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FURTHER MATHEMATICS TRIAL EXAMINATION 2 SOLUTIONS 2009

SECTION A

Core - solutions

Question 1

a. 6 boys are in the range 9% - 15%.

$$\left(\frac{6}{15}\times\frac{100}{1}\right)\% = 40\%$$

So 40% of boys in the sample are considered healthy.

(1 mark)

b. **i.** The mode is the most popular (frequently occurring) piece of data.
The mode is 16.

(1 mark)

ii. The mean is 16.2.

(1 mark)

c. $\text{Mean} = 16\cdot 2$
$\text{standard deviation} = 4\cdot 3$

$$\begin{aligned} z &= \frac{19 - 16\cdot 2}{4\cdot 3} \\ &= 0\cdot 7 \text{ (correct to 1 decimal place)} \end{aligned}$$

(1 mark)

Question 2

a. The median is the middle score.
It is in the $\left(\frac{n+1}{2}\right)$th position.
Since $n = 18$ it is in the 9.5$^{\text{th}}$ position i.e. between the 9$^{\text{th}}$ and 10$^{\text{th}}$ scores. So the median is between 16 and 17.
The median is 16.5.

(1 mark)

b. The median is a better measure of the centre of the distribution than the mean because

1. the distribution is positively skewed.

(1 mark)

2. there are two outliers (the scores 31 and 40).

(1 mark)

Question 3

a.

BFP	Gender	
	female	male
acceptable	40	24
unacceptable	12	19
total	52	43

(2 marks)

b. Yes the data supports the hypothesis because $\left(\frac{19}{43}\times 100\right)\% = 44.2\%$ of men have an unacceptable BFP compared with $\left(\frac{12}{52}\times 100\right)\% = 23.1\%$ of women. If there was no relationship between BFP and gender we would expect those percentages to be similar.

(1 mark) quoting correct %'s
(1 mark) quoting % in correct context

Question 4

a. It is not appropriate because from the scatterplot we see that the relationship between age and BFP is not linear.

(1 mark)

b. There are two possibilities.
The variable "age" could be replaced with "(age)^2" or the variable "BFP" could be replaced with "$\frac{1}{BFP}$".

(1 mark)

c. $\log(BFP) = 0\cdot 0061 \times \text{age} + 1\cdot 0947$

(1 mark) correct coefficient of age
(1 mark) correct equation including 1.0947
Total 15 marks

SECTION B

Module 1: Number patterns

Question 1

a. Three weeks after logging commences there will be $96-3\times 6=78$ hectares of trees left.

(1 mark)

b. $96\div 6=16$
It will take 16 weeks.

(1 mark)

c. $m=-6$
Check.
$T_{10}=96+10\times -6$
$=36$
There are 36 hectares left after 10 weeks since 60 hectares have been logged during that time.

(1 mark)

d. $c=6$ since 6 hectares are subtracted from last week's total T_{n-1} to find this week's total T_n that is left.

(1 mark)

Question 2

a. $r=\dfrac{76}{80}=\dfrac{72\cdot 2}{76}$
$=0\cdot 95$

(1 mark)

b. The geometric sequence is 80, 76,72.2, 68.59, 65.1605, 61.902475,…
At the end of the sixth week there were 61.90 hectares of timber left.

(1 mark)

c. At the end of the 10th week there will be 50.41995 hectares left and at the end of the 11th week there will be 47.89895 hectares left. So there are $50\cdot 41995-47\cdot 89895=2\cdot 52$ (correct to 2 decimal places) left.

(1 mark)

Use your calculator to generate the sequence up to the 11th term.

d. Again use your calculator to generate the sequence. It takes 42 weeks of logging until the number of hectares of timber remaining first drops below 10.

(1 mark)

e. $S_n=80\times(0\cdot 95)^{n-1}$

(1 mark)

Question 3

a. $W_n = 0 \cdot 97W_{n-1} + 2 \cdot 5, \quad W_0 = 120$

$W_1 = 118 \cdot 9$

$W_2 = 117 \cdot 833$

$W_3 = 116 \cdot 79801$

So there are 116.80 hectares of timber (old and new) on the block 3 weeks after the logging/replanting process begins.

(1 mark)

b. From the difference equation

$W_n = 0 \cdot 97W_{n-1} + 2 \cdot 5, \quad W_0 = 120$

we see that 2.5 hectares of trees are being replanted each week.

(1 mark)

c. $W_0 = 120$

$W_1 = 118 \cdot 9$

$W_2 = 117 \cdot 833$

At the end of the second week there were 117.833 hectares of trees (old and new) remaining. In the third week, the number of hectares of trees that remained before any more were planted was $0 \cdot 97 \times 117 \cdot 833 = 114 \cdot 29801$.

So $117 \cdot 833 - 114 \cdot 29801 = 3 \cdot 53499$ or $3 \cdot 53$ hectares (correct to 2 decimal places) of trees were logged in the third week.

(1 mark)

Question 4

a. $E_n = 0 \cdot 94E_{n-1}, E_0 = 285$

This difference equation describes a geometric sequence because each term is obtained by multiplying the previous term by the common ratio 0.94.

(1 mark)

b. $E_n = 0 \cdot 94E_{n-1}, \quad E_0 = 285$

$W_n = 0 \cdot 97W_{n-1} + 2 \cdot 5, \quad W_0 = 120$

Use a calculator to generate the sequences defined by these difference equations. We see from the table of terms of each of the sequences that at the end of the 16th week of logging the number of hectares of trees remaining is equal (correct to 1 decimal place). There are 105.9 hectares remaining.

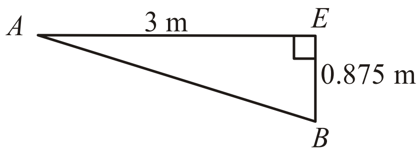
(1 mark) – for 16th week
(1 mark) – for 105.9 hectares
Total 15 marks

Module 2: Geometry and trigonometry

Question 1

a. In ΔABE,

$$(AB)^2 = 3^2 + 0 \cdot 875^2$$
$$AB = \sqrt{3^2 + 0.875^2}$$
$$= 3 \cdot 125\text{m}$$



(1 mark)

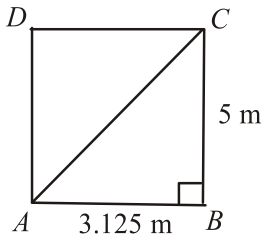
b. Again in ΔABE,

$$\tan(\angle BAE) = \frac{\text{opp}}{\text{adj}}$$
$$= \frac{0 \cdot 875}{3}$$
$$\angle BAE = \tan^{-1}\left(\frac{0 \cdot 875}{3}\right)$$
$$= 16 \cdot 3^\circ \quad \text{(correct to 1 decimal place)}$$

(1 mark)

c. In ΔABC, we have

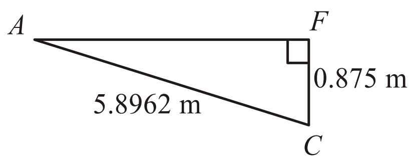
$$(AC)^2 = 3 \cdot 125^2 + 5^2$$
$$AC = \sqrt{3.125^2 + 5^2}$$
$$= 5 \cdot 8962\text{m} \quad \text{(correct to 4 decimal places)}$$



(1 mark)

d. In ΔACF

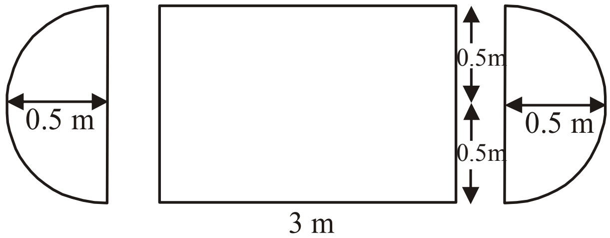
$$\sin(\angle CAF) = \frac{\text{opp}}{\text{hyp}}$$
$$= \frac{0 \cdot 875}{5 \cdot 8962}$$
$$\angle CAF = \sin^{-1}\left(\frac{0 \cdot 875}{5 \cdot 8962}\right)$$
$$= 8 \cdot 53^\circ \quad \text{(correct to 2 decimal places)}$$



(1 mark)

Question 2

a.



$$\begin{aligned}\text{S.A.} &= \text{area of circle} + \text{area of rectangle}\\ &= \pi r^2 + 3\times 1\\ &= \pi \times 0\cdot 5^2 + 3\\ &= 3\cdot 7854\text{m}^2\end{aligned}$$

(Note that the width of the rectangle is equal to $0\cdot 5 + 0\cdot 5 = 1$)

(1 mark)

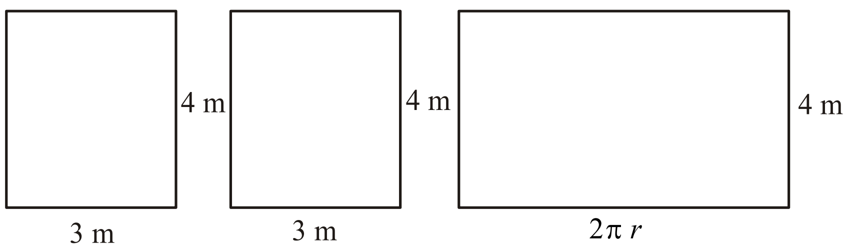
b. $V = \text{area of cross-section} \times \text{height}$
$= 3\cdot 7854 \times 4$ (from part **a.**)
$= 15\cdot 14\text{m}^3$ (correct to 2 decimal places)

(1 mark)

c. From part **b.**, $V = 15\cdot 14\text{m}^3$ so the tank can hold $15\cdot 14 \times 1000 = 15\,140$ litres.

(1 mark)

d. From part **a.**, the surface area of the top (and therefore the base) of the tank is $3\cdot 7854\text{m}^2$ (to 4 decimal places).
The surface area of the sides can be broken up into 2 rectangles and the sides of a cylinder with radius $0\cdot 5$m.



$\text{S.A. of sides} = 4\times 3 + 4\times 3 + 2\pi r \times 4$
$= 24 + 8\pi \times 0\cdot 5$
$= 24 + 4\pi$ **(1 mark)**

$\text{Total SA} = 24 + 4\pi + 2\times 3\cdot 7854$ (top and base)
$= 44\cdot 14\text{m}^2$ (correct to 2 decimal places)

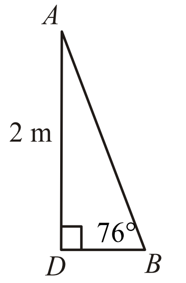
(1 mark)

Question 3

a. In ΔABD

$$\sin(76°) = \frac{2}{AB}$$

$$AB = \frac{2}{\sin(76°)}$$
$$= 2.0612 \text{ m (correct to 4 decimal places)}$$

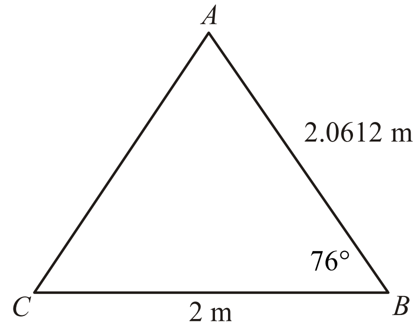


(1 mark)

b. In ΔABC,

$$(AC)^2 = 2^2 + 2\cdot0612^2 - 2\times2\times2\cdot0612\times\cos(76^\circ)$$

$$AC = 2\cdot5007...$$
$$= 2.5 \text{ m} \quad \text{(correct to 1 decimal place)}$$



(1 mark)

c. Again in ΔABC,

$$\frac{\sin(\angle ACB)}{2\cdot0612} = \frac{\sin(76^\circ)}{2\cdot5007}$$

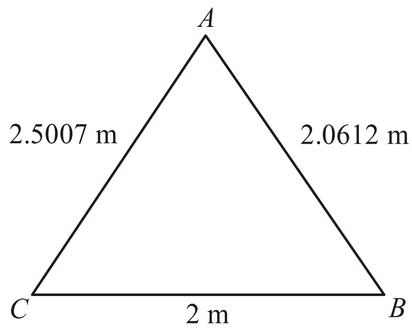
$$\angle ACB = 53\cdot1077...^\circ$$
$$= 53^\circ \quad \text{(to the nearest degree)}$$

(1 mark)

d. Heron's formula.

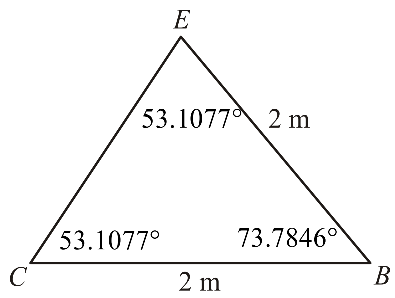
$$s = \frac{1}{2}(2\cdot0612 + 2\cdot5007 + 2)$$
$$= 3\cdot28095$$
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= 1.999945...$$
$$= 2\text{m}^2$$

(to the nearest square metre)



(1 mark)

e. ΔBCE is an isosceles triangle since $BC = BE = 2\text{ m}$.
So, $\angle BEC = \angle BCE = 53.1077°$ from part **c.**



(1 mark)

<u>Method 1</u> – using the sine rule

$$\frac{CE}{\sin(73.7846°)} = \frac{2}{\sin(53.1077°)}$$

$$CE = \frac{2}{\sin(53.1077°)} \times \sin(73.7846°)$$

$$CE = 2.40125...$$

$$= 2.4 \text{ metres (correct to 1 decimal place)}$$

(1 mark)

<u>Method 2</u> – using the cosine rule

$$(CE)^2 = 2^2 + 2^2 - 2 \times 2 \times 2 \times \cos(73.7846^\circ)$$

$$CE = 2 \cdot 40125...$$

$$= 2.4 \text{ metres (correct to 1 decimal place)}$$

(1 mark)

Total 15 marks

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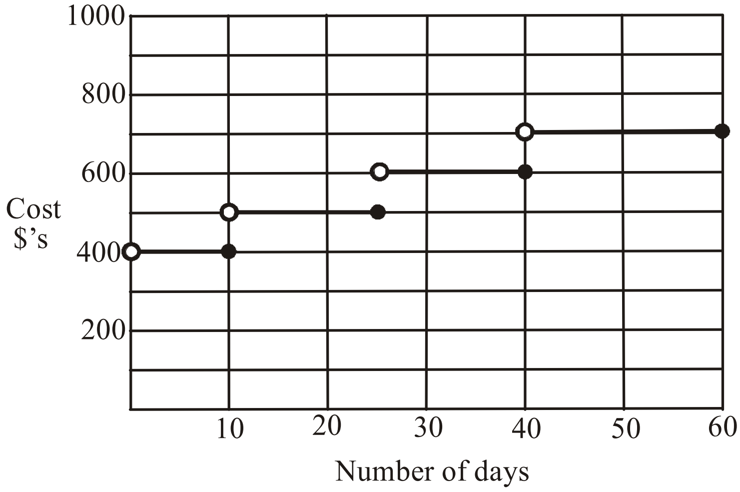
Module 3: Graphs and relations.

Question 1

a. From the graph, it costs $500.

(1 mark)

b.



(1 mark)

c. People would hire for exactly 10, 25 or 40 days because on the 11th, 26th and 41st day of hiring prices go up. People are getting the most number of days for the money they have paid.

(1 mark)

Question 2

a. $R = 1\,200x$

(1 mark)

b. $C = 80\;000 + 400x$

(1 mark)

c. At break even point, $R = C$

$$1\,200x = 80\;000 + 400x$$
$$800x = 80\,000$$
$$x = 100$$

Kate needs 100 clients to break even.

(1 mark)

d. If she has 60 clients she will make a loss.

If $x = 60$, $R = 1\,200 \times 60 = 72\,000$ $C = 80\,000 + 400 \times 60 = 104\,000$

$$\text{Loss} = 104\,000 - 72\,000 = \$32\,000$$

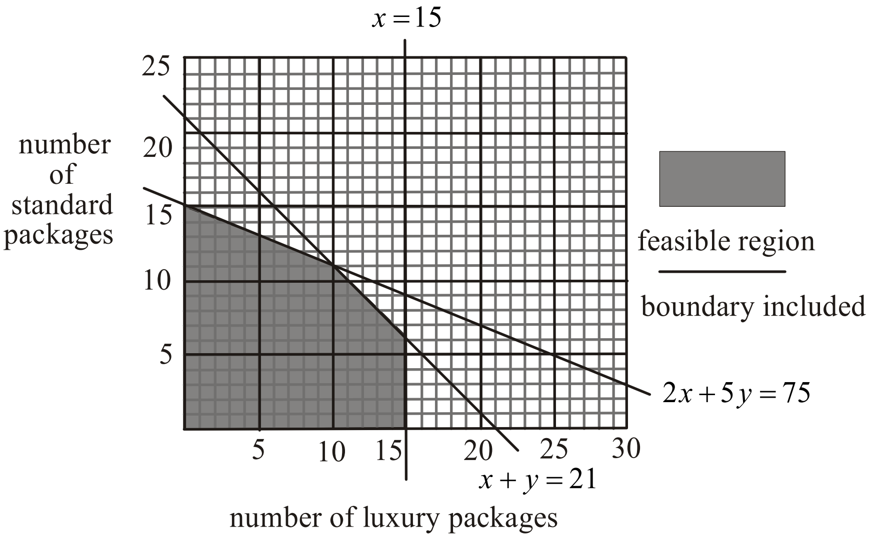
(1 mark)

Question 3

a. Constraint 5 is concerned with furniture availability.
It contains only the variable x which represents the number of luxury packages which are limited to 15.
So there are no constraints on the amount of furniture available for standard packages.

(1 mark)

b. c. and **d.**



(1 mark) for the line $x+y=21$
(1 mark) for the line $x=15$
(1 mark) for the feasible region

e. $P=800x+500y$

(1 mark)

f. A maximum will occur at one of the corner points of the feasible region. From the graph the corner points are $(0,0),(0,15),(10,11),(15,6),(15,0)$.

At (0,0) $P=0$
At (0,15) $P=800\times 0+500\times 15$
$=7\,500$
At (10,11) $P=800\times 10+500\times 11$
$=13\,500$
At (15,6) $P=800\times 15+500\times 6$
$=15\,000$
At (15,0) $P=800\times 15+500\times 0$
$=12\,000$

Maximum profit is \$15 000.

(1 mark)

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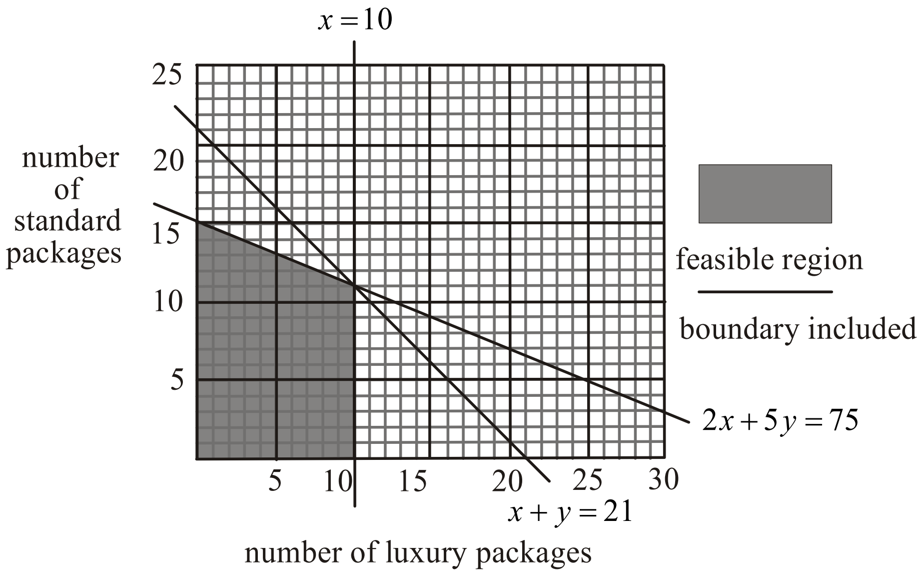
g. The line $x = 15$ moves left to $x = 10$ so the constraint $x + y \le 21$ now has no effect on the feasible region since only the point $(10,11)$ on the line $x + y = 21$ lies in the feasible region and it is the corner point created by the intersection of the lines $2x + 5y = 75$ and $x = 10$.

(1 mark)



h. The corner points that now remain are $(0,0), (0,15), (10,11)$ and $(10,0)$.

At $(0,0)$ $P = 0$ from part **f.**

At $(0,15)$ $P = 7500$ from part **f.**

At $(10,11)$ $P = 13500$ from part **f.**

At $(10,0)$ $P = 800 \times 10 + 0$

$= 8000$

The maximum profit is now $13 500.

(1 mark)
Total 15 marks

Module 4: Business-related mathematics

Question 1

a.

$$\begin{aligned}\text{amount withdrawn} &= \$35\,246\cdot 28 - \$34\,618\cdot 28\\ &= \$628\end{aligned}$$

(1 mark)

b.

$$\begin{aligned}\text{Interest for April} &= \frac{PrT}{100} \qquad \text{(simple interest)}\\ &= \frac{15246\cdot 28\times 2\cdot 4\times \frac{1}{12}}{100}\\ &= \$30\cdot 49\end{aligned}$$

(1 mark) correct balance
(1 mark) correct answer

c.

$$A = PR^n \qquad\qquad R = 1+\frac{r}{100} = 1+\frac{1}{100} = 0.01$$

$$\begin{aligned}&= 20000\times(1\cdot 01)^{12}\\ &= \$22536\cdot 50\end{aligned}$$

(1 mark)

So John earned $\$22\,536\cdot 50 - \$20\,000 = \$2\,536\cdot 50$ in interest.

(1 mark)

Question 2

a. To find the GST included in a price, divide by 11.
So $\$20\,000 \div 11 = \$1\,818\cdot 18$
John paid $\$1\,818\cdot 18$ in GST.

(1 mark)

b.

$$\begin{aligned}\%\ \text{discount} &= \left(\frac{22\,000-20\,000}{22\,000}\times\frac{100}{1}\right)\%\\ &= \left(\frac{2\,000}{22\,000}\times\frac{100}{1}\right)\%\\ &= 9\cdot 09\%\end{aligned}$$

(1 mark)

Question 3

a. $\$20\,000 - \$12\,000 = \$8\,000$

$\$8\,000 \div \$0 \cdot 05 = 160\,000$

So 160 000 lifts will have been performed.

(1 mark)

b. 7% of \$20 000

$= \$1\,400$

Book value after 5 years

$= \$20\,000 - 5 \times \$1\,400$

$= \$13\,000$

(1 mark)

c. At the end of the third year of operation the value of the fork lift is given by

$$V = 20\,000 \times \left(1 - \frac{8}{100}\right)^3$$
$$= 15\,573 \cdot 76$$

(1 mark)

The value at the end of the fourth year is given by

$$V = 20\,000 \times \left(1 - \frac{8}{100}\right)^4$$
$$= 14\,327 \cdot 8592$$

The fork lift depreciates by $\$15\,573 \cdot 76 - \$14\,327 \cdot 86 = \$1\,245 \cdot 90$ during its fourth year of operation.

(1 mark)

Question 4

a. Using *TVM* solver

$$N = 12$$
$$I\% = 6$$
$$PV = 80\,000 \quad \text{(positive because John receives this from the bank)}$$
$$PMT = ?$$
$$FV = 0$$
$$P/Y = 4$$
$$C/Y = 4$$
$$PMT = -7\,334 \cdot 3994... \quad \text{(negative because John has to pay the bank)}$$

John's quarterly repayments will be \$7 334.40.

(1 mark)

b. John will pay in total

$$12 \times \$7\,334 \cdot 40$$
$$= \$88\,012 \cdot 80$$
$$\text{Interest} = \$88\,012 \cdot 80 - \$80\,000$$
$$= \$8\,012 \cdot 80$$

(1 mark)

c. Use *TVM* solver to find how much John still owes two years into the loan.

$$N = 8$$
$$I\% = 6$$
$$PV = 80\,000$$
$$PMT = -7\,334 \cdot 3994$$
$$FV = ?$$
$$P/Y = 4$$
$$C/Y = 4$$

So John owes \$28 269.59657 two years into the loan.

(1 mark)

Again using *TVM* solver.

$$N = ?$$
$$I\% = 6$$
$$PV = 28\,269.59657$$
$$PMT = -5000$$
$$FV = 0$$
$$P/Y = 4$$
$$C/Y = 4$$
$$N = 5 \cdot 9524$$

It will take John $5 \cdot 9524 - 4 = 1 \cdot 9524$ extra quarters or 2 quarters correct to the nearest quarter.

(1 mark)

Total 15 marks

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Module 5: Networks and business mathematics

Question 1

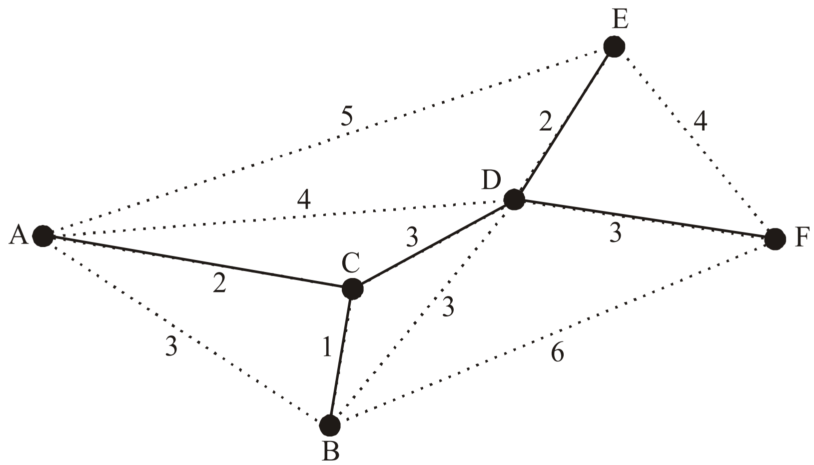
a. By inspection of the graph the shortest distance is along the path *ADF* and is 7km.

(1 mark)

b. A tree is a connected graph with no circuits. Since *E A B F E* is a circuit *E A B F E D C* cannot be a tree.

(1 mark)

c. i.



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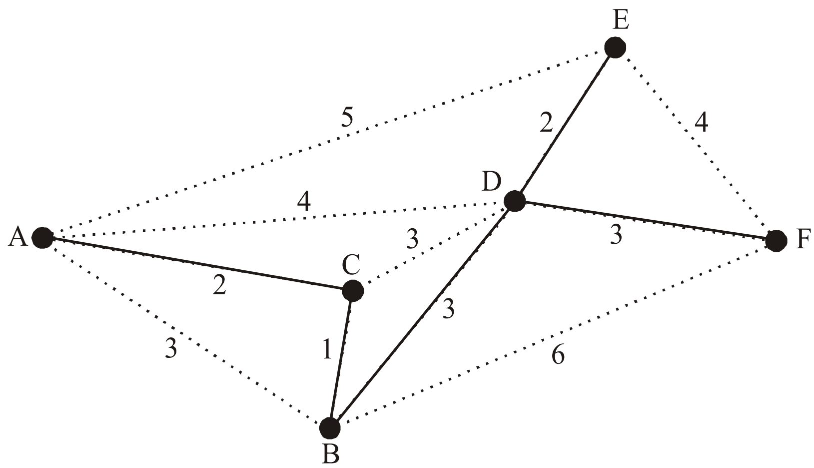


(1 mark) for one of these

ii. There are 2 minimal spanning trees that exist.

(1 mark)

iii. The minimum length of cable required is the sum of the distances on the minimum spanning tree which is $1+2+3+3+2=11$km.

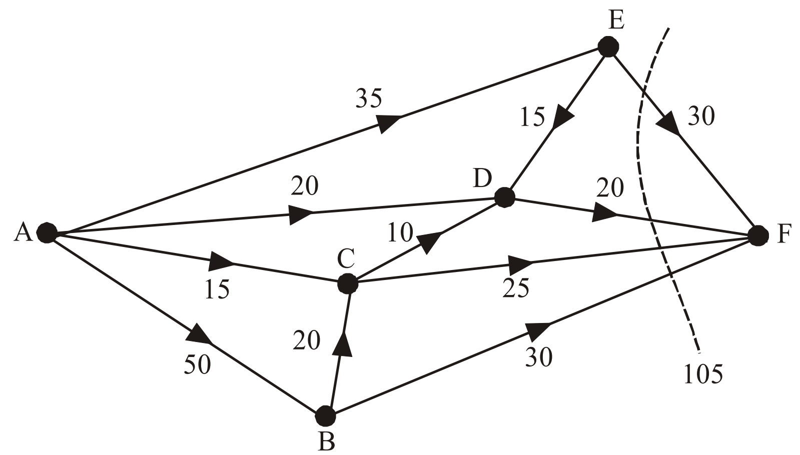
(1 mark)

Question 2

a. The capacity of the cut is $35+20+25+30=110$.
Note that since the edge with a capacity of 15 crosses from right to left and not left to right ($A-F$) it is not counted.

(1 mark)

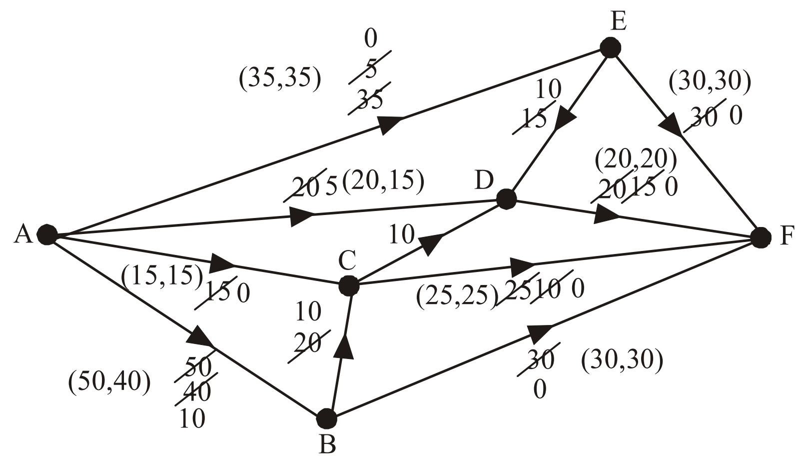
b. Method 1 – trial cuts



The minimum cut is 105 so the maximum number of people is 105.

(1 mark)

Method 2



Each bracket contains (initial capacity, final flow)

Number of passengers from $A = 35+15+15+40$

$=105$

Number of passengers into $F = 30+20+25+30$

$=105$

The maximum number of people who can travel from A to F is 105.

(1 mark)

Question 3

a. For one-step reachability we have

$A \to D$
$A \to C$
$B \to A$
$C \to D$
$D \to B$
$D \to E$
$E \to F$
$F \to D$

Looking at the destination islands, we see for example that island A is reached once.
The one-step reachability score for each island is

$A - 1$
$B - 1$
$C - 1$
$D - 3$
$E - 1$
$F - 1$

So D is the most reachable island.

(1 mark)

b. For two-step reachability we have

$A \to B$
$A \to D$
$A \to E$
$B \to C$
$B \to D$
$C \to B$
$C \to E$
$D \to A$
$D \to F$
$E \to D$
$F \to B$
$F \to E$

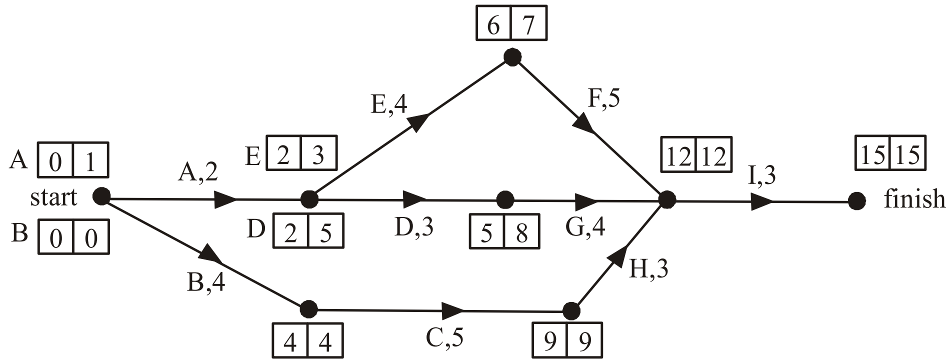
The two-step reachability score for each island is

$A - 1$
$B - 3$
$C - 1$
$D - 3$
$E - 3$
$F - 1$ **(1 mark)**

Adding one-step and two-step we have $A - 2$, $B - 4$, $C - 2$, $D - 6$, $E - 4$, $F - 2$.
So D is the most reachable island and then B and E are the next most reachable and are equally reachable.

(1 mark)

Question 4



a. Using our earliest start time and latest start time boxes we see that the shortest time in which the upgrade can be completed is 15 days.

(1 mark)

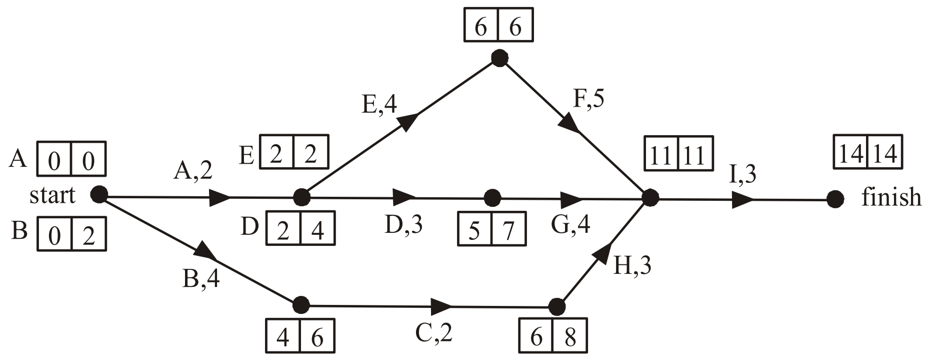
b. Again using our boxes we see that activities *D* and *G* have the greatest slack time (3 days).

(1 mark)

c. Latest start time for activity *E* is day 3.

(1 mark)

d.



The three activities that now lie on the critical path (and didn't previously) are *A*, *E* and *F*.

(1 mark)

e. By reducing activity *C* by 3 days a new critical path *A*, *E*, *F*, *I* has emerged and the new completion time is 14 days. It would only have been necessary to reduce activity *C* by 1 day to achieve this. So two days of money spent have been wasted. It was therefore not worth reducing activity *C* by 3 days.

(1 mark)
Total 15 marks

Module 6: Matrices

Question 1

a. The order of J is 4×1.
(4 rows and 1 column)

(1 mark)

b.

$$JD = \begin{bmatrix} 100 \\ 140 \\ 120 \\ 80 \end{bmatrix} \begin{bmatrix} 0 \cdot 55 & 0 \cdot 45 \end{bmatrix}$$

$$= \begin{bmatrix} 55 & 45 \\ 77 & 63 \\ 66 & 54 \\ 44 & 36 \end{bmatrix}$$

(1 mark)

c. $X = JD$
The element $X_{3,2}$ is 54 and it describes the number of players playing in the U/12 division 2 competition.

(1 mark)

Question 2

a. Enter the matrix into your calculator.
The determinant is – 40.

(1 mark)

b. Matrix A has an inverse because det (A) does not equal zero.

(1 mark)

c. Using your calculator,

$$A^{-1} = \begin{bmatrix} -0 \cdot 2 & 2 & -2 \\ 0 \cdot 975 & -7 \cdot 125 & 6 \cdot 875 \\ -0 \cdot 675 & 4 \cdot 625 & -4 \cdot 375 \end{bmatrix}$$

(1 mark)

d.

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

Enter matrix B on your calculator.
Multiply $A^{-1} \times B$ to get X.

$$X = \begin{bmatrix} 65 \\ 80 \\ 120 \end{bmatrix}$$

So $b = 65$, $d = 80$ and $r = 120$.

(1 mark)

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Question 3

a.

$$P_{2010} = QP_{2009} + R$$
$$= \begin{bmatrix} 0\cdot92 & 0 \\ 0 & 0\cdot85 \end{bmatrix} \begin{bmatrix} 15 \\ 10 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 13\cdot8 \\ 8\cdot50 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 15\cdot80 \\ 10\cdot50 \end{bmatrix}$$

The profit predicted to be made on the sale of a new uniform is \$15.80 in 2010.
On a secondhand uniform it is \$10.50.

(1 mark)

b.

$$P_{n+1} = QP_n + R$$

$$P_{2011} = QP_{2010} + R$$
$$= \begin{bmatrix} 0\cdot92 & 0 \\ 0 & 0\cdot85 \end{bmatrix} \begin{bmatrix} 15\cdot80 \\ 10\cdot50 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad \text{from part } \mathbf{a.}$$
$$= \begin{bmatrix} 14.536 \\ 8\cdot925 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 16.536 \\ 10\cdot925 \end{bmatrix}$$

(1 mark)

$$P_{2012} = QP_{2011} + R$$
$$= \begin{bmatrix} 0\cdot92 & 0 \\ 0 & 0\cdot85 \end{bmatrix} \begin{bmatrix} 16\cdot536 \\ 10\cdot925 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 15\cdot21312 \\ 9.28625 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$
$$= \begin{bmatrix} 17\cdot21312 \\ 11.28625 \end{bmatrix}$$

The profit made on a new uniform is predicted to be \$17.21 and on a secondhand uniform it is predicted to be \$11.29.

(1 mark)

Question 4

a. The proportion is 0.18.

(1 mark)

b.

$$\text{Let } S_{2008} = \begin{array}{c} \begin{bmatrix} 46 \\ 6 \\ 12 \end{bmatrix} \end{array} \begin{array}{c} \text{coach} \\ \text{didn't coach} \\ \text{joint} \end{array}$$

(1 mark)

$$\begin{aligned} S_{2010} &= T^2 S_{2008} \\ &= \begin{bmatrix} 52\cdot 5562 \\ 3\cdot 3282 \\ 8\cdot 1156 \end{bmatrix} \end{aligned}$$

In 2010, 3 rep. players won't coach.

(1 mark)

c. To find a steady state choose any two consecutive numbers for the power of T.

eg. $$T^{20} S_{2008} = \begin{bmatrix} 52\cdot 5425 \\ 3\cdot 3338 \\ 8\cdot 1237 \end{bmatrix}$$

$$T^{21} S_{2008} = \begin{bmatrix} 52\cdot 5425 \\ 3\cdot 3338 \\ 8\cdot 1237 \end{bmatrix}$$

(1 mark)

The steady state matrix has been found. Note that it may have been found earlier (i.e. for values less than 20). The steady state matrix tells us that in the long term 53 rep. players (out of the 64) will still be coaching. This represents $\left(\frac{53}{64}\times 100\right)\% =$ 82.8125% of rep. players coaching. So the club believes that the payment scheme has been successful.

(1 mark)
Total 15 marks

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