**MAV Trial Examination Paper 2009**
**Further Mathematics**
**Examination 1 – SOLUTIONS**

## Core: Data Analysis

### Answers

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | **E** | **2.** | **D** | **3.** | **B** | **4.** | **E** | **5.** | **D** | **6.** | **E** | **7.** | **C** |
| **8.** | **D** | **9.** | **B** | **10.** | **B** | **11.** | **A** | **12.** | **E** | **13.** | **A** | | |

### Solutions

**Question 1** **Answer E**

Height is presented as a *categorical* variable (3 levels)
Long jump is presented a *numerical* variable
A back to back stemplot displays a categorical variable (2 levels) with a numerical variable ∴ **not A**

A percentaged two way frequency table displays two categorical variables ∴ **not B**

A scatterplot displays two numerical variables ∴ **not C**

A histogram displays only *one* type of data which is numerical continuous ∴ **not D**

Parallel boxplots display a categorical variable (at least 2 levels) with a numerical variable.

**Question 2** **Answer D**

The five number summary for this data is
$Minimum = 42$
$Q_1 = 45.5$ **This means E is True**
$Median = 50$ **This means B is True**
$Q_3 = 59.5$
$Maximum = 79$
$\therefore IQR = 59.5 - 45.5 = 14$ **This means C is True**

$$\text{Upper Boundary} = Q_3 + 1.5 \times IQR$$
$$= 59.5 + 1.5 \times 14$$
$$= 80.5$$

$79 <$ Upper Boundary therefore 79 is not an outlier. i.e. **D is False**

**Question 3** **Answer B**

10 students out of 40 received at least 20 text messages.
i.e. $\frac{10}{40} \times 100\% = 25\%$

**Question 4** **Answer E**

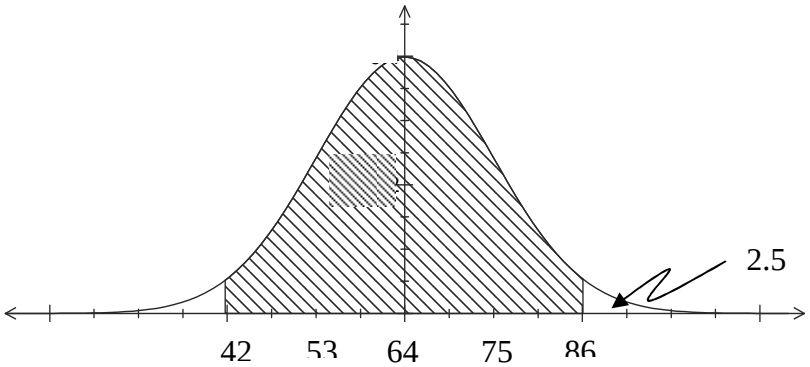
The shape is positively skewed. **This eliminates A, B and C**
The median is between the $20^{th}$ and $21^{st}$ student.
These students fall between 10 and 15 text messages.

**Question 5** **Answer D**

From the values marked on the graph

$\bar{x} = 64$ and $s = 64 - 53 = 11$

Using the 95% rule $\bar{x} + 2s = 64 + 2 \times 11 = 86$




**Question 6** **Answer E**

$$z = \frac{x - \bar{x}}{s}$$
$$= \frac{80 - 64}{11}$$
$$= \frac{16}{11}$$
$$= 1.45 \approx 1.5$$

**Question 7** **Answer C**

$$gradient = b = \frac{rS_y}{S_x}$$
$$= \frac{-0.6 \times 2}{3}$$
$$= -0.4$$

$$y - \text{intercept} = a = \bar{y} - b\bar{x}$$
$$= 7 - (-0.4) \times 10$$
$$= 11$$

The equation is $y = 11 - 0.4x$

**Question 8** **Answer D**

$$Weight = -102 + 0.968 \times height$$
$$= -102 + 0.968 \times 160$$
$$= 52.88$$

$$\text{Residual} = \text{actual weight} - \text{predicted weight}$$
$$= 58 - 52.88$$
$$= 5.12$$

**Question 9** **Answer B**

To linearise the data

- The $\log(y)$ and $\frac{1}{y}$ transformations will compress the upper end of the $y$ scale.
- The $\log(x)$ and $\frac{1}{x}$ transformations will compress the upper end of the $x$ scale.

Therefore the $y^2$ transformation is not likely to linearise the data.

**Question 10** **Answer B**

There are 16 points so they are grouped Left 5 points Middle 6 points Right 5 points

The gradient is determined by

the median point in the left group (17, 38) and
the median point in the right group (28, 8)

$$m = \frac{8-38}{28-17}$$
$$= \frac{-30}{11}$$
$$= -2.7$$

**Question 11** **Answer A**

$r = -0.7650$ will change to a value that is closer to $-1$ when the outlier is removed.

Therefore the $r$ value will decrease.

**Question 12** **Answer E**

The seasonal indices for the quarterly sales have a sum total of 4

$$x + 0.8 + 0.35 + 1.2 = 4$$
$$x + 2.35 = 4$$
$$\therefore x = 1.65$$

**Question 13** **Answer A**

Quarter 16 represents Spring 2009. Substituting 16 in the trend line gives:

$$\text{Deseasonalised no.of air conditioners} = 2300 + 2.215 \times 16$$
$$= 2335.44$$

Seasonalise this figure to find the actual amount

$$\text{Actual no.of air conditioners} = 2335.44 \times \text{S.I.}$$
$$= 2335.44 \times 1.2$$
$$= 2802.5 \approx 2803$$

**END OF CORE SOLUTIONS**

## Module 1: Number patterns

**Answers**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | **A** | **2.** | **D** | **3.** | **B** | **4.** | **D** | **5.** | **C** |
| **6.** | **D** | **7.** | **C** | **8.** | **A** | **9.** | **D** | | |

## Solutions

**Question 1** **Answer A**

Substituting $t_1 = a = 38$ into $t_3 = a + 2d = 10$ gives

$38 + 2d = 10$

$2d = -28$

$d = -14$

To find $S_6$ substitute $a = 38,\ d = -14,$ and $n = 6$ into $S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_6 = \frac{6}{2}[2 \times 38 + 5 \times -14]$$
$$= 3[76 - 70]$$
$$= 3 \times 6$$
$$= 18$$

*Check by adding the first 6 terms $38,\ 24,\ 10,\ -4,\ -18,\ -32$

**Question 2** **Answer D**

Substituting $t_1 = a = 5$ into $t_5 = ar^4 = 80$ gives

$5r^4 = 80$

$r^4 = 16$

$r = 2 \text{ or } -2$

So the sequence could be 5, 10, **20**, 40, 80, … or 5, -10, **20**, -40, 80,…

Either way $t_3 = 5 \times 2^2 = 20$

**Question 3** **Answer B**

Using the difference equations $t_{n+1} = 5 - t_n;\ t_1 = 2$

$$t_2 = 5 - t_1$$
$$= 5 - 2$$
$$= 3$$

**This eliminates C and E**

$$t_3 = 5 - t_2$$
$$= 5 - 3$$
$$= 2$$

**This eliminates A**

$$t_4 = 5 - t_3$$
$$= 5 - 2$$
$$= 3$$

Sequence generated by this difference equation is 2, 3, 2, 3, 2, 3, 2,….

**Question 4** **Answer D**

The oil leak sequence is 50, 45, 40.5, . . .

This is a geometric sequence with $a = 50$ and $r = \frac{45}{50} = \frac{40.5}{45} = 0.9$

The total amount of oil leaked is found by

$$S_\infty = \frac{a}{1-r}$$
$$= \frac{50}{1-0.9}$$
$$= 500$$

This means that $2000 - 500 = 1500$ ml of oil remains in the car.

**Question 5** **Answer C**

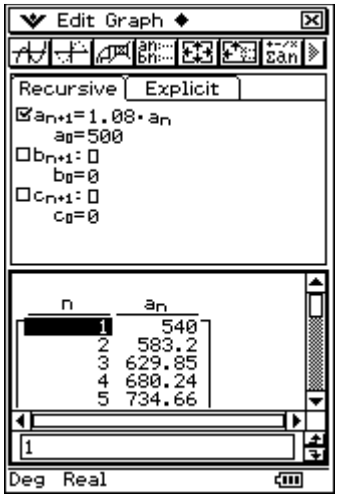
Number of shoppers increase by 8% means that previous number is multiplied by 1.08

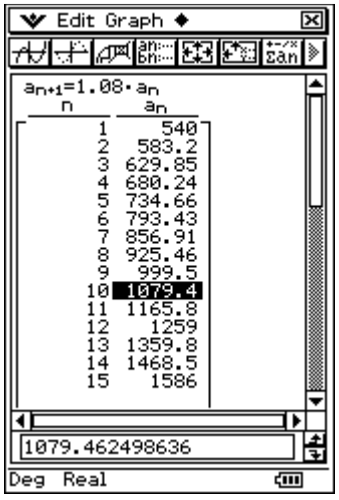
Number of shoppers initially is $P_0 = 500$

i.e. $P_n = 1.08 \times P_{n-1}$ where $P_0 = 500$

**Question 6** **Answer D**

Entering the correct difference equation in the calculator generates the sequence.
Scroll down until the sequence first exceeds 1000.




This occurs after 10 hours.

**Question 7** **Answer C**

To find $t_2$

Work backwards, subtract 3 from $t_3$ then multiply by -2

$$t_3 = -\frac{1}{2}t_2 + 3$$

$$-8 = -\frac{1}{2}t_2 + 3$$

$$-11 = -\frac{1}{2}t_2$$

$$t_2 = 22$$

To find $t_1$

Work backwards, subtract 3 from $t_2$ then multiply by -2

$$t_2 = -\frac{1}{2}t_1 + 3$$

$$22 = -\frac{1}{2}t_1 + 3$$

$$19 = -\frac{1}{2}t_1$$

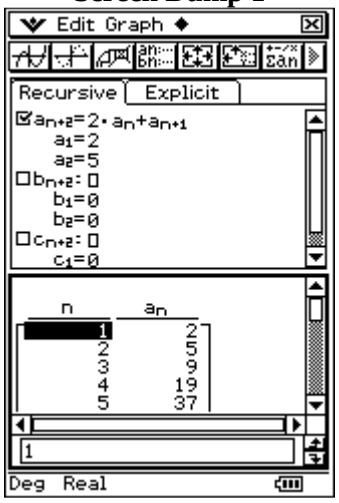
$$t_1 = -38$$

**Question 8** **Answer A**

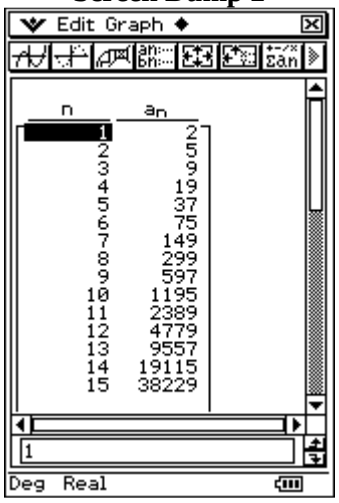
Entering the difference equation, $J_{n+2} = 2J_n + J_{n+1}$ where $J_1 = 2$ and $J_2 = 5$ in your calculator (see screen dump 1) will generate the sequence 2, 5, 9, 19, 37 ,… (see screen dump 2).

Find the sum of the first five numbers by adding them.

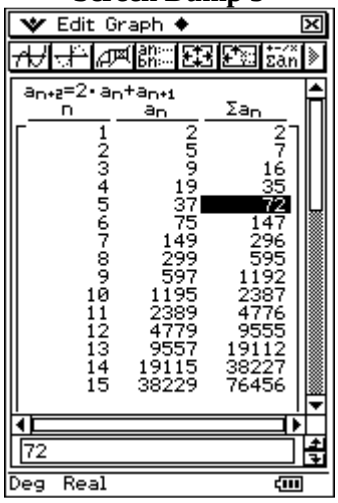
**Screen Dump 1**




**Screen Dump 2**




**Screen Dump 3**




Alternatively, set Σ display on (see screen dump 3) and read the answer from the table.

**Question 9** **Answer D**

A constant sequence will always generate the same value i.e 10, 10, 10, …

$10 = 0.8 \times 10 + k$

$10 = 8 + k$

$\therefore k = 2$

**END OF MODULE 1 SOLUTIONS**

## Module 2: Geometry and trigonometry

**Answers**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | **D** | **2.** | **C** | **3.** | **A** | **4.** | **D** | **5.** | **C** |
| **6.** | **B** | **7.** | **E** | **8.** | **C** | **9.** | **A** | | |

## Solutions

**Question 1** **Answer D**

Using triangle ABD

$\angle BAD = 180^0 - 143^0 = 37^0$

$\therefore \sin 57^0 = \dfrac{5}{x}$

$x = \dfrac{5}{\sin 37^0}$

Not listed as answer

Alternatively,

$\angle ABD = 90^0 - 37^0 = 53^0$

$\therefore \cos 53^0 = \dfrac{5}{x}$

$x = \dfrac{5}{\cos 53^0}$

**Question 2** **Answer C**

To find AD

$\tan 37^0 = \dfrac{5}{AD}$

$AD = \dfrac{5}{\tan 37^0}$

$= 6.64$

Area of $\Delta ABC$ = Area of $\Delta ABD$ + Area of $\Delta BCD$

$= \dfrac{1}{2} \times AD \times 5 + \dfrac{1}{2} \times 3 \times 5$

$= \dfrac{1}{2} \times (6.64 + 3) \times 5$

$= 24.1 \text{ m}^2$

**Question 3** **Answer A**

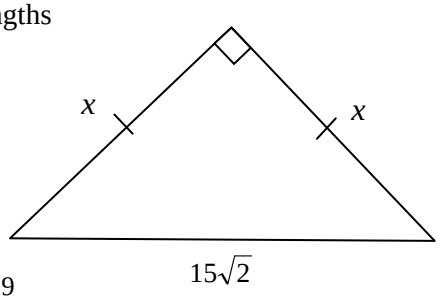
Use Pythagoras to find the unknown side lengths

$x^2 + x^2 = (15\sqrt{2})^2$

$2x^2 = 450$

$x^2 = 225$

$\therefore x = 15$



Perimeter $= 15 + 15 + 15\sqrt{2}$

$= 30 + 15\sqrt{2}$

$= 15(2 + \sqrt{2})$

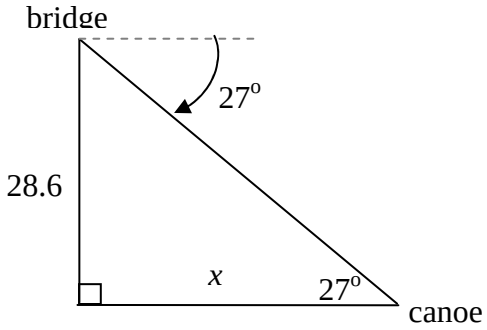
**Question 4** **Answer D**

To find horizontal the distance, $x$

$\tan 27^0 = \dfrac{28.6}{x}$

$x = \dfrac{28.6}{\tan 27^0}$

$= 56.1 \text{ m}$




**Question 5** **Answer C**

The smallest angle, $\theta$, is found opposite the shortest side length.

Using the cosine rule

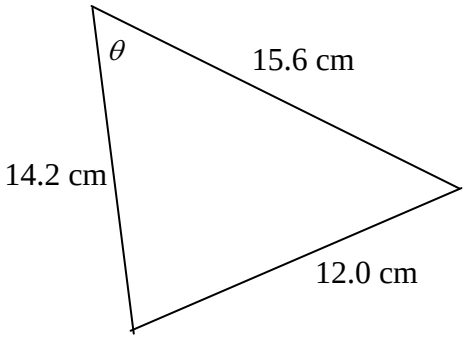
$\cos\theta = \dfrac{14.2^2 + 15.6^2 - 12.0^2}{2(14.2)(15.6)}$

$= \dfrac{301}{443.04}$

$= 0.6794$

$\therefore \theta = \cos^{-1}(0.6794)$

$= 47.2^0$




**Question 6** **Answer B**

$V = \dfrac{1}{3}\pi r^2 h$

$8000 = \dfrac{1}{3}\pi r^2 h$

$24\,000 = \pi r^2 40$

$r = \sqrt{\dfrac{24\,000}{40\pi}}$

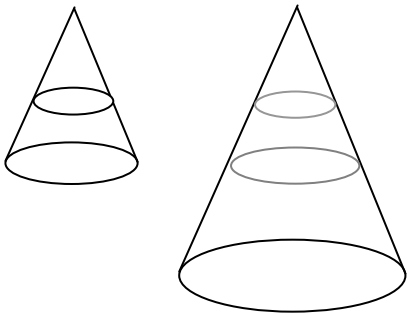
$= 13.8$

The diameter is $2 \times 13.8 = 27.6 \approx 28 \text{ cm}$

**Question 7** **Answer E**

Using similar shapes



The height ratio for

Not green : Total height

25 : 40

= 5 : 8

The area ratio for

Not green icing : total icing

$5^2 : 8^2$

= 25 : 64

Therefore the area ratio for

Green icing : total icing

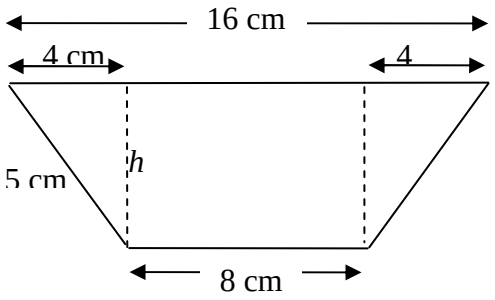
64 – 25 : 64

= 39 : 64

**Question 8** **Answer C**

The side of the bowl is in the shape of a trapezium

The height of the trapezium can be found using symmetry and Pythagoras' Theorem




$h^2 + 4^2 = 5^2$

$h^2 = 9$

$\therefore h = 3$

(This is not the height of the bowl)

Area of Trapezium is given by

$A = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(8+16)\times 3$

$= 36 \text{ cm}^2$

The Surface Area inside the bowl = Area of 4 sides + Area of square base

$= 4\times 36 + 8^2$

$= 144 + 64$

$= 208 \text{ cm}^2$

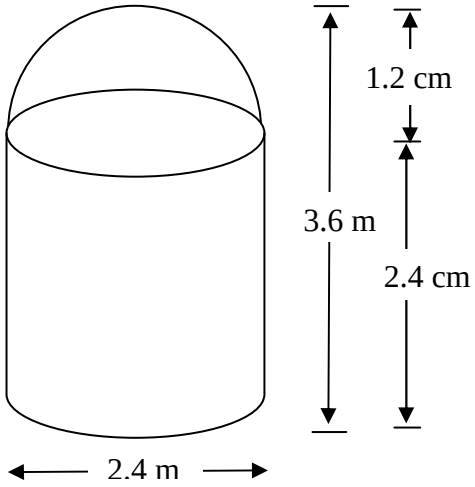
TSA to be glazed (inside and outside) = $2\times 208 = 416 \text{cm}^2$

**Question 9** **Answer A**

Radius of cylinder = Radius of hemisphere = height of hemisphere $= \frac{2.4}{2} = 1.2\,\text{m}$

Height of cylinder = $3.6 - 1.2 = 2.4\,\text{m}$




Total Volume
= Volume of cylinder + Volume of hemisphere

$= \pi r^2 h + \frac{1}{2} \times \frac{4}{3} \pi r^3$

$= \pi \times 1.2^2 \times 2.4 + \frac{2}{3} \times \pi \times 1.2^3$

$= 10.857 + 3.619$

$= 14.48$

Closest to 14.5

**END OF MODULE 2 SOLUTIONS**

## Module 3: Graphs and relations

### Answers

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | **D** | **2.** | **E** | **3.** | **D** | **4.** | **B** | **5.** | **C** |
| **6.** | **E** | **7.** | **D** | **8.** | **E** | **9.** | **C** | | |

### Solutions

**Question 1** **Answer D**

Find equation $y = mx + c$

To find gradient, use (0, 7) and (2, 0)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - 7}{2 - 0}$$

$$= \frac{-7}{2}$$

From the graph, the y-intercept, $c = 7$

$$\therefore y = \frac{-7}{2}x + 7$$

$$2y = -7x + 14$$

$$2y + 7x = 14$$

**Question 2** **Answer E**

The line $3x - 5y = 30$ can be written as

$$-5y = -3x + 30$$

$$y = \frac{-3}{-5}x + \frac{30}{-5}$$

$$y = \frac{3}{5}x - 6$$

This means that $m = \frac{3}{5}$ and $c = -6$

- The gradient is positive ( i.e. as $x$ increases, $y$ increases) $\therefore$ **A is false**
- The $y$-intercept is $-6$ $\therefore$ **B is false**
- To find $x$-intercept sub $y = 0$ in $3x - 5y = 30$ This gives $3x = 30$ i.e. $x = 10$, so the $x$-intercept is 10 $\therefore$ **C is false**.
- The gradient is $\frac{3}{5}$ $\therefore$ **D is false**.
- When $x = 5$, $15 - 5y = 30$

  $-5y = 15$

  $y = -3$

  This means $(5,-3)$ is on the line $3x - 5y = 30$

**Question 3** **Answer B**

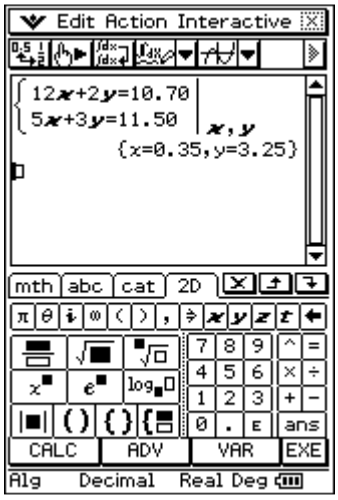
Let $x$ = the cost of a bread roll and $y$ = the cost of a loaf of bread

Peter's purchase gives the equation $12x + 2y = 10.70$ (1)

Harry's purchase gives the equation $5x + 3y = 11.50$ (2)

(where the amount paid = \$20 – \$8.50 = \$11.50)

At this point the equations can be entered into the calculator to solve simultaneously.




**Alternatively**, using algebra
Eliminate $y$ by
multiplying equation (1) by 3 $\quad 36x + 6y = 32.10$

$-$

multiplying equation (2) by 2 $\quad 10x + 6y = 23.00$

Subtracting the new equations gives $\quad 26x = 9.10$

$$x = \frac{9.10}{26} = 0.35$$

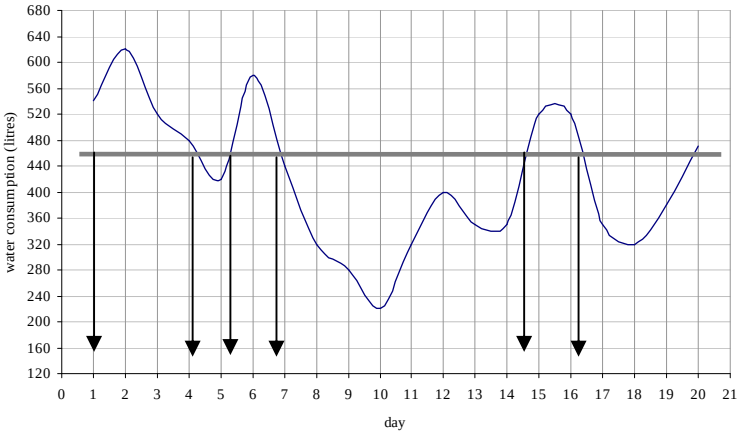
If one roll costs 35 cents, then ten rolls cost \$3.50

**Question 4** **Answer D**

The recommended maximum daily water consumption for the Smith family is $3 \times 155 = 465$ Litres
Placing a ruler horizontally at approximately 465 Litres shows that the Smith family consumed more than their maximum daily limit

- between Day 1 to Day 4 (3 days),
- between Day 5 to Day 7 (2 days), and
- between Day 14.5 to Day 16.5 (2 days)

This gives a total of 7 days.



**Question 5** **Answer C**

Two people are employed for 4 hours. This means that there is a total of 8 hours of donut preparation time.
i.e. $8 \times 60 = 480$ minutes are available to prepare the donuts.
If it takes 10 minutes to prepare one dozen jam donuts, then it takes $10x$ minutes to prepare $x$ dozen jam donuts.
If it takes 8 minutes to prepare one dozen iced donuts, then it takes $8y$ minutes to prepare $y$ dozen iced donuts
This means $10x + 8y \leq 480$

**Question 6** **Answer E**

Let $x$ = the no. of **dozens** of donuts sold
$\text{Cost} = 300 + 1.5x$
$\text{Revenue} = 6x$
To break even $\text{Revenue} = \text{Cost}$

$$6x = 300 + 1.5x$$
$$4.5x = 300$$
$$x = 66.667 \text{ dozen donuts}$$

Therefore $66.667 \times 12 = 800$ donuts must be sold to break even.

**Question 7** **Answer D**

Mark is charged \$7 for his late book fine
Melanie is charged \$2 for her late book plus $\$3 \times 3 = \$9$ for each of the two CDs

Total late fee = \$(7 + 2 + 9 +9) = \$27

**Question 8** **Answer E**

Equation joining (-1, 0) and (0, 1) is $y = x + 1$

Equation joining (0, 6) and (4, 0) is $y = -\frac{3}{2}x + 6$ , i.e. $2y + 3x = 12$

Solving simultaneously (sub equation 1 in equation 2) $2(x+1) + 3x = 12$

$$2x + 2 + 3x = 12$$
$$5x = 10$$
$$\therefore x = 2$$

sub $x = 2$ in $y = x + 1$, gives $y = 3$

Point B is (2, 3)

Each point A, B and C is substituted in the objective functions listed.

| Objective Function | A (0,1) | B(2, 3) | C(0, 6) | Maximum Point |
|---|---|---|---|---|
| $M = x + 3y$ | $M = 0 + 3 \times 1 = 3$ | $M = 2 + 3 \times 3 = 11$ | $M = 0 + 3 \times 6 = 18$ | C |
| $M = x - 3y$ | $M = 0 - 3 \times 1 = -3$ | $M = 2 - 3 \times 3 = -7$ | $M = 0 - 3 \times 6 = -18$ | A |
| $M = x + y$ | $M = 0 + 1 = 1$ | $M = 2 + 3 = 5$ | $M = 0 + 6 = 6$ | C |
| $M = x + 2y$ | $M = 0 + 2 \times 1 = 2$ | $M = 2 + 2 \times 3 = 8$ | $M = 0 + 2 \times 6 = 12$ | C |
| $M = 2x + y$ | $M = 2 \times 0 + 1 = 1$ | $M = 2 \times 2 + 3 = 7$ | $M = 2 \times 0 + 6 = 6$ | B |

**Question 9** **Answer C**

$y = \frac{k}{x^2}$ where on the linearised graph $k = \text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{24}{2} = 12$

The graph has the equation $y = \frac{12}{x^2}$

When $x = 2$, $y = \frac{12}{2^2} = \frac{12}{4} = 3$ and when $x = 1$, $y = \frac{12}{1^2} = 12$

So the original graph contains the points (2, 3) and (1, 12)

**END OF MODULE 3 SOLUTIONS**

## Module 4: Business related mathematics

**Answers**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | **D** | **2.** | **C** | **3.** | **D** | **4.** | **C** | **5.** | **B** |
| **6.** | **E** | **7.** | **B** | **8.** | **D** | **9.** | **D** | | |

## Solutions

**Question 1** **Answer D**

| Credit | Debit | Balance |
|---|---|---|
| | | 360.22 |
| 1450.60 | | 1810.82 |
| | 2000.0 | -189.18 |
| 900.00 | | 710.82 |
| | 26.50 | 684.32 |

**Question 2** **Answer C**

$$\text{Original price} = \frac{\$550}{1.1}$$
$$= \$500$$
$$\therefore \text{GST} = \$50$$

**Question 3** **Answer D**

Price increase = 2.65 – 0.90
= \$1.75 over 8 yrs

$$\%\text{increase} = \frac{1.75}{0.90} \times \frac{100}{1}$$
$$= 194.44\%$$

$$\text{Average annual inflation rate} = \frac{194.44}{8}\%$$
$$= 24.31\%\text{p.a.}$$

**Question 4** **Answer C**

$$P = \frac{100Q}{r}$$
$$= \frac{100 \times 2000}{5}$$
$$= \$40\,000$$

**Question 5** **Answer B**

$A = PR^n$ where

$$R = 1 + \frac{7.6}{400}$$
$$= 1.019$$

A = 10000 x $1.019^{32}$

= $18,263.07 (using compound interest)

∴ Interest = $ 8,263.07

$$\text{Simple Interest} = \frac{\text{Pr T}}{100}$$
$$= \frac{1000 \times 9 \times 8}{100}$$
$$= \$7200$$

∴ Compound interest better by $1063.07

**Question 6** **Answer E**

$$\text{Bookvalue} = \text{PR}^{\text{T}}$$
$$= 7000 \times 0.88^{\text{T}}$$
$$7000 \times 0.88^{\text{T}} > 1000$$
$$0.88^{\text{T}} > \frac{1000}{7000}$$
$$\text{T} \times \log(0.88) > \log\left(\frac{1}{7}\right)$$
$$\text{T} > 15.2$$
$$\therefore \text{T} = 16 \text{ years}$$

**Question 7** **Answer B**

$$\text{R} = 1 + \frac{\text{r}}{100\text{n}}$$
$$= 1 + \frac{6}{100 \times 12}$$
$$= 1.005$$

Loan for 6 yrs compounded monthly

∴ n = 72

**Question 8** **Answer D**

Cost Price = $25,000

Depr. = $12^{c}$ / ream (500 sheets)

Usage = 20 million sheets

= 20 000 000/500

= 40 000 reams

Depr. Cost = 40 000 x 0.12

= $4800

Book value = 25 000 - 4800

= $20 200

**Question 9** **Answer D**

Cost price = \$9900
Deposit = \$1500
Payments = \$425/month for 3 yrs
Total payment = 1500 + 425 x 36
= \$16 800
Interest paid = 16 800 – 9900
= \$6900

$$\text{Interest rate} = \frac{100\text{I}}{\text{PT}}$$
$$= \frac{100 \times 6900}{8400 \times 3}$$
$$= 27.38\%\text{p.a.}$$

$$\text{Effective rate} = \frac{2\text{n}}{\text{n}+1} \times \text{flat rate}$$
$$= \frac{2 \times 36}{36+1} \times 27.38\%$$
$$= 53.28\%$$

**END OF MODULE 4 SOLUTIONS**

## Module 5: Networks and decision mathematics

**Answers**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | **B** | **2.** | **B** | **3.** | **C** | **4.** | **D** | **5.** | **A** |
| **6.** | **E** | **7.** | **D** | **8.** | **B** | **9.** | **D** | | |

## Solutions

**Question 1** **Answer B**

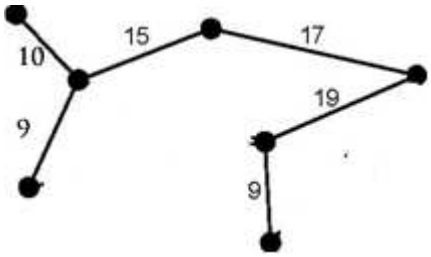
3 enclosed areas + 1 infinite space = 4 faces

**Question 2** **Answer B**

The bipartite graph presents both media and history nominated three times each more than any other subjects as given by option B.

**Question 3** **Answer C**

The minimum spanning tree is as shown and so the minimum weight is
10 + 15 + 9 + 17 + 19 + 9 = 79




**Question 4** **Answer D**

Team A defeated teams B, C and D. (0,1,1,1)
Team B defeated team D. (0,0,0,1)
Team C defeated team B. (0,1,0,0)
Team D defeated team C. (0,0,1,0)

The 1st dominance matrix becomes

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{bmatrix} & & & \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

**Question 5** **Answer A**

Euler circuit = only even degree on all vertices. $\therefore$ A

**Question 6** **Answer E**

| | A | B | C | D |
|---|---|---|---|---|
| J | 2 | 4 | 5 | 5 |
| A | 3 | 4 | 7 | 6 |
| R | 4 | 6 | 6 | 3 |
| W | 4 | 3 | 2 | 3 |

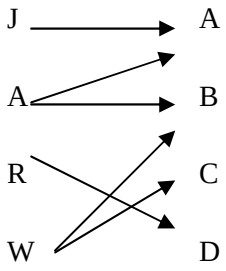
Subtract minimum value from each row

| | A | B | C | D |
|---|---|---|---|---|
| J | 0 | 2 | 3 | 3 |
| A | 0 | 1 | 4 | 3 |
| R | 1 | 3 | 3 | 0 |
| W | 2 | 1 | 0 | 1 |

Subtract minimum value for column with no "0" entry

| | A | B | C | D |
|---|---|---|---|---|
| J | 0 | 1 | 3 | 3 |
| A | 0 | 0 | 4 | 3 |
| R | 1 | 2 | 3 | 0 |
| W | 2 | 0 | 0 | 1 |

So we have:

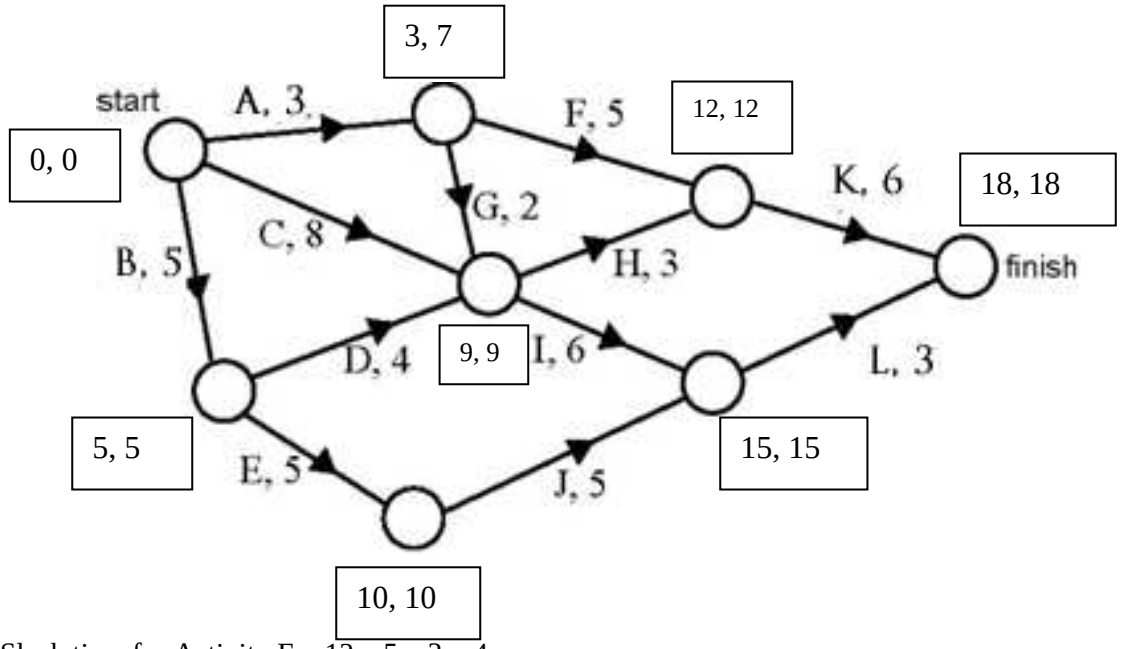



**Question 7** **Answer D**

Refer to network in Question 8. Minimum time to complete the project is 18 hrs.

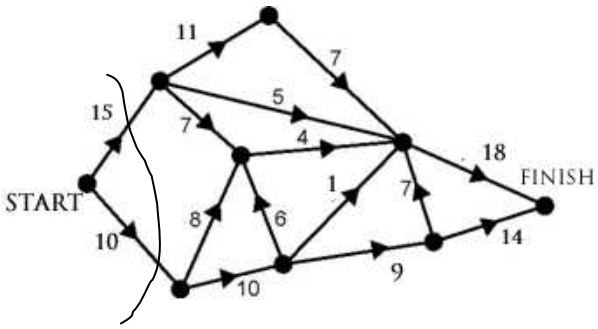
**Question 8** **Answer B**




Slack time for Activity F = 12 – 5 – 3 = 4

**Question 9** **Answer D**

Use the minimum cut to find the maximum flow of 25




Maximum flow is 15+10 = 25 through the network given.

**END OF MODULE 5 SOLUTIONS**

## Module 6: Matrices

**Answers**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | **B** | **2.** | **B** | **3.** | **E** | **4.** | **D** | **5.** | **E** |
| **6.** | **C** | **7.** | **C** | **8.** | **D** | **9.** | **C** | | |

## Solutions

**Question 1** **Answer B**

Order of matrix A is (3 x 2)
Order of matrix B is (1 x 3)
So the product is only defined for BA.
= (1 x 3)x(3 x 2)
$\Rightarrow$ ( 1 x 2 )

**Question 2** **Answer B**

$-9 + b = 2, \quad a + 2 = 0$
$\therefore b = 11 \quad \therefore a = -2$

**Question 3** **Answer E**

For singular matrix the determinant = 0

$$\begin{bmatrix} 6 & -3 \\ -10 & 5 \end{bmatrix}$$

Determinant = ad - bc
= 30 - 30
= 0

**Question 4** **Answer D**

$$\begin{bmatrix} 550 & 60 \\ 750 & 120 \\ 990 & 150 \end{bmatrix} \times \begin{bmatrix} 0.9 & 0 \\ 0 & 1.1 \end{bmatrix} \Rightarrow \begin{bmatrix} 550\times 0.9 & 60\times 1.1 \\ 750\times 0.9 & 120\times 1.1 \\ 990\times 0.9 & 150\times 1.1 \end{bmatrix}$$

Reduce by 10% - use 0.9.
Increase by 10% - use 1.1
So **A and B** are out.
**C** is not defined.
**E** gives the combined new prices.
**D** gives matrix showing new prices separate.

**Question 5** **Answer E**

Addition of matrices can be done in any order.

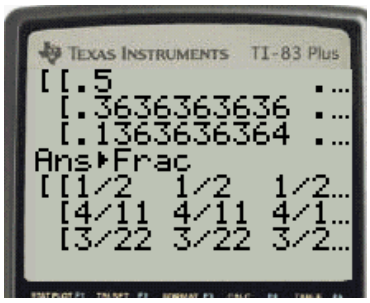
 

**Question 6** **Answer C**

For steady state $n$ is set as a large number say 50. Using graphics calculator gives

$$\begin{bmatrix} 0.8 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.7 \\ 0.1 & 0.2 & 0.1 \end{bmatrix}^{50} = \begin{bmatrix} 1/2 & 1/2 & 1/2 \\ 4/11 & 4/11 & 4/11 \\ 3/22 & 3/22 & 3/22 \end{bmatrix}$$

Note: Answer B is also correct but it needed to be raised to a larger power and as the **best** answer is required then option C is required.



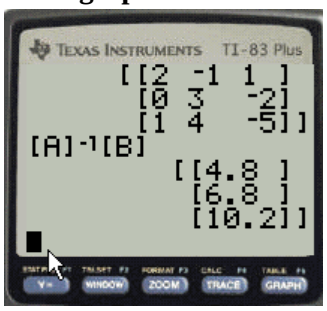

**Question 7** **Answer C**

$$\begin{bmatrix} 2 & -1 & 1 \\ 0 & 3 & -2 \\ 1 & 4 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 0 \\ -19 \end{bmatrix}$$

$$A \times X = B$$

$$X = A^{-1}B$$

**Use a graphics calculator to evaluate as follow.**




Gives $x = 4.8 \;\; y = 6.8 \;\; z = 10.2$

**Question 8** **Answer D**

$$\begin{bmatrix} 0.7 & 0.15 & 0.1 & 0.05 \\ 0.1 & 0.7 & 0.15 & 0.05 \\ 0.15 & 0.1 & 0.7 & 0.05 \\ 0.05 & 0.05 & 0.05 & 0.85 \end{bmatrix}^{2} \begin{bmatrix} 330 \\ 400 \\ 350 \\ 120 \end{bmatrix} = \begin{bmatrix} 329.975 \\ 352.125 \\ 333.1 \\ 184.8 \end{bmatrix}$$

Use a graphics calculator to evaluate as follows.

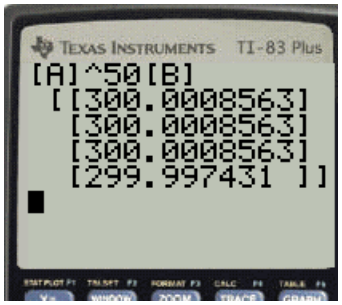

$\therefore$ total = 329.975 + 184.8

= 514.775

$\Rightarrow$ 515 students will be expected to do Number applications and Matrices modules.

**Question 9** **Answer C**

For long term set *n* to a large number say 50.




So the rounded values gave the result that **all** modules will have equal number of students.

**END OF SOLUTIONS**