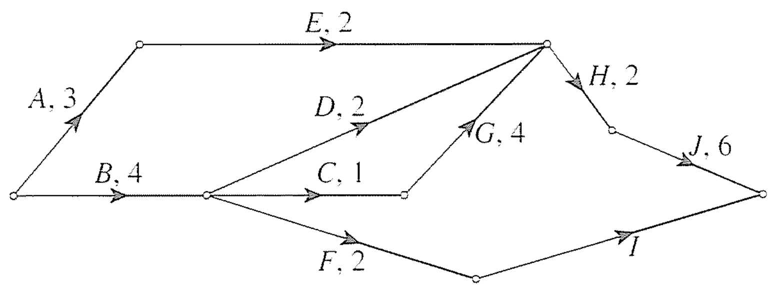
a.



B, C, G, J A1

b. $4 + 1 + 4 + 6 = 15$ hours A1

c.



A2
1 mark for activity G
1 mark for activities H, I and J

d. original critical path $= 4 + 1 + 4 + 6 = 15$ hours

revised critical path $= 4 + 1 + 4 + 2 + 6 = 17$ hours M1

Therefore, 2 extra hours are required to complete the project. A1

e. latest finish time of activity $D = 9$ hours

earliest start time of activity $D = 4$ hours

duration of activity $D = 2$ hours

maximum delay $= (9 - 4) - 2 = 3$ hours A1

f.

Reduction of activity G (hours)	Cost ($)	Critical Path	Project completion time (hours)
0	0	B, C, G, H, J	17
1	450	B, C, G, H, J	16
2	900	B, C, G, H, J	15
3	1350	B, C, G, H, J and B, D, H, J	14 14
4	1800	B, D, H, J	14

From the table, it can be seen that $1350 should be spent as this reduces the project completion time to 14 hours. Spending $1800 does not generate any further reduction in project completion time. A1

Question 4

$$D_1 = \begin{matrix} & S & B & G & A \\ S & 0 & 1 & 1 & 0 \\ B & 0 & 0 & 0 & 0 \\ G & 0 & 1 & 0 & 0 \\ A & 1 & 1 & 1 & 0 \end{matrix} \quad D_2 = \begin{matrix} & S & B & G & A \\ S & 0 & 1 & 0 & 0 \\ B & 0 & 0 & 0 & 0 \\ G & 0 & 0 & 0 & 0 \\ A & 0 & 2 & 1 & 0 \end{matrix} \quad D_3 = \begin{matrix} & S & B & G & A \\ S & 0 & 0 & 0 & 0 \\ B & 0 & 0 & 0 & 0 \\ G & 0 & 0 & 0 & 0 \\ A & 0 & 1 & 0 & 0 \end{matrix}$$

$$D_{1+2+3} = \begin{matrix} S \\ B \\ G \\ A \end{matrix} \begin{bmatrix} 2+1+0 \\ 0+0+0 \\ 1+0+0 \\ 3+3+1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 1 \\ 7 \end{bmatrix}$$

Final rankings – 1st Anthony
2nd Sam
3rd Geoff
4th Brian

M1

Geoff was ranked third. A1

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Module 6: Matrices

Question 1

a. $A = \begin{bmatrix} 250 & 0.3 \\ 150 & 0.4 \\ 300 & 0.2 \\ 250 & 0.1 \end{bmatrix}$ A1

b. **i.** $AB = \begin{bmatrix} 250 & 0.3 \\ 150 & 0.4 \\ 300 & 0.2 \\ 250 & 0.1 \end{bmatrix} \begin{bmatrix} 1.5 & 0.7 & 0.9 & 1.6 \\ 600 & 480 & 900 & 890 \end{bmatrix}$

$= \begin{bmatrix} 555 & 319 & 495 & 667 \\ 465 & 297 & 495 & 596 \\ 570 & 306 & 450 & 658 \\ 435 & 223 & 315 & 489 \end{bmatrix}$ A1

ii. The matrix AB shows the total water saved for each combination of brand and household. A1

Each row represents a different brand and each column represents a separate household. A1

iii. For Weather-watch, mean savings $= \frac{(555 + 319 + 495 + 667)}{4} = 509$ litres

For Irrigator, mean savings $= \frac{(465 + 297 + 495 + 596)}{4} = 463.3$ litres

For Dry-days, mean savings $= \frac{(570 + 306 + 450 + 658)}{4} = 496.0$ litres

For Eco-bounty, mean savings $= \frac{(435 + 223 + 315 + 489)}{4} = 365.5$ litres

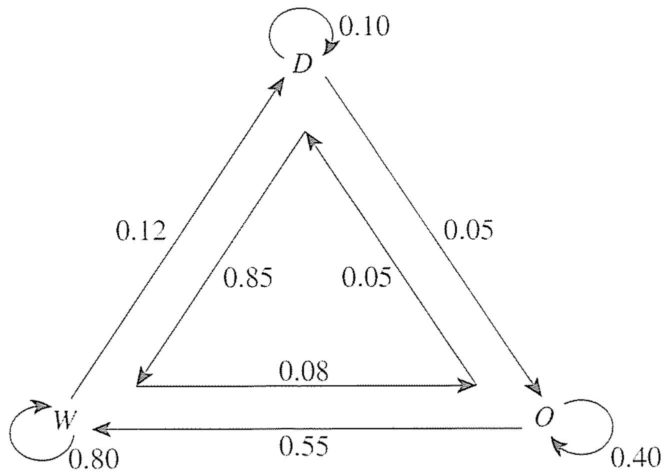
Weather-watch offers the best mean savings. A1

iv. Matrix BA is defined as the number of columns of B matches the number of rows of A. A1

Consideration of the first element in the first row of the matrix reveals that BA is not meaningful. The element is the sum of the rainwater captured for household one by brand one, household two by brand two, household three by brand three and household four by brand four. Thus BA makes no sense. A1

Question 2

a.



The diagram above shows the transitions between the three states of watered (W), dry (D) and over-watered (O).

M1

Any other similar diagram or description is satisfactory

$$T = \begin{bmatrix} 0.10 & 0.12 & 0.05 \\ 0.85 & 0.80 & 0.55 \\ 0.05 & 0.08 & 0.40 \end{bmatrix}$$

A2

1 mark for one incorrect matrix element

b. Determine the proportions in each category for each week until the 75% watered is reached.

$$S_1 = TS_0 = \begin{bmatrix} 0.069 \\ 0.630 \\ 0.301 \end{bmatrix} \qquad S_2 = TS_1 = \begin{bmatrix} 0.098 \\ 0.728 \\ 0.174 \end{bmatrix} \qquad S_3 = TS_2 = \begin{bmatrix} 0.106 \\ 0.761 \\ 0.133 \end{bmatrix}$$

M1

It takes three weeks to achieve the 75% watered state in the garden. A1

c. For one month, the matrix is T^4, which is the weekly transformation applied four times instead of once.

$$M = T^4 = \begin{bmatrix} 0.110 & 0.110 & 0.108 \\ 0.779 & 0.778 & 0.769 \\ 0.111 & 0.112 & 0.123 \end{bmatrix}$$

A1

d. Check the proportions after a large number of months to see if they remain constant.

$$M^{20}S_0 = \begin{bmatrix} 0.110 \\ 0.777 \\ 0.113 \end{bmatrix}$$

$$M^{21}S_0 = \begin{bmatrix} 0.110 \\ 0.777 \\ 0.113 \end{bmatrix}$$

M1

Since these results are constant, we can say that a steady state is achieved. It involves the watering level reaching a maximum value of 0.777. A1