# INSIGHT

*Trial Exam Paper*

# 2010

# FURTHER MATHEMATICS

## Written examination 1

# *Solutions book*

**This book presents:**

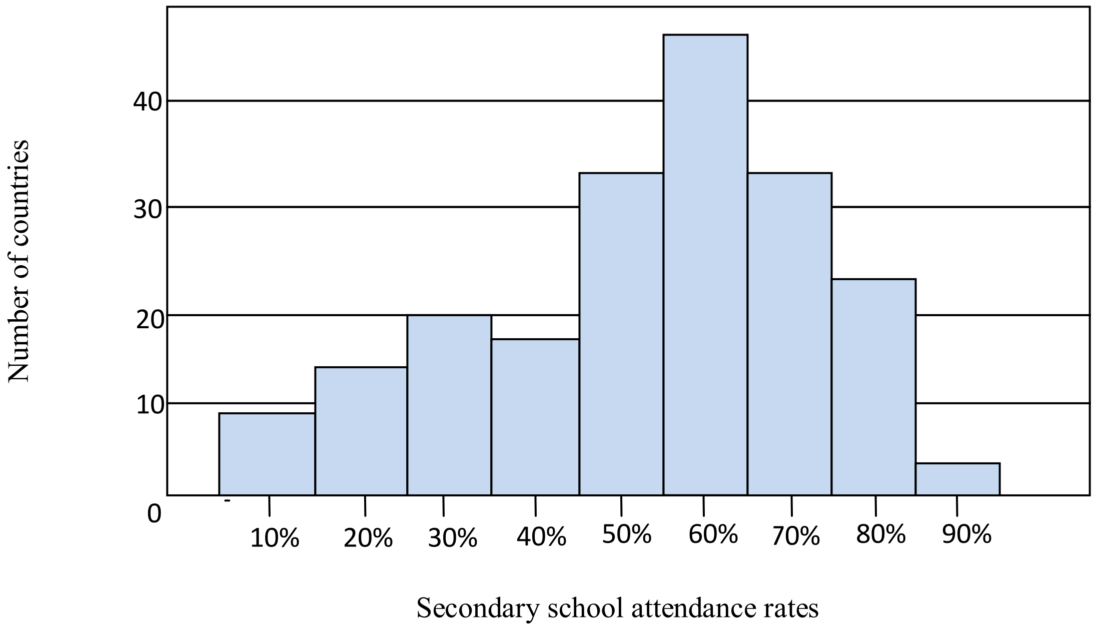
- worked solutions, giving you a series of points to show you how to work through the questions.
- tips and guidelines

## SECTION A

## Core: Data analysis

*The following information relates to Questions 1 to 3.*

The histogram below shows the distribution of secondary school attendance rates (in percentages) for 200 countries between the years 2000 and 2007.



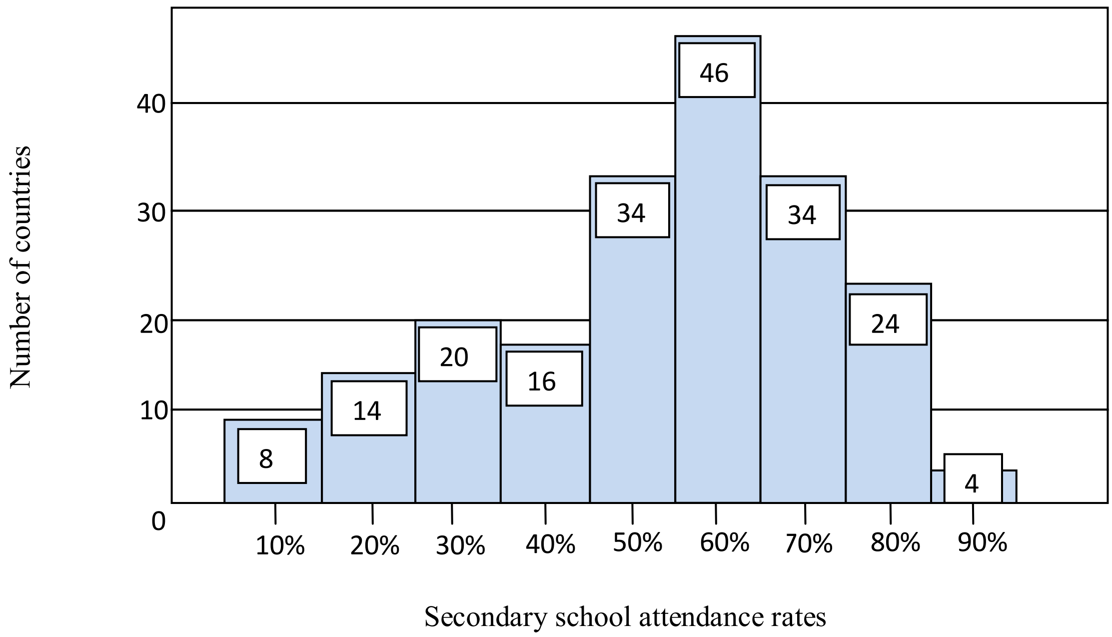

**Question 1**

The percentage of countries with secondary school attendance rates of 55% or smaller was closest to

**A.** 29%
**B.** 35%
**C.** **46%**
**D.** 54%
**E.** 92%

***Answer is C***

**Worked Solution**




Number of countries with secondary school attendance rates of 55% or smaller:

8+14+20+16+34=92. $\boldsymbol{Percentage}$: $\frac{\mathbf{92}}{\mathbf{200}} = \mathbf{46\%}$

**Note:** If you obtained an answer of E, the number of countries was found, not the percentage.

**Tip**

- *Please note that although secondary school attendance rates are given as percentages, their frequencies (the number of countries) are not given as percentages, i.e. the graph is not a percentage histogram.*

**Question 2**

For these 200 countries, secondary school attendance rates were most frequently

**A.** less than 25%
**B.** **between 25% and 55%**
**C.** between 55% and 65%
**D.** between 75% and 95%
**E.** greater than 75%

***Answer is B***

**Worked Solution**

Let's find the frequencies of all five options.
Option A, less than 25: 8+14=22
**Option B, between 25 and 55: 20+16+34=70**
Option C, between 55 and 65: 46
Option D, between 75 and 95: 24+4=28
Option E, greater than 75: 24+4=28
The most frequent values of secondary school attendance rates were between 25% and 55%.

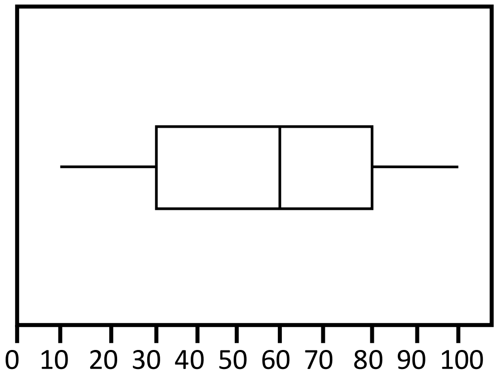
**Note:** If you obtained an answer of C, the mode was found. The question is not asking the mode, it's asking the most frequent interval.

**SECTION A** – continued

**Question 3**

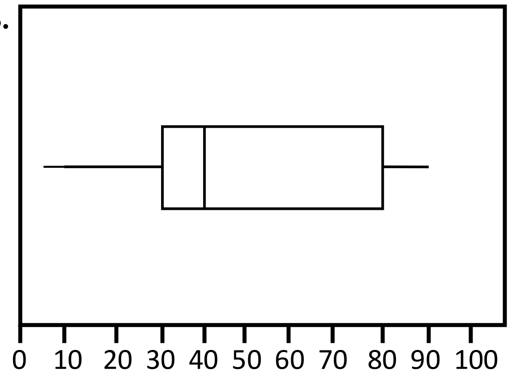
Which one of the boxplots below could best be used to represent the same secondary school attendance rate data as displayed in the histogram?

**A.**



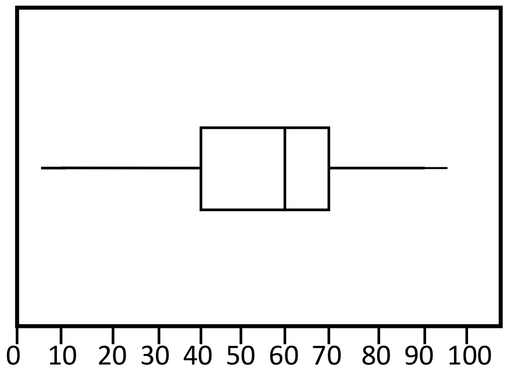

*Secondary school attendance rates (in percentages)*

**B.**



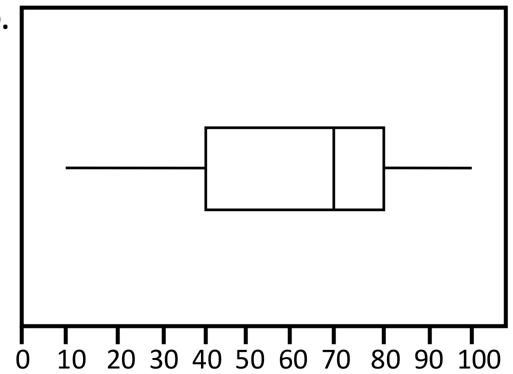

*Secondary school attendance rates (in percentages)*

**C.**



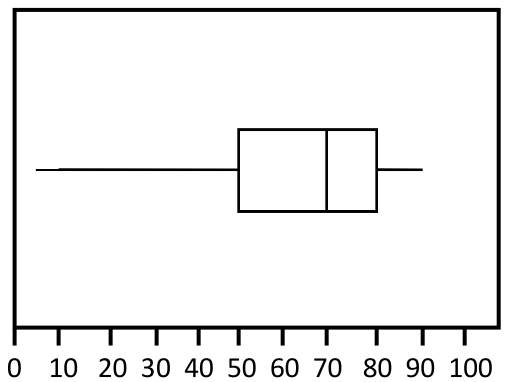

*Secondary school attendance rates (in percentages)*

**D.**




*Secondary school attendance rates (in percentages)*

**E.**




*Secondary school attendance rates (in percentages)*

***Answer is C***

**Worked Solution**

First find five figure summary statistics of the secondary school attendance rates from the histogram.

$Minimum\ value\ is\ 5\%,$

$Q_1\ is\ between\ 35\%\ and\ 45\%,$

$Median\ is\ between\ 55\%\ and\ 65\%,$

$Q_3\ is\ between\ 65\%\ and\ 75\%,$

$Maximum\ value\ is\ 95\%.$

The only boxplot that shows these 5 figure summary statistics is given in Option C.

*The following information relates to Questions 4 to 6.*

The ordered, back-to-back stemplot below shows exclusive breastfeeding rates for six months, expressed as a percentage, in 22 developed and developing countries.

Exclusive breastfeeding rates for 6 months (%)

| developed countries | | | | | | | | developing countries | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 9 | 9 | 7 | 4 | 1 | | | | | | | | |
| | 8 | 7 | 2 | 2 | 1 | 0 | 2 | 7 | 8 | | | | | | |
| 9 | 7 | 7 | 5 | 3 | 3 | 2 | 3 | 4 | 7 | | | | | | |
| | | | | 5 | 5 | 1 | 4 | 0 | 1 | 1 | 2 | | | | |
| | | | | | 4 | 2 | 5 | 1 | 2 | 3 | 3 | 6 | 7 | 8 | 9 |
| | | | | | | | 6 | 1 | 3 | 4 | 6 | | | | |
| | | | | | | | 7 | 3 | | | | | | | |
| | | | | | | | 8 | 2 | | | | | | | |

**Question 4**

For these 22 developed countries, the highest exclusive breastfeeding rates for 6 months is

**A.** 28%
**B.** 45%
**C.** **54%**
**D.** 73%
**E.** 82%

***Answer is C***

**Worked Solution**

The maximum value of the developed countries (left side of the stemplot) is 54.

**Note:** If you obtained an answer of B, the order of the stem and leaf must have been confused.
If you obtained an answer of E, the highest exclusive breastfeeding rates of the developing countries were found.

**Question 5**

For these 22 developing countries, the interquartile range of exclusive breastfeeding rates for 6 months is

**A.** 18
**B.** **20**
**C.** 21.5
**D.** 25
**E.** 28

***Answer is B***

**Worked Solution**

First find quartile 1 and quartile 3. Quartile 1 is the $6^{th}$ value from the start and Quartile 3 is the $6^{th}$ value from the end.
$Q_1 = 41 \; and \; Q_3 = 61$
$IQR = Q_3 - Q_1 = 61 - 41 = 20$

**Note:** If you obtained an answer of A, the interquartile range of exclusive breastfeeding rates of the developing countries was found.

**Question 6**

For these 22 developing countries, exclusive breastfeeding rates for 6 months are generally

**A.** **higher and more variable than the exclusive breastfeeding rates of the developed countries**
B. higher and less variable than the exclusive breastfeeding rates of the developed countries
C. about the same as the exclusive breastfeeding rates of the developed countries
D. lower and more variable than the exclusive breastfeeding rates of the developed countries
E. lower and less variable than the exclusive breastfeeding rates of the developed countries

***Answer is A***

**Worked Solution**

Let's compare exclusive breastfeeding rates of the developed and developing countries in terms of centre and spread.

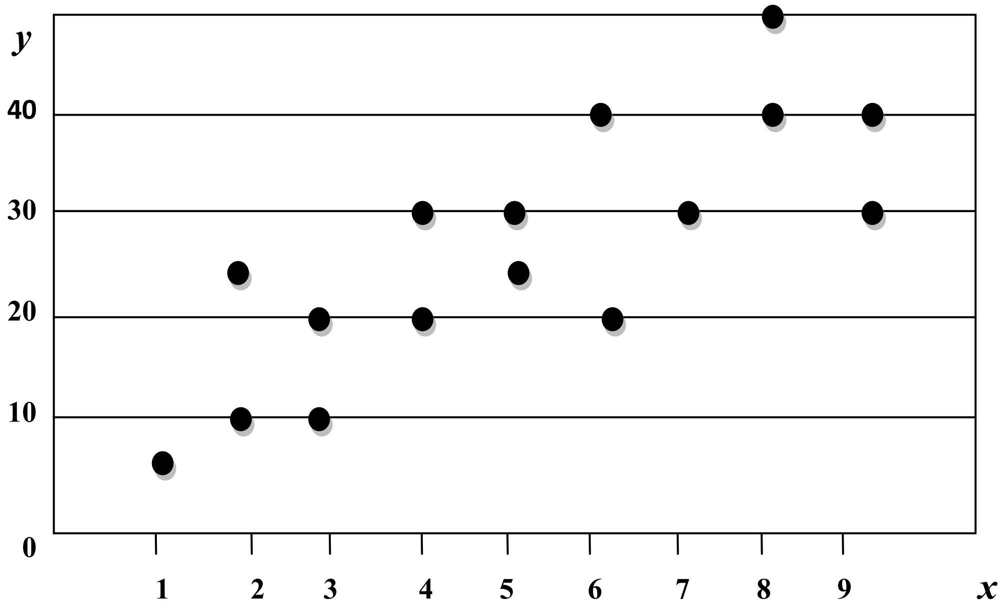
**Central tendency:** The median of the data of developing countries is 53 whereas the median of the data of developed countries is 32.5. This shows that for these 22 developing countries, exclusive breastfeeding rates for 6 months are generally higher than exclusive breastfeeding rates of developed countries.

**Spread:** The range of the data of developing countries is 55 whereas the range of the data of developed countries is 40. This shows that for these 22 developing countries, exclusive breastfeeding rates for 6 months are generally more variable than exclusive breastfeeding rates of developed countries.

**Question 7**

Consider the graph below. The gradient of the 3- median regression line is

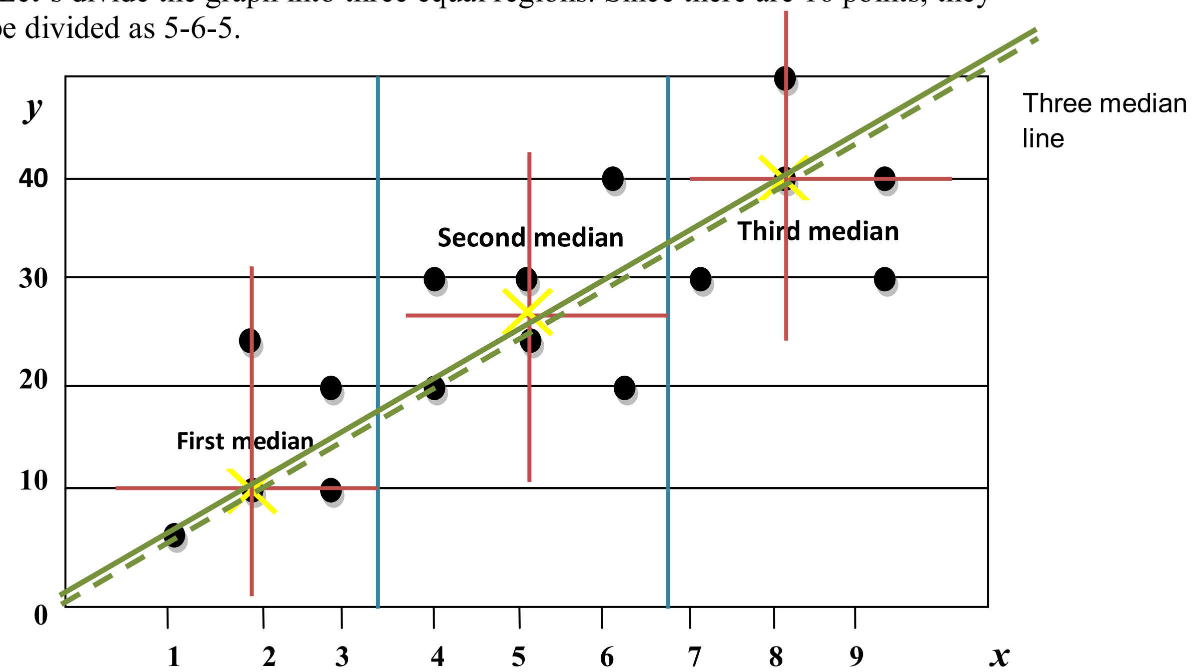
**A.** 3
**B.** 3.7
**C.** 4
**D.** 4.5
**E.** **5**




***Answer is E***

**Worked Solution**

**Step 1:** Let's divide the graph into three equal regions. Since there are 16 points, they should be divided as 5-6-5.




**SECTION A** – continued

**Step 2:** Let's find the median points of every region. In the first and last regions we have 5 points. The median of 5 points is $\frac{5+1}{2} = 3$ meaning that it's the 3rd point when we count horizontally and vertically.
In the middle region we have 6 points. The median of 6 points is $\frac{6+1}{2} = 3.5$ meaning that it's the point between the 3rd and the 4th points when we count horizontally and vertically.

**Step 3:** Now let's start moving from up towards down by counting points of the first region. We'll need to make a horizontal line where we find the third point. Then we'll move from left to right by counting points, we'll need to make a vertical line where we find the third point.

**Step 4:** The intersection point of the horizontal and vertical lines is the first median.
You need to repeat step 3 for all of the three regions to find the second and the third medians.

**Step 5:** Join the first and the third medians by a straight line (the dashed line on the above graph). Then move this line towards the second median by one third of the distance and the line you got is the 3-median regression line (the straight line parallel to the dashed line on the above graph).

**Step 6:** The gradient of the 3-median line can be found by using the first and the third medians which are (2, 10) and (8, 40) respectively. $m = \frac{40-10}{8-2} = \frac{30}{6} = 5.$

**Note:** Step 5 is not needed to answer this question. It's only given to provide more information about drawing 3-median lines. You can directly go from step 4 to step 6 to solve the question.

**Question 8**

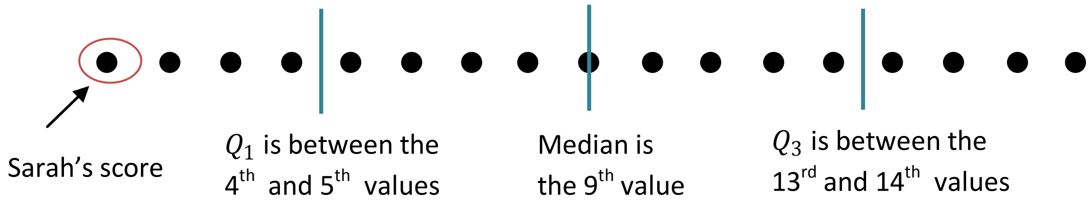
17 students sat a geometry exam and their teacher calculated some statistics for the scores of the students who sat for this exam, noting that no two of the students had the same score. Sarah scored the lowest mark in the class. The teacher then recalculated the statistics excluding Sarah's score.

Which one of the following statistics would not change when Sarah's score is excluded?

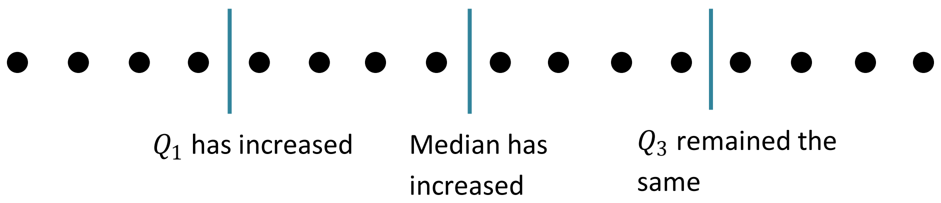
**A.** the median
**B.** the mean
**C.** the standard deviation
**D.** the lower quartile
**E.** **the upper quartile**

***Answer is E***

**Worked Solution**




Now let's remove Sarah's score and check again.



*The following information relates to Questions 9 and 10.*

The distribution of times for the 500 metres running championship amongst the high school students is bell shaped with a mean of 92 seconds and a standard deviation of 4.5 seconds.

**Question 9**

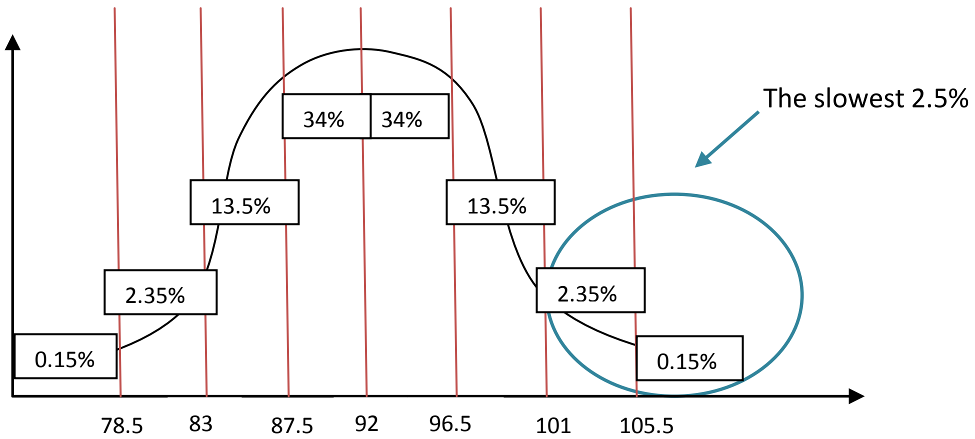
The slowest 2.5% of the athletes would have run 500 metres in

**A.** less than 83 seconds
**B. more than 101 seconds**
**C.** between 83 and 101 seconds
**D.** more than 96.5 seconds
**E.** less than 78.5 seconds

***Answer is B***

**Worked Solution**

We'll start with making a bell shaped graph with the given mean and standard deviation values.




The slowest 2.5% of the athletes would have run 500 metres in more than 101 seconds.

**Note:** If you obtained an answer of A, indirect proportion between time and running fast or slow is taken as direct proportion.

**Tip**

- *The athletes taking more time to run 500 metres are the slower ones. The ones who run quicker are the faster ones (indirect proportion).*

**Question 10**

Jordan is a high school student who attended the 500 metres running championship. If she ran 500 metres in 85.25 seconds, her standardised score (z-score) is closest to

**A.** -1
**B.** **-1.5**
**C.** -1.75
**D.** 1.75
**E.** 1.5

***Answer is B***

**Worked Solution**

$$z - score = \frac{x - \bar{x}}{S_x} = \frac{85.25 - 92}{4.5} = -1.5$$

**Question 11**

The dam levels and average rainfall (in cm) at several different locations in a particular region of Australia is shown below.

| **Dam level (%)** | 25 | 67 | 48 | 85 | 13 | 55 | 61 | 70 | 34 |
|---|---|---|---|---|---|---|---|---|---|
| **Average rainfall (in cm)** | 2.3 | 4.5 | 3.6 | 8.8 | 1.1 | 3.3 | 4.1 | 4.5 | 2.6 |

Using *dam level* as the dependent variable, a least squares regression line is fitted to the data. The equation of the least squares regression line is closest to

**A.** $\text{Average rainfall} = 0.083 - 0.3653 \times \text{dam level}$
**B.** $\text{Average rainfall} = -0.3653 + 0.083 \times \text{dam level}$
**C.** $\text{dam level} = -0.3653 + 0.083 \times \text{average rainfall}$
**D.** $\mathbf{dam\ level = 13.76 + 9.6 \times average\ rainfall}$
**E.** $\text{dam level} = 9.6 + 13.76 \times \text{average rainfall}$

***Answer is D***

**Worked Solution**

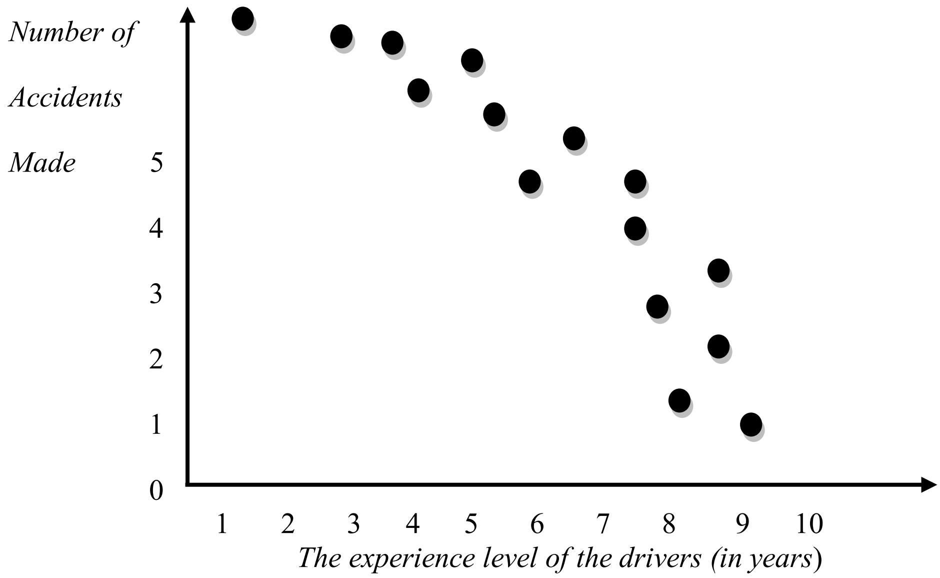
This is a calculator exercise (Stat→Calc→Linear Regression or Linreg). It must be noted that average rainfall is the independent variable which is x and dam level is the dependent variable which is y.
$\text{dam level} = 13.76 + 9.6 \times \text{average rainfall}$

**Note:** If you obtained an answer of E, the gradient and y-intercept of the equation were swapped. If you obtained an answer of B, dependent and independent variables were swapped.

**Question 12**

The relationship between two variables ***number of accidents made*** and ***the experience level of the drivers (in years)***, as shown on the scatterplot below, is nonlinear.

Which one of the following would be the most likely transformation to linearise the data?




**A.** *number of accidents made* against *the experience level of the drivers*.

**B.** $(\textbf{number of accidents made})^2$ **against *the experience level of the drivers.***

**C.** *number of accidents made* against log (*the experience level of the drivers)*.

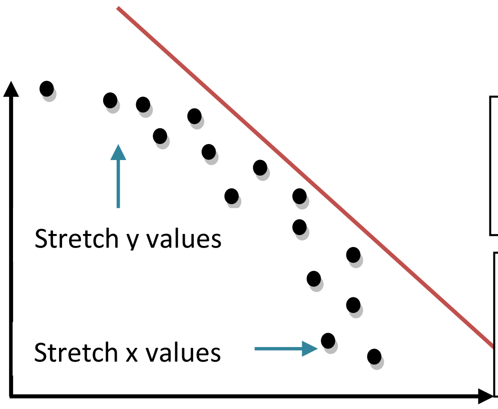
**D.** *number of accidents made* against $\dfrac{1}{\textit{the experience level of the drivers}}$ .

**E.** log (*number of accidents made)* against *the experience level of the drivers*.

***Answer is B***

**Worked Solution**

Let's find the possible transformations that can be applied to the graph to linearise the data.




We can stretch x values by $x^2$ transformation meaning ***number of accidents made* against (*the experience level of the drivers)²*.**

We can stretch y values by $y^2$ transformation meaning $(\textbf{number of accidents made})^2$ **against *the experience level of the drivers.***

**Question 13**

A line of best fit for deseasonalised data was given as

*Deseasonalised Monthly Sales* = $2500 - 1.2 \times timecode$ where for December 2009, t = 1.

Predict the actual expected sales figures for January 2012 if the January seasonal index is 1.15.

**A.** 2470
**B.** 2468.8
**C.** **2839.12**
**D.** 2840.5
**E.** 2146.78

***Answer is C***

**Worked Solution**

In December 2009 t=1,
In December 2010 t=1+12=13,
In December 2011 t=13+12=25,
In January 2012 t=25+1=26.

Substitute t=26 into the equation
$Deseasonalised\ Monthly\ Sales = 2500 - 1.2 \times timecode$
$Deseasonalised\ Monthly\ Sales = 2500 - 1.2 \times 26$
$Deseasonalised\ Monthly\ Sales = 2468.8$

Now we need to find the actual expected sales for January 2012.

$Actual\ monthly\ sales = Deseasonalised\ monthly\ sales \times seasonal\ index$
$Actual\ monthly\ sales = 2468.8 \times 1.15$
$Actual\ monthly\ sales = 2839.12$

**Note:** If you obtained an answer of B, deseasonalised monthly sales were found, not the actual ones.

**END OF SECTION A**

## SECTION B

### Module 1: Number Patterns

#### Question 1

The second and fourth terms of an arithmetic sequence are 24 and 42 respectively. The sum of the first 10 terms of the sequence is

**A.** 9
**B.** 15
**C.** **555**
**D.** 890
**E.** 1550

***Answer is C***

**Worked Solution**

Write arithmetic sequence rule $t_n = a + (n-1)d$ and substitute two given values, $t_2 = 24$ and $t_4 = 42$ into the rule.

$t_2 = 24 \rightarrow t_2 = a + (2-1)d \rightarrow a + d = 24\ (first\ equation)$
$t_4 = 42 \rightarrow t_2 = a + (4-1)d \rightarrow a + 3d = 42\ (second\ equation)$

Now solve these two equations simultaneously. The first term and the common difference of the sequence are 15 and 9, respectively.

Now use $S_n = \frac{n}{2} \times (2a + (n-1)d)$ formula to find the sum of the first 10 terms of the sequence.

$$S_{10} = \frac{10}{2} \times (2 \times 15 + (10-1)9) = 555$$

#### Question 2

For the sequence 53, 49, 45, 41,...the first term that will be negative will be the term

**A.** 23
**B.** 10
**C.** 14
**D.** **15**
**E.** -3

***Answer is D***

**Worked Solution**

The sequence is an arithmetic sequence with the common difference of -4.

53, 49, 45, 41, 37, 33, 29, 25, 21, 17, 13, 9, 5, 1, -3.
-3 is the first negative term and it's the term 15.

**Note:** An answer of E would have been obtained if the question was "the first term that will be negative will be".

**Question 3**

A certain geometric sequence has common ratio of 0.85. The sum of the infinite number of terms of this sequence is 20. The fourth term of this sequence is closest to

**A.** 3
**B.** 2.55
**C.** 2.17
**D.** **1.84**
**E.** 1.95

***Answer is D***

**Worked Solution**

We are going to use $S_\infty = \frac{a}{1-r}$ formula to find the first term, a.

$$20 = \frac{a}{1-0.85}$$

$a = 3$

Now we need to use geometric sequence rule $t_n = a \times r^{n-1}$ to eliminate $t_4$.

$t_4 = 3 \times 0.85^{4-1} = 1.842375 \cong 1.84$

**Question 4**

The first four terms of a geometric sequence are 7 200, $t_2$,16 200,-24 300. The value of the second term is

**A.** 8 100
**B.** -8 100
**C.** **-10 800**
**D.** 10 800
**E.** None of the above

***Answer is C***

**Worked Solution**

We'll find the common difference r by dividing the fourth term by the third term.

$$\frac{t_4}{t_3} = \frac{-24\,300}{16\,200} = -1.5$$

$t_2 = 7200 \times -1.5 = -10\,800$

**Question 5**

In a Fibonacci sequence, $t_{n+2} = t_n + t_{n+1}$. If $t_{16} = 221\ 250$ and $t_{18} = 579\ 240$, the 17th number in this Fibonacci sequence is

**A.** 84 510
**B.** 125 670
**C.** 569 140
**D.** 456 000
**E.** **357 990**

***Answer is E***

**Worked Solution**

$t_{18} = t_{16} + t_{17}$
$579\ 240 = 221\ 250 + t_{17}$
$t_{17} = 579\ 240 - 221\ 250$
$t_{17} = 357\ 990$

**Question 6**

Members of a gymnastic club have been decreasing by 20% every year since 2007. In order to reduce the loss, the club management decided to increase the yearly membership fees by $200 every year. The club had 8 000 members and membership fee was $540 in 2007. How much less total fees will the club collect in 2010 than 2008?

**A.** $85 560
**B.** $47 000
**C.** **$66 560**
**D.** $123 450
**E.** $150 000

***Answer is C***

**Worked Solution**

The members of the gymnastic club form a decreasing geometric sequence where $a = 8\ 000$ and $r = 1 - \frac{20}{100} = 0.8$.

The gym membership fees form an increasing arithmetic sequence where $a = 540$ and $d = 200$.

The total fees that will be collected in 2010
$$= (8\ 000 \times 0.8^{4-1}) \times [540 + (4-1) \times 200]$$
$$= \$4\ 669\ 440$$
The total fees that will be collected in 2008
$$= (8\ 000 \times 0.8^{2-1}) \times [540 + (2-1) \times 200]$$
$$= \$4\ 736\ 000$$

$The\ difference = 4\ 669\ 440 - 4\ 736\ 000 = -\$66\ 560$

The club will collect $66 560 less total fees in 2010 than 2008.

**Question 7**

In a chicken farm, the number of chickens is naturally increasing by 42% per year. The farm started with 400 chickens and at the end of each year 120 chickens are sold.

The difference equation that represents the number of chickens $C_n$ at the start of $n^{th}$ year is

**A.** $C_{n+1} = 1.42 \times (C_n - 120) \quad C_1 = 400$
**B.** $\boldsymbol{C_{n+1} = 1.42 \times C_n - 120 \quad C_1 = 400}$
**C.** $C_{n+1} = 0.58 \times C_n - 120 \quad C_1 = 400$
**D.** $C_{n+1} = 0.58 \times (C_n - 400) \quad C_1 = 120$
**E.** $C_{n+1} = 400 \times C_n - 120 \quad C_1 = 42$

***Answer is B***

**Worked Solution**

$$common\ ratio = 1 + \frac{42}{100} = 1.42$$
$$common\ difference = -120, \quad First\ term = 400$$
$$\therefore C_{n+1} = 1.42 \times C_n - 120 \quad C_1 = 400$$

**Note:** If you obtained an answer of C, the common ratio is calculated with 42% decrease.

**Question 8**

A cactus was 32 cm high initially. It grew 1.2 cm during the first week, 0.96 cm during the second week and 0.768 cm during the third week. If this cactus continues to grow in this manner, it will reach the maximum height of

**A.** 6
**B.** 35
**C.** 37
**D.** **38**
**E.** 41

***Answer is D***

**Worked Solution**

The growth sequence: 1.2cm, 0.96cm, 0.768cm, ...
This sequence is geometric with a common ratio of $r = \frac{0.96}{1.2} = 0.8$

We need to use $S_\infty = \frac{a}{1-r}$ formula to calculate the maximum growth of the cactus.

$$\boldsymbol{S_\infty = \frac{1.2}{1-08}}$$

$$\boldsymbol{S_\infty = 6}$$

We now need to add the initial height and the maximum growth of the cactus to find its maximum height.

$$Maximum\ height = 32 + 6 = 38\ cm.$$

**Note:** If you obtained an answer of A, the maximum growth of the cactus was found.

**Question 9**

The first four terms of a sequence are: 3, 5, 9, 17. The difference equation of the sequence could be

**A.** $t_{n+2} = 3t_n - 2t_{n+1}$ $t_1 = 3, \quad t_2 = 5$
**B.** $t_{n+1} = 3t_n - 4$ $t_1 = 3$
**C.** $t_{n+1} = t_n + 2$ $t_1 = 3$
**D.** $t_{n+1} = 4t_n - 7$ $t_1 = 3$
**E.** $\boldsymbol{t_{n+2} = 3t_{n+1} - 2t_n}$ $\boldsymbol{t_1 = 3, \quad t_2 = 5}$

***Answer is E***

**Worked Solution**

Generate the sequences in the options from the difference equations.

**Option A:** $\boldsymbol{t_3 = 3t_1 - 2t_2 = 3 \times 3 - 2 \times 5 = -1}$

So option A cannot be the correct answer.

**Option B:** $\boldsymbol{t_2 = 3t_1 - 4 = 3 \times 3 - 4 = 5}$
$\boldsymbol{t_3 = 3t_2 - 4 = 3 \times 5 - 4 = 11}$

So option B also cannot be the correct answer.

**Option C:** $\boldsymbol{t_2 = t_1 + 2 = 3 + 2 = 5}$
$\boldsymbol{t_3 = t_2 + 2 = 5 + 2 = 7}$

So option C also cannot be the correct answer.

**Option D:** $\boldsymbol{t_2 = 4t_1 - 7 = 4 \times 3 - 7 = 5}$
$t_3 = 4t_2 - 7 = 4 \times 5 - 7 = 13$

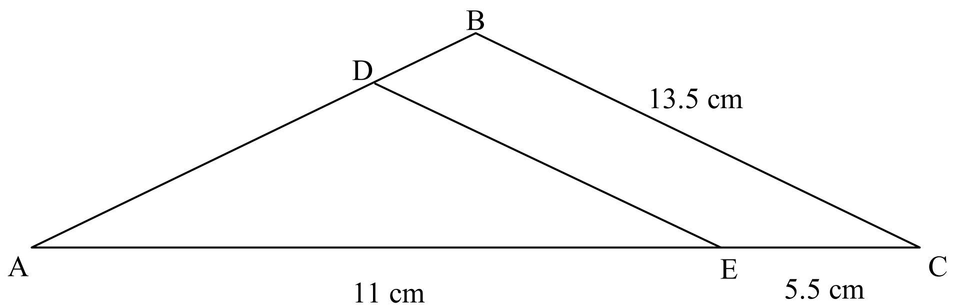
So option D also cannot be the correct answer.

**Option E:** $\boldsymbol{t_3 = 3t_2 - 2t_1 = 3 \times 5 - 2 \times 3 = 9}$
$\boldsymbol{t_4 = 3t_3 - 2t_2 = 3 \times 9 - 2 \times 5 = 17}$

So option E is the correct answer.

**Module 2: Geometry and trigonometry**

**Question 1**




DE ∥ BC. If AE= 11 cm, EC= 5.5 cm and BC= 13.5 cm, the length of DE, in cm, is

**A.** **9**
**B.** 6.75
**C.** 20.5
**D.** 13.5
**E.** 27

***Answer is A***

**Worked Solution**

ADE triangle is similar to ABC triangle. So $\frac{AE}{AC} = \frac{DE}{BC}$

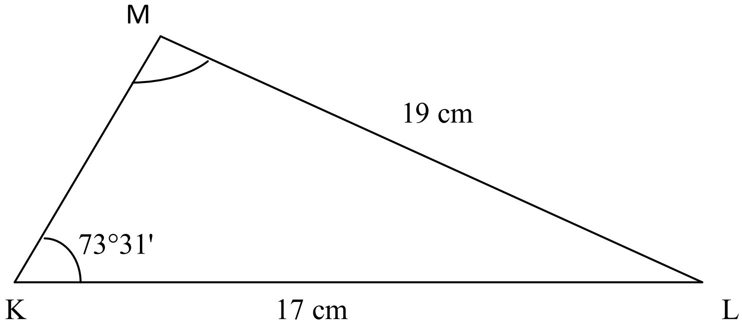
$$\frac{11}{11+5.5} = \frac{DE}{13.5}$$

$$Length\ of\ DE = 9\ cm$$

**Tip**

- *A very common mistake is writing the equation as* $\frac{AE}{EC} = \frac{DE}{BC}$ *. You would obtain an answer of B if you made that mistake.*

**Question 2**




In the triangle above, the angle LMK is closest to

**A.** 45°23'
**B.** 49°41'
**C.** **59°5'**
**D.** 63°9'
**E.** 73°51'

***Answer is C***

**Worked Solution**

We will use the sine rule to evaluate the angle M.

$$\frac{19}{\sin 73°31'} = \frac{17}{\sin M}$$

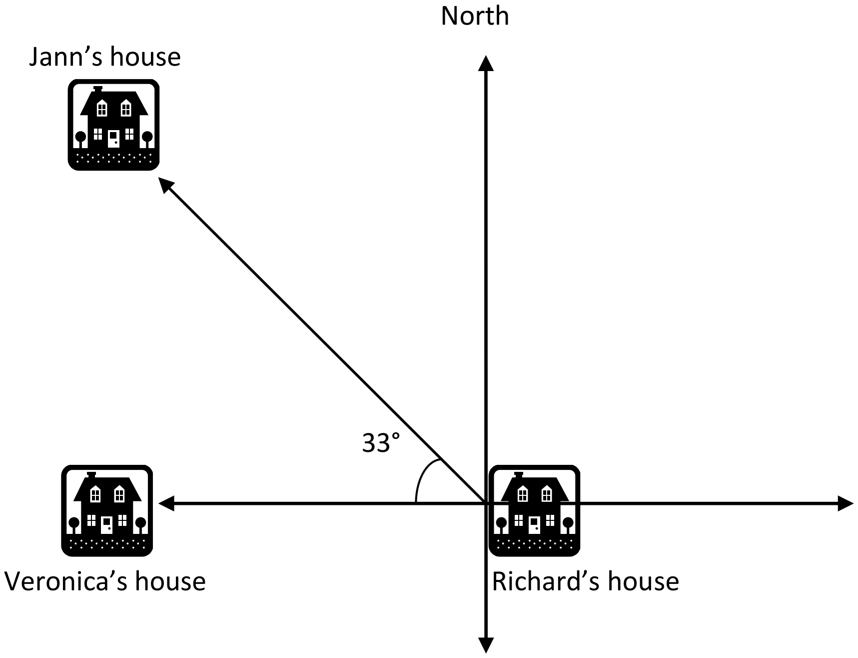
$$\sin M = \frac{17 \times \sin 73°31'}{19}$$

$$\sin \mathrm{M} = 0.8579652256$$

$$M = \sin^{-1} 0.8579652256$$

After doing the conversion from the decimal degree to degrees, minutes and seconds, we obtained M as $\mathbf{59°5}'$

**Question 3**

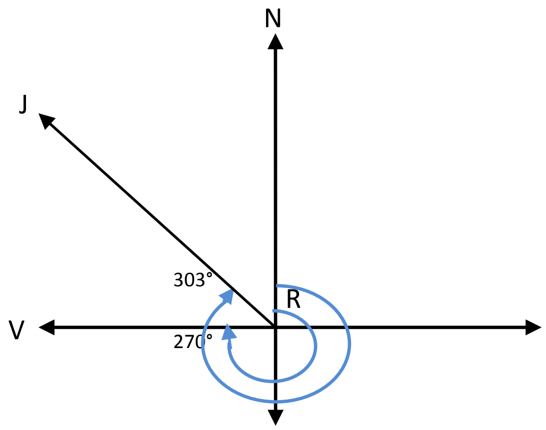



The locations of three houses, Jann's, Veronica's and Richard's houses are shown in the diagram above. Jann's house is due north of Veronica's house. The bearings of Jann's and Veronica's houses from Richard's house are

**A.** 303° and 180° respectively
**B.** 123° and 0° respectively
**C.** 33° and 0° respectively
**D.** 47° and 180° respectively
**E.** **303° and 270° respectively**

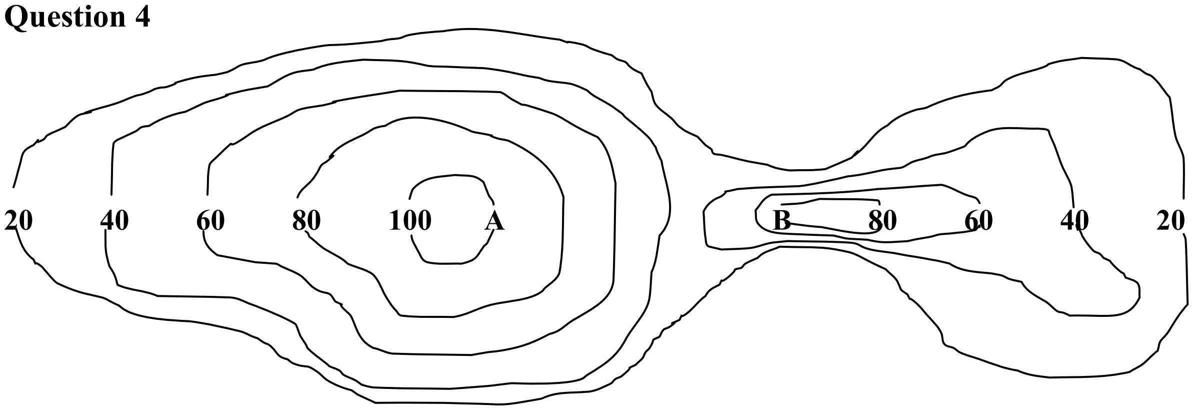
***Answer is E***

**Worked Solution**




The bearing of Veronica's house from Richard's house is 180°+ 90°=270°.
The bearing of Jann's house from Richard's house is 270°+33°=303°.

**Question 4**

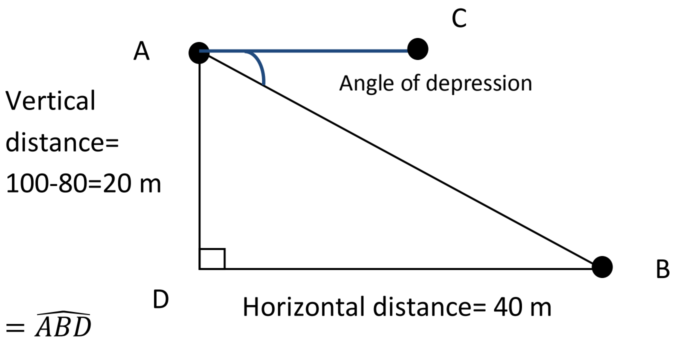



A contour map of a region is shown above. The contour interval is 20 m. The actual horizontal distance between points A and B is 40 m. The angle of depression of B from A is closest to

**A.** **27°**
**B.** 63°
**C.** 72°
**D.** 47°
**E.** 85°

***Answer is A***

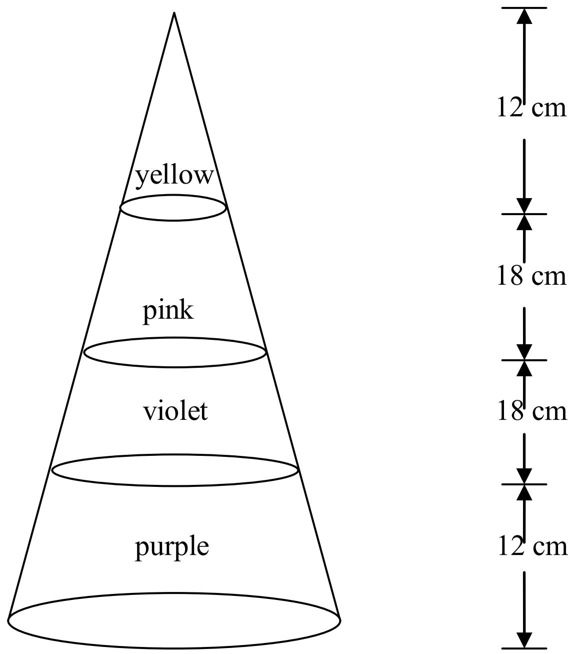
**Worked Solution**




$\widehat{CAB} = \widehat{ABD}$
$\tan\widehat{ABD} = \dfrac{20}{40}$
$\widehat{ABD} = \tan^{-1} 0.5$
$\widehat{ABD} = 26.565°$
$\widehat{ABD} \cong 27°$

*The following information relates to the questions 5 and 6*

A 60 cm high wedding cake has yellow, pink, violet and purple icing as shown in the diagram below



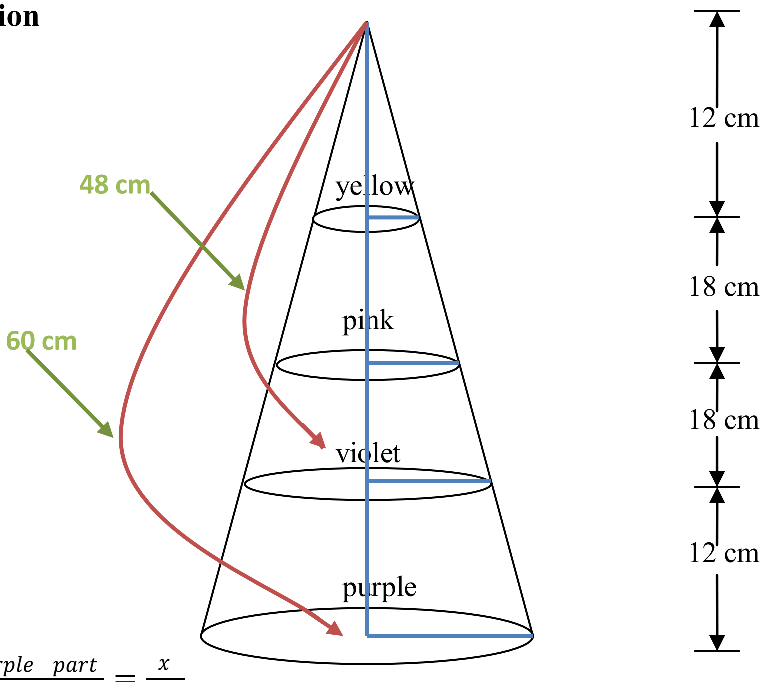

**Question 5**

If the volume of the cake is found to be 625 $cm^3$ , then the volume of the part covered in purple icing is closest to

**A.** **305 *cm³***
**B.** 105 $cm^3$
**C.** 5 $cm^3$
**D.** 425 $cm^3$
**E.** 610 $cm^3$

***Answer is A***

**Worked Solution**




$\frac{Volume\ \ of\ nonpurple\ \ part}{Volume\ \ of\ the\ whole\ cake} = \frac{x}{625}$

$\left(\frac{48}{60}\right)^3 = \frac{x}{625}$

$\left(\frac{4}{5}\right)^3 = \frac{x}{625}$

$\frac{64}{125} = \frac{x}{625}$

$x = \frac{64 \times 625}{125} = 320\ cm^3$

Volume of the purple part of the cake

= volume of the whole cake − volume of the nonpurple part of the cake

Volume of the purple part of the cake = 625 − 320 = 305 $\text{cm}^3$

**Question 6**

The area of the base, in $cm^2$, is closest to

**A.** 25
**B.** **31.25**
**C.** 45.5
**D.** 62.5
**E.** 125

***Answer is B***

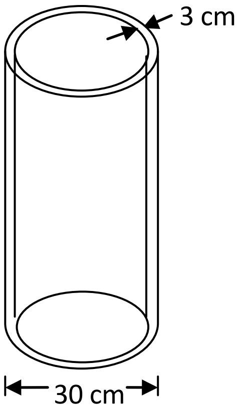
**Worked Solution**

$Volume\ of\ the\ cake = \frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi r^2 \times 60 = 625$

$r = 3.153915653\ \ \therefore The\ area\ of\ the\ base = \pi r^2 = \pi \times 3.153915653^2 = 31.25\ \text{cm}^2$

**Question 7**



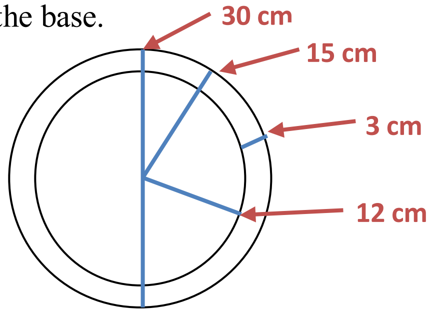

A cylindrical vase is 70 centimetres long. The outside diameter of the vase is 30 centimetres. Its walls are 3 centimetres thick. One litre of water occupies 1000 $cm^3$. When the vase is full of water, the volume it holds is closest to

**A.** 5 L
**B.** 15 L
**C.** 18 L
**D.** 22 L
**E.** **32 L**

***Answer is E***

**Worked Solution**

Let's find the area of the base.

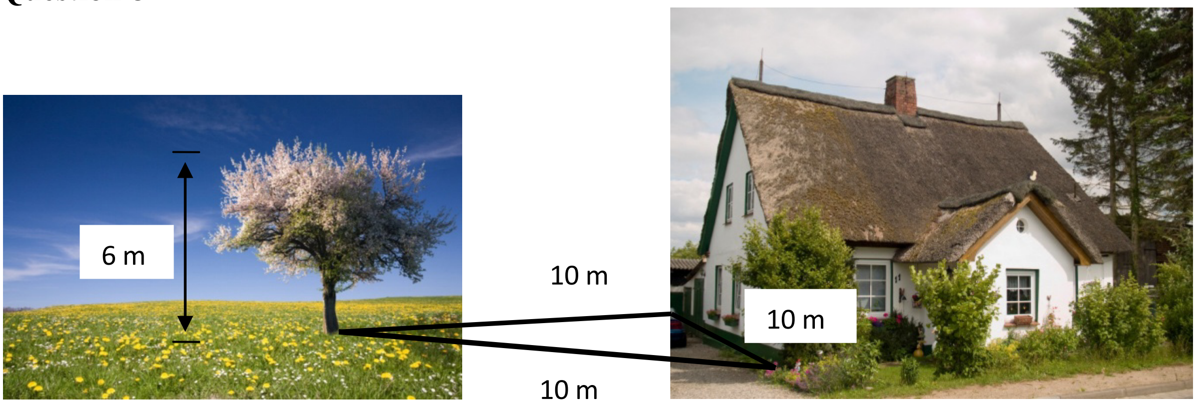



We can see from the above figure that the radius of the inner circle is 12 cm.

$$Volume\ of\ the\ vase = base\ area \times height$$
$$= \pi r^2 \times \mathrm{h}$$
$$= \pi \times 12^2 \times 70$$

$= 31\,667.25\ cm^3 = 32\ L$

**Question 8**




A tree is 6 metres tall and stands near a house. The base of the tree is 10 metres from both ends of the nearest wall of the house which is also 10 metres long.

The minimum distance between the top of the tree and the house when the tree falls is closest to

**A.** 10 m

**B.** 8.66 m

**C.** 4 m

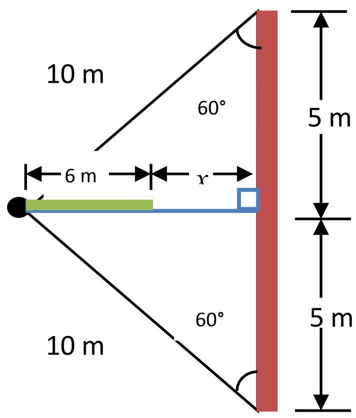
**D.** **2.66 m**

**E.** 1.26 m

***Answer is D***

**Worked Solution**

The tree and two sides of the house's wall form an equilateral triangle. The shortest distance between the top of the tree and the house is the perpendicular distance.




$$\tan 60^\circ = \frac{6 + x}{5}$$

$$x = 5\tan 60^\circ - 6$$

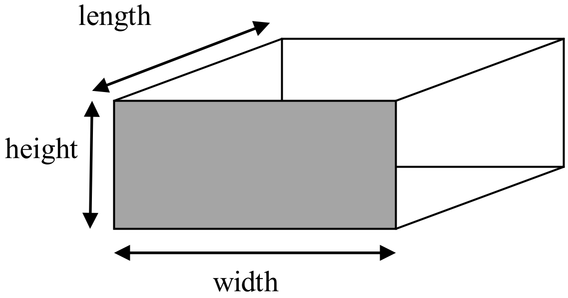
$$x = 2.66 \; metres$$

The minimum distance between the top of the tree and the house when the tree falls is 2.66 metres.

**Tip**

- *Students could also use the Pythagoras theorem to obtain the same answer.*

**Question 9**




A rectangular prism has a volume of 240 cm³. A second rectangular prism is made with the same length, half the width and four times the height of the prism shown.

The volume of the second prism is

**A.** 240 $cm^3$
**B.** **480 $cm^3$**
**C.** 120 $cm^3$
**D.** 360 $cm^3$
**E.** 540 $cm^3$

***Answer is B***

**Worked Solution**

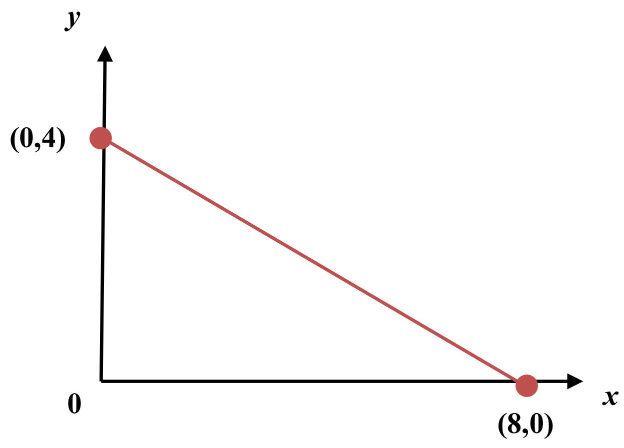
$$Volume\ of\ the\ first\ prism = l \times w \times h = 240$$

$$Volume\ of\ the\ second\ prism = l \times \frac{w}{2} \times 4h = 2 \times l \times w \times h$$

So the volume of the second prism is twice as much as the volume of the first prism. The answer is 480 $cm^3$.

**Module 3: Graphs and relations**

**Question 1**




The equation of the straight line shown is

**A.** $x = 8$

**B.** $y = 4$

**C.** $2y - x = 8$

**D.** $4x + 8y = 2$

**E.** $\boldsymbol{x + 2y = 8}$

***Answer is E***

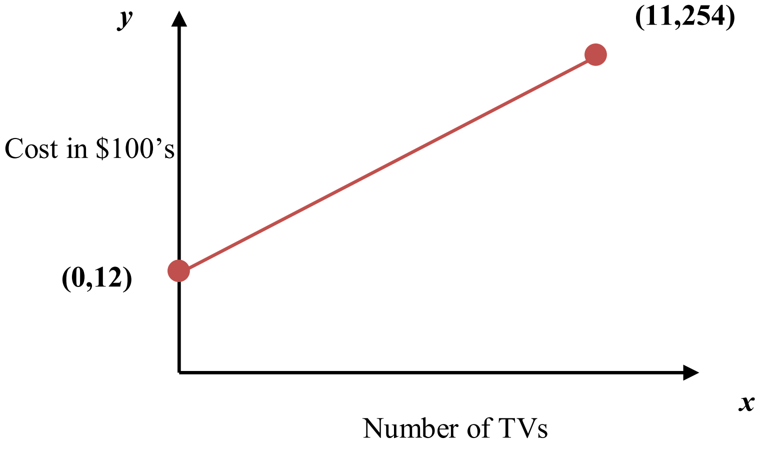
**Worked Solution**

A line equation is of the form $y = mx + c$, where m is the gradient and c is the y-intercept.


$$\boldsymbol{m = \frac{rise}{run} = -\frac{4}{8} = -\frac{1}{2}}$$
$$\boldsymbol{c = 4}$$
$$\boldsymbol{y = -\frac{1}{2}x + 4}$$
$$\boldsymbol{x + 2y = 8}$$

**Question 2**

Alice's father Danny manufactures LCD televisions in his electronic company's factory . The graph below shows the cost (in hundreds of dollars) of producing the LCD televisions in Danny's company.




The increase in cost that occurs from producing one LCD television is cost of producing each LCD television is**:**

**A.** 22

**B.** 23

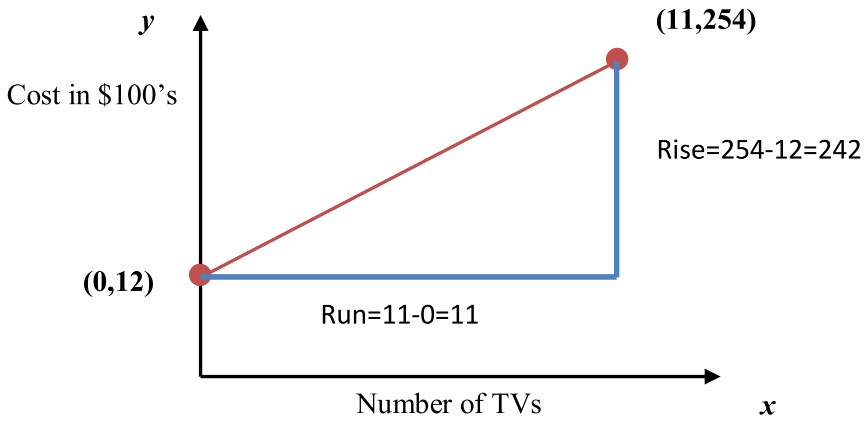
**C.** **2 200**

**D.** 23 000

**E.** 254

***Answer is C***

**Worked Solution**



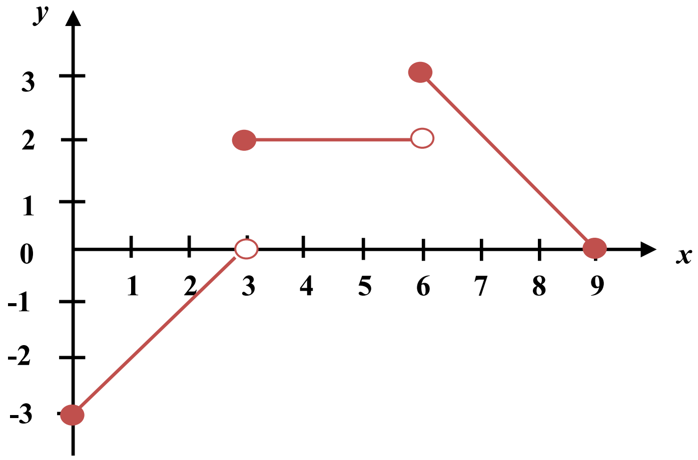

$\boldsymbol{m = \frac{rise}{run} = \frac{242}{11} = 22}$

The increase in cost that occurs from producing one LCD television is $2 200.

**Tip**

- *The reason why we need to multiply 22 by 100 is because the cost is given in hundreds of dollars. If you obtained the answer of A, you must have ignored this detail.*

**Question 3**




Which of the following rules would best describe the graph above?

**A.** $\begin{cases} \boldsymbol{y = x - 3}, & \boldsymbol{0 \leq x} < 3 \\ \boldsymbol{y = 2}, & \boldsymbol{3 \leq x} < 6 \\ \boldsymbol{y = -x + 9}, & \boldsymbol{6 \leq x \leq 9} \end{cases}$

**B.** $\begin{cases} y = x + 3, & 0 \leq x < 3 \\ y = 2, & 3 \leq x < 6 \\ y = -x + 9, & 6 \leq x < 9 \end{cases}$

**C.** $\begin{cases} y = x - 3, & 0 \leq x < 3 \\ y = 2, & 3 \leq x \leq 6 \\ y = x + 9, & 6 \leq x \leq 9 \end{cases}$

**D.** $\begin{cases} y = x - 3, & 0 < x \leq 3 \\ y = 2, & 3 < x \leq 6 \\ y = -x + 9, & 6 < x < 9 \end{cases}$

**E.** $\begin{cases} y = x - 3, & 0 \leq x < 2 \\ y = 3, & 2 \leq x < 6 \\ y = -x + 9, & 6 \leq x \leq 9 \end{cases}$

***Answer is A***

**Worked Solution**

Let's start by determining the equation of the first line. $y = mx + c, where\ m = 1\ and\ c = -3$.
So the first equation is $y = x - 3$, where $0 \leq x < 3$.
The second equation is $y = 2, where\ 3 \leq x < 6$.
The gradient of the third equation is -1 and its y-intercept is 9, so the third equation is $y = -x + 9, where\ 6 \leq x \leq 9$.

∴The correct answer is $\begin{cases} y = x - 3, & 0 \leq x < 3 \\ y = 2, & 3 \leq x < 6 \\ y = -x + 9, & 6 \leq x \leq 9 \end{cases}$

**Tip**

- *Please note that solid circles mean that those points are included in the domain, i.e.* $\leq or\ \geq signs$ *should be used.*

**Question 4**

The cost of hiring 7 overnight and 6 weekly DVDs is \$106.60. At the same shop, the cost of hiring 5 overnight and 2 weekly DVDs is \$70.20.
Let x be the cost of hiring a weekly DVD and y be the cost of hiring an overnight DVD.
The set of simultaneous equations that can be solved to find the cost of hiring a weekly and an overnight DVD is

**A.** $6x + 7y = 70.20$
$2x + 5y = 106.60$

**B.** $7x + 6y = 106.60$
$5x + 2y = 70.20$

**C.** $7x + 5y = 106.60$
$6x + 2y = 70.20$

**D.** $6x + 5y = 70.20$
$7x + 2y = 106.60$

**E.** $\mathbf{2x + 5y = 70.20}$
$\mathbf{6x + 7y = 106.60}$

***Answer is E***

**Worked Solution**

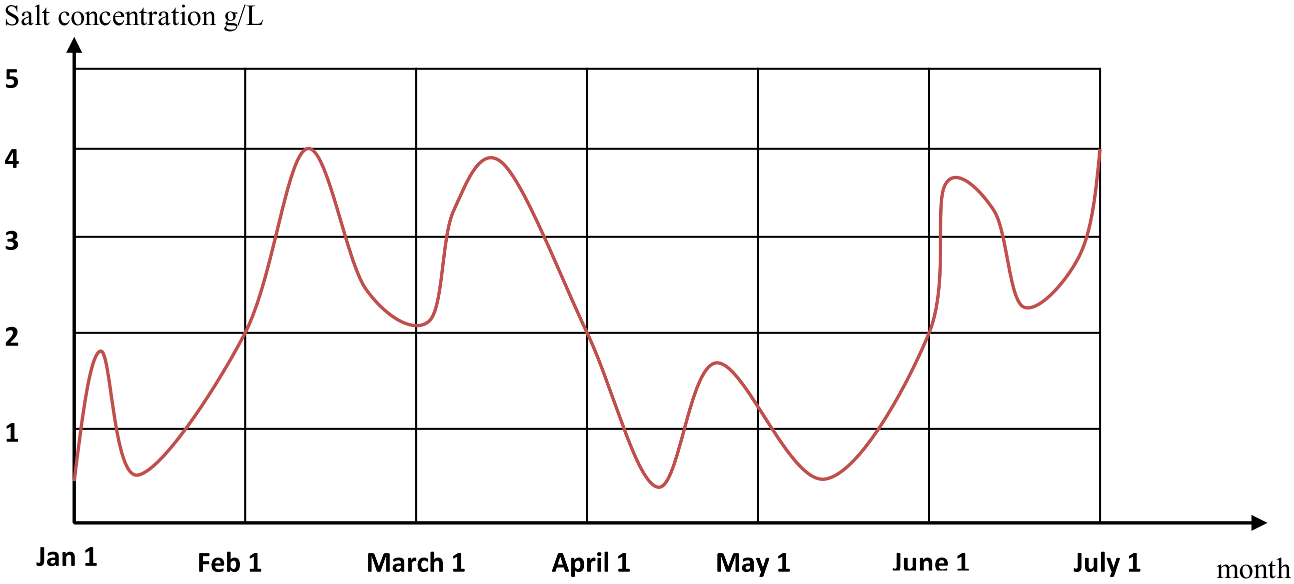
The answer is $2x + 5y = 70.20$
$6x + 7y = 106.60$

**Tip**

- *Although the cost of hiring a weekly DVD is x and the cost of hiring an overnight DVD is y, the cost sentence starts with the cost of hiring an overnight DVD which is represented by y, not x. A very common mistake is obtaining the answer of B.*

**Question 5**

The salt concentration (in g/L) in a particular pond is graphed over a six-month period.




For this 6-month period, the salt concentration of the pond was between 2 g/L and 4 g/L for

**A.** between two and three months

**B.** two months

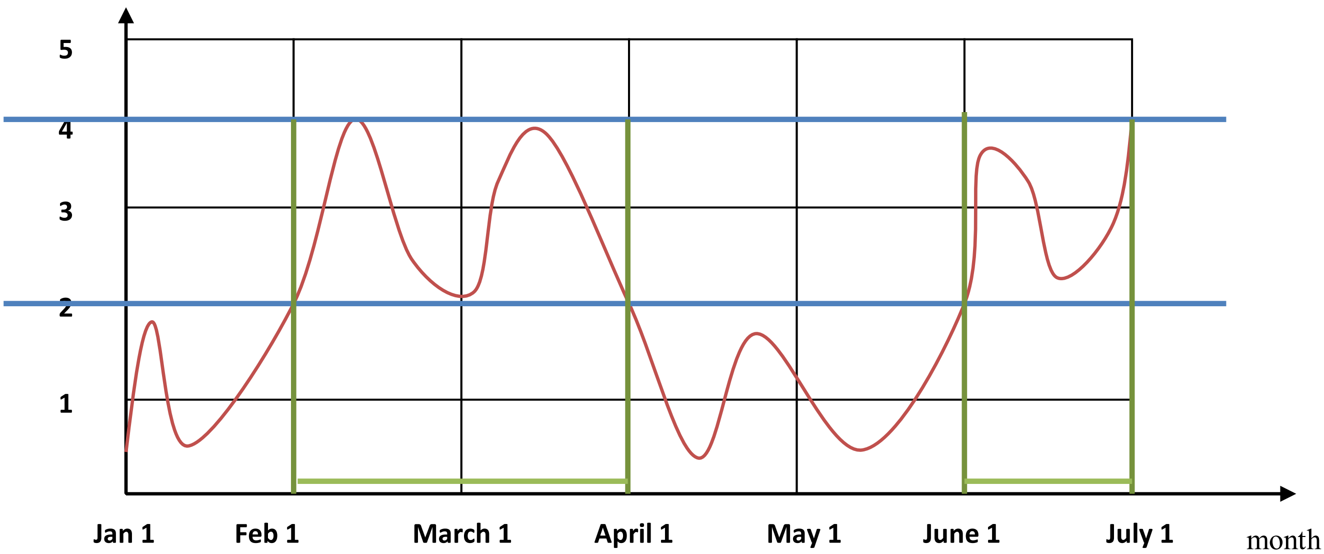
**C.** **three months**

**D.** between three and four months

**E.** four months

***Answer is C***

**Worked Solution**




As we can clearly see from the figure above, the salt concentration of the pond was between 2 g/L and 4 g/L for 3 months.

**Question 6**

"Tasty world" chocolate company produces two different types of handmade chocolates in five factories Each factory has the capacity to produce 500 boxes of each type of chocolates every day. The factories will produce chocolates for spring festival over a 10 day period. The company had to spend $20 000 as fixed cost of the preparation for the manufacture of the chocolate and the cost of producing one box of chocolate is $5.
To break even, the selling price of one box of chocolate should be

**A.** **$5.40**
**B.** $6.20
**C.** $5.80
**D.** $10.50
**E.** $70

***Answer is A***

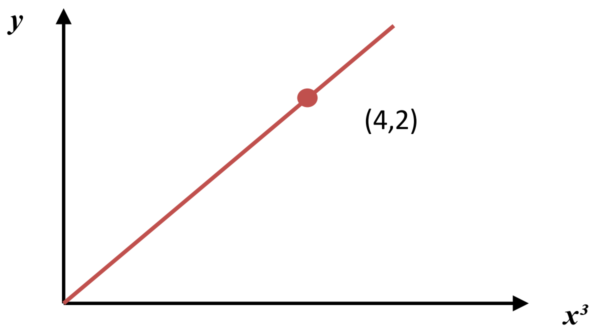
**Worked Solution**

The cost function is $C = 5x + 20\ 000$.
$The\ number\ of\ \ boxes\ of\ chocolates = 2 \times 5 \times 500 \times 10 = 50\ 000$.
$Cost\ of\ 50\ 000\ boxes\ of\ chocolates = 5 \times 50\ 000 + 20\ 000 = 270\ 000$.
To be able to break even, the selling price of one box of chocolate should be 270 000÷50 000= $5.40.

**Note:** If you obtained an answer of C, you must have forgotten multiplying the number of boxes by 2 because there are two different types of chocolates.

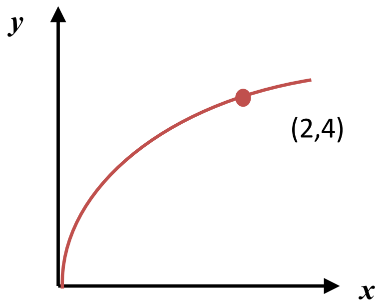
**Question 7**

The graph of y versus $x^3$ is shown

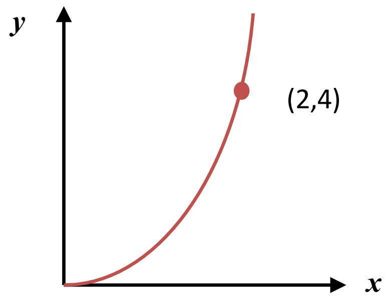



The graph of y versus x is

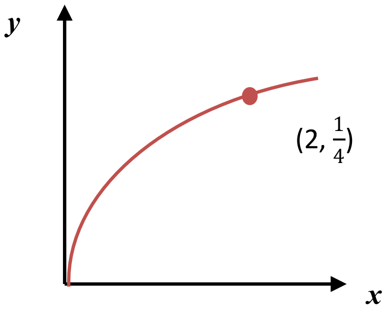
**A.**



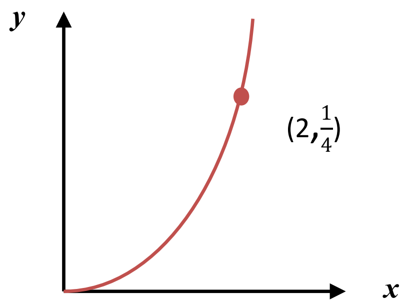

**B.**



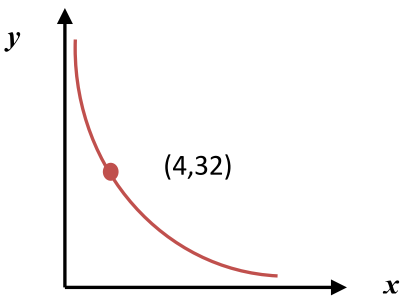

**C.**




**D.**

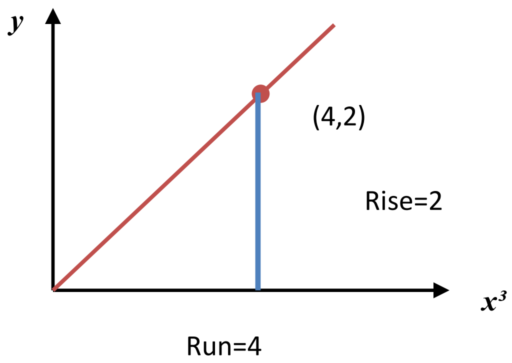



**E.**




***Answer is B***

**Worked Solution**



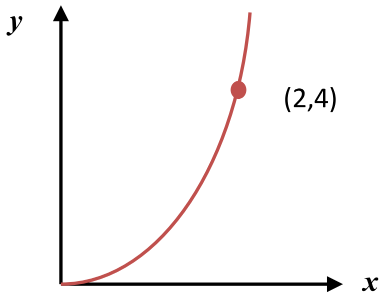

The equation is $y = kx^3$ where

$k = \frac{rise}{run} = \frac{2}{4} = \frac{1}{2}$

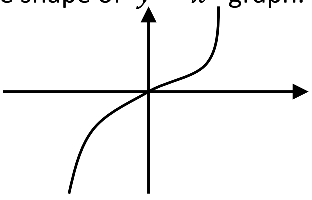
$So\ y = \frac{1}{2}x^3$

Substitute x=2 into this equation. $\boldsymbol{y = \frac{1}{2} \times 2^3 = 4}$.

So the answer is




**Hint:** The graph should have this shape because of the shape of $y = x^3$ graph.



**Question 8**

Amber is a year 12 student who spends at least 14 hours a week studying further mathematics and at least 9 hours a week studying chemistry. She can devote a maximum of 42 hours in total to studying further mathematics and chemistry in a week. She always makes sure that she spends at least three times as many hours studying further mathematics than studying chemistry.

Let $\boldsymbol{x}$ be the number of hours per week Amber studies further mathematics.

Let $\boldsymbol{y}$ be the number of hours per week Amber studies chemistry.

The constraints that apply to Amber's studying further mathematics and chemistry are described by

**A.** $x \le 14, y \le 9, x + y \ge 42, x \ge 3y$

**B.** $x \ge 14, y \ge 9, x + y \le 42, y \ge 3x$

**C.** $x \ge 14, y \ge 9, x + y \le 42, 3x \ge y$

**D.** $x \le 14, y \le 9, x + y \ge 42, y \ge 3x$

**E.** $\mathbf{x \ge 14, y \ge 9, x + y \le 42, x \ge 3y}$

***Answer is E***

**Worked Solution**

**Constraint 1:** Amber spends at least 14 hours a week studying further mathematics, i.e. $x \ge 14$.

**Constraint 2:** She also spends at least 9 hours a week studying chemistry, i.e. $y \ge 9$.

**Constraint 3:** She can devote a maximum of 42 hours in total to studying further mathematics and chemistry in a week, i.e. $\boldsymbol{x} + y \le 42$

**Constraint 4:** She always makes sure that she spends at least three times as many hours studying further mathematics than studying chemistry, i.e. $\boldsymbol{x \ge 3y}$.

**Tip**

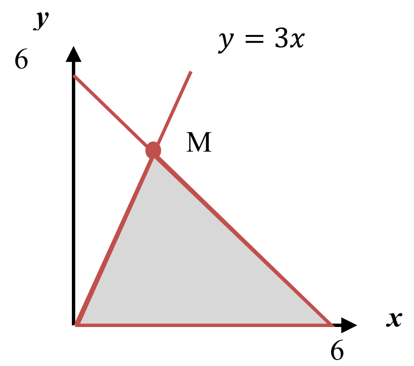
- *A very common mistake is writing the last constraint as* $y \ge 3x$.

**Question 9**

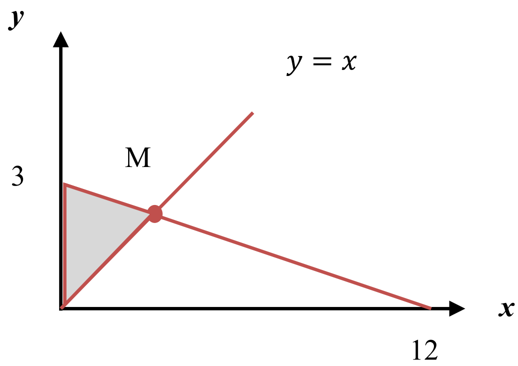
Two car production companies, “Fast Car” and “Luxury Car” decided to merge in order to decrease the production costs. Let x be the number of cars produced weekly by the “Fast Car” company and y be the number of cars produced weekly by the “Luxury Car” company.

In which one of the following graphs does the point M, the intersection point of two lines, minimise the cost function, $C = 3x + 2y$ for the given shaded region?

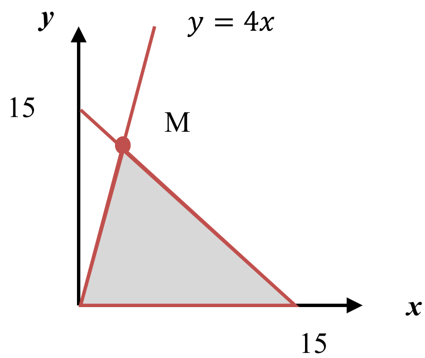
**A.**



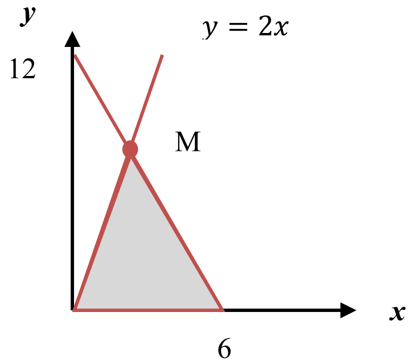

**B.**



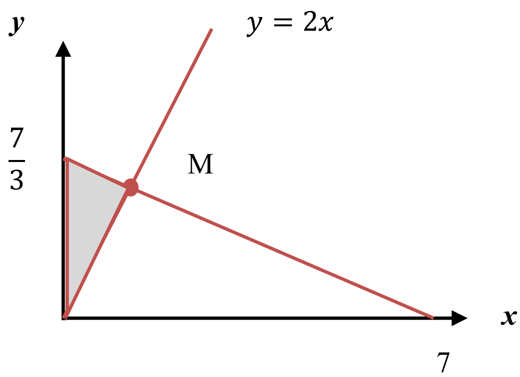

**C.**




**D.**




**E.**




***Answer is C***

**Worked Solution**
Let's find the intersection points of all the graphs and check if they minimise the shaded inequations.

**Option A:** Solve the equations $x + y = 6 \; and \; y = 3x$ simultaneously. The intersection point is (1.5, 4.5) which cannot be the answer of a discrete problem.

**Option B:** Solve the equations $x + 4y = 12 \; and \; y = x$ simultaneously. The intersection point is (2.4, 2.4) which cannot be the answer of a discrete problem.

**Option C:** Solve the equations $x + y = 15 \; and \; y = 4x$ simultaneously. The intersection point is (3,12). Now we need to check if the point (3,12) minimises the shaded inequation or not.

Let's substitute the vertices of the boundary of the region into the cost equation $C = 3x + 2y$.
*For the point (3,12)* $Cost = 3 \times 3 + 2 \times 12 = 33.$
*For the point (15,0)* $Cost = 3 \times 15 + 2 \times 0 = 45.$
The minimum cost is \$33 with the intersection point (3,12).
We found the right answer and we do not need to check options D and E. However, we'll show you their solutions as well.

**Option D:** Solve the equations $2x + y = 12 \; and \; y = 2x$ simultaneously. The intersection point is (3,6). Now we need to check if the point (3,6) minimises the shaded inequation or not.

Let's substitute the vertices of the boundary of the region into the cost equation $C = 3x + 2y$.
*For the point (3,6)* $Cost = 3 \times 3 + 2 \times 6 = 21.$
*For the point (6,0)* $Cost = 3 \times 6 + 2 \times 0 = 18.$
The minimum cost is \$18 with the point (6,0). The intersection point does not yield the minimum cost so option D is not the answer.

**Option E:** Solve the equations $x + 3y = 7 \; and \; y = 2x$ simultaneously. The intersection point is (1,2). Now we need to check if the point (1,2) minimises the shaded inequation or not.

Let's substitute the vertices of the boundary of the region into the cost equation $C = 3x + 2y$.
*For the point (1,2)* $Cost = 3 \times 1 + 2 \times 2 = 7.$
*For the point* $(0,\frac{7}{3})$ $Cost = 3 \times 0 + 2 \times \frac{7}{3} = 4.67.$
The minimum cost is \$4.67 with the point $(0,\frac{7}{3})$. The intersection point does not yield the minimum cost so option E is not the answer.

**Tip**

- *Solving the simultaneous equations by calculator will save you so much time and will help you minimise the number of calculation errors.*

## Module 4:Business-related mathematics

### Question 1

Jane invested her money in an account earning simple interest at a rate of 5.2% per annum in January 2005. When she withdrew all her money in January 2010 to buy a car, she had earned $2600 in interest. The total amount of money she withdrew to buy a car is

**A.** 10 000
**B.** 11 600
**C.** **12 600**
**D.** 14 400
**E.** 15 000

***Answer is C***

**Worked Solution**

$I = \frac{Prt}{100}$ (simple interest formula)

$$2\ 600 = \frac{P \times 5.2 \times 5}{100}$$
$P = 10\ 000$
$A = P + I$
$A = 10\ 000 + 2600 = \$12\ 600$

**Question 2**

Anthony wants to invest $300 000 in a bank for a period of 4 years. He can either invest his money in an account earning compound interest at a rate of 9% per annum compounding monthly or in an account earning simple interest at a rate of 10.7% per annum. The best option and amount extra in interest earned is given by

**A.** Simple interest is the best option by $15 650.20
**B.** Simple interest is the best option by $128 400
**C.** Compound interest is the best option by $301 021.60
**D.** Simple interest is the best option by $1248.80
**E.** **Compound interest is the best option by $1021.60**

***Answer is E***

**Worked Solution**

**Compound interest:**

$A = PR^n \textit{ where } R = 1 + \frac{9 \div 12}{100} = 1.0075$ (compounding monthly)
$A = 300\,000 \times 1.0075^{4\times12}$
$A = 429\,421.60$
$I = A - P$
$I = 429\,421.60 - 300\,000 = \$129\,421.60$

**Simple interest:**

$I = \frac{Prt}{100}$ (simple interest formula)

$I = \frac{300\,000 \times 10.7 \times 4}{100}$
$I = \$128\,400$

$\textit{Compound interest} - \textit{Simple interest} = \$129\,421.60 - \$128\,400 = \$1021.60$
$\therefore$ *Compound interest is the best option by* $1021.60

**Question 3**

Peter and Mary-Ellen borrowed $428 000 to purchase a house and land package. The loan was due to run for 30 years and attract interest at 6.5% per annum, debited quarterly on the outstanding balance. After 12 years the rate changed to 9.5% per annum. Peter and Mary-Ellen decided to change their payment method to monthly payments where interest is calculated monthly on the reducing balance of the loan. Their new repayments were $3100 per month. The total time it will take Peter and Mary-Ellen to pay off the loan completely is closest to

**A.** 30 years
**B.** 28 years
**C.** 36 years
**D.** **34 years**
**E.** 32 years

***Answer is D***

**Worked Solution**

We'll use *TVM* solver to find the quarterly repayments Peter and Mary-Ellen have to make originally.

N=120
I%=6.5
PV=428000
**PMT= -8129.954607**
FV=0
P/Y=4
C/Y=4

Now we'll calculate the amount outstanding when the rate changed at the end of the 12$^{th}$ year.

N=48
I%=6.5
PV=428000
PMT= -8129.954607
**FV= -343559.5958**
P/Y=4
C/Y=4

Now we'll find the term of the loan with 9.5% interest rate

**N=266.1320887**
I%=9.5
PV=343559.5958
PMT= -3100
FV=0
P/Y=12
C/Y=12

266.1320887÷12=22.2 years

**The total term of the loan**=12+22.2=34.2 years which is closest to **34 years**.

**Question 4**

Rob and Jane bought their cars at the same time from different car dealers. John's car costs \$53 760 and depreciates by the flat rate method. The depreciation rate of John's car is 12% of the prime cost price each year and its scrap value is \$2150.40. Jane's car costs \$32 000 and depreciates by the reducing balance method. The depreciation rate of Jane's car is 12% and it has a scrap value of \$1309. Which car will be written off first and how much later would the other car need to be replaced?

**A.** **Rob's car will be written off first and Jane's car will need to be replaced 17 years later**
**B.** Rob's car will be written off first and Jane's car will need to be replaced 25 years later
**C.** Both Rob's and Jane's cars will be written off at the same time
**D.** Jane's car will be written off first and Rob's car will need to be replaced 2 years later
**E.** Jane's car will be written off first and Rob's car will need to be replaced 8 years later

***Answer is A***

**Worked Solution**

**Rob's car depreciates by flat rate method.**

Total depreciation = Purchase price – Scrap value
= 53 760 – 2150.40
= 51 609.60

$$\text{Annual depreciation} = 53\ 760 \times \frac{12}{100} = 6451.20$$

$$\text{Number of years} = \frac{\text{total depreciation}}{\text{annual rate of depreciation}}$$

$$\text{Number of years} = \frac{51\ 609.60}{6451.20} = \mathbf{8\ years}$$

**Jane's car depreciates by reducing balance method.**

We'll find the total depreciation time by using *TVM solver*.

| |
|---|
| **N=25.00499187** |
| I%= -12 |
| PV= -32000 |
| PMT= 0 |
| FV=1309 |
| P/Y=1 |
| C/Y=1 |

Jane's car will depreciate for a total of 25 years and at the end of 25 years it will be written off.

∴ **Rob's car will be written off first and Jane's car will need to be replaced 17 years later.**

**Question 5**

Dr. Susan Lane is aged 47 and is planning to retire at 68 years of age. She estimates that she needs $1 200 000 to provide for her retirement. Her current superannuation fund has a balance of $150 000 and is delivering 8.4% per annum compounding monthly. The monthly contributions needed to meet her retirement lump sum target is closest to

**A.** $153
**B.** $277
**C.** $346
**D.** **$481**
**E.** $785

***Answer is D***

**Worked Solution**

Dr. Susan Lane has 21 years to grow her current superannuation fund of $150 000 to $ 1 200 000.

$$A = PR^n + \frac{Q(R^n - 1)}{R - 1} \; where \; R = 1 + \frac{8.4 \div 12}{100} = 1.007 \text{ (compounding monthly)}$$

$$1\,200\,000 = 150\,000 \times 1.007^{21\times 12} + \frac{Q(1.007^{21\times 12} - 1)}{1.007 - 1}$$

$\boldsymbol{Q = 481.2560244 \approx 481.26}$

Alternative using *TVM Solver*

N=252
I%= 8.4
PV= -150000
**PMT= -481.2560244**
FV=1200000
P/Y=12
C/Y=12

**Question 6**

Jasmine saw a laptop in an electronic shop and wanted to buy it. Although its marked price was $3200, after Jasmine's negotiation the shop owner offered her a discount of 12%. She paid $316 deposit and signed a hire-purchase agreement that she would pay the balance of the laptop's cost at 22% per annum flat rate with 24 equal monthly instalments.

The total price that Jasmine paid for the laptop is closest to

**A.** $3200
**B.** **$3916**
**C.** $3366
**D.** $4055
**E.** $4469

***Answer is B***

**Worked Solution**

$\text{Discounted price} = 3200 - 3200 \times \frac{12}{100} = 2816$

$\text{Amount owed} = 2816 - 316 = 2500$

$\text{Interest that will accumulate over 2 years} = \frac{2500 \times 22 \times 2}{100} = 1100$

$\textbf{Total price paid for the laptop} = \mathbf{2816 + 1100 = \$3916}$

**Question 7**

Serra invests \$290 000 in an annuity that earns 4.9% per annum interest compounded quarterly and pays her a quarterly sum of \$5890. The number of years for which this annuity continues is closest to

**A.** 10
**B.** 17
**C.** **19**
**D.** 21
**E.** 22

***Answer is C***

**Worked Solution**

We'll find the number of years for which the annuity continues by using *TVM solver.*

| |
|---|
| **N=75.90392609** |
| I%= 4.9 |
| PV= -290000 |
| PMT= 5890 |
| FV=0 |
| P/Y=4 |
| C/Y=4 |

$\textbf{The number of years} = \mathbf{75.90392609 \div 4 \cong 19}\ \textbf{years}$

**Tip**

- *PV is negative because Serra paid \$290 000 to the bank. PMT is positive because the bank paid \$5890 to Serra.*

**Question 8**

At the start of 2009 the price of a basketball was \$45.60 while Josh's annual salary was \$46 000. At the start of each subsequent year his annual salary increases by 3.42%. The predicted annual inflation rate between the years 2009 and 2013 is 6.4%.

In 2013 the predicted price of the basketball and Josh's annual salary will be closest to

**A.** **\$58.40 and \$52 623 respectively**
**B.** \$54.90 and \$50 883 respectively
**C.** \$57.30 and \$52 293 respectively
**D.** \$54.40 and \$50 720 respectively
**E.** \$52.20 and \$58 956 respectively

***Answer is A***

**Worked Solution**

We'll first calculate the predicted price of a basketball in 2013.

$$\text{The predicted price of a basketball} = PR^n \ where\ R = 1 + \frac{6.4}{100} = 1.064$$
$$= 45.60 \times 1.064^4$$
$$= \$58.44284571 \cong \$58.40$$

We'll now calculate Josh's annual salary in 2013.

$$\text{Josh's annual salary} = PR^n \ where\ R = 1 + \frac{3.42}{100} = 1.0342$$
$$= 46\ 000 \times 1.0342^4$$
$$= \$52\ 623.04388 \cong \$52\ 623$$

**Question 9**

John receives the following statement from his bank. Due to a technical error the interest and balances were not calculated.

| Date | Debit | Credit | Balance |
|---|---|---|---|
| 1 January | | | 25 156.70 |
| 5 January | | 2156.80 | |
| 14 January | | 175.45 | |
| 19 January | 218.65 | | |
| 21 January | | 612.15 | |
| 8 February | 4231.70 | | |
| 15 February | 105.20 | 55.20 | |
| 27 February | 67.90 | | |
| 3 March | | 125.50 | |
| 7 March | 762.00 | | |
| 18 March | | 235.10 | |
| 24 March | | 120.00 | |
| 31 March | 105.50 | | |

Interest is accrued at the end of March at a rate of 5.25% per annum calculated monthly on her minimum monthly balance. The total interest that John received in this statement is closest to

**A.** $252.50

**B.** $146.60

**C.** $453.40

**D.** $510.90

**E.** **$313.20**

***Answer is E***

**Worked Solution**

| Date | Debit | Credit | Balance |
|---|---|---|---|
| 1 January | | | 25 156.70 |
| 5 January | | 2156.80 | **27 313.50** |
| 14 January | | 175.45 | **27 488.95** |
| 19 January | 218.65 | | **27 270.30** |
| 21 January | | 612.15 | **27 882.45** |
| 8 February | 4231.70 | | **23 650.75** |
| 15 February | 105.20 | 55.20 | **23 600.75** |
| 27 February | 67.90 | | **23 532.85** |
| 3 March | | 125.50 | **23 658.35** |
| 7 March | 762.00 | | **22 896.35** |
| 18 March | | 235.10 | **23 131.45** |
| 24 March | | 120.00 | **23 251.45** |
| 31 March | 105.50 | | **23 145.95** |

Interest for January= $25\,156.70 \times \frac{5.25 \div 12}{100} = 110.0605625$

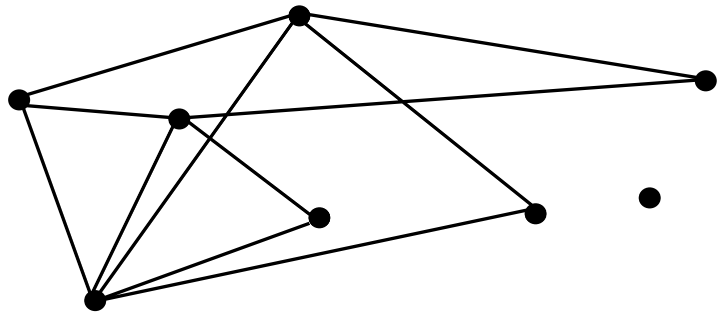
Interest for February= $23\,532.85 \times \frac{5.25 \div 12}{100} = 102.9562188$

Interest for March= $22\,896.35 \times \frac{5.25 \div 12}{100} = 100.1715313$

Total interest=110.0605625+102.9562188+100.1715313=313.1883126 ≈ $313.20

**Module 5: Networks and decision mathematics**

**Question 1**



The number of vertices with an odd degree in the graph above is

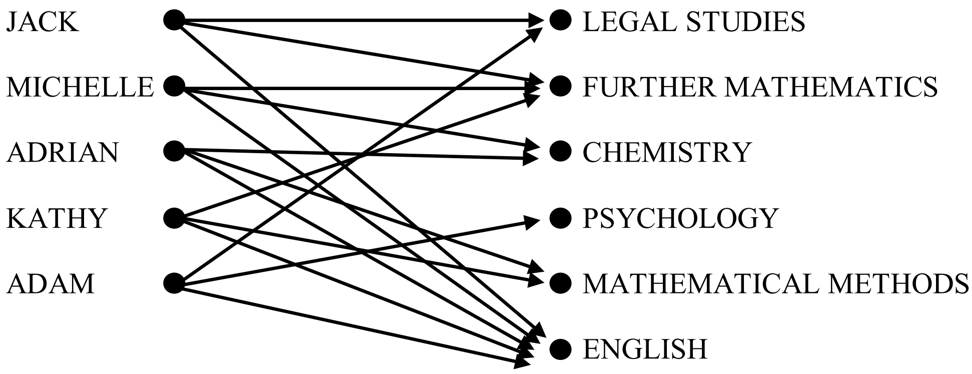
**A.** 1
**B.** **2**
**C.** 3
**D.** 4
**E.** 5

***Answer is B***

**Worked Solution**

Only two vertices have an odd degree.

**Question 2**
The bipartite graph below shows VCE subject preferences of five VCE students in a government school.




Which one of the following statements is not supported by the graph?

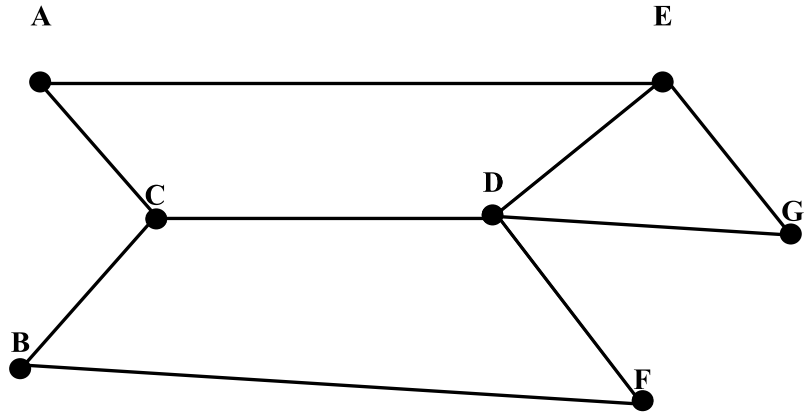
**A.** All five students have equal number of VCE subject preferences
**B.** English is the most popular subject
**C.** Further mathematics is the second mostly preferred subject
**D.** **Mathematical methods and psychology are equally preferred by students**
**E.** Michelle and Adrian both preferred to study chemistry

*Answer is D*

**Worked Solution**

Mathematical methods is preferred by 2 students while psychology is preferred by only one student.

**Question 3**




For the network shown above, a Hamiltonian circuit can be created by

**A.** adding an edge between A and B

**B.** adding an edge between F and G

**C.** removing the edge between B and C

**D.** removing the edge between D and G

**E.** **not changing the network, it is already a Hamiltonian circuit**

*Answer is E*

**Worked Solution**

We don't need to change the network because it's already a Hamiltonian circuit as stated in option E.

**Question 4**
A connected planar graph has 12 edges and 8 vertices. A further 3 vertices and 5 edges were added to the graph. The number of faces

**A.** **increased by 2**
**B.** decreased by 3
**C.** increased by 5
**D.** decreased by 8
**E.** remained the same

***Answer is A***

**Worked Solution**

Because the graph is connected and planar we can use Euler's formula $v + f = e + 2$.

$$Originally, \quad 8 + f = 12 + 2$$

$$f = 6$$

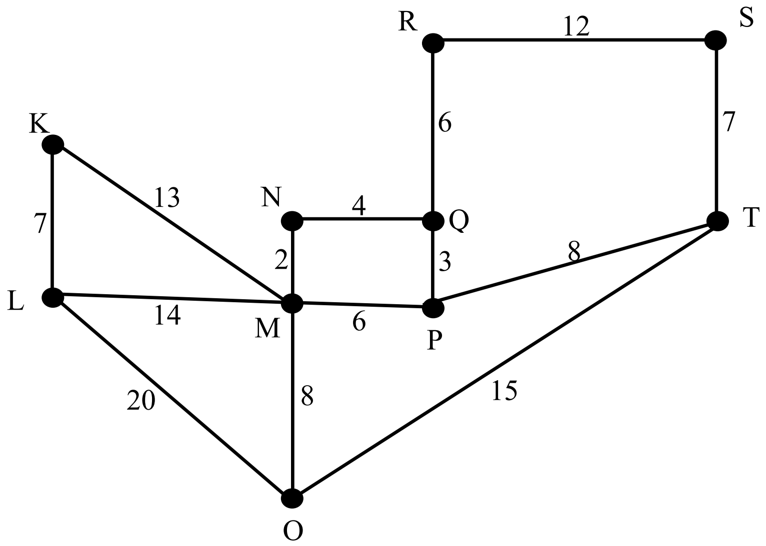
After the addition of 3 vertices and 5 edges,

$$11 + f = 17 + 2$$

$$f = 8$$

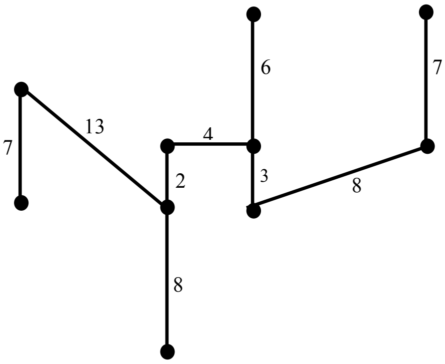
The number increased by 2.

**Question 5**

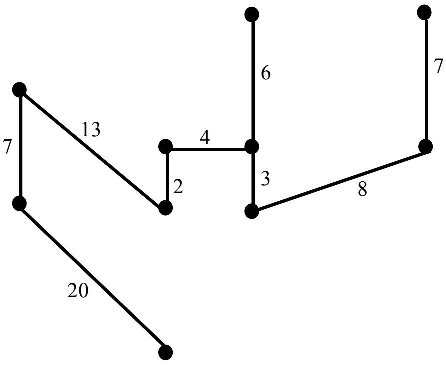


The minimum spanning tree for the above network is

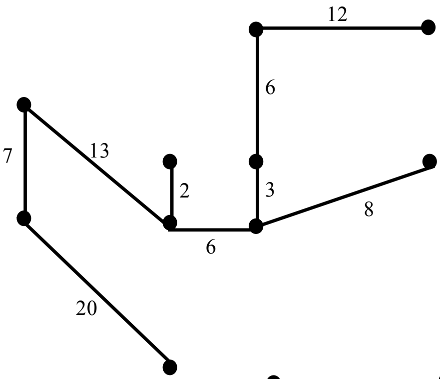
**A.**



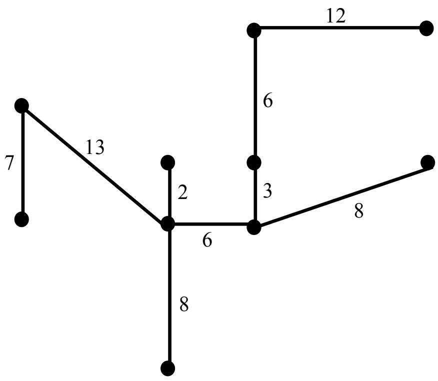

**B.**



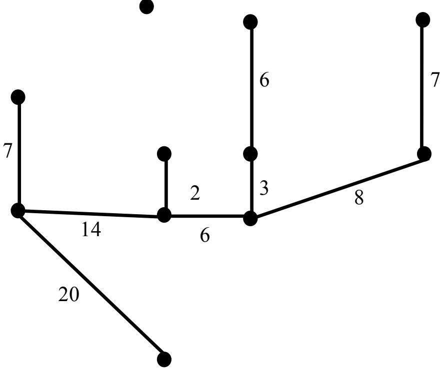

**C.**




**D.**




**E.**



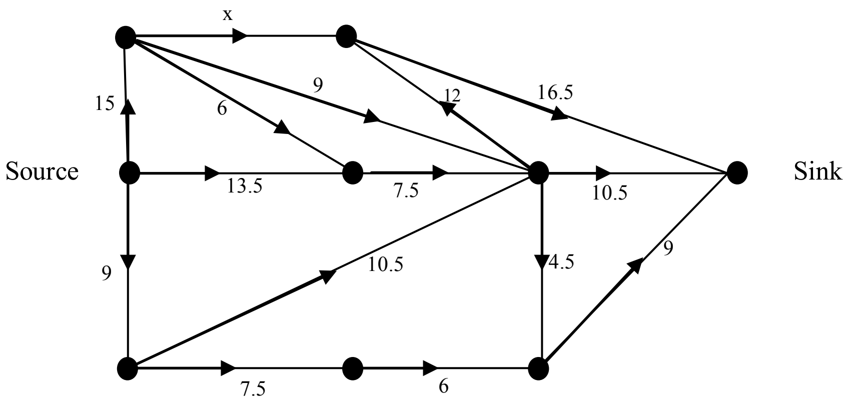

***Answer is A***

**Worked Solution**

Find the shortest edge: M-N (2). Add the next shortest edge to either M or N: M-N-O (4+2=6). Add the next shortest edge to M or O: M-N-O-P (4+2+3=9). Add the next shortest edge to M, O or P: O-R (6). Add the next shortest edge to M, R or P: P-T (8). Add the next shortest edge to M, R or T: P-T-S (8+7=15). Add the next shortest edge to M: M-O (8). Add the next shortest edge to M or O: M-K (13). Add the next shortest edge to K or O: K-L (7).

The minimum spanning tree has a length of 2+4+3+6+8+7+8+13+7=58.

**Question 6**



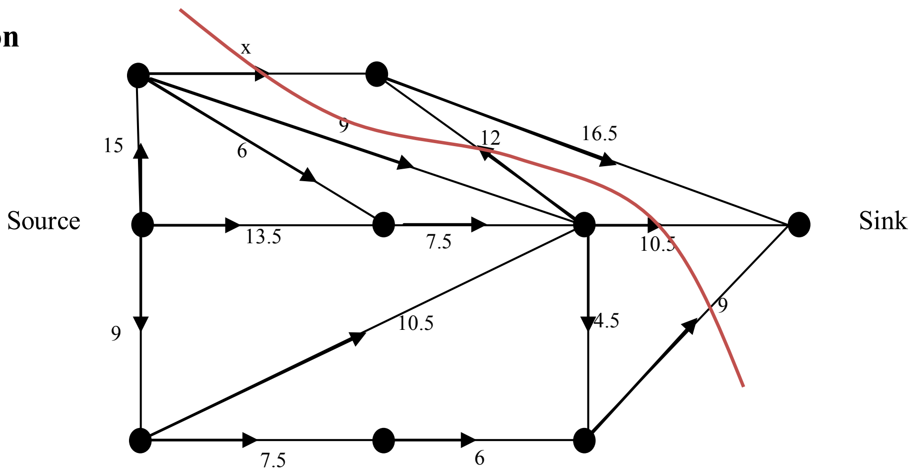

On the directed graph above the numbers on the edges give the maximum flow possible through each of those edges. The maximum flow possible through the network is 27.

The value of $x$ is

**A.** 12
**B.** 18
**C.** **7.5**
**D.** 9
**E.** 15

***Answer is C***

**Worked Solution**




There are no cuts possible with a capacity of 27 that don't include side $x$.

The cut shown has a capacity of

$$9 + 10.5 + x = 27$$

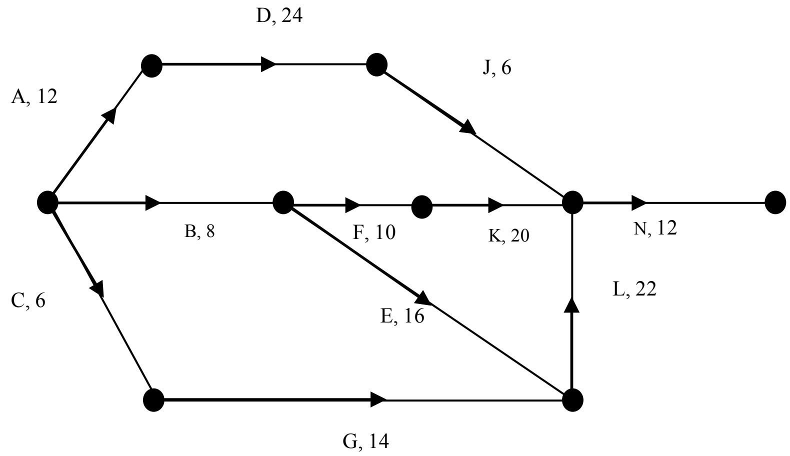
$$x = 7.5$$

**Tip**

- *The side with a flow of 12 is flowing from finish to start across the cut and so is counted as 0.*

**Question 7**

The network below shows 11 activities required to complete a project, together with the time it takes in days to complete each activity.

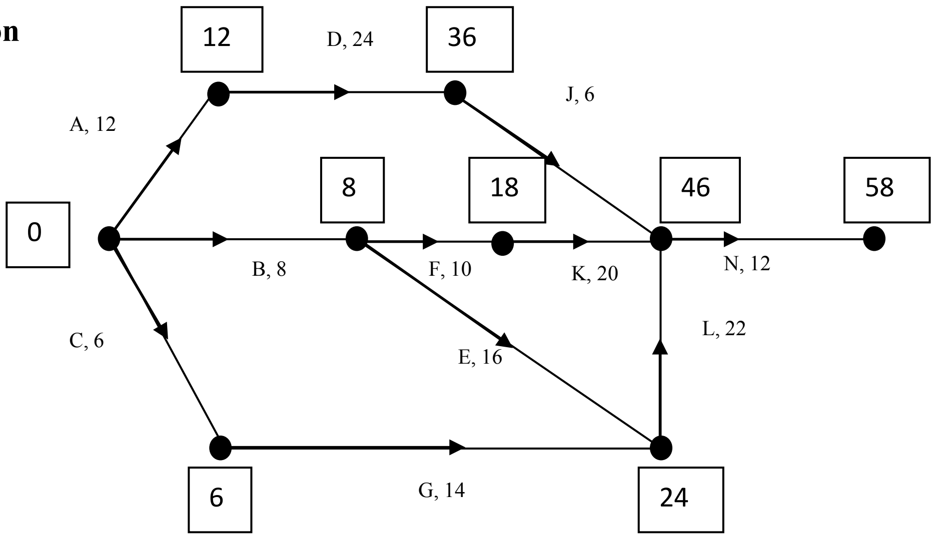



The critical path for this project has a length of

**A.** 54 days
**B.** 50 days
**C.** 44 days
**D.** **58 days**
**E.** 66 days

***Answer is D***

**Worked Solution**




The earliest start times are on the box. The critical path is *B,E,L,N*.

The length of the critical path is $8 + 16 + 22 + 12 = 58\ days$.

**Question 8**

Four different tutors have different hourly prices for tutoring different subjects which is shown on the table below.

Amy wants to get tutoring from all these subjects.

| | ***Tutors*** | | | |
|---|---|---|---|---|
| ***Subjects*** | June | Sharon | Sydney | Max |
| further mathematics | 32 | 25 | 12 | 27 |
| English | 45 | 26 | 30 | 50 |
| specialist mathematics | 22 | 34 | 41 | 23 |
| chemistry | 33 | 30 | 42 | 45 |

If she chooses to get each subject from a different tutor, which of the following allocations provides Amy's minimum cost for getting tutoring from all four subjects?

**A.** June →further maths, Sharon →English, Sydney →specialist maths, Max →chemistry

**B.** June →English, Sharon →further maths, Sydney →specialist maths, Max →chemistry

**C.** June →English, Sharon →specialist maths, Sydney →further maths, Max →chemistry

**D.** June →specialist maths, Sharon →chemistry, Sydney →further maths, Max →English

**E.** **June →chemistry, Sharon →English, Sydney →further maths, Max →specialist maths**

***Answer is E***

**Worked Solution**

Step 1: Perform row reduction.

$$\begin{bmatrix} 32 & 25 & 12 & 27 \\ 45 & 26 & 30 & 50 \\ 22 & 34 & 41 & 23 \\ 33 & 30 & 42 & 45 \end{bmatrix} \rightarrow \begin{bmatrix} 20 & 13 & 0 & 15 \\ 19 & 0 & 4 & 24 \\ 0 & 12 & 19 & 1 \\ 3 & 0 & 12 & 15 \end{bmatrix}$$

Only 3 lines- cannot continue allocation.

Step 2: Perform column reduction.

$$\begin{bmatrix} 20 & 13 & 0 & 15 \\ 19 & 0 & 4 & 24 \\ 0 & 12 & 19 & 1 \\ 3 & 0 & 12 & 15 \end{bmatrix} \rightarrow \begin{bmatrix} 20 & 13 & 0 & 14 \\ 19 & 0 & 4 & 23 \\ 0 & 12 & 19 & 0 \\ 3 & 0 & 12 & 14 \end{bmatrix}$$

Only 3 lines- cannot continue allocation.

Step 3: Perform the Hungarian algorithm. Find the smallest uncovered number from step 2. Add this number to all covered numbers. At the intersections of straight lines, add this number twice. Then subtract the overall smallest number from all the numbers in the matrix.

$$\begin{bmatrix} 20 & 13 & 0 & 14 \\ 19 & 0 & 4 & 23 \\ 0 & 12 & 19 & 0 \\ 3 & 0 & 12 & 14 \end{bmatrix} \rightarrow \begin{bmatrix} 20 & 16 & 3 & 14 \\ 19 & 3 & 7 & 23 \\ 3 & 18 & 25 & 3 \\ 3 & 3 & 15 & 14 \end{bmatrix} \rightarrow \begin{bmatrix} 17 & 13 & 0 & 11 \\ 16 & 0 & 4 & 20 \\ 0 & 15 & 22 & 0 \\ 0 & 0 & 12 & 11 \end{bmatrix}$$

There are 4 lines so we can now start our allocations. Sydney should be allocated to further mathematics. (She is the only one at FM) Sharon should be allocated to English (She is the only one at English) Sharon and June can be allocated to chemistry. Since Sharon is allocated to English, June must be allocated to chemistry. Max should be allocated to specialist mathematics.

**Question 9**

Six teams, B, C, D, E, F, G and H have played in a round-robin tournament where each team plays each other team only once. The results of the games are represented by the dominance matrix M, below, where a "1" in the matrix represents a win and a "0" represents a loss.

$$A= \begin{array}{c} \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \begin{array}{c} \begin{array}{cccccc} B & C & D & E & F & G \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Using A and $A^2$ to rank the teams, the ranking, first to last will be

**A.** **BCEGDF**
**B.** CBEGFD
**C.** GBCFDE
**D.** BEDCGF
**E.** CBDFGE

***Answer is A***

**Worked Solution**

$$A + A^2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 2 & 1 & 3 & 0 & 2 \\ 0 & 2 & 1 & 3 & 0 & 3 \\ 1 & 2 & 0 & 1 & 0 & 1 \\ 2 & 2 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 2 & 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 2 & 4 & 0 & 3 \\ 1 & 2 & 2 & 4 & 0 & 3 \\ 1 & 2 & 0 & 2 & 0 & 2 \\ 2 & 3 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 2 & 2 & 2 & 2 & 0 & 1 \end{bmatrix}$$

Determine the sum of the each row of the resultant matrix.

B → 2+3+2+4+0+3=14, C → 1+2+2+4+0+3=12, D → 1+2+0+2+0+2=7, E → 2+3+1+3+0+2=11, F → 0+0+0+0+2+0=2, G → 2+2+2+2+0+1=9. Thus the winning team is B, the rest are CEGDF.

**Module 6: Matrices**

**Question 1**

If $5\begin{bmatrix}-4x & -6 \\ 6 & 2y\end{bmatrix} - 3\begin{bmatrix}y & 5 \\ 7 & 4x\end{bmatrix} = \begin{bmatrix}-3 & -45 \\ 9 & 10\end{bmatrix}$, then the values of x and y are

**A.** $x = 2, y = 1$
**B.** $\boldsymbol{x = 0, y = 1}$
**C.** $x = 2, y = -1$
**D.** $x = -1, y = 2$
**E.** $x = 0, y = 0$

***Answer is B***

**Worked Solution**

$$\begin{bmatrix}-20x - 3y & -30 - 15 \\ 30 - 21 & 10y - 12x\end{bmatrix} = \begin{bmatrix}-3 & -45 \\ 9 & 10\end{bmatrix}$$

Solve two equations simultaneously.

Equation 1: $-20x - 3y = -3$

Equation 2: $10y - 12x = 10$

$x = 0, y = 1$

**Question 2**

If $A = \begin{bmatrix}1 & -3 & 2 & 5 & 0 \\ 0 & 2 & 6 & 7 & 2 \\ 3 & 4 & 5 & 1 & 0\end{bmatrix}$ and $B = \begin{bmatrix}5 \\ 7 \\ 9 \\ 1 \\ 8\end{bmatrix}$, then the order of the matrix product A×B is

**A.** (3×5)
**B.** (5×3)
**C.** **(3×1)**
**D.** (1×3)
**E.** Undefined

***Answer is C***

**Worked Solution**

*A* is a (3×5) matrix. *B* is a (5×1) matrix. The matrix product *AB* is of order **(**3×1).

**Question 3**

Given $= \begin{bmatrix} 2 & 1 & 3 & 0 \\ 4 & 2 & 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ 4 & -3 \end{bmatrix}$ and $C = \begin{bmatrix} 5 & 1 \\ 1 & 3 \\ 2 & 6 \\ 2 & 2 \end{bmatrix}$

The evaluation of 3(A×C-B) is

**A.** $\begin{bmatrix} 48 & 70 \\ 98 & 81 \end{bmatrix}$

**B.** $\begin{bmatrix} 42 & 66 \\ 90 & 69 \end{bmatrix}$

**C.** $\begin{bmatrix} 48 & 68 \\ 98 & 75 \end{bmatrix}$

**D.** $\begin{bmatrix} \mathbf{42} & \mathbf{72} \\ \mathbf{90} & \mathbf{87} \end{bmatrix}$

**E.** cannot be evaluated

***Answer is D***

**Worked Solution**

$$3\left(\begin{bmatrix} 2 & 1 & 3 & 0 \\ 4 & 2 & 1 & 5 \end{bmatrix} \times \begin{bmatrix} 5 & 1 \\ 1 & 3 \\ 2 & 6 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 4 & -3 \end{bmatrix}\right) = 3\left(\begin{bmatrix} 17 & 23 \\ 34 & 26 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 4 & -3 \end{bmatrix}\right) = 3 \times \begin{bmatrix} 14 & 24 \\ 30 & 29 \end{bmatrix} = \begin{bmatrix} 42 & 72 \\ 90 & 87 \end{bmatrix}$$

**Question 4**

The determinant of $A = \begin{bmatrix} -2 & -5 \\ x & 7 \end{bmatrix}$ is equal to 1.

The determinant of $B = \begin{bmatrix} 6 & -2y \\ -2 & 3 \end{bmatrix}$ is equal to 2. The value of $\frac{8x}{y}$ is

**A.** -4
**B.** 3
**C.** 1.2
**D.** **6**
**E.** 4.16

***Answer is D***

**Worked Solution**

$\det(A): -2(7) + 5x = 1 \rightarrow x = 3$

$\det(B):\ 6(3) - 4y = 2 \rightarrow y = 4$

$\frac{8x}{y} = \frac{8(3)}{4} = 6$

**Question 5**

Joan, Brian and Harry went to the vegetable market for their weekly shopping. Joan bought 3 kg of cucumber, 4 kg of broccoli and 1 kg of tomatoes and paid \$19.60. Brian bought 5 kg of cucumber, 2 kg of broccoli and 1.5 kg of tomatoes and paid \$21.80. Harry bought 2 kg of cucumber, 2 kg of broccoli and 4.5 kg of tomatoes and paid \$23.50. They all shopped from the same stall. Let x, y, z represent the cost of one kg of tomatoes, cucumber and broccoli respectively. A matrix equation to calculate the cost of each item (per kg) is

**A.** $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left(\begin{bmatrix} 3 & 4 & 1 \\ 5 & 2 & 1.5 \\ 2 & 2 & 4.5 \end{bmatrix}\right)^{-1} \times \begin{bmatrix} 19.60 \\ 21.80 \\ 23.50 \end{bmatrix}$

**B.** $\begin{bmatrix} 3 & 4 & 1 \\ 5 & 2 & 1.5 \\ 2 & 2 & 4.5 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 19.60 \\ 21.80 \\ 23.50 \end{bmatrix}$

**C.** $\begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & 1.5 \\ 2 & 2 & 4.5 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 23.50 \\ 21.80 \\ 19.60 \end{bmatrix}$

**D.** $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left(\begin{bmatrix} 3 & 1 & 4 \\ 5 & 1.5 & 2 \\ 2 & 4.5 & 2 \end{bmatrix}\right)^{-1} \times \begin{bmatrix} 19.60 \\ 21.80 \\ 23.50 \end{bmatrix}$

**E.** $\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{bmatrix} = \left(\begin{bmatrix} \mathbf{1.5} & \mathbf{5} & \mathbf{2} \\ \mathbf{1} & \mathbf{3} & \mathbf{4} \\ \mathbf{4.5} & \mathbf{2} & \mathbf{2} \end{bmatrix}\right)^{-1} \times \begin{bmatrix} \mathbf{21.80} \\ \mathbf{19.60} \\ \mathbf{23.50} \end{bmatrix}$

***Answer is E***

**Worked Solution**

x: the cost of a kg of tomatoes, y: the cost of a kg of cucumber, z: the cost of a kg of broccoli

$\begin{bmatrix} 1 & 3 & 4 \\ 1.5 & 5 & 2 \\ 4.5 & 2 & 2 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 19.60 \\ 21.80 \\ 23.50 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left(\begin{bmatrix} 1 & 3 & 4 \\ 1.5 & 5 & 2 \\ 4.5 & 2 & 2 \end{bmatrix}\right)^{-1} \times \begin{bmatrix} 19.60 \\ 21.80 \\ 23.50 \end{bmatrix}$ which is the same as

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left(\begin{bmatrix} 1.5 & 5 & 2 \\ 1 & 3 & 4 \\ 4.5 & 2 & 2 \end{bmatrix}\right)^{-1} \times \begin{bmatrix} 21.80 \\ 19.60 \\ 23.50 \end{bmatrix}$$

**Tip**

- *Be very careful with the order of items. Although x represents the cost of 1 kg of tomatoes, it's given as the third item in the question. The first item in the question is given as cucumber which is represented by y. Moreover in option E, which is the right option, Joan's and Brian's equations were swapped over.*

*The following information relates to Questions 6 to 7.*

An ice cream factory conducted a survey in December 2009 on the ice cream sales of their three shops. The following information relates to this survey.

- 15% of shop A customers will move to shop B the following month.
- 37% of shop A customers will move to shop C the following month.
- 48% of shop B customers will move to shop C the following month.
- 42% of shop B customers will remain at shop B the following month.
- 5% of shop C customers will move to shop B the following month.
- 10% of shop C customers will move to shop A the following month.

The market share at the time of the survey showed that shop A had 1800 customers, shop B had 1000 customers and shop C had 1300 customers.

**Question 6**

The transition matrix showing the way customers move between the ice cream shops is given by

**A.** This month

$$\text{Next month}\quad \begin{array}{c} A \\ B \\ C \end{array}\overset{\begin{array}{ccc} A & B & C \end{array}}{\begin{bmatrix} 0.48 & 0.1 & 0.1 \\ 0.15 & 0.42 & 0.05 \\ 0.37 & 0.48 & 0.85 \end{bmatrix}}$$

**B.** This month

$$\text{Next month}\quad \begin{array}{c} A \\ B \\ C \end{array}\overset{\begin{array}{ccc} A & B & C \end{array}}{\begin{bmatrix} 0.48 & 0.42 & 0.05 \\ 0.15 & 0.01 & 0.85 \\ 0.37 & 0.48 & 0.1 \end{bmatrix}}$$

**C.** This month

$$\text{Next month}\quad \begin{array}{c} A \\ B \\ C \end{array}\overset{\begin{array}{ccc} A & B & C \end{array}}{\begin{bmatrix} 0.15 & 0.48 & 0.05 \\ 0.37 & 0.42 & 0.1 \\ 0.48 & 0.1 & 0.85 \end{bmatrix}}$$

**D.** This month

$$\text{Next month}\quad \begin{array}{c} A \\ B \\ C \end{array}\overset{\begin{array}{ccc} A & B & C \end{array}}{\begin{bmatrix} 15 & 48 & 5 \\ 37 & 42 & 10 \\ 48 & 10 & 85 \end{bmatrix}}$$

**E.** This month

$$\text{Next month}\quad \begin{array}{c} A \\ B \\ C \end{array}\overset{\begin{array}{ccc} A & B & C \end{array}}{\begin{bmatrix} 48 & 1 & 10 \\ 15 & 42 & 5 \\ 37 & 48 & 85 \end{bmatrix}}$$

***Answer is A***

**Worked Solution**

Set up the transition matrix by writing the percentages in decimal form and complete any missing value knowing the columns must add up to 1.

**Question 7**

The number of customers expected to remain at each shop in September 2010 is

**A.** Shop A: 1094 customers, Shop B: 755 customers, Shop C: 2251 customers
**B.** **Shop A: 661 customers, Shop B: 431 customers, Shop C: 3008 customers**
**C.** Shop A: 1024 customers, Shop B: 1071 customers, Shop C: 1027 customers
**D.** Shop A: 638 customers, Shop B: 857 customers, Shop C: 2606 customers
**E.** Shop A: 537 customers, Shop B: 1220 customers, Shop C: 2056 customers

***Answer is B***

**Worked Solution**

Initial state matrix: $S_0 = \begin{bmatrix} 1800 \\ 1000 \\ 1300 \end{bmatrix}$. (in December 2009)

$S_9 = T^9 \times S_0 = \begin{bmatrix} 0.48 & 0.1 & 0.1 \\ 0.15 & 0.42 & 0.05 \\ 0.37 & 0.48 & 0.85 \end{bmatrix}^{\mathbf{9}} \times \begin{bmatrix} 1800 \\ 1000 \\ 1300 \end{bmatrix} = \begin{bmatrix} 661.48 \\ 430.84 \\ 3007.68 \end{bmatrix}$. (in September 2009, n=9)

∴Shop A is expected to have 661 customers, shop B is expected to have 431 customers and shop C is expected to have 3008 customers in September 2010.

**Question 8**

The long-term share of customers shopping at each shop is expected to be

**A.** Shop A: 755 customers, Shop B: 896 customers, Shop C: 2341 customers
**B.** Shop A: 645 customers, Shop B: 428 customers, Shop C: 3003 customers
**C.** Shop A: 1065 customers, Shop B: 1056 customers, Shop C: 1035 customers
**D.** **Shop A: 661 customers, Shop B: 430 customers, Shop C: 3008 customers**
**E.** Shop A: 587 customers, Shop B: 1343 customers, Shop C: 2234 customers

***Answer is D***

**Worked Solution**

We need to find the steady state matrix.

$S_{30} = T^{30} \times S_0 = \begin{bmatrix} 0.48 & 0.1 & 0.1 \\ 0.15 & 0.42 & 0.05 \\ 0.37 & 0.48 & 0.85 \end{bmatrix}^{\mathbf{30}} \times \begin{bmatrix} 1800 \\ 1000 \\ 1300 \end{bmatrix} = \begin{bmatrix} 661.29 \\ 430.36 \\ 3008.35 \end{bmatrix}$.

$S_{50} = T^{50} \times S_0 = \begin{bmatrix} 0.48 & 0.1 & 0.1 \\ 0.15 & 0.42 & 0.05 \\ 0.37 & 0.48 & 0.85 \end{bmatrix}^{\mathbf{50}} \times \begin{bmatrix} 1800 \\ 1000 \\ 1300 \end{bmatrix} = \begin{bmatrix} 661.29 \\ 430.36 \\ 3008.35 \end{bmatrix}$.

Since $S_{30}$ and $S_{50}$ are equal, we can say that the matrix has reached its steady state position. ∴the long-term share of customers shopping at each shop is Shop A: 661 customers, Shop B: 430 customers, Shop C: 3008 customers.

**Question 9**

The movement of university students doing biomedical science course between two universities, university $A$ and university $B$ can be predicted by the transition matrix $T$ where

$$\begin{array}{cc} & \text{this year} \\ & \begin{array}{cc} A & B \end{array} \\ \text{next year} \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0.3 & 0.1 \\ 0.7 & 0.9 \end{bmatrix} \end{array}$$

The number of biomedical science students in university A was twice as much as the number of biomedical science students in university B in 2009. In 2010 there were 115 students doing biomedical science in university B. The number of biomedical science students in university A in 2009 was

**A.** 50
**B.** 65
**C.** 82
**D.** 112
**E.** **100**

***Answer is E***

**Worked Solution**

Let the number of biomedical science students in university $B$ in 2009 be $x$ so the number of biomedical science students in university $A$ in 2009 is $2x$. Let the number of biomedical science students in university $A$ in 2010 be $y$. So,

$$\begin{bmatrix} 0.3 & 0.1 \\ 0.7 & 0.9 \end{bmatrix} \times \begin{bmatrix} 2x \\ x \end{bmatrix} = \begin{bmatrix} y \\ 115 \end{bmatrix}$$

Multiplying second row of the first matrix by the second matrix, we get

$0.7 \times 2x + 0.9 \times x = 115$

$1.4x + 0.9x = 115$

$2.3x = 115$

$x = 50$

The number of biomedical science students in university $A$ in 2009 was 2×50=100.

**Tip**

- *We multiplied 50 by 2 because the number of biomedical students in university A in 2009 is 2x. We are not asked to evaluate the value of y.*

**END OF SOLUTIONS PAPER**