Neap:

Trial Examination 2010

VCE Further Mathematics Units 3 & 4

Written Examination 1

Suggested Solutions

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SECTION A – Core: Data analysis

Question 1 **E**

The 194 figure is an outlier which makes mean, range and standard deviation not appropriate.

Question 2 **B**

There are twenty pieces of data on both sides so the median is between the 10th and 11th value, 9.85 for both.

Question 3 **B**

Calculate the five-figure summary for each set. Options **A** and **C** have the wrong Q_3, **D** has the incorrect minimum value, and **E** has the incorrect maximum value.

Question 4 **C**

$$\frac{(3 \times 47 + 8 \times 48 + 10 \times 49 + 12 \times 50 + 6 \times 51 + 1 \times 52)}{40}$$

median = 49
mean = 49.3
$Q_1 = 48$
$Q_3 = 50$
IQR = 2
range = 5

Question 5 **D**

The answer is **D** as the correlation is negative, so an increase in one decreases the other.

Question 6 **D**

50% + 34% = 84%

Question 7 **B**

Points are (3, 5) and (20, 15), gradient $= \frac{10}{17}$.

Question 8 **D**

The depth of snow will depend on the time of year.

Question 9 **B**

	Q_1
2008	$\frac{42\ 000}{25\ 000} = 1.68$
2009	$\frac{52\ 000}{40\ 000} = 1.3$
Seasonal index	$\frac{(1.68 + 1.3)}{2} = 1.49$

Option **A** gives the figure for $Q_1 2008$, **C** gives the figure for $Q_1 2009$, **D** gives the mean sales in Q_1 over two years, **E** gives total sales in Q_1 over two years.

Question 10 **D**

$\frac{32\ 000}{0.82} = 39\ 020$

Option **A** gives the incorrect answer by using the 2008 figure and multiplying, **B** gives the incorrect answer by using the 2008 figure, **C** gives the incorrect answer by multiplying by the SI instead of dividing and **E** gives the incorrect answer by adding both years' figures before dividing.

Question 11 **E**

$49 \times 30 + 763 = 2233$

Answer is **E** by substitution of 30 into the formula for the hours worked.

Question 12 **A**

Using the regression to predict the volume for 35 hours and then applying the formula:

residual = actual – predicted

$= 2400 - (763 + 49 \times 35)$

$= -78$

Question 13 **A**

Answer is **A** by using values for the log of hours worked and applying a linear regression.

SECTION B

Module 1: Number patterns

Question 1 **E**

Students can achieve this result by three possible methods. Those familiar with logs could calculate in that manner. The second method is to use a calculator: $3(2)^{n-1} = 1000$. Both of these methods would achieve a result of $n = 9.3808$. It would be necessary to round up to 10 as it is not possible to have non-integer term numbers. Finally, students could just calculate terms until 1000 is exceeded.

Question 2 **D**

This is a straight calculation question. It is necessary however, to note that the common ratio is $\frac{-1}{2}$ not $\frac{1}{2}$.

$$S_\infty = \frac{a}{1-r} = \frac{16}{1+0.5}$$
$$= \frac{32}{3}$$

Question 3 **E**

To be an arithmetic sequence, the graphed points must form a straight line if joined. In the case of this sequence, no single straight line could connect the points. Thus the sequence concerned is not arithmetic.

To be geometric, successive points must be in a common ratio. Thus there will never be a common difference and points cannot form a straight line in any section. The points on the graph shown however, do form straight lines in two sections, so geometric sequence formation is also untrue. Option **E** is all that remains.

Question 4 **C**

This can be done logically or computationally. The mean term is 10 and there are 5 terms. Thus the sum is $5 \times 10 = 50$.

Alternatively:

$a = 20$; $d = -5$

$$S_5 = \frac{5}{2}[2(20) + 4(-5)]$$
$$= \frac{5}{2}(20) = 50$$

Question 5 **C**

The sequence of events here is crucial. The interest calculation occurs prior to the repayment, so the existing term is first multiplied by 1.05 (105%) to add interest to principle. The remaining options either calculate interest after the payment is made or only calculate the interest and neglect to add to principle.

Question 6 **C**

$a = 27$

$S_5 = 25$

$\frac{5}{2}[54 + 4d] = 25$

$54 + 4d = 10$

$-44 = 4d$

$d = -10$

$t_2 = 27 - 11 = 16$

Question 7 **B**

This is a reference to a difference equation:

$t_{n+2} = 0.6(t_n + t_{n+1})$

$t_0 = 1280$

$t_1 = 1620$

$t_2 = 0.6(1280 + 1620) = 1740$

Question 8 **A**

$S_6 = \frac{a(r^6 - 1)}{r - 1} = 0$

$a(r^6 - 1) = 0$

Thus $a = 0$ or $r^6 = 1$.

We know the sequence is non-zero so that rules out $a = 0$ as a solution. If $r = 1$ then all terms are equal. For a non-zero sequence, obtaining a zero sum would be impossible. Thus $r = -1$.

Question 9 **D**

$t_{n+2} = 3t_n - t_{n+1}$, $t_1 = 2$, $t_2 = 5$

$t_3 = 3(5) - 2 = 1$

$t_4 = 3(13) - 5 = 14$

The process is one of straight calculation.

Module 2: Geometry and trigonometry

Question 1 **B**

$\frac{\text{opp}}{\text{hyp}} = \sin\theta$

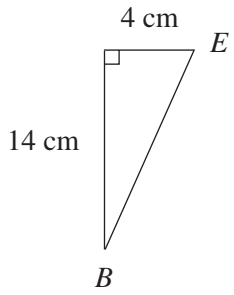
$\frac{0.7}{1.3} = \sin\theta$

$32.579° \ldots = \theta$

Question 2 **A**

$100 \text{ m} - 100 \text{ m} = 0 \text{ m}$

Question 3 **B**



$h^2 = a^2 + b^2$

$h^2 = 4^2 + 14^2$

$h^2 = 212$

$EB = 14.5602 \ldots \text{cm}$

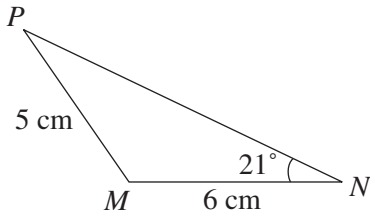
$EB \approx 15 \text{ cm}$

Perimeter = 12 + 14 + 16 + 15

= 57 cm

Question 4 E

Distance MP is 5 cm. This can be found by pythagoras' theorum or by recognition of a 3–4–5 triangle.



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{6}{\sin MPN} = \frac{5}{\sin 21°}$$

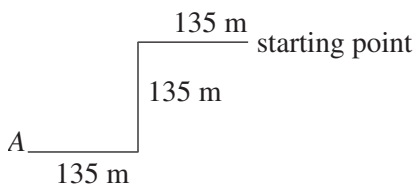
$$\frac{\sin MPN}{6} = \frac{\sin 21°}{5}$$

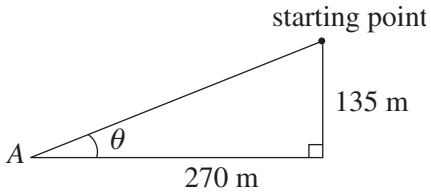
$$\sin MPN = \frac{6 \times \sin 21°}{5}$$

$$\sin MPN = 0.430042\ldots$$

$$\text{angle } MPN = 25.4702$$

Question 5 B





$$\frac{\text{opp}}{\text{adj}} = \tan\theta$$

$$\frac{135}{270} = \tan\theta$$

$$26.5651° \ldots = \theta$$

$$27° \approx \theta$$

$$\therefore \text{bearing } A \rightarrow \text{starting point}$$

$$90 - 27° = 63°$$

Question 6 **D**

Use of centimetres

$$S = \frac{a + b + c}{2}$$

$$S = \frac{92 + 115 + 193}{2}$$

$$S = \frac{400}{2}$$

$$S = 200$$

Question 7 **A**

The calculation for surface area must **not** combine different units, in this case centimetres and metres.

Each option from **B** to **E** combines centimetres and metres, for example **B** includes 0.4 m with 100 cm.

Question 8 **D**

The dimensions of the square end must be $9 \text{ cm} \times 9 \text{ cm}$. Therefore the length of the prism must be 18 cm.

The length of the diagonal of a $9 \text{ cm} \times 9 \text{ cm} \times 18 \text{ cm}$ rectangular prism is found

by $\sqrt{9^2 + 9^2 + 18^2} = 22.0454\ldots \text{cm}$

Question 9 **E**

$$\frac{\text{distance large}}{\text{distance small}} = k = \frac{140}{70} = 2$$

$$\frac{\text{volume of large}}{\text{volume of small}} = k^3 = 2^3 = 8$$

This tells us that the entire cone is larger than the cone removed by a factor of 8.

Examples include: 50 cm^3 and 400 cm^3, 100 cm^3 and 800 cm^3, etc.

It is not necessary to know the actual volume of either cone to recognise that the fraction remaining is $\frac{7}{8}$ of the original cone.

Module 3: Graphs and relations

Question 1 **E**

Firstly, it is necessary to determine the gradient of the line concerned.

$$m = \frac{5 - -1}{-6 - 3} = -\frac{2}{3}$$

Options **A**, **B** and **C** all clearly have incorrect gradients and are thus invalid. The gradient of the equation in option **D** can be calculated and is also seen to be incorrect. Option **E** is the only equation with the correct gradient.

Question 2 **A**

There are two sections involved. The first has a gradient of one, and passes through the origin, thus it is $P = V$. This extends from $V = 0$ to $V = 3$.

The second section extends from $V = 3$ and passes through (7, 15).

$$m = \frac{15 - 3}{7 - 3} = 3$$

$$P = 3V + c$$

Use (3, 3):

$$3 = 9 + c$$

$$c = -6$$

$$P = 3V - 6$$

Question 3 **B**

The easiest method to apply is the elimination method.

$2x + 5y = 8 \quad \ldots 1$

$4x + 3y = 2 \quad \ldots 2$

$(2 \times (1)) \quad :$

$4x + 10y = 16 \quad \ldots 1a$

(subtract (2) from (1a))

$7y = 14$

$y = 2$

(sub in (1))

$2x + 10 = 8$

$x = -1$

Question 4 **C**

The equation that we require is one that states that the cost of producing each page is equal to the revenue resulting from it:

$$20x + 2000 = 200x$$

Question 5 **D**

The relationship is clearly not linear. The values of y are decreasing toward zero so quadratic and cubic relationships are also impossible. Thus options **A**, **B** and **C** are ruled out. Students can test option **D** by plotting a few points on axes y against $\frac{1}{x}$. The result is a straight line and is clearly the correct response.

Question 6 **B**

Option **C** is easily seen to be false as it is virtually the opposite of that which is required. Option **E** is also incorrect for the same reason. It specifies that the required region is located below the boundary with the negative slope, when in fact, we require the region above this line. Options **A** and **D** also represent incorrect sides of a line – the boundary with a positive slope. Option **B** designates the correct (below) side of this boundary.

Question 7 **C**

It is necessary to identify the corners of the unshaded region and to calculate the objective function at each. The corners are $P(0, 18)$ $[C = 54]$; $Q(3, 9)$ $[C = 39]$; $R(8, 4)$ $[C = 44]$ and $S(16, 0)$ $[C = 64]$. The minimum value is thus 39.

Question 8 **B**

Inequations must be produced for each of the three conditions specified in the question. Firstly, for protein – the total units of protein in x P tablets is $20x$ and there is $30y$ among the y Q tablets. Since a minimum of 120 units is required, we achieve the inequation $20x + 30y \geq 120$. Constraints on the vitamins required a likewise result of $20x + 30y \geq 100$. Finally, it is required that x exceeds y. Thus option **B** has all 3 correct inequations.

Question 9 **D**

The task is to determine the month for which the graph is steepest upward or downward. This is August. Students may erroneously choose the month for which the price is highest (September).

Module 4: Business-related mathematics

Question 1 **D**

$18 \times 1.15 = \$20.70$

Question 2 **C**

$$\left(\frac{15\,000 \times 10 \times 3}{100} + 15\,000\right) \div 36 = \$541.67$$

Question 3 **D**

$$\frac{\frac{6.5}{100} \times 80}{12} = 43 \text{ cents } (+\$580) = \$580.43$$

Question 4 **D**

Repayments are $24 \times 55.47 = 1331.28$. After the deposit is paid, the amount owing is $1280 \times 0.8 = 1024$. The interest charged is $1331.28 - 1024 = 307.28$ The simple interest formula gives an answer for *r*.

$$r = \frac{100I}{PT} = \frac{100 \times 307.28}{1024 \times 2} = 15\%$$

Question 5 **A**

$$38\,000 \times \left(1 + \frac{1.8}{10}\right)^{10} = 45\,421.49$$

Question 6 **C**

$$A = PR^n$$

$$9000 = 7500\left(1 + \frac{5.6}{\left(\frac{100}{12}\right)}\right)^x$$

$$1.2 = 1.00467^x$$

$$\log 1.2 = x \log 1.00467$$

$$x = \frac{\log 1.2}{\log 1.00467}$$

$$x = 39.1$$

Question 7 **C**

$$\frac{(280\,000 - 45\,000)}{10\,000\,000}$$

$= \$0.0235$ per copy

$= 2.35$ cents per copy

Question 8 **E**

$$2400 \times \left(1 + \frac{8.2}{1200}\right)^{30} = 29\ 440$$

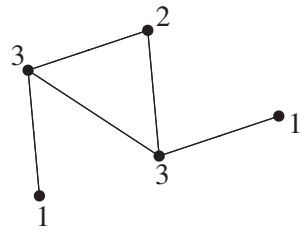
Question 9 **D**

Using TVM solver or equivalent:

$N = 60$, PV is –220 000, FV is the question, PMT is 1868, $P/Y = 12$, $C/Y = 12$.

Module 5: Networks and decision mathematics

Question 1 **B**



As shown above, one vertex has even degree.

Question 2 **E**

After one edge is added, the number of edges is six. Therefore $6 \times 2 = 12$ shows the sum of the degrees of every vertex.

Question 3 **C**

The sum of all entries in the adjacency matrix equals twice the number of edges in the network. Only options **C** and **E** have four edges. Option **E** is incorrect, it does not show an edge between vertices **A** and **B**.

Question 4 **E**

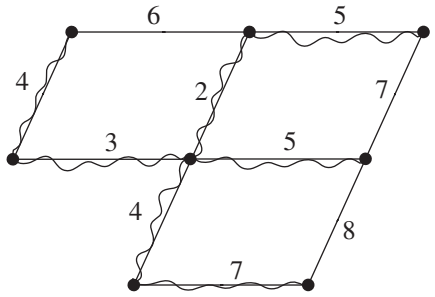
The maximum flow is equal to the minimum cut.

Cut	Capacity of Cut
F	16
G	Does not prevent all flow from source to sink.
H	4 + 7 + 4 = 15
I	5 + 3 + 4 + 4 = 16
J	5 + 6 = 11

Question 5 **D**

Activity *R* has an earliest start time of 1 hour and a latest finish time of 17 hours. It has a duration of 4 hours. Therefore, the float time is found by:
float $= 17 - (1 + 4) = 12$ hours.

Question 6 **D**

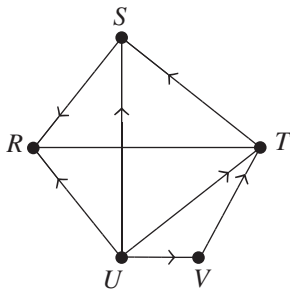


The minimal spanning tree is shown above. It does not contain an edge of weight 6.

Question 7 **C**

A complete graph with five vertices will contain ten edges. The graph contains four edges, therefore six edges are required to represent the completion of the tournament.

Question 8 **D**



There is enough information to determine that player *U* is undefeated. This guarantees that no other player has equal or higher 2-step dominance.

Question 9 **A**

$v + f = e + 2$

$12 + f = e + 2$

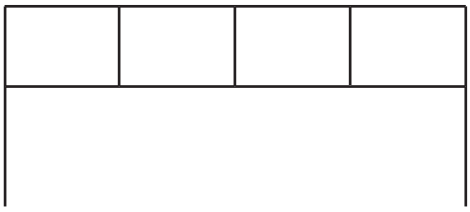
$10 + f = e$

$10 + f = 3f$

$10 = 2f$

$5 = f$

For example:



12 vertices, 15 edges, 5 faces

Module 6: Matrices

Question 1 **E**

For matrices to be added or subtracted, it is necessary for them to be of the same order. It is not sufficient for them to satisfy the multiplication condition (columns on left = rows on right).

Question 2 **D**

Firstly it must be checked whether the matrices allow multiplication. Since addition is defined, both P and Q are of the same order. Since they are square matrices, the number of rows and columns must all match. Multiplication will thus be defined.

It should be obvious that reversing the order in which addition occurs retains the same sum. Thus options **B** and **D** are the only credible responses.

Finally we will need to check the effect of reversing multiplication order.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

$$\begin{bmatrix} e & f \\ g & h \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ae+fc & eb+fd \\ ga+hc & gb+hd \end{bmatrix}$$

As can be seen, reversing the order does alter the result, therefore **D** is correct.

Question 3 **C**

Equation in matrix form is

$AX = B$

where $A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 1 & 0 \\ 2 & -3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$

Thus $X = A^{-1}B$

$$X = \frac{1}{4}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 4 & -1 \\ 5 & -12 & 1 \end{bmatrix}\begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$$

Question 4 **E**

Matrix A is singular because $\det(A) = 0$, thus no inverse can be found. Option **B** is false as it is $\det(A)$, not A^{-1} that is zero. Likewise it is A^{-1} which is undefined, not $\det(A)$ so **C** is also false. This is a question that requires careful reading and an understanding of the terms 'singular' and 'undefined'.

Question 5 **C**

The first conclusion that should be drawn about this question is that a process of multiplication will be required since addition of any other vector matrix will increase prices by a certain dollar amount, rather than by a percentage. This limits the available options to **C**, **D** and **E**.

We require that the new prices of fiction, non-fiction and magazines be 82%, 88% and 91% of the old price respectively. Option **D** fails to do this – it makes the new prices 18%, 12% and 9% of the old price, rather than increasing by these amounts.

Option **E** allows the new prices to be influenced by the prices of the other publication types, in addition to the price of the type being calculated. This is also incorrect.

Question 6 **D**

Firstly it must be possible to perform the multiplication concerned. Options **A** and **D** are the only matrices that work for this. Option **D** multiplies points per event by the event occurrence for each team, and then totals this for each team. This is what is required. **A**, in contrast, multiplies points per event by the team occurrence for each event. This is meaningless.

Question 7 **B**

It is not true to say that the market share of *X* decreases by 20% every year. 20% of *X* is lost every year but this is offset by gains from both *Y* and *Z*. Whether *Y* increases or decreases over time will depend on its initial value. Likewise it is not possible to know if the market share of *X* is largest of all three at all times. It could be initially the smallest.

That leaves only options **B** and **C**. The influence of *Z* on the following value of the *Y* market share is in $T^{2,3}$. The influence of *Y* on the following value of *Z* is in $T^{3,2}$. The former is 0.1 while the latter is 0.05.

Question 8 **A**

The process is simply one of calculating T^{50} and T^{51}. They are virtually identical and so a steady state has been attained.

Question 9 **C**

Students should calculate T^2 and T^3. From this they will see that T^3 is an identity matrix. Thus, results repeat every three rounds. This contradicts options **A**, **B** and **E**.

To obtain winnings from any three rounds we could perform the calculation:

$$\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \right) S_n = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} S_n$$

Thus all three have identical winnings.

A look at *T* and T^2 shows that it is simply a case of taking turns to win, regardless of the initial state.