The Mathematical Association of Victoria
Trial Examination 2011
Further Mathematics Exam 2--SOLUTIONS

SECTION A: Core--Data analysis

Question 1

a. The median is the middle value when the data is ordered from smallest to largest.

There are 12 data points, so the median will be the $\frac{12+1}{2} = 6.5^{\text{th}}$ score.

Taking the mean of the 6th and 7th scores give $\frac{0.886+0.861}{2} = 0.874$ **A1**

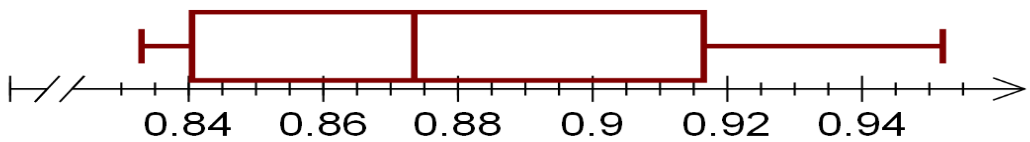
b. $\frac{5}{12} \times 100\% = 42\%$ **A1**

c. Range = highest value – lowest value
= 0.952 – 0.833
= 0.119 **A1**

Question 2

a i. US dollar: min = 0.833, Q_1 = 0.841, median = 0.874, Q_3 = 0.917, max = 0.952 **A1**

a ii. Outlier if less than $Q_1 - 1.5$ IQR, or greater than Q_3 + 1.5 IQR, i.e. < 0.727 or >1.031
There are no outliers present in the data



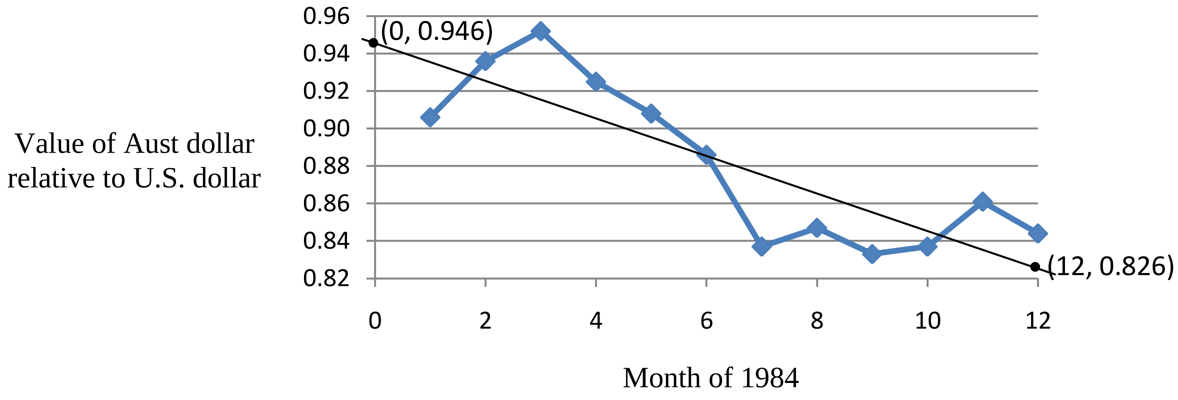
A1

Value of Australian dollar relative to the US dollar

b. The boxplot of the Australian dollar relative to the Japanese yen has a slight negative skew with an outlier at 219.70. **A1**
(Accept approximately symmetrical with an outlier at 219.70)

Question 3

a. The regression line drawn on the time series plot should pass through the points (0, 0.946) and (12, 0.826). Any two correct points on the line are acceptable.



A2

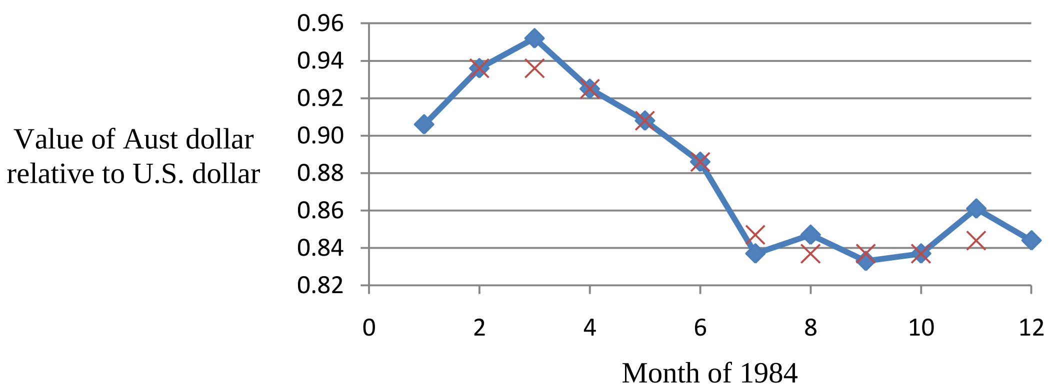
b. In November 1984 the value of the Aust dollar relative to the U.S. dollar was 86 cents.
Residual = actual value – predicted value
= 0.861 – [0.946 – 0.01 × 11]
= 0.861 – 0.836
= 0.025 **A1**

c. Month for July 1985: 12 + 7 = 19 **A1**
Aust dollar value = $0.946 - 0.01 \times 19$
= 0.756 **A1**

d. The linear model assumes the value of the Australian dollar relative to the U.S. dollar will continue to fall at a constant rate. Extrapolating outside the data set is not appropriate. Predicting long term values with this model will be very unreliable. **A1**

e. $r = -0.834$
The coefficient of determination $r^2 = 0.695$ (accept $r^2 = (-0.834)^2 = 0.696$)
69.5% (or 69.6%) of the variation in the value of the Australian dollar can be explained by the variation in the month of the year. **A1**

f. Three median smoothing shown with cross x in graph below **A2**



Module 1: Number patterns

Question 1

a. Arithmetic sequence: a = 11, d = 2, $t_n = a + (n-1)d$

$t_{12} = 11 + 11(2) = 33$ **A1**

b. Arithmetic series: $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$S_{15} = \frac{15}{2}\left[22 + (14)\,2\right] = 375$ **A1**

c i. $t_n = 55$ $11 + (n-1)2 = 55$

$2(n-1) = 44$

$n = 23$ **A1**

c ii. $S_{23} = \frac{23}{2}(11+55)$ OR $S_{23} = \frac{23}{2}\left[22 + (23-1)\,2\right]$

$= 759$ $= 759$ **A1**

Question 2

a. $r = \frac{2.2}{2} = 1.1$ **A1**

b. $t_7 = 2 \times 1.1^6 = 3.5$ metres **A1**

c. Geometric series: $S_n = \frac{a(r^n - 1)}{r-1}$

$\frac{2(1.1^n - 1)}{1.1 - 1} = 150$ **M1**

$2(1.1^n - 1) = 15$

$1.1^n - 1 = 7.5$

$1.1^n = 8.5$

$n = 22.45$ using the calculator

There are 22 trees planted in a row that is 150 metres long. **A1**

Question 3

a.

t_0	10
t_1	1.1 ×10 + 5 = 16
t_2	1.1 ×16 + 5 = 22.6
t_3	29.86
t_4	37.85
t_5	46.63
t_6	56.29
t_7	66.92
t_8	78.62
t_9	91.48

t_5 = 46.63 kg **A1**

b. $\frac{t_1}{t_0} = \frac{16}{10} = 1.6$ $\frac{t_2}{t_1} = \frac{22.6}{16} = 1.41$ **M1**

Since $\frac{t_1}{t_0} = 1.6 \neq 1.41 = \frac{t_2}{t_1}$ then sequence is not geometric (values must be shown)

c. From table $t_8 = 78.62$ and $t_9 = 91.48$

Nine years after ''first yield' the average yield will exceed 80 kg per tree. **A1**

d. $t_6 - t_5 = 56.29 - 46.63 = 9.66$ kg **A1**

e. $t_n = 5 \times 1.3^n$ **A1**

f. The Corregiola olive trees starts with a higher yield, but the yield of the Paragon olive trees increases at a faster rate.
The greater increase in the yield of the Paragon olive trees is due to the coefficient of p_n being 1.3, whereas for the Corregiola olive trees the coefficient of t_n is 1.1. **A1**

g. Paragon yield exceeds Corregiola yield after 14 years. **A1**

Years after first yield (n)	Corregiola $t_{n+1} = 1.1t_n + 5$	Paragon $p_{n+1} = 1.3p_n$
0	10	5
1	16	6.5
2	22.6	8.45
3	29.86	10.99
4	37.85	14.28
5	46.63	18.56
6	56.29	24.13
7	66.92	31.37
8	78.62	40.79
9	91.48	53.02
10	105.62	68.93
11	121.18	89.61
12	138.3	116.49
13	157.13	151.43
14	177.85	196.87

Module 2 Geometry and Trigonometry

Question 1

a. 10° **A1**

b. $\sin(10^\circ) = \dfrac{2000}{PA}$

$PA = \dfrac{2000}{\sin(10^\circ)} = 11\,517.54$

$PA = 11\,518$ metres **A1**

c. i. Horizontal distance from A to $P = \dfrac{2000}{\tan(10^\circ)} = 11\,342.56$ metres

$AM = 11\,342.56 - 3000 = 8343$ m **M1**

c. ii. $\tan\theta = \dfrac{2000}{8342.56}$

$\theta = 13.5^\circ$ **A1**

Question 2

a. i. Using the cosine rule: $\cos(\angle AOB) = \dfrac{OA^2 + OB^2 - AB^2}{2 \times OA \times OB}$

$\angle AOB = \cos^{-1}\left(\dfrac{32^2 + 45^2 - 67^2}{2 \times 45 \times 32}\right)$ **M1**

$\angle AOB = 120^\circ$

a. ii. $\text{Area} = \dfrac{1}{2} \times OA \times OB \sin(\angle AOB)$

$\text{Area} = \dfrac{1}{2} \times 32 \times 45 \times \sin(120^\circ)$

Area = 623.5 km^2 **A1**

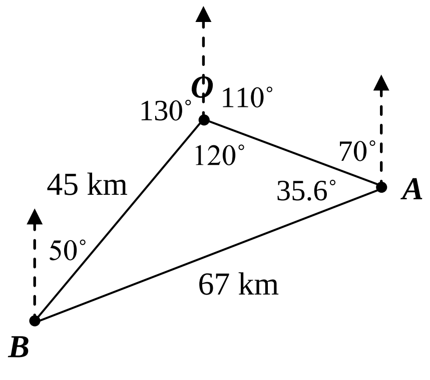
b. Bearing of B from O = 360° – 130° = 230° **A1**

c. Finding $\angle OAB$

Using the sine rule: $\dfrac{OB}{\sin(\angle OAB)} = \dfrac{AB}{\sin(\angle AOB)}$

$\dfrac{45}{\sin(\angle OAB)} = \dfrac{67}{\sin(120^\circ)}$

$\angle OAB = 35.6^\circ$ **A1**



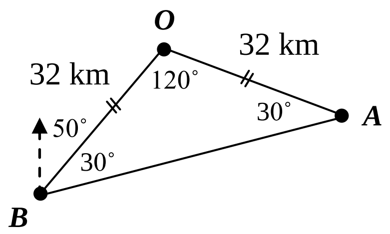
Bearing of B from A = 360° – (35.6° + 70°) = 254.4° **M1**

d i. $AB^2 = OA^2 + OB^2 - 2 \times OA \times OB \cos(\angle AOB)$

$AB^2 = 32^2 + 32^2 - 2 \times 32 \times 32 \cos(120°)$

$AB = \sqrt{3072}$

$AB = 55.4$ metres **A1**



d. ii. $OA = OB = 32$ km

Triangle OAB is isosceles

$\angle OBA = 30°$

Bearing of A from B is (50° + 30°) = 80° **A1**

Question 3

a. Radius of oil barrel = 50 cm

Cylinder: $V = \pi r^2 h$

$V = \pi \times 50^2 \times 85$ **M1**

$V = 667588.4 \text{ cm}^3$

$V = 667.5884$ litres

The cylindrical oil barrel contains 668 litres of oil when full **A1**

b. Containers X and Y are similar in shape, therefore the corresponding length dimensions are in the same ratio.

Volume container Y = 3.375 × Volume container X

$\Rightarrow$ Length container Y = $\sqrt[3]{3.375}$ × Length container X **M1**

Length container Y = $1.5 \times$ Length container X

$\Rightarrow$ Surface Area container Y = 1.5^2 × Surface Area container X

$13.95 = 1.5^2 \times$ Surface Area container X

$\Rightarrow$ Surface Area container $X = \dfrac{13.95}{1.5^2} = 6.2 \text{ m}^2$ **A1**

Module 3: Graphs and relations

Question 1

a. $0 \le x \le 15$

$0 \le y \le 8$ **A1**

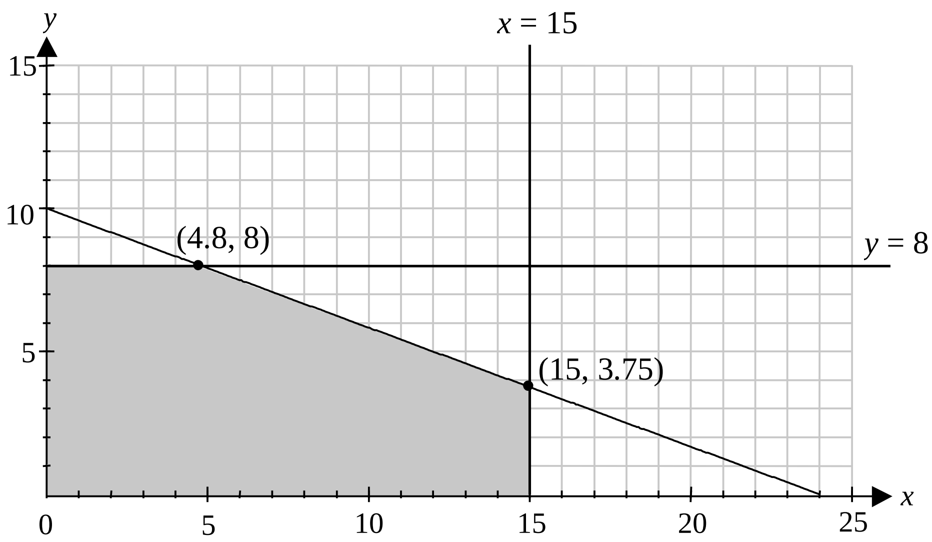
Accept $x \le 15$ and $y \le 8$

b. Both lines correctly labelled with the equation **A1**

c. Line $5x + 12y = 120$ drawn with intercepts (0, 10) and (24, 0) **A1**

or, the points of intersection (4.8, 8) and (15, 3.75) marked

Feasible region shaded **A1**



d. Objective to maximize $= 4x + 10y$

Point	Objective $= 4x + 10y$	Number of people
(4.8, 8)	$4 \times 4.8 + 10 \times 8 = 99.2$	99
(15, 3.75)	$4 \times 15 + 10 \times 3.75 = 97.5$	97
(0, 8)	$4 \times 0 + 10 \times 8 = 80$	80
(15, 0)	$4 \times 15 + 10 \times 8 = 60$	60

M1

The greatest number of people Millie can assist is 99. **A1**

This will occurs with 4.8 small tables and 8 large tables as shown by the calculations above.

It assumes she has a fraction of a table which would be possible in a restaurant.

Question 2

$5 = \frac{k}{120} \quad \Rightarrow \quad k = 600$ **A1**

Manager's equation is $n = \frac{600}{t}$

When $t = 75$, $n = \frac{600}{75} = 8$ staff **A1**

Question 3

a. $C = 2000 + 35x$ **A1**

b. The gradient of the line represents the cost to the restaurant per person. The gradient is \$35.
If 10 extra people attend the additional cost to the restaurant will be $10 \times 35 = \$350$ **A1**

c. Revenue $= 75x$ **A1**
Breakeven point occurs when Revenue = Costs
$75x = 2000 + 35x$
$40x = 2000$
$x = 50$
Fifty people would need to attend the function for the restaurant to breakeven. **A1**

d. Profit = Revenue – Costs
$P = 75x - (2000 + 35x)$
$3480 = 40x - 2000$
$x = \frac{5480}{40} = 137$ people **A1**

e. Let the *m* be the amount charged per person to make a profit of at least \$4000
$P = mx - (2000 + 35x)$
When $x = 125$
$P = 125m - (2000 + 35 \times 125)$
$P = 125m - 6375$
$125m - 6375 \geq 4000$ **M1**
$125m \geq 10375$
$m \geq 83$
The lowest charge is \$83 per person for profit of at least \$4000 **A1**

Module 4 Business related mathematics

Question 1

a. $\text{Value} = 490\ 000 \times (1.06)^3 = 583\ 597.84$
$\text{Value} = \$583\ 600$ **A1**

b. $700\ 000 = 490\ 000 \times (1.06)^n$ **M1**
$n = 6.1$ years **A1**

c. $\text{Capital Gains Tax} = 0.45 \times \$150\ 000 = \$67\ 500$ **A1**

Question 2

a i. Finance Solver
$N = 60$
$I(\%) = 10.4$
$PV = -35\ 000$
$Pmt = ?$
$FV = 0$
$PpY = 12$
$CpY = 12$
Monthly Payment is \$750.55 **A1**

a. ii. Total Interest = 750.55 × 60 – 35 000
Total Interest = \$10 033 **M1**

b. If the monthly payment is \$800 per month for two years.
Finance Solver
$N = 24$
$I(\%) = 10.4$
$PV = -35\ 000$
$Pmt = 800$
$FV = ?$
$PpY = 12$
$CpY = 12$
Amount outstanding after two years is \$21 812.91 **A1**

If the monthly payment is reduced to \$600 for another three years
Finance Solver
$N = 36$
$I(\%) = 10.4$
$PV = -21\ 812.91$
$Pmt = 600$
$FV = ?$
$PpY = 12$
$CpY = 12$
Amount outstanding at the end of five years is \$4537.66
\$4538 correct to the nearest dollar **A1**

Question 3

In Flat Rate Depreciation, the annual depreciation is constant.

This will be a percentage of the initial value of the item. **A1**

In Reducing Balance Depreciation, the annual depreciation is a percentage of the current value of the item. The depreciation decreases from one year to the next. **A1**

Question 4

a. Annual flat rate depreciation = 15% of $20 000 = $3000 **A1**

b. Depreciation over four years = 3 × $3000 = $9000 **A1**

c. Value of tractor after six years = $20 000 – 6 × $3000 = $2000 **A1**

Question 5

Bank statement for October

Date	**Transaction**	**Debit**	**Credit**	**Balance**
01 October	OPENING BALANCE			$1677.76
07 October	Cheque		$520.00	$2197.76
15 October	EFT		$1359.05	$3556.81
24 October	Automatic payment	$1506.93		$2049.88
31 October	CLOSING BALANCE			**$ 2049.88**

Work backwards to find missing balances

Minimum Monthly balance is $1677.76 **A1**

$$\text{Interest} = \frac{1}{12} \times \frac{3.75}{100} \times 1677.76$$

$$\text{Interest} = \$5.24$$ **A1**

Module 5: Networks and decision mathematics

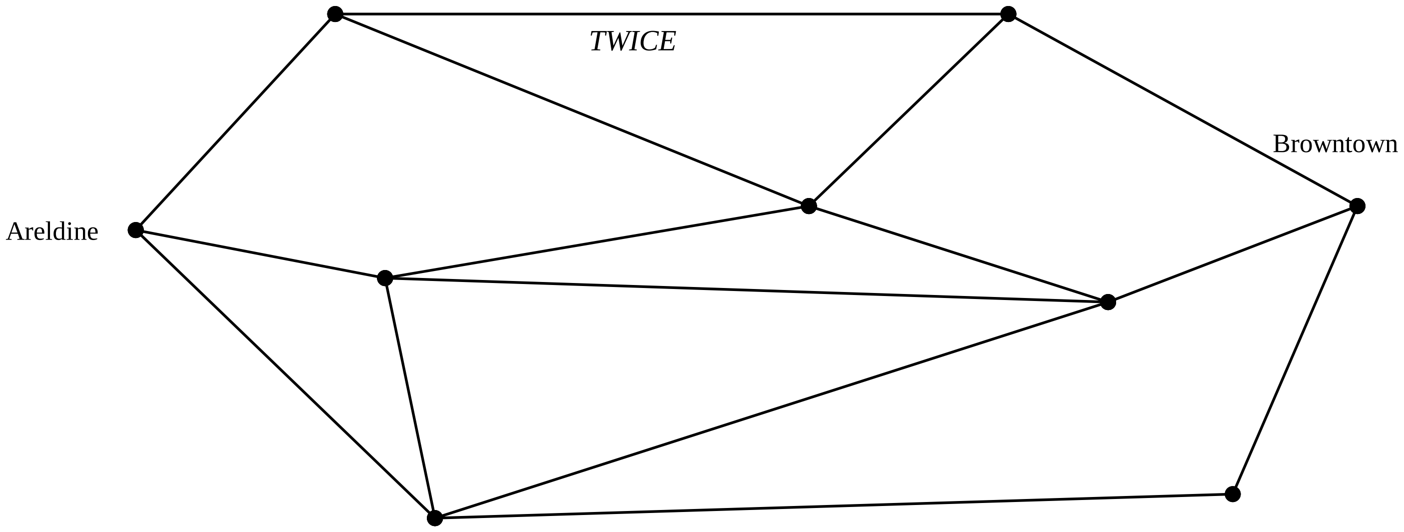
Question 1

a. Path **A1**

b. Euler path **A1**

c. There are four vertices of odd degree.
An Euler Path can have at most two vertices of odd degree. **A1**

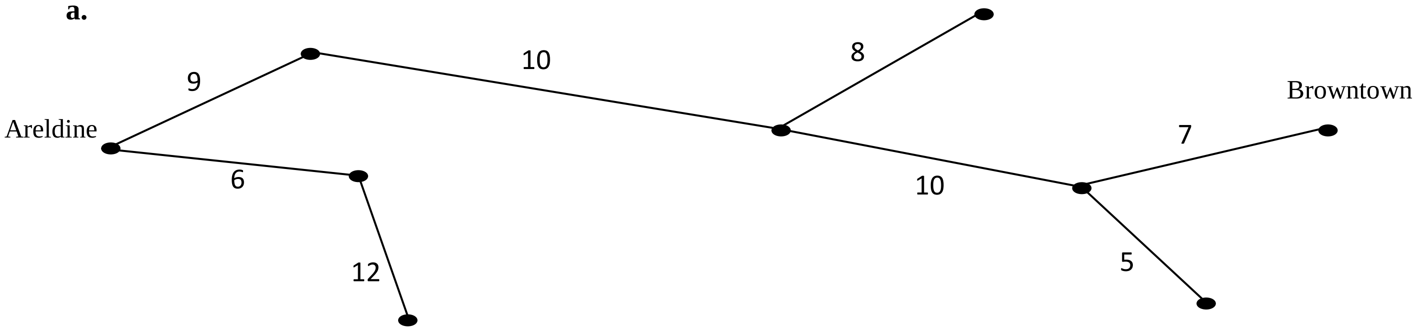
d.



A1

Question 2

a.



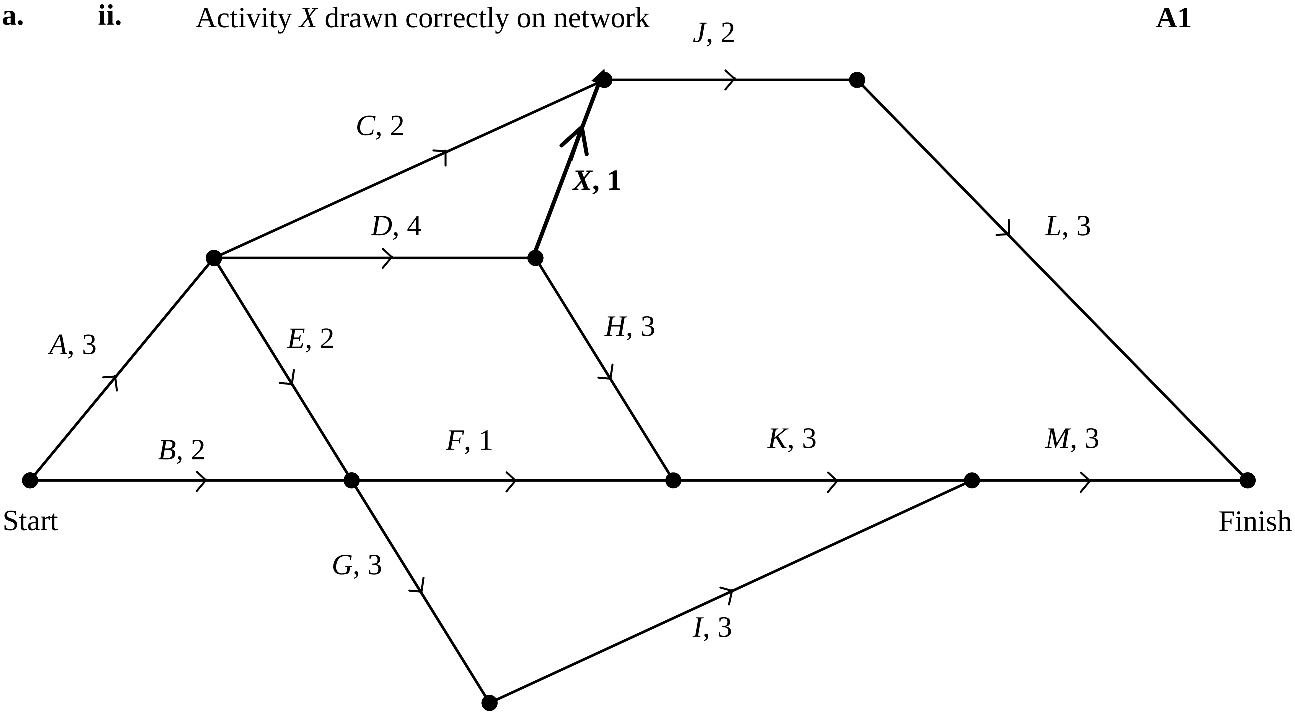
A1

b. $12+6+9+10+8+10+7+5=67$ **A1**
Minimum cost is $67 000

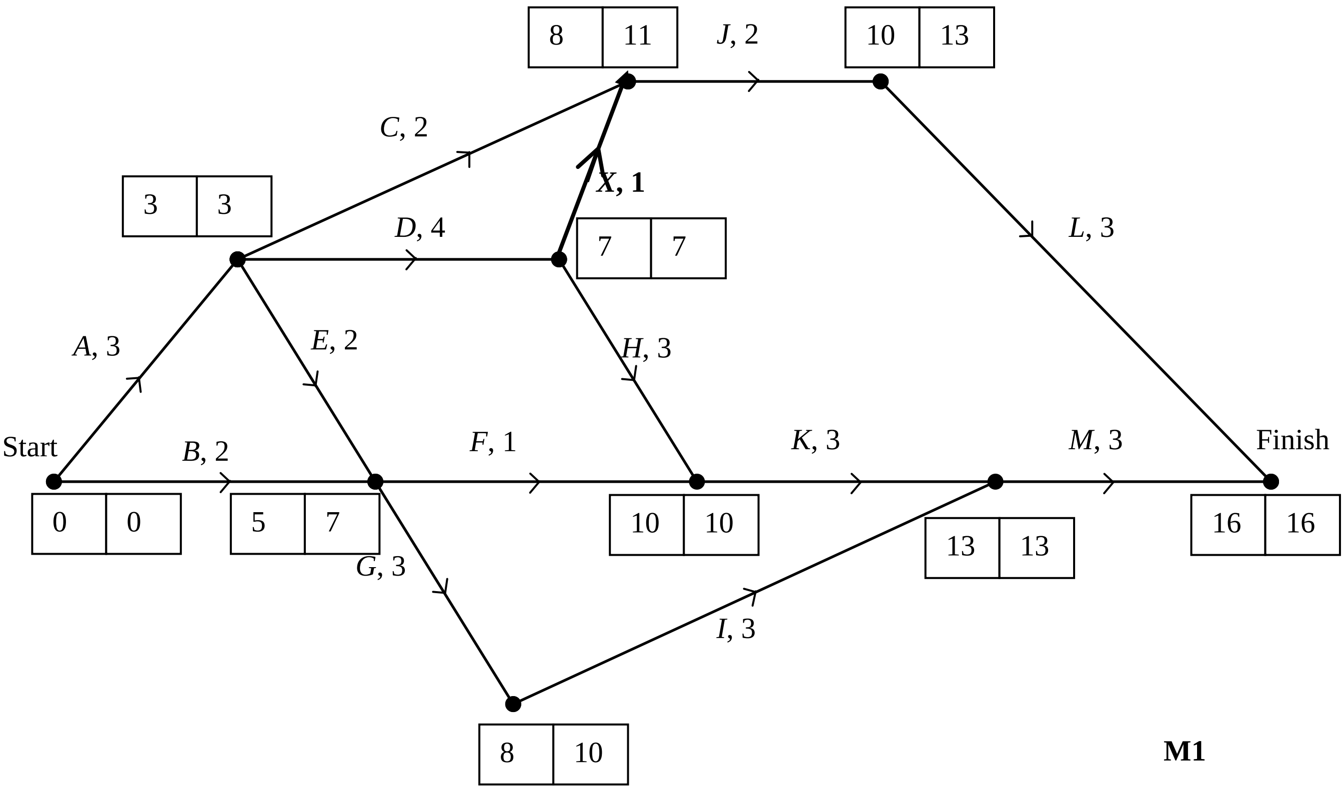
Question 3

a. **i.** The predecessor of task *E* is task *A*. **A1**

a. **ii.** Activity *X* drawn correctly on network **A1**



b.



M1

Task	Time to complete task (Days)	Earliest Start Time (Days)	Latest Start Time (Days)
G	3	5	7
L	3	10	13
M	3	13	13
X	1	7	10

A1

c. Critical path is *ADHKM* **A1**

d. Earliest completion time: $3+4+3+3+3=16$ **A1**

e. Float = latest start time – earliest start time
For activity *F*: $9-5=4$ **A1**

Question 4

a. There is little point in reducing the completion time of task *J* because it is not on the critical path either before or after reducing tasks *H* and *K*.
By either reducing *H* by 1 day and *K* by 1 day, or, *K* by 2 days the minimum completion time for the project will be 14 days.
However, it costs more per day to reduce task *K* than task *H.*
The minimum cost will be $3000 **A1**

b. This minimum cost occurs when *H* is reduced by 1 day and *K* is reduced by 1 day.
The completion time of task *J* will not be reduced. **A1**

Module 6: Matrices

Question 1

a. $\begin{bmatrix} 3.95 \\ 6.50 \\ 10.00 \end{bmatrix} \begin{matrix} B \\ R \\ P \end{matrix}$ **A1**

b. $\begin{bmatrix} 3 & 1 & 5 \end{bmatrix}$ **A1**

c. i. $\begin{bmatrix} 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} 3.95 \\ 6.50 \\ 10.00 \end{bmatrix}$ **M1**

c. ii. $\begin{bmatrix} 68.35 \end{bmatrix}$ Must be written as matrix **A1**

Question 2

a. Joan bought the most punnets of seedlings.
In total she bought 7 punnets of seedlings (3 bean and 4 lettuce) **A1**

b. The element in the second row and third column of the matrix is zero.
This means Doris bought no punnets of lettuce seedlings. **A1**

c. i.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 5 & 0 \\ 3 & 0 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 20 & -4 & -5 \\ -4 & 1 & 1 \\ -15 & 3 & 4 \end{bmatrix}$$

The missing element is 20. **A1**

c. ii. Solving the matrix equation to find the cost of one punnet of each type of seedling.

$$\begin{bmatrix} b \\ t \\ l \end{bmatrix} = \begin{bmatrix} 20 & -4 & -5 \\ -4 & 1 & 1 \\ -15 & 3 & 4 \end{bmatrix} \begin{bmatrix} 16.95 \\ 37.85 \\ 36.45 \end{bmatrix} = \begin{bmatrix} 5.35 \\ 6.50 \\ 5.10 \end{bmatrix}$$ **A1**

Kate's seedling purchase can be written as $\begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$

The total cost will be $\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 5.35 \\ 6.50 \\ 5.10 \end{bmatrix} = \begin{bmatrix} 34.15 \end{bmatrix}$ **A1**

Kate pays $34.15 for her seedlings.

Question 3

a. 20% **A1**

b.
$$\begin{bmatrix} 0\cdot 3 & 0\cdot 4 & 0\cdot 2 \\ 0\cdot 4 & 0\cdot 5 & 0\cdot 7 \\ 0\cdot 3 & 0\cdot 1 & 0.1 \end{bmatrix}^3 \begin{bmatrix} 0 \\ 0 \\ 1000 \end{bmatrix} = \begin{bmatrix} 336 \\ 492 \\ 172 \end{bmatrix} \begin{matrix} O \\ I \\ N \end{matrix}$$

336 of these gardeners will prefer to use organic fertilizer in three year's time. **A1**

c.
$$\begin{bmatrix} 0\cdot 3 & 0\cdot 4 & 0\cdot 2 \\ 0\cdot 4 & 0\cdot 5 & 0\cdot 7 \\ 0\cdot 3 & 0\cdot 1 & 0.1 \end{bmatrix}^{20} \begin{bmatrix} 0 \\ 0 \\ 1000 \end{bmatrix} = \begin{bmatrix} 333.\dot{3} \\ 500 \\ 166.\dot{6} \end{bmatrix} \begin{matrix} O \\ I \\ N \end{matrix} \quad \text{and}$$

$$\begin{bmatrix} 0\cdot 3 & 0\cdot 4 & 0\cdot 2 \\ 0\cdot 4 & 0\cdot 5 & 0\cdot 7 \\ 0\cdot 3 & 0\cdot 1 & 0.1 \end{bmatrix}^{21} \begin{bmatrix} 0 \\ 0 \\ 1000 \end{bmatrix} = \begin{bmatrix} 333.\dot{3} \\ 500 \\ 166.\dot{6} \end{bmatrix} \begin{matrix} O \\ I \\ N \end{matrix}$$

The steady state matrix is $\begin{bmatrix} 333.\dot{3} \\ 500 \\ 166.\dot{6} \end{bmatrix} \begin{matrix} O \\ I \\ N \end{matrix}$ **A1**

d. Half **A1**

Question 4

$$T = \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.5 \end{bmatrix}$$

$$D_2 = TD_1 + \begin{bmatrix} 4 \\ 4 \end{bmatrix} \quad \Rightarrow \quad D_2 = \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.5 \end{bmatrix} \times \begin{bmatrix} 10 \\ 20 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 20 \\ 18 \end{bmatrix} \begin{matrix} H \\ L \end{matrix}$$ **A1**

$$D_3 = TD_2 + \begin{bmatrix} 4 \\ 4 \end{bmatrix} \quad \Rightarrow \quad D_2 = \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.5 \end{bmatrix} \times \begin{bmatrix} 20 \\ 18 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 25 \\ 21 \end{bmatrix} \begin{matrix} H \\ L \end{matrix}$$

On the third weekend 25 people will attend the Horticulture demonstration **A1**

END OF SOLUTIONS

