Mathematical Association of Victoria
Trial Examination 2009

STUDENT NAME ____

FURTHER MATHEMATICS

Written Examination 2

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Core			
	Number of questions	*Number of questions to be answered*	*Number of marks*
	5	5	15
Module			
	Number of modules	*Number of modules to be answered*	*Number of marks*
	6	3	45
			Total 60

• Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
• Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
• Question and answer book of 31 pages, with a detachable sheet of miscellaneous formulas at the back.
• Working space is provided throughout the book.

Instructions
• Detach the formula sheet from the back of this book during reading time.
• Write your **student name** in the space provided above on this page.
• All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π, surds or fractions.
Diagrams are not to scale unless specified otherwise.

TURN OVER

Core

Question 1

Allen Key is a keen hobby tomato grower. His most recent crop of 11 tomatoes was weighed (in grams) and is listed as follows.

85 74 110 98 97 92 88 105 101 96 82

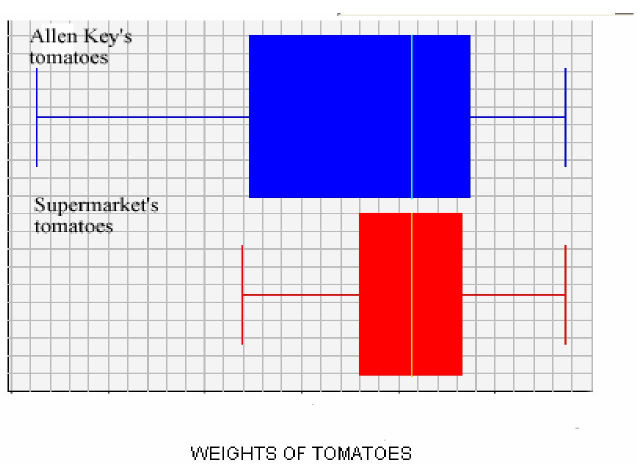
a. Complete the stemplot by recording the values of the weights of the 11 tomatoes.

<u>Stemplot of Allen Key's 11 tomato plants' weights in grams</u>

STEM	LEAF

2 marks

Alan was given a boxplot display of his crop of tomatoes compared to those from a large supermarket as follows.



b. From the above boxplots state the **two most noticeable** differences.

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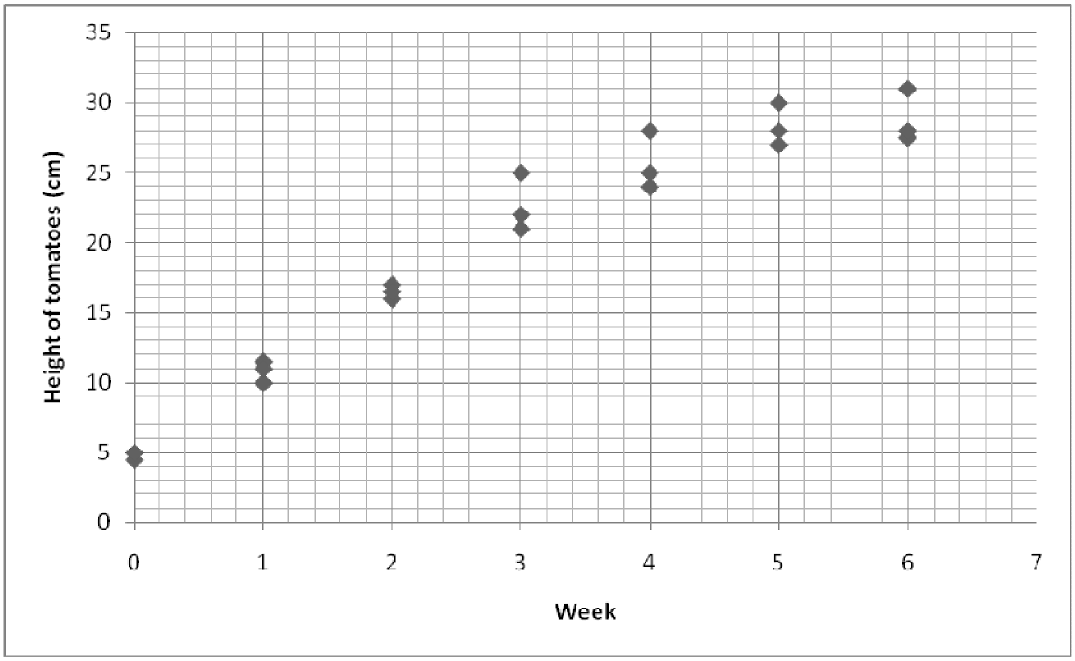
2 marks

Core – continued

Question 2

Allen recorded the heights of three tomato plant seedlings over a six week period and plotted them on a scatterplot as shown below.

Scatterplot of Heights of 3 tomatoes plants vs. Weeks since seedlings planted



a. **Draw** a suitable **linear** line of best **fit by eye**.

2 marks

b. Show with appropriate workings, the linear equation of the line drawn using the variable names **plant height** and **weeks.**

3 marks

c. A **least squares regression** analysis gave a correlation coefficient of $r = 0.45$ which Allen Keys interpreted as being a **weak positive relationship** between height of tomato plants and weeks. Explain what the term **positive** means in the context of this bivariate analysis.

1 mark

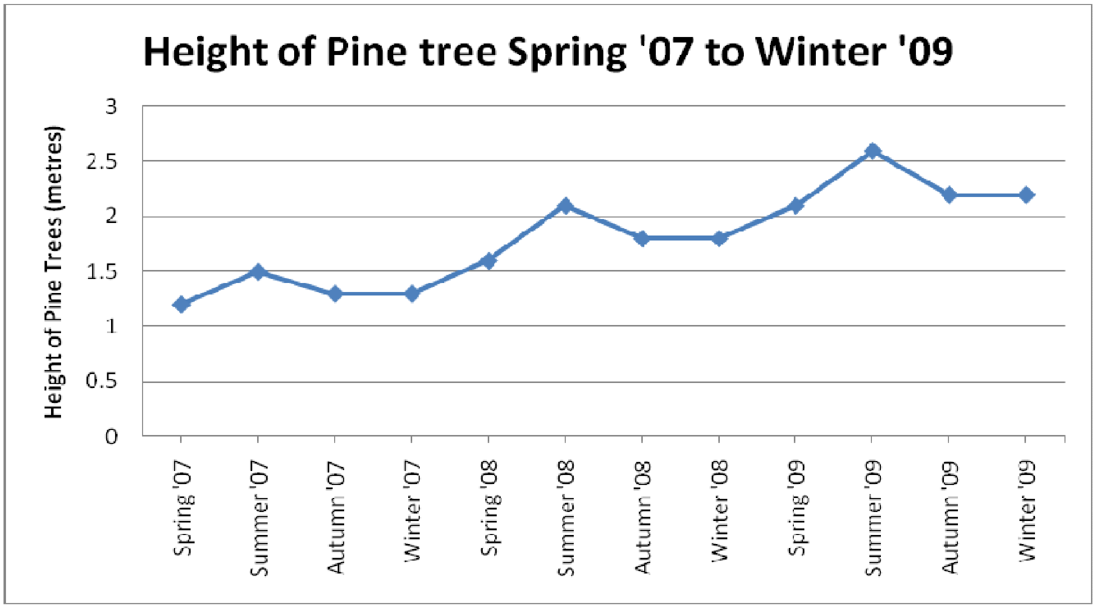
d. Given the low r value and thus a very low r^2 value, what type of transformation of **plant height** data would be suitable for the above scatterplot?

1 mark

Core – continued **TURN OVER**

Question 3

Moe Fitzy, another keen grower recorded the height of a pine tree over a period of three years since originally planting the seedling tree at the end of Winter 2007. The time series plot below recorded the height of the pine tree at the **end of each season** which included the pruning of the tree each autumn.



Moe Fitzy calculated the seasonal indices of each of the seasons and arrived at four values but only recorded three of the values as

0.90 0.98 1.14

a. Complete the table below as a summary of the seasonal indexes.

Season	Spring	Summer	Autumn	Winter
Seasonal index	0.90			

2 marks

The equation of the least squares regression line of the deseasonalised data is

$$\text{deseasonalised plant height (m)} = 1.1 + 0.1 \times \text{time period}$$

b. In the equation the value 1.1 can be interpreted as ________________________________

1 mark

c. In the equation the value 0.1 can be interpreted as ________________________________

1 mark

Total 15 marks

END OF CORE

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TURN OVER

Module 1: Number Patterns

Question 1
Tammy is learning to drive. The distance she travels during her first three lessons is shown in the table below.

Lesson	1	2	3
Distance (kilometres)	3	4.5	6

If this pattern continues

a. Find the distance Tammy travels during her 4^{th} driving lesson.

1 mark

b. Show that the distance travelled follows an arithmetic sequence.

1 mark

c. Determine the total distance that Tammy has driven in 10 lessons. Give your answer in kilometres correct to one decimal place.

1 mark

d. Tammy must complete a total of 120 km to be eligible for her driver's licence. After which lesson would she be eligible?

1 mark

Module 1: Number patterns - continued

Question 2

Mark, the driving instructor, has found that the average number of errors his students make each lesson generally decreases by 8% a lesson.

Mark observes that his students make an average of 20 errors during their first lesson.

a. How many errors on average do the students make in their second lesson? Give your answer correct to 1 decimal place.

1 mark

b. The average number of errors, E_n, made for the n^{th} lesson is specified by the difference equation

$$E_{n+1} = aE_n, \quad \text{where } E_1 = 20$$

Show that $a = 0.92$

1 mark

c. A student is ready for their driver's licence test when less than 4 errors are made on average. In which lesson would this first occur?

1 mark

d. Write an expression that will determine the total number of errors made for *n* lessons.

1 mark

e. A student completed 28 lessons. Find the total number of errors made during the last 5 lessons. Give your answer correct to the nearest whole number.

2 marks

Module 1: Number patterns – continued
TURN OVER

Question 3

It is found that the number of probationary (P plate) drivers in Melbourne during the month of January 2009 is 4000. Each month thereafter the number of probationary drivers increase by 6% and a total of 180 either lose their licence or are promoted to experienced drivers.

The number of P plate drivers in Melbourne, M_n, for the n^{th} month is specified by the difference equation

$$M_{n+1} = 1.06M_n - 180 \quad \text{, where } M_1 = 4000$$

a. Find the number of probationary drivers in April 2009. (When $n = 4$)

1 mark

b. Show that the number of probationary drivers at the beginning of each month does not follow an arithmetic or a geometric sequence.

1 mark

c. Instead of the 6% increase, what percentage increase would result in the number of probationary drivers remaining stable, that is, so that there are 4000 P plate drivers every month?

1 mark

The number of probationary (P plate) drivers in Sydney during the n^{th} month can be specified by the second order difference equation

$$S_{n+2} = 1.10S_{n+1} - 0.12S_n - 80 \text{ where } S_1 = 4800,\ S_2 = 4840$$

where S_1 represents number of P plate drivers in Sydney for January 2009

d. Find the number of probationary drivers in April 2009.

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____

____

1 mark

e. Determine when the number of P plate drivers in Melbourne will first exceed the number of P plate drivers in Sydney.

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1 mark

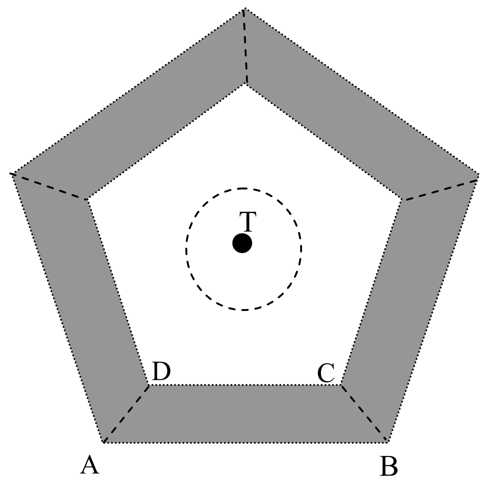
Total 15 marks

END OF MODULE 1
TURN OVER

Module 2: Geometry and trigonometry

Five bench seats are combined to form the shape of a regular pentagon. These seats are to be fixed around an old tree located near a riverbank.

The diagram below shows the pentagonal bench seat (shaded) centred around the tree trunk.



Question 1

a. Show that the angle at the centre, $\angle ATB$, is 72º.

1 mark

The length of AT is 2.55 metres.

b. **i.** Find the length of AB to the nearest metre.

2 marks

ii. Find the length of AD, if the length ratio of AD:AT is 11:51.
Give your answer in metres correct to 2 decimal places.

1 mark

c. The length of TC is 2 metres.

i. Find the area of the triangle TCD. Give your answer in square metres correct to three decimal places.

2 marks

ii Hence, determine the area of the top of the five bench seats. (i.e. The shaded region) Give your answer in square metres correct to two decimal places.

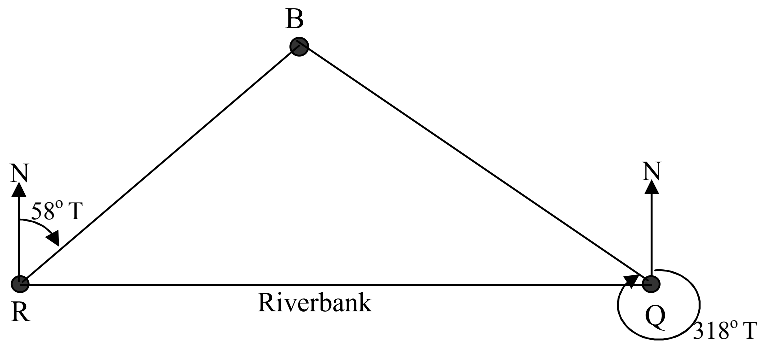
2 marks

Question 2

A woman in a canoe travelling along a nearby river at the point, Q, observes a man sitting on the pentagonal bench, B, on a bearing 318° T.

The woman in the canoe travels a further 12 metres in a Westerly direction and observes the man from a point, R, on a bearing 58° T.

This information is shown on the diagram below.



a. Show that the magnitude of angle RBQ is 100°.

1 mark

b. Find the distance, in metres, correct to one decimal place from the point Q to B.

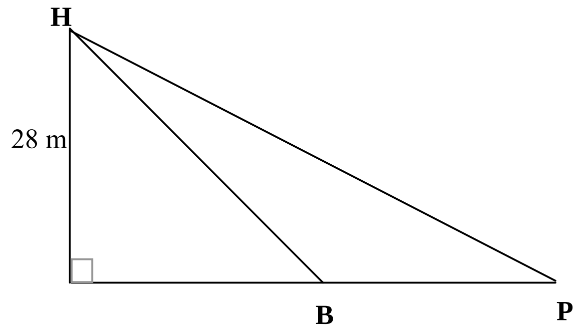
2 marks

c. Find the shortest distance the man at the bench, B, will need to walk to reach the bank of the river. Give your answer in metres correct to two decimal places.

1 mark

A hiker, H, on the top of a nearby hill also sights the same man sitting on the pentagonal bench B at an angle of depression of 43^{o}. He looks further, beyond the man, and observes a pelican, P, on the opposite bank of the river at an angle of depression of 26^{o}.

The hiker is 28 metres above the bank of the river.



d. **i**. Calculate the distance BP, in metres correct to two decimal places.

2 marks

ii. Hence, find the width of the river, in metres correct to two decimal places.

.

1 mark

Total 15 marks

END OF MODULE 2
TURN OVER

Module 3: Graphs and relations

Question 1

Zoe decides to organize an indoor soccer competition every Friday night. There is a $55 entry fee for each team each night they play.

a. Write an equation that gives the total amount, R dollars, collected each night when there are n teams entering in the competition.

1 mark

b. The Gym hire costs Zoe $240 for a night plus $5.00 per team that plays a game.

i. Write an equation that gives the total cost each night, C, in dollars, of the competition when there are n teams playing.

1 mark

ii. Determine the number of teams required for Zoe to break even.

1 mark

c. How many teams entered the competition if $910 profit was made?

1 mark

Question 2
Zoe enters a number of the indoor soccer competitors to try out for a position in the state team.

The state coach is interested in two main criteria.

- The time, in seconds, to run 100m (x) and
- The point average score for ball control (y).

To aid in the selection process a list of constraints are given.

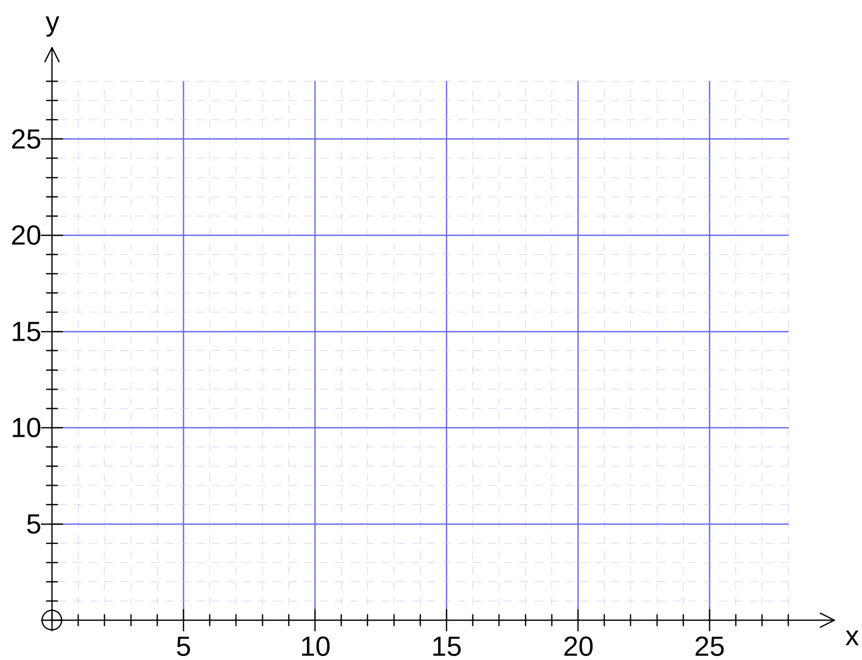
Inequality 1; $x \le 18$

Inequality 2: $y \ge 12$

Inequality 3: $2y - 2.4x \ge 3$

a. Explain the meaning of **Inequality 1** in terms of the context of the problem.

1 mark



b. On the graph above,

i. **Draw** and **label** the lines $x = 18$, $y = 12$ and $2y - 2.4x = 3$

ii. Clearly shade the feasible region defined by **Inequalities 1 to 3**.

3+1 = 4 marks

Module 3: Graphs and relations - continued
TURN OVER

c. A particular competitor scores 14 points for ball control. What is the slowest time she can run in order to satisfy the state team selection criteria?

1 mark

d. In addition to the state squad, an elite squad was also selected to represent the national team.

These elite competitors must also satisfy the inequality $y - 4x \geq 0$

Zoe also trialled and the coach gave her a score of (6, 23). Determine whether she made the state squad or both the elite and state squad, justify your answer.

1 mark

Question 3
Zoe has decided to host the annual regional competition. To cover costs she must charge an entrance fee to spectators. The amount of the fee depends on the number of spectators.

Spectators (S)	400	350	280	250	195
Entrance Fee (F)	\$1.75	\$2.00	\$2.50	\$2.80	

The entrance fee, F, is given by the relationship $F = kS^n$, where S is the number of spectators.

a. Find the value of n and k

2 marks

b. Find the Entrance Fee (to the nearest 10 cents) for 195 spectators and complete the table.

1 mark

c. How many spectators will attract an entrance fee of \$4.00?

1 mark

Total 15 marks

END OF MODULE 3
TURN OVER

Module 4: Business-related mathematics

Question 1

A community primary school is purchasing a small 12 seater school bus. As it is a non-profit organisation it is exempted from paying the GST. The 12-seater bus has a GST of $3800.

a. Find the retail price of the 12 seater bus.

1 mark

The owner of the car dealership has a child at the school and offers the bus for a special price of $33 000. He offers a hire purchase loan with $3000 deposit and 3% pa with monthly repayments over three years.

b. Calculate the monthly payment to the nearest cent.

2 marks

c. Calculate the effective interest rate for the hire purchase agreement (to one decimal place).

1 mark

Module 4: Business-related mathematics - continued

The stamp duty on used cars in Victoria summarised in the table below.

State Revenue Office of Victoria – Stamp Duty Schedule for Transfer of Vehicle Ownership			
Dutiable Value of Vehicle	**New Vehicle**		**Used Vehicle**
	Non Passenger (commercial)	**Passenger**	
Less than $35 000	$5 per $200 or part thereof	$5 per $200 or part thereof	$8 per $200 or part thereof
Exceeding $35 000 But not over $45 000	$5 per $200 or part thereof	$8 per $200 or part thereof	$8 per $200 or part thereof
Exceeding $45 000	$5 per $200 or part thereof	$10 per $200 or part thereof	$8 per $200 or part thereof

d. Calculate the stamp duty on the $33 000 commercial school bus.

1 mark

Question 2

An alternative is for the school to take out a loan from the state education department. The current rate is 5% pa compounded monthly with monthly regular payments.

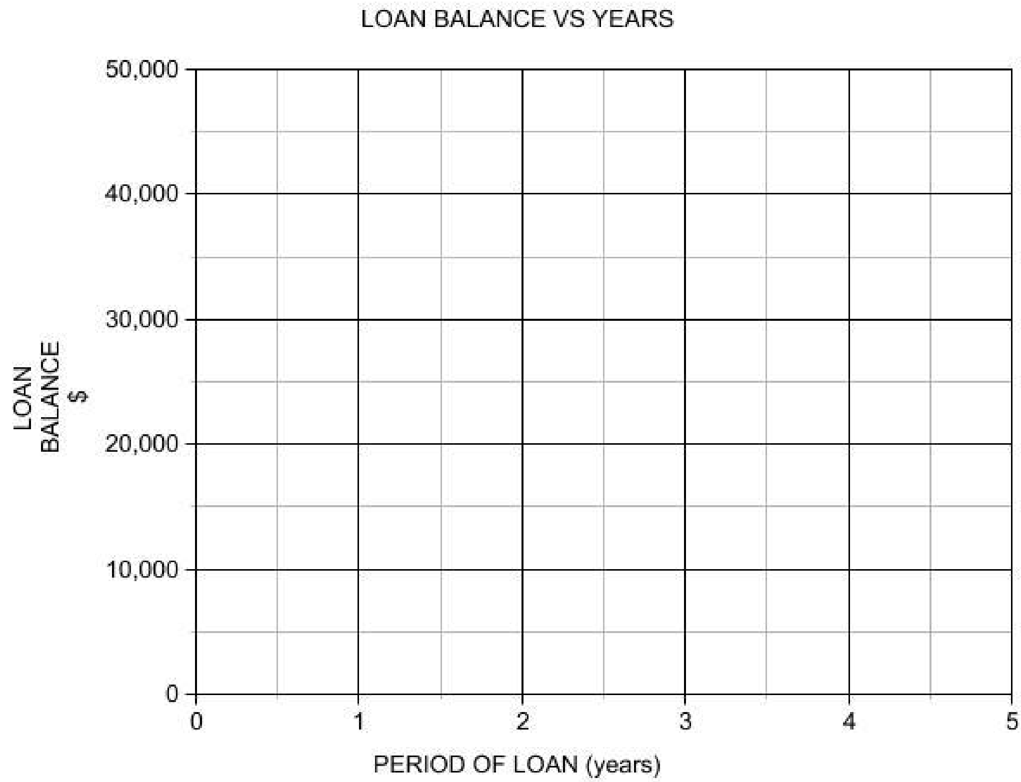
a. Explain why the 5% pa compound interest loan is preferable compared to the hire purchase plan (in Question 1) at 3% pa flat rate.

1 mark

b. Calculate the monthly payment for a 5% pa compound interest loan of $33 000 to be paid in full over 3 years paid in equal monthly instalments.

1 mark

c. On the graph below **sketch** a graph of the loan balance of the hire purchase loan arrangement compared to the state education compound interest annuity loan. Clearly label each of the curves as HIRE PURCHASE and COMPOUND.



2 marks

Module 4: Business-related mathematics - continued

Question 3
The school needs to consider which method of depreciation to report to the school board and its finance committee.

One option is a **reducing balance** model. If the value of the bus is to be depreciated from its purchase price of \$33 000 to a scrap value of \$8000 over 5 years

a. Calculate the **reducing balance** rate of depreciation as a percentage correct to one decimal place.

2 marks

b. What is the bookvalue of the bus after 2 years (to the nearest dollar)?

1 mark

c. Another method is to depreciate using the Unit Cost Depreciation method. If in the 5 years it is expected to have travelled 80 000 kilometres, calculate the rate of depreciation if it will be depreciated from \$33 000 down to \$8000. Give your answer in cents per 100 kilometres.

2 marks

Question 4
With the \$8000 from the sale of the bus after it is scrapped, a scholarship fund is to be set up. A scholarship amount of \$1200 is to be paid annually from the \$8000 and will be set up with the local bank offering a guaranteed interest rate of 4%pa compounded annually. For how many years will the scholarship be able to be granted before the funds are exhausted?

1 mark

Total 15 marks

END OF MODULE 4
TURN OVER

Module 5: Networks and decision mathematics

Question 1

With the prolonged drought period and water shortage, a new network of pipes from an old reservoir (node A) which has been reopened, is connected to several small towns (nodes B, C, D and E) and finally a regional city (node F). **Table 1** below shows the network of pipes joining the towns and city from the reservoir.

From	To	Flow Capacity (megalitres)
A (reservoir)	B	1000
A (reservoir)	C	600
A (reservoir)	D	800
B	E	1100
C	E	400
C	F (regional city)	100
D	F (regional city)	600
E	F (regional city)	1600

Table 1

a. Using the flow Table 1, complete the unfinished network diagram below which has some of the flow (edges) missing.

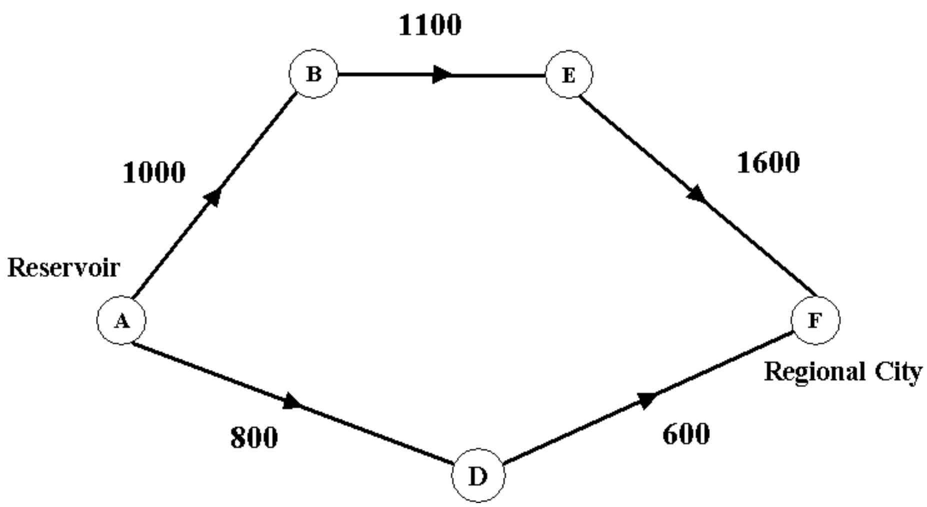


Figure 1.

2 marks

b. Comparing the flow from the reservoir (A) and the flow into the regional city (F), what is the maximum possible flow?

___

1 mark

Module 5: Network and decision mathematics – continued

c. Using the minimum cut method on the network from completed **Figure 1**, determine the actual maximum flow capacity. Use the **Figure 1.** diagram below.

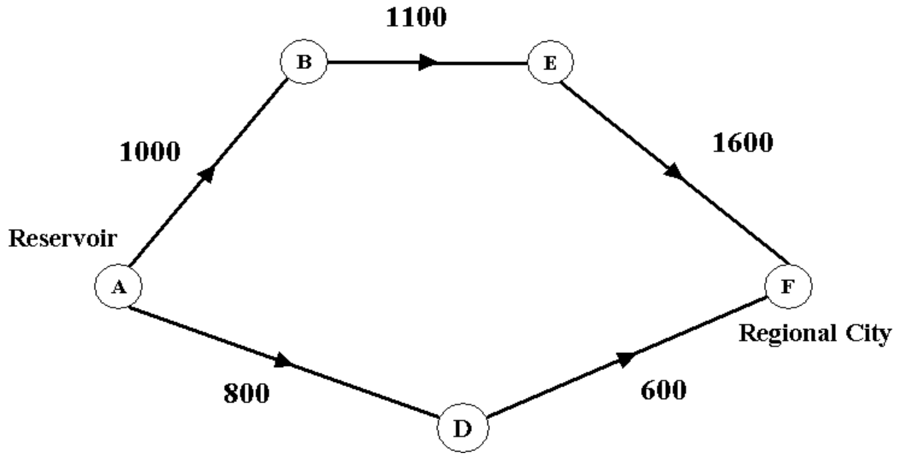


Figure 1.

1 mark

Question 2

To reopen the old reservoir some major earthworks and pipe laying are required. The project requires 10 major activities. The network diagram below gives the completion time **in weeks** and the immediate predecessors for each of these activities.

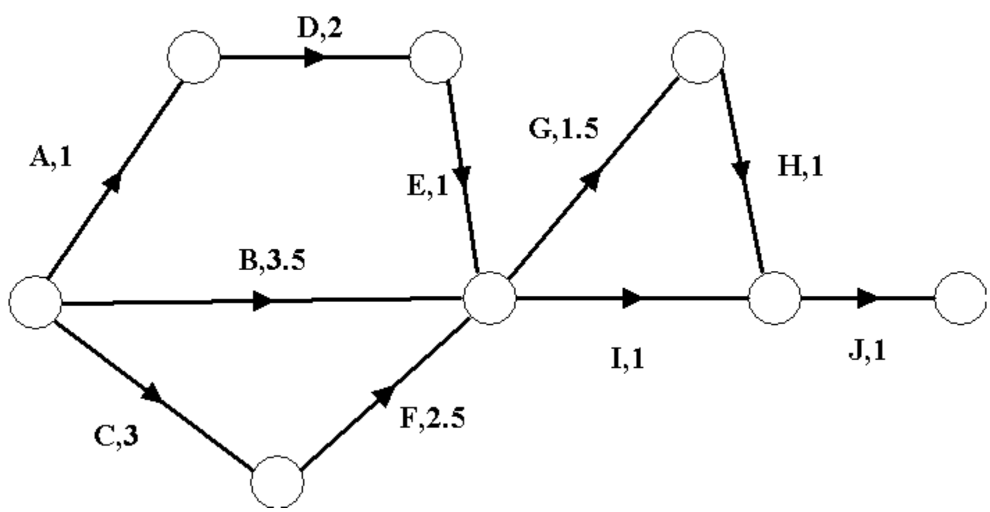


Figure 2.

a. Using the network diagram in **figure 2**, determine the earliest completion time for the whole project.

1 mark

b. State the critical path.

1 mark

c. Write down the float times for the non-critical activities A and D in weeks.

2 marks

Question 3

Seven worksites are set up for the reopening program for the reservoir and are placed nearby the reservoir and the five towns and regional city as shown in the undirected network diagram shown below in **Figure 3.**

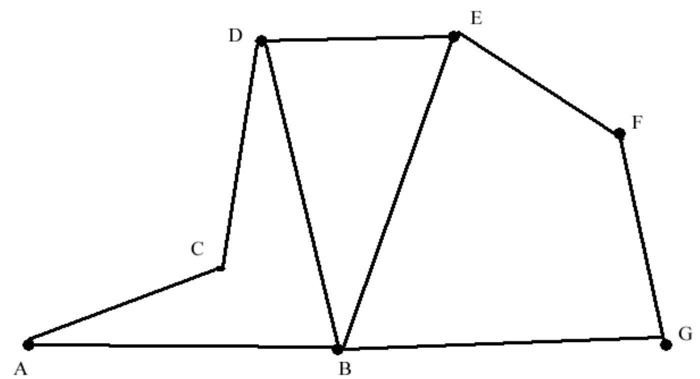


Figure 3

a. Which vertex or vertices could be the start of an Euler path?

1 mark

b. Using your answer from **part a.** explain which vertex could be the end of an Euler path.

1 mark

c. Determine an Euler **path**.

1 mark

d. Which edge would need to be removed to obtain an Euler circuit?

1 mark

e. A courier is to make a daily visit to each site. State whether it is a **circuit** or **path** and whether it should be **Euler** or **Hamiltonian** as the preferred delivery route. **Justify** your choice**.**

3 marks

Total 15 marks
END OF MODULE 5
TURN OVER

Module 6: Matrices

Question 1

FM Cruises sail several 7-day cruises from Brisbane to the South Pacific islands. They have set prices for four types of rooms/cabins (stateroom, balcony room, outer-cabin and inner-cabin). There are three deck level types (upper decks, middle decks and lower decks). The set prices are summarized in **Table 1** below.

Table 1

	Stateroom	Balcony room	Outer cabins	Inner cabins
Upper Decks	\$3200	\$2600	\$1700	\$1400
Middle Decks	\$2800	\$2400	\$1600	\$1300
Lower Decks	\$2400	\$2100	\$1400	\$1200

a. A special is offered for bookings made 12 months in advance. All prices are discounted by 50%. Show a suitable scalar matrix equation that would give the appropriate 3×4 matrix of the sale price.

$$= \begin{bmatrix} 1600 & 1300 & 850 & 700 \\ 1400 & 1200 & 800 & 650 \\ 1200 & 1050 & 700 & 600 \end{bmatrix}$$

1 mark

b. Within 6 weeks of the date of sailing all un-booked rooms and cabins have their prices quoted in **Table 1** discounted as shown below.

- All state rooms are discounted to 90% of original price
- All Balcony rooms are discounted to 80% of original price
- All Outer cabins are discounted to 60% of original price
- All Inner cabins are discounted to 50% of original price

Represent the rates of discount as a suitable 4×4 matrix

1 mark

c. **Using matrices**, calculate the new discounted prices for the un-booked rooms and cabins.

1 mark

Module 6: Matrices - continued

d. From your answer in **Part c**. state the element's position in the matrix and the value of the discounted price for an unbooked **outer middle deck cabin.**

____

1 mark

Question 2

Marvey Travel Agency has recently made bookings for three sporting groups for a Rhine river cruise. All groups were booked in the Upper decks in a stateroom, balcony room or an inner cabin. Each sporting group was billed for the total amount as summarised in **Table 2**.

Table 2

Group	Stateroom	Balcony	Inner cabin	Total Group Cruise Price $
Football Group	2	5	10	44 000
Netball Group	0	4	5	22 000
Tennis Group	3	4	1	27 500

a. Set up three simultaneous equations to find the room/cabin prices, using the following

x = stateroom price
y = balcony prices
z = inner cabin prices

First Equation: ____

Second Equation: ____

Third Equation: ____

2 marks

b. Represent the simultaneous equations as matrices and **calculate** and **state** the prices of all three types of rooms/cabins.

____

____

2 marks

Module 6: Matrices - continued
TURN OVER

Question 3

A tourism inquiry into types of holidays focused on cruises, tours and resorts. Market research suggests that people will have a preferred type of holiday but there will be some changes as follows

- 90% of tourists who **cruised** had gone on a **cruise** on their next holiday.
- 8% of tourists who **cruised** had gone on a **tour** on their next holiday.
- 70% of tourists who **toured** had gone on a **tour** on their next holiday.
- 20% of tourists who **toured** had gone on a **cruise** on their next holiday.
- 60% of tourists who went to a **resort** had gone to **resort** on their next holiday.
- 30% of tourists who went to a **resort** had gone on a **tour** on their next holiday.

a. Enter this information into transition matrix *T* as indicated below (express percentages as proportions).

$$\begin{array}{ll} & \textbf{This holiday} \\ & \begin{array}{ccc} \text{Cruise} & \text{Tour} & \text{Resort} \end{array} \\ \textbf{Next Holiday} \begin{array}{l} \text{Cruise} \\ \text{Tour} \\ \text{Resort} \end{array} & \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} \end{array}$$

1 mark

In 2008 there were a total of 300 000 tourists of whom 50 000 went on cruises, 100 000 went on a tour and the remainder went to a resort.

b. Write this information as a column matrix, H_{2008}.

1 mark

c. Use T and H_{2008} to **write** and **evaluate** a matrix product that determines the number of tourists expected at each of the types of holidays during the **2009 year**.

2 marks

d. Show by calculating at least two appropriate state matrices that, in the long term, the number of tourists expected on cruises is 190 141, on tours is 80 282 and on resorts is 29 577.

2 marks

Total 15 marks

END OF MODULE 6

END OF QUESTION AND ANSWER BOOK

Exam 1 & 2 Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET