2011

VCE
Further Mathematics
Trial Examination 1

Suggested Solutions

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Core: Data analysis

Question 1 E

$$\text{Mean} = \frac{(24 \times 16) + (15 \times 18)}{(24 + 15)} = 16.8$$

Question 2 D

Number of pets =

$$(3 \times 0) + (5 \times 1) + (7 \times 2) + (6 \times 3) + (2 \times 4) + (1 \times 5)$$

$$= 50$$

Question 3 B

30% 0f 70000 females voted for the greens = 21000.

10% 0f 85000 males voted for the greens = 8500

21000 –8500 = 12500

There were 12500 more females than males

Question 4 C

$$Z = \frac{x - \bar{x}}{s} = \frac{35 - \bar{x}}{5} = -1$$

$$\bar{x} = 40$$

Question 5 B

From the last column, the total number who drink less than once a week

= 1020 – (204 + 359 + 158) = 299

From the third last row, the number of students who are at least 15 but less than 18 and drink alcohol less than once a week

= 299 – (84 + 97 + 46) = 72

Number in this age group who drink more than once a week = 204 – (25 + 41 + 90) = 48

Total in this age group who drink alcohol

= 48 + 94 + 72 = 214

$$\% = \frac{214}{1020} \times 100 = 21\%$$

Question 6 B

164 is one standard deviation below the mean and 176 is two standard deviations above the mean.

68% within ± 1 s. d.

So between -1 and 0 there are 34%

95% within ± 2 s. d.

So between 0 and 2 there are 47.5%

Within -1 and 2 standard deviations

= 34 + 47.5 = 81.5%

81.5% of 300 = 0.815 × 300 = 244.5

This is closest to 245 students.

Question 7 A

$$y = a + bx$$

$$b = r\frac{s_y}{s_x} = 0.93 \times \frac{6}{7} = 0.797$$

$$a = \bar{y} - b\bar{x} = 60 - 0.797 \times 175 = -79$$

$$y = -79 + 0.8x$$

$$Weight = 0.8 \times height - 79$$

Question 8 C

$r^2 = 0.86$

∴ 86% of the variation in the weight can be explained by the variation in the height since height is the independent variable.

Core: Data analysis

Question 9 D

The given table gives a graph like

This can be linearised by using an x^2 transformation

Question 10 C

Equation of regression line is

$y = mx + 10$

$m = \frac{4-10}{8-0} = -\frac{3}{4}$

$y = -\frac{3}{4}x + 10$

When $x = 6, y = -\frac{18}{4} + 10 = 5.5$

Predicted value = 5.5

Actual value = Predicted value + Residual value

Actual value = 5.5 + (-2.5) = 3

Coordinate is (6,3)

Question 11 D

The sales figures for September, October and November are \$50000, \$90000and \$65000. The median of these values is \$65,000

Question 12 E

Using the values for June, July, August, September and October

Average = $\frac{70+60+75+50+90}{5} = \$69000$

Question 13 B

Use calculator with the first given table of values

to find the regression line for the deseasonalised values.

This gives

$Deseasonalised\ \text{Price} = 0.3857 \times day + 100.214$

For day 11, deseasonalised price

$= 0.3857 \times 11 + 100.214 = 104.4567$

Predicted price = 104.4567 × 0.7 = 73 cents.

Module 1 Number patterns

Question 1 C
An arithmetic sequence has a common difference.
For 6, 2, -2, -6
$2-6 = -2-2 = -6--2 = -4$
None of the other alternatives have a common difference.

Question 2 B
This is a geometric sequence with a common ratio of $1.25 \div 5 = 0.25$.
This means that each term equals the previous term multiplied by 0.25.
The fourth term is 0.3125×0.25
The fifth term is $0.3125 \times 0.25 \times 0.25 = 0.0195$
This is closest to 0.02

Question 3 E
Term 1 = 1
term 2 = 3
term 3 = $2 \times 3 + 1 = 7$
term 4 = $2 \times 7 + 3 = 17$
term 5 = $2 \times 17 + 7 = 41$
term 6 = $2 \times 41 + 17 = 99$

Question 4 D
Common ratio = $192 \div 96 = 2$
$t_6 = ar^5$
$a \times 2^5 = 96$
$a \times 32 = 96$
$a = 3$
$t_3 = ar^2$
$3 \times 2^2 = 12$

Question 5 B
This is an arithmetic sequence with $a = 80000$, and $d = -8000$
$t_n = a + (n-1)d$
$t_{10} = 80000 + 9 \times (-8000) = 8000$

Question 6 D
This is the addition of an arithmetic sequence where $a = 6$ and $d = 3$.
$S_n = \frac{n}{2}[2a + (n-1)d] = 357$
$\frac{n}{2}[12 + (n-1)3] = 357$
$\frac{n}{2}[12 + 3n - 3] = 357$
$\frac{n}{2}[9 + 3n] = 357$
$n[9 + 3n] = 714$
$3n^2 + 9n - 714 = 0$
Use calculator to solve this equation
$n = 14$
OR on Casio Classpad
In sequence from menu screen, select "ExplicitTab" and enter $\frac{n}{2}(12 + (n-1)3)$ and fill down the table. This gives
$a_nE = 375$ when $n = 14$.

Module 1 Number patterns

Question 7 A

When $n = 2$

$3a - b = 1 \quad (1)$

When $n = 3$

$a - b = -3 \quad (2)$

$(1) - (2) \Rightarrow 2a = 4 \Rightarrow a = 2$

From equation (2), $2 - b = -3 \Rightarrow b = 5$

Question 8 D

Number of koalas present in 10 year's time

$= t_{10} = ar^9 = 320(0.85)^9 = 74$

Number that have disappeared = $320 - 74 = 246$

Question 9 B

$$S_4 = \frac{a(1-r^4)}{1-r} = 65 \Rightarrow \frac{a}{1-r} \times (1-r^4) = 65$$

$$S_\infty = \frac{a}{1-r} = 81$$

$$S_4 = 81 \times (1-r^4) = 65$$

$$(1-r^4) = \frac{65}{81}$$

$$r^4 = \frac{16}{81}$$

$$r = \pm\frac{2}{3}$$

If $r = -\frac{2}{3}$

$$\frac{a}{1+\frac{2}{3}} = 81 \Rightarrow a = 135 \text{ km which is not possible}$$

since Ruby only rides a total of 65 km in 4 days.

If $r = \frac{2}{3}$

$$\frac{a}{1-\frac{2}{3}} = 81 \Rightarrow a = 27 \text{ km}$$

Module 2 Geometry and trigonometry

Question 1 E

$\tan\theta = \frac{17}{12}$

$\theta = \tan^{-1}\left(\frac{17}{12}\right) = 54.78^0$ which is closest to 55^0

Question 2 A

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{1}{2}(7+9+12) = 14$

$A = \sqrt{14(14-7)(14-9)(14-12)}$

$A = 31 \text{ cm}^2$

Question 3 D

1 cm : 20000 cm
3 cm : 60000 cm
60000 cm ÷ 100 = 600 m
600 m ÷ 1000 = 0.6 km

Question 4 D

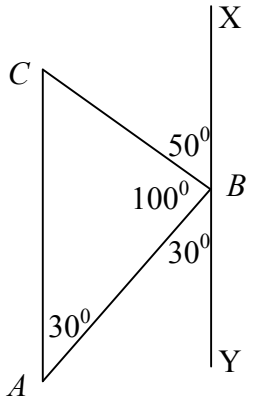
Triangles, ABX and CDX are similar (AAA)

$\frac{AX}{1.8} = \frac{5}{3}$

$3AX = 9$

$AX = 3$ cm

Question 5 E



$\angle CAB = 30^0$ (N30^0E)

$\angle ABY = 30^0$ (alternate angle)

$\angle CBX = 50^0$ (N50^0W)

$\angle ABY + \angle ABC + \angle CBX = 180^0$

(angles in a straight line)

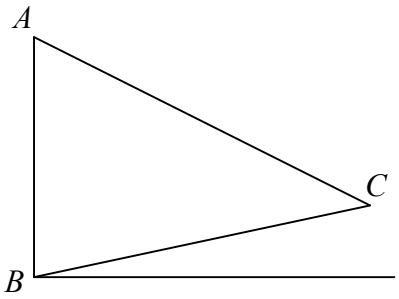
$\angle ABC = 180 - (30 + 50) = 100^0$

Question 6 B

Volume of cylinder B = $\pi \times 1^2 \times 3 = 3\pi$
Ratio of sides is 3 : 1
Ratio of volumes is 27 : 1
Volume of cylinder A = $27 \times 3\pi = 81\pi$
Volume of cylinder A = 254.47 which is closest to 254 cubic cm.

Module 2 Geometry and trigonometry

Question 7 A



$\angle ABC = 90 - 10 = 80^0$

$\angle ACB = 28^0$

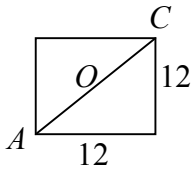
$\angle BAC = 180 - (80 + 28) = 72^0$

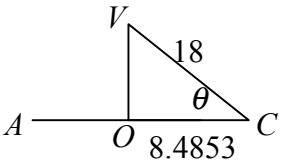
$BC = 40$

$$\frac{40}{\sin 72^0} = \frac{h}{\sin 28^0}$$

$$h = \frac{40 \sin 28^0}{\sin 72^0} = 19.7$$

Question 8 D





$AC = \sqrt{12^2 + 12^2} = 16.97$

$OC = \frac{1}{2} AC = 8.4853$

$$\cos\theta = \frac{8.4853}{18}$$

$$\theta = \cos^{-1}\left(\frac{8.4853}{18}\right) = 62^0$$

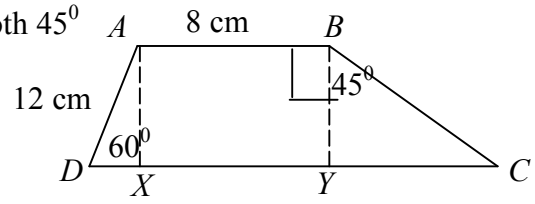
Question 9 C

Area of trapezium = Area triangle ADX + Area rectangle $ABXY$ + Area triangle BYC.

$$\sin 60^0 = \frac{AX}{12} \Rightarrow AX = 12 \sin 60^0 = 10.392$$

$$\cos 60^0 = \frac{DX}{12} \Rightarrow DX = 12 \cos 60^0 = 6$$

BYC is an isosceles triangle because the base angles are both 45^0



$$\text{Area} = \frac{6 \times 10.392}{2} + (8 \times 10.392) + \frac{10.392 \times 10.392}{2} = 168.3 \text{ cm}^2$$

Module 3 Graphs and relations

Question 1 E

$y = 3x + 2$

$2y = ax + 15$

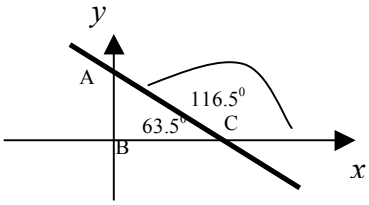
$y = \frac{a}{2}x + 7.5$

Parallel lines have the same gradient, which is the coefficient of the x. Hence

$\frac{a}{2} = 3$

$a = 6$

Question 2 B



Gradient

$= \frac{Rise}{Run}$

$= -\frac{AB}{BC}$

$= -\tan 63.5^o$

$= -2$

Question 3 A

$x - 3y \geq 6$

$-3y \geq -x + 6$

$y \leq \frac{1}{3}x - 2$

$6y \leq 2x - 12$

Question 4 D

(1,3) must lie on both lines.

For $y = 2x + p$, when $x = 1, y = 3$

$\Rightarrow 2 + p = 3$

$\Rightarrow p = 1$

For $y = q - x$, when $x = 1, y = 3$

$\Rightarrow 3 = q - 1$

$\Rightarrow q = 4$

Question 5 B

Distance from A to B:

60 km in 60 mins $\Rightarrow$ 20 km in 20 min.

Distance from H to B = 50 + 20 = 70 km

Distance from H to C = 70 km

This is the distance travelled in the first hour

Question 6 D

Speed from H to A = $\frac{50}{0.5} = 100$ km/hr

Speed from A to B = 60 km/hr

Speed from B to C = 0 km/hr

Speed from C to D = $\frac{115 - 70}{0.5} = 90$ km/hr

Maximum speed = 100 km/hr

Module 3 Graphs and relations

Question 7 C

This is a linear graph passing through origin

$\therefore$ equation is of the form

$y = m\dfrac{1}{x}$

When $\dfrac{1}{x} = 4, y = \dfrac{1}{2}$

$\dfrac{1}{2} = m \times 4$

$m = \dfrac{1}{8}$

$y = \dfrac{1}{8} \times \dfrac{1}{x} = \dfrac{1}{8x}$

Question 8 A

The region is above the x axis and between the y axis and the other vertical line.

It is below the line $y = -x + 6$

and it is above the line $y = \dfrac{3}{2}x - 3$

Question 9 B

Let x be the number of long stem roses sold and

let y be the number of short stem roses sold

$5x + 3y = 850 \quad (1)$

$x + y = 230$

$\Rightarrow 3x + 3y = 690 \quad (2)$

$(1) - (2) \rightarrow 2x = 160$

$x = 80$

Module 4 Business-related mathematics

Question 1 C
Superannuation = $0.09 \times 1200 = 108$
Tax = $0.32 \times 1200 = 384$
Total reductions = 384 + 108 = 492
Take home pay = 1200 – 492 = \$708.

Question 2 B
Amount of depreciation each year
$= 0.13 \times 1500 = 195$
Depreciation after two years = $2 \times 195 = \$390$.

Question 3 D
Amount Josh paid = $2000 + 500 \times 36 = 20000$
Interest = 20000 – 13000 = \$7000.

Question 4 C
Total value of investment = $P\left(1+\frac{r}{100}\right)^n$
$= 20000\left(1+\frac{8}{100}\right)^4$
$= 20000(1.08)^4$

Question 5 A
Interest = 39000 – 30000 = \$9000
$I = \frac{PRT}{100}$
$R = \frac{100I}{PT} = \frac{100 \times 9000}{30000 \times 5} = 6\%$ per annum
$= \frac{6}{12} = 0.5\%$ per month

Question 6 A
The monthly repayments repay principal and interest. Each month that goes by means that less and less of the repayment goes to pay interest and more and more of the repayment goes to pay principal. So the amount of principal repaid each month increases and the amount of interest repaid each month decreases.

Question 7 E
Let the original price be \$100
Price after first increase = 110% of 100 = \$110
Price after second increase = 108% of 110
= \$118.80
Price after third increase = 105% of 118.80
= \$124.74
Increase from original price = 124.74 – 100
= 24.74
% increase = $\frac{24.74}{100} \times 100 = 24.74\%$

Module 4 Business-related mathematics

Question 8 D
Using TI-84
Use TVM solver
$N = 15 \times 12 = 180$
$I = 6.4$
$PV = 250000$
$PMT = -1850$
$FV = ?$
$P/Y = 12$
$C/Y = 12$
End
Alpha solve gives $FV = 94511.4886$
This is closest to \$94512

Question 9 A
Interest per quarter = 2%
2% of 35000 = 700
At the end of the first quarter, Graham will pay \$1000. \$700 will pay interest and the remainder, \$300 will pay principal.

Question 8 D

Using TI-Nspire CAS
Select Financial Solver
$N = 180$
$I(\%) = 6.4$
$PV = 250000$
$PMT = -1850$
$FV = ?$
$P_pY = 12$
$C_pY = 12$
END
Move cursor to FV box and press enter.
FV=94511.4886
This is closest to \$94512

Question 8 D

Using Casio Classpad
Select the Financial application on the menu screen.
Select "Compound Interest"
$N = 180$
$I\% = 6.4$
$PV = -1850$
$FV = ?$
$P/Y = 12$
$C/Y = 12$
Tap the FV box
$\Rightarrow FV = 94511.4886$
This is closest to \$94512

Module 5 Networks and decision mathematics

Question 1 E
There are three lines coming from C and the loop counts as another two.

Question 2 B
Since the vertices are not all even, it is not an Eulerian circuit.
There are more than 2 vertices of odd degree, so not an Eulerian circuit.
It is a Hamiltonian circuit because you can start and finish at the one vertex and visit each vertex once.

Question 3 E
P to P is 2, so not A or B.
Q to Q is 0 so not C or D

Question 4 C

From A you can reach B and D. Hence, 2.

Question 5 A
A planar graph is a graph whose edges intersect only at the vertices.

Question 6 D
Geology and psychology are both studied by just one of the students, so D is not true.

Question 7 D

Capacity of cut = 6 + 7 + 0 + 4 = 17

Question 8 B

$F \to E = 10$

$F \to D = 12$

$F \to D \to B = 24$

$F \to D \to C = 22$

$F \to E \to C = 24$

$F \to D \to E \to C = 35$

$F \to D \to C \to A = 31$

$F \to D \to B \to A = 36$

Shortest path is 31

Question 9 A

The critical path is the longest path.

$B - D - E - H - J$

Module 6 Matrices

Question 1 B

$4a = -2 \Rightarrow a = -\frac{1}{2}$

$4 \times 3 = b \Rightarrow b = 12$

$4 \times 2 = c \Rightarrow c = 8$

$4 \times -1 = d \Rightarrow d = -4$

Question 2 A

The answer matrix when a 3×1 matrix is multiplied by a 1×3 matrix is a matrix where the number of rows equal the number of rows of the first matrix and the number of columns equal the number of columns of the second matrix. Hence, a 3×3 matrix

Question 3 E

$\Delta = ps - qr$

$\frac{1}{\Delta} = \frac{1}{ps - qr}$

$A^{-1} = \frac{1}{ps - qr}\begin{bmatrix} s & -q \\ -r & p \end{bmatrix}$

Question 4 C

For no solution, $3t - 4 = 0$

So $t = \frac{4}{3}$

Question 5 D

$PA = C - B = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} -3 & 5 \\ -3 & 4 \end{bmatrix}$

$P\begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ -3 & 4 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$

$A^{-1} = -1\begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$

$P\begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} -3 & 5 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$

$P = \begin{bmatrix} 11 & -25 \\ 10 & -23 \end{bmatrix}$

Question 6 B

AC does not exist.

B^{-1} does not exist.

CA does exist.

AB does exist.

BA does not exist.

BC does not exist.

BA does not exist.

Question 5 D

Using a calculator

Define $A = \begin{bmatrix} 2 & 5 \\ 1 & 2 \end{bmatrix}$

Then $PA = \begin{bmatrix} -3 & 5 \\ -3 & 4 \end{bmatrix}$

Post multiply by A^{-1}

$PAA^{-1} = \begin{bmatrix} -3 & 5 \\ -3 & 4 \end{bmatrix}A^{-1}$

$P = \begin{bmatrix} -3 & 5 \\ -3 & 4 \end{bmatrix}A^{-1} = \begin{bmatrix} 11 & -25 \\ 10 & -23 \end{bmatrix}$

Module 6 Matrices

Question 7 A

The matrix for Monday when Martha is on time is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Tuesday is $\begin{bmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Wednesday is $\begin{bmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{bmatrix}^2\begin{bmatrix} 1 \\ 0 \end{bmatrix}=\begin{bmatrix} 0.85 \\ 0.15 \end{bmatrix}$

0.15 is the probability that Martha will be late on Wednesday

This is the element in the second row and the first column.

Question 8 E

$$\begin{bmatrix} 200 & 150 & 300 \\ 300 & 80 & 250 \end{bmatrix}\begin{bmatrix} 2.00 \\ 1.50 \\ 1.80 \end{bmatrix}$$

$$=\begin{bmatrix} 1165 \\ 1170 \end{bmatrix}$$

total sales for two weeks = 1165 + 1170 = \$2335

Question 9 E

The equations are

$20x + 8y = 40900$

$\Rightarrow 5x + 2y = 10225$

$x + y = 3200$

$$\begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 3200 \\ 10225 \end{bmatrix}$$

$$-\frac{1}{3}\begin{bmatrix} 2 & -1 \\ -5 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=-\frac{1}{3}\begin{bmatrix} 2 & -1 \\ -5 & 1 \end{bmatrix}\begin{bmatrix} 3200 \\ 10225 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}=-\frac{1}{3}\begin{bmatrix} 2 & -1 \\ -5 & 1 \end{bmatrix}\begin{bmatrix} 3200 \\ 10225 \end{bmatrix}$$

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