INSIGHT

Trial Exam Paper

2009

FURTHER MATHEMATICS

Written examination 1

Worked solutions

This book presents:

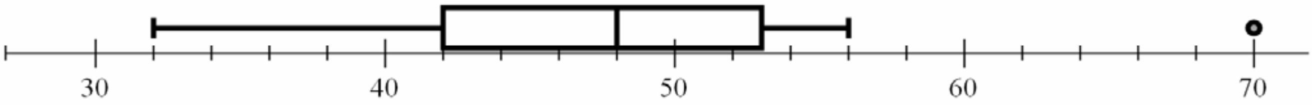
- worked solutions, giving you a series of points to show you how to work through the questions.
- tips and guidelines

SECTION A

Core: Data Analysis

Question 1

For the data represented by the boxplot, the five-figure summary statistics are



A. 32, 42, 48, 53 and 56
B. 32, 40, 47, 50 and 56
C. 35, 40, 47, 53 and 56
D. 32, 42, 48, 51 and 70
E. **32, 42, 48, 53 and 70**

Answer is E

Worked Solution

Reading from the scale, Minimum = 32, Q_1 = 42, Median = 48, Q_3 =53, Maximum = 70

Tip

- *A common error is to read the maximum at the Upper Whisker (56). The outlier (70) is still considered to be the maximum value in the data set.*

The following information relates to Question 2 and 3.

The following stem and leaf plots show the distribution of marks for 2 classes sitting the same test.

Class A		Class B
	1	3
0	2	12388
9820	3	34
53	4	9
1	5	2
0	6	
4	7	
0	8	

Question 2

The mean mark for Class B is

A. **30.3**
B. 3
C. 8.5
D. 4.3
E. 28

Answer is A

Worked Solution

Mean is the sum of the marks divided by the number of students:

$$\frac{(13+21+22+23+28+28+33+34+49+52)}{10} = 30.3$$

Tip

- *Using a graphics calculator enter the data in List1 and find 1 Var statistics* $\bar{x}$ *= 30.3*

Question 3

Which of the following marks would be considered an outlier for a student in Class A?

A. 51, 60, 74 and 80
B. 60, 74 and 80
C. 74 and 80
D. **80**
E. 20

Answer is D

Worked Solution

$Q_1 = 32$

$Q_3 = 51$

IQR = $Q_3 - Q_1$

= 19

Upper fence = Upper Quartile (Q_3) $+1.5 \times$ IQR

Lower fence = Lower quartile (Q_1) $-1.5 \times$ IQR

Any value above the upper fence or below the lower fence is considered to be a statistical outlier.

Upper fence = 79.5

Lower fence = 3.5

Therefore, only the mark of 80 is above the upper fence, so this mark would be considered an outlier. There is no lower outlier.

Tip

- *The data for class A could have been entered into a List on a graphics calculator and a boxplot could have been drawn. By your boxplot, any outliers would have been displayed.*

Question 4

The distribution of scores from a sample of 45 students on an algebra test was found to be normally distributed with a mean of 23 and a standard deviation of 6. The number of students who obtained a mark greater than 29 would be approximately

A. 16

B. 34

C. **7**

D. 15

E. 3

Answer is C

Worked Solution

From the 68–95–99.7% rule, 1 standard deviation either side of the mean is in the range 17 to 29. This is the middle 68% of the data. The question is concerned with the top 16% of the sample, as we need the number of students who obtained greater than 29.

Find 16% of 45 students

$$\frac{16}{100}\times 45 = 7.2 \text{ students}$$

Tip

- *Take care with these questions. Make sure you draw a diagram of the normal distribution and make sure you understand what the question is asking. Are they after a percentage, or a value above or below a certain point?*

Question 5

Which one of the following numerical variables is discrete?

A. The weights of three newly born monkeys at the local zoo

B. The lengths of a sample of nails are taken

C. **The number of siblings of a group of students**

D. The average time taken for 10 students to complete a writing task

E. The heights of 3 plant species undergoing an experiment

Answer is C

Worked Solution

A discrete numerical variable is a counting number. Option A, B, D and E are all numerical variables, but are continuous, as they may take any value within the range of data (time, length weight or height).

Question 6

The following table shows the results of Raj's mid-year examinations for 5 subjects. Also given is the class mean and standard deviation for each subject.

Subject	Mean	Standard Deviation	Raj's mark
Science	45	2.5	47
English	33	6	39
French	28	4.4	35
Geography	27	3.8	29
Maths	55	6	55

The subject in which Raj performed the best in relation to his classmates is

A. Science
B. English
C. **French**
D. Geography
E. Maths

Answer is C

Worked Solution

A z score comparison is for comparing scores from different populations with different means and standard deviations.

$$\text{z score} = \frac{\text{score} - \text{mean}}{\text{standard deviation}}$$

Subject	Mean	Standard Deviation	Raj's mark	z score
Science	45	2.5	47	$z = \frac{47-45}{2.5}$ $z = 0.8$
English	33	6	39	$z = \frac{39-33}{6}$ $z = 1$
French	28	4.4	35	$\boldsymbol{z = 1.59}$
Geography	27	3.8	29	$z = 0.526$
Maths	55	6	55	$z = 0$

Raj's French result has the highest z score, meaning that this result is the best result compared to his other subjects, in relation to his classmates.

The following information relates to Question 7 and 8.

Question 7

The owner of a clothes shop has noticed, over a number of years, that weekly sales exhibit highly seasonal characteristics:

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Seasonal Index	0.55	0.91		1.03	0.64

The seasonal index for Wednesday is

A. **1.87**
B. –0.23
C. –1.13
D. 1.13
E. Not enough information

Answer is A

Worked Solution

The sum of the seasonal indices will sum to the number of seasons. In this case there are 5 seasons.

If x is the value of the seasonal index for Wednesday,

$0.55 + 0.91 + x + 1.03 + 0.64 = 5$

$x = 1.87$

Question 8

If the deseasonalised amount for Monday is \$32 000, then the actual sales for Monday are closest to

A. \$58 180
B. \$27 840
C. \$16 000
D. **\$17 600**
E. \$17 100

Answer is D

Worked Solution

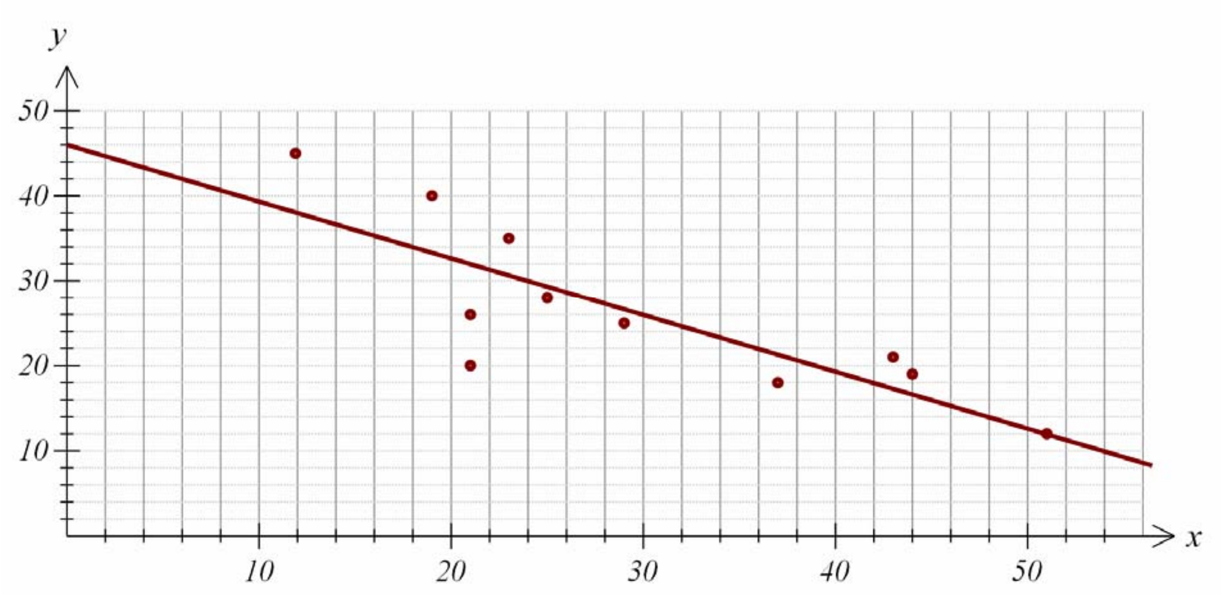
$$\text{Deseasonalised value} = \frac{\text{actual value}}{\text{Seasonal Index}}$$

$$32000 = \frac{x}{0.55}$$

$$x = 32000 \times 0.55 = 17600$$

The following information relates to Questions 9, 10 and 11.

The scatterplot below shows the relationship between 2 variables, x and y.



The line shown on the graph is the least squares regression line. The equation of this line is:

$y = 46 - 0.67x$

The coefficient of determination is 0.8234.

Question 9

The Pearson's product moment correlation co-efficient is closest to

A. 0.82

B. 0.91

C. -0.94

D. $-$**0.91**

E. -0.82

Answer is D

Worked Solution

Pearson's product moment correlation co-efficient (r) is found by taking the square root of the coefficient of determination (r^2)

r = $\sqrt{0.8234}$

r = 0.91 (2 d.p.)

As the slope of the line is negative, the value of r must be negative. This gives a value of -0.91.

Question 10

Fitting this regression line to the data, the value of the residual of the point (37, 18) is

A. -4.5

B. 3.21

C. 4.5

D. $-$**3.21**

E. 0.82

Answer is D

Worked Solution

Residual = actual y value – predicted y value

When $x = 37$, actual $y = 18$

When $x = 37$, using the equation of the least squares regression line:

$y = 46 - 0.67 \times 37$

$y = 21.21$

Residual $= 18 - 21.21$

$= -3.21$

Tips

- *From the graph the point (37, 18) is below the least squares regression line, giving it a negative residual. This means that the only correct answer is A or D.*
- *The residual can also be read directly off the graph, finding the distance between the actual y value and the predicted y value.*

Question 11

The rate of decrease of y compared to x is

A. -0.67

B. 46

C. -46

D. -45.33

E. **0.67**

Answer is E

Worked Solution

The rate of increase or decrease is a way of describing the gradient of the line. The line has a negative slope. The rate of **decrease** is therefore 0.67.

Question 12

The number of hoses sold at the local hardware store for each month in 2008 is shown below:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Number of hoses	41	15	19	25	35	38	44	32	30	18	32	39

If a 4-point moving-mean smoothing with centring is applied, the number of hoses sold for the month of September is

A. 29
B. 28
C. 31
D. **29.5**
E. 28.5

Answer is D

Worked Solution

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
41	15	19	25	35	38	44	32	30	18	32	39

31 28

29.5

First find the mean of sales for July, August, September and October:

$$\frac{44+32+30+18}{4}=31$$

Next, find the mean of August, September, October and November:

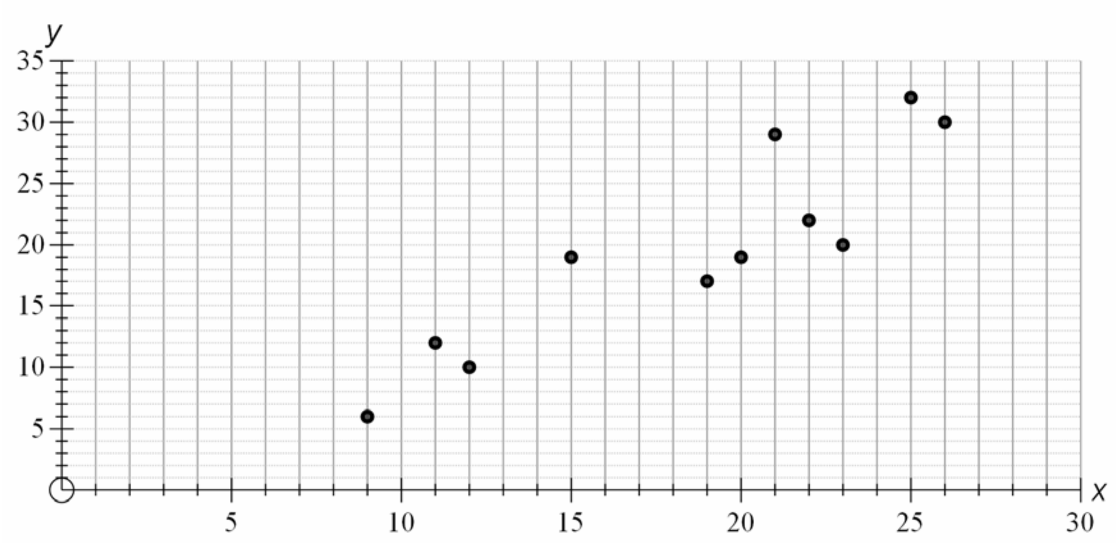
$$\frac{32+30+18+32}{4}=28$$

Using the process of centring, we now find the average of 31 and 28, giving 29.5.

SECTION A – continued

Question 13

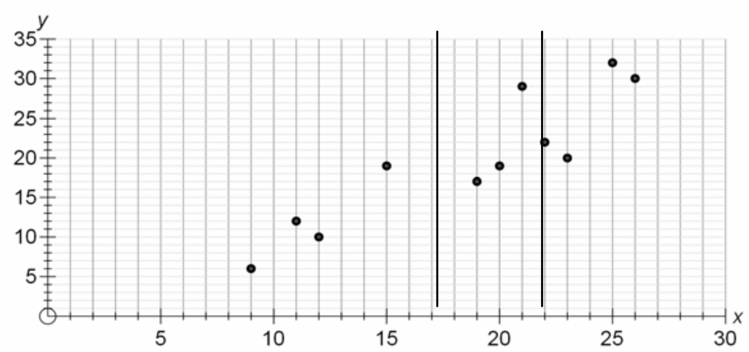
The gradient of the 3 median regression line for the following scatterplot is closest to



A. 1.29
B. **1.2**
C. 1.1
D. −1.2
E. −1.29

Answer is B

Worked Solution

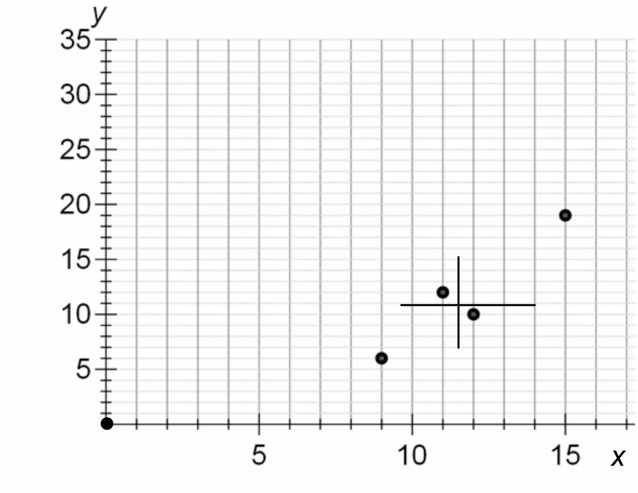


The data has been divided into 3 'even' sections – Upper, Middle and Lower. There needs to be the same number of points in the Upper and Lower Sections. This gives 4 points, 3 points and 4 points in each of the 3 sections. Taking each section in turn, the median point is found graphically.

Lower

Median = 2.5th score. The middle x value and the middle y value are found.

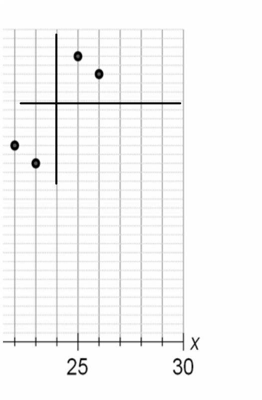
Median (11.5, 11)



Upper

Median = 2.5th score. The middle x value and middle y values are found

Median = (24, 26)



The gradient can now be found between (24, 26) and (11.5, 11)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{11 - 26}{11.5 - 24}$$

$$m = 1.2$$

Tip

- *The gradient of the line of best fit is clearly a positive slope. The Middle section does not need a median value, because a line is drawn between the Upper and Lower section, and while* ***maintaining the same gradient*** *the line is moved* $\frac{1}{3}$ *closer to the middle median.*

END OF SECTION A

SECTION B

Module 1: Number Patterns

Question 1

The first 4 terms of a sequence are 4, 6, 9, and 13. The next 3 terms, in order, are

A. 17, 21, 25
B. **18, 24, 31**
C. 19, 25, 32
D. 18, 25, 32
E. 18, 23, 30

Answer is B

Worked Solution

This sequence is neither arithmetic nor geometric, but a sequence of numbers maintaining some pattern. The difference between successive terms increases by 1, which means the next difference is 5, the one after that being 6 and the last being 7.

Question 2

Given that an arithmetic sequence had a first term of 100 and a common difference of -15, the term which is first negative is

A. **8th**
B. 9th
C. 10th
D. 11th
E. 12th

Answer is A

Worked Solution

$\mathrm{t}_{\mathrm{n}} = a + (n-1)d \qquad a = 100,\ d = -15$

$100 + (n-1) \times -15 < 0$

$100 - 15n + 15 < 0$

$-15n < -115$

$n > 7.67$

(NB: sign has changed as we have divided by a negative value)

This means that the 8th term will be the first term that is negative.

Tips

- *Trial and error could be used to substitute into the rule* $\mathrm{t}_{\mathrm{n}} = a + (n-1)d$, *to find the first value that was negative.*
- *Also a TABLE on a graphics calculator could have been used to find the first negative value.*

Question 3

The first and third terms of a geometric sequence are 27 and 12 respectively. The sum of the first 5 terms of this sequence is

A. **70 $\frac{1}{3}$**

B. $52\frac{1}{3}$

C. 39

D. 65

E. 56

Answer is A

Worked Solution

For a geometric sequence,

$\ldots x, y, z \ldots$

$y^2 = xz$

$t_1 = 27$ and $t_3 = 12$, $\therefore t_2 = 18$

The sequence is now 27, 18, 12 a = 27 and $r = \frac{2}{3}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_5 = \frac{27[1-\left(\frac{2}{3}\right)^5]}{1-\frac{2}{3}}$$

$$S_5 = \frac{23.44}{\frac{1}{3}}$$

$$S_5 = 70\frac{1}{3}$$

Question 4

An athlete's training regime has the following conditions. During the first week, she is required to run 6 km. Each week the distance is increased by 2.5 km. An expression for the **total** distance run in *n* weeks is

A. $n(2.5n + 9.5)$ km

B. $\frac{n(2.5n + 4.5)}{2}$ km

C. $\frac{(2.5n + 9.5)}{2}$ km

D. $\boldsymbol{\frac{n(2.5n + 9.5)}{2}}$ **km**

E. $4.5 + 2.5n$ km

Answer is D

Worked Solution

This situation represents an arithmetic series (indicated by the use of the word 'total') with a first term of 6 and a common difference of 2.5.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}[2 \times 6 + (n-1)2.5]$$

$$S_n = \frac{n}{2}(12 + 2.5n - 2.5)$$

$$S_n = \frac{n}{2}(9.5 + 2.5n)$$

or

$$S_n = \frac{n(2.5n + 9.5)}{2}$$

Tip

- *A common error is to apply the arithmetic sequence rule. Look for words referring to total, sum and cumulative to represent an arithmetic series.*

Question 5

The first term of a geometric sequence is 26. Each term is $\frac{2}{3}$ of the previous term. The sum of the infinite sequence is

A. $17\frac{1}{2}$

B. 39

C. **78**

D. $43\frac{1}{3}$

E. 31

Answer is C

Worked Solution

For an infinite arithmetic series

$$S_\infty = \frac{a}{1-r}$$

(a = 26 and r = $\frac{2}{3}$)

$$S_\infty = \frac{26}{1-\frac{2}{3}}$$

$$S_\infty = \frac{26}{\frac{1}{3}}$$

$$S_\infty = 78$$

Question 6

The 10th term of an arithmetic sequence which had a first term of 1.7 and the sum of its first 6 terms being 30 is

A. **13.58**

B. 9.62

C. 51

D. 12.8

E. 12.26

Answer is A

Worked Solution

$a = 1.7$ and $S_6 = 30$

$S_6 = \frac{n}{2}(2a + (n-1)d) = 30$

$3(2 \times 1.7 + (6-1)d) = 30$

$3.4 + 5d = 10$

$5d = 6.6$

$d = 1.32$

Find the 10th term, given that a = 1.7 and d = 1.32

$t_{10} = a + (n-1)d$

$t_{10} = 1.7 + 9 \times 1.32$

$t_{10} = 13.58$

Question 7

The first 4 terms of a sequence of numbers are

0.2, 1.64, 2.648, 3.3536...

The difference equation that describes this sequence is

A. $t_{n+1} = 0.2t_n + 1.44;\ t_1 = 0.2$

B. $\boldsymbol{t_{n+1} = 0.7t_n + 1.5;\ t_1 = 0.2}$

C. $t_{n+1} = 0.8t_n + 1.2;\ t_1 = 0.2$

D. $t_{n+1} = 0.9t_n + 1.5;\ t_1 = 0.2$

E. $t_n = 0.8t_{n-1} + 1.2;\ t_1 = 0.2$

Answer is B

Worked Solution

Using trial and error, the difference equation which describes the sequence of numbers is

$t_{n+1} = 0.7t_n + 1.5;\ \mathrm{t}_1 = 0.2$

Generating the 2nd term in the sequence would have been enough to determine the correct difference equation.

Tips

- *Make sure that the notation is correct, referring to Option E.*
- *The TABLE on a graphics calculator could have been used quickly to generate each sequence.*

Question 8

How many of the following first order difference equations rules form a geometric sequence?

$t_{n+1} = 5t_n - 2$

$t_{n+1} = t_n + 5$

$t_{n+1} = -4t_n$

$t_n = t_{n-1} + 4$

A. 0

B. **1**

C. 2

D. 3

E. 4

Answer is B

Worked solution

The first difference describes a sequence that is neither arithmetic nor geometric, the second describes an arithmetic sequence, the third is a geometric sequence with a common ratio of -4 and the fourth is an arithmetic sequence.

Question 9

Two animals are introduced into a nature reserve. The first, a Spotted Yap, will increase its numbers from an initial value of 85, with a population increase of 25 each year. The second, a Striped Yap, will increase its numbers by 9.8% each year with 65 in the first year. Assuming that all animals survive, the year in which there will be double the number of Striped Yaps as Spotted Yaps is the

A. 42nd
B. 31st
C. 50th
D. **38th**
E. 54th

Answer is D

Worked solution

The population growth of the Spotted Yaps will follow a arithmetic sequence, with a first term of 85 and a common difference of 25. The population growth of a Striped Yap will follow a geometric sequence with a first term of 65 and a common ratio of 1.098 $(1+\frac{9.8}{100})$

The rule for the Spotted Yap will be $t_n = 85 + (n-1)25$

The rule for the Striped Yap will be $t_n = 65 \times 1.098^{n-1}$

For the Striped variety to be double the Spotted variety, the equation becomes

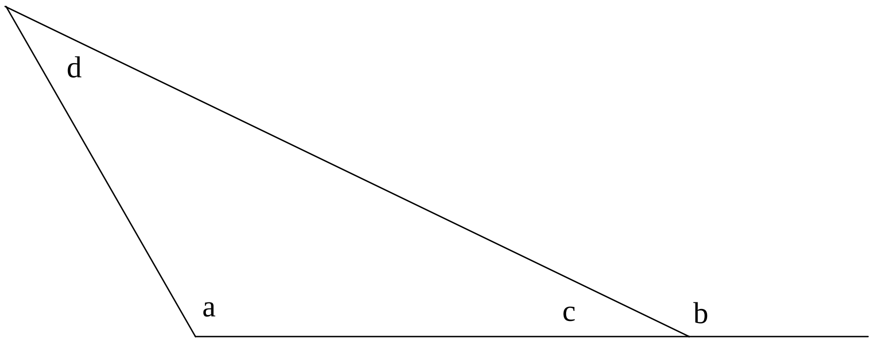
$2(85 + 25(n-1)) \geq 65 \times 1.098^{n-1}$

The easiest way to solve this is to either graph the 2 equations on a calculator to find the year that the 2 lines intersect, or to construct a table of values to find when the number of Striped Yaps is greater than twice the number of Spotted Yaps.

Module 2: Geometry and Trigonometry

Question 1

For the diagram below, the statement which is always true is



A. a + b + c = 180

B. a + b + c + d = 360

C. d + c = b

D. **a + d = b**

E. a + c = b

Answer is D

Worked Solution

The exterior angle rule states that the sum of 2 interior angles is equal to the opposite exterior angle.

Therefore, a + d = b

Question 2

A new chocolate bar, in the shape of a triangular prism, has been introduced to the market. The amount of wrapping that will be required to cover all faces, assuming no overlap, is closest to

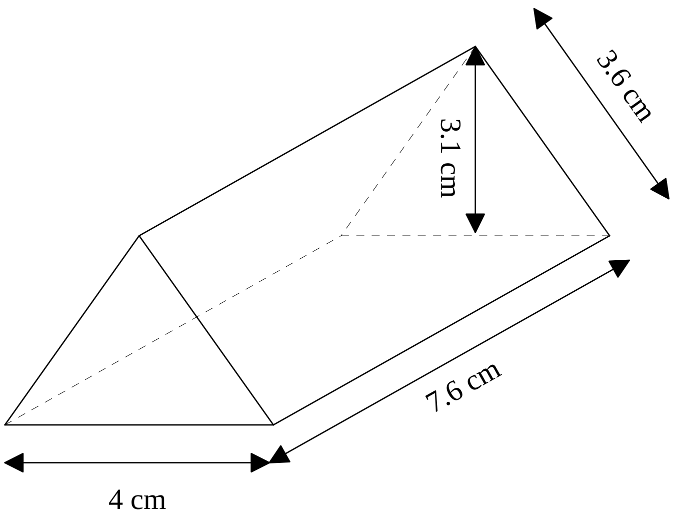
A. 85 cm^2

B. 12.4 cm^2

C. 61 cm^2

D. 67 cm^2

E. **98** $\mathbf{cm^2}$



Answer is E

Worked Solution

The total surface area of a triangular prism can be found using a formula, or the area can be found by summing the faces of the shape.

Total Surface Area = (Area of rectangular side × 2) + (Area of triangle × 2) + Base of prism

$$TSA = 2(length \times sloping\ \ height) + 2(\frac{1}{2} \times base \times height) + (base \times length)$$

$$TSA = 2(7.6 \times 3.6) + 2(\frac{1}{2} \times 4 \times 3.1) + (4 \times 7.6)$$

$$TSA = 97.52\ cm^2$$

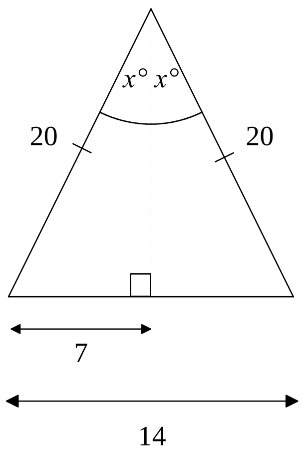
Question 3

An isosceles triangle has a base length of 14 cm and equal sides of 20 cm. The magnitude of the smallest angle, correct to 1 decimal place is

A. 20.4°
B. 20.5°
C. 40.8°
D. **41.0°**
E. 0.4°

Answer is D

Worked Solution



The perpendicular bisector divides the base in half and creates a right-angled triangle. The smallest angle is opposite the smallest side.

$$\sin x = \frac{7}{20}$$

$$x = \sin^{-1}\frac{7}{20}$$

$$x = 20.49°$$

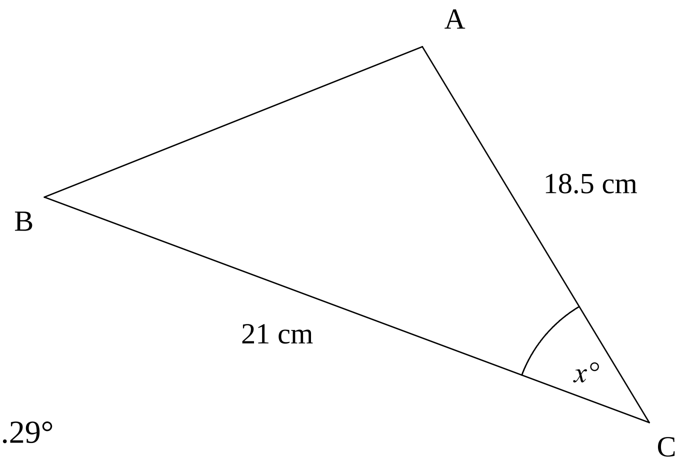
$$\therefore 2x = 40.98°$$

Tip

- *The angles at the base of the triangle can be found and with this information known, the desired angle can be found.*

Question 4

Given that the area of triangle ABC is 56 cm^2 the value of angle x is



A. 0.29°
B. 18°
C. **16.8°**
D. 21.4°
E. Not enough information

Answer is C

Worked Solution

Using $Area = \frac{1}{2} \times a \times b \times$ Sin C

$$56cm^2 = \frac{1}{2} \times 21 \times 18.5 \times \text{Sin C}$$

$$56\text{cm}^2 = 194.25 \times \text{Sin C}$$

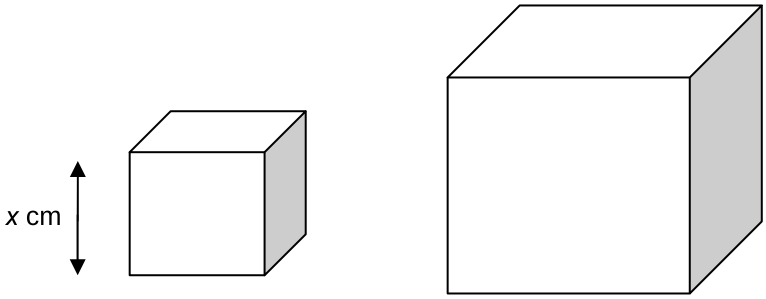
$$\frac{56\text{cm}^2}{194.25} = \text{Sin C}$$

$$\sin^{-1} \frac{56\text{cm}^2}{194.25} = \text{C}$$

$\therefore$ C = 16.76°

Question 5

A child has two similar rectangular blocks.



The volume of the small block is 27 cm^3 and the volume of the large block is 64 cm^3. The height of the small block is x cm. An expression for the height of the large block is

A. $\frac{3}{4}x$

B. $\frac{8}{3}x$

C. $\frac{64}{27}x$

D. $\boldsymbol{\frac{4}{3}x}$

E. $\frac{27}{64}x$

Answer is D

Worked Solution

Volume scale factor (k^3) = $\frac{64}{27}$

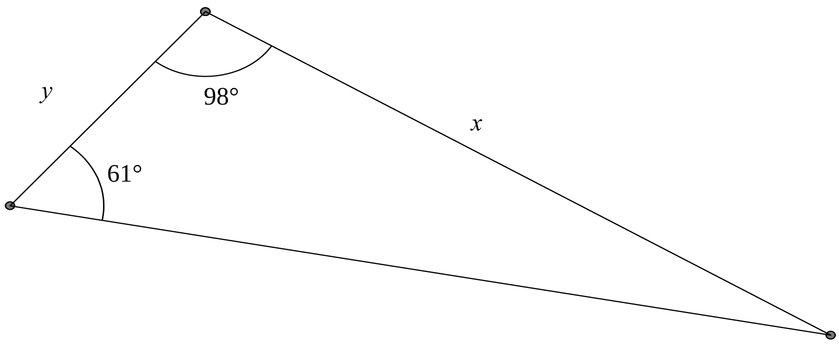
$$\therefore k = \sqrt[3]{\frac{64}{27}}$$

$$k = \frac{4}{3}$$

As the larger block is $\frac{4}{3}$ larger than the small block, the solution is $\frac{4}{3}x$.

Question 6

The length of the side marked x in the given triangle, in centimetres, is



A. $\dfrac{y \times \sin 98°}{\sin 21°}$

B. $\dfrac{y \times \sin 61°}{\sin 61°}$

C. $\dfrac{\sin 61°}{\sin 21°}$

D. $\dfrac{y \times \sin 61°}{\sin 98°}$

E. $\mathbf{\dfrac{y \times \sin 61°}{\sin 21°}}$

Answer is E

Worked Solution

The missing angle is 21° (angle sum of a triangle is 180°)

Using the sine rule

$$\frac{a}{Sin\ A} = \frac{b}{Sin\ B} = \frac{c}{Sin\ \mathrm{C}}$$

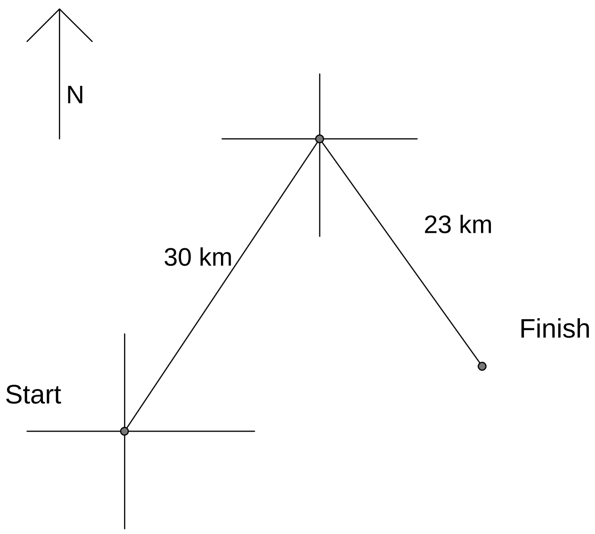
and finding an expression for x,

$$\frac{x}{Sin\ 61°} = \frac{y}{Sin\ 21°}$$

$$x = \frac{y \times Sin\ 61°}{Sin\ 21°}$$

The following information relates to Question 7 and 8.

A plane flies on a bearing of 046°T for 30 km and then changes direction flying for 23 km on a bearing of 149°T, as indicated in the diagram.



Question 7

The distance from the plane's final position to its starting point is closest to

A. **33 km**

B. 421 km

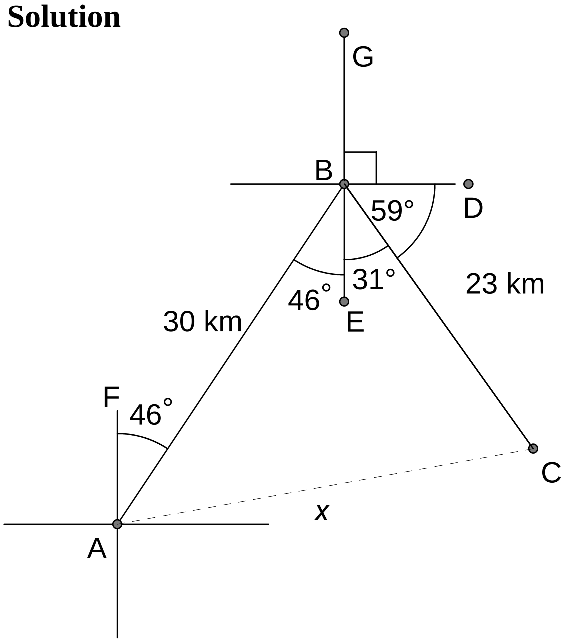
C. 1119 km

D. 51 km

E. 32 km

Answer is A

Worked Solution



$\angle ABE = 46°$ (vertically opposite to $\angle BAF$)

$\angle GBC = 149°$ (As $\angle GBD = 90° \therefore \angle DBC = 59$ and $\angle CBE = 31°$)

$\therefore \angle ABC = 77°$

Using the Cosine rule

$a^2 = b^2 + c^2 - 2bc \times \cos A$

$a^2 = 30^2 + 23^2 - 2 \times 30 \times 23 \times \cos 77°$

$a^2 = 1118.57$ km

$a = 33.44$ km

Tips

- *Ensure your calculator is set to Degree mode during Geometry and Trigonometry.*
- *Be careful to give the solution for a, not a^2, which is a common error.*

Question 8

The total area enclosed by the pilot's journey, having returned to the starting point is closest to

A. 344 km^2
B. 340 km^2
C. 348 km^2
D. 347 km^2
E. **336 km^2**

Answer is E

Worked Solution

Using Heron's formula, as all 3 sides are known:

$Area = \sqrt{s(s-a)(s-b)(s-c)}$

where

$s = \dfrac{a+b+c}{2}$

$s = \dfrac{30+23+33.44}{2}$

$s = 43.22$

$Area = \sqrt{43.22(43.22-30)(43.22-23)(43.22-33.44)}$

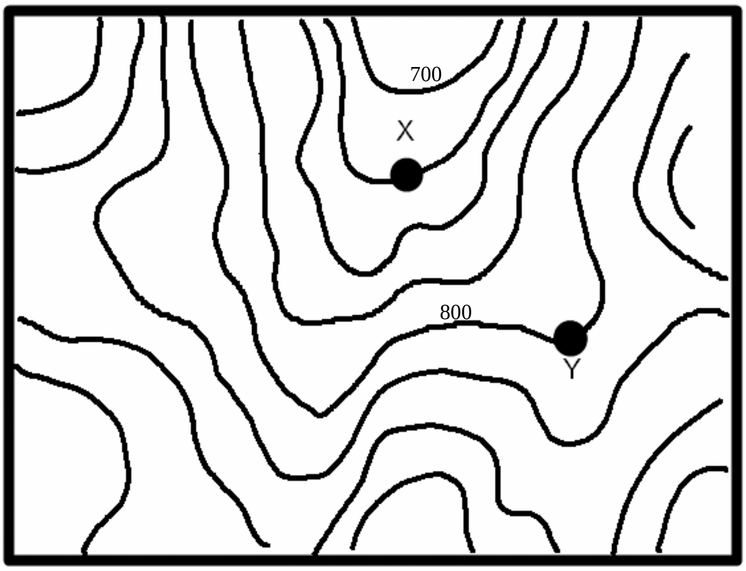
$Area = 336.16$ km^2

Tips

- *Alternatively, the formula* $Area = \frac{1}{2} \times b \times c \times \sin A$ *could be used. Ensure that the correct angles and sides are used.*
- *When dealing with a semi-perimeter(s), it can be stored as a constant on the calculator using STO→. It is then easier to substitute a constant, especially when using it multiple times. Take care when rounding in the middle of a question, rather than at the end. Accuracy may be lost.*

Question 9

For the following contour map, the distance on the map between X and Y is 4.5 cm. The direct distance joining these points is closest to



Scale:

1:10 000

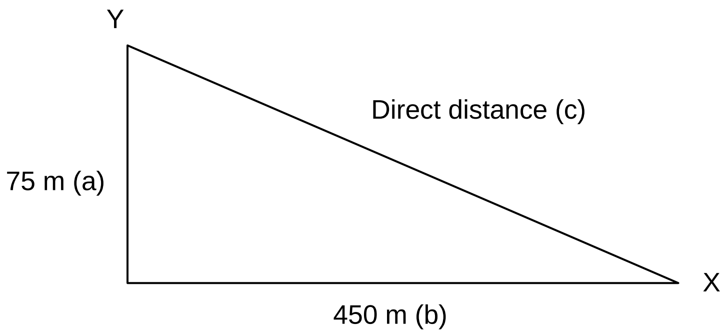
Contours in metres

A. 20.8 km
B. **456 m**
C. 4.56 km
D. 450 m
E. 9.46 m

Answer is B

Worked Solution

The direct distance refers to the hypotenuse of a right angle triangle.



The height difference between the contours is 75 m (100 m − 25 m) – the contours are in 25 m intervals.

The horizontal distance between X and Y is 450 m.

Using the scale 1:10 000, 4.5 cm of the map represents 45 000 cm in real life. Converted to metres the result is 450 m.

Pythagoras’ theorem can now be used

$c^2 = a^2 + b^2$

$c^2 = 450^2 + 75^2$

$c^2 = 208\,125$ m

$c = 456.2$ m

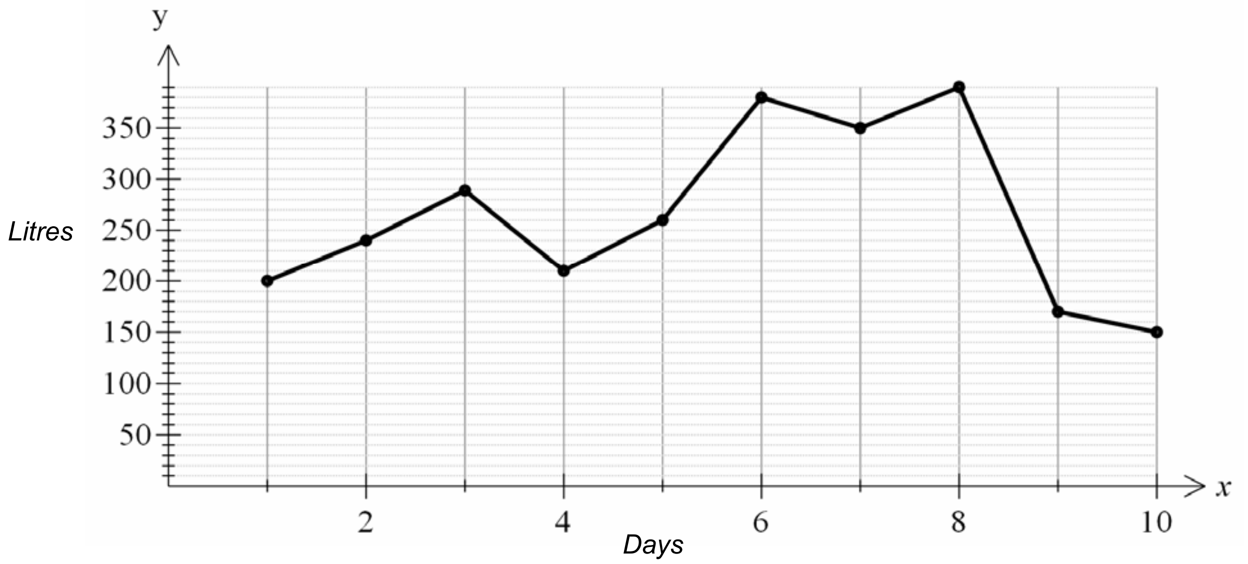
Tip

- *Ensure that scale is correctly calculated and that the term ‘direct distance’ is correctly interpreted.*

Module 3: Graphs and Relations

Question 1

The graph below shows the amount of water (in litres) in a water tank over a period of 10 days.



Between which two days did the greatest increase occur?

A. Day 2 and day 3

B. Day 4 and day 5

C. **Day 5 and day 6**

D. Day 8 and day 9

E. Day 1 and day 2

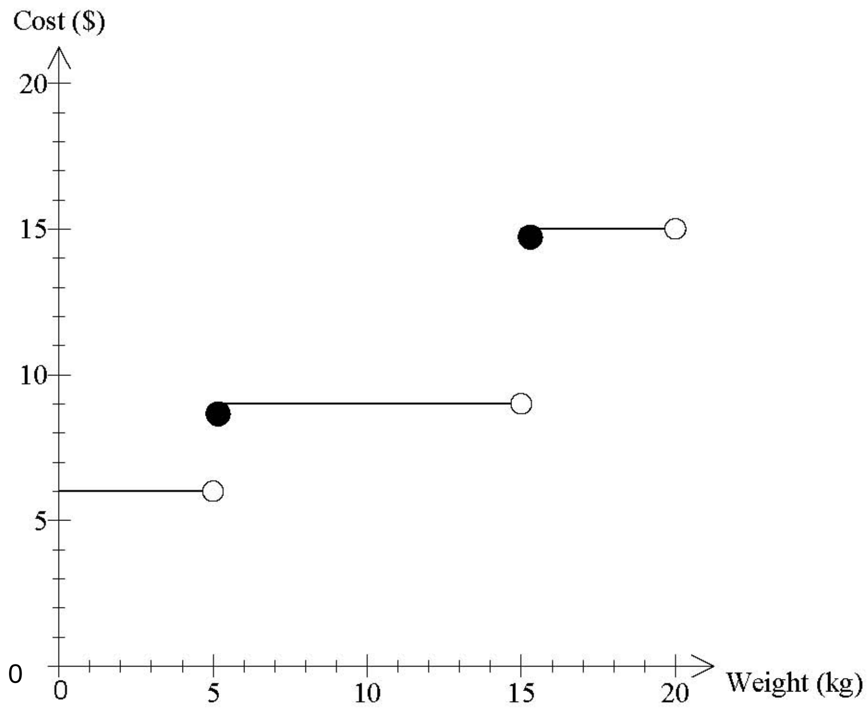
Answer is C

Worked Solution

A line segment with a positive gradient represents an increase and the greatest increase is the line with the steepest gradient.

Question 2

The fee for sending parcels interstate with a certain courier company is determined by the weight according to the graph shown.



If parcel A weighs 8.5 kg and parcel B weighs 15 kg, then the total cost of the 2 parcels will be

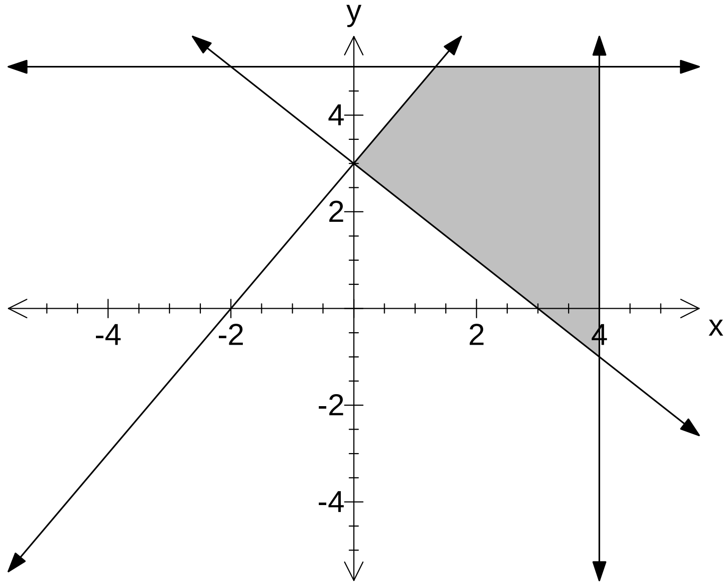
A. $18
B. $9
C. **$24**
D. $25
E. $30

Answer is C

Worked Solution

Reading from the graph, the cost of parcel A is $9 and the cost of parcel B is $15. Parcel B is not defined for a cost of $9, as indicated by the open circle at the right hand edge of each weight range. (The closed circle at the left hand edge of each weight range means that the value is defined – for example, a parcel weighing 15 kg will cost $15, not $9).

The following information is related to Questions 3 and 4.



Question 3

The shaded region on the graph is defined by $y \le 5, 2y \le 3x + 6,$

A. $x \le 4$ and $x + y \le 3$

B. $\boldsymbol{x \le 4}$ **and** $\boldsymbol{x + y \ge 3}$

C. $x \ge 4$ and $2x + y \le 3$

D. $x \ge 4$ and $x + y \le 3$

E. $x \ge 4$ and $x + y \ge 3$

Answer is B

Worked Solution

The vertical line has the equation of $x = 4$ and as the shading is on the left of the line, the inequality is $x \le 4$.

The second unknown line passes through the points (0,3) and (3,0). The y-intercept is 3 and the gradient is found by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

giving $m = -1$

Therefore the equation is $y = -x + 3$ or $x + y = 3$. Choosing a test point (1,3) and substituting this into the rule

$1 + 3 \ge 3$ which is true, therefore the inequality is $x + y \ge 3$

Question 4

An objective function, F, where $F = 2x + 4y$, has been formed. The maximum value of F to the above constraints is

A. 16
B. 30
C. 32
D. **28**
E. 25

Answer is D

Worked Solution

Intersection of points:

$x + y = 3$ and $2y = 3x + 6$

Rearranging the first equation $x = 3 - y$

Using the substitution method

$2y = 3(3 - y) + 6$
$2y = 9 - 3y + 6$
$5y = 15$
$y = 3$
$\therefore x = 0$
$(0,3)$

$x = 4$ *and* x + y = 3
$4 + y = 3$
$y = -1$
$(4,-1)$

$2y = 3x + 6$ *and* y = 5
2(5) = $3x + 6$

$4 = 3x$
$x = \dfrac{4}{3}$
$y = 5$
$\left(\dfrac{4}{3}, 5\right)$

Feasible end points	Value of $F = 2x + 4y$
(4,5)	$F = 2(4) + 4(5)$ $F = 28$
(0,3)	$F = 2(0) + 4(3)$ $F = 12$
$\left(\frac{4}{3}, 5\right)$	$F = 2\left(\frac{4}{3}\right) + 4(5)$ $F = \frac{68}{3}$
$(4, -1)$	$F = 2(4) + 4(-1)$ $F = 8 - 4$ $F = 4$

Maximum value is 28.

The following information relates to Question 5 and 6.

The hire company ‘Company A’, loans out trailers during January for a special deal. The cost is $25.40 per hour plus an initial charge of $40. A second company, ‘Company B’ offers a competing deal on its trailers, which has an hourly rate of $29 plus an initial charge of $25.

Question 5

If Nigel budgets for only $200 for the hire of a trailer from Company A, the number of whole hours he will be able to use the trailer is

A. 5
B. **6**
C. 7
D. 8
E. 9

Answer is B

Worked Solution

The rule for Company A is $Cost = 25.40 \times hours + 40$

Let $Cost = 200$

$200 = 25.40 \times hours + 40$

$160 = 25.40 \times hours$

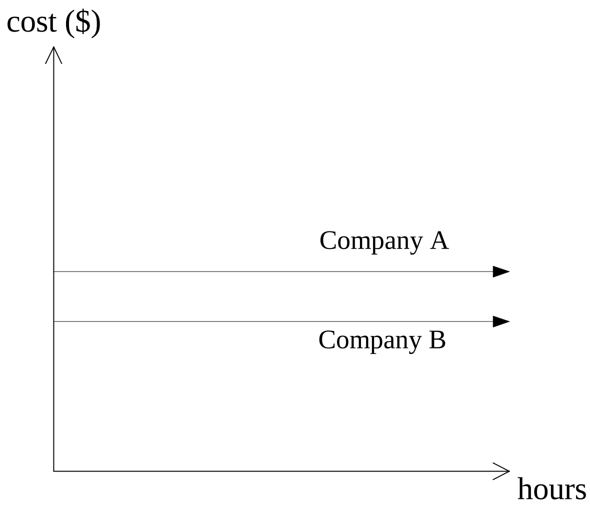
$hours = 6.3 \text{ } (1 \; d.p.)$

As the question asked for the number of whole hours, 6 hours will be required for $200.
7 hours will cause the cost to be over $200.

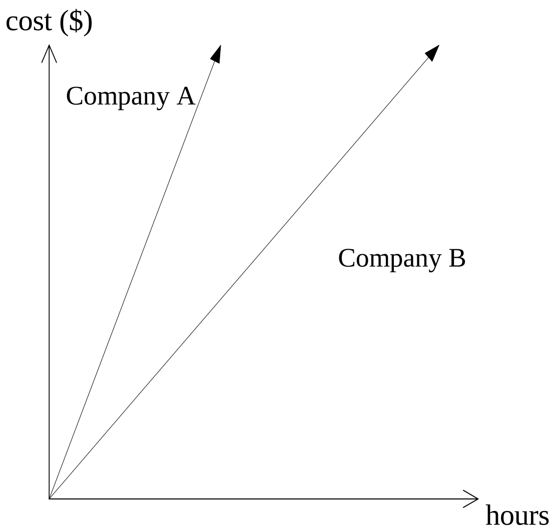
Question 6

The graph which best describes the fees charged by these companies for x hours is

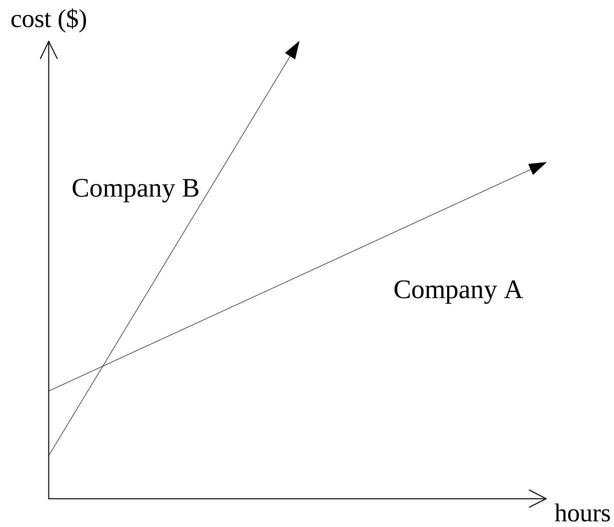
A.



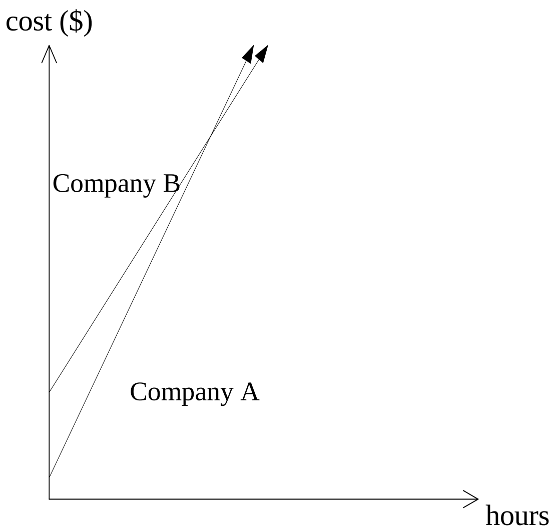
B.



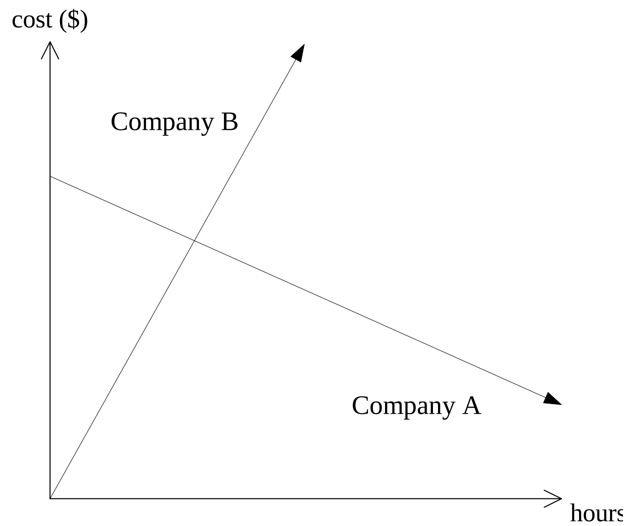
C.



D.



E.



Answer is C

Worked Solution

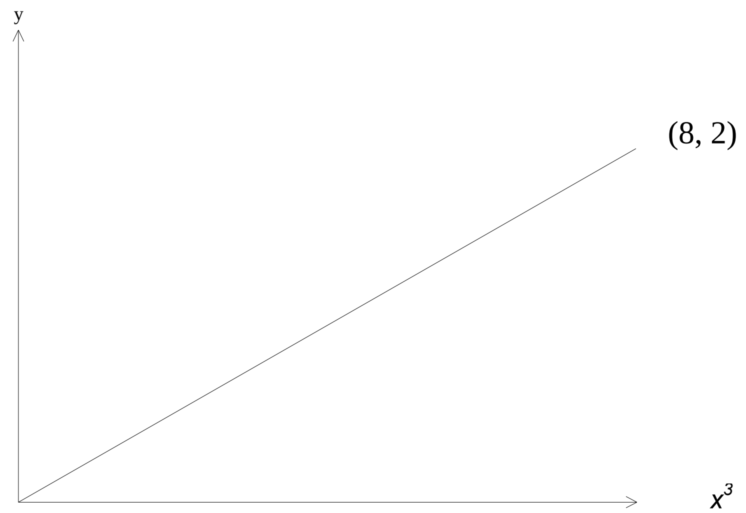
Both lines will have a positive gradient as the hourly rate represents the gradient of the line.

As the flat rate for Company A was larger than Company B, the graph of Company A will have a larger y–intercept than Company B.

As the gradient of the line represents the hourly charge, the gradient of Company A will be less steep than Company B. Using these 2 pieces of information, the only feasible option is C.

Question 7

The graph of y against x^3 is shown. The rule connecting x and y is



A. $y = \frac{1}{4}x$

B. $y = 4x^3$

C. $\boldsymbol{y = \frac{x^3}{4}}$

D. $y = 4x$

E. $y = \frac{1}{4}$

Answer is C

Worked Solution

The rule is in the form $y = kx^3$

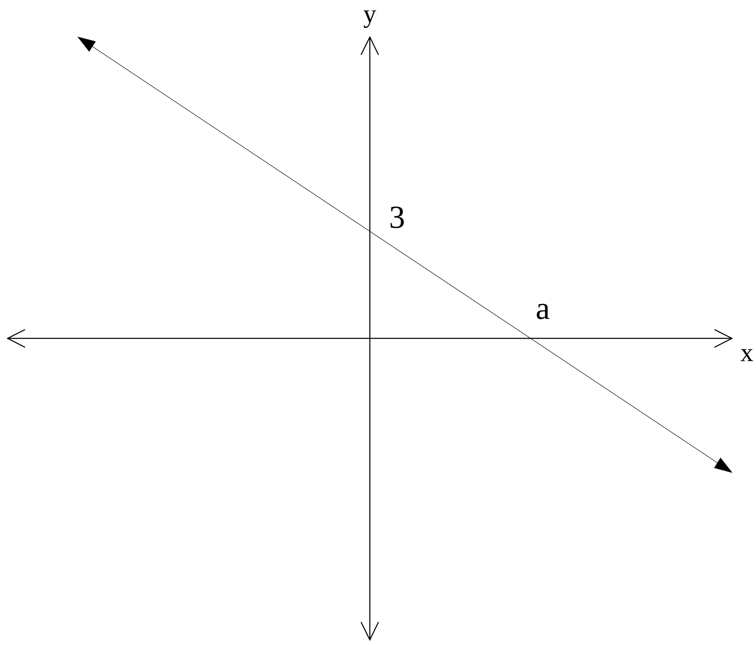
Using the point (8, 2)

$2 = k \times 8, \quad \therefore k = \frac{1}{4}$

The rule then is $y = \frac{1}{4}x^3$ or $y = \frac{x^3}{4}$

Question 8

The equation of the straight line below is



A. $y = \frac{3}{a}x + 3$

B. $y = -\frac{ax}{3} + 3$

C. $y = \frac{ax}{3} + 3$

D. $y + 3 = \frac{3}{a}x$

E. $\boldsymbol{y = -\frac{3x}{a} + 3}$

Answer is E

Worked Solution

The function has a y–intercept of 3 and passes through the point (a, 0)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0 - 3}{a - 0}$$

$$m = \frac{-3}{a}$$

$\therefore\ y = -\frac{3}{a}x + 3$ or $y = -\frac{3x}{a} + 3$

Question 9

The number of solutions for the set of simultaneous equations $\begin{array}{l} 2y = 2mx + 18 \\ -mx + y = 9 \end{array}$ is

A. 0
B. 1
C. 2
D. **Infinite**
E. 3

Answer is D

Worked Solution

If both equations are rearranged to make y the subject:

$y = mx + 9$

$y = mx + 9$

As both equations have a gradient of m and a y-intercept of 9, they are the same line. Therefore, they have an infinite number of solutions.

Module 4: Business-related Mathematics

Question 1

An account paid interest of $382 over a period of 6 years on a principal of $3 490. The annual rate of simple interest correct to one decimal place is

A. 7.1%
B. 18.5%
C. 9.1%
D. **1.8%**
E. 4.2%

Answer is D

Worked solution

$I = \frac{PRT}{100}$, $P = principle$, $R = rate$, $T = term$, $I = interest$

$382 = \frac{3490 \times R \times 6}{100}$

$38200 = 20940 \times R$

$R = 1.82\%$

Question 2

Find the value of the pronumerals in the following bank statement:

Date	Debit	Credit	Balance
12th March			$1 540
13th March		$200	x
18th March	$340		y
28th March		z	$1 775

A. $\mathbf{x = \$1740, \quad y = \$1400, \quad z = \$375}$
B. $x = \$1740, \quad y = \$2080, \quad z = \$375$
C. $x = \$1340, \quad y = \$1400, \quad z = \$375$
D. $x = \$1740, \quad y = \$1400, \quad z = \$275$
E. $x = \$1340, \quad y = \$1400, \quad z = \$375$

Answer is A

Worked solution

$x = 1540 + 200$ (Credit)

$x = 1740$

$x - 340 = y$ (Debit)

$1740 - 340 = y$

$y = 1400$

$y + z = 1775$ (Credit)

$\therefore z = 375$

Question 3

Madeline's investment of $2000 is earning interest at a rate of 8% p.a. compounding monthly. In 5 years, Madeline's investment will be worth

A. $202 514
B. $2 468
C. $3 100
D. **$2 980**
E. $2 101

Answer is D

Worked solution

Compound Interest: $A = P \times R^{n-1}$ where $R = 1 + \frac{r}{100}$

Rate = 8% per annum over 60 months

Rate $= \frac{8}{12}\%$ per month

$A = 2000(1.0066\dot{6})^{60}$
$A = \$2979.69$

Or using TVM Solver

N = 60

I = 8%

PV = $2000

PMT = 0

FV = -2979.60 (ALPHA Solve)

P/Y = 12

C/Y = 12

Question 4

If a hire purchase agreement offers a flat rate of interest of 9% over 180 repayments, then the effective rate of interest (correct to 1 decimal place), is

A. **17.9%**
B. 18%
C. 18.2%
D. 17.8%
E. 17.5%

Answer is A

Worked solution

The effective interest rate is approximately double the flat rate. This is not enough to be able to calculate the effective rate.

$$r_e = r_f \times \frac{2n}{n+1}$$

Where n = number of repayments and $r_f =$ flat rate

$$r_e = 9 \times \frac{2 \times 180}{181}$$

$$r_e = 17.9\%$$

Question 5

The average price of 1L of milk in Victoria during 2008 was $2.45. The price of the same product in 15 years' time, assuming the annual inflation rate is 1.8%, will be

A. $3.76
B. $29.45
C. $4.25
D. **$3.20**
E. $1.70

Answer is D

Worked solution

Assuming annual inflation

$$Cost = P \times (1 + \frac{r}{100})^t$$

$$Cost = 2.45\left(1 + \frac{1.8}{100}\right)^{15}$$

$$Cost = \$3.20$$

Or using TVM Solver

N = 15

I = 1.8%

PV = −2.45

PMT = 0

FV = 3.20 (ALPHA Solve)

P/Y = 1

C/Y = 1

Question 6

A shirt in a sale offering a 30% discount is priced at $35.20. The original cost of the shirt was

A. $24.64
B. $45.76
C. $59.84
D. $65.20
E. **$50.29**

Answer is E

Worked solution

$$Original\ price = \frac{100}{(100 - p)} \times\ new\ price$$

$$Original\ price = \frac{100}{70} \times \$35.20$$

$$Original\ price = \$50.29$$

Question 7

A heavy vehicle bought for $28 900 has a book value of $13 500 after 6 years of reducing balance depreciation. The depreciation rate is closest to

A. 9.8%
B. **11.9%**
C. 9.2%
D. 5.6%
E. 7.7%

Answer is B

Worked solution

$$Value = Purchase\ Price \times (1 - \frac{r}{100})^t$$

$$13500 = 28900 \times \left(1 - \frac{r}{100}\right)^6$$

$$\frac{13500}{28900} = (1 - \frac{r}{100})^6$$

$$\sqrt[6]{\frac{13500}{28900}} = 1 - \frac{r}{100}$$

$$r = 100 \times \left(1 - \sqrt[6]{\frac{13500}{28900}}\right)$$

$$r = 11.9\%$$

Or using TVM Solver

N = 6

I %= -11.91 (ALPHA Solve)   A negative value due to depreciation

PV = −$28900

PMT = 0

FV = 13500

P/Y= 1

C/Y= 1

Question 8

Jennifer takes out a loan of $150 000 and will repay the loan over 25 years paying an interest rate of 4.8% on the reducing balance of the loan compounded monthly. If she wants to repay the loan in 25 years, her monthly repayments will be

A. $6 317.00
B. $288.50
C. **$859.50**
D. $1 250.40
E. $731.60

Answer is C

Worked solution

Using TVM Solver   25 years × 12 months

N = 300

I = 4.8%

PV = −150000

PMT = 859.50 (ALPHA Solve)

FV = 0

P/Y = 12   A final value of 0 (i.e. the loan is completely paid off)

C/Y = 12

Question 9

Jill borrows $15 000 to finance an extension to her house. She plans to make monthly repayments of $220 with an interest rate of 9.5% calculated monthly on the reducing balance of the loan. What is the total interest that she will have paid after 4 years, to the nearest dollar?

A. **$4 676**
B. $5 134
C. $9 116
D. $4 400
E. $10 560

Answer is A

Worked Solution

After 4 years, she will still owe $9 115.59 (including interest). Her repayments in the first 4 years will total $10 560 (48 months × 220)

After 4 years, $15 000 – $10 560 = $4 440 would be owed if no interest was paid.

Therefore total interest paid is $9 115.59 − $4 440 = $4 676

Calculation of amount still owing after 4 years using TVM Solver

N = 48

I = 9.5%

PV = 15000

PMT = −220

FV = −9115.59 (ALPHA Solve)

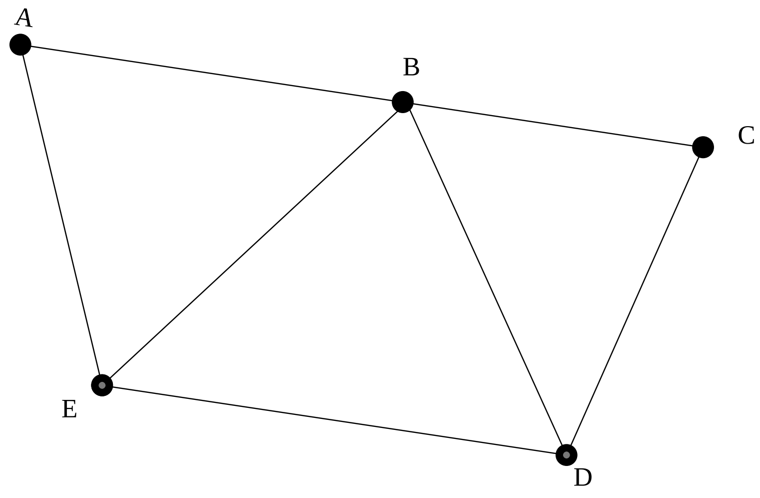
P/Y= 12

C/Y= 12

Module 5 – Networks and Decision Mathematics

Question 1

The degree of Vertex B is



A. 1
B. 2
C. 3
D. **4**
E. 5

Answer is D

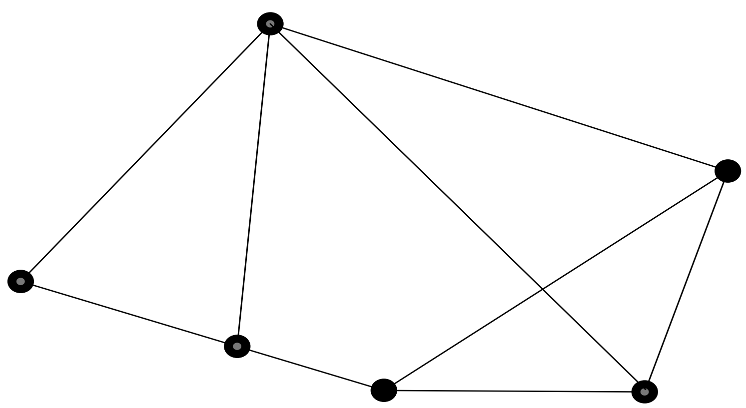
Worked Solution

The degree is the number of edges connecting a vertex.

Question 2

If the graph below was a connected planar graph, the number of edges, vertices and faces is respectively

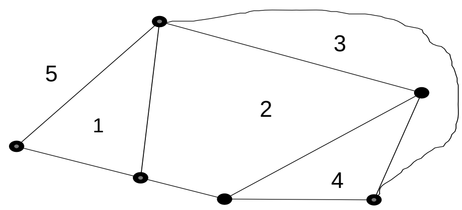
A. 9, 6 and 6
B. 9, 7 and 5
C. **9, 6 and 5**
D. 9, 7 and 4
E. 8, 7 and 5



Answer is C

Worked Solution

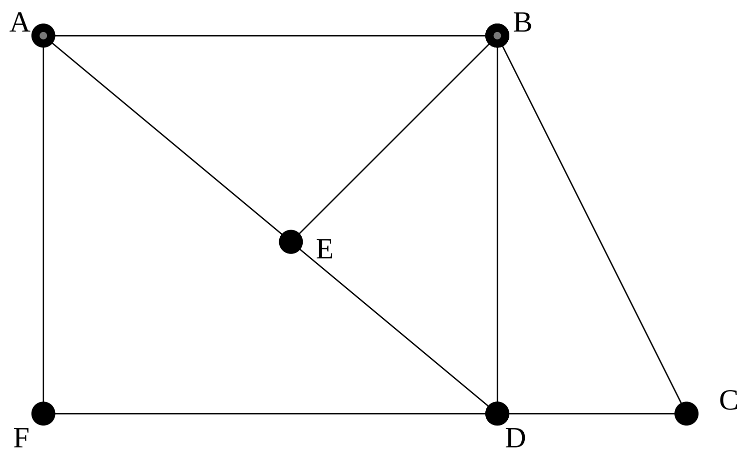
By redrawing the network as a planar graph, i.e. a graph with no overlapping edges.



The planar graph now has 9 edges and 6 vertices; and 4 finite faces and 1 infinite face, giving a total of 5 faces.

Question 3

A suitable Euler path for the planar graph below is

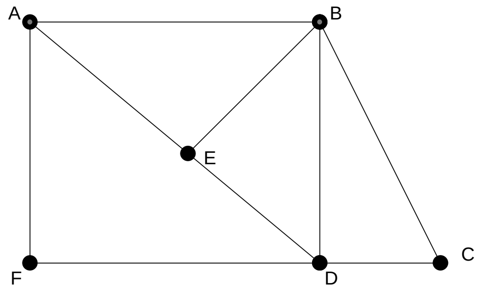


A. A–B–A
B. **A–F–D–C–B–D–E–B–A–E**
C. A–F–D–C–B–E
D. C–B–D–F–A–E–B–A
E. E–B–C–D–B–A–F–D

Answer is B

Worked Solution

An Euler path is a path that covers each edge once and only once without returning to the starting vertex.



As each edge needs to be traversed and there are 9 edges in the network, the only possible solution is the one that has 9 connections, i.e. option B.

Starting from A and finishing at E.

Tip

- *An Euler path can pass through a vertex more than once, but not an edge. A Hamiltonian path passes through each vertex only once.*

Question 4

Four jobs – planning, researching, writing and filing – are to be done by 4 people – Paul (P), Therese (T), David (D) and Jennifer (J). The times taken to perform the various tasks are shown in the matrix below.

	P	T	D	J
Planning	25	30	40	30
Researching	85	80	90	95
Writing	100	125	135	130
Filing	20	20	25	15

If an allocation has been made that will take the shortest amount of time to complete all tasks, the people allocated to Writing and Researching will, respectively, be

A. Jennifer and Paul

B. Paul and Jennifer

C. Therese and Paul

D. **Paul and Therese**

E. Therese and David

Answer is D

Worked Solution

By taking the lowest value in each column and row and subtracting it from each row and column, the matrix below will be achieved.

0	5	5	5
5	0	0	15
0	25	25	30
5	5	0	0

We are looking for zeros, since this means that the best allocation of jobs can be made. Cover all of the zeros by straight lines using the smallest number of horizontal or vertical lines. If we require four lines, then it is possible to find the best allocation of the four jobs to the four people.

In this case, only three lines are required. So we need some further steps. These extra steps are called the Hungarian algorithm.

Select the smallest number that is not covered (5).

Subtract this number from everything uncovered.

Add this number to anywhere the lines intersect.

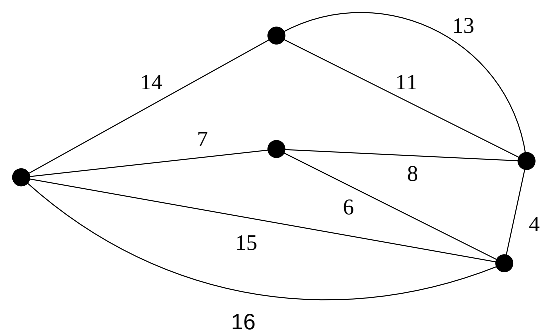
To cover all the zeros, we need four lines. So an optimal allocation can be made.

0	0	0	0
10	0	0	15
0	20	20	25
10	5	0	0

Paul is the best person to complete the Writing task; this will save 20 minutes. Researching should be completed by Therese or David.

Question 5

The length of the maximal spanning tree for the following network is



A. **51**

B. 48

C. 68

D. 50

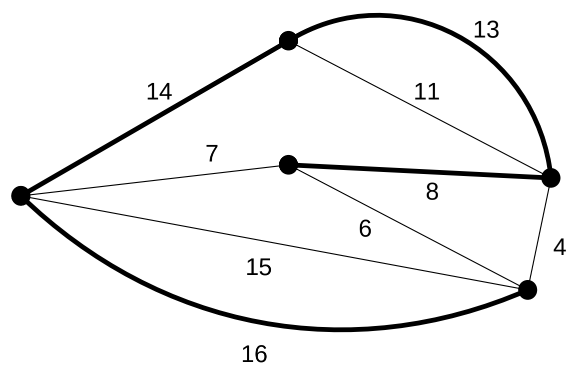
E. 31

Answer is A

Worked Solution

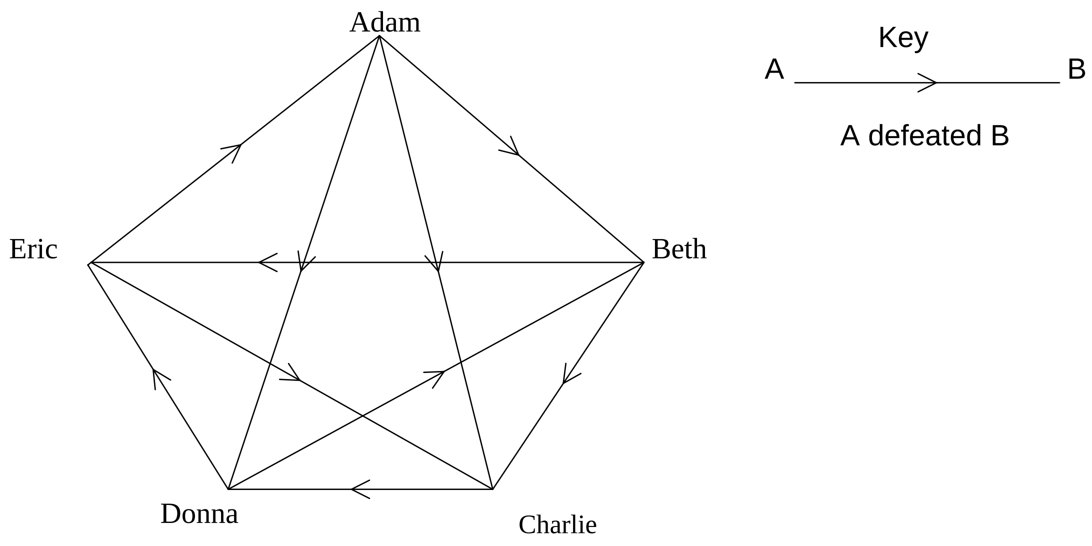
16+ 14 + 13 + 8 = 51

Prim's algorithm is used to find the maximum spanning tree. We start with the largest edge and keep choosing the largest remaining edge, remembering to always maintain a tree (no loops or circuits). The maximum spanning tree is shown below.



Question 6

Five friends, Adam, Beth, Charlie, Donna and Eric, play a game of chess against each other. The results of the game are summarised in the complete graph below.



Which of the following statements is true?

A. Donna beat Charlie

B. Beth has a two step dominance over Eric

C. Adam's only loss was to Charlie

D. **Beth won more games than Charlie**

E. Donna lost 3 games

Answer is D

Worked Solution

Beth has a two step dominance over Adam (Beth beat Eric, who beat Adam). Beth has a one step dominance over Eric.

Charlie beat Donna.

Adam beat Charlie and Donna and Donna won 2 games and lost 2 games. This leaves option D as the only correct solution; Beth won 2 games, while Charlie only won 1 game.

The information below relates to Question *7 and 8.*

Task	Predecessor	Activity Time (weeks)
A	-	5
B	-	7
C	A	4
D	B	6
E	C, D	3
F	B	1
G	E, F	2
H	E, F	6
J	G	1
K	H, J	5

Question 7

The table above lists the 10 activities in a project, the immediate predecessor and the activity time. The latest start time for activity F is

A. 8

B. 10

C. 16

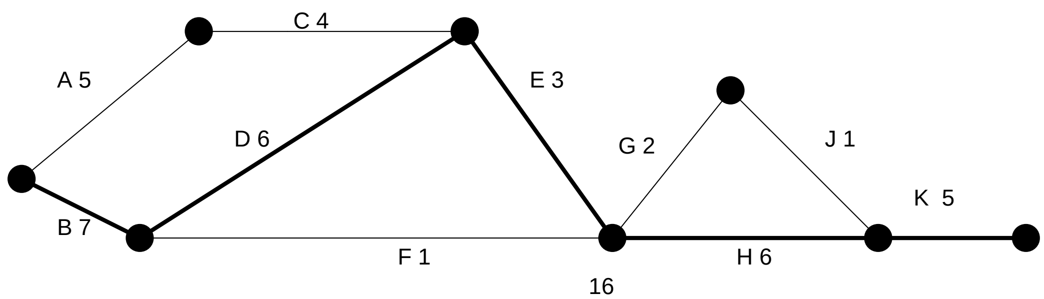
D. 9

E. **15**

Answer is E

Worked Solution

The latest start time is the time at which F has to start without delaying the entire project.



As the critical path is B–D–E–H–K and the latest start time for Activity H is 16, without delaying the entire project, Activity F has to start after 15 (16 – Activity F = 15)

Question 8

The project is modified and the following changes are made:

Activity A is delayed by 5 weeks, Activity F is delayed by 5 weeks and Activity G is delayed by 4 weeks.

The new critical path for the project is

A. A – C –E – H – K

B. B – F – H – K

C. B – D – E – G – J – K

D. B – F – H – K

E. **A – C – E – G – J – K**

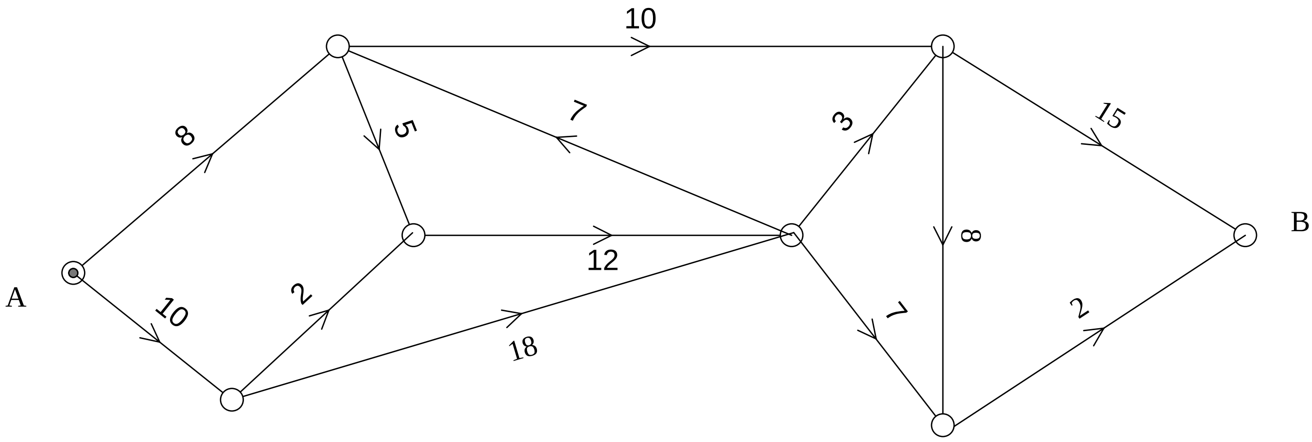
Answer is E

Worked Solution

Making the changes to the directed graph and considering that the critical path is the longest path through the project, the critical path is A–C–E–G–J–K
As the changes were made, the critical path has changed from the previous question.

Question 9

Water flows from a source at point A to a sink at Point B. Measurements are in litres. Find the maximum amount of water that is able to travel from Point A to Point B.



A. 18 litres

B. 17 litres

C. 20 litres

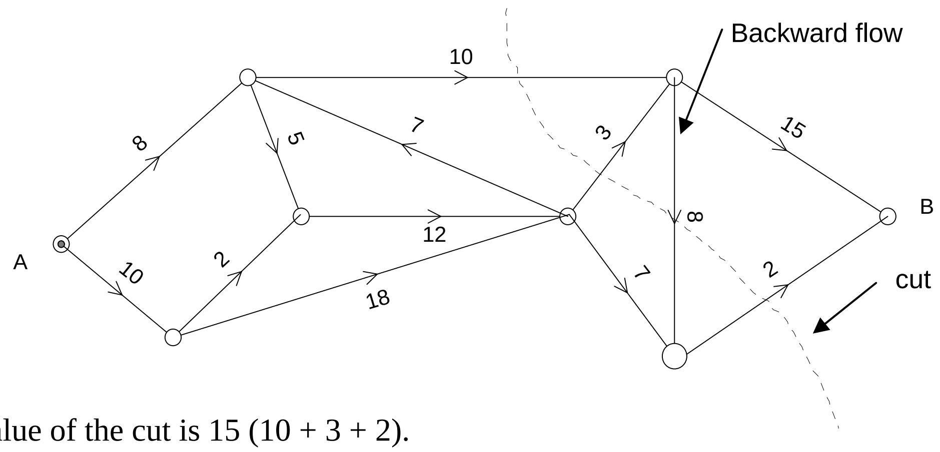
D. **15 litres**

E. 23 litres

Answer is D

Worked Solution

The maximum flow of a network is the value of the minimum cut. A cut through a directed network stops all flow from the source (A) to the sink (B).



The value of the cut is 15 (10 + 3 + 2).

The edge with a capacity of 8 is considered to be backward flow, as the flow across the cut is from the right to the left, not left to right.

Module 6 – Matrices

Question 1

Given that $A = \begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix}$ and that $B = \begin{bmatrix} 2 & -5 \\ 1 & 2 \end{bmatrix}$

The value of $A^2 + B$ is

A. $\begin{bmatrix} 6 & 4 \\ 2 & 11 \end{bmatrix}$

B. $\begin{bmatrix} \mathbf{9} & \mathbf{10} \\ \mathbf{6} & \mathbf{14} \end{bmatrix}$

C. $\begin{bmatrix} 1 & -5 \\ 2 & 2 \end{bmatrix}$

D. $\begin{bmatrix} 4 & -2 \\ 2 & 5 \end{bmatrix}$

E. $\begin{bmatrix} 7 & 15 \\ 5 & 12 \end{bmatrix}$

Answer is B

Worked Solution

$$\begin{bmatrix} 2 & 3 \\ 1 & 3 \end{bmatrix}^2 + \begin{bmatrix} 2 & -5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 10 \\ 6 & 14 \end{bmatrix}$$

This calculation can be done by hand or using the calculator.

Question 2

If the order of matrix CD is (3 × 4) and the order of matrix D is (2 × 4) then the order of matrix C is

A. (2 × 4)
B. **(3 × 2)**
C. (3 × 4)
D. (4 × 4)
E. (4 × 2)

Answer is B

Worked Solution

If C is an ($x \times y$) matrix then

$(x \times y) \times (2 \times 4) = (3 \times 4)$

For matrix C to be defined, y must be 2 and as CD has 3 rows, x must be 3, giving C an order of (3×2).

Question 3

Which statement is **false** relating to the matrices X, Y and Z?

$$X = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \quad Y = \begin{bmatrix} -1 & -2 \\ 0 & 0 \end{bmatrix} \quad Z = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

A. XY is defined

B. Z is a column matrix

C. The order of X is (2×2)

D. **The inverse of X is $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$**

E. The value of $2X$ is $\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$

Answer is D

Worked Solution

XY is defined as X is a (2×2) matrix and Y is a (2×2) matrix.

Z has a single column making it a column matrix.

The value of $2X$ is $\begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$

This leaves D as the incorrect solution: the inverse of X is $\begin{bmatrix} 0.5 & 0.5 \\ -0.5 & 0.5 \end{bmatrix}$

Question 4

The matrix that has a determinant of 5 is

A. $\begin{bmatrix} 5 & 5 \\ 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$

E. $\begin{bmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{3} \end{bmatrix}$

Answer is E

Worked Solution

Given a square matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ the determinant is ad – bc.

The determinant of matrix E is (2 × 3) – (1 × 1) = 5

Tips

- *Once a matrix has been defined using a calculator, the command det(A) will give you the determinant of matrix A.*

Question 5

Which of the following represents the solution to the simultaneous equation?

$2x + 5y = 9$

$-x - y + 2 = 0$

A. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ -1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 9 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 9 \\ -1 & -2 \end{bmatrix}$

C. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 5 & -1 \end{bmatrix}^{2} \begin{bmatrix} 9 \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{2} & \mathbf{5} \\ \mathbf{-1} & \mathbf{-1} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{9} \\ \mathbf{-2} \end{bmatrix}$

E. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ -1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 9 \\ 0 \end{bmatrix}$

Answer is D

Worked Solution

Rearranging the equations to $2x + 5y = 9$ and $-x - y = -2$

$$\begin{bmatrix} 2 & 5 \\ -1 & -1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ -1 & -1 \end{bmatrix}^{-1} \times \begin{bmatrix} 9 \\ -2 \end{bmatrix}$$

Question 6

The matrix that is equivalent to $\begin{bmatrix} 1 & 4 & 2 \\ 2 & -6 & 0 \end{bmatrix}$ is

A. $\begin{bmatrix} 1 & 4 & 0 \\ 2 & -6 & 2 \end{bmatrix}$

B. $\mathbf{2}\begin{bmatrix} \mathbf{0.5} & \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{-3} & \mathbf{0} \end{bmatrix}$

C. $\frac{1}{2}\begin{bmatrix} 2 & 8 & 4 \\ 4 & 3 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 1 & 4 \\ 2 & 3 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 3 & -2 \\ 0 & -3 & 3 \end{bmatrix}$

E. $\begin{bmatrix} 2 & -6 & 0 \\ 1 & 4 & 2 \end{bmatrix}^{-1}$

Answer is B

Worked Solution

For 2 matrices to be equivalent, they must have the same order and elements.

Using scalar multiplication

$$2\begin{bmatrix} 0.5 & 2 & 1 \\ 1 & -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 2 \\ 2 & -6 & 0 \end{bmatrix}$$

Question 7

A fieldtrip counting 2 species of birds, A and B at 3 separate locations, X, Y and Z can be summarised by the matrix, P.

$$P = \begin{bmatrix} 52 & 61 & 21 \\ 40 & 32 & 24 \end{bmatrix}$$

It is predicted that in the next 3 years there will be a 15% increase in the numbers of species A and an 18% decrease in the numbers of species B. The matrix calculation that best describes this situation is

A. $\begin{bmatrix} 15 \\ 82 \end{bmatrix} \begin{bmatrix} 52 & 61 & 21 \\ 40 & 32 & 24 \end{bmatrix}$

B. $\begin{bmatrix} 52 & 61 & 21 \\ 40 & 32 & 24 \end{bmatrix} \begin{bmatrix} 1.15 \\ 0.82 \end{bmatrix}$

C. $\begin{bmatrix} 1.15 & 0.82 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 52 & 61 & 21 \\ 40 & 32 & 24 \end{bmatrix}$

D. $\begin{bmatrix} \mathbf{1.15} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.82} \end{bmatrix} \begin{bmatrix} \mathbf{52} & \mathbf{61} & \mathbf{21} \\ \mathbf{40} & \mathbf{32} & \mathbf{24} \end{bmatrix}$

E. $1.15\begin{bmatrix} 52 & 61 & 21 \\ 40 & 32 & 24 \end{bmatrix} + 0.82 \begin{bmatrix} 52 & 61 & 21 \\ 40 & 32 & 24 \end{bmatrix}$

Answer is D

Worked Solution

Options A and B are not defined.

$$\begin{bmatrix} 1.15 & 0 \\ 0 & 0.82 \end{bmatrix} \begin{bmatrix} 52 & 61 & 21 \\ 40 & 32 & 24 \end{bmatrix} = \begin{bmatrix} 1.15\times 52 + 0\times 40 & 1.15\times 61 + 0\times 32 & 1.15\times 21 + 0\times 24 \\ 0\times 52 + 0.82\times 40 & 0\times 61 + 0.82\times 32 & 0\times 21 + 0.82\times 24 \end{bmatrix}$$

$$= \begin{bmatrix} 59.8 & 70.15 & 24.15 \\ 32.8 & 26.24 & 19.68 \end{bmatrix}$$

This represents an increase of 15% in species A and a decrease of 18% in species B.

The following information is related to question 8 and 9.

Three theatres in the same town compete for a fixed number of patrons. The matrix below shows the movements of patrons from one month to the next.

A particular month

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.38 & 0.41 & 0.3 \\ 0.26 & 0.5 & 0.24 \\ 0.36 & 0.09 & 0.46 \end{bmatrix} \end{array} \text{ The following month}$$

There are 180 patrons who regularly attend the theatres. Initially, half of these patrons attend Venue C, and the other half attends either Venue A or Venue B. Twice as many patrons attend Venue A as attend Venue B.

Question 8

The number of patrons that will attend Venue A in the 3rd month is closest to

A. **66**
B. 53
C. 54
D. 47
E. 78

Answer is A

Worked Solution

The initial matrix is $S_0 = \begin{bmatrix} 60 \\ 30 \\ 90 \end{bmatrix}$ following the conditions placed on the 3 venues, A, B, C and the number of patrons at each venue.

$$S_1 = = \begin{bmatrix} 0.38 & 0.41 & 0.3 \\ 0.26 & 0.5 & 0.24 \\ 0.36 & 0.09 & 0.46 \end{bmatrix} \times \begin{bmatrix} 60 \\ 30 \\ 90 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 0.38 & 0.41 & 0.3 \\ 0.26 & 0.5 & 0.24 \\ 0.36 & 0.09 & 0.46 \end{bmatrix} \times S_1 \text{ ... etc} \qquad \text{OR}$$

Using Markov chains:

$$S_3 = \begin{bmatrix} 0.38 & 0.41 & 0.3 \\ 0.26 & 0.5 & 0.24 \\ 0.36 & 0.09 & 0.46 \end{bmatrix}^3 \times \begin{bmatrix} 60 \\ 30 \\ 90 \end{bmatrix}$$

$$= \begin{bmatrix} 65.56 \\ 59.58 \\ 54.86 \end{bmatrix}$$

At Theatre A, there will be closest to 66 patrons.

Question 9

If this pattern continues, the number of patrons at venue C in the long term will be closest to

A. 53
B. 50
C. 52
D. 51
E. **54**

Answer is E

Worked Solution

To find the long term steady state, the transition matrix is raised to a large n (between 16 and 50).

$$S_{\text{large n}} = \begin{bmatrix} 0.38 & 0.41 & 0.3 \\ 0.26 & 0.5 & 0.24 \\ 0.36 & 0.09 & 0.46 \end{bmatrix}^{\text{large } n} \times \begin{bmatrix} 60 \\ 30 \\ 90 \end{bmatrix}$$

Try n = 10

$$S_{10} = \begin{bmatrix} 0.38 & 0.41 & 0.3 \\ 0.26 & 0.5 & 0.24 \\ 0.36 & 0.09 & 0.46 \end{bmatrix}^{10} \times \begin{bmatrix} 60 \\ 30 \\ 90 \end{bmatrix}$$

$$= \begin{bmatrix} 65.888 \\ 60.159 \\ 53.952 \end{bmatrix}$$

n = 20

$$S_{20} = \begin{bmatrix} 0.38 & 0.41 & 0.3 \\ 0.26 & 0.5 & 0.24 \\ 0.36 & 0.09 & 0.46 \end{bmatrix}^{20} \times \begin{bmatrix} 60 \\ 30 \\ 90 \end{bmatrix}$$

$$= \begin{bmatrix} 65.888 \\ 60.159 \\ 53.952 \end{bmatrix}$$

As $S_{20} = S_{10}$ the matrix has reached a steady state.

There will be 54 patrons (to the nearest whole patron).

END OF SOLUTIONS BOOK