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Student Name……………………………………

FURTHER MATHEMATICS

TRIAL EXAMINATION 1

2009

Reading Time: 15 minutes
Writing time: 1 hour 30 minutes

Instructions to students

This exam consists of Section A and Section B.
Section A contains 13 multiple-choice questions from the core, 'Data Analysis'.
Section A is compulsory and is worth 13 marks.
Section B consists of 6 modules each containing 9 multiple-choice questions. You should choose 3 of these modules and answer every question in each of your chosen modules.
Each of the modules is worth 9 marks.
Section B begins on page 8 of this exam.
There is a total of 40 marks available for this exam.
Unless otherwise stated the diagrams in this exam are not drawn to scale.
Students may bring one bound reference into the exam.
An approved graphics or CAS calculator may be used in the exam.
An answer sheet appears on page 37 of this exam.
Formula sheets can be found on pages 35 and 36 of this exam.

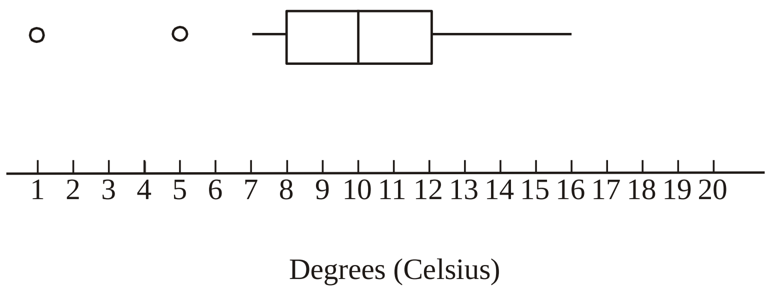
SECTION A

CORE – Data Analysis

This section is compulsory.

The following information relates to questions 1 and 2.

The distribution of the minimum overnight temperature, in °C, in a town during Autumn is displayed on the boxplot below.



Question 1

The range of the distribution is

A. 4
B. 5
C. 7
D. 10
E. 15

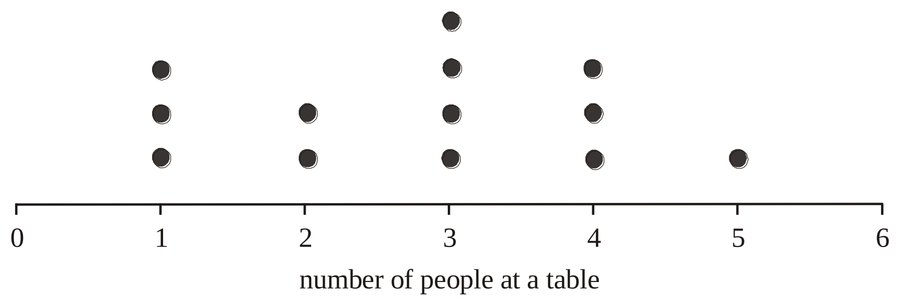
Question 2

The score 1 is an outlier because it is less than

A. 2
B. 2.5
C. 4
D. 5
E. 7

Question 3

The distribution of the number of people seated at each of the 13 tables in a café one lunchtime is displayed on the dot plot below.



The interquartile range of the distribution is

A. 1.5
B. 2.5
C. 2.8
D. 3
E. 4

Question 4

The height (in cm) of a large population of fifteen year old girls is normally distributed with a mean of 162cm and a standard deviation of 4.5cm. Two hundred of these girls are selected at random. The number with a height of less than 153cm is closest to

A. 0
B. 5
C. 10
D. 32
E. 50

The information below relates to questions 5 – 7.

In the log book of a small aircraft is recorded the number of passengers together with their total weight (in kg) for each flight.

The entries for the previous eight flights are shown in the table below.

number of passengers	2	4	3	3	5	4	2	4
total weight (kg)	158	300	215	242	392	347	165	341

Question 5

Which one of the following statements is true?

A. The variable "number of passengers" is discrete and is the independent variable.
B. The variable "number of passengers" is continuous and is the dependent variable.
C. The variable "total weight" is discrete and is the independent variable.
D. The variable "total weight" is continuous and is the independent variable.
E. The variable "total weight" is discrete and is the dependent variable.

Question 6

The value of *r*; Pearson's product moment correlation coefficient, for this data correct to four decimal places is closest to

A. 0.9217
B. 0.9431
C. 0.9578
D. 0.9787
E. 0.9892

Question 7

The equation of the least squares regression line for this data is

$$\text{total weight} = 81\cdot 1429 \times \text{number of passengers} - 3\cdot 8571$$

Based on this, the pilot could assume that for every extra one passenger on board, the extra weight (in kg) on board the plane is

A. 73.4287
B. 77.2858
C. 81.1429
D. 83.4287
E. 85

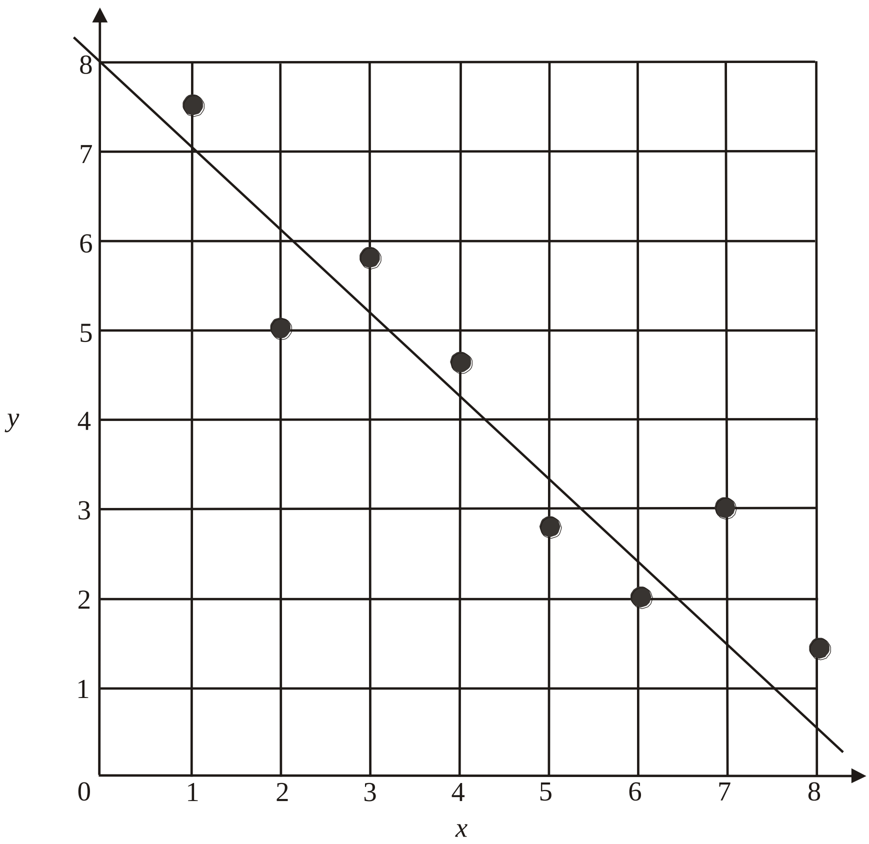
Question 8

A study involving a large sample of experienced runners showed that there is a positive correlation between the average number of kilometers run each day and the number of injuries incurred over a year. The correlation coefficient is 0.7.
The most accurate statement relating to this study is:

A. Seventy percent of injuries are caused by running.
B. An increase in the average number of kilometers run in a week is associated with a decrease in the number of injuries.
C. Seventy percent of runners are unable to run for at least one week a year because of injuries.
D. Forty-nine percent of the variation in the number of injuries can be accounted for by the variation in the average number of kilometres run.
E. Running fewer kilometers causes fewer injuries.

Question 9

For a set of bivariate data involving the variables *x* and *y* a trend line is fitted.
A scatterplot showing the data and the trend line is shown below.

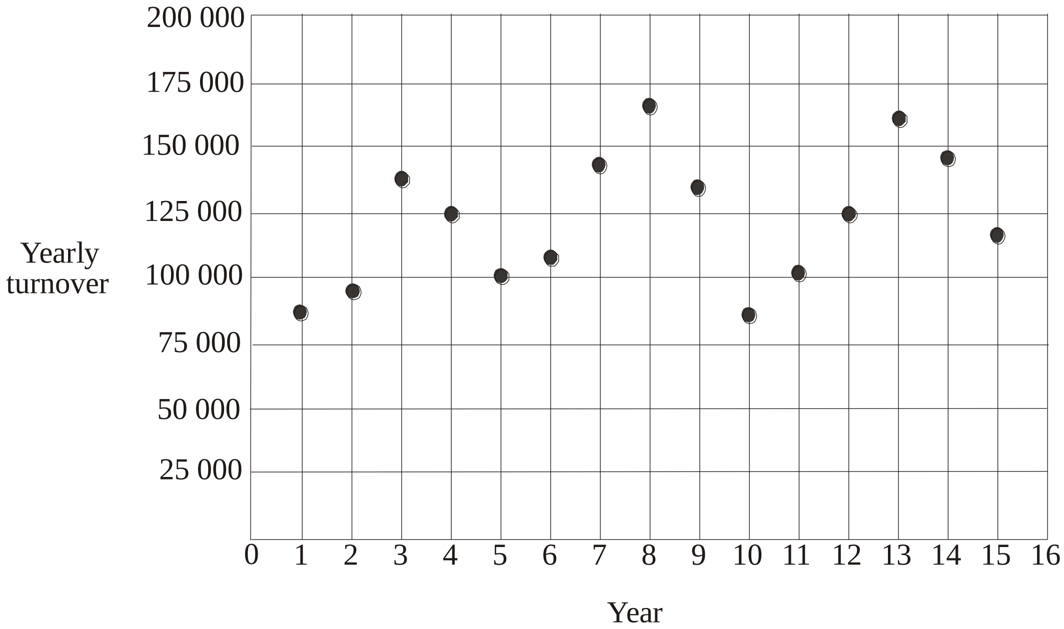


The largest residual value for the eight pieces of data is closest to

A. 1
B. 1.5
C. 3
D. 4
E. 7

The information below relates to Questions 10 and 11.

The time series plot below shows the yearly turnover (in dollars) of a small business over a 15 year period.



Question 10

The time series plot shows

A. no trend
B. a seasonal trend
C. a cyclic trend
D. a decreasing trend
E. a random trend.

Question 11

A three median trend line is fitted to the data. The slope of this trend line (expressed in dollars per year) is

A. 55
B. 750
C. 500
D. 1500
E. 2500

Question 12

The number of invoices paid each month by a company, are given in the table below.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Number of invoices paid	12	20	28	32	30	25	23	25	30	35	21	24

A three mean moving average is used to smooth the data. The smoothed value for June is

A. 23
B. 25
C. 26
D. 26.5
E. 27

Question 13

The table below shows the seasonal indices for quarterly sales (in dollars) at a factory outlet shop.

Quarter	1	2	3	4
Seasonal index	0.94	0.97	1.23	0.86

The deseasonalised sales for the factory outlet shop in Quarter 3 are $325 000. The actual sales, in dollars, for the shop in Quarter 3 are

A. 164 228
B. 279 500
C. 315 250
D. 335 051
E. 399 750

SECTION B

Module 1: Number patterns

If you choose this module all questions must be answered.

Question 1

For the arithmetic sequence 4, 11, 18, 25, … the twentieth term is

A. 83
B. 116
C. 130
D. 137
E. 144

Question 2

A geometric sequence is given by 3, *m*, 48,…
The value of *m* could be

A. 4
B. 6
C. 8
D. 12
E. 16

Question 3

A Fibonacci sequence is defined by the difference equation $t_n = t_{n-2} + t_{n-1}$, $t_1 = -3$, $t_2 = 1$.
The sixth term of this sequence is

A. – 10
B. – 4
C. – 3
D. 4
E. 14

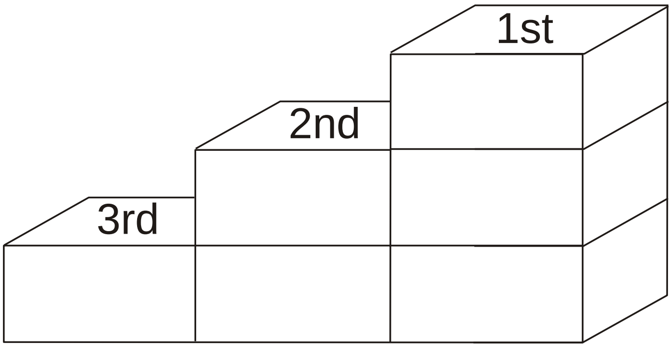
Question 4

A meat processing machine increases its output by 2% each hour. It produced 18 kg in its first hour of operation.
The total output, in kg, of this machine after nine hours is closest to

A. 154.49
B. 175.58
C. 198.72
D. 900
E. 9198

Question 5

The first three place getters in a race stand on a dais during the presentation of medals. The dais is made up of six identical rectangular boxes as shown in the diagram below.



A dais for the first twenty place getters in a marathon race is to be similarly constructed. The number of these rectangular boxes required would be

A. 20
B. 120
C. 198
D. 200
E. 210

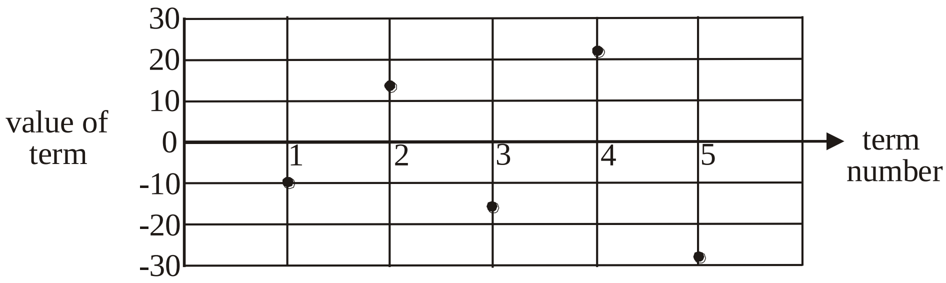
Question 6

The annual budget allocated to a museum is made up of the previous year's budget plus 3.5% of the previous year's budget plus an annual grant of $6000.
The museum had an initial budget of $42 000.
A difference equation that gives the budget allocation for the museum in its n^{th} year of operation is

A. $B_{n+1} = 3\cdot 5B_n + 6000 \qquad B_1 = 42\,000$
B. $B_{n+1} = 3\cdot 5(B_n + 6000) \qquad B_1 = 42\,000$
C. $B_{n+1} = 1\cdot 035B_n + 6000 \qquad B_1 = 42\,000$
D. $B_{n+1} = 1\cdot 035(B_n + 6000) \qquad B_1 = 42\,000$
E. $B_{n+1} = 1\cdot 35(B_n + 6000) \qquad B_1 = 42\,000$

Question 7

The first five terms of a sequence are plotted on the graph below.



A possible first order difference equation that could describe this sequence is

A. $t_n = -1 \cdot 3t_{n-1}, \quad t_1 = -10$
B. $t_n = 1 \cdot 3t_{n-1}, \quad t_1 = -10$
C. $t_n = t_{n-1} - 23, \quad t_1 = -10$
D. $t_n = t_{n-1} + 23, \quad t_1 = -10$
E. $t_n = 1 \cdot 3t_{n-1} - 23, \quad t_1 = -10$

Question 8

The initial dose of a prescription drug contains 10mg of the active ingredient.
Each subsequent dose contains just 96% of the active ingredient contained in the previous dose.
Over the course of this drug being administered to a patient, the total amount of active ingredient in mg taken by the patient will be

A. 40
B. 83.8
C. 96
D. 176.2
E. 250

Question 9

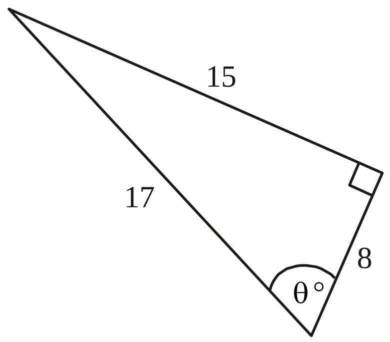
The difference equation $t_{n+1} = 0 \cdot 5t_n - 1, \quad t_1 = 1000$, describes a sequence.
Which one of the following statements about this sequence is **not** true?

A. The sequence is decreasing.
B. The sequence is not arithmetic.
C. The sequence has only positive numbers.
D. The sequence is not geometric.
E. $t_3 = 248 \cdot 5$.

Module 2: Geometry and trigonometry

If you choose this module all questions must be answered.

Question 1



In the triangle above $\cos\theta^\circ$ is equal to

A. $\frac{8}{15}$

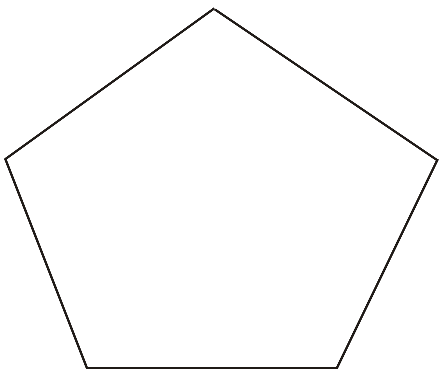
B. $\frac{8}{17}$

C. $\frac{15}{17}$

D. $\frac{15}{8}$

E. $\frac{17}{15}$

Question 2

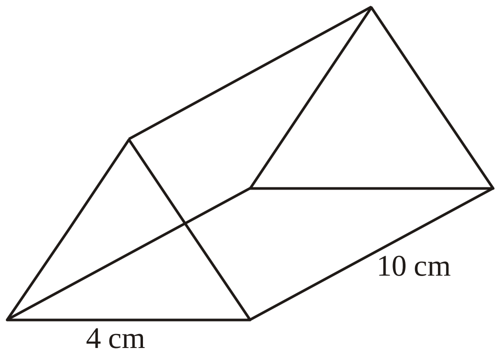


The diagram above shows a regular pentagon.
The sum of the interior angles of this pentagon is

A. 108°
B. 280
C. 360°
D. 540°
E. 720°

Question 3

A triangular prism with a cross-section of an equilateral triangle is shown below.

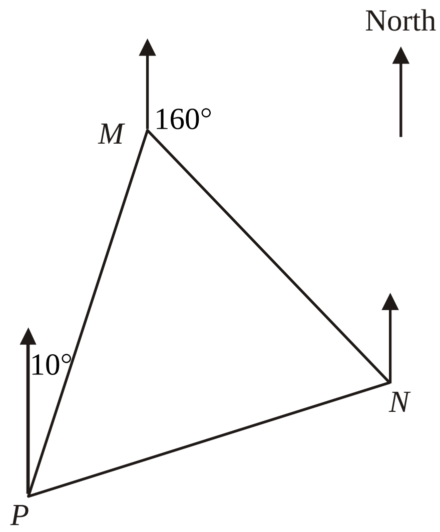


The sidelengths of the triangle are 4cm and the length of the prism is 10cm.
The surface area in cm^2 is

A. 46.93
B. 80
C. 93.86
D. 126.93
E. 133.86

Question 4

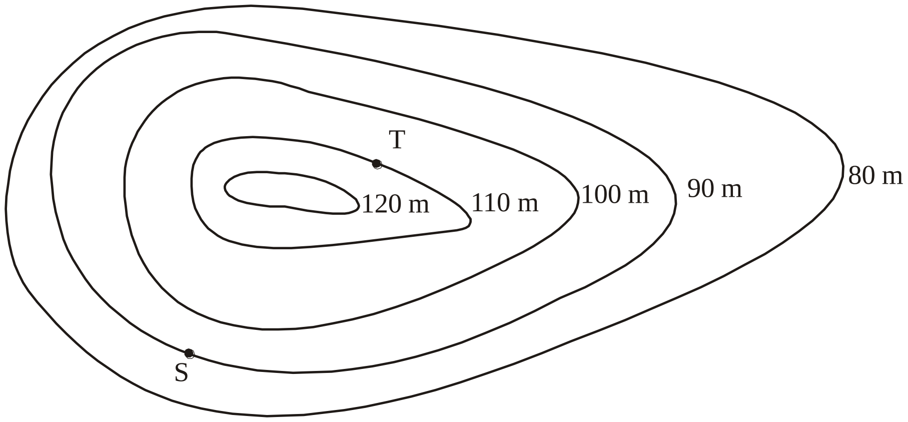
Three landmarks *M*, *N* and *P* are shown on the diagram below.



The bearing of *M* from *P* is 010° and the bearing of *N* from *M* is 160°.
The bearing of *P* from *M* is

A. 010°
B. 080°
C. 170°
D. 190°
E. 205°

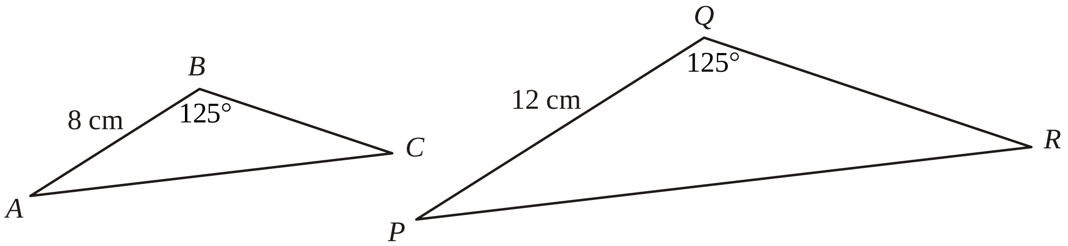
Question 5



The contour map above has contour intervals of 10m.
The horizontal distance from point T to point S is 180m.
The average slope between point T and point S is

A. $\frac{1}{18}$
B. $\frac{1}{6}$
C. $\frac{1}{9}$
D. $\frac{2}{11}$
E. $\frac{9}{11}$

Question 6

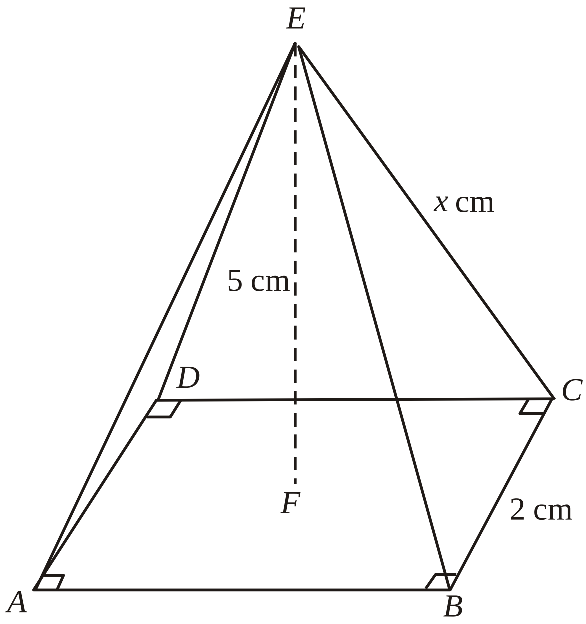


Triangles ABC and PQR are similar.
The area of triangle ABC is 30cm^2.
The area of triangle PQR, in square centimetres, is

A. 13.3
B. 20
C. 45
D. 67.5
E. 101.25

Question 7

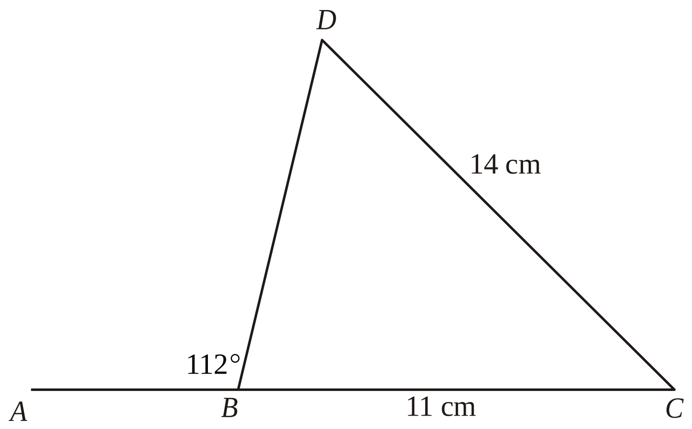
The diagram below shows the right, square-based pyramid *ABCDE*.
The point *F* is the centre of the base *ABCD*.



The side lengths of the base are 2cm and the length of *EF* is 5cm.
One of the slant sides *CE* has a length of *x* cm.
The value of *x* is

A. $\sqrt{21}$
B. $\sqrt{23}$
C. $\sqrt{27}$
D. $\sqrt{29}$
E. $\sqrt{33}$

Question 8



In the diagram above, angle $ABD = 112^{\circ}$, $CD = 14\text{cm}$ and $BC = 11\text{cm}$.
The angle *BCD* is closest to

A. 46.8°
B. 50.2°
C. 61.8°
D. 65.2°
E. 86.5°

Question 9

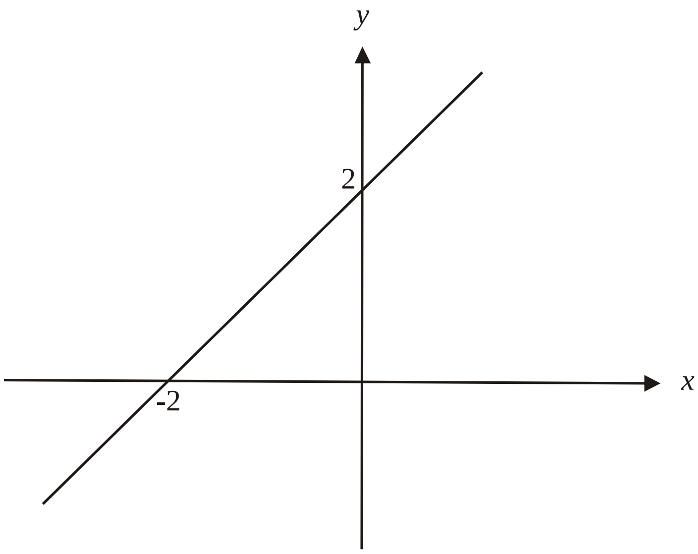
A walking party start at a carpark and walk 6km on a bearing of 050°. They stop at this point for morning tea. They then walk on a bearing of 195° for a distance of 8km. They stop at this point for lunch. After lunch they walk in a straight line back to the carpark. This final distance, in km, to be covered after lunch is closest to

A. 4.62
B. 6.18
C. 7.79
D. 13.29
E. 21.36

Module 3: Graphs and relations

If you choose this module all questions must be answered.

Question 1



The line above passes through the points (- 2, 0) and (0, 2).
The gradient of this line is

A. – 2
B. – 1
C. 0
D. 1
E. 2

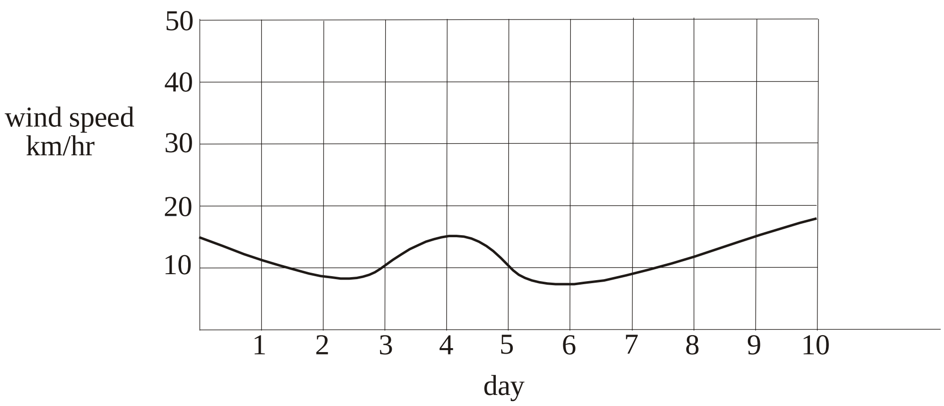
Question 2

A phone company has a plan for making international calls where the cost *C*, in dollars, for making *n* calls in a month is given by the rule $C = 30 + 4n$.
For each call made under this plan, the monthly cost will increase by

A. $0.13
B. $4
C. $7.50
D. $30
E. $34

The following information relates to questions 3 and 4.

The prevailing wind speed in km/hr recorded over a ten day period at a coastal recording station is shown on the graph below.



Question 3

Over this ten day period, the wind speed was less than 10km/hr for

A. less than 2 days
B. approximately 2 days
C. approximately 3 days
D. approximately 4 days
E. more than six days

Question 4

The rate at which the wind speed is increasing is greatest between

A. 0 – 1.5 days
B. 3 – 4 days
C. 4 – 5 days
D. 6 – 10 days
E. 8 – 10 days

Question 5

The cost of hiring 2 overnight and 3 weekly DVD's is \$24.40. At the same shop the cost of hiring 3 overnight and 5 weekly DVD's is \$38.50.
The cost of one overnight and one weekly DVD at this shop would be

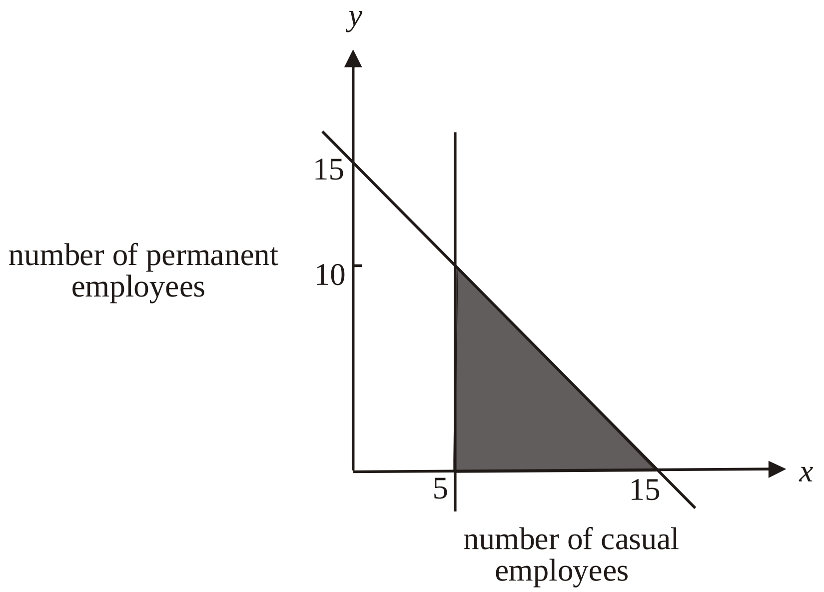
A. \$3.80
B. \$9.70
C. \$10.30
D. \$12.40
E. \$14.50

Question 6

In a linear programming problem involving the number of casual and permanent staff employed at a fast food outlet,

$$x = \text{the number of casual employees}$$
$$y = \text{the number of permanent employees.}$$

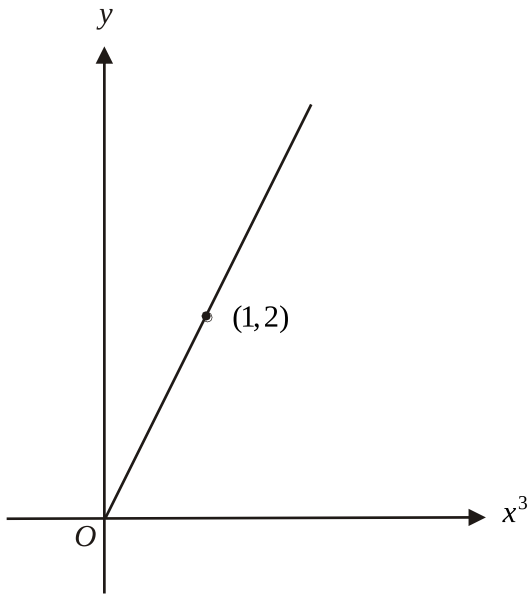
The feasible region for this problem is represented below by the shaded region with boundaries included.



Which one of the following statements is **not** true?

A. There are three constraints in this problem.
B. There cannot be less than 5 casual employees.
C. There can only be a maximum of 15 permanent employees.
D. There can only be a maximum of 15 casual staff.
E. There can be 10 casual staff and 3 permanent staff employed.

Question 7



The graph above shows the relationship between y and x^3.
The relationship between y and x is

A. $y = x$
B. $y = 2x$
C. $y = \frac{1}{2}x^3$
D. $y = 2x^3$
E. $y = 8x^3$

Question 8

A cabinet maker produces television stands at a fixed cost of $2800 plus $70 per stand.
When he sells 30 of these stands to a retailer he makes a profit of $1100.
The price that the cabinet maker charged the retailer for each stand was

A. $36.67
B. $100
C. $126.67
D. $163.33
E. $200

Question 9

A clothing manufacturer produces shirts and dresses.

- a shirt requires 20 minutes of labour and 80cm of fabric
- a dress requires 15 minutes of labour and 120cm of fabric
- a total of 2400 minutes of labour is available in a day
- a total of 14 400cm of fabric is available in a day
- x is the number of shirts produced in a day
- y is the number of dresses produced in a day

The constraints due to labour and fabric availability that affect the number of shirts and dresses that can be produced in a day are

A. $20x + 80y \le 2400$
$15x + 120y \le 14400$

B. $20x + 80y \le 14400$
$15x + 120y \le 2400$

C. $20x + 15y \le 2400$
$80x + 120y \le 14400$

D. $20x + 15y \le 14400$
$80x + 120y \le 2400$

E. $80x + 20y \le 14400$
$120x + 15y \le 2400$

Module 4: Business-related mathematics

If you choose this module all questions must be answered.

Question 1

Kim earned $40 over two years on an amount she invested that earned 2.5% per annum simple interest. The amount that Kim invested was

A. $800
B. $1 250
C. $2 000
D. $12 500
E. $20 000

Question 2

An amount of $13 000 is invested for 4 years and earns compound interest of 3.2% per annum calculated each 6 months.
The value of the investment at the end of 4 years is

A. $13 852.18
B. $13 419.33
C. $14 760.23
D. $16 725.57
E. $42 619.39

Question 3

Gwen receives a weekly pension of $350 from an amount she invested in an ordinary perpetuity. The interest rate on this investment is 5% per annum.
The amount Gwen invested in the perpetuity was

A. $7 000
B. $84 000
C. $175 000
D. $256 000
E. $364 000

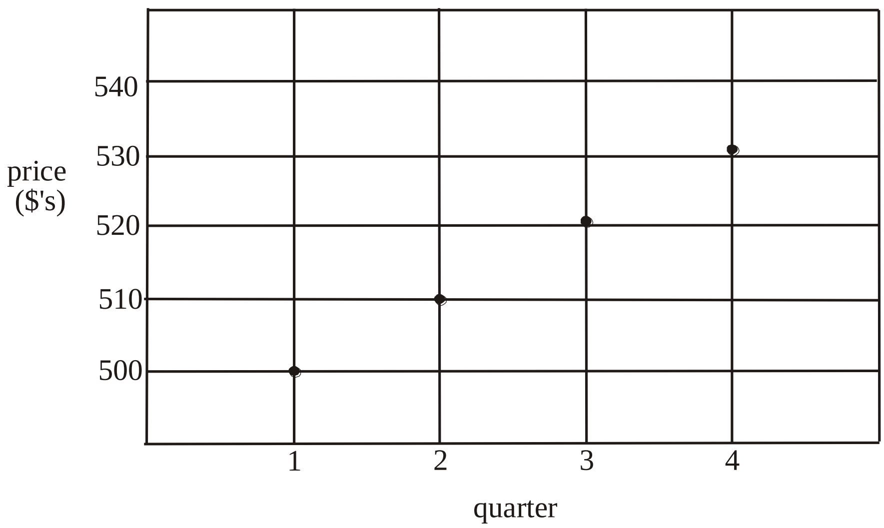
Question 4

A fax/scanner/copier machine is depreciated each year for taxation purchases using the reducing balance method. The machine was purchased for $1 500 and is valued at $800 after 3 years. The annual rate of depreciation used is closest to

A. 7%
B. 15%
C. 17%
D. 19%
E. 23%

Question 5

A bicycle retailer increases the prices of his bikes each quarter in line with inflation. The graph below shows the price over 4 quarters of one popular model of bike sold by the retailer.



If inflation was constant during these 4 quarters then the rate of inflation per quarter was

A. 1.25%
B. 1.96%
C. 2%
D. 2.5%
E. 10%

The information below relates to questions 6 and 7.

Georgia purchases a dining setting valued at $3 500 under a hire purchase agreement. She pays a deposit of $1 000 and makes equal fortnightly payments for 26 fortnights.
Georgia is charged a flat interest rate of 5% per annum.

Question 6

Her fortnightly payments are

A. $96.15
B. $100.96
C. $102.88
D. $105.77
E. $108.34

Question 7

The effective rate of interest per annum under this agreement is closest to

A. 7.26%
B. 9.63%
C. 10%
D. 12.52%
E. 52.92%

Question 8

Nigel invests $120 000 in an annuity that earns 5.5% per annum interest compounded monthly and pays him a monthly sum of $750. The number of years for which this annuity continues is closest to

A. 10
B. 12
C. 16
D. 22
E. 24

Question 9

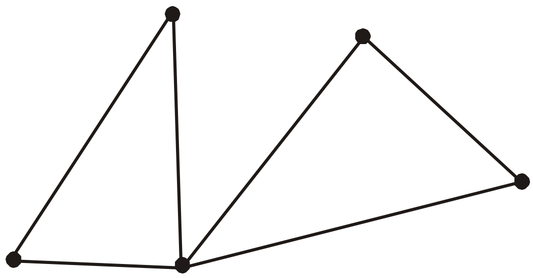
Paul borrowed $60 000. He will make equal quarterly repayments over the next 4 years and will be charged 6% interest per annum, calculated quarterly on the reducing balance.
After the eighth repayment, the amount of principal that Paul has paid off is

A. $26 032.76
B. $28 215.48
C. $30 000.00
D. $31 784.52
E. $33 967.24

Module 5: Networks and decision mathematics

If you choose this module all questions must be answered.

Question 1

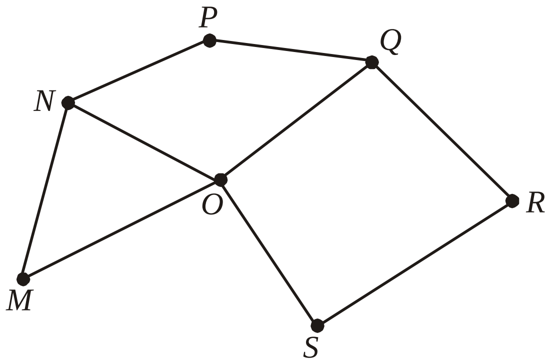


The number of vertices with an even degree in the graph above is

A. 1
B. 2
C. 3
D. 4
E. 5

The information below relates to questions 2 and 3.

A graph with vertices *M* - *S* is shown below.



Question 2

A Hamiltonian path for the graph above is

A. *P N O Q R S*
B. *QRSOQPNOMN*
C. *P Q R S O M N P*
D. *P Q R S O N M*
E. *S R Q P N M O Q*

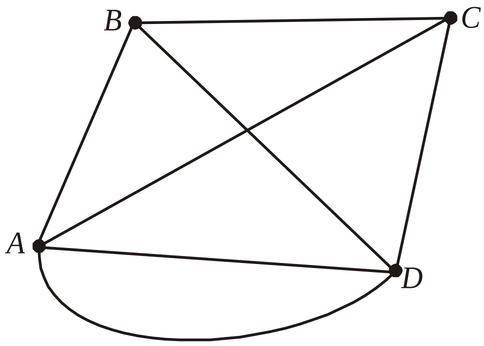
Question 3

Which one of the following statements is **true** for the graph shown?

A. The graph is a tree.
B. The graph is complete.
C. The graph has at least one loop.
D. The graph contains an Euler path.
E. The graph contains an Euler circuit.

Question 4

The undirected graph with vertices *A* - *D* is shown below.



An adjacency matrix that could be used to represent this graph is

A. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

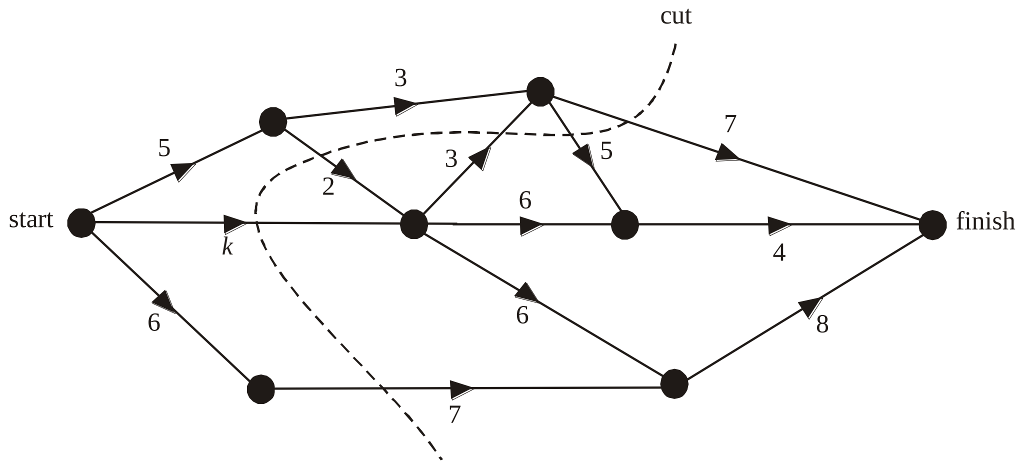
B. $\begin{bmatrix} 0 & 2 & 2 & 2 \\ 2 & 0 & 2 & 2 \\ 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 2 & 1 & 1 & 1 \end{bmatrix}$

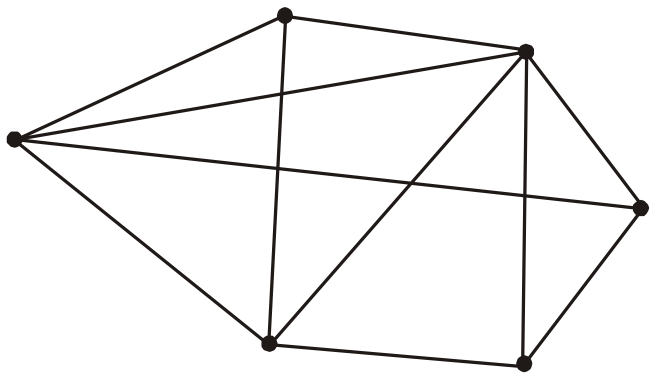
Question 5



On the directed graph above the capacity of each edge is given. The capacity of the cut shown is 28.
The value of *k* is

A. 4
B. 6
C. 7
D. 8
E. 10

Question 6



When drawn as a planar graph, the number of faces on the graph above is

A. 5
B. 7
C. 8
D. 10
E. 11

Question 7

A company operates four factories $F1$ to $F4$ and each factory can produce four different products $P1 – P4$. The cost in dollars to produce each of the products at each of the factories is given in the table below.

	P1	*P2*	*P3*	*P4*
F1	3	4	1	1
F2	4	7	3	6
F3	6	10	6	8
F4	8	5	9	4

If the company decides to assign the production of one product to one factory then the assignment that minimises cost is

A. $F1$ produces $P4$
$F2$ produces $P3$
$F3$ produces $P1$
$F4$ produces $P2$

B. $F1$ produces $P3$
$F2$ produces $P4$
$F3$ produces $P2$
$F4$ produces $P1$

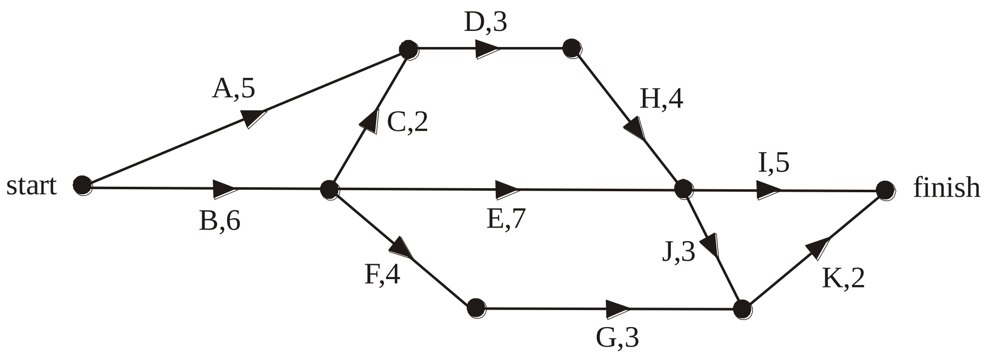
C. $F1$ produces $P4$
$F2$ produces $P2$
$F3$ produces $P3$
$F4$ produces $P1$

D. $F1$ produces $P3$
$F2$ produces $P1$
$F3$ produces $P4$
$F4$ produces $P2$

E. $F1$ produces $P2$
$F2$ produces $P4$
$F3$ produces $P1$
$F4$ produces $P3$

The information below relates to questions 8 and 9.

The network below shows the eleven activities needed to complete a project and the time in days that it takes for each activity to be completed.



Question 8

The number of critical paths is

A. 1
B. 2
C. 3
D. 4
E. 5

Question 9

The total amount of slack time; in days, in the project is

A. 2
B. 3
C. 5
D. 10
E. 15

Module 6: Matrices

If you choose this module all questions must be answered.

Question 1

The matrix expression

$\begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$ is equal to

A. $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 5 & 6 \\ 7 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 5 & 6 \\ 8 & 3 \end{bmatrix}$

Question 2

If $A = \begin{bmatrix} 2 & -5 & 7 & 2 \\ 3 & 4 & 6 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 9 \\ 7 \\ 5 \\ 4 \end{bmatrix}$, the order of the matrix product AB is

A. not defined
B. (1×2)
C. (2×1)
D. (1×4)
E. (2×4)

Question 3

At the footy, Pete bought 3 pies, 2 hot dogs and 4 buckets of chips.
At the footy, pies cost $4.50, hot dogs cost $3.70 and chips cost $4.20.
The matrix product that gives the total amount Pete spent on food at the footy is

A. $\begin{bmatrix} 3 & 2 & 4 \end{bmatrix}\begin{bmatrix} 4\cdot 50 & 3\cdot 70 & 4\cdot 20 \end{bmatrix}$

B. $\begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}\begin{bmatrix} 4\cdot 50 \\ 3\cdot 70 \\ 4\cdot 20 \end{bmatrix}$

C. $\begin{bmatrix} 3\times 4\cdot 50 & 2\times 3\cdot 70 & 4\times 4\cdot 20 \end{bmatrix}$

D. $\begin{bmatrix} 3 & 2 & 4 \end{bmatrix}\begin{bmatrix} 4.50 \\ 3.70 \\ 4.20 \end{bmatrix}$

E. $\begin{bmatrix} 4\cdot 50 \\ 3\cdot 70 \\ 4\cdot 20 \end{bmatrix}\begin{bmatrix} 3 & 2 & 4 \end{bmatrix}$

Question 4

The system of simultaneous linear equations

$$\begin{aligned} 2x + y - z &= 3 \\ 3y - z &= 4 \\ 5x + 3z &= 8 \end{aligned}$$

can be represented in matrix form as

A. $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -1 \\ 0 & 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -1 \\ 5 & 0 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

C. $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -1 \\ 5 & 3 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

D. $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -1 \\ 5 & 3 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix}$

E. $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -1 \\ 5 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix}$

Question 5

The determinant of the matrix $\begin{bmatrix} a & a \\ 2 & 3 \end{bmatrix}$ is 5.

The value of a is

A. – 5
B. – 1
C. 1
D. 5
E. 6

Question 6

Four sets of simultaneous equations are shown below.

$x+y=5$ $x+y=3$	$x=2$ $3x-y=1$
$x-y=4$ $y=3$	$2x-y=0$ $3x+y=0$

The number of these four sets that have a unique solution is/are

A. 0
B. 1
C. 2
D. 3
E. 4

Question 7

The O'Sullivan's have a family gathering each year in December. They either go to Cath's house (*C*), Gwen's house (*G*) or Dot's house (D).
No one else in the family has a house big enough to hold the gathering. They never go to the same person's house two years in a row.
A transition matrix that could be used to model this situation is

A.
$$\begin{array}{c} \text{this year} \\ \begin{array}{ccc} C & G & D \end{array} \\ \begin{bmatrix} 0 & 0\cdot6 & 0\cdot9 \\ 0\cdot7 & 0 & 0\cdot1 \\ 0\cdot3 & 0\cdot4 & 0 \end{bmatrix} \begin{array}{l} C \\ G \\ D \end{array} \text{next year} \end{array}$$

B.
$$\begin{array}{c} \text{this year} \\ \begin{array}{ccc} C & G & D \end{array} \\ \begin{bmatrix} 0\cdot4 & 0\cdot3 & 0\cdot2 \\ 0\cdot4 & 0\cdot5 & 0\cdot3 \\ 0\cdot2 & 0\cdot2 & 0\cdot5 \end{bmatrix} \begin{array}{l} C \\ G \\ D \end{array} \text{next year} \end{array}$$

C.
$$\begin{array}{c} \text{this year} \\ \begin{array}{ccc} C & G & D \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{l} C \\ G \\ D \end{array} \text{next year} \end{array}$$

D.
$$\begin{array}{c} \text{this year} \\ \begin{array}{ccc} C & G & D \end{array} \\ \begin{bmatrix} 0 & 0\cdot5 & 0\cdot8 \\ 0\cdot2 & 0 & 0\cdot2 \\ 0\cdot3 & 0\cdot4 & 0 \end{bmatrix} \begin{array}{l} C \\ G \\ D \end{array} \text{next year} \end{array}$$

E.
$$\begin{array}{c} \text{this year} \\ \begin{array}{ccc} C & G & D \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{array}{l} C \\ G \\ D \end{array} \text{next year} \end{array}$$

Question 8

The movement of mining workers between two mining towns *A* and *B* can be predicted by the transition matrix *T* where

$$\begin{array}{c} \text{this year} \\ \begin{array}{cc} A & B \end{array} \\ T = \begin{bmatrix} 0\cdot6 & 0\cdot8 \\ 0\cdot4 & 0\cdot2 \end{bmatrix} \begin{array}{l} A \\ B \end{array} \text{next year} \end{array}$$

There were equal numbers of mining workers in towns *A* and *B* in 2008. In 2009 there were 560 mining workers in town *A*. The number of mining workers in town *B* in 2008 was

A. 112
B. 224
C. 400
D. 448
E. 784

Question 9

The price of three different models of a certain brand of vacuum cleaner; each with a turbo or non-turbo motor, is given in the table below.

Model	Turbo	Non-turbo
XC 12	$980	$920
XC 15	$860	$810
XC 20	$750	$720

The store selling the vacuum cleaners discounts the non-turbo models by 20%. The price of the turbo models remains the same.
A matrix product that gives the prices of the 6 different vacuum cleaners after the discount is

A. $\begin{bmatrix} 980 & 920 \\ 860 & 810 \\ 750 & 720 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 \cdot 8 & 0 \cdot 8 \end{bmatrix}$

B. $\begin{bmatrix} 980 & 920 \\ 860 & 810 \\ 750 & 720 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \cdot 2 \end{bmatrix}$

C. $\begin{bmatrix} 980 & 920 \\ 860 & 810 \\ 750 & 720 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \cdot 8 \end{bmatrix}$

D. $\begin{bmatrix} 980 & 920 \\ 860 & 810 \\ 750 & 720 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 \cdot 8 & 0 \cdot 8 & 0 \cdot 8 \end{bmatrix}$

E. $\begin{bmatrix} 980 & 920 \\ 860 & 810 \\ 750 & 720 \end{bmatrix} \begin{bmatrix} 1 & 0 \cdot 2 & 0 \\ 1 & 0 \cdot 2 & 0 \end{bmatrix}$

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index $= \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n-1)d) = \dfrac{n}{2}[2a + (n-1)d] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, \ r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, \ |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base $\times$ height

volume of a pyramid: $\dfrac{1}{3}$ area of base $\times$ height

Pythagoras' theorem $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: $\text{effective rate of interest} \approx \frac{2n}{n+1} \times \text{flat rate}$

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET

FURTHER MATHEMATICS

TRIAL EXAMINATION 1

MULTIPLE- CHOICE ANSWER SHEET

STUDENT NAME:..

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: A ■ C D E

The answer selected is B. Only one answer should be selected.

Section A - Core

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E
11. A B C D E
12. A B C D E
13. A B C D E

Section B - Modules

Module Number ___

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E

Module Number ___

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E

Module Number ___

1. A B C D E
2. A B C D E
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4. A B C D E
5. A B C D E
6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E

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FURTHER MATHEMATICS TRIAL EXAMINATION 1 SOLUTIONS 2009

Section A – answers **Section B – answers**

Core	Module 1 Number patterns	Module 2 Geometry & trig	Module 3 Graphs & relations	Module 4 Business related maths	Module 5 Networks & decision maths	Module 6 Matrices
1. E	**1.** D	**1.** B	**1.** D	**1.** A	**1.** E	**1.** B
2. A	**2.** D	**2.** D	**2.** B	**2.** C	**2.** D	**2.** C
3. B	**3.** B	**3.** E	**3.** D	**3.** E	**3.** D	**3.** D
4. B	**4.** B	**4.** D	**4.** B	**4.** D	**4.** C	**4.** E
5. A	**5.** E	**5.** C	**5.** C	**5.** C	**5.** C	**5.** D
6. D	**6.** C	**6.** D	**6.** C	**6.** B	**6.** B	**6.** D
7. C	**7.** A	**7.** C	**7.** D	**7.** B	**7.** A	**7.** A
8. D	**8.** E	**8.** D	**8.** E	**8.** E	**8.** B	**8.** C
9. B	**9.** C	**9.** A	**9.** C	**9.** B	**9.** E	**9.** C
10. C						
11. E						
12. C						
13. E						

Core - solutions

Question 1

The minimum value is 1 (the outlier).
The maximum value is 16.
The range is $16-1=15$.
The answer is E.

Question 2

For this distribution, a score is an outlier if it is less than

$$Q_1 - 1\cdot 5 \times IQR$$
$$= 8 - 1\cdot 5 \times (12-8)$$
$$= 8 - 1\cdot 5 \times 4$$
$$= 8 - 6$$
$$= 2$$

The answer is A.

Question 3

<u>Method 1</u>

1 1 1 | 2 2 3 (3) 3 3 4 | 4 4 5

Q_1 median Q_3

$IQR = Q_3 - Q_1$
$= 4 - 1\cdot 5$
$= 2\cdot 5$

The answer is B.

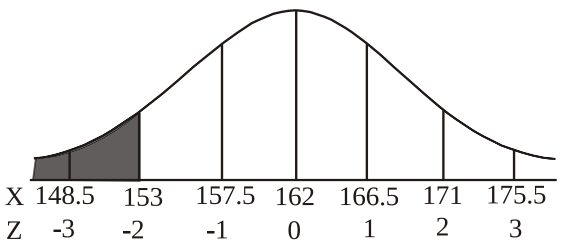
<u>Method 2</u> – use a calculator

Enter the 13 numbers.
Calculate 1 – variable stats.

$IQR = Q_3 - Q_1$
$= 4 - 1\cdot 5$
$= 2\cdot 5$

The answer is B.

Question 4



$\Pr(X < 153) = 0\cdot 025$

Note that 95% of heights lie between 2 standard deviations either side of the mean; that is between 153cm and 171cm. So 5% lie outside this; half below 153cm and the other half above 171cm.
Now $0\cdot 025 \times 200 = 5$.
The answer is B.

Question 5

The variable "number of passengers" is a discrete variable; that is you <u>count</u> the number of passengers you don't measure them. It is also the independent variable; that is, it does not depend on the total weight.
The variable "total weight" is a continuous variable; that is you <u>measure</u> it; you don't count it. It is the dependent variable; that is, it is dependent on how many passengers there are.
The answer is A.

Question 6

Enter the data into a calculator.
$r = 0 \cdot 9787$ correct to 4 decimal places.
The answer is D.

Question 7

For every extra passenger on board there will be an increase of 81.1429kg on board the plane.
The answer is C.

Question 8

The correlation coefficient *r*, is 0.7.
This suggests that an increase in the average number of kilometers run in a week is associated with an increase in the number of injuries.
Options A, B, C are incorrect.
Option E is incorrect because whilst an association between the two variables has been established, it does not prove that running causes injuries.
Since $r = 0 \cdot 7, r^2 = 0 \cdot 49$.
This is the coefficient of determination. It is true to say that 49% of the variation in the number of injuries can be accounted for by the variation in the average number of kilometers run.
The answer is D.

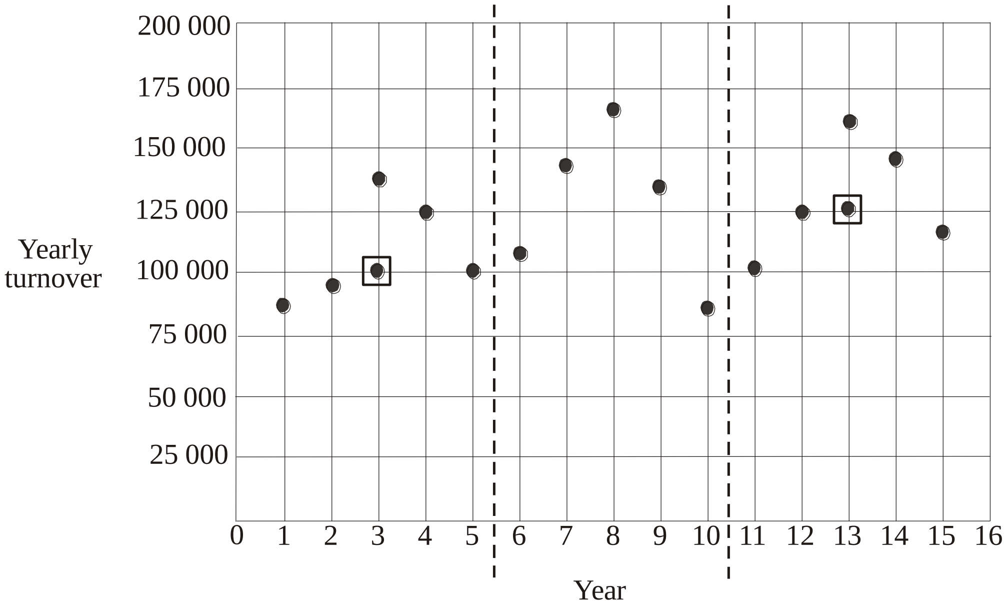
Question 9

Residual value = actual value – predicted value
The largest residual value occurs for the piece of data for which $x = 7$. That is, it is the piece of data that is furthest from the trend line.
The residual value is $3 - 1 \cdot 5 = 1 \cdot 5$.
The answer is B.

Question 10

There is a clear pattern in the turnover of the small business. It is a cyclic trend because each cycle occurs over the course of 5 years (unlike a seasonal trend which occurs over the course of 1 year).
The answer is C.

Question 11



The median in the bottom one third of data points is (3,100 000). The median in the top one third of data points is (13,125,000).
The slope is given by

$$\frac{125000-100000}{13-3}$$

$$=\frac{25000}{10}$$

$$=2500$$

The answer is E.

Question 12

The three mean moving average for June is given by $\frac{30+25+23}{3}=\frac{78}{3}=26$.

The answer is C.

Question 13

$$\text{deseasonalised sales} = \frac{\text{actual sales}}{\text{seasonal index}}$$

$$325000=\frac{\text{actual sales}}{1\cdot 23}$$

$$\text{actual sales} = 1\cdot 23\times 325000$$

$$=399750$$

The answer is E.

SECTION B

Module 1: Number patterns

Question 1

Method 1 – using a calculator
Generate the sequence on your calculator. The twentieth term is 137.
The answer is D.

Method 2

$$t_n = a + (n-1)\,d$$
$$t_{20} = 4 + (20-1) \times 7$$
$$= 137$$

The answer is D.

Question 2

Method 1 – trial and error

If $r = 2,\ 3 \times 2 = 6,\ 6 \times 2 = 12$

If $r = 3,\ 3 \times 3 = 9,\ 9 \times 3 = 27$

If $r = 4,\ 3 \times 4 = 12, 12 \times 4 = 48$

So $m = 12$

The answer is D.

Method 2
Because the sequence is geometric,

$$\frac{t_2}{t_1} = \frac{t_3}{t_2}$$

So, $\frac{m}{3} = \frac{48}{m}$ (cross multiply)

$$m^2 = 144$$
$$m = \pm 12$$

Only the option 12 is offered.
The answer is D.

Question 3

$$t_n = t_{n-2} + t_{n-1}, \quad t_1 = -3, t_2 = 1$$
$$t_1 = -3$$
$$t_2 = 1$$
$$t_3 = -3 + 1 = -2$$
$$t_4 = 1 - 2 = -1$$
$$t_5 = -2 - 1 = -3$$
$$t_6 = -1 - 3 = -4$$

The answer is B.

Question 4

We have a geometric sequence with $a = 18$ and $r = 1 \cdot 02$.
We want the sum of the first nine terms.

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{since } r > 1$$

$$S_9 = \frac{18(1 \cdot 02^9 - 1)}{0 \cdot 02}$$

$$= 175 \cdot 5833...$$

The closest answer is 175.58.
The answer is B.

Question 5

We need to find the sum of the first twenty terms of the arithmetic sequence 1,2,3,4,…

Method 1

$$S_n = \frac{n}{2}(a + l)$$

$$S_{20} = \frac{20}{2}(1 + 20)$$

$$= 210$$

The answer is E.

Method 2

$$S_n = \frac{n}{2}(2a + (n - 1)\ d)$$

$$S_{20} = \frac{20}{2}(2 \times 1 + (20 - 1) \times 1)$$

$$= 10(2 + 19)$$

$$= 210$$

The answer is E.

Question 6

For the budget for the $(n + 1)$th year, the previous year's budget is B_n.

3.5% of the previous year's budget is $\frac{3.5}{100} B_n = 0 \cdot 035 B_n$.

The previous year's budget plus 3.5% of the previous year's budget is given by
$B_n + 0 \cdot 035 B_n = 1 \cdot 035 B_n$

So $B_{n+1} = 1 \cdot 035 B_n + 6000 \quad B_1 = 42\,000$

The answer is C.

Question 7

The sequence is not arithmetic because there is no common difference between successive terms.
This eliminates options C and D.
Since the first term is negative, the second positive, the third negative, option B is not correct.
Similarly with option E since,

$$\begin{aligned} t_2 &= 1\cdot 3 \times -10 - 23 \\ &= -36 \end{aligned}$$

From the graph, $t_2 > 0$.
The answer is A.

Question 8

We have a geometric sequence with $a = 10$ and $r = 0\cdot 96$.

$$\begin{aligned} S_\infty &= \frac{a}{1-r} \\ &= \frac{10}{1-0\cdot 96} \\ &= \frac{10}{0\cdot 04} \\ &= 250 \text{ mg} \end{aligned}$$

The answer is E.

Question 9

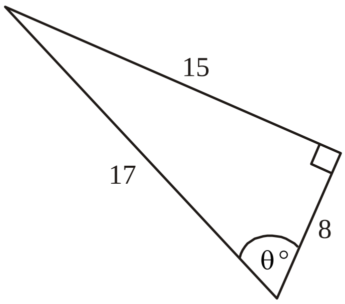
$t_{n+1} = 0\cdot 5t_n - 1 \quad t_1 = 1000$
Generate the sequence on your calculator.

1000, 499, 248.5, 123.25, 60.625, …

The sequence is neither arithmetic nor geometric so options B and D are true.
The sequence is decreasing so option A is true.
$t_3 = 248\cdot 5$ so option E is true.
The sequence has positive and negative numbers. The tenth term, the 11th term and the rest of the terms are negative.
So option C is incorrect.
The answer is C.

Module 2: Geometry and trigonometry

Question 1



$$\cos\theta^\circ = \frac{\text{adj}}{\text{hyp}}$$
$$= \frac{8}{17}$$

The answer is B.

Question 2

The sum of the interior angles of a rectangular polygon with *n* sides is given by $S = \{180(n-2)\}^\circ$.

For a regular pentagon, $n = 5$

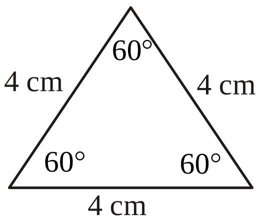
$$\text{so } S = \{180(5-2)\}^\circ$$
$$= 540^\circ$$

The answer is D.

Question 3

Because the cross-section is an equilateral triangle with sidelength 4cm, we have

$$\text{Area} = \frac{1}{2}bc\sin A$$
$$= \frac{1}{2} \times 4 \times 4 \times \sin 60^\circ$$
$$= 6\cdot 9282...$$



Area of rectangular face $= 10 \times 4 = 40\text{cm}^2$

$$\text{Total surface area} = 2 \times 6\cdot 9282... + 3 \times 40$$
$$= 133\cdot 8564...$$

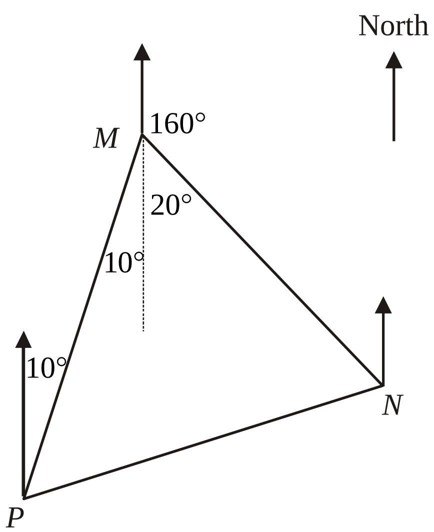
The closest answer is 133.86.

The answer is E.

Question 4

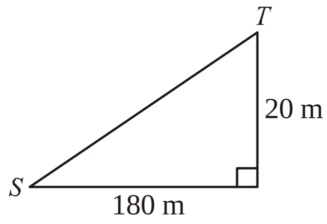
The bearing of *P* from *M* is 190°.

The answer is D.



Question 5

$$\text{average slope} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{20}{180}$$
$$= \frac{1}{9}$$



The answer is C.

Question 6

The triangles are similar.

$$\text{Scale factor} = k$$
$$= \frac{12}{8}$$
$$= 1 \cdot 5$$

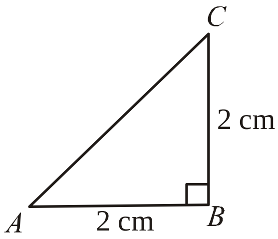
$$\text{So, area of } \Delta PQR = 1 \cdot 5^2 \times \text{area of } \Delta ABC$$
$$= 2 \cdot 25 \times 30$$
$$= 67 \cdot 5 \text{cm}^2$$

The answer is D.

Question 7

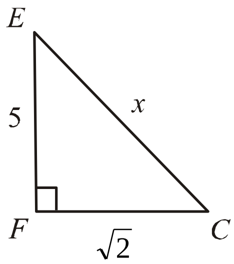
In ΔABC,

$$(AC)^2 = 2^2 + 2^2$$
$$= 8$$
$$AC = 2\sqrt{2}$$
$$\text{So } CF = \sqrt{2}$$



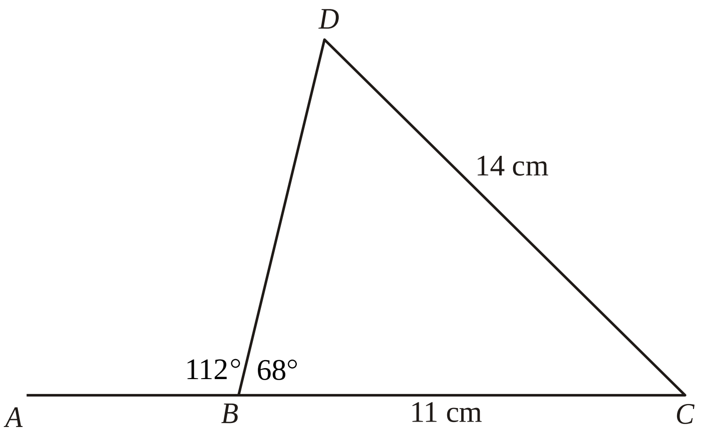
In ΔCEF

$$x^2 = 5^2 + \left(\sqrt{2}\right)^2$$
$$= 27$$
$$x = \sqrt{27}$$



The answer is C.

Question 8

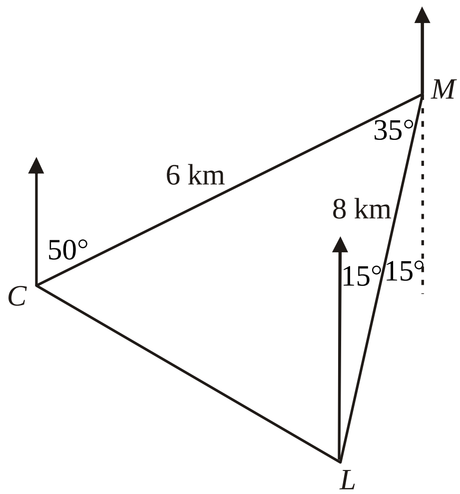


$\angle CBD = 180^\circ - 112^\circ$
$= 68^\circ$

$\frac{\sin(\angle BDC)}{11} = \frac{\sin(68^\circ)}{14}$
$\sin(\angle BDC) = 0 \cdot 7285...$
$\angle BDC = \sin^{-1}(0 \cdot 7285...)$
$= 46 \cdot 7609...$
So $\angle BCD = 180^\circ - 68^\circ - 46 \cdot 7609...^\circ$
$= 65 \cdot 2390...^\circ$
The closest answer is 65.2°.
The answer is D.

Question 9

Draw a diagram.
Use the bearing to find $\angle CML$
(since $35^\circ + 15^\circ = 50^\circ$ alternate angles).
$(CL)^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos(35^\circ)$
$= 21 \cdot 3614...$
$CL = 4 \cdot 6218...$
The closest answer is A.
The answer is A.



Module 3: Graphs and relations

Question 1

Method 1

$$\text{From the graph, gradient} = \frac{\text{rise}}{\text{run}} = \frac{2}{2} = 1$$

The answer is D.

Method 2

The line passes through $(-2,0)$ and $(0,2)$.
Let $(-2,0) = (x_1, y_1)$ and $(0,2) = (x_2, y_2)$.

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{0-{}^{-}2} = \frac{2}{2} = 1$$

The answer is D.

Question 2

$C = 30 + 4n$
For each call made, the monthly cost is increased by $4.
The answer is B.

Question 3

From the graph, the line drops below 10km/hr between 1.5 and 3 days and again between 5 and 7.5 days. In total it is below 10km/hr for approximately $1.5 + 2.5 = 4$ days.
The answer is D.

Question 4

We are looking for not just an increase in wind speed but the steepness of the increase. The steepest increase occurs between 3 – 4 days.
The answer is B.

Question 5

Let $x = \text{cost of hiring an overnight DVI}$
Let $y = \text{cost of hiring a weekly DVI.}$

$$\begin{array}{lrl} & 2x+3y=24\cdot 40 & -(1)\\ & 3x+5y=38\cdot 50 & -(2)\\ (1)\times 3 & 6x+9y=73\cdot 20 & -(3)\\ (2)\times 2 & 6x+10y=77\cdot 00 & -(4)\\ (4)-(3) & y=3\cdot 80 & \\ \text{In }(1) & 2x+3\times 3\cdot 80=24\cdot 40 & \\ & 2x+11\cdot 40=24\cdot 40 & \\ & 2x=13\cdot 00 & \\ & x=6\cdot 50 & \end{array}$$

So one overnight and one weekly DVD would cost $\$3\cdot 80+\$6\cdot 50=\$10\cdot 30$.
The answer is C.

Question 6

Option A is correct.
Option B is correct.
Option C is incorrect because in the feasible region, the maximum value of *y* is 10 (occurs at the corner point $(5,10)$).
Options D and E are both correct.
The answer is C.

Question 7

The gradient of the straight line is $\frac{2}{1}=2$.
The relationship between *y* and *x* is therefore given by $y=2x^3$.
The answer is D.

Question 8

Let $C = \text{total costs for the cabinet maker in dolla}$

$$\begin{aligned} C &= 2\,800+70\times 30\\ &= 4\,900 \end{aligned}$$

The cabinetmaker sells the 30 stands to the retailer for a total of \$6 000 (\$4 900 in costs and \$1 100 in profit).
For each stand, the retailer pays $\$6\,000\div 30=\$200$.
The answer is E.

Question 9

The constraint due to labour is $20x+15y\le 2\,400$.
The constraint due to fabric is $80x+120y\le 14\,400$.
The answer is C.

Module 4: Business-related mathematics

Question 1

$$\begin{aligned}\text{simple interest} &= \frac{PrT}{100}\\ 40 &= \frac{P\times 2\cdot 5\times 2}{100}\\ 40\times 100 &= 5P\\ \frac{4000}{5} &= P\\ P &= 800\end{aligned}$$

The answer is A.

Question 2

Compound interest.

$$\begin{aligned}A &= PR^n\\ &= 13000\times 1\cdot 016^8\\ &= 14760\cdot 23\end{aligned}$$

$$R = 1+\frac{r}{100} = 1+\frac{1.6}{100} = 1.016$$

Note that the rate per annum is 3.2% so the rate per 6 months is 1.6%.

The answer is C.

Question 3

For a perpetuity

$$P = \frac{100Q}{R}$$

where Q is the amount paid per annum.

Gwen receives \$350 per week which is $\$350\times 52 = \$18\,200$ per year. The interest per annum is 5%.

$$\begin{aligned}\text{So }\ P &= \frac{100\times 18\,200}{5}\\ &= \$364\,000\end{aligned}$$

The answer is E.

Question 4

Using the reducing balance method,

$$V = P \times \left(1 - \frac{r}{100}\right)^t$$

$$800 = 1500 \times \left(1 - \frac{r}{100}\right)^3$$

<u>Method 1</u> – trial and error

$$\text{If } r = 7\%, \quad 1500 \times \left(1 - \frac{7}{100}\right)^3 = 1206.5355$$

$$\text{If } r = 15\%, \quad 1500 \times \left(1 - \frac{15}{100}\right)^3 = 921.1875$$

$$\text{If } r = 17\%, \quad 1500 \times \left(1 - \frac{17}{100}\right)^3 = 857.6805$$

$$\text{If } r = 19\%, \quad 1500 \times \left(1 - \frac{19}{100}\right)^3 = 797.1615$$

$$\text{If } r = 23\%, \quad 1500 \times \left(1 - \frac{23}{100}\right)^3 = 684.7995$$

The closest answer is 19.
The answer is D.

<u>Method 2</u> – using indices

$$800 = 1500 \times \left(1 - \frac{r}{100}\right)^3$$

$$\frac{800}{1500} = \left(1 - \frac{r}{100}\right)^3$$

$$\left(\frac{8}{15}\right)^{\frac{1}{3}} = \left(\left(1 - \frac{r}{100}\right)^3\right)^{\frac{1}{3}}$$

$$0 \cdot 8109\ldots = 1 - \frac{r}{100}$$

$$0 \cdot 8109\ldots - 1 = -\frac{r}{100}$$

$$-0 \cdot 1890\ldots = -\frac{r}{100}$$

$$0 \cdot 1890\ldots = \frac{r}{100}$$

$$r = 100 \times 0 \cdot 1890\ldots$$

$$= 18 \cdot 90\ldots$$

The closest answer is 19.
The answer is D.

Question 5

Between quarter 1 and 2 the price of the bicycle increased by \$10.

This represents a percentage increase of $\left(\frac{10}{500} \times \frac{100}{1}\right)\% = 2\%$.

The answer is C.

Question 6

Method 1

For flat interest we have

$$r = \frac{100I}{Pt}$$

Now, $r = 5$

$$\begin{aligned} I &= \text{total interest paid} \\ &= 1000 + 26 \times x - 3500 \\ &= 26x - 2500 \end{aligned}$$

where x is the fortnightly payment.

$$\begin{aligned} P &= 3500 - 1000 \\ &= 2500 \end{aligned}$$

So, $5 = \frac{100(26x - 2500)}{2500 \times 1}$

$$\begin{aligned} \frac{5 \times 2500}{100} &= 26x - 2500 \\ 125 &= 26x - 2500 \\ 2625 &= 26x \\ x &= 100 \cdot 96 \end{aligned}$$

The answer is B.

Method 2

Amount owing $= \$3500 - \$1000 = \$2500$.

Fortnightly payment is given by

$$\frac{\$2500 \times 1.05}{26} = \$100.96$$

The answer is B.

Question 7

$$\begin{aligned} \text{the effective rate of interest} &= \frac{2n}{n+1} \times \text{flat rate} \\ &= \frac{2 \times 26}{26+1} \times 5\% \\ &= 9 \cdot 63\% \end{aligned}$$

The answer is B.

Question 8

Use *TVM* solver

$N = ?$

$I\% = 5 \cdot 5$

$PV = -120000$ (Negative because Nigel gave this money to the bank.)

$PMT = 750$ (Positive because the bank gives this money each month to Nigel.)

$FV = 0$

$P/Y = 12$

$C/Y = 12$

$N = 289 \cdot 0434...$

This is the number of months that the annuity will last for.

So $289 \cdot 0434... \div 12 = 24 \cdot 08....$

The annuity will last for 24.08…years.

The closest answer is 24.

The answer is E.

Question 9

Use *TVM* solver to find the quarterly repayments Paul has to make.

$N = 16$

$I\% = 6$

$PV = 60000$

$PMT = ?$

$FV = 0$

$P/Y = 4$

$C/Y = 4$

$PMT = -4\,245 \cdot 90$ (Negative because Paul has to pay it to the bank).

Use TVM solver to find the future value (i.e. the principal remaining to be paid off on the loan) of the loan after 8 repayments.

$N = 8$

$I\% = 6$

$PV = 60\,000$

$PMT = -4\,245 \cdot 90$

$FV = ?$

$P/Y = 4$

$C/Y = 4$

$FV = -31\,784 \cdot 52$

So after 8 repayments Paul still owes $\$31\,784 \cdot 52$.

This means he has paid $\$60\,000 - \$31\,784 \cdot 52 = \$28\,215 \cdot 48$ off the principal.

The answer is B.

Module 5: Network and decision mathematics

Question 1

All 5 vertices have an even degree.
The answer is E.

Question 2

A Hamiltonian path passes through each vertex once starting and finishing on a different vertex.
Only option D offers this.
The answer is D.

Question 3

A tree contains no circuits. The graph is not a tree. Option A is incorrect.
A complete graph has each pair of vertices connected by an edge.
Not all edges are connected by and edge eg. *M* and *S* or *P* and *R*. Option B is incorrect.
The graph has plenty of circuits but no loops. Option C is incorrect.
The graph does contain an Euler path since exactly two vertices *N* and *Q* have odd degrees and the rest have even degrees. Option D is correct.
The answer is D.

Question 4

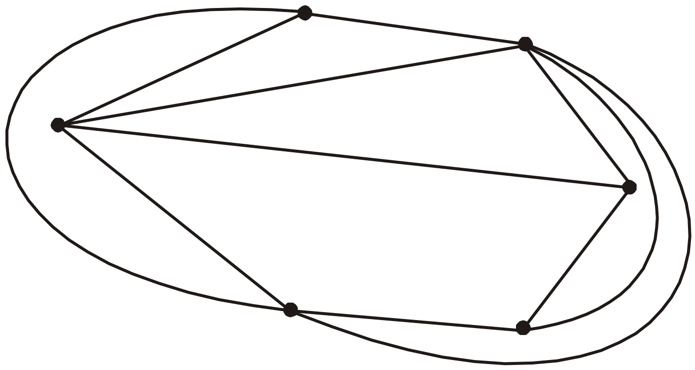
There are no loops so the leading diagonal of the matrix contains only zeroes.
Only vertices *A* and *D* have two vertices connecting them.
All other vertices are connected to each other vertex just once.
Matrix C represents this.
The answer is C.

Question 5

The directed edges running across the cut in the direction from start to finish are
$7+5+2+k+7=28$. So $k=7$.
The answer is C.

Question 6

The graph can be redrawn



to reveal a planar graph.
Since the graph is a connected, planar graph, Euler's formula can be applied.

$$v - e + f = 2$$

where $v = 6$

$e = 11$

So $6 - 11 + f = 2$

$f = 7$

The answer is B.

Question 7

	P1	*P2*	*P3*	*P4*
F1	3	4	1	1
F2	4	7	3	6
F3	6	10	6	8
F4	8	5	9	4

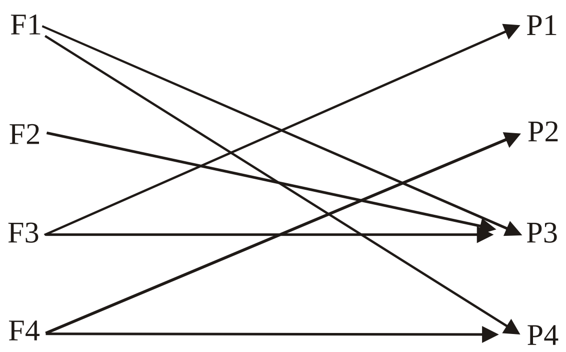
Use the Hungarian algorithm.
Subtract the minimum element in each row from each of the elements in that row.

	P1	*P2*	*P3*	*P4*
F1	2	3	0	0
F2	1	4	0	3
F3	0	4	0	2
F4	4	1	5	0

Do the same for column 2 because it has no zeroes.

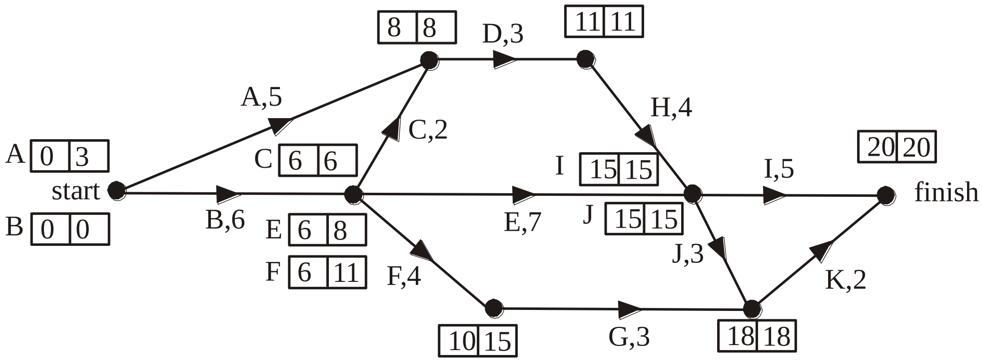
	P1	*P2*	*P3*	*P4*
F1	2	2	0	0
F2	1	3	0	3
F3	0	3	0	2
F4	4	0	5	0

Use a bipartite graph.



The allocation is *F*3 produces *P*1, *F*4 produces *P*2, *F*1 produces *P*4 and *F*2 produces *P*3.
The answer is A.

Question 8



There are two critical paths *B*, *C*, *D*, *H*, *I* and *B*, *C*, *D*, *H*, *J*, *K*.
The answer is B.

Question 9

From Question 8 we saw that there were 7 activities that were critical. For the other 4, *A* has 3 days of slack time, *E* has 2, *F* has 5 and *G* has 5.
In total there are 15 days of slack time in the project.
The answer is E.

Module 6: Matrices

Question 1

$$\begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 5 \\ 2 & 1 \end{bmatrix}$$

The answer is B.

Question 2

A is a (2×4) matrix.
B is a (4×1) matrix.
The matrix product *AB* is of order (2×1).
The answer is C.

Question 3

The matrix product in Option A is not defined $(1\times 3)\times(1\times 3)$.
Similarly the matrix product in option B is not defined $(3\times 1)\times(3\times 1)$.
The matrix in option C has 3 elements not one and is not really a matrix product (or product of matrices).
Option D is correct; $(1\times 3)\times(3\times 1)$. This product gives a (1×1) matrix which gives the total that Pete spent.
Option E; $(3\times 1)\times(1\times 3)$ whilst defined, gives a (3×3) matrix as an answer.
The answer is D.

Question 4

The system of equations

$$\begin{aligned} 2x+y-z&=3 \\ 3y-z&=4 \\ 5x+3z&=8 \end{aligned}$$

can be written as

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -1 \\ 5 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 8 \end{bmatrix}$$

The answer is E.

Question 5

$$\det\begin{bmatrix} a & a \\ 2 & 3 \end{bmatrix} = 3a - 2a$$

$$= a$$

So $a = 5$.
The answer is D.

Question 6

Express each one as a matrix equation in the form

$$AX = B$$

The first set

$$x + y = 5$$

$$x + y = 3$$

can be expressed as the matrix equation

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$\det A = 0$ so this set has no unique solution.

For the second set, $A = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$

$$\det A = 1 \times -1 - 3 \times 0 = -1$$

A unique solution exists.

For the third set, $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$$\det A = 1 \times 1 - 0 \times -1 = 1$$

A unique solution exists.

For the fourth set, $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$

$$\det A = 2 \times 1 - 3 \times {}^{-}1 = 5$$

A unique solution exists.

There are 3 unique solutions.
The answer is D.

Question 7

First, a transition matrix has column totals of 1. We can eliminate options D and E immediately.
Options B and C can be eliminated because the leading diagonal of both matrices doesn't contain zeroes meaning that they can return to the same place as last year.
The answer is A.

Question 8

Let the number of mining workers in town *A* in 2008 be *n* so the number of mining workers in town *B* in 2008 is also *n*. Let the number of mining workers in town *B* in 2009 be *x*.
So,

$$\begin{bmatrix} 0\cdot 6 & 0\cdot 8 \\ 0\cdot 4 & 0\cdot 2 \end{bmatrix}\begin{bmatrix} n \\ n \end{bmatrix} = \begin{bmatrix} 560 \\ x \end{bmatrix}$$

Multiplying, we get

$$\begin{aligned} 0\cdot 6\times n + 0\cdot 8\times n &= 560 \\ 1\cdot 4n &= 560 \\ n &= \frac{560}{1\cdot 4} \\ &= 400 \end{aligned}$$

The number of mining workers in town *B* (and town *A*) in 2008 was 400.
Note that we're not interested in finding the value of *x*, it was just introduced to fill a hole in the matrix.
The answer is C.

Question 9

The price matrix $\begin{bmatrix} 980 & 920 \\ 860 & 810 \\ 750 & 720 \end{bmatrix}$ is a (3×2) matrix.

We want to produce another (3×2) matrix so we need to multiply $(3\times 2)\times(2\times 2)$ to get a (3×2) matrix.
We can therefore eliminate options D and E.
We want the first column of our price matrix to remain the same and the second to be multiplied by 0.8 (20% reduction in price). Only option C offers this.
For example $980\times 1+920\times 0=980$ is element$_{1,1}$. Also $980\times 0+920\times 0\cdot 8=736$ is element$_{1,2}$.
The answer is C.